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Contents

Chapter 1

General Introduction

This dissertation consists of three self–contained papers, which contribute to different strands of the literature on industrial organization and microeconomic theory. In Chapter 2, I analyze an incentive problem within a principal–agent employment relationship when the principal has better information about the job offered to the agent. Chapter 3 examines market outcomes when consumers are loss averse. It contributes to the literature on behavioral industrial organization. Chapter 4 studies the allocation of ownership and control rights within industries and its implication on competition. It links the literature on industrial organization with the corporate finance literature. The appendix contains the appendices of the papers in which proofs and tables are presented. References of the papers can be found in the last chapter of the thesis.

1.1 Informed Principal with Moral Hazard

In Chapter 2, I study the design of employment contracts when an employer (=principal) is better informed about a job offered to an employee (=agent). I consider how the principal optimally incentivizes the agent given this information asymmetry by offering information–specific wage schemes to the agent.

While the optimal design of employment contracts, in which the agent makes an unobservable effort choice (=moral hazard), is well understood under standard assumptions, much less is known about optimal contracting if the commonly made assumption of symmetric information about job environment or job difficulty is not met. This is particularly true if the principal (rather than the agent) holds an informational advantage. However, situations like this can be frequently observed in reality. For instance, consider new employment contracts after early resignation or dismissal of professional conductors in an orchestra, coaches at sports teams, CEOs, or politicians. In such settings principals can access information about the circumstances which initiated predecessor's replacement more easily than external successors and, therefore, are better informed about the quality of the work environment or the difficulty of the job. There is also a clear moral hazard problem with respect to successor's future effort decision. Furthermore, since replacement decisions are very urgent, principals often do not face other means than wage offers to convince successors of the high quality of the work environment although other means as offering time for consideration or talks with other employees to potential successors could be less costly to the principal if there was no urgency. Finally, a potential external successor receives some information open to the public about the replacement, namely whether the orchestra or sports team was successful lately before the replacement took place or not. In my model this kind of public information turns out to be crucial for the optimal incentive scheme given to the agent.

In the main part of Chapter 2, I first consider contracting under moral hazard but full symmetric information as a benchmark case. Here, the agent observes directly whether the work environment is favorable or not, but agent's effort choice remains unobservable to the principal. In the example from above, it can be thought of an internal successor who already knows whether the current orchestra or sports team is of high or low quality. There arises a trade–off for the agent between spending high effort for favorable work environments if this increases the expected wage payment a lot and spending low effort for favorable work environments if the expected wage payment is already very high relative to unfavorable work environments.

Then, I turn to the setting in which only the principal is informed about the quality of the work environment. I analyze how a principal with favorable information optimally signals his information via wage offers to the agent and how this affects the agent's effort choice. Surprisingly, I find that in this case contracting can become more efficient with respect to agent's effort choice than under full symmetric information. This states a novel efficiency result and is the main finding of Chapter 2.

At the end of the main part of Chapter 2, I show that the principal with favorable information prefers to pool with the one with unfavorable information if the probability of facing favorable information becomes very high.

1.2 Pricing and Information Disclosure in Markets with Loss–Averse Consumers

In Chapter 3, the impact of the behavioral bias "loss aversion" on market outcomes as pricing and advertising levels is analyzed. Here, consumers are considered to be loss averse with respect to two dimensions—expected purchase prices and expected taste from product characteristics. Loss–averse consumers put a higher weight on losses (in the price or taste dimension) than on gains of equal size relative to their reference point (=expectation about future outcomes).

There is a growing literature in behavioral industrial organization that analyzes market outcomes when consumers show specific behavioral biases as unwareness about product add–ons, naivety, overconfidence, or loss aversion. As is shown by various field studies and experiments, those behavioral biases play an important role in daily consumer behavior. It is therefore interesting to ask, under which circumstances firms might want to exploit boundedly rational consumers and when there might be a need for consumer protection policies.

The existing literature on consumer loss aversion predicts higher prices in markets with horizontally differentiated products when firms do not have knowledge of their rivals' costs (cf. the paper by Heidhues and Koszegi (2008)). The model presented in Chapter 3 of this thesis considers a similar market environment but allows firms' costs to be common knowledge since market participants know each other already. This applies to markets in which costs are determined by firm size due to scale effects. Moreover, we focus on products for which price information is more easily accessible than information about product characteristics as for instance clothing or electronic devices. As a novelty, we highlight that information prior to the moment of purchase matters if consumers are loss–averse, since product information plays an important role already at the stage at which loss–averse consumers form expectations about future transactions (=form their reference point).

We postulate that, to make their consumption choices, loss–averse consumers form their probabilistic reference point based on expected future transactions which are confirmed in equilibrium. Here, a consumer's reference point is her probabilistic belief about the relevant consumption outcome held between the time she first focused on the decision determining the consumption plan and the moment she actually makes the purchase.

We distinguish between "informed" and "uninformed" customers at the moment consumers form their reference point. Informed consumers know their ideal taste ex ante and will perfectly foresee which product they will buy. Therefore they will not face a loss or gain in product satisfaction beyond their intrinsic valuation.

Uninformed consumers, by contrast, are uncertain about their ideal product characteristic: they form expectations about the difference between ideal and actual product characteristic which will serve as a reference point when evaluating a product along its taste dimension. They will also face a gain or a loss relative to their expected distributions of purchase price after learning the taste realization. Since we assume that all consumers become fully informed before they have to make their purchasing decision, we can isolate the effect of consumer loss aversion on consumption choices and abstract from the effects of different information at the moment of purchase.

Our main finding is that loss aversion—or, more precisely, the presence of more ex ante uninformed, loss averse consumers—may lead to lower prices which is in stark contrast to the existing literature on consumer loss aversion. Moreover, the standard result that more informed consumers (or more consumers without a behavioral bias) lead to lower prices is challenged in our model when firms are strongly asymmetric (=cost differences between firms are large). The driving force behind this result is that loss aversion in the price dimension has a pro–competitive effect while the effect of loss aversion in the taste dimension is anti–competitive. The pro–competitive effect dominates the anti–competitive effect if the size of loss aversion in the price dimension becomes sufficiently large. This occurs if the price difference becomes large, which is caused by strong cost asymmetries. In this situation uninformed consumers are very reluctant to buy the expensive product and rather accept a large reduction in taste when buying the low–price product.

We also link this result to firms' private incentives to disclose information about product characteristics at an early stage, i.e. to firms' private incentives to advertise. We find that firms want to advertise in strongly asymmetric markets, while the reverse holds true in rather symmetric markets. Moreover, in symmetric markets we predict the need for consumer protection policy to be highest.

1.3 Ownership and Control in Differentiated Product Markets

In Chapter 4, we study the equilibrium allocation of ownership and control rights within horizontal industries and its implication on competition.

Cross ownership arrangements between firms are widespread, particularly in Europe and Japan (cf. Allen and Gale, 2000). In Chapter 4, we therefore ask how incentives, to undertake (partial) financial investments, depend on market parameters and what the influence of ownership structures on allocative decisions is. We are also interested in the question whether it can ever be optimal to acquire cash flow rights without control rights in a competitor although horizontal integration increases profits. Moreover, we study whether a direct financial investment by an investor can ever be preferable to a financial investment via a firm controlled, but not fully owned by the investor.

We analyze the equilibrium allocation of ownership and control rights in a static duopoly. After acquisitions took place, firms sell a horizontally differentiated product and simultaneously set prices. Initially, an investor I_1 holds a controlling stake in one firm and decides whether to acquire a stake in the other firm (=target firm) and/or an additional stake in the initially controlled firm. Moreover, the investor can initiate a cross investment of the initially controlled firm in the target firm to indirectly participate in the target firm's profits.

The acquisition of shares has two effects. First, the acquisition is associated with cash flow rights on the target firm's profits. I_1 will internalize their effect by appropriately setting the initially controlled firm's price. Second, if I_1 acquires enough shares, she gains control in the target firm as well. She then sets both prices so as to maximize her portfolio return. The threshold of shares to gain control is assumed to be exogenous. We think of it as being the lower, the more dispersed the remaining ownership in the firm or, alternatively speaking, the less shareholders are able to coordinate their votes against decisions favorable to our investor I_1.

Against the standard idea that under no restraint the raider would want to overtake the target firm, we find that both partial and full acquisitions may arise, and control of the target firm is not always desirable. In some cases, cross ownership arrangements between firms are undertaken, whereas in others they are dominated by a direct investment of investor I_1.

Our qualitative results are robust to extensions of the model to more than two firms and settings in which the investor controls a second instrument as e.g. cost reducing investments, while the price instrument of a firm is under managerial control.

Chapter 2

Informed Principal with Moral Hazard

2.1 Introduction

2.1.1 Motivation

New employment contracts, particularly those after early resignation or dismissal of predecessors, show a specific kind of information asymmetry, namely private information on the principal's side, which is distinct from the one in standard problems. In such settings principals can access information about the circumstances which initiated predecessor's replacement more easily than external successors and, therefore, are better informed about the quality of the work environment or the difficulty of the job. Moreover, if concerns about successors' ability do not arise due to track records, private information on the principal's side constitutes the main source of information asymmetry. However, moral hazard with respect to the successor's future effort decision is likely to demonstrate a second restriction on efficient contracting in this context. Finally, since replacement decisions are very urgent, principals often do not face other means than wage offers to convince successors of the high quality of the work environment although other means as offering time for consideration or talks with other employees to potential successors could be less costly to the principal if there was no urgency.

Models with informed principals and adverse selection have been studied intensively by Maskin and Tirole (1990, 1992) and others. However, much less is said on informed principal models in which moral hazard is the main limitation on contracting. In the following I consider an informed principal model with moral hazard on the agent's side. Here, only the principal knows the project type he offers to an agent who is supposed to exert unobservable effort on the project. I want to show how the agent's equilibrium effort decision is altered, if the principal holds private information about the expected outcome of projects and the principal's wage offer is the only means to signal the project type to the agent. Different pairs of project types are considered and welfare implications are drawn about how contracting is affected by the degree of complementarity between effort and project type.

I define the degree of complementarity to be high if the good project, whose success probability function I assume to have a steeper slope in effort than bad projects in general, additionally shows a sufficiently low intercept of the success probability function. The degree of complementarity is low, or equivalently effort and project type are substitutes here, if the intercept of the good project is much higher than the one of the bad project. A high degree of complementarity following this definition corresponds to a higher effort elasticity of the good project's success probability function. Moreover, the degree of complementarity describes whether it is easier to motivate the agent to spend high effort levels for good projects than for bad projects. If the degree of complementarity between effort and project type is particularly high, then shirking at a good project is relatively unattractive for the agent, while the reverse holds true for effort and project type being substitutes.

My main finding is that contracting under moral hazard and private information on the principal's side leads to a more efficient effort choice than contracting under moral hazard and symmetric full information if effort and project type are complements, i.e. if the success probability function of good projects shows a steeper slope and a not too high intercept. The reverse holds true for effort and project type being substitutes, i.e. if the success probability function of good projects shows a steeper slope and a sufficiently higher intercept than bad projects. In the latter case the additional information asymmetry distorts contracting under moral hazard even further with respect to effort choice.[1] I also identify cutoff levels of slopes and intercepts for good and bad projects such that effort choice is not altered by the principal being endowed with private information.

With high complementarities, the principal with the good project separates from the one with the bad project in equilibrium by increasing the premium above its level with project observability. This also increases the agent's effort choice above its level when projects are fully

[1]This finding is in line with Inderst (2001) who also predicts incremental distortions if moral hazard and private information on the principal's side are combined.

observable ex ante. The intuition behind this main efficiency result is that mimicking a good project might become less profitable for a principal endowed with a bad project if the wage contracts for good projects are relatively high–powered, i.e. induce the agent to choose a relatively high effort level. This requires, however, that high–powered wage contracts are sufficiently profitable for good projects. This is the case if shirking at a good project is less attractive for the agent since success probabilities are sufficiently low at zero effort, which corresponds to effort and project type being complements. Note that the principal with the good project is not better off here although total efficiency increases. The reason for this is that preventing the principal with the bad project from mimicking makes the principal with the good project weakly worse off. Thus, the agent solely obtains the efficiency gain.

This setup applies to new employment contracts for professional conductors, coaches, CEOs, or politicians after early resignation or dismissal of the predecessor. External successors for these kinds of jobs clearly face an informational disadvantage since they cannot fully access what caused these incidents. Were problems purely predecessor–specific or were they also related to the quality of the orchestra, the sports team, the firm or the difficulty of the mission? On the other hand, the problem of private information about successor's ability is mitigated by the importance of reputation for those kinds of professions. I assume a potential successor provides sufficient track record. Moreover, moral hazard with respect to the successor's future effort decision is a big issue for those jobs. The distinction between effort and project type being complements or substitutes can be linked to information open to the public about the replacement. I will focus on the conductor–orchestra example from now on.

For complements consider a situation in which the orchestra attracted less audience lately and the predecessor was dismissed due to bad performance or quit by herself. In this situation a successor has to initiate drastic modifications to make the potentially good orchestra successful again. It can be thought of a higher practice frequency and the rehearsal of different musical scores. However, spending effort will be less attractive for a new conductor if the orchestra turns out to be limited in quality of musicians, i.e. being of bad type at least in the short run.

Substitutability between effort and project type occurs if the predecessor resigned early from her contract due to private reasons or due to an offer from another potentially better orchestra although the current orchestra performed well lately. Another situation of this kind is dismissal due to other non–performance related reasons as e.g. political incorrectness of the previous conductor although the orchestra was successful in the past. In these situations the successor can potentially benefit from the preparatory work of the previous conductor if the orchestra turns out to be of high quality. In a situation in which the predecessor was dismissed due to rumors between conductor and musicians, the successor might free–ride on musicians returning to their average—possibly high—performance after the replacement occurred. In a bad orchestra, however, shirking will be relatively less beneficial for the new conductor in all of these situations.

My main results translate to new employment contracts after early resignation or dismissal in the following way. If the previous conductor was dismissed due to bad performance (=effort and project type are complements), I predict more high–powered contracts for external successors (=private information on principal's side) than for internal successors (=symmetric full information) of similar track record if the orchestra is of high quality. But there is no contractual wage difference between compatible external and internal successors if the orchestra is of low type. The wage offers from good orchestras are always higher here than the ones from bad orchestras. Moreover, there is a one–to–one mapping between wage differentials and differences in effort levels for a specific type of orchestra. In a case in which a conductor resigned for private reasons (=effort and project type are substitutes), I find the external successor's wage contract to be less high–powered than the internal successor's wage contract if the orchestra is of high type. Again, both wage contracts are identical for low quality orchestras. But the wage offer for an external successor from a good orchestra can be lower here than the one from bad orchestra. The same applies even to an internal successor if the free–riding potential in good orchestras is huge, i.e. if effort and project type are extreme substitutes. The different results for both cases can be explained by the varying incentives of bad orchestras to pretend to be of good type. I provide a broader discussion of this in the main part of the paper.

This paper contributes to the existing literature on contracting under moral hazard and asymmetric information by providing a novel efficiency result: Contracting under less public information can increase efficiency with respect to effort choice. I also identify critical conditions under which this efficiency result vanishes or even is reversed.[2] Moreover, I provide a new application for informed principal models with moral hazard for which signaling of principal's type is purely wage–based, namely new employment contracts after early dismissal or resignation of predecessors. This application also allows to distinguish between conditions that are efficiency–increasing or –decreasing.

[2]The reversed result was shown by Inderst (2001) in a more specific setting.

The plan of this chapter is as follows. In Section 2.2, I first introduce the general setting and then analyze contracting under full symmetric information and under private information on the principal's side. In the latter case least–cost separating equilibria and optimal pooling equilibria for the high–type principal are derived and compared with the equilibria that arise under full symmetric information. This delivers the main efficiency result of this paper. Section 2.3 discusses robustness of the main efficiency result and Section 2.4 concludes.

2.1.2 Related Literature

Myerson (1983) analyzes a general informed–principal problem but focuses on the interrelation between different solution concepts. The characterization of equilibrium mechanisms is limited in his paper. Maskin and Tirole (1992) offer a detailed characterization of equilibrium contracts in an informed–principal problem with common values but consider problems of adverse selection rather than moral hazard problems. Beaudry (1994) focuses exclusively on risk–neutral agents who do not face limited liability constraints. He emphasizes that informed principals transfer rents to agents in equilibrium to reveal their type. The author relates this finding to the appearance of efficiency wages. A solution to the moral hazard problem with an informed principal and discrete effort can be found in Chade and Silvers (2002) who also show that there may arise a downward distortion of effort in equilibrium. Mezzetti and Tsoulouhas (2000) introduce an information collection stage by the uninformed party, i.e the agent, before she decides whether to accept or reject the principal's wage offer. The principal's private information is of private value in this paper. This means that it only contains knowledge about the agent's costs of effort. The separation of the good type of principal becomes only possible by offering an option contract which allows the agent to reject the initial offer after detection of the bad type of principal.

The paper closest to mine is Inderst (2001) who analyzes an informed principal problem with moral hazard on the agent's side in which players are risk–neutral. The principal's private information is about project type which positively affects the success probability of the high outcome. In Inderst (2001) a principal with a good project can separate from one with a bad project by increasing the share of project revenue he retains after wage payments are made. Separation is possible because expected revenue is more valuable for projects with the high success probability than for those with the lower one. However, not participating in the full revenue of the project destroys the agent's first–best incentives to exert effort which distorts effort for high projects downward in separating equilibrium. To show that equilibrium effort decision can be altered in either way I need two assumption which differ from those of Inderst (2001): In my model the agent is protected by limited liability of wages at zero such that even under symmetric full information about project type a distortion in effort arises due to moral hazard. Moreover, the specification of expected outcome differs since in my paper the marginal success probability of effort is not independent of the project type.

2.2 The Model

2.2.1 General Setting

Consider a principal–agent moral hazard problem with discrete outcomes and continuous effort choice by the agent.[3] Let the principal privately observe whether the job or project the agent works on is of good or bad type. I call a "good" project one in which the probability function of high outcome (=success probability function) is always "steeper" in effort than for a bad project, while the intercept of the success probability might be weakly lower or higher than for a bad project.

There are two players, a risk–neutral principal, and a risk–neutral agent who is protected by limited liability of transfers at zero. This means that wage payments are non–negative. The agent's outside option is normalized to zero. The outcome of a project is stochastically dependent on the agent's effort choice $e \in [0, \overline{e}]$, which is non–contractible, and on the project's type $i \in I = \{L, H\}$, which is the principal's private information. I will henceforth use "project type" and "principal's type" as synonyms. Each type of project refers to a technology parameter $\theta_i \in \{\theta_L, \theta_H\}$ with $\theta_H > \theta_L = 0$.[4] The agent's prior beliefs about the project type are described by $\beta = Prob\{i = H\}$ with $\beta \in (0, 1)$. The outcome of both project types may take two realizations $y \in \{0, \overline{y}\}$ with $\overline{y} > 0$.[5] But projects differ in their probability of high outcome

[3]For simplicity I focus on a setting with binary outcomes. I will show in Section 2.3 that results carry over to the continuous–outcome case. Continuous effort choice allows for a proper analysis of efficiency implications of this model.

[4]This setup resembles mixed models of moral hazard and adverse selection examined in Laffont and Martimort (2002), Chapter 7. However, in this model the informed rather than the uninformed party proposes the contract.

[5]It is shown below that restricting $\underline{y}$ to zero implies that the limited liability constraint at zero becomes binding.

(=success probability) which is a function of agent's effort level e,

$$p_i(e) \equiv p(\theta_i, e) = Prob\{y = \overline{y}|\theta_i, e\} \quad \text{with } e \in [0, \overline{e}] \text{ and } 0 < p_i(\overline{e}) < 1 \; \forall i \in I. \tag{2.1}$$

Furthermore, I specify $p_i(e)$ to be an affine function of θ_i represented by the following functional form,

$$p_i(e) = \begin{cases} p_L(e) = r_L + s_L p(e), & \text{if } i = L, \\ p_H(e) = (r_L + \Delta r \theta_H) + (s_L + \Delta s \theta_H) p(e), & \text{if } i = H, \end{cases} \tag{2.2}$$

with $p(e) \in C^3$, $p'(e) > 0$, $p''(e) \leq 0, \forall e \in [0, \overline{e}]$, and $p(0) = 0$. Typical examples for $p(e)$ are linear and root functions, i.e. $p(e) = e^x$, $0 < x \leq 1$.[6] r_L and s_L depict the intercept and the slope of the success probability function of the bad project, while $r_H = r_L + \Delta r \theta_H$ and $s_H = s_L + \Delta s \theta_H$ represent the counterparts of the good project. $\theta \in [0, \theta_H]$ describes the transition of intercept and slope from bad to good projects. Moreover, I assume that $r_L, s_L, \Delta s > 0$ and $\Delta r \geq -r_L/\theta_H$, i.e. both projects show a strictly positive intercept and a strictly positive slope with the slope of the good project being always higher than the one of the bad project and its intercept being either weakly lower or higher than the one of the bad project. This representation allows me to compare projects whose success probability functions belong to the same functional family but differ in slope and axis intercept in general terms.[7] It follows that $p_i(e)$ is element of C^3, strictly increasing and concave in effort, i.e. $p_i'(e) > 0$ and $p_i''(e) \leq 0 \; \forall e \in [0, \overline{e}]$, and that $p_i(0) \geq 0 \; \forall i$.[8] Additionally, the good project shows a higher marginal success probability of effort and a weakly stronger concavity for any given effort level, so that $p_H'(e) > p_L'(e)$ and $p_H''(e) \leq p_L''(e) \; \forall e \in [0, \overline{e}]$.[9] The latter two properties are necessary to obtain a convex optimization problem in the signaling game. Let $c(e)$ be the agent's cost of effort function, which is element of C^3, strictly increasing, and strictly convex in effort, i.e. $c'(e) > 0$ and $c''(e) > 0 \; \forall e \in [0, \overline{e}]$ and $c(0) = 0$ and $c'(0) = 0$. I will focus on interior solutions in the

[6]In contrast to Inderst (2001), I do not assume specific functional forms for the success probability function.

[7]In principle, comparisons of projects of different functional families are compatible with this model but the general proof of optimality of least–cost separating becomes intractable without specifying explicit functional forms (cf. Lemma 2.4). In addition, the given specification of $p_i(e)$ is a sufficient condition for preserving the ordering of the effort elasticity of the success probability functions for project L and H at any effort level (cf. Corollary 2.2.1). This property turns out to be of importance for the main efficiency result of this paper.

[8]These are the standard properties which together with the assumptions on the agent's cost of effort function yield concavity of the principal's utility function. Continuity of the third derivative of $p_i(e)$ I need to solve the principal's problem in the separating equilibrium.

[9]The former property does not rule that total revenue of effort for the good project is lower than for the bad project for some (low) effort levels.

following.

By conditioning wage payments on outcome (which is contractible), the principal fully describes a contract in this setting.[10] I denote the principal's transfer by $w(0) = a$ (=fixed wage) in case of failure and by $w(\overline{y}) = a + b$ (= fixed wage plus premium) in case of success. Let $w = (a, b)$ with $w \in W = \mathbb{R}^2$ state a contract the principal offers to the agent. The agent can either accept the principal's offer and exert a non–negative effort level, e, or reject it without exerting effort. For given output realization y, wage payment $w(y)$, and effort level e the principal receives the utility $v(w(y), y) = y - w(y)$ and the agent the utility $u(w(y), e) = w(y) - c(e)$.

For any accepted wage offer $w = (a, b)$ the agent maximizes her expected wage payment minus her costs of effort over e. This effort decision is dependent on the agent's conditional belief $\mu(i|w)$ about project type i, where μ maps W into the simplex Δ_2. Given my assumption on the concavity of $p_i(e)$ and the strict convexity of $c(e)$ there exists a unique solution to the agents problem. The first–order necessary condition of the agent's problem for a given premium b and beliefs $\mu(i|w)$ are as follows,

$$\sum_{i\in\{L,H\}} \mu(i|w)p_i'(e) \cdot b - c'(e) = 0. \tag{FOC_A}$$

Denote the solution to the agent's problem by $\hat{e} = e(b, \mu(.|w))$. The agent's effort choice is only indirectly affected by fixed wage a via her beliefs about the project type i. I next incorporate the underlying moral hazard problem by defining the following *indirect utility functions*:

- Principal:

$$V_i(w, \mu(i|w)) \quad = \quad p_i(\hat{e}) \cdot (\overline{y} - b) - a,$$

- Agent:

$$U_i(w, \mu(i|w)) \quad = \quad p_i(\hat{e}) \cdot b + a - c(\hat{e}),$$

  where $i \in \{L, H\}$ and $\mu(i|w) \in [0, 1]$.

I henceforth abbreviate $V_i(w, 1)$ by $V_{ii}(w)$ and $V_i(w, 0)$ by $V_{ij}(w)$ for all $i, j \in \{L, H\}, i \neq j$. I

[10] Note that in a model with binary outcomes the optimal contract is also binary.

proceed analogously for the agent's utility, i.e. $U_{ii}(w) \equiv U_i(w,1)$ and $U_{ij}(w) \equiv U_i(w,0)$.[11] Given the structure on $p_i(e)$ and $c(e)$, I obtain strict concavity of V and U in b (resp. e).

The timing and the information structure of the game are as follows. At date zero the principal learns the project type and makes a take–it–or–leave–it wage offer to the agent. The agent updates her beliefs about the project type conditional on the offered wage contract and then decides whether to accept the contract or not. If the agent accepts the offer, she chooses the optimal effort level given her beliefs. Then the outcome is realized and the principal makes the scheduled wage payment to the agent. If the agent rejects the offer, then zero effort is chosen and outcome is realized. The agent gets her outside option which is equal to zero and the principal receives the outcome which can be positive since the success probability function is strictly positive at zero effort. The solution concept is a Perfect Bayesian Nash Equilibrium (=PBE). Denote the principal's strategy by σ_P, where for each i, $\sigma_P(.|i)$ is a probability distribution over W and the agent's strategy by σ_A, where for each w, $\sigma_A(.|w)$ is a probability distribution over the set $\{0, 1\}$. Here, 1 represents acceptance by the agent, while 0 represents rejection. Thus, an equilibrium is determined by a vector of strategies and beliefs $\sigma = (\sigma_P, \sigma_A, \mu)$.

2.2.2 Observable Project Type

If project type i is observable (=symmetric full information), then the setting reduces to a pure moral hazard problem. The first–best effort level of project i, e_i^{FB}, is determined by

$$p_i'(e)\overline{y} = c'(e) \quad \forall i \in \{L, H\}. \tag{2.3}$$

Since the agent is protected by limited liability at zero transfer, the moral hazard problem generates a distortion even if project type i is observable.[12] Applying the implicit function theorem (=IFT) to (FOC_A), the first–order necessary condition of the agent's problem for

[11] In this model private information is in common values. This means that the success probabilities enter the utility functions of both players, the principal and the agent since wage payments are conditioned on outcomes.

[12] Suppose not, then given risk neutrality of both players agent's effort choice must be first–best efficient. From (2.3) it follows that in this case the premium b must be equal to $\overline{y}$. This corresponds to selling the entire project to the agent. At this the principal optimally chooses a fixed wage payment $a = -(p_i(e_i^{FB})\overline{y} - c(e_i^{FB}))$ to leave the agent without rent (= the agent's individual rationality condition is binding). But since a is strictly negative for positive NPV projects the limited liability constraint will be binding and a distortion arises.

$\mu(i|w_i) = 1$, I find a positive relationship between effort level e and premium b_i

$$\frac{de}{db_i} > 0 \quad \forall i. \tag{2.4}$$

I therefore conclude that an increase in the premium leads to a higher effort level for given beliefs.

For realized project type i the principal's problem is expressed by

$$\max_{\{w_i\}} p_i(\hat{e}) \cdot (\overline{y} - b_i) - a_i \tag{2.5}$$

$$s.t.$$

$$a_i \geq 0 \tag{LL_i}$$

$$U_{ii}(w_i) \geq 0 \tag{IR_i}$$

$$\hat{e} = \arg\max_{e} p_i(e) \cdot b_i + a_i - c(e). \tag{IC_i}$$

The following lemma shows the optimal effort level of (2.5) for interior solutions.

LEMMA 2.1: *Suppose project type i is observable. Then the optimal effort level for the principal's problem* (2.5), e_i^*, *is specified by*

$$p_i'(e)(\overline{y} - b_i(e)) - p_i(e)b_i'(e) = 0, \tag{2.6}$$

with $b_i(e) = c'(e)/p_i'(e)$ *and* $b_i'(e) = 1/(de/db_i)$ *from* (2.4)*. Moreover, the principal offers the contract* $w_i^* = (a_i^*, b_i^*)$ *with* $a_i^* = 0$ *and* $b_i^* = b_i(e_i^*)$.

I henceforth will refer to the moral hazard problem with symmetric full information as the benchmark case. In my conductor–orchestra example this translates to employment contracts offered to an internal successor who knows ex ante whether the orchestra is of high or low quality.

Note that (IR_i) is satisfied with strict inequality at e_i^*, i.e. the agent receives a positive rent. Furthermore, from (2.6) it can be seen that $e_H^* > e_L^*$ does not follow directly from the assumption that $p_H'(e) > p_L'(e)$ for all e. E.g. for $p_H(0) - p_L(0) = \Delta r \theta_H$ large and $p_H'(e) - p_L'(e) = \Delta s \theta_H p'(e)$ relatively low for all e (=effort and project type are substitutes) it could be the case that the second–best effort level is higher for the bad project. Here, the expected wage payment for the good project is much larger at higher effort levels than its expected outcome because the probability of paying the premium is very high and a compensating fixed wage is ruled out by limited liability at zero. In my conductor–orchestra example this means that if the previous conductor left a good orchestra e.g. for private reasons, then an internal successor is able to free–ride a lot on the good performance of the orchestra. She actually free–rides so much that less incentives will be given to her than to an internal successor in a bad orchestra.

From the derivation of $b_i(e)$ it follows that $e^*_H > e^*_L$ implies $b^*_H > b^*_L$. However, the reverse is not true in general. From $p'_H(e) > p'_L(e)$ for all e it follows that $b_H(e) < b_L(e)$ for any e, this means that inducing a specific effort level requires a lower premium payment for the good project. This creates incentives to mimic the good project for low–type principals. The next section deals with unobservable project types and with the incentives to mimic which arise is this setting.

2.2.3 Unobservable Project Type

Separating Equilibria

If the project type is private information to the principal and a principal with the good project offers the second–best contract from the previous section, then he might be mimicked by a principal with the bad project. The reason for this is that being perceived as the principal with the good project by the agent reduces the required premium per effort level, i.e. $b_H(e) < b_L(e)$ $\forall e$ feasible. By the same argument mimicking in the opposed direction is never profitable. However, in some settings it might be too costly for a principal with the good project to separate from principals with the bad project. This rises the issue of optimality (or existence) of separating equilibria.

In the following I consider PBE in pure strategies and focus on the least–cost separating contract. The refinement I choose is the intuitive criterion of Cho and Kreps (1987). If the principal with the good project (=H–type principal) wants to separate from the principal with the bad project (=L–type principal), then he can increase the differences between the premia of the two contracts by adjusting b_H or introduce a positive fixed wage for the good project, $a_H > 0$. The interplay of these two means of mimicking prevention is crucial for the main efficiency result in Lemma 2.3 and Proposition 2.2.

To solve for the least–cost separating equilibrium with $(w_L^s, w_H^s) = ((a_L^s, b_L^s), (a_H^s, b_H^s))$, I use the following strategy. I first show that in any separating equilibrium the L–type principal will offer the second–best contract, i.e. $w_L^s = w_L^* = (0, b_L^*)$ (see Lemma 2.2). This is feasible although the Spence–Mirrlees condition is not generally satisfied in this model. In Lemma 2.3 I then independently derive the unique least–cost separating contract w_H^s for the H–type principal subject to the incentive constraint of the L–type principal given $w_L^s = w_L^*$. Next, it is shown that the least–cost separating contract is more profitable for H–type principals than the best non–separation contract $(0, b_{HL}^*)$ for a huge set of parameters (see Lemma 2.4). This step is non–redundant due to the potential violation of the Spence–Mirrlees condition in this setup.

In Proposition 2.1 I state existence of the least–cost separating equilibrium (w_L^s, w_H^s) and uniqueness under the intuitive criterion. The properties of the the least–cost separating equilibrium and the subgame perfect Nash equilibrium under symmetric full information from Lemma 2.1 are compared in Proposition 2.2 which delivers the main efficiency result of this paper. In the subsequent example with linear success probability functions and quadratic costs I show that the necessary and sufficient condition for existence of a least–cost separating equilibrium are always met under the assumptions of this model.

I first determine w_L^s if w_H^s is such that mimicking the H–type is not profitable for the L–type principal, i.e. if the L–type incentive constraint is satisfied and potentially binding (=LCS allocation). The next lemma shows that in this case the L–type principal will always offer the second–best contract from above, $w_L^s = w_L^* = (0, b_L^*)$.

Lemma 2.2: *Suppose project type i is unobservable and the separating contract for the H–type principal w_H^s is such that the L–type's incentive constraint is satisfied. Then,* $w_L^s = w_L^* = (0, b_L^*)$.

The least–cost separating contract for the H–type w_H^s can now be derived from the H–type principal's problem given that the incentive constraint for the L–type, (IC_{P_L}) is satisfied,

$$\max_{\{w_H^s \in W\}} p_H(\hat{e}) \cdot (\overline{y} - b_H^s) - a_H^s \tag{2.7}$$

$$s.t.$$

$$a_H^s \geq 0 \tag{LL_H}$$

$$U_{HH}(w_H^s) \geq 0 \tag{IR_H}$$

$$\hat{e} = \arg\max_e p_H(e) \cdot b_H^s + a_H^s - c(e). \tag{IC_H}$$

$$V_{LH}(w_H^s) \leq V_{LL}(w_L^*). \tag{IC_{P_L}}$$

In Lemma 2.3 the unique least–cost separating contract for the H–type principal's problem is derived.

Lemma 2.3 (Least–Cost Separation): *Suppose project type i is unobservable. There exists a unique least–cost separating contract w_H^s for the H–type principal. The induced effort level e_H^s, the fixed–wage payment a_H^s, and the premium b_H^s are characterized as follows*

1. *if (IC_H) and (IC_{P_L}) are binding (=interior solution),*
   $e_H^s \in \{e \in [0, \overline{e}] \,|\, \left(p'_H(e) - p'_L(e)\right)(\overline{y} - b_H(e)) - \left(p_H(e) - p_L(e)\right) b'_H(e) = 0\}$,
   $a_H^s = \tilde{a}(e_H^s)$,
   $b_H^s = b_H(e_H^s)$,

2. *if (LL_H), (IC_H), and (IC_{P_L}) are binding (=corner solution),*
   $e_H^s \in \{e \in [0, \overline{e}] \,|\, \max_e V_H((0, b_H(e)))$ *s.t.* $p_L(e)(\overline{y} - b_H(e)) = V_{LL}(w_L^*)\}$,
   $a_H^s = 0$,
   $b_H^s = b_H(e_H^s)$,

3. *if (IC_{P_L}) is trivially satisfied (=trivial solution),*
   $e_H^s = e_H^*$,
   $a_H^s = 0$,
   $b_H^s = b_H^*$,

with $b_H(e) = c'(e)/p'_H(e)$ and $\tilde{a}(e) = p_L(e)(\overline{y} - b_H(e)) - V_{LL}(w_L^*)$. Moreover, the agent receives a positive rent.

Given the generalized specification of $p_i(e)$ and $c(e)$, solutions are interior if $p_H(e)$ and $p_L(e)$ are sufficiently similar such that $|b^*_H - b^*_{LH}|$ is small. This translates to $|\Delta s/\Delta r - s_L/r_L|$ being sufficiently small.[13] Corner solutions in the sense of Lemma 2.3 occur for intermediate values of $|\Delta s/\Delta r - s_L/r_L|$ for which the distance between b^*_H and b^*_{LH} becomes large enough for pure–premium separation to be optimal. If $|\Delta s/\Delta r - s_L/r_L|$ increases further I receive trivial solutions in which separation is costless since mimicking is not profitable. With complements this resembles a parameter setting with $\Delta r \to -r_L/\theta_H$ and r_L being positive and large. Here, the premium of the H–type project is too high for L–type principals although they receive higher effort levels per premium by misleading agents. With substitutes this is reversed. Here, L–type principals find the premium for H–type projects too low. Parameters are such that Δr is positive and large and $r_L \to 0$. In the following I will focus on interior and corner solutions.

