Essays in Industrial Organisation

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Hiermit erkläre ich, daß ich die Dissertation selbständig angefertigt habe und mich anderer als der in ihr angegebenen Hilfsmittel nicht bedient habe, insbesondere, daß aus anderen Schriften Entlehnungen, soweit sie in der Dissertation nicht ausdrücklich als solche gekennzeichnet sind und mit Quellenangaben versehen sind, nicht stattgefunden haben.

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Contents

Chapter 1

General Introduction

This dissertation contains three essays in industrial organisation. Chapter 2 analyses firms' incentives to preannounce innovations when consumers have costs of switching from one product to another. Chapter 3 deals with the emergence and consequences of price leadership in two different industry settings. And chapter 4 investigates the interaction between firms' R&D decisions and locational choices.

Each chapter is an independent piece of work, and can be read separately. The chapters contain their own introduction that raises the issues studied, relates them to the literature in the area, and highlights the contributions made. Here, I will therefore confine myself to provide the reader a short outline of each essay of this thesis.

In many markets consumers who have previously purchased from one firm have costs of switching to a competitor's product. These switching costs imply that especially in industries with fast technological progress buyers are confronted with an intertemporal trade-off between buying the presently available technology with the risk of economic obsolescence due to the arrival of an innovation and waiting for the new technology without locking into the old one. In **Chapter 2** we study the incentives of innovating firms to preannounce new technologies in order to increase the expected value of waiting for consumers. In particular, we will point out that this kind of advance information might not always be entirely beneficial for innovators. Announcing a new developed product might help to prevent the lock-in of potential customers into the old technology before the launch of the new product. But, at the same time, the information about future entry spills over to the incumbent firm and gives it the opportunity to take preemptive actions against the innovative firm. In this work we want to show how this trade-off influences the preannouncement behaviour of innovative entrants and derive conditions under which preannouncements are more likely to be observed. The welfare analysis of the model is supposed to give us indications for a sensible competition policy regarding innovation announcements.

For this purpose we make use of a two period model of vertical product differentiation with overlapping consumer generations and analyse intertemporal consumer choice under uncertainty and imperfect competition in the product market.

To sum up, we find that innovative firms may not always have an incentive to preannounce a new product generation. They might prefer not to inform their potential future clientele in order to avoid the information spillover and a tug-of-war with the incumbent. In this vein, preannouncements are more likely in industries where the innovation step of the new technology is relatively high, the time between announcement and launch is short and consumers are not to heterogenous. The welfare analysis shows that, from the point of view of the consumers, there might be too few or too many announcements and depending on the characteristics of the industry, announcement bans or enforcements might improve on the free market outcome.

Chapter 3 contains two duopolistic models that challenge commonly held views on the emergence and consequences of endogenous price leadership in industries. In the first part, we investigate again an industry with vertical product differentiation. In particular, we analyse the typical two stage setting in which firms first set qualities and then engage in price competition. But instead of assuming simultaneous choices we endogenise the timing in the price game and explicitly allow price leadership to emerge. In a first step, we demonstrate that price leadership is actually the equilibrium outcome of the second stage with the high-quality firm as the industry leader. More interestingly, the emergence of price leadership affects the quality choices in the first stage of the game. That means, firms anticipate that price leadership reduces the intensity of price competition in the short run and this gives the low-quality firm a stronger incentive to decrease product differentiation and to invest more in quality. By consequence, price leadership leads to higher prices for given qualities but it also implies a higher average product quality in the industry. Taken together, we can show that in this sense price leadership can actually be beneficial for consumers and hurt firms in the overall game.

In the second part of this chapter, we reconsider a model by Deneckere and Kovenock and Lee (1992) who study incentives for price leadership in markets with consumers' switching costs. Previous studies have demonstrated that when one identifies firm size with capacity then price leadership of the large firm should arise because the smaller firm stands to lose more by moving first and being undercut from its rival. Deneckere et al. argue that the same holds true if one measures firms size with the size of a firm's locked-in customer base. The model presented in Chapter 3 demonstrates that this claim crucially hinges on the size of switching costs for the consumers. In particular, they suppose that switching costs are prohibitive, i.e. once a consumer has bought a firm's product she can not switch afterwards. We relax this assumption and show that for positive but finite consumers' switching costs the opposite result holds. In contrast to their model, here the large firm stands to lose a lot if it leads and is undercut by its rival since in our model consumers can switch and the incentive to cut prices for the small firm is the larger, the bigger the base of its rival. In addition to this, the follower role makes the large firm tame since it would have to apply any price cut to his large customer base. Anticipating this, the small firm is willing to move first and this coincidence of incentives gives rise to situations in which one firm strictly prefers to lead while the other one strictly prefers to follow. Interestingly, our welfare considerations indicate that over a large range of the parameter space the endogenously determined price leadership pattern maximises social welfare.

Finally, in **Chapter 4**, which is joint work with Thomas Rønde and Konrad Stahl, we investigate the interaction between firms' R&D decisions and their location choices in product and/or geographical space. In the vein of the seminal paper of Hotelling (1929) firms' location choices follow the trade-off between two by now standard effects: a *demand effect* that induces the individual firm to move towards the center of the market, and a *competition effect* that drives the firms away from each other. It was Hotelling's belief that the former is stronger and firms tend to supply identical products. However, d'Aspremont and Gabszewicz and Thisse (1979) showed fifty years later that Hotelling's analysis was wrong and that for symmetric product qualities and quadratic transportation costs, the „principle of maximum differentiation" holds, i.e. the competition effect always dominates the market effect.

In our benchmark model, we introduce stochastic R&D in the Hotelling framework and show that this can restore Hotelling's initial result even in a model with quadratic transportation costs of consumers. The intuition for this result is that if market entry (or product quality) depends on the stochastic outcome of firms' R&D activities, a firm meets a successful competitor in the product market only with a certain probability. This weakens the competition effect while the demand effect remains unchanged. In modifications of this benchmark model we look at the impact of R&D spillovers and patent protection on firms' location choices and show that the former has a deglomerating effect while the latter has an agglomerating effect.

In the second part of this work, we extend our framework and allow firms to choose their R&D technology together with their location. More specifically, firms can adopt either a safe R&D project yielding a low-quality product or a risky project aiming at a large innovation step. We show that for a large range of the parameter space the following three types of equilibria can emerge. Either firms choose dispersed locations and adopt the safe R&D technology. Or, they agglomerate in the center and one of the firms opts for the risky technologies. Or, finally, they agglomerate and both choose the risky R&D technology. This result hints at a strong complementarity between risk taking in R&D and geographical concentration of firms. Finally, our welfare analysis gives a rather diverse picture. There may be excessive differentiation and concentration in product space and too less or too much risk taking in the choice of the R&D technology.

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Chapter 2

Innovation Preannouncement in a Vertically Differentiated Industry

2.1 Introduction

In industries with consumers' switching costs and fast technological progress, buyers face an intertemporal choice between subsequent product generations. Buying the presently available product entails the risk of early economic obsolescence due to an immediately afterwards upcoming innovation. This potential lock-in has to be traded off with the expected cost of waiting for a better technology. Product preannouncements are an appropriate and widely observed strategy for innovating firms to increase the expected value of waiting for the consumers and prevent the loss of potential future demand. For example, Volkswagen announced the arrival of its new beetle car three years in advance and one year before its availability, they already had 70.000 orders from waiting customers.

Nevertheless, such advance communications may not always be entirely beneficial to the innovative firm. Although they are directed to inform potential customers, the message will obviously reach incumbent competitors, too. And as many markets with fast technological progress are dominated by firms with a certain degree of temporary market power, strategic interaction is most probable as the following examples from the video game market demonstrates.

In 1988, Sega introduced its 16-bit-Mega Drive home video system which was at that time a large innovation step beyond the existing 8-bit systems. They sold the console for $190 and games were priced between $40 and $70. Nintendo, Sega's closest competitor, reacted and gave its customers a reason to wait by preannouncing their own new 16-bit system. As an immediate response to this, Sega started offering their system in bundle with one game for $150. One year later, Nintendo entered the market and soon prices dropped under $100.[1]

The 32-bit generation of game consoles was announced by the new entrant Sony in 1994. The year before its actual launch in 1995 was the poorest in terms of sales figures of the whole industry history because consumers were waiting for the new technology to arrive.[2]

Finally, in the end of 1995, Nintendo announced the launch of its 64-bit console machine in autumn 1996. From the day of the announcement until its introduction the prices of Sega's and Sony's 32-bit systems dropped from $299 to $149.[3]

Another well-known and well-reported preannouncement story, the Control Data anti-trust case in 1967 is another example for the severity of strategic reactions. The sales of the computer manufacturer Control Data suffered tremendously when IBM announced the arrival of its new and largely superior System/360 model in 1964 which was finally not available before 1967. But the immense price cuts that Control Data had to offer to attract customers, induced them to bring an anti-trust charge against IBM.[4]

[1] For more details see Brandenburger and Nalebuff (1996), p. 237-242.

[2] see 'Power games', Marketing Week: London; May 19, 1995.

[3] see 'Sony and Sega plan price cuts to torpedo Nintendo 64 launch', *Marketing Week*: London; July 26, 1996.

[4] Fisher and McGowan and Greenwood (1985) devote a whole book to the IBM/Control Data

With the use of preannouncements, firms can retain customers from buying substitute products and thus preserve their own potential demand until the period of product introduction. At the same time, as the examples above show, this advance information spills over to the incumbent competitor and gives him the opportunity for an additional strategic move before the innovative firm's entry.

The aim of the present paper is to analyse the strategic role of innovation preannouncements in an imperfectly competitive market setting. It will be shown how the described trade-off affects the announcement behaviour of an innovative firm in the context of a vertically differentiated market with overlapping consumer generations. We analyse situations in which the preannouncement of a new, superior technology by an outside firm gives the incumbent monopolist incentives for preemptive price cuts in the pre-entry period in order to attract consumers before the availability of the new product. The main results can be summarised as follows. The probability that an innovating firm will preannounce its product is high, if

- the innovation step beyond the existing technology is sufficiently large,
- time between preannouncement and launch is short (or the consumers are impatient),
- the average consumers' valuation for quality is rather low and/or
- consumers are more heterogeneous with respect to their valuation of quality.

At a first glance, the welfare results of the model are somewhat surprising. From the consumers' point of view, the market can produce *too few* or *too many* preannouncements. While the under-provision of information seems obvious given a preemptive reaction by an incumbent, the over-provision result stems from the consumers' trade-off between efficient *information transmission* and *market contestability*. Preannouncements of new products entail efficient information transmission and

case taking place between 1960 and 1980.

an efficient matching between consumers and product generations over time. However, with the entrant's use of preannouncements, markets lose the threat of entry in periods when there is no innovation and the incumbent regains market power. We show that consumers would prefer a ban on preannouncements in situations in which the upcoming innovation step is of an intermediate size and ex ante expectations about its introduction are rather high.

The previous work dealing with preannouncements of strategies mainly analyses problems related to commitment effects. Henkel (1996) studies games in which in an announcement stage each player commits partly to a strategy. The degree of commitment is endogenous and individually chosen and later deviation from the announcement is costly. The author finds that in the case of strategic complements in the basic game the introduction of the announcement stage induces the players to commit partly and thus supports collusion. Crawford and Sobel (1982) show that announcements without any direct influence on the payoffs ('cheap talk') can be relevant in games with private information. Farrell (1987) does the same for games in which there is a coordination problem.

The present paper departs from this strand of literature with the somewhat extreme assumption that the preannouncement of the innovation is fully credible, i.e. the innovative firm can perfectly commit to timing and quality of the new product. In other words, we completely abstract from any kind of untruthful preannouncement that might lead to 'vaporware'-products. We think that this is appropriate since it allows us to concentrate on the 'strategic reaction' effect of preannouncements without affecting the qualitative nature of our results (see the last section for a further discussion).

With this assumption, our work is much closer to some other studies. Farrell and Saloner (1985) analyse the effect of preannouncements on the adoption of a new incompatible good in the presence of network externalities. In this context, preannouncements make the adoption of the new technology more likely which *can* be socially desirable. A major drawback of their model is that it does not consider strategic pricing but assumes a competitive supply of the old and the new technology. Yin (1995) presents a study, in which an innovating firm can announce the quality of its product early or late. Delaying the announcement means that the competitor has to set its own quality on the basis of the distribution of the possible innovation outcomes. In this model, the innovator has no incentive to announce early and the results are straightforward: Although the early announcement policy is socially desirable, it is not supported in equilibrium. Gerlach (1999) analyses the preannouncement behaviour of a monopolist. Announcing a superior technology cannibalises the present sales of the old technology if consumers have some kind of switching costs. The author shows that monopolists may have an incentive not to preannounce in order to make consumers buy the old *and* switch to the new product afterwards. Furthermore, this work analyses the rationale behind 'vaporware', i.e. products that are announced although they will knowingly not be available at the promised date.

Our work also relates to the literature on information exchange among firms and their impact on competition. Kühn and Vives (1995) provide quite general conditions including the type of competition and the nature of uncertainty under which firms have an incentive to share information about cost or demand. The scenario that comes closest to our model is Bertrand competition with private value cost uncertainty in which firms typically have no incentives to share information with their competitor. In our framework, however, innovations are preannounced because firm want to influence *consumers'* decisions.

The organisation of the paper is along the dynamic structure of the presented game-theoretical model starting from backwards. The following section presents the basic assumptions of the model. Section 2.3 is devoted to the price quilibria in the second period in the case with and without an innovation. In section 2.4 we look at the first period pricing behaviour of the incumbent and the purchase decision of the consumers. The following two sections respectively analyse the preannouncement behaviour of the innovating firm and its welfare implications. Finally, section 2.7 concludes with the discussion of the proposed framework, some robustness checks and possible extensions. Note that all proofs are delegated to the appendix.

2.2 The Model

We consider a simple two period model of a vertically differentiated industry in the spirit of Gabszewicz and Thisse (1979) and Shaked and Sutton (1982, 1983). In every period, $t = 1, 2$, a cohort of consumers who only differ in their taste for quality θ, will enter the market. Though the valuation of a consumer is only known to himself, it is common knowledge that the taste parameter is uniformly distributed on $[a - h, a + h]$. The parameter a can be interpreted as the average valuation of consumers for quality, while h reflects the degree of heterogeneity of the consumer population. We normalise the mass of consumers in each cohort to 1 and we will submit the distribution parameters to the following restriction:

$$a \geq 5h. \tag{2.1}$$

This condition states that consumers' tastes are not too heterogeneous with respect to the average valuation of the population and it ensures us covered market equilibria in both periods. In the considered time span, every consumer can afford to buy at most one unit of the durable good. This means for cohort 1 consumers that switching from a product they bought in the first period to another in period 2 is prohibitively costly.

On the supply side, we consider two firms, an incumbent monopolist, referred to as firm 1, and a potentially innovating outside firm 2. While the incumbent offers in both periods product 1 of quality q_1, firm 2 conducts R&D to develop a superior technology. This innovation process is stochastic, though the R&D investment decision is not explicitly modelled. With probability ρ_0, the next step on an imaginary quality ladder is done and the resulting product is of a given quality $q_2 > q_1$. In the "no innovation" event (with probability $1 - \rho_0$), the only available quality is q_1, which prevents firm 2 from entering the market as there is some small entry cost $\varepsilon > 0$. For simplicity, both firms are assumed to produce at zero marginal costs. Firms and consumers have rational expectations, are risk neutral and discount future revenues using discount factors $0 \leq \delta \leq 1$ per period. Finally, it will be convenient to use $\Delta \equiv \frac{q_2 - q_1}{q_1}$ as a measure for the size of the upcoming innovation step.

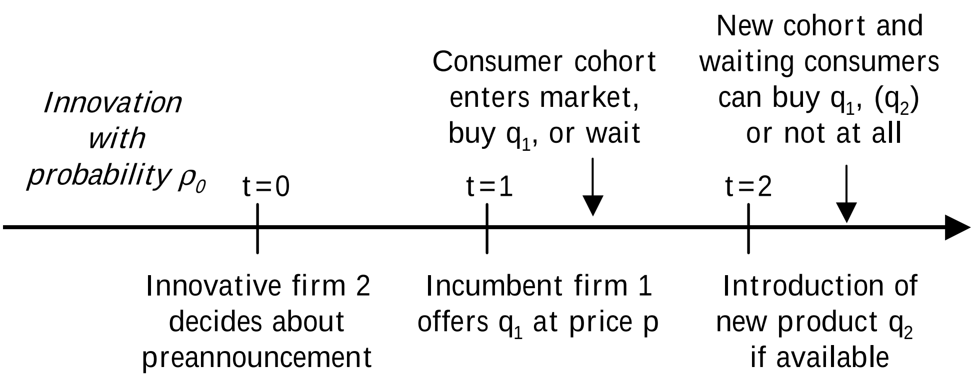

Figure 2.1: *Time Structure of the Model.*

The time structure of the model is as follows (cf. Figure 2.1). Before period 0, nature moves and firm 2 succeeds with probability ρ_0 in developing the new product which can be introduced to the market as soon as period 2.[5] In this case, firm 2 can decide whether to preannounce the product or not.[6] In period 1, the incumbent firm offering a product of quality q_1 and the first cohort consumers use the preannouncement signal to update their prior beliefs and form expectations about the market outcome in

[5]Think of the delay as the time for testing the product, preparing mass production and negotiate distribution channels.

[6]Notice that, this choice is only to be made in the innovation subgame since in this simple two period framework, untruthful preannouncements make no sense for firm 2. See the last section for a discussion of this assumption.

period 2. After the incumbent monopolist has set a price p, consumers can either purchase good q_1 in $t = 1$, or wait for the second period. In period $t = 2$, two constellations are possible. If there is no innovation to be introduced, firm 1 keeps on selling the old product at a price p_1^M. Otherwise, if firm 2 launches the innovation, we have a duopolistic market in which firms simultaneously set their prices p_1^D and p_2^D, respectively. Finally, the waiting first cohort and the entering second cohort consumers can buy product q_1, q_2 (if available) or not all (the value of the outside option is normalised to 0). In order to keep matters as simple as possible, we exclude transactions via a resale market and any depreciation of the durable good.

With these assumptions, a consumer θ, who buys product q_1 in period 1, derives an overall net utility of

$$U_{11}(\theta) \equiv \theta q_1 - p + \delta\theta q_1, \tag{2.2}$$

where U_{ij} denotes the total net utility of consuming product i in period 1 and j in period 2. The expected net utility of waiting (this first period option is denoted by 0) and buying either product q_1 or q_2 in period 2 is

$$E[U_0(\theta)] \equiv \delta\rho \ Max\{\theta q_1 - p_1^D, \theta q_2 - p_2^D\} + \delta(1-\rho)(\theta q_1 - p_1^M). \tag{2.3}$$

We will look for Perfect Bayesian equilibria of this game and start our analysis by looking at the second period market outcome taking into consideration the first cohort consumers that decided to wait. Then we turn to the first period and assume that the market participants rationally anticipate the future market constellations. In the case of an innovation announcement, priors are updated to one and there is complete certainty about the upcoming launch of a new product. If there is no announcement, beliefs can also be reconsidered since no communication can mean that either there is no innovation or that it is not profitable for the firm to preannounce it. Finally, we look at the announcement decision of the innovating firm who rationally expects the behaviour of her competitor and the consumers.

2.3 Second Period

In the last period, the entering second cohort consumers join the waiting first cohort consumers which results in some aggregate distribution function. As it will turn out in the next section, we can confine ourselves to two possible outcomes. Throughout the paper, we will refer to the situation in which low-valuation consumers in the interval $[a-h, x]$, with $a-h \leq x \leq a+h$, decide to wait as constellation I. The aggregate consumer density function in the second period is then simply

$$f_I^{agg}(\theta, x) \equiv \begin{cases} 2 & \textit{if } a-h \leq \theta \leq x, \\ 1 & \textit{if } x < \theta \leq a+h. \end{cases}$$

In constellation II, high-valuation consumers in $[y, a+h]$, with $a-h \leq y < a+h$, stay in the market until period 2, which results in

$$f_{II}^{agg}(\theta, y) \equiv \begin{cases} 1 & \textit{if } a-h \leq \theta \leq y, \\ 2 & \textit{if } y < \theta \leq a+h, \end{cases}$$

Given these two possible consumer populations (x or y are endogenously determined in the first period), we have to consider two possible market structures depending on the success of the R&D activity of firm 2. If the new product is not available, firm 1 keeps its monopolistic position and offers a product of quality q_1 at a price p_1. As the second period net utility of a consumer θ is given by $\theta q_1 - p_1$, the consumer who is indifferent between buying and the outside option of value 0 is at $\frac{p_1}{q_1}$. With a consumer population given by $f_I^{agg}(\theta, x)$, the monopolist faces a second period demand

$$D_I^M(p_1, x) \equiv \begin{cases} (x-(a-h))+2h & \textit{if } p_1 \leq (a-h)q_1, \\ (x-\frac{p_1}{q_1})+(a+h-\frac{p_1}{q_1}) & \textit{if } (a-h)q_1 \leq p_1 \leq xq_1, \\ Min\{a+h-\frac{p_1}{q_1}, 0\} & \textit{if } p_1 \geq xq_1. \end{cases}$$

For low prices the monopolist serves all or at least some of the waiting consumers. If $p_1 > xq_1$ he only sells to cohort 2 consumers with a high valuation. The demand function is continuous, piecewise linear and convex since its slope in $[(a-h)q_1, xq_1]$ is higher than in the regime with $p_1 > xq_1$. These properties yield a profit function $p_1 D_I^M(p_1, x)$ which may have two local maximisers. Nevertheless, it is straightforward to show that the slope of the profit function is negative for all $p_1 \geq (a-h)q_1$ as long as $a \geq 3h$ (see proof of Lemma 1 in the appendix). Thus, we have

Lemma 1 *Given demand $D_I^M(p_1, x)$ and for all $a \geq 3h$, a monopolist offering a product of quality q_1, optimally chooses $p_1^M = (a-h)q_1$ and serves all consumers in the market.*

If the consumer population is given by $f_{II}^{agg}(\theta, y)$, the demand function is

$$D_{II}^M(p_1, y) \equiv \begin{cases} (a+h-y)+2h & if\ p_1 \leq (a-h)q_1, \\ (a+h-y)+(a+h-\frac{p_1}{q_1}) & if\ (a-h)q_1 \leq p_1 \leq yq_1, \\ Min\{2(a+h-\frac{p_1}{q_1}), 0\} & if\ p_1 \geq yq_1. \end{cases}$$

If the waiting consumers are on the upper end of the taste scale, the demand has a piecewise linear but concave shape and incentives to set low prices become weaker since they would have to be applied over a larger mass of high valuation consumers. The corresponding profits are single-peaked and it can be shown that

Lemma 2 *For all $a \geq 5h$, a monopolist that faces $D_{II}^M(p_1, y)$ sets $p_1^M = (a-h)q_1$ and serves all consumers in the market.*

The intuition for these two lemmas is straightforward. In Lemma 1, there is an additional mass of low valuation consumers that gives the monopolist a strong incentive to decrease the price in order to serve all consumers. On contrary, if the bulk of consumers is on the upper end of the taste scale, firm 1 is inclined to raise its price and give up some of the low-valuation consumers. But as Lemma 2 shows, this is not optimal as long as (2.1) holds.

Let us now turn to the case in which the outside firm introduces the new product of quality q_2 in the second period and both firms play a simultaneous Nash equilibrium in prices. As long as all consumers derive a positive net utility, the indifferent consumer between buying q_1 at p_1 and q_2 at p_2 is at $\frac{p_2-p_1}{q_2-q_1}$. Consider first a population $f_I^{agg}(\theta, x)$ and denote the demand for firm 1 as $D_{I,1}^D(p_1, p_2, x)$. For sufficiently low prices p_1, firm 1 attracts all cohort 1 consumers and high valuation consumer of cohort 2. For higher prices some waiting cohort 1 consumers start to switch to the high quality product. Define for notational convenience $\Delta q \equiv (q_2 - q_1)$ and $\Delta p \equiv (p_2 - p_1)$, then

$$D_{I,1}^D(p_1, p_2, x) = \begin{cases} (x - a + h) + 2h & \text{if } \Delta p > (a + h)\Delta q, \\ (x - a + h) + (\frac{\Delta p}{\Delta q} - a) & \text{if } x\Delta q \leq \Delta p \leq (a + h)\Delta q, \\ Min\{2(\frac{\Delta p}{\Delta q} - a + h), 0\} & \text{if } \Delta p < x\Delta q. \end{cases}$$

Note again that this demand schedule is piecewise linear and concave because firm 1 serves with a relatively high price the high-density segment of the consumer distribution. The corresponding profits are therefore single-peaked and one gets five candidate solutions to the profit maximisation problem given a price p_2 of the high-quality firm: three corner solutions and two interior solutions. Accordingly, the best response function for firm 1, $R_{I,1}^D(p_2, x)$, consists of five parts

$$R_{I,1}^D(p_2, x) = \begin{cases} 0 & \text{if } p_2 \leq \widetilde{p}_1, \\ \frac{1}{2}[p_2 - (a - h)\Delta q] & \text{if } \widetilde{p}_1 \leq p_2 \leq \widetilde{p}_2, \\ p_2 - x\Delta q & \text{if } \widetilde{p}_2 \leq p_2 \leq \widetilde{p}_3, \\ \frac{1}{2}[p_2 - (2a - 2h - x)\Delta q] & \text{if } \widetilde{p}_3 \leq p_2 \leq \widetilde{p}_4, \\ p_2 - (a + h)\Delta q & \text{if } p_2 > \widetilde{p}_4, \end{cases}$$

with $\widetilde{p}_1 \equiv (a-h)\Delta q, \widetilde{p}_2 \equiv (2x-a+h)\Delta q$, $\widetilde{p}_3 \equiv (2a-2h-3x)\Delta q$ and $\widetilde{p}_4 \equiv (4h+x)\Delta q$. This reaction functions is continuous, piecewise linear and monotonically increasing in p_2.

The demand of the high-quality firm faced with a consumer distribution $f_I^{agg}(\theta, x)$ is

$$D_{I,2}^D(p_1, p_2, x) = \begin{cases} Min\{(a+h-\frac{\Delta p}{\Delta q}), 0\} & if\ \Delta p > x\Delta q, \\ (x-\frac{\Delta p}{\Delta q}) + (a+h-\frac{\Delta p}{\Delta q}) & if\ (a-h)\Delta q \leq \Delta p \leq x\Delta q, \\ (x-a+h)+2h & if\ \Delta p < (a-h)\Delta q. \end{cases}$$

This function is convex and the corresponding profit function can have two peaks. Nevertheless, as long as $a \geq 3h$, the marginal profits are negative for all $p_2 \geq p_1 + (a-h)(q_2 - q_1)$. Thus, the best response function simplifies to

$$R_{I,2}^D(p_1, x) = p_1 + (a-h)(q_2 - q_1).$$

Solving for the Nash equilibrium, we get

Lemma 3 *Consider a simultaneous price setting duopoly with a high quality firm q_2, a low quality firm q_1 and the consumer population $f_I^{agg}(\theta, x)$ given above. For $a \geq 3h$, the unique Nash equilibrium in prices is given by $p_1^D = 0$ and $p_2^D = (a-h)(q_2 - q_1)$. The high-quality firm serves all consumers in the market.*

This equilibrium with one active firm is mainly due to the assumption that the consumers are not too heterogeneous with respect to the taste parameter. Additionally, if the mass of low-valuation consumers is large, the firm with the high quality good has an even stronger incentive to serve all consumers by driving the low-quality supplier out of the market.

