Why "Peaches" Must Circulate Longer than "Lemons"*

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Abstract

This paper considers a competitive search market where sellers have private information about a good's quality. It is shown that separation of types may arise naturally if high-quality sellers derive a greater utility from search than low-quality sellers. For instance, sellers of high-quality goods may have to invest less time and money on repairs or spare parts than low-quality sellers or simply face a lower probability that the good breaks down before it is sold. In equilibrium, high-quality goods sell at a higher price, but also circulate longer than low-quality goods to ensure that low-quality sellers do not enter the market for high-quality goods. This holds even if an explicit sorting variable (e.g. warranties, advertising) does not exist. Moreover, all members of the short side of the market engage in trade, which is in contrast to the standard analysis where part of the short side of the market may not be served despite the presence of gains from trade.

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1 Introduction

Since the seminal contribution by Akerlof (1970), economists have suggested various ways to deal with the lemons problem. Typically, these suggestions involve the use of a sorting variable such as education (Spence 1973), deductibles in insurance policies (Rothschild and Stiglitz 1976), or warranties (Grossman 1980). If the marginal rate of substitution between money and the sorting variable differs between different types of informed agents ("single-crossing property"), the sorting variable can be used to separate agents according to their types. In many practical situations of interest, however, explicit sorting variables either do not exist or are not employed for whatever reasons. For instance, in the market for second-hand cars, warranties typically play no role, except perhaps when the seller is a professional car dealer. Nevertheless, it appears that cars of different qualities are frequently traded at different prices. In this connection, owners of used cars face a tradeoff between asking for a high price and selling the car fast. More precisely, if the asking price is low, car owners can usually sell their car within a shorter time since it is easier to find a potential buyer. In this paper, we argue that the decision whether to ask for a high or low price depends on the utility which the current owner derives from using the car until the car is sold. In particular, we argue that owners of high-quality cars derive a greater utility and are thus less eager to sell than owners of low-quality cars. As an illustration, consider the following example.

> Example 1. Unlike reliable cars, unreliable cars frequently require an engine overhaul, repair, or replacement of parts. Accordingly, owners of unreliable cars need to spend more time and money on keeping their car running than owners of reliable cars.

Clearly, the idea that agents derive different (flow) utilities until trade takes place is not restricted to the market for second-hand cars. Consider the following example.

> Example 2. More able workers typically perform better in their job than less able workers, which results in higher pay, more appreciation, and greater job satisfaction. Consequently, more able workers are less eager to find a new job and can devote more time to on-the-job search.

Sometimes, owners of low-quality goods are less patient not because they derive a lower utility, but because it is more likely that the good breaks down before it is sold.

> Example 3. Machines that are older and/or have been used more heavily are more likely to break down than equivalent machines that are new or have been used less. Consequently, the likelihood that the machine breaks down before it is sold is greater for old or more heavily used machines.

In all three cases, the marginal rate of substitution between money and the time it takes until trade occurs is greater for "good" types than for "bad" types. Thus, the single-crossing property holds even if an explicit sorting variable does not exist. This implies that in markets where buyers and sellers engage in search before trade takes place, separation of types may arise naturally. While low-quality goods sell fast but at a low price, high-quality goods sell at a higher price but circulate longer.

In our model, we consider a search market that is divided into various submarkets. Each submarket is characterized by a unique price. There are three types of agents: potential buyers (i.e. agents who are not initially endowed with a good), potential sellers of high-quality goods, and potential sellers of low-quality goods. Whether a potential seller is endowed with a high- or low-quality good is private information. The model is set in continuous time. In line with the above argument, we assume that high-quality sellers derive a greater (flow) utility during their search than low-quality sellers. Potential buyers and sellers must make two decisions: i) whether to enter the market at all, and if yes, ii) which submarket to enter. When a potential buyer is matched with a potential seller, the buyer pays the seller the price prevailing in the respective submarket. Subsequently, both the buyer and the seller leave.

Which goods are traded in equilibrium depends on relation between the total number of potential buyers and sellers in the economy. If there are more low-quality sellers than buyers, only low-quality goods are traded. Hence, this case is similar to the standard competitive analysis where the high-quality good is driven out of the market. Conversely, if the number of buyers exceeds the number of low-quality sellers, both low- and high-quality goods are traded. Moreover, each type of good is traded in a different submarket and thus commands a different price, which implies that the equilibrium is fully separating. To ensure that low-quality sellers do not enter the submarket for high-quality goods, goods in the high-quality submarket must circulate longer before they are sold. Additionally, all members of the short side of the market (i.e. potential sellers if the number of potential sellers exceeds the number of potential buyers, and vice versa) engage in trade. This is in contrast to the standard competitive analysis, where part of the short (!) side of the market is sometimes not served.

This paper is not the first to consider markets where trade takes place in different submarkets. Similar settings are examined in Gale (1992), Peters (1997), and Moen (1997). Unlike our model, however, Gale and Peters consider a static setting where all trade takes place instantaneously. More related to our paper is the work by Moen. In particular, Moen also assumes that each submarket constitutes a separate search environment. Besides, the equilibrium conditions in Moen's model are similar to the ones employed here. In contrast to our paper, however, Moen does not consider private information. Our paper is also related to work by Evans (1989). In a sequential bargaining game with incomplete information and correlated values, he shows that a buyer can use delay to screen different types of sellers. This only holds if the buyer is strictly more patient than the seller though. If this is not the case, price discrimination does not occur.[1] Finally, our paper is related to Wilson (1980), who considers a version of Akerlof's model where buyers differ in the value they attach to cars of the same quality. Like the original model by Akerlof, Wilson's model is static. Wilson shows that the nature of equilibrium is extremely sensitive to the prevailing price-setting convention, i.e. whether prices are set by buyers, sellers, or a Walrasian auctioneer. In particular, there may exist multiple equilibria, some of which exhibit price dispersion.

The rest of the paper is organized as follows. Section 2 introduces the model and defines the equilibrium concept. As a benchmark, Section 3 derives the set of competitive equilibria if trade takes place instantaneously. Section 4 contains the main analysis. Section 5 discusses some extensions, and concluding remarks are offered in Section 6.

2 The Model

2.1 Agents and Preferences

Agents are either endowed with zero or one unit of an indivisible good. We frequently refer to agents who are endowed with zero units as potential buyers and agents who are endowed with one unit as potential sellers. The good comes in two different qualities $q \in Q = \{l, h\}$, where l stands for low quality and h stands for high quality. All agents are risk neutral and discount future payoffs with the same rate $r > 0$. The model is set in continuous time with an infinite time horizon. Potential sellers derive a constant flow utility v_q from using the good. If the good is sold, the flow utility is zero. Likewise, the flow utility of potential buyers before and after the purchase is zero and u_q, respectively.

