

**Aging and Economic Growth:  
The Role of Factor Markets and of  
Fundamental Pension Reforms**

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# Aging and Economic Growth: The Role of Factor Markets and of Fundamental Pension Reforms\*

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**Preliminary. Comments Welcome**

## **Abstract**

This paper analyzes the implications of demographic change for economic growth in different countries. Quantitative projections are based on a multi-country overlapping generations model that is augmented with actual demographic data and projections for different OECD regions. According to the simulation results, per capita incomes decline substantially when aging processes peak in some regions and differences between regions are quite large. Additional capital formation and increases in labor supply resulting from a fundamental pension reform are found to mitigate but not to solve the problem.

*JEL classification:* E27; E62; F21; H55; J11

*Keywords:* OLG models; demographic change; growth; labor supply; capital markets; capital mobility; pension reform

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# 1 Introduction

Global aging will be a major determinant of long run economic development in industrial and developing countries. The extent of predicted demographic changes will be dramatic and will deeply affect future labor, capital and consumption goods markets. The expected strain on public budgets, especially on social security systems, has already received prominent attention both among researchers and policy makers, but demographic change also poses many other economic challenges that threaten productivity and growth if they remain unaddressed. The quantitative implications of demographic change for economic growth are the focus of this paper.

The probably most well-known strand of the academic literature focuses on the pressure induced by demographic change on social security systems and the welfare consequences of social security reforms that can be quantified with the use of large-scale OLG models (Samuelson 1958; Diamond 1965) of the Auerbach-Kotlikoff type (Auerbach, Kotlikoff, and Skinner 1983; Auerbach and Kotlikoff 1987), see, e.g., Kotlikoff, Smetters, and Walliser (1999), Storesletten (2000), Kotlikoff, Smetters, and Walliser (2001), Börsch-Supan, Ludwig, and Winter (2004), Fehr, Jokisch, and Kotlikoff (2004). A common finding of these studies is that there is no painful way out of the social security dilemma, but that welfare losses of currently old generations are small relative to the large gains for currently young and future generations.

This paper addresses the central question of how demographic change will affect growth rates of output and national incomes across countries.<sup>1</sup> Will per-capita incomes - expressed in terms of today's purchasing power - increase or decrease across countries and how large will be the effects? The contribution of the paper is thereby not the analysis of the question itself, but the careful distinction between the role played by various growth channels that are simultaneously at work: How much of the changes in growth rates can be attributed to the production factors of capital and labor? How does a fundamental pension reform - a shift from mainly PAYG financed pension systems, prevalent in most industrialized countries, to partial pre-funding - affect growth? What is the role of flexible labor markets in this context?

One way to think about these questions is to start from the fundamental components that determine a nation's output and income. Let domestic output  $Y$  (GDP) of a country with  $N$  inhabitants and  $W$  inhabitants in prime working age (the working age population) be given by  $Y = AF(l^W \cdot W, K)$ , where  $A$  is aggregate productivity,  $l^W$  is aggregate labor supply (employment) as a share of the working age population, and  $K$  is the capital stock. From a macroeconomic point of view, the main effect of aging is to reduce the size of the working age population,  $W$ , as a share of total population,  $N$ . In some countries, the working age population,  $W$ , will even decline in absolute size. Unless this is compensated by an increase in total factor productivity,  $A$ , and/or an increase in the capital stock,  $K$ , and/or an increase in employment rates,  $l^W$ , national output will decline, hence growth rates will decrease. Since  $W$  is changing quite differently across countries, the growth of

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<sup>1</sup>Schmidt (2004) analyzes similar questions focusing on the German economy.

output,  $Y$ , will reflect these differences.

Absolute size may not be the relevant measure since the total “pie”  $Y$  will have to be shared by fewer people if  $N$  declines as a consequence of demographic change. The relevant measure is therefore per capita output given by  $Y/N = AF(l \cdot W/N, K/N)$ . Furthermore, not all income needs to come from domestic production. In addition to wage and capital income from domestic production, equivalent to  $Y$ , incomes from abroad - either in the form of wage or in the form of capital income - need to be taken into account.

To quantify the differential effects of demographic change across world regions on economic growth in these regions, this paper develops a large-scale Auerbach-Kotlikoff type overlapping generations model that is based on the multi-country model version developed in Börsch-Supan et al. (2004) and in Ludwig (2005). Here, a four-countries / four-world-regions model is used consisting of (i) the United States, (ii) the region France-Germany-Italy as a region consisting of three core European economies that are enormously affected by the impact of demographic change and with pension systems in an ongoing reform process, (iii) Japan and (iv) all other OECD countries.

The key features of the simulation model and the formal calibration methodology are described in Section 2. The paper proceeds by presenting the main results in Section 3 as follows. First, estimation results on structural model parameters are presented and the back-fit of the model is discussed. Second, predicted growth rates of per capita output (and income) are shown for the four world regions if the current generosity of pension systems were maintained in the future. Per-capita output growth is further decomposed into its various components. Since the model is neo-classical in nature and since the multi-country framework abstracts from any technology spill-overs across countries, it is assumed that the growth rate of total productivity,  $A$ , is constant throughout. The analysis therefore focuses on the impact of demographic change on the growth rates of  $N$  and  $W$  as the exogenous driving forces and the endogenous reaction of the growth rates of capital and labor supply in the general equilibrium context. Third, the impact of fundamental pension reforms across countries - characterized by “frozen” contribution rates at year 2006 levels and corresponding decreases in (net) replacement rates - on economic growth and the associated welfare consequences are analyzed. Forth, and in order to shed some light on the role of flexible labor markets, the labor supply channel is discussed. These analytical steps provide some answers to the questions posed above. Section 4 summarizes the key findings of the paper and concludes the analysis.

## 2 The Overlapping Generations Model

The OLG model used to quantify the effects of demographic change on economic growth is a variant of the model used in Börsch-Supan, Ludwig, and Winter (2004) and in Ludwig (2005). It is a multi-country extension of the standard OLG model by Auerbach and Kotlikoff (1987) which is augmented with realistic demographic data across these countries. The model has three building blocks: a demographic projection, a stylized pension system, and a macroeconomic overlapping generations model to calculate the general equilibrium

of internationally linked economies. The following sections contain a detailed description of all these elements. Readers familiar with large-scale multi-country OLG models may skip the following Subsections 2.1 through 2.3. Subsection 2.4 describes how structural model parameters are determined and is less standard.

## 2.1 The Demographic Model

Detailed demographic projections form the background of the analysis. Demography is taken as exogenous and represents the main driving force of the simulation model.<sup>2</sup> In each country  $i$ , the size of population of age  $j$  in period  $t$ ,  $N_{t,j,i}$ , is given recursively by

$$N_{t,j,i} = \begin{cases} \sum_{j=15}^{50} f_{t-1,j,i} N_{t-1,j,i} & \text{for } j = 0 \\ N_{t-1,j-1,i} (s_{t-1,j-1,i} + m_{t-1,j-1,i}) & \text{for } j > 0, \end{cases}$$

where  $s_{t,j,i}$  denotes the age-specific conditional survival rate,  $m_{t,j,i}$  the net migration ratio, and  $f_{t,j,i}$  the age-specific fertility rate.

Individuals in the model economies enter economic life at the age of 20 which is denoted by  $a = 1$ . The maximum age as implied by the demographic projections is 104 years. Accordingly the maximum economic age, denoted by  $Z$ , is 85. To simplify calculations of the economic model, it is assumed that all migration takes place at the initial age of 20. This simplifying assumption allows to treat all “newborns” - immigrants and natives - in the economic model alike, see below.<sup>3</sup>

## 2.2 The Pension Model

Each region  $i$  is assumed to have a two-tier pension system. The first tier represents a conventional public pay-as-you-go (PAYG) system characterized by a country-specific contribution and replacement rate. More precisely, for each region  $i$ , the exogenous policy variable is the time-specific gross replacement rate,  $\gamma_{t,i}$ , defined as the ratio of average gross pension to average gross wage income at time  $t$ . The budget of the PAYG pension system is balanced at any time  $t$  and determines the contribution rate,  $\tau_{t,i}$ , by

$$\tau_{t,i} \sum_{a=1}^Z w_{t,a,i}^g l_{s_{t,a,i}} N_{t,a,i} = \sum_{a=1}^Z p_{t,a,i} (1 - l_{s_{t,a,i}}) N_{t,a,i}, \quad (1)$$

where pension benefits  $p_{t,a,i}$  of a household of age  $a$  in time period  $t$  are calculated by

$$p_{t,a,i} = \gamma_{t,i} \lambda_{t,a,i} w_{t,a,i}^g$$

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<sup>2</sup>Assuming exogenous demographic processes is of course a simplifying assumption since, in the long run, neither fertility nor mortality and of course not migration is exogenous to economic growth.

<sup>3</sup>Both groups, newborns and immigrants, enter the economic model with zero assets. Furthermore, there are no skill differences between the two groups as analyzed by, e.g., Razin and Sadka (1999) and Storesletten (2000).

On the revenue side,  $w_{t,a,i}^g$  denotes age-specific gross wages. Net wages are given by  $w_{t,a,i}^n = w_{t,a,i}^g(1 - 0.5\tau_{t,i})$  under the assumption that half of contributions are paid by the employee and the other half by the employer. This latter half will be taken into account when firms maximize profits.  $l_{t,a,i}^d$  denotes age specific labor supply shares resulting from optimal household decisions. The use of index  $d$  will be explained below.

On the benefit side of the budget equation, pensions are defined by the general replacement rate and by a “point system” that credits  $\lambda_{t,a,i}$  times the gross wage earned at age  $a$ . This is an approximation to the actual computation of pension benefits. Benefits are not taxed and interactions with other social protection systems are ignored. It is further assumed that all persons in each region participate in the same pension system.

## 2.3 The Macroeconomic Model

The two core elements of the macroeconomic general equilibrium model are the production and the household sector. They are presented separately here, although they are linked through several channels, in particular through the household’s labor supply and savings decisions. The production sector in each country consists of a representative firm that uses a Cobb-Douglas production function given by

$$Y_{t,i} = F(\Omega_i, K_{t,i}, L_{t,i}) = \Omega_i K_{t,i}^{\alpha_i} L_{t,i}^{1-\alpha_i}, \quad (2)$$

where  $K_{t,i}$  denotes the capital stock and  $L_{t,i}$  aggregate labor input of country  $i$  at time  $t$ .<sup>4</sup> Labor supply is measured in efficiency units and  $\alpha_i$  denotes the capital share.

Production efficiency of a household of age  $a$  at time  $t$  in country  $i$  has a factorial structure with three elements, relating to age, time and country. On the micro level, where households are distinguished by their age, labor productivity changes over the life-cycle according to age-specific productivity parameters  $\epsilon_a$ . Hence, the age-specific gross wage is  $w_{t,a,i}^g \epsilon_a$  and the aggregate labor supply is  $L_{t,i} = \sum_{a=1}^Z \epsilon_a l_{t,a,i} N_{t,a,i}$ , where  $l_{t,a,i}$  denotes a single household’s labor supply. Second, aggregate and individual labor supply ( $L_{t,i}$  and  $l_{t,a,i}$ ) are measured in efficiency units relative to a time endowment  $E_{t,i}$ . Age specific labor supply which corresponds to what is observed in the data is therefore given by  $L_{t,a,i}^d = l_{t,a,i} N_{t,a,i} / E_{t,i}$ . The index  $d$  is henceforth used to denote “de-trended” effective labor supply. The time endowment increases over time according to  $E_{t,i} = E_{0,i} \exp(g_{t,i}t)$ . This “growth in time endowment” specification is equivalent to the standard labor augmenting technological change specification for the production sector and has useful properties for the specification of the household sector, see below. Note that the growth rate is also indexed by time. This allows to capture decreases in the trend growth rate of technological progress over the observation period due to a (potential) productivity slowdown, also see below. Third,  $\Omega_i$  is the technology level of country  $i$  which is held constant over time.

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<sup>4</sup>The literature often examines the more general case of a CES production function (Altig et al. 2001; Börsch-Supan et al. 2004). Estimation of the elasticity of substitution between capital and labor however results in a coefficient close to one and therefore the simpler case of a Cobb-Douglas production function is used here.

