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Shortfall Risks and Excess Chances of Option-Based Rollover Hedge-Strategies with Respect to Alternative Target Returns: Empirical Evidence from the German Stock Market

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Abstract

Continuing the research of an earlier AFIR-paper, we examine on the basis of a (partially) historical simulation approach return and risk of various rollover option strategies (put hedge; covered short call; collar). In addition to measures of shortfall risks we propose measures of excess returns (with respect to the same target) and analyse as well the consequences of alternative target returns on the shortfall risks resp. the excess chances. Finally we try to identify dominance relations between the different types of option strategies.

Résumé

Les auteurs continuent leurs recheres, présenté au dernier colloque d'AFIR. Sur la base d'une simulation (partiellement) historique, ils evaluent le possibilités et risques des stratégies de hedging par options (put hedge; covered short call; collar). Les critères d'evaluation des risques mentionnés sont: shortfall probabilité, shortfall moyenne et shortfall variance. Les critères d'evaluation des possibilités sont:: rendement moyen, excess probabilité, excess moyenne et excess variance. Enfin les auteurs essaient d'identifier des relations déterminantes entre les diffèrents types des stratégies de hedging par options.

Keywords

Shortfall risk, excess chance, rollover hedge strategies, target returns.

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1 Introduction

The introduction of options markets provides a significant expansion in the patterns of portfolio returns that were previously unachievable by conventional investment vehicles, e.g. using combinations of stocks and fixed-income instruments. To be able to profit from additional investment opportunities provided by options, investors must be informed about the historical risk and return characteristics of various option strategies. In the present paper additional information on the risk-return characteristics of option strategies is given for a variety of combined rollover option strategies based on (partially) historical simulations, thus continuing the studies by *Merton/Scholes/Gladstein* 1978, 1982 and *Albrecht/Maurer/Stephan* 1995. We examine the return characteristics of option strategies over a number of market cycles and varying market environments and demonstrate how option strategies can be used to transform the original risk return patterns of an underlying stock (index) portfolio.

Hitherto, a still unanswered question is, how risk should be measured adequately for combined stock and option positions. The adequacy of the traditional measures of risk in portfolio theory, the variance resp. the standard deviation, is criticized to an increasing extent. Especially when analyzing positions with options, symmetrical measures of risk, like the variance or the standard deviation, are not very suitable, cf. already *Bookstaber/Clarke* 1985, as option positions typically generate an asymmetrical risk-return-profile. Looking at the put hedge for example, the downside risk of the investor is limited to an absolute extent. On the other hand the investor participates in increases of the price of the underlying object to an unlimited extent (only reduced by the option premium). It is almost evident that the variance in case of a put hedge is not a measure of risk but a measure of investment chances (in the sense of "upward" volatility). In the papers of *Albrecht* 1994, *Albrecht/Maurer/Timpel* 1995 and *Albrecht/Maurer/Stephan* 1995 attention has been drawn to a general class of shortfall risk measures with the known special cases shortfall probability, shortfall expectation and shortfall variance. Shortfall-risk denotes the risk that the realized return of a financial investment is below a specific

minimum target return.

While in decision theory and financial literature the adequate measure of risk has been extensively discussed for a long time, there have been no discussions at all about the expected return as an adequate measure of chance. As *Albrecht* 1994 pointed out, the expected return does not seem to be a consistent approach for measuring chances within a target return framework, because possible returns below the target are taken into consideration as well. Therefore we propose in complete analogy to the construction of shortfall-risk measures to use excess returns (returns exceeding the target return) as alternative measures of chance.

The selection of the target return, i.e. the reference point for measuring risk resp. chance, is a central point when analyzing shortfall risks resp. excess chance. Performing a sensitivity-analysis of the various risk-/chance-measures with respect to two different target returns is an additional object of our study. Finally we focus on risk-value-models, as discussed by *Sarin/Weber* 1993, and try to identify strategies which are dominated by other strategies within this framework.

2 Data basis

The data basis in our earlier study (*Albrecht/Maurer/Stephan* 1995) was the 34-year interval from January 1960 to December 1993. For the present study, it has been extended to the 36-year interval from January 1960 to December 1995. As in the earlier study, all the strategies examined have monthly holding periods between portfolio revisions. Therefore, there are a total of 432 basic observation subperiods in the simulations. This time series covers a sufficient number of different market enviroments as well as special events as the October Crash in 1987.

To perform the simulations of the various option strategies, we choose the German stock-index DAFOX (cf. *Göppl/Schütz* 1992) as the relevant underlying object on which options are either written or purchased. The stocks in the DAFOX span a wide range of risk levels and dividend yields, therefore portfolios of these stocks are expected to be well diversified. Furthermore the DAFOX is designed as a performance index in the sense of *Laspeyres*, i.e. all cash flows from each stock are (instantly) reinvested according to an operation blanche in the specific stock. Accordingly, the DAFOX time series mirrors the performance of an unprotected index strategy with respect to a highly diversified portfolio of German stocks.

