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Why do experimental subjects choose an equilibrium which is neither risk nor payoff dominant

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Why do experimental subjects choose an equilibrium which is neither payoff nor risk dominant?*

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Abstract

In an experimental 2x2 coordination game with two strict equilibria we observe that, in contrast to equilibrium selection theory (Harsanyi and Selten 1988), only half of the subjects choose the strategy that relates to the payoff- and risk-dominant equilibrium. We propose modified risk dominance as an explanation for the observed deviations from payoff and risk dominance.

JEL classification: C72, C90

Keywords : equilibrium selection, modified risk dominance, prominence theory, experimental economics

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1. Introduction

Coordination games are symmetric games with multiple Pareto-rankable equilibria. In such games equilibrium behavior requires to know which of the equilibria the other players are aiming at. Thus, we need a theory selecting a unique equilibrium. The most well-known equilibrium selection theory is the one by Harsanyi and Selten (1988). Their theory is based on two criteria, payoff dominance and risk dominance. These criteria might have opposite implications. In that case Harsanyi and Selten favor payoff dominance, which is based on the assumption of collective rationality. However, there is no general consensus on this issue. While, for example, also Anderlini (1990) chooses the payoff-dominant equilibrium, Carlsson and van Damme (1993) and Harsanyi (1995) choose the risk-dominant equilibrium.

There exist many experimental studies examining this conflict case.[1] The major results to be drawn out of these experiments are that pre-play communication, the number of players, the time horizon and the structure of interaction matters. In contrast to these studies, we present an experiment in which we investigate equilibrium selection by subjects when payoff and risk dominance predict the same equilibrium.

We present an experiment that is based on a simple two-player coordination game in which each player chooses between two strategies. The game has two equilibria; of which one is both payoff and risk dominant. Subjects played this game once. We observe a deviation from the payoff- and risk-dominant equilibrium that is too great to be downgraded to noise.

Note that our experiment is one-shot and, thus, does not allow for learning. In application to real world situations one might be also interested in repeated play and the process of learning. In our study, however, we are interested in the self-enforcement nature of equilibria in one-shot games.

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[1] Among them are Van Huyck, Battalio, and Beil (1990, 1991), Van Huyck, Gillette, and Battalio (1992), Cooper, DeJong, Forsythe, and Ross (1992), Van Huyck, Battalio, Beil (1993), Mehta, Starmer, and Sugden (1994), Brandts and MacLeod (1995), Cachon and Camerer (1996), Clark, Kay, and Sefton (1996), Berninghaus and Ehrhart (1996), Croson and Marks (1996), Keser, Ehrhart, and Berninghaus (1998).

We understand our experimental result as a warning that we should investigate more on how subjects actually choose among strategies in coordination games. Probably there are criteria other than risk dominance and payoff dominance that play an important role in equilibrium selection. For example, the security principle according to which players choose the strategy that maximizes their minimum possible payoff is proposed by Van Huyck, Battalio, and Beil (1990) as a selection criterion. Vogt and Albers (1997) present modified risk dominance as a selection criterion using a numerical transformation in the perception of payoffs. This transformation is modeled by prominence theory (Albers 1997). We argue that modified risk dominance is a convincing explanation for the large number of observed deviations from the risk- and payoff-dominant equilibrium.

2. The game

We consider a symmetric two-player coordination game, in which each player chooses among strategies X and Y. The payoff matrix is presented in Figure 1. The game has two strict equilibria in pure strategies, (X,X) and (Y,Y). The (Y,Y)-equilibrium is payoff dominant as $50 < 70$ and risk-dominant as $(50-5) < (70-20)$. Furthermore, the game has an equilibrium in mixed strategies where each player chooses strategy X with probability 0.526.

	X	Y
X	50 50	20 5
Y	5 20	70 70

Figure 1: Payoff table of the symmetric coordination game

3. The experiments

3.1 Experimental design

The experiments were run at the University of Karlsruhe. After participation in a three-player experiment on decentralized or collective bargaining, subjects were invited to stay another ten minutes in our (very different) experiment. Instructions (see appendix) were distributed and read aloud. Subjects played the game once. They were not allowed to communicate and made their decision independently at their computer terminals. The pairing of subjects was random and anonymous. When all subjects had made their decisions, each participant was informed about his or her payoff. Payoffs were in Deutsche Mark (DM). Thus, subjects could earn a relatively important amount of money in a very short time. However, only one subject pair out of six pairs was randomly chosen for cash payment. In total, forty-eight subjects participated in this experiment.