If firms are confronted with a $f_{II}^{agg}(\theta, y)$ population, the respective demand function of firm 1 is

$$D_{II,1}^D(p_1, p_2, y) = \begin{cases} (a+h-y)+2h & if\ \Delta p > (a+h)\Delta q, \\ (\frac{\Delta p}{\Delta q} - y) + (\frac{\Delta p}{\Delta q} - a + h) & if\ (a+h)\Delta q \geq \Delta p \geq y\Delta q, \\ Min\{(\frac{\Delta p}{\Delta q} - a + h), 0\} & if\ \Delta p < y\Delta q. \end{cases}$$

This convex demand schedule implies a profit function that has two peaks for certain values of p_2. And this is reflected in a discontinuity in the best response pattern of firm 1:

$$R^D_{II,1}(p_2, y) = \begin{cases} 0 & if\ p_2 < \widetilde{p}_1 \\ \frac{1}{2}[p_2 - (a-h)\Delta q] & if\ \widetilde{p}_1 \leq p_2 \leq \widetilde{p}_5, \\ \frac{p_2}{2} - \frac{y+a-h}{4}\Delta q & if\ \widetilde{p}_5 \leq p_2 \leq \widetilde{p}_6, \\ p_2 - (a+h)\Delta q & if\ p_2 > \widetilde{p}_6, \end{cases}$$

with $\widetilde{p}_5 \equiv (y + \frac{y-a}{\sqrt{2}})\Delta q$ and $\widetilde{p}_6 \equiv \frac{1}{2}(3a+5h-y)\Delta q$. This best response has a discontinuity at $p_2 = \widetilde{p}_6$, where the low-quality firm is indifferent between serving some of the bigger mass of high-valuation consumers at a low price and serving only low-valuation consumers from cohort 2 at a rather high price.

Finally, if high-valuation consumers wait for the second period, the demand of firm 2 is

$$D^D_{II,2}(p_1, p_2, y) = \begin{cases} Min\{2(a+h-\frac{\Delta p}{\Delta q}), 0\} & if\ \Delta p > y\Delta q, \\ (a+h-y) + (a+h-\frac{\Delta p}{\Delta q}) & if\ y\Delta q \geq \Delta p \geq (a-h)\Delta q, \\ (a+h-y) + 2h & if\ \Delta p < (a-h)\Delta q. \end{cases}$$

When looking at the corresponding profits of firm 2, we can again show that as long as (2.1) holds marginal profits are negative for all $p_2 \geq p_1 + (a-h)(q_2 - q_1)$ and positive for lower p_2. Hence the reaction function is

$$R^D_{II,2}(p_1, y) = p_1 + (a-h)(q_2 - q_1).$$

Solving for the Nash equilibrium gives

Lemma 4 *Consider a simultaneous price setting duopoly with the consumer population $f^{agg}_{II}(\theta, y)$ given above. For $a \geq 5h$, the unique Nash equilibrium in prices is given by $p^D_1 = 0$ and $p^D_2 = (a-h)(q_2 - q_1)$. The high-quality firm serves all consumers in the market.*

As for the monopolist, the duopolist has an incentive to serve even the consumer with the lowest valuation as long as average valuation is sufficiently large relative to consumer heterogeneity. Therefore, condition (2.1) ensures us that in the second period only one firm will be active in the market, the incumbent in the no innovation case or the innovator otherwise. Furthermore, all waiting consumers will be served in equilibrium at a price that equals the valuation (or difference in valuation in the duopoly case) of the consumer with the lowest θ.

2.4 First Period

After firm 2 had the opportunity to preannounce the new product in $t = 0$, the incumbent firm and the cohort 1 consumers update their initial belief ρ_0 to $\widetilde{\rho}$. And with this additional information consumers decide whether to lock in product q_1 or to wait for the next period. The net utility of the first alternative is given by $U_{11}(\theta)$ from (2.2). Plugging the second period equilibrium values in (2.3), we get for the expected utility of waiting

$$
\begin{aligned}
E[U_0(\theta)] &= \widetilde{\rho}[\delta\theta q_2 - \delta p_2^D] + (1-\widetilde{\rho})[\delta\theta q_1 - \delta p_1^M] \qquad (2.4) \\
&= \widetilde{\rho}\delta[\theta q_2 - (a-h)(q_2 - q_1)] + (1-\widetilde{\rho})\delta[\theta q_1 - (a-h)q_1].
\end{aligned}
$$

Note that both in the innovation and the no innovation event, all consumers expect a positive net utility in the second period. Further, although the duopoly price of the high quality good increases with the quality difference $(q_2 - q_1)$, every consumer benefits from a larger innovation step. Figure 2.2 depicts the indirect utility of the two alternatives as a function of the consumers' taste parameter θ. Two parameter regimes have to be distinguished. If $\Delta \leq \frac{1}{\widetilde{\rho}\delta}$, then the slope of $U_{11}(\theta)$ is steeper than the slope of $E[U_0(\theta)]$, implying that high-valuation consumers are more inclined to buy in the first period than low-valuation consumers. If $\Delta > \frac{1}{\widetilde{\rho}\delta}$, then the expected

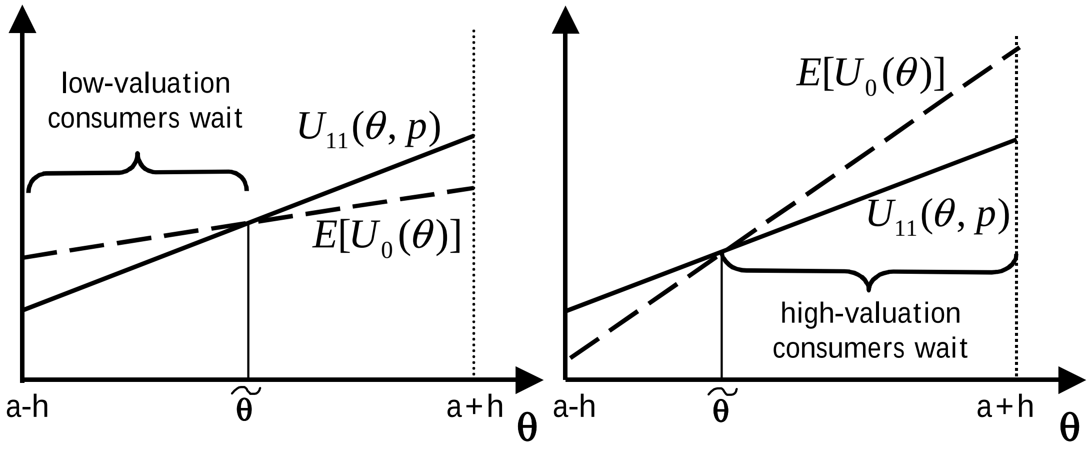

Figure 2.2: *(Indirect) utility functions for* $\Delta \leq \frac{1}{\widetilde{\rho}\delta}$ *(left) and for* $\Delta > \frac{1}{\widetilde{\rho}\delta}$ *(right)*

quality of the second period purchase is sufficiently high to make the high-valuation consumers more patient than the low-valuation consumers.

In both cases, one gets the position of the consumer who is just indifferent between buying q_1 in period 1 and waiting for the second period, by setting equal (2.2) and (2.4). This yields

$$\widetilde{\theta}(p) \equiv \frac{p - (a-h)\widetilde{\rho}\delta(q_2 - q_1) - (1-\widetilde{\rho})(a-h)\delta q_1}{q_1 - \widetilde{\rho}\delta(q_2 - q_1)}. \tag{2.5}$$

This threshold value determines the composition of the demand for the incumbent's product. If $\Delta \leq \frac{1}{\widetilde{\rho}\delta}$, all consumers in $[\widetilde{\theta}(p), a+h]$ will buy q_1 in the first period and and leave the market. The remaining first cohort consumers wait for the second period, generating a consumer population $f_I^{agg}(\theta, x = \widetilde{\theta}(p))$. Thus, the incumbent's demand from cohort 1 consumers is split into a certain first period demand and an expected second period demand that depends on the launch of the new technology. Therefore, the incumbent's expected profit function for this parameter regime[7] takes

[7]Notice that our notation is in accordance with the two different demand constellations discussed in Section 2.3.

the following form

$$E[\Pi^I(p,\widetilde{\rho})] \equiv p[a+h-\widetilde{\theta}(p)] + \delta(1-\widetilde{\rho})(a-h)q_1[(\widetilde{\theta}(p)-(a-h))+2h],$$

which he maximises for given (updated) expectations $\widetilde{\rho}$ with respect to the first period price p. Note that the incumbent serves the whole market in the first period if and only if $(1+\delta)(a-h)q_1 - p \geq E[U_0(a-h)]$ or

$$p \leq (1+\delta-\widetilde{\rho}\delta)(a-h)q_1. \tag{2.6}$$

For $\Delta > \frac{1}{\widetilde{\rho}\delta}$, the discounted, expected value of the new product is higher than the 'buy and keep' value of the incumbent's product. For this reason, consumers with a higher marginal utility of quality are more willing to wait for the second period. This also means that for a given first period price p, all consumers in $[a-h,\widetilde{\theta}(p)]$ will buy q_1 and leave the market, while the upper part of the cohort will wait for the second period and generate a $f_{II}^{agg}(\theta, y=\widetilde{\theta}(p)])$ population. In this parameter regime (II), the incumbent maximises

$$E[\Pi^{II}(p,\widetilde{\rho})] \equiv p[\widetilde{\theta}(p)-(a-h)] + \delta(1-\widetilde{\rho})(a-h)q_1[(a+h-\widetilde{\theta}(p))+2h], \tag{2.7}$$

and serves all cohort 1 consumers in the first period if and only if $(1+\delta)(a+h)q_1 - p \geq E[U_0(a+h)]$ or

$$p \leq (a+h+(a-h)\delta(1-\widetilde{\rho})q_1 - \widetilde{\rho}\delta 2h(q_2-q_1). \tag{2.8}$$

Proposition 1 summarises the solution to the maximisation problem of the incumbent in both parameter regimes. Define

$$\widehat{\Delta} \equiv \frac{a+3h}{4h\widetilde{\rho}\delta},$$

then

Proposition 1 *For given expectations $\widetilde{\rho}$, the incumbent's optimal price in the first period is:*

$$p^* = \begin{cases} (1+\delta-\widetilde{\rho}\delta)(a-h)q_1 & \text{if } 0 \le \Delta \le \frac{1}{\widetilde{\rho}\delta}, \\ (1+\delta-\widetilde{\rho}\delta)(a-h)q_1 - 2h(\widetilde{\rho}\delta(q_2-q_1)-q_1) & \text{if } \frac{1}{\widetilde{\rho}\delta} \le \Delta \le \widehat{\Delta}, \\ (1+\delta-\widetilde{\rho}\delta)(a-h)q_1 - \frac{(a-h)q_1}{2} & \text{if } \Delta > \widehat{\Delta}. \end{cases}$$

For innovation steps $\Delta \le \frac{1}{\widetilde{\rho}\delta}$, the incumbent's product has a higher expected quality than the new technology which makes it optimal for firm 1 to preempt the market with the price that makes the consumer with the lowest valuation indifferent between buying and waiting. For intermediate values of Δ (later referred to as parameter regime *II.1*), the expected value of the new technology is higher than the quality of the existing one but they are still close substitutes which implies that the incumbent has relatively low costs (in terms of a relatively high first period price p) to attract the whole market. However, if Δ exceeds the threshold value $\widehat{\Delta}$ (region *II.2*), it is no longer optimal to serve the highest valuation consumers with a lower price and the incumbent only sells to low-valuation consumers, while all consumers θ in $[\widetilde{\theta}(p^*), a+h]$ wait for the second period. The optimal price p^* decreases with a higher quality q_2 and higher expectations $\widetilde{\rho}$ since these variables make the consumers more patient and require stronger price cuts from the incumbent in order to retain demand.

To conclude this section, we will look at the impact of the incumbent's pricing strategy on the residual demand RD from cohort 1 in period 2. Plugging the optimal price from Proposition 1 into (2.5), one obtains

Corollary 1 *The residual demand from cohort 1 consumers is given by*

$$RD^*(\widetilde{\rho}, .) \equiv \begin{cases} 0 & \text{if } 0 \le \Delta \le \widehat{\Delta}, \\ 2h - \frac{(a-h)q_1}{2[\widetilde{\rho}\delta(q_2-q_1)-q_1]} & \text{if } \Delta > \widehat{\Delta}. \end{cases}$$

In fact, in the present model, firm 2 is solely interested in the mass of cohort 1 consumers that is waiting since the price in the second period is independent of the composition of the consumer population. Corollary 1 shows that the incumbent prefers to preempt the market as long as the upcoming innovation step is not too large. For sufficiently high Δ, the mass of waiting cohort 1 consumers increases with a higher q_2, $\widetilde{\rho}$, h and δ. It decreases with a higher a.

2.5 The Preannouncement Decision

When considering the preannouncement decision, the innovating entrant rationally anticipates the behaviour of the incumbent in the pre-entry period and knows that his signal is used by the market participants to update their priors. In this section, we will look for Perfect Bayesian equilibria of this game, in which firm 2 chooses a cohort 1 demand maximising announcement strategy for beliefs that are updated according to Bayes' rule. Let us denote β as the probability that firm 2 preannounces an innovation. Two types of equilibria can be distinguished: no announcement ('pooling') equilibria and announcement equilibria.

In an *announcement equilibrium* (β^*=1)[8], consumers and the incumbent know that it is profitable for an entrant to preannounce its new product, i.e. they can infer from the absence of an announcement that there has been no innovation, i.e.

$$\widetilde{\rho} = E[\text{innovation} \mid \text{no announcement}] = 0. \tag{2.9}$$

This Bayesian updating is anticipated by the entrant who has an incentive to preannounce whenever the following condition holds

$$RD^*(\rho = 1, .) > RD^*(\rho = 0, .). \tag{2.10}$$

In a *pooling equilibrium* ($\beta^* = 0$), the entrant prefers not to announce the innovation. Thus, consumers and firm 1 can interpret the absence of an announcement either

[8]Since we do not consider the case of untruthful preannouncements, any preannouncement changes the priors to $\widetilde{\rho} = 1$.

with the possibility that there is no innovation or with the event that the innovation is not preannounced. Their priors remain unchanged,

$$\widetilde{\rho} = E[\text{innovation} \mid \text{no announcement}] = \rho_0. \tag{2.11}$$

This argument yields the following necessary and sufficient condition for pooling equilibria:

$$RD^*(\rho = \rho_0, .) \geq RD^*(\rho = 1, .). \tag{2.12}$$

Proposition 2 gives the different equilibria regimes.

Proposition 2 *Depending on the size of the innovation step* Δ*, we get the following two types of Perfect Bayesian equilibria:*

1. $0 \leq \Delta \leq \Delta^A \equiv \frac{a+3h}{4\delta h}$*: The innovating firm does not preannounce (*β^**=0) and market participants hold the belief given in (2.11).*

2. $\Delta > \Delta^A$*: It is always profitable for the innovating firm to preannounce (*β^**=1); belief updating follows (2.9).*

Figure 2.4 in the next section shows the graph of the preannouncement probability β^* as a function of the new product's quality. The main comparative statics of these equilibria are summarised in the next corollary.

Corollary 2 *Comparative statics of the equilibria described in Proposition 2 yield:*

$$\frac{\partial \Delta^A(.)}{\partial a} > 0, \frac{\partial \Delta^A(.)}{\partial h} < 0, \frac{\partial \Delta^A(.)}{\partial \delta} < 0.$$

The rationale behind Corollary 2 is rather simple. Preannouncements will take place for parameter constellations at which preemption is most expensive for the incumbent. Ceteris paribus, this is true whenever the upcoming innovation step is sufficiently large or if the discount rate is high, which is equivalent to saying that time between launch and preannouncement is short for a given discount rate. Furthermore, there are more preannouncements in industries with a more heterogenous population of consumers. This holds because inframarginal rents for consumers are higher when tastes are more dispersed and this increases the value of waiting for higher quality. In the limiting case, where $h \to 0$, the incentive to preannounce disappears completely because consumers foresee that the innovating firm can appropriate all the innovation rents and this makes it easy for the incumbent to preempt the market in the pre-entry period.

By contrast, the average consumer valuation has a negative impact on the occurrence of an announcement because it raises disproportionately the value of today's purchase option compared to any expected value of future product generations. Interestingly, the initial innovation probability ρ_0 has no impact on the preannouncement behaviour of an entrant. On the one hand, a higher innovation probability increases the incentives for consumers to wait, but on the other, it makes the incumbent more aggressive in the pre-entry period. Taken together, these effects cancel out.

2.6 Welfare Implications

Obviously, there are two potential welfare distortions in this model. First, we have imperfect competition in both periods and second, there is an asymmetric information constellation between the potentially innovating firm and the other market participants. This section will show that in some sense our social planner has to trade off these two imperfections.

In order to analyse the welfare effects of the preannouncement behaviour in this second-best world, we will take the market structure as given and concentrate on the efficiency of the information transmission in the economy. Our welfare measure will be the *ex ante* expected total consumer surplus of cohort 1 which is the sum of the expected net utility of the consumers who buy product q_1 in period 1 and the expected net utility of cohort 1 consumers who buy a product of quality $q_i, i = 1, 2$, in period 2.[9] For the computations of these measures, we have to return to the parameter regimes in Proposition 1, since each of them corresponds to a different allocation of consumers to products and periods and to a different first period price of the incumbent. Denote $E[CS_r^k(\widetilde{\rho})]$ the interim consumer surplus with $k = Inno$ (NoIn) standing for the (no) innovation subgame and $r \in \{I,\ II.1,\ II.2\}$ standing for the respective parameter regime.[10]

If $0 \leq \Delta \leq \frac{1}{\widetilde{\rho}\delta}$,all consumers in $[a-h, a+h]$ buy the incumbent's product in the first period. Therefore, for given (updated) expectations $\widetilde{\rho}$, the *interim* consumer surplus for cohort 1 is the same whether there is an innovation or not, i.e.

$$E[CS_I^{Inno}(\widetilde{\rho})] = E[CS_I^{NoIn}(\widetilde{\rho})] = \frac{1}{2h}\int_a^b ((1+\delta)\theta q_1 - (1+\delta-\widetilde{\rho}\delta)(a-h)q_1)d\theta.$$

For $\frac{1}{\widetilde{\rho}\delta} < \Delta \leq \widehat{\Delta}$, all consumers of cohort 1 buy product q_1 and pay $p^* = (1+\delta-\widetilde{\rho}\delta)(a-h)q_1 - 2h(\widetilde{\rho}\delta(q_2-q_1)-q_1)$, this yields

$$E[CS_{II.1}^{Inno}(\widetilde{\rho})] = E[CS_{II.1}^{NoIn}(\widetilde{\rho})] = \frac{1}{2h}\int_a^b ((1+\delta)\theta q_1 - p^*)d\theta.$$

Finally, in the second part of parameter region II the incumbent's optimal first period price is $(1+\delta-\widetilde{\rho}\delta)(a-h)q_1 - \frac{(a-h)q_1}{2}$ and consumers with a valuation in $[\widetilde{\theta}(p^*), a+h]$ decide to wait for the second period. Thus, the launch of a new product generates a consumer surplus of

$$E[CS_{II.2}^{Inno}(\widetilde{\rho})] = \frac{1}{2h}\int_{a-h}^{\widetilde{\theta}(p^*)} ((1+\delta)\theta q_1 - p^*)d\theta + \frac{1}{2h}\int_{\widetilde{\theta}(p^*)}^{a+h} \delta(\theta q_2 - (a-h)(q_2-q_1))d\theta.$$

[9] Note that consumers of cohort 2 are not affected by any preannouncements and can thus be left out of the welfare considerations.

[10] The term 'interim' refers to the moment after the signalling of the entrant but before the possible introduction of a new product.

If the innovation is not introduced, we have

$$E[CS_{II.2}^{NoIn}(\widetilde{\rho})] = \frac{1}{2h}\int_{a-h}^{\widetilde{\theta}(p^*)}((1+\delta)\theta q_1 - p^*)d\theta + \frac{1}{2h}\int_{\widetilde{\theta}(p^*)}^{a+h}\delta(\theta q_1 - (a-h)q_1)d\theta.$$

Before proceeding to the *ex ante* measures, some comments on the *interim* surplus functions are in order. First, it is straightforward to show that if there is a new product, more certainty about it is always beneficial to the consumers, i.e. $E[CS^{Inno}(\rho'')] > E[CS^{Inno}(\rho')]$ for $\rho'' > \rho'$. Higher innovation expectations entail a higher threat of entry and thus a lower first period price of the incumbent, an effect that in the following will be referred to as the *contestability effect*. Moreover, in the innovation event a higher $\widetilde{\rho}$ means that the purchase decision is based on better information (since the true probability of the launch of a new product is 1) and there is less scope for a mismatch between consumers and products.

Nevertheless, in the no innovation event, better information (a lower $\widetilde{\rho}$) is not always beneficial. Figure 2.3 below depicts the interim consumer surplus in the innovation and in the no innovation event. High expectations in the no innovation event lead to a mismatch of consumers to periods and a loss of consumer rent due to waiting. But once this mismatch is eliminated (which is the case in the regimes I and $II.1$ since the incumbent preempts the market), the negative effect of the decreasing threat of entry (implying a higher first period price) is dominating and the consumer surplus decreases for lower $\widetilde{\rho}$.

With this in mind, let us now turn to the calculation of the optimal preannouncement probability β^{opt}. Assume β^{opt} can be implemented exogenously by a social planner that is maximising the *ex ante* expected consumer surplus. To compute this measure, one has to take into account three different events and the respective beliefs held by the incumbent and the consumers. First, with probability $\rho_0\beta$, there will be an innovation that is preannounced in period 0. Secondly, with probability $\rho_0(1-\beta)$, a new product is introduced but not preannounced and finally, with the remaining

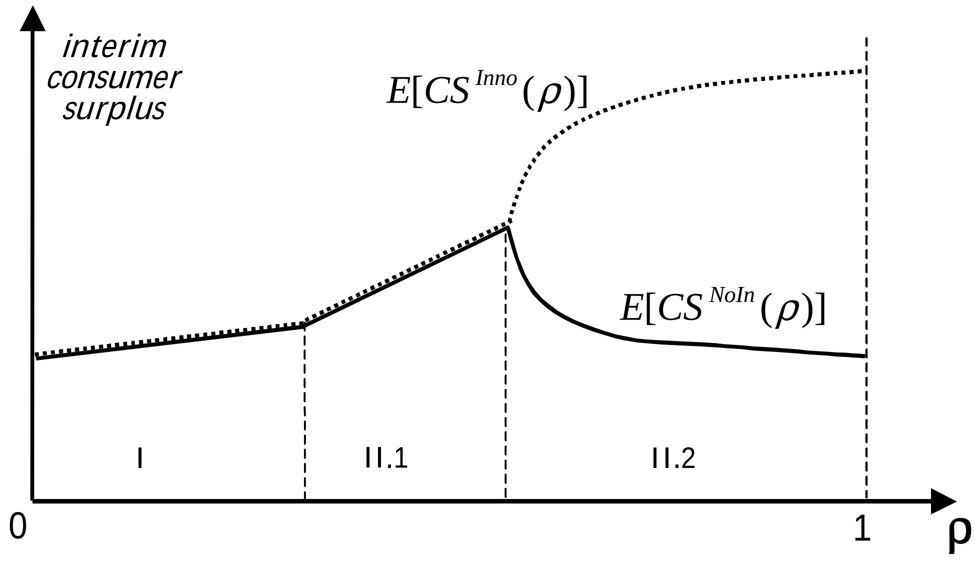

Figure 2.3: *Expected (interim) consumer surplus*

probability $(1-\rho_0)$ there will be no innovation. The absence of a preannouncement given any β leads to the following Bayesian' belief updating

$$\begin{aligned}\widetilde{\rho}(\beta) &= \frac{prob(innovation/no\ announcement)}{prob(innovation/no\ announcement)+prob(no\ innovation)} \\ &= \frac{\rho_0(1-\beta)}{\rho_0(1-\beta)+1-\rho_0} = \frac{\rho_0(1-\beta)}{1-\rho_0\beta}.\end{aligned}$$

Thus, the *ex ante* expected consumer surplus can be written as follows

$$E[CS(\beta)] \equiv \rho_0\beta E[CS^{Inno}(1)] + \rho_0(1-\beta)E[CS^{Inno}(\frac{\rho_0(1-\beta)}{1-\rho_0\beta})] + (1-\rho_0)E[CS^{NoIn}(\frac{\rho_0(1-\beta)}{1-\rho_0\beta})]. \tag{2.13}$$

The announcement probability β enters this surplus function in two ways. In case of an innovation, it influences the relative weight of preannouncement versus pooling equilibria. In this respect, the effect of increasing β is - ceteris paribus - always positive. But it also appears as a measure of the contestability of the market if no preannouncement occurs, since it determines the belief updating. A higher β implies that consumers and the incumbent ascribe the absence of an announcement more to the possibility that there is no innovation since $\widetilde{\rho}(\beta)$ decreases in β. And - as discussed above - lower innovation expectations might have a negative impact on $E[CS^{NoIn}]$.

We demonstrate in the appendix that these two opposed effects generate a surplus function that has two local maxima over a large range of parameters. One at $\beta = 1$ which takes advantage of the benefits of an announced innovation and one at a lower level of β which relies on keeping up the threat of entry and thus reducing the market power of the incumbent in the case of no innovation. The result of the social planner solving the programme $max_\beta E[CS(\beta)]$ is explicitly derived in the appendix and given in the following proposition. Define

$$\begin{aligned}
\Delta_0 &\equiv \frac{1}{\delta}, \\
\Delta_1 &\equiv \frac{4h + \sqrt{\rho_0^2(a-h) + 16h^2(1-\rho_0)^2}}{4\rho_0\delta h} \text{ and} \\
\Delta_2 &\equiv \frac{a+3h}{4\rho_0\delta h}.
\end{aligned}$$

Then,

Proposition 3 *If $\Delta \leq \Delta_0$, consumers are indifferent between all β in [0,1]. Otherwise, the preannouncement probability that maximises the expected consumer surplus (2.13), is given by*

$$\beta^{opt} = \begin{cases} 1 & \text{if } \Delta_0 \leq \Delta \leq \Delta_1, \\ 0 & \text{if } \Delta_1 < \Delta \leq \Delta_2, \\ 1 - \frac{(1-\rho_0)(a+3h)q_1}{\rho_0[4\delta h(q_2-q_1)-(a+3h)q_1]} & \text{if } \Delta > \Delta_2. \end{cases}$$

Figure 2.4 below sketches the graph of the efficient preannouncement probability β^{opt} as a function of the innovation step size Δ. For $\Delta \leq \Delta_0$, the overall value of the new technology is too small to make a difference. Consumers will choose the old product independently of their beliefs about an innovation. By contrast, for larger innovation steps, the result of Proposition 3 reflects the trade off between information transmission leading to a better match between consumers and products and the contestability of the market implying low first period prices in the no innovation event. For rather low values of Δ, the threat of entry is not sufficient to force the incumbent to set a low first period price. On the other hand, a high Δ increases the value of a good match between products and consumers. Therefore, only intermediate values of Δ make it optimal to choose a low announcement probability β in order to let consumers benefit from a contestable market.