In this version of the model, adjustment costs in the firm sector are not considered. Hence, profit maximization is static and the only constraint to firm maximization is given by the capital accumulation condition

$$K_{t+1,i} = K_{t,i} + I_{t,i} - D_{t,i} = (1 - \delta_i)K_{t,i} + I_{t,i}, \quad (3)$$

where  $I_{t,i}$  is gross investment,  $D_{t,i}$  is depreciation and  $\delta_i$  is the country-specific depreciation rate. The first-order conditions resulting from profit maximization give standard expressions for equilibrium wages

$$w_{t,i}^g(1 + 0.5\tau_{t,i}) = (1 - \alpha_i) \frac{Y_{t,i}}{L_{t,i}} \quad (4)$$

and interest rates

$$r_{t,i} = \alpha_i \frac{Y_{t,i}}{K_{t,i}} - \delta_i, \quad (5)$$

Equation 5 is the familiar arbitrage condition for the rate of return on financial and physical investment: The return on financial investment,  $r_{t,i}$ , must be equal to the return on one unit of physical investment in each country. The latter equals the marginal product of capital minus depreciation.

In order to determine aggregate consumption, savings and wealth, optimal household behavior derived from inter-temporal utility maximization is considered next. By choosing an optimal consumption path, each cohort born in time period  $t$  maximizes at any point in time  $t + a$  and age  $a$  the sum of discounted future utility. The within-period utility function exhibits constant relative risk aversion, and preferences are additive and separable over time. Cohort  $t$ 's maximization problem at  $a = 1$  is given by

$$\max_{\{C_{t,a,i}, l_{t,a,i}\}_{a=1}^Z} \left( \frac{1}{1 + \rho_i} \right)^{a-1} \pi_{t,a,i} U(C_{t,a,i}, l_{t,a,i}), \quad (6)$$

where  $\rho_i$  is the pure time discount rate. In addition to pure discounting, households discount future utility with their unconditional survival probability,  $\pi_{t,a,i} = \prod_{j=1}^a s_{t,j-1,i}$ .  $C_{t,a,i}$  denotes consumption. Remember that a single household's labor supply  $l_{t,a,i}$  is measured in efficiency units relative to time endowment,  $E_{t,i}$ .

It is assumed that the period specific utility function is of the standard CES form given by

$$U(C_{t,a,i}, L_{t,a,i}) = \frac{1}{1 - \sigma_i} \left( \left( [\phi_{a,i} C_{t,a,i}^{-\gamma_i} + (1 - \phi_{a,i})(E_{t+a} - l_{t,a,i})^{-\gamma_i}]^{-\frac{1}{\gamma_i}} \right)^{1 - \sigma_i} - 1 \right).$$

$\sigma_i$  is the coefficient of relative risk aversion,  $\phi_{a,i}$  the consumption share parameter, i.e. the weight of consumption relative to leisure in household's utility and  $\xi_i = 1/(1 + \gamma_i)$  the intra-temporal substitution elasticity between consumption and leisure.<sup>5</sup> Note that the

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<sup>5</sup>In contrast to the use of Cobb-Douglas utility, the more general CES utility function is used here. While the elasticity of substitution between capital and labor is close to one, which justifies the use of a Cobb-Douglas production technology in equation 2, compare footnote 4, there is a large agreement in the literature that the elasticity of substitution between consumption and leisure is significantly below one (Altig et al. 2001).

consumption share parameter is also indexed by age  $a$ . This it to account for the observed strong decreases in age-specific labor supply shares after the age of 55, see below.

A complication arises because households face the risk of prematurely dying with positive wealth. For simplification, the assumption of perfect annuity markets is made which implies that accidental bequests are distributed implicitly, as in the life-insurance framework by Yaari (1965). As shown in Börsch-Supan et al. (2004), this assumption does not affect simulation results on the aggregate level much.

Denoting total wealth by  $A_{t,a,i}$ , maximization of the household's intertemporal utility is subject to a dynamic budget constraint given by

$$A_{t,a+1,i} = \frac{1}{s_{t,a,i}} \left( A_{t,a,i}(1 + r_{t+a+1,i}) + l_{t,a,i}w_{t,a,i}^n + (E_{t+a} - l_{t,a,i})p_{t,a,i} - C_{t,a,i} \right) \quad (7)$$

The term  $1/s_{t,a,i}$  reflects how accidental bequests are dissipated through the annuity market. Income consists of asset income, net wages, and pensions. The corresponding present value budget constraint is given by

$$\sum_{a=1}^Z \pi_{t,a,i} y_{t,a,i}^n \prod_{j=1}^a (1 + r_{t+j-1,i})^{-1} - \sum_{a=1}^Z \pi_{t,a,i} C_{t,a,i} \prod_{j=1}^a (1 + r_{t+j-1,i})^{-1} \geq 0, \quad (8)$$

where the short hand notation  $y_{t,a,i}^n = l_{t,a,i}w_{t,a,i}^n + (E_{t+a} - l_{t,a,i})p_{t,a,i}$  is adopted to denote after-contributions non-asset income.

Maximization is also subject to the constraint that leisure may not exceed time endowment (and may not be negative)

$$0 \leq l_{t,a,i} \leq E_{t+a}. \quad (9)$$

The solution to the intertemporal optimization problem is characterized by two first-order conditions. First, the inter-temporal Euler equation describes the consumption growth rate of each household given by

$$C_{t,a+1,i} = C_{t,a,i} \left( \frac{1 + r_{t+a+1,i}}{1 + \rho} \frac{v_{t,a+1,i}}{v_{t,a,i}} \right)^{1/\sigma_i}, \quad (10)$$

where  $v_{t,a,i} = (\phi_i + (1 - \phi_i)clr_{t,a,i}^{-\gamma_i})^{-(1+\gamma-\sigma)/\gamma}$ .  $clr_{t,a,i}$  is the consumption-leisure ratio defined below.

Second, the intra-temporal Euler equation relates current period consumption to current period leisure choice by

$$E_{t+a} - l_{t,a,i} = \left( \frac{1 - \phi_i}{\phi_i} \frac{1}{w_{t,a,i}^n + \mu_{t,a,i} - p_{t,a,i}} \right)^{1/(1+\gamma_i)} C_{t,a,i} = clr_{t,a,i} C_{t,a,i}, \quad (11)$$

where  $\mu_{t,a,i} \geq 0$  is the shadow value of leisure. The ‘‘growth in time endowment’’ specification of the production function insures that steady state labor force participation is



constant even if the elasticity of substitution between consumption and leisure is not equal one, i.e. if  $\gamma_i \neq 0$  (Auerbach and Kotlikoff 1987; Altig et al. 2001; Börsch-Supan et al. 2004).

For given factor prices (i.e., wages and interest rates), shadow wage rates and the parameters of the public pension system (i.e., contribution and replacement rates), the life-time consumption paths of all generations can be computed using the Euler equations 10 and 11 as well as the budget constraints.

The dynamic general equilibrium of the model economy is defined sequentially.<sup>6</sup>

**Definition 1:** A competitive equilibrium of the economy is defined as a sequence of disaggregate variables,  $\{C_{t,a,i}, l_{t,a,i}, A_{t,a,i}\}$ , aggregate variables,  $\{C_{t,i}, L_{t,i}, K_{t,i}\}$ , wage rates,  $\{w_{t,i}\}$  in each country  $i$  and a common world interest rate,  $\{r_t\}$  such that

- The allocations are feasible, i.e.

$$\begin{aligned} Y_{t,i} + r_t F_{t,i} &= S_{t,i}^n + C_{t,i} + D_{t,i} = S_{t,i}^g + C_{t,i} \\ &= \sum_{a=1}^Z (s_{t-a,a,i} A_{t+1-a,a+1,i} - A_{t-a,a,i}) N_{t,a,i} + \sum_{a=1}^Z C_{t-a,a,i} N_{t,a,i} + \delta_i K_{t,i}, \end{aligned}$$

where  $F_{t,i}$  is the amount of foreign assets and  $D_{t,i}$  is depreciation of capital and  $S_{t,i}^n (S_{t,i}^g)$  is net (gross) savings.

- Factor prices equal their marginal productivities as given in equations 4 and 5.
- Firms and households behave optimally, i.e., firms maximize profits subject to the capital accumulation constraint given in equation 3 and households maximize life-time utility given in equation 6 subject to the constraints in equations 7 through 9.
- All markets clear. Market clearing on national markets requires that

$$S_{t,i}^n = \sum_{a=1}^Z S_{t-a,a,i}^n \quad C_{t,i} = \sum_{a=1}^Z C_{t-a,a,i} \quad A_{t,i} = \sum_{a=1}^Z A_{t-a,a,i} \quad L_{t,i} = \sum_{a=1}^Z L_{t-a,a,i}.$$

Market clearing on the international capital market and the assumption of perfect capital mobility across regions requires that the rate of return on financial investment is equalized across all countries,

$$r_{t,i} = r_t, \tag{12}$$

and that the sum of all foreign assets, defined as the difference between home assets and the home capital stock,  $F_{t,i} = A_{t,i} - K_{t,i}$ , across all world regions

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<sup>6</sup>The definition of equilibrium as sequential coincides with the computational solution method (Ludwig 2004). It can be numerically computed since the model economy converges to a steady state and becomes a well-behaved system with a small number of equations.

equals zero, i.e.

$$\sum_{i=1}^R F_{t,i} = 0,$$

where  $R$  is the total number of regions. International capital flows are defined by the difference between foreign assets in two successive periods,

$$CA_{t,i} = F_{t+1,i} - F_{t,i} = S_{t,i}^g - I_{t,i}.^7$$

The time line of the model has four periods: a phase-in period,  $t = -T^S, \dots, 0$ , a calibration period (1960-2003),  $t = 1, \dots, T$ , a projection period (2004-2100),  $t = T + 1, \dots, T^P$  and a phase-out period,  $t = T^P + 1, \dots, T^E$ , that lasts until 2300 when the model reaches a final steady state.

## 2.4 Calibration

Calibration of the model requires (i) determination of values for several structural model parameters and (ii) data for the exogenous demographic processes and pension system replacement and/or contribution rates. The determination of structural model parameters follows the procedure recently developed by Ludwig (2005) who suggests to estimate *free* model parameters by a formal method of moments methodology. Before proceeding it is useful to describe the functional form assumptions on the time dependence of the trend growth rate of technological progress,  $g_{t,i}$ , and on the age-dependence of the consumption share parameter,  $\phi_{a,i}$ .

The process of efficiency units,  $E_{t,i}$  is specified as

$$E_{t,i} = \begin{cases} E_{0,i} \exp(g_i^1 t) & \text{for } t = -T^S, \dots, 0 \\ E_{0,i} \exp((g_i^1 + g_i^2 t)t) & \text{for } t = 1, \dots, T \\ E_{T,i} \exp((g_i^1 + g_i^2 T)(t - T)) = E_{T,i} \exp(g_i^S (t - T)) & \text{for } t = T, \dots, T^E. \end{cases}$$

In other words, the growth rate of efficiency units,  $g_{t,i} = \ln E_{t,i} - \ln E_{t-1,i}$  equals  $g_i^1$  during the “face in” period and is equal to  $g_i^1 + g_i^2(2t - 1)$  during the calibration period lasting from  $t = 1$  to  $t = T$ . As shown below,  $g_2 < 0$  and thereby captures decreases in the trend growth rate of technological progresses, the slowdown of productivity growth. During the projection and phase-out periods,  $t = T + 1, \dots, T^E$ , the growth rate is assumed to remain constant at the level reached in period  $T$ ,  $g_i^S = g_i^1 + g_i^2 T$ .

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<sup>7</sup>Throughout, implicit use of the simplifying assumption that all migration is concentrated at age  $a=1$  was made. Since initial wealth at  $a = 1$  is assumed zero, transfers of assets due to migration does not need to be taken into account.

The functional form of the consumption share parameters,  $\phi_{a,i}$ , is given by

$$\phi_{a,i} = \begin{cases} \bar{\phi}_i & \text{for } a \leq \underline{A} \\ \phi_{a,i} = \bar{\phi}_i - \Delta\phi_i(a - \underline{A}) & \text{for } \underline{A} < a < \bar{A} \\ \phi_{a,i} = \underline{\phi}_i = \bar{\phi}_i - \Delta\phi_i(\bar{A} - \underline{A}) & \text{for } a \geq \bar{A}, \end{cases}$$

in order to account for the fact that labor supply shares decrease sharply between the ages  $\bar{A} = 35$ , corresponding to the actual age of 54, and  $\underline{A} = 61$ , corresponding to the actual age of 80.

