Common Extensions of Positive Vector Measures

Klaus D. Schmidt and Gerd Waldschaks

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Klaus D. Schmidt and Gerd Waldschaks, Mannheim

We prove a general theorem on the existence of a positive common extension of a family of positive vector measures in an order complete Riesz space. Our theorem gives an easy access to vector-valued versions of results due to Guy, Horn and Tarski, and Marczewski.

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1. Introduction

The following result was proposed by Guy [3]:

1.1. Proposition.

Let Ω be a set, let M and N be algebras of subsets of Ω, and let $\mu : M \longrightarrow \mathbb{R}$ and $\nu : N \longrightarrow \mathbb{R}$ be positive additive set functions. Then the following are equivalent:

(a) $\mu(A) \leq \nu(B)$ and $\nu(C) \leq \mu(D)$ holds for all $A, D \in M$ and $B, C \in N$ satisfying $A \subseteq B$ and $C \subseteq D$.

(b) There exists a positive additive set function $\varphi : 2^\Omega \longrightarrow \mathbb{R}$ satisfying $\varphi(A) = \mu(A)$ for all $A \in M$ and $\varphi(A) = \nu(A)$ for all $A \in N$.

This result is remarkable since it characterizes the existence of a positive common extension in terms of the set functions alone; by contrast, no such result is known in the case of bounded additive set functions, where some condition on the algebras seems to be indispensable; see Lipecki [8] and Schmidt and Waldschaks [12]. Moreover, Proposition 1.1 cannot be extended to more than two set functions; see Bhaskara Rao and Bhaskara Rao [2; Example 3.6.3].

Unfortunately, the proof of Proposition 1.1 given by Guy [3] is incorrect, as will be made precise in Section 4 of this paper, and the proofs given by Bhaskara Rao and Bhaskara Rao [2; Theorem 3.6.1] and Kindler [5,6] are rather extensive. In the present paper we prove a general extension theorem for families of positive vector measures which gives an easy access to vector-valued versions of Proposition 1.1 and results due to Horn and Tarski [4] and Marczewski [10,11].

Throughout this paper, let Ω be a set and let $\mathbb{G}$ be an order complete Riesz space. Let us first recall some definitions and facts which will be needed in the sequel:

For a Riesz space $\mathbb{H}$, a linear operator $T : \mathbb{H} \longrightarrow \mathbb{G}$ is <u>positive</u> if $Tx \in \mathbb{G}_+$ holds for all $x \in \mathbb{H}_+$. For further details on Riesz spaces and linear operators, see [1].

For an algebra $\mathcal{F}$ of subsets of Ω , a vector measure $\varphi : \mathcal{F} \longrightarrow \mathbb{G}$ is <u>positive</u> if it maps $\mathcal{F}$ into $\mathbb{G}_+$. Let

$$\mathbb{E}(\mathcal{F}) := \text{lin}\{ \chi_A \mid A \in \mathcal{F} \}$$

and define $\chi : \mathcal{F} \longrightarrow \mathbb{E}(\mathcal{F})$ by letting

$$\chi(A) := \chi_A \quad ,$$

where χ_A denotes the indicator function of $A \in \mathcal{F}$. Then $\mathbb{E}(\mathcal{F})$ is a Riesz space with order unit χ_Ω , and χ is a positive vector measure. Moreover, each vector measure $\varphi : \mathcal{F} \longrightarrow \mathbb{G}$ defines its representing linear operator $T : \mathbb{E}(\mathcal{F}) \longrightarrow \mathbb{G}$, given by

$$T\left(\sum_{i=1}^{n} \alpha_i \chi_{A_i} \right) := \sum_{i=1}^{n} \alpha_i \varphi(A_i) \quad ,$$

and each linear operator $T : \mathbb{E}(\mathcal{F}) \longrightarrow \mathbb{G}$ defines a vector measure $\varphi : \mathcal{F} \longrightarrow \mathbb{G}$, given by

$$\varphi := T \circ \chi \quad .$$

Obviously, φ is positive if and only if T is positive.

