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Information Acquisition in Double Auctions

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Abstract

This paper analyses information acquisition in the Reny and Perry (2006) type double auction environment and shows that an efficient and fully revealing equilibrium may fail to exist if information is endogenous and costly. As the number of traders increases, the equilibria are inefficient even though the information cost is very large. Because of endogenous noise trading, the price is also not fully revealing. This paper provides a strategic foundation for Grossman and Stiglitz (1980) and discusses some market microstructure implications.

Journal of Economic Literature Classification Numbers: C78, D82, D83

Key words: double auction, efficiency, endogenous lemons problem, information acquisition

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1. Introduction

Agents trading financial assets typically face common values uncertainty because the cash flow stream of a financial asset is risky. In secondary markets the seller does not necessarily posses better information about the value of the asset than a potential buyer but the agents on both sides of the market can acquire information before they trade. This paper analyzes information acquisition and trade in the Reny and Perry (2006) type double auction environment.[1]

Reny and Perry (2006) analyze information aggregation and efficiency in a large double auction and show that a Bayesian Nash equilibrium (BNE) in pure monotone bidding functions exits, the equilibrium outcome is arbitrarily close to an efficient and fully revealing rational expectations equilibrium. In their model the diverse information of the traders are exogenous. This paper assumes that all traders have symmetric information ex ante but both the buyers and the sellers can acquire information about the value of the asset before they trade. This paper analyses the implications of endogenous information acquisition for allocative and informational efficiency in small and large double auctions where all liquidity traders behave strategically. This paper contains two parts.

The first part analyses information acquisition in a double auction with one liquidity buyer and one liquidity seller and shows that two types of inefficiencies may arise.[2] (1) If the information cost is low, then in a BNE with trade both traders acquire socially useless information. The motive for information acquisition is driven by the desire to trade without being exploited. In such an equilibrium the price is fully revealing. There is no inverse relation between informationally efficient prices and the compensation for information cost because the traders receive enough compensation when they realize the trading gains.

(2) If the information cost is in an intermediate range, then no pure strategy BNE with efficient trade exists although the traders maintain symmetric information in equilibrium and the trading gains are common knowledge. This paper shows that an *endogenous* lemons problem, i.e. the concern of suffering a potential speculative loss due to

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[1] Liquidity motives such as portfolio rebalance needs, tax-induced trades, and dividend-captured trades give rise to mutually beneficial transactions. The demand for financial analysts' coverage and Bloomberg's and Reuters' financial services suggest that information acquisition is a prevalent activity on financial markets.

[2] A small double auction or simultaneous offer bargaining can be interpreted as a model of over-the-counter-trading. Bargaining is a standard feature in many decentralized markets, such as those for corporate bonds, derivatives, mortgage-backed securities, currencies, and real estate. See Duffie et al. (2005).

the mere possibility of information acquisition by the other trader, can already render efficient trade unattractive.[3]

If the traders randomize information acquisition and their offer prices, then trade occurs with positive probability. The outcome in a mixed strategy BNE has the following properties. (1) The motive for information acquisition is to exploit the other trader. If trade occurs, then an informed trader makes some speculative profits. In particular, no trade occurs if both traders are informed in a mixed strategy equilibrium. (2) The behavior of an uninformed trader exhibits endogenous noise trading in the sense that he proposes with positive probability an offer which is prone to speculation and may suffer a speculative loss. (3) The equilibrium payoff of an uninformed trader is non-negative because there is a positive probability that his trading partner is also uninformed so that he realizes the trading gain without suffering a speculative loss. (4) The equilibrium price is not fully revealing. If the price was fully revealing, then an uninformed trader would know for sure that he suffers a speculative loss given that his offer is vulnerable to speculation.

The second part of the paper extends the two trader case to a double auction with many traders and shows that a large double auction market may not be able to mitigate the inefficiencies in the small double auction, but it may even perform worse. In decentralized trading, if the information cost is large, then the buyer and the seller trade without costly information acquisition. In centralized trading on a large market, this efficient equilibrium fails to exist even though the information cost is very large.

As the number of buyers and sellers increases, if a trader can trade several units in a large market, then the potential speculative profits of an informed trader increases because there are potentially more uninformed traders to exploit. Therefore, the potential speculative threat an uninformed trader faces exists not only for low information costs, but also for very large information cost. In such a case only mixed strategy equilibria exist in which a fraction of liquidity traders does not trade and the price is not fully revealing. This paper shows that an efficient and fully revealing equilibrium in the Reny and Perry (2006) type double auction environment may fail to exist if information is endogenous and costly.[4]

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[3] This no trade result is different from Myerson and Satterthwaite (1983) because the gains from trade are common knowledge in the present model. This result is also different from Akerlof (1970) and Gresik (1991) since the traders possess symmetric information about the common valuation in the no trade equilibrium.

[4] The reason why Reny and Perry (2006) result at the double auction stage does not apply to this setting is the following. In their model it is common knowledge that the traders have private information and a pure strategy BNE exists. In this model information acquisition is not observable. From a strategic perspective, this model can be interpreted as one-stage game with two action variables, namely choosing information acquisition and

This model can be regarded as a strategic version of Grossman and Stiglitz (1980) who analyze information acquisition within the noisy rational expectations equilibrium framework with no explicit market mechanism. They assume that the liquidity traders neither consider information acquisition nor care about prices but just want to trade. The rational players in their model are the speculators without real trading motive. They acquire information to exploit the noisy liquidity traders.[5] This paper endogenizes their noise trading assumption as well as provides a strategic foundation for their so-called impossibility result of informationally efficient prices.

In the present model there are no noise traders in the market initially since all liquidity traders behave strategically. The intuition why no pure strategy BNE exists and the price is not fully revealing is the following. Suppose the price is fully revealing. Then there is free-riding in the sense that not acquiring information is a profitable deviation. As the number of informed traders decreases, the remaining informed traders can move prices and makes speculative profits. Therefore, no pure strategy BNE exists. As in a small double auction, there is endogenous noise trading and the price is not fully revealing in a mixed strategy BNE.

This paper is also related to the literature on information acquisition in auctions. Milgrom (1981), Matthews (1984), Hausch and Li (1993), and Persico (2000) analyse information acquisition in auctions where only the buyers' side considers information acquisition while the seller is non-strategic or noisy. Since the seller just wants to sell his asset holding, trade always occurs but the bidders acquire (socially wasteful) information in equilibrium.[6] In contrast, this model assumes that all traders behave strategically and can acquire information. A strategic buyer (seller) may not want to buy (sell) and is willing to forgo the gain due to the endogenous lemons problem. The two-sided strategic behavior (even with exogenous private information) gives rise to some technical difficulties since the random variables or so-called order statistics are not affiliated. Reny and Perry (2006) provide a new technique to solve for an equilibrium in the large limit market.

offer prices. In a mixed strategy equilibrium a trader is not able to infer the information of the other traders from observing the price. On the other hand, if information is costless then as in Reny and Perry (2006), an efficient and fully revealing BNE exists.

[5] Under these assumptions they show that the equilibrium price is not fully revealing. If noise vanishes, then the market breaks down. See also Hellwig (1982), Verrecchia (1982) and Barlevy and Veronesi (2000).

[6] Bergemann and Valimäki (2002) employ a local efficiency concept and show that any ex post efficient allocation mechanism causes an ex ante information acquisition inefficiency.

The remainder of the paper is organized as follows. The next section introduces the basic model. Section 3 analyzes information acquisition in a small double auction. Section 4 analyses information acquisition in large double auctions. Section 5 discusses the results. Section 6 concludes with a discussion of some market microstructure implications.

2. The Basic Model

Two risk neutral traders seek to agree on a price p at which to trade one unit of an asset. It is common knowledge that the asset is worth $v+\Delta$ to the buyer and $v-\Delta$ to the seller where $\Delta>0$ is a constant and v is the uncertain common value component which is either v_L or v_H with equal probability and $v_H>v_L>\Delta$. In addition, $\Delta<\frac{1}{8}(v_H-v_L)$ (Assumption 1).[7]

In the first stage, a trader can learn about the true value by incurring the cost $c>0$. The information acquisition decision of the buyer and the seller is denoted with $n_B,n_S\in\{0,1\}$ and is not observable by the other trader. In the second stage the traders play a double auction, i.e. the buyer proposes a bid price b and the seller proposes an ask price s simultaneously. If $b\geq s$, then the asset changes hand at the mean price $p=\frac{1}{2}(b+s)$ and $U^B=v+\Delta-p$ and $U^S=p-(v-\Delta)$. Otherwise no trade occurs and the payoff is normalized to zero.

A strategy of the buyer and the seller is denoted with $B=(n_B,b(\cdot))$ and $S=(n_S,s(\cdot))$, respectively. If $n_B=1$, then the buyer can submit state depend offers, i.e. $b=(b_L,b_H)$ where b_L and b_H denote the buyer's offer in the state L and H, respectively. Analogously for the seller. An equilibrium in which trade occurs with probability one is called a *full trade* equilibrium. An equilibrium is called a *k-sharing* equilibrium if the buyer's expected payoff is $\Delta-k$ and the seller's expected payoff is $\Delta+k$. The solution concept is Bayesian Nash equilibrium (BNE).

Remark 1

A set of no-trade equilibria is given by $B=(0,b)$ and $S=(0,s)$ with $b\leq v_L-\Delta$ and $s\geq v_H+\Delta$. These equilibria always exist.

[7] A low valuation $(v-\Delta)$-trader may have low liquidity (that is a need for cash), hedging reasons to sell, or a relative tax disadvantage. The total trading surplus is therefore 2Δ. Assumption 1 makes the problem interesting and is assumed to hold throughout the paper. If Δ is large, the potential lemons problem has no adverse allocative consequences which will become clear in the analysis.

