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The consistency of a noninterleaving and an interleaving model for full TCSP

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1. Introduction

Various formalisms have been proposed in the past for the description of nondeterministic concurrent systems, the most well- known of which are CCS [14,15] ,ACP [2] and TCSP [6,13,17] . These languages or calculi have been given a variety of semantical descriptions [1,2,3,4,5,6,7,8,10,11,12,18,19,20,21]. A first classification of this semantics distinguishes between interleaving and noninterleaving models .
In noninterleaving models as [3,4,8,10,11,12,18,21,23] an attempt is made to capture ' true parallelism ' where as interleaving models as [1,2,5,6,7,19] somehow reduce concurrency to nondeterministic sequential behaviour by arbitrary interleaving of atomic actions,
e.g. the process $\alpha.stop \parallel \beta.stop$ 'behaves' like $\alpha.\beta.stop \,\Box\, \beta.\alpha.stop$, if $\alpha, \beta \neq \tau$.
In this paper we compare an interleaving semantics of full TCSP based on a transition system with a noninterleaving model based on labelled event structures [16,22,23,24] .
In an earlier paper [11] have shown for **finite** TCSP processes without recursion and *div* that the interleaving transition system based description and the respective event structure semantics are consistent . As recursion is a very powerful tool to build concurrent systems , it is an interesting question if this result carries over to **full** TCSP. We show here that this question has a positiv answer. The result is in particular interesting, as it not only relates an interleaving specification with a noninterleaving but also relates at the same time an operational specification with a compositional one, that provides semantic operators for all syntactical constructs including fix.

2. The syntax of guarded TCSP

Let *Comm* be the set of possible communications. A special action τ, as in CCS, is introduced to describe internal actions which may not communicate. For notational convenience, we allow τ to occur syntactially in expressions denoting processes.
So let the set *Act* of *actions* be defined as

$$Act \;:=\; Comm \cup \{\, \tau \,\}.$$

Let Idf be a set of identifiers which will serve as variables for programs. The set $TCSP$ of $TCSP$ *terms* is defined by the following production system :

$$\begin{aligned} P \;:=\; & stop \mid \alpha.P \mid div \mid P \text{ or } Q \mid P \,\Box\, Q \mid \\ & P \parallel_A Q \mid P \setminus \beta \mid x \mid fix\ x.P, \end{aligned}$$

where $\alpha \in Act$, $\beta \in Comm$, $A \subseteq Comm$, $x \in Idf$.

2.1 Definition :
An occurence of an identifier x is called *free* in a term $P \in TCSP$ iff it does not occur within a subterm of the form $fix\ x.Q$. A TCSP term P is said to be *closed* iff it does not contain identifiers which occur free in P.

An identifier x is *guarded* in a term $P \in TCSP$ iff each free occurence of x in P is in the scope of a prefixing operation $Q \mapsto \alpha.Q$.

A term $P \in TCSP$ is called *guarded* iff in each subterm $fix\ x.Q$ of P the identifier x is guarded in Q.
Let $GTCSP$ be the set of all guarded TCSP terms.

A *GTCSP process* is a closed, guarded TCSP term.

2.2 Definition :
Let $P, A_1, \dots, A_n \in GTCSP$ and $x_1, \dots, x_n \in Idf$ pairwise distinguished identifiers. The GTCSP term

$$P[A_1/x_1, \dots, A_n/x_n] \text{ or shortly } P[\tilde{A}/\tilde{x}]$$

arises from P by substituting each free occurrence of the identifiers $x_1, \dots, x_n$ in P simultaneously by the GTCSP terms $A_1, \dots, A_n$.

3. Transition systems

3.1 Definition :
$A = (S, L, \rightarrow, q_0)$ is called a *(labelled) transition system* iff

(a) S is a set of *states*.

(b) L is a set of *labels*.

(c) $\rightarrow \subseteq S \times L \times S$, where we will write $p \xrightarrow{\alpha} q$ instead of $(p, \alpha, q) \in \rightarrow$.

(d) $q_0 \in S$, q_0 is called the *initial state* of A.

3.2 Definition :
Two equally labelled transition systems $A_i = (S_i, L, \rightarrow_i, q_i)$, $i = 1, 2$, are *bisimular* $(A_1 \approx A_2)$ if there exists a *bisimulation* R between A_1 and A_2, i.e. a relation $R \subseteq S_1 \times S_2$ with $(q_1, q_2) \in R$ and, for all $(p, q) \in R$:

1. Whenever $p \xrightarrow{\alpha}_1 p'$ for some $p' \in S_1$ then there exists some $q' \in S_2$ with $(p', q') \in R$ and $q \xrightarrow{\alpha}_2 q'$

and symmetrically

2. whenever $q \xrightarrow{\alpha}_2 q'$ for some $q' \in S_2$ then there exists some $p' \in S_1$ with $(p', q') \in R$ and $p \xrightarrow{\alpha}_1 p'$.

