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A Data-Analytic Examination of the Risk in Hedge Funds Returns: The g-and-h Distribution

von
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Introduction

In the recent years the alternative asset class of hedge funds is widely discussed in financial literature. On the one hand this is caused by an increasing demand on these unregulated investments[1] and on the other hand it is caused by a lack of transparency concerning their investment strategies and investment instruments used. Especially this lack of knowledge gives reason to an unreflected adaption of the standard arguments of the attractiveness of a hegde fund investment, for example like a high expected return is going together with a low to a moderate risk.[2] Therefore this lack of knowledge gives reason to hope for a lasting substantial improvement of the return/risk-profile through the incorporation of such an investment.

In exploring hedge fund return patterns there are a variety of problems concerning the quality of the data and their structural behaviour. Regarding the quality of the data one can for example distinguish well documented biases, as the self-selection bias, the survivorship bias, the backfill bias and the liquidity bias.[3] In attaining satisfactory results concerning the quality of the data the following analysis is based on a subset of hedge fund index data chosen from the established data vendor Credit Suisse/Tremont from January 1994 till June 2005.[4]

Summarizing the problems concerning the structure of the data one can first mention the comparatively small amount of return data. Moreover numerous empirical studies lead to a substantial body of evidence that hedge fund return data rarely exhibit the ideal behaviour postulated by the Gaussian distribution.[5] Generally such results are obtained by using the following procedure: First of all the hypothesis of normality is rejected qualitatively by arguments concerning special style characteristics of a hedge fund investment. In a second step this qualitative result is affirmed empirically based on quantitative methods. To sum up the studies men-

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[1] See *Eling* 2006, p. 19 et sqq.
[2] See the empirical results supporting the above argumentation for a specified time-frame in *Liang* 1999 and *Kazemi/Schneeweis* 2004, p. 5.
[3] See *Lhabitant* 2004, pp. 88-95, *Bessler/Drobetz/Henn* 2005, p. 24 et sqq.
[4] The selection of this database was mainly driven by its size, by the asset weighted index construction and the effort of mitigate biases like the survivorship bias. A detailed overview of existing databases can be found in *Lhabitant* 2004, p. 88 and *Bessler/Drobetz/Henn* 2005, p. 22.
[5] Regarding the above mentioned index data *Eling* 2006, Appendix C. Regarding individual hedge funds see the empirical results in *Agarwal/Naik* 2000a, *Agarwal/Naik* 2000b; *Fung/Hsieh* 1997 and *Fung/Hsieh* 1999. A detailed overview can be found in *Liang/Park* 2007.

tioned above are based on specific characteristics of the empirical distribution. Most publications uniformly relied on classical measures of skewness and kurtosis which provide only a single scalar representation of the distributional shape aspects present in the data.[6] Using the third and fourth moments of a distribution or a sample distribution gives rise to some points of critique: First of all it is straightforward to show the lack of robustness of the estimators of higher moments with respect to extreme realisations.[7] Secondly the single scalar representation seems to be an inappropriate simplification concerning the complex structural shape of hedge fund return data.[8] In attempting to resolve the missing robustness and the single scalar representation, further analysis is based on special data analytic techniques using order statistics or quantiles exploring departure from symmetry (skewness) and heavier tails (elongation). Analysing the elongation of an empirical or theoretical distribution means comparing their tail strength to the strength of a given standard.[9] Assuming symmetry usually the normal distribution serves as the standard defining neutral elongation. After this qualitative analysis the next step is to fit a special distribution which leads to a reliable framework of exploring skewness and elongation in situations when skewness and elongation are jointly considered. In this joint case another standard defining neutral elongation has to be established.

Besides the difficulties mentioned above there are methodical problems in using traditional methods of analysing risk in the context of hedge funds. Although it is a clear fact that traditional methods like the standard deviation are likely to fail in the context of alternative investments, establishing an adequate conceptional framework seems to be an unsolved problem. Though measures of risk are intensively studied in the light of postulated features and are controversially discussed in the presence of non-normal risk distributions, the modelling of return patterns of alternative investments is still in its infants. In their work on the return/risk profile of a hedge fund investment *Gupta/Liang* 2005 used methods from extreme value theory and the risk measures Value-at-Risk (VaR) and Tail Conditional Loss. Based on their model they judged the adequacy of the capital reserve of individual funds. They found the capital reserve to be predominantly adequate. Recapitulatory *Gupta/Liang* 2005 modelled the extreme left tail of the return distribution using special central limit theorems for extreme re-

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[6] See *Badrinath/Chatterjee* 1988, p. 453.

[7] See *Badrinath/Chatterjee* 1988, p. 453; *Hoaglin* 1985a, p. 427.

[8] Although it is common practice to use the standardised third and fourth moments of the empirical data or an underlying distribution to characterise skewness and kurtosis *Tukey* 1977 showed that the concepts of skewness and elongation commonly used in statistical measurement are vague concepts and the results are a matter of the measurement method. *Dutta/Perry* 2006 p. 8.

[9] Elongation is a relative concept of tail strength which is often referred to in interpretations of the moment based definition of kurtosis. *Balanda/MacGillivray* 1990, p. 17.

alisations, which in general seems to be inappropriate in the case under consideration. Beyond that critique special notice should be given to the results of *Degen/Embrechts/Lambrigger* 2006. They explore the characteristics of a special distribution model, the g-and-h distribution, which has received special attention in modelling operational risk, in the light of the extreme value theory. They show that in situations in which the g-and-h model is an appropriate description for the data at hand, the use of methods based on extreme value theory leads to nonsatisfying results. Also *Liang/Park* 2007 explore diverse loss based risk measures. Their application was based on parametric as well as on non parametric methods. The *Cornish/Fisher*-Expansion played a special role in their analysis.

The present paper supports in a first step the result of non normal hedge fund returns based upon robust exploratory data analytic techniques. In addition the g-and-h distribution proposed by *Tukey* 1977 is fitted to the data. This approach exhibits a clear and straight structural form, is easy to apply and moreover provides an excellent fit for the entire data set. In addition this parametric model offers a high flexibility in modelling the shape of the distribution. For example the fitted distribution is allowed to have different structures in the two tails.[10] But one has to be aware of the problems and consequences of over fitting the model. In general the g-and-h distribution offers a genuine and flexible way of detailed exploration of skewness and elongation which is somewhat superior to classical moment based measures. The second contribution of this paper regarding existing hedge fund literature like *Gupta/Liang* 2005 and *Liang/Park* 2007 is an enhancement of existing methods quantifying risk in that it provides a new VaR-approach based on the g-and-h distribution.

Exploratory Data Analysis

CS/Tremont offers an aggregate hedge fund index based on 437 representative individual funds as well as 13 indices representing individual hedge fund styles respectively strategies. The styles can for example be divided into the three following groups: Market Neutral, Event Driven and Opportunistic. An important criterion concerning a classification system of hedge fund styles is their sensitivity to changes in the financial markets. Figure 1 summarises the chosen classification system for chosen style indices in short.

[10] See *Badrinath//Chatterjee* 1988, p. 454.