The total set of structural model parameters can now be collected in the following vectors

$$\begin{aligned} \text{Production Sector: } \Psi^{PS} &= (\{\delta_i\}_{i=1}^R, \{\alpha_i\}_{i=1}^R, \{g_i^1\}_{i=1}^R, \{g_i^2\}_{i=1}^R, \{\Omega_i\}_{i=1}^R)' \\ \text{Household Sector: } \Psi^{HS} &= (\{\rho_i\}_{i=1}^R, \{\sigma_i\}_{i=1}^R, \{\xi_i\}_{i=1}^R, \{\bar{\phi}_i\}_{i=1}^R, \{\Delta\phi_i\}_{i=1}^R)'. \end{aligned}$$

For reasons of data constraints and inconsistencies between the theoretical model and capital stock data (if they were available) discussed in Ludwig (2005), not all parameters are assumed to vary across countries. The following restrictions are imposed:

$$\begin{aligned} \delta_i &= \delta_1; \alpha_i = \alpha_1; g_{1,i} = g_{1,1}; g_{2,i} = g_{2,1} \quad \forall i \text{ and} \\ \rho_i &= \rho_1; \sigma_i = \sigma_1; \xi_i = \xi_1; \Delta\phi_i = \Delta\phi_1 \quad \forall i. \end{aligned}$$

In other words, most of the parameters are estimated only for country  $i = 1$ , which is the United States.

In addition, a subset,  $\Psi^p$ , of the remaining calibration parameters are regarded as predetermined (i.e. as fixed by reference to other studies). Specifically, the elasticity parameters  $1/\sigma_1$  and  $\xi_1$  are treated as predetermined since estimated values of these parameters would be outside ranges regarded as reasonable in the literature (Ludwig 2005). Furthermore, due to its minor role and in order to simplify computations,  $\Delta\phi_1$  is fixed (predetermined) here.

To summarize, predetermined parameters,  $\Psi^p$ , and estimated (free) parameters,  $\Psi^e$ , are given as follows:

$$\begin{aligned} \Psi^p &= (\sigma_1, \xi_1, \Delta\phi_1)' \\ \Psi^e &= (\delta_1, \alpha_1, g_{1,1}, g_{2,1}, \{\Omega_i\}_{i=1}^R, \rho_1, \{\bar{\phi}_i\}_{i=1}^R)' \end{aligned}$$

According to these assumptions, only the structural model parameters  $\Omega_i$  and  $\bar{\phi}_i$  vary across countries. These parameters determine the effective “size” of each country in terms of technology levels (aggregate output, GDP) and in terms of the size of the aggregate labor force. All calibration parameters are collected as  $\Psi = ((\Psi^p)', (\Psi^e)')$ .

Estimation of the free parameters,  $\Psi^e$ , is by matching simulated moments to actual moments of the data. Let

$$u_t(\Psi^e) = y_t^a - y_t^s \quad \forall t = 1, \dots, T$$

be an  $e \times 1$  vector. The method of moments (MM) error,  $u_t$ , measures the discrepancy between actual ( $y^a$ ) and predicted (simulated,  $y^s$ ) values. The dimensions of the MM error,  $u_t$ , and the parameter vector,  $\Psi^e$ , coincide by definition of (exactly) matching moments.

The MM error,  $u_t$ , has the property that

$$\mathbb{E}[u_t(\Psi^{e,0})] = 0,$$

where  $\Psi^{e,0}$  denotes the true values of the parameters of the underlying data generating process. The method of moments estimator, denoted by  $\hat{\Psi}_T^e$ , is given by

$$h_T[\hat{\Psi}_T^e] = 0, \quad h_T[\Psi_T^e] \equiv \frac{1}{T} \sum_{t=1}^T u_t(\Psi^e) = 0.$$

As Hansen (1982) shows,  $\hat{\Psi}_T^e$  is asymptotically distributed as normal if it satisfies certain regularity conditions. Most importantly,  $u_t(\Psi^e)$  must be a strictly stationary stochastic process not only for  $\Psi^{e,0}$  but for all possible  $\Psi^e$ . This implies that all regressors need to be strictly stationary stochastic processes as well. For the case of deterministic trends, Andrews and McDermott (1995) establish that consistent estimation may still be possible if the deterministic trend is of a particular structure relative to the econometric model. Model parameters and the asymptotic variance-covariance matrix can then be estimated with the same procedures as in the case of strictly stationary regressors. This framework is adopted here.

Since the model is deterministic, Ludwig (2005) further suggests to HP-filter (Hodrick and Prescott 1997) the data and to compare the model simulated trends with the trend components of the data.<sup>8</sup> The MM error then reflects measurement and specification error. Intrinsic stochastic components, e.g., due to productivity shocks as studied in the RBC literature, are absent from the model and are - by the application of the filter - assumed to be filtered out from the data.

With these assumptions, moment conditions for identification of the structural model parameters  $\Psi^e$  follow directly from the above relationships of the theoretical model. Let lower case letters denote the ln of the HP-filtered and possibly trending data. From equation 3 identification of  $\delta_1$  is achieved by

$$\mathbb{E} [d_{t,1}^a - k_{t,1}^a - \ln \delta_1] = 0,$$

and of  $\alpha_1$  by transforming equation 4 as

$$\mathbb{E} [w_{t,1}^a + y_{t,1}^a - l_{t,1}^a - \ln(1 - \alpha_1)] = 0.$$

The growth rate of output is given by

$$\gamma_{t,i}^Y - \alpha_i \gamma_{t,i}^K - (1 - \alpha_1)(\gamma_{t,i}^L + g_1 + g_2(2t - 1)) = 0.$$

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<sup>8</sup>Note that this procedure is just opposite to the standard approach used in the RBC literature.

Identification of the parameters  $g_1$  and  $g_2$  is by the moment conditions (OLS normal equations)

$$\begin{aligned}\mathbb{E} \left[ \left( \gamma_{t,i}^{a,Y} - \alpha_i \gamma_{t,i}^{a,K} - (1 - \alpha_1)(\gamma_{t,i}^{a,L} + g_1 + g_2(2t - 1)) \right) \right] &= 0 \\ \mathbb{E} \left[ \left( \gamma_{t,i}^{a,Y} - \alpha_i \gamma_{t,i}^{a,K} - (1 - \alpha_1)(\gamma_{t,i}^{a,L} + g_1 + g_2(2t - 1)) \right) (2t - 1) \right] &= 0.\end{aligned}$$

Identification of the  $\Omega_i$ 's is achieved by

$$\mathbb{E} \left[ y_{t,i}^a - \ln \Omega_i - \alpha_1 k_{t,i}^a - (1 - \alpha_1)(l_{t,i}^a + g_{t,1}t) \right] = 0 \quad \forall i = 1, \dots, R.$$

Identification of structural model parameters of the household sector closes the model but requires simulation since no closed form solution exists. While the above moment conditions for the production sector imply stationarity of the MM error  $u_t$  at  $\Psi^{e,0}$ , this may not be the case for the household sector if only one RHS variable is used. For instance, the endogenous labor supply model may fail - and also does in some cases - to replicate the growth rate of actual labor supply. Matching simulated to actual labor supply on average would then result in a non-stationary MM error even at the true value  $\Psi^{e,0}$  and the estimated variance-covariance matrix would be inconsistent. For these reasons, the (log) capital output ratio (and not the (log) capital stock itself) is matched to the data and labor supply shares are de-trended.

Identification of  $\rho_1$  is achieved by requiring the model to match the average capital output ratio

$$\mathbb{E} \left[ k_{t,1}^a - y_{t,1}^a - \left\{ \ln \left( \sum_{t,1}^Z H_{t-a,a,1}^s(\Psi, X) N_{t-a,a,1} \right) - y_{t,1}^s \right\} \right] = 0,$$

where  $H_{t,a,1}^s$  are simulated age-specific holdings of home assets by households of age  $a$  living in country 1.

Identification of the  $\phi_i$ 's is by a similar condition on labor supply given by

$$\mathbb{E} \left[ l_{t,i}^a - \gamma_i^L t - \left\{ \ln \left( \sum_{a=1}^Z \frac{l s_{t-a,a,i}^s(\Psi, X) N_{t-a,a,i}}{E_{t,i}} \right) - \gamma_i^{L,s} t \right\} \right] = 0 \quad \forall i = 1, \dots, R.$$

Division by  $E_{t,i}$  is necessary since individual simulated labor supply is measured in efficiency units, see Section 2. Average growth rates of labor supply,  $\gamma_i^{L^d}$  are jointly estimated with the other parameters,  $\Psi^e$ , by the moment conditions

$$\mathbb{E} \left[ \gamma_i^L - \gamma_{i,t}^{L,a} \right] = 0 \quad \forall i = 1, \dots, R.$$

Data to estimate all these parameters is constructed using national accounts data for the United States provided by the Bureau of Economic Analysis (BEA), United Nations (UN) population data and labor supply data provided by the OECD and the World Bank for all regions. A description of the data set and of the construction of the remaining input

data - demographic projections and the pension system's replacement and contribution rates - is contained in Ludwig (2005).

Solution of the simulation model is as follows. For a given set of parameters, the general equilibrium of the model is calculated iteratively by using the Gauss-Seidel-Quasi-Newton procedure developed in Ludwig (2004). In addition, outer loop iterations solve the above non-linear system of moment conditions using Broyden's method.

### 3 Results

Estimation results for structural model parameters are presented in Subsection 3.1. Subsections 3.2 and 3.3 contain the main simulation results. Growth projections are calculated for two pension policy scenarios, (i) the "old system Pure PAYG" which maintains countries' current generous public pension systems (Subsection 3.2), and (ii) the "Freezing" reform scenario which introduces a transition to a funded pension system by simultaneously freezing contribution rates at year 2006 levels in three countries: in the United States, in the France-Germany-Italy region and in Japan. The other OECD countries' pension systems remain unchanged (Subsection 3.3).

In order to solve the pension system equation 1 for each country in the "Pure PAYG" pension system scenario, net replacement rates - given by  $\gamma_{t,i}^n = \gamma_{t,i}/(1 - 0.5\tau_{t,i})$  - are assumed constant over time at current levels. The associated paths of contribution rates are endogenously calculated taking data on the pension system's replacement rates as given. The second tiers of the stylized pension systems represent pre-funded components of private pensions. These funded components are not modelled explicitly. Rather, they consist of voluntary private savings resulting from households' optimal life-cycle decisions.

The pension reform of the "Freezing" type implies that replacement rates are endogenously determined. They will decrease due to the projected increases in economic dependency ratios - the ratios of pensioners to workers - implied by the dynamics of demographic change. The additional funded components that arise from this reform - households are projected to actually save more and not just to substitute old savings for new ones - are again modelled implicitly.

Both pension system scenarios are counterfactual. First, actual pension reforms passed in some of the countries (e.g. in Germany) already depart from the assumed status quo of the model, the "Pure PAYG" pension system. Second, these reforms / reform proposals are not as "deep" as the model's "Freezing" reform implies. But focusing on these two polar and counterfactual scenarios allows to separate the *direct* effects of demographic change on growth - in the "Pure PAYG" scenario as a benchmark for the remainder of the analysis - from the *additional* feedback effects of fundamental pension reforms. The analysis of simulation results concludes with a discussion on the role of flexible labor markets in the context of demographic change (Subsection 3.4).

### 3.1 Estimation Results

Table 1 shows the calibration parameters. Values of predetermined elasticity parameters ( $\sigma_1 = 2$ ,  $\xi_1 = 0.8$ ) are considered as reasonable in the literature. As discussed above, the predetermined parameter  $\Delta\Phi_1 = 0.01$  is of minor importance and set such as to account for decreases in age-specific labor supply rates at higher ages. The sensitivity of simulation results with regard to the values of predetermined parameters is low (results not shown).

Parameters  $\Psi^e$  of the endogenous labor supply model are estimated with the procedure described in Section 2.4. Estimated parameter values are also within ranges considered as reasonable in the literature. The estimates on the two growth factors  $g^1$  and  $g^2$  imply that the exogenous growth rate of technology levels decreases from about 2 percentage points in 1960 to about 1.7 percentage points in 2003. As described above, the growth rate is assumed constant at the level of 1.7 percentage points for the remaining projection period.

The estimate of the discount rate  $\rho = 0.011$  is identical to the point estimate of Hurd (1989). Its exact value however strongly depends on the choice of  $\sigma$ . E.g., increasing the coefficient of relative risk aversion,  $\sigma$ , to 3 would result in an estimate for  $\rho$  below 0.005, the value used by Altig et al. (2001).

