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# Capital Accumulation, Factor Prices and Endogenous Labor-Saving Technical Change

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# Capital Accumulation, Factor Prices, and Endogenous Labor-Saving Technical Change

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**Abstract**

The process of capital accumulation understood as a rise in the capital-labor ratio steadily raises the scarcity of labor with respect to capital and leads to a rise in the cost of labor relative to the cost of capital. This movement in relative factor prices may act as an incentive for profit-maximizing firms to direct innovations towards labor saving technologies. We make this point in a model of endogenous technical change that relates the neoclassical growth paradigm to the concept of induced innovation. These ingredients suggest an economic development of economies characterized by an endogenous "run through stages." In early stages, the driving force of economic growth is capital accumulation because the return to physical capital is high and labor is cheap. In mature stages, however, labor is expensive so that firms invest in new technologies that economize on labor. Thus, economies may evolve from a regime of pure capital accumulation into one with capital accumulation and endogenous technical change.

**Keywords:** endogenous technical change, neoclassical growth model, induced innovation, productivity growth

**JEL Classification:** D24, J30, O33, O41

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## 1. Introduction

This paper argues that the process of capital accumulation steadily raises the scarcity of labor with respect to capital and leads to a rise in the cost of labor relative to the price of capital. This movement in relative factor prices acts as an incentive for profit maximizing firms to invest in labor-saving technologies. The growth performance of countries may then be characterized by an endogenous run through stages of development. In early stages, the driving force of economic growth is capital accumulation because the return to physical capital is high and labor is cheap. In mature stages, however, labor is sufficiently expensive so that firms invest in new technologies that economize on labor. Thus, the economies may evolve from a regime of pure capital accumulation into one with capital accumulation and endogenous technical change.

The central idea of this paper relates the neoclassical growth paradigm (Solow (1956), Swan (1956)) to the concept of induced innovation enunciated by e.g. Hicks (1932) who emphasizes the role of factor prices as an incentive for technical change.[1] In a famous passage of his *Theory of Wages* (p. 124-125) he writes:

> *"A change in the relative prices of the factors of production is itself a spur to invention, and to invention of a particular kind – directed to economising the use of a factor which has become relatively expensive. The general tendency to a more rapid increase of capital than labour which has marked European history during the last few centuries has naturally provided a stimulus to labour-saving inventions."*

The present study provides *a* formal interpretation of Hicks' observation in an intertemporal general equilibrium setting with endogenous technical change. The two building blocks of our analysis account for Hicks' suggested causality: the neoclassical process of capital accumulation leads to changes in relative factor prices and, in response, profit-maximizing firms direct resources towards labor-saving inventions.

It has been pointed out by e.g. Scott (1989) and Howitt and Aghion (1998) that orthodox growth theories tend to regard scientific discovery or invention as the ultimate determinants of the rate of productivity growth.[2] This view accords only a minor role to capital accumulation as a source of economic growth. It can be based on a version of the neoclassical growth model with exogenous technical change where the growth rate of capital is a function of the exogenous growth rate of a productivity indicator and, due to diminishing returns, adjusts to this rate along the transition to the steady state. Models with endogenous innovation and capital accumulation like Romer (1990) or Grossman and Helpman (1991), Chap.

[1] Ruttan (2001) provides a comprehensive up-to-date survey of the literature on induced innovation.

[2] This view has also been expressed in leading textbooks. See e.g. Jones (1998), p. 162, and the quotes of Romer (1996) and Blanchard (1997) in Howitt and Aghion (1998), p. 112.

5, seem to support this view, too. Here, capital is inessential for the economy's steady-state growth rate because it leaves the incentives for innovation unaffected.

This paper departs from this view and emphasizes a way how capital accumulation affects the incentive of profit-maximizing firms to engage in innovation. On the one hand, technical change typically requires an innovation investment which must be financed on the capital market because the proceeds of the investment materialize only later. Then, capital accumulation under diminishing returns implies a lower interest rate, thus reducing the cost of innovation. On the other hand, capital accumulation raises the relative scarcity of inputs other than capital which tends to raise wages and the price of other inputs relative to the price of capital. Thus, the gains from an innovation that allows to save on these factors increases. Capital accumulation may then reduce the cost and increase the gain associated with such innovation investment.

Under constant returns of the economy's aggregate production technology and perfect foresight these two effects are essentially two sides of the same coin. We make this point in a neoclassical growth framework extended to allow for endogenous labor-saving technical change. Here, capital accumulation may *cause* productivity growth as it guides the economy from a *stationary regime* without technical change into an *innovation regime* with capital accumulation and endogenous productivity growth. In the stationary regime, the economy grows only through capital accumulation. Factor markets clear and optimizing behavior on the side of firms imply that factor prices *adjust* to the value of the marginal product of the respective factor at their supplied quantities. The evolution of factor prices during the process of capital accumulation is then linked to the evolution of marginal products which in turn reflects the relative scarcity because inputs are complements in the aggregate production function. Thus, the marginal productivity of labor and the price of labor increase whereas the marginal productivity of capital and the price of capital fall.

Such factor price movements have no bearing on the development of real magnitudes in the stationary regime of the economy.[3] Yet, they may ignite a switch into the innovation regime if the *expected* relative price of labor reaches a given threshold level. The expectation of a high relative price of labor leads profit-maximizing firms to undertake an innovation investment today to save on their labor input tomorrow. It is in this sense that innovations are *induced*.[4] In the innovation regime capital accumulation remains essential to sustain productivity growth because a high growth rate of capital implies a high relative price of labor.

The extension of the neoclassical growth model that we suggest allows for a final-good sector and an intermediate-good sector. Both sectors are competitive.

[3]The stationary regime will later be shown to coincide with the neoclassical growth model. It is well known that the core of this model can be stated without reference to factor markets. In its simplest version it boils down to studying a differential equation that links changes in the aggregate stock of capital to aggregate savings.

[4]The idea that labor-saving inventions occur in circumstances where real wages are expected to rise relative to the real interest rate already appears in an informal note by Fellner (1961).

Final-good production requires capital and an intermediate good as inputs. The final good is used for consumption, capital, or as an input in innovation.

Innovation occurs at the level of competitive intermediate-good firms. The specification of the intermediate-good sector is based on Bester and Petrakis (1998) and Hellwig and Irmen (2001). The technology of such firms makes use of two inputs to be delivered at different dates. To produce in period $t$ an intermediate-good firm must employ labor in period $t$. The productivity of labor depends on whether and to what extend this firm engaged in innovation in period $t-1$. A firm that does not innovate in $t-1$ can avail itself for production in $t$ of the labor productivity used in $t-1$. Innovation improves on this level and requires an innovation investment of the final good in period $t-1$.

The incentive to innovate is located in the input market of intermediate-good firms. It stems from inframarginal rents which in turn depend on the level of the expected relative price of labor. According to the level of this expectation firms may or may not innovate. In equilibrium the expected relative price of labor is linked to the economy's expected capital intensity so that under perfect foresight the evolution of the economy can be characterized in terms of the evolution of its capital intensity.

Our analysis suggests that the induced-innovation argument allows for the integration of endogenous technical change in the neoclassical growth model while preserving its main dynamic properties. Indeed, we find that the dynamical system of the economy has a unique stable steady state. Yet, depending on the efficiency of an economy's research technology and/or its assessment of future consumption this steady state is stationary or exhibits endogenous technical change. In the former case, the evolution of the economy is as in the neoclassical growth model. Diminishing returns guide the economy towards a steady state with constant per-capita output. In the latter case we observe "growth through stages." An economy starting in the stationary regime will at some point in time switch into the innovation regime. This switch is brought about by the expectation of a high capital intensity which can be sustained even if some current savings are directed towards innovation investments. Once the innovation regime is reached the economy stays there forever and converges towards a steady state with a constant capital intensity and a constant growth rate of labor productivity. Capital, per-capita consumption and output of the final good grow this rate.

The related literature includes studies that emphasize other potential links between the economic development of countries, the accumulation of physical capital and/or factor prices. For instance, Galor and Moav (2001) share our view that the evolution of relative factor prices may ignite a regime switch that fosters economic growth. They emphasize a falling profit rate as an incentive to augment labor via human capital accumulation and interpret the establishment of public education in the second half of the 19th century as a cooperative endeavor made by workers and capitalists which let to the eventual demise of class structure. According to Galor and Moav, capitalists were ready to financially support public education as education augments labor which in turn raised their profit rates due to the complementarity between physical and human capital in aggregate production.

A similar effect on the profit rate is present in our model. While technical change is labor-saving at the level of the individual firm it is labor-augmenting at the aggregate level. Accordingly, technical change implies more output of the intermediate good which raises the marginal productivity of capital and the profit rate. Then, the owners of capital may be the prime beneficiaries of labor-saving technical change. Yet, augmenting labor is here the result of individual profit-maximizing behavior.

Howitt and Aghion (1998) consider a Schumpeterian model of endogenous growth with capital accumulation. Capital accumulation matters for the economy's long-run growth rate through a scale effect that augments the profit of innovating firms. Similar to our setup, research and the production of capital use the same inputs. However, in our model the scale effect is absent and capital accumulation nevertheless matters for innovation incentives through its effect on relative factor prices.