3. Information Acquisition in a Small Double Auction

The socially efficient outcome is trade without costly information acquisition. The set of strategies leading to the set of acceptable prices $p\in[E[v]-\Delta,E[v]+\Delta]$ for an uninformed buyer and uninformed seller is $B=(0,b)$ and $S=(0,s)$ with $b=s=E[v]+k$ and $k\in[-\Delta, \Delta]$. In the k-sharing outcome the buyer gets $EU^B=\Delta-k$ and the seller gets $EU^S=\Delta+k$. When do these strategies constitute best responses and an efficient equilibrium?

Given the strategy S of the seller, suppose the buyer acquires information and speculates. In the low state he chooses a bid price $b_L<s$ and no trade occurs. In the high state he chooses $b_H=s$ and makes some speculative profits since he pays less than the true value of the asset. This response yields $EU^B=\frac{1}{2}[(v_H+\Delta)-(E[v]+k)]-c=\frac{1}{4}(v_H-v_L)+\frac{1}{2}(\Delta-k)-c$. So if $\frac{1}{4}(v_H-v_L)+\frac{1}{2}(\Delta-k)-c>\Delta-k$, then speculation is the best response, and the seller suffers an endogenous lemons problem since $EU^S=\frac{1}{2}(\Delta+k)-\frac{1}{4}(v_H-v_L)<0$. Analogously, if $\frac{1}{4}(v_H-v_L)+\frac{1}{2}(\Delta+k)-c>\Delta+k$, then the seller's best response to $B=(0,b)$ with $b=E[v]+k$ is to choose $S=(1,s_L,s_H)$ with $s_L=b$ and $s_H>b$. Therefore, if $c<\max\{\pi-\frac{1}{2}(\Delta-k),\pi-\frac{1}{2}(\Delta+k)\}=\pi-\frac{1}{2}(\Delta-|k|)$ where $\pi\equiv\frac{1}{4}(v_H-v_L)$, then no efficient k-sharing BNE exists.[8]

The economic intuition for this condition is simple. π is the expected speculative profit an informed trader makes and $\frac{1}{2}(\Delta-|k|)$ is the expected opportunity cost of speculation. In the k-sharing outcome $EU^B=\Delta-k$ and $EU^S=\Delta+k$. If the buyer acquires information and speculates, he does not trade in state L and ex ante he forgoes the surplus $(\Delta-k)$ with probability 0.5. If the seller speculates, his opportunity cost of speculation is $\frac{1}{2}(\Delta+k)$. So if the information cost is smaller than the speculative profit net the opportunity cost of speculation, a trader has an incentive to speculate and trade at the price $p=E[v]+k$ is not an equilibrium outcome.

Proposition 1

If $\Delta<c<\frac{1}{4}(v_H-v_L)-\frac{1}{2}\Delta$, then no pure strategy BNE with trade exists.

[8] If $c\geq\frac{1}{4}(v_H-v_L)$, then any k-sharing outcome is attainable as a BNE. As in the "standard" double auction, a continuum of trading equilibria exists. However, the set of efficient equilibria "shrinks" as the information cost decreases. If $c=\frac{1}{4}(v_H-v_L)-\frac{1}{2}\Delta$, then only the equal-split (k=0) outcome is attainable as an efficient equilibrium, i.e. the efficient BNE is unique.

In order to complete the proof of Proposition 1, it remains to show that a pure strategy trading BNE with one-sided or two-sided information acquisition also fails to exist. It is easy to see that if $c>\Delta$, then no pure strategy equilibrium exists in which both traders acquire information. Suppose only the seller acquires information. This causes a lemons problem. Assumption 1 implies that $v_H-\Delta>E[v]\pm\Delta>v_L+\Delta$. A standard lemons argument shows that the buyer offers at most $v_L+\Delta$. Trade only occurs in state L and the seller's payoff is at most $EU^S=\Delta-c<0$. (Analogously for the case $n_B=1$ and $n_S=0$.) In such a case, no trader acquires too expensive and non-exploitable information but in order to account for the endogenous lemons problem the buyer proposes $b\leq v_L+\Delta$ and the seller proposes $s\geq v_H-\Delta$. Therefore, no pure strategy BNE with trade exists.

This inefficiency result is different from Myerson and Satterthwaite (1983) since the gain from trade is common knowledge. It is also different from Akerlof (1970) and Gresik (1991) since there is no asymmetric information about the common valuation in equilibrium. The reason is an endogenous lemons problem caused by the mere possibility of information acquisition. Dang (2006) shows that this no trade result under symmetric information also arises in ultimatum and alternating offer bargaining. The next proposition shows that a mixed strategy equilibrium is as inefficient as the no trade equilibrium.

Proposition 2

Suppose that $\Delta<c<\frac{1}{4}(v_H-v_L)-\frac{1}{2}\Delta$. The outcome in a (non-degenerated) mixed strategy BNE has the following properties. (i) Both traders get zero expected payoff. (ii) If trade occurs, the price is not fully revealing.

Proof: See Appendix.

The following example highlights the intuition behind Proposition 2. Suppose that the traders are only allowed to choose three offer prices, namely $b,s\in\{v_L,E[v],v_H\}$. The Appendix shows that in such a case in the unique (non-degenerated) mixed strategy equilibrium the buyer randomizes over the following three pure strategies: $(0,v_L)$, $(0,E[v])$ and $(1,v_L,E[v])$. The seller randomizes over the strategies $(0,v_H)$, $(0,E[v])$ and $(1,E[v],v_H)$. Trade only occurs at the following events. (i) Both traders do not acquire information and choose $E[v]$. (ii) The seller is uninformed and chooses $s=E[v]$ and the buyer is informed and the true state is H. (iii) The buyer is uninformed and chooses $b=E[v]$ and the seller is informed and the true state is L. In particular, no trade occurs if both traders are informed. The probability of trade is $\frac{16c^2}{(v_H - v_L)^2 - 4\Delta^2}$ and the price is p=E[v] and not fully revealing.

Some further comments might be at place. (a) Why is the price not fully revealing? Suppose the seller does not acquire information and observes trade at p=E[v]. In this case he does not know whether the buyer chooses (0,E[v]) or $(1,v_L,E[v])$. Although his posterior belief for $v=v_H$ increases, it is strictly below one. Otherwise he would know for sure that he has made a bad deal and this cannot be an equilibrium outcome.

(b) In the mixed strategy equilibrium an uninformed trader proposes the offer price E[v] with positive probability, i.e. his behavior exhibits endogenous noise trading. In other words, an uninformed trader proposes an offer which is prone to speculation and he may suffer a speculative loss. However, his equilibrium payoff is non-negative since he meets an uninformed trader with positive probability. In such a case he realizes the trading gain without suffering a speculative loss.

(c) Why is there no trade if both traders are informed? Equivalently, why does an informed buyer choose b=E[v] in state H with probability 1 rather than randomizing over b=E[v] and $b=v_H$, i.e. why does he choose $(1,v_L,v_H)$ with probability zero? If he chooses $b=v_H$ in state H, then trade also occurs in the event where the seller is informed since an informed seller choose $s=v_H$ in state H. The "problem" is that if the buyer is indifferent between the strategies $(1,v_L,E[v])$ and $(1,v_L,v_H)$, then both strategies with information acquisition is strictly dominated by the strategy $(0,v_L)$. If the informed buyer trades at the price v_H in state H, then his expected payoff net information cost is negative. So not acquiring information would be a best response. In other words, if a trader acquires information in a mixed strategy equilibrium, then he speculates since an informed trader expects to meet an endogenous noise trader with positive probability.

(d) Why is the equilibrium payoff non-positive? Put it differently, since the minimum price the seller demands is s=E[v], why does the buyer choose $(0,v_L)$ with positive probability? The "problem" with a mixed strategy equilibrium is that the buyer has to randomize such that the seller is indifferent between information acquisition and no information acquisition. If the buyer only randomizes over the strategies (0,E[v]) and $(1,v_L,E[v])$, some probability is "left" and this has to be put on the strategy $(0,v_L)$ although $EU^B(0,v_L)=0$. Since the buyer chooses $(0,v_L)$ with positive probability, the (overall) expected payoff of the buyer is zero. Therefore, the mixed strategy equilibrium is as inefficient as the no trade equilibrium. (This result also holds if the set of offer prices is continuous.)

(e) The difference $v_H - v_L$ captures the importance of the endogenous lemons problem. As this difference and the endogenous lemons problem increases, this exerts two effects. (i) The information cost range which implies no pure strategy trading equilibrium increases, i.e. even for high information cost no efficient BNE exists. (ii) The equilibrium probability of trade $\frac{16c^2}{(v_H - v_L)^2 - 4\Delta^2}$ decreases because the probability that an uninformed trader chooses the offer price E[v] decreases.

Proposition 3

If $c \leq \Delta - |k|$ where $k \in [-\Delta, \Delta]$, then a full trade k-sharing BNE exists. In any such BNE both traders acquire information, the price is fully revealing, $EU^B = \Delta - k - c$ and $EU^S = \Delta + k - c$.

Proof: See Appendix.

Proposition 3 shows that if the information cost is low, the traders face an information acquisition dilemma but trade occurs with probability one and the price is fully revealing.[9] The intuition is simple. The desire of the liquidity traders to trade without being exploited induces information acquisition. There is no inverse relation between informationally efficient prices and the compensation for information cost since the trading surplus is fixed and larger than the information cost.[10]

This result can be interpreted as an endogenous justification of the symmetric information assumption of the over-the-counter-trading model in Duffie et al. (2005). If the information cost is low, both traders acquire information and have symmetric information before they meet and bargain over the trading surplus. If the cost is high, no trader acquires information and they also have symmetric information.

[9] If $c = \Delta$, then $B = S = (1, v_L + \Delta, v_H - \Delta)$ is the unique pure strategy BNE in which trade occurs with probability one. There also exist equilibria in which one trader acquires information and trade occurs with probability 0.5. From a social point of view, a full trade BNE dominates a partial trade BNE if $c < \Delta$.

