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A Note on Case-Based Optimization with a Non-Degenerate Similarity Function

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I am indebted to my advisor Juergen Eichberger for his helpful guidance and to Alexander Zimper for his helpful suggestions and illuminating discussions. I would like to thank Hans Haller, Clemens Puppe, Itzhak Gilboa, Hans Gersbach, Klaus Ritzberger, as well as the articipants of the RUD conference in Evanston, of the FUR XI conference in Paris and of the PhD seminar at the University of Heidelberg for helpful discussions and comments. Financial support from the DFG is greatfully acknowledged.

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The paper applies the ”realistic-ambitious” rule for adaptation of the aspiration level suggested by Gilboa and Schmeidler (1996) to a situation in which the similarity between the available acts is represented by a non-degenerate function. The paper shows that the optimality result obtained by Gilboa and Schmeidler (1996) in general fails. With a concave similarity function, the best corner act is chosen in the limit. Introducing convex regions into the similarity function improves the limit choice. A sufficiently fine similarity function allows to approximate optimal behavior with an arbitrary degree of precision.

Keywords: Case-Based Decision Theory, similarity, optimal behavior.

JEL classification: D81, D83

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1 Introduction

The case-based decision theory was proposed by Gilboa and Schmeidler (1995) as an alternative theory for decision making under uncertainty. It models decisions in situations of structural ignorance, in which neither states of the world, nor their probabilities can be derived from the description of the problem. A decision-maker can, therefore, only learn from experience, by evaluating an act based on its past performance in similar circumstances.

This behavior deviates significantly from the one of a Bayesian expected utility maximizer. Nevertheless, Gilboa and Schmeidler (1996) show that a case-based decision-maker learns to choose the optimal (expected utility maximizing) act if the same problem is repeated an infinite number of times. This result relies on a rule for adapting the aspiration level that combines ”realism”, i.e. updating the aspiration level towards the highest average payoff achieved, with ”ambitiousness”, i.e. updating the aspiration level upwards on an infinite but sparse subset of periods.

However, the result Gilboa and Schmeidler (1996) depends on the assumption of a specific similarity function: two acts are similar if and only if they are identical.

In this note, I explore whether case-based decisions lead to expected utility maximization in the limit if more general similarity functions are considered. I assume that the set of acts is the one-dimensional simplex and define the similarity function as decreasing in the Euclidean distance. Following results obtain:

1. Similarity functions which are convex over some range lead a case-based decision-maker to behave as if she were Bayesian;
2. For concave similarity functions, a case-based decision-maker either chooses the better of the two corner acts 0 and 1 or switches constantly between these two acts. She may, therefore, fail to behave as a Bayesian.

2 The Model

I use the model of Gilboa and Schmeidler (1996). A decision-maker faces an identical decision problem p in each period $t = 1, 2....$ $A \equiv [0;1]$ denotes the set of available acts. The utility resulting from the choice of $a \in A$ is an i.i.d. random variable $\mathfrak{U}_a$ with a continuous distribution function $(\Pi_a)_{a \in A}$. The distributions $(\Pi_a)_{a \in A}$ have finite expectations μ_a, finite variance σ_a and bounded and convex supports Δ_a.

The decision-maker's perception of similarity is described by a function $s : A \times A \to [0;1]$:

$$\begin{aligned} s(a;a) &= 1 \\ s(a;a') &= s(a';a) \\ s(0;1) &= 0. \end{aligned}$$

s depends only on the distance between a and a'.

The memory of the decision-maker is represented by a set of cases. A case is a triple of a problem encountered, an act chosen and a utility realization achieved. Since the problem is identical in each period of time, a case is characterized by an act and a utility realization. As in Gilboa and Schmeidler (1996), the memory M_t contains only cases actually encountered by the decision-maker until period t:

$$M_t = ((a_\tau; u_\tau))_{\tau=1,2...t}\,.$$

The aspiration level of the decision-maker in period t is $\bar{u}_t$.