[1] A similar point is also made by Vincent (1989).

For both types of agents, we assume that the flow utility from a high-quality good is greater than from a low-quality good, i.e. $v_h > v_l > 0$ and $u_h > u_l > 0$. Moreover, we assume that there are strictly positive gains from trade for both types of goods, i.e. $u_h > v_h$ and $u_l > v_l$. Finally, any agent can derive utility from at most one unit of the good at any point in time. For instance, if the good is a car, this means that an agent only derives utility from driving the car, not from owning it per se.

The asset value of a good is defined as the discounted stream of utilities derived from using the good until the indefinite future. Accordingly, the asset value of a good of quality q for a potential seller is $V_q = v_q/r$, and the corresponding asset value for a potential buyer is $U_q = u_q/r$.

During one unit of time (what constitutes a time unit is implicitly defined by the discount rate r), the measure one of agents appears at the market fringe.[2] Of these agents, a fraction $b \in (0, 1)$ are potential buyers. The fraction $1 - b = s$ of potential sellers is divided further into a fraction $s_h > 0$ of owners of the high-quality good and a fraction $s_l > 0$ of owners of the low-quality good. The quality of a good is private information. For convenience, we restrict attention to generic parameter values $b \neq s$, $b \neq s_q$ for all $q \in Q$, and $v_h \neq u_l$. Agents appearing at the market fringe can choose whether to enter or stay outside. If they do not enter, their utility is determined by their initial endowment. To avoid that the market fringe clogs up over time (which may be the case when agents are indifferent between entering the market and staying outside), agents must decide immediately whether to enter or not.

2.2 Matching Technology

The good is traded in a search market where potential buyers are matched with potential sellers. While searching, agents incur a time-invariant search cost $c > 0$. The market consists of a continuum of potential buyers and sellers, which implies that the probability of trade is determined by the respective measures of agents engaging in search. We restrict attention to stationary equilibria where the stock of agents is constant over time (see condition (E.2) below). The measure of potential buyers in the market is denoted by β, and the measure of potential sellers of type q is denoted by σ_q. The measure of buyer-seller matches per unit of time is expressed by the matching function $x(\beta, \sigma)$, where

[2] By using potential entrants as primitives, we follow Gale (1987) and Peters (1992). In contrast, Rubinstein and Wolinsky (1985) use the stock of agents in the market as primitives and adjust the flow of entrants to preserve stationarity. On this point, see also Osborne and Rubinstein (1992, Ch.7).

$\sigma = \sigma_l + \sigma_h$. Following standard assumptions, the matching function is continuous and homogeneous of degree one in both arguments. The transition rates for potential buyers and sellers are then $x(\beta, \sigma)/\beta = x(1, 1/k) = f(k)$ and $x(\beta, \sigma)/\sigma = f(k)k$, respectively, where $k = \beta/\sigma$ denotes the "market tightness" from the perspective of potential buyers.[3] Define $g(k) = f(k)k$. Following again standard assumptions, $f(k)$ is strictly decreasing in k with limits $\lim_{k\to 0} f(k) = \infty$ and $\lim_{k\to\infty} f(k) = 0$, and $g(k)$ is strictly increasing in k with limits $\lim_{k\to 0} g(k) = 0$ and $\lim_{k\to\infty} g(k) = \infty$. The search market is fully characterized by the tightness k, the distribution of offered qualities $\pi(q) = \sigma_q/\sigma$, and the prevailing price p. For convenience, we assume that p lies in a sufficiently large compact interval. Note that by restricting attention to prices, we implicitly rule out more complicated contractual arrangements, e.g. where buyers are given an option to sell back the good after they have learned its quality.

2.3 Asset Value Equations

Denote by $V_q^M(k, p)$ the utility of a potential seller of a good of quality q in a market with tightness k and price p. The asset value equation for $V_q^M(k, p)$ is

$$rV_q^M(k, p) = -c + v_q + g(k)\left[p - V_q^M(k, p)\right],$$

which can be rearranged as

$$V_q^M(k, p) = \frac{v_q - c + g(k)p}{r + g(k)}. \tag{1}$$

Hence, potential sellers of a high-quality good derive a greater utility from search than potential sellers of a low-quality good, which implies that they are less eager to sell and more ready to accept a longer search time.

Likewise, denote by $U^M(\pi, k, p)$ the (expected) utility of a potential buyer engaging in search. For obvious reasons, this utility depends on the distribution of qualities in the market. The asset value equation for $U^M(\pi, k, p)$ is

$$rU^M(\pi, k, p) = -c + f(k)\left[\sum_{q\in Q} \pi(q)U_q - p - U^M(\pi, k, p)\right],$$

[3] The market tightness is an indirect measure of the expected time which goods must circulate before trade takes place. As the tightness is defined from the perspective of potential buyers, a greater tightness implies a shorter circulation time.

which can be rearranged as

$$U^M(\pi, k, p) = \frac{-c + f(k)\left[\sum_{q \in Q} \pi(q) U_q - p\right]}{r + f(k)}. \tag{2}$$

2.4 Equilibrium Conditions

The "grand" market consists of several submarkets, which are indexed by natural numbers $n \in N = \{1, 2, ..., \bar{n}\}$. Each submarket constitutes an independent search environment, which implies that a submarket is fully characterized by the triple (π^n, k^n, p^n) representing the distribution of offered qualities, the tightness, and the prevailing price. Both potential sellers and buyers must decide i) whether or not to enter the grand market, and ii) if entry occurs, which of the n submarkets to choose.[4] If both decisions are made optimally, the utility of a potential seller is

$$V_q^* = \max\left\{V_q, \max_{n \in N} V_q^M(k^n, p^n)\right\}.$$

Likewise, the utility of a potential buyer is

$$U^* = \max\left\{0, \max_{n \in N} U^M(\pi^n, k^n, p^n)\right\}.$$

Given that search is costly, it is optimal for both parties to leave the grand market once trade has occurred. Moreover, since the stock of potential buyers and sellers in the market is stationary and the (expected) number of matches is constant, entry flows must also be stationary. The measure of potential buyers and sellers of a good of quality q entering submarket n is denoted by b^n and s_q^n, respectively. For all $n \in N$, define

$$V^*_{q, N\setminus\{n\}} = \max\{V_q, \max_{n' \in N\setminus\{n\}} V_q^M(k^{n'}, p^{n'})\}$$

and

$$U^*_{N\setminus\{n\}} = \max\{0, \max_{n' \in N\setminus\{n\}} U^M(\pi^{n'}, k^{n'}, p^{n'})\}.$$

A search market equilibrium is characterized by three conditions: i) optimality, ii) stationarity, and iii) competitiveness.