From the previous Lemma and Lemma 2.1 it becomes apparent that the relation of the optimal effort levels for good projects under least–cost separation, e^s_H, and under full symmetric information, e^*_H, crucially depends on the properties of the success probability function $p_i(e)$ $\forall i$. This relation will be analyzed more closely in Proposition 2.2.

I next identify conditions for the least–cost separating contract w^s_H to be optimal for H–type principals relative to the best non–separating contract $(0, b^*_{HL})$. This is important since the Spence–Mirrlees condition is not globally satisfied in this setup. Therefore the H–type principal can have an incentive to deviate from the the least–cost separating contract w^s_H. His best deviation under pessimistic beliefs by the agent, $\mu(H|w \neq w^s_H) = 0$, is given by $(0, b^*_{HL})$ with $b^*_{HL} = \arg\max_b V_{HL}((0, b))$.[14] Deviating becomes more attractive if the zero–effort success probability for H–type project is much higher than for the L–type project, i.e. Δr being relatively large. The H–type's optimal premium is rather low in this case compared to the one of the L–type although the marginal success probability is higher for the H–type project (cf. Lemma 2.1 with $p_H(0) = (r_L + \Delta r\theta_H)$ being large and $p_L(0) = r_L$ being small.) Then, reducing the premium to b^*_{HL} and being perceived as a L–type principal might be less utility decreasing than paying a positive fixed wage $a^s_H > 0$ to separate at a higher premium b^s_H. The following lemma shows that conditions to rule out such deviations are rather mild.

[13] Cf. Proposition 2.2 and Corollary 2.2.1.

[14] $b^*_{HL} = b^s_H$ is unproblematic for interior solutions in the sense of Lemma 2.3 since $a^*_{HL} = 0$ and $a^s_H > 0$. For corner solutions separation is always preferred if $b^*_{HL} = b^s_H$ since the agent chooses a higher effort level for H–type projects.

LEMMA 2.4 (Optimality of Least–Cost Separation): *Suppose project type i is unobservable and the agent holds pessimistic beliefs with respect to the least–cost separating contract,* $\mu(H|w \neq w_H^s) = 0$. *Then the H–type principal weakly prefers the least–cost separating contract* w_H^s *to the optimal non–separating contract* $(0, b_{HL}^*)$, *i.e.* $V_{HH}(w_H^s) \geq V_{HL}((0, b_{HL}^*))$, *if the difference in the zero–effort success probability for the H–type project,* Δr, *is weakly lower than the critical level* Δr^c *with* $\Delta r^c > r_L \cdot \Delta s / s_L$.
Moreover, $\Delta r^c \equiv \min\{\Delta r_1^c, \Delta r_2^c\}$ *with* Δr_1^c *s.t.* $b_{HL}^*(\Delta r) \geq 0$ *for* $\Delta r \leq \Delta r_1^c$ *and* Δr_2^c *s.t.* $(V_{HH}^* - V_{HL}^* - V_{LH}^* + V_{LL}^*)|_{\Delta r} \geq 0$ *for* $\Delta r \leq \Delta r_2^c$ *and* $V_{ij}^* \equiv V_{ij}((0, b_{ij}^*))$.

For $\Delta r_1^c > \Delta r_2^c$ there might be true non–optimality of least–cost separation for extremely high levels of Δr, while for $\Delta r_1^c \leq \Delta r_2^c$ this is never the case since parameters would have to be such that second–best solutions (=observable project type) are not interior which is not considered in this paper. Optimality of least–cost separation, $V_{HH}(w_H^s) \geq V_{HL}((0, b_{HL}^*))$, is necessary and sufficient for existence of a least–cost separating equilibrium. This condition, however, is hard to verify without specific functional forms of $p_i(e)$ and $c(e)$ because the relevant effort levels and wage payments are only implicitly determined by Lemma 2.3 and no general single–crossing property is available in this setup. Before providing a proof of this lemma I transform the necessary and sufficient condition and derive a sufficient condition for existence.

$$
\begin{aligned}
V_{HH}(w_H^s) &\geq V_{HL}((0, b_{HL}^*)) \\
V_{HH}((0, b_H^s)) - a_H^s &\geq V_{HL}((0, b_{HL}^*)) && \text{by separability} \\
V_{HH}((0, b_H^s)) - (V_{LH}((0, b_H^s)) - V_{LL}((0, b_L^*))) &\geq V_{HL}((0, b_{HL}^*)) && \text{by Lemma 2.3} \\
V_{HH}((0, b_H^s)) - V_{HL}((0, b_{HL}^*)) &\geq V_{LH}((0, b_H^s)) - V_{LL}((0, b_L^*)). && (2.8)
\end{aligned}
$$

If the H–type principal sets b_H^s equal to b_H^*, which is obviously suboptimal in general, separation will be purely fixed–wage based, i.e. $a_H^s = V_{LH}((0, b_H^*)) - V_{LL}((0, b_L^*))$. Thus, $V_{HH}((0, b_H^*)) - V_{HL}((0, b_{HL}^*)) \geq V_{LH}((0, b_H^*)) - V_{LL}((0, b_L^*))$ suffices for existence. I obtain the following second sufficient condition for existence, since $V_{LH}((0, b_H^*)) \leq V_{LH}((0, b_{LH}^*))$ with $b_{LH}^* = \arg\max_{b_{LH}>0} V_L((0, b_{LH}), \mu(H|.) = 1)$, being the L–type's hypothetically optimal premium if L–type principal is perceived as the H–type principal w.p.o.,

$$V_{HH}((0, b_H^*)) - V_{HL}((0, b_{HL}^*)) \geq V_{LH}((0, b_{LH}^*)) - V_{LL}((0, b_L^*)). \tag{2.9}$$

It can be seen that this condition compares the optimal contracts for each type i of principal being perceived as type j w.p.o. by the agent.[15] The proof of Lemma 2.4 is presented in the appendix.

I finally conjecture that even without $p_i(e)$ being an affine function of θ_i there will be optimality of least–cost separation for $p_H(0)$ not too high relative to $p_L(0)$. However, this is much harder to show without using specific functional forms. In a subsequent example I provide evidence that the necessary and sufficient condition for existence (2.8) is always satisfied under the standard assumptions of this model if explicit functional forms for $p_i(e)$ and $c(e)$ are considered. This translates into $\Delta r_1^c < \Delta r_2^c$ which means that the condition for second–best solutions being interior is reached at a lower level of Δr than the condition for least–cost separation being optimal.

The implementation of the least–cost separating contract is presented next.

Proposition 2.1: *Suppose project type i is unobservable and* Δr *is weakly lower than* $\Delta r^c > r_L \cdot \Delta s / s_L$*. Then, there exists an equilibrium* $\sigma = (\sigma_P, \sigma_A, \mu)$ *which implements the least–cost separating contract, i.e.* $\sigma_P(w_L^*|L) = 1, \sigma_P(w_H^s|H) = 1$, $\sigma_A(1|w_L^*) = 1, \sigma_A(1|w_H^s) = 1$, *and* $\mu(H|w_H^s) = 1$*. Moreover,* any *equilibrium that satisfies the intuitive criterion implements the least–cost separating contract.*

I now turn to the comparison of equilibrium properties between the game with private information and symmetric full information. The following proposition states the main efficiency result of this paper, namely that in the game with less public information efficiency can increase for the high–type principal.

Proposition 2.2: *Suppose* $p_i(e)$ *and* $c(e)$ *are such that the incentive constraint of the L–type principal is not trivially satisfied by* w_H^**. Then in the game with* ***less*** *prior information, efficiency with respect to the agent's effort choice* ***increases*** *in the contract of the H–type principal, (i.e. the equilibrium effort level* e_H^s *in the least–cost separating contract* w_H^s *is* ***higher***

[15] This condition can be related to supermodularity of V_{ij} in true type i and perceived type j with $i, j \in \{L, H\}$. But supermodularity of V_{ij} is not sufficient to show that the condition holds true since each V_{ij} is additionally maximized over e.

than the second–best effort level e_H^* under symmetric full information), if and only if

$$\eta_H(e_H^*) > \eta_L(e_H^*). \tag{2.10}$$

Here, $\eta_i(e) \equiv e \cdot p_i'(e)/p_i(e)$ is defined as the effort elasticity of project i's success probability function at effort e. The corresponding fixed–wage and premium payments a_H^s and b_H^s are given in Lemma 2.3.

The effort elasticity of the success probability function reflects the principal's trade–off between a higher marginal success probability, which rises principal's revenue, and a higher marginal premium payment if the principal increases the induced effort level at a certain point.[16] If the net effect of this is larger for the H–type principal than for the mimicking L–type principal at the second–best effort level for good projects e_H^*, then the H–type principal will increase the induced effort level in the least–cost separating contract. This demonstrates a situation with a high degree of complementarity between effort and project type. For the conductor–orchestra example this implies that after dismissal of the previous conductor due to bad performance an external conductor will receive a higher premium from a good orchestra than an internal or informed successor of similar track record. Thus, the third–best effort level is higher than the second–best effort level. For a situation with effort and project type being substitutes I predict the reverse result.

The previous proposition uses a local property of the success probability functions $p_H(e)$ and $p_L(e)$. However, it can be shown that the sign of $\eta_H(e) - \eta_L(e)$ is constant globally under given assumptions.

Corollary 2.2.1: *For $p_i(e) = (r_L + \Delta r\theta_i) + (s_L + \Delta s\theta_i) \cdot p(e)$ with $p(0) = 0$, $\theta_L = 0$ and $\theta_H > 0$ the sign of $(\eta_H(e) - \eta_L(e))$ is constant for all e and θ_H. It is positive if and only if $\Delta s/\Delta r > s_L/r_L$.*

This means that for $\Delta r < r_L\Delta s/s_L$ effort and project type are complements for all effort levels, while they are substitutes for $\Delta r \geq r_L\Delta s/s_L$. A proof is provided in the appendix.

I next show a closed–form solution of w_H^s in a parametric example. Consider a success proba-

[16] Cf. the first–order condition of the H–type principal's problem in case 1 of Lemma 2.3.

bility function with $p(e)$ being linear, i.e. $p_i(e) = (r_L + \Delta r\theta_i) + (s_L + \Delta s\theta_i)\cdot e$, and a quadratic cost of effort function so that $c(e) = \gamma/2 \cdot e^2$ with $\gamma > 0$. Then the least–cost separating contracts $(w_L^s, w_H^s) = ((0, b_L^*), (a_H^s, b_H^s))$ for both types of principal are described by

$$b_L^* = \frac{1}{2}\overline{y} - \frac{\gamma r_L}{2s_L^2},$$

$$a_H^s = \begin{cases} \frac{s_L \Delta s\theta_H \overline{y}^2}{4\gamma} - \frac{\gamma\left(\Delta r^2 s_L^3 - 2r_L\Delta r\Delta s s_L^2 + r_L^2\Delta s^2(s_L+\Delta s\theta_H)\right)}{4s_L^2\Delta s^2(s_L+\Delta s\theta_H)}, & \text{if only } (IC_H) \text{ and } (IC_{P_L}) \text{ bind} \\ 0, & \text{otherwise} \end{cases}$$

$$b_H^s = \begin{cases} \frac{1}{2}\overline{y} - \frac{\gamma\Delta r}{2\Delta s(s_L+\Delta s\theta_H)}, & \text{if } (IC_H) \text{ and } (IC_{P_L}) \text{ bind} \\ \tilde{b}, & \text{if } (LL_H), (IC_H), \text{ and } (IC_{P_L}) \text{ bind} \\ b_H^* = \frac{1}{2}\overline{y} - \frac{\gamma(r_L+\Delta r\theta_H)}{2(s_L+\Delta s\theta_H)^2}, & \text{if is } (IC_{P_L}) \text{ trivially satisfied} \end{cases}$$

with

$$\tilde{b}_{1/2} = \frac{(s_L + \Delta s\theta_H)\overline{y}s_L^3 - \gamma r_L s_L^2 \pm \sqrt{s_L^3\Delta s\theta_H\left(s_L^3(s_L + \Delta s\theta_H)\overline{y}^2 - \gamma^2 r_L^2\right)}}{2s_L^3(s_L + \Delta s\theta_H)}$$

and $\tilde{b} = \tilde{b}_1$ if $\Delta s/\Delta r > s_L/r_L$ and $\tilde{b} = \tilde{b}_2$ otherwise. These results are directly derived from Lemma 2.3. The corresponding effort levels can be determined by $e_i^s = b_i^s \cdot (s_L + \Delta s\theta_i)/\gamma$, which directly follows from the first–order condition of the agent's problem (FOC_A).

It can be shown here that (IC_{P_L}) is trivially satisfied for r_L being very large or Δr being very large. Moreover, if (IC_{P_L}) is not trivially satisfied, then $b_H^s > b_H^*$ if and only if $\Delta s/\Delta r > s_L/r_L$, which follows directly from Proposition 2.2 and Corollary 2.2.1. It can also be shown here by rearranging $b_H^s = \frac{1}{2}\overline{y} - \frac{\gamma\Delta r}{2\Delta s(s_L+\Delta s\theta_H)} > b_H^* = \frac{1}{2}\overline{y} - \frac{\gamma(r_L+\Delta r\theta_H)}{2(s_L+\Delta s\theta_H)^2}$. Furthermore, $\tilde{b}_1 > b_H^*$ and $\tilde{b}_2 < b_H^*$ for $\theta_H > 0$, and $\tilde{b}_1 = \tilde{b}_2 = b_H^*$ for $\theta_H \to 0$.

For the interior solution case of Lemma 2.3, the necessary and sufficient condition for optimality of least–cost separation, $V_{HH}(w_H^s) \geq V_{HL}((0, b_{HL}^*))$, becomes equivalent to

$$s_L\Delta s\theta_H^2\left(s_L\Delta s^2\overline{y}^2(s_L + \Delta s\theta_H) + \gamma^2\frac{(r_L\Delta s - \Delta r s_L)^2}{s_L\Delta s\theta_H} - \gamma^2\Delta r^2\right) \geq 0. \tag{2.11}$$

It collapses to $(s_L\Delta s(s_L + \theta_H\Delta s)(s_L + 2\theta_H\Delta s)\overline{y}^2)/\theta_H$ for $\Delta r = 0$ and to $\theta_H s_L\Delta s^3\overline{y}^2$ for $\Delta r =$

$r_L \cdot \Delta s/s_L$, both of which are strictly positive. For $\Delta r > r_L \cdot \Delta s/s_L$ it can be shown that (2.11) remains strictly positive if $b^*_{HL}(\Delta r) = 1/2\left(\bar{y} - \gamma(r_L + \Delta r\theta_H)/((s_L + \Delta s\theta_H)s_L)\right)$ becomes negative, i.e. least–cost separation is always optimal here (cf. Lemma 2.4).

A similar argument can be made for the corner solution case of Lemma 2.3.

Pooling Equilibria

A disadvantage of the intuitive criterion applied in Proposition 2.1 is that its selection is not sensitive to the prior distribution of types. This is particularly worrisome if the probability of H–type projects β is close to one and therefore costly separation is very likely from an ex ante point of view. In this situation offering a pooling contract can be beneficial for the H–type principal since for $\beta \to 1$, his optimal pooling contract approaches the second–best contract under observable project type that is strictly more profitable than any nontrivial least–cost separating contract. This is true since nontrivial separation adds another binding constraint to the H–type principal's problem.

To allow for optimal pooling equilibria of this kind I depart from the intuitive criterion in the next proposition. A different selection procedure—namely, lexicographical maximum selection—is introduced. This concept is borrowed from Inderst (2001) and initiates a weakly stronger selection than the undefeated equilibrium concept by Mailath, Okuno-Fujiwara, and Postlewaite (1993).[17] The set of lexicographical maximum equilibria is defined by $M^* \equiv M_L(M_H(\Sigma))$ with Σ being a compact subset of the set of PBE and $M_i(\Sigma)$ the set of equilibria maximizing the payoff of type i.

The following proposition shows that for given priors β all equilibria in M^* implement a unique allocation. There exists a cutoff level $\beta^c \in (0, 1)$ such that all equilibria in M^* implement the least–cost separating allocation for $\beta < \beta^c$, while for $\beta \geq \beta^c$ all equilibria in M^* implement the unique optimal pooling allocation for the H–type principal.

Let $b^P(e|\beta)$ depict the premium payment such that the agent exerts effort e for given priors

[17] In contrast to Mailath's undefeated equilibrium concept the lexicographical maximum equilibrium concept selects a unique equilibrium in this game.

$\beta \in [0,1]$. $b^P(e|\beta)$ is derived from the agent's problem and can be expressed by

$$b^P(e|\beta) = \frac{c'(e)}{\mathbb{E}_i[p_i'(e)]} = \frac{c'(e)}{\Big(\beta p_H'(e) + (1-\beta)p_L'(e)\Big)}. \tag{2.12}$$

Let $e^{PO}(\beta)$ describe the effort level induced in the H–type's optimal pooling equilibrium which is determined by the first–order condition of the H–type's problem given prior beliefs β,

$$p_H'(e)\Big(\bar{y} - b^P(e|\beta)\Big) - p_H(e)b'^P(e|\beta) = 0, \tag{2.13}$$

for all $\beta \in [0,1]$. $e^{PO}(\beta) \in [e^*_{HL}, e^*_H]$ since by construction $e^{PO}(0) = e^*_{HL}$ and $e^{PO}(1) = e^*_H$. Analogously, the optimal pooling premium for the H–type principal, $b^{PO}(\beta) \equiv b^P(e^{PO}(\beta)|\beta)$, satisfies $b^{PO}(\beta) \in [b^*_{HL}, b^*_H]$ with $b^{PO}(0) = b^*_{HL}$ and $b^{PO}(1) = b^*_H$. The following proposition shows that there always exists a cutoff level for the prior probability of facing a good project β such that the optimal pooling equilibrium for the H–type principal generates a higher expected surplus for the H–type principal than the least–cost separating equilibrium.

Proposition 2.3: *Suppose project type i is unobservable, Δr is lower than $\Delta r^c > r_L \cdot \Delta s/s_L$, and the incentive constraint of the L–type principal is not trivially satisfied by w^*_H. Then there exists a unique value $\beta^c \in (0,1)$ such that for $\beta < \beta^c$ all equilibria $\sigma \in M^*$ specify the least–costs separating contract (w^s_H, w^*_L), while for $\beta \geq \beta^c$ they specify the optimal pooling contract of the high–type principal $w^{PO}(\beta) = (0, b^{PO}(\beta))$ with $b^{PO}(\beta) \equiv b^P(e^{PO}(\beta)|\beta)$.*

The next proposition describes the motion of the effort level induced by the H–type principal as a function of the prior probability of facing a H–type principal β for the lexicographical maximum equilibrium.

Proposition 2.4: *Consider the lexicographical maximum equilibrium from above. For $\beta < \beta^c$ the effort level induced by the H–type principal is constant at a level of e^s_H, while at $\beta = \beta^c$ there is a discontinuity in e_H:*

a.) *If $\Delta r < r_L \Delta s/s_L$ (=effort and project type are complements), then e_H jumps downward to $e^{PO}(\beta^c)$ and for $\beta > \beta^c$ is strictly increasing up to $e^{PO}(1) = e^*_H < e^s_H$.*

b.) *If $\Delta r \geq r_L \Delta s/s_L$ (=effort and project type are substitutes), then e_H jumps downward to*

$e^{PO}(\beta^c)$ for Δr close to $r_L \Delta s/s_L$ and upward to $e^{PO}(\beta^c)$ forΔr much larger than $r_L \Delta s/s_L$. For $\beta > \beta^c$, e_H is strictly increasing up to $e^{PO}(1) = e^*_H > e^s_H$.

Analogously, the effort level induced by the L–type principal is constant at e^*_L for $\beta < \beta^c$ and for $\beta \geq \beta^c$ is identical to the effort level induced by the H–type principal. If $\beta \geq \beta^c$, then the induced effort level is always weakly lower than the second–best effort level for the H–type principal, which states a negative efficiency result for high levels of β. The induced effort level can be higher or lower than the second–best effort level for the L–type principal dependent on the the sign of $e^*_{HL} - e^*_L$. E.g. for extreme substitutes with Δr being much larger than $r_L \Delta s/s_L$, it holds that $e^*_{HL} < e^*_H < e^*_L$. Here, $e^{PO}(\beta)$ is lower than e^*_L for all $\beta \geq \beta^c$ because it always holds that $e^{PO}(\beta) \leq e^*_H$. For the conductor–orchestra example the previous proposition implies that there shouldn't arise wage differences for external successors between good or bad orchestras if the probability of facing a good orchestra is very high after early dismissal or resignation of a conductor. However, the offered wage premium will be lower than the one for internal successors in good orchestras.

In the parametric example with linear success probability, $b^{PO}(\beta)$ is determined by

$$b^{PO}(\beta) = \frac{1}{2}\bar{y} - \frac{\gamma(r_L + \Delta r \theta_H)}{2(s_L + \Delta s \theta_H)(s_L + \beta \cdot \Delta s \theta_H)}.$$

The corresponding effort level follows from $e^{PO}(\beta) = b^{PO}(\beta) \cdot (s_L + \beta \cdot \Delta s \theta_H)/\gamma$.

2.3 Discussion

Proposition 2.2 states that if effort and project type are complements, then the principal's private information about the project type increases the induced effort level for good projects above its level under full observability of project type. This rather surprising efficiency result is very general with respect to the functional form of the success probability function. In fact, the proof of Proposition 2.2 does not even require the used affine representation of $p_i(e)$ in θ_i. This representation is only necessary to be able to show optimality of least–cost separation without using specific functional forms.

The positive efficiency result relies on the assumption that contracting under full symmetric information is already second–best with respect to effort choice, i.e. the sole moral hazard problem between the principal and the agent already generates a distortion of the effort choice by itself. Such distortions of the effort choice can be caused by limited liability or risk aversion on the agent's side. In my setup the positive efficiency result disappears, if the limited liability constraint is relaxed. To see this more clearly, consider the modified limited liability condition

$$a \geq -l \quad \text{with } l \geq 0. \tag{LL'}$$

For $l \to \infty$, the effort induced by the H–type's least–cost separating contract $e_H^s(l)$ remains constant, while his second–best effort level $e_H^*(l)$ converges to first–best, i.e. to e_H^{FB} with $e_H^{FB} > e_H^s(l)\ \forall l$.

Furthermore, the positive efficiency result is robust to an extension to continuous outcome. This can be shown, for instance, in a setup with CARA utility and normally distributed outcome following Holmstrom and Milgrom (1994). Here, the agent's effort choice determines the mean of the outcome distribution. Let the bad project's mean of outcome be equal to the agent's effort choice, while the good project's mean of outcome is an affine function of effort with a positive slope larger than one. Then the positive efficiency result for good projects can be reproduced under a negative relationship between the intercept and the slope of the agent's affine production function, if the degree of the agent's risk aversion and the variance of the outcome distribution are sufficiently high. Again, there is a trade–off between marginal expected revenue of effort and expected revenue at zero effort. Moreover, the distortion that arises in the pure moral hazard problem must be sufficiently large.

2.4 Conclusion

In this paper I consider a one–shot moral hazard problem between a risk–neutral principal and a risk–neutral agent protected by limited liability in which the principal holds private information that is output–relevant and thus of common value. Surprisingly, this simple combination of moral hazard and asymmetric information on the principal's side has not been studied intensively in the literature. I analyze the game by deriving separating equilibria in which the principal signals his type purely via wage offers to the agent if other means of information disclosure are not at hand. The lack of a single crossing property constitutes a special difficulty of this model. Therefore it is much harder to show optimality of separation under favorable information and thus existence of a separating equilibrium. In contrast to the standard problem with moral hazard and private information on the agent's side as well as in contrast to the related informed principal model of Inderst (2001), I find that in certain setups contracting will become more efficient with respect to agent's effort choice if output–relevant information becomes private to the principal. This states a novel efficiency result in which the third–best effort level dominates the second–best one.[18]

The intuition behind this result is that in certain setups the principal with favorable information wants to recoup efficiency relative to the pure moral hazard problem under full symmetric information to prevent the principal with unfavorable information from mimicking. However, the effect can be directed in either way depending on the ratio of principal's marginal expected revenue to principal's total expected revenue of effort for both kinds of information. This ratio can be expressed by the effort elasticity of expected revenue. My prediction is that efficiency is affected positively through private information on the principal's side if the effort elasticity of expected revenue is higher under favorable information. I also provide evidence that this result is robust to an extension to continuous outcome.

This model applies to new employment contracts after early dismissal or resignation of professional conductors, coaches, CEOs, or politicians. Here, urgency of replacement often prevents other means of information disclosure as e.g. offering time for consideration or talks with other employees to potential successors. Thus, signaling takes place via wage offers. There is also a clear moral hazard problem with respect to successor's future effort decision and successor's ability is observable due to his track record. Moreover, external successors for these kinds of jobs clearly face an informational disadvantage, since they cannot fully access whether the early replacement of the predecessor was also related to the quality of the work environment. On the other hand, a potential external successor receives some information open to the public about the replacement. If e.g. an orchestra performed badly before the dismissal of the predecessor, then there is no space for free–riding in the new position even if the orchestra is of high quality. This is reversed, if the orchestra performed well lately before the predecessor resigned. Thus in the former case, I predict higher premium payments for external successors (=private information on the principal's side) than for internal ones of similar track record (=full symmetric information) and lower premium payments for external successors than for internal ones in the latter case.

[18] The first–best effort level is never reached due to limited liability on the agent's side.

Chapter 3

Pricing and Information Disclosure in Markets with Loss–Averse Consumers

3.1 Introduction

Consumer information about price and match value of products is a key ingredient in determining market outcomes. Previous work has emphasized the role of consumer information at the moment of purchase.[1] If consumers are loss–averse information prior to the moment of purchase matters: Product information plays an important role at the stage at which loss–averse consumers form expectations about future transactions. Our analysis applies to inspection goods with the feature that consumers readily observe prices in the market but have to inspect products before knowing the match value between product characteristics and consumer tastes.

Loss–aversion in consumer choice has been widely documented in a variety of laboratory and field settings starting with Kahneman and Tversky (1979). Loss–averse consumers have to form expectations about product performance. We postulate that, to make their consumption choices, loss–averse consumers form their probabilistic reference point based on expected future transactions which are confirmed in equilibrium. Here, a consumer's reference point is her probabilistic belief about the relevant consumption outcome held between the time she first focused on the decision determining the consumption plan—i.e., when she heard about the products, was informed about the prices for the products on offer, and formed her

[1] See e.g. Varian (1980), Janssen and Moraga-Gonzlez (2004), and Armstrong and Chen (2008).

expectations—and the moment she actually makes the purchase.[2]

We distinguish between “informed” and “uninformed” customers at the moment consumers form their reference point. Informed consumers know their taste ex ante and will perfectly foresee their equilibrium utility from product characteristics. Therefore they will not face a loss or gain in product satisfaction beyond their intrinsic valuation.

Uninformed consumers, by contrast, are uncertain about their ideal product characteristic: they form expectations about the difference between ideal and actual product characteristic which will serve as a reference point when evaluating a product along its taste or match value dimension. They will also face a gain or a loss relative to their expected distributions of price after learning the taste realization. Since all consumers become fully informed before they have to make their purchasing decision, we isolate the effect of consumer loss aversion on consumption choices and abstract from the effects of differential information at the moment of purchase.[3]

In this paper, we study the competitive effects of firm asymmetry and consumer loss aversion in duopoly markets. Consumers are loss–averse with respect to prices and match value and have rational expectations about equilibrium outcomes to form their reference point, as in Heidhues and Koszegi (2008). Firms are asymmetric due to deterministic cost differences and this is common knowledge among the firms when the game starts.[4] Firms compete in prices for differentiated products. Prices are deterministic and possibly asymmetric. Consumers observe equilibrium prices before forming their reference point. Note that if prices are asymmetric, uninformed consumers will face either a loss or a gain in the price dimension depending on which product they buy. Hence, an (ex ante) uninformed consumer’s realized net utility depends not only on the price of the product she buys but also on the price of the product she does not buy.

Our theory applies to a number of inspection good industries in which some consumers form expectations before knowing the match value a particular product offers. Let us provide some examples. First, prices of clothing and electronic devices are easily accessible (and are often

[2]For evidence that expectation–based counterfactuals can affect the individual’s reaction to outcomes, see Breiter, Aharon, Kahneman, Dale, and Shizgal (2001), Medvec, Madey, and Gilovich (1995), and Mellers, Schwartz, and Ritov (1999). The general theory of expectation–based reference points and the notion of personal equilibrium have been developed by Koszegi and Rabin (2006) and Koszegi and Rabin (2007).

[3]Our model can alternatively be interpreted as one in which consumers know their ideal taste ex ante but are exposed to uncertainty about product characteristics when they form their reference point.

[4]In the extension section we show that our analysis also applies to products of different qualities.

advertised) in advance while, for inexperienced consumers, the match quality between product and personal tastes is impossible or difficult to evaluate before actually seeing or touching the product. A related example is high–end hifi–equipment and, in particular, loudspeakers. Price tags are immediately observed but it may take several visits to the retailers (on appointment) or even trials at home to figure out the match value of the different products under consideration—for example, because people differ with respect to the sound they like. In these markets potential cost differences may arise from size differences of producers and product–specific costs (or, as we allow in our extension, from different ex ante observable quality differences). Second, the housing market has the feature that the price is listed (and, in some countries, not negotiable) whereas the match value is only found out after visiting the flat. Third, price information on products sold over the internet—for example, CDs of a particular classical concert—is immediately available, while the match value is often determined only after listening to some of the material that is provided online. Fourth, competing services such as long–distance bus rides and flights are differentiated by departure times. Here consumers are perfectly aware of the product characteristics ex ante—i.e.,price and departure time—but learn their preference concerning their ideal point of departure only at some later stage (after forming their probabilistic reference point but before purchase).

Our first main result is that, in asymmetric markets, price variation is increased, relative to the scenario without loss–averse consumers. This is in stark contrast to the focal price result by Heidhues and Koszegi (2008).[5]

Our second main result is that loss aversion—or, more precisely, the presence of more ex ante uninformed, loss averse consumers—may lead to lower prices. Hence, the standard result that more informed consumers (or more consumers without a behavioral bias) lead to lower prices is challenged in our model when firms are strongly asymmetric. The driving force behind this result is that loss aversion in the price dimension has a pro–competitive effect while the effect of loss aversion in the taste dimension is anti–competitive.[6] The pro–competitive effect dominates the anti–competitive effect if the size of loss aversion in the price dimension becomes sufficiently large. This occurs if the price difference is large, which is caused by

[5]In a related setting to ours, Heidhues and Koszegi (2008) show that consumer loss aversion can explain the empirical observation that firms often charge the same price in differentiated product markets even if they have different costs. One of the distinguishing features of our model is that realized costs are public information and consumers observe prices before forming their reference point.

[6]Note that this is different from Heidhues and Koszegi (2008) where loss aversion has an anti–competitive effect in both dimensions.

strong cost asymmetries. In this situation uninformed consumers are very reluctant to buy the expensive product and rather accept a large reduction in match value when buying the low–price product.

This paper contributes to the understanding of the effect of consumer loss aversion in market environments and is complementary to Heidhues and Koszegi (2008). More broadly, it contributes to the analysis of behavioral biases in market settings, as in Eliaz and Spiegler (2006), Gabaix and Laibson (2006), and Grubb (forthcoming). An important issue in our paper, as also in Eliaz and Spiegler (2006), is the comparative statics effects in the composition of the population. However, whereas in their models this composition effect is behavioral in the sense that the share of consumers with a behavioral bias changes, we do not need to resort to this interpretation although our analysis is compatible with it: We stress the composition effect to be informational in the sense that the arrival of information in the consumer population is changed (while the whole population is subject to the same behavioral bias).

The informational interpretation lends itself naturally to address questions about the effect of early information disclosure to additional consumers. We analyze information disclosure policies by firms and public authorities in the context of a behavioral industrial organization framework. We thus demonstrate the possible use of behavioral models to address policy questions in industrial organization. As stated above, our model has the feature that, absent behavioral bias, information disclosure policies are meaningless. Thus the behavioral bias is essential in our model to address these issues. In particular, we show that private and social incentives to disclose information early on are not aligned. We also show that the more efficient and thus larger firm discloses information if firms have conflicting interests.

Our analysis contributes to the literature on the economics of advertising (see Bagwell (2007) for an excellent survey). It uncovers the role of advertising as consumer expectation management. Note that at the point of purchase consumers are fully informed so that there is no role for informative advertising. However, since consumers are loss–averse, educating consumers about their preferences or, alternatively, about product characteristics, makes these consumers informed in our terminology. Advertising thus can remove the uncertainty consumers face when forming their reference point. This form of advertising can be seen as a hybrid form of informative and persuasive advertising because it changes preferences at the point of purchase—this corresponds to the persuasive view of advertising—, albeit due to information that is received ex ante—this corresponds to the informative view of advertising. It also points to the importance of the timing of advertising: for expectation management it is important to inform consumers early on.

Other marketing activities can also be understood as making consumers informed at the stage when consumers form their reference point. For instance, test drives for cars or lending out furniture, stereo equipment, and the like make consumers informed early on. Arguably, in reality uncertainty would otherwise not be fully resolved even at the purchasing stage. However, to focus our minds, we only consider the role of marketing activities on expectation formation before purchase. In short, in our model firms may use marketing to manage expectations of loss–averse consumers at an early stage.[7]

Our paper can be seen as complementary to the work on consumer search in product markets (see e.g. Varian (1980), Anderson and Renault (2000), Janssen and Moraga-Gonzlez (2004), Armstrong and Chen (2008)). Whereas that literature focuses on the effect of differential information (and consumer search) at the purchasing stage, our paper abstracts from this issue and focuses on the effect of differential information at the expectation formation stage which is relevant if consumers are loss aversion.

We will discuss the connections to a number of the above cited contributions in more detail in the main text. The plan of this chapter is as follows. In Section 3.2, we present the model. Here, we have to spend some effort to determine the demand of uninformed consumers. In Section 3.3, we establish equilibrium uniqueness and equilibrium existence. Our existence proof requires to bound the parameters of our model, in particular, the two firms cannot be too asymmetric for equilibrium existence to hold. In Section 3.4, we obtain comparative statics results. First, we characterize equilibrium under cost symmetry and, secondly, analyze the impact of the degree of asymmetry on equilibrium outcomes. Thirdly and most importantly, we analyze the effect of changing the share of ex ante informed consumers on market outcomes. In Section 3.5 we provide two extensions. Section 3.6 concludes.

[7] For a complementary view see Bar-Isaac, Caruana, and Cunat (2007).

3.2 The Model

3.2.1 Setup

Consider a market with two asymmetric firms, A and B, and a continuum of loss–averse consumers of mass 1. The firms' asymmetry consists of differences in marginal costs. Here, the more efficient firm is labeled to be firm A—i.e., $c_A \leq c_B$. Firms are located on a circle of length 2 with maximum distance, $y_A = 0$, $y_B = 1$. Firms announce prices p_A and p_B and product locations to all consumers. Consumers of mass one are uniformly distributed on the circle of length 2. A consumer's location x, $x \in [0, 2)$, represents her taste parameter. Her taste is initially, i.e., before determining her reference point, known only to herself if she belongs to the set of informed consumers. Note that consumers' differential information here applies to the date at which consumers determine their reference point and not to the date of purchase: at the moment of purchase all consumers are perfectly informed about product characteristics, prices, and tastes. However, a fraction $(1-\beta)$ of loss–averse consumers, $0 \leq \beta \leq 1$, is initially uninformed about their taste. As will be detailed below, they endogenously determine their reference point and then, before making their purchasing decision, observe their taste parameter (which is private information of each consumer). All consumers have reservation value v for an ideal variety and have unit demand. Their utility from not buying is $-\infty$ so that the market is fully covered.