The case-based decision-rule prescribes choosing the act with maximal cumulative utility in each period of time. The cumulative utility of an act a at time t is given by:

$$U_t(a) = \sum_{\tau=1}^{t} s(a;a_\tau)(u_\tau - \bar{u}_t)\,.$$

The set of all possible decisions paths that can be observed can be written as

$$S_0 = \left\{ \omega = (a_t; u_t; \bar{u}_t)_{t=1,2...} \mid a_t \in A,\, u_t \in \Delta,\, \bar{u}_t \in \mathbb{R} \right\},$$

where $\Delta = \cup_{a \in A} \Delta_a$ denotes the set of possible utility realizations. Let S_1 be the set of those paths on which the decision-maker chooses $\arg\max_{a \in A} U_t(a)$ in each period:

$$S_1 = \left\{ \omega \in S_0 \mid a_t = \arg\max_{a \in A} U_t(a) \text{ for all } t = 1, 2... \right\}.$$

As well as a_t, u_t and $\bar{u}_t$ all variables introduced below depend on the path ω. I neglect this dependence in the notation for simplicity of exposition.

$C_t(a)$ denotes the set of periods preceding t in which a has been chosen:

$$C_t(a) = \{\tau < t \mid a_\tau = a\}$$

Let

$$X_t(a) = \frac{\sum_{\tau \in C_t(a)} u_\tau}{|C_t(a)|}$$

denote the average utility obtained by choosing a until period t if $|C_t(a)| > 0$. $X_t =: \max_{a \in A} X_t(a)$ stays for the maximal achieved average utility until time t.

The two adaptation rules proposed by Gilboa and Schmeidler (1996) are:

$$\begin{array}{l} \bar{u}_1 = \bar{u} \\ \bar{u}_t = \beta \bar{u}_{t-1} + (1-\beta) X_t \text{ for } t \geq 2 \end{array} \tag{1}$$

and

$$\begin{array}{l} \bar{u}_1 = \bar{u} \\ \bar{u}_t = \beta \bar{u}_{t-1} + (1-\beta) X_t \text{ for } t \geq 2, t \notin N \\ \bar{u}_t = X_t + h \text{ for } t \geq 2, t \in N, \end{array} \tag{2}$$

where $N \subset \mathbb{N}$ is a sparse set, $\beta \in (0;1)$ describes the speed of updating of the aspiration level and $h > 0$ is a constant by which the aspiration level is increased in a period $t \in N$.

Finally, denote by S and S' the set of paths, on which the case-based rule is applied in combination with (1) and (2), respectively:

$$S = \left\{ \omega \in S_1 \mid \begin{array}{l} \bar{u}_1 = \bar{u}_1 \\ \bar{u}_t = \beta \bar{u}_{t-1} + (1-\beta) X_t \text{ for } t \geq 2 \end{array} \right\}$$

$$S' = \left\{ \omega \in S_1 \mid \begin{array}{l} \bar{u}_1 = \bar{u}_1 \\ \bar{u}_t = \beta \bar{u}_{t-1} + (1-\beta) X_t \text{ for } t \geq 2, t \notin N \\ \bar{u}_t = X_t + h \text{ for } t \geq 2, t \in N, \end{array} \right\}.$$

Let P and P' be probability measures on S and on S', respectively which are consistent with $(\Pi_a)_{a \in A}$, as in Gilboa and Schmeidler (1996, p.11).

Denote by

$$\pi(a) = \lim_{t \to \infty} \frac{|C_t(a)|}{t}$$

the frequency with which a is chosen, if the limit on the right hand side exists. Usually, this frequency will be path-dependent.

Optimal behavior in the limit means that

$$\pi\left(\arg\max_{a \in A} \mu_a\right) = 1$$

almost surely holds. For A finite and a similarity function:

$$\begin{aligned} s(a;a') &= 1, \text{ if } a = a' \\ s(a;a') &= 0, \text{ else}, \end{aligned} \tag{3}$$

Gilboa and Schmeidler (1996) show that (2) implies optimal behavior in the above sense. For

(1), Gilboa and Schmeidler (1996) demonstrate that for each $\varepsilon > 0$, there is a $\bar{u}_0$, so that for all $\bar{u} \geq \bar{u}_0$, expected utility maximization obtains with probability of at least $(1 - \varepsilon)$.

The following two sections analyze the implications of a non-degenerate similarity function on learning.

3 Learning with a Concave Similarity Function

Assumption 1

$$s\left(a; a'\right) = f\left(\|a - a'\|\right),$$

with $f' < 0$ *and* $f'' < 0$*, as illustrated in figure* 1.