Scale effects are also at the heart of Matsuyama (1999) who studies a variant of Rivera-Batiz and Romer's (1990) lab equipment model. He finds that capital accumulation may give rise to cyclical growth with an economy switching between a 'Solow' (stationary) and a 'Romer' (innovation) regime. Contrary to our study factor price movements do not affect the incentives of innovating firms and are therefore inessential for his findings. However, the present paper adds a new argument to Matsuyama's critique of Krugman's (1994) pessimistic prediction concerning the economic performance of the East-Asian Tigers. Indeed, if capital accumulation along neoclassical lines is a satisfactory explanation of the past growth performance of the Tigers (Lau and Kim (1994), Young (1995)) the present study suggests that the Tigers may switch into a regime of endogenous technical change as past capital accumulation has rendered labor sufficiently expensive.

The role of factor prices has also been stressed in the recent literature on technology adoption and economic growth. Zeira (1998) argues that the adoption of technical innovations replacing labor with machines depends on relative factor prices which in turn reflect country-specific parameters such as the productivity of the aggregate production technology and intertemporal preferences. Countries that differ only slightly in this respect may follow different paths of technology adoption and economic growth staying either at the technological frontier or remain technologically backward.

Among others, these parameters also account for different growth performances of countries in our model. For instance, countries equipped with a good infrastructure have a high aggregate income, thus savings are high. Therefore, these countries reach the innovation regime earlier and converge towards a steady state with a higher capital intensity and a higher growth rate of labor productivity.

The paper is organized as follows. In section 2 we present the details of the model. Section 3 studies the intertemporal general equilibrium, characterizes the dynamical system, and analyzes possible equilibrium paths. In Section 4 we discuss some extensions. Section 5 concludes.

## 2. The Model

We study an economy comprising a household sector, a final-good sector, and an intermediate-good sector in an infinite sequence of periods $t = 1, 2, ...$ There are four objects of exchange, a manufactured final good, a manufactured intermediate good, labor, and bonds. We call 'final good' a commodity which serves for consumption as well as for investment. If invested, this commodity is either used as future capital in the final-good sector or as an immediate input into innovation undertaken by firms of the intermediate-good sector.

In each period $t$, there are markets for all four objects of exchange. Treating the final good as the numéraire, we let $p_t$ denote the real price of the intermediate good, $w_t$ the real wage, and $p_t^b$ the real bond price at $t$. A bond at $t$ is a claim on one unit of the final good at $t+1$. Working with real interest rates rather than bond prices, we write $p_t^b = 1/(1 + r_{t+1})$ where $r_{t+1}$ is the real interest rate from $t$ to $t+1$.

### 2.1. The Household Sector

The household sector has an initial endowment of $B_1$ bonds coming due at $t = 1$ and owns the shares of all firms in the economy. Moreover, it supplies $L$ units of labor in each period. The allocation of per-period income to consumption and savings is subject to the budget constraint

$$C_t + \frac{B_{t+1}}{1 + r_{t+1}} = w_t L + B_t + \Pi_t, \tag{2.1}$$

where $C_t$ is consumption of the final good, $B_{t+1}$ is bond demand in $t$, $w_t L$ is wage income, $B_t$ capital income from the repayment of bonds due in $t$, and $\Pi_t$ denotes the aggregate dividend distribution.

As to the consumption-savings decision of the household sector we take a behavioristic point of view and assume that real aggregate savings in $t$ is a fixed fraction of aggregate income in $t$, i.e.,[5]

$$\frac{B_{t+1}}{1 + r_{t+1}} = s\left(w_t L + B_t + \Pi_t\right), \tag{2.2}$$

with $s \in (0, 1)$ denoting the marginal and average propensity to save.

[5] The main results of the analysis can also be obtained using a standard two-period lived overlapping generations framework.

## 2.2. The Final-Good Sector

The final-good sector produces according to the production function

$$Y_t = F(K_t, X_t); \tag{2.3}$$

here $Y_t$ is aggregate output of the final good, $K_t$ is capital input in $t$, and $X_t$ denotes the amount of the intermediate good used in period-$t$ production. The function $F : \Re_+^2 \to \Re_+$ exhibits constant returns to scale, satisfies standard concavity, differentiability as well as Inada conditions. The latter are often sufficient but not necessary for our results.

One may want to think of the final-good sector as comprising three different sectors, i.e. a consumption-good sector, a capital-good sector, and an investment-good sector, each endowed with the same production technology (2.3). Because this production function has constant returns to scale, output in the final-good sector can be described in terms of the actions of a single, aggregate firm.

Capital in $t$ must be installed one period before its use in production and fully depreciates after being used.[6] A capital investment of $K_t$ units undertaken in period $t-1$ is financed by an issue of $(1+r_t)K_t$ bonds.

In terms of the final good of period $t$ as numéraire the profit in $t$ of the final-good sector is

$$Y_t - (1+r_t)K_t - p_t X_t, \tag{2.4}$$

where $(1+r_t)K_t$ is capital service payments and $p_t X_t$ is the cost of the intermediate-good input. The price for $K_t$ in units of the final good in $t$ is $(1+r_t) > 1$. This reflects the fact that $K_t$ must be carried over from period $t-1$ before its use in production in period $t$.

The final-good sector takes the sequence $\{p_t, r_t\}$ of prices and interest rates as given and maximizes the sum of the discounted present values of profits in all periods. Since it simply buys capital and intermediate goods for each period, its maximization problem is equivalent to a series of one-period maximization problems. The respective first-order conditions for capital and intermediate goods state that the value of the respective marginal product must be equal to the input price. Define the period-$t$ capital intensity in the final good-sector as

$$k_t \equiv \frac{K_t}{X_t}. \tag{2.5}$$

[6] Clearly, one may argue that a defining property of capital is that it depreciates at a rate $\delta \in [0, 1)$. The assumption $\delta = 1$ is only made because it simplifies the exposition. For it guarantees that the intertemporal price of the final good is independent of its use as capital in the final-good sector or as an input into innovation. None of our results is affected by this assumption.

Then, for $t = 1, 2, ...$ with $f(k_t) \equiv F(k_t, 1)$ the first-order conditions are

$$K_t \quad : \quad f'(k_t) = 1 + r_t \tag{2.6}$$

$$X_t \quad : \quad f(k_t) - k_t f'(k_t) = p_t \tag{2.7}$$

Initially, the final-good sector has $K_1 = \overline{K}_1$ units of capital at its disposal.[7] It stems from investment decisions prior to period $t = 1$ and causes outstanding debt obligations equal to $(1 + r_1)\overline{K}_1$. This is the counterpart of the initial bond holdings of the household sector:

$$B_1 = (1 + r_1)\overline{K}_1. \tag{2.8}$$

### 2.3. The Intermediate-Good Sector

The set of all intermediate-good firms is represented by the set $\Re_+$ of nonnegative real numbers.

**Technology** With respect to the production of output in periods $t = 1, 2, ...,$ all firms have the same technology. Each firm has a capacity limit of one unit of output per period.[8] Its output in period $t$ is given as

$$x_t = \min\{1, a_t l_t\}, \tag{2.9}$$

where $a_t$ is the firm's labor productivity in period $t$ and $l_t$ its labor input. The firm's labor productivity $a_t$ is equal to

$$a_t = A_{t-1}(1 + q_t); \tag{2.10}$$

here $A_{t-1}$ is an indicator of the economy-wide labor productivity in period $t - 1$, and $q_t$ is an indicator of productivity growth at this firm.

To achieve a productivity growth rate $q_t > 0$ from period $t - 1$ to period $t$, the firm must invest $i(q_t)$ units of the final good in period $t - 1$. The function $i(.)$ satisfies

$$i(0) = 0; \;\; i'(q_t) > 0 \text{ and } i''(q_t) \geq 0 \text{ for } q_t \geq 0. \tag{2.11}$$

If the firm decides not to innovate in period $t-1$ it can avail itself for production in $t$ of the production technique of period $t - 1$ in which case $a_t = A_{t-1}$.

[7] Condition (2.6) for $t = 1$ implies that $\overline{K}_1$ is consistent with profit-maximizing behavior.

[8] The analysis is easily extended to a more general specification involving variable capacity based on prior capacity investments, with investment outlays a strictly convex function of capacity. In such a setting profit-maximizing behavior implies that a large innovation investment is accompanied by a large capacity investment. Thus, the simpler specification treated here abstracts from effects on firm size in an environment with changing levels of innovation investments.

Following Hellwig and Irmen (2001) we assume that the resulting innovation is proprietary knowledge of the firm only in period $t$, i.e., the period when it is made. Subsequently, the innovation becomes embodied in the economy-wide productivity indicators $A_t, A_{t+1}, ...$, with no further scope for proprietary exploitation.

