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CREDIT PORTFOLIO RISK

Modelling, Estimation and Backtesting

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Abbreviations

A	Rating grade A
AA	Rating grade AA
AR	Accuracy Ratio
AR($\cdot$)	auto-regressive
ASSM	Approximative state-space model
AUT	Automobile sector
BBB	Rating grade BBB
BBG	Bloomberg
BCBS	Basel Committee on Banking Supervision
BHHH	Estimator of Berndt, Hall, Hall and Hausman (1974)
BMA	Basic material sector
bps	basis points
CAP	Cumulative Accuracy Profile
CCF	Credit conversion factor
CCY	Cyclic consumer sector
cdf	cumulative distribution function
CDS	Credit Default Swap
CIER	Conditional Information Entropy Ratio
CIR	Cox, Ingersoll and Ross (1985)
CNC	Consumer, non-cyclic sector
COM	Communication sector
CON	Construction sector
c.p.	ceteris paribus
CR	Capital requirement
DtD	Distance-to-Default
EAD	Exposure-at-Default
ECY	Early cyclic sector
EDF	Expected Default Frequency
EKF	Extended Kalman-Filter
EL	Expected loss
EMU	European Monetary Union

ENY	Energy sector
EUR	Euro
FIN	Financial sector
FPT	First-passage-time
IAS	International Accounting Standards
IEKF	Iterative Extended Kalman-Filter
IFRS	International Financial Reporting Standards
i.i.d.	identically independently distributed
IND	Industrial sector
IRB	Internal rating-based
LCY	Late-cyclic sector
LEE	Loan equivalent exposure
LGD	Loss-given-Default
MAD	Mean absolute deviation
MED	Media sector
ML	Maximum likelihood
MLE	Maximum likelihood estimator
NCY	Non-cyclic sector
NF	Non-financial sector
OTC	Over-the-counter
PD	Probability of Default
PIT	Point-in-time
pdf	Probability density function
pgf	Probability generating function
QML	Quasi maximum likelihood
QMLE	Quasi maximum likelihood estimator
RAD	Risk-adjusted discounting
RNV	Risk-neutral valuation
ROC	Receiver Operating Characteristic
RR	Recovery rate
RWA	Risk-weighted asset
SDE	Stochastic Differential Equation
SSM	State-space-model
TEC	Technology sector
TRA	Transportation sector
TTC	Through-the-cycle
UL	Unexpected loss
UTY	Utility sector
VaR	Value-at-Risk

List of Symbols

$\mathbb{1}_{\tau>t}$	Indicator of non-default by time t
$\mathbb{1}_{\tau\leq t}$	Indicator of default by time t
$\mathbf{B}$	Matrix of systematic factor coefficients of a portfolio of exposures
$\mathbf{B}^m$	Coefficient matrix of risk class model m
$\mathbf{B}_k^m$	k^{th} row vector of factor coefficient matrix of model m
B_t	Value of a portfolio of synthetic riskless exposures at time t
B_t^i, B_t^j	Value of synthetically riskless exposures i, j at time t
B_t^T	Value of a riskless zero-coupon bond with unity face value by time t
$\bar{b}$	Vector of normalized specific factor coefficients of exposures in a portfolio
$\mathbb{C}$	Copula
$\mathbb{C}_{i,j}$	Bi-variate copula of default times of exposures i, j
$C_{t,j}$	Coupon of exposure j at time t
$CF^i(t)$	Vector of cash flows of exposure i at time t
$CF_{t_j}^i$	Cash-flow of exposure i at time t_j
$Cov(\cdot)$	Covariance function
$CCVAR_\alpha$	Conditional Credit-VaR with confidence level α
$CVAR_\alpha$	Credit-VaR with confidence level α
c_{t_i}	Rate of coupon number i at time t_i
cys_t^i	Credit yield spread of exposure i at time t
$\overline{cys}_t^{k_j}$	Average credit yield spread of exposures in k_j
$D(\cdot)$	Value of defaultable exposure with constant discrete coupon at time t
$D_C(\cdot)$	Value of defaultable coupon payment at time t
$D_{Merton}(\cdot)$	Value of a defaultable zero bond according to Merton (1974)
$D_Z(\cdot)$	Time-t value of defaultable principal payment
D_t	Mark-to-model valuation of a portfolio of exposures $i = 1, ..., N$ at time t
D_t^i	Observed price of bond i at time t
$D_{t,j}$	Value of exposure j at time t
$D_{t,j}^s$	Simulated value of exposure j at time t
D_i^t	Mark-to-model valuation of exposure i at time t
$D(\mu, \sigma)$	General distribution with expected value μ and covariance matrix σ
$\widehat{D}_t^f$	Vector of filtered factor-implied value of exposures at time t

$\widehat{D}^f_{t,i}$	Vector of filtered factor-implied value of exposure j at time t
$\widehat{D}^f_{t\|\Delta t}$	Vector of predicted factor-implied value of exposures at time t
$\widehat{D}_{t\|t-\Delta t,j}$	Predicted value of credit exposure at time t
$\widehat{D}^f_{t\|\Delta t,i}$	Predicted factor-implied value of exposure j at time t
$\widehat{D}^i_t(\tau_i, CF^i(t); \beta^c_t)$	Value of exposure i by risk-adjusted discounting at time t
$\widehat{D}^V_t$	Vector of filtered value of exposures at time t
$\widehat{D}^V_{t,j}$	Filtered value of exposure j at time t
$\widehat{D}^V_{t\|t-\Delta t}$	Vector of predicted value of exposures at time t
$\widehat{D}^V_{t\|t-\Delta t,j}$	Prediction of value of exposure j at time t
d_1, d_2	Auxiliary variables
dt	Time differential
d_t	Number of defaults in period t
d^{rc}_t	Number of defaults in risk class rc in period t
$\mathbb{E}[\cdot]$	Expectation operator
$F(t)$	Distribution function of the default time
$F_A(\cdot), F_B(\cdot)$	Distribution function of default time of obligor A and B
$F_{LN}(S_{t,j}; \gamma_{t,j}, \delta_{t,j})$	Log-normal distribution function of yield spread $S_{t,j}$ given $\gamma_{t,j}$ and $\delta_{t,j}$
$F_V(\cdot)$	Distribution function of asset value V_t
$F_i(\tau_i)$	Distribution function of the default time of exposure i
F_t	Vector of systematic factors at time t
F_t	Systematic factor at time t
F^j_t	Systematic factor j at time t
$F_\tau(\cdot)$	Distribution function of default time τ
$\widehat{F}(\cdot)$	Estimate of distribution function F
$\widehat{F}^{\epsilon,rc}_T$	Filtered normalized factor of risk class rc
$\widehat{\mathbf{F}}^{\epsilon,rc}_T$	Time Series of filtered normalized factor of risk class rc
$\widehat{F}_t$	Filtered value of factor F_t at time t
$\widehat{F}^\epsilon_t$	Normalized filtered systematic factor at time t
$\widehat{F}_{t\|t-\Delta t}$	Prediction of factor F_t at time t
$\overline{F}_{V,\tau}(\cdot)$	Complementary joint distribution of asset value V_t and time t
$f(\cdot), \overline{f}(\cdot)$	Density of portfolio credit loss under $\mathcal{H}$ and $\overline{\mathcal{H}}$
$f_V(\cdot)$	Density function of asset value V_t
$f_V(F, t; \widehat{F}_{t-\Delta t}, \phi)$	Transition density of factor $\widehat{F}_{t-\Delta t}$ at time $t-\Delta t$
$f_{X,i}(X_{t,i}\|X_{t-\Delta t,i}; \psi)$	Discrete-time transition density of state variable $X_{t,i}$
$f_{\eta,i}(\eta_{t,i}\|X_{t-\Delta t,i}; \psi)$	Discrete-time transition density of state disturbance $\eta_{t,i}$
$f_\tau(\cdot)$	Density function of default time τ
$f_{\tau\|\tau\leq\bar{t}}(\cdot)$	Density of the default time conditional on default by time $\bar{t}$
$\overline{f}_{V,\tau}(\cdot)$	Complementary joint density of asset value V_t and time t
$\overline{f}_{V\|\tau}(\cdot)$	Complementary density of asset value V_t conditional on default by time t

$g(L - l^{ref})$	Risk-defining function of portfolio loss
$\mathcal{H}_0, \overline{\mathcal{H}}_0$	Null hypotheses of backtesting the credit portfolio model and its alternative
$h_j(\cdot)$	Auxiliary vairable
$\mathcal{I}$	Identity matrix
I_c	Set of exposures in risk class c
I_s	Set of exposures in sector s
$I^s_{k_j}$	Set of sectors included in sector-class k_j in clustering iteration$j$
i	Exposure index
i_t	Index of bond in a risk class at time t
j	Observation index, clustering iteration
K	Face value of exposure
K_i	Face value of exposure i
K_t	Kalman matrix at time t
k_1, k_2	Parameters
k_j	Classes of sectors in clustering iteration j
$\mathcal{L}$	Likelihood function
L	General portfolio credit loss
$L(D_0)$	Portfolio credit loss referring to the value of a credit portfolio
$L(B_1)$	Portfolio credit loss referring to the value of the synthetic riskless portfolio
$L(\mathbb{E}[D_1])$	Portfolio credit loss referring to the expected value of the portfolio
$\mathcal{LL}_t(S_t; \psi)$	Log-Likelihood of observation error at time t
l_j	Class of sectors in clustering iteration j
l^{max}	Maximum portfolio credit loss according to the definition of loss
l^{min}	Minimum portfolio credit loss according to the definition of loss
l^{ref}	Reference value of portfolio loss in the definition of portfolio risk
M_t	Minimum of the log-return of the asset value by time t
m	Model index
N	Number of exposures in portfolio
$\mathcal{N}(0, \mathcal{I})$	Multi-variate standard normal distribution
$\mathcal{N}(\mu, \sigma)$	Normal distribution with expected value μ and standard deviation σ
n	Number of obligor observations across periods
n	Number of uniform random variables
n_F	Number of factors
n_S	Number of observations
n_V	Number of state variables
n_X	Number of state variables
n_m	Number of risk classes of model m
$n^i_{s,t}$	Number of exposures in sector s at time t
n^{sc}_j	Number of sector-classes in clustering iteration j

n_t^{rc}	Number of obligors in risk class rc in period t
n_t	Number of obligors in period t
n_t	Number of bonds in a risk class at period or time t
n_{rc}	Number of risk classes
$P[\cdot]$	Probability function under the real-world probability measure
$\mathbf{P}_t$	Covariance matrix of state filtering error
$\mathbf{P}_{t\|t-\Delta t}$	Prediction of covariance matrix of filtering error
p	Unconditional one-year probability of default
p_{AB}	Probability of a joint default of obligor A and B to time t
p_i	One-year probability of default of exposure i
p_t	Probability of default in period t
p_t	Probability of default until time t
p_t	Probability of default in time period t
$p_{t\|s}$	Probability of default in a period of length t conditional on non-default at time s
$p_{t\|z_t}$	Probability of default in period t conditional on factor z_t
p^{rc}	Pooled one-year probability of default of risk class rc
$\widehat{p}$	Default rate
$\widehat{p}_t$	Default rate in time period t
$\overline{p}, \overline{p}_i$	Default probability of exposures in the alternative model
Q	Unique equivalent Martingale measure
$\mathcal{Q}(X_{t-\Delta t};\psi)$	Covariance matrix of state vector X_t disturbance
$\mathcal{Q}_{ii}(X_{t-\Delta t};\psi)$	Conditional variance of state disturbance
$\mathcal{QLL}_F$	Quasi-Log-likelihood of factor process estimation
$\mathcal{QLL}_V$	Quasi-Log-likelihood of asset value process estimation
$\mathcal{QLL}_{\widehat{T}}(\mathbf{S}_{\overline{T}};\psi)$	Quasi log-Likelihood of the aggregate observation error
q_h	Scaling factor
q_t	Probability of survival until time t
$q_{t\|s}$	Survival Probability Probability of default in a period of length t conditional on non-default at time s
$q_{1-\alpha}, \overline{q}_{\overline{\alpha}}$	Quantiles of portfolio loss under hypothesis $\mathcal{H}$ and $\overline{\mathcal{H}}$
$\mathcal{R}(\cdot)$	Risk measure
$R_t^c(\tau;\beta_t^c)$	Zero-coupon rate of class $c \in \{RC, rl\}$ for time-to-maturity τ at time t
$R_t^c(\tau;\beta_t^c)$	Spot rate of risk class rc at time t
$R_{t,j}$	Random variable for credit spread of exposure j at time t
$R_t^{rl}(\tau;\beta_t^{rl})$	Riskless rate of risk class rc at time t
RC	Set of risk class models
RC^m	Set of risk classes of risk class model m
r	Constant continuous-compounding riskless rate

rc	Risk class index
rc_i	Risk class of exposure i
$rcys^{k_j}$	Time series of log-returns of yield spreads of sector-class k_j for interval t
$rcys_t^{k_j}$	Log-return of average credit yield spreads of sector-class k_j for interval t
$S_\tau(\cdot)$	Survival function
SIM_{k_j,l_j}	Similarity measure for a join of sector-classes k_j, l_j in clustering iteration j
$\mathbf{S}_t$	Time series of yield spread observation vectors by time t
S_t	Vector of yield spread observations at time t
$S_{t,j}$	Spread observation of exposure j at time t
$S_{j,t}$	Spread observation j at time t
S_{t,i_t}^{rs}	Residual yield spread of bond i_t at time t
$\widehat{S}_t^f$	Vector of filtered factor-implied yields spreads at time t
$\widehat{S}_{t,i}^f$	Vector of filtered factor-implied yield spread of exposure j at time t
$\widehat{S}_{t\|\Delta t}^f$	Vector of predicted factor-implied yields spreads at time t
$\widehat{S}_{t\|\Delta t,i}^f$	Predicted factor-implied yield spread of exposure j at time t
$\widehat{S}_t^{rc}$	Vector of filtered yield spread of exposures at time t
$\widehat{S}_{t,j}^{rc}$	Filtered yield spread of exposure j at time t
$\widehat{S}_{t\|t-\Delta t}^{rc}$	Vector of predicted yield spread of exposures at time t
$\widehat{S}_{t\|t-\Delta t,j}^{rc}$	Prediction of yield spread of exposure j at time t
s	Sector index
$\overline{T}$	Length of estimation period
T_i	Maturity of exposure i
$T(X_{t-\Delta t};\psi)$	Transition function vector
TtM	Time-to-Maturity
$T_X(X_{t-\Delta t};\psi)$	Transition function
t, t'	Time index
$t_1(V,t), t_2(V,t)$	Auxiliary variables
$\bar{t}$	Time horizon of Credit-VaR
$\rfloor t \lfloor$	Previous cash flow date operator
$\rceil t \lceil$	Next coupon date operator
$\mathcal{U}(0,1)$	Standard uniform distribution
U_j	Uniform random variable j
u_j	Representation of uniform random variable j
V	Representation of the stochastic asset value at time t
V_t	Asset value at time t
V_t	Vector of asset values
$V_{t,j}$	Asset values of exposure j at time t
V_t^ϵ	Vector of normalized asset value of exposures at time t
$V_{t,j}^\epsilon$	Normalized asset value of exposure j at time t

$\widehat{V}_t$	Vector of filtered asset value of exposures at time t
$\widehat{V}_{t,j}$	Filtered asset value of exposure j at time t
$\widehat{V}_{t\|t-\Delta t}$	Vector of predicted asset value of exposures at time t
$\widehat{V}_{t\|t-\Delta t,j}$	Prediction of asset value of exposure j at time t
$\widehat{V}_t^\epsilon$	Vector of filtered normalized asset value of exposures at time t
$\widehat{V}_{t,j}^\epsilon$	Filtered normalized asset value of exposure j at time t
$\widehat{V}_{t\|t-\Delta t}^\epsilon$	Vector of predicted normalized asset value of exposures at time t
$\widehat{V}_{t\|t-\Delta t,j}^\epsilon$	Prediction of normalized asset value of exposure j at time t
$\overline{V}$	Constant default threshold
$Var(\cdot)$	Variance function
v_t	Prediction error of observation
W_t	Standard Brownian motion under the physical measure
W_t^Q	Standard Brownian motion under the risk-neutral measure
X	General Random Variables
$\mathbf{X}_t$	Time series of state vector by time t
X_t	General State Vector at time t
$X_{t,i}$	General state variable i at time t
$\widehat{X}_t$	Filtered state vector at time t
$\widehat{X}_{t,i}$	Filtered state variable i at time t
$\widehat{X}_{t\|t-\Delta t}$	Prediction of filtered state vector X_t
$\widehat{X}_{t\|t-\Delta t,i}$	Prediction of state variable $X_{t,i}$
$x_1(t), x_2(t)$	Auxiliary variable
Y	General Random Variables
Y_t^{pc}	Yield vector of synthetic credit exposures at time t
Y_t^{rl}	Yield vector of synthetic riskless exposures at time t
$\mathbf{Y}_t^{pc}$	Time series of credit yield vectors by time t
$\mathbf{Y}_t^{rl}$	Time series of riskless yield vectors by time t
Y_{t,i_t}	Internal rate of return of bond i_t at time t
$Y_{t,j}^{pc}$	Yield-to-maturity of synthetic par-exposure j at time t
$Y_{t,j}^{rl}$	Yield-to-maturity of riskless synthetic par-exposure j at time t
$Y_{t,j}^s$	Simulated internal rate of return of exposure j at time t
$ytm(Y_{t,j}^{pc}, D_{t,j}; \phi_j)$	Internal rate of return of exposure j at time t
$\widehat{Y}_t^f$	Vector of filtered factor-implied yields at time t
$\widehat{Y}_{t,j}^f$	Filtered factor-implied yield of exposure j at time t
$\widehat{Y}_{t\|\Delta t}^f$	Vector of predicted factor-implied yields at time t
$\widehat{Y}_{t\|\Delta t,j}^f$	Prediction of factor-implied yield of exposure j at time t
$\widehat{Y}_t^{rc}$	Vector of filtered yield of exposures at time t
$\widehat{Y}_{t,j}^{rc}$	Filtered yield of exposure j at time t
$\widehat{Y}_{t\|t-\Delta t}^{rc}$	Vector of predicted yield of exposures at time t

$\widehat{Y}^{rc}_{t\|t-\Delta t,j}$	Prediction of yield of exposure j at time t
$Z(X_{t-\Delta t}, t_t; \psi)$	Measurement function vector
$Z_\tau(X_{t-\Delta t}, t_t; \psi)$	Measurement function
$Z_V(\epsilon_t, \widehat{F}_t; \psi_F; \psi_V)$	Measurement function of the ASSM of the asset value process estimation
$ZCR^c_t(\tau; \beta^{rc}_t, \beta^{rl}_t)$	Synthetic credit-risky spot rate of risk class rc at time t
ZCR^{rl}_t	Riskless continuously-compounding spot rate at time t
$ZCS^c_t(\tau; \beta^{rc}_t, \beta^{rl}_t)$	Credit spot spread of risk class rc at time t
z_t	Standard-Gaussian single-factor
$\widehat{\nabla T}_t$	Predicted Gradient Vector of Transition function $T(\widehat{X}_{t-\Delta t}; \psi)$
$\widehat{\nabla Z}_t$	Predicted Gradient Vector of observation function $Z(\widehat{X}_{t\|t-\Delta t}, t_t; \psi)$
$\Delta\widehat{F}^\epsilon_t$	Change of the normalized filtered systematic factor at time t
$\Delta\widehat{F}^{\epsilon,rc}_T$	Filtered normalized factor returns of risk class rc
$\boldsymbol{\Delta}\widehat{\mathbf{F}}^{\epsilon,rc}_T$	Time Series of filtered normalized factor returns of risk class rc
Δl	Deviation of portfolio loss from a reference value
$\Delta S^{rs}_{t,i_t}$	Change of residual yield spread of bond i_t at time t
Δt	Discrete time interval
$\Delta\widehat{V}^\epsilon_{t,j}$	Change of the normalized asset value of exposure j at time t
Σ_t	Prediction error covariance matrix at time t
$\Phi(x)$	Standard Gaussian distribution function
$\Phi_2(x,y)$	Standard bi-variate Gaussian distribution function
$\Omega(\psi)$	Covariance matrix of the observation vector disturbance
α	Confidence level of portfolio loss in backtesting of VaR and Credit-VaR
$\alpha_{\mathcal{H}}, \alpha_{\overline{\mathcal{H}}}$	Significance level of hypothesis $\mathcal{H}$ and $\overline{\mathcal{H}}$
$\overline{\alpha}$	Confidence level of the acceptance barrier in the alternative model
$\beta^c_j(t)$	Nelson-Siegel parameter j of class $c \in \{RC, rl\}$ at time t
β^c_t	Vector of Nelson-Siegel parameters of class $c \in \{RC, rl\}$ at time t
$\beta_{rc_i,j}$	Coefficient of systematic factor j in risk class rc_i
β_V	Systematic factor coefficient
β^{rc}_V	Systematic factor coefficient of risk class rc
$\gamma_{t,j}$	Distribution parameter of exposure j at time t
$\delta_{t,j}$	Distribution parameter of exposure j at time t
ϵ_t	Vector of specific factors at time t
ϵ_t	Vector of normalized specific factor components of exposures
ϵ^i_t	Specific factor of exposure i at time t
$\epsilon_{t,j}$	Normalized specific factor component of asset value $V_{t,j}$ of exposure j
$\widehat{\epsilon}_t$	Vector of filtered normalized specific factor components of exposures
$\widehat{\epsilon}_{t,j}$	Filtered normalized specific factor of asset value $V_{t,j}$ of exposure j
$\widehat{\epsilon}_{t\|t-\Delta t}$	Vector of predicted normalized specific factor components of exposures
$\widehat{\epsilon}_{t\|t-\Delta t,j}$	Prediction of normalized specific factor of asset value $V_{t,j}$ of exposure j

ε_t^i	Standard Gaussian white noise process
ε_t	Vector of Standard Gaussian white noise process of exposure i at time t
ε_t	Vector of observation disturbances at time t
$\varepsilon_{j,t}$	Disturbances of observation j at time t
η_t^f	Factor disturbance at time t
η_t	Vector of state disturbances at time t
$\eta_{t,i}$	Disturbances of state variable i at time t
$\eta_{t,j}$	Disturbance of state variable j at time t
θ	Auxiliary variable
ϑ	Auxiliary vairable
$\underline{\lambda}$	Coefficient of lower tail dependence
λ	Time-homogenous default intensity
$\lambda(t)$	Time-inhomogenous default intensity at time t
μ	Drift rate of asset value
μ_F	Drift rate of systematic factor
μ_V	Drift rate of asset value process
$\mu_t^{\Delta S}$	Average change of residual yield spreads at time t
μ_i	Asset value drift rate of exposure i
$\mu_{t,j}$	Expected yield spread of exposure j at time t
$\overline{\mu}, \overline{\mu}_i$	Asset value drift rate of exposures in the alternative model
ν	Auxiliary variable
ρ	General correlation structure
ρ_V	Inner-class asset correlation
ρ^a	Correlation of normalized asset returns (asset correlation)
ρ^a_{AB}	Asset correlation of obligor A and B
ρ^a_{ij}	Asset correlation of exposures i, j
$\rho^{a,m}$	Asset correlation matrix of risk class model m
$\rho^{a,m}_{rc_k,rc_l}$	Asset correlation of exposures in risk classes $rc_k, rc_l \in RC^m$
ρ^{def}	Correlation of default indicating random variables (default correlation)
$\rho^{f,m}$	Matrix of factor correlations
$\rho^{f,m}_{rc_k,rc_l}$	Correlation of normalized factor returns of risk classes $rc_k, rc_l \in RC^m$
ρ_{rc}	Asset correlation between any pair of exposures in risk class rc (inner-class correlation)
ρ_{rc_i,rc_h}	Asset correlation between any pair of exposures of risk classes rc_i and rc_h (inter-class correlation)
$\rho^{sc}_{k_j,l_j}$	Correlation between log-returns of spreads of sector-classes k_j and l_j
$\overline{\rho^a}, \overline{\rho^a}_{i,j}$	Asset correlation of exposures in the alternative model
$\overline{\rho}^{sc}_{k_j,l_j}$	Average of spread-return correlations $\rho^{sc}_{r,s}$ of sectors $r, s \in \{I^s_{k_j}, I^s_{l_j}\}$

$\overline{\overline{\rho}}^{sc}_{k_j,l_j}$	Average of the spread-return correlation of joined sector-classes k_j,l_j with remaining sector-classes
ϱ, ϱ_i	Recovery rate of exposures
σ	Standard deviation of asset value
σ_F	Diffusion rate of systematic factor
σ_V	Diffusion rate of asset value process
$\sigma_t^{\Delta S}$	Standard deviation of residual yield spreads at time t
σ_i	Asset value diffusion rate of exposure i
$\sigma_{t,j}$	Standard deviation of yield spread of exposure j at time t
$\overline{\sigma}, \overline{\sigma}_i$	Asset value diffusion rate of exposures in the alternative model
τ	Default time
τ	Time-to-maturity
τ_A, τ_B	Default time of obligor A and B
τ_i	Time of default of exposure i
τ_j	Time-to-maturity of observation j
ϕ	Parameter set for the valuation of a credit exposure
ϕ_0	Parameter set of credit valuation
$\phi_{t,j}$	Parameter set of exposure j at time t
$\varphi(x)$	Standard normal density function
ψ	Parameter set of state-space model
ψ_0	Initial parameter set of state-space model
ψ_F	Parameter set of the ASSM of the factor process
ψ_V	Parameter set of the ASSM of asset value processes
ψ^*	Optimal parameter set of state-space model
$\widehat{\psi}^*, \widehat{\psi}^*_T$	QML estimate of optimal parameter set of state-space model
$\omega_{j,k}$	Covariance of observation prediction error of exposure j and k

Chapter 1

Introduction

Credit business is the core business of banks. In general, credit risk represents the highest risk exposures of a bank, compared to liquidity risk and market risk. Accordingly, accurate measurement and effective management of credit risk is the major challenge in the attempt to ensure the solvency and profitability of a bank. Financial institutions employ credit portfolio models for credit risk measurement and management, risk-adjusted performance measurement and economic capital allocation.

Within recent years, credit markets have experienced a dynamic evolution characterized by a rich availability of funds to finance banks' credit involvements, rising real estate and equity prices as well as lending volumes, the emergence and exceptional growth of markets for instruments of credit risk mitigation, the introduction of new risk-adjusted capital standards for credit exposures, and the wide spread implementation and use of complex credit risk models.

In the first years of this century, some central banks have eased the availability of funds excessively by setting moderate interest rates and by increasing the money supply to support economic growth. Combined with increasing asset prices, especially in many real estate markets around the world, this expansion of money supply was accompanied by increased volumes of banks' lending activities.

The increase in lending volumes was accompanied by an exceptional growth in credit assurance and credit securitization markets. Within the last decade, the volume of outstanding credit derivatives has increased from almost zero to a notional amount of 62.17 trillion USD by the end of 2007, compared to equity derivatives of 10 trillion USD of outstanding notional amount and 382 trillion USD of interest rate and currency derivatives. In comparison, the total amount outstanding of the US bond market including Treasury, municipal, corporate, money market, MBS, ABS and agency securities, amounts to 30.14 trillion USD, with the European market being approximately of the same size.

The existence of active credit derivative markets and primary markets for credit securitization has enabled banks to sell, buy, hedge and restructure their credit portfolio risk using credit default swaps and structured asset-backed securities. From an ex post perspective, the complex credit risk modelling techniques available and the approved regulatory standards used to determine risk-adjusted capital requirements seem to be responsible for the overconfidence of banks in their ability to assess and manage the credit risk inherent in their portfolios. Inadequate calibration, application and interpretation of credit risk models caused by the complexity of models, methodological problems in the estimation and quantification of dependencies of credit exposures, and, finally, a high tolerance for credit risk has resulted in the accumulation of portfolio credit risk in banks' portfolios that is not properly reflected in credit risk measurement and management.

In recent years, many banks have eased their standards for granting loans. The possibility of removing credit exposures from the balance sheet and of earning additional fee income by marketing credit securitizations has encouraged some banks to abandon prudent standards in granting loans, especially in some mortgage and consumer-financing markets. Fostered by incentive systems based on an appraisal of short-term performance, the objective of banks' credit business activities has shifted from the origination and management of on-balance held-to-maturity credit exposures to short-term transaction-focussed origination and securitization activities. Furthermore, communication barriers within banks and the functional separation of credit origination functions and credit analysis and monitoring functions seems to have precluded a consistent assessment of credit risk exposures. Paradoxically, some credit exposures, which have been securitized and marketed, have been removed from the balance sheet, while at the same time different exposures of a similar risk profile have been added to bank portfolios from the credit derivative market and the secondary credit markets.

The objective of financial markets' supervisory agencies is to maintain the stability of the financial system by imposing adequate capital requirements on financial institutions. The revised regulatory framework on the "International Convergence of Capital Measurement and Capital Standards" BCBS (2006c) adjusts the fixed credit risk capital charges originally imposed by the Basel Committee BCBS (1988) to more risk-adequate capital requirements. It follows the successful implementation of a supervision process for banks' market risk by switching from a quantitative type of banking regulation to a process-oriented banking regulation based on banks' internal models. If this change of paradigm is conducted analogously for the capital requirements induced by credit risk, adequate procedures and methodologies for the accurate modelling and estimation of credit portfolio risk must be available.

The Basel Committee requires an adequate portfolio credit risk model to be conceptually sound, empirically validated, and to produce capital requirements that are comparable across institutions. Sound methodological concepts such as Creditmetrics^{TM} described by Gupton, Finger and Bhatia (1997), CreditRisk+^{TM} CSFP (1997), CreditMonitor^{TM} and LossCalc^{TM}, (Vasicek (1984) and Kealhofer (1998)), and CreditPortfolioView^{TM} by Wilson (1997a, 1997b) are readily available. Methodological comparisons of different credit portfolio models as provided by BCBS (1999a), Crouhy, Galai and Mark (2000), Jarrow and Protter (2004), and Eberlein, Frey, Kalkbrenner and Overbeck (2007). However, there is still a fundamental need for reliable backtesting procedures to determine the adequacy of internal credit risk models, both from an internal perspective and from the perspective of the supervisory agencies.

This need is the starting point of this dissertation. In contrast to tests of the accuracy of a models' parameters, the backtesting of credit portfolio models assesses the adequacy of a model to determine reliable capital requirements. The major and most important contribution of this thesis to the literature is the development and empirical analysis of a backtesting procedure to assess the adequacy of a credit portfolio model. This goal is attained in three major steps. First, a firm-value-based risk-class model of credit portfolio risk is presented. In the second step, an estimation procedure to determine the parameters of the portfolio model is introduced. In the third and most important step, a simple backtesting approach to assess the adequacy of the parameterized credit portfolio model is developed and analyzed in an extensive simulation study.

A structural first-passage credit-valuation model based on the firm value model proposed by Black and Cox (1976) is introduced for the valuation of defaultable exposures with fixed periodic interest payments. In this model, a constant default threshold allows for the predictable default of a loan at any time until maturity, so that credit default indicates over-indebtedness and insolvency of obligors. For the first time in the literature ,a structural first-passage credit-valuation model is supplemented by a factor model to create a risk-class-based credit portfolio model.

The estimation of the credit portfolio model encounters the problem that firms' asset values, the underlying of firms' credit liabilities, cannot be observed at any point in time. Because of their latent nature, asset values are estimated implicitly from observable market prices of credit-risky instruments using a latent variable estimation approach.

The estimation of the credit portfolio model is based on weekly data from the European corporate bond market. A clustering procedure is employed to group exposures into risk classes. For each risk class, the term structures of defaultable zero-coupon bonds' yield-to-maturities are fitted using the Nelson and Siegel (1987) parametric form.

A two-stage quasi-maximum likelihood estimator based on an Extended Kalman-Filter is applied to estimate the time series of latent systematic and specific factors and the parameters of the corresponding stochastic processes using two non-linear state-space models. In the first estimation step, process parameters and time series of an implicit systematic factor are estimated for each risk class. In the pivotal second estimation step, the process parameters of the asset value process and the coefficients of the systematic factor process are simultaneously estimated for each risk class using the implicit systematic factor series from the previous step.

For three variants of the credit portfolio model, the coefficients of orthogonal statistic risk factors and, accordingly, the correlations of the asset values of different risk classes are calculated using the coefficients of the systematic factors and the correlations between the implied factor return series. The estimated asset value correlations are in line with alternative estimates of asset correlations presented by Akhavein, Kocagil and Neugebauer, Dietsch and Petey (2002, 2004) as well as Düllmann and Scheule (2003) under comparable model assumptions.

With respect to the empirical validation of credit portfolio models, specification tests of parameter estimates must be differentiated from approaches that are used for backtesting the adequacy of a model to achieve specified risk management objectives, such as ensuring the solvency of a bank at a specific level of significance. Apart from this thesis, there are only few comprehensive studies on backtesting the overall adequacy of credit portfolio models to set capital requirements, such as the studies provided by Koyluoglu and Hickman (1998b), Gordy (2000), Kern and Rudolph (2001), Frerichs and Löffler (2002), Emmer and Tasche (2005), Nickell, Perraudin and Varotto (2001a), and Tasche (2006). For general considerations regarding the backtesting of the adequacy of risk models, confer Kupiec (2001, 2002, 2004).

Both, credit cycles and credit involvements of banks typically span periods of several years, so that banks' credit exposures often do not reveal their ultimate profitability until years after the origination. Therefore, credit risk forecasts typically use long-term time horizons and as a consequence, only few independent historical data on credit portfolio performance are available for a statistical backtesting of credit risk models. However, within a given time periods rich cross-sectional information is available on the performance of single credit exposures.

The pivotal problem that has prevented the development of reliably backtesting procedures in the past is this lack of sufficient independent time series data on portfolios' credit performance. Observations of the credit performance are only independent if they refer to different time periods, because within a given time period all credit exposures are influenced by joint background factors that result in cross-sectional observations of credit performance being dependent. Since conventional statistical inference relies on independent observations, model adequacy can only be tested conditionally on actual factor values. Furthermore, the backtesting of a conditional credit risk on the basis of the conditional loss rates would omit that part of the credit portfolio model that controls for the dependence of exposures and the variation of loss rates in time. In consequence, due to this lack of time series data on credit portfolio performance, the conventional statistical test theory is not applicable for the backtesting of credit portfolio models.

In the following, the development of techniques for backtesting the adequacy of unconditional Credit-VaR using a time series of few independent observations of credit portfolio loss is omitted, because small sample inference on the basis of conventional statistical test theory has proven to result only in a test of low power. Instead, the classical statistical test paradigm which requires independent observations is dropped.

A simple traffic-light approach to backtesting the adequacy of credit portfolio models is suggested. This approach relies only on the rich cross-sectional information of a credit portfolio's performance within a single time period as a test statistic for backtesting the unconditional Credit-VaR.

The zone approach to the backtesting of market risk models is transferred to the backtesting of credit risk models by replacing information in time with cross-sectional information. A two-hypotheses test is formulated to check whether the observation of credit portfolio loss complies with the assumptions of the model used by a bank, which must not be rejected, and a more prudent alternative model, which must be rejected by the test. Three zones of model adequacy for portfolio credit loss are defined: a green zone of confidence that the model being used is adequate, a yellow zone of indetermination regarding the model adequacy, and a red zone of model rejection. The location of the green zone of credit portfolio loss is critically determined by the applied level of significance and the specification of the alternative model, particularly its correlation assumption.

The backtesting procedure is analyzed on the basis of a synthetic model setting with respect to a variation of the model structure, the portfolio characteristics and some technical assumptions made by the risk model. The examinations concerning the model structure include the assessment of effects on the adequacy zones by a change of the default model, the number of risk classes and the number of rating classes. Portfolio characteristics are analyzed for variations in the time-to-maturity and default probability of loans, the size of asset correlations, and the drift and volatility parameters of asset value processes. The technical assumptions made by the risk model examined include the number of simulation sub-intervals and the time horizon of the Credit-VaR forecast. The Credit-VaR and the zone locations are examined based on two different definitions of portfolio loss, which refer either to the unexpected credit loss as prescribed by the New Capital Adequacy framework, or to the value of exposures at the time of the risk forecast, equal to the market risk definition of loss.

A pivotal result of this dissertation is the application of the backtesting procedure to the three variants of the credit portfolio model that have been estimated using credit market data. The resulting capital requirements comply with the core capital ratios typically maintained by banks only for a significance level of Credit-VaR that is substantially lower than the level implicitly presumed by the revised capital adequacy framework.

The results of the simulation study show that the structure and parameterization of the portfolio model and the characteristics of the portfolio have a significant effect on Credit-VaR and the zones of model adequacy. The location of the adequacy zones depends primarily on the asset correlation and the granularity of exposures. Furthermore, the definition of credit portfolio loss impacts Credit-VaR and the zone locations.

The robustness of the zone locations is examined with respect to variations in the time-to-maturity, face value and default probability of exposures. The impact of drift rate, asset volatility and the structure of risk classes is assessed. A comparison of the backtesting zones for a homogenous portfolio and for heterogenous portfolios of loans reveals that the diversification of the portfolio characteristics and increasingly detailed risk class structures improve the discriminatory power of the backtesting.

The risk of a deterioration in credit quality and a corresponding decline in the market value of credit exposures is included in the definition of the credit performance. As a consequence, the Credit-VaR exceeds the capital requirements of the IRB approach of the revised Capital Standards.

The proposed test is easily applicable and computationally feasible. It requires minimal data and can be used in principle independently of the credit risk model, as it does not refer to parametric or structural model assumptions. In contrast to the backtesting of market risk models, an unambiguous location of test zones independent of portfolio characteristics and model specifications cannot be achieved when backtesting the adequacy of portfolio credit risk models.

The dissertation is organized as follows. In Chapter 2, fundamental concepts, components and methodologies of credit risk models are introduced. Particularly, different mark-to-model-based definitions of credit portfolio loss are discussed and a simple traffic-light approach to backtesting the adequacy of credit portfolio models is suggested.

Chapter 3 starts with a review of the academic literature on single-name credit-valuation models and credit portfolio models. A structural first-passage credit valuation model for defaultable claims with fixed deterministic cash flows is presented, and the characteristics of the model are assessed using a comparative-static analysis. The structural credit portfolio model used in subsequent chapters is defined as a multi-variate risk-factor-based extension of the single-name credit valuation model introduced above, in which credit dependence is modelled by a system of systematic and specific factors.

Chapter 4 includes the clustering of risk classes, the fitting risk classes' term structures of interest rates, and the Kalman Filter-based quasi-maximum likelihood estimation of the systematic and specific factors of risk classes. After a calibration of the coefficients of the risk-class factor model, estimation results for three representations of the credit portfolio model are discussed.

Finally, in Chapter 5, the backtesting approach is applied based on simulated distributions of credit portfolio loss. The backtesting procedure is specified by setting the level of significance of the two test hypotheses and by defining the alternative model to be rejected. Credit-VaR and the adequacy zones of the credit portfolio loss are examined for different discretionary specifications of the credit portfolio model and for the different characteristics of the portfolio. Credit-VaR and the adequacy zones of the backtesting are discussed for the portfolio models whose parameters have been estimated in the preceding chapter. In Chapter 6, the fundamental results are summarized and evaluated. Prospective research topics are recommended.

Chapter 2

Introduction to Credit Risk Management

Financial institutions use a variety of different concepts to manage credit portfolio risk. A general overview of concepts, methodologies, procedures, systems and standards of credit risk modelling is presented in accordance with the definitions and requirements of the revised capital adequacy framework set forth by the Basel Committee (BCBS (2006a)).

2.1 General Framework of Credit Risk Management

2.1.1 Procedural and Organizational Standards

The implementation of an appropriate and reliable credit risk management framework is the prerequisite of a reliable credit risk measurement. In the following, best practice procedural and organizational standards in the credit risk management of banks are presented. These standards represent the basis for the modelling, estimation and backtesting of portfolio credit risk models in subsequent chapters. The following delineation is based on the requirements defined by the Basel Committee.[1]

The Basel Committee on Banking Supervision requires banks to maintain appropriate information systems and processes to identify, measure, monitor and control credit risk exposures in size, quality and composition. By definition, credit exposures comprise all contractual arrangements on and off the balance sheet that involve deterministic or contingent future payments to be received from a contractual partner, such as bonds, loan arrangements, credit facilities, credit card obligations and contingent claims with counterparty risk arising from OTC derivative contracts in the trading book.

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[1] Cf. BCBS (1999b, 2000a, 2000b, 2005c, 2005d, 2006b, 2006d, 2006e).

A credit risk strategy of a bank is set and approved by the board of directors and implemented by senior management. Credit risk strategies, policies, processes and limits should be documented, regularly reviewed, updated and communicated throughout the bank. Credit risk systems provide timely and accurate information for the analysis, accounting, provisioning and capital requirements of credit exposures.

The credit risk management of a bank covers the ongoing identification, origination, administration, measurement and monitoring of all on- and off-balance sheet credit exposures. An effective credit management involves lending controls and limits as well as a comprehensive reporting process. Credit risk management implements an adequate diversification of credit portfolios and ensures that exposure limits to single counterparties, groups of connected counterparties, particular industries or economic sectors, geographic regions and specific products are set and complied with. Concentrations of credit risk are managed by means of concentrations limits and risk mitigation techniques such as collateralization, third-party guarantees, credit derivatives, securitization programmes and secondary credit markets. All credit-risk- related processes are performed in a timely, periodical and consistent way. Risk assessment, monitoring and control functions are clearly separated from risk-taking functions of the bank.

Credit origination includes the analysis, granting and approval of credit exposures. A formal process to decide on and approve the origination or the renewal of credit exposures on an arm's length basis is defined. Policies for the origination of credit exposures determine functional and personal responsibilities in terms of amount and product type. Senior management approves large credit involvements that exceed a certain amount or percentage of banks' capital.

The main objective of credit analysis is to assess the risk-reward profile of prospective and current credit engagements to foreclose that loans are granted or extended on a subjective basis or non-risk-adequate credit pricing due to personal or commercial affiliations to borrowers takes place. The effectiveness of credit analysis is based on the quality, detail and timeliness information is recorded and processed. Often, a comprehensive initial credit analysis is limited by time constraints caused by competitive pressure. An ongoing credit re-assessment implements the early identification of deteriorating assets, is used to determine loan loss provisions in a timely manner, and monitors problem assets and collections on past-due obligations.

Credit analysis accounts for business-cycle effects using stress tests and scenario analysis, that examine economic or industry downturns, market-risk events and liquidity conditions. The counterparty risk of market-risk sensitive exposures, particularly of derivatives that do not constitute original credit exposures, are analyzed for the counterparties' willingness and ability to pay. Third-party guarantees or credit facilities that are sensitive to the liquidity of credit markets are analyzed for the borrower's vulnerability to financial stress, which may threaten its debt-serving capabilities as well as its financing needs.

Credit policies specify the information and methodologies used to assess the credit risk of exposures. Methodologies such as internal rating systems quantify and classify the credit risk of individual exposures. The valuation, classification and provisioning of large credit risk exposures is conducted on an individual basis considering all available information on the obligor and the engagement itself, including credit covenants and means of credit risk mitigation. Periodical re-assessments of credit involvements ensure that specific and general loan loss provisions and write-offs adequately absorb expected credit losses and reflect realistic repayment and recovery expectations.

Internal credit risk rating systems support the origination, risk measurement, monitoring and administration of individual credit exposures by assessing the ability and willingness of borrowers to meet contractual financial obligations. Rating systems differentiate the degree of credit risk of exposures and allow for a more accurate control of problem exposures, risk concentrations, capital allocation, pricing of credit exposures and determine risk-adjusted performance of exposures, adequacy and loan loss provisions. Typically, credit exposures are categorized into classes of different risk levels considering all relevant indicators of an actual or a potential deterioration of the credit risk of the exposure and the borrower. For each risk class estimates of he probability of default (PD), the exposure-at-default (EAD) and the percentage loss-given-default (LGD) of exposures are provided and reviewed at least annually. Rating migration, default and loss experience of the bank's own credit portfolio and credit market data from rating agencies as well as market observed credit spread data are used to estimate these parameters. Rating systems provide historical information on credit exposures and information that indicate the solvency of borrowers for a time period of several years, ideally spanning a complete economic cycle. If pooled data is used, methodological differences in the definition of data is taken into account. Structural changes in credit markets and the time-inhomogeneity of market-derived parameters are addressed by a frequent updating of the estimates of the relevant parameters. Means of credit risk mitigation including guarantees and collateral is re-assessed periodically.

Portfolio credit risk models provide measures of portfolio credit risk that reflect the overall credit risk of exposures more accurately than the aggregate of single-exposure capital charges. The advantages of credit portfolio models compared to a single-exposure-based credit risk assessment include: (1) the implementation of a centralized exposure management, (2) the analysis of marginal and absolute contributions of single exposures to portfolio credit risk, (3) portfolio-specific estimates of unexpected credit loss, including the quantification of concentration effects in credit risk, (4) improvements in system and data collection efforts, (5) the consideration of portfolio risk in setting limits, reserves and risk-return-based credit pricing, (6) the implementation of a company-wide consistent economic capital allocation, (7) improved correspondence of regulatory and economic capital requirements.

Banks are assumed to be proficient in the methodologies, capabilities and limitations of the credit risk model used, especially if it is a vendor model. The definition of key model parameters and the sensitivity of credit risk measures to changes in parameters are assumed to be understood and considered in operational credit risk assessments. Reasonable structural consistency between model setup and data used for model estimation and the bank's portfolio must be ensured. Since model estimation often remains proprietary to vendors, banks typically face the challenge of adapting the bank's credit portfolio to the structural requirements of an external model.

Banks' business strategies often aim for a specialization in narrow segments of credit markets, which inherently leads to concentrations of credit risk. Apart from risk concentrations the correlation of seemingly unrelated risk components can pose a threat to the solvability of banks, for example, the financial strength of a borrower and the liquidity of the market for its collateral assets may be correlated. Policies on the acceptability of various forms of collateral, procedures for the ongoing valuation of such collateral and a process to ensure that collateral is and continues to be adequate, enforceable and realizable are required to be in place.

In the process of credit monitoring credit exposures are segmented according to the grade of credit risk, type of loan, geographical location, collateral type and past-due status. The composition and quality of the overall credit position, including the identification of risk concentrations, is monitored. For each exposure, the monitoring system provides the current financial conditions of the borrower or counterparty, compliance with existing covenants, an assessment of collateral coverage, the identification of contractual payment delinquencies, loss provisions incurred and a classification of potential problem exposures. Effective exposures are monitored against established limits. Meaningful exposure limits are set on the basis of forward-looking stress tests and effective measures of potential future exposures. Limit monitoring involves netting agreements with specific counterparties.

All functions in credit risk origination, analysis, management and monitoring are required to be subject to periodical controls by operational credit risk review functions, an independent internal audit department and external supervisors. The criteria used to evaluate the quality of the credit functions are effectiveness, accuracy, timeliness and documentation of policy compliance, model estimation, exposure administration and information processing.

2.1.2 Conceptual Standards in Credit Risk Modelling

Fundamental concepts of credit risk modelling differ between (1) a credit risk assessment conditional and unconditional on economic conditions, (2) an actuarial versus a mark-to-market consideration of credit risk, (3) expected and unexpected loss of defaultable exposures, and (4) the concepts of risk-adjusted discounting and risk-neutral valuation.

Conditional vs. Unconditional Credit Risk Models

Reflecting the background conditions on credit risk a general framework for modelling credit portfolio risk involves:[2]

- conditional credit performance
- dependence model of credit risk
- unconditional credit performance

The performance of a credit portfolio as measured in terms of default rate, credit loss, total net revenues or changes in portfolio value over a time interval, fluctuates in time, reflecting the variation of economic background factors that jointly affect the economic prospects of all obligors in the portfolio. Conditional on effective economic conditions, credit performance such as credit defaults of individual obligors, are assumed to be independent in a period as economic background factors jointly affect the variation of obligors' credit risk only in time. Conditional credit risk measures account for the effective economic conditions and represent the credit risk of obligors in a specific period.

The dependence of exposures' credit risk is typically incorporated by a parametric model of factors that jointly control for an exposure's credit risk indicating variables. Factors may be of statistical, macro-economic or business-specific nature. The strength of the dependence between the credit risk of single exposures is reflected by the strength of the variation of default rates in time. Credit portfolio models incorporate the dependence of exposures' credit quality in predicting credit portfolio risk.

The expected unconditional credit performance represents a long-run average of credit performance across the full range of probable economic conditions. A distribution of unconditional credit performance is obtained by aggregating the respective conditional credit performance multiplied with the probability of the conditioning state-of-economy over the range of possible economic conditions. Risk forecasts generated by unconditional models do not explicitly refer to current economic conditions, so that risk parameters involve a through-the-cycle representation. However, risk models used in the banking sectors typically adapt the estimation of through-the-cycle parameters by emphasizing

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[2] Cf. Koyluoglu and Hickman (1998a, 1998b)

more recent market conditions, thus taking a hybrid approach concerning the conditioning of risk forecasts.

Through-the-cycle credit risk considerations are not meaningful if a short-term credit risk forecast is required. Instead, conditional credit risk models are forward-looking in nature, as they rely on current or predicted future economic conditions up to the time horizon of the risk forecast. Clearly, the economic conditioning of parameter estimates must coincide with the assumption of the respective risk model application. Conditional risk forecasts are typically performed by macroeconomic factor models as presented in Section 3.4.1. The ability of conditional models to provide adequate forecasts of credit risk is closely related to the accurate prediction of future economic conditions, so that prediction failures or unexpected changes in business prospects result in biased credit risk projections.

Actuarial vs. Mark-to-market based Credit Risk Models

The actuarial concept of credit risk recognition considers a discrete state-space of credit quality that refers to a real-world probability measure. In actuarial models state-specific amounts of exposure and credit loss are typically specified, but no explicit valuation of credit exposures takes place. Mostly a default-only paradigm is implemented, where credit loss is only incurred if a borrower defaults on its contractual obligations, and the effective loss is defined as the difference between the bank's exposure set in terms of the notional amount of the outstanding claim, and the present value EAD(1-LGD) of expected net recoveries. Since, the actuarial definition of credit loss does not take into account a deterioration of credit quality, unrealized economic losses from an adverse change of mark-to-market values of credit involvements are ignored and may accumulate in credit portfolios.

In the mark-to-market framework, additionally, deteriorations in the credit quality of exposures unequal to a credit default are reflected. Credit-risky exposures are explicitly valued using a valuation model that is calibrated either to reproduce market-observed values of credit exposures or to provide mark-to-model values for exposures without observable market indications of the credit risk in question. The value of a credit exposure depends either on a continuous credit-risk-indicating state variable or a discrete multinomial credit score, such as a rating. Credit performance is defined on the basis of exposures' change in credit value within a specified period. Rating-based valuation models derive credit values from the rating of an exposure and consider obligor default as a specific rating state.

Expected vs. Unexpected Credit Loss

The expected credit loss $EL = PD \cdot EAD \cdot LGD$ of an exposure is defined by the estimates of the probability of default, the exposure-at-default and the loss-given-default. The unexpected credit loss (UL) of an individual exposure or a credit portfolio is typically defined as the standard deviation of the (portfolio) credit loss.[3] In the determination of unexpected credit portfolio loss, mutual correlations of default events, exposures and loss rates must be considered for each borrower as well as between borrowers. Expected and unexpected loss typically refer to a one-year time horizon. Obviously, the time horizon and the definition of default must coincide in the estimation of PD, EAD and LGD for loss estimates to be conclusive.

Risk-adjusted Discounting vs. Risk-neutral Valuation

Credit valuation models either involve the risk-adjusted discounting (RAD)[4] of contractual cash flows under a real-world probability measure using credit-risk adjusted rates or the risk-neutral valuation (RNV) of defaultable cash flows under a risk-neutral probability measure to determine current and prospective future credit values.

RAD of non-publicly traded credit exposures relies on the classification of exposures with homogenous credit risk characteristics into risk classes and requires discount rates of a class-specific term structures that represents the average credit risk of the class. Risk classes are typically defined on the basis of the rating of exposures or obligors. However, exposures of the same rating grade may vary substantially with respect to the expected LGD of exposures, the migration probabilities or the sensitivity to changes in systematic risk factors, so that valuation errors may arise by using a term structure of discount factors that is homogenous for all exposures of a risk class.

Risk-neutral models may circumvent this short-coming. Risk-neutral valuation (RNV) refers to models of a state- and time-continuous credit risk indicating state variable under a risk-neutral probability measure. From the stochastic properties of this credit risk indicating variable risk-neutral survival and default probabilities can be derived that enable, in a complete market setting, the discounting of default state dependent cash flows using a risk-free interest rate. Models that incorporate risk-neutral valuation include structural firm-value models and reduced-form models of an exponential-affine default intensity. The way credit events are triggered constitutes the elementary difference between the two model types.

Structural models incorporate a microeconomic interpretation of the firm, with default only triggered if the value of a firm's assets is not sufficient to serve its financial obligations. From the dynamics of the firm value risk-neutral probabilities of default and survival are

[3] Cf. Ong (1999), p. 113.

[4] Cf. Gupton et al. (1997).

calculated that enable the risk-neutral discounting of probability-weighted cash flows.[5]

In the reduced-form framework, credit default is typically triggered by an exponentially distributed default time with an instantaneous default intensity parameter that has no economic analogy. From the distributional properties of default intensities discount factors that incorporate default risk of an exposure can be derived.

The data used for model estimation substantially affects the appropriateness of both model types. Whereas structural models are more convenient for credit risk applications which are based on macroeconomic and fundamental firm data, intensity models are more suited to a model calibration from market prices of credit risk. A detailed comparison of the competing classes of credit valuation models is given in Section 3.1.

With respect to application issues, RAD and RNV mainly differ in the way discount factors are calculated. RAD models explicitly use market-derived discount factors, whereas RNV is based on parametric models which need to be calibrated to reproduce observed credit spreads, for example in the credit default swaps (CDS) market or the corporate bond markets. RNV models are vulnerable to erroneous specifications and estimation errors and strongly rely on time series of credit market data for the estimation of the model, whereas the non-parametric RAD approach makes minimal use of modelling assumptions but presents estimation problems in the portfolio context.

2.2 Supervisory Framework of Credit Risk Management

2.2.1 Capital Adequacy Framework

Supervisory agencies set minimum capital adequacy requirements to ensure an adequate capital endowment of financial institutions. Banks are required to calculate and consistently maintain a minimum capital adequacy ratio. A revised framework for capital measurement and capital standards was passed by the Basel Committee in 2004 to improve the coherence of capital requirements with the risk inherent in the financial positions of financial institutions and to specify balance sheet items as regulatory capital according to their ability to absorb losses. Banks are required to adopt a forward-looking approach to capital management and to set capital levels in anticipation of possible adverse events or changes in market conditions.

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[5] Structural models of credit risk typically incorporate the principle of no-arbitrage valuation, which involves the dynamical replication of uncertain state-dependent cash flows of an asset in a complete market using a self-financing trading strategy. By generation of a dynamically riskless position consisting of the risky asset and its replication portfolio, resulting cash flows are discounted at a riskless rate. The basic principles of no-arbitrage theory and risk-neutral valuation are outlined in Neftci (1996) and Baxter and Rennie (1996). A more rigorous treatment is found in Karatzas and Shreve, (1988, 1998) and Musiela (1998).

The definition of bank capital eligible for regulatory purposes is defined in the Basel Capital Accord BCBS (1988) and its Market Risk Amendment BCBS (1996a). Regulatory eligible capital is divided into core capital (tier-I capital) and supplementary capital (tier-II capital). Core capital includes equity capital and disclosed reserves from post-tax retained earnings and is required to constitute at least 50% of regulatory capital. Supplementary capital amounts to the size of tier-I capital at most, and includes undisclosed reserves, revaluation reserves, general loan loss provisions, hybrid debt capital instruments and subordinated term debt. Goodwill and investments in non-consolidated financial subsidiaries and significant corporate investments are deducted from core capital.[6] The Market Risk Amendment establishes an additional tier-III capital consisting of short-term subordinated debt, which is exclusively determined to cover market risk.[7] Regulatory capital is measured against risk weighted assets. This total capital ratio must not fall below the level of 8%; for the core capital ratio a minimum of 4% is obligatory.[8] Specific capital requirements such as capital multipliers or position limits can be imposed by supervisory agencies on all material exposures.

Capital standards differ financial instruments into banking book exposures and trading book exposures. Banks must have clearly defined policies and procedures to determine which exposure to include and exclude from the trading book for capital adequacy purposes and risk management. The trading book consists of positions in financial instruments and commodities held either with the intent of trading or as a hedge of other positions in the trading book. Financial instruments constitute either assets or liabilities and involve primary financial claims and derivative instruments. Financial assets include cash, the right to receive cash or another financial asset, equity investments or the contractual right to exchange financial assets at potentially favorable terms. Financial liabilities subsume contractual obligations to deliver cash or another financial asset or the exchange of financial liabilities under conditions that are potentially unfavorable. Positions held with the intent of trading are projected for short-term resale to profit from actual or expected short-term price movements or to lock-in arbitrage profits and include proprietary positions and positions arising from client servicing and market making.

The extent to which an exposure can be marked-to-market on a daily basis by means of reference to an active, liquid two-way market determines the eligibility of an exposure as trading position. Basic requirements to qualify positions for trading book capital

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[6] Cf. BCBS (1988), p. 3ff and BCBS (2005b), p. 10

[7] Cf. BCBS (1988), p. 3ff including Annex I, p. 17ff, as well as BCBS (1996a), p. 7f. for a rigorous definition of eligible capital.

[8] The calculation of risk-weighted assets and the regulatory requirements for the use of internal models in the calculation of the market risk and the specific market risk charge are outlined in BCBS (1988) and BCBS (1996a), respectively. Numerous amendments and newsletters further detail the applications of risk weights.

treatment include (1) a clearly documented trading strategy, (2) position limits appropriately set and monitored, (3) daily mark-to-market or at least a daily assessment of model parameters in the case of mark-to-model valuations, (4) positions actively monitored, managed and reported on a daily basis, (5) ability of the bank to identify and hedge the material risks of the exposure, (6) external validation of a bank's own valuation of mark-to-model exposures.

Positions in the bank's own eligible regulatory capital instruments are deducted from capital. Trading positions in other financial institutions' eligible regulatory capital instruments will be deducted from capital at the discretion of supervisory agencies. Internal hedges of banking book credit exposure using trading book credit derivatives do not qualify for the mitigation of capital requirements unless the bank purchases credit protection from an eligible third party. Interest rate risk in the banking book is subject to the supervisory review of pillar-II of the revised standards.

Banking book positions subject to the treatment of the revised capital standards are basically categorized into five classes of corporate, sovereign, bank, retail and equity exposures. Within the corporate asset class, there are five sub-classes of specialized lending (project finance, object finance, commodities finance, income-producing real estate and high-volatility commercial real estate). Bank exposures include exposures to banks and securities firms.

The capital standards differ between expected loss (EL) and unexpected loss (UL) of exposures.[9] Unexpected loss is defined as the amount by which the incurred credit loss exceeds the expected loss. The amount of available regulatory (economic) capital is specified by supervisory agencies (banks) to cover unexpected credit loss. In order to assess the adequacy of capital, the capital endowment of financial institutions is compared to the experienced credit loss. Economic capital constitutes the capital available to achieve a target insolvency rate and to cover the predicted unexpected losses.[10] The majority of banks handle economic and regulatory capital requirements independently, however, a few banks include the cost of regulatory capital in their credit pricing methodology, thereby increasing the expected loss.

The revised capital standards enable banks to choose between four approaches to determine the capital requirements for their credit risk. First, financial institutions may stick to the established capital requirements set forth in the original Basel Accord of 1988 and its amendments. Second, under a standardized approach, risk weights of defaultable exposures refer to obligor ratings provided by external rating agencies. The two internal-rating-based (IRB) approaches rely on banks' internal assessment of the risk components

[9] Cf. BCBS (2006c), p. 86f.

[10] Cf. BCBS (1999a), p. 14.

PD, LGD and EAD in determining the capital charges of exposures. The foundation IRB approach requires that banks use internal PD estimates derived for obligors of a particular rating class, whereas supervisory agencies provide estimates of LGD, EAD and the effective maturity of exposures. A reference LGD of 45% for senior unsecured claims and 75% for subordinated claims is mandatory. The effective LGD of collateralized exposures is calculated by adjusting the reference LGD to the exposure net of the collateral. Under the advanced IRB approach, the bank's own estimates of PD, LGD and EAD from an internal credit assessment process are used and an effective maturity is calculated for each exposure. PD must refer to a one-year time period and will be set to 100% for obligors in default. LGD estimates must refer to specific EAD estimates.

Partial use of the IRB approach to cover only a few particular asset classes is permissible. The percentage capital requirement (CR) of an exposure is defined as the eligible capital required per monetary unit of a single exposure and takes into account PD, LGD, the correlation of defaults and a maturity adjustment.[11] The risk-weighted assets (RWA) are calculated by $RWA = CR \cdot EAD \cdot 12.5$. For small and medium size corporate borrowers, an RWA discount is incorporated using a reduced correlation assumption. The total risk-weighted assets aggregate the market risk, operational risk and credit risk RWA of exposures. An additional scaling factor at the discretion of supervisory agencies generates the aggregate level of minimum capital requirements.

Risk-weight functions provide capital requirements for UL, whereas EL is covered by credit loss provisions. A loss provisioning methodology identifies exposures to be evaluated for impairment on an individual basis. Impairments of individual loans result in specific loan-loss provisions, whereas collective impairments induce general-loss provisions. Loss charge-offs and recoveries must be carried out in accordance with the applicable accounting framework.

The total eligible provisions include specific provisions, partial write-offs, discounts on defaulted assets and portfolio-specific provisions, such as country risk or general-loss provisions. A positive difference between total eligible provisions and total expected loss can be allotted to tier-II capital under the IRB approach, whereas the standardized approach allows only the to inclusion of general provisions in tier-II capital.

Loans not subject to individual impairment must be grouped and impaired collectively. Factors likely to alter credit loss as compared to historical loss experience such as (1) changes in lending policy, (2) changes in relevant economic, business or market conditions, (3) changes in trend, volume and severity of past-due and low-quality loans, and (4) changes in the quality of the loan review system, should be considered in collective loss provisions.

[11] Cf. BCBS (2005b), p. 60.

Instruments of credit risk mitigation can be used to adjust PD, LGD or EAD estimates in a consistent way to lower capital requirements. Instruments of credit risk mitigation include third-party guarantees, collateral, securitization, credit derivatives and netting agreements and may generate types of risk that make risk reduction less effective. Credit mitigation risks include the inability to seize pledged collateral in a timely manner, refusal or delay by a guarantor to pay as well as legal, documentation and liquidity risk. The counterparty risk of trading exposures is covered as specific market risk under provisions of the market risk amendment (BCBS (1996a)).

On- and off-balance sheet exposures must be measured gross of specific provisions or partial write-offs. The EAD of facilities must not be less than the amount currently drawn. For on-balance sheet items, the netting of loans and deposits is recognized, with special adjustments for currency and maturity mismatches.[12] For off-balance-sheet items, the exposure is calculated as the committed but undrawn amount multiplied by a credit conversion factor depending on the type of instrument.[13] Effective maturity is set to 2.5 years using the foundation approach. Using the advanced IRB approach, the effective maturity calculates as McCauley-Duration given a zero yield-to-maturity. If contracted payments are not specified explicitly, the time-to-cash flow of the last payment is used as an alternative.

2.2.2 IRB Minimum Requirements

Minimum requirements for the use of the IRB approach include, amongst others, corporate governance, organizational, procedural and system standards, the methodology and operation of a rating system, the validation of procedures and system calibration including the estimates of risk quantities, as well as disclosure requirements.

As part of the loan approval process, a rating is assigned to each (potential) borrower or exposure. Rating assignments and periodic rating reviews are carried out by a credit analysis function that does not directly profit from taking credit risk exposures. An effective process must be in place for obtaining and updating all credit-risk-relevant and material obligor- and exposure-specific information in a timely way. Ratings must be reviewed and updated whenever relevant new information is received, at least annually.

A bank must have specific rating definitions, processes and criteria for assigning exposures to classes within a rating system. The term rating systems includes all methods, processes, controls, and data collection and IT systems that support the assessment of credit risk, the assignment of internal ratings and the quantification of default and loss estimates.

[12]BCBS (2006d), p. 45ff.

[13]BCBS (2005b), p. 22f.

Theory, assumptions and methodologies of the rating process must be documented as well as the empirical basis and statistical and mathematical methods used. Documented class descriptions and criteria must be clear and detailed enough to make possible a consistent assignment of rating grades to borrowers of similar risk. The rating criteria must be consistent with the bank's lending standard and must be applied consistently across business lines and geographical regions. All relevant and material information available must be used in the rating process.

A rating system which qualifies for the IRB approach must consider obligor-specific as well as transaction-specific risk factors. Obligor-specific risk factors determine an identical obligor rating for any transaction against the obligor, irrespective of its specific nature. Transaction-specific factors include collateral, seniority, product type and third-party guarantees. Under the advanced IRB approach, transaction-specific factors must be reflected exclusively in the LGD estimates, whereas PD estimates incorporate the obligor-specific default risk, so that a cross-default clause is incorporated. Under the foundation approach, LGD are equal for each obligor in a risk class.

A minimum of seven non-default grades and one default grade is mandatory. The specification of risk classes must avoid a concentration of the credit portfolio in a particular market segment or range of default risk. For LGD estimates, there is no specific number of LGD classes that is obligatory under the advanced IRB approach, however, grouping exposures into a single risk class with a widely varying LGD must be avoided.

Banks must have independent credit risk control units responsible for the design, selection, implementation and performance of internal rating systems. Credit risk control must be functionally and personally independent from management functions responsible for originating exposures. Credit risk control includes the testing and monitoring of internal ratings, the production and analysis of summary reports including historical default statistics, rating migration analysis, controls to ensure the consistent application of the rating process across departments and geographic entities, as well as the review and documentation of changes in the rating process. Furthermore, the credit risk control unit must actively participate in the development, selection, implementation and validation of rating models.

The variables of a credit scoring model must be reasonable predictors of credit risk. The model must be accurate on average across the range of bank's obligors and foreclose any material bias. A process must be in place to secure the quality of data inputs for its accuracy, completeness and appropriateness must be in place. The data used for model estimation must be representative of the population of banks' actual borrowers and exposures. Regular model validations include the monitoring of the model's performance, a stability review of model relationships and the testing of model outputs against realized outcomes. A rigorous statistical validation process, including out-of-time and out-of-sample tests, must be established.

Although default probabilities refer to a one-year period, a longer time horizon is to be considered in rating assignments. A conservative through-the-cycle rating standard is required, where a borrower rating represents the bank's assessment of the borrower's ability and willingness to fulfil its contractual obligations under stress scenarios of unfavorable economic conditions. The range of economic conditions included in the calculation must be likely to occur during a business-cycle in the industry sector and/or geographic region in question and be consistent with the current state of the economy. Credit scoring models and other standardized rating procedures generally utilize only a subset of the relevant information. Mechanical credit scoring is another source of rating errors. Experienced skills are required to ensure that all relevant and material information, including information outside the scope of the scoring model, is appropriately taken into account.

Banks using the advanced IRB approach must estimate a PD for each risk class of corporate, sovereign and bank exposures. Default probabilities must represent a long-run estimate of the one-year default rates in each class. Additionally, an appropriate LGD and long-run default-weighted EAD must be estimated. Internal estimates of PD, LGD and EAD must incorporate all relevant, material and available information from internal and external sources. The population of exposures in the estimation data should closely match the current exposure of the bank in terms of the relevant characteristics. Estimates must be based on historical experience and sufficient empirical data for the bank to be confident of the estimates' accuracy and robustness.

Exposures limits for single counterparties or groups of related counterparties must be established and monitored on a frequent basis. Aggregate limits of large exposures are recommended.

Credit risk concentrations arise from (1) exposures to counterparties in the same economic sector or geographic region, (2) exposures to counterparties whose financial performance is dependent on the same activity, and (3) credit risk mitigation involving a single collateral type or a single provider of credit protection. Concentration limits should be defined in relation to the bank's capital, total assets or overall risk level. Periodic stress tests of major credit risk concentrations should be conducted. Counterparty risk management should include the identification, measurement, management, approval and internal reporting of counterparty credit risk. CCR must take into account market, liquidity, legal and operational risk factors.

Credit risk management involves credit scoring, estimating and measuring of credit risk, as well as stress testing and the validation of the quality of credit risk assessment models.

An independent credit review process that involves a credit evaluation system must be consistently applied and identifies changes of credit risk characteristics in a timely manner. Independent internal audit functions review the estimations of PD, LGD and EAD on at least an annual basis. Rating and estimation systems designed and implemented exclusively to qualify for the advanced IRB approach are not accepted by supervisory agencies. A three-year track record is mandatory to gain supervisory recognition of a rating system for the advanced IRB approach.

Banks must have a robust system in place to validate the accuracy and consistency of rating systems, processes, and the estimation of all relevant risk components. Banks must regularly compare actual default rates with estimated PD for each risk class and be able to demonstrate that the actual default rates are within the expected rage for that class. Under the IRB approach banks, must complete such analysis for their estimates of LGD and EAD using historical data. The assessment of the performance of banks' own rating systems must be based on long data histories, covering a range of economic conditions and, ideally, one or more complete business-cycles. Banks must demonstrate that quantitative testing methods and other validation methods do not vary systematically with the economic cycle.

Stress tests must be used to assess capital adequacy. Stress testing must involve the identification of possible future changes in economic conditions with possible unfavorable effects on credit exposures. Appropriate scenarios include economic or industry downturns, market risk events and market illiquidity. The objective of stress tests is not to consider worst-case scenarios, but rather, scenarios of mild recession to assess their effect on PD, LGD and EAD estimates. The effect of stress conditions on regulatory capital adequacy must be assessed. Effects of the rating migration of obligors and the worsening of the bank's own rating should be considered in stress tests as well.

2.2.3 IFRS Accounting Framework

Accounting standards affect the earnings, risk and capital adequacy ratios of financial institutions and thus interact with capital requirements. The International Financial Reporting Standards (IFRS) are mandatory for financial institutions within the European Union. The supervisory definition of eligible regulatory capital refers to balance sheet items that are subject to accounting standards. Capital standards rely on a definition of loss that should not contradict the value credit exposures are recognized in the financial statement. Accounting standards relevant to the evaluation of credit exposures include IAS 39 which regulates the "Recognition and Measurement of Financial Instruments", whereas IFRS 7 stipulates disclosures about risk and performance of financial instruments.

IAS 39 permits the use of a fair value option in the valuation of financial instruments, if a fair value can reliably be obtained directly from observable market prices or from a robust valuation technique. Fair value accounting requires that a risk management system is in place which incorporates appropriate valuation methods to calculate reliable fair values.[14] The fair value option of IAS 39 requires a firm to decide irrevocably at the initiation of an exposure, whether a financial asset or liability will be measured at fair value to determine profit and loss. Criteria for determining the eligibility of exposures for fair value accounting are stated in paragraph IAS 39.9-39.11A.[15] The objective of fair value accounting is to avoid significant mismatches between the value of an exposure under accounting standards and economic criteria.[16] The exclusion of loans and receivables from fair value accounting supports the transparency and reliability of financial statements, as it is difficult to determine and validate reliable fair values of financial instruments without an observable market price from an active market. Furthermore, applying fair value accounting to liabilities without an observable market price would permit institutions to report profits from a deterioration of their own creditworthiness. Banks would be enabled to strategically manage earnings report and to misreport financial statements.

Banking book assets are mostly qualified as held-to-maturity and accounted for at accrued costs, whereas, trading book assets are generally assumed to be available-for sale and accounted for at fair value.

Supervisory agencies decline the use of fair value accounting for illiquid financial instruments to prevent unrealized gains of credit exposures from being recognized as regulatory capital.[17] Deficiencies in banks' risk management must not result in recognizing unrealized gains in the regulatory capital or deriving understated unrealized losses from unreliable fair values. The exclusion of loans and receivables from fair value accounting implements the supervisory view. Corporate loans of banking book exposures are accounted for at accrued costs so that unrealized gains in fair value are omitted from being recognized as capital, whereas unrealized losses are covered, establishing adequate loss provisions.

The Basel Committee recommends recognizing gains and losses which result from fair value accounting as tier-I capital, with the exception of gains and losses arising from changes in a bank's credit risk of liabilities. However, unrealized gains or losses resulting from exposures subject to fair value accounting must not alter regulatory capital in a way that would distort the economic condition of a bank. Supervisors agencies control for the level of cumulative unrealized gains attributable to the fair value option in relation to

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[14] Supervisory agencies encourage the use of systems that integrate accounting, risk assessment and capital adequacy functions for credit exposures.

[15] Effects of accounting standards on capital requirements will not be addressed for host contracts with embedded derivatives, hedge accounting or derivative contracts of production assets.

[16] Cf. BCBS (2005e), p. 7.

[17] In contrast, supervisory agencies consider the use of mark-to-model valuations to be an indispensable requirement for an adequate valuation and economic performance measurement of credit portfolios in risk management applications.

equity and regulatory capital.

The expected loss of credit exposures differ from credit loss provisions reported in financial statements for methodological reasons. Accounting standards allow the identified but not yet incurred credit loss of exposures recognized at amortized costs to be considered in loss provisions. IFRS fair value accounting does not allow for loss provisions. Differences between the level of loss provisions and expected losses under the Basel II framework partially result from the exclusion of recently originated loans and the fact that expected loss only covers default risk for a one-year time horizon. Under the revised capital standards, the eligible regulatory capital is adapted for any difference between credit loss provisions and expected credit loss of exposures.

For credit exposures carried at amortized cost, a methodology must be in place to specify loan loss provisions, if (1) bankruptcy or financial reorganization of the borrower is probable, (2) the borrower suffers from significant financial difficulties, (3) a breach of contract occurs, such as default or delinquency on interest or principal payments, or (4) the lender has granted concessions to the obligors to circumvent financial difficulties. The methodology for determining loan provisions is accepted by supervisory agencies, if the bank (1) maintains effective systems and controls for identifying, monitoring and addressing asset quality problems in a timely manner, (2) has analyzed all factors that significantly affect the collection of obligations, (3) has established an appropriate loan provisioning process.

After an identification of exposures, banks assess loans for impairment. Financial instruments carried at amortized costs are impaired individually by discounting expected outstanding cash flows using the original effective interest rate of the instrument. If an impairment on the basis of an individual loan assessment is denied, a loan is included in a group of loans with similar credit risk characteristics and the group is assessed for a collective impairment. Segmentation of loans for collective impairment is typically based on the type of loan, credit risk class, geographical location, collateral type and past-due status.

Under the IFRS framework, impairments are incurred as a result of effective events that impact the estimated future cash flows of the asset, whereas likely losses expected as a result of future events are not recognized. The definition of impairment events is based on objective and subjective criteria. Objective criteria refer to actions that are beyond the control of the bank, such as payments that are overdue by a minimum amount or by a minimum number of days. Subjective criteria depend on the bank's assessment of exposures and bank-initiated actions such as the granting of a payment delay. Events that trigger an impairment of exposures include (1) significant financial difficulties of the obligor, (2) a breach of contract such as default or failure to make in interest or principal payment, (3) concessions granted by the lender in response to the borrower's financial situation, (4) a probable default, (5) a change of the obligor's PD, which may coincide with a change of external rating, (6) a change in the LGD estimate, (7) a change in credit spread given a constant expected loss, (8) a change in the usage of a particular credit facility, (9) the disappearance of an active market, or (10) an expected decrease in estimated future cash flows of a group of exposures, for example due to unfavorable national or sectoral economic conditions.

It is assumed that a secondary markets exists for any credit corporate exposure considered in the prediction of credit portfolio risk, so that fair values reliably represent the economic risk to be covered by regulatory capital. The quantification of credit portfolio risk described in Section 5 relies on the assumption that the credit risk valuation model of Section 3 will receive the approval of supervisory agencies as an appropriate valuation method to provide fair values of credit exposures.

2.3 Fundamental Components of Credit Risk Models

2.3.1 Rating Methodology

A rating system involves the classification of borrowers or exposures into disjoint risk classes of homogenous credit quality estimates. Risk classes are typically defined by rating, country of risk and industrial sector of the exposure. The credit quality is defined either by the probability of default, expected loss or, more generally, by a credit score. Credit scoring models determine a credit quality score on the basis of quantitative and qualitative characteristics of a (potential) exposure, including PD, transition risk, LGD and EAD. Some credit scoring models provide an automatic credit assessment process for retail exposures, whereas other systems have a strong focus on expert judgement in the assessment of obligor and exposure characteristics.

With respect to the rating methodology, point-in-time (PIT) ratings and through-the-cycle (TTC) ratings must be differentiated. PIT ratings respond to changes in current business conditions and focus on the current economic perspectives of obligors, whereas TTC ratings tend to be stable throughout the business-cycle and assess obligors' performance during the entire business-cycle. A PIT rating system uses static and dynamic obligor-specific and aggregate macroeconomic information. PIT ratings of obligors adjust quickly to changing economic prospects. Overall PIT ratings of obligors are positively correlated with the economic cycle and tend to rise (fall) during economic expansions (downturns).

A TTC rating system uses static and dynamic obligor characteristics, but does not adjust ratings in response to changes in macroeconomic conditions, so that TTC ratings of obligors tend to be stable throughout an economic cycle and the distribution of TTC ratings of a fixed credit portfolio is not expected to change significantly during the business-cycle. During times of economic expansions, the mean quality of dynamic obligor-specific information in a PIT class tends to fall, compensating for improved macroeconomic conditions. A TTC rating migration occurs only if the change in dynamic obligor-specific characteristics deviates significantly from the average change of the class.

It is at the discretion of national supervisory agencies whether to allow for PIT ratings or TTC ratings, or both, under the IRB approach. However, the rating methodology should reflect the business model of a bank. For banking book exposures TTC ratings are more favorable, whereas PIT ratings are more suited to the pricing of credit-risky trading positions and the tracking of credit portfolio risk PIT ratings are suited. Treacy and Carey (1998) find that the rating systems of US commercial banks conform more closely to a PIT standard. The frequent allegation levelled at rating agencies, that they fail to provide timely PIT ratings that reflect current market conditions, has to be considered with respect to the rating model used. Rating agencies do not claim to provide PIT ratings and mostly employ hybrid rating models with characteristics of PIT and TTC rating methodology that incorporate cyclical changes in obligor ratings according to macroeconomic conditions with a long-run outlook.[18]

In conclusion, PIT rating systems are suited for mark-to-model valuation, economic capital allocation and short-term credit risk management of credit risky assets available for sale. In contrast, TTC rating systems inherently implement a hold-to-maturity approach and qualify for the credit provisioning of exposures held at accrued cost. In practice, most rating systems do not incorporate a TTC or PIT methodology in a pure form as outlined by the Basel Committee.[19]

The calculation of corporate exposures' capital charges as well as typical credit portfolio management applications refer to pooled unconditional PD. The pooled PD of a risk class designates the default probability assumed for each obligor in the class. The minimum standards for the internal rating process of IRB banks outline permissible approaches to estimate pooled PD, however, no single approach to the estimation and validation of pooled PD is prescribed. The pooled PD of risk classes can be estimated using the default experience of internal risk classes and an external risk class mapping. Since the capital requirements imposed by supervisory agencies rely on unconditional PD estimates, credit portfolio models intended to determine capital requirement must also perform unconditional credit risk forecasts on the basis of unconditional PD estimates.

[18]Cf. Moody's (1999), p. 6f, and Standard and Poor's (2002), p. 41ff.

[19]Cf. BCBS (2005d), p. 14f.

2.3.2 Probability of Default

The probability of default of a particular obligor is a forward-looking assessment of the likelihood that the obligor will fail to meet its contractual obligations or file for bankruptcy during a fixed time interval which is conventionally set at one year. A default rate is defined as the number of defaults in a risk class within a specified time interval of one year, in general, divided by the total number of obligors in the class at the beginning of the interval. Unlike the PD, the default rate is an ex post measure of the number of actual default events and refers to a set of obligors rather than to a single obligor.[20]

The definition of default is typically based on subjective conditions established in the loan agreements for corporate portfolios, whereas objective conditions are predominant in retail portfolios. A credit default of a borrower is triggered, if either or both of the following apply: (1) it is unlikely that the obligor will pay its obligations in full, or (2) any material financial obligation owed by the borrower is more that 90 days past due. Indicators for an unlikely payment are the borrower's filing for bankruptcy, non-accrued status of debt, charge-offs or specific provisions, sale of the credit obligation at a material credit-related economic loss, or consent of banks to a distressed restructuring. Typically, cross-default clauses of credit arrangements synchronize the default events of all exposures against a particular counterparty, so that the default probabilities of an obligor and its exposures are considered to be equal. The definition of default must be used consistently when PD, LGD and EAD are being estimated.

Transition or migration probabilities designate the likelihood that an obligor or exposure will migrate from one class of a rating system to another within one year's time. Probabilities of default and transition probabilities can refer to a risk-neutral probability measure or to a real-world probability measure. Risk-neutral default or transition probabilities can be derived from no-arbitrage credit pricing models that are calibrated using time series of cross-sectional price data of defaultable securities from efficient credit markets. In estimating the real-world default and migration probabilities of obligors, one can distinguished between direct and indirect methods. Direct methods provide a credit score that represents a single-obligor PD and includes statistical default prediction models such as Logit, Probit or Hazard Rate models. If credit scores of a rating model do not represent default probabilities as in the case of the default-prediction by discriminant analysis, indirect methods estimate the pooled PD of risk classes and use either historical migration rates and the default experience of obligors or external risk class mapping. Direct methods are typically connected to PIT ratings, whereas indirect methods mostly refer to TTC rating systems.

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[20]Cf. BCBS (2005a)

Methodologically, stressed and unstressed PD can be differentiated according to the economic state and the time period on which the credit risk assessment is conditioned. Stressed PD indicate the default probability of an obligor, conditioned on a specified stress scenario of unfavorable economic conditions, whereas unstressed PD comprise probabilities of default which are either conditional or unconditional on actual economic conditions.

Conditional PD indicate the likelihood that an obligor will default, assuming an extrapolation of current economic conditions. Conditional PD incorporate static and dynamic credit-quality characteristics of the obligor and current aggregate information. By virtue of the dependence on macroeconomic variables, conditional PD are negatively correlated to the credit cycle and tend to fall (rise) during economic upturns (contractions), so that, in principle, the deviations of an obligor's conditional PD from its long-run average are exclusively caused by the economic cyclic only. Unconditional PD indicate the likelihood of default under long-run average economic conditions, incorporate only obligor-specific information, are expected to remain stable throughout the business-cycle, and do not show a significant correlation with the economic cycle.

Pooled PD reflect the central tendency (mean or median) of the individual PD of obligors assigned to a risk class. The pooled PD of a risk class change during a business-cycle, where the dynamic properties of the fluctuation depend on methodology and stress characteristics of the PD considered. The utilization of the direct method of PD estimation dominates, if pooled PD condition on the business-cycle or are intended to incorporate stress scenarios. The estimation of pooled PD by statistical default prediction models involves (1) the estimation of individual default probabilities for each obligor, (2) the classification of obligors into segments of homogenous credit risk, and (3) the derivation of the pooled PD of a risk class that reflects the individual PD of all obligors in the class. Single-obligor PD can be estimated regardless of the rating methodology applied. According to the minimum requirements of the IRB approach, banks are allowed to calculate a simple average of single-obligor PD estimates to estimate the pooled PD of a class. Testing the accuracy of pooled PD estimates involves validating the risk class assignment as well as the estimation model of single-obligor PD.

The estimation of single-obligor PD by statistical default risk models incorporates obligor-specific information as well as aggregate information on the economic environment the obligor operates in to assess the obligor's ability and willingness to repay its debt. Obligor-specific information is unique to a particular obligor and can be static or dynamic, such as economic sector affiliation or financial leverage. Aggregate information refers to the time a PD is estimated, affects many obligors jointly and typically includes macroeconomic variables such as exchange rates, unemployment rates or GDP growth. Aggregate and dynamic obligor-specific information are often highly correlated, such as GDP growth and increased of revenues.

The default experience of proprietary corporate loan portfolios or public credit markets is used to estimate pooled unstressed PD of PIT risk classes. Default events in a risk class are typically correlated to joint background factors, so that the default rates of risk classes deviate from the pooled unstressed PD due to unexpected changes in economic conditions. Averaging one-year default rates of a risk class gives the long-run default frequency. Over time, differences between pooled unstressed PD and observed default rates cancel out and the long-run average default rate is expected to converge toward the average pooled unstressed PD of the class. In the calculation of default rates, changes in rating methodology, underwriting standards and default definitions must be reflected and, according to the minimum requirements of the IRB approach, the length of the observed default experience must cover all obligors of a risk class for a period of at least five years. In principle, the pooling of data across institutions is permitted.

Finally, the pooled PD of risk classes can be determined from external risk class mappings. Using a mapping of the internal and external rating scale, the pooled PD estimates of external risk classes are assigned to obligors of the corresponding internal risk class. However, the suitability of the external mapping relies on the consistency of the internal and external rating methodology. The mapping of rating systems must involve a comparison of the internal and external rating criteria and a comparison of assigned internal and external rating classes of any common borrower. Rating agencies mostly derive estimates of the pooled PD from public credit markets, so that the conformity of PD methodology, sample population and default definition with internal risk classes must explicitly be ensured.

Despite its simplicity, the external mapping poses some difficulties with respect to rating validation. The validation of pooled PD estimates from an external risk class mapping involves the validation of the accuracy of pooled PD of the external rating system and the validation of the mapping itself. For external rating systems, the estimation of pooled PD pose the same challenges as it does for internal rating systems. If historical default experience is used, pooled PD must be checked against long-run default rates, and for an external statistical default risk model the same validation procedures apply as for an internal model, so that the main benefit of using external PD estimates is the availability of a more extensive data set in the time- and cross-sectional dimension. The mapping of risk classes is stable in time, if the bank and the external rating provider use unchanged rating methodologies. If this is not the case, the mapping is time-inhomogenous and eventually the pooled PD estimates of external risk classes will need to be adapted to the internal rating system.

In a PIT risk class, obligors share similar conditional PD, and the PIT ratings of obligors change if obligors' conditional PD change, so that the volatility of obligor ratings is higher than in a TTC rating system. Conditional PD decrease if business conditions improve and obligors tend to migrate upwards out of a PIT risk class, whereas previously lower-quality obligors migrate into that class. The reverse applies for adverse business conditions. In contrast, the variation in the pooled unconditional PD of a PIT risk class is positively correlated to changes in economic conditions. This seemingly paradoxical effect is caused by obligors with currently improved conditional PD and high unchanged unconditional PD who migrate into a PIT risk class, while obligors with lower unconditional PD migrate upwards. In summary, the pooled conditional PD of a PIT risk class remain stable throughout the economic cycle, while pooled unconditional PD tend to rise as business conditions improve and slump during recessions.

In a TTC risk class, all obligors share a similar unconditional PD. The TTC ratings as well as the unconditional PD of obligors can fluctuate over time, however, changes are uncorrelated to the economic cycle. Obligors migrate in and out of the TTC risk class as their particular business prospects change beyond current economic conditions, so that strong cyclical migration patterns do not occur. In contrast, the conditional PD of obligors in a TTC risk class decline (rise) when business prospects improve (deteriorate). In summary, the pooled unconditional PD of a TTC risk class remain stable as economic conditions change, whereas the variation of pooled conditional PD is negatively correlated to changes in the economic cycle and pooled conditional PD of TTC rating classes rise (decrease) during economic recessions (upswings).

Methods used for the estimation of single-obligor PD must reflect the type of rating system and the usage of PD estimates. Econometric models based on macroeconomic, statistic or fundamental factor models are prevalent in estimating conditional PD.[21] Since conditional PD are not suited to determine capital requirements under the IRB approach, further considerations are restricted to the estimation and validation of unconditional PD estimates from the historical default rates of risk classes.

A natural estimator for the pooled conditional default probability p_{t+1} of a risk class in period $t+1$ is the default rate $\hat{p}_t = d_t/n_t$ of n_t obligors at the beginning of the preceding period t with d_t defaults observed. However, as pointed out by Bühler, Engel, Korn and Stahl (2002), the default rate $\hat{p}_t$ of a risk class is a biased estimator for the pooled unconditional PD p with strictly positive approximative variance $p(1-p)\rho^{def}$ for $n_t \to \infty$, if a positive homogenous correlation $\rho^{def} > 0$ is present in the class, so that the central limit theorem does not apply and the default rate $\hat{p}_t$ is not a consistent estimator of p. An unbiased estimator of p is the mean default rate $\hat{p} = \sum_{t=1}^{T} \hat{p}_t/T$ of the risk class across $t = 1, ..., T$ periods.[22] Assuming a time-invariant default correlation ρ^{def}, the mean default rate is consistent if $n = \sum_{t=1}^{T} n_t \to \infty$ and $n_t/n \to 0, \forall t$, which is equivalent to

[21] Cf. Hamerle, Liebig and Scheule (2002, 2004).

[22] Cf. Huschens and Stahl (2004), p. 6.

$T \to \infty$.[23] In most practical applications, however, the number of observation periods is typically small, and due to the default correlation ρ^{def} unconditional PD estimates include large estimation errors if the default rates of only a few periodic observations of a credit portfolio are available. To cope with this problem, Koyluoglu and Hickman (1998b) as well as Gordy and Heitfield (2000) additionally account for the correlation of credit defaults in the estimation of pooled unconditional PD of a single risk class. With default probabilities and inner-class default correlations being unobservable, Gordy and Heitfield estimate both parameters simultaneously using a multi-period maximum-likelihood-estimator (MLE). Using a one-factor asset value default model with a standard-Gaussian single factor z_t and time-invariant asset correlation ρ^a in a risk class with n_t^{rc} obligors, the pooled conditional probability of default $p_{t|z_t}$ is calculated as follows:

$$p_{t|z_t} = p_t(z_t; p, \rho^a) = \Phi\left(\frac{\Phi^{-1}(p) - \sqrt{\rho^a} z_t}{\sqrt{1-\rho^a}}\right). \tag{2.1}$$

Conditional on the factor return z_t, default events in period t are stochastically independent and the number of defaults d_t is binomially distributed with probability $p_{t|z(t)}$. This conditional independence of defaults results in the likelihood function

$$L(p, \rho^a; d_t, n_t) = P(\hat{p}_t = d_t/n_t) = \int_{-\infty}^{\infty} \binom{n_t}{d_t} p_{t|z_t}^{d_t} \left(1 - p_{t|z_t}\right)^{n_t - d_t} \varphi(z_t) dz \tag{2.2}$$

of the unconditional default probability p, which is equal to the probability function of the default rate $\hat{p}_t = d_t/n_t$ and constitutes a mixture of a binomial distribution with probability $p_{t|z_t} = p_t(z_t; p, \rho^a)$ and the standard normal distribution of the factor z_t. Enhancements of the basic likelihood function in (2.2) incorporate multiple time periods, risk classes and refined correlation structures into the estimation of unconditional PD.[24] For example, Huschens, Vogl and Wania (2003) propose a simultaneous multi-period MLE with the likelihood function

$$\mathcal{L}(p) = \prod_{t=1}^{T} \int_{-\infty}^{\infty} \prod_{rc=1}^{n_{rc}} \binom{n_t^{rc}}{d_t^{rc}} p_{t,rc|z_t^{rc}}^{d_t^{rc}} \left(1 - p_{t,rc|z_t^{rc}}\right)^{n_t^{rc} - d_t^{rc}} \varphi(z_t^{rc}) dz_t^{rc} \tag{2.3}$$

being a convolution of mixture distributions for the vector $p = (p^1, ..., p^{n_{rc}})$ of the unconditional default probabilities of n_{rc} risk classes with the conditional default probability $p_{t,rc|z_t^{rc}} = p(z_t^{rc}; p^{rc}, \rho_{rc}^a)$ and given the number of defaults d_t^{rc} and the number of obligors n_t^{rc} of risk class rc in period $t = 1, ..., T$. Note that the risk class factors z_t^{rc} in (2.3) are assumed to be independent. Estimation results for (2.2) and (2.3) are not satisfying,

[23] Cf. Höse and Huschens (2003a), p. 158.

[24] Cf. Gordy and Heitfield (2002), Höse and Huschens (2003b).

however, as the identification of parameters is insufficient, especially if different factors are involved and the asymptotic properties of the MLE cannot be determined analytically. Simulation results reveal that the MLE perform well in reproducing unconditional PD, whereas estimated asset correlations are biased downward and produce large standard errors.[25]

In summary, the single-period default rate $\hat{p}_t$ may be used to estimate pooled conditional PD of a PIT rating system, whereas the average default rate of risk classes in time is more suited for estimating unconditional PD of TTC ratings systems. In a situation in which historical default experience is scarce or there are structural breaches in the rating methodology or default definition, the unconditional PD of risk classes can be estimated from the default rates of risk class using the MLE estimator in (2.2) and (2.3).

2.3.3 Loss-Given-Default and Recovery Values

Loss-given-default is a measure of the loss severity in the case of the default of an exposure, expressed as a percentage of the EAD.[26] LGD estimates represent the ex-ante expectation of the loss conditional on the default of an exposure that is yet non-defaulted, whereas the realized LGD or "loss rate"-given-default is an ex post measure of the loss severity of a defaulted exposure. The LGD of a particular exposure is assumed to depend on a limited set of characteristics such as the type of credit product, estimated PD or risk grade, seniority, collateral and country of origination. Complementary, LGD = 1 - RR can be expressed in terms of a recovery rate RR, which is defined as the expected or actual discounted value of uncertain or actual recoveries net of workout costs at the time of default divided by the EAD amount.

In credit risk modelling, the LGD is assumed to be either deterministic or random. In the latter case, the LGD of exposures with equal properties are typically assumed to be i.i.d. In the specification of LGD distributions, pooled information from a bank's or public loan loss experience from rating agencies or supervisory authorities are often used, as well as rating agencies' corporate bond LGD data. Obviously, for reasons of estimation conformity, the type of instrument, default definition, reference value, risk grade, regional and seasonal origin of pooled LGD data must coincide.

For the estimation of LGD, there are subjective methods, such as expert judgement, and objective, mostly quantitative methods. Objective methods are subdivided into implicit and explicit methods. Implicit methods do not take into account single-exposure loss rates, but use model-derived expected credit loss of single exposures or the historical aggregate

[25] Cf. Gordy and Heitfield (2000) and Düllmann and Scheule (2002).

[26] Cf. BCBS (2005b), paragraph 468–473.

loss of credit portfolios for LGD estimates. Implicit market LGD methods derive LGD estimates from market prices of non-defaulted credit exposures, such as corporate bonds or credit default swaps (CDS), using an asset pricing model. Though market-implied LGD estimates typically suffer from a lack of identification in the decomposition of credit spreads into PD and LGD components, they are considered to be the only manageable way to estimate appropriate LGD for large, "too-big-to-fail" corporate exposures. For retail portfolios implicit historical LGD estimates are derived from PD estimates and the aggregate loss experience of a portfolio.

Explicit methods derive LGD estimates for non-defaulted exposures from loss rates of defaulted claims. The actual credit loss of single exposures can be computed either from market prices of defaulted bonds or loans (explicit market LGD) or by discounting the expected cash flows, including workout cost from the date of default to the end of the recovery process (workout LGD). Factors that affect the workout LGD are (1) the amount and date of cash and non-cash recoveries, (2) direct and indirect workout cost, (3) the definition of workout completion, (4) the treatment of recovery profits and (5) the discount factor, as well as (6) the reference value of the exposure.

Cash recoveries are easy to handle in LGD calculations. If all cash flows from the date of default to the end of the recovery process are known with certainty, to obtain actual LGD one subtracts the value of discounted net recoveries from a reference value conventionally set to the face value of the debt. Direct workout costs, such as legal costs or the costs of the appraisal of collateral are associated with a particular exposure. Indirect costs, such as the office and staff costs of the workout department emerge from the recovery process itself, and the allocation of indirect costs to defaulted exposures affects the estimation of workout LGD.

The recovery process is definitely complete when all non-cash recoveries, such as collateral, repossessions or restructured claims, have been sold to a third party. However, it seems more appropriate to consider the recovery process complete when the non-cash recoveries are transferred, because the change in the management of seized assets may impact valuations, and because costs attributable to the workout process can no longer be unambiguously identified. In principle, the exposure to credit risk is terminated at the time of recovery transfer, and other sort of risks, such as market risk become relevant afterwards. Furthermore, the time lag between the time of transfer and the disposal to a third party can be considerable, which prevents recently defaulted exposures with uncompleted workout and distorts effective LGD estimates from being taken into account. Hence, non-cash recoveries are often transformed into artificial cash recoveries including a haircut to the book value of repossessed goods which is carried out to be prudent. Obviously, workout and collection expertise significantly impacts recovery rates and LGD estimates.

Supervisory agencies require LGD estimates to be non-negative. In this context, censoring recovery profits does not interfere with the definition of default in PD estimates, whereas the truncation of recovery profits biases LGD estimates.

Discount rates must reflect the uncertainty of the recoveries to be received over a workout period of unknown length. The recovery value can be computed by discounting expected net recoveries using a risk-adjusted discount rate or by discounting certainty-equivalent cash flows at a riskless rate. The impact of the discount rate on LGD estimates is particularly important if the recovery period is long. Theoretically, appropriate risk-adjusted discount rates should be derived from liquid markets for recovery claims. However, as such markets typically do not exist, the historical or current rates of conventional credit markets are used. Historical discount rates are fixed at the date of default, and typically, either the contractual rate of the original exposure or a suitable rate for assets with a similar risk to that of the recovery claim is used. Current discount rates are fixed at each date on which recoveries are valued and effectively assess the marketability of the recovery claim, which facilitates the comparison of LGD estimates of different exposures.

The loss specified by the LGD represents the economic loss incurred by the lender from the default of an exposure, which may differ from the credit loss considered in financial accounting. For example, explicit market LGD typically compare the market price of a credit-risky asset shortly before the default event with the market price of the asset 30 days after the date of default. Compared to workout LGD, the use of explicit market LGD is straightforward because neither allocations of costs nor discount rates need to be taken into account. Though observable prices of defaulted assets are scarce, most rating agencies apply this approach.

In credit pricing applications, regression models are used for LGD forecasts that are conditional on current economic conditions. Though the predictive power of regression-based LGD estimates can be readily assessed by out-of-time and out-of-sample tests, the advanced IRB approach requires LGD estimates to represent long-run default-weighted average loss rates instead of point-in-time estimates.

The revised capital standards require LGD estimates under the advanced IRB approach to represent conditions during an economic downturn. Downturn conditions can be characterized by periods of expected negative GDP growth, increased unemployment rate or credit default rates, or by periods in which other risk factors, such as collateral values, are expected to jointly affect default and recovery rates. Loss rates are positively correlated to default rates, so that LGD estimates, which are assumed to be fixed or stochastically independent from default rates, will result in an underestimation of Credit-VaR. To qualify for the advanced IRB approach, LGD estimates must not be lower than the long-run average loss rate of defaulted exposures during periods of increased credit loss for the exposure type in question. The loss experience must span a complete credit cycle of at least seven years and contain all defaults in a bank's credit portfolios within this time frame. Obviously, the definition of default must be the same for PD and LGD estimates. No particular method is prescribed for the validation of LGD estimates by supervisory agencies,[27] but an assessment of LGD estimates under conditions of an economic downturn, along with a comparison to external LGD estimates and a backtesting of actual LGD against a bank's LGD estimates is recommended.

Empirical LGD studies are available for corporate bond and syndicated loan markets and include a wide range of actual LGD.[28] Different studies find average actual LGD for the overall US corporate bond market of between 58% and 78%, whereas for senior secured (senior subordinated, subordinated) corporate bonds average loss rates fall into the intervals from $42\% - 47\%$ ($58\% - 66\%$, $61\% - 69\%$). For senior secured (senior unsecured, commercial) loans, average actual LGD in the interval from $13\% - 38\%$ ($21\% - 48\%$, $31\% - 40\%$) have been observed. Obviously, loans suffer from lower actual LGD than corporate bonds, which can be attributed to stricter covenants and higher collateral pledged in loan contracts.

The distribution of single-exposure LGD is mostly found to be unimodal, highly dispersed and skewed to low LGD. Factors found to affect actual LGD are predominantly seniority and the type of collateralization of claims, macroeconomic conditions at default, industry affiliation of the obligor and the liquidity of collateral. In periods of high default frequency, loss rates are found to be higher than in periods of infrequent defaults. Liquid collateral such as cash and accounts receivable yield lower loss rates than illiquid collateral such as property, plant and equipment. Industries that provide less liquid collateral show higher actual LGD. The size of the borrower does not seem to affect LGD, whereas loss rates increase with the amount of other outstanding debt, especially for unsecured loans. The effect of loan size is therefore unclear.