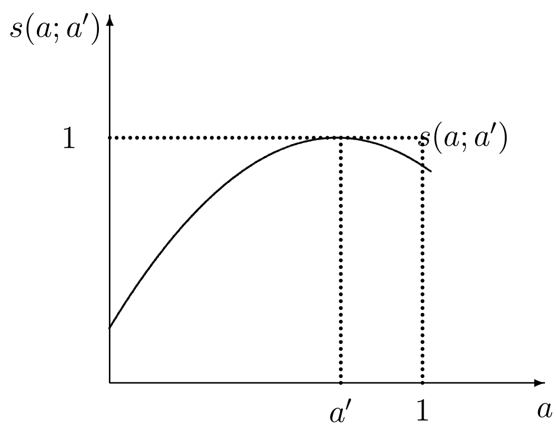

Figure 1

The concavity of s implies that the greater the distance of two acts a' and a'' from the reference act a, the more the decision-maker distinguishes between a' and a'' with respect to their similarity to a.

Let $a_1 = \bar{a}$ and let $u_1(\bar{a})$ denote the utility realization of portfolio $\bar{a}$ in period 1.

Proposition 1 *Let assumption 1 hold. Define $\tilde{S}$ as*

$$\tilde{S} = \{\omega \in S \mid u_1(\bar{a}) \leq \max\{\mu_0; \mu_1\}\}.$$

For each $\varepsilon > 0$ there exists a $\bar{u}_0$ such that for any $\bar{u}_1 > \bar{u}_0$:

$$P\left\{\omega \in S \mid \exists \pi \left(\arg\max_{a \in \{0;1\}} \mu_a\right) = 1\right\} \geq (1-\varepsilon) P\left(\tilde{S}\right)$$

$$P\left\{\omega \in S \mid \begin{array}{ll} \text{for each } a \in A & \exists \pi(a) \text{ such that} \\ \frac{\pi(0)}{\pi(1)} = \frac{\mu_1 - u_1(\bar{a})}{\mu_0 - u_1(\bar{a})} & \text{and} \\ \pi(a) = 0 & \text{for } a \notin \{0;1\} \end{array}\right\} = 1 - P\left(\tilde{S}\right),$$

holds.

Hence, for concave similarity functions optimal behavior fails to emerge under rule (1). Only the corner acts 1 and 0 are chosen infinitely often. On $\tilde{S}$, the better of these two acts is satisfactory in the limit and is chosen with frequency 1 with arbitrarily high probability. On $S \backslash \tilde{S}$, the limit aspiration level exceeds the mean utilities of 1 and 0. Both acts are therefore chosen with positive frequencies.

Proposition 2 *Let assumption 1 hold. Assume that either*

$$\bar{u} > \max_{u \in \Delta_{\bar{a}}} u$$

or $1 \in N$. Define $\tilde{S}'$ as

$$\tilde{S}' = \{\omega \in S' \mid u_1(a) \leq \max\{\mu_0; \mu_1\}\}.$$

Then:

$$P\left\{\omega \in S' \mid \exists \pi \left(\arg\max_{a \in \{0;1\}} \mu_a\right) = 1\right\} = P\left(\tilde{S}'\right)$$

$$P\left\{\omega \in S' \mid \begin{array}{ll} \text{for each } a \in A & \exists \pi(a) \text{ such that} \\ \frac{\pi(0)}{\pi(1)} = \frac{\mu_1 - u_1(\bar{a})}{\mu_0 - u_1(\bar{a})} & \text{and} \\ \pi(a) = 0 & \text{for } a \notin \{0;1\} \end{array}\right\} = 1 - P\left(\tilde{S}'\right),$$

holds.

Two effects prevent efficient learning. First, the strict monotonicity of the similarity function and the initially high aspiration level imply that $\bar{a}$ is abandoned in the second period. Since s is concave, only corner acts are chosen after $t = 2$.

Second, although $\bar{a}$ is never chosen again, its initial realization influences the evolution of the aspiration level. Especially, if $u_1(\bar{a}) > \max\{\mu_0; \mu_1\}$,

$$\lim_{t \to \infty} \bar{u}_t = u_1(\bar{a}) > \max\{\mu_0; \mu_1\}$$

and both $a = 0$ and $a = 1$ seem unsatisfactory in the limit.

