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**Cooperative Prisoners and Aggressive Chickens:
Evolution of Strategies and Preferences in 2x2
Games**

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Abstract

By means of simulations I investigate a two-speed dynamic on strategies and preferences in the prisoners' dilemma and in the chicken game. Players learn strategies according to their preferences while evolution leads to a change in preference composition. With complete information cooperation in the prisoners' dilemma is often achieved, with "reciprocal" preferences. In the chicken game a symmetric correlated strategy profile is played that is as efficient as the symmetric equilibrium. Among preferences only pure "hawkish" preferences and "selfish" preferences survive. With incomplete information, the symmetric equilibrium of the material payoff game is played. All types of preferences are present in the population in the medium run.

Keywords: two-speed evolution; simulations; replicator dynamic.

JEL Code: C72

1 Introduction

Suppose that pairwise interactions in a large population can be represented by a certain game, payoffs in which have a direct influence on fitness of individuals. Individuals are randomly matched to interact. Individuals can be driven by idiosyncratic factors other than fitness, like sympathy towards the opponent, altruism, biases towards a particular action, or spitefulness. They change their actions to achieve higher subjective utility as represented by these factors, while evolution changes the proportions of preferences in the population according to the underlying fitness. This paper analyzes what preferences and what actions survive in the medium to long run when the underlying fitness game is either a prisoners' dilemma or a chicken (hawk-dove) game.

There are two distinct dynamic processes in the model. One is a learning process on strategies based on the subjective preferences of players. The other is an evolutionary process on preferences based on fitness. Arguably, learning is faster than evolution. This idea is taken to extreme in the indirect evolution approach, initiated in Güth and Yaari (1992), where learning is infinitely faster than evolution, and the play is assumed to converge to an equilibrium of the game with subjective preferences before evolution proceeds. The main differences of the model in this paper from the indirect evolution approach are the following. First, learning, though being faster, is not assumed to converge before evolution operates. Second, evolution is modeled explicitly, and not through the static evolutionary concepts of evolutionary or neutrally stable strategy.

It has been shown that the results of indirect evolution are sensitive to the set of admissible preferences (see Bester and Güth (1998), Bolle (2000) and Possajennikov (2000)). Therefore I consider as large a set of admissible preferences as possible. Ideally, all (von Neumann-Morgenstern) preferences should be allowed. However, due to computational limitations only a finite subset of all preferences can be considered. Also, in large games the set of admissible preferences is too large, so I focus on symmetric 2×2 games, prisoners' dilemmas and chicken games being in this class of games. In these games all important types of preferences are covered by the finite set of preferences I consider.

I look at two informational models. In one model the individuals know the preferences of the opponent in a match, so they can condition their strategy on those preferences. In the other model the individuals do not

observe the preferences of the opponent, therefore they have to use the same strategy in every match. The second model also requires a special model of imitation learning, since it is logical to assume that individuals cannot observe preferences when imitating either.

There have been theoretical models of indirect evolution with all preferences allowed. Ely and Yilankaya (2001) analyze such a model with a continuous preference distribution, a setting that is not possible to simulate. Dekel et al. (1998) and Possajennikov (2002) analyze stability of finite distribution of preferences by means of static stability concepts. It is shown that with complete information only efficient symmetric strategy profiles can be stable, but there is no stable strategy profiles in two-speed evolution of strategies and preferences in prisoners' dilemmas and in chicken games. In this paper I simulate an explicit dynamic model to get additional insight into the situation. Though there are no completely stable outcomes in these games, some outcomes are more unstable than others, and simulations can reveal which outcomes are least unstable. The results of the simulations partially support the superior stability of efficient outcomes: in the prisoners' dilemma mutual cooperation is observed most often, while in the chicken game the observed correlated strategy profile is as efficient as the symmetric equilibrium, and making the grid on the space of preferences finer leads to higher efficiency.

With incomplete information it is shown that only Nash equilibria of the underlying fitness game can be stable when learning have converged to a Bayesian equilibrium. A corollary of this result is that only preferences that can play this equilibrium survive in the long run. Since I model the learning process explicitly I consider an imitation learning model that does not require the unknown information about the preferences of the player that gets imitated. Despite of this limitation, the play observed in the long run is in a Nash equilibrium, but preferences change very little. In prisoners' dilemma, for example, even the individuals who strictly prefer to cooperate eventually imitate the population to play the dominant strategy "defect".

The next section introduces the model. Section 3 discusses the results of the simulations, and Section 4 concludes.

2 The Model of Learning and Evolution

2.1 Games

A symmetric 2×2 material payoff game is given by $G = (N, S, u)$ where $N = \{1, 2\}$ is the set of players, $S = \{C, D\}$ is the set of strategies, the same for both players, and $u : S \times S \rightarrow \mathbb{R}$ is the symmetric material payoff function, $u(s_i, s_j) = u_1(s_i, s_j) = u_2(s_j, s_i) \forall s_i, s_j \in S$. The function u represents *fitness* on which evolution works. The mixed strategy extension of the strategy set is $\Sigma = \Delta S$. A mixed strategy can be represented by the probability p of playing strategy C . The material payoff function extends to the set of mixed strategy profiles $\sigma = (p^1, p^2)$ by $u(\sigma) = p^1 p^2 u(C, C) + p^1 (1 - p^2) u(C, D) + (1 - p^1) p^2 u(D, C) + (1 - p^1)(1 - p^2) u(D, D)$. A strategy profile σ is symmetric if $p^1 = p^2$. A symmetric strategy profile σ is *efficient* if $u(\sigma) \geq u(\sigma')$ for any symmetric σ' . A *correlated* strategy profile σ_c specifies the probability with which each pair of pure strategies is played, i.e. $\sigma_c \in \Delta(S \times S)$ while a usual strategy profile belongs to $\Delta S \times \Delta S$.

