Discussion Paper No. 03-36

The Connection of Stock Markets Between Germany and the USA

New Evidence From a Co-integration Study

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Non-technical summary

Although aggregated stock markets show strong evidence for a random walk in the short run, they are known to have predictable components during longer periods. In the descriptive part of this paper a close long-term interdependence between the MSCI USA gross dividend index divided by the US consumer price index for all urban consumers and the DAX for the German stock market divided by the price index for the living standard of all private households in the earlier federal territory is shown. The idea of this paper is that both real total return indices are systematically linked together by a cointegrating relation, i.e., by a common stochastic trend. In fact, this hypothesis can be statistically confirmed. However, it is discovered that the definition of returns in highly volatile markets is essential: Cumulated discretely compounded real stock market returns (simple returns) are co-integrated. Thereby, co-integration even supplies an explanation for their well-known skewness to the left. Cumulated continuously compounded returns (log returns) in opposite – which are frequently used in time-series analysis – are not co-integrated.

Central theme of this paper are the advantages of extending the random walk for forecasting purposes by a cointegrating relation. Thus, multi-period forecast performance of the co-integration model and a corresponding random walk model are compared. In the former approach, dynamic effects of unique shocks to the German respectively the US stock market occur in difference to the classical random walk hypothesis. These are illustrated by a brief impulse responses study. Investigations of the estimated cointegrating relation indicate its robustness over time, although its influence for forecasting future real stock market returns seems to lose strength. Nevertheless, the dynamic model is connected with new insights especially into predictability of real returns of the German stock market in the future. Due to the stabilizing cointegrating relation forecasts of German stock market returns are more accurate than reflected by the

random walk approach. This does not only hold in the short term, but particularly with increasing forecasting horizon in the strategic analysis. In difference, forecasting uncertainty for real US stock market returns in the dynamic model rises super-proportionally with forecasting horizon. The US-American real stock market index reacts – in opposite to the German – sensitively and durably to exogenous shocks. Actually, this can be explained by a leading role of the US-market, whereas the German stock market is strongly orientated on global stock market behaviour in the long run.

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Abstract

This paper uses an empirical connection between real stock market indices of Germany and the USA for forecasting corresponding returns. We are starting from the random walk as the traditional forecasting model in stock market applications, extending it by co-integration. Since the co-integrating relation considers information about a systematic link between the stock market indices, containing a common stochastic trend of both, differences from the random walk occur particularly in the long run. Thus, the estimation period shows that with increasing forecasting horizon predictability of simple real returns of the German stock market gets more accurate than reflected traditionally.

Keywords: Co-integration of international stock markets; random walk; discretely and continuously compounded returns; impulse responses; inequality coefficient of Theil

JEL-Classification: C 52/53; F 36; G 12

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I. Introduction

The systematic predictability of financial asset returns represents one of the first and most persistent questions of financial econometrics [see Campbell, Lo and MacKinlay (1997), p. 27]. The stock market premium is a central, although very elusive quantity [see Minister and Stambaugh (2001), p. 1207]. For a long time the Capital Asset Pricing Model (CAPM) was considered as being indicative of a risk-conform evaluation of shares. Nevertheless, with growing empirical evidence it was replaced increasingly by multi-factor models, which explain returns on shares by risk premiums of different investment factors [see the large literature of capital market anomalies and the influential essays of Fama and French (1993, 1996) and Davis, Fama and French (2000)]. Likewise, the postulate of independent identically distributed stock market returns is no longer taken as being self-evident. Even aggregated stock markets, which show strong evidence following a random walk in the short run, have predictable components during longer periods [see Cochrane (1999a), p. 36; Barberis (2000), p. 225]. This degree of long-term predictability of stock markets is attributed to return expectations varying systematically over time [see Campbell and Lo and MacKinlay (1997), p. 80]. However, the approach of conditional volatility based on Merton (1980) did not lead to the desired clear results. Research in the last decade has been extended to the question of co-integration of national stock markets. Evidence of numerous applications is mixed [see, e.g., Richards (1995), p. 638]. Initially, Kasa (1992) presents results using monthly data from 1974 to 1990 which suggest the presence of a co-integrating relation between the most important national stock markets. In contrast, Richards (1995) uses logarithms of stock market indices of different countries and finds little empirical evidence for co-integration. All studies which focus on the most important stock market indices so far have in common that they do not pay attention to the precise definition of stock market returns and the implications of co-integration results for the classical random walk hypothesis.

If prices do not follow a first-differences-stationary process like a random walk, the investment horizon becomes substantial for portfolio decisions. The usually high risk premium on stock markets is partly connected to compensate for the risk of negative development in times of adverse financial market conditions [see Cochrane (1999b), p. 59]. Negative intertemporaneous interdependence causes diversification effects over time, so in respect to a corresponding random walk investment in stock markets is comparatively safer in the long run than in the short. The idea of this paper is that these effects depend on stabilizing interdependence of international stock markets. Concretely, the empirical connection between real stock market indices of Germany and the USA is analyzed for new insights into predictability of German stock market returns.

The remainder of this paper is organized as follows: In section I, indicators of German and US-stock market returns are analyzed as well as the interconnection of the price indices. Subsequently, in section II the traditional random walk model is extended by stable structures between the real stock market indices; it is specified empirically, estimated and then evaluated. Special attention is given to the advantages of the co-integration approach for forecasting purpose in relation to a pure random walk. In final section III the main results of the co-integration study are summarized and a view on a possible extension of the analysis is given.

II. Descriptive Analysis of the Connection of Stock Markets

For compressing information about stock market trends in one single indicator, frequently stock market indices on total return basis are used. These underlie the fiction of immediate reinvestment of dividend and bonus payments. Gross dividends are included without accounting for later tax rebates, so that total return indices describe average taxed returns of the stock market. Being constructed consistently with each other, these indices are suitable for comparison of the development of different national markets on average, particularly because international influences affect single titles less than indices.

In respect of share capital and number of stock companies, the German stock market internationally still stands on narrow grounds. However, with its electronic commercial platform, Xetra, the German stock exchange has developed to the second largest all-electronic stock exchange in the world. In this paper the US-stock market is used to describe interdependence of the German and the global stock market, because the USA possess the largest stock market all over the world with a high reference function for individual national stock markets. The capitalisation-weighted national total return indices of Morgan Stanley Capital International (MSCI) are suited to comparison purposes, because they are constructed uniformly (with ultimo 1969 = 100). Since the German MSCI total return index hardly differs structurally from the Deutscher Aktienindex (DAX), the latter one as the usual indicator for the German stock market is used nevertheless [see MSCI (1998); Richard (1992); Janssen and Rudolph (1992)]. Standardized to an uniform initial value at a fixed time both runs are very similar – with the German MSCI index slightly above DAX. At the same time with the DAX traced back by Stehle, Huber and Maier (1996) comparatively reliable, carefully examined data are available. Therefore, further stock market investigations are based on the DAX for Germany and the MSCI gross dividend index for the USA (MSCI-USA).

DAX and MSCI-USA are based on different national currencies. Therefore, the MSCI-USA has to be converted additionally into Deutsche Marks (DM). Since the official collapse of the system of Bretton Woods in March 1973, a system of flexible rates of exchange has existed between the Federal Republic of Germany and the USA. Accordingly, for further analyses the period from March 1973 to August 2002 is used, remembering that from January 1999 the DM/EURO-relationship has been fixed at 1.95583. Due to the recent German reunification, there is no long-term data history for all Germany. We implicitly assume that all-German financial markets in the future will develop to a large extent in compliance with the history of the Federal Republic. Therefore, the MSCI-USA in US dollars (USD) is converted into an appropriate DM index in each point of time as follows [see also MSCI (1998), p. 47]:

$$\text{MSCI-USA [DM]} = \text{MSCI-USA [USD]} * (100 / 127.862) * (\text{Exchange Rate [DM/USD]} / 2.8132) \quad (1)$$

The multiplier in the second line relates the rate of exchange of each month to the rate of exchange from March 1973. The bracketed term in the first line rebases the MSCI-USA [USD] computationally on the initial value 100 instead of 127.862 at the end of March 1973. Accordingly, the DAX has to be multiplied by (100 / 371.65). The MSCI-USA before and after this currency conversion from USD in DM and the DAX are illustrated in figure 1, all starting in March 1973 with an initial value of 100.

