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Neurons, Knowledge Hypernets, and Information Flow

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Contents

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Abstract

In [VS2007] we described a model of a neuron and the G/<V, E>/F system for modelling nets of neurons. There we also described information flow in the invariant knowledge hypernet (KH) subnets of the hypernet <V, E>. We introduced weights for the edges of the KH in <V, E>, and weights and capacities for the derivation paths in such a KH. We mentioned that this could lead to application of Menger's Theorem to information flow through a KH, and in this paper we deal briefly with that topic in the G/<V, E>/F system model. We deduce the transport network max-flow, min-cut theorem of Ford and Fulkerson for information flow in <V, E> - [FF1962] - and discuss the meaning of information flow. All the required background can be found in [GVS1999], [VGS2004], and [VS2007].

1. Introduction

In [VS2007] we described a simple model of a neuron, invariant KH subnets of the hypernet <V, E> in which V is the set of nuclei of the neurons, information flow through invariant KH subnets, and the teaching/training of the G/<V, E>/F system model. Information flow is induced by activation sweeps in the "hardware" digraph G, and is controlled via the feedback F from the output node O to the input node I through the environment/programmer, and then on through G and <V, E> back to O. <V, E> is the entirely theoretical interpretational model in the system.

An edge for a and b in <V, E>, with derivation direction from a to b, is labelled with a minimal set of nuclei, of V, that must fire, in G, simultaneously, to fire a, and a, and a minimal set of nuclei that must fire, later, simultaneously, to fire b, and b. An edge that has this set of vertices will "conduct" flow if these nuclei fire appropriately. Each such edge is entered individually onto a line between a and b in a diagram of <V, E>. A line in the diagram of a hypernet can have more than one label – see [VGS2004].

By the flow capacity, or simply the capacity, of a line we mean the number of labels on that line. The weight of a line – see [VS2007] – is at least the capacity of that line. If we delete any one or more members of an edge then that edge disappears and the capacity of the relevant line decreases by 1. If all the labels disappear, or if, for an edge from a to b, a or b or both are deleted, then that line disappears. Strong vertex vulnerability means that several edges, and possible lines as well, can disappear on the deletion of even just one vertex/nucleus from V, because all the edges in the context schema of each deleted vertex are then deleted – see [VGS2004]. Every edge lies on at least one derivation path, from a primary to a goal, in each KH in <V, E>. If we delete an edge then we interrupt all those derivation paths which use that edge. This leads to the notion of a separation, also called a cut.

2. Separations and Menger's Theorem

By an edge separation in a KH we mean a set of edges which, if all deleted, will cut all derivation paths, from primaries to goals, in that KH. We will be concerned mainly with minimal separations, i.e. separations which are such that if we join any one edge of the separation back in to the hypernet then there will be at least one derivation path in the resulting hypernet.

Flow from the set of all primaries, *P*, to the set of all goals, *G*, in a KH <A, D> follows paths in the diagram of <A, D>. We develop a line separation for *P* and *G* in <A, D> i.e. for <A, D>, as follows. Choose any path from a member of *P* to a member of *G* in the diagram of <A, D>. Choose any line q_o on that path, and find the set S_o of all the paths from *P* to *G* on which that line lies. Delete all the lines of each member of S_o from <A, D>.Consider now the remaining hypernet. Choose any *P* to *G* path in it, if any remain, and choose any line q_1 on that path. Find the set of S_1 of all paths from primaries to goals, in that hypernet, on which q_1 lies. Delete every line on each member of S_1. Continuing in this manner we find a set $q_o, q_1, ..., q_r$ of lines and the corresponding sets S_o, S_1, ..., S_r such that after deleting all the lines of each member of S_r no more paths from a member of P to a member of *G* are left in the resulting sub-hypernet of <A, D>.

q_o, q_1, ... , q_r constitute a line separation for <A, D>, and it is easy to show that the sets S_o, S_1, ..., S_r generate a partition of all the *P* to *G* paths in the diagram of <A, D>.