For a given strategy p^j of player j the best response $BR^i(p^j)$ of player i is the set of strategies p^i such that $\forall q^i \in \Sigma u(p^i, p^j) \geq u(q^i, p^j)$. For a given strategy profile $\sigma = (p^1, p^2)$ the best response correspondence $BR(\sigma)$ is the product set $BR^1(p^2) \times BR^2(p^1)$. A strategy profile σ is a *Nash equilibrium* if $\sigma \in BR(\sigma)$. A Nash equilibrium σ is symmetric if strategy profile σ is symmetric.

I consider two types of symmetric 2×2 games, both represented by the same matrix

	C	D
C	1, 1	b, c
D	c, b	0, 0

The games are characterized by the relative values of the payoffs:

- Prisoners' dilemmas: $b < 0, c > 1, b + c < 2$;
- Chicken games: $b > 0, c > 1, b + c > 2$.

In prisoners' dilemmas the unique Nash equilibrium is $(D, D) = (0, 0)$ with fitness $(0, 0)$, while strategy profile $(C, C) = (1, 1)$ with fitness 1 is the unique symmetric efficient profile. In a chicken game the unique symmetric equilibrium is $(\frac{b}{b+c-1}, \frac{b}{b+c-1})$, while the unique symmetric efficient profile is

$\left(\frac{b+c}{2(b+c-1)}, \frac{b+c}{2(b+c-1)}\right)$. The symmetric equilibrium and the symmetric efficient profile coincide if and only if $b = c$.

2.2 Preferences

Individuals have subjective preferences over strategy profiles of the game, represented by a von Neumann-Morgenstern utility functions $v_i : S \times S \rightarrow \mathbb{R}$. The utility functions do not necessarily coincide with the material payoff function. The utility functions extend to mixed strategy profiles $\sigma = (p^1, p^2)$ in a straightforward way $v_i(\sigma) = p^1 p^2 v_i(C, C) + p^1(1 - p^2)v_i(C, D) + (1 - p^1)p^2 v_i(D, C) + (1 - p^1)(1 - p^2)v_i(D, D)$.

I identify preferences with the utility function representing them. I consider as admissible all utility functions. Since a utility function is determined by its values on the four pure strategy combinations, the set of admissible preferences is equivalent to \mathbb{R}^4 .

It is convenient to divide the set of admissible preferences into following types:

1. (St1): $v_i(C, C) > v_i(D, C), v_i(C, D) > v_i(D, D)$;
2. (CO): $v_i(C, C) \geq v_i(D, C), v_i(C, D) \leq v_i(D, D)$, at least one inequality is strict;
3. (NC): $v_i(C, C) \leq v_i(D, C), v_i(C, D) \geq v_i(D, D)$, at least one inequality is strict;
4. (St2): $v_i(C, C) < v_i(D, C), v_i(C, D) < v_i(D, D)$;
5. (BB): $v_i(C, C) = v_i(D, C), v_i(C, D) = v_i(D, D)$.

Preferences v_i belong to type k if v_i satisfies the inequalities for type k . Players with type (St1) preferences perceive the game as having dominant strategy C , while players of type (St2) think that D is dominant. Type (CO) players (COordinators or CONformists) perceive that C is a best reply to C and D is to D , while type (NC) (NonConformists) players prefer to play C against D and D against C . Finally, there are preferences of type (BB) ("Big Bores") for which the strategies are equivalent. The players with such preferences are indifferent between the strategies for any strategy of the opponent.

An interpretation of having different preferences can be seen on the example of the prisoners' dilemma. Agents with type (St2) preferences have "selfish" preferences in the sense that they rightly perceive D as the dominant strategy. Other agents might not like to let their opponents down and therefore have a higher subjective utility from mutual cooperation than from defecting against a cooperator (type (CO) preferences). Yet others can be heroic unconditional cooperators who derive a higher utility even from being defected upon, that is, they prefer to sacrifice themselves in favor of the other player (type (St1) preferences).

Though the agents know their preferences, they do not need to know what the material payoffs are. Learning leads to strategies that are better with respect to the subjective preferences. Evolution chooses those preferences that have higher fitness.

2.3 Evolution and Learning

There is a large (infinite) population of agents who are randomly matched to play a given material payoff game. Agents do not distinguish the roles of players (Player 1 or 2). Agents in the population are characterized by the preferences they have and the strategy they play. They change strategy due to learning, and the distribution of preferences in the population changes due to evolution.

2.3.1 Evolution

Suppose that, out of potentially infinite space of preferences, only a finite number $\{v_1, \dots, v_n\}$ is present in the population. The state of the population is described by the proportions $\mu_1, \dots, \mu_n, \mu_i > 0, \sum_{i=1}^n \mu_i = 1$ of these preferences. Agents with given preferences v_i form an infinite subpopulation. Though individuals in the subpopulation can use different strategies and so have different fitness, evolution depends solely on the average expected fitness of each subpopulation. Denote the average expected fitness in subpopulation i from an encounter with a player with preferences v_j by u_{ij} . Then the average expected fitness of subpopulation i over all matches is $u_i = \sum_{j=1}^n \mu_j u_{ij}$. I assume that the proportions change according to the *replicator dynamic*:

$$\dot{\mu}_i = \rho_e \mu_i \left(u_i - \sum_j \mu_j u_j \right). \quad (1)$$

Evolutionary justification of replicator dynamic can be found in Weibull (1995, Ch.3). The proportion of players with preferences v_i grow (resp. decline) if the average expected fitness of these players is higher (resp. lower) than the average average expected fitness in the whole population.

2.3.2 Learning

With respect to learning I consider two informational setups: complete information and incomplete information.

