Bidding for Unit-Price Contracts - How Craftsmen Should Bid

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Abstract

We analyse the bidding for unit-price contracts, a very common procurement auction. With a unit price contract, not the provision of the good but the employment of several kinds of inputs is priced. The seller charges a unit price for the employed quantity of each input. To select one seller, a linear selection rule is used to rank submitted lists of unit prices.

In this paper, we model heterogeneous technologies of craftsmen: firms differ in their requirement of input-quantities.

An equilibrium of this model is found. The composition of submitted lists does not mirror the cost structure and the selection probability is not monotone in the type. Sometimes the "lamer" of two craftsmen is selected, enhancing all but the very lame types to bid very aggressively. Caused by this, unit-price bidding can be cheaper (require a lower expected payment) than standard auctions like the first price auction.

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1 Introduction

This paper analyses a common way public agencies select and pay the constructor of public buildings, highways and other infrastructure projects. This procurement auction is called the **Unit-Price-Contract Auction** (UPC-auction). UPC-auctions are also often used by firms or private persons to select and pay a craftsmen who offers some service. The volume of jobs allocated via UPC-auctions amounts to many billion DM per year, since only in public procurement of construction projects in Germany 100 billion DM per year are paid[1], most of it using UPC-auctions.

A unit-price contract (UPC) between a buyer and a seller prescribes the payment to the seller. The buyer does not pay an all-inclusive fee for the provision of the good as a whole. Instead, he compensates the employment of several kinds of inputs, works or services. For each input, the seller charges a unit-price for the employed quantity of this input. For example, consider the excavation of material out of a mountain to build a tunnel. The material consists of rock and soil, altogether 1000 m^3. The UPC specifies a unit-price p_1 for each unit of the work "excavation of rock" (measured in m^3), and another unit-price p_2 for each m^3 of the work "excavation of soil". If actually 600 m^3 of rock and 400 m^3 of soil are excavated, then the seller pays $600m^3 \cdot p_1 + 400m^3 \cdot p_2$. Of course, actual quantities have to be publicly observable after completion. Hence the payment makes use of so called "ex-post observable information".

In UPC-*auctions* several sellers compete for a UPC. Every seller submits a sealed envelope, containing a list of unit prices, one unit-price for each input. Hence the bid is multidimensional. To rank the submitted lists, the buyer weights the inputs and selects the seller with the lowest weighted sum of unit-prices, called the "sum" of the bid. Weights - called assessments - are already announced in the invitation for tenders. They sometimes reflect the auctioneer's assessment of actual quantities, but might be chosen arbitrarily.

We focus on the provision of craftsmen's services and address the following questions:

1.) How do experienced bidders bid and why do bidders fail who are not used to UPC-auctions? Companies facing a UPC-auction for the first time are often unsuccessful, as the example of American firms bidding for

[1] In 1995, 110.5 billion DM were spent on public construction projects in Germany. Source: Institut der Deutschen Wirtschaft, Zahlen zur Wirtschaftlichen Entwicklung der Bundesrepublik Deutschland 1996, Table 71.

the construction of the airport in Hong Kong shows: To explain their lack of success, a Hong Kong competitor explains

"American companies just don't know how to operate in this kind of environment in Asia. In a project like this, you have to bid low on certain projects, take a loss, and make it up on pieces elsewhere."[2]

This paper shows that rational sellers indeed follow this pattern. They charge cost-unsufficient unit-prices for some inputs and make it up on other services with very high unit prices.

2.) Is the UPC-auction efficient? We show that sometimes a seller wins who bears higher total costs than his competitors.

3.) Are UPC-auctions "cheaper" than other allocation mechanisms? Do we expect the payment in a UPC-auction to be lower than the payment in other auctions like the english auction or the first price sealed bid auction?[3] We show that the UPC-auction can be cheaper than many other auctions.

4.) Does a buyer earmarking the sum of the winning bid in his budget underestimate the payment? He does!

We also raise the question of bribery, a relevant problem in procurement. If the execution of the auction is delegated to an agent, how easy is it to prevent preferential treatment of a bidder who bribes the agent?

The paper is organised as follows: Chapter 2 motivates the assumptions and refers to the literature. In chapter 3 the model is presented. Chapter 4 shows a property of rational bidding in this model, the one-sidedness. Chapter 5 deals with equilibrium analysis. Properties of this equilibrium are analysed in chapter 6. Chapter 7 concludes the paper. Proofs are relegated to the appendix.

2 Modelling Approach and Literature

A sketch of the model presented in chapter 3 is as follows: Risk-neutral craftsmen compete in a UPC-auction. The quantity of one of the two inputs is deterministic. The actual quantity a craftsman employs of the other input is his type. It is private information and distributed independently with the same distribution for every craftsman. Craftsman's costs are a linear function of actual quantities. Every craftsman submits a list of two unit-prices, one

[2] Quoted from the International Herald Tribune, Monday, October 30, 1995

[3] This question is motivated by the interest of the auctioneer, for example a government asking for the construction of a highway or a private person requiring craftsmen's services.

for each input. The craftsman with the lowest weighted sum of unit-prices is selected. After completion of the job the actual quantities of the selected bidder are observed. According to the UPC, this bidder is compensated for the actual quantities with his submitted unit prices.

The Mechanism The model differs from most papers in the auction literature in two aspects[4]: First, some information about the utility of the bidders is publicly observable ex-post, after the completion of the job. Second, bids are multidimensional.

Actual costs are hard to verify. If actual costs were verifyable, the buyer could extract all the rent. However, actual quantities of the inputs employed are often publicly observed after completion and used to determine the payment. Publicly **ex-post observable information** enables auctions where the payment is contingent not only on the bids but also on this information. This can raise the expected revenue of the auctioneer: Hansen (1985) showed this for three special auctions in a SIPV-framework. Riley (1988) found that making the payment contingent on the ex-post observable oil-production (for example) instead of agreeing a fixed fee can raise auctioneer's revenue in an offshore oil auction where types are correlated estimations of the oil-production. These papers on ex-post observable information only consider auctions with one-dimensional bids.

The present paper considers an auction with **two-dimensional bids**. With multidimensional bids, in equilibrium typically every fixed winning probability can be attained by a continuum of bids. Che (1993) analyses a procurement auction with multidimensional bids (*without* ex-post observable information). The bid is two-dimensional, consisting of the price and the quality of the good. In his model, the *derivatives* of both the iso-winner's-payoff curve and the iso-winning-probability curve in the price-quality space only depend on quality, not on price. Hence for a given type the maximisation problem: "maximise the winner's payoff under the constraint of attaining a given winning probability" always yields the same optimal quality (characterised by the tangential point of both curves), regardless of the winning probability. This reduces the strategic decision of the bidder to a one-dimensional problem. In the model analysed in the present paper, a similar reduction occurs.

The bidding behaviour in the UPC-auction with multidimensional bids

[4]For a survey on auction literature refer to McAfee, McMillan (1987).

making use of ex-post observable information differs from one-dimensional bidding with ex-post observable information as well as from other multidimensional auctions. If the payment can be made contingent on ex-post observable information, bidder's incentive constraints in the auctioneer's mechanism design problem not necessarily result in monotone winning probabilities. In the above mentioned auctions with one-dimensional bids using ex-post observable information, monotonicity of the winning probability w.r.t. the type still holds. However, in the UPC-auction with multidimensional bids, the type space splits up in two parts with different kinds of payment discrimination. As a result, the function mapping the types to the equilibrium-probabilities of award is not monotone anymore.

Two other papers consider the UPC-auction. Stark 1974 conducts a decision theoretic analysis of the UPC-auction, not considering strategic interaction. Samuelson 1986 is the model nearest to ours. He describes a more general class of mechanisms under risk aversion and the same informational assumptions. He allows for a more general scoring function, including the UPC-auction (with a linear weighting rule) as a special case. He finds equilibria for some very special scoring functions, all of them involving monotone winning probabilities. However, Samuelson 1986 does not analyse the UPC-auction.

Informational Assumptions To keep the problem simple and to abstract from effects due to multidimensional *types*, I assume that the quantity of one input is common knowledge and deterministic[5]. Only the other input is subject to uncertainty. The outcome of the auction depends on the nature of the quantities of this input.

The UPC-auction is commonly used in many different situations. Sometimes the actual quantities do not depend on the characteristics of the seller or the buyer. For example the amount of rock burried in a tunnel only depends on God's choice when creating the mountain. If sellers have some private "estimation" about the true quantity of rock, modelling it as common value distributed is a realistic assumption[6].

[5]However, making use of techniques used in this paper the model can be extended to two stochastic inputs.

[6]In a common value distribution, the realised quantity of an input is unknown before completion but it is the same ex post for all firms. Hence it is a "true value". Every firm receives an independent private signal which is correlated with the true value.

Despite the suitability of common value models for many situations, these models are difficult to analyse. I confine myself to a simpler model instead. In the present paper I assume that craftsmen are competing for a repair job. The orderer arbitrarily asks a small number of craftsmen out of the large set of all craftsmen in his town to submit unit prices for material and hours for a specified project. Craftsmen have different technologies or skills and therefore differ in their requirement of one input, called "hours": The type of a craftsman is the number of working-hours or machine hours he requires to complete a specific order. Ex-ante, all craftsmen have the same distribution of their type. The realisation of a craftsman's type is independent from his competitors and it is private information to him. Hence the quantities are **symmetric independent private values** (SIPV).

To simplify notation, this paper focusses on the symmetric two-bidder case. Since the proofs easily extend to a general number of bidders, the equilibrium of the symmetric n-bidder case is also stated.

Moreover, I abstract from collusion, which is a main feature of the construction industry. The industry conception is that collusion is common in boom times, but it doesn't work in recession, i.e. when demand is low. With respect to collusion in the presence of a cartel-enforcement problem, the UPC-auction has the same properties as the standard first price auction. Without transfers, maintaining collusion faces the same enforcement problem: loosers have an incentive to overbid the designated winner. Hence the UPC-auction is less viable to collusion than a second price auction.[7]

Costs per unit of one input are assumed to be the same across all bidders. This can be justified by the fact that many inputs are bought from the same suppliers at announced prices (at least for "small" bidders) and the market for labour is gouverned by industry-wide agreements.