At least until 1999 and thus until the fixing of rates of exchange among the members of the European monetary union (EWU), the development of the MSCI-USA [DM] resembles to a large extent to that of the DAX. However, already since the middle of the nineties, the DAX has been almost constantly below the MSCI-USA even after currency conversion. Finally, at least since 1999, uniform development of the German and the US-American stock market

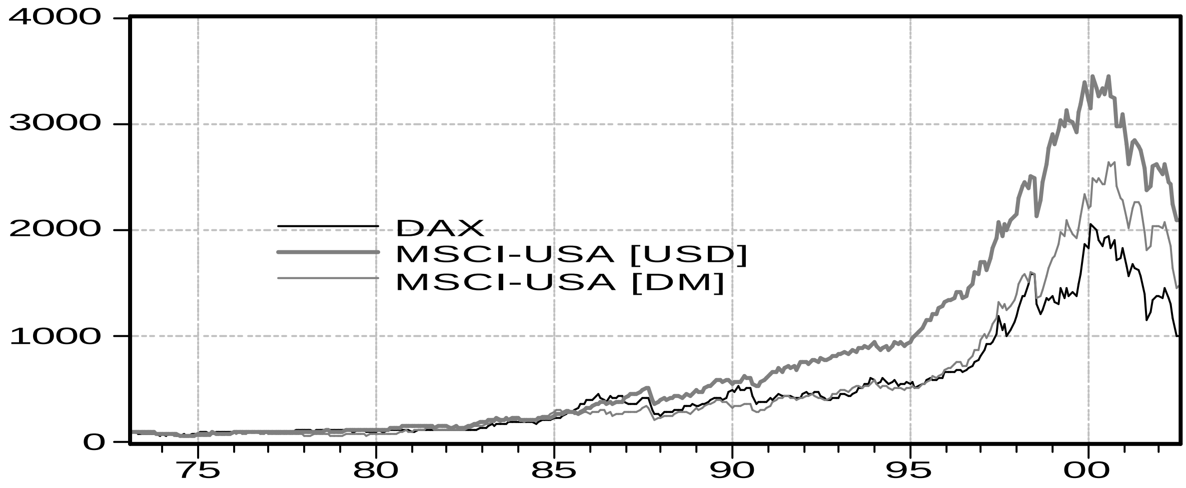

Fig. 1: Stock market indices for Germany and the USA (March 1973 = 100)

index seems to have loosened (possibly caused by additional influences from other EWU countries). The period from 1999 to 2002 – with its enormous and continuous stock market crash – should not be excluded from analyses, since additional variation in the data can improve the discrimination of statistical tests. Due to the Euro-problem, instead of the currency conversion of the MSCI-USA according to (1) real national stock market indices are used below. Focusing on real returns in respect of long-term decisions investors underlie no illusion of money [see also Albrecht, Maurer and Ruckpaul (2001) who investigate the long-term performance of the real stock market return]. Besides, according to the theory of relative purchasing power parity, inflation differences in two countries during a certain period should exactly represent the change of the flexible rate of exchange.

For measuring real effects, total return indices are divided by the respective national indices of general price level. The usual indicator of general price level for the Federal Republic of Germany is the price index for the living standard of all private households. For the earlier federal territory it is available on monthly basis starting from January 1962. For the new countries including East Berlin a respective index exists from 1990; for all Germany from 1991. The increase of the price level in the eastern part was super-proportionally high immediately after the German reunification because of the development of free market prices and because of dammed up demands [see Statistisches Bundesamt (1997), pp. 333-334]. In the meantime, inflation rates calculated from the west and from the all-German consumer price indices have adapted to each other. Therefore, the development of the price index in the earlier federal territory is used (CPI-D-West). Direct US-American equivalent to the German price index for the living standard of all private households is the seasonally unadjusted consumer price index for all urban consumers (CPI-USA). Compared with the rates of exchange, the quotient of both national consumer price indices is clearly less volatile. Smoothing effects refer to different inquiry methods. While rates of exchange are measured as daily averages at the end of month, consumer prices are derived from a representative monthly survey [see Statistisches Bundesamt (1997), p. 331]. In figure 2, both national real stock market indices are illustrated – i.e., DAX divided by CPI-D-West and MSCI-USA divided by CPI-USA. For better comparability, again, both rows are standardized in such a way that they begin in March 1973 with an initial value of 100.

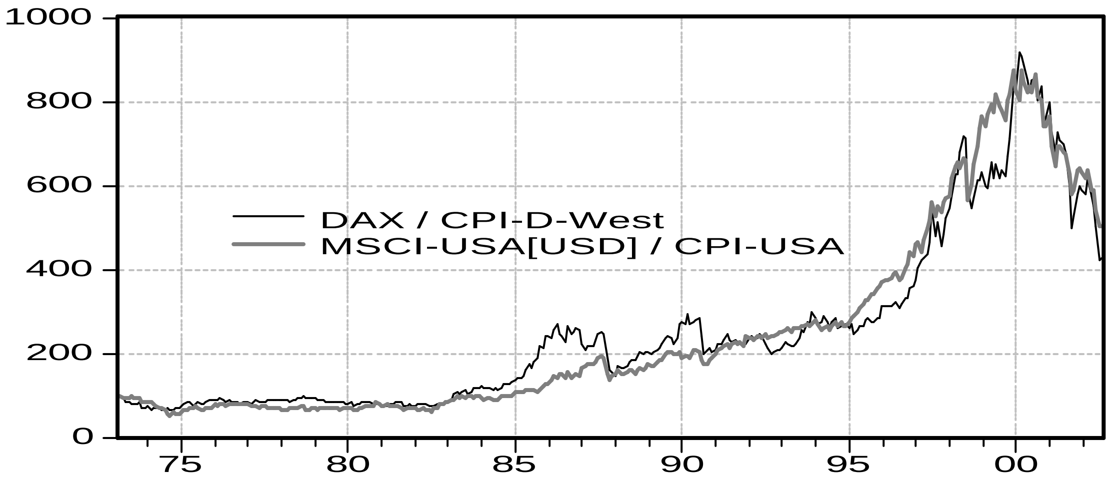

Fig. 2: Real stock market indices for Germany and the USA

Similar development of the national stock market indices becomes clearer after adjusting for average national price levels. In particular, there has been no obvious structural break since the end of the nineties and the fixing of exchange rates within the EWU although Germany is no longer protected against inflation influences from other EWU-countries. Thus, the connection of the stock market indices between Germany and the USA reflects the intuitive concept of integrated stock markets. The following section will clarify how this can be evaluated and used within a co-integration analysis.

III. Co-integration Analysis of Stock Markets

A. Empirical Model Specification

Model specification starts with the issue of whether there exists a co-integration relation between the DAX and MSCI-USA [USD] after adjusting by the respective national consumer price index. However, instead of real indices, in figure 3 the developments of their natural logarithms are considered first.