4. An interleaving transition system based description for guarded TCSP

4.1 Definition :

Let $\rightarrow$ be the binary relation on TCSP that is defined as follows :

(a) **Prefixing**
$\alpha.P \xrightarrow{\alpha} P$

(b) **Internal nondeterminism**
$P \text{ or } Q \xrightarrow{\tau} P$, $P \text{ or } Q \xrightarrow{\tau} Q$

(c) **External nondeterminism**
External choice : $\dfrac{P \xrightarrow{\alpha} P'}{P \,\Box\, Q \xrightarrow{\alpha} P'}$, $\dfrac{Q \xrightarrow{\alpha} Q'}{P \,\Box\, Q \xrightarrow{\alpha} Q'}$, where $\alpha \neq \tau$.

Internal choice : $\dfrac{P \xrightarrow{\tau} P'}{P \sqcap Q \xrightarrow{\tau} P' \sqcap Q}$, $\dfrac{Q \xrightarrow{\tau} Q'}{P \sqcap Q \xrightarrow{\tau} P \sqcap Q'}$.

(d) **Parallel composition**

Synchronisation case : $\dfrac{P \xrightarrow{\alpha} P' \ , \ Q \xrightarrow{\alpha} Q'}{P \parallel_A Q \xrightarrow{\alpha} P' \parallel_A Q'}$, where $\alpha \in A$.

Independent execution (modelled by interleaving):

$\dfrac{P \xrightarrow{\alpha} P'}{P \parallel_A Q \xrightarrow{\alpha} P' \parallel_A Q}$, $\dfrac{Q \xrightarrow{\alpha} Q'}{P \parallel_A Q \xrightarrow{\alpha} P \parallel_A Q'}$, where $\alpha \notin A$.

(e) **Hiding**

$\dfrac{P \xrightarrow{\beta} P'}{P \setminus \beta \xrightarrow{\tau} P' \setminus \beta}$, $\dfrac{P \xrightarrow{\alpha} P'}{P \setminus \beta \xrightarrow{\alpha} P' \setminus \beta}$, where $\alpha \neq \beta$.

(f) **Recursion**

$\dfrac{P[fix\ x.P/x] \xrightarrow{\alpha} Q}{fix\ x.P \xrightarrow{\alpha} Q}$.

(g) **Divergence**

$div \xrightarrow{\tau} div$.

An interleaving model of a closed GTCSP term P is the transition system

$$A(\ P\ )\ =\ (\ GTCSP,\ Act,\ \rightarrow,\ P\ ).$$

4.2 Definition :

For $P, Q \in GTCSP$ and $\omega \in Comm^*$, we define :

$P \overset{\omega}{\Rightarrow} Q$, iff there exists a sequence

$$P = P_1 \xrightarrow{\alpha_1} P_2 \xrightarrow{\alpha_2} \ldots \xrightarrow{\alpha_n} P_{n+1} = Q$$

where $n \geq 0$ and ω results from $\alpha_1 \ldots \alpha_n \in Act^*$ by skipping all occurrences of τ.

We call Q a *derivative* of P .

Let P be a closed GTCSP term. Then, the transition system

$$O(\ P\ )\ =\ (\ GTCSP, Comm^*, \Rightarrow, P\ )$$

gives an operational semantics for P that specifies only the observable behaviour of the process P.

5. Labelled event structures

5.1 Definition :

$\varepsilon\ =\ (E, \leq, \#, l)$ is called a *(labelled) event structure* iff

(a) E is a set (of *events*),

(b) $\leq$ is a partial order on E,

(c) $\#$ is an irreflexive, symmetric relation on E , called *conflict relation* , with :

$$\forall e_1, e_2, e_3 \in E :\ (e_1 \leq e_2 \text{ and } e_1 \# e_3\ ) \implies e_2 \# e_3,$$

(d) $l : E \rightarrow Act$, where Act is the alphabet of *actions (labelling functions)*.

5.2 Definition :
Let $\varepsilon = (E, \leq, \#, l)$ be an event structure, $E' \subset E$, $e \in E$.

(a) $\#(e) := \{ e' \in E : e' \# e \}$.

(b) $\#(E') := \bigcup_{e \in E'} \#(e)$.

(c) $\downarrow e := \{ e' \in E : e' \leq e \text{ and } e' \neq e \}$ is called the *preset of e* .

5.3 Definition :
Let $\varepsilon = (E, \leq, \#, l)$ be an event structure, $e \in E$.

$$depth(e) = \begin{cases} 1 & : \text{ if } \downarrow e = \emptyset \\ max\{depth(e') : e' \in \downarrow e\} + 1 & : \text{ if } \downarrow e \text{ is finite} \\ \infty & : \text{ otherwise} \end{cases}$$

5.4 Definition :
An event structure $\varepsilon = (E, \leq, \#, l)$ is called *(finitely) approximable* iff

(a) for each $e \in E$, $depth(e)$ is finite and

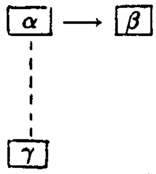
(b) for each $n \in N, \{e \in E : depth(e) = n\}$ is finite.