[4] We implicitly assume that once a submarket is chosen, agents must stay in this submarket until trade occurs. This assumption is only restrictive if agents are indifferent between different submarkets or between entering and not entering the grand market, in which case they may want to switch back and forth between submarkets or between the grand market and their outside option. Allowing for this possibility is straightforward but does not yield any additional insight.

(E.1) Optimality. The decision to enter a submarket must be optimal for all agents. Accordingly, the measure of potential sellers of a good of quality q entering submarket n must satisfy

$$s_q^n = \begin{cases} 0 & \text{if } V_q^M(k^n, p^n) < V^*_{q,N\setminus\{n\}} \\ s_q & \text{if } V_q^M(k^n, p^n) > V^*_{q,N\setminus\{n\}} \\ \in [0, s_q] & \text{if } V_q^M(k^n, p^n) = V^*_{q,N\setminus\{n\}} \end{cases}$$

for all $n \in N$, where $\sum_{n\in N} s_q^n \leq s_q$.

Likewise, the measure of potential buyers entering submarket n must satisfy

$$b^n = \begin{cases} 0 & \text{if } U^M(\pi^n, k^n, p^n) < U^*_{N\setminus\{n\}} \\ b & \text{if } U^M(\pi^n, k^n, p^n) > U^*_{N\setminus\{n\}} \\ \in [0, b] & \text{if } U^M(\pi^n, k^n, p^n) = U^*_{N\setminus\{n\}} \end{cases}$$

for all $n \in N$, where $\sum_{n\in N} b^n \leq b$.

Note that (E.1) contains the obvious requirement that the measure of agents entering the grand market cannot exceed the measure of agents arriving at the market fringe.

(E.2) Stationarity. In all submarkets, the flow of entries must equal the flow of exits, i.e. $s_q^n = \sigma_q^n g(k^n)$ for $q \in Q$, and $b^n = \beta^n f(k^n)$ for all $n \in N$.

As the number of buyer-seller matches is constant, (E.2) automatically implies that the stock of agents in each submarket must be stationary.

Finally, we come to our last condition, which states that in equilibrium, it must not be profitable to open a new submarket, i.e. to offer a price that is different from the already existing prices. Below, we only consider the case where buyers can open new submarkets. In Section 5, we show that the model can be straightforwardly extended to the case where submarkets can be opened by either buyers or sellers. Whether opening a new submarket is profitable depends on the price p, the ensuing tightness k, and the attracted distribution of qualities π. Given a pair of equilibrium utilities $\{V_q^*\}_{q\in Q}$ and a "deviating" offer $p \notin \{p^n \mid n \in N\}$, where $p > \max_{q\in Q} V_q^*$, any pair (k, π) that is consistent with $(p, \{V_q^*\}_{q\in Q})$ must satisfy $V_q^M(k, p) = V_q^*$ if $\pi(q) > 0$ and $V_q^M(k, p) \leq V_q^*$ if $\pi(q) = 0$. In words, if a particular type of seller is attracted by the new submarket, the circulation time must adjust until he is indifferent between the new submarket and the set of existing submarkets. Conversely, any type of seller that is not attracted by the new submarket must weakly prefer the set of existing submarkets to the new one. The set of all pairs (k, π) that are consistent with $(p, \{V_q^*\}_{q\in Q})$ is denoted by $F(p, \{V_q^*\}_{q\in Q})$.

Deviations of the form $p \leq \max_{q\in Q} V_q^*$ can be safely ruled out as they lead to infinite delay and thus to infinite search cost. Thus, for all prices $p \leq \max_{q\in Q} V_q^*$, we set $k = \infty$.

(E.3) Competitiveness. In equilibrium, it must not be profitable to open new submarkets in addition to the N existing submarkets. More precisely, given the equilibrium utilities U^* and $\{V_q^*\}_{q\in Q}$, there must not exist a price $p \notin \{p^n \mid n \in N\}$ such that $U^M(\pi, k, p) > U^*$ for all $(k, \pi) \in F(p, \{V_q^*\}_{q\in Q})$.

Observe that the requirement that a deviation must be profitable for *all* $(k, \pi) \in F(p, \{V_q^*\}_{q\in Q})$ is more restrictive than the alternative requirement that a deviation must only be profitable for *some* $(k, \pi) \in F(p, \{V_q^*\}_{q\in Q})$. In principle, destroying an equilibrium is much easier under the latter requirement, which implies that Lemma 2 below would be less meaningful. Notwithstanding this argument, however, the equilibria characterized in Propositions 1-3 all hold under this alternative requirement.

To summarize, a search market equilibrium consists of a finite set N of submarkets with characteristics (π^n, k^n, p^n) and entry flows b^n and s_q^n satisfying conditions (E.1)-(E.3). As a benchmark, we first derive the set of competitive equilibria for a static version of our model where prices are set by a Walrasian auctioneer to equate supply and demand. We then proceed with the analysis of search market equilibria.

3 Standard Competitive Analysis

Consider a static version of our model in which trade takes place instantaneously. The economy is populated by a measure one of agents, of which a fraction $b \in (0, 1)$ constitutes potential buyers. The fraction $1 - b = s$ of potential sellers is divided further into a fraction $s_h > 0$ of owners of a high-quality good and a fraction $s_l > 0$ of owners of a low-quality good. The utilities of potential sellers and buyers from using the good are v_q and u_q, respectively, where $q \in Q$. The supply correspondence is

$$S(p) = \begin{cases} 0 & \text{if } p < v_l \\ \in [0, s_l] & \text{if } p = v_l \\ s_l & \text{if } v_l < p < v_h \\ \in [s_l, s] & \text{if } p = v_h \\ s & \text{if } p > v_h. \end{cases}$$

Clearly, whenever a positive fraction of the high-quality good is supplied, the entire fraction of the low-quality good must be supplied as well. For a given value $S > 0$, the distribution of qualities is therefore $\pi(h,S) = \max\{0, S - s_l\}/S$ and $\pi(l,S) = 1 - \pi(h,S)$. Hence, the demand correspondence is

$$D(p,\pi) = \begin{cases} 0 & \text{if } \pi(l)u_l + \pi(h)u_h < p \\ \in [0,b] & \text{if } \pi(l)u_l + \pi(h)u_h = p \\ b & \text{if } \pi(l)u_l + \pi(h)u_h > p. \end{cases}$$

A *competitive equilibrium* is a triple (D, S, p) such that $D = S \geq 0$, $S \in S(p)$, and $D \in D(p, \pi(S))$. Recall that we restrict attention to generic parameter values. We can then distinguish between three cases. The first two cases represent buyer markets ($s > b$), whereas the third case represents a seller market ($b > s$). Since the analysis is standard, we confine ourselves to summarizing the results.