Two remarks about our modeling choice are in order: First, we could alternatively work with the Hotelling line. Results directly carry over to the Hotelling model in which consumers are uniformly distributed on the $[0, 1]$–interval. Second, the circle model allows for an alternative and equivalent interpretation about the type of information some consumers initially lack: at the point in time consumers form their reference point distribution, they all know their taste parameters but only a fraction $(1-\beta)$ does not know the location of the high– and the low–cost firm. These uninformed consumers only know that the two firms are located at maximal distance and that one is a high– whereas the other is a low–cost firm.

To determine the market demand faced by the two firms, let the informed consumer type in $[0, 1]$ who is indifferent between buying good A and good B be denoted by $\hat{x}_{in}(p_A, p_B)$. Correspondingly, the indifferent uninformed consumer is denoted by $\hat{x}_{un}(p_A, p_B)$. Since market shares on $[0,1]$ and $[1,2]$ are symmetric, the firms' profits are:

$$\pi_A(p_A, p_B) = (p_A - c_A)[\beta \cdot \hat{x}_{in}(p_A, p_B) + (1-\beta) \cdot \hat{x}_{un}(p_A, p_B)]$$
$$\pi_B(p_A, p_B) = (p_B - c_B)[\beta \cdot (1 - \hat{x}_{in}(p_A, p_B)) + (1-\beta) \cdot (1 - \hat{x}_{un}(p_A, p_B))].$$

The timing of events is as follows:

Stage 0.) Marginal costs (c_A, c_B) realize (and become common knowledge among firms)

Stage 1.) Firms simultaneously set prices (p_A, p_B)

Stage 2.) All consumers observe prices and

a) informed consumers observe their taste x (for them uncertainty is resolved)

b) uninformed consumers form reference point distribution over purchase price and match value, as detailed below

Stage 3.) Inspection stage: Entering the shop also uninformed consumers observe their taste x (uncertainty is resolved for **all** consumers)

Stage 4.) Purchase stage: Consumers decide which product to buy:

a) informed consumers make rational purchase decision (≡ benchmark case)

b) (ex ante) uninformed consumers compare price and match value (of each product) with the reference point distribution and choose the most appealing product

At stage 1 we solve for subgame perfect Nash equilibrium, where firms foresee that uninformed consumers play a personal equilibrium at stage 2b. Personal equilibrium in our context simply means that consumers hold rational expectation about their final purchasing decision; for the general formalization see Koszegi and Rabin (2006). Without loss of generality we consider realizations $c_A \le c_B$.

3.2.2 Demand of informed consumers

Let us first consider informed consumers. They ex ante observe prices and their taste parameter and therefore do not face any uncertainty when forming their reference point. Hence, their behavior is the same as the behavior of unboundedly rational consumers in a classical Salop model. For prices p_A and p_B an informed consumer located at x obtains the following indirect utility from buying product i

$$u_i(x, p_i) = v - t|y_i - x| - p_i,$$

where t scales the disutility from distance between ideal and actual taste on the circle. The expression $v - t|y_i - x|$ then captures the match value of product i for consumer of type x. Denote the indifferent (informed) consumer between buying from firm A and B on the first half of the circle by $\hat{x}_{in} \in [0, 1]$ and solve for her location given prices. The informed indifferent consumer is given by

$$\hat{x}_{in}(p_A, p_B) = \frac{(t + p_B - p_A)}{2t}. \tag{3.1}$$

Symmetrically, a second indifferent (informed) consumer type is located at $2 - \hat{x}_{in}(p_A, p_B) \in [1, 2]$. Without loss of generality we focus on demand of consumers between 0 and 1 and multiply by 2. Cost differences influence the location of indifferent consumers via prices: If asymmetric costs lead to asymmetric prices in equilibrium, then the indifferent informed consumer will also be located apart from 1/2 (resp. 3/2), the middle between A and B.[8]

3.2.3 Demand of uninformed consumers

Uninformed consumers do not know their ideal taste x ex ante. Since they cannot judge which product they will buy before they inspect products and learn their ideal taste x, they ex ante face uncertainty about their match value and purchase price (although they know firms' prices already). With regard to this uncertainty uninformed consumers form reference point distributions over match value and purchase price. Following Heidhues and Koszegi (2008) they will experience gains or losses in equilibrium depending on their realized taste and their purchase decision. These gains and losses occur in two dimensions, in a taste dimension (as determined by the fit between idiosyncratic taste and product characteristics) and in a price dimension. In both dimensions losses are evaluated at a rate λ and gains at a rate 1 with $\lambda > 1$. This reflects widespread experimental evidence that losses are evaluated more negatively than gains.

[8]E.g. if there are only informed consumers, $\hat{x}_{in} = 1/2 + (c_B - c_A)/(6t)$ in equilibrium . This is closer to B for $c_B > c_A$. Thus, the low–cost firm serves a larger market share.

Three properties of this specification are worthwhile pointing out. First, consumers have gains or losses not about net utilities but about each product "characteristic", where price is then treated as a product characteristic. This is in line with much of the experimental evidence on the endowment effect; for a discussion see e.g. Koszegi and Rabin (2006). Second, consumers evaluate gains and losses *across* products.[9] This appears to be a natural property for consumers facing a discrete choice problem: they have to compare the merits of the two products to each other. In other words, consumers view the purchasing decision with respect to these two problems as a single decision problem. Third, to reduce the number of parameters, we assume that the gain/loss parameters are the same across dimensions. This appears to be the natural benchmark.

While our setting is related to Heidhues and Koszegi (2008) (see also Heidhues and Koszegi (2005) for a related monopoly model) our model has three distinguishing features. First, firms' deterministic costs are known by their competitor. This property is in line with a large part of the industrial organization literature on imperfect competition and is approximately satisfied in markets in which firms are well–informed not only about their own costs but also about their relative position in the market. Second, prices are already set before consumers form their reference point.[10] This property applies to markets in which consumers are from the start well–informed about the price distribution they face in the market. This holds in markets in which firms inform consumers about prices (but consumers are initially uncertain about the match value and thus their eventual purchasing decision) or in which prices are publicly posted.[11] Third, there is a fraction of $(1-\beta)$ of uninformed consumers who face uncertainty about their ideal taste x and a fraction of β informed consumers who know their ideal taste ex ante. As motivated in the introduction, various justifications for differential information at the ex ante stage can be given. Consumers differ by their experience concerning the relevant product feature. Alternatively, a share of consumers know that they will be subject to a taste shock between forming their reference point and making their purchasing decision. These consumers then do not condition their reference point on the ex ante taste parameter, whereas

[9] Gains and losses also matter in the price dimension because, even though prices are deterministic, they are different across firms. Hence, a consumer who initially does not know her taste parameter is uncertain at this point in time about the price at which she will buy.

[10] This is particularly appropriate in market environments in which price information has been provided from the outset, while uninformed (or inexperienced) consumers observe the match value only when physically or virtually inspecting the product.

[11] Note that in an asymmetric market firms set different prices. Hence, although prices are deterministic, a consumer who does not know her taste parameter is uncertain about the price she will pay for her preferred product.

those belonging to the remaining share do.

Consider an uninformed consumer who will be located at x after her ideal taste is realized. Suppose firms set prices p_A and p_B in equilibrium. Then the uninformed consumer will buy from firm A if $x \in [0, \hat{x}_{un}(p_A, p_B)] \cup [2 - \hat{x}_{un}(p_A, p_B), 2]$, where $\hat{x}_{un}(p_A, p_B)$ is the location of the indifferent (uninformed) consumer we want to characterize. Hence, the uninformed consumer at x will pay p_A in equilibrium with $Prob[x < \hat{x}_{un}(p_A, p_B) \vee x > 2 - \hat{x}_{un}(p_A, p_B)]$ and p_B with $Prob[\hat{x}_{un}(p_A, p_B) < x < 2 - \hat{x}_{un}(p_A, p_B)]$. Since x is uniformly distributed on $[0, 2]$ we obtain that $Prob[x < \hat{x}_{un}(p_A, p_B) \vee x > 2 - \hat{x}_{un}(p_A, p_B)] = \hat{x}_{un}(p_A, p_B)$, i.e., from an ex ante perspective p_A is the relevant price with probability $\text{Prob}[p = p_A] = \hat{x}_{un}$. Correspondingly, the purchase at price p_B occurs with probability $\text{Prob}[p = p_B] = 1 - \hat{x}_{un}$.

The reference point with respect to the match value is the reservation value v minus the expected distance between ideal and actual product taste times the taste parameter t. The distribution of the expected distance is denoted by $G(s) = Prob(|x - y_\sigma| \leq s)$, where $s \in [0, 1]$, the location of the firm $y_\sigma \in \{0, 1\}$, and the consumer x's purchase strategy in equilibrium for given prices is denoted by $\sigma \in \{A, B\}$, $\sigma \in \arg\max_{j \in \{A,B\}} u_j(x, p_j, p_{-j})$.

Since $c_A \leq c_B$, we restrict attention to the case $\hat{x}_{un} \geq 1/2$, i.e., firm A has a weakly larger market share than firm B also for uninformed consumers. Given that some uninformed consumers will not buy from their nearest firm, $G(s)$ will be kinked. This kink is determined by the maximum distance $|x - y_B|$ that consumers are willing to accept buying the more expensive product B, $s = 1 - \hat{x}_{un}$ because $s \leq 1 - \hat{x}_{un}$ holds for consumers close to either A or B, while $s > 1 - \hat{x}_{un}$ only holds for the more distant consumers of A. Hence, the distribution of s is

$$G(s) = \begin{cases} 2s & \text{if } s \in [0, 1 - \hat{x}_{un}] \\ s + (1 - \hat{x}_{un}) & \text{if } s \in (1 - \hat{x}_{un}, \hat{x}_{un}] \\ 1 & \text{otherwise.} \end{cases}$$

Note that if the indifferent uninformed consumer is located in the middle between A and B, $\hat{x}_{un} = 1/2$, the expected distance between ideal and actual product taste, $\mathbb{E}[s]$, is minimized and equal to $1/4$.

Following Koszegi and Rabin (2006), after uncertainty is resolved consumers experience a gain–loss utility: the reference distribution is split up for each dimension at the value of realization in a loss part with weight $\lambda > 1$ and a gain part with weight 1. In the loss part the realized value is compared to the lower tail of the reference distribution; in the gain part it is compared to the upper tail of the reference distribution.

Consider the gain–loss utility of an uninformed consumer located at x, at the moment she decides whether to purchase the product. Recall that at this point she knows her taste parameter x. The initially uninformed consumer now decides which product to buy taking into account her intrinsic utility from a product and her gain–loss utility when she compares the price–taste combination of a product with her two–dimensional reference point distribution.

First, consider the utility of an uninformed consumer from a purchase of product A when this consumer is located at $x \in (1 - \hat{x}_{un}, 1]$.[12]

$$\begin{aligned} u_A(x, p_A, p_B) \quad &= (v - tx - p_A) - \lambda \cdot \text{Prob}[p = p_A](p_A - p_A) + \text{Prob}[p = p_B](p_B - p_A) \\ &- \lambda \cdot t \int_0^x (x - s)dG(s) + t \int_x^1 (s - x)dG(s), \end{aligned} \tag{3.2}$$

where the first term is the consumer's intrinsic utility from product A. The second term is the loss in the price dimension from not facing a lower price than p_A. This term is equal to zero because p_A is the lowest price offered in the market place. The third term is the gain from not facing higher price than p_A, which is positive. The last two terms correspond to the loss (gain) from not facing a smaller (larger) distance in the taste dimension than x. An uninformed consumer's utility from a purchase of product B is derived analogously,

$$\begin{aligned} u_B(x, p_A, p_B) \quad &= \underbrace{v - t(1 - x) - p_B}_{\text{Intrinsic utility}} \underbrace{-\lambda \cdot \text{Prob}[p = p_A](p_B - p_A)}_{\text{Loss from facing a higher } p \text{ than } p_A} \\ &\underbrace{-\lambda \cdot t \int_0^{1-x} ((1 - x) - s)dG(s)}_{\text{Loss from facing larger distance than 0}} + \underbrace{t \int_{1-x}^1 (s - (1 - x))dG(s)}_{\text{Gain from facing smaller distance than 1}} \end{aligned} \tag{3.3}$$

This allows us to determine the location of the indifferent uninformed consumer $\hat{x}_{un}$.

[12]The indifferent uninformed consumer will be located at $x = \hat{x}_{un}$, therefore $(1 - \hat{x}_{un}, 1]$ is the relevant interval for determining $\hat{x}_{un}$.

Lemma 3.1: *Suppose that* $\hat{x}_{un} \in [1/2, 1)$. *Then* $\hat{x}_{un}$ *is given by*

$$\hat{x}_{un}(\Delta p) = \frac{\lambda}{(\lambda-1)} - \frac{\Delta p}{4t} - \underbrace{\sqrt{\frac{\Delta p^2}{16t^2} - \frac{(\lambda+2)}{2t(\lambda-1)}\Delta p + \frac{(\lambda+1)^2}{4(\lambda-1)^2}}}_{\equiv S(\Delta p)}. \tag{3.4}$$

where $\Delta p \equiv p_B - p_A$.

The square root, $S(\Delta p)$, is defined for $\Delta p \in [0, \Delta\overline{p}]$ with

$$\Delta\overline{p} \equiv \frac{2t}{(\lambda-1)}\left(2(\lambda+2) - \sqrt{(2(\lambda+2))^2 - (\lambda+1)^2}\right), \tag{3.5}$$

which is strictly positive for all $\lambda > 1$. It can be shown that for $\lambda \geq 3 + 2\sqrt{5} \approx 7.47$, $\hat{x}_{un}(\Delta p) \in [1/2, 1]$ for all $\Delta p \in [0, \Delta\overline{p}]$. Given monotonicity $\hat{x}_{un}(\Delta\overline{p})$ expresses the upper bound on firm A's demand from uninformed consumers for $\beta = 0$. If the degree of loss aversion is smaller, $\lambda < 3 + 2\sqrt{5}$, $\hat{x}_{un}(\Delta\overline{p})$ rises above one. Hence, we define another upper bound on the price difference, $\Delta\tilde{p}$, with $\Delta\tilde{p} < \Delta\overline{p}$ by the solution to $\hat{x}_{un}(\Delta\tilde{p}) = 1$. We can solve explicitly,

$$\Delta\tilde{p} = \frac{(\lambda+3)t}{2(\lambda+1)}. \tag{3.6}$$

The location of the indifferent uninformed consumer, $\hat{x}_{un}$, has a number of properties. Clearly, $\hat{x}_{un}(0) = 1/2$, i.e. market splits equally under symmetric prices. Another obvious property is that $\hat{x}_{un}(\Delta p)$ is equal to the demand of firm A if only a measure zero set of consumers is informed, i.e. $\beta = 0$.

It can be shown that the first derivative of $\hat{x}_{un}(\Delta p)$ with respect to Δp, $\hat{x}'_{un}(\Delta p)$, is strictly positive for all $\Delta p \in [0, \Delta\overline{p}]$:

$$\hat{x}'_{un}(\Delta p) = -\frac{1}{4t} - \frac{1}{2 \cdot S(\Delta p)} \cdot \left(\frac{\Delta p}{8t^2} - \frac{(\lambda+2)}{2t(\lambda-1)}\right)$$

At $\Delta p = 0$ the first derivative of $\hat{x}_{un}(\Delta p)$ is equal to

$$\hat{x}'_{un}(0) = -\frac{1}{4t} + \frac{(\lambda+2)}{2t(\lambda+1)}.$$

$\hat{x}'_{un}(0)$ is approaching $1/(2t)$ from below for $\lambda \to 1$ and $1/(4t)$ from above for $\lambda \to \infty$. This implies that, evaluated at $\Delta p = 0$, demand of uninformed consumers reacts less sensitive to price changes than demand of uninformed consumers—we return to this property in the following section. Moreover, $\hat{x}_{un}(\Delta p)$ is strictly convex for all $\Delta p \in [0, \Delta\overline{p}]$.

$$\hat{x}''_{un}(\Delta p) = \frac{(3+\lambda)(5+3\lambda)}{64t^2 \cdot (S(\Delta p))^3} > 0$$

Finally, it can be shown that the level of convexity of $\hat{x}_{un}(\Delta p)$ is strictly increasing in λ.

3.2.4 Demand comparison between informed and uninformed consumers

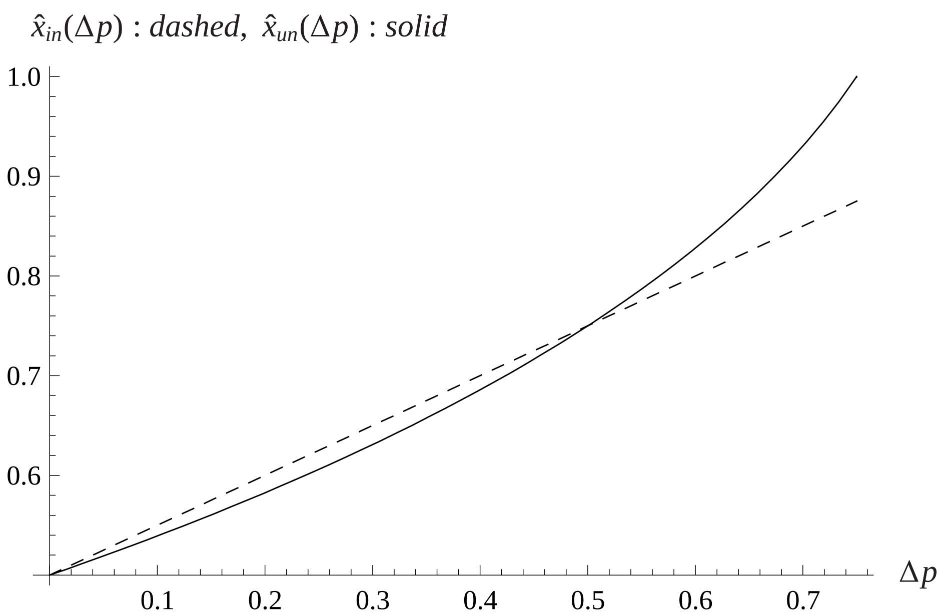
In this subsection we establish a number of properties when comparing market demand for uninformed relative to informed consumers, i.e. we compare $\hat{x}_{un}(\Delta p)$ and $\hat{x}_{in}(\Delta p)$ with one another.

The first property is a continuity property. For $\lambda \to 1$, the indirect utility function of uninformed consumers differs from the one of informed consumers only by a constant (this can be called a level effect). Equation (B.3) collapses to a linear equation and we receive $\hat{x}_{un}(\Delta p) = \hat{x}_{in}(\Delta p)$ as a solution in this case. This means that if consumers put equal weights on gains and losses, the effect of comparing expectations with realized values exactly cancels out when a choice between two products is made.

The next properties refer to the sensitivity of demand with respect to price. The first derivative of $\hat{x}_{in}(\Delta p)$ w.r.t. Δp is equal to $1/(2t)$ for all Δp. Therefore $\hat{x}'_{in}(0)$ is strictly larger than $\hat{x}'_{un}(0)$. This implies that the demand of uninformed consumers, evaluated at equal prices reacts less sensitive to price changes than the demand of informed consumers.

Evaluated at large price differences, this relationship is possibly reversed: for $\Delta p \to \Delta\overline{p}$ the square root, $S(\Delta p)$, becomes zero and $\hat{x}'_{un}(\Delta p)$ rises to infinity. Thus, $\hat{x}'_{un}(\Delta\overline{p}) > \hat{x}'_{in}(\Delta\overline{p}) = 1/(2t)$. Demand of uninformed consumers, evaluated at a large price difference reacts more sensitive to an increase in the price difference than the demand of informed consumers. (This property is satisfied if the indifferent consumer at this price difference is strictly interior; otherwise some more care is needed, as is done in the following section.)

Due to monotonicity of $\hat{x}'_{un}(\Delta p)$ and applying the mean value theorem, there exists an intermediate price difference $\Delta\hat{p} \in [0, \Delta\overline{p}]$ such that $\hat{x}'_{un}(\Delta\hat{p}) = \hat{x}'_{in}(\Delta\hat{p}) = 1/(2t)$. This critical price



The Figure shows the location of the indifferent consumer (= demand of firm A) for informed and uninformed consumers as a function of price difference Δp for parameter values of $t = 1$ and $\lambda = 3$: $\Delta\overline{p} = 0.8348$, $\Delta\tilde{p} = 3/4$ and $\Delta\hat{p} = 0.2789$.

Figure 3.1: Demand of informed and uninformed consumers

difference can be explicitly calculated as

$$\Delta\hat{p} = \frac{t\left(2\sqrt{2}\cdot(2(\lambda+2)) - 3\cdot\sqrt{(2(\lambda+2))^2 - (\lambda+1)^2}\right)}{\sqrt{2}(\lambda-1)},$$

which is strictly positive for all $\lambda > 1$ since $\Delta\hat{p}(\lambda = 1) = 0$ and $\Delta\hat{p}'(\lambda) > 0$.

Hence, we find that the demand of uninformed (or loss–averse) consumers is less price sensitive than the demand of informed consumers if price differences are small, $\Delta p < \Delta\hat{p}$. The underlying intuition is that for small price differences loss–averse consumers are harder to attract by price cuts because their gain from lower prices is outweighed by their loss in the taste dimension if they change producers. Thus, demand of loss–averse consumers reacts less sensitive to price in this range. For large price differences, however, their gain from lower prices starts to dominate their loss in the taste dimension if consumers switch to the cheaper producer. Therefore loss–averse consumers are more price–sensitive than informed (or classical Hotelling) consumers for $\Delta p > \Delta\hat{p}$. In section 4 it becomes apparent that this demand characteristic is a driving force for our comparative static results.

3.3 Market Equilibrium

In this section we focus on the market equilibrium of the firms' price–setting game. We derive market conditions under which equilibrium exists and under which it is unique. We start by showing some properties of market demand which will be needed later to prove some of the results. We then give an equilibrium characterization before turning to uniqueness and existence.

3.3.1 Properties of market demand

For notational convenience we first define an upper bound for the price difference (which depends on the parameters t and λ):

$$\Delta p^{max} \equiv \begin{cases} \Delta\tilde{p}, & \text{if } 1 < \lambda \leq \lambda^c; \\ \Delta\overline{p}, & \text{if } \lambda > \lambda^c. \end{cases} \tag{3.7}$$

with $\lambda^c \equiv 3 + 2\sqrt{5} \approx 7.47$. Note that $\Delta\tilde{p} \in [t \cdot (\sqrt{5} - 1)/2, t) \approx [0.618t, t)$ for $1 < \lambda \leq \lambda^c$ and $\Delta\overline{p} \in (t \cdot 2(\sqrt{3} - 2), t \cdot (\sqrt{5} - 1)/2) \approx (0.536t, 0.618t)$ for $\lambda > \lambda^c$. Using results from Section 2.4, we define the upper bound of firm A's demand of uninformed consumers as[13]

$$\hat{x}_{un}(\Delta p^{max}) \equiv \begin{cases} \hat{x}_{un}(\Delta\tilde{p}) = 1, & \text{if } 1 < \lambda \leq \lambda^c, \\ \hat{x}_{un}(\Delta\overline{p}) < 1, & \text{if } \lambda > \lambda^c. \end{cases} \tag{3.8}$$

[13] $\hat{x}_{un}(\Delta\overline{p}) = \frac{\lambda}{\lambda-1} - \frac{2(\lambda+2)-\sqrt{4(\lambda+2)^2-(\lambda+1)^2}}{2(\lambda-1)} \in (\sqrt{3}/2, 1)$ for $\lambda > \lambda^c$,

i.e. $\hat{x}_{un}(\Delta\overline{p})$ is lower than one for $\lambda > \lambda^c$. This leads to a jump in demand of uninformed consumers at $\Delta\overline{p}$ from $\hat{x}_{un}(\Delta\overline{p})$ to one (see the definition of $q_A(\Delta p;\beta)$), as $\hat{x}'_{un}(\Delta\overline{p}) \to \infty$.

Combining (3.1) and (3.4), we obtain the market demand of firm A as the weighted sum of the demand by informed and uninformed consumers,

$$
\begin{aligned}
q_A(\Delta p;\beta) &= \beta \cdot \hat{x}_{in}(\Delta p) + (1-\beta)\cdot \begin{cases} \hat{x}_{un}(\Delta p), & \text{if } 0 \leq \Delta p < \Delta p^{max} \\ 1, & \text{if } t \geq \Delta p \geq \Delta p^{max} \end{cases} \\
&= \begin{cases} \frac{1}{2} - \frac{1}{4t}(1-3\beta)\Delta p + (1-\beta)\frac{(\lambda+1)}{2(\lambda-1)} - (1-\beta)S(\Delta p), & \text{if } 0 \leq \Delta p < \Delta p^{max} \\ \beta \cdot \frac{t+\Delta p}{2t} + (1-\beta), & \text{if } t \geq \Delta p \geq \Delta p^{max} \end{cases} \\
&\equiv \begin{cases} \phi(\Delta p;\beta), & \text{if } 0 \leq \Delta p < \Delta p^{max} \\ \beta \cdot \frac{t+\Delta p}{2t} + (1-\beta), & \text{if } t \geq \Delta p \geq \Delta p^{max} \end{cases} \qquad (3.9)
\end{aligned}
$$

where

$$
S(\Delta p) = \sqrt{\frac{\Delta p^2}{16t^2} - \frac{(\lambda+2)}{2t(\lambda-1)}\Delta p + \frac{(\lambda+1)^2}{4(\lambda-1)^2}}.
$$

The demand of firm A is a function in the price difference Δp, which is kinked at Δp^{max} and for $\Delta p^{max} = \Delta\bar{p}$ additionally discontinuous at Δp^{max}. It approaches one for $\Delta p = t$.[14] Firm B's demand is determined analogously by $q_B(\Delta p;\beta) = 1 - q_A(\Delta p;\beta)$. In the following we are interested in interior equilibria in which products are bought by a positive share of uninformed consumers, i.e. Δp is lower than Δp^{max}.[15] We next state properties of $\phi(\Delta p;\beta)$, the demand of firm A in this case:[16]

Lemma 3.2: *For $0 \leq \Delta p < \Delta p^{max}$, the demand of firm A, $q_A(\Delta p;\beta) = \phi(\Delta p;\beta)$ is strictly increasing and convex in Δp.*

We note that also the third derivative, ϕ''', is greater than zero. However, the derivative of ϕ with respect to β can be positive or negative. The first derivative of the demand of A w.r.t. β is

[14]At $\Delta p = t$ firm A serves also all distant informed consumers which are harder to attract than distant uninformed consumers because the latter face a loss in the price dimension if buying from the more expensive firm B. For $\Delta p > t$ demand of firm A shows a second kink. This region we ignore since we are interested in cases in which both firms face a positive demand.

[15]This corresponds to industries in which firms are not too asymmetric.

[16]We will use ϕ as a short–hand notation for $\phi(\Delta p;\beta)$.

the difference of the demand of informed and uninformed consumers:

$$\frac{\partial\phi(\Delta p;\beta)}{\partial\beta} \equiv \phi_\beta = \hat{x}_{in}(\Delta p) - \hat{x}_{un}(\Delta p) = \frac{3}{4t}\Delta p - \frac{\lambda+1}{2(\lambda-1)} + S(\Delta p) \gtrless 0$$

with $\phi_\beta = 0$ at $\Delta p = 0$ and $\Delta p = t/2$.

This expression is of ambiguous sign, as has been pointed out in the previous section. We also note that cross derivative of the demand of A w.r.t. Δp and β,

$$\frac{\partial\phi'}{\partial\beta} \equiv \phi'_\beta = \hat{x}'_{in}(\Delta p) - \hat{x}'_{un}(\Delta p) = \frac{3}{4t} + \frac{1}{2S(\Delta p)} \cdot \left(\frac{\Delta p}{8t^2} - \frac{(\lambda+2)}{2t(\lambda-1)}\right),$$

is of ambiguous sign. This derivative has the boundary behavior that $\phi'_\beta = 0$ at $\Delta\hat{p}$. and $\phi'_\beta \to \infty$ for $\Delta p = \Delta\overline{p}$; the latter holds because $S(\Delta\overline{p}) = 0$.

3.3.2 Equilibrium characterization

We next turn to the equilibrium characterization. At the first stage firms foresee consumers' purchase decisions and set prices simultaneously to maximize profits. This yields the following first–order conditions:

$$\frac{\partial\pi_i}{\partial p_i} = \quad q_i + (p_i - c_i)\tfrac{\partial q_i}{\partial p_i} = 0 \quad \forall i \in \{A, B\}$$

If the solution has the feature that demand of each group of consumers, informed and uninformed, is positive, then first–order conditions can be expressed by

$$\frac{\partial\pi_A}{\partial p_A} = \phi - (p_A - c_A)\phi' = 0 \qquad (FOC_A)$$

$$\frac{\partial\pi_B}{\partial p_B} = (1-\phi) - (p_B - c_B)\phi' = 0. \qquad (FOC_B)$$

In this case concavity of the profit functions would assure that the solution characterizes an equilibrium.

$$\frac{\partial^2\pi_A}{\partial p_A^2} = -2\phi' + (p_A - c_A)\phi'' < 0 \qquad (SOC_A)$$

$$\frac{\partial^2\pi_B}{\partial p_B^2} = -2\phi' - (p_B - c_B)\phi'' < 0. \qquad (SOC_B)$$

Given the properties of ϕ—particularly that ϕ is strictly increasing and convex for $\beta < 1$—SOC_B holds globally, while SOC_A is not necessarily satisfied. Using that $(p_A - c_A) = \phi/\phi'$ by FOC_A, SOC_A can be expressed as follows

$$-2(\phi')^2 + \phi\phi'' < 0. \tag{3.10}$$

It can be shown that (3.10) is satisfied for small Δp (and λ) while it is violated for $\Delta p \rightarrow \Delta\overline{p}$ as ϕ'' goes faster to infinity in Δp than $(\phi')^2$.[17] This violation reflects that firm A has an increasing interest to non–locally undercut prices to gain the entire demand of uninformed consumers when Δp is large. The driving force behind this is that loss aversion in the price dimension dominates loss aversion in the taste dimension if price differences are large. Moreover, large losses in the price dimension if buying the expensive product B makes far–distant consumers of A more willing to opt for product A.

We will discuss the issue of non–interior solutions and non–existence in Proposition 3.2, but focus next on interior solutions. We denote an equilibrium with prices (p_A^*, p_B^*) that is determined by an interior solution as an interior equilibrium.

Lemma 3.3: *In an interior equilibrium with equilibrium prices* (p_A^*, p_B^*)*, the price difference* $\Delta p^* = p_B^* - p_A^*$ *satisfies*

$$\Delta p^* \quad = \quad \Delta c + f(\Delta p^*; \beta) \quad \forall \beta \in [0,1], \Delta p \textit{ feasible}, \tag{3.11}$$

with $\Delta c = c_B - c_A$ *and* $f(\Delta p; \beta) = (1 - 2\phi)/\phi'$.

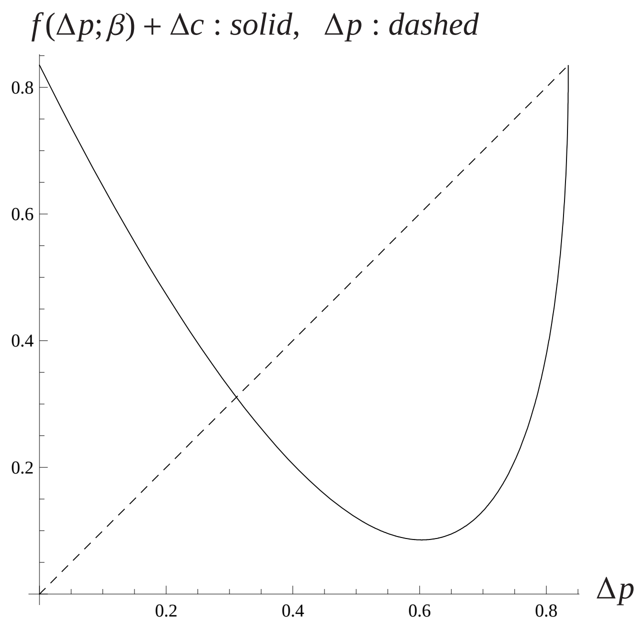
Thus, (3.11) implicitly defines the optimal Δp as a correspondence of Δc, β, λ, and t.[18]

3.3.3 Equilibrium uniqueness

In Proposition 3.1 we state conditions under which an interior equilibrium is unique. Given parameters λ and t, the condition states that the cost asymmetry between firms is not too large.

[17] This implies that π_A is not globally concave. We will show later that it is neither globally quasi–concave. Moreover, the non–concavity of π_A becomes more severe as Δp (resp. $-p_a$) increases.

[18] Besides β the latter two parameters affect the functional form of f via ϕ.



The Figure shows the equilibrium condition (3.11) at $\Delta c = \Delta\overline{p}$ for parameter values of $\beta = 0$, $t = 1$, and $\lambda = 3$: $\Delta\tilde{p} = 0.75$, $\Delta\overline{p} = 0.8348$.

Figure 3.2: Two potential interior equilibria

Proposition 3.1: *An interior equilibrium is unique if*

$$\Delta c < \Delta\overline{p} = \frac{2t}{(\lambda - 1)}\left(2(\lambda + 2) - \sqrt{(2(\lambda + 2))^2 - (\lambda + 1)^2}\right), \tag{3.12}$$

where $\Delta\overline{p}$ depicts the critical value of Δp such that the $S(\Delta p)$ in $\hat{x}_{un}(\Delta p)$ is equal to zero.[19]

3.3.4 Equilibrium existence

The next proposition clarifies the issue of equilibrium existence. It deals with the non–concavity of firm A's profit function by determining critical levels for firm A's incentive to non–locally undercut prices. Moreover, it is shown that non–interior equilibria fail to exist.

Proposition 3.2: *An interior equilibrium with prices (p_A^*, p_B^*)* ***exists*** *if and only if*

[19]Cf. equation (3.5).

1. *Δc satisfies*

$$\Delta c \leq \Delta c^{nd} \equiv \max\{\Delta p^{nd} - f(\Delta p^{nd};\beta), 0\}, \tag{3.13}$$

with Δp^{nd} being implicitly determined by the following non–deviation condition

$$\Delta p^{nd} = \left\{\Delta p \mid \Delta p = \Delta p^{max} - \frac{\phi \cdot \left(\phi(\Delta p^{max};\beta) - \phi\right)}{\phi' \cdot \phi(\Delta p^{max};\beta)}, \Delta p \neq \Delta p^{max}\right\}, \tag{3.14}$$

where $\phi(\Delta p^{max};\beta) = \beta \cdot \hat{x}_{in}(\Delta p^{max}) + (1-\beta) \leq 1$,

2. *and if $\Delta p^{nd} < 0$, β additionally satisfies*

$$\beta \geq \beta^{crit}(\lambda), \tag{3.15}$$

with $\beta^{crit}(\lambda)$ being an increasing function in λ which is expressed by

$$\beta^{crit}(\lambda) \equiv \begin{cases} 0, & \text{if } \lambda \in (1, 1+2\sqrt{2}]; \\ \beta_0^{crit}(\lambda) \in (0, 0.349], & \text{if } \lambda \in (1+2\sqrt{2}, \lambda^c]; \\ \beta_1^{crit}(\lambda) \in (0.349, 0.577), & \text{if } \lambda > \lambda^c. \end{cases} \tag{3.16}$$

Moreover, any equilibrium is interior.

Before turning to the proof, let us comment on this proposition. The result shows that an equilibrium exists if firm A has no incentive to non–locally undercut prices. In fact, the incentive to undercut prices increases in more asymmetric industries or for more loss–averse consumers. For a low degree of loss aversion ($1 < \lambda < 1 + 2\sqrt{2} \approx 3.828$) equilibrium exists if the cost difference between firms is not too large (see (3.13)).[20] In this case, an equilibrium exists for all values of β. However, if the degree of loss aversion rises further, equilibria only exist if there is a sufficiently large share of informed consumers. Such a large share of informed consumers reduces the undercutting incentive of firm A. The possible non–existence due to undercutting even holds for symmetric industries. Again, if the share of informed consumers is sufficiently large, an equilibrium exists; e.g. if 60% (which is greater than 57.7%) of the

[20]Note that according to experimental work on loss aversion λ takes the value of approximately 3, which is within this range.

consumers are informed then an equilibrium exists in symmetric industries for any level of loss aversion $\lambda > 1$.

