Essays on Coalition Formation

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1 Introduction

In democratic countries economic decisions are made through the political process. Electoral results do not often generate a majority, which makes coalition politics crucial. The coalition formation and the bargaining inside the coalition directly influence a policy decision, given the election results. Who holds the government positions is one of the most important predictors of what policies will be adopted. In addition, expectations regarding potential future coalitions have an influence on the voters' choice at the election stage.

The existing literature on bargaining in legislatures and coalition formation is abundant. Most of the studies generally assume that if decision makers do not reach a decision, it is postponed until the next period and a certain default policy is implemented. Two kinds of a default policy are generally considered: the policy that gives exogenous benefits to all decision makers or a stochastic default policy. However, the policy decisions usually have a prolonged effect and become the default policies in future legislations. Nevertheless, there exist only a few attempts to introduce an endogenous default policy in bargaining problems. The first three chapters of my dissertation consider the endogenous default policy. By the endogenous default policy I shall basically mean the situation when the default policy in the absence of a decision is determined by the policy last implemented.

Furthermore, my dissertation considers the coalition size. Minimal winning coalitions (MWC) have appeared as a key prediction of various models of coalition formation and vote buying where the MWC is defined as the cheapest coalition which induces the acceptance. But as an empirical matter, supermajority coalitions appear at least as prevalent as minimal winning coalitions. I investigate conditions under which the MWC and supermajority coalitions emerge as an equilibrium outcome.

Chapter 1 incorporates an endogenous default policy into the distribution model of legislative bargaining. For more than three risk neutral players, the stationary Markov perfect equilibrium yields a minimum winning coalition under congressional and parliamentary systems. The resulting equilibrium allocation will be generally more unequal than with the exogenous default policy.

In Chapter 2 the endogenous and exogenous default policy rules are compared in the distribution model of legislative bargaining. If the initial default policy is symmetric, players are indifferent between the two rules. Players with a relatively high initial default payoff always support the endogenous default policy, which leads to an extreme allocation and cycling majorities. Players in the "middle" support the exogenous system, because they are always included into the coalition. Players with relative low initial default payoff favor the endogenous rule, if their default payoff is extremely low, and prefer the exogenous rule otherwise. If I allow for negative payments, players with an extremely low default payoff may change their preferences and support the exogenous system, which guarantees more stability. Moreover, if players decide about a policy rule by a simple majority voting, both rules can be chosen, depending on the initial allocation.

As shown in Chapter 1, an endogenous default policy in the "divide the dollar" game leads to an extreme allocation when politicians are risk neutral. In Chapter 3 I show that if voters prefer more stability in the allocation, they can influence politicians via elections. This chapter shows that a symmetric allocation is obtained for a high enough discount rate if voters can reelect all politicians. If voters can elect only the agenda setter, only limited stability can be achieved. Maximal outcome stability requires political stability. Limited political competition can be introduced at the cost of the outcome stability.

Chapter 4 studies coalition formation under asymmetric information. An outside party offers private transfers to members of a committee in order to influence its decision. The willingness to accept such transfers is private information. The paper demonstrates that a supermajority coalition induces truth-telling equilibrium and secures the implementation of the decision, desirable by an outside party, for a price close to the minimal winning coalition price.

2 Endogenous default policy in political games

2.1 Introduction

Most of the existing literature on bargaining in legislatures and coalition formation generally assumes that when decision makers do not reach a decision, it is postponed till the next period and a certain default policy is implemented. Two kinds of a default policy are generally considered: the policy that gives symmetric constant benefits to all decision makers or a stochastic default policy. This paper considers the endogenous default policy: the default policy in the absence of a decision is determined by the policy last implemented.

One example of a model in which a constant default policy is applied is Persson, Roland and Tabellini (2000). There the default policies are constant and give the same utility to all participants in a complex framework that includes taxation, redistribution and public good decisions as well as elections.

Classical examples of decision postponing in the distribution problem with a symmetric zero default policy can be found in Rubinstein (1982) and Baron and Ferejohn (1989) articles. The decision process is characterized by an infinite horizon where a single proposal is made in each period and is either accepted, terminating the process, or rejected, moving the decision process to the next period. An additional example is presented in Diermeier and Feddersen (1998), where the authors investigate the influence of the voice of confidence procedure on the decision making process.

The stochastic default policy is considered by Diermeier and Merlo (2000). In a two period model of government formation and termination, a policy shock determines the default policy that will be implemented if no government is formed during that period.

There are few attempts to introduce an endogenous default policy in bargaining problems. Baron (1996) considers a dynamic problem of collective goods programs, where the players have different preferences regarding the amount of the public good characterized by a one-dimensional ideal point. Baron shows that when the default policy is the decision from the previous period, the amount of the public good converges to the median politician's ideal point. Baron and Diermeier (1998) also use an endogenous default policy in a two period model that incorporates elections, governmental formation and legislative bargaining in a two-dimensional spatial environment.

Shepsle and Weingast (1984) investigate strategic voting and agenda setting in a dynamic framework and in the contest of amendment agendas. Differing from the current study, they assume that there is a final number of voting rounds and that players are interested only in the final result.

The current paper investigates a classical "divide the cake"-problem with the endogenous default policy as introduced above. Politicians have to decide how to divide an exogenously given budget under two alternative systems: the parliamentary and the congressional. As in Helpman and Persson (2001), I define the political systems in the following way: under the congressional system, an agenda setter is chosen randomly in each period and proposes a budget allocation to the congress. The proposal is accepted and implemented if the majority supports it; otherwise the default policy is implemented. Under the parliamentary system a randomly chosen agenda setter has to obtain the support of a randomly chosen coalition.

This modeling of the systems is a simplified way to reflect the following difference between them: under the congressional system the legislative cohesion is relatively low. The direct election of an executive makes it unnecessary to form a stable majority to support the cabinet, nothing then limits the variety of coalitions that can be formed. Such a regime captures some of the features of the presidential-congressional regime, like that of the US. On the contrary, the government survival plays a crucial role in the formation of a stable parliamentary majority and the legislative cohesion emerges as an endogenous outcome under the parliamentary system,. The assumption of the model reflects the case when each coalition partner has a veto right. The veto can be thought of as a vote of confidence on the government.

The main results of the paper are the following: when politicians are risk neutral the equilibrium under both systems is characterized by a minimal winning coalition strategy. This result is proved for a two period game as well as for an infinite horizon game.

I show that under the congressional system such an equilibrium does no longer exist for a two period game when the politicians are sufficiently risk averse. The equilibrium is characterized by risk sharing allocations and the degree of risk sharing depends on the number of politicians and the level of risk aversion. In the parliamentary system the equilibrium strategy is characterized by a minimal winning coalition independently of the players' risk attitudes.

In the simultaneous research Kalandrakis (2002, 2004) investigates an infinite game under what I call a congressional system. He obtains the same result as in the current paper for a more general utility function, but he uses a more complex proof.

The paper is structured in the following way: the next section describes the model. Section 2.3 considers myopic players. Section 2.4 presents a two period game under the assumption of risk neutrality. The section considers both systems: the congressional and the parliamentary system. Section 2.5 presents an infinite horizon version of the same game. Section 2.6 considers risk aversion in the two period framework. Section 2.7 concludes.

2.2 Model

The political system consists out of the set of players $N = \{1, ..., n\}$ where $n \geq 3$ and odd. Each period politicians are to allocate a unit of a perfectly divisible cake among themselves. Let X denote the set of feasible allocations in each period, that is, $X = \left\{x \in \Re^n_+ : \sum_{i=1}^n x^i_t = 1\right\}$, where x^i_t denotes the payoff of player i at period t.

At this stage I assume that each player has a linear utility function that depends on her payoff only. I also assume that the players discount the future by a common discount factor $\delta < 1$. The preferences are separable over time, so the utility function of player i is

$$U_i = \sum_{t=1}^{T} \delta^{t-1} x^i_t. \tag{1}$$

The game is played as follows: each period nature randomly selects one player to be the agenda setter. The agenda setter proposes an allocation in X, and each player responds by either accepting or rejecting the proposal.

Under the congressional system the decision rule in each period is a simple majority rule. If at least $\frac{n+1}{2}$ members of the congress support proposal $x_t \in X$ in period t, it is implemented. Otherwise, the endogenous default policy is implemented, which is the allocation in X that was implemented in the previous period, namely $x_{t-1} \in X$. $x_0 \in X$ is the default policy in the first period and is given exogenously.

Following Helpman and Persson (2001), I assume that the parliamentary system is characterized by nature choosing the agenda setter as well as coalition members. I assume for simplicity that the size of an exogenously given coalition is exactly the simple majority needed for a decision. If all coalition members in period t support proposal x_t, it is implemented. Otherwise, the endogenous default policy, x_{t-1}, is implemented. It would be more realistic to extend the model to an initial "government formation" stage, when nature chooses the agenda setter – the prime minister – who afterwards proposes to some of the politicians to form a coalition. But in the simple framework of this paper such a game will be identical to the congressional regime game.

Let h_t denote the past history (identity of previous agenda setters, proposals they made, how each player voted for those proposals, allocations implemented in each period) together with the identity of the current agenda setter. A feasible action for player i at date t is denoted by $a^i_t(h_t)$. When i is the agenda setter, $a^i_t(h_t) \in X$ denotes the proposal offered by i at date t, when the history is h_t.

When i is not the agenda setter, $a_t^i(h_t) \in \{accept, reject\}$ denotes the decision rule by i at date t. Strategy s^i for player i is a sequence of actions $\left\{a_t^i(h_t)\right\}_{t=1}^T$, and the strategy profile s is an n-tuple of strategies, one for each player.

Whenever a player is to take an action, she knows which history has occurred, the rules of the game and the preferences of the other players; so the game is of perfect information.

The solution concept is a sub-game perfect Nash equilibrium. I focus on a particular class of equilibria called stationary. In such an equilibrium a player chooses a history-independent strategy in each strategically identical sub-game. In this model, all sub-games with the same default policy are strategically identical, since their action sets and the future sequences of moves are the same. Thus, the equilibrium considered is a "Markov" or a "state-space" equilibrium in which the past history influences the current play only through its effect on a state variable that summarizes the direct effect of the past on the current environment. Alternatively, the strategies depend only on the payoff-relevant information. Then the equilibrium strategy profile can be defined as an n-tuple of strategies s^i, that depend only on the actual default policy: $\left\{a_t^i(x_{t-1})\right\}_{t=1}^T$in each period. Using this specific type of equilibrium I rule out a possible punishment strategy profile which typically leads to a multiplicity of sub-game perfect equilibria (see Baron and Ferejohn (1989)).

Since no player can change the outcome if a proposal is supported by more than simple majority, I restrict attention to equilibria in which weakly dominated strategies are eliminated. Otherwise any allocation can be supported as an equilibrium.

2.3 Myopic case

As a benchmark case consider myopic players who maximize their current payoff in each period, without taking into consideration how their action will influence the future periods' outcomes. This case also corresponds to the one period game.

To begin with, I define the minimal winning coalition (MWC) for myopic (one period) game:

Definition 1 *The agenda setter forms the MWC if any coalition member is an element of MWC, where $MWC \subset N$ is nonempty set which satisfies*

i. $|MWC| = \frac{n-1}{2}$

ii. $\forall\ j \in MWC\ x_t^j = \min\left\{ \widehat{x}_t^j \in x_t \middle| a_t^j(x_{t-1}, x_t) = accept\right\}$

$\forall\ j \notin MWC\ \ x_t^j = 0$

iii. $\forall l \in MWC$ *and* $\forall k \notin MWC: \quad \sum\limits_{j \in MWC \setminus \{l\}} x_t^j + x_t^k \geq \sum\limits_{j \in MWC} x_t^j$

where $x_t^k = \min\left\{ \widehat{x}_t^k \in x_t \middle| a_t^k(x_{t-1}, x_t) = accept\right\}$

Thus, in each period the agenda setter chooses exactly $\frac{n-1}{2}$ coalition members such that the sum of coalition payments, which induce the proposal acceptance, is minimal.

The equilibrium strategy profile and payoffs in the myopic case are characterized as follows[1]:

Proposition 1 *a strategy profile is an equilibrium of myopic (one period) game if it has the following form:*

Let $i \in N$ denote the agenda setter

1. The decision rule for player j is as follows:
accept proposal x_t iff

$$x_t^j \geq x_{t-1}^j, \ \forall \ \ j \in N, j \neq i$$

2. The agenda setter forms the MWC and coalition payoffs are

$$x_t^j = x_{t-1}^j \ \forall \ j \in MWC$$

Proof. Given a proposal by player i, player $j \in MWC$ will accept if he is at least as good as under the default policy[2]. Since the agenda setter's utility is strictly increasing in her share, she proposes exactly x_{t-1}^j to player j in order to buy his vote. In order to get the proposal accepted, the agenda setter needs the support of $\frac{n-1}{2}$ players. Thus, forming the MWC is equivalent to maximizing the agenda setter's current period utility.

Note, that the minimum value of the MWC proposal for the agenda setter is always at least as high as her own default policy payoff :

$$x_t^i = 1 - \frac{1 - x_{t-1}^i}{n-1}\frac{n-1}{2} = \frac{1}{2} + \frac{x_{t-1}^i}{2}\begin{cases} > x_{t-1}^i & if \quad x_{t-1}^i < 1 \\ = x_{t-1}^i & if \quad x_{t-1}^i = 1 \end{cases}, \qquad (2)$$

where $\frac{1-x_{t-1}^i}{n-1}$ is the maximal payoff of a MWC member, which occurs when all other players, apart from i, have an equal default policy. Therefore, the agenda setter always proposes such an allocation that will be accepted. ∎

Note that when the default policy is different among potential coalition members, the agenda setter will always play a pure strategy and there is unique equilibrium allocation. However, if there are several players with equal default policies, then she is indifferent whom to include into the coalition and, for simplicity, I assume that she includes them with equal probabilities.

2.4 Congressional system

2.4.1 Two period game

I start with considering the two periods version, $T = 2$, of a simple congressional system, and investigate how an assumption of the endogenous default policy influences the results of legislature bargaining. The game is solved by backward induction.

First result considers the second period of the game:

[1]As it is standard in the literature, I assume that a player supports the proposal whenever he is indifferent between accepting or rejecting it.

[2]I refer to the current agenda setter as "she" and to other players as "he"

Corollary 1 *The equilibrium strategy profile in the second period is described by Proposition 1.*

Proof. Follows from Proposition 1. ■

Next, I define the MWC strategy profile for the first period:

Definition 2 *The MWC strategy profile of the two period dynamic game under congressional system is defined as follows: let $i \in N$ denote the agenda setter in period 1.*

1. For each player j the decision rule in period 1 is as follows: accept proposal x_1 iff

$$x_1^j + \delta V_2^j(x_1) \geq x_0^j + \delta V_2^j(x_0), \text{ for all } j \in N,\, j \neq i, \tag{3}$$

where $V_2^j(x)$ denotes the continuation value of player j and is defined as the expected second period payoff when the default policy vector in period two is x. The LHS is the expected utility of coalition member j if he accepts proposal x_1^j, and RHS is his expected utility if he rejects it.

2. In period 1 the agenda setter forms the MWC and coalition payoffs are

$$x_1^j = \max\left[-\delta V_2^j(x_1) + x_0^j + \delta V_2^j(x_0), 0\right] \ \forall \ j \in MWC \tag{4}$$

Now I turn to the main result of this section, namely that the MWC is the equilibrium strategy profile for any initial allocation. In order to prove it I use the following strategy: to begin with, I calculate the lower bound of the agenda setter's utility under the MWC strategy profile, denoted by U_{MWC}^i. Then, I find the upper bound of the agenda setter's utility if he deviates from the MWC strategy. Finally, I show that there are no profitable deviations from the MWC strategy.

Lemma 1 *Under congressional system the lower bound of the first period agenda setter's utility if she plays according to the MWC strategy profile is equal to*:

$$\inf U_{MWC}^i = \frac{1}{2} - \delta\frac{1}{4n} + \delta\frac{1}{n}$$

Proof. Let i be the agenda setter in period one. Given the proposal x_1^j by player i, player j will accept if he is at least as happy as under keeping the default policy. Following the MWC strategy, the agenda setter will propose to player j exactly $\max\left[0, -\delta V_2^j(x_1) + x_0^j + \delta V_2^j(x_0)\right]$, in order to buy his vote. The agenda setter proposes such payoffs to $\frac{n-1}{2}$ players and they all accept. All other players obtain zero.

I split the payoff of the first period's coalition members into the component, denoted by $Q^j(x_0)$, that depends only on the initial default policy, and the component that depends on proposal x_1, denoted by $f^j(x_1)$ (for details see

Appendix):

$$\begin{aligned} x_1^j &= \max\left[0, -\delta V_2^j(x_1) + x_0^j + \delta V_2^j(x_0)\right] = \quad (5)\\ &= \max\left[0, x_0^j + \delta V_2^j(x_0) - \delta\left(\frac{1}{n} + \frac{n-1}{2n} f^j(x_1)\right)\right] \\ &\equiv \max\left[0, Q^j(x_0) - \delta \frac{n-1}{2n} f^j(x_1)\right], \end{aligned}$$

$$Q^j(x_0) \equiv x_0^j + \delta V_2^j(x_0) - \delta \frac{1}{n},$$

where

$$f^j(x_1) = \begin{cases} 0 & \text{if} \quad \exists l : x_1^l < x_1^j, \quad l, j \in MWC \\ x_1^j & \text{if} \quad \forall l: \ x_1^l > x_1^j, l \neq j; l, j \in MWC \\ \frac{x_1^j}{|U|+1} & \text{if } \exists U = \left\{ \begin{array}{c} x_1^l : x_1^l = x_1^j, l \neq j \text{ and } \ \forall m : x_1^l < x_1^m \\ l \neq m \neq j; \ l, j, m \in MWC \end{array} \right\} \end{cases}. \quad (6)$$

Next, I characterize the most expensive MWC. I denote as coalition cost the sum of coalition members' payoffs $\sum_{j \in MWC} x_t^j$. The maximal cost of the MWC is bounded by the following expression:

$$\begin{aligned} \sum_{j \in MWC} x_t^j &\leq \frac{\frac{n-1}{2}}{n-1} \sum_{j=1, j \neq i}^{n} x_1^j \leq \frac{\frac{n-1}{2}}{n-1} \sum_{j \neq i} \max\left[0, Q^j(x_0)\right] \\ &= \frac{1}{2} \sum_{j=1, j \neq i}^{n} \max\left[0, Q^j(x_0)\right]. \quad (7) \end{aligned}$$

The logic behind this expression is the following: the agenda setter has to choose $\frac{n-1}{2}$ members of her coalition from $n-1$ players, where $n-1$ is an even number. The maximal cost of the coalition with $\frac{n-1}{2}$ members cannot be higher than half of the cost of the coalition with $n-1$ members. The next step is to observe that $f^j(x_1) \geq 0$. Therefore, $x_1^j \leq Q^j(x_0)$ for all $j \in N$.

Then, the upper bound of the MWC cost is obtained when $Q^j(x_0) > 0$ for all $j \in N$[3], and we can neglect the max operator. Substituting for $Q^j(x_0)$, I obtain:

$$\begin{aligned} \sum_{j \in MWC} x_t^j &\leq \frac{1}{2} \sum_{j=1, j \neq i}^{n} x_1^j \leq \frac{1}{2} \sum_{j=1, j \neq i}^{n} Q^j(x_0) \\ &= \frac{1}{2}\left(\sum_{j=1, j \neq i}^{n} x_0^j + \delta \sum_{j=1, j \neq i}^{n} V_2^j(x_0) - \delta \frac{n-1}{n}\right). \quad (8) \end{aligned}$$

[3] For example, this would be the case if the initial default policy is symmetric.

The maximal value of the $\sum_{j=1,j\neq i}^{n} x_0^j$ is equal to one. The sum of the continuation values of all players other than the agenda setter is equal to $\sum_{j=1,j\neq i}^{n} V_2^j(x_0) = 1 - V_2^i(x_0)$. This expression is bounded from above by $1 - \frac{1}{n}\left(1 - \frac{1}{2}\right)$, because, first, in the last period the maximal cost of the MWC is bounded by $\frac{1}{2}$ (see equation (2)). Second, the agenda setter in period t will be in the coalition in period $t+1$ only in case she is again chosen to be the agenda setter, because her default policy will be maximal comparing to other players[4].

As a result, the upper bound of the coalition cost in the first period is:

$$\begin{aligned} \sum_{j\in MWC} x_t^j &\leq \frac{1}{2}\sum_{j=1,j\neq i}^{n} x_1^j \leq \frac{1}{2}\sum_{j=1,j\neq i}^{n}\left[x_0^j + \delta V_2^j(x_0) - \delta\frac{1}{n}\right] \\ &\leq \frac{1}{2}\left[1 + \delta\left(1 - V_2^i(x_0)\right) - \delta\frac{n-1}{n}\right] \\ &\leq \frac{1}{2}\left[1 + \delta\left(1 - \frac{1}{n}\left(1 - \frac{1}{2}\right)\right) - \delta\frac{n-1}{n}\right] = \frac{1}{2} + \delta\frac{1}{4n}. \end{aligned} \tag{9}$$

Then the lower bound of the expected utility of the first period's agenda setter is equal to:

$$\inf U^i = \frac{1}{2} - \delta\frac{1}{4n} + \delta\frac{1}{n}. \tag{10}$$

■

Next step is to describe the potentially profitable deviations from the MWC strategy profile in the first period. Lemma 2 shows that only two proposals has to be considered.

Lemma 2 *Under congressional system the upper bound of the first period agenda setter's utility if she deviates from the MWC strategy profile is achieved when:*

(i) If $n \leq \frac{2(1+\delta)+\sqrt{4+8\delta+5\delta^2}}{\delta}$, *the agenda setters plays according to* $A0$ *strategy profile, where* $A0$ *is characterized by the following allocation: the agenda setter and any* $\frac{n-1}{2}$ *other players obtain* $\frac{2}{n+1}$, *all the rest obtain zero.*[5]

(ii) If $n > \frac{2(1+\delta)+\sqrt{4+8\delta+5\delta^2}}{\delta}$, *the agenda setters plays according to* $A1$ *strategy profile, where* $A1$ *is characterized by the following allocation: the agenda setter and any* $\frac{n+1}{2}$ *other players obtain* $\frac{2}{n+3}$, *all the rest obtain zero.*

Proof. The utility of the agenda setter is increasing in her first period payoff and in the probability to be in the second period coalition. Moreover, the probability to be in the second period coalition is decreasing in her first period payoff but not monotonically. In the first period, under the MWC the agenda setter obtains the highest payoff comparing to other players and she will never be the coalition member in the second period. Decreasing her payoff

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[4] I show in Appendix that this is indeed the case.

[5] The number in the notation of the allocation defines how many players are included into the coalition above the $\frac{n-1}{2}$ required majority.

by a small amount, the agenda setter does not influence her chance to be in the second period coalition. Therefore, it is not worth for the agenda setter to deviate "a bit" from the allocation specified by the MWC strategy.

In order to have a positive chance to be in the second period coalition the first period agenda setter has to propose an allocation that determines her second period default policy to be not higher than the default policy of at least $\frac{n-1}{2}$other players. Otherwise she obtains a positive payoff in the second period only if she becomes the agenda setter again.

In order to achieve a positive chance to be in the second period coalition at a minimum cost, she allocates $\frac{2}{n+1}$ to herself and to $\frac{n-1}{2}$ other players ($A0$). I can assume that her payoff is by ε less than payoffs of coalition members, where ε is a very small number. As a result, she will be in the winning coalition with probability $\frac{n-1}{2n}$.

Moreover, if she allocates $\frac{2}{n+3}$ to herself and to $\frac{n+1}{2}$ other players ($A1$), she will be a coalition member for sure[6]. Hence there is no sense for her to allocate positive payoffs to more than $\frac{n+1}{2}$ players .

Therefore, if the agenda setter deviates form the MWC strategy, she has to choose between these two allocations, $A0$ and $A1$. The following trade off takes place: under $A0$ the probability to be in the second period coalition is lower comparing to $A1$, but the payoff in the first period, and, hence, the default policy in the second period, is higher.

The expected utilityof the agenda setter under $A0$ is

$$EU_i^{A0} = \frac{2}{n+1} + \delta \left(\frac{1}{n} + \frac{2}{n+1} \frac{n-1}{2n} \right). \tag{11}$$

The expected utility of the agenda setter under $A1$ is

$$EU_i^{A1} = \frac{2}{n+3} + \delta \left(\frac{1}{n} \left(1 - \frac{2}{n+3} \right) + \frac{2}{n+3} \frac{n-1}{n} \right). \tag{12}$$

The difference between these expressions is equal to

$$EU_i^{A0} - EU_i^{A1} = \frac{4n - \delta n^2 + 4\delta n + \delta}{(n+1)(n+3)n} \begin{cases} \geq 0 \text{ iff } n \leq \frac{2(1+\delta)+\sqrt{4+8\delta+5\delta^2}}{\delta} \\ < 0 \text{ iff } n > \frac{2(1+\delta)+\sqrt{4+8\delta+5\delta^2}}{\delta} \end{cases} \tag{13}$$

■

The intuition for this result is that with an increase in n the agenda setter's payoff in the first period decreases, and she is more willing to trade it for the higher chances to be in the second period coalition, therefore she prefers $A1$. Moreover, for a higher δ the maximal value of n that will be consistent with the optimal $A0$ will be lower because the agenda setter will strongly prefer a higher chance to be in the second period coalition over a higher payoff in the first one.

In this lemma I do not consider the feasibility of allocations $A0$ and $A1$, in the sense that they will be accepted, once proposed by the agenda setter. I do

[6] I assume again that agenda setter's payoff is by ε less than the payoffs of the other coalition members.

not have the formal proof, but I have a strong intuition that these allocations are feasible for any initial default policy. Nevertheless, I do not need the proof of the feasibility in order to obtain the main results of this section.

Combining the results of Lemmas 1 and 2, I can state the following result:

Proposition 2 *Under congressional system the MWC strategy profile is the unique equilibrium of two periods game if* $n \geq 5$ *,.*

Proof. In the second period the MWC strategy profile is the equilibrium due to Proposition 1.

In the first period all coalition members accept the MWC proposal due to the construction of payoffs. The agenda setter cannot propose to coalition members the payoffs lower than under the MWC strategy, because then the proposal would be rejected.

The possible candidates deviations from the MWC are described in Lemma 2. For $n \geq 7$ the lower bound of the expected utility from playing according to the MWC strategy (8) is strictly higher than the expected utility of playing according to the deviation strategy (9) and (10). When $n = 5$, the numerical simulations for various initial default policy vectors show the same result.

The uniqueness follows from a simple observation that the MWC strategy profile maximizes an expected utility of the agenda setter in both periods. Because the agenda setter has the advantage of the proposal, she sets which strategy profile will be realized. ■

As mentioned above, I do not need to prove that allocations $A0$ and $A1$ are feasible. The agenda setter does not deviate, hence the question of feasibility of such allocations is not relevant.

For $n = 3$ there will be cases when it will be profitable for the agenda setter to deviate to $A0$. It happens when the following conditions are satisfied:

$$\left(1 - x_0^j\right) + \delta\frac{1}{3} < \frac{1}{2}\left(1 + \delta\right) \qquad if \quad x_0^j < x_0^l,$$

$$\left(1 - \frac{3}{3+\delta}x_0^j\right) + \delta\frac{1}{3} < \frac{1}{2}\left(1 + \delta\right) \qquad if \quad x_0^j > x_0^l,$$

$$\left(1 - \frac{6+\delta}{6+2\delta}x_0^j\right) + \delta\frac{1}{3} < \frac{1}{2}\left(1 + \delta\right) \qquad if \quad x_0^j = x_0^l,$$

where i is the agenda setter, and l and j are two other players. These conditions will hold if the initial default policy is low for the agenda setter and if the default policies of two other players are relatively close.