Ev denotes the set of all finetely approximable event structures where we abstract from the names of the events, i.e. we will not distinguish isomorphic event structures. Two event structures $\varepsilon_i = (E_i, \leq_i, \#_i, l_i)$, $i = 1,2$ are *isomorphic* if there exists a bijective mapping $f : E_1 \rightarrow E_2$ so that

1. $e_1 \leq_1 e_2 \iff f(e_1) \leq_2 f(e_2) \;\; \forall e_1, e_2 \in E_1$,
2. $e_1 \#_1 e_2 \iff f(e_1) \#_2 f(e_2) \;\; \forall e_1, e_2 \in E_1$ and
3. $l_2(f(e)) = l_1(e) \;\; \forall e \in E_1$.

Event structures can be depicted graphically by representing events as boxes (inscribed with the event label) and connecting them with their direct predecessors and successors.
A conflict between two events is a *direct* conflict if no predecessors of the events are in conflict. Direct conflicts are depicted graphically by a broken line .

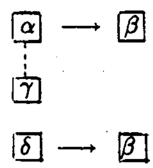
Example :
The event structure $\varepsilon = (E, \leq, \#, l)$ with
$E = \{e_1, e_2, e_3\}$, $e_1 \leq e_2$, $e_1 \# e_3$, $e_2 \# e_3$ and
$l(e_1) = \alpha$, $l(e_2) = \beta$, $l(e_3) = \gamma$ is shown as



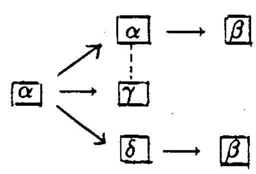
6. Composition operations for event structures

The event structure semantics for GTCSP to be defined is compositional, which means that composition operators corresponding to the syntactical operators prefix, *or* , $\Box$, $\|_A$, $\backslash\beta$ and fix have to be defined . This has been done in [11], we will here explain examples only and refer for the precise definitions to the appendix.

6.1 Example : Prefixing
$\alpha.\varepsilon$ describes a process that first performs α and then behaves like ε. If ε is

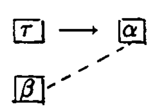


then $\alpha.\varepsilon$ is



6.2 Example : $\Box$ - choice

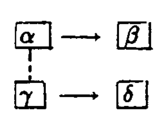
Let ε_1 be $\boxed{\tau} \longrightarrow \boxed{\alpha}$ and ε_2 be $\boxed{\beta}$. Then $\varepsilon_1 \Box \varepsilon_2$ is given by



which describes that ε_1 may perform its τ-actions independently and that a decision has to take place as soon as communications are involved.

6.3 Example : $\Box$ - choice

Let ε_1 be $\boxed{\alpha} \longrightarrow \boxed{\beta}$ and ε_2 be $\boxed{\gamma} \longrightarrow \boxed{\delta}$, then $\varepsilon_1 \Box \varepsilon_2$ is



describing external choice.

6.4 Example : *or*-choice
The *or*-choice reflects internal nondeterminism .

Let ε_1 be $\boxed{\tau} \longrightarrow \boxed{\alpha}$ and ε_2 be $\boxed{\beta}$.

Then ε_1 *or* ε_2 is given by

$$\begin{tikzcd}
\boxed{\tau} \arrow[r] \arrow[d, dash, dashed] & \boxed{\tau} \arrow[r] & \boxed{\alpha} \\
\boxed{\tau} \arrow[r] & \boxed{\beta}
\end{tikzcd}$$.

The internal character of the *or* -choice is modelled by prefixing the respective event structures with internal actions and by imposing a conflict between these internal actions .

6.5 Example : Parallel composition $||_A$

Let ε_1 be $\boxed{\beta} \longrightarrow \boxed{\alpha} \longrightarrow \boxed{\gamma}$ and ε_2 be $\boxed{\delta} \longrightarrow \boxed{\alpha} \longrightarrow \boxed{\rho}$, then $\varepsilon_1 \; ||_{\{\alpha\}} \; \varepsilon_2$ is given by

$$\begin{tikzcd}
\boxed{\beta} \arrow[dr] & & \boxed{\gamma} \\
& \boxed{\alpha} \arrow[ur] \arrow[dr] & \\
\boxed{\delta} \arrow[ur] & & \boxed{\rho}
\end{tikzcd}$$

6.6 Example : Hiding

Let ε be

$$\begin{tikzcd}
\boxed{\alpha} \arrow[r] \arrow[d, dash, dashed] & \boxed{\beta} \\
\boxed{\beta} &
\end{tikzcd}$$,

then $\varepsilon \setminus \beta$ is

$$\begin{tikzcd}
\boxed{\alpha} \arrow[r] \arrow[d, dash, dashed] & \boxed{\tau} \\
\boxed{\tau} &
\end{tikzcd}$$,

i.e. hiding transforms actions labelled by β into actions labelled by τ .

7. The metric space of finite approximable event structures

In this section we will define a metric d on finite approximable event stuctures. [15] have shown that (Ev, d) is a complete ultametric space. Thus, every Banach-contractive mapping $\Phi : Ev \rightarrow Ev$ has a unique fixpoint in Ev.