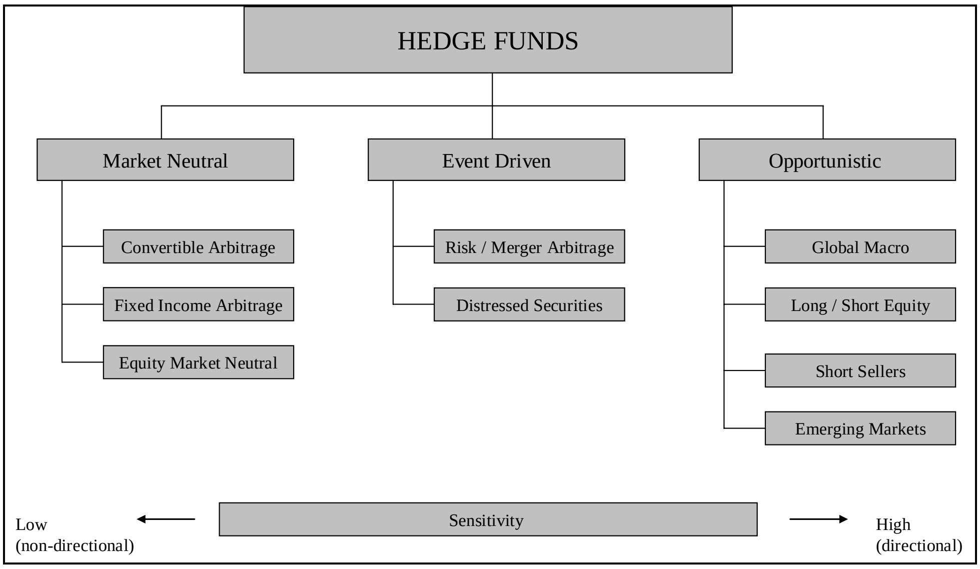


figure 1

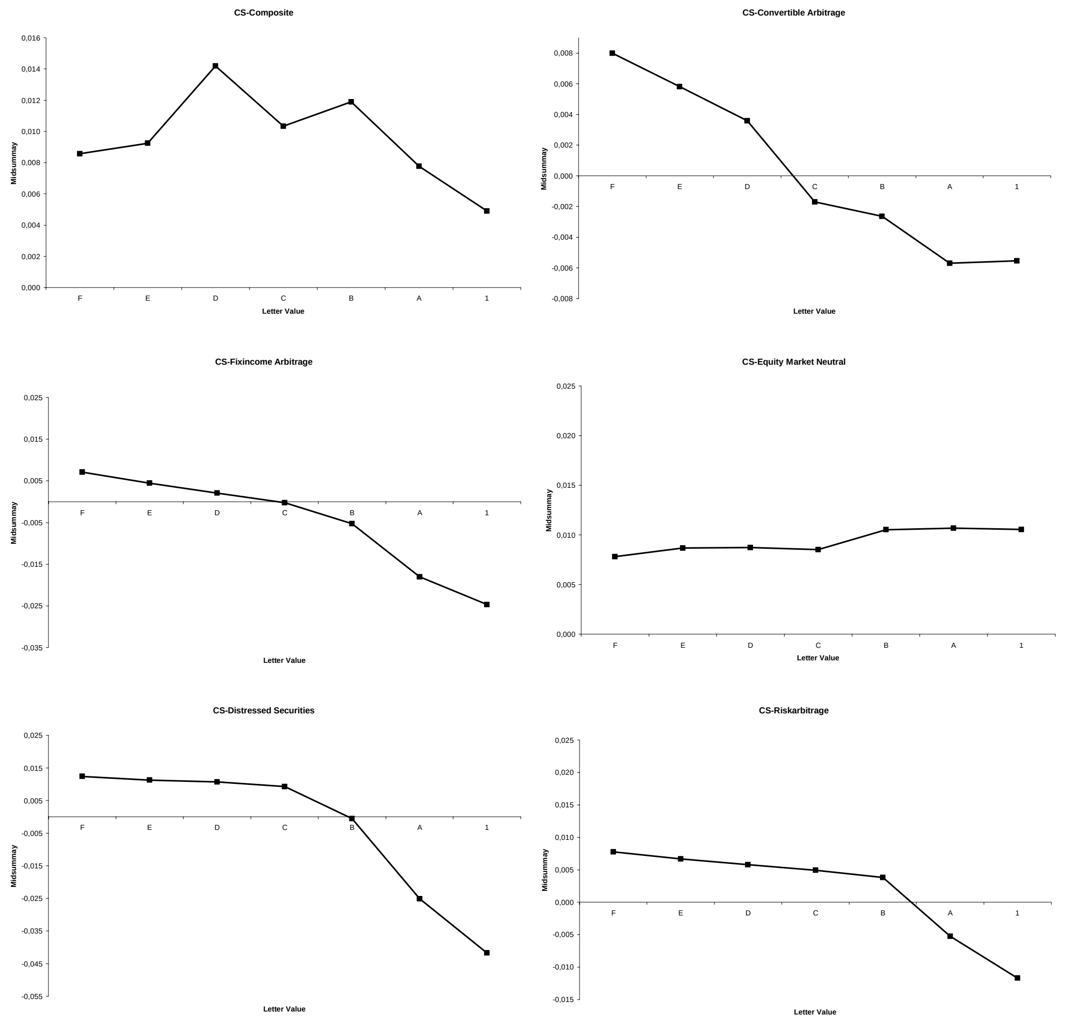
Prior to fitting the parametric model, a first analysis of skewness and elongation is based on special data analytic techniques using selected quantiles. These techniques measure skewness and elongation of the data with respect to the skewness and elongation of the Gaussian distribution. A common approach in choosing these quantiles mentioned above is to use letter values first proposed by *Tukey* 1977 and incorporated in *Hoaglin* 1985a. In doing so one has to choose the sequence of α-quantiles so that the probability α successivley always halves, i.e. $\alpha = 1/4, 1/8, 1/16, ...,$. For each α there is a corresponding pair of quantiles, the α-quantile (lower quantile) and the (1−α)-quantile (upper quantile). As a consequence of successively halving α the selected quantiles come more from the tails than from the body of the data.[11] Therefore further analysis can be based on a set of quantiles instead of one special pair alone to get a closer and more robust look at skewness and elongation. In doing so asymmetry and heavy tails can be considered graphically as follows.

In a first step the skewness of the data is judged by the midsummary statistic. Intuitively, if the situation at hand is symmetric, each corresponding pair of quantiles must be symmetrically placed around the median, which is the point of symmetry. Following this intuition the midsummary statistic for the data $\{X_i\}_{i=1}^{N}$ is calculated as:

[11] See *Hoaglin* 1985a, p. 419. The optimal spacing of the quantiles usually depends on the correlation structure of the order statistics. Typically less information in the form of quantiles is needed to model the body of the distribution than to model the tails of the distribution. *Dutta/Perry* 2006, p. 77.

$$Mid_{\alpha} = 0.5(Q_{\alpha} + Q_{(1-\alpha)}) \, , \qquad \textbf{(1)}$$

in which Q_{α} represents the above defined α-quantile of the data. The plot of the midsummaries versus various levels of α can help judging whether there is a systematic pattern in the data.[12] If there is a regular or a moderately regular steady increase or decrease from one midsummary to the next one can talk about a systematic skewness.[13] Summing up regular patterns in the midsummaries reveal the presence of skewness and moreover their direction. Following figure 2 illustrates the results concerning the CS-style indices.



[12] *Brizzi* 2003 explores the statistic „letter coefficient of skewness“ as a quantitative method of judging skewness. This statistic is also based on the above defined midsummaries using letter values. In spite of basically positive characteristics of the statistic one can find a lack of robustness in the presence of extreme realisations. As a consequence the letter value of skewness is not applied to the present data.

[13] See *Hoaglin* 1985a, p. 425; *Dutta/Perry* 2006, p. 9.