4 Introducing Convexities into the Similarity Function

The proof of proposition 1 heavily relies on the concavity of the similarity function. I, therefore, explore how results change if the similarity function is convex over some range of values.

Assumption 2 *Let*

$$s(a;a') = f(\|a-a'\|),$$

with $f' < 0$, $f'' < 0$ *for* $\|a-a'\| \leq \frac{1}{l}$ *and some* $l > 1$. *Let*

$$f(\|a-a'\|) = 0,$$

for $\|a-a'\| \geq \frac{1}{l}$.

A convex similarity function implies that the greater the distance of two acts a' and a'' from the referential act a, the less is the decision-maker able to distinguish between a' and a'' with respect to their similarity to a. When $l \to \infty$, s approaches the similarity function (3) considered by Gilboa and Schmeidler (1996).

Assumption 3 *Let* $a_1 = 0$ *and let*

$$a_t = \arg \min_{a \in \arg\max U_t(a)} \left\{ \left| \arg \max_{a \in [0;1]} \{U_t(a)\} - a_{t-1} \right| \right\}.$$

Assumption 3 says that when indifferent among several acts, the decision-maker chooses the act closest to the act chosen last.

Figure 2 illustrates assumptions 2 and 3.

The set of paths consistent with (2) and assumptions 2 and 3 is denoted by:

$$S'' = \left\{ \omega \in S' \mid \begin{array}{l} a_1 = 0 \\ a_t = \arg\min_{a\in\arg\max U_t(a)} \left\{ \left|\arg\max_{a\in[0;1]} \{U_t(a)\} - a_{t-1}\right| \right\} \end{array} \right\}.$$

Proposition 3 *For all* $\bar{u}_1 \in \mathbb{R}$ *and all* $\beta \in (0;1)$,

$$P\left\{ \omega \in S'' \mid \exists \pi \left(\arg\max_a \left\{ \mu_a \mid a \in \left\{ 0; \frac{1}{l}; \frac{2}{l} ... \frac{l-1}{l}; 1 \right\} \right\} \right) = 1 \right\} = 1.$$

Introducing convexities into the similarity function obviously improves the limit choice. Note, however that although

$$\lim_{l\to\infty} \left(\arg\max_a \left\{ \mu_a \mid a \in \left\{ 0; \frac{1}{l}; \frac{2}{l} ... \frac{l-1}{l}; 1 \right\} \right\} \right) = \arg \max_{a\in[0;1]} \mu_a,$$

optimal learning cannot obtain because A is uncountable. However, for sufficiently large l, the investor's limit choice approximates expected utility maximization with an arbitrary degree of accuracy.

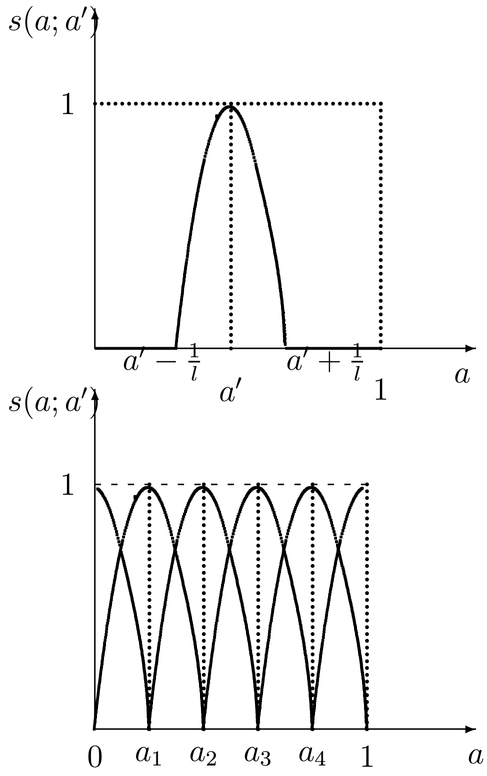

Figure 2

5 Conclusion

It is beyond the scope of this note to explore whether similarity is better represented by a convex or concave function. Intuitively, it seems that similarity perceptions are more vague for distant objects, hence that convexity is a more appropriate assumption. If convexity of the similarity function is indeed the more relevant case, then my findings would clearly support Gilboa and Schmeidler's claim that case-based decision-makers behave optimally in the limit.