Parameter estimates for  $\Omega_i$  and  $\bar{\phi}_i$ ,  $i = 1, \dots, R$  vary quite considerably across countries and capture the different sizes of GDP levels and aggregate labor forces, respectively. The estimates for  $\bar{\phi}_i$ ,  $i = 1, \dots, R$  imply that households spend roughly 60 to 70 percent of their time on leisure related activities. Their exact magnitude however depends on the value of the predetermined elasticity parameter  $\xi$ . Decreasing  $\xi$  to 0.6 results in estimates of  $\bar{\phi}$  in the order of magnitude of 0.4 to 0.5 and increasing it to 1 (Cobb-Douglas utility) in values of 0.7 to 0.8 (results not shown).

Results on the backward fit of the model's predicted growth rates for GDP levels, the aggregate capital stock and labor supply in the United States are shown in Figure 1. Given the small number of parameters and the simple structure of the economic model, these back fitting results are quite encouraging. From period 1970 on, predicted and actual GDP growth rates roughly match the data (first panel of the figure). Actual and predicted capital stock growth rates are of the same order of magnitude as found in the data (second panel of the figure). The model however fails to capture a large proportion of the upward and downward swings of the capital stock growth rate observed in the data. This is due to the fact that the open economy, endogenous labor supply model washes out a large part of actual fluctuations of aggregate labor supply and of the aggregate capital stock. As the third panel of the figure shows, the model also captures a good proportion of the observed downward trends of aggregate employment growth rates (again from period 1970 on).

Figure 2 depicts the corresponding levels of GDP, the capital stock and of the aggregate employment. These findings again reveal that the model underestimates the level of the capital stock between 1975 and 1990 and overestimates the level since 1990. Predicted labor supply is lower than the actual data after 1990. A more extensive treatment on how well Auerbach-Kotlikoff type OLG models match the data is given in Ludwig (2005).

### 3.2 The “Pure PAYG” Pension System

This section focuses on growth and its components under the assumption that countries’ currently generous pension systems are maintained in the future. Recall that output (GDP) in country  $i$  is given by the relationship

$$Y_{t,i} = \Omega_i K_{t,i}^{\alpha_i} (E_{t,i} L_{t,i}^d)^{1-\alpha_i},$$

where  $L_{t,i}^d$  denotes de-trended labor supply that corresponds to the respective actual labor supply data  $L_{t,i}^a$ , see above. It follows that the growth rate of output is given by

$$\gamma_{t,i}^Y = \alpha_i \gamma_{t,i}^K + (1 - \alpha_i) g^S + (1 - \alpha_i) \gamma_{t,i}^{L^d} \quad \forall t \geq T,$$

where  $g^S = g^1 + g^2 T$  is the constant growth rate of technological progress during the projection period, see above. The per capita growth rate is accordingly given by

$$\gamma_{t,i}^{Y/N} = \alpha_i \gamma_{t,i}^K + (1 - \alpha_i) g^S + (1 - \alpha_i) \gamma_{t,i}^{L^d} - \gamma_{t,i}^N \quad \forall t \geq T.$$

The steady state growth rates are hence given by  $\gamma^{Y,*} = \gamma^{K,*} = n + g^S$  and  $\gamma^{Y/N,*} = \gamma^{K/*} = g^S$ .

Figure 3 shows predicted per capita GDP (panel a) and GNP (panel b) growth rates for the four world regions. The estimated exogenous steady state growth rate of per capita output,  $g^S$ , is shown as a flat line. The United States, the France-Germany-Italy region and all other OECD countries experience above steady state growth rates at the beginning of the simulation period. Over time, growth rates in these regions decrease and fall below the trend growth rate until they reach a trough at the peak of the aging process around 2030-2040. The US per capita growth rate is predicted to remain relatively close to the steady state growth rate whereas Europe and even more so the group of other OECD countries are projected to experience strong decreases in growth rates. The Japanese economy is predicted to experience growth rates which are lower than steady state levels throughout the entire simulation period.

Projected GNP growth rates show roughly the same pattern and do not differ much from projected GDP growth rates. This reflects that capital flows and foreign asset positions - according to the model’s assumptions foreign asset income is the only income from abroad, see above - are within reasonable ranges. Results on international capital flows between the world regions, the projected current account to output ratios, are shown in Figure 4, panel a. As Japanese capital outflows, capital flows from the France-Germany-Italy region to the rest of the world decrease but the decrease is not as strong. The US is projected to remain a capital import region whereas the ROECD region changes its relative position. A fundamental pension reform significantly alters magnitudes of international capital flows (panel b of the figure) which is discussed below.

The corresponding levels of growth rates and their decreases relative to the levels of year 2005 are shown in Tables 2 (for GDP) and 3 (for GNP). Maxima of decreases in per capita GDP growth rates are at 0.5 percentage points for the US (in around 2025)



and Europe (in around 2030) and at 0.85 percentage points in the other OECD countries (around 2040). Decreases in GNP growth rates are similar for these countries. Japan however experiences much stronger decreases of GNP growth rates than of GDP growth rates with a maximum decrease of 0.4 percentage points (around 2045). This difference is due to the strong decreases of Japanese predicted capital exports.

Figure 5 shows the different elements of growth - the endogenously determined growth rates of capital stocks and aggregate labor supply as well as the exogenous population growth rates. In all regions, the fluctuations of predicted per capita growth rates more or less parallel the fluctuations of predicted labor supply growth rates. This is unsurprising since labor supply determines overall wage income and also capital formation.

For the United States, the population growth rate remains positive during the entire simulation period which is largely explained by migration and higher fertility rates of migrants. The positive population growth rate also explains the high growth rate of the aggregate capital stock. During the period 2015 to 2040, labor supply growth rates are below population growth rates and therefore per capita output growth is also below the steady state level.

In the France-Germany-Italy region, population (and labor supply) growth rates are about zero towards the beginning of the simulation period. The slight deviation of the growth rate of the capital stock above the trend growth rate is hence due to the relatively high savings of the baby boom cohorts. As populations are aging, the gap between predicted labor supply growth rates widens and the decrease of the capital stocks' growth rates is significantly stronger than the decrease of population growth rates. Therefore per-capita GDP growth is significantly below steady state levels. Its rebound after 2030 is due to the corresponding rebound of labor supply and due to the negative population growth rate - the total "pie" has to be shared by fewer and fewer people.

The Japanese economy is predicted to experience a strong gap between population and labor supply growth rates. This explains why the growth rate remains below the steady state level throughout the entire simulation period. For the remaining OECD countries the predicted decrease in the level of the labor supply growth rate is strongest - from a positive value of about one percentage point to negative at about minus 0.3 percentage points. This explains why the decrease in output growth rates is strongest in these regions.

The accumulated effects of these patterns are depicted in Figure 6. In each country  $i$ , simulated per capita GDP and GNP values are de-trended with the level of labor productivity,  $E_{t,i}$ , which allows for the focus on the "pure" effects of demographic change (trending patterns would be meaningless). Furthermore, values are normalized to 100 in the year 2005. The resulting indices can be interpreted as the value of per capita GDP in year  $t$  relative to per capita GDP in year 2005 in terms of purchasing power in year 2005. Initially - as the dynamics of capital accumulation dominate the effects of decreasing labor supply - the index-values increase in all economies but Japan. For the France-Germany-Italy region the index then decreases to slightly below 95 until 2050. Matters are worse for the Japanese economy where it decreases to about 87.

### 3.3 Fundamental Pension Reform

The macroeconomic and distributional effects of fundamental pension reforms have received enormous attention among academic researchers during the past years. Börsch-Supan et al. (2004) provide an overview of the literature that focuses primarily on the (international) capital market effects of such reforms. To the best of my knowledge the literature has so far only implicitly analyzed consequences of such fundamental reforms on economic growth.<sup>9</sup>

Table 4 presents predicted time paths of pension system contribution rates in the four different world regions. These contribution rates reflect *direct* equilibrium pension system contribution rates. *Indirect* contributions by general taxes are not included. The increase in contribution rates is strongest in Japan (from about 14 percent in 2005 to about 25 percent in 2050), but the France-Germany-Italy region and the other OECD countries are also projected to experience strong increases of contribution rates of about 7 percentage points until 2050. For the US, the increase in contribution rates is less pronounced.

The fundamental pension reform by the “Freezing” scenario fixes contribution rates at year 2006 levels in the US, in the France-Germany-Italy region and in Japan. The resulting impact, the difference in contribution and replacement rates between the “Pure PAYG” and the “Freezing” scenarios, is therefore strongest for Japan.

Households react to this pension reform in two ways. First, they increase retirement savings in order to make up for the gap in retirement income resulting from a decrease of pension system replacement rates relative to the “status quo”. Second, households increase labor supply by working more hours and by retiring later. Börsch-Supan et al. (2004) discuss the relevance of these two reaction channels for the predicted time path of the rate of return to capital. The reaction of capital flows is shown in Figure 4. As the effects of the pension reform for the France-Germany-Italy region and Japan are stronger than for the US, their savings rates increase and hence also their external positions. Due to the large inflows from these countries, US saving rates respond in the opposite direction and therefore the US current account position becomes even more negative.

Table 5 summarizes the resulting differences in per capita growth rates. In Japan, the strength of the fundamental pension reform may add up to 0.2 percentage points to the predicted GNP growth rate. In the France-Germany-Italy region, increases in output growth rates are up to 0.18 percentage points when aging processes peak in these regions around the year 2030.

The spill-overs to the no-reform region ROECD are interesting. As Börsch-Supan et al. (2004) discuss, the endogenous increase of labor supply induced by the pension reform initially outweighs the endogenous increase of capital formation such that the rate of return to capital initially increases and the gross wage rate initially declines (the effects are however small). World gross wages determine households net wages in the no-reform countries and households react to this change in relative prices by decreasing labor supply. The effect is small, see Table 6.<sup>10</sup> This explains why output growth is slightly lower in the new system

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<sup>9</sup>An explicit treatment is given in Futagami and Nakajima (2001) who only analyze the effects of increasing retirement ages though.

<sup>10</sup>Results for the “Pure PAYG, constant labor supply” scenario shown in the table are discussed in the

for the ROECD countries. In the other regions, the reform regions, households increase their labor supply to make up for the resulting pension income gap by an additional 1 (2, 3) percentage point(s) in the United States (the France-Germany-Italy region, Japan).

While the numbers shown in Table 5 may be regarded as small, the accumulated impact of the reform on GDP and GNP levels is strong, see Figure 7. As expected, gains in terms of per capita GDP (GNP) levels are particularly strong for Japan and per capita incomes in the France-Germany-Italy region would roughly remain at current relative levels if these regions were to simultaneously implement the stylized reform.

Evaluating pension reforms on the basis of per capita income growth rates is however not the most relevant measure as far as the welfare consequences of such reforms are concerned. Figure 8 shows the effects of the fundamental pension reform on remaining lifetime utilities for different cohorts in the different world regions. Following Altig et al. (2001) the change in remaining lifetime utility is calculated as the equivalent variation of full lifetime income. The resulting welfare index measures the present value of remaining life-time resources relative to current full life-time resources a household would have to receive (pay) under the new system to make him indifferent between the old and the new system. Therefore, an index number greater (smaller) than one has to be interpreted as loss (gain) in remaining life-time utility.

Remaining life-time utilities of a large number of generations decreases as a consequence of the fundamental pension reforms. The number of cohorts experiencing losses of life-time utility is lower in those regions where the impact of the pension reform is strongest, and hence where future gains are highest. Therefore, Japan experiences the strongest losses, but for the least number of cohorts, and accordingly also the strongest gains in remaining life-time utilities of young and future cohorts. Interestingly, remaining life-time utilities of future cohorts also increase in the no-reform region consisting of the remaining OECD countries.

### 3.4 The Role of Flexible Labor Markets

How important are flexible labor markets in the context of demographic change and fundamental pension reform? How would growth rates change if age-specific labor supply is held constant at levels reached in year 2005, i.e. at levels reached at the beginning of the projection period? Table 6 shows aggregate labor supply shares for three different scenarios: the “Pure PAYG, constant labor supply” scenario in which age-specific labor supply are held constant, the “Pure PAYG, endogenous labor supply” scenario in which labor supply is determined by optimal life-cycle decisions and the “Freezing, endogenous labor supply scenario” where households also react to the fundamental pension reform by increasing labor supply and savings.