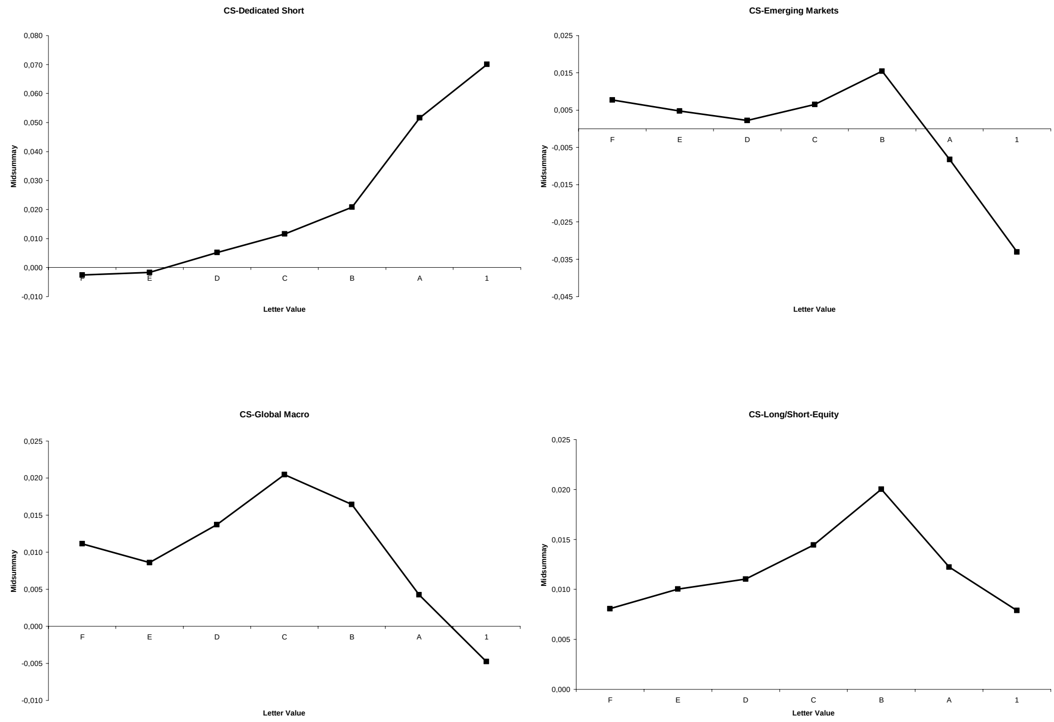


figure 2

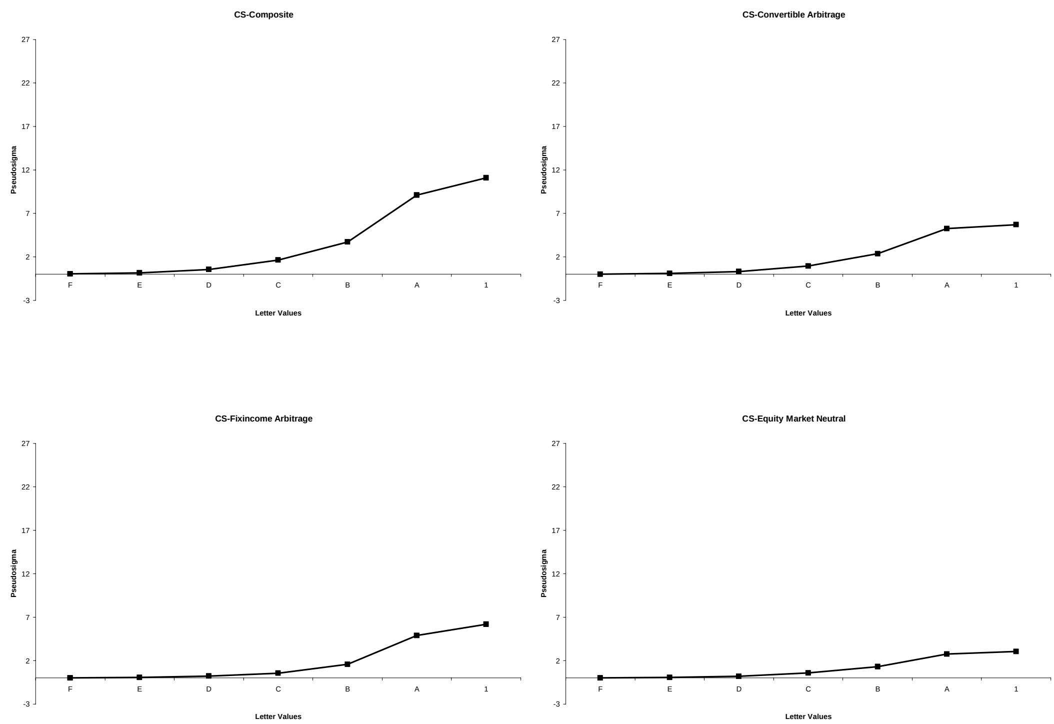
The Composite Index reveals only unsystematic variation of the midsummaries around the median, which argues against a systematic skewness at least for the body of the data. Patterns suggesting systematic left skewed data can be found for the Convertible Arbitrage, the Fixed Income Arbitrage, the Distressed Securities and the Risk Arbitrage Index. The Dedicated Short Index obviously exhibits a right skewed pattern. On the other hand the Equity Market Neutral Index reveals midsummaries at the same level of the median and can hence be seen as symmetric. The residual indices from the set of opportunistic strategies, the Emerging Markets, the Global Macro and the Long/Short Equity Index show similar patterns in the sense that there are, but different, systematic patterns for the body and tail areas of the data. Summing up there is evidence for all indices, that symmetry cannot be rejected at least for the body of the data. Therefore a normal approximation for the body of the data seems to be appropriate.

Regarding the classification system in figure 1 the following statements can be made: The Equity Market Index reveals to be symmetric and can be considered as the odd one out in the set of Market Neutral Indices. The others show a systematic skewness to the left, whereas the plot of midsummaries follows a regular pattern for all levels of α. The indices building up the set of Event Driven strategy reveal a systematic skewness to the left as well, but display a more complex structure of decrease. This manifests itself by means of a slight decrease of the midsummaries for the more central quantiles under consideration and a notable stronger decrease of the midsummaries for the extreme quantiles. Just as the latter, the plot for the opportunistic, except for the Dedicated Short Index, follows a complex but similar pattern. All in all these results under consideration, with the exceptions mentioned, support the used classification.

In the following elongation is measured. For this purpose the pseudosigma statistic is defined:[14]

$$Pseudo_\alpha = \frac{X_{1-\alpha} - X_\alpha}{2Z_P}, \tag{2}$$

whereas Z_P is the α-quantile of the standard normal distribution. This measure relates the positive distance of corresponding α-quantiles (upper and lower) to the standardnormal counterpart. If there is a systematic increasing (decreasing) behaviour in the plot of the pseudosigmas versus different levels of α, this is indicative for positive (negative) elongation.[15] Figure 3 illustrates the pseusosigmas of the stated indices:



[14] See *Hoaglin* 1985a, p. 426.
[15] See *Hoaglin* 1985a, p. 427.