Case 1 ($s_l > b$). There exists a unique equilibrium where all potential buyers purchase a low-quality good at the price $p = v_l$.

Case 2 ($s > b > s_l$). We can distinguish between three subcases. If $v_h < u_l$, there are gains from trade regardless of the distribution of qualities. In this case, there exists a unique equilibrium where both low- and high-quality goods are traded at $p = v_h$. In this equilibrium, the measure s_l of buyers purchases the low-quality good, and the measure $b - s_l$ of buyers purchases the high-quality good. If $v_h > u_l$ and $s_l u_l + (b - s_l)u_h < bv_h$, there exists a unique equilibrium where only the low-quality good is traded at the price $p = u_l$. The measure $b - s_l$ of potential buyers does not trade despite the presence of high-quality sellers in the market. Finally, if $v_h > u_l$ and $s_l u_l + (b - s_l)u_h \geq bv_h$, there exist two equilibria. Either only the low-quality good is traded at the price $p = u_l$, or both the low- and high-quality good are traded at the price $p = v_h$.

Case 3 ($b > s$). The analysis is similar to Case 2. We can again distinguish between three subcases. If $v_h < u_l$, there exists a unique equilibrium where all goods are traded at the price $p = (u_l s_l + s_h u_h)/s$, which implies that potential buyers are forced down to their reservation utility of zero. Second, if $v_h > u_l$ and $s_l u_l + s_h u_h < sv_h$, there exists a unique equilibrium where only the low-quality good is traded at the price $p = u_l$. In this case, the measure s_h of high-quality sellers does not trade despite the presence of potential buyers in the market. Finally, if $v_h > u_l$ and $s_l u_l + s_h u_h \geq sv_h$, there exist two equilibria. Either only the low-quality good is traded at the price $p = u_l$, or both the low- and high-quality good are traded at the price $p = (u_l s_l + s_h u_h)/s$.

To summarize, for most parameter values there exist gains from trade which remain unexhausted in equilibrium. In particular, if $v_h > u_l$, a strictly positive fraction of the short side of the market is not served (Cases 2 and 3). This is different if we consider search markets. As is shown in the following section, all members of the short side of the market then engage in trade if search costs are sufficiently low.

4 Search Market Equilibria

We begin with a characterization of the equilibrium utilities of the long side of the market. From (1), it immediately follows that

$$\left(V_l^M(k,p) - V_l\right) - \left(V_h^M(k,p) - V_h\right) = \frac{g(k)\,(v_h - v_l)}{r(r + g(k))} > 0, \tag{3}$$

which implies that if high-quality sellers weakly prefer to enter the (grand) market, low-quality sellers must strictly prefer to enter. Since potential buyers and sellers must exit the market in pairs, the long side of the market (or at least part of it) must be indifferent between entering and not entering to ensure that the stock of agents in the market remains constant. We therefore have the following result.

Lemma 1: *In equilibrium, the following must hold*:

Case 1 ($s_l > b$). *Potential sellers of the low-quality good are indifferent between entering and not entering the grand market, whereas potential sellers of the high-quality good strictly prefer not to enter. Consequently, all potential sellers must receive their reservation utility* V_q, *where* $q \in Q$.

Case 2 ($s > b > s_l$). *Potential sellers of the low-quality good strictly prefer to enter the grand market, whereas potential sellers of the high-quality good are indifferent between entering and not entering. Consequently, potential sellers of the high-quality good must receive their reservation utility* V_h.

Case 3 ($b > s$). *Potential buyers are indifferent between entering and not entering the grand market, which implies that they must receive their reservation utility of zero.*

Next, let us derive the indifference curve of a potential seller of type q. Solving (1) for p and setting the seller's utility equal to V, we obtain

$$p = V + \frac{rV - v_q + c}{g\,(k)}. \tag{4}$$

Recall that k represents the market tightness from the perspective of potential buyers. From the perspective of potential sellers, an increase in k is thus associated with a decrease in the circulation time. Differentiating (4) with respect to k yields[5]

$$p'(k) = -\frac{rV - v_q + c}{[g(k)]^2} g'(k), \tag{5}$$

which implies that the marginal rate of substitution between the price and the time it takes until the good is sold is greater for high-quality sellers than for low-quality sellers. In the literature, (5) is called single-crossing condition. Given that (5) holds, it is now straightforward to show that no submarket attracts more than one type of seller.

Lemma 2: *In equilibrium, there exists no pooling submarket.*[6]

Furthermore, define for all utilities $V \geq V_q$ and prices $p > V$ the tightness $k_q^S(p, V)$ that ensures that each type of seller remains on his indifference curve $V_q^M(k, p) = k$. By (4), $k_q^S(p, V)$ is implicitly defined

$$g(k_q^S(p, V)) = \frac{rV + c - v_q}{p - V}.$$

Observe that $k_q^S(p, V)$ is unique and continuous in both arguments. For all prices $p \leq V$, set $k_q^S(p, V) = \infty$. Given this definition, we can define for all qualities $q \in Q$ and utilities $V \geq V_q$ the program $P_q(V)$ in which p is chosen to maximize the utility of potential buyers $U^M(\pi, k, p)$ subject to $k = k_q^S(p, V)$ and $\pi(q) = 1$. Note that in this program, the quality of the good is known. As search costs are strictly positive, the optimal solution must satisfy $p > V$. Moreover, the value function $\bar{U}_q(V)$ is continuous (by the maximum theorem) and decreasing in both q and V for all $V \geq 0$.[7]

We now proceed by formally characterizing the set of equilibria for Cases 1-3. For the sake of clarity, this is done by means of three separate propositions. Incidentally, if engaging in search is too costly, there may be no trade at all in equilibrium. In all three propositions, we therefore assume that search costs are sufficiently low.

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[5] The assumption that $g(k)$ is differentiable is only for convenience. The same argument can be made without this assumption by comparing the utility of potential sellers for different values of p and k.

[6] Formally, a submarket is pooling if $\pi(q) > 0$ for all $q \in Q$.

[7] The fact that $\bar{U}_q(V)$ is strictly decreasing in q is obvious. To see that $\bar{U}_q(V)$ is also strictly decreasing in V, consider a decrease (!) in V such that the tightness $k_q^S(p, V)$ remains constant. From the definition of $k_q^S(p, V)$, it follows that the price p must also strictly decrease, which implies that $U^M(\cdot)$ must be strictly decreasing in V.

