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The determinants of BUND-future price changes: An ordered probit analysis using DTB and LIFFE data

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Abstract: This paper investigates the determinants of transaction price changes during BUND-future trading at Deutsche Terminbörse (DTB) and London International Financial Futures Exchange (LIFFE). The analysis uses the ordered probit model, which is an econometric tool that is comparatively new to the econometrics of financial markets. It is especially valuable with respect to high frequency data which are used for the empirical analysis of this paper. Although the ordered probit model is nonstructural, it allows to test the validity of market microstructure literature. A comparison of BUND-futures trading at DTB and LIFFE is also conducted.

Zusammenfassung: In dieser Arbeit werden die Determinanten von Transaktionspreisveränderungen während des BUND-Future Handels an der Deutschen Terminbörse (DTB) und der London International Financial Futures Exchange (LIFFE) untersucht. Die Analyse wird mit Hilfe eines geordneten Probitmodells durchgeführt, das bisher kaum für Finanzmarktdaten verwendet wurde. Obwohl das geordente Probitmodell nichtstrukturell ist, erlaubt es, einige mikroökonomische Ansätze zur Erklärung bestimmter Marktphänomena auf ihre Richtigkeit hin zu überprüfen. Die Untersuchung verwendet dabei Hochfrequenzdaten, deren Eigenschaften das geordnete Probitmodell in besonderem Maße Rechnung trägt. Außerdem wird ein Vergleich zwischen den beiden Börsen DTB und LIFFE angestellt.

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This paper examines the determinants of BUND–future transaction price changes using the ordered probit model. Microeconometric tools like the ordered probit model are comparatively new to the analysis of financial data. Ordered probit models were originally developed to study opinion surveys, voting behavior,[1] and labor market data. In recent years, ordered probit models became more and more popular in the analysis of financial time series. Hausman et al. (1992) were the first to introduce such approaches to financial econometrics when they studied the determinants of transaction price changes of U.S. stock data. Other authors like Bollerslev and Melvin (1994) and Bollerslev and Domowitz (1993) picked up their idea and used an ordered probit model to explain exchange rates. The current paper is to some extent based on the article of Hausman et al.

Another interesting aspect of this paper is that it uses *intradaily* data. These high frequency data are gaining importance in financial econometrics since the loss of information when almost any trade is recorded is substantially lower than if, say, simply daily highs or lows were taken instead.

Section 1 briefly outlines the econometric theory of ordered probit models. It gives the motivation for the use of ordered probit models in financial markets, illustrates how ordered probits work, and concludes with the estimation procedure. Section 2 describes the dataset and points out differences and similarities between the two competing exchanges Deutsche Terminbörse (DTB) and London International Financial Futures Exchange (LIFFE). Section 3 suggests a basic specification and interprets first estimation results. This section also includes applications of the ordered probit results such as tests for order flow dependence and for differences of Open Outcry (OOC) and the computerized Automated Pit Trading (APT) at LIFFE. The following section 4 makes clear that although the ordered probit model is nonstructural, it can be used to find evidence in favor or against market microstructure theory. Section 5 summarizes the empirical findings and gives suggestions for further reserarch.

1 Motivation, model setup and estimation

What do we gain from using the ordered probit approach instead of standard time series methods? What are the advantages over the "classical" methods like ARMA, ARIMA, ARMAX or state–space models? Hausman et al. (1992, p. 320) find three major reasons to apply the ordered probit model:

(1) **Discreteness:** Stock and future market prices are usually measured in a *discrete* way, in *ticks*, the smallest possible unit of a price change. At both DTB and LIFFE, the minimum price movement is 0.01 % which corresponds – with the BUND–future's nominal value of DM 250,000 – to a tick size of DM 25.[2] [3] Discreteness cannot be modelled with the "classical" approaches but is a neccessary condition for the appropriate use of ordered probit models.

(2) **Timing:** The timing of transactions is assumed to be irregular and random. Thus, transaction price[4] changes should only be modelled by discrete–time processes – like for example ARMA approaches –, if the information that determines

[1]The initial paper on ordered probit models by McKelvey and Zavoina (1975) analyzed congressional voting behavior on the 1965 U.S. medicare bill.

[2]Details on the BUND–future are to be found in DTB (1990) and LIFFE (1994).

[3]Glosten and Harris (1988, pp. 134) find more accurate estimates after discreteness is taken into account in their empirical investigation. Hausman et al. (1992, pp. 362) confirm this finding by a comparison of ordered probit results with OLS estimates (it should be noted that applying OLS was a bit unfair since the error term was heteroscedastic).

[4]The notation "transaction price" is used in order to stress the difference to bid and ask prices.

the waiting interval between trades can be neglected.[5] The ordered probit specification discussed below explicitly models time between trade.

(3) **Conditionality:** Recent studies have mainly focused on the *unconditional* distribution of transaction price changes whereas economic interest should be on the *conditional* distribution. Both the mean and the variance in ordered probit models are calculated conditioned on explanatory variables.

Three important topics should be added:

(4) **Tractability:** The ordered probit estimation results can be interpreted almost like a linear regression.