Complete Information When matched, agents can observe the preferences of the opponent and so condition their strategy on those preferences. The appropriate description of the state of each subpopulation should include the possibility that they use different strategies against opponents with different preferences. This can be done by a vector $x_i = (x_{i1}, \dots, x_{in})$, where x_{ij} is the proportion of the subpopulation of players with preferences v_i that play strategy C against players with preferences v_j (and the remaining proportion $1 - x_{ij}$ play D).

The state of the whole population is given by the n vectors x_i . Since matching is random, it is as if a player with preferences v_i , when matched with a player with preferences v_j , faces mixed strategy x_{ji} . The average expected fitness of a player with preferences v_i in a match with preferences v_j is $u_{ij} = u(x_{ij}, x_{ji}) = x_{ij}x_{ji}u(C, C) + x_{ij}(1 - x_{ji})u(C, D) + (1 - x_{ij})x_{ji}u(D, C) + (1 - x_{ij})(1 - x_{ji})u(D, D)$, so evolution is well defined now.

Since players can condition their strategy on the preferences of the opponent, learning processes against opponents with different preferences are independent of each other. I postulate the following learning process. Suppose a player with preferences v_i has just been matched with a player with preferences v_j , and received subjective utility \bar{v}_i . The player knows his utility function and so can calculate his *dissatisfaction* $\gamma - \delta\bar{v}_i$ with the current utility. The probability of revising the strategy is proportional to the degree of dissatisfaction. When the player revises the strategy he samples at random another player with preferences v_i that has just played against a player with preferences v_j and copies the strategy the sampled player used. In Weibull (1995, Ch.4) it is shown that such learning behavior gives rise to the dynamic

$$\dot{x}_{ij} = \rho_l \mu_j x_{ij} (v_i(C, x_{ji}) - v_i(x_{ij}, x_{ji})) \quad (2)$$

which is also a replicator dynamic but now on strategies and using subjective utility functions instead of on preferences and using material payoffs like in evolution. The speed of the dynamic is affected by how often a player is matched with a player with given preferences v_j , which is reflected in the equation by μ_j .

Learning is normally faster than evolution, therefore $\rho_l \geq \rho_e$. The indirect evolution approach is the limit case when $\frac{\rho_l}{\rho_e} \rightarrow \infty$, i.e. learning is infinitely faster than evolution. It also assumes that learning converges to equilibrium while I do not make this assumption here.

Incomplete Information Assume now that when matched, agents do not know the preferences of the opponent and so cannot condition their strategy on this. Then they can only use the same strategy for every match. The state of each subpopulation can be described by the scalar x_i , which represents the proportion of individuals in the subpopulation using strategy C , with the remaining proportion $1 - x_i$ using strategy D .

The state of the population, apart from the proportions of preferences μ_i , is given by the n scalars x_i . The average expected fitness in subpopulation i from an encounter with a player with preferences v_j is $u_{ij} = u(x_i, x_j) = x_i x_j u(C, C) + x_i(1 - x_j)u(C, D) + (1 - x_i)x_j u(D, C) + (1 - x_i)(1 - x_j)u(D, D)$, which determines evolution.

Again, when learning, players revise their strategy according to dissatisfaction with current utility. However, since players cannot observe the preferences of the opponent, it would be unnatural to assume that they can observe preferences when imitating. Therefore they imitate "the first man on the street", and they adopt new strategy according to the frequency of strategies in the whole population $x = \sum_{i=1}^n \mu_i x_i$. Along the lines of Weibull (1995, Ch.4) the learning process is then given by

$$\dot{x}_i = \rho_l x_i \left[\frac{\gamma}{\delta} \left(\frac{x}{x_i} - 1 \right) + \left(v_i(C, x) - \frac{x}{x_i} v_i(x_i, x) \right) \right]. \quad (3)$$

There are a few features that make this process different from the usual replicator dynamic. First, the comparison of performance of a strategy is against the whole population, not against the distribution in a subpopulation. Second, there is a bias towards more popular strategy in the whole population since players imitate a random strategy in the whole population. Note that

players do not need to know the μ 's, and they do not attempt to learn them, but they are implicitly taken into account by using x .

Again, normally learning is faster than evolution, $\rho_l \geq \rho_e$.

2.4 Some Basic Observations

Complete Information In the complete information case the combined dynamic is

$$\dot{\mu}_i = \rho_e \mu_i \left(u_i - \sum_{j=1}^n \mu_j u_j \right), i = 1, \dots, n; \quad (4)$$

$$\dot{x}_{ij} = \rho_l \mu_j x_{ij} (v_i(C, x_{ji}) - v_i(x_{ij}, x_{ji})), i, j = 1, \dots, n, \quad (5)$$

where $u_i = \sum_{j=1}^n \mu_j u(x_{ij}, x_{ji})$. There are, of course, many monomorphic steady states, since the replicator dynamic cannot bring new preferences or strategies into the population without perturbation. "Monomorphic" here means that either there is only one type of preferences present in the population, or that all types use the same strategy, or both. Polymorphic steady states are also possible, in which for any $\mu_i, \mu_j > 0$ it holds that $u_i = u_j$ ($= \sum_{k=1}^n \mu_k u_k$), and for any $x_{ij} \in (0, 1)$ it holds that $v_i(C, x_{ji}) = v_i(x_{ij}, x_{ji})$ ($= v_i(D, x_{ji})$). If $x_{ij}, x_{ji} \in (0, 1)$, then in a steady state subpopulations i and j play a Nash equilibrium of the subjective game.