As the numbers show, aggregate labor supply increases in the “Pure PAYG” scenario relative to the constant labor supply shares case. The reason is twofold: first, households live longer and therefore decide to delay retirement which has very small effects on aggre-

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next Subsection.

gate labor supply. Second, and more importantly, as a consequence of demographic change the rate of return to capital falls relative to the net wage rate which makes life-time income generation via the labor supply channel relatively more attractive than capital accumulation. Aggregate labor supply therefore increases relative to the case of constant age-specific labor supply shares by 2 to 2.5 percentage points until 2050 in all regions.

To calculate the effects of these increases in labor supply note that the growth accounting relationships imply that

$$\Delta\gamma_{t,i}^{Y/N} = (1 - \alpha_i)\Delta\gamma_{t,i}^{L^d}.$$

and therefore that

$$\gamma_{t,i}^{Y/N,CLS} = \gamma_{t,i}^{Y/N} - (1 - \alpha_i)(\gamma_{t,i}^{L^d} - \gamma_{t,i}^{L^d,CLS}),$$

where  $\gamma^{Y/N,CLS}$  ( $\gamma^{L^d,CLS}$ ) denotes the growth rate of per capita output (labor supply) if age-specific labor supply shares are held constant. The corresponding per capita GDP and GNP indices can then be constructed recursively.

Figure 9 shows the de-trended aggregate GDP and GNP levels for the “Pure PAYG, Constant Labor Supply” scenario. A comparison with Figure 6 highlights the accumulated potential gains in per capita output and incomes that arise from increases in aggregate labor supply by the above mentioned 2 to 2.5 percentage points as a consequence of the *direct* effects of demographic change. In particular, the index value for Japan is below 85 in the constant labor supply scenario. Compared with the corresponding value in the endogenous labor supply scenario, the gain from endogenous labor supply is at about 3 percentage points. Gains for the other regions are of similar magnitudes. Table 7 summarizes the numbers.<sup>11</sup>

## 4 Conclusions

Will demographic change lead to a decrease of per-capita incomes - expressed in terms of today’s purchasing power? How do the effects vary across countries? How large will the effects be? How much of the changes in growth rates can be attributed to the production factors of capital and labor? How does a fundamental pension reform - a shift from mainly PAYG financed pension systems, prevalent in most industrialized countries, to partial pre-funding - affect growth? What is the role of flexible labor markets in this context?

The above analysis provides answers to these questions with the use of a large-scale multi-country overlapping generations model for (i) the United States, (ii) the region consisting of France, Germany and Italy, (iii) Japan and (iv) all other OECD countries. It is shown that the effects of demographic change on economic growth are substantial. Initially, and if countries maintain their currently generous public pension systems, the dynamics of capital accumulation dominate the decrease in labor supply caused by demographic change in most regions but in Japan. Therefore, per capita GDP and income (GNP) growth rates are initially above steady levels in these countries. As a consequence, per

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<sup>11</sup>Note that this analysis is in partial equilibrium. Results using an exogenous labor supply version of the OLG model are similar but involve other confounding factors.

capita incomes - expressed in terms of today's purchasing power - initially increase as well. However, in around 2010, per capita income growth rates are projected to decrease in the US, the France-Germany-Italy region and other OECD countries. The overall population aging-induced decrease of growth rates is up to 0.5 percentage points. While per capita incomes fall below the constant trend growth path in the France-Germany-Italy region by around 2020, the United States' per capita incomes are projected to remain above (on) the constant trend growth path for the entire projection period (until 2050).

These substantial international differences reflect the differences in the speed and the extent of demographic change across countries. While the US experiences declining but positive population growth until 2050, population growth rates are negative in Japan and the France-Germany-Italy region. The even stronger decreases in the growth rates of the labor force lead to the decreases of per capita outputs and incomes. While the growth rates of labor supply are initially slightly higher than population growth rates for the US, the France-Germany-Italy region and other OECD countries, respectively, labor supply growth is significantly lower than population growth in Japan. The relative decrease in capital accumulation due to dis-saving by the baby-boom cohorts adds to these demographic and labor supply effects.

A fundamental pension reform - a shift from "pure PAYG" financed to partially funded pensions - is found to unambiguously increase capital accumulation and labor supply in the reform countries. An about seven (eleven) percentage points decrease in pension system contribution rates relative to the model's "status quo" of a "pure PAYG" pension system in year 2050 leads to an increase of per capita income growth rates of up to 0.18 (2) percentage points in the France-Germany-Italy region (in Japan). As a consequence, per capita incomes in the France-Germany-Italy region are shown to be substantially higher than without the reform: they only slightly fall below the constant trend level after 2035. Gains for Japan are also remarkable: The overall decrease in Japanese per capita incomes is about 6 percentage points lower than without such a reform. The pension reform further implies small welfare losses for a relatively large number of cohorts during the transition but strong welfare gains for young and unborn cohorts. Future cohorts in no-reform countries are also found to benefit from pension reforms in other countries.

Flexible labor markets play an important role. Until 2050, the increase in age-specific labor supply rates in the "pure PAYG" pension system scenario leads to an increase of aggregate labor supply of about 2 percentage points relative to a scenario with constant age-specific labor supply rates. Without this increase, per capita output levels would be about 3 to 4 percentage points lower. These findings highlight that both capital accumulation on the international capital market (Börsch-Supan et al. 2004) as well as flexible labor supply adjustments are important mechanisms to master the macroeconomic consequences of demographic change. The labor supply channel has so far not received sufficient attention in the literature. Its relevance is not only due to the direct effects studied here but also due to more indirect effects on labor markets that may result from aging-induced changes of consumption demand (Lührmann 2005).

If maintaining per capita GNP growth rates and levels are political targets, then the capital accumulation and labor supply channels may not suffice to reach this target. While

fundamental pension reforms are shown to increase capital and aggregate labor supply and thereby lead to substantial increases of per capita incomes, actual pension reform proposals are not as “deep” as the stylized pension reform analyzed in this paper. Therefore, the projected gains in per capita incomes are higher than what can actually be expected. This suggests that other factors such as total factor productivity and human capital formation that remained unaddressed in the above analysis also play important roles in the future. How technological change is affected by demographic change - and this is by no means unambiguous - and whether higher spending on education of younger (but smaller) cohorts helps solving the difficulties caused by demographic change are open research questions.

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Table 1: Parameter Values of Endogenous and Exogenous Labor Supply Models

<b>Predetermined Parameters, <math>\Psi^p</math></b>		
$\sigma_1$		2
$\xi_1$		0.8
$\Delta\phi_1$		0.01
<b>Estimated Parameters, <math>\Psi^e</math></b>		
	$\hat{\Psi}_i^e$	(t-Statistic)
$\delta_1$	0.0371	19.1144
$\alpha_1$	0.3290	84.4969
$g^1$	0.0209	4.0242
$g^2$	-0.0001	-1.0835
$\Omega_1$	0.0753	23.6277
$\Omega_2$	0.0749	16.0564
$\Omega_3$	0.0808	9.7099
$\Omega_4$	0.0470	18.9042
$\rho_1$	0.0112	0.1871
$\bar{\phi}_1$	0.6329	6.8439
$\bar{\phi}_2$	0.5502	5.0450
$\bar{\phi}_3$	0.7057	6.6965
$\bar{\phi}_4$	0.5688	5.6974
<b>Other Estimated Parameters</b>		
$\gamma_1^L$	0.0169	13.7206
$\gamma_2^L$	0.0027	3.2018
$\gamma_3^L$	0.0082	5.2455
$\gamma_4^L$	0.0125	17.1420

*Notes:* This table shows values of predetermined ( $\Psi^p$ ) and free (estimated,  $\Psi^e$ ) calibration parameters. Free model parameters are estimated by matching of moments. *t*-Statistics are calculated using the Hansen-Hodrick-White (HHW) estimator with a bandwidth parameter equal to 4.

*Source:* Own calculations, based on demographic projections of the United Nations (2002).

Table 2: Predicted per Capita GDP Growth Rates - Pure PAYG

Year	$\frac{Y/N}{\gamma_{US}}$	$\Delta^05 \frac{Y/N}{\gamma_{US}}$	$\frac{Y/N}{\gamma_{FGI}}$	$\Delta^05 \frac{Y/N}{\gamma_{FGI}}$	$\frac{Y/N}{\gamma_{JAP}}$	$\Delta^05 \frac{Y/N}{\gamma_{JAP}}$	$\frac{Y/N}{\gamma_{ROECD}}$	$\Delta^05 \frac{Y/N}{\gamma_{ROECD}}$
2005	0.0206	0.0000	0.0186	0.0000	0.0144	0.0000	0.0223	0.0000
2010	0.0200	-0.0006	0.0181	-0.0005	0.0137	-0.0007	0.0212	-0.0011
2015	0.0182	-0.0024	0.0170	-0.0016	0.0144	0.0001	0.0193	-0.0030
2020	0.0165	-0.0041	0.0156	-0.0030	0.0153	0.0010	0.0173	-0.0051
2025	0.0157	-0.0049	0.0142	-0.0044	0.0156	0.0012	0.0156	-0.0067
2030	0.0159	-0.0047	0.0136	-0.0050	0.0150	0.0006	0.0145	-0.0079
2035	0.0165	-0.0041	0.0140	-0.0046	0.0138	-0.0006	0.0139	-0.0084
2040	0.0170	-0.0036	0.0150	-0.0036	0.0125	-0.0019	0.0138	-0.0085
2045	0.0171	-0.0035	0.0159	-0.0027	0.0122	-0.0022	0.0140	-0.0083
2050	0.0167	-0.0039	0.0164	-0.0022	0.0133	-0.0010	0.0144	-0.0079

*Notes:* This table shows predicted per capita GDP growth rates ( $\gamma^{Y/N}$ ) and the differences of time  $t$  growth rates relative to the year 2005 growth rates ( $\Delta^05$ ) for the United States (US), the France-Germany-Italy region (FGI), Japan (JAP) and all other OECD countries (ROECD).  
*Source:* Own calculations, based on demographic projections of the United Nations (2002).

Table 3: Predicted per Capita Income Growth Rates - Pure PAYG

Year	$\gamma_{US}^{GNP/N}$	$\Delta^{05} \gamma_{US}^{GNP/N}$	$\gamma_{FGI}^{GNP/N}$	$\Delta^{05} \gamma_{FGI}^{GNP/N}$	$\gamma_{JAP}^{GNP/N}$	$\Delta^{05} \gamma_{JAP}^{GNP/N}$	$\gamma_{ROECD}^{GNP/N}$	$\Delta^{05} \gamma_{ROECD}^{GNP/N}$
2005	0.0199	0.0000	0.0185	0.0000	0.0167	0.0000	0.0223	0.0000
2010	0.0195	-0.0004	0.0180	-0.0005	0.0154	-0.0013	0.0215	-0.0008
2015	0.0179	-0.0019	0.0169	-0.0017	0.0152	-0.0016	0.0200	-0.0023
2020	0.0163	-0.0036	0.0156	-0.0030	0.0152	-0.0015	0.0182	-0.0041
2025	0.0154	-0.0045	0.0143	-0.0042	0.0152	-0.0016	0.0166	-0.0057
2030	0.0154	-0.0044	0.0136	-0.0049	0.0148	-0.0020	0.0156	-0.0067
2035	0.0160	-0.0039	0.0139	-0.0046	0.0140	-0.0027	0.0150	-0.0073
2040	0.0167	-0.0032	0.0147	-0.0039	0.0132	-0.0035	0.0147	-0.0076
2045	0.0171	-0.0028	0.0155	-0.0030	0.0128	-0.0040	0.0146	-0.0077
2050	0.0171	-0.0028	0.0160	-0.0025	0.0132	-0.0036	0.0147	-0.0076

*Notes:* This table shows predicted per capita income (GNP) growth rates ( $\gamma^{GNP/N}$ ) and the differences of time  $t$  growth rates relative to the year 2005 growth rates ( $\Delta^{05}$ ) for the United States (US), the France-Germany-Italy region (FGI), Japan (JAP) and all other OECD countries (ROECD).

*Source:* Own calculations, based on demographic projections of the United Nations (2002).