2.4.2 Infinite game

In this section I consider the infinite game under a congressional system. The preference function of player i now is given as

$$U_i = \sum_{t=1}^{\infty} \delta^{t-1} x_t^i. \tag{14}$$

This game satisfies the property of continuity in the infinity: the discounted value of a game is limited in any period, because the payoffs are limited in each period and the players discount the future. Hence, for the strategy profile to be a sub-game perfect, it has to satisfy the one-stage deviation principle. It is enough to show that no player wants to deviate in period t, if in all subsequent periods all players play according to the equilibrium strategy (see Fudenberg and Tirole (1998)).

First, I define the MWC strategy profile. It is similar to Definition 2:

Definition 3 *The MWC strategy profile of infinite game and congressional system is defined as follows: let $i \in N$ denote the agenda setter in period t.*

1. For each player j the decision rule in period t is as follows:
accept the proposal x_t iff

$$x_t^j + \delta V_{t+1}^j(x_t) \geq x_{t-1}^j + \delta V_{t+1}^j(x_{t-1}), \text{ for all } j \in N,\, j \neq i, \tag{15}$$

where the LHS is the expected utility of coalition member j if he accepts proposal x_t^j, and RHS is his expected utility if he rejects. $V_{t+1}^j(x_t)$ denotes the continuation value of player j when the default policy in period $t+1$ is vector x_t. It is defined recursively as

$$V_{t+1}^j(x_t) = E\left[x_{t+1}^j(x_t) + \delta V_{t+2}^j(x_{t+1})\right].$$

2. The agenda setter forms the MWC and coalition payoffs are

$$x_t^j = \max\left[\left(-\delta V_{t+1}^j(x_t) + x_{t-1}^j + \delta V_{t+1}^j(x_{t-1})\right), 0\right] \;\; \forall \;\; j \in MWC \tag{16}$$

I begin with some characteristics of the MWC:

Lemma 3 *If the default payoff of player j is zero, then he supports the MWC proposal even if his payoff is zero.*

Proof. Consider player j in period t such that $x_{t-1}^j = 0$. If $x_t^j = 0$ and the MWC proposal is accepted , then $V_{t+1}^j(x_t) = \frac{1}{n} + \frac{1}{n}\sum_{\tau=1}^{\infty}\delta^\tau$. If the MWC proposal is rejected then $V_{t+1}^j(x_{t-1}) = \frac{1}{n}\left(1 - C_{t+1}^j(x_{t-1})\right) + \frac{1}{n}\sum_{\tau=1}^{\infty}\delta^\tau$, where $C_{t+1}^j(x_{t-1}) \geq 0$ is the coalition cost if j is the agenda setter in period $t+1$. Then $-\delta V_{t+1}^j(x_t) + \delta V_{t+1}^j(x_{t-1}) \leq 0$ and, according to (22):

$$x_t^j = \max\left[\left(-\delta V_{t+1}^j(x_t) + 0 + \delta V_{t+1}^j(x_{t-1})\right), 0\right] = 0 \tag{17}$$

■

The next lemma follows directly from Lemma 3 and equation (23):

Lemma 4 *The allocation is an extreme one starting from period $t+2$: the agenda setter obtains the whole budget, while other players obtain zero.*

The next lemma considers players with equal default policies:

Lemma 5 *Consider two players $l, k \in N$ such that $x_{t-1}^k = x_{t-1}^l$. Then the agenda setter includes them into the coalition in period t with equal probabilities.*

Proof. (i) Suppose that both players are included into coalition with probability 1 or 0. Then proof is obvious.

(ii) Consider the case where the agenda setter has to choose one coalition member between players k and l. Note that if they are included into a the coalition with equal probability $\frac{1}{2}$, then $x_t^k = x_t^l$. Suppose that both players have a positive inclusion probability, but l is included with higher probability than k. This implies that both players support the proposal for the same payoff $x_t^k = x_t^l$, otherwise the agenda setter would include only the player with lower coalition payoff. Then $V_{t+1}^k(x_{t-1}) < V_{t+1}^l(x_{t-1})$, while, first, player l has a higher probability to be next period coalition member and, second, both of them face the same probability to be the agenda setter and the same coalition costs. $x_{t-1}^k = x_{t-1}^l$ and $x_t^k = x_t^l \Rightarrow V_{t+1}^k(x_t) = V_{t+1}^l(x_t)$. Therefore, $x_t^k > x_t^l$, which is a contradiction.

Suppose that only l is included into a coalition, which implies that player l will support the proposal for the payoff $x_t^l \leq x_t^k$. Then $V_{t+1}^k(x_{t-1}) < V_{t+1}^l(x_{t-1})$ and $V_{t+1}^k(x_t) < V_{t+1}^l(x_t)$. But $-V_{t+1}^l(x_t) + V_{t+1}^l(x_{t-1}) > -V_{t+1}^k(x_t) + V_{t+1}^k(x_{t-1})$, which implies $x_t^l > x_t^k$ and is again an contradiction.

This Lemma is easily extended to any number of players with identical initial default policies. ■

The proof of the main result in this section follows the same steps as the proof of Proposition 2: first, I calculate the lower bound of the agenda setter's utility under the MWC strategy profile, denoted by U_{MWC}^i. Second, I find the upper bound of the agenda setter's utility if she deviates from the MWC strategy. And finally, I show that there are no profitable deviations from the MWC strategy.

Lemma 6 *Under congressional system the lower bound of the agenda setter utility if she plays according to the MWC strategy is equal to:*

$$\frac{1}{2} + \frac{1}{n}\sum_{\tau=1}^{\infty}\delta^{\tau}$$

Proof. Suppose all players play according to the MWC strategy. Let i denote the agenda setter in period t. The payoff to the coalition member j in period t has to satisfy the condition (21):

$$x_t^j + \delta\left(\frac{1}{n} + \frac{1}{n}\sum_{as_{t+1}, as_{t+1}\neq j}\left[p_{t+1}^j(x_t, as_{t+1})\, x_{t+1}^j(x_t, as_{t+1})\right]\right) + \frac{1}{n}\sum_{\tau=2}^{\infty}\delta^{\tau} \geq$$

$$x_{t-1}^j + \delta\left(\frac{1}{n}\left(1 - C_{t+1}^j(x_{t-1})\right) +\right.$$

$$\left.\frac{1}{n}\sum_{as_{t+1}, as_{t+1}\neq j}\left[p_{t+1}^j(x_{t-1}, as_{t+1})\, x_{t+1}^j(x_{t-1}, as_{t+1})\right]\right) +$$

$$+\delta^2\frac{1}{n}\sum_{as_{t+1}}\left(\frac{1}{n}\left(1 - C_{t+2}^j(x_{t+1})\right) +\right.$$

$$\frac{1}{n}\sum_{as_{t+2},as_{t+2}\neq j}\left[p_{t+2}^{j}\left(x_{t+1},as_{t+2}\right)x_{t+2}^{j}\left(x_{t+1},as_{t+1}\right)\right]\Bigg)+$$

$$+\frac{1}{n}\sum_{\tau=3}^{\infty}\delta^{\tau}, \tag{18}$$

where $as_t \in N$ is the identity of the agenda setter in period t. $p_{t+1}^{j}\left(x_t, as_{t+1}\right)$ is the probability for player j to be in the coalition in period $t+1$. $C_t^j\left(x_{t-1}\right)$ is the cost of the MWC in period t for player j, if j is the agenda setter.

In period $t+1$ the payoff of coalition member j has to satisfy the following condition:

$$x_{t+1}^{j}+\frac{1}{n}\sum_{\tau=1}^{\infty}\delta^{\tau}\geq x_{t}^{j}+ \tag{19}$$

$$+\delta\left(\frac{1}{n}+\frac{1}{n}\sum_{as_{t+2},as_{t+2}\neq j}\left[p_{t+2}^{j}\left(x_{t},as_{t+2}\right)x_{t+2}^{j}\left(x_{t},as_{t+2}\right)\right]\right)+\frac{1}{n}\sum_{\tau=2}^{\infty}\delta^{\tau}$$

When the MWC proposal is accepted in period t, the coalition member j's payoff in period $t+1$ depends only on x_t^j and does not depend on the identity of the agenda setter. Therefore $x_{t+2}^{j}\left(x_t, as_{t+2}\right) = x_{t+1}^{j}$. Hence, according to (22):

$$x_{t+1}^{j}=\max\left[\frac{1}{1-\delta\frac{1}{n}\sum_{as_{t+2},as_{t+2}\neq j}\left[p_{t+2}^{j}\left(x_{t},as_{t+2}\right)\right]}x_{t}^{j},0\right]. \tag{20}$$

Player's j payoff in period $t+1$ will be positive if and only if his payoff in period t is positive, otherwise it would be zero. Moreover, the player's payoff in $t+1$ is the increasing function of his payoff in period t.

To begin with, I assume that the coalition payoffs of all potential coalition members in period t, x_t^j, is positive. Then, I can rewrite condition (22) in period t as:

$$x_{t}^{j}+\delta\frac{1}{n}\sum_{as_{t+1},as_{t+1}\neq j}\left[p_{t+1}^{j}\left(x_{t},as_{t+1}\right)\frac{1}{1-\delta\frac{1}{n}\sum_{as_{t+2},as_{t+2}\neq j}\left[p_{t+2}^{j}\left(x_{t},as_{t+2}\right)\right]}x_{t}^{j}\right]=$$

$$x_{t-1}^{j}+\delta\left(-\frac{1}{n}C_{t+1}^{j}\left(x_{t-1}\right)+\frac{1}{n}\sum_{as_{t+1},as_{t+1}\neq j}\left[p_{t+1}^{j}\left(x_{t-1},as_{t+1}\right)x_{t+1}^{j}\left(x_{t-1},as_{t+1}\right)\right]\right)+$$

$$+\delta^{2}\frac{1}{n}\sum_{as_{t+1}}\Bigg(-\frac{1}{n}C_{t+2}^{j}\left(x_{t+1}\right)+$$

$$\frac{1}{n}\sum_{as_{t+2},as_{t+2}\neq j}\left[p^j_{t+2}\left(x_{t+1},as_{t+2}\right)x^j_{t+2}\left(x_{t+1},as_{t+1}\right)\right]\Bigg). \tag{21}$$

Suppose that the condition (28) also holds for player i for some payoff $\widetilde{x}^i_t$. Then, if I sum condition (28) over the players, the second and the third expressions on the RHS are canceled out, because they are equal to the sum of coalition members' payoffs minus the sum of coalition costs, for all MWCs. As a result, the sum of the coalition payoffs of all players has to satisfy the following condition:

$$\sum_{k=1}^{n}\left(1+\delta\frac{1}{n}\sum_{as_{t+1},as_{t+1}\neq j}\left[p^j_{t+1}\left(x_t,as_{t+1}\right)\right]\frac{1}{1-\delta\frac{1}{n}\sum\limits_{as_{t+2},as_{t+2}\neq j}\left[p^j_{t+2}\left(x_t,as_{t+2}\right)\right]}\right)x^k_t$$

$$= \sum_{k=1}^{n}x^k_{t-1},$$

where $\sum\limits_{k=1}^{n}x^k_t = \sum\limits_{j=1,j\neq i}^{n}x^j_t+\widetilde{x}^i_t$. Note that

$$1+\delta\frac{1}{n}\sum_{as_{t+1},as_{t+1}\neq j}\left[p^j_{t+1}\left(x_t,as_{t+1}\right)\right]\frac{1}{1-\delta\frac{1}{n}\sum\limits_{as_{t+2},as_{t+2}\neq j}\left[p^j_{t+2}\left(x_t,as_{t+2}\right)\right]}>1.$$

Then

$$\sum_{j\neq i}x^j_t \leq \sum_{j\neq i}x^j_t+\widetilde{x}^i_t \leq \sum_{j=1}^{n}x^j_{t-1}=1.$$

For that reason, the cost of the MWC is bounded from above by $\frac{1}{2}$. Then, the lower bound of the expected utility of the agenda setter is:

$$\inf EU^{MWC}_i = \frac{1}{2}+\frac{1}{n}\sum_{\tau=1}^{\infty}\delta^{\tau}. \tag{22}$$

The case, when RHS of condition (28) is negative or zero is similar and considered in Appendix ■

As a next step, I describe probable deviations from the MWC strategy:

Lemma 7 *Under congressional system the upper bound of the expected utility of the agenda setter if she deviates from the MWC is:*

$$\sup U^i_t\left(x_{t-1}\right)=\frac{2}{n+1}+\delta\left(\frac{1}{n}+\frac{n-1}{2n}\frac{2n}{2n-\delta\left(n-1\right)}\frac{2}{n+1}\right)+\frac{1}{n}\sum_{\tau=2}^{\infty}\delta^{\tau}$$

I outline the main ideas of the proof, the technical details are presented in Appendix.

Proof. According to the one stage deviation principle, I assume that the agenda setter deviates only in the current period, and after that the MWC strategy is played.

For the agenda setter there is no sense to allocate any positive payoff to less than $\frac{n-1}{2}$ players and to more than $\frac{n+1}{2}$ players due to the same reasoning as in Lemma 2. As a result I need only to consider two types of coalitions, with $\frac{n-1}{2}$ and with $\frac{n+1}{2}$ players. Depending on coalition members' payoffs, the first type yields for the agenda setter the probability to be in the next period coalition $\frac{1}{n} \leq p^c \leq \frac{n-1}{2n}$ and the second type $\frac{n-1}{2n} < p^c \leq 1$.

The analysis is simplified if one notes the following: it is impossible that the agenda setter has a higher payoff than any other player in period t and has a positive probability to be in the winning coalition in period $t+1$ (Equations (42) and (43), see Appendix).

Then, the upper bound to the agenda setter's utility if she deviates from the MWC and wants to achieve at least probability $\frac{n-1}{2n}$ to be in the next coalition, is the allocations $A0$ described in Lemma 2: the agenda setter and other $\frac{n-1}{2}$ players obtain $\frac{2}{n+1}$, all the rest obtain zero. The upper bound of the expected utility for the agenda setter is then as follows:

$$\sup U^i_{A0t}\left(x_{t-1}\right) = \frac{2}{n+1} + \delta\left(\frac{1}{n} + \frac{n-1}{2n}\frac{2n}{2n-\delta\left(n-1\right)}\frac{2}{n+1}\right) + \frac{1}{n}\sum_{\tau=2}^{\infty}\delta^{\tau} \quad (23)$$

The calculations show that expected utility of the agenda setter under the strategies which guarantee higher probability of the coalition membership will always be lower than the upper bound value for $A0$ allocation.

I consider only the case when the agenda setter tries to influence her chances to be in the winning coalition in the following period, and not in the more remote ones. The agenda setter in period t can influence her chances to be in the winning coalition in period $t+2$ only if she chooses the allocation in which her payoff is lower than payoffs of at least $n-3$ other players. Such allocations require a crucial decrease in the agenda setter payoff in period t and the resulting expected utility will be lower than $\sup U^i_{A0t}\left(x_{t-1}\right)$, for all x_{t-1}. Under all other possible allocations in period t, the agenda setter will obtain zero in period $t+2$ and in all subsequent periods, unless she is the agenda setter again. Moreover, she can not influence her chances to be in the winning coalition in the period more remote than $t+2$ at all. ■

Now I will turn to the main result of this section:

Proposition 4 *Under congressional system the MWC strategy profile is an equilibrium of infinite game if* $n \geq 5$.

Proof. The agenda setter cannot propose to coalition members payoffs lower than the MWC proposal, because then the proposal will be rejected. All coalition members accept the MWC proposal due to the payoffs construction.

For $n \geq 7$ the lower bound of the expected utility from playing according to the MWC strategy (30) is strictly higher than the upper bound of the expected utility of playing according to the deviation strategy (38). When $n = 5$, the numerical simulations for various initial default policy vectors show the same result. ■

2.5 Parliamentary system

2.5.1 Two period game

In this section I consider a parliamentary system. The agenda setter has to buy an exogenously given coalition and she can include additional members into such a coalition. This leads to the following behavior in the two periods game:

Proposition 3 *Under parliamentary system, the strategy profile is an equilibrium of two periods game if it has the following form:*

let i be the agenda setter in period t and $EWC_t \subset N$ be a set of $\frac{n-1}{2}$ members of the exogenous coalition in period t. Then

i. For each player j, $j \neq i$, the decision rule in period t is as follows: accept proposal x_t iff

$$x_t^j + \delta V_2^j(x_t) \geq x_{t-1}^j + \delta V_2^j(x_{t-1}), \text{ for all } j \in N, j \neq i, t = 1,2 \tag{24}$$

ii The agenda setter proposes the following allocation

$$x_t^j = x_{t-1}^j \quad \forall j \in EWC_t \tag{25}$$

$$x_t^l = 0 \quad \forall l \notin EWC_t \tag{26}$$

Proof. For the second period the proof is obvious and similar to Proposition 1.

In the first period the agenda setter again faces the exogenously given coalition, so she has to propose to coalition member j at least

$$x_1^j = -\delta V_2^j(x_1) + x_0^j + \delta V_2^j(x_0), \tag{27}$$

where $V_2^j(x_1)$ is the continuation value of player j, as in Definition 1. All players will accept such a proposal.

The continuation value can be written in the following way: note, that there are $k = \frac{n!}{\left(\frac{n-1}{2}\right)!\left(n-\frac{n-1}{2}\right)!}$ possible exogenous coalitions for a given agenda setter and each player appears exactly in half of the coalitions. Then player j's continuation value $V_2^j(x_1)$ is:

$$\begin{aligned} V_2^j(x_1) &= \frac{1}{n}\frac{1}{k}\left(k - \frac{k}{2}\left(x_1^1 + x_1^2 + ... + x_1^{j-1} + ... + x_1^n\right)\right) + \frac{n-1}{n}\frac{1}{2}x_1^j = \\ &= \frac{1}{n}\left(1 - \frac{1}{2}\left(1 - x_1^j\right)\right) + \frac{n-1}{n}\frac{1}{2}x_1^j = \frac{1}{2n} + \frac{1}{2}x_1^j. \end{aligned}$$

Then

$$\begin{aligned} x_1^j &= x_0^j + \delta V_2^j(x_0) - \delta V_2^j(x_1) = x_0^j + \delta\left(\frac{1}{2n} + \frac{1}{2}x_0^j\right) - \delta\left(\frac{1}{2n} + \frac{1}{2}x_1^j\right) \Rightarrow \\ x_1^j &= x_0^j. \end{aligned}$$

Note that payoffs in the first period does not affect the agenda setter's chances to be in the winning coalition in the second period. Therefore, the agenda setter's expected utility is a strictly positive function of her payoff in the first period:

$$U^i = x_1^i + \delta\left(\frac{1}{2n} + \frac{1}{2}x_1^i\right). \tag{28}$$

Thus, the agenda setter has no reasons to pay coalition members more than $-\delta V_2^j(x_1) + x_0^j + \delta V_2^j(x_0)$. For the same reasons she does not have incentives to increase the number of coalition members by proposing a positive payoff to the players who are not in the EWC. As a result, the strategy profile described in the proposition is an equilibrium. Moreover, the coalition formed in equilibrium is the MWC: there is no cheaper coalition, which induces the acceptance of the agenda setter's proposal. ■

This proposition shows that the parliamentary as well as the congressional systems are characterized in equilibrium by the MWC strategy profile. The difference between these systems is that under the parliamentary system the agenda setter will not necessarily obtain the maximal payoff: for example if the initial default policy allocates 1 to some member of exogenous coalition, the payoff of this player will be equal to one, and the agenda setter's to zero.

2.5.2 Infinite game

Under the parliamentary system there is no apparent reason for the agenda setter to deviate from the MWC strategy. If the MWC strategy is played in all subsequent periods, any deviation from the MWC by the agenda setter in the current period will not change the probability to be in the next period coalition and will decrease her payoff via a decrease in her default policy and an increase in the default policy of other players. Hence, she will always choose the MWC strategy. More formally:

Proposition 5 *Under the parliamentary system the strategy profile is an equilibrium of infinite game if it has the following form:*

Let i be the agenda setter in period t and $EWC_t \subset N$ be a set of $\frac{n-1}{2}$ members of the exogenous coalition in period t. Then

i. The acceptance rule in period t is as follows: accept iff

$$x_t^j + \delta V_{t+1}^j(x_t) \geq x_{t-1}^j + \delta V_{t+1}^j(x_{t-1}), \textit{ for all } j \in N, j \neq i, \tag{29}$$

ii. The agenda setter proposes the following allocation

$$x_t^j = x_{t-1}^j \quad \forall\, j \in EWC_t \tag{30}$$

$$x_t^l = 0 \quad \forall l \notin EWC_t \tag{31}$$

Proof. As in the case of a congressional system, in order to show that a strategy profile is an equilibrium, I have to show that it is not profitable to make a one period deviation.

Assume that all players play according to the strategy profile which is described in the proposition. Due to the same calculations as in Proposition 4, the continuation value of player j in period t is:

$$V_{t+1}^{j}\left(x_{t-1}\right)=\frac{1}{2n}+\frac{1}{2}x_{t-1}^{j}+V_{t+2}^{j}\left(x_{t+1}\left(x_{t-1}^{j}\right)\right),$$

$$V_{t+1}^{j}\left(x_{t}\right)=\frac{1}{2n}+\frac{1}{2}x_{t}^{j}+V_{t+2}^{j}\left(x_{t+1}\left(x_{t}^{j}\right)\right).$$

According to (30), the agenda setter proposes to a coalition member $x_t^j = x_{t-1}^j$. Then the following condition is satisfied:

$$x_{t}^{j}=x_{t-1}^{j}+\delta V_{t+1}^{j}\left(x_{t-1}\right)-\delta V_{t+1}^{j}\left(x_{t}\right).$$

Therefore, such payoff makes a coalition member indifferent between accepting and rejecting the proposal, hence all coalition members will accept the proposal. When only members of exogenous coalition $j \in WC_t$ obtain positive payoffs $x_t^j = x_{t-1}^j$, the coalition is the MWC.

Note, that the expected payoff in each period is an increasing function of the players own default policy and does not depend on the distribution of payoffs among other players. Then, like in Proposition 4, the expected utility of the agenda setter is a strictly increasing function of her payoff in period t. Hence, she will not deviate from the strategy profile described in (30) and (31). Therefore, the MWC strategy profile is an equilibrium. ■

There is still a difference between the congressional and the parliamentary systems: under the congressional system the equilibrium allocation converges to the extreme allocation at most after three periods. Under the parliamentary system the convergence may take longer time, if it happens at all. There is always a positive probability that a player with an original positive default policy appears period after period in the winning coalition implying that she obtains positive payoffs all the time. Another difference between the two systems is that under the parliamentary system the agenda setter will not always obtain the highest payoff among other players.

2.6 Risk aversion

In this section I consider only two period games. For simplicity, I consider a specific example with the symmetric initial default policy vector. Moreover, I assume that each player i has the following utility function of player i:

$$U_t^i = \left(x_t^i\right)^{1-\gamma}, \quad \forall t, \qquad 0<\gamma<1 \tag{32}$$

2.6.1 Congressional system

Under the congressional system, the agenda setter in the first period faces the following problem:

$$\max_{S_1,x_1}\left(1-S_1x_1^j\right)^{1-\gamma}+\delta E\left(x_2^i\left(x_1\right)\right)^{1-\gamma}$$

$$s.t.\left(x_1^j\right)^{1-\gamma}+\delta E\left(x_2^j\left(x_1\right)\right)^{1-\gamma}\geq\left(\frac{1}{n}\right)^{1-\gamma}+\delta\left(\frac{1}{n}\left(1-\frac{n-1}{2}\frac{1}{n}\right)^{1-\gamma}+\right.$$

$$\left.+\frac{n-1}{2}\frac{1}{2}\left(\frac{1}{n}\right)^{1-\gamma}\right),$$

where S is the size of a coalition.

The numerical solutions for the agenda setter maximization problem are presented in Table 1. The results show that with low levels of risk aversion, the agenda setter plays according to the MWC strategy. With an increase in risk aversion parameter γ, the agenda setter will generally increase risk sharing between herself and coalition members under the influence of two forces: first, she herself will prefer to increase her chances to be in the winning coalition in the future. Second, coalition members will demand a higher payoff in order to support the MWC allocation because they will trade off higher payoff in the first period for the lower chances to be in the coalition in the second period. Risk sharing is composed of more equal distribution of the budget in the first period which implies more equal distribution of coalition payments and more equal chances to coalition membership in the second period. With an increase in the risk aversion, the agenda setter will have to increase the payoffs of coalition members or to increase the number of coalition members in order to increase their probability to be in the winning coalition in the second period. Eventually, when γ is sufficiently high, the agenda setter proposes symmetric allocation equal to the initial default policy.

As it can be seen from Table 1, the agenda setter chooses different patterns of the equilibrium allocation, depending on n and γ. With an increase in n the agenda setter has to be less risk averse in order to switch from the MWC to some risk sharing allocation or to propose the symmetric allocation. In general, the number of coalition members increases with an increase in γ, but not always monotonically: see for example $n = 9$ and $n = 11$. Moreover, the results show that a number of politicians may have an influence on the policy outcomes even in such simple setting.

Table 1 Equilibrium allocation

n	5	7	9	11	13	17	21	37	45
$\gamma = 0.1$	MWC	MWC	MWC	MWC	MWC	MWC	MWC	MWC	MWC
0.2	MWC	MWC	MWC	MWC	MWC	MWC	MWC	MWC	MWC
0.3	MWC	MWC	MWC	MWC	MWC	MWC	MWC	WC1	WC2
0.35	MWC	A1	MWC	MWC	MWC	MWC	WC1	WC2	WC3
0.4	A0	B1	A1	A1	A1	A1	WC1	WC3	WC4
0.45	A0	B1	B1	B1	B1	A1	A1	A1	WC6
0.5	B0	B0	B2	B2	B2	A3	A3	A4	A4
0.55	B0	MWC	B2	B3	B3	B4	B5	B7	B8
0.6	MWC	WC1	WC1	WC2	WC3	B5	B7	B13	B15
0.65	C1	C2	C2	C3	C4	C6	C8	C15	C19
0.7	C1	C2	C3	sd	sd	sd	sd	sd	sd
0.8	sd	sd	sd	sd	sd	sd	sd	sd	sd
0.9	sd	sd	sd	sd	sd	sd	sd	sd	sd

MWC - the minimal winning coalition.

WCk - allocation characterized by $\frac{n-1}{2}+k$ coalition members. Coalition members obtain their continuation values.

Ak - allocation characterized by $\frac{n-1}{2}+k$ coalition members. The agenda setter's payoff is by ε less than payoff of coalition members, where ε is arbitrarily small.

Bk - allocation characterized by $\frac{n-1}{2}+k$ coalition members. The agenda setter's payoff is significantly lower than payoff of coalition members.

Ck - allocation characterized by $\frac{n-1}{2}+k$ coalition members. The agenda setter's payoff is equal to payoff of coalition members.

sd - the initial symmetric default policy.

2.6.2 Parliamentary system

Consider a simple example with three players and a symmetric initial default policy. The maximization problem of the agenda setter in the first period is:

$$\max \left(x_1^i\right)^{1-\gamma} + \delta\left[\frac{1}{3}\left(\frac{1}{2}\left(x_1^j\right)^{1-\gamma} + \frac{1}{2}\left(x_1^l\right)^{1-\gamma}\right) + \frac{1}{3}\left(x_1^i\right)^{1-\gamma}\right]$$

$$s.t. \left(x_1^j\right)^{1-\gamma} + \delta\left[\frac{1}{3}\left(\frac{1}{2}\left(x_1^i\right)^{1-\gamma} + \frac{1}{2}\left(x_1^l\right)^{1-\gamma}\right) + \frac{1}{3}\left(x_1^j\right)^{1-\gamma}\right] \geq$$

$$\left(\frac{1}{3}\right)^{1-\gamma} + \delta\left(\frac{1}{3}\left(\frac{2}{3}\right)^{1-\gamma} + \frac{1}{3}\left(\frac{1}{3}\right)^{1-\gamma}\right)$$

where x_1^i, x_1^j and x_1^l are the first period payoffs of the agenda setter, the coalition member and the opposition member respectively.