To generate a time series of returns for the examined option strategies it is necessary to have put- and call-options premiums on the DAFOX. Because options on the DAFOX are not traded, all option prices are derived from the *Black-Scholes*-option pricing model. For a number of reasons we think that the *Black/Scholes* premiums are reasonable estimates. First, the *Black/Scholes*-formula assumes an underlying like the DAFOX with no cash dividends until time to expiration. Because in our study we often use in-the-money options, and the chosen time to expiration is relatively short, in this case the inner values of the options constitute an essential part of the total option premiums. All option positions are maintained until expiration, hence, the option pricing model is needed only to estimate the premium at the time the position is established.

To estimate the volatility parameter for the DAFOX we use - as proposed in *Hull* (1993, p. 215) - in each case the sample variance of monthly logarithmic price changes of the DAFOX over the 11 previous months. This takes into consideration (in a special way) the possibility of a changing volatility parameter in the considered subperiods.

As the riskless interest rate per month we use the one month money rates at the beginning of the corresponding month that are published by the German Federal Bank. Transaction costs of 1 % of the option premium are assumed when buying or selling an option. When exercising an option at the end of the month we additionally assume 0.2 % of the inner value as transaction costs.

3 The strategies

In a similar fashion to our earlier study, at the beginning of each month (date of rollover) one European one-month-put or/and one European one-month-call on the DAFOX is bought resp. sold and is maintained until expiration. The monthly cash flows received in form of option premiums for the short calls resp. payed for the long puts and the transaction costs are financed or invested according to an operation blanche by additionally investing eventual proceeds in the DAFOX and by financing capital requirements by selling a portion of the DAFOX-portfolio. Specifically, we consider the following option-based hedging strategies:

I. Rollover (Fixed-Percentage) Put Hedge Strategies: At every rollover date we buy one put on the basis of a fixed-percentage strategy, i.e. the exercise prices X_t are corresponding to a fixed percentage rate p of the price S_t of the DAFOX being the underlying security, i.e. $X_t = (p/100)S_t$. We consider one in-the-money strategy ($p = 102$), one at-the-money strategy ($p = 100$) as well as three out-of-the-money strategies ($p = 94, 96, 98$).

II. Rollover (Fixed Percentage) Covered Short Call Strategies: Following this strategy the investor sells a call at each rollover date, the exercise price being a fixed percentage of the price of the underlying. Again we consider one in-the-money strategy ($p = 98$), one at-the-money strategy ($p = 100$) as well as three out-of-the-money strategies ($p = 102, 104, 106$).

III. Rollover (Fixed Percentage) Collar Strategies: Following this strategy at each rollover date one put is bought and one call with a higher excercise price as the put is sold simultaneously. The exercise prices of both the put and the call follow fixed percentage strategies and only the symmetrical strategies (94, 106), (96, 104) and (98, 102) are considered.

4 Measures of Risk and Chance of Combined Option Strategies

Taking into account the asymmetrical nature of positions with options we will measure risk using three measures of shortfall risk. As shown in *Albrecht* (1994, p. 93) a general class of risk measures with respect to a *deterministic* target return r_Z can be obtained by using the *lower partial moments* of degree $n \geq 0$ of the random return R:

$$\mathrm{LPM}_n(R; r_Z) = \mathrm{E}[\max(r_Z - R,\ 0)^n] \quad . \tag{1}$$

Only realisations of R below the target return are taken into consideration when using these risk measures.

To be able to cope with a *random* target return R_Z, as e.g. the DAFOX, we use the following device. We take the random variable $L_Z := R_Z - R$ as the basis variable and use the lower partial moments $\mathrm{LPM}_n(L_Z) = \mathrm{LPM}_n(L_Z;\ 0)$ of L_Z with respect to the deterministic target return 0:

$$\mathrm{LPM}_n(L_Z) = \mathrm{E}[\max(R_Z - R,\ 0)^n] \quad . \tag{2}$$

Obviously (1) is a special case of (2), so (2) is a proper generalisation of measures of shortfall risk with respect to a deterministic target return.

By conditioning we obtain another useful relation. We have

$$\begin{aligned} \mathrm{LPM}_n(L_Z) &= \mathrm{E}[\max(L_Z, 0)^n \mid L_Z > 0]\ P(L_Z > 0) \\ &\quad + \mathrm{E}[\max(L_Z, 0)^n \mid L_Z \leq 0]\ P(L_Z \leq 0) \\ &= \mathrm{E}[L_Z^n \mid L_Z > 0]\ P(L_Z > 0) \quad . \end{aligned} \tag{3}$$

This means that the resulting shortfall risk measures can be represented as the product of a conditional n-th moment of the random variable L_Z and the properly generalized shortfall probability.