To distinguish among various steady states, stability notions are used. A state is Lyapunov stable if every neighborhood U of it contains another neighborhood U' such that if the dynamic starts in the smaller neighborhood U' , it stays in the larger neighborhood U . A state is asymptotically stable if it is Lyapunov stable and every U contains U^* such that if the dynamic starts in U^* its limit is the steady state.

Possajennikov (2002), using techniques of Dekel et al. (1998), shows that no population state is *indirectly stable* in prisoners' dilemmas and chicken games. Indirect stability concept stems from the usual indirect evolution approach, when learning is infinitely faster than evolution. In the present case, the relative speed of learning with respect to evolution is given by $\frac{\rho_l}{\rho_e}$, and this relative speed may influence stability. But even in the limit $\frac{\rho_l}{\rho_e} \rightarrow \infty$, a steady state that is not indirectly stable can be asymptotically stable in the replicator dynamic, since despite a pull away from the steady state in one direction, the dynamic may converge back to it from another direction.

Incomplete Information Here the dynamic is

$$\dot{\mu}_i = \rho_e \mu_i \left(u_i - \sum_{j=1}^n \mu_j u_j \right), i = 1, \dots, n; \quad (6)$$

$$\dot{x}_i = \rho_l x_i \left[\frac{\gamma}{\delta} \left(\frac{x}{x_i} - 1 \right) + \left(v_i(C, x) - \frac{x}{x_i} v_i(x_i, x) \right) \right], i = 1, \dots, n; \quad (7)$$

where $u_i = \sum_{j=1}^n \mu_j u(x_i, x_j)$ and $x = \sum_{i=1}^n \mu_i x_i$. Since $u(\cdot, \cdot)$ is bilinear, $\sum_j \mu_j \sum_k \mu_k u(x_j, x_k) = \sum_j \mu_j u(x_j, \sum_k \mu_k x_k) = u(\sum_j \mu_j x_j, x) = u(x, x)$. The first n equations of the system can be rewritten as $\dot{\mu}_i = \rho_e \mu_i (u(x_i, x) - u(x, x)), i = 1, \dots, n$. The second set of equations can be rewritten as $\dot{x}_i = \rho_l \left[\frac{\gamma}{\delta} (x - x_i) + (x_i v_i(C, x) - x v_i(x_i, x)) \right], i = 1, \dots, n$.

As in the complete information case, there are many monomorphic steady states. In polymorphic steady states, if $\mu_i, \mu_j > 0$, then $u(x_i, x) = u(x_j, x) = u(x, x)$. Then either $x_i = x_j = x$, or x is an interior symmetric Nash equilibrium of the material payoff game. If $x_i = x_j = x$, the second n equations reduce to $\dot{x}_i = \rho_l x_i (v_i(C, x) - v_i(x_i, x))$. Then an interior x_i has to be a Bayesian-Nash equilibrium for preferences v_i . If x is an interior symmetric Nash equilibrium of the material payoff game, then if it is possible to find μ_i 's, x_i 's such that $\sum_{i=1}^n \mu_i x_i = x$ and $\frac{\gamma}{\delta} (x - x_i) + (x_i v_i(C, x) - x v_i(x_i, x)) = 0$ for any i , the system is in a steady state. In this case it is not necessary that x_i is a v_i -best response to x . Thus, it is not necessary that in a steady state the play is in Bayesian-Nash equilibrium, like in standard models of indirect evolution.

3 Numerical Analysis

The results of Dekel et al. (1998) and Possajennikov (2002) indicate that in games under consideration with complete information no state is completely stable but some states are upset by very improbable perturbations only. With incomplete information only Nash equilibria of the material payoff game can be stable. I choose a numerical approach of computer simulations to get additional insight into the situation.

3.1 Simulations

To perform simulations, additional assumptions are needed. The preference space needs to be discretized, as well as the dynamics.

I consider one utility function of types (St1), (St2) (since they all will have learned to use their dominant strategy anyway), and (BB) (since they all are indifferent among strategies and so will not learn) and six utility functions of types (CO) and (NC) corresponding to different mixed strategy equilibria they would play among themselves. All the utilities are normalized so that $v_k(C, C) = 1$ and $v_k(D, D) = 0 \forall k$. The utility function of type (St1) is given by $v_k(C, D) = 1, v_k(D, C) = 0$; that of type (St2) by $v_k(C, D) = -1, v_k(D, C) = 2$; and that of type (BB) is given by $v_k(C, D) = 0, v_k(D, C) = 1$. Utility functions of type (CO) are given by $v_k(C, D) = -0.2k, v_k(D, C) = 0.2k, k = 0, \dots, 5$. The symmetric mixed equilibrium of a game between two players with the same preferences k of type (CO) is thus $(p, p) = (0.2k, 0.2k)$. For type (NC) utility functions are given by $v_k(C, D) = 0.2k, v_k(D, C) = 2 - 0.2k, k = 0, \dots, 5$. Again, the symmetric mixed equilibrium of the game between players with each particular preferences k is $(p, p) = (0.2k, 0.2k)$.

For evolution I take the discrete time version of the replicator dynamic

$$\mu_i^{t+1} = \mu_i^t \frac{d + u_i}{d + u}, \quad (8)$$

where u_i is the average expected fitness in subpopulation i given the state of the population, and $u = \sum_j \mu_j u_j$ is the average average expected fitness in the population.