2.3.4 Exposure-at-Default

The revised capital standards define the exposure-at-default of on-balance-sheet and off-balance-sheet items as the respective expected net credit exposure of the obligor upon default. For on-balance sheet facilities, the EAD estimate must not be lower than the amount currently drawn, subject to on-balance-sheet netting. Off-balance-sheet items are converted into loan equivalent exposures (LEE) using mandatory credit conversion factors (CFF) under the standardized approach, whereas the advanced IRB approach requires the estimate of a credit conversion factor for each off-balance-sheet item, based on the expo-

[27] Cf. BCBS (2005d), p. 71f.

[28] For a comprehensive overview of empirical LGD studies, see BCBS (2005d), p. 77ff.

sure type. EAD estimates must represent the long-run default-weighted average exposure of similar facilities and borrowers, including a security margin subject to estimation error. The time period of EAD estimates must ideally cover a complete economic cycle, but in any case should not be less than seven years. If a positive correlation is expected between the default frequency and the magnitude of the EAD, the margin of conservatism inherent in the EAD estimate must be larger. EAD estimates of exposure types that are volatile over the economic cycle, must incorporate economic-downturn conditions.

The EAD estimate of a credit exposure is either determined by fixed contractual payments to be received or it is uncertain and depends on future random events as in the case of credit facilities or credit-risky OTC derivatives. The uncertainty about the exposure at a potential default event can have two reasons:

First, the future usage of a borrower's credit facility is uncertain. Typically, credit facilities are used to a low extent and draw-down rates increase not before the credit quality of the obligor deteriorates considerably, which reflects the reduced availability or higher costs of alternative funding. Conceptually, EAD estimates for credit facilities include an estimate of additional future draw-downs on the basis of the current usage of the agreed commitment.[29] Second, the market value of derivative exposures with counterparty risk is uncertain, with the LEE imposed being equal to the instrument's current market value plus a surcharge for potential future exposure.

Given a mark-to-market recognition of credit exposures, no-arbitrage credit pricing models make it possible to derive EAD estimates from model-inherent assumptions on the recovery claim at the time of default. Four definitions of the recovery claim can be distinguished: (1) recovery-of-face-value specifies the recovery claim as the face value of the debt; (2) recovery-of-market-value claims the market value of the debt immediately before the default occurs; (3) recovery-of-treasury value defines the recovery claim as the present value of the remaining contractual payments of the debt discounted at default-free rates; (4) recovery-of-firm-value applies to firm value models, where the value of the firm's assets at the time of default determines the recovery claim of creditors. The choice of the definition of the recovery claim is often a question of model consistency, since the definitions of EAD and LGD are interrelated.

Public empirical evidence on EAD estimates are scarce. The current usage of a commitment is a leading indicator of the expected EAD. The type of the credit exposure, the credit characteristics and the type of borrower typically impact EAD.[30] In contrast to fixed rate facilities, the usage of floating commitments depends on the variation of the

[29]Frequently, the draw-down rate of a credit facility is assumed to be a deterministic function of the obligor rating, so that the expected usage of a credit facility aggregates the draw-down rates of rating classes multiplied by the migration probability during a risk period.

[30]Cf. BCBS (2005d), p. 94f.

underlying benchmark rate. The usage of revolving and non-revolving facilities differs and the time-to-maturity of a commitment is supposed to be positively related to the EAD. The borrowers' access to alternative sources and forms of funding reduces the EAD. Empirical findings indicate that the usage of credit facilities at the time of default increases with the rating of the obligor at the time the commitment was granted. Apparently, banks require more restrictive covenants for obligors with low credit quality which constrains draw-downs if the obligor faces financial difficulties. In this context, restrictive covenants may reduce the EAD at the cost of a higher PD.

2.3.5 Definition of Credit Loss

The definition of credit loss affects the distribution of portfolio credit loss, Credit-VaR and the actual portfolio performance used in the validation of portfolio credit risk models. Without restriction of generality, credit portfolio loss aggregates the individual credit losses of N credit exposures in the interval from forecasting time $t = 0$ to forecasting horizon $t = 1$. In market risk management, VaR refers to a synthetic performance measure that is used only in market risk measurement and backtesting of the VaR model, and does not enter into financial accounting or internal performance measurement. This 'buy-and-hold-performance' considers a trading portfolio held constant during the forecasting interval, so that profits from intra-day trading do not pollute the change-of-portfolio value. Likewise, synthetic credit performance measures are defined for a capital-related credit risk management.

The criteria for a definition of credit loss that are appropriate for a model-based determination of capital requirements include:

- consistency of performance and risk definition
- robustness across valuation models and model configurations
- independence of accounting standards applied
- prudence

Obviously, the definition of credit performance used in the calculation of Credit-VaR and the measurement of actual portfolio performance must be the same. Omitting market risk, liquidity risk and operational risk, the performance measure must reflect only changes in the value of credit portfolios that result from changes in the credit risk of exposures or from changes in time.

Second, the applied risk model affects how credit performance is measured. Default-only models recognize loss due to credit default, omit the deterioration of exposures' credit quality and exclude positive credit performance. The difference between the notional value and the recovery payment defines the credit loss of an exposure. Mark-to-market models derive credit values either from a rating of the obligor or exposure itself, or alternatively from continuous credit-risk-indicating state variables, using a parametric valuation model. Changes in exposures' credit quality apart from default should be recognized and positive credit performance be allowed, if it is convenient. The robustness of a performance measure requires that Credit-VaR and the credit performance of exposures are comparable across credit risk models of different model categories, even for extreme parameter specifications and portfolio compositions.

Third, the recognition of credit performance differs under different accounting categories. Book value accounting at amortized costs is linked to a held-to-maturity horizon, whereas fair value accounting incorporates the pre-mature resolution of credit exposures. Book value accounting at amortized costs acknowledges effective net cash flows and non-cash-related impairments in profit and loss. Fair value accounting recognizes positive and negative changes in the mark-to-market value of exposures differently. Although the Basel Committee disclosed "Sound Practices for Loan Accounting" (BCBS 1998), the valuation of credit exposures without observable market prices in financial accounting is, to a certain extent, a political question and provisioning rules provide some leeway to influence the recognized profit and loss. Capital standards require credit performance to represent economic loss. For marketable exposures, adverse changes in fair value constitute economic loss, whereas for exposures held-at-amortized-costs, changes in book value are not adequate to represent economic performance. A performance measure used in credit risk management must therefore be independent of the credit assessment of the relevant financial accounting standards.

Fourth, supervisory agencies require credit performance to be measured in a conservative way. Therefore, netting of unrealized mark-to-model profits of exposures without observable market price against credit loss from marketable exposures or incurred by obligor default should be prevented. It is controversial whether and how interest payments enter into credit performance. Interest received from credit exposures includes refinancing and administration costs, a compensation for bearing the default risk and a profit margin. Credit risk compensation and profit margins are proprietary-information of a bank, vary across institutions and should not be subject to supervisory recognition in setting capital requirements. A conservative measure of credit performance used to set adequate capital requirements must therefore omit unrealized changes in mark-to-model values and interest income.

In principle, the credit loss L^i of a credit exposure $i = 1, ..., N$ with fixed periodic interest payments is derived from the difference between a reference value and the mark-to-model valuation D_t^i of the credit exposure at time $t = 1$ net of accrued or paid interest. The value of the credit portfolio is set to be $D_t = \sum_{i=1}^{N} D_t^i$ at time t. Additionally, the value of a portfolio of synthetic loans with identical but non-defaultable cash flows is defined by $B_t = \sum_{i=1}^{N} B_t^i$, with the individual default-free bond value B_t^i calculated by discounting the contractual cash flows of exposure i using the default-free term structure.

The adaption of the loss definition in market risk modelling by defining credit loss $L^i = D_0^i - D_1^i$ of exposure i as the change of credit valuations during the forecasting horizon is not adequate for the setting of conservative capital requirements, because marketable exposures and exposures without observable market prices are taken into account and unrealized profits would enter into portfolio performance.

Three definitions of credit loss will be considered below. At first, in line with supervisory agencies' objection of fair value accounting of non-marketable exposures, credit loss

$$L^i(D_0^i) = \frac{(D_0^i - D_1^i)^+}{D_0^i}, i = 1, ..., N \tag{2.4}$$

is defined in percentage terms of the original credit valuation D_0^i, with positive changes in the mark-to-market value of exposures being excluded. Correspondingly, portfolio credit loss is defined by $L(D_0) = \sum_{i=1}^{N} D_0^i L^i(D_0^i)/D_0$. The percentage notation facilitates the comparison of portfolio credit loss to capital requirements, and loss distributions of different portfolios can easily be compared. Notice that interest accrued or paid during the risk interval is not included in $D_t^i, t = 0, 1$.

Second, the reference value of credit performance is set as the value B_1^i of the non-defaultable contractual cash flows of exposure i. The percentage credit risk discount

$$L^i(B_1^i) = \frac{B_1^i - D_1^i}{D_0^i}, i = 1, ..., N \tag{2.5}$$

in terms of D_0^i omits interest accrued or paid until $t = 1$. The portfolio performance $L(B_1) = \sum_{i=1}^{N} D_0^i L^i(B_1^i)/D_0$ aggregates the individual credit risk discounts. Since D_t^i never exceeds B_t^i, a positive credit performance is by definition impossible. The reference value of B_1^i is justified by the objective to ensure adequate capital to cover credit risk, with credit risk understood to be the difference between the mark-to-model value of credit exposures and the value of its non-defaultable equivalent.

Even if the credit quality of an exposure remains constant, credit valuations change in time, because the constant interest paid, in general, does not incorporate a fair compensation of default risk for any interval. To capture this effect, the credit valuation expected in the forecasting horizon is taken into account in a third definition of credit loss:

$$L^i(\mathbb{E}[D_1^i]) = \frac{(\mathbb{E}[D_1^i] - D_1^i)^+}{D_0^i}, i = 1, ..., N \tag{2.6}$$

Setting the reference value equal to $\mathbb{E}[D_1^i]$ corresponds to the loss definitions preferred by Gupton et al. (1997), Ong (1999) and Crouhy et al. (2000). Positive credit performance and interest accrued or paid is excluded and portfolio loss is defined by $L(\mathbb{E}[D_1]) = \sum_{i=1}^{N} D_0^i L^i(\mathbb{E}[D_1^i])/D_0$. Furthermore, loss definition $L(\mathbb{E}[D_1])$ corresponds to the supervisory agencies' objective that unexpected loss be covered by capital requirements. In Chapter 5, the loss definitions $L(D_0)$ and $L(\mathbb{E}[D_1])$ are used to derive loss distributions for the calculation of Credit-VaR and the backtesting of the credit portfolio model in 3.5.

2.3.6 Concentration and Dependence of Credit Risk

Credit risk concentrations constitute a serious threat to the solvency of a bank. A risk concentration is defined as a single large exposure or a group of exposures with the potential to induce losses, which are large relative to a bank's capital, its total assets or risk limits, and that threaten the solvency and core operations of a bank.[31] Credit risk concentrations arise from direct exposures to obligors and guarantees of third-party protection, and comprise material exposures against (1) a single counterparty, (2) a group of connected counterparties, (3) a particular industry or economic sector, (4) a geographic region, (5) an individual foreign country or a group of countries, whose economies are closely interrelated, (6) a certain type of credit risk related products, (7) a certain type of collateral, or (8) exposures of the same maturity.

In situations of market stress, credit risk concentrations materialize by a joint adverse effect on the credit quality of each exposure that is part of the concentration. Risk components affected by stress scenarios are default events, rating migrations, or more generally, a change in credit quality, LGD and EAD. Empirical evidence confirms, that the cyclical pattern of default rates coincides with the economic cycle, and from the business interactions of firms, one can conclude that conditional default probabilities vary in time in a common way, so that obligor defaults are stochastically dependent in time.[32]

A dependence concept incorporates the joint variation of credit risk components in portfolio credit risk modelling. Credit risk dependence is typically implemented by models of orthogonal or correlated risk factors that represent either macroeconomic, obligor-specific or latent statistical factors and affect the credit quality of any exposure in a risk class in a common way. Though intuitive in nature and empirically validated, risk models typically neglect the dependencies between PD, LGD and EAD estimates of exposures, whereas

[31] BCBS (2005b) defines a single large exposure to comprise ten percent or more of a bank's capital and requires a concentration exposure not to exceed more than twenty-five percent of capital.

[32] Löffler (2001) separates the cyclicity of default probabilities from the random noise included in default rates.

the dependence of credit risk components across borrowers is well-established.[33]

Copulas provide a flexible way to represent the dependence structure of multi-variate random variables independently of their marginal distributions. For a general discussion of dependence concepts, see Joe (1997) and Mari and Kotz (2001). A basic introduction to copulas is provided by Nelson (1999). The univariate marginal distributions and the correlation structure of random variables are not sufficient to unambiguously determine the joint distribution function denoted as copula.[34] Instead, the copula and the marginal distributions must be given explicitly to specify the distribution of a multi-variate random variable. In general, correlation structure and marginal distributions can be determined from a multi-variate distribution, but not vice versa. Formally, a copula $\mathbb{C}$ of n uniform random variables $U_1, ..., U_n$ is defined by the joint distribution function

$$\mathbb{C}(u_1, ..., u_n; \rho) = P[U_1 \leq u_1, ..., U_n \leq u_n]. \tag{2.7}$$

where ρ represents a general concept of correlation that is not necessarily restricted to typical coefficients of correlation such as Pearson's linear correlation, Spearman's Rho or Kendall's τ. Replacing $U_1, ..., U_n$ with univariate marginal distribution functions $F_1(\tau_1), ..., F_n(\tau_n)$ yields the copula

$$\mathbb{C}(F_1(\tau_1), ..., F_n(\tau_n); \rho) = F(\tau_1, ..., \tau_n) \tag{2.8}$$

in terms of the multivariate distribution $F(\cdot)$. Sklar's Theorem proves the existence of a unique copula for any multivariate distribution F with continuous univariate marginal distributions $F_i, i = 1, ..., n$.[35] In contrast, given a set of univariate distributions, the existence of a suitable copula to attain any arbitrary correlation structure is not ensured, whereas marginal distributions $F_i(\tau_i) = \mathbb{C}(F_1(\infty), ..., F_i(\tau_i), ..., F_n(\infty); \rho)$ can easily be derived from the copula $\mathbb{C}$. Copulas are independent under increasing and continuous transformations of the marginals. In the Gaussian case, any m-dimensional marginal of an n-variate Gaussian copula is m-variate normal for $m \leq n$.

Different measures of credit dependence can be differentiated. The default correlation is defined as the correlation between two dichotomous variables of credit default and compares the probability of a joint default of two obligors to the default probabilities of individual obligor defaults.[36] Based on the random default times τ_A and τ_B of two

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[33]Recent approaches such as Düllmann and Trapp (2005) consider the relation of PD and LGD.

[34]Cf. Embrechts, McNeil and Straumann (2001), p. 28 for an intuitive example.

[35]Cf. Nelson (1999), p. 15.

[36]For a comprehensive comparison of the different concepts of credit correlation see Erlenmaier and Gersbach (2000), and Erlenmaier (2001).

obligors A and B with individual default probabilities $p_A = P[\tau_A \leq t]$, $p_B = P[\tau_B \leq t]$ in a time interval $[0,t]$, the joint default probability in the interval $[0,t]$ is defined by $p_{AB} = P[\tau_A \leq t, \tau_B \leq t]$. This results in a default correlation

$$\rho_{AB} = \frac{p_{AB} - p_A p_B}{\sqrt{p_A(1-p_A)p_B(1-p_B)}} \tag{2.9}$$

for the time interval $[0,t]$.[37] Default correlations can be obtained under a risk-neutral probability measure or obey to the real-world measure if default histories are used for estimation. A default correlation always refers to a specified time interval and takes only joint credit default of exposures into account, but it omits the joint dynamics of obligors' credit quality irrespective of default, which is incorporated in the correlations of default times, asset values or default intensities.

Another measure of credit dependence is the default time correlation between the random default times τ_A and τ_B of two obligors A and B.[38] Default-time correlation always refers to a default model that permits the determination of the time of default of the obligors. Suspending the reference to a specific time interval and using $Cov(\tau_A, \tau_B) = \mathbb{E}[\tau_A, \tau_B] - \mathbb{E}[\tau_A]\mathbb{E}[\tau_B]$, the default time correlation for obligor A and B is defined by

$$\rho_{AB} = \frac{E[\tau_A, \tau_B] - \mathbb{E}[\tau_A]\mathbb{E}[\tau_B]}{\sqrt{Var(\tau_A)Var(\tau_B)}}. \tag{2.10}$$

Asset correlations and intensity correlations refer to the separate classes of credit valuation models discussed in Chapter 3. Intensity correlations determine the co-movement of stochastic default intensities of inhomogenous Poisson processes, whereas asset correlations define the correlation between the normalized asset returns of obligors. The asset correlation ρ^a_{AB} between the normalized asset returns of two obligors A and B with a bi-variate standard normal joint distribution function $\Phi_2(\cdot)$ and marginal distribution functions $F_A(1)$ and $F_B(1)$ of default by time $t = 1$ is implicitly given by the joint probability

$$P[\tau_A \leq 1, \tau_B \leq 1] = \Phi_2(\Phi^{-1}(F_A(\overline{V})), \Phi^{-1}(F_B(\overline{V})); \rho^a_{AB}), \tag{2.11}$$

of default in a one-year time interval where $\Phi(\cdot)$ denotes the standard normal distribution function. Within the asset value and within the default intensity framework, default correlations can be derived from the expectation of the joint evolution of asset values or intensities which is typically incorporated by factor models. Asset and intensity cor-

[37] Cf. Li (1999a).

[38] Li (1999a) shows that for a given default time correlation the default correlation increases monotonically with the length of the time interval considered.

relations determine the strength of the variation in the default rates of a risk class in time. Empirical estimates of asset and intensity correlations vary in time, however, it remains controversial whether this variation is just a sample effect, or it is due to the time-inhomogeneity of correlations.

Despite its widespread use in credit risk modelling and analysis, linear correlation is not a conceptually consistent measure of the dependence of random default variables or default times, for several reasons. First, linear correlation is not defined if variances are not finite.[39] Second, the independence of two random variables implies a zero correlation, whereas the reverse is generally not true, so that particular patterns of credit risk dependence are not captured. Third, linear correlation is only suited to multi-variate elliptical distributions. Finally, linear correlation is not invariant under non-linear increasing transformations. With respect to VaR forecasts, Embrechts, McNeil and Straumann (2001) show that the maximal VaR of a bivariate linear portfolio does not necessarily correspond to the maximal linear correlation.

Concordance (rank correlation), quadrant dependence and tail dependence provide alternative dependence concepts.[40] Concordance measures transfer the concept of linear correlation to ordinal variables and imply invariance under monotonic transformations, but cannot be extended to more than the bivariate case. The most prominent concordance measures are Kendall's τ and Spearman's ρ. Positive quadrant dependence of default times exists, if

$$P[\tau_A \leq t_A, \tau_B \leq t_A] \geq P[\tau_A \leq t_A] \cdot P[\tau_B \leq t_B] \tag{2.12}$$

for any arbitrary time frame t_A, t_B. Orthant dependence denotes the multivariate analogue of quadrant dependence. The coefficient of (lower) tail dependence

$$\underline{\lambda} = \lim_{\alpha \to 0} P[\tau_A < F_A^{-1}(\alpha) | \tau_B < F_B^{-1}(\alpha)] \tag{2.13}$$

measures the asymptotic dependence of bivariate default events and is defined as the probability of a random variable takes an extreme value, conditional on another variable being extreme as well. If $\underline{\lambda} > 0$, default times are said to be asymptotic dependent. Under a Gaussian copula, default times are asymptotic independent, i.e. they have zero pairwise tail dependence, so that joint defaults within a short period of time are a rare event. In contrast, normal mean-variance mixtures such as multivariate hyperbolic and t-copulas of default times exhibit strictly positive (lower) tail dependence even for nega-

[39]For bivariate t-distributed default time correlation is not defined, if the degree of freedom is smaller than 3.

[40]For a definition and a rigorous discussion of aforementioned dependence measures cf. Embrechts, McNeil and Straumann (2001), p. 195ff and Malevergne and Sornette (2006), p. 154ff.

tive or zero linear correlations of default times and show more joint default events than under a Gaussian copula for equal time periods. For multivariate t-distributions, the tail dependence increases as the degree of freedom decreases, i.e. the marginal becomes more heavy-tailed.

2.3.7 Measures of Risk

For a probable supervisory recognition of model-based credit risk capital charges, the measure of risk to apply must be defined. A measure $\mathcal{R}(\cdot)$ of portfolio credit risk is defined as a function of random portfolio credit losses $X, Y \geq 0$. Axiom systems as provided by Artzner, Delbaen, Eber and Heath (1999) and Albrecht (2003) define the properties of financial risk measures. Axioms that characterize the properties of risk measures include:

(A1)non-negativity:	$\mathcal{R}(X) \geq 0$
(A2)positive homogeneity:	$\mathcal{R}(cX) = c\mathcal{R}(X), c \geq 0$
(A3)sub-additivity:	$\mathcal{R}(X+Y) \leq \mathcal{R}(X) + \mathcal{R}(Y)$
(A4)shift invariance:	$\mathcal{R}(X+c) \leq \mathcal{R}(X), \forall c$
(A5)translation invariance:	$\mathcal{R}(X+c) = \mathcal{R}(X) - c, \forall c$
(A6)monotonicity:	$X \leq Y \Longrightarrow \mathcal{R}(X) \leq \mathcal{R}(Y)$
(A7)expectation-boundedness:	$\mathcal{R}(X) > E(-X)$
(A8)comonotone-additivity:	$\mathcal{R}(X+Y) = \mathcal{R}(X) + \mathcal{R}(Y)$

Table 2.1: Axioms of Risk Measures

Non-negativity of the portfolio credit risk measure requires the exclusion of unrealized profits in credit loss. Positive homogeneity implies that a multiple of a credit exposure induces an identical multiple of risk. Sub-additivity ensures the existence of diversification effects when sub-portfolios are combined. Shift-invariance turns a risk measure invariant to the addition of a constant, such as expected loss. Positive homogeneity and sub-additivity together imply that zero risk is assigned to a constant loss. Positive homogeneity and sub-additivity imply the convexity of the risk measure.[41] Translation invariance ensures that adding a fixed amount to a random credit loss reduces the risk measure by the same amount. Monotonicity implements, that X is less risky than Y if $X(\omega) \leq Y(\omega)$ for any state ω. A risk measure satisfying (A2, A3, A5, A6) is termed coherent.

Credit risk measures assess either or both the dispersion of portfolio credit loss L or its deviation from a reference value. A general form of a risk measure is defined by

$$\mathcal{R}(L) = \mathbb{E}[g(L - l^{ref})^{k_1}]^{1/k_2}, \tag{2.14}$$

[41] Cf. Albrecht (2003), p. 12.

with reference value z and parameters k_1 and k_2. The risk-defining function $g(\Delta l)$ of $\Delta l = L - l^{ref}$ represents either (1) the identity function $g(\Delta l) = \Delta l$ to calculate higher (centralized) moments, (2) the maximum function to obtain lower partial moments, (3) the absolute value of an absolute deviation measure, or (4) a utility function as used in expected utility theory.

Setting $g(\Delta l) = \Delta l$, $k_2 = 1$ and the reference value $l^{ref} = \mathbb{E}[L]$ in (2.14), we obtain two-sided risk measures that assess the dispersion of loss from its expected value. We get the k_1-th central moment, i.e. variance ($k_1 = 2$), skewness ($k_1 = 3$), or kurtosis ($k_1 = 4$). Adapting k_2, we obtain the standard deviation ($k_2 = 2$) and the respective roots of higher moments. Distributions of portfolio credit loss are typically skewed and leptokurtic, i.e. fat-tailed, so that variance is not an appropriate risk measure for its negligence of extreme value properties. Skewness and kurtosis seem more appropriate for portfolio credit risk applications, however, two-sided risk measures oppose the perception that a risk measure must only refer to downside risk.

Measures of shortfall risk are one-sided risk measures that consider the downside risk of credit loss relative to a reference value. Shortfall risk measures basically involve lower partial moments $\mathbb{E}[\max(L - lref, 0)^{k_1}]^{1/k_2}$. Setting $k_2 = 1$, we get the shortfall probability ($k_1 = 0$), shortfall expectation ($k_1 = 1$) and shortfall variance ($k_1 = 2$) with respect to a fixed benchmark. The standard deviation of shortfall results for $k_1 = 2$ and $k_2 = 2$. Regarding capital requirements, the shortfall probability with respect to the available economic or regulatory capital is of interest. If $l^{ref} = \mathbb{E}[L]$, it results lower-semi-absolute deviation, semi-variance and semi-standard deviation of the shortfall, respectively.

Other risk-defining functions $g(\Delta l)$ that rely on utility theory are omitted here. Complementary to the general form in (2.14), conditional measures of shortfall risk provide worst-case risk measures. Conditional expected shortfall, for example, is defined as mean excess loss $\mathbb{E}(L - l^{ref} | L \geq l^{ref})$.

Supervisory agencies prefer Credit-Value-at-Risk (Credit-VaR) to other measures for the quantification of portfolio credit risk. The Credit-VaR $CVAR_\alpha$ at a confidence level α designates the maximum credit loss of a portfolio that is not exceeded with probability α and is defined by

$$P(L \leq CVAR_\alpha) = \alpha, \tag{2.15}$$

so that $CVAR_\alpha$ is equal to the $\alpha-$quantile of the distribution of portfolio credit loss L. Credit-VaR is monotone, positive homogenous, translation-invariant and comonotone-additive. Since it is not sub-additive, Credit-VaR is not generally coherent, but it is only coherent for certain well-behaved distribution classes, such as normal distributions. Furthermore, $CVAR_\alpha$ does consider extreme tail properties. A remedy is provided by the

Conditional-Credit-VaR $CCVAR_\alpha(L) = \mathbb{E}[L|L > CVAR_\alpha]$, defined by the expected loss exceeding $CVaR_\alpha$. Albrecht (2004) shows that Conditional-Credit-VaR equals $CVAR_\alpha$ plus the conditional mean excess loss $\mathbb{E}[L - CVAR_\alpha|L > CVAR_\alpha]$. The Conditional-Credit-VaR is coherent if a closed-form density function of portfolio credit loss exists.

In accordance with the preference of supervisory agencies and risk management practice, Credit-VaR is considered as the relevant risk measure to determine capital charges for credit portfolios. Credit-VaR is typically measured at quantiles $1-\alpha$ higher than those in market risk management. However, extreme-tail properties of credit loss distributions differ considerably across different credit portfolio models. Since parametric representations of portfolio loss are typically not available and the estimation error increases dramatically the more extreme quantiles are considered, the adequate measurement and validation of Credit-VaR presents a special challenge.

Reflecting the possible classification of balance-sheet items under IFRS accounting standards, the time horizon of Credit-VaR can be set according to a liquidation period or a hold-to-maturity period approach. Under the liquidation period approach, a one-year time horizon of Credit-VaR has become the general standard, because it is a time interval which is sufficiently long, so that the following measures can be implemented: (1) a capital raise can be completed, (2) loss mitigation actions and default workouts can typically be completed, (3) new obligor information for the adaption credit ratings reveal, (4) calibrations of credit risk parameters can be updated, (5) internal capital budgeting is planned, and (6) credits are reviewed for prolongation. From a supervisory and risk management perspective, a hold-to-maturity time horizon of Credit-VaR and credit loss recognition is not appropriate, because it makes possible the accumulation of unperceived credit loss. Because recalculations of market value-at-risk are required by supervisors for a ten-day time period and recalculations are usually performed on a daily basis by banks, the typical recalculation frequency of Credit-VaR range from weekly to quarterly intervals.

The risk measure which is most relevant to ensure the capital adequacy of financial institutions is the unexpected loss of the credit portfolio. Huschens et al. (2003) show that the distribution of expected conditional loss rates of a credit portfolio approximate the distribution of the random unconditional loss rate if the portfolio is infinitely granular. Given a credit portfolio with exposures of different size, loss distributions can be approximated by means of a concentration-equivalent portfolio with exposures of equal size that is constructed using a concentration index.[42] Assuming infinite granularity, an asymptotical distribution of portfolio loss with given density can be determined under certain conditions. Using the equivalent homogenous-portfolio approximation a Credit-VaR that incorporates concentration effects can be determined.

[42] Gordy (2001) uses the Herfindahl index in this context.

2.4 Backtesting of Credit Risk Models

After a review of supervisory agencies requirements on the validation of credit risk models, methods for the validation of default prediction models and for the backtesting of default probability estimates of rating models are discussed. A parsimonious approach to backtesting the adequacy of portfolio credit risk models is introduced.

2.4.1 General Standards of Model Validation

The Basel Committee requires banks to establish robust procedures and methodologies to validate if internal risk models are conceptually suited and if they accurately and adequately represent the material risk.[43] Supervisory recognition of risk models under the IRB approach and internal market risk models of the revised capital standards require: (1) an initial approval of the model based on specified minimum requirements, (2) an independent review process established to assess the risk systems and processes, and (3) a supervisory review of the validation process.

The initial approval of risk models takes into account the documentation of risk procedures, models and systems, the consistency of type and source of data, the methodological and statistical concept of the risk model, procedural requirements on the estimation and validation of risk measures as well as the independence and qualification of staff engaged in developing and operating risk models.[44] Guidelines for the model approval are industry standards and established results from academic research.

Banks take on the primary responsibility for the review of risk measurement systems and processes. The independent review of the internal risk measurement systems for market risks involves the verification of approval, documentation, a change-control for all risk measurement processes and systems, the integration of risk measurement into the risk management function, the consistency, timeliness and reliability of independent data sources, the accuracy and completeness of exposures, the accuracy in parameters in estimation, risk measurement and validation models, and finally, the quantitative model validation using backtesting.[45] Backtesting involves all validation techniques that compare estimates of risk components to actual outcomes using statistical test theory. Supervisors do not stipulate specific techniques or definite criteria for the validation of risk models, but they require banks to develop the expertise for a self-reliant qualitative and quantitative assessment of applied risk models.

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[43] Cf. BCBS (2006c), p. 109ff and p. 202.

[44] Cf. BCBS (2006c), p. 88ff. and p. 254ff.

[45] Cf. BCBS (2006c), p. 193.

The review of internal rating models for the estimation of PD, LGD and EAD is divided into a procedural validation of the rating process and a methodological validation of the risk model. The evaluation of the rating process is qualitative in nature and involves data quality, internal reporting, problem identification and handling, system usage, staff training, and consistent application of the risk model throughout business lines and geographic regions. The methodological validation consists of an assessment of model design and a quantitative assessment of the quality of predictions. The examination of the model design includes a qualitative review of the statistical concept, the relevance of the input data, the way risk factors were selected and whether they are economically meaningful. Quantitative validation involves backtesting and the comparison of estimates to external data sources. Quantitative tests must not vary systematically during the economic cycle.

Requirements similar to those specified for market risk measurement systems and models for the estimation of risk components under the IRB approach are assumed to apply to the validation of internal credit portfolio models as well, although, the use of internal models to set capital requirements for credit risk has not yet been approved and standards for the validation of credit portfolio models are not specified.

The validation procedures conducted by supervisory agencies are specified on a general level to avoid a limitation of the competence of supervisors. Supervisory agencies review the adequacy of risk assessments and derived capital requirements, the compliance with minimum standards and qualifying criteria and the effectiveness of the review of the risk assessment processes.[46]

The quantitative validation of risk models has two objectives: (1) testing the accuracy of risk measures and risk parameter estimates, and (2) assessing the adequacy of a model to accomplish the overall objectives of the risk measurement process.[47]

Risk parameters that are typically controlled for the accuracy of estimates are PD, LGD, EAD and credit correlations of individual exposures or exposure classes. Accurate risk models implement a timely and accurate estimation of risk measures. However, statistical tests are limited in their ability to distinguish between accurate and inaccurate models or risk estimates, since the power of a hypothesis test to avoid committing a type II error, i.e. to reject the null hypothesis when it is actually false, cannot be calibrated to unity by a suitable specification of the test. The Basel Committee acknowledges that it is not possible to define a statistical test to correctly identify inaccurate models that prevents giving erroneous negative indications on other accurate models.[48] According to the limitations of hypothesis tests, statistical evidence on the accuracy of a risk model is

[46] Cf. BCBS (2006c), p. 209ff.

[47] Cf. BCBS (2006c), p. 193.

[48] Cf. BCBS (1996b), p. 5.

not mandatory for a model to receive approval from supervisory agencies, because even risk models whose accuracy cannot be verified can nevertheless be considered to provide adequate risk forecasts. The current backtesting procedure for internal market risk models implements this view.

Model adequacy refers to the ability of a model to reach an appropriate decision under uncertainty to meet pre-specified objectives. Pesaran and Smith (1985) stipulate the following criteria to assess the adequacy of a risk model:

- Does the model comply with the requirements of the user? (relevance criteria)
- Does the model contradict to secured knowledge? (consistency criteria)
- Is reliability ensured on a satisfactory statistical level? (statistical adequacy)

Credit portfolio models are required to ensure capital adequacy of a bank with respect to the credit risk of the bank's credit portfolio. The consistency of a model is assumed to be given, if the conceptual design implements established methodologies of risk modelling and if the definition of model parameters coincide with those of parameter estimates. Statistical adequacy requires that the relevance criteria be met on a specified confidence level.

Quantitative methods of assessing the adequacy of credit risk models are sensitivity analysis, stress tests and backtesting. Sensitivity tests of credit portfolio models examine the dependence of capital requirements and risk measures on a change in PD, LGD, EAD, correlations and credit spreads. Furthermore, the impact of a change of risk concentrations within particular economic sectors, regions or rating classes is assessed. For mark-to-market credit portfolio models, the effect of a change in drift, reversion, volatility or correlation parameters is of interest.

Stress tests examine the effect of adverse circumstances on risk measures and capital adequacy. Stress tests typically take the form of a scenario analysis and represent either an absolute or a relative change of several model parameters and market factors. Instead of the directional change of risk measures as observed in sensitivity analysis, stress tests consider the absolute outcome of a risk measure.

Regarding validation methodologies different approaches are distinguished for default prediction models, rating models, and portfolio credit risk models. Default prediction models discriminate between prospectively defaulting and non-defaulting exposures to support credit approval and prolongation decisions. Rating models assess effective and potential credit exposures on a fine-grained scale for risk-adequate pricing, assessment of general and specific credit loss provisions, determination of impairments, economic loss and capital requirements. Risk parameters estimated by rating models comprise PD, LGD, EAD or correlations. Credit portfolio models provide risk measures to assess the unexpected loss of portfolios and the capital requirements of banks.

Backtesting the statistical adequacy of credit risk models either involves testing the discriminatory power of default prediction models, the accuracy of risk parameter estimates, or the adequacy of portfolio credit risk measures to specify capital requirements. Techniques for validating the effectiveness of default prediction models are presented in Section 2.4.2. Backtesting risk parameter estimates involves hypothesis tests as outlined in Section 2.4.3 for the validation of PD estimates. Drift, recursion and diffusion parameters of credit portfolio models have not been considered in backtesting studies yet. Backtesting the adequacy of a credit portfolio model involves verifying whether the actual portfolio credit loss is in line with unexpected loss and corresponding economic and regulatory capital.[49]

Data limitations are a key impediment to the estimation and validation of credit risk models. The scarcity of data is due to the infrequent nature of default events and long-term time horizons in risk measurement. In consequence, data pooling, proxy data and low-frequency data are frequently used in model estimation and validation. Compared to market risk models, the one-year holding period and the higher quantile of Credit-VaR complicates backtesting. A statistical confidence equal to the backtesting of market risk models would require the excessive number of 250 observations of actual one-year portfolio outcome in backtesting credit portfolio models. To qualify for the advanced IRB approach, banks are required to establish a track record of historical estimates and actual rates of PD, LGD, EAD, as well as rating histories including the dates of any rating review for a time interval of at least five years.[50]

Corporate loans are typically not marked-to-market, so that risk models cannot be estimated from time series of loan valuations and the predictive quality of portfolio models cannot be compared to observed portfolio outcomes. Instead, it is assumed that mark-to-model valuations of credit exposures without observable market prices are derived from updated ratings to determine the credit loss of exposures. Thus, the validation of the credit portfolio model relies on the accuracy of the internal rating system, which is itself subject to a separate validation process. In this context, the inaccurate recognition of significant credit losses that accumulate in the banking book unnoticed due to misspecified PD estimates is the major threat to an adequate capital endowment and affects rating-based capital standards as well as credit portfolio risk measures.

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[49] The comparison of expected and actual portfolio credit loss does not address the capital adequacy purposes but only credit provisioning and pricing.

[50] Cf. BCBS (2006c), p. 102.

2.4.2 Validation of Default Prediction Models

The effectiveness of default prediction models depends on the accurate discrimination between obligors predicted to default (defaulters) and those predicted not to default (non-defaulters) on an ordinal or continuous rating scale that represents the default risk of obligors. Common techniques used to assess the discriminatory power of default predictions are the Cumulative Accuracy Profile (CAP), the related Accuracy Ratio (AR), the Receiver Operating Characteristic (ROC), the ROC measure and the related Pietra index, the Bayesian error rate, Conditional Information Entropy, Information Value, Kendall's τ, Somers' D and the Brier score.[51]

The CAP is defined as the percentage of defaulters per portion of obligors ranked by decreasing riskiness, i.e. obligors are ordered from risky to safe according to their rating score from . A perfect model will assign the lowest scores to defaulting obligors, whereas all obligors above a threshold score will be non-defaulting. For a random model without discriminatory power, any equal-sized portion of obligors will contain the same proportion of defaulting obligors. Typically, rating systems perform in between a perfect and a random rating model. The Accuracy Ratio aggregates the information of the CAP about the discriminatory power of the scoring function into a single number by setting the surface between the CAP of the perfect model and the randomly discriminating model in relation to the surface between the CAP of the current model and the randomly discriminating model.[52]

The Receiver Operating Characteristic compares the distribution of rating scores for defaulted and non-defaulted obligors. Assuming that all obligors with a rating score below a certain threshold will default, a false alarm rate is defined as type-I-error of erroneously qualifying a non-defaulting loan as prospectively defaulting, and a hit rate specifies the number of obligors, whose default has been correctly predicted. The ROC curve now gives the ratio of the hit rate to the false alarm rate per applied threshold score, and the ROC measure is defined as the area under the ROC curve. AR and ROC measure are linearly related, and confidence intervals can be derived analytically for both measures.[53]

The Pietra index is derived from the maximum distance of the ROC curve to the diagonal of the random scoring model. The Bayesian error rate considers the default-frequency-weighted type-I-errors and type-II-errors of the prediction model for any threshold score. Under specified conditions, it is equivalent to the Pietra Index and the Kolmogorov-Smirnov test statistic. Kendall's τ and Somers' D are rank-order statistics used to evaluate

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[51] A comprehensive overview about statistical methodologies to assess the discriminatory power of default prediction processes is given in BCBS (2005d), p. 36–46.

[52] Cf. Sobehart, Keenan and Stein (2000), p. 13.

[53] Cf. BCBS (2005d), p. 39ff.

out-of-sample out-of-time rating systems. Brier scores are typically used to compare the effectiveness of rating systems, but lack statistical tests which can be used for validation purposes.

The information-theoretic concept of entropy considers how much randomness is included in the observation of an obligor's default status. Entropy measures assess the information on credit scores or PD estimates by observing the ultimate default status of the obligor. Entropy measures are Information Entropy, Conditional Entropy, Kullback-Leibler Distance, Conditional Information Entropy Ratio (CIER) and Information Value.[54] Minimum values of entropy measures indicate a high discriminatory power of a rating system, but, as distributions of entropy-based measures cannot be derived in parametric form, they are not useful for hypothesis tests.

On the basis of the aforementioned statistics, the Validation Group of the Basel Committee considers Accuracy Ratio (AR) and ROC measure to be most appropriate for the favorable properties of their confidence intervals.[55]

2.4.3 Validation of Internal Rating Systems

The ability of internal rating models to provide accurate estimates of risk parameters is assessed by backtesting techniques. With respect to the estimation of PD, backtesting compares pooled PD estimates for risk classes of assumed homogenous credit quality with default rates of the class. A major obstacle to the backtesting of PD estimates is the scarcity and infrequency of default events and the impact of default correlation on confidence intervals. The explanatory power of statistical tests is further limited if data are restricted to the five year interval required to qualify for the IRB approach.

In a PIT rating system, pooled conditional PD of a risk class are stable in time and expected to equal the default rates of the class. A substantial deviation of default rates of a PIT risk class from its pooled PD estimates indicates either an inaccurate consideration of systematic and obligor-specific effects in PD estimates, a time delay in the assignment of obligors to risk buckets, or an inconsistency of the rating methodology, such as processing a hybrid or TTC methodology instead of assigning PIT ratings. For an accurate PIT rating system, the long-run average of default rates must converge towards the long-run average of conditional pooled PD of the PIT risk classes, as the number of observation periods approaches to infinity.

Ratings of a TTC system express the long-run average default expectation. The TTC rating of an obligor remains unchanged throughout the cycle, unless obligor-specific effects

[54] For a detailed discussion of entropy measures in credit scoring confer Keenan and Sobehart (1999).

[55] Cf. BCBS (2005d), p. 32.

distort the credit quality as compared with the remaining obligors of the class. Pooled unconditional PD estimates of TTC risk classes are stable in time and do not refer to the prospective state of the economy in the prediction period, whereas pooled conditional PD estimates of a TTC risk class and observed default rates are not expected to stay constant in time, but may vary substantially throughout the credit cycle. Unconditional PD estimates are not expected to match periodic default rates, and the long-run average default rate of a TTC risk class will converge towards the pooled unconditional PD if the TTC rating system is accurate. The conditional pooled PD of a TTC risk class is negatively correlated to economic conditions and cannot be compared to the long-run average default frequency, but must be compared to the periodic default rates under consideration of prevailing economic conditions.[56] The validation of conditional PD estimates tests two properties of a TTC rating system. First, the ability of the estimator to predict systematic factor effects on the default risk of obligors during the prediction period, and second, the ability to assess the specific credit risk of obligors given the predicted state of the credit cycle.

Stressed PD estimates typically exceed observed default rates and can only be assessed using default rates that refer to economic conditions inherent in these PD estimates, whereas long-run average default rates are not suited to the backtesting of stressed PD.

In summary, the long-run average default rate of a PIT rating class must converge to the average pooled conditional PD of the class. For a TTC rating class, the average long-run default rate is expected to converge to the long-run average of pooled unconditional PD. Single one-year default rates of risk classes are not meaningful in testing pooled unconditional PD estimates without a model assumption of the cross-sectional dependence of credit defaults that relates the variation of default rates to the unconditional PD estimate. Estimates of stressed PD can only be validated using default rates under infrequent stress conditions. Consistency in the rating methodology and validation methods is an important prerequisite for an effective backtesting of rating systems. In principle, credit dependence measures and unconditional PD estimates must be tested simultaneously, however, most approaches to the validation of unconditional PD estimates fail to fulfil this requirement. Techniques for backtesting the accuracy of rating models can be divided into

- parameter tests,
- distribution tests,
- resampling methods, and
- traffic light procedures.

[56] Cf. BCBS (2005d), p. 24ff.

Parameter tests evaluate the accuracy of estimates of single or groups of risk parameters predicted by a rating system such as pooled PD, EL loss or correlation estimates. Distribution tests assess if the empirical distribution of parameter values matches the predicted distribution of the parameter. Resampling methods enlarge the statistical basis for parameter and distribution tests and increase the number of observations available for testing using simulation techniques. Traffic-light procedures specify different zones of model acceptance for a test statistic, with zone ranges typically specified on the basis of statistical test theory.

A binomial test can be applied to test the pooled conditional PD estimate for a PIT risk class of independent exposures with homogenous credit risk, using the one-period default rate as a test statistic. However, the binomial test cannot be applied to the backtesting of pooled unconditional PD estimates if default correlation across periods is present, so that default rates typically differ substantially from unconditional PD estimates and exceed the critical values of a two-sided binomial test. In this case, the true type-I error is much larger than it is specified in the binomial test. In consequence, tests of unconditional PD estimates based on the independence assumption are rather conservative in nature, whereas binomial tests of conditional PD estimates that inherently consider default correlation enable only to detect obvious cases of estimation bias.

Another approach to the validation of the pooled conditional PD estimate of a risk class is a multi-period normal test, which is based on a normal approximation of the distribution of the mean default rate and makes use of the central limit theorem. Default events are assumed to be independent in time and cross-sectional dependence is permitted. Several studies show that the power of the normal test is modest but that it exhibits a conservative bias.[57] The true type-I-error is lower than presumed by the test and it is robust against a violation of the assumed independence of credit defaults in time. However, approximating the asymptotic distribution of conditional PD by a normal distribution allows in principle for negative PD.

The binomial test of PD estimates refers to a single risk class only. If the conditional PD estimates of several risk classes are tested using separate binomial tests with typical confidence intervals, it is probable that the null hypothesis of a correct PD estimate will be erroneously rejected for at least one risk class. A simultaneous validation of the PD estimates of all risk classes of a rating system is achieved using a chi-square test based on the assumptions of independent credit defaults within and in-between risk classes and assuming that the differences in default rates from conditional PD are approximatively normal due to the central limit theorem.[58] However, in practice, the factual dependence of

[57]Cf. BCBS (2005d), p. 53ff. and Höse and Huschens (2003a), p. 159f.

[58]Cf. BCBS (2005d), p. 52f.

default events and the low frequency of default events results in single-period chi-squared tests underestimating the true type-I-error.

Pooled unconditional PD estimates of TTC risk classes are not expected to equal the observed default rates of a risk class, due to the dependence of exposures' credit quality on common background factors that fluctuate during a credit cycle. Given a positive time-invariant default correlation ρ^{def} between n_t obligors of a TTC risk class in period t, the one-period default rate $\hat{p}_t$ represents an inconsistent estimator for the unconditional PD with variance

$$Var(\widehat{p}_t) = \frac{p_t(1-p_t)}{n_t} + \frac{n_t - 1}{n_t}\rho^{def} p_t(1-p_t), \tag{2.16}$$

that converges to the strictly positive asymptotic variance $\rho^{def} p_t(1-p_t)$ if the number of obligors tends to infinity. In consequence, the default rate $\hat{p}_t$ does not converge towards the unconditional PD, if $n_t \to \infty$, but rather, it converges towards an asymptotic distribution.[59] Taking the long-run average default rate as an estimator of the pooled unconditional PD instead, the asymptotic variance decreases proportional to the number of observation periods.[60]

Another approach to the backtesting of pooled unconditional PD estimates of TTC risk classes is the estimation of conditional PD using a one-factor model according to (2.1), so that the default rate $\hat{p}_t$ converges towards the random conditional PD $p_{t|z_t}$ in distribution. Given the inner-class asset correlation ρ^a, the probability distribution

$$P(p_{t|z_t} \leq x) = \Phi\left(\frac{\sqrt{1-\rho^a}\Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho^a}}\right) \tag{2.17}$$

of the conditional PD as calculated by Vasicek (1991) results in the asymptotic standard Gaussian test statistic

$$\frac{\sqrt{1-\rho^a}\Phi^{-1}(\hat{p}_t) - \Phi^{-1}(p)}{\sqrt{\rho^a}} \xrightarrow{D} \mathcal{N}(0,1) \tag{2.18}$$

for the hypothesis of equality between the default rate and the model-derived conditional PD.[61] Asymptotic confidence intervals for the unconditional PD estimate p can be specified as a function of default rate $\hat{p}_t$ and asset correlation ρ^a.[62] The range of non-rejection of PD estimates is essential for the acceptance and suitability of the test in the banking practice, so that the specification of the significance level becomes a meaningful deci-

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[59] Cf. Höse and Huschens (2002), p.12.

[60] Cf. Huschens and Locarek-Junge (2000), p. 21.

[61] Höse and Huschens (2003a), p. 151f.

[62] Cf. Höse and Huschens (2003b).

sion to be taken by supervisory agencies. However, when typical default rates and asset correlations are used, analytical results exhibit only a modest power for typical levels of confidence.[63]

The random conditional PD involved in the backtesting of unconditional PD estimates is a function of asset correlation ρ^a. If ρ^a is derived in a simultaneous estimation together with the unconditional PD estimate p, e.g. using the MLE in (2.2), both parameters must, in principle, be validated in a simultaneous test using a joint confidence interval for the unconditional PD and the asset correlation. This issue is yet still unresolved.

Simultaneous tests of unconditional PD estimates of several risk classes are based on the joint asymptotic distribution of default rates.[64] In an attempt to derive a lower bound of a joint confidence interval Huschens (2004) as well as Höse and Huschens (2003b) provide different statistics to test the joint pooled unconditional PD of risk classes, given a one-factor default model with known asset correlations.[65] Extensions of the factor-based backtesting of unconditional PD estimates by Huschens and Stahl (2004) include more detailed factor structures and tests of unconditional portfolio loss that use concentration indices to generate synthetic credit portfolios of infinite granularity to adjust for differing face values of exposures.

Distribution tests not only consider the accuracy of unconditional PD estimates, they also test the shape of the distribution function of conditional PD estimates. Frerichs and Löffler (2002) apply an approach, originally proposed by Berkowitz (1999) to evaluate VaR forecasts, to test conditional PD and asset correlation estimates of a one-factor default model. Given the model-implied estimate of the distribution function $\hat{F}(\hat{p}_t)$ of default rate $\hat{p}_t$, the transformation $\Phi^{-1}(\hat{F}(\hat{p}_t))$ results in a series of transformed observations that must be i.i.d. standard-Gaussian if the estimated distribution of conditional PD is equal to the true one. A standard likelihood ratio test with a joint hypothesis of zero mean and unit variance is applied to test the normality of the transformed observations. In a simulation study, the power of the test is assessed for different hypotheses of unconditional PD and asset correlation estimates. At a moderate level of significance, the power of the test is suitable for a time series of ten default rates, but the power of the test decreases if distributions of portfolio loss rates are considered instead.

In a resampling method proposed by Lopez and Saidenberg (2000), the number of observations available for backtesting is multiplied by cross-sectional resampling credit loss from a panel data set. Assuming that exposures have equal face values and omitting re-

[63] Cf. Höse and Huschens (2003a), p. 155.

[64] Cf. Höse and Huschens (2003c), p. 553.

[65] The alternative use of the Bonferroni inequality to derive a lower bound of the joint confidence level of unconditional PD estimates results in a low power of the test, cf. Höse and Huschens (2003c), p. 553.

covery rates, the approach corresponds to the backtesting of unconditional PD estimates. Within each period of the sample, a number of sub-portfolios is determined at random. Assuming default rates of any sub-portfolio to be independent in-between and within each time period, Mincer-Zarnowitz regressions and likelihood ratio tests are proposed to test expected loss or PD estimates, Credit-VaR and the shape of loss distributions. However, Bühler et al. (2002) as well as Frerichs and Löffler (2002) show that the assumption of independent observations is invalid and tests that rely on this assumptions will lead to erroneous statistical inference.[66]

Traffic light approaches define zones of a test statistic that may lead to escalating supervisory actions.[67] The traffic light approach has established as a methodology for the supervisory backtesting of internal market risk models, where three zones are defined for the number of daily portfolio performances that exceed the VaR forecast within a one-year period. Within a green zone, no action is taken. In the yellow zone, the market risk capital multiplier for capital requirements is incremented and the VaR model has to be reviewed. The red zone triggers a maximum multiplier and fundamental changes to the market risk model. Bühler et al. (2002), Blochwitz and Hohl (2003) and Tasche (2003) transfer the concept of the traffic light approach to the validation of PD estimates and consider the actual default rate compared to the estimated PD as a test statistic.

Blochwitz, Hohl, Tasche and Wehn (2004) as well as Blochwitz, Hohl and Wehn (2005) propose a multi-period traffic light approach to the validation of unconditional PD estimates of a single risk class with conditional PD estimates derived from a default-only single-factor model with time-invariant asset correlation. The number of defaults in a single year is assumed to be normally distributed and the one-period default experience is mapped to one out of four color-grade zones according to the deviation of the default rate from the pooled unconditional PD of the respective risk class. This mapping results in a multinomial distribution of frequencies with which the particular zones are hit in time and which are used to infer on the accuracy of PD estimates. In contrast to the traffic light approach to backtesting market risk models, no alternative model is considered so that the definition of zones is judgemental. The extended traffic light approach appears to be conservative, with type-I-errors being higher than expected, whereas the frequency of false alerts is controllable, as simulative results indicate.

In summary, no powerful test of the accuracy of unconditional PD estimates exists due to the cross-sectional dependence of default events in time, nor do supervisory agencies

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[66] If the number of defaults in the portfolio is elevated in a particular period, any resampled sub-portfolio will reflect the enlarged default rate. In consequence, tests are biased towards rejection as the adverse information content of a single period is overestimated. Cf. Frerichs and Löffler (2002), p. 37ff for a formal proof.

[67] Cf. Kupiec (1995), Kupiec and O'Brien (1997).

expect such a test to be developed in the future. Existing tests, such as the binomial test and the chi-squared test are rather conservative or detect only the most obvious cases of estimation inaccuracies, i.e. the discriminatory power is not satisfactory at typical levels of significance. Distributional tests are promising in terms of their general requirements, but suffer from the modest number of available observations in time, so that the power of these tests is low. The examined technique for the resampling of cross-sectional information has been proven to result in invalid statistical inference. Traffic light approaches appear to be a promising tool, due to their easy adaptability and interpretation, though the definition of zones is judgemental.

Effects that are typically not considered in the estimation and validation of default probabilities are methodologically inconsistent mixtures of TTC- and PIT-rating systems, and granularity effects of differing exposure sizes in a portfolio. Meyer zu Selhausen (2004b) fundamentally disputes that credit portfolio models can be tested at all, because time-homogenous sets of obligors are not available for model estimation or validation, and the number of available observations does not enable the calculation of prediction errors. The information used by the rating models is typically not complete and in part represents inappropriate proxy data. Credit experience used for PD estimation represents the filtered data of creditors that passed the loan-granting process, so that it is questionable, if rating models can be applied to assess new loan applicants. Changes in the credit quality of obligors short of credit defaults, e.g. rating migrations, are neglected in the estimation and validation of rating models, so that the information contained in the majority of non-defaulted exposures is omitted. Finally, according to Meyer zu Selhausen (2004a), structural breaches of economic background factors, credit granting policy, insolvency rulings or variables used in credit scoring models prevent the time-homogeneity of risk parameters and turn long-term time series of credit data inappropriate for the estimation and validation of credit risk models.

Due to these limitations of statistical backtesting, supervisory agencies additionally recommend the benchmarking of banks' ratings and PD estimates against equivalent data from external sources as a complementary tool for PD validation, provided that appropriate adaptions for methodological differences of the ratings systems are conducted. With no unambiguously superior test available to adequately validate the accuracy of internal rating systems, supervisory agencies resort to a mixture of quantitative and qualitative validation procedures.

2.4.4 Backtesting of Credit Portfolio Models

In this section, a traffic light approach to the backtesting of the adequacy of a credit portfolio model is suggested that is based on the binomial tests supervisory agencies use in the backtesting of market risk models (VaR backtesting).[68] VaR considers changes in the mark-to-market value of a static portfolio that result from changes in market risk factors during the VaR horizon. Correspondingly, portfolio performance is given by a synthetic buy-and-hold-performance measure that excludes intra-day trading positions, fee income and the mark-to-market performance of positions entered into during the VaR horizon. The mark-to-market performance of trading positions liquidated during the VaR horizon is replaced by the hypothetical outcome of the position up to the end of the VaR horizon.

In the VaR backtesting, the time horizon of the VaR is synchronized with the daily calculation frequency of VaR and portfolio performance, though a one-day VaR horizon differs from the 10-day period prescribed by supervisory agencies to determine capital requirements. With respect to the backtesting of credit portfolio models, a time horizon of Credit-VaR shorter than the one-year horizon of typical Credit-VaR calculations and prescribed by the revised capital standards may be used if the credit risk of any exposure in the portfolio has been re-assessed during that period. Typically, Credit-VaR is calculated on a quarterly or monthly basis, however, re-ratings of exposures are not so frequent.

VaR backtesting assesses if the confidence level of the VaR model is adequate, i.e. daily $VaR_{0.99}$ can be expected to cover 99% of the daily portfolio loss observations. Given stationary market factors, it can be assumed that the observations of portfolio performance are independent and the number of VaR outliers is binomially distributed. Two binomial tests for the risk models and an alternative model are defined by hypothesis $\mathcal{H}_0$ and $\overline{\mathcal{H}}_0$:

$$\mathcal{H}_0 : \alpha \geq 99\% \qquad \mathcal{H}_1 : \alpha < 99\% \tag{2.19}$$

$$\overline{\mathcal{H}}_0 : \overline{\alpha} \leq 95\% \qquad \overline{\mathcal{H}}_1 : \overline{\alpha} > 95\% \tag{2.20}$$

Under the assumption that the calculated VaR figures do not underestimate the market risk of the portfolio, the hypothesis $\mathcal{H}_0$, which assumes that the risk model indicates VaR at a confidence level $\alpha \geq 99\%$, is tested using a binomial test. Accordingly, given the conjecture that the portfolio market risk is underestimated and that the probability of observing a portfolio loss above the reported VaR is at least five times higher, the hypothesis $\overline{\mathcal{H}}_0$, which assumes that the calculated VaR figure in fact refers to an alternative confidence level of $\overline{\alpha} \leq 95\%$, is tested. The test statistic for both hypotheses is the number of portfolio loss observations caused by a change of market factors within the last 250 trading days, which exceed the VaR of the respective day (outlier). Significance level $\alpha_{\mathcal{H}} = 0.1\% (\alpha_{\overline{\mathcal{H}}} = 0.5\%)$ is set for the binomial test of hypothesis $\mathcal{H}_0(\overline{\mathcal{H}}_0)$.

The green, yellow and red zones of model adequacy are defined for the number of outliers,

[68] Cf. BCBS (1996b) in conjunction with BCBS (1996a).

according to the range of outliers that induces a rejection of hypotheses $\mathcal{H}_0$ and $\overline{\mathcal{H}}_0$. The green zone comprises the outlier interval $[0, 4]$, where $\mathcal{H}_0$ is not rejected, $\overline{\mathcal{H}}_0$ is rejected, the market risk model is qualified as adequate, and supervisors allow its use to calculate capital requirements. The interval $[10, 250]$ defines the red zone. Observing 10 or more outliers, $\mathcal{H}_0$ is rejected, $\overline{\mathcal{H}}_0$ is not rejected and the market risk model is qualified as inadequate for VaR calculation. In case the number of outliers hits the yellow zone $[5, 9]$, both null hypotheses cannot be rejected, it exists no confidence in the ability of the model to determine the VaR adequately, and supervisors increase the regulatory capital requirement multiplier unless the bank can prove the adequacy of its model by additional analysis. Possible explanations for an increased number of VaR outliers and a related failure of backtesting concern the basic integrity of the model (positions are considered incorrectly or parameter estimates are inaccurate), insufficient model precision, random chance or structural breaks in market movements. It is at the discretion of supervisory agencies to react to failures in risk measurement in a judgemental way, taking the severity of the model deficit into account.

With reduced significance levels, the binomial tests could unambiguously decide between the risk model and the specified alternative. However, it is not possible to define zone locations so that inadequate models are correctly indicated at a sufficient level of significance without triggering a false rejection for many adequate models. Consequently, a yellow zone of uncertainty regarding model adequacy is established due to the comparatively high levels of significance required for the rejection of $\mathcal{H}_0$ and $\overline{\mathcal{H}}_0$.

Backtesting using a two-hypotheses tests has to balance two types of inference error: (1) the erroneous rejection of an adequate risk model (error-of-rejection, type-I-error). (2) the erroneous non-rejection of an inadequate risk model (error-of-non-rejection, type-II-error). In contrast to the classical hypotheses tests, the two types of inference error refer to different hypotheses.[69]

It is assumed that Credit-VaR and the backtesting of credit portfolio models use one of the measures of credit portfolio loss defined in Section 2.3.5. In the backtesting of credit portfolio models, the prospects of time series inference are restricted due to the lack of a sufficient number of independent portfolio observations. In VaR backtesting the daily VaR and portfolio loss is used, so that the required 250 observations accrue within a one-year time interval. In contrast, Credit-VaR typically refers to a one-year time horizon, so that 250 yearly observations of portfolio loss are necessary to apply the binomial tests of VaR backtesting to the backtesting of portfolio credit risk models equivalently. Furthermore, changes in the credit scoring and loan granting process, the conceptual design of the risk model and the parameter estimation, as well as structural

[69] In a hypothesis test a type-I-error is committed, if a true null hypothesis is incorrectly rejected. A type-II-error occurs, if the test fails to reject a false null hypothesis.

breaches in market data are likely to occur over multiple-year time horizons and challenge the methodological consistency of the risk measurement and backtesting. In practice, only a small number of independent observations of credit portfolio loss in time are available, whereas many observations of the credit loss of single exposures exist in cross-sectional data.[70] However, the credit loss of exposures depends on common risk factors in time, so that single exposures are independent only within a single period conditional on the actual factor values. Since conventional statistical inference relies on independent observations, model adequacy can only be tested conditionally on actual factor values. Furthermore, the backtesting of a conditional credit risk on the basis of the conditional loss rates would omit that part of the credit portfolio model that controls for the dependence of exposures and the variation of loss rates in time.

The development of techniques for backtesting the adequacy of unconditional Credit-VaR using a time series of few independent observations of credit portfolio loss is omitted, because small sample inference on the basis of conventional statistical test theory is expected to result in a test of moderate power that suffers from the fundamental obstacles enumerated in the previous section. Instead, the classical statistical test paradigm which requires independent observations is dropped, and only one observation of credit portfolio loss which includes the cross-sectional information of only a single period is considered as a statistic to test the adequacy of the corresponding unconditional Credit-VaR.

By omitting the time-dimension of a series of independent portfolio loss observations and restricting the test statistic to the cross-sectional information of a single period, a backtesting approach is proposed that uses two-hypotheses tests to validate if the unconditional portfolio credit loss observed is distributed as presumed by the credit portfolio model, and if a distribution of portfolio loss given by an alternative model can be rejected.[71] The test statistic is defined as single-period portfolio credit loss according to one of the definitions presented in Section 2.3.5. In contrast to standard hypothesis tests, the true distribution of the test statistic is unknown. The central limit theorem and the law of large numbers no longer apply with respect to the number of exposures in the portfolio or the number of portfolio loss observations available. The dependence concept and the strength of credit dependence incorporated by the credit portfolio model and its alternative are not revealed by a single period loss observation, but by the loss distributions assumed under the different hypotheses. The green, yellow and red zones of model

[70] Resampling of cross-sectional data results in invalid statistical inference as outlined in Bühler et al. (2002).

[71] The portfolio credit loss converges in distribution against the unconditional distribution of loss provided by the model, if the credit portfolio model is accurate, however this convergence cannot be tested meaningful due to the lack of observations.

adequacy are defined on the basis of the rejection range of the two-hypotheses tests

$$\mathcal{H}_0 : L \leq q_{1-\alpha} \qquad \mathcal{H}_1 : L > q_{1-\alpha} \tag{2.21}$$

$$\overline{\mathcal{H}}_0 : L \geq \overline{q}_{\overline{\alpha}} \qquad \overline{\mathcal{H}}_1 : L < \overline{q}_{\overline{\alpha}}, \tag{2.22}$$

where loss L is specified according to one of the definitions of portfolio credit loss in Section 2.3.5 and refers to the distributions of loss provided by the credit portfolio model to be tested under hypothesis $\mathcal{H}_0$ and to the loss distribution given by an alternative model $\overline{\mathcal{H}}_0$. The level of significance $\alpha(\overline{\alpha})$ of the hypotheses tests and correspondingly the zone locations of the backtesting can be adapted according to the required level of statistical confidence and discriminatory power of the backtesting given model specification and portfolio composition.

The conjecture that credit risk is underestimated by the model and that the probability of observing losses higher than Credit-VaR is higher than the confidence level of the Credit-VaR predicts, is validated by testing $\mathcal{H}_0$. Hypothesis $\overline{\mathcal{H}}_0$ is rejected if the portfolio loss L is above the α-quantile $q_{1-\alpha}$ of the unconditional loss rate with density $\bar{f}(L)$ as provided by the credit portfolio model. A red zone of model rejection is defined by the interval $[q_{1-\alpha},\ l^{max}]$, where l^{max} designates the maximum loss given definition L of portfolio credit loss. If the significance level of the test is smaller than the confidence level of Credit-VaR, i.e. if $q_{1-\alpha}$ is smaller than Credit-VaR, a portfolio loss that exceeds Credit-VaR is automatically attributed to the red zone of model rejection. The alternative

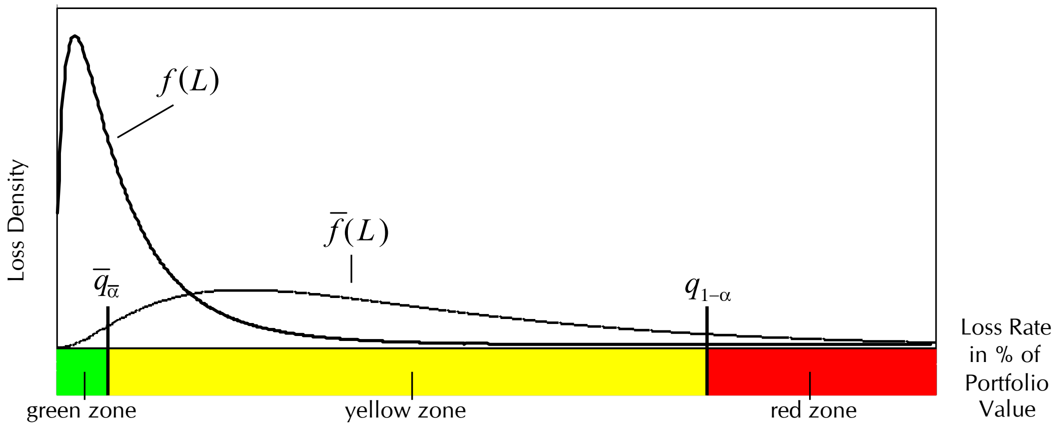

Figure 2.1: Principle of Zone Definition

model tested by hypothesis $\overline{\mathcal{H}}_0$ represents a more conservative specification of the same credit portfolio model, depending on the conceptual design and the model parameters of the model in question. Given deterministic exposure amounts, the main risk drivers of Credit-VaR are PD, LGD and correlations. Relevant model parameters which control for the main risk drivers are (depending on the type of model) asset values, default intensities, drift, diffusion, mean-reversion and recovery parameters as well as factor coefficients. A consideration of the definition of the alternative model to a structural credit portfolio model is given in Section 5.3.2.

Under hypothesis $\overline{\mathcal{H}}_0$, it is assumed that the alternative model adequately measures the credit risk of the portfolio. $\overline{\mathcal{H}}_0$ is rejected, if the portfolio loss L is below the $\overline{\alpha}$-quantile $\overline{q}_{\overline{\alpha}}$ of the unconditional loss rate with density $\bar{f}(L)$ as provided by the alternative model. The green zone of model adequacy is defined by the interval $[l^{min}, \overline{q}_{\overline{\alpha}}]$ of portfolio loss, where l^{min} designates the minimum loss given the definition L of portfolio credit loss.

The credit portfolio model is considered to be adequate, i.e. there is no indication that Credit-VaR is underestimated, if hypothesis $\mathcal{H}_0$ is not rejected and hypothesis $\overline{\mathcal{H}}_0$ is rejected, which is equivalent to $L \leq \min\{q_{1-\alpha}, \overline{q}_{\overline{\alpha}}\}$. The yellow zone, in which the adequacy of the model is indeterminate, is located in-between the green and the red zone and set to the interval $[\min\{q_{1-\alpha}, \overline{q}_{\overline{\alpha}}\},\, q_{1-\alpha}]$. Typically, it is $\overline{q}_{\overline{\alpha}} < q_{1-\alpha}$, and the yellow zone is of a strictly positive size. Figure 2.1 represents a principle depiction of the location of the green, yellow and red zone.

The backtesting approach suggested here does not depend on the particular credit portfolio model under examination, although zone locations and the discriminatory power of the tests depend on the conceptual design and specification of the model and the composition of the credit portfolio as will be seen in Chapter 5. In most practical applications, there is no closed-form representation available for the distribution of portfolio loss under hypotheses $\mathcal{H}_0$ and $\overline{\mathcal{H}}_0$, so that approximate loss distributions are generated using a simulative exercise to determine the zone locations.

Chapter 3

Portfolio Credit Risk Modelling

Portfolio credit risk models involve a model for the valuation of individual defaultable exposures and a dependence model that incorporates the co-movement of credit risk of exposures in time. This chapter starts with a general overview of models for the valuation of individual defaultable claims. Next, a structural first-passage credit valuation model for defaultable exposures with constant periodic interest is introduced. The mechanics of the credit valuation model are examined using a comparative-static analysis. A systematic overview of credit portfolio models is provided and a credit portfolio factor model based on the structural first-passage credit valuation model for individual exposures is introduced.

3.1 Single-name Credit Risk Pricing Models

Structural models and intensity models are distinguished for the pricing of defaultable claims. Structural models use the evolution of a firm's asset value to define the default event and to determine the value of credit exposures. The term 'structural model' refers to the capital structure of a firm that can be derived if the market value of the firm's assets and the market value of its debt is known. Structural credit valuation models are also referred to as asset value models, firm value models, or generally as ability-to-pay model. In contrast to the structural approach, the intensity-based pricing framework does not incorporate an economic intuition for the default of a financial claim. Instead, a purely statistical concept of credit default is applied, so that intensity based credit pricing models are often alternatively referred to as reduced-form models. For reasons of completeness, a brief overview of intensity models found in the credit risk literature is provided to describe the advantages and deficiencies of the competing approaches. Subsequently, the structural valuation framework is reviewed and an asset value model for the valuation of defaultable debt with discrete interest payment is introduced.

3.1.1 Intensity Models

The fundamental concept of intensity models for the valuation of defaultable claims is the modelling of the time of default as stopping or arrival time τ of the first jump of a non-explosive counting or point process. In their seminal work, Jarrow and Turnbull (1995) propose the use of a homogenous Poisson counting process with a constant intensity parameter, so that inter-arrival times are independent and exponentially distributed. The non-negative default intensity of the Poisson process represents the instantaneous probability of default. Hull and White (2000a, 2000b) consider an inhomogenous Poisson process, in which the intensity is a deterministic function of time. Given the term structure of the riskless zero bond prices, piecewise constant risk-neutral default intensities are fitted from prices of traded defaultable instruments of a particular obligor. Other approaches consider the default intensity itself to be stochastic, so that default events are triggered by a doubly stochastic Poisson process, referred to as Cox process.[1]

In a finite state space Jarrow, Lando and Turnbull (1997) as well as Kijima and Komoribayashi (1998) consider rating classes of defaultable claims with an absorbing default state, and model rating transitions of an obligor using a time-homogenous discrete-time Markov chain with constant transition intensities, where the risk of a change in credit spreads is neglected, if it does not coincide with a change of the rating. Das and Tufano (1996) extend the model of Jarrow, Lando and Turnbull and implement obligor-specific random recovery rates in a discrete-time Heath-Jarrow-Morton framework, so that the risk of a change in credit spreads is considered along with to the risk of a rating change. This results in a better fit of empirical credit spreads than the approach of Jarrow, Lando and Turnbull and enables obligor-specific pricing.

Extending the framework of Jarrow, Lando and Turnbull, Lando (1997) considers default intensities following a multi-dimensional Cox process. The process of rating transitions is implemented using a homogenous continuous-time Markov chain with finite state space and an absorbing default state, and incorporates the correlation between the riskless rate and default intensities, so that empirically observed co-movements of interest rates and credit spreads can be fitted.

In contrast to structural models, the default time of intensity models is not predictable.[2] A stopping time is predictable only if a continuous-state variable of credit risk refers to a default-triggering barrier function, which excludes defaults triggered by jumps events. The unpredictability of defaults in the intensity based framework challenges the arbitrage-free pricing framework, since defaultable bonds that are prone to unpredictable jumps cannot

[1] Cf. Lando (1998), p. 101.

[2] For a formal definition of the concept of predictability see Duffie (1996),p. 357ff.

be duplicated by assets that follow a predictable process, i.e. the defaultable bond cannot be hedged perfectly and the market for the obligor's default risk is incomplete if there is not at least one other traded asset with payoffs contingent on the same default process.[3]

The pricing framework of intensity models for the valuation of defaultable claims is analogous to affine diffusion models of the default-risk-free instantaneous short rate. Seminal affine term structure models include the model of Vasicek (1977) with a Gaussian diffusion of the riskless short rate[4], the CIR model presented by Cox et al. (1985), which prevents negative short rates using a noncentral-χ^2-distributed diffusion term, and the Gaussian forward rate model introduced by Heath, Jarrow and Morton (1992). Term structure models with a log-normal dynamic of the short rate as suggested by Black and Karasinski (1991), or the market model by Brace, Gatarek and Musiela (1997), and Miltersen, Sandmann and Sondermann (1997) have not yet been incorporated into closed-from credit pricing formulas of the exponential-affine type.

The exponential-affine pricing framework for short rate models defined by Duffie and Kan (1996) can be extended to cover interest-risky defaultable claims, if riskless short rates and default intensities follow a general affine jump-diffusion process.[5] In the Martingale representation of a zero-coupon bond price, an instantaneous mean-loss spread is added to the riskless short rate in an extended risk-neutral probability space[6], with the mean-loss short spread defined as a product of a stochastic instantaneous intensity factor and a loss rate expressed in percentage of the market value of the claim immediately before default.[7] Given a zero-recovery assumption short spread and default intensity coincide. If a fractional recovery of the market value is assumed in the case of default, the price of a defaultable zero-coupon bond is given by an exponential-affine function of instantaneous short rate and short spread.[8] The implied term structure of defaultable zero-coupon bond yields (zero yield curve) is derived using an affine function of short spread and short rate. Dependence between interest rate risk and credit risk and an improved flexibility of the implied defaultable zero yield curve is incorporated by decomposing the short rate and short spread into functions of orthogonal or correlated latent factors that follow basic

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[3] In an arbitrage free market, the price of an asset cannot perform unpredictable jumps with known jump size that do not correspond to a cash outflows.

[4] Cf. Hull and White (1990) for an extended version of the Vasicek model with time-dependent parameters.

[5] Cf. Duffie and Singleton (1999a), p. 688ff. An affine process is a jump-diffusion process with drift vector, covariance matrix and jump-arrival intensities, which have affine dependence on a state vector, cf. Duffie and Singleton (2003), p. 346.

[6] See Duffie and Singleton (1997), p. 1291.

[7] By use of the Doob-Meyer decomposition, loss arrival is conditioned on the default event only (cf. Duffie, Schroder and Skiadas (1996), as well as Duffie and Singleton (1999a)).

[8] Cf. Duffie (1999), p. 79, and Duffie (2005), p.2756ff.

affine processes.[9] Factor dynamics are calibrated to reproduce the joint evolution of the empirical riskless yield curve and the term structure of the obligor's credit spreads. For the empirical fitting of intensity models, the time-discrete affine expectation of instantaneous factors is used to constitute the transition function of a state-space model as defined in Harvey (1989), with the affine function (exponential-affine) of zero yields or zero spreads (zero-coupon bond prices) representing the measurement equation.

Hybrid models approximate intensity models towards the structural valuation framework. Belanger, Shreve and Wong (2004) suggest a jump-diffusion model of the asset value that determines the jump component using an intensity based point process, whereas intensity models proposed by Madan and Unal (1998, 2000) incorporate asset values as a Markovian factor that affects default intensity.

Among many other studies on the fitting of empirical credit spreads using reduced-form models, Duffie and Singleton (1997), Duffee (1999), and Liu, Longstaff and Mandell (2001) present a satisfying fitting quality, especially with regard to empirical short-term spreads.

3.1.2 Structural Models

In the structural framework, credit events are triggered if the firm value falls below a certain threshold or barrier. In contrast to reduced form models, probabilities of credit default are not given exogenously, but have to be derived endogenously, taking the dynamics of the value of firm assets and the definition of default into account. [10]

Credit default due to insolvency and default due to over-indebtedness are distinguished. Insolvency designates the inability of a firm to fulfil its financial obligations in time, also referred to as cash-flow-based default. Over-indebtedness occurs if the market value of a firm's assets no longer exceeds the market value of its obligations.[11] First-passage models trigger credit default at the first time the asset value hits a lower default threshold and can be interpreted as causing over-indebtedness as well as insolvency due to the inability to raise additional capital to meet financial obligations. Constitutive characteristics of first-passage models that influence the ability to find a closed-form valuation for defaultable claims include the dynamics of the asset value and the riskless short rate, the definition of the default barrier, the timing and seniority of the cash flows, and the recovery claim.

The structural approach to value defaultable claims was originated by Merton (1974), who considers corporate debt to be a contingent claim on the value of a firm, that is financed by equity and a defaultable zero(-coupon) bond with face value K and maturity

[9] Cf. Lando (1998), p. 383ff as well as Duffie, Pan and Singleton (2000), p. 1351ff.

[10] The notion of firm value and asset value are considered to be synonymous in the remainder.

[11] The jurisdictional definition of over-indebtedness refers to the accounting value of assets and debt.

at time T. The value V_t of the firm at time t follows a geometric Brownian motion with constant instantaneous drift rate μ and standard deviation σ. If the value V_T of the firm at maturity T of the debt falls below its face value, the firm defaults on its debt. In case of default, the firm is over-indebted and illiquid at the same time, and the bondholders take over the firm with the equity holders receiving nothing. Summarized, bondholders receive the amount $\min\{V_T, K\}$ at the debt's maturity, and the value of a defaultable zero bond equals the value of a non-defaultable zero bond less a put option on the firm value at any time $t \leq T$.

The pricing formula for an equity put option provided by Black and Scholes (1973) is used to derive the value

$$D_{Merton}(V_t, K, \sigma, r, T-t) = Ke^{-rT}\Phi(d_1) + V_t\Phi(d_2) \tag{3.1}$$

of a defaultable zero bond at time $t \leq T$, with constant instantaneous riskless rate r, standard Gaussian distribution $\Phi(\cdot)$ and auxiliary variables

$$d_1 = \frac{\ln(V_t/K)+(r-\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} d_2 = \frac{\ln(K/V_t)-(r+\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}. \tag{3.2}$$

Although it is elegant in its consideration of corporate debt, the applicability of the Merton model is limited by its restrictive assumptions. The consideration of a zero bond prevents credit defaults prior to maturity of the debt, and default probabilities can only be expressed for the entire lifetime of the bond. The omission of interest rate risk and the simplistic capital structure are further aspects which are subject to criticism.

Subsequent structural models relax the restrictive assumptions of the Merton model in several ways. First, different types of debt are taken into account, including perpetual and finite-maturity coupon bonds, capital structures of senior and junior debt as well as defaultable bonds with embedded options and credit derivatives.

Second, default can be triggered by events other than the firm's failure to timely fulfil its obligations from debt contracts. Over-indebtedness and the inability to fulfil third-party financial commitments such as wages, accounts payable or social security payments can also force a credit default. First-passage models implement the premature default of debt contracts by triggering the default event if the asset value of a firm hits a deterministic or stochastic lower threshold for the first time. With default events enabled continuously during the lifetime of the debt, models differ between cash-flow-based default and default from over-indebtedness. First-passage models with exogenous and endogenous default barriers can be distinguished. An exogenous default barrier is determined by a bankruptcy code or by calibrating the firm value to reproduce a specific term structure of default probabilities or loss expectation. Furthermore, bond indenture and loan provisions often include safety covenants that grant to the lender the right to foreclose on the debt or to reorganize the firm. In contrast, models with an endogenous definition of default allow the firm's stockholders to decide at their own discretion and in their own interest, whether and when the firm will default on its obligations.[12]

Third, stochastic models of the riskless interest rates are incorporated, so that credit valuations involve the prevailing term structure of riskless market rates. Finally, recovery in the case of default is no longer derived from the relation of the firm value to the contractual payments of the debt. Exogenous LGD estimates are used instead. Alternatively, Hsu, Saa-Requejo and Santa-Clara (2003) differentiate between the firm value as the going-concern value of the firm and the liquidation value of its assets.

Geske (1977) considers a defaultable coupon bond to be a compound option on the firm assets. In his model, default occurs at coupon dates and results from the equity holder's strategic decision whether to foreclose the debt contract immediately or to receive the coupon payment and maintain a claim on the assets of the firm for another period,along with the recursive option to extend the claim on the firm's assets at each coupon date before the maturity of the debt. A term structure of periodic default probabilities can be derived, however, the calibration to bond market credit spreads is restricted due to the endogenous default decision. Furthermore, computational requirements restrict the applicability of the approach within large portfolio applications.

Structural first-passage models with an exogenous default barrier will be examined in detail, because of their well-founded economic theory, combined with their convenient characteristics in credit portfolio applications. Fundamental properties of major structural first-passage models with an exogenous default barrier are exhibited in 3.1. A comprehensive overview of structural pricing models of defaultable claims is provided by Uhrig-Homburg (2002). For a rigorous treatment of structural first-passage models, confer Harrison (1985), Karatzas and Shreve (1988), and Bielecki and Rutkowski (2002).

Black and Cox (1976) developed the first-passage approach to the valuation of defaultable claims. Closed-form solutions are provided for the valuation of senior and subordinated defaultable zero bonds with a deterministic default barrier that grows monotonously at a constant rate. Black and Cox allow for default at maturity, enabling the default barrier at maturity of the debt to be smaller than its face value. Premature default caused by hitting the barrier function is considered to indicate that the firm is over-indebted, whereas default at maturity represents insolvency. Furthermore, Black and Cox provide

[12] A list of structural models that incorporate strategic behavior of equity holders and an endogenous definition of the default threshold includes (in chronological order) the contributions of Black and Cox (1976), Leland (1994), Leland and Toft (1996), Anderson and Sundaresan (1996), Anderson, Breedon, Deacon, Derry and Murphy (1996), Mella-Barral and Perraudin (1997), Mella-Barral (1999), Anderson and Sundaresan (2000), Ericsson (2000), Fan and Sundaresan (2000), Goldstein, Ju and Leland (2001), as well as Francois and Morellec (2004).

an extended model for the valuation of defaultable consol bonds with endogenous default barrier and strategic behavior by equity holders. Bielecki and Rutkowski (2002) provide a corrected version of the original Black and Cox model and propose a model extension for corporate coupon bonds that pay interest continuously at a constant coupon rate.

The system application Creditgrade proposed by Finger (2002) is based on the model presented by Black and Cox (1976) and involves a structural first-passage model with a random log-normal barrier function that represents the debt per share. With parameters estimated from stock market data and the asset value calibrated to CDS data, the model is used for credit pricing, where a constant riskless rate is assumed.

Mella-Barral and Tychon (1996) assume an exogenous default-triggering state variable with geometric Brownian dynamics that does not represent the value of the firm's assets, but reflects its economic state in general. A constant default threshold is assumed, and the prices of discount and perpetual bonds are calculated along the line presented by Black and Cox. For the valuation of a defaultable coupon bond with finite maturity, periodic coupon payments are transferred into periodical continuous coupon rates. The use of a general state variable allows the inclusion of more sophisticated structures of several senior and subordinated debt contracts under a cross-default clause, with each defaultable claim being evaluated independently.

Briys and de Varenne (1997) incorporate interest rate risk into the Black and Cox model in the form of a generalized one-factor Vasicek short rate model. An exogenous recovery rate is assumed, and correlation between the asset value and the riskless rate is considered. The default barrier is defined by a fixed default point at the maturity of the debt, discounted with the random riskless rate, so that the default barrier becomes stochastic.[13] Apart from the groundbreaking work of Black and Cox (1976), the approach of Briys and de Varenne is the only first-passage model to provide a closed-form solution for the valuation of defaultable claims with an exogenous default barrier. Bielecki and Rutkowski (2002) introduce time-dependent deterministic parameters of the asset value process into the set-up of Briys and de Varenne (1997). In a Gaussian HJM setting, a quasi-closed-form solution for the forward value of a defaultable zero bond is presented under the Forward-martingale measure. An explicit pricing formula, however, is only provided for the case of a zero payout rate of the asset.

Longstaff and Schwartz (1995) incorporate a stochastic riskless short rate according to the one-factor Vasicek model into a first-passage model with a constant default barrier and allow for correlation between the asset value and the riskless short rate. Longstaff and Schwartz show that the ratio of the asset value to the default barrier is sufficient

[13] The model of Schöbel (1999) presents a simplified version of the Briys and de Varenne model using a standard Vasicek short rate with time-stationary parameters and with default at maturity precluded.

to specify the default time of a defaultable claim, so that both quantities need not be considered explicitly in the valuation of individual claims. Coupon bonds with finite time to maturity are considered as a portfolio of defaultable discount bonds, and the value additivity of cash flows is valid, which presents a major benefit compared to the compound option approach. The capital structure is exogenous and allows for more complex debt structures that include bonds of different coupon rates, maturity dates, and seniority. Net-worth-based default and cash-flow-based default can be distinguished. A cross-default clause ensures that the firm defaults on all its obligations simultaneously. The strict absolute priority of claims with respect to seniority at default is revoked by the specification of exogenous recovery rates. Recovery is paid at the original maturity of the debt and formally, a recovery-of-face-value assumption defines the recovery amount, however, the exogenous recovery rates enables to specify discretionary recovery claims. Under the forward-risk-neutral measure, only a quasi-closed-form solution is provided for the valuation of defaultable discount bonds, because the default time value of the recovery payment at maturity weighted by the density of the default time must be approximated numerically. It appears that no closed-form solution can be derived for the valuation of defaultable claims with finite maturity by a first-passage model with constant default barrier and correlated stochastic riskless short rate. [14] Therefore, it is conjectured that a stochastic short rate in general precludes a closed-form valuation of defaultable claims if a structural first-passage model with constant exogenous default barrier is considered. Although, the model of Longstaff and Schwartz (1995) matches the methodological requirements for a structural credit valuation model, it will be omitted in the remainder for its lack of analytical tractability.

Kim, Ramaswamy and Sundaresan (1993) consider a first-passage model that involves a one-factor CIR model of the riskless short rate to incorporate interest rate risk. Correlation between riskless rate and asset value is permitted, however, the risk premium of interest rate risk is set to zero, so that the short rate has identical dynamics under the risk-neutral and the real-world probability measure. Credit default occurs if the payout of the assets, modelled by a constant continuous payout rate, is not sufficient to cover the debt service for a bond with finite maturity and a constant continuous coupon rate. The constant default threshold is defined by the ratio of the payout rate of the assets to the coupon rate of the bond, with the fraction of the payout available for debt service considered to be the net cash flow of the firm's production and investment decisions.[15] The partial differential equation of the bond price is solved numerically, which renders

[14] See Bielecki and Rutkowski (2002), p. 100ff for a discussion of this topic.

[15] It is controversial why a firm should default due to a pure cash flow shortage, since equity holders might be willing to provide additional capital to avoid bankruptcy costs as long as the firm is not over-indebted.

the model infeasible for portfolio applications.[16]

Nielsen, Saa-Requejo and Santa-Clara (1993) as well as Saa-Requejo and Santa-Clara (1999) extend the model presented by Longstaff and Schwartz (1995) by assuming a stochastic default barrier, which is assumed to represent the exogenous liquidation value of the firm. The liquidation value determines the willingness of equity holders to provide additional capital to prevent default. The resulting bond price PDE must be solved by numerical techniques. Hsu et al. (2003) generalize the model of Nielsen, Saa-Requejo and Santa-Clara to a Gaussian HJM framework with deterministic volatility of default-free zero bond prices. A quasi-analytical formula for the price of a defaultable discount bond is derived under the risk-neutral forward measure.

Cathcart and El-Jahel (1998) relax the assumption that the default-triggering variable represents the asset value of a firm and propose a structural model with a signaling process and a mutually independent one-factor CIR short rate process. A constant default threshold is assumed and a quasi-analytical solution to the fundamental PDE of a defaultable coupon bond is provided in terms of the inverse Laplace transform. Lo and Hui (1999) modify the model of Cathcart and El-Jahel using a generalized one-factor CIR short-rate process and a default barrier that is a deterministic function of the default-signalling variable.

Collin-Dufresne and Goldstein (2001) propose a first-passage model with a Vasicek short-rate model that is correlated to the asset value. A stable capital structure is implemented by a stochastic mean-reverting default barrier. The definition of the default barrier assumes that the firm issues additional debt if the leverage ratio falls below a target value, and that it reduces debt financing if the ratio is above the target. A quasi-analytical pricing formula is provided for discount and coupon bonds under the forward measure at the bond maturity and an approximation scheme for the distribution function of the first-passage time is proposed to solve the probabilistic pricing equation of the bond price.

Default events triggered by diffusion processes are inherently predictable. Credit spreads implied by first-passage models therefore involve the empirical unsupported property converging to zero if the time-to-maturity of the claim approximates zero. This problem can be circumvented if asset values follow a jump-diffusion process as proposed by Zhou (1997). In contrast to intensity models, the jump of the asset value does not automatically trigger a credit default, because the default barrier is not necessarily hit. Since the default threshold may be reached either by a jump or by diffusion, the predictability of default events does no longer apply unambiguously. In the estimation of jump-diffusion models of the asset value, the specification of the jump size is critical. Zhou assumes a constant

[16]For the analytical intractability of the pricing equation the same arguments apply as for the model of Longstaff and Schwartz (1995).

jump intensity and a log-normal distributed jump size, and solves the resulting pricing PDE numerically.

The first-passage jump-diffusion model presented by Cathcart and El-Jahel (2002) involves a geometric Brownian signalling variable, an independent Gaussian short-rate process, and additionally, a doubly stochastic jump process. Since the jump component is not interrelated to the signalling variable, a jump unequivocally triggers a default event. A stochastic default threshold is modelled along the lines pretended by Briys and de Varenne (1997), and the pricing PDE of a defaultable coupon bond is solved by numerical integration on the basis of the value additivity of cash flows.

The extension of first-passage models with a jump component effects model-implied credit spreads, especially in the short term, whereas for long-term credit spreads, default induced by the asset diffusion is prevalent. However, for the unresolved identification of jump size and jump frequency in model estimation, first-passage jump-diffusion models are excluded from further consideration in this study.

The evidence on the power of structural models to fit empirical credit spreads is mixed. Helwege and Turner (1999) examine fundamental properties of the empirical term structures of bond yield spreads. Sarig and Warga (1989) find that the empirical default risk premia resemble the term structure of spreads implied by the Merton model. Wei and Guo (1997) compare Merton's model to the model of Longstaff and Schwartz in its ability to fit Eurodollar rates by minimizing the mean squared yield errors along the line suggested by Brown and Dybvig (1986), and receive a sufficient quality of the fit. However, if a term structure of empirical credit yields is expected to results in a decrease of the fitting quality, since existing estimates of model parameters imply hump-backed term structures of credit spreads for long-term maturities.

Strictly positive short-term yield spreads underpin the conjecture that either structural models are systematically flawed, or empirical yield spreads contain factors other than credit risk, such as a margin for liquidity, taxes, or trading costs. Low trading volumes of bonds with maturities of less than one year support the assumption the a liquidity premium is included in the short-term yield spreads. In a comparative analysis Huang and Huang (2002) use bond market data as well as observed default rates to examine to which extent the empirical bond yield spreads can be attributed to credit risk for a variety of structural credit pricing models. The results reveal that structural models provide a better fit for junk bonds, whereas proportionally higher fitting errors are observed for investment grade credit spreads, which gives further support to the assumption that empirical bond yield spreads incorporate factors other than credit risk. Eom, Helwege and Huang (2002) compare several structural models in their ability to fit US corporate bond market yield data and conclude that structural models do not underestimate yield spreads systematically, with yield and pricing errors considered to be satisfying.

First-Passage Model	Default-free Interest Rate Model	Correlation of Asset value and Short Rate	Continuous Asset Dividend	Continuous Coupon Rate	Default at Maturity	Barrier Growth Rate	Claim at Default	Premature Recovery Rate	Pricing Formula Solution	Special Properties
Black/Cox (1976)	constant	—	yes	no	yes	constant	asset value	—	analytical	
Nielsen/Saá-Requejo/ Santa-Clara (1993)	Vasicek	const.	yes	yes	no	stochastic	treasury value	const.	numerical	
Kim/Ramaswamy/ Sundaresan (1993)	Cox-Ingersoll-Ross	const.	yes	yes	yes	—	Minimum of asset and treasury value	deterministic game-theoretic	numerical	
Longstaff/Schwartz (1995)	Vasicek	const.	no	no	no	—	treasury value	const.	numerical	
Briys/de Varenne (1997)	Extended-Vasicek	const.	no	no	yes	short rate	asset value	const.	analytical	
Zhou (1997)	Vasicek	const.	no	no	no	—	treasury value	const.	numerical	ind. jump-diffusion with homogenous intensity
Cathart/El-Jahel (1998)	Cox-Ingersoll-Ross	zero	no	no	no	—	treasury value	const.	numerical	
Mella-Barral/Tychon (1999)	constant	—	yes	no	yes	—	face value	—	analytical	
Schöbel (1999)	Vasicek	const.	no	no	no	short rate	asset value	const.	analytical	constant capital structure
Collin-Dufresne/ Goldstein (2001)	Vasicek	const.	yes	no	no	stochastic	face value	const.	numerical	constant capital structure
Bielecki/Rutkowski (2002), p. 79	constant	—	yes	yes	yes	constant	asset value	const.	analytical	
Bielecki/Rutkowski (2002), p. 91	Gaussian-HJM	const.	yes	no	yes	stochastic	asset value	const.	numerical	
Cathart/El-Jahel (2002)	Gaussian spot rate	const.	no	no	yes	short rate	treasury value	const.	analytical	Ind. jump-diffusion with non-homog. intensity
Hsu/Saá-Requejo/ Santa-Clara (2003)	Gaussian spot rate	const.	yes	yes	no	stochastic	treasury value	const.	numerical	
Lo/Hui/Tsang (2005)	Extended-Vasicek	time-dependent	no	no	yes	short rate	asset value	const.	analytical	asset volatility affects default barrier
Bühler/Engel(2006)	constant	—	no	no	no	—	face value	const.	analytical	explicit coupon payments

Table 3.1: Single-name First-Passage Models with Exogenous Default

On one hand, a structural model used for the valuation of defaultable exposures in a credit portfolio model must sufficiently reproduce the cross-sectional and inter-temporal properties of empirical credit spreads, on the other, it should be as parsimonious as possible. With respect to model estimation and simulative portfolio analysis, analytical tractability and economical computing times are required. Specific model components that cannot be generalized within a risk class, or which develop problems, qualify for its exclusion from a model. Therefore, models that include jump components or information on the capital structure of firms are discarded. Despite the methodological suitability, the analytical intractability and extensive computational requirements prevent the use of structural models with stochastic interest rates within portfolio applications.

3.2 A Structural First-Passage Credit Risk Model

A structural first-passage model is presented for the valuation of defaultable claims with finite maturity and fixed coupons. The objective of the model is to provide mark-to-model valuations of credit exposures in corporate loan or bond portfolios to be used in the calculation of portfolio credit risk.

Conventionally, structural models consider a credit exposure to be a contingent claim on the assets of a firm and involve assumptions regarding the firm's capital structure and the trading of assets.[17] The assumption that the structural variable of a first-passage model represents the value of firm assets, available to settle the claims of borrowers, is relaxed in the remainder. Instead, the asset value is supposed to represent a latent default-signalling variable without an economic correspondence that indicates the ability and willingness of an obligor to timely fulfill the financial obligations induced by the exposure and that can be calibrated to approximate a specific term structure of credit spreads or default probabilities. Credit default is assumed to include over-indebtedness as well as insolvency. The no-arbitrage requirement that the underlying be a traded asset is circumvented by assuming the existence of a secondary market for any single-name credit risk, such as credit insurance or CDS markets, which makes enables the replication of the exposure's contingent cash flows and ensure a dynamically complete market. Furthermore, the notion of obligor default and exposure default are used interchangeably and the ratings of the obligor and the exposure are assumed to be equal.

[17] Assumptions necessary to enforce arbitrage-free pricing comprise: firm assets are traded in a continuous-time frictionless and efficient market, shares of the firm assets are infinitely divisible, short selling is not restricted, and the borrowing and lending of unrestricted amounts is possible at a constant instantaneous riskless rate.