In our study we consider as risk measures the *shortfall probability*, which corresponds to the case $n = 0$, the *shortfall expectation*, which corresponds to the case $n = 1$ and the *shortfall variance* which corresponds to the case $n = 2$. These measures are studied in the context of utility theory especially in *Fishburn* 1977. With the shortfall probability, risk is measured as the absolute danger to stay below the pre-determined target return. It makes no difference to which extent the possible realisations are below the target return, since all return-realisations below the target are only evaluated with their probability of occurrence. Due to this, the shortfall probability is criticized. No matter whether the extent of staying below the target return is small or high, both cases are evaluated in the same way. The shortfall expectation tries to compensate this problem of the shortfall probability by looking at the average amount of possible returns below the target. It takes into consideration both the probability and the amount of below target returns. Finally, the shortfall variance measures the squared below target returns. With this risk measure large below targets returns are weighted higher than small ones so that the shortfall variance is the suitable measure in case the control of large deviations is of interest.

In investment practice as well as in financial theory the expectation $\mathrm{E}(R)$ is a broadly accepted measure of value (average return). However, a measure of value is not necessarily suitable to measure chances (returns exceeding the target) as in its calculations it also considers realisations below the target. Therefore we propose in complete analogy to the construction of shortfall risk measures to use excess returns as measures of chance. A general class of chance measures with respect to a *deterministic* target return r_Z can be obtained by using the *upper partial moments* of degree $n \geq 0$ of the random return R:

$$\mathrm{UPM}_n(R; r_Z) = \mathrm{E}[\max(R - r_Z,\ 0)^n] \quad . \tag{4}$$

To be able to cope with a *random* target return R_Z we take the random variable $U_Z := R - R_Z$ as the basic variable and use the upper partial moments $\mathrm{UPM}_n(U_Z) = \mathrm{UPM}_n(U_Z,\ 0)$ with respect to the deterministic target return 0 as the proper generalization of (4):

$$\mathrm{UPM}_n(U_Z) = \mathrm{E}[\max(R - R_Z, 0)^n] \quad . \tag{5}$$

For the special cases $n = 0$ resp. $n = 1$ resp. $n = 2$ we obtain the following measures of chance: excess probability resp. excess expectation resp. excess variance. By conditioning we finally obtain a relation similar to (3):

$$\mathrm{UPM}_n(U_Z) = \mathrm{E}[U_Z^n \mid U_Z > 0]\ P(U_Z > 0) \quad , \tag{6}$$

thus connecting the general excess chance measure with the (properly generalized) excess probability.

The setting of the target return is of decisive importance when calculating the various shortfall-risk and excess-chance measures. Although maybe somewhat arbitrarily, we have chosen in this study of what we believe to be reasonable candidates for a target return: namely, we take into consideration the monthly money market returns as a proxy for a riskless investment opportunity and in addition the one-month-return of the unprotected DAFOX-portfolio being the "neutral" benchmark portfolio.

5 Statistical estimation of the measures of risk and chance

Starting point of the statistical estimation of the considered measures of risk resp. chance is the sequence $\{r_t\}_{t=1,\dots,T}$ of monthly returns of the various (protected or unprotected) strategies, being a realization of a corresponding sequence $\{R_t\}_{t=1,\dots,T}$ of random variables. All monthly returns are calculated on a *continuous* basis, i.e. let V_t denote the value of the financial asset at time t, R_t becomes $\ln V_t - \ln V_{t-1}$. In case the $\{R_t\}$ would be an independent and identically (according to R) distributed sequence of random variables the sample estimator

$$\bar{R} := \frac{1}{T}\sum_{t=1}^{T} R_t \tag{7}$$

is the well known distribution free and unbiased estimator of $\mathrm{E}(R)$.

We estimate the n-th absolute moment of the random variable $\max(L_Z, 0)$, on the

basis of $L_{Z,t} := R_{Z,t} - R_t$. The corresponding sample counterpart is $\frac{1}{T}\sum_{t=1}^{T} \max(L_{Z,t}, 0)^n$. Defining the indicator variable $I_{(a,b)}(R) = 1$, in case $R \in (a, b)$ and $I_{(a,b)}(R) = 0$, in case $R \notin (a, b)$, the last expression is equivalent to

$$\hat{\mathrm{LPM}}_n(L_Z) := \frac{1}{T}\sum_{t=1}^{T} (L_{Z,t})^n\, I_{(0,\infty)}(L_{Z,t}) \quad . \tag{8}$$

In case of independent and identically distributed $\{L_{Z,t}\}$ expression (8) gives us a distribution free and unbiased estimator of the n-th moment of $\max(L_Z, 0)$. The cases $n = 0, 1, 2$ give the corresponding estimators for the risk measures shortfall probability, shortfall expectation and shortfall variance.