With an increase in risk aversion, the coalition member will demand a higher MWC. Therefore, the agenda setter may want to give some positive payoff to the opposition member, decreasing by this action the payoff required by the coalition member in order to support the proposal, and smoothing her own payoff in the case she will be the agenda setter in the second period.

The numerical solutions for this problem for different values of the risk aversion parameter show that the agenda setter will always prefer the MWC: she will allocate zero payoff to the opposition member and the minimal feasible payoff to the coalition member.

2.7 Conclusions

This paper demonstrates that the introduction of the endogenous default policy into a dynamic distributive model of legislature bargaining among risk neutral players does not change the general results obtained in models with the exogenous default policy: in the two-period and the infinite framework the equilibrium is characterized by the minimal winning coalition for the congressional and the parliamentary systems. Moreover, under the congressional system the equilibrium allocation becomes extreme at most after two periods: the agenda setter gets the whole budget.

For risk averse players in the two period game the results are different for the two systems: under the parliamentary regime the equilibrium is characterized by the minimal winning coalition, while under the congressional system risk sharing takes place if the risk aversion is strong enough. The general pattern of the equilibrium under the congressional system is the following: with an increase in the risk aversion the number of the players in a coalition increases, creating a supermajority coalition. Inside the coalition different patterns of the allocation can exist depending on a degree of risk aversion and the number of politicians.

If the players are risk averse, then under the congressional system the agenda setter may propose an allocation which gives positive payoffs to more than a simple majority of players. On the contrary, under the parliamentary system, the agenda setter will prefer to allocate payoffs according to the MWC even with a high degree of risk aversion. These results predict that we will obtain broader programs under the congressional system than under the parliamentary system and will also obtain more "supermajority" coalitions. The empirical validation of this prediction can be an interesting extension of this research.

Kalandrakis (2002) considers a more general framework of the model for the system named congressional in the current paper. He considers a general class of concave utility functions under the infinite horizon. He also accounts for the possibility that the probabilities of being the agenda setter can be asymmetric across players. His main result is that if the players are not much risk averse and their number is higher than 5 the equilibrium allocation converges to an extreme one within at most three periods, which is the same result as presented in the current paper for a risk neutral case. Because of a more general framework he uses much more complex proof techniques.

All results concern only specific kinds of policy programs, namely the allocation of an exogenously given budget. Under an endogenously given budget the results may be quite different and it is possible to model more differences between the parliamentary and the congressional systems reflecting their specific features and making the model more realistic.

Another possible extension of the present paper may examine a game in which players' utility depends not only on redistribution but also on ideological variable. For example, the distribution of the ideological ideal points within the one dimensional space usually creates "natural" coalitions, and combining this with the redistribution policy and the endogenous default policy, we may be able to shed the light on a possible trade-off in the process of the coalition formation.

Still another possible extension can model endogenous agenda setting: the probability to be the agenda setter in the next period may depend on the allocation in the current period. This can be modeled by introducing explicit elections, adding the general public as an additional player.

Certain attention should also be devoted to the endogenous institutions problem: implementation of the endogenous versus exogenous default policy might be itself the endogenous choice of politicians. It is interesting to investigate under what conditions they will prefer one of the two regimes.

2.8 Appendix

2.8.1 Appendix to Lemma 1

1. The continuation value $V_2^j(x_1)$ is calculated as follows: according to Proposition 1, in the second period a player supports the proposal, if his payoff is at least his default policy. Then, if period one's coalition member j is the agenda setter in period two then $x_2^j = 1$, because there will be $\frac{n-1}{2}$ players with the default payoff equal to zero. If the second period agenda setter is a first period coalition member other than j, then $x_2^j = 0$. If the second period agenda setter is the player who was not a coalition member in the first period the expected second period payoff of player j is described by function $f(x_1)$: if there is a first period coalition member whose second period default policy is lower than j's, then $x_2^j = 0$. If j is the cheapest one among the first period coalition members, then $x_2^j = x_1^j$. If there are other players with the default policy equal to that of j, and this default policy is the lowest among members of the first period coalition, then j obtains his default policy with probability equal to $\frac{1}{|U|+1}$, where $|U|$ is the number of such players.

For example, if $n = 5$, the coalition size will be 2, and the first period payoffs of coalition members j and l are :

$$x_1^j = \max\left[Q^j(x_0) - \delta\frac{2}{3}x_1^j, 0\right] \textit{ and } \; x_1^l = \max\left[Q^l(x_0), 0\right] \textit{ iff } x_1^l > x_1^j,$$

$$x_1^l = \max\left[Q^l(x_0) - \delta\frac{2}{3}x_1^l, 0\right] \textit{ and } x_1^j = \max\left[Q^j(x_0), 0\right] \textit{ iff } x_1^l < x_1^j,$$

$$x_1^l = x_1^j = \max\left[Q^l(x_0) - \delta\frac{2}{3}\frac{1}{2}x_1^l, 0\right] \textit{ iff } x_1^l = x_1^j.$$

2. The last step is to show that the following assumption is indeed the case: the agenda setter's default policy in the beginning of the second period is the highest comparing to all other players. The minimal payoff of the agenda setter in the first period is $\frac{1}{2} - \delta\frac{1}{4n}$. Suppose that there is another player l that demands at least the same payoff in the first period in order to support the agenda setter's proposal. Then the sum of demands of all other $n-2$ players is equal to $\frac{1}{2} + \delta\frac{1}{4n}$. Then there are possible alternative coalitions to build without need to pay such high payoff to player l.

2.8.2 Appendix to Lemma 6

Consider the case, when RHS of condition (28) is negative or zero. Hence, $x_t^k = 0$ for some $k \in N$. Then I can rewrite the condition (21) in the following way:

$$x_t^j\left(1 + \delta\frac{1}{n}\sum_{as_{t+1}, as_{t+1}\neq j} p_{t+1}^j(x_t, as_{t+1})\frac{1}{1 - \delta\frac{1}{n}\sum\limits_{as_{t+1}, as_{t+1}\neq j}\left[p_{t+1}^j(x_t, as_{t+1})\right]}\right) =$$

$$\max\left[0, x_{t-1}^j + \delta\left(\begin{array}{c} -\frac{1}{n}C_{t+1}^j(x_{t-1}) + \\ +\frac{1}{n}\sum\limits_{as_{t+1}, as_{t+1}\neq j}\left[p_{t+1}^j(x_{t-1}, as_{t+1})\, x_{t+1}^j(x_{t-1}, as_{t+1})\right]\end{array}\right) + \right.$$

$$\left. + \delta^2\frac{1}{n}\sum_{as_{t+1}}\left(\begin{array}{c} -\frac{1}{n}C_{t+2}^j(x_{t+1}) + \\ +\frac{1}{n}\sum\limits_{as_{t+2}, as_{t+2}\neq j}\left[p_{t+2}^j(x_{t+1}, as_{t+2})\, x_{t+2}^j(x_{t+1}, as_{t+1})\right]\end{array}\right)\right]. \tag{33}$$

If there is at least one player in period t, who is ready to support the agenda setter's proposal for a zero payoff, then he will be in the coalition for sure. Moreover, if the MWC proposal is accepted, the allocation in the next period, $t+1$, will be extreme and none of the players obtains a positive payoff apart from the agenda setter. If some coalition member deviates in period t and rejects the proposal, after two periods the allocation will also converge to an extreme one. This allows me to rewrite the condition (28) as:

$$x_t^j = \max\left[0, x_{t-1}^j + \delta\left(-\frac{1}{n}C_{t+1}^j(x_{t-1}) + P_{t+1}^j\right)\right], \tag{34}$$

where $P_{t+1}^j \equiv \frac{1}{n}\sum\limits_{as_{t+1}, as_{t+1}\neq j}\left[p_{t+1}^j(x_{t-1}, as_{t+1})\, x_{t+1}^j(x_{t-1}, as_{t+1})\right].$

Assume for simplicity that there is only one player, denoted k, for whom $x_t^k = 0$ (the result is easily extended to more players). It implies that $x_{t-1}^k - \delta \frac{1}{n} C_{t+1}^k(x_{t-1})$ is either zero or negative. (20) implies that if $x_t^k = 0$ then $x_{t+1}^k = 0$ and $P_{t+1}^k = 0$.

Note that for all j the following is satisfied:

$$C_t^j(x_{t-1}) = C_{t+1}^j(x_{t-1}) \leq \frac{1}{2} \sum_{j=1}^{n} x_t^j. \tag{35}$$

Suppose that (34) is satisfied for player i for some $\widetilde{x}_t^i$. Summing (34) over all players I obtain:

$$\sum_{j=1..N} x_t^j = \sum_{j \neq k} \left[x_{t-1}^j + \delta \left(-\frac{1}{n} C_{t+1}^j(x_{t-1}) + P_{t+1}^j \right) \right] + 0. \tag{36}$$

Adding and subtracting $C_{t+1}^k(x_{t-1})$ I obtain

$$\sum_{j=1..N} x_t^j = \sum_{j \neq i,k} x_{t-1}^j + \frac{\delta}{n} C_{t+1}^k(x_{t-1}). \tag{37}$$

All other expressions are canceled out due to the same reasons as in (21). Using (35) I can rewrite 37 as

$$\begin{aligned} \sum_{j=1..N} x_t^j - \frac{\delta}{n} C_{t+1}^k(x_{t-1}) &= \sum_{j \neq i,k} x_{t-1}^j \\ \Rightarrow \sum_{j=1..N} x_t^j - \frac{\delta}{2n} \sum_{j=1..N} x_t^j &\leq \sum_{j \neq k} x_{t-1}^j \leq 1. \end{aligned} \tag{38}$$

Then

$$\sum_{j=1..N} x_t^j \ \leq \frac{1}{\left(1 - \frac{\delta}{2n}\right)}. \tag{39}$$

Player k will be in the coalition for sure. Hence, some player with the highest coalition payoff x_t^l, x_t^l at least $\frac{1}{n}$, will not be in the coalition. Therefore, the upper bound for the coalition cost in period t is

$$\sup C_t^i(x_{t-1}) = \frac{1}{2} \sum_{j \neq i} x_t^j - \frac{1}{n} - \widetilde{x}_t^i \leq \frac{1}{2} \left(\frac{1}{\left(1 - \frac{\delta}{2n}\right)} - \frac{1}{n} \right) = \frac{1}{2} \frac{2n^2 - 2n + \delta}{(2n - \delta) n} < \frac{1}{2}.$$

2.8.3 Appendix to Lemma 7

The Lemma is proved in three steps. First, I calculate the upper bound for the agenda setter utility, if her probability to be in the next period coalition, denoted by p^c, is $\frac{n-1}{2n} \frac{2}{n-1} \leq p^c \leq \frac{n-1}{2n}$. Second, I consider the strategy which secure the next period coalition place for the agenda setter, namely $p^c = 1$. Last, I consider the strategy which yields $\frac{n-1}{2n} < p^c \leq 1$.

1. Consider the coalition with $\frac{n-1}{2}$ members. To start with, I investigate the case where the agenda setter aims to achieve probability $\frac{n-1}{2n}$ to be in the next coalition. Denote the agenda setter in period t by i, coalition members by j, and other players by l. Player i tries to allocates the budget in the way that guarantees her a place in $t+1$ coalition in case the agenda setter will be player l.

Starting from period $t+1$ the MWC is played and in period $t+2$ the allocation will always be an extreme one. In the period $t+1$ only one coalition member will obtain a positive payoff when the agenda setter will be player l: either player i or one of the players j. When player i is the coalition member in period $t+1$, her payoff has to satisfy:

$$x^i_{t+1} + \frac{1}{n}\sum_{\tau=1}^{\infty}\delta^\tau \geq x^i_t + \delta\left(\frac{1}{n} + \frac{n-1}{2n}x^i_{t+1}\right) + \frac{1}{n}\sum_{\tau=2}^{\infty}\delta^\tau$$

$$\Rightarrow x^i_{t+1} = x^i_t + \delta\frac{n-1}{2n}x^i_{t+1}\ . \tag{40}$$

The payoffs of players l in period $t+1$ are equal to zero. The payoff of player j, when he is a coalition member, has to satisfy

$$x^j_{t+1} + \frac{1}{n}\sum_{\tau=1}^{\infty}\delta^\tau \geq x^j_t + \delta\left(\frac{1}{n} + \frac{n-1}{2n}\frac{2}{n-1}x^i_{t+1}\right) + \frac{1}{n}\sum_{\tau=2}^{\infty}\delta^\tau$$

$$\Rightarrow x^j_{t+1} = x^j_t + \frac{\delta}{n}x^j_{t+1}\ . \tag{41}$$

In order to be in the second period coalition, player i has to allocate x^j_t and x^i_t in the way that implies $x^i_{t+1} < x^j_{t+1}$by an arbitrary small amount. Assume that $x^i_{t+1} = x^j_{t+1}$.

As a result, there are four conditions which define players' payoffs:

$$x^i_{t+1} = x^i_t + \delta\frac{n-1}{2n}x^i_{t+1} \Rightarrow x^i_{t+1} = \frac{1}{1-\delta\frac{n-1}{2n}}x^i_t, \tag{42}$$

$$x^j_{t+1} = x^j_t + \frac{\delta}{n}x^j_{t+1} \Rightarrow x^j_{t+1} = \frac{1}{1-\frac{\delta}{n}}x^j_t, \tag{43}$$

$$x^i_{t+1} = x^j_{t+1}, \tag{44}$$

and a budget constraint

$$x^i_t = 1 - \frac{n-1}{2}x^j_t. \tag{45}$$

The resulting payoffs are

$$x_t^j = 2\frac{-n+\delta}{-n+2\delta n-2\delta-n^2}, \tag{46}$$

$$x_t^i = \frac{-2n+\delta n-\delta}{-n+2\delta n-2\delta-n^2}. \tag{47}$$

The main difference between the infinite game versus allocation $A0$ under the two period framework is as follows: in order to achieve $p^c = \frac{n-1}{2n}$, it is not enough that the agenda setter's payoff in period t is only by ε smaller than that of other coalition members, where ε is arbitrarily small. Her payoff in period t has to be significantly lower than that of other coalition members. For example, if $n = 5$, and $\delta = 0.5$, the agenda setter obtains 0.3077 in period t, while coalition members payoff is 0.3462. If the agenda setter proposes allocation $A0$, then $p^c = \frac{n-1}{2n}\frac{2}{n-1} = \frac{1}{n}$. Varying coalition payoffs between payoffs specified in (46)–(47) and $A0$ payoffs, the agenda setter can obtain $\frac{n-1}{2n}\frac{2}{n-1} \leq p^c \leq \frac{n-1}{2n}$.

Therefore, the upper bound to the agenda setter's utility is as follows. Payoffs are consisted with allocation $A0$ described in Lemma 2: the agenda setter and other $\frac{n-1}{2}$ players obtain $\frac{2}{n+1}$, all the rest obtain zero. But the probability to be in the next coalition $p^c = \frac{n-1}{2n}$. Then, the upper bound of the expected utility for the agenda setter is

$$\sup U_{A0t}^i(x_{t-1}) = \frac{2}{n+1} + \delta\left(\frac{1}{n} + \frac{n-1}{2n}\frac{2n}{2n-\delta(n-1)}\frac{2}{n+1}\right) + \frac{1}{n}\sum_{\tau=2}^{\infty}\delta^\tau. \tag{48}$$

2. Next, I consider the strategy which guarantees for the agenda setter a place in the next period coalition. The number of coalition members in this case is $\frac{n+1}{2}$. If the agenda setter is the coalition member in period $t+1$, her payoff has to satisfy

$$x_{t+1}^i + \frac{1}{n}\sum_{\tau=1}^{\infty}\delta^\tau \geq x_t^i + \delta\left(\frac{1}{n}\left(1 - x_{t+1}^j\right) + \frac{n-1}{n}x_{t+1}^i\right) + \frac{1}{n}\sum_{\tau=2}^{\infty}\delta^\tau \tag{49}$$

$$\Rightarrow x_{t+1}^i = x_t^i + \delta\left(-x_{t+1}^j\frac{1}{n} + \frac{n-1}{n}x_{t+1}^i\right)$$

$$\Rightarrow x_{t+1}^i = \frac{n}{n-\delta(n-1)}\left(x_t^i - x_{t+1}^j\frac{\delta}{n}\right), \tag{50}$$

where

$$x_{t+1}^j + \frac{1}{n}\sum_{\tau=1}^{\infty}\delta^\tau \geq x_t^j + \delta\left(\frac{1}{n}\left(1 - x_{t+1}^i\right) + \frac{n-1}{2n}\frac{2}{n+1}x_{t+1}^j\right) + \frac{1}{n}\sum_{\tau=2}^{\infty}\delta^\tau$$

$$\Rightarrow x_{t+1}^j = x_t^j + \delta\left(-x_{t+1}^i\frac{1}{n} + \frac{n-1}{2n}\frac{2}{n+1}x_{t+1}^j\right)$$

$$\Rightarrow x_{t+1}^{j}=\frac{n\left(n+1\right)}{n\left(n+1\right)-\delta\left(n-1\right)}\left(x_{t}^{j}-\frac{\delta}{n}x_{t+1}^{i}\right). \tag{51}$$

By analogy with (43–46), there are four conditions which define $A1$ payoffs

$$x_{t+1}^{i}=\frac{n}{n-\delta\left(n-1\right)}\left(x_{t}^{i}-x_{t+1}^{j}\frac{\delta}{n}\right), \tag{52}$$

$$x_{t+1}^{j}=\frac{n\left(n+1\right)}{n\left(n+1\right)-\delta\left(n-1\right)}\left(x_{t}^{j}-\frac{\delta}{n}x_{t+1}^{i}\right), \tag{53}$$

$$x_{t+1}^{i}=x_{t+1}^{j}, \tag{54}$$

$$x_{t}^{i}=1-\frac{n+1}{2}x_{t}^{j}. \tag{55}$$

The resulting payoffs are

$$x_{t+1}^{i}=x_{t+1}^{j}=2\frac{n}{n^{2}+\left(3-2\delta\right)n+6\delta}, \tag{56}$$

$$x_{t}^{j}=2\frac{n^{2}+n+2\delta}{\left(n^{2}+\left(3-2\delta\right)n+6\delta\right)\left(n+1\right)}, \tag{57}$$

$$x_{t}^{i}=1-\frac{n+1}{2}2\frac{n^{2}+n+2\delta}{\left(n^{2}+\left(3-2\delta\right)n+6\delta\right)\left(n+1\right)}=2\frac{\left(1-\delta\right)n+2\delta}{n^{2}+\left(3-2\delta\right)n+6\delta}. \tag{58}$$

The expected utility of the agenda setter is

$$U_{A1t}^{i}\left(x_{t-1}\right)=2\frac{\left(1-\delta\right)n+2\delta}{n^{2}+\left(3-2\delta\right)n+6\delta}+\delta\frac{3n^{2}-n-2n\delta+6\delta}{n\left(n^{2}+3n-2n\delta+6\delta\right)}+\frac{1}{n}\sum_{\tau=2}^{\infty}\delta^{\tau} \tag{59}$$

Note that

$$\sup U_{A0t}^{i}\left(x_{t-1}\right)-U_{A1t}^{i}\left(x_{t-1}\right)=2n\frac{n^{2}\delta+\left(-4\delta+4\right)n+11\delta}{\left(\left(2-\delta\right)n+\delta\right)\left(n+1\right)\left(n^{2}+\left(3-2\delta\right)n+6\delta\right)}>0$$

The upper bound of the agenda setter's utility under $A0$ allocation is higher than under $A1$ allocation for the relevant parameters.

3. The last step is to consider the strategies which secure for the agenda setter the place in the next coalition with $\frac{n-1}{2n}<p^{c}<1$. Consider the coalition with $\frac{n+1}{2}$ members. The agenda setter can allocate the payoffs which are equal to her own to some coalition members. If there are k such players from $\frac{n+1}{2}$ coalition members, then the probability to be in the next coalition for the agenda setter is

$$p^c = \frac{n-3}{2n}\frac{2}{k+1} + \frac{n+3-2\,(k+1)}{2n}\frac{1}{k+1} + \frac{k}{n}\frac{1}{k} = \frac{3}{2}\frac{n-1}{n\,(k+1)}$$

Under $A0$ the probability to be in the coalition is $\frac{n-1}{2n}$. Hence $k<2$, and I have to investigate only the case where one player obtains the payoff equal to that of the agenda setter. In this case $p^c = \frac{3}{2}\frac{n-1}{2n}$. Note that other values of $\frac{n-1}{2n} < p^c < 1$ are not obtainable for the agenda setter.

Conditions which define payoffs are as follows (by analogy to (43–46):

$$x^i_{t+1} + \frac{1}{n}\sum_{\tau=1}^{\infty}\delta^\tau \geq x^i_t + \delta\left(\frac{1}{n}\left(1-x^i_{t+1}\right) + \frac{3}{4}\frac{n-1}{n}x^i_{t+1}\right) + \frac{1}{n}\sum_{\tau=2}^{\infty}\delta^\tau$$

$$\Rightarrow x^i_{t+1} = x^i_t + \delta\left(-\frac{1}{n}x^i_{t+1} + \frac{3}{4}\frac{n-1}{n}x^i_{t+1}\right)$$

$$\Rightarrow x^i_{t+1} = -4x^i_t\frac{n}{-4n-7\delta+3\delta n}. \tag{60}$$

$$x^j_{t+1} + \frac{1}{n}\sum_{\tau=1}^{\infty}\delta^\tau \geq x^j_t + \delta\left(\frac{1}{n}\left(1-x^j_{t+1}\right) + \right.$$

$$\left.\left(\frac{2}{n}\frac{2}{n-1} + \frac{n-3}{2n}\frac{4}{n-1} + \frac{n-3}{2n}\frac{2}{n-3}\right)x^j_{t+1}\right) + \frac{1}{n}\sum_{\tau=2}^{\infty}\delta^\tau$$

$$\Rightarrow x^j_{t+1} = x^j_t + \delta\left(-\frac{1}{n}x^j_{t+1} + \left(\frac{2}{n}\frac{2}{n-1} + \frac{n-3}{2n}\frac{4}{n-1} + \frac{n-3}{2n}\frac{2}{n-3}\right)x^j_{t+1}\right)$$

$$\Rightarrow x^j_{t+1} = x^j_t\frac{n}{n-2\delta}, \tag{61}$$

$$x^i_{t+1} = x^j_{t+1}, \tag{62}$$

$$2x^i_t = 1 - \frac{n-1}{2}x^j_t. \tag{63}$$

The resulting payoffs are

$$x^i_{t+1} = x^j_{t+1}\frac{2}{n^2+(-5\delta+3)\,n+9\delta}n,$$

$$x^i_t = -\frac{1}{2}\frac{-4n-7\delta+3\delta n}{n^2-5\delta n+3n+9\delta},$$

$$x^i_{t+1} = \frac{2}{n^2+(-5\delta+3)\,n+9\delta}n.$$

The expected utility of the agenda setter is

$$
\begin{aligned}
&U^i_{A1t}\left(x_{t-1}\right) = -\frac{1}{2}\frac{-4n-7\delta+3\delta n}{n^2-5\delta n+3n+9\delta}+\frac{1}{n}\sum_{\tau=2}^{\infty}\delta^\tau \\
&+\delta\left(\frac{1}{n}\left(1-\frac{2}{n^2+\left(-5\delta+3\right)n+9\delta}n\right)+\frac{3}{4}\frac{n-1}{n}\frac{2}{n^2+\left(-5\delta+3\right)n+9\delta}n\right) \\
= \quad &\frac{\left(2+\delta\right)n^2+\left(3\delta-5\delta^2\right)n+9\delta^2}{n\left(n^2+\left(-5\delta+3\right)n+9\delta\right)}+\frac{1}{n}\sum_{\tau=2}^{\infty}\delta^\tau.
\end{aligned}
$$

The upper bound of the agenda setter's utility under $A0$ allocation is higher for the relevant parameters than $U^i_{A1t}\left(x_{t-1}\right)$:

$$
\sup U^i_{A0t}\left(x_{t-1}\right)-U^i_{A1t}\left(x_{t-1}\right) = 2n\frac{17\delta-10\delta n+4n+\delta n^2}{\left(2n-\delta n+\delta\right)\left(n+1\right)\left(n^2-5\delta n+3n+9\delta\right)} > 0.
$$

3 Choice of default policy rules

3.1 Introduction

The number of papers devoted to political games with an endogenous default policy has been constantly growing recently. By the endogenous default policy I shall basically mean the situation when the default policy in the absence of a decision is determined by the policy last implemented. This definition contrasts with the two kinds of exogenous default policy which are generally considered: the policy that gives constant benefits to all decision makers and the stochastic default policy.

The current paper investigates a classical "divide the cake" problem. Politicians have to decide how to divide an exogenously given budget. An agenda setter is chosen randomly in each period and proposes a budget allocation. The proposal is accepted and implemented if a simple majority supports it; otherwise the default policy is implemented. Two rules for default policies are compared: the dynamic endogenous rule and the exogenous rule. Under the exogenous rule, the default policy is given and is constant in each period. The paper considers the question which default policy rule will be installed if the players can decide about it in advance.

The main result of the paper are as follows: if the decision rule is a simple majority rule, then any of both default policy rules can be chosen. Moreover, it is possible to characterize what default policy rule would be preferred by each player depending on his initial default policy.

This paper closely relates to the literature on endogenous political institutions. This literature considers two stages of the analysis, as suggested by Buchanan (1987): at the first stage a constitution is designed, which sets the rules by which policy decisions are made. At the second stage the policies are chosen. Aghion, Alesina and Trebbi (2004) focus upon the endogenous choice of the majority needed to implement a policy. A number of papers (Aghion and Bolton (1997), Maskin and Tirole (2001) Barbera and Jackson (2001)) investigate "choosing how to choose", i.e. voting on voting rules. Besley and Case (2003) provide literature review as well as empirical evidence using variations in institutional rules across the United States. Eicher and García-Peñalosa (2003) incorporate endogenous institutes into the R&D based growth model.

Furthermore, this paper is related to the literature on dynamic political games with an endogenous default policy. Baron (1996) considers the dynamic problem of collective goods programs, where the players have different preferences regarding the amount of the public good characterized by a one-dimensional ideal point. Baron shows that the amount of the public good converges to the median politician's ideal point when the default policy is the decision made in the previous period. Baron and Diermeier (1998) also use the endogenous default policy in the two period model which incorporates elections, governmental formation and legislative bargaining in the two-dimensional spatial environment. Kalandrakis (2002, 2004) considers the similar distributive model with endogenous default policy. He investigates a more general framework and obtains the same results as in Winschel (2004). Fong (2004) considers a two-dimensional spatial model of government formation and the policy choice with three parties. In addition to the policy position, parties care about political rents collected in the process. Rents come from the division of pork in the office and side payments within the government. He shows that if the default policy is endogenous, policies are far away from the center of preferences. They even move outside the Pareto set of the game. Fong concludes that the exogenous default policy rule can lead to a higher social welfare.

The paper is organized as follows. The next section presents the model. Section 3.3 describes the equilibrium of the game under alternative rules. Section 3.4 considers the symmetric initial default policy. Section 3.5 calculates the payoffs under the exogenous rule and Section 3.6 under the endogenous rule. Section 3.7 discusses the results and concludes.