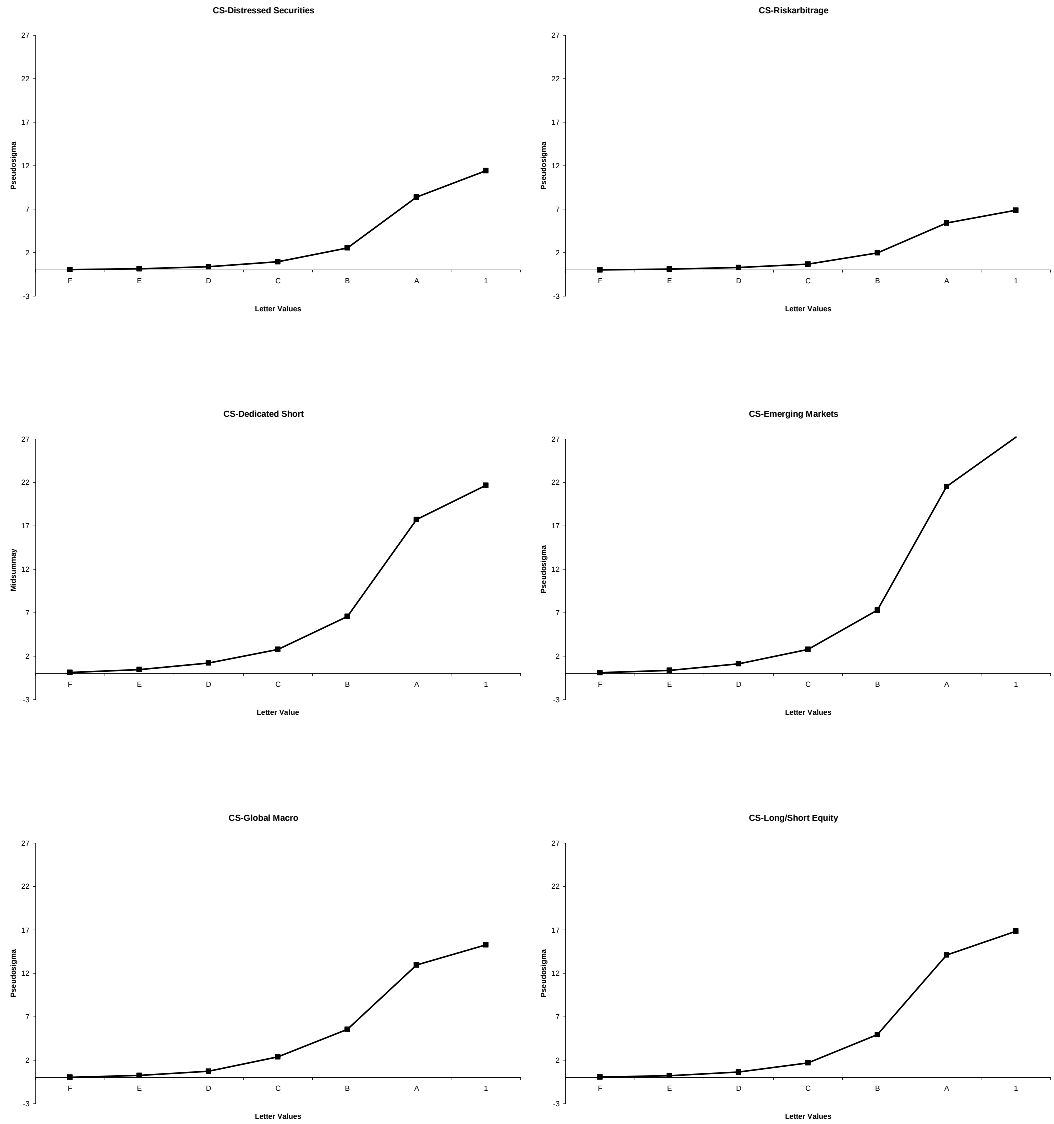


figure 3

Figure 3 illustrates the systematic elongation inherent in the data. Indeed for all strategies there is the same structural behaviour of the pseudosigmas though in a different magnitude. The most noticeable patterns are those of the Dedicated Short Index and the Emerging Markets Index. As in the context of skewness the elongation of the Equity Market Neutral Index is the fewest of all. However one thing has to be pointed out about the correct use of the peseudosigma statistic. Usually judging elongation using pseudosigmas is based on the working hypothesis of a symmetric distribution. For this very reason, having the midsummary patterns in mind, elongation can be confounded with the effects of skewness and moreover a distinct analysis of skewness-induced elongation and further elongation is not feasible.[16] Summarising fitting a model to the data at hand requires an approach flexible enough in describing the pre-

[16] As in the symmetric case neutral elongation is defined by the normal distribution, but in the presence of skewness a new standard defining neutral elongation has to be established.

vailing patterns of skewness and elongation in situations in which they occur jointly in the data.

The g-and-h distribution

The g-and-h distribution was first proposed by *Tukey* 1977 and is further analyzed by *Hoaglin* 1985b. This type of distribution is flexible enough to model a variety of different skewness and elongation patterns.[17] The ability of approximating a wide variety of distributions, commonly used in modelling financial data, only by means of a specific choice of a few parameters is the advantage of the g-and h-distribution.[18] The estimation of parameters can basically be made on the basis of the maximum likelihood-method, the method-of-moments approach or it can be based on quantiles.[19] Robust methods have, as already explained, favourable characteristics concerning the estimation of the tails, therefore they will be applied in the following. Furthermore, the structural form of the transformation of a standard normal distributed random variable is particularly suitable for a quantile-based estimation.[20] For a more detailed presentation of the quantile-based estimation of parameters see *Hoaglin* 1985b, *Dutta/Perry* 2006 and *Mills* 1995.

Currently the g-and-h distribution is being used successfully in order to quantify operational risks in the context of banking and insurance.[21] In addition, *Mills* 1995 and *Badrinath/Chatterjee* 1988 successfully modelled stock market returns using this distribution. *Babbel/Dutta* 2002 model skewness and elongation of the LIBOR and in *Babbel/Dutta* 2005 the g-and-h distribution serves as a starting point in option pricing.

Formally, the g-and-h distribution results from a non-linear transformation of a standard normal distributed random variable with real valued parameters a, b, g and h. In this case the pa-

[17] In addition there are other distributions, which were proposed for modeling skewness and elongation. See *Fischer/Horn/Klein* 2003. They compare different distributions in the presence of skewness and positive elongation and get compelling results using the g-and-h distribution.
[18] See *Martinez/Iglewicz* 1984, p. 363 et sqq.; *Dutta/Perry* 2006, p. 14.
[19] See *MacGillivray/Rayner* 2002a; *MacGillivray/Rayner* 2002b; *Badrinath/Chatterjee* 1988, p. 455.
[20] See *Dutta/Perry* 2006, p. 17.
[21] See *Perry/Dutta* 2006, *Albrecht/Schwake/Winter* 2007.

rameter a accounts for the location and b accounts for the scale.[22] The skewness and elongation are controlled by the parameters g and h, what ensures the desired flexibility.[23]

Assuming that Z is a standard normal distributed random variable, X follows a g-and-h distribution if X satisfies:[24]

$$X = a + b(e^{gZ} - 1)\frac{e^{(hZ^2/2)}}{g} \tag{3}$$

From this relation two different distributions arise for $g = 0$ or $h = 0$ as special cases:[25]

$$X = a + b\frac{(e^{gZ} - 1)}{g} \text{ the g-distribution in the first case and} \tag{4}$$

$$X = a + Zbe^{(hZ^2/2)} \text{ the h-distribution in the second case.}^{26} \tag{5}$$

At this point it is evident, that the measuring of the elongation always requires a reference distribution. Specifically the normal distribution as a particular g-and-h distribution $(g = 0, h = 0)$ serves as a standard for symmetric distributions and the lognormal distribution as a particular g-and-h distribution $(g \neq 0, h = 0)$ for asymmetric distributions.

A simple structural analysis results for the g-and-h distribution in case of $h > 0$, since this is the case of a strictly monotone transformation of a standard normal distributed random variable. Therefore, arbitrary quantiles of the g-and-h distribution emerge according to the following relation: Assuming

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[22] See *Mills* 1995, p. 325. To be precise the location and the scale are determined by a simple linear transformation of $Y_{g,h}(Z) = (e^{gZ} - 1)\frac{e^{(hZ^2/2)}}{g}$.

[23] See *Albrecht/Schwake/Winter* 2007, p. 31.