When facing a mechanism where the payment depends on the realised inputs, especially labour, one would expect a craftsman to be tempted to manipulate the quantities of these inputs. I do not model **moral hazard** of this kind. In the model, the realised input-quantities are not choice variables of the firm[8]. This is to keep the model simple, but it is not innocuous. In a common value-framework, abstracting from moral hazard seems to be

[7] For a comparison of first and second price auction see Robinson (1985).

[8] Moral hazard is analysed in the literature on the auctioning of incentive contracts, see for example McAfee, McMillan (1986). However, their model abstracts from selection problems by assuming ex-post observability of total costs. In our paper, the auctioneer does not observe the total costs ex-post, only a cost parameter is observable.

justifiable. There realised inputs (like volume of rock excavated) are assumed to be completely determined by nature, and therefore not influenced by the firm.

3 The Model

3.1 The Framework

Each of two risk-neutral craftsmen requires 1 washing barrel (a normalisation) and h hours to repair a damaged washing machine. The type h is private information and drawn from a continuously differentiable distribution $F(h)$ with density $f(h)$, and with support $\left[\underline{h}, \overline{h}\right] \subseteq [0, \infty)$. $F(h)$ is independent of the opponent's type. The type h (i.e. the number of hours required) of the craftsman who carries out the repair is observed during the repair. I will use bold letters to denote vectors, starting with the vector $\mathbf{h} = \begin{pmatrix} 1, & h \end{pmatrix}$.

Input unit costs are the same for both craftsmen. A washing barrel costs a craftsman $c_W \geq 0$ and an hour costs him $c_H \geq 0$. Let $\mathbf{c} = \begin{pmatrix} c_W, & c_H \end{pmatrix}$. The total cost of carrying out the job for type h is $C(\mathbf{h}) = \mathbf{c} \cdot \mathbf{h}$, which is strictly monotone increasing in the type.[9]

3.2 The Auction Rules

The craftsman is selected and the order is paid using a "Unit-Price Contract Auction" (UPC-auction). First, each craftsman learns his type. Then each of the two craftsmen submits a bid, which is a vector or list of unit prices $\mathbf{p} = \begin{pmatrix} p_W, & p_H \end{pmatrix}$, consisting of an unit price $p_W \geq 0$ for every washing barrel and an unit price $p_H \geq 0$ for every hour. The UPC-auction uses exogeneous weights $\mathbf{s} = \begin{pmatrix} 1, & s \end{pmatrix}$ with $s \in \left[\underline{h}, \overline{h}\right]$ to evaluate bids by the formula $S(\mathbf{p}) = p_W + p_H s = \mathbf{p} \cdot \mathbf{s}$. I call s the "assessment" of hours and $S(\mathbf{p})$ the "sum" of bid $\mathbf{p}$. The parameter s is common knowledge and characterises the auction. The job is placed to the craftsman whose bid has the lowest sum $S(\mathbf{p})$. Ties are resolved at random. After completion of the job, the realised quantities of inputs are observed and the craftsman i to whom the job was placed is paid $\mathbf{p}_i \cdot \mathbf{h}_i$.

Hence a type h_i bidding $\mathbf{p}_i$ gets a payoff of $\mathbf{p}_i \cdot \mathbf{h}_i \Leftrightarrow \mathbf{c} \cdot \mathbf{h}_i$ if he wins. Let $\pi(\mathbf{p}_i, h_i) = \mathbf{p}_i \cdot \mathbf{h}_i \Leftrightarrow \mathbf{c} \cdot \mathbf{h}_i$ and call it winner's payoff. Let

[9] Adding fixed costs would not affect the character of the results.

$$\Pi\left(\mathbf{p}_i, h_i\right) = Prob\left\{h_j : S\left(\mathbf{p}_j\left(h_j\right)\right) \geq S\left(\mathbf{p}_i\right)\right\} \cdot \pi\left(\mathbf{p}_i, h_i\right)$$

be the expected payoff of type h_i bidding $\mathbf{p}_i$ when the opponent j uses strategy $\mathbf{p}_j(h_j)$. $\Pi\left(\mathbf{p}_i, h_i\right)$ is the product of the probability of winning and the winner‘s payoff. The multiplicative separability of the winning-probability and the winner's payoff is a property of this model with independent private quantities. It vanishes in a model with correlated quantities, which makes those models much more difficult.

Define $Q\left(S\left(\mathbf{p}_i\right)\right) = Prob\left\{h : S\left(\mathbf{p}_j\left(h\right)\right) \geq S\left(\mathbf{p}_i\right)\right\}$. Hence $\Pi\left(\mathbf{p}, h\right)$ can be written as

$$\Pi\left(\mathbf{p}, h\right) = Q\left(S\left(\mathbf{p}\right)\right) \cdot \pi\left(\mathbf{p}, h\right)$$

4 One-Sided Bidding

Submitting One-Sided Bids - An Example In reality, one often observes a peculiar pattern in the composition of unit-price lists. The list of a seller often contains some unit-prices which are lower than his own average costs for this input while some other unit prices are much higher than his costs. Sometimes unit-prices in bids for construction projects even drop to zero. Sometimes firms don't charge for their work, only for the material ("service included"). The present paper shows that equilibrium strategies in the UPC-auction indeed involve this patterns.

To illustrate this, consider an example. A craftsman is bidding for the repair of a washing machine. This requires two inputs, washing barrels and hours. The orderer assesses that it will take 1 washing barrel and 4 hours to repair the washing machine. He uses these numbers to weight unit-prices and compute the sum of a bid. The bidder knows he will indeed require 1 washing barrel, but with his equipement it will take 6 hours to get it done. He has costs of 400DM per washing barrel and 80DM per hour. He might mirror his costs in the bids, submitting prices per unit of 400DM per washing barrel and 80DM per hour, resulting in a sum of $1 \cdot 400DM + 4 \cdot 80DM = 720DM$. If he wins, he is paid $1 \cdot 400DM + 6 \cdot 80DM = 880DM$. However, if he is smart, he might as well submit a bid of 0DM for washing barrels and 180DM per hour (a "material included" bid), resulting in the same sum of $720DM$ and in the same odds of winning. However, in case of winning his payment now is $1 \cdot 0DM + 6 \cdot 180DM = 1080DM$, which is much more than 880DM. Note that the payment he gets for washing barrels does not cover the cost of washing barrels. The craftsman takes a loss on washing barrels and makes it up on hours. Of course, this does not completely characterise equilibrium bidding. Additional to considerations about the *composition* of his bid, a bidder also considers opponents' strategies in the determination of the *level* of his bid, characterised by the sum.

One-Sided Bidding In this introducing example, we have seen a craftsman bidding in a very extreme way: he charges a price of zero for washing barrels. The following lemma states that bidding "material-included" or "service-included" (charging an unit-price of 0 for hours) is a general property of rational bidding.

Definition 1 *A **service-included** bid is a bid where $p_H = 0$. A **material-included** bid has $p_W = 0$. An **one-sided bid** is a bid $\mathbf{p} = \begin{pmatrix} p_W, & p_H \end{pmatrix}$ where either $p_W = 0$ or $p_H = 0$, i.e. it is either a service-included or a material-included bid. A **disproportionate** bid is a bid that is not parallel to the cost-vector. A **lame** type is a type which requires $h > s$ hours. A **quick** type requires $h < s$ hours.*

Lemma 2 *In a UPC-auction, let s be the assessment of hours. Assume that bidder j uses strategy $\mathbf{p}_j(.)$. Then every best reply of bidder $i \neq j$ involves the following:*

> *1.) Quick types (i.e. types $h < s$) make service-included bids.*
>
> *2.) Lame types (i.e. types $h > s$) make material-included bids.*

Proof. See the appendix.

Strategies involving this pattern (1.) and 2.)) are called **one-sided bidding functions**.

If a lame type charges a material-included bid, it obtains a high payment. A quick type gets a lower payment from the same bid, it will be better of charging a service-included bid with the same sum.

The intuition of the proof is as follows: The problem of choosing two unit-prices is equivalent to the choice of two other variables: the *sum* of a bid and the *composition* $\frac{p_W}{p_W + s p_H} \in [0, 1]$ of a bid. Only the sum has a strategic meaning: it determines the odds of winning. The composition only influences the ex-post payment. For a given sum, one can compute the optimal (payment-maximising) composition. It turns out that the payment is linear in the composition, hence the optimal composition of a bid for a given sum is either 0 (material is not charged) or 1 (all the payment comes from material). Which of these extremes is optimal does not depend on the sum of the bid. Working backwards, this reduces every type's strategic problem to the choice of only one variable, the sum.

Types $h = s$ are indifferent between all compositions, hence we can not say anything a priori about their bidding behaviour. The composition of best reply bids with $h \neq s$ does not depend on the opponent's strategy. Nevertheless we can not speak of dominant strategies: One of the unit-prices is always positive. The optimal *value* of this price depends on the opponent's strategy.

One-sidedness is not restricted to the presented setting. It holds true for every cost function and every number of bidders. For auctions with a higher number of inputs and a higher dimensionality of bids, one-sidedness generalises: all but one of the unit-prices equal zero. Increasing the dimensionality of uncertainty, i.e. introducing multidimensional type-spaces, doesn't affect the result, either. I could even change the information assumptions by the introduction of common or affiliated quantities. In general, the one-sidedness property holds as long as both the sum and the payment are additively separable functions of multidimensional bids and the bidders are risk-neutral.

One-sidedness simplifies the problem. However, as we will see, the auction is not reduced to a standard one-dimensional auction, because in equilibrium the winning-probability $Q\left(S\left(\mathbf{p}^*\left(h\right)\right)\right)$ is not monotone in h.