3.2 Model

The political system consists out of a set of players $N = \{1, ..., n\}$ where $n \geq 3$ and odd. Each period, politicians are to allocate a unit of a perfectly divisible cake among themselves. Let X denote the set of feasible allocations in each period, that is, $X = \left\{x \in \Re^n_+ : \sum_{i=1}^n x_t^i = 1\right\}$, where x_t^i denotes the payoff of player i at period t.

Each player has a linear utility function that depends on her payoff only. Players discount the future by a common discount factor $\delta < 1$. The preferences are separable over time, so the utility function of player i is

$$U_i = \sum_{t=1}^{\infty} \delta^{t-1} x_t^i. \tag{1}$$

The game is played as follows: each period nature randomly selects one player to be the agenda setter. The agenda setter proposes an allocation in X, and each player responds by either accepting or rejecting the proposal.

The decision rule in each period is the simple majority rule. If at least $\frac{n+1}{2}$ players support proposal x_t in period t, it is implemented. Otherwise, the default policy is implemented.

I consider two types of default policy:

Definition 1 *The endogenous default policy in period t is the allocation in X that was implemented in the previous period, namely x_{t-1}. $x_0 \in X$ is the default policy in the first period and is given exogenously.*

Definition 2 *The exogenous default policy is allocation $\widetilde{x} \in X$, which is given exogenously and is constant over time.*

Let h_t denote the history of the game up to period t. A feasible action for player i at date t is denoted by $a_t^i(h_t)$. When i is the agenda setter, $a_t^i(h_t) \in X$ denotes the proposal offered by i at date t, when the history is h_t. When i is not the agenda setter, $a_t^i(h_t) \in \{accept, reject\}$ denotes the decision rule by i

at date t. Strategy s^i for player i is a sequence of actions $\left\{a_t^i\left(h_t\right)\right\}_{t=1}^{\infty}$, and the strategy profile s is an n-tuple of strategies, one for each player.

Whenever a player is to take an action, she knows what history has occurred, the rules of the game and the preferences of other players; so the game is of perfect information.

The solution concept is a sub-game perfect Nash equilibrium. I focus on a particular class of equilibria called stationary. In such an equilibrium a player chooses a history-independent strategy in each strategically identical subgame. In this model, all subgames with the same default policy are strategically identical, since their action sets and the future sequences of moves are the same. Thus, the equilibrium considered is a "Markov" or a "state-space" equilibrium in which the past history influences the current play only through its effect on a state variable that summarizes the direct effect of the past on the current environment, or, alternatively, the strategies depend only on the payoff-relevant information. Then the equilibrium strategy profile can be defined as an n-tuple of strategies s^i, that depend only on the actual default policy: $\left\{a_t^i\left(x_{t-1}\right)\right\}_{t=1}^{\infty}$. Using this specific type of equilibrium I rule out a possible punishment strategy profile which typically leads to a multiplicity of subgame perfect equilibria (see Baron and Ferejohn (1989)).

Since no player can change the outcome if a proposal is supported by more than a simple majority, I restrict attention to equilibria in which weakly dominated strategies are eliminated. Otherwise any allocation can be supported as equilibrium.

3.3 Equilibrium

In this section I consider the game which is investigated in details in Winschel (2004) (Section 2). I will provide just a summary of the results which are useful for the current paper. I keep numeration of propositions and definitions in accordance with Winschel (2004).

3.3.1 Exogenous rule

Under the exogenous default policy all subgames in each period are strategically identical, since their action sets and the future sequences of moves are the same. In addition, actions taken in any period do not influence the future periods at all. As a result, I can summarize the game as a sequence of identical one shot games. The equilibrium of such game is described in Proposition 1, Section 2:

Proposition 1 *a strategy profile is an equilibrium if it has the following form:*

Let $i \in N$ denote the agenda setter

1. The decision rule for player j is as follows:
accept proposal x_t iff

$$x_t^j \geq \widetilde{x}^j, \ \forall \ \ j \in N, j \neq i.$$

2. The agenda setter forms a minimal winning coalition (MWC) which is characterized as follows: player j is a member of the MWC iff $j \in MWC$, where $MWC \subset N$ is nonempty set which satisfies

i. $|MWC| = \frac{n-1}{2}$,

ii. $x_t^j = \widetilde{x}^j \ \forall \ j \in MWC$,

$x_t^j = 0 \ \ \forall \ j \notin MWC$,

iii. $\forall l \in MWC$ *and* $\forall k \notin MWC$: $\sum_{j \in MWC \setminus \{l\}} \widetilde{x}^j + \widetilde{x}^k \geq \sum_{j \in MWC} \widetilde{x}^j$.

Proof. see Section 2. ■

3.3.2 Endogenous rule

First, I define the MWC strategy profile for the endogenous default policy:

Definition 4 *The MWC strategy profile is defined as follows: let $i \in N$ denote the agenda setter in period t.*

1. The acceptance rule in period t is as follows: accept if

$$x_t^j + \delta V_{t+1}^j (x_t) \geq x_{t-1}^j + \delta V_{t+1}^j (x_{t-1}), \text{ for all } j \in N, j \neq i, \tag{2}$$

where the LHS is the expected utility of the coalition member j if he accepts the proposal x_2^j, and the RHS is his expected utility if he rejects. $V_{t+1}^j (x_t)$ denotes the continuation value of player j when the default policy in period $t+1$ is vector x_t. It is defined recursively as

$$V_{t+1}^j (x_t) = E \left[x_{t+1}^j (x_t) + \delta V_{t+2}^j (x_{t+1}) \right].$$

2. In each period the agenda setter forms the MWC which is characterized as follows: player j is a member of the MWC iff $j \in MWC$, where $MWC \subset N$ is nonempty set which satisfies:

i. $|MWC| = \frac{n-1}{2}$

ii.

$$x_t^j = \max \left[\left(-\delta V_{t+1}^j (x_t) + x_{t-1}^j + \delta V_{t+1}^j (x_{t-1}) \right), 0 \right] \ \forall \ \ j \in MWC, \tag{3}$$

$$x_t^j = 0 \ \ \forall \ j \notin MWC. \tag{4}$$

iii. $\forall l \in MWC$ *and* $\forall k \notin MWC$

$$\sum_{j \in MWC \setminus \{l\}} x_t^j + \max \left[\left(-\delta V_{t+1}^k (x_t) + x_{t-1}^k + \delta V_{t+1}^k (x_{t-1}) \right), 0 \right] \geq \sum_{j \in MWC} x_t^j. \tag{5}$$

Thus, in each period the agenda setter chooses exactly $\frac{n-1}{2}$ coalition members among the cheapest to buy and proposes the minimal payoff so that coalition members are indifferent between accepting and rejecting the proposal.

The following proposition is from Section 2

Proposition 4 *For $n \geq 5$ the MWC strategy profile is an equilibrium.*

Proof. See Section 2. ■

The following corollary follows from the calculation of the equilibrium payoffs:

Corollary 1 *(i) After at most two periods the MWC payoffs are extreme: the agenda setter obtains the whole budget.*

(ii) In all periods the payoff of the agenda setter is higher than the payoff of any other player.

(iii) If the default payoff of player j is zero, then his MWC payoff is zero and he supports the MWC proposal.

(iiii) If there are two players $l, k \in N$ such that $x_{t-1}^k = x_{t-1}^l$, the agenda setter includes them into the coalition in period t with equal probabilities.

3.4 Symmetric default policy

In this section I consider the symmetric exogenous default policy vector $\widetilde{x}$ and symmetric initial endogenous default policy vector x_0. The expected payoff of each player in each period for exogenous default policy is:

$$Ex_t^{i,EX} = \frac{1}{n}\left(1 - \frac{1}{n}\frac{n-1}{2}\right) + \frac{n-1}{n}\frac{1}{2}\frac{1}{n} = \frac{n+1}{2n^2} + \frac{n-1}{2n^2} = \frac{1}{n}, \forall i \in N, t.$$

Hence, the expected utility is

$$EU_i^{EX} = \sum_{t=1}^{\infty} \delta^{t-1}\frac{1}{n}, \forall i \in N. \tag{6}$$

For endogenous default policy the expected payoff of player $i \in N$ is:

$$\begin{aligned} Ex_t^{i,END} \quad &= \quad \frac{1}{n}\left(1 - \frac{n-1}{2}x_1\right) + \frac{n-1}{n}\frac{1}{2}x_1 + \\ &\quad \delta\left(\frac{1}{n}\left(1 - \frac{n-1}{n}\frac{1}{2}x_2\right) + \frac{n-1}{n}\frac{1}{2}\frac{n-1}{2n}\frac{2}{n-1}x_2\right) + \frac{1}{n}\sum_{\tau=2}^{\infty}\delta^{\tau} \end{aligned}$$

$$\Rightarrow EU_i^{END} = \frac{1}{n}(1+\delta) + \frac{1}{n}\sum_{\tau=2}^{\infty}\delta^{\tau}. \tag{7}$$

As a result, players are indifferent between exogenous and endogenous rules. This result shows that if players decide about the rule before they observe their initial default policy and if the expected default policy is $\frac{1}{n}$ for all players, then they will be indifferent between two rules.

3.5 Exogenous default policy

In this section I concentrate on the case when all players have different default policies. Then the probability to be in the coalition depend only on the number of players who have lower default payoffs. If there are players with the same default payoffs, I have to consider not only their relative ranking, but also how many they are. This would complicate the computations severely without adding inside to the results.

If all players have a different default payoffs, I can order them from the minimal initial default payoff to the maximal. Then the players numbered from 1 to $\frac{n-1}{2}$ are for sure either coalition members or the agenda setter. The players numbered from $\frac{n+3}{2}$ to n are for sure not included into the coalition. Player $\frac{n+1}{2}$ is in the coalition with probability $\frac{1}{2}$: if the agenda setter has a higher default payoff than player $\frac{n+1}{2}$ then he is not included into the coalition. Otherwise, player $\frac{n+1}{2}$ is included into the coalition.

Denote set of players numbered from 3 to $\frac{n-1}{2}$ as $J \subset N$ with typical member j. Then the expected payoff of player j in each period is

$$Ex^j(\widetilde{x}) = \frac{1}{n}\left(1 - Q^J(\widetilde{x}) - \widetilde{x}^j\right) + \frac{n-1}{n}\widetilde{x}^j, \tag{8}$$

where

$$Q^J(\widetilde{x}) = \sum_{j\in J}\widetilde{x}^j + \widetilde{x}^{\frac{n+1}{2}} + \widetilde{x}^1 + \widetilde{x}^2.$$

Denote set of players numbered from $\frac{n+3}{2}$ to n as $L \subset N$ with typical member l. Then the expected payoff of player l is

$$Ex^l(\widetilde{x}) = \frac{1}{n}\left(1 - Q^L\right), \tag{9}$$

where

$$Q^L(\widetilde{x}) = \sum_{j\in J}\widetilde{x}^j + \widetilde{x}^1 + \widetilde{x}^2.$$

The expected payoff of player 1 is

$$Ex^1(\widetilde{x}) = \frac{1}{n}\left(1 - \sum_{j\in J}\widetilde{x}^j - \widetilde{x}^{\frac{n+1}{2}} - \widetilde{x}^2\right) + \frac{n-1}{n}\widetilde{x}^1. \tag{10}$$

The expected payoff of player 2 is

$$Ex^2(\widetilde{x}) = \frac{1}{n}\left(1 - \sum_{j\in J}\widetilde{x}^j - \widetilde{x}^{\frac{n+1}{2}} - \widetilde{x}^1\right) + \frac{n-1}{n}\widetilde{x}^2. \tag{11}$$

The expected payoff of player $\frac{n+1}{2}$ is

$$Ex^{\frac{n+1}{2}}(\widetilde{x}) = \frac{1}{n}\left(1 - \sum_{j\in J}\widetilde{x}^j - \widetilde{x}^1 - \widetilde{x}^2\right) + \frac{n-1}{2n}\widetilde{x}^{\frac{n+1}{2}}. \tag{12}$$

Note that all payoffs are non-negative. Due to this I can ignore the non-negative payoffs condition, which follows from (3). The expected utility for T-periods game for each player $i \in N$ is

$$EU_i^{EX} = \sum_{t=1}^{\infty} \delta^{t-1} x^i. \tag{13}$$

All expected payoffs can be easily calculated and depend only on the default policy vector.

3.6 Endogenous default policy

Under the MWC equilibrium strategy profile, the allocation will be extreme at most after two periods. Hence, I have to calculate only the payoffs in the first and in the second period. In the first period at most $\frac{n+1}{2}$ players obtain positive payoffs and in the second period at most 2 players obtain positive payoffs. The expected utility in the later periods is identical for all players and is equal to $\frac{1}{n}$ in each period, regardless of the initial default policy.

As in the previous section, I order players according to their default policies. In the first period, players from 1 to $\frac{n-1}{2}$ will be for sure coalition members. Players from $\frac{n+3}{2}$ to n will be for sure not. Player $\frac{n+1}{2}$ will be in the coalition with probability $\frac{1}{2}$, depending on whether the agenda setter' default payoff is higher or lower than his. The chances to be in the second period coalition will have either player 1 or player 2, if 1 is the agenda setter in the first period.

Denote by Q^i the price that player i has to pay to his coalition in case he is the agenda setter. Denote set of players numbered from 3 to $\frac{n-1}{2}$ as $J \subset N$ with typical member j. Then the expected utility of such a player is

$$U_j^{END} = \frac{1}{n}\left(1 - Q^j\right) + \frac{n-1}{n} x_1^j + \frac{1}{n}\sum_{\tau=1}^{\infty} \delta^{\tau}. \tag{14}$$

The expected payoff of player j in the second period is $\frac{1}{n}$. The payoff of player j in the first period is calculated from the following condition

$$\begin{aligned} x_1^j + \frac{1}{n}\sum_{\tau=1}^{\infty}\delta^{\tau} &\geq x_0^j + \delta\left(\frac{1}{n}\left(1 - Q^j\right) + \frac{n-1}{n}x_1^j\right) + \frac{1}{n}\sum_{\tau=2}^{\infty}\delta^{\tau}, \\ s.t.\ x_1^j &\geq 0, \end{aligned} \tag{15}$$

where

$$Q^j = \sum_{j \in J} x_1^j + x_1^{\frac{n+1}{2}} + x_1^1 + x_{1a}^2 - x_1^j.$$

According to (3), the payoff is

$$x_1^j = \max\left(0, \frac{1}{1-\delta}\left(x_0^j - \frac{\delta}{n}\left(\sum_{j \in J} x_1^j + x_1^{\frac{n+1}{2}} + x_1^1 + x_{1a}^2\right)\right)\right). \tag{16}$$

Denote set of players numbered from $\frac{n+3}{2}$ to n as $L \subset N$ with typical member l. Then the expected utility of such a player is

$$U_l^{END} = \frac{1}{n}\left(1 - Q^l\right) + \delta \frac{1}{n}\left(\frac{1}{n} + \frac{n-1}{n}\left(1 - \frac{1}{n}x_{2b}^2 - \frac{n-1}{n}x_2^1\right)\right) + \frac{1}{n}\sum_{\tau=2}^{\infty}\delta^\tau, \tag{17}$$

where

$$Q^L = \sum_{j\in J} x_1^j + x_1^1 + x_{1a}^2 + x_1^{\frac{n+1}{2}}, Q^l = Q^L \forall l \in L.$$

Next, I calculate the expected utilities of players 1 and 2. These two players are the only ones who can expect positive coalition payoffs in the second period.

The expected utility of player 1 is

$$\begin{aligned} U_1^{END} &= \frac{1}{n}\left(1 - \sum_{j\in J} x_1^j - x_1^{\frac{n+1}{2}} - x_{1b}^2 + \frac{1}{n}\sum_{\tau=1}^{\infty}\delta^\tau\right) \\ &\quad + \frac{n-1}{n}\left(x_1^1 + \delta\left(\frac{1}{n} + \frac{n-1}{2n}x_2^1\right) + \frac{1}{n}\sum_{\tau=2}^{\infty}\delta^\tau\right). \end{aligned} \tag{18}$$

The second period's payoff of player 1 is calculated according to (2) and (3):

$$\begin{aligned} x_2^1 + \frac{1}{n}\sum_{\tau=1}^{\infty}\delta^\tau &\geq x_1^1 + \delta\left(\frac{1}{n} + \frac{n-1}{2n}x_2^1\right) + \frac{1}{n}\sum_{\tau=2}^{\infty}\delta^\tau, \\ s.t.\ x_2^1 &\geq 0 \end{aligned} \tag{19}$$

$$\Rightarrow x_2^1 = \frac{2n}{2n - \delta(n-1)}x_1^1 \geq 0. \tag{20}$$

Then the first period's payoff of player 1 has to satisfy:

$$\begin{gathered} x_1^1 + \delta\left(\frac{1}{n} + \frac{n-1}{2n}x_2^1\right) + \frac{1}{n}\sum_{\tau=1}^{\infty}\delta^\tau \geq x_0^1 + \\ \delta\left(\frac{1}{n}\left(1 - \sum_{j\in J} x_1^j - x_1^{\frac{n+1}{2}} - x_{1b}^2 + +\frac{1}{n}\sum_{\tau=1}^{\infty}\delta^\tau\right) + \right. \\ \left.\frac{n-1}{n}\left(x_1^1 + \delta\left(\frac{1}{n} + \frac{n-1}{2n}x_2^1\right) + \frac{1}{n}\sum_{\tau=1}^{\infty}\delta^\tau\right)\right), \\ s.t.\ x_1^1 \geq 0. \end{gathered} \tag{21}$$

From (3) the payoff is:

$$x_1^1 + \delta \frac{n-1}{2n} x_2^1 = x_0^1 + \delta \left(\begin{array}{c} \frac{1}{n} \left(-\sum_{j \in J} x_1^j - x_1^{\frac{n+1}{2}} - x_{1b}^2 \right) \\ + \frac{n-1}{n} \left(x_1^1 + \delta \frac{n-1}{2n} x_2^1 \right) \end{array} \right) \Rightarrow \tag{22}$$

$$x_1^1 = \max \left(0, \frac{1}{2} \frac{n(2-\delta)+\delta}{n(1-\delta)+\delta} \left(x_0^1 + \delta \frac{1}{n} \left(-\sum_{j \in J} x_1^j - x_1^{\frac{n+1}{2}} - x_{1b}^2 \right) \right) \right). \tag{23}$$

In the similar way the expected utility of player 2 is calculated:

$$\begin{aligned} U_2^{END} &= \frac{1}{n} \left(1 - \sum_{j \in J} x_1^j - x_1^{\frac{n+1}{2}} - x_1^1 + \frac{1}{n} \sum_{\tau=1}^{\infty} \delta^\tau \right) \\ &+ \frac{n-2}{n} \left(x_{1a}^2 + \frac{1}{n} \sum_{\tau=1}^{\infty} \delta^\tau \right) + \\ &+ \frac{1}{n} \left(x_{1b}^2 + \delta \left(\frac{1}{n} + \frac{n-1}{2n} x_2^2 \right) + \frac{1}{n} \sum_{\tau=2}^{\infty} \delta^\tau \right). \end{aligned} \tag{24}$$

The second period's payoff of player 2 is then

$$x_2^2 = \frac{2n}{2n - \delta(n-1)} x_{1b}^2 \geq 0. \tag{25}$$

Again, there are two cases to be considered separately: the case when the first period agenda setter is player 1 (case b) and otherwise (case a).

Payoff x_{1a}^2 has to satisfy the following condition:

$$\begin{aligned} & x_{1a}^2 + \frac{1}{n} \sum_{\tau=1}^{\infty} \delta^\tau \geq x_0^2 + \delta \left(\frac{1}{n} \left(1 - \sum_{j \in J} x_1^j - x_1^{\frac{n+1}{2}} - x_1^1 + \frac{1}{n} \sum_{\tau=1}^{\infty} \delta^\tau \right) + \right. \\ & \left. + \frac{n-2}{n} \left(x_{1a}^2 + \frac{1}{n} \sum_{\tau=1}^{\infty} \delta^\tau \right) + \frac{1}{n} \left(x_{1b}^2 + \delta \left(\frac{1}{n} + \frac{n-1}{2n} x_2^2 \right) + \frac{1}{n} \sum_{\tau=2}^{\infty} \delta^\tau \right) \right), \end{aligned} \tag{26}$$

$$s.t. x_{1a}^2 \geq 0.$$

From (3) the payoff is

$$x^2_{1a} = x^2_0 + \delta \left(\frac{1}{n} \left(-\sum_{j \in J} x^j_1 - x^{\frac{n+1}{2}}_1 - x^1_1 \right) + \right. \tag{27}$$

$$\left. + \frac{n-2}{n} x^2_{1a} + \frac{1}{n} \left(x^2_{1b} + \delta \frac{n-1}{2n} x^2_2 \right) \right) \Rightarrow$$

$$x^2_{1a} = \max \left(0, \frac{n}{n(1-\delta) + 2\delta} \left(x^2_0 + \delta \left(\frac{1}{n} \left(-\sum_{j \in J} x^j_1 - x^{\frac{n+1}{2}}_1 - x^1_1 \right) + \frac{2x^2_{1b}}{n(2-\delta) + \delta} \right) \right) \right). \tag{28}$$

Payoff x^2_{1b} has to satisfy the following condition:

$$x^2_{1b} + \delta \left(\frac{1}{n} + \frac{n-1}{2n} x^2_2 \right) + \frac{1}{n} \sum_{\tau=2}^{\infty} \delta^\tau \geq x^2_0 +$$

$$\delta \left(\frac{1}{n} \left(1 - \sum_{j \in J} x^j_1 - x^{\frac{n+1}{2}}_1 - x^1_1 + \frac{1}{n} \sum_{\tau=1}^{\infty} \delta^\tau \right) + \frac{n-2}{n} \left(x^2_{1a} + \frac{1}{n} \sum_{\tau=1}^{\infty} \delta^\tau \right) + \right.$$

$$\left. + \frac{1}{n} \left(x^2_{1b} + \delta \left(\frac{1}{n} + \frac{n-1}{2n} x^2_2 \right) + \frac{1}{n} \sum_{\tau=2}^{\infty} \delta^\tau \right) \right), \tag{29}$$

$$s.t.\ x^2_{1b} \geq 0.$$

From (3) the payoff is

$$x^2_{1b} + \delta \frac{n-1}{2n} x^2_2 = x^2_0 + \delta \left(\frac{1}{n} \left(-\sum_{j \in J} x^j_1 - x^{\frac{n+1}{2}}_1 - x^1_1 \right) + \right.$$

$$\left. + \frac{n-2}{n} x^2_{1a} + \frac{1}{n} \left(x^2_{1b} + \delta \frac{n-1}{2n} x^2_2 \right) \right) \Rightarrow$$

$$x^2_{1b} = \max \left(0, \frac{n(2-\delta) + \delta}{2(n-\delta)} \left(x^2_0 + \delta \left(\frac{1}{n} \left(-\sum_{j \in J} x^j_1 - x^{\frac{n+1}{2}}_1 - x^1_1 \right) + x^2_{1a} \frac{n-2}{n} \right) \right) \right). \tag{30}$$

The expected utility of player $\frac{n+1}{2}$ is

$$U^{END}_{\frac{n+1}{2}} = \frac{1}{n} \left(1 - \sum_{j \in J} x^j_1 - x^1_1 - x^2_{1a} \right) + \frac{1}{2} x^{\frac{n+1}{2}}_1 + \frac{1}{n} \sum_{\tau=1}^{\infty} \delta^\tau. \tag{31}$$

The first period payoff of player $\frac{n+1}{2}$ has to satisfy the following condition

$$\begin{aligned} x_1^{\frac{n+1}{2}} + \frac{1}{n}\sum_{\tau=1}^{\infty}\delta^{\tau} &\geq x_0^{\frac{n+1}{2}} + \delta\left(\frac{1}{n}\left(1-\sum_{j\in J}x_1^j - x_1^1 - x_{1a}^2\right) + \frac{1}{2}x_1^{\frac{n+1}{2}} + +\frac{1}{n}\sum_{\tau=1}^{\infty}\delta^{\tau}\right), \\ s.t.\ x_1^{\frac{n+1}{2}} &\geq 0. \end{aligned}$$

From (3) the payoff is

$$x_1^{\frac{n+1}{2}} = x_0^{\frac{n+1}{2}} + \delta\left(\frac{1}{n}\left(-\sum_{j\in J}x_1^j - x_1^1 - x_{1a}^2\right) + \frac{1}{2}x_1^{\frac{n+1}{2}}\right)$$

$$\Rightarrow x_1^{\frac{n+1}{2}} = \max\left(0, x_0^{\frac{n+1}{2}} + \delta\left(\frac{1}{n}\left(-\sum_{j\in J}x_1^j - x_1^1 - x_{1a}^2\right) + \frac{1}{2}x_1^{\frac{n+1}{2}}\right)\right)$$

$$\Rightarrow x_1^{\frac{n+1}{2}} = \max\left(0, \frac{2}{2-\delta}\left(x_0^{\frac{n+1}{2}} + \delta\frac{1}{n}\left(-\sum_{j\in J}x_1^j - x_1^1 - x_{1a}^2\right)\right)\right). \tag{32}$$

As a result, the coalition members' payoffs in the first period are calculated from the following system of $\frac{n+3}{2}$ non linear equations:

$$x_1^1 = \max\left(0, \frac{1}{2}\frac{n(2-\delta)+\delta}{n(1-\delta)+\delta}\left(x_0^1 + \delta\frac{1}{n}\left(-\sum_{j\in J}x_1^j - x_1^{\frac{n+1}{2}} - x_{1b}^2\right)\right)\right), \tag{33}$$

$$x_{1a}^2 = \max\left(0, \frac{n}{n(1-\delta)+2\delta}\left(x_0^2 + \delta\left(\frac{1}{n}\left(-\sum_{j\in J}x_1^j - x_1^{\frac{n+1}{2}} - x_1^1\right) + x_{1b}^2\frac{2}{n(2-\delta)+\delta}\right)\right)\right), \tag{34}$$

$$x_{1b}^2 = \max\left(0, \frac{n(2-\delta)+\delta}{2(n-\delta)}\left(x_0^2 + \delta\left(\frac{1}{n}\left(-\sum_{j\in J}x_1^j - x_1^{\frac{n+1}{2}} - x_1^1\right) + \frac{n-2}{n}x_{1a}^2\right)\right)\right), \tag{35}$$

$$x_1^j = \max\left(0, \frac{1}{1-\delta}\left(x_0^j - \frac{\delta}{n}\left(\sum_{j\in J}x_1^j + x_1^{\frac{n+1}{2}} + x_1^1 + x_{1a}^2\right)\right)\right) \quad j = 3...\frac{n-1}{2}, \tag{36}$$

$$x_1^{\frac{n+1}{2}} = \max\left(0, \frac{2}{2-\delta}\left(x_0^{\frac{n+1}{2}} + \delta\frac{1}{n}\left(-\sum_{j\in J} x_1^j - x_1^1 - x_{1a}^2\right)\right)\right). \tag{37}$$

Consider for example the case $n = 5$. Then the coalition members' payoffs in the first period are calculated from the following equations:

$$x_1^1 = \max\left(0, \frac{1}{2}\frac{n(2-\delta)+\delta}{n(1-\delta)+\delta}\left(x_0^1 + \delta\frac{1}{n}\left(-x_1^3 - x_{1b}^2\right)\right)\right),$$

$$x_{1a}^2 = \max\left(0, \frac{n}{n(1-\delta)+2\delta}\left(x_0^2 + \delta\left(\frac{1}{n}\left(-x_1^3 - x_1^1\right) + x_{1b}^2\frac{2}{n(2-\delta)+\delta}\right)\right)\right),$$

$$x_{1b}^2 = \max\left(0, \frac{n(2-\delta)+\delta}{2(n-\delta)}\left(x_0^2 + \delta\left(\frac{1}{n}\left(-x_1^3 - x_1^1\right) + \frac{n-2}{n}x_{1a}^2\right)\right)\right),$$

$$x_1^3 = \max\left(0, \frac{2}{2-\delta}\left(x_0^3 + \delta\frac{1}{n}\left(-x_1^1 - x_{1a}^2\right)\right)\right).$$

3.7 Results and discussion

In this section I compare the two alternative default policy rules. I have to relay on the numerical solution for the equations' system (33)-(37). The payoffs and utilities are calculated for various initial default policy vectors using Matlab software.