**Profit Maximization** The innovation investment $i(q_t)$ in period $t-1$ is financed by an issue of $(1+r_t)\ i(q_t)$ bonds. In terms of the final good of period $t$ as numéraire, a production plan $(q_t, l_t, x_t)$ for period $t$ thus yields the profit

$$\pi_t = p_t x_t - w_t l_t - (1+r_t) i(q_t), \tag{2.12}$$

where $p_t x_t = p_t \min\{1,\ A_{t-1}(1+q_t) l_t\}$ is the firm's revenue from output sales, $w_t l_t$ its wage bill at the real wage rate $w_t$, and $(1+r_t)i(q_t)$ its debt service.

We assume that the firm takes the sequence $\{p_t, w_t, r_t\}$ of real prices as well as the sequence $\{A_t\}$ of aggregate productivity indicators as given and chooses its production plan so as to maximize the sum of the discounted present values of its profits in all periods. Because production choices for different periods are independent of each other, for each period $t$, it will in fact choose the plan $(q_t, l_t, x_t)$ to maximize the profit $\pi_t$ from this plan in period $t$.

If the firm innovates, it incurs an investment cost $(1+r_t)i(q_t)$ that is associated with a given innovation rate $q_t > 0$ and is independent of the output $x_t$. This introduces a positive scale effect, namely if the firm innovates, then it wants to apply the innovation to as large an output as possible and to produce at the capacity limit $x_t = 1$. The choice of $(q_t, l_t)$ must then minimize the costs of producing the capacity output.

Suppose $w_t > 0$, then an input combination $(q_t, l_t)$ that minimizes unit costs must satisfy

$$l_t = \frac{1}{A_{t-1}(1+q_t)}, \tag{2.13}$$

and

$$q_t \in \arg\min_{q \geq 0} \left[ \frac{w_t}{A_{t-1}(1+q)} + (1+r_t) i(q) \right]. \tag{2.14}$$

Given the differentiability and convexity of the innovation cost function $i(.)$, (2.14) actually determines $q_t$ *uniquely* as the solution to the first-order condition

$$\begin{gathered} \frac{w_t}{A_{t-1}(1+q_t^*)^2} \leq (1+r_t) i'(q_t^*), \\ \text{with strict inequality } \textit{only} \text{ if } q_t^* = 0. \end{gathered} \tag{2.15}$$

The latter relates the marginal reduction of the firm's wage bill to the marginal increase in its investment costs. As both marginal effects are proportional to the respective factor price condition (2.15) implies

**Lemma 1** The unit-cost-minimizing growth rate of labor productivity can be expressed in terms of a map $q : \Re_{+} \rightarrow \Re_{+}$,

$$q_t^* = q\left(\frac{w_t}{A_{t-1}(1+r_t)}\right), \tag{2.16}$$

with

$$\begin{aligned} q_t^* &= 0 \text{ for } \frac{w_t}{A_{t-1}(1+r_t)} \leq i'(0), \\ q_t^* &> 0 \text{ otherwise.} \end{aligned} \tag{2.17}$$

Moreover,[9]

$$q'(.) \geq 0 \text{ with strict inequality only if } \frac{w_t}{A_{t-1}(1+r_t)} \geq i'(0).$$

Hence, for any $A_{t-1}$ the chosen growth rate of labor productivity is an increasing function of relative factor prices. More precisely, given that the innovation decision is made in period $t-1$ the choice of $q_t$ depends on the *expected relative factor price ratio.* The higher this ratio the more pronounced is the incentive to engage in labor saving innovation in $t-1$. It is in this sense that an innovation is *induced.* Clearly, the evolution of factor prices to be determined by general equilibrium conditions will play a crucial role for whether and to what extend intermediate-good firms engage in innovation investment.

Lemma 1 also emphasizes the role of the plausible assumption $i'(0) > 0$ which we made in (2.11). Roughly speaking, it means that the first unit of $q$ is *not* costless. Instead, had we assumed $i'(0) = 0$ any equilibrium with a strictly positive real wage would imply a strictly positive growth rate of labor productivity as the unit-cost minimizing choice.

A choice of $q_t^*$ may not be profit-maximizing if the prices $p_t$, $w_t$, $r_t$, and the productivity index $A_{t-1}$ are such that the profit associated with a production plan $(q_t, l_t, x_t)$ is negative. In this case the firm will prefer not to produce any output at all, i.e. it chooses the production plan $(0, 0, 0)$. Any profit-maximizing production plan with output $x_t > 0$ must therefore satisfy

$$\pi_t^* := \pi\left(q_t^*; p_t, w_t, r_{t-1}, A_{t-1}\right) \geq 0. \tag{2.18}$$

We assume that if intermediate-good firms choose to be active, they *always* plan to produce the capacity output $x_t = 1$. This assumption simplifies the exposition because it implies that all active intermediate-good firms choose the

[9]The last result follows immediately from total differentiation of (2.15) in conjunction with (2.11).

same production plan $(q_t, l_t, x_t)$ for period $t$.[10] No significant loss of generality is involved because in circumstances where intermediate-good firms do plan to produce some output $x_t \neq 1$, their maximized profits as well as their innovation investments are zero, and they would be just as willing to choose the production plan $(0, 1/A_{t-1}, 1)$ or the production plan $(0, 0, 0)$. It would therefore be possible to rearrange profit-maximizing production plans across firms so that all firms plan to have output equal to either zero or one and moreover the aggregate impact of the intermediate-good sector on markets is unchanged.

**The Aggregate Intermediate-Good Sector** The set of all active firms in period $t$ has Lebesgue measure $n_t$ and generates

- an aggregate investment demand in period $t-1$ of $n_t i(q_t^*)$,
- a corresponding aggregate supply of bonds in $t-1$ of $(1+r_t)\, n_t i(q_t^*)$,
- an aggregate labor demand in period $t$ of $n_t l_t$,
- an aggregate intermediate-good supply in period $t$ of $n_t x_t = n_t$.

**Zero-Profits** In representing the set of all intermediate-good firms by $\Re_+$ with Lebesgue measure, we implicitly introduce a zero-profit condition. Given that labor supply in each period is bounded, in any equilibrium the set of intermediate-good firms employing more than some $\varepsilon > 0$ units of labor must have bounded measure and hence must be smaller than the set of all intermediate-good firms. Given that inactive intermediate-good firms must be maximizing profits just like the active ones, this implies that in any equilibrium in any period $t$, $t = 1, 2, \ldots$, maximum profits of intermediate-good firms at equilibrium prices must be equal to zero.

**Initial Conditions** As to intermediate-good production in period $t = 1$ we assume that *none* of the intermediate-good firms active in this period has made an innovation investment prior to period 1. This reflects our intention to study the evolution of an economy that starts in an environment without technical change.

**Economy-wide Productivity Indicators** To conclude the account of the intermediate-good sector, we turn to the evolution of the economy-wide productivity indicators $A_0, A_1, A_2, \ldots$ As mentioned above we assume that all innovations are publicly available after one period. Anybody can then incorporate them into

[10] Indeed, for any constellation of the parameters $p_t$, $w_t$, $r_t$, and $A_{t-1}$, there may be more than one profit-maximizing production plan. In particular, if we have $\pi_t^* = 0$, maximum profits are zero, and this maximum is attained at both, the plan $(q_t, l_t, x_t)$ satisfying (2.13), (2.16), and $x_t = 1$, *and* the plan $(0, 0, 0)$ providing for inactivity of the firm in period $t$. If in addition $q^* = 0$, profits are maximized by *any* production plan of the form $(0, x/A_{t-1}, x)$.

their production processes or take them as a basis for additional innovations. Proprietary use of innovations is thus limited to the period in which they occur. Given that for any $t$ all firms that are active at $t$ choose the same innovation rate $q_t^*$ and attain the same labor productivity $a_t = A_{t-1}(1+q_t^*)$, we identify $A_t$ with $a_t$ and write

$$A_t = A_{t-1}(1+q_t^*) \tag{2.19}$$

for $t = 1, 2, ...$, with $a_1 = \overline{A}_0 > 0$ given by initial conditions.

## 3. Intertemporal General Equilibrium

### 3.1. Definition

Turning to the behavior of the economy as a whole, we refer to a sequence $\{p_t, w_t, r_t\}$ of real prices for the intermediate good, real wages, and real interest rates for periods $t = 1, 2, ...$ as a *price system.* By an *allocation* we understand a sequence $\{C_t, L, B_t, Y_t, K_t, X_t, n_t, q_t, l_t\}$ that comprises a strategy $\{C_t, L, B_t\}$ for the household, a strategy $\{Y_t, K_t, X_t\}$ for the final-good sector, and, for each $t$, a measure $n_t$ of intermediate-good firms active at $t$ producing the capacity output $x_t = 1$ with input choices $(q_t, l_t)$.

An *equilibrium* will correspond to a price system, an allocation, and a sequence $\{\Pi_t, A_t\}$ of distributed aggregate profits, and productivity indicators that satisfy the following conditions:

(E1) Given the initial bond endowment $B_1$ and the sequence $\{w_t, r_t, \Pi_t\}$ of real wages, interest rates, employment, and dividend distributions $\Pi_t$, the household sector saves according to (2.2) and supplies $L$ units of labor in all periods.