[24] See *Degen/Embrechts/Lambrigger* 2006, p. 5.

[25] See *Degen/Embrechts/Lambrigger* 2006, p. 5; *Martinez/Iglewicz* 1984, p. 354.

[26] A detailed overview of the g-distribution and the h-distribution can be found in *Badrinath/Chatterjee* 1988, p. 454 et sqq.; *Hoaglin* 1985b, p. 462 et sqq. An approach for modelling the departure from symmetry based on different h-transformations belonging to different parts of the data can be found in *Fischer/Horn/Klein* 2003.

$$k(x) = a + b(e^{gx} - 1)\frac{e^{(hx^2/2)}}{g} \quad \textbf{(6)}$$

describes the transformation function and ϕ the cumulated distribution function of a standard normal random variable:

$$F_X(x) = \phi(k^{-1}(x)), \quad \textbf{(7)}$$

holds.[27]

Consequently this implies for the VaR of a g-and-h distributed random (loss) variable:[28]

$$VaR_{1-\alpha}(X) = F_X^{-1}(1-\alpha) = k(\phi^{-1}(1-\alpha)),\ 0 < \alpha < 1. \quad \textbf{(8)}$$

The papers of *Babbel/Dutta* 2002 and *Martinez/Iglewicz* 1984 exhibit basic features of the g-and-h distribution, whereas the latter mentioned also examine the case of a non-strictly monotonic increasing transformation. They determine analytically the cumulative distribution function for the case $h < 0$.

Of particular importance concerning the estimation of the parameters is at first the characteristic, that the location parameter a represents the median of the data. Analogue to the preceding data analysis by using (3) for every particular level of α by using (3) an estimate of parameter g, g_α, is resulting which is independent of h due to its structural design.[29] In this context all that remains to be clarified is which estimate of g is best for the given situation. Relying on these results, estimations of the parameters b and h are received conditional on g.[30] The possibility of analysing skewness and elongation depending on particular levels of probability is one of the advantages of this approach. Furthermore, generalisations of (3) are possible. To be more precise, the parameters g and h cannot only be constants but can also alternatively be polynomials in Z^2. This naturally enlarges the flexibility but it also enlarges the number of parameters to be estimated.[31] Finally a Monte Carlo simulation is particularly easy to be car-

[27] See *Martinez/Iglewicz* 1984, p. 354.
[28] See *Martinez/Iglewicz* 1984, p. 355.
[29] See *Hoaglin* 1985b, p. 486.
[30] See *Hoaglin* 1985b, p. 487.
[31] See *Dutta/Perry* 2006, p. 16

ried out since the g-and-h distribution is based on a transformation of a standard normal distributed random variable.

Results

In the first instance the g-and-h distribution will be fitted to the empirical data. In a first analysis a quantile-quantile-plot (QQ-Plot) is being used to assess the goodness of fit. Since the sample is being compared to a theoretical distribution the corresponding quantiles have to be determined.[32] As long as the transformation (6) is strictly monotone the quantiles arise from relation (7). In the case of a violated monotonicity, the quantiles can be determined analytically using simple arguments[33] or by simulation. To clarify the fundamental approach of parameter estimation, the procedure is being demonstrated exemplary with the CS-Distressed Securities Strategy:

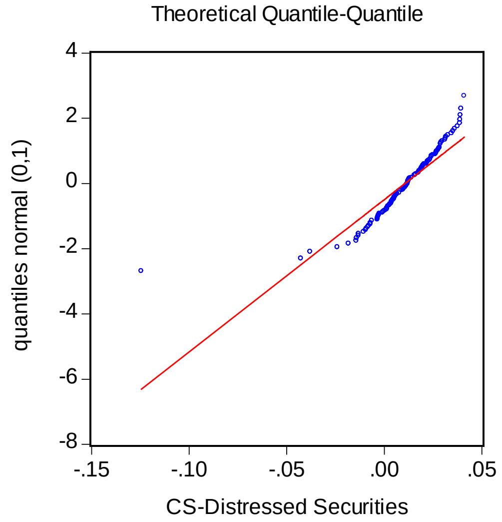

figure 4

As the QQ-Plot versus the standard Gaussion distribution exhibits, the Distressed Securities Index reveals departure from symmetry as well as heavy tails, since clear deviations from the idealization, a straight line which has slope one, are detectable. In this situation at first, as

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[32] Based on an order statistic the probability p results form $(i-\frac{1}{3})/(N+1/3)$, whereas i describes the index of the order statistic and n describes the sample size. See *Hoaglin* 1985a, p. 435.

[33] See *Martinez/Iglewicz* 1984, p. 354 et sqq.

already outlined, an adjustment of skewness takes place, before the tails conditional on this adjustment, are being examined.[34]

Using (3) and for each above defined level of α the corresponding upper and lower quantiles, whereby a<0 applies, leads to a set of estimates for the parameter g, each of them denoted by g_α:[35]

$$g_\alpha = -\frac{1}{Z_\alpha}\ln\left(\frac{X_{1-\alpha} - X_{0.5}}{X_{0.5} - X_\alpha}\right) \qquad \textbf{(9)}$$

After all it remains to resolve, which estimate provides for the best adjustment of skewness. If the g_α values display no systematic structure in a plot versus the corresponding squared quantiles of standard Gaussion distribution, *Hoaglin* 1985b suggests as best alternative the value which halves the set of classified values (median). This situation can often be found in distributions with a simple skewness-pattern, as for example the lognormal distribution.[36] In this respect it seems to be appropriate to examine at first the structure of the g_α values in dependence of the corresponding squared quantiles of the standard Gaussion distribution.

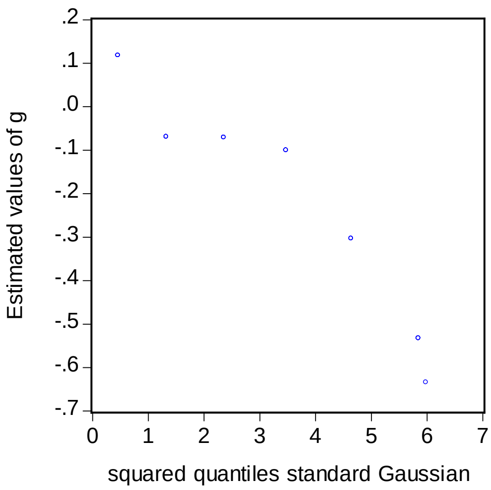

figure 5

[34] See *Hoaglin* 1985b, p. 486.
[35] The estimator directly results from (3) in combination with the symmetry of the normal distribution.
[36] See *Badrinath/Chatterjee* 1988, p. 460 in connection with *Dutta/Babbel* 2002, p.4. The g-distribution with constant and positive g is a lognormal distribution, whereas the constant parameter g fully describes the skewness of the distribution. *Hoaglin* 1985b, p. 475.