In the proof we first provide the critical level of Δc for which the equilibrium condition in (3.11) is satisfied for *potentially* interior equilibria. We next identify the set of interior equilibria which locally satisfy the SOC's and which are robust to non–local price deviations of firm A. Finally, the existence of non–interior equilibria is refuted.

We conclude this section by a numerical example. For $\lambda = 3$, $t = 1$ and $\beta = 0$, the following price differences arise $\Delta p^{nd} = 0.27889$, $\Delta p^{ta} = 0.69532$, $\Delta p^{max} = \Delta\tilde{p} = 3/4$, and $\Delta\bar{p} = 0.83485$.[21] Moreover, Δc^{nd} is equal to $(\Delta p^{nd} - f(\Delta p^{nd}; 0)) = 0.75963$, i.e. an equilibrium exists for $\Delta c < 0.75963$. Compare table B.1 and B.2 in the appendix with $\Delta c = 0.25$ and 0.75 at $\beta = 0$. For non–existence at $\beta = 0$ consider Figure 3.2 and B.1 with $\Delta c = \Delta\bar{p}$ and 1.

Table 3.1 depicts the critical level of price differences and cost differences for non–deviation for $\beta \geq 0$ and $\lambda \geq 3$. It can be seen that a sufficiently large share of informed consumers dampens firm A's incentive to deviate even if the degree of loss aversion becomes high.[22]

Table 3.1: Non–deviation condition

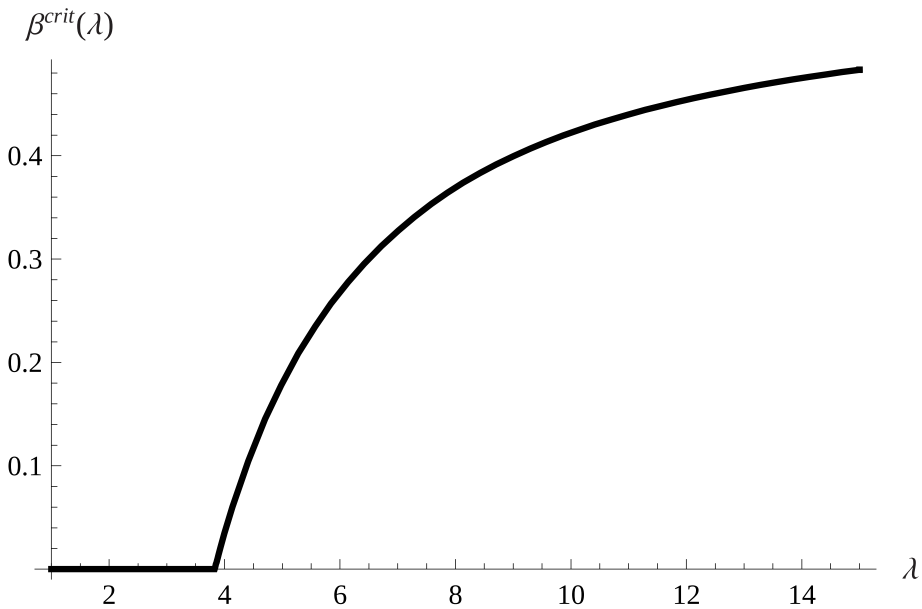
The table shows the variation of Δp^{nd} and Δc^{nd} in β and λ.

	$\lambda = 3$		$\lambda = 6$		$\lambda = 9$	
β	$\Delta p^{nd}(\beta)$	$\Delta c^{nd}(\beta)$	$\Delta p^{nd}(\beta)$	$\Delta c^{nd}(\beta)$	$\Delta p^{nd}(\beta)$	$\Delta c^{nd}(\beta)$
1.0	-	-	-	-	-	-
0.8	0.648337	1.75869	0.372669	1.07069	0.294726	0.857815
0.6	0.543254	1.45317	0.23824	0.686206	0.150303	0.440498
0.4	0.459237	1.22329	0.107415	0.314749	0.000320	0.000959
0.2	0.377489	1.00993	-0.0719496	-	-0.229582	-
0.0	0.278889	0.75963	-0.521395	-	-1.0704	-

Finally, the critical β for existence of symmetric equilibria ($\beta \geq \beta^{crit}(\lambda)$) is depicted in Figure 3.3.

[21] Figure B.2 in the appendix depicts the determination of Δp^{nd} for these parameter values.

[22] Note that for $\Delta c^{nd}(\beta) > \Delta\bar{p}$ potential second equilibria can arise (=second intersection of Δp and $\Delta c + f(\Delta p;\beta)$, compare Figure 3.2). However, those equilibria can be ruled out by the non–deviation condition since $\Delta p^{**} > \Delta p^{nd}(\beta)$. This means that by combining uniqueness and existence conditions equilibrium uniqueness can be granted for a broader class of industries.



The Figure shows the critical amount of informed consumers, $\beta^{crit}(\lambda)$, for which symmetric equilibria exist as a function of the degree of loss aversion $\lambda > 1$. Parameter values are $\Delta c = 0$ and $t = 1$: $\Delta p^{nd}(\Delta c = 0, \beta = \beta^{crit}(\lambda)) = 0$. non–deviation for $\beta \geq \beta^{crit}(\lambda)$.

Figure 3.3: Non–deviation for symmetric industries

3.4 Comparative Static Analysis

In this section we focus on comparative static properties of the equilibrium. As a starting point, we analyze comparative statics properties of symmetric markets, i.e., markets in which $c_A = c_B$. We then investigate the role of cost asymmetries and then turn to the role of the degree of initial information disclosure (captured by the share of informed consumers) in asymmetric markets. Finally, we investigate the effect of various demand characteristic on equilibrium outcomes.

3.4.1 Symmetric Market

In contrast to Heidhues and Koszegi (2008) our framework allows us to explicitly solve for equilibrium markup in our model. The following result characterizes the symmetric equilibrium.

Proposition 3.3: *For $\Delta c = 0$, any equilibrium is unique and symmetric. Equilibrium prices* *are given by*

$$p_i^* = c_i + \frac{t}{1 - \frac{(1-\beta)}{2}\frac{(\lambda-1)}{(\lambda+1)}}, i = A, B. \tag{3.17}$$

For $\Delta p^*(\beta) = 0$ loss aversion about prices is irrelevant even for uninformed consumers. In this situation uninformed consumers exclusively try to avoid losses in the taste dimension. This reduces the attractiveness of a lower–priced firm and thus the price elasticity of demand. This can be exploited by the firms the higher the degree of loss aversion and the higher the share of uninformed consumers. Since firms apply a markup over marginal costs equilibrium profits are independent of the level of marginal costs.[23]

Three comparative statics results are immediate.

Corollary 3.3.1: *For $\Delta c = 0$ and $\lambda > 1$, equilibrium markup is decreasing in the share of informed consumers β.*

This follows directly from differentiating (3.17) with respect to β and means that as the share of informed consumers increases the firms' markup decreases. In other words, informed consumers exert a positive externality on uninformed consumers. This prediction is in line with alternative models from the search literature, where a larger share of consumers who do not know some products exert a negative externality on those who do. Nevertheless our framework is substantially different since all consumers are fully informed at the moment of purchase. Here, an externality also arises due to uncertainty at the moment consumers form their reference points. With respect to recent work with behavioral biases, our result is of interest in the light of claims that better informed consumers are cross–subsidized at the cost of less informed consumers. This, for instance, holds in Gabaix and Laibson (2006) where only a fraction of consumers are knowledgeable about their future demand of an "add–on service", while other consumers are "naively" unaware of this. This shows that the particular type of behavioral bias is central to understand the competitive effect of changes in the composition of the consumer population.

Our first comparative statics result in the symmetric setting implies that firms do not have

[23] This is a standard property of models with demand aggregated over the two products that is perfectly price inelastic (more specifically of spatial models with full coverage).

an incentive to inform consumers at an early stage. However, there is a potential role of public authorities to inform consumers about their match value at an early point in time so that all uncertainty is resolved early on. This increases competitive pressure and thus lead to higher consumer surplus. As we already pointed out in the introduction, it is not required that public authorities aim at eliminating the behavioral bias directly (and thus to manipulate consumer preferences) but rather to disclose information at an early stage. This neutralizes the behavioral bias (but does not change the consumers' utility function). This insight provides a novel rational for information disclosure by public authorities due to behavioral biases in the consumer population.

Secondly, equilibrium markup is increasing in the degree of loss aversion, λ. For $\lambda \to 1$ firms receive the standard Hotelling markup of t. Thirdly, equilibrium markup is increasing in the inverse measure of industry competitiveness, t. For $t \to 0$ firms face full Bertrand competition and markups converge zero for all levels of loss aversion. This shows that consumer loss aversion does not affect market outcomes in perfectly competitive environments and our results rely on the interaction of imperfect competition and behavioral bias. The second and third comparative statics results are rather obvious but still noteworthy.

Table 3.2: Symmetric Equilibrium: Equilibrium Markups

The table shows the variation of $m_i^*(\Delta c = 0, \beta, \lambda) \equiv p_i^*(\Delta c = 0, \beta, \lambda) - c_i$ for all $i \in \{A, B\}$ in β and λ.

β \ λ	1	2	3	3.8284	5	7	9	∞
1	1	1	1	1	1	1	1	1
0.8	1	1.03448	1.05263	1.06222	1.07143	1.08108	1.08696	1.11111
0.6	1	1.07143	1.11111	1.1327	1.15385	1.17647	1.19048	1.25
0.4	1	1.11111	1.17647	1.2132	1.25	1.29032	1.31579	-
0.2	1	1.15385	1.25	1.30602	1.36364	-	-	-
0	1	1.2	1.33333	1.41421	-	-	-	-

Table 3.2 shows the variation of equilibrium markups in the share of informed consumers β and the degree of loss aversion λ for fully symmetric markets ($\Delta c = 0$). We make the following observations: (1) The highest markup is reached when all consumers are uninformed and the degree of loss–aversion approaches its critical level for existence in symmetric markets $\lambda = 1 + 2\sqrt{2} \approx 3.82843$.[24] (2) If the share of informed consumers is sufficiently large (above 57.7%)

[24]Compare Figure 3.3.

symmetric equilibria exist for all $\lambda > 1$. With such a large share of informed consumers the equilibrium markup is below its maximum level since the demand of informed consumers is more elastic and thus dampens the firms' incentives to set higher prices.

3.4.2 The role of cost asymmetries

In this subsection we take a first look at comparative statics properties of the asymmetric market. Here we focus on the degree of cost asymmetry, i.e. the level of $\Delta c = c_B - c_A$.

PROPOSITION 3.4: *In equilibrium, the price difference $\Delta p^*(\Delta c, \beta)$ is an increasing function in the cost asymmetry between firms Δc. Moreover, $\Delta p^*(\Delta c, \beta) \geq 1/3$.*

This result says that the more pronounced the cost asymmetry the larger the price difference between high–cost and low–cost firm. This result shows that standard comparative statics result with respect to cost difference are qualitatively robust to consumers being loss averse. However, in our model the marginal effect of an increase in cost differences on price variation is much stronger if some consumers are loss averse. To see this, note that $d\Delta p^*(\Delta c)/d\Delta c$ is equal to $1/3$ for $\beta = 1$, i.e. if all consumers are informed. This coincides with the standard Hotelling case. By contrast, for $\beta < 1$ our model predicts exacerbated price variation in markets with cost asymmetries.

This is in stark contrast to Heidhues and Koszegi (2008) who found that price variation is reduced in markets with loss–averse consumers. This difference arises because in our model prices are set early and become transparent before consumers form their reference point distributions. Consumers in our setup therefore incorporate the realized level of price variation into their reference point distribution instead of forming expectations about the future level of price variation: they do not form beliefs about firms' price setting strategy but only about their own product choice for given observed prices. This product choice is uncertain due to the uncertainty about ideal tastes. Consumers therefore correctly identify high–price firms before forming their reference point distributions. This affects firm behavior. They condition their price–setting behavior on the cost difference since they are informed about own and rival's costs. It follows that high–cost firms have less incentives to pool with more efficient firms in our setup than in Heidhues and Koszegi (2008).

Let us now look at the individual prices set by the two firms. For comparative statics we use markups $m_i^* \equiv p_i^* - c_i$, $i \in \{A, B\}$ instead of prices because markups are net of individual costs and depend solely on cost differences.[25] At the same time we could use individual prices but focus on changes in rival's costs only.

First, we observe that the low–cost firm's markup is increasing or decreasing depending on the degree of market asymmetries (=cost differences) and the share of uninformed consumers in the market.

Proposition 3.5: *For $\beta < 1$ and $\lambda > 1$, the equilibrium markup charged by the low–cost firm $m_A^*(\Delta c) \equiv p_A^*(\Delta c, c_A) - c_A$ is either first monotonously increasing and then decreasing in the cost difference if the share of informed consumers β is high, or always monotonously decreasing if β is sufficiently low. For $\beta = 1$ or $\lambda \to 1$, $m_A^*(\Delta c)$ is always monotonously increasing.*

In the latter case when all consumers are informed or the behavioral bias vanishes we receive the standard Hotelling result that the low–cost firm faces a larger markup in more asymmetric markets.

Note that, for $\beta = 1$, $dm_A^*/d\Delta c$ collapses to $1/3$. This implies that in the standard Hotelling world without behavioral biases ($\beta = 1$) the markup of the more efficient firm is increasing in the cost difference. The proposition thus shows that a local increase of the cost difference may have the reverse effect under consumer loss aversion ($\beta < 1$, $\lambda > 1$). If the degree of loss aversion and the share of uninformed consumers are high, firms obtain much higher markups under symmetric costs than in the standard Hotelling world (compare table 3.2). This leads to a level effect due to high markups if cost differences increase: Firm A decreases its markup to gain more consumers already in slightly asymmetric markets. It does so although in these markets price sensitivity of demand is lower than in the standard Hotelling world due to the dominating loss in the taste dimension. Here, the effect of a high markup level dominates the effect of a low price sensitivity of demand. For intermediately and strongly asymmetric markets firm A decreases its markup even further since in these markets the price sensitivity of demand becomes even larger than in the standard Hotelling world due to the dominating loss in the price dimension. Under very large cost differences firm A's markup might even fall

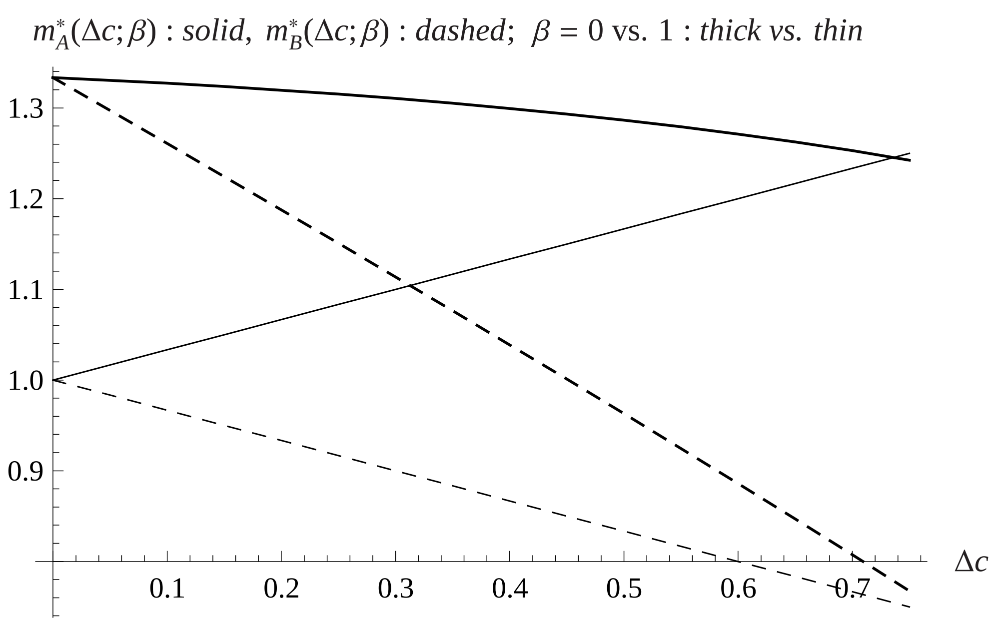
[25]This follows directly from firms' first–order conditions. Δc affects $p_i - c_i = \phi(\Delta p)/\phi'(\Delta p)$ via Δp.

below its level in the standard Hotelling case (compare Figure 3.4).

Second, we consider the markup of firm B.

PROPOSITION 3.6: *The equilibrium markup charged by the high–cost firm* $m_B^*(\Delta c) \equiv p_B^*(\Delta c, c_B) - c_B$ *is always decreasing in the cost difference.*

Note that for $\beta = 1$, $dm_B^*/d\Delta c$ is equal to $-1/3$. Thus the qualitative finding that the equilibrium markup of the high–cost firm is decreasing in the cost difference is preserved under consumer loss aversion. Due to a level effect of high markups we find that firm B's markup is decreasing more strongly than in the standard Hotelling world without behavioral bias. However, the critical market asymmetry for which its markup drops below its Hotelling level has to be larger than for firm A. This is presented in Figure 3.4.



The Figure shows the equilibrium markups of firm A and B for markets in which either all consumers are uninformed ($\beta = 0$) or informed (=benchmark case, $\beta = 1$) as a function of cost differences Δc for parameter values of $t = 1$ and $\lambda = 3$: $\Delta c^{nd}(\beta = 0) = 0.75963$.

Figure 3.4: Equilibrium markup of both firms

3.4.3 The role of information

In this subsection we focus on comparative statics results with respect to β, the share of initially informed consumers. These results are relevant to evaluate information disclosure policies by public authorities and firms. The latter provide new insights into the firms' advertising and marketing activities. Our first result concerns the equilibrium price difference.

PROPOSITION 3.7: *The equilibrium price difference $\Delta p^*(\beta)$ is decreasing in β.*

The above proposition says that prices become more equal as the share of initially informed consumers increases, or, in other words, that the population average becomes less loss–averse. Put differently, more loss–averse consumers lead to larger price differences. This is in stark contrast to one of the main findings in Heidhues and Koszegi (2008) who show in their setting that consumers loss aversion is a rationale for focal prices compared to a setting without behavioral biases in which firms would set different prices (using our terminology they compare a setting with mass 1 of uninformed consumers, i.e. $\beta = 0$, to a setting with mass 0 of uninformed consumers, which corresponds to a world without behavioral bias). Their message is that consumer loss aversion tends to lead to the (more) equal prices; our finding says that consumer loss aversion leads to larger price differences of asymmetric firms.

Let us now look at the individual prices set by the two firms. We first observe that the low–cost firm's price is monotone or inverse U–shaped in β depending on the parameter constellation.

PROPOSITION 3.8: *The equilibrium price charged by the low–cost firm $p_A^*(\beta)$ may be increasing or decreasing in the share of informed consumers β: $p_A^*(\beta)$ is monotonously increasing, monotonously decreasing or first increasing and then decreasing in β. It tends to be decreasing for small and increasing for large cost differences.*

The critical price difference (which implies the critical cost difference) at which price locally does not respond to β (c.p. Δp, i.e. partial effect) can be solved for analytically. The critical Δp, which is a function of λ and t and is independent of β:

$$\Delta p^{crit\,\partial p_A/\partial\beta}(\lambda, t) = \frac{t}{4(3+5\lambda)}\left((9-(26-15\lambda)\lambda) + \sqrt{3}\cdot|-1+5\lambda|\sqrt{(2(\lambda+2))^2-(\lambda-1)^2}\right)$$

For example, for parameters $\lambda = 3$ and $t = 1$ the critical price difference, at which the price of the low–cost firm reaches its maximum, satisfies $\Delta p^{crit\,\partial p_A/\partial\beta}(3,1) = 0.2534$. It is also insightful to evaluate the derivative in the limes as β turns to 1. In this case we can also solve analytically for a critical Δp at which the total derivative of p_A is zero, i.e. $\frac{dp_A^*(\Delta p^*(\beta);\beta)}{d\beta} = 0$:

$$\Delta p^{crit\,dp_A/d\beta}(\lambda,t) \;=\; t\frac{3(\lambda(31\lambda+42)-41)-\sqrt{21}\cdot|7-11\lambda|\sqrt{(\lambda+3)(3\lambda+5)}}{2(\lambda-3)(9\lambda-1)} \quad \text{at } \beta = 1$$

For example, $\Delta p^{crit\,dp_A/d\beta}(3,1) = 7/26 = 0.2692$ at $\beta = 1$. This means that, given parameters $\lambda = 3$ and $t = 1$, if we observe $\Delta p^*(1) = A < 0.2692$ a small increase in the share of informed consumers leads to a lower price of the more efficient firm, $dp_A/d\beta < 0$ (this confirms our numerical results in table B.1 and B.2), while for $\Delta p^*(1) > 0.2692$ the opposite holds, i.e. $dp_A/d\beta > 0$. (this confirms our numerical results in B.3).

The previous proposition implies that consumers who end up buying from the low–cost firm may actually be worse off when additional consumers become informed ex ante. Consider a change in policy from β to β' with $\beta' > \beta$. This parameterizes the market environment. Some consumers buy from the low–price firm in both market environments. For a sufficiently large cost asymmetry, the equilibrium price of the low–cost firm is locally increasing for all environments between β and β'. Hence, all those consumers of the low–cost firm whose ex ante information is constant across the two market environments are worse off from information disclosure to a share of $\beta' - \beta$ of consumers. This tends to occur in markets in which the initial share of informed consumers is small and in which the asymmetry (i.e. cost difference) between firms is large.

What is the effect on the price of the high–cost firm? Here our result is qualitatively similar: The price tends to be decreasing in β for small cost differences and increasing for large cost differences.

Proposition 3.9: *In equilibrium, the price of the high–cost firm $p_B^*(\beta)$ may be increasing or decreasing in the share of informed consumers β: $p_B^*(\beta)$ is monotonously increasing, monotonously decreasing or first increasing and then decreasing in β. It tends to be decreasing for small cost differences and increasing for large cost differences.*

We can also solve for critical values at which the comparative statics effect changes sign:

$$\Delta p^{crit\,\partial p_B/\partial\beta}(\lambda, t) = \frac{t}{2(\lambda+1)(\lambda+7)}\left((-23+(\lambda-10)\lambda)+|5-\lambda|\sqrt{(2(\lambda+2))^2-(\lambda-1)^2}\right)$$

For instance, $\Delta p^{crit\,\partial p_B/\partial\beta}(3,1) = 0.3201$. At $\beta = 1$ we can also solve analytically for a critical Δp at which the total derivative of p_B is zero, i.e. $\frac{dp_B^*(\Delta p^*(\beta);\beta)}{d\beta} = 0$:

$$\Delta p^{crit\,dp_B/d\beta}(\lambda, t) = \frac{t\left(3(\lambda(17\lambda+6)-55)-\sqrt{15}\cdot|11-7\lambda|\sqrt{(\lambda+3)(3\lambda+5)}\right)}{4\lambda(3\lambda-11)}$$

For instance, $\Delta p^{crit\,dp_B/d\beta}(3,1) = 1/2\cdot(5\sqrt{35}-29) = 0.2902$ at $\beta = 1$. This means that for $\Delta p^*(1) < 0.2902$ we expect $dp_B/d\beta < 0$ at $\beta = 1$ (compare table B.1 and B.2), while for $\Delta p^*(1) > 0.2902$ we expect $dp_A/d\beta > 0$ at $\beta = 1$ (compare table B.3). Thus, for this set of parameter values the overall effect of a marginal increase in β can indeed become positive if price differences (resp. cost asymmetries) become large enough.

Let us distinguish consumer groups by the product they consume. We observe that $\Delta p^{crit\,dp_B/d\beta}(\lambda, t) > \Delta p^{crit\,dp_A/d\beta}(\lambda, t)\ \forall \lambda, t$. Hence, for a larger range of cost parameters the price of the high–cost firm is locally decreasing (compared to the low–cost firm). This implies that, focusing on the consumers whose ex ante information remains unchanged, there exists an intermediate range of values of β under which consumers of the low–cost product lose whereas consumers of the high cost product gain from an increase in β. This means that in such cases additional information in the population benefits those consumers who purchase the high–cost product. Since the high–cost product only serves a niche market we may call these consumers niche consumers. Hence, informed niche consumers are more likely to benefit from an increase in β than the other informed consumers.[26]

The above observation helps us to shed some light on information acquisition by consumers. A particular application are consumer clubs that provide early information on match value to its members. Whether existing club members have an incentive to attract additional members depends on the market environment. Our above observation also indicates, that consumer clubs may be more likely to be formed by niche consumers. We also note that a forward–looking club may be willing to cope with increasing prices for a while with the understanding that, as the club further increases in size (reflected by an increase in β) prices will eventually

[26]The effect on uninformed consumers is ambiguous from an ex ante perspective since they buy the low–cost and the high–cost product with positive probability.

fall.

With respect to equilibrium demand our model generates the following predictions.

$$\begin{aligned}\frac{dq_A(\Delta p^*(\beta);\beta)}{d\beta} &= \beta\frac{d\hat{x}_{in}(\Delta p^*)}{d\Delta p^*}\cdot\frac{d\Delta p^*}{d\beta}+\hat{x}_{in}(\Delta p^*)+(1-\beta)\frac{d\hat{x}_{un}(\Delta p^*)}{d\Delta p^*}\cdot\frac{d\Delta p^*}{d\beta}-\hat{x}_{un}(\Delta p^*)\\ &= \underbrace{\frac{\partial q_A(\Delta p^*)}{\partial\Delta p^*}}_{\oplus}\cdot\underbrace{\frac{d\Delta p^*}{d\beta}}_{\oplus}+(\hat{x}_{in}(\Delta p^*)-\hat{x}_{un}(\Delta p^*))\gtrless 0,\end{aligned}$$

which is positive for small cost (resp. price) differences and negative for large cost (resp. price) differences (consider also Figure 3.5). Hence, in rather symmetric markets the demand of the more efficient firm rises, as the share of informed consumers increases (compare Table B.1 in the appendix). This implies that with consumer loss aversion (and a positive share of uninformed consumers) firm A's equilibrium demand is lower than in the standard Hotelling case.[27] Our result is reversed in strongly asymmetric markets in which the demand of the more efficient firm decreases in the share of informed consumers (compare Table B.3 in the appendix).

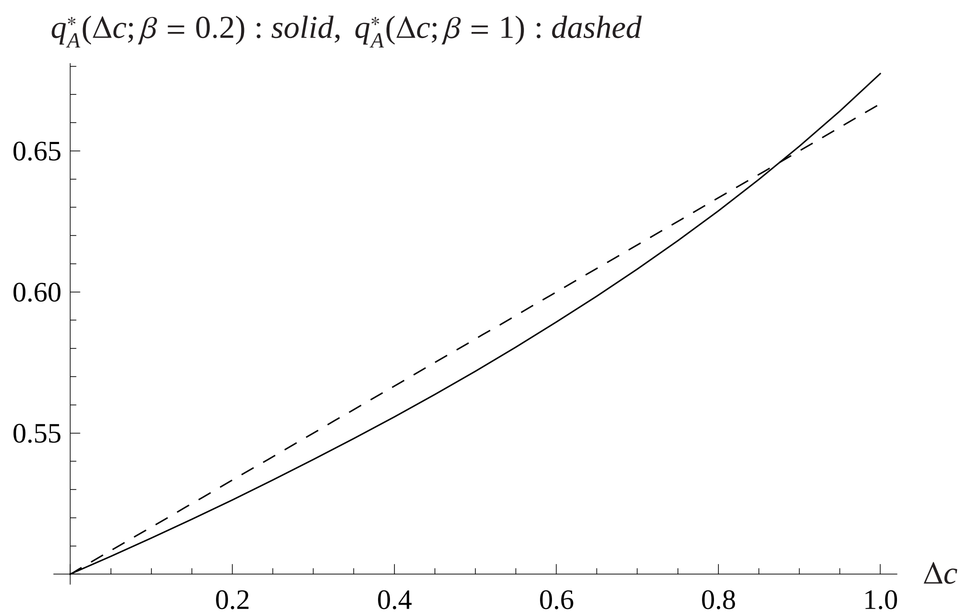
What about private incentives to disclose information? To address this question we will have to investigate the effect on profits. Here private information disclosure can be seen as the firms' management of consumer expectations (i.e. reference points). Note that in our simple setting information disclosure by one firm fully discloses the information of both firms since consumers make the correct inferences from observing the match value for one of the two products.[28]

$$\frac{d\pi_A(\Delta p^*(\beta),p_A^*(\beta);\beta)}{d\beta} = \frac{dp_A^*(\Delta p^*;\beta)}{d\beta}\cdot q_A(\Delta p^*;\beta)+\Big(p_A^*(\Delta p^*;\beta)-c_A\Big)\cdot\frac{dq_A(\Delta p^*;\beta)}{d\beta}\lesseqgtr 0$$

$$\begin{aligned}\frac{d\pi_B(\Delta p^*(\beta),p_B^*(\beta);\beta)}{d\beta} &= \frac{dp_B^*(\Delta p^*;\beta)}{d\beta}\cdot\Big(1-q_A(\Delta p^*;\beta)\Big)\\ &\quad -\Big(p_B^*(\Delta p^*;\beta)-c_B\Big)\cdot\frac{dq_A(\Delta p^*;\beta)}{d\beta}\lesseqgtr 0\end{aligned}$$

[27] This is qualitatively in line with Heidhues and Koszegi (2008) who predict equal splits of demand between firms in asymmetric markets.

[28] This is due to our assumption that firms necessarily locate at distance 1 from each other. It applies to either the setting in which uninformed consumers do not know their type before forming their reference point or they do not know the locations of firms in the product space.



The Figure shows the equilibrium demand of firm A for markets with either many uninformed consumers ($\beta = 0.2$) or only informed consumers (=benchmark case, $\beta = 1$) as a function of cost differences Δc for parameter values of $t = 1$ and $\lambda = 3$: $\Delta c^{nd}(\beta = 0.2) = 1.00993$.

Figure 3.5: Equilibrium demand of firm A

It is of interest to compare the size of the price effect to the size of the quantity effect for different degrees of market asymmetry. Numerical simulations suggest that the price effect dominates the quantity effect for all $\lambda > 1$. Thus, profits closely follow prices. Here, we confine attention to a single numerical example. The critical value of Δp such that $d\pi_A(.)/d\beta = 0$ at $\beta = 1$ and $\lambda = 3$ and $t = 1$, $c_A = 0.25$, and $c_B = 1$ is $\Delta p = 0.2581$. The critical values of Δp s.t. $d\pi_B(.)/d\beta = 0$ at the same values as above is $\Delta p = 0.2870$.[29] For comparison, we take a look at table B.2 in the appendix: The critical value at $\beta = 1$ is $\Delta p^*(1) = 0.25$. Hence, the critical values of Δp at $\beta < 1$ are larger than $\Delta p^*(1)$. Moreover, $\Delta p_B^{crit} > \Delta p_A^{crit}$.

Our numerical example also suggests that increasing the initial share of ex ante informed consumers first none, then one and then both firms gain from information disclosure. In case of conflicting interests it is the more efficient firm which locally gains from information disclosure as an expectation management tool.

Our numerical finding has direct implication for the observed advertising strategy of the firm.

[29]Note that we have problems to obtain an analytical solution as a function of λ and t or c_B even for the special case $\beta = 1$.

Our model predicts that it is rather more efficient firms that advertise product features and price and run promotions that allow consumers test–drives etc. This means that one should observe a positive correlation between efficiency level and advertising and marketing activities of the above mentioned form. We would like to stress that although all consumers will be fully informed at the moment of purchase, advertising content and price matters for firms if consumers are loss–averse. Without this behavioral bias it would be irrelevant whether or not a firm advertises price and characteristics.

How are the different consumer groups doing after an increase of the share of informed consumers? Let us first consider informed consumers. Their change in consumer surplus is simply a weighted average of price changes. To show this we next derive the aggregate consumer surplus for informed consumers.

$$CS_{in}(p_A(\beta), p_B(\beta)) = \int_0^{\hat{x}_{in}(\Delta p(\beta))} u_A(x, p_A(\beta))dx + \int_{\hat{x}_{in}(\Delta p(\beta))}^1 u_B(x, p_B(\beta))dx$$

We thus receive

$$\begin{aligned}\frac{dCS_{in}}{d\beta} &= \int_0^{\hat{x}_{in}(\Delta p(\beta))} \underbrace{\frac{\partial u_A(x, p_A(\beta))}{\partial p_A(\beta)}}_{=-1} \cdot \frac{dp_A}{d\beta} \cdot dx + \int_{\hat{x}_{in}(\Delta p(\beta))}^1 \underbrace{\frac{\partial u_B(x, p_B(\beta))}{\partial p_B(\beta)}}_{=-1} \cdot \frac{dp_B}{d\beta} \cdot dx \\ &= -\hat{x}_{in}(\Delta p)\frac{dp_A}{d\beta} - (1 - \hat{x}_{in}(\Delta p))\frac{dp_B}{d\beta} \gtrless 0.\end{aligned}$$

Consumer surplus of informed consumers may increase or decrease in the share of informed consumers. The sign of the derivative is determined by the weighted marginal price changes $dp_i/d\beta$ of the two products. If the two prices respond in different directions some informed consumers are better off whereas others are worse off in response to a increase in the share of informed consumers.

Evaluating the ex ante effect on uninformed consumers is more involved because gains and losses relative to their reference point have to be taken into account.

$$
\begin{aligned}
CS_{un}(p_A(\beta), p_B(\beta)) = \quad & \Big(\int_0^{1-\hat{x}_{un}(\Delta p(\beta))} \tilde{u}_A(x, p_A(\beta), p_B(\beta), \hat{x}_{un}(\Delta p(\beta)))dx \\
& + \int_{1-\hat{x}_{un}(\Delta p(\beta))}^{\hat{x}_{un}(\Delta p(\beta))} u_A(x, p_A(\beta), p_B(\beta), \hat{x}_{un}(\Delta p(\beta)))dx \Big) \\
& + \int_{\hat{x}_{un}(\Delta p(\beta))}^{1} u_B(x, p_A(\beta), p_B(\beta), \hat{x}_{un}(\Delta p(\beta)))dx,
\end{aligned}
$$

where $u_A(x, .)$ and $u_B(x, .)$ represent uninformed consumers' gain/loss utility for distant consumers of A and nearby consumers of B derived in (B.1) and (B.2), and

$$
\begin{aligned}
\tilde{u}_A(x, p_A(\beta), p_B(\beta), \hat{x}_{un}(\Delta p(\beta))) \quad &= (v - tx - p_A) + (1 - \hat{x}_{un})(p_B - p_A) \\
& - \lambda \cdot tx^2 + \frac{t}{2}\Big((1 - \hat{x}_{un})^2 - 2(1 - x)x + \hat{x}_{un}^2\Big),
\end{aligned}
$$

which demonstrates the gain/loss utility for nearby uninformed consumers of A. $\tilde{u}_A(x, .)$ differs from $u_A(x, .)$ only in the taste dimension of the gain/loss utility.