Appendix

Proof of Proposition 1

Denote

$$V_t(a) =: \sum_{\tau \in C_t(a)} \left[u_\tau(a) - \bar{u}_t\right],$$

with

$$\sum_{\tau\in C_t(a)} [u_\tau(a) - \bar{u}_t] = 0, \text{ if } C_t(a) = \emptyset.$$

The proof proceeds in three steps. First, it is shown that $a_2 \in \{0;1\}$. Second, it is demonstrated that $a_t \in \{0;1\}$ for all $t \geq 2$. In step 3, applying theorem 1 of Gilboa and Schmeidler (1996) shows that optimal behavior obtains on $\tilde{S}$ and theorem 1 of Gilboa and Pazgal (2001) allows to derive the result on $S\backslash\tilde{S}$.

Step 1

In $t = 2$,

$$U_2(\bar{a}) = V_2(\bar{a}) = u_1(\bar{a}) - \beta\bar{u}_1 - (1-\beta)u_1(\bar{a}) = \beta[u_1(\bar{a}) - \bar{u}_1] < 0,$$

whereas:

$$U_2(a) = V_2(\bar{a})\, s(a;\bar{a}) < 0.$$

Since $s(a;\bar{a})$ is strictly decreasing,

$$\arg\max_{a\in[0;1]} U_2(a) \in \{0;1\}.$$

Remark 1 *Let $C_\tau(a) \geq 1$. Let $\bar{u}_\tau > X_\tau(a)$ and $a_\tau \neq a$.*

$$\bar{u}_{\tau+1} = \beta X_{\tau+1} + (1-\beta)\bar{u}_\tau \geq \beta X_\tau(a) + (1-\beta)\bar{u}_\tau$$

and $\bar{u}_\tau > X_\tau(a)$ imply

$$\begin{aligned} \bar{u}_{\tau+1} &> X_\tau(a)\text{, or} \\ V_t(a) &< 0 \end{aligned}$$

for each $t \geq \tau$ such that $a_k \neq \bar{a}$ for all $\tau \leq k < t$.

Step 2

Assume w.l.g. that $a_2 = 0$, i.e.

$$\min_{a\in[0;1]} s(a;\bar{a}) = 0$$

If $a_\tau = 0$ for all $t > \tau \geq 2$,

$$U_t(a) = s(a;\bar{a})\, V_t(\bar{a}) + s(a;0)\, V_t(0)$$

holds. If $V_t(0) \geq 0$, $a_t = 0$, since

$$\begin{aligned} U_t(a) &= s(a;\bar{a})\, V_t(\bar{a}) + s(a;0)\, V_t(0) \leq \\ &\leq s(0;\bar{a})\, V_t(\bar{a}) + V_t(0) = U_t(0). \end{aligned}$$

If $V_t(0) < 0$,

$$U_t(a) = s(a;\bar{a})\, V_t(\bar{a}) + s(a;0)\, V_t(0)$$

is convex, since $V_t(\bar{a}) < 0$, $V_t(0) < 0$ and s is concave. Therefore,

$$\arg\max_{a \in [0;1]} U_t(a) \in \{0; 1\}.$$

Let $a_{\bar{t}'} = 1$. $\bar{t}'$ is a.s. finite. If $a_\tau = 1$ for all $t > \tau \geq \bar{t}'$,

$$U_t(a) = s(a; \bar{a}) V_t(\bar{a}) + s(a; 0) V_t(0) + s(a; 1) V_t(1).$$

$V_{\bar{t}'}(0) < 0$ implies $V_t(0) < 0$, see remark 1.

If $V_t(1) \leq 0$ holds, $U_t(a)$ is convex and $a_t \in \{0; 1\}$.

Let $V_t(1) > 0$. Since

$$V_t(1) > 0 > \max\{V_t(\bar{a}); V_t(0)\},$$

$X_t = X_t(1)$. Let

$$\tilde{t} = \max\{\bar{t}'; \tau < t \mid V_\tau(1) \leq 0, a_\tau = 1, V_{\tau+1}(1) > 0\}. \tag{4}$$

Lemma 4 $\bar{u}_t > \bar{u}_{\tilde{t}}$.