[10] Jackson (1991) shows that with imperfect competition fully revealing prices exist despite costly information. In his model there are three players (an informed rational traders, an uninformed rational trader, and a noise trader) and the motive for information acquisition is to exploit the noise trader. In contrast, Proposition 3 shows that the traders acquire information to avoid being exploited if the information cost is low.

4. Information Acquisition in Large Double Auctions

This section extends the two trader case to a setting with 2N traders. All assumptions of the basic model in section 2 hold. In addition, the following assumptions are made.

Assumptions and Notations

(1) There are N traders who each has a valuation v+Δ for the first unit and a valuation v for the other units. There are N traders who each has a valuation v−Δ for the first unit and a valuation v for the other units. These traders possess one unit of the asset each.[11]

(2) A trader is allowed to buy and sell m units. Short selling is allowed. A strategy of a buyer i and seller i consists of three components and is denoted with $B_i=(n^B_i,b_i(\cdot),u^B_i(\cdot))$ and $S_i=(n^S_i,s_i(\cdot),u^S_i(\cdot))$, respectively, where u_i denotes the units of the asset a trader wants to trade. An informed trader can submit state-contingent orders, i.e. $u_i=(u_{iL},u_{iH})$.

(3) The price is set to equalized total demand and total supply. If there is excess demand (supply), then the buyers (sellers) with the highest bid prices (lowest ask price) are served first. The remaining units are allocated to the traders who propose the same offer with equal probability.[12]

A motivation for Assumption (1) is the following. Suppose the traders are funds managers who follow different trading styles, have opposite trading needs, and want to rebalance their portfolio by trading one unit of the asset with each other. A buyer is willing to pay (up to Δ) more than the value v, while a seller is willing to accept less than v. Having rebalanced their positions and met their liquidity needs they are not willing to pay more or less than the value v. The assumption is employed to replicate the traders' valuations in Reny and Perry (2006). Since short-selling is allowed, there is no build-in asymmetry between buyers and sellers. If short-selling is not allowed, all following results also hold qualitatively. (See footnote 15.)

Corollary

Suppose there are N traders where a trader is allowed to buy and sell m=1 unit each and $\Delta<c<\frac{1}{2}(v_H-v_L)-\frac{1}{2}\Delta$. (i) No pure strategy BNE with trade exists. (ii) A mixed strategy BNE is inefficient and the price is not fully revealing.

[11] To simplify the wording, a low (high) valuation trader is called seller (buyer).

[12] This allocation rule is adopted from Reny and Perry (2006). See their section 4.1 for a detailed description.

Part (i) of the Corollary follows almost directly from Proposition 1. Suppose p=E[v]. This price is less prone to speculation because the opportunity cost of speculation is highest for *both* the buyers and sellers. Since there are additional buyers and sellers, an informed trader can potentially make more profit in a large market although he can only buy or sell one unit. For example, an informed seller becomes a speculator in state L and buys one unit of the asset which is not possible in a small (N=1) double auction. In state H, as in a small double auction, he sells one unit of the asset. Therefore, his expected payoff with information acquisition is $\frac{1}{2}(v_H-v_L)+\frac{1}{2}\Delta-c$. If $c<\frac{1}{2}(v_H-v_L)-\frac{1}{2}\Delta$, this strategy dominates trade at p=E[v] with expected payoff Δ, and no pure strategy trading BNE without information acquisition exists. In a pure strategy BNE with trade, at least one buyer and one seller acquire information. Since $c>\Delta$, a pair of informed traders cannot jointly cover their information cost by trading one unit each. No pure strategy trading BNE with information acquisition exists.

Part (ii) follows from Proposition 2. The price is not fully revealing. If $p=v\pm\Delta$, then either the informed buyer or the informed seller has a negative payoff since $c>\Delta$. In a mixed strategy BNE there is a positive probability that some uninformed traders do not trade. Since a fraction of liquidity traders does not trade, the equilibrium is inefficient.

This Corollary provides the simplest example which shows that an efficient and fully revealing BNE in a large double auction market may fail to exist if information is endogenous and costly. Similar to Reny and Perry (2006), this example also assumes that a trader is only allowed to trade one unit.

This Corollary shows that having a common market place may not be able to resolve the inefficiency that arises in the small double auction. In contrast, the next proposition shows that a large double auction market where a trader can trade m>1 units may perform worse. In the small (N=1) double auction, if the information cost is large, i.e. $c\geq\frac{1}{4}(v_H-v_L)-\frac{1}{2}\Delta$, then an efficient equilibrium exists. If the number of traders increases, an efficient equilibrium fails to exist although the information cost is very large.

Proposition 4

Suppose the number N of traders is large and $c<\frac{1}{2}m(v_H-v_L)-\frac{1}{2}\Delta$. There exists no pure strategy trading BNE without information acquisition.

Proof

The proof for m=1 follows directly from the previous analysis, but the proof for m>1 is based on three arguments. (a) If a pure strategy trading BNE without information acquisition exists, then N units are traded, i.e. all traders are “satisfied”. (b) If a pure strategy trading BNE without information acquisition exists, then trade is executed at the price $p=E[v]$. (c) No pure strategy trading BNE without information acquisition exists where trade is executed at the price $p=E[v]$.

The following arguments prove claim (a). Suppose all traders do not acquire information, and each low-valuation trader (buyer) submits an order to buy one unit and each high-valuation trader (seller) submits an order to sell one unit and their offer prices are summarized by the offer price profile $B=(b_1,\ldots.b_N)$ and $S=(s_1,\ldots.s_N)$. Given (B,S), suppose the resulting market (clearing) price is $p\in(E[v]-\Delta,E[v]+\Delta)$. Suppose $b_i<p$ and buyer i does not get the asset. Given (B,S), buyer i can do better by choosing $b_i\geq p$ and gets one unit with positive probability and $EU>0$. Any buyer or seller who does not get to buy or sell one unit of the asset at the resulting price, has not played a best response. (If $p=E[v]-\Delta$, “unsatisfied” buyers will deviate. If $p=E[v]+\Delta$, “unsatisfied” sellers will deviate.) This reasoning implies that if a trading equilibrium without information acquisition exists, then all traders are “satisfied”, i.e. N units are traded. Therefore, a candidate offer profile (B,S) for being part of a pure strategy BNE must have $b_i\geq p$ and $s_i\leq p$ $(i=1,\ldots.N)$ where p is the resulting market price given (B,S).

The proof of claim (b) is as follows. Suppose each trader trades one unit and the offer profile (B,S) gives rise to the price $p\in[E[v]-\Delta,E[v]+\Delta]$. Case (i) Suppose $p<E[v]$. Consider buyer i who chooses b_i and gets one unit of the asset. For example, a profitable deviation is to choose the same offer price but submits an order of m units. The expected payoff is $EU=(E[v]+\Delta-p)+prob(trade)(m-1)(E[v]-p)>\Delta$. Therefore, if $p<E[v]$, then a buyer who gets one unit has not played a best response. In some sense there is incentive to buy more and overbid the other buyers. (ii) If $p>E[v]$, then a seller who sells one unit has not played a best response. There is incentive to sell short and underbid the other sellers. Consequently, only if $p=E[v]$, then an uninformed trader who trades one unit has no profitable deviation.

The proof of claim (c) is similar to the proof of Proposition 1. Suppose that no trader acquires information and the offer profile (B,S) yields the market price $p=E[v]$ and all traders trade one unit each. Then some traders have a profitable deviation. For example, buyer i acquires information. In state H, he chooses $b_i=E[v]+\gamma_b$ to buy m units where γ_b is chosen such that b_i is larger than the m-th highest bid prices given $B=(b_1,..,b_N)$. The (additional) demand of buyer i may increase the market clearing price from $p=E[v]$ to at most $p=E[v]+\gamma_b$. Suppose $p=E[v]+\gamma_b$. His payoff in this state is $(v_H+\Delta)+(m-1)v_H-mp-c=\frac{1}{2}m(v_H-v_L-2\gamma_b)$

$+\Delta-c$.[13] In state L he forgoes the surplus Δ and chooses $b_i=E[v]-\gamma_s$ to sell short m units where γ_s is chosen such that b_i is smaller than the m-th lowest ask prices given $S=(s_1,..,s_N)$. Suppose $p=E[v]-\gamma_s$. His payoff in this state is $\frac{1}{2}m(v_H-v_L-2\gamma_s)-c$.

If the number N of traders is large, then the price impact is zero in the following sense. For a given offer profile (B,S) and the market clearing price $p=E[v]$, there always exists the same market clearing price such that buyer i buys or sells m units.[14] Therefore, the expected payoff of buyer i with information acquisition is $EU_i^B=\frac{1}{2}m(v_H-v_L)+\frac{1}{2}\Delta-c$. Speculation is the best response if $\frac{1}{2}m(v_H-v_L)+\frac{1}{2}\Delta-c>\Delta$. Analogously for a seller. Consequently, if $c<\frac{1}{2}m(v_H-v_L)-\frac{1}{2}\Delta$, there exists no pure strategy trading equilibrium without information acquisition.[15] **QED**

Proposition 5

Suppose the number N of traders is large and $c<\frac{1}{2}m(v_H-v_L)-\frac{1}{2}\Delta$. The outcome in a BNE with trade has the following properties (i) A strictly positive fraction of liquidity traders does not trade. (ii) The price is not fully revealing.

Proof: See Appendix.

A notion of a competitive or close to competitive market is that a trader can trade as many units of the asset as he likes without having much price impact. Proposition 5 can be interpreted as saying that such a large double auction market may be both allocatively and informationally inefficient. These inefficiencies do not disappear when the number of traders increases but arise even though the information cost is very large. Therefore, an efficient and fully revealing equilibrium in the Reny and Perry (2006) type large double auction market may fail to exist if information is endogenous and costly. On the other hand, the next Proposition shows that if c=0, then this model generates an equilibrium which is consistent with Reny and Perry (2006) result.