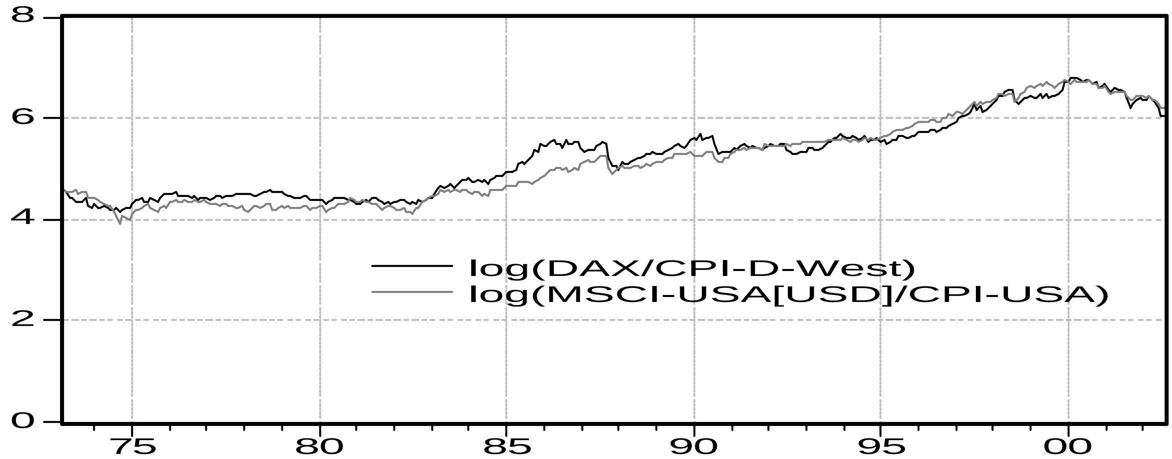

Fig. 3: Logarithms of real stock market indices for Germany and the USA

The time series processes look more linear after taking logarithms. The synchronisation of the two indices is strengthened. Indeed, for financial time series analyses continuously compounded returns (log returns) are usually consulted [for discrete and continuously compounded returns see Dorfleitner (2002) and Eberts (2002), pp. 11-15, for example]. These are computed as absolute changes of the index logarithms, thereby representing continuously paid interest during the period under consideration. With continuously compounded returns, the random walk model of the German and the US-American stock market can be specified as follows (t = 1,2,…,T):

$$\log Y_t = \gamma + I \bullet \log Y_{t-1} + U_t \qquad (2)$$

Y_t is a two-dimensional vector with the value of the real DAX at time t in first place and the real MSCI-USA as second component. Thus, $\log Y_t$ describes the vector of corresponding logarithms at time t. In addition, γ represents a two-dimensional coefficient vector and U_t a two-dimensional vector of stochastic error terms. The multiplication with unity matrix I of order 2 in (2) serves as illustration, but it is however redundant. The random vectors U_t are independent of $\log Y_{t-i}$ (i = 1,2....,t). Usually, it is also assumed that all U_t are independent identically distributed over time with expectation vector zero and covariance matrix Σ. For continuously compounded stock market returns for one period it follows directly from (2)

$$\nabla \log Y_t := \log Y_t - \log Y_{t-1} = \gamma + U_t \qquad (3)$$

Thus, ∇ represents the one-step backwards difference operator. Based on the assumptions for the error terms, the continuously compounded returns in the random walk model are independent identically distributed with expectation vector γ and covariance matrix Σ. Thus, they follow a static stationary development over the time [see the so-called submartingal characterisation of Fama (1970), p. 386]. The only linkage of the endogenous variables in $\nabla \log Y_t$ are contemporaneous correlations, which however are little helpful for forecasting individual markets. In order to model additionally the connection of the underlying indices, the following extension offers:

$$\log Y_t = \gamma + (I + \Pi) \log Y_{t-1} + U_t \quad (4)$$

So for continuously compounded stock market returns for one period

$$\nabla \log Y_t = \gamma + \Pi \log Y_{t-1} + U_t \quad (5)$$

holds. The additional (2x2)-dimensional matrix Π is called co-integration matrix. If $\log Y_t$ is non-stationary (i.e., afflicted with trend) Π does not possess full rank r, $0 \le r < 2$. If both components of $\log Y_t$ as well as each linear combination of them are not stationary – such as in the classical case of random walk (2) – then $\Pi = 0$ and thus r = 0. However, if r = 1 applies, then at most one component of $\log Y_t$ is stationary. If under r = 1 no component is stationary, there must be linear combinations $\beta' \log Y_t$ of both non-stationary components, which are stationary. In that case (4) respectively (5) is called co-integrated in the sense of Engle and Granger (1987). Therefore, non-stationary co-integrated variables have a common stochastic trend, so that certain linear combinations follow a stable regularity which corresponds with the theoretical concept of a long-term statistic equilibrium [see Enders (1995)]. In the two-dimensional case $\Pi := \alpha\beta'$ can be multiplicatively separated into the two-dimensional coefficient vectors α and β, both with full column rank. For identification of β usually one of its components is normalized to unity. From this point, the coefficients of α can be interpreted as measures for the adjustment of the i-th model variable (i = 1, 2) to the cointegrating relation $\beta' \log Y_t$.

Instead of referring the random walk characteristic to $\log Y_t$, it is frequently also formulated in respect to Y_t. Therefore, (4) and (5) are replaced by

$$Y_t = \tilde{\gamma} + (I + \tilde{\Pi}) Y_{t-1} + \tilde{U}_t \quad (6)$$

respectively

$$\nabla Y_t = \tilde{\gamma} + \tilde{\Pi}\; Y_{t-1} + \tilde{U}_t \tag{7}$$

In order to mark differences to coefficients and error terms in (4) and (5) tilde quotations are used here. In their general characteristics $\tilde{\gamma}$, $\tilde{\Pi}$, and $\tilde{U}_t = (\tilde{U}_{t,1}; \tilde{U}_{t,2})'$ correspond perfectly to γ, Π, and U_t. In table I results of the λ_{trace} tests of the null hypothesis H_0: $r = r_0$ versus H_1: $r > r_0$ as well as the results of the λ_{max} tests of H_0: $r = r_0$ versus H_1: $r = r_0 + 1$ are summarized ($r_0 = 0, 1$) – both in respect of $\log Y_t$ and Y_t. The test statistics and their asymptotic distributions depend – similar to univariate unit root tests – on the specification of the constants γ respectively $\tilde{\gamma}$. So (5) and (7) are separated as follows with $\gamma := \gamma_0 + \alpha\beta_0$ and $\Pi := \alpha\beta'$ respectively with $\tilde{\gamma} := \tilde{\gamma}_0 + \tilde{\alpha}\,\tilde{\beta}_0$ and $\tilde{\Pi} := \tilde{\alpha}\,\tilde{\beta}'$:

$$\nabla \log Y_t = \gamma_0 + \alpha\;(\beta_0 + \beta' \log Y_{t-1}) + U_t \tag{8}$$

and

$$\nabla Y_t = \tilde{\gamma}_0 + \tilde{\alpha}(\tilde{\beta}_0 + \tilde{\beta}' Y_{t-1}) + \tilde{U}_t \tag{9}$$

The investigation depends on 353 German and US-American observations of real stock market indices in the period from March 1973 to August 2002, although theoretically annual returns are ideal for the analysis of long-term behaviour of stock markets [fading out seasonal effects which are caused by high dividend payments between May and July in Germany for example, see Richard (1992), p. 180]. However with a maximum of 29 observations per time series on annual basis one cannot trust in sufficiently discriminating statistical tests – in particular for the context of the unit root problem [also note that overlapping annual returns as weighed averages of the 29 non-overlapping observations in fact do not widen the database]. To that extent, the observation of stock markets at the end of the month is a compromise.

Tab. I: Johansen Rank Tests

The table shows values of the test statistics under three alternative constant specifications. In parentheses, the simulated lower critical borders of Osterwald-Lenum (1992) at 5% significance level are reported. Values of test statistic, for which the null hypothesis has to be rejected, are marked by *. However, for $\gamma = \gamma_0 + \alpha\beta_0$ the test of H_0: r = 1 against H_1: r = 2 is unreasonable [see Lütkepohl (2001), p. 688].