(E2) For any $t$, the profit distribution $\Pi_t$ which the household sector expects to receive at $t$ is equal to the actual aggregate of the profits that accrue at $t$ in the final-good sector and the intermediate-good sector, i.e.,

$$\Pi_t = Y_t - (1+r_t)K_t - p_t X_t + n_t \left[p_t - w_t l_t - (1+r_t) i\,(q_t)\right].$$

(E3) Given $p_t$ and $r_t$ for all $t \geq 1$ the final-good sector produces according to (2.3). Its profit-maximizing behavior is characterized by conditions (2.6) and (2.7).

(E4) Given the productivity indicator $A_{t-1}$, the real output price $p_t$, the real wage rate $w_t$, and the real interest rate $r_t$, for any $t \geq 1$, the input choice $(q_t, l_t, 1)$ minimizes the unit cost of production of an intermediate-good firm active at $t$. By assumption $q_1 = 0$.

(E5) Given the productivity indicator $A_{t-1}$, the real output price $p_t$, the real wage rate $w_t$, and the real interest rate $r_t$, for any $t \geq 1$,

$$\pi^* \leq 0, \tag{3.1}$$

with a strict inequality only if $n_t = 0$.

(E6) (final-good market) For any $t$,

$$Y_t = C_t + K_{t+1} + n_{t+1} i\left(q_{t+1}\right). \tag{3.2}$$

(E7) (intermediate-good market) For any $t$,

$$X_t \leq n_t, \tag{3.3}$$

with a strict inequality only if $p_t = 0$.

(E8) (bonds market) For any $t \geq 1$,

$$B_{t+1} = \left(1 + r_{t+1}\right)\left(K_{t+1} + n_{t+1} i\left(q_{t+1}\right)\right).$$

(E9) (labor market) For any $t$,

$$n_t l_t \leq L, \tag{3.4}$$

with a strict inequality only if $w_t = 0$.

(E10) For any $t$, the indicators $A_t$ satisfy the updating condition (2.19).

In specifying a consistent circular flow of income, condition (E2) ensures that in equilibrium aggregate income equals total output in the final good sector. To see this use (E2), (E7) and (E9) for period $t$, and (E8) for $t-1$ to find

$$\begin{aligned} w_t L + B_t + \Pi_t &= w_t L + B_t + Y_t - (1+r_t)K_t - p_t X_t \\ &\quad + n_t\left[p_t - w_t l_t - (1+r_t) i\left(q_t\right)\right] \\ &= Y_t + p_t\left[n_t - X_t\right] + w_t\left[L - n_t l_t\right] \\ &\quad + \left[B_t - (1+r_t)\left(K_t + n_t i\left(q_t\right)\right)\right] \\ &= Y_t. \end{aligned}$$

Two implications are immediate. First, given our assumption that intermediate-good firms active in $t=1$ will not have innovated and with (2.8) stating the analogue of (E8) for $t=0$ we find that aggregate savings as specified by (2.2) is

$$\frac{B_{t+1}}{1+r_{t+1}} = sY_t \text{ for } t = 1, 2, .... \tag{3.5}$$

so that the bond market equilibrium condition (E8) can be stated as

$$sY_t = K_{t+1} + n_{t+1} i\left(q_{t+1}\right) \text{ for } t = 1, 2, ... \tag{3.6}$$

The second implication concerns Walras' Law. Indeed, from the household sector's budget constraint (2.1), (3.5), and (3.6) it follows that for all $t$ (E6) holds if (E8) does. The equilibrium condition of the final-good market is therefore redundant.

### 3.2. The Dynamical System

The purpose of this section is to establish the dynamical system which allows us to study the evolution of the economy for given parameter constellations and initial conditions. We choose the capital intensity in the final-good sector as the state variable of the system. We begin with the implications of the equilibrium conditions (E1) - (E10) for the price system $\{p_t, w_t, r_t\}$ and the state variable $k_t$.

**Lemma 2** Let $\{p_t, w_t, r_t\}$, $\{C_t, L, B_t, Y_t, K_t, X_t, n_t, q_t, l_t\}$, $\{\Pi_t, A_t\}$ be an equilibrium. Then, for $t = 1, 2, ...$ $p_t > 0, w_t > 0$, and $1 + r_t > 0$.

**Proof.** From (2.6), (2.7), and the fact that the function $f(k_t)$ satisfies standard Inada conditions it follows that $p_t > 0$ and $1+r_t > 0$. If $w_t \leq 0$, intermediate-good firms chose $q_t^* = 0$, $x_t = 1$, and earned strictly positive profits at $p_t > 0$ thus violating (E5).

QED.

**Lemma 3** Let $\{p_t, w_t, r_t\}$, $\{C_t, L, B_t, Y_t, K_t, X_t, n_t, q_t, l_t\}$, $\{\Pi_t, A_t\}$ be an equilibrium. Then,

$$k_t = \frac{K_t}{A_t L} \text{ for } t \geq 1 \tag{3.7}$$

with $k_1 = \overline{K}_1 / \ A_1 L > 0$ given by initial conditions.

**Proof.** By Lemma 2 all prices are strictly positive. Consider (E9) for any $t$, condition (2.13) in conjunction with the updating condition (2.19) and (E7) to find successively that

$$L = n_t l_t = n_t / A_{t-1} (1 + q_t) = n_t / A_t = X_t / A_t \Leftrightarrow X_t = A_t L. \tag{3.8}$$

Then, the lemma follows from the definition of $k_t$ and the fact that $\overline{K}_1$ and $A_1 = \overline{A}_0$ are given by initial conditions.

QED.

Lemma 3 points to the close link between the labor market, the measure of active intermediate-good firms producing one unit of output, and the equilibrium in the intermediate-good market. Indeed, with each intermediate-good firm employing $l_t = 1/a_t = 1/A_t$ units of labor, we have in equilibrium for all $t$

$$A_t L = n_t = X_t \tag{3.9}$$

so that the capital intensity in the final-good sector is equal to capital per unit of efficient labor. Moreover, output of the final-good sector (2.3) has the familiar form

$$Y_t = F(K_t, A_t L).$$

We may use (3.9) to disentangle the notion of '*labor-saving* technical change' at the level of the individual firm from the notion of '*labor-augmenting* technical change' at the level of economic aggregates. When an intermediate-good firm innovates between period $t-1$ and $t$ its labor productivity in $t$ is $a_t = A_{t-1}(1+q_t)$ and employment per firm shrinks by the factor

$$\frac{l_t}{l_{t-1}} = \frac{a_{t-1}}{a_t} = \frac{1}{1+q_t}.$$

At the same time free entry in intermediate-good production assures full employment and with (3.9)

$$\frac{A_t L}{A_{t-1} L} = \frac{n_t}{n_{t-1}} = \frac{X_t}{X_{t-1}} = 1 + q_t.$$

Hence, at the level of economic aggregates technical change augments the measure of intermediate-good firms and the output of intermediate goods by a factor $1+q_t$.

Next, we turn to the evolution of $k_t$ for $t > 1$ :

**Proposition 1** Let $\{p_t, w_t, r_t\}$, $\{C_t, L, B_t, Y_t, K_t, X_t, n_t, q_t, l_t\}$, $\{\Pi_t, A_t\}$ be an equilibrium. Then, for $t = 2, 3, ...$ $k_t$ evolves according to

$$k_t = \begin{cases} sf(k_{t-1}) & \text{if } q_t = 0, \\ \frac{s}{1+q_t} f(k_{t-1}) - i(q_t) & \text{if } q_t > 0. \end{cases} \tag{3.10}$$

**Proof.** Consider (3.6) with (2.3):

$$sF(K_{t-1}, X_{t-1}) = K_t + n_t i(q_t).$$

Use Lemma 2, (E7) for periods $t-1$ and $t$, and the definition of $k_t$ to write the latter as

$$k_t = \frac{n_{t-1}}{n_t} sf(k_{t-1}) - i(q_t).$$

From (3.9) we have for all $t$ that $n_t = A_t L$. If $q_t > 0$, then $n_t = (1+q_t) n_{t-1}$, and if $q_t = 0$, then $n_t = n_{t-1}$. In view of $i(0) = 0$ (3.10) follows.

QED.

The evolution of $k_t$ depends on the amount of aggregate savings in $t-1$ and on the growth rate of labor productivity between $t-1$ and $t$. The term $sf(k_{t-1})$ is $(t-1)$-savings per unit of efficient labor in $t-1$. Without technical change the amount of efficient labor in the economy remains constant over time. Changes in $k_t$ come about through a rise or a fall of the capital input employed in the final-good sector. Indeed, if $q_t = 0$ the capital intensity $k_t$ evolves as in the neoclassical growth model and (3.10) is a special case of Solow's famous equation.

The presence of technical change between period $t-1$ and $t$ modifies the expression for $k_t$ in two ways. First, technical change 'augments' labor in $t$ by a factor $1+q_t$. Therefore, $sf(k_{t-1})$ must be divided by this factor. Second, $i(q_t)$ is simply innovation investment per unit of efficient labor in $t$, i.e., the amount of $(t-1)$-savings per unit of efficient labor in $t$ which is no longer available as period-$t$ capital in the final-good sector.

Proposition 1 treats $q_t$ as a parameter. In order to endogenize it we have to embed the intermediate-good firms' innovation decision in the general equilibrium framework. The following proposition establishes that in equilibrium the chosen growth rate of labor productivity depends on $k_t$.