[13] The buyer's valuation for the first unit is $v+\Delta$ and v for the other units.

[14] Consider the two extreme cases. (i) All sellers choose $E[v]-\Delta$ and all buyers choose $E[v]+\Delta$ and the price is $E[v]$. If the true value is high, an informed buyer chooses $E[v]+\Delta+\varepsilon$ and buys m units. For $m<N$, there exists a market clearing price $p=E[v]$. (ii) All $b=s=E[v]$. If the buyer chooses $E[v]+\Delta+\varepsilon$, a market price $p=E[v]$ exists.

[15] If short-selling is not allowed, then the condition is $c<\frac{1}{4}m(v_H-v_L)-\frac{1}{2}\Delta$.

Proposition 6

If c=0, then there exists a BNE with the following properties. (i) All traders acquire information and trade one unit each. (ii) If m=1, the equilibrium price is $p \in [v-\Delta, v+\Delta]$. (iii) If m>1, then the equilibrium price is p=v.

It is easy to see that (a) for m=1, $B_i=S_i=(1,(v_L+k,v_H+k),(1,1))$ where $k\in[-\Delta,\Delta]$ and i=1,..,N constitute a BNE where p=v+k and $EU^B=\Delta-k$ and $EU^S=\Delta+k$. (b) For m>1, $B_i=S_i=(1,(v_L,v_H),(1,1))$ constitute the unique pure strategy trading BNE and $EU^B=EU^S=\Delta$. Note if $p\neq v$, then a trader may want to trade more units. There is overbidding and underbidding which can not be an equilibrium. (See the proof of Claim B in Proposition 4.). In both cases an efficient and fully revealing BNE exists.[16]

5. Discussion

(a) This paper assumes that the signal a trader acquires is perfect. Under the assumption of risk neutrality this assumption is not crucial. If the signal is noisy, it only changes the expected potential speculative profit and the critical value of the information cost but not the qualitative implications.[17] The results also hold if the signals are noisy and only a few traders are risk neutral. Then the risk neutral traders (such as hedge funds) are willing to take large position and exert a speculative threat that "destroys" the efficient equilibrium. This paper implies that as the market size N increases, the risk neutral traders have the highest incentive to acquire information and exert a speculative threat although the information cost is high.

(b) This paper assumes that the value v of the asset is a binary random variable. If v is a continuous random variable and the information of the traders is represented as a partition, then the qualitative results hold. Other versions of modeling information give rise to technical difficulties. See footnote 17.

(c) This paper assumes a minimum heterogeneity between traders. Each buyer and each seller just has a trading need of one unit. Each seller possesses one unit of the asset.

[16] Since all traders receive the same signal, this model is not able to discuss the aggregation of diverse information. However, for the case c=0, Reny and Perry (2006) have established an important result.

[17] Consider the following two versions of a noisy signal. (a) With probability q the trader learns the true value and with probability 1−q he learns the wrong value. If the traders receive perfectly correlated signals, all results go through smoothly. (b) If trader i receives the signal $z_i=v+\psi_i$ where ψ_i is a zero mean random variable, then this gives rise to some technical difficulties in calculating expected payoff conditional on trade and therefore the equilibrium offer strategies.

Modifying the assumption by assuming different endowments and a more complicated individual demand and supply schedule is equivalent to assuming small and large liquidity traders, i.e. heterogeneous Δ_i. This would only change the critical information cost ranges.

(d) The analysis is based on the assumption that information acquisition is not observable. The next proposition shows that if information acquisition is observable by the other traders in the auction stage, then endogenous information acquisition has no adverse allocative consequences.

Proposition 7

Suppose N=1, information acquisition is observable and $k\in[-\Delta, \Delta]$. Any efficient k-sharing outcome is attainable as a perfect BNE irrespective of information cost.

The intuition is as follows.[18] Since information acquisition is observable, a trader can also condition his offer strategy on the fact whether the other party is better informed or not. Suppose ex ante the traders "agree" to trade at p=E[v], but the buyer acquires information. In the auction stage the seller knows that the buyer is informed. So the seller does not submit the offer s=E[v] anymore but a high offer. Since the buyer anticipates the lemons problem he himself creates by acquiring information, his best response is not to acquire information. No trader has an incentive to acquire more information than the counter party. However, if information acquisition is not observable, the traders cannot target their offer strategies appropriately and are concerned about the endogenous lemons problem.[19]

6. Conclusion

This paper analyses information acquisition in a financial market setting where all liquidity traders behave strategically and shows that an efficient and fully revealing equilibrium in the Reny and Perry (2006) type double auction market may fail to exist if information is endogenous and costly. For a given number of traders there exists a range for the information

[18] Formally, it is to show and easy to see that for $k\in[-\Delta,\Delta]$, $r\leq k$, and $t\geq k$, the strategies B=(0,b) with b=E[v]+k if $n_S=0$, and $b=v_L+r$ if $n_S=1$; and S=(0,s) with s=E[v]+k if $n_B=0$, and $s=v_H+t$ if $n_B=1$ constitute a perfect BNE with $EU^B=\Delta-k$ and $EU^S=\Delta+k$. Analogously, one can show that for the N>1, an efficient equilibrium exists in which no trader acquires information, and all traders chooses E[v] in the auction stage if no trader has acquired information.

[19] Proposition 7 is similar in flavor to Perry and Reny (2002) who show that two-stage bidding can achieve efficiency in common values auctions with (exogenous) private information. See also Mezzetti (2004).

cost such that only mixed strategy trading equilibria exist. The more traders and the more units a trader can trade, the larger this range with inefficient equilibria.

In a mixed strategy equilibrium the uninformed strategic liquidity traders exhibits endogenous noise trading in the sense that they propose offers that are prone to speculation and suffer a speculative loss with positive probability. Therefore, this paper provides a strategic foundation for both the noise trading assumption in Grossman and Stiglitz (1980) and their impossibility result of informationally efficient prices.

In addition, this paper shows that a centralized market may perform worse than bilateral trading and identifies a potential benefit of over-the-counter markets.[20] In decentralized trading, if the information cost is large, then no socially wasteful information is acquired and trade occurs in equilibrium. In centralized trading, if the information cost is large, then information is acquired and some liquidity traders do not trade in equilibrium.

However, this paper ignores the potential cost of finding a trader with the opposite trading need. An important function of a large market may be the bundling of liquidity. In addition, this paper analyzes a one-period trading game and highlights a potential inefficiency in a static large double auction. A second important function of a centralized market is the transmission of information through prices and sequential trading. If the traders are heterogeneous in liquidity and urgency, i.e. they have different trading needs or different discounting of their trading gains, then it is likely that the large liquidity traders with a continuous inflow of trading needs (such as the execution of incoming orders of costumers) are the ones who acquire information and determine prices. The small and more patient traders wait, observe the price and trade without costly information acquisition. Further research may provide additional insights on the working of different trading institutions when the information of the traders is endogenous.

Appendix

Proof of Proposition 2

The set of mixed strategies may be very large. A trader randomizes over two actions. First, he randomizes over the information acquisition decision. Then he randomizes over choosing offer prices. The proof contains two parts and proceeds as follows. Part A derives a mixed

[20] Common stocks are traded in centralized markets with high transparency (opened order book) and relatively high daily trading volume, while other financial instruments are traded in decentralized dealership markets with low transparency and relatively low daily trading volume. For example, the trading of corporate bonds, derivatives, and mortgage-back securities in over-the-counter markets are said to be opaque.

strategy BNE under the assumption that the traders can only choose three offer prices. Part B shows that the equilibrium properties in part A also hold without this restriction.

Part A

Assumption A

The traders can only choose offer prices from the set $b,s\in\{v_L,E[v],v_H\}$.[21]

Remark A1

From the discussion in Section 3, an informed buyer does not choose $b>v_L+\Delta$ at v_L while an informed seller does not choose $s<v_H-\Delta$ at v_H. An uninformed buyer (seller) does not choose $b>E[v]+\Delta$ ($s<E[v]-\Delta$). These actions are dominated choices.[22]

Step 1

(a) Given Assumption A and Remark A1, one can focus on the following mixed strategies. The buyer chooses $n_B=1$ with probability α_1, and $n_B=0$ with probability $1-\alpha_1$. An *uninformed* buyer chooses $b=v_L$ with probability α_2, and $b=E[v]$ with probability $1-\alpha_2$. An *informed* buyer chooses $b=v_L$ at $v=v_L$; and at $v=v_H$ he chooses $b=E[v]$ with probability α_3, and $b=v_H$ with probability $1-\alpha_3$.[23] The seller chooses $n_S=1$ with probability β_1, and $n_B=0$ with probability $1-\beta_1$. An *uninformed* seller chooses $s=E[v]$ with probability β_2; and $s=v_H$ with probability $1-\beta_2$. An *informed* seller chooses $s=v_H$ at $v=v_H$; and at $v=v_L$ he chooses $s=E[v]$ with probability β_3, and $s=v_L$ with probability $1-\beta_3$. (See Figure 1.)