5 Equilibrium Analysis

The "Worst" Type The next step is to identify the type with the lowest probability of getting the contract in equilibrium. In both standard and UPC-auctions, there is only one type with this property. It is called worst type and denoted by h_0. In standard procurement auctions, the strategic position of a bidder strictly monotone worsens in the type (i.e. the total costs). Hence the worst type is always the highest type, i.e. the one with the highest costs. However, in the UPC-auction a bidder has a higher degree of freedom when choosing his bid. We have already shown that this causes the composition of bids to be different for lame and quick types if the payment makes use of ex-post observable cost information. Due to this asymmetric one-sidedness, both parts of the type space face different forces determining the strategic position of a type, as will be shown below. Hence there is no reason to expect the worst type to be the highest type. We now state that instead the worst type is the type $h_0 = s$, which is in the interior of the type-space.

Lemma 3 *Let* $\mathbf{p}^*(h)$ *be a symmetric equilibrium bid function in pure strategies. Then the unique worst type is* $h_0 = s$.

Proof. See the appendix.

Iso-Sum-Types Both parts of the type space differ in the forces determining the strategic position of the bidder (i.e. the winning probability of a type). In the following I give an intuition of the way these forces work, starting with quick types.

Quick types (i.e. types $h < s$) are in a better strategic position than type $h_0 = s$:

For a craftsman bidding service-included ($p_H = 0$), the payment always equals the sum of his bid. The payment to type h_0 also always equals the sum of his bid, since he is indifferent between bidding service-included and any other composition. A quick type bids service-included. If a quick type would like to achieve the same sum as h_0, his bid would yield the same payment as h_0's bid. However, he has lower costs than h_0. Hence he has an strategic advantage compared to h_0. This advantage increases, the quicker the type is, i.e. the lower h is. Hence focusing on quick types, one should expect an equilibrium to involve higher odds of winning for "quicker" quick types than for "lamer" quick types. In other words: one should expect the equilibrium winning probability to decrease strictly on $[\underline{h}, s]$. This is analogous to standard auctions.

However, contrary to standard auctions, lame types (i.e. $h > s$) are also in a better strategic position than h_0:

A lame type bids material-included. If he would like to achieve the same sum as h_0, he would yield a higher payment than h_0. He also has higher costs. However, the cost effect is of less extent than the payment effect. He can apportion the costs of the washing barrel on more hours. This advantage increases, the lamer the type is. Hence the equilibrium winning probability should increase strictly on $\left[s, \overline{h}\right]$.

This intuition will become the equilibrium hypothesis: $Q(h)$ is strictly decreasing on $[\underline{h}, s]$ and strictly increasing on $\left[s, \overline{h}\right]$. In standard auctions, bids are strictly monotone, and hence the probability of winning is strictly monotone. Therefore every type has a different probability of winning in equilibrium. In the present model, there are pairs consisting of one quick type and one lame type, both having the same odds of winning. Hence to compute the winning-probability of a type, one needs to know the set of all types that achieve the same sum in equilibrium.

The following lemma characterises pairs consisting of one quick type and one lame type whose bids have the same sum. For the moment, assume that if there is more than one type attaining a distinct sum in equilibrium, for all types attaining this sum the following holds: For quick types, the equilibrium p_W maximising the expected payoff $Q\left(S\left(\begin{array}{cc} p_W, & 0 \end{array}\right)\right)(p_W \Leftrightarrow \mathbf{c}\cdot\mathbf{h})$ satisfies the first order condition of this expected payoff with respect to p_W. For lame types, p_H satisfies the first order condition of the expected payoff $Q\left(S\left(\begin{array}{cc} 0, & p_H \end{array}\right)\right)(p_H h \Leftrightarrow \mathbf{c}\cdot\mathbf{h})$ with respect to p_H. I will later show that the equilibrium analysed in Proposition 5 satisfies this.

Lemma 4 *Let $\mathbf{p}^*(h)$ be a symmetric equilibrium bidding function of the UPC-auction, where $p_W^*(h)$ on $[\underline{h}, s]$ and $p_H^*(h)$ on $\left(s, \overline{h}\right]$ satisfy the first order conditions of the maximisation of the expected payoff w.r.t. p_W or p_H, respectively. Then for every sum S the set of all types whose bids have this sum has at most two elements. Pairs (h_W, h_H) of types with the same sum consist of a quick type h_W and a lame type h_H, characterised by:*

$$\frac{c_W + c_H h_W}{c_W + c_H h_H} = \frac{s}{h_H}$$

Proof. See the appendix.

I call the condition $\frac{c_W+c_H h_W}{c_W+c_H h_H} = \frac{s}{h_H}$ the "iso-sum condition", because types with the same sum must satisfy it. It enables us to deal with the same winning probability for a pair caused by different forces on both sides of the type space. Note that this lemma doesn't predict the specific sum of a type. The lemma also states that there can not be two quick types or two lame types achieving the same sum. Note that an equilibrium with a non-monotone winning-probability is not efficient: sometimes the lamer bidder wins.

The intuition of the proof is that for types with the same equilibrium sum, the winning probability is the same. Using this, both types' first order conditions result in the iso-sum condition.

Because there may be quick types without corresponding iso-sum lame type (and vice versa), I now define iso-sum functions taking care of this.

Iso-Sum-Functions Define the function $\varphi(h_W)$ mapping a quick type $h_W \in [\underline{h}, s]$ to the lame type having the lowest sum not lower than the sum of h_W:

$$\varphi(h_W) = \sup\left\{h \in \left[s, \overline{h}\right] : S(\mathbf{p}^*(h)) \geq S(\mathbf{p}^*(h_W))\right\}$$

If a quick type h_W is matched to a corresponding lame type $h_H \in \left(s, \overline{h}\right]$ by the iso-sum-condition (Lemma 4), the value of $\varphi(h_W)$ is this corresponding lame type, i.e. $h_H = \frac{c_W s}{c_W + c_H(h_W - s)}$. However, the iso-sum condition may as well yield a matching type not in the support of the type space. Denote by μ_W that type which is matched to the highest lame type, $h_H = \overline{h}$. For quick types $h < \mu_W$ (if they exist), $\varphi(h)$ is the highest lame type at all.

Figure 3

Solving the iso-sum condition for h_W and inserting $\overline{h}$ for h_H, I obtain:

$$\mu_W = s + \frac{c_W}{c_H}\left(\frac{s}{\overline{h}} \Leftrightarrow 1\right)$$

Now $\varphi(h)$ takes the following form:

$$\varphi(h) = \begin{cases} \overline{h} & \text{for } h \in [\underline{h}, s] \text{ and } h \leq \mu_W \\ \frac{c_W s}{c_W + c_H(h - s)} & \text{for } h \in [\underline{h}, s] \text{ and } \mu_W < h \end{cases}$$

One can obtain a similar function

$$\psi(h_H) = \sup\{h \epsilon [\underline{h}, s] : S(\mathbf{p}^*(h)) \geq S(\mathbf{p}^*(h_H))\}$$

mapping a lame type on a corresponding quick type. For pairs (h_W, h_H) out of $[\underline{h}, s] \times \left[s, \overline{h}\right]$ who are matched by the iso-sum condition, $\psi(h_H)$ is the inverse function of $\varphi(h_W)$.

Denote by μ_H the lame type which achieves the same sum as the lowest quick type $\underline{h}$. Solving the condition for h_H and inserting $\underline{h}$ for h_W, I obtain

$$\mu_H = \frac{c_W s}{c_W + c_H(\underline{h} \Leftrightarrow s)}$$

Now $\psi(h)$ takes the following form.

$$\psi(h) = \begin{cases} s + \frac{c_W}{c_H}\left(\frac{s}{h} \Leftrightarrow 1\right) & \text{for } h \in \left[s, \overline{h}\right] \text{ and } h \leq \mu_H \\ \underline{h} & \text{for } h \in \left[s, \overline{h}\right] \text{ and } \mu_H < h \end{cases}$$

$\varphi(h)$ and $\psi(h)$ are continuous and monotone decreasing functions. $\varphi(h)$ is differentiable on its range apart from the value μ_W. $\psi(h)$ is differentiable on its range apart from the value μ_H. Their derivatives are not continuous at μ_W and μ_H, respectively.

Existence of a Symmetric Equilibrium

Proposition 5 (Existence) *For differentiable distributions $F(.)$, the following is a symmetric equilibrium of the UPC-auction with two bidders*

$$p_W^*(h) = \begin{cases} \int_h^s \mathbf{c}\cdot\mathbf{g}\frac{f(g)-f(\varphi(g))\varphi'(g)}{F(\varphi(h))-F(h)}dg & \text{for } h \in [\underline{h}, s) \\ \mathbf{c}\cdot\mathbf{s} & \text{for } h = s \\ 0 & \text{for } h \in \left(s, \overline{h}\right] \end{cases}$$

$$p_H^*(h) = \begin{cases} 0 & \text{for } h \in [\underline{h}, s] \\ \int_s^h \frac{\mathbf{c}\cdot\mathbf{g}}{g}\frac{f(g)-f(\psi(g))\psi'(g)}{F(h)-F(\psi(h))}dg & \text{for } h \in \left(s, \overline{h}\right] \end{cases}.$$

Proof. See the appendix.

The worst type in this Bayes Nash equilibrium is $h_0 = s$. For quick types, i.e. on $[\underline{h}, s)$, the relevant unit price for washing barrels $p_W^*(h)$ is a continuous function and it is strictly montone increasing, just like the bid in a standard auction. For lame types, i.e. on $\left(s, \overline{h}\right]$, the relevant unit price for hours $p_H^*(h)$ is continuous and strictly monotone decreasing. Hence also the sum $S(\mathbf{p}^*(h))$ is a piecewise monotone function of the type h. On $[\underline{h}, s]$, $S(\mathbf{p}^*(h))$ strictly increases, and on $\left[s, \overline{h}\right]$ it strictly decreases. Furthermore, the sum is continuous on $\left[\underline{h}, \overline{h}\right]$ and it is differentiable on $\left\{\left[\underline{h}, \overline{h}\right] \setminus \{s, \mu_W, \mu_H\}\right\}$. Figure 2 sketches the unimodal shape of the sum as a function of the type.