Rank of P in respect to $\log Y_t$						
constant term	$\gamma = 0$ ($\gamma_0 = \beta_0 = 0$)		$\gamma = \alpha\beta_0$ ($\gamma_0 = 0$)		$\gamma = \gamma_0 + \alpha\beta_0$	
λ_{trace} tests (H_0: $r = r_0$ versus H_1: $r > r_0$)						
$r_0 = 0$	9.45	(12.53)	13.02	(19.96)	9.39	(15.41)
$r_0 = 1$	3.18	(3.84)	3.55	(9.24)	0.001	(3.76)
λ_{max} tests (H_0: $r = r_0$ versus H_1: $r = r_0+1$)						
$r_0 = 0$	6.27	(11.44)	9.47	(15.67)	9.39	(14.07)
$r_0 = 1$	3.18	(3.84)	3.55	(9.24)	0.001	(3.76)
Rank of $\tilde{P}$ in respect to Y_t						
constant term	$\tilde{\gamma} = 0$ ($\tilde{\gamma}_0 = \tilde{\beta}_0 = 0$)		$\tilde{\gamma} = \tilde{\alpha}\tilde{\beta}_0$ ($\tilde{\gamma}_0 = 0$)		$\tilde{\gamma} = \tilde{\gamma}_0 + \tilde{\alpha}\tilde{\beta}_0$	
λ_{trace} tests (H_0: $r = r_0$ versus H_1: $r > r_0$)						
$r_0 = 0$	17.50 *	(12.53)	25.26 *	(19.96)	23.50 *	(15.41)
$r_0 = 1$	0.29	(3.84)	1.38	(9.24)	0.23	(3.76)
λ_{max} tests (H_0: $r = r_0$ versus H_1: $r = r_0+1$)						
$r_0 = 0$	17.21 *	(11.44)	23.88 *	(15.67)	23.27 *	(14.07)
$r_0 = 1$	0.29	(3.84)	1.38	(9.24)	0.23	(3.76)

The results are independent of the selected specification of the constant term and independent of the formulation of the alternative hypothesis H_1. There seems to be no evidence for a cointegrating relation between the logarithms of the two real stock market indices. Interestingly, a differing result occurs if Y_t instead of $\log Y_t$ is considered. In the same sample period a cointegrating rank of r = 1 between the components of Y_t can be proven at the level of significance of 5% and r = 2, which means that the chosen indices are not afflicted with trend, can be excluded independently of the selected specification of the constant term.

The absolute index differences ∇Y_t represent discretely compounded returns (so-called simple returns) of stock markets only after the transformation $\nabla Y_t / Y_{t-1}$. Due to the statistic elegance of continuously compounded returns in time series applications, it is a disappointment that only for the indices without logarithms a significant linear cointegrating relation can be found. However, this result may not be interpreted as an indication that the influence of the cointegrating relation is weak and has no considerable effect on long-term predictability of stock markets. Particularly in the case of stock markets with occasionally unusual high as well as exceptional low returns, continuously compounded returns are often substantially smaller than corresponding discrete ones. Due to enormous volatility, differences of both return definitions are thereby no longer negligible. Accordingly, evidences for $\Pi = 0$ and $\tilde{\Pi} \neq 0$ are quite compatible with each other. So in the remainder of this paper we focus on the effect of (7) for forecasting discretely compounded real stock market returns of Germany and the USA in comparison to the traditional random walk model.

B. Model Estimation

We take the unrestricted approach in respect of the constant term $\tilde{\gamma}$,

$$\nabla Y_t = \tilde{\gamma}_0 + \tilde{\alpha}\,(\tilde{\beta}_0 + \tilde{\beta}' Y_{t-1}) + \tilde{U}_t \tag{10}$$

For estimation it is conditioned on a cointegrating rank of r = 1. In order to identify $\tilde{\alpha}$ and $\tilde{\beta}$, the first component of the cointegrating vector $\tilde{\beta}$ is restricted to 1. For the absolute differences of the normalized real stock market indices for Germany ($\nabla Y_{t,1}$) and the USA ($\nabla Y_{t,2}$), the system estimation procedure following the work of Johansen (1988) and Johansen and Juselius (1990) leads to the following coefficient estimators of (10) [see also Johansen (1995)]:

$$\nabla Y_{t,1} = 0.9191 - 0.1374\,(Y_{t-1,1} - 0.8941\ Y_{t-1,2} - 29.6619) + \tilde{U}_{t,1}$$
$$\nabla Y_{t,2} = 1.1402 - 0.0675\,(Y_{t-1,1} - 0.8941\ Y_{t-1,2} - 29.6619) + \tilde{U}_{t,2} \qquad (11)$$

The asymptotic distribution of the vector $\tilde{\gamma}_0$ is non-standard. However, H_0: $\tilde{\gamma}_0 = 0$ – which excludes a linear deterministic trend in the stock market indices – can be examined by a likelihood ratio type test. Here the restriction $\tilde{\gamma}_0 = 0$ cannot be rejected at the 5% level of significance, which is quite intuitive for the time series processes illustrated in figure 2. The bracketed term in both equations of (11) represents the estimated cointegrating relation. With same signs of estimated coefficients in both equations both indices develop similar. Although the estimation $\hat{\alpha}$ of $\tilde{\alpha}$ depends on the normalization of $\tilde{\beta}$, it is possible to test whether the components of the loading vector $\tilde{\alpha}$ are zero based on the asymptotic normal distribution of $\sqrt{353}(\hat{\alpha} - \tilde{\alpha})$. With coefficients of –0.1374 in relation to a standard deviation of 0.0281 and –0.0675 in relation to 0.0208 the cointegrating relation is significant at 5% for describing both the increases of the German and the US-American real stock market index. The cointegrating coefficients follow a mixed normal distribution asymptotically. Again it can be examined by a likelihood ratio type test whether the estimated value of –0.8941 differs significantly from -1. Thereby, the test statistic and its asymptotic χ_1^2-distribution is conditioned on $\tilde{\gamma} = \tilde{\gamma}_0 + \tilde{\alpha}\,\tilde{\beta}_0$ and r = 1. With a test statistic of 6.05, the assumption β = (1; -1) does not hold.

The estimated cointegrating relation in (11) allows for statements about the intertemporaneous connection of the two real stock market indices: If deviation from the equilibrium relationship is very high in the preliminary period – as with the real German stock market index being high compared to the US-American – the adjustment process causes a super-proportionally low change of the German index in the subsequent period. Turned around, if the real German stock market index is comparatively low, the absolute difference of the index will be tendentiously high in the subsequent period. This result is also reflected in the orthogonalized impulse responses presented in figure 4. These illustrate how unique (positive) shocks in $\tilde{U}_{t,1}$ respectively $\tilde{U}_{t,2}$ – at an amount of one standard deviation – affect $\nabla Y_{t,1}$ and $\nabla Y_{t,2}$ in the 60 subsequent periods. In contrast to the static random walk, unique shocks in (10) have dynamic effects on stock markets based upon the cointegrating relation. In the two-dimensional model without autoregressive components, the only lagged effect is based upon the single cointegrating relation.

