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Infinite Possible Words for Process Algebras
(Extended Abstract)

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Infinite Possible Worlds for Process Algebras

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Abstract. [VD98] propose to view a *finite* nondeterministic process as a specification for a set of deterministic implementations: its possible worlds or model space. Refinement amounts to inclusion of possible worlds. We consider here the extension to infinite processes. We study the properties of possible worlds semantics, answer in particular an open question concerning the relation between bisimulation and possible worlds equivalence and discuss operational aspects.

keywords: denotational semantics, concurrency, specification.

1 Introduction

In a recent paper [VD98] Veglioni and De Nicola propose to view a nondeterministic process as a set of deterministic ones: its possible worlds. In this view nondeterminism is understood as underspecification. Nondeterministic processes are considered to be specifications and the possible worlds represent the model space. Refinement can be modelled by inclusion between sets of possible worlds. This notion of refinement is commonly used for the algebraic specifications of abstract data types [ONS96,HB85]. It is an interesting concept, as it is consistent with the program design methodology of stepwise refinement where more concrete specifications have less 'implementation freedom'. In [VD98] the authors consider a simple language **BP** of *finite* processes and present a denotational semantics in terms of finite sets of finite deterministic trees as well as an axiomatic characterization of the possible worlds semantics. The location of the possible worlds equivalence in the spectrum of equivalence notions as given e.g. in [vG90a,vG90b] is determined. We consider here the extension to a wider class **RBP** of processes including recursion. We give an (infinite) possible worlds semantics for **RBP** and study its properties. In particular we deal with the relation between possible worlds equivalence and bisimulation. [VD98] show that bisimulation implies possible worlds equivalence for *finite* processes and conjecture that this is not true for infinite processes. In contrast to that we show that bisimulation implies possible worlds equivalence also for infinite processes. In addition we discuss problems related with operational possible worlds semantics.

2 Definitions and elementary facts about metric spaces

2.1 Processes

In this paper we consider sequential nondeterministic processes that are able to perform actions from a given set **Act**. An action represents any activity of a system at a chosen level of abstraction.
A domain **A** of sequential nondeterministic processes can be represented as a *labelled transition system*, i.e. a pair $(\mathbf{A}, \rightarrow)$, where **A** is the class of processes and $\rightarrow \subseteq \mathbf{A} \times \mathbf{Act} \times \mathbf{A}$ is the action relation. We write $p \stackrel{a}{\rightarrow} q$ for $(p, a, q) \in \rightarrow$.
The set of initial actions of p is

$$I(p) = \{a \in \mathbf{Act} : p \stackrel{a}{\rightarrow} q \quad \text{for some q}\}$$

On **A** a variety of semantic equivalences have been investigated. [VD98] show how possible worlds refinement for *finite* processes fits into the spectrum of equivalences as e.g. presented in [vG90a,vG90b].

We extend the class **BP** of [VD98] by recursion to model infinite behaviour and thus obtain the class **RBP** of processes given by

- $0 \in \mathbf{RBP}$
- $a.P \in \mathbf{RBP}$ (prefix) for all $a \in \mathbf{Act}$, $P \in \mathbf{RBP}$
- $X \in \mathbf{RBP}$ for all $X \in Idf$
- $P + Q \in \mathbf{RBP}$ (sum) for all $P, Q \in \mathbf{RBP}$
- $fix(X = P)$ for all $X \in Idf$, $P \in \mathbf{RBP}$ such that X is guarded in P

Here Idf is a set of identifiers. An occurrence of $X \in Idf$ is free in P iff it does not occur within a subterm of the form $fix\ X = Q$. $X \in Idf$ is *guarded* in P iff each free occurrence of X in P is in the scope of a prefix operation. For $P, Q \in \mathbf{RBP}$, $X \in Idf$, $P[X/Q]$ denotes the process where each free occurrence of X in P is substituted by Q. P is closed if it is without free occurrences of variables. **RBP** will be regarded as a labelled transition system with the transitions $a.P \stackrel{a}{\rightarrow} P$, $P+Q \stackrel{a}{\rightarrow} P'$ if $P \stackrel{a}{\rightarrow} P'$ or $Q \stackrel{a}{\rightarrow} P'$, $fix(X = P) \stackrel{a}{\rightarrow} P'$ if $P[X/fix(X = P)] \stackrel{a}{\rightarrow} P'$.
Let M be a set and $\sigma : Idf \rightarrow M$ a map, $U \in M$

$$\sigma[X/U]Y = \begin{cases} \sigma(Y) & \text{if } Y \neq X \\ U & \text{if } Y = X \end{cases}$$