(5) **Volatility:** Another interesting feature of the ordered probit model is that it simultaneously determines both the conditional mean and the conditional variance of transaction price changes. Moreover, it allows to correct for heteroscedasticity.

(6) Although the ordered probit model is **nonstructural**, it is possible to draw firm conclusions from its empirical findings for the validity of microtheoretical models. This topic be will be thoroughly discussed in section 4.

The ordered probit approach studies *discrete* dependent variables. It is just one among several models which take into account this special feature that data sometimes exhibit.[6]
Ordered probit models differ from other approaches to discrete dependent variables in two aspects. The first is that they are concerned with *categorial* data and the second is that these categories are *naturally ordered.* The ordered probit is a model of *qualitative choice*, it detects why a decision is reached when multiple alternatives exist.
Models of qualitative choice assume that individuals are faced with choices between two or more alternatives and that their final decision is reached *conditional* on the characteristics of the individual. The conditionality of these approaches is crucial. A binary choice model, like a binomial logit or probit, which investigates for example the voting behavior on a senate election, estimates the conditional probability that an individual chooses one or the other candidate given the voters personal characteristics and socio–economic variables. Thus, a binary choice model estimates the conditional probability that the dependent variable takes on a certain value, say, one for voting "yes" and zero for voting "no", given other exogenous variables.
This is exactly the same what the ordered probit model does. The only difference is that more than two choices exist and that these choices are naturally – as opposed to arbitrarily – ordered.
The dependent variable that is examined in the following is the transaction price change, Z_k. It is computed as $Z_k = P(t_k) - P(t_{k-1})$, where $P(t_k)$ denotes the transaction price at time t_k. The subscript $k \in \mathbb{K}$ indicates the kth transaction of a particular trading day, while $t \in \mathbb{T}$ indicates the tth unit of actual time of a particular trading day, e.g., seconds. The index set $\mathbb{K}$ is hence called *transaction time* and the index set $\mathbb{T}$ is hence called *clocktime.* Since the clocktiming of transactions is irregular (more than than one or less than one transaction may take place within a single unit $t \in \mathbb{T}$), clocktime cannot uniquely order the available observations. A unique ordering

[5] The information content of time elapsing between trade is discussed in lenght in section 4.

[6] Details on ordered probit models are provided by Amemiya (1985, ch. 9), Greene (1993, ch. 22), Maddala (1983, ch. 2) and Ronning (1991, ch. 2). Becker and Kennedy (1992) graphically explain the ordered probit model.

is achieved by using transaction time.[7] However, since the clocktime elapsed between successive trades is important for expected transaction price changes and volatility, both indices are used to characterize observations. Clearly, transaction time may move faster (more than one transaction per second) or slower (no transaction per second) than clocktime. Transaction time and clocktime are thus not equivalents as could be assumed at first sight.

Without the concept of transaction time, trading days are - like in ARMA models — divided up into clocktime intervals of equal length. It is clear that this causes a substantial loss of information since one does not know exactly where in clocktime an observation was originally situated. The interval just carries an "aggregate" information. The observations are not exactly defined.

A model that is set up in transaction time does not need such even spacing. There is no loss of information since any observation is uniquely defined. This is an important feature especially with respect to the availabilty of intraday data. Both the LIFFE and DTB dataset contains many observations that take place at the very same second.[8]

Now, since the transaction price change Z_k is discrete and naturally ordered, traditional regression models do not apply here. These methods would treat the difference between a price changes of -2 and -1 ticks in the same way as the difference between a price change of +3 and +4 ticks, while in fact these numbers represent only a ranking. What econometricians are interested in is to find out what variables determine to what extent the transaction price change Z_k. The ordered probit model assumes an underlying, latent variable Z_k^* that influences Z_k. Z_k^* itself is driven by an $q \times 1$ vector of exogenous variables, $\boldsymbol{X_k}$, and an error term ϵ_k, which is generally assumed to be independently identical normal distributed.[9] Here, it is stayed away from the assumption that the errors are identically distributed here since heteroscedasticity is suspected. In section 3, a time-dependent variance specification will be introduced.

In order to simplify the calculation of the second derivatives of the log likelihood function, a Gaussian specification of ϵ_k is taken:

$$Z_k^* = \boldsymbol{X_k}'\boldsymbol{\beta} + \epsilon_k, \quad E[\epsilon_k|\boldsymbol{X_k}] = 0, \quad \epsilon_k \sim i.n.i.d. \; N(0, \sigma_k^2) \tag{1}$$

The assumption that the ϵ_k's are conditionally independent but not identical (*i.n.i.d.*) normal distributed, conditioned on $\boldsymbol{X_k}$ and other economic variables $\boldsymbol{W_k}$ which influence the conditional variance σ_k^2, allows for clocktime effects similar to the case of an arithmetic Brownian motion.