In contrast to intrinsic utility the gain/loss utility also depends on reference point distributions which require knowledge of all prices and the location of the indifferent uninformed consumer. Taking derivatives with respect to β we obtain

$$
\begin{aligned}
\frac{dCS_{un}}{d\beta} = & \int_0^{\hat{x}_{un}(\Delta p(\beta))} \left(\frac{\partial u_A(x, .)}{\partial p_A} \cdot \frac{dp_A}{d\beta} + \frac{\partial u_A(x, .)}{\partial p_B} \cdot \frac{dp_B}{d\beta} \right) \cdot dx \\
& + \Big(\int_0^{1-\hat{x}_{un}(\Delta p(\beta))} \left(\frac{\partial \tilde{u}_A(x, .)}{\partial \hat{x}_{un}} \cdot \frac{d\hat{x}_{un}(\Delta p)}{d\Delta p} \cdot \frac{d\Delta p}{d\beta} \right) \cdot dx \\
& + \int_{1-\hat{x}_{un}(\Delta p(\beta))}^{\hat{x}_{un}(\Delta p(\beta))} \left(\frac{\partial u_A(x, .)}{\partial \hat{x}_{un}} \cdot \frac{d\hat{x}_{un}(\Delta p)}{d\Delta p} \cdot \frac{d\Delta p}{d\beta} \right) \cdot dx \Big) \\
& + \int_{\hat{x}_{un}(\Delta p(\beta))}^{1} \left(\frac{\partial u_B(x, .)}{\partial p_A} \cdot \frac{dp_A}{d\beta} + \frac{\partial u_B(x, .)}{\partial p_B} \cdot \frac{dp_B}{d\beta} + \frac{\partial u_B(x, .)}{\partial \hat{x}_{un}} \cdot \frac{d\hat{x}_{un}(\Delta p)}{d\Delta p} \cdot \frac{d\Delta p}{d\beta} \right) \cdot dx.
\end{aligned}
$$

Beside consumers' intrinsic utility a price change also affects consumers' gains/losses with respect to the price dimension via the varying price difference. A change of the location of the indifferent uninformed consumer $\hat{x}_{un}$ has an impact on consumers' gains/losses in both dimensions. The taste dimension is affected since an increase of $\hat{x}_{un}$ shifts mass of the reference point distribution to the upper tail.[30] An impact on the price dimension occurs since the prob-

[30]It can be easily shown that $G(s|\hat{x}'_{un})$ first–order stochastically dominates $G(s|\hat{x}_{un})$ for all $\hat{x}'_{un} > \hat{x}_{un}$ feasible.

ability of buying at a specific price depends on the location at which consumers are indifferent between two products. The equation of $dCS_{un}/d\beta$ can be further simplified to

$$\begin{aligned}\frac{dCS_{un}}{d\beta} = &- \hat{x}_{un} \cdot \frac{dp_A}{d\beta} - (1-\hat{x}_{un}) \cdot \frac{dp_B}{d\beta} \\ &+ \left((\lambda-1)\hat{x}_{un}(1-\hat{x}_{un}) + \Delta p\,(\hat{x}_{un} + \lambda(1-\hat{x}_{un})) \cdot \frac{d\hat{x}_{un}}{d\Delta p} \right) \cdot (-\frac{d\Delta p}{d\beta}) \\ &- t\left(\frac{1}{2}(2\hat{x}_{un}-1)\Big((\lambda-1)(2\hat{x}_{un}-1)+2\Big) \right) \cdot \frac{d\hat{x}_{un}}{d\Delta p} \cdot (-\frac{d\Delta p}{d\beta}) \gtreqless 0, \end{aligned} \tag{3.18}$$

where the first line shows marginal effect of β on intrinsic utility (compare CS_{in}). This effect is positive in markets with small cost differences in which prices decrease in the share of informed consumers ($dp_i/d\beta < 0$) and negative in markets with large cost differences in which the reverse is true.

In the second line of equation (3.18) the marginal effect of β on the price dimension of consumers' gain/loss utility is depicted. An increase of the share of informed consumers has a positive overall impact on CS_{un}. This holds true for two reasons. Firstly, from Proposition 3.7 we obtain that the price difference is a decreasing function in the share of informed consumers. It turns out that a lower price difference (=seize of gains and losses in the price dimension) always reduces the losses for B consumers more in total terms than the gains for A consumers (consider the first term in second line). Secondly, a downward shift of the location of the indifferent uninformed consumer (caused by an reduction of the price difference) makes uninformed consumers of both firms better off with respect to gains/losses in the price dimension since the reference point distribution becomes skewed towards gains. This means that the probability of facing a loss in the price dimension decreases (for B consumers), while the probability of facing a gain in the price dimension increases (for A consumers).

The third line shows that the marginal effect of β on the match value dimension of consumers' gain/loss utility is always negative. A downward shift of the location of the indifferent uninformed consumer (caused by an increase in β) decreases the probability of large taste differences ($s \in (1-\hat{x}_{un}, \hat{x}_{un}]$) keeping the probability of small taste differences ($s \in [0, 1-\hat{x}_{un}]$) constant.[31] Since remaining uninformed consumers of firm B are located on the interval with small taste differences, they feel the same losses but lower gains. They are clearly worse off with respect to the the match value dimension of their gain/loss utility. The same holds true

[31] This argument also relies on the FOSD property of $G(s|\hat{x}_{un})$.

for nearby uninformed consumers of firm A. On top of lower gains, more distant consumers of A experience higher losses due to the downward shifted reference point distribution for the taste dimension. Thus, the overall effect of β on the taste dimension of consumers' gain/loss utility must be negative indeed.

The overall effect of β on CS_{un} is positive in rather symmetric markets since the effect of β on individual prices p_i is negative in these markets (compare CS_{in} and the tables in the appendix). By the same argument, the effect is negative in more asymmetric markets. Hence, the result from informed consumers qualitatively carries over to uninformed consumers. The reason for this that the sign of the effect of β on both dimensions of consumers' gain/loss utility does not change in market asymmetries. Moreover, it can be shown that for all $\lambda > 1$ and Δc feasible the sum of the second and the third line of (3.18) is negative, i.e. the marginal effect of β on the taste dimension dominates its effect on the price dimension of consumers' gain/loss utility. Unfortunately, this does not suffice to predict that the sign of $dCS_{un}/d\beta$ is changing for a higher level of β in intermediately asymmetric markets since the price changes, which determine the sign change of consumer surplus, are weighted by different means between informed and uninformed consumers. Table B.2 demonstrates the effect of the weight difference dominates the negative effect of β on the both dimensions of consumers' gain loss utility, i.e. the critical β at which the marginal consumer surplus of uninformed consumers switches sign is lower than the critical β for informed consumers.

To determine the overall effect of β on aggregate consumer surplus of both consumer groups, an additional decomposition effect has to be taken into account. This effect reflects the consumer surplus of the group of formerly uninformed consumers which become informed. The overall effect of β on aggregate consumer surplus is determined by the first derivative of $CS(\beta) = \beta \cdot CS_{in}(p_A(\beta), p_B(\beta)) + (1-\beta) \cdot CS_{un}(p_A(\beta), p_B(\beta))$ with respect to β, which yields the following expression

$$
\begin{aligned}
\frac{dCS}{d\beta} &= \beta \cdot \frac{dCS_{in}}{d\beta} + CS_{in} + (1-\beta) \cdot \frac{dCS_{un}}{d\beta} - CS_{un} \\
&= \beta \cdot \frac{dCS_{in}}{d\beta} + (1-\beta) \cdot \frac{dCS_{un}}{d\beta} + (CS_{in} - CS_{un}).
\end{aligned}
$$

It can be shown that the decomposition effect represented by $(CS_{in} - CS_{un})$ is always strictly positive, which is intuitive since the group of uninformed consumers faces a lower average utility level due to the higher weight on losses than on gains. Although some uninformed consumers which receive high match value at low price are better off than their informed counterparts, the average utility of uninformed consumers is lower due to the losses in the taste dimension of consumers located apart from the product they purchase and the losses in the price dimension of B consumers (consider the tables in the appendix). It turns out that the decomposition effect always dominates the group–specific effect of β on consumer surplus. This means that the group of consumers who becomes informed is so much better off that its surplus increase always dominates the surplus change of the remaining uninformed consumers and the old informed consumers. This holds even in strongly asymmetric markets in which remaining uninformed and old informed consumers are worse off if the share of informed consumers increases.

3.5 Extensions

3.5.1 Relative weight on gain–loss utility

Consider next consumer preferences for which the intrinsic utility is weighted by one, while the gain–loss utility has a weight of $\alpha > 0$.[32] It could now be asked whether a change of the relative weight on the gain–loss utility has a different influence on the location of the indifferent uninformed consumer than a change in the degree of loss aversion λ. The next proposition shows that this is not the case.

Proposition 3.10: *Suppose the utility function of uninformed consumers shows an additional weight, $\alpha > 0$, on the gain–loss utility, i.e. all terms except for the intrinsic utility term in* (B.1) *(resp.* (B.2)*) are pre–multiplied by α.*
Then, $\forall \lambda' > 1, \alpha' > 0 \; \exists \lambda > 1$ such that

$$\hat{x}_{un}(\Delta p; \lambda, \alpha = 1) = \hat{x}_{un}(\Delta p; \lambda', \alpha'), \tag{3.19}$$

where $\hat{x}_{un}(\Delta p; \lambda, \alpha)$ is the location of the indifferent uninformed consumer given α–extended preferences. Moreover, $\lambda \geq \lambda'$ for $\alpha' \geq 1$ and $\lambda < \lambda'$ for $\alpha' < 1$.

[32]For $\alpha = 0$ we are obviously situated in a standard Salop world.

The previous proposition points out that for any change of the relative weight on gain–loss utility apart from one, there is an equivalent change of the degree of loss aversion, λ, which shows the same sign.

3.5.2 Asymmetric product quality

Our model is easily extended to allow for differences in product quality which are known to consumers at the beginning of the game. An informed consumer's utility function is $u_i(x, p_i) = (v_i - p_i) - t|y_i - x|$. We then distinguish between a quality–adjusted price dimension, which includes easily communicated product characteristics which are of unambiguous value to consumers and a taste dimension which includes those product characteristics whose value depends on the consumer type. We define quality–adjusted (or hedonic) prices $\tilde{p}_i = p_i - v_i$, $i \in \{A, B\}$ for all consumers and consider those to be relevant for consumers' purchase decision. The main difference arises for uninformed consumers when building their reference point distribution with respect to prices. Here, only the gain/loss in quality–adjusted prices $\Delta\tilde{p} = \Delta p - \Delta v$ matters, $\Delta v \equiv v_B - v_A$. We label firms such that $\Delta c - \Delta v > 0$ and call firm A the more efficient firm. In the following proposition we show that any market with asymmetric quality is equivalent to a market with symmetric quality and more asymmetric costs.

Proposition 3.11: *For any market with asymmetric quality represented by a vector* $(\Delta v, \Delta c)$ *with* $\Delta c - \Delta v > 0$ *there exists a market with symmetric quality represented by a vector* $(\Delta v', \Delta c')$ *with* $\Delta v' = 0$, $\Delta c' > 0$ *such that market equilibria of both markets are the same, i.e.* $\Delta p^* - \Delta v = \Delta p'^*$. *Moreover,* $\Delta c' = \Delta c - \Delta v$.

As a special case, it can be thought of all asymmetry in the first market being generated by quality differences. This means that firm A delivers higher quality in a market with symmetric costs, $\Delta v < 0$ and $\Delta c = 0$. Then, the costs asymmetry in the second market shows the same size in absolute terms as the quality difference in the first market, $\Delta c' = -\Delta v$.
In the proof we show that the optimization problems of the two consumer groups and the firms are the same in both markets.

3.6 Conclusion

This paper has studied the impact of consumer loss aversion on market outcomes in asymmetric imperfectly competitive markets. Consumer loss aversion only makes a difference compared to a market in which consumers lack this behavioral bias if they are uncertain about product characteristics or associated match value at an initial stage where they form expectations. Early information disclosure can thus be interpreted as expectation management. Such information disclosure can be achieved through advertising campaigns and promotional activities which do not generate additional information at the moment of purchase (at this point consumers would be informed in any case) but make consumers informed much in advance of their actual purchasing decision.

We followed Heidhues and Koszegi (2008) and modeled the market as a Salop circle. Our framework, however, has notable differences to their work: consumers and firms know the market environment; in particular, they know the actual (asymmetric) cost realizations. Consumers also observe prices from the outset. Our model is enriched by considering a heterogenous population which differs according to their knowledge of their preferences at the initial point when they form their (probabilistic) reference point. Our model delivers remarkably different results compared to Heidhues and Koszegi (2008): while they obtained focal pricing as a consequence of the presence of loss–aversion in the population, we show that the price difference *increases* in the share of uninformed loss averse consumers. We also show that prices and profits *decrease* if the cost asymmetry is large.

Our results have implications for public policy and firms' advertising strategies. There are instances in which consumers would gain from more information whereas both firms would refrain from early information disclosure, namely when the market is symmetric or moderately asymmetric. In these markets public information disclosure (which allows consumers to learn the products' match values) would enhance consumer surplus. Moreover, our model predicts that advertising and other marketing instruments that allow for early information disclosure about match value are more prevalent in markets characterized by large asymmetries between firms. In these asymmetric markets one or both firms gain from information disclosure because this leads to higher prices. Whenever firms have conflicting interests with respect to information disclosure, it is the more efficient firm that discloses information.

We have analyzed industries that are characterized by cost asymmetries. Alternatively, asymmetries with respect to observed product quality may be introduced. Since there is a one–to–one relationship between these two models our insights are directly applicable to a model in which firms differ in observed product quality.

Chapter 4

Ownership and Control in Differentiated Product Markets

4.1 Introduction

4.1.1 Motivation

In this paper, we analyze the equilibrium allocation of ownership and control rights in a simple static economy consisting of two competing firms, A and B. Initially, an investor I_1 holds a controlling stake in A and decides whether to acquire a stake in B and/or an additional stake in A. Moreover, he can initiate a cross investment of A in B to indirectly participate in B's profits.

The acquisition of shares has two effects. First, the acquisition is associated with cash flow rights on firm B's profits. I_1 will internalize their effect by appropriately setting firm A's price. Second, if I_1 acquires enough shares, she gains control in B as well. She then sets both prices so as to maximize her portfolio return. The threshold of shares to gain control is assumed to be exogenous. We think of it as being the lower, the more dispersed the remaining ownership in the firm or, alternatively speaking, the less shareholders are able in coordinating their votes against decisions favorable to our investor I_1.

As to the initial ownership structure we consider two polar cases in each firm, i.e. four cases in total. In the first two cases, initial ownership of the shares in B is dispersed, but the shares in

A net of I_1's stake are either owned by a single investor, or ownership is dispersed as well. In both cases, initial owners in *B* can free–ride on the increased value of *B* that may result from an acquisition of shares directly by I_1 or indirectly via firm *A*, so that I_1 will profit only from the increased value of her stake in *A*. This value is negatively related to the level of her direct and indirect investment in *B*. In equilibrium, she therefore acquires only a minimal controlling stake in *B*. If I_1 is not able to acquire all of *A*'s shares, her payoff is maximized if that stake is acquired via a cross investment of firm *A* in firm *B*. This constitutes an important leverage in our model.

In the remaining two cases, firm *B* is initially owned by a large investor. If the remaining ownership in *A* is dispersed, the remaining owners of *A* can again free–ride on the benefits of control over both firms. We show that in this case, a cross ownership arrangement is never optimal for I_1 unless she initially owns all of *A*. She instead prefers to directly acquire all of firm *B*'s cash flow rights and to set both prices as to maximize a weighted sum of profits, with the weight on *B*'s profit higher than the weight on *A* 's profit. This is so because when buying out a blockholder, I_1 can benefit from acquiring *B* at a low price and absorbing all benefits of control herself. In the last case, both the remaining ownership in *A* and ownership in *B* are concentrated. Again, I_1 will acquire a controlling stake in *B* as well. She then sets both prices as to maximize the joint profits of both firms, which results in monopoly prices.

In this paper we aim at analyzing the interplay between investment decisions, the attainment of control over two competing firms and product market outcomes. Clear limitations of our model are that (i) it is static, (ii) debt finance is not modeled (Jensen, 1989), and that (iii) we abstract from agency costs (Manne, 1965; Jensen and Meckling, 1976; Shleifer and Vishny, 1997) and other benefits of control such as efficiency gains. These are important aspects which we leave for future research.

Nevertheless, we are able to make predictions over and above the ones that can be obtained in models with passive investments in, or controlling takeovers of rivals, as well as in models with homogeneous product markets. Most importantly, we are able to relate the evolution of ownership and control structures to initial conditions and properties of the relevant product market.

4.1.2 Related Literature

We are aware of only one paper, Charlty-Lepers, Fagart, and Souam (2002), that studies acquisition decisions in a similar context. Their work differs from ours in at least two important ways. First, they don't consider the possibility that the investor might want to adjust her investment in the firm already controlled by her. Second, they consider a Cournot industry with homogeneous goods. As suggested by Salant, Switzer, and Reynolds (1983), in such an industry, the acquisition of a controlling stake *always* goes along with a lower aggregate profit of both firms. Therefore, after the acquisition took place, the investor will *always* shut down the firm in which her investment is smaller.[1] Similar conclusions arise in the case of Bertrand competition in homogeneous product markets, where in the case of a take over it is optimal to shut down one of the two firms.

Within the context of pure mergers much simpler than the one considered by us, Salant, Switzer, and Reynolds (1983) find that in a Cournot model, the joint profits of two merging parties decrease due to the merger, yet Deneckere and Davidson (1985) show that they increase if the two firms engage in price competition in a market with differentiated products. Similarly, Flath (1991) shows that if stock markets are efficient in the sense that share prices reflect post–share trading product market equilibria, acquiring shares in rivals is not rational in Cournot industries, but can be so in Bertrand duopolies. This is related to an article by Fudenberg and Tirole (1984) who find that only those investments are made that yield toughness. This is not the case in Cournot models where quantities are strategic substitutes but in models where prices are strategic complements.

At any rate, we wish to make use of the fact that under price competition there is an incentive to invest in rivals even if the investment incentive is only related to cash flows. Such investment is observed in many industries (Gilo, 2000).

There is a sizeable literature on the competitive effect of passive investments in rivals in a static context. In general, competition is reduced (Reynolds and Snapp, 1986; Bolle and Gth, 1992; Flath, 1992; Reitman, 1994; Dietzenbacher, Smid, and Volkerink, 2000). O'Brien and Salop (2000) distinguish in addition several control scenarios and derive comparative static results in a Cournot framework. However, they do not model the acquisition stage.

[1] In related work, Schwartz and Thompson (1986) and Baye, Crocker, and Ju (1996) show that firms have an incentive to divisionalize, keeping two firms and letting each of them maximize their own profit. This can, however, give rise to a commitment problem because both managers still belong to the same firm.

In their empirical work, Dorofeenko, Lang, Ritzberger, and Shorish (2005) assume that there is a controlling group of shareholders in each company. They then use German data to identify control scenarios consistent with the observed ownership structure.

In a dynamic context, Malueg (1992) and Gilo, Moshe, and Spiegel (2006) examine the effects of exogenously given passive investments in rivals on the incentives of firms to engage in tacit collusion. The effect is ambiguous in Cournot industries because, relative to the case without cross holdings, competitors act less aggressively if collusion breaks down. In contrast, when price competition takes place, under general conditions, collusion is facilitated.

There are also important links to the literature in corporate finance. In particular, for the case of initially dispersed ownership in firm B, our findings are related to the free–rider effect that is studied in Grossman and Hart (1980), namely that in the case of dispersed ownership the value of the firm after the acquisition of shares determines the acquisition price. In our model, the firm value is a function of control and ownership arrangements. In that, our paper is related to Burkart, Gromb, and Panunzi (1998). They argue that bidders cannot commit *ex ante* not to extract private benefits *ex post*. Therefore, the more shares they buy, the higher the acquisition price. Consequently, the investor acquires as few shares as necessary to gain control, either by directly investing in firm B or by initiating a cross holding of A in B, thereby maximizing *ex post* moral hazard.

For the case in which the shares of firm B are initially held by a large block holder, our results are related to pivotal shareholder models of takeovers Bagnoli and Lipman (1988); Bebchuk (1989); Holmstrom and Nalebuff (1992). As such block holders cannot free–ride on the benefits of a takeover, such a takeover will always occur in our model and is then used to fleece consumers.

Our paper is furthermore related to the literature which deals with the separation between ownership and control. Such a separation occurs in a number of countries including Germany (La Porta, Lopez-de Silanes, Shleifer, and Vishny, 1998) which results from a high concentration of cash flow rights (La Porta, Lopez-de Silanes, and Shleifer, 1999; Franks and Mayer, 2000; Faccioa and Lang, 2002).

Finally, there is a literature on alternative uses of cross holdings.[2] First, cross holdings can be used as a monitoring device when banks provide debt. By investing in the lender, banks

[2] See Becht and Boehmer (2001, 2003) for descriptive evidence. Streek and Höpner (2003) is a collection of case studies concerned with the recent development of the “Deutschland AG”.

become represented in the supervisory board of the firm which is the main monitoring institution within the firm (Böhm, 1992). Second, cross ownership arrangements could be used as a means against takeovers (Hellwig, 2000). For instance, a ring structure could be used by a group of firms to prevent outsiders from buying a controlling stake in each of the inside firms. Third, a pyramidal structure could be built. Almeida and Wolfenzon (2005) argue that firms can thereby acquire new firms without external funding. Riyanto and Toolsema (2004) present a formal model in which controlling shareholders can shift resources from one firm into another. Then, so–called tunneling within pyramidal ownership structures can be used to save the receiving firm from bankruptcy.[3]

4.2 The Model

Consider an industry involving two firms $i \in \{A, B\}$ selling differentiated products that are substitutes to each other. Their reduced form payoffs $\pi_i(p_A, p_B), i = A, B$ are supposed to be twice differentiable. In addition, the payoffs are supposed to satisfy the following assumptions:

(i) $\pi_A(x, y) = \pi_B(y, x) \forall x, y \geq c$,

(ii) $\frac{\partial \pi_i}{\partial p_j} \geq 0, \left|\frac{\partial \pi_i}{\partial p_i}\right| \geq \frac{\partial \pi_i}{\partial p_j}$,

(iii) $\frac{\partial^2 \pi_i}{\partial p_i^2} < 0, \frac{\partial^2 \pi_i}{\partial p_j^2} \leq 0$,

(iv) $\frac{\partial^2 \pi_i}{\partial p_i \partial p_j} > 0, \left|\frac{\partial^2 \pi_i}{\partial p_i^2}\right| > \frac{\partial^2 \pi_i}{\partial p_i \partial p_j}, i, j = A, B, j \neq i.$

Assumption (i) ensures complete symmetry between the two firms, which allows us to concentrate on the effects of ownership arrangements on allocative decisions. The first part of Assumption (ii) is standard. The second part of Assumption (ii) limits the cross price effect by the own price effect on profits. The first part of Assumption (iii) is also standard. The second part of Assumption (iii) is needed to satisfy the second order conditions for optimization in the interactive situation considered here. Finally the first part of Assumption (iv) is again standard. Its second part states that the effect of a change in its own price p_i on the marginal profits of firm i is stronger than the effect of a price change in the competing firm.

[3] See also Chapelle and Szafarz (2005) who develop a model for measuring integrated ownership and threshold–based control.

In our acquisition subgame specified later, we consider only one active agent named investor I_1. Initially, that investor controls firm A with an initial stake $\alpha_A = \alpha_A^0 \in (0, 1]$ of its cash flow rights. Neither investor I_1 nor firm A hold stakes in firm B.

Firm B is assumed to be either owned by a unit mass of atomless investors, or owned (and controlled) by another large investor I_2. To simplify matters, neither investor I_2 nor firm B are initially invested nor will invest in firm A.

Investor I_1 can either *directly* acquire a stake α_B in firm B, or, by virtue of controlling firm A, induce it to acquire a stake γ in firm B. Investor I_1's *indirect* acquisition in B is then of size $\alpha_A^0\gamma$. Naturally, all magnitudes involved here as well as $\alpha_B + \gamma$ take values in the unit interval.

If a firm is owned by dispersed shareholders, we assume its management to maximize its profit by controlling its price. By contrast, if controlling shares of that firm are owned by a block holder, she decides about the direct or the indirect acquisition of stakes in both firms, and if controlling a firm sets its product price, taking into account her interests in both firms. In particular, I_1 is supposed to control firm B if she acquires at least a fraction $\hat{\alpha}_B$ of firm B's shares. [4] She can do so by directly buying sufficiently many shares herself, or by indirectly initiating a cross holding of firm A in firm B, or a convex combination thereof. Her financial interest in B is denoted by $\tilde{\alpha}_B \equiv \alpha_B + \alpha_A\gamma$, where α_A denotes the quantity of shares she ultimately acquires in firm A. However, since I_1 has a controlling interest already in firm A, I_1 controls B if $\alpha_B + \gamma \geq \hat{\alpha}_B$, i.e. even if $\tilde{\alpha}_B < \hat{\alpha}_B$

The cases involving firm ownership patterns considered here are collected in the following table.

	Shares of B dispersed	Shares of B concentrated
Remaining shares of A dispersed	1	3
Remaining shares of A concentrated	2	4

The time structure in our model is as follows.

1. Investment: I_1 decides whether or not to buy additional stakes $\alpha_A - \alpha_A^0$ in firm A, and

[4] A natural sufficient condition for control is that she owns more than 50 per cent of the shares. We have conducted field studies suggesting that the percentage of shares sufficient for control tends to be much smaller. In general, the controlling stake size depends on the distribution of a firm's ownership. If it is dispersed, then a much smaller percentage (sometimes as small as 5 per cent) is sufficient for control.

non–controlling or controlling direct stakes α_B, or indirect stakes γ via firm A, in firm B, respectively.

2. Pricing: If $\alpha_B + \gamma < \hat{\alpha}_B$ so that I_1 does not control B, then she sets p_A so as to maximize $\alpha_A \pi_A(p_A, p_B) + \tilde{\alpha}_B \pi_B(p_A, p_B)$ for given p_B . In turn, firm B's management or controlling owner sets price p_B so as to maximize $\pi_B(p_A, p_B)$.

   If I_1 does control B, i.e. if $\alpha_B + \gamma \geq \hat{\alpha}_B$, she sets both p_A and p_B so as to maximize $\alpha_A \pi_A(p_A, p_B) + \tilde{\alpha}_B \pi_B(p_A, p_B)$.

3. Payoff: I_1 obtains $\alpha_A \pi_A + \tilde{\alpha}_B \pi_B$, less the acquisition price of her additional stake in A and her stake in B. The remaining owners of A and B obtain their fraction of π_A and π_B, respectively.

Our equilibrium concept is subgame perfection, so that this game can be solved by backward induction. In Section 4.3 we characterize the Nash equilibria in the product market stage, that arise for any given initial allocation of ownership and control rights. In the ensuing Section 4.4 we incorporate product market outcomes to analyze I_1's acquisition of cash flow and control rights, and to relate it to the initial ownership structure in our economy.

4.3 Product Market Stage

In this section, we characterize product market equilibrium prices and profits as a function of direct and indirect investment of I_1 in B, separately for the case in which I_1 does not control firm B but enjoys cash flow rights in it, vs. the case where she controls firm B.

Towards this analysis, consider the case in which I_1 holds controlling shares $\alpha_A \geq 0$ in firm A, and non–controlling cash flow rights $\tilde{\alpha}_B \geq 0$ in firm B. Let $\omega \in [0, \infty)$ denote investor I_1's share of cash flow rights in B relative to A, so that $\omega \equiv \tilde{\alpha}_B / \alpha_A \in [0, 1/\alpha_A]$.

4.3.1 Firm B uncontrolled by I_1

If I_1 owns α_A controlling shares in firm A and α_B non–controlling shares in firm B she solves

$$\max_{p_A} \pi_A(p_A, p_B) + \omega \pi_B(p_A, p_B).$$

Firm B's price p_B is set such that it maximizes B 's profits, hence

$$\max_{p_B} \pi_B(p_A, p_B).$$

The respective best responses are given by

$$BR_A(p_B) = \left\{ p_A(p_B) \mid \frac{\partial \pi_A}{\partial p_A}(p_A, p_B) + \omega \frac{\partial \pi_B}{\partial p_A}(p_A, p_B) = 0 \right\} \tag{4.1}$$

and

$$BR_B(p_A) = \left\{ p_B(p_A) \mid \frac{\partial \pi_B}{\partial p_B}(p_A, p_B) = 0 \right\}. \tag{4.2}$$

Assumptions (iv) guarantee that the second order conditions

$$\frac{\partial^2 \pi_A}{\partial p_A^2} + \omega \frac{\partial^2 \pi_B}{\partial p_A^2} < 0 \tag{4.3}$$

and

$$\frac{\partial^2 \pi_B}{\partial p_B^2} < 0 \tag{4.4}$$

are satisfied for all $\omega \in [0, \frac{1}{\alpha_A}]$. Assumptions (iv) and (v) guarantee that the best responses are both positively sloped, as both

$$\frac{\partial BR_A(p_B)}{\partial p_B} = -\frac{\frac{\partial^2 \pi_A}{\partial p_A \partial p_B} + \omega \frac{\partial^2 \pi_B}{\partial p_A \partial p_B}.}{\frac{\partial^2 \pi_A}{\partial p_A^2} + \omega \frac{\partial^2 \pi_B}{\partial p_A^2}} \tag{4.5}$$

and

$$\frac{\partial BR_B(p_A)}{\partial p_A} = -\frac{\frac{\partial^2 \pi_B}{\partial p_A \partial pB}.}{\frac{\partial^2 \pi_B}{\partial p_B^2}} \tag{4.6}$$

are strictly positive. Denote by $(p_A^O(\omega), p_B^O(\omega))$ a Nash equilibrium price vector. We assume this to be unique. The equilibrium is stable if

$$\frac{\partial (BR_A)^{-1}}{\partial p_A}(p_A^O(\omega)) > \frac{\partial BR_B}{\partial p_A}(p_A^O(\omega)) \tag{4.7}$$

which we assume henceforth.

With the following Proposition we characterize Nash equilibrium prices and profits as a function of ω, the relative share of cash flow rights held by investor I_1 in firm B over firm A.

Proposition 4.1 (Equilibrium Prices and Profits under Separate Control): *Let I_1 control firm A only, so that the two firms compete with each other à la Bertrand. Then*

(i) $p_A^O(\omega) \geq p_B^O(\omega)$ *for all* $\omega > 0$,

(ii) $\pi_A(p_A^O(\omega), p_B^O(\omega)) \leq \pi_B\,(p_A^O(\omega), p_B^O(\omega))$ *for all* $\omega > 0$,

(iii) $p_A^O(\omega)$ *and* $p_B^O(\omega)$ *increase in* ω, *with* $\frac{\partial p_A^O}{\partial \omega} > \frac{\partial p_B^O}{\partial \omega}$,

(iv) $\pi_A(p_A^O(\omega), p_B^O(\omega))$ *increases for small* ω *up to some* ω^O, *and strictly decreases thereafter.* $\pi_B(p_A^O(\omega), p_B^O(\omega))$ *increases for all* $\omega > 0$.

The main effect of an investment of I_1 in B under separate control is that I_1 uses p_A to soften competition and thus to increase relative profits π_B, and this the more, the larger ω. As a direct corollary it emerges that, provided that demand is downward sloping, consumer welfare as measured by consumers's surplus decreases with an increase in ω, as long as no controlling stake is associated with that increase. We cannot say much about changes in total welfare, as this would necessitate a direct comparison of negative changes in consumer, and positive changes in producer surplus.[5]

4.3.2 Firm B controlled by I_1

We now consider the case in which I_1 holds controlling shares α_A in firm A and $\tilde{\alpha}_B$ in firm B, respectively. Consider first profit maxima involving both firms active. Then I_1 solves

$$\max_{p_A, p_B} \pi_A(p_A, p_B) + \omega \pi_B(p_A, p_B). \tag{4.8}$$

The necessary conditions are given by

$$\frac{\partial \pi_A}{\partial p_A}(p_A, p_B) + \omega \frac{\partial \pi_B}{\partial p_A}(p_A, p_B) = 0 \tag{4.9}$$

[5] In a Hotelling example of our product market specification we obtained a decrease also in total welfare with an increase in ω.

and

$$\frac{\partial \pi_A}{\partial p_B}(p_A, p_B) + \omega \frac{\partial \pi_B}{\partial p_B}(p_A, p_B) = 0. \tag{4.10}$$

The second order conditions are satisfied by invoking Assumptions (iii) and (iv). Denote the optimal choice by $(p_A^M(\omega), p_B^M(\omega))$.

Proposition 4.2 (Optimal Prices and Profits under Joint Control): *Let I_1 control both firms A and B. Then*

(i) $p_A^M(\omega) \leq p_B^M(\omega)$ *if* $\omega < 1$, $p_A^M(\omega) = p_B^M(\omega)$ *if* $\omega = 1$, *and* $p_A^M(\omega) \geq p_B^M(\omega)$ *if* $\omega > 1$

(ii) $\pi_A(p_A^M(\omega), p_B^M(\omega)) \gtreqless \pi_B\,(p_A^M(\omega), p_B^M(\omega))$ *for all* $\omega \lesseqgtr 1$

(iii) $p_A^M(\omega)$ *strictly increases and* $p_B^M(\omega)$ *strictly decreases in* ω

(iv) $\pi_A(p_A^M(\omega), p_B^M(\omega))$ *decreases and* $\pi_B(p_A^M(\omega), p_B^M(\omega))$ *increases in* ω.

Observe in particular that if the controlling stakes α_A, α_B and γ are such that $\omega = 1$, the monopoly solution obtains, no matter how small the stakes actually are.

4.4 Acquisition Decision

We now look at I_1's acquisition decisions. We relate the equilibrium allocation of ownership and control rights to the initial ownership structure in *A* and *B*. Again, we consider I_1 and firm *A* to be the only active investors in our model and thus exclude competitive bidding. I_1 and *A* are assumed to be able to buy any stake in firm *B* out of current profits obtained. The opportunity costs of their investments are normalized to zero. In line with the approach taken here, firm values are supposed to be solely determined by the profits obtained from product market activity.

In the following we analyze acquisition decisions separately for the four cases introduced in the model specification, that related to the structure of initial ownership of the remaining shares in *A*, and that of the shares in *B*.

We use dispersed ownership in the sense of Grossman and Hart (1980), so that every shareholder perceives herself as being non-pivotal. In particular, she believes that her decision of whether or not to accept an offer neither influences the overall stake I_1 holds in A or B, nor whether or not I_1 gains control in B.

If initial ownership in firm B is dispersed, then I_1 and A can acquire shares in B via a tender offer. Towards this, an offer price $P_B(\omega)$ per share is announced which then attracts a fraction $\alpha_B + \gamma$ of all shares outstanding. We normalize to unity the total mass of (infinitely divisible) shares so that $P_B(\omega)$ is the firm value. Following Burkart, Gromb, and Panunzi (1998), we use the concept of a rational expectations equilibrium in which all shareholders behave symmetrically, each shareholder tendering her shares with probability $\alpha_B + \gamma$ and retaining them with probability $1 - \alpha_B - \gamma$.

In equilibrium, every single atomistic shareholder is indifferent between tendering or not, and believes that both the acquisition of cash flow and/or control rights in firm B by I_1 does not depend on her decision. Therefore, the offer price for α_i shares in firm i is equal to $\alpha_i \pi_i(p_A^k(\omega), p_B^k(\omega)), k = O, M$ which—as shown in Section (4.3)—depends on I_1's stakes in A and B, and on whether she gains control in B. Formally, letting $\pi_i^k(\omega) \equiv \pi_i(p_A^k(\omega), p_B^k(\omega)), i = A, B; k = O, M$

$$\begin{aligned} P_A^k(\omega) &= \pi_A^k(\omega) + \gamma(\pi_B^k(\omega) - P_B^k(\omega)) \\ P_B^k(\omega) &= \pi_B^k(\omega). \end{aligned} \tag{4.11}$$

This specification reflects the free–rider problem discussed by Grossman and Hart (1980) faced by I_1 when acquiring shares from dispersed owners: Because shareholders rationally expect the consequences of that acquisition on firm profits, the acquisition price fully incorporates the allocative gains to firm B. Hence I_1 can never gain directly from acquiring (additional) cash flow rights when (remaining) ownership is dispersed, as the acquisition price is always equal to the profits she will earn. However, if investing in firm B, I_1 may benefit from an increased value of her initial stake in firm A, which we have modeled in Section 4.3. In all, if the (remaining) ownership is dispersed, our acquiring investor I_1 has *de facto* no bargaining power.

By contrast, if the target shares of one of the firms $i, i = A, B$ are held by one investor, then we suppose that all bargaining power rests with the acquiring investor I_1, so she can absorb all the surplus generated from that acquisition. Accordingly, the acquisition price per share is determined by equalizing the seller's payoff obtained when selling some of his shares to I_1

and enjoying the profits from his remaining shares to his outside option, which is the payoff generated when selling no shares at all.

To make our arguments transparent, we henceforth sequence investor I_1's decisions. Within all cases, 1 through 4, we first consider the optimal specification of the relative weight ω, and then how this ω is optimally reached; that is, the trade–off between directly acquiring α_B shares vs. indirectly acquiring γ shares via firm A, together with the option of increasing I_1's stakes in firm A over and above α_A^0. In all this we analyze first the acquisition of cash flow rights separately for the cases where no control rights and where control rights go with them, and only thereafter whether the acquisition of control rights in B at the exogenously specified level $\hat{\alpha}_B$ is profitable to I_1.