The function giving the set of initial actions for a process P is defined as follows: let $INIT = \{\sigma | \sigma : Idf \rightarrow \mathcal{P}(\mathbf{Act})\}$. Then $I : \mathbf{RBP} \rightarrow (INIT \rightarrow \mathcal{P}(\mathbf{Act}))$ is given by

$I(0)(\sigma) = \emptyset$ $\qquad I(X)(\sigma) = \sigma(X)$
$I(a.P)(\sigma) = \{a\}$ $\qquad I(P + Q)(\sigma) = I(P)(\sigma) \cup I(Q)(\sigma)$
$I(fix(X = P))(\sigma) = lf\Phi_{P,X}(\sigma)$

where $lf\Phi_{P,X}(\sigma)$ is the least fixed point of $\Phi_{P,X}(\sigma) : \mathcal{P}(\mathbf{Act}) \to \mathcal{P}(\mathbf{Act})$ given by $\Phi_{P,X}(\sigma)(U) = I(P)(\sigma)[X/U]$.
Let $P \in \mathbf{RBP}$, $\sigma \in INIT$. The relation $\delta \subset \mathbf{RBP} \times INIT$ is given as follows

$(0, \sigma) \in \delta, \quad (X, \sigma) \in \delta, \quad (a.P, \sigma) \in \delta$ if $(P, \sigma) \in \delta$
$(P_1 + P_2, \sigma) \in \delta$ if $I(P_1)(\sigma) \cap I(P_2)(\sigma) = \emptyset$ and $(P_i, \sigma) \in \delta,\ i = 1, 2$
$(fix(X = P), \sigma) \in \delta$ if $(P, \sigma) \in \delta$

We call a process P *deterministic* if $(P, \sigma) \in \delta$ for all σ. In [VD98] the meaning of **BP** is given in terms of the algebra of labelled trees $\mathbf{T} = (\mathbf{Act}, \mathbf{T}, \circ, \cdot, +)$ where

$\circ$ is the empty tree
$\cdot : \mathbf{Act} \times \mathbf{T} \to \mathbf{T}$ is the prefixing operator
$+ : \mathbf{T} \times \mathbf{T} \to \mathbf{T}$ is the operator joining two trees at the root

$\mathbf{T}_{finbran}$ is the subclass of finitely branching trees. The meaning function $\langle\langle \cdot \rangle\rangle : \mathbf{BP} \to \mathcal{P}(\mathbf{T})$ is given by [VD98]

$\langle\langle 0 \rangle\rangle = \{\circ\}$ $\qquad \langle\langle a.P \rangle\rangle = \{a \cdot x : x \in \langle\langle P \rangle\rangle\}$
$\langle\langle P + Q \rangle\rangle = \langle\langle P \rangle\rangle \star \langle\langle Q \rangle\rangle$

where $\star : \mathcal{P}(\mathbf{T}) \times \mathcal{P}(\mathbf{T}) \to \mathcal{P}(\mathbf{T})$ is given by

$T_1 \star T_2 = \bigcup_{t_i \in T_i} t_1 \star t_2$
$t_1 \star t_2 = \{t_1 + t_2\}$ if $I(t_1) \cap I(t_2) = \emptyset$, I gives the branches of the root.
$(a \cdot t_1' + t_1'') \star (a \cdot t_2' + t_2'') = (a \cdot t_1' \star (t_1'' \star t_2'')) \cup (a \cdot t_2' \star (t_2'' \star t_1''))$

2.2 Metric Spaces

We recall some basic facts from (metric) topology. We presuppose the notions of *metric space*, *isometry*, *Cauchy sequence* in a metric space, *limit*, *compactness*, *completeness* of a metric space and the theorem that each metric space has a unique completion. A n-ary function $f : M \times \ldots \times M \to N$ is called *non-distance-increasing* iff

$$d_N(f(x_1, \ldots, x_n), f(y_1, \ldots, y_n)) \leq \max_{i=1 \ldots n} d_M(x_i, y_i)$$