For estimating the n-th moment of the random variable $\max(U_Z, 0)$, we define $U_{Z,t} = - L_{Z,t}$ which gives us the following expression

$$\hat{\mathrm{UPM}}_n(U_Z) := \frac{1}{T}\sum_{t=1}^{T} (U_{Z,t})^n\, I_{(0,\infty)}(U_{Z,t}) \quad . \tag{9}$$

As can be verified mathematically (cf. Appendix), however, even in case of independent and identically distributed returns $\{R_t\}$ of the underlying, the DAFOX, the corresponding sequence of returns for rollover option strategies are *neither independent nor identically distributed*. *Figlewski* et al. (1993) characterize rollover option strategies on a more intuitive basis as *path-dependent strategies*. This consequently does imply, that the estimators (7), (8) and (9) are loosing their properties in the i.i.d.-case. The estimators now are only simple *descriptive* statistical measures and no more unbiased estimators for the moments of a parent distribution. This has to be kept in mind, when interpreting the various estimators used in sequel.

6 The average and excess returns of rollover hedge strategies

The average returns of the underlying, the DAFOX, as well as the alternative rollover hedge strategies are estimated on the basis of (7). The excess quantities are estimated on the basis of (9).

The following table 1 contains the values of the average returns, the excess frequencies, the average excess and the excess standard deviations for the one-month money market return (MMR) and the unprotected DAFOX-portfolio resp. for the rollover put hedge strategies for the different exercise prices expressed as a percentage of the price of the underlying, the DAFOX:

	Exercise Price				
	94	96	98	100	102
Average Return	0,5945	0,5787	0,5482	0,5287	0,4690
Excess Frequency (Target MMR)	50,59	49,42	44,08	38,75	28,07
Excess Frequency (Target DAFOX)	5,80	11,83	18,79	37,79	38,51
Average Excess (Target MMR)	1,78	1,68	1,49	1,20	0,83
Average Excess (Target DAFOX)	0,22	0,38	0,62	0,95	1,25
Excess Std. (Target MMR)	3,31	3,19	2,97	2,61	2,14
Excess Std. (Target DAFOX)	1,34	1,68	2,10	2,52	2,89

Table 1: Put Hedge Fixed-Percentage

First one recognizes that higher levels of protection, i.e. higher exercise prices, are corresponding to lower average returns which is according to intuition as well as theory. Remarkably, however, in case of deep-out-of-the-money puts the average return (0,5945 %) is slightly higher compared to the unprotected position (0,5896 %), the DAFOX (although we have included transaction costs!). This is opposed to theory and we will analyze this phenomenon at the end of this paragraph.

Second table 1 shows that the asymmetrical excess-measures are very sensitive to a change of the excercise-price. By using the target MMR all three excess-measures, the excess frequency, the average excess and the excess standard deviation, are falling with higher levels of protection. This result is consistent to the use of the average return as a chance-measure.

Applying the unprotected DAFOX-portfolio as the target, we recognize the inverse phenomenon, i. e. the excess-measures are increasing with higher levels of protection. But this appearently surprising result is consistent with theory too: it is a fact, that in case of increasing DAFOX-quotations the put hedge strategy will never reach the level of the unprotected position, because in this case the costs of the put-options reduce the profit margin. Therefore a positive excess-position can only be realized by decreasing DAFOX-quotations. In this case, a high exercise-price, and therefore a high level of protection, is more favourable than a lower one, because in case of increasing quotations a higher level of protection leads faster and in a higher extent to a positive excess-position.

The corresponding table for the covered short call is:

	Exercise Price				
	98	100	102	104	106
Average Return	0.4931	0.5041	0.5457	0.5882	0.6152
Excess Frequency (Target MMR)	79,81	67,98	62,02	57,77	56,15
Excess Frequency (Target DAFOX)	56,15	61,48	71,46	82,90	89,79
Average Excess (Target MMR)	0,66	1,00	1,35	1,60	1,75
Average Excess (Target DAFOX)	1,40	1,12	0,80	0,52	0,31
Excess Std. (Target MMR)	0,94	1,37	1,89	2,39	2,78
Excess Std. (Target DAFOX)	2,10	1,58	1,11	0,76	0,52

Table 2: Covered Short Call Fixed-Percentage

Again we notice a trade-off between an increasing level of protection, i. e. decreasing excercise-price, and average return as in case of the put hedge strategies. Compared to the unprotected position, the average return is also reduced with the exception of a deep-out-of-the-money call. The excess frequency falls with increasing exercise-price by using the target MMR and rises by using the target DAFOX.

This result can be interpreted from two different perspectives. First for the target MMR the probability, that the complete position beats the target, is higher for a relativly low exercise-price than for a relativly high one. For the target DAFOX we get the opposite effect. From an economical point of view this result shows, that already for a small profit the sum of the positive DAFOX-return and the received option premium is higher than the payment of interest as given by the riskless interest rate. By construction the amount of the possible profit is not considered by the excess frequency.

According to expression (6) the excess expectation quantities are the product of a conditioned expected profit and the excess probability. In accordance to the explanations above the excess frequency increases in case the exercise-price falls. As the average excess is increasing with an increasing exercise-price, the conditioned expected profit has to grow stronger than the excess frequency. Because the excess standard deviation considers probable profits as squared numbers, the basic effect is even more obvious in this case.