Distributions of Asset Value and Default Time

The asset value V_t of an obligor at time t is supposed to follow a geometric Brownian motion

$$dV_t = \mu V_t dt + \sigma V_t dW_t \tag{3.3}$$

with constant instantaneous drift μ, standard deviation $\sigma > 0$ and standard Brownian motion W_t. Setting time origin $t = 0$ with boundary asset value V_0, a unique solution of (3.3) for the asset value at time $t > 0$ is given by

$$V_t = V_0 e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma W_t}, \tag{3.4}$$

where the asset value V_t is distributed log-normally with $\ln V_t \sim \mathcal{N}(\ln V_0 + (\mu - \frac{1}{2}\sigma^2)t, \sigma\sqrt{t})$, expected value $\mathbb{E}(V_t|V_0;\phi) = V_0 e^{\mu t}$, variance $Var(V_t|V_0;\phi) = V_0^2 e^{2\mu t}(e^{\sigma^2 t} - 1)$, density function

$$f_V(v_t, t) = \begin{cases} \frac{1}{\sqrt{2t\pi}\sigma}\frac{1}{v_t} e^{\frac{\left(\ln v_t - \ln V_0 - (\mu - \frac{1}{2}\sigma^2)t\right)^2}{2\sigma^2 t}} & , v_t > 0 \\ 0 & , v_t \leq 0 \end{cases} \tag{3.5}$$

and distribution function $F_V(v_t, t) = P[V_t \leq v_t] = P[V_0 e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma W_t} \leq v_t]$ given by

$$F_V(v_t, t) = \int_0^{v_t} f_V(v, t) dv = \frac{1}{\sqrt{2t\pi}\sigma} \int_0^{v_t} \frac{1}{v} e^{-\frac{\left(\ln v - \ln V_0 - (\mu - \frac{1}{2}\sigma^2)t\right)^2}{2\sigma^2 t}} dv. \tag{3.6}$$

The first-passage model triggers default of an exposure with face value K, if asset value V_t hits a constant default threshold $\overline{V} = K$ before maturity T of the exposure, i.e. $\min_{t<T} V_t \leq \overline{V}$.[18] Thus, the minimum

$$M_t = \min_{0<t' \leq t} \left((\mu - \frac{1}{2}\sigma^2)t' + \sigma W_{t'} \right) \tag{3.7}$$

of the normalized asset values $\ln(V_{t'}/V_0)$ in the interval $(0, t]$ triggers the default of an obligor. The default time or first-passage time

$$\tau = \inf\left\{t > 0 : V_t \leq \overline{V}\right\} = \inf\left\{t > 0 : M_t = \ln(\overline{V}/V_0)\right\}. \tag{3.8}$$

of a first-passage default model is a continuous random variable defined by the cumulative

[18] Setting $\overline{V} < K$ involves the distinction of premature default at a time $t < T$ due to V_t hitting the barrier $\overline{V}$, and default at maturity by the asset value $V_T \in [\overline{V}, K)$ falling short on redemption K.

default time probability

$$\begin{aligned} P[\tau \le t] &= P[\min_{t' \le t} V_{t'} \le \overline{V}] \\ &= P[\min_{t' \le t} V_0 e^{(\mu - \frac{1}{2}\sigma^2)t' + \sigma W_{t'}} \le \overline{V}] \\ &= P[M_t \le \ln(\overline{V}/V_0)]. \end{aligned} \tag{3.9}$$

Harrison (1985) shows that default time τ is distributed inverse-Gaussian with cumulative probability

$$P[\tau \le t] = N\left(\frac{\ln(\overline{V}/V_0) - \nu t}{\sigma\sqrt{t}}\right) + \left(\frac{\overline{V}}{V_0}\right)^{2\theta-2} N\left(\frac{\ln(\overline{V}/V_0) + \nu t}{\sigma\sqrt{t}}\right), \tag{3.10}$$

where auxiliary variables $\nu = \mu - \frac{1}{2}\sigma^2$ and $\theta = (\mu + \frac{1}{2}\sigma^2)/\sigma^2$ are used, and $\ln(\overline{V}/V_0)$ is designated as adjusted default threshold. The distribution function of default time τ is defined by $F_\tau(t) = P[\tau \le t], t \ge 0$ and the corresponding survival function is given by $S_\tau(t) = 1 - F_\tau(t) = P[\tau > t]$. The conditional default probability $p_{t|s} = P[\tau \le s+t | \tau > s]$ denotes the probability that an obligor, who survived until time $s \ge 0$, defaults before time $s + t, t \ge 0$. Accordingly, $q_{t|s} = 1 - p_{t|s} = P[\tau > s + t | \tau > s]$ is the conditional survival probability for the interval $[s, t + s]$. For the time origin $s = 0$, $p_t = P[\tau \le t] = F(t)$ is the unconditional probability of default, $q_t = 1 - p_t$ is the unconditional survival probability, and $p = p_1$ $(q = q_1)$ is the one-year probability of default (survival). Conditional one-year default probabilities $p_{1|t+s} = (p_{t+1|s} - p_{t|s})/(1 - p_{t|s})$ result iteratively from multi-period equivalents, so that a term structure of one-year default probabilities is defined by the sequence $\{p_1, p_{1|1}, p_{1|2}, ...p_{1|n}\}$, $n = 1, ..., T$. From the distribution function $F_\tau(t) = \int_0^t f_\tau(u)\, du$ of the default time, the density function

$$f_\tau(t) = \frac{\partial P[\tau \le t]}{\partial t} = -\frac{1}{2t\sqrt{2\pi}}\left(x_2(t)e^{-\frac{1}{2}(x_1(t))^2} + x_1(t)(\overline{V}/V_0)^{2\theta-2}e^{-\frac{1}{2}(x_2(t))^2}\right) \tag{3.11}$$

of the default time results by differentiating (3.10), with

$$x_1(t) = \frac{\ln(\overline{V}/V_0) - (\mu - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}, \qquad x_2(t) = \frac{\ln(\overline{V}/V_0) + (\mu - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}.$$

The distribution function of the first-passage time conditional on $\tau \in (0, \overline{t}]$ is given by

$$P[\tau \le t | \tau \le \overline{t}] = \frac{P[\tau \le t]}{P[\tau \le \overline{t}]}, \tag{3.12}$$

with density $f_{\tau|\tau \le \overline{t}}(t) = f_\tau(t)/F_\tau(\overline{t})$ of the first-passage time conditional on default up to time horizon $\bar{t}$. In this context, the instantaneous conditional default probability

$$\lim_{\Delta t \to 0^+} \frac{P[t < \tau \leq t + \Delta t | \tau > t]}{\Delta t} = \lim_{\Delta t \to 0^+} \frac{F_\tau(t + \Delta t) - F_\tau(t)}{(1 - F_\tau(t))\Delta t} \approx \frac{f_\tau(t)}{1 - F_\tau(t)} \tag{3.13}$$

can be compared to the time-inhomogenous intensity of reduced-form models.[19]

The distribution and density function of the first-passage time is calculated under the real-world probability measure, whereas the valuation of defaultable cash flows is performed using risk-neutral default probabilities $p_t^Q = F^Q(t) = P^Q[\tau \leq t]$ under the unique Martingale measure Q derived from (3.10) using Girsanov's theorem. Risk-neutral default probabilities result from (3.10) if the drift parameter μ is replaced by the constant instantaneous riskless rate r. The density of the first-passage time under the risk-neutral measure is defined by $f_\tau^Q(t) = \partial P^Q[\tau \leq t]/\partial t$.[20]

The complementary joint cumulative probability $\overline{F}_{V,\tau}(v_t, t) = P[V_t > v_t, \tau > t]$ is provided by Bielecki and Rutkowski (2002):[21].

$$\overline{F}_{V,\tau}(v_t, t) = N\left(\frac{\ln(V_0/v_t) + \nu t}{\sigma\sqrt{t}}\right) - \left(\frac{\overline{V}}{V_0}\right)^{2\theta-2} N\left(\frac{2\ln\overline{V} - \ln(v_t V_0) + \nu t}{\sigma\sqrt{t}}\right), \tag{3.14}$$

and the complementary joint density $\overline{f}_{V,\tau}(v_t, t)$ of the non-defaulted asset value is obtained by differentiation:

$$\overline{f}_{V,\tau}(v_t, t) = \begin{cases} t_1(v_t, t) - t_2(v_t, t)\left(\frac{\overline{V}}{V_0}\right)^{2\theta-2} & , v_t > \overline{V} \\ 0 & , v_t \leq \overline{V} \end{cases} \tag{3.15}$$

with auxiliary variables

$$t_1(v_t, t) = \frac{1}{\sqrt{2\pi t}\sigma}\frac{1}{v_t}e^{-\frac{(\ln(V_0/v_t)+\nu t)^2}{2t\sigma^2}}, \qquad t_2(v_t, t) = \frac{1}{\sqrt{2\pi t}\sigma}\frac{1}{v_t}e^{-\frac{(2\ln\overline{V}-\ln(v_t V_0)+\nu t)^2}{2t\sigma^2}}, \tag{3.16}$$

is constituted by two log-normal density functions. Using the law of total probability, the joint cumulative distribution function of asset values without default until time t is

[19] The intensity $\lambda(t)$ used by reduced-form models in the specification of default probabilities $p_t = 1 - e^{-\int_0^t \lambda(u)du}$ also represents a hazard rate. For a constant intensity λ, the default time density $f(t) = \lambda e^{-\lambda t}$ is exponentially distributed with default probability $p_t = 1 - e^{-ht}$. First-passage model, however, do not assume credit events to be Poisson distributed, so that the exponential distribution is not applicable to determine default probabilities within structural models.

[20] For f_τ and f_τ^Q to represent probability densities of the default time in the strict sense, it is additionally defined: $P[\tau = \infty] \equiv 1 - \lim_{t\to\infty} P[\tau \leq t]$, if $\lim_{t\to\infty} P[\tau \leq t] < 1$, and $P^Q[\tau = \infty] \equiv 1 - \lim_{t\to\infty} P^Q[\tau \leq t]$, if $\lim_{t\to\infty} P^Q[\tau \leq t] < 1$, so that the law of total probability yields $P[\tau \leq t] + P[\tau > t] = 1$.

[21] Note, that $P[V_t > \overline{V}, \tau > t] = P[\tau > t] = 1 - P[\tau \leq t]$.

derived:

$$P[V_t \leq v_t, \tau > t] = 1 - P[\tau \leq t] - P[V_t > v_t, \tau > t] \tag{3.17}$$

The distribution of the complementary asset value conditional on survival by time t is

$$P[V_t > v_t | \tau > t] = \frac{P[V_t > v_t, \tau > t]}{1 - P[\tau \leq t]} \tag{3.18}$$

with density $\overline{f}_{v_t|\tau}(V) = \overline{f}_{v_t,\tau}(v_t)/(1 - F_\tau(t))$, that reveals to be

$$\overline{f}_{V|\tau}(V_t, t) = \frac{t_1(v_t,t) - t_2(v_t,t)\left(\frac{\overline{V}}{V_0}\right)^{2\theta-2}}{N\left(\frac{\ln(V_0/\overline{V})+\nu t}{\sigma\sqrt{t}}\right) - \left(\frac{\overline{V}}{V_0}\right)^{2\theta-2} N\left(\frac{\ln(\overline{V}/V_0)+\nu t}{\sigma\sqrt{t}}\right)} \tag{3.19}$$

using (3.10) and (3.15). The distribution of the asset value conditional on survival by time t is given by

$$P[V_t \leq v_t | \tau > t] = 1 - P[V_t > v_t | \tau > t], \tag{3.20}$$

while the joint distribution function $F(v_t, t) = P[\tau \leq t, V_t \leq v_t]$ of asset value and default time is derived using the law of total probability twice:

$$P[V_t \leq v_t, \tau \leq t] = P[V_t \leq v_t] - P[V_t \leq v_t, \tau > t] \tag{3.21}$$

$$= P[V_t \leq v_t] - (1 - P[\tau \leq t] - P[V_t > v_t, \tau > t]). \tag{3.22}$$

Finally, the distribution of the default time conditional on $V_t \leq v_t$ is given by

$$P[\tau \leq t | V_t \leq v_t] = \frac{P[V_t \leq v_t] - (1 - P[\tau \leq t] - P[V_t > v_t, \tau > t])}{P[V_t \leq v_t]}. \tag{3.23}$$

Credit Valuation

The value of a non-defaultable zero bond with maturity T and unity face value at time $t \in [0, T]$ is defined by $B_t^T = e^{-r(T-t)}$, and $1/B_0^t$ is the value of the money market account at time t. Using a constant instantaneous riskless rate r, the interest rate risk is omitted. In the case of default, the borrower owns a claim on the firm's assets equal to the face value of its exposure, whereas accrued interest are not considered in the recovery claim. Accrued interest and bankruptcy costs such as work-out cost, legal costs or the difference between the going-concern value of the firm and the liquidation value of its assets are subsumed under a recovery rate ϱ, and it is assumed that an obligor receives a recovery of ϱK at time $\tau \leq T$. For a credit exposure with redemption of face value K at maturity T and interest $c_{t_i} K$ paid at a constant rate $c_{t_i} = c$ at time $t_i = 1, ..., T$, a general valuation formula is given by

$$D(V_t, t; \phi) = \mathbb{E}^Q[K(B_t^T \mathbb{1}_{\{\tau > T\}} + B_t^\tau \varrho \mathbb{1}_{\{\tau \leq T\}}) + cK \sum_{t_i = \rceil t \lceil}^{T} B_t^{t_i} \mathbb{1}_{\{\tau > t_i\}}], \tag{3.24}$$

where $\mathbb{E}^Q[\cdot]$ represents the expectation under risk-neutral measure Q, $\phi = \{K, T, c, \varrho, \overline{V}, r, \mu, \sigma\}$ aggregates the constitutive parameter set of the exposure, $\mathbb{1}_{\{\tau \leq T\}}$ denotes the default indicator, $\mathbb{1}_{\{\tau > T\}}$ indicates the non-default of the exposure during its lifetime, and $\rceil t \lceil$ specifies the next date when interest is paid. The present value

$$D(V_t, t; \phi) = D_Z(V_t, t; \phi) + D_C(V_t, t; \phi) \tag{3.25}$$

of the credit exposure at time t is decomposed into the value of the principal component

$$D_Z(V_t, t; \phi) = E^Q[K(B_t^T \mathbb{1}_{\{\tau > T\}} + B_t^\tau \varrho \mathbb{1}_{\{\tau \leq T\}})] \tag{3.26}$$

represented by a defaultable zero bond of face value K, and the value of the interest component

$$D_C(V_t, t; \phi) = E^Q[cK \sum_{t_i = \rceil t \lceil}^{T} B_t^{t_i} \mathbb{1}_{\{\tau > t_i\}}] \tag{3.27}$$

of non-recoverable interest payments. A closed-form expression for the value of the defaultable zero bond is derived from Bielecki and Rutkowski (2002):

$$\begin{aligned} D_Z(V_t, t; \phi) = {} & K e^{-r(T-t)} \left(\Phi(h_1(V_t, t; \phi)) - (\overline{V}/V_t)^{2\vartheta - 2} \Phi(h_2(V_t, t; \phi)) \right) \\ & + \varrho V_t \left((\overline{V}/V_t)^{2\vartheta} \Phi(h_3(V_t, t; \phi)) - \Phi(h_4(V_t, t; \phi)) \right), \end{aligned} \tag{3.28}$$

with auxiliary variables $\vartheta = (r + \frac{1}{2}\sigma^2)/\sigma^2$, and

$$h_1(V_t, t; \phi) = \frac{\ln(V_t/K) + (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}}, \; h_2(V_t, t; \phi) = \frac{2\ln \overline{V} - \ln(KV_t) + (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}},$$

$$h_3(V_t, t; \phi) = \frac{\ln(\overline{V}/V_t) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}}, \; h_4(V_t, t; \phi) = \frac{\ln(\overline{V}/V_t) - (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}}.$$

Setting $\varrho = 1$, $D_Z(V_t, t; \phi)$ converges to the value of the zero bond from the Merton (1974) model for $\overline{V} \to 0$. It is assumed that the claim on accrued and future interest does not enter into the recovery claim and the recovery rate ϱ, and the value of the interest component is given by

$$D_C(V_t, t; \phi) = \sum_{t_i = \rceil t \lceil}^{T} e^{-r(t_i - t)} cKP^Q[\tau > t_i], \tag{3.29}$$

where the risk-neutral probability of non-default until time $t_i = \rceil t \lceil, ..., T$ is defined by $P^Q[\tau > t_i] = 1 - P^Q[\tau \leq t_i]$. The present value $D(V_t, t; \phi)$ performs jumps at each time interest is paid and represents a so-called dirty price. In contrast, the clean price defined by

$$D^{clean}(V_t, t; \phi) = D(V_t, t; \phi) - cK(t - \rfloor t \lfloor), \tag{3.30}$$

is continuous in time and differs from $D(V_t, t; \phi)$ by the amount of the interest $cK(t - \rfloor t \lfloor)$ accrued since the last time $\rfloor t \lfloor < 0$ interest was paid.

3.3 Comparative-static Analysis

A comparative-static analysis of credit value $D(V_t, t; \phi)$ is conducted with respect to the basis case of a par-value exposure with face value $K' = 100$, time-to-maturity $T' = 10$, drift rate $\mu' = 8\%$, volatility $\sigma' = 10\%$, default threshold $\overline{V'} = K'$, and recovery rate $\varrho' = 50\%$. The constant instantaneous interest rate is set to $r' = 5\%$. A one-year default probability $p' = p_1 = 1\%$ is presumed and the asset value is calibrated by solving $F_\tau(V_0', 1) = p'$ to obtain $V_0' = 121.39$. Par value $D(V_0', 0; \phi') = 100$ implies a coupon rate $c' = 6.18\%$ in the basis parameter set $\phi' = \{K', T', c', \varrho', \overline{V'}, r', \mu', \sigma'\}$. The abbreviation $D_t = D(V_t, t; \phi), \forall t > 0$ applies.

After a consideration of the default time τ, the asset value V_t, that corresponds to a given one-year default probability p is examined. Second, the coupon rate c that calibrates D_0 to par is investigated for different values of p. Third, a term structure of the credit value is studied for different values of p, r, μ, and σ. Finally, the term structures of the partial derivatives of $D(V_0, 0; \phi)$ with respect to a change in p, r, μ, and σ are analyzed for different values of p.

3.3.1 Default Time and Calibrated Asset Value

The first-passage distribution $F_\tau(V_0, t)$ of the default time τ is displayed in the left part of Figure 3.1 for asset values $V_0 \in \{110, 115, 120, 125\}$, given μ', σ', and $\overline{V'}$. The cumulative default probabilities p_t decrease in V_0 and increase in t, as expected. The distribution function of τ is convex for small t and becomes concave later on, converging to a specified limit for $t \to \infty$, with default time density $f_\tau(V_0, t)$ reaching a maximum in the short term and converging to zero afterwards. Consequently, the periodic default risk in form

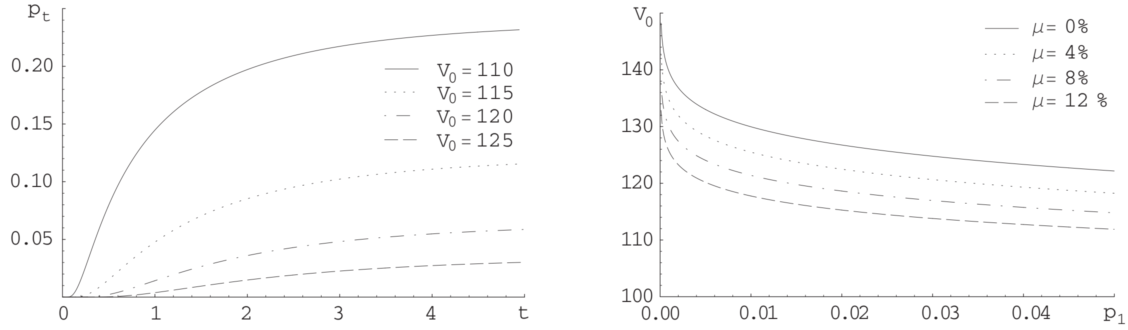

Figure 3.1: Firm Value V_0 and Default-Time τ

The left graph shows the probability p of an early default by time t for firm values $V_0 \in \{110, 115, 120, 125\}$. The right graph indicates the firm value V_0, that corresponds to default probability p for drift rate $\mu \in \{0\%, 4\%, 8\%, 12\%\}$.

of conditional one-year default probabilities is not spread homogenously in time, which affects the credit risk premium, i.e. the interest rate c of par-exposures, as will be seen in Section 3.3.2. The low level of default probabilities for leverage ratios $V_0/\overline{V'}$ that are high compared to real-world benchmarks can be attributed to the considerable drift rate and the relatively low level of the diffusion parameter compared to quotations of stock price volatilities.

In the right part of Figure 3.1, the calibrated asset value V_0 is depicted for different drift rates $\mu \in \{0\%, 4\%, 8\%, 12\%\}$ depending on the one-year default probability p. Ceteris paribus V_0 declines in p and in μ. Noted without graphical depiction, p declines with an increasing drift rate and decreasing volatility for a constant asset value and parameter set. Correspondingly, for fixed p, the calibrated asset value V_0 decreases in μ and increases in σ. The SDE of the asset value in (3.3) can be calibrated to approximate a pre-specified term structure of default probabilities by adjusting V_0, μ, and σ, whereas in Section 4.4, V_0, μ, and σ are estimated from market data of credit spreads.

3.3.2 Term Structure of the Par-Coupon Rate

The coupon rate c of par-exposures is examined for different one-year default probabilities $p \in \{0\%, 0.5\%, 1\%, 3\%, 5\%, 10\%\}$. The calibrated interest rate of a dirty-price par-exposure jumps each time T is increased across a full-year maturity, so that the par rate c is calculated for full-year maturities only and interpolated in-between to facilitate interpretation. In the left part of Figure 3.2, the par rate increases with p, with the bottom line indicating the periodic interest rate of a default-free exposure. The par rate takes a maximum for short-term maturities and declines for $T > 2$. This effect is inferred from the term structure of default probabilities. If the maturity of an exposure is extended, the default risk of the additional period is compensated for by an additional interest payment. Induced by the distribution function of the first-passage time, the probability $P[\tau \leq 1]$ of default in the first year is smaller than the probability $P[1 < \tau \leq 2]$ of default in the second period for small values of p, so that the interest rate required to compensate for the default risk of a one-year par exposure is higher than for a two-year par exposure, and the par rate increases in T in the short-term. For long-term maturities the default risk $P[T < \tau \leq T + 1]$ of an additional period is smaller than the previous average default probability $P[0 < \tau \leq T]/T$. Consequently, less additional default compensation is required than provided by the former interest rate and the par rate decreases. For exposures with $p = 5\%$ and above, periodic default probabilities $P[T < \tau \leq T + 1]$ monotonously decrease in T and par rates decline accordingly.

In the right half of Figure 3.2, the par rate of an exposure with maturity T' is displayed for different default probabilities depending on the default-free market rate r. For a non-defaultable exposure, par rates increase linearly at the market rate. Furthermore, the par rates rises in p ceteris paribus. An increase of the market rate has two contradictory effects on exposure value and par rate. First, the risk-neutral density of the default time decreases, so that expected payments under the risk-neutral measure increase in r. Second, the discount factors decrease in r, so that the present value of future cash flows drops. For high default probabilities p, the first effect dominates if r is small and the par rate is required to compensate for default risk declines in the market rate until both effects are equal and a minimum par rate is reached. If r is raised further, the decline of the discount factor outweighs the decrease of the risk-neutral probability and par rates increase in r. For small values of p, the discount effects dominate the change in expected cash flows and par rates grow monotonously at the market rate.

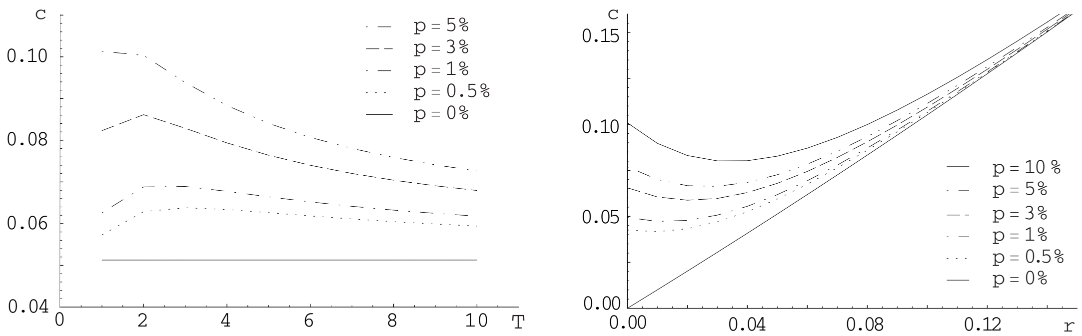

Figure 3.2: Par-Coupon Rate depending on T and r.

Calibrated coupon rates c are displayed for various $p \in \{0\%, 0.5\%, 1\%, 3\%, 5\%, 10\%\}$. In the left graph, c depends on T, and the right graph indicates c for increasing r.

3.3.3 Loan Value

The term structure of loan values D_0 of the basis case exposure are considered for different p, r, μ, and σ. For a change of default probability p', drift μ', or volatility σ', asset value V_0 is adjusted to keep p constant for the different maturity. In the left part of Figure 3.3, the value of a non-defaultable exposure grows monotonously in the time-to-maturity, because interest rate c' increasingly overcompensates for the interest required by a non-defaultable par-exposure for rising maturities. Ceteris paribus, D_0 decreases in p. For $p \geq 0.5\%$, D_0 reaches a minimum for mid-term maturities, which is explained by coupon rate c' compensating for a non-homogenous default risk as introduced in the preceding paragraph. For small T, additional interest does not fully compensate for the additional default risk if time-to-maturity is extended. After reaching its peak, the density of the default time converges to zero for $t \to \infty$, so that additional interest overcompensates for the additional default risk and the credit value improves if T is increased.

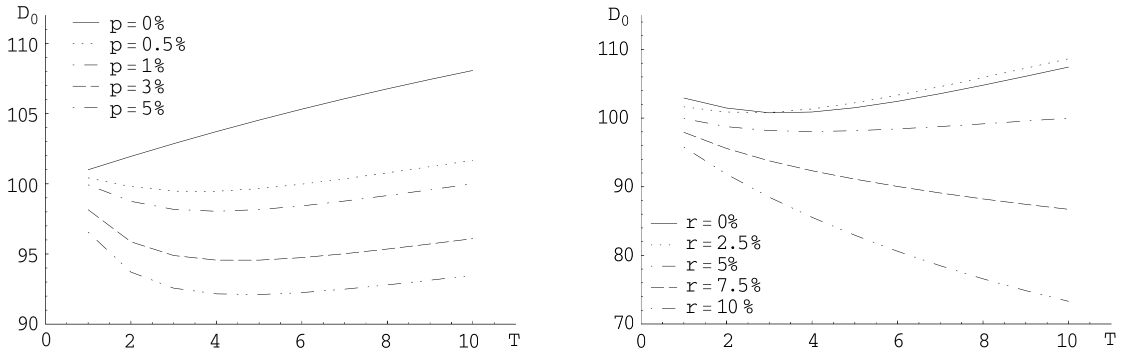

Figure 3.3: Term Structure of Credit Value D_0 for different p and r

The left graph depicts D_0 for $p \in \{0\%, 0.5\%, 1\%, 2\%, 3\%, 5\%\}$ depending on T. The right graph shows D_0 for $r \in \{0\%, 2.5\%, 5\%, 7.5\%, 10\%\}$ over T.

In the right half of Figure 3.3, the credit value of the basis exposure is shown for different market rates $r \in \{0\%, 2\%, 4\%, 6\%, 8\%, 10\%\}$ depending on T. For $r > c'$, the interest $c'K'$ excluding default risk compensation is not adequate to cover the market rate and D_0 decreases monotonously in T. An increase of r reduces the risk-neutral default probabilities and the expected future payments rise, whereas discount factors fall. Given p' and a small value of T, the second effect dominates and the loan value drops in r. In contrast, for high T and small r the first effect outweighs the decrease in the discount factors and the credit value increases in the market rate. The convex shape of D_0 contributes to the evolution of the default intensity in t. For short maturities, the default risk exceeds the default compensation included in the additional interest payment $c'K'$, if T is extended by one period. For a long time-to-maturity the opposite holds. The inhomogenous default compensation effects a minimum credit value for small values of r, whereas D_0 monotonously decreases in T for high market rates.

In the left part of Figure 3.4, the credit value D_0 is examined for different $\mu = \{0\%, 4\%, 8\%, 12\%\}$ depending on T. Drift rate μ does not appear directly in the valuation, but affects the probability of default under the real-world measure. For the different values of μ, default probability p' is held constant for any maturity of the bond, while V_0 is recalibrated.[22] Altering μ, the distribution of default times $P[\tau \leq t], t \neq 1$, the limiting default probability $\lim_{t\to\infty} P[\tau \leq t]$, and the risk-neutral default probability $P^Q[\tau \leq t]$ change. If μ is lifted, V_0 is adjusted downward and an elevated risk-neutral one-year default probability p^Q is obtained. Accordingly, D_0 falls, all else being unchanged. The

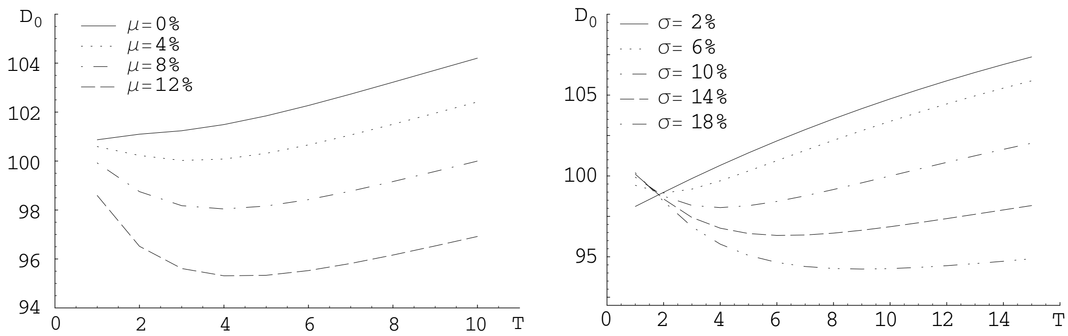

Figure 3.4: Term Structure of the Credit Value D_0 for different μ and σ

The left graph depicts D_0 for $\mu = \{0\%, 4\%, 8\%, 12\%\}$, the right graph shows D_0 for $\sigma = \{2\%, 6\%, 10\%, 14\%, 18\%\}$ depending on T.

convex shape of D_0 with minimum credit value at a medium-term maturity attributes to the relation of additional default risk and additional default compensation by the constant interest rate c', if T is enhanced. Setting $\mu = 8\%$, the difference of the additional default compensation exceeds the additional default risk for maturities greater than $T = 4$. In contrast, D_0 increases monotonously in T for $\mu = 0\%$, because the risk-neutral default probabilities decrease from the re-calibration of asset values and periodic default risk is overcompensated for by $c'K'$ in any period.

Finally, the term structure of credit value is considered for volatilities $\sigma = \{2\%, 6\%, 10\%, 14\%, 18\%\}$. If σ is increased for a fix value of V_0, default probabilities rise under both the real-world and the risk-neutral measure, so that D_0 deteriorates in σ. In contrast, if default probability p' is fixed and if σ is changed, two contradictory effects appear. First, an increase of σ leads to an increased calibrated asset value V_0, and credit values increase. Second, risk-neutral default probabilities p_t^Q rise for a given V_0. As can be seen in the right-hand graph of Figure 3.4, the first effect dominates for $T < 2$ and D_0 increases in σ, whereas for $T > 2$, the second effect outweighs the first, and

[22]Alternatively, μ can be changed holding V_0 constant with an implicit change of p.

D_0 declines in σ for a constant default probability p' and constant T. For $\sigma = 2\%$, the reduction of p_t^Q compared to the basis case dominates the decrease in the adjusted V_0 for any maturity, so that D_0 rises monotonously in T. For all other volatilities considered, D_0 takes a minimum value, when the additional default risk excluding net compensation from an increase in maturity, changes its sign.

3.3.4 Sensitivity of the Loan Value

The partial derivatives of D_0, with respect to a change in p, r, μ, and σ, are only piecewise continuous in T. Due to the jumps of the present value D_0, when interest is paid, partial derivatives of the loan value are only piecewise continuous in T. For the partial derivative $\partial D_0/\partial r$ a closed-form solution is derived, whereas a right-sided differential quotient is used to determine sensitivity of D_0 to a change in p, μ, and σ.

In the left part of Figure 3.5 the term structure of sensitivity $\partial D_0/\partial p$ is given for default probabilities $p \in \{0.5\%, 1\%, 3\%, 5\%, 10\%\}$. This p-sensitivity of D_0 can be interpreted as

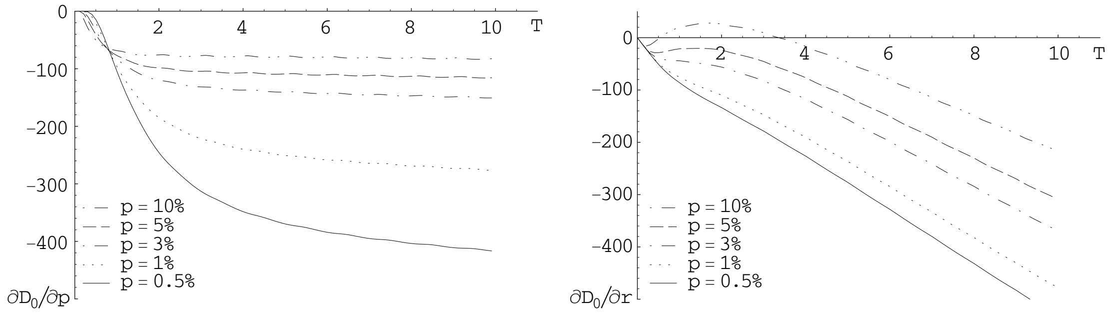

Figure 3.5: Sensitivity of D_0 to a change in p and r.

The graphs depict term structures of partial derivatives $\partial D_0/\partial p$ in the left part and of $\partial D_0/\partial r$ in the right part for $p = \{0.5\%, 1\%, 3\%, 5\%, 10\%\}$.

an analogy to the dollar duration, which considers credit risk instead of interest rate risk. The p-sensitivity for the basis case exposure indicates, for example, a decrease in D_0^t of 277 basis points if the one-year default probability is increased to $p = 2\%$ by adjustment of V_0. The absolute p-sensitivity of D_0 monotonously increases in T. For $T \gtrsim 0.8$, the absolute p-sensitivity of the loan value is higher for exposures with small values of p, whereas the absolute p-sensitivity of D_0 otherwise increases with p. The p-sensitivity turns from a concave to a convex shape if the maximum of the default density is reached.

In the right part of Figure 3.5, term structures of the partial differential $\partial D_0/\partial r$ are calculated for default probabilities $p \in \{0.5\%, 1\%, 3\%, 5\%, 10\%\}$ of the basis case exposure. The r-sensitivity of defaultable exposures evolves in a similar way to those of a default-free coupon bond for small values of p: it is negative and its absolute value increases with T. Surprisingly, the r-sensitivity is positive for $p = 10\%$ and short time-to-maturity, which is attributed to the fact that the decrease in p^Q and the corresponding increment of the expected future payments, caused by an up-shift of r, outweighs the reduction of the discount factor. For long maturities, this effect vanishes. Choosing $r = 0\%$ and a high p instead, the r-sensitivity of D_0 monotonously increases in T.

The partial derivative $\partial D_0/\partial\mu$ is analyzed for the previous default probabilities of the basis case exposure. The drift rate does not affect D_0 directly, but through the change in the calibrated asset value, so that a closed-form μ-sensitivity cannot be derived. Instead a two-fold first order approximation of $\partial D_0/\partial\mu$ is performed. A small change of μ is transferred into a change of V_0 and the differential quotient of D_0 is calculated with respect to the change of V_0. An upward shift of μ negatively affects V_0, p^Q is enhanced and it results a negative μ-sensitivity of D_0, as can be seen in the left part of Figure 3.6. The absolute μ-sensitivity increases in T, however, the trend is not monotonous, because the μ-sensitivity is not continuous in T. The sensitivity $\partial D_0/\partial\sigma$ is examined in the right

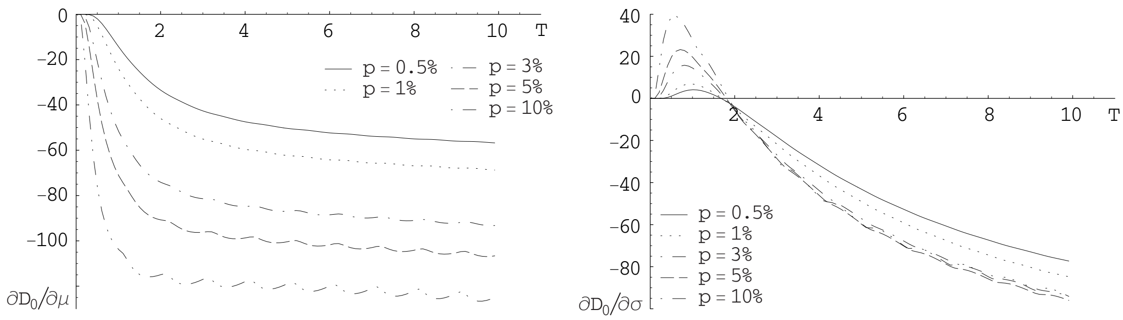

Figure 3.6: Sensitivity of D_0 to a change in μ and σ.

The figures depict term structures of the partial derivatives $\partial D_0/\partial\mu$ on the left side and of $\partial D_0/\partial\sigma$ on the right side for $p \in \{0.5\%, 1\%, 3\%, 5\%, 10\%\}$ depending on T.

part of Figure 3.6 for the basis case exposure with $p \in \{0.5\%, 1\%, 3\%, 5\%, 10\%\}$. As before, increasing σ affects D_0 due to the adverse effects on V_0 and p_t^Q. The σ-sensitivity of D_0 is positive for small maturities, reaches a maximum approximately at $T = 1$, and decreases in T to negative σ-sensitivities afterwards. Obviously, the increase of V_0 has only a short-term effect, with the rise of p_t^Q dominating for longer maturities. In contrast to previous sensitivities, the variation of the σ-sensitivity is less pronounced with respect to a change in p.

3.4 Credit Portfolio Models

Credit portfolio models are used for pricing, risk management and capital allocation of credit portfolios and multi-name credit derivatives. Credit portfolio models generate distributions and derived statistics of portfolio value, portfolio loss or (percentage) number of defaulted exposures in a credit portfolio.

From a mathematical viewpoint, any model of dependent credit defaults involves a combination of a credit default mixture model, a factor model of default probabilities, and a concept of default dependence.[23] In a generalized form, the defaults of individual obligors are typically modelled by Bernoulli- or Poisson-distributed default variables. Bernoulli and Poisson mixture models aggregate individual default variables to determine a (default) distribution for the number of defaulting obligors.[24] The dependence between obligor defaults is a result of the joint co-movement of state variables that represent the credit risk of obligors and typically depend on a set of observable or latent factors. Given a realization of the factor set, the defaults of obligors are independent and determined by conditional default probabilities, so that credit dependence is controlled by the joint evolution of factors that is defined by a copula. Frey and McNeil (2001) affirm that, given an appropriate calibration, models using the same mixture type of individual default events and the same dependence concept, i.e. the same copula, are equivalent.[25] However, the mathematical generalization of credit portfolio models using a default mixture and a copula fails to consider model characteristics and objectives with respect to (1) data used for model estimation, (2) credit risk definition, (3) type of variable that represents credit risk, (4) risk segmentation, (5) factors used, (6) dependence concept of credit risk, and (7) time-invariance of parameters.

Estimation data comprise either market price information or historical rating experience. Time series of market information enable to determine the real-world and the risk-neutral dynamics of credit risk driving state variables, whereas the information content of rating and default data is restricted to real-world probability measures. The marketability of the credit exposures in the portfolio determines whether market price information or historical credit experience is more suited to model estimation.

The notion of credit risk refers either to the number or percentage of default events, to pre-specified or stochastic default loss amounts in terms of $EAD \times LGD$, or to a definition

[23] Cf. Frey and McNeil (2001, 2003), whose generalized view of joint credit default coincides with the framework of credit portfolio modelling presented by Koyluoglu and Hickman (1998a, 1998b) in Section 2.1.2.

[24] Cf. Frey and McNeil (2001), p. 7 and Bluhm, Overbeck and Wagner (2003), p. 56ff.

[25] In contrast, Bluhm, Overbeck and Wagner (2001) contest that mixture models of the Bernoulli and the Poisson type are compatible.

of credit loss that involves changes in the valuations of exposures. Components of credit risk are default risk, risk of rating migration and changes in credit spreads, as well as systematic spread risk associated with a change in the risk perception and the credit risk premium required by market participants. In the portfolio context, default distributions and loss distributions of credit risk are distinguished.

Variables that represent credit risk comprise either multinomial rating variables, asset or firm value, including other latent structural variables that describe the ability-to-pay of an obligor, stochastic or homogenous default intensities, or stochastic default times.

Models of credit dependence include multi-name credit risk models and credit portfolio models with a risk segmentation of exposures. Multi-name credit risk models are used for the valuation of multi-name credit derivatives, counterparty risk or third-party guarantees and consider the risk characteristics and the credit dependence of each individual exposure explicitly. Risk segmentation is used for portfolios that contain a large number of exposures against different obligors and involves the classification of exposures with perceived similarity of credit risk characteristics and a homogenous dependence on a set of common factors.

Factors that determine the credit risk of obligors represent either observable macroeconomic industry and country-specific factors, fundamental obligor-specific factors, or latent unobservable statistical factors. The dependence concept, i.e. the joint distribution or copula of a model, affects portfolio credit risk in the form of fat tail effects or tail dependence of loss distributions. Finally, assumptions regarding the time-homogeneity of model parameters must be aligned to the length of the time series used for model estimation, since the dynamics of empirical data are typically time-inhomogenous.

With respect to the aforementioned characteristics of credit portfolio models, econometric default models, asset value models, actuarial models, intensity models and default time copula models are distinguished. Table 3.2 categorizes common credit portfolio models from science and banks and compares the models' basic properties. Methodological particularities of the categories are subsequently highlighted.

Comprehensive comparative examinations of credit portfolio models are provided by Koyluoglu and Hickman (1998a), Crouhy et al. (2000), Gordy (2000), Bluhm et al. (2003), and Schönbucher (2003). Studies by Gordy (1998) and Nickell, Perraudin and Varotto (2001b) compare credit portfolio models with respect to capital requirements. Fundamental characteristics of credit dependence are pointed out by Gersbach and Lipponer (2000) and Erlenmaier (2001).

3.4.1 Econometric Factor Models of Credit Default

Econometric factor models use observable macroeconomic and fundamental factors to forecast conditional point-in-time default or rating migration probabilities. Macroeconomic factors include level and changes in production, consumption, income, employment, inflation, capital market, trade balance and government activities. Fundamental factors typically include efficiency, solvability, earnings, and balance sheet ratios, as well as management quality and the market prospects of obligors.

Econometric factor models consider the default probabilities of single obligors or risk classes as relevant credit risk indicators and use either lagged factors or inherently predict the evolution of factors for the period of the risk forecast. Since factor models rely on the historically observed relation between factor values and default rates, they are vulnerable to shifts in the periodicity of the business-cycle, which may bias established relations between factors and default rates.

Credit risk factor models are structurally similar to factor models of stock returns.[26] A first seminal credit risk factor model was suggested by Altman (1968), who predicts corporate bankruptcy using a discriminant analysis on the basis of fundamental factors.

Credit Portfolio View, a credit portfolio model proposed by Wilson (1997a, 1997b), is a two-step procedure that predicts conditional independent point-in-time credit default and rating migration probabilities for the obligors of a risk class. Using a logit regression, the default probability of a risk class is predicted from macroeconomic factors which themselves are predicted by an AR(2) factor model. Risk classes represent industries, countries or both. In a second step, conditional rating migration matrices are derived from unconditional migration probabilities by considering the ratio of the predicted conditional PD to the long-term default rate of the class. Conditional distributions of portfolio default rates are simulated for single or multiple risk classes. Distributions of the conditional portfolio loss are derived either by assigning explicit loss amounts to simulated default events or from portfolio values calculated by risk-adjusted discounting of exposures with respect to simulated ratings. Unconditional loss distributions are received by averaging the conditional loss distributions given simulated values of factors.

Among many others, Altman and Suggitt (2000) provide empirical results on default rates throughout the business-cycle for the US syndicated loan market. Recent studies by Hamerle, Liebig and Rösch (2003, 2004) as well as Hamerle, Liebig and Scheule (2004) incorporate macroeconomic, fundamental and latent statistical factors in probit and logit

[26] Macroeconomic factor models of stock returns are based on the arbitrage pricing theory introduced by Ross (1976) or, more generally, on the CAPM presented by Sharpe (1964) and Lintner (1965). Constitutive fundamental factor models are proposed by Fama and French (1973) and Fama and French (1992). Statistical factor models of stock returns often rely on principle component analysis.

representations of a linear random-effect panel data model for the simultaneous estimation of conditional default probabilities and credit correlations. Lucas, Klaassen, Spreji and Straetsmans (2001) derive an analytic approximation of the credit loss distribution for an infinite number of exposures in a factor model of conditional independent defaults. Koopman, Lucas and Klaasen (2003) use credit risk factor models for the examination of business-cycle effects of credit risk. The evolution and correlation of default intensities of business sectors is examined by Das, Freed, Geng and Kapadian (2002) and Hamerle, Liebig and Scheule (2002). Factor models that explain the evolution of credit spreads are proposed by Duffee (1998), Pedrosa and Roll (1998), Collin-Dufresne and Goldstein (2001) and Collin-Dufresne, Goldstein and Martin (2001).

3.4.2 CreditRisk+

In an actuarial framework, CreditRisk+ presents a default-only model with closed-form solutions for unconditional default and loss distributions.[27] Credit risk of an obligor is represented by independent random credit default and loss severity as it is typical for actuarial models. Changes in credit quality other than default are omitted. Loss severity is specified as difference between an obligor's exposure, given by the face value or a mark-to-model value, and the present value of expected recoveries.

In a basic setting, the defaults of individual obligors are assumed to be Poisson-distributed with time-homogenous intensity and grouped to a single risk class.[28] Individual intensities are fitted to coincide with exogenously estimated default probabilities of obligors.[29] Obviously, in the single-class model, intensities can differ considerably between obligors. Given obligor-specific intensities, default events are assumed to be independent, which enables to aggregate the probability-generating functions (pgf)[30] of individual obligors, so that the default rate of the risk class is a Poisson-distributed mixture of independent Poisson-distributed default variates.

Creditrisk+ may be calibrated to produce unconditional as well as conditional credit risk forecasts. Fitting intensities to point-in-time PD estimates yields in conditional default rate distributions adapted to current economic conditions. In contrast, unconditional default rate distributions reflect the dependence of credit events imposed by a variation

[27] Cf. CSFP (1997).

[28] Poisson default models allow multiple defaults to be assigned to a single obligor, however, the effect on credit risk is considered to be immaterial in the Creditrisk+ framework.

[29] CreditRisk+ is not an intensity model in the sense of Section 3.1.1 as it does not involve risk-neutral valuation.

[30] Cf. Bosch (1992), p. 162ff. for the definition and properties of probability generating functions.

of default rates, which is assumed to be gamma-distributed in time.[31]

A generalized multiple risk class approach incorporates background factors of specific sectors on the variation of obligor-specific intensities. Sectors represent a single obligor, a rating class, business sector or country with gamma-distributed default intensities. Gamma-distributed obligor-specific intensities result from the aggregation of weighted sector intensities, and the default rate distribution of the aggregate portfolio is negative-binomially distributed. The extreme quantiles of the portfolio default rate increase with the volatility of the pooled intensity, however, since sector intensities are assumed to be independent, the probability mass of the distributional tail of the default rate tends to be biased downwards.

For the deduction of loss distributions, exposures are grouped into exposure bands of similar expected loss. Assuming time-homogenous loss severities loss distributions are deduced from default distributions. The pgf of the portfolio loss is given by the product of the loss pgf of each exposure band, and the loss distribution is approximated for single exposure bands as well as for the entire portfolio using a recurrence algorithm. For the single-class case, Bluhm et al. (2003, p. 100) show that the portfolio loss converges toward a negative-binomial distribution with gamma-distributed default intensity for an infinite number of obligors.

Several comparative studies and extensions of the original CreditRisk+ framework exist. Gordy (2000) compares CreditRisk+ to a restricted Creditmetrics model. Bluhm et al. (2001, p. S35) and Bluhm et al. (2003, p. 102) find that Bernoulli mixture models have systematically fatter tails than Poisson mixture models. Bühler, Uhrig-Homburg, Walter and Weber (1999) suggest incorporating correlation between sector intensities in the multi-class case. Haaf and Tasche (2002) calculate single-exposure risk contributions induced by CreditRisk+. Several contributions in Gundlach (2004) provide model extensions to incorporate multi-period credit risk forecasts, rating migrations, dependent sectors as well as simulation and estimation techniques, and a methodology for capital allocation.

3.4.3 Structural Credit Portfolio Models

Structural credit portfolio models used in banking practice include Creditmetrics, proposed by Gupton et al. (1997), and the KMV approach presented by Kealhofer (1995) and Vasicek (1984).[32]

[31] Cf. CSFP (1997), p. 44f.

[32] Gordy (2000) shows that Creditmetrics and the KMV model represent a Bernoulli mixture models, if asset returns follow a common factor structure.

Creditmetrics is a latent-variable approach[33] that considers credit risk in the form of a change in the value of an exposure incurred due to a change in the obligor rating. Under the assumptions of the Merton model, the continuous latent state-variable can be interpreted as a firm's asset value. A mapping translates the normalized asset returns into a multinomial rating migration. The domain of the normalized asset return of an obliger is partitioned into ranges that represent rating migrations over a Credit-VaR horizon in ascending order, where for each migration range the increase of the Gaussian distribution function of the asset return equals the probability of a migration of an obligor's current rating. The rating migration of an obligor is considered to be a credit event where credit default is defined as a special grade of the rating system. Probabilities of rating migrations across the Credit-VaR horizon are given exogenously by rating transition matrices provided either by rating agencies from historical transition experience or estimated based on default intensities using a Markov process.[34]

The use of a structural model for the valuation of an exposure with a forecasted asset value at the Credit-VaR horizon as introduced in Section 3.1.2 is omitted. Instead, exposures are valued on the basis of the corresponding rating forecast at the Credit-VaR horizon by discounting the contractual cash flows of the exposure using a risk-adjusted term structure of forward rates for the respective rating grade. Credit loss is defined as the difference of the exposure value from its expected value. In the case of default, the exposure value is set to the time value of expected recovery payment, which is assumed to be beta-distributed.

The dependence of obligors' asset returns and rating migrations is incorporated by aggregating the asset value of any obligor from a set of correlated systematic and specific factors. Gupton, Finger and Bhatia suggest using country-sector equity indices as systematic factors and attributing factor weights in a judgemental procedure, according to the "participation" of the obligor in the respective segment. No defined procedure is provided for determining the weights of systematic and specific factors. Given the standard deviations and correlations of normalized factors as well as the factor weights for any obligor, the asset correlation between any pair of obligors is specified. Loss distribution and related credit risk measures of a credit portfolio are determined by deriving the asset returns and rating migrations of all obligors in a portfolio from simulated scenarios of systematic and specific factors.

Creditmetrics' use of average historical transition rates relies on two critical assumptions: (1) all obligors within a rating class have the same default and migration probabilities, and (2) current migration probabilities are equal to historical average transition rates. However, empirical evidence produced by the KMV corporation shows that historical

[33] Cf. Frey and McNeil (2001), as well as Bluhm et al. (2003) for the notion of the latent-variable approach.

[34] Cf. Jarrow et al. (1997).

tabulations of rating transitions do not provide a powerful estimator for the migration and default probabilities of obligors. The average default probabilities of a typical firm in a rating class are overstated due individual PD being skewed. Furthermore, transition matrices overstate the probability of a rating staying constant, since agencies are slow to adjust ratings. In consequence, the probabilities of rating migrations are underestimated.[35] Further criticism focusses on the credit spread, used in discounting an exposure's cash flows, which is only determined by the rating of the obligor, thus neglecting an explicit inclusion of the capital structure and balance sheet ratios of the firm. Another question is, whether equity indices appropriately reflect the credit risk of unlisted obligors. Finally, unadjusted equity index returns are expected to exceed asset correlations, which is confirmed by alternative estimates of asset correlations.[36]

The KMV model as introduced by Kealhofer (1995, 1998) and Crosbie (1999) is an asset value approach that differs from Creditmetrics in that default and transition probabilities are calibrated by thresholds of a structural variable. The default threshold at a time horizon is derived from the book value of short-term and long-term debt, adjusted for expected growth of the debt. The initial asset value of an obligor and its diffusion parameter are estimated iteratively using an extended version of the Black-Scholes model[37] for the valuation of common stock under more complex capital structures.[38] For obligors without common stock, parameters are derived from comparables. The drift parameter of the asset value is estimated using a linear regression of asset returns against the returns of a market index. Instead of calculating default probabilities using the distributional assumptions of the diffusion process, default risk is indicated by the normalized distance-to-default (DtD) of the expected asset value at the time horizon to the default threshold. Obligors are classified according to their distance-to-default, and historical default rates, referred to as Expected Default Frequency (EDF), are calculated for each DtD class. The default probability of an obligor is assumed to equal the EDF of its DtD class instead of referring to its rating. The valuation of exposures is based on the risk-neutral default probabilities of cash flows, derived from the EDF term structure of the DtD class. Credit risk dependence is incorporated using a model of global, regional, sector and country factors similar those used in the Creditmetrics approach, which decomposes the asset returns into a systematic and obligor-specific component. Simulated distributions of credit portfolio loss refer to the value of the non-defaultable portfolio. Kealhofer and Kurbat (2001) show that the KMV approach of assigning default probabilities to obligors discriminates defaulting obligors more accurately than rating-based PD estimates, and that the time

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[35] Cf. Crouhy et al. (2000), p. 95.

[36] Cf. Düllmann and Scheule (2003) as well as Dietsch and Petey (2002).

[37] Cf. Black and Scholes (1973), p. 649ff

[38] Cf. Vasicek (1984)

series of expected default frequencies can be useful in the prediction of obligor defaults. However, only discretion-based criteria are provided for the aggregation of asset returns to weight factors according to an attribution of sales and assets to factors, which raises the question of whether Credit-VaR is a reliable risk measure in the portfolio context.

Further structural portfolio models expand single-obligor first-passage models to incorporate credit dependence. Zhou (2001) provides a closed-form solution for the joint default probability in a bivariate first-passage model with deterministic default thresholds and correlated asset values, however, it appears that no generalized closed-form solution for the joint default probabilities in a multi-obligor case exists.

Hull and White (2000b) suggest a discrete-time approximation of a first-passage model with time-dependent default barriers and correlated asset values for the valuation of Basket-CDS. Default events occur if the asset value falls below the default barrier at a discrete time grid. Given a constant recovery rate, the level of the default barrier is calibrated to reproduce market-observed credit spreads. Since default barriers are not constant, the calibration of default dependencies depend not only on the specification of a factor structure, it also requires the simulation of default times and the derivation of joint default probabilities from the samples, which is computationally expensive. Non-zero short-term credit spreads require the barrier to have an infinitely negative slope at the valuation date for an infinitesimal time interval. For the valuation of Basket-CDS Overbeck and Schmidt (2003) introduce a transformation of the first-passage time distribution to improve the calibration of first-passage models with correlated asset values.

Hull, Predescu and White (2005) propose a structural first-passage model with a constant default barrier, constant drift and diffusion parameters and a constant riskless rate similar to the default model described in Section 3.2. Credit dependence is introduced by a factor representation of normalized asset returns. Diffusion parameters are calibrated to cross-sectional term structures of CDS indices, while asset correlations are fitted to reproduce the market values of CDO first loss tranches. Empirical experience reveals that the calibration of CDO valuation models with respect to different tranches leads to differing dependence parameters. Furthermore, the calibration using cross-sectional term structures omits the estimation of drift rate of asset values.

3.4.4 Intensity-based Portfolio Models

Intensity-based portfolio models incorporate credit dependence in the form of correlation of obligors' default intensities. Concepts which implement correlated default intensities include basic-affine factor models, joint jump processes of intensities and contagion models, where infectious default events affect the intensities of economically related obligors.

Duffie and Garleanu (2001) transfer the general multi-factor model of stochastic short rates presented by Duffie and Kan (1996) to derive the joint evolution of default intensities of multiple obligors from a latent factor model of orthogonal systematic and obligor-specific basic affine intensity processes. Conditional on systematic factors, the default intensities, default events and default times of obligors are independent. Schirm (2004) estimates a restricted version of the Duffie and Garleanu model in which obligor-specific basic affine factors are omitted, whereas Duffee (1999), Driessen (1999) and Zhang (2003) propose similar default intensity factor models.

Factor models of the default intensity that use observable real-world indices as proxy factors of obligors' credit quality are subject to criticism because they generate low default correlations. According to Yu (2002), low default correlations typically arise from an insufficient specification of the common factor structure that fails to capture all sources of common variation of intensities and results in correlated obligor-specific intensities. In fact, Duffee (1999) finds that idiosyncratic factors, though statistically significant, are correlated across obligors.

Several extensions of basic affine factor models of default intensities increase default correlations by introducing additional sources of credit risk dependence. Duffie and Singleton (1999b) propose a multi-variate intensity model with a deterministic mean reversion model and correlated jumps of default intensity.[39] The intensity jump of each obligor consists of a joint and an obligor-specific jump component. Obligors may not default simultaneously in this case. Alternatively, in addition to individual Poisson-distributed defaults with independent obligor-specific intensities, Duffie and Singleton (1999b) and Kijima (2000) introduce point processes that simultaneously trigger joint credit events, a subset of which consists of obligors defaults who will default with a positive probability. Joint shocks can be fatal, resulting in unit default probabilities, or non-fatal in which case default probabilities are smaller than one.[40] However, multiple point processes of joint default intensity are troublesome in terms of notation and calibration, because a joint intensity must be specified for every joint default process, so that the number of model parameters becomes gets unmanageably large, even for portfolios which contain few obligors.

Default events are often triggered by the default of a related firm, and default times tend to show clusters in time. Contagion models of infectious defaults incorporate joint credit events to reflect the business relations of firms. Default intensities of obligors are assumed to follow correlated jump-diffusion processes, with default intensities jointly jumping upwards by a discrete amount in the case of a credit event. David and Lo

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[39] A generalized version of a multi-variate affine jump-diffusion intensity model is introduced by Duffie et al. (2000).

[40] Giesecke (2002), p. 4, shows that the Marshall-Olkin copula represents the bivariate survival probabilities in this case.

(1999, 2000) suggest that in case a firm defaults, the intensities of all remaining firms are elevated by an enhancement factor for an exponentially distributed period of time. Jarrow and Yu (2000) take counterparty risk into account more specifically, i.e. default intensities increase only for obligors which have commercial or financial relations with the defaulting firm. The default intensity of a firm depends on the default status of all related firms, which implements a symmetric dependence of default intensities. [41] Primary and secondary firms are introduced to prevent looping defaults. Default intensities of primary firms follow a factor model and do not depend on the default status of any other firm. If a primary firm defaults the intensities of secondary firms increase, so that the default correlation between secondary firms and between primary and secondary firms is enhanced, while default correlations between primary firms are substantially lower.

With respect to the real-world economic interrelations of firms, contagion models implement credit risk dependencies most realistically, however, the number of parameters rises dramatically for large sets of obligors, which negatively affects model calibration. Intensity models such as provided by Schirm (2004) or Jarrow et al. (1997) and its continuous-time generalization by Lando (1998) are less suited to credit portfolio risk applications because they neglect obligor-specific effects in the evolution of credit risk and extreme events of joint credit default or joint loss receive insufficient attention. In general, intensity-based credit portfolio models require the estimation of an extensive number of model parameters, which results in identification problems. For example, jump-diffusion processes suffer from difficulties in the estimation of size and probability of the jump component, even in the univariate case.

3.4.5 Default-time Copula Models

Credit portfolio models contain a copula to incorporate credit dependence by definition. Although, the notion of credit risk copula models is typically used in conjunction with models that explicitly consider the random joint default time of obligors. Li (1999b) is one of the first to relate the default time of exposures as explicit state variables of credit risk by differing copulas to represent the dependence of individual default risks. A general discussion of copula applications in finance is provided by Embrechts, Lindskog and McNeil (2001), Cherubini, Luciano and Vecchiato (2004), Malevergne and Sornette (2006) and McNeil, Frey and Embrechts (2005), including an overview of different concepts and measures of dependence as well as different families of copulas and copula mixture models. The use of copulas in credit risk modelling is promoted by Embrechts, McNeil

[41] Symmetric dependence introduces circularity of defaults into the model. This effect is termed as looping defaults by Jarrow and Yu and aggravates the determination of the joint distribution of default times.

and Straumann (2001), Frey and McNeil (2001) and Schönbucher and Schubert (2001).[42] Hull and White (2004) present a default time model with a Gaussian copula for the pricing of nth-to-default basket derivatives as well as for the valuation of CDO. Fermanian (2003) provides a goodness-of-fit test for copulas.

In the absence of an economic theory on the joint default time, no copula can be favored unequivocally. Default time copulas are default-only models that neglect the joint evolution of credit quality apart from default, and are typically used for the pricing of multi-name credit derivatives, like Basket-CDS and CDO, contingent only on the joint default-state of obligors. Within conventional credit portfolio models that consider changes in credit quality irrespective of default, default times can be derived from underlying state variables of exposures' credit risk and the default time copula is not explicitly specified, but results from the copula of state variables. In particular, default time copula representations exist for factor models of structural and intensity-based credit portfolio models. For example, within the Creditmetrics framework, a Gaussian copula of default times can be derived from asset returns with a Gaussian copula that specifies the joint default events within a homogenous time interval, but omits the determination of the exact joint default time. Frey and McNeil (2001) show that Creditmetrics, KMV and any structural latent-variable model with a default threshold and a Gaussian Copula are equivalent if the marginal distributions of rating migrations are appropriately calibrated and the copulas coincide.

Credit-VaR and the tail properties of default rate, default time and loss distributions are affected by the type of copula used and the respective dependence parameterization of default times. The selection and calibration of an appropriate copula is the major challenge in the application of default time copula methods. Frey and McNeil (2001) and Frey, McNeil and Nyfeler (2001) quantify the impact of lower tail dependence on the default rate of a credit portfolio for a structural one-factor model with multi-variate normal distributed asset returns and for a respective model with asset returns that follow a multi-variate t-distribution.

The estimation of default time copula models typically refers to the joint default rate of obligors, so that the copula is not specified on the basis of the joint default risk across the complete lifetime of exposures, but with respect to a fixed time interval. Furthermore, the calibration of models for the pricing of nth-to default baskets and ABS typically refers to the first loss tranche, whereas model calibration on the basis of another tranche often results in a differing credit dependence. Alternatively to the use of copulas to incorporate lower-tail dependence of default times, Andersen and Sidenius (2005) introduce random recovery and random factor coefficients to achieve the same effect.

[42] Mashal and Zeevi (2001) and Malevergne and Sornette (2006) use different copulas with lower tail dependence in a general model of the joint returns of financial assets.

	Credit Portfolio View	**KMV Model**	**Creditmetrics**	**CreditRisk+**	**Intensity Models**	**Default time Copula Models**
Risk Driver	macro-economic factors	asset value process	asset value process	stochastic default intensity	intensity process	stochastic default time
Credit Risk Indicator	mark-to-model value of loan	distance-to-default	mark-to-model value of loan	default state	mark-to-model value of loan	default state
Risk Scale	multinomial	continuous	multinomial	dichotomous	continuous	dichotomous
Migration Probabilities	endogenous, conditional on factors	endogenous	exogenous	exogenous default probabilities	endogenous default probabilities	exogenous default probabilities
Credit Risk Dependence	correlated factor returns	asset return factor model	equity return factor model	stochastic intensity model	intensity factor model	default time factor model
Recovery/LGD	stochastic	deterministic or beta-distributed	deterministic or beta-distributed	deterministic	deterministic or stochastic	deterministic or stochastic

Table 3.2: Basic Properties of Credit Portfolio Models

3.5 A Structural Credit Portfolio Model

A convenient model of credit dependence should be parsimonious in the use of parameters, model the number and the timing of defaults, consider joint changes in credit risk supplemental to credit defaults, permit default clustering, produce default correlations of a realistic magnitude and should be easy to calibrate.

A drawback of structural credit valuation models is their limited ability to fit empirical spread curves. The multi-factor decomposition of default intensities enable a comprehensive fit of the term structure dynamics of single obligor credit spreads and spread volatilities. In an extended multi-factor multi-obligor setting, the decomposition of default intensities into latent factors enables to approximate the joint evolution of the credit spreads for several obligors.

In contrast to exponential-affine intensity models, structural models include the asset value as a single state variable, so that multi-factor decompositions of the asset value along the lines presented by Duffie and Kan (1996) which improve the fitting of the term structure of credit spreads, are foreclosed for the valuation model presented in Section 3.2.

With respect to the credit dependencies within a portfolio, multi-obligor models and risk class models are distinguished. Multi-obligor models enable to specify the individual correlation between the state variables of any pair of obligors. The number of factors equals the number of obligors at least, and any discretionary credit correlation matrix can be specified.

For a large number of obligors in a credit portfolio, the specification of any pairwise credit dependence is inappropriate for two reasons. First, the robustness of the model estimation is affected by the non-homogenous quality of the estimation data that typically differ due to the unequal number, type and maturity of instruments, in the length of sample data or due to the general absence of data for a specific obligor. Second, the factor model might become over-parameterized for a complete specification of the correlation structure and problems with the identification of parameters in simultaneous estimation procedures may arise.[43]

Risk class models restrict the dependence structure by grouping obligors into risk classes, and assume that stochastic properties of normalized asset values coincide for all obligors of a class, although asset values do not evolve identically. In principle, several corporate liabilities of a single obligor with different seniority and different time to maturity are permitted. Correlation is performed by decomposing asset values into a restricted number

[43]The number of factor coefficients increases quadratically in the number of obligors, so that $n(n+1)/2$ factor coefficients and $2n$ time-invariant parameters of the asset value process had to be determined.

of common systematic factors and a supplementary obligor-specific factor. Systematic factors control for effects on asset values and credit spreads, which are common to all members of a class. Specific factors add information how obligors' credit spreads deviate from the term structure of the risk class's credit spreads induced by systematic factors. Within each risk class, the innovations of specific factors are equally weighted and i.i.d. among themselves and towards systematic factors.

In conclusion, in a risk class model the weightings of systematic and specific factors are identical for all members of a class. The correlation of normalized asset returns is equal (1) for any pair of obligors within a class (inner-class correlation), and (2) for any pair of obligors belonging to two different classes (inter-class correlation).[44] The ability to specify any arbitrary structure of inner-sector and inter-sector correlations requires, that the number of risk classes does not exceed the number of systematic factors.

The factor system constitutes either of independent abstract factors or correlated real-world factors.[45] Orthogonal factors facilitate the estimation of factor coefficients, whereas the abstract nature turns the interpretation of factors impractical.

Risk class models that decompose intensities exclusively into systematic factors and omit obligor-specific factors assume that the credit risk of all obligors in a class evolves identically and will mis-fit the distribution of credit loss and underestimate Credit-VaR.

The dependence of credit exposures subject to the first-passage default model described in Section 3.2 is incorporated by a risk class factor model of normalized asset returns. The asset value of the obligor $i = 1, ..., n_i$ assigned to a risk class $rc_i \in \{1, ..., n_{rc}\}$ is assumed to follow the SDE

$$dV_t^i = \mu_{rc_i} V_t^i dt + \sigma_{rc_i} V_t^i dW_t^i \tag{3.31}$$

with standard Brownian motion W_t^i, homogenous constant instantaneous drift μ_{rc}, and volatility σ_{rc} in each risk class $rc = 1, ..., n_{rc}$. Modifying of (3.4) yields the normalized asset return

$$\varepsilon_t^i = \frac{\ln(V_t^i / V_0^i) - (\mu_{rc_i} - \frac{1}{2}\sigma_{rc_i}^2)t}{\sigma_{rc_i}\sqrt{t}} \tag{3.32}$$

which represents a standard Gaussian white noise process $d\varepsilon_t^i = dW_t^i / \sqrt{dt} \sim \mathcal{N}(0,1)$ of (3.31) that is decomposed into a systematic factor component and a specific factor

[44] Creditmetrics and the KMV model classify obligors with respect to individual credit risk but allow for the obligor-specific weighting of factors and avoid the classification of obligors based on their credit dependence.

[45] Any system of weighted correlated normal factors can be transferred into a system of weighted uncorrelated normal factors that maintains inner-class as well as inter-class correlations of asset returns and significantly reduces the number of free coefficients.

component:

$$d\varepsilon_t^i = \sum_{j=1}^{n_{rc}} \beta_{rc_i,j} dF_t^j + \sqrt{1 - \sum_{j=1}^{n_{rc}} \beta_{rc_i,j}^2} d\epsilon_t^i, \tag{3.33}$$

where the systematic factors $dF_t^j \sim \mathcal{N}(0,1), j = 1, ..., n_{rc}$ and the specific factors $d\epsilon_t^i \sim \mathcal{N}(0,1), i = 1, ..., n$ are assumed to be independent processes of standard Gaussian white noise with covariances $dF_t^j dF_t^k = 0, \forall j \neq k, dF_t^j d\epsilon_t^i = 0, \forall j, i$ and $d\epsilon_t^i d\epsilon_t^h = 0, \forall i \neq h$. The coefficients $\beta_{rc_i,j}$ determine the sensitivity of the normalized asset returns of obligor $i = 1, ..., n$ in risk class $rc_i \in \{1, ..., n_{rc}\}$ with respect to an instantaneous change of the systematic factor F_t^j and involve restriction $\sum_{j=1}^{n_{rc}} \beta_{rc_i,j}^2 < 1$. In the model, aggregate vectors of normalized asset values ε_t, systematic factors F_t and specific factors ϵ_t are defined by

$$d\varepsilon_t = \begin{pmatrix} d\varepsilon_t^1 \\ \vdots \\ d\varepsilon_t^i \\ \vdots \\ d\varepsilon_t^n \end{pmatrix}, \qquad dF_t = \begin{pmatrix} dF_t^1 \\ \vdots \\ dF_t^j \\ \vdots \\ dF_t^{n_{rc}} \end{pmatrix}, \qquad d\epsilon_t = \begin{pmatrix} d\epsilon_t^1 \\ \vdots \\ d\epsilon_t^i \\ \vdots \\ d\epsilon_t^n \end{pmatrix}, \tag{3.34}$$

and the time-homogenous $n \times n_{rc}$-matrix of systematic factor coefficients $\mathbf{B}$ and the n-vector of specific factor coefficients $\bar{b}$ shall be given by

$$\mathbf{B} = \begin{pmatrix} \beta_{rc_1,1} & \dots & \beta_{rc_1,j} & \dots & \beta_{rc_1,n_{rc}} \\ \vdots & \ddots & & & \vdots \\ \beta_{rc_i,1} & & \beta_{rc_i,j} & & \beta_{rc_i,n_{rc}} \\ \vdots & & & \ddots & \vdots \\ \beta_{rc_n,1} & \dots & \beta_{rc_n,j} & \dots & \beta_{rc_n,n_{rc}} \end{pmatrix}, \qquad \bar{b} = \begin{pmatrix} \sqrt{1 - \sum_{j=1}^{n_{rc}} \beta_{rc_1,j}^2} \\ \vdots \\ \sqrt{1 - \sum_{j=1}^{n_{rc}} \beta_{rc_i,j}^2} \\ \vdots \\ \sqrt{1 - \sum_{j=1}^{n_{rc}} \beta_{rc_n,j}^2} \end{pmatrix}. \tag{3.35}$$

A vector notation of the risk class factor model of normalized asset returns in (3.33) yields

$$d\varepsilon_t = \mathbf{B} dF_t + (\mathcal{I}\bar{b}) d\epsilon_t, \tag{3.36}$$

with n-dimensional identity matrix $\mathbf{I}$. The risk class factor model will be used to simulate credit portfolio loss distributions in Section 5. Regarding the instantaneous (asset) correlations of normalized asset returns, homogenous inner-class correlations $\rho_{rc} = \rho_{i,h} = Cov(d\varepsilon_t^i, d\varepsilon_t^h) = \sum_{j=1}^{n_{rc}} \beta_{rc,j}^2, \forall i, h : rc = rc_i = rc_h$ and inter-class correlations $\rho_{rc_i,rc_h} = \rho_{i,h} = Cov(d\varepsilon_t^i, d\varepsilon_t^h) = \sum_{j=1}^{n_{rc}} \beta_{rc_i,j} \beta_{rc_h,j}, \forall i, h : rc_i \neq rc_h$ are differed between any pair of obligors in the respective risk classes.

In a generalized two-obligor case of the first-passage default model defined in (3.10), with asset values correlated according to (3.36), Zhou (2001) as well as Overbeck and Schmidt (2003) derive a quasi-closed form expression for the distribution function of the joint first-passage-time $P[\tau_i \leq t, \tau_h \leq t] = P[\min_{s \leq t} V_s^i < \overline{V}^i, \min_{s \leq t} V_s^h < \overline{V}^h]$ of obligors i and h. Thereby, the joint first-passage-time is formulated in terms of the modified Bessel function with constant default thresholds $\overline{V}^i$ and $\overline{V}^h$, which represents a unique copula $\mathbb{C}_{ih}(P[\tau_i \leq t], P[\tau_h \leq t])$ of the individual default times. Since no closed-form representation of a multi-variate first-passage time distribution is known, a generalized distribution function of the joint default times of n obligors cannot be determined. Furthermore, in credit risk assessments, not only the distribution of joint default times is of interest, but also the joint evolution of non-defaulted asset values, for which a closed-form representation is as well not specified and which is determined by simulative exercise in Chapter 5. Finally, the estimation of model parameters, in principle, requires a simultaneous estimation of asset value process parameters and the factor coefficient matrix B, thereby considering the fitting quality of the cross-section of credit spreads derived from implied asset values, as well as the concordance of the joint evolution of asset values in time with the distributional assumptions.

Chapter 4

Model Estimation

The estimation of credit portfolio models is based either on historical default and loss experience of credit portfolios, or market price information from equity, corporate bond or CDS markets is used either. In this chapter, the credit portfolio model defined by the single-name credit valuation model of Section 3.2 and the factor model of Section 3.5 is estimated. In the first section, the corporate bond data used for the model estimation are analyzed. Risk classes of a credit portfolio are specified in Section 4.2. A parametric fit of non-defaultable zero curves and of class-specific credit-risky zero curves is performed in Section 4.3. The pivotal Section 4.4 presents a three-step procedure for the estimation of the asset value dynamics of risk classes and the factor correlation structure of the portfolio model. First, process parameters and time series of latent systematic risk class factors are estimated on the basis of class-specific credit-risky yield curves using a Non-Gaussian Extended Kalman-Filter (EKF). Second, process parameters, factor decomposition and the time series of the latent individual asset values of risk classes are estimated analogously by an EKF using bootstrapped credit yield spreads and the factor series described above. Subsequently, asset correlations are derived from the coefficients and the trajectories of systematic factors. Finally, the estimation results are analyzed.

4.1 Government and Corporate Bond Data

General requirements to the data used for model estimation comprise:

- obligors included in the estimation data must structurally coincide with obligors in the portfolio the model is applied to
- price quotations used instead of transaction prices must reflect market conditions and must be systematically updated
- corporate bond data must exclusively reflect changes in credit spreads and must

omit effects not related to credit risk

- estimation data must be available in equally spaced and frequent time intervals and must be rich enough for a robust fitting of yield curves and for a reliable determination of the stochastic properties of yield residuals for each risk class
- risk classes can be defined to cover the entire market, distinct rating classes and industry sectors or both of the latter
- estimation data must span a maximum of non-overlapping one-year estimation periods and must refer to a constant set of obligors for each period

Conventional approaches for the estimation of structural credit models are based on equity data and focus obligor-specific risk assessments.[1] Corporations typically have only a single type of ordinary shares outstanding, so that equity market data of a corporation is widely homogenous. Furthermore, stock prices are public information, available free of charge on a continuous basis and easily accessible from stock exchanges.

In contrast, the estimation approach presented throughout is based on data from European government and corporate bond markets. Corporate bond market data suffers from infrequent trading and low volumes; it refers to heterogenous instruments and is time-consuming to process. OTC trading dominates exchange traded volumes, so that transaction prices are private information mostly and public market data typically refers to less-reliable price quotations. Differing bond features preclude a comparison of bond prices across issues and costly recording and processing of bonds' specification data is required. Bond-specific day-count conventions must be used to calculate the present value from price quotations. Nevertheless, the estimation procedure is based on bond market data, because data from stock or CDS markets involve some inconveniences:

- The asset value as defined before is considered to be a pure statistical concept of credit risk that has no economic causality to the value of equity. Furthermore, the stock prices of large corporations are considered to be an inappropriate indicator for the dynamics of credit values of small and medium-size companies, while corporate bond data is conjectured to provide a better indication of loan valuations.
- Equity-based estimation approaches typically take book values of corporate debt into account and do not incorporate changes of the debt value that coincide with changes in the value of equity, which contradicts the objective of estimating a credit valuation model.

[1] Cf. Gupton et al. (1997), Kealhofer (1998) or Hahnenstein (2004).

- CDS data is increasingly used for the estimation of multi-obligor credit valuation models. In principle, CDS data are advantageous if compared to bond data, because reliable quotations are typically available on a daily basis and refer to standardized contracts. However, CDS reference entities do not cover many small and medium size obligors that are dominant in banks' loan portfolios. Furthermore, at the time of this study, CDS quotations were not available across industry sectors and rating classes for a sample period of sufficient length.

Single national corporate bond markets in Europe do not provide sufficient data for the estimation of a portfolio model with risk classes defined across rating and industry sectors. However, the European monetary union and the European economic convergence which came along with a joint European monetary policy and the termination of mutual foreign exchange risk has made it possible to consider the aggregate corporate bond market of the Euro-zone since the introduction of the Euro. A sample period from 01/01/1999 to 31/12/2003 is used. Euro-denominated corporate bonds issued by Swiss and Swedish companies are included to enlarge the sample, because systematic factors of credit risk in these countries are assumed to coincide with those in the Euro zone, while issuers from the UK are excluded, since the British business-cycle and monetary policy differ significantly from that of the Euro zone.

Euro-denominated corporate and government bonds with deterministic cash flows and full redemption at a fixed maturity are selected using bond specification data from the Bloomberg system (BBG). Bond issues with embedded options, partial amortization or non-fixed coupon rates are excluded, as are bond issues with multi-currency, amortizing, index-linked, exchangeable, convertible or securitization features. Issues from governmental or multi-national agencies, regional or municipal authorities and from cooperative and savings banks are disregarded, while bonds issued by regional banks and central institutions of the cooperative banking sector are included. All in all, bond specification data, agency ratings and the Bloomberg sector classification data were collected for 9,487 eligible corporate bond issues and 2,517 government bonds.

Daily bond price information is obtained from the German exchange, Bloomberg and Datastream. Price information includes exchange-traded transaction prices, average and individual mid quotations contributed by market participants and indicative price quotations from the proprietary bond valuation model of Bloomberg. In case of competing or missing price information a priority list of data sources was used for the data selection. From clean price observations, present values (dirty prices) are calculated by adding accrued interest according to the interest conventions 30/360, 30/365, 30/ACT, ACT/360, ACT/365 and ACT/ACT. Corrupt data is corrected for outliers and repeating price information. Finally, weekly price information is gathered as of Wednesday, and continuous-compounding bond yields are calculated.

The classification of bonds into rating classes refers to the rating of a bond issue at the start of an estimation period. Ratings available in BBG are provided by different rating agencies, differ in methodology and refer either to the issuer of the debt (issuer-type rating) or to specific bond issues (issue-type rating). Issuer-type ratings assess the financial strength of a company in general, while issue-type ratings refer to debt classes characterized by seniority, currency-denomination and tenor. Rating grades are assigned to bonds prices according to the rating type and the corporate entity that a rating refers to. A corporate entity subsumes all bond issuing entities subject to a cross-default assumption throughout the sample period, including holding companies, corporate affiliations and financial services subsidiaries. Corporate mergers trigger a reassignment of the corporate entity of a debt issue. The rating type is specified by (1) the rating agency, (2) a flag to differentiate between issuer-type and issue-type ratings, and (3) in case of issue-type ratings, the category of debt. Rating notches are neglected. Different ratings are assigned to bonds issued by the same corporate entity, if corporate debt is of a different issue-type, e.g. senior and subordinated debt.

A priority scheme of rating types is applied if ratings of different type or assigned by different agencies are available for a particular bond issue. Ratings from Moody's and Fitch are preferred to those from Standard & Poor's (S&P) for methodological reasons. The former two agencies provide predominantly issue-type ratings that classify corporate bonds based on homogenous expected loss. Ratings that involve homogenous EL translate into homogenous PD estimates within a rating grade, given the assumption of a fixed LGD, and obligors can be grouped into classes of homogenous default risk. In contrast, S&P's issuer-type ratings assess the probability of default and neglect the LGD estimate of bond issues. The credit spreads of bonds with equal issuer-type ratings may differ due to divergent LGD estimates, which must be taken into account in the definition of risk classes to receive homogenous credit risk. In practice, however, the issuer-type and issue-type ratings of obligors tend to be equal, so that issuer-type ratings are also used if no more appropriate rating type is available. If several rating types are equally appropriate for a bond issue, the time span of rating histories and the frequency of ratings reassignments are taken into account in the selection of a rating type.

In line to the conventional time horizon of Credit-VaR, estimation samples are specified for five consecutive non-overlapping one-year estimation periods and for an additional sample spanning the entire five-year period. In each estimation, sample bond issues are assigned to risk classes according to the BBG sector affiliation and to the selected rating at the beginning of the sample period. The risk class affiliation of issues is held constant during each estimation period, so that rating changes do not involve a reclassification of the bond issue for the time remaining in the estimation period. The sector affiliations of bonds are kept constant throughout the entire sample period, while a change of the rating involves a reclassification for the next one-year estimation period. With rating changes during an estimation period being permitted within a risk class, migration risk would be included.

For the estimation of riskless term structures 27,911 bond prices of 161 German and 246 French government bond issues are used as indicated in Table A.4. Issuance activity in the European corporate bond market across economic sectors is presented in Table A.2-A.3. Abbreviations for the 13 economic sectors are introduced in the left column of Table A.2. The number of issues in the European corporate bond market increased from 272 in 1998 to 345 issues in 1999, the year after the EMU took effect, and the numbers peaked in 2001 at 465 bonds, which can be attributed to the attractive conditions for debt financing including low interest rates and improved access to investors caused by the ongoing EMU bond market integration. In the financial sector, the peak at 334 bond issues in 2001 is caused by increased liquidity needs in the direct aftermath of 9/11 and a restricted loan granting in the years that followed. The total number of issues by non-financial issuers more than doubled from 58 in 1998 to 131 in 2001 and remained at this level until the end of the sample period. The dip in issuance activity in 2000 is attributable to attractive conditions in equity financing and to high interest rates, while the large number of issues from 2001 to 2003 was caused by corporations' need for liquidity to cover operational loss. With an increasing number of issues outstanding, improved price availability is observed throughout the sample period. Although, this fact is only of minor importance for the supply of price data, the average issue amount of bonds steadily increased from EUR 213.7mn in the last year before the EMU took effect to EUR 530.9mn in 2003, while average issue amounts in the corporate segment jumped from EUR 322.6mn in 1998 to EUR 554.5mn in 1999, and remained stable afterwards.

Table A.4 provides detailed statistics on the estimation samples. The entire sample is made up of 312,799 weekly price data from 2,817 bonds issued by 355 corporate issuers in 13 sectors. Investment-grade classes AA, A and BBB are taken into account individually, NI aggregates all non-investment grade ratings, while NR denotes not-rated issues and obligors. The financial sector FIN, including merchant and retail banks as well as insurance companies, dominates with 223,461 price observations from 2001 bonds issued by 119 issuers. Regarding the non-financial sectors, only the AUT, NCC and UTY sectors are data-rich enough to promise a robust estimation of asset value dynamics. The restricted availability of data makes a robust estimation for the TEC, MED and TRA sector impossible, although price availability, in general, improves throughout the sample period.[2] Restricted price availability also prevents a model estimation for single non-financial sectors in 1999, and a non-investment grade rating class cannot be considered.

[2] The number of issuers across rating classes does not add up to the total number of issuers in a given year, because particular issuers have outstanding debt of different ratings.

In the next section, sectors will be clustered into sector classes of homogenous credit risk, concentrating the data to enable a more robust estimation and to reduce the complexity of the risk class model.

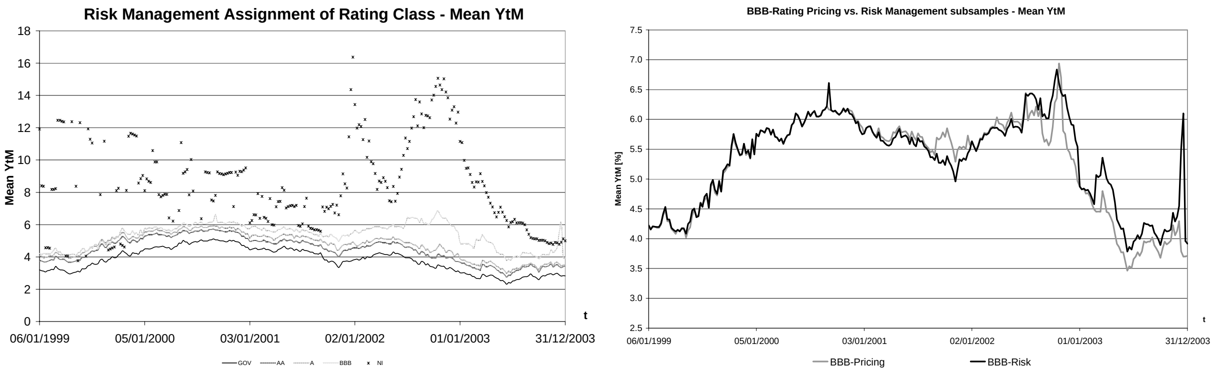

Figure 4.1: Average Yield per Rating-Class and Sampling Effect

Observed rating migrations are summarized on a yearly basis in Table A.5. Along with the number of price observations, the number of rating migrations increases during the sample period. Constant ratings dominate, and downgrades occur with significantly greater frequency than upgrades. The percentage of downgrades reaches a peak in 2003 and ratings are withdrawn in three cases. Bond defaults occur for a total of 15 issues by Parmalat and its Financial subsidiaries (12 issues), KPNQwest (2) and Global Telesystems Europe. Especially the credit default of Parmalat has seriously affected the term structure of credit spreads of BBB issues in the CNC sector in 2003 as will be seen in Section 4.3.2.[3]

For a brief overview of bond yields, the left-hand graph of Figure 4.1 shows the average yield-to-maturity of rating classes AA, A and BBB during the sample period. Average yields in investment-grade classes rise until autumn 2000, reflecting the increase in riskless rates. Later, a steady decline is observed for ratings AA and A, while the BBB spreads widen significantly and reach a peak in October 2002. Non-investment grade yields confirm this effect, although insufficient data and discontinuous price observation prevent a further investigation of the NI class.

Risk management applications imposes different requirements on the assignment of issues to risk classes than bond pricing applications. For bond pricing, the benchmark curves used refer to a particular sector and rating class, and observed bond prices currently attached to the sector and rating are used to fit the benchmark curve. In a risk management setting, the dynamics of a portfolio with a constant composition during the time horizon of a risk forecast are taken into account. Consequently, yield curves used for bond pricing

[3] For other corporate bond defaults in the sample period bond prices could not be observed. A detailed coverage of defaults in the European corporate bond markets between 1998 and 2003 is presented by Hamilton (2002) and Hamilton, Cantor, Ou and Varma (2004).

are not applicable to the estimation of a credit risk model, due to the time-variant composition of bond samples used in curve fitting. This sampling effect is illustrated in the right-hand graph of Figure 4.1, which compares the average yield of the BBB bond sample to a sample adjusted for rating migrations. The average yields of both samples clearly differ, especially in the second half of each year. Since yield and price effects induced by rating downgrades exceed those from upgrades, sticking to pricing curves is expected to underestimate the credit risk of a portfolio.

4.2 Clustering of Risk-Classes

In the risk class factor model described in Section 3.5, exposures of a risk class are assumed to follow identically specified asset value processes and have homogenous asset correlation to other exposures. Attributes used to specify the risk class affiliation of exposures are either its rating or economic sector, or both.

A brief look at the data set reveals that rating-sector-based risk classes subject to the BBG sector classification do not provide enough bond price observations for a robust fitting of yield curves. To concentrate the available data, sectors are grouped into sector-classes of assumed homogenous credit risk using an agglomerative cluster procedure based on a credit spread related similarity measure.[4]

Definition of Similarity Measure

The objective of clustering is to achieve a classification that groups bonds exhibiting the most homogenous evolution of credit risk within sector-classes and that provides most heterogenous spreads between classes. The indicator for the evolution of credit risk in a sector is set to be the time series of log-returns of the average credit yield spread in a sector, denoted as log-returns of sector spreads. Log-returns are preferred to the first differences of average credit yield spreads because they are less sensitive to outliers. Thus, they smooth out extreme obligor-specific effects and better represent the dependence of the sectors' credit risk, as can be seen in the case of the Parmalat default in December 2003 in the CNC sector.

A hierarchical iterative clustering procedure is used. The prospective joining of two sector-classes k_j and l_j in iteration step j is assessed using a similarity measure which is based on an indicator for the homogeneity of credit risk within the joint sector-class and an indicator to assess the heterogeneity of credit risk against the remaining classes. Both indicators rely on the time series $rcys^{k_j} = (rcys_t^{k_j})_{t=2,\ldots,T}$ of log-returns $rcys_t^{k_j} = \ln(\overline{cys}_t^{k_j}/\overline{cys}_{t-1}^{k_j})$ of the average credit yield spreads $\overline{cys}_t^{k_j} = (\sum_{s\in I_{k_j}^s}\sum_{i\in I_s} cys_t^i)/\sum_{s\in K_j} n_{s,t}^i$ in a sector-class

[4] For a brief introduction into cluster analysis, confer Everitt (1993).

k_j including set $I^s_{k_j}$ of sectors. In each iteration j, $n^i_{s,t}$ denotes the number of bond yield spreads available in sector s at time t, $n^s_{k_j}$ gives the number of sectors included in sector-class k_j and $n^{sc}_j = 13 - j$ represents the number of sector-classes. I_s denotes the set of bonds in sector s. The credit yield spread cys^i_t is defined as the difference between the internal yield-to-maturity of bond i, derived from the observed bond price and the yield of an equivalent non-defaultable bond determined using the term structure of riskless rates fitted in Section 4.3. The dependence of credit spreads of two different sector-classes k_j and l_j is measured by the correlation

$$\rho^{sc}_{k_j l_j} = \frac{Cov(rcys^{k_j}, rcys^{l_j})}{\sqrt{Var(rcys^{k_j})}\sqrt{Var(rcys^{l_j})}} \tag{4.1}$$

of the time series of log-returns of average sector-class spreads. For a join of sector-classes k_j and l_j, the homogeneity of credit spreads within the joint class is assessed by averaging the correlations $\rho^{sc}_{s_1 r_1}$ of spread returns of included sectors $s_1, r_1 \notin \{I^s_{k_j}, I^s_{l_j}\}, s_1 \neq r_1$:

$$\overline{\rho}^{sc}_{k_j l_j} = \frac{\sum_{s_1 \epsilon \{I^s_{k_j}, I^s_{l_j}\}} \sum_{r_1 \epsilon \{I^s_{k_j}, I^s_{l_j}\}; r_1 \neq s_1} \rho^{sc}_{s_1 r_1}}{(n^s_{k_j} + n^s_{l_j})(n^s_{k_j} + n^s_{l_j} - 1)} \tag{4.2}$$

The heterogeneity of credit spreads between a joint sector-class of k_j and l_j and the remaining sector-classes is measured by the average

$$\overline{\overline{\rho}}^{sc}_{k_j l_j} = \frac{\sum_{h_j \neq k_j, l_j} \rho^{sc}_{k_j h_j} + \rho^{sc}_{l_j h_j}}{2(n^{sc}_j - 2)} \tag{4.3}$$

of all correlations $\rho^{sc}_{k_j h_j}$ and $\rho^{sc}_{l_j h_j}$ between the joining sector-classes and all remaining classes $h_j \in \{1, ..., n^{sc}_j\} \backslash \{k_j, l_j\}$ of the current classification at cluster level j. Note that in each iteration j, the inner-class credit risk homogeneity $\overline{\rho}^{sc}_{k_j l_j}$ is derived from correlations $\rho^{sc}_{s_1 r_1}$ of sector spread returns $\mathbf{rcys}^{s_1}$ and $\mathbf{rcys}^{r_1}$, which do not change throughout the clustering, while for the calculation of heterogeneity $\overline{\overline{\rho}}^{sc}_{k_j l_j}$ series $\mathbf{rcys}^{k_j}$ and $\mathbf{rcys}^{l_j}$ of average sector-class spread returns need to be aggregated from sample data according to the current classification of sectors. Ultimately, the similarity measure

$$SIM_{k_j l_j} = \frac{\overline{\rho}^{sc}_{k_j l_j}}{\overline{\overline{\rho}}^{sc}_{k_j l_j}} \tag{4.4}$$

is used to determine which sector-classes to join at iteration level j.

Clustering Procedure

For the classification of sectors, an agglomerative hierarchical clustering procedure is used. The financial sector provides by far the largest number of price observations and is therefore set to be a sector-class on its own in the final cluster. The other 12 BBG sectors are partitioned into three sector-classes to provide a sufficient number of price observations in each sector-class without smoothing out diversification effects imposed by the heterogenous evolution of the credit spreads of different sector-classes. The sector clustering is accomplished in a four-step iterative procedure:

Step 1: Initialize sector-classes to represent individual sectors. Set iteration count $j = 1$.
Step 2: Calculate $SIM_{k_j l_j}$ for each eligible pair of sector-classes $(k_j, l_j), k_j < l_j$.
Step 3: Join pair of sector-classes (k_j^*, l_j^*) with similarity $SIM_{k_j^* l_j^*} = \max_{k_j < l_j \leq n_j^{sc}} SIM_{k_j l_j}$
Step 4: If $n_j^{sc} > 3$, then set $j = j + 1$ and proceed with step 2, else end.

Results of Clustering

Using the clustering procedure described above, the sector-classification received after $j = 10$ iterations is given in the right column of Table 4.1[5]. The denomination of sector-classes in Table 4.1 is set based on the cyclicity of sectors. Market participants typically consider the cyclicity of business sectors on the basis of lead-lag effects of stock returns relative to the economic cycle. A similar cyclicity is assumed to determine the evolution of credit risk markets. It is conjectured that the basic material (BMA) sector and the technology sector (TEC) lead the business-cycle, while the industrial sector (IND) and the construction sector (CON) lag behind. The non-cyclic consumer (NCC), utility (UTY) and energy (ENY) sectors do not show a distinct cyclicity. Correspondingly, sector-classes are designated as early-cyclic (ECY), late-cyclic (LCY), and non-cyclic (NCY). The

$\overline{\rho}^{sc}_{k_{10}l_{10}} \backslash \rho^{sc}_{k_{10}l_{10}}$	**ECY**	**FIN**	**LCY**	**NCY**	**Sector Set**
ECY	50.4	41.6	73.3	57.5	{AUT, BMA, COM, MED, TEC}
FIN	34.7	100.0	46.4	29.3	{FIN}
LCY	37.7	24.6	39.9	40.0	{CCY, CON, IND}
NCY	41.0	41.2	29.2	45.2	{CNC, ENY, TRA, UTY}

Table 4.1: Sector Classification

correlation matrix in Table 4.1 represents inter-class correlations $\rho^{sc}_{k_{10}l_{10}}$ above the diagonal, average inner-class sector correlations $\overline{\rho}^{sc}_{k_{10}k_{10}}$ within sector-classes on the diagonal, and the average inter-class sector correlations $\overline{\rho}^{sc}_{k_{10}l_{10}}$ of sectors classes below the diagonal. Inner-class sector correlations $\overline{\rho}^{sc}_{k_{10}k_{10}}$ with an average of 45.17%, excluding the FIN class, are higher than the 34.73% average of inter-class correlations $\overline{\rho}^{sc}_{k_{10}l_{10}}$, which confirms that the clustering procedure effectively classifies sectors, so that the evolution of credit spreads is more heterogenous between sector-classes than within classes. Considering the correlation of log-returns of average credit yield spreads of rating class in Table B.1, correlations are high for neighboring ratings and decrease with enhanced rating distance, which suggests

[5] An analogous clustering procedure was used to generate risk classes specified by the sector and rating of bonds, however, no intuitive classification emerged, so that a sector-rating classification was omitted.

the existence of common background factors not included in the rating. In contrast, no clear-cut pattern of sector spread correlations reveals from Table 4.1, so that it is assumed that sector affiliations present an appropriate classification of obligors to ensure a maximum heterogeneity of credit risk between risk classes and to implement a maximum diversification of credit risk in a portfolio.

p-values	**ECY**	**FIN**	**LCY**	**NCY**
ECY	–	0.3899	0.1469	0.0457
FIN	0.0521	–	0.9242	0.4126
LCY	0.0281	0.5903	–	0.5215
NCY	0.0165	0.7415	0.7866	–

Table 4.2: Granger Test of Sector-Class Causality

Table 4.2 presents the p-values of a 2-lag Granger causality test of the column sector-class leading the row class on the basis of monthly log-returns of average sector-class spreads. For example, the p-value of 0.3899 indicates that non-causality of sector-class FIN to sector-class ECY is not rejected at a 5% level of significance, so that a causality of sector-class FIN to sector-class ECY in the Granger-sense is denied. The sector-class causalities ECY→LCY, ECY→NCY, NCY→ECY are significant at a 5% level. The fact that monthly spread returns of the ECY sector-class lead those of the LCY and NCY sectors gives additional support to the designation of the ECY sector as early-cyclic, even though the direction of the causality between the ECY and NCY classes is not unambiguous.

Causalities on a sector level provide additional insights. In Table B.3, results of Granger tests for sector causalities of monthly spread returns are presented. Figure B.1 graphs sector causalities that are significant at a 5%-level.

Avg. no. of causalities per sector	**ECY**	**FIN**	**LCY**	**NCY**
Lag causalities	2.80	2.00	3.67	2.75
Lag causalities from different class	1.00	2.00	3.00	1.75
Lead causalities	4.80	0.00	1.67	2.25
Lead causalities to different class	3.00	0.00	1.00	1.25
Net lead-lag causality per sector	2.00	-2.00	-2.00	-0.50

Table 4.3: Causality Analysis per Sector-Class

Summary statistics on sector causalities in Table B.3 reveal that lead causalities and lag causalities exist predominantly for sectors of a different sector-class. Netting the number of the average lead and lag causalities per sector for each sector-class reveals that sectors of the ECY class show two more lead causalities than lag causalities, whereas the net lead-lag causality is negative for the LCY class. The NCY sectors show an almost neutral net causality. Thus, net causalities confirm that the ECY sector-class is early-cyclic, the LCY class is late-cyclic and the NCY class is almost neutral with respect to the serial cross-dependencies of the sectors. In the following, sector-classes will be used instead of sectors to specify risk classes.

The increased number of price observations in sector-classes will affect the fitting of yield curves in the next section. Four effects are expected: (1) a reduced variation of sector-class yield curves in time, (2) residuals of yields against sector-curve-induced yields will show wider spreading, (3) a more synchronous co-movement of sector-class spreads, i.e. a higher dependence of the credit risk of sector-classes as compared to a sector setting, so that systematic factors show elevated correlations. The conjectured effects, however, will not be subject to an empirical assessment, since the fitting of yield curves for sectors is not possible due to lack of data.

4.3 Term Structures of Credit-risky Interest Rates

4.3.1 Parametric Fitting Method

Two major approaches for fitting the term structures of interest rates can be differentiated.[6] McCulloch (1971a, 1971b) introduced spline methods to approximate the discount function using a continuous piecewise polynomial or exponential function.[7] Spline methods provide sufficient fitting quality and require acceptable computational effort, however, they are omitted below due to their sensitivity to outliers, especially when data is scarce as in this case.

Alternatively, parametric models define the term structure of spot or forward rates as a functional form specified by a set of parameters. Parametric models are stable with respect to outliers and provide sufficient variation in the shape of fitted term structures. For its parsimonious parameterization with only four free parameters and its ability to fit normal, inverse and humped-back term structures, the exponential form provided by Nelson and Siegel (1987) is used to fit the term structures of riskless rates and for the term structures of risk classes of defaultable bonds. Nelson and Siegel propose the functional

[6] A comprehensive overview of procedures for the fitting of term structures of interest rates is provided by Anderson et al. (1996), p. 57-64.

[7] Cf. Vasicek and Fong (1982), p. 344ff.

form

$$R_t^c(\tau;\beta_t^c) = \beta_0^c(t) + (\beta_1^c(t) + \beta_2^c(t))\frac{1 - e^{\frac{-\tau}{\beta_3^c(t)}}}{-\frac{\tau}{\beta_3^c(t)}} - \beta_2^c(t)e^{\frac{-\tau}{\beta_3^c(t)}} \tag{4.5}$$

for the time t yield-to-maturity $R_t^c(\tau, \beta_t^c)$ of a zero-coupon bond with time-to-maturity τ, termed as spot rate or zero rate, and the parameter vector $\beta_t^c = (\beta_0^c(t), \beta_1^c(t), \beta_2^c(t), \beta_3^c(t))$. The short rate and the zero rate for an infinite time-to-maturity are directly related to parameters by the convergence characteristics:

$$\lim_{\tau->0} R_t^c(\tau;\beta_t^c) = \beta_0^c(t) + \beta_1^c(t) \tag{4.6}$$

$$\lim_{\tau->\infty} R_t^c(\tau;\beta_t^c) = \beta_0^c(t), \tag{4.7}$$

so that $\beta_0^c(t)$ represents the long rate and $\beta_1^c(t)$ indicates the term spread of the term structure. Furthermore, $\beta_2^c(t) > 0$ ($\beta_2^c(t) < 0$) controls for the upward (downward) hump of the term structure in the short to medium term of the curve, while $\beta_3^c(t)$ characterizes the curvature.

An extension of the Nelson-Siegel form proposed by Svensson (1994) enables an even more flexible fit of term structures using an additional exponential term with two more parameters to set. However, limitations of data on defaultable bonds, especially at the beginning of the sample period, turn the fitting process unstable and result in term structures that are not robust. Even if a robust fitting of riskless term structures is possible, the Svensson model is not used, because the increase in fitting quality is negligible compared to the Nelson-Siegel model, so that the use of different parametric forms for the fitting of riskless curves and risk class curves is avoided.

The Nelson-Siegel term structures of spot rates at time t is specified by the parameter set β_t^{c*} which minimizes the sum of the squared differences

$$\sum_{i \in RC} \left(\widehat{D}_t^i(\tau_i, CF^i(t); \beta_t^c) - D_t^i \right)^2 \tag{4.8}$$

of the present value $\widehat{D}_t^i(\tau_i, CF^i(t); \beta_t^c) = \sum_{t_j=\rceil t \lceil}^{t+\tau_i} e^{-R_t^c(t_j - t;\beta_t^c)} \cdot CF_{t_j}^i$ of the coupon bond $i \in I^c$ in class $c \in \{RC, rl\}$ with deterministic cash flows $CF^i(t) = (CF_{\rceil t \lceil}^i, ..., CF_{t+\tau_i}^i)$ and time-to-maturity τ_i to the observed dirty price D_t^i of the bond.

Alternatively to (4.8), the Nelson-Siegel term structure can be fitted to minimize differences in bond yields. Since the sensitivity of bond prices to a change in the term structure increases in the time-to-maturity of bonds, the fitting of bond prices implicitly accentuates observations of distant maturities, while yield fitting does not involve an implicit maturity-dependent weighting of observations. However, due to the illiquidity of bond markets, yields show an increased spreading close to maturity which negatively affects the stability of term structures, so that price fitting is preferred.

Before riskless yield curves are fitted, mis-specified prices of government bonds are corrected according to the method proposed by Schwartz (1998). An outlier correction for credit curves is not feasible, because considerable changes in credit yields cannot easily be separated into pricing errors and actual changes in credit risk. In the fitting of risk class curves, the short rate of a risk class is fixed to the short rate of the riskless yield curve to stabilize credit curves in the short term and to ensure that short spreads converge to zero if the remaining time to maturity converges to zero, in line with the structural credit valuation model.

Fitted term structures of risk classes do not incorporate information on the evolution of residuals of obligor-specific bond yields from term-structure-implied yields. Since the fitting of obligor-specific yield curves is precluded due to data limitations, the yield curves of risk classes only qualify to estimate systematic factor processes.