Complete Information For the learning with complete information I take the discrete time linear model of imitation by dissatisfaction. This dissatisfaction and so the probability to change the strategy is measured by $\gamma - \delta v(s, x)$, where $v(s, x)$ is the utility strategy s gets against relevant population profile x . Suppose that the population size is fixed to N . The number n_{sij}^t of players in population i playing strategy s against population j is changing according to $n_{sij}^{t+1} = n_{sij}^t + \sum_s n_{sij}^t (\gamma - \delta v(s, x_{ji}^t)) \frac{n_{sij}^t}{N} - n_{sij}^t (\gamma - \delta v(s, x_{ji}^t))$. Here the number of players that change to s is given by the sum of the products of the probabilities that players are dissatisfied with their current strategies and they imitate s . The number of players that switch away from s is given by the probability that they are dissatisfied with it. The induced dynamic on population shares when $N \rightarrow \infty$, taking into account the assumption that the speed of learning depends on how often interaction takes place is

$$x_{ij}^{t+1} = x_{ij}^t [1 + \mu_j \delta (v_i(C, x_{ji}^t) - v_i(x_{ij}^t, x_{ji}^t))]. \quad (9)$$

Incomplete Information Learning with incomplete information, as argued above, has to take into account also the impossibility of observing preferences when imitating. The discrete time imitation by dissatisfaction process described above leads to the dynamic

$$x_i^{t+1} = x_i^t \left[1 + \gamma \left(\frac{x^t}{x_i^t} - 1 \right) + \delta \left(v_i(C, x^t) - \frac{x^t}{x_i^t} v_i(x_i^t, x^t) \right) \right]. \quad (10)$$

In both cases in the beginning a random selection of preference proportions and strategy proportions takes place, so that in the initial state all preferences and all strategies are represented. To represent different speed of processes, I allow learning to take place for L periods ($L = 100$ in simulations). Then one period of evolution passes, using the average payoff of players with each utility function over the 100 periods of learning. Then learning continues from the learned place further, though with probability 0.05 a random shock takes place, with the distribution similar to the truncated normal one with the mean on the current strategy (offspring do not imitate perfectly parent's strategy). Furthermore, a mutation takes place with probability 0.025 (offspring do not inherit perfectly parent's preferences). One utility function is chosen at random, and its share is increased by a random number from interval $(0, 0.1)$, after which all proportions are adjusted so that they add up to 1. The newly arrived mutants choose their strategy at random. This model of mutations is the one behind the evolutionary stability analysis of Dekel et al. (1998) and Possajennikov (2002).

The parameters has to be chosen in such a way that the dissatisfaction probability is between zero and one and that the replicator dynamic in evolution is well defined. Given the utility functions above, it suffices to have $\gamma = 0.5, \delta = 0.25$ to keep the learning dynamics in the simplex. Given the material payoff games below, $d = 2$ is enough to keep evolution in the simplex. Initial proportions of preferences and strategies used by different preferences are chosen randomly, uniform on types of preferences and on strategies. For each game, 100 simulations are run for 1000 evolutionary periods each.

3.2 Results

3.2.1 Prisoners' Dilemma

Complete Information Simulations are done for the prisoners' dilemma with $b = -1, c = 2$. The game has the unique symmetric Nash equilibrium

	Period 1	Period 10	Period 100	Period 1000
(St1)	0.213 [0.009-0.575]	0.019 [0.000-0.104]	0.012 [0.000-0.606]	0.018 [0.000-0.968]
(St2)	0.187 [0.005-0.458]	0.489 [0.030-0.865]	0.060 [0.000-0.757]	0.111 [0.000-0.946]
(BB)	0.190 [0.010-0.432]	0.094 [0.006-0.410]	0.067 [0.000-0.856]	0.051 [0.000-0.792]
(CO)	0.201 [0.000-0.748]	0.261 [0.001-0.801]	0.829 [0.013-1.000]	0.765 [0.000-1.000]
(NC)	0.209 [0.007-0.490]	0.137 [0.010-0.536]	0.032 [0.000-0.934]	0.054 [0.000-0.995]
(C,C)	0.257 [0.110-0.398]	0.059 [0.001-0.408]	0.755 [0.003-1.000]	0.638 [0.000-1.000]
(C,D)	0.238 [0.187-0.274]	0.110 [0.032-0.276]	0.033 [0.000-0.250]	0.034 [0.000-0.248]
(D,C)	0.238 [0.187-0.274]	0.110 [0.032-0.276]	0.033 [0.000-0.250]	0.034 [0.000-0.248]
(D,D)	0.266 [0.165-0.418]	0.721 [0.283-0.935]	0.179 [0.000-0.991]	0.295 [0.000-0.992]
Fitness	0.495 [0.346-0.606]	0.169 [0.033-0.465]	0.788 [0.066-1.000]	0.672 [0.004-1.000]

Table 1: Prisoners' Dilemma with complete information

(0,0) with fitness 0, while the unique efficient (also among correlated profiles) symmetric strategy profile is (1,1) with fitness 1. The results of the simulations are in Table 1. Mean values as well as minimum and maximum values over 100 simulations are reported.

Starting from uniformly distributed initial conditions in period 1, in period 10 one observes the growth of the average proportions of types (St2) and (CO), with type (St2) growing more. Among strategy profiles, (D,D) is most often played and average fitness is rather low.

This bleak picture changes dramatically by period 100. In this period the average proportion of type (St2) has dropped below 0.1 and the average proportion of type (CO) has grown above 0.8. Among strategies, (C,C) is now played more often, and the average fitness is well above 0.5, the random initial fitness. How does this change happen? A sort of secret handshake

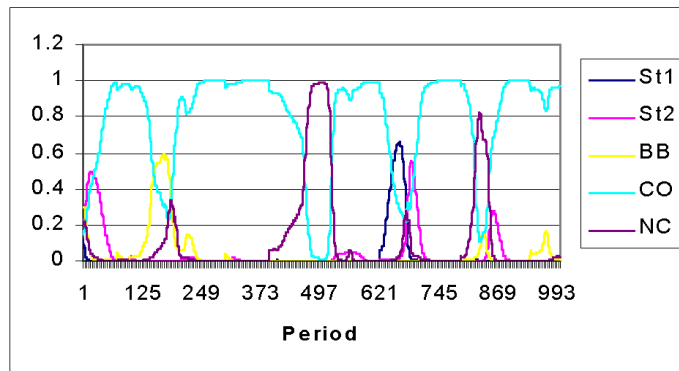


Figure 1: Evolution of types of preferences

story (Robson, 1990) explains it. Players of type (CO) are reciprocators in that they play (D, D) against players of type (St2), but may play (C, C) among themselves, which is also an equilibrium for them and by period 100 it is learned. Among types (CO) the most popular are the types for whom the efficient equilibrium (C, C) has a larger basin of attraction, but on the other hand, not too large, since otherwise such types can be exploited for their inclination to cooperate.