4.4.1 Case 1: Remaining Shares in A Dispersed and Ownership of B Dispersed

Investor I_1's overall payoff from acquiring cash flow rights in firms A and B is given by

$$\Pi_1^k(\omega) = \alpha_A[\pi_A^k(\omega) + \gamma(\pi_B^k(\omega) - P_B^k(\omega))] - (\alpha_A - \alpha_A^0)P_A^k(\omega) + \alpha_B\left[\pi_B^k(\omega) - P_B^k(\omega)\right], k = O, M, \tag{4.12}$$

where $\omega = \frac{\alpha_B+\alpha_A\gamma}{\alpha_A}$, $\alpha_A \in [\alpha_A^0, 1]$, $\alpha_B \in [0, 1]$, $\gamma \in [0, 1]$, and $\alpha_B + \gamma \leq 1$.

The first term reflects her share α_A of the payoffs obtained from her interest in firm A, including cross holding acquisitions taken by that firm; the second term denotes the acquisition costs of an additional stake $(\alpha_A - \alpha_A^0)$; and the third term reflects payoffs after acquisition costs from a stake α_B in firm B.

Using (4.11) specifying the acquisition price when the (remaining) ownership is dispersed, investor I_1's overall acquisition payoff (4.12) reduces to

$$\Pi_1^k(\omega) = \alpha_A^0\pi_A^k(\omega), k = O, M. \tag{4.13}$$

All payoff increases generated through the acquisition go to the atomless owners of A and B. Now, result (iv) from Proposition 4.1 states that $\pi_A^O(\omega)$ increases for small ω. From Proposition 4.1 we know that $\omega^O = \arg\max \pi_A^O(\omega)$. By contrast, result (iv) from Proposition 4.2 states that $\pi_A^M(\omega)$ is a strictly decreasing function, so given full control, investor I_1's maximal payoff is trivially equal to $\Pi_1^M(0) = \alpha_A^0 \pi_A^M(0)$. This is so because any increase in her stakes in firm A leaves I_1 indifferent, whilst any direct or indirect increase in her stakes in firm B would, due to the allocative effects, result in a decrease in profits obtained from firm A.

Since $\pi_A^O(\omega^O)$ is the equilibrium payoff with positive weight on firm B and thus less than full weight on firm A, whilst $\pi_A^M(0)$ is the maximal monopoly payoff with complete weight on firm A , it must hold that $\pi_A^O(\omega^O) < \pi_A^M(0)$, so there must be an $\omega^M > 0$ with $\pi_A^O(\omega^O) = \pi_A^M(\omega^M)$ such that if the minimal controlling share $\hat{\alpha}_B \leq \alpha_B(\omega^M) + \gamma(\omega^M)$, then I_1 prefers to acquire a controlling stake $\hat{\alpha}_B$ in B. The payoff maximizing stake must be minimal because $\pi_A^M(\omega)$ is a strictly decreasing function.

Towards determining the mode of acquisition, observe that two alternative acquisition modes have differing allocative effects and thus are not payoff neutral. In fact, the direct acquisition of minimal shares $\hat{\alpha}_B$ results in some $\omega = \frac{\hat{\alpha}_B}{\alpha_A^0}$, whilst the indirect acquisition via firm A results in a smaller $\omega = \hat{\alpha}_B < \frac{\hat{\alpha}_B}{\alpha_A^0}$. Hence I_1 acquires the minimal controlling cash flow rights through firm A, so that $\gamma = \omega^M$ and $\alpha_B = 0$. Only if she would fully own firm A, i.e. $\alpha_A^0 = 1$, would she be indifferent between direct and indirect acquisition.

However, if $\hat{\alpha}_B > \omega^M$, then I_1 will acquire non–controlling cash flow rights in firm B such that $\frac{\alpha_B + \alpha_A^0 \gamma}{\alpha_A^0} = \omega^O$. In view of the fact that all acquisition rents are dissipated to the dispersed owners of A and B, she is indifferent between the direct and the indirect form of acquisition.

We summarize in

Proposition 4.3: *Suppose that initial ownership in both firms A and B is dispersed. Then there exists ω^M such that*

1. *if $\hat{\alpha}_B \leq \omega^M$, then it is optimal for I_1 to acquire a minimal controlling stake $\hat{\alpha}_B$ in B via a cross holding, so that $\gamma = \hat{\alpha}_B$ and $\alpha_B = 0$. In the product market, this results in prices $p_A^M(\gamma) < p_B^M(\gamma)$ and profits $\pi_A^M(\gamma) > \pi_B^M(\gamma)$.*

2. *If $\hat{\alpha}_B > \omega^M$, then investor I_1 acquires non–controlling shares so that $\frac{\alpha_B + \alpha_A \gamma}{\alpha_A} = \omega^O$. In this she is indifferent between the direct and the indirect mode of acquisition. In the product market, this results in prices $p_A^O(\omega^O) > p_B^O(\omega^O)$ and profits $\pi_A^O(\omega^O) < \pi_B^O(\omega^O)$.*

4.4.2 Case 2: Remaining Shares in *A* Concentrated and Ownership of *B* Dispersed

Now we study the case in which initial ownership of the remaining shares in A is concentrated, i.e. held by one investor called I_3. In line with our earlier discussion, the acquisition price for $\alpha_B - \alpha_A^0$ shares in firm A is determined so that I_3 is indifferent between selling and keeping them. Hence the acquisition price for $\alpha_A - \alpha_A^0$ additional shares in A, $P_A(\omega)$, conditional on investor I_1's acquisition of (non–) controlling shares in B must satisfy

$$(\alpha_A - \alpha_A^0)P_A^k(\omega) + (1-\alpha_A)[\pi_A^k(\omega) + \gamma(\pi_B^k(\omega) - P_B^k(\omega))] = (1-\alpha_A^0)\cdot\pi_A^k(\omega), k = O, M \quad (4.14)$$

where, as determined in Proposition 4.3, $\omega^O = \omega^O$ and $\omega^M = \gamma$. Thus the left hand side of (4.14) is the payoff to I_3 in case he sells to I_1 a fraction $(\alpha_A - \alpha_A^0)$ of firm A's shares, and the right hand side is his payoff if he does not sell. However, that payoff eventually reflects I_1's engagement in firm B, on which I_3 is able to free–ride.

If ownership of B is dispersed we have $P_B(\omega) = \pi_B(\omega^k)$ so that

$$P_A^k(\omega) = \frac{(1-\alpha_A^0)\cdot\pi_A^k(\omega^k) - (1-\alpha_A)\cdot\pi_A^k(\omega)}{\alpha_A - \alpha_A^0} = \pi_A^k(\omega).$$

Since the purchase price of shares exactly reflects the payoffs generated from an engagement in firm B, I_1's choice as to that remains unchanged with the structure of the remaining ownership in firm A.

Proposition 4.4: *Suppose that ownership of the remaining shares in A is concentrated and initial ownership in firm B is dispersed. Then the results of Proposition 4.3 carry over.*

The reason for this surprising result is that the choice of α_A has no influence, no matter whether I_1chooses to obtain controlling cash flow rights in firm B via cross ownership rather than direct investment, or to stick to her initial engagement.

4.4.3 Case 3: Remaining Shares in A Dispersed and Ownership of B Concentrated

We now consider the case in which firm B is initially held by one investor, I_2 say, who is not invested in A. In line with before, the acquisition price for $\alpha_B+\gamma$ shares in firm B is determined so that I_2 is indifferent between selling and keeping them. It thus satisfies

$$(\alpha_B+\gamma)P_B^k(\omega)+(1-\alpha_B-\gamma)\pi_B^k(\omega)=\pi_B^O(0), k=O,M, \tag{4.15}$$

where $\omega=\frac{\alpha_B+\alpha_A\gamma}{\alpha_A}$. Notice that I_2's outside option on the right hand side is to obtain $\pi_B^O(0)$, the profit of B without any investment of I_1 in B. From this we get

$$P_B^k(\omega)=\frac{1}{\alpha_B+\gamma}\cdot[\pi_B^O(0)-(1-\alpha_B-\gamma)\cdot\pi_B^k(\omega)]. \tag{4.16}$$

In contrast to the cases in which ownership in firm B is dispersed, the benefits of control over B are shared with I_2 only if $\alpha_B+\gamma<1$, because of the dependence of the post acquisition firm value on the ownership structure. Therefore, if no full acquisition takes place, the acquisition price depends on both ω, and on whether I_1 gains control in B.

As before, the acquisition price for shares in firm A in the case the remaining shares are under dispersed ownership is equal to the overall value of A. Using (4.16),

$$P_A^k(\omega)=\pi_A^k(\omega)+\gamma\{\pi_B^k(\omega)-\frac{1}{\alpha_B+\gamma}\cdot[\pi_B^O(0)-(1-\alpha_B-\gamma)\cdot\pi_B^k(\omega)]\}, k=O,M.$$

Investor I_1's overall payoff is now

$$\begin{aligned}\Pi_1^k(\omega)=\alpha_A\cdot\Big(\pi_A^k(\omega)+\gamma\cdot\{\pi_B^k(\omega)-\frac{1}{\alpha_B+\gamma}\cdot[\pi_B^O(0)-(1-\alpha_B-\gamma)\cdot\pi_B^k(\omega)]\}\Big)\\ +\alpha_B\cdot\{\pi_B^k(\omega)-\frac{1}{\alpha_B+\gamma}\cdot[\pi_B^O(0)-(1-\alpha_B-\gamma)\cdot\pi_B^k(\omega)]\}-(\alpha_A-\alpha_A^0)P_A^k(\omega), k=O,M.\end{aligned}$$

Since investor I_1 is indifferent between acquisition and non–acquisition from dispersed owners of A, we can rewrite this W.L.O.G. as

$$\Pi_1^k(\omega)=\alpha_A^0\cdot\pi_A^k(\omega)+\frac{\alpha_B+\alpha_A^0\gamma}{\alpha_B+\gamma}\cdot[\pi_B^k(\omega)-\pi_B^O(0)], k=O,M. \tag{4.17}$$

The first term specifies the value of I_1's initial stake in A. The second term refers to I_1's net benefit from directly or indirectly acquiring $\alpha_B + \gamma$ shares in B.

In the following proposition we establish that $\Pi_1^k(\omega)$ is maximized for $k = M$, $\alpha_B = 1$ and $\gamma = 0$, no matter the level of α_A^0. This reflects I_1's interest in fully internalizing the positive acquisition gains via $\alpha_B = 1$, whilst otherwise these acquisition gains would have to be shared with the owners of the remaining shares in A.
Towards establishing that proposition, we introduce Assumption

(v) $\pi_A^M(\omega) + \pi_B^M(\omega)$ is maximal at $\omega = 1$.

This assumption furthers the symmetry in the two firms' payoffs. It rules out that investor I_1 prefers to shut down one of the two firms in order to reduce fixed costs when controlling both of them with symmetric weights. The issue of firm shut down could arise otherwise if products are close substitutes and fixed costs are sufficiently high.[6]

Proposition 4.5: *Suppose that ownership of the remaining shares in A is dispersed, and initial ownership in firm B is concentrated. Then it is optimal for I_1 to always acquire a full controlling direct investment in firm B, such that $\alpha_B = 1$ and $\gamma = 0$. However, she is indifferent between selling and not selling shares in firm A. In the product market, this results in asymmetric monopoly prices $p_A^M(1/\alpha_A^0) > p_B^M(1/\alpha_A^0)$ and monopoly profits $\pi_A^M(1/\alpha_A^0) < \pi_B^M(1/\alpha_A^0)$.*

4.4.4 Case 4: Remaining Shares in A Concentrated and Ownership of B Concentrated

We finally study the case in which the shares in B are initially held by I_2 as in Subsection (4.4.3) but at the same time the remaining shares in A are held by investor I_3 as in Subsection (4.4.2).

The acquisition price $P_A^k(\omega)$ for additional shares in A must satisfy

$$(\alpha_A - \alpha_A^0) \cdot P_A^k(\omega) + (1 - \alpha_A) \cdot (\pi_A^k(\omega) + \gamma[\pi_B^k(\omega) - P_B^k(\omega)] = (1 - \alpha_A^0) \cdot \pi_A^k(1/\alpha_A^0), i = O, M,$$

where the right hand side follows from Proposition 4.5 because the relevant payoff function

[6]We will analyze the impact of product substitutability and fixed costs on investors' acquisition decision extensively in a future version of this paper. All over the current version we focus on interior solutions in which both firms remain active.

for I_1 is equal to the one in (4.17) if I_3 does not sell his shares in A.

We know from Proposition 4.5 that irrespective of the level of α_A^0 (controlling) shares held in firm A, I_1 will acquire all shares in B by a direct investment, i.e. $\alpha_B = 1$.

$P_B(\omega)$ is given by (4.16) so that

$$\begin{aligned}(\alpha_A - \alpha_A^0) \cdot P_A^k(\omega) &= (1-\alpha_A^0)\cdot \pi_A^k(1/\alpha_A^0) - (1-\alpha_A)\cdot\{\pi_A^k(\omega) + \gamma[\pi_B^k(\omega) - P_B^k(\omega)]\} \\ &= (1-\alpha_A^0)\cdot \pi_A^k(1/\alpha_A^0) - (1-\alpha_A)\cdot\Big(\pi_A^k(\omega) + \gamma\pi_B^k(\omega) \\ &\quad + \frac{\gamma}{\alpha_B+\gamma}\cdot((1-\alpha_B-\gamma)\cdot\pi_B^k(\omega) - \pi_B^O(0))\Big).\end{aligned} \tag{4.18}$$

Hence, we have

$$\begin{aligned}\Pi_1 &= \alpha_A[\pi_A^k(\omega) + \gamma(\pi_B^k(\omega) - P_B^k(\omega))] + \alpha_B\pi_B^k(\omega) - (\alpha_A-\alpha_A^0)\cdot P_A^k(\omega) \\ &\quad - \alpha_B P_B^k(\omega)\end{aligned}$$

which is, after some algebra,

$$\Pi_1 = \pi_A^k(\omega) + \pi_B^k(\omega) - (1-\alpha_A^0)\cdot\pi_A^k(1/\alpha_A^0) - \pi_B^O(0), k = O, M. \tag{4.19}$$

This shows that I_1's payoff is given by the sum of profits of both firms less the outside options of I_2 and I_3 which are independent of I_1's choice of ω. However, the latter's outside option is weighted by $1-\alpha_A^0$, which is minimized by acquiring all of firm A and choosing α_A^0.

In all, investor I_1's payoff is maximized for $\omega = 1$. She is indifferent between achieving this via a cross holding, a direct investment, or any convex combination of the two.

Proposition 4.6: *Suppose that ownership of the remaining shares in A and of B is concentrated. Then it is optimal for I_1 to acquire a controlling investment in firm B with symmetric weights on A and B, i.e. to choose for any $\alpha_A \geq \alpha_A^0$ a convex combination between a full cross holding, $\alpha_B = 0, \gamma = 1$ and a controlling direct investment $\alpha_B = \alpha_A, \gamma = 0$ in firm B s.t. $\omega = 1$. In the product market, this results in symmetric monopoly with prices $p_A^M(1) = p_B^M(1)$ and monopoly profits $\pi_A^M(1) = \pi_B^M(1)$.*

4.5 Conclusion

In this paper, we analyze the optimal allocation of ownership and control rights in two competing firms, *A* and *B*, for an investor who initially holds controlling cash flow rights in firm *A*. Cash flow and control rights in firm *B* can be acquired by that investor either directly by buying shares herself, or indirectly by having the controlled firm *A* buy shares. We distinguish between four cases of initial ownership structures generated by combining the alternatives that the (remaining) shares in firm *A*, and all shares in firm *B* may be held by atomless dispersed owners, or by one block holder.

We find that if the initial ownership of the target firm *B*, is dispersed, I_1 acquires a minimal controlling investment in *B* if the (exogenous) critical level of controlling shares is small enough. Otherwise she will buy a non–controlling (smaller) share in firm *B*. The reason for the former is free–riding by the remaining shareholders in *B*. In the latter case a trade–off arises from the fact that the acquisition of a small non–controlling stake in *B* allows I_1 to benefit from the strategic complementarity involved in price increases. As long as I_1 does not own all of firm *A* 's cash flow rights, this acquisition is accomplished via a cross holding. I_1 is indifferent between cross holding and direct investment only if she is able to acquire all of *A*'s remaining shares.

If the initial ownership in the target firm *B* is concentrated, the equilibrium allocation of ownership rights depends on the initial ownership structure of the remaining shares in *A* as well. If it is concentrated, I_1 will invest symmetrically in both firms and control both of them. Here, she is indifferent between drawing on a cross ownership arrangement or investing herself in *B*. By contrast, if the remaining ownership in *A* is dispersed, I_1 will directly invest in *B* rather than drawing on cross holdings. The reason is that by investing directly I_1 does not have to share the acquisition gain in *B* with the remaining shareholders in *A*.

The present model setup is limited in several respects. Firstly, our industry consists of two firms only, so the acquisition of control rights by our investor leads immediately to monopolistic control. Secondly, one might argue that investors even if holding controlling cash flow rights typically do not exercise control on prices, but on strategic variables such as product quality enhancing, or product portfolio widening, or cost reducing investment. Thirdly, the acquisition of cash flow or control rights may be contested by competitors.

Towards these extensions, we have analyzed the numerical version of a model involving three symmetric competing specialized firms, in which investors with controlling cash flow rights exercise control over cost reducing investment (and firm acquisitions), while leaving the pricing decision to an independent management. We again consider an investor holding controlling cash flow rights in some firm A and investing in firm B, so the third firm C including its owners, while active in the product market, is passive in the market for shares and control rights.

Surprisingly little changes in this extended set up. Essentially all that happens is that the effects on product market prices and profits of the two firms A and B active in the market for acquisitions are weakened due to both, the impact of outside competition from firm C and the impact of less direct control, namely on cost reducing investment rather than prices, by the controlling investor. In view of this, the above analysis of the acquisition stake should carry through these generalizations. In view of this we should emphasize, that with the above results we do provide very clearly testable empirical predictions.

Yet one important generalization, that includes contests between block holders involved in more than two firms, is much more involved, and must be left for further research.

Appendix A

Appendix of Chapter 2

A.1 Proofs of Section 2.2

Proof of Lemma 2.1. I first show that the agent's individual rationality constraint (IR_i) is not binding and that the limited liability constraint (LL_i) is binding instead. Suppose by contradiction (IR_i) is binding. Then solving (IR_i) for $a_i(e)$ and substituting a_i in the principal's objective function by $a_i(e)$ yields $p_i(e)\overline{y} - c(e)$ which is maximized at the first–best effort level. As mentioned above $a_i(e)$ will be negative at e_i^{FB} which violates (LL_i). Thus, (IR_i) cannot be binding. If a_i is positive, however, decreasing a_i to zero is beneficial for the principal by additive separability of V in a and keeps (LL_i) satisfied. Thus, (LL_i) is binding at $a_i = 0$, while (IR_i) is not binding.

Using $a_i = 0$ and $b_i(e) = c'(e)/p_i'(e)$ from (FOC_A) (=first–order approach) the principal's objective function can be expressed by $p_i(e)(\overline{y} - b(e))$. Taking the first derivative w.r.t. e yields the required condition. □

Proof of Lemma 2.2. Suppose not, then there exists a profitable deviation for the L–type since $b_L^* = \arg\max_{b_L > 0} V_L((a, b_L), \mu(L|.) = 1)$ for all $a \geq 0$ and V_L is strictly decreasing in a (=additive separability of b and a). □

Proof of Lemma 2.3. As shown in the proof of Lemma 2.1 (IR_H) will never be binding under limited liability at zero under given assumptions. If (IC_{P_L}) is not trivially satisfied at w_H^*, i.e. $V_{LL}(w_L^*) < V_{LH}(w_H^*)$, then ($IC_{P_L}$) will be binding in the least–cost separating contract

w_H^s, i.e. $V_{LL}(w_L^*) = V_{LH}(w_H^s)$. Suppose not, then $V_{LL}(w_L^*) > V_{LH}(w_H^s)$. If $a_H^s > a_H^* = 0$, decreasing a_H^s raises both, $V_{LH}(w_H^s)$ and $V_{HH}(w_H^s)$ keeping the agent's incentive constraint (IC_H) binding. This states a contradiction to least–cost separation for the interior solution case. If $a_H^s = 0$ instead (=corner solution case), then decreasing the distance between b_H^s and b_H^* is strictly profit–enhancing for the H–type principal and reduces the slackness of the L–type's incentive constraint by strict concavity of V. To see this more clearly define the L–type's hypothetically optimal premium if being perceived as the H–type principal with probability one, $b_{LH}^* = \arg\max_{b_{LH}>0} V_L((0, b_{LH}), \mu(H|.) = 1)$, and note that (IC_{P_L}) is violated at b_{LH}^* since $V_{LL}(w_L^*) < V_{LH}(w_{LH}^*)$ by $b_H(e) < b_L(e)$ for all e. By continuity of V there exists a $\tilde{b}$ with $|\tilde{b} - b_{LH}^*| < |\tilde{b} - b_H^s|$ and $|\tilde{b} - b_H^*| > 0$ such that (IC_{P_L}) is binding. Moreover, by monotonicity and concavity of V choosing any premium b_H^s further apart from b_H^* than $\tilde{b}$ makes the H–type principal worse off. Thus, $(0, \tilde{b})$ states a profitable deviation from any contract $w_H^s = (0, b_H^s)$ with $V_{LL}(w_L^*) > V_{LH}(w_H^s)$ for the corner solution case. Hence, (IC_{P_L}) is binding in the least–cost separating contract w_H^s.

If (IC_H) and (IC_{P_L}) are binding, then b and a in (2.7) can be replaced by $b_H(e)$ rearranging (IC_H) and by $\tilde{a}(e)$ rearranging (IC_{P_L}). The transformed objective function equals $(p_H(e) - p_L(e)) \cdot (\overline{y} - b_H(e)) + V_{LL}(w_L^*)$ which is strictly concave by the assumptions on $p_i(e)$ that $p_H''(e) - p_L''(e) \leq 0$ and $p_H'(e) - p_L'(e) \geq 0$ for all e. Maximizing this objective function over e yields the required condition for case 1 which uniquely identifies the least–cost separating effort level.

If (LL_H), (IC_H), and (IC_{P_L}) are binding, then the interior solution of the H–type principal's transformed objective function would specify a negative fixed-wage payment a_H^s which is ruled out by (LL_H). Substituting $a_H^s = 0$ from (LL_H) and $b_H(e)$ from (IC_H) into (IC_{P_L}) gives the constraint of the condition in case 2. The constraint yields up to two solutions of which the one that maximizes the H–type principal's expected utility is selected by the condition in case 2.

Finally, if (IC_{P_L}) is trivially satisfied by w_H^*, then w_H^* states the unique least–cost separating contract. □

Proof of Lemma 2.4. To provide a proof for Lemma 2.4, the 4 functions in equation (2.8) (resp. (2.9)) have to be compared at 3 (resp. 4) points although no explicit values of neither premia nor expected utilities can be determined. Moreover, since $b_H(e) < b_L(e)$ $\forall e$ it always holds that $b_H^* > b_{HL}^*$ and $b_{LH}^* > b_L^*$ but the ordering of b_H^* and b_{LH}^* and of b_{HL}^* and b_L^* depends on parameters and specific functional forms of $p(e)$ and $c(e)$. To provide a general proof in this setting I will make use of the fact that for a specific combination of parameters of $p_i(e)$ the 4 points in equation (2.9) partially coincide and at the same time equation (2.8) and (2.9) become equivalent.[1]

First note that if $b^*_H = b^*_{LH}$ (or equivalently $e^*_H = e^*_{LH}$), it follows from Lemma 2.3 (interior solution case) that $b^s_H = b^*_H = b^*_{LH}$. It is shown in Proposition 2.2 and Corollary 2.2.1 that this is case if $\Delta s/\Delta r = s_L/r_L$. Moreover, for $\Delta s/\Delta r = s_L/r_L$ it also holds that $b^*_{HL} = b^*_L$ since $p'_H(e)/p_H(e) = s_L p'(e)/(r_L + s_L p(e)) = p'_L(e)/p_L(e)$ by the first–order conditions of the principal's problem under project observability in Lemma 2.1. Hence, the sufficient condition in equation (2.9) becomes also necessary here. Next, I show that the necessary and sufficient condition for existence (2.8) is always fulfilled for $\Delta s/\Delta r = s_L/r_L$. Define $\Delta V^*_{\cdot,H} \equiv V^*_{HH} - V^*_{LH}$ and $\Delta V^*_{\cdot,L} \equiv V^*_{HL} - V^*_{LL}$. Now, using that $e^s_H = e^*_H = e^*_{LH}$ and $e^*_L = e^*_{HL}$ if $\Delta s/\Delta r = s_L/r_L$ yields $\Delta V^*_{\cdot,H} = (\Delta r\theta_H + \Delta s\theta_H p(e^*_H))(\bar{y} - b_H(e^*_H))$ and $\Delta V^*_{\cdot,L} = (\Delta r\theta_H + \Delta s\theta_H p(e^*_{HL}))(\bar{y} - b_L(e^*_{HL}))$. Equation (2.8) will be satisfied if $\Delta V^*_{\cdot,H} - \Delta V^*_{\cdot,L} \geq 0$. I next show that this holds true here. The trick I will use takes into account that $\Delta V^*_{\cdot,H}$ (resp. $\Delta V^*_{\cdot,L}$) are equal to V^*_{HH} (resp. V^*_{HL}) up to a constant if $\Delta s/\Delta r = s_L/r_L$.

$$\begin{aligned}
\Delta V^*_{\cdot,H} &= \Big(\Delta r\theta_H + \Delta s\theta_H p(e^*_H)\Big)\Big(\bar{y} - b_H(e^*_H)\Big) \\
&= \Big(\Delta r\theta_H + \frac{\Delta r s_L}{r_L}\theta_H p(e^*_H)\Big)\Big(\bar{y} - b_H(e^*_H)\Big) && \text{by } \Delta s = \frac{\Delta r s_L}{r_L} \\
&= \frac{\Delta r\theta_H}{r_L}\Big(r_L + s_L p(e^*_H)\Big)\Big(\bar{y} - b_H(e^*_H)\Big).
\end{aligned}$$

[1] It is shown in Proposition 2.2 that this specific combination of parameters is also the cutoff point for the main efficient result of this paper.

Analogously, we receive $\Delta V^*_{\cdot,L} = \frac{\Delta r\theta_H}{r_L}\Big(r_L + s_L p(e^*_{HL})\Big)\Big(\overline{y} - b_L(e^*_{HL})\Big)$.

$$\begin{aligned}
V^*_{HH} &= \Big((r_L + \Delta r\theta_H) + (s_L + \Delta s\theta_H)p(e^*_H)\Big)\Big(\overline{y} - b_H(e^*_H)\Big) \\
&= \Big((r_L + \Delta r\theta_H) + (s_L + \frac{\Delta r s_L\theta_H}{r_L})p(e^*_H)\Big)\Big(\overline{y} - b_H(e^*_H)\Big) && \text{by } \Delta s = \frac{\Delta r s_L}{r_L} \\
&= \Big((r_L + \Delta r\theta_H) + \frac{s_L}{r_L}(r_L + \Delta r\theta_H)p(e^*_H)\Big)\Big(\overline{y} - b_H(e^*_H)\Big) \\
&= \Big((r_L + \Delta r\theta_H)(1 + \frac{s_L}{r_L}p(e^*_H))\Big)(\overline{y} - b_H(e^*_H)) \\
&= \frac{(r_L + \Delta r\theta_H)}{r_L}\Big(r_L + s_L p(e^*_H)\Big)\Big(\overline{y} - b_H(e^*_H)\Big).
\end{aligned}$$

$$\begin{aligned}
V^*_{HL} &= \Big((r_L + \Delta r\theta_H) + (s_L + \Delta s\theta_H)p(e^*_{HL})\Big)\Big(\overline{y} - b_L(e^*_{HL})\Big) \\
&= \frac{(r_L + \Delta r\theta_H)}{r_L}\Big(r_L + s_L p(e^*_{HL})\Big)\Big(\overline{y} - b_L(e^*_{HL})\Big) && \text{analogously to } V^*_{HH}.
\end{aligned}$$

Hence, the maximizer of $V_{HH}(e)$, e^*_H, is also a maximizer of $\Delta V_{\cdot,H}(e)$. The same argument applies to $V_{HL}(e)$ and $\Delta V_{\cdot,L}(e)$. Moreover, $\Delta V_{\cdot,H}(e)$ and $\Delta V_{\cdot,L}(e)$ inherit strict concavity in e from $V_{HH}(e)$ and $V_{HL}(e)$.

Thus, it finally suffices to show that $\Delta V_{\cdot,H}(e)$ is weakly larger than $\Delta V_{\cdot,L}(e)$ at $e = e^*_{HL}$.

$$\begin{aligned}
\Delta V_{\cdot,H}(e^*_{HL}) - \Delta V_{\cdot,L}(e^*_{HL}) &= \frac{\Delta r\theta_H}{r_L}\Big(r_L + s_L p(e^*_{HL})\Big)\Big(\overline{y} - b_H(e^*_{HL}) - (\overline{y} - b_L(e^*_{HL}))\Big) \\
&= \frac{\Delta r\theta_H}{r_L}\Big(r_L + s_L p(e^*_{HL})\Big)\Big(b_L(e^*_{HL}) - b_H(e^*_{HL})\Big),
\end{aligned}$$

which is strictly positive since $b_H(e) < b_L(e)\ \forall e$. By strict concavity of $\Delta V_{\cdot,H}(e)$ and $\Delta V_{\cdot,L}(e)$ I receive that $\Delta V^*_{\cdot,H} = \Delta V_{\cdot,H}(e^*_H) > \Delta V_{\cdot,H}(e^*_{HL}) > \Delta V_{\cdot,L}(e^*_{HL}) = \Delta V^*_{\cdot,L}$, which completes this first step of the proof.

If $\Delta r \neq r_L\Delta s/s_L$, $\Delta V^*_{\cdot,H}$ and $\Delta V^*_{\cdot,L}$ can be expressed as follows

$$\begin{aligned}
\Delta V^*_{\cdot,H} &= V_{HH}(e^*_H) - V_{LH}(e^*_{LH}) \\
&= \Big((r_L + \Delta r\theta_H) + (s_L + \Delta s\theta_H)p(e^*_H)\Big)\Big(\overline{y} - b_H(e^*_H)\Big) - V^*_{LH} \\
\Delta V^*_{\cdot,L} &= V_{HL}(e^*_{HL}) - V_{LL}(e^*_L) \\
&= \Big(\Delta r\theta_H + \Delta s\theta_H p(e^*_{HL})\Big)\Big(\overline{y} - b_L(e^*_{HL}))\Big) - V^*_{LL}.
\end{aligned}$$

The sufficient condition for existence (2.9) will be met if $\Delta V^*_{\cdot,H} - \Delta V^*_{\cdot,L} \geq 0$. In a last step it will be shown that (2.9) is always satisfied if $\Delta r < r_L \Delta s / s_L$, while (2.9) (and therefore potentially (2.8)) might be violated if Δr is much larger than $r_L \Delta s / s_L$. Given that $(\Delta V^*_{\cdot,H} - \Delta V^*_{\cdot,L})|_{\Delta r = r_L \Delta s / s_L} > 0$ it suffices to show that

$$
\begin{aligned}
(\Delta V^*_{\cdot,H} - \Delta V^*_{\cdot,L})|_{\Delta r' < r_L \Delta s / s_L} &> (\Delta V^*_{\cdot,H} - \Delta V^*_{\cdot,L})|_{\Delta r = r_L \Delta s / s_L} && \text{and} \\
(\Delta V^*_{\cdot,H} - \Delta V^*_{\cdot,L})|_{\Delta r'' > r_L \Delta s / s_L} &< (\Delta V^*_{\cdot,H} - \Delta V^*_{\cdot,L})|_{\Delta r = r_L \Delta s / s_L} && \forall \Delta r', \Delta r'' \text{ feasible.}
\end{aligned}
$$

From the first–order conditions for $V_{ij}(e)$, it can be seen that V^*_{LH} and V^*_{LL} are independent of Δr. Hence, it is sufficient to show that $dV_{HH}(e^*_H(\Delta r), \Delta r)/d\Delta r - dV_{HL}(e^*_{HL}(\Delta r), \Delta r)/d\Delta r < 0$ for all Δr feasible.

$$
\begin{aligned}
\frac{dV_{HH}(e^*_H(\Delta r), \Delta r)}{d\Delta r} &= \frac{\partial V_{HH}(e^*_H(\Delta r), \Delta r)}{\partial \Delta r} + \frac{\partial V_{HH}(e^*_H(\Delta r), \Delta r)}{\partial e} \cdot \frac{de^*_H(\Delta r)}{d\Delta r} \\
&= \frac{\partial V_{HH}(e^*_H(\Delta r), \Delta r)}{\partial \Delta r} + 0 \quad \text{by the envelope theorem} \\
&= \theta_H \Big(\bar{y} - b_H(e^*_H(\Delta r)) \Big) > 0 \\
\frac{dV_{HL}(e^*_{HL}(\Delta r), \Delta r)}{d\Delta r} &= \frac{\partial V_{HL}(e^*_{HL}(\Delta r), \Delta r)}{\partial \Delta r} + \frac{\partial V_{HL}(e^*_{HL}(\Delta r), \Delta r)}{\partial e} \cdot \frac{de^*_{HL}(\Delta r)}{d\Delta r} \\
&= \theta_H \Big(\bar{y} - b_L(e^*_{HL}(\Delta r)) \Big) > 0
\end{aligned}
$$

It now holds that $dV_{HH}(e^*_H(\Delta r), \Delta r)/d\Delta r < dV_{HL}(e^*_{HL}(\Delta r), \Delta r)/d\Delta r$ for all Δr feasible since $b_H(e^*_H(\Delta r)) = b^*_H(\Delta r) > b^*_{HL}(\Delta r) = b_L(e^*_{HL}(\Delta r))$ for all Δr feasible.

By applying the implicit function theorem to the first–order condition of $V_{H,\cdot}(e)$, it can be easily shown that $de^*_{H,\cdot}(\Delta r)/d\Delta r < 0$. Since $e^*_H(\Delta r) > e^*_{HL}(\Delta r)$ (or equivalently $b^*_H(\Delta r) > b^*_{HL}(\Delta r)$) for all Δr feasible there exists a $\Delta r^c_1 > r_L \cdot \Delta s / s_L$ s.t. $b^*_{HL}(\Delta r) \leq 0$ for $\Delta r \geq \Delta r^c_1$ and $b^*_H(\Delta r^c_1) > 0$. For Δr sufficiently large there exists a $\Delta r^c_2 > r_L \cdot \Delta s / s_L$ s.t. $(\Delta V^*_{\cdot,H} - \Delta V^*_{\cdot,L})|_{\Delta r} \leq 0$ for $\Delta r \geq \Delta r^c_2$. Finally, I define Δr^c as $\min\{\Delta r^c_1, \Delta r^c_2\}$.