In Section 4.4, the dynamics of risk class factor processes and the trajectories of systematic factors will be estimated based on yield series of coupon bonds which are derived from synthetic term structures of credit spot rates. The synthetic nature of the risk class curves is caused by the constant riskless short rate which is mandatory in the credit valuation model. The term structure of synthetic spot rates of a risk class is calculated by adding the riskless short rate $R_t^{rl}(0; \beta_t^{rl})$ to the credit spot spreads $S_t^{rc}(\tau; \beta_t^{rc}, \beta_t^{rl}) = R^{rc}(\tau; \beta_t^{rc}) - R^{rl}(\tau; \beta_t^{rl})$, defined as difference between the spot rate $R_t^{rc}(\tau; \beta_t^{rc})$ of risk class rc and the riskless spot rate $R_t^{rl}(\tau; \beta_t^{rl})$. In order to regain the lost information about specific yield variations in the estimation of specific factor weights of the risk class factor model, synthetic credit spot rates are used in Section 4.4.6 for bootstrapping obligor-specific changes in credit spreads.

4.3.2 Results of Term Structure Fitting

The effects of a rating-based versus a sector-rating-based specification of risk classes is examined using descriptive statistics and by a consideration of the evolution of fitted term structures of credit spot spreads in time. Autocorrelations and Unit Root tests are used to investigate if the time series of fitted credit spreads are stationary. A correlation analysis evaluates alternative definitions of risk classes. A residual analysis is used to infer on the homogeneity of credit risk in risk classes and to conclude on the strength of asset correlations induced by systematic factors.

4.3.2.1 Descriptive Statistics

Descriptive statistics on riskless spot rates and credit spot spreads fitted from year-based bond samples are given in Table 4.4. The information on year, rating and sector-class indicate the term structures, where RL refers to the riskless term structure, sector-class NF includes bonds of all non-financial sectors and risk class ALL contains all bonds of the respective rating class. Empirically, normal-shaped term structures of credit spot spreads are prevalent. In the years 2001–2003 increased short-term credit spreads led to hump-shaped term structures, especially for risk classes with a BBB rating.[8] The shift from normal-shaped to hump-shaped term structures during a sample period is due either to an increase of the pooled PD or LGD of bonds, or it reflects a higher market price being asked for bearing the credit risk. Assuming a constant LGD and a constant riskless rate in the credit valuation model of Section (3.2), the evolution of the fitted term structures can only represent effects on risk-neutral pooled PD that are caused either by changes in asset values induced by systematic risk factors, or by a change in the volatility of asset values, which raises concerns about the time-homogeneity of the parameters. In both cases, real-world PD change accordingly, so that a change in credit spreads always indicates a change in the real-world pooled probability of obligors' default.

The term structure of riskless rates is predominantly of a normal shape, indicated by average spot rates in Table 4.4 increasing in the time-to-maturity and by the shape of term structures derived from Figure 4.2. S-shaped term structures are defined by a local maximum of medium-term rates and a positive slope of long-term rates. This effect typically accompanies a flattening of the term structure if central bank rates are expected to rise.

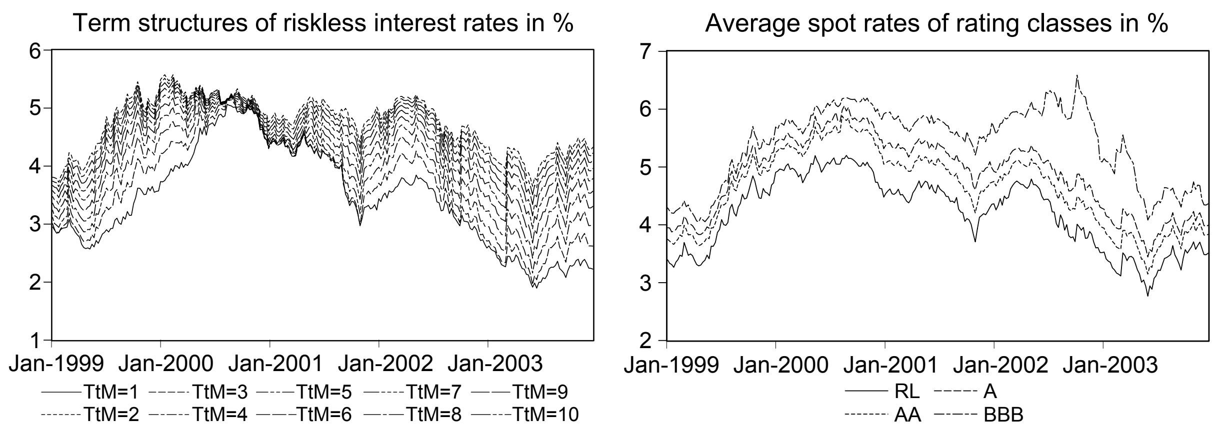

Figure 4.2: Spot Rates of Riskless-Class and Rating-Classes

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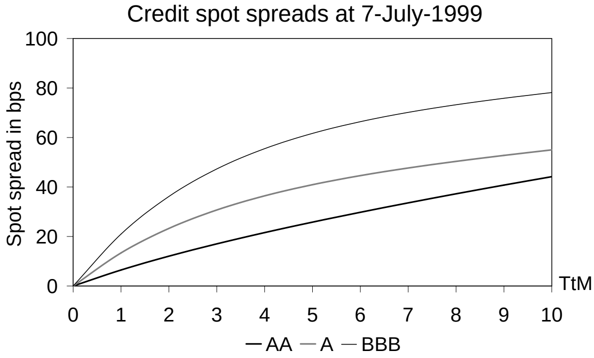
8 Hump-shaped term structures of credit spreads are typically observed for low-quality obligors, cf. Sarig and Warga (1989) as well as Fons (1994).

Systematic effects on the evolution of the level of interest rates and credit spreads, defined as average of full-year-maturity rates, are analyzed using the right-hand graph in Figure 4.2. At the beginning of the sample period, interest rates rise due to a prosperous economic outlook, concurrent fears of inflation and increasing ECB rates. As is typical for the turning point of an economic cycle, this effect is accompanied by a flattening of the riskless term structure; however, it is not accentuated enough to result in inverse term structures. Riskless rates reach a maximum of 5.199% on 30/08/2000 and decline afterwards due to the crisis of the technology sector and the corresponding recession. The 9/11 event and two subsequent ECB rate cuts of 50 basis points (bps) each on 18/09/2001 and 9/11/2001 mark a preliminary minimum rate of 3.704% on the 7/11/2001. Expectations of a fast economic recovery, spurred by low central bank rates, pushed interest rates up to 4.79% on 15/05/2002, but impairments and the negative operating results of major European companies, especially in the telecommunication, technology and financial sectors led to an economic downturn in the second half of 2002. Enforced by political uncertainties the recession culminated in minimum rates of 2.767% on 11/06/2003. The subsequent economic recovery in the second half of 2002 came along with a rebound in interest rates to 3.509% at the end of the sample period.

The credit spreads of the rating classes contract during a booming economical period, while a spread widening is observed during the technology crisis and the recession of 2002. Spreads increase until October 2001, fall back up to March/April 2002 and spike afterwards until the fourth quarter of 2002. However, the evolution of credit spreads differs between rating classes. Credit spreads are closest for AA (A) borrowers on 7/07/1999 at 26.8 bps (39.5 bps) and for BBB borrowers on 23/06/1999 with 58.5 bps. The AA rating class displays its maximum spread of 73.5 bps on 16/08/2000 during the technology crisis, and does not have credit spreads of more than 56.1 bps after the 9/11 attacks, which can be attributed to bond investors seeking a safe haven. In contrast, A and BBB ratings show maximum spreads of 94.3 bps and 257.7 bps in October 2002.[9]

Figure 4.3 illustrates the normal-shaped term structures of credit spreads on 07/07/1999, when average spot spreads of AA and A ratings reached their minimum, and the credit spread structure on 9/10/2002, when the average spreads of ratings A and BBB reached a maximum. On 9/10/2002, the term structure is hump-shaped for the BBB class, while it is normal-shaped for the better credit qualities. For rating BBB, 89 humped-shaped spread structures are found throughout the sample period, with only one (two) humped term structures in 2000 (2001), but 50 occurrences in 2003. Hump shapes are rare, with only three occurrences in 2001 and 2003 in rating class A and no occurrence for rating AAA.

[9] Note that no jumps in credit-risk rates appear in Figure 4.2 at the beginning of any year, which might have been expected due to the resampling of data.



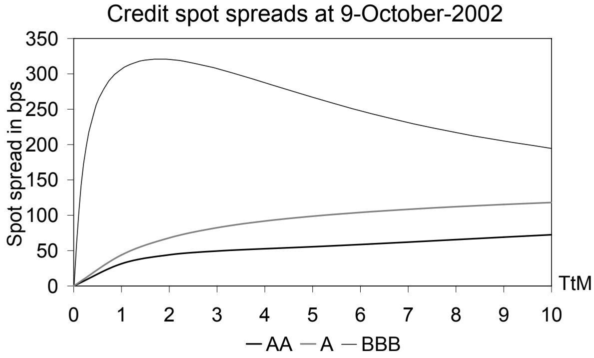


Figure 4.3: Term Structures of Credit Spot Spreads

With respect to the term structures of sector-rating classes, hump-shaped term structures are much more frequent, especially for non-financial sector classes. This can be attributed to the small sample size of the risk classes. Outlier effects from obligor-specific credit events and correlation effects in response to credit events of related issuers result in a more pronounced variation of term structures to credit events for small bond samples. In contrast, there is a sufficient average of 47 (36.6) different obligors in the financial classes FIN-AA and FIN-A with an average of 16.7 (14.8) bonds outstanding, so that short-term obligor-specific spread outliers and correlation effects are better absorbed and hump-shaped term structures are prevented.

In Table 4.4, the credit spreads of sector-rating classes with a common rating show similar properties in the years 1999 and 2000, but differ substantially for the years 2001 to 2003. In 2001, for example, the average and the standard deviation of credit spreads of risk class LCY-AA are located considerably above the average spreads of other AA classes. This effect is explained by specific credit events that only enter the spreads of the aggregate rating class in a smoothed form, while the particular credit events do not affect other sector-rating classes. The same effects appear for classes ECY-A and ECY-BBB in 2001 and 2002, LCY-A in 2002, and LCY-AA and NCY-BBB in 2003.

Standard deviations of credit spreads typically increase with worsening credit quality. In case of a credit event, the increase in volatility is more pronounced for short-term spreads, mirroring the tendency towards hump-shaped term structures, while long-term spreads remain more stable. For ratings AA and A, spread volatilities either increase monotonically with time-to-maturity, or they show a hump-shaped term structure. In times of credit distress, the term structures of spread volatility turn inverse, as can be observed for the risk classes LCY-A, ECY-BBB and LCY-BBB in 2002. The credit spreads of non-financial sectors are in general more volatile than those of the financial sector for any rating class. This effect is attributed to the higher number of observations available in the financial sector, which reduces the variation of term structures.

	Risk-Class		Term structure type			Average in bps per TtM				Std. dev. in bps per TtM				Skewness per TtM			
Year	Rating	Sector	normal	humped	s-shape	1	3	5	10	1	3	5	10	1	3	5	10
1999	RL		39	0	13	304.7	357.1	397.2	457.6	33.4	54.5	59.4	53.7	0.53	0.23	0.13	-0.07
	AA	FIN	52	0	0	11.5	26.8	36.9	52.1	3.4	3.9	4.7	5.6	0.73	0.27	0.13	0.04
		NF	45	0	7	15.1	30.4	37.7	48.9	4.9	4.5	5.1	6.5	0.79	0.32	0.47	0.39
		ALL	52	0	0	12.4	27.4	36.8	52.4	3.7	3.7	4.5	5.4	1.01	0.14	0.14	0.12
	A	FIN	52	0	0	24.2	47.5	56.8	66.8	5.6	6.4	5.7	5.8	0.50	-0.38	-0.69	-0.31
		NF	52	0	0	20.7	44.2	56.7	73.6	5.4	5.2	4.4	7.4	1.02	0.31	-0.25	-0.34
		ALL	52	0	0	23.2	46.5	56.6	68.1	5.4	6.0	5.3	5.7	0.95	-0.17	-0.79	-0.29
	BBB	NF	46	6	0	27.2	57.7	75.2	109.6	5.9	8.8	9.9	28.8	0.00	0.15	-0.31	0.55
		ALL	42	10	0	31.2	63.9	79.9	105.8	6.0	7.3	7.7	27.2	0.13	-0.21	-0.50	0.26
2000	RL		41	0	11	452.9	483.9	501.5	525.0	44.0	20.3	15.3	16.6	-0.52	-0.19	-1.50	-0.04
	AA	FIN	52	0	0	14.5	37.6	51.7	68.7	4.5	9.6	10.8	12.0	-0.31	0.52	0.18	-0.10
		LCY	52	0	0	13.7	38.1	53.9	74.3	4.7	9.7	12.3	17.0	0.69	0.23	-0.13	0.03
		NCY	52	0	0	17.3	44.0	59.0	70.5	7.0	12.1	12.2	9.1	0.23	-0.22	-0.54	-0.27
		NF	52	0	0	18.0	45.6	61.0	75.3	7.4	13.8	14.8	12.6	0.36	-0.07	-0.18	0.04
		ALL	52	0	0	15.1	38.8	53.4	71.2	5.0	10.3	11.5	12.9	0.37	0.54	0.12	-0.10
	A	ECY	52	0	0	17.9	52.0	77.3	115.6	6.1	13.2	17.6	29.1	0.30	0.39	0.11	0.41
		FIN	52	0	0	20.0	50.7	69.5	94.3	4.3	10.1	12.2	17.6	-0.10	1.33	0.63	-0.13
		LCY	52	0	0	22.3	56.7	77.1	102.1	4.9	11.3	15.7	27.8	0.15	-0.27	-0.42	0.53
		NCY	52	0	0	20.0	51.2	70.9	95.8	6.1	11.2	12.5	16.8	0.27	0.18	0.02	0.56
		NF	52	0	0	20.8	54.7	76.9	105.7	6.6	13.5	16.4	22.4	0.28	0.01	-0.14	0.48
		ALL	52	0	0	20.0	51.6	71.5	96.5	3.9	9.9	12.8	17.8	0.44	0.80	0.20	-0.12
	BBB	ECY	50	2	0	36.9	80.9	102.0	120.9	10.4	16.2	16.7	18.0	0.18	-0.20	-0.17	0.85
		LCY	48	4	0	35.2	82.3	105.3	123.4	9.8	11.7	11.0	13.9	0.45	0.29	0.14	-1.29
		NCY	49	3	0	32.6	77.5	102.7	130.1	14.9	25.0	25.4	25.1	1.28	0.86	0.37	-0.16
		NF	52	0	0	35.0	80.1	103.3	125.1	9.6	15.6	17.0	18.7	0.20	0.15	0.16	0.44
		ALL	51	1	0	36.1	81.4	103.6	123.4	9.3	15.0	16.4	17.6	0.25	0.25	0.14	0.20
2001	RL		6	0	46	400.0	410.3	435.8	488.8	48.7	34.3	23.3	16.5	-0.74	-0.89	-0.96	-0.31
	AA	FIN	52	0	0	20.6	37.8	45.9	62.3	4.4	4.0	4.9	7.9	0.34	-0.45	-0.11	0.61
		LCY	36	12	4	32.1	56.3	63.5	78.5	20.0	22.4	15.4	12.5	0.75	1.10	0.75	-0.30
		NCY	45	6	1	17.3	41.6	53.5	60.1	7.3	6.6	7.1	4.7	0.64	0.10	-0.45	-0.43
		NF	22	30	0	15.6	46.2	59.8	58.4	8.7	7.2	5.3	6.8	1.31	0.64	0.66	1.07
		ALL	52	0	0	22.2	39.2	46.6	64.8	4.6	3.7	4.9	6.9	0.12	-0.32	0.06	0.34
	A	ECY	42	10	0	54.2	105.6	128.7	166.3	25.0	36.2	29.8	15.7	1.22	1.32	1.39	0.57
		FIN	52	0	0	26.6	49.8	63.8	104.7	4.8	4.9	5.8	7.0	0.81	-0.24	0.20	0.11
		LCY	52	0	0	29.2	63.1	88.3	158.8	10.9	18.9	24.1	24.0	2.00	1.73	1.16	0.66
		NCY	52	0	0	22.9	50.3	71.0	117.4	8.3	5.2	7.3	11.1	1.07	0.51	0.34	0.50
		NF	48	4	0	43.9	90.1	113.1	142.5	20.1	27.1	22.0	9.2	1.05	1.27	1.25	0.76
		ALL	50	2	0	27.5	61.3	81.2	109.2	5.8	8.2	7.6	7.3	0.94	1.11	1.17	0.37
	BBB	ECY	30	22	0	64.8	120.7	137.6	146.5	22.0	28.3	21.1	19.4	1.01	0.78	0.51	0.25
		LCY	50	2	0	50.0	97.5	118.0	147.6	10.8	12.9	14.0	23.2	1.04	1.02	0.70	0.68
		NCY	52	0	0	47.0	96.1	126.7	206.0	12.1	15.2	11.5	30.4	0.96	0.51	0.22	-0.39
		NF	51	1	0	51.4	101.4	126.5	178.1	14.9	19.5	15.5	20.5	0.92	0.67	0.32	-0.33
		ALL	50	2	0	51.5	101.6	126.9	178.2	14.5	18.9	15.2	24.6	0.93	0.58	0.17	-0.37
2002	RL		34	0	18	338.7	389.1	429.2	484.4	32.5	45.4	41.3	27.6	-0.45	-0.29	-0.23	-0.20
	AA	FIN	51	0	0	19.8	35.0	41.4	54.3	7.2	6.2	4.3	7.1	0.48	0.50	0.50	0.37
		LCY	33	19	0	20.0	42.8	51.8	57.3	13.3	14.4	10.0	8.7	1.13	0.51	0.30	0.40
		NCY	50	0	2	22.9	43.2	50.8	58.0	7.3	7.6	5.9	6.6	0.67	0.27	0.34	0.48
		NF	46	6	0	18.4	42.2	52.0	55.5	3.9	5.8	6.6	5.8	0.15	0.44	0.38	0.61
		ALL	52	0	0	19.9	35.8	42.7	54.8	6.6	6.1	4.6	7.0	0.49	0.54	0.51	0.31
	A	ECY	42	10	0	47.3	85.7	96.3	100.3	10.5	13.8	13.6	13.3	0.59	0.50	0.40	0.36
		FIN	52	0	0	20.0	46.1	63.5	90.4	5.8	9.1	10.1	11.9	0.72	0.71	0.55	0.36
		LCY	4	48	0	133.0	159.4	143.2	104.5	86.2	55.3	41.9	24.8	2.29	0.91	0.80	1.02
		NCY	52	2	0	26.6	56.5	73.0	95.3	11.2	15.1	12.2	9.0	0.41	0.46	0.60	0.12
		NF	33	18	1	49.4	87.7	96.1	95.6	15.2	19.4	16.0	10.8	0.64	0.63	0.54	0.70
		ALL	52	0	0	29.1	59.0	73.6	90.8	8.7	12.2	11.8	11.4	0.71	0.47	0.34	0.43
	BBB	ECY	3	49	0	308.3	254.5	217.5	164.8	225.5	95.3	54.5	20.2	0.54	0.15	0.17	1.03
		LCY	26	26	0	72.8	121.3	129.1	122.2	35.8	33.9	22.5	16.3	1.94	1.00	0.47	0.16
		NCY	50	2	0	64.7	133.4	166.4	201.3	20.7	36.0	43.1	59.4	0.56	0.54	0.44	0.53
		NF	26	26	0	147.0	185.6	181.2	158.9	91.1	65.8	42.6	17.3	0.58	0.29	0.26	1.14
		ALL	26	26	0	147.3	185.8	181.3	158.9	91.6	66.3	43.0	17.7	0.59	0.31	0.28	1.17
2003	RL		26	0	27	227.6	280.5	334.9	417.7	17.7	25.2	26.1	19.3	-0.07	-0.54	-0.53	-0.78
	AA	FIN	51	0	2	21.1	32.2	35.8	49.7	3.3	4.0	4.6	7.0	-0.22	0.18	0.56	0.14
		LCY	31	20	2	22.0	42.4	48.3	51.7	9.9	6.5	6.6	12.4	0.25	0.28	-0.02	0.89
		NCY	40	6	7	23.3	40.9	45.9	49.0	7.2	7.5	5.8	5.7	0.23	0.50	0.64	0.84
		NF	36	13	4	22.3	40.4	45.6	49.2	6.9	5.0	3.7	8.6	-0.54	-0.06	0.01	0.55
		ALL	46	0	7	22.6	33.5	36.4	51.8	4.2	3.8	3.7	6.9	-0.08	0.08	0.51	0.00
	A	ECY	53	0	0	36.2	63.3	72.2	86.4	9.0	12.5	13.6	21.0	0.60	-0.13	-0.19	0.79
		FIN	51	2	0	32.1	60.3	70.3	78.9	3.6	7.9	10.6	11.5	0.37	-0.25	-0.10	0.32
		LCY	35	16	2	33.8	59.5	67.5	91.9	11.4	10.2	12.4	48.5	0.08	0.12	0.01	0.72
		NCY	53	0	0	24.8	46.9	56.3	69.9	5.5	7.9	10.5	13.7	1.28	0.44	0.24	0.53
		NF	53	0	0	32.6	56.9	65.0	76.0	6.0	10.0	12.2	17.2	0.62	0.21	0.10	0.36
		ALL	52	1	0	32.0	58.9	68.5	77.7	3.3	8.2	11.2	13.0	-0.11	-0.19	-0.04	0.24
	BBB	ECY	16	35	2	68.4	115.5	122.8	122.1	13.2	26.1	33.7	51.4	0.57	0.56	0.55	0.75
		LCY	0	53	0	84.9	141.4	139.0	103.9	15.9	27.1	29.4	28.1	0.75	0.27	0.14	0.10
		NCY	0	53	0	159.5	219.9	201.7	146.6	73.6	75.4	62.5	49.6	1.27	1.31	1.30	1.02
		NF	3	50	0	96.9	152.7	149.6	118.4	23.2	36.9	39.0	39.7	1.26	0.89	0.66	0.62
		ALL	3	50	0	97.2	153.1	150.0	118.5	23.4	37.2	39.3	39.9	1.24	0.87	0.65	0.61

Table 4.4: Descriptive Statistics of Credit Spot Spreads

A skew of credit spreads is considered to be an indicator for the one-year trend of the systematic factor that determines the credit spreads of a risk class. The credit spreads of a risk class are assumed to be governed by a geometric Brownian factor with an absorbing lower threshold where the distribution density of the factor is skewed to the right. Since credit spreads are not linearly related to the systematic factor, the factor density should translate into a distribution of observed credit spreads that is skewed to the right. The skew of spot spreads presented in Table 4.4 is significant at a level of 1% (5%) for 1 (2) out of 54 spot rates with a negative skew, while positive skewness is significant for 26 (43) out of 234 observations. The dominating positive skew of the observed spreads supports the assumption of a positively skewed factor with an absorbing barrier. Furthermore, a negative skew of credit spreads indicates a positive evolution of the respective factor in a particular period, as can be seen in the years 1999 and 2000, while significant positive skews of credit spreads are most frequent in the years 2001 and 2002 for the lower ratings A and BBB, that were hit strongest by the deteriorating credit conditions at that time.

The credit spreads of a risk class, defined by either a rating or a sector-rating attribution, are expected to represent a typical obligor of the class and should be of similar size for risk classes of equal rating. However, even spreads of risk classes with an equal rating differ at specific points in time, as exhibited in Figure 4.4.

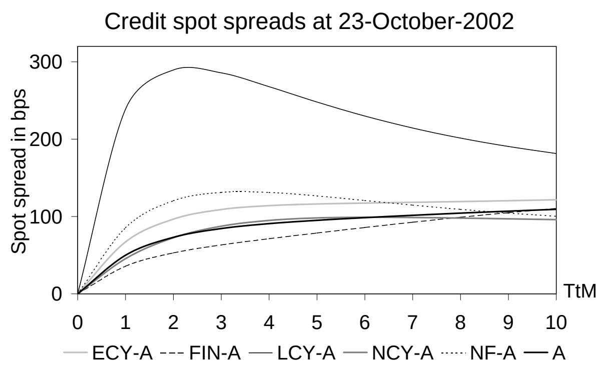


Figure 4.4: Variation of Credit Spot Spreads at Selected Credit Events

Joint events that impact the evolution of risk class spreads are therefore examined using the time series of credit spreads in Figures 4.5−4.7. The variation of the credit spreads of a risk class can be attributed either to rating-specific effects, to a general or a sector-specific systematic effect on the risk class, or to obligor-specific effects. In the sample period from 1999 to 2003, four systematic events can be identified: (1) the technology crisis in the last three quarters of 2000, (2) the attacks of 9/11, (3) the loan and impairment crisis in the 3rd quarter of 2002, and (4) the start of the second Gulf war in March 2003. These systematic events, however, affected the spread curves of sector-rating classes in different ways.

The technology crisis caused a modest increase in credit spreads during the year 2000 for all sectors and ratings and is therefore assumed to result from a general systematic factor that affected the obligors of all risk classes in a comparable way.

In contrast, the 9/11 attacks affected risk classes in different ways. The credit spreads of the financial sector show no significant reaction, which may be explained by the fast reaction of central banks in providing liquidity and lowering funding rates. In the non-financial sectors, short-term credit spreads rise considerably across all ratings, however, no distinct jump in spreads can be observed. Obligors from cyclic sectors bear the most pronounced expansion of spreads due to their high sensitivity towards an expected recession.

Corporate impairments in combination with high leverage and gloomy operational results affected the credit markets of particular sectors and rating grades in the second half of 2002, representing sector-related and obligor-specific effects. In consequence, time intervals can be observed in which the spreads of risk classes with equal ratings differ distinctly. In 2001 and 2002, for example, cyclical sectors show higher spreads than the financial sector in the 5-year term for AA and A ratings. These differences do not only result from a potentially dissimilar evolution of obligors' creditworthiness in both years, but they also appear at the start of both estimation periods, when ratings definitely coincide across classes of equal rating. Finally, the second Gulf War is a typical example of a general systematic effect.

Some peaks of risk class spreads are remarkable. The peak in short-term credit spreads of the LCY-AA class in Figure 4.5 can be explained by the systematic sector effects of the 9/11 attacks, which affected the late-cyclic sector to a much higher extent than the FIN-AA and NCY-AA risk classes, while the extreme peaks of the LCY-A, ECY-BBB and NCY-BBB classes in 2002 and 2003 predominantly reflect the impact of obligor-specific events.

Furthermore, the peaks of risk class spreads are compared across the remaining ratings of the sector. Figure 4.6 shows, that the September 2002 peak in the ECY-BBB class is not observed for the ECY-A spreads, which leads to the conclusion that the peak was caused by an obligor-specific credit event that translates into the spread structure of the class. Accordingly, the first LCY-A spread peak in 2002 cannot be identified as systematic, while the LCY-A peak in late 2002 partially translates into the LCY-BBB class. Apart from that, there are no simultaneous changes in spreads that indicate credit events systematic to a particular sector only. For the non-cyclic sector-class, the spread peaks of the NCY-BBB class in 2002 and 2003 are only partially reflected in the NCY-A class. A comparison of the different sectors also reveals the increase and elevated variation of sector spreads during the technology crisis in 2000 and preceding the second Gulf War.

All peaks of sector spreads are reflected in the spreads of respective NF sector-rating

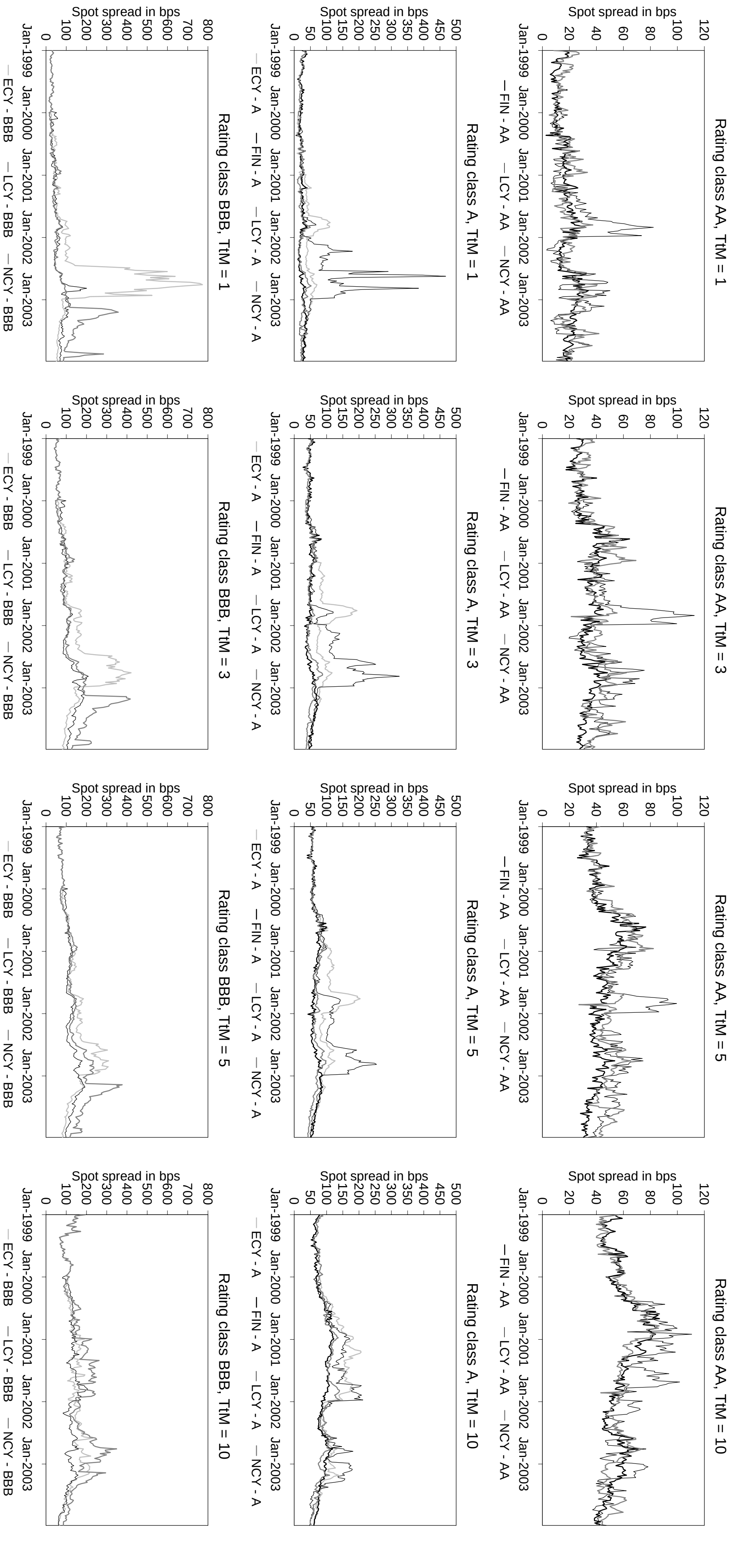


Figure 4.5: Credit Spreads of Rating-Classes

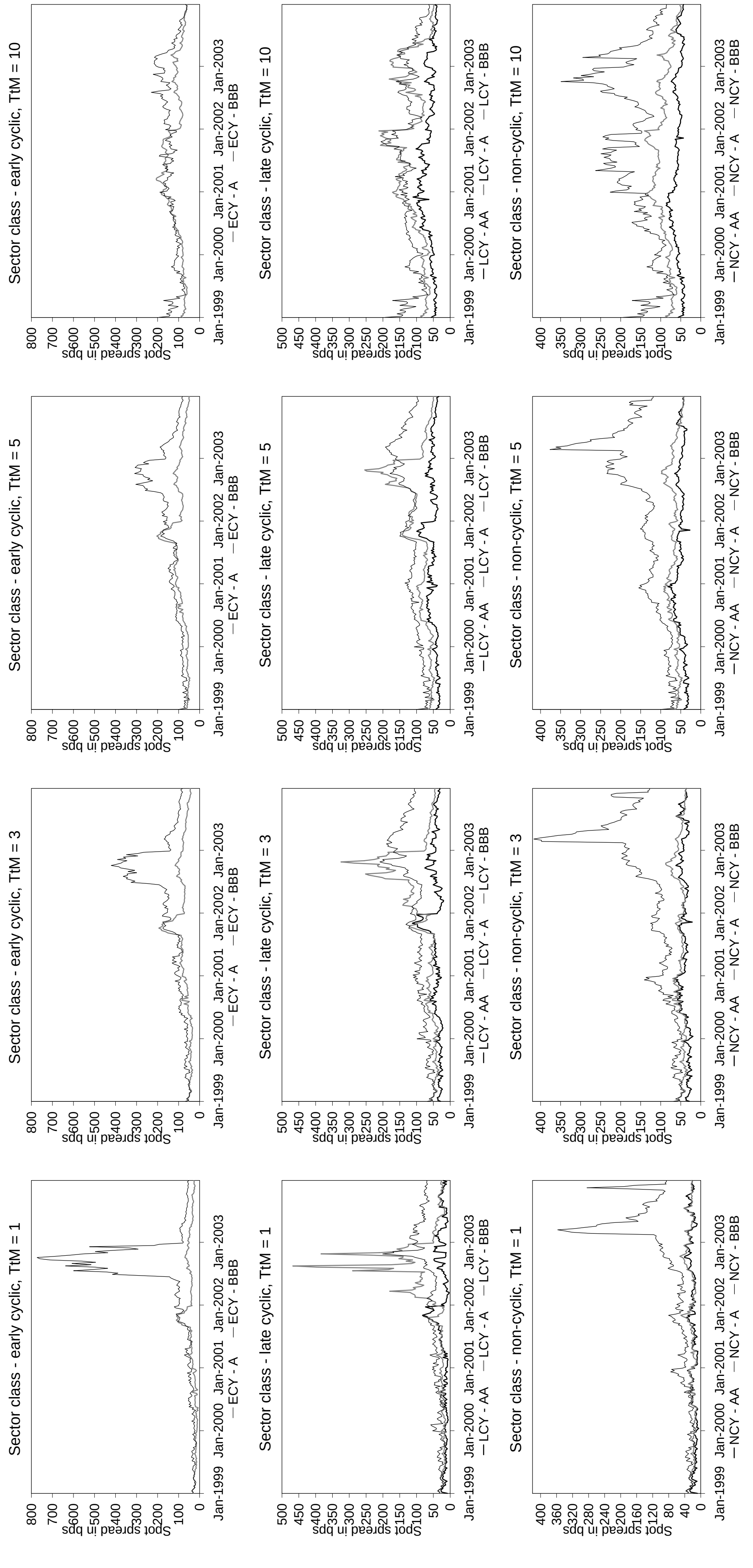


Figure 4.6: Credit Spreads of Risk-Classes I

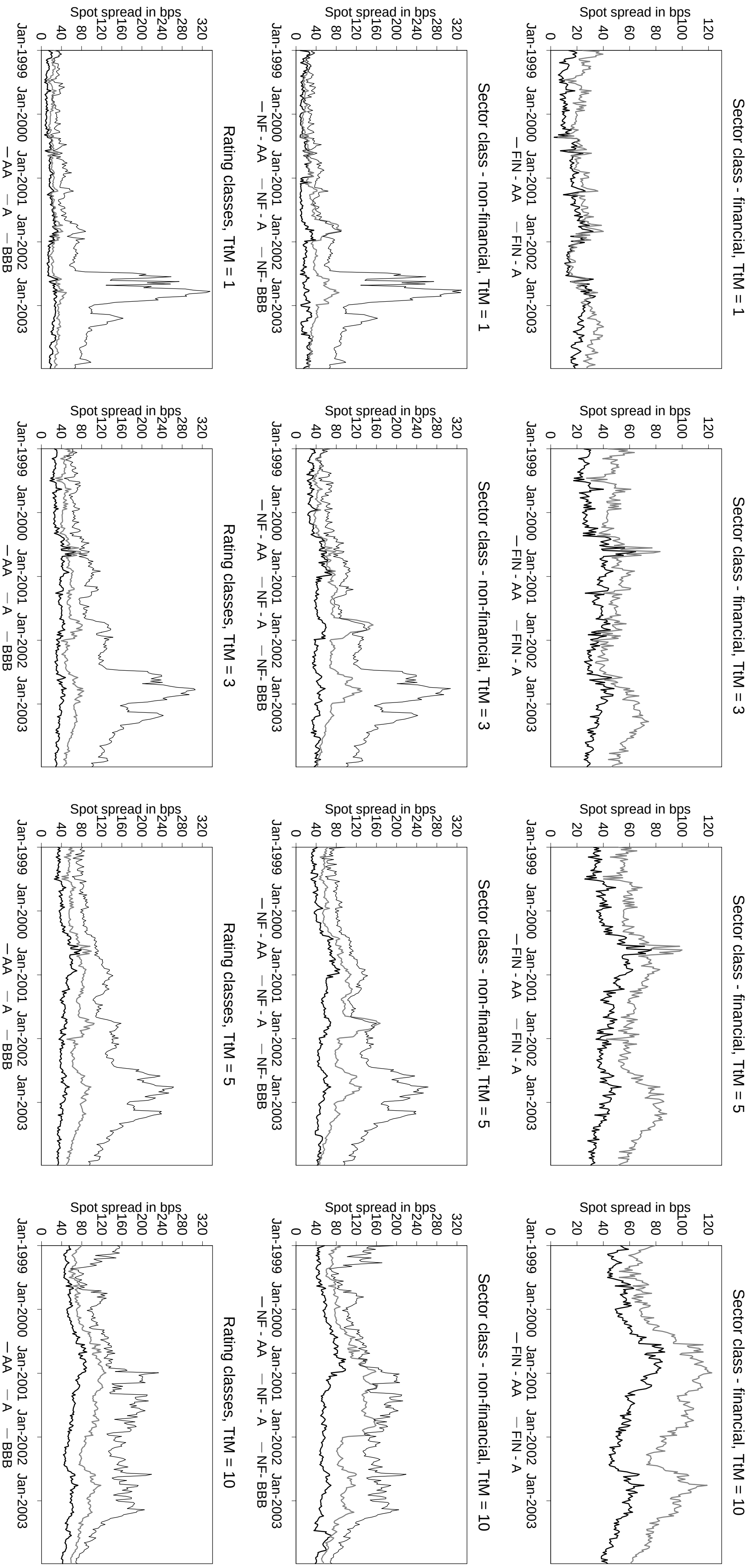


Figure 4.7: Credit Spreads of Risk-Classes II

classes in a smoothed form, with credit events caused by several bonds being more persistent. The peaks of the BBB class are still considerable, because the financial sector does not contribute many observations to smooth out obligor-specific effects in the aggregate BBB class. In contrast, the spread peaks of the LCY-A class are dampened in the non-financial sector-class and vanish almost completely in the rating A class, which is dominated by observations from the financial sector.

A further examination of the peaks of the ECY-BBB, LCY-A, and LCY-BBB classes in 2002 as well as the NCY-BBB class in 2003 reveals obligor-specific events causing excessive credit spreads.[10] A sudden simultaneous increase in the bond spreads of telecommunication firms Ericsson and Alcatel triggers an increase of the ECY-BBB spreads from 143 bps on the 19/06/02 to a maximum of 773 bps on the 9/10/02 and spreads stayed elevated for the remainder of the year. A rating downgrade occurs not before the third quarter of 2002, so that both issuers disappear from the BBB sample in 2003, which reveals that ratings lag behind the credit risk assessment of the market, and that rating-derived spreads may be biased if used for pricing exposures of a typical obligor of a rating class. Because no other BBB-bonds from the technology sector exists within the ECY-BBB class, it cannot be concluded whether the spread increase is due to an effect that is common to all BBB issues of the technology sector. For single-A bonds of the technology sector, yield spreads rise from 66.4 bps to 96.2 bps at the same time.

Considering other sectors of the ECY-BBB class, the average yield spreads of the automotive sectors rise from 152 bps on the 19/06/02, the last observation date before the spread increase starts, to 336 bps at its peak. The mean yield spreads of the basic material (communications, media) sector increases from 167 (182, 120) bps to 241 (214, 328) bps, respectively. The reaction of other sectors included in the ECY class is most pronounced for the automotive and the media sectors, while the economically closely related communications sector, dominated by phone companies, shows only a modest increase of yield spreads of 32 bps. It appears that the spread jump was caused by a systematic shock to the technology sector that partially affected other sectors of the same rating class, as well as single-A bonds of the technology sector.

In the LCY-A class, both jumps in credit spreads result from the bonds of the Swiss engineering company ABB that incurred financial difficulties in 2002, leading to three peaks of credit spreads in the LCY-A class on 27/03/02 (21/08/02, 23/10/02), with one-year spot spreads reaching a level of 179 (460, 384) bps in response to the spreads of the ABB bond with ISIN XS0108171298 and maturity date on 8/03/04 spiking up to 767

[10]Certainly, jumps in credit spreads that do not trigger a default event contradict to the assumption of credit risk to be represented by a state-continuous asset value.

Type			Credit Spot Spreads				First Diff. of Spreads				Credit Spot Spreads		First Diff. of Spreads	
Time-to-maturity			1	1	10	10	1	1	10	10	1	10	1	10
Lag			1	10	1	10	1	10	1	10	1	1	1	1
Year	Rating	Sector	**Autocorrelation**								**Augmented Dickey-Fuller Test**			
1999	RL		94.9	33.3	94.3	51.2	-1.7	5.4	-3.5	26.5	-0.94	-0.07	-4.12***	-5.35***
	AA	FIN	60.3	11.8	73.8	12.7	-41.6	31.3	-36.8	-6.1	-2.61*	-1.84	-6.22***	-6.95***
		NF	59.7	33.3	74.4	26.2	-18.8	20.2	-29.1	3.8	-2.75*	-1.56	-7.11***	-8.24***
		ALL	64.0	17.8	73.2	12.9	-40.0	11.2	-36.4	-8.0	-2.35	-1.88	-6.42***	-7.38***
	A	FIN	60.7	1.6	46.5	3.7	-46.0	12.6	-48.5	3.9	-2.25	-2.97**	-6.01***	-6.80***
		NF	75.2	24.9	75.9	3.9	-28.3	5.1	-33.7	-11.7	-2.31	-2.07	-5.86***	-6.36***
		ALL	69.1	6.9	57.7	5.1	-46.3	25.4	-44.1	5.1	-2.03	-2.67*	-5.26***	-6.66***
	BBB	NF	54.9	-17.2	72.3	15.4	-26.6	4.1	-21.6	-4.4	-3.13**	-2.26	-7.35***	-8.30***
		ALL	63.0	-35.1	82.8	9.6	-14.2	-9.6	-34.9	0.0	-3.05**	-2.05	-5.97***	-7.27***
2000	RL		94.9	44.4	84.1	11.1	-17.6	-9.4	-8	4.9	-1.14	-2.60*	-3.55**	-5.43***
	AA	FIN	48.2	12.2	89.7	43.4	-40.1	-13.0	-33.6	-2.1	-2.91*	-1.07	-7.41***	-7.58***
		LCY	38.3	-0.4	82.3	38.8	-40.2	3.5	-33.7	9.7	-3.38**	-1.28	-8.78***	-8.00***
		NCY	78.0	36.2	88.2	39.1	-30.3	12.1	-34.1	17.5	-1.37	-1.14	-7.04***	-5.94***
		NF	75.4	35.6	89.8	36.4	-29.9	-5.9	-32.7	-2.3	-1.15	-0.72	-7.76***	-5.61***
		ALL	58.1	17.3	88.7	42.8	-23.7	-5.4	-39.8	7.9	-3.00**	-0.85	-9.58***	-8.21***
	A	ECY	53.7	14.2	91.4	41.4	-18.4	-11.0	4.7	-5.4	-3.58***	0.86	-9.20***	-6.43***
		FIN	34.8	-6.8	90.9	44.2	-34.8	-10.7	-32.9	-2.0	-3.45**	-0.65	-9.38***	-8.19***
		LCY	39.1	8.3	89.7	30.0	-32.4	-4.5	-28.8	6.0	-3.58***	0.92	-9.54***	-5.70***
		NCY	64.9	30.6	89.0	22.2	-26.2	10.0	-19.5	17.7	-2.39	0.63	-8.14***	-6.45***
		NF	67.0	33.6	87.1	30.4	-37.1	-0.3	-22.4	-11.9	-1.49	0.24	-8.39***	-7.28***
		ALL	43.5	9.9	90.1	43.5	-20.6	-0.2	-44.5	17.5	-3.97***	-0.48	-10.93***	-6.68***
	BBB	ECY	65.3	-2.9	74.4	26.3	-51.4	3.5	-43.1	-3.5	-1.99	-0.34	-5.99***	-6.78***
		LCY	54.3	13.9	79.0	18.4	-24.3	9.2	-40.0	-6.1	-3.13**	-1.77	-6.70***	-6.75***
		NCY	83.1	27.2	90.0	23.0	-17.5	-5.8	-26.3	-3.3	0.20	-1.45	-6.67***	-5.36***
		NF	72.5	8.7	81.6	27.2	-27.0	5.3	-20.7	8.3	-2.18	-0.68	-6.08***	-5.66***
		ALL	62.8	14.1	80.6	30.8	-39.8	15.4	-28.3	2.9	-2.10	-1.40	-7.06***	-7.00***
2001	RL		95.3	36.4	87.6	0.6	12.8	8.6	-16.4	8.1	-1.96	-0.61	-5.37***	-4.95***
	AA	FIN	48.7	10.3	83.7	31.7	-42.5	-10.4	-28.2	-8.2	-2.70*	-1.86	-7.03***	-5.81***
		LCY	89.3	30.1	78.3	-24.0	-22.6	-3.1	-21.5	-28.4	-0.08	-1.77	-4.35***	-5.87***
		NCY	72.3	12.3	51.3	18.5	-22.3	13.3	-49.6	7.5	-2.59	-4.34***	-5.70***	-7.39***
		NF	77.5	17.1	79.9	23.8	-61.6	-9.7	-32.9	7.3	-1.25	-5.03***	-7.20***	-9.29***
		ALL	58.8	21.7	81.3	34.0	-41.6	-9.9	-36.2	-7.5	-2.35	-2.77*	-7.06***	-6.19***
	A	ECY	95.2	9.6	85.3	-16.9	23.7	-37.8	25.4	-3.8	-1.65	-2.53	-4.42***	-4.44***
		FIN	35.2	3.2	79.7	7.1	-33.6	3.6	-22.9	15.4	-3.83***	-1.77	-6.45***	-5.26***
		LCY	77.4	-25.7	79.2	17.8	-13.2	-4.1	-13.3	-14.4	-2.11	-1.22	-5.39***	-5.85***
		NCY	80.8	14.8	88.1	8.4	-25.3	4.3	1.9	3.6	-1.82	-1.02	-5.83***	-3.03**
		NF	94.2	12.8	83.7	-22.9	12.5	-27.8	31.8	-6.8	-1.27	-3.37**	-4.23***	-3.89***
		ALL	51.9	18.0	78.3	15.6	-35.8	15.5	-10.9	13.6	-2.80*	-2.09	-6.82***	-5.44***
	BBB	ECY	81.9	19.8	53.9	10.7	-8.1	-5.0	-2.7	-1.9	-0.80	-3.49**	-5.07***	-6.21***
		LCY	77.2	10.6	84.1	20.2	-2.3	2.1	-26.5	-3.8	-2.52	-1.44	-6.71***	-5.20***
		NCY	83.7	19.1	61.5	18.9	-15.7	-11.4	-3.9	3.2	-2.66*	-4.03***	-6.12***	-6.95***
		NF	87.0	18.4	65.7	-10.1	0.0	-11.5	-15.8	5.7	-1.21	-2.54	-5.08***	-5.80***
		ALL	84.1	11.1	59.0	-22.8	-4.0	-17.4	-18.6	0.7	-1.78	-2.92*	-6.01***	-8.01***
2002	RL		92.5	39.1	90.8	40	-11.5	-18.3	1.5	-13.2	-0.41	0.61	-5.15***	-6.78***
	AA	FIN	79.2	33.5	85.5	37.1	-22.1	4.6	-20.7	2.9	-1.63	-1.51	-7.48***	-6.68***
		LCY	70.5	28.6	74.8	4.3	-7.4	12.4	-6.6	14.7	-4.48***	-2.49	-5.53***	-5.03***
		NCY	81.8	41.5	91.8	29.8	-2.0	27.2	9.1	-0.1	-1.81	-1.51	-6.67***	-5.25***
		NF	37.8	28.9	89.2	16.5	-28.4	21.9	8.9	11.1	-5.68***	-1.83	-7.18***	-5.60***
		ALL	76.9	35.2	87.9	34.7	-23.7	6.2	-17.9	8.0	-1.77	-1.47	-7.51***	-6.47***
	A	ECY	90.5	38.2	90.3	40.9	-12.5	1.7	8.9	13.1	-1.66	-3.93***	-7.22***	-6.16***
		FIN	79.0	27.4	92.8	31.5	-14.1	5.1	-12.6	-13.1	-2.37	-1.23	-6.94***	-5.78***
		LCY	61.4	27.0	84.1	20.7	-7.1	33.9	-27.1	11.5	-3.61***	-4.07***	-8.08***	-6.52***
		NCY	92.6	45.6	83.8	-20.5	-10.5	-12.0	-0.6	7.1	-0.88	-4.39***	-6.22***	-6.88***
		NF	93.1	40.3	88.6	17.3	-10.7	3.7	13.4	18.7	-0.94	-4.93***	-4.46***	-7.66***
		ALL	82.8	32.4	93.5	33.3	-42.1	-19.2	-1.9	-6.7	-1.21	-1.52	-7.42***	-5.89***
	BBB	ECY	92.0	35.5	65.1	1.8	-11.3	11.1	2.1	3.6	-1.28	-4.20***	-5.79***	-7.15***
		LCY	88.9	19.6	80.3	-1.2	17.9	10.3	24.0	4.5	-1.85	-4.11***	-3.78***	-5.00***
		NCY	92.3	38.9	92.9	46.0	-7.4	-2.8	-24.2	-0.2	-0.54	-0.78	-6.75***	-6.11***
		NF	89.0	32.4	59.7	23.0	-25.9	-12.3	6.0	2.4	-1.38	-4.29***	-5.11***	-7.34***
		ALL	89.0	32.2	59.0	27.1	-26.9	-11.0	6.8	5.8	-1.38	-4.46***	-4.81***	-7.59***
2003	RL		87.2	6.2	85.3	-5.2	4.5	-7.3	-2.2	7.2	-1.93	-2.55	-5.15***	-5.05***
	AA	FIN	74.1	27.3	90.3	34.1	-15.9	-3.3	-36.7	-6.0	-2.80*	-1.10	-5.43***	-6.99***
		LCY	76.4	-22.5	89.6	18.2	-30.1	7.7	-18.3	-15.1	-2.13	-1.51	-5.17***	-4.79***
		NCY	73.9	-11.2	87.1	-8.7	-6.8	12.3	9.2	-3.2	-3.16**	-2.15	-7.14***	-4.26***
		NF	64.5	-22.3	83.8	-21.3	-25.8	12.8	-2.7	-2.0	-2.70*	-1.81	-6.98***	-4.70***
		ALL	83.6	26.0	90.5	19.5	-5.8	12.1	-15.8	-0.1	-1.90	-0.77	-5.07***	-5.63***
	A	ECY	91.7	26.6	92.3	37.3	-27.3	-2.4	-5.0	-3.9	-1.92	-0.91	-6.22***	-4.24***
		FIN	63.6	23.5	91.4	33.3	-23.0	6.3	-26.1	-7.0	-3.15**	-0.32	-6.29***	-8.81***
		LCY	89.5	17.6	94.7	42.0	-5.9	-0.3	-7.5	4.1	-7.88***	-0.75	-13.02***	-6.21***
		NCY	67.2	-31.5	90.3	20.1	-12.7	-16.1	-13.5	9.8	-3.62***	-1.01	-7.70***	-5.61***
		NF	91.4	31.2	94.8	37.2	-35.6	-8.4	-3.4	-5.5	-5.32***	-0.70	-11.98***	-4.39***
		ALL	69.2	36.1	93.3	37.0	-31.6	13.3	-24.6	-3.0	-2.58	-0.19	-7.48***	-6.95***
	BBB	ECY	93.6	43.0	95.6	32.4	-21.3	24.4	7.2	-33.4	-8.70***	-0.61	-25.38***	-5.04***
		LCY	85.9	34.6	92.7	35.6	-25.1	13.1	-28.7	15.4	-1.66	-0.28	-5.58***	-6.54***
		NCY	85.8	-9.2	88.0	29.7	22.8	-12.3	-27.0	-7.1	-2.45	-2.13	-4.56***	-5.96***
		NF	91.6	13.6	95.7	36.8	28.5	-15.0	-4.8	-17.9	-1.94	-1.01	-4.83***	-6.72***
		ALL	91.8	14.4	95.7	36.9	27.9	-15.2	-4.9	-18.8	-1.89	-0.99	-4.78***	-6.66***

Table 4.5: Autocorrelation and Unit Root Test of Fitted Credit Spreads

(2163, 4513) bps at that time.[11] It is possible that the evolution of the credit spreads from Alcatel and ABB were driven by the same factors, since their business portfolios both include power generation activities, though Alcatel is assigned to the technology sector for its predominant business in telecommunication infrastructure, whereas ABB with its focus on engineering, power generation and power transmission is part of the industrial sector.

The spread events of ABB and Alcatel appear simultaneously to spread peaks in the LCY-BBB class, which where driven by bonds of Dyckerhoff, Printemps, and Sol Melia Europe, so that the one-year spot spreads of the LCY-BBB class reached a 200 bps high on 30/10/02. Since these issuers come from the industrial and the cyclical-consumer sectors, systematic sector effects related to the evolution of the ABB credit spreads can be ruled out. In conclusion, the simultaneous spread widening in the ECY-BBB, LCY-A and LCY-BBB classes must be attributed either to simultaneous specific effects or to a general macroeconomic shock that affected all bonds in the market at that time.

In 2003, two spread peaks occured in the NCY-BBB class, attributed to credit events involving the Dutch food retail company Ahold and the Italian food producer Parmalat. Yield spreads of 7 Ahold bonds spiked to a level up to 1617 bps on the 26/02/03, driving fitted one-year spreads of the NCY-BBB class to 322 bps. The Parmalat default on 10 bond issues on the 8/12/03 forced the NCY-BBB spreads to a maximum of 964 bps on the 19/11/03.[12]

In summary, general and sector-specific systematic factors, as well as obligor-specific factors, affect the credit spreads of risk classes, where the particular type of factor must be determined on a case by case basis. Systematic factors represent the fundamental effects in the change of risk class spreads and must be included in the fitting of term structures. Obligor-specific spread events instead can not only have serious effects on the fit of the credit spreads of single risk classes, but may also alter the correlations of risk classes' spreads and should in principle be filtered in the process of curve fitting. Furthermore, it must be ensured that the bond sample and the fitted credit spreads of risk classes are representative of the credit portfolio under consideration.

Autocorrelations with a lag of one and ten years are presented in Table 4.5 on a level and a first-difference basis for one-year and ten-year riskless rates, as well as for the credit spreads of risk classes. As expected, high positive lag-one autocorrelations are observed on a level basis for riskless rates of one-year and ten-year tenor, that decrease substantially if the lag size is increased to ten.

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[11] At the second and third peak yield spreads of the bond with ISIN XS0143736162, issued by Investor Group and maturing on 5/03/12 were affected, since Investor was a major shareholder of ABB.

[12] Yield spreads reached an even higher level on a daily basis immediately before the time of default, but did not enter the data set due to the weekly sample interval.

Lag-one autocorrelations of credit spreads compare to those of the riskless rate, while autocorrelations with a lag size of ten are considerably smaller and may even become negative. Due to their lower volatility, ten-year spreads mostly show higher positive autocorrelations for lag size one. For the years 2001-2003 of credit distress, lag-one autocorrelations of one-year spreads are elevated in contrast to the less exposed years 1999 and 2000, but apart from that, on a level basis there are no systematic effects on autocorrelations apparent for different sectors, rating classes and in time.

In contrast, lag-one autocorrelations of the first differences of credit spot spreads are predominately negative, which indicates a mean reversion of short-term spreads. However, autocorrelations of lag size ten are close to zero and do not enable to conclude, if spread differences are autocorrelated positively or negatively. For the riskless rate, the sign of first-difference autocorrelations cannot be unambiguously determined. Interpretations apply analogously to the first differences of ten-year credit spreads.

A lag-one Augmented Dickey-Fuller (ADF) test is applied to test the stationarity of riskless rates and of credit spreads on a level basis and a first difference basis. The results in Table 4.5 indicate that the unit root hypothesis cannot be rejected on a level basis for riskless rates. For the credit spreads the unit root is rejected for 11 (18, 26) out of 72 one-year spread series on a 1% (5%, 10%) level of significance and for 11 (14, 17) ten-year spreads, respectively. On a first-difference level non-stationarity of rates and spreads is overwhelmingly rejected, so that credit spreads can be assumed to be difference-stationary or integrated of order one.

4.3.2.2 Analysis of Credit Spread Correlations

Analyzing the correlations of the changes in credit spreads reveals the variability of the co-movement structure of credit spreads in time. Three structures of risk classes are used for model estimation and simulations in the subsequent chapters. Risk classes are defined either by the rating or by a sector-rating attribution. The first risk class structure considers rating classes AA, A, and BBB. Second, risk classes are detailed additionally by a sector differentiation with sector-rating classes either from the financial sector (FIN) or from the aggregate non-financial sector (NF). Third, all sector-rating classes of the four sector-classes are taken into account. In sectors LCY and NCY, three rating classes are available, whereas risk classes ECY-AA and FIN-BBB are omitted due to data limitations.

Spread correlations are considered on the basis of one-year and 10-year credit spot spreads to carve out differences in the co-movement of short-term and long-term credit spreads. Furthermore, correlations are examined for yearly and quarterly updates of the bond assignment to rating classes. Results are presented in Tables 4.6 and 4.7

$\tau = 10 \setminus \tau = 1$	ECY-A	ECY-BBB	FIN-AA	FIN-A	LCY-AA	LCY-A	LCY-BBB	NCY-AA	NCY-A	NCY-BBB	NF-AA	NF-A	NF-BBB	AA	A	BBB	RL
ECY-A		27.2 17.5	52.0 18.5	47.0 22.4	36.3 19.5	29.7 33.6	29.3 18.5	34.7 17.9	42.7 18.6	38.5 19.6	32.8 32.2	77.2 4.1	34.0 29.7	55.7 16.5	48.4 22.4	29.4 28.8	-22.7 13.4
		-1.0 47.5	32.6 78.7	29.5 85.0	3.7 53.2	-15.5 65.8	11.1 59.7	5.7 50.2	28.6 68.5	12.8 60.6	-11.2 66.5	71.8 81.3	1.3 69.0	39.4 81.1	34.2 87.4	-5.6 59.0	-32.2 0.1
ECY-BBB	26.1 27.3		18.3 20.5	20.0 18.5	23.5 16.8	26.7 5.3	17.7 17.1	10.6 12.5	25.8 8.6	14.7 17.4	19.5 14.3	34.0 8.7	58.7 10.7	17.3 12.5	16.8 13.9	55.9 10.0	-17.1 15.3
	-12.6 60.9		-4.4 40.3	-3.4 36.5	0.3 43.9	17.9 30.6	4.7 42.4	-11.6 17.6	14.4 37.2	-2.1 36.7	1.5 41.0	22.1 44.0	44.0 69.0	-0.4 33.6	-0.6 32.8	44.0 71.7	-31.6 8.7
FIN-AA	38.0 18.8	26.9 15.9		73.9 16.3	28.0 24.8	20.7 24.9	24.7 20.4	31.5 10.4	36.6 19.9	37.5 35.3	36.1 19.7	48.5 17.0	27.9 36.9	91.3 7.9	69.3 10.0	21.4 33.6	-33.2 13.9
	18.7 63.6	15.5 54.8		52.0 91.3	-8.4 54.2	-21.9 42.5	2.0 55.1	14.1 39.6	10.6 56.6	-21.2 69.1	13.3 57.9	32.6 76.1	-21.1 65.5	81.1 98.5	56.5 82.2	-22.4 53.7	-54.9 -20.4
FIN-A	33.9 19.0	26.3 17.7	66.1 15.5		31.1 28.5	22.4 30.3	25.2 22.1	32.3 10.0	33.4 26.3	32.7 37.1	31.5 22.5	45.0 19.7	32.8 32.9	72.0 14.4	85.9 11.2	30.7 29.5	-29.5 11.3
	5.1 56.6	8.9 45.9	42.9 85.9		-0.7 56.9	-29.3 50.5	0.8 58.1	20.0 43.8	-5.1 60.7	-28.9 69.0	8.9 63.6	29.5 77.7	-14.5 68.7	52.2 90.2	66.2 93.0	-14.6 59.0	-41.1 -17.0
LCY-AA	43.2 17.9	14.7 16.4	32.4 18.7	29.6 23.1		30.0 25.6	17.0 16.8	9.0 12.1	28.5 24.8	15.5 23.4	13.1 18.8	30.1 20.1	22.7 36.8	28.3 23.9	25.9 27.7	26.5 29.8	-13.3 12.1
	25.6 69.4	-5.4 29.4	10.0 57.2	-4.2 52.1		-1.1 59.4	2.4 45.5	-7.5 21.7	0.3 55.7	-7.3 54.6	-4.6 38.3	-3.8 45.3	-20.3 77.4	-4.4 54.4	-9.7 54.2	-12.8 56.9	-25.4 5.7
LCY-A	22.5 17.8	23.3 13.2	31.4 22.5	22.6 21.0	29.7 18.3		23.7 13.7	26.5 22.6	20.6 25.0	22.3 27.0	21.3 38.1	41.0 16.3	29.6 13.2	24.6 28.5	32.9 21.1	28.1 13.2	-11.4 16.7
	3.7 45.3	4.3 40.2	0.5 63.6	-12.3 40.5	5.2 51.7		-0.8 30.6	-9.0 47.9	-5.9 54.4	-19.9 47.9	-33.9 60.3	25.8 62.6	7.9 43.3	-22.7 52.3	-0.4 57.9	6.0 40.8	-29.7 6.3
LCY-BBB	30.1 15.2	16.7 11.3	18.6 12.9	24.6 11.8	25.2 18.6	15.0 10.5		4.6 10.7	12.5 22.8	18.0 23.5	13.9 13.0	32.6 22.6	32.6 24.5	26.9 19.0	27.8 18.9	32.2 20.3	-10.2 22.0
	16.3 51.9	6.1 31.6	-0.1 34.4	8.8 38.6	4.3 46.7	-2.5 26.0		-12.1 15.7	-17.8 38.7	0.9 52.4	0.2 28.9	2.6 62.5	14.6 68.8	8.8 56.8	-0.7 51.8	11.3 54.0	-34.3 15.1
NCY-AA	36.8 19.0	16.3 23.2	44.7 17.0	44.3 12.7	28.0 23.5	29.7 17.9	31.5 15.3		59.8 22.0	21.2 19.0	49.6 42.3	41.4 23.3	11.0 18.6	38.8 16.3	28.8 14.0	15.5 27.4	-19.4 13.3
	12.7 57.8	-8.8 48.4	21.4 64.9	32.6 61.2	-2.5 51.1	4.6 51.7	13.8 53.7		31.3 85.4	-7.8 41.9	-12.6 81.9	15.4 72.5	-15.9 35.7	14.1 57.9	6.1 40.8	-17.4 47.6	-40.4 -6.1
NCY-A	40.7 16.9	25.8 9.1	33.0 22.4	24.6 20.2	24.3 22.2	26.2 19.1	33.6 11.3	29.5 24.0		34.3 21.8	42.1 30.8	51.5 26.7	24.4 26.0	45.0 14.6	29.3 29.8	23.6 24.5	-19.9 20.3
	15.7 51.3	15.0 37.1	8.3 63.6	-2.3 45.6	-8.1 51.7	2.9 42.3	18.1 44.7	2.3 53.0		-0.2 54.9	-7.9 76.3	15.8 80.3	-10.4 48.8	29.8 64.2	-10.4 60.6	-11.6 49.6	-42.3 2.9
NCY-BBB	30.0 31.2	20.4 22.4	14.2 30.7	16.1 31.0	26.6 15.5	6.8 27.5	21.0 15.5	18.1 20.2	25.4 15.4		17.5 19.2	32.3 17.8	50.9 37.5	42.2 33.5	32.6 32.1	48.6 34.0	-21.2 13.0
	-8.3 72.4	-3.5 41.7	-30.6 54.2	-29.5 56.9	10.3 52.5	-35.1 38.3	1.9 34.5	0.8 51.2	4.0 41.7		-3.1 43.2	8.3 49.8	-1.0 79.4	-12.4 68.8	-21.1 61.0	-5.3 77.4	-33.0 -4.5
NF-AA	40.2 19.4	22.2 23.9	43.7 20.5	40.2 16.7	29.5 29.2	29.0 19.3	32.3 15.0	71.3 25.2	33.6 26.8	17.8 17.8		41.4 33.2	15.1 23.0	41.5 23.0	25.6 26.8	19.3 33.2	-15.7 15.5
	6.6 52.7	-3.5 46.5	14.8 67.8	17.7 64.9	-13.0 53.1	5.0 51.7	11.4 47.2	34.0 89.8	0.8 60.0	3.6 46.8		-5.1 84.8	-6.9 53.1	16.4 71.6	-8.6 55.9	-8.7 62.5	-41.4 -0.7
NF-A	78.7 8.0	30.3 31.2	36.3 24.6	27.4 27.9	33.6 12.8	30.3 16.6	32.8 12.8	37.2 16.1	59.7 16.4	30.9 22.2	40.1 16.5		35.8 23.2	52.8 11.1	50.7 13.1	32.7 26.1	-31.2 10.9
	68.9 88.2	-10.2 63.3	-3.6 63.6	-22.4 42.5	20.5 51.7	5.5 40.7	18.1 52.7	16.9 55.4	37.8 77.5	-2.3 47.5	13.9 55.6		4.9 62.6	39.4 69.3	36.6 71.8	3.3 70.5	-44.0 -19.7
NF-BBB	32.4 40.8	56.4 20.1	16.5 18.2	13.7 25.8	28.5 13.4	22.8 18.8	36.6 12.2	26.0 22.6	28.8 13.4	51.7 13.4	21.5 27.9	35.4 36.8		27.9 31.2	30.4 28.8	84.2 18.2	-13.6 15.0
	-31.3 72.0	37.9 76.3	-0.9 45.0	-17.5 44.9	11.8 46.4	-7.2 41.2	22.0 51.5	1.2 55.3	11.7 43.1	39.8 67.1	-17.8 47.9	-20.5 68.3		-12.2 63.8	-12.5 57.4	53.9 99.8	-31.8 5.6
AA	39.2 20.8	26.1 14.8	92.7 8.6	66.6 10.5	35.1 22.8	32.0 23.8	19.4 12.1	54.3 14.3	31.8 26.4	17.7 31.7	53.4 18.2	37.5 25.1	20.0 17.5		72.7 12.0	21.2 27.6	-30.0 22.5
	15.9 67.1	13.7 50.4	77.6 98.2	54.7 81.8	12.1 68.5	1.5 67.1	3.3 33.3	39.8 69.7	0.4 67.1	-30.6 56.3	29.2 72.3	-0.2 67.1	1.9 45.5		58.1 87.5	-13.7 46.3	-52.1 7.8
A	51.1 13.1	31.5 23.3	65.4 11.1	91.1 6.2	32.5 21.8	32.5 20.4	29.3 15.5	50.3 15.8	37.3 23.7	21.2 32.4	45.0 17.8	45.6 28.1	21.7 33.0	67.3 8.8		25.5 28.8	-31.7 23.9
	33.3 70.2	1.6 55.2	49.7 75.4	82.3 98.3	2.4 58.6	5.7 51.9	8.0 49.2	34.0 76.6	7.3 57.0	-22.6 65.1	31.9 74.7	-3.4 68.4	-26.4 58.7	55.9 77.0		-13.1 52.0	-56.2 7.2
BBB	30.1 38.9	55.8 21.1	19.2 20.7	15.4 24.9	21.6 15.0	21.1 12.9	35.1 7.1	18.9 22.9	27.7 16.2	49.8 18.8	20.3 29.5	35.7 34.7	75.0 27.5	20.7 21.9	23.2 30.4		-8.7 14.6
	-27.0 71.7	32.6 77.5	-10.3 45.0	-16.1 45.1	8.6 45.1	4.4 39.0	25.3 43.8	-3.9 55.6	7.0 45.9	26.5 67.3	-16.0 53.8	-13.8 68.2	32.6 99.9	-13.7 45.7	-15.1 59.0		-26.9 6.9
RL	-29.4 16.7	-27.9 10.9	-56.6 12.8	-54.2 11.5	-28.4 11.2	-16.3 10.0	-23.6 8.1	-30.3 25.7	-22.0 11.9	-19.1 23.8	-28.6 26.1	-25.7 20.7	-24.1 13.1	-52.0 11.3	-54.6 8.2	-24.7 13.6	
	-49.8 -7.0	-39.8 -17.4	-69.5 -37.5	-63.2 -35.9	-44.5 -15.3	-33.5 -7.0	-34.8 -12.4	-52.0 10.8	-39.1 -7.0	-49.1 15.6	-48.8 16.5	-44.5 0.2	-44.0 -8.8	-67.1 -39.2	-66.3 -46.7	-41.8 -3.9	

Table 4.6: Correlations of First Differences of Credit Spreads on an Annual Basis

For each pair of sector-rating classes, four statistics are calculated. The upper left figure presents the average correlation of credit spreads' first-differences for the five one-year estimation periods. The upper right figure indicates the standard deviation of the correlations. The lower left (right) figure gives the minimum (maximum) correlation of the estimation period. Both tables present the results for the correlations of one-year spreads above the diagonal and for ten-year spreads below.

The correlations of the first differences of spreads are high between close ratings and decrease if ratings become more distant, an effect well-known from other studies. Furthermore, the high-quality rating classes AA and A show an average correlation of 72.7% that is higher than for rating classes A and BBB with 25.5%. The average correlation is smallest between AA and BBB classes, with 21.2%, and the range of correlations is considerably higher for BBB. Obviously, spreads of high-quality obligors evolve in a comparatively homogenous way, while the spreads of BBB obligors are more heterogenous, thereby reflecting more frequent obligor-specific credit events. Negative correlations between ratings AA and BBB appear in the years 2002 and 2003, which can be explained by investors' flight to quality as outlined before. Obviously, one-year spread correlations between AA and BBB classes change in time and depend on prevailing market conditions.

The first-differences of rating class spreads mostly show a negative correlation with the first differences of riskless rates, except for the year 2000, when one-year AA and A spreads as well as riskless rates both increased, resulting in positive correlations. In contrast, long-term credit spreads are negatively correlated with riskless rates, revealing a discrepancy between a favorable long-term economic outlook and short-term concerns about the creditworthiness of obligors. A similar effect can be observed in the year 2003 for the BBB class, where positive correlations of one-year BBB spreads and riskless rates are explained by a recovery in the short-term economic outlook after the second Gulf War was accomplished, coinciding with a decrease in riskless interest rates.

Compared to short-term equivalents, the correlations of 10-year credit spot spreads between rating classes are mostly lower. Obviously, the short-term assessments of creditworthiness in time are more homogenous across rating classes than the long-term consideration of credit risk. It is concluded that short-term credit spreads are notably more sensitive to general systematic factors, while long-term credit spreads are influenced by sector-related or obligor-specific considerations. Additionally, correlations of 10-year spread differences are less volatile, as indicated by standard deviations and the range of correlations. The average correlation between long-term spread differences and the changes in riskless rates with -52.0% (-54.6%, -24.7) for rating class AA (A, BBB) is distinctly negative, when compared to one-year correlations. As before, the standard deviation and range of correlations decrease, which accentuates the higher stability of dependence compared to short-term spreads.

$\tau = 10 \setminus \tau = 1$	ECY-A	ECY-BBB	FIN-AA	FIN-A	LCY-AA	LCY-A	LCY-BBB	NCY-AA	NCY-A	NCY-BBB	NF-AA	NF-A	NF-BBB	AA	A	BBB	RL
LCY-A		35.1 31.3 -25.5 87.0	46.2 35.5 -51.9 86.9	41.5 34.8 -37.8 91.0	35.5 34.6 -45.0 81.0	34.9 46.4 -43.3 94.4	32.4 26.0 -6.4 85.1	15.0 39.0 -45.0 90.2	40.0 30.8 -13.1 95.1	36.0 40.8 -54.2 93.7	17.7 36.5 -55.4 84.2	70.5 22.6 19.3 97.0	30.2 36.8 -62.8 91.4	49.2 33.4 -49.1 86.8	50.9 34.9 -20.2 91.5	30.3 38.0 -62.7 86.1	-19.4 46.0 -84.8 83.0
LCY-BBB	47.3 37.1 -32.8 89.2		32.7 25.9 -8.2 75.3	27.2 26.4 -22.3 70.4	28.6 37.2 -34.4 89.1	24.6 40.2 -51.4 79.7	19.8 38.2 -44.9 83.4	19.7 36.3 -62.7 74.3	23.3 32.5 -46.6 86.2	17.2 35.9 -80.5 77.2	21.0 33.6 -31.0 73.7	32.1 32.2 -54.2 80.2	50.7 24.7 20.6 94.8	32.8 26.2 -14.3 81.0	28.7 29.6 -40.9 76.8	54.3 25.9 19.4 94.8	-15.9 29.8 -70.1 42.9
FIN-AA	36.8 32.7 -28.3 93.9	30.1 25.5 -8.2 81.5		72.0 20.4 24.1 96.5	34.3 38.6 -50.8 84.1	33.5 37.7 -45.3 90.1	30.9 29.3 -24.9 90.5	22.9 38.8 -48.7 76.8	38.7 41.4 -44.3 90.8	44.1 32.9 -37.4 92.8	30.4 35.0 -61.8 82.0	45.1 42.1 -36.3 93.3	25.6 36.1 -34.3 76.9	96.3 4.7 80.8 99.8	69.4 26.1 -15.1 95.4	36.0 34.3 -35.0 78.5	-27.6 41.6 -89.5 51.7
FIN-A	38.6 27.0 -5.0 82.5	36.5 31.1 -34.5 88.8	61.7 19.4 21.3 89.9		33.3 32.9 -29.7 81.2	27.4 41.4 -39.5 94.4	24.5 30.1 -37.7 83.0	17.0 37.2 -39.2 81.0	28.5 40.8 -60.7 83.7	29.5 35.2 -50.7 90.5	23.5 29.7 -31.8 76.1	39.1 34.6 -25.2 80.6	23.5 33.2 -34.2 78.1	72.3 18.5 40.6 96.5	87.7 23.8 -6.1 98.6	32.6 35.2 -41.9 76.7	-26.1 32.6 -74.0 59.4
LCY-AA	40.2 26.3 -20.5 88.4	32.3 32.7 -56.8 68.9	34.7 32.7 -39.4 93.3	33.0 35.5 -38.2 80.5		34.1 41.8 -45.0 96.6	18.6 31.4 -37.4 73.7	12.2 45.3 -64.5 84.9	25.7 44.5 -66.4 86.8	23.8 39.9 -66.0 77.6	20.1 39.2 -35.5 88.9	27.7 36.4 -45.0 90.1	20.6 39.6 -55.5 81.4	36.2 36.7 -55.3 84.9	31.3 33.3 -26.6 84.5	30.5 37.9 -41.4 89.8	-9.8 42.8 -59.6 76.6
LCY-A	18.7 41.1 -64.7 81.6	22.6 32.7 -37.3 81.7	31.1 37.6 -52.5 93.9	19.5 35.4 -48.6 82.5	32.1 36.6 -61.1 88.4		19.1 33.4 -51.5 74.1	14.5 45.1 -70.8 90.6	18.3 40.6 -66.2 90.2	28.6 37.1 -62.5 91.4	14.1 39.3 -58.7 82.2	33.3 44.7 -66.6 88.9	19.9 37.1 -54.8 83.8	34.9 38.8 -49.7 88.5	22.8 42.8 -42.4 88.2	18.8 38.0 -54.8 76.8	-13.2 44.5 -77.6 74.9
LCY-BBB	39.2 30.1 -17.9 81.7	26.9 33.3 -30.9 79.2	24.7 30.5 -21.4 81.5	27.0 32.4 -32.9 88.8	30.6 30.5 -35.2 75.6	23.2 37.6 -45.0 81.7		12.2 36.0 -39.1 77.9	13.0 35.9 -50.6 89.3	17.7 33.5 -49.7 76.0	11.6 36.9 -56.6 79.5	23.5 32.5 -36.1 82.4	35.2 38.9 -38.2 94.0	29.5 27.9 -24.0 93.1	24.3 29.9 -35.2 89.1	36.0 36.8 -38.2 94.0	-4.3 31.0 -54.2 60.1
NCY-AA	29.6 36.6 -64.4 88.4	22.4 41.8 -62.3 76.0	42.1 27.5 -6.6 93.3	41.5 32.8 -58.9 87.4	37.1 27.0 -24.4 74.3	27.7 30.3 -38.5 88.4	19.5 37.1 -55.7 68.9		32.3 41.4 -45.0 94.6	24.1 30.8 -19.4 92.2	49.0 43.4 -25.8 99.1	22.1 35.7 -45.0 86.1	-3.1 40.6 -74.0 73.7	29.2 34.7 -46.7 83.6	24.3 32.2 -44.9 78.9	4.2 41.5 -58.0 89.8	-23.1 31.6 -64.6 64.9
NCY-A	39.0 36.5 -20.4 91.1	37.1 28.7 -7.5 81.7	36.2 27.6 -17.4 93.9	30.6 29.9 -33.8 82.5	26.3 40.9 -42.9 88.4	14.2 30.9 -48.4 57.6	32.0 33.0 -21.5 81.7	29.0 33.4 -48.2 88.4		30.9 36.8 -40.8 91.2	19.9 48.2 -56.0 88.1	51.6 32.1 -25.5 95.0	22.5 34.3 -49.6 90.7	42.5 37.8 -32.2 91.7	38.2 34.3 -43.5 85.5	21.6 40.3 -55.9 91.9	-24.4 38.3 -77.6 61.8
NCY-BBB	31.3 46.0 -69.9 95.5	32.5 34.9 -18.1 75.9	28.2 34.5 -39.6 81.5	25.6 37.5 -62.6 88.8	28.7 34.0 -34.3 83.4	27.1 31.4 -30.4 81.7	12.7 36.8 -42.3 67.8	26.4 31.1 -31.0 72.0	33.8 31.9 -18.5 88.9		19.2 30.0 -42.1 73.7	33.8 35.7 -45.4 79.5	38.0 45.4 -74.1 94.1	42.3 33.3 -38.1 90.7	34.7 32.4 -52.0 83.5	39.5 39.6 -76.0 94.1	-14.6 32.7 -75.3 42.7
NF-AA	31.4 37.4 -66.1 88.4	26.2 39.3 -61.2 73.4	42.6 30.4 -14.2 93.3	40.0 34.4 -60.0 85.7	48.8 29.8 -21.1 87.7	26.2 32.5 -51.4 88.4	24.9 35.2 -32.7 76.2	79.7 27.1 0.2 99.5	28.8 39.1 -47.2 88.4	26.1 36.0 -44.5 80.5		25.4 37.8 -51.2 96.0	6.6 32.2 -51.9 73.7	38.9 33.0 -59.0 83.3	21.5 31.5 -50.4 78.9	13.4 33.3 -33.0 89.8	-9.5 37.3 -78.0 73.6
NF-A	84.4 7.6 68.8 94.6	44.8 35.3 -32.0 82.5	38.3 35.7 -25.0 93.9	34.9 30.4 -34.1 85.6	35.7 31.6 -44.2 88.4	28.3 40.5 -49.1 81.1	39.0 28.6 0.2 81.7	32.2 35.2 -73.8 88.4	55.0 34.5 0.2 92.3	32.4 42.7 -70.8 83.0	32.7 34.6 -62.1 88.4		28.7 31.7 -48.5 83.6	47.9 40.2 -36.1 91.5	51.7 35.0 -29.4 88.2	24.6 39.9 -55.5 84.6	-29.6 37.8 -80.7 64.8
NF-BBB	41.5 40.2 -40.7 89.2	62.6 39.1 -28.1 98.3	30.5 32.8 -30.4 81.5	31.0 38.6 -56.9 88.8	37.6 26.9 -22.6 81.5	19.4 37.1 -55.8 81.7	40.5 29.1 -8.2 84.6	27.4 33.5 -33.3 68.9	33.0 32.7 -30.1 81.7	45.2 34.8 -30.1 82.7	28.8 37.4 -53.8 70.5	43.0 34.4 -18.6 81.7		22.5 35.7 -38.0 73.8	23.6 30.2 -43.7 75.2	77.6 29.8 19.4 93.6	-6.4 31.5 -71.2 44.1
AA	37.9 32.5 -27.3 93.7	31.1 24.6 -0.8 79.4	97.6 2.5 90.3 99.7	62.1 18.8 12.9 84.5	40.8 28.7 -16.7 95.6	33.8 35.6 -55.0 93.7	24.8 30.3 -23.5 79.4	52.3 24.8 -9.2 95.6	37.6 27.6 -3.0 93.7	29.3 33.1 -25.8 83.6	50.9 27.7 -10.4 95.6	40.3 33.8 -19.5 93.7	33.3 27.8 -16.3 79.4		69.9 25.4 -18.0 94.5	32.6 36.3 -37.8 78.4	-30.1 38.0 -87.7 55.8
A	53.1 24.0 10.1 91.1	42.0 31.9 -36.5 90.0	64.6 18.6 16.5 98.6	94.1 7.1 70.1 99.7	38.1 32.4 -44.8 80.1	27.1 36.6 -49.9 89.5	31.5 29.6 -27.8 90.0	44.2 30.9 -50.2 88.6	41.0 30.1 -36.8 89.5	31.6 37.7 -46.0 90.0	41.3 34.2 -50.6 87.4	52.4 28.7 -8.2 94.2	39.9 35.9 -48.3 90.0	66.1 19.0 10.8 97.4		29.2 37.0 -41.1 78.9	-27.9 32.6 -73.1 62.0
BBB	41.0 40.9 -53.7 92.4	65.7 30.4 1.6 98.3	29.6 29.0 -24.9 78.0	26.8 35.3 -49.7 73.6	30.3 35.4 -49.4 81.5	16.3 33.8 -61.7 60.5	36.9 31.1 -14.6 85.2	20.4 39.0 -51.3 63.9	28.0 34.4 -30.2 76.7	42.6 31.7 1.6 90.0	22.0 42.8 -53.8 70.9	42.6 36.7 -42.4 80.4	74.4 34.8 -4.8	30.7 26.1 -9.0 78.3	36.1 33.7 -35.3 83.9		-15.3 29.7 -70.7 45.5
RL	-27.8 28.9 -70.1 47.0	-32.0 26.9 -70.9 23.0	-59.0 17.2 -89.9 -30.8	-58.4 20.2 -87.4 -6.4	-30.3 29.1 -77.5 52.4	-19.8 30.4 -55.1 59.2	-23.9 23.7 -59.7 14.1	-33.9 29.7 -86.2 44.5	-28.4 22.8 -65.2 4.0	-27.1 30.5 -72.6 40.6	-33.8 30.7 -71.2 47.6	-26.7 28.7 -62.9 23.7	-33.0 25.3 -67.0 26.8	-56.3 18.6 -87.3 -19.1	-59.0 18.5 -81.4 -6.8	-32.6 26.7 -73.1 29.2	

Table 4.7: Correlations of First Differences of Credit Spreads on a Quarterly Basis

If correlations for year-based periods are compared to those for quarterly periods, the average spread correlation remains almost unaltered between rating classes AA and A for one-year and ten-year spreads. In contrast, correlations between BBB and AA (A) increase considerably, on average from 21.2% (25.5%) to 32.6% (29.2%) for one-year spreads and from 20.7% (23.2%) to 30.7% (36.1%) for 10-year spread differences. Furthermore, the variation of correlations increases substantially for all pairs of risk classes if quarterly correlations are taken into account. The strong variation of the co-movement structure of credit spreads in time raises doubts about the assumption of time-homogenous correlations in credit portfolio models.

With respect to the influence of the economic sector on the dependence structure of credit spreads, the two-sector case with five sector-rating classes FIN-AA, FIN-A, NF-AAA, NF-A, NF-BBB is examined. The average correlation of spread differences between FIN-AA and FIN-A resemble those between the corresponding rating classes, independent of the maturity of spreads or sample frequency. Clearly, this effect is caused by the dominance of observations from the financial sector within the high-quality rating classes.

Comparing the credit spreads of FIN risk classes to those of NF classes reveals more ambiguous results. Average short-term correlations between FIN-AA and risk classes FIN-A and NF-A are higher than correlations between FIN-AA and NF-AA for yearly and quarterly periods. This conflicts with the assumption that the rating is the prevailing attribute to determine the co-movement of spreads. Correlations of the FIN-A risk class, however, show the pattern already observed between rating classes, so that correlations to the NF-A risk class are higher than those to the NF-AA and NF-BBB classes. Long-term correlations on a yearly and quarterly basis show the usual pattern: average correlations between risk classes of different sectors but equal rating classes exceed correlations between risk classes of different sectors and rating classes. Within the NF sector, average correlations between risk classes of adjoint ratings are observed to be higher than those between classes of more distant ratings. However, correlations between NF-AA and NF-A are considerably lower than those between FIN-AA and FIN-A. This is attributed to the smaller sample size of the NF sector, which gives additional emphasis to changes in credit spreads imposed by non-systematic effects. The standard deviation and the range of year-based short-term correlations increase with the distance of the ratings in most cases, no matter whether the sectors involved are different or not. Long-term correlations show a distinctly lower variation, indicating a more stable co-movement structure for long-term spreads.

On a quarterly basis, the average short-term correlations are lower than year-based equivalents, while for long-term correlations the reverse effect is prevailing. Quarterly resampling turns obligors within risk classes more homogenous and more heterogenous between risk classes, since non-representative obligor-specific spreads are excluded from a risk class sooner. The effect of more homogenous bond samples concerning the rating assignment should lead to an increase in correlations between risk classes of the same rating. An increase in correlations is, however, only observed for long-term spreads, while quarter-based short-term correlations between risk classes of equal rating are lower. It is concluded that the co-movement of spot spreads is robustly indicated only for long term spreads that reflect effects of systematic factors to a smaller extent. Analogously, quarter-based spread correlations between rating classes show a higher variation of considerable size.

With respect to year-based correlations between first differences of credit spreads and riskless rates, negative correlations are observed as before, with correlations being lower for ten-year spreads. If a resampling is carried out on a quarterly basis, correlations show a higher variation, and long-term correlations mostly become even more negative.

4.3.2.3 Residual Analysis

The analysis of price and yield residuals of the term structure fitting in Table 4.8 reveals further insights into the suitability of the Nelson-Siegel functional form and the performed fitting process as well as into price and yield effects which have not been captured by the risk class structures used. Price residuals are calculated as present values obtained from fitted yield curves less the market-observed dirty prices of bonds. Average price residuals are calculated across all bond observations and term structures of a risk class in a given year. For the riskless term structures, average price residuals vary between -0.3 bps and 0.8 bps, while credit spread structures range from -16.2 bps to 26 bps. The average price residuals are considered to represent the potential fitting error that could be reduced if a better-suited functional form of the term structure or a more efficient fitting algorithm were used.

Riskless term structures are based on a set of French and German government bonds and resulting price errors are attributed to the fitting procedure only. For corporate term structures, fitting errors are notably smaller for data-rich risk classes such as rating classes, financial sector classes, or aggregate non-financial risk classes.

In contrast, the mean absolute price deviation (MAD) reveals effects from the heterogeneity of spreads within the bond sample, which could be eliminated by defining more homogenous risk classes. The MAD of riskless term structures varies between 5 bps and 16 bps and indicates the price variability attributed to effects other than credit risk, such as illiquidity, coupon effects or errors in market prices. The average median price residual is smaller than the average price error for 63 out of 72 fitting scenarios. The price residuals are in most cases skewed to the right, but 5%- and 95%-quantiles show that the skew is not considerable and may be attributed to the fact that the short spread is fixed at zero.

The minimization of price residuals in (4.8) ensures that long-term and short-term bond

	Risk-Class		Price-Residuals in %						Yield-Residuals in bp					
Year	Rating	Sector	Average	MAD	Std. Dev.	Median	Q05	Q95	Average	MAD	Std. Dev.	Median	Q05	Q95
1999	RL		0.007	0.16	0.24	0.03	-0.47	0.33	-1.53	5.64	7.81	-1.56	-11.79	9.64
	AA	FIN	0.016	0.55	0.86	-0.04	-1.05	1.75	-2.42	11.77	18.69	0.91	-38.23	17.19
		NF	-0.020	0.63	0.94	0.05	-1.57	1.31	-2.60	12.90	18.75	-1.21	-31.72	22.22
		ALL	0.004	0.56	0.88	-0.04	-1.13	1.71	-2.32	11.92	18.80	0.88	-37.31	18.18
	A	FIN	-0.030	1.20	1.51	-0.54	-1.69	2.92	-0.25	21.56	28.13	9.55	-52.03	28.62
		NF	0.004	0.61	0.83	0.04	-1.26	1.49	-2.73	14.16	18.71	-1.10	-37.86	23.99
		ALL	-0.023	1.08	1.40	-0.37	-1.66	2.73	-0.75	20.00	26.44	6.75	-48.52	27.95
	BBB	NF	-0.050	0.78	1.01	-0.03	-1.88	1.68	-3.55	17.76	23.27	-0.53	-49.28	30.60
		ALL	-0.028	0.88	1.17	-0.09	-2.00	1.92	-3.27	19.24	25.53	0.35	-52.86	32.27
2000	RL		0.008	0.14	0.20	0.02	-0.35	0.29	-0.81	5.71	8.75	-1.09	-11.73	12.28
	AA	FIN	-0.039	0.51	0.75	-0.08	-0.99	1.28	-0.87	12.02	16.99	1.95	-30.95	18.29
		LCY	-0.024	0.65	0.81	-0.06	-1.27	1.16	0.34	14.73	18.18	0.51	-25.97	28.70
		NCY	-0.013	0.67	0.97	0.05	-1.60	1.53	-1.64	14.81	19.73	-1.28	-32.92	27.82
		NF	-0.021	0.77	1.10	-0.03	-1.75	1.77	-1.10	16.25	21.73	0.71	-36.16	29.94
		ALL	-0.059	0.58	0.84	-0.11	-1.14	1.42	-0.42	13.15	18.21	2.70	-32.31	20.79
	A	ECY	-0.033	0.49	0.70	0.00	-1.09	0.92	-1.58	12.96	19.74	0.13	-32.62	21.61
		FIN	-0.088	1.08	1.34	-0.42	-1.85	2.32	1.16	22.10	27.44	10.89	-46.60	30.87
		LCY	0.021	0.58	0.81	0.00	-1.68	1.20	-1.68	13.12	18.44	0.47	-33.15	33.22
		NCY	-0.010	0.51	0.74	-0.07	-1.11	1.25	-2.11	12.44	18.10	1.78	-42.63	18.07
		NF	-0.051	0.55	0.79	-0.05	-1.28	1.13	-0.75	13.51	20.02	1.41	-32.15	23.56
		ALL	-0.077	0.98	1.25	-0.27	-1.86	2.12	0.61	20.47	26.28	6.70	-43.58	31.28
	BBB	ECY	0.023	0.62	0.86	0.13	-1.91	1.26	-1.04	14.74	19.66	-3.06	-32.82	39.35
		LCY	0.037	0.62	0.86	-0.06	-1.06	1.62	-3.31	16.41	26.12	1.23	-56.61	26.00
		NCY	-0.034	1.03	1.64	-0.13	-2.55	3.43	0.21	23.80	36.13	3.27	-73.99	54.28
		NF	0.007	0.74	1.22	-0.04	-1.87	1.84	-1.43	17.66	30.46	1.16	-40.29	39.93
		ALL	0.008	0.75	1.22	-0.04	-1.92	1.89	-1.23	17.72	30.26	1.21	-41.32	40.30
2001	RL		-0.003	0.12	0.18	-0.01	-0.26	0.35	-0.06	4.34	6.27	0.51	-8.20	7.82
	AA	FIN	0.002	0.49	0.78	-0.05	-0.99	1.32	-1.01	11.42	16.84	1.71	-29.59	18.83
		LCY	0.031	0.69	0.90	-0.05	-1.11	1.77	-2.06	16.90	21.41	1.62	-41.50	26.73
		NCY	0.013	0.61	0.84	-0.07	-1.24	1.60	-0.91	13.29	17.27	1.99	-30.59	20.03
		NF	0.007	0.69	0.97	-0.14	-1.31	2.12	-1.01	15.62	21.22	3.71	-43.07	23.69
		ALL	-0.012	0.53	0.83	-0.08	-1.08	1.44	-0.57	12.15	17.74	2.35	-31.14	20.43
	A	ECY	0.026	1.68	2.66	-0.45	-2.67	6.33	-2.40	42.12	66.18	14.40	-181.08	57.40
		FIN	-0.060	1.07	1.45	-0.25	-2.27	2.12	0.56	22.47	29.32	7.87	-40.93	35.79
		LCY	0.002	1.02	1.64	-0.12	-2.27	4.14	-0.69	21.70	32.49	3.58	-79.38	43.71
		NCY	0.004	0.53	0.76	-0.01	-1.21	1.44	-0.92	12.20	16.33	0.35	-32.56	22.26
		NF	0.008	1.57	2.55	-0.44	-2.70	4.65	-1.61	38.21	61.39	13.66	-117.51	55.32
		ALL	-0.032	1.30	2.08	-0.19	-2.78	2.92	-0.36	29.46	47.13	5.24	-63.98	47.48
	BBB	ECY	0.264	1.55	2.49	-0.36	-2.42	5.42	-8.01	41.40	62.27	9.84	-157.64	52.79
		LCY	-0.003	0.60	0.84	-0.15	-1.02	1.66	0.13	14.23	20.10	3.95	-39.18	25.02
		NCY	0.063	1.81	2.60	-0.42	-2.97	4.26	-4.64	40.75	58.77	11.35	-90.56	56.41
		NF	0.088	1.39	2.25	-0.37	-2.25	3.95	-4.55	33.17	52.54	9.43	-95.77	44.57
		ALL	0.059	1.40	2.25	-0.38	-2.31	3.80	-3.85	33.41	52.69	9.58	-93.07	46.64
2002	RL		0.000	0.08	0.12	0.00	-0.22	0.19	-0.57	4.29	6.48	-0.22	-9.32	7.16
	AA	FIN	0.020	0.55	0.92	-0.02	-1.25	1.48	-2.27	13.95	19.67	0.89	-34.64	22.26
		LCY	-0.073	0.55	0.82	-0.21	-1.18	1.39	-0.64	12.78	18.43	3.51	-37.94	20.56
		NCY	0.004	0.69	0.94	-0.14	-1.22	1.71	-1.19	15.43	20.50	4.08	-41.05	22.63
		NF	0.031	0.67	0.94	-0.13	-1.09	1.84	-1.82	15.03	20.53	3.74	-43.29	21.76
		ALL	0.020	0.57	0.93	-0.03	-1.26	1.56	-2.13	14.12	19.89	1.26	-35.78	22.88
	A	ECY	0.022	0.87	1.24	-0.01	-1.88	2.17	-3.39	25.65	33.70	-0.07	-70.23	41.75
		FIN	0.014	0.97	1.50	-0.04	-2.15	2.48	-3.07	23.05	38.38	2.08	-55.42	37.40
		LCY	0.179	3.81	6.08	-1.84	-3.66	13.90	-30.16	138.05	266.27	62.44	-769.56	109.99
		NCY	0.010	0.75	1.17	-0.10	-1.46	2.43	-1.50	19.27	27.62	3.96	-51.28	30.15
		NF	0.042	1.26	2.67	-0.34	-2.10	2.83	-6.08	38.98	109.78	11.46	-71.53	49.18
		ALL	0.014	1.10	1.99	-0.17	-2.28	2.64	-3.17	29.29	72.44	7.04	-62.18	43.17
	BBB	ECY	0.020	4.21	5.57	-1.55	-5.90	12.05	-14.18	148.17	214.77	45.13	-504.57	223.27
		LCY	-0.023	1.23	1.82	-0.56	-1.79	3.49	-2.19	37.51	63.12	17.21	-90.22	44.16
		NCY	0.124	2.24	3.20	0.01	-4.45	7.03	-14.46	61.84	85.94	-0.57	-217.66	85.80
		NF	0.070	3.26	4.68	-1.24	-4.45	10.02	-17.52	106.10	177.53	39.56	-390.34	114.49
		ALL	0.064	3.27	4.68	-1.25	-4.45	10.01	-17.30	106.20	177.53	39.71	-390.49	114.75
2003	RL		0.000	0.05	0.08	0.00	-0.14	0.11	-1.14	4.01	7.53	-0.24	-12.39	5.93
	AA	FIN	0.007	0.50	0.83	-0.05	-1.12	1.62	-3.46	15.05	29.41	2.65	-39.59	20.10
		LCY	-0.162	0.78	1.30	-0.08	-3.21	1.64	2.51	10.74	14.45	1.96	-19.21	30.25
		NCY	0.024	1.12	2.09	-0.34	-1.60	4.23	-2.32	24.26	42.12	9.23	-116.86	30.40
		NF	-0.020	1.06	1.98	-0.30	-1.70	3.56	-1.45	22.08	39.13	8.04	-97.31	29.76
		ALL	-0.010	0.57	1.14	-0.09	-1.25	1.56	-2.70	15.96	31.72	3.80	-38.92	21.86
	A	ECY	0.022	0.64	0.93	0.02	-1.38	1.69	-4.58	19.47	24.59	-1.09	-47.24	28.20
		FIN	0.030	0.73	1.31	0.01	-1.81	1.80	-6.12	22.08	34.44	-0.44	-60.33	33.69
		LCY	0.004	0.38	0.52	-0.02	-0.86	1.03	-2.17	11.90	16.19	1.73	-31.26	17.00
		NCY	-0.007	0.51	0.79	0.03	-1.30	1.38	-2.90	14.32	20.20	-0.77	-38.31	23.91
		NF	0.018	0.61	0.92	-0.01	-1.33	1.70	-3.64	17.46	23.33	0.74	-45.94	28.03
		ALL	0.025	0.70	1.21	0.00	-1.64	1.75	-5.44	20.80	31.78	0.09	-56.09	31.66
	BBB	ECY	0.112	1.51	2.33	-0.24	-2.55	3.71	-10.54	46.42	83.52	8.12	-103.34	55.66
		LCY	-0.069	2.26	3.87	-1.05	-2.88	8.13	3.07	66.00	104.82	40.45	-214.09	70.21
		NCY	-0.024	4.55	5.98	-2.11	-5.91	12.46	-6.18	139.46	185.94	64.30	-384.50	158.05
		NF	0.024	2.68	4.32	-1.14	-3.56	9.85	-5.27	81.56	133.06	40.33	-289.95	86.90
		ALL	0.023	2.66	4.29	-1.12	-3.58	9.59	-5.19	80.65	131.87	39.71	-286.79	87.20

Table 4.8: Price and Yield Residuals

prices show equal fitting quality. In contrast, residuals on a yield basis can be better compared across bonds, so that yield residuals are more appropriate for use in assessing the fitting performance. Average yield residuals are calculated as yields derived from fitted bond prices less market-observed yields, and vary between -1.53 bps and -0.06 bps for riskless term structures, and between -30.16 bps and 3.07 bps for the credit-risky term structures. The considerable overestimation of market-observed bond yields in some periods coincides with fitted bond prices being to small on average, which is attributed to outlier effects. The credit distress of specific bonds moves term structures upward, leading to an underpricing of the remaining bonds in the sample. This misrepresentation of typical bond prices in a risk class in case of credit distress is a property specific to term structures which were determined for risk management purposes and clearly reveals the difference from term structures used for bond pricing that are gained from bond samples of instantaneous homogenous credit risk.

In general, market-observed yields in a risk class are positively skewed, due to the downward bounding of bond yields by the riskless term structure and an upward potential of yields that is typically more pronounced and limited only by the recovery value of bonds. This effects results in an underpricing and a systematic overestimation of yields for most bonds. Accordingly, average median yield residuals exceed the average yield residuals in Table 4.8 for 70 fitting scenarios, suggesting that yield residuals are negatively skewed. The skewness is more pronounced for risk classes that suffer credit distress during the estimation period, as can be seen in the case of risk classes LCY-A, ECY-BBB, NF-BBB in 2002 and all BBB classes in 2003. The average standard deviation of yield residuals increases with deteriorating credit quality, while no tendency of the average standard deviation of price residuals can be identified.