The core of an ordered probit analysis is the assumption that the (continuous) latent variable Z_k^* is related to the dependent variable Z_k in the following manner:

$$Z_k = \begin{cases} -4 & \text{if} \quad Z_k^* \leq \alpha_1 \\ -3 & \text{if} \quad \alpha_1 < Z_k^* \leq \alpha_2 \\ \vdots & \vdots \qquad\quad \vdots \\ +4 & \text{if} \quad Z_k^* > \alpha_8. \end{cases} \tag{2}$$

If the unobservable latent variable Z_k^* lies below the boundary α_1, the corresponding transaction price change Z_k is -4 ticks. If Z_k^* lies in the interval $(\alpha_1, \alpha_2]$, Z_k is -3 ticks, and so on. It is crucial to note that Z_k^* cannot be directly observed. It can, due to the

[7]The concept of transaction time was adopted by several authors in financial literature. See, i.e. Hasbrouck (1992, p. 582), Lo and MacKinlay (1988, p. 44) Hausman et al. (1992, p. 323) and Tauchen and Pitts (1983, p. 488).

[8]On the average of the 21 trading days of the sample in question, there are 244 trades at DTB and 278 trades at LIFFE each day that occur at the very same second.

[9]Vectors and matrices are printed in boldface hereafter.

relation given by equation (2), be infered from the observable variable Z_k to Z_k^*.
Ordered probit models estimate the conditional probability of the dependent variable falling into one of the several categories. The conditional probability to observe a transaction price change of s_i ticks is given by

$$Pr(Z_k = s_i|\boldsymbol{X_k}) \quad = \begin{cases} Pr(\boldsymbol{X_k}'\boldsymbol{\beta} + \epsilon_k \leq \alpha_1|\boldsymbol{X_k}) & \text{if} \quad i = 0, \\ Pr(\alpha_{i-1} < \boldsymbol{X_k}'\boldsymbol{\beta} + \epsilon_k \leq \alpha_i|\boldsymbol{X_k}) & \text{if} \quad 1 < i < m \\ Pr(\alpha_{m-1} < \boldsymbol{X_k}'\boldsymbol{\beta} + \epsilon_k|\boldsymbol{X_k}) & \text{if} \quad i = m. \end{cases} \tag{3}$$

The ϵ_k's are assumed to be Gaussian distributed. Therefore, equation (3) can be rewritten as[10]

$$Pr(Z_k = s_i|\boldsymbol{X_k}) = \begin{cases} \Phi\left(\dfrac{\alpha_1 - \boldsymbol{X_k}'\boldsymbol{\beta}}{\sigma_k}\right) & \text{if} \quad i = 0 \\ \Phi\left(\dfrac{\alpha_i - \boldsymbol{X_k}'\boldsymbol{\beta}}{\sigma_k}\right) - \Phi\left(\dfrac{\alpha_{i-1} - \boldsymbol{X_k}'\boldsymbol{\beta}}{\sigma_k}\right) & \text{if} \quad 1 < i < m \\ 1 - \Phi\left(\dfrac{\alpha_{m-1} - \boldsymbol{X_k}'\boldsymbol{\beta}}{\sigma_k}\right) & \text{if} \quad i = m, \end{cases} \tag{4}$$

where $\boldsymbol{\Phi}(...)$ denotes the standard normal cumulative distribution function.
It can be seen from equation (4), that the normality assumption is not a strong one since shifting the boundaries α_i can make ordered probit fit any arbitrarily chosen multinominal distribution.
Equation (4) highlights that the ordered probit model estimates the conditional probability of the dependent variable to fall into one of the categories, conditional on the vector of exogenous variables $\boldsymbol{X_k}$.
In order to keep the number of states, m, to be estimated finite, transaction price changes Z_k are restricted to take on three different distinct values. A price change $Z_k = -1$ represents a price change of -1 ticks *or less* and a price change $Z_k = +1$ represents a price change of +1 ticks *or more*.
Although the ordered probit model distinguishes between price changes equal to +1 and price changes larger than +1, an enlargement of m would not affect the estimated vector of coefficients, $\hat{\boldsymbol{\beta}}$, asymptotically.[11] Furthermore, the most often observed price changes are -1, 0 and +1, so that the model captures all relevant information even if this restriction is imposed.[12]

Equation (4) is estimated with nonlinear maximum-likelihood. Maximum-likelihood estimation works as follows. If the variables in the sample are indepedently distributed and discrete, the joint probability to observe a vector of price changes $\boldsymbol{Z}= (\boldsymbol{Z_1}, \boldsymbol{Z_2}, \ldots, \boldsymbol{Z_n})'$, given the vector of explanatory variables $\boldsymbol{X}= (\boldsymbol{X_1}, \boldsymbol{X_2}, \ldots, \boldsymbol{X_n})'$, is

$$Pr(\boldsymbol{Z}|\boldsymbol{X}) = \prod_{i=1}^{m} Pr(s_i \mid \boldsymbol{X_k}, \boldsymbol{\theta}), \tag{5}$$

where, in the current example, $\boldsymbol{\theta}$ consists of the vector of slope parameters $\boldsymbol{\beta}$ and the partition boundaries α_i. The maximum likelihood estimate (MLE) of $\boldsymbol{\theta}$ is the vector of parameters that maximizes the probability to obtain the observed vector of price

[10] Since, e.g., $Pr\left(\frac{\epsilon_k}{\sigma_k} \leq \frac{\alpha_1 - \boldsymbol{X_k}'\boldsymbol{\beta}}{\sigma_k}\right) = \boldsymbol{\Phi}\left(\frac{\alpha_1 - \boldsymbol{X_k}'\boldsymbol{\beta}}{\sigma_k}\right)$.