Two *P* to *G* paths in the diagram of <A, D> are said to be independent iff they belong to two different S_i in a line separation partition of the *P* to *G* paths in that diagram. The measure of a flow is defined to be the number of pairwise independent *P* to *G* paths in the diagram, and this number is dependent upon the line separation developed in each case. It is then easy to show that the following holds.

a) Menger's Theorem for flow in a KH diagram.

The maximum measure of a flow in a KH diagram, i.e. the maximum number of pairwise independent *P* to *G* paths in that diagram = the minimum number of lines in a line separation for that diagram.

Proof. No two independent "flow paths" from *P* to *G* can use the same line. Suppose the maximum number of pairwise independent *P* to *G* paths is greater than the number of S_i sets in a minimum partition. Then there would have to be a set such as S_i that is not in that minimum partition, so that partition is not a minimum one. Suppose that the maximum number of pairwise independent *P* to *G* paths is less than the number of S_i sets in a minimum partition. Then that partition is not minimum.

Each of the independent paths in a flow corresponds with a derivation path family from the relevant member of *P* to the relevant member of *G*, where a derivation path family is the set of all distinct derivation paths that use edges which label the lines on the relevant *P* to *G* path in the diagram of <A, D>. The measure of an information flow from *P* to *G* in <A, D> is defined to be the number of pairwise edge-disjoint *P* to *G* derivation paths in the diagram of <A, D>. Again, this number depends on the line separation chosen, and we have the following. Notice that independent implies edge-disjoint, but the converse is not generally true: the measure of a flow ≤ the measure of an information flow for a given separation.

b) Menger's theorem for information flow in a KH

The maximum measure of an information flow in a KH = the minimum number of pairwise edge-disjoint derivation paths in that KH ≥ the minimum number of lines in a line separation for the diagram of that KH.

Proof: Follows at once from the theorem above.

We can go a little further. Consider a set S_i in a minimum line partition. The edge-disjoint derivation paths in it will all have to use an edge that labels q_i. It follows that the number of edge-disjoint derivation paths in S_i is equal to the number of edges that label q_i. Every one of those derivation paths is edge-disjoint from every derivation path in every other member S_j of the minimum line partition.

We can thus state another version of Menger's Theorem.

Given the lines q_o, q_1, ..., q_r of a separation for KH <A, D>, the capacity of q_i is the number of edges that label it, and the capacity of the separation is the sum of the line capacities over the lines in the separation. We then have the following.

c) Menger's theorem for edge-disjoint derivation paths in a KH.

The maximum number of pairwise edge-disjoint derivation paths in a KH = the minimum capacity of a separation for that KH.

Proof: Follows by a similar argument.

The max-flow, min-cut form for transport networks, a version of Menger's Theorem, was proved by Ford and Fulkerson in 1955 – see [FF1962].Here we put some new faces on it. Some other versions of Menger's Theorem are presented in [GVS1999] and [VGS2004].

By a vertex separation for *P* and *G* in a KH we mean a set of vertices which, if deleted, will cut every derivation path from *P* to *G*. We then have the following version of Menger's Theorem.

The maximum number of edge-disjoint derivation paths in a KH ≥ the minimum number of vertices in a vertex separation for *P* and *G* in that KH.

Proof: Given two edge-disjoint *P* to *G* derivation paths, they can be cut either by one or by two vertices of a minimum vertex separation.

If the two derivation paths are vertex-disjoint but perhaps for their primary and goal, and thus also edge-disjoint, then we have equality in the theorem. (Notice that a vertex belongs to a derivation path iff it is a member of at least one edge on that path; "vertex-disjoint" is a strong condition in a hypernet!)

Consider an edge separation for a KH. Deletion of one vertex from each of it's edges deletes all those edges. Since deletion of a single vertex could delete more than one edge, we have the following for a KH.

d) Menger's Theorem for deletion of vertices that induce an edge separation.

The minimum number of vertices that, if deleted, will cause a minimum edge separation is ≤ the minimum number of edges in an edge separation.