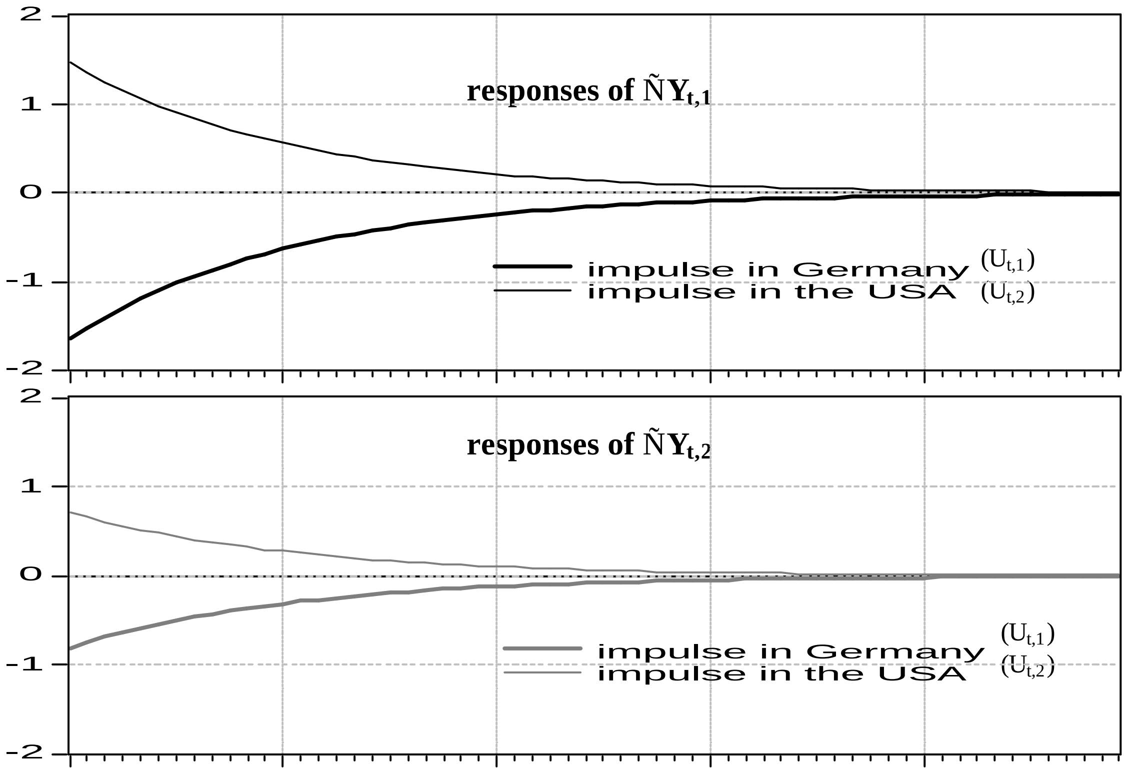

Fig. 4: Impulse responses for ÑY_t over 60 periods

Since the estimated co-integration coefficient –0.8941 is similar to 1 in absolute term, impulse responses to $\tilde{U}_{t,1}$ respectively $\tilde{U}_{t,2}$ graphically are nearly symmetrically around the zero line. By the German and the US-American real stock market index, not departing arbitrarily far from each other in the long term, a positive impulse on the German stock market leads to fallings of real indices in the subsequent periods, thus the contemporaneous effect is waived over time. In contrast, a positive impulse on the US stock market causes comparably high, but positive real effects. After an US-shock both the German and the US-American real stock market index are raised durably on a higher level, whereby the total effect is substantially stronger than the contemporaneous.

C. Discretely Compounded Real Stock Market Returns

The influence of the estimated stock market approach (11) to the descriptive statistics of discretely compounded real returns is interesting to evaluate. For this purposes, the 353 observations of $\nabla Y_t / Y_{t-1}$ on the one hand and their stochastic error terms $\tilde{U}_t / Y_{t-1}$ on the other are compared in sample. Table II shows the corresponding descriptive statistics.

Tab. II:
Descriptive Statistics of Monthly Discretely Compounded Real Stock Market Returns and their Stochastic Components for Germany (1) and the USA (2)

	$\nabla Y_{t,1} / Y_{t-1,1}$	$\nabla Y_{t,2} / Y_{t-1,2}$	$\tilde{U}_{t,1} / Y_{t-1,1}$	$\tilde{U}_{t,2} / Y_{t-1,2}$
mean	0.0057	0.0057	-0.0058	-0.0042
standard deviation	0.0568	0.0462	0.0605	0.0503
skewness	-0.45	-0.30	0.01	-0.14
curtosis	4.59	4.63	3.99	3.86

Because of high monthly volatility all mean values are not significantly different from zero. For the stochastic terms, the standard deviations are even higher than for the corresponding real returns. This can be partly attributed to smaller skewness to the left and less over-curtosis of the error distributions compared with real return distributions [whereas $\tilde{U}_{t,i}$ in contrast has fewer standard deviation, skewness and lepto-curtosis than $\nabla Y_{t,i}$ (i = 1, 2)]. The combination of lepto-curtosis and skewness to the left in fact is a central characteristic of the risk of stock returns. While discretely compounded real returns are significantly skewed asymptotically at the 5% level, with absolute values smaller $1.96\sqrt{6/353} > 0.25$, the opposite is true for the components of the vector $\tilde{U}_t/Y_{t-1}$. Therefore, co-integration of the indices can be seen as an explanation for real return distributions being skewed to the left. Although with 0.99 respectively 0.86 higher than $2 * 1.96\sqrt{6/353}$, curtosis still deviates significantly from the value 3 of Gaussian variables, the error distributions are substantially less curved than the corresponding distributions of real returns. Figure 5 illustrates the development of $\tilde{U}_{t,1}/Y_{t-1,1}$ and $\tilde{U}_{t,2}/Y_{t-1,2}$.

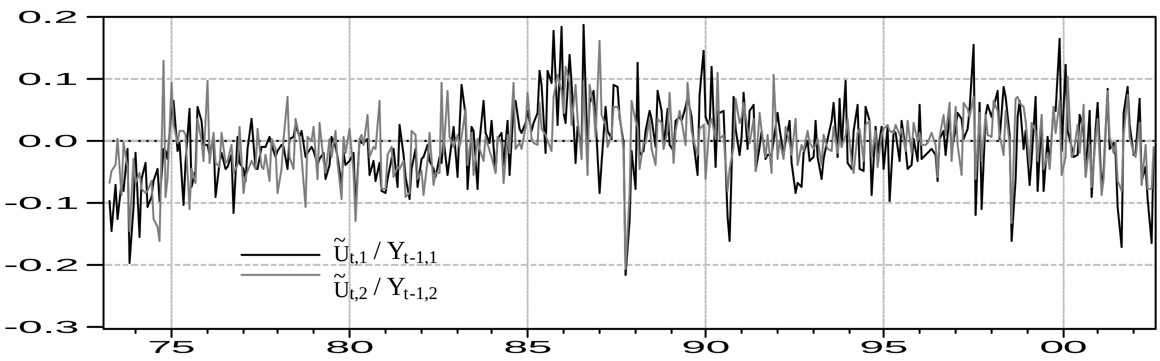

Fig. 5: Development of stochastic components of the discrete real returns

D. Forecasting Characteristics

Explorative time series approaches describe intertemporaneous correlation. Therefore, they are especially suited for forecasting purpose in financial market applications. In that sense, also (11) represents nothing but a description of historical interdependencies of German and US American real stock market returns. Thus, without an explanation model standing behind, it remains unclear when forecasts are more or less good. However, due to the systematic use of the historical cointegrating relation under constant economic conditions, the dynamic model should at least work better than the usual random walk. Therefore, conditioned forecasts of real returns in-sample are looked at first. The forecast mean squared error (FMSE) serves as criterion of forecast performance. Thereby, consideration of the standard deviation of coefficient estimators acquires higher forecast accuracy with respect to the estimation period [see appendix A]. To that extent it seems favourable to neglect deviations between estimated values and true but unknown coefficients within the estimation period. Thus, it also becomes possible to determine analytically conditioned multi-step ahead FMSE in the dynamic time series model. If standard deviations of coefficient estimators remain unconsidered, then under standard assumptions the conditioned one step ahead FMSE is equal to the estimated residual variance of the respective process. Multi-step ahead random walk forecasting then further implies linear growth of the h-step-FMSE in respect of $\nabla_h Y_{t,i} = Y_{t,i} - Y_{t-h,i}$ (i = 1,2) with forecast horizon h. Neglecting the intrinsic inaccuracy of coefficient estimation, for the conditioned h-step-FMSE of the co-integration model (10) the term $\sum_{j=0}^{h-1} \psi_{(i\bullet)}^{(j)} \Sigma_U \left(\psi_{(i\bullet)}^{(j)}\right)'$ results. $\psi_{(i\bullet)}^{(j)}$ marks line i of the j-th power of the matrix $\Psi := (I + \tilde{\Pi})$. $\tilde{U}_t$ from (10) are independent identically distributed over time with expectation vector zero and covariance matrix Σ_U, while the covariance matrix of the random walk model is represented by Σ with elements $\Sigma_{(ii)}$ on the main diagonal.