Proposition 1: *If* $s_l > b$ *and search costs are sufficiently low, an equilibrium exists, and any equilibrium exhibits the following characteristics*:

i) Only low-quality goods are traded. More precisely, $\sum_{n\in N} b^n = b$, $\sum_{n\in N} s_l^n = b < s_l$, *and* $\sum_{n\in N} s_h^n = 0$.

ii) In any submarket, the price p^n *for the low-quality good solves the program* $P_l(V_l)$, *and the corresponding tightness is* $k^n = k_l^S(p^n, V_l)$.

By Proposition 1, high-quality goods are never traded in equilibrium if the supply of low-quality goods already exceeds the potential demand. Hence, the outcome is the same as in the standard competitive analysis. This is no longer true if the measure of potential buyers exceeds the measure of potential sellers of the low-quality good. As is shown below, both low- and high-quality goods are then traded in equilibrium.

To characterize submarkets where high-quality goods are traded, it is necessary to define a new program. For all utilities $V(q) \geq V_q$, where $q \in Q$, define by $P^C(V(l), V(h))$ the program in which p is chosen to maximize the utility of potential buyers $U^M(\pi, k, p)$ subject to $k = k_h^S(p, V(h))$, $\pi(h) = 1$, and the incentive compatibility constraint for low-quality sellers $V(l) \geq V_l^S(k, p)$. The corresponding value function is denoted by $\bar{U}^C(V(l), V(h))$. Incidentally, note that the solution to $P^C(V(l), V(h))$ may be empty (to see this, take the case where $V(l) = V_l$ and $V(h) > V_h$).

Proposition 2: *If* $s > b > s_l$ *and search costs are sufficiently low, an equilibrium exists, and any equilibrium exhibits the following characteristics*:

i) Both low- and high-quality goods are traded. More precisely, $\sum_{n\in N} b^n = b$, $\sum_{n\in N} s_l^n = s_l$, *and* $\sum_{n\in N} s_h^n = b - s_l < s_h$.

ii) The set of submarkets is fully separating. In submarkets where the low-quality good is traded, the price p^n *solves the program* $P_l(V_l^*)$, *while in submarkets where the high-quality good is traded,* p^n *solves the program* $P^C(V_l^*, V_h)$. *Moreover, the tightness in a submarket where a good of quality* q *is traded is* $k^n = k_q^S(p^n, V_q^*)$. *The equilibrium utility* V_l^* *of low-quality sellers satisfies* $V_l^* > V_l$ *and is determined by*

$$\bar{U}^C(V_l^*, V_h) = \bar{U}_l(V_l^*). \tag{6}$$

Finally, high-quality sellers receive exactly their reservation utility $V_h^* = V_h$.

While Case 2 still represents a buyer market, the measure of low-quality sellers no longer exceeds the measure of potential buyers. In contrast to Case 1, both low- and high-quality goods are now traded in equilibrium. More importantly, however, low-quality goods are traded in different submarkets and at different prices than high-quality goods. In particular, high-quality submarkets exhibit higher prices, but also longer circulation times to ensure that low-quality sellers do not enter. The circulation time (more precisely: the tightness) that is necessary to achieve separation is implicitly determined by the single-crossing condition (5). Finally, unlike in the standard analysis, the short side of the market (here: potential buyers) is fully served. More precisely, the measure s_l of potential buyers purchases the low-quality good, while the high-quality good is purchased by the remaining measure $b - s_l$.

Denote by $\hat{V}$ the utility of low-quality sellers if potential buyers receive their reservation utility. Thus, $\hat{V}$ satisfies $\bar{U}_l(\hat{V}) = 0$.

Proposition 3: *If $b > s$ and search costs are sufficiently low, an equilibrium exists, and any equilibrium exhibits the following characteristics*:

i) Both low- and high-quality goods are traded. More precisely, $\sum_{n \in N} b^n = s < b$ and $\sum_{n \in N} s_q^n = s_q$, where $q \in Q$.

ii) The set of submarkets is fully separating. In submarkets where the low-quality good is traded, the price p^n solves the program $P_l(\hat{V})$, while in submarkets where the high-quality good is traded, p^n solves the program $P^C(\hat{V}, V_h^*)$. Moreover, the tightness in a submarket where a good of quality q is traded is $k^n = k_q^S(p^n, V_q^*)$. The equilibrium utility V_h^* of high-quality sellers satisfies $V_h^* > V_h$ and is determined by

$$\bar{U}^C(\hat{V}, V_h^*) = 0. \tag{7}$$

Finally, low-quality sellers receive a utility of $V_l^* = \hat{V} > V_l$.

Proposition 3 shows that Case 3 is very similar to Case 2. Again, low-quality goods are traded in different submarkets and at different prices than high-quality goods. The intuition for this is exactly the same as in Case 2. Also, in contrast to the standard analysis, the short side of the market (here: potential sellers) is again fully served.

5 Extensions

In this section, we extend the model in two different directions. First, we show that all our results remain unchanged if goods differ with regard to their risk of breakdown instead of their flow utility. Second, we argue that it is inessential whether only buyers (as was assumed in the model) or both buyers and sellers can open up new submarkets.

5.1 Goods Differ with Regard to Their Risk of Breakdown

Suppose both types of goods yield the same flow utility, but differ in their (constant) risk of breakdown, which is expressed by the rate s_q. More precisely, assume that $s_l > s_h \geq 0$, i.e. the risk of breakdown is greater for low-quality goods than for high-quality goods. As in the previous model, we assume that potential sellers derive a constant flow utility v from using the good. If the good is sold, the flow utility is zero. Likewise, the flow utility of potential buyers before and after the purchase is zero and u, respectively, where $u > v$. If the good is broken, its flow utility is also zero. Accordingly, the asset value of a good of quality q to an initial owner and non-owner is $V_q = v/(s_q + r)$ and $U_q = u/(s_q + r)$, respectively. While $U^M(\pi, k, p)$ is still given by (2), the expected utility of a potential seller engaging in search is now defined by the asset value equation

$$(r + s_q)V_q^M(k, p) = -c + v + g(k)\left[p - V_q^M(k, p)\right],$$

which can be rearranged as

$$V_q^M(k, p) = \frac{v - c + g(k)p}{r + s_q + g(k)}. \tag{8}$$

Observe that the risk of breakdown enters negatively in the seller's utility.

By going through the same steps as in Section 4, it is now easy to check that all our results continue to hold under the alternative assumption that goods only differ in their risk of breakdown. To see why, consider the indifference curve of a potential seller of type q. Solving (8) for p and setting the seller's utility equal to V, we have

$$p = V + \frac{V(r + s_q) - v + c}{g(k)}. \tag{9}$$

Moreover, differentiating (9) with respect to k gives

$$p'(k) = -\frac{V(r + s_q) - v + c}{[g(k)]^2} g'(k),$$

which implies that the single-crossing condition is again satisfied. Thus, to overcome the lemons problem, it does not matter whether goods differ in their flow utility or their constant risk of breakdown.