Proof of lemma 4:

Since $V_{\tilde{t}}(1) \leq 0$, $X_{\tilde{t}}(1) - \bar{u}_{\tilde{t}} \leq 0$ holds. $V_{\tilde{t}+1}(1) > 0$ implies

$$X_{\tilde{t}+1}(1) - \bar{u}_{\tilde{t}+1} > 0 > \max\{X_{\tilde{t}+1}(0) - \bar{u}_{\tilde{t}+1}; X_{\tilde{t}+1}(\bar{a}) - \bar{u}_{\tilde{t}+1}\}.$$

Hence, $X_{\tilde{t}+1} = X_{\tilde{t}+1}(1)$ and

$$\bar{u}_{\tilde{t}+1} = \beta\bar{u}_{\tilde{t}} + (1-\beta) X_{\tilde{t}+1}(1).$$

Since $X_{\tilde{t}+1}(1) - \bar{u}_{\tilde{t}+1} > 0$,

$$\bar{u}_{\tilde{t}+1} > \bar{u}_{\tilde{t}}$$

follows.

At $\tilde{t}+2$, $V_{\tilde{t}+2}(1) > 0$ holds, hence $X_{\tilde{t}+2} = X_{\tilde{t}+2}(1) > \bar{u}_{\tilde{t}+2}$ and

$$\bar{u}_{\tilde{t}+2} = \beta\bar{u}_{\tilde{t}+1} + (1-\beta) X_{\tilde{t}+2}(1).$$

Therefore,

$$\bar{u}_{\tilde{t}+2} > \bar{u}_{\tilde{t}+1}.$$

Hence, by induction, $\bar{u}_t > \bar{u}_{\tilde{t}}$.■

At t,

$$U_t(1) = s(1; \bar{a}) V_t(\bar{a}) + V_t(1).$$

Hence, $a_t = 1$ if

$$s(1;\bar{a})\,V_t(\bar{a}) + V_t(1) \geq s(a;\bar{a})\,V_t(\bar{a}) + s(a;0)\,V_t(0) + s(a;1)\,V_t(1) \tag{5}$$

for all $a \in [0;1]$. Rewrite (5) as

$$V_t(1)(1 - s(a;1)) - s(a;0)\,V_t(0) + [s(1;\bar{a}) - s(a;\bar{a})]\,V_t(\bar{a}) \geq 0$$

$$V_t(1)(1 - s(a;1)) - s(a;0)(\bar{t}' - 2)[X_{\tilde{t}}(0) - \bar{u}_{\tilde{t}}] + [s(1;\bar{a}) - s(a;\bar{a})][u_1(\bar{a}) - \bar{u}_{\tilde{t}}]$$

$$+ [\bar{u}_t - \bar{u}_{\tilde{t}}][s(a;0)(\bar{t}' - 2) - s(1;\bar{a}) + s(a;\bar{a})] \geq 0.$$

Since $s(\cdot;\cdot)$ is concave,

$$\arg\min_{a\in[0;1]} s(a;0)(\bar{t}' - 2) + s(a;\bar{a}) \in \{0;1\}.$$
$$s(1;0)(\bar{t}' - 2) + s(1;\bar{a}) = s(1;\bar{a})$$

and

$$s(0;0)(\bar{t}' - 2) + s(0;\bar{a}) = (\bar{t}' - 2) + s(0;\bar{a}) > 1 + s(0;\bar{a}) > s(1;\bar{a}).$$

It follows that:

$$\min_{a\in[0;1]} [\bar{u}_t - \bar{u}_{\tilde{t}}][s(a;0)(\bar{t}' - 2) - s(1;\bar{a}) + s(a;\bar{a})] = 0.$$

Therefore,

$$\begin{aligned} U_t(1) - U_t(a) &\geq V_t(1)(1 - s(1;a)) - s(a;0)\,V_{\tilde{t}}(0) + [s(1;\bar{a}) - s(a;\bar{a})]\,V_{\tilde{t}}(\bar{a}) \\ &> V_{\tilde{t}}(1)(1 - s(a;1)) - s(a;0)\,V_{\tilde{t}}(0) + [s(1;\bar{a}) - s(a;\bar{a})]\,V_{\tilde{t}}(\bar{a}) \\ &= U_{\tilde{t}}(1) - U_{\tilde{t}}(a) \geq 0 \end{aligned}$$

for each $a \in [0;1]$, since $V_{\tilde{t}}(1) \leq 0$ and $a_{\tilde{t}} = 1$. Hence, $a_t = 1$.