A non-distance-increasing function $f : M \times \ldots \times M \to N$, where (N, d_N) is a complete metric space has a unique extension to the completion of $M \times \ldots \times M$. $f : M \times \ldots \times M \to N$ is called *contractive* iff there exists a constant c, $0 \leq c < 1$ such that $d_N(f(x_1, \ldots, x_n), f(y_1, \ldots, y_n)) \leq c \cdot \max_{i=1 \ldots n} d_M(x_i, y_i)$.
The fixed point theorem by Banach/Cacciopoli states that every contractive function $f : M \to M$ on a complete metric space M has a *unique fixed point* in M. We will make use of several constructions involving metric spaces.
If A is a set and (M, d_M) a metric space then

- $(A \times M, d_{A\times M})$ with $d_{A\times M}((a,x),(b,y)) = \begin{cases} 1 & \text{if } a \neq b \\ \frac{1}{2} \cdot d_M(x,y) & \text{if } a = b \end{cases}$ for all $(a,x),(b,y) \in A \times M$

- $(\mathcal{P}_{nco}(M), d_H)$ with
  $\mathcal{P}_{nco}(M) = \{U \subseteq M : U \neq \emptyset,\ U \text{ compact}\}$ and the Hausdorff metric

$$d_H(X,Y) = \max\{\sup_{x\in X}\inf_{y\in Y} d(x,y),\ \sup_{y\in Y}\inf_{x\in X} d(x,y)\}$$

  for $X, Y \in \mathcal{P}_{nco}(M)$

If (M, d_M) is a discrete space then $\mathcal{P}_{nco}(M) = \mathcal{P}_{nf}(M)$ where

$$\mathcal{P}_{nf}(M) = \{U \subseteq M,\ U \neq \emptyset,\ U \text{ finite}\}$$

Our intention is to associate with each process $P \in \mathbf{RBP}$ a *compact* set of deterministic trees. The following theorems turn out to be useful.

Theorem 1. *Let (M,d) be a metric space. If $X \subseteq \mathcal{P}_{nco}(M)$ is compact then $\bigcup_{A\in X} A \in \mathcal{P}_{nco}(M)$.*

Theorem 2. *Let $(M,d_M),(N,d_N)$ be metric spaces. $f : M \times M \to \mathcal{P}_{nco}(N)$ a non-distance-increasing function. We put for $U,V \in \mathcal{P}_{nco}(M)$*

$$\hat{f}(U,V) = \bigcup_{u\in U, v\in V} f(u,v)$$

then

i) $\hat{f}(U,V)$ is a nonempty compact subset of N for all $U,V \in \mathcal{P}_{nco}(M)$.
ii) $d_H(\hat{f}(U,V), \hat{f}(U',V')) \leq \max(d_H(U,U'), d_H(V,V'))$
for all $U,V \in \mathcal{P}_{nco}(M)$, i.e. $\hat{f} : \mathcal{P}_{nco}(M) \times \mathcal{P}_{nco}(M) \to \mathcal{P}_{nco}(N)$ is non-distance-increasing.

Proof. i) we first observe that $\{f(u,v) | u \in U,\ v \in V\}$ is a nonempty compact set for $U,V \in \mathcal{P}_{nco}(M)$, from where the statement follows by theorem 1.
ii) standard calculations.

3 A domain of trees

The mapping $\langle\langle\cdot\rangle\rangle$ can be viewed as a mapping from **BP** to $\mathcal{P}_{nf}(\mathbf{T}_{finbran}/_\sim)$ where $\sim$ denotes bisimulation. We propose to model **RBP** by $\mathcal{P}_{nco}(\mathbf{D})$ where $\mathbf{D}$ is the completion of the metric space $\mathbf{T}_{finbran}/_\sim$: The natural metric on $\mathbf{T}_{finbran}$, $d_T(t_1,t_2) = \inf\{\frac{1}{2^n} : t_1^{(n)} = t_2^{(n)}\}$ where $t^{(n)}$ denotes the n-cut of t, carries over to $\mathbf{T}_{finbran}/_\sim$ and yields an incomplete metric space. The completion (Δ, δ) of $(\mathbf{T}_{finbran}/_\sim, d_T)$ is a suitable basis for a semantics of **RBP**. (Δ,δ) can be given an alternative, more flexible characterization as follows. Let **CMS** be the category where the objects are complete metric spaces and the arrows are non-distance-increasing functions. The functor $\mathcal{F} : \mathbf{CMS} \to \mathbf{CMS}$ given by $\mathcal{F}(M) = \{\emptyset\} \cup \mathcal{P}_{nco}(\mathbf{Act} \times M)$ and $\mathcal{F}(f) = \lambda U.\{(a, f(m)) : (a,m) \in U\}$. It is a well known fact that $\mathcal{F}$ has a unique fixed point in **CMS** [dBZ82,MCZ91] that can be obtained as the metric completion $(\mathbf{D}, d)$ of $\bigcup_{i\geq 0} D_i$ where $D_0 = \{\emptyset\}$, $D_{i+1} = \mathcal{F}(D_i) \quad i \geq 0$. As D_i is discrete for $i \geq 0$, we have $D_{i+1} = \{\emptyset\} \cup \mathcal{P}_{nf}(\mathbf{A} \times D_i)$ for $i \geq 0$. $\bigcup_{i\geq 0} D_i$ consists of finitely branching trees of finite height.