So, everybody is happy, then? Not quite. At period 1000 the average proportion of cooperators of type (CO) has dropped, as well as the average proportion of (C, C) plays. To see how this could happen, look at an individual simulation.

In Figure 1 one can see that most often preferences of type (CO) are prevalent. Mutants from other types, however, sometimes invade them, though not for long. Among strategy profiles, in Figure 2, (C, C) is most often on the top, though (D, D) also has its peaks. Thus, at any given period it is possible that mutants have just taken over from the coordinating types, and mutual defection prevails. The secret handshake story, however, again comes into play, and the coordinating types return to dominate.

It is possible that mutual defection survives for a long time. The simulations indicate, however, that at any given moment it is more likely to observe coordinating types cooperate and dominate. Thus the basic conclusion is that secret handshake works.

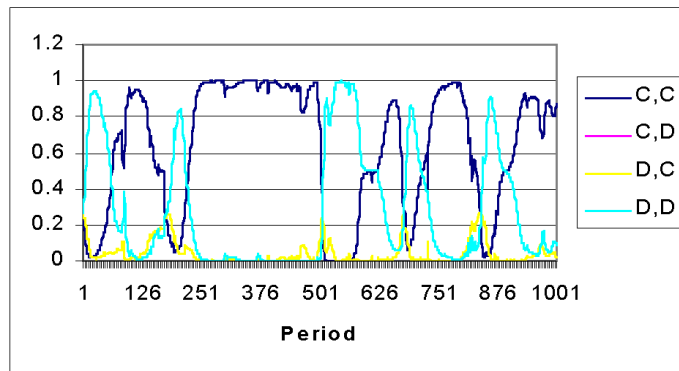


Figure 2: Evolution of strategies

In Dekel et al. (1998) preferences of the type that has D as weakly dominant strategy but that somehow nevertheless plays the (weakly dominated) equilibrium (C, C) are stable. This equilibrium, however, is not stable with respect to learning, and in Possajennikov (2002) it is shown that there is no stable preferences, or stable strategy profiles in prisoners' dilemma with infinite number of admissible preferences. The simulations support this claim, but they reveal more: at any given moment mutual cooperation is more probable than mutual defection. In dynamic perspective, longer spells of cooperation are interrupted by shorter spells of defection.

Incomplete Information Table 2 presents the results of simulations for the same prisoners' dilemma game, but with incomplete information.

As can be seen from the table, from uniformly random initial conditions the play converged to the unique Nash equilibrium (D, D) of the material payoff game. Already in period 10 strategy D is an overwhelming choice though in some simulations C is played by a majority. Later, though, D wins in all simulations. The proportions of types change little, mostly in the beginning, when the play did not converge yet, and later due to mutations. Though most of the types are dissatisfied with playing D , since it is the only strategy played in the population, they do not have an opportunity to change to C .

The conclusion is that though the proportions of types change little, the

	Period 1	Period 10	Period 100	Period 1000
(St1)	0.212 [0.003-0.507]	0.128 [0.003-0.416]	0.069 [0.002-0.149]	0.062 [0.009-0.133]
(St2)	0.209 [0.006-0.570]	0.293 [0.017-0.580]	0.355 [0.097-0.637]	0.387 [0.164-0.555]
(BB)	0.201 [0.009-0.447]	0.200 [0.009-0.444]	0.196 [0.008-0.460]	0.173 [0.030-0.368]
(CO)	0.198 [0.003-0.437]	0.200 [0.003-0.438]	0.219 [0.003-0.477]	0.254 [0.044-0.492]
(NC)	0.181 [0.010-0.574]	0.179 [0.010-0.572]	0.162 [0.013-0.441]	0.125 [0.026-0.358]
C	0.500 [0.243-0.770]	0.207 [0.000-0.962]	0.002 [0.000-0.033]	0.003 [0.000-0.205]
D	0.500 [0.230-0.757]	0.793 [0.038-1.000]	0.998 [0.967-1.000]	0.997 [0.795-1.000]
Fitness	0.500 [0.243-0.770]	0.207 [0.000-0.962]	0.002 [0.000-0.033]	0.003 [0.000-0.205]

Table 2: Prisoners' Dilemma with incomplete information

play quickly and without exception converges to the dominant strategy D , in contrast with the complete information case, where most often the play was around C . The simulations of the incomplete information case validate the result of Dekel et al. (1998): only Nash equilibria of the material payoff game are played in the long run.

3.2.2 Chicken Game

Complete Information I perform simulations for the chicken game with $b = 0.8, c = 2.2$. The symmetric Nash equilibrium of such a game is $(0.4, 0.4)$ with fitness 0.88, and the efficient symmetric strategy profile is $(0.75, 0.75)$ with fitness 1.125. The results of the simulations are presented in Table 3. Recall that the numbers in each cell of the table are mean values and minimum-maximum intervals of values over 100 simulations.

