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Half-Optimal Error Diffusion for Binary Fourier Transform Holograms

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The error diffusion method was investigated with novel diffusion coefficients for generating binary Fourier transform holograms. By defining an error measure, the quality of the reconstruction from such holograms was estimated. Computer simulated reconstructions are presented.

1. Introduction

There are many methods to generate binary holograms. They can be divided into two groups, i.e., noniterative and iterative methods [1]. Hard clipping and error diffusion [3-5] belong to the former, while direct binary search [7-9], iterative stepwise quantization [10,11], and gradual and random binarization [12] belong to the latter. At first glance, they seem to be completely different. However, from the point of view of their working fashion, they are same. Noniterative methods diffuse the binarization error at once, while iterative method in several steps [1]. We restrict our work here to the noniterative methods, particularly to error diffusion. Because this method has a linear computational complexity (the time needed for generating a gray-scale hologram is excluded from the process), it is widely used today when the demand on the reconstruction is not very rigorous.

Normally, we assign a random phase to the object when generating its hologram. Because of this, a uniformly distributed spectrum of the object on the hologram plane is obtained so that the quantization error is minimized. This is particularly important for Fourier transform holograms. In the reconstruction, only a certain sub-region, which we call reconstruction region, is of interest. The aim is to generate a binary hologram, which reconstructs the object in the reconstruction region as well as possible.

If gray-scale holograms are regarded as images, they can be binarized using the error diffusion method. Originally, error diffusion was developed for displaying gray-scale images on a black-white screen using half-toning technique [2]. It was then introduced into computer holography [3-6]. In this field however, error diffusion works in a completely new fashion. This can be seen in the following three aspects: a) The binarization of gray-scale images is performed in the object domain and the binarization result is also estimated in this domain; while the binarization of gray-scale holograms is performed in hologram domain, whereas the binarization result is estimated in object domain. b) The binarization of gray-scale images is a pure empirical process. It is difficult to define a criterion to assess the result, mostly this is done by humans visually. While the binarization of gray-scale holograms is a theoretical process. One can exactly define a criterion to measure the result of the reconstruction produced by such a binarized hologram. c) The diffusion coefficients used in the binarization of images usually remain unchanged, while in the binarization of gray-scale holograms the diffusion coefficients change from application to application [1].

In the following sections we investigate this method in detail for generating binary Fourier transform holograms, find some skills in using this technique and give the limitations of this method.

2. Error diffusion and optimal diffusion coefficient matrix

The basic concept of the original error diffusion method for the binarization of gray-scale images can be described as follows [2] (see Fig. 1):

The first pixel I(0,0) of the image I(x,y) is compared to a threshold T, and a new pixel value $I_{out}(0,0)$ is produced. If $I(0,0) \geq T$, then $I_{out}(0,0)=1$, otherwise $I_{out}(0,0)=0$. This operation introduces an error $E(0,0)=I_{out}(0,0)-I(0,0)$, which is diffused to other unprocessed pixels. The new value $I'(x,y)=I(x,y)-W_{x0y0}E(0,0)$ is assigned to the pixel at (x,y), (x,y)=(1,0), (1,1), (0,1), (-1,1). The values W_{x0y0} are called diffusion coefficients. Fig. 1 shows the diffusion coefficients $W_{xx'yy'}$, where (x',y') is the pixel being binarized. This algorithm works sequentially, i.e., line by line, from left to right. Note that the local average of the binarized pixel $I_{out}(x',y')$ and its modified neighbors I'(x,y) approximates that of the original pixel since $\Sigma_{(x,y)} W_{xx'yy'} = 1$, (x,y)=(x'+1,y'), (x'+1,y'+1), (x',y'+1), (x'-1,y'+1).

→ x

↓ y

B	B	B
B	(x',y')	7/16
3/16	5/16	1/16

Fig. 1. Error diffusion coefficients used by Floyd and Steinberg [2]. Pixels that have already been processed are marked by B. The pixel being processed is marked by (x',y').

The diffusion coefficient matrix of Fig. 1 for the binarization of images proposed by Floyd and Steinberg was determined empirically. For computer holography many different variations of this algorithm have been suggested later [3-6]. They include the use of non-adjacent pixels, the use of different diffusion matrices etc.. However, questions as why the algorithm works, which variation is applicable for what applications, have been answered only recently by Eryi Zhang *et al* using an exact mathematical analysis of this problem [1]. From this analysis a new diffusion coefficient matrix was derived which depends on the application, i.e., on the size of the hologram, the size of the reconstruction region, and the position where the reconstruction region is located. The problem of finding optimal diffusion coefficients for computer holography was solved and the questions to each variation were answered satisfactorily.