The results of numerical simulations can be summarized as follows: players with relative high default policies always favor the endogenous rule. Players who are in the middle of the initial default policy distribution prefer the exogenous rule. Players with low initial default policies favor the endogenous rule, if their default policy is low in absolute terms, for example, close to zero. But they prefer the exogenous rule if their default policy is low only in comparison with other players. For example, if $n = 5$ and the initial default policy is $(0.05, 0.1, 0.15, 0.3, 0.4)$ then the first three players prefer the exogenous rule and the last two prefer the endogenous rule. If the initial default policy is $(0.01, 0.14, 0.15, 0.3, 0.4)$ then the first player and two last players favor the endogenous rule and the two "middle" players favor the exogenous rule.

As a result, if there exists a relative equal initial distribution on the "poor" side of the income distribution then the majority supports the exogenous rule. The exogenous rule induces a more stable redistribution pattern, but mostly to the same hands, because the coalition members are mostly the same people (the middle class and the poor). Good example would be a society with a large middle class and without an extremely poor population: the middle class supports the exogenous rule because it guarantees a stable allocation which allows them to keep their position in time.

If there exists an initially extremely poor population, then this population forms a coalition with the rich population and supports the endogenous rule. The result is an extreme redistribution pattern after at most two periods, but all players have equal chances to participate in the coalition. In this case players with high initial allocation, who are repeatedly left outside the coalition under the exogenous rule and do not enjoy redistribution, prefer a more extreme allocation which equalizes the chances among the players. The initially poor, despite being constant coalition members under the exogenous rule, prefer an extreme allocation as well. The reason is that having low initial default policy leads to a permanently low coalition payoff under the exogenous rule. The poor are ready to trade off safe but low coalition payments in order to get chance to obtain the whole budget under the endogenous rule.

If I allow for negative payments, the negative payoffs arise in the equilibrium under the endogenous rule: players with low default policies may have negative payoffs in the first period. It happens because such players are ready to pay in order to move to an extreme allocation. Under the exogenous rule all payoffs are always non-negative. This changes the trade off between the two rules, and, ex ante, the poor population have more incentives to support the exogenous rule, in order to avoid expropriation. As a result, players with low initial default policies may prefer the exogenous rule if negative payments are allowed, even if they prefer the endogenous rule otherwise. For example, if the initial default policy is $(0.03, 0.06, 0.11, 0.3, 0.5)$ and the rule is endogenous, then the first player's coalition payoff is 0, if negative payoffs are not allowed, and -0.0124 otherwise. Therefore such player will prefer the endogenous rule if negative payoffs are not allowed and the exogenous one otherwise. As a result, compared to the situation where only non-negative payoffs are allowed, the exogenous rule has more chances to be chosen.

These results can be summarized as follows:

Corollary *Suppose that players can choose the default policy rule. Then*

(i) if the decision rule is a simple majority rule, then any of both default policy rules can be chosen depending on the initial default policy.

(ii) Players with relative high default policies always favor the endogenous rule.

(iii) Players, who are in the middle of the initial default policy distribution, always prefer the exogenous rule.

(iv) Players with a low initial default policy may prefer either the endogenous or the exogenous rule, depending on the default policy and on whether the negative payoffs are allowed.

The model suggests the following empirical prediction: the countries with more equal income distribution, without extreme poor population, would tend to use exogenous default policy rules to a greater extent compared with countries with less unequal distribution. Composing the data set on the default policy rules and checking this prediction would be an useful extension of the current paper.

4 Political stability

4.1 Introduction

Recent results concerning the distribution model of legislative bargaining with an endogenous default policy (Kalandrakis 2002, Winschel 2003) predict extremely instable outcomes. In a simple dynamic model where n politicians divide an exogenously given budget under the closed agenda and a simple majority rule, the agenda setter obtains the whole budget after two periods at most. Yet, we usually do not observe such a discriminatory behavior in a modern democratic system.

This paper extends the framework above in order to provide an explanation for the observable lack of discrimination. A simple distributional model reflects only one side of a modern political process, namely legislative bargaining and coalition formation. The other crucial side of a representative democratic system is absent: voters and an election mechanism are not modeled.

This paper introduces risk averse voters into a dynamic redistribution model with the endogenous default policy given by the policy last implemented. I investigate the following political game: at the first stage voters elect the agenda setter among a group of politicians. At the second stage the agenda setter proposes an allocation of the budget and all other politicians vote. If a simple majority of politicians support the proposal then it is implemented. Otherwise the default policy is implemented. This game is repeated infinitely and all politicians stay for ever in office. This reflects an empirical observation that usually only the political leadership is changed (namely the agenda rights) but the majority of people in politics stay for considerably long time and political parties stay even longer. The reason for this situation, not explicitly modelled, can be that the reelection of politicians is costly for voters or that there is uncertainty concerning preferences and abilities of new, unknown candidates. I compare this game with the benchmark scenario which represents an ideal situation: voters can reelect their representatives in each period.

The main idea of the paper is that the extreme budget allocation is undesirable for risk averse voters. Rational voters should then coordinate their strategies and restrict policy proposals in order to obtain the stable budget allocation. Stability is defined as an absence of cycling majorities and discrimination against single groups in favor of all others. Full stability yields the symmetric allocation of the budget in all periods.

I assume that the politicians are risk neutral while the voters are risk averse. The idea that the politicians are on average less risk averse than the general population goes back to Tullock. In *The Politics of Bureaucracy* (1967) he emphasizes how the internal political structure of any organization is manifested in promoting policies that will systematically emphasize certain personal characteristics while discouraging others. He observes, for example, that successful politicians will tend to exhibit below average levels of personal integrity simply as a result of the process through which they achieve their positions. Daly (1981) develops Tullock ideas further. He describes the political process as a series of 'winner takes all' contests and concludes that not only the political system tends to promote less risk averse individuals, but the potential suppliers of politicians' services are, on average, likely to be less risk averse than the public they seek to represent. In other words, a self-selection process takes place in the market for political candidates. Recently, Fatas, Neugebauer and Tamborero (2004) tested this hypothesis in the experimental study using real-world politicians as subjects. They found out that politicians appear to be less risk averse than student subjects. Moreover, this hypothesis can be supported by the following observation: politicians tend to be older, more affluent, white and male than their electorate. Assuming non linear utility function, it implies that politicians may be less risk averse just because they have higher than average income and wealth. Moreover, there is evidence that men are less risk averse than women (see Hartog et al. 2002).

My objective is to determine the most stable budget allocation that can be sustained as an equilibrium allocation via a simple trigger strategy. In order to avoid the situation when nearly any allocation can be supported in the equilibrium I do not consider punishments where subsets of voters are punished by other voters. Instead, I concentrate on punishments which imply a return to a non-cooperative strategy in the future. It can be justified by the assumption that the vote is secret; i.e., only the outcome of the vote but not the actions of individual voters are observable.

The main results of the paper are as follows: coordination among voters can explain observed stability. I show that full outcome stability is not possible and the limit to outcome stability is characterized. The limit is achieved under full political stability in the following sense: the same player is elected as the agenda setter during the whole game. Moreover, the agenda setter invites the same players to join the coalition in each period. Political mobility in the sense that different politicians obtain agenda rights and/or participate in the coalition, comes at the cost of stability. On the contrary, in the ideal situation when all politicians can be reelected, the symmetric budget allocation can be supported under full political mobility.

This work is closely related to the paper by Artale and Grüner (2000). The main result is very similar: the theory of repeated games provides a straightforward explanation for political stability. But the frameworks in the two papers are quite different. In contrast to the redistributional model investigated in the current paper, Artale and Grüner (2000) consider a two-dimensional political space. Moreover, in their model the politicians are purely power-seeking and new politicians appear each period, so their behavior is myopic by definition. As a consequence, Artale and Grüner (2000) have no results concerning how the cooperation among voters can influence the dynamic behavior of politicians and the pattern of political mobility.

This paper is also related to Epple and Riordan (1987). They consider repeated majority voting with an exogenous default policy and show that a wide range of budget allocations can be sustained as equilibria by the threat of political banishment. Baron and Ferejohn (1989) show a similar result. In their paper the budget is allocated only once, terminating the game.

In the next section I present the model. In Section 4.3 I describe the equilibrium of the political game when no election is held. In Section 4.4 I consider the main election mechanism: the election is held periodically and voters elect the agenda setter among politicians. In Section 4.5 I introduce the benchmark scenario. Section 4.6 concludes.

4.2 Model

In this section I extend the framework, discussed in Winschel (2004), by adding voters and specifying an election mechanism.

4.2.1 Preferences

The political system consists out of the set of politicians $N = \{1, ..., n\}$ where $n \geq 5$ and an odd number. The politicians are to allocate a unit of a perfectly divisible cake in each period. Let X denote the set of feasible allocations in each period, that is, $X = \left\{x \in \Re^n_+ : \sum_{i=1}^n x^i_t = 1\right\}$, where x^i_t denotes the payoff for politician i at period t.

Each politician has a linear utility function that depends on her payoff only and discounts the future by a common discount factor $\delta < 1$:

$$W^i = \sum_{\tau=0}^{\infty} \delta^\tau x^i_\tau \tag{1}$$

Each politician represents some district populated by homogenous voters. I assume, for simplicity, that the coordination problem is solved inside the district so that each district's population acts as one single player. Further, I assume that all districts are of an equal size. Voters in each district have a common von Neumann Morgerstern utility function which is concave and monotonically increasing in budget share x^i_t. Voters discount future by a common discount factor $\delta < 1$:

$$U^i = \sum_{\tau=0}^{\infty} \delta^\tau U\left(x^i_\tau\right), \quad U'\left(x^i_\tau\right) > 0, \quad U''\left(x^i_\tau\right) < 0 \tag{2}$$

If the budget is allocated only once then the myopic interests of the population are the same as of their representatives: their utility is increasing in the budget share. But, in contrast to the risk neutral politicians, voters are risk averse and hence may have different preferences in the dynamic framework.

4.2.2 Timing

Consider the temporal sequence within each period t. To simplify the discussion I divide each period t into two stages denoted as the voting stage and the political stage. Briefly, the temporal sequence is as follows:

1. Voting stage: voters elect the agenda setter among the given set of politicians.

2. Political stage:

    (a) the agenda setter elected during the voting stage proposes an allocation,

    (b) all politicians cast a vote for or against the proposed allocation,

    (c) if a simple majority of the politicians accepts the proposal, it is implemented, otherwise the default policy is implemented.

This game is repeated infinitely. Each district life and each politicians life are endless.

More detailed, during the voting stage at the beginning of each period the election takes place and voters have to take a decision. Abstention is not allowed. In each period voters from all districts simultaneously vote on an agenda setter. Remember that each district's population acts as one single player, so I assume that each district has one vote. Each district can cast a vote for their own representative, as well as for any other. The winner of the election is the politician who obtains the most votes, and subsequently he is elected as the agenda setter. If two or more politicians tie for the first place then each wins with equal probability.

After the election stage, the political stage takes place. It is played as follows: first the agenda setter, elected at the voting stage, proposes an allocation in X, x_t. Then all politicians simultaneously respond by either accepting or rejecting the proposal. The decision rule in each period is a simple majority rule. So, if at least $\frac{n+1}{2}$ politicians support the proposal x_t in period t, it is implemented. Otherwise, the endogenous default policy is implemented, which is the allocation in X that was implemented in the previous period, namely x_{t-1}. $x_0 \in X$ is the default policy in the first period and is given exogenously.

4.2.3 Strategies and equilibrium concept

Let h_t denote the history at time t. A feasible action for player i at date t is denoted by $a_t^i(h_t)$. Whenever a player is to take an action, she knows what history has occurred, the rules of the game and the preferences of other players; so the game is one of perfect information. The feasible action for voters from each district is $a_t(h_t) \in \{i \in N\}$. When politician i is the agenda setter, $a_t^i(h_t) \in X$ denotes the proposal offered by i at date t, when the history is h_t. When politician i is not the agenda setter, $a_t^i(h_t) \in \{accept, reject\}$ denotes the decision rule by i at date t. I denote the politicians who support the proposal as coalition members. Strategy s^i for player i is a sequence of actions $\left\{a_t^i(h_t)\right\}_{t=1}^{\infty}$, and the strategy profile s is an $2n$-tuple of strategies, one for each politician and for each district.

The equilibrium concept is a sub-game perfect Nash equilibrium. My objective is to determine the most stable allocation that can be sustained as an equilibrium allocation via a simple trigger strategy. Stability is defined as an absence of cycling majorities and discrimination against single groups in favor of all others. Voters cooperate and coordinate their strategies in order to induce the stable budget allocation. They also punish deviations from the cooperative behavior. In order to avoid the situation when nearly any allocation can be supported in the equilibrium I, as in Artale and Grüner (2000), do not consider punishments where subsets of voters are punished by other voters. Instead, I concentrate on punishments which consist of returning to a non-cooperative strategy in the future.

For simplicity I consider only the symmetric initial default policy: $x_0^i = \frac{1}{n}$, $\forall i$, so I can rephrase that a stable allocation induces a minimal deviation from the initial symmetric payoff vector in each period. Then I can state that a maximal degree of outcome stability is achieved if the following function, denoted by $L(x_t)$, is minimized in each period:

$$L(x_t) = \sum_{j=1}^{n} \left(\frac{1}{n} - x_t^j \right)^2 \quad \forall t \tag{3}$$

$$s.t. \sum_{j=1}^{n} x_t^j = 1$$

Given the budget constraint, function $L(x_t)$ is minimized under the symmetric allocation and maximized under the extreme allocation ($x_t^i = 1$, $x_t^j = 0$ $\forall j \neq i$). Then, for any two allocations x_t^* and x_t, if $L(x_t) > (<) L(x_t^*)$ then allocation x_t is less (more) stable than allocation x_t^*.

As it is standard in the literature, I do not consider the equilibrium strategy profile where no player (a district or a politician) deviates just because he is not pivotal and does not influence the decision. Otherwise any allocation can be sustained as an equilibrium.

4.3 Game without elections

Suppose that there are no voters. Politicians play the following repeated game: preferences and payoffs of politicians are the same as in Section 2.1, timing of the game and feasible strategies are identical to the timing and strategies of the political stage in Section 2.2. The agenda setter is chosen randomly in the beginning of each period. This game is investigated in detail in Winschel (2004). I focus on a particular class of equilibria called stationary in which the past history influences the current play only through its effect on a state variable that summarizes the direct effect of the past on the current environment, or, alternatively, the strategies depend only on the payoff-relevant information. Then a player chooses a history-independent strategy in each strategically identical subgame. In this game, any two subgames with identical default policies are strategically identical. Using this specific type of equilibrium I rule out a possible punishment strategy profile which typically leads to a multiplicity of subgame perfect equilibria (see Baron and Ferejohn (1989)). I will provide just the summary of the results which are useful for the current paper.

To begin with, I define the minimal winning coalition (MWC).

Definition 1 *The agenda setter forms the MWC if any coalition member is an element of MWC, where $MWC \subset N$ is a nonempty set which satisfies*

i. $|MWC| = \frac{n-1}{2}$

ii. $\forall\ j \in MWC\ x_t^j = \min \left\{ \widehat{x}_t^j \in x_t \middle| a_t^j(x_{t-1}, x_t) = accept \right\}$

$\forall\ j \notin MWC\ \ x_t^j = 0$

iii. $\forall l \in MWC$ *and* $\forall k \notin MWC: \sum\limits_{j \in MWC \setminus \{l\}} x_t^j + x_t^k \geq \sum\limits_{j \in MWC} x_t^j$

where $x_t^k = \min \left\{ \widehat{x}_t^k \in x_t \middle| a_t^k(x_{t-1}, x_t) = accept \right\}$

Thus, in each period the agenda setter chooses exactly $\frac{n-1}{2}$ coalition members among the cheapest to buy and proposes the minimal payoff so that coalition members are indifferent between accepting and rejecting the proposal. Thus, in each period the agenda setter chooses exactly $\frac{n-1}{2}$ coalition members such that the sum of coalition payments which induce the proposal acceptance is minimal.

Next, I define the MWC strategy profile:

Definition 2 *The MWC strategy profile is defined as follows: let $i \in N$ denote the agenda setter in period t.*

1. For each player j the decision rule in period t is as follows:
accept the proposal x_t iff

$$x_t^j + \delta V_{t+1}^j(x_t) \geq x_{t-1}^j + \delta V_{t+1}^j(x_{t-1}),\ \textit{for all } j \in N,\ j \neq i, \tag{4}$$

where the LHS is the expected utility of coalition member j if he accepts the proposal x_t^j, and the RHS is his expected utility if he rejects. $V_{t+1}^j(x_t)$ denotes the continuation value of player j when the default policy in period $t+1$ is vector x_t. It is defined recursively as

$$V_{t+1}^j(x_t) = E\left[x_{t+1}^j(x_t) + \delta V_{t+2}^j(x_{t+1})\right].$$

2. The agenda setter forms the MWC and coalition payoffs are

$$x_t^j = \max\left[\left(-\delta V_{t+1}^j(x_t) + x_{t-1}^j + \delta V_{t+1}^j(x_{t-1})\right), 0\right]\ \forall\ \ j \in MWC \tag{5}$$

The following proposition is from Winschel (2004)

Proposition 4 (from Winschel 2004) *For $n \geq 5$ the MWC strategy profile is the equilibrium.*

Proof. See the paper. ■

The following corollary follows from the calculation of the equilibrium payoffs:

Corollary 1 *(i) After at most two periods the MWC payoffs are extreme: the agenda setter obtains the whole budget.*

(ii) In all periods the payoff of the agenda setter is higher than the payoff of any other player.

(iii) If the default payoff of player j is zero, then he supports the MWC proposal even if his payoff is zero.

(iiii) If there are two players $l, k \in N$ such that $x_{t-1}^k = x_{t-1}^l$, the agenda setter includes them into the coalition in period t with equal probabilities.

4.4 Limit to stability

In this section I analyze the game introduced in section 2. As I have already mentioned, my objective is to determine the most stable allocation that can be sustained as an equilibrium allocation through a simple trigger strategy. In other words, I want to determinate the limit to stability. I concentrate on punishments which consist of returning to a non-cooperative strategy in the future and I do not consider punishments where subsets of voters are punished by other voters.

To begin with, I characterize the non-cooperative equilibrium strategy profile. In this case the players' strategies are restricted to stationary strategies. Then I, firstly, introduce the trigger strategy profile. Secondly, I construct the payoffs which are compatible with this trigger strategy profile being an equilibrium of the dynamic game. Thirdly, I show that the proposed trigger strategy profile yields maximal available stability of the allocation. Fourthly, I show that voters indeed profit from the cooperation compared with non-cooperation.

4.4.1 Non-cooperative equilibrium

Consider the case where voters do not cooperate. Then they prefer to vote for the candidate from their own district:

Proposition 1 *The following strategy profile is an equilibrium: voters always vote for their representative and politicians play according to the MWC strategy profile.*

Proof. Assume that all players play according to the strategy described in the proposition. Then all politicians obtain the same amount of votes and the agenda setter is elected randomly. According to Proposition 4 (Winschel 2004), the equilibrium strategies of politicians are identical to the MWC strategy profile.

Consider now one period deviation of some district: voters from district i vote for representative $j \neq i$. Then j is certainly elected to be the agenda setter and proposes according to the MWC strategy. In all subsequent periods all districts vote again for their representatives. It is obvious that the deviation is costly for district i, because the payoff of the agenda setter and his district is the highest compared to other politicians and districts (see Corollary 1).

The question arises if voters can profit from the deviation. The potentially profitable deviation is possible only in the first period: if the voters deviate they have a higher probability that their representative will be a coalition member in the next period. The sufficient condition for non-deviation is:

$$U\left(1-\frac{n-1}{2}\frac{n-\delta}{n^2}\right)-\frac{1}{2}\left[U\left(\frac{n-\delta}{n^2}\right)+\delta\frac{1}{n}U\left(\frac{1}{n}\right)\right]+\left[\delta\frac{n^2+1}{2n^2}-\frac{1}{2n}\right]U\left(0\right)>0$$

This condition is easily satisfied, especially for large n, if voters are not extremely risk averse and $U(0)$ is non-negative. I assume that this is the case.

In all other periods voters will never profit from the deviation due to the extreme allocation: all coalition members will support the MWC proposal for a zero payoff. If in period $t > 1$ politician j is chosen as the agenda setter, district i, which supported j rather than their own representative, looses in the current period and does not gain anything in future periods (for details see Appendix). ■

It may be not the unique equilibrium, but it is the simplest and the most intuitive one. In addition, the strategy profile where voters vote for their representative and politicians play according to the MWC strategy, is an unique equilibrium of the one period game.

Now I have to calculate the MWC payoffs and utilities. Denote the agenda setter by i, the coalition by $J \subset N$ with typical coalition member j and the opposition by $L \subset N$ with typical opposition member l. After two periods an allocation is always extreme and the agenda setter obtains the whole budget. The expected utility of the agenda setter in the first period from playing according to the MWC strategy is (for details see Appendix)

$$EW^i = 1 - \frac{n-1}{2}\frac{1}{n^2}(n-\delta) + \sum_{\tau=1}^{\infty}\delta^{\tau}\frac{1}{n}. \tag{6}$$

The expected utility of a coalition member in the first period is

$$EW^j = \frac{1}{n^2}(n-\delta) + \delta\left(\frac{1}{n} + \frac{1}{n^2}\right) + \sum_{\tau=2}^{\infty}\delta^{\tau}\frac{1}{n} = \frac{1}{n} + \sum_{\tau=1}^{\infty}\delta^{\tau}\frac{1}{n}. \tag{7}$$

The expected utility of an opposition member in the first period is

$$EW^l = 0 + \delta\frac{1}{n}\left(1 - \frac{1}{n}\right) + \sum_{\tau=2}^{\infty}\delta^{\tau}\frac{1}{n}. \tag{8}$$

4.4.2 The trigger strategy

In this section I introduce the trigger strategy. I denote the strategy profile which yields the most stable outcomes as the SP (stable payoffs) strategy profile and payoffs which are generated by the SP strategy profile as SP payoffs. I also use the terms the SP allocation, which is the vector of SP payoffs, and the SP proposal which is the shortcut to "the agenda setter proposes the SP allocation".

Consider the following strategy of players:

Definition 3 *voters and politicians play according to the SP strategy if they behave as follows:*

1. In the first period :
(i) All voters vote for their representative
(ii) The agenda setter is selected randomly and proposes the SP allocation.
In all consequent periods:
2. If the policy implemented in the last period is consistent with the SP allocation:
(i) All voters vote for the first period agenda setter.

(ii) If the first period agenda setter is chosen as the agenda setter, she proposes the SP allocation.

(*iii) If any other politician is chosen as the agenda setter, he proposes the MWC allocation.*

3. If the implemented policy in the last period is not consistent with the SP allocation

(i) voters vote for their own representative.

(ii) the agenda setter is chosen randomly and proposes the MWC allocation.

4. Politicians always vote sincerely for the alternative they prefer.

Note, that voters will switch to the punishment strategy only if the implemented allocation is not consistent with the SP allocation and not if some players do not play according to SP strategy profile. The interpretation is that voters are interested only in the actually implemented allocation and not in the details of the political process.

Next I will construct the payoffs which are compatible with the SP strategy being an equilibrium of the dynamic game and will show that the SP strategy yields maximal stability of the allocation.

I again denote the agenda setter by i, the coalition by $J \subset N$ with typical coalition member j and the opposition by $L \subset N$ with typical opposition member l. The SP payoff of player k is denoted as x_{SP}^{k} .

Election of the agenda setter In this section I show that in order to achieve maximal available stability, the same politician has to be elected as the agenda setter in all periods.

Note that playing according to the MWC strategy, the agenda setter allocates the highest payoff to herself, and the MWC equilibrium is characterized by an extreme allocation after at most two periods. Then, with a more stable allocation than the MWC allocation, the payoff of the agenda setter decreases. As a result, if voters want a more stable proposal, they need to compensate the agenda setter. This can only be done by influencing her chances to be the agenda setter in the future. As a result, if voters want to achieve maximal stability they have to elect the same agenda setter each period:

Lemma 1 *Maximal allocation stability is achieved if the same player is always reelected as the agenda setter.*

Proof. The agenda setter's expected utility under the SP strategy profile has to be at least as high as the expected utility from playing according to the MWC strategy.

In each period, the agenda setter's payoff from playing according to the MWC strategy is higher than the payoff of any other politician. Together with the budget constraint it implies that $x_t^i > \frac{1}{n}\ \forall t$. Moreover, if the agenda setter proposes the MWC allocation, then her expected utility is higher than $\frac{1}{n}\sum_{\tau=1}^{\infty}\delta^{\tau}$. In order to compensate the agenda setter for not deviating from the SP proposal her SP payoff has to guarantee that $x_{SP}^{i} > \frac{1}{n}$. This implies that $x_{SP}^{i} > x_{SP}^{k}$, $k \in N$, $k \neq i$. Thus, the player's expected utility increases in the probability to be the agenda setter in future periods. The conclusion is rather trivial: if one player is always reelected as the agenda setter, her SP payoff, which guarantees non deviation, is minimal and results in a larger budget to be allocated among other players and, hence, results in higher outcome stability. ■

Coalition composition In this section I construct the coalition and calculate SP coalition payoffs in a way that maximal stability of the allocation is achieved.

In order to discuss coalition members' payoffs, two issues are to be considered. First the coalition composition: either the SP proposal has to be supported by the stable coalition, in the sense that the same politicians are included into the coalition in each period, or there is a possibility for a coalition members rotation. If payoffs to coalition members and opposition members are different, it will be less discriminating if there is a rotation among coalition members. Second, the coalition size has to be considered.

Suppose that all players play according to the SP strategy. Moreover, suppose that a coalition members rotation exists. Then each politician can be a coalition member with some probability less than one, denoted by p_k^c for politician k. Note, that maximal allocation stability requires equal coalition and opposition payoffs for all coalition and opposition members respectively: $x_{SP}^{l_1} = x_{SP}^{l_2}\ \forall l_1, l_2 \in L$ and $x_{SP}^{j_1} = x_{SP}^{j_2}\ \forall j_1, j_2 \in J$. According to Lemma 1, the payoff of the agenda setter is higher than $\frac{1}{n}$ in each period. Together with the budget constraint it implies that $Ex_{SP}^k = x_{SP}^l\left(1 - p_k^c\right) + x_{SP}^j p_k^c < \frac{1}{n}$. Hence, if there is a coalition members rotation, the expected payoff in each period for all politicians is lower than $\frac{1}{n}$, apart from the agenda setter. All politicians vote sincerely for the alternative which guarantees the highest expected utility, so they would never support the SP proposal in the first period because their expected utility from switching to the MWC strategy profile, given the initial default policy, is $\frac{1}{n}\sum_{\tau=1}^{\infty}\delta^\tau$. Then the SP allocation has to yield the expected utility at least $\frac{1}{n}\sum_{\tau=1}^{\infty}\delta^\tau$ for each coalition member. From this reasoning it follows directly that the coalition composition has to be stable. Thus, if we want to achieve maximal stability, the payoff for each coalition member has to be $\frac{1}{n}$ in all periods. This is a minimal payoff which guarantees the acceptance of the SP proposal in the first period. In all later periods the default policy of coalition members equals $\frac{1}{n}$ and all coalition members will support the SP proposal.