Table 4: PAYG Contribution Rates

Years	Pure PAYG				Freezing			
	US	FGI	JAP	ROECD	US	FGI	JAP	ROECD
2005	0.1001	0.1767	0.1443	0.1301	0.1001	0.1767	0.1443	0.1301
2010	0.1017	0.1841	0.1609	0.1353	0.1003	0.1781	0.1475	0.1353
2015	0.1052	0.1923	0.1769	0.1421	0.1003	0.1781	0.1475	0.1421
2020	0.1108	0.2016	0.1906	0.1510	0.1003	0.1781	0.1475	0.1510
2025	0.1176	0.2122	0.2015	0.1619	0.1003	0.1781	0.1475	0.1619
2030	0.1241	0.2236	0.2103	0.1738	0.1003	0.1781	0.1475	0.1738
2035	0.1287	0.2336	0.2187	0.1856	0.1003	0.1781	0.1475	0.1856
2040	0.1309	0.2408	0.2281	0.1963	0.1003	0.1781	0.1475	0.1963
2045	0.1314	0.2448	0.2393	0.2053	0.1003	0.1781	0.1475	0.2053
2050	0.1316	0.2466	0.2502	0.2125	0.1003	0.1781	0.1475	0.2125

*Notes:* This table shows predicted PAYG pension system contribution rates in the “Pure PAYG” as well as the “Freezing” scenarios for the United States (US), the France-Germany-Italy region (FGI), Japan (JAP) and all other OECD countries (ROECD).  
*Source:* Own calculations, based on demographic projections of the United Nations (2002).

Table 5: Differences in Predicted per Capita Growth Rates - Freezing versus Pure PAYG

Year	per Capita GDP				per Capita GNP			
	$\Delta^P \gamma_{US}^{Y/N}$	$\Delta^P \gamma_{FGI}^{Y/N}$	$\Delta^P \gamma_{JAP}^{Y/N}$	$\Delta^P \gamma_{ROEC}^{Y/N}$	$\Delta^P \gamma_{US}^{GNP/N}$	$\Delta^P \gamma_{FGI}^{GNP/N}$	$\Delta^P \gamma_{JAP}^{GNP/N}$	$\Delta^P \gamma_{ROEC}^{GNP/N}$
2005	-0.0001	0.0002	0.0001	0.0005	0.0002	0.0000	-0.0001	0.0004
2010	0.0009	0.0009	0.0025	-0.0005	0.0006	0.0008	0.0021	0.0000
2015	0.0014	0.0012	0.0021	-0.0005	0.0010	0.0011	0.0020	0.0000
2020	0.0017	0.0015	0.0014	-0.0004	0.0013	0.0013	0.0016	0.0000
2025	0.0017	0.0018	0.0009	-0.0003	0.0014	0.0016	0.0013	0.0000
2030	0.0013	0.0018	0.0008	-0.0001	0.0012	0.0017	0.0012	0.0000
2035	0.0008	0.0013	0.0013	0.0001	0.0008	0.0015	0.0014	0.0000
2040	0.0005	0.0007	0.0017	0.0002	0.0005	0.0010	0.0018	0.0000
2045	0.0003	0.0002	0.0015	0.0002	0.0003	0.0007	0.0018	-0.0001
2050	0.0004	0.0001	0.0006	0.0002	0.0003	0.0006	0.0014	-0.0001

*Notes:* This table shows differences in predicted per capita GDP and national income (GNP) growth rates ( $\gamma^{Y/N}$  and  $\gamma^{GNP/N}$ ) between the “Freezing” and the “Pure PAYG” scenarios ( $\Delta^P$ ) for the United States (US), the France-Germany-Italy region (FGI), Japan (JAP) and all other OECD countries (ROECD).

*Source:* Own calculations, based on demographic projections of the United Nations (2002).

Table 6: Labor Supply Shares in Different Systems

Years	Pure PAYG, Const. LS			Pure PAYG, End. LS			Freezing, End. LS					
	US	FGI	JAP	ROECD	US	FGI	JAP	ROECD	US	FGI	JAP	ROECD
2005	0.459	0.428	0.522	0.450	0.460	0.429	0.522	0.451	0.461	0.429	0.522	0.451
2010	0.464	0.426	0.509	0.456	0.468	0.430	0.511	0.460	0.468	0.432	0.517	0.460
2015	0.465	0.421	0.496	0.458	0.471	0.429	0.500	0.464	0.473	0.434	0.512	0.464
2020	0.461	0.414	0.484	0.455	0.470	0.426	0.492	0.464	0.474	0.433	0.509	0.463
2025	0.453	0.404	0.475	0.447	0.465	0.419	0.486	0.459	0.471	0.430	0.506	0.458
2030	0.445	0.393	0.466	0.438	0.460	0.411	0.481	0.453	0.468	0.426	0.502	0.451
2035	0.440	0.383	0.456	0.429	0.457	0.404	0.475	0.446	0.466	0.422	0.498	0.443
2040	0.438	0.376	0.445	0.421	0.456	0.399	0.466	0.439	0.465	0.418	0.492	0.437
2045	0.437	0.372	0.433	0.413	0.456	0.396	0.457	0.433	0.465	0.416	0.486	0.431
2050	0.436	0.370	0.423	0.407	0.456	0.395	0.448	0.428	0.466	0.415	0.478	0.425

*Notes:* This table shows predicted aggregate labor supply rates for three different scenarios: the “Pure PAYG, constant labor supply (Const. LS)” scenario in which age-specific labor supply shares are held constant, the “Pure PAYG, endogenous labor supply (End. LS)” scenario in which labor supply is determined by optimal life-cycle decisions and the “Freezing, endogenous labor supply scenario” where households also react to the fundamental pension reform. Data are shown for the United States (US), the France-Germany-Italy region (FGI), Japan (JAP) and all other OECD countries (ROECD).

*Source:* Own calculations, based on demographic projections of the United Nations (2002).

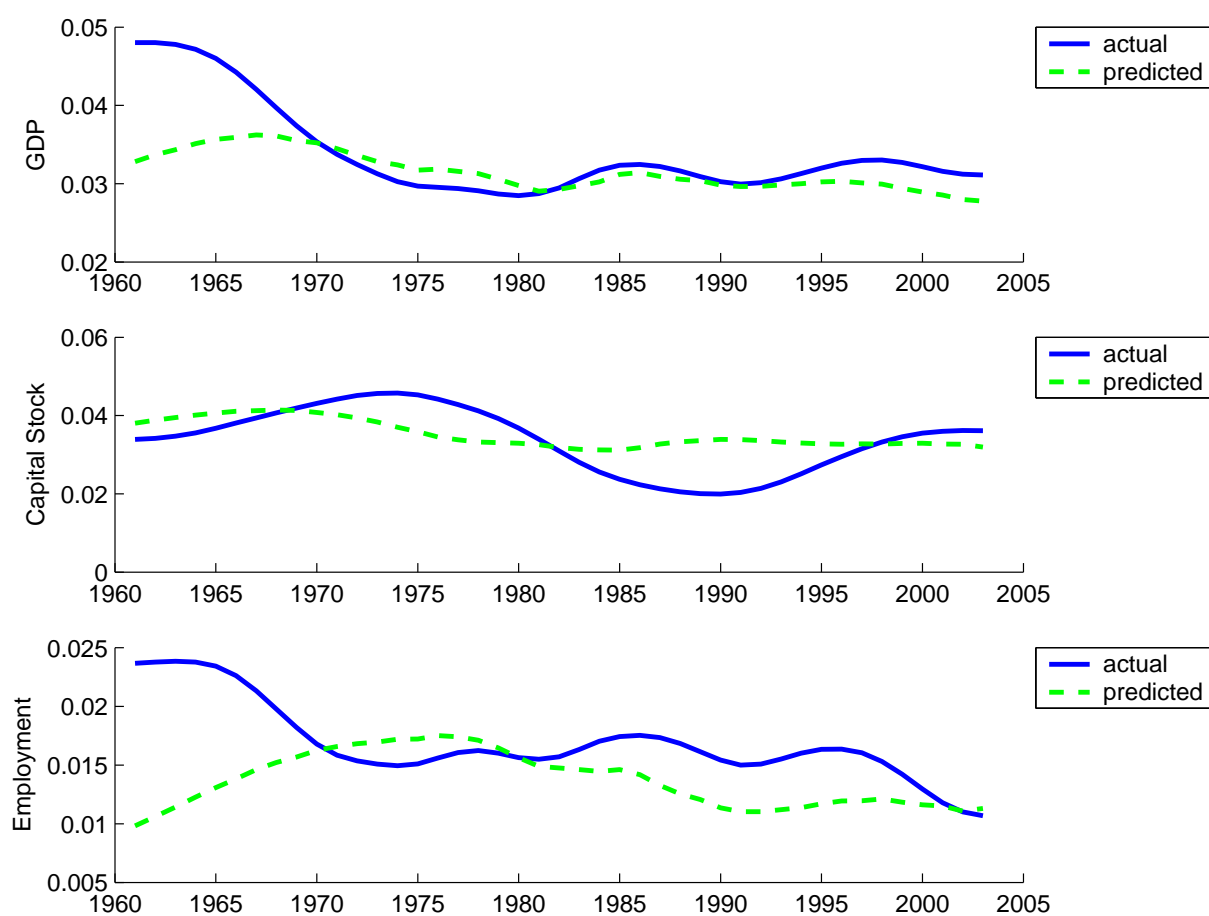
Table 7: De-trended per Capita GDP Levels in Different Systems [Indices, 2005=100]

Years	Pure PAYG, Const. LS			Pure PAYG, End. LS			Freezing, End. LS			
	US	FGI	JAP	US	FGI	JAP	US	FGI	JAP	ROECD
2005	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
2010	101.0	100.1	99.3	102.1	101.4	99.6	102.5	101.6	100.9	100.3
2015	101.6	99.8	98.2	103.7	102.3	98.7	104.5	103.0	101.7	100.4
2020	101.2	98.9	96.9	104.4	102.4	97.8	105.6	103.6	101.9	100.5
2025	100.2	97.3	95.5	104.2	101.8	96.9	105.9	103.7	101.6	100.3
2030	99.0	95.3	94.1	103.3	101.0	96.0	105.4	103.5	100.9	99.9
2035	98.0	93.3	92.4	102.1	100.3	96.5	104.5	103.4	100.1	99.3
2040	97.5	91.7	90.5	100.7	99.9	95.1	103.4	103.3	99.3	98.4
2045	97.2	90.5	88.3	99.3	99.9	94.2	102.2	103.4	98.8	97.4
2050	97.1	89.7	86.4	97.9	99.9	93.6	101.0	103.6	98.4	96.2

*Notes:* This table shows predicted pre capita GDP levels for three different scenarios: the “Pure PAYG, constant labor supply (Const. LS)” scenario in which age-specific labor supply shares are held constant, the “Pure PAYG, endogenous labor supply (End. LS)” scenario in which labor supply is determined by optimal life-cycle decisions and the “Freezing, endogenous labor supply scenario” where households also react to the fundamental pension reform. Data are shown for the United States (US), the France-Germany-Italy region (FGI), Japan (JAP) and all other OECD countries (ROECD).

*Source:* Own calculations, based on demographic projections of the United Nations (2002).