Information flow "chokes" at the lines of a separation of minimum capacity if we regard information flow as proceeding along edge-disjoint derivation paths through those lines. One has to try to deal with these “blockages”. The weight of a line - see [VS2007] – is greater than or equal to the capacity of that line, so if one or all of the edges on a line from vertex a to vertex b are destroyed in a KH one can use other, non-unit length, derivation paths from a to b to try to rehabilitate the KH by enhancing the relationships of the edges on the line in CRKS interpretations. The damage to a KH subnet of <V, E> occurs due to strong vulnerability in a KH. If a vertex/nucleus, and thus a neuron, is damaged or malfunctions or does not fire, i.e. is deleted from the KH, then it deletes every edge of the context schema of that vertex of the KH – see [VGS2004]. Before returning to rehabilitation briefly in section 4, we deal with the question of what information flow in a KH means.

3. The meaning of information flow in a KH

Consider a minimum line partition in a KH. Choose one derivation path from each set in it. We get a set C_0 of pairwise edge-disjoint derivation paths. Delete every edge of each of those paths, and consider what remains in each of the sets of the line partition. Choose, in what remains, a set C_1 as for C_0. Continuing, we get a collection C_0, C_1, ..., C_S that partitions the set of all pairwise edge-disjoint derivation paths in the KH into sets of pairwise edge-disjoint derivation paths. The sets C_0, C_1, ..., C_S carry the information flow for that minimum separation, and thus characterise information flow.

The information flow from *P* to *G* in a KH is carried by the pairwise edge-disjoint derivation paths in that KH. What is the meaning of information flow? In the context of G/<V, E>/F it represents that part of an activation sweep, in G, which “activates” the KH in question – [VS2007]. Now what does that mean? Here, as well as in learning/teaching the knowledge stored in the CRKS interpretations of the KH – see [VGS2004] -, it is the progression of teaching/learning along derivation paths from *P* to *G*. This takes place via presentation strategies – [GVS1999] and [VGS2004] – as these divide the information flow into teachable/learning sections that constitute a hierarchy; for example following a limited access cascade from *P* to *G* in steps, teaching one, or all, the derivation paths in each S_i of a line separation in turn, teaching/learning clusters one at a time and associating them, and so on – see [GVS1999], [VGS2004], [VS2007]. Measures of complexity allow one to find those derivation paths along which the information flow is “simplest”; a “least resistance” approach to presentation of information flow in an interpreted KH. [VGS2004] – see path tree and gauges of complexity.

4. Rehabilitation

In training/teaching/re-training a KH – see [VS2007] – in <V, E> by trial-and-error, examples, mimicry and the establishment of invariance, bearing in mind the role of CRKS abstraction, interpretations and isomorphisms, the edges of a minimum capacity line separation would be critical. In the case of damage to those edges, due to damage or malfunction of nuclei in them, other derivation paths that bypass the damaged edges, or now non-existent lines, must be “taught”, i.e. the relevant memory gauge values increased, or we must do this for still existing derivation path bypasses.

Destruction of nuclei, and thus neurons, may induce a separation of a KH. Major rehabilitation is then required. This may be achievable by patching together any remaining invariant sub-KH’s and noise generated by successive sweeps, by starting over from scratch, or by adapting some other partially isomorphic KH in <V, E> with its CRKS interpretations using formal analogical reasoning – a presentation strategy, see [GVS1999] and [VGS2004]. (CRKS isomorphism is defined via hypernet isomorphism in [VGS 2004]: CRKS1 → abstraction → KH1 → hypernet isomorphism → KH2 → interpretation → CRKS2; where we could have isomorphic sub – KH’s of either KH1 or KH2 or both, and we could have KH1 = KH2.) In the first case we have a situation of remnant memories of certain invariant sub-KH’s of the original invariant KH, together with interfering and confusing noise which we may want to damp, or even partially use, in rehabilitation - noise is essential for emphasising invariance.