In case the DAFOX falls, all excess-measures at all exercise-prices are higher than the DAFOX-benchmark because of the received option premium. But in case the DAFOX increases, an investor will profit longer from an increasing market if the exercise-price is relatively higher. The excess probability must increase when the exercise-price increases. This result is empirically confirmed in table 2.

Because the excess frequency increases if the exercise-price rises and the average excess simultaneously falls, this means that with an increasing exercise-price the conditioned expected excess falls to a greater extent than the excess frequency is increasing. Falling exercise-prices that lead to decreasing prices on the market and relatively high excess-profits, have a greater effect than a long participation in an increasing market resulting from rising exercise-prices. The declining of the excess standard deviation can be interpreted analogously.

Finally we have the following table for the collar strategy:

	Exercise Price		
	94/106	96/104	98/102
Average Return	0.6216	0.5783	0.4992
Excess Frequency (Target MMR)	53,60	54,52	55,22
Excess Frequency (Target DAFOX)	89,10	79,12	65,66
Average Excess (Target MMR)	1,62	1,34	0,83
Average Excess (Target DAFOX)	0,34	0,59	1,02
Excess Std. (Target MMR)	2,61	2,05	1,18
Excess Std. (Target DAFOX)	1,44	1,89	2,53

Table 3: Collar Fixed-Percentage

According to the design of the collar the effects of the put hedge and the covered short call are reflected simultaneously. Narrowing the collar to (98 %, 102 %) implies an almost dramatic reduction of the average return so that an extreme strategy of this kind has to be thought about well.

When interpreting the excess frequency on the basis of a riskless interest rate as benchmark, two opposite effects occur: due to the put hedge strategy on the one hand, the excess frequency is falling with an increasing exercise-price. On the other hand, the excess frequency is increasing with falling exercise-price, because of the covered short call strategy. As table 3 shows, the effect of the covered short call strategy slightly dominates under the condition of the chosen data and constellation of parameters.

Regarding the two other excess-measures, both strategies have parallel effects. The closer the exercise-prices are, the smaller the average excess and the excess standard deviation are.

Taking the DAFOX-return as benchmark, we have the following results for the excess-measures: the closer the exercise-prices are, the smaller the excess frequency and the higher the average excess and the excess standard deviation are. For the purpose of interpretating these results, one can argue in the same way as in the case of the riskless money market return as the target return.

Finally we have to analyze the "deep-out-of-the-money phenomenon" resulting in average returns which - although transaction costs are included - are higher than in the unprotected case, which is contrary to theory. An explanation for this anomaly is given by analyzing the deviation of the empirical distribution function of the continuous returns of the underlying from the normal distribution. The normal distribution for the continuous returns is implied, cf. *Hull* (1993, p. 212), by the assumption of a geometrical *Brown*ian motion process for the stock prices in the *Black/Scholes*-model. Using the *Anderson-Darling* goodness-of-fit test for the normal distribution, cf. *D'Agostino* (1986), the (composite) hypothesis of normally distributed (continuous) DAFOX-returns is very clearly rejected. The test statistic gives a value of 1.871 which distinctly exceeds the critical value even in case of a very low level of significance of 0.5 %, which gives a critical value of 1.159.

The calculation of the skewness and the curtosis brings further information about the spread of the empirical distribution of the DAFOX-returns from the normal distribution. Assuming a normal distribution one should get a skewness of zero and a curtosis of three. But the analysis of the DAFOX-return time series results in a skewness of -0,346 and a curtosis of 5,585. A negative skewness-ratio implies, that the probability-mass on the negative return-axis is higher if you take the empirical DAFOX-distribution compared to the normal distribution. A curtosis-value higher than three implies that, compared to a normal distribution, the empirical return distribution has more probability-mass at the end of the distribution (fat tails).

The reason of this "non-normality" is given by the sharp breaks in prices, e.g. the crash in October 1987. By eliminating the four lowest returns from the sample, however, the *Anderson-Darling*-test (value of the test statistic: 0.787) now does not reject the normal distribution at a level of significance of 10 % (critical value: 0.873) anymore. For the skewness resp. the curtosis we obtain the values 0,236

resp. 3,856.

This leads to the following explanation of the observed phenomenon. The *buyer* of deep-out-of-the-money puts takes profit from the shortfall risks which are empirically higher than implicitely assumed by the *Black/Scholes*-formula. The *writer* of deep-out-of-the-money calls takes profit from the lower return chances of the underlying compared to the normal distribution because of the skewness to the left of the empirical returns.

7 The shortfall risk of rollover option strategies

In general, theoretical considerations about the capital market predict increasing chances of a set of strategies when simultaneously increasing risks. In our empirical examination the postulated trade-off between chance and risk is confirmed in case of using excess- and shortfall expectations resp. excess- and shortfall variances as measures for chance and risk. See tables 4-6 for that.