Finally, the effect of rating, sector and bond liquidity on yield residuals was examined. In general, price and yield residuals increase with deterioration of the credit quality, as observed for different rating classes of a particular period, and also observed from comparing residuals for the years 1999 and 2000 to those of the recession years 2001-2003. The hypothesis that trading activity, indicated by the issue amount of bonds as a liquidity proxy, is negatively related to the absolute size of yield residuals was not supported by a regression analysis. Data analysis reveals that corporate bonds from the non-financial sectors have on average a substantially higher issue amount of EUR 556.96*mn* (421.14*mn*, 514.17*mn*) in the ECY (LCY, NCY) sector, compared to EUR 175.88*mn* in the financial sector, which may explain the fact that the average absolute yield residuals for non-financial sectors are not higher when compared to the more homogenous financial sector. However, unambiguous sector-related effects in the fitting quality could not be identified across different periods, which confirms that the Nelson-Siegel functional form and the fitting procedure are suited for a wide range of risk classes specifications.

4.4 Estimation of the Credit Portfolio Model

4.4.1 Estimation Approach

The latent nature and the dependence of asset values within and between risk classes challenge the estimation of the structural credit portfolio model introduced in Chapter 3. Exponential-affine multi-factor term structure models are exposed to similar estimation requirements, although the two model types seem unrelated at first. The structural default variable in (3.4) evolves according to a Geometric Brownian motion with constant drift, while default intensities follow a mean-reverting diffusion process. Concerning the default model, the default time of an exposure is predictable from a time-continuous structural variable that represents either the value of a firm's assets or an abstract default indicator that triggers obligor default if a constant absorbing default barrier is hit, while default times from exponential-affine intensity models are not predictable and the default intensity represents a random parameter of a Poisson distributed default variate.[13]

Despite these differences, the information content of both concepts is similar. Both model types have a cross-sectional and a time-series dimension, i.e. the term structure of credit spreads and its dynamic evolution in time is specified by the current value and the diffusion process of Markovian state variables. In reverse, the cross-sectional and the time series dimension of panel data that indicate the dynamics of spread structures in time must be exploited to estimate both kind of models. Exponential-affine term structure models can be disaggregated into an affine function of a set of latent state variables, similar to the risk class factor model in Section 3.5.[14] Consequently, the estimation procedure to be proposed will adapt techniques for the estimation of exponential-affine multi-factor term structure models, where two major approaches can be distinguished:[15]

The inversion approach fits unobserved state variables in order to reproduce exactly market-observed rates conditional on factor dynamics, and typically makes use of a maximum likelihood estimator to determine the parameter set. Chen and Scott (1993), Pearson and Sun (1994), Duan (2004), as well as Düllmann and Windfuhr (2000) pursue this approach.

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[13]Cf. Section 3.1.1 as well as Duffie and Singleton (1997, 1999a) and Lando (1998) for the default concept of intensity-based default models.

[14]Duffie and Kan (1996) show that a necessary and sufficient condition for the zero-coupon bond price to be an exponential-affine function of factors requires drift and diffusion terms of the Itô processes of factors to be affine functions of an underlying state vector.

[15]Further approaches include proxy models of unobserved state variables by Marsh and Rosenfeld (1983) as well as Chan, Karolyi, Longstaff and Sanders (1992), a cross-sectional restriction of term structure estimation as applied by Brown and Dybvig (1986), Titman and Torous (1989), or De Munnik and Schotman (1994), as well as the generalized method of moments with restricted conditional moments proposed by Gibbons and Ramaswamy (1993).

The maximum likelihood estimation of the parameter set and state vector series of a state-space model using the Kalman-Filter[16] (KF) has become the prevalent approach to specify exponential-affine term structure models. In principle, the Kalman-Filter is a Bayesian estimator for the joint evolution of a set of latent state variables during a sample period. State variables are combined to a state vector. Basic components of the Kalman-Filter are the transition equation and the measurement equation of a state-space model (SSM). The transition equation determines the conditional expectation of the state vector over a discrete time interval, taking into account the joint dynamics of state variables. The measurement equation defines the functional relation between the state vector and a vector of cross-sectional observations. In contrast to the inversion approach, the measurement equation does not fit observations exactly, so that empirical observations face a residual error to the filtered-implied observation estimates. In a sequential procedure, the evolution of the latent state variables during the sample period is estimated. At each point in time, a prediction step and an update or filtering step are performed. The prediction step determines (1) an ex-ante forecast of the state vector, and (2) the covariance matrix of the state vector prediction error, conditional on the information set at the time of the forecast. The update step adjusts the prediction of the state vector and the covariance matrix of state prediction errors using the measurement error of model-derived observations based on the state prediction. For a general in-depth examination of the Kalman-Filter, see Harvey (1989), Hamilton (1994) and Gouriéroux and Montfort (1997).

The measurement equation of state-space models can be adapted to implement different functional relations between latent state variables and observation data, and the transition equation offers a maximal variability concerning the factor model assumed. State space models are used for exponential-affine term structure models of riskless short rate and credit spreads as well as for multi-obligor and risk class models. The exponential-affine form results in an affine transition equation for the logarithm of factors. For the measurement equation to be linear, observations must be given by continuously compounded yields of zero-coupon bonds.[17]

With a Gaussian discrete-time transition density of the state vector, as included in the model of Vasicek (1977), the standard Kalman-Filter with Gaussian disturbances applies. Given a Non-Gaussian transition density, as imposed by the CIR model, the filtering steps remain unchanged, but optimization has to rely on the quasi-maximum likelihood (QML) concept, with the quasi-maximum likelihood estimator (QMLE) being consistent only for linear measurement equations. However, a simulation study by Lund (1997a) shows

[16]See Kalman (1960).

[17]Duan and Simonato (1995) show that all exponential-affine models can be put in a vector-autoregressive VAR(1) form and that the conditional covariance matrix of the state prediction error is an affine function of the lagged state vector estimate.

that the Kalman-filtering is comparatively well-behaved with regard to the distributional assumptions of the state variables. Among many others, Chen and Scott (1993), Duan and Simonato (1995), Lund (1997a, 1997b), De Jong (1999), Geyer and Kossmeier (2001, 2003), and Schirm (2004) are proponents of this estimation approach.

Furthermore, Cumby and Evans (1997) compare Kalman-Filter applications for the estimation of several discrete-time default intensity models, while Claessens and Pennacchi (1996) apply the Kalman-filtering of state-space models to a structural two-factor credit-pricing model for Brady bonds. Baadsgaard and Madsen (2000) use coupon bond prices to estimate an exponential-affine term structure model using a state-space model with a non-linear measurement equation. In all three of the models, the measurement equation is non-linear and, as a result, the Extended Kalman-Filter (EKF) as described in Harvey (1989) is used to approximate conditional expectations of the state vector and the covariance matrix of the state prediction error.

Due to its similar informational requirements, the KF-based estimation of exponential-affine intensity models is adapted to the estimation of the structural risk class based credit portfolio models. A sequential two-step QMLE of the credit portfolio model introduced in Chapter 3 is proposed, whose distinctive characteristics comprise:

- A consecutive EKF-based QML estimation of latent systematic and specific factors in a state-space model with non-linear transition and measurement equations,
- the assumption of normal distributed measurement disturbances and log-normal transition density of factors,
- observation data that represent yield spreads of synthetic par-coupon bonds based on fitted term structures of risk classes for systematic factors, and based on bootstrapped yield spreads for specific factors, and
- asset correlations that are derived from filtered time series of systematic factors and estimated systematic factor coefficients.

Sequential Kalman-Filter Estimation

The estimation of the structural credit portfolio model involves specifying the asset value processes that represent the risk classes, and the co-movement structure of asset values. The estimation of process parameters and times series of latent systematic and specific factors of risk classes is separated into two consecutive estimation steps which enforces the identification of parameter and factor estimates. The estimations of systematic and specific factors are conducted for each risk class individually.

In a first step, a non-linear state-space model relates a single systematic risk class factor to a panel data set of coupon bond yield spreads derived from the fitted term structure of a risk class. The state variable represents the systematic factor of the class. The non-linear measurement equation relates the benchmark-curve-derived yield spreads of synthetic par-coupon bonds to yield spreads implied by the credit valuation model of (3.25), subject to the systematic factor. The non-linear transition equation specifies the conditional expectation of the systematic factor based on a log-normal transition density. The Extended Kalman-Filter, as described in Harvey (1989, p. 160ff.), is used to approximate linearized transition and measurement equations by means of a first-order Taylor approximation. A maximum likelihood (ML) estimation of EKF-based prediction errors involves the estimation of process parameters μ_F and σ_F of the latent systematic factor and the filtering of a factor series.

On the basis of the process specification and the time series of systematic factors, a second non-linear state-space model is used to estimate the parameters of the asset value processes, and the coefficients and time series of obligor-specific factors. The explicit estimation of the obligor-specific factor dynamics of a risk class model based on corporate bond data represents an innovation when compared to conventional exponential-affine credit portfolio models, which mostly do not consider obligor-specific factors.[18] In contrast to the systematic-factor state-space model, each observation included in the observation vector refers to a separate i.i.d. state variable that represents the specific factor.[19] The transition equation is given by the multi-variate log-normal distribution of specific factors. Observations are given by bootstrapped yield spreads. The measurement function calculates yield spreads on the basis of asset values derived from the filtered systematic factor and a specific factor using the systematic factor coefficient and the parameters of the asset value process. The different specific factors of the state vector share equal coefficients and distributional properties but take different realizations across obligors.

Observation Data

The credit valuation model used implies a constant and flat term structure of riskless interest rates, while empirical bond market data is subject to a dynamic non-flat term structure of riskless rates. Furthermore, the valuation model is not designated to value zero-coupon bonds, which means that zero rates cannot be used as observation data. The observations used to resolve these detriments represent credit yield spreads of synthetic par-coupon bonds with constant time-to-maturity. The use of synthetic bond price observations is omitted due to their maturity-dependent sensitivities to a change in state variables. In the case of the systematic-factor state-space model, yield spreads are derived from bond valuations that are subject to synthetic credit-risky term structures of

[18] Cf. Geyer and Kossmeier (2001), Schirm (2004), whereas Duffie and Garleanu (2001) include specific factors, but fail to provide model estimates.

[19] Lund (1997b) points out that for a multi-variate transition density of dependent state variables, conditional expectation and prediction errors cannot easily be calculated, which turns state-space models with dependent factors inapplicable in practice.

zero rates made up of a constant riskless rate and the empirically fitted credit spreads of the respective risk class.

Observations used for the estimation of the specific-factor process are required to represent the typical variation of coupon bond yield spreads around the spread derived from the synthetic term structure of the risk class. The reason for using bootstrapped yield spreads in the estimation of specific factors is the inhomogeneity of empirical bond market data of the risk classes, which are in general not evenly spread with respect to the number of observations per risk class and obligor, the time-to-maturity of bonds and the availability of observations in time.[20] With no unambiguous criteria for the selection of representative bond issues at hand, and due to the change of the time-to-maturity of empirical bonds a robust estimation of specific factors is ensured best if bootstrapped times series of observations are used, which incorporate the distributional properties of the discrete-time changes of empirical bond yield spreads in a risk class.

Distribution Assumptions

Systematic and specific factors are assumed to follow a geometric Brownian motion with log-normal discrete-time transition densities of state variables in both state-space models. Although the observation data for the specification of systematic-factor processes is derived from defaultable bond prices, the risk class itself, represented by the fitted term structure of spreads, is non-defaultable, so that the systematic factor incorporates credit risk but precludes the default of the risk class by definition.[21] The assumption of a log-normal transition density for the systematic factor involves the event of a risk class default with all obligors of the class going bankrupt at the same time as the systematic factor hits the default barrier. In principle, a factor consistent with economic reality must preclude a first-passage default of all obligors in a class by using the bounded transition density of (3.19) at the cost of the state disturbance being non-Gaussian.[22]

Despite intensive research, no closed-form decomposition or aggregation of variables that follow a diffusion process with an absorbing barrier could be found in the literature on stochastics.[23] In consequence, the barriers of different bounded factors cannot be added to receive the default threshold of the aggregate factor and, in reverse, if the barrier of

[20] Reasons for the discontinuity of bond prices are the illiquidity of the market, the maturing of bond issues, technical problems, traders' negligence or the exclusion of improper price quotations.

[21] Within intensity models, default events are not triggered directly by the state variable, so that the existence of credit defaults in the observation data does not disturb the assumptions on the transition density of the state vector, but it may affect estimation results.

[22] Restrictions to ensure that factor values stay above the default threshold can be compared to restrictions in the Kalman-Filtering of exponential-affine term structure models that guarantee the non-negativity of state variables and that keep the covariance matrix of disturbances positive definite. Cf. Lund (1997a), Chen and Scott (1993) as well as Duan and Simonato (1995) for a discussion.

[23] In contrast, default intensities of exponential-affine models can easily be aggregated from latent factor models.

an factor aggregate is given, individual factor barriers and the first-passage time of single factors cannot be derived. Due to these considerations, the assumption of a bounded transition density of systematic factors is omitted.

Correlation Model

The factor coefficients of the specific-factor state-space model determine the inner-class asset correlations of each risk class. Inter-class asset correlations are estimated using the correlation of normalized systematic factor series. Aggregating the systematic and specific factors of decomposed asset values from all risk classes results in a multi-factor model of asset values with correlated systematic factors, where asset returns in each risk class depend on only one class-specific systematic factor and exposure-specific factors. Using the Cholesky decomposition, a risk class model with independent abstract factors is derived with factor coefficients adapted to reproduce the estimated inner-class and inter-class correlations.

Procedural Outline

The model estimation begins with the methodological introduction of the state-space model, the quasi-likelihood function and the Extended Kalman-Filter representation used. Implementation notes on the EKF-based QML estimation of systematic and specific factor processes are provided. The correlation structure of the credit portfolio multi-factor risk class model is calibrated from coefficient estimates and time series of systematic factors for different sample periods and risk class structures. The results of the estimations are presented.

4.4.2 General State-Space Model

The Kalman-Filtering of a vector of unobservable state variables exploits cross-sectional and dynamic information of observations dependent on a vector of state variables. A time series of the latent state vector and a set of parameters that determine the joint dynamics of state variables are estimated using a QML estimation. For the Kalman-Filtering of systematic factors and asset values, the non-linear state-space model

$$X_t = T(X_{t-\Delta t};\psi) + \eta_t, \qquad \eta_t \sim \mathcal{D}(0, \mathcal{Q}(X_{t-\Delta t};\psi)), \quad t = 1, ..., \overline{T} \quad (4.9)$$

$$S_t = Z(X_t, t_t;\phi_t) + \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0, \mathbf{\Omega}(\psi)), \quad t = 1, ..., \overline{T} \quad (4.10)$$

is defined by the transition equation (4.9) and measurement equation (4.10).

The transition equation specifies the discrete-time series $\mathbf{X}_{\overline{T}} = (X_t)_{t=1,...,\overline{T}}$ of a state vector X_t=$(X_{t,1}, ..., X_{t,i}, ..., X_{t,n_X})'$ of latent orthogonal state variables $X_{t,i}$, i=1,...,n_X, that represent either a single risk class factor or obligor-specific asset values with parameter set ψ and continuous-time dynamics according to the SDE in (3.3). The non-linear transition function $T(X_{t-\Delta t};\psi) = (T_X(X_{t-\Delta t,1};\psi), ..., T_X(X_{t-\Delta t,i};\psi), ..., T_X(X_{t-\Delta t,n_X};\psi))'$ is a first-order Markov process defined elementwise by conditional expectations $T_X(X_{t-\Delta t,i};\psi) = \mathbb{E}[X_{t,i}|X_{t-\Delta t,i}]$, i=1,...,n_X for time interval Δt=7/365, and it results vector of expectations $T(X_{t-\Delta t};\psi) = \mathbb{E}[X_t|X_{t-\Delta t}] = (\mathbb{E}[X_{t,1}|X_{t-\Delta t,1}], ..., \mathbb{E}[X_{t,i}|X_{t-\Delta t,i}], ..., \mathbb{E}[X_{t,n_X}|X_{t-\Delta t,n_X}])'$.

The disturbance vector $\eta_t=(\eta_{t,1}, ..., \eta_{t,i}, ..., \eta_{t,n_X})'$ contains innovations $\eta_{t,i}=X_{t,i} - \mathbb{E}[X_{t,i}|X_{t-\Delta t,i}]$, which are assumed to be mutually, serially and cross-serially uncorrelated, i.e. $\mathbb{E}[\eta_{t,i}\eta_{t,h}]$=0, $i \neq h$ and $\mathbb{E}[\eta_{t,i}\eta_{u,h}]$=0; i,h=1,...,n_X;$t \neq u$. From $\eta_t \sim \mathcal{D}(0, \mathcal{Q}(X_{t-\Delta t};\psi))$, the vector of expected disturbances $\mathbb{E}[\eta_t|X_{t-\Delta t}] = 0$ is defined and the covariance matrix $\mathcal{Q}(X_{t-\Delta t};\psi)$ is assumed to be conditionally heteroscedastic with respect to $X_{t-\Delta t}$ and diagonal with conditional variances $\mathcal{Q}_{ii}(X_{t-\Delta t,i};\psi) = X^2_{t-\Delta t,i}e^{2\mu\Delta t}(e^{\sigma^2\Delta t}-1)$; $\mu, \sigma \in \psi$. The marginal transition density $f_{X,i}(X_{t,i}|X_{t-\Delta t,i};\psi)$ is given by (3.5) and the marginal discrete-time density of disturbance $\eta_{t,i}$ is defined by $f_{\eta,i}(\eta_{t,i}|X_{t-\Delta t,i};\psi) = f_{X,i}(X_{t,i}|X_{t-\Delta t,i};\psi) - T_X(X_{t-\Delta t,i};\psi)$.

The measurement equation (4.10) defines the functional dependence of the time series $\mathbf{S}_{\overline{T}} = (S_t)_{t=1,...,\overline{T}}$ of observation vectors on the state vector series $\mathbf{X}_{\overline{T}}$. Observation vector $S_t = (S_{t,1}, .., S_{t,n_S})'$ of dimension $n_S = 4$ aggregates the time-t yield spreads $S_{t,j}$ of synthetic credit-risky coupon bonds with time-to-maturity $T_j \in \{1, 3, 5, 10\}$ and is decomposed into the n_S-vector $Z(X_t;\phi) = (Z_\tau(X_t;\phi_{t,1}), ..., Z_\tau(X_t;\phi_{t,j}), ..., Z_\tau(X_t;\phi_{t,n_S}))'$ of yield spread predictions calculated using the non-linear time-invariant measurement function $Z_\tau(X_t;\phi_{t,j})$,[24] and the n_S-vector $\varepsilon_t = (\varepsilon_{t,1}, ..., \varepsilon_{t,j}, ..., \varepsilon_{t,n_S})'$ of measurement disturbances $S_{t,j} - Z_\tau(X_t;\phi_{t,j})$, j=1,...,n_S. The vector $\phi_t = (\phi_{t,1}, ..., \phi_{t,j}, ..., \phi_{t,n_S})$ contains the parameter sets $\phi_{t,j} = \{K, T_j, c_{t,j}, \varrho, \overline{V}, r\} \bigcup \psi$ of bond yield spread observation j at time t, where $c_{t,j}$ denotes the coupon rate of a par-bond with time-to-maturity T_j. Measurement errors $\varepsilon_{t,j}$ are assumed to be i.i.d. with a time-invariant covariance matrix $\mathbf{\Omega}(\psi)$. The disturbances of measurement and state prediction are assumed to be mutually and serially independent, i.e. $\mathbb{E}[\eta_{t,i}\varepsilon_{s,j}] = 0$; t,s=1,...,$\overline{T}$; i=1,...,n_X; j=1,...,n_S.

In contrast to state-space models of exponential-affine term structure models, here, the transition and the measurement equations are non-linear, so that the distributional assumptions for ε_t and η_t will adversely affect the properties of the QML estimation.

4.4.3 Extended Kalman-Filter

The Kalman-Filter in its basic form does not perform an optimal filtering of state variables for non-linear state-space models in general. Instead, the Extended Kalman-Filter (EKF) provides an approximate optimal filtering recursion by linearizing the transition

[24] The non-linear time-invariant measurement functions $Z_\tau(X_t;\phi_{t,j})$ will be defined in Sections 4.4.5 and 4.4.6.

and measurement functions.[25] The non-linear measurement function $Z(X_t;\phi_t)$ and transition function $T(X_{t-\Delta t};\psi)$ are recursively approximated by a first-order Taylor expansion around the estimates $\widehat{X}_{t|t-\Delta t}$ and $\widehat{X}_{t-\Delta t}$ of the state vector to obtain the approximative state-space model (ASSM)

$$X_t \simeq T(\widehat{X}_{t-\Delta t};\psi) + \frac{\partial T(\widehat{X}_{t-\Delta t};\psi)}{\partial X'_{t-\Delta t}}(X_{t-\Delta t} - \widehat{X}_{t-\Delta t}) + \eta_t, \qquad \eta_t \sim \mathcal{D}(0, \mathcal{Q}(\widehat{X}_{t-\Delta t};\psi)) \tag{4.11}$$

$$S_t \simeq Z(\widehat{X}_{t|t-\Delta t};\phi_t) + \frac{\partial Z(\widehat{X}_{t|t-\Delta t};\phi_t)}{\partial X'_t}(X_t - \widehat{X}_{t|t-\Delta t}) + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \mathbf{\Omega}(\psi)). \tag{4.12}$$

With state variables being orthogonal, and observations being dependent on one state variable only, the first-order derivative of transition and measurement function is defined by

$$\frac{\partial T(\widehat{X}_{t-\Delta t};\psi)}{\partial X'_{t-\Delta t}} = \widehat{\nabla T}_t = \begin{pmatrix} \frac{\partial T_X(\widehat{X}_{t-\Delta t,1};\psi)}{\partial X_{t-\Delta t,1}} \\ \vdots \\ \frac{\partial T_X(\widehat{X}_{t-\Delta t,n_X};\psi)}{\partial X_{t-\Delta t,n_X}} \end{pmatrix}, \text{and} \tag{4.13}$$

$$\frac{\partial Z(\widehat{X}_{t|t-\Delta t};\phi_t)}{\partial X'_t} = \widehat{\nabla Z}_t = \begin{pmatrix} \frac{\partial Z_\tau(\widehat{X}_{t|t-\Delta t,1};\phi_{t,1})}{\partial X_{t,1}} \\ \vdots \\ \frac{\partial Z_\tau(\widehat{X}_{t|t-\Delta t,n_S};\phi_{t,n_S})}{\partial X_{t,n_S}} \end{pmatrix}, \tag{4.14}$$

where $\widehat{\nabla T}_t$ and $\widehat{\nabla Z}_t$ are introduced for ease of notation. The filtering recursion of the EKF consists of a prediction and an update step for time $t = 1, ..., \overline{T}$:

Prediction Step

The prediction step provides the estimate $\widehat{X}_{t|t-\Delta t}$ of the state vector and the $n_X \times n_X$-covariance matrix $\mathbf{P}_{t|t-\Delta t}$ of the state prediction error $X_t - \widehat{X}_t$ conditional on the filtered estimate $\widehat{X}_{t-\Delta t}$ of the state vector. The prediction

$$\widehat{X}_{t|t-\Delta t} = \mathbb{E}[X_t|\widehat{X}_{t-\Delta t}] = T(\widehat{X}_{t-\Delta t};\psi) \tag{4.15}$$

of the state vector is defined as conditional expectation of the state vector. The prediction of the covariance matrix $\mathbf{P}_t = \mathbb{E}[(X_t - \widehat{X}_t)(X_t - \widehat{X}_t)']$ of the state prediction error is given by

$$\mathbf{P}_{t|t-\Delta t} = \widehat{\nabla T}_t \mathbf{P}_{t-\Delta t} \widehat{\nabla T}'_t + \mathcal{Q}(\widehat{X}_{t-\Delta t};\psi), \tag{4.16}$$

[25] Cf. Harvey (1989), p. 160ff.

where the conditional variances $\mathcal{Q}_{ii}(\widehat{X}_{t-\Delta t,i};\psi)$ are defined analogously to $\mathcal{Q}_{ii}(X_{t-\Delta t,i};\psi)$ to approximate the covariance matrix $\mathcal{Q}(X_{t-\Delta t};\psi) \simeq \mathcal{Q}(\widehat{X}_{t-\Delta t};\psi)$ of state vector disturbances.

Update Step

The update step adapts $\widehat{X}_{t|t-\Delta t}$ and $\mathbf{P}_{t|t-\Delta t}$ to the extended information set S_t at time t and is considered either as a linear projection of $\widehat{X}_{t|t-\Delta t}$ or as conditional expectation of X_t given S_t. For the approximative non-linear state-space model, the update step is the equivalent of solving the non-linear generalized least-square problem

$$\widehat{X}_t = \underset{X}{\operatorname{argmin}}(X - \widehat{X}_{t|t-\Delta t})'\mathbf{P}^{-1}_{t|t-\Delta t}(X - \widehat{X}_{t|t-\Delta t}) + (S_t - Z(X;\phi_t))'\Omega^{-1}(\psi)(S_t - Z(X;\phi_t)), \tag{4.17}$$

which needs to be minimized at each observation time using the optimal candidate parameter set ψ. The estimator $\widehat{X}_{t|t-\Delta t}$ of X_t is updated when observation S_t becomes available. The prediction error

$$v_t = S_t - Z(\widehat{X}_{t|t-\Delta t};\phi_t) \tag{4.18}$$

with error covariance matrix

$$\mathbf{\Sigma}_t = \widehat{\nabla Z}_t \mathbf{P}_{t|t-\Delta t} \widehat{\nabla Z}'_t + \mathbf{\Omega}(\psi) \tag{4.19}$$

is defined with respect to the measurement equation being linearized around $\widehat{X}_{t|t-\Delta t}$. The Kalman matrix

$$\begin{aligned}\mathbf{K}_t &= \mathbf{P}_{t|t-\Delta t}\widehat{\nabla Z}'_t\mathbf{\Sigma}^{-1}_t \\ &= \mathbf{P}_{t|t-\Delta t}\widehat{\nabla Z}'_t(\widehat{\nabla Z}_t\mathbf{P}_{t|t-\Delta t}\widehat{\nabla Z}'_t + \mathbf{\Omega}(\psi))^{-1}\end{aligned} \tag{4.20}$$

enables to specify the Kalman gain $\mathbf{K}_t v_t$, which represents the information gain contributed by observation S_t to the estimation of X_t.[26] The state update equation is

$$\begin{aligned}\widehat{X}_t &= \widehat{X}_{t|t-\Delta t} + \mathbf{K}_t v_t \\ \widehat{X}_t &= \widehat{X}_{t|t-\Delta t} + \mathbf{P}_{t|t-\Delta t}\widehat{\nabla Z}'_t\mathbf{\Sigma}^{-1}_t(S_t - Z_t(\widehat{X}_{t|t-\Delta t};\phi_t)),\end{aligned} \tag{4.21}$$

with $\widehat{X}_t$ being the optimal estimator of X_t based on the observations up to and including

[26] The Markov property of the transition equation implies, that only the prediction error v_t of the current observation S_t contributes additional information at time t.

S_t. The update of the covariance matrix[27] is given by

$$\mathbf{P}_t = \mathbf{P}_{t|t-\Delta t} - \mathbf{K}_t \widehat{\nabla Z}_t' \mathbf{P}_{t|t-\Delta t} \tag{4.22}$$
$$\mathbf{P}_t = \mathbf{P}_{t|t-\Delta t} - \mathbf{P}_{t|t-\Delta t} \widehat{\nabla Z}_t' \mathbf{\Sigma}_t^{-1} \widehat{\nabla Z}_t \mathbf{P}_{t|t-\Delta t}$$

Approximate filtering techniques are differ primarily with respect to the implementation of the update step. For example, Lund (1997b) proposes an Iterative-Extended Kalman-Filter (IEKF), that employs a first-order linear Taylor approximation based on a Newton-Raphson iteration scheme for closed-form derivatives. Lund reports the IEKF to be more efficient than the EKF, though the unbiasedness and consistency of the QMLE is not proven and the IEKF is computationally more demanding. For the application of the approximative SSM in the next section, no closed-form derivatives of the transition and measurement function will be available, so that $\widehat{\nabla T}_t$ and $\widehat{\nabla Z}_t$ are approximated using differential quotients, which generates computation times of an iterative EKF that are even longer. Since, further on, the convergence of Lund's iteration scheme is not proven, the EKF approach used throughout this study is a restricted single-iteration version of the IEKF.

4.4.4 Quasi-Maximum Likelihood Estimator

A maximum likelihood estimator (MLE) is typically employed to estimate a parameter set $\widehat{\psi}^*$, that is optimal with respect to the minimization of the EKF-implied prediction error v_t of a state-space model. The substantial properties of MLE are determined by distributional considerations. According to Bollerslev and Wooldridge (1992), MLE are asymptotically normal if the first and second derivatives of the log-likelihood function are well-defined and the Fisher information matrix is non-zero. Asymptotic normality induces the MLE to be consistent as well as asymptotically unbiased and efficient. With the distribution of the EKF prediction error v_t of the ASSM being undetermined, the prerequisite required for the MLE to be consistent and efficient cannot be fulfilled. In consequence, a quasi-maximum likelihood estimation[28] is applied with the log-likelihood contributions

$$\mathcal{LL}_t(S_t;\psi) = -\frac{1}{2}\log(2\pi) - \frac{1}{2}\log|\mathbf{\Sigma}_t| - \frac{1}{2}v_t'\mathbf{\Sigma}_t^{-1}v_t \tag{4.23}$$

[27] Using the Riccati equation as described in Harvey (1989, p. 106), the recursion of the predicted covariance matrix $\mathbf{P}_{t+\Delta t|t}$ can be calculated directly from $\mathbf{P}_{t|t-\Delta t}$, so that the update of $\mathbf{P}_t$ can be skipped.

[28] Cf. White (1982), Gallant and White (1988) and White (1994) for an in-depth exposition of the quasi-maximum likelihood principle.

provided recursively by the EKF, based on parameter set ψ, observation S_t, filter-implied state vector $\widehat{X}_t$ and prediction error covariance $\mathbf{\Sigma}_t$.[29] Aggregating the log-likelihood contributions of (4.23) yields the quasi-log-likelihood function

$$\mathcal{QLL}_{\widehat{T}}(\mathbf{S}_{\overline{T}};\psi) = \frac{1}{n}\sum_{t=1}^{\overline{T}} \mathcal{LL}_t(S_t;\psi), \tag{4.24}$$

denoted as prediction-error decomposition. The QML estimate $\widehat{\psi}^*_{\overline{T}}$ of the optimal parameter set is obtained by maximizing (4.24).

The consistency of the QMLE in (4.24) involves $\widehat{\psi}^*_{\overline{T}}$ converge towards the unknown true parameter set ψ^* and requires the prediction error v_t to have a conditional normal distribution with a specified expected value and covariance matrix Σ_t. The use of coupon-bond yields as observations precludes to derive a closed-form formula of the prediction error and the asymptotic distribution of the QMLE is not known, so that the requirements specified by Bollerslev and Wooldridge for the QMLE to be consistent and asymptotically normal are not fulfilled.[30] In the absence of formal proof, it remains undetermined, whether the QMLE is consistent.[31]

In conclusion, implied by the absence of an exact Kalman-Filter for non-linear non-Gaussian state-space models, QMLE are inconsistent due to the estimation error being calculated by approximative filtering techniques.[32] Comparable QMLE for EKF-based non-linear non-Gaussian state-space models provided by Claessens and Pennacchi (1996), Cumby and Evans (1997) and Lund (1997b) have the same problem, but the finite sample bias is found to be negligible, so that the QMLE in (4.24) are considered to be appropriate to determine an optimal parameter set $\widehat{\psi}^* = \widehat{\psi}^*_{\overline{T}}$.

[29]The information filter presents an alternative to the Kalman-Filter that avoids calculating the inversion of the prediction error covariance matrix $\mathbf{\Sigma}_t$ for the log-likelihood using the information matrix $\mathbf{P}_t^{-1}$.

[30]Lund (1997a, p. 11ff) shows that the KF-based QMLE of a non-Gaussian state-space model are consistent with respect to the estimation of parameter set ψ if the measurement equation is linear.

[31]In a similar application, Lund (1997b, p. 13), principally challenges the notion that a non-linear, non-Gaussian state-space model can be consistently estimated if an EKF approximation is involved. The approximation error introduced into the log-likelihood by the approximate filtering converges to zero in the number of observations. However, Lund explains that this property does not ensure the overall consistency of the QMLE.

[32]Lund (1997b) conjectures that a consistent and computationally tractable estimation method for non-linear non-Gaussian state-space models categorically does not exist.

4.4.5 Estimation of the Systematic Factor Process

4.4.5.1 Observation Data

The approximative state-space model of (4.11-4.12) is used to relate market-derived observations of factor risk to a systematic factor process using the credit-valuation model of Section 3.2. Pre-processing is required to derive usable observation data from the term structures of risk classes, for two reasons. First, the credit valuation model considers defaultable claims with fixed periodic interest, whereas empirically fitted term structures of risk classes represent zero rates. Second, market-observed bond prices and derived term structures refer to a non-flat and time-variant term structure of riskless interest rates, whereas the credit pricing model assumes a flat and time-invariant term structure of a continuous riskless rate.[33]

The factor filtering of the EKF is conditioned on the riskless rate of the credit valuation model. Using original credit yield observations of the risk class structures, either the systematic factor or the constant riskless rate of the model have to be recalibrated with respect to the term structure of the risk class at each time interval. However, in both cases, the distribution function of the first-passage time changes under the risk-neutral measure and, accordingly, credit valuations and prediction errors change, which distorts the EKF. In consequence, the risk class structures are transformed from a setting of dynamic riskless rates so that observations represent: (a) yield spreads of synthetic coupon bonds with respect to (b) a flat term structure of the riskless rate, that is constant throughout the estimation period.

The data setup refers to each risk class analogously, although a risk class index is omitted for ease of notation. In each risk class, the term structure of zero spreads at time t is defined by

$$ZCS_t(T;\beta_t^c,\beta_t^{rl}) = R_t^c(T;\beta_t^c) - R_t^{rl}(T;\beta_t^{rl}), \tag{4.25}$$

given term structure $R_t^c(T;\beta_t^c)$ of credit-risky zero rates and the term structure $R_t^{rl}(T;\beta_t^{rl})$ of the riskless zero rates, as specified by the Nelson-Siegel parametric form in 4.5. The 10-year riskless zero rate $R_1^{rl}(10;\beta_1^{rl})$ at the beginning of an estimation period is taken to specify the term structures of synthetic credit-risky zero rates

$$ZCR_t^c(T;\beta_t^c,\beta_t^{rl}) = ZCR_t^{rl} + ZCS_t(T;\beta_t^c,\beta_t^{rl}) \tag{4.26}$$

---

[33] A flat term structure requires the parameter functions $A(r_t,T)=0$ and $B(r_t,T)=1$ of an exponential-affine term structure model to be independent of maturity T, which is not possible in common short rate models, e.g. by Vasicek (1977) or by Cox et al. (1985). In effect, a flat term structure can only be calibrated by a stochastic short rate model with time-dependent parameters in the SDE.

that refer to a flat and time-invariant riskless zero rate $ZCR_t^{rl} = R_1^{rl}(10;\beta_1^{rl}), t = 1,...,\overline{T}$ and incorporate the empirical zero spreads of a risk class.

Since the credit valuation model used does not enable the valuation of zero-coupon bonds, the synthetic credit-risky zero curves are transferred to obtain yield spreads of $n_S = 4$ synthetic defaultable par-bonds $j = 1,...,n_S$ with time-to-maturity $T_j \in \{1,3,5,10\}$ assigned in ascending order. With par-value $D_{t,j} = 100$ and par-coupon

$$c_{t,j} = \frac{(1 - e^{-ZCR_t^c(T_j;\beta_t^c,\beta_t^{rl})T_j})}{\sum_{t_j=\rceil t\lceil -t}^{T_j} e^{-ZCR_t^c(tj;\beta_t^c,\beta_t^{rl})t_j}}, \tag{4.27}$$

the implicit yield pricing equation

$$ytm(Y_{t,j}^{pc}, D_{t,j};\phi_j) \equiv c_{t,j}\sum_{t_j=\rceil t\lceil -t}^{T_j} e^{-ytm_{t,j}^c t_j} + e^{-Y_{t,j}^{pc}T_j} - D_{t,j} = 0 \tag{4.28}$$

is solved for the continuously-compounded yield-to-maturity $Y_{t,j}^{pc}$ of the synthetic defaultable par-bond with time-to-maturity τ_j. A riskless reference yield to determine par-bond yield spreads is obtained from the value

$$B_t^j = 100(c_t^j \sum_{t_j=\rceil t\lceil -t}^{\tau_j} e^{-R_t^{rl}(t_j;\beta_t^{rl})t_j} + e^{-R_t^{rl}(t_j;\beta_t^{rl})T_j}) \tag{4.29}$$

of the corresponding riskless par-bond with par-coupon-rate c_t^j and time-to-maturity T_j. Solving the yield pricing equation $ytm(Y_{t,j}^{rl}, B_t^j;\phi_{t,j})$ for the riskless yield $ytm_{t,j}^{rl}$ gives the yield spread $S_{t,j} = Y_{t,j}^{pc} - Y_{t,j}^{rl}$ of a synthetic defaultable par-bond as observation data of the ASSM. The time series

$$\mathbf{Y}_{\overline{T}}^{pc} = (Y_t^{pc})_{t=1,...,\overline{T}}, \tag{4.30}$$

$$\mathbf{Y}_{\overline{T}}^{rl} = (Y_t^{rl})_{t=1,...,\overline{T}}, \tag{4.31}$$

$$\mathbf{S}_{\overline{T}} = (S_t)_{t=1,...,\overline{T}} \tag{4.32}$$

of vectors

$$Y_t^{pc} = (Y_{t,1}^{pc},...,Y_{t,n_S}^{pc}))', \tag{4.33}$$

$$Y_t^{rl} = (Y_{t,1}^{rl},...,Y_{t,n_S}^{rl}))', \tag{4.34}$$

$$S_t = (S_{t,1},...,S_{t,n_S}))' \tag{4.35}$$

collect the synthetic yields and yield spreads of a risk class for $t = 1,...,\overline{T}$.

4.4.5.2 Implementation of the Extended Kalman-Filter

The EKF described in Section 4.4.3 is used for a QML estimation of the process parameters of a systematic factor. Individual factor processes are estimated for the risk classes defined by rating and rating-sector affiliation for one-year estimation periods from 1999 to 2003, as well as for the entire sample period. With only one single factor F_t considered as a state variable, the dimension of the transition function $T(F_t;\psi_F)$ and disturbance $\varepsilon_t \sim \mathcal{D}(0, \mathcal{Q}_t(F_{t-\Delta t};\psi_F))$ with parameter set $\phi_F = \{\mu_F, \sigma_F\}$ reduces to $n_F = 1$.

Initialization

The observations at time $t = 1$ are used to specify an initial parameter set ψ_0 and an initial factor value $\widehat{X}_1$ with variance P_1, that is, filtering does not start before $t = 2$. Kalman-Filter applications of mean-reverting short-rate models typically set initial state variables to unconditional expected values, which represent long-term expectations induced by the initial parameter set. Given a factor process according to (3.3), the unconditional expectation of the factor tends either to infinity or to the default threshold, depending on the drift term and presuming that the variation of the factor is not bounded. In consequence, no constant long-term unconditional expectation exists to serve as an initial factor value. Given the initial parameter set ψ_0, the initial factor value

$$\widehat{F}_1 = \underset{F_1}{\operatorname{argmin}} \sum_{j=1}^{n_S} \Big(Y_{1,j} - ytm(\widehat{Y}^f_{1,j}, D(F_1, 1, \phi_{1,j}); \phi_{1,j}) \Big)^2 \tag{4.36}$$

is set to minimize the sum of squared measurement errors between the par-bond yields $Y^{pc}_{t,i}$ and factor-dependent yield $\widehat{Y}^f_{1,j}$. As ψ_F is optimized, the initial factor value will not be altered to ensure the continuity of the log-likelihood function. The variance of the factor disturbance and the variance of the state prediction error are initialized by $\mathcal{Q}_1(\widehat{F}_1;\psi_F) = \widehat{P}_1 = \widehat{F}_1^2 e^{2\mu_F \Delta t}(e^{\sigma_F^2 \Delta t} - 1)$. Given parameter set ψ_F, the covariance matrix $\Omega(\psi_F)$ of measurement disturbances is calculated from the covariance matrix of filter-implied prediction errors that result from an initial EKF run with $\omega_{j,j} = 0.0001$, j=1,...,n_S.

Drift μ and variation σ of the factor are initialized by searching within a two-dimensional grid for the maximum log-likelihood provided by the EKF according to (4.24) and with the initial factor set by (4.36).

Prediction Step

The factor prediction $\widehat{F}_{t|t-\Delta t}$ is given by the expectation of the factor conditional on the filtered factor estimate $F_{t-\Delta t}$ and transition density $f_V(F, t; \widehat{F}_{t-\Delta t}, \psi_F)$ at time $t - \Delta t$:

$$\widehat{F}_{t|t-\Delta t} = T(\widehat{F}_{t-\Delta t};\psi_F) = \mathbb{E}[F_t|\widehat{F}_{t-\Delta t};\psi_F] = \widehat{F}_{t-\Delta t} e^{\mu_F \Delta t} \tag{4.37}$$

The yield pricing equation $ytm(\widehat{Y}^f_{t,j}, \widehat{D}_{t|t-\Delta t,j};\phi_{t,j})$ is solved for the predicted yield-to-maturity $\widehat{Y}^f_{t,j}$ of observation $j = 1, ..., 4$ with coupon rate $c_{t,j}$ and time-to-maturity T_j and predicted bond value $\widehat{D}_{t|t-\Delta t,j}$=$D(\widehat{F}_{t|t-\Delta t}, t; \phi_{t,j})$, given the predicted factor $\widehat{F}_{t|t-\Delta t}$.[34] From vector $\widehat{Y}^f_{t|t-\Delta t}$=$(\widehat{Y}^f_{t|t-\Delta t,1}, ..., \widehat{Y}^f_{t|t-\Delta t,4})$ of predicted factor yields the vector $\widehat{S}^f_{t|t-\Delta t}$=$(\widehat{S}^f_{t|t-\Delta t,1}, ..., \widehat{S}^f_{t|t-\Delta t,4}))'$ of predicted factor yield spreads

$$\widehat{\mathbf{S}}^f_{t|t-\Delta t} = Z(\widehat{F}_t; \phi_t) = \widehat{Y}^f_{t|t-\Delta t} - Y^{rl}_t \tag{4.38}$$

is derived and prediction error $v_t = \widehat{S}^f_{t|t-\Delta t} - S^f_t$ is obtained. First-order derivatives $\widehat{\nabla T}_t$ and $\widehat{\nabla Z}_t$ are approximated using a one-sided first-order differential quotient of $T(\widehat{F}_{t-\Delta t}; \psi_F)$ and $Z(\widehat{F}_{t-\Delta t}; \phi_t)$, respectively. The variance $\mathcal{Q}(F^f_{t-\Delta t}; \psi_F) \simeq \mathcal{Q}(\widehat{F}^f_{t-\Delta t}; \psi_F) = \widehat{F}^2_t e^{2\mu_F \Delta t}(e^{\sigma^2_F \Delta t} - 1)$ of factor disturbance η^f_t conditional on the unobserved factor $F_{t-\Delta t}$ is assumed to equal the second central moment of the factor conditioned on the filtered factor estimate $\widehat{F}_{t-\Delta t}$ at time $t - \Delta t$.

Update Step

The update proceeding as described in Section 4.4.3 gives the filtered factor $\widehat{F}_t$ with the filtered yield $ytm(\widehat{Y}^f_{t,j}, \widehat{D}^f_{t,j}; \phi_{t,j})$, j=1,...,4 calculated from the bond price $\widehat{D}^f_{t,j} = D(\widehat{F}_t, t; \phi_{t,j})$.

The QML optimization of the parameter set ψ_F applies an iterative search routine on a discretely spaced grid of parameter sets for the log-likelihood of the EKF until a convergence criterion is fulfilled. With respect to standard errors of the parameter estimates, the asymptotic covariance matrix of the maximum likelihood estimator must be approximated, since the Hessian matrix of the log-likelihood with respect to changes in the parameter set ψ_F is not available in closed-form. Following Greene (2003, p. 480f), asymptotic standard errors are calculated from the outer product of gradients known as the BHHH-estimator.[35]

4.4.6 Estimation of the Asset Value Process

The dynamics of obligor-specific asset values and their dependence within and across risk classes have yet to be specified. Yield spreads that incorporate a systematic and an obligor-specific component of credit risk are bootstrapped to represent exposure-specific observation data. An EKF-based QML estimation is performed to specify process parameters and filtered times series of asset values, as well as the dependence of the asset values on the systematic factor for single risk classes.

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[34] Neither $\widehat{F}_{t|t-\Delta t,j}$ nor $\widehat{Y}^f_{t,j}$ represent the expectation but instead the prediction of credit value and yield-to-maturity respectively, conditional on the filter factor $\widehat{F}_{t-\Delta t}$ with transition density $f_V(F; \widehat{F}_{t-\Delta t}; \phi_F)$.

[35] Cf. Berndt et al. (1974).

4.4.6.1 Bootstrapping Specific Yield Spreads

The decomposition of asset values into systematic and specific factor components requires considering that part of a bond yield that is residual to the yield implied by the term structures of the risk class in question. In principle, residual spreads of bonds with observed market prices could be used. However, to resolve discontinuities in the data series and to avoid including effects induced by the inhomogeneity of the bond sample, the time series of obligor-specific yield spreads for the par-coupon bonds of the factor-process estimation are bootstrapped on the basis of empirical spread dynamics.[36] The first differences of residual yield spreads at time t are assumed to be i.i.d. across all obligors within a class and independent of the maturity of the bonds. To ensure that spreads are strictly positive, a log-normal distribution is used to bootstrap yield spreads on a weekly basis for all risk classes. As before, a risk class indicator is omitted below to simplify the notations.

For any bond $i_t = 1, ..., n_t$ of a risk class with market-implied yield Y_{t,i_t} at time t and $t - \Delta t$, the residual yield spread $S^{rs}_{t,i_t} = Y_{t,i_t} - Y^c_{t,i_t}$ with respect to the yield Y^c_{t,i_t} implied by the term structure of the class is used to calculate the first difference of the residual spread $\Delta S^{rs}_{t,i_t} = S^{rs}_{t,i_t} - S^{rs}_{t-\Delta t,i_t}$ with mean $\mu^{\Delta S}_t = 1/n_t \sum_{i_t=1}^{n_t} \Delta S^{rs}_{t,i_t}$ and standard deviation $\sigma^{\Delta S}_t = 1/n_t \sum_{i_t=1}^{n_t} (\Delta S^{rs}_{t,i_t} - \mu^{\Delta S}_t)^2$.

Specific yield spreads $S_{t,j}$ for class-implied par-coupon bonds $j = 1, ..., 4$ with time-to-maturity $T_j \in \{1, 3, 5, 10\}$ are simulated successively across the estimation period, starting with the class-implied yield spread $S_{1,j} = Y^c_{1,j} - Y^{rl}_{1,j}$, so that the residual spreads $S_{t,j} = S_{t,j} + Y^{rl}_{t,j} - Y^c_{t,j}$ are initially set at $S^{rs}_{0,j} = 0$. Given, the residual spread $S^{rs}_{t-\Delta t,j}$, the specific yield spread $S^{rs}_{t,j}$ of the par-coupon bond at time t is simulated using a log-normal distribution with expected value $\mu_{t,j} = Y^c_{t,j} - Y^{rl}_{t,j} + S^{rs}_{t-\Delta t,j} + \mu^{\Delta S}_t$ and homogenous standard deviation $\sigma_{t,j} = \sigma^{\Delta S}_t$ for each maturity T_j at time t. The log-normal distribution of yield spreads is calibrated by solving the non-linear equation system

$$\mu_{t,j} = e^{\gamma_{t,j} + \frac{1}{2}\delta^2_{t,j}} \tag{4.39}$$

$$\sigma_{t,j} = e^{2\gamma_{t,j} + \delta^2_{t,j}} (e^{\delta^2_{t,j}} - 1) \tag{4.40}$$

with respect to parameters $\gamma_{t,j}$ and $\delta_{t,j}$. Using random variables $R_{t,j} \sim \mathcal{U}(0, 1)$, yield spreads are simulated by solving the log-normal distribution function $F^{LN}(S_{t,j}; \gamma_{t,j}, \delta_{t,j}) = R_{t,j}$ for $S_{t,j}$.

[36] In contrast, CDS data refer to homogenous contracts, constant tenors and time-continuous quotations typically prevail, so that CDS prices qualify even better for the estimation of risk class processes, if series of sufficient length are available.

4.4.6.2 Implementation of the Extended Kalman-Filter

The ASSM is adapted for an EKF estimation of process and dependence parameters of a vector $V_t = (V_{t,1}, ..., V_{t,n_S})$ of obligor-specific asset values in a common risk class across estimation period $t = 1, ..., \overline{T}$. According to (3.31), the SDE of asset values $V_{t,j}$ are assumed to be homogenous across all obligors in a class. Within a risk class, asset values are dependent and decomposed into a common systematic factor and specific factors as presented in the risk class factor model discussed in Section 3.5, with SDE and the filtered time series of the systematic factor resulting from the systematic factor estimation of the previous section. The parameter set $\psi_V = \{\mu_V, \sigma_V, \rho_V\}$ to be estimated includes drift rate μ_V, diffusion parameter σ_V and inner-class asset correlation ρ_V, represented by the systematic factor coefficient $\beta_V = \sqrt{\rho_V}$. Filtered time series of obligor-specific asset values and specific factors present a side product of the EKF-based QML estimation.

State variables $\epsilon_{t,j}$ represent the specific-factor component of asset value $V_{t,j}$. The state vector $\epsilon_t = (\epsilon_{t,1}, ..., \epsilon_{t,4})$ implements multi-variate standard Gaussian noise in the form of transition equation $\epsilon_t = \epsilon_{t-\Delta t} + \eta_t$ with transition function $T(\epsilon_{t-1}; \psi_V) = \epsilon_{t-\Delta t}$ and state disturbance vector $\eta_t \sim \mathcal{N}(0, \mathcal{I}), t = 2, ..., \overline{T}$, 4×4 identity matrix $\mathcal{I}$ and disturbances $\epsilon_{t,j}$ assumed to be serially and cross-serially i.i.d.

Observations are given by bootstrapped yield spreads $S_{t,j}$ of bond $j = 1, ..., n_S$ with class-implied par-coupon $C_{t,j}$ and time-to-maturity $T_j \in (1, 3, 5, 10)$. Apart from the systematic factor and parameter set ψ_V, the evolution of spread $S_{t,j}$ depends on the single state variable $\epsilon_{t,j}$ only. Since observations refer to defaultable bonds, default thresholds need not be taken into account in the transition density of specific factors or asset values.

The measurement function $Z_V(\epsilon_t; F_t, \phi_t, \psi_V)$ implements the functional relation between state variable $\epsilon_{t,j}$ and spread observation $s_{t,j}$ conditional on systematic factor F_t. On the basis of the normalized discrete-time return

$$\Delta\widehat{F}_t^{\epsilon} = \frac{\ln(\widehat{F}_t/\widehat{F}_{t-\Delta t}) - (\mu_F - \frac{1}{2}\sigma_F^2)\Delta t}{\sigma_F\sqrt{\Delta t}} \tag{4.41}$$

of the filtered systematic factor, the normalized discrete-time asset return

$$\Delta V_{t,j}^{\epsilon} = \beta_V \Delta\widehat{F}_t^{\epsilon} + \sqrt{1 - \beta_V^2}\eta_{t,j} \tag{4.42}$$

is derived from disturbance $\eta_{t,j} = \Delta\epsilon_t = \epsilon_t - \epsilon_{t-\Delta t}$, and asset value $V_{t,j}$ is calculated iteratively using equation

$$V_{t,j} = V_{t-\Delta t,j} \cdot \exp^{(\mu_V - \frac{1}{2}\sigma_V^2)\Delta t + \Delta V_{t,j}^{\epsilon}\sigma_V\sqrt{\Delta t}} \tag{4.43}$$

for $j = 1, ..., n_V$ and $t = 1, ..., \overline{T}$. Analogous to the specific factor $\epsilon_{t,j}$, the time series of the normalized asset value $V^{\epsilon}_{t,j} = V^{\epsilon}_{t-\Delta t,j} + \Delta V^{\epsilon}_{t,j}$ and the series of the normalized systematic factor $F^{\epsilon}_{t} = F^{\epsilon}_{t-\Delta t} + \Delta F^{\epsilon}_{t}$ are defined iteratively for $j = 1, ..., n_S$.

Initialization

Asset values are initialized at $t = 1$ by present value $D^s_{1,j} = c_{1,j} 100 \sum_{t_i=1}^{T_j} e^{-Y^s_{1,j} t_i} + 100 e^{-y^s_{1,j} T_j}$ of the defaultable par-coupon bond, given yield observation $Y^s_{1,j}$ and by solving the credit valuation equation $D(V_{1,j}, 1; \phi_{1,j}, \psi_V) = D^s_{1,j}$ for the implicit asset value $V_{1,j}$. The normalized asset and factor values are initially set at $V^{\epsilon}_{1,j} = 0$, $\widehat{F}^{\epsilon}_1 = 1$ and $\epsilon_{1,j} = 0$, $j = 1, ..., n_S$.

The vector of measurement disturbances $\varepsilon \sim \mathcal{D}(0, \Omega)$ is assumed to be multivariate-normal with time-homogenous zero expectation. The time-invariant covariance matrix Ω is set equal to the covariance matrix of measurement errors of an initial EKF run with start parameter set ψ_V and is held constant throughout the optimization. Accordingly, the $n_S \times n_S$ covariance matrix of state prediction errors $\mathbf{P}_1$ at time $t = 1$ is set equal to the EKF-implied covariances of the state variable disturbances.

Prediction Step

The EKF start with a state prediction at time $t = \Delta t$. The conditional expectation of state vector ϵ_t is a Martingale, so that the prediction of state vector

$$\widehat{\epsilon}_{t|t-\Delta t} = \mathbb{E}[\epsilon_t | \widehat{\epsilon}_{t-\Delta t}] = \widehat{\epsilon}_{t-\Delta t} \tag{4.44}$$

at time $t = \Delta t, ..., \overline{T}$ is given by the filtered state vector $\widehat{\epsilon}_{t-\Delta t}$. Using the filtered systematic factor $\widehat{F}_t$, the normalized factor returns $\Delta \widehat{F}^{\epsilon}_t$ are calculated according to 4.41 to obtain the normalized filtered factor $\widehat{F}^{\epsilon}_t = \widehat{F}^{\epsilon}_{t-\Delta t} + \Delta \widehat{F}^{\epsilon}_t$ and the predicted normalized asset value

$$\widehat{V}^{\epsilon}_{t|t-\Delta t,j} = \frac{\beta \cdot \widehat{F}^{\epsilon}_t + \sqrt{1-\beta^2}\,\widehat{\epsilon}_{t|t-\Delta t,j}}{\sqrt{t}} \tag{4.45}$$

for bond $j = 1, ..., n_S$. Reversing the normalization yields the predicted asset values

$$\widehat{V}_{t|t-\Delta t,j} = \widehat{V}_{t-\Delta t,j} \cdot \exp^{(\mu_V - \frac{1}{2}\sigma^2_V)\Delta t + (\widehat{V}^{\epsilon}_{t|t-\Delta t,j} - \widehat{V}^{\epsilon}_{t-\Delta t,j})\sigma_V \sqrt{\Delta t}} \tag{4.46}$$

and predicted bond price $\widehat{D}^s_{t|t-\Delta t,j} = D(\widehat{V}_{t|t-\Delta t,j}, t; \phi_{t,j}, \psi_V)$ of par-coupon bond $j = 1, ..., n_S$. Solving the implicit yield pricing equation

$$ytm(\widehat{Y}^s_{t|t-\Delta t,j}; B^j_t, \phi_{t,j}) \equiv 100(c_{t,j} \sum_{t_j=1}^{T_j} e^{-\widehat{y}^s_{t|t-\Delta t,j} t_j} + e^{-y^s_{t,j} T_j}) - \widehat{D}^s_{t|t-\Delta t,j} = 0 \tag{4.47}$$

for predicted bond yield $\widehat{Y}^c_{t|t-\Delta t,j}$ and subtracting the yield $Y^{rl}_{t,j}$ of the equivalent riskless bond, the prediction of spread vector $\widehat{S}_{t|t-\Delta t}$ of yield spreads $\widehat{S}_{t|t-\Delta t,j} = \widehat{Y}^s_{t|t-\Delta t,j} - Y^{rl}_{t,j}$ and the vector of prediction errors $v_t = S_t - \widehat{S}_{t|t-\Delta t}$ is obtained.

Gradient vectors $\widehat{\nabla T}_t$ and $\widehat{\nabla Z}_t$ are approximated using elementwise one-sided first order differential quotients. The predicted covariance matrix of the state prediction error

$$\mathbf{P}_{t|t-\Delta t} = q_h \widehat{\nabla T}_t \mathbf{P}_{t-\Delta t} \widehat{\nabla T}_t' + (1 - q_h)\mathcal{I} \tag{4.48}$$

is derived from the updated covariance matrix $\mathbf{P}_{t-\Delta t}$ and the identity matrix $\mathcal{I}$ of the state disturbance covariances with a scaling factor q_h according to Harvey (1989, p. 107), set at $q_h = 0.5$ after experimental considerations.

Update Step

The EKF filtering update of state vector $\widehat{\epsilon}_t$ and covariance matrix $\mathbf{P}_t$ is carried out as presented in Section 4.4.3. For an assessment of the Kalman-Filter, the updated spread vector $\widehat{S}_t^s$ is derived from the updated vector of asset values $\widehat{V}_t^\epsilon$ and $\widehat{V}_t$ and, bond prices $\widehat{D}_t^s$ and yield-to-maturities $\widehat{Y}_t^s$, calculated analogously to the prediction step based on the filtered state vector.

A numerical gradient approach is applied to the QML optimization of EKF, as conducted for the factor process estimation. Concerning the impact of the initialization of ASSM components on the estimation result, it turns out that covariance matrix Ω has considerable impact on the estimation result and the pre-qualification of Ω, with EKF-implied covariances of measurement disturbances, significantly improves the optimal log-likelihood. With respect to the free parameters, the drift parameter μ_V has the least influence on the likelihood, while the optimization is more sensitive to a variation of σ_V and the dependence parameter β_V. Asymptotic standard errors for parameter set ψ_V are derived using the BHHH-estimator as in the factor estimation.

4.4.7 Calibration of the Risk-Class Factor Model

From the series of filtered systematic risk class factors and the factor coefficients determined in the estimation of asset value process estimation, inner-class and inter-class asset correlations and orthogonal factor coefficients are derived for three representations of the general risk class factor portfolio models in Section 3.5:

- a rating class model with three risk classes $RC^{rating} \in$ {AA, A, BBB}
- a two-sector model with five risk classes $RC^{2-sector} \in$ {FIN-AA, FIN-A, NF-AA, NF-A, NF-BBB},
- a four-sector model with ten risk classes $RC^{4-sector} \in$ {ECY-A, ECY-BBB, FIN-AA, FIN-A, NCY-AA, NCY-A, NCY-BBB, LCY-AA, LCY-A, LCY-BBB}

Compared to the rating class model, the two-sector model introduces an additional differentiation between obligors of the financial and the non-financial sector. The four-sector model adds additional details to the risk classes by splitting the non-financial classes into more granular sector-rating classes of the early-cyclic, non-cyclic and late-cyclic sectors. The risk classes ECY-AA and FIN-BBB could not be considered due to data limitations.

For the specification of the risk-class factor model $m \in$ RC={rating, two-sector, four-sector} with risk class count $n_m \in \{n_{rating}, n_{2-sector}, n_{4-sector}\} = \{3, 5, 10\}$, the $n_m \times n_m$-correlation matrix $\rho^{f,m} = (\rho^{f,m}_{rc_k,rc_l})_{k,l=1,...,n_m}$ of the time series $\mathbf{\Delta\widehat{F}}^{\epsilon,rc}_{\overline{T}} = (\Delta\widehat{F}^{\epsilon,rc}_t)_{t=\Delta t,...,\overline{T}}$ of log-returns of the normalized filtered factor series $\mathbf{\widehat{F}}^{\epsilon,rc}_{\overline{T}} = (\widehat{F}^{\epsilon,rc}_t)_{t=0,...,\overline{T}}$ is defined by correlation $\rho^{f,m}_{k,l} = Corr(\mathbf{\Delta\widehat{F}}^{\epsilon,rc_k}_{\overline{T}}, \mathbf{\Delta\widehat{F}}^{\epsilon,rc_l}_{\overline{T}}), rc_k,\ rc_l \in RC^m,\ k, l = 1, ..., n_m$.

Inner-class asset correlations $\rho^{a,m}_{rc,rc} = \beta^{rc2}_V$, $rc = 1, ..., n_m$ are determined using systematic factor coefficients β^{rc}_V from the asset process estimation. For inter-class asset correlations, the factor correlation matrix $\rho^{f,m}$ is required as well. The matrix of risk class asset correlations $\rho^{a,m} = (\rho^{a,m}_{rc_k,rc_l})_{k,l=1,...,n_m}$ is defined by

$$\rho^{a,m}_{rc_k,rc_l} = \beta^{rc_k}_V \beta^{rc_l}_V \rho^{f,m}_{rc_k,rc_l},\ \ m \in RC, rc_k, rc_l \in RC^m, k, l = 1, ..., n_m. \tag{4.49}$$

The Cholesky decomposition $\rho^{a,m} = B^m(B^m)'$ results in a coefficient matrix $\mathbf{B}^m$ for a system of abstract orthogonal factors that reproduce the inner-class and inter-class asset correlations $\rho^{f,m}_{rc_k,rc_l}$ for exposures of risk class $rc_k, rc_l \in RC_m$ if used to define the coefficient matrix of a credit portfolio in 3.35. With B^m_k indicating the $k^{th}-$row vector of $\mathbf{B}^m$, the specific factor coefficients of exposures in risk class $k \in RC^m$ is defined by $\sqrt{1 - \mathbf{B}^m_k\mathbf{B}^{m'}_k}$.

4.4.8 Estimation Results

4.4.8.1 Parameter Estimates

Estimates of parameter sets ψ_F and ψ_V for a five-year estimation period from 1999 to 2003 are presented in Table 4.9 for the 16 risk classes involved. The optimal log-likelihood of factor process and asset value process estimation according to 4.23 is denoted by $\mathcal{LL}_F$ and $\mathcal{LL}_V$, respectively. The BHHH-estimator of asymptotic standard errors is given in parentheses. Parameter estimates and standard errors for annual estimation periods are given in Table D.1 of Appendix D.

Risk classes with few observation data and considerable spread peaks,[37] such as the BBB classes and the LCY-A class, achieve a comparatively small maximum log-likelihood in the factor and asset value estimation. Estimated drift rates of systematic factors (asset values) are close to zero, with a minimum of -1.18% (-2.67%) for the risk class NCY-

[37] Cf. Section 4.3.2.1

Risk-Class	$\mathcal{LL}_F$	μ_F	σ_F	$\mathcal{LL}_V$	μ_V	σ_V	ρ_V	β_V
AA	11,380.3	-0.0007	0.1274	13,086.1	-0.0028	0.0535	0.0849	0.2913
		(0.1407)	(0.0156)		(0.0046)	(0.0018)	(0.0366)	
A	10,661.5	-0.0015	0.1435	12,743.0	0.0062	0.0561	0.0830	0.2880
		(0.1505)	(0.0112)		(0.0043)	(0.0012)	(0.0182)	
BBB	9,067.7	-0.0071	0.1139	11,782.0	-0.0081	0.0952	0.1142	0.3379
		(0.1088)	(0.0093)		(0.0065)	(0.0018)	(0.0162)	

Table 4.9: Parameter Estimates

AA (NCY-A) and a maximum of 1.02% (0.75%) for the risk class NCY-A (NF-BBB). The fact that drift rates are close to zero throughout the five-year period is explained by the cyclicity, i.e. an empirical mean-reversion of credit spreads throughout the economic cycle. Although the length of the economic cycle cannot be determined unambiguously a drift rate of zero can be expected across the five-year estimation period.

The drift rates for annual estimations in Table D.1 show a considerable variation across risk classes for each year, as well as within a single risk class across annual periods, thereby reflecting the cyclical patterns of credit spreads from 1999 to 2003. In conclusion, a model specification based on one-year estimation periods is considered to be suited only to conditional Credit-VaR forecasts based on the current state of the economy, including a conditional variability of asset values and conditional PD assumed for exposures.

In every risk class except for NCY-A, the factor volatility σ_F exceeds the asset volatility σ_V. This effect is attributed to the fact that the asset volatility in a risk class is a weighted mean of factor volatility and residual volatility, with the latter being comparatively small and empirically mean-reverting when compared to systematic spread variation. Estimated factor volatilities range from 10.09% to 16.05%, while asset volatilities span the range from 3.45% to 24.81%.

The size of the bond sample affects the factor volatility σ_F, since classes with few bond observations exhibit a higher sensitivity of fitted credit spreads to jumps of obligor-specific spreads. The bootstrapping of the specific spread residuals reflects changes in empirical spread volatilities, so that the maximum estimate of σ_V is seen in the data-scarce NCY-A class, where considerable jumps in credit spreads are observed in 2002 and 2003. Volatility estimates σ_F and σ_V are smaller than common historical equity volatilities, which is attributable to a smoothing effect of the capital structure on the variation of the asset values. Equity proxies must therefore be corrected for capital structure effects when asset volatilities are being estimated.

Inner-class asset correlations reach a minimum of 0.74% for risk class NCY-AA and a maximum of 46.4% for risk class ECY-A. Increased correlation estimates $\rho_V > 20\%$ only occur for the data-scarce risk classes ECY-A, LCY-BBB, NCY-A, and NF-A. Obviously, high asset correlations coincide with high volatility estimates, which is attributed to the fact that in data-scarce risk classes, obligor-specific jumps in credit spreads are clearly reflected in the fitted term structures, so that filtered systematic factors have a greater impact on the variation of asset values than the specific factor component implied by bootstrapped yield residuals. In contrast, inner-class asset correlations of data-extensive rating and financial sector classes do not exceed a maximum of $\rho_V = 11.42\%$, with no unambiguous relationship between the estimates of asset volatility and asset correlation.

With respect to the different risk class factor models considered, the average inner-class asset correlation increases with the number of risk classes, while the average asset volatility remains stable. However, the increased number of inter-class correlations in the correlation matrix $\mathbf{B}'\mathbf{B}$ of normalized asset returns must be considered, if a conclusion is to be reached on the correlation effects of a more detailed set of risk classes

The estimated drift rates of the factors are not significant, since the hypothesis $H_0 : \mu_F = 0$ is not rejected by the asymptotic standard errors $\sigma_\epsilon(\mu_F)$ that reach a minimum of 6.81% for risk class LCY-A. Though asymptotic standard errors $\sigma_\epsilon(\mu_V)$ are clearly smaller than $\sigma_\epsilon(\mu_F)$, the significance of drift μ_V is mostly rejected. The asymptotic standard errors of drift rate estimates therefore support the supposition of a zero drift in line with the empirically observed cyclicity of credit spreads.

The maximum asymptotic standard errors $\sigma_\epsilon(\sigma_F) = 1.56\%$, $\sigma_\epsilon(\sigma_V) = 2.82\%$, and $\sigma_\epsilon(\rho_V) = 3.66\%$ of volatility and asset correlation estimates support the zero drift hypothesis of significant parameter estimates except for the asset correlation of risk class NCY-AA, so that the specification of an alternative parameter set to backtesting the credit portfolio model will incorporate a variation of estimated asset volatilities and correlations.

Since bond-market-induced estimations of structural portfolio models are not common, a comparison of estimation results from affiliated studies is not possible.

4.4.8.2 Probability of Default Estimates

For an assessment of the fitting quality of the factor estimation, credit spreads and corresponding risk-neutral default probabilities of risk classes are examined. Table 4.10 reveals that average par-bond yield spreads derived from fitted term structures exceed average yield spreads implied by filtered factors for maturities of one and ten years, while factor-implied spreads surpass empirically derived spreads for maturities of three and five years.

Even though the average of fitted spreads increases monotonically in the time-to-maturity for all risk classes, hump-backed term structures of average factor-implied yield spreads are observed in most risk classes, except for the data-rich low-risk classes FIN-AA, FIN-A, AA and A, where factor spreads ascend with maturity. The average risk-neutral cumulative

Risk-Class	Empirical Yield Spread of Risk-Class [bps]				Yield Spread of Factor [bps]				Periodic risk-neutral Factor-PD				Cumulative risk-neutral Factor-PD			
	T=1	T=3	T=5	T=10	T=1	T=3	T=5	T=10	T=1	T=3	T=5	T=10	T=1	T=3	T=5	T=10
ECY-A	0.35	0.69	0.84	1.04	0.13	0.83	1.02	0.98	0.26	2.73	2.47	1.33	0.26	4.85	9.71	17.24
ECY-BBB	1.01	1.25	1.30	1.32	0.78	1.49	1.43	1.11	1.47	3.41	2.19	0.82	1.47	8.34	12.81	18.05
FIN-AA	0.18	0.33	0.41	0.55	0.02	0.34	0.50	0.54	0.03	1.34	1.50	0.87	0.03	2.02	4.98	10.15
FIN-A	0.25	0.50	0.63	0.83	0.03	0.52	0.77	0.84	0.06	2.03	2.29	1.47	0.06	3.08	7.51	15.52
LCY-AA	0.21	0.41	0.50	0.60	0.05	0.47	0.60	0.57	0.10	1.66	1.53	0.75	0.10	2.79	5.90	10.59
LCY-A	0.48	0.76	0.85	1.02	0.31	0.93	0.97	0.80	0.59	2.53	1.81	0.72	0.59	5.36	9.11	13.84
LCY-BBB	0.54	0.99	1.12	1.19	0.39	1.23	1.27	1.05	0.74	3.34	2.34	0.97	0.74	7.08	11.83	17.80
NCY-AA	0.19	0.39	0.48	0.56	0.04	0.45	0.58	0.56	0.08	1.61	1.51	0.75	0.08	2.65	5.72	10.41
NCY-A	0.23	0.49	0.64	0.86	0.03	0.51	0.77	0.86	0.05	2.02	2.34	1.55	0.05	3.04	7.54	15.86
NCY-BBB	0.67	1.16	1.33	1.53	0.47	1.40	1.48	1.29	0.90	3.78	2.85	1.35	0.90	7.95	13.57	21.20
NF-AA	0.18	0.40	0.50	0.56	0.04	0.46	0.60	0.58	0.08	1.65	1.55	0.77	0.08	2.71	5.86	10.69
NF-A	0.33	0.66	0.80	0.95	0.13	0.80	0.96	0.91	0.25	2.61	2.29	1.17	0.25	4.68	9.23	16.08
NF-BBB	0.72	1.14	1.26	1.35	0.54	1.41	1.42	1.16	1.03	3.59	2.46	1.01	1.03	8.01	12.97	19.11
AA	0.18	0.35	0.42	0.57	0.02	0.34	0.51	0.56	0.03	1.35	1.57	0.95	0.03	2.01	5.08	10.63
A	0.26	0.55	0.69	0.85	0.05	0.61	0.83	0.86	0.10	2.25	2.30	1.35	0.10	3.61	8.11	15.67
BBB	0.73	1.16	1.27	1.35	0.53	1.40	1.42	1.17	1.02	3.60	2.49	1.04	1.02	7.98	13.00	19.29

Table 4.10: Average Factor Spread and Default Probability

default probabilities in Table 4.10 are calculated using the distribution function of the first-passage time in (3.10) on the basis of filtered factor values. Periodic risk-neutral factor-PD represent the average factor-implied one-year probability of default during the indicated year, conditional on the absence of a previous default and decline from a medium-maturity maximum with increasing maturity for any estimation period.[38]

Fitted one-year spreads exceed factor-implied spreads because cumulative default probabilities converge to zero with declining time-to-maturity and because of restrictions implied by the functional form of the first-passage time distribution function, which result in a slow ascent of the cumulative PD for maturities below one year to reach a proper fit of spreads for long-term maturities. The steep increase in cumulative and periodic default probabilities for medium-range maturities, typical for first-passage time distribution functions, yield factor-implied spreads of three-year and five-year maturities that exceed empirically derived yield spreads. The subsequent flattening of the cumulative PD curve for maturities greater than five years leads to a compensating under-fitting of empirical ten-year spreads. Since there is no real-world equivalent to the default probability of a risk class, factor implied cumulative real-world first-passage times have been omitted.

Default probabilities derived from obligor-specific asset values are shown in Table 4.11. Cumulative PD are calculated on the basis of asset values calibrated to reproduce the yield spreads of par-bonds using the optimal parameter set ψ_V, because filtered asset values refer to bootstrapped spreads and do not represent the typical default risk implied by

[38]Cf. Table D.3 for results of the examination of spread and PD for annual estimation periods.

the asset value process. Periodic PD generate default probabilities for one-year intervals, conditional on previous non-default as before. Calibrating asset values with respect to the

Risk-Class	Risk-neutral periodic PD of Obligor				Risk-neutral cumulative PD of Obligor				Periodic PD of Obligor				Cumulative PD of Obligor			
	T=1	T=3	T=5	T=10	T=1	T=3	T=5	T=10	T=1	T=3	T=5	T=10	T=1	T=3	T=5	T=10
ECY-A	0.68	1.91	2.17	2.16	0.68	4.07	8.11	17.84	1.54	5.70	7.08	7.35	1.54	11.02	22.76	47.56
ECY-BBB	1.90	2.52	2.14	1.58	1.90	6.76	10.90	18.44	10.14	20.58	16.69	9.58	10.14	42.47	61.12	79.39
FIN-AA	0.34	0.88	0.98	1.04	0.34	1.94	3.82	8.71	3.88	15.23	14.83	9.97	3.88	28.21	48.39	72.45
FIN-A	0.48	1.39	1.61	1.59	0.48	2.95	5.97	13.37	2.20	9.39	11.02	9.03	2.20	17.33	34.26	61.30
LCY-AA	0.40	1.14	1.21	1.09	0.40	2.44	4.78	10.08	2.18	9.97	11.66	9.78	2.18	18.11	35.82	63.65
LCY-A	0.92	1.75	1.67	1.65	0.92	4.23	7.45	14.84	6.92	18.95	16.62	10.37	6.92	37.06	57.07	78.04
LCY-BBB	1.04	2.56	2.45	1.89	1.04	5.72	10.39	19.39	2.50	8.04	8.62	7.15	2.50	16.04	29.86	53.33
NCY-AA	0.37	1.05	1.07	0.83	0.37	2.26	4.37	8.73	4.28	15.43	13.01	7.04	4.28	29.44	47.62	67.35
NCY-A	0.44	1.45	1.90	2.42	0.44	2.94	6.40	16.54	0.90	3.87	5.54	7.42	0.90	7.23	16.58	41.56
NCY-BBB	1.28	2.67	2.49	2.12	1.28	6.28	11.00	20.58	9.25	22.20	18.07	10.47	9.25	44.03	63.50	81.80
NF-AA	0.35	1.18	1.28	0.98	0.35	2.40	4.88	10.03	1.28	6.48	8.08	7.38	1.28	11.68	24.99	50.15
NF-A	0.65	1.76	1.84	1.51	0.65	3.81	7.33	14.75	3.04	11.49	12.23	8.79	3.04	21.65	39.72	64.84
NF-BBB	1.37	2.65	2.36	1.81	1.37	6.37	10.90	19.39	4.57	11.72	10.62	6.56	4.57	24.35	40.14	60.59
AA	0.36	0.92	1.05	1.14	0.36	2.03	4.02	9.31	2.47	10.67	12.41	10.41	2.47	19.58	38.01	66.31
A	0.51	1.52	1.68	1.43	0.51	3.19	6.38	13.41	2.12	8.70	9.52	6.71	2.12	16.33	31.52	54.63
BBB	1.39	2.81	2.63	2.07	1.39	6.62	11.59	21.21	3.95	11.20	11.97	10.16	3.95	22.48	39.87	66.46

Table 4.11: Average Obligor-Specific Default Probability

prices of par-bonds, risk-neutral obligor-specific PD dissolve the fitting error contained in factor PD. In consequence, periodic risk-neutral PD of asset values (asset-PD) are strictly higher than periodic risk-neutral factor PD for one-year and ten-year maturities and smaller for medium-term maturities. The periodic risk-neutral asset-PD mostly decline from a medium-maturity maximum with increasing time-to-maturity, however, the hump-back is not as pronounced when compared to the factor PD and monotonically increasing term structures of periodic PD appear for risk class FIN-AA, NCY-A, and AA. The cumulative risk-neutral PD of asset values reflect the evolution of periodic asset-PD, so that five-year cumulative PD are always smaller than factor equivalents, whereas no systematic effect can be identified for ten-year cumulative asset-PD.

Obligor-specific real-world PD significantly exceed risk-neutral PD, since near-zero drift rate estimates are generally smaller than the riskless rate.[39] Furthermore, real-world asset-PD exceed real-world factor-PD due to lower volatility estimates for the asset values.

For all sectors, risk-neutral PD increase as ratings decrease, while different volatility estimates distort a monotonous increase in real-world PD with declining credit quality, which is equivalent to the assumption of different market prices of credit risk for different risk classes.

[39] Average risk-neutral default probabilities exceed their real-world counterparts for estimation periods with riskless rates smaller than the drift rate of the asset value, as can be seen in Table D.4.

Cumulative factor-implied PD by far exceed cumulative default frequencies published by rating agencies.[40] Though model-implied default expectations do not reproduce historical default rates in the long-run, estimated parameter sets will be used, since a one-year time horizon of Credit-VaR neglects distant default events from cumulative multi-year PD.

4.4.8.3 Explanatory Power of Estimates

The standard deviations of par-bond yield spreads derived from the fitted term structure of risk classes and the standard deviation of yield spreads induced by the filtered systematic factor in Table 4.12 show only tendentious similarity, which once again illustrates the limited flexibility of the structural credit valuation model in fitting empirical term structures of credit spreads. In contrast to the factor estimation, the asset values filtered by the EKF provide a better explanatory power. This is indicated by similar standard deviations of bootstrapped and filtered yield spreads. The spread variation intensifies with decreasing ratings for risk class spreads of all sectors and for the asset-specific spreads of most sectors. Asset-specific spreads show a notably higher variation with standard deviations rising threefold for risk class FIN-AA. The spread variations of annual estimation periods in Table D.2 show equivalent results.

Risk-Class	Std. Dev. of emp. Risk class Spreads				Std. Dev. of Factor Spreads				Std. Dev. of Bootstrap Spreads				Std. Dev. of Asset Spreads			
	T=1	T=3	T=5	T=10	T=1	T=3	T=5	T=10	T=1	T=3	T=5	T=10	T=1	T=3	T=5	T=10
ECY-A	19.4	29.2	30.0	34.8	15.9	39.1	35.3	25.7	37.8	30.7	39.4	41.5	37.7	30.4	38.8	41.5
ECY-BBB	145.3	84.8	60.6	37.8	116.5	97.5	72.6	45.6	112.0	80.5	60.6	61.3	113.3	81.8	59.5	60.8
FIN-AA	6.1	7.1	8.4	10.3	1.0	8.0	8.6	6.8	32.7	35.8	30.1	29.4	32.5	35.6	29.8	28.9
FIN-A	6.7	9.2	10.1	15.6	1.4	10.4	11.0	8.6	24.4	28.8	33.2	25.4	24.5	28.8	32.0	24.8
LCY-AA	13.6	15.5	13.3	15.5	6.3	19.0	17.4	12.3	16.8	26.6	30.4	29.8	17.5	26.3	30.0	29.4
LCY-A	57.9	50.0	39.0	37.1	47.9	57.2	45.2	29.2	51.3	46.3	29.4	40.8	54.0	46.6	30.3	40.9
LCY-BBB	28.8	35.9	29.3	24.7	26.4	39.1	31.6	20.6	51.3	83.3	36.5	46.3	52.2	83.3	36.3	44.0
NCY-AA	7.5	9.3	10.2	10.2	2.3	11.2	11.0	8.1	13.4	34.5	16.7	23.9	13.4	34.3	16.3	23.6
NCY-A	7.9	10.4	12.0	19.0	1.9	13.3	14.1	11.2	23.3	21.9	26.6	23.3	23.2	21.5	26.4	22.2
NCY-BBB	60.2	69.6	58.5	52.2	58.2	75.7	61.5	41.5	50.4	55.9	61.5	62.9	50.6	56.3	63.4	63.0
NF-AA	7.0	9.6	11.6	12.2	2.7	12.4	12.0	8.9	20.4	20.1	17.3	21.1	20.2	19.7	15.9	20.6
NF-A	16.8	24.8	25.3	27.5	13.2	33.8	30.3	21.8	42.3	62.7	38.1	48.7	43.7	65.1	38.7	48.3
NF-BBB	61.8	59.1	47.0	35.3	52.3	61.8	48.7	31.9	73.0	85.2	51.2	78.9	76.5	86.4	53.7	77.9
AA	6.3	7.3	8.8	10.7	0.9	8.2	9.0	7.2	31.0	27.7	25.5	21.9	30.7	27.7	25.2	21.0
A	7.1	10.5	12.5	17.3	2.8	14.4	14.4	11.0	20.2	25.0	36.0	32.1	20.0	24.4	35.8	32.2
BBB	61.3	57.9	46.0	36.1	55.8	65.0	51.1	33.4	85.6	72.6	46.3	64.4	87.0	73.2	45.9	64.5

Table 4.12: Standard Deviation of Spreads

In Tables 4.13 and 4.14, the explanatory power of EKF estimations is assessed examining the yield residuals induced by the optimal EKF filtering of factors and asset values.

[40] Cf. Hamilton (2002) and Hamilton et al. (2004)

Residual Analysis of the Factor Estimation

In the factor estimation, the credit valuation model of Section 3.2 exhibits a systematic pattern of mis-fitting factor spreads for particular maturities due to its functional restrictions. In accordance with the comparison of average empirical and average factor-derived yield spreads, the analysis of factor-implied yield-spread residuals in Table 4.13 reveals that one-year and ten-year empirical spreads exceed filtered spreads, while the negative residuals for three-year and five-year filtered yield spreads indicate filtered spreads which exceed empirical spreads. The delayed range of residuals with respect to zero-expectation supports the hypothesis of a systematical mis-fitting of yield spreads, and the systematically positive or negative residuals prevent short-lag auto correlations from becoming insignificant.

A slight variation in factor yield-spread residuals is observed for the risk classes AA and A, indicating an improvement of fitting ability of the EKF if the level of spreads is low and the term structure steepening is moderate. However, mean absolute deviation (MAD) and the standard deviation of residuals reveal that, for risk classes with a high variation of empirical spreads, the filter update has difficulties in keeping up with the variation of spreads. Accordingly, the standard deviations of yield spread residuals for annual estimation periods in Table D.5 shows that the variation of residuals was only elevated in 2002 and 2003 for BBB risk classes and data-scarce risk classes, which suffer from jumps in credit spreads in the respective period. Meanwhile, for the same risk classes, filtering is more effective in periods with a more typical spread variation.

The homogenous occurrence of negative and positive skewness, which is small in absolute terms, and an excess kurtosis close to zero (except for NCY-AA, NCY-A and BBB) support the assumption of residuals being distributed approximately normal. The correlations of factor spread residuals in Table D.6 show distinct positive correlations of spread residuals for neighboring maturities, while residual correlations between one-year and ten-year maturities are mostly negative.

Residual Analysis of the Asset Value Estimation

In the asset value estimation the EKF demonstrates better fitting abilities due to the fitting of yield spreads by individual state variables. The average yield-spread residuals exceed two basis points in absolute terms only for the ten-year residuals of the ECY-BBB class, and the medium spread does not amount to more than one basis point in absolute terms. The mean absolute deviation of asset yield-spread residuals exceed four basis points only for the one-year residuals of class LCY-A, where yield spread residuals show a wide variation due to jumps in empirical spreads.

With the exception of the ten-year FIN-A residuals, average residuals are negative throughout, which can be attributed to the mostly negative skewness. Skewness and a notably positive kurtosis contradict the normality assumption of residuals. Autocor-

Risk-Class	Tenor	Residual Statistics on Factor Estimation										Autocorrelation				
		Mean	Min	Max	Med.	$q_{0.05}$	$q_{0.95}$	MAD	σ	Skewn.	Kurt.	ρ_1	ρ_2	ρ_5	ρ_{10}	ρ_{20}
ECY-A	**1**	21.9	1.2	44.3	22.4	10.0	35.0	1.6	7.9	0.08	-0.42	0.8452	0.7513	0.6891	0.6260	0.5675
	3	-13.6	-56.9	12.0	-10.9	-35.0	2.8	4.2	12.7	-0.67	0.14	0.9249	0.8784	0.7807	0.6252	0.3910
	5	-17.3	-52.2	5.5	-16.5	-39.4	-1.1	3.5	11.5	-0.56	-0.06	0.9320	0.8862	0.7672	0.6355	0.4335
	10	6.6	-26.5	66.0	-0.5	-17.7	48.2	12.1	21.6	0.84	-0.23	0.9730	0.9442	0.8782	0.8325	0.6531
ECY-BBB	**1**	22.6	-91.2	213.5	15.5	-5.9	112.1	39.4	38.9	2.67	9.61	0.7789	0.6680	0.5422	0.3627	-0.1152
	3	-23.9	-114.7	32.0	-20.9	-69.6	-0.5	11.0	20.6	-1.65	4.24	0.8512	0.7758	0.5396	0.3869	0.1988
	5	-12.0	-100.7	27.7	-8.1	-55.6	8.1	10.2	19.8	-2.26	6.36	0.8962	0.8163	0.6542	0.3869	0.0839
	10	20.9	-88.8	99.0	25.3	-31.8	61.0	21.2	28.6	-1.02	2.07	0.9026	0.8242	0.6662	0.3733	-0.1035
FIN-AA	**1**	15.8	1.5	31.0	14.9	7.5	25.8	0.8	5.7	0.30	-0.48	0.7802	0.7256	0.6740	0.6006	0.3562
	3	-0.4	-13.7	10.3	-0.1	-7.5	5.2	0.4	3.9	-0.39	0.73	0.4854	0.4052	0.3441	0.3076	0.0841
	5	-8.8	-21.6	4.1	-8.4	-15.6	-1.9	0.5	4.4	0.05	0.13	0.6770	0.6519	0.5354	0.5131	0.2516
	10	1.5	-8.5	16.2	0.3	-6.4	11.2	0.8	5.5	0.46	-0.59	0.8591	0.8194	0.7017	0.6611	0.5170
FIN-A	**1**	21.7	4.1	37.2	21.6	13.0	31.8	0.9	5.9	0.15	-0.30	0.7207	0.6422	0.5405	0.4695	0.1867
	3	-1.6	-16.8	15.1	-1.8	-9.7	6.9	0.7	5.3	0.22	0.34	0.5294	0.4829	0.3839	0.3710	0.2759
	5	-13.2	-26.5	7.9	-13.5	-22.7	-4.2	0.9	5.8	0.27	0.00	0.6753	0.6528	0.5405	0.4848	0.2579
	10	-0.7	-25.1	23.6	0.2	-19.4	16.1	3.9	12.2	-0.16	-1.26	0.9526	0.9368	0.8861	0.8032	0.6159
LCY-AA	**1**	15.3	1.1	42.5	13.0	4.2	34.8	2.3	9.4	0.84	-0.03	0.7249	0.5997	0.3419	0.3120	-0.0281
	3	-5.7	-48.7	12.5	-4.3	-18.4	5.9	1.8	8.2	-1.22	3.72	0.7779	0.6616	0.4840	0.4093	0.0941
	5	-10.2	-46.0	8.4	-9.1	-27.3	1.9	2.1	8.9	-1.24	2.57	0.8486	0.7798	0.6051	0.3844	0.0688
	10	2.9	-40.3	41.8	2.6	-23.1	27.3	5.2	14.2	-0.36	0.69	0.9138	0.8535	0.6455	0.3835	0.3051
LCY-A	**1**	17.0	-31.6	183.3	13.9	-7.1	44.2	10.5	20.1	3.79	25.08	0.5065	0.2714	0.1661	0.2695	0.0647
	3	-17.2	-115.1	7.6	-13.2	-42.5	-2.1	6.2	15.4	-2.27	8.94	0.8137	0.6445	0.2838	0.2477	0.4913
	5	-11.8	-112.5	15.5	-9.3	-37.0	4.1	5.6	14.7	-2.39	10.69	0.8424	0.6772	0.3893	0.3659	0.3906
	10	21.9	-87.9	100.4	18.3	-24.0	69.5	26.7	32.1	0.06	-0.20	0.9403	0.8887	0.7607	0.5822	0.2457
LCY-BBB	**1**	15.6	-9.6	54.1	15.7	2.1	28.8	1.8	8.3	0.54	1.98	0.6300	0.3944	0.1515	0.1318	-0.0027
	3	-23.9	-59.6	10.8	-24.6	-36.0	-9.5	2.0	8.7	-0.04	2.49	0.7175	0.5600	0.2724	0.2770	0.1753
	5	-15.5	-57.4	8.0	-13.9	-34.9	-1.3	2.9	10.5	-0.83	1.12	0.8277	0.7295	0.4425	0.3833	0.1286
	10	13.9	-58.4	92.1	18.7	-36.8	48.2	16.9	25.5	-0.41	0.07	0.9192	0.8607	0.6976	0.5659	0.2978
NCY-AA	**1**	15.1	2.5	36.6	14.5	5.7	26.7	1.1	6.4	0.50	0.10	0.7468	0.5942	0.3750	0.4114	0.3101
	3	-5.1	-26.9	10.5	-4.9	-13.5	3.7	0.7	5.3	-0.20	1.02	0.6950	0.5527	0.2734	0.3530	0.1325
	5	-9.9	-35.2	4.9	-9.3	-19.1	-2.1	0.8	5.5	-0.48	1.07	0.7941	0.7034	0.4790	0.3745	0.1801
	10	-0.1	-17.3	14.6	0.0	-11.3	11.0	1.2	6.8	-0.07	-0.67	0.8980	0.8579	0.7597	0.6460	0.4923
NCY-A	**1**	20.3	5.9	44.5	19.4	11.4	33.8	1.2	6.8	0.76	0.61	0.7918	0.6794	0.4652	0.3067	0.0481
	3	-1.9	-18.1	13.6	-1.6	-12.9	6.8	0.9	6.0	-0.33	-0.04	0.8014	0.7138	0.5736	0.4565	0.1406
	5	-12.8	-31.5	2.7	-12.2	-25.7	-4.4	1.0	6.3	-0.71	0.56	0.8757	0.8044	0.5796	0.4153	0.0432
	10	0.3	-27.2	31.1	-0.7	-19.3	21.0	4.6	13.3	0.03	-0.90	0.9509	0.9195	0.8395	0.7048	0.5484
NCY-BBB	**1**	19.3	-11.6	150.9	18.8	2.9	33.8	4.6	13.3	4.52	39.86	0.6404	0.3311	0.1326	0.0397	-0.0408
	3	-23.9	-88.5	64.3	-24.9	-45.0	0.3	6.7	16.0	0.43	3.44	0.7695	0.6675	0.4740	0.4116	0.3676
	5	-15.7	-88.8	74.3	-15.2	-45.9	7.3	7.9	17.5	-0.47	4.08	0.8296	0.7765	0.6453	0.5759	0.3853
	10	24.6	-76.2	142.3	26.0	-47.2	88.1	43.7	41.0	-0.21	-0.21	0.9244	0.8786	0.7802	0.6971	0.3407
NF-AA	**1**	13.6	0.8	29.6	12.8	5.4	24.5	0.9	6.0	0.43	-0.49	0.6857	0.5897	0.4140	0.3395	0.1791
	3	-5.3	-25.0	5.4	-4.5	-14.6	3.2	0.8	5.5	-0.73	1.13	0.7455	0.6722	0.4511	0.3636	0.0337
	5	-9.6	-27.1	2.7	-9.2	-18.7	-1.6	0.8	5.5	-0.33	-0.19	0.8412	0.7628	0.5428	0.3077	-0.0652
	10	-1.4	-17.9	19.0	-1.9	-14.7	12.3	1.7	8.2	0.13	-0.70	0.9072	0.8512	0.7446	0.6352	0.4522
NF-A	**1**	20.5	5.9	39.9	21.1	10.1	32.0	1.2	6.9	0.20	-0.41	0.8245	0.7478	0.6153	0.5712	0.4588
	3	-13.9	-49.4	6.6	-11.9	-37.1	0.2	3.3	11.3	-0.76	0.00	0.9384	0.8990	0.7771	0.6184	0.3423
	5	-16.5	-47.3	4.0	-15.0	-40.5	-3.1	2.8	10.4	-0.88	0.49	0.9527	0.9124	0.8012	0.6747	0.4677
	10	4.8	-32.6	46.0	2.8	-18.1	35.0	7.9	17.5	0.26	-0.61	0.9628	0.9302	0.8670	0.7809	0.6418
NF-BBB	**1**	17.5	-17.9	100.9	13.8	-5.1	51.9	9.5	19.1	1.82	5.04	0.6793	0.5699	0.4257	0.2222	0.0366
	3	-26.2	-56.6	18.6	-25.9	-50.2	-3.9	4.9	13.7	0.05	-0.09	0.8366	0.7337	0.5126	0.3903	0.1943
	5	-15.9	-44.6	27.9	-15.1	-32.1	-3.1	2.7	10.2	-0.04	1.44	0.7538	0.5954	0.3892	0.1423	-0.1123
	10	19.7	-31.7	88.2	23.6	-25.3	63.5	17.9	26.2	-0.05	-0.58	0.9062	0.8376	0.7567	0.5850	0.4108
AA	**1**	16.9	4.8	31.5	15.9	8.5	27.0	0.9	6.0	0.26	-0.89	0.8106	0.7447	0.6846	0.6402	0.3949
	3	1.0	-12.8	13.4	1.3	-7.2	6.1	0.4	4.0	-0.52	1.11	0.5054	0.4009	0.3080	0.3491	0.1021
	5	-8.9	-20.1	5.1	-8.7	-15.9	-1.8	0.5	4.5	0.20	-0.03	0.6947	0.6538	0.5214	0.5288	0.2917
	10	0.4	-9.3	15.8	-0.5	-7.2	10.9	0.8	5.6	0.51	-0.34	0.8594	0.8167	0.6876	0.6115	0.5001
A	**1**	21.4	7.5	37.9	20.3	13.9	30.9	0.8	5.5	0.31	-0.27	0.7045	0.6324	0.5670	0.4997	0.3066
	3	-6.2	-24.7	12.2	-6.7	-17.3	3.3	1.1	6.6	-0.14	-0.30	0.7635	0.7200	0.5950	0.5241	0.2354
	5	-14.6	-36.8	5.7	-14.1	-25.0	-4.4	1.1	6.5	-0.22	0.33	0.7755	0.7167	0.5781	0.5112	0.2751
	10	-0.6	-21.7	24.5	-2.3	-17.2	18.5	3.3	11.3	0.30	-0.92	0.9471	0.9277	0.8690	0.7885	0.6603
BBB	**1**	19.3	-21.8	93.2	17.5	1.4	43.4	5.6	14.6	1.72	6.85	0.5483	0.4387	0.2349	0.0789	-0.0270
	3	-24.1	-66.2	13.7	-23.4	-45.9	-5.6	4.2	12.6	-0.35	0.66	0.8120	0.6887	0.5043	0.3616	0.2248
	5	-15.0	-59.0	28.6	-12.4	-37.9	-0.6	4.0	12.3	-0.72	1.41	0.8193	0.6832	0.5616	0.3019	0.0848
	10	18.0	-40.2	95.7	22.2	-32.4	68.4	22.8	29.6	-0.02	-0.55	0.9077	0.8487	0.7725	0.5745	0.4099

Table 4.13: Statistics on KF-induced Spread Residuals of Filtered Systematic Factors

Risk-Class	Tenor	Residual Statistics on Asset Value Estimation										Autocorrelation				
		Mean	Min	Max	Med.	$q_{0.05}$	$q_{0.95}$	MAD	σ	Skewn.	Kurt.	ρ_1	ρ_2	ρ_5	ρ_{10}	ρ_{20}
ECY-A	**1**	-0.6	-20.4	0.7	-0.2	-2.4	0.2	0.1	1.7	-7.63	74.69	0.1287	0.0585	0.1102	0.0756	0.0042
	3	-0.4	-21.0	8.2	-0.1	-4.3	2.7	0.2	2.5	-2.28	16.52	0.3009	0.1787	-0.0123	-0.0388	0.0814
	5	-0.4	-19.1	13.7	0.0	-11.2	8.3	0.9	5.8	-0.36	0.40	0.5682	0.4007	0.0924	-0.1837	-0.1886
	10	0.0	-17.2	19.1	-0.3	-9.5	9.3	0.7	5.3	0.30	1.26	0.4983	0.2340	-0.1025	-0.0794	-0.0060
ECY-BBB	**1**	-1.4	-50.5	17.7	-0.3	-6.2	1.0	0.8	5.7	-5.27	37.42	0.1591	-0.0065	0.0995	-0.1396	0.1288
	3	-1.5	-40.1	25.3	-0.3	-7.5	1.3	1.0	6.2	-3.82	21.92	0.3606	0.1256	0.0841	-0.1294	0.1398
	5	-1.8	-47.3	8.4	-0.2	-11.2	1.3	1.0	6.3	-4.42	22.43	0.6670	0.6571	0.1666	0.1430	-0.0442
	10	-2.2	-130.3	3.1	-0.4	-8.7	0.7	2.3	9.4	-10.45	134.81	0.1568	0.0520	0.1906	-0.0002	0.1455
FIN-AA	**1**	-0.8	-28.2	0.3	-0.3	-2.9	0.1	0.1	2.1	-9.66	120.99	0.1171	-0.0007	-0.0351	0.0030	0.0031
	3	-0.5	-14.7	15.6	-0.4	-5.5	5.6	0.3	3.3	0.46	4.60	0.3739	0.1793	-0.0431	-0.0629	0.0331
	5	-0.4	-12.8	27.1	-0.6	-7.6	7.7	0.6	4.8	1.25	5.67	0.4590	0.2467	0.0074	-0.1070	0.0375
	10	-0.6	-11.5	22.5	-0.2	-6.1	2.6	0.2	3.0	1.29	15.36	0.2591	0.1317	-0.2598	0.1781	-0.0589
FIN-A	**1**	-1.0	-14.8	0.4	-0.4	-3.7	0.0	0.1	1.6	-4.04	24.80	0.0120	0.0121	0.0337	0.0586	-0.0026
	3	-0.7	-8.8	0.8	-0.3	-2.7	0.4	0.0	1.2	-3.08	14.74	0.1384	-0.0686	0.1044	-0.0565	-0.0251
	5	-0.9	-18.0	24.8	-1.0	-9.2	7.2	0.7	5.1	0.20	3.07	0.5387	0.3251	-0.0091	-0.0676	-0.0813
	10	0.1	-22.4	28.5	-0.3	-9.3	11.9	1.1	6.6	0.77	2.64	0.4578	0.2312	-0.0113	0.0470	-0.1047
LCY-AA	**1**	-0.9	-56.9	2.0	-0.3	-3.2	0.4	0.4	3.9	-12.19	169.59	0.2434	-0.0366	0.1295	0.0107	0.0229
	3	-0.4	-15.4	13.8	-0.4	-7.2	6.0	0.3	3.7	-0.07	2.13	0.4948	0.2177	-0.1354	-0.0300	0.0346
	5	-0.3	-19.5	17.1	-0.1	-4.5	3.8	0.2	3.1	-0.55	11.62	0.4725	0.2871	-0.0439	-0.0859	0.1572
	10	-0.2	-26.2	11.5	-0.2	-5.6	5.6	0.4	3.9	-1.50	9.28	0.3574	0.2036	-0.0410	-0.0012	-0.1222
LCY-A	**1**	0.8	-296.8	152.9	-1.0	-37.6	58.9	31.5	34.9	-1.98	22.32	0.5257	0.3604	0.2742	0.1639	-0.1655
	3	-1.4	-63.6	6.4	-0.2	-7.9	1.6	1.2	6.8	-6.47	48.45	0.3187	0.2525	0.2857	0.1406	-0.0305
	5	-1.0	-76.3	2.7	-0.1	-4.6	0.3	0.8	5.4	-11.14	146.30	0.3247	0.2216	0.0952	0.0444	-0.0282
	10	-1.1	-33.9	7.0	-0.3	-5.2	2.3	0.5	4.5	-4.46	24.73	0.1995	0.1956	0.2998	0.1392	-0.1217
LCY-BBB	**1**	-0.9	-73.1	12.7	-0.3	-6.4	2.8	0.7	5.4	-9.51	128.19	0.1265	0.0634	-0.0055	0.0090	-0.0414
	3	-0.4	-9.8	0.8	-0.1	-2.1	0.1	0.0	1.0	-4.99	32.62	0.2968	0.0188	0.0924	0.0865	0.0401
	5	-0.3	-13.4	10.3	-0.1	-3.6	2.3	0.1	2.2	-1.64	11.68	0.4344	0.1144	0.1720	0.0824	-0.1838
	10	-1.1	-41.7	29.6	-0.8	-14.3	11.8	2.2	9.2	-0.57	3.24	0.5163	0.2458	0.0341	0.0790	-0.2255
NCY-AA	**1**	-0.9	-12.3	1.6	-0.4	-4.0	0.2	0.1	1.6	-3.27	14.83	0.1727	0.0305	-0.0700	0.1262	0.0477
	3	-0.5	-9.1	9.4	-0.2	-3.3	2.2	0.1	1.8	0.11	6.42	0.2750	0.0052	-0.0374	0.1617	-0.0143
	5	-0.1	-14.1	14.9	-0.1	-5.9	5.6	0.4	3.7	0.30	3.38	0.4360	0.1250	0.0254	-0.0415	-0.0001
	10	-0.4	-9.0	9.0	-0.2	-3.9	2.2	0.1	2.0	-0.21	3.89	0.1969	0.0007	0.0674	-0.0167	0.0029
NCY-A	**1**	-0.6	-35.2	3.4	-0.1	-2.7	0.9	0.2	2.6	-10.07	130.77	0.2846	-0.0277	-0.0118	-0.0523	0.0239
	3	-0.2	-18.9	8.2	0.0	-4.6	3.0	0.1	2.3	-2.27	16.86	0.4112	0.1218	-0.0887	0.0543	-0.0267
	5	-0.1	-18.0	15.0	0.0	-4.9	4.5	0.2	3.0	-0.04	6.96	0.2086	0.0856	-0.0597	0.0658	-0.0350
	10	0.1	-35.6	20.2	-0.2	-8.8	10.2	1.1	6.4	-0.57	4.20	0.5032	0.2932	-0.0014	0.1485	-0.0487
NCY-BBB	**1**	-1.4	-75.3	6.6	-0.3	-5.5	1.1	0.8	5.7	-9.69	115.66	0.3957	0.0338	-0.0180	0.0123	-0.0191
	3	-1.1	-22.8	6.0	-0.3	-6.2	0.7	0.3	3.1	-4.12	20.22	0.2008	0.0155	0.0437	0.1588	-0.0173
	5	-1.4	-68.9	17.6	-0.2	-6.8	2.3	1.3	7.0	-6.91	61.19	0.4223	0.0104	-0.0444	0.1334	0.0115
	10	-1.3	-46.3	7.5	-0.2	-6.4	1.0	0.5	4.4	-6.14	52.39	0.3047	0.0190	0.1228	0.0468	0.0052
NF-AA	**1**	-0.5	-5.5	0.5	-0.3	-2.4	0.2	0.0	0.9	-2.74	9.57	0.1336	0.1296	0.0873	0.0596	0.1021
	3	-0.4	-14.2	5.5	-0.1	-3.9	2.5	0.1	2.2	-2.00	9.36	0.2376	0.2291	-0.0288	-0.0492	-0.1372
	5	-0.3	-16.8	15.7	0.1	-8.7	6.8	0.5	4.6	-0.20	1.35	0.5539	0.3026	-0.0841	-0.2028	0.0270
	10	-0.1	-13.5	14.3	-0.2	-5.8	6.1	0.3	3.6	0.18	1.50	0.2658	0.1802	0.0228	-0.1042	-0.0211
NF-A	**1**	-0.8	-79.2	3.4	-0.2	-3.4	1.3	0.7	5.2	-13.70	207.14	0.1451	0.0233	0.0020	0.0115	-0.0233
	3	-1.1	-124.9	7.9	-0.3	-7.5	4.4	1.9	8.6	-11.85	169.82	0.2600	0.1579	0.0337	0.0359	-0.0097
	5	-0.8	-36.3	4.1	-0.1	-4.1	1.5	0.3	3.5	-6.10	48.20	0.2067	0.1127	0.1093	-0.0346	0.0625
	10	-0.4	-30.3	12.8	0.0	-4.6	3.7	0.4	3.7	-3.22	22.03	0.1653	0.0254	0.0038	0.1234	-0.0097
NF-BBB	**1**	-1.1	-155.3	28.0	-0.2	-9.2	5.3	3.1	10.9	-10.89	155.46	0.1508	0.1129	-0.0396	0.1024	-0.1460
	3	-1.5	-56.2	2.8	-0.2	-7.7	0.7	0.8	5.6	-6.75	54.99	0.1841	0.4551	0.2812	0.2230	-0.0598
	5	-1.3	-107.6	29.5	0.0	-11.0	5.3	2.4	9.6	-6.55	64.51	0.4011	0.0526	-0.0333	0.1287	0.0585
	10	-1.4	-26.9	12.3	-0.3	-9.1	1.7	0.4	3.9	-2.21	9.92	0.3050	0.1126	0.1687	0.0926	-0.1467
AA	**1**	-1.1	-16.2	0.9	-0.4	-4.2	0.1	0.1	2.0	-4.21	23.36	0.2048	0.0877	0.0471	0.0342	0.0758
	3	-0.4	-7.7	7.7	-0.2	-3.8	2.6	0.1	2.0	-0.01	1.88	0.0935	-0.0164	0.0505	-0.0239	-0.0605
	5	-0.3	-12.0	18.1	-0.3	-5.9	4.4	0.3	3.6	0.86	6.43	0.3986	0.2477	-0.1839	-0.1243	0.0105
	10	-0.5	-19.6	15.2	-0.2	-5.9	5.2	0.4	3.8	-0.51	3.54	0.3400	0.1259	0.0005	-0.1843	0.1244
A	**1**	-1.4	-26.7	1.8	-0.5	-5.8	0.1	0.2	2.7	-5.23	39.23	0.0924	0.0258	0.0122	0.0446	-0.0103
	3	-0.7	-21.1	13.9	-0.2	-5.4	2.9	0.2	2.9	-1.29	11.91	0.1300	0.0921	0.0015	-0.0727	-0.0097
	5	-0.7	-13.8	1.7	-0.2	-3.1	0.6	0.1	1.6	-4.04	25.49	0.1793	0.1920	-0.0038	0.2063	-0.0261
	10	-0.5	-9.7	2.4	-0.2	-2.5	1.3	0.1	1.5	-2.56	11.35	0.1670	0.0123	0.1993	0.0073	-0.0910
BBB	**1**	-1.6	-36.2	1.8	-0.4	-5.8	0.5	0.5	4.2	-5.51	37.02	0.2371	0.1417	0.1625	0.2573	-0.0018
	3	-1.2	-30.2	4.0	-0.3	-4.3	0.9	0.3	3.4	-5.53	38.04	0.1679	0.3132	0.0504	0.1071	-0.0490
	5	-0.7	-16.2	25.2	-0.1	-7.8	5.5	0.5	4.3	0.17	6.50	0.1482	-0.0364	0.0045	-0.0003	0.0415
	10	-1.0	-40.5	32.2	-0.5	-12.2	5.6	1.1	6.6	-1.04	10.75	0.2819	0.0119	-0.0100	-0.0588	0.0881

Table 4.14: Statistics on KF-induced Spread Residuals of Filtered Asset Values

relations of residuals strongly converge to zero in the correlogram for most risk classes, underscoring the fitting capability of the Kalman-Filter.