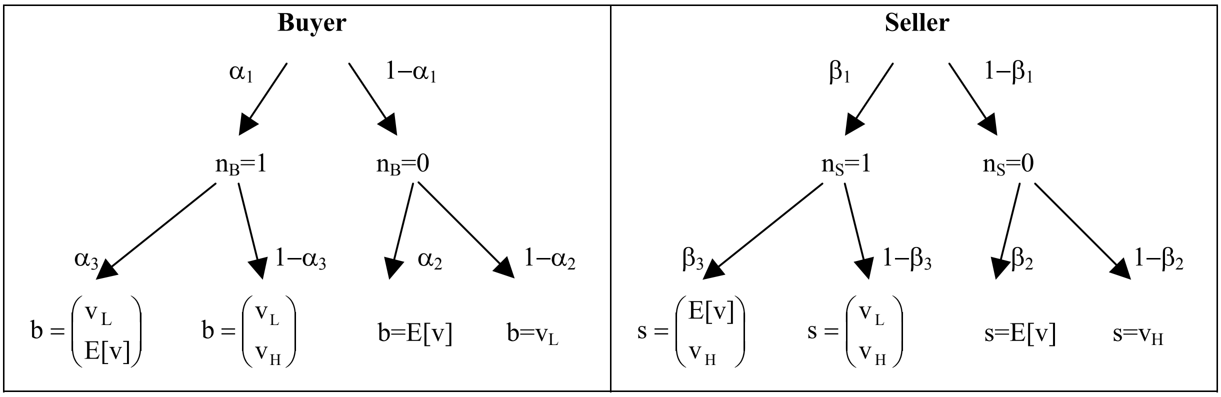
In other words, the buyer considers a randomization over the following pure strategies: He chooses the strategy $(0,v_L)$ with probability $\sigma_{B1}=(1-\alpha_1)(1-\alpha_2)$, the strategy $(0,E[v])$ with probability $\sigma_{B2}=(1-\alpha_1)\alpha_2$, $(1,v_L,E[v])$ with $\sigma_{B3}=\alpha_1\alpha_3$, and $(1,v_L,v_H)$ with $\sigma_{B4}=\alpha_1(1-\alpha_3)$. The seller considers a randomization over the following pure strategies: He chooses $(0,v_H)$, $(0,E[v])$, $(1,E[v],v_H)$ and $(1,v_L,v_H)$ with the probabilities $\sigma_{S1}=(1-\beta_1)(1-\beta_2)$, $\sigma_{S2}=(1-\beta_1)\beta_2$, $\sigma_{S3}=\beta_1\beta_3$, and $\sigma_{S4}=\beta_1(1-\beta_3)$, respectively.

[21] If Δ is small, then the discrete offer set is a good approximation.

[22] Suppose the buyer chooses $(0, v_H-\Delta)$. His expected payoff is negative although the seller is "honest" and "generous" in the sense that he chooses $(0, E[v]-\Delta)$ or $(1, v_L+\Delta,v_H-\Delta)$.

[23] The informed buyer may also choose $b=v_L$ in the state H. However, there is never trade since an informed seller does not choose $s<v_H$ and an uninformed seller does not choose $s<E[v]$.

Figure 1



(b) The expected payoffs of the buyer are given as follows.

$EU^B(0,v_L) \quad = \frac{1}{2}\beta_1(1-\beta_3)\Delta$

$EU^B(0,E[v]) \quad = \frac{1}{2}[\beta_1(1-\beta_3)(v_L+\Delta-\frac{1}{2}(E[v]+v_L))+\beta_1\beta_3(v_L+\Delta-E[v])+$

$(1-\beta_1)\beta_2(v_L+\Delta-E[v])] + \frac{1}{2}(1-\beta_1)\beta_2(v_H+\Delta-E[v])$

$= \frac{1}{2}\beta_1(1-\beta_3)(\Delta-\frac{1}{4}(v_H-v_L))+\frac{1}{2}\beta_1\beta_3(\Delta-\frac{1}{2}(v_H-v_L))+(1-\beta_1)\beta_2\Delta$

$EU^B(1,v_L, E[v]) \quad = \frac{1}{2}\beta_1(1-\beta_3)\Delta+\frac{1}{2}(1-\beta_1)\beta_2(v_H+\Delta-E[v])-c$

$= \frac{1}{2}\beta_1(1-\beta_3)\Delta+\frac{1}{2}(1-\beta_1)\beta_2(\Delta+\frac{1}{2}(v_H-v_L))-c$

$EU^B(1,v_L,v_H) \quad = \frac{1}{2}\beta_1(1-\beta_3)\Delta+\frac{1}{2}[\beta_1\Delta+(1-\beta_1)(1-\beta_2)\Delta+(1-\beta_1)\beta_2(v_H+\Delta-\frac{1}{2}(v_H+E[v])]-c$

$= \frac{1}{2}\beta_1(1-\beta_3)\Delta+\frac{1}{2}\beta_1\Delta+\frac{1}{2}(1-\beta_1)(1-\beta_2)\Delta+\frac{1}{2}(1-\beta_1)\beta_2(\Delta+\frac{1}{4}(v_H-v_L))-c$

To save on notation, the subsequent analysis is done in terms of σ_B and σ_S. The expected payoffs of the buyer are given as follows.

$EU^B(0,v_L) \quad = \frac{1}{2}\sigma_{S4}\Delta$

$EU^B(0,E[v]) \quad = \frac{1}{2}\sigma_{S4}(\Delta-\frac{1}{4}(v_H-v_L))+\frac{1}{2}\sigma_{S3}(\Delta-\frac{1}{2}(v_H-v_L))+\sigma_{S2}\Delta$

$EU^B(1,v_L,E[v]) \quad = \frac{1}{2}\sigma_{S4}\Delta+\frac{1}{2}\sigma_{S2}(\Delta+\frac{1}{2}(v_H-v_L))-c$

$EU^B(1,v_L,v_H) \quad = \frac{1}{2}\sigma_{S4}\Delta+\frac{1}{2}(\sigma_{S3}+\sigma_{S4})\Delta+\frac{1}{2}\sigma_{S1}\Delta+\frac{1}{2}\sigma_{S2}(\Delta+\frac{1}{4}(v_H-v_L))-c$

(c) The expected payoffs of the seller are given as follows.[24]

$EU^S(0,v_H) \quad = \frac{1}{2}\sigma_{B4}\Delta$

$EU^S(0,E[v]) \quad = \frac{1}{2}\sigma_{B4}(\Delta-\frac{1}{4}(v_H-v_L))+\frac{1}{2}\sigma_{B3}(\Delta-\frac{1}{2}(v_H-v_L))+\sigma_{B2}\Delta$

[24] Suppose the seller chooses $(1,E[v],v_H)$. His payoff is $\sigma_{B2}(E[v]-(v_L-\Delta))$ if the state is L, and $\sigma_{B4}\Delta$ if the state is H. Analogously for the other strategies.

$\mathrm{EU}^S(1, \mathrm{E}[v], v_H) \quad = \frac{1}{2}\sigma_{B4}\Delta + \frac{1}{2}\sigma_{B2}(\Delta + \frac{1}{2}(v_H - v_L)) - c$

$\mathrm{EU}^S(1, v_L, v_H) \quad = \frac{1}{2}\sigma_{B4}\Delta + \frac{1}{2}(\sigma_{B3} + \sigma_{B4})\Delta + \frac{1}{2}\sigma_{B1}\Delta + \frac{1}{2}\sigma_{B2}(\Delta + \frac{1}{4}(v_H - v_L)) - c$

<u>Step 2</u>

(a) This step analyses the best responses of the buyer.

(i) The strategy $(1,v_L,\mathrm{E}[v])$ weakly dominates $(1,v_L,v_H)$ if $\mathrm{EU}^B(1,v_L,\mathrm{E}[v]) \geq \mathrm{EU}^B(1,v_L,v_H)$

$\Leftrightarrow \quad \frac{1}{2}\sigma_{S2}(\Delta + \frac{1}{2}(v_H - v_L)) - c \geq \frac{1}{2}(\sigma_{S3} + \sigma_{S4})\Delta + \frac{1}{2}\sigma_{S1}\Delta + \frac{1}{2}\sigma_{S2}(\Delta + \frac{1}{4}(v_H - v_L)) - c$

$\Leftrightarrow \quad \frac{1}{8}\sigma_{S2}(v_H - v_L)) \geq \frac{1}{2}(\sigma_{S1} + \sigma_{S3} + \sigma_{S4})\Delta$

$\Leftrightarrow \quad \sigma_{S2}(v_H - v_L)) \geq 4(1 - \sigma_{S2})\Delta$

$$\Leftrightarrow \quad \sigma_{S2} \geq \frac{4\Delta}{(4\Delta + v_H - v_L)} \equiv I^{(v_L,\mathrm{E}[v])}_{(v_L,v_H)}$$

(ii) The strategy $(1,v_L, \mathrm{E}[v])$ weakly dominates the strategy $(0,v_L)$ if

$$\sigma_{S2} \geq \frac{4c}{(2\Delta + v_H - v_L)} \equiv I^{(v_L,\mathrm{E}[v])}_{v_L}$$

(iii) The strategy $(1,v_L, \mathrm{E}[v])$ weakly dominates the strategy $(0,\mathrm{E}[v])$ if

$$\sigma_{S2} \geq \frac{8c - (\sigma_{S3} + \sigma_{S4})(v_H - v_L) - \sigma_{S3}(v_H - v_L - 4\Delta)}{2(v_H - v_L - 2\Delta)} \equiv I^{(v_L,\mathrm{E}[v])}_{\mathrm{E}[v]}$$

(iv) The strategy $(0,\mathrm{E}[v])$ weakly dominates the strategy $(0,v_L)$ if

$$\sigma_{S2} \geq \frac{(2\sigma_{S3} + \sigma_{S4})(v_H - v_L)}{8\Delta} - \frac{1}{2}\sigma_{S3} \equiv I^{\mathrm{E}[v]}_{v_L}$$

(v) The strategy $(1,v_L, v_H)$ weakly dominates the strategy $(0,v_L)$ if

$$\sigma_{S2} \geq \frac{4(2c - \Delta)}{v_H - v_L} \equiv I^{(v_L,v_H)}_{v_L}$$

(vi) The strategy $(1,v_L, v_H)$ weakly dominates the strategy $(0,\mathrm{E}[v])$ if

$$\sigma_{S2} \geq \frac{8c - 4\sigma_{S1}\Delta + 2\sigma_{S3}(v_H - v_L)}{(v_H - v_L - 4\Delta)} + \sigma_{S4} \equiv I^{(v_L,v_H)}_{\mathrm{E}[v]}$$

(b) Analogously for the seller. E.g., if $\sigma_{B2} \geq I^{(v_L,\mathrm{E}[v])}_{(v_L,v_H)}$, then $\mathrm{EU}^S(1,\mathrm{E}[v],v_H) \geq \mathrm{EU}^S(1,v_L,v_H)$.