Consider now the coalition size. It is obvious that the coalition should consists of $\frac{n-1}{2}$ members: this maximizes the surplus which can be distributed among the opposition.

The result is summarized in the following lemma:

Lemma 2 *the SP strategy profile can be supported as the equilibrium and yields maximal output stability iff :*

(i) the coalition consists of $\frac{n-1}{2}$ *members,*

(ii) the payoff to a coalition member equals to $\frac{1}{n}$ *in each period,*

(iii) the coalition composition is stable.

Equilibrium payoffs Having described the way the coalition is constructed,

I can calculate how the rest of the budget is allocated between the agenda setter and opposition members. The payoffs are derived from the agenda setter's incentive constraints. As I argued above, the agenda setter has to obtain at least the same expected utility as under the MWC strategy in all periods. Hence, we need to consider two incentives constrains: one for the first period, when the default policy is symmetric, and the other one for all consequent periods, when the default policy is the SP allocation $\left(x^i_{SP}, \frac{1}{n}, x^l_{SP}\right)$.

The incentive constraint for the first period is

$$W^i_{SP} = \left(1 - \frac{n-1}{2}\frac{1}{n} - \frac{n-1}{2}x^l_{SP}\right)\sum_{\tau=0}^{\infty}\delta^\tau \geq$$

$$\geq 1 - \frac{n-1}{2}\frac{1}{n^2}(n-\delta) + \sum_{\tau=1}^{\infty}\delta^\tau\frac{1}{n} = EW^i_{MWC}, \tag{9}$$

and for all other periods is

$$W^i_{SP} = \left(1 - \frac{n-1}{2n} - \frac{n-1}{2}x^l_{SP}\right)\left(1 + \sum_{\tau=1}^{\infty}\delta^\tau\right) \geq 1 + \sum_{\tau=1}^{\infty}\delta^\tau\frac{1}{n} = EW^i_{MWC} \tag{10}$$

In order to support the SP strategy profile as an equilibrium strategy profile, an opposition member's payoff has to satisfy the following condition (for the details see Appendix)

$$x^l_{SP} \leq \frac{2\delta - 1}{n}. \tag{11}$$

In order to obtain maximal output stability the payoff for an opposition member has to be equal to $x^l_t = \frac{2\delta-1}{n}$.

Summarizing the results:

Proposition 2 *Suppose that voters play according to the SP strategy. Then the SP strategy profile can be*

(1) supported as the equilibrium of the cooperation game and (2) yields maximal output stability iff

(i) $\delta > 0.5$.

(ii) It is characterized by the following payoffs:

$$x^i_{SP} = 1 - \frac{n-1}{2}\frac{1}{n} - \frac{n-1}{2}\frac{2\delta-1}{n} = \frac{n(1-\delta)+\delta}{n}$$

$$x^j_{SP} = \frac{1}{n}$$

$$x^l_{SP} = \frac{2\delta - 1}{n}$$

Proof. follows directly from the construction of the payoffs. ■

This proposition implies that full stability $\left(x_{SP}^k = \frac{1}{n},\ \forall k \in N\right)$ cannot be achieved.

Comparing the results with the MWC expected utilities we see that the agenda setter i is better off under the SP

$$EW_{MWC}^i - W_{SP}^i = 1 - \frac{n-1}{2}\frac{1}{n^2}(n-\delta) + \frac{\delta}{1-\delta}\frac{1}{n} - \left(\frac{\delta}{1-\delta}+1\right)\frac{n(1-\delta)+\delta}{n} =$$

$$= -\frac{1}{2}\frac{n^2-\delta n-n+\delta}{n^2} < 0, \tag{12}$$

coalition members are indifferent

$$EW_{MWC}^j - W_{SP}^j = \frac{1}{n^2}(n-\delta) + \delta\frac{1}{n^2} + \frac{\delta}{1-\delta}\frac{1}{n} - \frac{1}{n} - \frac{\delta}{1-\delta}\frac{1}{n} = 0, \tag{13}$$

and opposition members loose

$$\begin{aligned} EW_{MWC}^l - W_{SP}^l &= -\frac{\delta}{n^2} + \frac{\delta}{1-\delta}\frac{1}{n} - \frac{2\delta-1}{n} - \frac{\delta}{1-\delta}\frac{2\delta-1}{n} = \\ &= \frac{1}{n^2}(n-\delta) > 0. \end{aligned} \tag{14}$$

Ex ante, prior to the identities of the agenda setter and coalition members are known, the politicians are indifferent between cooperative and non cooperative equilibria.

Voters Till now it was only an assumption, that voters cooperate in order to reduce discrimination. The following Lemma verifies that there are benefits from this political consensus:

Lemma 3 *Assume that voters are risk averse. Then the following is true:*

(i) ex ante, before the identities of the agenda setter and coalition members are known, all voters prefer the SP allocation over the MWC allocation,

(ii) ex post the majority of voters prefer the SP payoffs,

(iii) if risk aversion is strong enough, all voters prefer the SP payoffs ex post.

Proof. (i) Follows from the fact that the ex ante distribution of the SP payoffs, compared to the MWC payoffs, is a mean preserving spread with a lower variance.

(ii) Voters from the agenda setter's district obviously prefer the SP payoffs: they obtain a deterministic payoff which is higher than the expected payoff under the MWC. Voters from the coalition districts obtain a deterministic payoff which is equal to the MWC expected payoff. Together they form a majority.

(iii) Voters from the opposition districts obtain a payoff which is lower than the expected payoff under the MWC but they bear no risk. If risk aversion is strong enough, they prefer the SP allocation. ■

Lemma 3 states that a restricted political competition is ex ante desirable from the voters' point of view, which guarantees participation. Ex post it is desirable for at least the majority of voters, which guarantees the implementation of the SP allocation in all periods.

4.4.3 Limited political competition

In the previous section I concentrated on the most stable allocation that can be sustained as an equilibrium allocation and I calculated the limit to outcome stability. The main problem concerning these results is an absolute absence of political competition: the agenda rights and the coalition composition are completely stable which contradicts to what we observe in reality. In this section I consider an equilibrium which incorporates some limited political mobility and show that it can be introduced only at the expense of outcome stability.

Political mobility can be introduced in two different manners: via coalition members rotation and through the agenda setters rotation. To start with, note that the coalition non-rotation result depends on the symmetric default policy assumption. By voting against the agenda setter's proposal in the first period, all politicians obtain an expected payoff $\frac{1}{n}$ in each period. If the initial allocation is not symmetric this does not have to be the case. The coalition members rotation is also possible if the following conditions are satisfied:

Lemma 4 *Consider allocation* $\left(x_{SP}^i, x_{SP}^j, x_{SP}^l\right)$ *described in Proposition 2. Suppose that starting from the second period each politician can be a coalition member with some probability less than one, denoted by* p_k^c *for politician* k, $k \neq i$. *Then this allocation can be supported as a SP equilibrium with the coalition members rotation if*

(i) The following condition is satisfied:

$$x_{SP}^i \sum_{\tau=1}^{\infty} \delta^\tau \frac{1}{n} \geq \left(1 - MWC^i(x_0)\right) + \sum_{\tau=1}^{\infty} \delta^\tau \frac{1}{n}, \tag{15}$$

where $MWC^i(x)$ *is the cost of the MWC coalition given the default policy* x *if the agenda setter is player* i.

(ii) In the first period there are at least $\frac{n-1}{2}$ *players whose expected utility from the MWC is weakly lower than the expected utility under the SP.*

(iii) In the first period the agenda setter and coalition members are chosen in such a way that (i) and (ii) are satisfied.

Proof. *(i)* This condition guarantees that the agenda setter does not deviate in the first period. The utility from the SP proposal is higher than the expected utility from the MWC. In the later periods the SP payoffs are the equilibrium payoffs due to the construction, identical to the one in the previous section.

(ii) At least the majority of politicians have to support the allocation in the first period. If the SP proposal in the first period is rejected by the coalition,

then in the next period a non-cooperative game is played. Coalition member k supports the SP proposal in the first period if

$$x_{SP}^{j}+\sum_{\tau=1}^{\infty}\delta^{\tau}\left(p_{k}^{c}x_{SP}^{j}+p_{k}^{c}x_{SP}^{l}\right)\geq x_{0}^{k}+V_{MWC}^{k}\left(x_{0}\right), \tag{16}$$

where $V_{MWC}^{k}\left(x_{0}\right)$ is k's continuation value if the MWC strategy is played in all periods, given the initial default policy.

In the first period the agenda setter proposes the SP allocation and invites into the coalition $\frac{n-1}{2}$ politicians for whom condition (19) is satisfied. Then the SP allocation is accepted. In the later periods the SP payoffs are the equilibrium payoffs due to the construction, identical to the one in the previous section.

(iii) obvious. ■

The main difference from the symmetric default policy case, where these conditions are not satisfied, is that in the first period the agenda setter and coalition members may not necessarily be picked randomly from the general population but chosen from some subgroup of the politicians.

In the following example the initial default policy is such that all conditions from Lemma 4 are satisfied and the SP allocation can be supported as an equilibrium allocation:

Example 1 *Consider the following initial default policy: one player obtains the whole budget and all the others obtain zero. Then the SP payoffs as described in Proposition 2 can be supported as equilibrium payoffs with the coalition members rotation if the agenda setter is chosen randomly but the coalition in the first period has to consist of the players with the default policy zero.*

The alternative way to extend the model for limited political competition is the agenda setters rotation. One can construct an equilibrium where several players replace each other in the role of the agenda setter. Consider the symmetric initial default policy. Suppose that two politicians replace each other in subsequent periods as the agenda setter. The coalition payoffs are $\frac{1}{n}$ each period and there is no coalition members rotation. The agenda setter's payoff is calculated from the two non-deviation conditions for the first and for further periods, as in the previous section. Such an agenda setter's rotation implies lower payoffs for opposition members, but when voters are sufficient risk averse they can still be better off compared with non cooperation. Moreover, one can mix these two approaches and obtain limited rotation among the agenda setters as well as in the coalition.

This section shows that for some specific initial default policies the coalition members rotation can be introduced without reducing outcome stability. But for the arbitrary default policy one should expect a lower outcome stability if the coalition is not stable. Some limited political mobility in the agenda setting can be introduced only at the expense of outcome stability.

4.5 Ideal scenario: reelection of all politicians

In this section I introduce an ideal voting scenario: voters can reelect their representatives in each period. I will show that voters can induce symmetric payoffs in each period if they coordinate their strategies.

The preferences and payoffs of all players, politicians as well as voters, are described in section 2.1. The temporal sequence within single period t is as follows:

1. Voting stage:voters in each district elect their representative (one per district),
   (a) voters in each district elect their representative, one per district,
   (b) the agenda setter is elected randomly among n politicians elected in (a).
2. Political stage:
   (a) the agenda setter, elected during the voting stage, proposes an allocation,
   (b) all politicians vote for or against the proposed allocation,
   (c) if a simple majority of politicians accepts the proposal, it is implemented, otherwise the default policy is implemented.

During the voting stage, each district elects their own representative in each period. All districts act simultaneously. After the election has taken place, the agenda setter is chosen randomly[7]. The feasible action for the voter is $a_t(h_t) \in \{$*reelect an old representative, elect a new representative*$\}$. I assume that in each district there is always a large pool of identical potential politicians, and once out of the office the representative cannot be reelected again. The political stage is described in section 2.2.

This game is infinitely repeated. Each voter's life is endless. Each politician's life starts in the period when he is elected and ends in the period when he is replaced by a new representative. Note that each politician's life may be endless if he is always reelected.

4.5.1 Non-cooperative equilibrium

If voters can reelect their representatives they can influence the political process even without coordinating their strategies by threatening not to reelect their representative. As a result, they may generally achieve a higher allocation stability, compared to the MWC. Nevertheless, in the absence of cooperation it is impossible to support the symmetric allocation as an equilibrium allocation, even if voters are extreme risk averse:

[7] I could add the election of the agenda setter as well, but it would not change the results.

Lemma 5 *In the absence of cooperation the payoff profile* $x_t^k = \frac{1}{n}\ \forall k \in N, \forall t$ *cannot be the equilibrium payoff.*

Proof. Suppose that there is such an equilibrium, where the agenda setter proposes $x_t^k = \frac{1}{n} \forall k \in N$, $\forall t$, the majority of politicians supports this proposal and all politicians are reelected by their districts.

Suppose that agenda setter i deviates and proposes the MWC allocation in period t. In all consequent periods the equilibrium strategy is played. In this case i's payoff in period t is $1 - \frac{n-1}{2n} > \frac{1}{n}$. Her district profits from the deviation and coalition members' districts are indifferent. As a result, i is reelected and profits from the deviation. Hence, the proposed strategy profile cannot be an equilibrium. ■

4.5.2 The trigger strategy

According to Definition 1, the ideal situation for voters is a symmetric payoffs' vector in each period. In this section I aim to investigate if voters can achieve such stability via cooperation.

Define the SP strategy as follows:

Definition 4 *voters and politicians play the SP strategy if they behave as follows:*

1. In each period the agenda setter proposes symmetric allocation $x_t^k = \frac{1}{n} \forall k \in N, \forall t$.

2. If the implemented policy in the last period is the symmetric allocation, voters reelect their representatives.

3. If the implemented policy in the last period is not the symmetric allocation, voters elect new representatives and all players play according to strategy a^* *which is an equilibrium of a non-cooperative game.*

4. Politicians always vote sincerely for the alternative they prefer.

The punishment strategy profile a^* can be any strategy profile which constitutes the equilibrium of a non-cooperative game. As shown in Lemma 5 no such equilibrium profile can induce the symmetric payoffs vector.

If the agenda setter deviates from the equilibrium strategy, she faces the inevitable reelection[8]. Then she considers the deviation period as the last period of the game and makes the MWC proposal. Moreover, the agenda setter has to compensate coalition members for not being reelected.

If the allocation is symmetric in all periods then each politician's utility is

$$W_{SP} = \frac{1}{n} + \frac{1}{n}\frac{\delta}{1-\delta}. \tag{17}$$

Hence, this is also the payoff that the agenda setter has to pay to any coalition member if she makes the MWC proposal. Then she obtains:

[8] I consider only the situation when, deviating, the agenda setter proposes an allocation which is accepted. The situation when the allocation is rejected and all players are reelected again is possible but not interesting.

$$W_{MWC}^{i} = 1 - \frac{n-1}{2}\left(\frac{1}{n} + \frac{1}{n}\frac{\delta}{1-\delta}\right) = \frac{1}{2}\frac{n(1-2\delta)+1}{n(1-\delta)}. \tag{18}$$

The agenda setter deviates if

$$W_{MWC}^{i} - W_{SP}^{i} = \frac{1}{2}\frac{n(1-2\delta)+1}{n(1-\delta)} - \frac{1}{n} - \frac{1}{n}\frac{\delta}{1-\delta} = \tag{19}$$

$$= \frac{1}{2}\frac{n(1-2\delta)-1}{n(1-\delta)} > 0$$

$$\Rightarrow \delta < \frac{1}{2}\frac{n-1}{n}. \tag{20}$$

If $\delta \geq \frac{1}{2}\frac{n-1}{n}$ the agenda setter will never deviate. Coalition members will always support the symmetric allocation, because if they deviate they obtain the same payoff. As a result, the symmetric allocation can be supported as the equilibrium allocation.

This result is summarized in the following proposition:

Proposition 4 *The symmetric allocation can be supported as an equilibrium allocation of the cooperative game if all politicians can be reelected and* $\delta \geq \frac{1}{2}\frac{n-1}{n}$.

4.6 Conclusions

In this paper I follow Artale and Grüner (2000) and show how the theory of repeated games provides a straightforward explanation for political stability in a different framework. By coordinating their actions, voters can induce a stable political outcome in the model which is otherwise characterized by an extreme instability. I defined stability as a minimal deviation from the initial symmetric payoff vector or, in other words, as minimal discrimination against some population groups.

The main conclusion of the paper is that maximal outcome stability is achieved under the highest political stability meaning a permanent agenda setter and a stable coalition. Political mobility in the agenda setting or/and the coalition can be introduced at the expense of outcome stability. Since the agenda setter and the coalition are chosen arbitrary in the beginning of the game, the political stability induces the persistence of different political outcomes in otherwise similar countries.

In a certain sense I have obtained controversial results: the goal of voters' cooperation is to minimize discrimination, but, as a consequence, persistent, though not extreme, discrimination takes place in favor of some groups. This result introduces a trade-off between static discrimination and dynamic stability: the absence of extreme discrimination in one particular period turns out to be at the cost of permanent inequality in the dynamic framework. A society where voters cooperate secures stability but implies that some groups are permanently worse off than others. A society without cooperation produces extreme dynamic instability but equal chances for the whole population. Further research is required to clarify this trade off, in particular a more complicated model which could reflect aspects of the political process in more detail.

An interesting observation is that under the benchmark ideal democracy, where all politicians can be costlessly reelected in each period such a trade-off does not arise: full outcome stability is easily achieved under full political mobility and does not imply permanent discrimination. Then the costs induced by the trade-off between stability and equal chances can be the result of failures in the political process.

The model suggests the following empirical predictions: a higher political stability induces less discrimination and more stable political outcomes. The verification of this prediction would be an useful extension of the current paper.

4.6.1 Appendix

4.6.2 Appendix to Section 4.4.1

First, I calculate the MWC payoffs.

In period two, if the agenda setter is the player with a zero payoff in the first period, she allocates a positive payoff only to one first period coalition member. According to (4), such payoff has to satisfy:

$$x_2^j + \delta\frac{1}{n} + \frac{1}{n}\sum_{\tau=2}^{\infty}\delta^{\tau} \geq x_1^j + \delta\left(\frac{1}{n} + \frac{n-1}{2n}\frac{2}{n-1}x_2^j\right) + \frac{1}{n}\sum_{\tau=2}^{\infty}\delta^{\tau}. \tag{21}$$

From this condition together with (5) it follows that

$$x_2^j = x_1^j\frac{n}{n-\delta}. \tag{22}$$

The first period coalition payoff x_1^j has to satisfy the following condition:

$$x_1^j + \delta\left(\frac{1}{n} + \frac{1}{n}x_2^j\right) + \frac{1}{n}\sum_{\tau=2}^{\infty}\delta^{\tau} \geq \frac{1}{n} + \delta\left(\frac{1}{n}\left(1 - \frac{n-1}{2}x_1^j\right) + \frac{n-1}{n}\frac{1}{2}x_1^j\right) +$$

$$+\delta^2\left(\frac{1}{n}\left(1 - \frac{n-1}{2n}x_2^j\right) + \frac{n-1}{2n^2}x_2^j\right) + \frac{1}{n}\sum_{\tau=3}^{\infty}\delta^{\tau}, \tag{23}$$

$$\Rightarrow x_1^j + \delta\frac{1}{n}x_2 \geq \frac{1}{n},$$

and, according to (5),

$$\Rightarrow x_1^j = \frac{1}{n^2}(n-\delta), \tag{24}$$

and

$$x_2^j = x_1^j\frac{n}{n-\delta} \Rightarrow x_2^j = \frac{1}{n^2}(n-\delta)\frac{n}{n-\delta} = \frac{1}{n}. \tag{25}$$

Then the expected utility of the first period agenda setter from playing according to the MWC strategy is

$$EW^i = 1 - \frac{n-1}{2}\frac{1}{n^2}(n-\delta) + \sum_{\tau=1}^{\infty}\delta^\tau \frac{1}{n}. \tag{26}$$

The expected utility of the first period coalition member is

$$EW^j = \frac{1}{n^2}(n-\delta) + \delta\left(\frac{1}{n} + \frac{1}{n^2}\right) + \sum_{\tau=2}^{\infty}\delta^\tau \frac{1}{n} = \frac{1}{n} + \sum_{\tau=1}^{\infty}\delta^\tau \frac{1}{n}. \tag{27}$$

The expected utility of the first period opposition members is

$$EW^l = 0 + \delta\frac{1}{n}\left(1 - \frac{1}{n}\right) + \sum_{\tau=2}^{\infty}\delta^\tau \frac{1}{n}. \tag{28}$$

Then if all players play according to the MWC strategy in the first period then the expected utility of each voter in district i is:

$$EU^i = \frac{1}{n}\left[U\left(1 - \frac{n-1}{2}\frac{n-\delta}{n^2}\right) + \delta\left(\frac{1}{n}U(1) + \frac{n-1}{n}U(0)\right)\right] +$$

$$\frac{n-1}{2n}\left[U\left(\frac{1}{n^2}(n-\delta)\right) + \delta\left(\frac{1}{n}U(1) + \frac{1}{n}U\left(\frac{1}{n}\right) + \frac{n-2}{n}U(0)\right)\right] + \tag{29}$$

$$+\frac{n-1}{2n}\left[U(0) + \delta\left(\frac{1}{n}U\left(1 - \frac{1}{n}\right) + \frac{n-1}{n}U(0)\right)\right] + \sum_{\tau=2}^{\infty}\delta^\tau\left[\frac{1}{n}U(1) + \frac{n-1}{n}U(0)\right].$$

If voters deviate in the first period then their expected utility is:

$$EU^i = \frac{1}{2}\left[U\left(\frac{1}{n^2}(n-\delta)\right) + \delta\left(\frac{1}{n}U(1) + \frac{1}{n}U\left(\frac{1}{n}\right) + \frac{n-2}{n}U(0)\right)\right] + \tag{30}$$

$$+\frac{1}{2}\left[U(0) + \delta\left(\frac{1}{n}U\left(1 - \frac{1}{n}\right) + \frac{n-1}{n}U(0)\right)\right] + \sum_{\tau=2}^{\infty}\delta^\tau\left[\frac{1}{n}U(1) + \frac{n-1}{n}U(0)\right].$$

The sufficient condition for non-deviation is then

$$U\left(1 - \frac{n-1}{2}\frac{n-\delta}{n^2}\right) - \frac{1}{2}\left[U\left(\frac{n-\delta}{n^2}\right) + \delta\frac{1}{n}U\left(\frac{1}{n}\right)\right] + \left[\delta\frac{n^2+1}{2n^2} - \frac{1}{2n}\right]U(0) > 0$$

If voters do not deviate in the second period, then their expected utility is

$$EU^i = \frac{1}{n}\left[U\left(1-\frac{n-1}{2}\frac{n-\delta}{n^2}\right)+\delta\left(\frac{1}{n}U(1)+\frac{n-1}{n}U(0)\right)\right]+$$

$$+\frac{n-1}{2n}\left[U\left(\frac{1}{n^2}(n-\delta)\right)+\delta\left(\frac{1}{n}U(1)+\frac{1}{n}U\left(\frac{1}{n}\right)+\frac{n-2}{n}U(0)\right)\right]+ \quad (31)$$

$$+\frac{n-1}{2n}\left[U(0)+\delta\left(\frac{1}{n}U\left(1-\frac{1}{n}\right)+\frac{n-1}{n}U(0)\right)\right]+\sum_{\tau=2}^{\infty}\delta^{\tau}\left[\frac{1}{n}U(1)+\frac{n-1}{n}U(0)\right].$$

If voters deviate in the second period, then their expected utility is

$$\begin{aligned} EU^i &= \frac{1}{n}\left[U\left(1-\frac{n-1}{2}\frac{n-\delta}{n^2}\right)+\delta\left(U(0)\right)\right]+ \\ &+\frac{n-1}{2n}\left[U\left(\frac{1}{n^2}(n-\delta)\right)+\delta\left(\frac{1}{n}U\left(\frac{1}{n}\right)+\frac{n-1}{n}U(0)\right)\right]+ \\ &+\frac{n-1}{2n}\left[U(0)+\delta\left(U(0)\right)\right]+\sum_{\tau=2}^{\infty}\delta^{\tau}\left[\frac{1}{n}U(1)+\frac{n-1}{n}U(0)\right], \end{aligned} \quad (32)$$

and is strictly lower than the expected utility if they do not deviate. The same is true for the deviation in any later period.

4.6.3 Appendix to Section 4.2.3

The incentive constrain for the first period is:

$$\begin{aligned} EW^i_{SP} &= \left(1-\frac{n-1}{2}\frac{1}{n}-\frac{n-1}{2}x^l_{SP}\right)\sum_{\tau=0}^{\infty}\delta^{\tau} \geq \\ &\geq 1-\frac{n-1}{2}\frac{1}{n^2}(n-\delta)+\sum_{\tau=1}^{\infty}\delta^{\tau}\frac{1}{n} = EW^i_{MWC}. \end{aligned} \quad (33)$$

Rewriting condition this condition I obtain an expression for the lower bound on an opposition member payoff:

$$1-\frac{n-1}{2}\frac{1}{n^2}(n-\delta)+\frac{1}{n}\frac{\delta}{1-\delta} \leq \left(1-\frac{n-1}{2}\frac{1}{n}-\frac{n-1}{2}x^l_{SP}\right)\left(\frac{\delta}{1-\delta}+1\right) \Rightarrow$$

$$\Rightarrow x^l_{SP} \leq \max\left((n-1+\delta)\frac{\delta}{n^2}, 0\right) = (n-1+\delta)\frac{\delta}{n^2}. \quad (34)$$

The next step is to calculate the agenda setter's SP payoff that prevents her from deviating in periods $t \geq 2$. Playing according to the MWC strategy, she takes into the coalition either SP coalition members or SP opposition members. Intuitively, such an incentive constrain is stricter than condition (33): the MWC is cheaper while it includes the players who stay opposition members under the SP and whose default policy is lower than $\frac{1}{n}$. It is plausible that they agree to support the MWC allocation for a lower payoff than the SP coalition members.

For now I assume that SP opposition members are invited into the MWC in period t, namely that $x_t^l \leq x_t^j$. Later I will check this assumption.

Note, that if any coalition member rejects the proposal made by the agenda setter, the agenda setter is nevertheless reelected, because the implemented policy is consistent with the SP allocation.

According to (4), payoff x_t^l for which player l supports the MWC proposal in period t, $t > 1$, can be calculated from the following condition:

$$x_t^l + \delta\left(\frac{1}{n} + \frac{n-1}{2n}\frac{2}{n-1}x_{t+1}^l\right) + \sum_{\tau=2}^{\infty}\delta^\tau\frac{1}{n} \geq x_{SP}^l +$$

$$+\delta x_t^l + \delta^2\left(\frac{1}{n} + \frac{n-1}{2n}\frac{2}{n-1}x_{t+1}^l\right) + \sum_{\tau=3}^{\infty}\delta^\tau\frac{1}{n}. \tag{35}$$

If politician l supports the MWC allocation proposal in period t, then the punishment strategy is played, and x_{t+1}^l is calculated according to (5):

$$x_{t+1}^l + \delta\frac{1}{n} = x_t^l + \delta\left(\frac{1}{n} + \frac{n-1}{2n}\frac{2}{n-1}x_{t+1}^l\right) \Rightarrow$$

$$\Rightarrow x_{t+1}^l = x_t^l\frac{n}{n-\delta}. \tag{36}$$

Then, substituting for x_{t+1}^l in (35)

$$x_t^l+\delta\left(\frac{1}{n} + \frac{n-1}{2n}\frac{2}{n-1}x_t^l\frac{n}{n-\delta}\right)+\delta^2\frac{1}{n} \geq x_{SP}^l+\delta x_t^l+\delta^2\left(\frac{1}{n} + \frac{n-1}{2n}\frac{2}{n-1}x_t^l\frac{n}{n-\delta}\right) \Rightarrow$$

$$x_t^l + \delta\left(\frac{1}{n} + \frac{1}{n}x_t^l\frac{n}{n-\delta}\right) = x_{SP}^l + \delta x_t^l + \delta^2\frac{1}{n}x_t^l\frac{n}{n-\delta} \Rightarrow$$

$$\Rightarrow x_t^l = \max\left[\frac{x_{SP}^l n-\delta}{n^2(1-\delta)}(n-\delta), 0\right]. \tag{37}$$

Expression $\frac{x_{SP}^l n-\delta}{n^2(1-\delta)}(n-\delta)$ can be negative as well as positive. But if we take into consideration that from (34) follows that $x_{SP}^l \leq (n-1+\delta)\frac{\delta}{n^2}$, then:

$$x_t^l = \max\left[\frac{(n-1+\delta)\frac{\delta}{n^2}n-\delta}{n^2(1-\delta)}(n-\delta), 0\right] = \max\left[-(n-\delta)\frac{\delta}{n^3}, 0\right] = 0. \tag{38}$$

In other words, opposition members always support the MWC proposal, even if they obtain a zero payoff. This means that the agenda setter obtains the whole budget playing according to the MWC strategy.