2. Positive operators

The following extension theorem is a consequence of the Hahn-Banach theorem for linear operators; for a proof, see [1; Theorem 2.8]:

2.1. Proposition.

Let $\mathbb{E}$ be a Riesz space with order unit $e \in \mathbb{E}_+$, let $\mathbb{F}$ be a subspace of $\mathbb{E}$ satisfying $e \in \mathbb{F}$, and let $S : \mathbb{F} \longrightarrow \mathbb{G}$ be a positive operator. Then there exists a positive operator $T : \mathbb{E} \longrightarrow \mathbb{G}$ satisfying $Tx = Sx$ for all $x \in \mathbb{F}$.

For a Riesz space $\mathbb{E}$ and a family $\{ \mathbb{E}_\delta \mid \delta \in \Delta \}$ of subspaces of $\mathbb{E}$, let $\Phi(\mathbb{E}_\delta \mid \delta \in \Delta)$ denote the collection of all families $\{ x_\delta \in \mathbb{E}_\delta \mid \delta \in \Delta \}$ satisfying $x_\delta \neq 0$ for at most finitely many $\delta \in \Delta$. A family $\{ T_\delta : \mathbb{E}_\delta \longrightarrow \mathbb{G} \mid \delta \in \Delta \}$ of linear operators has a common extension if there exists a linear operator $T : \mathbb{E} \longrightarrow \mathbb{G}$ satisfying $Tx = T_\delta x$ for all $\delta \in \Delta$ and $x \in \mathbb{E}_\delta$.

2.2. Theorem.

Let $\mathbb{E}$ be a Riesz space with order unit $e \in \mathbb{E}_+$ and let $\{ \mathbb{E}_\delta \mid \delta \in \Delta \}$ be a family of subspaces of $\mathbb{E}$ satisfying $e \in \bigcap_\Delta \mathbb{E}_\delta$. For a family $\{ T_\delta : \mathbb{E}_\delta \longrightarrow \mathbb{G} \mid \delta \in \Delta \}$ of positive operators, the following are equivalent:

(a) $\Sigma_\Delta T_\delta x_\delta \in \mathbb{G}_+$ holds for each family $\{x_\delta\} \in \Phi(\mathbb{E}_\delta \mid \delta \in \Delta)$ satisfying $\Sigma_\Delta x_\delta \in \mathbb{E}_+$.

(b) The family $\{T_\delta\}$ has a positive common extension $T : \mathbb{E} \longrightarrow \mathbb{G}$.

Proof. It is sufficient to prove that (a) implies (b). Define $\mathbb{F} := \mathrm{lin}\,(\bigcup_\Delta \mathbb{E}_\delta)$. Then the mapping $S : \mathbb{F} \longrightarrow \mathbb{G}$, given by

$$Sx := \Sigma_\Delta T_\delta x_\delta$$

for all $x \in \mathbb{F}$ and arbitrary $\{x_\delta\} \in \Phi(\mathbb{E}_\delta \mid \delta \in \Delta)$ satisfying $x = \Sigma_\Delta x_\delta$, is well-defined and linear, and it is also positive. Now the assertion follows from Proposition 2.1. □

Theorem 2.2 is due to Maharam [9].

3. Positive vector measures

For a family $\{ F_\delta \mid \delta \in \Delta \}$ of algebras of subsets of Ω, a family $\{ \varphi_\delta : F_\delta \longrightarrow \mathbb{G} \mid \delta \in \Delta \}$ of vector measures has a <u>common extension</u> if there exists a vector measure $\varphi : 2^\Omega \longrightarrow \mathbb{G}$ satisfying $\varphi(A) = \varphi_\delta(A)$ for all $\delta \in \Delta$ and $A \in F_\delta$.