<u>Step 3</u>

<u>Claim:</u> There exists no mixed strategy equilibrium in which the buyer and the seller choose the strategy $(1,v_Lv_H)$ with positive probability.

<u>Proof:</u> For the buyer, $(1,v_L,v_H)$ and $(1,v_L,\mathrm{E}[v])$ are the two potential strategies with information acquisition for being a candidate in a mixed strategy equilibrium. The buyer

does not choose $(1,v_L,v_H)$ with positive probability if it is strictly dominated by $(1,v_L,E[v])$. Suppose that the strategy $(1,v_L,v_H)$ weakly dominates $(1,v_L,E[v])$, i.e. $\sigma_{S2} \le I^{(v_L,E[v])}_{(v_L,v_H)}$. It is easy to see that $I^{(v_L,E[v])}_{(v_L,v_H)} < I^{(v_L,v_H)}_{v_L}$. Consequently, if the strategy $(1,v_L,v_H)$ weakly dominates $(1,v_L,E[v])$, then $(1,v_L,v_H)$ is strictly dominated by the strategy $(0,v_L)$ because in this case $\sigma_{S2} < I^{(v_L,v_H)}_{v_L}$.[25] Therefore, if the seller randomizes such that the buyer is indifferent between $(1,v_L,v_H)$ and $(1,v_L,E[v])$; or $(1,v_L,v_H)$ dominates $(1,v_L,E[v])$, then the buyer chooses $\sigma_{B1}=1$, i.e. he does not acquire information. Analogously for the seller, if the strategy $(1,v_L,v_H)$ weakly dominates $(1,E[v],v_H)$, then $(1,v_L,v_H)$ is strictly dominated by the strategy $(0,v_H)$.

Consequently, in a mixed strategy equilibrium (where information must be acquired with positive probability), the strategy $(1,v_L,v_H)$ must be a strictly dominated strategy and one must have $\sigma_{B4}=\sigma_{S4}=0$ (or $\alpha_3=\beta_3=1$).[26]

Step 4

Claim: In a mixed strategy equilibrium the traders get zero expected payoff.

Proof: For $\sigma_{B4}=\sigma_{S4}=0$, $EU^B(0,v_L)=EU^S(0,v_H)=0$. In other words, if the buyer is indifferent between $(1,v_L,E[v])$ and $(0,v_L)$ or indifferent between $(0,E[v])$ and $(0,v_L)$, then his expected payoff is zero. In order to find a mixed strategy equilibrium in which the traders get positive expected payoffs, the following is required: For the buyer, he should be indifferent between $(1,v_L,E[v])$ and $(0,E[v])$; and $(0,E[v])$ should strictly dominate $(0,v_L)$, i.e. the buyer chooses $\sigma_{B1}=0$.[27] The buyer is indifferent between $(1,v_L,E[v])$ and $(0,E[v])$ if $\sigma_{S2} = I^{(v_L,E[v])}_{E[v]}$. For $\sigma_{S4}=0$,

$$\sigma_{S2} = \frac{8c - \sigma_{S3}(v_H - v_L) - \sigma_{S3}(v_H - v_L - 4\Delta)}{2(v_H - v_L - 2\Delta)}$$

[25] The intuition is that the buyer does not get enough compensation for the information cost if he is "honest", i.e. when trade occurs at $p=v_H$ in the state H. Therefore, not acquiring information is a best response. If the buyer is indifferent between $(1,v_L,E[v])$ and $(1,v_L,v_H)$, then $(0,v_L)$ strictly dominates both $(1,v_L,E[v])$ and $(1,v_L,v_H)$ since $\sigma_{S2} = I^{(v_L,E[v])}_{(v_L,v_H)}$ and $c>\Delta$ imply $\sigma_{S2} < I^{(v_L,E[v])}_{v_L}$. If the buyer is indifferent between $(1,v_L,E[v])$ and $(0,v_L)$, then $(1,v_L,v_H)$ is a strictly dominated strategy because $\sigma_{S2} = I^{(v_L,E[v])}_{v_L}$ implies $\sigma_{S2} > I^{(v_L,E[v])}_{(v_L,v_H)}$.

[26] Suppose the buyer just randomizes over $(0,v_L)$ and $(0,E[v])$. If the seller also does not acquire information, then he chooses $s=E[v]$ with probability 1. Given the seller's response, the buyer's best response is $(1,v_L,E[v])$. So there exists no mixed strategy equilibrium in which information is acquired with zero probability.

[27] For the seller, he should be indifferent between $(1,E[v],v_H)$ and $(0,E[v])$; and $(0,E[v])$ should strictly dominate $(0,v_H)$, i.e. he chooses $\sigma_{S1}=0$.

$$\Leftrightarrow \qquad \sigma_{S2} = \frac{4c}{(v_H - v_L - 2\Delta)} - \sigma_{S3}.$$

Since $c<\frac{1}{4}(v_H-v_L)-\frac{1}{2}\Delta$, this implies that $\frac{4c}{v_H-v_L-2\Delta}<1$. Therefore $\sigma_{S2}+\sigma_{S3}<1$, which means that there is some probability "left", i.e. σ_{S1} must be larger than zero. In order to make the buyer indifferent between $(1,v_L,E[v])$ and $(0,E[v])$, the seller must choose $(0,v_H)$ with positive probability.

On the other hand, if the seller chooses $(0,v_H)$ with positive probability he must be indifferent between $(0,E[v])$ and $(0,v_H)$. Since $EU^S(0,v_H)=0$, $EU^S(0,E[v])$ must be zero, too. Otherwise, the seller is not indifferent. Consequently, the expected payoff of the seller must be zero in a mixed strategy equilibrium.[28]

Step 5

Claim: In the unique (non-degenerated) mixed strategy equilibrium the buyer randomizes over $(0,v_L)$, $(0,E[v])$ and $(1,v_L,E[v])$ according to σ_B and the seller randomizes over $(0,v_H)$, $(0,E[v])$ and $(1,E[v],v_H)$ according to σ_S where[29]

$$\sigma_B = \sigma_S = \left(1 - \frac{4c}{v_H - v_L - 2\Delta}, \frac{4c}{v_H - v_L + 2\Delta}, \frac{16c\Delta}{(v_H - v_L)^2 - 4\Delta^2}\right).$$

Proof:

In order to make the buyer indifferent between $(1,v_L,E[v])$ and $(0,v_L)$, the seller chooses σ_{S2} such that $\sigma_{S2} = I_{v_L}^{(v_L,E[v])}$; and to make the buyer indifferent between $(1,v_L,E[v])$ and $(0,E[v])$, the seller chooses σ_{S2} and σ_{S3} such that $\sigma_{S2} = I_{E[v]}^{(v_L,E[v])}$.[30] So $I_{v_L}^{(v_L,E[v])} = I_{E[v]}^{(v_L,E[v])}$ implies

$$\frac{4c}{v_H - v_L + 2\Delta} = \frac{4c}{v_H - v_L - 2\Delta} - \sigma_{S3}$$

$$\Leftrightarrow \qquad \sigma_{S3} = \frac{16c\Delta}{(v_H - v_L + 2\Delta)(v_H - v_L - 2\Delta)} = \frac{16c\Delta}{(v_H - v_L)^2 - 4\Delta^2}.$$ [31]

[28] Analogously for the buyer, he must choose $(0,v_L)$ with positive probability in order to make the seller indifferent between $(1,E[v],v_H)$ and $(0,E[v])$, i.e. his expected payoff is zero in a mixed strategy equilibrium.

[29] Equivalently, the traders choose $\alpha_1 = \beta_1 = \frac{16c\Delta}{(v_H-v_L)^2-4\Delta^2}$, $\alpha_2 = \beta_2 = \frac{4c(v_H-v_L-2\Delta)}{(v_H-v_L+2\Delta)(v_H-v_L-2\Delta)-16c\Delta}$, and $\alpha_3 = \beta_3 = 1$. Note, $\sigma_{B3}=\alpha_1\alpha_3$ and $\sigma_{B2}=(1-\alpha_1)\alpha_2$.

[30] Alternatively, the buyer should be indifferent between $(0,E[v])$ and $(0,v_L)$. This requires $\frac{4c}{(2\Delta+v_H-v_L)} = \frac{2\sigma_{S3}(v_H-v_L)}{8\Delta} - \frac{1}{2}\sigma_{S3}$ and yields the same condition.

[31] Note, $\sigma_{S3}<1$, since $c\Delta<c^2<\frac{1}{4}(v_H-v_L-2\Delta)\frac{1}{4}(v_H-v_L-2\Delta)<\frac{1}{16}(v_H-v_L-2\Delta)(v_H-v_L+2\Delta)$.

In addition, the seller chooses

$$\sigma_{S1} = 1 - \sigma_{S2} - \sigma_{S3} = 1 - \frac{4c}{v_H - v_L - 2\Delta}.$$

Step 6

Claim: The outcome in a mixed strategy BNE has the following properties. (i) The probability of trade is $\frac{16c^2}{(v_H - v_L)^2 - 4\Delta^2}$. (ii) The only trading price is E[v] and not fully revealing.

Proof :

The buyer randomizes over $(0,v_L)$, (0, E[v]) and $(1,v_L, E[v])$.

The seller randomizes over $(0,v_H)$, (0, E[v]) and $(1, E[v], v_H)$.