5.2 Submarkets Can be Opened by Either Buyers or Sellers

If submarkets can be opened by sellers, the decision of buyers to enter such submarkets depends on their beliefs regarding the sellers' types. Denote these beliefs by $\mu(q \mid p)$. For all prices $p \in \{p^n \mid n \in N\}$, consistency requires that $\mu(q \mid p) = \pi^n(q)$. For all prices not observed in equilibrium, there are no restrictions on beliefs.[8] Given some price p and beliefs $\mu(q \mid p)$, the tightness k can then be derived from (2) as follows.

$$f(k) = \begin{cases} \infty & \text{if } \sum_{q \in Q} \mu(q \mid p) U_q - U^* - p \leq 0 \\ \frac{rU^* + c}{\sum_{q \in Q} \mu(q|p) U_q - U^* - p} & \text{otherwise.} \end{cases} \tag{10}$$

We now extend the set of equilibrium conditions by adding the requirement that in equilibrium, it must not be profitable for potential sellers to open new submarkets.

(E.3') Competitiveness. In equilibrium, it must not be profitable to open new submarkets in addition to the N existing submarkets. More precisely, given the equilibrium utilities U^* and $\{V_q^*\}_{q \in Q}$, there must exist (out-of-equilibrium) beliefs $\mu(q \mid p)$ such that for all prices $p \notin \{p^n \mid n \in N\}$ and types $q \in Q$ it holds that $V_q^* \geq V_q^M(k, p)$, where k is implicitly defined by (10).

Given our freedom to specify out-of-equilibrium beliefs, it should be obvious that adding (E.3') has no effect on the existing set of equilibria.[9]

Proposition 4: *The set of equilibria satisfying* (E.1)-(E.3) *also satifies* (E.3').

6 Conclusion

In this paper, we consider a search market where sellers have private information about a good's quality. We show that if sellers of high-quality goods derive a greater utility from search than sellers of low-quality goods, separation of types may occur even if an explicit sorting variable (e.g. warranties or advertising) does not exist. For instance, in the market for second-hand cars, owners of reliable cars need to spend less time and money on repairs and spare parts than owners of unreliable cars. Consequently, owners of reliable cars are less eager to sell fast and can thus ask for a higher price, even if this means that they have to wait longer (in expected terms) until the car is sold. Based

[8] This is analogous to signalling games, where out-of-equilibrium beliefs can be specified freely within the bounds imposed by the respective refinement concept.

[9] For instance, the existing set of equilibria can be supported by pessimistic beliefs $\mu(l \mid p) = 1$ if $p \notin \{p^n \mid n \in N\}$. Pessimistic beliefs are not necessary to support the existing set of equilibria though. For instance, if we employ a refinement concept along the lines of Cho and Kreps' (1987) "intuitive criterion", the set of equilibria remains still unchanged. This follows from the fact that all equilibria in Section 4 are least-cost separating from the perspective of potential sellers.

on these considerations, we derive an equilibrium for a competitive search market where goods of different qualities are traded at different prices. In equilibrium, high-quality goods sell at a higher price, but also circulate longer to ensure that low-quality sellers have no incentive to enter the (sub-)market for high-quality goods. Moreover, it is shown that all members of the short side of the market (i.e. sellers if the number of sellers in the economy exceeds the number of buyers, and vice versa) engage in trade. This is in contrast to the standard analysis, where part of the short side of the market is sometimes not served despite the presence of potential gains from trade. Finally, it is shown that our results continue to hold if the utility from search is the same for both goods, but high-quality goods have a lower risk of breaking down before they are sold.

As in models of signalling and screening, the equilibrium allocation is only second-best efficient. More precisely, high-quality goods must circulate inefficiently long to ensure that low-quality sellers have no incentive to mimic sellers of high-quality goods. Incidentally, the low-quality sellers' incentive compatibility constraint (which causes the inefficiency) can be relaxed by subsidizing low-priced goods. If the fraction of low-quality sellers in the economy is sufficiently low, the subsidy can be chosen to make both low- and high-quality sellers strictly better off, even if high-quality sellers have to pay for it. In a competitive market, however, such a cross-subsidization is not possible as it gives rise to cream-skimming by buyers which try to attract high-quality sellers.

7 Appendix

Proof of Lemma 2: Suppose in equilibrium, a pooling submarket with characteristics (p, k, π) exists. Consider now a new ("deviating") offer $p' > p$. We claim that for all pairs $(k', \pi') \in F(p', \{V_q^*\}_{q \in Q})$, the distribution is $\pi'(h) = 1$, i.e. the new submarket attracts only high-quality sellers. Suppose not. If $\pi'(l) > 0$, the tightness k' in the new submarket must adjust to make low-quality sellers indifferent between the pooling submarket and the new submarket. Hence $V_l^M(k', p') = V_l^M(k, p)$, where $k' < k$ (recall that k represents the tightness from the buyers' persepctive). However, by (5),

$$V_h^M(k', p') - V_h^M(k, p) > V_l^M(k', p') - V_l^M(k, p),$$

which implies that $V_h^M(k', p') > V_h^M(k, p)$, contradicting the assumption that $(k', \pi') \in F(p', \{V_q^*\}_{q \in Q})$. Next, we claim that if p' is sufficiently close to p, the deviation is strictly profitable. Consider a deviating offer $p' = p + \varepsilon$, where $\varepsilon > 0$. To make high-quality sellers indifferent between the pooling submarket and the new submarket, the tightness $k'(\varepsilon)$ must satisfy $V_h^M(k'(\varepsilon), p + \varepsilon) = V_h^M(k, p)$. Obviously, if $\varepsilon = 0$, we have $U^M(\pi', k, p) > U^M(\pi, k, p)$. From continuity of k' in ε and $U^M(\pi', k', p')$ in both k' and p', it then follows that for sufficiently small values of ε, $U^M(\pi', k'(\varepsilon), p + \varepsilon) > U^M(\pi, k, p)$, i.e. the deviation is strictly profitable. ■

Proof of Proposition 1: We first show that any equilibrium set of submarkets must satisfy i) and ii). By Lemma 1 and (E.2), it must be true that $V_l^* = V_l$, $\sum_{n \in N} s_h^n = 0$, and $\sum_{n \in N} s_l^n < s_l$. The fact that $k^n = k_l^S(p^n, V_l)$ for all $n \in N$ follows from the definition of $k_q^S(p, V)$ and the fact that low-quality sellers are indifferent between choosing their outside option and entering any particular submarket. Moreover, by the construction of the program $P_q(V)$, all prices p^n must solve $P_l(V_l)$, or else potential buyers can make an offer $p \neq p^n$ that makes them strictly better off. Finally, for sufficiently small values of c, there exists a pair (k, p) such that $g(k)(p - V_l) \geq c$ and $f(k)(U_l - p) > 0$. From the properties of the value function $\bar{U}_q(\cdot)$, it then follows that $\bar{U}_l(V_l) > 0$, i.e. all potential buyers strictly prefer to enter. In conjunction with (E.2), this implies that $\sum_{n \in N} b^n = b = \sum_{n \in N} s_l^n$.