[11] There are far over 1,000 observation in the sample so that asymptotic properties are preserved.

[12] For example, for Aug. 08 and DTB (LIFFE) data 92 % (91 %) of the price changes were -1, 0 or +1 tick.

changes $\boldsymbol{Z}$ given $\boldsymbol{X}$. The likelihood function is the joint probability distribution of the sample written as a function of $\boldsymbol{\theta}$.
The MLE has many desirable properties. Given correct specification, it is asymptotically unbiased, consistent and efficient.[13] [14]
As soon as equations (4) and (5) is specified, it is straightforward to derive the loglikelihood function $\mathcal{L}(\boldsymbol{Z}|\boldsymbol{X_k})$:

$$
\begin{aligned}
\mathcal{L}(\boldsymbol{Z}|\boldsymbol{X_k}) \;=\; & \sum_{k=1}^{K}\left\{Y_{1k}\cdot\log\Phi\left(\frac{\alpha_1-\boldsymbol{X_k}'\boldsymbol{\beta}}{\sigma_k}\right)\right. \\
&+\sum_{i=2}^{m-1} Y_{ik}\cdot\log\left[\Phi\left(\frac{\alpha_i-\boldsymbol{X_k}'\boldsymbol{\beta}}{\sigma_k}\right)-\Phi\left(\frac{\alpha_{i-1}-\boldsymbol{X_k}'\boldsymbol{\beta}}{\sigma_k}\right)\right] \qquad (6)\\
&+\left. Y_{mk}\cdot\log\left[1-\Phi\left(\frac{\alpha_{m-1}-\boldsymbol{X_k}'\boldsymbol{\beta}}{\sigma_k}\right)\right]\right\},
\end{aligned}
$$

where Y_{ik} is an indicator variable that is coded one if the kth transaction price change Z_k equals s_i in state i and zero otherwise. K denotes the total number of transactions at a particular trading day.
Since the log-likelihood function (6) is highly nonlinear, numerical optimization is required. For the GAUSS implementation of the ordered probit model which was used for estimation, the Newton-Raphson algorithm is chosen. One of the major advantages of Newton-Raphson is that it calculates the Hessian - which can be used as an estimator for the information matrix and thus for the variance-covariance matrix - at any iteration, and hence the Hessian at the very last iteration as well.[15]
All ordered probit estimations were run using the GAUSS program while the software package STATA was used for descriptive statistics.

2 The dataset

The BUND-future has recently attracted much attention in literature. One of the major reasons why this financial product inspires so many researchers is probably that the BUND future is traded in almost identical design at two entirely different trading systems. Furthermore, the two markets have almost the same trading hours. Thus, differences in the pricing of the BUND future at the two exchanges should be due to the trading method. DTB is an electronic exchange while at LIFFE trading is, despite a short computer trading period, conducted via floor trading.
It is worthwhile to take down some of the major similarities and differences between the two competing systems. The BUND-future is an agreement between buyer and seller to exchange a DM 250,000 nominal value notional long-term (8 1/2 - 10 years) German government bond with 6 % coupon, at a fixed date, for cash with delivery four times a year.[16] LIFFE was the firstcomer in the BUND-future market. It commenced trading in September 1988, DTB followed in November 1990. LIFFE opens trading at 07:30 London time (GMT). Until 16:15 (GMT), trading is conducted via a continuous floor auction called Open Outcry (OOC). After this period and a five-minute break, a computerized screen-based version of OOC called "Automated Pit Trading" (APT),

[13] The variance of an MLE is given by the Cramer-Rao lower bound asymptotically.
[14] A more thorough discussion of the MLE and its utilization in microeconometrics is provided, e.g., by Ronning (1991, ch. 1.3).
[15] See Hamilton (1994, pp. 138) for details.
[16] See LIFFE (1994) and DTB (1990) for details.