7.1 Definition :
Let $\varepsilon, \varepsilon' \in Ev, n \in N, \varepsilon = (E, \leq, \#, l)$.

(a) The *truncation* of ε (of the depth n) is defined as follows :
$$\varepsilon^n \;:=\; (E^n, \leq|_{E^n}, \#|_{E^n}, l|_{E^n})$$
where $E^n \;:=\; \{e \in E : depth(e) \leq n\}$.

(b) The distance between the event structures $\varepsilon, \varepsilon'$ is defined by
$$d(\varepsilon, \varepsilon') = 0 \quad :\Longleftrightarrow \quad \varepsilon = \varepsilon'$$
$$d(\varepsilon, \varepsilon') = \tfrac{1}{2^n} \quad :\Longleftrightarrow \quad \varepsilon \neq \varepsilon' \text{ and } n = max\{i : \varepsilon^i = \varepsilon'^i\}.$$
We recall that we deal with isomorphism class of event structures, i.e. we abstract of the names of the events $e \in E$. It is clear that the distance $d(\varepsilon, \varepsilon')$ is independent of chosen representatives.

7.2 Definition :
Let $Env := \{\sigma : \; \sigma : Idf \rightarrow Ev\}$ the set of *environments*. These are mappings which assign a meaning to free identifiers of a term.

For $\varepsilon_1, \ldots, \varepsilon_n \in Ev$, we define $\sigma[\varepsilon_1/x_1, \ldots, \varepsilon_n/x_n] : Idf \rightarrow Ev$ by
$$x_i \mapsto \varepsilon_i, \qquad i = 1, \ldots, n,$$
$$y \mapsto \sigma(y) \qquad \text{if } y \notin \{x_1, \ldots, x_n\}.$$

Let $\Phi : \; GTCSP \times Env \times Id \;\rightarrow\; (Ev \rightarrow Ev)$ be given by
$$\Phi(P, \sigma, x)(\varepsilon) \;:=\; M[P]\sigma[\varepsilon/x],$$

where M is the meaning function
$$M \;:\; GTCSP \times Env \;\rightarrow\; Ev$$

given by :

Let $\sigma \in Env,\; \alpha \in Act,\; \beta \in Comm,\; A \subseteq Comm,\; P, P_1, P_2 \in GTCSP$.

(a) $M[x]\sigma \;:=\; \sigma(x)$ where $x \in Idf$.

(b) $M[\alpha.P]\sigma \;:=\; \alpha.M[P]\sigma$.

(c) $M[P \setminus \beta]\sigma \;:=\; M[P]\sigma \setminus \beta$.

(d) $M[P_1 \Box P_2]\sigma \;:=\; M[P_1]\sigma \;\Box\; M[P_2]\sigma$.

(e) $M[P_1 or P_2]\sigma \;:=\; M[P_1]\sigma \; or \; M[P_2]\sigma$.

(f) $M[P_1 \|_A P_2]\sigma \;:=\; M[P_1]\sigma \;\|_A\; M[P_2]\sigma$.

(g) $M[fix\; x.P]\sigma \;:=\; fix\; \Phi(P, \sigma, x)$
where $fix\; \Phi(P, \sigma, x)$ denotes the unique fixpoint of the Banach - contractive mapping $\Phi(P, \sigma, x)$. See [11],where it has been shown that $\Phi(P, \sigma, x)$ is Banach - contractive .

Lemma 1 :
Let $x \in Idf$ be guarded in $P \in GTCSP$.

(a) $\sigma_1, \sigma_2 \in Env, \sigma_1(y) = \sigma_2(y) \forall y \in Idf \setminus \{x\} \implies fix\Phi(P, \sigma_1, x) = fix\Phi(P, \sigma_2, x)$.

(b) If P is closed then $fix\Phi(P, \sigma, x)$ is independent of the environment σ.

(c) Let $x_1, \ldots, x_n$ be pairwise different identifiers, $A_1, \ldots, A_n \in GTCSP$, then :

$$M[P[A_1/x_1, \ldots, A_n/x_n]]\sigma = M[P]\sigma[\,M[A_1]\sigma/x_1, \ldots, M[A_n]\sigma/x_n\,].$$

Proof:

(a) follows immediately from the definition of Φ.

(b) is clear .

(c) By structural induction on the syntax of P.

Lemma 2 :
Let $P, B, A_1, \ldots, A_n \in GTCSP$ and let $x_1, \ldots, x_n, y \in Idf$ be pairwise different identifiers,so that y does not occur free in $A_1, \ldots, A_n$. Then,

$$P[A_1/x_1, \ldots, A_n/x_n, B[\tilde{A}/\tilde{x}]/y] = P[B/y][\tilde{A}/\tilde{x}].$$

Proof :
By induction on the syntax of P .

Lemma 3 :
Let $P \in GTCSP$. Then, for all $x_1, \ldots, x_n \in Idf$ pairwise different identifiers, which are guarded in P , and for all $A_1, \ldots, A_n \in GTCSP$:
If $P[A_1/x_1, \ldots, A_n/x_n] \xrightarrow{\alpha} Q$, then there exists $P' \in GTCSP$ with

1. $P \xrightarrow{\alpha} P'$ and
2. $P'[A_1/x_1, \ldots, A_n/x_n] = Q$.