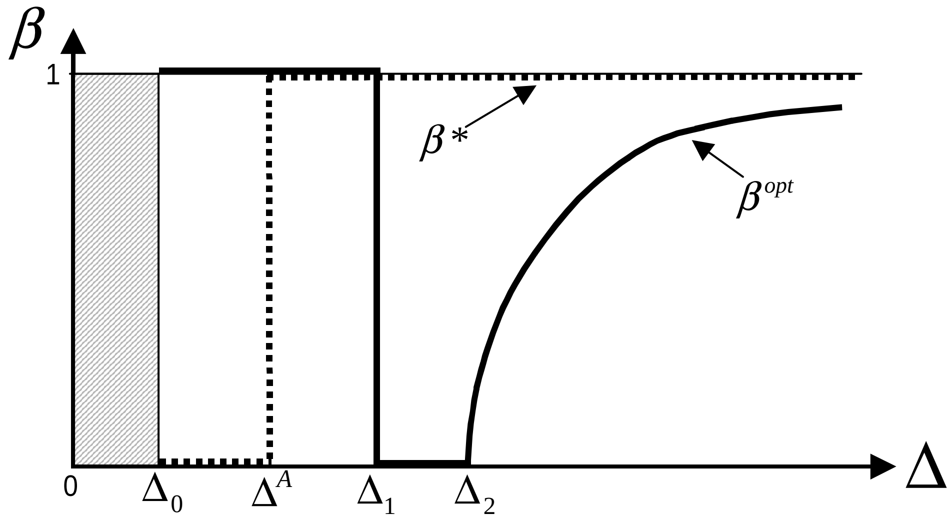

Figure 2.4: *Equilibrium (β^*) and efficient (β^{opt}) announcement probability*

Now, we are in the position to compare the efficient preannouncement with the market outcome described in Proposition 2. Since it turns out that $\Delta_0 \leq \Delta^A \leq \Delta_1$ we obtain

Corollary 3 *The market outcome, as described by Proposition 2, can yield efficient preannouncement, excessive announcement and excessive pooling.*

This overprovision result is due to the fact that from an *ex ante* point of view, the innovative firm only considers the impact of the preannouncement on the market outcome if the new product can actually be launched. She does not take into account the effect of her preannouncement behaviour if there is no innovation, i.e. the potential entrant is not concerned about the contestability of the market. In a situation, in which a preannouncement is optimal, its absence generates the certainty that there is no upcoming innovation and without the threat of entry, the incumbent firm regains market power and can extract more consumer surplus. Thus, the innovating firm confers a negative externality to the consumers and tends to make too many preannouncements.

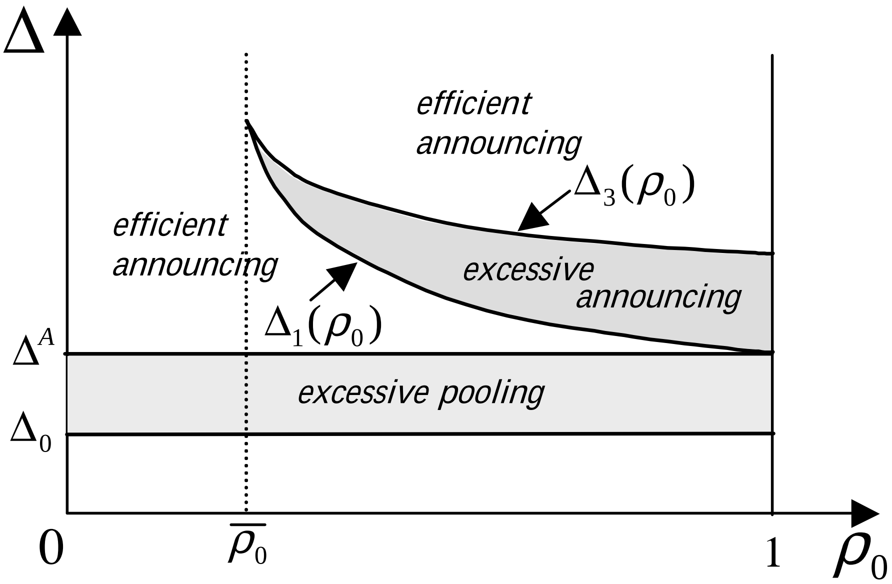

Figure 2.5: *Excessive announcing and pooling in the* $\Delta - \rho_0$*-space*

Finally, let us compare the market outcome with two scenarios that are somehow more realistic than the randomising choice of a social planner considered above. First, consider a full information scenario (FIS), in which the result of the R&D process is common knowledge in the economy. One might think, for example, of a law that forces innovating firms to preannounce their new product at least a minimum time before the actual launch. Or perhaps of an omniscient innovation agency that is able to gather and spread all information about upcoming product launches. Formally, this means that the incumbent and the consumers can update their priors correctly and we get

$$E[CS^{FIS}] = E[CS(1)] = \rho_0 E[CS^{Inno}(1)] + (1-\rho_0)E[CS^{NoIn}(0)]. \tag{2.14}$$

Further, consider a scenario, in which a law prohibits preannouncements of any kind. In such a no information scenario (NIS), consumers would derive a surplus of

$$E[CS^{NIS}] = E[CS(0)] = \rho_0 E[CS^{Inno}(\rho_0)] + (1-\rho_0)E[CS^{NoIn}(\rho_0)]. \tag{2.15}$$

The next corollary compares the consumers' surplus in the full information and the no information scenario. Define

$$\Delta_3 := \frac{2\sqrt{(a-h)^2+16h^2}}{\delta(1+\rho_0)\sqrt{(a-h)^2+16h^2} - \delta\sqrt{(1+\rho_0)^2(a-h)^2+16h^2(1-\rho_0)^2}},$$

then

Corollary 4 *If* $\Delta_1 < \Delta < \Delta_3$, *then* $E[CS^{NIS}] > E[CS^{FIS}]$, *else* $E[CS^{NIS}] \leq E[CS^{FIS}]$.

Figure 2.5 illustrates this result in the $\Delta - \rho_0-$parameter space and relates the efficient consumer policy to the market outcome of Proposition 2. Consumers would be best off if product announcements were banned for all Δ in $[\Delta_1, \Delta_3]$, that means if the upcoming innovation step is intermediate but the innovation probability is rather high. By contrast, if Δ is in $[\Delta_0, \Delta^A]$ the best consumer policy is to enforce preannouncements in order to prevent excessive pooling of the innovating firm. Eventually, in the remaining parameter space, the market outcome coincides with the consumers' optimal choice between FIS and NIS.

2.7 Concluding remarks

This paper analyses the role of innovation preannouncements in markets with imperfect competition and rational, forward looking consumers. For innovative firms, preannouncements are a way to induce potential customers to wait for the innovation instead of buying the presently available technology. But in the presence of an incumbent firm with market power, this advantage has to be traded off with the possibility of a strategic reaction of the latter in the form of preemptive pricing in the pre-entry period. Our model incorporates the intertemporal product choice of consumers under uncertainty into the framework of a vertically differentiated industry and enables us to rationalise some of the findings of an empirical study on product preannouncements by Eliahsberg and Robertson (1988). In analysing interview data of 75 firms, they concluded that preannouncements are more likely if the measure for competitive environment is low, if the upcoming innovation step is large and if consumers are sufficiently forward-looking.

More surprisingly, the welfare analysis of our model shows that, from the *ex ante* point of view of the consumers, the signalling equilibria of the market can lead to under- *and* overprovision of preannouncements. This result is due to the fact that a policymaker has to trade off transmission of information in the economy and the contestability of the pre-innovation market. In this vein, a ban on preannouncements is the best consumer policy in constellations in which the industry is expected to grow fast and the next innovation step is neither too large nor too small. Innovation preannouncements have to be enforced when the new technology is not much better than the old one.

To conclude, let us discuss some limitations of this work that could serve as possible starting point for extensions of the framework. First, we confined our analysis to the case in which the incumbent can not introduce the new technology. This extension would enrich the possible strategic interaction of the model, in particular, it would allow for the use of product preannouncements as a means to deter entry into the industry, a strategy that is often mentioned in relation with Microsoft's product introductions.

A second issue is the exclusion of untruthful preannouncement which enabled us to concentrate on the 'strategic reaction' argument. In the absence of a contract between consumer and preannouncing firm, the costs of waiting for any new technology will be sunk as soon as the firm is not able to introduce the innovation at the promised date. Thus, in a setting with more than two periods, the innovating firm could be tempted to preannounce the product too early to make consumers wait and postpone the launch afterwards. Rational consumers would anticipate this and would no longer believe in announcements without commitment. Therefore, in order to make trustable announcements the innovative firm would additionally need some commitment mechanisms (like reputation or sunk costs) to make consumers wait and this might dilute the communication process between the firm and the market to some degree but the qualitative nature of the effects discussed in this work would not change.

2.8 Appendix

Proof of Lemma 1

The profit function of the monopolist is piecewise concave and continuous. For Lemma **1** to hold it suffices to show that the marginal profit is negative for all $p_1 \geq (a-h)q_1$. For $(a-h)q_1 \leq p_1 \leq xq_1$, the marginal profit is $a+h+x-\frac{4p_1}{q_1}$, which is negative for all $p_1 > \frac{q_1}{4}(a+h+x)$. This threshold is smaller than $(a-h)q_1$ for all x in $[a-h, a+h]$ if and only if $a > 3h$. For $xq_1 \leq p_1 \leq (a+h)q_1$, the marginal profit is $a+h-\frac{2p_1}{q_1}$, which is negative for all $p_1 > \frac{q_1}{2}(a+h)$. This threshold is smaller than xq_1 for all x in $[a-h, a+h]$ if and only if $a > 3h$. Thus, $p_1^* = (a-h)q_1$.¥

Proof of Lemma 2

For $(a-h)q_1 \leq p_1 \leq yq_1$, the marginal profit is $2(a+h)-x-\frac{2p_1}{q_1}$, which is negative for all $p_1 > \frac{q_1}{2}(2a+2h-x)$. This threshold is smaller than $(a-h)q_1$ for all y in $[a-h, a+h]$ if and only if $a > 5h$. For $p_1 > yq_1$, the marginal profit is $2(a+h)-\frac{4p_1}{q_1}$, which is negative for all $p_1 > \frac{q_1}{2}(a+h)$. This threshold is smaller than yq_1 for all y in $[a-h, a+h]$ if and only if $a > 3h$. Thus, if $a > 5h$, $p_1^* = (a-h)q_1$.¥

Proof of Lemma 3

Given the piecewise linear and globally concave demand schedule, the profit function of firm 1 is always single-peaked. The interior maximum for $x\Delta q \leq \Delta p \leq (a+h)\Delta q$ is then defined by $\partial[p_1((x-a+h)+(\frac{\Delta p}{\Delta q}-a))]/\partial p_1 = 0$, which yields

$$p_1 = \frac{1}{2}[p_2 - (2a-2h-x)\Delta q].$$

It is straightforward to check that this value is in the regime range whenever $\widetilde{p}_3 \leq p_2 \leq \widetilde{p}_4$,with $\widetilde{p}_2$, $\widetilde{p}_3$ defined in the text. Equivalently, the interior solution for $\Delta p < x\Delta q$ is given by $\partial[p_1 2(\frac{\Delta p}{\Delta q} - a + h)]/\partial p_1 = 0$ or

$$p_1 = \frac{1}{2}[p_2 - (a-h)\Delta q].$$

This solution holds for all p_2 in $[\widetilde{p}_1, \widetilde{p}_2]$. Finally, taking into account the three possible corner solutions, one can compose the reaction function $R^D_{I,1}(p_2, x)$.

Firm 2's profit function is continuous. In order to show that $R^D_{I,2}(p_1, x) = p_1 + (a - h)\Delta q$, it suffices to show that the marginal profit is negative for $p_2 \geq p_1 + (a-h)\Delta q$. For $(a-h)\Delta q \leq \Delta p \leq x\Delta q$ the marginal revenue is negative if $p_1 > \frac{1}{2}[5h - 3a + x]\Delta q$, which is negative and holds for all non-negative p_1 and all x whenever $a > 3h$. For $\Delta p > x\Delta q$ the marginal revenue is negative if $p_1 > \frac{1}{2}[a + h - 2x]\Delta q$, which is negative and holds for all non-negative p_1 and all x whenever $a > 3h$.

Hence, for $a > 3h$, it is easy to check that $(p_1^* = 0, p_2^* = (a-h)\Delta q)$ is the unique solution to the equation system

$$p_1 = R^D_{I,1}(p_2, x) \text{ and } p_2 = R^D_{I,2}(p_1, x).$$

which proves the lemma.¥

Proof of Lemma 4

Consider firm 1's maximisation problem. The interior maximum for $(a+h)\Delta q \geq \Delta p \geq y\Delta q$ is given by

$$p_1 = \frac{p_2}{2} - \frac{y + a - h}{4}\Delta q$$

and the interior maximum for $\Delta p < y\Delta q$ by

$$p_1 = \frac{1}{2}[p_2 - (a-h)\Delta q].$$

Check that firm 1's profit function has two peaks for p_2 in $[\frac{1}{2}(3x - a - h)\Delta q, (2x - a - h)\Delta q]$. The local maximum value of the smaller maximiser is larger than the local maximum of the larger one whenever $p_2 > \widetilde{p}_6$, with $\widetilde{p}_6$ given in the text.

Consider the maximisation problem of firm 2. For $y\Delta q \geq \Delta p \geq (a-h)\Delta q$, its marginal revenue is negative whenever $p_1 > (4h - y)\Delta q$, which is negative and holds for all non-negative p_1 and all y whenever $a > 5h$. For $\Delta p > y\Delta q$ the marginal revenue is negative if $p_1 > [3h - a]\Delta q$, which is negative and holds for all non-negative p_1 and all x whenever $a > 3h$.

Hence, for $a > 5h$, we have the reaction functions given in the text and from this, it is straightforward to check that $(p_1^* = 0, p_2^* = (a-h)\Delta q)$ is the unique mutually best reply.¥

Proof of Proposition 3

The analysis of the consumer surplus $E[CS(\beta)]$ is rather tedious because dependent on beliefs and other parameters the economy can be in one of the three regimes identified in Proposition 1. We will proceed in two steps. First, we will identify local maxima over the whole parameter range. And then, for all regions where there are more than one, we will pick the global maximiser.

Given a preannouncement probability β, the economy is in region $II.2$ whenever $0 \leq \beta \leq \beta_1 \equiv \frac{\rho_0\delta(q_2-q_1)-q_1}{\rho_0[\delta q_2-(1+\delta)q_1]}$, it is in region $II.1$ if $\beta_1 < \beta \leq \beta_2 \equiv 1 - \frac{(1-\rho_0)(a+3h)q_1}{\rho_0[4\delta h(q_2-q_1)-(a+3h)q_1]}$ and in region I for $\beta_2 < \beta \leq 1$. Note that $\beta_1 > 0$ if $\Delta > \Delta_2$ and $\beta_2 > 0$ if $\Delta > \widehat{\Delta} > \Delta_0$ (Δ_0, $\Delta_2, \widehat{\Delta}$ are defined in the text). Some calculations give the slopes of the ex ante consumer surplus in these three parameter ranges. For region I, one gets

$$\frac{\partial E[CS_I(\beta)]}{\partial \beta} = \begin{cases} 0 & \textit{if } 0 \leq \Delta \leq \Delta_0 \\ 2h[\rho_0[\delta q_2 - (1+\delta)q_1] > 0 & \textit{if } \Delta_0 \leq \Delta \leq \Delta^A \\ \frac{16h^2(\delta\Delta q - q_1)+(a-h)^2}{16h(\delta\Delta q - q_1)} > 0 & \textit{if } \Delta > \Delta^A. \end{cases}$$

Region $II.1$ only exists if $\Delta > \widehat{\Delta}$ thus,

$$\frac{\partial E[CS_{II.1}(\beta)]}{\partial \beta} = \begin{cases} 0 & \textit{if } \widehat{\Delta} \leq \Delta \leq \Delta^A \\ \frac{[4h\delta\Delta q+(a-3h)q_1][(a+3h)q_1-4h\delta\Delta q]\rho_0}{16h(\delta\Delta q-q_1)} < 0 & \textit{if } \Delta > \Delta^A. \end{cases}$$

Finally, in region $II.2$ which exists for $\Delta > \Delta_2$,

$$\frac{\partial E[CS_{II.1}(\beta)]}{\partial \beta} = \frac{\delta^2(a-h)^2(\Delta q)^2 q_1^2(1-\rho_0)^2\rho_0}{16h(\delta\Delta q - q_1)[q_1(1+\rho_0\delta - (1+\delta)\rho_0\beta) - (1-\beta)\rho_0\delta q_2]^2} > 0.$$

At least four conclusions can be drawn from this. First, for $0 \leq \Delta \leq \Delta_0$, the only relevant region is I and in this region the slope is 0, i.e. consumers are indifferent. Second, for $\Delta_0 \leq \Delta \leq \Delta^A$, there is only local maximum at $\beta = 1$ since the slope in region $II.1$ is 0 and positive for higher β in region I. Third, for $\Delta^A \leq \Delta \leq \Delta_2$, two local maxima exist, one at $\beta = 0$ and one at $\beta = 1$. Finally, if region $II.2$ exists for $\Delta > \Delta_2$, the two local maxima are $\beta = \beta_2$ and $\beta = 1$ because $\frac{\partial E[CS(\beta)]}{\partial \beta} > 0$ for all β in $[0, \beta_2]$.

To conclude, we look for the global maximiser in all cases in which we have more than one local maximiser. For $\Delta^A \leq \Delta \leq \Delta_2$, the choice is between $\beta = 0$ and $\beta = 1$. Some calculations yield that $E[CS(1)] \geq E[CS(0)]$ if $\Delta \leq \Delta_1$ with Δ_1 given in the text. For $\Delta > \Delta_2$ we get that $E[CS(\beta_2)] \geq E[CS(1)]$. This completes the proof.¥

Proof of Corollary 3

The only change to the previous proof of Proposition **3** is that for $\Delta > \Delta_2$ we have to compare the local maxima at $\beta = 0$ and $\beta = 1$. Straightforward calculations show that $E[CS(1)] \geq E[CS(0)]$ if and only if $\Delta > \Delta_3 (> \Delta_2)$.¥

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Chapter 3

Two Notes on Models with Endogenous Price Leadership

3.1 Introduction

Price leadership in markets is an ever recurring topic in industrial organisation and competition policy since its first discussion by Stigler (1947) and Markham (1951). These two authors assumed dominant firm markets with one firm controlling at least 50 percent of the industry's output and a price taking fringe of small firms. They concluded that the large firm should have an incentive to commit first to an "umbrella" price for the industry with all small firms following suit. All later work concentrated on extending this dominant firm paradigm to oligopolistic industries where firms realise their interdependence, act strategically and where leadership arises endogenously. Our work takes on this line of research and challenges two of their main findings.

In the first part, we analyse a vertically differentiated industry in which two firms first set qualities and then prices. We show that when one allows for endogenous price leadership it is the high-quality firm that takes the lead and that equilibrium prices are higher compared to simultaneous price setting. Nevertheless, the emergence of price leadership affects the quality choices of firms in the first stage. In particular, less aggressive sequential price setting in the short run increases the incentive of the low-quality firm to invest in quality and therefore to decrease product differentiation in the long run. Consequently, price leadership implies higher prices and higher average product quality which in sum may lead to a net gain for consumers.

In a second model, we investigate the role of firm size as a determinant of price leadership in industries. Previous studies showed that if firm size is measured by capacity then price leadership of the *large* firm arises endogenously because the small capacity firm stands to lose more by being undercut by a large rival. Consequently, the smaller firm should have a stronger preference for assuming the followership role. Deneckere and Kovenock and Lee (1992) claim that the same holds when one identifies firm size with the base of loyal costumers. Our note demonstrates that this result crucially depends on the size of switching costs for the consumers. In Deneckere et al. (1992) it is assumed that switching costs are prohibitive, i.e. once a consumer has bought a brand he will never switch afterwards. Thus, demand consists of consumers that are either locked in at firm 1 or at firm 2 or are not brand loyal at all. The endogenous determination of moves emerges from the fact that the small base firm has a greater incentive to follow in order to undercut and grab the unlocked customers that are in the market (while the large base firm would stand to lose more from undercutting since a low undercutting price would also have to be applied over the larger loyal base). In this work, we reconsider the framework of this model but allow for non-prohibitive switching costs. Once consumers are able to switch the supplier at some finite cost, incentives change because both firms can undercut as well as being undercut. We will show that the threat of being undercut is higher for the firm with the large customer base and that endogenous price leadership arises with the smaller firm as leader. Moreover, we will demonstrate that consumer switching costs give rise to what has been called - but to the best of our knowledge never been found -

in this literature a "marriage in heaven": one firm strictly prefers to lead, the other one to follow. Finally, welfare considerations indicate that over a large range of the parameter space the endogenously emerging price leadership pattern maximises total welfare and that only for rather symmetric customer bases simultaneous price setting would be welfare improving.

Both parts of this paper are related to the steadily growing body of literature on industry leadership that analyses under which conditions firms have incentives to lead, to follow or to move simultaneously. First results were concerned with the nature of competition between firms. It was found that quantity competition induces a struggle for industry leadership while price competition gives firms a preference for followership (cf. Gal-Or (1985), Dowrick (1986)). When the strategy space is extended to quantity-price pairs firms prefer to follow rather than to lead (Boyer and Moreaux (1987)). While we assume price competition in both models below, it is only in the first one that firms display a strict preference to follow.

A second issue is the identity of the industry leader. It has been argued that the firm with

- the larger capacity (Deneckere and Kovenock (1992)),
- the higher costs (Ono (1972), Deneckere and Kovenock (1992)),
- the better information about market conditions (Rotemberg and Saloner (1990), Cooper (1996)) and
- the higher cost variance (Albaek (1990))

can be expected to be the industry leader. In this paper, we add two more firm characteristics to this list, namely, a high product quality and a small loyal customer base.

Finally, from a methodological point of view, the literature on price leadership can be divided in two different approaches to the endogenisation of the leadership structure. The "choice of role" approach systematically introduced by Hamilton and Slutsky (1990) extends the original basic game by adding a preplay stage at which firms simultaneously decide whether to move early or late in the basic game. The basic game is then played according to these timing decisions. The second approach is to explicitly model the dynamic game of price setting as it is demonstrated in Deneckere and Kovenock (1992) or Maskin and Tirole (1988). In this paper, we follow the first approach and claim that our results should also hold in a dynamic version of the model.

This work is organised as follows. In the next section we analyse the "classical" model of vertically differentiated industry. In section 3 we look at the model with consumers' switching costs while the last section is meant for some concluding remarks and comments on the empirical relevance of the two models. The inner organisation of section 2 and 3 is identical. We first characterise price equilibria, then investigate the incentives for an endogenous determination of the timing of price announcements and finally compare the equilibrium outcome with the social optimum. All relevant proofs are delegated to the appendix.

3.2 Price leadership and vertical differentiation

3.2.1 The Model

Consider the following model of vertical product differentiation introduced by Shaked and Sutton (1982). A consumer's utility is described by $U = \theta q_i - p_i$ if he consumes a good of quality q_i and pays price p_i and by 0 otherwise. The parameter θ which measures the taste for quality is uniformly distributed across the population of consumers in $[0, 1]$. Total mass of consumers is normalised to 1.

Suppose there are two firms, 1 and 2, that produce and offer products of quality q_1 and q_2 at prices p_1 and p_2. To simplify the exposition, assume that firms have no production costs. Suppose further that only product qualities in $[0, \overline{q}]$ can be implemented but that this choice is costless.[1]

The time structure of the model is as follows. In the first stage, firms simultaneously choose the quality q_i of their product. In the second stage, firms will set their prices p_i. In order to endogenise the timing of (committed) price quotes we will consider a simple timing subgame in which firms are assumed to choose the point in time of their price announcement. Afterwards, all prices are set accordingly and consumers choose whether to buy product 1, 2 or not all. This overall structure insinuates that quality choices are a firm's long term variable whereas prices can be changed more often. We look for subgame perfect equilibria of the game and solve the model by backward induction.

3.2.2 Price competition

In the last stage, qualities have already been chosen and the sequence of price quotes is agreed upon. In order to analyse the price competition between the two firms, we will proceed in two steps. In the spirit of subgame perfectness, we will first derive the demand functions for any given pair of prices and then solve for a Nash equilibrium in prices. Assume without loss of generality $q_1 > q_2$.

The consumer $\widetilde{\theta}$ who is indifferent between buying product 1 or 2 can be found by solving $\widetilde{\theta}q_1 - p_1 = \widetilde{\theta}q_2 - p_2$, which yields $\widetilde{\theta} = \frac{p_1 - p_2}{q_1 - q_2}$. The consumer that is indifferent between buying the low quality or not all is at $\frac{p_2}{q_2}$. Thus, the demand for the high quality good q_1 consists of all high valuation consumers in $[\widetilde{\theta}, 1]$ while consumers in

[1] This formulation concentrates on the pure product differentiation effects and allows for explicit solutions of all subgames. Introducing linear quality costs does not qualitatively affect any of our results.

$[\frac{p_2}{q_2}, \widetilde{\theta}]$ choose q_2. Hence, demand functions are given by

$$\begin{aligned} D_1(p_1, p_2) &= (1 - \frac{p_1 - p_2}{q_1 - q_2}), \\ D_2(p_1, p_2) &= (\frac{p_1 - p_2}{q_1 - q_2} - \frac{p_2}{q_2}). \end{aligned} \tag{3.1}$$

With no marginal costs, firms maximise $p_i D_i(p_1, p_2)$ with respect to p_i taking the other firm's choice as given. Simple calculations yield the respective reaction functions

$$p_1^R(p_2) = \frac{p_2 + q_1 - q_2}{2} \tag{3.2}$$

and

$$p_2^R(p_1) = \frac{p_1 q_2}{q_1}. \tag{3.3}$$

As one can easily check, prices are strategic complements, i.e. a firm's optimal price increases in its rival's price. Note however, that the low-quality firm's incentive to follow price increases of the high-quality supplier is lower the greater the quality differential.