Without loss of generality, we suppose that the gray-scale hologram H(μ,v) has N×N pixels, and the object u(x,y), which will be reconstructed in the reconstruction region $\mathcal{W}$ centered at (x_0,y_0), has M×M (M<N) pixels. Then the optimal diffusion coefficient matrix is given by [1]

$$C_{\mu\mu'vv'} = \begin{cases} 1-\left(\frac{M}{N}\right)^2 & \text{if } \mu=\mu' \text{ and } v=v' \\ -\left(\frac{M}{N}\right)^2 \operatorname{sinc}\left(\frac{M(\mu-\mu')}{N}\right) \operatorname{sinc}\left(\frac{M(v-v')}{N}\right) \cos\left(\frac{2\pi[x_0(\mu-\mu')+y_0(v-v')]}{N}\right) & \text{otherwise} \end{cases} \tag{1}$$

where (μ',v') denotes the binarized pixel.

Eq. (1) indicates that the diffusion coefficients $C_{\mu\mu'vv'}$ depend only on the parameters M/N, x_0/N and y_0/N, on the distance |μ-μ'| and |v-v'|. They are independent of which pixel is being binarized.

To get an intuition about $C_{\mu\mu'vv'}$, we have calculated its numerical values for the example M/N=30/128, $x_0/N=y_0/N=1/4$ in 3 adjacent pixels of the binarized pixel (μ',v'). Table I gives these values. The sum to the diffusion coefficients for μ=μ'-3, ..., μ'+3, v=v'-3, ..., v'+3 of table I gives $\Sigma_{(\mu,v)} C_{\mu\mu'vv'} = 0.9932$.

Table I. The diffusion coefficients calculated with Eq. (1) (M/N=30/128, $x_0/N=y_0/N=1/4$)

→ μ

↓	+0.00726	0	-0.01822	0	+0.01822	0	-0.00726
v	0	-0.02509	0	+0.03712	0	-0.02509	0
	-0.01822	0	+0.04569	0	-0.04569	0	+0.01822
	0	+0.03712	0	0.9451 (μ',v')	0	+0.03712	0
	+0.01822	0	-0.04569	0	+0.04569	0	-0.01822
	0	-0.02509	0	+0.03712	0	-0.02509	0
	-0.00726	0	+0.01822	0	-0.01822	0	+0.00726

Obviously, the binarization error is mainly diffused to the binarized pixel (μ',v').

We define the Mean Square Error (MSE) between the original signal f and the reconstruction g in the reconstruction region $\mathcal{W}$ as measure of the reconstruction quality:

$$\mathrm{MSE} = \sum_{xy \in \mathcal{W}} \left| \frac{f_{xy} - \bar{f}}{\sigma_f} - \frac{g_{xy} - \bar{g}}{\sigma_g} \right|^2 \tag{2}$$

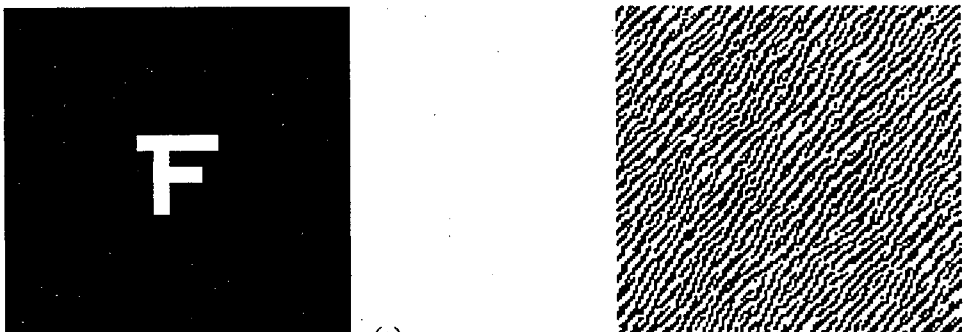
$$\bar{f} = \frac{1}{M^2} \sum_{xy \in \mathcal{W}} f_{xy}, \quad \bar{g} = \frac{1}{M^2} \sum_{xy \in \mathcal{W}} g_{xy}, \quad \sigma_f^2 = \frac{1}{M^2} \sum_{xy \in \mathcal{W}} \left| f_{xy} - \bar{f} \right|^2, \quad \sigma_g^2 = \frac{1}{M^2} \sum_{xy \in \mathcal{W}} \left| g_{xy} - \bar{g} \right|^2$$

Note that MSE calculated with (2) is independent of the actual values of the reconstruction g and any constant added to it.

In the following we investigate the influences of the diffusion size and the parameters (M/N, x_0/N and y_0/N) in Eq. (1) on the reconstruction, and discuss the efficiency of this algorithm. We choose the capital letter 'F' in three different sizes as test patterns. Object 1 shown in Fig. 2(a) has 30×30 pixels, object 2 shown in Fig. 8(a) has 45×45 pixels, and object 3 shown in Fig. 10(a) has 20×20 pixels.