(i) Trade occurs in the following events: (a) both the buyer and the seller actually choose (0,E[v]); (b) the buyer chooses (0,E[v]) and the seller chooses $(1,E[v],v_H)$ and the true state is L; and (c) the buyer chooses $(1,v_L,E[v])$, the seller chooses (0,E[v]) and the true state is H. The probability of trade is given as follows:

$$\text{prob(trade)} = \sigma_{B2}\sigma_{S2} + \tfrac{1}{2}\sigma_{B2}\sigma_{S3} + \tfrac{1}{2}\sigma_{B3}\sigma_{S2} = \sigma_{B2}\sigma_{S2} + \sigma_{B2}\sigma_{S3}$$

$$\Leftrightarrow \qquad \text{prob(trade)} = \frac{16c^2}{(v_H - v_L + 2\Delta)^2} + \frac{64c^2\Delta}{(v_H - v_L + 2\Delta)^2(v_H - v_L - 2\Delta)}$$

$$\Leftrightarrow \qquad \text{prob(trade)} = \frac{16c^2}{(v_H - v_L)^2 - 4\Delta^2}.$$

(ii) The buyer bids at most E[v] and the seller demands at least E[v]. Therefore, no trade occurs at the price v_L and v_H. Trade only occurs if at least one trader is uninformed. If the uninformed trader observes trade, he cannot distinguish whether he makes a fair deal and realizes Δ, or suffers a speculative loss. Although the uninformed trader updates his belief, he does not know the true state when observing p=E[v].

Remark A2

σ_{i3} is the equilibrium probability of information acquisition. (See also footnote 29.) It decreases in $v_H - v_L$ and increases in the information cost c which seems unintuitive. As the potential speculative profit increases and the cost of becoming informed decreases, one might expect that a trader has a higher incentive to acquire information. But in order to make the other trader indifferent between pure strategies, equilibrium randomization requires it.

Remark A3

If $c=\frac{1}{4}(v_H-v_L)-\frac{1}{2}\Delta$, then in the non-degenerated mixed strategy BNE the buyer randomizes over $(0,E[v])$ and $(1,v_L,E[v])$ according to σ_B and the seller randomizes over $(0,E[v])$ and $(1,E[v],v_H)$ according to σ_S where

$$\sigma_B=\sigma_S=\left(\frac{v_H-v_L-2\Delta}{v_H-v_L+2\Delta},\ \frac{4\Delta}{v_H-v_L+2\Delta}\right),$$

and trade occurs with probability σ_{i1}, and both traders have zero expected payoff.[32]

Remark A4

(a) For $c=\frac{1}{4}(v_H-v_L)-\frac{1}{2}\Delta$, as $(v_H-v_L)\rightarrow\infty$, then $c\rightarrow\infty$ and the probability that both traders choose $(0,E[v])$ converges to one and yet $EU^B=EU^S=0$.

(b) For $c<\frac{1}{4}(v_H-v_L)-\frac{1}{2}\Delta$, as $(v_H-v_L)\rightarrow\infty$, then the probability that the buyer chooses $(0,v_L)$ and the seller chooses $(0,v_H)$ converges to one.

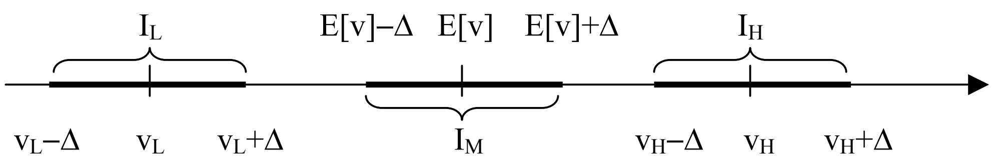
Part B

Now it is assumed that the traders can choose any real number as an offer price. The buyer (and seller) randomizes over pure offer strategies by choosing a distribution with positive density f (and g) over the real line. Assumption 1 and Remark A1 imply that in search for a candidate for an equilibrium offer randomization with positive payoffs, it suffices to focus on densities f (and g) with the following properties:

$$f(x)=\begin{cases} f^L & \text{for } x\in I_L \\ f^M & \text{for } x\in I_M \\ f^H & \text{for } x\in I_H \\ 0 & \text{otherwise}\end{cases}$$

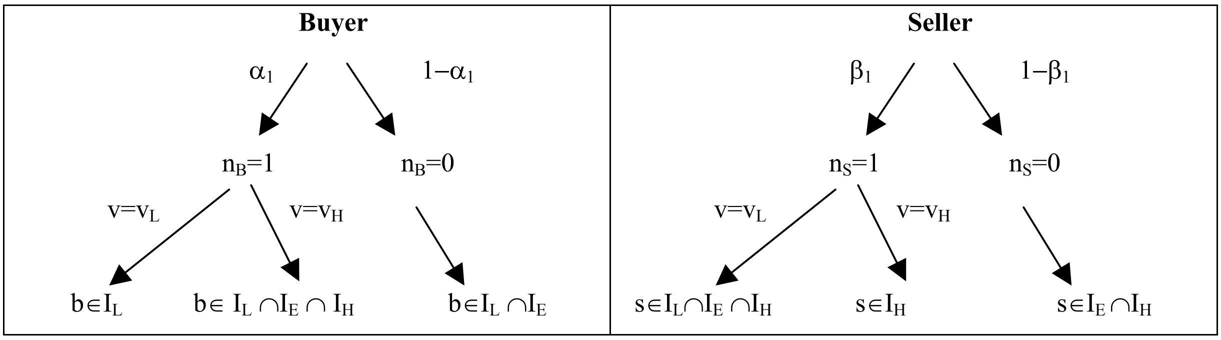
where $f^L,f^M,f^H\geq 0$. In particular, $f_0^H=0$ ($g_0^L=0$) if the buyer (seller) does not acquire information. If the buyer acquires information and $v=v_L$, then $f_{1,L}^M=f_{1,L}^H=0$. If the seller acquires information and $v=v_H$, then $g_{1,H}^L=g_{1,H}^M=0$. (See Figures 2 and 3).

Figure 2



[32] In the unique pure strategy trading BNE both traders choose $(0,E[v])$ and $EU^B=EU^S=\Delta$.

Figure 3



<u>Claim 1:</u> The equilibrium payoff is zero.

<u>Proof:</u>

Observation

$$\int_{b\in I_L} f^L(b)db = \text{prob}(b \in I_L) = \text{prob}(v_L - \Delta \le b \le v_L + \Delta) \text{ which is “equivalent” to } \alpha_3.$$

Analogously for the other cases. The extension of the strategy where an uninformed buyer plays $b=v_L$ with probability α_2, $b=E[v]$ with probability $1-\alpha_2$ and $b=v_H$ with probability zero is choosing f with $f_0^L, f_0^M>0$ and $f_0^H=0$, i.e. the uninformed buyer only chooses a positive density over the interval I_L and I_M.

So if the seller actually chooses $n_S=0$ and $s=v_H$ as the randomization outcome, then

$$EU^S(0,v_H)= \alpha_1\cdot\left[\tfrac{1}{2}\int_{\substack{b\in I_H\\ b>v_H}}\left(f_{1,L}^H(b)\cdot(\tfrac{1}{2}(b-v_H)+\Delta)\right)db + \tfrac{1}{2}\int_{\substack{b\in I_H\\ b>v_H}}\left(f_{1,H}^H(b)\cdot(\tfrac{1}{2}(b-v_H)+\Delta)\right)db\right]$$

$$+ (1-\alpha_1)\cdot \int_{\substack{b\in I_H\\ b>v_H}}\left(f_0^H(b)\cdot(\tfrac{1}{2}(b-v_H)+\Delta)\right)db.$$

Since $f_{1,L}^H = f_0^H = 0$,

$$EU^S(0,v_H)=\tfrac{1}{2}\alpha_1 \int_{\substack{b\in I_H\\ b>v_H}}\left(f_{1,H}^H(b)\cdot(\tfrac{1}{2}(b-v_H)+\Delta)\right)db,$$

which is analogous to $EU^S(0,v_H)=\frac{1}{2}\alpha_1(1-\alpha_3)\Delta= \frac{1}{2}\sigma_{B4}\Delta$.

From Step 3, a similar argument will show that in no mixed strategy equilibrium does an *informed* buyer choose a positive density over $[v_H-\Delta, v_H+\Delta]$, i.e. $f_{1,H}^H=0$. If he is indifferent between $(1,b_L',b_H')$ and $(1,b_L'',b_H'')$ where $b_L', b_L''\in I_L$, $b_H'\in I_M$ and $b_H''\in I_H$, then $(0,b)$ with $b\in I_L$ strictly dominates the both strategies. Therefore $EU^S(0,v_H)=0$.

From Step 4, in order to make the buyer indifferent between n_B=0 and n_B=1, the seller must choose n_S=0 and $\int_{s\in I_H} g_0^H(s)ds > 0$. Therefore, the equilibrium payoff of the seller is zero.

Claim 2: If trade occurs, then the price is $p\in(E[v]-\Delta,E[v]+\Delta)$ and not fully revealing.

Proof: Remark A1 shows that an informed buyer chooses $f_{1,L}^H = f_{1,H}^H = 0$ and an informed seller chooses $g_{1,L}^L = g_{1,H}^L = 0$. For the buyer, instead of randomizing over (1,v_L, E[v]), (0,v_L) and (0, E[v]), he randomizes over information acquisition and then chooses a density f^L and f^M. Instead of randomizing over (1, E[v], v_H), (0,v_H) and (0, E[v]), the seller randomizes over information acquisition and then chooses a density g^M and g^H. The previous steps show that trade can only occur if the buyer actually chooses $b\in I_M$ and the seller actually chooses $s\in I_M$ and $b\geq s$. In any case $p\in I_M$. **QED**

Proof of Proposition 3

Claim A: Suppose $r,t\in[-\Delta, \Delta]$. If $c\leq\frac{1}{2}(\Delta+\min\{r,-t\})$, then $B^*=S^*=(1, v_L+r, v_H+t)$ are best responses and $EU^B=\Delta-\frac{1}{2}(r+t)-c$ and $EU^S=\Delta+\frac{1}{2}(r+t)-c$.