In context of the inequality coefficient of Theil as relative criterion of comparison, the conditioned h-step-FMSE of a dynamic model is set in relation to corresponding one of a static random walk forecast. In the following, the inequality coefficient of Theil, briefly U, is specified as the square root of the conditioned h-step-FMSE of the time series model in relation to that of the random walk:

$$U = \sqrt{\frac{\sum_{j=0}^{h-1} \psi_{(i\bullet)}^{(j)} \Sigma_U \left(\psi_{(i\bullet)}^{(j)}\right)'}{h\, \Sigma_{(ii)}}} \tag{12}$$

If instead of $\nabla_h Y_{t,i} = Y_{t,i}\text{-}Y_{t\text{-}h,i}$ discretely compounded returns are focused on. Their h-step-FMSE conditioned on the information at time t-h result by division by the value of Y_{t-h}^2. However, if for discrete returns only the inequality coefficient of Theil is of interest, then the divisor in counter and denominator shortens itself, so that (12) already describes the relative forecasting performance concerning discretely compounded returns.

In table III the development of the inequality coefficient of Theil over different forecasting horizons between one and 60 months is summarized for the in-sample period.

Tab. III: The inequality coefficient of Theil

The conditioned h-step-FMSE of the co-integration model in the estimate period from March 1973 to August 2002 is documented in relation to the corresponding conditioned h-step-FMSE of a static random walk model.

	$\nabla Y_{t,1} / Y_{t\text{-}1,1}$	$\nabla Y_{t,2} / Y_{t\text{-}1,2}$
h = 1	0.9676	0.9854
h = 3	0.9012	0.9883
h = 6	0.8271	1.0001
h = 12	0.7461	1.0357
h = 18	0.7192	1.0733
h = 24	0.7174	1.1065
h = 30	0.7256	1.1340
h = 36	0.7367	1.1564
h = 48	0.7581	1.1893
h = 60	0.7746	1.2113

As a cumulating size, the conditioned FMSE rises with increasing forecasting horizon h. The increasing uncertainty of long-term forecasts only depends on the stochastic error terms. Because of higher parameterisation in the case of monthly in-sample forecasts, values of conditioned one-step-FMSE in the time series model are systematically smaller than in the random walk. Thus U is smaller than unity for h = 1. Since real indices do not follow any stationary process, however, neither the cointegrating approach nor the random walk necessarily has smaller conditioned h-step-FMSE for h > 1. This depends on a false specification of intertemporaneous covariance structures in the random walk approach if in fact the co-integration model applies. Here for discretely compounded real returns of the German stock market in-sample, U decreases on values smaller than 0.72 with increasing h. Only for horizons of approximately two years does it rise a bit again. However, it must be noted that with 353 monthly observations the conditioned FMSE for h = 24 is calculated based upon only 14 lap-free two-annual returns. Accordingly, with increasing h the reliability of estimation continues to sink. Nevertheless, it can be observed that the stabilizing cointegrating relation causes more accurate forecastability of the discretely compounded real return of the German stock market than suggested by the static forecasting approach. Due to international stock market connections this does not only hold in the short term, but also in the strategic analysis [see also Granger (1986), p. 219].

For discretely compounded real returns of the US-American stock market, different conclusions appear. The values of the inequality coefficient of Theil grow with increasing length of forecast horizon on values exceeding unity for h ≥ 6. Therein, forecasting uncertainty rising super-proportionally with horizon is expressed, which is ignored in the static case. Thus, the random walk model overestimates short term risk of return slightly, but underestimates the long-term risk of return. Increasing forecast uncertainty corresponds with the evidence from the impulse response analysis: The US-American real stock market index reacts – differently than the German – sensitively and durably to unforeseeable exogenous shocks. Actually, this can be explained by a leading role of the US-market, whereas the German stock market is strongly orientated on global stock market behaviour in the long run.

E. Structural Break Test

Systematically in the estimate period, the dynamic model specification (10) leads to more precise specifying of forecast uncertainty. However, it is not certain that the co-integration model out of sample is more accurate than the static random walk. Therefore, as a convenient albeit not sufficient tool, structural break tests are finally inspected. If no structure break becomes obvious in the estimation period, then there is at least no hint that the time series model collapses in the future.

Since statistical inferences in cointegrating context are based upon asymptotic distributions, small sample periods are little trustworthy. Partitioning the investigation period into relatively long estimation periods and according relatively short out ofsample periods is also unsatisfactory. So with substantial deviations from the random walk arising particularly with increasing forecast horizon in the estimation period, long-term stock market projections move in the center of interest. Here, the following procedure is selected: The total period is divided into two subperiods and model estimation in both periods is compared. However, no unique breakpoint between both periods is specified ad hoc. Instead, model coefficients for all divisions of the total period into two subperiods are estimated. The results are represented in appendix B in graphic form separately for each model coefficient, whereby only estimation results for such subperiods are shown, which do not fall below a certain minimum length. Both edges of the six charts in appendix B represent estimations during the entire investigation period. Therefore, they show accurately the respective coefficient values indicated in (11). It is noticeable that the recursive coefficient estimators develop more smoothly for increasing estimation periods ending August 2002 than those starting April 1973. This may indicate that stock market development at the beginning of the investigation period no longer completely corresponds with today's. Actually, stock markets are more volatile today – not at least depending on regular electronic snapshots which were technically impossible some years ago. Outliers in December 1996 concerning the cointegrating coefficients are particularly remarkable. Apart from this, coefficient estimators of the cointegrating relation resemble each other in principle in the subperiods – at least with breakpoints since 1983. To that extent the estimated cointegrating relation seems to be of a robust nature and thus trustworthy also for the future. A clear structural break since 1999 cannot be discovered, which suggests that the discretely compounded real return seems suitable for the comparison of the German and the US-American stock market. For the remaining regression coefficients subperiod results deviate remarkably. The estimation vector of $\tilde{\gamma}_0$ describes a volatile process with clear upward trend for more current estimation samples. But on average over the entire time period, these constant effects are not significantly different from zero [see section 3.2]. In contrast, the loading coefficients for both the German and US-American stock markets have presently decreased. Therefore, it can be concluded that long-term adjustment effects lose meaning. However, that does not mean that traditional random walk forecasts are an appropriate approximation for the future. Altogether it can be concluded that (11) is a solid forecasting approach especially for real German stock market returns. However, for consideration of sinking loading coefficients and increasing time constant effects it is worthwhile adjusting regression coefficients for forecasting purposes.

IV. Results and View

Figure 2 illustrates an empirical connection between the real total return indices of the German and the US-American stock market. Its statistical significance is confirmed by formal co-integration tests. Therefore, the traditional random walk model for forecasting discretely compounded stock market returns of Germany and the USA is extended by a corresponding cointegrating relation in this paper. Without limitation of generality, the constant term remains unrestricted, although no evidence for a linear time trend in real stock market indices exists.