3.1. Theorem.

Let $\{ F_\delta \mid \delta \in \Delta \}$ be a family of algebras of subsets of Ω. For a family $\{ \varphi_\delta : F_\delta \longrightarrow \mathbb{G} \mid \delta \in \Delta \}$ of positive vector measures, the following are equivalent:

(a) $\sum_{i=1}^{m} \varphi_{\delta(i)}(A_i) \leq \sum_{i=m+1}^{m+n} \varphi_{\delta(i)}(A_i)$ holds for all $m, n \in \mathbb{N}$, all $A_1, \ldots, A_{m+n} \in \bigcup_\Delta F_\delta$ satisfying $\sum_{i=1}^{m} \chi_{A_i} \leq \sum_{i=m+1}^{m+n} \chi_{A_i}$, and all $\delta(1), \ldots, \delta(m+n) \in \Delta$ satisfying $A_i \in F_{\delta(i)}$ for all $i \in \{1,\ldots,m+n\}$.

(b) The family $\{\varphi_\delta\}$ has a positive common extension $\varphi : 2^\Omega \longrightarrow \mathbb{G}$.

Proof. It is sufficient to prove that (a) implies (b). For all $\delta \in \Delta$, define $\mathbb{E}_\delta := \mathbb{E}(F_\delta)$ and let $T_\delta : \mathbb{E}_\delta \longrightarrow \mathbb{G}$ denote the representing linear operator of φ_δ. We claim that

$$0 \leq \Sigma_\Delta T_\delta g_\delta$$

holds for each family $\{g_\delta\} \in \Phi(\mathbb{E}_\delta \mid \delta \in \Delta)$ satisfying $0 \leq \Sigma_\Delta g_\delta$. Indeed, this is obvious for families of simple functions taking their values in $\mathbb{Z}$, by the assumption on $\{\varphi_\delta\}$, and hence for families of simple functions taking their values in $\mathbb{Q}$. Consider now an arbitrary family $\{g_\delta\} \in \Phi(\mathbb{E}_\delta \mid \delta \in \Delta)$ satisfying $0 \leq \Sigma_\Delta g_\delta$, and let m denote the number of $\delta \in \Delta$ for which $g_\delta \neq 0$. For each $k \in \mathbb{N}$ and $\delta \in \Delta$ choose $g_{\delta,k} \in \mathbb{E}_\delta$ such that each $g_{\delta,k}$ takes its values in $\mathbb{Q}$ and satisfies $g_{\delta,k} = 0$ if $g_\delta = 0$ and

$$g_{\delta,k} - \frac{1}{km}\chi_\Omega \leq g_\delta \leq g_{\delta,k} \ .$$

Then we have, for all $k \in \mathbb{N}$,

$$0 \leq \Sigma_\Delta \, g_{\delta,k}$$

and hence

$$0 \leq \Sigma_\Delta \, T_\delta g_{\delta,k} \leq \Sigma_\Delta \, T_\delta g_\delta + \frac{1}{k} T\chi_\Omega \ .$$

Since $\mathbb{G}$ is order complete and hence Archimedean, we obtain

$$0 \leq \Sigma_\Delta \, T_\delta g_\delta \ ,$$

which proves our claim. Define now $\mathbb{E} := \mathbb{E}(2^\Omega)$. By what we have shown and Theorem 2.2, the family $\{T_\delta\}$ has a positive common extension $T : \mathbb{E} \longrightarrow \mathbb{G}$, and it is then clear that the vector measure $\varphi : 2^\Omega \longrightarrow \mathbb{G}$, given by

$$\varphi := T \circ \chi \ ,$$

is a positive common extension of the family $\{\varphi_\delta\}$. □

In the case $\mathbb{G} = \mathbb{R}$, Theorem 3.1 is equivalent to a result of Lembcke [7].

As a consequence of Theorem 3.1 we obtain the following vector-valued version of Proposition 1.1:

<u>3.2. Corollary.</u>

Let F_1 and F_2 be algebras of subsets of Ω . For positive vector measures $\varphi_1 : F_1 \longrightarrow \mathbb{G}$ and $\varphi_2 : F_2 \longrightarrow \mathbb{G}$, the following are equivalent:

(a) $\varphi_i(A_i) \leq \varphi_j(A_j)$ holds for all $i,j \in \{1,2\}$ and all $A_i \in F_i$ and $A_j \in F_j$ satisfying $A_i \subseteq A_j$.