If one compares excess- and shortfall probabilities, one obtains a different result. Due to the identity $LPM_0 = 1 - UPM_0 - P(L_Z = 0)$ there is no positive trade-off between chance and risk. If a riskless financial investment is used as a benchmark, for put hedge strategies low exercise-prices are favourable. Considering covered short calls one would prefer lower exercise-prices to higher ones. But if the DAFOX-return is taken as a benchmark, the results are vice versa. Because of the above mentioned opposite effects there is no clear-cut result for the collar.

	Exercise Price				
	94	96	98	100	102
Shortfall Frequency (Target MMR)	49,41	50,58	55,92	61,25	71,93
Shortfall Frequency (Target DAFOX)	94,20	88,17	81,21	62,21	61,49
Average Shortfall (Target MMR)	1,65	1,57	1,41	1,11	0,825
Average Shortfall (Target DAFOX)	0,21	0,38	0,66	1,01	1,37
Shortfall Std. (Target MMR)	2,82	2,52	2,11	1,60	1,14
Shortfall Std. (Target DAFOX)	0,39	0,62	0,94	1,39	1,92

Table 4: Put Hedge Fixed-Percentage

	Exercise Price				
	98	100	102	104	106
Shortfall Frequency (Target MMR)	20,19	32,02	38,98	42,23	43,85
Shortfall Frequency (Target DAFOX)	43,85	38,52	28,54	18,10	10,21
Average Shortfall (Target MMR)	0,63	0,96	1,27	1,48	1,60
Average Shortfall (Target DAFOX)	1,50	1,21	0,84	0,517	0,285
Shortfall Std. (Target MMR)	2,12	2,54	2,91	3,15	3,30
Shortfall Std. (Target DAFOX)	2,98	2,64	2,17	1,69	1,27

Table 5: Covered Short Call Fixed-Percentage

	Exercise Price		
	94/106	96/104	98/102
Shortfall Frequency (Target MMR)	46,40	45,48	44,78
Shortfall Frequency (Target DAFOX)	10,90	20,88	34,34
Average Shortfall (Target MMR)	1,47	1,23	0,80
Average Shortfall (Target DAFOX)	0,31	0,60	1,11
Shortfall Std. (Target MMR)	2,59	2,07	1,27
Shortfall Std. (Target DAFOX)	1,35	1,86	2,57

Table 6: Collar Fixed-Percentage

8 Option Strategies within the Context of Risk-Value-Models

It should be noted that there is of course no single best strategy for all investors. However, this does not imply that an investor should be indifferent in his choice among various option strategies (cf. *Merton/Scholes/Gladstein* 1978, p. 184), because investors have different objectives resulting in an individual trade-off between risk and chance. Hence one strategy may be preferred by one investor while a different strategy is preferred by an other one.

On a more formal level the decision behaviour of investors can be analysed on the basis of a preference functional Φ. The preference functional is inducing an order relation between different random varables X and Y given by $X \succeq Y \Leftrightarrow \Phi(X) \geq \Phi(Y)$. On the basis of our approach in the previous chapters we investigate preference functionals of the following type:

$$\Phi(X) = H[R(X), C(X)] \ . \tag{10}$$

The preference functional (10) falls under the class of *risk-value models* (cf. *Sarin/Weber* 1993) where the valuation of a random variable depends on a measure of risk R(X) on one hand and a measure of chance C(X) on the other. The function H quantifies the trade-off between risk and chance. An economically plausible requirement for H is the property, that H must be monotonically increasing in the second component as well as monotonically decreasing in the first component. This means that H is consistent with the following definition of dominance:

> A random variable X is dominating the random variable Y if and only if $C(X) \geq C(Y)$ and $R(X) \leq R(Y)$, where one of these two inequalities must be sharp.

A random variable is called efficient in case it will not be dominated (in the above sense) by any other random variable.

In the following we look at different pairs of excess chance- resp. shortfall risk-measures (in each case with an identical target) and try to separate inefficient from efficient option strategies. The question of an explicit trade-off based on a special functional form of H will not be considered in the present contribution. As we have discussed four measures of chance and three measures of risk, these are in total twelve possibilities for constructing a preference functional. However, in the following we will discuss only three of them.

Considering at first the pair excess- resp. shortfall probability there is - as already mentioned - no positive trade-off. If we choose the riskless money market return as the relevant target return the covered short call with exercise-price 98 is dominating all the other strategies. Choosing the unprotected DAFOX position this will be the covered short call with exercise-price 106.

Considering the preference functional $H(LPM^1_Z(X), UPM^1_Z(X))$ a property of dominance does not show up (see figure 1 and 2), neither using the riskless money market return neither using the DAFOX as target return. However, in case an investor makes his decisions on the basis of a preference functional of type $H(LPM^2_Z(X), UPM^2_Z(X))$ there are clear relations of dominance between the different classes of option strategies.