Proof :
By induction on the syntax of P .

Remark :
Let $P, Q, A_1, \ldots, A_n \in GTCSP$ and $x_1, \ldots, x_n \in Idf$ be pairwise different identifiers which are guarded in P so that $P[\tilde{A}/\tilde{x}] \xrightarrow{\alpha} Q$.
Then, there exists $P' \in GTCSP$ with

1. $P \xrightarrow{\alpha} P'$ and
2. $P'[\tilde{A}/\tilde{x}] = Q$.

It is easy to see that for all terms $B_1, \ldots, B_n \in GTCSP$:

$$P[\tilde{B}/\tilde{x}] \xrightarrow{\alpha} P'[\tilde{B}/\tilde{x}].$$

Remark :
If $A \in GTCSP$ is closed then

$$M[A]\sigma_1 = M[A]\sigma_2 \quad \forall\, \sigma_1, \sigma_2 \in Env.$$

So, we can define

$$M[A] := M[A]\sigma \quad \text{where } \sigma \in Env.$$

7.3 Definition :
For $\omega = \alpha_1 \ldots \alpha_n \in Act^*$, we define $\hat{\omega}$ to be the word in $Comm^*$ which arises from ω by eliminating all actions labelled by τ.
I.e., $\hat{\omega} = \alpha_{i_1} \ldots \alpha_{i_k}$ where $1 \le i_1 < \ldots < i_k \le n$ are the indices $i \in \{1, \ldots, n\}$ with $\alpha_i \in Comm$.

7.4 Definition :

(a) Let $\mu \in Act$, ε, $\varepsilon' \in Ev$, $\varepsilon = (E, \le, \#, l)$. The transition relation $\rightarrow \subseteq Ev \times Act \times Ev$ on event structures is defined by :
$\varepsilon \xrightarrow{\mu} \varepsilon'$ iff there exists some event $e \in E$ with $depth(e) = 1$, $l(e) = \mu$ and $\varepsilon' = (E', \le|_{E'}, \#|_{E'}, l|_{E'})$ where $E' = E \setminus (\{e\} \cup \#(e))$.

(b) When we abstract from τ-events we get the transition relation $\Rightarrow \subseteq Ev \times Comm^\star \times Ev$:
$\varepsilon \overset{\omega}{\Rightarrow} \varepsilon'$ iff there exists a sequence

$$\varepsilon = \varepsilon_1 \xrightarrow{\mu_1} \varepsilon_2 \xrightarrow{\mu_2} \ldots \xrightarrow{\mu_n} \varepsilon_{n+1} = \varepsilon'$$

where $n \ge 0, \mu_1, \ldots, \mu_n \in Act$ and $\omega \in Comm^\star$ results from $\mu_1\mu_2 \ldots \mu_n$ by removing all $\mu_i = \tau$.

(c) The *(observable) interleaving semantics* of $\varepsilon \in Ev$ is defined as the transition system

$$O(\varepsilon) = (Ev, Comm^\star, \Rightarrow, \varepsilon).$$

7.5 Definition :
The event structures $\varepsilon_1, \varepsilon_2$ are called *τ-equivalent* , written $\varepsilon_1 \approx_\tau \varepsilon_2$, iff there exists event structures ε, Int_1, Int_2 , where all events in Int_1, Int_2 are labelled by τ , with $\varepsilon_i = \varepsilon \,||_\emptyset\, Int_i$, $i = 1, 2$.

It is easy to see that τ-equivalence is an equivalence relation on Ev. [11] have shown that if $\varepsilon_1 \approx_\tau \varepsilon_2$ and $\varepsilon_1 \overset{\omega}{\Rightarrow} \varepsilon_1'$ then there exists $\varepsilon_2' \in Ev$ with $\varepsilon_1' \approx_\tau \varepsilon_2'$ and $\varepsilon_2 \overset{\omega}{\Rightarrow} \varepsilon_2'$.

Lemma 4 :
Let $P \in GTCSP$, $\alpha \in Act$, $\sigma \in Env$.

(a) If $P \xrightarrow{\alpha} P'$ then $M[P]\sigma \overset{\hat{\alpha}}{\Rightarrow} M[P']\sigma$.

(b) If $x_1, \ldots, x_n$ be the pairwise different identifiers that occur free in P and if $\sigma(x_i) = M[A_i]$ where A_i is a closed GTCSP term, $i = 1, \ldots, n$, then, for all event structures $\varepsilon' \in Ev$ with $M[P]\sigma \xrightarrow{\alpha} \varepsilon'$, there exists a term $P' \in GTCSP$ with
1. $P[\, A_1/x_1, \ldots, A_n/x_n \,] \overset{\hat{\alpha}}{\Rightarrow} P'$ and
2. $M[P']\sigma \approx_\tau \varepsilon'$.