(b) φ_1 and φ_2 have a positive common extension $\varphi : 2^\Omega \longrightarrow \mathbb{G}$.

Proof. Consider $m, n \in \mathbb{N}$ and $A_1, \ldots, A_{m+n} \in \mathcal{F}_1 \cup \mathcal{F}_2$ satisfying

$$\sum_{i=1}^{m} \chi_{A_i} \leq \sum_{i=m+1}^{m+n} \chi_{\overline{A}_i}$$

and hence

$$\sum_{i=1}^{m+n} \chi_{A_i} \leq n\chi_\Omega \ .$$

For $i \in \{1,\ldots,m+n\}$, define

$$C_i := \begin{cases} A_i , & \text{if } A_i \in \mathcal{F}_1 \\ \emptyset , & \text{otherwise} \end{cases}$$

and

$$D_i := A_i \setminus C_i \ .$$

Define now

$$g := \sum_{i=1}^{m+n} \chi_{C_i} \quad \text{and} \quad h := \sum_{i=1}^{m+n} \chi_{D_i} \ .$$

Then we have $g \in \mathbb{E}(\mathcal{F}_1)$ and $h \in \mathbb{E}(\mathcal{F}_2)$, and $g + h \leq n\chi_\Omega$. For $k \in \{1,\ldots,n\}$, define

$$M_k := \{ \omega \in \Omega \mid k\chi_\Omega(\omega) \leq g(\omega) \}$$

and

$$N_k := \{ \omega \in \Omega \mid k\chi_\Omega(\omega) \leq n\chi_\Omega(\omega) - h(\omega) \}$$

Then we have $M_k \in \mathcal{F}_1$ and $N_k \in \mathcal{F}_2$, and we also have $M_k \subseteq N_k$ and thus

$$\varphi_1(M_k) \leq \varphi_2(N_k) \ .$$

Using

$$\sum_{i=1}^{m+n} \chi_{C_i} = g = \sum_{k=1}^{n} \chi_{M_k}$$

and

$$\sum_{k=1}^{n} \chi_{N_k} = n\chi_\Omega - h = n\chi_\Omega - \sum_{i=1}^{m+n} \chi_{D_i} ,$$

and the previous inequality, we obtain

$$\sum_{i=1}^{m+n} \varphi_1(C_i) = \sum_{k=1}^{n} \varphi_1(M_k)$$

$$\leq \sum_{k=1}^{n} \varphi_2(N_k)$$

$$= n\varphi_2(\Omega) - \sum_{i=1}^{m+n} \varphi_2(D_i) \quad ,$$

hence

$$\sum_{i=1}^{m+n} (\varphi_1(C_i)+\varphi_2(D_i)) \leq n\varphi_2(\Omega) \quad ,$$

whence

$$\sum_{i=1}^{m+n} \varphi_{j(i)}(A_i) \leq n\varphi_2(\Omega) = n\varphi_1(\Omega) \quad ,$$

and thus

$$\sum_{i=1}^{m} \varphi_{j(i)}(A_i) \leq \sum_{i=m+1}^{m+n} \varphi_{j(i)}(\overline{A}_i) \quad ,$$

for all $j(1), \ldots, j(m+n) \in \{1,2\}$ satisfying $A_i \in F_{j(i)}$ for all $i \in \{1,\ldots,m+n\}$. The assertion now follows from Theorem 3.1. □

We now record two further applications of Theorem 3.1:

<u>3.3. Corollary.</u>

Let $\mathcal{C}$ be a collection of subsets of Ω satisfying $\emptyset, \Omega \in \mathcal{C}$. If $\zeta : \mathcal{C} \longrightarrow \mathbb{G}$ is a set function such that

$$\sum_{i=1}^{m} \zeta(C_i) \leq \sum_{i=m+1}^{m+n} \zeta(C_i)$$

holds for all $m, n \in \mathbb{N}$ and $C_1, \ldots, C_{m+n} \in \mathcal{C}$ satisfying

$$\sum_{i=1}^{m} \chi_{C_i} \leq \sum_{i=m+1}^{m+n} \chi_{C_i} \quad ,$$

then there exists a positive vector measure $\varphi : 2^{\Omega} \longrightarrow \mathbb{G}$ satisfying $\varphi(C) = \zeta(C)$ for all $C \in \mathcal{C}$.