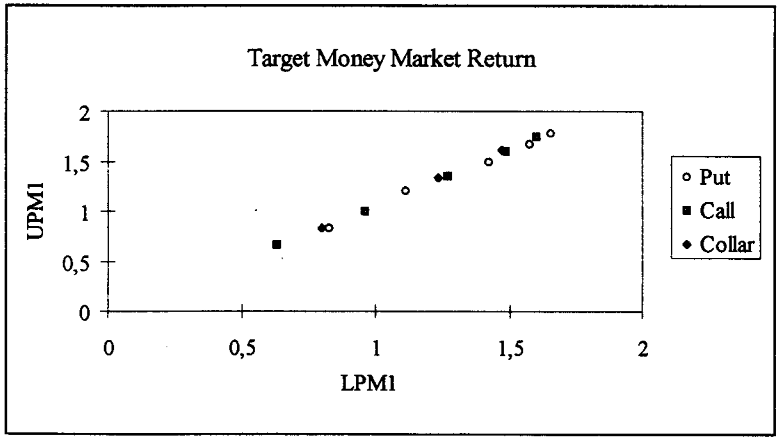

Figure 1: Option Strategies Evaluated by $H(LPM^1_Z, UPM^1_Z)$ and Target Money Market

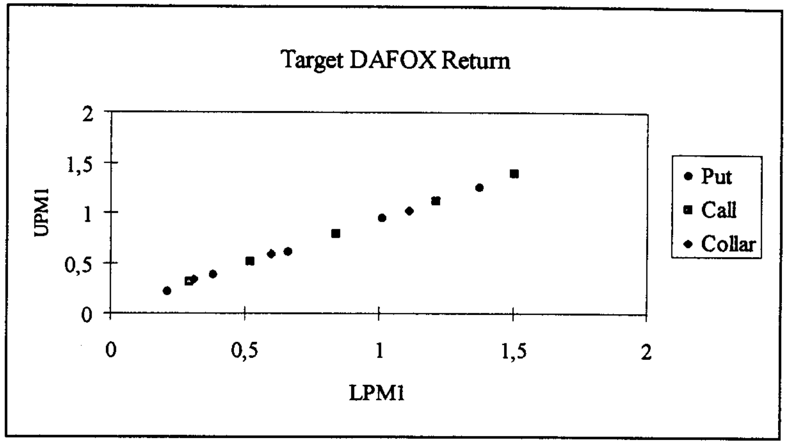

Figure 2: Option Strategies Evaluated by $H(LPM^1_Z, UPM^1_Z)$ and Target unprotected DAFOX-portfolio

The upper row of points in figure 3 and 4 is corresponding to the different put hedge strategies, the intermediate row to the collar strategies and the lower row to the covered short call strategies. This holds for the case of choosing the one-month money market return as a target as well as for the DAFOX. The clear dominance of put hedge strategies is theoretically consistent as using the (LPM^2, UPM^2)-functional large deviations from the target are given a higher weight. Considering the put hedge strategies high negative returns are excluded whereas in case of high positive return of the DAFOX the investor is participating. Considering the covered short call the situation is exactly inverse: high negative returns are still possible whereas a participation in sharply rising returns is excluded. Considering the collar neither highly positive neither highly negative returns are possible. This results in an intermediate position for the collar.

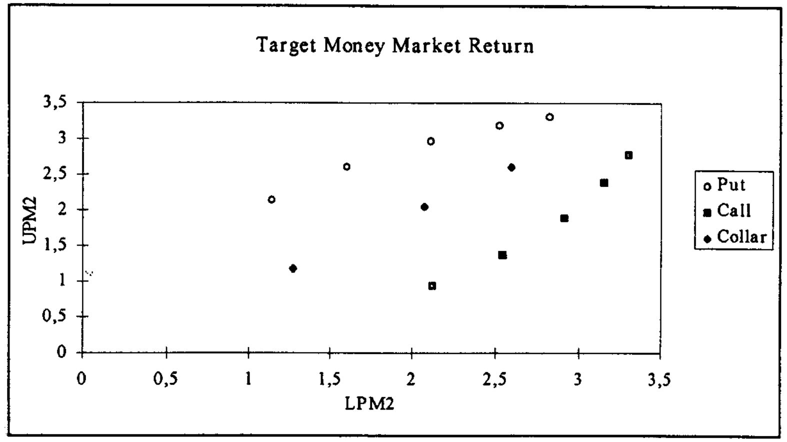

Figure 3: Option Strategies Evaluated by H(LPM^2_Z, UPM^2_Z) and Target Money Market