When considering the endogenous timing of price announcements in this duopoly, three situations can be distinguished. First, firms may set prices simultaneously. Solving the equation system given by (3.2) and (3.3) with respect to (p_1, p_2), we obtain the following Bertrand-Nash-equilibrium in prices

$$p_1^{sim}(q_1, q_2) = \frac{2q_1(q_1 - q_2)}{4q_1 - q_2}, \; p_2^{sim}(q_1, q_2) = \frac{q_2(q_1 - q_2)}{4q_1 - q_2} \tag{3.4}$$

and the corresponding profits

$$\Pi_1^{sim}(q_1, q_2) = \frac{4q_1^2(q_1 - q_2)}{(4q_1 - q_2)^2}, \; \Pi_2^{sim}(q_1, q_2) = \frac{q_1 q_2(q_1 - q_2)}{(4q_1 - q_2)^2}. \tag{3.5}$$

It is obvious that the price and the profits of the high-quality firm increase in the quality differential $(q_1 - q_2)$. By contrast, an increase in product differentiation has a non-monotonous impact on price and profits of the low-quality firm 2, which first increase due to stronger product differentiation but then decrease.

Secondly, if the high-quality firm commits to a price before firm 2, it takes the latter's optimal reaction into account and maximises $p_1 D_1(p_1, p_2^R(p_1))$ with respect to p_1. This yields

$$p_1^{12}(q_1, q_2) = \frac{q_1(q_1 - q_2)}{2q_1 - q_2},\ p_2^{12}(q_1, q_2) = \frac{q_2(q_1 - q_2)}{4q_1 - 2q_2} \tag{3.6}$$

as equilibrium prices and

$$\Pi_1^{12}(q_1, q_2) = \frac{q_1(q_1 - q_2)}{4q_1 - 2q_2},\ \Pi_2^{12}(q_1, q_2) = \frac{q_1 q_2(q_1 - q_2)}{4(2q_1 - q_2)^2} \tag{3.7}$$

as equilibrium profits.

Finally, if the low-quality firm takes the price leadership it maximises $p_2 D_2(p_1^R(p_2), p_2)$ which results in an equilibrium given by

$$p_1^{21}(q_1, q_2) = \frac{(4q_1 - q_2)(q_1 - q_2)}{8q_1 - 4q_2},\ p_2^{21}(q_1, q_2) = \frac{q_2(q_1 - q_2)}{4q_1 - 2q_2} \tag{3.8}$$

and

$$\Pi_1^{21}(q_1, q_2) = \frac{(4q_1 - q_2)^2(q_1 - q_2)}{16(2q_1 - q_2)^2},\ \Pi_2^{21}(q_1, q_2) = \frac{q_2(q_1 - q_2)}{16q_1 - 8q_2}. \tag{3.9}$$

Let us compare prices and returns of the two firms in the different settings

Remark 1 *For given qualities* (q_1, q_2)*, we obtain*

$$p_1^{12}(q_1, q_2) > p_1^{21}(q_1, q_2) > p_1^{sim}(q_1, q_2)$$

and

$$p_2^{12}(q_1, q_2) = p_2^{21}(q_1, q_2) > p_2^{sim}(q_1, q_2).$$

For the firms' profits, i=1, 2, j=3-i, we get:

$$\Pi_i^{ji}(q_1, q_2) > \Pi_i^{ij}(q_1, q_2) > \Pi_i^{sim}(q_1, q_2).$$

The high-quality firm can charge the highest price when it moves first and the lowest when prices are set simultaneously. The low-quality firm sets the same price as leader or follower but a lower price whenever they set prices simultaneously. By consequence, we get, when comparing equilibrium profits, the well-known result that firms prefer to follow rather than to take the lead under price competition (Gal-Or (1985), Dowrick (1986)). Nevertheless, they also prefer both to take the lead instead of setting prices simultaneously.

3.2.3 Endogenous Price Leadership

With the results from the previous section, we can now investigate the endogenous timing of price quotes. The standard approach to this problem is to introduce a preliminary stage in which firms are assumed to choose their "roles" in the pricing game. Assume that only two dates of price quoting are possible, t_0 and t_1, and that firms simultaneously commit to one of these two dates (see Hamilton and Slutsky (1990) or Robson (1990) for a more detailed description of this game). Figure 3.1 gives the payoff matrix of this coordination game.

firm 1 \ *firm 2*	*sets price in t_0*	*sets price in t_1*
sets price in t_0	$\Pi_1^{sim}(q_1,q_2)$ $\Pi_2^{sim}(q_1,q_2)$	$\Pi_1^{12}(q_1,q_2)$ $\Pi_2^{12}(q_1,q_2)$
sets price in t_1	$\Pi_1^{21}(q_1,q_2)$ $\Pi_2^{21}(q_1,q_2)$	$\Pi_1^{sim}(q_1,q_2)$ $\Pi_2^{sim}(q_1,q_2)$

Figure 3.1: *Pay-off matrix of the timing game*

It is then straightforward to check that

Lemma 1 *Sequential price moves are the only two Nash equilibria in pure strategies of the timing game.*

The proof of this lemma is obvious from Remark 1 in the preceding section. Firms would never choose the same date since waiting or postponing the price quote always yields higher profits. Further, any unilateral deviation from leadership or followership is never profitable. Therefore, this game has two strict Nash equilibria for all possible product qualities. Standard refinements like perfectness, properness, or strategic stability do not select among strict Nash equilibria. In addition to this, each firms prefers the equilibrium in which it follows, thus none of the two equilibria Pareto dominates the other one. Nevertheless, there is one solution concept that is typically used to select among equilibria in this kind of situation, the *risk dominance criterion* introduced by Harsanyi and Selten (1988). This criterion selects equilibria by defining a measure for the "riskiness" of equilibrium points. This is done by calculating for each equilibrium the gains each player can make by predicting correctly that the other player will play the respective equilibrium strategy instead of predicting wrongly (and reacting optimally to this false prediction). Then, the risk dominance criterion states that the equilibrium with the highest product of the players' gains is to be chosen. Besides the intuition and axiomatisation provided by Harsanyi and Selten (1988), there are several reasons given in the literature why risk dominance could be considered a good equilibrium selection criterion. Perhaps the most persuasive one is that it has well performed in experimental analysis (see Cabrales and Garcia-Fontes and Motta (2000) and references therein). The proof of Proposition 1 is given in the appendix.

Proposition 1 *For all qualities q_1, q_2, the risk dominance criterion selects the equilibrium with the high-quality firm setting its price at t_0 and the low-quality firm setting its price in t_1.*

Before moving to the quality choices in the first stage of the game, some comments on the coordination game that we used above are in order. We want to argue that this coordination game should be considered as a strategic equilibrium selection game. To understand why, take the above model but look at a two stage 'waiting game' for the two firms. In the first stage, firms have the choice between either setting a price that they can not change afterwards or waiting for the second period. In $t = 2$, all firms that have not set a price in the first stage can do so in this period. After stage 2, consumers get to know the prices and decide which product to buy. Finally, profits are realised. This type of waiting game seems to be a natural candidate for modelling price leadership but it has the inconvenient property that all possible orders of moves can be sustained as an equilibrium outcome, namely simultaneous price setting in period 1, the high-quality firm as price leader or the low-quality firm as price leader. Moreover, it turns out that prices and profits in these equilibria coincide with the price equilibria of the previous section. Therefore, given that equilibrium selection is needed anyway, we chose to do it in the strategical model proposed by Hamilton and Slutsky (1990).

3.2.4 Quality competition

In the first stage of the game, firms simultaneously choose their qualities without further costs and anticipate the endogenous price setting order in the second stage. It is easy to check that the high quality supplier always gains more than the low quality firm. Thus, in our symmetric setting both firms would like to choose the highest possible quality $\overline{q}$. But since identical qualities would entail zero profits, one gets two perfectly symmetric Nash equilibria. One in which firm 1 offers $\overline{q}$ and firm 2 a best response to it and one with reversed roles. Let us assume, without loss of generality that firm 1 is the high-quality supplier and offers the highest possible quality $\overline{q}$. Firm 2 responds optimally to this quality by maximising $\Pi_2^{12}(q_1 = \overline{q}, q_2)$

from (3.7) with respect to q_2. It follows that

Proposition 2 *In a subgame perfect Nash equilibrium, firm 1 chooses* $q_1^* = \overline{q}$ *and firm 2 chooses* $q_2^* = \frac{2}{3}\overline{q}$.

The low-quality firm has an incentive to evade price competition by increasing product differentiation. In the subgame perfect equilibrium in which the high-quality firm is price leader in the second stage, the low-quality firm optimally chooses a quality that is $\frac{2}{3}$ of the highest possible quality. In order to compare this equilibrium outcome with situations in which another price setting order would be implemented in the second period, we will give the optimal quality choices for these subgames in

Remark 2 *If firms were to choose prices simultaneously in the second stage, the low-quality firm would choose* $q_2^{sim} = \frac{4}{7}\overline{q} \approx 0,571\overline{q}$ *and if the low quality supplier would act as price leader, it would set* $q_2^{21} = (2 - \sqrt{2})\overline{q} \approx 0,585\overline{q}$ *in the first stage.*

Thus, with simultaneous price setting in the short run, firms choose the highest degree of product differentiation since price competition is most intense. If they anticipate that they will set prices sequentially, they will decrease product differentiation with the low-quality firm choosing a higher quality. In this sense, average product quality is maximised when firms can agree on price leadership of the high-quality firm.

Interestingly, the different quality choices under the different price setting orders in the second period lead to a change in the price and profits ranking of Remark 1. Plugging the optimal qualities into the second period prices, we now get

Remark 3 *Comparing equilibrium prices under the different price setting regimes with endogenous qualities yields*

$$p_1^{12}(1, q_2^{12}) = p_1^{21}(1, q_2^{21}) = p_1^{sim}(1, q_2^{sim})$$

and

$$p_2^{21}(1, q_2^{21}) > p_2^{12}(1, q_2^{12}) > p_2^{sim}(1, q_2^{sim}).$$

Now compare Remark 1 and 3. For given exogenous qualities the high-quality charged higher prices under sequential moves. But this price difference is outweighed when quality is endogenised and firms anticipate the price equilibrium they play in the short run. This effect beomces important when looking at welfare in the next section.

3.2.5 Welfare

Let us finally assess the consequences of the above results for the economy's welfare. In particular, we are interested in comparing the welfare levels for the overall game for the different orders of move in the second stage. Define consumer surplus CS as the sum of all consumers' utility. Thus,

$$CS(p_1, p_2, q_1, q_2) = \int_{\frac{p_2}{q_2}}^{\widetilde{\theta}} (\theta q_2 - p_2) d\theta + \int_{\widetilde{\theta}}^{1} (\theta q_1 - p_1) d\theta. \tag{3.10}$$

The total welfare W of the economy is the sum of consumer surplus and the firms' profits,

$$W = CS + \Pi_1 + \Pi_2 \tag{3.11}$$

Proposition 3 compares these measures for the cases of sequential and simultaneous price setting in the second period.

Proposition 3 *It holds that*

$$CS^{1,2} > CS^{sim} > CS^{2,1}$$

and

$$W^{2,1} > W^{sim} > W^{1,2}.$$

Proposition 3 states our main result. Price leadership in the short run can be beneficial to consumers and society when it affects firms' long term variables which are in this case product qualities. More specifically, consumers are best off under the derived subgame perfect Nash equilibrium, i.e. in the equilibrium in which the high-quality firm takes the lead in the price setting game. The loss in the intensity of price competition through sequential moves is made up by the increased average product quality in the economy. It is easy to see from Proposition 3 (but also derived in the appendix) that the sum of the firms' profits is higher in the two other price subgames. With equal weights for consumers' and firms' rents, the firms' losses outweigh the consumers' gains and the overall economy would be better off if firms chose their prices simultaneously or if the low quality firm took the lead.

3.3 Price leadership in markets with consumers' switching costs

3.3.1 The Model

We consider an industry with two (active) firms, $i = 1, 2$, that produce a non-storable good at a constant and identical unit cost which, for simplicity, is assumed to be zero. Demand is derived from a continuum of consumers with mass 1 who are differentiated along two dimensions, purchase history and brand switching costs. In particular, we will assume that a fraction α of all consumers has previously bought firm 1's product while the remaining $(1 - \alpha)$ consumers purchased firm 2's product. In all what follows, we will consider firm 1 as the firm with the large customer base, thus we assume that $\frac{1}{2} \leq \alpha < 1$. Although the two products are functionally identical, previous consumption of one of the goods creates some kind of lock-in and we assume that consumers differ in their costs s of switching from the previously bought brand to the new one. More precisely, let us suppose that the switching costs of consumers are uniformly distributed on the interval $[0, \sigma]$, with $\sigma > 0$. Hence, for any s' in $[0, \sigma]$, the mass of brand 1 consumers whose switching costs are not larger than s' is given by $\frac{1}{\sigma}\alpha s'$ and $\sigma/2$ is the average switching cost in the industry. Figure 3.2 below illustrates the initial consumer distributions.

In the considered time span, each consumer buys one unit of the product. If he opts to buy his previous brand again, he gets a net utility of

$$U_{ii} = -p_i, \tag{3.12}$$

where $i = 1, 2$ and p_i is the price that firm i charges. If a consumer with switching costs s who has previously bought brand i switches to brand j, $i \neq j$, he gets

$$U_{ij}(s) = -p_j - s. \tag{3.13}$$

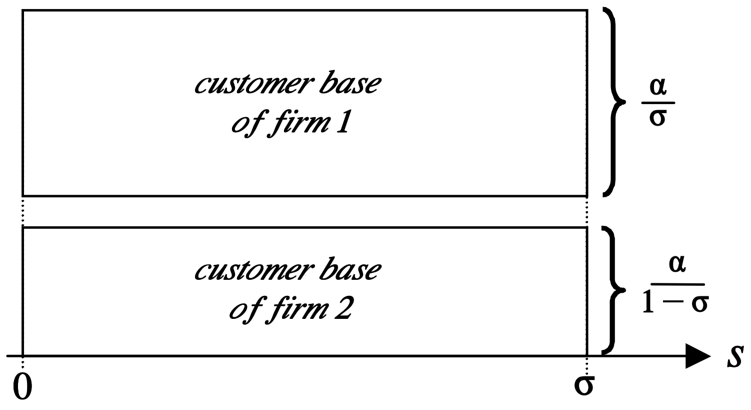

Figure 3.2: *Initial consumer distribution*

We assume that α, σ are common knowledge but that firms are unable to determine to which group any given consumer belongs, i.e. interpersonal price discrimination with respect to s is not possible.

The time structure of the model is as follows. First, we endogenise the price leadership by assuming a timing subgame in which firms choose the time of their price commitment. Then, firms set their prices accordingly and finally, consumers choose one of the two brands or the outside option of value 0. We look for subgame perfect Nash equilibria of this game and solve the model backwards.

We will start by deriving the demand for both firms for any given prices p_1, p_2. If one firm sets a lower price than its rival, it will attract some or all of its rival's previous customers. The size of this flow depends on the price difference, the average switching costs and the size of the customer base of the firm with the higher price. Suppose that $p_1 < p_2$, then firm 1 will induce all his old customers to repeat purchases and get all customers from firm 2's base with switching costs $s \leq p_2 - p_1$. Equivalently, if firm 1 charges a higher than firm 2, it will lose customers from his base with switching costs lower than $p_1 - p_2$. Thus, firm 1's demand is

$$D_1(p_1, p_2) = \begin{cases} 1 & \textit{if } p_1 < p_2 - \sigma, \\ \alpha + (1-\alpha)\frac{p_2 - p_1}{\sigma} & \textit{if } p_2 - \sigma \leq p_1 \leq p_2, \\ \alpha - \alpha\frac{p_1 - p_2}{\sigma} & \textit{if } p_2 \leq p_1 \leq p_2 + \sigma, \\ 0 & \textit{if } p_1 > p_2 + \sigma, \end{cases} \tag{3.14}$$

while firm 2 serves the remaining customers and has a demand of

$$D_2(p_1, p_2) = \begin{cases} 1 & \textit{if } p_2 < p_1 - \sigma, \\ (1-\alpha) + \alpha\frac{p_1 - p_2}{\sigma} & \textit{if } p_1 - \sigma \leq p_2 \leq p_1, \\ (1-\alpha)(1 - \frac{p_2 - p_1}{\sigma}) & \textit{if } p_1 \leq p_2 \leq p_1 + \sigma, \\ 0 & \textit{if } p_2 > p_1 + \sigma. \end{cases} . \tag{3.15}$$

Note that both firms' demand schedules are continuous and piecewise linear. Interestingly, firm 1's demand is concave while firm 2's demand is convex. This is due to the fact that at equal prices $p_1 = p_2$, a price cut from firm 1 attracts less consumers from the small base firm than a similar price cut of firm 2 would do. Or, equivalently, price increases have a stronger effect on both firms' demand as long as the marginal consumer belongs to the large firm's customer base.

3.3.2 Simultaneous Price Setting

With the above demand functions, we can now proceed to determine the optimal price $R_i(p_j)$ that firm i sets in response to any price p_j of its rival. Thus, we have to solve for both firms, $i = 1, 2$, the following maximisation problem

$$\max_{p_i} p_i D_i(p_i, p_j). \tag{3.16}$$

Consider the problem of firm 1 and check that it follows from (3.14) that we have four (possible) local maxima that can qualify for a global optimum. The interior solution for all $p_2 - \sigma \leq p_1 \leq p_2$ is given by

$$R_1^1(p_2) \equiv \frac{\alpha\sigma + (1-\alpha)p_2}{2(1-\alpha)},$$

with a maximum value of $\frac{(\alpha\sigma+(1-\alpha)p_2)^2}{4(1-\alpha)\sigma}$. The interior, local maximiser for all $p_2 \leq p_1 \leq p_2 + \sigma$ is at

$$R_1^2(p_2) \equiv \frac{\sigma + p_2}{2},$$

which implies profits of $\frac{\alpha(\sigma+p_2)^2}{4\sigma}$. Further, we have two (candidate) corner solutions at $p_1 = p_2 - \sigma$, i.e. when firm 1 grabs the whole market and at $p_1 = p_2$ where no consumers switches. It follows from straight comparison of the respective local maximum values that firm 1's reaction function consists of four parts. If $p_2 \leq \sigma$, firm 1 optimally chooses a price that is higher than the price of his rival and lets some of his previous consumers switch. For intermediate values of p_2, it just meets the price of firm 2 and keeps all his previous customers. If firm 2's price is higher than $\frac{\alpha\sigma}{(1-\alpha)}$, then it undercuts its rival and attracts some customers from its base and finally, if p_2 exceeds $\frac{2-\alpha}{1-\alpha}\sigma$, firm 1 grabs the whole market. Hence, we have

$$R_1(p_2) = \begin{cases} R_1^2(p_2) & \textit{if } p_2 \leq \sigma, \\ p_2 & \textit{if } \sigma < p_2 \leq \frac{\alpha}{(1-\alpha)}\sigma, \\ R_1^1(p_2) & \textit{if } \frac{\alpha}{(1-\alpha)}\sigma < p_2 \leq \frac{2-\alpha}{1-\alpha}\sigma, \\ p_2 - \sigma & \textit{if } p_2 > \frac{2-\alpha}{1-\alpha}\sigma. \end{cases} \tag{3.17}$$

Firm 2's maximisation problem is similar, although the solution will be qualitatively different. The local maximiser for p_2 in $[p_1 - \sigma, p_1]$, i.e. if firm 2 undercuts its rival is at

$$R_2^1(p_1) \equiv \frac{(1-\alpha)\sigma + \alpha p_1}{2\alpha}$$

and the local maximiser for $p_1 \leq p_2 \leq p_1 + \sigma$ is given by

$$R_2^2(p_1) \equiv \frac{\sigma + p_1}{2}.$$

Together with the corner solutions $p_2 = p_1 - \sigma$ and $p_2 = p_1$, we again have four candidates for the global maximiser. Straightforward computations give the local maximum values for the interior solutions as $\frac{((1-\alpha)\sigma+\alpha p_1)^2}{4\alpha\sigma}$ and $\frac{(1-\alpha)(\sigma+p_1)^2}{4\sigma}$ respectively. Comparing them with the corner maxima yields

$$R_2(p_1) = \begin{cases} p_{2b}^R(p_1) & \textit{if } p_1 \leq \sqrt{\frac{1-\alpha}{\alpha}}\sigma, \\ p_{2a}^R(p_1) & \textit{if } \sqrt{\frac{1-\alpha}{\alpha}}\sigma < p_2 \leq \frac{1+\alpha}{\alpha}\sigma, \\ p_1 - \sigma & \textit{if } p_1 > \frac{1+\alpha}{\alpha}\sigma. \end{cases} \tag{3.18}$$

By contrast to firm 1's reaction function, the firm with the small customer base has a disontinuity in its best response schedule. At $p_1 = \sqrt{(1-\alpha)/\alpha}\sigma$, it is indifferent between charging a low price and cutting into firm 1's customer base and a rather high price which would entail the loss of some of its own customers. This discontinuity stems from the fact that firm 2's demand is convex in p_2. At equal prices $p_2 = p_1$, a marginal price decrease attracts α consumers from firm 1's base while a marginal price increases makes only 1-α of its own consumers switch. Thus, the kink in $D_2(p_1, p_2)$ makes both the low- and the high-price strategy attractive for the firm with the small customer base. This discussion is summarised with the plot of the reaction functions in Figure 3.3.

Let us now analyse the price equilibrium when firms post their prices simultaneously.

Proposition 4 *When the firms post their prices simultaneously, the unique equilibrium is given by*

$$p_1^{sim} = \frac{1+\alpha}{3\alpha}\sigma \textit{ and } p_2^{sim} = \frac{2-\alpha}{3\alpha}\sigma.$$

The corresponding equilibrium profits are

$$\Pi_1^{sim} = \frac{(1+\alpha)^2}{9\alpha}\sigma \textit{ and } \Pi_2^{sim} = \frac{(2-\alpha)^2}{9\alpha}\sigma.$$

As illustrated in Figure 3.3, the unique intersection of the firms' reaction functions is in the regime where firm 1's price is higher than firm 2's price. This follows directly

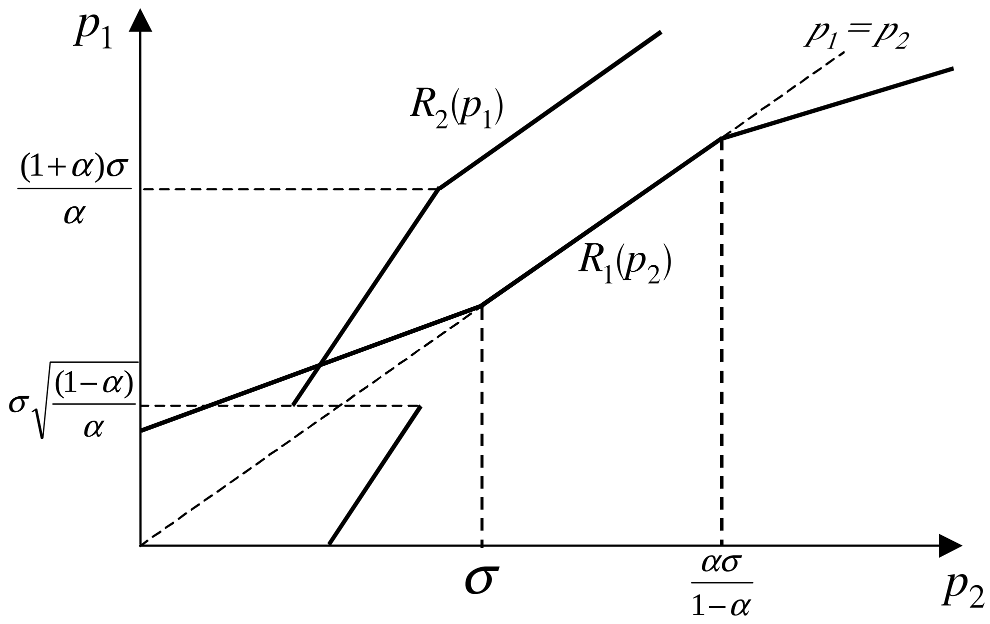

Figure 3.3: *Firms' reaction functions*

from two observations. First, at its discontinuity $R_2(p_1)$ jumps down to a price that is smaller than p_1 but always larger than the value at this point of the inverted reaction function of firm 1. And second, for all $p_1 > p_1^{sim}$, both functions are continuous, with $R_2(p_1)$ never having a steeper slope than the inverted reaction function of firm 1. Therefore, in any equilibrium with simultaneous price moves, $p_1 > p_2$ and consumers with low switching costs will change from firm 1 to firm 2.

Let us look at the interesting comparative statics of this equilibrium. First, notice that equilibrium prices and profits of both firms decrease as the base of the large firm increases. With customer bases of almost equal size, both firms have a stronger incentive to exploit their own base with rather high prices. This strategy becomes less profitable for the smaller firm when markets become unequal and it is more and more tempting to cut into the large customer base of its rival. Hence, unequal market shares foster price competition and hurt both firms.

Second, the difference in equilibrium prices between the large and the small firm is

$$p_1^{sim} - p_2^{sim} = \frac{2\alpha - 1}{3\alpha}\sigma$$

and *increases* when more customers are initially attached to firm 1. This relies on two effects. A larger customer base turns firm 1 into a "fat cat" in the sense that it is more inclined to exploit its locked-in customers with a high price. At the same time, the small firm becomes increasingly aggressive and cuts its price. Taken together, the more unequal the two firms are, the higher the price difference and the more consumers will switch from the large to the small firm. Consequently, the size of the customer bases converges. Nevertheless, check that for all $\alpha \geq 1/2$, the profits of the large base firm are always higher than those of the small base firm and that this difference is also increasing in α.

By contrast, it is straightforward to verify that higher consumers' switching costs hamper competition and help firms to sustain higher prices and profits. And, last but not least, it is noteworthy that we get equilibria in pure strategies for all values of α and σ, which is by no means common in models with simultaneous pricing and consumers' switching costs.

3.3.3 Firm 1 as price leader

When the firm with the larger customer base posts its price first, it takes into account the optimal reaction of his rival from (3.18). If it sets a price lower than $\sqrt{(1-\alpha)/\alpha}\sigma$, the firm with the smaller customer base will respond with a higher price and give up some of its previous consumers with low switching costs. If the posted price of firm 1 exceeds this threshold, its rival will respond with price cutting and consumers switch in the opposite direction. Plugging (3.18) into (3.16) gives the following profit function for firm 1,

$$\Pi_1^{lead}(p_1) \equiv p_1 \begin{cases} \frac{(1+\alpha)\sigma-(1-\alpha)p_1}{2\sigma} & \textit{if } p_1 \leq \sqrt{\frac{1-\alpha}{\alpha}}\sigma, \\ \frac{(1+\alpha)\sigma-\alpha p_1}{2\sigma} & \textit{if } p_1 > \sqrt{\frac{1-\alpha}{\alpha}}\sigma. \end{cases} \tag{3.19}$$

Some remarks on this function which is depicted in Figure 3.4 are in order. First, the demand is more price elastic in the second part of this function, i.e. when firm 1's price will be undercut by its rival. Second, this function has a discontinuity at $p_1 = \sigma\sqrt{(1-\alpha)/\alpha}$, where the follower's price cut leads to a downward jump in demand and profits of the price leader. Finally, note that $\Pi_1^{lead}(p_1)$ has two local maxima, one at a price below or equal to the threshold price, at which the small base firm would start cutting into firm 1's customer base and one for a relatively high price, at which firm 1 would lose some of its customers with low switching costs.