3. Influence of the diffusion size on the reconstruction

Now we investigate the influence of the diffusion size on the reconstruction. As example we use object 1. This object is the capital letter 'F'. It has 30×30 pixels and three gray values. It is centrally embedded in a zero array of 128×128 pixels as shown in Fig. 2(a). Random phases are assigned to it to generate its hologram. Fig. 2(b) shows this gray-scale hologram with $(x_0,y_0)=(N/4,N/4)$.

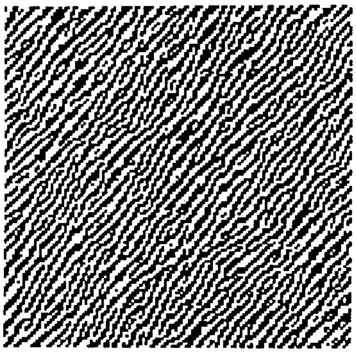


(a) (b)

Fig. 2. Object 1 (capital letter 'F') with 30×30 pixels (a) and its gray-scale hologram (256 gray values) (b).

To binarize the gray-scale hologram of Fig. 2(b), we use the diffusion coefficients of Table I because object 1 has the same parameters as those of Table I. Three different diffusion sizes were tested, which are 0-adjacent neighborhood, 1-adjacent neighborhood, and 15-adjacent neighborhood.

In the first test, using diffusion size of 0-adjacent neighborhood, we diffused the binarization error only to the binarized pixels themselves. This corresponds to the hard-clipping method. The binary hologram generated using this diffusion size is shown in Fig. 3(a). $\Sigma_{(\mu,v)}c_{\mu\mu'vv'}=0.9451$ for $(\mu,v)=(\mu',v')$. Its computer simulated reconstruction is given in Fig. 3(b) with MSE=1.26×10^{-1}. The diffraction efficiency is given by η=8.93%.


(a)

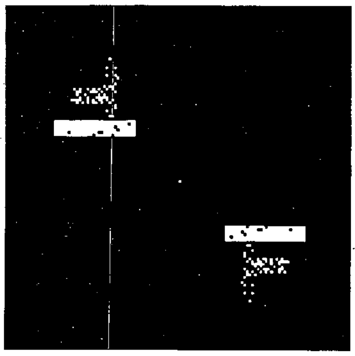

(b)

Fig. 3. Binary hologram generated using a diffusion size of 0-adjacent neighborhood (a), and its computer simulated reconstruction (b).

The second test, using diffusion size of 1-adjacent neighborhood, is similar to the original error diffusion algorithm of section 2, but the diffusion coefficient matrix is completely different from that. In Fig. 4 we rewrite the matrix of Table I with this diffusion size.

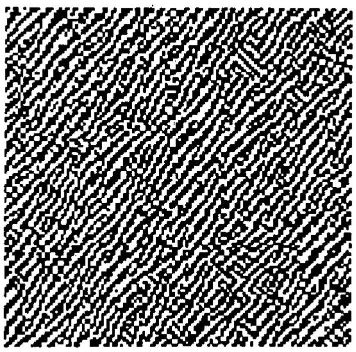
$\rightarrow\ \mu$

$\downarrow\ v$

B	B	B
B	0.9451 $\underline{(\mu',v')}$	0
-0.04569	0	+0.04569

Fig. 4. Error diffusion coefficients in 1-adjacent neighborhood. Pixels that have been processed are marked by B. The pixel being processed is marked by $\underline{(\mu',v')}$.

The binarization error was diffused to the unprocessed pixels, i.e., to the pixels which are right or below of the binarized pixel (see Fig. 4). The sum to all 5 diffusion coefficients gives $\Sigma_{(\mu,v)}c_{\mu\mu'vv'}=0.9451$, the same as that of the first test. However, a different binary hologram was generated, and the reconstruction has been improved. Fig. 5(a) shows this binary hologram, Fig. 5(b) its computer simulated reconstruction with MSE=7.28×10^{-2}. The diffraction efficiency is given by η=7.79%.


(a)

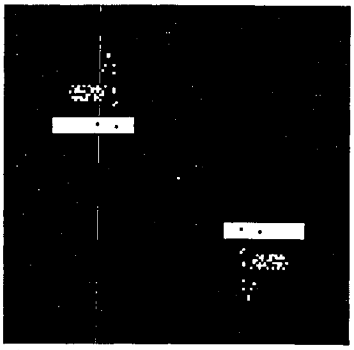

(b)

Fig. 5. Binary hologram generated using the diffusion size of 1-adjacent neighborhood (a), and its computer simulated reconstruction (b).

The third test uses a diffusion size of 15 adjacent neighborhoods. The sum of all diffusion coefficients gives $\Sigma_{(\mu,v)} c_{\mu\mu'vv'} = 0.9721$, which is very close to 1. The binary hologram generated using this diffusion size is shown in Fig. 6(a), its computer simulated reconstruction is given in Fig. 6(b) with MSE=4.08×10^{-2}. The diffraction efficiency is η=5.61% (the diffusion coefficient matrix for this test is omitted here).