Next, we show that an equilibrium with the above properties exists. Existence of a solution to $P_l(V_l)$ follows from the continuity of the objective function and the fact that prices lie in a compact interval. Furthermore, by the construction of the program, (E.1) is satisfied for potential buyers and low-quality sellers. Since $v_h > v_l$, it must be true that $V_h^M(k, p) \leq V_h$ for all p solving $P_l(V_l)$ with corresponding tightness $k = k_l^S(p, V_l)$. Consequently, (E.1) is also satisfied for high-quality sellers. The fact that (E.2) holds is obvious. Finally, by (3), there cannot exist a profitable deviation attracting high-quality sellers, which proves that (E.3) is also satisfied. ■

Proof of Proposition 2: We first prove an auxiliary claim.

Claim 1: *There exists a critical value $\bar{c}$ such that for all $c < \bar{c}$, the following holds:*

I) $\bar{U}_l(V_l) > 0$.

II) The program $P^C(\hat{V}, V_h)$, where $\hat{V}$ is defined by $\overline{U}_l(\hat{V}) = 0$, has a solution $\bar{U}^C(\hat{V}, V_h) > 0$.

Proof: Part I) was already shown in the proof of Proposition 1. In particular, this implies that there exists a pair (Δ, c_1) such that for all $c < c_1$, it holds that $\hat{V} - V_l > \Delta > 0$, where $\hat{V}$ is uniquely defined from the continuity and strict monotonicity of $\bar{U}_l(\cdot)$ and the fact that $\bar{U}_l(V_l) > 0$ and $\overline{U}_l(V) < 0$ for sufficiently large V. Inserting $g(k) = c/(p - V_h)$ in (1), we can write the low-quality sellers' incentive compatibility constraint as

$$\hat{V} \geq \frac{v_l (p - V_h) + cV_h}{r(p - V_h) + c}. \tag{11}$$

Moreover, the utility of a potential buyer from purchasing the low-quality good is strictly positive if

$$f(k)(U_h - p) > c. \tag{12}$$

Clearly, if $c < c_1$, there exists a value c_2 such that (11)-(12) are satisfied for all $c < c_2$ and prices $V_h < p < U_h$, which proves part II) of the claim. Defining $\bar{c} = \min[c_1, c_2]$ completes the proof. ■

In what follows, we assume that $c < \bar{c}$, implying that parts I) and II) of Claim 1 hold. We first show that any equilibrium set of submarkets must satisfy parts i) and ii) of the proposition. With regard to part i), suppose that only low-quality goods are traded in equilibrium. In this case, (E.2) implies that $U^* = 0$, i.e. potential buyers receive exactly their reservation utility. But this implies that there exists a profitable deviation which attracts only high-quality sellers and makes potential buyers strictly better off. To see this, suppose that (p, k) solves $P^C(\hat{V}, V_h)$ such that $U^M(\pi, k_h^S(p, V_h), p) = \bar{U}^C(\hat{V}, V_h) > 0$, where $\pi(h) = 1$. By part II) of Claim 1, such a pair (p, k) exists. Since p must be strictly less than

V_h, potential buyers can then offer a new ("deviating") price $p' > p$ which attracts only high-quality sellers by (5). The tightness k' associated with p' is unique and satisfies $k' = k_h^S(p', V_h)$. Clearly, if p' is chosen sufficiently close to p, the deviation is strictly profitable since $U^M(\pi, k_h^S(p, V_h), p)$ is strictly positive and continuous in p, contradicting the assumption that only low-quality goods are traded in equilibrium. But if high-quality goods are traded in equilibrium, Lemma 1 implies that all low-quality goods must also be traded, which implies that $\sum_{n\in N} s_l^n = s_l$. Additionally, from part I) of Claim 1 it follows that all potential buyers strictly prefer to enter. In conjunction with (E.2) and the fact that $b > s_l$, this then implies that $\sum_{n\in N} b^n = b$ and $\sum_{n\in N} s_h^n = b - s_l < s_h$.

Regarding part ii), we know from Lemma 2 that the equilibrium set of submarkets must be separating. Moreover, by Lemma 1, we have $V_l^* > V_l$ and $V_h^* = V_h$. It remains to be shown that the prices for low- and high-quality submarkets solve the respective programs. With regard to low-quality submarkets, this is obvious (see the proof of Proposition 1). With regard to high-quality submarkets, suppose p^n does not solve $P^C(V_l^*, V_h)$. By the construction of $P^C(V_l^*, V_h)$, potential buyers can then realize a strictly higher utility by offering a price $p \neq p^n$ which solves $P^C(V_l^*, V_h)$.[10] Finally, equation (6) and the fact that $k^n = k_q^S(p^n, V_q^*)$ follow immediately from the construction of $P^C(\cdot)$.

To prove existence, we proceed in two steps. First, we show that there exists a unique value V_l^* such that (6) is satisfied and the respective programs have a solution. Recall from the proof of Claim 1 that $\bar{U}_l(V_l) > 0$ and $\bar{U}_l(\hat{V}) = 0$, where $\hat{V} > V_l$, and where $\bar{U}_l(V)$ is continuous and strictly decreasing in V. If $P^C(V, V_h)$ has a solution (in some neighborhood of V), $\bar{U}^C(V, V_h)$ must be continuous (by the maximum theorem) and nondecreasing in V. Moreover, by part II) of Claim 1, it must be true that $\bar{U}^C(\hat{V}, V_h) > 0$. To see that $P^C(V, V_h)$ has a solution for all $V > V_l$, note that for any given difference $V - V_l > 0$, the set of feasible prices is non-empty. This is due to the fact that by choosing p sufficiently high, the sellers' transition rate

$$g(k_h^S(p, V_h)) = \frac{c}{p - V_h}$$

can be rendered sufficiently small such that

$$\hat{V} \geq \frac{v_l - c + g(k)p}{r + g(k)} = \frac{v_l(p - V_h) + cV_h}{r(p - V_h) + c}$$

holds, where $k = k_h^S(p, V_h)$. Since the set of prices ensuring incentive compatibility is compact, existence of a solution to $P^C(V, V_h)$ follows from the continuity of the objective function. Next, note that the maximum value $\bar{U}^C(V, V_h)$ becomes negative as $V - V_l \to 0$, and that $p > V_h$ since $c > 0$. Given our results on existence, continuity, and (strict) monotonicity of both $\overline{U}_l(\cdot)$ and $\overline{U}^C(\cdot)$, it is now immediate that there exists a unique value V_l^* solving (6) such that the respective programs have a solution.