Proof : We will prove the statements by induction on the structure of P .

(a) We assume that $P \xrightarrow{\alpha} P'$. Basis of induction :
1. $P = stop$ has no derivatives.
2. $P = div$, then $\alpha = \tau$ and $P' = div$, $M[P]\sigma = M[P']\sigma$.
3. $P = x \in Idf$, then P has no derivatives.

Induction step :

The most interesting operator is the fix- operator.
$P = fix\ x.Q$, then $Q[fix\ x.Q/x] \xrightarrow{\alpha} P'$.
By Lemma 3, there exists $Q' \in GTCSP$ with $Q \xrightarrow{\alpha} Q'$ and $Q'[fix\ x.Q/x] = P'$.
By induction hypothesis :

$$M[Q]\sigma[\ {\scriptstyle M[P]\sigma/x}\ ] \overset{\hat{\alpha}}{\Rightarrow} M[Q']\sigma[\ {\scriptstyle M[P]\sigma/x}\ ]$$

On the other side, we have :

$$M[P]\sigma \;=\; fix\Phi(Q,\sigma,x) \;=\; M[Q]\sigma[\ {\scriptstyle M[P]\sigma/x}\ ]$$

and

$$M[P']\sigma \;=\; M[Q'[fix\ x.Q/x]]\sigma \;=\; M[\ Q'[P/x]\ ]\sigma \;=\; M[Q']\sigma[\ {\scriptstyle M[P]\sigma/x}\ ]$$

(Lemma 1c). Then, $M[P]\sigma \overset{\hat{\alpha}}{\Rightarrow} M[P']\sigma$.

(b) Induction step :

Again, we only consider the fix - operator : $P = fix\ x.Q$.
The identifiers occuring free in Q are $x_1, \ldots, x_n$ and x . We get :

$$M[P]\sigma \;=\; fix(Q,\sigma,x) \;=\; \Phi(Q,\sigma,x)(\ M[P]\sigma\ ) \;=\; M[Q]\sigma[\ {\scriptstyle M[P]\sigma/x}\ ]$$

and

$$M[P]\sigma \;=\; M[\ P[A_1/x_1,\ldots,A_n/x_n]\ ]$$

(by Lemma 1c).

Hence

$$\sigma[\ {\scriptstyle M[P]\sigma/x}\ ](x) \;=\; M[P]\sigma \;=\; M[\ P[\tilde{A}/\tilde{x}]\ ] \quad \text{and}$$
$$\sigma[\ {\scriptstyle M[P]\sigma/x}\ ](x_i) \;=\; \sigma(x_i) \;=\; M[A_i]\ ,\ i = 1,\ldots,n.$$

Since $M[P]\sigma \xrightarrow{\alpha} \varepsilon'$, we get by Lemma 1c and Lemma 2 :

$$M[Q]\sigma[\ {\scriptstyle M[P]\sigma/x}\ ] \xrightarrow{\alpha} \varepsilon'.$$

By induction hypothesis, there exists $P' \in GTCSP$ with

$$Q[\ A_1/x_1,\ldots,A_n/x_n, P[\tilde{A}/\tilde{x}]/x\ ] \overset{\hat{\alpha}}{\Rightarrow} P' \quad \text{and} \quad M[P']\sigma \approx_\tau \varepsilon'.$$

Since the terms $A_1, \ldots, A_n$ are closed, we get :

$$Q[\tilde{A}/\tilde{x}][\ P[\tilde{A}/\tilde{x}]/x\ ] \;=\; Q[\ A_1/x_1,\ldots,A_n/x_n, P[\tilde{A}/\tilde{x}]/x\ ].$$
$$\Longrightarrow \quad Q[\tilde{A}/\tilde{x}][\ fix\ x.Q[\tilde{A}/\tilde{x}]/x\ ] \overset{\hat{\alpha}}{\Rightarrow} P'$$
$$\Longrightarrow \quad P[\tilde{A}/\tilde{x}] = fix\ x.Q[\tilde{A}/\tilde{x}] \overset{\hat{\alpha}}{\Rightarrow} P' \quad .$$

Corollary :

Let $R := \{ (P,\varepsilon) : P \in GTCSP, P$ closed, $\varepsilon \in Ev,\ \varepsilon \approx_\tau M[P] \}$.
Then R is a bisimulation.

Proof :
Let $(P,\varepsilon) \in R$.
1. When $P \xrightarrow{\alpha} P'$, so we have by Lemma 4a :

$$M[P] \stackrel{\hat{\alpha}}{\Rightarrow} M[P'].$$

Since $\varepsilon \approx_\tau M[P]$, there exists $\varepsilon' \in Ev$ with

$$\varepsilon \stackrel{\hat{\alpha}}{\Rightarrow} \varepsilon' \text{ and } \varepsilon' \approx_\tau M[P'].$$

Then $(P',\varepsilon') \in R$.