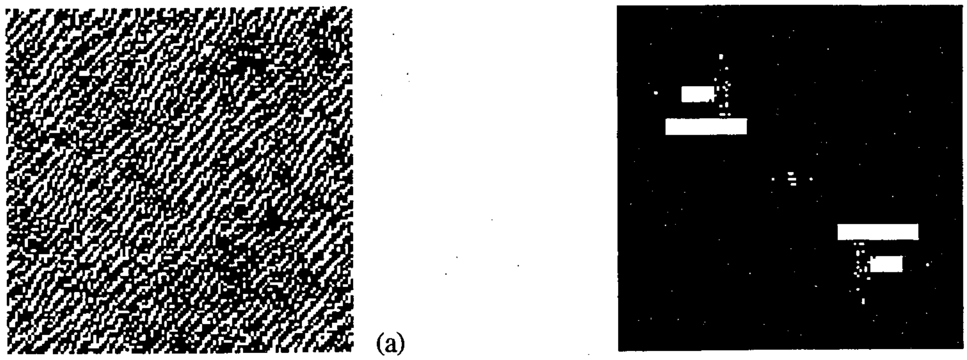


Fig. 6. Binary hologram generated using the diffusion size of 15 adjacent neighborhoods (a), and its computer simulated reconstruction (b).

More tests using larger diffusion size have been performed. However, no distinct improvement has been achieved because for larger diffusion size the diffusion coefficients become negligibly small. F … he MSE as a function of the incremental diffusion size.

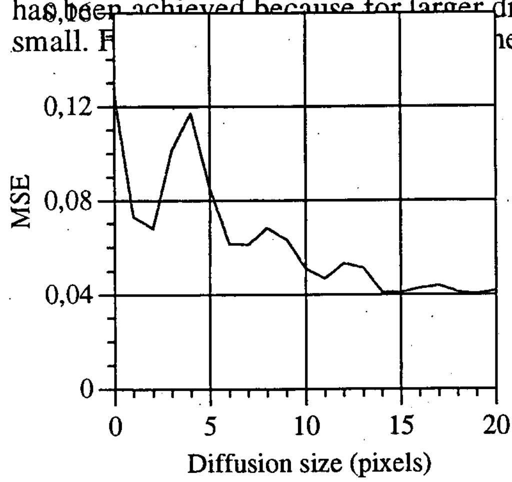


Fig. 7. MSE vs. incremental diffusion size.

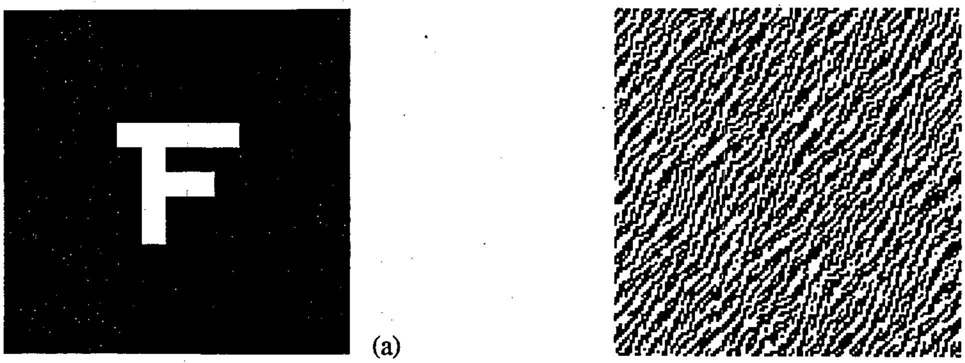
Fig. 7 shows that the MSE decreases oscillatingly and that the oscillation amplitude diminishes with increasing diffusion size. Further tests showed that the oscillating period depends on the size of the reconstruction region. This property is expressed by Eq. (1) which gives periodic diffusion coefficients due to the sinc function. From the results above we conclude that error diffusion method is available to computer holography. Using the optimal diffusion coefficients of Eq. (1) and a proper diffusion size, which is determined by the size of the reconstruction region, a better reconstruction can be achieved.

4. Influences of parameters M×M and (x_0,y_0) on the reconstruction

In the following we investigate the size of the reconstruction region M×M on the reconstruction, and the influence of the position (x_0,y_0), i.e., where the reconstruction region is located.