**Proposition 2** Suppose (E3) - (E5) hold and let

$$\lim_{k_t \to \infty} \frac{f - k_t f'}{f'} > i'(0). \tag{3.11}$$

Then, there is $k_c > 0$ and a map $g : \Re_+ \to \Re_+$ such that

$$q_t^* = g(k_t) \text{ with } \begin{cases} g(k_t) = 0 & \text{for } k_t \leq k_c, \\ g(k_t) > 0 & \text{for } k_t > k_c. \end{cases} \tag{3.12}$$

Moreover, the equilibrium wage is

$$w_t = A_{t-1}(1+g)\left(f - k_t f' - f' i(g)\right), \tag{3.13}$$

where the argument of $f$ and $g$ is $k_t$.

**Proof.** See the Appendix.

Proposition 2 shows that the equilibrium conditions of both production sectors imply a relationship between the rate of productivity growth chosen by intermediate-good firms and the capital intensity in the final-good sector. This relationship is summarized in the function $g(k_t)$ which is piecewise defined. There is a critical level $k_c$ so that in period $t$ the economy must be in one of two regimes depending on how $k_t$ relates to $k_c$. If $k_t > k_c$ we say that the economy is in the *innovation regime* because intermediate-good firms choose a strictly positive growth rate of labor productivity. If instead $k_t \leq k_c$ no innovation occurs and the economy is said to be in the *stationary regime*.

The intuition behind the proposition is the following. The technology of the final-good sector implies that the marginal productivity of capital falls in $k_t$ whereas the marginal productivity of intermediate goods rises. Accordingly, the real interest rate falls and the price of the intermediate good rises in $k_t$. These price changes feed back onto wages through the zero-profit condition of the intermediate-good sector. The *expected relative factor price ratio* which we found to determine the intermediate-good sector's innovation decision can then be expressed as a increasing function of the capital intensity in the final good sector.

Indeed, for $k_t \leq k_c$ intermediate-good firms break even at a wage $w_t = A_{t-1}(f - k_t f')$ so that with (2.6) we find

$$\frac{w_t}{A_{t-1}(1+r_{t-1})} = \frac{f - k_t f'}{f'}. \tag{3.14}$$

The right-hand side of (3.14) is the technical rate of substitution (TRS) between capital and the intermediate good of the final-good sector's production function at some $(K_t, X_t)$. The TRS increases in $k_t$ so that under (3.11) there is a critical capital intensity $k_t = k_c$ which satisfies

$$\frac{f - k_t f'}{f'} = i'(0). \tag{3.15}$$

In general, $k_c$ depends on the assumed degree of substitutability in the final-good production and on $i'(0)$, the amount of the final-good necessary to obtain the first unit of $q_t$. We may interpret $i'(0)$ as an indicator of the efficiency of the available innovation technology. Then $k_c$ is lower the more efficient this technology is.

For $k_t > k_c$, intermediate-good firms innovate and break even at a wage $w_t = A_{t-1}(1+q_t)(f - k_t f' - f' i(q_t))$. At this wage and an interest rate given by (2.6) the first-order condition for unit-cost minimization in (2.17) determines a unique growth rate of labor productivity $q_t^* = g(k_t)$ that satisfies

$$\frac{f - k_t f'}{f'} = i(q_t) + (1+q_t)\, i'(q_t). \tag{3.16}$$

For the general equilibrium, condition (3.16) assures the static efficiency of the allocation in $t$ when some foregone $(t-1)$-consumption is channeled towards innovation.[11] To see this, write (3.16) as

$$\frac{\partial F(K_t, X_t)}{\partial X_t} = \frac{\partial F(K_t, X_t)}{\partial K_t}\left[i(q_t) + (1+q_t)\, i'(q_t)\right] \tag{3.17}$$

and consider a marginal increase $dX_t = 1$. The left-hand side is the marginal productivity of this additional unit of the intermediate good in final-good production in $t$. The right-hand side is the marginal productivity of the amount of foregone capital necessary to provide for this unit of $X_t$. Indeed, given $q_t$ the equilibrium in the intermediate-good market requires $dX_t = dn_t = 1$ which creates additional

[11] Static efficiency means that the allocation in $t$ maximizes final-good output in $t$ for a given level of $(t-1)$-savings subject to the equilibrium in the intermediate-good market and the labor market as expressed in (3.9).

investment needs of $i\left(q_t\right)$. Moreover, given $n_t$ the link between the intermediate good-market and the labor market featured in (3.9) with $a_t L = A_{t-1}(1+q_t)L$ implies $1 = dX_t = A_{t-1}Ldq_t$. Hence, each intermediate-good firm has to increase its growth rate of labor productivity by $dq_t = 1/A_{t-1}L$ in order to produce the additional unit of output. In the aggregate, this augments intermediate-good firms' investment demand in $t-1$ by $n_t i'\left(q_t\right) dq$. In view of the labor-market equilibrium in $t$ we have $n_t = A_t L$ and the additional investment demand is

$$n_t i'\left(q_t\right) dq = \frac{A_t L}{A_{t-1}L} i'\left(q_t\right) = \left(1+q_t\right) i'\left(q_t\right).$$

Hence, the term in brackets on the right-hand side of (3.17) is the foregone final-good output in $t-1$ necessary to raise $X_t$ marginally.

**Corollary 1** It holds that

$$\frac{dq_t^*}{dk_t} = \frac{dg\left(k_t\right)}{dk_t} \begin{cases} = 0 & \text{for } k_t \leq k_c \\ > 0 & \text{for } k_t \geq k_c. \end{cases}$$

**Proof.** Follows immediately from total differentiation of (3.16).

The intuition behind Corollary 1 is easily seen from equation (3.17). Consider an increase in $K_t$ holding $X_t$ constant. The left-hand side rises because inputs into final-good production are complements. On the right-hand side, the marginal productivity of capital falls so that the term in brackets must increase. These additional innovation needs are rising in $q_t$ so that Corollary 1 follows.

We noticed following Lemma 1 that the intermediate-good firms' innovation decision relies on the *expected* relative factor price ratio. By Proposition 2, the relative factor price ratio depends on $k_t$ which, from the vantage point of period $t-1$, is to be interpreted as the *expected* capital intensity. In equilibrium under perfect foresight actual and anticipated developments must coincide. This requirement leads to the characterization of the dynamical system in

**Proposition 3** Denote

$$\underline{k}_c := f^{-1}\left(\frac{k_c}{s}\right). \tag{3.18}$$

The dynamical system of the economy for $t = 2, 3, ...$ is characterized by a continuous, monotonically increasing map $k_t = \Psi\left(k_{t-1}\right)$ where $\Psi : \Re_+ \rightarrow \Re_+$ and satisfies

$$k_t = \begin{cases} sf\left(k_{t-1}\right) & \text{if } k_{t-1} \leq \underline{k}_c, \\ \frac{s}{1+g(k_t)} f\left(k_{t-1}\right) - i\left(g\left(k_t\right)\right) & \text{if } k_{t-1} > \underline{k}_c. \end{cases} \tag{3.19}$$

**Proof.** See the Appendix.

Proposition 3 shows the existence of a unique level of the capital intensity $\underline{k}_c$ that separates the stationary and the innovation regime from an ex ante point of view. The intuition is as follows. If $k_{t-1} \leq \underline{k}_c$, then $k_t \leq k_c$ even if none of the intermediate-good firms innovates. Next period's relative factor price ratio is too low for $k_t$ to make innovation a profitable endeavor. On the other hand, if $k_{t-1} > \underline{k}_c$ then $k_t > k_c$ even if all intermediate-good firms active in $t$ innovate so that the expectation of a high relative factor price ratio is fulfilled.

Observe that $\underline{k}_c$ reflects both the preferences and the technology of an economy. From (3.18) we readily verify that $\underline{k}_c$ is smaller the larger $s$ and the smaller $i'(0)$. In other words, for an economy with a high propensity to save and/or an efficient innovation technology $\underline{k}_c$ is low.

### 3.3. The Equilibrium Path

This section studies the evolution of the economy that starts in the stationary regime. According to the following proposition the equilibrium path can take two distinct forms. Figures 3.1 and **??** illustrate these cases.

**Proposition 4** Let $0 < k_1 < k_c$. Then, the economy may evolve in two different ways:

- If $\underline{k}_c \geq k_c$ then the economy starts in the stationary regime and remains there forever. It converges towards the unique steady state defined by

$$k^* = sf(k^*). \tag{3.20}$$

- If $k_c > \underline{k}_c$ then the economy starts in the stationary regime, switches at some $t > 1$ into the innovation regime, and converges towards a unique steady state which satisfies

$$k^{**} = \frac{s}{1 + g(k^{**})} f(k^{**}) - i(g(k^{**})). \tag{3.21}$$

**Proof.** See the Appendix.

As $0 < k_1 < k_c$, the economy starts in the stationary regime in $t = 1$. This is consistent with our previous assumption that none of the intermediate-good firms active in $t = 1$ will have innovated. Then, the equilibrium paths depends on how the critical values $\underline{k}_c$ and $k_c$ relate to each other. It is not difficult to see that $\underline{k}_c$ is more likely to exceed $k_c$ if $s$ is low and $i'(0)$ is large. Therefore, an economy for which $\underline{k}_c \geq k_c$ saves little in relation to the efficiency of its innovation technology. According to Proposition 4 such economy cannot reach the innovation regime.