Second, we show that any set of submarkets satisfying i)-ii) must also satisfy (E.1)-(E.3). For potential buyers and low-quality sellers, (E.1) is satisfied by the construction of $P_l(\cdot)$ and $P^C(\cdot)$. Moreover, by the single-crossing condition (5), (E.1) is satisfied for high-quality sellers. The fact that (E.2) holds

[10] Strictly speaking, this is only necessarily true if the low-quality sellers' incentive compatibility constraint is not binding at p. If it is binding, potential buyers can still profitably deviate by offering $p' = p + \varepsilon$, where ε is small. By the single-crossing property (5), the set $F(p', \{V_q^*\}_{q\in Q})$ has a unique element $(k', \pi'(h) = 1)$, which implies that only high-quality sellers are attracted by p'. From the continuity of the buyers' utility function, it then follows that such a deviation is strictly profitable.

is again obvious. With regard to (E.3), we can restrict ourselves to pooling deviations. However, any price p such that $\pi(h) > 0$ and $(k, \pi) \in F(p, \{V_q^*\}_{q \in Q})$, where $\pi(h) > 0$, must also satisfy the program $P^C(V_l^*, V_h)$, which implies that a pooling deviation cannot be strictly profitable. ■

Proof of Proposition 3: As in the proof of Proposition 2, we only consider sufficiently low values of c such that $\bar{U}_l(V_l) > 0$ and $\bar{U}^C(\hat{V}, V_h) > 0$. We first show that any equilibrium set of submarkets must satisfy i) and ii). By Lemma 2, the set of submarkets must be separating. Furthermore, Lemma 1 implies that $U^* = 0$, i.e. potential buyers receive exactly their reservation utility. Next, consider the equilibrium utility V_l^* of low-quality sellers. Clearly, in any separating set of submarkets, V_l^* cannot exceed $\hat{V}$. On the other hand, if $V_l^* < \hat{V}$, it is strictly profitable for potential buyers to deviate. To see this, suppose that $V_l^* < \hat{V}$. Since $P_l(V)$ is strictly monotonic in V and $\bar{U}_l(\hat{V}) = 0$, it follows that $\bar{U}_l(V_l^*) > 0$, i.e. opening a new submarket is strictly profitable even if only low-quality sellers show up. Hence, it must be true that $V_l^* = \hat{V} > V_l$, which implies that $\sum_{n \in N} s_l^n = s_l$. Next, consider the equilibrium utility V_h^* of high-quality sellers. If $V_h^* = V_h$, it is strictly profitable for potential buyers to deviate and open a submarket that attracts only high-quality sellers. The reasoning is exactly the same as in the proof of Proposition 2. Accordingly, it must be true that $V_h^* > V_h$, which implies that $\sum_{n \in N} s_h^n = s_h$. From (E.2), it then immediately follows that $\sum_{n \in N} b^n = s < b$. Thus, both low- and high-quality goods are traded in equilibrium. The rest is analogous to the proof of Proposition 2.

To prove existence, we first show that there exists a unique value V_h^* such that the program $P^C(\cdot)$ has a solution and (7) is satisfied (the fact that $P_l(\cdot)$ has a solution was already shown in the proof of Proposition 2). It turns out that the incentive compatibility constraint is easier to handle if we consider an alternative program. Denote by $k_h^B(p)$ the tightness that ensures that $U^M(\pi, k_h^B(p), p) = 0$ for all $p < U_h$, where $\pi(h) = 1$. From (2), it follows that $k_h^B(p)$ is uniquely defined by

$$f(k_h^B(p)) = \frac{c}{U_h - p}.$$

If $p \geq U_h$, we set $f(k_h^B(p)) = \infty$. Define now by $P_S^C(\hat{V})$ the program in which p is chosen to maximize the utility of high-quality sellers $V_h^M(k, p)$ subject to $k = k_h^B(p)$ and the low-quality sellers' incentive compatibility constraint $V_l^M(k, p) \leq \hat{V}$. Denote the maximum value of the objective function by $\widetilde{V}$. To see that $P_S^C(\hat{V})$ has a solution, note that the optimal price must be strictly bounded away from U_h. It therefore remains to show that the set of prices satisfying $V_l^M(k, p) \leq \hat{V}$ is non-empty. As in the proof of Proposition 2, this follows from the fact that the sellers' transition rate can be rendered arbitrarily small through an appropriate choice of p. We now show that the value of p that solves $P_S^C(\hat{V})$ also solves $P^C(\hat{V}, \widetilde{V})$, which proves that a solution to $P^C(\cdot)$ exists. To see this, suppose p solves $P_S^C(\hat{V})$, but not $P^C(\hat{V}, \widetilde{V})$. By the construction of $P^C(\cdot)$, there then exists a price p' such that $V_h^M(k', p') = \widetilde{V}$, where $k' = k_h^S(p', \widetilde{V})$, $V_l^M(k', p') \leq \hat{V}$, and $U^M(\pi, k', p') > 0$, where $\pi(h) = 1$. But this implies that we can find a pair (p'', k''), where $p'' > p'$ and $k'' = k_h^S(p'')$, such that $U^M(\pi, k'', p'') = 0$ at $\pi(h) = 1$ (this follows from the definition of $k_h^S(\cdot)$), $V_l^M(k'', p'') < \hat{V}$, and $V_h^M(k'', p'') > \widetilde{V}$ (this follows from continuity and the single-crossing condition (5)), contradicting the assumption that p solves $P_S^C(\hat{V})$. Given that p solves $P^C(\hat{V}, \widetilde{V})$, the corresponding tightness is $k = k_h^S(p, \widetilde{V})$, which, in conjunction with the fact that $\widetilde{V} = V_h^*$, implies that $\bar{U}^C(\hat{V}, V_h^*) = 0$. Uniqueness of V_h^* follows then immediately from the fact that $\bar{U}^C(\hat{V}, V)$ is strictly decreasing in V.[11]

The argument that any set of submarkets satisfying i)-ii) must also satisfy (E.1)-(E.3) is exactly the same as in the proof of Proposition 2. ■

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[11] The same argument that was used to show that $\bar{U}_q(\cdot)$ is strictly decreasing can be used to show that $\bar{U}^C(\cdot)$ must be strictly decreasing (cf. footnote 7).