Proof. For each $C \in \mathcal{C}$, let $\mathcal{F}_C$ denote the algebra generated by C and define a positive vector measure $\varphi_C : \mathcal{F}_C \longrightarrow \mathbb{G}$ by letting $\varphi_C(C) := \zeta(C)$ and $\varphi_C(\overline{C}) := \zeta(\Omega) - \zeta(C)$; note that the assumption on ζ yields $\varphi_C(\overline{C}) = \zeta(\overline{C})$ for all $C \in \mathcal{C}$ satisfying $\overline{C} \in \mathcal{C}$. Consider $m, n \in \mathbb{N}$ and $A_1, \ldots, A_{m+n} \in \bigcup_{\mathcal{C}} \mathcal{F}_C$ satisfying

$$\sum_{i=1}^{m} \chi_{A_i} \leq \sum_{i=m+1}^{m+n} \chi_{\overline{A}_i}$$

and thus

$$\sum_{i=1}^{m+n} \chi_{A_i} \leq n\chi_{\Omega} .$$

Relabelling the A_i if necessary, we obtain

$$A_1, \ldots, A_p, \overline{A}_{p+1}, \ldots, \overline{A}_{m+n} \in \mathcal{C}$$

for some $p \in \{0,1,\ldots,m+n\}$. Then we have

$$\sum_{i=1}^{p} \chi_{A_i} + (m+n-p)\chi_{\Omega} \leq \sum_{i=p+1}^{m+n} \chi_{\overline{A}_i} + n\chi_{\Omega} ,$$

hence

$$\sum_{i=1}^{p} \zeta(A_i) + (m+n-p)\zeta(\Omega) \leq \sum_{i=p+1}^{m+n} \zeta(\overline{A}_i) + n\zeta(\Omega) ,$$

whence

$$\sum_{i=1}^{m+n} \varphi_{C(i)}(A_i) \leq n\zeta(\Omega) ,$$

and thus

$$\sum_{i=1}^{m} \varphi_{C(i)}(A_i) \leq \sum_{i=m+1}^{m+n} \varphi_{C(i)}(\overline{A}_i) ,$$

for all $C(1), \ldots, C(m+n) \in \mathcal{C}$ satisfying $A_i \in \mathcal{F}_{C(i)}$ for all $i \in \{1,\ldots,m+n\}$. The assertion now follows from Theorem 3.1 □

In the case $\mathbb{G} = \mathbb{R}$, Corollary 3.3 is due to Horn and Tarski [4]; see also Lembcke [7].

3.4. Corollary.

Let $\mathcal{C}$ be a collection of subsets of Ω such that

$$(\bigcap_{\mathcal{D}} D) \cap (\bigcap_{\mathcal{E}} \bar{E}) \neq \emptyset$$

holds for any two disjoint finite subcollections $\mathcal{D}$ and $\mathcal{E}$ of $\mathcal{C}$. If $\zeta : \mathcal{C} \longrightarrow \mathbb{G}$ is a set function which maps $\mathcal{C}$ into an order bounded subset of $\mathbb{G}_+$, then there exists a positive vector measure $\varphi : 2^{\Omega} \longrightarrow \mathbb{G}$ satisfying $\varphi(C) = \zeta(C)$ for all $C \in \mathcal{C}$.