Analogously, if $a_{t-1} = 0$ and $V_t(0) < 0$, $a_t \in \{0;1\}$. For $V_t(0) \geq 0$,

$$U_t(0) - U_t(a) = V_t(0)(1 - s(a;0)) - s(a;1)\,V_t(1) + [s(0;\bar{a}) - s(a;\bar{a})]\,V_t(\bar{a}) \geq 0,$$

since $V_t(0)(1 - s(a;0)) \geq 0$, $V_t(1) < 0$, $V_t(\bar{a}) < 0$ and $s(0;\bar{a}) - s(a;\bar{a}) < 0$ for all $a \in [0;1]$. Hence, $a_t = 0$.

Reasoning by induction implies

$$P\{\omega \in S \mid a_t \in \{0;1\} \text{ for each } t > 1\} = 1. \tag{6}$$

Step 3

On $\tilde{S}$, theorem 1 of Gilboa and Schmeidler (1996) applies. Hence, there exists a $\bar{u}_0$ such that for each $\bar{u}_1 \geq \bar{u}_0$:

$$P\left\{\omega \in S \mid \exists\pi \left(\arg\max_{a\in\{0;1\}} \mu_a\right) = 1\right\} \geq (1 - \varepsilon)\,P\left(\tilde{S}\right).$$

If $u_1(\bar{a}) > \max\{\mu_1; \mu_0\}$, $\lim_{t\to\infty} \bar{u}_t = u_1(\bar{a})$ almost surely. Hence, on almost each ω, there is a $T(\omega)$ such that

$$\bar{u}_t > \max\{\mu_1; \mu_0\} + \xi$$

for each $t > T(\omega)$. Therefore, $a = 0$ and $a = 1$ are almost surely chosen infinitely often. Theorem 1 in Gilboa and Pazgal (2001) implies:

$$\frac{\pi(0)}{\pi(1)} = \frac{\mu_1 - u_1(\bar{a})}{\mu_0 - u_1(\bar{a})}$$

$S \backslash \tilde{S}$-a.s.. By (6), $\pi(a) = 0$ for $a \notin \{0; 1\}$.■

Proof of Proposition 2

The proof proceeds in three steps. Step 1 demonstrates that $a_2 \in \{0; 1\}$. Step 2 shows that $a_t \in \{0; 1\}$ for $t \geq 2$. Step 3 uses theorem 2 of Gilboa and Schmeidler (1996) and theorem 1 of Gilboa and Pazgal (2001) to derive the result.

Step 1

At $t = 2$, either

$$U_2(\bar{a}) = V_2(\bar{a}) = u_1(\bar{a}) - \beta\bar{u}_1 - (1-\beta)u_1(\bar{a}) = \beta[u_1(\bar{a}) - \bar{u}_1] < 0$$

or

$$U_2(\bar{a}) = V_2(\bar{a}) = u_1(\bar{a}) - X_2 - h = -h < 0$$

holds. The strict monotonicity of s implies $a_t \in \{0; 1\}$.

Remark 2 *Remark 1 applies, since for* $(\tau + 1) \notin N$, $\bar{u}_{\tau+1}$ *is adapted as under (1), whereas for* $0 \in N$,

$$\bar{u}_{\tau+1} = X_{\tau+1} + h > X_{\tau+1} \geq X_{\tau+1}(a)$$

holds.

Step 2

As in the proof of proposition 1, it can be shown that

$$P\{\omega \in S_1 \mid a_t \in \{0; 1\} \text{ for each } t > 1\} = 1.$$

This follows from remark 2 and from the definition of $\tilde{t}$ in (4). Note that (4) implies

$$\tilde{t} + 1, ...\tilde{t}' \notin N,$$

since for $t \in N$, $V_t(a_{t-1}) < 0$.