Initial conditions are uniformly distributed. Consider now period 10. Already some growth of types (St2) and (NC) is observed. This trend can clearly be seen in later periods, where the average proportions of both types

	Period 1	Period 10	Period 100	Period 1000
(St1)	0.195 [0.009-0.464]	0.149 [0.010-0.397]	0.010 [0.000-0.166]	0.011 [0.000-0.078]
(St2)	0.200 [0.011-0.659]	0.250 [0.022-0.545]	0.499 [0.133-0.618]	0.485 [0.085-0.618]
(BB)	0.201 [0.004-0.489]	0.165 [0.006-0.456]	0.008 [0.000-0.619]	0.003 [0.000-0.037]
(CO)	0.200 [0.014-0.614]	0.136 [0.014-0.458]	0.023 [0.001-0.190]	0.031 [0.000-0.384]
(NC)	0.204 [0.001-0.604]	0.300 [0.001-0.578]	0.460 [0.225-0.671]	0.470 [0.353-0.713]
(C,C)	0.266 [0.176-0.514]	0.171 [0.055-0.401]	0.020 [0.005-0.242]	0.013 [0.003-0.055]
(C,D)	0.239 [0.163-0.300]	0.281 [0.150-0.367]	0.292 [0.239-0.339]	0.283 [0.243-0.339]
(D,C)	0.239 [0.163-0.300]	0.281 [0.150-0.367]	0.292 [0.239-0.339]	0.283 [0.243-0.339]
(D,D)	0.255 [0.139-0.354]	0.267 [0.108-0.440]	0.397 [0.281-0.477]	0.421 [0.307-0.504]
Fitness	0.984 [0.870-1.157]	1.014 [0.765-1.218]	0.895 [0.781-1.050]	0.862 [0.739-1.032]

Table 3: Chicken game with complete information

(St2) and (NC) almost reach 0.5. Of the remaining preferences types (St1) and (BB) virtually disappear, while a higher average proportion of type (CO) remains. A closer inspection of individual simulations reveals that this average proportion comes from some simulations where type (CO) with $k = 5$, i.e. preferences that regard strategy D as weakly dominant, survived. These preferences are on the border of types (CO) and (St2) and are in this case better considered as belonging to type (St2). Among type (NC) most often preferences with low k have the largest proportion. Such preferences are indifferent between strategies when the opponent plays a strategy close to D , so such preferences play equilibrium (D, C) against other preferences of type (NC). Due to their aggressiveness they have higher material payoff.

Given that types (St2) and (NC) survive most often, it is not surprising the in the medium run (D, D) and the off-diagonal strategy profiles (C, D)

and (D, C) are played most often. In a game between (St2) and (NC) off-diagonal cells are played, as well as most probably in a game among different preferences of type (NC). Among (St2) (D, D) is played. The result for fitness falls short of the fitness achieved with the efficient symmetric strategy profile. Even the maximal fitness over simulations does not reach efficiency. The average fitness in period 1000 is even smaller than the equilibrium fitness 0.88, though not by much.

The conclusions one can draw from the results: most often types (St2) and (NC) survived, and this result is quite robust to initial conditions and perturbations; a certain correlated strategy profile is played with approximate weights 0.4 on (D, D) and 0.3 on $(C, D), (D, C)$; players achieve fitness close to the equilibrium fitness and short of fitness of the symmetric efficient strategy profile. The results are in contrast with the results of Dekel et al. (1998) and Possajennikov (2002) who show that no outcome is stable in the chicken game, and, more generally, inefficient strategy profiles are not stable. I show that though nothing is stable, the instability rests on a very unlikely event, and most likely a mixture of "hawkish" and "selfish" agents will survive in the population. Aggressiveness pays in chicken games, at least for a part of the population.

Incomplete Information Table 4 presents the results for the same game with incomplete information.

The average proportions of types (St2) and (NC) grow a bit in the beginning but then stabilize. There is very little change in the proportion of types. This is not surprising since the distribution of strategies becomes quickly as in the mixed equilibrium and therefore both strategies have the same fitness. In most simulations the distribution of strategies converges to the mixed symmetric equilibrium of the material payoff game. In some simulations, however, the strategy distribution stays long time on D . In all of these simulations the initial proportion of type (St2) is large relative to (St1). Players of type (St2) learn to play D , and then the "popularity bias" leads to the adoption of D by all other players, before mutations and trembles tip the play over to the equilibrium strategy.

The conclusion is that an equilibrium of the material payoff game is achieved (cf. Dekel et al.,1998). It is possible, however, that the play locks in on a pure strategy for some time, because of the "popularity bias". The proportions of types do not change because any strategy has the same fitness

	Period 1	Period 10	Period 100	Period 1000
(St1)	0.207 [0.007-0.637]	0.186 [0.007-0.504]	0.163 [0.008-0.318]	0.167 [0.020-0.259]
(St2)	0.206 [0.006-0.474]	0.220 [0.011-0.457]	0.231 [0.034-0.432]	0.224 [0.110-0.353]
(BB)	0.197 [0.001-0.476]	0.197 [0.001-0.477]	0.196 [0.001-0.470]	0.202 [0.042-0.392]
(CO)	0.194 [0.002-0.438]	0.191 [0.002-0.415]	0.190 [0.002-0.440]	0.188 [0.059-0.356]
(NC)	0.196 [0.004-0.448]	0.206 [0.004-0.456]	0.219 [0.004-0.483]	0.219 [0.013-0.395]
C	0.496 [0.212-0.764]	0.437 [0.000-0.887]	0.381 [0.000-0.467]	0.401 [0.366-0.460]
D	0.504 [0.236-0.788]	0.563 [0.113-1.000]	0.619 [0.533-1.000]	0.599 [0.540-0.634]
Fitness	0.975 [0.546-1.125]	0.849 [0.000-1.125]	0.840 [0.000-0.965]	0.881 [0.830-0.957]

Table 4: Chicken game with incomplete information

in the mixed equilibrium, and so all types have the same fitness, irrespective of their actual strategy.