As figure e (5) reveals a complex form of skewness in the sense of a systematic pattern of the g_α, it seems indicated to use the following polynomial describing this:

$$g(z^2) = g_0 + g_1 z^2 . \qquad \textbf{(10)}$$

In figure (6) two QQ-Plots are being contrasted. The left QQ-Plot visualizes the goodness of fit for the case of constant parameters g and h, while the right plot illustrates the goodness of fit quality when using (10) jointly with a constant parameter for h.[37]

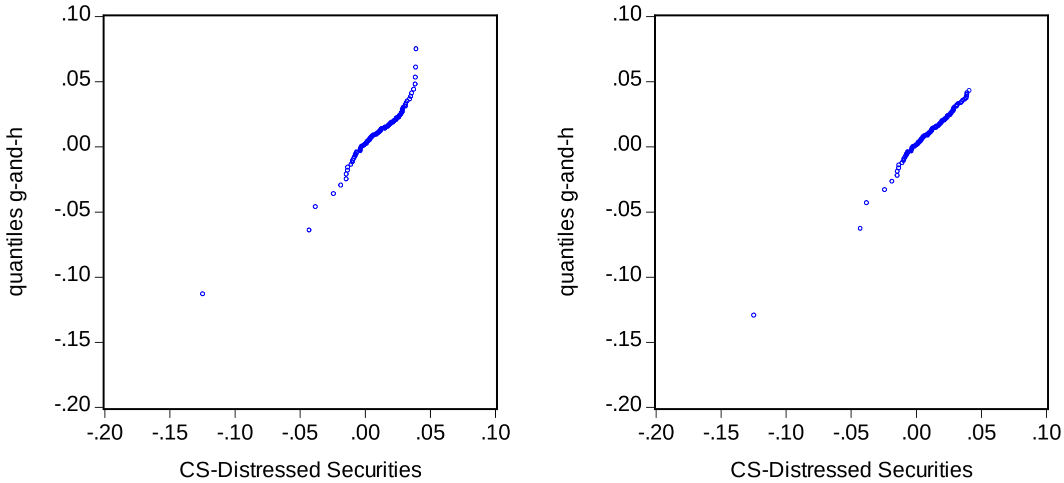

figure 6

Noticeable is the improvement of fit in the central areas of data by the usage of (10). Moreover an extremely good fit in the left and right tail of the distribution is perceivable. For illustrative purposes the consequences of an over fitting of the model by using a polynomial for h will be described in the following. As the QQ-Plot figures, there is a notable misspecification in the left and right extreme part of the tails.

[37] Since the quantiles of the g-and-distribution cannot be calculated analytically any more, they were gained from a Monte Carlo simulation.

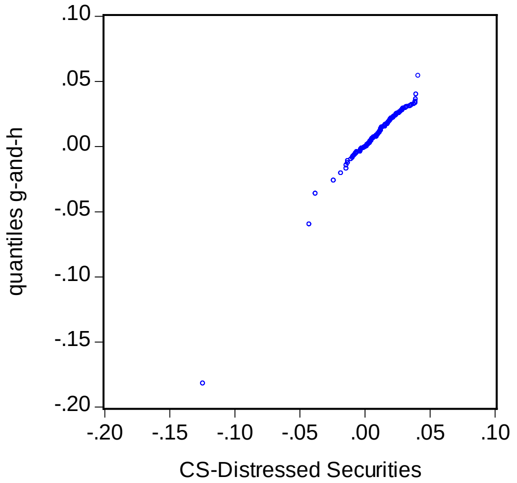

figure 7

In estimating g it was unavoidable to use information from both the lower and upper tails. For the estimation of h conditional on g the total data can be used or the data set can be divided relatively to the median of the data.[38] Hence the elongation of both tails can be modelled separately.[39] This can prove to be a great advantage within the VaR model, since a better fit of distribution can often be attained.

In the case of a considering the whole data set the estimation of h is generally based on the positive quantile distance of α– and their corresponding (1–α)–quantiles.[40] The quantile distance to a fixed level α, corrected for asymmetry, is being called the correct full spread (CFS_α). If the quantile distances are being calculated relative to the median separately, this results in, again corrected for asymmetry, lower half spreads (LHS_α) and upper half spreads (UHS_α). In this way three measures are available to examine the elongation of both tails simultaneously or separately from each other. It remains to remark that the usage of half spreads in modelling the complete data set can be advised by the complexity of real life data.[41] Given the data are skewed to the left (right) and positively elongated it seems to be

[38] See *Mills* 1995, p. 326. Beyond that, other separations are possible. *Badrinath/Chatterjee* 1988 fit two g-and-h models to their data set. The first one for the central part of the data, while the second one models the extreme realisations.
[39] See *Dutta*/*Babbel* 2002, p. 5.
[40] It is to note that, different from the examination of the tails in the data-analytical part, by using pseudosigmas, the asymmetry of the data is taken into consideration while estimating h. See *Badrinath/Chatterjee* 1988, p. 457.
[41] See *Hoaglin* 1985b, p. 487 in connection with *Badrinath/Chatterjee* 1988, p. 457.

more appropriate to use the LHS (UHS) measure.[42] In particular the specifications above can be expressed formally using (3) in conjunction with different sets of quantiles as follows:[43]

$$CFS = g\left(\frac{X_{1-\alpha} - X_{\alpha}}{e^{(-gZ_{\alpha})} - 1}\right) = be^{hZ_{\alpha}^{2}/2}, \tag{11}$$

$$LHS = g\left(\frac{X_{0.5} - X_{\alpha}}{1 - e^{gZ_{\alpha}}}\right) = be^{hZ_{\alpha}^{2}/2}, \tag{12}$$

$$UHS = g\left(\frac{X_{1-\alpha} - X_{0.5}}{e^{-gZ_{\alpha}} - 1}\right) = be^{hZ_{\alpha}^{2}/2} \tag{13}$$

By taking the logarithm the parameters b and h can be estimated based on robust regression methods.

Tabel 1 displays the results for the CS-Indices, the stock index S&P 500 and the bond index Citigroup Broad Investmentgrade Bond Index concerning their distributional shape and the goodness of fit. After a first assessment of the goodness of fit by using a qq-Plot two standard tests for normality are applied. The monotonicity of the transformation function hereby is crucial, since in this case, the inverse function exists and more over under the null distribution, g-and-h, the inverse $k^{-1}(X)$ is a standard Gaussian random variable. Therefore the *Jarque-Bera* (JB) as well as the *Anderson-Darling* (AD) test are performed. The JB test measures departure from normality based on moment-based definitions of skewness and kurtosis. Besides the general critique concerning the usage of moments, it remains to criticise that the test is a large sample test. Therefore it is questionable in how far the present sample fulfils these conditions. The AD test is a modification of the *Kolmogrov-Smirnov* test, whereby more weight is given to the tails.[44] Beyond this the AD test can also be used in small samples. As the main disadvantage one can count the fact that the critical values depend on the used null distribution. In the case the g-and-h distribution is the null distribution this would be quite remarkable. For a standard Gaussian distribution serving as the null distribution this should not be a problem, since tables of critical values exists. Generally one has to keep in mind, that even the case $k^{-1}(X)$ exists, no explicit solution for $Z - k^{-1}(X) = \mathbf{0}$ can be found, so it has to be determined numerically. In the following table the results are summed up and the P-Values of the relevant test statistics are given.

[42] See *Dutta/Perry* 2006, p. 75.
[43] See *Mills* 1995, p. 326.
[44] See *Dutta/Perry* 2006, p. 82.