Theorem 3. $(\mathbf{D}, d)$ *and* (Δ, δ) *are isometric.*

Proof. Show that (Δ, δ) is fixed point of $\mathcal{F}$.

By standard arguments **D** can be turned into a Σ-algebra. Σ consists of the operators $\circ$, $+$, and $\cdot$ as follows

- $\circ$: corresponds to the empty tree $\emptyset$
- $+$: $\bigcup_{i\geq 0} D_i \times \bigcup_{i\geq 0} D_i \to \bigcup_{i\geq 0} D_i$
  $t_1 + t_2 := t_1 \cup t_2$
- $\cdot$: $\mathbf{Act} \times \bigcup_{i\geq 0} D_i \to \bigcup_{i\geq 0} D_i$
  $a \cdot t := \{(a, t)\}$

$+$ and $\cdot$ are non-distance -increasing on $\bigcup_{i\geq 0} D_i$ and may hence be uniquely extended to **D**. The initial actions function I will also be used for trees in **D** and is given by $I(\emptyset) = \emptyset$ and $I(t) = \{a : (a, x) \in t \text{ for some } x \in \bigcup_{i\geq 0} D_i\}$ for $t \in \bigcup_{i\geq 0} D_i$. For $t = \lim t_n \in \mathbf{D}$ we choose some $\varepsilon < \frac{1}{2}$ and determine N such that $d(t_n, t) < \frac{1}{2}$ for all $n \geq N$. We put $I(t) = \bigcup_{k\geq N} I(t_k)$. The subset $\mathbf{D}^d \subseteq \mathbf{D}$ of *deterministic trees* is given by $D_0^d = \{\emptyset\}$ and for $i \geq 0$
$D_{i+1}^d = \{\emptyset\} \cup \{U \in \mathcal{P}_{nf}(A \times D_i^d) : \forall a \forall b \forall x \forall y \ (a, x) \in U \wedge (b, y) \in U \Rightarrow a \neq b\}$

$\mathbf{D}^d$ is the completion of $\bigcup D_i^d$. $t \in \bigcup_{i\geq 0} D_i$ is *root deterministic* iff $\forall a \forall b \forall x \forall y \ (a, x) \in t \wedge (b, y) \in t \Rightarrow a \neq b$.

4 Denotational infinite possible worlds semantics for RBP

In [VD98] the denotational possible worlds semantics for $P \in \mathbf{BP}$ is a (finite) set of (finite) deterministic trees in **T**, its possible worlds. We propose $(\mathcal{P}_{nco}(\mathbf{D}), d_H)$ as semantic domain for possible worlds semantics of **RBP**. The semantics of **BP** is given in [VD98] using the mapping $\star$, see section 2. We use here a slightly modified operator $\circledast$ that coincides with $\star$ in the case of (sets of) deterministic trees.

Definition 1. *Let* $t \in \bigcup_{i\geq 0} D_i$. *We put*
$rdet(\emptyset) = \{\emptyset\}$ *and* $rdet(t) = \{ \ \bigcup_{a\in I(t)} \{(a, \tau) : (a, \tau) \in t\} \ \} \quad t \neq \emptyset$

Definition 2. *Let $t_1, t_2 \in \bigcup_{i\geq 0} D_i$ be root deterministic.*

$$t_1 \circledast t_2 = \{ \bigcup_{a\in I(t_1)\cup I(t_2)} \{(a,x) : (a,x) \in t_1 \vee (a,x) \in t_2\} \}$$

For $t_1, t_2 \in \bigcup_{i\geq 0} D_i$ we put $t_1 \circledcirc t_2 = \bigcup_{\substack{t\in rdet(t_1)\\ t'\in rdet(t_2)}} t \circledast t'$.