Then, the expected utility of the agenda setter if she deviated from the SP strategy is

$$EW^i_{MWC} = 1 + \sum_{\tau=1}^{\infty} \delta^{\tau} \frac{1}{n} = 1 + \frac{1}{n}\frac{\delta}{1-\delta}. \tag{39}$$

In order to prevent the agenda setter from deviation, the following condition has to be satisfied:

$$EW^i_{MWC} = 1 + \frac{1}{n}\frac{\delta}{1-\delta} \leq \left(1 - \frac{n-1}{2}\frac{1}{n} - \frac{n-1}{2}x^l_{SP}\right)\left(1 + \frac{\delta}{1-\delta}\right) = W^i_{SP} \Rightarrow$$

$$\Rightarrow x^l_{SP} \leq \frac{2\delta - 1}{n} \tag{40}$$

It follows that the SP strategy can be supported as an equilibrium only for $\delta > 0.5$.

Finally, I have to check the assumption that $x^l_t \leq x^j_t$. Suppose that SP coalition members are included in the MWC, namely that $x^l_t > x^j_t$. Then:

$$x^j_t + \delta\left(\frac{1}{n} + \frac{n-1}{2n}\frac{2}{n-1}x^j_t\frac{n}{n-\delta}\right) + \delta^2\frac{1}{n} = \frac{1}{n} + \delta x^j_t + \delta^2\left(\frac{1}{n} + \frac{n-1}{2n}\frac{2}{n-1}x^j_t\frac{n}{n-\delta}\right) \Rightarrow$$

$$\Rightarrow x^j_t = \frac{n-\delta}{n^2} > 0 = x^l_t, \tag{41}$$

which is a contradiction.

5 Coalition formation with private information

5.1 Introduction

Minimal winning coalitions appeared as a key prediction of various models of coalition formation and vote buying (Baron and Ferejohn 1989, Denzau and Munger 1986, Shepsle 1974, Riker 1962). But as an empirical matter, oversized coalitions appear at least as prevalent as minimal winning coalitions (see discussion in Groseclose and Snyder (1996), Diermeier et all (2002, 2003)).

In this paper I propose a new explanation to the supermajority coalition existence. I investigate whether there are incentives to form a supermajority coalition in the presence of private information.

The direct explanation of the supermajority existence in an uncertain environment is intuitive: it serves to increase the probability of a desirable decision (Koehler 1972, 1975, Riker 1962). Another possible reason is the norm of universalism (Klingaman 1969, Weingast 1979). Groseclose and Snyder (1996) suggest that supermajority coalitions may be cheaper than minimal winning coalitions if there is a competition among vote buyers who move sequentially. Diermeier and Merlo (2000) analysis accounts for surplus and minority governments in a two periods model of the government formation and termination with a stochastic default policy. Axelrod (1970) suggests that supermajority coalitions arise when small, ideologically centrist parties are included to reduce the conflict of interest among parties in the government. Baron and Diermeier (2001) believe that supermajority coalitions form when the status quo policy is ideologically extreme. Crombez (1996) predicts that the largest party will propose an supermajority coalition when it holds few seats and is ideologically extreme. Carrubba and Volden (2000) propose that supermajority coalitions come about when policy logrolls are difficult to achieve and sustain over time. Lijphart (1984) and Sjolin (1993) argue that supermajority coalitions are formed in order to ensure control of the upper chamber in bicameral systems[9].

In the current paper I consider the case of coalition formation in the presence of private information. Consider the following model: some payer who is interested in the implementation of a particular project tries to influence a political body which is responsible for the decision. This player uses private payments in order to influence the politicians. Such a player can be an interest group, which tries to influence the politicians using illegal bribes or legal campaign contributions. The alternative interpretation is that this player is the government which tries to influence the decision of the parliament or some committee. In this case the payments can be governmental portfolios or the implementation of other projects in which the politicians might be interested. Throughout the paper I will call this player a lobby.

Politicians decide about the implementation of the project by the simple majority voting rule. In order to make the game interesting, I assume that the project will not be implemented without the influence from the lobby. Moreover,

[9] See Carrubba and Volden (2004) for a summary of theoretical explanations for the supermajority existence.

for simplicity, in case there are no private payments, all politicians are better off if the project is not implemented.

I assume that politicians bear some costs if they accept private payments[10]. Moreover, I assume that politicians differ with respect to these costs and that a politician's cost is his private information. The costs can be interpreted in a number of ways. Firstly, there is a risk of punishment if the lobbying activity is illegal. In this case the politician can be punished for obtaining illegal private payments. Secondly, when private payments take the form of a campaign contribution, a realistic interpretation for costs can be the reduction in the reelection chances (Prat 2004, Prat, Puglisi and Snyder, Jr. 2005). Thirdly, the costs can take a form of a private "scruple": a politician suffers if he is associated with the undesired decision and obtains a private benefit from it.

I investigate the closed voting procedure, advocated by Dal Bo (2004) and Grüner and Felgenhauer (2003)[11], where the actual vote of a politician cannot be observed and private payments can be conditioned only on the project implementation. Accordingly, I define the politicians who obtain positive private payments as coalition members. The private costs are associated with being a coalition member and obtaining private benefits rather than with the actual vote. This is different for opposition members, who lose only from the implementation of the undesirable project and bear no additional costs.

A good example for this model is the recent political process in Israel, where some politicians did not want to be included in the coalition in order not to be associated with the disintegration process, regardless of all other issues. The following citation introduces the concept of the "collective responsibility" and explains the spirit of my model:

"Worst of all is the way the politicians – government ministers, mainly – think they can have their cake and eat it, too. They want to keep their seats in a government that voted for the pullout, but without voicing solidarity for its decisions. They serve in a government that elected to leave Gaza, but they don't understand that even if they voted "no," they are partners in the decision. Because there is a principle known as collective responsibility. Sharon's success will be their success. Yet no one has come out in defense of Sharon - or in

[10] The assumption that the politicians have costs while obtaining private payments resembles a vote related costs assumption in Dal Bo (2006). In his paper, politicians care about how they vote per se, beside caring about private payments and the final decision. Under the open voting rule, where payments are conditioned on a vote, vote related costs are equivalent to private payments related costs in the current paper.

[11] Both papers belong to the recent literature that compares open and closed voting rules. Dal Bo (2004) and Felgenhauer and Grüner (2003) found that under an open rule an outside party can manipulate the committee decision at very small costs or even at no cost at all. As a result both papers advocate for a closed voting procedure.

Dal Bo presents a complete information model where all committee members are to some extent against the approval of the project desired by an interest group.

Felgenhauer and Grüner discuss the case where committee members have private information about what might be the best policy. Moreover, they are influenced by an external interest group, which favors a certain policy.

defense of the decision reached by his government, to be more precise." ("Silence of the lambs as Israel burns" Yoel Marcus, 01.07.2005 Haaretz).

A part of Sharon's government remained government members, receiving all office "perks", despite voting against the disintegration plan. But others considered the damage of the "collective responsibility" for their future political carrier as too high and left the government. For example, the financial minister and Sharon's main rival in the Likud party Benjamin Netanyahu, whose supporters considered the disintegration plan as the opposite to "success", left the government shortly before the disintegration's plan implementation. In other words, a coalition member cannot simultaneously enjoy private payments associated with the coalition membership and the political capital associated with "no" vote.

The main result of this paper is that if the supermajority coalition is formed, a credible exchange of information can take place between the lobby and the politicians. There is a fully revealing equilibrium and the lobby can efficiently buy the cheapest players. On the contrary, if the lobby forms the minimal winning coalition, no fully revealing equilibrium is possible.

An additional interesting result is that with an increase in the number of politicians, a fully revealing equilibrium is more likely to exist and , hence, coalitions are more likely to be oversized. Therefore, the possibility of the undesired project's implementation increases. Thus, the society or representative political institutions may prefer to reduce the number of politicians in the decision body. This result contradicts the famous Condorcet's jury theorem which states that the delegation of a decision to a bigger committee yields better results.

This paper is closely related to Tsai and Yang (2006, 2007). In these two papers Tsai and Yang introduce asymmetric information into the coalition formation models proposed by Baron and Ferejohn (1989) and Persson, Roland and Tabellini (2000) respectively. The main result is that both supermajority coalitions and minimal winning coalitions may arise in the equilibrium. This is in contrast to the certain world in which only minimal winning coalitions appear. But in Tsai and Yang (2006, 2007) there is no possibility of information exchange between the players, so the incentives to form the supermajority coalition are quite different from the current paper.

The paper is structured in the following way: the next section describes the model. Section 5.3 considers the complete information case. Section 5.4 discusses coalition formation under asymmetric information: section 5.4.2 presents minimal winning coalition and section 5.4.3 investigates the supermajority coalition. Section 5.5 discusses the optimal coalition size. Section 5.6 considers continuous types. Section 5.7 concludes.

5.2 The model

There is a group of n politicians who decide about the implementation of some project. Let n be an odd number. The decision rule is a simple majority voting rule, so the project is implemented if at least $\frac{n+1}{2}$ politicians support it. If the project is not implemented, a default policy takes place.

Assume that there is some player, called the lobby, who is interested in the implementation of the project and has the ability to influence the decision by proposing private payments to the politicians. I denote the vector of payments by t, $t_i \geq 0$, where t_i is the payment to politician i.

5.2.1 Preferences

The politicians derive utility from either the implemented project or the default policy as well as from private payments. I assume that all politicians obtain a utility normalized to zero, if the project is implemented. Otherwise it is equal to some constant $p > 0$. Therefore, without additional incentives, all politicians vote against the project.

I assume that politicians bear some costs if they obtain private payments. The politicians differ with respect to these costs. The cost for politician i is denoted by d_i and is distributed independently on the interval $[d_L, d_H]$ according to some distribution function D. Moreover, I assume that d_i is private information and is observed only by politician i. From now on, I refer to d_i as to the type of politician i. Note, that costs are realized only if $t_i > 0$. If $t_i = 0$ then politician i's cost is zero.

I consider the closed voting rule, so the payments are conditioned on the implementation of the project and not on the individual votes. As a result, the utility obtained by politician i is:

$$U_i\left(d_i, t_i\right) = \begin{cases} t_i - d_i & \text{if the project is implemented and } t_i > 0 \\ 0 & \text{if the project is implemented and } t_i = 0 \\ p & \text{if the project is not implemented} \end{cases} . \tag{1}$$

Following Dal Bo (2006), I do not explicitly model the lobby's utility, but I assume that it decreases in the sum of payments $\sum\limits_i t_i$ and increases in the probability of the project's approval. I further assume that the value of the project for the lobby is relatively high and that she faces no budget constraint, so she can credibly commit to large private payments.

5.2.2 Timing

The timing of the game is as follows:

1. The lobby announces her payments strategy.
2. The politicians announce their types simultaneously.
3. The lobby observes the announcements and proposes payments which are consistent with her announcement at the first stage.
4. The voting takes place, the decision is implemented and payments are paid in a pre-specified way.

The lobby's strategy includes the structure of the payments as a function of the announced types of politicians, in particular the number of the coalition members. As I have already mentioned, I define the players who obtain a positive private payment as the coalition members. As it is traditional in the lobbying literature and along the lines of Grossman and Helpman (2001), I assume that the lobby is able to commit to the proposed payments. On the contrary, the politicians' announcements do not need to be true and the politicians cannot commit to any specific voting behavior. The lobby's proposal is a take-it-or-leave-it offer, so there is no bargaining between the lobby and the politicians.

5.2.3 Strategies and the equilibrium concept

Feasible action a_l of the lobby is payments' vector $t\left(\widetilde{d}\right) \in \Re^n_+$, where $\widetilde{d}_i$ is a type announcement of player i. I restrict the announcements to the space of feasible types. Therefore, the feasible action of politician i at the second stage of the game is the type announcement $\widetilde{d}_i\left(d_i, t\right) \in \left[d_L, d_H\right]$. The feasible action of politician i at the voting stage is $a_i\left(d_i, \widetilde{d}_i, t\right) \in \{accept, reject\}$.

Strategy s_i for player i is a sequence of actions and the strategy profile s is an $(n+1)$-tuple of strategies, one for each player.

The game is solved by backward induction. The solution concept is subgame perfect Bayesian Nash equilibrium. Since no player can change the outcome if a proposal is supported by more than a simple majority, I restrict attention to the equilibria in which weakly dominated strategies are eliminated.

There exist a number of potential strategies for the lobby. The lobby can always propose the payment equal to $p + d_H$ to a simple majority. Such a proposal will guarantee the project implementation, but it can be quite expensive for the lobby. The lobby can also propose lower payments to some players and face a certain risk of the project rejection. The optimal strategy will depend on the exact form of the lobby's utility function and on the types' distribution. In this paper I assume that the lobby is extremely risk averse and strongly prefers the guaranteed implementation of the project. For that reason, I concentrate mainly on the question of whether the lobby can induce politicians to reveal their true types. Therefore, I restrict the lobby behavior to the following strategy: she takes the announced types as true types and optimizes over the number of the coalition members and the coalition payments. The minority coalition case is trivial, so I consider the minimal winning coalition (MWC) and the supermajority coalition.

I define the truth-telling equilibrium in the following way:

Definition 1 *The strategy profile is called a truth-telling equilibrium if it fulfills the following conditions:*

1. The strategy profile is a subgame-perfect Bayesian Nash equilibrium.

2.Politicians believe that all other politicians make truthful announcements.

3. Politicians make truthful announcements: all politicians announce their true types, $\widetilde{d}_i = d_i$ *for* $\forall i$

4. The lobby takes the announcements as truthful and proposes private payments in accordance with that.

5.3 Complete information

As a benchmark, I look at the case where the types of politicians are common knowledge. Under complete information, the standard result is that the lobby chooses $\frac{n+1}{2}$ politicians with the lowest d_i and forms the MWC:

Definition 2 *The agenda setter forms the MWC if any coalition member is an element of MWC, where $MWC \subset N$ is nonempty set which satisfies*

i. $|MWC| = \frac{n+1}{2}$

ii. $\forall\ j \in MWC \quad t_j(d_j) = \min\left\{\widehat{t}_j(d_j) \in \Re \middle| a^j(\widehat{t}_j(d_j)) = accept\right\}$
$\forall\ j \notin MWC \quad t_j = 0$

iii. $\forall l \in MWC \text{ and } \forall k \notin MWC: \quad \sum\limits_{j \in C \setminus \{l\}} t_j(d_j) + t_k(d_k) \geq \sum\limits_{j \in C} t_j(d_j)$

where $t_k = \min\left\{\widehat{t}_k(d_k) \in \Re \middle| a^k\left(\widehat{t}_k(d_k)\right) = accept\right\}$

According to this definition, each politician is pivotal and the payment to each coalition member has to equalize the utility from the project implementation and the coalition membership with the default option[12]:

$$\begin{aligned} t_i - d_i &= p \\ &\Rightarrow \quad t_i = p + d_i > 0 \end{aligned} \tag{2}$$

Note that the payment increases in the politicians' type.

The utility of politician i is:

$$U_i\left(d_i, t_i\right) = \begin{cases} p & \text{if the project is implemented and } t_i > 0 \\ 0 & \text{if the project is implemented and } t_i = 0 \\ p & \text{if the project is not implemented} \end{cases} \tag{3}$$

5.4 Asymmetric information

5.4.1 Coalition formation and voting

Throughout this section and the rest of the paper I assume that d_i is private information and only its distribution is common knowledge.

According to Definition 1, the lobby's strategy is as following: at the first stage of the game she commits to the number of coalition members, denoted by c. Afterwards she observes the vector of announcements $\widetilde{d}$, takes it is as truthful announcement, and proposes membership in the coalition to c players with the minimal announcements, in order to minimize the aggregate transfer:

[12] As it is customary in the literature, I assume that if a politician is indifferent, he will vote for the project.

Definition 3 *The agenda setter forms the following coalition: each coalition member is an element of C, where $C \subset N$ is nonempty set which satisfies*

i. $|C| = c$

ii. $\forall\ j \in C\ t_j(\widetilde{d}_i) = \min\left\{ \widehat{t}_j(\widetilde{d}_j) \in \Re \middle| a^j(\widehat{t}_j(\widetilde{d}_j)) = accept\ if\ \widetilde{d}_j \geq d_j \right\}$

$\forall\ j \notin C\ \ t_j(\widetilde{d}_j) = 0$

iii. $\forall l \in C\ and\ \forall k \notin C: \quad \sum_{j \in C \setminus \{l\}} t_j(\widetilde{d}_j) + t_k(\widetilde{d}_k) \geq \sum_{j \in C} t_j(\widetilde{d}_j)$

$where\ \ t_k = \min\left\{ \widehat{t}_k(\widetilde{d}_k) \in \Re \middle| a^k\left(\widehat{t}_k(\widetilde{d}_k)\right) = accept\ if\ \widetilde{d}_j \geq d_j \right\}$

According to Definition 3, the proposed coalition payment to politician i, t_i, makes him indifferent between accepting or rejecting the proposal if his announcement type is his true type. Then the payments are similar to the complete information case: the lobby minimizes coalition costs and the payments are

$$t_i = p + \widetilde{d}_i. \tag{4}$$

In this case the payment does not depend on the politician's real type but increases in the announced type $\widetilde{d}_i$. Then the utility of politician i with type d_i who announces type $\widetilde{d}_i$ is

$$U_i\left(d_i, \widetilde{d}_i\right) = \begin{cases} p + \widetilde{d}_i - d_i & \text{if the project is implemented and } t_i > 0 \\ 0 & \text{if the project is implemented and } t_i = 0 \\ p & \text{if the project is not implemented} \end{cases} \tag{5}$$

As I have mentioned earlier, the lobby announces her strategy to all politicians already in the beginning of the game. Since the lobby is restricted to only one feasible strategy, throughout the rest of the paper I concentrate only on the politicians' strategies. First I introduce some lemmas which describe the voting behavior of politicians at the last stage of the game and which will be helpful throughout the paper:

Lemma 1 *All opposition members vote against the project.*

The proof follows directly from the assumption that $0 < p$.

From Lemma 1 the following results follow directly:

Corollary 1 *If the coalition is a minority coalition, the project is never implemented.*

Thus, the lobby never forms a minority coalition.

Lemma 2 *If the announcement of coalition member i is truthful, then he votes for the project.*

Proof. If the announcement is truthful, then the coalition member is indifferent between the implementation of the project and the default. Hence, by assumption, he votes for the project (see footnote3). ∎

It follows from (5) that for $\widetilde{d}_i \leq d_i \Rightarrow U_i\left(\widetilde{d}_i\right) \leq U_i\left(d_i\right)$ and $\widetilde{d}_i > d_i \Rightarrow U_i\left(\widetilde{d}_i\right) > U_i\left(d_i\right)$. From this result the next lemma follows directly:

Lemma 3 *(i) If coalition member i's announced type is higher than the true one,* $\widetilde{d}_i > d_i$*, then he votes for the project.*

(ii) If coalition member i's announced type is lower than the true one, $\widetilde{d}_i < d_i$ *then he votes against the project.*

Note that these three lemmas fully characterize the politician's voting behavior at the last stage of the game.

5.4.2 Minimal winning coalition

In this section I investigate what happens when the lobby forms the MWC. According to Definition 3, the lobby observes the vector of announcements $\widetilde{d}$ and proposes membership in the coalition to $\frac{n+1}{2}$ players with the minimal announcements.

The first result is that if the lobby forms the MWC, she cannot induce the truth-telling equilibrium:

Proposition 1 *Under the MWC, there are no truthful announcements in the equilibrium.*

Proof. Suppose, contrary to proposition, that there exists an equilibrium with truthful announcements. Then all politicians announce their true types. Consider now the politician with true type d_H. He either announces his true type or he can deviate and make an announcement $\widetilde{d}_i < d_H$. If he is a coalition member, his minimal utility is p, regardless of his announcement. This is because all coalition members are pivotal, hence, if any coalition member obtains less than p as a result of the project implementation, he can always induce the project rejection and obtain p as default payment. If he is an opposition member, his payment is 0 because the project is always implemented. It follows that he will always announce to be d_L type in order to increase his chances to be in the coalition and his announcement will never be truthful.

As a result, there is no truthful announcement in the equilibrium. ■

Since the MWC is a key prediction of various models for coalition formation, it may be interesting to investigate what announcements are made if the lobby forms the MWC:

Proposition 2 *If the lobby forms the MWC and proposes payments according to (4), then the announcements strategies are equilibrium strategies only if:*

(i) all players announce $\widetilde{d}_L$*. If politicians' types are continuous, it is a unique announcement.*

(ii) If politicians' types are discrete, then there exists type d^* *such that all politicians with true type* $d_i \leq d^*$ *announce* $\widetilde{d}_i > d_i$ *and all politicians with true type* $d_i \geq d^*$ *announce* $\widetilde{d}_L$*.*

This proposition implies an even stronger result than just lack of a truthful announcement: the lobby cannot sort politicians according to their types and induce an informational announcement where for each two politicians i and j $\widetilde{d}_i > \widetilde{d}_j \Leftrightarrow d_i > d_j$ and $\widetilde{d}_i = \widetilde{d}_j \Leftrightarrow d_i = d_j$. As a result, if the lobby believes the announcements and forms the MWC, the probability that the project is implemented is equal to zero for a continuous type. For discrete types such probability would be positive but can be rather small.

5.4.3 Supermajority coalition

In this section I show that the truth-telling equilibrium exists if the lobby forms the supermajority coalition. In this model, the supermajority comprises $\frac{n+3}{2}$ or more politicians. I consider the case of $\frac{n+3}{2}$ coalition members, the "minimal" supermajority coalition and I discuss larger coalitions in the next section. Therefore, according to Definition 3, the lobby observes the vector of announcements $\widetilde{d}$ and proposes membership in the coalition to $\frac{n+3}{2}$ players with the minimal announcements. I assume, for simplicity, that there are only two possible types of politicians $\{d_L, d_H\}$ and the probability of type d_H is h. The private payments are then as follows

$$t_H = p + d_H, \tag{6}$$

$$t_L = p + d_L. \tag{7}$$

The important feature of a supermajority coalition and the main difference from the MWC is that no coalition member is pivotal. If all but one coalition member j, announce their true type then the project is implemented, regardless of the announcement, the private payment and the voting behavior of j. Hence, if a single player deviates from the truthful announcement strategy, his utility depends only on his announced and true type and his coalition status, but not on his voting behavior. Therefore, the minimal utility of the coalition member depends on the announced type and can be smaller than p, while it is equal to p under the MWC. This would changes the incentives during the announcement stage. The question arises if there is a possibility for truth-telling equilibrium and under what conditions.

The following proposition presents the main result of the paper:

Proposition 3 *If the lobby forms a supermajority coalition, the truthful announcement is an equilibrium strategy if*

$$\left(1 - \frac{1}{a(n,h)}\right) p \leq d_H - d_L \leq (a(n,h) - 1)\, p \tag{8}$$

where

$$a(n,h) \equiv \frac{1 - \sum_{k=\frac{n+3}{2}}^{n-1} \frac{(n-1)!}{(n-1-k)!k!} h^{n-k-1} (1-h)^k \left(1 - \frac{n+3}{2(k+1)}\right)}{\sum_{k=0}^{\frac{n+1}{2}} \frac{(n-1)!}{(n-1-k)!k!} h^{n-k-1} (1-h)^k \frac{n+3-2k}{2(n-k)}}.$$

Proof. At the first stage of the game the lobby commits to a coalition size $c = \frac{n+3}{2}$ and to private payments (6) and (7). At the last stage of the game all politicians vote according to Lemmas 1-3.

Consider now the second stage of the game. Suppose that all politicians announce their true types. If player i is of type d_L and he announces his true type, then his expected utility is:

$$EU_i\left(d_L,\widetilde{d}_L\right)=p+\sum_{k=\frac{n+3}{2}}^{n-1}\frac{(n-1)!}{(n-1-k)!k!}h^{n-1-k}\left(1-h\right)^k\left(\frac{n+3}{2\left(k+1\right)}-1\right)p, \tag{9}$$

and if he deviates and announces type d_H, then his expected utility is:

$$EU_i\left(d_L,\widetilde{d}_H\right)=\sum_{k=0}^{\frac{n+1}{2}}\frac{(n-1)!}{(n-1-k)!k!}h^{n-k-1}\left(1-h\right)^k\frac{n+3-2k}{2\left(n-k\right)}\left(p+d_H-d_L\right). \tag{10}$$

If player i is of type d_H and he announces his true type, his expected utility is

$$EU_i\left(d_H,\widetilde{d}_H\right)=\sum_{k=0}^{\frac{n+1}{2}}\frac{(n-1)!}{(n-1-k)!k!}h^{n-k-1}\left(1-h\right)^k\frac{n+3-2k}{2\left(n-k\right)}p. \tag{11}$$

If he deviates

$$EU_i\left(d_H,\widetilde{d}_L\right)=\left(p+d_L-d_H\right)+ \tag{12}$$

$$\sum_{k=\frac{n+3}{2}}^{n-1}\frac{(n-1)!}{(n-1-k)!k!}h^{n-k-1}\left(1-h\right)^k\left(\frac{n+3}{2\left(k+1\right)}-1\right)\left(p+d_L-d_H\right).$$

The truth-telling equilibrium exists if there are no incentives to deviate for both types, namely

$$U_i\left(d_L,\widetilde{d}_L\right)\geq U_i\left(d_L,\widetilde{d}_H\right), \tag{13}$$

$$U_i\left(d_H,\widetilde{d}_H\right)\geq U_i\left(d_H,\widetilde{d}_L\right), \tag{14}$$

or

$$\left[\frac{1-\sum_{k=\frac{n+3}{2}}^{n-1}\frac{(n-1)!}{(n-1-k)!k!}h^{n-k-1}\left(1-h\right)^k\left(1-\frac{n+3}{2(k+1)}\right)}{\sum_{k=0}^{\frac{n+1}{2}}\frac{(n-1)!}{(n-1-k)!k!}h^{n-k-1}\left(1-h\right)^k\frac{n+3-2k}{2(n-k)}}-1\right]p+d_L\geq d_H, \tag{15}$$

$$\left[1-\frac{\sum_{k=0}^{\frac{n+1}{2}}\frac{(n-1)!}{(n-1-k)!k!}h^{n-k-1}\left(1-h\right)^k\frac{n+3-2k}{2(n-k)}}{1-\sum_{k=\frac{n+3}{2}}^{n-1}\frac{(n-1)!}{(n-1-k)!k!}h^{n-k-1}\left(1-h\right)^k\left(1-\frac{n+3}{2(k+1)}\right)}\right]p+d_L\leq d_H. \tag{16}$$

One can rewrite these conditions as

$$\left(1-\frac{1}{a\left(n,h\right)}\right)p\leq d_H-d_L\leq\left(a\left(n,h\right)-1\right)p \tag{17}$$

where

$$a(n,h) \equiv \frac{1-\sum_{k=\frac{n+3}{2}}^{n-1} \dfrac{(n-1)!}{(n-1-k)!k!} h^{n-k-1}(1-h)^k \left(1-\frac{n+3}{2(k+1)}\right)}{\sum_{k=0}^{\frac{n+1}{2}} \dfrac{(n-1)!}{(n-1-k)!k!} h^{n-k-1}(1-h)^k \frac{n+3-2k}{2(n-k)}}.$$

Moreover, for a relevant parameters range ($n > 3$, $0 < h < 1, p > 0$), it can be shown that

$$(a(n,h)-1)p > \left(1-\frac{1}{a(n,h)}\right)p. \tag{18}$$

Therefore, for a relevant parameters range, there are always d_H and d_L such that conditions (15) and (16) are satisfied and the truth-telling equilibrium exist[13]. ■

This proposition shows that the truth-telling equilibrium exists when the difference between d_H and d_L is considerable, but not extreme. The range of the parameters for which the truth-telling equilibrium exists is determined by conditions (15) and (16). The incentive to deviate for type d_L is to increase the private payment in the case of coalition membership, at the expense of the probability of coalition membership. If the difference between d_H and d_L is too large, type d_L deviates. Type d_H may deviate in order to increase the coalition membership probability at the expense of the private payment in case of coalition membership, he will deviate if the difference d_H and d_L is too small.