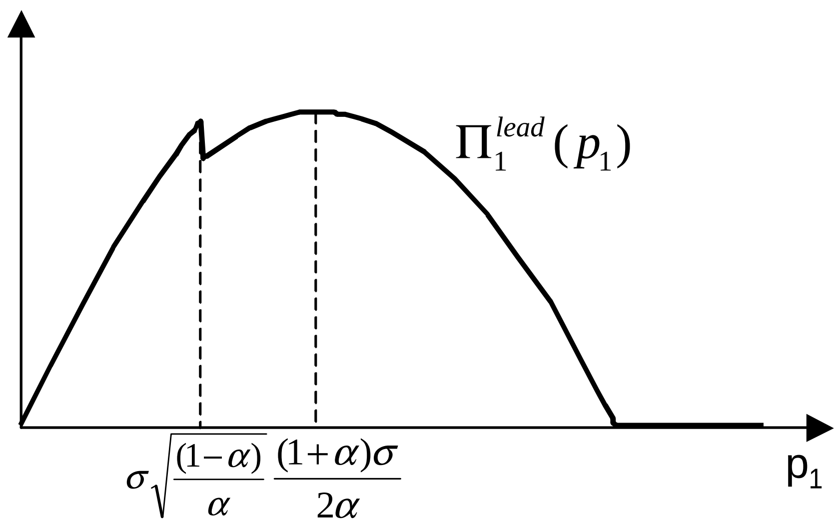

Figure 3.4: *Profits of firm 1 as price leader for* $\alpha = 0,6$ *and* $\sigma = 0,4$.

This trade-off between exploiting its more attached customers with a high price and expanding its customer base with a low price is resolved in the following proposition. The complete proof can be found in the appendix.

Proposition 5 *Assume price leadership of the firm with the larger customer base. Then, the unique price equilibrium is*

$$p_1^{lead} = \frac{1+\alpha}{2\alpha}\sigma \text{ and } p_2^{follow} = \frac{3-\alpha}{4\alpha}\sigma,$$

and the corresponding equilibrium profits are

$$\Pi_1^{lead} = \frac{(1+\alpha)^2}{8\alpha}\sigma \text{ and } \Pi_2^{follow} = \frac{(3-\alpha)^2}{16\alpha}\sigma.$$

Like in the case of simultaneous price setting, the equilibrium is in the regime where the large base firm sets a higher price than the smaller base firm. Hence, consumers switch from firm 1 to firm 2. Moreover, this equilibrium displays similar comparative statics, i.e. prices and profits fall if the base of the larger firm increases and they rise with higher average switching costs of consumers. The price difference is

$$p_1^{lead} - p_2^{follow} = \frac{3\alpha - 1}{4\alpha}\sigma,$$

which again increases with α. Note however, that under price leadership of the large firm, the smaller one can earn higher profits than its rival whenever $\alpha < 4\sqrt{2} - 5 \approx 0.6568$. This is due to the fact that being undercut as a price leader is the more detrimental, the less attached consumers a firm has.

3.3.4 Firm 2 as price leader

Finally, we look at the case where the firm with the smaller installed base posts its price first. Again, the leader takes into account the reaction of the follower. From (3.17) we know that for small prices $p_2 \leq \sigma$, firm 1 will react with a higher price and exploit its old customers with high switching costs. For intermediate prices, it will exactly meet the leader's price and induce repeat purchases from its customer base. For sufficiently high prices of the leader, the large base firm 1 will undercut and attract new customers. Therefore, firm 2's profit function as a price leader is

$$\Pi_2^{lead}(p_2) = p_2 \begin{cases} \frac{(2-\alpha)\sigma-\alpha p_2}{2\sigma} & \textit{if } p_2 \leq \sigma, \\ 1-\alpha & \textit{if } \sigma < p_2 \leq \frac{\alpha}{(1-\alpha)}\sigma, \\ \frac{(2-\alpha)\sigma-(1-\alpha)p_2}{2\sigma} & \textit{if } p_2 > \frac{\alpha}{(1-\alpha)}\sigma. \end{cases} \tag{3.20}$$

As depicted in Figure 3.5, this function has no discontinuity and is concave. Further it is easy to check that, by contrast to (3.19), the small firm's demand as price leader is more price elastic when its own price is low, i.e. when firm 2 cuts into its rival's large customer base and consumers switch from 1 to 2. For higher prices, price increases induce the loss of firm 2's own, small customer base.

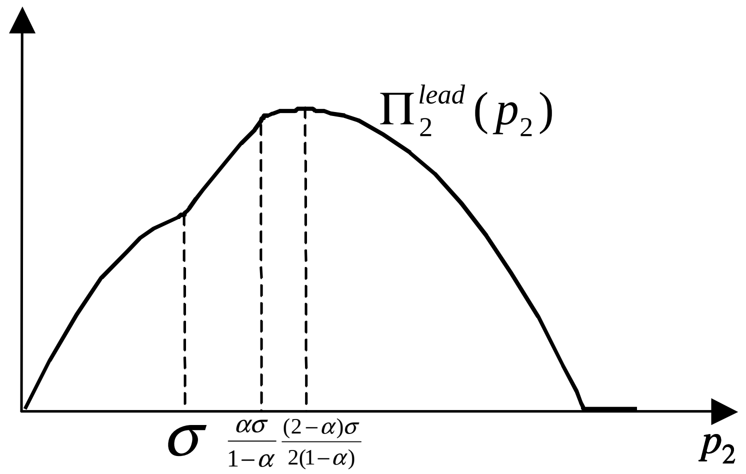

Figure 3.5: *Profits of the small base firm for* $\alpha = 0,6$ *and* $\sigma = 0,6$.

Again, the price leader considers two possible strategies. Either quoting a low price and forcing the follower to yield some of its customers or to post a high price and giving up its own customers with low switching costs. Solving the corresponding maximisation problem for firm 2 entails the characterisation of local maxima and the identification of a global maximum. This is delegated to the appendix, the result is given in

Proposition 6 *Assume price leadership of the firm with the smaller customer base. Then, we get as unique price equilibria*

$$p_1^{follow} = \begin{cases} \frac{(2+\alpha)}{4(1-\alpha)}\sigma & \text{if } \alpha \leq 2/3, \\ \frac{\alpha}{(1-\alpha)}\sigma & \text{if } \alpha > 2/3 \end{cases} \quad \text{and } p_2^{lead} = \begin{cases} \frac{(2-\alpha)}{2(1-\alpha)}\sigma & \text{if } \alpha \leq 2/3, \\ \frac{\alpha}{(1-\alpha)}\sigma & \text{if } \alpha > 2/3. \end{cases}$$

The corresponding equilibrium profits are

$$\Pi_1^{follow} = \begin{cases} \frac{(2+\alpha)^2}{16(1-\alpha)}\sigma & \text{if } \alpha \leq 2/3, \\ \frac{\alpha^2}{(1-\alpha)}\sigma & \text{if } \alpha > 2/3 \end{cases} \quad \text{and } \Pi_2^{lead} = \begin{cases} \frac{(2-\alpha)^2}{8(1-\alpha)}\sigma & \text{if } \alpha \leq 2/3, \\ \alpha\sigma & \text{if } \alpha > 2/3. \end{cases}$$

The equilibrium with the small firm as a price leader is qualitatively different from the two others described above. First, in equilibrium, the large base firm never sets a higher price than its rival, i.e. the large firm will not lose customers. While for rather small differences in the size of the customer base (an α close to 1/2), the following, large firm has an incentive to cut into the small firm's base, a large initial base makes firm 1 "fat" and he will just meet the leader's price to retain his customers. By this, the price difference is in this case

$$p_1^{follow} - p_2^{lead} = \begin{cases} \frac{(3\alpha-2)}{4(1-\alpha)}\sigma & \text{if } \alpha \leq 2/3, \\ 0 & \text{if } \alpha > 2/3, \end{cases}$$

which is decreasing in absolute terms if the customer base advantage of firm 1 increases. However, notice that increasing firm 1's base α and increasing the switching costs σ have a positive effect on prices and profits of both firms. In the following section, we will compare prices and profits of the three pricing subgames and analyse the endogenous choice of price leadership.

3.3.5 Endogenous Price Leadership

In this section, we will examine the endogenous determination of the timing of price announcements. But let us first compare the results of the three price subgames derived in the previous section. The following lemma looks at firms' prices

Lemma 2 *If* $\frac{1}{2} \leq \alpha < \frac{\sqrt{7}-1}{3} \simeq 0.5486$, *we get for the firm with the larger customer base that*

$$p_1^{lead} > p_1^{follow} > p_1^{sim}$$

else it holds that

$$p_1^{follow} > p_1^{lead} > p_1^{sim}.$$

For the firm with the smaller base, it always holds that

$$p_2^{lead} > p_2^{follow} > p_2^{sim}.$$

Note first that simultaneous price setting always entails the lowest prices in the economy. Second, being a follower in the price moves makes firms aggressive since they have the last say and can grab the other firm's market. Nevertheless, this price cutting is anticipated by the leader and the more "hungry" the follower, the more cautious the leader will be. Therefore, the small firm will set a higher price as leader than as follower for two reasons: first, it expects a "fat" follower when it leads and second, it is itself aggressive when it follows. For the large base firms, we have to distinguish two cases. For α close to $1/2$, we get the same result as for the small firm, the large firm is aggressive when it follows and sets a higher price when it leads. However, a larger customer base makes the large firm softer and the smaller firm aggressive and the price ranking is just reversed.

This argumentation also carries over to the comparison of the price difference under the different scenarios. Please check that for all relevant α and σ, the price difference is largest in the subgame where firm 1 acts as price leader. In the two remaining cases, the (absolute) price difference is higher under simultaneous price setting whenever α is sufficiently large, otherwise it is higher in the subgame with the small firm as price leader.

As a next step, we compare the firms' profits.

Lemma 3 *For the firm with the larger customer base it always holds that*

$$\Pi_1^{follow} > \Pi_1^{lead} > \Pi_1^{sim}.$$

If $\frac{1}{2} \leq \alpha < 0.5942$, *firm 2 has the following ranking:*

$$\Pi_2^{follow} > \Pi_2^{lead} > \Pi_2^{sim},$$

otherwise it holds that

$$\Pi_2^{lead} > \Pi_2^{follow} > \Pi_2^{sim}.$$

Lemma 3 states that both firms prefer sequential timing to the simultaneous move equilibrium. For a rather small α, both firms would prefer to follow since both of them fear the aggressivity of the other one as a second-mover. But, as mentioned before, a large costumer base makes firm 1 soft and allows the small firm to set a sufficiently high price as first mover without being undercut. Thus, for $\alpha \geq 0.5942$, the small firm strictly prefers to lead in the price game and the large firm strictly prefers to follow.

Let us now proceed to the analysis of the endogenous timing of price announcements as we did in the first part of this chapter. We look at the same coordination subgame in which firms are assumed to choose their "roles". Suppose again that only two dates of price quoting are possible, t_0 and t_1 and that firms simultaneously commit to one of these two dates. It is obvious from the previous lemma that firms would never choose the same date since waiting or postponing the price quote always yields higher profits. But although we have "a marriage in heaven" for high values of α, we will have two strict Nash equilibria. This is due to the fact that we allow in our simple timing game for simultaneous moves which gives firms no incentive to deviate from a sequential equilibria and thus both of them qualify for Nash equilibria.

Nevertheless, we can resort again to the *risk dominance criterion* to select between these two equilibria. The following proposition summarises the equilibrium analysis. The proof of this proposition is given in the appendix.

Proposition 7 *For all* $\alpha \in [\frac{1}{2}, 1]$, *the risk dominance criterion selects the equilibrium with the small customer base firm as price leader.*

The risk dominance criterion unambiguously selects the equilibrium with the small firm as a price leader for both regimes of Lemma 3, i.e. for the case where both firms prefer to follow and for the case where they agree on price leadership of the small firm.

3.3.6 Welfare

Let us finally have a look at welfare in this economy under the different price leadership scenarios. More specifically, we will investigate what price quoting order a social planner would choose and compare it to the market outcome of the previous section. Define consumer utility CS as the sum of all consumers' utility. If $p_1 < p_2$, some previous consumers of firm 2 will switch to firm 1 and CS is

$$\begin{aligned} CS(p_1, p_2)\mid_{p_1<p_2} &= \tfrac{1-\alpha}{\sigma} \int_0^{p_2-p_1} (-p_1 - s)ds + \tfrac{1-\alpha}{\sigma} \int_{p_2-p_1}^{\sigma} (-p_2)ds - \alpha p_1 \\ &= \tfrac{(p_2-p_1)}{2\sigma}(2\alpha\sigma + (1-\alpha)(p_2 - p_1)) - p_2. \end{aligned} \tag{3.21}$$

If $p_1 \geq p_2$ consumers switch from firm 1 to firm 2 and consumer surplus is defined as

$$\begin{aligned} CS(p_1, p_2)\mid_{p_1\geq p_2} &= \tfrac{\alpha}{\sigma} \int_0^{p_1-p_2} (-p_2 - s)ds + \tfrac{\alpha}{\sigma} \int_{p_1-p_2}^{\sigma} (-p_1)ds - (1-\alpha)p_2 \\ &= \tfrac{(p_1-p_2)}{2\sigma}(-2\alpha\sigma + \alpha(p_1 - p_2)) - p_2. \end{aligned} \tag{3.22}$$

We define the total welfare W of the economy as the unweighted sum of consumer surplus and the firms' profits. Since consumers' expenditures become firms' revenues, all what accounts for social welfare is the sum of consumers' switching costs. Thus

W simplifies to

$$W(p_1, p_2) = CS(p_1, p_2) + \Pi_1(p_1, p_2) + \Pi_2(p_1, p_2)$$
$$= \begin{cases} -\frac{(1-\alpha)(p_2-p_1)^2}{2\sigma} & \text{if } p_1 < p_2, \\ -\frac{\alpha(p_1-p_2)^2}{2\sigma} & \text{if } p_1 \geq p_2. \end{cases} \quad (3.23)$$

Define $CS^{sim}(W^{sim})$, $CS^{1leads}(W^{1leads})$ and $CS^{2leads}(W^{2leads})$ as consumer surplus (social welfare) under simultaneous pricing, price leadership of firm 1 and price leadership of firm 2, respectively. Plugging the results from Proposition 4, 5 and 6 into (3.21) or (3.22), we get for the consumer surplus under simultaneous price moves

$$CS^{sim} = -\frac{\sigma(11 - 8(1-\alpha)\alpha)}{18\alpha}.$$

When firm 1 acts as price leader we have

$$CS^{1lead} = -\frac{\sigma(23 - 5(2-3\alpha)\alpha)}{32\alpha}$$

and with 2 as price leader

$$CS^{2lead} = \begin{cases} -\frac{\sigma(28-5(4-3\alpha)\alpha)}{32(1-\alpha)} & \text{if } \alpha \leq 2/3, \\ -\frac{\alpha\sigma}{(1-\alpha)} & \text{if } \alpha > 2/3. \end{cases}$$

The total consumer surplus is determined by the absolute price level and the relative price level which induces costly switching. In this respect, it is obvious from our previous analysis of the different equilibria that in all three cases, consumer surplus decrease when average switching costs increase. By contrast, increasing asymmetry between firms' customer bases, i.e. a higher α, lowers prices and thus raises consumer welfare in the setting with simultaneous price quotes and with the large firm as leader but raises prices and lowers surplus when the large firm follows.

Equivalently, one can compute the measures for total welfare in the economy which corresponds to the switching costs that consumers have to incur in the different price equilibria. Simple algebra gives

$$\begin{aligned} W^{sim} &= -\frac{\sigma(1-2\alpha)^2}{18\alpha}, \; W^{1lead} = -\frac{\sigma(1-3\alpha)^2}{32\alpha} \text{ and} \\ W^{2lead} &= \begin{cases} -\frac{\sigma(2-3\alpha)^2}{32(1-\alpha)} & \text{if } \alpha \leq 2/3, \\ 0 & \text{if } \alpha > 2/3. \end{cases} \end{aligned} \tag{3.24}$$

Proposition 8 compares the measures for consumer surplus and total welfare.

Proposition 8 *Consumers have the ranking*

$$CS^{sim} > CS^{1leads} > CS^{2leads}.$$

The socially efficient ranking for $\frac{1}{2} \leq \alpha < 0,5963$ *is*

$$W^{sim} > W^{2leads} > W^{1leads}$$

and

$$W^{2leads} > W^{sim} > W^{1leads}$$

otherwise.

The maximisation of consumers' surplus has to take into account the price difference and the price levels of the different scenarios. From the above discussion of the corresponding equilibrium prices, it is easy to see that simultaneous price setting offers consumers the best deal with respect to price *levels* and the second best with respect to price *differences,* which translate into switching costs that consumers have to incur. Nevertheless, the absolute price levels dominate this ranking which is just the reverse order of the firms' price ranking of Lemma 2.

Contrarily to that, all what counts for total welfare is the price *difference* in the three pricing scenarios. The higher the price difference, the more consumers will incur switching costs and change to the low price firm. Thus, social welfare is maximised either under simultaneous price setting (for α close to $1/2$) or under leadership of the small firm (for sufficiently large α). This is simply due to the fact that - as we already argued - the large firm has less of an incentive to undercut when it follows because it has to apply a low price to its locked-in customers, too. The higher α, the stronger the incentive to meet the posted price of the small firm and the less consumers have to switch.

Finally, it follows directly from Proposition 4 and 5 that for rather symmetric customer bases price leadership arises although simultaneous pricing would be socially optimal. However, for sufficiently large α, the endogenisation of price moves generates the socially optimal order for $\alpha > 0,5963$.

3.4 Conclusions

This chapter contains two notes on models of endogenous price leadership. In the first one, we analysed a vertically differentiated industry and found that under price competition the high and the low-quality supplier would both prefer to follow but that the risk dominance criterion unambiguously selects the high quality firm as a price leader. The reason for this is that the high quality firm has a higher potential to undercut its low quality rival and this latter has therefore more of an incentive to outwait the price quote of the former. More importantly and completely independent of the price equilibrium selection we showed that the emergence of price leadership softens the competitive pressure in the industry with the result that in the long run firms would choose products of higher quality and decrease product differentiation. Taken together, we demonstrated that price leadership may actually hurt firms but benefit consumers.

Notice that the results of the first note are confirmed by at least two empirical studies on price leadership. Roy and Hanssens and Raju (1994) examined the mid-size sedan segment of the US automobile market and found evidence that the high end car Ford Thunderbird acted as a price leader to Chrysler's New Yorker for the duration of the study (28 years). Kadiyali and Vilcassim and Chintagunta (1996) investigated the market for liquid laundry detergents and showed that Procter & Gamble and Lever used their respective upper quality brands as price leaders for the low quality brands of the industry.

In the second model, we took a closer look at price leadership in markets with consumers' switching costs. Deneckere et al. (1992) argued that the size of a firm's customer base has the same impact on endogenous price leadership as firms' production capacity has. In our model, we show that this conclusion crucially hinges on their assumption of prohibitive switching costs. If one allows consumers to switch suppliers at some positive, but finite cost, the firm with the larger segment of loyal customers stands to lose more by being undercut since its smaller rival faces a higher (absolute) demand elasticity and less own customers over which to spread the lower price when it follows. In this vein, we demonstrate that industries with consumers' switching costs provide a natural example for a constellation that has - to the best of our knowledge - not yet been found in the literature: one firm strictly prefers to lead and the other one strictly prefers to follow. Moreover, we can show that the market outcome, which is price leadership of the small firm, minimises consumers' switching costs and is the socially efficient order of price quotes over a large range of parameters.

Finally, from a conceptual point of view, we presented a model of an industry with consumers' switching costs that explicitly allows for closed form solutions and unique equilibria in pure strategies. Both properties are rather the exception in this strand of literature.

To conclude, notice that a relevant market for which there have been empirical studies on price leadership is the US cigarette market. Scherer and Ross (1990) report that during 1921 and 1965 market share leader Reynolds (Camel) seemed to be more reluctant to lead price decreases than its rivals American and Liggett & Myers.

3.5 Appendix

Proof of Proposition 1

From Lemma 1, we know that only two equilibria in pure strategies can exist, (t_0, t_1) and (t_1, t_0), where the dates denote the point in time at which firm 1 and firm 2 will quote their price. Following Harsanyi & Selten (1988), we will say that the former equilibrium risk dominates the latter whenever $G_1^{12}G_2^{12} > G_1^{21}G_2^{21}$, with $G_1^{12} \equiv \Pi_1^{12} - \Pi_1^{sim}, G_2^{12} \equiv \Pi_2^{12} - \Pi_2^{sim}, G_1^{21} \equiv \Pi_1^{21} - \Pi_1^{sim}$ and $G_2^{21} \equiv \Pi_2^{21} - \Pi_2^{sim}$. Tedious calculations show that

$$G_1^{12}G_2^{12} - G_1^{21}G_2^{21} = \frac{(q_1 - q_2)^2 q_1^4 (8q_1 - q_2)}{128(2q_1 - q_2)^3(4q_1 - q_2)^2} > 0.$$

Hence, (t_0, t_1) is the risk dominant equilibrium in the above sense. ¥

Proof of Proposition 3

Consumer surplus is defined in (3.10). With (3.4), we get the consumer surplus when prices are set simultaneously

$$CS^{sim}(q_1, q_2) = \int_{\frac{q_1 - q_2}{4q_1 - q_2}}^{\frac{2q_1 - q_2}{4q_1 - q_2}} (\theta q_2 - p_2) d\theta + \int_{\frac{2q_1 - q_2}{4q_1 - q_2}}^{1} (\theta q_1 - p_1) d\theta = \frac{q_1^2(4q_1 + 5q_2)}{2(4q_1 - q_2)^2}.$$

If the low quality firm would be the second stage price leader, one would have, using (3.8),

$$\begin{aligned} CS^{2,1}(q_1, q_2) &= \int_{\frac{q_1 - q_2}{4q_1 - 2q_2}}^{\frac{4q_1 - 3q_2}{8q_1 - 4q_2}} (\theta q_2 - p_2) d\theta + \int_{\frac{4q_1 - 3q_2}{8q_1 - 4q_2}}^{1} (\theta q_1 - p_1) d\theta \\ &= \frac{4q_1^2(4q_1 + 3q_2) + 3q_2^2(q_2 - 5q_1)}{32(2q_1 - q_2)^2}. \end{aligned}$$

and similarly with the high quality firm as price leader and (3.6)

$$CS^{1,2}(q_1, q_2) = \int_{\frac{q_1 - q_2}{4q_1 - 2q_2}}^{\frac{1}{2}} (\theta q_2 - p_2) d\theta + \int_{\frac{1}{2}}^{1} (\theta q_1 - p_1) d\theta = \frac{q_1(4q_1 + q_1 q_2 - q_2^2)}{8(2q_1 - q_2)^2}.$$

Plugging in the optimal qualities from Proposition 2 and Remark 2 yields

$$CS^{sim} = 0,29166, CS^{2,1} = 0,28888, CS^{1,2} = 0,296875.$$

For the firms profits, one gets

$$\begin{aligned}\Pi^{21} = &\quad \Pi_1^{21} + \Pi_2^{21} = (7 - 3\sqrt{2}) = 0,17233, \\ \Pi^{12} = &\quad \Pi_1^{12} + \Pi_2^{12} = \tfrac{1}{8} + \tfrac{1}{32} = 0,15625, \\ \Pi^{sim} = &\quad \Pi_1^{sim} + \Pi_2^{sim} = \tfrac{7}{48} + \tfrac{1}{48} = 0,1\overline{6}.\end{aligned}$$

And finally, it is straightforward to calculate the total welfare in the different regimes

$$W^{sim} = 0,45833, W^{2,1} = 0,4611, W^{1,2} = 0,4531. ¥$$

Proof of Proposition 5

First, check that (piecewise) concavity and the downward discontinuity of the profit function gives rise to three potential local maxima. First, the interior solution for $0 < p_1 \leq \sigma\sqrt{(1-\alpha)/\alpha}$, which is at $p_1 = \frac{1+\alpha}{2(1-\alpha)}\sigma$ and which is larger than the upper threshold value $\sigma\sqrt{(1-\alpha)/\alpha}$ of this region for all $\alpha \geq 1/2$. Therefore, the local maximum in this price regime is exactly at the threshold value $p_1 = \sigma\sqrt{(1-\alpha)/\alpha}$and takes a value of $(1+\alpha)\sigma\sqrt{\alpha(1-\alpha)}/2\alpha - \frac{(1-\alpha)^2}{2\alpha}$. The third local maximum can be found for $p_1 > \sqrt{(1-\alpha)/\alpha}\sigma$. Taking the first order condition of the second part of $\Pi_1^{lead}(p_1)$ yields $p_1 = \frac{1+\alpha}{2\alpha}\sigma$ and a maximum value of $\frac{(1+\alpha)^2}{8\alpha}\sigma$. Comparing the local maximum values and plugging the optimal price for firm 1 in $R_2(p_1)$ and in the profit functions gives Proposition 5. ¥

Proof of Proposition 6

Note first that the profit function is continuous and (piecewise) concave. Solving the maximisation problem of firm 2 again implies the identification of local maxima. First, consider the price regime $0 < p_2 \leq \sigma$. The first order condition of this first part of the profit function yields $p_2 = \frac{2-\alpha}{2\alpha}\sigma$, which is smaller than σ whenever $\alpha \geq 2/3$. Thus, for $1/2 < \alpha < 2/3$, the local maximum is the corner solution $p_2 = \sigma$, otherwise it is at $\frac{2-\alpha}{2\alpha}\sigma$ with a maximum value of $\frac{(2-\alpha)^2}{8\alpha}\sigma$.

For $\sigma < p_2 < \frac{\alpha}{(1-\alpha)}\sigma$, the profit function is strictly increasing in p_2, therefore a second local maximum may only be found for $p_2 > \frac{\alpha}{(1-\alpha)}\sigma$. Deriving the first order condition for the third part of $\Pi_2^{lead}(p_2)$ and solving it, gives $p_2 = \frac{(2-\alpha)}{2(1-\alpha)}\sigma$ with a maximum value of $\frac{(2-\alpha)^2}{8(1-\alpha)}\sigma$. This solution is interior as long as $\frac{(2-\alpha)}{2(1-\alpha)}\sigma > \frac{\alpha}{(1-\alpha)}\sigma$ or $1/2 < \alpha < 2/3$. Otherwise, for $\alpha > 2/3$, the local maximum is at $\frac{\alpha}{(1-\alpha)}\sigma$.