2. When $\varepsilon \xrightarrow{\alpha} \varepsilon'$, then there exists $\varepsilon'' \in Ev$ with

$$M[P] \stackrel{\hat{\alpha}}{\Rightarrow} \varepsilon'' \text{ and } \varepsilon' \approx_\tau \varepsilon''.$$

By Lemma 4b, it is easy to show that there exists $P' \in GTCSP$, P' closed, with

$$P \stackrel{\hat{\alpha}}{\Rightarrow} P' \ ,\ M[P'] \approx_\tau \varepsilon''$$

Then, $M[P'] \approx_\tau \varepsilon'$ and $(P',\varepsilon') \in R$.

Theorem :

For every closed $P \in GTCSP$(i.e. every guarded process), the transition systems $O(P)$ and $O(M[P])$ are bisimular.

8. Conclusion

We have shown that an interleaving specification of a GTCSP process P and a noninterleaving meaning of P are 'bisimular'. One difficulty in establishing such a result, in particular when including recursion via the fix-operator, is, that a compositional semantics that provides semantic operators for the syntactical constructs, is compared with an operational semantics using a transition system . Hence, in order to establish a relation between the two meanings of a process P we may not simply perform an induction on the structure of P . In particular, in the case of recursion, we have no operator that determines the 'meaning' of $fix\ x.Q$ from the 'meaning' of Q in the transition system case.
Our proof works by obtaining information on the behaviour of a process P from the knowledge of the behaviour of $P[\tilde{A}/\tilde{x}]$, see lemma 3 and lemma 4 .
The obtained theorem may be interpreted as a consistency result.
Consistency problems concerning noninterleaving and interleaving models are discussed in [9,18,20]. These investigations differ from the present work in particular in the noninterleaving model (petri nets, prime event structures) and / or in the language studied and in the proof method .

Appendix

This section gives the operations for finite approximable event structures modelling the operations of GTCSP as defined in [11] .

A.1 Definition :
Let $stop \in Ev$ to be defined as

$$stop := (\emptyset, \emptyset, \emptyset, \emptyset).$$

A.2 Definition :
Let $\varepsilon = (E, \leq, \#, l) \in Ev, \alpha \in Act, e_0 \notin E$. Then , the event structure $\alpha.\varepsilon$ will describe a process which first performs α and then behaves like ε.

$$\alpha.\varepsilon = (E', \leq', \#', l')$$

where

1. $E' = E \cup \{e_0\}$,
2. $e_1 \leq' e_2 \iff e_1 = e_0 \;\; or \;\; (e_1, e_2 \in E \;\&\; e_1 \leq e_2)$
3. $e_1 \#' e_2 \iff e_1, e_2 \in E \;\&\; e_1 \# e_2$
4. $l' : E' \rightarrow Act$ is defined by $l'(e) = l(e)$, if $e \in E$, and $l'(e_0) = \alpha$.

A.3 Definition :
For $\varepsilon = (E, \leq, \#, l) \in Ev$,we define the set of *initial internal events* by

$$In(\varepsilon) := \{e \in E : \forall e' \in E, e' \leq e : l(e') = \tau\}$$

.

A.4 Definition :
Let $\varepsilon_i = (E_i, \leq_i, \#_i, l_i) \in Ev, i = 1, 2$, w.l.o.g. $E_1 \cap E_2 = \emptyset$. The *conditional composition* of ε_1 and ε_2 is defined by

$$\varepsilon_1 \,\Box\, \varepsilon_2 := (E, \leq, \#, l)$$

where

1. $E = E_1 \cup E_2$
2. $\leq \; = \; \leq_1 \cup \leq_2$
3. $e_1 \# e_2 \iff (e_1, e_2 \in E_1 \;\&\; e_1 \#_1 e_2) \;\; or \;\; (e_1, e_2 \in E_2 \;\&\; e_1 \#_2 e_2) \;\; or$ $(e_1 \in In(\varepsilon_1) \;\&\; e_2 \in In(\varepsilon_2)) \;\; or \;\; (e_1 \in In(\varepsilon_2) \;\&\; e_2 \in In(\varepsilon_1))$
4. $l : E \rightarrow Act, l(e) = l_i(e)$ if $e \in E_i$, $i = 1, 2$.

$\varepsilon_1 \,\Box\, \varepsilon_2$ describes the process which behaves like one of the event structures ε_1 or ε_2 where the decision which alternative is left open as long as only internal actions are being performed.

A.5 Definition :
Let $\varepsilon_i = (E_i, \leq_i, \#_i, l_i) \in Ev, i = 1, 2$, w.l.o.g. $E_1 \cap E_2 = \emptyset$. The *nondeterministic combination* of ε_1 and ε_2 is defined by

$$\varepsilon_1 \; or \; \varepsilon_2 := (E, \leq, \#, l)$$

where

1. $E \;=\; E_1 \cup E_2 \cup \{f_1, f_2\}$, $f_1, f_2 \notin E_1 \cup E_2$
2. $e_1 \leq e_2 \iff (e_1, e_2 \in E_i \;\&\; e_1 \leq_i e_2,\; i = 1 \text{ or } i = 2)$ or
$(e_i = f_i \;\&\; e_2 \in E_i,\; i = 1 \textit{ or } i = 2) \textit{ or } e_1 = e_2$
3. $\#$ is the symmetric closure of $\#_1 \cup \#_2 \cup ((E_1 \cup \{f_1\}) \times (E_2 \cup \{f_2\}))$
4. $l : E \to Act$, $l(e) = l_i(e)$ if $e \in E_i$ and $l(f_i) = \tau, i = 1, 2$.