Lemma 1. *Let $t_1, t_2 \in \bigcup_{i\geq 0} D_i$ be deterministic trees. Then $t_1 \circledast t_2$ contains only deterministic trees and $t_1 \star t_2 = t_1 \circledast t_2$.*

Theorem 4. *Let $t_1, t_2, t_1', t_2' \in \bigcup_{i\geq 0} D_i$, then*

$$d_H(t_1 \circledcirc t_2, t_1' \circledcirc t_2') \leq \max\{d_H(t_1, t_1'), d_H(t_2, t_2')\}.$$

Hence $\circledcirc$ can be canonically extended to $\mathbf{D} \times \mathbf{D}$.

Theorem 5. *Let T_1, T_2 be nonempty compact sets of trees in $\mathbf{D}$ then*

i) $T_1 \circledcirc T_2 := \bigcup_{t_i\in T_i} t_1 \circledcirc t_2$ *is a nonempty compact subset of* $\mathbf{D}$
ii) $\circledcirc : \mathcal{P}_{nco}(\mathbf{D}) \times \mathcal{P}_{nco}(\mathbf{D}) \to \mathcal{P}_{nco}(D)$ *a non-distance-increasing function.*

Proof. By theorem 2 and theorem 4.

Let $\mathbf{ENV} = \{\sigma | \sigma : Idf \to \mathcal{P}_{nco}(\mathbf{D})\}$ be the set of environments. The meaning function $\langle\langle\cdot\rangle\rangle : \mathbf{RBP} \to \mathbf{ENV} \to \mathcal{P}_{nco}(\mathbf{D})$ is given by

$\langle\langle 0\rangle\rangle(\sigma) = \{\emptyset\}$ $\qquad$ $\langle\langle X\rangle\rangle(\sigma) = \sigma(X)$
$\langle\langle a.P\rangle\rangle(\sigma) = \{\{(a,t)\} : t \in \langle\langle P\rangle\rangle(\sigma)\}$ $\qquad$ $\langle\langle P_1 + P_2\rangle\rangle(\sigma) = \langle\langle P_1\rangle\rangle(\sigma) \circledcirc \langle\langle P_2\rangle\rangle(\sigma)$
$\langle\langle fix(X = P)\rangle\rangle(\sigma) = fix\ \Phi_{P,X}(\sigma)$

where $fix\ \Phi_{P,X}(\sigma)$ is the unique fixed point of the contractive mapping $\Phi_{P,X}(\sigma) : \mathcal{P}_{nco}(\mathbf{D}) \to \mathcal{P}_{nco}(\mathbf{D})$ defined by $\Phi_{P,X}(\sigma)(T) = \langle\langle P\rangle\rangle\sigma[X/T]$.

Remark 1. $\langle\langle P\rangle\rangle$ consists of deterministic trees for closed P.

Example 1. Let $P = fix(X = a.0 + a.X)$ and $Q = a.0 + fix(X = a.X)$ then $\langle\langle P\rangle\rangle$ consists of infinitely many worlds while $\langle\langle Q\rangle\rangle$ has two possible worlds as shown in figure 1.

Definition 3. *Let $P, Q \in$ **RBP** be closed processes. Q is a possible worlds refinement of P, written $P \leq_D Q$ iff $\langle\langle Q\rangle\rangle \subseteq \langle\langle P\rangle\rangle$. P and Q are possible worlds equivalent, $P =_D Q$, iff $\langle\langle P\rangle\rangle = \langle\langle Q\rangle\rangle$.*

Example 2. $P \leq_D Q$ where P, Q are taken from example 1.

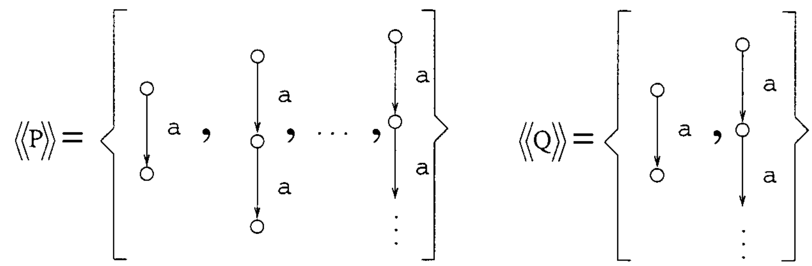


Fig. 1.