To sum up, for $1/2 < \alpha < 2/3$, the unique local maximum is $\frac{(2-\alpha)}{2(1-\alpha)}\sigma$. For $\alpha \geq 2/3$, the profits at $\frac{2-\alpha}{2\alpha}\sigma$ are always larger than those at $\frac{\alpha}{(1-\alpha)}\sigma$. Plugging these optimal prices into the reaction function of firm 1 and into the profit functions gives Proposition 6.¥

Proof of Proposition 7

Following Harsanyi and Selten (1988), we will say that the equilibrium with firm 1 as follower risk dominates the equilibrium with 1 as price leader whenever $G_1^{follow}G_2^{lead} \geq G_1^{lead}G_2^{follow}$, with $G_1^{lead} \equiv \Pi_1^{lead} - \Pi_1^{sim}, G_2^{follow} \equiv \Pi_2^{follow} - \Pi_2^{sim}, G_1^{follow} \equiv \Pi_1^{follow} - \Pi_1^{sim}$ and $G_2^{lead} \equiv \Pi_2^{lead} - \Pi_2^{sim}$. Tedious calculations show that

$$G_1^{follow}G_2^{lead} - G_1^{lead}G_2^{follow} =$$
$$Z(\alpha,\sigma) \equiv \begin{cases} \frac{(1-2\alpha)(24\alpha^5-45\alpha^4-86\alpha^3+103\alpha^2+140\alpha-55)}{1152\alpha^2(1-\alpha)^2}\sigma^2 & \text{if } \alpha \leq 2/3, \\ \frac{(1+\alpha)(1137\alpha^4-454\alpha^3-168\alpha^2-58\alpha+55)}{1152\alpha^2(1-\alpha)}\sigma^2 & \text{if } \alpha > 2/3. \end{cases}$$

Finally, check that $Z(1/2, \sigma) = 0$, that it is monotonically increasing for $a \in [1/2, 1]$ and that $Z(\alpha, \sigma) \to \infty$ for $\alpha \to 1$. Thus, $Z(\alpha, \sigma) \geq 0$ for all σ and $a \in [1/2, 1]$. ¥

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Chapter 4

Agglomeration in R&D Intensive Industries

4.1 Introduction

When selecting amongst portfolios of alternative product specifications, under which conditions do firms tend to agglomerate and thus seek competition instead of looking for a market niche they can supply monopolistically? Similarly, what are the characteristics of firms and products that lead firms to agglomerate in geographical space, vs. those that lead firms to seek a solitary location? How do the incentives to agglomerate in product, or geographical spaces change with exogeneities, such as transaction costs and differences in consumers' taste? Are the tendencies towards agglomeration (or dispersion) in harmony, or in disharmony with those leading the social planner to propose agglomeration vs. dispersion?

Questions of this nature have been addressed in a long string of literature. Within the new industrial economics emerging from the seventies, many authors have analysed variants of Hotelling's seminal (1929) paper. The question of agglomeration also features centrally in the New Economic Geography. Yet, to our surprise, while there is ample empirical documentation at least of agglomeration in geographical space with Silicon Valley and Route 128 as the most prominent examples for the agglomeration of innovative hi-tech industry, little has been said to date in the theoretical literature about the relationship between firms' innovative activity and their incentives towards agglomeration.

The interest in this issue is far from being only of a theoretical nature - in particular when it comes to the geographical interpretation of these questions. Geographical agglomerations of innovative industrial activity are thought to contribute particularly much to the generation of employment, and indeed, to lead the growth path of entire national economies. Take again Silicon Valley as an example. In spite of ups and downs in employment demand during the nineties of the last century, the employment growth rate in Silicon Valley outpaced with an impressive 15 per cent the U.S. national employment growth rate and the mean personal income per head was up to 50 per cent higher than the corresponding national income figure (Audretsch, 1998). Yet it is fair to ask whether a dispersion of these activities could induce not only a more equitable allocation, but could also increase the efficiency in the allocation of economic activity. It is thus of utmost importance to investigate, and to evaluate in detail the reasons for such agglomerations to form.

More specifically, the question is as to the strategic forces that lead these innovative firms to choose locations close to each other, and thus to opt for aggressive competition in either input, or output markets or both, rather than to evade it. The story in favor of agglomeration in the geographical case proposed by data analysts e.g. by Saxenian (1994), Harhoff (1995), Audretsch and Feldman (1995, 1996) or Audretsch (1998) is that firms seek to partake in an "information rich environment": firms benefit from clustering together because tacit knowledge is transmitted either in informal contacts between employees of different firms, or via the movement of these employees across firms.[1]

[1]Knowledge spillovers through employee mobility have been studied by Fosfuri et al. (1999) and

Our explanation of agglomeration in innovative industries does not build on spillovers but on the inherent stochasticity of R&D activities. In the vein of Hotelling's classical paper firms' location choices are governed by the trade-off between two by now standard effects: a *demand effect* that induces the individual firm to move towards the center of the market, and a *competition effect* that drives the firms away from each other. For symmetric and certain product qualities and quadratic transportation costs, we know from the „principle of maximum differentiation" established by d'Aspremont and Gabszewicz and Thisse (1979) that the competition effect always dominates and that firms locate at the opposite ends of the line. However, if market entry (or product quality) depends on the stochastic outcome of firms' R&D activities, a firm meets a successful competitor in the market only with a certain probability. This weakens the competition effect while the demand effect remains unchanged. In the first part of this work, we show in a simple variant of the Hotelling framework that this can actually lead to complete agglomeration of firms in the center of the market. From a welfare point of view, firms choose too much concentration (dispersion) if innovation probabilities are low (high). Moreover, we investigate the impact of R&D spillovers and patent protection on the location equilibrium. We show that the former is actually working against agglomeration because it increases the probability that firms end up in a duopoly while patent protection leads to more monopolistic outcomes and therefore constitutes an agglomerative force in our model.

In the second part, we extend our framework and allow firms to choose their R&D technology together with their location. More specifically, firms can adopt either a safe R&D project yielding a low-quality product or a risky project aiming at a large innovation step. We show that for a large range of the parameter space the following three types of equilibria can emerge. Either firms choose dispersed locations and adopt the safe R&D technology. Or, they agglomerate in the center and one of the firms opts for the risky technologies. Or, finally, they agglomerate and both choose

Rønde (2000), among others.

the risky R&D technology. Following the intuition of the first part, this result hints at a strong complementarity between risk taking in R&D and geographical concentration of firms. Our welfare analysis gives a rather diverse picture. There may be excessive differentiation and concentration in product space and too less or too much risk taking in the choice of the R&D technology.

Finally, our model allows an interesting reinterpretation in terms of the labour market pooling argument forwarded by Marshall (1920). He argued that firms might have incentives to locate in the same region when they face stochastic labour demands that are imperfectly correlated. By this, the firm with the high demand can draw skilled labour at low cost from the common local labour market since the labour demand of the other firm is low. In a straightforward reformulation of our product market model, firm-specific shocks to the labour demand are generated by the stochastic R&D technology. And when firms choose locations they have to trade off labour market competition and access to labour (which is cheapest in the center of the line). To clearify this reinterpretation, we will explicitly formulate the model variant of the first part of our work from this input market perspective.

To the best of our knowledge, there are only two papers linking location decisions in the geographical interpretation to innovation issues and both have spillovers as the unique agglomerative force. Gersbach and Schmutzler (1999) analyse agglomeration due to localised technological spillovers within a discrete location framework. They explicitly look at a model in which firms are not only recipients of positive spillovers, but they may also be the victim of undesired information leakages. Their focus is on the interplay between internal interplant spillovers as well as external interfirm spillovers from cost reducing R&D efforts on the plant location decisions of duopolistic firms. Abstracting from the price competition stage, they compare in detail the reduced form profits net of the fixed costs of establishing a plant and of (certain) innovation, to arrive at what they call research centre, or alternatively, technology sourcing equilibria that differ by the fact that in the former, both firms exercise innovation efforts, while in the latter, only one firm does this.

Mai and Peng (1999) concentrate on an explicit form of co-operation between firms as the explicit reason for agglomerations to form. They modify the Hotelling-framework by assuming that distance dependent communication between competitors reduces production costs. In their model, the firms will symmetrically move the closer to each other, the stronger the cost reducing effect of communication, and the more drastic the increase in communication costs with distance. However, the firms will fully agglomerate only if the positive externality is infinitely large.

The literature on clustering in product space is much richer. For instance, Bester (1998) and Vettas (1999) look at a situation where firms signal the quality of their products to imperfectly informed consumers and show how this might lead to agglomeration. The authors use a set-up similar to ours, with horizontal product differentiation as well as vertical quality differences. Vettas looks at a location model in which one firm sells an exogenously specified low quality product, whilst the other one sells a high quality product. He shows that by locating close to the low quality firm, the high quality firm can signal the superiority of the quality offered, whilst the low quality firm seeks a niche location. Bester looks at model where it is more costly to produce high quality than low quality. He shows that this leads to a lower bound on price that the firms can charge and still (credibly) commit to supplying high quality. Therefore, firms do not compete away all profits as they move close to each other in the horizontal dimension, and agglomeration might occur in equilibrium. In our model, consumers can perfectly observe the quality of the products, so signalling does not play a role. The effects leading to agglomeration in our model are thus quite different from those in Bester and Vettas.

Within a modified Hotelling framework, Neven and Thisse (1993) and Irmen and Thisse (1998) demonstrate clustering effects in one of two dimensions of product differentiation, where one is vertical and the other horizontal, and both are horizontal, respectively. Their central point is on sufficient conditions under which (maximal) product differentiation in one dimension suffices for firms to agglomerate in the center of the market in another dimension, without the effect that profits evaporate. Echoing somewhat this type of results, we show in our model that there exists an equilibrium where the firms choose the same horizontal product characteristic but different R&D strategies: one firm pursues a high risk-high return project whilst the other one chooses a more conservative R&D strategy.

There are also models on competitive R&D portfolio choices that demonstrate clustering effects, by authors such as Bhattacharya and Mokherjee (1986), Dasgupta and Maskin (1987), and Cardon and Sasaki (1998), among others. Bhattacharya and Mokherjee compare private and social incentives towards risk in R&D projects with winner-takes-all outcomes enforced by a patent mechanism. Their primary interest is in the exploration of relative levels of risk taking and in correlation choices. Our focus is different. We are interested in exploring the trade off between fierce competition that arises when firms locate close to each other and are equally (un-)successful; and dominant firm profits if only one succeeds. However, in second part of our work, we include in a rudimentary way the choice amongst research portfolios according to risk/return relationships. In doing so, we are able to analyse asymmetric choices between firms, which is not possible in the otherwise more general models by Dasgupta and Maskin, and Cardon and Sasaki.

There is a large literature analysing market structure in industries with network externalities. It is shown how network externalities create 'bandwagon' effects that make firms and consumers choose products that are standardized or compatible across firms; see, e.g., Katz and Shapiro (1985) and Farrell and Saloner (1986). Our paper is most closely related to Katz and Shapiro (1986), who introduce a stochastic R&D technology into a network industry. Two generations of homogenous consumers choose which technology to buy, and the firms decide whether to make their proprietary technologies compatible. The decision to make the products compatible resembles the choice of agglomerating in product space, as the products become closer substitutes. Katz and Shapiro show that the winner in the R&D race benefits from compatibility, as the consumers have a higher willingness to pay for the product. However, the loser is better off under incompatibility, as she might avoid being pushed out of the market.

Finally, in models in the "herding" tradition, authors such as Banerjee (1992) or Hirshleifer et al. (1992) demonstrate imitative behavior within a dynamic framework. Unlike in our model, imitation is driven here by consumer biases.

There is finally a particularly small literature related to the second interpretation of our model, namely firms' location decisions in geographical space, relative to localised labor markets. While authors such as Topel (1986), Baumgardner (1988), or most recently Picard and Toulemonde (2000) and Combes and Duranton (2000) all focus on different issues such as workers' migration incentives, division of labor as changing with labor market size, agglomeration as a result of supply elasticity of labor, and the advantages of labor market pooling vs. the disadvantages of labor poaching, respectively.

The remainder of the paper is organised as follows: In the next section, we present our benchmark model, in which an unsuccessful firm, while *ex ante* active, will be inactive *ex post* in both, its product market and labor market interpretations. In sections 3 and 4, we determine and characterise the price equilibrium, and the location equilibrium and welfare outcomes, respectively. In Section 5, we endogenise R&D decisions and again derive and characterise equilibria and welfare outcomes. In the concluding section, we summarise our results and speculate about possible extensions.

4.2 The Benchmark Model

The Product Market Interpretation We employ the standard Hotelling (1929) duopoly model in the version of d'Aspremont, Gabszewicz & Thisse (1979) with quadratic consumer transportation costs. The market area is described by the unit interval M=[0,1]. There is a unit mass of consumers whose locations are uniformly distributed over M.

We consider an industry with two firms $i = A, B$ potentially active in the market, that choose locations $a, b \in M$, $a \leq b$, at which they offer their good. Each consumer buys at most one unit of the product and incurs costs of overcoming space that are quadratic in the distance travelled. Thus a consumer located at y derives net utility

$$U_A(a, q_A, p_A, y) = q_A - p_A - t(a - y)^2$$

when buying good A and

$$U_B(b, q_B, p_B, y) = q_B - p_B - t(y - b)^2$$

when buying B. q_i and p_i are the quality and the price of firm i's product, respectively. $t > 0$ reflects the degree of consumer heterogeneity or horizontal product differentiation. We assume that all variables are common knowledge in the economy.

We assume that the firms costlessly invest in R&D. With probability ρ a firm's project succeeds. Then the firm sells a product of quality q. The successes of the firms are uncorrelated . In order to make the model tractable, we assume that the firms have no fall-back quality. Therefore, if the project is unsuccessful, the firm is, while in the market, not active. One can think of this as a situation where there are significant retooling or marketing costs, so a quick change in the production schedule if the R&D project is a failure is not possible. We will later consider variations of these assumptions, but these will be explained when introduced.

The timing is the following: First, firms simultaneously choose their locations (a, b). Second, the outcomes of their R&D investments are realised. Finally, firms set prices simultaneously, consumers buy one of the available products, and profits are realised.

The Labor Market Interpretation The presentation of the model was thus far held in the context of horizontal product differentiation. Yet there is another fundamentally different, but rather realistic interpretation of the model, running as follows. Suppose that the firms have to locate somewhere in a valley of length one. At each point in the valley lives one worker endowed with utility function

$$U(w, x) = w - t(x)^2$$

where w is the wage earned and x is the distance travelled to work. Our two firms hire the workers to produce a vertically differentiated commodity with a one-to-one production function. The firms engage in wage competition once the outcome of R&D is known. The commodity is sold on a competitive world market, and Firm i sells its product at price $p_i = q_i$.

The product differentiation and local labor market formulations are formally equivalent and thus give rise to precisely the same equations. When interpreting our results we will refer to the product market interpretation of the model, and come back later to its labor market interpretation.

4.3 Price Equilibrium for given qualities and locations

Depending on the outcome of the R&D process after the location is fixed, the typical firm may produce and sell a product at positive quality, or it may remain inactive if unsuccessful. If successful, it may either be a monopolist or a duopolist, depending on the success or failure of its competitor.

In the present stage, the firms' locations and product qualities are known and taken as given when they set their prices. First, we derive the monopoly price, and afterwards the duopoly ones. For future reference, we calculate equilibrium prices and profits allowing for quality differences between the firms' products.

4.3.1 The Monopoly Outcome

Suppose that only one firm, say firm A, located at $a \leq \frac{1}{2}$ succeeded in developing a marketable product of quality q_A. The consumers indifferent between buying the product at a price p_A and not purchasing at all (at opportunity utility of 0) are located at

$$\widetilde{y}_1 = Max\{0, a - \sqrt{\frac{q_A - p_A}{t}}\} \text{ and } \widetilde{y}_2 = Min\{0, a + \sqrt{\frac{q_A - p_A}{t}}\}.$$

With this, we can derive firm A's demand as monopolist, $D^M(q_A, a, p_A)$, which is given by

$$D^M(q_A, a, p_A) = \begin{cases} 0 & \text{if } p_A > q_A \\ 2\sqrt{\frac{q_A - p_A}{t}} & \text{if } q_A \geq p_A > q_A - ta^2 \\ a + \sqrt{\frac{q_A - p_A}{t}} & \text{if } q_A - ta^2 \geq p_A > q_A - t(1-a)^2 \\ 1 & \text{if } p_A \leq q_A - t(1-a)^2. \end{cases}$$

The monopolist maximizes his profits, $pD^M(q_A, a, p)$. Lemma 1, which follows from simply calculating the monopolist's profit maximum, characterises the solution to this problem under an assumption that leads the market to be covered under any market arrangement.

Lemma 1 *Let* $q_A \geq 3t$. *Then, for all* $a \in [0, \frac{1}{2}]$, *the monopolist charges the price* $p_A^M = q_A - t(1-a)^2$ *and covers the market.*

For simplicity, we will in the sequel only consider that situation:

A. 1. *The market is covered,* $q_i \geq 3t$.

Since the monopolist serves all consumers and the total mass of consumers is 1, the monopoly profit $\Pi^M(q_A, a)$ is equivalent to the monopoly price, $p_A^M = q_A - t(1-a)^2$. Thus,

$$\Pi_A^M(q_A, a) = q_A - t(1-a)^2.$$

Suppose instead that firm B, located at $b \in [\frac{1}{2}, 1]$, is a monopolist in the market. It follows from symmetry that under assumption A.1. firm B charges the price $p_B^M = q_B - t(b)^2$ and covers the market.

4.3.2 The Duopoly Outcome

This is the case where both firms conduct successful R&D, resulting in qualities q_A and q_B, respectively. Let the consumer located at $\widetilde{y}$ be indifferent between buying from between firm A or firm B. $\widetilde{y}$ is given as the solution to the following equation:

$$q_A - p_A - (\widetilde{y} - a)^2 = q_B - p_B - (b - \widetilde{y})^2.$$

Solving for $\widetilde{y}$, we obtain:

$$\widetilde{y} = Max\left\{0, Min\left\{\frac{a+b}{2} - \frac{q_B - q_A + p_A - p_B}{2t(b-a)}, 1\right\}\right\}. \tag{4.1}$$

By assumption A.1., the market is covered, so firm A's demand is $\widetilde{y}$ and firm B's demand is $1 - \widetilde{y}$. Firms simultaneously set prices and the corresponding Bertrand-Nash equilibrium is given in Lemma 2.

Lemma 2 *(i) For $q_A - q_B < -t(b-a)(2+a+b)$, firm B is the only firm with positive market share. The unique price equilibrium is*

$$p_A^D(a, b, q_A, q_B) = 0 \text{ and } p_B^D(a, b, q_A, q_B) = q_B - q_A - t(b^2 - a^2). \tag{4.2}$$

(ii) For $-t(b-a)(2+a+b) < q_A - q_B < t(b-a)(4-a-b)$*, the firms share the market. The unique price equilibrium is*

$$\begin{aligned} p_A^D(a,b,q_A,q_B) &= \frac{1}{3}(q_A - q_B + t(b-a)(2+a+b)) \text{ and} \\ p_B^D(a,b,q_A,q_B) &= \frac{1}{3}(q_B - q_A + t(b-a)(4-a-b)). \end{aligned} \tag{4.3}$$

(iii) For $q_A - q_B > t(b-a)(4-a-b)$*, firm A is the only firm with positive market share. The unique price equilibrium is*

$$p_A^D(a,b,q_A,q_B) = q_A - q_B - t(b-a)(2-a-b) \text{ and } p_B^D(a,b,q_A,q_B) = 0. \tag{4.4}$$

Proof: See Appendix.

In the proof we derive the reaction functions and show that the price equilibrium is unique. Not unexpectedly, Lemma 2 demonstrates that if there are quality differences, the low quality firm is dominated by the high quality one if the quality difference is large relative to the transportation cost t.

Using Lemma 2, we can derive the equilibrium profits. In case (i) where firm A is inactive, the profits are given as

$$\begin{aligned} \Pi_B^D(a,b,q_A,q_B) &= q_B - q_A - t(b^2 - a^2) \text{ and} \\ \Pi_A^D(a,b,q_A,q_B) &= 0. \end{aligned} \tag{4.5}$$

Similarly, if firm B is inactive as in case (iii), the profits are

$$\begin{aligned} \Pi_A^D(a,b,q_A,q_B) &= q_A - q_B - t(b-a)(2-a-b) \text{ and} \\ \Pi_B^D(a,b,q_A,q_B) &= 0. \end{aligned} \tag{4.6}$$

Finally, in the case where the firms share the market, they earn profits

$$\begin{aligned} \Pi_A^D(a,b,q_A,q_B) &= \frac{(q_A - q_B + t(b-a)(2+a+b))^2}{18t(b-a)} \text{ and} \\ \Pi_B^D(a,b,q_A,q_B) &= \frac{(q_B - q_A + t(b-a)(4-a-b))^2}{18t(b-a)}. \end{aligned} \tag{4.7}$$

This completes the analysis of price competition in the market place.

4.4 Location under Stochastic R&D Outcomes

4.4.1 The Equilibrium

We now turn to the firms' location decisions. They have to take place before the outcomes of their R&D efforts are known. Each of the firms innovates with probability ρ. If both firms are successful, they produce and sell a product of the same quality, $q_A = q_B = q$ .Therefore, if actively competing in the market, the firms have the same quality commodity. For given locations, the expected profits of the firms are:

$$\begin{aligned} E(\Pi_A(a,b,q_A,q_B,\rho)) &= \rho(\rho\Pi_A^D(a,b,q,q) + (1-\rho)\Pi^M(q,a)) \text{ and} \\ E(\Pi_B(a,b,q_A,q_B,\rho)) &= \rho(\rho\Pi_B^D(a,b,q,q) + (1-\rho)\Pi^M(q,b)). \end{aligned}$$

Using Lemma 1 and equation (4.7), we obtain the first order condition for firm A when choosing its location a:

$$\frac{\partial E(\Pi_A(a,b,q_A,q_B,\rho))}{\partial a} = \frac{t}{18}\left(36(1-a) - (40 + 3a^2 - 2a(14-b) - b^2)\rho\right).$$

It can be shown that firm A's problem is concave, so solving the equation $\frac{\partial E(\Pi_A(a,b,\rho))}{\partial a} = 0$, we find the optimal location of firm A as a function of the location of firm B. The reaction function of firm A is given as:

$$R_A(b,\rho) = \frac{-36 + 28\rho - 2b\rho - \sqrt{(36 - 28\rho + 2b\rho)^2 - 12\rho(-36 + 40\rho - b^2\rho)}}{6\rho}.$$

The reaction function of firm B is derived the same way.

Figure 4.1 shows the reaction functions of the two firms for $\rho = 0.8$ (solid) and for $\rho = 0.9$ (dashed). It illustrates that the firms agglomerate closer to the center of the market for low values of ρ and separate for higher values. The equilibrium is derived formally in Proposition 1.

Proposition 1 *Consider the choice of location in the first stage of the game.*
i) For $\rho \leq \frac{2}{3}$, *the unique equilibrium locations are* $a^* = b^* = \frac{1}{2}$.

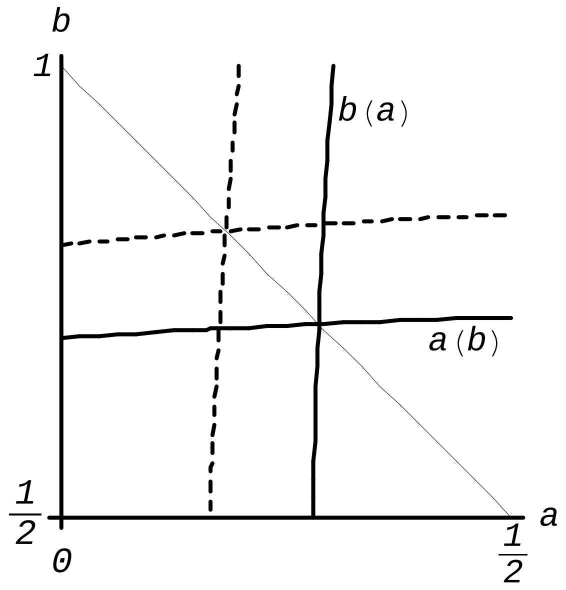

Figure 4.1: *The reaction functions of firm A and B.*

ii) For $\frac{2}{3} < \rho \leq 1$*, the unique equilibrium locations for* $a \leq b$ *are*

$$a^* = Max\left\{0, \frac{1}{2} - \Gamma\right\} \;\; and \; b^* = 1 - a^*, .$$

where $\Gamma \equiv \frac{3(3\rho-2)}{4(3-2\rho)}$.

Proof: See appendix.

4.4.2 Discussion

The equilibrium outcome can easily be understood in terms of the effects of the model. On one hand, the typical firm wishes to choose a location that captures as many consumers as possible, as this increases the sales and profits for given prices. This 'demand' effect tends to make the firm locate in the centre of the market, as pointed out by Hotelling (1929). On the other hand, moving towards the centre of the market also means moving closer to the other firm. This intensifies competition, which reduces profits.

D'Aspremont, Gabszewicz and Thisse (1979) show that if both firms are active with certainty, the 'competition' effect dominates, so firms locate as far as possible from each other. However, in our model, the firms foresee that if they enter the market, they will only meet an active competitor with probability ρ. This weakens the competition effect but not the demand effect, as a firm benefits from a central location as a monopolist, and this the more the smaller ρ. Therefore, there can be agglomeration in equilibrium. Complete agglomeration can only occur for $\rho \leq 2/3$ where the probability of ending up in a duopoly situation is low. For $\rho > 2/3$, the duopoly outcome becomes so likely that the competition effect starts to dominate and firms fragment in equilibrium. Indeed, as in d'Aspremont, Gabszewicz and Thisse, that outcome does not depend on t.

Technological Spillovers and Patent Protection

As emphasized in the introduction, technological spillovers are often cited as one of the main reasons why firms in R&D intensive industries agglomerate. It is thus worth investigating in our model the effect spillovers have on firms' locations. We model spillovers the following way: If one of the firms innovates, but the other one does not, the unsuccessful firm receives a spillover that allows it to become active with probability σ. Solving for the equilibrium locations in the proof of Proposition 1 by allowing for positive technological spillovers, we obtain

Corollary 1 *Consider the choice of location in the first stage of the game when there are positive spillovers.*

i) The optimal locations are given as

$$a^* = Max\left\{0, Min\left\{\frac{1}{2}, \frac{1}{2} - \Psi\right\}\right\} \text{ and } b^* = 1 - a^*,$$

where $\Psi \equiv \frac{3(-2+3\rho+4(1-\rho)\sigma)}{2(6-4\rho-2(1-\rho)\sigma)}$.

ii) Technological spillovers are a deglomerating force, i.e.

$$\frac{\partial a^*}{\partial \sigma} = -\frac{\partial b^*}{\partial \sigma} \leq 0.$$

Proof: See proof of Proposition 1.

The intuition behind this result is straightforward: Spillovers make it more likely that the firms end up in duopoly. This strengthens the competition effect and makes agglomeration less attractive. Suppose that these spillovers are localized in the sense that spillovers increase with the proximity of the firms. An empirical example for the geographical interpretation of the model is given by Jaffe et al. (1993). This, however, only reinforces the deglomerating effect of spillovers. The reason is that moving closer in product space becomes more costly marginally, as this increases the probability that the firms end up in a duopoly. The model thus provides an interesting contrasting view on the effects of spillovers to the standard argument suggesting that agglomeration in R&D intensive industries is due to knowledge spillovers.