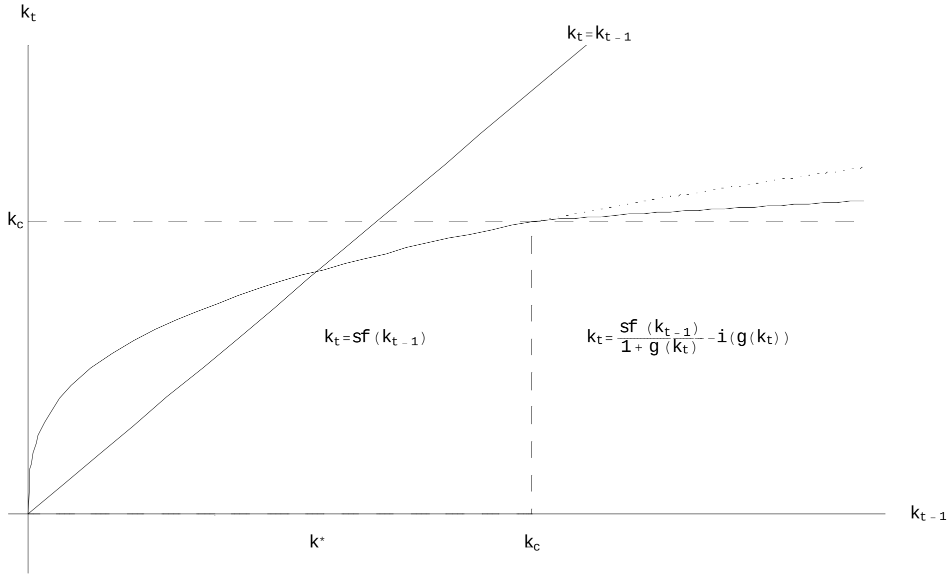


Figure 3.1: $k_c \leq \underline{k}_c$ : Steady State in the Stationary Regime.

The convergence towards the steady state is as in the neoclassical model without technical change. The steady state is stationary in the sense that per-capita output and consumption is constant.

For $k_c > \underline{k}_c$ we observe "growth through stages." The economy starts in the stationary regime, accumulates a sufficient amount of capital which induces at some $t$ expectations about relative factor prices that lead to innovation. Following the switch into the innovation regime, there is both capital accumulation and innovation. More precisely, for $\underline{k}_c < k_{t-1} < k^{**}$, the capital intensity grows and from (3.7) it must hold that

$$\frac{K_t}{K_{t-1}} > \frac{A_t}{A_{t-1}} = (1 + q_t),$$

i.e., capital grows faster than labor productivity. Moreover, from Corollary 1 it follows that $q_t$ rises over time. Productivity growth is endogenous and responds to a rising capital intensity in final-good production. Indeed, the *expected factor price ratio* can be written as

$$\frac{w_t}{1 + r_t} = A_{t-1}(1 + g)\left(\frac{f - k_t f'}{f'} - i(g)\right) \tag{3.22}$$

and increases in $k_t$.[12]

[12]Using (3.16) the derivative of the right-hand side of (3.22) with respect to $k_t$ is $-A_{t-1}(1 + g) f'' f / (f')^2 > 0$.

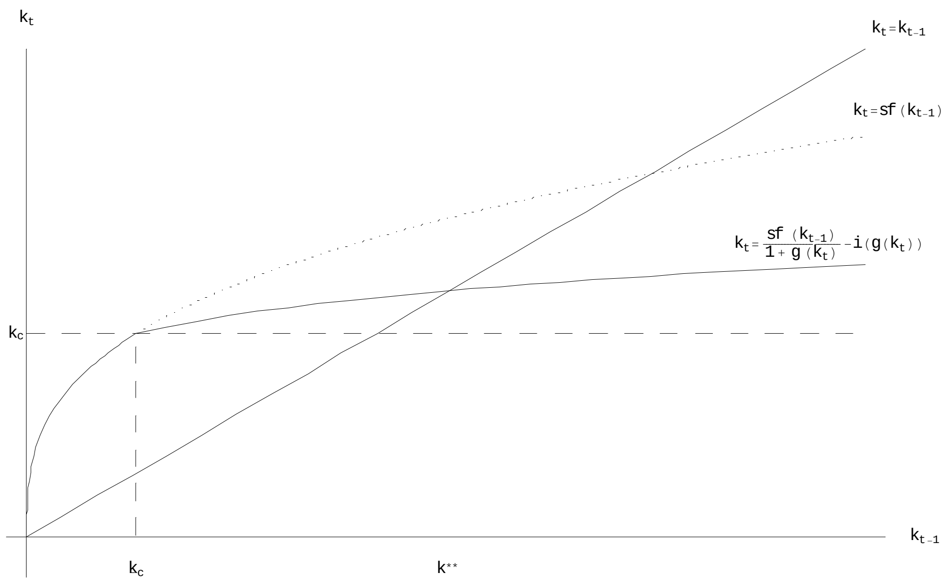


Figure 3.2: $k_c > \underline{k}_c$: Steady State in the Innovation Regime.

Observe that the evolution of $k_t$ is governed by diminishing returns. When $k_{t-1} > \underline{k}_c$, then (3.19) defines $k_t = \Psi(k_{t-1})$ implicitly through

$$(1 + g(k_t))[k_t + i(g(k_t))] = sf(k_{t-1}), \tag{3.23}$$

which in conjunction with the Inada condition $\lim_{k\to\infty} f'(k) = 0$ implies that in the innovation regime growth of $k_t$ cannot be sustained forever. Each new unit of foregone $(t-1)$-consumption generates less additional output of the final-good. Thus, less savings result from the marginal unit than from inframarginal units. Then, the next addition to investment is even smaller than the last, which generates still less additional output.... Therefore, in the long run, capital per effective labor approaches a constant. Capital and labor productivity grow at the same rate which satisfies

$$q^{**} = g(k^{**})$$

and is susceptible to changes in parameters like $s$.

**Corollary 2** It holds that

$$\frac{dk^{**}}{ds} > 0, \frac{dq^{**}}{ds} > 0. \tag{3.24}$$

**Proof.** See the Appendix.

As expected, increasing the propensity to save raises the steady-state capital intensity so that the relative price of labor and the growth rate of labor productivity rises.

## 4. Discussion and Extensions

### 4.1. Subsidizing Capital in the Final-Good Sector

Suppose the final-good sector receives a subsidy $\sigma > 0$ per unit of capital $K_t$ employed in $t$ which is financed by a tax on the household sector's labor income. An immediate implication is that the analogue of the first-order condition (2.6) is now

$$1 + r_t = f'(k_t) + \sigma. \tag{4.1}$$

This affects the equilibrium incentives to innovate in the following way.

**Proposition 5** Let

$$\lim_{k_t \to \infty} \frac{f - k_t f'}{f' + \sigma} > i'(0). \tag{4.2}$$

Then, in equilibrium there is $k_{c\sigma} = k_c(\sigma) > 0$ and a map $g_\sigma : \Re_+^2 \to \Re_+$ such that

$$q_t^* = g_\sigma \text{ with } \begin{cases} g(\sigma, k_t) = 0 & \text{for } k_t \leq k_{c\sigma}, \\ g(\sigma, k_t) > 0 & \text{for } k_t > k_{c\sigma} \end{cases} \tag{4.3}$$

and the equilibrium wage is

$$w_t = A_{t-1}(1+g)\left(f - k_t f' - \left(f' + \sigma\right) i(g)\right), \tag{4.4}$$

where the argument of $f$ and $g$ is $k_t$.

Moreover,

$$\frac{dk_{c\sigma}}{d\sigma} > 0 \text{ and, for } k_t > k_{c\sigma}, \frac{dg_\sigma}{d\sigma} < 0. \tag{4.5}$$

**Proof.** See the Appendix.

Proposition 5 extends Proposition 2 for $\sigma > 0$. The upshot is that a subsidy reduces the incentive to innovate. At a given level of $k_t$ the interest rate rises due to (4.1) so that the price of an innovation investment in the intermediate-good sector increases. This has two implications stated in (4.5). First, increasing

$\sigma$ raises $k_{c\sigma}$. Second, for $k_t > k_{c\sigma}$ the growth rate of labor productivity $g_\sigma$ is lower the higher $\sigma$. The latter captures the fact that the final-good sector and the intermediate-good sector compete for scarce resources. Subsidizing one activity leads in equilibrium to an expansion of this activity at the expense of the other.