5 Properties of the infinite possible worlds refinement

5.1 Axioms

Lemma 2. *The possible worlds semantics satisfies the following axioms: Let P, Q be closed processes*

A0: $a.P + a.Q \leq a.P$ *A1:* $P + Q = Q + P$
A2: $P + P = P$ *A3:* $(P + Q) + R = P + (Q + R)$
A4: $(P + Q) = P$
A5: $a.(b.P + b.Q + R) = a.(b.P + R) + a.(b.Q + R)$
A6: $fix(X = R) = R[X/fix(X = R)]$

where $R \in \mathbf{RBP}$ and X is the only variable occuring free in R.

Proof. The axioms A1 to A5 were already established by [VD98] and carry over to infinite processes. A6 is established by uniqueness of fixed points of contractive maps in $\mathcal{P}_{nco}(\mathbf{D})$.

5.2 $\langle\langle\rangle\rangle$ and $\langle\rangle$

We associate with each closed $P \in \mathbf{RBP}$ a meaning in $\mathbf{D}$, i.e. $\langle P\rangle$, and relate the set of trees $\langle\langle P\rangle\rangle$ with the tree $\langle P\rangle$.

Definition 4. *Let* $\mathbf{Env} = \{\sigma : Idf \to \mathbf{D}\}$
$\langle\rangle : \mathbf{RBP} \to \mathbf{Env} \to \mathbf{D}$ *is given by*

$\langle 0\rangle(\sigma) = \emptyset$ $\langle X\rangle(\sigma) = \sigma(X)$
$\langle a.P\rangle(\sigma) = a \cdot \langle P\rangle(\sigma)$ $\langle P_1 + P_2\rangle(\sigma) = \langle P_1\rangle(\sigma) + \langle P_2\rangle(\sigma)$
$\langle fix(X = P)\rangle(\sigma) = fix\ \varphi_{P,X}(\sigma)$

where $fix_{P,X}(\sigma)$ is the unique fixed point of the contractive mapping $\varphi_{P,X}(\sigma) : \mathbf{D} \to \mathbf{D}$ where $\varphi_{P,X}(\sigma)(t) = \langle P\rangle\sigma[X/t]$

Theorem 6. *Let P be a deterministic process, $\sigma \in \mathbf{Env}$.*

$$\langle\langle P\rangle\rangle(\tilde{\sigma}) = \{\langle P\rangle(\sigma)\}$$

where $\tilde{\sigma}(X) = \{\sigma(X)\}$ for all $X \in Idf$. In particular $\langle\langle P\rangle\rangle = \{\langle P\rangle\}$ for all closed deterministic P.

Proof. by structural induction and uniqueness of fixed points for contractive maps.

Definition 5. *Let* $t \in \bigcup_{i\geq 0} D_i$, $I(t) = \{a_1, \dots, a_n\}$.

$$t(a) := \{t' : (a, t') \in t\} \quad a \in Act$$

$$det(t) := \{\bigcup_{i=1}^{n}\{(a_i, x) \text{ where } x \in det(t'), t' \in t(a_i)\} \quad \}$$

$det(t)$ is non-distance-increasing on $\bigcup_{i\geq 0} D_i$ and can hence be extended to D

Theorem 7. *Let* $P \in \mathbf{RBP}$, $\sigma \in \mathbf{Env}$.

$$\langle\langle P\rangle\rangle\tilde{\bar{\sigma}} = det(\langle P\rangle\sigma)$$

in particular $\langle\langle P\rangle\rangle = det(\langle P\rangle)$ *for all closed processes* P *where* $\tilde{\bar{\sigma}}(X) = det(\sigma(X))$

Proof. by structural induction and uniqueness of fixed points of $\Phi_{P,X}(\bar{\sigma})$.

5.3 Possible worlds and bisimulation

In [VD98] it is shown that two *finite* processes P, Q are possible worlds equivalent if they are bisimular. [VD98] conjecture that this result does not hold in the infinite case. We show that for all $P, Q \in \mathbf{RBP}$ $P \sim Q$ implies $P =_D Q$.