There is another interpretation for the spillover parameter. Reinterpret the inverse of σ as the time period elapsing between marketing the original innovation, and marketing its clone. Then, in the original model, that time period is infinitely long (zero spillovers) whilst in the version of this section, it is finitely long and decreasing in σ. Clearly, the innovator's tendency to choose a central location increases in the time he has monopolistic control over his innovation. Conversely, an increase in the adoption lag with the distance from the innovator's location increases the tendency towards deglomeration.

Following the same logic, it is clear that any factor that prevents the duopoly outcome from occurring tends to make firms agglomerate. The best example is probably patent protection. If the innovators are sufficiently close, it is possible that only one patent would be granted. This, of course, would prevent the duopoly outcome from occurring. We model patent protection by assuming that if both firms innovate, there is a probability γ that only one of the firms can enter the market (the firms are equally likely to be excluded from the market). Corollary 2 summarises the analysis of this case:

Corollary 2 *Consider the choice of location in the first stage of the game when there is patent protection.*
i) The optimal locations are given as

$$a^* = Max\left\{0, Min\left\{\frac{1}{2}, \frac{1}{2} - \Phi\right\}\right\} \text{ and } b^* = 1 - a^*,$$

where $\Phi = \frac{3(-2+3\rho-2\gamma\rho)}{2(6-4\rho+\gamma\rho)}$.
ii) Patent protection is an agglomerating force, i.e.

$$\frac{\partial a^*}{\partial \gamma} = -\frac{\partial b^*}{\partial \gamma} \geq 0.$$

Proof: See proof of Proposition 1.

Finally, notice that positive or negative correlation in R&D outcomes would have an effect similar to spillovers and patent protection, respectively.[2] Hence, a positive correlation between R&D outcomes would lead to less agglomeration in equilibrium compared to the benchmark model, and negative correlation to more.

4.4.3 Welfare

In the previous section we have derived the firms' equilibrium locations. As discussed in the introduction, it is important to know whether there is too much, or too little

[2] Yet the effects are not identical. Take the case of spillovers. With spillovers, the fact that a firm fails to innovate does not contain information about the other firm's likelihood of failing, as it would if projects were positively correlated. A postive correlation in R&D outcomes might occur for exogenous reasons (for example, the next step forward is evident to participants in the industry) or might be the conscious choice of firms, see also Cardon and Sasaki (1998).

agglomeration as compared to the locational choice of a hypothetical social planner. The following two observations greatly simplify the calculation of the welfare maximising locations. First, having assumed that the market is always covered, we do not need to worry about how many consumers buy the product. Second, as the consumers exercise unit demand, there is no deadweight loss due to monopoly pricing. Therefore, the welfare optimal locations are simply those that minimise consumers' expected transportation costs. The next proposition, in which we again allow for spillovers, states the welfare maximising locations of the two firms.

Proposition 2 *The welfare maximising locations are:*

$$a^W = Max\left\{0, Min\left\{\frac{1}{2}, \frac{1}{2} - \Omega\right\}\right\} \text{ and } b^W = Min\left\{Max\left\{\frac{1}{2}, \frac{1}{2} + \Omega\right\}, 1\right\},$$

where $\Omega \equiv \frac{\rho + 2(1-\rho)\sigma}{4(2-\rho)}$.

Proof: See appendix.

In interpreting the welfare result and comparing it with the equilibrium locations derived in proposition 1, let us first concentrate on the situation without spillovers. In stage 3 of our game, i.e. after the R&D outcomes are realised, the optimal locations depend on whether there are one or two firms active in the market. For the monopolist, the welfare maximising location is 1/2. The duopolists' optimal locations are $a = \frac{1}{4}$ and $b = \frac{3}{4}$, respectively. These locations all minimise the welfare loss due to transportation costs. What does this imply for the firms' welfare maximising locations in stage 1, i.e. before the outcomes of the R&D efforts are revealed?

Suppose first that there are no spillovers between the firms. Let ρ, the probability that a firm innovates, become very small. In this case, if there is an innovation, the innovator will tend to be a monopolist. For $\rho \to 0$, the optimal location is thus $a = b = \frac{1}{2}$. As ρ increases, the probability that a duopoly arises increases. Therefore, the welfare maximising locations are such that the firms are located symmetrically

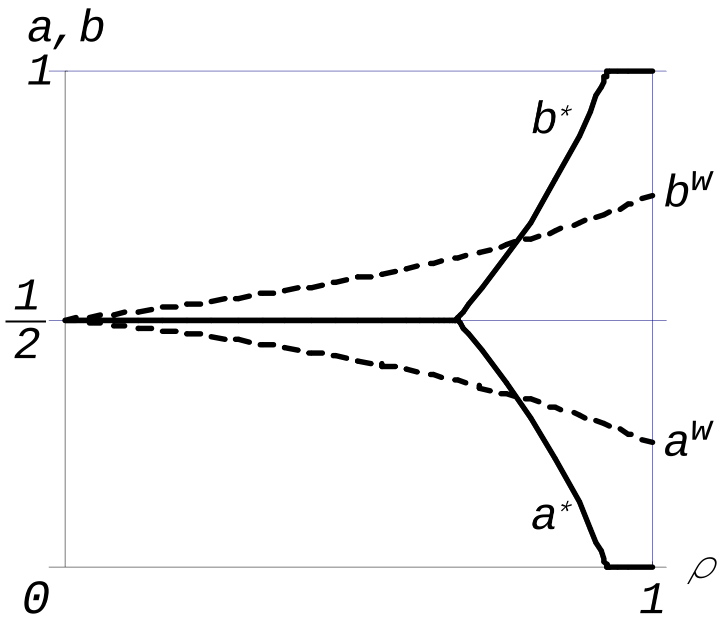

Figure 4.2: *The equilibrium and the welfare maximising locations.*

around $\frac{1}{2}$ and with a positive distance (smaller than $\frac{1}{2}$) between them. Finally, for $\rho = 1$ there will be a duopoly with certainty, so the welfare maximising locations are $a = \frac{1}{4}$ and $b = \frac{3}{4}$, respectively.

Comparing Proposition 1 and Proposition 2, we obtain immediately

Corollary 3 *Consider the model without spillovers. There exists a unique value* $\widetilde{\rho}$ *such that*

i) $\rho < \widetilde{\rho}$ *implies* $a^* < a^w$ $(b^* > b^w)$

ii) $\rho > \widetilde{\rho}$ *implies* $a^* > a^w$ $(b^* < b^w)$.

Figure 4.2 illustrates the welfare maximising locations (dashed) as well as the equilibrium locations (solid). The figure shows how there is excessive spatial concentration for low ρ, and excessive spatial dispersion for high ρ.

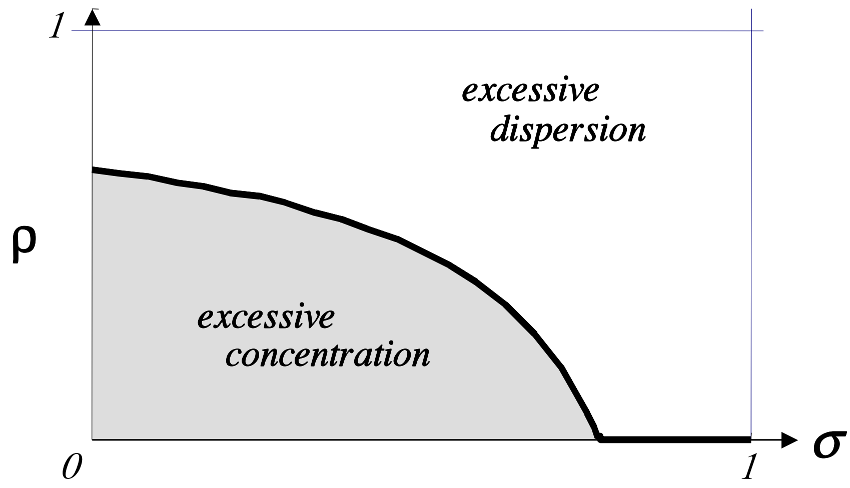

Figure 4.3: *The regions with excessive concentration and dispersion.*

Consider now the situation with positive spillovers ($\sigma > 0$). Spillovers imply that the success of one firm increases the other firm's probability of success. Compared to the situation with no spillovers, the duopoly outcome is more likely to occur (and monopoly less likely). Hence the social planner would opt for relatively more dispersion. In equilibrium, spillovers lead also to more dispersion. Figure 4.3 illustrates how, as a result of these two effects, the region with excessive spatial concentration decreases as the technological spillovers between firms increase. Indeed, the line segmenting the two regions is given by the function $\rho(\sigma)$. This is summarised in

Corollary 4 *There exists a monotonically declining function* $\rho(\sigma)$ *such that for any* $\sigma \leq \overline{\sigma}$, $\rho < \rho(\sigma)$ *implies excessive concentration, whence* $\rho > \rho(\sigma)$ *implies excessive dispersion. For* $\sigma > \overline{\sigma}$, *there is excessive dispersion for any* ρ.

4.5 Endogenous R&D Decisions

In the benchmark model, the firms had no choice of how to pursue their R&D activities. In this section, we allow the firms to choose between two R&D strategies. They can costlessly follow either a 'safe' and well-known path ('S') or a riskier and more innovative path ('R'). Following the safe path, the firm develops with certainty a product of quality q_L. If instead the firm follows the riskier path, it develops, with probability ρ, a product of higher quality q_H. We denote $\Delta \equiv q_H - q_L$. The outcomes of risky R&D efforts are again uncorrelated. We assume that the firms can only follow one of the paths. Hence, if a firm tries to develop the high quality product but fails, it cannot switch to the low risk strategy, and thus must stay inactive *ex post*.

The firms simultaneously choose their location and R&D strategies. We already have determined the equilibrium prices for given location and qualities, so we only have to look for a Nash-equilibrium jointly in locations and R&D choices. We denote firm A's strategy by $s_A = (a, z)$, where a is the location and $z \in \{S, R\}$ is the R&D project. Firm B's strategy is denoted in the same manner.

4.5.1 The Equilibrium

Our procedure for determining the equilibria in this game is as follows. In the ensuing two lemmata, we specify the locational equilibria conditional on chosen R&D strategies. These lemmata are used in the final proposition to show that payoffs in the equilibrium candidates, in which both R&D and locational strategies are jointly determined, remain undominated.

Proposition 1 specifies the equilibrium location conditional upon firms both choosing the risky R&D path. The equilibrium location of the firms if they choose the safe technology is already known, so we will just state the result.

Lemma 3 *Consider a candidate equilibrium, in which both firms choose the safe technology. Then, each firm's optimal location, given any location of the other firm, is the one that is furthest away from its competitor's location.*

Proof: See d'Aspremont, Gabszewicz & Thisse (1979).¥

Consider now locational equilibria conditional when one firm chooses the safe technology and the other the risky one. In this case the reaction functions are highly non-linear. Towards solving the model closed form we need to make restrictions on some parameters. The following assumption states sufficient (but not necessary) conditions on these:

A. 2. $\Delta \geq \underline{\Delta} = t$ and $\rho \leq \frac{2}{3} = \overline{\rho}$.

Assumption 2 excludes technologies that improve little on the safe technology and/or are successful with a very high probability.

In the next lemma, we specify the relevant equilibrium locations..

Lemma 4 *Let Assumption 2 hold and consider a candidate equilibrium, in which one firm (say, A) chooses the safe technology while the other (B) chooses the risky one. Then, firm A's optimal location for any b is $a = 1/2$. Firm b's optimal location given a is $b = a$.*

Proof: See appendix.¥

The reaction functions of the two firms illustrate how differently they weigh the possible outcomes. If the risky R&D project is successful, the firm that has chosen the safe technology is not going to earn much profits no matter where it locates, as it supplies an inferior product. Therefore, it chooses the central location, because that maximizes its profits when it is alone in the market. The firm that chooses the risky

R&D project only earns profit if it is successful. It also prefers to go to the centre of the market, as this results in the highest possible price obtainable, whilst the low quality firm is driven out of the market.

The fact that the high quality firm drives the low quality firm out of the market is a consequence of Assumption 2 that rules out very small quality differences. However, it is important to notice that the possibility that both firms stay in the market is not ruled out by Assumption 2. From Lemma 2, it follows that if the firms would locate at the opposite ends of the line, the low quality firm would stay in the market as long as $\Delta \leq 3t$.

Using Lemma 3 and 4 and Proposition 1, it is now possible to derive the equilibria of the full game. We will denote the possible equilibria by $\{s_A, s_B\}$ where the two entries refer to firm A's firm B's strategies respectively. Abusing notation slightly, we will denote by $\Pi_i^D(s_A, s_B)$ firm i's expected profit as a function of the two firms' strategies.

Proposition 3 *i)* $\{(\frac{1}{2}, R), (\frac{1}{2}, R)\}$ *is a unique equilibrium if and only if*

$$\Delta \geq \frac{(1-\rho)(4q_L - t)}{4\rho}. \tag{4.8}$$

ii) $\{(\frac{1}{2}, S), (\frac{1}{2}, R)\}$ *and* $\{(\frac{1}{2}, R), (\frac{1}{2}, S)\}$ *are equilibria that (modulo symmetry) are unique if and only if*

$$\frac{25t}{144\rho} \leq \Delta \leq \frac{(1-\rho)(4q_L - t)}{4\rho}. \tag{4.9}$$

iii) $\{(0, S), (1, S)\}$ *is a unique equilibrium if and only if*

$$\Delta \leq \frac{t}{2\rho}. \tag{4.10}$$

Proof: *Part i)* Consider the strategy choice $\{(\frac{1}{2}, R), (\frac{1}{2}, R)\}$. We know from Proposition 1 that if both firms choose the risky technology, the unique equilibrium locations are $a = b = \frac{1}{2}$ under Assumption 2. Therefore, we only need to check for deviations involving the safe technology. It follows from Lemma 4 that the optimal deviation for firm A (B,respectively) would be to the strategy $(\frac{1}{2}, S)$. Therefore, that strategy choice is an equilibrium iff. $\Pi_A^D((\frac{1}{2}, R), (\frac{1}{2}, R)) \geq \Pi_A^D((\frac{1}{2}, S), (\frac{1}{2}, R))$, which reduces to equation (4.8).

Part ii) Consider instead the strategy choice $\{(\frac{1}{2}, S), (\frac{1}{2}, R)\}$ (the analysis for $\{(\frac{1}{2}, R),$ $(\frac{1}{2}, S)\}$ is analogous) It follows from Lemma 4 that if the firms choose different technologies, the only candidate equilibrium locations are both firms choosing $\frac{1}{2}$. Again, we consider deviations to a different technology. Proposition 1 implies that the optimal deviation for firm A is $(\frac{1}{2}, R)$, while Lemma 3 implies that the optimal deviation for firm B is $(1, S)$ (or, $(0, S)$). It follows that strategy choice is an equilibrium iff $\Pi_A^D((\frac{1}{2}, S), (\frac{1}{2}, R)) \geq \Pi_A^D((\frac{1}{2}, R), (\frac{1}{2}, R))$ and if $\Pi_B^D((\frac{1}{2}, S), (\frac{1}{2}, R)) \geq \Pi_B^D((\frac{1}{2}, S), (1, S))$, which reduce to equation (4.9), respectively.

Part iii) Consider the strategy choice $\{(0, S), (1, S)\}$. From Lemma 3 it follows that this is the candidate equilibrium given the technology choice. Suppose that firm B (or, alternatively, A) would choose the technology R. From Lemma 4 it follows that the optimal deviation would be to $(0, R)$. Therefore, the strategy choice is an equilibrium iff. $\Pi_A^D((0, S), (1, S)) \geq \Pi_A^D((0, S), (0, R))$, which reduces to equation (4.10).¥

4.5.2 Discussion

Figure 4.4 summarises the equilibrium outcomes. For low values of both Δ and ρ, the expected return on the risky technology is so low that the firms prefer choosing the safe technology, in which case the firms locate as far as possible from each other. By contrast, if both, Δ and ρ exhibit high values, the risky technology is more attractive. Under Assumption 2, the firms thus locate together in the middle and choose the risky

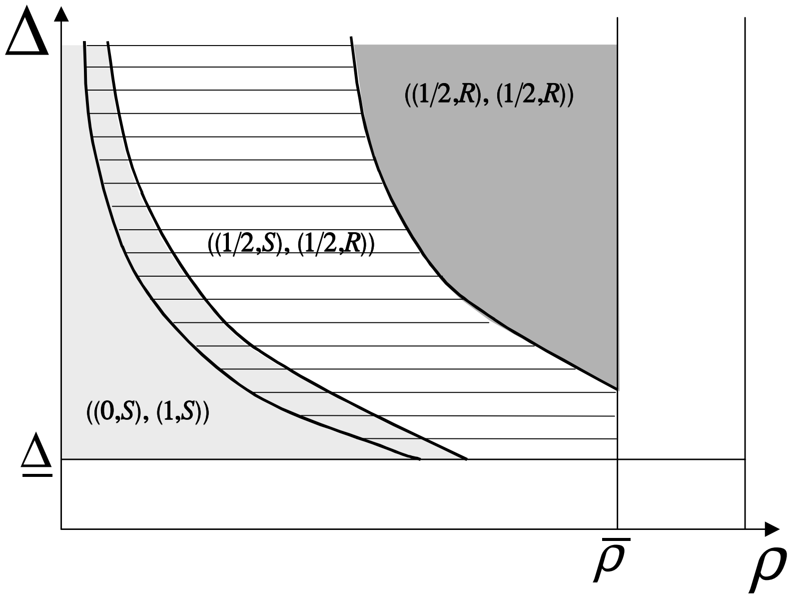

Figure 4.4: *The equilibrium outcome with endogenous R&D choices.*

technology. The most interesting equilibria arise for intermediate values of Δ and ρ. As shown in the figure, we find equilibria where the firms agglomerate in the centre, but choose different technologies. In these equilibria, the firms do not differentiate themselves by choosing different horizontal characteristics, but instead by choosing different R&D strategies which, in our model, invariably lead to differing outcomes: one of the firms will dominate the market, whilst the other will remain inactive.

This result provides an interesting link to different branches of the literature. For instance, in the literature on multi-dimensional horizontal differentiation, it has been shown how firms differentiate themselves maximally in one dimension and minimally in the other one, see Irmen and Thisse (1998). This is similar to what happens in our model for low and intermediate values of Δ and ρ. For low values, they differ maximally in locations but minimally in the vertical dimension, whilst for intermediate values, locational differentiation is minimal and vertical differentiation is maximal.

Yet this cannot happen for high values: Since consumers are willing to pay more for higher quality, both firms differentiate minimally in space, and prefer the risky technology if it has a high expected pay-off.

The risk-return trade-off in R&D has been studied by Dasgupta and Maskin (1987) and Bhattacharya and Mookherjee (1986), among others. They used very general risk-return functions. However, to make the analysis tractable, the authors restricted their attention to symmetric R&D choices by the firms in the industry. In a less general model, we demonstrate that imposing symmetry might be quite restrictive. Indeed, our results suggest that firms have incentives to differentiate themselves also along R&D dimension, in order to compete less fiercely in the product market.

The following corollary looks at the equilibrium outcome under a specific subset of the risky innovation technologies, namely all mean-preserving spreads of the safe innovation technology.

Corollary 5 *If the firms can choose between two technologies with the same mean but a different spread, i.e.* $q_L = \rho q_H$*, the equilibrium outcome under Assumption 2 is* $\{(\frac{1}{2}, R), (\frac{1}{2}, R)\}$.

Proof: Using $q_L = \rho q_H$, we can express the quality difference as $\Delta = \frac{1-\rho}{\rho} q_L$, which is strictly greater than $\frac{(1-\rho)(4q_L - t)}{4\rho}$. It follows from Proposition 3 that the unique equilibrium under Assumption 2 is $\{(\frac{1}{2}, R), (\frac{1}{2}, R)\}$.¥

In the Hotelling model considered here, competition between the firms dissipates aggregate rents.[3] However, if possible, the firms prefer to participate in a lottery where they sometimes win and be alone in the market, and sometimes loose (with the same probability) and be inactive. If both firms choose the risky technology,

[3] In most standard models of competition, like, e.g., Cournot or Bertrand competition with homogenous goods, competition destroys rents compared to a situation of monopoly. In this case, 'the efficiency effect' or 'the joint profit effect' is said to hold.

they are essentially partaking in such a lottery. The cost of the lottery, however, is that if neither of the firms innovates, they cannot serve the market. It turns out, as Corollary 5 shows, that the benefit from gambling in the technology choice outweighs the cost if the risky technology has the same mean quality as the safe one.

4.5.3 Welfare

Let us now assess the welfare properties of the equilibrium derived in the previous section. In order to find the welfare maximising pairs of locations and innovation technologies for both firms, we will proceed as follows. First, we will derive the socially efficient locations for all three possible innovation patterns in the industry. Then, we will compare the attainable welfare levels under these innovation patterns given optimal locations and determine the global maximum. And finally, we compare these results with the equilibrium outcome.

Suppose first that both firms were to employ the risky technology. Then the results of Proposition 2 apply. The corresponding expected welfare level for these locations is derived in the appendix. Second, if both firms are given the safe technology, then, as argued in the benchmark model, the transportation costs are minimised and welfare maximised for $a = \frac{1}{4}$ and $b = \frac{3}{4}$.

The last possibility is that one firm chooses the risky R&D strategy and the other the safe one. Here, we need to take into account not only the transportation costs, but also the quality of the products bought by the consumers. Assumption 2 implies that no matter how far the firms are located from each other, it is welfare maximising that all consumers buy the high quality product whenever it is developed. This is so, as, by Assumption 2 the maximal travelling cost (t) is lower than the quality difference (Δ). The welfare maximising locations are thus $a = b = \frac{1}{2}$ because this ensures that the firm with the highest available quality serves the whole market at the lowest possible transportation cost. This is shown formally in the following lemma.

Lemma 5 *Suppose that Assumption 2 holds and that one firm is endowed with the safe technology and the other with the risky one. Then the welfare maximising locations are* $a{=}b{=}\frac{1}{2}$.

Proof: See appendix.

We are now in the position to compare the welfare outcomes under the different technology choices and corresponding optimal locations of firms. The next proposition states the main welfare result. Define

$$\Delta_1^W \equiv \frac{t}{16\rho} \text{ and } \Delta_2^W \equiv \frac{1-\rho}{\rho} q_L + \frac{t(\rho^3 - 16\rho^2 + 20\rho - 8)}{48t(1-\rho)(2-\rho)},$$

then

Proposition 4 *Suppose that Assumption 2 holds. Then the welfare maximising strategies* $\{s_A^W, s_B^W\}$ *are*

$$\{s_A^W, s_B^W\} = \begin{cases} \{(\frac{1}{4}, S), (\frac{3}{4}, S)\} & \text{for } 0 < \Delta \leq \Delta_1^W, \\ \{(\frac{1}{2}, S), (\frac{1}{2}, R)\} \text{ and } \{(\frac{1}{2}, R), (\frac{1}{2}, S)\} & \text{for } \Delta_1^W < \Delta \leq \Delta_2^W, \\ \{(a^W, R), (b^W, R)\} & \text{for } \Delta > \Delta_2^W. \end{cases}$$

where a^W *and* b^W *are as in Proposition 2.*

Proof: See appendix.

The welfare maximising technology and location choice is illustrated in Figure 4.5. First, for low values of Δ and ϱ where the expected return of risky R&D strategy is low, it is optimal that both firms choose the safe technology. In order to minimise the transportation costs, the firms should then locate at $\frac{1}{4}$ and $\frac{3}{4}$. For intermediate values of Δ and ϱ, it is optimal that one firm tries to develop the high quality product while the other firm develops the safe, low quality product that is then available if the risky R&D strategy fails. Here, as discussed above, the optimal locations are $a = b = \frac{1}{2}$. Finally, if Δ and ϱ are high, the risky technology is so attractive that both firms should choose it from point view of welfare. The optimal locations are then symmetric around $\frac{1}{2}$ and situated between $\frac{1}{4}$ and $\frac{3}{4}$ as discussed in section 4.

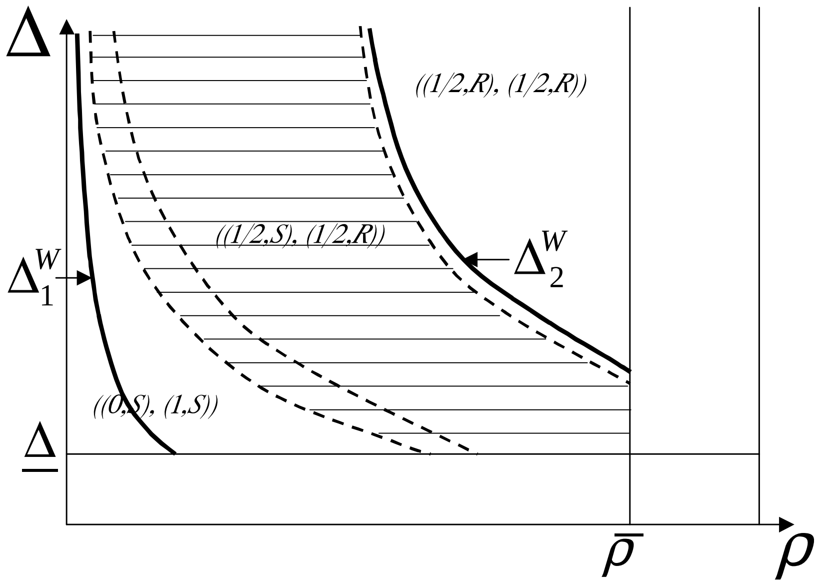

Figure 4.5: *Welfare properties of the equilibrium.*

Finally, the last corollary compares the socially efficient allocation with the equilibrium outcome.

Corollary 6 *Comparing Proposition 3 and Proposition 4, we get*
(i) If $0 < \Delta \leq \Delta_1^W$*, firms excessively differentiate in product space but the optimal innovation techniques.*
(ii) If $\Delta_1^W < \Delta \leq \frac{t}{2\rho}$*, firms excessively differentiate in product space and one firm inadequately chooses the safe production strategy.*

(iii) If $\frac{25}{144\rho} < \Delta \leq \frac{(1-\rho)(4q_L - t)}{4\rho}$*, firms choose the welfare maximizing allocation.*

(iv) If $\frac{(1-\rho)(4q_L - t)}{4\rho} < \Delta \leq \Delta_2^W$*, firms choose optimal locations but one firm inadequately adopts the risky innovation strategy.*

(v) If $\Delta > \Delta_2^W$*, firms excessively concentrate in product space but select the optimal innovation techniques.*

Figure 4.5 shows that for low values of Δ and ϱ (point (i) and (ii) in the corollary), the firms adopt the safe technology and differentiate themselves in the horizontal dimension. In equilibrium, there is too much horizontal differentiation given the safe technology choice. Furthermore, there is a region, described in point (ii), where there is too little R&D differentiation. For intermediate values Δ and ϱ (point (iii)), the firms choose the optimal location as well as technology. For high values of Δ and ϱ (point (v)), the firms adopt the risky technology, which is the welfare maximising choice. However, when choosing their location the firms put too much weight on the monopoly outcome relative to duopoly. The reason is that the firms extract all social surplus under monopoly but not under duopoly. Therefore, there is too little horizontal differentiation in equilibrium, as the firms agglomerate in the middle (the optimal location under monopoly). Finally, there is a small region where the firms choose the optimal horizontal product characteristics, but end up with too much R&D differentiation (point (iv)).