where the names of the traders appear on the screen is in place and lasts until 17:55 (GMT). Bouwman et al. (1994, p. 16) argue that APT trading is merely used to offset positions at the end of a trading day.
DTB opens at 8:00 and trading continuous until 17:00 (Frankfurt time) via a fully automated electronic system. The fixing period[17] is from 12:30 to 14:30 (Frankfurt time). In this paper, time is measured according to Franfurt time.
The two exchanges closely follow each other. Moreover, Bouwman et al. (1994, p. 23), running Granger–Sims causality tests, state that no hint for a lead of either of the exchanges is given. The two time series deviate, if at all, by just one tick in both directions.
DTB and LIFFE differ sharply in the way trading is conducted. It will be drawn on Franke and Hess's (1995) overview of the similarities and differences between the two exchanges. The electronic screen-based trading at DTB is anonymous, the names of traders do not appear on the screen. If names appeared on the screen, market participants could assess the trustworthiness of their prospective counterparts. Trustworthiness in this sense means that a trader may not take advantage of superior information. That is, it is easier to infer whether an order was placed by an insider or if it was motivated by liquidity needs.[18] Floor trading reveals more information about who is trading. Although it is not obvious whether a trader is privately informed, floor trading may support longstanding relationships between exchange members which cause the participants to have incentives to signal if their trade is private–information motivated or not. If a single trader received good news about an asset's value, she would be tempted to buy a large share at the current price. The other market participants would then punish her by charging bad prices for future transactions. According to Benviste et al. (1992, p. 63), identification and sanctioning is easier in a dealership market like OOC than in electronic exchanges.
On the other hand, in an electronic market valuable information is provided by the limit order book, where traders specify a maximum price they are willing to pay on a purchase and a minimum price they are willing to accept on a sale. If the order book indicates, e.g., many sell orders, traders can expect a trend towards price detoriation. Other similarities and differences between the two competing trading systems are discussed in Pagano and Röll (1992) and Madhavan (1992).
The comparison of DTB and LIFFE is concluded with an overview on recent empirical findings on the advantages of the one or the other trading system. According to Pirrong (1996), the DTB Bund market is both more liquid and deeper than OOC at LIFFE. Franke and Hess (1995) state that in periods of low information intensity, the insight into the order book of DTB is more valuable with respect to its information content than floor trading, while the reverse holds if the information intensity is high. Bouwman et al. (1994) come to the conclusion that LIFFE tends to be the dominant exchange and that the computerized systems DTB and APT are hurt by large bid–ask spreads.

The data of both DTB and LIFFE include the time when the transaction took place (rounded to the nearest second), the transaction price (measured in percentage points of the BUND future's nominal value) and trading volume (measured in number of traded shares). The LIFFE data also provide bid and ask prices, but trading volume is not correctly taken down. Transactions are manually recorded at LIFFE. The person who is in charge of noting bid, ask and transaction prices as well as trading

[17] In this period the underlying asset itself is traded.
[18] The difference between liquidity and noise traders is pointed out in section 4.

volume may at times not be able to take down all these information immediately and correctly. In these cases she simply takes down a "40" for trading volume. Although the individual trading volume is not reliable, so is the total trading volume for each trading day. Things change as soon as APT is in operation where trades are conducted electronically and traders have to enter their names, their desired trading volume and their ask and bid prices.

TABLE 1

DESCRIPTIVE STATISTICS FOR A REPRESENTATIVE TRADING DAY

	Variable	Obs.	Mean	std. dev.	Min.	Max.	Median
DTB:	p_k	2,801	9,282	12.6655	9,255	9,318	9,280
	Z_k	2,800	-0.1264	0.8738	-9	5	0
	Δt_k	2,800	5.3979	10.7370	0	203	3
	vol_k	2,801	21.2738	32.1680	1	687	11
LIFFE:	p_k	2,750	9,282	12.9847	9,258	9,313	9,279
	Z_k	2,749	0.1127	0.9053	-3	4	0
	bas_k	2,750	1.0469	0.4370	0	4	1
	Δt_k	2,749	8.7822	16.0999	0	295	5
	vol_k	2,750	35.8847	25.1965	1	500	40
	ss_k	2,750	0.5884	0.4922	0	1	1
	bb_k	2,750	0.0680	0.2518	0	1	0

Table 1 gives some descriptive statistics for Aug. 09, 1994. p_k denotes the transaction price of the BUND–future measured in percentage points of the nominal value times 100. Z_k denotes the transaction price change (in ticks), Δt_k the time elapsed between transactions (in seconds), vol_k the trading volume (in number of traded shares), bas_k the bid–ask spread (in ticks). The dummy variables ss_k and bb_k denote sell/sell and buy/buy trade sequences that will be described in section 3. The subscript k indicate the kth transaction and $k = 1, \ldots, K$, where K is the total number of transactions.

Therefore, as long as the bid/ask–spread is of interest, the LIFFE dataset is taken. The DTB data are used to draw conclusion on the influence of trading volume on the conditional mean and the conditional variance of the latent variable Z_k^*.
The dataset contains 21 trading days ranging from August 01 to August 30, 1994.[19]
On the average of the 21 trading days, trades occur in almost any three minutes interval at both exchanges. At DTB, trades are observed in 97.3 % of the three minutes intervals while at the LIFFE trades take place in 98.0 % of the intervals.
The average number of contracts per trade is 13.49 at DTB and 35.61 at LIFFE. LIFFE – the firstcomer in the BUND–futures market – still attracts most of the trading volume. In the sample period, its average market share per trading day was 68.06 %. On average, 1.1 times more transactions occur at LIFFE than at DTB.
The average of 2.06 trades (DTB) and 2.13 trades (LIFFE) per minute exemplifies the high liquidity of both markets.[20]
Table 1 presents some summary statistics for Aug. 9, 1994 and the most important variables. The bid-ask spread is the difference between the ask price – the price at which a trader is willing to sell shares – and the bid price – the price at which a trader

[19]Due to a banking holiday in Great Britain on August 29, only 21 instead of 22 trading days are present.