Figure 1

The structure of this equilibrum bidding function resembles but is not in complete analogy to the structure of the standard first price auction. In our setting, the equilibrium bid of a first price standard auction is

$$p^*(h) = \int_h^{\overline{h}} \mathbf{c}\cdot\mathbf{g}\frac{f(g)}{1 \Leftrightarrow F(h)}dg$$

As in the standard first price auction, the bid of type h in the UPC-auction is an expected value where the expectation is taken over all types worse than h, worse in the sense of having a lower winning probability. However, in the UPC-auction both the interval of worse types and the terms integrated over differ from the standard first price auction.

In the following, I analyse bids and resulting payments of quick types and lame types, and I compare it to the standard first price auction.

First, consider a quick type $h \in [\underline{h}, s)$. He bids service-included, hence $p_W^*(h)$ is positive. $p_W^*(h)$ can be decomposed and transformed to

$$p_W^*(h) = \int_h^s \mathbf{c} \cdot \mathbf{g} \frac{f(g)}{F(\varphi(h)) \Leftrightarrow F(h)} dg + \int_s^{\varphi(h)} \mathbf{c} \cdot \mathbf{g} \frac{s}{g} \frac{f(g)}{[F(\varphi(h)) \Leftrightarrow F(h)]} dg$$

The integration is on types $[h, \varphi(h)]$. Hence it is on all types having a lower winning probability than type h. When integrating over all quick types being worse (i.e. "lamer") than the considered quick type, the total costs of these quick types are the integrand, as in a standard first price auction. However, the integrand for the worse lame types is a little bit different. Their costs are corrected by a term $s/g < 1$, and this is less than their costs. Therefore the equilibrium bid of a quick type is *less* than the expected value of the costs of all worse types. This is in contrast to the standard first price auction, where the bid is the expected value of the total costs of all worse types. In addition, the worse types have even lower costs than in the standard first price auction, because the most lame types are excluded. Since quick types bid service-included and one washing barrel is required, the payment is the price for washing barrels. Hence both effects result in a lower payment to winning quick types than in the standard first price auction.

This can be explained by two effects: First, the immediate neighbours of the worst type have to bid very aggressively (compared to their bidding in the standard first price auction) to have a positive probability of winning.[10] This is taken into account by all other types to the left, resulting in more aggressive biddding by everyone. The second effect is the fact that bidders face higher competition: In standard auctions (with two bidders) a bidder only looses against opponents of quicker type. However, in the UPC-auction, a bidder of quick type in addition also looses against very lame types of the opponent.

[10]The same effect is also present in a standard first price auction if a reserve price is introduced, moving the worst participating type to the interior of the type space, resulting in more aggressive bidding by his immediate neighbours.

Hence he faces more competition, resulting in more aggressive bidding. Both effects go in the same direction, hence quick types bid considerably more aggressive in the UPC-auction.

Now consider a lame type $h \in \left(s, \overline{h}\right]$. His payment is $hp_H^*(h)$. It can be decomposed and transformed to:

$$hp_H^*(h) = \int_{\psi(h)}^{s} \mathbf{c} \cdot \mathbf{g} \frac{h}{s} \frac{f(g)}{F(h) \Leftrightarrow F(\psi(h))} dg + \int_{s}^{h} \mathbf{c} \cdot \mathbf{g} \frac{h}{g} \frac{f(g)}{F(h) \Leftrightarrow F(\psi(h))} dg$$

The intervals of worse types only contain types quicker than h (i.e. lower cost types), as opposed to the standard first price auction, where the integration is only on types lamer than h. However, this adverse integration over cost terms does not necessarily induce lower payment, because total-costs are corrected by terms $\frac{h}{s} > 1$ and $\frac{h}{g} > 1$, respectively. For very lame types h, types near to $\overline{h}$, the correction terms are relatively high, resulting in a higher payment than in the standard first price auction. This can be seen using the Mean-Value Theorem:

In the UPC-auction, type s is paid its costs. This is **lower** than in the first price auction, where type s makes positive profit. Consider type $h = \overline{h}$. Using the formula from Proposition 5, integration is over terms $\mathbf{c} \cdot \mathbf{g} \frac{\overline{h}}{g}$. This term is decreasing in g and has a value of $\mathbf{c} \cdot \overline{\mathbf{h}}$ at its end-point $\overline{h}$. Since $\frac{f(g)-f(\psi(g))\psi'(g)}{F(h)-F(\psi(h))}$ is a density, the payment has a value **higher** than $\mathbf{c} \cdot \overline{\mathbf{h}}$, which is the payment to type $\overline{h}$ in the standard first price auction. By continuity and the Mean-Value Theorem, there is a type $h \in \left(s, \overline{h}\right]$ for whom both auctions have the same payment. Because both payments are strictly monotone in h, this type is unique. "Moderate" lame types obtain a lower payment than in a standard auction, whereas very lame types obtain a higher payment.

This can be explained as follows: As for quick types, a lame type has additional competition by lamer types. However, competition by quicker types is lower than in a standard auction. For lame types, competition decreases with the type. In effect, "moderate" lame types have higher competition than in a standard auction, but very lame types have less competition and therefore bid less aggressively.

To summarise: All quick types and some moderate lame types obtain a lower payment than in the standard first price auction. This hints that the UPC-auction might result in a lower expected payment than standard auctions. I will return to this point later.

Apart from this equilibrium with a continuous sum, there is a multitude of discontinuous equilibria, which do only differ from the continuous one in the bid of the worst type. Every such strategy-profile where the worst type h_0 is bidding a bid not lower than $\mathbf{c} \cdot \mathbf{s}$ in any arbitrary composition is an equilibrium, as long as all other types bid according to Proposition 5.

Generalisation to N Symmetric Bidders Proposition 5 easily extends to the case of n symmetric bidders. Denote the vector $(h_1, ..., h_{i-1}, h_{i+1}, ..., h_n)$ by $\mathbf{h}_{-i}$. The generalised winning probability then becomes

$$\widetilde{Q}\left(S(\mathbf{p}_i)\right) = Prob\left\{\mathbf{h}_{-i} : S\left(\mathbf{p}_j(h_j)\right) \geq S(\mathbf{p}_i) \forall j \neq i\right\}.$$

Lemmata 2, 3 and 4 still hold. Using the generalised winning probability, I obtain the following equilibrium bidding function applying the same arguments as in Proposition 5:

$$p_W^*(h) = \begin{cases} \int_h^s (\mathbf{c} \cdot \mathbf{g})\,(n \Leftrightarrow 1)\, \frac{[f(g)-f(\varphi(g))\varphi'(g)][F(\varphi(g))-F(g)]^{n-2}}{[F(\varphi(h))-F(h)]^{n-1}} dg & \text{for } h \in [\underline{h}, s) \\ \mathbf{c} \cdot \mathbf{s} & \text{for } h = s \\ 0 & \text{for } h \in \left(s, \overline{h}\right] \end{cases}$$

$$p_H^*(h) = \begin{cases} 0 & \text{for } h \in [\underline{h}, s] \\ \int_s^h \frac{\mathbf{c} \cdot \mathbf{g}}{g}\,(n \Leftrightarrow 1)\, \frac{[f(g)-f(\psi(g))\psi'(g)][F(g)-F(\psi(g))]^{n-2}}{[F(h)-F(\psi(h))]^{n-1}} dg & \text{for } h \in \left(s, \overline{h}\right] \end{cases}$$

6 Properties of the auction

Efficiency The equilibrium considered in Proposition 5 is not efficient. To see this, remember that total costs $c_W + c_H h$ strictly monotone increase in the type h. However, the sum and, thus, the winning probability are not monotone functions of the type. Hence not always the bidder with the lowest type and lowest costs wins. If, for example, both bidders are lame types, then the one with the higher type wins, even though this is not efficient, because he has higher costs.

Payment Ranking If a procurement agency only cares about the expected payment to the bidders, it should choose a mechanisms keeping the payment low. In this sense, does the UPC-auction perform better than standard auctions like the classical first price or second price auction (with one-dimensional bids and no contingency on ex-post observable information)?

The usual analytical methods to compare auctions (for example in Riley (1988)) only apply to auctions where the equilibrium winning probability is strictly monotone increasing in the type. Hence they don't apply to the UPC-auction. Instead I analyse this question numerically.

Let $EP^U(s)$ be the expected payment the orderer has to bear in the equilibrium of the UPC-auction (U labels the UPC-auction). $EP^U(s)$ is

$$\int_{\underline{h}}^{s} p_W^*(h)\, 2f(h) \left(F(\varphi(h)) \Leftrightarrow F(h)\right) dh + \int_{s}^{\overline{h}} p_H^*(h) h 2 f(h) \left(F(h) \Leftrightarrow F(\psi(h))\right) dh$$

The functions $p_W^*(h)$, $p_H^*(h)$, $\varphi(h)$ and $\psi(h)$ all depend on the auction parameter s. I have no explicit expression for the value s^* which minimizes $EP^U(s)$. However, if each bidder's type is drawn from the unit-interval $[0,1]$ with density $f(h) = 1$, numerical examples (for different cost parameters c_W and c_H) show that s^* lies in the interior of the interval $\left(\frac{c_W}{c_W + c_H}, 1\right)$ and it increases with the cost ratio $\frac{c_W}{c_H}$. Hence the more influence hours have on total costs of an average bidder, the smaller is the optimal s^*, i.e. the smaller is the weight of hours in the selection. The values of the expected payment decrease with s from $\underline{h}$ to s^*, and they increase to the right of s^*.

space

Table I: expected payment of the optimal UPC-auction as compared to standard auctions

Model	c_W	c_H	s^*	$EP^U(s^*)$	EP^F
1	1	5	.485	3.7601	4.3333
2	1	2	.6	2.1547	2.3333
3	1	1	.7	1.5997	1.6666
4	2	1	.796	2.6211	2.6666
5	5	1	.9	5.6432	5.6666

space

For the uniform distribution on $[0,1]$, 2 bidders, and for different cost-ratios $\frac{c_W}{c_H}$, table I compares the expected payment of the UPC auction, parametrised by s^*, to the expected payment of the standard first-price auction. For all considered cost-parameters, the UPC-auction with s^* produces a lower expected payment than both the standard first price and the standard second price auction[11], both producing EP^F. Hence the UPC-auction is better than

[11] Restricting the class of mechanisms to mechanisms not making use of ex-post observable information and payment-ceilings, the framework becomes the standard (procurement-) setting, where the standard first price and standard second price auc-

standard auctions for all considered parameters.[12] This is especially distinct if the proportion of the stochastic input is high. And it is robust in s, since it holds for all $s \in \left(\frac{c_W}{c_W + c_H}, 1\right)$.