4.1. Influence of the size of the reconstruction region

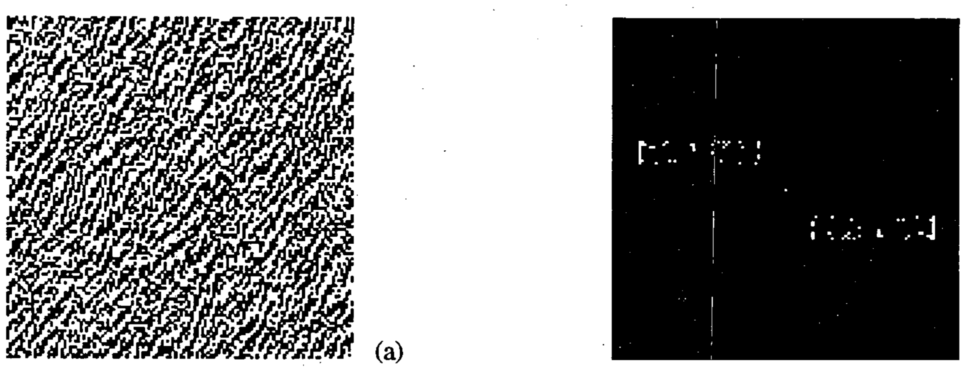
Two objects with different sizes are used. One is the object 2 shown in Fig. 8(a), which has the same pattern as object 1 but a larger size (45×45 pixels). Another one, object 3, has also the same pattern, but 20×20 pixels. Object 3 is shown in Fig. 10(a). Fig. 8(b) and Fig. 10(b) show their gray-scale holograms with $(x_0,y_0)=(N/4,N/4)$.



(a) (b)

Fig. 8. Object 2 (capital letter 'F') with 48×48 pixels (a) and its gray-scale hologram (256 gray values) (b).

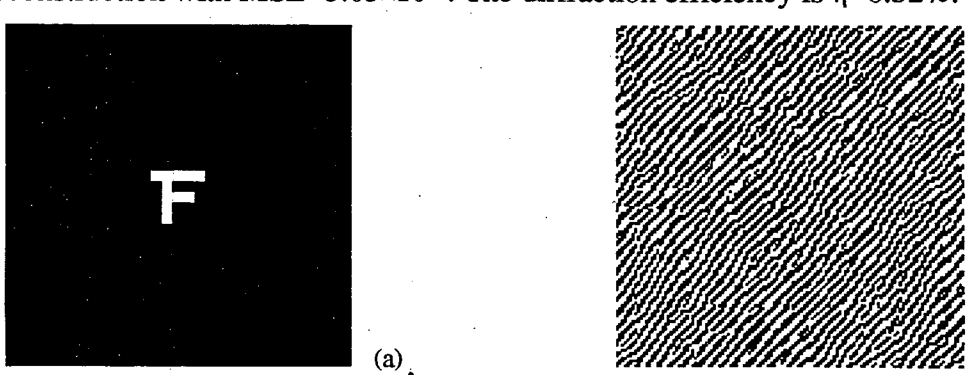
Using the diffusion coefficients calculated with Eq. (1) for object 2 and the diffusion size of 15 adjacent neighborhoods, we have generated the binary hologram of Fig. 8(b). The sum of all diffusion coefficients gives $\Sigma_{(\mu,v)} c_{\mu\mu' vv'} = 0.9375$. Fig. 9(a) shows this binary hologram, next to it is its computer simulated reconstruction with MSE=6.06×10^{-2}. The diffraction efficiency is $\eta=6.38\%$.



(a) (b)

Fig. 9. Binary hologram of object 2 generated using a diffusion size of 15 adjacent neighborhoods (a), and its computer simulated reconstruction (b).

The same process is performed for object 3. The sum of all diffusion coefficients gives $\Sigma_{(\mu,v)} c_{\mu\mu' vv'} = 0.9876$. Fig. 11(a) shows this binary hologram, Fig. 11(b) its computer simulated reconstruction with MSE=5.05×10^{-2}. The diffraction efficiency is $\eta=6.52\%$.



(a) (b)

Fig. 10. Object 3 (capital letter 'F') with 20×20 pixels (a) and its gray-scale hologram (256 gray values) (b).

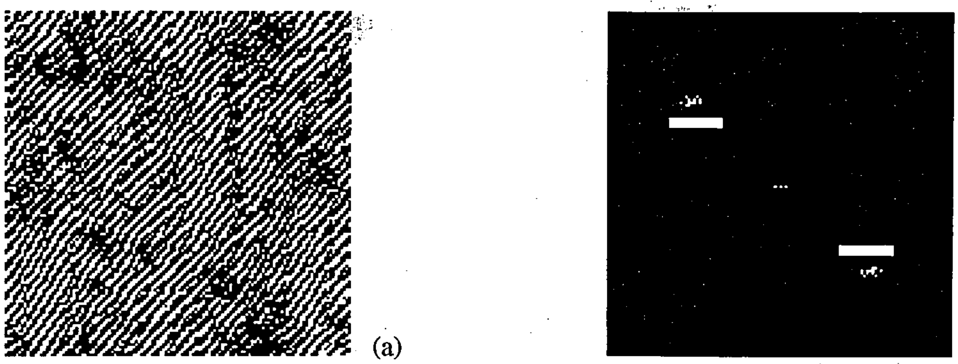


Fig. 11. Binary hologram of object 3 generated using a diffusion size of 15 adjacent neighborhoods (a), and its computer simulated reconstruction (b).