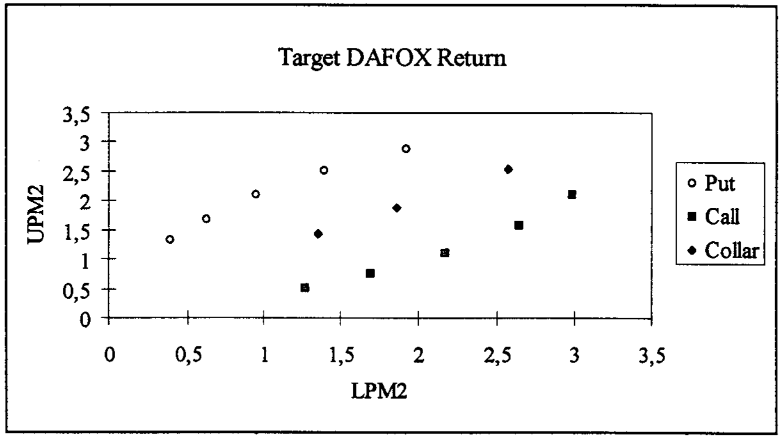

Figure 4: Option Strategies Evaluated by H(LPM^2_Z, UPM^2_Z) and Target unprotected DAFOX-portfolio

Appendix: Statistical problems of rollover options strategies

This appendix has the purpose to analyze the statistical problems of rollover option-strategies on a rigorous basis. For this purpose we are modelling the price S_t of the underlying according to a geometrical *Brown*ian motion with constant drift μ and constant volatility σ. At the points in time $t = 1,\ldots,T$ we obtain the following prices:

$$S_t = S_{t-1}\exp[(\mu - \frac{1}{2}\sigma^2) + \sigma(W_t - W_{t-1})] \,, \qquad \textbf{(A.1)}$$

where W_t stands for the standardized *Wiener*-Process. For the continuous one period returns $R_t = \ln S_t - \ln S_{t-1}$ it follows, that they are stochastically independent and identically distributed to

$$R \sim N(\mu - \frac{1}{2}\sigma^2, \sigma^2) \,. \qquad \textbf{(A.2)}$$

Nevertheless we will demonstrate in the following that neither independence nor the identical distribution will be valid anymore in general for rollover 1:1 fixed percentage put option strategies.

As an illustration we assume that at the beginning of each period t ($t = 1, 2, \ldots, T$) one European one period put of the underlying is bought. The exercise price of the put always corresponds to a fixed percentage of p of S_t. Putting $p = a/100$ leads to an exercise price of $K_t = aS_t$. It is assumed, that the prices of the European puts P_t are determined by the *Black/Scholes*-formula with maturity in $t + 1$, an exercise price of X_t and a riskless rate of return r_f

$$P_t = X_t e^{-r_f} N[-d_2(t)] - N[-d_1(t)]\, S_t \,, \qquad \textbf{(A.3)}$$

where

$$d_1(t) = \frac{\ln\left[\frac{S_t}{X_t}\right] + (r_f + \frac{1}{2}\sigma^2)}{\sigma} \tag{A.4}$$

$$d_2(t) = d_1(t) - \sigma .$$

We obtain

$$d_1(t) = \frac{1}{\sigma}\left[\ln\left[\frac{1}{a}\right] + (r_f + \frac{1}{2}\sigma^2)\right] \tag{A.5}$$

and due to this we can conclude:

$$P_t = [ae^{-r_f}\mathrm{N}(-d_2) - \mathrm{N}(-d_1)]S_t = AS_t . \tag{A.6}$$

Under these circumstances the put price corresponds at any time to the same percentage 100 A of the former price of the basis object at the beginning of the period. For the development of the values V_0, V_1, . . V_T of the fixed percentage strategy we obtain

$$V_t = H_t S_t \quad ; \quad t = 0, \ldots, T-1 \tag{A.7}$$

with $H_t := V_t / S_t$. At the beginning of the period the aggregated assets are often higher than the price of the underlying. In $t = 0$ we have $H_0 = 1$ and in the following periods the realisations h_t of H_t can obtain values of less or in excess of 1, depending on the basic price development. The total assets at the beginning of a determined period now can be divided in a part that is necessary for financing the put $A\ S_{t-1}$ and a remaining part of $(h_{t-1} - A)$. At the end of the period we obtain

$$V_t = (h_{t-1} - A)S_t + \max(a\ s_{t-1} - S_t, 0) . \tag{A.8}$$

Thus, the continuous return OR_t of the option strategy is given by

$$\begin{aligned} OR_t &= \ln\left[\frac{V_t}{V_{t-1}}\right] \\ &= \ln\left[\frac{(h_{t-1} - A)S_t + \max(as_{t-1} - S_t, 0)}{h_{t-1}\ s_{t-1}}\right] \\ &= \ln\left[\left[1 - \frac{A}{h_{t-1}}\right]\frac{S_t}{s_{t-1}} + \frac{1}{h_{t-1}}\max\left[a - \frac{S_t}{s_{t-1}}, 0\right]\right] . \end{aligned} \tag{A.9}$$

Each h_t now represents a realization of the random variable H_t. H_t however is determined by the development of the basis object in the previous period. We would obtain for example $H_1 = (1 - A) + \max(a\ s_0/S_1 - 1, 0)$. This leads to the result that the continuous one period return of a rollover fixed percentage strategy is neither stochastically independent nor identically distributed.

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