3.2.3 Robustness Check

To see how robust the results are with respect to a change in the parameters, I perform also simulations with different values of them. In prisoners' dilemma with complete information with $b = -2$, thus more risky cooperation, the results were less in favor of cooperative behavior: only in about half of the simulations mutual cooperation with types (CO) is observed in period 1000. Thus the likelihood of cooperation may depend on the incentives to defect (or on punishment of being a sucker, as in this case). When material payoffs are changed in the chicken game to $c = 4.2$, the results are similar to the ones above: most often types (St2) and (NC) survive, and the played strategy profile is a correlated one with weights on (C, D) , (D, C) and (D, D) . The fitness in period 1000 is between the equilibrium fitness 0.84 and the efficient fitness 1.5625, and closer to the former. This confirms that efficiency is not

easy to achieve in chicken games. With incomplete information the change in the material payoff parameters did not influence the results: in a prisoners' dilemma with $b = -\frac{1}{2}$, despite the lower punishment for cooperation against a defector, mutual defection is established, while all types of preferences are present; in a chicken game with $c = 4.2$ in most simulations the equilibrium is played, and all types are present in period 1000.

The increase in the number of preferences of types (St1), (St2), and (BB) changes the results neither in the prisoners' dilemma nor in the chicken game. The enlargement of the set of preferences of types (CO) and (NC) increases the proportion of mutual coordination but mutual defection is also observed in the prisoners' dilemma with complete information. In the chicken game with complete information the average fitness is higher and closer to the efficient fitness, due to better (mis)coordination on off-diagonal payoffs. The distribution of types in the long run is similar, with large proportions of types (St2) and (NC) but, surprisingly, there is a larger proportion of (St2) type preferences. Thus it is possible that efficiency is (almost) achieved though not with all present types playing the same strategy but with a certain correlated profile that does not place (almost) any weight on (C, C) . In games with incomplete information the results do not change. Allowing players to use (some) mixed strategies does not change the results much either. (Though there is more cooperation in the prisoners' dilemma with complete information, and there is more variance in strategies in period 1000 in the chicken game with incomplete information.)

The learning model for the incomplete information case takes into account that players do not observe preferences of the person they are imitating. I also run simulations with two alternative models: one when players do observe the preferences of the person they are imitating, and so they imitate only strategies used by players with the same preferences as their own; the other model is not imitating at all but picking a strategy randomly instead. When players imitate within their own preferences only, the strategies played in the long run are the equilibrium of the material payoff game, like in the main learning model. Players with given preferences, however, learn their 'preferred' strategy and so can survive only if such a strategy is a part of the equilibrium. For example, in the prisoners' dilemma only preferences of types (St2) and (CO) survive because for them (D, D) is an equilibrium. When players choose a strategy randomly, not always the Nash equilibrium is reached: in the prisoners' dilemma, though only preferences of type (St2) survived, the average strategy is such that dissatisfaction is offset by random

choice. Thus, changing the learning model does change the results. Nevertheless, I think that the learning model I employ in Section 2 is the most natural one.

A change in the parameters d, γ, β does not produce any significant change in the results. In simulations without perturbations in learning or/and without mutations, there is more mutual cooperation in the prisoners' dilemma with complete information. The "secret handshake" works initially; since there is no mutations and/or perturbations in learning it could be upset more seldom. In the chicken game the fitness is higher despite of larger proportion of (St2) players. A closer look reveals that these results can be due to the fact that without permanent perturbations convergence to an equilibrium in which one of the players uses weakly dominant strategy is more likely. Then a mixture of cooperation and defection is possible in a stationary state in the prisoners' dilemma, and it is not necessary that more hawkish (NC) preferences get their preferable equilibrium. In games with incomplete information the results are the same as with perturbations.

It seemed interesting for me to see how the speed of learning influence the results. With 10 periods of learning (instead of 100), there is less cooperation in the prisoners' dilemma with complete information, and more surviving (St2) type players. In other games the results do not change much, only in the prisoners' dilemma with incomplete information the proportions of types change more. With equal speed of learning and evolution ($L = 1$) there is very little change both in the proportions of preferences and in the proportions of strategies in all games. On the other hand, with $L = 200$, the results are the same as in the basic model, which indicates that $L = 100$ is about right for learning to converge.

Finally, as the time horizon of the simulations is increased to $T = 2000$, the proportions of the preferences and strategies are approximately the same as in period 1000, which shows that the original time horizon is enough for the model to reach some sort of 'stable' distribution.

4 Conclusion

The results presented in this paper give mixed support for the previous analysis of models of indirect evolution for the complete information case, and vindicate those results for the incomplete information case. Efficiency in the complete information case is not always achieved in chicken games, while

in prisoners' dilemmas it is achieved only temporarily. Cooperation in prisoners' dilemma can be sustained by reciprocity with the observability of preferences. In chicken games, however, a higher efficiency is achieved by specializing, with some proportion of the population being "hawks", and the remaining proportion being "selfish", but this specialization does not lead to fully efficient outcome.

In the incomplete information case, even with a learning model that does not reflect the rationality assumptions usually put in the indirect evolution approach, two-speed evolution of preferences and strategies leads to the Nash equilibrium of the material payoff game. This equilibrium, however, is supported by preferences that are very diverse. This result is interesting, agents with cooperative tendencies can survive, but they defect because everybody else defects.

The most obvious extension of the model proposed in this paper is with respect to the informational assumptions. The complete and incomplete information cases represent two extremes of information the players possess. It would be interesting to see what happens with intermediate values of information, and with different information technologies.

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