For example, the truth-telling equilibrium for $n = 7$, $h = 0.5$ and $p = 1$ exists if the following conditions are satisfied:

$$.542\,14 < d_H - d_L < 1.184\,1.$$

The following results show how the change in the parameters p, n and h influences the likeliness of the existence of the truth-telling equilibrium.

Corollary 2 *The range of parameters d_H and d_L, for which the truth-telling equilibrium exists, increases in p.*

Proof. From (15) and (16) it follows that the range of parameters d_H and d_L, for which the truth-telling equilibrium exists, increases in

$$\left((a(n,h)-1)-\left(1-\frac{1}{a(n,h)}\right)\right)p.$$

$(a(n,h)-1)-\left(1-\frac{1}{a(n,h)}\right) > 0$, therefore it increases in p. ■

This corollary states that the higher the utility from default policy is, the higher is the chance that the lobby can implement the truth-telling equilibrium.

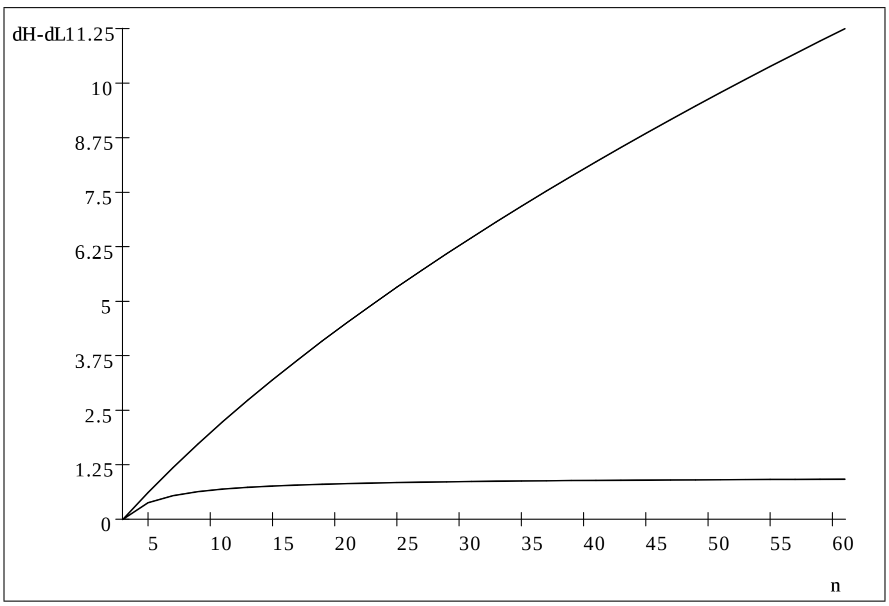
[13] It is easy to extend this model for more types. Like in a two types case, the truth-telling equilibrium exists if the distance between types is not too large and not too small. Unfortunately, the number of the conditions cannot be reduced. The truth-telling equilibrium exists if for k types $k(k-1)$ conditions are satisfied.

The conditions under which the truth-telling equilibrium exists are too complicated to investigate analytically, but numerical simulations provide the following results:

Corollary 3 *The range of parameters d_H and d_L, for which the truth-telling equilibrium exists increases with n .*

This result is represented in Figure 1. Assume that $p = 1$ and $h = 0.5$. The upper line corresponds to the LHS of condition (15) and the lower line to the LHS of condition (16). Therefore, the truth-telling equilibrium exists if the value of $d_H - d_L$ is between these two lines. With an increase in n the bounds on $d_H - d_L$, implied by conditions (15) and (16), relax considerably upwards and tighten slightly downwards. The first effect is dominant, so the total effect is an increase of the parameter range for which the equilibrium exists.

Figure 1

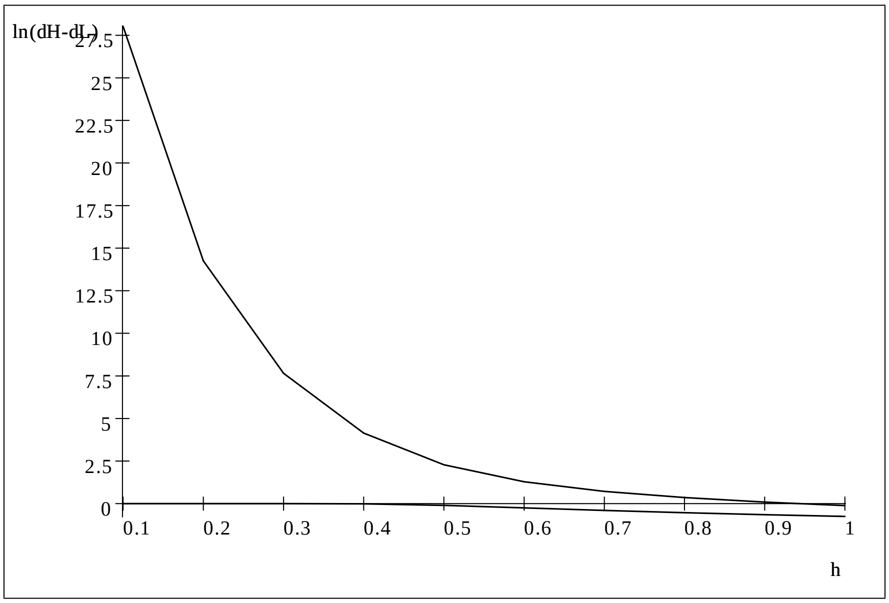


As a result, the society or representative political institutions may prefer to reduce the number of politicians in the decision body in order to prevent the existence of the truth-telling equilibrium and to reduce the probability of the project implementation. This result seems counter-intuitive, because it is usually more expensive to buy the majority in a large decision body. But buying the winning coalition under truth-telling in a large decision body can still be cheaper for the lobby than dealing with asymmetric information in a small decision body. Furthermore, this result contradicts the famous Condorcet's jury theorem which states that the delegation of a decision to a bigger committee yields better results.

Corollary 4 *If the probability for the high type h increases, the condition under which type d_L announces the truth becomes more restrictive and the condition under which type d_H announces the truth becomes less restrictive.*

This result is represented in Figure 2[14]. Assume that $n = 51$. The upper line corresponds to the LHS of condition (15) and the lower line to the LHS of condition (16). Therefore, the truth-telling equilibrium exists if the value of $d_H - d_L$ is between these two lines. With an increase in h the bounds on $d_H - d_L$, implied by conditions (15) and (16), tighten.

Figure 2



5.5 Optimal supermajority size

In the previous section I analyzed the minimal supermajority: simple majority plus one politician. In this section I investigate how many coalition members the lobby should include into the coalition. Including an additional member into the coalition is costly for the lobby, and *ceteris paribus* she will prefer the "minimal" supermajority, with $\frac{n+3}{2}$ members. But one also has to consider the parameter restrictions under which the truth-telling equilibrium exists. If they change, it can be the case that the supermajority with more than $\frac{n+3}{2}$ members is preferable.

Assume that lobby commits to include $n > c \geq \frac{n+5}{2}$ politicians into the coalition. Suppose that all politicians announce their true type.

Then the politician with type d_L announces his true type if the following condition is satisfied:

[14] For the sake of convenience of the grafical representation I used $\ln(d_H - d_L)$.

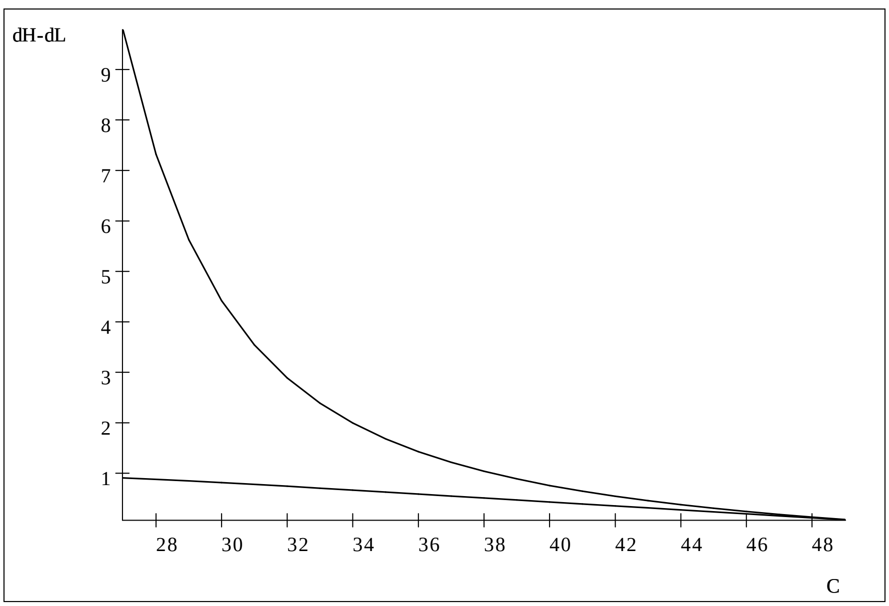
$$\left(\frac{1+\sum_{k=c}^{n-1} \dfrac{(n-1)!}{(n-1-k)!k!} h^{n-1-k} (1-h)^k \left(\frac{c}{(k+1)} - 1 \right)}{\sum_{k=0}^{c-1} \dfrac{(n-1)!}{(n-1-k)!k!} h^{n-k-1} (1-h)^k \frac{c-k}{(n-k)}} - 1 \right) p \geq d_H - d_L \tag{19}$$

The politician with type d_H announces his true type if the following condition is satisfied:

$$\left(1 - \frac{\sum_{k=0}^{c-1} \dfrac{(n-1)!}{(n-1-k)!k!} h^{n-k-1} (1-h)^k \frac{c-k}{(n-k)}}{1+\sum_{k=c}^{n-1} \dfrac{(n-1)!}{(n-1-k)!k!} h^{n-k-1} (1-h)^k \left(\frac{c}{(k+1)} - 1 \right)} \right) p \leq d_H - d_L \tag{20}$$

Assume that $n = 51$. Conditions (19) and (20) are represented in Figure 3. The upper line corresponds to the LHS of condition (19) and the lower line to the LHS of condition (20). Therefore, the truth-telling equilibrium exists if the value of $d_H - d_L$ is between these two lines.

Figure 3



This example shows that big supermajorities can emerge in the equilibrium. This will occur if the parameters are such that the truth-telling equilibrium does not exist for $\frac{n+3}{2}$-members coalition. In this particular example it happens if $d_H - d_L < 1$. Inviting additional players into the coalition is costly for the lobby, but it can still be profitable, compared to the situation when the truth-telling equilibrium cannot be achieved.

Finally, note that:

Remark 1 *If all players are coalition members, they will announce the highest type. Therefore, there will be no truth-telling equilibrium in this case.*

5.6 Continuous distribution

In this section I consider continuous types. I assume that d_i is distributed independently on the interval $[d_L, d_H]$. I show that politicians do not announce their true types, even if a supermajority is formed.

The intuition for the non-existence of truthful announcements is that high type players expect that they will not be in the coalition with a high probability. Moreover, the proposal will always be accepted if all players announce their true types. Hence, the expected utility of the players with high types is close to zero. This means that it will be profitable for some players to deviate from the truthful announcement and to announce a lower type. Opposite to a discrete case when the announcement of the lower type implies a considerable change in the coalition payment (from p to $p + d_L - d_H$) if the types are continuous there is always announcement $\widetilde{d} < d_H$ such that deviation is profitable. Two main differences between discrete and continuous types explain these results. If the types are discrete, first, the probability of the coalition participation does not go to zero even if the true type is maximal. Second, the politician cannot announce the arbitrary type $d_i - \varepsilon$, he is limited to feasible types. More formally:

Proposition 4 *There is no truth-telling equilibrium under the supermajority coalition, if the types are continuous.*

Proof. Assume, contrary to the proposition, that all players announce their true types and the lobby forms supermajority. At the last stage of the game all politicians vote according to Lemmas 1-3.

Consider player i: if he announces his true type his expected utility is:

$$EU_i\left(d_i, d_i\right) = P_i(d_i, d_{-i}) * p + (1 - P_i(d_i, d_{-i})) * 0, \tag{21}$$

where $P_i(d_i, d_{-i})$ is the probability that player i is a coalition member, which is a weakly negative function of i's announcement.

If a player deviates and announces $d_i - \varepsilon$, then his expected utility is:

$$EU_i\left(d_i, \widetilde{d}_i\right) = P_i(d_i - \varepsilon, d_{-i}) * (p + (d_i - \varepsilon) - d_i) + (1 - P_i(d_i - \varepsilon, d_{-i})) * 0 \tag{22}$$

$$= P_i(d_i - \varepsilon, d_{-i}) * (p - \varepsilon)\,.$$

If a player with type d_H announces his true type then the probability that he is in the coalition, $P_i(d_H, d_{-i})$, is equal to zero. Hence, his expected utility is zero as well. Therefore, there always exists such ε that

$$P_i(d_H - \varepsilon, d_{-i}) * (p - \varepsilon) > 0. \tag{23}$$

As a result, the player with true type d_H never announces his true type and truth-telling cannot be equilibrium strategy. ■

This result follows logically from the equilibrium existence conditions (15) and (16): if the types are continuous, they can never be satisfied and the lobby cannot induce the truth-telling equilibrium.

In order to deal with nonexistence of the truth-telling equilibrium, the lobby can discretize the announcements. namely, she can adopt similar to the Crawford and Sobel (1982) partition strategy. Then the lobby divides announcements space into intervals and asks players to announce in what intervals they are. The existence of truthful announcement in Crawford Sobel type equilibrium for this model can easily be shown.

5.7 Conclusions

This paper presented a model of influence over the group decision under asymmetric information. I considered the secret voting, so that the private payments can be conditioned only on the project implementation. The politician bears costs if he is associated with the project via the private payments, regardless of his actual vote.

In this paper I present an explanation to the fact that the supermajority coalitions are often observed in reality even if the standard theory prediction is the MWC. Under asymmetric information, by forming supermajority coalition the lobby can induce the politicians to reveal their true types, if the type space is discrete. This leads to a sure implementation of a generally undesired project at a cost close to that of the complete information MWC.

The truth-telling equilibrium exists already for the minimal supermajority: simple majority plus one politician. The bigger coalition can be formed by the lobby in spite of the increase in the coalition cost: introducing additional members into the coalition leads to the existence of the truth-telling equilibrium for a broader range of parameters.

Adding new politicians to the decision body (an increase in n) relaxes the conditions for the existence of a truth-telling equilibrium. Therefore, the society or representative political institutions may prefer to reduce the number of politicians in the decision body. This result seems counter-intuitive, because it is usually more expensive to buy the majority in a large decision body. But buying the winning coalition under truth-telling in a large decision body can still be cheaper for the lobby than facing false announcements in a small decision body. Furthermore, this result contradicts the famous Condorcet's jury theorem which states that the delegation of a decision to a bigger committee yields better results. This result is tested and found to have strong support in Carrubba and Volden (2004). Carrubba and Volden (2000) predict that coalitions are more likely to be oversized when the number and diversity of actors in a legislative chamber is greater. In order to test this predictions they use the number of actors in a legislative chamber is measured by the number of seats and as a function of the number of parties. They state that coalitions will be more oversized in legislative chambers comprised of more decisive actors.

A possible extension of the present paper may examine the multidimensional policy space. It is interesting to investigate if the supermajority coalitions will be formed in this case as well.

5.8 Appendix

5.8.1 Proof of Proposition 2

Note that at the last stage of the game all politicians vote according to Lemmas 1-3. Consider politician i, $d_L \leq d_i \leq d_H$. If politician i is an opposition member, his utility is 0 in the case of project implementation and p in the case of project rejection, regardless of his announcement and the announcement of any other player. According to Lemma 1, he always votes against the project.

If politician i is a coalition member, his minimal utility is p. This is because all coalition members are pivotal, hence, if politician i obtains less than p as a result of the project implementation, he can always induce the project rejection and obtains p as a default payment. Then it follows from (utility under asymmetric information), that all announcements $\widetilde{d_i} \leq d_i$ secure the same utility for coalition member i. Moreover, announcement $\widetilde{d_i} = \widetilde{d_L}$ weakly maximizes the chances to be in the coalition. Thus, in any equilibrium each politician i announces either $\widetilde{d_i} = \widetilde{d_L}$ or $\widetilde{d_i} > d_i$. Hence, a player with true type d_H always announces $\widetilde{d_L}$.

Player i, given the expected types and the actions of the other players, faces the following problem: if he announces $\widetilde{d_i} > d_i$ then his expected utility is

$$EU_i\left(\widetilde{d_i}, d_i, \widetilde{d}_{-i}, d_{-i}\right) = \left(1 - P_c\left(\widetilde{d_i}, \widetilde{d}_{-i}\right)\right)\left(p_o\left(\widetilde{d}_{-i}, d_{-i}\right) * 0 + \left(1 - p_o\left(\widetilde{d}_{-i}, d_{-i}\right)\right)p\right) +$$

$$P_c\left(\widetilde{d_i}, \widetilde{d}_{-i}\right)\left(p_c\left(\widetilde{d}_{-i}, d_{-i}\right)\left(p + \widetilde{d_i} - d_i\right) + \left(1 - p_c\left(\widetilde{d}_{-i}, d_{-i}\right)\right)p\right) \qquad (24)$$

$$s.t.\ \widetilde{d_i} > d_i$$

where $\widetilde{d}_{-i}$ is the announcements of all other players, $P_c\left(\widetilde{d_i}, \widetilde{d}_{-i}\right)$ is the probability that player i is a coalition member, $p_c\left(\widetilde{d}_{-i}, d_{-i}\right)$ is the probability of a project's implementation, given that player i is a coalition member and $p_o\left(\widetilde{d}_{-i}, d_{-i}\right)$ is the probability of a project's implementation given that player i is an opposition member. Note that if player i is a coalition member, he always supports the project and if he is an opposition member he votes against it. Hence, the probabilities of the project's implementation $p_c\left(\widetilde{d}_{-i}, d_{-i}\right)$ and $p_o\left(\widetilde{d}_{-i}, d_{-i}\right)$ depend only on actions of other players.

If player i announces $\widetilde{d_L}$, then his expected utility is as follows:

$$EU_i\left(\widetilde{d}_L,\widetilde{d}_{-i},d_{-i}\right)=P_c\left(\widetilde{d}_L,\widetilde{d}_{-i}\right)p+ \tag{25}$$

$$\left(1-P_c\left(\widetilde{d}_L,\widetilde{d}_{-i}\right)\right)\left(p_o\left(\widetilde{d}_{-i},d_{-i}\right)*0+\left(1-p_o\left(\widetilde{d}_{-i},d_{-i}\right)\right)p\right)$$

Note that if player i is a coalition member, then the project is not implemented, because player i votes against it.

Player i maximizes his expected utility over $\widetilde{d}_i$, given d_i and the announcements of the other players. Note, first, that if the player announces $\widetilde{d}_L$ then his expected utility does not depend on his true type. Secondly, if the player is an opposition member, his expected utility does not depend on his announcement. Thus I can denote the opposition member's expected utility as

$$C\left(\widetilde{d}_{-i},d_{-i}\right)\equiv p_o\left(\widetilde{d}_{-i},d_{-i}\right)*0+\left(1-p_o\left(\widetilde{d}_{-i},d_{-i}\right)\right)p \tag{26}$$

Player i announces $\widetilde{d}_i>d_i$ if and only if

$$EU_i\left(\widetilde{d}_i,d_i,\widetilde{d}_{-i},d_{-i}\right)>EU_i\left(\widetilde{d}_L,\widetilde{d}_{-i},d_{-i}\right)\Leftrightarrow \tag{27}$$

$$P_c\left(\widetilde{d}_i,\widetilde{d}_{-i}\right)\left(p_c\left(\widetilde{d}_{-i},d_{-i}\right)\left(p+\widetilde{d}_i-d_i\right)+\left(1-p_c\left(\widetilde{d}_{-i},d_{-i}\right)\right)p\right)-P_c\left(d_L,\widetilde{d}_{-i}\right)p+$$

$$+\left(P_c\left(d_L,\widetilde{d}_{-i}\right)-P_c\left(\widetilde{d}_i,\widetilde{d}_{-i}\right)\right)C\left(\widetilde{d}_{-i},d_{-i}\right)>0 \tag{28}$$

The probability to be a coalition member weakly decreases in the announcement. Hence $P_c\left(d_L,\widetilde{d}_{-i}\right)>P_c\left(\widetilde{d}_i,\widetilde{d}_{-i}\right)$, and the third term in inequality (28) is positive. Therefore, it is enough to prove that the first two terms are positive in order to show that politician i announces $\widetilde{d}_i>d_i$:

$$P_c\left(\widetilde{d}_i,\widetilde{d}_{-i}\right)\left(p_c\left(\widetilde{d}_{-i},d_{-i}\right)\left(p+\widetilde{d}_i-d_i\right)+\left(1-p_c\left(\widetilde{d}_{-i},d_{-i}\right)\right)p\right)-P_c\left(d_L,\widetilde{d}_{-i}\right)p>0 \tag{29}$$

$$\Rightarrow EU_i\left(\widetilde{d}_i,d_i,\widetilde{d}_{-i},d_{-i}\right)>EU_i\left(\widetilde{d}_L,\widetilde{d}_{-i},d_{-i}\right). \tag{30}$$

Consider two types $d_{i1}>d_{i2}$. Suppose that for d_{i1} the optimal announcement is $\widetilde{d}_i>d_{i1}$:

$$EU_i\left(\widetilde{d}_i,d_{i1},\widetilde{d}_{-i},d_{-i}\right)>EU_i\left(\widetilde{d}_L,\widetilde{d}_{-i},d_{-i}\right). \tag{31}$$

Then, $\widetilde{d}_i>d_{i1}>d_{i2}$ and

$$P_c\left(\widetilde{d}_i,\widetilde{d}_{-i}\right)p_c\left(\widetilde{d}_{-i},d_{-i}\right)\left(p+\widetilde{d}_i-d_{i1}\right)<P_c\left(\widetilde{d}_i,\widetilde{d}_{-i}\right)p_c\left(\widetilde{d}_{-i},d_{-i}\right)\left(p+\widetilde{d}_i-d_{i2}\right). \tag{32}$$

Together with (11) it implies

$$EU_i\left(\widetilde{d}_i, d_{i1}, \widetilde{d}_{-i}, d_{-i}\right) < EU_i\left(\widetilde{d}_i, d_{i2}, \widetilde{d}_{-i}, d_{-i}\right) \tag{33}$$

$$\Rightarrow EU_i\left(\widetilde{d}_i, d_{i2}\right) > EU_i\left(\widetilde{d}_L, \widetilde{d}_{-i}, d_{-i}\right)$$

Hence, if for true type d_{i2}, $d_{i2} > d_{i1}$, the player announces $\widetilde{d}_i > d_{i2}$, then for true type d_{i1} the announcement is also higher than the true type: $\widetilde{d}_i > d_{i1}$. Taking into account that a player with true type d_H always announces $\widetilde{d}_L$, I obtain that there are only two possible equilibrium announcement strategies for player i:

(a) player i always announces $\widetilde{d}_L$ regardless of his true type.

(b) there is type d^* such that player i announces $\widetilde{d}_i > d_i$ $\forall d_i < d^*$ and $\widetilde{d}_L$ otherwise.

The next step is to show that there is an equilibrium where all politicians follow the strategy (a) and announce $\widetilde{d}_L$. If the project is rejected each player obtains p. If the project is accepted, all coalition members obtain p and the opposition members zero. If one player deviates and announces $\widetilde{d}_i > d_L$, he stays in opposition. As a result, he obtains a lower payment in the case of project acceptance and is indifferent in the case of project rejection. Hence, all players will announce $\widetilde{d}_L$.

Now I shall argue for discrete types, for some type distributions, there is an equilibrium where all players follow strategy (b). For example, assume that there are only two types, d_H and d_L, and three players. The probability for type d_H is 0.5. Suppose that $0.5p + d_L < d_H$. Then there is an equilibrium where the players of type d_L announce d_H and vice versa.

The last step is to show that if the type/announcement space is continues, then there is no equilibrium where all players follow strategy (b).

Assume that all players follow strategy (b) and announce $\widetilde{d}_i > d_i$ $\forall d_i < d^*$. Consider the incentives of player j with true type very close to d^*. According to strategy (b) he announces $\widetilde{d}_j > d_j$. Such player expects that his announcement is a maximal among all other announcements. Then the expected probability that he is a coalition member is very close to zero. If politician j is an opposition member, his utility is 0 in the case of project implementation and p in the case of project rejection. If politician j is a coalition member, his minimal utility is p. Then, for all $\widetilde{d}_j > d_j$, there always exists ε such that

$$\left(p_o\left(\widetilde{d}_{-i}, d_{-i}\right) * 0 + \left(1 - p_o\left(\widetilde{d}_{-i}, d_{-i}\right)\right) p\right) < \left(1 - P_c\left(\widetilde{d}_i - \varepsilon, \widetilde{d}_{-i}\right)\right)\left(1 - p_o\left(\widetilde{d}_{-i}, d_{-i}\right)\right) p +$$

$$+P_c\left(\widetilde{d}_i - \varepsilon, \widetilde{d}_{-i}\right)\left(p_c\left(\widetilde{d}_{-i}, d_{-i}\right)\left(p + \widetilde{d}_i - \varepsilon - d_i\right) + \left(1 - p_c\left(\widetilde{d}_{-i}, d_{-i}\right)\right) p\right)$$

$$\Rightarrow EU_i\left(\widetilde{d}_j, d_j \lesssim d^*, \widetilde{d}_{-i}, d_{-i}\right) < EU_i\left(\widetilde{d}_j - \varepsilon, d_j, d_i, \widetilde{d}_{-i}, d_{-i}\right).$$

Therefore, player j deviates and announces $\widetilde{d}_j - \varepsilon$ in order to improve his chances to be in the coalition. As a result, strategy (b) cannot be the equilibrium one. Hence, if the type/announcement space is continuous, the unique equilibrium is that all politicians announce $\widetilde{d}_L$.

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Ehrenwörtliche Erklärung

Hiermit erkläre ich ehrenwörtlich, dass ich diese Dissertationsschrift selbständig angefertigt habe und mich anderer als der in ihr angegebenen Hilfsmittel nicht bedient habe. Entlehungen aus anderen Schriften sind ausdrüklich als solche gekennzeichnet und mit Quellenangaben versehen.

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