Figure 1: Actual and Predicted Growth Rates of GDP, Capital and Employment for the United States

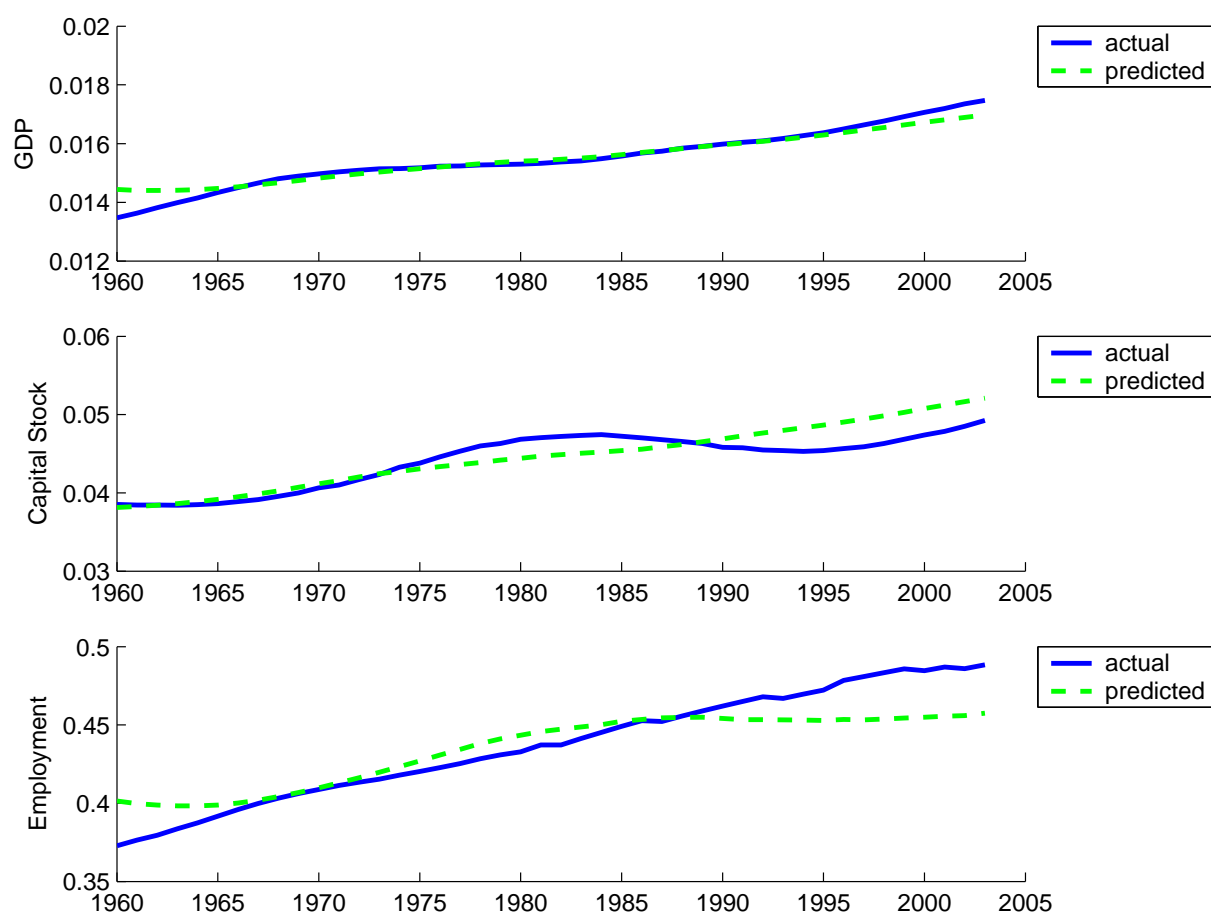


*Notes:* This figure shows actual (solid) and predicted (dashed) growth rates of GDP, the capital stock and aggregate employment using the endogenous labor supply model for the United States. Actual data are HP-filtered using a penalty factor for annual data of  $\lambda = 100$  (Ludwig 2005).

*Source:* Own calculations, based on demographic projections of the United Nations (2002).



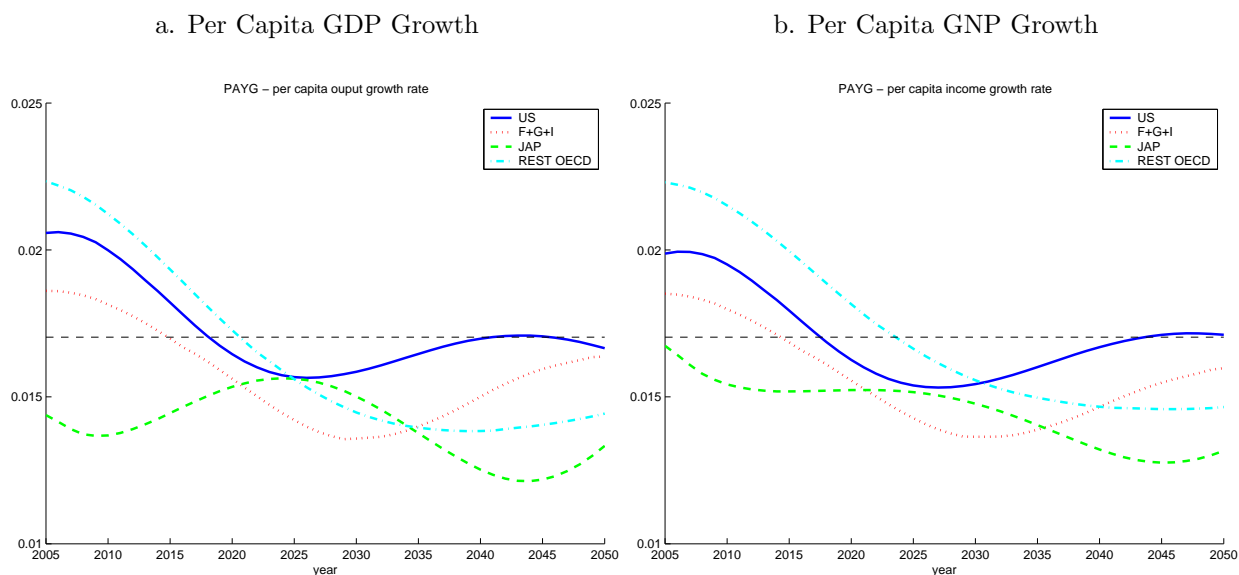
Figure 2: Actual and Predicted per Capita GDP, Capital and Employment for the United States



*Notes:* This figure shows actual (solid) and predicted (dashed) per capita levels of GDP, the capital stock and aggregate employment using the endogenous labor supply model for the United States. GDP and capital stock data are detrended by the technology level. Actual data are HP-filtered using a penalty factor for annual data of  $\lambda = 100$  (Ludwig 2005).

*Source:* Own calculations, based on demographic projections of the United Nations (2002).

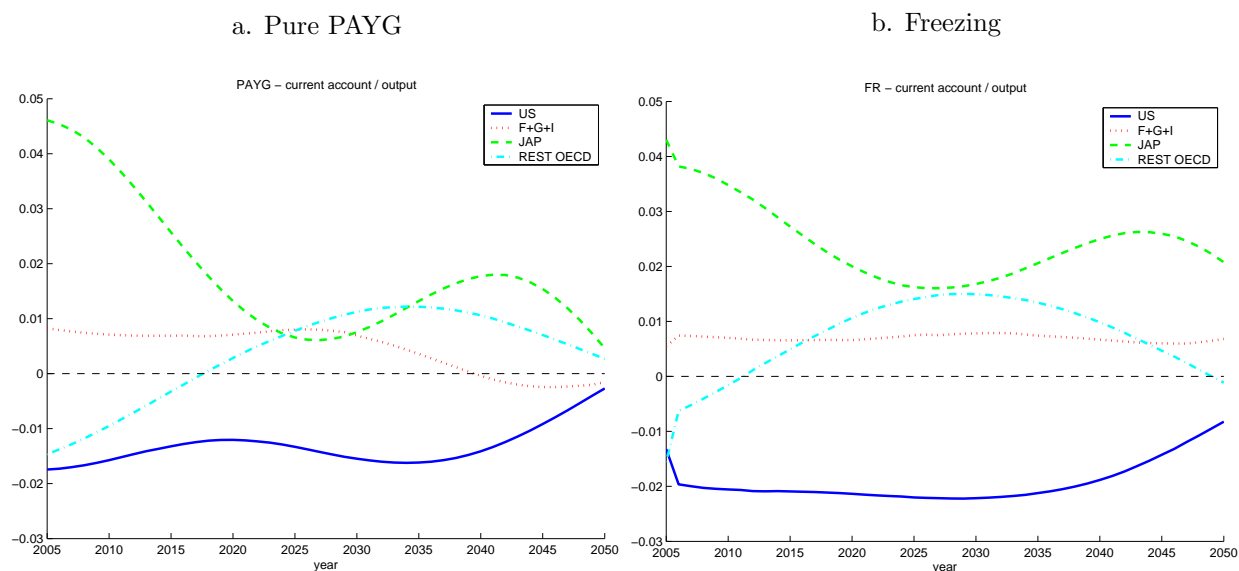
Figure 3: Per Capita GDP and GNP Growth Rates - Pure PAYG



*Notes:* This figure shows per capita GDP and income (GNP) growth rates if current generous PAYG pension systems were maintained for the United States (US), the region France-Germany-Italy (FGI), Japan (JAP) and all other OECD countries (ROECD).

*Source:* Own calculations, based on demographic projections of the United Nations (2002).

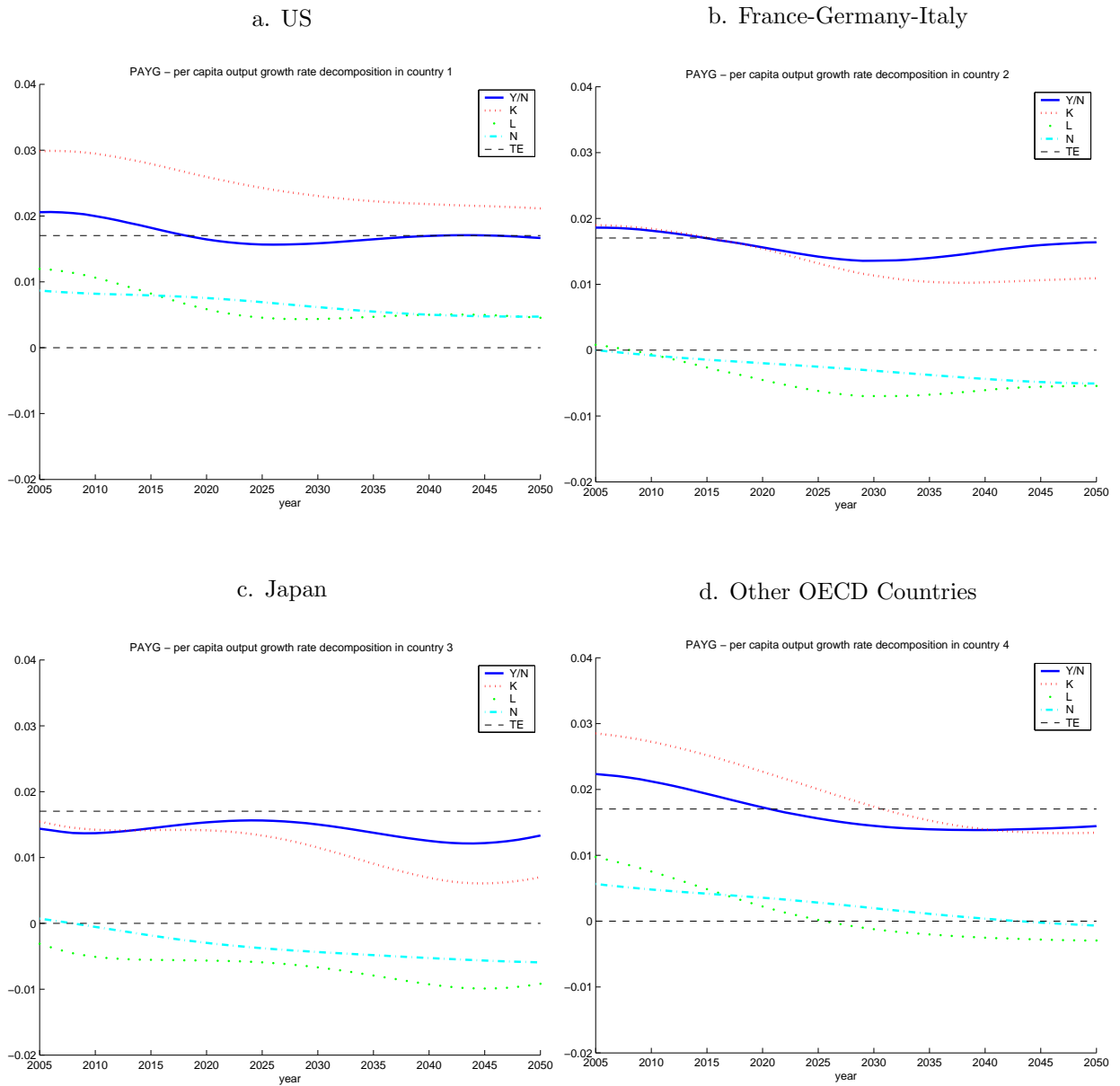
Figure 4: Current Account to Output Ratios - Pure PAYG and Freezing



*Notes:* This figure shows current account to output ratios for the “Pure PAYG” and the “Freezing” scenarios for the United States (US), the region France-Germany-Italy (FGI), Japan (JAP) and all other OECD countries (ROECD).

*Source:* Own calculations, based on demographic projections of the United Nations (2002).