4.6 Conclusions

In this paper, we analyse the interaction between firms' R&D decisions and their location choices in product or geographical space which have not been analysed heretofore. In a benchmark model, we introduce stochastic R&D in the classical Hotelling model and show that this might restore the principle of minimum differentiation even in a model with quadratic transportation costs of consumers. The intuition for this result is that if R&D success is stochastic, a firm only meets a competitor in the product market with a certain probability and this weakens the centrifugal 'competition' effect that normally dominates the centripetal 'demand' effect in the Hotelling model.

Modifications of the model to include R&D spillovers or patenting show clearly that spillovers exercise a deglomerating and patents an agglomerating effect. The reason is that spillovers align the firms' R&D successes and make them more competitive when they are close to each other. Whereas patent protection lead one of the firms to win, at the cost of the other which decreases the competitive pressure.

In the second part of the paper, we combine locational choices of firms with an endogenisation of their R&D technologies in the sense that they can choose between riskless production of a low quality good or a risky development of a high quality good. Our results hint at a strong complementarity between risk taking in R&D and clustering in product or geographical space. We find three different types of equilibria: (i) firms stay apart from each other and choose the safe innovation technology, (ii) they cluster in the center and choose the risky technology or (iii) they cluster in the center and differentiate themselves along the R&D dimension.

As far as welfare comparisons is concerned, we obtain excessive dispersion (yet compliance with the welfare allocation with respect to the chosen R&D technique) for low quality differences, and similarly excessive concentration for high quality differences. For intermediate differences, we obtain either the welfare result or deviations with respect to the chosen R&D strategy.

There is an utterly clear labor market interpretation for the model results for the case where both firms choose to agglomerate whilst employing the risky R&D technology: Firms agglomerate to share the same (large) labor market when chances are high that only one of the two firms (at a time) benefits from it. To us, this most clearly reflects the hearsay about Silicon valley dynamics. According to that, 'firms come and go, labor stays'.

It remains to speculate about possible extensions of the model. A particularly interesting, yet difficult one seems to be to consider that firms produce several products and compete only in a subset of these, yet benefit from a central location in two ways. First, from participation in a large output (or labor) market; and second, from shared knowledge that helps improving also on the product lines in which they don't compete.

Another interesting extension, especially within the geographical interpretation of the model would be to explicitly account for the localized cumulative effects of R&D. Empirical research suggests that knowledge seems not to spread as easily as one would expect. It is not only embedded in human beings, but also in, possibly informal, institutions. It seems that this feature also enhances the spatial concentration of firms in R&D intensive industries. Yet, as usual, this, as well as other potential extensions of our model, must be left for further consideration.

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4.8 Appendix

Proof of Lemma 2

The demand of firm A can is given as

$$\widetilde{y} = Max\left\{0, Min\left\{\frac{a+b}{2} - \frac{\widetilde{p}_A - \widetilde{p}_B}{2t(b-a)}, 1\right\}\right\},$$

where $\widetilde{p}_i = q_i - p_i$ is the quality adjusted price of firm i. We know from d'Aspremont, Gabszewicz & Thisse (1979) that $\widetilde{y}$ is continuous in $(\widetilde{p}_A, \widetilde{p}_B)$, so it follows immediately that it is also continuous in (p_A, p_B). Quality differences do thus not create problems for the existence of a price equilibrium.

Define $\overline{p}_A \equiv p_B + q_A - q_B + (b^2 - a^2)t$ and $\underline{p}_A \equiv p_B + q_A - q_B - (b-a)(2-a-b)$. The profit function of firm A (for given a, b, q_A, and q_B) can be written as:

$$\Pi_A^D(p_A, p_B) = \begin{cases} 0 & \text{for } p_A > \overline{p}_A \\ p_A\left(\frac{a+b}{2} - \frac{p_A - p_B}{2t(b-a)}\right) & \text{for } \overline{p}_A \geq p_A \geq \underline{p}_A \\ p_A & 0 \leq p_A < \underline{p}_A. \end{cases}$$

It is easy to verify that $\Pi_A^D(p_A, p_B)$ is continuous in p_A. Furthermore, $\Pi_A^D(p_A, p_B)$ consists of concave segments and the left hand side derivative (LHS) at $\underline{p}_A$ is larger than the right hand side (RHS) derivative. Therefore, $\Pi_A^D(p_A, p_B)$ is concave for $p_A \leq \overline{p}_A$, so whenever the exists an optimal price, p_A^*, such that $\Pi_A^D(p_A^*, p_B) > 0$, p_A^* is unique. The profit function of firm B is derived analogously. Suppose first that there exists an interior solution where both firms have a positive market share. The first order conditions of the two firms are given as:

$$\begin{aligned} \frac{\partial \Pi_A^D(p_A, p_B)}{\partial p_A} &= \frac{a+b}{2} - \frac{2p_A - p_B - q_A + q_B}{2(b-a)t} = 0 \\ \frac{\partial \Pi_B^D(p_A, p_B)}{\partial p_B} &= (1 - \frac{a+b}{2}) - \frac{2p_B - p_A - q_B + q_A}{2(b-a)t} = 0. \end{aligned}$$

Solving for (p_A, p_B), we obtain (4.3). We need to check for corner solutions. Take the case of $q_A - q_B > t(b-a)(4-a-b)$. We will now verify that prices given by (4.4) constitute an equilibrium. For firm A, the LHS derivative at $\underline{p}_A$ is 1, while the RHS derivative is $\frac{\partial \Pi_A^D(p_A,p_B)}{\partial p_A}|_{(p_A,p_B)=(\underline{p}_A,0)} < 0$. Hence, the optimal price is $p_A^* = \underline{p}_A = q_A - q_B - t(b-a)(4-a-b)$. For firm B, $\overline{p}_B < 0$, so firm B earns zero profits. Thus, $p_B^* = 0$ is an optimal price. The other corner solution, given by (4.2), can be verified in a similar way. Finally, it can be shown that under the assumption $p_A, p_B \geq 0$, the price equilibrium is unique.¥

Proof of Proposition 1

We we will here solve the model allowing for spillovers. If there are spillovers, the profit of firm A is given as:

$$E(\Pi_A(a,b,q_A,q_B,\rho)) = \rho(\rho(1-\sigma)\Pi_A^D(a,b,0) + (1-\rho(1-\sigma))\Pi^M(q,a)).$$

Solving the first-order condition, and excluding solutions that do not satisfy $0 \leq a \leq 1$, we obtain the reaction function:

$$R_A(b) = \frac{\left(-36 - 4\left(-5+b\right)\sigma - \rho\left(-28+2b+20\sigma-4b\sigma\right) + \sqrt{V+W}\right)}{6\rho + 12\left(1-\rho\right)\sigma},$$

where

$$V := \left(36 + 2\left(-14+b\right)\rho - 4\left(5-b\right)\left(1-\rho\right)\sigma\right)^2$$

and

$$W := 4\left(3\rho + 6\left(1-\rho\right)\sigma\right)\left(36 - 40\rho + b^2\rho - 2\left(22-b^2\right)\left(1-\rho\right)\sigma\right)$$

.

Deriving the reaction function of firm B, we have:

$$R_B(a) = \frac{36 - 4\left(1+a\right)\sigma - 2\rho\left(10+a-2\left(1+a\right)\sigma\right) - \sqrt{X-Y}}{6\rho + 12\left(1-\rho\right)\sigma},$$

where

$$X := \left(36 - 2\left(10 + a\right)\rho - 4\left(1 + a\right)\left(1 - \rho\right)\sigma\right)^2$$

and

$$Y := 12\left(16 - a^2\right)\left(\rho + 2\sigma - 2\rho\sigma\right)^2$$

.

It can be verified that $R_B(1 - x) = 1 - R_A(x)$. Using the symmetry, we solve $R_A(b) = 1 - b$ and $R_B(a) = 1 - a$ for (a, b), which proves Corollary 1. Proposition 1 follows directly by setting $\sigma = 0$. Corollary 2 is proved in a similar way.¥

Proof of Lemma 4

We will without loss of generality focus on situations where $a \leq b$ and firm A has chosen the safe technology and firm B the risky.

The reaction function of firm A

>From equation (4.7) it follows that firm A is driven out the market when firm B innovates iff. $t(b - a)(2 + a + b) - \Delta \leq 0$. Therefore, there exists a value of a, $\overline{a}$, such that Firm A stays in the market if Firm B innovates iff. $a \leq \overline{a}$. $\overline{a}$ is gives as:

$$\overline{a} \equiv Max\left\{0, -1 + \frac{\sqrt{(1+b)^2 t - \Delta}}{\sqrt{t}}\right\}.$$

Using equation (4.7), the profit function of firm A can be written as:

$$\Pi_A^D((a, S), (b, R) = \begin{cases} \rho\frac{(t(b-a)(2+a+b)-\Delta)^2}{18t(b-a)} + (1-\rho)(q_L - t(Max\{a, 1-a\})^2) & \text{for } a \leq \overline{a} \\ (1-\rho)(q_L - t(Max\{a, 1-a\})^2) & \text{for } a > \overline{a}. \end{cases}$$

Case 1: $\overline{a} = 0$

Maximizing the profits wrt. a yields:

$$\frac{\partial \Pi_A^D((a,S),(b,R))}{\partial a} = \begin{cases} 2(1-\rho)(1-a)t \text{ for } a \leq \frac{1}{2}. \\ -2(1-\rho)at \text{ for } a > \frac{1}{2}. \end{cases} \tag{4.11}$$

It follows that the optimal location is $a^* = \frac{1}{2}$.

Case 2: $0 < \overline{a} \leq \frac{1}{2}$

It follows from (4.11) that the $\Pi_A^D((a,S),(b,R))$ is increasing for $\frac{1}{2} \geq a > \overline{a}$ and decreasing after $\frac{1}{2}$. We now look at $a \leq \overline{a}$. Maximizing the profits wrt. a yields:

$$\frac{\partial \Pi_A^D((a,S),(b,R))}{\partial a} =$$
$$\frac{1}{18}\left(-2(\Delta)\rho + \frac{(\Delta)^2\rho}{(a-b)^2 t} - \left(36(-1+a) + \left(40 + 3a^2 + 2a(-14+b) - b^2\right)\rho\right)t\right)$$

We have that

$$\frac{\partial^2 \Pi_A^D((a,S),(b,R))}{\partial a \partial b} = \frac{t\rho}{9}\left((b-a) - \frac{1}{(b-a)^3}\right) < 0.$$

This implies that $\frac{\partial \Pi_A^D((a,S),(b,R))}{\partial a} \geq \frac{\partial \Pi_A^D((a,S),(1,R))}{\partial a}$ for all (a,b). We want to show that $\frac{\partial \Pi_A^D((a,S),(b,R))}{\partial a} \geq 0$ and it is thus sufficient to show that $\frac{\partial \Pi_A^D((a,S),(1,R))}{\partial a} \geq 0$ for all $a \leq \overline{a}$. As a first step, we look at the $\frac{\partial \Pi_A^D((a,S),(1,R))}{\partial a}$ evaluated at $a = 0$. From Assumption 2 it follows that

$$\frac{\partial \Pi_A^D((0,S),(1,R))}{\partial a} = \frac{1}{18}\left(-2\Delta\rho + \frac{\Delta^2\rho}{t} + (36 - 39\rho)t\right) \geq 0.$$

Second, we show that $\frac{\partial \Pi_A^D((a,S),(1,R))}{\partial a}$ is positive at $a = \frac{1}{2}$ (the maximal value of a). Straight forward calculations show that $\frac{\partial \Pi_A^D((\frac{1}{2},S),(1,R))}{\partial a}$ is positive if Assumption 2 holds:

$$\frac{\partial \Pi_A^D((\frac{1}{2},S),(1,R))}{\partial a} = \frac{1}{18}\left(-2\Delta\rho + \frac{4\Delta^2\rho}{t} + \left(18 - \frac{107\rho}{4}\right)t\right) \geq 0.$$

We have now shown that $\frac{\partial \Pi_A^D((a,S),(1,R))}{\partial a}$ is positive at the minimal and the maximal value of a. What is left to show is that $\frac{\partial \Pi_A^D((a,S),(1,R))}{\partial a}$ cannot take on negative values between 0 and $\frac{1}{2}$. We show this in two steps: First, we show that $\frac{\partial^2 \Pi_A^D((\frac{1}{2},S),(1,R))}{\partial a^2}$ is negative at $a = 0$ and $a = \frac{1}{2}$. Afterwards, we show that $\frac{\partial \Pi_A^D((\frac{1}{2},S),(1,R))}{\partial a}$ is convex, i.e. $\frac{\partial^3 \Pi_A^D((\frac{1}{2},S),(1,R))}{\partial a^3} \geq 0$. This imply that the first order condition is positive for all $0 \leq a \leq \frac{1}{2}$.

We have:

$$\frac{\partial^2 \Pi_A^D((0,S),(1,R))}{\partial a^2} = -\frac{2}{9}(9 - 7\rho) < 0 \text{ and}$$

$$\frac{\partial^2 \Pi_A^D((\frac{1}{2},S),(1,R))}{\partial a^2} = (-2 + \frac{13}{6}\rho) < 0,$$

where $\frac{\partial^2 \Pi_A^D((\frac{1}{2},S),(1,R))}{\partial a^2} < 0$ holds because of Assumption 2. Finally, using Assumption 2, we have:

$$\frac{\partial^3 \Pi_A^D((a,S),(1,R))}{\partial a^3} = \frac{\rho}{3}\left(\frac{\Delta^2}{t(b-a)^4} - t\right) > 0.$$

Hence, we conclude that if $\overline{a} \leq \frac{1}{2}$, the first order condition is positive for all $a \leq \frac{1}{2}$ and negative for $\frac{1}{2} < a \leq b$. It follows that $a^* = \frac{1}{2}$.

Case 3: $\overline{a} > \frac{1}{2}$

It follows from the analysis of Case 2 that the first order condition is positive for $a \leq \frac{1}{2}$. Consider now $\frac{1}{2} < a \leq \overline{a} < b$. Here, the first order condition is given as

$$\frac{\partial \Pi_A^D((a,S),(b,R))}{\partial a} = \\ \frac{\rho(\Delta - t(b-a)(2+a+b)t)(\Delta + t(b-a)(2+3a-b)t)}{18(b-a)^2} + (-2(1-\rho)at) < 0.$$

It follows form the analysis of Case 1 that $\frac{\partial \Pi_A^D((a,S),(1,R))}{\partial a} < 0$ for $\overline{a} < a \leq b$. Hence, $a^* = \frac{1}{2}$. This proves part i) of Lemma 4.

The reaction function of firm B

If firm B innovates, it drives firm A out the market $t(b-a)(2+a+b) - \Delta \leq 0$. Therefore, there exists a value of b, $\overline{b}$, such that Firm A stays in the market if Firm B innovates iff. $b \geq \overline{b}$. $\overline{b}$ is gives as:

$$\overline{b} \equiv Min\left\{1, -1 + \frac{\sqrt{(1+a)^2 t + \Delta}}{\sqrt{t}}\right\}.$$

Using equation (4.7), the profit function of firm B can be written as:

$$\Pi_B^D((a,S),(b,R) = \begin{cases} \rho(\Delta - t(b^2 - a^2)) & \text{for } b \leq \overline{b} \\ \rho\frac{(t(b-a)(4-a-b)-\Delta)^2}{18t(b-a)} & \text{for } b > \overline{b}. \end{cases}$$

First, we consider the consider the case of $b \leq \overline{b}$. Here, we have

$$\frac{\partial \Pi_B^D((a,S),(b,R))}{\partial b} = -2\rho bt < 0 \text{ for } b \leq \overline{b}.$$

Next, we consider $b > \overline{b}$:

$$\frac{\partial \Pi_B^D((a,S),(b,R))}{\partial b} = -\frac{\rho(\Delta - (b-a)(4+a-3b)t)(\Delta + (b-a)(4-a-b)t)}{18(b-a)^2} \text{ for } b > \overline{b}.$$

As a first step, we show that this function is convex in the relevant range.

$$\frac{\partial^2 \Pi_B^D((a,S),(b,R))}{\partial b^2} = \rho\frac{\left(-\frac{\Delta^2}{(a-b)^3} - t(8-a-3b)t^2\right)}{9t}$$

It can be shown that the $\frac{\partial^3 \Pi_B^D((a,S),(b,R))}{\partial b^3} > 0$. Hence, in order to show that the function is convex, it is enough to show that it is convex at the lowest possible value of b, $\bar{b}$. Calculations show that $\frac{\partial^3 \Pi_B^D((a,S),(b,R))}{\partial b^2 \partial a}\mid_{b=\bar{b}} > 0$ under Assumption 2. To show convexity, it is thus sufficient to show that $\frac{\partial^2 \Pi_B^D((a,S),(b,R))}{\partial b^2}\mid_{(a,b)=(0,\bar{b})} > 0$. We have that

$$\frac{\partial^2 \Pi_B^D((a,S),(b,R))}{\partial b^2}\mid_{(a,b)=(0,\bar{b})} = \frac{4\rho}{9\Delta}\left(t(t-2\Delta)+\sqrt{t}(t+\Delta)^{3/2}\right).$$

Analysis of this function shows that it has a global minimum at $\Delta = 3t$ where it takes on the value $\frac{4\rho t}{9}$. Hence, we have $\frac{\partial^2 \Pi_B^D((a,S),(b,R))}{\partial b^2} > \frac{\partial^2 \Pi_B^D((a,S),(b,R))}{\partial b^2}\mid_{(a,b)=(0,\bar{b})} > 0$ for (a,b) in the relevant range, so the $\Pi_B^D((a,S),(b,R))$ is convex. In order to show that $\frac{\partial \Pi_B^D((a,S),(b,R))}{\partial b} < 0$ for all $b > \bar{b}$, it is thus enough to show that $\frac{\partial \Pi_B^D((a,S),(b,R))}{\partial b}\mid_{b=1} < 0$. We have that

$$\frac{\partial^2 \Pi_B^D((a,S),(1,R))}{\partial b \partial \Delta} < 0.$$

Hence, the first order condition is maximized for the minimal value of Δ. Under Assumption 2, this is $\Delta = t$. Therefore, we have

$$\frac{\partial \Pi_B^D((a,S),(1,R))}{\partial b} < \frac{\partial \Pi_B^D((a,S),(1,R))}{\partial b}\mid_{\Delta=t} = -\frac{(-2+a)^2 a^2 \rho t}{18(1-a)^2} < 0.$$

We conclude that $\frac{\partial \Pi_B^D((a,S),(b,R))}{\partial b} < 0$ for all $b > \bar{b}$. Hence, the $\frac{\partial \Pi_B^D((a,S),(b,R))}{\partial b} < 0$ for all $b \geq a$, so $b^* = a$. This proves part ii) of Lemma 4.¥

Proof of Proposition 2

Total welfare in a monopoly where a firm offers a product of quality q at location a, $0 \leq a \leq 1$ is given by

$$W^M := \int_0^1 (q - t(y-a)^2)dy = q - \frac{t}{2} + at(1-a).$$

In a duopoly with firms located at a and b, $0 \leq a \leq b \leq 1$ and offering a product of quality q, welfare is

$$\begin{aligned} W^D \quad : \quad &= \int_0^{\frac{2+a+b}{6}} (q - t(a-y)^2)dy + \int_{\frac{2+a+b}{6}}^1 (q - t(b-y)^2)dy \\ &= q - \frac{t}{3} + bt(1-b) + \frac{t}{36}(b-a)(2+a+b)(5b+5a-2). \end{aligned}$$

Expected welfare in the economy, $EW(a,b)$, is then defined as

$$EW(a,b) := \rho^2 W^D + \rho(1-\rho)W^M(a) + (1-\rho)\rho W^M(b).$$

Taking the first derivatives of this function with respect to the locations a and b yields the following two necessary conditions:

$$\frac{(4 - a\ (16 + 15\,a) - 10\,a\,b + 5\,b^2)\ \rho^2\,t}{36} + (1 - 2\,a)\ (1-\rho)\ \rho q = 0$$

and

$$\frac{(32 - 56\,b + 5\ (a+b)\ (-a + 3\,b))\,\rho^2\,t}{36} + (1 - 2\,b)\ (1-\rho)\ \rho q = 0.$$

These conditions are also sufficient for a maximum since

$$\frac{\partial^2 EW(a,b)}{\partial a^2} = \frac{-\left((8 + 15\,a + 5\,b)\ \rho^2\,t\right)}{18} - 2\ (1-\rho)\ \rho\,q < 0$$

and

$$\frac{\partial^2 EW(a,b)}{\partial b^2} = -2\,q\ (1-\rho)\,\rho - \frac{(28 - 5\,a - 15\,b)\,\rho^2\,t}{18} < 0.$$

Thus, solving the two first order conditions above for (a,b) yields the welfare maximising locations given in the proposition.¥

Proof of Lemma 5

We assume as in the proof of Lemma 4 that firm A chooses the safe technology and firm B the risky. $\overline{a}$ and $\overline{b}$ are defined in Lemma 4. If firm A is alone in the market, the welfare under Assumption 1 is given as:

$$W_A^M(a) = \int_0^1 \left(q_L - t(a-x)^2\right) dx = q_L - \frac{t}{3} + at(1-a).$$

If firm B innovates and $b \geq \overline{b}$, so firm A stays active in the market, the welfare is given as:

$$W_{A,B}^D(a,b) = \int_0^{\widetilde{y}} \left(q_L - t(a-x)^2\right) dx + \int_{\widetilde{y}}^1 \left(q_H - t(b-x)^2\right) dx$$

where $\widetilde{y}$ is given by 4.1. Integrating this expression, we obtain:

$$W_{A,B}^{D}(a,b) = \frac{1}{36}(2(4+5a+5b)q_L + 28q_H - 10(a+b)q_H - \frac{5(q_L-q_H)^2}{(a-b)t} - \left(12+5a^3-(-4+b)b(-8+5b)+a^2(8+5b)-a4+5b^2)\right)t).$$

Finally, if firm B innovates, and $b < \overline{b}$, the expected welfare is given by:

$$W_B^D(b) = \int_0^1 \left(q_H - t(b-x)^2\right)dx = q_H - \frac{t}{3} + bt(1-b).$$

The expected ex-ante welfare is then given as:

$$E(W(a,b)) = \begin{cases} \rho W_{A,B}^{D}(a,b) + (1-\rho)W_A^M(a) & \text{if } b \geq \overline{b} \\ \rho W_B^D(b) + (1-\rho)W_A^M(a) & \text{if } b < \overline{b}. \end{cases}$$

We first the optimal location of firm B. It is easily shown that for $b < \overline{b}$, $E(W(a,b))$ is maximized for $b = Min\{1/2, \overline{b}\}$. Consider now $b \geq \overline{b}$. The first-order condition wrt. to b is given as:

$$\frac{\partial E(W(a,b))}{\partial b} = \frac{r}{36}\left((32-56b-5(a-3b)(a+b))t - 10\Delta - \frac{5\Delta^2}{(a-b)^2 t}\right).$$

We want to show that $\frac{\partial E(W(a,b))}{\partial b} < 0$ in the relevant area. Since $\frac{\partial^2 E(W(a,b))}{\partial b \partial a}, \frac{\partial^2 E(W(a,b))}{\partial b \partial \Delta} < 0$, $\frac{\partial E(W(a,b))}{\partial b}$ takes on the highest value for the lowest admissible values of Δ and a: $\Delta = t$ and $a = 0$. Plugging these values into the first-order condition, we obtain:

$$\frac{\partial E(W(0,b))}{\partial b} = \frac{rt}{36b^2}\left(-5+b^2(22+b(-56+15b))\right)$$

In order to find the maximal value of $\frac{\partial E(W(0,b))}{\partial b}$, we solve:

$$\frac{\partial^2 E(W(0,b))}{\partial b^2} = \frac{(5+b^3(-28+15b))rt}{18b^3} = 0.$$

We find that there is only one extremum in $[0,1]$ at $b \approx 0,6494$. Furthermore, as $\frac{\partial^3 E(W(0,b))}{\partial b^3} < 0$, this is a maximum. Finally, we show that $\frac{\partial E(W(0,b))}{\partial b} |_{b=0,6494} < 0$.

Hence, $\frac{\partial E(W(a,b))}{\partial b} < 0$ for all (a,b) st. $b \geq \overline{b}$. It follows that the optimal b is given by $b^* = Min\{1/2, \overline{b}\}$.

$b = Min\{1/2, \overline{b}\}$ implies that firm A is out of the market whenever firm B innovates. This implies readily that $a^* = \frac{1}{2}$. Finally, $a^* = \frac{1}{2}$ implies that $\frac{1}{2} < \overline{b}$, so $b^* = \frac{1}{2}$.¥

Proof of Proposition 4

Expected welfare if both firms have the safe technology and are located at $\frac{1}{4}$ and $\frac{3}{4}$, respectively is denoted by W_{SS} and equals

$$W_{SS} = q_L - \frac{t}{48}.$$

If both firms adopt the risky technology and locate as stated in Proposition *2 at* (a^W, b^W), the expected welfare, W_{RR}, is given by

$$W_{RR} = (q_L + \Delta)\rho(2-\rho) - \frac{\rho t}{48(2-\rho)}\left(16(1-\rho)+\rho^2\right).$$

Finally, if firms have different innovation technologies and are located at the center of the line, we get

$$W_{RS} = q_L + \rho\Delta - \frac{t}{12}.$$

>From this, if follows that

$$W_{RR} > W_{SS} \iff \Delta > \Delta_3^W := \frac{q_L(1-\rho)^2}{(2-\rho)\rho} - \frac{(1-\rho)(2-(15-\rho)\rho)t}{48(2-\rho)^2\rho}$$

and that

$$W_{RS} > W_{SS} \iff \Delta > \Delta_1^W$$

and finally

$$W_{RR} > W_{RS} \iff \Delta > \Delta_2^W.$$

We will now proceed by showing that under our assumptions A.1. and A.2., the ranking of these threshold values is unambiguous. First, notice that the following ordering holds. If $q_L > \widetilde{q}_L := \frac{(14-29\rho+19\rho^2-\rho^3)t}{48(2-\rho)(1-\rho)}$, then $\Delta_1^W < \Delta_3^W < \Delta_2^W$, otherwise we have $\Delta_2^W < \Delta_3^W < \Delta_1^W$. Verify that $\widetilde{q}_L$ is increasing in ρ. Further check that $\widetilde{q}_L$ is smaller than $3t$ as long as $\rho < 0.8756$. Hence, we have that for all parameter values that satisfy A.1. and A.2., the ranking is $\Delta_1^W < \Delta_3^W < \Delta_2^W$. This means that if $W_{RR} > W_{RS}$, then it also holds that $W_{RR} > W_{SS}$. And, from $W_{SS} > W_{RS}$ it follows that $W_{SS} > W_{RR}$. We thus get the result stated in Proposition 4.¥