[20]Franke and Hess (1995, pp. 16), Bouwman et al. (1994, pp. 10) and Franses et al. (1994, pp. 2) have, although using data from 1992, find quite similar descriptive statistics on DTB and LIFFE trading.

is willing to buy. A transaction is said to be seller (buyer)-initiated if the price was settled above (below) the midpoint of the quoted bid-ask spread.
The only data manipulation that took place with the LIFFE data, was that all observation with a negative bid-ask spread were deleted. No manipulation was done with the DTB data.

3 A basic specification and estimation results

As it was mentioned in section 1, the error term ϵ_k is assumed to be not identically distributed. Therefore, both a mean and a variance function have to be specified. In addition, since there are two different datasets containing different variables, two different models for the competing exchanges have to be found.
It will be shown in section 4 that the specifications described below are strictly motivated by economic theory. The present section aims to provide some intuition on how ordered probit models work and on what variables generally determine transaction price changes and volatilities. The later presentation of economic theory is purely didactical.

The conditional mean of Z_k^* for the DTB model

The DTB dataset contains transaction prices, correct trading volume and the corresponding clocktime. The basic specification tries to capture three major effects suspected to have an impact on the mean function. These are time elapsed between successive trades, Δt_k, lagged price changes Z_{k-l} ($l = 1,2,3$) and the natural logarithm of trading volume vol_{k-l} ($l = 1,2,3$). Logarithms are taken to allow volume to enter the conditional mean and the conditional variance nonlinearly. This follows Hasbrouck (1988, p. 249), while Hausman et al. (1992, p. 342) use a Box-Cox transformation of trading volume.[21] Hence, the vector of exogenous variables $\boldsymbol{X_k}^{DTB}$ that governs the conditional mean of Z_k^* in the DTB model is given by

$$\boldsymbol{X_k}^{DTB} = \Big(\Delta t_k, Z_{k-1}, Z_{k-2}, Z_{k-3}, log(vol_{k-1}), log(vol_{k-2}), log(vol_{k-3})\Big)'. \qquad (7)$$

The conditional mean of Z_k^* for the LIFFE model

Since trading volume is not reliable at LIFFE, it is not included it in the mean function, here. Instead, bid and ask prices are used which are taken down properly at LIFFE. Hausman et al. (1992, p. 341)[22] generate a buy/sell indicator variable IBS_k which is coded 1 (-1) if the corresponding transaction price is larger (smaller) than the average of the quoted bid-ask spread and zero if it is equal to the bid-ask spread:

$$IBS_{k-l} = \begin{cases} 1 & \text{if} \quad Z_{k-l} > \frac{Z_{k-l}^a + Z_{k-l}^b}{2} \\ 0 & \text{if} \quad Z_{k-l} = \frac{Z_{k-l}^a + Z_{k-l}^b}{2} \\ -1 & \text{if} \quad Z_{k-l} < \frac{Z_{k-l}^a + Z_{k-l}^b}{2}, \end{cases} \qquad (8)$$

where Z_{k-l}^a is the ask price at $k-l$ and Z_{k-l}^b is the associated bid price ($l = 1,2,3$). These buy/sell indicators are not specified here. Instead, trade sequence dummies are included in the conditional mean of Z_k^*. The sell/sell trade sequence dummy ss_k is

[21] The Box-Cox parameter λ proved to be zero in the Hausman et al. paper, suggesting that taking logarithms is appropriate anyway.

[22] Their idea can be traced back to Hasbrouck (1988, p. 247).

coded one (and zero otherwise) if the last two trades are seller-initiated ($IBS_{k-1} = -1$ and $IBS_{k-2} = -1$) while a buy/buy sequence bb_k is coded one (and zero otherwise) if these transactions are buyer-initiated ($IBS_{k-1} = 1$ and $IBS_{k-2}=1$).[23] Other trade sequences are not included. $\boldsymbol{X_k}$ in the LIFFE model thus consists of the elements:

$$\boldsymbol{X_k}^{LIFFE} = \left(\Delta t_k, Z_{k-1}, Z_{k-2}, Z_{k-3}, ss_{k-1}, bb_{k-1}\right)'. \tag{9}$$