Proof: Given S^*, if the buyer chooses B^*, then $EU^B=\frac{1}{2}[v_L+\Delta-(v_L+r)]+\frac{1}{2}[v_H+\Delta-(v_H+t)]-c=\Delta-\frac{1}{2}(r+t)-c\geq 0$ since $\Delta-\frac{1}{2}(r+t)\geq\frac{1}{2}(\Delta+\min\{r,-t\})\geq c$. If the buyer chooses (0,b) with $b=v_H+t$ then $EU^B=\frac{1}{2}[v_L+\Delta-\frac{1}{2}(v_L+r+v_H+t)]+\frac{1}{2}[v_H+\Delta-(v_H+t)]=\Delta-\frac{1}{4}(v_H-v_L)-\frac{1}{4}(r+3t)$. For $r=t=-\Delta$, the buyers' payoff is maximal and yet $EU^B=-\frac{1}{4}(v_H-v_L)+2\Delta<0$ (Assumption 1). If the buyer chooses (0,b) with $b=v_L+r$ then $EU^B=\frac{1}{2}(\Delta-r)$. This response is weakly dominated by response B^* since $c\leq\frac{1}{2}(\Delta-t)$. So B^* is a best response to S^*.[33]

Analogously for the seller, if he chooses (0,s) with $s=v_L+r$ then $EU^S=\Delta+\frac{1}{4}(3r+t)-\frac{1}{4}(v_H-v_L)<0$. If the seller chooses (0,s) with $s=v_H+t$ then $EU^S=\frac{1}{2}(\Delta+t)\leq\Delta+\frac{1}{2}(r+t)-c$ since $c\leq\frac{1}{2}(\Delta+r)$. So S^* is a best response to B^*. For $c\leq\min\{\frac{1}{2}(\Delta-t),\frac{1}{2}(\Delta+r)\}=\frac{1}{2}(\Delta+\min\{r,-t\})$, (B^*,S^*) constitutes a BNE equilibrium.

Claim B: If $c\leq\Delta-|k|$ for $k\in[-\Delta,\Delta]$, there exists a two-sided information acquisition and full trade equilibrium with the payoffs $EU^B=\Delta-k-c$ and $EU^S=\Delta+k-c$.

[33] It is easy to see that B^* also dominates the strategies (0,b) where $b<s_L$, $b\in(s_L,s_H)$ or $b>s_H$.

Proof: For a fixed $k\in[-\Delta,\Delta]$, define $k=\frac{1}{2}(r+t)$ then $r=2k-t$ and $t=2k-r$. From Claim A, the maximal allowable information cost c_k for the payoffs $EU^B=\Delta-\frac{1}{2}(r+t)-c$ and $EU^S=\Delta+\frac{1}{2}(r+t)-c$ being equilibrium payoffs is $c_k=0.5\cdot(\Delta+\min\{r,-t\})$. The maximum allowable cost is given by

$$\max_{\substack{r,t\in[-\Delta,\Delta]\\ \text{s.t. } r+t=2k}} 0.5\cdot(\Delta+\min\{r,-t\})=\max_{\substack{r,t\in[-\Delta,\Delta]\\ \text{s.t. } r+t=2k}} 0.5\cdot(\Delta+\min\{2k-t,\ -2k+r.\}). \qquad (*)$$

Since the $\min\{r,-t\}$ in (*) is non-decreasing in r and $-t$, set $r=\Delta$ and $t=-\Delta$. Then

$$\max_{\substack{r,t\in[-\Delta,\Delta]\\ \text{s.t. } r+t=2k}} 0.5\cdot(\Delta+\min\{2k-t,\ -2k+r.\})=0.5\cdot(\Delta+\min\{2k+\Delta,\ -2k+\Delta\})=0.5\cdot(2\Delta-2|k|).$$

For $c=c_k=\Delta-|k|$, *one* equilibrium strategies pair $B=S=(1,v_L+r,\ v_H+t)$ leading to the payoffs $EU^B=\Delta-k-c$ and $EU^S=\Delta+k-c$ is given as follows: If $k\in[-\Delta,0]$ then set $r=\Delta+2k$ and $t=-\Delta$. If $k\in[0,\Delta]$ then set $r=\Delta$ and $t=-\Delta+2k$.

Claim C: If $c\leq\Delta-|k|$, no full trade equilibrium exists in which (i) no trader acquires information or (ii) only one trader acquires information.

Proof: (i) Assumption 1 implies that $\Delta-|k|<\frac{1}{4}(v_H-v_L)-\frac{1}{2}\Delta$. So no efficient equilibrium exists. (ii) Suppose that only the buyer acquires information. He is willing to choose any (b_L,b_H) with $b_L=v_L+r$ and $b_H=v_H+t$ where $r,t\in[-\Delta,\ \Delta]$. If there is to be full trade the seller must choose $(0,s)$ with $s=v_L+r$. For any $r,t\in[-\Delta,\ \Delta]$, $EU^S=\Delta+\frac{1}{4}(3r+t)-\frac{1}{4}(v_H-v_L)<0$. Analogously for $n_B=0$ and $n_S=1$. So no full trade occurs if only one trader acquires information. **QED**

Proof Proposition 5 (ii)

Case (a): $c<\Delta$.

Since $c<\Delta$, all informed trader can cover their information cost by trading just one unit. Suppose the price is fully revealing and all traders realize their trading gains. (Otherwise some traders have a profitable deviation.) Then the only candidate price for an equilibrium price is $p=v$. If $p<v$ ($p>v$), then both an informed and uninformed buyer (seller) has a profitable deviation by overbidding (underbidding) the other traders and buy (sell) m units instead of one unit.

Suppose k buyers and r sellers acquire information and the traders choose the following strategies: $B_i=(1,(v_L,v_H),(u^B_{iL},u^B_{iH}))$, $S_j=(1,(v_L,v_H),(u^S_{jL},u^S_{jH}))$, $B_m=(0,v_H,u^B_m)$, $S_n=(0,\ v_L,u^S_n)$ where $i=1,..,k$; $j=1,..,r$; $m=k+1,..,N$ and $n=r+1,..,N$ and trade occurs at $p=v$.

(i) Suppose k=r=1. Consider the following strategies: $B_1=S_1=(1,(v_L,v_H),(1,1))$, $B_i=(0,v_H,1)$, $S_i=(0,v_L,1)$. The market clearing price is $p=v_L$ in state L and $p=v_H$ in state H and each trader trades one unit. The following arguments show that these strategies do not constitute a BNE. A profitable deviation of Buyer 1 is $B'=(1,(v_L,v_L+\varepsilon),(1,m))$. Suppose $v=v_H$. If $p=v_H$ is the price, then (N−1) units are traded. The order of Buyer 1 is not executed. If $p=v_L$ is the price, then also (N−1) units are traded. Buyer 1 receives m units. (Seller 1 does not trade). So there is equal probability that trade is executed at one of these prices. It is easy to see that $EU_1^B(B_1')>EU_1^B(B_1)$. Analogously, the informed seller has the same incentive tp speculate.[34]

(ii) So for any given number (k,r)-informed traders, if an informed trader can move the price (with positive probability), then the informed trader speculates. If the price is fully revealing, the uninformed traders on the other side of the market know that they suffer a speculative loss. This cannot be an equilibrium outcome.

(iii) Suppose that the number (k,r)-informed traders is such that an informed trader cannot influence the price. If the price is fully revealing, then e.g. an informed buyer deviates to $B_i=((0,v_H),1)$. He realizes his trading needs without paying the information cost.

Consequently, there exists no pure strategy trading BNE as well as no BNE where the price is fully revealing. If trade occurs, then the price $p\in(E[v]-\Delta,E[v]+\Delta)$.

Case (b): $c>\Delta$.

Suppose the price is fully revealing. In a pure strategy BNE with trade, at least one buyer and one seller acquires information. Since $c>\Delta$, this pair of informed traders cannot jointly cover their information cost by trading one unit each. At least one informed trader must trade more than one unit of the asset. This means that some uninformed traders do not realize their trading needs. Suppose the offer profiles (B,S) yields a fully revealing price $p=v\pm\Delta$. An argument similar to the one given in the proof of Proposition 4 will show that an uninformed trader who does trade has not played a best response. Given the profile (B,S), an "unsatisfied" buyer (seller) would choose a sufficiently high (low) offer price so as to get one unit for sure without being concern about the lemons problem since the price is fully revealing. So no pure strategy BNE with fully revealing prices exists. Case (a) shows that the price is not fully revealing in a mixed strategy BNE.

[34] The same arguments apply if the traders choose any offer $v\pm\Delta$.

Proof Proposition 5 (i)

Suppose the equilibrium is efficient. If a pure strategy BNE exists, then the price is fully revealing since there is no uncertainty about the offer of all traders in equilibrium. (In a pure strategy BNE an uninformed trader can infer the information of other traders from observing the price.) The proof of part 5 (ii) implies that the equilibrium price is no fully revealing. Therefore, no pure strategy trading BNE exists. In a mixed strategy BNE some traders do not trade. Denote the fraction λ of uninformed liquidity traders who do not trade. Given m, if λ is bounded away from zero as N converges to infinity, then a BNE is neither efficient nor asymptotically efficient. The following arguments show that this is the case.

In a mixed strategy equilibrium the expected payoff of an informed and an uninformed trader must be the same. Otherwise either an informed or uninformed trader has a profitable deviation. (In the two trader case, their equilibrium payoffs are zero.)

An uninformed trader only trades if he proposes an offer $E[v]\pm\Delta$. As the fraction $(1-\lambda)$ of uninformed traders with this offer increases, the probability that an informed trader makes speculative profits increases. Consequently, their expected payoff increases while the these uninformed traders decreases. In a mixed strategy BNE λ is strictly bounded away from zero.[35] **QED**

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[35] If c is large, then an informed trader must trade more units and make more speculative profits in order to cover the information cost. Therefore, even more orders of uninformed traders are not executed.

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