CS	a	b	g0	g1	h0	h1	JB	AD
Composite	0.008	0.015	0.057		0.275		0.74	0.71
Convertible Arbitrage	0.010	0.012	-0.401		0.070		0.20	0.10
Fixed Income Arbitrage	0.008	0.006	-0.544		0.278		0.25	0.36
Equity Market Neutral	0.008	0.008	0.093		0.031		0.92	0.86
Distressed Securities	0.012	0.013	0.119	-0.106	0.185		0.34	0.38
Risk Arbitrage	0.006	0.008	0.311	-0.098	0.250		0.58	0.40
Dedicated Short	-0.004	0.042	0.176		0.138		0.18	0.02
Emerging Markets	0.012	0.035	-0.143		0.203		0.84	0.84
Global Macro	0.011	0.021	0.014		0.287		0.87	0.73
Long Short Equity	0.008	0.019	0.066		0.296		0.46	0.60
S&P 500	0.014	0.042	-0.134	-0.018	0.005		-	-
Citigroup Bond Index	0.007	0.010	-0.210		0.037		0.32	0.94

table 1

Table 1 describes the Composite Index as approximately symmetric with a simple form of asymmetry. With exception of the almost symmetric Equity Market Neutral Index all Market Neutral strategies show a simple form of negative skewness. Notable is the complex structure of asymmetry in the Distressed Securities strategies, which is left skewed except for the body of the distribution. On the contrary, the opportunistic strategies show only simple asymmetries, whereas the strategies Global Macro and Long Short Equity are to a great extent symmetric. This result is essentially driven by the establishment of a model for the entire data. A closer examination of the g_α-values reveals an image analogue to the analysis of the mid-summaries, which suggests different models for different subsets of data. As expected, the Dedicated Short Index shows a simple structure of positive skewness. The S&P 500 and the bond index reveal negative skewness, whereby the stock index presents a complex structure of asymmetry.

With the exception of the model for S&P 500 for which no inverse transformation function exists all models based on the JB-statistic are significant on a high level of confidence. The same is true for the AD-test excepting the model of the Dedicated Short Index. Since h is estimated after adjusting for skewness, the elongation is measured relative to the neutral elongation defined by the g-distributions.[45] The analysis exhibits for all hedge fund strategies heavy tails relative to neutral elongation, which means extra risk. As to the structure of h one receives clearly a more simple picture. Remarkable is again the Equity Market Neutral Index which shows a parameter h at the same level as the parameter of the bond index. Further the Convertible Arbitrage strategy is noticeable with a low parameter h. For the rest the estimations of parameters for the other indices show a similar level. The stock index S&P 500 does

[45] See *Badrinath/Chatterjee* 1988, S. 464.

not show heavy tails induced by the g-distribution. To assess the single strategy-indices finally with respect to their chance/risk-potential, the location a and the scale b of the distribution have to be taken into account. The following section attends to a quantile based risk analysis of the CS-indices by using the preceding distribution models.

Risk Analysis

Since the preceding analysis identified the g-and-h distribution as a satisfactory model of the empirical data, the estimation of extreme quantiles and thus the VaR can be performed on this basis. This displays an important extension compared to the traditional parametric and non-parametric VaR models, the so-called normal VaR or the variance-covariance-approach as well as the historic simulation. The modified VaR, based on the *Cornish/Fisher*-Expansion, VaR models using the general student t distribution or VaR Models based on the extreme value theory are earlier parametric approaches trying to incorporate departure from normalty. In this context the g-and-h VaR can be understood as an independent and genuine VaR model as follows:[46]

$$VaR_{\alpha}(X) = -\left[a + b(e^{gZ_{1-\alpha}} - 1)\frac{e^{(hZ_{1-\alpha}{}^{2}/2)}}{g} \right],\ 0 < \alpha < 1. \tag{14}$$

Particular importance is inherent to the strict monotonicity of the transformation function. If the monotonicity is violated, this leads to a restricted domain of the VaR. After all, from the view of a reasonable risk analysis, extremely unfavourable scenarios have to be possible, respectively a total loss, defined as -100% return, has to be an element of the domain.

Again, this highlights the dangers, which come along with an over-fitted model restricting the possible set of VaR realisations. The following image displays the VaR of individual styles, combined in Market Neutral, Event Driven und Opportunistic strategies, where α varies from 90 % to 99,9 %.

[46] The random variable X is measuring returns not losses.