Proof. Let $u := \sup_{\mathcal{C}} \zeta(C)$. For each $C \in \mathcal{C}$, let $\mathcal{F}_C$ denote the algebra generated by C and define a positive vector measure $\varphi_C : \mathcal{F}_C \longrightarrow \mathbb{G}$ by letting

$$\varphi_C(C) := \zeta(C) \quad \text{and} \quad \varphi_C(\bar{C}) := u - \zeta(C) .$$

Consider $m, n \in \mathbb{N}$ and $A_1, \ldots, A_{m+n} \in \bigcup_{\mathcal{C}} \mathcal{F}_C$ satisfying

$$\sum_{i=1}^{m} \chi_{A_i} \leq \sum_{i=m+1}^{m+n} \chi_{\bar{A}_i}$$

and thus

$$\sum_{i=1}^{m+n} \chi_{A_i} \leq n\chi_{\Omega} .$$

We now reduce the previous inequality by subtracting

$$\chi_{A_i} = 0 \quad \text{if } A_i = \emptyset ,$$
$$\chi_{A_i} = \chi_{\Omega} \quad \text{if } A_i = \Omega , \text{ and}$$
$$\chi_{A_i} + \chi_{A_j} = \chi_{\Omega} \quad \text{if } A_i \text{ and } A_j \text{ are complementary.}$$

Relabelling the A_i if necessary, we thus obtain

$$\sum_{i=1}^{p+q} \chi_{A_i} \leq k\chi_{\Omega} ,$$

for suitable $p, q, k \in \mathbb{N} \cup \{0\}$ satisfying $p+q \leq m+n$ and $k \leq n$ as well as

$$A_1, \ldots, A_p, \bar{A}_{p+1}, \ldots, \bar{A}_{p+q} \in \mathcal{C} .$$

By the assumption on $\mathcal{C}$, we have

$$\left(\bigcap_{i=1}^{p} A_i \right) \cap \left(\bigcap_{i=p+1}^{p+q} A_i \right) \neq \emptyset \quad .$$

This yields $p+q \leq k$, hence

$$\sum_{i=1}^{p+q} \varphi_{C(i)}(A_i) \leq (p+q)\varphi_{C(i)}(\Omega) \leq ku \quad ,$$

whence, reversing the previous reduction,

$$\sum_{i=1}^{m+n} \varphi_{C(i)}(A_i) \leq nu \quad ,$$

and thus

$$\sum_{i=1}^{m} \varphi_{C(i)}(A_i) \leq \sum_{i=m+1}^{m+n} \varphi_{C(i)}(\overline{A}_i) \quad ,$$

for all $C(1), \ldots, C(m+n) \in \mathcal{C}$ satisfying $A_i \in \mathcal{F}_{C(i)}$ for all $i \in \{1,\ldots,m+n\}$. The assertion now follows from Theorem 3.1. □

In the case $\mathbb{G} = \mathbb{R}$, Corollary 3.4 is due to Marczewski [10,11]; see also Lembcke [7].

4. Remark

The following example shows that the proof of Proposition 1.1 given by Guy [3] is incorrect:

In the notation of [3], define $X := [0,1)$ as well as $F_1 := [0,\frac{1}{2})$, $F_2 := [\frac{1}{2},1)$, $G_1 := [0,\frac{1}{3})$, $G_2 := [\frac{1}{3},\frac{2}{3})$, $G_3 := [\frac{2}{3},1)$, let $\mathcal{R}$ and $\mathcal{S}$ denote the algebras generated by the sets F_1, F_2 and G_1, G_2, G_3, respectively, let $\lambda : \mathcal{R} \longrightarrow \mathbb{R}$ and $\mu : \mathcal{S} \longrightarrow \mathbb{R}$ denote the restrictions of the Lebesgue measure to these algebras, and define $N := 2$ and $M := 3$ as well as $a_1 := -1$, $a_2 := -1$, $b_1 := 1$, $b_2 := 1$, $b_3 := 1$. With these definitions, the final equality in the formula following (12) is false, and this is also true for the inequality by which it may be replaced.

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Authors' address:

Fakultät für Mathematik und Informatik
Universität Mannheim
A 5
6800 Mannheim
West Germany