The UPC-auction fares better than standard auctions. At first glance this is surprising, since standard auctions are optimal in a wide class of mechanisms. However, this class includes only mechanisms where both allocation and payment are contingent only on bids. The UPC-auction is not a member of this class, here the payment also is contingent on ex-post observable information. The optimal auction problem in the class of mechanisms containing also mechanisms like this has not been solved yet. However, it is not surprising that there really are better mechanisms in this broader class[13].

It also puzzles that a non-efficient auction like the UPC-auction outperformes the efficient standard auctions. However, introducing a reserve price into a standard first price auction raises its performance even though it causes inefficencies.[14] Hence inefficiencies do not necessarily hamper the performance of an auction. However, the inefficiencies in both auctions are of different kind: In the case of the reserve price, there still is (weak) monotonicity: sometimes nobody wins, but if somebody wins, it is always the quickest bidder. In the UPC-auction, even weak monotonicity fails. Sometimes the job is allocated to the lamest bidder. This inefficiency is due to ex-post observable information. The ex-post observability of types enables direct discrimination between types in the payment function. This discrimination is not monotone, instead in one subset of the type space (the set of lame types) there is a different kind of price discrimination than in its complement, the set of quick types. The resulting non-monotone winning probability alters competition. This causes some types to bid more aggressive, making up for the inefficiency (as long as $s \in \left(\frac{c_W}{c_W + c_H}, 1\right)$).

tion both are optimal and revenue equivalent (see Myerson (1981)). Hence both auctions obtain the same expected payment and I can restrict attention to only one of them.

[12]I haven't yet found an example where the UPC-auction with s^* fares worse than standard auctions.

[13]For literature on ex-post observable information see chapter 2.

[14]If the reserve price in the standard first price procurement auction is lower than the auctioneer's reservation costs and also lower than the highest type's costs, not all types participate and hence sometimes nobody wins, even though conducting the job would cause a Pareto improvement.

Three Bidders If there are more than two competing bidders, the auction ranking result of the two bidder case above still holds true, as table II shows. However, the effect is less pronounced, because an increase in the number of bidders already reduces information rents.

space

Table II

No.	c_W	c_H	s^*	$EP^U(s^*)$	EP^F
1	1	5	.515	3.2829	3.5
2	1	2	.62	1.9317	2
3	1	1	.715	1.4744	1.5
4	2	1	.81	2.4828	2.5
5	5	1	.902	5.4912	5.5

space

Comparison of Sums and Payment A buyer, earmarking the sum of the winning bid in his budget, often faces a bill higher than this earmark. With this interpretation of the sum of a bid, the next proposition can explain why procurement agencies often lament about payments exceeding the estimated price. The proposition directly follows from Lemma 2.

Proposition 6 *For $s < \overline{h}$, the payment by the orderer in the UPC-auction never falls short of the sum of the winning bid and with positive probability it exceeds the sum.*

Proof. Quick types $h \leq s$ have a sum $S(\mathbf{p}^*(h)) = 1 \cdot p_W^*(h) + s \cdot 0$ (using the one-sidedness). The payment equals $p_W^*(h) + h \cdot 0$, which is the same as the sum. However, lame types $h > s$ attain a sum $S(\mathbf{p}^*(h)) = 1 \cdot 0 + s \cdot p_H^*(h)$, which is smaller than their payment $1 \cdot 0 + h \cdot p_H^*(h)$ since $h > s$. ■

Figure 2

Hence, in the UPC-auction, the price per unit of one input is lower than costs, but the total payment is higher than the total costs. An orderer should not regard the sum as an estimator of the payment, he should instead correct it upwards when computing the expectation of the payment.

Bribery Bribery is a relevant problem in procurement. From time to time, newspapers report about the detection of cases where employees in public procurement departments or in private purchase departments are bribed by a seller. The present model doesn't capture bribery. However, it is clear that

UPC-auctions are more viable to manipulations than standard auctions, because UPC-auctions are parametrised auctions: The selection of the winning seller depends on the choice of the parameter s, the assessment of hours. A type's odds of winning (and also his winner's profits) can be changed by a change in the assessment, hence a seller has an incentive to influence the choice of the assessment. Consider the case of a buyer's employee choosing the assessment[15]. Typically, an employee has less incentive to choose the optimal assessment than the buyer. Hence for some types of sellers, bribing the buyer's employee to choose a "nice" assessment might increase both seller's and employee's utility in an apropriate model.

7 Concluding Remarks

This paper studies the UPC-auction, a frequently used procurement auction. This mechanism differs from standard auctions in having two-dimensional bids and a payment-function being contingent on a cost-parameter (representing the type) publicly observable after the auction. This enables to discriminate types in the payment function, favouring some high cost types (lame types). An equilibrium of the UPC-auction in a SIPV cost setting is found. The equilibrium involves one-sided bidding, splitting the type space into two areas with different compositions of bids: quick firms charge "service included" price structures (they do not charge service, only material) and lame firms choose "material included"-tariffs. Caused by the payment-discrimination favouring lame types, the winning probability is not a monotone function of the type. Instead, lame types have a better strategic position than in standard auctions, enhancing competition for most of the types, especially for quick types. This results in more aggressive bidding by most of the types. Even though this is counteracted by the inefficiency of the allocation, the UPC-auction fares better than standard auctions (from the point of view of a risk-neutral auctioneer), at least for all numerically analysed parameter-constellations.

The model doesn't capture all aspects of procurement with this kind of auctions.

First, we assume that craftsmen don't have influence on the speed of their work. They are not allowed to work lame by purpose. This is a severe

[15]For example because of having better information about the distribution of sellers' types.

restriction, because in reality, there often is room for undetectable excess employment of inputs, raising the payment. On the other hand, many situations call for a model where the auctioneer influences the quantities.

Second, the present model is restricted to symmetric independent private values. However, often the influence of the characteristics of the bidder on the quantities is small. Instead, nature plays a crucial role in determining realised quantities. This should be modelled using **common value distributions**. With common value quantities, I expect UPC-auctions to perform even better, compared to standard auctions, because there are no efficiency distortions.

A Appendix

A.1 Proof of Lemma 2

Assume that $\mathbf{p}_i^*(h) = \begin{pmatrix} p_{iW}^*(h), & p_{iH}^*(h) \end{pmatrix}$ is a best reply to the opponent's strategy. Consider type L. Let $S^* = S(\mathbf{p}_i^*(h))$. By the optimality of $\mathbf{p}_i^*(h)$ there can be no $\mathbf{p} = \begin{pmatrix} p_W, & p_H \end{pmatrix}$ with $\Pi(\mathbf{p}, h) > \Pi(\mathbf{p}_i^*(h), h)$. In particular, there can be no $\mathbf{p} = \begin{pmatrix} p_W, & p_H \end{pmatrix}$ achieving the same sum S^* such that $\Pi(\mathbf{p}, h) > \Pi(\mathbf{p}_i^*(h), h)$. Consider the set T of all bids $\mathbf{p}$ achieving S^*: Since $p_W, p_H \geq 0$ and $S^* = S(\mathbf{p}) = p_W + p_H s$ for $\mathbf{p} \in T$, we obtain that

$$T = \left\{ \begin{pmatrix} t \\ \frac{S^*-t}{s} \end{pmatrix} : 0 \leq t \leq S^* \right\}$$

For $\mathbf{p} \in T$, $\Pi(\mathbf{p}, h) = Q(S^*) \left(t + \frac{S^*-t}{s} h \Leftrightarrow C(\mathbf{h})\right)$. The price of washing barrels corresponding to the optimal bid $\mathbf{p}_i^*(h)$ must satisfy:

$$t^* = \arg \max_{t \in [0, S^*]} Q(S^*) \left(t + \frac{S^* \Leftrightarrow t}{s} h \Leftrightarrow C(\mathbf{h})\right)$$

$$\Leftrightarrow t^* = \arg \max_{t \in [0, S^*]} \left(t + \frac{S^* \Leftrightarrow t}{s} h\right)$$

$$\Leftrightarrow t^* = \arg \max_{t \in [0, S^*]} \left(t \left(1 \Leftrightarrow \frac{h}{s}\right) + \frac{S^*}{s} h\right)$$

Since this maximand is a linear function in t, we obtain

$$
\begin{array}{ll}
t^* = S^* & \forall h < s \\
t^* = 0 & \forall s < h \\
t^* \in [0, S^*] & \forall h = s
\end{array}
$$

Hence $\mathbf{p}_i^*(h)$ is service included for $h < s$, and it is material-included for $s < h$, while $\mathbf{p}_i^*(h)$ can be either for $h = s$. ■

A.2 Proof of Lemma 3

First I show that in a symmetric equilibrium with pure strategies there can not be a set of types with positive mass attaining the same sum and hence having the same winning-probability.

Suppose that in equilibrium there were a set of types with positive mass having the same winning-probability $\widetilde{Q}$. Denote this set by A. Because the job is always awarded and there is positive probability that the types of all bidders are element of A, $\widetilde{Q} > 0$. The bid of a type h with positive winning-probability has to ensure a non-negative winner's payoff $\pi(\mathbf{p}^*(h), h)$. Otherwise he would have a negative expected payoff $\Pi(\mathbf{p}^*(h), h)$, contradicting the optimality of $\mathbf{p}^*(h)$, because there always exists a bid high enough to ensure a non-negative expected payoff.

Because costs are strictly monotone increasing in the type, for a given bid the winner's payoff is strictly monotone increasing in the type. Because of one-sidedness there are only two different bids attaining the same sum, an service-included-bid and an material-included bid, neglecting type s. Therefore not all elements of A have zero winner's payoff. Hence there are types in A having a positive winner's payoff.