Correlations of asset-spread residuals in Table D.6 do not show a distinct pattern with respect to the sign of correlations. Although specific components of asset values are assumed to be independent, significant residual correlations may appear due to sampling effects of the bootstrap, systematic fitting deficiencies or due to the non-linear functional relation between state variables and yield spread residuals.

4.4.8.4 Specification of the Dependence Model

For the rating class model, the two-sector model and the four-sector model, dependence parameters are presented in Tables 4.15-4.17. For each model, correlations of systematic factors, asset values and yield spreads are considered. Factor correlations $\rho^f_{rc_i,rc_j}$ are calculated from normalized time series of systematic factor returns of risk class $rc_i, rc_j = 1, ..., n_m$, while the correlations of normalized assets returns of risk classes rc_i and rc_j are given by $\rho^a_{ij} = \beta_{rc_i}\beta_{rc_j}\rho^f_{ij}$. The coefficients of the orthogonal abstract factors are used to specify the row vectors of the factor coefficient matrix $\mathbf{B}$ in (3.35) in the risk class factor model of (3.36), according to the risk class affiliation of exposures. For the factor estimation, average correlations of first differences of empirical and filtered yield spreads are compared, and for the asset value estimation average correlations of first differences of bootstrapped and filtered spreads are compared.

Factor Correlations					**Correlations of Factor Spreads**			
Rating	**AA**	**A**	**BBB**	**Avg. diagonal** **Avg. off-diag.**	**empirical** / **filtered**	**AA**	**A**	**BBB**
AA	100.0	79.6	26.8	100.0	**AA**	100.0	73.6	20.7
A		100.0	36.6	47.7	**A**	73.1	100.0	21.2
BBB			100.0		**BBB**	18.0	28.1	100.0

Asset Correlations					**Correlations of Asset Spreads**			
Rating	**AA**	**A**	**BBB**	**Avg. diagonal** **Avg. off-diag.**	**bootstrap** / **filtered**	**AA**	**A**	**BBB**
AA	8.5	6.7	2.6	9.4	**AA**	100.0	8.1	7.5
A		8.3	3.6	4.3	**A**	7.3	100.0	5.0
BBB			11.4		**BBB**	8.6	3.4	100.0

Latent Factor Coefficients			
Rating	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_3}$
AA	29.1		
A	22.9	17.4	
BBB	9.1	8.5	31.4

Table 4.15: Credit Dependence of Rating-Class Model

Rating-Class Model

Correlations of rating class factors in Table 4.15 are higher for affiliate rating classes as compared to the correlation between AA and BBB. Asset correlations are smaller than factor correlations by definition. The average inner-class asset correlation of 9.4% approximately doubles the average inter-class correlation of 4.3%, which indicates that the evolution of credit risk within rating classes is recognized to be more homogenous than the evolution of spreads of different rating classes. However, the average asset correlation of risk classes cannot easily be used as an indicator of the correlation risk in a portfolio, since the distribution of exposures across risk classes affects the average of exposures' asset correlations. Alternative studies provide asset correlation estimates for a rating class structure of risk classes in the range between close to zero and 30%, so that estimated asset correlations are not implausible.[41]

Factor and asset correlations of annual estimations in Table D.11 differ considerably from year to year and mostly exceed correlation estimates of the overall period except for 2003. Although correlations calculated from filtered factor series may vary due to the sampling effect, without a structural change in the co-movement of factors, credit market events of the years from 2000 to 2003 suggest correlations throughout the five-year period to be time-inhomogenous. Since Credit-VaR forecasts of the risk class factor model are unconditional on realized factor values, the estimation period should span a full factor cycle, which implies that annual estimation periods are methodologically insufficient.

The correlations of the empirical par-bond yield spreads of rating classes are always smaller than factor correlations, and the correlations of filtered factor-implied yield spreads are similar to their empirical counterparts, which indicates the empirical co-movement of spreads implied from individually filtered factor series are satisfactorily reproduced. Accordingly, asset correlations are, on average at about the same level as the correlations of bootstrapped yield spreads, which are sufficiently reproduced by the correlations of asset-implied spreads.

A comparison of factor correlations with correlations of yield spreads for single maturities in Table D.7 reveals that correlations correspond best for the three-year and five-year maturities. Although the correspondence is rather rough, the tendency of correlations between rating classes is identical, and single-maturity spread correlations are sufficiently reproduced by filtered spreads.

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[41] Cf. Akhavein, Kocagil and Neugebauer, Dietsch and Petey (2002, 2004), Düllmann and Scheule (2003), Hamerle, Liebig and Rösch (2004).

Factor Correlations						
Risk-Class	**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**	**Avg. diagonal / Avg. off-diag.**
FIN-AA	100.0	82.2	47.1	52.8	29.9	100.0
FIN-A		100.0	38.4	46.5	26.9	46.0
NF-AA			100.0	53.6	32.0	
NF-A				100.0	50.9	
NF-BBB					100.0	

Correlations of Factor Spreads					
empirical / filtered	**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**
FIN-AA	100.0	78.1	36.8	41.2	22.8
FIN-A	82.3	100.0	33.2	36.7	18.1
NF-AA	42.6	35.1	100.0	43.8	15.7
NF-A	41.8	34.8	44.5	100.0	34.1
NF-BBB	22.5	17.4	25.5	44.3	100.0

Asset Correlations						
Risk-Class	**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**	**Avg. diagonal / Avg. off-diag.**
FIN-AA	4.2	4.7	4.5	3.1	1.8	10.1
FIN-A		7.9	5.0	3.7	2.2	4.1
NF-AA			21.7	7.1	4.4	
NF-A				8.2	4.3	
NF-BBB					8.5	

Correlations of Asset Spreads					
empirical / filtered	**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**
FIN-AA	100.0	15.3	5.9	6.3	4.1
FIN-A	17.4	100.0	9.2	10.7	7.7
NF-AA	4.6	6.7	100.0	7.3	7.4
NF-A	6.4	7.8	5.6	100.0	9.5
NF-BBB	3.2	5.4	6.9	7.1	100.0

Factor Coefficients					
Risk-Class	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_3}$	$\mathbf{F_4}$	$\mathbf{F_5}$
FIN-AA	20.5				
FIN-A	23.0	16.0			
NF-AA	21.9	-0.3	41.1		
NF-A	15.1	1.6	9.3	22.4	
NF-BBB	8.7	1.2	5.9	10.6	25.1

Table 4.16: Credit Dependence of Two-Sector Model

Two-Sector Model

Factor correlations of the two-sector model in Table 4.16 amount to 46% on average and show the highest correlations between the affiliate rating classes of a sector. Inter-sector correlations between risk classes with investment grade ratings are considerably higher than cross-sector correlations between investment-grade classes and the NF-BBB class.

Asset correlations between exposures from the same sector are higher than inter-sector asset correlations, except for the FIN-A sector. The average inner-class asset correlation of 10.1% is more than double the average inter-class correlation of 4.1%. Inner-class asset correlations in the two-sector model exceed those of the rating class model, while inter-class correlations have diminished. This effect suggests that the exposures of a risk class are more homogenous with respect to credit risk in the more granular two-sector-class model, whereas the heterogeneity of different risk classes is more pronounced.

Factor correlations exceed the corresponding average of empirical factor spread correlations in the left part of Table 4.15. Analogously, factor spread correlations show the highest inner-sector correlations between affiliate rating classes, and inter-sector correlations between investment-grade classes are higher than correlations with the NF-BBB class. The relational pattern of average empirical spread correlations and factor correlations is almost identical. Analogously to the rating class model, average asset spread correlations exceed inter-sector asset correlations. A comparison of the average empirical spread correlations to the average filtered spread correlations for the factor and asset value estimation confirms there is a sufficient fit of spread co-movements for filtered factor series and for filtered asset values.

The correlations of yield spreads for single maturities in Table D.8 show the best correspondence to factor correlations for three-year and five-year maturities. Since asset spreads are bootstrapped on the basis of independent specific spread movements, the single-maturity asset-implied spread correlations coincide only roughly with asset correlations due to the sampling effect. Empirical and bootstrapped correlations of factor and asset yield spreads are sufficiently reproduced by correlations of filter-derived spreads.

With regard to the factor and asset correlations of annual estimations in Table D.12, the same considerations regarding the time-inhomogeneity of spreads throughout the overall estimation apply as they do for the rating class model. Annual correlation estimates generally exceed their full sample equivalents in every year except 2003. High inner-class asset correlations indicate risk classes and estimation periods with severe credit events.

Four-Sector Model

The credit dependence of the four-sector model with 10 risk classes is presented in Table 4.17. Factor correlations between high-grade classes (AA and A rating) amount to 42.1% on average, while correlations between high-grade classes and BBB classes average out at 30.5% and correlations between BBB classes exhibit a 25.5% average. Analogously to former risk class models, factor correlations are considerably larger than the 8.0% average of asset correlation between high-grade risk classes, the 4.1% average between high-grade and BBB classes and the 7.8% average between BBB classes. Inner-class asset correlations are comparatively high for the data-scarce sectors ECY-A, LCY-BBB and NCY-A, which suffer from notable obligor-specific credit events. The average of inner-class (inter-class) asset correlations is 15.1% (4.6%), which confirms the suitability of the classification of exposures with respect to the homogeneity of credit risk. Factor and asset correlations of annual estimations in Table D.13 show a high variation in time. However, no systematic pattern of factor and asset correlations is obvious in time and across risk classes.

The average empirical spread correlation between high-grade classes, (between high-grade and BBB classes, between BBB classes) amounts to 33.0% (18.2%, 27.9%), while filter-implied factor spreads result in correlations of 34.7% (22.6%, 16.8%), which is considered to be an acceptable fit, except in the case of BBB inner-class correlations, where the EKF suffers from a lower fitting power due to the high variation of spreads. Equivalent average correlations of bootstrapped yield spreads amount to 7.3% (6.5%, 3.7%), close to asset-implied correlations of 6.1% (6.2%, 3.3%). Obviously, the fit of filtered spread correlations is better for asset spread correlations than for factor-spread correlations. Spread correlations of annual estimations in Table D.9 and D.10 are attached for reasons of completeness.

Factor Correlations

Risk-Class	ECY-A	ECY-BBB	FIN-AA	FIN-A	LCY-AA	LCY-A	LCY-BBB	NCY-AA	NCY-A	NCY-BBB	Avg. diagonal Avg. off-diag.
ECY-A	100.0	40.8	51.7	49.6	52.5	32.0	42.9	39.6	64.1	27.5	100.0
ECY-BBB		100.0	25.7	14.3	35.5	40.5	29.9	26.8	45.4	22.4	35.6
FIN-AA			100.0	82.2	36.6	21.3	33.9	43.6	57.8	22.4	
FIN-A				100.0	32.6	17.2	36.3	33.3	54.0	22.1	
LCY-AA					100.0	13.8	14.6	29.3	44.4	19.2	
LCY-A						100.0	44.7	27.7	39.0	16.5	
LCY-BBB							100.0	25.5	48.1	24.1	
NCY-AA								100.0	60.9	19.9	
NCY-A									100.0	38.6	
NCY-BBB										100.0	

Asset Correlations

Risk-Class	ECY-A	ECY-BBB	FIN-AA	FIN-A	LCY-AA	LCY-A	LCY-BBB	NCY-AA	NCY-A	NCY-BBB	Avg. diagonal Avg. off-diag.
ECY-A	40.0	8.6	6.7	8.8	10.0	4.1	13.1	2.2	27.6	3.5	15.1
ECY-BBB		11.0	1.7	1.3	3.5	2.7	4.8	0.8	10.2	1.5	4.6
FIN-AA			4.2	4.7	2.3	0.9	3.4	0.8	8.1	0.9	
FIN-A				7.9	2.8	1.0	4.9	0.8	10.3	1.2	
LCY-AA					9.1	0.8	2.1	0.8	9.1	1.2	
LCY-A						4.1	4.4	0.5	5.4	0.7	
LCY-BBB							23.3	1.1	15.8	2.3	
NCY-AA								0.7	3.6	0.3	
NCY-A									46.4	5.3	
NCY-BBB										4.1	

Latent Factor Coefficients

Risk-Class	ECY-A	ECY-BBB	FIN-AA	FIN-A	LCY-AA	LCY-A	LCY-BBB	NCY-AA	NCY-A	NCY-BBB
ECY-A	63.3									
ECY-BBB	13.5	30.2								
FIN-AA	10.6	1.0	17.5							
FIN-A	13.9	-1.8	18.6	15.5						
LCY-AA	15.8	4.7	3.0	0.5	25.0					
LCY-A	6.5	6.1	0.8	0.3	-2.0	18.1				
LCY-BBB	20.7	6.5	6.3	6.4	-6.7	13.5	39.4			
NCY-AA	3.4	1.0	2.3	-0.5	0.4	1.1	0.2	7.4		
NCY-A	43.6	14.4	18.8	6.4	3.7	8.8	8.4	19.2	40.0	
NCY-BBB	5.5	2.5	1.8	1.2	0.4	0.9	1.9	1.1	4.1	18.5

Factor Spread Correlations

empirical / filtered	ECY-A	ECY-BBB	FIN-AA	FIN-A	LCY-AA	LCY-A	LCY-BBB	NCY-AA	NCY-A	NCY-BBB
ECY-A	100.0	22.8	46.5	45.6	39.1	22.7	28.2	27.2	57.0	19.2
ECY-BBB	25.7	100.0	17.8	12.4	19.9	32.8	35.0	10.8	19.4	24.1
FIN-AA	38.3	19.1	100.0	78.1	29.7	12.2	21.9	33.1	42.3	14.8
FIN-A	34.6	4.2	82.3	100.0	27.4	11.8	22.7	29.4	39.6	11.6
LCY-AA	51.4	25.2	29.7	27.0	100.0	10.7	15.1	28.0	26.7	12.2
LCY-A	17.1	26.0	8.9	6.7	5.7	100.0	26.6	13.0	25.4	6.6
LCY-BBB	35.5	20.2	24.9	27.0	11.7	45.5	100.0	13.2	24.2	24.8
NCY-AA	34.3	25.5	42.2	30.8	22.7	16.7	22.3	100.0	47.1	9.5
NCY-A	51.0	37.7	51.0	46.8	40.3	28.7	46.1	61.9	100.0	19.5
NCY-BBB	15.4	13.4	11.0	11.2	11.4	7.8	16.9	13.7	26.6	100.0

Asset Spread Correlations

bootstrap / filtered	ECY-A	ECY-BBB	FIN-AA	FIN-A	LCY-AA	LCY-A	LCY-BBB	NCY-AA	NCY-A	NCY-BBB
ECY-A	100.0	7.5	6.5	8.0	13.7	5.7	3.4	5.8	11.0	10.6
ECY-BBB	7.9	100.0	6.0	10.9	10.3	16.1	0.4	6.5	5.5	6.1
FIN-AA	5.6	6.1	100.0	15.3	0.8	-0.5	0.2	6.2	16.0	0.2
FIN-A	8.5	12.3	17.4	100.0	8.0	5.2	4.8	13.6	7.3	5.2
LCY-AA	15.5	9.7	-2.3	5.9	100.0	-2.4	1.4	6.1	11.5	10.8
LCY-A	5.6	13.4	-0.8	5.0	-4.5	100.0	14.5	5.3	2.9	-0.4
LCY-BBB	4.8	-1.3	3.0	2.9	3.8	11.8	100.0	2.7	3.0	4.5
NCY-AA	5.6	6.7	6.4	10.9	4.0	2.7	3.5	100.0	7.5	5.5
NCY-A	11.9	3.7	9.9	6.0	6.2	1.4	4.3	7.7	100.0	12.1
NCY-BBB	11.0	4.8	-0.1	2.7	11.3	-2.0	6.3	3.4	11.0	100.0

Table 4.17: Credit Dependence of Four-Sector Model

4.4.8.5 Conclusion of Estimation

For the estimation of the structural risk class factor model price data from the European corporate bond market from 01/01/1999 to 31/12/2003 was used. Term structures of interest rates for rating and sector-rating classes were fitted using the Nelson-Siegel functional form. In a two-step procedure, optimal factor coefficients, process parameter and filtered series of systematic factors and specific asset values are determined by an EKF-based QML estimation. Observation data for the estimation of the asset value process were bootstrapped to ensure a homogenous quality of yield spreads.

The observation sample of obligors is held constant throughout annual intervals, so that credit spreads of different ratings may intercept in the course of the estimation period. The fitting of riskless term structures results in average price residuals between -0.3 and 0.8 bps. Term structures of credit risk classes show average price residuals in the range of -16.2 to 26.4 bps. Term structures, standard deviations and correlations of empirical and bootstrapped yield spreads are sufficiently reproduced by factor- and asset-implied yield spreads.

Structural credit valuation models with premature default triggered at a constant default threshold show a distinct pattern of fitting errors for the term structure of par-bond yield spreads, due to functional restrictions in the shape of the first-passage time distribution. While an underfitting is observed for one-year and ten-year yield spreads, three-year and five-year spreads are overfitted. Relaxing the assumption of a constant default threshold, as it is done in the model presented by Hull et al. (2005), will provide a better fit of individual term structures. However, the accompanying increase of model complexity renders the model inappropriate for large portfolio applications.

Estimated asset correlations range from 0.3% to 46.4%, but have an average value of 9.4% (10.1%, 15.1%) for inner-class correlations and 4.3% (4.1%, 4.6%) for inter-class correlations of the rating class (two-sector, four-sector) model. A comparison of factor and asset correlations of annual estimates with those of the total estimation period reveals a considerable variation of correlations in time. However, it is unclear, whether the variation of correlations represents a sampling effect or results from the time-inhomogeneity of correlations.

Estimation periods that span a complete credit cycle result in drift-rate estimates close to zero. In connection with strictly positive riskless rates it yields real-world probabilities of default that exceed risk-neutral default probabilities and that deviate considerably from historical default rates as determined by rating agencies. The development of credit valuation models that incorporate mean-reversion of asset values may provide a better concordance of real-world and risk-neutral default probabilities.

Chapter 5

Simulation Results

The backtesting approach introduced in Section 2.4.4 is assessed in a simulation study based on the structural risk class factor model from Chapter 3 and the definition of credit loss in Section 2.3.5. Different specifications of the adequacy zones of portfolio credit loss are examined. The upper bound of the green zone of model adequacy (acceptance barrier) is based on an alternative parameter specification of the credit portfolio model under hypothesis $\overline{H}_0$ in (2.22), which is more conservative from a regulatory point of view. The lower bound of the red zone of model rejection (rejection barrier) represents a quantile of the loss distribution provided by the model to be tested under hypothesis $\overline{H}_0$ in (2.21). The range between the acceptance barrier and the rejection barrier is termed as the yellow zone of model indetermination. After the general decisions on the backtesting hypotheses and the related specification of backtesting zones, the simulated distributions of credit portfolio loss, the resulting capital charges and the presumed supervisory judgement on the model adequacy are examined for different portfolios, parameter specifications and structural variations of the portfolio model.

The composition of a basis portfolio of homogenous loans and a diversified portfolio of heterogenous loans that resembles the characteristics of real-word corporate loan portfolios is detailed in Section 5.1. Section 5.2 outlines the simulation model. In order to gain insights for the specification of the backtesting procedure the impact of the

- level of significance,
- definition of credit loss, and
- alternative model

on the location of the adequacy zones of credit loss is examined in Section 5.3. In Section 5.4 a basis case scenario is examined to identify the effects of a variation of model structure, model parameterization and portfolio characteristics on portfolio credit loss and the location of zones of model adequacy. The basis case is specified by the basis portfolio of homogenous loans, a single risk class, a time horizon of one year and four intermediate simulation intervals. For the basis case, the model components examined with regard to their effect on backtesting are:

- asset correlation,
- default model (Merton-type vs. premature default),
- number of simulation intervals,
- length of time horizon,
- homogenous probability of default,
- homogenous time-to-maturity,
- granularity of principal values,
- risk class model, and
- drift and volatility parameters.

In Section 5.5, a diversified portfolio with heterogenous probabilities of default, maturities and principal value of exposures is considered, so that the portfolio better reflects the characteristics of a real-world corporate loan portfolio. Furthermore, the risk class structure distinguishes between two economic sectors with different inner-sector and inter-sector asset correlations. This diversified portfolio case is examined with regard to a variation in

- the number of simulation intervals,
- the length of the forecast period,
- drift and volatility parameter

affecting portfolio credit loss and zone locations. Finally, in Section 5.6, the diversified portfolio is used in combination with the parameter estimates from Chapter 4 to determine the zone locations of the backtesting for a real-world model specification. Compared to the synthetic model parameterization described in Section 5.5 the diversified portfolio will refer to empirically calibrated rating-class, two-sector and four-sector models that specify credit dependence on the basis of risk classes defined by the rating and sector affiliation of exposures.

5.1 Portfolio Composition

Simulations are based on a portfolio of $N = 900$ exposures $i = 1, ..., N$, characterized by a one-year probability of default $p_i = P[T_i \leq 1]$, coupon rate c^i, maturity T_i and principal amount K_i as well as drift parameter μ_i and volatility σ_i of asset value V_t^i. Omitting the interest rate and recovery risk, the riskless rate of each exposure i is fixed at $r_i = r = 5\%$ and a homogenous recovery rate $\varrho_i = \varrho = 50\%$ is set for any simulation scenario. The Parameter variations considered include maturity, face value, PD, drift and volatility parameters, as well as asset correlations either for a subset of exposures or for the entire portfolio.

Variations in the characteristics of exposures and in the underlying portfolio model are analyzed based on a homogenous basis portfolio and a diversified basis portfolio. The homogenous basis portfolio consists of exposures with identical time to maturity $T_i = T = 5$, principal value $K_i = K = 1$, default threshold $\overline{V}^i = \overline{V} = K$ and one-year probability of default $p_i = p = 1\%$ for any exposure $i = 1, ..., N$. The dynamics of asset values V_t^i are specified by a homogenous drift $\mu_i = \mu = 0\%$ and standard deviation $\sigma_i = \sigma = 10\%$. Portfolio characteristics closer to real-world credit portfolios are received by relaxing

Parameter	**Homogenous Basis Portfolio**	**Diversified Basis Portfolio**	**Parameter Set**
r	5%	5%	$\{5\%\}$
μ_i	0%	0%	$\{0\%, 8\%\}$
σ_i	10%	10%	$\{10\%\}$
p_i	1%	$\{0.5\%, 1.5\%\}$	$\{0.5\%, 1\%, 1.5\%\}$
T_i	1	$\{1, 5, 10\}$	$\{1, 5, 10\}$
K_i	1	$\{1, 10, 100\}$	$\{1, 10, 100\}$
ϱ_i	50%	50%	$\{50\%\}$
ρ_{ij}^a			$\{0\%, 5\%, 10\%, 15\%, 20\%, 25\%, 30\%\}$

Table 5.1: Parameter Sets

the requirement of homogeneity for face value, PD, maturity and sector affiliation of exposures. The diversified basis portfolio divides exposures into one half with probability of default $p_i = 0.5\%, i \leq 450$ and a second half of exposures with $p_i = 1.5\%, i > 450$, with the average PD of exposures remaining unchanged at 1%. A face value of $K_i = 1(10, 100)$ is assigned to $563(225, 112)$ exposures, which approximately represents $4\%(16\%, 80\%)$ of the total face value of the portfolio. A maturity of $T_i = 1(5, 10)$ is set to one third of the exposures, so that the average maturity of the diversified portfolio increases to 5.33 years. Finally, a second risk class assumed to represent a different economic sector is introduced and exposures are divided by half between sectors. The correlation structure of exposures is now characterized by asset correlations between exposures of the same sector (inner-sector correlation) and by asset correlations between exposures of different sectors (inter-sector correlation). The inner-sector correlations are set equal for both sectors and the homogenous inter-sector correlations are considered to be lower than inner-sector correlations for any simulation scenario. Conditional on the restrictions above, the attributes of face value, PD, time-to-maturity and sector affiliation are distributed equally across exposures. A summary of the considered variation of parameters for both the homogenous and the diversified basis portfolio is given in Table 5.1.

5.2 Simulation Procedure

The risk class factor model of Section 3.5 based on the structural first-passage credit valuation model of Section 3.2 is used to repeatedly simulate scenarios of jointly moving asset values in order to derive distributions of credit portfolio value and portfolio credit loss for different portfolios and model settings.

In the basis case, a one-year holding period between the time of risk consideration $\underline{t} = 0$ and time horizon $\bar{t} = 1$ is used in accordance with common practice in banking. The initial asset value V_0^i and coupon rate c^i of exposures $i = 1, ..., N$ are calibrated in a two-stage process. First, given drift μ_i and volatility σ_i, asset value V_0^i is calibrated, so that the distribution function of the first-passage time $F_T(\bar{t}) = P[\tau_i \leq \bar{t}] = p_i$ equals the exogenous one-year default probability. Second, using the valuation model presented in Chapter 3, coupon rate c^i is adapted to calibrate the present value $D(V_0^i, 0; \phi_i) = D_0^i = K_i$ of exposure i to equal par value K_i. With exposure characteristics restricted to full-year maturities T_i and annual interest frequency, the interest $c^i K_i$ of any exposure is not paid not before time horizon $\bar{t}$. The simulation of the joint evolution of asset values proceeds as follows:

1. For interval h of the holding period a multi-variate standard-normal vector of independent discrete-time factor returns $\Delta F_{t_h^\epsilon}$ until t_h is drawn by random

2. The N-dimensional multi-variate standard-normal vector $\boldsymbol{\Delta\epsilon_{t_h}}$ of independent specific factor returns of exposures is drawn.

3. The vector of normalized asset returns $\Delta V_{t_h}^\epsilon$ is calculated using a discrete-time form of (3.33). The vector V_{t_h} of asset values at the end t_h of sub-period h is derived using (4.43).

4. If $V_{t_h}^i \leq K_i$, exposure i has unambiguously defaulted during sub-period h. If $V_{t_h}^i > K_i$, the premature default in interval h is simulated by a random draw of a Bernoulli default variable with conditional default probability $P[\tau_i < 1/h | V_{t_h}^i]$. In case of default, the value $D_{t_h}^i$ of the exposure is set to recovery value βK_i and the default time is set at $\tau_i = t_h$.

5. For exposures not defaulted up to time t_h, steps $1-4$ are repeated until $t_h = \bar{t}$.

6. At time horizon $\bar{t}$, the values of credit exposures are calculated. In case of non-default, it is $D_{\bar{t}}^{i} = D(V_{\bar{t}}^{i}, t, \phi_i)$, if $\tau_i \leq \bar{t}$, it is $D_{\bar{t}}^{i} = K_i \beta e^{(\bar{t}-\tau^i)r}$.

7. The credit loss of exposures and the aggregate portfolio credit loss are calculated according to the loss definitions $L(\mathbb{E}[D_1]) - L(D_0)$.

8. Distributions of portfolio credit loss, portfolio value and default rate are generated by repeating steps 1-7 for $50,000$ times.

First-passage time default models allow continuous-time credit default within any pre-mature time interval. However, as outlined before, a multi-variate distribution of asset value-derived continuous-time default times is not available in closed form. Instead, the simulation of joint asset value-derived default times is performed in a two-stage procedure. At first, correlated asset values at the end of sub-intervals of the holding period are simulated by steps $1-3$ without consideration of the absorbing default barrier being hit in the mean time. Exposures with $V_{t_h}^{i} \leq \overline{V}^{i}$ are set to credit default status at time t_h. In the case that an unrestricted asset value $V_{t_h}^{i} > \overline{V}_i$ exceeds the default barrier at time t_h, the occurrence of a pre-mature default event is simulated in a second step using a Bernoulli default variable with conditional default probability $P[\tau_i < t_h | V_{t_h-1/h}^{i}, V_{t_h}^{i}]$ derived in Appendix E.[1]

In any case of default in interval h, the default time τ^i is set equal to t_h. The specification of default times is motivated by the monotonous increase of the first-passage density in the short term, which makes defaults at the end of a sub-interval more likely. The measurement error introduced by the discretization of default times is considered to be small and is therefore neglected. In any case, the specification of default times is conservative, since a deferral of the default time reduces accruals on the recovery received up to the time $\bar{t}$ and accordingly reduces the portfolio value.

The value $D_{\bar{t}}^{i} = D(V_{\bar{t}}^{i}, \bar{t}; \phi_i)$ of a non-defaulted exposure is derived from the simulated asset value $V_{t_h}^{i}$ at time horizon $\bar{t}$, and the credit loss of exposures is calculated according to the loss definitions $L^i(D_0^i)$ and $L^i(\mathbb{E}[D_1^i])$ from Chapter 2.3.5.

With respect to backtesting, each simulation run is executed twice, once using the model specification to be tested and once for the alternative model in question. Estimation errors for portfolio loss quantiles cannot be determined analytically, and the determination of approximative standard errors of portfolio loss quantiles is omitted because of the disproportionate efforts associated with a repeated simulation of loss distributions.

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[1] Drawing exact default times of exposures is too expensive computationally, since it involves the inversion of the distribution function of conditional default times in Appendix E.

5.3 Specification of Backtesting

5.3.1 Loss Definition

The definition of credit loss is critical in a mark-to-model portfolio valuation. For the decision on the credit losses measure used in backtesting, the location of adequacy zones is analyzed for different definitions of credit loss. The major characteristics that determine the properties of a credit loss definition are:

- inclusion of interest income,
- applied reference value, and
- consideration of non-realized mark-to-model profits.

Admittedly, a shift in the location of adequacy zones caused by a different definition of credit loss coincides with an according shift in portfolio credit loss observation, so that the decision on model adequacy will generally not be altered. However, the applied credit loss definitions, differ in the variation of zone locations with respect to different portfolio characteristic and model specifications.

In Table 5.2, the $1\%-$, $50\%-$, and $99\%-$quantiles of portfolio credit loss are given as a percentage of portfolio value D_0 for different asset correlations. For the basis case, realized gains do not appear due to a homogenous maturity $T = 5$ of exposures beyond the time horizon of risk, so that loss definitions including and excluding unrealized mark-to-model gains do not differ.

For the diversified case, a positive credit performance results either from exposures matured at $\bar{t}$ or from a positive change in the mark-to-model values of exposures. In the lower part of Table 5.2 the credit loss of the diversified case portfolio is compared with respect to exclusion vs. inclusion of unrealized mark-to-model profits. If unrealized profits are excluded, only exposures with non-positive credit performance, i.e. a positive difference between the reference value and the exposure value at time $\bar{t}$, are considered in portfolio loss. Including mark-to-market profits instead, positive credit performance compensates for credit losses of other exposures, so that loss quantiles shift marginally to the left and are even negative for $q_{0.01}$ for $L(\mathbb{E}(D_1^i))$.

There are two reference values considered: the par value D_0^i of exposures at time $t = 0$ and the expected value $\mathbb{E}(D_1^i)$ of the exposure at the risk horizon. Portfolio loss is considered for both reference values including and excluding interest $K_i c^i$ paid at the risk horizon. Asset correlations are considered in a range between 0%, which the represents independence of exposures, and 30%. The former inner-sector correlation is set equal

Interest income		**included**						**not included**					
Loss Reference		D_0			$\mathbb{E}(\mathbf{D_1})$			D_0			$\mathbb{E}(\mathbf{D_1})$		
Scenario	$\rho^{\mathbf{a}}$	$q_{0.01}$	$q_{0.5}$	$q_{0.99}$	$q_{0.01}$	$q_{0.5}$	$q_{0.99}$	$q_{0.01}$	$q_{0.5}$	$q_{0.99}$	$q_{0.01}$	$q_{0.5}$	$q_{0.99}$
Basis case	0%	2.74	3.30	3.93	1.93	2.44	3.03	2.99	3.54	4.15	1.91	2.40	2.96
	5%	1.18	3.14	6.97	0.74	2.29	5.63	1.35	3.39	7.21	0.74	2.26	5.46
	10%	0.71	2.97	9.08	0.40	2.13	7.51	0.83	3.22	9.26	0.40	2.10	7.26
	15%	0.43	2.78	10.82	0.22	1.95	9.10	0.52	3.04	10.97	0.22	1.94	8.77
	20%	0.26	2.62	12.47	0.12	1.80	10.60	0.33	2.89	12.56	0.12	1.79	10.21
	25%	0.16	2.45	14.10	0.07	1.65	12.15	0.21	2.73	14.15	0.07	1.64	11.66
	30%	0.09	2.28	15.71	0.03	1.48	13.66	0.13	2.56	15.68	0.03	1.48	13.11
Diversified case excl. realized mtm profits	0% - 0%	1.39	2.40	3.88	0.87	1.79	3.19	1.54	2.53	3.92	0.87	1.75	3.05
	5% - 5%	0.70	2.28	5.59	0.39	1.67	4.65	0.81	2.41	5.62	0.39	1.64	4.44
	10% - 10%	0.40	2.16	7.13	0.19	1.55	6.04	0.48	2.29	7.14	0.20	1.53	5.79
	15% - 15%	0.25	2.03	8.43	0.10	1.43	7.22	0.31	2.17	8.38	0.10	1.41	6.90
	20% - 20%	0.15	1.90	9.76	0.05	1.30	8.46	0.20	2.05	9.65	0.05	1.29	8.08
	25% - 25%	0.09	1.77	11.09	0.02	1.18	9.71	0.12	1.92	10.91	0.02	1.18	9.26
	30% - 30%	0.05	1.64	12.13	0.01	1.06	10.70	0.08	1.80	11.89	0.01	1.06	10.16
Diversified case incl. realized mtm profits	0% - 0%	1.34	2.35	3.83	0.68	1.60	3.00	1.54	2.53	3.92	0.70	1.58	2.89
	5% - 5%	0.66	2.23	5.54	0.20	1.48	4.47	0.81	2.41	5.62	0.22	1.47	4.29
	10% - 10%	0.36	2.11	7.09	0.00	1.37	5.86	0.48	2.29	7.14	0.02	1.36	5.63
	15% - 15%	0.20	1.98	8.39	-0.09	1.24	7.05	0.31	2.17	8.38	-0.07	1.24	6.74
	20% - 20%	0.11	1.85	9.72	-0.14	1.12	8.29	0.20	2.05	9.65	-0.12	1.12	7.93
	25% - 25%	0.04	1.72	11.05	-0.17	1.00	9.55	0.12	1.92	10.91	-0.15	1.01	9.10
	30% - 30%	0.01	1.59	12.09	-0.18	0.88	10.54	0.08	1.80	11.89	-0.16	0.89	10.00

Table 5.2: Quantiles for Different Loss Definitions

to the latter inter-sector correlation for the diversified portfolio. With increasing asset correlation, the variation of loss distributions increases and the median portfolio loss decreases. Since interest income of the first period represents additional loss potential, the distances between the quantiles $q_{0.01}$ and $q_{0.99}$ increase with the inclusion of interest at time $\bar{t}$ in portfolio value D_1. In practical applications, the negligence of interest income will mean the exclusion of calculatory effects such as margin requirements, administration costs and refinancing conditions, as outlined in Section 2.3.5. Overall, omitting interest paid at t_1 reduces the spreading of portfolio loss and will enhance the discriminatory power of the backtesting.

Bühler and Engel (2006) reveal that loss distributions that refer to D_0^i are especially sensitive towards a change in the drift rate μ, whereas credit portfolio loss is more robust with respect to a change in parameters if it refers to the expected values $\mathbb{E}(D_1^i)$ of exposures. Furthermore, considering reference values $\mathbb{E}(D_1^i)$, the distance between the quantiles $q_{0.01}$ and $q_{0.99}$ is smaller for both the homogenous and the diversified basis case portfolio, and the location of quantiles is more robust, so that derived zones of model adequacy will be less sensitive to portfolio composition and model specification.

In the following, portfolio loss $L(\mathbb{E}[D_1])$, defined according to (2.6), is preferred as definition of portfolio loss, as it excludes interest income and unrealized mark-to-model and obeys reference values $\mathbb{E}(D_1^i)$. Furthermore, $L(\mathbb{E}[D_1])$ is in line with the objective of the Basel Committee in that it only takes unexpected loss for the determination of capital charges into account, while the consideration of reference values D_0^i in loss definition $L(D_0)$ according to (2.6), does not consider the expected dynamics of default risk throughout the lifetime of exposures, so that portfolio loss is not restricted to unexpected loss. In conclusion, loss definition $L(\mathbb{E}[D_1])$ fits best the criteria of methodological consistency, robustness, independence of the accounting regime and prudence. However, adequacy zones defined on the basis of $L(D_0)$ will be considered additionally as a reference, because $L(D_0)$ is defined as being equivalent to market VaR.

5.3.2 Alternative Model

Backtesting VaR models, the location of the green zone is defined by the 0.5%−quantile of an alternative model, whose 95%−quantile of portfolio market rate loss equals the VaR predicted at a 99% level of confidence. From Section 2.4.4, it is known that the binomial test of the VaR backtesting is meaningfully applicable only if a sufficient number of independent observations is available, for example if the conditional pooled PD of a risk class is to be tested using a cross-section of single-exposure default observations of a credit portfolio. For the backtesting of the unconditional probability of default or the portfolio credit loss, the binomial test is dismissed due to a lack of sufficient independent observations of annual portfolio performance required for a satisfactory significance of the test.

The backtesting approach suggested in Section 2.4.4 is based on a single observation of portfolio credit loss with zones of model adequacy defined by quantiles of the loss distribution of the model to be tested and of an alternative more conservative model. Although the backtesting approach is defined in principle, the specification of the alternative model to be rejected is still undetermined. Model and portfolio characteristics that inter-relatedly impact portfolio credit risk and that are considered to be changed in the alternative model are the drift and diffusion rates of asset values, correlations, the average and concentration of face values, PD, recovery rates and the time-to-maturity of the exposures. Distributions of credit loss are examined for more conservative specifications of exposures' (1) drift rate, (2) asset volatility, (3) probability of default, and (4) asset correlation to decide on the alternative model for backtesting. Model alternatives considered include combinations of the following changes in portfolio and model specification:

- drift rate $\overline{\mu} \in \{0\%, -8\%\}$,
- volatility $\overline{\sigma} \in \{\sigma, \sigma + 10\%\}$,
- default probability $\overline{p} \in \{p, p + 1\%\}$, and
- asset correlation $\overline{\rho}^a = \rho^a + 5\%$.

For the distribution of default rate and portfolio loss $L(\mathbb{E}[D_1])$ and $L(D_0)$, the 0.5%−, 1%−, and 2.5%−quantiles, which potentially qualify as acceptance barrier, are presented in Table 5.3, where the left four columns indicate the respective parameter set. An additional indicator for the appropriateness of an alternative model is the median portfolio loss for the model to be tested, which is required to be lower than the acceptance barrier to ensure that at least half of the portfolio observations of a correct model fall into the green zone.

For any parameter set, quantiles of the default rate are not convenient to define the green zone of model adequacy, because the absence of defaults is required for a model to qualify as adequate in most cases. Only for an asset correlation $\overline{\rho}^a \leq 10\%$ do the parameter sets with $\overline{p} = 2\%$ allow for a default rate acceptance barrier $\overline{q}_{\overline{\alpha}} > 0$ for level $\overline{\alpha} \leq 97.5\%$ of confidence. The parameter set $\{\overline{\mu} = 0\%, \overline{\sigma} = 10\%, \overline{p} = 1\%\}$ constitutes the basis

Parameter set				Default rate				$L(\mathbb{E}[D_1])$				$L(D_0)$			
$\overline{\mu}$	$\overline{\sigma}$	$\overline{p}$	$\rho^a/\overline{\rho}^a$	$\overline{q}_{0.005}$	$\overline{q}_{0.01}$	$\overline{q}_{0.025}$	$\overline{q}_{0.5}$	$\overline{q}_{0.005}$	$\overline{q}_{0.01}$	$\overline{q}_{0.025}$	$\overline{q}_{0.5}$	$\overline{q}_{0.005}$	$\overline{q}_{0.01}$	$\overline{q}_{0.025}$	$\overline{q}_{0.5}$
0%	10%	1%	0%	0.22	0.33	0.44	1.00	1.86	1.91	1.99	2.40	2.93	2.99	3.08	3.54
			5%	0.00	0.00	0.11	0.89	0.65	0.74	0.90	2.26	1.20	1.35	1.58	3.39
			10%	0.00	0.00	0.00	0.67	0.33	0.40	0.54	2.10	0.71	0.83	1.05	3.22
			15%	0.00	0.00	0.00	0.56	0.17	0.22	0.33	1.94	0.40	0.52	0.72	3.04
			20%	0.00	0.00	0.00	0.44	0.08	0.12	0.20	1.79	0.25	0.33	0.50	2.89
			25%	0.00	0.00	0.00	0.33	0.04	0.07	0.12	1.64	0.15	0.21	0.35	2.73
			30%	0.00	0.00	0.00	0.22	0.02	0.03	0.07	1.48	0.08	0.13	0.23	2.56
			35%	0.00	0.00	0.00	0.22	0.01	0.02	0.04	1.34	0.05	0.08	0.16	2.41
		2%	0%	0.89	1.00	1.11	2.00	3.01	3.07	3.17	3.72	4.37	4.44	4.55	5.14
			25%	0.00	0.00	0.00	0.89	0.10	0.15	0.27	2.74	0.29	0.40	0.63	4.16
	20%	1%	0%	0.22	0.33	0.44	1.00	4.58	4.65	4.75	5.30	6.58	6.65	6.75	7.34
			25%	0.00	0.00	0.00	0.33	0.33	0.45	0.70	4.43	0.89	1.14	1.59	6.55
		2%	0%	0.89	1.00	1.11	2.00	6.43	6.51	6.63	7.29	8.67	8.75	8.88	9.58
			5%	0.22	0.22	0.44	1.78	3.12	3.44	3.91	7.11	4.82	5.20	5.77	9.41
			10%	0.00	0.11	0.11	1.56	2.02	2.32	2.83	6.92	3.42	3.81	4.48	9.23
			15%	0.00	0.00	0.00	1.33	1.34	1.62	2.11	6.73	2.54	2.92	3.60	9.06
			20%	0.00	0.00	0.00	1.11	0.91	1.15	1.61	6.55	1.90	2.28	2.94	8.91
			25%	0.00	0.00	0.00	0.89	0.59	0.82	1.21	6.34	1.41	1.79	2.40	8.72
			30%	0.00	0.00	0.00	0.78	0.42	0.58	0.91	6.17	1.11	1.41	1.98	8.57
			35%	0.00	0.00	0.00	0.56	0.24	0.38	0.66	5.97	0.77	1.06	1.59	8.40
-8%	10%	1%	0%	0.22	0.33	0.44	1.00	2.09	2.14	2.22	2.66	3.23	3.29	3.38	3.87
			25%	0.00	0.00	0.00	0.33	0.05	0.08	0.15	1.86	0.18	0.25	0.40	3.03
		2%	0%	0.89	1.00	1.11	2.00	3.37	3.44	3.55	4.11	4.82	4.89	5.01	5.62
			25%	0.00	0.00	0.00	0.89	0.12	0.19	0.32	3.10	0.35	0.48	0.74	4.62
	20%	1%	0%	0.22	0.33	0.44	1.00	4.81	4.87	4.98	5.53	6.83	6.91	7.02	7.62
			25%	0.00	0.00	0.00	0.33	0.36	0.49	0.76	4.66	0.95	1.21	1.69	6.82
		2%	0%	0.89	1.00	1.11	2.00	6.74	6.83	6.95	7.62	9.02	9.11	9.24	9.94
			25%	0.00	0.00	0.00	0.89	0.66	0.88	1.32	6.69	1.53	1.91	2.56	9.10

Table 5.3: Quantiles for Alternative Parameter Sets (Basis Case)

case scenario with a homogenous portfolio, so that the choice of an alternative model is restricted to parameter sets that differ by elevated asset correlations. Loss quantiles and accordingly, the extension of a green zone decrease with increasing correlation, which is unfavorable with respect to the discrimination power of backtesting.

The remaining parameter sets represent a change in the basis case portfolio with respect to the drift rate, volatility and PD assumed for the parameter set of the alternative model. For all parameter sets loss distributions shift to the right, so that the quantiles considered as potential acceptance barrier increase, while the elevation of the asset correlation is accompanied by a left shift in quantiles and a decrease in the discriminatory power.

If changes in the drift rate, volatility and PD are combined, the right shift in loss distributions is more pronounced and the potential extension of the green zone is enlarged, with the maximum effect occurring for the parameter set $\{\overline{\mu} = -8\%, \overline{\sigma} = 20\%, \overline{p} = 2\%\}$.

Defining the alternative model on the basis of an increased asset correlation provides a two-fold incentive for banks to underestimate asset correlation. First, capital charges derived from Credit-VaR decrease with the estimated asset correlation. Second, the green zone will simultaneously expand which reduces the risk of model rejection. In contrast, the underestimation of PD lowers capital charges, but at the same time increases the risk of model rejection caused by a reduced green zone. In consequence, a fixed asset correlation $\overline{\rho}^a = 25\%$ will be set for the alternative model, which seems to be conservative with respect to the estimates of asset correlations.

Furthermore, negative drift rates will be omitted, because the corresponding increase in calibrated asset values at $t = 0$ will elevate cumulative probabilities of default in the long term only, which is considered not to be relevant for examinations of annual credit loss. Additionally, the alternative model must provide a conservative representation of a through-the-cycle credit portfolio model, which is not appropriately represented by negative drift rates as affirmed in the previous chapter by the close-to-zero drift rate estimates for the five-year estimation period.

The alternative default probability $\overline{p}$ represents an inadequate specification of exposures' default probability by the rating model, while asset volatility $\overline{\sigma}$ refers to an error in the estimation of asset value processes. A reasonable backtesting of a model's ability to set adequate capital requirements will consider both sources of modelling error, so that setting

$$\overline{\mu_i} = \mu_i, \qquad \overline{\sigma_i} = \sigma_i + 10\%, \qquad \overline{p_i} = p_i + 1\%, \qquad \overline{\rho}^a_{i,j} = 25\% \tag{5.1}$$

for exposure $i, j = 1, ..., N$ will define the alternative model of backtesting.[2]

[2] Setting $\{\overline{\mu} = 0\%, \overline{\sigma} = 20\%, \overline{p} = 2\%, \overline{\rho}^a = 25\%\}$ stipulates an acceptance barrier that covers the credit loss incurred by financial institutions in the distressed credit markets between 2001 to 2003. In this consideration, $L(D_0)$ is assumed to approximate the definition of credit loss used by financial institutions and differences in quality and composition of credit portfolios are neglected.

5.3.3 Level of Significance

The objective of backtesting is to ensure with a specified level of confidence that the credit portfolio models employed by financial institutions are adequate to set capital requirements. Setting the significance level of backtesting, therefore, has to consider the level of confidence required for an adequate model and the size of credit loss must be specified, which is considered to be immaterial for the assumption of an appropriate credit risk model. Quantiles of the loss distributions of the model to be tested and its alternative

Dist. Type	$\overline{\rho}^{a}$	$\overline{q}_{0.0001}$	$\overline{q}_{0.001}$	$\overline{q}_{0.005}$	$\overline{q}_{0.01}$	$\overline{q}_{0.025}$	$\overline{q}_{0.05}$	$\overline{q}_{0.1}$	$\overline{q}_{0.5}$
Default Rate	25%	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.89
$\mathbf{L}(\mathbb{E}[\mathbf{D_1}])$	25%	0.14	0.30	0.59	0.82	1.21	1.69	2.34	6.34
$\mathbf{L}(\mathbf{D_0})$	25%	0.48	0.86	1.41	1.79	2.40	3.10	3.98	8.72

Dist. Type	ρ^{a}	$q_{0.5}$	$q_{0.9}$	$q_{0.95}$	$q_{0.975}$	$q_{0.99}$	$q_{0.995}$	$q_{0.999}$	$q_{0.9999}$
Default Rate	0%	1.00	1.44	1.56	1.67	1.89	2.00	2.11	2.44
	5%	0.89	1.89	2.33	2.78	3.33	3.78	4.78	6.78
	10%	0.67	2.22	3.00	3.78	4.89	5.78	8.00	11.22
	15%	0.56	2.44	3.44	4.56	6.22	7.56	11.44	17.78
	20%	0.44	2.56	3.78	5.33	7.56	9.56	14.33	21.67
	25%	0.33	2.67	4.22	6.11	8.89	11.22	18.33	29.56
	30%	0.22	2.67	4.44	6.78	10.33	13.67	22.22	34.22
$\mathbf{L}(\mathbb{E}[\mathbf{D_1}])$	0%	2.40	2.70	2.79	2.87	2.96	3.02	3.16	3.28
	5%	2.26	3.76	4.30	4.83	5.46	5.94	7.09	8.73
	10%	2.10	4.37	5.25	6.11	7.26	8.07	9.90	12.39
	15%	1.94	4.76	5.98	7.18	8.77	9.84	12.80	16.77
	20%	1.79	5.19	6.66	8.20	10.21	11.81	15.32	20.15
	25%	1.64	5.52	7.31	9.21	11.66	13.42	18.10	23.90
	30%	1.48	5.73	7.84	10.08	13.11	15.32	20.57	26.84
$\mathbf{L}(\mathbf{D_0})$	0%	3.54	3.87	3.97	4.06	4.15	4.22	4.36	4.52
	5%	3.39	5.22	5.87	6.48	7.21	7.74	9.02	10.78
	10%	3.22	5.98	7.01	7.98	9.26	10.16	12.16	14.82
	15%	3.04	6.49	7.89	9.20	10.97	12.13	15.32	19.38
	20%	2.89	7.02	8.69	10.38	12.56	14.27	17.96	22.93
	25%	2.73	7.46	9.46	11.51	14.15	15.96	20.79	26.72
	30%	2.56	7.75	10.08	12.49	15.68	18.02	23.36	29.73

Table 5.4: Quantiles of Significance Test (Basis Case)

determine the significance level of the backtesting and the location of the green, yellow and red zones of portfolio loss that prompt supervisory action in terms a potential add-on to the capital charge or a detailed supervisory examination of Credit-VaR related processes and models.

For different levels $1-\alpha$ and $\overline{\alpha}$ of test significance with respect to the model to be tested and the alternative model defined in Section 5.3.2, the quantiles of default rate, $L(\mathbb{E}[D_1])$ and $L(D_0)$ for the homogenous and the diversified basis case portfolios are depicted in Tables 5.4 and 5.5.

The difference between the acceptance barrier $\overline{q}_{\overline{\alpha}}$ and the rejection barrier $q_{1-\alpha}$ determines the yellow zone of model indetermination, which is tightened if $\overline{\alpha}$ or α is increased. The median $q_{0.5}$ of portfolio loss for a tested model must be smaller than the acceptance barrier to ensure that at least half of the portfolio observations of a correct model are in the green zone.

For the homogenous basis case, both $L(\mathbb{E}[D_1])$ and $L(D_0)$ provide $\overline{q}_{0.01} < q_{0.5}$ for any asset correlation considered. For $L(\mathbb{E}[D_1])$, loss quantile $\overline{q}_{0.1}$ supersedes $q_{0.5}$ for any of the indicated correlations $\rho^a > 5\%$, whereas an acceptance bound $\overline{q}_{0.05}$ exceeds the median of $L(\mathbb{E}[D_1])$ only for $\rho^a > 25\%$. Accordingly, for $L(D_0)$, it is $\overline{q}_{0.1} > q_{0.5}$ for any positive asset correlation, whereas $\overline{q}_{0.05}$ exceeds $q_{0.5}$ only for ρ^a of 15% and above. For the diversified

Dist. Type	$\overline{\rho}^{\mathbf{a}}$	$\overline{\mathbf{q}}_{\mathbf{0.0001}}$	$\overline{\mathbf{q}}_{\mathbf{0.001}}$	$\overline{\mathbf{q}}_{\mathbf{0.005}}$	$\overline{\mathbf{q}}_{\mathbf{0.01}}$	$\overline{\mathbf{q}}_{\mathbf{0.025}}$	$\overline{\mathbf{q}}_{\mathbf{0.05}}$	$\overline{\mathbf{q}}_{\mathbf{0.1}}$	$\overline{\mathbf{q}}_{\mathbf{0.5}}$
Default Rate	25% - 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.89
$\mathbf{L}(\mathbb{E}[\mathbf{D_1}])$	25% - 25%	-0.02	0.22	0.55	0.74	1.09	1.47	2.04	5.19
$\mathbf{L}(\mathbf{D_0})$	25% - 25%	0.53	1.03	1.54	1.83	2.33	2.84	3.54	7.07

Dist. Type	$\rho^{\mathbf{a}}$	$\mathbf{q_{0.5}}$	$\mathbf{q_{0.9}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.975}}$	$\mathbf{q_{0.99}}$	$\mathbf{q_{0.995}}$	$\mathbf{q_{0.999}}$	$\mathbf{q_{0.9999}}$
Default Rate	0% - 0%	1.00	1.44	1.56	1.67	1.89	2.00	2.11	2.44
	10% - 0%	0.89	1.89	2.44	2.89	3.56	4.00	5.11	6.44
	10% - 5%	0.78	2.11	2.67	3.22	4.00	4.67	6.33	9.11
	10% - 10%	0.78	2.22	2.89	3.67	4.67	5.44	7.56	10.78
	20% - 0%	0.67	2.33	3.11	4.00	5.22	6.22	8.78	12.67
	20% - 10%	0.56	2.44	3.33	4.56	6.00	7.44	10.56	14.67
	20% - 20%	0.44	2.56	3.67	5.11	7.22	9.00	13.44	20.00
$\mathbf{L}(\mathbb{E}[\mathbf{D_1}])$	0% - 0%	1.58	2.25	2.46	2.66	2.89	3.04	3.43	3.88
	10% - 0%	1.47	2.84	3.33	3.79	4.41	4.87	5.71	7.15
	10% - 5%	1.42	3.01	3.61	4.20	5.02	5.56	6.82	8.81
	10% - 10%	1.36	3.18	3.90	4.62	5.63	6.36	7.91	10.33
	20% - 0%	1.32	3.25	4.04	4.85	5.84	6.56	8.24	11.02
	20% - 10%	1.23	3.51	4.52	5.48	6.77	7.88	9.77	13.96
	20% - 20%	1.12	3.73	4.89	6.09	7.93	9.21	12.24	16.86
$\mathbf{L}(\mathbf{D_0})$	0% - 0%	2.53	3.24	3.47	3.68	3.92	4.09	4.47	4.88
	10% - 0%	2.41	4.00	4.56	5.05	5.73	6.25	7.16	8.70
	10% - 5%	2.36	4.22	4.90	5.55	6.45	7.05	8.37	10.52
	10% - 10%	2.29	4.43	5.25	6.04	7.14	7.90	9.62	12.11
	20% - 0%	2.26	4.50	5.38	6.24	7.32	8.06	9.84	12.90
	20% - 10%	2.16	4.85	5.96	7.02	8.39	9.56	11.59	15.85
	20% - 20%	2.05	5.14	6.42	7.71	9.65	11.02	14.12	18.88

Table 5.5: Quantiles of Significance Test (Diversified Case)

basis case portfolio, the quantiles of portfolio loss under the alternative model in Table 5.5 perform better with respect to model discrimination compared to the homogenous portfolio. For $L(\mathbb{E}[D_1])$, it is $\overline{q}_{0.05} \geq q_{0.5}$ if the average asset correlation is 5% or above. For $L(D_0)$, it is $\overline{q}_{0.05} > q_{0.5}$ for all correlation structures taken into account.

A level of significance $\overline{\alpha} = 5\%$ is considered to be adequate to stipulate the rejection of the alternative hypothesis $\overline{\mathcal{H}}_0$ in (2.22), and the acceptance barrier is set to $\overline{q}_{0.05}$, so that the alternative model can be expected to be erroneously rejected in one out of twenty cases, while given a correct model a realized portfolio loss given a correct model is expected to appear outside the green zone for not more than half of the portfolio observations. Although, the backtesting performs better in terms of the type-I-error for the diversified portfolio, the extension of the green zone is smaller than for the homogenous basis case portfolio, which reveals that Credit-VaR and the variation in portfolio loss are smaller for the diversified portfolio.

The rejection barrier of the yellow zone is defined by a quantile of the loss distribution itself. In contrast to market-risk backtesting, the confidence level of hypothesis $\mathcal{H}_0$ in 2.22 is set to be lower than the presumed quantile of Credit-VaR, because, in market-risk backtesting, the test statistic is not directly linked to the VaR or to the capital charge, so that model rejection and the level of test significance are not related to the capital charge. A model that incurs credit loss above the average regulatory capital charge of 8% must be considered as inadequate. If the confidence level of $\mathcal{H}_0$ were set higher than the confidence level of Credit-VaR, the credit risk model would not be rejected before the bank ran out of capital, so that the 99.5% confidence level of credit risk, employed to determine the Basel II capital requirements, is considered to be a natural upper level for the significance level of $\mathcal{H}_0$. If the rejection barrier and, thus, the level of confidence in $\mathcal{H}_0$ is decreased, the type-I-error of rejecting a correct model increases and supplemental supervisory actions to evaluate model adequacy are prompted sooner, so that banks' credit risk models will be examined more frequently.

A significance level of $\alpha = 5\%$ is defined for the rejection of hypothesis $\mathcal{H}_0$, and the rejection barrier of the yellow zone is specified by the loss quantile $q_{0.95}$. This specification has been chosen, because simulation results for the diversified portfolio reveal that otherwise the model will be rejected with certainty for any asset correlation considered, if the percentage credit loss is lower than the average 8% capital requirement.

5.4 Simulation of Homogenous Portfolios

5.4.1 Basis Case with a Homogenous Portfolio

In the basis case of a portfolio of homogenous exposures, the distributions of loss and portfolio value are simulated with risk horizon $\bar{t} = 1$ and four sub-intervals. Considering a single risk class, all exposures refer to one systematic factor in a common way, so that asset correlations $\rho_{i,j} = \rho^a \in \{0\%, 5\%, 10\%, 15\%, 20\%, 25\%, 30\%\}$ between any pair of exposures $i \neq j; i, j{=}1, ..., N$ are equal. Asset values V_0^i and coupon rates c^i are calibrated, so that $D_0^i = K_i, i = 1, ..., N$. Table 5.6 provides location and dispersion statistics on the distribution of percentage credit default, credit loss $L(\mathbb{E}[D_1])$ and $L(D_0)$, and portfolio value D_1 for the basis case scenario with homogenous exposures and different ρ^a.

For $\rho^a = 0$, asset values and credit defaults are independent and the quantiles of the default rate $\widehat{p} \sim Bin(N, p)$ are specified using a binomial distribution. An increase of ρ^a

results in a decrease of $q_{0.01}$ for the default rate, whereas $q_{0.99}$ increases up to 10.33% for $\rho^a = 30\%$, i.e. the distribution of $\widehat{p}$ becomes more leptokurtic and skewed to the right, which is also confirmed by the decrease in the median.

Due to the homogenous time-to-maturity $T > \bar{t}$ adverse changes in credit values are not only incurred due to credit default but also due to adverse changes in the mark-to-model values of non-defaulted exposures. In accordance, if the $q_{0.5}$-quantile of $\widehat{p}$, $L(\mathbb{E}[D_1])$, and $L(D_0)$ decreases, then the median of the portfolio value D_1 increases with ρ^a, with the size of the change in portfolio loss being equal to the change in portfolio value. In Table

	Default Rate $\widehat{p}$									**Portfolio Value D_1**								
$\rho^{\mathbf{a}}$	**Min.**	$q_{0.01}$	$q_{0.5}$	$q_{0.99}$	**Max.**	$\mu(\widehat{p})$	$\sigma(\widehat{p})$	**Skew**	**Kurt.**	**Min.**	$q_{0.01}$	$q_{0.5}$	$q_{0.99}$	**Max.**	$\mu(D_1)$	$\sigma(D_1)$	**Skew**	**Kurt.**
0%	0.00	0.33	1.00	1.89	2.89	1.00	0.33	0.33	0.11	95.82	96.42	97.05	97.62	98.07	97.04	0.26	-0.11	0.01
5%	0.00	0.00	0.89	3.33	7.78	1.00	0.71	1.55	4.13	88.88	93.08	97.20	99.62	100.70	97.04	1.40	-0.70	0.82
10%	0.00	0.00	0.67	4.89	13.56	1.01	1.02	2.48	10.43	84.16	90.93	97.35	100.29	101.36	97.03	2.01	-0.98	1.45
15%	0.00	0.00	0.56	6.22	24.56	1.00	1.29	3.48	21.24	76.64	89.18	97.52	100.75	101.63	97.05	2.46	-1.24	2.49
20%	0.00	0.00	0.44	7.56	31.78	1.01	1.56	4.14	29.27	72.54	87.53	97.67	101.05	101.77	97.04	2.88	-1.42	3.11
25%	0.00	0.00	0.33	8.89	31.44	1.01	1.84	4.75	36.63	72.05	85.90	97.82	101.27	101.94	97.03	3.26	-1.58	3.82
30%	0.00	0.00	0.22	10.33	48.89	1.00	2.14	5.72	53.11	64.73	84.36	97.98	101.44	101.99	97.06	3.59	-1.77	4.83
25%	0.00	0.00	0.89	14.89	55.00	1.99	3.04	3.72	21.75	59.78	75.77	91.34	98.76	101.04	90.55	5.02	-0.89	0.95
	$\mathbf{L(\mathbb{E}[D_1])}$									$\mathbf{L(D_0)}$								
$\rho^{\mathbf{a}}$	**Min.**	$q_{0.01}$	$q_{0.5}$	$q_{0.99}$	**Max.**	$\mu(L)$	$\sigma(L)$	**Skew**	**Kurt.**	**Min.**	$q_{0.01}$	$q_{0.5}$	$q_{0.99}$	**Max.**	$\mu(L)$	$\sigma(L)$	**Skew**	**Kurt.**
0%	1.49	1.91	2.40	2.96	3.45	2.41	0.23	0.15	0.02	2.54	2.99	3.54	4.15	4.71	3.55	0.25	0.12	0.01
5%	0.24	0.74	2.26	5.46	9.21	2.41	1.01	0.98	1.58	0.58	1.35	3.39	7.21	11.30	3.55	1.26	0.82	1.09
10%	0.06	0.40	2.10	7.26	13.48	2.42	1.47	1.38	2.90	0.20	0.83	3.22	9.26	15.92	3.56	1.81	1.15	1.96
15%	0.02	0.22	1.94	8.77	20.65	2.41	1.82	1.77	4.96	0.08	0.52	3.04	10.97	23.39	3.54	2.23	1.46	3.32
20%	0.00	0.12	1.79	10.21	24.62	2.42	2.15	2.02	6.19	0.04	0.33	2.89	12.56	27.47	3.56	2.62	1.65	4.12
25%	0.00	0.07	1.64	11.66	25.11	2.42	2.46	2.26	7.58	0.01	0.21	2.73	14.15	27.95	3.57	2.98	1.84	5.03
30%	0.00	0.03	1.48	13.11	32.36	2.40	2.73	2.53	9.49	0.00	0.13	2.56	15.68	35.28	3.54	3.29	2.06	6.29
25%	0.06	0.82	6.34	21.32	37.27	7.27	4.54	1.12	1.51	0.26	1.79	8.72	24.23	40.22	9.55	4.93	0.94	1.04

Table 5.6: Basis Case Statistics

5.7, the adequacy barriers and the 99.5%−Credit-VaR of the indicated distributions are presented for different asset correlations, with the alternative model defined by $\overline{p} = p+1\%$, $\overline{\sigma} = \sigma + 10\%$ and homogenous asset correlation $\overline{\rho} = 25\%$, as outlined in Section 5.3.2.

In the course of the simulation, exposure values D_1^i and default times τ_i are derived from $V_{t_h}^i$, $t_h = \bar{t}/h, ..., \bar{t}$ and $i = 1, ..., N$ to determine the distributions of the default rate $\widehat{p}$, portfolio loss $L(\mathbb{E}[D_1])$ and $L(D_0)$, and portfolio value D_1 for the model to be tested and the alternative model. The acceptance barrier of the green zone equals the quantile $\overline{q}_{0.05}$ of the respective loss distribution under the alternative model, whereas the rejection barrier of the red zone is given by the quantile $q_{0.95}$ of the original model. For $\rho_a = 0\%$, asset values and credit defaults are independent. The acceptance and rejection barriers do not intersect even for independent asset returns, so that the specification of backtesting is more restrictive compared to the approach proposed by Bühler and Engel (2006)[3], because the existence of a yellow zone of model indetermination prevents an unambiguous model

[3] The intersection of acceptance and rejection barriers is enabled for independent assets by setting $\overline{p} = 5p$.

	$\widehat{p}$				$\mathbf{L}(\mathbb{E}[\mathbf{D_1}])$				$\mathbf{L}(\mathbf{D_0})$				D_1			
$\rho^{\mathbf{a}}$	$\overline{q}_{0.05}$	$q_{0.5}$	$q_{0.95}$	$q_{0.995}$	$\overline{q}_{0.05}$	$q_{0.5}$	$q_{0.95}$	$q_{0.995}$	$\overline{q}_{0.05}$	$q_{0.5}$	$q_{0.95}$	$q_{0.995}$	$\overline{q}_{0.05}$	$q_{0.5}$	$q_{0.95}$	$q_{0.995}$
0%		1.00	1.56	2.00		2.40	2.79	3.02		3.54	3.97	4.22		97.05	96.61	96.35
5%		0.89	2.33	3.78		2.26	4.30	5.94		3.39	5.87	7.74		97.20	94.50	92.53
10%		0.67	3.00	5.78		2.10	5.25	8.07		3.22	7.01	10.16		97.35	93.28	90.00
15%		0.56	3.44	7.56		1.94	5.98	9.84		3.04	7.89	12.13		97.52	92.33	87.98
20%		0.44	3.78	9.56		1.79	6.66	11.81		2.89	8.69	14.27		97.67	91.48	85.82
25%	0.00	0.33	4.22	11.22	1.69	1.64	7.31	13.42	3.10	2.73	9.46	15.96	97.23	97.82	90.67	84.09
30%		0.22	4.44	13.67		1.48	7.84	15.32		2.56	10.08	18.02		97.98	90.02	82.00

Table 5.7: Basis Case

discrimination. With increasing asset correlation ρ^a, the probability of joint defaults and the probability of joint non-defaults increases, and as does the dispersion of portfolio loss, Credit-VaR and the rejection barrier.

The acceptance barrier is defined by the quantile $\overline{q}_{0.05}$ given $\overline{\rho}^a = 25\%$ and does not depend on the asset correlation ρ^a of the original model. Setting asset correlation $\rho^a = 20\%$ equal to its upper bound in the New Capital Adequacy framework,[4] the yellow zone $[1.69\%, 6.66\%]$ ($[3.1\%, 8.69\%]$) of model indetermination on loss definition $L(\mathbb{E}[D_1])$ ($L(D_0)$), does not reach the 8% target capital level and allows, from a banking perspective, an acceptable but noncritical portfolio loss before triggering regulatory actions. $L(D_0)$ indicates higher acceptance and rejection barriers compared to $L(\mathbb{E}[D_1])$, because its reference values D_0^i turn out to be higher than the expected values $\mathbb{E}(D_1^i)$ of exposures.

5.4.2 Default-at-Maturity Model

The effects of the type of the default model used are examined by comparing the first-passage default model with a constant default threshold defined in Section 3.2 to a Merton-style default-at-maturity model for an adapted basis portfolio with homogenous maturity $T_i = T = \bar{t} = 1$ of exposure $i = 1, ..., N$. The Credit-VaR and zone locations of $L(\mathbb{E}[D_1])$, $L(D_0)$ are presented in Table 5.8 for the usual asset correlations. The acceptance and rejection barriers as well as median and Credit-VaR are defined as before. A comparison of the loss quantiles of the homogenous portfolio with one-year maturities to the basis case portfolio reveals that adverse changes in the mark-to-model values of exposures exceed credit loss from default by far and account for more than half of the Credit-VaR.

The reduced maturity $T = \bar{t}$ of exposures results in a negative acceptance barrier of credit loss $L(\mathbb{E}[D_1])$ for the adapted portfolio, because the redemption of face values at time $\bar{t}$ overcompensates the default expectation in the low quantiles of loss distribution, while the par reference values ensure that portfolio loss $L(D_0)$ stays positive.

In the right tail of both distributions, the portfolio loss is higher for the Merton-style

[4] BCBS (2006c), p. 63ff.

Default Type		**Basis Case**				**First Passage Model**				**Default-at-Maturity Model**			
Loss Type	ρ^{a}	$\overline{q}_{0.05}$	$q_{0.5}$	$q_{0.95}$	$q_{0.995}$	$\overline{q}_{0.05}$	$q_{0.5}$	$q_{0.95}$	$q_{0.995}$	$\overline{q}_{0.05}$	$q_{0.5}$	$q_{0.95}$	$q_{0.995}$
	0%		2.40	2.79	3.02		0.00	0.28	0.50		0.00	0.29	0.51
	5%		2.26	4.30	5.94		-0.05	0.67	1.39		-0.06	0.70	1.51
	10%		2.10	5.25	8.07		-0.16	1.00	2.38		-0.17	0.99	2.47
$\mathbf{L}(\mathbb{E}[\mathbf{D_1}])$	15%		1.94	5.98	9.84		-0.22	1.21	3.21		-0.23	1.26	3.35
	20%		1.79	6.66	11.81		-0.27	1.39	4.25		-0.29	1.44	4.48
	25%	1.69	1.64	7.31	13.42	-0.50	-0.33	1.55	5.16	-0.52	-0.34	1.66	5.62
	30%		1.48	7.84	15.32		-0.38	1.71	6.28		-0.40	1.85	6.64
	0%		3.54	3.97	4.22		0.50	0.78	0.99		0.51	0.81	1.03
	5%		3.39	5.87	7.74		0.44	1.16	1.89		0.45	1.22	2.02
	10%		3.22	7.01	10.16		0.33	1.49	2.87		0.35	1.50	2.98
$\mathbf{L}(\mathbf{D_0})$	15%		3.04	7.89	12.13		0.28	1.71	3.71		0.29	1.78	3.86
	20%		2.89	8.69	14.27		0.22	1.88	4.75		0.23	1.96	5.00
	25%	3.10	2.73	9.46	15.96	0.00	0.17	2.05	5.65	0.00	0.17	2.17	6.14
	30%		2.56	10.08	18.02		0.11	2.21	6.77		0.12	2.36	7.16

Table 5.8: Default-at-Maturity Model

model. Although, the asset values are calibrated to a homogenous annual default probability $p = 1\%$ for both models, default events occur earlier under the first-passage model, so that recovery values accrue until risk horizon $\bar{t}$ and reduce portfolio loss compared to the default-at-maturity model. As expected, this accrual effect is more pronounced for the high quantiles $q_{0.95}$ and $q_{0.995}$ than for the median.

Furthermore, simulated default rates in the right tail of the default rate distribution are slightly higher for the Merton-style default model, while the two models do not differ significantly in the mean or in the median of the default rate. This is explained by the calibration of the asset values, which are set higher under the first-passage model, so that the negative returns of the systematic factor necessary to trigger the simultaneous default of numerous exposures is more pronounced and therefore more unlikely under the first-passage model. With respect to simulated default rates and the accrual effect of recoveries, it can be stated that default-at-maturity models imply a higher likelihood of simultaneous default events and yield higher Credit-VaR predictions than first-passage models given identical default probabilities and asset correlations.

5.4.3 Variation of Simulation Intervals

The simulation procedure outlined in Section 5.2 divides the holding period into sub-intervals to approximate the joint dynamics of asset values. In Table 5.9, Credit-VaR and zone locations of simulations of the homogenous basis case model with four and twelve sub-intervals, i.e. quarterly and monthly sub-periods, are compared to assess the impact of a more precise modelling of the joint dynamics of asset values. The 99.5%−quantiles of $\widehat{p}$ ($L(\mathbb{E}[D_1])$, $L(D_0)$) are higher with 12 intervals for 2 (5, 5) out of 7 correlation scenarios, where changes in Credit-VaR mostly coincide to the changes of the respective quantiles of the default rate. Additionally, with 12 sub-intervals, default events occur earlier and

Interval Count		h = 4			h = 12			Difference in %		
Loss Type	ρ^a	$\overline{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$	$\overline{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$	$\overline{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$
	0%		2.79	3.02		2.79	3.02		0.11	-0.21
	5%		4.30	5.94		4.30	5.93		-0.08	0.18
	10%		5.25	8.07		5.22	8.08		0.50	-0.16
$\mathbf{L}(\mathbb{E}[\mathbf{D_1}])$	15%		5.98	9.84		5.97	9.99		0.11	-1.53
	20%		6.66	11.81		6.64	11.77		0.32	0.40
	25%	1.69	7.31	13.42	1.67	7.30	13.60	1.40	0.08	-1.36
	30%		7.84	15.32		7.87	15.37		-0.38	-0.32
	0%		3.97	4.22		3.97	4.23		0.06	-0.24
	5%		5.87	7.74		5.87	7.72		0.04	0.27
	10%		7.01	10.16		6.98	10.17		0.32	-0.07
$\mathbf{L}(\mathbf{D_0})$	15%		7.89	12.13		7.87	12.30		0.23	-1.37
	20%		8.69	14.27		8.66	14.21		0.36	0.40
	25%	3.10	9.46	15.96	3.07	9.44	16.17	0.85	0.19	-1.34
	30%		10.08	18.02		10.12	18.06		-0.31	-0.19

Table 5.9: Simulation Intervals (Basis Case)

the accrual of recoveries is enforced, so that Credit-VaR is expected to fall. Although, it appears from the percentage changes of Credit-VaR that Credit-VaR increases marginally with the number of sub-intervals. From the results, it cannot be proven unambiguously that credit dependence is more pronounced and loss distributions will change systematically if the number of sub-intervals is increased. Since the 95%-quantiles of the default rate are equal for both simulation variants, and the rejection barriers $q_{0.95}$ of portfolio loss do not reveal a significant difference resulting from an increase of sub-intervals, four sub-intervals are considered to be sufficient for an accurate determination of the rejection barriers.

Analogously, simulations with quarterly and monthly sub-intervals are compared in Table F.2 for the diversified portfolio described in Section 5.5. With 12 sub-intervals, the 99.5%-quantile of the default rate, ($L(\mathbb{E}[D_1])$, $L(D_0)$), is higher for 6 (6,6) out of 12 correlation scenarios, so that the results do not confirm a change in credit dependence when the number of intervals is altered.

5.4.4 Variation of Holding Period

With an enhanced reassessment frequency of obligors' credit quality and the growing securitization of credit portfolios, the consideration of sub-annual holding periods of Credit-VaR is appropriate. Analogously to shortening the Credit-VaR horizon for risk management purposes, VaR predictions are conducted and backtested on a daily basis, whereas the determination of market-risk capital requirements refers to a ten-day holding period. The effect of the use of a quarterly or semi-annual holding period on credit risk predictions and backtesting is examined in Table 5.10 for the homogenous basis case model. If the holding period is shortened, a sub-linear reduction of Credit-VaR defined

Holding Period		$\bar{t} = 0.25$			$\bar{t} = 0.5$			$\bar{t} = 1$		
Loss Type	ρ^a	$\bar{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$	$\bar{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$	$\bar{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$
$L(\mathbb{E}[D_1])$	0%		0.89	0.95		1.49	1.60		2.79	3.02
	5%		1.33	1.76		2.25	3.10		4.30	5.94
	10%		1.57	2.31		2.72	4.16		5.25	8.07
	15%		1.78	2.80		3.12	5.18		5.98	9.84
	20%		1.98	3.22		3.41	6.08		6.66	11.81
	25%	3.30	2.13	3.72	2.38	3.69	6.89	1.69	7.31	13.42
	30%		2.31	4.21		4.00	7.79		7.84	15.32
$L(D_0)$	0%		1.19	1.26		2.09	2.21		3.97	4.22
	5%		1.71	2.20		3.02	3.99		5.87	7.74
	10%		2.00	2.80		3.58	5.17		7.01	10.16
	15%		2.24	3.33		4.05	6.27		7.89	12.13
	20%		2.46	3.78		4.40	7.23		8.69	14.27
	25%	3.91	2.64	4.30	3.41	4.73	8.10	3.10	9.46	15.96
	30%		2.84	4.82		5.08	9.03		10.08	18.02

Table 5.10: Holding Period (Basis Case)

by $L(\mathbb{E}[D_1])$ ($L(D_0)$) is observed, e.g. from 8.07% (10.16%) to 2.31%(2.8%) of D_0 for $\rho^a = 10\%$. Moreover, the acceptance barriers increase considerably. For $\bar{t} = 0.25$, the intersection of the acceptance and rejection barriers allows perfect discrimination between the model and its alternative for both loss definitions. Even with a semi-annual holding period, perfect discrimination is still possible for asset correlations of 5% and below. The improved discriminatory power provides some leeway in the definition of the alternative model or enables the tightening the significance level of backtesting.

5.4.5 Variation of Default Probability

The acceptance and the rejection barriers depend on the characteristics of the portfolio that is considered, and particularly on the default probability of exposures. Tables 5.11 and 5.12 indicate that Credit-VaR and the rejection barrier increase in the one-year probability of default p of the homogenous basis portfolio. For default probability $p > 1\%$, Credit-VaR exceeds the regulatory capital target of 8% even for small values of ρ^a, which supports a calculation of model-derived capital requirements on the basis of a Credit-VaR with a holding period $\bar{t} < 1$.

In contrast, the acceptance barriers of $L(\mathbb{E}[D_1])$ ($L(D_0)$) reach a minimal value of $\bar{q}_{0.05} = 1.65\%$ ($\bar{q}_{0.05} = 2.96\%$) at $\bar{p} = 2.5\%$ ($\bar{p} = 1.5\%$),[5] which can be explained by a two-fold effect. First, the sensitivity of the exposure value to a change in the asset value increases as the default probability decreases, so that credit loss resulting from a change in mark-to-model values is comparatively high for small values of $\bar{p}$. Second, credit default is more probable for high values of $\bar{p}$. In summary, credit loss from adverse changes in mark-to-

[5] For the indicated values of p, default probabilities $\bar{p} = p + 1\%$ of the alternative model vary between 1.05% and 11%.

$\mathbf{L(\mathbb{E}[D_1])}$ in [%]		**Homogenous Probability of Default**										
Quantile	ρ^{a}	**0.05%**	**0.10%**	**0.25%**	**0.50%**	**1.00%**	**0.5% & 1.5%**	**1.5%**	**2.00%**	**2.50%**	**5.00%**	**10.00%**
$\overline{\mathbf{q}}_{\mathbf{0.05}}$	25%	2.42	2.20	1.93	1.76	1.69	1.70	1.65	1.66	1.68	1.82	2.15
	0%	0.97	1.23	1.68	2.16	2.79	2.71	3.26	3.66	4.00	5.34	7.28
	5%	1.51	1.90	2.60	3.33	4.30	4.16	4.99	5.56	6.06	7.96	10.52
	10%	1.86	2.33	3.19	4.06	5.25	5.06	6.05	6.73	7.32	9.53	12.42
$\mathbf{q_{0.95}}$	15%	2.14	2.68	3.67	4.65	5.98	5.76	6.90	7.72	8.38	10.86	14.05
	20%	2.38	2.99	4.08	5.20	6.66	6.47	7.73	8.60	9.32	12.01	15.41
	25%	2.59	3.26	4.45	5.63	7.31	7.02	8.38	9.38	10.18	13.14	16.79
	30%	2.77	3.49	4.78	6.17	7.84	7.67	9.19	10.11	10.97	14.13	17.99
	0%	1.08	1.35	1.84	2.35	3.02	2.94	3.52	3.94	4.30	5.70	7.71
	5%	2.18	2.71	3.67	4.66	5.94	5.72	6.82	7.55	8.18	10.50	13.47
	10%	3.02	3.76	5.04	6.32	8.07	7.71	9.11	10.04	10.80	13.63	16.98
$\mathbf{q_{0.995}}$	15%	3.79	4.70	6.30	7.83	9.84	9.47	11.15	12.29	13.18	16.34	19.85
	20%	4.67	5.77	7.67	9.40	11.81	11.32	13.20	14.71	15.72	19.08	22.71
	25%	5.39	6.67	8.86	10.85	13.42	13.04	15.17	16.75	17.89	21.56	25.27
	30%	6.03	7.44	9.93	12.64	15.32	15.11	17.50	18.51	19.64	23.44	27.20

Table 5.11: Variation of Default Probability for $L(\mathbb{E}[D_1])$

model values dominates $\overline{q}_{0.05}$ for small values of $\overline{p}$, whereas loss from default is prevalent for high values of $\overline{p}$.[6]

With respect to the different definitions of portfolio loss, the acceptance barrier of $L(\mathbb{E}[D_1])$ is comparatively steady for $\overline{p} > 2\%$, whereas for $L(D_0)$ the location of the green zone is more stable for $\overline{p} < 2\%$. Considering exposures in two rating classes with differ-

$\mathbf{L(D_0)}$ in [%]		**Homogenous Probability of Default**										
Quantile	ρ^{a}	**0.05%**	**0.10%**	**0.25%**	**0.50%**	**1.00%**	**0.5% & 1.5%**	**1.5%**	**2.00%**	**2.50%**	**5.00%**	**10.00%**
$\overline{\mathbf{q}}_{\mathbf{0.05}}$	25%	3.16	3.07	2.97	2.96	3.10	3.07	3.19	3.32	3.45	4.02	4.94
	0%	1.33	1.69	2.34	3.04	3.97	3.86	4.68	5.26	5.78	7.83	10.84
	5%	1.99	2.52	3.48	4.49	5.87	5.67	6.85	7.67	8.40	11.19	15.12
	10%	2.39	3.03	4.18	5.38	7.01	6.75	8.15	9.11	9.95	13.13	17.53
$\mathbf{q_{0.95}}$	15%	2.73	3.45	4.74	6.07	7.89	7.61	9.17	10.28	11.21	14.80	19.57
	20%	3.01	3.80	5.24	6.73	8.69	8.44	10.15	11.33	12.35	16.17	21.25
	25%	3.26	4.13	5.68	7.24	9.46	9.09	10.91	12.27	13.38	17.52	22.94
	30%	3.48	4.40	6.06	7.87	10.08	9.86	11.86	13.14	14.31	18.72	24.42
	0%	1.44	1.82	2.51	3.24	4.22	4.11	4.96	5.58	6.13	8.25	11.35
	5%	2.73	3.44	4.70	6.01	7.74	7.48	8.95	9.96	10.86	14.25	18.65
	10%	3.66	4.59	6.24	7.90	10.16	9.72	11.59	12.86	13.93	17.92	22.93
$\mathbf{q_{0.995}}$	15%	4.50	5.62	7.60	9.55	12.13	11.67	13.88	15.39	16.58	21.01	26.38
	20%	5.45	6.76	9.09	11.25	14.27	13.66	16.11	18.03	19.36	24.03	29.64
	25%	6.21	7.74	10.35	12.78	15.96	15.52	18.24	20.23	21.69	26.72	32.52
	30%	6.89	8.53	11.46	14.68	18.02	17.71	20.68	22.03	23.52	28.81	34.66

Table 5.12: Variation of Default Probability for $L(D_0)$

ent default probabilities $p_i \in \{0.5\%, 1, 5\%\}$ in a portfolio with average default probability $p = 1\%$, the standard deviation of the default rate and, correspondingly, the standard

[6] The acceptance barrier $\overline{q}_{0.05}$ of the default rate distribution does not include any default event for $\overline{p} < 3.5\%$.

deviation of portfolio loss declines, as previously observed by Bühler et al. (2002), so that the green zone expands marginally, while the yellow zone shrinks considerably by a decrease of the rejection barrier.

In conclusion, the acceptance barrier and especially the rejection barrier markedly depend on the default probabilities of exposures, so that the characteristics of the portfolio must definitely be considered when the zones of model adequacy are specified. An improved portfolio diversification with respect to exposure ratings will in general reduce the yellow zone and produce an improved model discrimination.

5.4.6 Variation of Time-to-Maturity

The prevalent portion of portfolio credit loss does not arise from credit default, rather it is due to changes in the mark-to-model valuations of exposures, as noted in Section 5.4.2. The sensitivity of a credit valuation to a change in the asset value increases with the time-to-maturity of the exposure. In Table 5.13, Credit-VaR and zone locations are examined for three basis-case portfolios with adapted homogenous maturities $T = 1, 5, 10$, and additionally for a portfolio of loans with a 5.33 year average maturity and equally distributed maturities $T_i \in \{1, 5, 10\}$. For $T = 1$, a negative portfolio credit loss is

Time-to-Maturity		**T=1**			**T=5**			$\mathbf{T_i \in \{1, 5, 10\}}$			**T=10**		
Loss Type	ρ^{a}	$\overline{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$	$\overline{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$	$\overline{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$	$\overline{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$
	0%		0.28	0.50		2.79	3.02		2.03	2.25		2.99	3.22
	5%		0.67	1.39		4.30	5.94		3.17	4.50		4.57	6.26
	10%		1.00	2.38		5.25	8.07		3.91	6.23		5.57	8.45
$\mathbf{L(\mathbb{E}[D_1])}$	15%		1.21	3.21		5.98	9.84		4.48	7.72		6.31	10.21
	20%		1.39	4.25		6.66	11.81		5.04	9.26		7.03	12.28
	25%	-0.50	1.55	5.16	1.69	7.31	13.42	1.51	5.47	10.78	3.36	7.63	13.90
	30%		1.71	6.28		7.84	15.32		5.98	12.57		8.27	15.81
	0%		0.78	0.99		3.97	4.22		3.03	3.26		4.30	4.55
	5%		1.16	1.89		5.87	7.74		4.42	5.91		6.29	8.22
	10%		1.49	2.87		7.01	10.16		5.29	7.83		7.48	10.71
$\mathbf{L(D_0)}$	15%		1.71	3.71		7.89	12.13		5.96	9.46		8.37	12.65
	20%		1.88	4.75		8.69	14.27		6.61	11.13		9.23	14.87
	25%	0.00	2.05	5.65	1.79	9.46	15.96	1.88	7.11	12.71	3.84	9.94	16.61
	30%		2.21	6.77		10.08	18.02		7.70	14.57		10.69	18.66

Table 5.13: Variation of Time-to-Maturity

obtained at the acceptance barrier for $L(\mathbb{E}[D_1])$, and compared to the basis portfolio with $T = 5$, the rejection bounds decrease sharply for $L(\mathbb{E}[D_1])$ and $L(D_0)$, irrespective of the asset correlation used. Credit-VaR declines correspondingly. This left shift of loss distributions is caused exclusively by the absence of adverse changes in mark-to-model credit valuations, since exposures either default or redeem at $T = \overline{t} = 1$.

Accordingly, the sensitivity of credit valuations to a change in the asset value increases with the time-to-maturity of exposures, which explains the distinct upward shift of loss distributions given $T = 10$ and arbitrary asset correlations. If $\rho^a = 10\%$ is considered as an example, the acceptance barriers for $L(\mathbb{E}[D_1])$ $(L(D_0))$ raise to 3.36% (3.84%), rejection barriers increase to 5.57% (7.48%) and the resulting Credit-VaR is above the 8% target capital ratio at 8.45% (10.71%).

If time-to-maturity increases, the yellow zone widens from a range of 1.5% (1.49%) for $L(\mathbb{E}[D_1])$ $(L(D_0))$, $T = 1$ and $\rho^a = 10\%$, to a range of 3.56% (5.22%) for $T = 5$, which reduces the discrimination power of the backtesting. However, for homogenous $T = 10$, the yellow zone shrinks again to a range of 2.21% (3.64%).

The acceptance barrier is more sensitive to a change in T than the rejection barrier, because credit values in $\overline{q}_{0.05}$ show a higher variation compared to $T = 5$ and loss definitions exclusively consider negative credit performance, whereas mark-to-model valuation profits are by definition not included in the portfolio loss. For example, with a positive factor realization, the predominant positive changes in credit valuations do not compensate for the losses suffered by a few loans, so that portfolio loss may be observed even for extreme positive states of the factor, which explains the increase in $\overline{q}_{0.05}$ for $T = 10$.

If the portfolio includes exposures of different maturities $T_i \in \{1, 5, 10\}$, the acceptance barriers decrease further to 1.51% (1.88%) and the rejection barriers for $\rho^a = 10\%$ decrease to 3.91% (5.29%), which indicates that the effect of short-maturity exposures on Credit-VaR and backtesting barriers overcompensates the increase of the high quantiles of portfolio loss induced by the distant-maturity exposures. Thus, a diversification of maturities will balance unfavorable maturity-derived effects on the location of the backtesting barriers, so that the acceptance zone is restricted to a range that might be acceptable with respect to supervisory requirements, and that nevertheless covers typical observations of portfolio credit loss throughout the credit cycle.

With respect objective of minimizing the capital requirements in a mark-to-model setting, banks have an incentive to shorten the average maturity of credit portfolios. This increases the vulnerability of portfolios to the cyclicity of credit markets, because redemption and prolongation of an enlarged portion of the portfolio will take place under adverse market conditions, so that the number of credit defaults will potentially increase and banks' capital endowments may be distorted.

5.4.7 Variation of Risk Concentration and Asset Correlation

Risk concentrations in a credit portfolio arise from exposures being heterogenous with respect to face value, correlation, rating or maturity. In Table 5.14, the effects of portfolio concentrations with respect to face value and economic sectors are examined. First, the case of a portfolio with 563, (225, 112) exposures of face value $K_i = 1$ (10,100), but otherwise equal to the homogenous basis portfolio, is examined, so that small (medium, large) exposures make up 4%, (16%, 80%) of the total portfolio value. Second, exposures of the homogenous basis portfolio are equally assigned to one out of two sectors, which are defined by equal inner-sector asset correlations and different inter-sector correlations, indicated by the pair of correlations in the second column of Table 5.14.

Loss Type		$\mathbf{L(\mathbb{E}[D_1])}$			$\mathbf{L(D_0)}$		
Face Value	$\rho^{\mathbf{a}}$	$\overline{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$	$\overline{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$
	0%		2.79	3.02		3.97	4.22
	5%		4.30	5.94		5.87	7.74
	10%		5.25	8.07		7.01	10.16
K=1	15%		5.98	9.84		7.89	12.13
	20%		6.66	11.81		8.69	14.27
	25%	1.69	7.31	13.42	1.79	9.46	15.96
	30%		7.84	15.32		10.08	18.02
	0%		3.33	3.92		4.56	5.20
	5%		4.48	6.25		6.04	8.04
	10%		5.34	8.26		7.10	10.39
$\mathbf{K_i \in \{1, 10, 100\}}$	15%		6.08	9.93		7.98	12.23
	20%		6.79	11.84		8.81	14.26
	25%	1.62	7.32	13.49	1.70	9.46	16.07
	30%		7.96	15.59		10.20	18.30
	0% - 0%		2.79	3.02		3.97	4.22
	5% - 5%		4.29	5.91		5.85	7.71
	10% - 0%		4.35	6.06		5.89	7.86
	10% - 5%		4.78	6.98		6.44	8.95
	10% - 10%		5.21	7.94		6.96	10.01
	15% - 5%		5.26	8.04		7.01	10.08
	20% - 0%		5.33	8.29		7.06	10.35
K=1	15% - 15%		5.93	9.92		7.82	12.22
	20% - 10%		5.99	9.94		7.88	12.23
	30% - 0%		6.25	10.48		8.11	12.65
	20% - 20%		6.61	11.68		8.64	14.10
	30% - 10%		6.76	11.87		8.76	14.23
	25% - 25%	1.68	7.26	13.45	1.80	9.39	16.01
	30% - 20%		7.28	13.42		9.41	15.96
	30% - 30%		7.75	15.12		9.99	17.78

Table 5.14: Variation of Risk Concentration

With heterogenous face values, both loss distributions exhibit a higher Credit-VaR and a lower skewness and kurtosis, so that the acceptance barriers decline to 1.62%(1.7%) for both $L(\mathbb{E}[D_1])$ and $L(D_0)$. In consequence, the range of the yellow zone increases and the discriminatory power of backtesting deteriorates.

For the case of two economic sectors, the alternative model is specified by homogenous inter- and inner-sector correlations $\rho^a_{i,j} = 25\%$, $\forall i \neq j$, which represent a conservative assumption on the diversification effects of different sectors.[7] The comparison of correlation structures with an equal average asset correlation, such as the correlation tuples

[7] Comparing the quantiles of the homogenous basis portfolio with those of the two-sector model with respective homogenous inner- and inter-sector correlations reinforces the stability of loss distributions with respect to simulation error.

$\mathbf{L(\mathbb{E}[D_1])}$		μ = **-8%**			μ = **0%**			μ = **8%**		
σ	$\rho^{\mathbf{a}}$	$\bar{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$	$\bar{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$	$\bar{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$
	0%		1.79	1.99		1.60	1.81		1.25	1.45
	5%		2.83	4.15		2.52	3.73		1.93	2.91
	10%		3.51	5.80		3.14	5.29		2.41	4.13
σ = **5%**	15%		4.03	7.42		3.62	6.69		2.77	5.39
	20%		4.53	9.13		4.09	8.13		3.12	6.76
	25%	2.06	4.98	10.49	2.06	4.45	9.68	2.25	3.50	8.01
	30%		5.40	12.09		4.86	11.47		3.67	9.44
	0%		2.88	3.11		2.79	3.02		2.67	2.91
	5%		4.43	6.16		4.30	5.94		4.14	5.71
	10%		5.36	8.17		5.25	8.07		5.00	7.89
σ = **10%**	15%		6.19	10.17		5.98	9.84		5.74	9.69
	20%		6.80	11.82		6.66	11.81		6.39	11.42
	25%	1.65	7.48	13.85	1.69	7.31	13.42	1.76	6.97	13.22
	30%		8.18	15.51		7.84	15.32		7.63	14.89
	0%		4.00	4.26		4.04	4.29		4.08	4.35
	5%		6.00	7.93		6.06	8.05		6.12	8.18
	10%		7.17	10.27		7.20	10.52		7.29	10.63
σ = **20%**	15%		8.09	12.44		8.20	12.81		8.35	12.88
	20%		8.98	14.72		9.02	14.65		9.28	15.01
	25%	1.52	9.79	16.24	1.54	9.89	16.80	1.60	10.07	17.23
	30%		10.55	18.27		10.81	18.74		10.85	19.52

Table 5.15: Variation of μ and σ for $L(\mathbb{E}[D_1])$ of the Homogenous Portfolio

(5%;5%) vs. (10%;0%), or (10%;10%) vs. (20%;0%), reveals that the rejection bound and Credit-VaR are lower for more homogenous asset correlations, which raises concerns of a potential underestimation of credit risk by conventional single-risk class models. Otherwise, the introduction of a second sector with reduced inter-sector correlations lowers rejection bounds and Credit-VaR, when compared to the respective one-sector scenario, while the acceptance barriers remain unchanged.