The conditional variance of Z_k^* for the DTB and LIFFE model

The variance σ_k^2 is conditioned upon a vector of exogenous variables $\boldsymbol{W_k}$. It is assumed to follow an arithmetic Brownian motion which means that it is made proportional to the time elapsed between two successive trades, Δt_k:

$$\sigma_k^2 \;=\; \gamma_1 \, \Delta t_k. \tag{10}$$

The inclusion of Δt_k is well motivated by literature.[24] Besides time between trade, there are a few other variables that, according to market microstructure literature and empirical investigations, determine the conditional variance of transaction price changes. The DTB specification includes lagged transformed trading volume, while the LIFFE model contains lagged bid-ask spread, BAS_{k-1}, instead and a dummy variable $DVOL_{k-1}$ which is coded one if trading volume is not equal to 40 or exceeds 99, that is, it is probably correctly noted. $DVOL_{k-1}$ is included in order to save some information about trading volume. The conditional variance σ_k^2 then is[25]

$$\sigma_k^2 \;=\; exp\Big(\gamma_1 \, \Delta t_k \;+\; \gamma_2 \, BAS_{k-1} \;+\; \gamma_3 \, DVOL_{k-1}\Big) \tag{11}$$

for LIFFE and

$$\sigma_k^2 \;=\; exp\Big(\widetilde{\gamma_1} \, \Delta t_k \;+\; \widetilde{\gamma_2} \, log(vol_{k-1})\Big) \tag{12}$$

for DTB.

The exponential function is taken to avoid negative variances. A constant is neither included in the conditional variance or in the conditional mean of Z_k^*. This is due to the identification problem that arises in models for qualitative dependent variables. The present ordered probit specification restricts transaction price changes to take on three different discrete values. It thus contains two boundries, α_1 and α_2. An inclusion of a constant in the conditional mean and the conditional variance would render these boundaries unidentifieable. Any linear transformation would yield the same probabilty that Z_k falls into one particular category. Two solutions apply here. The first one is to set one of the boundaries equal to zero. The second is to not include a constant in the specification. For current purposes the latter solution is chosen.

Due to the large sample sizes, only coefficients significantly different from zero at the 5 % level or better will be considered. The estimation results are presented in tables 2 (LIFFE) and 3 (DTB) for three representative days and can be summarized as follows:

[23] Regressions with sell/sell/sell and buy/buy/buy dummy variables were run as well. None of these sequences proved to be significantly different from zero for more than five trading days. This may be simply due to the fact that there are too few such sequences.

[24] Glosten and Harris (1988, p. 128) as well as Grammatikos and Saunders (1986, p. 322) include Δt_k in their variance specifications. Easley and O'Hara (1992) thoroughly discuss the impact of time between trade on transaction prices. Some of these approaches will be discussed in section 4.

[25] In earlier specifications an APT indicator variable was additionally included but it turned out to be highly correlated with Δt_k.

Δt_k : Time between trade affects the conditional mean of Z_k^* quite differently at both exchanges. Δt_k does not have explanatory power at LIFFE but turned out to be negative and significant at ten (out of 21) days at DTB. This implies that with much time elapsing between successive trades, the conditional mean of Z_k^* declines. This result could be due to a correlation between Δt_k and lagged transformed volume. A reestimation where volume was neglected in the conditional mean did not yield different results. Furthermore, the negative impact of Δt_k is not due to extraordinary trading patterns on the corresponding trading days. Although the significant impact of Δt_k contradicts economic intuition.[26] the coefficient of Δt_k is very small in magnitude compared with the other coefficients.
The impact of Δt_k on the conditional variance of Z_k^* also differs across both exchanges. At LIFFE, Δt_k is not important for the conditional variance whereas it turned out to be significantly positive at DTB for any trading day. Since there is strong theoretical evidence that time between trade affects the conditional variance - this point will be discussed in section 4 -, it was tested if the variable $DVOL_{k-1}$ could have caused Δt_k not to be significant in the variance of the LIFFE specification. Therefore, an estimation was run without inclusion of $DVOL_{k-1}$. This specification did not yield different results. The finding that Δt_k is significantly positive in the conditional variance of Z_k^* at DTB is consistent with Hausman et al. (1992, p. 344). Larger waiting intervals cause prices to be more volatile. This seems to be quite intuitive: if there is a break between successive trades, the next price might deviate from the former since new information concerning the asset's value could be available meanwhile.
Z_{k-l} : The impact of lagged transaction price changes is almost the same for both exchanges. The first lag of transaction price changes turned out to be highly significant and negative at DTB as well as at LIFFE. The second lag of Z_k also was significant for only eleven (DTB) and eight (LIFFE) trading days. The third lag proved to be - if significant at all - positive. At DTB, the third lag of Z_k was five times significantly positive and one time negative, whereas it does not influence the conditional mean of Z_k^* at LIFFE.
The negative signs suggest a tendency towards price reveasals which is sometime called the "bounce effect". If, e.g., there were three successive price changes of plus one tick on Aug. 08,[27] the conditional mean of Z_k^* would change by -0.445 -0.149 -0.082 = -0.676. Successive buys (price changes of +1 or more ticks) cause negative price changes and vice versa.
This finding also holds if the last lag of Z_k is significantly positive as at some DTB trading days. This is because the first lag of Z_k is 26 times larger than the second and the third on average of the 21 trading days at DTB. The last two lags are close to each other in magnitude. Thus, the impact of the first lag overweighs the impact of the last two ones.
Trade sequences : The findings for the trade sequences sell/sell and buy/buy support the evidence for price reveasals. Both the buy/buy and the sell/sell dummy variable in the LIFFE model turned out to be significant for six trading days. A sell/sell sequence causes the conditional mean of Z_k^* to rise while two successive buys have a negative impact.
$log(vol_{k-l})$: Lagged trading volume does not significantly affect the conditional mean of Z_k^* in the DTB model. This might be due to the fact that volume is associated

[26] A negative impact of Δt_k implies that a trader who expects increased asset prices simply has to wait with proceeding her transactions since with more time elapsing between transactions, prices will eventually fall.