W.l.o.g. consider a quick type $h \in A$. Since $\pi(\mathbf{p}^*(h), h)$ is continuous in p_W, a marginal decrease in p_W would only result in a marginal decrease of $\pi(\mathbf{p}^*(h), h)$. However, the winning-probability $Q(S(\mathbf{p}))$ would discontinuously increase by the mass of A. Hence with a slight decrease of p_W the expected payoff $\Pi(\mathbf{p}, h) = \pi(\mathbf{p}, h) \cdot Q(S(\mathbf{p}))$ would increase, contradicting the optimality of the original bid. The same holds for lame types. This proves that there can not exist a set of types with positive mass having the same winning-probability and hence attaining the same sum. Because the set of types with the lowest sum has zero mass, types being element of this set loose with probability 1. So there do exist types with zero winning-probability.

Next I show that there can not be a lame type with zero winning-probability. Suppose in equilibrium there would exist a lame type $h_0 > s$ with zero winning-probability. Because the set (s, h_0) has positive mass, there does exist a lame type $h \in (s, h_0)$ with positive winning-probability $Q\left(S\left(\mathbf{p}^*\left(h\right)\right)\right) > 0$. Remember that, in equilibrium, all types with positive probability of winning have non-negative winner's payoff. Because both h and h_0 are lame types and h_0 has zero winning-probability, $p_H^*(h) < p_H^*(h_0)$. Hence, using one-sidedness:

$$\begin{aligned} 0 &\leq \pi\left(\mathbf{p}^*\left(h\right), h\right) = \left(p_H^*(h) \Leftrightarrow c_H\right) h \Leftrightarrow c_W \\ &< \left(p_H^*(h_0) \Leftrightarrow c_H\right) h_0 \Leftrightarrow c_W = \pi\left(\mathbf{p}^*\left(h_0\right), h_0\right) \end{aligned}$$

Because his probability of winning is zero, h_0 has zero *expected* payoff. He could improve his expected payoff by bidding $\mathbf{p}^*(h) = (0, p_H^*(h))$.

$$0 \leq \pi\left(\mathbf{p}^*\left(h\right), h\right) < \pi\left(\mathbf{p}^*\left(h\right), h_0\right) < \pi\left(\mathbf{p}^*\left(h_0\right), h_0\right)$$

Then he still obtains a positive winner's payoff $\pi\left(\mathbf{p}^*\left(h\right), h_0\right)$, but now yields a positive probability of winning and hence a positive expected payoff, a contradiction to the optimality of $\mathbf{p}^*\left(h_0\right)$. Hence there can not be a lame type with zero winning-probability.

The same arguments apply when showing that there can not be a quick type with zero winning-probability. Type s is the only type who is neither lame nor quick. From above we know that there must exist a type with zero winning probability. Hence the worst type is $h_0 = s$. It has zero winning-probability. ■

A.3 Proof of Lemma 4

Consider a quick type h_W and a lame type h_H, both achieving the same sum in equilibrium. Denote this sum by $\widetilde{S} = S\left(\mathbf{p}^*\left(h_W\right)\right) = S\left(\mathbf{p}^*\left(h_H\right)\right)$. Since $S\left(\mathbf{p}\right) = p_W + p_H s$ the quick type h_W bids $p_W^*\left(h_W\right) = \widetilde{S}$, using one-sidedness. The lame type h_H bids $p_H^*\left(h_H\right) = \frac{\widetilde{S}}{s}$. When choosing $\mathbf{p}$, $\widetilde{S}$ maximises both type's expected payoffs, represented as a function of the sum and of the type:

$$\begin{cases} \widetilde{S} = \arg\max\limits_S \left\{Q\left(S\right) \cdot \left(S \Leftrightarrow c_W \Leftrightarrow c_H h_W\right)\right\} \\ \widetilde{S} = \arg\max\limits_S \left\{Q\left(S\right) \cdot \left(\frac{S}{s} h_H \Leftrightarrow c_W \Leftrightarrow c_H h_H\right)\right\} \end{cases}$$

Consider a quick type h_W and use the one-sidedness property. If the bid $\left(p_W^*\left(h_W\right), 0\right)$ maximises $\Pi\left(\mathbf{p}, h_W\right)$ then $S(p_W^*\left(h_W\right), 0)$ maximises the expected payoff as a function of S and h_W, because $S(p_W^*\left(h_W\right), 0)$ is injective in the first argument. If $p_W^*\left(h_W\right)$ is determined by a unique solution to the first order condition with respect to p_W, then the payoff-maximising S must satisfy the first order condition with respect to S, because $S(p_W, p_h)$ is differentiable in p_W. The same applies for lame types. Hence for a quick type h_W and a lame type h_H both scoring $\widetilde{S}$, the following first order conditions must be satisfied:

$$\left\{\begin{array}{c} \frac{dQ(S)}{dS}\mid_{S=\widetilde{S}} \cdot \left(\widetilde{S} \Leftrightarrow c_W \Leftrightarrow c_H h_W\right) + Q\left(\widetilde{S}\right) \stackrel{!}{=} 0 \\ \frac{dQ(S)}{dS}\mid_{S=\widetilde{S}} \cdot \left(\frac{\widetilde{S}}{s} h_H \Leftrightarrow c_W \Leftrightarrow c_H h_H\right) + \frac{h_H}{s} Q\left(\widetilde{S}\right) \stackrel{!}{=} 0 \end{array}\right.$$

$$\Leftrightarrow \left\{\begin{array}{c} \frac{dQ(S)}{dS}\mid_{S=\widetilde{S}} \cdot \left(\widetilde{S} \Leftrightarrow c_W \Leftrightarrow c_H h_W\right) + Q\left(\widetilde{S}\right) \stackrel{!}{=} 0 \\ \frac{dQ(S)}{dS}\mid_{S=\widetilde{S}} \cdot \left(\widetilde{S} \Leftrightarrow c_W \frac{s}{h_H} \Leftrightarrow c_H s\right) + Q\left(\widetilde{S}\right) \stackrel{!}{=} 0 \end{array}\right.$$

$$\Longrightarrow \left\{\begin{array}{c} \frac{dQ(S)}{dS}\mid_{S=\widetilde{S}} \cdot \left(\widetilde{S} \Leftrightarrow c_W \Leftrightarrow c_H h_W\right) + Q\left(\widetilde{S}\right) = \\ \frac{dQ(S)}{dS}\mid_{S=\widetilde{S}} \cdot \left(\widetilde{S} \Leftrightarrow c_W \frac{s}{h_H} \Leftrightarrow c_H s\right) + Q\left(\widetilde{S}\right) \end{array}\right.$$

$$\Leftrightarrow c_W + c_H h_W = c_W \frac{s}{h_H} + c_H s$$

Since the right hand side is strictly monotone increasing in h_W and the left hand side is strictly monotone decreasing in h_H, there is at most one quick type h_W suiting to a lame type h_H, and v.v.. There is no other type having the same sum as $h = s$. The higher h_H is, the lower is the corresponding h_W. Hence solving for h_W yields a strict monotone decreasing function of h_H (and vice versa).

Rewriting the iso-sum condition one can see that the cost relation of two types with the same winning probability is s/h_H.

$$\frac{c_W + c_H h_W}{c_W + c_H h_H} = \frac{s}{h_H}$$

Two different lame types can not have the same sum. This can be seen from equating the first order conditions of two lame types h and g. The only solution to the resulting equation is $h = g$. The same holds true for service-included types.

■

A.4 Proof of Proposition 5

The proof is done in several steps. First, I derive the winning probability of every type, using the iso-sum-functions. Analysing the first order condition of the direct mechanism then yields differential equations determining the equilibrium bidding function. At the end I check whether the resulting bidding function satisfies the equilibrium hypothesis stated in the following.

The Equilibrium Hypothesis Consider the following equilibrium hypothesis: Assume that, in equilibrium, $S(\mathbf{p}^*(h))$ is continuous and strictly monotone increasing in h for $h \leq s$ and continuous and strictly monotone decreasing in h for $s < h$.[16] This is consistent with the fact that the worst type is $h_0 = s$ (Lemma 3). Assume that in equilibrium for quick types the first order condition of the maximisation of the expected payoff w.r.t. p_W holds. For lame types, assume that the corresponding first order condition w.r.t. p_H holds in equilibrium.

The Winning Probability Under the equilibrium hypothesis the set of all types with sums higher than a given sum S is an interval. The pairs enclosing those intervals are given by the iso-sum functions $\varphi(h)$ and $\psi(h)$. The winning probability of a type h is

$$Q(S(\mathbf{p}^*(h))) = \begin{cases} F(\varphi(h)) \Leftrightarrow F(h) & \text{for } h \in [\underline{h}, s) \\ 0 & \text{for } h = s \\ F(h) \Leftrightarrow F(\psi(h)) & \text{for } h \in \left(s, \overline{h}\right] \end{cases}$$

Bidder's Problem A bidder of type h maximises $\Pi(\mathbf{p}, h) = Q(S(\mathbf{p})) \cdot \pi(\mathbf{p}, h) = Q(S(\mathbf{p}))(\mathbf{p}^* \cdot \mathbf{h} \Leftrightarrow \mathbf{c} \cdot \mathbf{h})$. From Lemma 2 we already now that his bid has to be one-sided. Hence he maximises

$$\Pi(\mathbf{p}, h) = \begin{cases} Q\left(S\left(\begin{array}{cc} p_W, & 0 \end{array}\right)\right)(p_W \Leftrightarrow \mathbf{c} \cdot \mathbf{h}) & \text{for } h \in [\underline{h}, s) \\ Q\left(S\left(\begin{array}{cc} 0, & p_H \end{array}\right)\right)(p_H h \Leftrightarrow \mathbf{c} \cdot \mathbf{h}) & \text{for } h \in \left[s, \overline{h}\right] \end{cases}$$

Looking at the corresponding direct mechanism[17], with the bidder announcing a type $\hat{h}$, he faces the problem:

[16] Note that I have not assumed continuity of $S(\mathbf{b}^*(L))$ at $L = s$.