This argument is given for interior solutions (case 1 of Lemma 2.3) but is also applicable to corner solutions (case 2 of Lemma 2.3). To see this note that the sufficient condition for existence (2.9) includes pure fixed–payment separation, which could have been chosen by the H–type principal if choosing pure premium separation (=corner solution). Thus, by revealed preferences the H–type principal in case 2 of Lemma 2.3 will not deviate from the least–cost separating contract if (2.9) holds. □

Proof of Proposition 2.1. For σ being a PBE, w_L^* (resp. w_H^s) must maximize the L–type's (resp. H–type's) expected utility given σ_A. For pessimistic out-of-the-equilibrium beliefs $\mu(H|w \neq w_H^s) = 0$, the L–type optimally chooses w_L^*, but the H–type might prefer not to separate from the L–type via w_H^s if this is very costly relative to being perceived as low-type principal (=potential non-existence). The best non-separating contract for H–type is equal to $w_{HL}^* = (0, b_{HL}^*)$ which maximizes $V_H(w, \mu(L|.) = 1)$. Hence, σ is an equilibrium if and only if $V_{HH}(w_H^s) \geq V_{HL}((0, b_{HL}^*))$ which is the case for $\Delta r \leq \Delta r^c$ (cf. Lemma 2.4).[2]

By applying the intuitive criterion of Cho and Kreps (1987), the least–cost separating contract is uniquely selected in equilibrium. This means that equilibria which allow credible deviations for the H–type are ruled out: Hence, there is no contract w with $\sigma_P(w|i) = 0$ for all i such that $\hat{\sigma}_A(.|w) \cdot V_{LH}(w) < V_{LL}(w_L^*)$ for all possible best responses $\hat{\sigma}_A(.|w)$ to w, while $\hat{\sigma}_A(.|w) \cdot V_{HL}(w) > V_{HH}(w_H^*)$ for all possible best responses $\hat{\sigma}_A(.|w)$ to w for which the agent's beliefs satisfy $\hat{\mu}(H|w) = 1$. □

Proof of Proposition 2.2. Firstly, consider the first–order condition of the H–type principal's problem for interior solutions (=case1 of Lemma 2.3) which determines e_H^s, the least–cost separating effort level. Suppose that $(p'_H(e) - p'_L(e))(\bar{y} - b_H(e)) - (p_H(e) - p_L(e))b'_H(e) \gtrless 0$ for $e = e_H^*$, the second-best effort level. Rearranging yields

$$\frac{(p'_H(e_H^*) - p'_L(e_H^*))}{(p_H(e_H^*) - p_L(e_H^*))} \gtrless \frac{b'_H(e_H^*)}{(\bar{y} - b_H(e_H^*))}.$$

From the first–order condition of the H–type's problem under project observability in (2.6), I receive analogously,

$$\frac{p'_H(e_H^*)}{p_H(e_H^*)} = \frac{b'_H(e_H^*)}{(\bar{y} - b_H(e_H^*))}.$$

Substituting the the last equation into the inequality from above and simplifying yields

$$\frac{p'_H(e_H^*)}{p_H(e_H^*)} \gtrless \frac{p'_L(e_H^*)}{p_L(e_H^*)}.$$

By multiplying with e_H^*, I get the elasticity condition in (2.10) (for case 1 of Lemma 2.3). I

[2] It turns out that relating the existence condition to properties of success probability function is difficult in the general framework since the model does not satisfy a single-crossing property. Moreover, the comparison between $V_{HH}(w_H^s)$ and $V_{HL}((0, b_{HL}^*))$ requires knowledge of the contracts (a_H^s, b_H^s) and $(0, b_{HL}^*)$ in absolute terms rather than implicit functional forms.

conclude that $(p'_H(e) - p'_L(e))(\bar{y} - b_H(e)) - (p_H(e) - p_L(e))b'_H(e) \gtrless 0$ at $e = e^*_H$ corresponds to $e^s_H \gtrless e^*_H$ by strict concavity of the principal's expected utility in the least–cost separation case (see proof of Lemma 2.3).

In case 2 of Lemma 2.3, i.e. if also (LL_H), besides (IC_H) and (IC_{P_L}), is binding, the interior solution from above does not exist. However, by continuity of the principal's expected utility function, the selection condition in case 2 picks the effort level with the corresponding properties, i.e. $e^s_H > e^*_H$ if and only if $\eta_H(e^*_H) > \eta_L(e^*_H)$. □

Proof of Corollary 2.2.1.

$$\begin{aligned}
\eta_H(e) &> \eta_L(e) \quad \Leftrightarrow \\
\frac{p'_H(e)}{p_H(e)} &> \frac{p'_L(e)}{p_L(e)} \quad \Leftrightarrow \\
\frac{(s_L + \Delta s\theta_H) \cdot p'(e)}{(r_L + \Delta r\theta_H) + (s_L + \Delta s\theta_H) \cdot p(e)} &> \frac{s_L \cdot p'(e)}{r_L + s_L \cdot p(e)} \quad \Leftrightarrow \\
\frac{r_L + s_L \cdot p(e)}{s_L} &> \frac{(r_L + \Delta r\theta_H) + (s_L + \Delta s\theta_H) \cdot p(e)}{(s_L + \Delta s\theta_H)} \quad \Leftrightarrow \\
\frac{r_L}{s_L} + p(e) &> \frac{(r_L + \Delta r\theta_H)}{(s_L + \Delta s\theta_H)} + p(e) \quad \Leftrightarrow \\
\frac{r_L}{s_L} &> \frac{(r_L + \Delta r\theta_H)}{(s_L + \Delta s\theta_H)} \quad \Leftrightarrow \\
r_L s_L + r_L \Delta s\theta_H &> r_L s_L + \Delta r s_L \theta_H \quad \Leftrightarrow \\
\frac{\Delta s}{\Delta r} &> \frac{s_L}{r_L},
\end{aligned}$$

which is independent of e and θ_H. □

Proof of Proposition 2.3. If an equilibrium $\sigma \in M_H(\Sigma)$ is separating, then the unique least–cost separating contract (w^s_H, w^*_L) will be selected. For $\sigma \in M_H(\Sigma)$ being pooling, both types of principal must choose this contract with probability one. A candidate for such a pooling contract is the optimal pooling contract for the H–type principal $w^{PO}(\beta) = (0, b^{PO}(\beta))$, which is unique. By construction of $w^{PO}(\beta)$, $V^P_H(w^{PO}(\beta)) \in [V^*_{HL}, V^*_{HH}]$ with $V^P_H(w^{PO}(\beta)) = p_H(e^{PO}(\beta))(\bar{y} - b^{PO}(\beta))$. By optimality of least–cost separation ($\Delta r < \Delta r^c$, cf. Lemma 2.4) it holds that $V^P_H(w^{PO}(0)) = V^*_{HL} < V_H(w^s_H)$. Moreover, $V^P_H(w^{PO}(1)) = V^*_{HH} > V_H(w^s_H)$, if the incentive constraint of the L–type principal is not trivially satisfied by w^*_H. Hence, by continuity of $V^P_H(w^{PO}(\beta))$ in β, there exists a $\beta^c \in (0, 1)$ such that $V^P_H(w^{PO}(\beta)) < V_H(w^s_H)$ for $\beta < \beta^c$ and

$V_H^P(w^{PO}(\beta)) \geq V_H(w_H^s)$ for $\beta \geq \beta^c$.

It is left to show that for $\beta \geq \beta^c$ there exists a $\sigma \in M_H(\Sigma)$ which implements the optimal pooling contract for the H–type principal $w^{PO}(\beta)$. This requires that it is also optimal for the L–type principal to offer $w^{PO}(\beta)$, i.e. $V_L^P(w^{PO}(\beta)) \geq V_L^*$ given pessimistic out–of–equilibrium beliefs. This condition is satisfied even with strict inequality since otherwise $w^{PO}(\beta) \neq w_H^s$ would satisfy the incentive constraint of L–type principal and make H–type principal weakly better off than w_H^s which contradicts optimality and uniqueness of least–cost separating contract w_H^s.

Finally, $M^* = M_L(M_H(\Sigma))$ selects the pooling contract for $\beta = \beta^c$ because $V_L^P(w^{PO}(\beta)) > V_L^*$. □

Proof of Proposition 2.4. The proof follows directly from the derivation of $b^{PO}(\beta)$ and Proposition 2.2. □

Appendix B

Appendix of Chapter 3

B.1 Proofs of Section 3.2

Proof of Lemma 3.1. Using the properties of the reference distributions, we rewrite the utility function further,

$$\begin{aligned}
u_A(x, p_A, p_B) \quad &= (v - tx - p_A) + (1 - \hat{x}_{un})(p_B - p_A) \\
&\quad - \lambda \cdot t\Big(\int_0^{1-\hat{x}_{un}} 2(x - s)\, ds + \int_{1-\hat{x}_{un}}^{x} (x - s)\, ds \Big) + t\Big(\int_x^{\hat{x}_{un}} (s - x)\, ds \Big) \\
&= (v - tx - p_A) + (1 - \hat{x}_{un})(p_B - p_A) \\
&\quad - \lambda \cdot \frac{t}{2}\Big(x^2 + 2x(1 - \hat{x}_{un}) - (1 - \hat{x}_{un})^2 \Big) + \frac{t}{2}(\hat{x}_{un} - x)^2
\end{aligned} \tag{B.1}$$

$$\begin{aligned}
u_B(x, p_A, p_B) \quad &= (v - t(1 - x) - p_B) - \lambda \cdot \hat{x}_{un}(p_B - p_A) - \lambda \cdot t \int_0^{1-x} 2((1 - x) - s)\, ds \\
&\quad + t\Big(\int_{1-x}^{1-\hat{x}_{un}} 2(s - (1 - x))\, ds + \int_{1-\hat{x}_{un}}^{\hat{x}_{un}} (s - (1 - x))\, ds \Big) \\
&= (v - t(1 - x) - p_B) - \lambda \cdot \hat{x}_{un}(p_B - p_A) - \lambda \cdot t(1 - x)^2 \\
&\quad + t\Big((x - \hat{x}_{un})^2 + (\frac{1}{2} - x - \hat{x}_{un} + 2x\hat{x}_{un}) \Big).
\end{aligned} \tag{B.2}$$

Next, we find the location of the indifferent uninformed consumer $x = \hat{x}_{un}$ by setting $u_A = u_B$,

where

$$u_A(\hat{x}_{un}, p_A, p_B) = v - t\hat{x}_{un} - p_A + (1-\hat{x}_{un})(p_B - p_A) - \lambda \cdot \frac{t}{2}\left(1 - 2(1-\hat{x}_{un})^2\right)$$
$$u_B(\hat{x}_{un}, p_A, p_B) = v - t(1-\hat{x}_{un}) - p_B - \lambda \cdot \hat{x}_{un}(p_B - p_A) - \lambda \cdot t(1-\hat{x}_{un})^2 + 2t(\frac{1}{2} - \hat{x}_{un})^2$$

If she buys product A the indifferent uninformed consumer will experience no gain but the maximum loss in the taste dimension. If she buys product B she will experience a gain and a loss because distance could have been smaller or larger than $1 - \hat{x}_{un}$. With respect to the price dimension the indifferent uninformed consumer (like all other consumers) faces only a loss when paying price p_B and only a gain when paying price p_A.

$u_A(\hat{x}_{un}, p_A, p_B) = u_B(\hat{x}_{un}, p_A, p_B)$ can be transformed to the following quadratic equation in $\hat{x}_{un}$,

$$0 = 2t(\lambda-1)\cdot\hat{x}_{un}^2 - \left((\lambda-1)(p_B-p_A) - 4t\lambda\right)\cdot\hat{x}_{un} + \left(2(p_B-p_A) + \frac{t}{2}(3\lambda+1)\right) \qquad \text{(B.3)}$$

Solving this quadratic equation w.r.t. $\hat{x}_{un}$ leads to the expression given in the lemma. □

B.2 Proofs of Section 3.3

Proof of Lemma 3.2.

$$\begin{aligned}
\phi' &= \frac{\partial q_A(\Delta p;\beta)}{\partial \Delta p} = -\frac{\partial q_A(\Delta p;\beta)}{\partial p_A} = -\frac{\partial q_B(\Delta p;\beta)}{\partial \Delta p} = -\frac{\partial q_B(\Delta p;\beta)}{\partial p_B} \\
&= \beta\cdot\hat{x}'_{in}(\Delta p) + (1-\beta)\cdot\hat{x}'_{un}(\Delta p) \\
&= -\frac{1}{4t}(1-3\beta) - \frac{(1-\beta)}{2(S(\Delta p))}\underbrace{\left(\frac{\Delta p}{8t^2} - \frac{(\lambda+2)}{2t(\lambda-1)}\right)}_{\ominus} > 0
\end{aligned}$$

$\phi' > 0 \quad \forall\Delta p$ feasible and $\forall\beta$. At the boundaries we have

$$\begin{aligned}
\phi'(0;\beta) &= -\frac{1}{4t}(1-3\beta) + (1-\beta)\frac{(\lambda+2)}{2t(\lambda-1)} > 0 \\
\phi'(\Delta p \to \Delta\overline{p}; \beta < 1) &\to \infty \quad \text{since } S(\overline{p}) = 0.
\end{aligned}$$

For $0 \leq \Delta p < \Delta p^{max}$ the demand of A is convex in Δp. At the boundaries we have

$$\phi''(\Delta p;\beta) \;=\; (1-\beta)\cdot\hat{x}''_{un}(\Delta p) = (1-\beta)\cdot\frac{(3+\lambda)(5+3\lambda)}{64t^2\cdot(S(\Delta p))^3} \geq 0$$

$\phi'' > 0 \quad \forall\Delta p$ feasible and $\forall\beta < 1$ since $S(\Delta p) \geq 0$:

$$\begin{aligned}\phi''(0;\beta) \;&=\; (1-\beta)\cdot\frac{(3+\lambda)(5+3\lambda)}{32t^2\cdot\frac{(\lambda+1)^3}{(\lambda-1)^3}} > 0\\ \phi''(\Delta p\to\Delta\bar{p};\beta<1) \;&\to\; \infty.\end{aligned}$$

□

Proof of Lemma 3.3. Combining (FOC_A) and (FOC_B) yields the required equilibrium condition as a function of price differences. □

Proof of Proposition 3.1. We first consider the case of $\lambda > \lambda^c$. We can derive a number of useful properties of $f(\Delta p;\beta) = (1-2\phi)/\phi'$:
$f(0;\beta) = 0/\phi'(0) = 0\;\forall\beta$, $f(\Delta\bar{p};\beta) \to 0$ since $\phi'(\Delta\bar{p}) \to \infty\;\forall\beta < 1$, and $f(\Delta\bar{p},1) = -2\Delta\bar{p} < 0$.

$$f'(\Delta p;\beta) \;=\; \frac{-2(\phi')^2-\phi''(1-2\phi)}{(\phi')^2} = -\left(2+\frac{\phi''(1-2\phi)}{(\phi')^2}\right) \lesseqgtr 0,$$

$f'(0;\beta) = -2 < 0 \quad \forall\beta$ and $f'(\Delta\bar{p};\beta) \to -(2+\frac{-\infty^{3/2}}{\infty^1}) \to \infty \quad \forall\beta < 1$, and $f'(\Delta p,1) = -2 \quad \forall\Delta p$.

It has to be shown that $f(\Delta p;\beta)$ is strictly convex in Δp for $\beta < 1$. We find that

$$f''(\Delta p;\beta) \;=\; -\frac{(\phi'\phi'''-2(\phi'')^2)(1-2\phi)-2(\phi')^2}{(\phi')^3} > 0.$$

If $\beta < 1$ by continuity of $f(\Delta p)$, $f(0;\beta) = 0$, $f(\Delta\bar{p};\beta) \to 0$, $f'(0;\beta) < 0$, $f'(\Delta\bar{p};\beta) \to \infty > 1$, and strict convexity of $f(\Delta p)$ for $\beta < 1$, we know that for $\Delta c = \Delta\bar{p}$ there are two potential interior equilibria. This is illustrated in Figure 3.2. The second equilibrium arises because $\Delta\bar{p}$ depicts a second solution to $\Delta p = f(\Delta p;\beta < 1) + \Delta\bar{p}$ since $f(\Delta\bar{p};\beta < 1) = 0$. Moreover, by continuity of $f(\Delta p)$ two potential equilibria occur for $\Delta c > \Delta\bar{p}$ (if any) because $\Delta\bar{p} < f(\Delta\bar{p};\beta < 1) + \Delta c$. For values of Δc lower than $\Delta\bar{p}$, $f(\Delta\bar{p};\beta < 1) + \Delta c$ is always smaller than $\Delta\bar{p}$ and no second equilibrium can arise.

If $\beta = 1$, $f(\Delta p;\beta)$ is strictly decreasing for all Δp and at most one intersection between $f(\Delta p; 1) + \Delta c$ and Δp exists (standard Hotelling case).[1]

Secondly, in the case of $1 \leq \lambda < \lambda^c$ there are corner solutions if $\Delta p > \Delta\tilde{p}$ because firm A's demand of uninformed consumers is bounded at one. This reduces firm A's incentives to set a very low p_A in equilibrium (that leads to $\Delta p > \Delta\tilde{p}$) because that would decrease the profit margin for all its consumers while only increasing firm A's demand of informed consumers. It can be shown that a Δp above $\Delta\tilde{p}$ is not optimal if the optimal price difference for informed consumers $\Delta p^* = \Delta c/3$ lies below $\Delta\tilde{p}$.[2] Thus, there exists no second equilibrium in this case. For $\Delta p^* = \Delta c/3 > \Delta\tilde{p}$ a higher price difference than $\Delta\tilde{p}$ can arise in equilibrium because attracting further informed consumers is profitable in this situation. But then $\Delta p^* = \Delta c/3$ describes the only potential equilibrium which is driven by the demand of informed consumers (standard Hotelling case). Hence, the uniqueness condition (3.12) also suffices to rule out second equilibria for $\lambda \in (1, \lambda^c]$. □

Proof of Proposition 3.2.

1. To find an upper bound on Δc for which the equilibrium condition (3.11) is satisfied we determine the point at which $f(\Delta p;\beta)$ is a tangent on the Δp-line.
   Tangent condition:

$$f'(\Delta p;\beta) = 1 \qquad \Leftrightarrow \qquad 3(\phi')^2 + \phi''(1 - 2\phi) = 0 \tag{B.4}$$

   An analytical solution to $3(\phi')^2 + \phi''(1 - 2\phi) = 0$ can be found for $\beta = 0$.[3] Denote this critical price difference as $\Delta p^{ta}(\lambda, t)$.[4]

   Then, the equilibrium condition in (3.11) can be fulfilled if and only if Δc satisfies the following condition

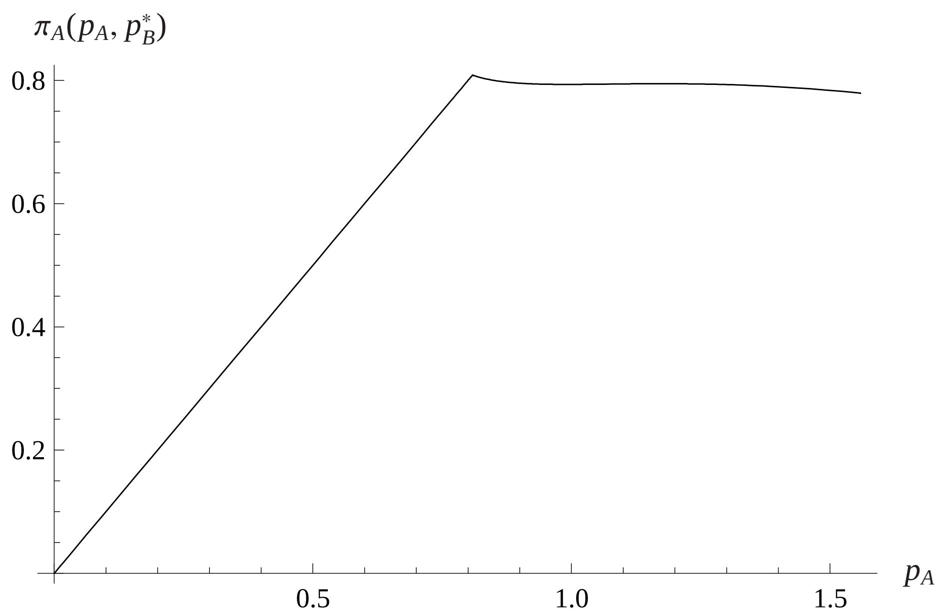
$$\Delta c \leq \Delta c^{ta} \equiv \Delta p^{ta}(\lambda, t) - f(\Delta p^{ta}(\lambda, t); \beta = 0). \tag{B.5}$$

2. We next rule out some *potentially* interior equilibria . First suppose $\Delta p'$ does not satisfy SOC_A, then $\Delta p'$ depicts a profit minimum for firm A. $\Delta p'$ cannot be an equilibrium.

[1] An analytical solution for (3.11) can be determined in this case: $\Delta p^* = \Delta c/3$.
[2] Under (3.12) Δc is weakly lower than $\Delta\overline{p}$ which can rise above $3\Delta\tilde{p}$ for $\lambda \to 1$.
[3] This is sufficient since $\beta = 0$ is the most critical case w.r.t. existence and uniqueness. The reason for this is that for $\beta > 0$ there is a positive weight on the demand of informed consumers which is purely linear.
[4] $\Delta p^{ta}(\lambda, t)$ is decreasing in λ.

Moreover, comparing (3.10) and (B.4) shows that the critical price difference for locally satisfying SOC_A is always lower than Δp^{ta}. Hence, a non–empty set of *potentially* interior equilibria is ruled out by local non-concavity.

Secondly, if a *potentially* interior equilibrium locally satisfies SOC_A but SOC_A is locally violated for some larger Δp, the profit function of firm A is strictly convex for a sufficiently large non-local price decrease p_A. If the convexity is sufficiently large the profit of firm A is increasing for large non-local price decreases. Thus, a non-local deviation becomes profitable for firm A.[5] Given the non-decreasing convexity of π_A in $-p_A$ the optimal deviation of firm A is such that firm A serves the entire demand of uninformed consumers, i.e. p_A^d s.t. $\Delta p^d = \Delta p^{max}$. Decreasing p_A^d further is not profitable since firm A only attracts informed consumers while its profit margin goes down for informed and uninformed consumers.[6] In the following we can restrict our attention to price deviations by firm A that steal the entire demand of uninformed consumers.



The Figure shows the profit of firm A, $\pi_A(p_A, p_B^*)$, as a function of its own price given $p_B = p_B^*$ for $\Delta c = 1$ ($c_A = 0, c_B = 1$) and parameter values of $\beta = 0$, $t = 1$, and $\lambda = 3$: $p_A^* = 1.17309$, $p_B^* = 1.55863$, $p_A^d = 0.80863$, $\Delta p^* = 0.385537$, and $\Delta p^{max} = \Delta\tilde{p} = 3/4$.

Figure B.1: Non-existence

[5]Figure B.1 shows an example of an *potentially* interior equilibrium in which deviating by firm A is profitable.
[6]For situations with $\lambda \to 1$, in which $\Delta p^* > \Delta p^{max}$ can arise, it can be shown that non-concavity of π_A is not a problem.

In such a situation firm A sets $p_A^d = p_B^* - \Delta p^{max}$. For $\beta = 0$ the firm A's deviation profit, π_A^d, is equal to $(p_A^d - c_A) \cdot 1$ while for $\beta \in (0, 1]$ it is equal to $(p_A^d - c_A) \cdot \phi(\Delta p^{max}; \beta)$ with $\phi(\Delta p^{max}; \beta) \equiv \beta \cdot \hat{x}_{in}(\Delta p^{max}) + (1 - \beta) \cdot 1$. Using that $p_A^d = p_B^* - \Delta p^{max}$ we receive

$$\begin{aligned} \pi_A^d = & \left(p_B^* - \Delta p^{max} - c_A\right) \cdot \phi(\Delta p^{max}; \beta) \\ = & \left(\frac{1-\phi}{\phi'} + \Delta c - \Delta p^{max}\right) \cdot \phi(\Delta p^{max}; \beta) & \text{by } FOC_B \\ = & \left(\Delta p^{nd} + \frac{\phi}{\phi'} - \Delta p^{max}\right) \cdot \phi(\Delta p^{max}; \beta) & \text{by (3.11)} \end{aligned} \tag{B.6}$$

For non–deviation, firm A's profit is equal to $\pi_A(\Delta p^*) = (p_A^* - c_A)\phi$, which is equivalent to ϕ^2/ϕ' by FOC_A.

Thus, deviation of firm A is not profitable if and only if $\pi_A(\Delta p^*) \geq \pi_A^d$.[7] Rearranging yields the required non–deviation condition

$$\Delta p \leq \Delta p^{nd} \equiv \Delta p^{max} - \frac{\phi \cdot (\phi(\Delta p^{max}; \beta) - \phi)}{\phi' \cdot \phi(\Delta p^{max}; \beta)}.$$

In Lemma B.1 in the appendix we show that Δp^{nd} is uniquely determined by this non–deviation condition if $\Delta p^{nd} \neq \Delta p^{max}$ and that the set of non–negative Δp^{nd} is non–empty.

Combining this with the equilibrium condition (3.11) we get that existence of interior equilibria is ensured for non–negative Δp^{nd} if and only if $\Delta c \leq \Delta c^{nd} \equiv \Delta p^{nd} - f(\Delta p^{nd})$. However, Δp^{nd} can become negative if the degree of loss aversion becomes too high. Here deviation is profitable even for symmetric settings ($\Delta c = 0$). But an upper limit on the amount of uninformed consumers can reinforce existence of symmetric equilibria in this case. In the second part of Lemma B.1 the critical level of loss aversion for which Δp^{nd} becomes negative is determined and the critical level of β as a function of λ for $\Delta c = 0, \beta^{crit}(\lambda)$, is defined.

3. Any equilibrium is interior because discontinuity of firm A's best response function rules out non–interior equilibria.

□

[7] We assume that firm A does not deviate from an interior strategy if it is indifferent between deviating and playing the interior best-response.

Existence result completed

LEMMA B.1: *1. For* $\lambda \in (1, 1+2\sqrt{2}]$, $\Delta p^{nd} \geq 0$ *is uniquely determined by the non–deviation condition in* (3.14),

$$\Delta p^{nd}(\Delta c \geq 0, \beta = 0) = \left\{\Delta p \mid \Delta p = \Delta p^{max} - \frac{\phi \cdot \left(\phi(\Delta p^{max};\beta) - \phi\right)}{\phi' \cdot \phi(\Delta p^{max};\beta)}, \Delta p \neq \Delta p^{max}\right\},$$

2. For $\lambda > 1+2\sqrt{2}$, $\exists\, \beta^{crit}(\lambda) \geq 0$ *s.t.* $\Delta p^{nd}(\Delta c = 0, \beta = \beta^{crit}(\lambda)) = 0$.

Proof of Lemma B.1. First note that the non–deviation condition is trivially satisfied at $\Delta p = \Delta p^{max}$ (see Figure B.2 below for a graphical illustration of the non–deviation condition). It can be shown that $\Delta p + \frac{\phi\cdot(\phi(\Delta p^{max};\beta)-\phi)}{\phi'\cdot\phi(\Delta p^{max};\beta)}$ approaches Δp^{max} from above for $\Delta p < \Delta p^{max}$. At $\Delta p = 0$, $\Delta p + \frac{\phi\cdot(\phi(\Delta p^{max};\beta)-\phi)}{\phi'\cdot\phi(\Delta p^{max};\beta)}$ is strictly increasing and strictly concave. Moreover, $\Delta p + \frac{\phi\cdot(\phi(\Delta p^{max};\beta)-\phi)}{\phi'\cdot\phi(\Delta p^{max};\beta)}$ is continues and exhibits at most one saddle point for $\Delta p \leq \Delta p^{max}$. Taken together, there exists a unique $\Delta p < \Delta p^{max}$ at which the non–deviation condition is satisfied. Denoting this Δp by Δp^{nd}, $\Delta p^{nd} \leq 0$ if and only if at $\Delta p = 0$, $\Delta p + \frac{\phi\cdot(\phi(\Delta p^{max};\beta)-\phi)}{\phi'\cdot\phi(\Delta p^{max};\beta)} \leq \Delta p^{max}$. It can be shown that $\forall t > 0$ and $\beta = 0$ this holds if and only if $\lambda \in (1, 1+2\sqrt{2}]$.

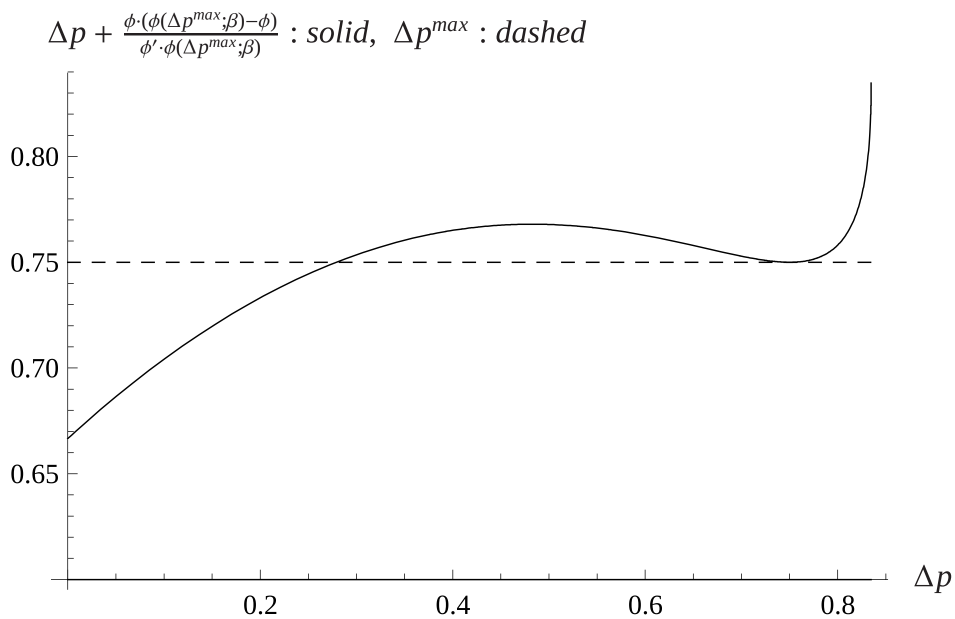
It can be shown that the non–deviation condition is continuous and monotonous in β.
For $\lambda > 1+2\sqrt{2}$ the non–deviation condition can be reinforced if $\beta > 0$. Solving for $\beta^{crit}(\lambda)$ in $\Delta p^{nd}(\Delta c = 0, \beta = \beta^{crit}(\lambda) > 0) = 0$ yields

$$\beta_0^{crit}(\lambda) \equiv 1 - \frac{-\lambda(5\lambda+14) + \sqrt{(3\lambda+5)(\lambda(11\lambda(\lambda+5)+113)+77)} - 13}{2(\lambda-1)(\lambda+3)}, \tag{B.7}$$

for $\lambda \in (1+2\sqrt{2}, \lambda^c]$ (i.e. $\Delta p^{max} = \Delta\tilde{p}$) and

$$\beta_1^{crit}(\lambda) \equiv 1 - \frac{37\lambda^3 - 21\Lambda\lambda^2 + 177\lambda^2 - 54\Lambda\lambda + 247\lambda - 21\Lambda - \Omega + 83}{2\,(12\lambda^3 - 7\Lambda\lambda^2 + 46\lambda^2 - 10\Lambda\lambda + 8\lambda + 17\Lambda - 66)} \tag{B.8}$$

with $\Omega \equiv (4\lambda^6 - 2\Lambda\lambda^5 + 1596\lambda^5 - 918\Lambda\lambda^4 + 19848\lambda^4 - 9316\Lambda\lambda^3 + 91384\lambda^3 - 31228\Lambda\lambda^2 + 197268\lambda^2 - 42618\Lambda\lambda + 201868\lambda - 20366\Lambda + 78880)^{1/2}$
and $\Lambda \equiv \sqrt{3\lambda^2 + 14\lambda + 15}$ for $\lambda > \lambda^c$ (i.e. $\Delta p^{max} = \Delta\bar{p}$). For $\lambda \to \infty$ it holds that $\beta_1^{crit}(\lambda) \to 1 - \frac{-37+21\sqrt{3}+\sqrt{4-2\sqrt{3}}}{-24+14\sqrt{3}} \approx 0.577$. Compare Figure 3.3. □



The Figure shows the non–deviation condition of firm A, as a function of the price difference Δp for $\Delta c = 0.25$ ($c_A = 0.25, c_B = 0.5$) and parameter values of $\beta = 0$, $t = 1$, and $\lambda = 3$: $\Delta p^{nd} = 0.27889$, $\Delta c^{nd} = (\Delta p^{nd} - f(\Delta p^{nd}; 0)) = 0.75963$, $\Delta p^{max} = \Delta\tilde{p} = 3/4$, and $\Delta\overline{p} = 0.83485$. non–deviation for $\Delta p \leq \Delta p^{nd} = 0.27889$.

Figure B.2: non–deviation for asymmetric industries

B.3 Proofs of Section 3.4

Proof of Proposition 3.3. For $\Delta c = 0$ we get by (3.11), (3.12), and $f(0;\beta) = 0$ that $\Delta p^*(\beta) = 0$ is the unique equilibrium $\forall\beta \in [0, 1]$ (provided it exists). Rearranging (FOC_i) and applying that $\phi(0,\beta) = 1/2$ for all β yields

$$p_i^* - c_i = \frac{\frac{1}{2}}{\phi'(0;\beta)} \quad \forall i \in \{A, B\},$$

where

$$\phi'(0;\beta) = -\frac{1}{4t}(1-3\beta) - \frac{(1-\beta)}{2(S(0))}\left(0 - \frac{(\lambda+2)}{2t(\lambda-1)}\right)$$

$$= -\frac{1}{4t}(1-3\beta) + \frac{(1-\beta)}{2\frac{\lambda+1}{2(\lambda-1)}}\left(\frac{(\lambda+2)}{2t(\lambda-1)}\right)$$

$$= -\frac{1}{4t}(1-3\beta) + \frac{(1-\beta)(\lambda+2)}{2t(\lambda+1)}$$

$$= \frac{1}{4t(\lambda+1)}\Big(2(\lambda+1) - (1-\beta)(\lambda-1)\Big).$$

This gives rise to (3.17). □

Proof of Proposition 3.4.

$$\frac{d\Delta p^*(\Delta c)}{d\Delta c} = -\frac{(\phi')^2}{3(\phi')^2 + \phi''(1-2\phi)} \cdot (-1) \tag{B.9}$$

$$= \frac{(\phi')^2}{3(\phi')^2 + \phi''(1-2\phi)}$$

Since ϕ' is strictly positive and denominator of $d\Delta p^*(\Delta c)/d\Delta c$ is equivalent to the tangent condition (B.4). We obtain that

$$\frac{d\Delta p^*(\Delta c)}{d\Delta c} > 0 \tag{B.10}$$

if $\Delta p < \Delta p^{ta}(\lambda, t)$. Moreover, since $\phi''(1-2\phi) = 0$ for $\Delta c = 0$ (i.e. $\Delta p = 0$, compare symmetric equilibrium) and $\phi''(1-2\phi) \leq 0$ for $\Delta c > 0$ it holds true that $d\Delta p^*(\Delta c)/d\Delta c \geq 1/3$. □

Proof of Proposition 3.5.

$$\frac{dm_A^*(\Delta p^*(\Delta c))}{d\Delta c} = \frac{\partial m_A^*}{\partial \Delta p^*} \cdot \frac{\partial \Delta p^*}{\partial \Delta c},$$

where by (FOC_A)

$$\frac{\partial m_A^*}{\partial \Delta p^*} = \frac{\partial p_A^*}{\partial \Delta p^*} = \frac{(\phi')^2 - \phi'' \cdot \phi}{(\phi')^2} \gtrless 0, \tag{B.11}$$

which may be positive or negative for $\beta < 1$. Firm A's markup is increasing in the price difference if the price difference is rather low and the share of uninformed consumers is not too high. It is decreasing for large price differences and/or if the share of uninformed consumers is high. Using (B.9) we receive that

$$\frac{dm_A^*(\Delta p^*(\Delta c))}{d\Delta c} = \frac{(\phi')^2 - \phi'' \cdot \phi}{3(\phi')^2 + \phi''(1-2\phi)} \gtrless 0. \tag{B.12}$$

Hence m_A^* is not strictly increasing in Δp^*. Firm A's markup decreases in the price difference if the price difference, i.e. if the cost asymmetries in the industry, and/or the share of uninformed consumers become too large. (Compare markup of B.) □

Proof of Proposition 3.6.

$$\frac{dm_B^*(\Delta p^*(\Delta c))}{d\Delta c} = \frac{\partial m_B^*}{\partial \Delta p^*} \cdot \frac{\partial \Delta p^*}{\partial \Delta c},$$

where by (FOC_B)

$$\frac{\partial m_B^*}{\partial \Delta p^*} = \frac{\partial p_B^*}{\partial \Delta p^*} = \frac{-(\phi')^2 - \phi'' \cdot (1-\phi)}{(\phi')^2} < 0, \tag{B.13}$$

which is always negative for all β. Using (B.9) we obtain that

$$\frac{dm_B^*(\Delta p^*(\Delta c))}{d\Delta c} = -\frac{(\phi')^2 + \phi'' \cdot (1-\phi)}{3(\phi')^2 + \phi''(1-2\phi)} < 0. \tag{B.14}$$

□

Proof of Proposition 3.7. Recall that the equilibrium is implicitly characterized by

$$\Delta p - \Delta c - \frac{1 - 2\phi(\Delta p;\beta)}{\phi'(\Delta p;\beta)} = 0$$

The equilibrium price difference then satisfies

$$\begin{aligned}
\frac{d\Delta p^*(\beta)}{d\beta} &= -\Big(1 - \frac{-2(\phi')^2 - \phi''(1-2\phi)}{(\phi')^2}\Big)^{-1}\Big(-\frac{-2\phi'\frac{\partial\phi}{\partial\beta} - \frac{\partial\phi'}{\partial\beta}(1-2\phi)}{\phi'^2}\Big) \\
&= -\frac{(\phi')^2}{3(\phi')^2 + \phi''(1-2\phi)} \cdot \Big(\frac{2\phi'\phi_\beta + \phi'_\beta - 2\phi'_\beta\phi}{(\phi')^2}\Big) \\
&= -\frac{2\phi'\phi_\beta + \phi'_\beta(1-2\phi)}{3(\phi')^2 + \phi''(1-2\phi)}
\end{aligned}$$

We show that the numerator of $\frac{d\Delta p^*(\beta)}{d\beta}$, denoted by $N(\Delta p^*;\beta) = -(2\phi'\phi_\beta + \phi'_\beta(1-2\phi))$ is negative: For all Δp with $0 \le \Delta p \le \Delta p^{max}$ and for all $\beta \in [0,1]$, we can rewrite

$$\begin{aligned}
N(\Delta p;\beta) &= -2\phi'\phi_\beta - \phi'_\beta(1-2\phi) = 2((1-\beta)\hat{x}'_{un} + \beta\frac{1}{2t}) \cdot (\hat{x}_{un} - \hat{x}_{in}) \\
&\quad +(\hat{x}'_{un} - \frac{1}{2t})(1 - 2(1-\beta)\hat{x}_{un} - 2\beta\hat{x}_{in}) \\
&= \frac{1}{t}(\hat{x}_{un} - \hat{x}_{in}) + (\hat{x}'_{un} - \frac{1}{2t})(1 - 2\hat{x}_{in}) \\
&= \frac{1}{t}\hat{x}_{un} + (\hat{x}'_{un})(1 - 2\hat{x}_{in}) - \frac{1}{2t} \\
&= \frac{1}{t}(\hat{x}_{un} + \frac{1}{2}) - \hat{x}'_{un}(2\hat{x}_{in} - 1) \\
&= -2t\hat{x}'_{un} \cdot (\hat{x}_{in} - \frac{1}{2}) + 1(\hat{x}_{un} - \frac{1}{2}) \\
&= -2t\hat{x}'_{un}(\Delta p)(\hat{x}_{in}(\Delta p) - \frac{1}{2}) + (\hat{x}_{un}(\Delta p) - \frac{1}{2})
\end{aligned}$$

Since $N(0;\beta) = 0$ and

$$\begin{aligned}
\frac{\partial N(\Delta p;\beta)}{\partial \Delta p} &= -\frac{1}{t}\Big(2t\hat{x}''_{un}(\Delta p)(\hat{x}_{in}(\Delta p) - \frac{1}{2}) + 2t(\hat{x}'_{un}(\Delta p))(\hat{x}'_{in}(\Delta p)) - \hat{x}'_{un}(\Delta p)\Big) \\
&= -\frac{1}{t}\Big(2t\hat{x}_{un}(\Delta p)(\hat{x}_{in}(\Delta p) - \frac{1}{2}) + 0 - 0\Big) < 0
\end{aligned}$$

it holds that $N(\Delta p^*;\beta) < 0$ for all admissible $\Delta p, \beta$.