In summary, a more comprehensive sector structure may result in a lower Credit-VaR and an improved model discrimination, while concentrations in the face values of exposures produce the opposite effect.

5.4.8 Variation of Asset Value Process

The homogenous drift rate μ and the homogenous diffusion rate σ of asset values impact the distributions of portfolio value, default rate and portfolio credit loss. In Tables 5.15 and 5.16, Credit-VaR and zone locations of portfolio credit loss $L(\mathbb{E}[D_1])$ and $L(D_0)$ are examined on the basis of the homogenous basis case portfolio for a variation of homogenous $\mu \in \{-8\%, 0\%, 8\%\}$ and homogenous $\sigma \in \{5\%, 10\%, 20\%\}$. Credit-VaR and rejection barriers increase with diffusion rate σ for both loss definitions independent of the drift rate, This effect results from the increased variation of asset values, even if calibration of asset values ensures that the default probability $p = 1\%$ remains unchanged for all loans. For the acceptance barrier, however, the effects of a change in σ are more complex. For

$L(\mathbb{E}[D_1])$, the acceptance barriers decrease σ as increases, whereas the acceptance barriers of $L(D_0)$ decrease (increase) in σ for $\mu = 0\%$ ($\mu = 8\%$) and yield an inner maximum of $\overline{q}_{0.05} = 4.25\%$ for $\mu = -8\%$. These mixed effects are due to the exclusion of positive credit performance in the loss measures, which results in the omission of a varying number of exposures in the portfolio loss at quantile $\overline{q}_{0.05}$.

$\mathbf{L(D_0)}$		μ = -8%			μ = 0%			μ = 8%		
σ	$\rho^{\mathbf{a}}$	$\overline{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$	$\overline{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$	$\overline{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$
	0%		2.55	2.77		2.09	2.31		1.07	1.26
	5%		3.85	5.34		3.21	4.55		1.64	2.53
	10%		4.65	7.20		3.93	6.25		2.06	3.63
σ = 5%	15%		5.29	8.97		4.49	7.80		2.38	4.79
	20%		5.86	10.79		5.03	9.36		2.69	6.01
	25%	3.71	6.40	12.26	3.30	5.44	10.99	1.42	3.00	7.22
	30%		6.88	13.95		5.94	12.86		3.16	8.49
	0%		5.11	5.37		3.97	4.22		2.50	2.72
	5%		7.23	9.30		5.87	7.74		3.89	5.41
	10%		8.45	11.70		7.01	10.16		4.71	7.51
σ = 10%	15%		9.51	13.98		7.89	12.13		5.42	9.26
	20%		10.26	15.79		8.69	14.27		6.04	10.97
	25%	4.25	11.10	17.98	3.10	9.46	15.96	1.55	6.60	12.71
	30%		11.95	19.79		10.08	18.02		7.23	14.38
	0%		6.89	7.19		5.70	5.99		4.44	4.73
	5%		9.61	11.99		8.21	10.51		6.61	8.75
	10%		11.14	14.80		9.59	13.31		7.84	11.30
σ = 20%	15%		12.33	17.29		10.78	15.88		8.95	13.61
	20%		13.43	19.83		11.75	17.86		9.93	15.79
	25%	3.84	14.45	21.47	2.78	12.75	20.17	1.86	10.76	18.06
	30%		15.38	23.67		13.82	22.21		11.57	20.39

Table 5.16: Variation of μ and σ for $L(D_0)$ of the Homogenous Portfolio

With $\mu = 8\%$ and $\sigma = 5\%$, $L(\mathbb{E}[D_1])$ even provides perfect discrimination for $\rho^a = 5\%$, just as $L(D_0)$ does for $\mu = 0\%$ and $\sigma = 5\%$. The rejection barrier increases beyond the target capital charges for $\sigma = 20\%$ even for moderate asset correlations. Overall, the discrimination power of backtesting improves with decreasing σ, while capital charges decline. This effect gives banks an incentive to underestimate the diffusion rate of asset values.

With respect to a change in the drift rate, acceptance barriers increase (decline) as μ increases for $L(\mathbb{E}[D_1])$ ($L(D_0)$), irrespective of the σ considered. The decrease in calibrated asset values for increasing μ dominates for $L(\mathbb{E}[D_1])$, whereas the decrease of $\overline{q}_{0.05}$ in μ can be attributed to a growing difference between the reference values D_0^i and the expected values $\mathbb{E}(D_1^i)$ of exposures. In contrast, the dependence of the rejection barrier and Credit-VaR on μ is not clear-cut. For $L(\mathbb{E}[D_1])$ the rejection barrier and Credit-VaR decrease in μ for small diffusion rates, but the effect for $\sigma = 20\%$ is opposite, whereas the rejection barrier and Credit-VaR of $L(D_0)$ decline in μ for any σ.

Overall, the acceptance barriers of $L(\mathbb{E}[D_1])$ are more robust against a mis-specification of μ, whereas acceptance barriers of $L(D_0)$ are more stable with respect to a change in σ.

5.5 Simulation of Diversified Portfolios

5.5.1 Basis Case with Diversified Portfolio

In this section, a generalized model for a diversified portfolio is analyzed. In a two-sector version of the risk class factor model in (3.36), the credit portfolio of diversified exposures, defined in Section 5.1 and assumed to represent the characteristics of a typical real-world corporate loan portfolio, is examined. The risk classes that determine asset correlations $\rho^a_{i,j}$ between exposures are assumed to be represented by the economic sectors of exposures irrespective of the rating, i.e. the default probability of exposures. Table 5.17 provides the backtesting barriers and Credit-VaR for default rate, portfolio loss $L(\mathbb{E}[D_1])$ and $L(D_0)$ and portfolio value D_1 of the diversified portfolio for different inner-sector and inter-sector correlations, where correlation scenarios are ordered according to an increasing average and increasing heterogeneity of asset correlations. Equal inter-sector and inner-sector correlations correspond to the one-sector case with homogenous asset correlation ρ^a.

Diversified Case	$\widehat{\mathbf{p}}$			$\mathbf{L(\mathbb{E}[D_1])}$			$\mathbf{L(D_0)}$			$\mathbf{D_1}$		
$\rho^{\mathbf{a}}_{\mathbf{i,j}}$	$\bar{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$	$\bar{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$	$\bar{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$	$\bar{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$
0% - 0%		1.56	2.00		2.46	3.04		3.47	4.09		96.94	96.29
5% - 0%		2.00	2.89		2.94	3.99		4.07	5.24		96.26	95.03
5% - 5%		2.33	3.78		3.27	4.72		4.49	6.09		95.78	94.10
10% - 0%		2.44	4.00		3.33	4.87		4.56	6.25		95.72	93.98
10% - 5%		2.67	4.67		3.61	5.56		4.90	7.05		95.35	93.13
15% - 0%		2.78	5.11		3.68	5.76		4.96	7.23		95.29	92.96
10% - 10%		2.89	5.44		3.90	6.36		5.25	7.90		94.97	92.22
15% - 5%		3.00	5.67		3.98	6.39		5.32	7.92		94.89	92.20
20% - 0%		3.11	6.22		4.04	6.56		5.38	8.06		94.85	92.09
15% - 10%		3.11	6.44		4.18	7.09		5.56	8.71		94.63	91.38
20% - 5%		3.22	6.78		4.28	7.16		5.68	8.77		94.51	91.34
25% - 0%		3.33	7.33		4.36	7.37		5.74	8.91		94.48	91.24
15% - 15%		3.33	7.22		4.46	7.69		5.90	9.38		94.26	90.70
20% - 10%		3.33	7.44		4.52	7.88		5.96	9.56		94.21	90.53
25% - 5%		3.56	8.00		4.55	8.02		5.98	9.65		94.20	90.47
30% - 0%		3.67	8.78		4.67	8.33		6.08	9.90		94.11	90.28
20% - 15%		3.56	8.22		4.73	8.54		6.22	10.24		93.93	89.80
25% - 10%		3.67	8.44		4.81	8.65		6.30	10.41		93.86	89.66
30% - 5%		3.78	9.22		4.89	8.83		6.37	10.61		93.79	89.50
20% - 20%		3.67	9.00		4.89	9.21		6.42	11.02		93.71	89.03
25% - 15%		3.78	9.11		4.97	9.26		6.49	11.04		93.64	89.00
30% - 10%		3.89	9.78		5.03	9.32		6.54	11.09		93.61	88.95
25% - 20%		3.89	9.89		5.19	10.02		6.76	11.82		93.36	88.23
30% - 15%		4.00	10.22		5.21	10.02		6.76	11.84		93.37	88.21
25% - 25%	0.00	4.11	10.78	1.47	5.38	10.66	2.84	7.00	12.55	97.31	93.10	87.49
30% - 20%		4.22	11.22		5.49	10.74		7.10	12.58		93.00	87.47
30% - 25%		4.22	11.89		5.62	11.37		7.24	13.27		92.85	86.74
30% - 30%		4.33	12.78		5.78	11.92		7.45	13.85		92.62	86.19

Table 5.17: Diversified Portfolio

The quantiles $q_{0.95}$ and $q_{0.995}$ of the default rate are equal to those of the homogenous portfolio presented in Table 5.7 for independent exposures and for homogenous asset correlation $\rho^a = 5\%$. However, as the asset correlation increases, the quantiles of the default rate reveal to be substantially lower compared to the homogenous portfolio case, with differences being more pronounced for $q_{0.995}$. This reduced dispersion of the default rate can be attributed to the diversification in default probabilities as observed analogously for loss distributions in Section 5.4.5.

Portfolio value and credit loss are sensitive to the frequency and the average of maturities in the portfolio. Compared to the homogenous portfolio in Table F.1 in the Appendix, the variation in portfolio values increases due to the elevated sensitivity of the values of ten-year credit exposures to a change in asset value and due to the increased risk concentration introduced by the variation in face values.[8] Reflecting the increase in Credit-VaR, the dispersion of D_1 decreases with increasing asset correlations.

The quantiles $q_{0.99}$ of D_1 in Table F.1 increase, because the missing potential of mark-to-model loss of exposures that mature at time $\bar{t}$ overcompensates for the additional potential of devaluation of the ten-year exposures. The median portfolio values are below par, because, in a typical state of factors at time $\bar{t}$, the outstanding contractual interest does not suffice to compensate for the default risk of exposures. This effect is less pronounced for the diversified portfolio, because exposures that redeem at $T = \bar{t} = 1$ are not subject to this effect and overcompensate for the additional decline in credit values of the ten-year exposures. Since D_1 is capped by the value of a corresponding riskless portfolio, quantiles $q_{0.01}$ of the value of a diversified portfolio do not exceed those of the homogenous portfolio.

For portfolio loss $L(\mathbb{E}[D_1])$ and $L(D_0)$ in Table F.1 acceptance and rejection barriers as well as Credit-VaR are lower for the diversified portfolio with comparable asset correlations. The considerable decrease in Credit-VaR can be explained by the reduced potential of a mark-to-model devaluation of exposures that mature at time $\bar{t}$, which overcompensates the increased loss potential of ten-year exposures. For independent exposures, the risk-smoothing effect of a variation of maturities is offset by the increased concentration risk caused by a variation of face values. Acceptance barriers decline because the definition of credit loss excludes the netting of positive credit performance. The diversified portfolio allows for fewer combinations of exposures that result in a considerable portfolio loss, in a state where factors perform positive in general.

Overall, Credit-VaR declines and the yellow zone contracts if the diversified portfolio is considered, giving banks an incentive to improve the diversification of their credit portfolios. For example, the rejection barrier is still below banks' typical 6% target core capital ratio for an average asset correlation of 15%. In the case of homogenous $\rho^a = 20\%$, the yellow zone shifts from $(1.69\%, 6.66\%)$ to $(1.47\%, 4.49\%)$ for $L(\mathbb{E}[D_1])$ and from $(3.10\%, 8.69\%)$ to $(2.84\%, 6.42\%)$ for $L(\mathbb{E}[D_1])$, so that the discriminatory power of backtesting improves if a diversified portfolio is examined.

[8] Cf. findings on Credit-VaR for different maturities in Section 5.4.6 and for different face values in Section 5.4.7.

5.5.2 Variation of Holding Period

The effects of a shortened quarterly or semi-annual holding period on the backtesting and Credit-VaR of a diversified portfolio are generally identical to those of the homogenous portfolio and are, for reasons of completeness, presented in Table 5.18.

Holding Period		$\bar{t} = 0.25$			$\bar{t} = 0.5$			$\bar{t} = 1$		
Loss Type	$\rho^a_{i,j}$	$\bar{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$	$\bar{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$	$\bar{q}_{0.05}$	$q_{0.95}$	$q_{0.995}$
$\mathbf{L}(\mathbb{E}[\mathbf{D_1}])$	0% - 0%		0.79	0.93		1.38	1.68		2.46	3.04
	10% - 0%		1.04	1.41		1.85	2.59		3.33	4.87
	10% - 5%		1.13	1.63		1.97	3.00		3.61	5.56
	10% - 10%		1.22	1.78		2.15	3.37		3.90	6.36
	20% - 0%		1.23	1.83		2.18	3.53		4.04	6.56
	20% - 10%		1.36	2.19		2.43	4.10		4.52	7.88
	30% - 0%		1.39	2.30		2.54	4.39		4.67	8.33
	20% - 20%		1.49	2.48		2.66	4.88		4.89	9.21
	30% - 10%		1.51	2.54		2.70	4.87		5.03	9.32
	25% - 25%	3.17	1.64	2.79	2.42	2.90	5.59	1.47	5.38	10.66
	30% - 20%		1.62	2.87		2.90	5.51		5.49	10.74
	30% - 30%		1.76	3.19		3.09	6.34		5.78	11.92
$\mathbf{L}(\mathbf{D_0})$	0% - 0%		1.01	1.16		1.82	2.12		3.47	4.09
	10% - 0%		1.32	1.72		2.38	3.22		4.56	6.25
	10% - 5%		1.43	1.97		2.55	3.67		4.90	7.05
	10% - 10%		1.52	2.13		2.75	4.07		5.25	7.90
	20% - 0%		1.53	2.18		2.78	4.21		5.38	8.06
	20% - 10%		1.69	2.58		3.08	4.86		5.96	9.56
	30% - 0%		1.71	2.66		3.17	5.09		6.08	9.90
	20% - 20%		1.84	2.89		3.36	5.70		6.42	11.02
	30% - 10%		1.86	2.93		3.39	5.66		6.54	11.09
	25% - 25%	3.60	2.00	3.22	3.20	3.64	6.47	2.84	7.00	12.55
	30% - 20%		1.98	3.30		3.63	6.36		7.10	12.58
	30% - 30%		2.15	3.64		3.86	7.22		7.45	13.85

Table 5.18: Holding Period of Diversified Portfolio

Shortening the holding period leads to a decrease in the rejection barrier and Credit-VaR, whereas the acceptance barrier increases. This yields a contracted yellow zone and enhances the discriminatory power of the backtesting. For an asset-correlation structure of (10%; 5%), the Credit-VaR of $L(\mathbb{E}[D_1])$ and $L(D_0)$ decreases to 1.63% (3.0%) and 1.97% (3.67%) credit loss for a quarterly (semi-annual) holding period. For a quarterly holding period, perfect model discrimination is achieved for any correlation structure considered. Even with a semi-annual horizon, perfect discrimination is obtained for an average asset correlation of 10% (15%) for $L(\mathbb{E}[D_1])$ ($L(D_0)$).

Compared to the homogenous portfolio case, the backtesting barriers and Credit-VaR are lower for the reasons outlined in the previous section, and the acceptance barriers of $L(\mathbb{E}[D_1])$ and $L(D_0)$ decrease from to 1.69% (2.38%,3.3%) and 3.1% (3.41%,3.91%) to 1.47% (2.42%, 3.17%) and 2.84% (3.2%, 3.6%) for the annual (semi-annual, quarterly) holding period. The improved discriminatory power allows either to define a less divergent alternative model or to set a higher significance level of backtesting.

5.5.3 Variation of Asset Value Process

Analogously to Section 5.4.8, the backtesting bounds and Credit-VaR of the diversified portfolio are presented in Table 5.19 and 5.20 for any combination of a homogenous $\mu \in \{-8\%, 0\%, 8\%\}$ and a homogenous $\sigma \in \{5\%, 10\%, 20\%\}$. Corresponding to the

$\mathbf{L(\mathbb{E}[D_1])}$		μ = -8%			μ = 0%			μ = 8%		
σ	$\rho^{\mathbf{a}}_{\mathbf{i,j}}$	$\overline{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$	$\overline{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$	$\overline{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$
σ = **5%**	0% - 0%		1.72	2.28		1.59	2.13		1.36	1.93
	10% - 0%		2.30	3.68		2.11	3.43		1.76	2.92
	10% - 5%		2.49	4.18		2.31	3.96		1.90	3.34
	10% - 10%		2.75	4.75		2.52	4.39		2.03	3.78
	20% - 0%		2.85	5.13		2.61	4.82		2.15	4.05
	20% - 10%		3.14	6.12		2.87	5.73		2.36	4.78
	30% - 0%		3.37	6.94		3.08	6.49		2.52	5.50
	20% - 20%		3.49	7.41		3.17	6.91		2.56	5.70
	30% - 10%		3.59	7.59		3.31	7.04		2.66	6.15
	25% - 25%	1.83	3.79	8.60	1.83	3.53	8.02	2.00	2.78	6.60
	30% - 20%		3.86	8.57		3.53	8.06		2.84	6.78
	30% - 30%		4.07	9.69		3.72	9.00		3.07	8.04
σ = **10%**	0% - 0%		2.50	3.06		2.46	3.04		2.42	2.99
	10% - 0%		3.39	4.90		3.33	4.87		3.26	4.84
	10% - 5%		3.69	5.55		3.61	5.56		3.53	5.45
	10% - 10%		3.96	6.43		3.90	6.36		3.79	6.13
	20% - 0%		4.10	6.61		4.04	6.56		3.96	6.41
	20% - 10%		4.59	7.93		4.52	7.88		4.37	7.75
	30% - 0%		4.77	8.38		4.67	8.33		4.60	8.21
	20% - 20%		4.97	9.34		4.89	9.21		4.82	8.99
	30% - 10%		5.14	9.42		5.03	9.32		4.97	9.32
	25% - 25%	1.43	5.53	10.73	1.47	5.38	10.66	1.59	5.22	10.52
	30% - 20%		5.58	10.74		5.49	10.74		5.30	10.70
	30% - 30%		5.89	12.09		5.78	11.92		5.67	12.05
σ = **20%**	0% - 0%		3.36	3.97		3.43	4.05		3.52	4.15
	10% - 0%		4.51	6.21		4.59	6.34		4.69	6.51
	10% - 5%		4.88	7.12		4.98	7.17		5.12	7.42
	10% - 10%		5.22	7.88		5.33	8.05		5.47	8.15
	20% - 0%		5.40	8.09		5.50	8.27		5.60	8.54
	20% - 10%		6.01	9.54		6.13	9.78		6.19	9.98
	30% - 0%		6.14	9.86		6.25	10.08		6.45	10.32
	20% - 20%		6.53	11.04		6.67	11.31		6.86	11.55
	30% - 10%		6.67	11.28		6.81	11.47		6.93	11.85
	25% - 25%	1.17	7.08	12.46	1.24	7.25	12.95	1.28	7.44	13.20
	30% - 20%		7.17	12.77		7.32	13.01		7.44	13.30
	30% - 30%		7.67	14.01		7.82	14.33		7.96	14.77

Table 5.19: Variation of μ and σ for $L(\mathbb{E}[D_1])$ of the Diversified Portfolio

homogenous portfolio, Credit-VaR and rejection barriers increase with diffusion rate σ for both loss definitions, independent of the drift rate. For the acceptance barrier, a pattern identical to the homogenous portfolio is found. This can be attributed to the exclusion of positive credit performance in portfolio loss. The acceptance barriers of $L(\mathbb{E}[D_1])$ decrease if σ is increased, whereas the acceptance barriers of $L(D_0)$ decrease (increase) in σ for $\mu = 0\%$ ($\mu = 8\%$) and reach an inner maximum of $\overline{q}_{0.05} = 3.80\%$ for $\mu = -8\%$. The variability of the acceptance barrier is more pronounced for $L(\mathbb{E}[D_1])$ than for $L(D_0)$. Overall, the yellow zone expands and discriminatory power and Credit-VaR rise as σ declines, which gives banks an incentive to underestimate asset volatilities.

With respect to a change in the drift rate, acceptance barriers increase (decrease) as μ

$\mathbf{L(D_0)}$		μ=-8%			μ=0%			μ=8%		
σ	$\rho^{\mathbf{a}}_{\mathbf{i,j}}$	$\overline{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$	$\overline{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$	$\overline{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$
	0% - 0%		2.42	3.00		2.10	2.65		1.39	1.93
	10% - 0%		3.14	4.62		2.72	4.12		1.73	2.82
	10% - 5%		3.38	5.20		2.95	4.70		1.85	3.21
	10% - 10%		3.67	5.81		3.20	5.18		1.97	3.60
	20% - 0%		3.75	6.17		3.29	5.59		2.08	3.87
σ = **5%**	20% - 10%		4.12	7.30		3.60	6.64		2.25	4.52
	30% - 0%		4.36	8.03		3.80	7.28		2.41	5.25
	20% - 20%		4.53	8.68		3.94	7.88		2.44	5.38
	30% - 10%		4.63	8.82		4.08	7.94		2.53	5.82
	25% - 25%	3.21	4.90	9.92	2.91	4.35	9.04	1.38	2.62	6.26
	30% - 20%		4.96	9.92		4.34	9.08		2.67	6.42
	30% - 30%		5.22	11.09		4.57	10.05		2.87	7.52
	0% - 0%		4.32	4.94		3.47	4.09		2.40	2.96
	10% - 0%		5.54	7.27		4.56	6.25		3.18	4.72
	10% - 5%		5.95	8.07		4.90	7.05		3.43	5.31
	10% - 10%		6.30	9.05		5.25	7.90		3.68	5.95
	20% - 0%		6.42	9.17		5.38	8.06		3.84	6.24
σ = **10%**	20% - 10%		7.06	10.71		5.96	9.56		4.22	7.50
	30% - 0%		7.17	11.00		6.08	9.90		4.46	8.00
	20% - 20%		7.56	12.29		6.42	11.02		4.64	8.70
	30% - 10%		7.69	12.25		6.54	11.09		4.80	9.03
	25% - 25%	3.80	8.23	13.75	2.84	7.00	12.55	1.45	5.02	10.19
	30% - 20%		8.26	13.79		7.10	12.58		5.11	10.36
	30% - 30%		8.69	15.19		7.45	13.85		5.45	11.70
	0% - 0%		5.72	6.41		4.81	5.49		3.88	4.51
	10% - 0%		7.29	9.26		6.24	8.17		5.08	6.95
	10% - 5%		7.79	10.36		6.72	9.14		5.54	7.86
	10% - 10%		8.23	11.26		7.13	10.11		5.90	8.63
	20% - 0%		8.39	11.43		7.29	10.30		6.03	9.04
σ = **20%**	20% - 10%		9.19	13.12		8.04	11.97		6.64	10.49
	30% - 0%		9.23	13.34		8.12	12.20		6.89	10.81
	20% - 20%		9.85	14.73		8.69	13.62		7.33	12.09
	30% - 10%		9.98	14.97		8.83	13.74		7.40	12.38
	25% - 25%	3.34	10.54	16.29	2.47	9.36	15.33	1.58	7.92	13.76
	30% - 20%		10.64	16.62		9.43	15.43		7.93	13.83
	30% - 30%		11.26	17.92		10.02	16.81		8.47	15.34

Table 5.20: Variation of μ and σ for $L(D_0)$ of the Diversified Portfolio

increases for $L(\mathbb{E}[D_1])$ ($L(D_0)$), irrespective of the asset volatility. In contrast, the change in the rejection barriers and Credit-VaR from a change of μ is special. For $L(\mathbb{E}[D_1])$, the rejection barriers and Credit-VaR decrease in μ for small diffusion rates. However, the opposite is true for $\sigma = 20\%$. In contrast, the rejection barriers as well as Credit-VaR of $L(D_0)$ decline in μ for any σ.

Comparing the results for the diversified portfolio with homogenous correlations to the homogenous portfolio, acceptance and rejection barriers of $L(\mathbb{E}[D_1])$ and $L(D_0)$ are lower, predominantly due to the impact of matured exposures in the portfolio. As before, the specification of adequacy zones for $L(\mathbb{E}[D_1])$ is more robust against a mis-specification of μ, whereas zone locations for $L(D_0)$ are more stable with respect to a change in σ.

For low correlations and $\sigma = 5\%$, both loss measures even allow for perfect discrimination between the model and its alternative. Compared to the homogenous case, the rejection barriers and Credit-VaR of the diversified portfolio are considerably lower. Overall, the discriminatory power of backtesting improves as σ decreases, while capital charges decline, which gives banks an incentive to underestimate the diffusion rate of asset values.

5.6 Empirical Model Specification

Subsequent to the examination of portfolio credit loss and the backtesting bounds of synthetic model and portfolio characteristics, a diversified portfolio based on the empirically estimated risk class factor models described in Section 4.4 is examined. The model structures considered, comprise the

- rating class model,
- two-sector model with five financial and non-financial rating classes, and
- four-sector model with ten sector-rating risk classes

introduced in Section 4.4.7.

Each simulation scenario is characterized by its risk class model, the holding period and the estimation period of parameters. For each simulation scenario, the distributions of default rate, portfolio value and portfolio credit loss $L(\mathbb{E}[D_1])$ $(L(D_0))$ are simulated for quarterly and annual holding periods, based on process parameters and correlation estimates for the annual and five-year estimation periods presented in Section 4.4. The credit portfolio consists of $N = 900$ exposures $k = 1, ..., N$ with maturities and face values distributed exactly as in the diversified portfolio case of the previous section, so that differences in the distributions of portfolio loss are entirely due to differing drift rates, volatilities, default probabilities of asset values, and changes in asset correlations.

The drift rate μ_{rc_k} and diffusion rate σ_{rc_k} of exposure $k = 1, ..., N$; in risk class $rc_k \in RC^m$ are specified by the parameter estimates in Table 4.9. In contrast to the diversified portfolio model each risk class consists only of homogenously rated exposures. Inner-class and inter-class asset correlations $\rho^{a,m}_{rc_k,rc_l}$; k,l=$1, ..., N$; $rc_k, rc_l \in RC^m$ of the risk class models $m \in RC$ and the corresponding matrix of factor coefficients $\mathbf{B}^m$ are set according to the estimation results presented in Section 4.4.8.4.

The asset values of the exposures are specified by the (end-of-period) average of the par-coupon bond-derived asset values of the risk class throughout the estimation period in question, as presented in Section Table 4.11. The risk-neutral and real-world default probabilities of the exposures are given in Table 4.11.

5.6.1 Rating-Class Model

The aggregation of exposures with a particular rating into a single risk class ignores diversification effects across economic sectors. Nevertheless, it is common practice in credit portfolio modelling to consider only the rating of exposures as class-defining property. In 5.21, backtesting barriers based on the default rate, $L(\mathbb{E}[D_1])$ and $L(D_0)$ and portfolio value, simulated for quarterly and annual holding periods, are compared for the six aforementioned parameter sets.

Rating-Class Model		$\widehat{\mathbf{p}}$			$\mathbf{L(\mathbb{E}[D_1])}$			$\mathbf{L(D_0)}$			$\mathbf{D_1}$		
Holding Period	**Estimation Period**	$\overline{\mathbf{q}}_{0.05}$	$\mathbf{q}_{0.95}$	$\mathbf{q}_{0.995}$	$\overline{\mathbf{q}}_{0.05}$	$\mathbf{q}_{0.95}$	$\mathbf{q}_{0.995}$	$\overline{\mathbf{q}}_{0.05}$	$\mathbf{q}_{0.95}$	$\mathbf{q}_{0.995}$	$\overline{\mathbf{q}}_{0.05}$	$\mathbf{q}_{0.95}$	$\mathbf{q}_{0.995}$
$\bar{\mathbf{t}} = \mathbf{0.25}$	1999	0.00	0.11	0.22	4.97	1.65	2.63	5.60	2.11	3.16	94.43	98.03	96.93
	2000	0.00	0.11	0.22	3.81	2.01	3.62	4.60	2.66	4.36	95.41	97.39	95.67
	2001	0.00	0.00	0.11	3.59	2.73	5.02	4.15	3.25	5.61	95.94	96.84	94.43
	2002	0.00	0.33	0.67	5.11	2.36	3.62	6.15	3.11	4.49	93.90	97.06	95.61
	2003	0.00	0.11	0.22	3.97	1.85	2.60	4.45	2.19	2.98	95.68	98.20	97.30
	1999-2003	0.00	0.00	0.00	4.09	1.28	1.80	4.60	1.61	2.19	95.43	98.59	97.95
$\bar{\mathbf{t}} = \mathbf{1}$	1999	0.22	10.33	16.78	1.45	6.38	10.15	4.47	9.01	13.12	95.61	91.16	86.99
	2000	0.44	14.44	28.56	0.64	9.02	16.00	4.24	13.05	20.54	95.86	86.99	79.49
	2001	0.11	9.78	22.78	1.23	8.30	16.11	3.30	10.70	18.76	97.04	89.41	81.29
	2002	0.89	19.22	27.22	0.43	8.41	12.35	5.21	12.84	17.26	94.84	87.33	82.85
	2003	0.11	6.78	10.11	1.44	4.81	7.10	3.24	6.33	8.77	97.04	94.23	91.66
	1999-2003	0.00	4.67	7.33	1.64	4.28	6.41	3.61	5.95	8.29	96.52	94.31	91.90

Table 5.21: Backtesting of Rating-Class Model

For annual holding periods, the Credit-VaR of the yearly estimation periods varies between 7.1% (6.33%) and 16.11% (20.54%) for $L(\mathbb{E}[D_1])$ $(L(D_0))$, while the Credit-VaR of 6.41% (8.29%) for the five-year estimation period is more moderate, due to smaller asset volatilities and drift rate estimates close to zero. The acceptance barriers for annual model estimations vary between 0.43% (3.24%) and 1.45% (5.21%) for $L(\mathbb{E}[D_1])$ $(L(D_0))$, and the rejection barrier is positioned at values between 4.81% (6.33%) and 9.02% (13.05%), representing a yellow zone between 3.37% (3.09%) and 8.38% (8.8%) for $L(\mathbb{E}[D_1])$ $(L(D_0))$. For the five-year estimation period, the acceptance and rejection barriers are set at 1.64% (3.61%) and 4.28% (5.95%) for $L(\mathbb{E}[D_1])$ $(L(D_0))$, which results in a yellow zone of 2.64% (2.35%).

Backtesting barriers as well as Credit-VaR that refer to the empirically estimated model parameters show a substantial variation, which is undesirable with respect to the robustness of the backtesting. The use of annual estimation periods implies the specification of a model used for point-in-time considerations of credit portfolio risk by parameter sets that condition on an incomplete (annual) period of a full credit cycle. Furthermore, Credit-VaR is close to the 8% target capital charge given the model specification derived from a five-year estimation period. However, additional capital requirements for market risk and operational risk will potentially raise the total capital charge above a level, that is still convenient for banks.

For the reason of model consistency and the robustness of backtesting, it is consequently recommended to use portfolio models specified on the basis of a long-term estimation period.

A shortening of the holding period results in a reduction of capital charges and an improved discriminatory power without altering the setting of the portfolio model, the sample period of data, parameter estimation, credit valuation and the backtesting procedure. When the minimum quarterly holding period is considered, a Credit-VaR of 1.8% (2.19%) for $L(\mathbb{E}[D_1])(L(D_0))$ is obtained. The acceptance and rejection barriers of 4.09% (4.6%) and 1.28% (1.61%) for $L(\mathbb{E}[D_1])$ $(L(D_0))$ intersect and allow for a perfect discrimination between the model alternatives, which are based on the five-year estimation period.

5.6.2 Two-Sector Model

The two-sector model with five risk classes defined by exposures' sector-rating attribute provides a more detailed model of credit-risk dependence, when compared to the rating class model in the previous section. The two-sector model is made up of five risk classes FIN-AA, FIN-A, NF-AA, NF-A, and NF-BBB, from the financial (FIN) and non-financial (NF) sector.

Compared to the rating-model, Credit-VaR and the rejection barrier for an annual holding period increase for both loss definitions, given a five-year estimation period. In contrast, both quantiles decrease in four cases if the model is based on an annual estimation period. Quantiles of default rate and portfolio value show analogous results. For a quarterly holding period, the same effects are observed expect for a marginal decrease in Credit-VaR for the five-year estimation, which can be explained by peculiarities in the simulation.

The effects, a more detailed risk class structure has on the acceptance barriers is less clear-cut. The acceptance barrier of $L(\mathbb{E}[D_1])$ $(L(D_0))$ is raised for 3(3) year-based simulations and the full-cycle-estimated model. Given quarterly risk horizons, the same effects are observed. The increase in the acceptance barriers enlarges the range of credit loss accepted for more fine-grained model of credit dependence, while the yellow zone of model indetermination remains more or less unchanged, and the increase in rejection barriers reduces the risk of model rejection at the cost of enlarged capital charges. However, the location of the backtesting barriers cannot be easily be interpreted without a detailed examination of the changes in the estimates of process parameters and asset correlations.

In the two-sector model, standard deviations and real-world default probabilities increase considerably on average, as can be seen in Table 4.9 and Table 4.11, so that an increase in the rejection barrier and Credit-VaR is expected. With respect to the correlation structure of the portfolio, the number of inner-sector correlations is decreased from 201,600 for

Two-Sector Model		$\widehat{\mathbf{p}}$			$\mathbf{L}(\mathbb{E}[\mathbf{D_1}])$			$\mathbf{L}(\mathbf{D_0})$			$\mathbf{D_1}$		
Holding Period	**Estimation Period**	$\overline{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$	$\overline{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$	$\overline{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$	$\overline{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$
$\bar{\mathbf{t}} = \mathbf{0.25}$	1999	0.00	0.11	0.22	4.57	1.46	2.24	5.17	1.87	2.70	94.86	98.28	97.41
	2000	0.00	0.00	0.11	3.45	1.73	2.99	4.11	2.28	3.59	95.91	97.78	96.44
	2001	0.00	0.00	0.22	3.59	2.34	3.99	4.22	2.89	4.62	95.85	97.22	95.44
	2002	0.00	1.00	1.56	5.57	2.40	3.49	6.76	3.10	4.30	93.27	97.05	95.81
	2003	0.00	0.11	0.33	4.23	1.79	2.75	4.75	2.14	3.14	95.34	98.17	97.11
	1999-2003	0.00	0.00	0.11	4.56	1.28	1.79	5.10	1.63	2.18	94.93	98.57	97.97
$\bar{\mathbf{t}} = \mathbf{1}$	1999	0.22	7.44	12.89	1.53	5.42	8.77	4.24	7.74	11.31	95.85	92.45	88.82
	2000	0.22	10.33	21.22	0.92	7.53	13.91	3.77	10.70	17.47	96.32	89.37	82.56
	2001	0.22	10.33	20.56	1.13	7.66	14.10	3.52	10.31	17.11	96.74	89.83	82.95
	2002	1.67	19.67	26.44	0.24	7.86	11.66	5.73	12.38	16.54	94.30	87.77	83.57
	2003	0.22	7.89	13.22	1.46	5.17	8.39	3.60	6.87	10.23	96.55	93.52	90.08
	1999-2003	0.11	6.11	9.11	1.68	4.43	6.51	4.04	6.27	8.52	96.03	93.99	91.65

Table 5.22: Backtesting of Two-Sector Model

the rating class model to 80,100 for the two-sector model, while the number of inter-sector correlations increase from 202,000 to 324,000. With inner-sector correlations exceeding inter-sector correlations on average,[9] the average asset correlation decreases with an increase in the number of risk classes. A dampening effect on Credit-VaR can be expected due to the improved level of detail in the modelling of credit dependence. This is, however, overcompensated for by the increase in asset volatilities.

5.6.3 Four-Sector Model

The four-sector model with ten sector-rating classes incorporates the maximal differentiation of risk classes that can be specified based on the estimation results presented in Section 4.4. The Credit-VaR and backtesting barriers are presented in Table 5.23, analogously to the previous sections.

Four-Sector Model		$\widehat{\mathbf{p}}$			$\mathbf{L}(\mathbb{E}[\mathbf{D_1}])$			$\mathbf{L}(\mathbf{D_0})$			$\mathbf{D_1}$		
Holding Period	**Estimation Period**	$\overline{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$	$\overline{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$	$\overline{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$	$\overline{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$
$\bar{\mathbf{t}} = \mathbf{0.25}$	1999	0.00	0.00	0.11	4.26	1.64	2.57	4.88	2.09	3.07	95.16	98.07	97.05
	2000	0.00	0.00	0.11	3.02	1.66	2.83	3.68	2.20	3.44	96.34	97.87	96.60
	2001	0.00	0.00	0.11	3.36	2.25	3.78	3.94	2.75	4.35	96.13	97.39	95.71
	2002	0.00	1.56	2.22	4.75	3.01	4.84	6.12	3.96	5.92	93.91	96.16	94.17
	2003	0.00	0.56	0.89	3.86	2.28	3.69	4.40	2.66	4.12	95.70	97.58	96.05
	1999-2003	0.00	0.11	0.33	5.03	1.42	1.99	5.67	1.81	2.43	94.36	98.39	97.72
$\bar{\mathbf{t}} = \mathbf{1}$	1999	0.11	7.11	13.67	1.43	5.73	9.93	3.98	8.11	12.54	96.13	92.08	87.58
	2000	0.11	7.89	16.22	0.98	6.78	12.14	3.45	9.72	15.42	96.68	90.37	84.61
	2001	0.11	8.56	16.22	1.19	6.95	12.07	3.27	9.33	14.78	97.07	90.87	85.31
	2002	1.89	20.44	31.78	0.25	9.19	15.07	5.44	14.47	20.67	94.66	85.63	79.40
	2003	0.56	7.22	12.11	1.36	5.40	9.22	3.36	7.19	11.14	97.03	93.16	89.07
	1999-2003	0.33	7.56	9.89	1.59	4.71	6.65	4.57	6.97	9.13	95.53	93.26	91.05

Table 5.23: Backtesting of Four-Sector Model

[9] Cf. Table 4.15 and Table 4.16

For the full-sample model estimation, Credit-VaR and the rejection barrier increase further when the maximum achievable level of detail in the risk class structure is examined, whereas the acceptance barrier declines for $L(\mathbb{E}[D_1])$ and increases for $L(D_0)$.

Although, inner-sector correlations increase to an average of 15.1%, the number of inner-sector correlations is further reduced to 39,600, so that the 364,500 inter-sector correlations with an average of 4.6% are the predominant factor that determining the correlation structure of the portfolio. A further increase in asset volatilities and default probabilities of the risk classes, however, compensates for the risk-reducing effects of the diversified credit dependence. With respect to the use of the one-factor model in portfolio credit risk assessments, it can be conjectured that one-factor models systematically overestimate the correlation effects of credit risk, but underestimate the effects of volatility on credit risk, if asset volatilities are empirically estimated.

For any of the risk class structures that have been examined, the backtesting barriers result in acceptance and indifference zones, that can be assumed to result in an approval of banks' established credit portfolio models and that enables a short-term supervisory actions in the case a credit loss occurs that endangers the solvency of a bank. However, for a definite assessment of the applicability of the proposed backtesting approach, a comparison of Credit-VaR and backtesting barriers with banks' actual credit portfolio loss is required, based on an analogous definition of credit risk.

Chapter 6

Conclusion

With respect to a prospective supervisory approval of calculating regulatory capital requirements based on banks' internal credit portfolio models, this dissertation addresses the question of accurately estimating and backtesting a structural credit portfolio model.

A structural first-passage model is suggested for the valuation of defaultable loans with deterministic interest payments and redemption of face value at a fixed maturity. In the credit valuation model credit default is triggered by a constant default threshold at any time during the lifetime of the loan. Accordingly, credit default events incorporate over-indebtedness and illiquidity of the obligor. Comparative statics are used to assess the properties of the credit valuation model. In the portfolio context, the dependence of credit exposures values is implemented by a risk class model of orthogonal standard normal systematic and specific factors that control for the normalized log-returns of the asset values of exposures.

With respect to the requirements of methodological consistency, robustness, independence of accounting standards and prudence, several definitions of portfolio credit loss are considered. Suggesting a two-hypotheses test, the zone approach used for the backtesting of market risk models is transferred to backtesting credit portfolio models' adequacy to assess the credit risk of a loan portfolio. As a test statistic a one-period observation of portfolio credit loss is used. Three zones of model adequacy are defined for portfolios' credit loss: a green zone, where an alternative more prudent model is rejected and the tested model is qualified as adequate to set capital requirements, a yellow zone, in which the model's adequacy cannot be determined, and a red zone, in which the model is rejected.

In an approach, which is unique to the literature on the estimation of structural credit portfolio models, a two-stage quasi-maximum likelihood estimation of a non-linear non-Gaussian state-space model based on the Extended Kalman-Filter is introduced. In the course of the estimation the process parameters, correlations and series of systematic factors and asset values the credit portfolio risk-class model are estimated

In a simulation study, first, the backtesting procedure is specified with respect to the definition of credit portfolio loss, the alternative model which must be rejected, and the significance level of the test. The prudent model alternative of backtesting is specified to incorporate a default probability add-on of one percent, an enhancement of asset volatilities by 5% for any exposure, and a fixed asset correlation of 25% between any pair of exposures. The asset correlations of the alternative model are independent of the correlation estimate of the model to be tested, in order to preclude any incentive to banks to underestimate the asset correlations of their credit portfolio models. The locations of zone-defining barriers are examined for differing default models, holding periods and for different numbers of simulation sub-intervals.

Second, the distributions of default rate and credit portfolio loss are examined for different specifications of the credit valuation model and the risk class factor model, as well as for different characteristics of the credit portfolio. Within these analyzes, two definitions of credit loss are considered that refer to the unexpected credit loss, as prescribed by the New Capital Adequacy Framework, and to the valuation of exposures at the time of the risk assessment, as implemented in market risk measurement. The impact of the portfolio characteristics on the backtesting are analyzed based on a portfolio of homogenous loans and a diversified portfolio of heterogenous loans. The effect of a variation of exposures' time-to-maturities, face values and default probabilities on the location of the backtesting zones is examined. The impact of drift rate and volatility of asset values and the risk-class structure specified by the model is assessed. The model's specification of the drift rate and volatility of asset values and the impact of the risk-class structure on the zone locations is assessed.

The specification of the portfolio model and the characteristics of the portfolio substantially affect the backtesting barriers. Acceptance (rejection) barriers decline (increase) as asset correlations increase and the granularity of exposures decreases. The diversification of the portfolio with respect to the maturities of exposures and the decrease of the holding period diminishes the yellow zone considerably, i.e. the discriminatory power of the backtesting is improved. With respect to the different loss definitions, acceptance barriers of $L(\mathbb{E}[D_1])$ are shown to be more robust against a mis-specification of the drift rate, while acceptance barriers of $L(D_0)$ are more stable with respect to a change in σ. In contrast to the backtesting of market risk models, no unambiguous location of zones of portfolio loss can be defined to backtest the adequacy of portfolio credit risk models and to prompt the supervisory interventions.

The proposed test is easily applicable, computationally feasible and requires minimal data. In principle, the backtesting approach can be used for any credit risk model. In particular, the specification of the alternative model can be adapted to any parametric or structural assumptions of a credit portfolio model. Considering typical amounts of loss provisions set aside by banks as an indicator for the incurred credit portfolio loss, the proposed backtesting approach defines adequacy zones that allow the acceptance of models at a significance level of $\overline{\alpha} = 5\%$ under standard credit market conditions.

Capital requirements derived from our model comply with the core capital ratios typically maintained by banks only for a confidence level of Credit-VaR that is substantially lower than the level implicitly pretended by the revised capital adequacy framework. Simulation results suggest that banks might have difficulties fulfilling capital requirements for default and devaluation risk derived from Credit-VaR for annual holding periods, however, a shortened quarterly or semi-annual holding period may provide a solution. Furthermore, a reduced holding period causes an increased discriminatory power as acceptance barriers rise and rejection barriers decline.

The implementation of model-based capital requirements to cover the risk of credit-quality-induced adverse changes in credit portfolio values, in addition to existing capital charges for market and operational risk, is likely to result in capital requirements that exceed those of the original Basel Capital Accord or imposed by the IRB approach of the revised Capital Standards. The calculation of integrated market and credit-risk measures of bank portfolios may solve the problem of the arbitrary aggregation of dependent credit exposures without considering the sub-additive nature of credit risk.

The requirement of a long-term estimation period, spanning at least a complete credit cycle, results in estimated drift rates close to zero. Strictly positive riskless rates result in real-word probabilities of default that exceed the risk-neutral default probabilities and that are not realistic in view of the historical default experience of banks and ratings agencies. There are two likely solutions to this problem: First, in simulating credit risk, asset values may be calibrated according to real-world default probabilities estimated for the portfolio and risk class structure in question by a supplemental rating system. Second, structural credit valuation models with mean reverting drift rate promise a better fit of the model to the cyclical nature of asset values throughout a credit cycle.

The data used in the estimation provides potential for improvement. Bond market data have been used, because CDS market data have not available for a complete credit cycle at the beginning of the examination. Although, bond market data turn the model estimation more involved because of the heterogeneity of data, and because of unstable, competing, or missing price information, while CDS prices of credit risk refer to standardized contracts of obligors and are subject to a time-continuous pricing. Furthermore, it is suspected that scarce data and intense spread movements of individual obligors are reflected in fitted term structures of risk classes to an extent that exaggerates the spread movements typical for a risk class. In this context, the separation of class-specific and obligor-specific effects in the fitting of risk class curves provides further potential for examination.

Estimation results reveal deficiencies in the fitting of empirical credit-spread structures, that are attributed to the use of a structural credit valuation model. With the riskless rate required to be constant and the drift rate coercively estimated as being close to zero, factually only the asset value and the diffusion rate remain as free parameters of model estimation, which seriously restricts the functional ability to fit empirical spread structures. The zero drift estimates result in real-world default probabilities that differ substantially from empirical default rates. With respect to the simulation of portfolio credit loss, the high long-term default probabilities are compensated for by the typically small increase in the cumulative distribution function of the default time in the short term, which is constitutive for achieving Credit-VaR figures acceptable to banks. The development of a structural credit valuation model that incorporates a mean reversion of the asset value is recommended as a subject of further research. As an alternative, intensity-based risk-class models are considered to be especially suited to integrated risk management for their well-behaved aggregation of credit and interest rate risk, the superior fit of empirical credit spreads, the existence of well-established estimation procedures, and the provision for mean-reverting credit spreads, which results in an improved fit of real-world default probabilities.

Beside the adequate modelling, estimation and backtesting of credit portfolio models, procedural aspects concerning the identification, measurement and disclosure of credit risk exposures are increasingly important. Extended disclosure standards with respect to contractual off-balance credit exposures and the composition of structured credit products are required for a reliable credit risk management. The compliance of the credit-granting policies with risk assessment standards represents another field of possible improvement.

Additional empirical research is required on the dependence between the credit risk of single exposures, on the pricing of multi-obligor credit derivatives, and the calibration of respective credit-risk models. Furthermore, supplementary research on the CDS-based estimation of the proposed credit portfolio model is required, especially with respect to the differentiation between systematic and specific risk factors. The estimation of single-obligor asset value processes using equity market data may be another field of future research. Finally, the development of benchmark correlation products, such as n^{th}-to-default basket derivatives for equity-index related credit baskets, which are traded in active markets, might foster the estimation of credit portfolio models.

Appendix A

Data Set

Issuer	Year	<1998	1998	1999	2000	2001	2002	2003	All
DE	No. of bond issues	70	11	17	13	10	19	21	161
	Avg. issue amount	5,217.7	7,931.8	7,899.8	9,359.2	10,946.1	10,661.6	9,062.5	7,520.5
	Non-matured bonds			95	83	78	80	83	161
	Avg. no. of prices			73.3	67.7	65.0	62.8	57.1	65.1
FR	No. of bond issues	32	11	11	29	48	52	63	246
	Avg. issue amount	10,212.0	8,457.1	7,223.7	4,842.2	3,395.3	5,572.6	5,736.2	5,912.0
	Non-matured bonds			52	71	93	96	110	246
	Avg. no. of prices			32.8	36.4	39.8	46.0	53.7	41.8
DE&FR	No. of bond issues	102	22	28	42	58	71	84	407
	Avg. issue amount	6,784.6	8,194.5	7,634.2	6,240.3	4,697.2	6,934.4	6,598.6	6,553.0
	Non-matured bonds			147	154	171	176	193	407
	Avg. no. of prices			106.2	104.1	104.8	108.8	110.8	106.9

Table A.1: Overview of Government Bond Data

Sectors	<1998	1998	1999	2000	2001	2002	2003	All
AUT - Automobile	30	3	13	8	17	22	30	123
BMA - Basic Materials	22	9	7	6	12	8	13	77
COM - Communications	12	8	10	9	12	15	12	78
CON - Construction	14	5	7	3	4	6	4	43
CCY - Consumer, cyclic	20	6	9	4	14	8	6	67
CNC - Consumer, non-cyclic	27	7	11	15	25	15	14	114
ENY - Energy	18	2	4	9	3	4	7	47
FIN - Financial	816	214	252	225	334	89	71	2001
IND - Industrial	6	2	10	5	12	7	4	46
MED - Media	12	5	3	3	2	1	3	29
TEC - Technology	15	–	3	1	6	2	2	29
TRA - Transportation	15	1	1	2	4	3	3	29
UTY - Utilities	28	10	15	14	20	19	28	134
Issues all	1035	272	345	304	465	199	197	2817
Issues all exFIN	219	58	93	79	131	110	126	816

Table A.2: Number of Bond Issues per Sector and Year

Sectors	<1998	1998	1999	2000	2001	2002	2003	All
AUT - Automobile	181.6	132.0	601.6	643.8	932.1	515.7	395.6	470.5
BMA - Basic Materials	106.0	198.5	144.3	533.3	314.2	1,012.5	409.2	331.4
COM - Communications	458.8	722.8	1,095.0	836.1	678.2	840.0	1,208.3	833.4
CON - Construction	180.3	173.8	500.9	733.3	443.8	356.7	600.0	358.5
CCY - Consumer, cyclic	168.7	89.2	362.2	275.0	445.7	412.5	496.7	310.3
CNC - Consumer, non-cyclic	166.1	217.3	438.2	510.0	508.2	357.4	590.6	393.1
ENY - Energy	135.5	152.4	1,006.3	305.0	281.7	32.5	521.4	300.8
FIN - Financial	130.5	183.6	172.4	179.5	181.3	275.9	453.3	172.2
IND - Industrial	92.4	75.9	369.4	550.0	604.2	297.9	400.0	399.8
MED - Media	201.4	192.4	483.3	316.7	500.0	30.0	383.3	274.5
TEC - Technology	181.2	–	756.7	1,000.0	645.0	200.0	750.0	405.4
TRA - Transportation	1,595.6	766.9	1,350.0	512.5	607.7	666.7	900.0	1,179.5
UTY - Utilities	354.8	563.3	494.3	385.7	657.8	959.5	568.7	564.9
Issues all	166.8	213.7	278.2	267.6	296.3	460.1	530.9	262.7
Issues all exFIN	300.8	322.6	554.5	515.1	588.0	597.5	570.3	479.9

Table A.3: Average Corporate Bond Issue Amount per Sector and Year in EUR

Bonds denominated in national currencies are converted at the official Euro conversion rates

Sector	Year	AA	A	BBB	NI	NR	All
AUT	1999		6 / 31 / 15.9	2 / 10 / 2.4		3 / 4 / 3.4	10 / 45 / 21.7
	2000		5 / 32 / 23.5	2 / 6 / 2.9		2 / 3 / 3.0	9 / 41 / 29.4
	2001		6 / 43 / 32.6	2 / 10 / 4.4		3 / 4 / 2.7	11 / 57 / 39.7
	2002		7 / 46 / 35.4	4 / 22 / 18.4		2 / 5 / 3.5	12 / 73 / 57.3
	2003		7 / 72 / 55.3	3 / 12 / 10.0	1 / 12 / 10.0	1 / 2 / 2.0	12 / 98 / 77.3
AUT all			8 / 102 / 32.6	4 / 28 / 7.6	1 / 12 / 2.0	5 / 8 / 2.9	12 / 123 / 45.2
BMA	1999		5 / 12 / 8.7	2 / 6 / 2.8		6 / 14 / 3.5	13 / 32 / 14.9
	2000	1 / 1 / 0.5	5 / 12 / 6.5	5 / 7 / 4.9		9 / 18 / 5.4	20 / 38 / 17.3
	2001	1 / 1 / 1.0	4 / 10 / 6.7	7 / 14 / 8.7	1 / 1 / 0.9	12 / 25 / 8.6	25 / 51 / 25.9
	2002	2 / 3 / 2.0	6 / 13 / 11.4	9 / 16 / 12.6	1 / 1 / 0.9	11 / 15 / 11.1	28 / 48 / 37.9
	2003	3 / 6 / 4.3	7 / 20 / 15.5	7 / 17 / 13.5	3 / 4 / 3.5	10 / 13 / 7.5	30 / 60 / 44.4
BMA all		3 / 6 / 1.6	8 / 25 / 9.8	9 / 24 / 8.5	3 / 4 / 1.1	16 / 30 / 7.2	31 / 77 / 28.2
COM	1999	5 / 17 / 12.8	4 / 6 / 3.5	2 / 2 / 1.1		1 / 1 / 1.0	11 / 26 / 18.3
	2000	6 / 20 / 13.5	5 / 11 / 9.8	2 / 2 / 1.8	1 / 1 / 0.0	1 / 1 / 0.3	14 / 35 / 25.4
	2001	2 / 4 / 3.7	9 / 35 / 29.1	2 / 2 / 1.9	1 / 2 / 0.1		14 / 43 / 34.8
	2002		8 / 28 / 23.6	7 / 27 / 21.1		2 / 2 / 0.4	17 / 57 / 45.1
	2003		8 / 28 / 24.4	9 / 37 / 32.0	1 / 1 / 1.0	1 / 1 / 0.5	19 / 67 / 57.9
COM all		6 / 23 / 6.0	12 / 54 / 18.1	10 / 40 / 11.7	3 / 4 / 0.2	3 / 3 / 0.4	22 / 78 / 36.4
CON	1999	1 / 1 / 0.2	2 / 12 / 5.0			7 / 12 / 5.9	9 / 25 / 11.1
	2000	1 / 4 / 3.7	2 / 12 / 7.3	1 / 4 / 2.8		5 / 6 / 3.2	9 / 26 / 17.0
	2001	1 / 5 / 3.2	1 / 4 / 4.0	3 / 13 / 9.4		5 / 6 / 2.7	10 / 28 / 19.3
	2002	1 / 5 / 3.7	2 / 7 / 6.7	5 / 17 / 13.3	1 / 1 / 0.6	4 / 5 / 1.9	12 / 35 / 26.1
	2003	1 / 6 / 5.2	2 / 8 / 7.3	4 / 16 / 13.9	2 / 3 / 2.5	3 / 4 / 2.4	12 / 37 / 31.4
CON all		1 / 6 / 3.2	3 / 18 / 6.1	5 / 18 / 7.9	2 / 3 / 0.6	9 / 15 / 3.2	13 / 43 / 21.0
CCY	1999	1 / 1 / 0.4	2 / 3 / 1.9	3 / 4 / 1.4		17 / 21 / 7.0	22 / 29 / 10.7
	2000	2 / 2 / 0.3	2 / 3 / 1.8	5 / 8 / 4.2		14 / 19 / 9.5	22 / 32 / 15.8
	2001	1 / 1 / 0.0	3 / 3 / 2.5	7 / 19 / 14.6		18 / 23 / 13.9	28 / 46 / 31.1
	2002	2 / 2 / 1.1	3 / 3 / 3.0	10 / 29 / 25.5		13 / 13 / 8.0	28 / 47 / 37.6
	2003	2 / 3 / 2.0	2 / 3 / 2.6	11 / 34 / 31.8		14 / 14 / 7.7	29 / 54 / 44.2
CCY all		3 / 5 / 0.8	4 / 6 / 2.4	11 / 35 / 15.5		28 / 35 / 9.2	34 / 67 / 27.9
CNC	1999	5 / 14 / 7.3	3 / 6 / 3.3	5 / 6 / 2.0		10 / 13 / 6.8	21 / 39 / 19.4
	2000	5 / 16 / 11.2	5 / 10 / 6.2	5 / 9 / 6.7		8 / 16 / 9.0	22 / 51 / 33.0
	2001	5 / 16 / 13.7	8 / 16 / 9.9	8 / 28 / 20.7		10 / 12 / 6.1	31 / 72 / 50.5
	2002	5 / 11 / 3.6	9 / 25 / 22.7	8 / 33 / 27.1	1 / 1 / 0.9	7 / 11 / 7.3	30 / 81 / 61.5
	2003	4 / 17 / 12.1	11 / 29 / 25.7	7 / 31 / 27.4	3 / 4 / 2.1	10 / 14 / 9.2	34 / 95 / 76.5
CNC all		7 / 31 / 9.6	13 / 39 / 13.6	10 / 40 / 16.8	3 / 4 / 0.6	18 / 30 / 7.7	38 / 114 / 48.3
ENY	1999	4 / 24 / 15.1					4 / 24 / 15.1
	2000	5 / 29 / 17.7	1 / 3 / 2.7				6 / 32 / 20.4
	2001	5 / 29 / 21.0	1 / 4 / 3.1				6 / 33 / 24.1
	2002	5 / 30 / 24.5		1 / 4 / 3.4			6 / 34 / 27.9
	2003	5 / 33 / 28.8		1 / 4 / 3.5		1 / 1 / 0.5	7 / 38 / 32.8
ENY all		6 / 43 / 21.4	1 / 4 / 1.2	1 / 5 / 1.4		1 / 1 / 0.1	7 / 47 / 24.1
FIN	1999	41 / 647 / 439.8	27 / 346 / 194.4	3 / 4 / 1.3	1 / 1 / 0.6	29 / 104 / 41.6	89 / 1,102 / 677.7
	2000	43 / 705 / 497.2	33 / 428 / 246.2	2 / 3 / 1.0	2 / 3 / 1.2	28 / 106 / 43.5	97 / 1,245 / 789.1
	2001	51 / 892 / 602.8	40 / 526 / 320.6	1 / 1 / 0.5	2 / 3 / 0.9	29 / 122 / 53.0	106 / 1,544 / 977.8
	2002	50 / 865 / 666.7	41 / 670 / 411.2		2 / 3 / 0.9	23 / 93 / 42.6	105 / 1,631 / 1,121.5
	2003	50 / 839 / 617.0	42 / 783 / 493.8	2 / 4 / 3.2	2 / 3 / 2.5	23 / 92 / 45.1	110 / 1,721 / 1,161.7
FIN all		60 / 1,241 / 564.9	53 / 907 / 333.9	5 / 9 / 1.2	2 / 3 / 1.2	40 / 158 / 45.2	119 / 2,001 / 946.4
IND	1999	2 / 5 / 4.2	1 / 1 / 0.8	1 / 1 / 0.3	1 / 1 / 0.5	8 / 9 / 1.7	13 / 17 / 7.5
	2000	2 / 8 / 6.2	3 / 4 / 1.9	2 / 2 / 1.2		9 / 10 / 4.6	16 / 24 / 13.9
	2001	2 / 10 / 8.4	4 / 7 / 5.6	3 / 4 / 2.2		10 / 12 / 6.3	19 / 33 / 22.5
	2002	1 / 4 / 3.8	7 / 17 / 14.9	3 / 4 / 4.0		12 / 14 / 7.4	23 / 39 / 30.1
	2003	1 / 3 / 3.0	6 / 12 / 10.8	6 / 9 / 7.1	1 / 6 / 4.5	10 / 11 / 5.3	23 / 41 / 30.7
IND		2 / 10 / 5.1	8 / 19 / 6.8	7 / 10 / 3.0	2 / 7 / 1.0	14 / 18 / 5.1	26 / 46 / 21.0
MED	1999			1 / 8 / 2.7		5 / 10 / 7.4	6 / 18 / 10.1
	2000		2 / 7 / 5.4	2 / 9 / 4.8		2 / 4 / 2.2	6 / 20 / 12.4
	2001		2 / 8 / 6.2	2 / 9 / 6.0	1 / 1 / 0.8	2 / 4 / 4.0	6 / 22 / 16.9
	2002		1 / 2 / 2.0	4 / 15 / 11.5	1 / 2 / 1.9	1 / 3 / 2.8	6 / 22 / 18.2
	2003		1 / 2 / 2.0	3 / 10 / 9.1	2 / 8 / 5.7	2 / 5 / 3.8	7 / 25 / 20.6
MED all			2 / 8 / 3.1	4 / 20 / 6.8	2 / 8 / 1.7	6 / 14 / 4.0	7 / 29 / 15.7
TEC	1999		3 / 14 / 9.5			1 / 2 / 1.4	4 / 16 / 11.0
	2000		3 / 13 / 10.4			1 / 2 / 1.2	4 / 15 / 11.6
	2001		3 / 18 / 13.4			2 / 3 / 1.5	5 / 21 / 15.0
	2002		1 / 9 / 7.3	2 / 9 / 7.6		3 / 4 / 2.5	6 / 22 / 17.4
	2003		2 / 11 / 10.1		2 / 8 / 6.6	3 / 4 / 2.1	7 / 23 / 18.7
TEC all			4 / 23 / 10.1	2 / 9 / 1.5	2 / 8 / 1.3	3 / 4 / 1.8	7 / 29 / 14.8
TRA	1999	1 / 2 / 2.0				3 / 15 / 12.8	4 / 17 / 14.8
	2000	2 / 4 / 2.9				3 / 15 / 13.2	4 / 19 / 16.1
	2001	4 / 15 / 11.8				2 / 6 / 4.2	6 / 21 / 16.0
	2002	5 / 18 / 14.3				1 / 4 / 4.0	6 / 22 / 18.3
	2003	4 / 17 / 14.7	1 / 1 / 1.0			1 / 5 / 3.6	6 / 23 / 19.3
TRA all		5 / 22 / 9.2	1 / 1 / 0.2			3 / 15 / 7.5	7 / 29 / 16.9
UTY	1999	7 / 21 / 12.4	3 / 8 / 4.5	1 / 1 / 0.2		5 / 8 / 5.0	13 / 38 / 22.1
	2000	8 / 29 / 23.3	4 / 14 / 9.7	1 / 2 / 1.9		4 / 5 / 3.1	16 / 50 / 38.1
	2001	11 / 36 / 25.3	8 / 25 / 19.0	2 / 4 / 3.2		2 / 2 / 0.9	20 / 67 / 48.4
	2002	12 / 40 / 34.9	12 / 46 / 30.9	2 / 5 / 5.0		2 / 2 / 0.5	26 / 93 / 71.4
	2003	8 / 22 / 19.4	16 / 80 / 60.0	5 / 15 / 10.9		2 / 2 / 0.9	31 / 119 / 91.3
UTY all		15 / 62 / 23.1	18 / 89 / 25.0	5 / 15 / 4.3		11 / 15 / 2.1	32 / 134 / 54.4
All sectors	1999	67 / 732 / 494.2	56 / 439 / 247.5	20 / 42 / 14.2	2 / 2 / 1.1	95 / 213 / 97.5	240 / 1,428 / 854.5
	2000	75 / 818 / 576.5	70 / 549 / 331.5	27 / 52 / 32.2	3 / 4 / 1.3	86 / 205 / 98.1	261 / 1,628 / 1,039.6
	2001	83 / 1,009 / 690.8	89 / 699 / 452.7	37 / 104 / 71.7	5 / 7 / 2.7	95 / 219 / 104.1	309 / 2,038 / 1,322.0
	2002	83 / 978 / 754.6	97 / 866 / 569.0	55 / 181 / 149.4	6 / 8 / 5.3	81 / 171 / 91.9	322 / 2,204 / 1,570.2
	2003	78 / 946 / 706.5	105 / 1,049 / 708.6	58 / 189 / 162.6	17 / 49 / 38.4	81 / 168 / 90.7	339 / 2,401 / 1,706.9
Overall		108 / 1,449 / 644.8	135 / 1,295 / 462.8	73 / 253 / 86.3	20 / 53 / 9.9	157 / 346 / 96.5	355 / 2817 / 1,300.2
exFIN	1999	26 / 85 / 54.4	29 / 93 / 53.1	17 / 38 / 12.8	1 / 1 / 0.5	66 / 109 / 56.0	139 / 326 / 176.8
	2000	32 / 113 / 79.3	37 / 121 / 85.3	25 / 49 / 31.2	1 / 1 / 0.0	58 / 99 / 54.6	153 / 383 / 250.4
	2001	32 / 117 / 88.1	49 / 173 / 132.1	36 / 103 / 71.2	3 / 4 / 1.8	66 / 97 / 51.1	186 / 494 / 344.3
	2002	33 / 113 / 87.8	56 / 196 / 157.8	55 / 181 / 149.4	4 / 5 / 4.3	58 / 78 / 49.3	206 / 573 / 448.8
	2003	28 / 107 / 89.5	63 / 266 / 214.8	56 / 185 / 159.4	15 / 46 / 35.9	58 / 76 / 45.6	220 / 680 / 545.2
exFIN all		48 / 208 / 79.9	82 / 388 / 128.9	68 / 244 / 85.1	18 / 50 / 8.6	117 / 188 / 51.3	236 / 816 / 353.8

Table A.4: Corporate Bond Data per Sector and Year

Available data indicated by no. of issuers / no. of issues / avg. no. of price observations.

Migration type		constant rating					upgrade			downgrade					withdrawn		new rating				default		
Sector	Year	AA ↓ AA	A ↓ A	BBB ↓ BBB	NI ↓ NI	NR ↓ NR	A ↓ AA	BBB ↓ A	NI ↓ BBB	AA ↓ A	AA ↓ NI	A ↓ BBB	A ↓ NI	BBB ↓ NI	AA ↓ NR	NI ↓ NR	NR ↓ AA	NR ↓ A	NR ↓ BBB	NR ↓ NI	BBB ↓ D	NI ↓ D	Overall
AUT	1999		31	10		3												1					45
	2000		32	6		3																	41
	2001		31	10		2						12						1	1				57
	2002		46	15		3								7				2					73
	2003		72	12	12	2																	98
AUT all			212	53	12	13						12		7				4	1				314
BMA	1999		12	6		14																	32
	2000	1	10	7		17						2								1			38
	2001	1	10	14	1	23												1	1				51
	2002	3	13	13	1	12								3				1	2				48
	2003	6	20	12	4	12		1						4				1					60
BMA all		11	65	52	6	78		1				2		7				3	3	1			229
COM	1999	17	6	2														1					26
	2000	4	11	2		1				16												1	35
	2001		18	2						4		17										2	43
	2002		21	26		2						7		1									57
	2003		21	37	1	1						7											67
COM all		21	77	69	1	4				20		31		1				1				3	228
CON	1999	1	12			8											3		1				25
	2000	4	5	4		5						7							1				26
	2001	5	4	13		4												2					28
	2002	5	7	16	1	4								1					1				35
	2003		8	12	3	3				6				4					1				37
CON all		15	36	45	4	24				6		7		5			3	2	4				151
CCY	1999	1	3	4		18											1		2				29
	2000	2	3	8		17													2				32
	2001	1	2	19		16						1					1	1	5				46
	2002	2	2	29		13						1											47
	2003	3	3	32		14								2									54
CCY all		9	13	92		78						2		2			2	1	9				208
CNC	1999	14	6	6		12													1				39
	2000	12	6	9		9				4		4					2		5				51
	2001	9	16	27		10				7				1			1		1				72
	2002	10	25	32	1	11								1	1								81
	2003	10	28	10	4	13		1			7		1	9						1	11		95
CNC all		55	81	84	5	55		1		11	7	4	1	11	1		3		7	1	11		338
ENY	1999	22								2													24
	2000	29	3																				32
	2001	29										4											33
	2002	30		4																			34
	2003	33		4		1																	38
ENY all		143	3	8		1				2		4											161
FIN	1999	640	338	3	1	95	8			7				1			2	7					1,102
	2000	698	409	1	3	95	19	2		7								11					1,245
	2001	831	518		3	107	8	1		61							1	14					1,544
	2002	723	633		3	92	36			142		1						1					1,631
	2003	771	593	4	1	84	182			68		8				2		8					1,721
FIN all		3,663	2,491	8	11	473	253	3		285		9		1		2	3	41					7,243
IND	1999	5	1	1		9			1														17
	2000	8	4	2		10																	24
	2001	5	7	4		11				5								1					33
	2002	4	10	4		12						2	5						2				39
	2003	3	12	9	6	10													1				41
IND		25	34	20	6	52			1	5		2	5					1	3				154
MED	1999			8		2												6	2				18
	2000		7	9		4																	20
	2001		2	9	1	3						6								1			22
	2002		2	9	2	3								6									22
	2003			10	8	1						2							4				25
MED all			11	45	11	13						8		6				6	6	1			107
TEC	1999	1	14			2																	17
	2000		13			2																	15
	2001		10			3						8											21
	2002		9	1		4								8									22
	2003		2		8	4						9											23
TEC all		1	48	1	8	15						17		8									98
TRA	1999	2				15																	17
	2000	4				6											9						19
	2001	15				6																	21
	2002	17				4				1													22
	2003	14	1			5				3													23
TRA all		52	1			36				4							9						102
UTY	1999	21	8	1		1											6	1					38
	2000	24	14	2		1				5							2	2					50
	2001	31	25	4		1				5									1				67
	2002	20	41	5		2				20		5											93
	2003	16	70	15		1				6		10						1					119
UTY all		112	158	27		6				36		15					8	4	1				367
All sectors	1999	724	431	41	1	179	8		1	9				1			12	16	6				1,429
	2000	786	517	50	3	170	19	2		32		13					13	13	8	1		1	1,628
	2001	927	643	102	5	186	8	1		82		48		1			3	20	9	1		2	2,038
	2002	814	809	154	8	162	36			163		16	5	27	1			4	5				2,204
	2003	856	830	157	47	151	182	2		83	7	36	1	19		2		10	6	1	11		2,401
Overall		4107	3,230	504	64	848	253	5	1	369	7	113	6	48	1	2	28	63	34	3	11	3	9,700
exFIN	1999	84	93	38		84			1	2							10	9	6				327
	2000	88	108	49	0	75				25		13					13	2	8	1		1	383
	2001	96	125	102	2	79				21		48		1			2	6	9	1		2	494
	2002	91	176	154	5	70				21		15	5	27	1			3	5				573
	2003	85	237	153	46	67		2		15	7	28	1	19				2	6	1	11		680
exFIN all		444	739	496	53	375		2	1	84	7	104	6	47	1		25	22	34	3	11	3	2,457

Table A.5: Rating Migrations in Sample

Appendix B

Risk class Clustering

ρ	AA	A	BBB	NI	NR
AA	100.0	83.0	33.5	11.3	78.5
A		100.0	42.8	16.3	78.5
BBB			100.0	25.7	43.1
NI				100.0	14.1
NR					100.0

Table B.1: Correlations of Spread Returns between Rating-Classes

ρ	AUT	BMA	COM	CON	CCY	CNC	ENY	FIN	IND	MED	TEC	TRA	UTY
AUT	100.0	54.8	59.9	51.4	44.9	56.4	37.6	42.3	49.5	55.2	47.5	49.6	56.7
BMA		100.0	44.0	34.4	32.0	43.9	26.4	44.6	44.0	46.7	43.0	33.4	42.5
COM			100.0	39.2	38.1	45.9	35.8	27.3	34.8	47.9	60.4	33.6	58.5
CON				100.0	48.6	29.7	23.7	23.6	41.0	34.1	30.5	24.9	36.9
CCY					100.0	29.8	23.9	15.3	30.1	24.2	32.2	15.3	26.8
CNC						100.0	39.1	34.4	39.4	43.5	41.5	44.6	57.9
ENY							100.0	29.0	25.7	30.8	26.4	34.2	39.6
FIN								100.0	35.0	36.7	22.5	54.1	47.5
IND									100.0	32.5	43.2	32.5	41.6
MED										100.0	44.9	38.7	43.9
TEC											100.0	27.0	48.9
TRA												100.0	55.6
UTY													100.0

Table B.2: Correlations of Spread Returns between Sectors

Sector	AUT	BMA	CCY	CNC	COM	CON	ENY	FIN	IND	MED	TEC	TRA	UTY	#Caus.
AUT	—	0.2093	0.6315	0.1817	0.0053***	0.7043	0.0010***	0.1925	0.2892	0.8636	0.0041***	0.3350	0.1242	**3**
BMA	0.2913	—	0.2599	0.2446	0.0271**	0.0351**	0.0526*	0.7759	0.2203	0.1440	0.0005***	0.7789	0.3174	**3**
CCY	0.7837	0.5296	—	0.9858	0.0137**	0.8447	0.0363**	0.9207	0.0405**	0.5157	0.0004***	0.7526	0.9262	**4**
CNC	0.5373	0.5368	0.7548	—	0.0226**	0.0820*	0.0244**	0.9548	0.1970	0.3099	0.0000***	0.6757	0.2190	**3**
COM	0.0062***	0.0001***	0.0095***	0.0053***	—	0.1470	0.0530*	0.4896	0.0208**	0.0346**	0.3538	0.4926	0.0750*	**6**
CON	0.3981	0.9494	0.2271	0.4746	0.0062***	—	0.0032***	0.2700	0.6947	0.3750	0.0073***	0.0931	0.1354	**3**
ENY	0.2082	0.0070***	0.4564	0.1364	0.0841*	0.9064	—	0.1338	0.9087	0.4870	0.0338**	0.0134**	0.6843	**3**
FIN	0.2457	0.1945	0.2542	0.7480	0.0333**	0.7466	0.0806	—	0.7549	0.8122	0.0078***	0.3856	0.1024	**2**
IND	0.7081	0.5272	0.4995	0.6308	0.0124**	0.0307**	0.0192**	0.4421	—	0.4090	0.0003***	0.5584	0.7802	**4**
MED	0.6684	0.1518	0.2588	0.7559	0.0033***	0.8854	0.1664	0.9640	0.7993	—	0.0003***	0.6301	0.7586	**2**
TEC	0.2459	0.0505*	0.2127	0.2893	0.2378	0.1921	0.1696	0.6194	0.3572	0.5298	—	0.9385	0.2862	**0**
TRA	0.8924	0.1413	0.3272	0.4060	0.7107	0.3016	0.0019***	0.4428	0.7027	0.5643	0.4774	—	0.9478	**1**
UTY	0.8789	0.0453**	0.4542	0.1009	0.0423**	0.3619	0.0228**	0.6264	0.6572	0.8652	0.0038***	0.5971	—	**4**
#Causalities	**1**	**3**	**1**	**1**	**9**	**2**	**7**	**0**	**2**	**1**	**10**	**1**	**0**	**38**

Table B.3: Granger Causality of Sector Spread Returns

The p-values of a 2-lag Granger causality test are based on monthly log-returns of average sector spread with 1%(5%, 10%) level of significance indicated by *** (**,*).

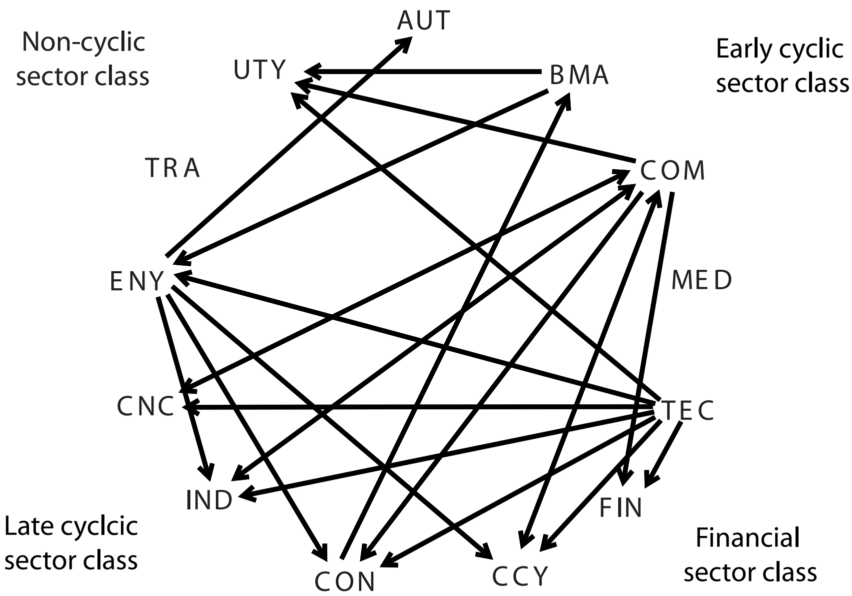

Figure B.1: Sector-Causality Analysis

Arrows indicate 2-lag Granger causality at a 5% level of significance

Appendix C

Non-Gaussian Kalman-Filter

In case of a state-space model with non-Gaussian measurement disturbances an optimal filtering of state series and a consistent ML estimation of parameters is achieved by a generalized Kalman-Filter.[1] The generalized Kalman-Filter involves a recursion to determine the density $f(X_t|S_t)$ of the state vector X_t conditional on the information set at time t, represented by observation vector $\mathbf{S}_t$. Given a non-Gaussian transition density $f(X_t|X_{t-\Delta t})$, the conditional expectation

$$\mathbb{E}[X_t|S_t] = \int X_t f(X_t|S_t) dX_t, \tag{C.1}$$

based on information set S_t, is the minimum mean squared estimator of X_t in the update step of the Kalman-Filter. Transforming the conditional update density

$$f(X_t|S_t) = \frac{f(S_t|X_t) f(X_t|S_{t-\Delta t})}{f(S_t|S_{t-\Delta t})} \tag{C.2}$$

using the conditional density

$$f(S_t|S_{t-\Delta t}) = \int f(S_t|X_t) f(X_t|S_{t-\Delta t}) dX_t \tag{C.3}$$

of the observation vector, the log-likelihood of the prediction-error decomposition

$$\mathcal{LL}(\mathbf{S}_{\overline{T}}; \psi) = \sum_{t=1}^{\overline{T}} \log f(S_t|S_{t-\Delta t}; \psi) \tag{C.4}$$

is obtained from the non-linear filter recursion of (C.2) and (C.3).[2]

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[1] Cf. Harvey (1989), p. 162ff. or Lund (1997b), p. 2ff.

[2] The conditional density $f(X_t|S_{t-\Delta t})$ is obtained from the density $f(X_t|X_{t-1})$ of the transition disturbance η_t and the update density $f(X_{t-\Delta t}|S_{t-\Delta t})$ of the previous recursion (cf. Harvey (1989), p. 163).

For the state-space model of Section 4.4.2, no closed-form solution of (C.3) is available.[3] Kitigawa (1987) suggests numerical integration to compute the densities in (C.3), however, Lund (1997b, p. 4) considers an exact filtering of non-linear state-space models using numerical integration to be computationally infeasible for state vectors of dimension $n_X > 1$. For a multi-factor state-space model with non-Gaussian density $f(S_t|X_t)$, Frühwirth-Schnatter (1994) proposes to approximate the update density $f(X_{t-\Delta t}|S_{t-\Delta t})$ of the state vector by a Gaussian density with identical mean and covariance matrix, with numerical integration only required to calculate mean and covariance matrix of the state vector. To circumvent the computational requirements of the exact filtering of non-linear and non-Gaussian state-space models, Lund (1997b) introduces an Iterative Extended Kalman-Filter (IEKF), but compared to the EKF, the IEKF imposes a loss of efficiency, unbiasedness and consistency of the resulting QML estimation is not guaranteed and the selection of the most efficient iteration scheme is not unambiguous.

Since no ambiguously favorable optimal filtering technique is available for the non-linear state-space model with non-Gaussian disturbances of Section 4.4.2, the approximate Extended Kalman-Filter (EKF) is employed to estimate state process and series.

[3] Closed-form solution of (C.3) are known only in the linear case and for some other exceptional state-space models (cf. Harvey (1989, chap. 6).

Appendix D

Estimation Results

Risk-Class	Year	$\mathcal{LL}_F$	$\mu_{\mathbf{F}}$	$\sigma_\varepsilon(\mu_{\mathbf{F}})$	$\sigma_{\mathbf{F}}$	$\sigma_\varepsilon(\sigma_{\mathbf{F}})$	$\mathcal{LL}_V$	$\mu_{\mathbf{V}}$	$\sigma_\varepsilon(\mu_{\mathbf{V}})$	$\sigma_{\mathbf{V}}$	$\sigma_\varepsilon(\sigma_{\mathbf{V}})$	$\rho_{\mathbf{V}}$	$\sigma_\varepsilon(\rho_{\mathbf{V}})$	$\beta_{\mathbf{V}}$
ECY-A	**2000**	2,221.2	-0.1152	0.3501	0.1778	0.0302	2,683.6	-0.0367	0.0174	0.1016	0.0127	0.3148	0.0560	0.5610
	2001	1,902.2	-0.0200	0.2704	0.1791	0.0236	2,419.6	0.0217	0.0110	0.0814	0.0041	0.1972	0.0587	0.4440
	2002	2,086.3	-0.0195	0.3511	0.1207	0.0287	2,631.4	-0.0025	0.0118	0.0794	0.0122	0.6017	0.0550	0.7757
	2003	2,297.9	0.0570	0.3495	0.1119	0.0091	2,837.5	0.0466	0.0323	0.1244	0.0191	0.5467	0.0475	0.7394
ECY-BBB	**2000**	2,095.1	-0.0836	0.4219	0.1902	0.0241	2,577.3	-0.0237	0.0148	0.1145	0.0122	0.5467	0.0232	0.7394
	2001	1,865.7	-0.0371	0.2930	0.1413	0.0307	2,435.7	-0.0086	0.0129	0.0995	0.0067	0.5016	0.0236	0.7083
	2002	1,523.1	-0.0356	0.2228	0.1082	0.0244	1,873.0	-0.0067	0.0070	0.0346	0.0014	0.0990	0.0480	0.3147
	2003	1,974.0	0.0586	0.4189	0.1226	0.0169	2,559.6	0.0568	0.0221	0.1056	0.0145	0.4232	0.0439	0.6505
FIN-AA	**1999**	2,398.1	-0.0135	0.3041	0.1321	0.0229	2,653.3	0.0058	0.0073	0.0303	0.0020	0.0594	0.0834	0.2437
	2000	2,358.0	-0.0854	0.3239	0.1815	0.0273	2,728.6	-0.0084	0.0109	0.0587	0.0041	0.0825	0.0609	0.2872
	2001	2,302.0	0.0322	0.3785	0.1381	0.0270	2,595.5	0.0023	0.0103	0.0621	0.0059	0.1957	0.0782	0.4424
	2002	2,377.3	-0.0302	0.2830	0.1306	0.0177	2,549.4	0.0072	0.0053	0.0281	0.0011	0.0239	0.0370	0.1545
	2003	2,443.5	0.0324	0.3651	0.1092	0.0220	2,562.5	0.0094	0.0055	0.0401	0.0025	0.1999	0.0727	0.4471
FIN-A	**1999**	2,230.6	0.0021	0.2476	0.1144	0.0130	2,600.7	-0.0059	0.0114	0.0617	0.0061	0.0787	0.0691	0.2806
	2000	2,243.6	-0.0691	0.3202	0.1811	0.0303	2,586.3	-0.0095	0.0078	0.0632	0.0037	0.1574	0.0459	0.3968
	2001	2,177.3	0.0223	0.6294	0.1843	0.0543	2,581.4	-0.0080	0.0113	0.0608	0.0043	0.2381	0.0783	0.4880
	2002	2,153.7	-0.0629	0.5456	0.1799	0.0381	2,507.9	-0.0001	0.0058	0.0296	0.0015	0.0209	0.0586	0.1445
	2003	2,206.4	0.0368	0.4344	0.1122	0.0213	2,459.3	0.0137	0.0106	0.0594	0.0027	0.1212	0.0492	0.3481
LCY-AA	**2000**	2,315.7	-0.0403	0.3539	0.1822	0.0267	2,812.1	-0.0369	0.0165	0.1048	0.0107	0.2748	0.0543	0.5243
	2001	2,116.6	-0.0565	0.2233	0.1152	0.0093	2,658.4	-0.0153	0.0102	0.0637	0.0044	0.2356	0.0470	0.4854
	2002	2,201.9	-0.0018	0.1650	0.1104	0.0106	2,693.0	-0.0078	0.0107	0.0567	0.0031	0.2439	0.0487	0.4939
	2003	2,386.6	0.0320	0.2533	0.1135	0.0091	2,820.5	0.0062	0.0157	0.0859	0.0152	0.5436	0.0617	0.7373
LCY-A	**2000**	2,109.0	-0.0900	0.4773	0.1768	0.0340	2,778.0	-0.0661	0.0173	0.1386	0.0234	0.6313	0.0352	0.7945
	2001	2,056.8	-0.0652	0.6994	0.3357	0.0511	2,634.4	0.0118	0.0277	0.1701	0.0113	0.3099	0.0288	0.5567
	2002	1,743.6	-0.0416	0.1751	0.1220	0.0149	2,087.8	0.0081	0.0054	0.0300	0.0011	0.0201	0.0176	0.1417
	2003	2,198.0	0.0966	0.5927	0.1412	0.0240	2,874.4	0.0488	0.0176	0.1225	0.0115	0.7666	0.0188	0.8755
LCY-BBB	**2000**	1,932.9	-0.0337	0.5397	0.1944	0.0322	2,650.8	-0.0245	0.0176	0.1408	0.0214	0.5255	0.0429	0.7249
	2001	2,005.2	-0.0138	0.4188	0.1772	0.0300	2,566.1	-0.0156	0.0184	0.1298	0.0177	0.3705	0.0375	0.6087
	2002	1,864.1	-0.0228	0.2350	0.1190	0.0233	2,396.1	-0.0439	0.0160	0.1057	0.0082	0.4828	0.0315	0.6949
	2003	2,005.2	0.0260	0.2527	0.1019	0.0104	2,413.9	0.0424	0.0248	0.1001	0.0102	0.2797	0.0539	0.5288
NCY-AA	**2000**	2,295.7	-0.0736	0.3466	0.1587	0.0160	2,671.8	-0.0342	0.0164	0.0988	0.0127	0.5036	0.0487	0.7096
	2001	2,266.9	0.0286	0.2430	0.1278	0.0137	2,718.6	-0.0092	0.0199	0.0913	0.0115	0.4116	0.0368	0.6416
	2002	2,313.4	-0.0360	0.4933	0.1806	0.0205	2,811.4	-0.0277	0.0148	0.1083	0.0239	0.5181	0.0600	0.7198
	2003	2,375.9	-0.0011	0.1264	0.0816	0.0075	2,830.1	0.0236	0.0095	0.0530	0.0040	0.0900	0.0541	0.3000
NCY-A	**2000**	2,164.7	-0.0855	0.4635	0.1805	0.0378	2,762.6	-0.0470	0.0236	0.1515	0.0360	0.6977	0.0293	0.8353
	2001	2,224.0	-0.0213	0.9391	0.2669	0.0469	2,799.6	0.0083	0.0399	0.1880	0.0893	0.4972	0.0727	0.7051
	2002	2,103.1	-0.0211	0.4091	0.1792	0.0230	2,759.7	-0.0208	0.0190	0.1474	0.0337	0.6582	0.0272	0.8113
	2003	2,338.0	0.0610	0.3952	0.1313	0.0147	2,782.9	-0.0020	0.0224	0.1433	0.0276	0.6217	0.0306	0.7885
NCY-BBB	**2000**	2,074.9	-0.1399	0.5878	0.1973	0.0223	2,670.9	-0.0850	0.0285	0.1650	0.0198	0.6492	0.0169	0.8058
	2001	1,985.1	0.0463	1.3671	0.3332	0.0483	2,523.2	0.0266	0.0325	0.2415	0.0268	0.5237	0.0242	0.7237
	2002	1,853.5	-0.0925	0.5331	0.1991	0.0664	2,393.4	-0.0560	0.0180	0.1219	0.0074	0.3815	0.0414	0.6177
	2003	1,747.6	0.0275	0.1425	0.1138	0.0176	1,964.2	-0.0034	0.0126	0.0383	0.0009	0.0963	0.0299	0.3103
NF-AA	**1999**	2,427.6	-0.0068	0.2312	0.1102	0.0156	2,628.6	-0.0186	0.0109	0.0557	0.0047	0.0832	0.0714	0.2885
	2000	2,261.5	-0.1260	0.4167	0.1788	0.0198	2,694.4	-0.0271	0.0122	0.0764	0.0076	0.3307	0.0545	0.5751
	2001	2,254.1	0.0042	0.3189	0.1190	0.0097	2,727.4	0.0056	0.0098	0.0595	0.0051	0.3608	0.0580	0.6007
	2002	2,331.7	-0.0142	0.2901	0.1255	0.0142	2,878.2	0.0048	0.0132	0.0848	0.0125	0.4856	0.0733	0.6969
	2003	2,401.1	0.0081	0.2166	0.1048	0.0104	2,734.3	0.0015	0.0089	0.0586	0.0027	0.1229	0.0467	0.3505
NF-A	**1999**	2,324.1	0.0155	0.3441	0.1299	0.0227	2,740.6	-0.0134	0.0115	0.0888	0.0192	0.4535	0.0490	0.6734
	2000	2,101.5	-0.1141	0.4887	0.1779	0.0505	2,718.3	-0.0541	0.0206	0.1424	0.0288	0.4981	0.0597	0.7058
	2001	2,026.1	-0.0226	0.2502	0.1788	0.0139	2,556.3	0.0040	0.0169	0.1115	0.0109	0.3753	0.0421	0.6127
	2002	2,011.5	-0.0224	0.2892	0.1121	0.0343	2,248.0	0.0022	0.0092	0.0445	0.0015	0.0916	0.0453	0.3027
	2003	2,264.4	0.0322	0.3736	0.1114	0.0139	2,854.8	0.0204	0.0142	0.0845	0.0149	0.5848	0.0484	0.7647
NF-BBB	**1999**	2,128.8	0.0559	0.3344	0.1788	0.0108	2,472.9	0.0025	0.0125	0.0646	0.0039	0.0908	0.0505	0.3013
	2000	2,008.1	-0.0844	0.6037	0.1987	0.0387	2,586.7	-0.0480	0.0189	0.1506	0.0173	0.6094	0.0276	0.7807
	2001	1,893.7	-0.0151	0.7139	0.2450	0.0540	2,459.3	-0.0211	0.0225	0.1945	0.0356	0.5038	0.0321	0.7098
	2002	1,664.0	-0.0355	0.2800	0.1197	0.0378	1,917.3	-0.0047	0.0071	0.0488	0.0010	0.0884	0.0309	0.2973
	2003	1,963.4	0.0247	0.2192	0.1032	0.0093	2,267.5	0.0221	0.0078	0.0576	0.0021	0.0748	0.0425	0.2735
AA	**1999**	2,400.6	-0.0188	0.3327	0.1313	0.0274	2,609.3	-0.0049	0.0074	0.0356	0.0031	0.0983	0.1068	0.3135
	2000	2,340.6	-0.0921	0.3301	0.1863	0.0265	2,632.8	-0.0117	0.0066	0.0472	0.0025	0.0896	0.0596	0.2994
	2001	2,281.9	0.0299	0.4200	0.1382	0.0285	2,608.8	-0.0012	0.0117	0.0605	0.0086	0.2389	0.1182	0.4888
	2002	2,297.8	-0.0264	0.3066	0.1316	0.0186	2,572.6	-0.0051	0.0068	0.0475	0.0025	0.0894	0.0389	0.2990
	2003	2,416.3	0.0289	0.3718	0.1092	0.0195	2,632.9	0.0197	0.0105	0.0587	0.0051	0.1626	0.0889	0.4032
A	**1999**	2,214.3	-0.0068	0.2647	0.1130	0.0165	2,643.1	0.0082	0.0082	0.0536	0.0049	0.0848	0.0703	0.2912
	2000	2,133.1	-0.0746	0.4131	0.1812	0.0415	2,611.6	-0.0437	0.0194	0.1005	0.0125	0.2786	0.0763	0.5278
	2001	2,110.7	0.0022	0.4946	0.1800	0.0387	2,467.4	0.0120	0.0108	0.0622	0.0030	0.2374	0.0409	0.4872
	2002	2,131.8	-0.0489	0.4462	0.1598	0.0285	2,512.6	0.0060	0.0057	0.0403	0.0024	0.0948	0.0882	0.3079
	2003	2,246.7	0.0314	0.4565	0.1122	0.0191	2,518.3	0.0087	0.0086	0.0557	0.0052	0.0832	0.0777	0.2885
BBB	**1999**	2,119.0	0.0215	0.4226	0.1607	0.0316	2,522.1	-0.0110	0.0155	0.0777	0.0076	0.2456	0.0683	0.4955
	2000	2,003.6	-0.0918	0.5992	0.2004	0.0383	2,632.2	-0.0399	0.0148	0.1244	0.0142	0.5920	0.0368	0.7694
	2001	1,900.5	0.0102	0.6372	0.2425	0.0438	2,454.6	0.0018	0.0262	0.2067	0.0362	0.6832	0.0173	0.8265
	2002	1,663.1	-0.0348	0.2794	0.1197	0.0376	1,949.9	-0.0186	0.0101	0.0752	0.0026	0.1383	0.0412	0.3719
	2003	1,962.7	0.0246	0.2194	0.1032	0.0092	2,359.9	0.0275	0.0116	0.0801	0.0044	0.0996	0.0480	0.3156

Table D.1: Parameter Estimates of Annual Estimation

Risk-Class	Year	Empirical Yield Spread of Risk-Class				Factor-implied Yield Spread				Bootstrapped Yield Spread				Asset-implied Yield Spread			
		T=1	T=3	T=5	T=10	T=1	T=3	T=5	T=10	T=1	T=3	T=5	T=10	T=1	T=3	T=5	T=10
ECY-A	**2000**	6.10	12.89	16.80	25.88	3.74	17.66	17.44	13.88	12.38	18.30	17.10	18.96	12.03	17.27	17.17	18.85
	2001	25.01	35.76	30.12	14.31	21.18	39.59	33.47	22.77	22.15	48.32	45.36	37.22	22.59	48.07	46.27	37.15
	2002	10.46	13.63	13.41	13.13	10.46	15.99	12.92	8.89	17.89	18.14	13.50	8.38	18.17	18.16	12.54	7.04
	2003	9.03	12.34	13.39	19.42	7.52	17.84	15.44	11.07	7.74	16.93	9.74	26.66	7.83	16.62	9.52	24.50
ECY-BBB	**2000**	10.37	15.94	16.34	16.80	6.22	20.24	18.71	14.01	17.87	15.75	12.80	19.63	16.98	14.33	11.72	16.92
	2001	21.99	28.10	21.51	15.22	17.84	24.44	19.56	12.39	35.96	49.44	24.72	21.54	36.25	48.83	24.81	18.64
	2002	225.51	100.83	62.09	26.83	161.96	102.99	72.89	44.21	253.59	96.71	109.08	54.87	258.09	99.80	111.58	53.43
	2003	13.15	25.58	32.63	47.52	22.53	32.23	26.23	19.33	15.17	27.13	24.66	49.92	16.33	26.96	24.34	49.16
FIN-AA	**1999**	3.42	3.85	4.52	5.26	0.16	2.68	3.26	2.78	9.81	13.73	9.43	10.79	9.81	13.73	9.43	10.79
	2000	4.55	9.42	10.52	11.48	1.13	10.72	11.94	9.81	6.82	10.36	14.14	16.81	6.64	9.35	14.08	16.54
	2001	4.40	3.95	4.63	7.10	0.44	3.03	3.14	2.50	13.07	21.13	13.20	13.27	13.25	20.30	12.13	12.45
	2002	7.23	6.30	4.43	6.56	1.06	6.03	6.02	4.58	11.48	27.16	15.65	10.17	12.41	27.12	15.31	10.38
	2003	3.31	3.94	4.52	6.49	0.78	4.79	4.84	3.68	10.04	12.46	16.62	22.83	9.57	11.39	15.96	22.08
FIN-A	**1999**	5.62	6.36	5.63	5.45	0.98	4.74	4.60	3.39	15.96	13.22	9.09	9.87	15.96	13.22	9.09	9.87
	2000	4.35	9.84	11.77	16.03	2.20	11.70	11.73	9.28	10.99	13.73	13.91	12.17	10.84	13.42	13.54	11.72
	2001	4.84	4.80	5.52	6.37	0.90	5.57	5.69	4.53	11.96	11.18	17.84	22.71	11.44	10.88	17.99	22.58
	2002	5.79	8.95	9.85	11.12	1.65	10.73	11.10	8.77	10.74	24.62	15.97	21.59	11.17	25.15	15.05	22.39
	2003	3.61	7.67	10.25	11.02	3.69	9.75	8.59	6.10	26.25	36.74	20.83	27.41	26.50	36.86	19.46	25.42
LCY-AA	**2000**	4.74	9.52	11.83	15.30	1.25	11.23	12.37	10.15	6.14	7.41	13.01	21.44	5.32	6.79	12.94	21.31
	2001	19.98	22.39	15.73	9.52	11.82	23.40	19.57	12.68	14.53	21.77	10.36	8.45	14.60	21.39	10.12	8.17
	2002	13.25	14.39	10.18	7.40	5.74	14.20	12.27	8.24	18.03	13.43	13.18	19.99	18.38	13.51	12.41	18.86
	2003	9.85	6.47	6.38	11.01	1.90	8.09	7.69	5.71	9.68	10.40	9.21	9.73	8.79	10.39	8.13	6.24
LCY-A	**2000**	4.91	11.01	14.99	24.52	3.57	16.26	15.90	12.54	14.44	10.45	16.16	22.82	13.91	9.96	16.45	22.23
	2001	10.90	18.57	23.16	22.51	1.20	17.62	20.97	18.31	14.01	22.83	23.26	19.88	14.29	23.18	21.82	17.45
	2002	86.17	56.03	43.35	27.51	59.74	56.10	42.31	26.99	80.00	54.47	38.54	32.25	79.27	52.75	39.00	33.73
	2003	11.39	10.15	12.12	41.60	5.24	21.18	20.19	16.14	10.37	6.27	7.01	35.97	10.14	5.92	7.21	32.36
LCY-BBB	**2000**	9.80	11.53	10.71	12.26	3.26	9.82	8.95	6.57	11.38	19.83	11.61	13.35	11.03	18.85	11.53	14.71
	2001	10.82	12.81	13.61	20.49	6.92	16.54	14.51	11.01	22.42	31.34	14.52	26.72	22.08	31.38	13.99	24.96
	2002	35.77	34.01	23.48	14.98	30.23	32.14	24.50	15.26	78.68	34.89	29.98	23.21	79.35	36.20	29.91	22.44
	2003	15.89	26.62	28.87	28.20	23.89	25.36	19.67	13.36	18.75	31.33	44.44	51.84	21.62	31.70	43.65	49.58
NCY-AA	**2000**	6.96	11.94	12.07	9.39	2.85	14.86	14.86	11.32	15.54	12.18	15.32	11.75	14.87	10.79	14.42	9.77
	2001	7.34	6.59	6.79	4.77	1.37	5.80	5.51	3.99	11.20	9.44	10.43	8.13	11.54	8.43	9.42	6.63
	2002	7.34	7.59	5.99	6.31	0.51	6.18	7.12	5.90	7.39	12.81	8.09	13.22	6.94	12.51	7.56	12.77
	2003	7.22	7.54	5.87	5.57	4.03	7.83	6.41	4.20	7.76	10.95	12.20	14.82	7.39	10.70	11.68	14.46
NCY-A	**2000**	6.08	11.00	12.20	15.35	2.77	14.25	14.21	11.12	8.54	13.67	11.61	15.38	7.36	13.05	10.95	12.27
	2001	8.28	5.23	6.78	9.56	0.32	5.81	7.15	6.41	10.30	7.64	14.33	13.56	10.02	7.20	13.17	12.55
	2002	11.19	14.97	12.33	8.44	3.91	18.58	18.21	13.56	7.77	12.88	13.03	17.35	7.60	11.39	14.01	15.58
	2003	5.48	7.77	10.17	12.84	2.04	10.62	10.55	8.07	8.18	7.34	14.86	13.84	7.87	6.43	13.58	8.32
NCY-BBB	**2000**	14.91	24.54	24.91	23.54	10.57	31.35	28.85	21.44	24.88	18.29	16.61	24.22	24.94	15.72	15.86	21.93
	2001	12.10	15.10	11.61	22.89	1.27	10.72	11.47	8.87	15.25	20.84	10.53	31.66	14.17	19.16	9.64	26.05
	2002	20.67	35.30	41.58	54.28	24.98	43.91	37.38	28.99	27.68	48.30	39.00	74.55	28.23	48.40	38.64	74.54
	2003	73.57	75.29	63.80	52.33	79.73	63.71	47.83	31.81	62.12	118.78	63.27	47.32	46.53	116.82	73.33	51.08
NF-AA	**1999**	4.92	4.48	4.85	6.07	0.30	2.53	2.72	2.13	10.07	21.84	19.15	15.44	10.07	21.84	19.15	15.44
	2000	7.41	13.53	14.47	12.61	2.70	17.19	17.94	14.13	11.72	17.01	13.00	17.25	11.35	16.79	12.65	16.62
	2001	8.70	7.22	5.16	6.12	3.35	9.94	8.84	6.09	9.60	10.70	7.87	12.00	9.52	10.10	7.42	10.56
	2002	3.88	5.73	6.41	5.70	1.67	6.26	5.84	4.21	5.99	11.35	9.41	5.99	4.92	10.90	9.09	4.69
	2003	6.94	5.02	3.70	7.62	1.24	4.99	4.66	3.39	8.37	17.03	12.29	10.81	7.76	16.42	11.86	9.28
NF-A	**1999**	5.42	5.24	4.30	6.77	0.68	4.70	4.87	3.77	8.89	9.16	9.34	9.00	8.89	9.16	9.34	9.00
	2000	6.62	13.21	15.83	20.55	4.19	18.81	18.39	14.32	16.86	11.24	18.96	21.79	16.64	10.55	17.74	20.40
	2001	20.06	26.86	22.24	10.49	14.68	33.32	28.96	20.30	30.93	25.42	25.07	15.32	31.29	25.79	24.64	13.68
	2002	15.21	19.23	16.16	11.32	14.90	19.96	15.74	10.34	32.72	83.09	56.64	22.64	32.19	81.78	58.67	21.95
	2003	6.01	9.88	11.94	16.23	5.58	14.84	13.08	9.38	12.68	17.80	8.38	18.84	12.32	17.82	7.90	16.78
NF-BBB	**1999**	5.87	8.68	9.59	24.14	1.62	11.25	11.70	9.89	18.31	17.90	14.33	35.76	18.31	17.90	14.33	35.76
	2000	9.61	15.36	16.59	17.55	6.22	20.33	18.85	14.24	21.56	19.16	17.02	17.68	20.81	17.75	13.79	19.38
	2001	14.86	19.32	15.61	15.15	4.23	16.69	15.84	11.26	13.52	18.23	29.82	24.15	13.16	18.43	27.91	19.34
	2002	91.07	66.85	45.63	20.95	78.07	65.41	48.53	30.56	93.92	39.66	61.25	56.79	94.39	39.82	66.88	54.99
	2003	23.21	36.36	38.40	39.24	33.91	33.11	25.52	17.49	40.62	41.99	59.85	18.80	42.84	41.83	59.41	19.10
AA	**1999**	3.65	3.67	4.38	5.04	0.14	2.33	2.83	2.42	15.08	12.67	13.66	23.98	15.08	12.67	13.66	23.98
	2000	4.97	10.12	11.25	12.28	1.19	11.62	13.01	10.74	10.54	14.99	14.80	14.92	10.53	14.51	14.80	14.75
	2001	4.59	3.73	4.58	6.21	0.39	2.60	2.69	2.12	31.55	11.40	9.46	12.93	31.54	9.40	7.83	10.48
	2002	6.58	6.13	4.72	6.53	1.32	6.91	6.80	5.11	14.73	17.77	10.68	8.54	17.03	17.81	9.91	6.74
	2003	4.18	3.83	3.64	6.30	0.76	4.41	4.41	3.33	9.10	14.40	11.76	22.70	8.61	13.33	8.22	21.63
A	**1999**	5.35	5.93	5.22	5.39	1.02	4.75	4.58	3.36	10.77	17.87	11.43	14.05	10.77	17.87	11.43	14.05
	2000	3.87	9.64	12.32	16.38	2.53	12.99	12.94	10.19	15.29	20.30	16.20	27.07	15.03	19.82	15.32	25.42
	2001	5.75	8.05	7.53	6.22	1.43	7.10	6.99	5.17	29.22	24.61	15.14	17.40	30.09	24.57	14.56	16.59
	2002	8.75	12.10	11.68	11.15	3.85	14.86	14.03	10.45	9.85	13.58	20.04	20.47	9.48	13.50	19.52	20.29
	2003	3.32	8.01	10.86	12.47	4.18	11.15	9.85	7.03	14.81	19.96	23.74	19.26	15.11	20.15	23.13	18.88
BBB	**1999**	6.01	7.24	7.43	23.04	0.90	5.50	5.59	4.93	15.98	21.57	20.67	26.56	15.98	21.57	20.67	26.56
	2000	9.27	14.77	15.97	16.61	5.90	19.44	18.04	13.61	13.86	13.55	17.40	24.28	13.73	12.18	16.04	20.70
	2001	14.49	18.74	15.24	18.60	4.51	17.18	16.21	11.57	15.91	24.87	15.45	21.69	15.53	24.12	12.03	16.52
	2002	91.62	67.31	45.97	21.32	78.69	65.80	48.82	30.75	110.29	60.15	39.65	56.59	111.79	61.47	43.87	56.13
	2003	23.37	36.63	38.70	39.43	34.21	33.37	25.72	17.62	37.64	54.65	41.39	51.09	37.63	56.38	41.21	50.83