Fig. 12 shows the influence of the size of the reconstruction region on the reconstruction for the

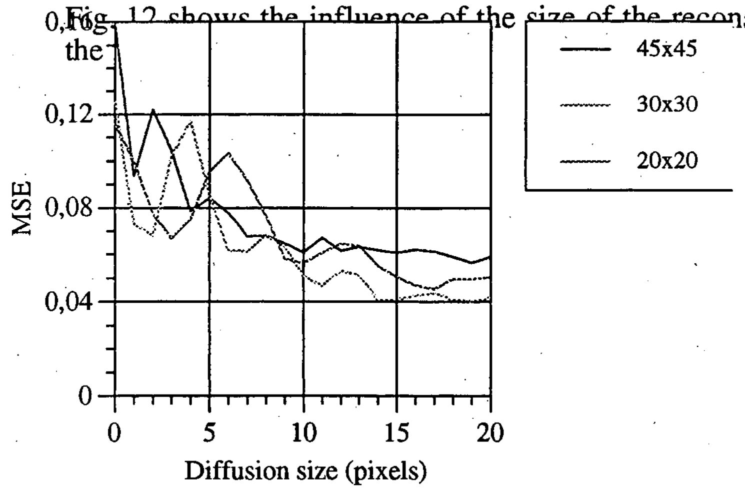


Fig. 12. MSEs of the three examples vs. the incremental diffusion size.

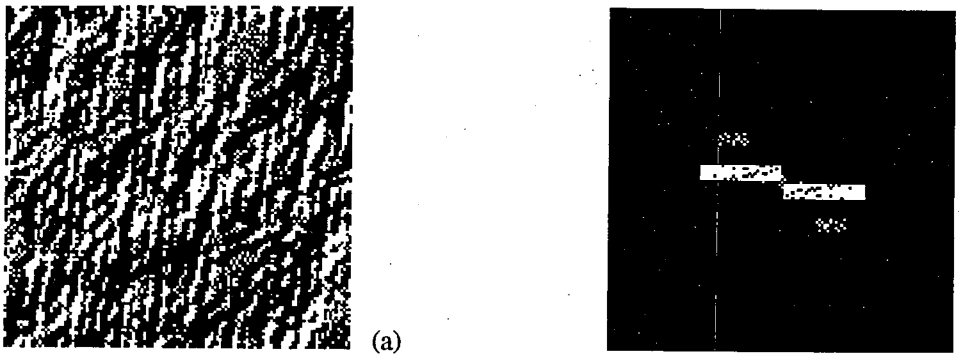
From Fig. 12 it has been shown that the larger the reconstruction region is (e.g., object 2 which has 45×45 pixels), the smaller an oscillating period the MSE has, and the more quickly the MSE is going to a steady value. In other words, for a larger size object we need only consider a smaller diffusion size to generate a binary hologram using error diffusion (it is enough for object 2, e.g., to consider up to 10 adjacent neighborhoods as diffusion size after 3 periods). On the other hand, for a smaller object (e.g., object 3 which has 20×20 pixels) we have to consider a relative large diffusion size (it is necessary for object 3, e.g., to consider up to 20 adjacent neighborhoods as diffusion size after 3 periods). Further tests showed that the reduction of the MSE depends also on the position where the reconstruction region is located. This will be investigated in section 4.2.

It is impossible to achieve the same reconstruction for the three objects so that they have the same diffusion size and the same reconstruction position (x_0,y_0). However, it is really possible for object 3 to achieve the same reconstruction as object 1 if the diffusion size is even larger, while this is not possible for object 2 due to the limitation of this method. This limitation is that for larger object the diffusion coefficient in the center (binarized pixel) has a smaller value, around the center (pixels around the binarized pixel) they have larger values. Thus, the error introduced by the binarized pixel and by the pixels that have already been processed is much larger, and these errors cannot be diffused further in noniterative method. This results in a larger reconstruction error. Theoretically, the smaller the reconstruction region is, the less binarization error is diffused to the neighbors, and the better reconstruction is achieved. In the extreme case $M \rightarrow 0$, hard clipping can achieve the same reconstruction as error diffusion does.

4.2. Influence of the position of the reconstruction region

Object 1 and object 3 are used for the investigation of the influence of the position of the reconstruction region. For each object two tests are carried out with parameter $(x_0,y_0)=(N/8,N/8)$, $(3N/8,3N/8)$ ($(x_0,y_0)=(N/4,N/4)$ has been tested in section 4.1).