Definition 6. $\mathbf{env} = \{\sigma : Idf \to \mathbf{T}_{finbran}\}$. *Let* $tr : \mathbf{RBP} \to \mathbf{env} \to \mathbf{T}_{finbran}$

$tr(0)(\sigma) = t_\emptyset \qquad tr(X)(\sigma) = \sigma(X) \qquad tr(a.P)(\sigma) = a \cdot tr(P)(\sigma)$
$tr(P_1 + P_2)(\sigma) = tr(P_1)(\sigma) + tr(P_2)(\sigma) \qquad tr(fix(X = Q))(\sigma) = fix\ f_{X,Q}(\sigma)$

where $f_{X,Q}(\sigma)(t) = tr(Q)\sigma[X/t]$

$\mathbf{T}_{finbran}$ can be viewed as a transition system in a straightforward way. Let $F : \mathbf{T}_{finbran}/_\sim \to \mathbf{D}$ be given by $F([t]_\sim) = \lim_{n\to\infty}(F_n[t^{(n)}])$ where $t^{(n)}$ is the n-cut of t, i.e. the subgraph of all nodes of depth $\leq n$ and F_n is defined by $F_n([t_\emptyset]) = \emptyset$, $F_n([t]_\sim) = \{(a, F_{n-1}[t']_\sim) : t \xrightarrow{a} t'\}$ for t with $1 \leq heigh(t) \leq n$. Please note that $F_n([t^{(n)}])$ is a Cauchy sequence in $\mathbf{D}$.

Lemma 3. *Let* $P \in \mathbf{RBP}$, $X_1, X_2, \dots, X_n \in Idf$ *the identifiers that occur free in* P *and* $P_1, \dots, P_n \in \mathbf{RBP}$ *be closed. Let* $\tau \in \mathbf{env}$ *with* $\tau(X_i) = tr(P_i)$ *and* $\sigma \in \mathbf{Env}$ *with* $\sigma(X_i) = F([tr(P_i)]_\sim)$. *Then* $\langle P\rangle\sigma = F([tr(P)\tau]_\sim)$

Lemma 4. *Let* $P, P_1, \dots, P_n \in \mathbf{RBP}$ *and* $X_1, \dots, X_n \in Idf$, $X_i \neq X_j$, *identifiers in* P, $a \in Act$. *If* $X_1, \dots, X_n$ *are guarded in* P *then* $P[X_1/P_1, \dots, X_n/P_n] \xrightarrow{a} Q$ *implies the existence of* $P' \in \mathbf{RBP}$ *s. t.* $P \xrightarrow{a} P'$ *and* $P'[X_1/P_1, \dots, X_n/P_n] = Q$

Proof. [BMC94]

Lemma 5. *Let $P \in \mathbf{RBP}$, $a \in \mathbf{Act}$, $\sigma \in \mathbf{Env}$. If $P \xrightarrow{a} P'$ then $tr(P)(\sigma) \xrightarrow{a} tr(P')(\sigma)$.*

Proof. structural induction and lemma 4.

Lemma 6. *Let $P \in \mathbf{RBP}$ and $X_1, \ldots, X_n$ the identifiers that occur free in P. Let $\sigma \in \mathbf{Env}$ such that $\sigma(X_i) = P_i$ where $P_i \in \mathbf{RBP}$ is closed. If $X_1, \ldots, X_n$ are guarded in P then for all $t' \in \mathbf{T}_{finbran}$ if $tr(P)\sigma \xrightarrow{a} t'$ then there exists $P' \in \mathbf{RBP}$, P' closed such that $P[X_1/P_1, \ldots, X_n/P_n] \xrightarrow{a} P'$ and $tr(P') = t'$*

Proof. structural induction and lemma 4

Lemma 7. *$R = \{(Q, tr(Q)) \quad Q \in \mathbf{RBP}$ closed $\}$ is a bisimulation between $(\mathbf{RBP}, \mathbf{Act}, \rightarrow, P)$ and $(\mathbf{T}_{finbran}, Act, \rightarrow, tr(P))$.*

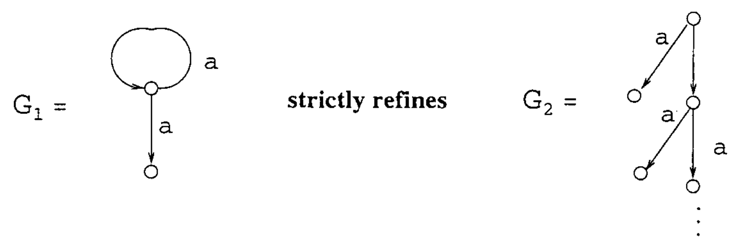
Proof. by theorem 5 and lemma 6.