The nondeterministic combination ε_1 *or* ε_2 behaves like ε_1 or like ε_2 where an internal decision choose the alternative .

A.6 Definition :
Let $\varepsilon_i = (E_i, \leq_i, \#_i, l_i) \in Ev,\; i = 1, 2$ and $A \subseteq Comm$.

1. The *syntactical communication* of ε_1 and ε_2 on A is defined by

$$\begin{aligned} Comm_A(\varepsilon_1, \varepsilon_2) := \{ (e, \star) :\; & e \in E_1 \;\&\; l_1(e) \notin A \\ \text{or } & e \in E_2 \;\&\; l_2(e) \notin A \} \\ \cup \{ (e_1, e_2) \in & E_1 \times E_2 : l_1(e_1) = l_2(e_2) \in A \} . \end{aligned}$$

There $\star$ is an auxiliary symbol, $\star \notin E_1 \cup E_2$.

2. Two communications $(e_1, e_2), (e'_1, e'_2) \in Comm_A(\varepsilon_1, \varepsilon_2)$ are *in conflict* iff they contain conflicting events, i.e. $e_1 \#_1 e'_1$ or $e_2 \#_2 e'_2$, or one event communicates with two distinct events, i.e. $(e_1 = e'_1 \wedge e_2 \neq e'_2)$ or $(e_2 = e'_2 \wedge e_1 \neq e'_1)$.
3. A subset C of $Comm_A(\varepsilon_1, \varepsilon_2)$ is *conflict-free* iff no two communications in C are in conflict.
4. Let $C \subseteq Comm_A(\varepsilon_1, \varepsilon_2)$ be conflict- free, $(e_1, e_2), (f_1, f_2) \in C$.

   (a) The relation $\prec$ is defined by

   $$(e_1, e_2) \prec (f_1, f_2) \iff ((e_1 \leq f_1) \wedge \neg(e_2 > f_2)) \text{ or } ((e_2 \leq f_2) \wedge \neg(e_1 > f_1)).$$

   We say (e_1, e_2) *precedes* (f_1, f_2) if $(e_1, e_2) \prec (f_1, f_2)$.

   (b) C is called *complete* iff
   $\forall (e_1, e_2) \in C, \forall f_1 \in E_1$ with $f_1 \leq_1 e_1$ there exists $(e_2, f_2) \in C$ with

   $$(f_1, f_2) \prec (e_1, e_2)$$

   and symmetrically
   $\forall (e_1, e_2) \in C, \forall f_2 \in E_2$ with $f_2 \leq_2 e_2$ there exists $(f_1, f_2) \in C$ with

   $$(f_1, f_2) \prec (e_1, e_2).$$

   (c) C is called *cycle-free* iff the transitive closure of $\prec$ is antisymmetric.
5. The *parallel composition* of ε_1 and ε_2 with communication on $A \subseteq Comm$ is given by

$$\varepsilon_1 \parallel_A \varepsilon_2 \;:=\; (E, \leq, \#, l)$$

where

   (a) $E = \{ C_{(e_1, e_2)} : C_{(e_1, e_2)} \subseteq Comm_A(\varepsilon_1, \varepsilon_2)$ is conflict-free, cycle-free, complete and $(e_1, e_2) \in C_{(e_1, e_2)}$ is the only maximal element (with respect to $\overset{*}{\prec}$) $\}$.

   (b) $\leq \;=\; \subseteq$

   (c) $\# \;=\; \{ (C_1, C_2) \in E \times E : \exists (e_1, e_2) \in C_1, (f_1, f_2) \in C_2$ with $(e_1, e_2), (f_1, f_2)$ in conflict $\}$

(d) $l : E \to Act$, $l(C_{(e_1,e_2)}) = label(e_1, e_2)$
where $label(e_1, e_2) = l_1(e_1)$ if $e_1 \in E_1$ and $label(e_1, e_2) = l_2(e_2)$ if $e_2 \in E_2$.

The parallel composition $\varepsilon_1 \ ||_A \ \varepsilon_2$ describes the independent execution of ε_1 and ε_2 where the actions of A may only be executed as joint actions by both processes together. In particular, $||_\emptyset$ stand for fully independent execution (without synchronisation), and on the other extrem , $||_{Comm}$ only allows actions which are performed in common.

A.7 Definition :
Let $\varepsilon = (E, \leq, \#, l) \in Ev, \beta \in Comm.$

$$\varepsilon \setminus \beta \ := \ (E, \leq, \#, l')$$

where $l' : E \to Act$, $l'(e) = l(e)$ if $l(e) \neq \beta$ and $l'(e) = \tau$ otherwise.

The hiding operator transforms the actions labelled by β into internal actions, i.e. τ-events.

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