As a neuron existing in G is just a particular subgraph of G that is activated by a sweep, neurons are induced by sweeps: the current set of neurons in G is dependent upon the current sweep! One implication of this, in our model, is that “new” neurons, and thus synapses, can arise. This, in turn, implies the possibility of rehabilitation due to substitution of malfunctioning neurons by “new” neurons and synaptic paths due to “new” sweeps: a form of learning/teaching that may be accomplished in the standard manner outlined in [VGS2004]. In contrast, malfunctioning, “dead” neurons may possibly be regenerated by a living system, or simply replaced in an artificial system. The point to note is that in our model neurons that are inherent in G can only be activated/generated by sweeps.

Primary and goal nuclei of a KH in <V, E> - see [VS2007] – will of course be extremely critical in the case of malfunction!

5. Interpretation and misconceptions

An observation may be regarded as an interpretation of a relationship among perceptual stimuli, or “perceptions”. It can be represented by an interpretation of a KH cluster, and such clusters can be associated, such associations then being seen as relationships among observations.

Given a repeatable situation, we will have observations with relationships among them for that situation or phenomena. During repetition, certain observations will remain invariant over repetition: we call them observables of the situation. Certain relationships among the observables will remain invariant over repetition, and the observables and those relationships, in an interpretation, are represented by a model, i.e. an isomorphic CRKS. By applying precisely specified reasoning to the model, we have a theory, i.e. model and reasoning together, that makes predictions of new observables and invariant relationships among them from the model by applying the reasoning to the model. Some predictions should be (roughly) empirically confirmed, the support for the theory, while at least one prediction should be empirically falsified; all this by use of the feedback circuit. Falsification implies that our interpretation, and thus the interpreted part of the relevant KH, is not satisfactory, and this implies that we must continue to learn: new or modified KH’s, and new or modified interpretations of them are required, and this entails learning/teaching/rehabilitation.

The observables are the data, the relationships among them (in interpretations) the information, and the patterns in those relationships the knowledge (which arises from investigation of the interpretations). Thus structured data is information, and structured information is knowledge.

On the large scale this is the way science proceeds; it encapsulates scientific method. On the smaller scale we refer to a falsified, or falsifiable, interpretation of part of, or all of, a KH in teaching as a misconception, or, more kindly, as an inappropriate conception. In a broad sense then, science, teaching, learning, training can all be seen as forms of rehabilitation; as forms of attempting to establish invariance of KH's and their interpretations, in the G/<V, E>/F model.

6. Conclusion

We have presented a bit more about flow, to support [VS2007]. We have dealt with some of the implications of separations and information flow, and looked at the meaning of information flow in a KH. We have arrived at max-flow, min-cut theorems for flow in a KH. Finally we examined some of the affects of damage to neurons, and have indicated some methods of rehabilitation by training/teaching. The underlying theme of the paper is teaching/learning, and we assume that knowledge is represented by CRKS's – see [GVS1999], [VGS2004].

References

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Appendix: Modifications of [VGS 2004]

pg. 13, line 9: … entails the deletion of all the tuples of the context…

pg. 67, line 5: … entails deleting all the edges of <A, E> [a].
line 6/7: … iff there is at least one derivation path on which every given pair of distinct non-primary, non-goal vertices of <A, E> lies.

pg. 89, line 8: CS 2.1.1 should be CS 1.2.7

pg. 92, line 2: … found in $<B_n, E_n>$, i.e. in the n'th step of the cascade.
(delete: i.e. $a \notin B_{n-1}$)

pg. 93, line 1: … n'th step of the cascade. (delete: i.e. $a \in (B_n - B_{n-1})$)
line 2: wd(a) should be wdd(a)
line 3: $wdd(a) = N_o + \Sigma_i |N_i|$, $i = 1,2, ---, n_i -1$
and the **accumulated deductive distance** by
$add(a) = N_o + (\Sigma_i\, i\, |N_i|) + n_i$, $i = 1,2, ---, n_i -1$.

pg. 120: -. 2. - becomes -. 1. - in each case, -. 3. - becomes -. 2. - in each case, 1.5.1. becomes 1.4.1