[17] Using the direct mechanism as a representation of the indirect mechanism of announcing a bid can be justified by the following arguments: For a wood-type, the problem of choosing an optimal bid (b_W, b_L) is equivalent to the problem of announcing an optimal

$$\begin{cases} \max\limits_{\hat{h}\in[\underline{h},s)} \left\{ Q\left(S\left(\ p_W^*\left(\hat{h}\right),\ \ 0\ \right)\right)\left(p_W^*\left(\hat{h}\right) \Leftrightarrow \mathbf{c}\cdot\mathbf{h}\right)\right\} & \text{for } h\in[\underline{h},s) \\ \max\limits_{\hat{h}\in\left[s,\overline{h}\right]} \left\{ Q\left(S\left(\ 0,\ \ p_H^*\left(\hat{h}\right)\ \right)\right)\left(p_H^*\left(\hat{h}\right)h \Leftrightarrow \mathbf{c}\cdot\mathbf{h}\right)\right\} & \text{for } h\in\left[s,\overline{h}\right] \end{cases}$$

Using the formula for the winning probability this becomes:

$$\begin{cases} \max\limits_{\hat{h}\in[\underline{h},s)} \left\{ \left[F\left(\varphi\left(\hat{h}\right)\right) \Leftrightarrow F\left(\hat{h}\right)\right]\left(p_W^*\left(\hat{h}\right) \Leftrightarrow \mathbf{c}\cdot\mathbf{h}\right)\right\} & \text{for } h\in[\underline{h},s) \\ \max\limits_{\hat{h}\in\left[s,\overline{h}\right]} \left\{ \left[F\left(\hat{h}\right) \Leftrightarrow F\left(\psi\left(\hat{h}\right)\right)\right]\left(p_H^*\left(\hat{h}\right)h \Leftrightarrow \mathbf{c}\cdot\mathbf{h}\right)\right\} & \text{for } h\in\left[s,\overline{h}\right] \end{cases}$$

The first order conditions are:

$$\begin{cases} \left[f(\varphi(\hat{h}))\varphi'(\hat{h}) \Leftrightarrow f(\hat{h})\right]\left(p_W^*(\hat{h}) \Leftrightarrow \mathbf{c}\cdot\mathbf{h}\right) + \left[F(\varphi(\hat{h})) \Leftrightarrow F(\hat{h})\right]\frac{dp_W^*\left(\hat{h}\right)}{dh} = 0 \\ \text{for } h\in[\underline{h},s) \\ \left[f(\hat{h}) \Leftrightarrow f(\psi(\hat{h}))\psi'(\hat{h})\right]\left(p_H^*(\hat{h})h \Leftrightarrow \mathbf{c}\cdot\mathbf{h}\right) + \left[F(\hat{h}) \Leftrightarrow F(\psi(\hat{h}))\right]\frac{dp_H^*\left(\hat{h}\right)}{dh}h = 0 \\ \text{for } h\in\left[s,\overline{h}\right] \end{cases}$$

We will later confirm that the first order conditions lead to a *global* maximum.

For $\mathbf{p}^*(h)$ to be an equilibrium bidding function, truthtelling (that is announcing $\hat{h}=h$), must be optimal, hence:

$$\begin{cases} \left[f(\varphi(h))\varphi'(h) \Leftrightarrow f(h)\right]\left(p_W^*(h) \Leftrightarrow \mathbf{c}\cdot\mathbf{h}\right) + \left[F(\varphi(h)) \Leftrightarrow F(h)\right]\frac{dp_W^*(h)}{dh} = 0 \\ \text{for } h\in[\underline{h},s) \\ \left[f(h) \Leftrightarrow f(\psi(h))\psi'(h)\right]\left(p_H^*(h)h \Leftrightarrow \mathbf{c}\cdot\mathbf{h}\right) + \left[F(h) \Leftrightarrow F(\psi(h))\right]\frac{dp_H^*(h)}{dh}h = 0 \\ \text{for } h\in\left[s,\overline{h}\right] \end{cases}$$

This becomes a first order linear differential equation:

$$\begin{cases} \frac{dp_W^*(h)}{dh} = \left(p_W^*(h) \Leftrightarrow \mathbf{c}\cdot\mathbf{h}\right)\frac{[f(h)-f(\varphi(h))\varphi'(h)]}{[F(\varphi(h))-F(h)]} & \text{for } h\in[\underline{h},s) \\ \frac{dp_H^*(h)}{dh} = \Leftrightarrow\frac{\left(p_H^*(h)h-\mathbf{c}\cdot\mathbf{h}\right)}{h}\frac{[f(h)-f(\psi(h))\psi'(h)]}{[F(h)-F(\psi(h))]} & \text{for } h\in\left[s,\overline{h}\right] \end{cases}$$

type $\hat{L}\in[\underline{L},s]$ and inserting it into an equilibrium $b^*()$ which is continuous on $[\underline{L},s]$ as well as on $\left(s,\overline{L}\right]$ and yields a continuous score. This equivalence is due to the fact that the subset of equilibrium bids $\{(b_W^*(L),0) \mid L\leq s\}$ contains his optimal bid: Because of the one-sidedness property, his optimal bid involves setting $b_L=0$. Bidding higher than $\inf\{b_W^*(L)\}$ results in zero expected payoff due to the zero winning probability, and bidding lower than $\sup\{b_W^*(L)\mid L\leq s\}$ yields less payoff than bidding $\sup\{b_W^*(L)\mid L\leq s\}$. Due to the one-sidedness property, he will never bid $b_L>0$. The same arguments apply for labour-types.

Boundary Conditions In the following I derive the boundary conditions using Lemma 3.

First I show that the worst type $h_0 = s$ can not have a positive winner's payoff: Suppose instead $\pi(\mathbf{p}^*(s), s) > 0$. Type s could improve his *expected* payoff $\Pi(\mathbf{p}, h) = Q(S(\mathbf{p})) \cdot \pi(\mathbf{p}, h)$ (which is zero due to zero probability of winning) by bidding slightly lower: By continuity of the sum of the equilibrium bidding function, there exists an ε such that $p_W^*(s) \Leftrightarrow \varepsilon$ results in $\pi((0, p_W^*(s) \Leftrightarrow \varepsilon), s) > 0$ and still $Q((0, p_W^*(s) \Leftrightarrow \varepsilon)) > 0$. This is a contradiction to the optimality of $p_W^*(s)$.

I show next that the sum must be continuous at s: Consider first the case $S(\mathbf{p}^*(s)) > \lim_{h \to s+} S(\mathbf{p}^*(s))$ (i.e. $p_W^*(s) > \lim_{h \to s+} p_H^*(s) \cdot s$). Every type can obtain a non-negative winner's payoff by bidding high enough. Hence a type with positive probability of winning does not have a negative winner's payoff in equilibrium. Especially the winner's payoff of every element of an arbitrary sequence $h \to s+$ is non-negative. At s the winner's payoff is the same for bids $(p_H s, 0)$ and $(0, p_H)$. Because $p_W^*(s) > \lim_{h \to s+} p_H^*(s) \cdot s$, a bid $(p_W^*(s), 0)$ would lead to a positive winner's payoff, which yields a contradiction. Because s is the worst type, the second case $S(\mathbf{p}^*(s)) = p_W^*(s) < \lim_{h \to s+} s p_H^*(s) = \lim_{h \to s+} S(\mathbf{p}^*(s))$ is excluded as well.

Now the continuity of the sum, the non-negativity of the winner's payoff for all types with positive winning probability and the non-positivity of the winner's payoff of the worst type yields the boundary condition $p_W^*(s) = \mathbf{c} \cdot \mathbf{s}$. By continuity of the sum, the other boundary condition is $\lim_{h \to s+} p_H^*(s) = \frac{\mathbf{c} \cdot \mathbf{s}}{s}$.

Solution Solving the differential equations yields the following equilibrium bidding-function:

$$p_W^*(h) = \begin{cases} \int_h^s \mathbf{c} \cdot \mathbf{g} \frac{f(g) - f(\varphi(g))\varphi'(g)}{F(\varphi(h)) - F(h)} dg & \text{for } h \in [\underline{h}, s) \\ \mathbf{c} \cdot \mathbf{s} & \text{for } h = s \\ 0 & \text{for } h \in \left(s, \overline{h}\right] \end{cases}$$

$$p_H^*(h) = \begin{cases} 0 & \text{for } h \in [\underline{h}, s] \\ \int_s^h \frac{\mathbf{c} \cdot \mathbf{g}}{g} \frac{f(g) - f(\psi(g))\psi'(g)}{F(h) - F(\psi(h))} dg & \text{for } h \in \left(s, \overline{h}\right] \end{cases}$$

Monotonicity To prove the monotonicity-assumption stated earlier in the proof, I now show that the equilibrium bidding function is strictly monotone increasing on $[\underline{h}, s)$ and strictly monotone decreasing on $\left[s, \overline{h}\right]$. Consider a quick type. Adjust the equilibrium bidding function downwards by replacing $\mathbf{c} \cdot \mathbf{g} = c_W + c_H g$ in the integrand by $c_W + c_H h$:

$$p_W^*(h) > \int_h^s \mathbf{c} \cdot \mathbf{h} \frac{f(g) \Leftrightarrow f(\varphi(g))\varphi'(g)}{F(\varphi(h)) \Leftrightarrow F(h)} dg$$

Because $\frac{f(g)-f(\varphi(g))\varphi'(g)}{F(\varphi(h))-F(h)}$ is a density on $[h, s)$ (this can be seen by a simple check), $p_W^*(h) > c_W + c_H h$ for all $h < s$. Hence the payment exceeds the costs and the bidder gains a positive winner's payoff. Using this in the differential equation we see that $\frac{dp_W^*(h)}{dh} > 0$. Applying the same arguments for $p_H^*(h)$, one obtains $\frac{dp_H^*(h)}{dh} < 0$. Because $S(\mathbf{p})$ is monotone, this confirms the assumptions on $S(\mathbf{p}(h))$.