The dynamical system is easily derived from replacing $g$ by $g_\sigma$ in (3.19) and noting that for $\sigma > 0$ we have $\underline{k}_{c\sigma} > \underline{k}_c$ because $k_{c\sigma} > k_c$. Accordingly, the equilibrium path has the same properties as stated in Proposition 4 for the case $\sigma = 0$. Yet, a subsidy on capital may have drastic consequences for the evolution of the economy. First, as $\underline{k}_{c\sigma} > \underline{k}_c$ it postpones the switch into the innovation regime. Second, a simple geometrical argument shows that a subsidy may prevent the switch altogether. Indeed, an economy for which the underlying parameters imply $k_c > \underline{k}_c$ may experience a change in the relation of these critical values to $k_{c\sigma} \le \underline{k}_{c\sigma}$ so that it converges towards the stationary steady state.

As to the steady state in the innovation regime one finds[13]

**Proposition 6** If the steady state is in the innovation regime then $k_\sigma^{**} = k^{**}(\sigma)$ and $q_\sigma^{**} = g(\sigma, k^{**}(\sigma))$ with

$$\frac{dk_\sigma^{**}}{d\sigma} > 0 \tag{4.6}$$

$$\frac{dq_\sigma^{**}}{d\sigma} < 0. \tag{4.7}$$

**Proof.** See the Appendix.

Hence, the subsidy raises the steady-state capital intensity. More interestingly, while economies with a high subsidy converge towards a steady state with a higher capital intensity in the final-good sector the steady-state growth rate of the labor productivity is lower. The latter is the result of two opposing effects:

$$\frac{dq_\sigma^{**}}{d\sigma} = \underset{(-)}{\frac{dg_\sigma}{d\sigma}} + \underset{(+)}{\frac{dg_\sigma}{dk_t}}\underset{(+)}{\frac{dk_\sigma^{**}}{d\sigma}} < 0.$$

First, there is a negative direct effect which captures competition among the two sectors for scarce resources. Second, there is an indirect effect through the complementarity of capital and labor in the reduced form of the final-good production function. A higher capital intensity in the aggregate raises the expected factor price ratio and thereby the incentive to innovate.[14]

[13] If the steady state is in the stationary regime it is unaffected by the subsidy as in equilibrium the tax on wage income is equal to the additional capital income that the household sector receives due to the rise in the equilibrium interest rate.

[14] Note that the sign of $dq_\sigma^{**}/d\sigma$ is actually sensitive to the underlying savings hypothesis. Had we endogenized the savings decision through an infinitely lived household and undertaken comparative statics along the Euler condition we would find $dq_\sigma^{**}/d\sigma > 0$.

Had we considered a subsidy on innovation investments instead of capital in the final-good sector the comparative static results (4.6) and (4.7) would be reversed. In particular, the direct effect of such a subsidy is positive and exceeds the effect through a lower steady-state capital intensity.

### 4.2. Population Growth

Allow for the labor force to grow at a constant rate $\lambda > 0$. As population growth does not interfere with the production technology all prices expressed as functions of the state variable $k_t$ remain unchanged and so does the incentive to innovate. However, aggregate $(t-1)$-savings per unit of efficient labor in $t$ falls with $\lambda$. Following the same procedure that led to Proposition 3 one finds the dynamical system of the economy as in (3.19) with $s$ replaced by $s' \equiv s/(1+\lambda)$ and $\underline{k}_c$ by

$$\underline{k}_{\lambda c} := f^{-1}\left(\frac{k_c}{s'}\right). \tag{4.8}$$

The following comparative static results obtain:

**Proposition 7** It holds that

$$\frac{d\underline{k}_{\lambda c}}{d\lambda} > 0. \tag{4.9}$$

If $k_c > \underline{k}_{\lambda c}$ we have

$$\frac{dk_{\lambda}^{**}}{d\lambda} < 0, \; \frac{dq_{\lambda}^{**}}{d\lambda} < 0. \tag{4.10}$$

**Proof.** (4.9) follows from the monotonicity of $f(.)$. The results of (4.10) follow immediately from the definition of $s'$ and Corollary 2.
QED.

Hence, we arrive at the interesting result that economies whose population grows faster reach the innovation regime later or may even fail to reach it if $\underline{k}_{\lambda c}$ becomes too high. The reason is simple. Population growth renders labor more abundant which diminishes the relative price of labor and thus the incentive to engage in labor-saving innovation. This force is also behind (4.10): higher population growth leads to a steady state with a lower capital intensity and a lower growth rate of labor productivity.

### 4.3. Other Country-Specific Factors

Countries differ with respect to climate or the quality of their infrastructure. These features may affect the aggregate production technology and thereby impinge on the growth performance. We follow Zeira (1998) and capture these differences through a simple parameterization. Assume that aggregate final-good production (2.3) takes the form

$$Y_t = \theta K_t^{\alpha} X_t^{1-\alpha} \text{ with } 0 < \alpha < 1,$$

where $\theta \geq 1$ is a country-specific productivity parameter.

It is then not difficult to see that

$$\frac{d\underline{k}_{\theta c}}{d\theta} < 0$$

and for $k_c > \underline{k}_{\theta c}$

$$\frac{dk_{\theta}^{**}}{d\theta} > 0, \; \frac{dq_{\theta}^{**}}{d\theta} > 0.$$

Intuitively, raising $\theta$ works like an increase in $s$. In other words, countries with a good infrastructure are more likely to reach the innovation regime and converge to a steady state with faster productivity growth. Allowing for $\theta$ to also affect intermediate-good production would strengthen these effects.

## 5. Concluding Remarks

This paper presents Hicks' idea according to which a tendency to a more rapid increase of capital than labor provides a stimulus to labor-saving inventions. We show that capital accumulation along neoclassical lines is accompanied by a continuous rise in the relative price of labor which indeed serves as an incentive for profit-maximizing firms to engage in labor-saving technical change. This seems to confirm Hicks' assessment about the European growth performance of the last few centuries.

Our analysis develops an argument why capital accumulation should be regarded as an *essential* ingredient of the process of economic growth. First, a sufficient amount of capital is necessary to induce a switch into the innovation regime. Second, as factor prices are determined by the evolution of economic aggregates capital accumulation must be maintained in the innovation regime as it feeds back on the incentives to innovate. Clearly, the second argument supports the view expressed by Howitt and Aghion (1998) that at the level of economic aggregates capital accumulation and innovation are complementary factors in long-run growth. Yet, at the level of individual firms both sectors compete for foregone consumption as an input. Therefore, subsidizing capital accumulation may reduce long-run growth.

The present paper also contributes to the understanding of large observed international differences in per-capita output that persist or even grow over time. On the one hand, we find that only small differences in preference parameters like the propensity to save, demographical parameters like the population growth rate, technological parameters, or geographical parameters such as climate may postpone or prevent an economy from reaching the innovation regime. Similar effects may be attributed to inadequate policy measures such as a subsidy on capital. On the other hand, these parameters also account for differences in steady-state growth rates.

Finally, let us summarize the main differences between the neoclassical growth model with exogenous technical change and the innovation regime of our model. First, the switch into the innovation regime is brought about by an endogenous movement of factor prices. Second, the growth of labor productivity accelerates along the transition to the steady state. Third, with growth being endogenous, changes in the savings rate or the population growth rate do not only affect the level but also the long-run growth rate of per-capita output.

# 6. Appendix

## 6.1. Proof of Proposition 2

Condition (E5) requires zero-profits of all active intermediate-good firms, i.e.

$$p_t - \frac{w_t}{A_{t-1}(1+q_t)} - (1+r_t)\, i(q_t) = 0. \tag{6.1}$$

By (2.6) and (2.7) $r_t$ and $p_t$ depend on $k_t$. Then (6.1) determines $w_t$ as a function of $(A_{t-1}, q_t, k_t)$ :

$$w_t = A_{t-1}(1+q_t)\left(f - k_t f' - f' i(q_t)\right). \tag{6.2}$$

If not indicated otherwise the argument of $f$ is $k_t$.

By (E4) the choice of labor productivity must minimize unit costs. Hence, for any $t$, $q_t$ must satisfy (2.15). With (2.6), (2.7), and (6.2) this requires

$$\begin{gathered}\frac{f - k_t f'}{f'} \leq \left[i'(q_t)(1+q_t) + i(q_t)\right] \\ \text{with strict inequality } \textit{only} \text{ if } q_t = 0.\end{gathered} \tag{6.3}$$

Due to the concavity of $f$, the left-hand side of (6.3) is strictly increasing in $k_t$. Moreover, Inada conditions imply that $\lim_{k_t \to 0} (f - k_t f')/f' = 0$. From (2.11), the right-hand side of (6.3) is strictly increasing in $q_t$. Moreover, $\lim_{q_t \to 0} i'(1+q_t) + i = i'(0) > 0$. Hence, (6.3) defines a map $g : \Re_+ \to \Re_+$ that associates a unique value $q_t \geq 0$ with each $k_t \geq 0$. Equation (3.11) is sufficient to guarantee the existence of a critical level $k_c > 0$ defined by

$$\frac{f(k_c) - k_c f'(k_c)}{f'(k_c)} = i'(0) \tag{6.4}$$

so that $q_t^* = g(k_t)$ has the properties stated in the proposition.

Insertion of $q_t^* = g(k_t)$ for $q_t$ into (6.2) gives (3.13).

QED.