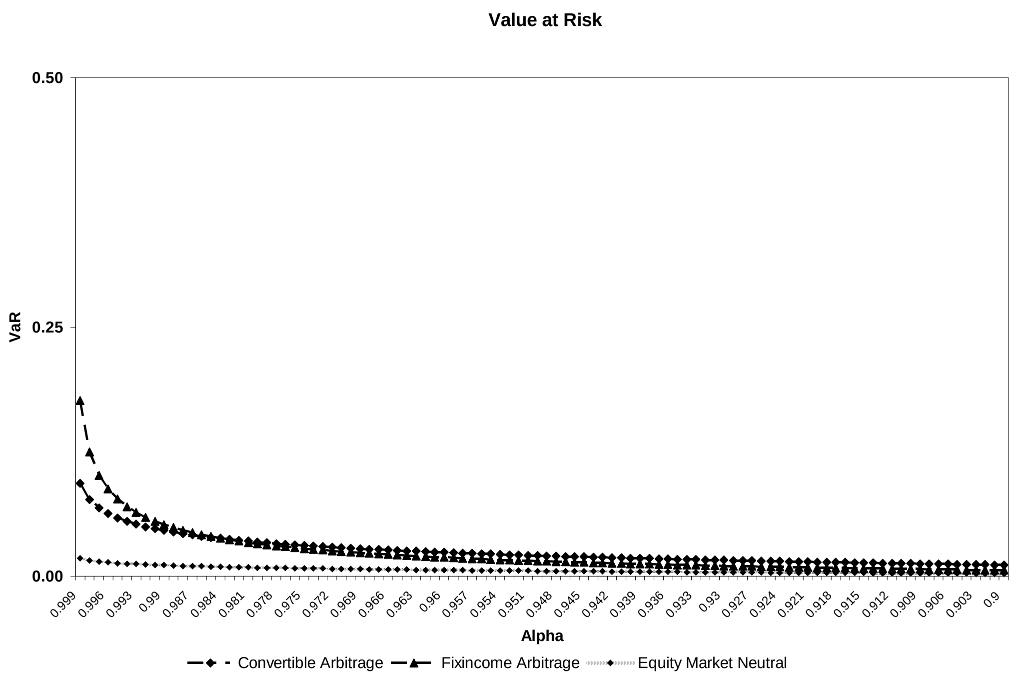

figure 8

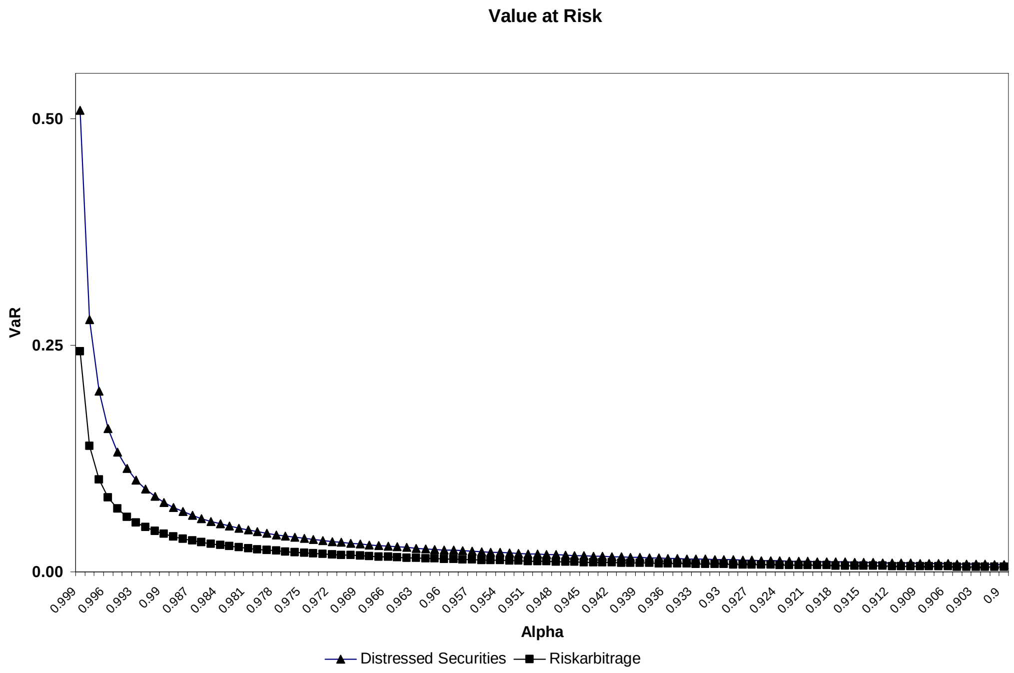

figure 9

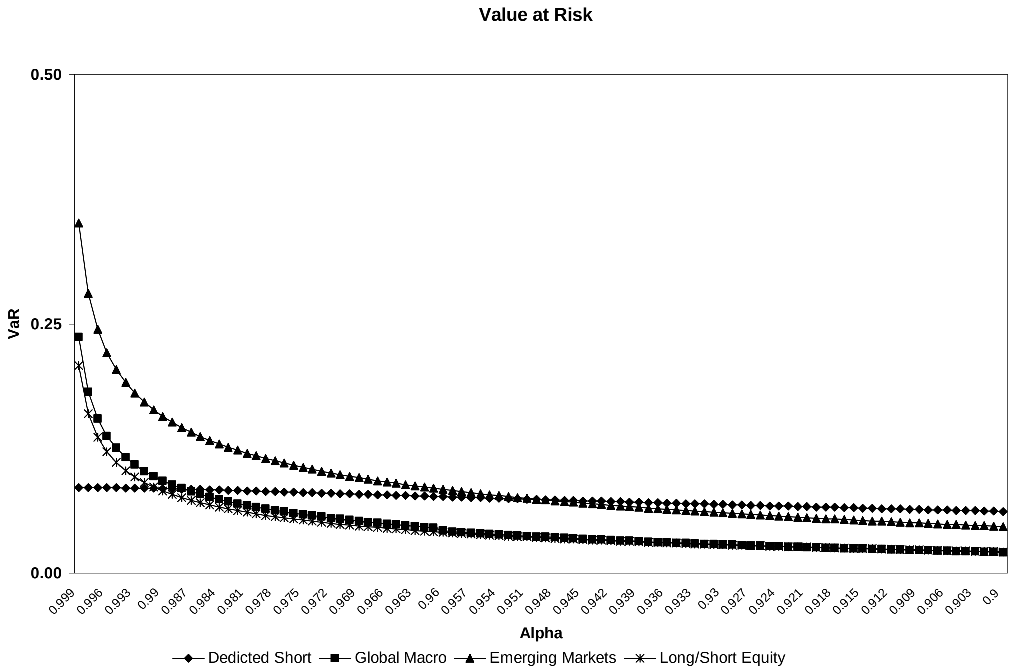

figure 10

In a first graphic analysis the Market Neutral strategies appear to have a relatively low risk in the most extreme situations, whereas the Equity Market Neutral strategy again takes on a special position. The Distressed Securities Index arises as the most dangerous, followed by the remaining indices on a comparable level. For selected levels of α the VaR of CS-strategies and capital market indices shows as follows:

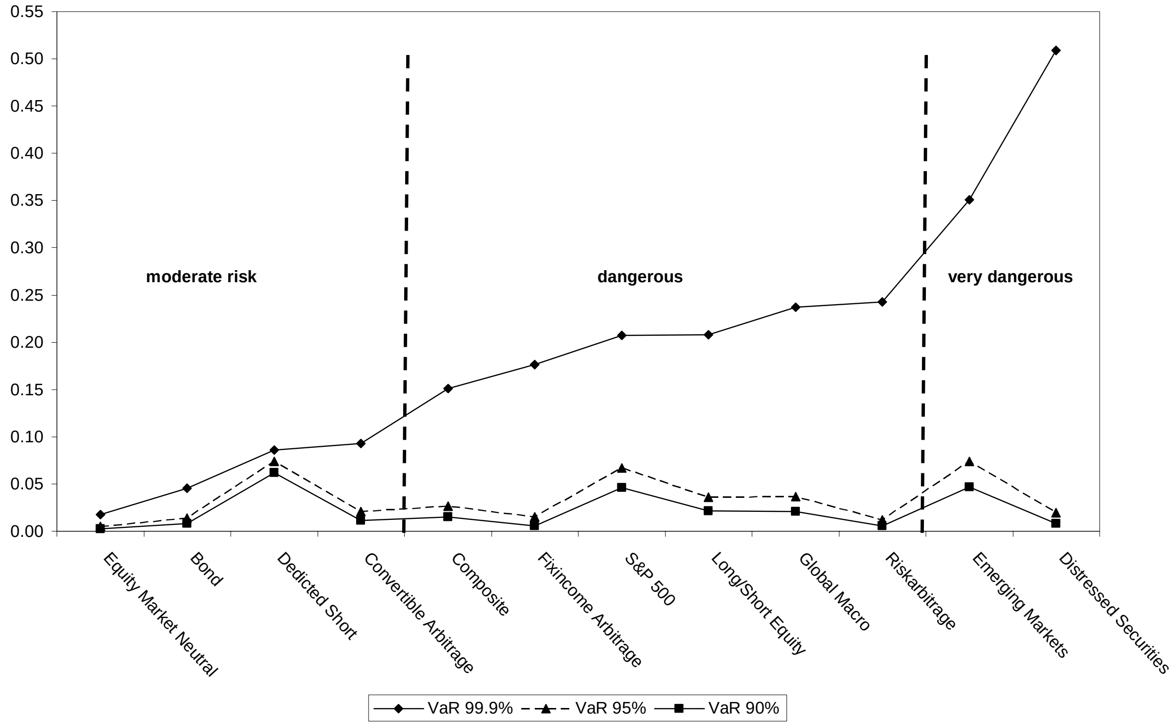

figure 11

With the three scenarios which were examined, basically three groups can be classified by a first subjective analysis: moderate risk, dangerous and very dangerous. Exemplary this was conducted on the scenario based on 99.9% in figure 11. However, it is easy to recognize, that the classification of indices to these groups depends on the selected scenario.

Hence the g-and-h VaR allows a more detailed location free risk analysis as for example the normal VaR, whose classification is invariant with respect to the selected scenario, i.e. completely determined by the scale parameter σ.

Summary

As revealed by the preceding application of exploratory data analytic techniques hedge fund investments require an adequate enhancement of the traditional methods of risk management. Fundamental is the non-normal modelling of empirical return data and especially the part which is considered to be particularly risky – the left tail. The preceding analysis identified the g-and-h distribution, which was established by *Tukey* 1977, as an appropriate model of the return patterns of several hedge fund strategies. Additionally the model offers the possibility to analyse aspects such as asymmetry and heavy tails in detail, to make their structure transparent and also to simplify comparative analysis of individual strategy indices. Beyond, this distribution model can be characterised as easy to implement in the context of a Monte Carlo Simulation because the standard normal distribution serves as the starting point.

A direct implementation of the results leads to a flexible VaR-model, the g-and-h VaR, which shows clear advantages compared to the traditional VaR-models and their currently existing modifications. The results of the new VaR model for changing levels of α from 90 % to 99,9% highlight the usage of this approach in the context of a meaningful risk analysis. The more complex modelling of asymmetry and heavier tails finally leads to the matter of fact, that the ranking of dangerousness measured by VaR depends on the chosen level of α.

Summing up the results of the present paper offer a range of areas for further analysis. At first it seems to be indicated to analyse the results mentioned above with respect to their stability over time. Moreover, the usefulness of a separate modelling of the left and right tail in the context of a risk and performance analysis can be examined. As the preceding results also support the modelling of the stock index and the bond index on the basis of the g-and-h distribution, a joint portfolio analysis is straightforward.

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