Figure 5: Decomposition of per Capita GDP and GNP Growth Rates

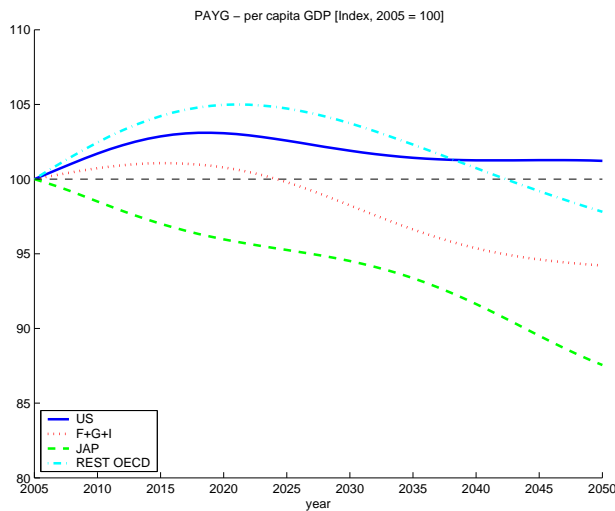


*Notes:* This figure shows the composition of per capita GDP growth rates into its components if current generous PAYG pension systems were maintained for the United States (US), the region France-Germany-Italy (FGI), Japan (JAP) and all other OECD countries (ROECD). Components shown are the per capita growth rate itself,  $Y/N$ , the trend growth rate of (exogenous) technological progress,  $TE$ , the growth rate of the capital stock,  $K$ , the growth rate of aggregate labor supply,  $L$ , and the (exogenous) growth rate of total population  $P$ .

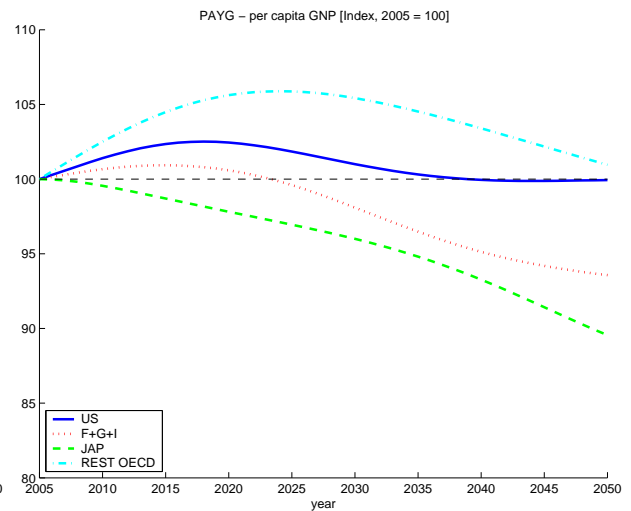
*Source:* Own calculations, based on demographic projections of the United Nations (2002).

Figure 6: Detrended per Capita GDP and GNP Levels - Pure PAYG

a. Per Capita GDP [Index, 2005=100]



b. Per Capita GNP [Index, 2005=100]

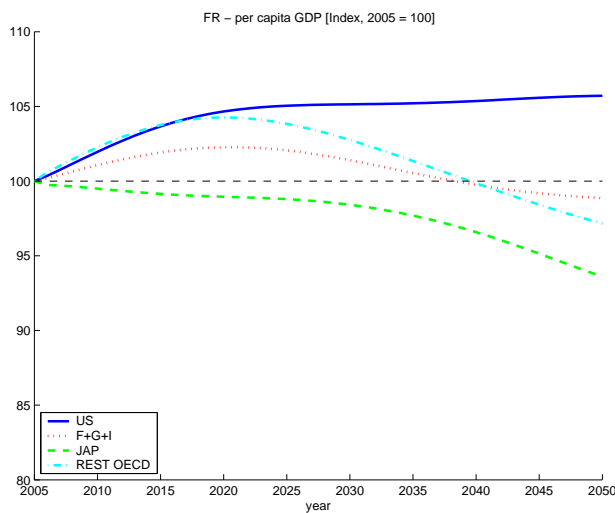


*Notes:* This figure shows detrended levels of per capita GDP and income (GNP) as indices if current generous PAYG pension systems were maintained for the United States (US), the region France-Germany-Italy (FGI), Japan (JAP) and all other OECD countries (ROECD).

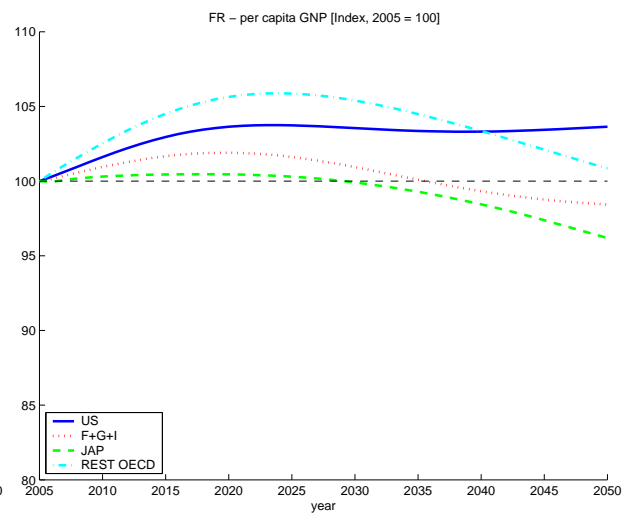
*Source:* Own calculations, based on demographic projections of the United Nations (2002).

Figure 7: Detrended per Capita GDP and GNP Levels - Freezing

a. Per Capita GDP [Index, 2005=100]



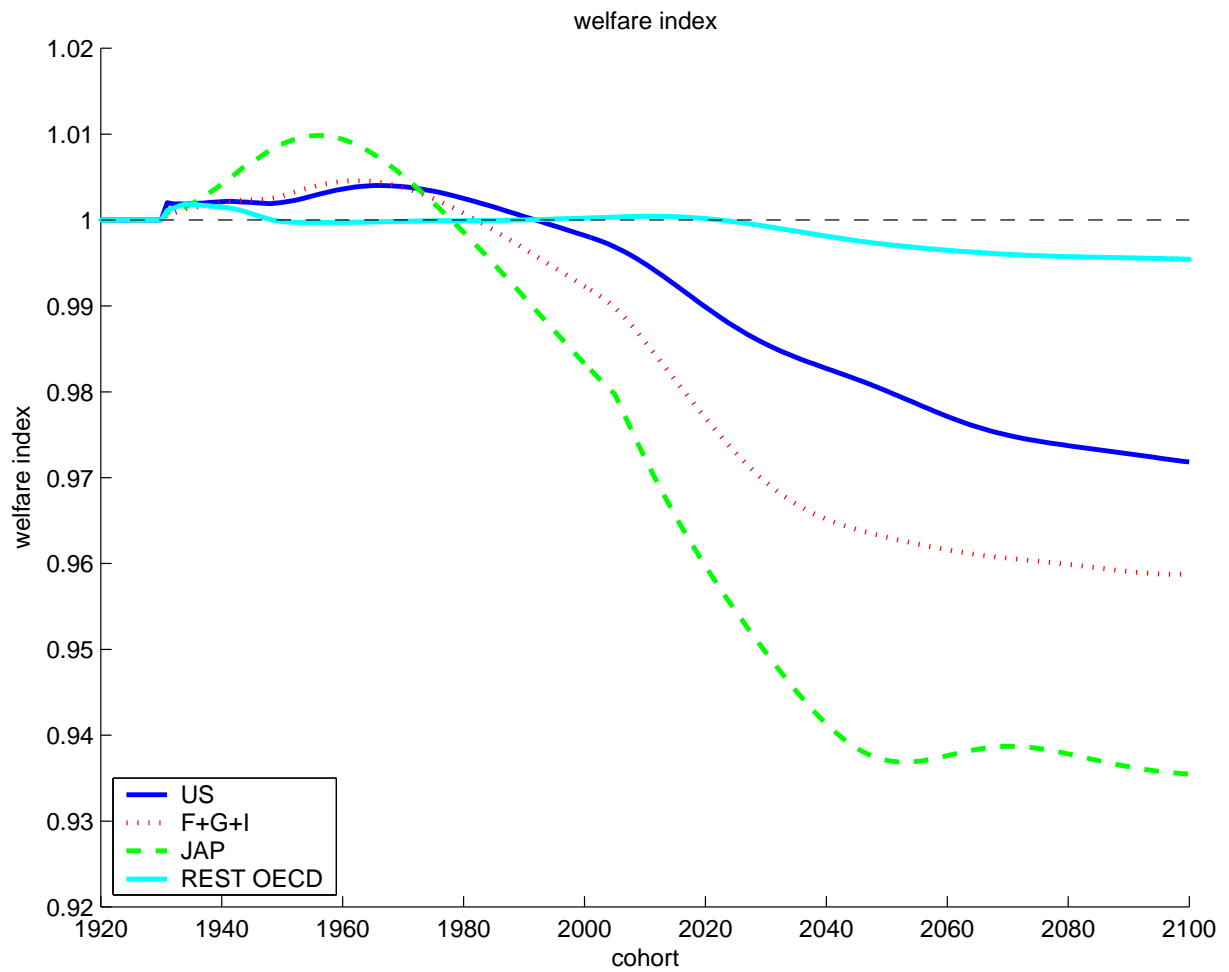
b. Per Capita GNP [Index, 2005=100]



*Notes:* This figure shows detrended levels of per capita GDP and income (GNP) as indices under the Freezing reform scenario for the United States (US), the region France-Germany-Italy (FGI), Japan (JAP) and all other OECD countries (ROECD).

*Source:* Own calculations, based on demographic projections of the United Nations (2002).

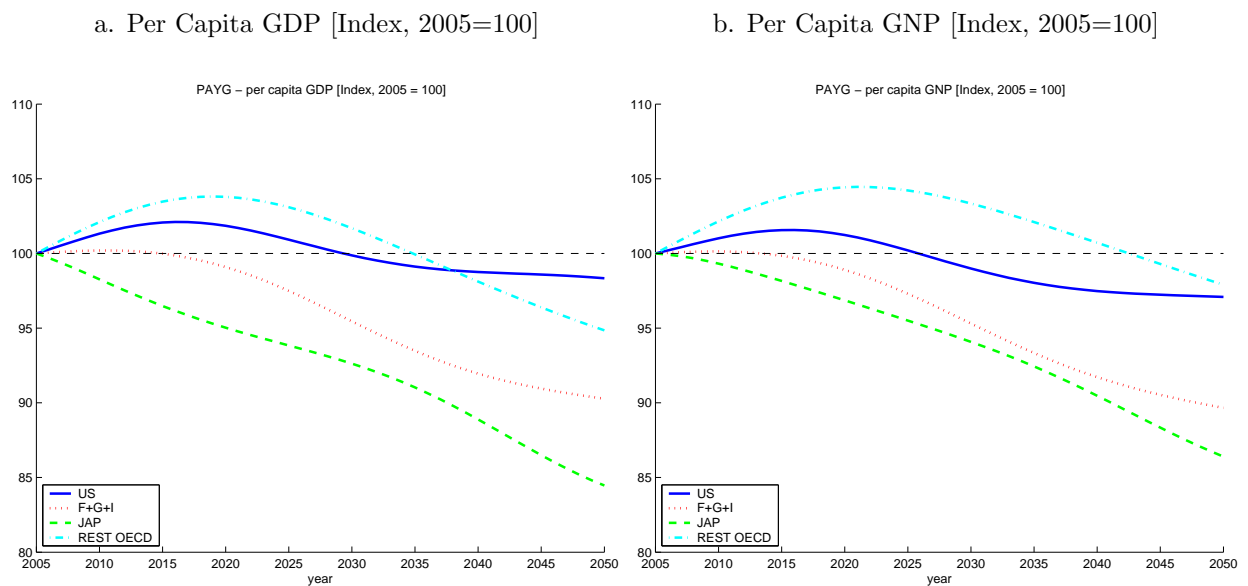
Figure 8: Welfare Index



*Notes:* This figure shows the welfare index for the United States (US), the region France-Germany-Italy (FGI), Japan (JAP) and all other OECD countries (ROECD).

*Source:* Own calculations, based on demographic projections of the United Nations (2002).

Figure 9: Detrended per Capita GDP and GNP Levels - Pure PAYG, Constant Labor Supply Scenario



*Notes:* This figure shows detrended levels of per capita GDP and income (GNP) as indices under the constant labor supply scenario if current generous PAYG pension systems were maintained for the United States (US), the region France-Germany-Italy (FGI), Japan (JAP) and all other OECD countries (ROECD).

*Source:* Own calculations, based on demographic projections of the United Nations (2002).



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