3.2 Experimental results

Table 1 shows the frequencies with which subjects chose strategies X and Y. Forty-two percent of the decisions were X-choices.

Table 1: Frequencies of X- and Y-choices

# subjects	# X-choices	# Y-choices
48	20	28

Consider the null hypothesis that subjects choose strategy Y and, in the case of an error occurring with probability ε, strategy X. For $\varepsilon \leq 0.25$ a one-sided Binomial test allows us to reject the null hypothesis at the 1 percent level. Thus we may conclude that the observed frequency of X-decisions cannot simply be due to errors.

4. Modified risk dominance

If subjects followed only risk and payoff dominance, they should have chosen strategy Y. We observe, however, that subjects significantly deviate from this prediction. Are there equilibrium selection theories that would predict the choice of X in our game? Let us consider modified risk dominance (Vogt and Albers 1997) which is based on prominence theory (Albers 1997).

4.1 Prominence Theory

Prominence theory is based on the empirical observation that some numbers are easier accessible than others. The most easily accessible numbers in the decimal system are called the prominent numbers P which are:

$$P = \{n*10^{z} | z \in \mathbf{Z}, n \in \{1,2,5\}\} = \{...,0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100,....\}.$$

The next accessible numbers are the spontaneous numbers S which are

$$S = \{n*10^{z} | z \in \mathbf{Z}, n \in \{-7, -5, -3, -2, -1.5, -1, 0, 1, 1.5, 2, 3, 5, 7\}\}.$$

The spontaneous numbers include the prominent numbers and one additional number between any two neighbored prominent numbers in the negative and positive range.

In prominence theory the perception of numbers (for example, the payoffs in a game) is described by "steps". By definition, the difference between any two neighbored prominent numbers (ordered according to their size) is one step. Accordingly, the difference between any two neighbored spontaneous numbers (ordered according to their size) is half a step.

In specific tasks (contexts) different smallest amounts of money are important. For example, in decisions in the context of the annual budget of a state, $1 billion might be perceived as the smallest „important" amount, whereas it might be between $1 to $5 for the price of a dinner. In prominence theory this is modeled by assuming a "finest perceived full step" unit Δ. The difference between zero and this number is perceived as one step. The step structure and the finest perceived full step defines, up to an additive constant, a perception function v_Δ. By normalizing $v_\Delta(0) = 0$, we get the function presented in Table 2 for $\Delta = 20$ and the spontaneous numbers between –100 and +100.

Table 2:

Transformation of the spontaneous numbers between -100 and 100 by the v_Δ-function for $\Delta = 20$

number:	-100,	-70,	-50,	-30,	-20,	-15,	-10,	-5,	0,	5,	10,	15,	20,	30,	50,	70,	100
v_{20} :	-3,	-2.5,	-2,	-1.5,	-1,	-0.75,	-0.5,	-0.25,	0,	0.25,	0.5,	0.75,	1,	1.5,	2,	2.5,	3

Note that for numbers $x > \Delta$ the function $v_\Delta(x)$ is (nearly) equal to $3*\log(x/\Delta)+1$. Below the smallest unit Δ the function is linear[2].

In a specific task (e.g. the experimental game) the finest perceived full step Δ has to be determined. Prominence theory proposes the following rule: Δ is the prominent number that is two steps below the smallest prominent number greater than or equal to the maximal payoff in the task.

4.2 The selection criterion

Let us consider the symmetric bimatrix game presented in Figure 2 of which the game examined in our experiment is a special case. The strategies are denoted by X and Y. For the payoffs it holds that $a > b > c > d$. The equilibrium points are (X,X) and (Y,Y); (Y,Y) is payoff dominant.

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[2] This description of the perception is similar to the Weber-Fechner law (for example, in G.T. Fechner 1968) which describes the perception of stimuli in psychophysics. Above a smallest unit, the perception is proportional to a logarithmic function.