Theorem 8. *Let $P, Q \in \mathbf{RBP}$ be closed processes. If $P \sim Q$ then $P =_D Q$*

Proof. by theorem 7, lemma 3 and lemma 7.

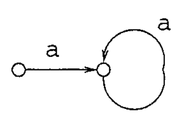
6 Operational characterization

[VD98] associate with $p \in \mathbf{BP}$ an operational meaning $PW(p)$ consisting of all graphs h that are isomorphic to a minimal deterministic graph g satisfying $R(g) = p$ and $q \xrightarrow{a} q'$, $q \in N(g) \Rightarrow \exists q'' \in N(g) : (q, a, q'') \in E(g)$ and $q \xrightarrow{a} q''$. Theorem 1. in [VD98] states: given processes $p, q \in \mathbf{BP}$ then $\langle\langle q \rangle\rangle \subseteq \langle\langle p \rangle\rangle$ iff $PW(q) \subseteq PW(p)$.
For infinite processes [VD98] remark:
'...that infinite processes are already considered in the operational characterization, in fact it is not restricted to finite transitions systems. ... . We also have that:

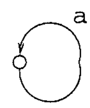


though they are bisimilar. Notice, in fact, that the process on the left admits only two possible worlds... . Unfortunately, Definition 3 (the definition of PW) *cannot be directly used for infinite processes; it is not sufficiently abstract for loops.*

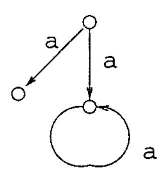
For example



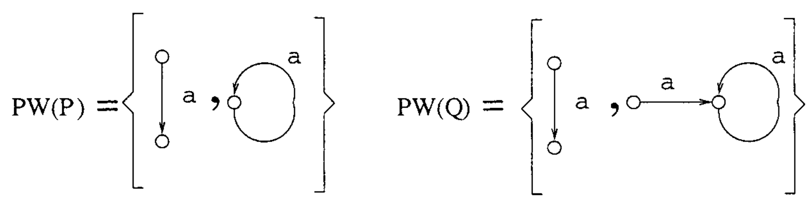
is not equivalent to



However, this can be easily resolved by choosing a graph equivalence weaker than isomorphism.'
Now consider e.g. the two processes P, Q from example 1. The process graph of P is G_1 whereas the process graph of Q is



Hence in the view of [VD98]



would coincide under the assumed weaker notion of graph equivalence, but $\langle\langle P\rangle\rangle \neq \langle\langle Q\rangle\rangle$. Hence such an operational semantics is not equivalent to the denotational semantics. Moreover, we claim that P and Q are inherently different. Q exhibits underspecification once, whereas P exhibits underspecification in every recursion step. See [MC98] for further discussion of operational semantics.

7 Extensions and related work

It is not difficult to see that concatenation can be easily incorporated into our setting. As [VD98] we considered a standard binary 'choice'. It should be noted that the approach can be extended to an infinite summation operator Σ. The language thus obtained would then include the coffee machine example of [VD98]: $cof + cof.cof + \ldots = \Sigma_{i\geq 1} cof^i$. More details are given in [MC98].

As already suggested in [VD98] the possible worlds concept can be used to obtain a whole spectrum of possible worlds notion, as e.g. trace possible worlds equivalence and so forth. Another track of transfer of the possible worlds idea leads to other models of computation as e.g. true concurrency models provided we extend the language **RBP** by concurrency features. More details can be found in [MC98].
The idea of interpreting certain choices by a set of trees instead of branching is not new. It can be found in [Rou85], where a CSP-type language with two versions of 'nondeterminism' is used. The one is interpreted by branching the other by yielding a set of trees. For a finite set of actions [Rou85] establish the relation between the Hennessy Milner Logic HML and $\mathcal{P}_c(\mathbf{T})$ where $\mathcal{P}_c(\mathbf{T})$ denotes the closed subsets of the pseudometric space $\mathbf{T}$ of trees.

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