Continuity The continuity-assumption in the equilibrium hypothesis is satisfied, because the bidding functions are integrals over bounded values and hence are continuous on $[\underline{h}, s)$ and $\left[s, \overline{h}\right]$

Due to the boundary conditions, the sum $S\left(\mathbf{p}^*\left(h\right)\right)$ is continuous at s. Hence it is continuous on $\left[\underline{h}, \overline{h}\right]$.

Differentiability I now show that at the equilibrium bid for a quick type the first order condition of the expected payoff w.r.t. p_W must be satisfied. This was part of the equilibrium hypothesis. Since we already know that in any equilibrium p_H of a quick type must be zero, it suffices to show that the expected payoff w.r.t. p_W is pseudoconcave and locally differentiable. For lame types, the expected payoff w.r.t. p_H has to be pseudoconcave and locally differentiable.

First, consider **quick** types with a **matching** lame type having the same sum. Those types are elements of $A \equiv [\underline{h}, s) \cap \{h > \mu_W\}$.

I first show that $p_W^*(h)$ is differentiable on A. Consider the differential equation characterising $p_W^*(h)$

$$\frac{dp_W^*\left(h\right)}{dh} = \left(p_W^*(h) \Leftrightarrow \mathbf{c} \cdot \mathbf{h}\right) \frac{\left[f(h) \Leftrightarrow f(\varphi(h))\varphi'(h)\right]}{\left[F(\varphi(h)) \Leftrightarrow F(h)\right]}$$

We have just proven that $p_W^*(h)$ is continuous on $[\underline{h}, s)$. $F(h)$ is differentiable by assumption and $\varphi'(h)$ is continuous on A. Using this, we see that $\frac{dp_W^*(h)}{dh}$ is continuous on A and therefore $p_W^*(h)$ is differentiable on A.

Because $p_W^*(h)$ is strictly monotone increasing on A, the inverse function $p_W^{*-1}(p_W)$ on $B \equiv \{p_W^*(h)|h \in A\}$ exists. Due to the differentiability of $p_W^*(h)$, its inverse is differentiable, too. Since $F(h)$ and $\varphi(h)$ are both differentiable on A, $F\left(\varphi\left(p_W^{*-1}(p_W)\right)\right) \Leftrightarrow F\left(p_W^{*-1}(p_W)\right)$ is differentiable in p_W on B as well. Therefore also the expected payoff

$$\Pi\left(\mathbf{p}^*\left(p_W^{*-1}(p_W)\right), h_i\right) = \left(F\left(\varphi\left(p_W^{*-1}(p_W)\right)\right) \Leftrightarrow F\left(p_W^{*-1}(p_W)\right)\right)\left(p_W \Leftrightarrow \mathbf{c} \cdot \mathbf{h}_i\right)$$

is differentiable in p_W on B.

It is left to check whether for quick types **not** being matched to a lame type with the same sum the expected payoff is differentiable and pseudoconcave. Those types are element of $\tilde{A} \equiv [\underline{h}, s) \cap \{h < \mu_W\}$. At $p_W^*(h)$ they have the expected payoff of

$$\Pi\left(\mathbf{p}^*\left(p_W^{*-1}(p_W)\right), h_i\right) = \left(1 \Leftrightarrow F\left(p_W^{*-1}(p_W)\right)\right)\left(p_W \Leftrightarrow \mathbf{c} \cdot \mathbf{h}_i\right)$$

Using the same arguments we easily see that this is differentiable, too.

For the quick type μ_W being matched to the highest lame type $\overline{h}$, the expected payoff is not differentiable. I will show below, that by pseudoconcavity at μ_W the equilibrium bidding function is continuous at μ_W as well. The same holds true for the lame type μ_H.

Using the same arguments, I can prove that for lame types the expected payoff is differentiable in p_H on $\left\{p_H^*(h)|h \in \left\{(s, \overline{h}]\backslash \mu_H\right\}\right\}$.

Pseudoconcavity It is left to check whether the first order conditions lead to *global* maxima. I do this by showing pseudoconcavity of $\Pi\left(\mathbf{p}^*(\hat{h}), h\right)$.

Because of the one-sidedness property, for quick types I only have to check whether announcing the true type $\hat{h} = h$ yields a higher expected payoff than announcing any other quick type $\hat{h} \leq s$. I do not have to consider announcements $\hat{h} > s$. Similar for lame types. A type h announcing type $\hat{h}$ obtains the following expected payoff:

$$\Pi\left(\mathbf{p}^*(\hat{h}), h\right) = \begin{cases} \left(p_W^*\left(\hat{h}\right) 1 \Leftrightarrow c_W \Leftrightarrow c_H h\right)\left(F(\varphi(\hat{h})) \Leftrightarrow F(\hat{h})\right) & \text{for } h, \hat{h} \in [\underline{h}, s] \\ \left(p_H^*\left(\hat{h}\right) h \Leftrightarrow c_W \Leftrightarrow c_H h\right)\left(F(\hat{h}) \Leftrightarrow F(\psi(\hat{h}))\right) & \text{for } h, \hat{h} \in \left(s, \overline{h}\right] \end{cases}$$

Inserting the equilibrium bidding function, the right hand side becomes:

$$
= \begin{cases} \left(\int_{\hat{h}}^{s} \mathbf{c}\cdot\mathbf{g}\frac{f(g)-f(\varphi(g))\varphi'(g)}{F(\varphi(\hat{h}))-F(\hat{h})}dg \Leftrightarrow \mathbf{c}\cdot\mathbf{h}\right)\left(F(\varphi(\hat{h})) \Leftrightarrow F(\hat{h})\right) & \begin{array}{c}\text{for } h\in[\underline{h},s]\,,\\ \hat{h}\in[\underline{h},s)\end{array} \\ \left(c_W + c_H\hat{h} \Leftrightarrow \mathbf{c}\cdot\mathbf{h}\right)0 & \begin{array}{c}\text{for } h\in[\underline{h},s]\,,\\ \hat{h}=s\end{array} \\ \left(\int_{s}^{\hat{h}} \frac{\mathbf{c}\cdot\mathbf{g}}{g}\frac{f(g)-f(\psi(g))\psi'(g)}{F(\hat{h})-F(\psi(\hat{h}))}dgh \Leftrightarrow \mathbf{c}\cdot\mathbf{h}\right)\left(F(\hat{h}) \Leftrightarrow F(\psi(\hat{h}))\right) & \text{for } h,\hat{h}\in\left(s,\overline{h}\right] \end{cases}
$$

$$
= \begin{cases} \begin{array}{c}\int_{\hat{h}}^{s} \mathbf{c}\cdot\mathbf{g}\left(f(g) \Leftrightarrow f(\varphi(g))\varphi'(g)\right)dg \\ \Leftrightarrow \mathbf{c}\cdot\mathbf{h}\left(F(\varphi(\hat{h})) \Leftrightarrow F(\hat{h})\right)\end{array} & \text{for } h\in[\underline{h},s]\,,\hat{h}\in[\underline{h},s) \\ 0 & \text{for } h\in[\underline{h},s]\,,\ \hat{h}=s \\ \begin{array}{c}\int_{s}^{\hat{h}} \frac{\mathbf{c}\cdot\mathbf{g}}{g}\left(f(g) \Leftrightarrow f(\psi(g))\psi'(g)\right)dgh \\ \Leftrightarrow \mathbf{c}\cdot\mathbf{h}\left(F(\hat{h}) \Leftrightarrow F(\psi(\hat{h}))\right)\end{array} & \text{for } h,\hat{h}\in\left(s,\overline{h}\right] \end{cases}
$$

Unfortunately, because of the discontinuity of $\varphi'(\hat{h})$ and $\psi'(\hat{h})$, $\Pi\left(\mathbf{p}^*(\hat{h}),h\right)$ is not differentiable at $\hat{h}=\mu_W$ and at $\hat{h}=\mu_H$. However, at these values it is continuous. Using Leibniz' rule, $\frac{\partial\Pi\left(\mathbf{p}^*(\hat{h}),h\right)}{\partial\hat{h}}$ equals:

$$
\begin{cases} \left(\Leftrightarrow\left(c_W+c_H\hat{h}\right)+(c_W+c_H h)\right)\left(f(\hat{h}) \Leftrightarrow f(\varphi(\hat{h}))\varphi'(\hat{h})\right) & \begin{array}{c}\text{for } h,\hat{h}\in[\underline{h},s]\,,\\ \hat{h}\neq\mu_W\end{array} \\ \left(\frac{c_W+c_H\hat{h}}{\hat{h}}h \Leftrightarrow (c_W+c_H h)\right)\left(f(\hat{h}) \Leftrightarrow f(\psi(\hat{h}))\psi'(\hat{h})\right) & \begin{array}{c}\text{for } h,\hat{h}\in\left(s,\overline{h}\right]\,,\\ \hat{h}\neq\mu_H\end{array} \end{cases}
$$

Because $\left(f(\hat{h}) \Leftrightarrow f(\varphi(\hat{h}))\varphi'(\hat{h})\right)$ and $\left(f(\hat{h}) \Leftrightarrow f(\psi(\hat{h}))\psi'(\hat{h})\right)$ are positive for all $\hat{h}$, we obtain:

$$
\frac{\partial\Pi\left(\mathbf{p}^*(\hat{h}),h\right)}{\partial\hat{h}} = \left\{\begin{array}{ll} >0 & \text{for } \hat{h}<h \\ =0 & \text{for } \hat{h}=h \\ <0 & \text{for } h<\hat{h} \end{array}\right\} \begin{array}{l} \text{if } \hat{h}\neq\mu_W, \hat{h}\neq\mu_H \\ \text{and } sign(h \Leftrightarrow s) = sign(\hat{h} \Leftrightarrow s) \end{array}
$$

Together with the continuity of $\Pi\left(\mathbf{p}^*(\hat{h}),h\right)$ at $\hat{h}=\mu_W$ and at $\hat{h}=\mu_H$, this prooves pseudoconcavity of $\Pi\left(\mathbf{p}^*(\hat{h}),h\right)$ in $\hat{h}$. Hence for the types μ_W and μ_H announcing their true type is optimal. Since the same holds true for all other types, the first order conditions characterise global maxima for these types.

■

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