	X	Y
X	b b	c d
Y	d c	a a

Figure 2: A symmetric 2x2 game

We apply the criterion of modified risk dominance proposed by Vogt and Albers (1997). For this symmetric game the criterion of modified risk dominance is obtained from the concept of risk dominance by the application of the v_Δ-function to the payoffs.

The criterion of risk dominance is:

(X,X) dominates (Y,Y) iff $b - d > a - c$.

Replacing the payoffs by the perceived payoffs (resulting from the application of the v_Δ-function) we obtain the criterion of modified risk dominance:

(X,X) dominates (Y,Y) iff $v_\Delta(b) - v_\Delta(d) > v_\Delta(a) - v_\Delta(c)$.

The application of the v_Δ-function does not effect the ordering of the payoffs as it is a monotonic transformation. Thus, the (Y,Y)-equilibrium remains payoff dominant.

Prediction

For the prediction of this model the finest perceived full step Δ has to be determined according to the rule of prominence theory: The maximal payoff is 70. The smallest prominent number greater than or equal to 70 is 100. Δ is two steps below 100. This results in $\Delta = 20$. Thus, Table 2 (presented in Section 4.1 above) can be used for the transformation of the payoffs. The transformed payoffs for the experimental game in Figure 1 are presented in Figure 3.

	X	Y
X	2 2	1 0.25
Y	0.25 1	2.5 2.5

Figure 3: The transformed payoffs

Applying the criterion of the modified risk dominance to this game leads to the following result.

(X,X) dominates (Y,Y),
since
$v_{\Delta}(b) - v_{\Delta}(d) = 2 - 0.25 = 1.75 \quad > \quad 1.5 = 2.5 - 1 = v_{\Delta}(a) - v_{\Delta}(c)$.

Thus, the criterion of the modified risk dominance predicts that the equilibrium (X,X) is selected in our game.

5. Discussion

In the literature we find other principles that might explain the choice of strategy X in our game. Among them are the security principle (Van Huyck, Battalio, and Beil 1990) and the selection of mixed strategies.

Subjects following the security principle always select the so-called secure action. The secure action is the strategy that maximizes the minimum possible payoff. In our game the secure action is strategy X. The choices of X observed in our experiment could be interpreted with the security principle. However, other studies show much less importance of this principle (Vogt and Albers 1997, Brandts and

MacLeod 1995). They observe that payoffs other than the minimal guaranteed one can have a great impact on the choices of subjects. Thus, we conclude that the security principle can be only part of an explanation. Note that the security principle can be regarded as an "ingredient" of modified risk dominance. Using the notation of Figure 3, the security principle compares the payoffs c and d in a similar way to modified risk dominance. However, modified risk dominance includes the additional comparison of payoffs a and b.

One might argue that subjects play mixed strategies. Following Nash (1950), mixed strategies should be interpreted in terms of a state in a population where each player chooses a pure strategy but the relative frequency with which each pure strategy is chosen corresponds to its probability in the mixed strategy equilibrium. This interpretation is not convincingly applicable to our one-shot game. It could probably make sense in repeated play or a learning environment (see, for example, Oechssler 1995). Although in the experiment subjects have to decide on a pure strategy, one might assume that each subject chooses a pure strategy based on the outcome of a lottery prior to the experiment. Given this assumption, which is strongly criticized in the literature, we might conclude that the probability of X in this lottery was 0.42, a number fairly close to the mixed strategy equilibrium prediction. It had to be explained, however, why they should select the mixed strategy equilibrium and not the payoff- and risk-dominant (Y,Y)-equilibrium. The mixed strategy equilibrium is payoff dominated even by the (X,X)-equilibrium.

6. Conclusion

In our experimental coordination game we observe that an equilibrium that is neither payoff dominant nor risk dominant is selected by almost fifty percent of the subjects. Among all equilibrium selection theories known to us only modified risk dominance predicts this dominated equilibrium. Given our experimental evidence and given that in previous studies (Vogt and Albers 1997) modified risk dominance appears to be a better predictor for the equilibrium selection in 2x2 games than risk dominance we conclude that the concept of modified risk dominance should be included in the on-going discussion on equilibrium selection in coordination games which is typically focussed on the notions of risk dominance and payoff dominance.

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