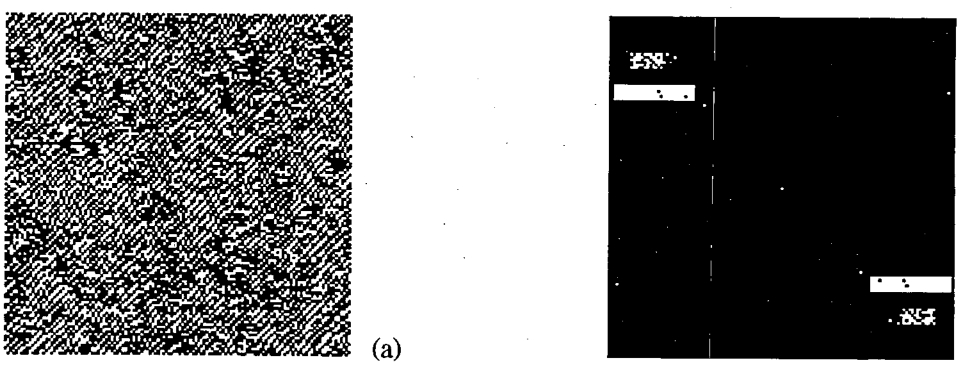
Fig. 13(a) shows the generated binary hologram using the diffusion coefficients for object 1 with $(x_0,y_0)=(N/8,N/8)$. The diffusion size is also 15 adjacent neighborhoods. The sum of all diffusion coefficients gives $\Sigma_{(\mu,v)} c_{\mu\mu' vv'} = 0.9373$. Fig. 13(b) shows its computer simulated reconstruction with $MSE=7.81\times10^{-2}$.



(a) (b)

Fig. 13. Binary hologram generated using a diffusion size of 15 adjacent neighborhoods for object 1 with $(x_0,y_0)=(N/8,N/8)$ (a), and its computer simulated reconstruction (b).

Fig. 14 shows another binary hologram for object 1 with $(x_0,y_0)=(3N/8,3N/8)$. Fig. 14(b) is its computer simulated reconstruction with $MSE=4.09\times10^{-2}$. The diffusion size is 15 adjacent neighborhoods too. The sum of all diffusion coefficients gives $\Sigma_{(\mu,v)} c_{\mu\mu' vv'} = 0.9723$.



(a) (b)

Fig. 14. Binary hologram generated using a diffusion size of 15 adjacent neighborhoods for object 1 with $(x_0,y_0)=(3N/8,3N/8)$ (a), and its computer simulated reconstruction (b).

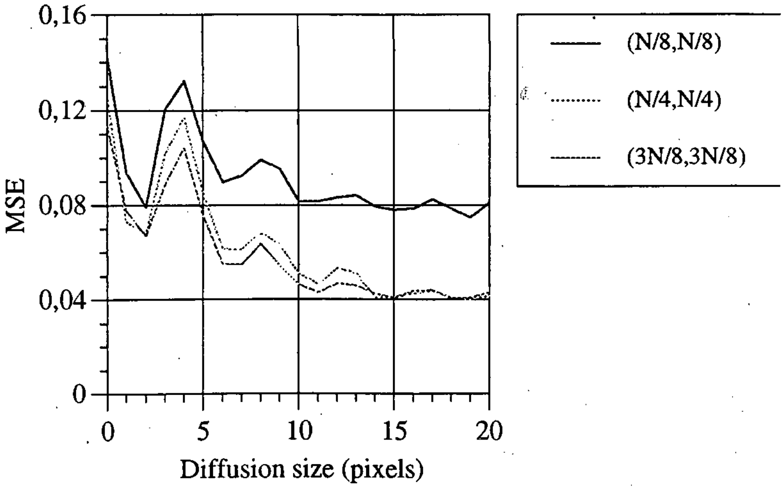


Fig. 15. MSE vs. diffusion size for object 1 in three reconstruction positions (N/8,N/8), (N/4,N/4), and (3N/8,3N/8).

Fig. 15 above shows the dependence of the MSE on the diffusion size for object 1 in three reconstruction positions.

The same process is performed for object 3. Fig. 16(a) and Fig. 17(a) show their binary holograms for the tests $(x_0,y_0)=(N/8,N/8)$ and $(x_0,y_0)=(3N/8,3N/8)$. Fig. 16(b) is the computer simulated reconstruction for $(x_0,y_0)=(N/8,N/8)$ with MSE=4.85×10^{-2}, and Fig. 17(b) for $(x_0,y_0)=(3N/8,3N/8)$ with MSE=4.49×10^{-2}. Fig. 18 shows the dependence of the MSE on the diffusion size for object 3 in three reconstruction positions.