The dynamic model is connected with new insights, in particular, for predictability of German stock market returns. The co-integration of non-stationary real indices of the German and the US-American stock market, i.e., a common stochastic trend, systematically links them together at least in the long-run. Accordingly, clear deviations from static random walk forecasts particularly appear in the strategic analysis. Thus, discretely compounded real returns of the German stock market on the one hand are clearly predictable more accurately in the estimation period than reflected by the traditional random walk – particularly with increasing forecasting horizons. On the other hand, for corresponding US American returns random walk extrapolations are shown to be dangerous in the long run since they underestimate the risk of return contained in the historical information.

Investigations of the estimated cointegrating relation indicate its robustness over time, although its influence for forecasting future real stock market returns seems to lose strength. Furthermore, it is shown that co-integration supplies an explanation for skewness to the left of discretely compounded real returns. In order to describe also the lepto-curtosis of the empirical distribution of stock market returns, the residual processes could be examined for possible GARCH effects in a more extensive analysis. With GARCH approaches the variance of the error terms, which illustrates short-term forecast risk, is considered more accurately with respect to current information. The existence of the fourth moments of the residual terms is, however, no longer secured with a description by GARCH processes and lepto-curtic distributions may arise. The extent to which current statistical restrictions on co-integration analysis remain thereby met, must be examined individually.

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Appendix A
Forecasting Returns

First, we focus on discretely compounded returns for one period $\nabla Y_{t,j} \,/\, Y_{t-1,j}$ with $\tilde{\Pi} = 0$, so $\nabla Y_{t,j} = \tilde{\gamma}_j + \tilde{U}_{t,j}$. Additionally, we have the assumption $\tilde{U}_{t,j}$ i.i.d. with expectation vector $E\tilde{U}_{t,j} = 0$ and covariance matrix $\mathrm{var}\,\tilde{U}_{t,j} = \Sigma_{(jj)}$. With the GLS estimator $\hat{\tilde{\gamma}}_j$ of $\tilde{\gamma}_j$ the conditioned forecast mean squared error of discrete returns over h periods, $\nabla_h Y_{t,j} / Y_{t-h,j} := (Y_{t,j} - Y_{t-h,j}) / Y_{t-h,j}$, for each period t within the estimate period (t = 1,2,...,T) results in

$$\mathrm{FMSE}_{t,j}(h)|Y_{t-h} = \mathrm{var}\left[h\left(\hat{\tilde{\gamma}}_j - \tilde{\gamma}_j\right) - \sum_{i=0}^{h-1} \tilde{U}_{t-i,j}\right] \cdot \frac{1}{Y_{t-h}^2} = h\,\Sigma_{(jj)}\left(1 - \frac{h}{T}\right) \cdot \frac{1}{Y_{t-h,j}^2} \qquad (A.1)$$

The FMSE is reduced by the adjustment of the coefficient estimators to the observations in sample. Out of sample, their standard deviation leads on the contrary to an increase of forecasting uncertainty – under the assumption of intertemporaneous independence of the residual terms – since the cross products in the variance term are zero in expectation

$$\mathrm{FMSE}_{T+h,j}(h)|Y_T = h\,\Sigma_{(jj)}\left(1 + \frac{h}{T}\right) \cdot \frac{1}{Y_{T,j}^2} \qquad (A.2)$$

Thus, under neglect of the variance of GLS estimators it arises for arbitrary t

$$\mathrm{FMSE}_{t,j}(h)|\,\tilde{\gamma}_j, Y_{t-h} = h\,\Sigma_{(jj)} \cdot \frac{1}{Y_{t-h,j}^2} \qquad (A.3)$$

The case becomes more complex, if restriction $\tilde{\Pi} = 0$ is given up and forecasts in the co-integration model are looked at. Since the estimators of the normalized cointegrating relation are super-consistent, their estimation uncertainty can be neglected. Therefore, $\tilde{\gamma}_0$ and $\tilde{\alpha}$ from (10), $\nabla Y_t = \tilde{\gamma}_0 + \tilde{\alpha}(\tilde{\beta}_0 + \tilde{\beta}'Y_{t-1}) + \tilde{U}_t$, can be calculated as usual GLS estimators using the estimated cointegrating relation as exogenous size. In contrast to the random walk model the covariance matrix of the intertemporaneous independent and homoskedastic $\tilde{U}_t$ with $E\tilde{U}_t = 0$ is marked by Σ_U, its main diagonal elements by $\Sigma_{U(jj)}$. For simplifying notation we define $X_{t,j} := (1; \tilde{\beta}_0 + \tilde{\beta}'Y_{t-1})$ and $X_j' := (X_{1,j}; X_{2,j}; ...; X_{T,j})$. Thus, for the conditioned one-step-FMSE of discretely compounded returns for each period t within the estimation period results(t = 1.2....,T)

$$FMSE_{t,j}(1)|Y_{t-1} = \Sigma_{U(jj)}\left(1 - X_{t,j}(X_j'X_j)^{-1}X_{t,j}'\right)\cdot\frac{1}{Y_{t-1,j}^2} \quad (A.4)$$

while one step forecasting out of sample leads to

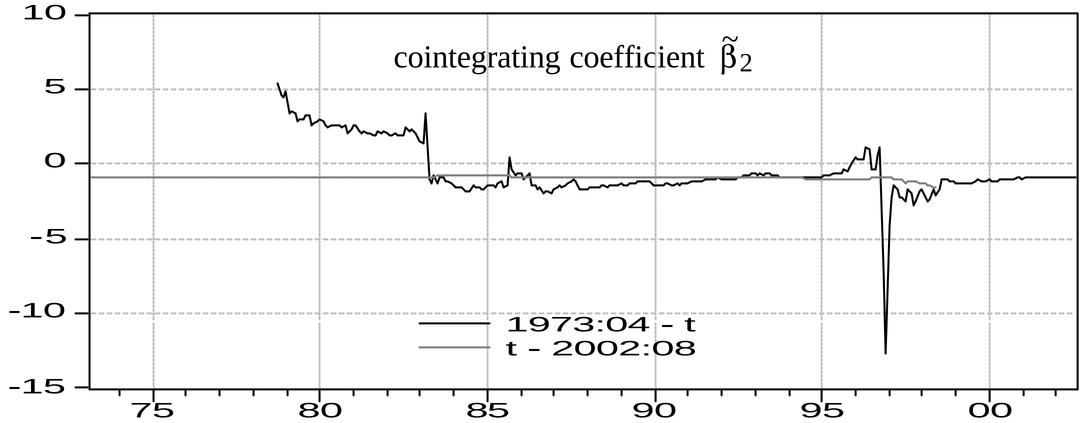
$$FMSE_{T+1,j}(1)|Y_T = \Sigma_{U(jj)}\left(1 + X_{T,j}(X_j'X_j)^{-1}X_{T,j}'\right)\cdot\frac{1}{Y_{T,j}^2} \quad (A.5)$$