Table D.2: Standard Deviation of Yield Spreads for Annual Estimation

Risk-Class	Year	Empirical Yield Spread of Risk class [bps]				Factor-implied Yield Spread [bps]				Periodic Risk-neutral Factor-PD				Periodic Factor-PD			
		T=1	T=3	T=5	T=10	T=1	T=3	T=5	T=10	T=1	T=3	T=5	T=10	T=1	T=3	T=5	T=10
ECY-A	**2000**	0.18	0.51	0.74	1.07	0.06	0.63	0.82	0.83	0.12	2.24	2.16	1.19	1.72	27.48	31.47	30.38
	2001	0.54	1.04	1.25	1.58	0.28	1.30	1.49	1.38	0.54	3.96	3.27	1.72	1.40	11.39	11.10	8.47
	2002	0.47	0.84	0.95	0.99	0.33	1.04	1.05	0.84	0.63	2.79	1.82	0.65	2.59	14.05	12.71	9.38
	2003	0.36	0.63	0.71	0.84	0.15	0.75	0.83	0.71	0.30	2.29	1.71	0.69	0.20	1.44	0.96	0.30
ECY-BBB	**2000**	0.37	0.79	0.99	1.16	0.13	0.92	1.13	1.09	0.25	3.08	2.75	1.48	1.77	21.10	22.86	20.68
	2001	0.65	1.19	1.35	1.43	0.49	1.48	1.51	1.26	0.93	3.93	2.71	1.14	3.74	18.21	16.75	13.08
	2002	3.08	2.58	2.24	1.77	2.78	2.91	2.37	1.64	5.09	4.42	2.13	0.57	17.77	25.85	21.03	15.63
	2003	0.68	1.14	1.21	1.21	0.47	1.37	1.39	1.14	0.90	3.60	2.44	0.98	0.65	2.36	1.43	0.44
FIN-AA	**1999**	0.12	0.26	0.36	0.50	0.01	0.25	0.43	0.50	0.01	1.08	1.43	0.96	0.05	4.65	6.87	6.49
	2000	0.15	0.37	0.50	0.65	0.02	0.38	0.57	0.63	0.03	1.51	1.76	1.10	0.41	16.19	21.12	21.00
	2001	0.21	0.37	0.45	0.59	0.02	0.38	0.53	0.54	0.04	1.45	1.50	0.80	0.07	2.22	2.45	1.55
	2002	0.20	0.34	0.41	0.52	0.03	0.36	0.48	0.47	0.05	1.33	1.28	0.62	0.36	10.10	12.13	10.77
	2003	0.21	0.32	0.35	0.47	0.02	0.31	0.42	0.41	0.04	1.17	1.13	0.53	0.06	1.74	1.79	1.00
FIN-A	**1999**	0.24	0.47	0.56	0.65	0.05	0.51	0.66	0.64	0.09	1.84	1.71	0.86	0.25	5.40	5.87	4.34
	2000	0.20	0.50	0.67	0.89	0.05	0.56	0.77	0.79	0.09	2.09	2.12	1.21	0.68	15.78	18.76	17.48
	2001	0.27	0.49	0.62	0.97	0.04	0.60	0.86	0.92	0.08	2.31	2.49	1.55	0.12	3.73	4.27	3.11
	2002	0.20	0.45	0.61	0.85	0.03	0.50	0.72	0.78	0.06	1.93	2.14	1.32	0.41	13.34	16.75	15.90
	2003	0.32	0.59	0.69	0.77	0.13	0.71	0.80	0.69	0.25	2.22	1.69	0.69	0.30	2.71	2.16	0.98
LCY-AA	**2000**	0.14	0.37	0.52	0.70	0.02	0.40	0.59	0.65	0.04	1.58	1.81	1.12	0.21	8.91	11.69	10.96
	2001	0.32	0.56	0.62	0.75	0.15	0.69	0.76	0.64	0.29	2.08	1.52	0.59	2.83	23.32	24.22	21.54
	2002	0.20	0.42	0.51	0.56	0.10	0.52	0.58	0.48	0.19	1.62	1.18	0.42	0.71	7.59	7.38	5.24
	2003	0.22	0.42	0.47	0.51	0.04	0.44	0.56	0.52	0.09	1.55	1.38	0.63	0.12	2.25	2.15	1.16
LCY-A	**2000**	0.22	0.55	0.74	0.96	0.07	0.65	0.84	0.84	0.13	2.30	2.19	1.19	1.32	21.75	24.84	23.37
	2001	0.29	0.62	0.85	1.44	0.01	0.56	1.04	1.42	0.02	2.48	3.82	3.47	0.08	7.49	11.66	12.11
	2002	1.33	1.59	1.45	1.12	0.94	1.74	1.60	1.19	1.78	3.78	2.21	0.74	8.00	23.06	20.36	15.95
	2003	0.34	0.59	0.66	0.87	0.07	0.62	0.80	0.79	0.14	2.19	2.06	1.10	0.04	0.46	0.30	0.07
LCY-BBB	**2000**	0.35	0.81	1.02	1.18	0.13	0.96	1.18	1.14	0.25	3.23	2.88	1.56	0.91	12.08	12.69	10.39
	2001	0.50	0.96	1.15	1.41	0.19	1.12	1.33	1.26	0.36	3.61	3.10	1.65	0.93	9.94	9.88	7.47
	2002	0.73	1.20	1.27	1.23	0.64	1.43	1.35	1.03	1.22	3.35	2.00	0.67	4.73	16.70	14.39	10.47
	2003	0.85	1.40	1.38	1.09	0.76	1.55	1.43	1.06	1.46	3.48	2.00	0.64	2.16	5.79	3.85	1.70
NCY-AA	**2000**	0.17	0.43	0.57	0.68	0.05	0.50	0.65	0.64	0.09	1.79	1.71	0.88	0.96	18.68	21.91	20.65
	2001	0.17	0.41	0.52	0.58	0.05	0.47	0.60	0.56	0.09	1.66	1.50	0.70	0.14	2.86	2.84	1.69
	2002	0.23	0.43	0.50	0.56	0.01	0.34	0.55	0.64	0.02	1.41	1.80	1.22	0.11	7.36	10.39	10.02
	2003	0.23	0.40	0.45	0.48	0.13	0.52	0.52	0.39	0.25	1.45	0.89	0.24	1.13	8.41	7.57	5.06
NCY-A	**2000**	0.20	0.50	0.68	0.90	0.05	0.58	0.78	0.80	0.10	2.14	2.14	1.21	0.94	19.39	22.82	21.62
	2001	0.23	0.49	0.69	1.08	0.01	0.45	0.82	1.09	0.02	2.00	2.98	2.51	0.05	4.97	7.66	7.54
	2002	0.27	0.56	0.71	0.91	0.06	0.63	0.85	0.88	0.11	2.30	2.34	1.38	0.36	8.21	9.57	8.10
	2003	0.25	0.46	0.55	0.67	0.04	0.48	0.64	0.63	0.08	1.76	1.71	0.90	0.05	1.02	0.89	0.36
NCY-BBB	**2000**	0.33	0.76	0.99	1.23	0.13	0.91	1.13	1.11	0.25	3.05	2.80	1.56	3.03	32.61	35.88	34.40
	2001	0.47	0.94	1.23	1.88	0.04	0.90	1.43	1.75	0.07	3.65	4.67	3.77	0.07	3.69	4.73	3.83
	2002	0.65	1.31	1.61	1.92	0.40	1.61	1.79	1.66	0.77	4.69	3.78	1.99	4.04	25.47	25.59	22.48
	2003	1.60	2.18	2.03	1.56	1.49	2.34	2.09	1.55	2.81	4.68	2.67	0.92	3.73	6.90	4.42	1.95
NF-AA	**1999**	0.15	0.30	0.37	0.47	0.01	0.29	0.43	0.45	0.03	1.17	1.27	0.69	0.11	5.15	6.60	5.61
	2000	0.18	0.45	0.59	0.72	0.04	0.49	0.68	0.71	0.07	1.84	1.93	1.13	1.31	27.89	33.37	33.13
	2001	0.16	0.45	0.58	0.58	0.08	0.55	0.64	0.57	0.15	1.80	1.46	0.61	0.44	6.11	6.09	4.23
	2002	0.18	0.41	0.51	0.54	0.06	0.48	0.58	0.53	0.11	1.64	1.39	0.60	0.51	8.75	9.43	7.50
	2003	0.22	0.40	0.45	0.48	0.05	0.43	0.52	0.47	0.10	1.48	1.24	0.51	0.27	4.81	4.92	3.35
NF-A	**1999**	0.21	0.44	0.55	0.71	0.03	0.45	0.64	0.67	0.05	1.72	1.84	1.06	0.09	3.26	3.78	2.72
	2000	0.21	0.53	0.74	0.99	0.07	0.65	0.84	0.84	0.13	2.29	2.19	1.20	1.80	27.43	31.25	30.07
	2001	0.44	0.89	1.10	1.36	0.19	1.09	1.30	1.24	0.36	3.51	3.06	1.65	1.03	10.89	11.11	8.76
	2002	0.49	0.86	0.95	0.95	0.39	1.06	1.03	0.80	0.75	2.68	1.64	0.53	3.45	15.79	14.10	10.49
	2003	0.33	0.56	0.64	0.74	0.12	0.67	0.76	0.66	0.23	2.11	1.63	0.67	0.31	2.99	2.48	1.22
NF-BBB	**1999**	0.27	0.57	0.73	1.04	0.03	0.62	0.92	1.03	0.07	2.43	2.80	1.89	0.05	1.67	1.82	1.08
	2000	0.35	0.78	1.00	1.19	0.12	0.92	1.15	1.13	0.23	3.12	2.87	1.60	1.55	20.25	22.21	20.16
	2001	0.51	1.00	1.23	1.66	0.10	1.08	1.46	1.55	0.20	3.89	4.01	2.65	0.42	8.40	9.36	7.61
	2002	1.47	1.84	1.81	1.63	1.39	2.11	1.87	1.38	2.61	4.15	2.29	0.73	9.98	22.53	19.05	14.34
	2003	0.97	1.51	1.49	1.23	0.89	1.69	1.54	1.14	1.70	3.68	2.10	0.68	2.55	6.25	4.15	1.86
AA	**1999**	0.12	0.27	0.36	0.50	0.01	0.25	0.42	0.50	0.01	1.07	1.42	0.94	0.06	5.28	7.82	7.52
	2000	0.15	0.38	0.51	0.67	0.02	0.39	0.59	0.66	0.04	1.56	1.83	1.17	0.43	17.04	22.23	22.21
	2001	0.22	0.39	0.46	0.61	0.03	0.39	0.54	0.55	0.05	1.49	1.52	0.81	0.07	2.42	2.66	1.71
	2002	0.20	0.35	0.42	0.52	0.03	0.38	0.51	0.49	0.06	1.41	1.34	0.64	0.37	9.60	11.33	9.85
	2003	0.23	0.33	0.36	0.49	0.03	0.33	0.44	0.42	0.05	1.23	1.16	0.54	0.08	2.05	2.11	1.22
A	**1999**	0.23	0.46	0.56	0.66	0.05	0.51	0.66	0.63	0.09	1.83	1.69	0.84	0.32	6.89	7.60	5.94
	2000	0.20	0.50	0.69	0.91	0.05	0.60	0.80	0.82	0.10	2.19	2.18	1.23	0.83	17.26	20.23	18.87
	2001	0.28	0.60	0.79	1.03	0.05	0.67	0.92	0.96	0.10	2.50	2.56	1.52	0.23	5.77	6.51	5.04
	2002	0.29	0.58	0.72	0.87	0.07	0.65	0.83	0.81	0.14	2.28	2.12	1.12	0.82	14.19	16.01	14.19
	2003	0.32	0.58	0.67	0.76	0.13	0.70	0.79	0.68	0.24	2.19	1.68	0.69	0.33	3.17	2.62	1.30
BBB	**1999**	0.31	0.63	0.78	1.01	0.04	0.65	0.91	0.98	0.08	2.45	2.63	1.64	0.12	3.52	3.95	2.79
	2000	0.36	0.80	1.00	1.18	0.12	0.93	1.16	1.14	0.23	3.15	2.90	1.63	1.67	21.63	23.75	21.73
	2001	0.52	1.00	1.23	1.66	0.11	1.10	1.47	1.56	0.21	3.94	4.01	2.63	0.33	6.35	6.84	5.17
	2002	1.47	1.85	1.81	1.63	1.39	2.12	1.87	1.39	2.61	4.15	2.29	0.73	9.90	22.31	18.84	14.14
	2003	0.97	1.51	1.49	1.23	0.90	1.69	1.55	1.14	1.71	3.68	2.10	0.68	2.57	6.28	4.17	1.88

Table D.3: Average Spread and Default Probability of Factors for Annual Estimation

Risk-Class	Year	Risk-neutral periodic Asset-PD				Risk-neutral cumulative Asset-PD				Periodic Asset-PD				Cumulative Asset-PD			
		T=1	**T=3**	**T=5**	**T=10**	**T=1**	**T=3**	**T=5**	**T=10**	**T=1**	**T=3**	**T=5**	**T=10**	**T=1**	**T=3**	**T=5**	**T=10**
ECY-A	**2000**	0.34	1.59	2.13	1.90	0.34	2.96	6.87	16.18	3.28	20.03	25.17	22.11	3.28	32.68	61.57	90.19
	2001	1.04	2.70	2.81	2.52	1.04	5.90	11.12	22.28	2.19	6.83	7.12	5.38	2.19	13.85	25.74	45.85
	2002	0.90	2.12	1.90	1.20	0.90	4.84	8.56	14.98	4.12	13.43	13.17	9.05	4.12	26.04	44.60	68.39
	2003	0.70	1.63	1.70	1.74	0.70	3.72	6.94	14.79	0.65	1.46	1.50	1.54	0.65	3.38	6.24	13.25
ECY-BBB	**2000**	0.70	2.22	2.41	1.70	0.70	4.60	9.13	17.91	3.59	15.42	17.76	14.73	3.59	27.55	50.71	79.63
	2001	1.22	3.03	2.84	1.92	1.22	6.72	12.10	21.57	4.30	13.95	14.38	10.87	4.30	26.82	46.51	72.27
	2002	5.70	3.31	2.08	1.05	5.70	12.71	16.75	22.17	38.97	33.84	22.49	12.80	38.97	76.67	86.82	94.37
	2003	1.32	2.80	2.39	1.63	1.32	6.60	11.23	19.23	0.96	1.78	1.43	1.00	0.96	4.42	7.30	12.41
FIN-AA	**1999**	0.22	0.74	0.92	0.94	0.22	1.51	3.25	7.80	3.21	12.84	10.83	5.84	3.21	24.44	40.86	59.98
	2000	0.28	1.07	1.22	0.97	0.28	2.11	4.47	9.56	3.96	19.69	19.74	12.88	3.96	33.98	57.96	81.80
	2001	0.39	0.97	1.06	1.11	0.39	2.18	4.21	9.39	2.47	9.83	10.79	8.34	2.47	18.51	35.09	60.52
	2002	0.38	0.80	0.83	0.87	0.38	1.90	3.50	7.62	9.32	15.94	10.06	5.11	9.32	38.11	51.27	65.38
	2003	0.41	0.72	0.72	0.96	0.41	1.84	3.23	7.42	3.16	9.00	7.80	5.28	3.16	18.96	31.64	50.42
FIN-A	**1999**	0.47	1.27	1.30	1.13	0.47	2.76	5.30	10.82	2.76	11.58	13.08	10.80	2.76	21.28	40.35	68.27
	2000	0.38	1.43	1.63	1.32	0.38	2.83	5.94	12.66	4.43	20.46	20.55	13.64	4.43	35.55	59.77	83.39
	2001	0.51	1.28	1.54	2.08	0.51	2.84	5.68	14.31	4.43	16.79	18.29	15.05	4.43	30.73	53.64	81.29
	2002	0.39	1.19	1.45	1.40	0.39	2.47	5.20	11.95	12.75	26.07	18.63	10.09	12.75	53.55	70.50	85.14
	2003	0.62	1.54	1.49	1.11	0.62	3.46	6.37	12.06	2.12	7.15	6.98	4.40	2.12	14.30	26.13	43.97
LCY-AA	**2000**	0.26	1.17	1.49	1.27	0.26	2.21	5.01	11.51	2.55	16.65	21.62	20.11	2.55	27.31	54.45	86.24
	2001	0.60	1.36	1.29	1.35	0.60	3.16	5.69	11.65	5.53	20.04	20.73	16.90	5.53	36.33	60.15	85.64
	2002	0.39	1.13	1.09	0.72	0.39	2.42	4.59	8.60	4.33	18.56	18.18	11.72	4.33	33.07	55.74	79.28
	2003	0.42	1.13	1.08	0.83	0.42	2.49	4.63	8.97	1.31	5.26	5.94	5.08	1.31	10.11	20.35	40.10
LCY-A	**2000**	0.42	1.68	2.07	1.79	0.42	3.26	7.12	15.92	3.67	20.27	25.77	25.54	3.67	33.56	62.49	91.82
	2001	0.57	1.81	2.59	4.13	0.57	3.67	8.24	23.69	0.99	3.85	5.66	8.34	0.99	7.35	16.79	43.86
	2002	2.54	2.66	1.52	0.59	2.54	8.21	11.43	15.05	20.95	18.66	9.35	3.16	20.95	52.67	62.65	70.78
	2003	0.65	1.53	1.59	2.04	0.65	3.49	6.50	14.96	0.58	1.27	1.28	1.71	0.58	2.97	5.43	12.52
LCY-BBB	**2000**	0.67	2.33	2.61	1.90	0.67	4.69	9.56	19.18	2.65	12.39	14.89	13.00	2.65	22.06	43.10	73.30
	2001	0.96	2.59	2.80	2.61	0.96	5.58	10.76	22.25	3.02	11.00	12.85	12.24	3.02	20.82	39.48	69.39
	2002	1.37	2.87	2.42	1.42	1.37	6.77	11.48	18.98	8.55	25.46	26.14	21.92	8.55	46.20	70.77	92.39
	2003	1.63	3.31	2.25	0.79	1.63	7.98	12.58	17.68	1.66	3.40	2.32	0.82	1.66	8.16	12.90	18.17
NCY-AA	**2000**	0.33	1.29	1.51	0.99	0.33	2.52	5.39	11.10	3.16	18.13	22.29	19.16	3.16	30.22	57.26	86.74
	2001	0.33	1.21	1.35	0.89	0.33	2.41	5.02	10.06	1.68	9.36	11.55	9.65	1.68	16.32	34.12	62.47
	2002	0.44	1.14	1.20	1.01	0.44	2.52	4.84	9.96	2.66	11.82	14.68	14.60	2.66	21.30	42.06	74.29
	2003	0.45	1.01	0.93	0.65	0.45	2.36	4.22	7.81	1.18	3.59	3.34	1.87	1.18	7.55	13.85	23.83
NCY-A	**2000**	0.38	1.54	1.99	1.83	0.38	2.96	6.63	15.49	2.15	12.66	17.27	17.83	2.15	21.40	45.11	79.90
	2001	0.44	1.47	2.16	3.07	0.44	2.94	6.78	19.25	0.77	3.26	4.96	6.88	0.77	6.09	14.47	38.59
	2002	0.51	1.60	1.96	1.94	0.51	3.28	6.91	15.92	1.74	7.97	10.71	11.42	1.74	14.43	30.96	62.61
	2003	0.48	1.27	1.44	1.56	0.48	2.77	5.49	12.56	1.11	4.13	5.29	6.21	1.11	8.04	17.11	39.21
NCY-BBB	**2000**	0.61	2.22	2.71	2.31	0.61	4.38	9.35	20.35	4.48	22.13	27.84	27.87	4.48	36.90	66.27	93.70
	2001	0.90	2.68	3.59	5.27	0.90	5.54	11.80	30.78	1.12	3.64	4.95	7.08	1.12	7.30	15.70	39.55
	2002	1.22	3.53	3.76	2.68	1.22	7.40	14.24	26.94	7.77	27.43	30.16	26.02	7.77	47.30	74.13	94.98
	2003	3.06	3.97	2.22	0.76	3.06	11.30	15.86	20.51	21.21	29.07	18.62	9.13	21.21	63.01	76.87	87.68
NF-AA	**1999**	0.29	0.82	0.91	0.94	0.29	1.77	3.54	7.95	3.74	18.21	21.51	19.44	3.74	31.28	57.21	86.50
	2000	0.34	1.30	1.46	1.01	0.34	2.57	5.38	10.97	4.56	23.52	26.13	20.37	4.56	39.13	66.72	90.88
	2001	0.29	1.37	1.34	0.53	0.29	2.61	5.32	9.12	1.81	10.97	10.71	5.28	1.81	19.13	36.14	56.00
	2002	0.35	1.19	1.21	0.74	0.35	2.43	4.83	9.14	1.45	7.14	8.17	5.94	1.45	12.98	26.52	48.63
	2003	0.43	1.02	0.95	0.71	0.43	2.34	4.23	7.98	2.55	9.83	10.17	7.14	2.55	18.72	34.58	57.77
NF-A	**1999**	0.40	1.25	1.49	1.49	0.40	2.59	5.40	12.41	1.82	8.75	11.50	11.77	1.82	15.68	33.21	64.84
	2000	0.39	1.64	2.15	1.94	0.39	3.14	7.08	16.48	2.72	16.08	21.43	21.49	2.72	26.78	53.72	86.74
	2001	0.84	2.44	2.75	2.33	0.84	5.13	10.21	21.12	2.16	8.11	9.59	8.18	2.16	15.45	30.56	56.63
	2002	0.94	1.99	1.61	1.00	0.94	4.75	8.00	13.35	8.68	20.25	14.71	7.46	8.68	41.67	58.97	75.33
	2003	0.63	1.45	1.46	1.30	0.63	3.32	6.13	12.42	1.26	3.78	4.11	3.51	1.26	7.88	15.23	30.29
NF-BBB	**1999**	0.53	1.60	1.91	2.13	0.53	3.33	6.87	16.14	2.10	8.90	10.59	9.02	2.10	16.48	32.88	60.20
	2000	0.66	2.27	2.61	2.05	0.66	4.58	9.42	19.48	3.44	16.06	19.72	18.60	3.44	28.00	52.93	84.06
	2001	0.97	2.75	3.30	4.01	0.97	5.81	11.70	27.26	2.28	8.31	10.58	12.44	2.28	15.88	32.04	64.22
	2002	2.78	3.38	2.38	1.30	2.78	9.61	14.26	20.98	17.61	27.14	19.71	10.86	17.61	57.32	73.70	87.31
	2003	1.87	3.28	2.10	0.81	1.87	8.34	12.68	17.60	4.05	8.24	5.50	2.07	4.05	19.24	28.95	39.16
AA	**1999**	0.24	0.76	0.91	0.98	0.24	1.57	3.30	7.92	4.65	20.18	19.45	13.06	4.65	35.67	58.85	82.15
	2000	0.29	1.07	1.21	0.99	0.29	2.14	4.47	9.60	7.52	28.94	25.55	16.36	7.52	50.07	73.23	90.93
	2001	0.43	0.98	1.04	1.21	0.43	2.26	4.26	9.65	3.09	11.78	12.61	10.08	3.09	22.23	40.60	67.24
	2002	0.38	0.87	0.89	0.86	0.38	2.01	3.75	7.90	5.70	19.61	17.85	11.73	5.70	36.56	57.91	80.08
	2003	0.44	0.76	0.77	1.10	0.44	1.96	3.44	8.13	1.25	3.39	3.61	3.52	1.25	7.37	13.85	28.62
A	**1999**	0.45	1.24	1.30	1.11	0.45	2.69	5.21	10.70	1.87	7.41	7.92	5.58	1.87	14.14	27.28	48.13
	2000	0.38	1.52	1.87	1.52	0.38	2.95	6.45	14.22	4.22	22.81	27.60	24.52	4.22	37.52	66.67	92.73
	2001	0.53	1.68	1.93	1.66	0.53	3.46	7.09	15.17	2.19	9.01	9.46	6.07	2.19	16.92	32.07	53.81
	2002	0.56	1.46	1.50	1.24	0.56	3.21	6.11	12.30	6.28	17.15	12.94	6.59	6.28	34.85	51.88	69.32
	2003	0.62	1.49	1.43	1.10	0.62	3.37	6.17	11.77	2.76	9.32	8.93	5.62	2.76	18.44	32.75	52.84
BBB	**1999**	0.61	1.75	1.98	1.98	0.61	3.70	7.40	16.33	2.84	11.90	14.23	12.86	2.84	21.72	41.95	72.31
	2000	0.68	2.27	2.51	1.81	0.68	4.62	9.32	18.57	4.13	18.47	21.66	19.04	4.13	32.16	57.94	86.58
	2001	0.98	2.76	3.32	4.08	0.98	5.83	11.76	27.52	1.69	5.73	7.24	8.71	1.69	11.31	23.17	50.34
	2002	2.78	3.60	2.60	1.36	2.78	9.92	14.97	22.08	13.76	26.30	22.94	15.65	13.76	52.30	72.41	89.85
	2003	1.87	3.41	2.28	0.90	1.87	8.52	13.17	18.62	2.88	5.98	4.33	1.90	2.88	14.02	22.03	31.73

Table D.4: Asset-implied Default Probability for Annual Estimation

Risk-Class	Year	Std. Dev. of Factor-implied Yield Spread Residuals				Std. Dev. of Asset-implied Yield Spread Residuals			
		T=1	T=3	T=5	T=10	T=1	T=3	T=5	T=10
ECY-A	**2000**	4.36	7.75	3.95	13.37	1.58	3.11	2.28	3.59
	2001	8.29	12.84	12.58	22.23	2.65	3.33	2.73	1.47
	2002	2.71	4.61	5.80	5.89	1.49	0.91	2.82	2.75
	2003	3.23	5.89	3.85	9.31	0.54	1.49	1.54	6.50
ECY-BBB	**2000**	6.84	7.99	4.55	8.59	3.62	4.25	4.07	7.12
	2001	8.90	9.75	8.56	19.61	3.76	2.33	2.06	7.77
	2002	89.30	21.13	20.47	28.15	16.12	12.04	17.15	11.04
	2003	10.06	8.32	8.16	29.49	2.44	1.75	2.03	7.89
FIN-AA	**1999**	3.36	2.88	3.20	3.42	0.96	4.18	3.20	1.19
	2000	3.90	5.17	4.08	3.33	1.19	1.89	0.78	1.41
	2001	4.45	3.68	2.80	5.14	1.51	3.23	2.62	3.39
	2002	6.29	2.63	3.41	2.98	2.93	3.23	1.55	1.19
	2003	2.77	2.08	2.11	3.18	2.18	2.05	2.23	4.13
FIN-A	**1999**	4.89	4.27	4.94	4.63	1.16	2.57	4.02	3.93
	2000	3.62	6.18	4.41	7.96	1.38	1.10	1.39	1.34
	2001	4.93	4.63	3.46	2.68	1.73	1.22	2.65	1.99
	2002	4.54	4.05	4.11	6.13	2.35	1.81	4.94	4.53
	2003	3.49	3.46	3.01	6.23	2.33	4.62	4.37	6.67
LCY-AA	**2000**	4.20	6.04	5.05	8.14	1.86	1.65	1.64	3.35
	2001	10.19	5.18	9.25	19.08	1.42	2.33	1.51	1.37
	2002	9.60	6.80	6.78	10.64	1.37	1.18	3.11	5.16
	2003	8.86	4.94	5.65	6.63	2.60	2.28	4.04	7.32
LCY-A	**2000**	3.80	7.90	4.04	13.69	1.77	1.55	2.15	6.66
	2001	10.56	12.64	8.56	9.62	3.06	1.30	4.79	6.99
	2002	56.31	19.07	15.13	16.21	53.83	10.02	5.90	8.12
	2003	7.42	11.69	9.12	26.27	0.74	1.75	3.01	8.64
LCY-BBB	**2000**	8.23	6.83	5.92	10.79	1.24	2.86	0.81	6.57
	2001	6.91	7.62	5.84	12.66	2.14	1.26	2.07	5.27
	2002	10.53	8.44	10.28	18.79	5.34	3.12	2.48	4.59
	2003	11.24	4.93	10.00	15.46	5.81	1.10	5.88	7.47
NCY-AA	**2000**	4.53	5.26	4.71	3.52	2.77	4.50	5.05	6.29
	2001	6.57	4.67	6.30	4.63	1.62	2.76	3.15	4.45
	2002	6.91	3.49	3.38	3.52	1.06	1.27	2.59	3.14
	2003	4.87	3.77	2.52	2.85	1.93	1.75	1.42	2.26
NCY-A	**2000**	4.17	6.19	4.03	6.83	2.67	1.80	5.55	8.60
	2001	8.13	3.72	5.90	3.96	1.47	2.27	5.74	4.45
	2002	7.74	5.98	8.79	13.26	1.57	4.05	3.22	3.99
	2003	4.83	4.12	2.44	5.90	1.68	1.83	3.73	13.41
NCY-BBB	**2000**	5.90	9.08	5.21	13.80	1.92	5.92	4.28	9.36
	2001	11.15	7.59	3.79	24.44	3.15	3.92	2.94	16.57
	2002	7.01	10.49	8.44	27.91	1.75	1.43	1.83	3.65
	2003	31.96	23.05	23.20	27.09	48.59	37.16	30.94	9.92
NF-AA	**1999**	4.85	3.42	3.50	5.12	1.43	1.94	4.32	1.28
	2000	5.08	5.91	4.91	3.69	1.88	1.38	1.40	3.75
	2001	5.68	5.48	7.93	7.96	2.42	2.28	1.07	3.24
	2002	3.37	2.78	2.92	3.42	2.65	1.75	1.43	2.35
	2003	6.17	3.33	2.92	5.64	2.09	2.58	2.40	3.70
NF-A	**1999**	4.89	2.76	3.72	5.66	0.86	2.11	3.09	2.95
	2000	3.70	7.46	4.09	8.45	2.21	2.30	3.48	6.10
	2001	8.35	11.24	11.42	14.39	2.20	2.62	2.73	3.72
	2002	3.12	4.39	5.73	7.19	4.60	12.48	7.48	4.61
	2003	2.07	5.17	2.06	7.35	2.19	1.45	2.87	4.78
NF-BBB	**1999**	6.06	15.55	14.56	16.05	5.12	0.80	2.73	4.32
	2000	6.14	8.07	3.89	7.67	2.72	3.23	8.34	6.08
	2001	11.73	9.09	6.93	20.37	1.96	1.24	5.80	10.03
	2002	30.81	12.70	11.15	19.09	7.98	7.80	15.15	8.95
	2003	13.50	7.19	14.18	23.60	6.75	2.69	7.53	3.48
AA	**1999**	3.62	2.92	3.06	3.41	3.33	1.90	6.14	2.38
	2000	4.16	5.28	4.35	3.62	1.32	1.65	0.87	1.17
	2001	4.66	3.46	2.78	4.90	0.95	5.51	3.24	5.10
	2002	5.45	3.13	3.89	3.19	5.48	1.29	3.40	3.43
	2003	3.63	2.05	2.28	3.37	2.62	2.72	5.76	3.54
A	**1999**	4.56	3.88	4.72	4.61	2.57	2.08	2.87	1.88
	2000	3.04	6.78	4.52	7.18	1.10	3.25	3.97	4.54
	2001	5.01	4.26	4.34	7.61	6.12	2.32	1.54	2.33
	2002	5.35	4.95	5.06	5.04	1.74	1.87	2.17	3.12
	2003	2.68	3.70	2.15	6.22	3.40	2.48	4.23	2.01
BBB	**1999**	5.94	8.37	6.84	19.12	1.68	1.02	1.79	7.38
	2000	6.09	7.95	3.64	7.19	1.46	4.05	4.03	12.00
	2001	11.32	9.43	7.16	22.66	1.74	2.41	7.65	12.84
	2002	31.04	12.87	11.26	19.25	9.85	4.93	7.94	8.53
	2003	13.59	7.10	14.26	23.63	1.99	4.85	2.27	10.50

Table D.5: Standard Deviation of Yield Spread Residuals for Annual Estimation

Correlations of Factor-implied Yield Spread Residuals					Correlations of Asset-implied Yield Spread Residuals				
ECY-A	**T=1**	**T=3**	**T=5**	**T=10**	**ECY-A**	**T=1**	**T=3**	**T=5**	**T=10**
T=1	100.0	78.4	37.3	-0.2	**T=1**	100.0	26.0	-14.2	-1.1
T=3		100.0	82.4	3.9	**T=3**		100.0	-1.5	-52.8
T=5			100.0	30.7	**T=5**			100.0	-16.7
T=10				100.0	**T=10**				100.0
ECY-BBB	**T=1**	**T=3**	**T=5**	**T=10**	**ECY-BBB**	**T=1**	**T=3**	**T=5**	**T=10**
T=1	100.0	33.9	-5.8	-30.8	**T=1**	100.0	41.3	-3.8	-16.6
T=3		100.0	80.9	-7.2	**T=3**		100.0	20.2	-2.4
T=5			100.0	41.3	**T=5**			100.0	18.0
T=10				100.0	**T=10**				100.0
FIN-AA	**T=1**	**T=3**	**T=5**	**T=10**	**FIN-AA**	**T=1**	**T=3**	**T=5**	**T=10**
T=1	100.0	80.4	44.6	30.1	**T=1**	100.0	-7.4	19.5	-0.1
T=3		100.0	87.2	42.5	**T=3**		100.0	-26.2	-10.6
T=5			100.0	52.6	**T=5**			100.0	-28.8
T=10				100.0	**T=10**				100.0
FIN-A	**T=1**	**T=3**	**T=5**	**T=10**	**FIN-A**	**T=1**	**T=3**	**T=5**	**T=10**
T=1	100.0	81.8	54.0	25.9	**T=1**	100.0	3.6	8.7	10.1
T=3		100.0	90.8	39.1	**T=3**		100.0	0.4	-15.9
T=5			100.0	52.1	**T=5**			100.0	-34.9
T=10				100.0	**T=10**				100.0
LCY-AA	**T=1**	**T=3**	**T=5**	**T=10**	**LCY-AA**	**T=1**	**T=3**	**T=5**	**T=10**
T=1	100.0	72.9	8.1	-21.4	**T=1**	100.0	15.8	5.7	2.5
T=3		100.0	69.6	-17.2	**T=3**		100.0	-39.9	-47.1
T=5			100.0	15.5	**T=5**			100.0	54.4
T=10				100.0	**T=10**				100.0
LCY-A	**T=1**	**T=3**	**T=5**	**T=10**	**LCY-A**	**T=1**	**T=3**	**T=5**	**T=10**
T=1	100.0	13.7	-29.5	-45.3	**T=1**	100.0	57.4	44.3	44.4
T=3		100.0	83.1	-2.6	**T=3**		100.0	59.1	83.6
T=5			100.0	34.4	**T=5**			100.0	44.1
T=10				100.0	**T=10**				100.0
LCY-BBB	**T=1**	**T=3**	**T=5**	**T=10**	**LCY-BBB**	**T=1**	**T=3**	**T=5**	**T=10**
T=1	100.0	70.6	19.4	-32.1	**T=1**	100.0	13.8	4.4	10.4
T=3		100.0	75.9	-9.3	**T=3**		100.0	20.9	22.1
T=5			100.0	42.4	**T=5**			100.0	-52.5
T=10				100.0	**T=10**				100.0
NCY-AA	**T=1**	**T=3**	**T=5**	**T=10**	**NCY-AA**	**T=1**	**T=3**	**T=5**	**T=10**
T=1	100.0	75.8	25.0	-7.7	**T=1**	100.0	8.6	6.6	10.9
T=3		100.0	77.9	3.1	**T=3**		100.0	-20.4	5.1
T=5			100.0	31.7	**T=5**			100.0	-31.5
T=10				100.0	**T=10**				100.0
NCY-A	**T=1**	**T=3**	**T=5**	**T=10**	**NCY-A**	**T=1**	**T=3**	**T=5**	**T=10**
T=1	100.0	76.1	14.2	4.2	**T=1**	100.0	-7.2	34.4	-14.8
T=3		100.0	69.7	-11.4	**T=3**		100.0	-25.2	0.3
T=5			100.0	4.7	**T=5**			100.0	-43.8
T=10				100.0	**T=10**				100.0
NCY-BBB	**T=1**	**T=3**	**T=5**	**T=10**	**NCY-BBB**	**T=1**	**T=3**	**T=5**	**T=10**
T=1	100.0	58.2	30.9	-6.3	**T=1**	100.0	4.5	18.4	7.6
T=3		100.0	87.1	16.9	**T=3**		100.0	49.9	46.4
T=5			100.0	52.6	**T=5**			100.0	38.2
T=10				100.0	**T=10**				100.0
NF-AA	**T=1**	**T=3**	**T=5**	**T=10**	**NF-AA**	**T=1**	**T=3**	**T=5**	**T=10**
T=1	100.0	72.2	19.2	-13.7	**T=1**	100.0	14.7	-22.9	-3.4
T=3		100.0	76.5	-16.5	**T=3**		100.0	-2.1	-45.3
T=5			100.0	12.2	**T=5**			100.0	-21.4
T=10				100.0	**T=10**				100.0
NF-A	**T=1**	**T=3**	**T=5**	**T=10**	**NF-A**	**T=1**	**T=3**	**T=5**	**T=10**
T=1	100.0	79.9	30.4	-2.2	**T=1**	100.0	6.1	29.2	24.9
T=3		100.0	74.1	-2.0	**T=3**		100.0	26.7	48.6
T=5			100.0	34.5	**T=5**			100.0	32.7
T=10				100.0	**T=10**				100.0
NF-BBB	**T=1**	**T=3**	**T=5**	**T=10**	**NF-BBB**	**T=1**	**T=3**	**T=5**	**T=10**
T=1	100.0	56.4	-4.3	-43.1	**T=1**	100.0	13.1	20.4	11.9
T=3		100.0	70.6	-8.5	**T=3**		100.0	20.1	11.1
T=5			100.0	51.1	**T=5**			100.0	1.4
T=10				100.0	**T=10**				100.0
AA	**T=1**	**T=3**	**T=5**	**T=10**	**AA**	**T=1**	**T=3**	**T=5**	**T=10**
T=1	100.0	82.1	48.4	27.2	**T=1**	100.0	20.0	-5.6	10.5
T=3		100.0	87.9	37.5	**T=3**		100.0	-5.5	-25.9
T=5			100.0	48.4	**T=5**			100.0	-21.3
T=10				100.0	**T=10**				100.0
A	**T=1**	**T=3**	**T=5**	**T=10**	**A**	**T=1**	**T=3**	**T=5**	**T=10**
T=1	100.0	70.8	35.5	17.1	**T=1**	100.0	12.6	-3.9	6.9
T=3		100.0	88.8	30.0	**T=3**		100.0	21.6	-24.2
T=5			100.0	45.4	**T=5**			100.0	24.8
T=10				100.0	**T=10**				100.0
BBB	**T=1**	**T=3**	**T=5**	**T=10**	**BBB**	**T=1**	**T=3**	**T=5**	**T=10**
T=1	100.0	41.6	-16.6	-45.8	**T=1**	100.0	1.8	-4.6	20.3
T=3		100.0	73.7	-9.1	**T=3**		100.0	34.3	-11.4
T=5			100.0	44.8	**T=5**			100.0	-10.9
T=10				100.0	**T=10**				100.0

Table D.6: Inner-Class Correlations of Yield Spread Residuals

	Correlations of Empirical Risk class Spreads					Correlations of Bootstrapped Spreads			
Correlations of Factor-implied Spreads	**τ=1**	**AA**	**A**	**BBB**	Correlations of Asset-implied Spreads	**τ=1**	**AA**	**A**	**BBB**
	AA	100.0	75.5	7.4		**AA**	100.0	4.6	9.2
	A	63.4	100.0	11.5		**A**	6.5	100.0	5.6
	BBB	10.2	22.7	100.0		**BBB**	8.4	3.7	100.0
	τ=3	**AA**	**A**	**BBB**		**τ=3**	**AA**	**A**	**BBB**
	AA	100.0	76.3	26.2		**AA**	100.0	11.0	23.6
	A	74.7	100.0	24.7		**A**	6.2	100.0	1.5
	BBB	17.4	27.8	100.0		**BBB**	24.4	1.8	100.0
	τ=5	**AA**	**A**	**BBB**		**τ=5**	**AA**	**A**	**BBB**
	AA	100.0	72.6	32.4		**AA**	100.0	16.5	7.1
	A	76.3	100.0	30.1		**A**	15.9	100.0	13.0
	BBB	20.1	29.5	100.0		**BBB**	6.8	11.1	100.0
	τ=10	**AA**	**A**	**BBB**		**τ=10**	**AA**	**A**	**BBB**
	AA	100.0	69.9	16.9		**AA**	100.0	0.1	-9.7
	A	78.0	100.0	18.6		**A**	0.4	100.0	-0.1
	BBB	24.3	32.4	100.0		**BBB**	-5.0	-2.9	100.0

Table D.7: Rating-Class Model Yield Spread Correlations

	Correlations of Empirical Risk class Spreads							Correlations of Bootstrapped Spreads					
Correlations of Factor-implied Spreads	**τ=1**	**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**	Correlations of Asset-implied Spreads	**τ=1**	**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**
	FIN-AA	100.0	77.4	34.0	51.6	12.2		**FIN-AA**	100.0	23.1	5.6	1.6	5.9
	FIN-A	81.7	100.0	31.9	45.0	18.0		**FIN-A**	21.7	100.0	6.8	16.6	7.9
	NF-AA	36.3	30.2	100.0	44.5	5.9		**NF-AA**	5.7	5.8	100.0	9.3	3.7
	NF-A	31.9	24.9	33.4	100.0	24.3		**NF-A**	0.4	10.0	11.9	100.0	5.8
	NF-BBB	15.8	11.0	18.0	39.6	100.0		**NF-BBB**	2.9	-2.0	2.3	-0.6	100.0
	τ=3	**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**		**τ=3**	**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**
	FIN-AA	100.0	80.7	38.9	46.6	29.5		**FIN-AA**	100.0	16.4	9.4	8.5	0.2
	FIN-A	82.1	100.0	36.6	41.3	20.0		**FIN-A**	16.9	100.0	4.8	1.6	9.7
	NF-AA	43.8	35.6	100.0	48.6	17.3		**NF-AA**	7.0	4.6	100.0	8.5	12.1
	NF-A	43.7	36.1	46.2	100.0	42.4		**NF-A**	6.0	-6.0	0.4	100.0	-2.7
	NF-BBB	22.2	17.6	26.0	44.8	100.0		**NF-BBB**	1.0	9.5	13.9	2.3	100.0
	τ=5	**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**		**τ=5**	**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**
	FIN-AA	100.0	80.4	35.4	37.0	30.4		**FIN-AA**	100.0	17.9	8.0	4.6	11.5
	FIN-A	82.3	100.0	31.5	34.9	19.4		**FIN-A**	17.1	100.0	10.9	12.9	1.9
	NF-AA	44.8	36.4	100.0	45.9	21.9		**NF-AA**	6.3	6.8	100.0	4.9	-0.9
	NF-A	45.5	38.1	48.1	100.0	45.9		**NF-A**	2.3	18.7	-1.2	100.0	11.9
	NF-BBB	24.5	19.2	28.0	46.4	100.0		**NF-BBB**	8.7	-0.7	-3.6	5.0	100.0
	τ=10	**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**		**τ=10**	**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**
	FIN-AA	100.0	73.8	38.8	29.5	19.0		**FIN-AA**	100.0	3.7	0.6	10.4	-1.3
	FIN-A	83.2	100.0	32.9	25.6	14.9		**FIN-A**	13.7	100.0	14.4	11.8	11.5
	NF-AA	45.6	38.0	100.0	36.1	17.8		**NF-AA**	-0.7	9.6	100.0	6.5	14.8
	NF-A	46.3	40.1	50.2	100.0	23.9		**NF-A**	17.0	8.7	11.1	100.0	23.0
	NF-BBB	27.5	21.9	29.9	46.3	100.0		**NF-BBB**	0.3	15.0	15.0	21.8	100.0

Table D.8: Two-Sector Model Yield Spread Correlations

	Correlations of Empirical Risk Class Spreads									
Correlations of Factor-implied Spreads										
τ=1	**ECY-A**	**ECY-BBB**	**FIN-AA**	**FIN-A**	**LCY-AA**	**LCY-A**	**LCY-BBB**	**NCY-AA**	**NCY-A**	**NCY-BBB**
ECY-A	100.0	8.2	55.7	51.0	38.3	0.7	23.1	27.3	49.5	14.9
ECY-BBB	15.8	100.0	4.4	5.5	16.9	31.9	15.3	9.7	13.1	1.6
FIN-AA	26.4	12.4	100.0	77.4	30.7	-7.5	20.8	29.3	41.4	11.1
FIN-A	22.0	-3.3	81.7	100.0	33.2	-7.4	20.2	30.3	38.3	6.8
LCY-AA	47.4	16.5	21.1	19.6	100.0	0.0	7.5	19.5	27.4	7.3
LCY-A	8.9	23.1	2.5	1.6	1.6	100.0	25.1	2.6	3.9	-1.7
LCY-BBB	25.0	10.8	14.6	17.0	5.8	44.8	100.0	3.7	8.5	8.6
NCY-AA	28.9	22.9	39.4	27.1	16.5	9.9	14.9	100.0	61.8	5.0
NCY-A	38.7	34.6	41.5	37.3	32.3	23.5	42.8	61.1	100.0	13.0
NCY-BBB	7.9	6.0	4.9	6.2	5.5	3.5	11.4	9.8	19.2	100.0
τ=3	**ECY-A**	**ECY-BBB**	**FIN-AA**	**FIN-A**	**LCY-AA**	**LCY-A**	**LCY-BBB**	**NCY-AA**	**NCY-A**	**NCY-BBB**
ECY-A	100.0	30.4	49.7	47.4	45.1	17.8	24.4	26.8	56.4	17.1
ECY-BBB	26.3	100.0	16.4	7.0	22.2	36.6	32.1	9.7	22.8	17.3
FIN-AA	40.0	18.0	100.0	80.7	30.2	12.0	20.7	31.7	47.3	16.4
FIN-A	35.9	3.1	82.1	100.0	27.3	9.7	19.1	27.9	43.4	12.9
LCY-AA	51.4	27.2	30.9	27.6	100.0	5.1	8.5	26.0	27.6	8.6
LCY-A	17.2	27.8	9.0	6.3	5.5	100.0	30.3	12.3	21.8	7.5
LCY-BBB	35.4	18.9	24.8	27.0	9.3	45.3	100.0	4.0	20.7	18.9
NCY-AA	34.8	25.1	42.9	31.1	23.3	17.0	21.6	100.0	52.8	7.7
NCY-A	52.3	37.6	53.2	48.5	40.8	28.5	45.5	62.4	100.0	20.1
NCY-BBB	15.5	11.3	11.0	11.4	11.2	7.1	15.4	13.9	27.2	100.0
τ=5	**ECY-A**	**ECY-BBB**	**FIN-AA**	**FIN-A**	**LCY-AA**	**LCY-A**	**LCY-BBB**	**NCY-AA**	**NCY-A**	**NCY-BBB**
ECY-A	100.0	36.3	43.6	44.6	39.2	29.9	32.1	21.8	60.1	21.4
ECY-BBB	29.1	100.0	21.9	13.7	24.4	50.1	49.0	9.5	27.7	37.4
FIN-AA	42.4	20.3	100.0	80.4	28.0	18.3	24.0	28.7	47.0	17.3
FIN-A	38.6	5.5	82.3	100.0	25.1	17.1	23.0	20.2	44.7	12.4
LCY-AA	51.9	28.9	32.5	29.0	100.0	16.0	22.3	28.7	33.6	14.8
LCY-A	19.3	28.4	10.9	7.8	6.8	100.0	26.2	16.6	31.6	22.1
LCY-BBB	37.4	22.0	27.1	29.0	11.2	45.6	100.0	16.4	30.9	35.0
NCY-AA	35.8	26.0	43.1	31.6	24.6	18.9	23.0	100.0	43.5	12.2
NCY-A	54.7	38.6	54.7	50.2	41.9	30.2	46.2	62.1	100.0	25.8
NCY-BBB	17.2	13.9	12.8	12.7	12.7	8.5	17.0	14.9	28.9	100.0
τ=10	**ECY-A**	**ECY-BBB**	**FIN-AA**	**FIN-A**	**LCY-AA**	**LCY-A**	**LCY-BBB**	**NCY-AA**	**NCY-A**	**NCY-BBB**
ECY-A	100.0	16.4	37.2	39.2	33.8	42.7	33.3	32.8	62.0	23.3
ECY-BBB	31.6	100.0	28.4	23.3	16.2	12.6	43.7	14.4	13.9	40.1
FIN-AA	44.4	25.8	100.0	73.8	29.7	26.2	22.3	42.8	33.3	14.5
FIN-A	42.0	11.6	83.2	100.0	24.1	28.0	28.7	39.4	31.9	14.2
LCY-AA	54.7	28.3	34.4	31.6	100.0	21.6	21.9	37.9	18.1	18.0
LCY-A	23.1	24.8	13.3	11.2	9.1	100.0	24.6	20.6	44.1	-1.4
LCY-BBB	44.0	28.9	32.8	34.8	20.6	46.5	100.0	28.5	36.9	36.5
NCY-AA	37.8	27.9	43.4	33.4	26.4	21.2	29.7	100.0	30.1	13.1
NCY-A	58.3	40.2	54.7	51.3	46.1	32.7	50.0	62.1	100.0	19.1
NCY-BBB	21.0	22.4	15.3	14.4	15.9	12.3	23.8	16.2	31.1	100.0

Table D.9: Four-Sector Model Yield Spread Correlations of Risk classes

		Correlations of Bootstrapped Spreads									
Correlations of Asset-implied Spreads	**τ=1**	**ECY-A**	**ECY-BBB**	**FIN-AA**	**FIN-A**	**LCY-AA**	**LCY-A**	**LCY-BBB**	**NCY-AA**	**NCY-A**	**NCY-BBB**
	ECY-A	100.0	5.4	3.4	13.2	0.8	-0.1	2.8	9.1	-0.6	9.9
	ECY-BBB	5.0	100.0	1.3	9.5	8.6	22.0	-12.1	8.6	2.8	4.0
	FIN-AA	1.8	2.3	100.0	23.1	1.7	2.4	7.1	-1.1	11.6	-2.2
	FIN-A	15.7	10.9	21.7	100.0	8.1	-0.6	3.7	15.8	6.3	4.5
	LCY-AA	5.9	4.2	3.5	5.1	100.0	-14.1	3.9	3.8	10.5	7.3
	LCY-A	0.4	9.9	1.3	3.3	-13.4	100.0	26.5	9.9	4.9	1.3
	LCY-BBB	5.5	-14.8	4.2	-7.8	4.1	19.0	100.0	-0.5	2.5	5.9
	NCY-AA	7.9	7.6	-3.3	14.1	-1.4	2.4	3.0	100.0	3.6	4.1
	NCY-A	-3.5	2.9	8.7	2.2	4.3	0.8	1.8	-0.1	100.0	10.4
	NCY-BBB	8.2	2.1	0.8	-1.5	4.3	3.9	9.1	2.9	6.6	100.0
	τ=3	**ECY-A**	**ECY-BBB**	**FIN-AA**	**FIN-A**	**LCY-AA**	**LCY-A**	**LCY-BBB**	**NCY-AA**	**NCY-A**	**NCY-BBB**
	ECY-A	100.0	12.2	10.5	9.3	26.4	6.9	5.6	12.7	18.3	12.9
	ECY-BBB	10.5	100.0	3.1	8.8	2.6	12.4	20.3	12.1	13.4	7.6
	FIN-AA	10.7	7.9	100.0	16.4	-0.1	-4.0	1.8	19.0	14.1	5.7
	FIN-A	9.7	9.3	16.9	100.0	14.5	3.5	8.4	5.7	11.2	3.2
	LCY-AA	33.0	1.7	0.1	11.0	100.0	-6.3	0.0	5.7	9.4	14.0
	LCY-A	4.2	5.9	-1.4	3.8	-9.8	100.0	10.6	2.6	2.6	-10.9
	LCY-BBB	5.2	14.8	3.8	7.6	1.8	9.9	100.0	5.6	-10.2	6.1
	NCY-AA	12.4	10.8	19.3	3.4	0.9	-0.3	4.0	100.0	8.1	10.7
	NCY-A	19.1	13.1	8.4	5.7	6.2	0.9	-9.6	5.8	100.0	10.7
	NCY-BBB	15.3	5.9	4.8	0.3	19.8	-13.3	4.7	10.0	8.7	100.0
	τ=5	**ECY-A**	**ECY-BBB**	**FIN-AA**	**FIN-A**	**LCY-AA**	**LCY-A**	**LCY-BBB**	**NCY-AA**	**NCY-A**	**NCY-BBB**
	ECY-A	100.0	5.8	2.8	9.5	6.1	6.8	1.4	2.9	4.3	9.2
	ECY-BBB	7.1	100.0	0.1	34.0	24.2	17.5	-0.8	5.6	-2.5	14.0
	FIN-AA	0.0	1.5	100.0	17.9	-2.5	-0.7	-1.1	4.3	14.6	3.5
	FIN-A	6.2	31.8	17.1	100.0	14.4	11.0	-2.8	16.1	5.4	7.6
	LCY-AA	2.5	25.2	-8.5	12.9	100.0	10.6	2.2	8.4	15.6	12.4
	LCY-A	3.8	9.0	-2.9	6.4	3.8	100.0	14.1	5.9	0.0	7.8
	LCY-BBB	4.2	-2.3	4.1	-1.4	3.6	19.9	100.0	-7.0	9.6	4.4
	NCY-AA	5.6	7.2	7.5	14.2	11.9	7.2	-6.9	100.0	14.0	5.1
	NCY-A	15.1	-5.5	9.6	7.0	7.1	3.4	11.9	19.5	100.0	11.3
	NCY-BBB	8.7	11.7	0.4	0.4	8.6	1.8	2.9	0.2	16.2	100.0
	τ=10	**ECY-A**	**ECY-BBB**	**FIN-AA**	**FIN-A**	**LCY-AA**	**LCY-A**	**LCY-BBB**	**NCY-AA**	**NCY-A**	**NCY-BBB**
	ECY-A	100.0	6.7	9.3	0.0	21.7	9.2	3.8	-1.3	22.0	10.6
	ECY-BBB	8.9	100.0	19.4	-8.8	6.1	12.4	-5.8	-0.4	8.2	-1.2
	FIN-AA	10.1	12.6	100.0	3.7	4.0	0.3	-6.9	2.6	23.6	-6.0
	FIN-A	2.5	-2.8	13.7	100.0	-5.3	6.8	10.0	16.8	6.4	5.5
	LCY-AA	20.6	7.8	-4.4	-5.5	100.0	0.2	-0.4	6.7	10.6	9.6
	LCY-A	14.2	28.9	-0.3	6.5	1.6	100.0	6.7	2.9	4.1	0.1
	LCY-BBB	4.1	-2.9	-0.1	13.2	5.6	-1.7	100.0	12.4	10.2	1.4
	NCY-AA	-3.4	1.4	2.0	11.9	4.5	1.6	13.8	100.0	4.3	2.1
	NCY-A	16.7	4.2	12.7	8.9	7.2	0.6	13.0	5.3	100.0	15.8
	NCY-BBB	11.6	-0.3	-6.4	11.4	12.6	-0.5	8.6	0.4	12.7	100.0

Table D.10: Four-Sector Model Yield Spread Correlations of Assets

	Factor Correlations				Asset Correlations				Betas		
1999	**AA**	**A**	**BBB**		**AA**	**A**	**BBB**		F_1	F_2	F_3
AA	100.0	82.9	60.4	100.0	9.8	7.6	9.4	14.3	31.4		
A		100.0	61.9	68.4		8.5	8.9	8.6	24.1	16.3	
BBB			100.0				24.6		29.9	10.5	38.1
2000	**AA**	**A**	**BBB**		**AA**	**A**	**BBB**		F_1	F_2	F_3
AA	100.0	87.4	39.0	100.0	9.0	13.8	9.0	32.0	29.9		
A		100.0	43.8	56.7		27.9	17.8	13.5	46.1	25.7	
BBB			100.0				59.2		30.0	15.4	69.2
2001	**AA**	**A**	**BBB**		**AA**	**A**	**BBB**		F_1	F_2	F_3
AA	100.0	76.1	49.4	100.0	23.9	18.1	20.0	38.6	48.9		
A		100.0	69.8	65.1		23.7	28.1	22.1	37.1	31.6	
BBB			100.0				68.3		40.8	41.0	59.0
2002	**AA**	**A**	**BBB**		**AA**	**A**	**BBB**		F_1	F_2	F_3
AA	100.0	79.8	27.8	100.0	8.9	7.3	3.1	10.7	29.9		
A		100.0	41.0	49.5		9.5	4.7	5.0	24.6	18.6	
BBB			100.0				13.8		10.3	11.6	33.8
2003	**AA**	**A**	**BBB**		**AA**	**A**	**BBB**		F_1	F_2	F_3
AA	100.0	59.3	3.8	100.0	16.3	6.9	0.5	11.5	40.3		
A		100.0	24.2	29.1		8.3	2.2	3.2	17.1	23.2	
BBB			100.0				10.0		1.2	8.6	30.3

Table D.11: Credit Dependence of Rating-Class Model with Annual Estimation

	Factor Correlations						Asset Correlations						Betas				
1999	**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**		**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**		F_1	F_2	F_3	F_4	F_5
FIN-AA	100.0	80.0	37.5	62.7	45.3	100.0	5.9	5.5	2.6	10.3	3.3	15.3	24.4				
FIN-A		100.0	40.1	59.0	42.9	48.7		7.9	3.2	11.1	3.6	6.1	22.5	16.8			
NF-AA			100.0	42.7	23.1				8.3	8.3	2.0		10.8	4.9	26.3		
NF-A				100.0	53.3					45.3	10.8		42.2	9.9	12.3	50.0	
NF-BBB					100.0						9.1		13.6	3.3	1.4	9.1	25.0
2000	**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**		**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**		F_1	F_2	F_3	F_4	F_5
FIN-AA	100.0	88.6	52.2	72.7	41.9	100.0	8.3	10.1	8.6	14.7	9.4	33.6	28.7				
FIN-A		100.0	46.8	69.7	41.1	58.7		15.7	10.7	19.5	12.7	16.6	35.2	18.4			
NF-AA			100.0	78.7	42.1				33.1	31.9	18.9		30.0	0.7	49.0		
NF-A				100.0	53.4					49.8	29.4		51.3	8.0	33.6	34.0	
NF-BBB					100.0						60.9		32.7	6.7	18.5	17.4	65.9
2001	**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**		**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**		F_1	F_2	F_3	F_4	F_5
FIN-AA	100.0	87.4	36.0	44.1	38.9	100.0	19.6	18.9	9.6	11.9	12.2	33.5	44.2				
FIN-A		100.0	41.0	45.0	47.8	51.0		23.8	12.0	13.4	16.6	16.5	42.6	23.7			
NF-AA			100.0	42.3	65.1				36.1	15.6	27.8		21.6	11.7	54.8		
NF-A				100.0	62.6					37.5	27.2		27.0	8.2	16.0	52.0	
NF-BBB					100.0						50.4		27.6	20.2	35.4	24.0	45.1
2002	**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**		**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**		F_1	F_2	F_3	F_4	F_5
FIN-AA	100.0	75.4	57.7	62.2	30.0	100.0	2.4	1.7	6.2	2.9	1.4	14.2	15.4				
FIN-A		100.0	45.2	59.7	26.6	52.4		2.1	4.5	2.6	1.1	4.9	10.9	9.5			
NF-AA			100.0	65.6	45.1				48.6	13.8	9.3		40.2	1.7	56.9		
NF-A				100.0	56.8					9.2	5.1		18.8	5.9	10.8	20.2	
NF-BBB					100.0						8.8		8.9	1.8	10.1	11.1	24.0
2003	**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**		**FIN-AA**	**FIN-A**	**NF-AA**	**NF-A**	**NF-BBB**		F_1	F_2	F_3	F_4	F_5
FIN-AA	100.0	66.4	27.9	11.7	-1.5	100.0	20.0	10.3	4.4	4.0	-0.2	22.1	44.7				
FIN-A		100.0	0.5	16.1	3.4	24.2		12.1	0.1	4.3	0.3	4.7	23.1	26.0			
NF-AA			100.0	30.5	23.3				12.3	8.2	2.2		9.8	-8.5	32.6		
NF-A				100.0	63.8					58.5	13.3		8.9	8.5	24.6	71.4	
NF-BBB					100.0						7.5		-0.4	1.6	7.4	16.0	20.8

Table D.12: Credit Dependence of Two-Sector Model with Annual Estimation

Factor Correlations

	ECY-A	ECY-BBB	FIN-AA	FIN-A	LCY-AA	LCY-A	LCY-BBB	NCY-AA	NCY-A	NCY-BBB	
1999											
ECY-A	100.0	53.3	62.7	59.0	42.7	100.0	53.3	42.7	100.0	53.3	100.0
ECY-BBB		100.0	45.3	42.9	23.1	53.3	100.0	23.1	53.3	100.0	54.2
FIN-AA			100.0	80.0	37.5	62.7	45.3	37.5	62.7	45.3	
FIN-A				100.0	40.1	59.0	42.9	40.1	59.0	42.9	
LCY-AA					100.0	42.7	23.1	100.0	42.7	23.1	
LCY-A						100.0	53.3	42.7	100.0	53.3	
LCY-BBB							100.0	23.1	53.3	100.0	
NCY-AA								100.0	42.7	23.1	
NCY-A									100.0	53.3	
NCY-BBB										100.0	
2000											
ECY-A	100.0	41.7	82.6	85.4	50.8	65.8	23.4	42.1	68.4	46.8	100.0
ECY-BBB		100.0	33.8	27.8	43.7	40.6	4.4	14.0	33.3	33.3	42.0
FIN-AA			100.0	88.6	61.2	50.1	21.7	42.0	64.9	38.0	
FIN-A				100.0	52.9	54.7	35.4	32.0	61.0	38.5	
LCY-AA					100.0	46.7	9.9	18.9	43.2	23.4	
LCY-A						100.0	17.6	35.2	53.5	36.1	
LCY-BBB							100.0	19.4	32.9	22.6	
NCY-AA								100.0	69.2	35.0	
NCY-A									100.0	47.5	
NCY-BBB										100.0	
2001											
ECY-A	100.0	62.5	38.8	40.6	55.3	53.8	70.7	41.7	56.4	54.3	100.0
ECY-BBB		100.0	30.4	48.0	68.0	61.8	68.1	23.3	68.4	45.2	53.0
FIN-AA			100.0	87.4	39.0	26.9	44.1	35.1	49.0	59.3	
FIN-A				100.0	51.0	36.5	52.9	40.9	54.8	57.4	
LCY-AA					100.0	67.5	72.8	32.4	73.3	52.2	
LCY-A						100.0	72.7	47.3	63.6	48.9	
LCY-BBB							100.0	46.6	74.5	65.4	
NCY-AA								100.0	49.8	30.9	
NCY-A									100.0	65.4	
NCY-BBB										100.0	
2002											
ECY-A	100.0	64.3	71.4	58.7	24.7	32.6	56.0	82.6	73.3	74.6	100.0
ECY-BBB		100.0	37.8	16.1	34.1	26.4	31.1	63.1	52.7	43.5	44.8
FIN-AA			100.0	75.4	25.2	15.2	40.9	65.1	43.6	70.5	
FIN-A				100.0	-0.6	25.2	44.4	55.4	48.1	66.1	
LCY-AA					100.0	-3.7	-11.8	17.1	7.0	24.0	
LCY-A						100.0	61.2	32.0	47.6	40.6	
LCY-BBB							100.0	43.9	60.4	54.1	
NCY-AA								100.0	75.3	78.5	
NCY-A									100.0	72.2	
NCY-BBB										100.0	
2003											
ECY-A	100.0	68.8	17.1	17.3	45.1	58.3	60.1	34.5	55.5	34.8	100.0
ECY-BBB		100.0	3.5	3.3	45.0	73.2	60.8	13.6	58.1	51.4	29.6
FIN-AA			100.0	66.4	9.7	18.3	8.0	28.8	18.6	-4.6	
FIN-A				100.0	-3.9	22.1	6.0	11.5	4.0	0.3	
LCY-AA					100.0	29.8	42.3	17.9	30.2	13.2	
LCY-A						100.0	49.8	7.9	38.3	42.4	
LCY-BBB							100.0	15.8	42.6	19.2	
NCY-AA								100.0	58.3	-0.1	
NCY-A									100.0	38.6	
NCY-BBB										100.0	

Asset Correlations

	ECY-A	ECY-BBB	FIN-AA	FIN-A	LCY-AA	LCY-A	LCY-BBB	NCY-AA	NCY-A	NCY-BBB	
1999											
ECY-A	45.3	10.8	10.3	11.1	8.3	45.3	10.8	8.3	45.3	10.8	19.4
ECY-BBB		9.1	3.3	3.6	2.0	10.8	9.1	2.0	10.8	9.1	9.6
FIN-AA			5.9	5.5	2.6	10.3	3.3	2.6	10.3	3.3	
FIN-A				7.9	3.2	11.1	3.6	3.2	11.1	3.6	
LCY-AA					8.3	8.3	2.0	8.3	8.3	2.0	
LCY-A						45.3	10.8	8.3	45.3	10.8	
LCY-BBB							9.1	2.0	10.8	9.1	
NCY-AA								8.3	8.3	2.0	
NCY-A									45.3	10.8	
NCY-BBB										9.1	
2000											
ECY-A	31.5	17.3	13.3	19.0	14.9	29.3	9.5	16.7	32.1	21.2	43.8
ECY-BBB		54.7	7.2	8.1	16.9	23.9	2.3	7.4	20.5	19.8	15.8
FIN-AA			8.3	10.1	9.2	11.4	4.5	8.6	15.6	8.8	
FIN-A				15.7	11.0	17.2	10.2	9.0	20.2	12.3	
LCY-AA					27.5	19.4	3.8	7.0	18.9	9.9	
LCY-A						63.1	10.2	19.8	35.5	23.1	
LCY-BBB							52.6	10.0	19.9	13.2	
NCY-AA								50.4	41.0	20.0	
NCY-A									69.8	32.0	
NCY-BBB										64.9	
2001											
ECY-A	19.7	19.7	7.6	8.8	11.9	13.3	19.1	11.9	17.7	17.5	34.8
ECY-BBB		50.2	9.5	16.6	23.4	24.3	29.4	10.6	34.2	23.2	17.9
FIN-AA			19.6	18.9	8.4	6.6	11.9	10.0	15.3	19.0	
FIN-A				23.8	12.1	9.9	15.7	12.8	18.8	20.3	
LCY-AA					23.6	18.2	21.5	10.1	25.1	18.3	
LCY-A						31.0	24.6	16.9	25.0	19.7	
LCY-BBB							37.1	18.2	32.0	28.8	
NCY-AA								41.2	22.5	14.3	
NCY-A									49.7	33.4	
NCY-BBB										52.4	
2002											
ECY-A	60.2	15.7	8.6	6.6	9.5	3.6	30.2	46.1	46.1	35.7	30.5
ECY-BBB		9.9	1.8	0.7	5.3	1.2	6.8	14.3	13.5	8.5	11.7
FIN-AA			2.4	1.7	1.9	0.3	4.4	7.2	5.5	6.7	
FIN-A				2.1	0.0	0.5	4.5	5.8	5.6	5.9	
LCY-AA					24.4	-0.3	-4.1	6.1	2.8	7.3	
LCY-A						2.0	6.0	3.3	5.5	3.6	
LCY-BBB							48.3	22.0	34.1	23.2	
NCY-AA								51.8	44.0	34.9	
NCY-A									65.8	36.2	
NCY-BBB										38.2	
2003											
ECY-A	54.7	33.1	5.7	4.5	24.6	37.7	23.5	7.7	32.3	8.0	36.9
ECY-BBB		42.3	1.0	0.7	21.6	41.7	20.9	2.7	29.8	10.4	11.5
FIN-AA			20.0	10.3	3.2	7.2	1.9	3.9	6.6	-0.6	
FIN-A				12.1	-1.0	6.7	1.1	1.2	1.1	0.0	
LCY-AA					54.4	19.2	16.5	4.0	17.5	3.0	
LCY-A						76.7	23.0	2.1	26.5	11.5	
LCY-BBB							28.0	2.5	17.8	3.2	
NCY-AA								9.0	13.8	0.0	
NCY-A									62.2	9.4	
NCY-BBB										9.6	

Abstract Factor Coefficients

	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}
1999										
ECY-A	42.2	9.9	12.3	50.0						
ECY-BBB	13.6	3.3	1.4	9.1	25.0					
FIN-AA	24.4									
FIN-A	22.5	16.8								
LCY-AA	10.8	4.9	26.3							
LCY-A	42.2	9.9	12.3	50.0						
LCY-BBB	13.6	3.3	1.4	9.1	25.0					
NCY-AA	10.8	4.9	26.3							
NCY-A	42.2	9.9	12.3	50.0						
NCY-BBB	13.6	3.3	1.4	9.1	25.0					
2000										
ECY-A	56.1									
ECY-BBB	30.8	67.2								
FIN-AA	23.7	-0.2	16.2							
FIN-A	33.9	-3.4	12.7	16.0						
LCY-AA	26.6	13.0	18.1	0.8	39.3					
LCY-A	52.3	11.6	-5.9	4.3	12.8	56.8				
LCY-BBB	17.0	-4.3	2.9	24.5	-2.3	2.1	65.8			
NCY-AA	29.9	-2.7	9.0	-14.6	-5.3	11.2	11.8	59.6		
NCY-A	57.2	4.4	12.5	-3.7	2.3	10.1	16.4	32.7	45.6	
NCY-BBB	37.7	12.2	-0.8	0.4	-4.1	4.3	10.7	12.2	8.6	67.4
2001										
ECY-A	44.4									
ECY-BBB	44.3	55.3								
FIN-AA	17.2	3.5	40.6							
FIN-A	19.8	14.1	36.9	20.8						
LCY-AA	26.8	20.8	7.5	5.1	33.5					
LCY-A	29.9	20.1	1.9	2.1	17.3	38.7				
LCY-BBB	43.1	18.6	9.5	5.1	15.3	13.2	31.3			
NCY-AA	26.7	-2.3	13.5	13.9	5.0	20.5	5.3	50.5		
NCY-A	39.8	29.9	18.2	0.0	20.3	8.3	10.7	13.6	37.1	
NCY-BBB	39.3	10.4	29.2	1.3	10.0	9.2	14.0	-6.3	15.7	45.4
2002										
ECY-A	77.6									
ECY-BBB	20.2	24.1								
FIN-AA	11.0	-1.7	10.7							
FIN-A	8.5	-4.1	6.4	8.9						
LCY-AA	12.2	11.7	7.2	-11.8	44.3					
LCY-A	4.6	1.0	-1.5	2.9	-1.1	12.9				
LCY-BBB	38.9	-4.5	0.2	10.8	-15.8	29.4	45.4			
NCY-AA	59.4	9.3	7.8	6.8	-4.6	2.3	-6.4	37.2		
NCY-A	59.4	5.9	-9.3	16.1	-5.8	15.5	8.8	20.6	43.5	
NCY-BBB	46.1	-3.6	14.8	10.1	5.1	11.2	3.3	16.7	8.2	29.2
2003										
ECY-A	73.9									
ECY-BBB	44.7	47.2								
FIN-AA	7.7	-5.1	43.8							
FIN-A	6.0	-4.1	22.1	25.9						
LCY-AA	33.2	14.2	3.1	-12.0	63.1					
LCY-A	51.0	40.0	12.1	10.2	-4.1	56.5				
LCY-BBB	31.8	14.2	0.4	-1.2	6.0	2.6	39.3			
NCY-AA	10.4	-4.2	6.5	-4.0	0.7	-3.3	-0.6	26.5		
NCY-A	43.7	21.7	9.8	-10.8	-2.7	-8.3	2.5	33.4	49.2	
NCY-BBB	10.8	11.7	-2.0	1.2	-3.2	2.3	-4.6	-1.5	6.5	24.9

Table D.13: Credit Dependence of Four-Sector Model with Annual Estimation

Appendix E

Conditional Default Probability

The conditional probability, that an asset value with dynamics according to (3.3) and value $V_t = V$ at time t has hit barrier $\overline{V}$ at time $\tau \le t$, is defined as $P[\tau \le t | V_t = V]$. The closed-form formula of $P[\tau \le t | V_t = V]$ is derived by a convergence consideration. For continuous V_t, the probability of an early default conditional on asset value $V_t \in (V - \Delta V, V + \Delta V]$ is defined by

$$P[\tau \le t | V - \Delta V < V_t \le V + \Delta V] = \frac{P[\tau \le t, V - \Delta V < V_t \le V + \Delta V]}{P[V - \Delta V < V_t \le V + \Delta V]}.$$

By use of the integral form of distributions $F_{V,\tau}(V,t) = P[V_t \le V, \tau \le t] = \int_0^t \int_0^V f_{V,\tau}(v,u) dv \; du$ with density $f_{V,\tau}(V,t)$, and $F_V(V,t) = P[V_t \le V] = \int_0^V f_V(v,t) dv$ with log-normal density $f_V(V,t)$ as defined in (3.6) and (3.5), it results the limit $P[\tau \le t | V_t = V]$, according to the rule of l'Hôspital:

$$\lim_{\Delta V \to 0} P[\tau \le t | V - \Delta V < V_t \le V + \Delta V] = \lim_{\Delta V \to 0} \frac{\int\limits_0^t \int\limits_{V-\Delta V}^{V+\Delta V} f_{V,\tau}(v,u) dv \; du}{\int\limits_{V-\Delta V}^{V+\Delta V} f_V(v,t) dv} = \frac{\int\limits_0^t f_{V,\tau}(V,u) du}{f_V(V,t)},$$

A close-form solution of $P[\tau \le t | V_t = V] = \int_0^t \frac{f(V,u)}{f_V(V,t)} du$ requires to express $\int_0^t f(u,V) du$ explicitly. Using the law of total probability, it is

$$\int\limits_0^t f_{V,\tau}(V,u) du = f_V(V,t) - \int\limits_t^\infty \overline{f}_{V,\tau}(V,u) du, .$$

With $\overline{f}_{V,\tau}(V,t)$ defined by (3.15), it results a closed-form solution for the probability

$$P[\tau \leq t | V_t = V] = \frac{\int\limits_0^t f_{V,\tau}(u,V)\ du}{f_V(V,t)} = \frac{f_V(V,t) - \int\limits_t^\infty \overline{f}_{V,\tau}(u,V)\ du}{f_V(V,t)},$$

of an early default in the interval $(0,t]$ conditional on $V_t = V$. [q.e.d.].

Appendix F

Simulation Results

Distribution Type		$\widehat{\mathbf{p}}$			$\mathbf{L}(\mathbb{E}[\mathbf{D_1}])$			$\mathbf{L}(\mathbf{D_0})$			$\mathbf{D_1}$		
Portfolio Type	$\rho^{\mathbf{a}}_{\mathbf{i,j}}$	$q_{0.01}$	$q_{0.5}$	$q_{0.99}$	$q_{0.01}$	$q_{0.5}$	$q_{0.99}$	$q_{0.01}$	$q_{0.5}$	$q_{0.99}$	$q_{0.01}$	$q_{0.5}$	$q_{0.99}$
Homogenous Portfolio	0%	0.33	1.00	1.89	2.99	3.54	4.15	1.91	2.40	2.96	97.62	97.05	96.42
	5%	0.00	0.89	3.33	1.35	3.39	7.21	0.74	2.26	5.46	99.62	97.20	93.08
	10%	0.00	0.67	4.89	0.83	3.22	9.26	0.40	2.10	7.26	100.30	97.35	90.93
	15%	0.00	0.56	6.22	0.52	3.04	10.97	0.22	1.94	8.77	100.75	97.52	89.18
	20%	0.00	0.44	7.56	0.33	2.89	12.56	0.12	1.79	10.21	101.05	97.67	87.53
	25%	0.00	0.33	8.89	0.21	2.73	14.15	0.07	1.64	11.66	101.27	97.82	85.90
	30%	0.00	0.22	10.33	0.13	2.56	15.68	0.03	1.48	13.11	101.44	97.98	84.36
Diversified Portfolio	0% - 0%	0.33	1.00	1.89	0.70	1.58	2.89	1.54	2.53	3.92	98.96	97.91	96.47
	5% - 0%	0.11	0.89	2.67	0.38	1.53	3.67	1.06	2.47	4.88	99.57	97.97	95.41
	5% - 5%	0.00	0.89	3.33	0.22	1.47	4.29	0.81	2.41	5.62	99.91	98.03	94.59
	10% - 0%	0.00	0.89	3.56	0.21	1.47	4.41	0.80	2.41	5.73	99.92	98.02	94.50
	10% - 5%	0.00	0.78	4.00	0.11	1.42	5.02	0.63	2.36	6.45	100.16	98.07	93.74
	15% - 0%	0.00	0.78	4.33	0.08	1.40	5.09	0.60	2.34	6.50	100.19	98.09	93.69
	10% - 10%	0.00	0.78	4.67	0.02	1.36	5.63	0.48	2.29	7.14	100.37	98.13	93.01
	15% - 5%	0.00	0.67	4.78	0.03	1.35	5.64	0.49	2.29	7.14	100.36	98.14	93.02
	20% - 0%	0.00	0.67	5.22	0.01	1.32	5.84	0.47	2.26	7.32	100.37	98.17	92.87
	15% - 10%	0.00	0.67	5.22	-0.02	1.29	6.13	0.40	2.22	7.71	100.51	98.20	92.41
	20% - 5%	0.00	0.67	5.67	-0.04	1.28	6.32	0.37	2.21	7.86	100.54	98.21	92.27
	25% - 0%	0.00	0.56	6.11	-0.05	1.26	6.51	0.36	2.19	8.06	100.53	98.23	92.10
	15% - 15%	0.00	0.56	5.89	-0.07	1.24	6.74	0.31	2.17	8.38	100.65	98.25	91.73
	20% - 10%	0.00	0.56	6.00	-0.07	1.23	6.77	0.31	2.16	8.39	100.65	98.26	91.71
	25% - 5%	0.00	0.56	6.44	-0.08	1.22	6.99	0.29	2.15	8.61	100.67	98.27	91.53
	30% - 0%	0.00	0.56	7.11	-0.09	1.19	7.20	0.28	2.12	8.80	100.67	98.29	91.36
	20% - 15%	0.00	0.56	6.56	-0.09	1.17	7.23	0.25	2.10	8.92	100.75	98.31	91.15
	25% - 10%	0.00	0.56	7.00	-0.10	1.17	7.44	0.24	2.09	9.11	100.76	98.32	90.98
	30% - 5%	0.00	0.44	7.44	-0.11	1.13	7.66	0.22	2.07	9.28	100.78	98.34	90.86
	20% - 20%	0.00	0.44	7.22	-0.12	1.12	7.93	0.20	2.05	9.65	100.85	98.36	90.42
	25% - 15%	0.00	0.44	7.22	-0.12	1.10	7.84	0.20	2.03	9.55	100.84	98.38	90.53
	30% - 10%	0.00	0.44	7.78	-0.12	1.10	8.09	0.19	2.02	9.78	100.84	98.39	90.31
	25% - 20%	0.00	0.44	8.00	-0.14	1.06	8.48	0.16	1.99	10.27	100.93	98.42	89.78
	30% - 15%	0.00	0.44	8.22	-0.14	1.05	8.55	0.16	1.97	10.29	100.92	98.43	89.78
	25% - 25%	0.00	0.33	8.67	-0.15	1.01	9.10	0.12	1.92	10.91	101.01	98.48	89.13
	30% - 20%	0.00	0.33	8.89	-0.15	1.00	9.08	0.12	1.92	10.88	101.01	98.49	89.17
	30% - 25%	0.00	0.33	9.22	-0.16	0.95	9.46	0.09	1.86	11.30	101.07	98.54	88.76
	30% - 30%	0.00	0.33	9.78	-0.16	0.89	10.00	0.08	1.80	11.89	101.11	98.59	88.14

Table F.1: Quantiles of Homogenous and Diversified Portfolio

Interval Count		**h = 4**			**h = 12**			**Difference in %**		
Loss Type	$\rho^{\mathbf{a}}_{\mathbf{i,j}}$	$\bar{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$	$\bar{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$	$\bar{\mathbf{q}}_{\mathbf{0.05}}$	$\mathbf{q_{0.95}}$	$\mathbf{q_{0.995}}$
	0% - 0%		2.46	3.04		2.45	3.04		0.24	-0.12
	10% - 0%		3.33	4.87		3.30	4.81		1.13	1.22
	10% - 5%		3.61	5.56		3.61	5.59		-0.06	-0.47
	10% - 10%		3.90	6.36		3.89	6.32		0.15	0.61
	20% - 0%		4.04	6.56		4.01	6.60		0.78	-0.66
	20% - 10%		4.52	7.88		4.46	7.82		1.43	0.78
$\mathbf{L}(\mathbb{E}[\mathbf{D_1}])$	30% - 0%		4.67	8.33		4.64	8.41		0.72	-0.99
	20% - 20%		4.89	9.21		4.95	9.23		-1.25	-0.22
	30% - 10%		5.03	9.32		5.02	9.46		0.16	-1.47
	25% - 25%	1.47	5.38	10.66	1.48	5.39	10.63	-1.01	-0.05	0.25
	30% - 20%		5.49	10.74		5.41	10.73		1.41	0.02
	30% - 30%		5.78	11.92		5.79	12.24		-0.17	-2.63
	0% - 0%		3.47	4.09		3.47	4.08		0.19	0.20
	10% - 0%		4.56	6.25		4.51	6.18		1.20	1.07
	10% - 5%		4.90	7.05		4.90	7.07		-0.03	-0.34
	10% - 10%		5.25	7.90		5.23	7.88		0.23	0.31
	20% - 0%		5.38	8.06		5.33	8.11		1.02	-0.58
	20% - 10%		5.96	9.56		5.89	9.48		1.21	0.88
$\mathbf{L}(\mathbf{D_0})$	30% - 0%		6.08	9.90		6.06	10.00		0.29	-0.99
	20% - 20%		6.42	11.02		6.47	11.00		-0.83	0.19
	30% - 10%		6.54	11.09		6.53	11.25		0.16	-1.37
	25% - 25%	2.84	7.00	12.55	2.85	6.99	12.51	-0.54	0.04	0.32
	30% - 20%		7.10	12.58		7.02	12.65		1.10	-0.54
	30% - 30%		7.45	13.85		7.47	14.19		-0.20	-2.44

Table F.2: Simulation Intervals of Diversified Portfolio

Div. Case	Default Rate									Portfolio Value								
Corr.	**Min.**	$q_{0.01}$	$q_{0.5}$	$q_{0.99}$	**Max.**	μ	σ	**Skew**	**Kurt.**	**Min.**	$q_{0.01}$	$q_{0.5}$	$q_{0.99}$	**Max.**	μ	σ	**Skew**	**Kurt.**
0% - 0%	0.00	0.33	1.00	1.89	2.67	1.00	0.33	0.31	0.07	95.28	96.47	97.91	98.96	99.55	97.88	0.54	-0.39	0.19
5% - 0%	0.00	0.11	0.89	2.67	5.44	1.00	0.55	1.07	2.06	91.90	95.41	97.97	99.57	100.32	97.87	0.90	-0.63	0.69
5% - 5%	0.00	0.00	0.89	3.33	8.00	1.00	0.69	1.51	4.04	90.55	94.59	98.03	99.91	100.84	97.88	1.15	-0.77	0.97
10% - 0%	0.00	0.00	0.89	3.56	8.22	1.00	0.73	1.61	4.30	90.23	94.50	98.02	99.92	100.71	97.87	1.16	-0.79	0.95
10% - 5%	0.00	0.00	0.78	4.00	12.11	1.00	0.84	2.01	7.23	86.02	93.74	98.07	100.16	101.03	97.87	1.37	-0.96	1.59
15% - 0%	0.00	0.00	0.78	4.33	14.22	1.00	0.91	2.25	9.20	87.39	93.69	98.09	100.19	101.24	97.88	1.40	-0.97	1.57
10% - 10%	0.00	0.00	0.78	4.67	14.11	1.00	0.98	2.43	10.37	83.51	93.01	98.13	100.37	101.16	97.88	1.56	-1.08	2.01
15% - 5%	0.00	0.00	0.67	4.78	15.44	1.00	1.01	2.40	9.57	85.48	93.02	98.14	100.36	101.17	97.87	1.57	-1.07	1.77
20% - 0%	0.00	0.00	0.67	5.22	14.67	1.00	1.09	2.75	12.62	85.51	92.87	98.17	100.37	101.06	97.88	1.60	-1.16	2.18
15% - 10%	0.00	0.00	0.67	5.22	15.89	0.99	1.11	2.88	14.46	83.91	92.41	98.20	100.51	101.29	97.89	1.73	-1.24	2.62
20% - 5%	0.00	0.00	0.67	5.67	17.11	1.00	1.17	2.84	13.37	82.84	92.27	98.21	100.54	101.28	97.88	1.76	-1.21	2.32
25% - 0%	0.00	0.00	0.56	6.11	18.67	1.00	1.26	3.19	16.49	83.34	92.10	98.23	100.53	101.18	97.88	1.78	-1.29	2.72
15% - 15%	0.00	0.00	0.56	5.89	26.67	1.00	1.23	3.21	18.57	78.58	91.73	98.25	100.65	101.36	97.87	1.89	-1.33	2.96
20% - 10%	0.00	0.00	0.56	6.00	21.11	1.00	1.26	3.21	17.39	82.59	91.71	98.26	100.65	101.27	97.88	1.90	-1.35	2.92
25% - 5%	0.00	0.00	0.56	6.44	26.22	1.00	1.34	3.45	20.36	78.16	91.53	98.27	100.67	101.54	97.87	1.94	-1.37	3.10
30% - 0%	0.00	0.00	0.56	7.11	28.44	1.00	1.45	3.69	22.41	79.09	91.36	98.29	100.67	101.53	97.87	1.97	-1.40	3.16
20% - 15%	0.00	0.00	0.56	6.56	22.78	1.00	1.37	3.64	22.48	81.25	91.15	98.31	100.75	101.32	97.88	2.05	-1.47	3.52
25% - 10%	0.00	0.00	0.56	7.00	26.33	1.00	1.43	3.58	21.49	79.75	90.98	98.32	100.76	101.45	97.87	2.07	-1.44	3.26
30% - 5%	0.00	0.00	0.44	7.44	29.22	1.00	1.52	3.78	23.01	78.12	90.86	98.34	100.78	101.33	97.88	2.09	-1.49	3.45
20% - 20%	0.00	0.00	0.44	7.22	35.11	1.00	1.49	4.13	32.13	73.90	90.42	98.36	100.85	101.47	97.89	2.19	-1.59	4.36
25% - 15%	0.00	0.00	0.44	7.22	26.56	1.00	1.52	3.99	26.74	78.45	90.53	98.38	100.84	101.48	97.89	2.20	-1.60	4.29
30% - 10%	0.00	0.00	0.44	7.78	31.89	1.00	1.59	4.09	27.73	75.28	90.31	98.39	100.84	101.47	97.88	2.22	-1.61	4.36
25% - 20%	0.00	0.00	0.44	8.00	44.67	1.00	1.63	4.48	38.51	68.11	89.78	98.42	100.93	101.61	97.87	2.34	-1.69	4.88
30% - 15%	0.00	0.00	0.44	8.22	38.11	0.99	1.68	4.42	33.39	73.13	89.78	98.43	100.92	101.52	97.88	2.34	-1.71	4.89
25% - 25%	0.00	0.00	0.33	8.67	39.00	1.00	1.77	4.77	39.37	69.79	89.13	98.48	101.01	101.55	97.88	2.47	-1.79	5.40
30% - 20%	0.00	0.00	0.33	8.89	37.33	1.01	1.79	4.57	34.52	71.52	89.17	98.48	101.01	101.57	97.87	2.49	-1.76	4.87
30% - 25%	0.00	0.00	0.33	9.22	55.00	1.00	1.90	5.33	53.59	65.32	88.76	98.54	101.07	101.65	97.88	2.60	-1.90	6.16
30% - 30%	0.00	0.00	0.33	9.78	42.11	0.99	2.02	5.41	47.93	68.72	88.14	98.59	101.11	101.67	97.88	2.72	-1.99	6.60
25% - 25%	0.00	0.00	0.89	14.67	46.44	1.99	3.01	3.73	22.10	64.25	80.46	92.95	98.41	100.65	92.31	3.83	-1.08	1.81

Table F.3: Default Rate and Portfolio Value of Diversified Portfolio

	$\mathbf{L}(\mathbb{E}[\mathbf{D_1}])$									$\mathbf{L}(\mathbf{D_0})$								
Corr.	**Min.**	$q_{0.01}$	$q_{0.5}$	$q_{0.99}$	**Max.**	μ	σ	**Skew**	**Kurt.**	**Min.**	$q_{0.01}$	$q_{0.5}$	$q_{0.99}$	**Max.**	μ	σ	**Skew**	**Kurt.**
0% - 0%	0.28	0.70	1.58	2.89	3.99	1.62	0.47	0.49	0.28	1.01	1.54	2.53	3.92	5.04	2.56	0.52	0.42	0.21
5% - 0%	-0.04	0.38	1.53	3.67	6.69	1.62	0.72	0.82	1.11	0.50	1.06	2.47	4.88	8.24	2.57	0.83	0.71	0.85
5% - 5%	-0.10	0.22	1.47	4.29	7.97	1.61	0.88	1.04	1.71	0.26	0.81	2.41	5.62	9.56	2.56	1.05	0.89	1.25
10% - 0%	-0.08	0.21	1.47	4.41	8.23	1.62	0.91	1.06	1.69	0.28	0.80	2.41	5.73	9.87	2.57	1.06	0.90	1.23
10% - 5%	-0.15	0.11	1.42	5.02	12.20	1.62	1.05	1.30	2.84	0.11	0.63	2.36	6.45	14.00	2.57	1.24	1.11	2.07
10% - 10%	-0.16	0.02	1.36	5.63	14.60	1.62	1.19	1.50	3.71	0.08	0.48	2.29	7.14	16.51	2.56	1.42	1.26	2.65
15% - 0%	-0.16	0.09	1.40	5.09	10.96	1.62	1.08	1.33	2.83	0.07	0.60	2.34	6.50	12.67	2.56	1.28	1.12	2.03
15% - 5%	-0.15	0.03	1.35	5.64	12.79	1.62	1.20	1.46	3.21	0.09	0.49	2.29	7.14	14.57	2.57	1.43	1.25	2.32
20% - 0%	-0.16	0.01	1.32	5.84	13.51	1.62	1.25	1.58	3.95	0.09	0.47	2.26	7.32	14.85	2.56	1.46	1.33	2.81
15% - 10%	-0.17	-0.02	1.29	6.13	14.09	1.61	1.32	1.72	4.83	0.03	0.40	2.22	7.71	16.09	2.55	1.57	1.45	3.43
20% - 5%	-0.16	-0.04	1.28	6.32	15.35	1.62	1.36	1.67	4.25	0.06	0.37	2.21	7.86	17.21	2.57	1.60	1.41	3.03
25% - 0%	-0.17	-0.05	1.26	6.51	14.82	1.62	1.40	1.77	4.91	0.04	0.36	2.19	8.06	16.68	2.56	1.64	1.49	3.49
15% - 15%	-0.17	-0.07	1.24	6.74	19.32	1.62	1.45	1.85	5.55	0.02	0.31	2.17	8.38	21.42	2.57	1.72	1.55	3.90
20% - 10%	-0.17	-0.07	1.23	6.77	15.56	1.62	1.46	1.86	5.37	0.06	0.31	2.16	8.39	17.43	2.56	1.73	1.56	3.81
25% - 5%	-0.17	-0.08	1.22	6.99	19.80	1.62	1.50	1.89	5.71	0.00	0.29	2.15	8.61	21.84	2.57	1.77	1.59	4.03
30% - 0%	-0.17	-0.09	1.19	7.20	18.95	1.62	1.55	1.93	5.74	0.00	0.28	2.12	8.80	20.92	2.57	1.81	1.61	4.04
20% - 15%	-0.17	-0.09	1.17	7.23	16.77	1.61	1.57	2.04	6.52	0.03	0.25	2.10	8.92	18.76	2.56	1.86	1.71	4.60
25% - 10%	-0.17	-0.10	1.17	7.44	18.25	1.62	1.60	1.99	5.96	0.02	0.24	2.09	9.11	20.27	2.57	1.89	1.67	4.23
30% - 5%	-0.17	-0.11	1.13	7.66	19.89	1.62	1.64	2.04	6.26	0.01	0.22	2.07	9.28	21.88	2.56	1.92	1.71	4.44
20% - 20%	-0.17	-0.12	1.12	7.93	23.98	1.61	1.69	2.23	8.28	0.01	0.20	2.05	9.65	26.10	2.55	1.99	1.86	5.73
25% - 15%	-0.17	-0.12	1.10	7.84	19.45	1.61	1.70	2.23	7.98	0.01	0.20	2.03	9.55	21.55	2.55	2.01	1.86	5.59
30% - 10%	-0.17	-0.12	1.10	8.09	22.63	1.61	1.73	2.23	7.97	0.01	0.19	2.02	9.78	24.72	2.56	2.03	1.86	5.62
25% - 20%	-0.17	-0.14	1.06	8.48	29.77	1.62	1.81	2.36	9.26	0.00	0.16	1.99	10.27	31.89	2.57	2.14	1.96	6.39
30% - 15%	-0.17	-0.14	1.05	8.55	24.76	1.61	1.83	2.37	9.04	0.00	0.16	1.97	10.29	26.87	2.56	2.15	1.98	6.33
25% - 25%	-0.17	-0.15	1.01	9.10	28.11	1.62	1.93	2.52	10.19	0.00	0.12	1.92	10.91	30.21	2.56	2.27	2.09	7.06
30% - 20%	-0.17	-0.15	1.00	9.08	26.38	1.63	1.95	2.44	8.99	0.00	0.12	1.92	10.88	28.48	2.57	2.28	2.04	6.31
30% - 25%	-0.17	-0.16	0.95	9.46	32.56	1.61	2.03	2.67	11.67	0.00	0.09	1.86	11.30	34.68	2.56	2.39	2.21	8.02
30% - 30%	-0.17	-0.16	0.89	10.00	29.16	1.61	2.13	2.79	12.33	0.00	0.08	1.80	11.89	31.28	2.56	2.50	2.31	8.53
25% - 25%	-0.11	0.74	5.19	17.43	33.63	5.91	3.61	1.25	2.31	0.22	1.84	7.07	19.54	35.75	7.73	3.80	1.11	1.89

Table F.4: Loss Rates of Diversified Portfolio

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