Step 3

On $\tilde{S}_1$, theorem 2 of Gilboa and Schmeidler (1996) applies. Therefore,

$$\pi\left(\arg\max_{a\in\{0;1\}}\mu_a\right)=1$$

almost surely obtains.

On $S_1\backslash\tilde{S}_1$, $\bar{u}_t\to u_1(\bar{a})$. Hence, a.s. there is a period $T(\omega)$ such that

$$|\bar{u}_t-u_1(\bar{a})|<\zeta$$

for each $t>T(\omega)$ except on a sparse set of periods for an arbitrary ζ. Hence, the acts 0 and 1 are chosen an infinite number of times. But since now

$$\lim_{t\to\infty}\bar{u}_t=u_1(\bar{a})>\max\{\mu_0;\mu_1\},$$

theorem 1 of Gilboa and Pazgal (2001) implies:

$$\frac{\pi(0)}{\pi(1)}=\frac{\mu_1-u_1(\bar{a})}{\mu_0-u_1(\bar{a})}.\blacksquare$$

Proof of Proposition 3

Rule (2) implies

$$U_{\bar{t}}(0)=V_{\bar{t}}(0)<0$$

for some finite $\bar{t}>0$. By assumption 2,

$$U_{\bar{t}}(a)=0>U_{\bar{t}}(a')\text{ for all }a\geq\frac{1}{l}\text{ and }a'<\frac{1}{l}.$$

Hence, $a_{\bar{t}+1}=\frac{1}{l}$ by assumption 3. Remark 2 applies here as well. Hence, for a finite $\hat{t}>\bar{t}$,

$$U_{\hat{t}}\left(\frac{1}{l}\right)<0$$

and

$$U_{\hat{t}}(a)=0>U_{\hat{t}}(a')\text{ for all }a\geq\frac{2}{l}\text{ and }a'<\frac{2}{l}$$

from assumption 2. Hence, $a_{\hat{t}}=\frac{2}{l}$, etc.

Once $C_t\left(\frac{k}{l}\right)\geq 1$ obtains for all $k=0...l$,

$$\begin{aligned}U_t(a)&=\sum_{i=0}^{l}V_t\left(\frac{i}{l}\right)s\left(a;\frac{i}{l}\right)\\&=V_t\left(\frac{k}{l}\right)s\left(a;\frac{k}{l}\right)+V_t\left(\frac{k-1}{l}\right)s\left(a;\frac{k-1}{l}\right),\end{aligned}$$

for $a\in\left[\frac{k-1}{l};\frac{k}{l}\right]$. According to remark 2, $V_t\left(\frac{k}{l}\right)>0$ can hold for at most one $k\in\{0;...l\}$. If

$$V_t\left(\frac{k}{l}\right)>0,$$

then $a_t = a_{t-1} = \frac{k}{l}$, since

$$V_t\left(\frac{k}{l}\right) > V_t\left(\frac{k}{l}\right) s\left(a; \frac{k}{l}\right) + V_t\left(\frac{k-1}{l}\right) s\left(a; \frac{k-1}{l}\right)$$

for all $a \in \left[\frac{k-1}{l}; \frac{k}{l}\right]$ and

$$V_t\left(\frac{k}{l}\right) > 0 \geq V_t\left(\frac{k'}{l}\right) s\left(a; \frac{k'}{l}\right) + V_t\left(\frac{k'-1}{l}\right) s\left(a; \frac{k'-1}{l}\right)$$

for all $k' \neq k$ and all $a \in [0; 1]$ hold. If

$$\max\left\{V_t\left(\frac{k}{l}\right); V_t\left(\frac{k-1}{l}\right)\right\} < 0,$$

then $U_t(a)$ is convex for $a \in \left[\frac{k-1}{l}; \frac{k}{l}\right]$. Hence,

$$\arg \max_{a \in \left[\frac{k-1}{l}; \frac{k}{l}\right]} U_t(a) \in \left\{\frac{k}{l}; \frac{k-1}{l}\right\}$$

and

$$\arg\max_{a \in A} U_t(a) \in \left\{0; \frac{1}{l}; ...\frac{k}{l}; ...1\right\} \text{ for every } t.$$

The result then follows from theorem 2 of Gilboa and Schmeidler (1996).■

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