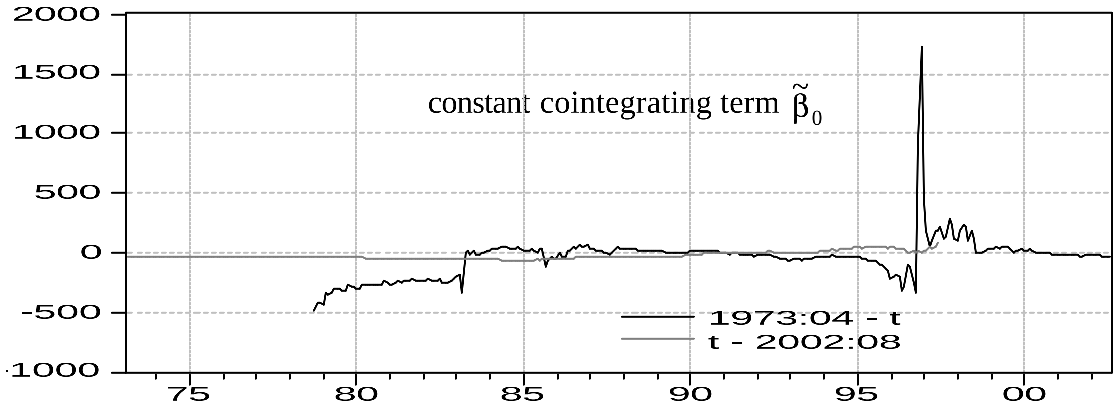
With $\nabla Y_t = \tilde{\Pi} Y_{t-1} + (\tilde{\gamma} + \tilde{U}_t)$ in general $\nabla_h Y_t = (\Psi^h - I)\, Y_{t-h} + \sum_{i=0}^{h-1} \psi^i\, (\tilde{\gamma} + \tilde{U}_{t-i})$ applies, $\Psi := (I + \tilde{\Pi})$. So h-step-forecasts depend nonlinearly on the coefficient estimators, and computation of the conditioned h-step-FMSE gets only possible if the variances of the GLS estimators are neglected. Then, it arises for arbitrary t

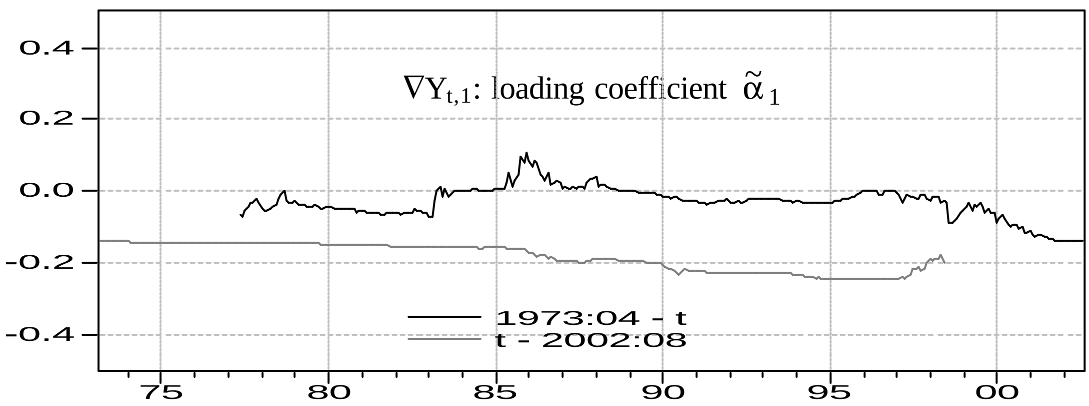
$$FMSE_{t,j}(h)|\ \tilde{\gamma}, \Psi, Y_{t-h} = \sum_{i=0}^{h-1} \psi_{(j\bullet)}^{(i)}\, \Sigma_U\, \left(\psi_{(j\bullet)}^{(i)}\right)' \cdot \frac{1}{Y_{t-h,j}^2} \quad (A.6)$$

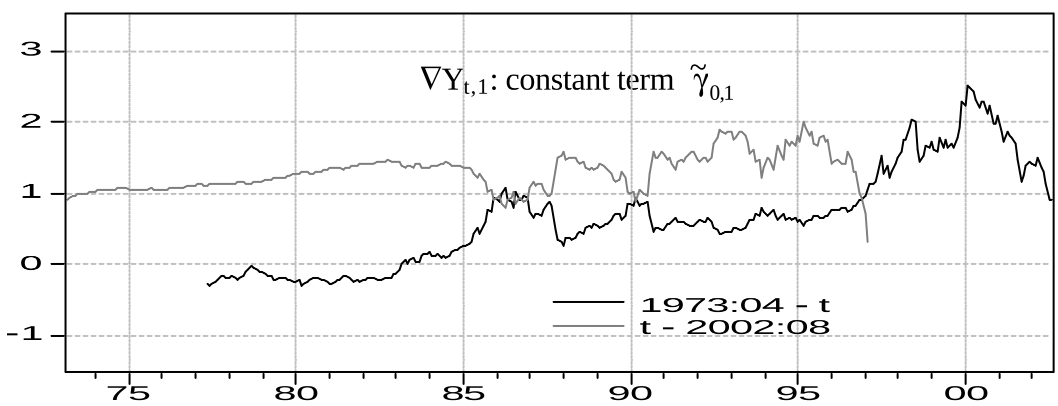
Appendix B

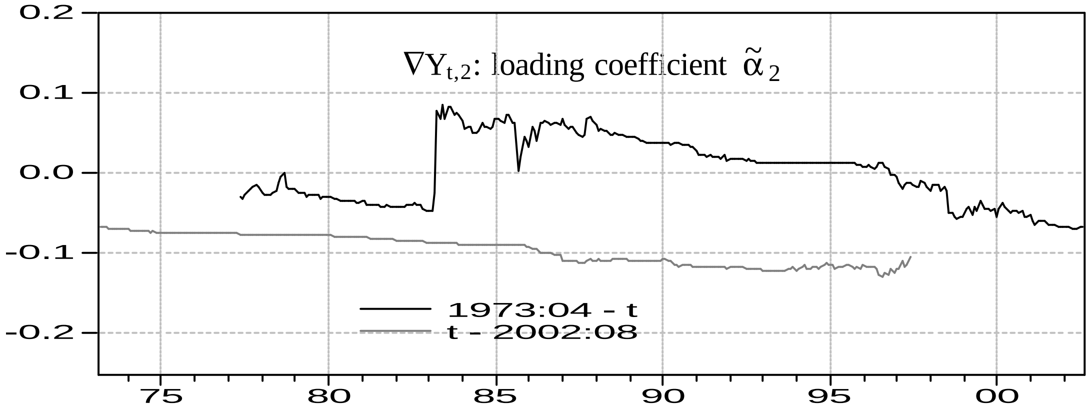
Subsample Analysis: Recursive VECM Estimation










∇Y_{t,2}: loading coefficient α̃_2
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t - 2002:08
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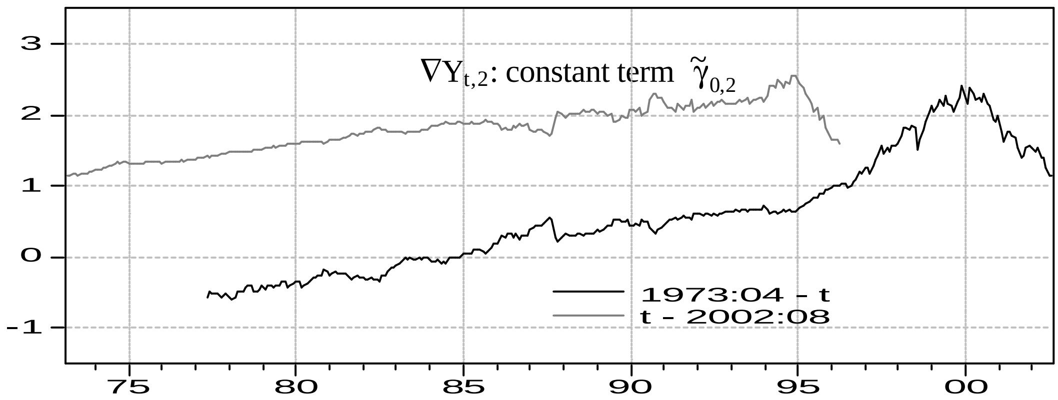
0.0
-0.1
-0.2
75
80
85
90
95
00


∇Y_{t,2}: constant term γ̃_{0,2}
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t - 2002:08
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2
1
0
-1
75
80
85
90
95
00