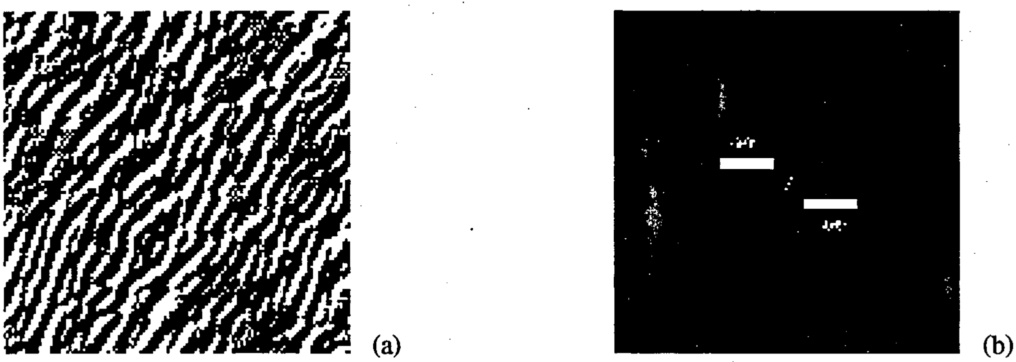


Fig. 16. Binary hologram generated using a diffusion size of 15 adjacent neighborhoods for object 3 with $(x_0,y_0)=(N/8,N/8)$ (a), and its computer simulated reconstruction (b).

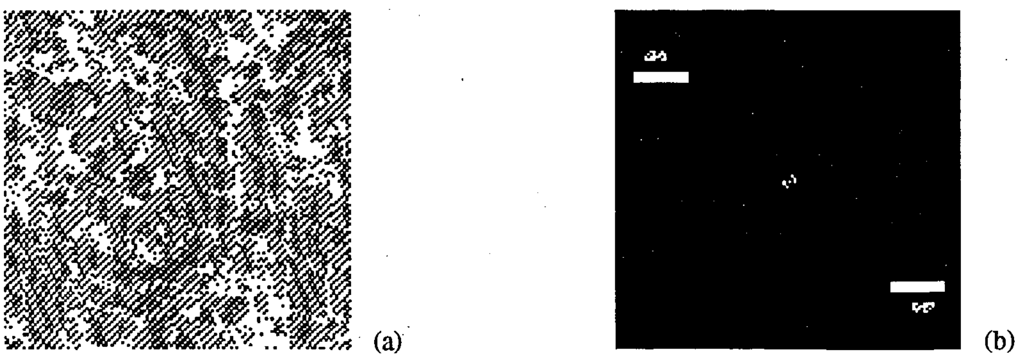


Fig. 17. Binary hologram generated using a diffusion size of 15 adjacent neighborhoods for object 3 with $(x_0,y_0)=(3N/8,3N/8)$ (a), and its computer simulated reconstruction (b).

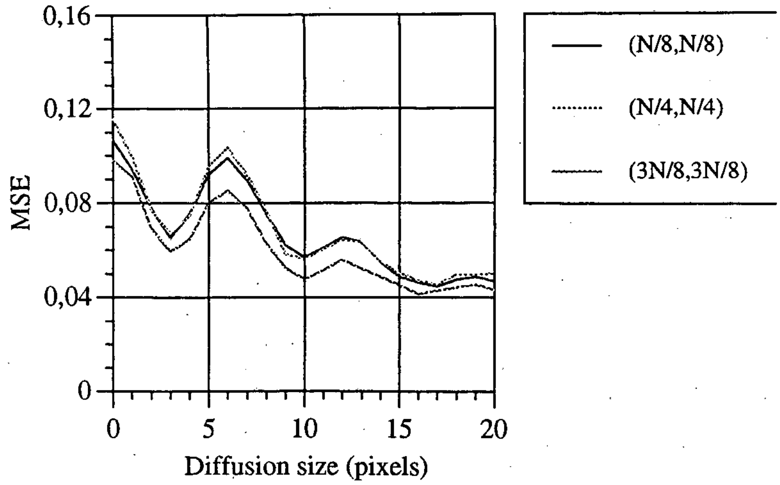


Fig. 18. MSE vs. diffusion size for object 3 in three reconstruction positions (N/8,N/8), (N/4,N/4), and (3N/8,3N/8).

There is no serious position influence on the reconstruction for smaller objects (see Fig. 18 for object 3). However, for larger objects the position influence may be considerable (see Fig. 15 for object 1). It is important to choose a proper reconstruction position. One can, e.g., choose the reconstruction region as far as possible from the center because the binarization error in the reconstruction is mainly in the vicinity of the center. Using this property we can get a higher quality reconstruction without additional computing requirement.

5. Conclusions

The error diffusion method is available for computer holography. Using diffusion coefficients given by Eq. (1), we can generate partly optimal binary holograms from their gray-scale holograms. The reconstruction can be improved further using a larger diffusion size. Of course, more computing time is required then. Choosing the reconstruction region as far as possible off-center, a better reconstruction is achieved without additional computing requirements. Since the binarization error can be diffused neither to the binarized pixel, nor to the pixels that have already been processed, the reconstruction cannot be improved arbitrarily, that is to say, the algorithm is in practice limited. We can conclude that the computational efficiency of the error diffusion algorithm, which is O(N) (N is the size of the hologram), is contrasted to its limited reconstruction quality.

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