Consider now the denominator of $\frac{d\Delta p^*(\beta)}{d\beta}$, denoted by $D(\Delta p^*;\beta) = 3(\phi')^2 + \phi''(1-2\phi)$. We show that on the relevant domain of price differences $D(\Delta p^*;\beta)$ is strictly positive. We have that

$$\begin{aligned} D(0;\beta) &= 3(\phi'(0;\beta))^2 + \phi''(0;\beta)\cdot 0 \\ &= 3(\phi'(0;\beta))^2 > 0 \end{aligned}$$

The sign of the derivative is of ambiguous sign:

$$\begin{aligned} \frac{\partial D(\Delta p;\beta)}{\partial \Delta p} &= 6\phi'\phi'' + \phi'''(1-2\phi) - 2\phi''\phi' \\ &= 4\phi'\phi'' + \phi'''(1-2\phi) \end{aligned}$$

Thus $D(\Delta p^*;\beta)$ is not necessarily non–negative. However, since $D(\Delta p^*;\beta)$ is equivalent to the tangent condition (B.4) which approaches zero at $\Delta p = \Delta p^{ta}(\lambda, t)$ we conclude that

$$\frac{d\Delta p^*(\beta)}{d\beta} < 0 \tag{B.15}$$

for $\Delta p < \Delta p^{ta}(\lambda, t)$, which is the relevant domain for equilibrium existence. □

Proof of Proposition 3.8.

$$\frac{dp_A^*(\Delta p^*(\beta);\beta)}{d\beta} = \frac{\partial p_A^*}{\partial \Delta p^*}\cdot\frac{\partial \Delta p^*}{\partial \beta} + \frac{\partial p_A^*}{\partial \beta},$$

where

$$\frac{\partial p_A^*}{\partial \Delta p^*} = \frac{(\phi')^2 - \phi''\cdot\phi}{(\phi')^2} \gtrless 0,$$

which may be positive or negative. Hence p_A^* is not strictly increasing in Δp^*. Firm A's prices goes down in the price difference if the price difference becomes too large, i.e. if the cost asymmetries in the industry or the share of uninformed consumers becomes too large.

(Compare price of B.)

$$\begin{aligned}
\frac{\partial p_A^*}{\partial \beta} &= \frac{\phi'\phi_\beta - \phi'_\beta\phi}{(\phi')^2} \\
&= -\Big[((1-\beta)\hat{x}'_{un} + \beta\hat{x}'_{in})(\hat{x}_{un} - \hat{x}_{in}) - (\hat{x}'_{un} - \hat{x}'_{in}) \cdot ((1-\beta)\hat{x}_{un} + \beta\hat{x}_{in})\Big] \cdot \frac{1}{\phi'^2} \\
&= -\Big[(1-\beta)(\hat{x}'_{un} - \frac{1}{2t})(\hat{x}_{un} - \hat{x}_{in}) - (1-\beta)(\hat{x}'_{un} - \frac{1}{2t})(\hat{x}_{un} - \hat{x}_{in})\frac{1}{2t}(\hat{x}_{un} - \hat{x}_{in}) - (\hat{x}_{un} - \frac{1}{2t})\hat{x}_{in}\Big] \cdot \frac{1}{\phi'^2} \\
&= -\Big[\frac{1}{2t}\hat{x}_{un} - \hat{x}'_{un}\hat{x}_{in}\Big] \cdot \frac{1}{\phi'^2}
\end{aligned}$$

The numerator of $\frac{\partial p_A^*}{\partial \beta}$ is independent of β.

$$\frac{\partial p_A^*}{\partial \beta}(\Delta p = 0) = -\frac{1}{2}\Big(\frac{1}{2t} - \hat{x}'_{un}(0)\Big) \cdot \frac{1}{\phi'(0)^2} < 0$$

$$\frac{\partial p_A^*}{\partial \beta}(\Delta p = \Delta\overline{p} - \epsilon) = -\frac{\Big(\frac{1}{2t}\hat{x}_{un} - \hat{x}'_{un}\hat{x}_{in}\Big)}{\phi'^2} > 0$$

for ϵ small because the numerator is positive for Δp slightly less than $\Delta\overline{p}$. This implies that $\frac{\partial p_A^*}{\partial \beta} = 0$ for some $\Delta p \in (0, \Delta p^{max}), \forall\beta$. □

Proof of Proposition 3.9.

$$\frac{dp_B^*(\Delta p^*(\beta);\beta)}{d\beta} = \frac{\partial p_B^*}{\partial \Delta p^*} \cdot \frac{\partial \Delta p^*}{\partial \beta} + \frac{\partial p_B^*}{\partial \beta},$$

$$\text{where} \quad \frac{\partial p_B^*}{\partial \Delta p^*} = \frac{-(\phi')^2 - \phi''(1-\phi)}{(\phi')^2} = -\Big(1 + \frac{\phi''(1-\phi)}{(\phi')^2}\Big) < 0$$

In contrast to A, the price of B is always decreasing in $\Delta p^*(\beta)$.

$$\begin{aligned}
\frac{\partial p_B^*}{\partial \beta} &= \frac{-\phi'\phi_\beta - \phi'_\beta(1-\phi)}{(\phi')^2} \\
&= -\Big[-((1-\beta)\hat{x}'_{un} + \beta\frac{1}{2t})(\hat{x}_{un} - \hat{x}_{in}) - (\hat{x}'_{un} - \frac{1}{2t})(1-(1-\beta)\hat{x}_{un} - \beta\hat{x}_{in})\Big] \cdot \frac{1}{(\phi')^2} \\
&= -\Big[-(1-\beta)(\hat{x}'_{un} - \frac{1}{2t})(\hat{x}_{un} - \hat{x}_{in}) + (1-\beta)(\hat{x}'_{un} - \frac{1}{2t})(\hat{x}_{un} - \hat{x}_{in}) \\
&\quad -\frac{1}{2t}(\hat{x}_{un} - \hat{x}_{in}) - (\hat{x}'_{un} - \frac{1}{2t})(1-\hat{x}_{in})\Big] \cdot \frac{1}{(\phi')^2} \\
&= -\Big[-\frac{1}{2t}(\hat{x}_{un}) - (\hat{x}'_{un} - \frac{1}{2t}) + \hat{x}'_{un}\hat{x}_{in}\Big] \cdot \frac{1}{(\phi')^2} \lesseqgtr 0
\end{aligned}$$

□

B.4 Proofs of Section 3.5

Proof of Proposition 3.10. The derivation of the indifferent uninformed consumer with α–extended preferences is analogous to the derivation of the indifferent uninformed consumer for $\alpha = 1$ provided in the proof of Lemma 3.1. With α–extended preferences the location equals

$$\hat{x}_{un}(\Delta p; \lambda, \alpha) = \frac{1 + \alpha(2\lambda - 1)}{2\alpha(\lambda - 1)} - \frac{\Delta p}{4t} - \sqrt{\frac{\Delta p^2}{16t^2} - \frac{(\alpha(2\lambda + 1) + 3)}{4\alpha t(\lambda - 1)}\Delta p + \frac{(\alpha\lambda + 1)^2}{4\alpha^2(\lambda - 1)^2}}. \quad \text{(B.16)}$$

By solving for λ in equation (3.19) we receive

$$\lambda(\lambda', \alpha') = \frac{1 + \alpha'(2\lambda' - 1)}{1 + \alpha'}. \quad \text{(B.17)}$$

Since $\lambda(\lambda', \alpha' = 1) = \lambda'$ and $\partial\lambda/\partial\alpha' = 2(\lambda' - 1)/(1 + \alpha')^2 > 0$, λ shows the required properties.

□

Proof of Proposition 3.11. First consider informed consumers' utility: We find $u_i(x, p_i) = (v_i - p_i) - t|y_i - x| = -\tilde{p}_i - t|y_i - x|$ for all $i \in \{A, B\}$ in the first market and $u_i(x, p'_i) = (v'_i - p'_i) - t|y_i - x|$ for all $i \in \{A, B\}$ in the second market. Since in the second market quality levels are identical ($\Delta v' = 0$), it holds true that $\hat{x}_{in}(\Delta\tilde{p}) = \hat{x}_{in}(\Delta p')$ for $\Delta p' = \Delta p - \Delta v$. If uninformed consumers use quality-adjusted prices for determining their reference point distribution in the price dimension we also receive $\hat{x}_{un}(\Delta\tilde{p}) = \hat{x}_{un}(\Delta p')$ for $\Delta p' = \Delta p - \Delta v$ by the same argument. Finally, compare firms' maximization problem for both markets. Firm A solves

$$\max_{\tilde{p}_A} \pi_A(\tilde{p}_A, \tilde{p}_B) = (\tilde{p}_A + v_A - c_A)[\beta \cdot \hat{x}_{in}(\tilde{p}_B - \tilde{p}_A) + (1-\beta) \cdot \hat{x}_{un}(\tilde{p}_B - \tilde{p}_A)] \quad \text{and}$$
$$\max_{p'_A} \pi_A(p'_A, p'_B) = (p'_A - c'_A)[\beta \cdot \hat{x}_{in}(p'_B - p'_A) + (1-\beta) \cdot \hat{x}_{un}(p'_B - p'_A)].$$

Firm A's equilibrium prices are identical iff markups in both markets are identical, i.e. $\tilde{p}_A + v_A - c_A = p'_A - c'_A$, and both demand functions are identical, i.e. $\Delta p' = \Delta p - \Delta v$. Analogously, for firm B this holds true iff $\tilde{p}_B + v_B - c_B = p'_B - c'_B$ and $\Delta p' = \Delta p - \Delta v$. Finally, taking markup differences between firms we get $\Delta\tilde{p} + \Delta v - \Delta c = \Delta p - \Delta c$ in first market and $\Delta p' - \Delta c'$ in the second market. For $\Delta p' = \Delta p - \Delta v$ both markup differences are the same iff $\Delta c' = \Delta c - \Delta v$. $\square$

B.5 Tables

Table B.1: Small Cost Differences:

The table shows the analytical solution of the market equilibria for parameter values of $t = 1$, $\lambda = 3$, $c_A = 0.25$, $c_B = 0.5$:

β	$p_A^*(\beta)$	$p_B^*(\beta)$	$\Delta p^*(\beta)$	$q_A(\Delta p^*)$	$\hat{x}_{in}(\Delta p^*)$	$\hat{x}_{un}(\Delta p^*)$	π_A^*	π_B^*	CS^*	CS_{in}^*	CS_{un}^*
1.0	1.33333	1.41667	0.0833333	0.541667	0.541667	0.532453	0.586806	0.420139	1.37674	1.37674	1.16648
0.8	1.37274	1.45643	0.0836887	0.539995	0.541844	0.532597	0.606272	0.439961	1.29508	1.33717	1.12672
0.6	1.41524	1.49932	0.0840806	0.538326	0.54204	0.532755	0.627281	0.461361	1.21022	1.29448	1.08382
0.4	1.46121	1.54572	0.0845149	0.536662	0.542257	0.532931	0.650008	0.484522	1.12178	1.24832	1.03742
0.2	1.51103	1.59603	0.0849986	0.535002	0.542499	0.533127	0.674653	0.509652	1.02934	1.19828	0.987112
0.0	1.56518	1.65072	0.0855405	0.533347	0.54277	0.533347	0.701446	0.536986	0.932421	1.14388	0.932421

Table B.2: Intermediate Cost Differences

The table shows the analytical solution of the market equilibria for parameter values of $t = 1$, $\lambda = 3$, $c_A = 0.25$, $c_B = 1$:
Prices of both firms are first increasing and then decreasing in β.

β	$p_A^*(\beta)$	$p_B^*(\beta)$	$\Delta p^*(\beta)$	$q_A(\Delta p^*)$	$\hat{x}_{in}(\Delta p^*)$	$\hat{x}_{un}(\Delta p^*)$	π_A^*	π_B^*	CS^*	CS_{in}^*	CS_{un}^*
1.0	1.5	1.75	0.25	0.625	0.625	0.605992	0.78125	0.28125	1.14063	1.14063	0.834921
0.8	1.5039	1.758	0.254109	0.62324	0.627054	0.60798	0.781477	0.285586	1.07357	1.13519	0.827071
0.6	1.50553	1.76414	0.25861	0.621651	0.629305	0.61017	0.780502	0.289112	1.00758	1.13188	0.821115
0.4	1.50448	1.76803	0.263546	0.62026	0.631773	0.612585	0.778104	0.29165	0.942908	1.13111	0.81744
0.2	1.50029	1.76925	0.26896	0.619097	0.63448	0.615251	0.774048	0.293008	0.879835	1.13332	0.816464
0.0	1.49248	1.76737	0.274896	0.618194	0.637448	0.618194	0.768092	0.292988	0.818625	1.13897	0.818625

Table B.3: Large Cost Differences:

The table shows the analytical solution of the market equilibria for parameter values of $t = 1$, $\lambda = 3$, $c_A = 0.25$, $c_B = 1.25$: Non–existence for $\beta = 0$ (see Figure B.1). $q_A(\Delta p^*)$ is decreasing in β, i.e. uninformed consumers are easier to attract than informed consumers. Reason: Due to large price differences loss aversion in price dimension dominates loss aversion in taste dimension. Uninformed consumers are more willing to buy the less expensive product.

β	$p_A^*(\beta)$	$p_B^*(\beta)$	$\Delta p^*(\beta)$	$q_A(\Delta p^*)$	$\hat{x}_{in}(\Delta p^*)$	$\hat{x}_{un}(\Delta p^*)$	π_A^*	π_B^*	CS^*	CS_{in}^*	CS_{un}^*
1.0	1.58333	1.91667	0.333333	0.666667	0.666667	0.648371	0.888889	0.222222	1.02778	1.02778	0.673468
0.8	1.5623	1.90417	0.341863	0.66734	0.670931	0.652973	0.875753	0.217615	0.974147	1.04598	0.686806
0.6	1.5361	1.88738	0.351282	0.668631	0.675641	0.658117	0.859926	0.211208	0.923306	1.06911	0.7046
0.4	1.5043	1.86596	0.361666	0.670654	0.680833	0.663868	0.841199	0.202865	0.87537	1.09757	0.727236
0.2	1.46663	1.83971	0.373075	0.673535	0.686538	0.670284	0.819444	0.192519	0.830299	1.13163	0.754968
0.0	-	-	-	-	-	-	-	-	-	-	-

Appendix C

Appendix of Chapter 4

C.1 Proofs of Section 4.3

Proof of Proposition 4.1. (i) Comparing the respective necessary conditions for profit maximization and invoking the assumption of symmetry between the two profit functions, we obtain

$\frac{\partial \pi_A}{\partial p_A}(p_A, p_B) = -\omega \frac{\partial \pi_B}{\partial p_A}(p_A, p_B) \leq \frac{\partial \pi_B}{\partial p_B}(p_A, p_B) = 0.$

Since the slope of π_A is smaller than that of π_B for any chosen $(p_A(\omega), p_B(\omega))$ given $\omega > 0$, it follows from the strict concavity of π_A in p_A that $p_A^O(\omega) \geq p_B^O(\omega)$.

(ii) It follows directly from the above argument and Assumption (i) invoking symmetric profit functions that $\pi_A(p_A^O(\omega), p_B^O(\omega)) \leq \pi_B(p_A^O(\omega), p_B^O(\omega))$ for all $\omega > 0$.

(iii) Differentiating twice the two necessary conditions and inverting the matrix of derivatives, we obtain

$$\begin{bmatrix} \frac{\partial p_A^O}{\partial \omega} \\ \frac{\partial p_B^O}{\partial \omega} \end{bmatrix} = \frac{-1}{a_4 a_1 - a_3 a_2} \begin{bmatrix} a_4 & -a_3 \\ -a_2 & a_1 \end{bmatrix} \begin{bmatrix} \frac{\partial \pi_B}{\partial p_A} \\ 0 \end{bmatrix}, \tag{C.1}$$

so that we have to evaluate

$$\frac{\partial p_A^O}{\partial \omega} = \frac{-a_4}{a_4 a_1 - a_3 a_2} \frac{\partial \pi_B}{\partial p_A} \tag{C.2}$$

and

$$\frac{\partial p_B^O}{\partial \omega} = \frac{a_2}{a_4 a_1 - a_3 a_2} \frac{\partial \pi_B}{\partial p_A}, \tag{C.3}$$

where

$$a_1 \equiv \frac{\partial^2 \pi_A}{\partial p_A^2} + \omega \frac{\partial^2 \pi_B}{\partial p_A^2}, \tag{C.4}$$

$$a_2 \equiv \frac{\partial^2 \pi_B}{\partial p_A \partial p_B}, \tag{C.5}$$

$$a_3 \equiv \frac{\partial^2 \pi_A}{\partial p_A \partial p_B} + \omega \frac{\partial^2 \pi_B}{\partial p_A \partial p_B}, \tag{C.6}$$

and

$$a_4 \equiv \frac{\partial^2 \pi_B}{\partial p_B^2}. \tag{C.7}$$

The denominator of the right hand fraction, $a_4 a_1 - a_3 a_2$, is positive under assumptions (iii) and (iv). Both numerators $-a_4$ and a_2 are positive by the same assumptions. Hence both (C.2) and (C.3) are positive, by Assumption (ii).

Towards seeing that $\frac{\partial p_A^O}{\partial \omega} > \frac{\partial p_B^O}{\partial \omega}$, observe that $\frac{\partial p_A^O}{\partial \omega} \gtrless \frac{\partial p_B^O}{\partial \omega}$ iff $-a_4 \gtrless a_2$. But $-a_4 > a_2$ by the second part of Assumption (iv).

(iv) Differentiating $\pi_A(p_A^O(\omega), p_B^O(\omega))$ and $\pi_B(p_A^O(\omega), p_B^O(\omega))$, we obtain

$$\frac{\partial \pi_A}{\partial \omega}(p_A^O(\omega), p_B^O(\omega)) = \frac{\partial \pi_A}{\partial p_A}\frac{\partial p_A}{\partial \omega} + \frac{\partial \pi_A}{\partial p_B}\frac{\partial p_B}{\partial \omega} \tag{C.8}$$

and

$$\frac{\partial \pi_B}{\partial \omega}(p_A^O(\omega), p_B^O(\omega)) = \frac{\partial \pi_B}{\partial p_A}\frac{\partial p_A}{\partial \omega} + \frac{\partial \pi_B}{\partial p_B}\frac{\partial p_B}{\partial \omega}. \tag{C.9}$$

In (C.8) $\frac{\partial \pi_A}{\partial p_A}$ tends to zero at $(p_A^O(\omega), p_B^O(\omega))$ when $\omega \to 0$, so that the first term is close to zero in that neighborhood. The second term is positive throughout, so that $\pi_A^O(\omega)$ increases up to some ω^O. By the second part of Assumption (ii) and the second part of (iii) above, the negative first term must eventually dominate the positive second one as ω increases, so that $\frac{\partial \pi_B}{\partial \omega} < 0$.

In (C.9) the two components of the first term are positive by Assumption (ii) and Proposition 4.1 (ii), respectively, whilst the first component of the second term is zero by the necessary condition, so that $\frac{\partial \pi_B}{\partial \omega} \geq 0$ for all positive ω.

□

Proof of Proposition 4.2. (i) Below, under (iii) we show that $p_A^M(\omega)$ increases and $p_B^M(\omega)$

decreases in ω. By Assumption (i) we know that $p_A^M(\omega) = p_B^M(\omega)$ at $\omega = 1$. To satisfy this equality, it must hold that $p_A^M(\omega) \leq p_B^M(\omega)$ for $\omega < 1$ and $p_A^M(\omega) \geq p_B^M(\omega)$ for $\omega > 1$.

(ii) Below, under (iv) we show that $\pi_A(p_A^M(\omega), p_B^M(\omega))$ decreases and $\pi_B(p_A^M(\omega), p_B^M(\omega))$ increases in ω. By Assumption (i) we know that $\pi_A(p_A^M(\omega), p_B^M(\omega)) = \pi_B(p_A^M(\omega), p_B^M(\omega))$ at $\omega = 1$. To satisfy this equality, it must hold that $\pi_A(p_A^M(\omega), p_B^M(\omega)) > \pi_B(p_A^M(\omega), p_B^M(\omega))$ for $\omega < 1$ and $\pi_A(p_A^M(\omega), p_B^M(\omega)) < \pi_B(p_A^M(\omega), p_B^M(\omega))$ for $\omega > 1$.

(iii) Differentiating twice the two necessary conditions and inverting the matrix of derivatives, we obtain

$$\begin{bmatrix} \frac{\partial p_A^M}{\partial \omega} \\ \frac{\partial p_B^M}{\partial \omega} \end{bmatrix} = \frac{-1}{b_4 b_1 - b_3 b_2} \begin{bmatrix} b_4 & -b_3 \\ -b_2 & b_1 \end{bmatrix} \begin{matrix} \frac{\partial \pi_B}{\partial p_A} \\ \frac{\partial \pi_B}{\partial p_B} \end{matrix} \tag{C.10}$$

where

$$b_1 \equiv \frac{\partial^2 \pi_A}{\partial p_A^2} + \omega \frac{\partial^2 \pi_B}{\partial p_A^2}, \tag{C.11}$$

$$b_2 = b_3 \equiv \frac{\partial^2 \pi_A}{\partial p_A \partial p_B} + \omega \frac{\partial^2 \pi_B}{\partial p_A \partial p_B}. \tag{C.12}$$

and

$$b_4 \equiv \frac{\partial^2 \pi_A}{\partial p_B^2} + \omega \frac{\partial^2 \pi_B}{\partial p_B^2}. \tag{C.13}$$

As before, $b_4 b_1 - b_3 b_2$ is positive under assumptions (iii) and (iv). Both numerators $-b_4$ and b_2 are positive by the same assumptions. Hence by (C.10) $\frac{\partial p_A^M}{\partial \omega} > 0$ and $\frac{\partial p_B^M}{\partial \omega} < 0$ if

$$b_4 \frac{\partial \pi_B}{\partial p_A} - b_3 \frac{\partial \pi_B}{\partial p_B} < 0 \tag{C.14}$$

and

$$-b_3 \frac{\partial \pi_B}{\partial p_A} + b_1 \frac{\partial \pi_B}{\partial p_B} > 0. \tag{C.15}$$

Assumption (ii) ensures that $\frac{\frac{\partial \pi_B}{\partial p_B}}{\frac{\partial \pi_B}{\partial p_A}} \leq -1$. To ensure the inequalities (C.14) and (C.15), respectively, we need that both $|b_4| > b_3$ and $|b_1| > b_3$. For given ω both inequalities are ensured by Assumption (iv).

(iv) Differentiating $\pi_A(p_A^M(\omega), p_B^M(\omega))$ and $\pi_B(p_A^M(\omega), p_B^M(\omega))$,

$$\frac{\partial \pi_A}{\partial \omega}(p_A^M(\omega), p_B^M(\omega)) = \frac{\partial \pi_A}{\partial p_A} \frac{\partial p_A}{\partial \omega} + \frac{\partial \pi_A}{\partial p_B} \frac{\partial p_B}{\partial \omega} \tag{C.16}$$

and

$$\frac{\partial \pi_B}{\partial \omega}(p_A^M(\omega), p_B^M(\omega)) = \frac{\partial \pi_B}{\partial p_A}\frac{\partial p_A}{\partial \omega} + \frac{\partial \pi_B}{\partial p_B}\frac{\partial p_B}{\partial \omega}, \tag{C.17}$$

we see that $\frac{\partial \pi_A}{\partial \omega} \leq 0$ follows directly from the fact that $\frac{\partial \pi_A}{\partial p_A} \leq 0$ at $(p_A^M(\omega), p_B^M(\omega))$, Assumption (i) and Proposition 4.2 (iii). $\frac{\partial \pi_B}{\partial \omega} \geq 0$ follows from the symmetric argument.

□

C.2 Proofs of Section 4.4

Proof of Proposition 4.5. First we show that for any given $\omega > 0$ it is optimal to choose a direct mode of acquisition because this maximizes I_1's participation in acquisition gains from an investment in B. Then we establish that given a direct mode of acquisition $\omega = 1/\alpha_A^0$ maximizes I_1's payoff.

1. Consider the second term of (4.17). For given ω and α_A^0, if $[\pi_B^k(\omega) - \pi_B^O(0)]$ is positive (resp. negative), investor I_1 wants the weight, $g(\alpha_B, \gamma; \alpha_A^0) \equiv (\alpha_B + \alpha_A^0\gamma)/(\alpha_B + \gamma)$, to be as large (resp. small) as possible. However, $g(\alpha_B, \gamma; \alpha_A^0)$ is maximized for $\alpha_B > 0$, $\gamma = 0$, i.e. $g(\alpha_B, 0; \alpha_A^0) = 1$, and minimized for $\alpha_B = 0$, $\gamma > 0$, i.e. $g(0, \gamma; \alpha_A^0) = \alpha_A^0$.

   Next, we have to distinguish the cases of control vs. no control:

   - $k = O$:
     $[\pi_B^O(\omega) - \pi_B^O(0)] > 0$ for $\omega > 0$ since $\pi_B^O(\omega)$ is strictly increasing in ω by Proposition 4.1. Hence, $\alpha_B > 0$, $\gamma = 0$ is optimal and we receive

$$\Pi_1^O(\omega) = \alpha_A^0 \cdot \pi_A^O(\omega) + [\pi_B^O(\omega) - \pi_B^O(0)]. \tag{C.18}$$

   - $k = M$:
     Since for $\omega = 0$ investor I_1 only cares about the profit in firm A, we cannot rule out that $[\pi_B^M(0) - \pi_B^O(0)] < 0$.
     By symmetry and no firm shut (Assumption (v)) $\pi_A^M(\omega) + \pi_B^M(\omega)$ is maximized at $\omega = 1$. By the principle of optimization we also know that $\pi_B^M(1) > \pi_B^O(0)$ because $\pi_B^M(1)$ is the profit resulting from maximizing the equally weighted sum of profits at coordinated prices, whilst $\pi_B^O(0)$ is the symmetric NE profit. Thus,

$[\pi_B^M(1) - \pi_B^O(0)] > 0$.

Since by Proposition 4.2 (iv) $\pi_B^M(\omega)$ strictly increases for ω, $\exists$ unique $\bar{\omega} \equiv \{\omega | \, [\pi_B^M(\omega) - \pi_B^O(0)] = 0\}$ with $\bar{\omega} \in [0, 1)$.

Hence, given maximization / minimization of $g(\alpha_B, \gamma; \alpha_A^0)$ for given ω

$$\Pi_1^M(\omega) = \begin{cases} \alpha_A^0 \cdot \pi_A^M(\omega) + \alpha_A^0 \cdot [\pi_B^M(\omega) - \pi_B^O(0)], & \text{if } \omega < \bar{\omega}; \\ \alpha_A^0 \cdot \pi_A^M(\omega) + [\pi_B^M(\omega) - \pi_B^O(0)], & \text{if } \omega \geq \bar{\omega}\,. \end{cases} \tag{C.19}$$

Now, since $\pi_A^M(\omega) + \pi_B^M(\omega)$ is maximized at $\omega = 1$, $\alpha_A^0 \cdot \pi_A^M(\omega) + \alpha_A^0 \cdot [\pi_B^M(\omega) - \pi_B^O(0)]$ is also maximized at $\omega = 1$. This implies $\omega > \bar{\omega}$ and therefore we leave the first case of (C.19).

Moreover, we can show that $\omega \geq 1$ in the optimum for the second case of (C.19), thus we stay in the second case. Suppose not, then $\exists \omega' < 1$ which maximizes $\alpha_A^0 \cdot \pi_A^M(\omega) + [\pi_B^M(\omega) - \pi_B^O(0)]$. Now, we use that the first derivative of $\alpha_A^0 \cdot \pi_A^M(\omega) + [\pi_B^M(\omega) - \pi_B^O(0)]$ w.r.t. ω can be expressed as

$$\alpha_A^0 \cdot [\frac{\partial \pi_A^M(\omega)}{\partial \omega} + \frac{\partial \pi_B^M(\omega)}{\partial \omega}] + (1 - \alpha_A^0) \cdot \frac{\partial \pi_B^M(\omega)}{\partial \omega}.$$

Since the sum of profits is maximized at $\omega = 1$ we must have that $[\partial \pi_A^M(\omega)/\partial \omega + \partial \pi_B^M(\omega)/\partial \omega]$ is positive for all $\omega < 1$. At the same time $\partial \pi_B^M(\omega)/\partial \omega$ is positive for all ω by Proposition 4.2. Thus, for $\alpha_A^0 < 1$, $\Pi_1^M(\omega)$ is strictly increasing in ω for all $\omega < 1$. This contradicts $\omega' < 1$ being optimal. This implies $\omega \geq 1 > \bar{\omega}$.

Hence, $\alpha_B > 0$, $\gamma = 0$ is optimal in the monopoly and the relevant payoff function is the payoff function of second case of (C.19)

$$\Pi_1^M(\omega) = \alpha_A^0 \cdot \pi_A^M(\omega) + [\pi_B^M(\omega) - \pi_B^O(0)]. \tag{C.20}$$

Combining the oligopoly case and monopoly case yields the following payoff function

$$\Pi_1^k(\omega) = \alpha_A^0 \cdot \pi_A^k(\omega) + [\pi_B^k(\omega) - \pi_B^O(0)], k = O, M. \tag{C.21}$$

2. We now derive the optimal ω.[1] We therefore show that if I_1 maximizes the payoff function in (C.21) over $\omega \in [0, 1/\alpha_A^0]$, then $\omega = 1/\alpha_A^0$ is a unique global maximizer and

[1]Due to free-riding of the remaining shareholders in A we can restrict to $\alpha_A = \alpha_A^0$ wlog.

$k = M$.

To provide this result we use a revelation principle argument for I_1's payoff function in the monopoly case and then receive global optimality by the principal of optimization.

First reconsider (4.8) in Section 4.3.2—the maximization problem of I_1 in product market stage for $k = M$ and for given ω. The arg max of this problem is denoted by $(p_A^M(\omega), p_B^M(\omega))$. (Cf. Proposition 4.2)

Next, turn back to case 3 in the acquisition stage:

Maximizing $\Pi_1^k(\omega)$ in (C.21) over ω in the monopoly case ($k = M$) is equivalent to

$$\max_{\omega \in [0, 1/\alpha_A^0]} \Pi_1(\omega) = \pi_A^M(\omega) + \frac{1}{\alpha_A^0} \pi_B^M(\omega).^2 \tag{C.22}$$

Moreover, (C.22) is equivalent to

$$\max_{p_A, p_B} \pi_A(p_A, p_B) + \frac{1}{\alpha_A^0} \pi_B(p_A, p_B) \quad \text{s.t. } (p_A, p_B) \in \{(p_A^M(\omega), p_B^M(\omega)) | \, \omega \in [0, 1/\alpha_A^0]\}. \tag{C.23}$$

The objective function of (C.23) is identical to the objective function of (4.8) for $\omega = 1/\alpha_A^0$. Since we know that $(p_A^M(1/\alpha_A^0), p_B^M(1/\alpha_A^0))$ is a maximizer of (4.8) for $\omega = 1/\alpha_A^0$, it is a maximizer of the unconstrained problem of (C.23) as well. $(p_A^M(1/\alpha_A^0), p_B^M(1/\alpha_A^0))$ is also a maximizer of the constrained problem of (C.23) because it lies in the constraint set $\{(p_A^M(\omega), p_B^M(\omega)) | \, \omega \in [0, 1/\alpha_A^0]\}$. Therefore we get that $\omega = 1/\alpha_A^0$ is a maximizer of (C.22). It is the unique maximizer of (C.22) by Proposition 4.2 (iii).

Finally, since the unique maximizer in the monopoly case, $\omega = 1/\alpha_A^0$, can always be achieved by the investor with an entire direct investment, $\alpha_B = 1$, $\gamma = 0$, we know by the principle of optimization that $\omega = 1/\alpha_A^0$ is the unique maximizer of (C.21). This implies that the investor will choose control in the optimum ($k = M$).

□

Proof of Proposition 4.6. The result follows directly from the assumption that the unweighed sum of profits is maximized at $\omega = 1$ (Assumption (v)). □

[2]The constant can be ignored.

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Eidesstattliche Erklärung

Hiermit erkläre ich, die vorliegende Dissertation selbständig angefertigt und mich keiner anderen als der in ihr angegebenen Hilfsmittel bedient zu haben. Insbesondere sind sämtliche Zitate aus anderen Quellen als solche gekennzeichnet und mit Quellenangaben versehen.

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