## 6.2. Proof of Proposition 3

The following two cases must be considered:

1. If intermediate-good firms expect some $k_t \leq k_c$ then from Proposition 2 none of them innovates in $t-1$ and from (3.10) and (3.12) $k_t$ is

$$k_t = \Psi(k_{t-1}) = sf(k_{t-1}). \tag{6.5}$$

The expected and the actual level of $k_t$ coincide if and only if

$$k_c \geq k_t = sf(k_{t-1}) \Leftrightarrow k_{t-1} \leq f^{-1}\left(\frac{k_c}{s}\right) = \underline{k}_c. \tag{6.6}$$

If instead $k_{t-1} > \underline{k}_c$ the economy cannot be in the Solow regime in $t$ because (6.5) yielded $k_t > k_c$ which is incompatible with initial expectations.

2. If intermediate-good firms expect some $k_t > k_c$ then from Proposition 2 all of them innovate. From (3.10) with (3.12) $\Psi$ is then implicitly defined by

$$k_t = \frac{s}{1+g(k_t)} f(k_{t-1}) - i(g(k_t)). \tag{6.7}$$

Perfect foresight requires

$$\Psi(k_{t-1}) > k_c. \tag{6.8}$$

From total differentiation of (6.7) we obtain

$$\frac{dk_t}{dk_{t-1}} = \frac{sf'}{g'(k_t + i) + (1+g)(1+i'g')} > 0, \tag{6.9}$$

where the argument of $g$ is $k_t$, the argument of $i$ is $g(k_t)$, and the argument of $f$ is $k_{t-1}$.

As

$$\frac{s}{1+g(k_c)} f(k_{t-1}) - i(g(k_c)) = sf(k_{t-1}),$$

the map $\Psi(k_{t-1})$ is continuous at $k_c$. Hence, with (6.9) it follows that (6.8) holds for $k_{t-1} > \underline{k}_c$.

QED.

## 6.3. Proof of Proposition 4

By Proposition 2 the economy starts in the stationary regime because $0 < k_1 < k_c$.

**Case 1:** $\underline{k}_c \geq k_c$

We have $\underline{k}_c > k_c > k_1 > 0$. By Proposition 3 the economy evolves for at least one period according to the difference equation $k_t = sf(k_{t-1})$. From $\underline{k}_c > k_c$ it follows that $\underline{k}_c > sf(\underline{k}_c) = k_c$. Moreover, $sf(k_{t-1})$ is continuous, strictly increasing and concave with $f(0) = 0$ and $f'(0) = \infty$. Therefore, it has two steady states on $[0, \underline{k}_c]$ which satisfy (3.20). The trivial steady state $k_1^* = 0$ is unstable whereas the second $k_2^* > 0$ is globally stable in the sense that for all $k_1 \in (0, k_c]$ and $k_1 \neq k^*$, $k_t \to k^*$ for $t \to \infty$. This follows from the fact that to the left (right) of $k^*$ we have $sf(k_{t-1}) > k_t$ $(sf(k_{t-1}) < k_t)$.

**Case 2:** $k_c > \underline{k}_c$

If $k_c > \underline{k}_c$ then $sf(\underline{k}_c) = k_c > \underline{k}_c$ and there is no steady state with $k^* > 0$ on $[0, \underline{k}_c]$. Hence, the economy reaches the innovation regime at some period $t' > 1$. The transition is described by

$$k_{t'} = \frac{s}{1+g(k_{t'})} f(k_{t-1}) - i(g(k_{t'})).$$

From period $t'+1$ onwards the economy remains in the innovation regime and converges for $t \to \infty$ towards the steady state that satisfies (3.21). The steady state is globally stable in the sense that for all $k_1 \in (0, k_c]$, $k_t \to k^{**}$ for $t \to \infty$. This follows from (6.9) and the fact that $0 < dk_t/dk_{t-1} < sf'(k_{t-1})$ for all $k_{t-1} > 0$.

QED.

## 6.4. Proof of Corollary 2

Total differentiation of (3.21) gives

$$\frac{dk^{**}}{ds} = \frac{f}{g'(k^{**} + i) + (1+g)(1+i'g') - sf'}, \tag{6.10}$$

where the argument of $f$ and $g$ is $k^{**}$ and the argument of $i$ is $g$. From the stability of the steady state and (6.9) it follows that

$$\left.\frac{dk_t(k_{t-1})}{dk_{t-1}}\right|_{k_{t-1}=k_t=k^{**}} = \frac{sf'}{g'(k^{**} + i) + (1+g)(1+i'g')} < 1$$

so that the denominator of (6.10) is positive. Hence, the first claim in (3.24) follows. With Corollary 1 we find the second claim:

$$\frac{dq^{**}}{ds} = \frac{dg\left(k^{**}\right)}{dk}\frac{dk^{**}}{ds} > 0.$$

QED.

## 6.5. Proof of Proposition 5

From the zero-profit condition of an intermediate-good firm, (4.1) and (2.7) we obtain the break-even wage as

$$w_t = A_{t-1}\left(1+q_t\right)\left(f - k_t f' - \left(f'\left(k_t\right)+\sigma\right) i\left(q_t\right)\right). \tag{6.11}$$

This leads to the analogue of (6.3) is:

$$\frac{f - k_t f'}{f' + \sigma} \leq \left[i'\left(q_t\right)\left(1+q_t\right) + i\left(q_t\right)\right]$$

$$\text{with strict inequality } \textit{only} \text{ if } q_t = 0 \tag{6.12}$$

where the argument of $f$ is $k_t$. (6.12) defines the map $g_\sigma$ which associates a unique $q_t \geq 0$ with each $(\sigma, k_t) \geq 0$. Condition (4.2) is sufficient to guarantee the existence of a critical level $k_{\sigma c} = k_c(\sigma) > 0$ defined by

$$\frac{f\left(k_{\sigma c}\right) - k_{\sigma c} f'\left(k_{\sigma c}\right)}{f'\left(k_{\sigma c}\right) + \sigma} = i'(0)$$

which is generalizes (6.4) for $\sigma > 0$. Upon total differentiation we obtain the first comparative-static result in (4.5). Similarly, total differentiation of (6.12) at $(\sigma, k_t)$ for $k_t > k_{\sigma c}$ gives

$$\frac{dg_\sigma}{d\sigma} = \frac{-\frac{f-k_t f'}{(f'+\sigma)^2}}{2i' + \left(1+g_\sigma\right) i''} < 0 \tag{6.13}$$

which shows the second comparative-static result in (4.5).

QED.

## 6.6. Proof of Proposition 6

The steady state in the innovation regime satisfies[15]

$$\left(1 + g\left(\sigma, k^{**}\right)\right)\left(k^{**} + i\left(g\left(\sigma, k^{**}\right)\right)\right) = sf\left(k^{**}\right) \tag{6.14}$$

which defines $k^{**}_\sigma = k^{**}(\sigma)$ with

$$\frac{dk^{**}_\sigma}{d\sigma} = -\frac{dg_\sigma}{d\sigma}\frac{i'\left(1+g_\sigma\right) + (k+i)}{\left(1+i'g'_\sigma\right)\left(1+g_\sigma\right) + g'_\sigma\left(k+i\right) - sf'} > 0 \tag{6.15}$$

at $(k^{**}, \sigma)$. Here, $g'_\sigma := dg_\sigma/dk_t$. From the stability of the steady state the denominator is positive. With (6.13) the sign in (4.6) follows.

Turning to the steady-state growth rate of labor productivity we note that

$$q^{**}_\sigma = g_\sigma\left(k^{**}_\sigma\right)$$

[15] Existence and uniqueness of a steady state in the innovation regime follow from the same arguments that are given in Proposition 4 for the case $\sigma = 0$.

so that at $(k_\sigma^{**}, \sigma)$

$$\frac{dq_\sigma^{**}}{d\sigma} = \frac{dg_\sigma}{d\sigma} + \frac{dg_\sigma}{dk_t}\frac{dk_\sigma^{**}}{d\sigma}.$$

One readily verifies with (6.13) and (6.15) that the right-hand side is smaller than zero iff

$$(1 + g(k_\sigma^{**}, \sigma)) > sf'(k_\sigma^{**}). \tag{6.16}$$

From (6.14) we know that at the steady state

$$(1 + g(k_\sigma^{**}, \sigma)) = \frac{sf(k_\sigma^{**})}{k_\sigma^{**} + i(g(k_\sigma^{**}, \sigma))}$$

so that (6.16) is satisfied at $(k_\sigma^{**}, \sigma)$ iff

$$\begin{aligned} \frac{sf}{k_\sigma^{**} + i(g)} &> sf' \\ \Leftrightarrow \frac{f - f'k_\sigma^{**}}{f'} &> i(g). \end{aligned}$$

Yet, from (6.12) at $(k_\sigma^{**}, \sigma)$ we have

$$\begin{aligned} \frac{f - f'k_\sigma^{**}}{f' + \sigma} &= i'(g)(1+g) + i(g) \\ \Leftrightarrow \frac{f - f'k_\sigma^{**}}{f'} &= \left(1 + \frac{\sigma}{f'}\right)\left[i'(g)(1+g) + i(g)\right] > i(g). \end{aligned}$$

QED.

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