[27] August 08 is chosen since at that day all lags of transaction price change Z_k are significant. Note that any interpretation of non-significant coefficients is misleading.

with both positive and negative price changes so that the "overall impact" of volume on transaction price changes is zero. Although transformed volume is not important for the conditional mean, it is for the conditional variance where it turned out to be highly significant and negative for 17 trading days. Large trading volume thus lowers the volatility of price changes. This finding is clearly contradictive to economic theory: a large volume usually indicates that a trader has a high incentive to buy or sell. The other market participants observe the large amount and believe that prices will move downwards if the large trade is a sell or upward if it is a buy. In any case, the other market participants will react causing prices to move and thus volatility to increase. Therefore, a positive sign of $log(vol_{k-1})$ in the variance would be expected.
An interpretation could be that DTB and LIFFE are both highly liquid markets. I.e., large volume can be traded without causing prices to alter much. Furthermore, about 90 % of the transactions are associated with a transaction price change of +1, 0 or -1 ticks. This explains why volume might not matter in the variance but it is not strong enough to explain why volume *decreases* volatility. An argument may be that large trades make traders more secure about the direction of future price changes. If a large trade is, e.g., a buy, they might expect increasing transaction prices and prices could thus settle earlier.
$\boldsymbol{BAS_{k-1}}$: The lagged bid-ask spread has a negative impact on the conditional variance of Z_k^*. This result somehow contradicts economic intuition since prices should only alter strongly if the bid-ask spread is wide. A negative sign of the bid-ask spread indicates that large spreads have a dampening effect on volatility. However, the bid-ask spread has almost no variation. For Aug. 08., the mean bid-ask spread is 1.149 ticks, the median is 1 tick and the standard deviation is 1.744 ticks. Thus, a bid-ask spread has to be labeled "large" if it exceeds one tick. An explanation for the negative impact then is that such a "large" bid-ask spread indicates the direction of future price changes without permitting prices to actually alter much. For example, if the last transaction price was 99.89, the current ask price is 99.90 and the current bid price is 99.89, then the quote indicates that prices will change in the upward direction without allowing large price changes.
$\boldsymbol{DVOL_{k-1}}$: The dummy variable $DVOL_{k-1}$ is created since at LIFFE volume is not correctly recorded. The interpretation of the variable is a bit difficult since it is not known if trading volume is actually correctly recorded or if these non-40 shares trades are simply used in order to gain a correct overall trading volume at a trading day. $DVOL_{k-1}$ turned out to be highly significant and negative for 17 trading days. This results supports the finding for lagged trading volume in the LIFFE specification since the volume-dummy captures information about trading volume although it does not carry it explicitely.

Table 2
Estimation Results for the Basic Specification of LIFFE

LIFFE, Aug. 08, 1994

Parameter	Estimates	Std. err.	Est./s.e	P-value
Δt_k	0.0004	0.0011	0.335	0.3690
Z_{k-1}	-0.3508	0.0370	-9.482	0.0000
Z_{k-2}	-0.0865	0.0389	-2.221	0.0132
Z_{k-3}	0.0270	0.0341	0.793	0.2140
ss_k	0.1368	0.0631	2.168	0.0151
bb_k	0.0146	0.0615	0.237	0.4063
α_1	-0.6571	0.0505	-13.001	0.0000
α_2	0.8364	0.0568	14.730	0.0000
Δt_k	0.0037	0.0010	3.648	0.0001
BAS_{k-1}	0.0171	0.0217	0.789	0.2150
$DVOL1_k$	-0.5796	0.0562	-10.313	0.0000

number of observations: 1,819

LIFFE, Aug. 11, 1994

Parameter	Estimates	Std. err.	Est./s.e	P-value
Δt_k	-0.0000	0.0011	-0.005	0.4979
Z_{k-1}	-0.0171	0.0049	-3.483	0.0002
Z_{k-2}	0.0046	0.0043	1.070	0.1423
Z_{k-3}	-0.0040	0.0043	-0.936	0.1745
ss_k	0.1411	0.0320	4.405	0.0000
bb_k	-0.1132	0.0355	-3.188	0.0007
α_1	-0.4140	0.0296	-13.972	0.0000
α_2	0.5082	0.0311	16.326	0.0000
Δt_k	0.0013	0.0015	0.875	0.1907
BAS_{k-1}	-0.0121	0.0147	-0.823	0.2052
$DVOL1_{k-1}$	-0.5572	0.0433	-12.855	0.0000

number of observations: 3,968

LIFFE, Aug. 18, 1994

Parameter	Estimates	Std. err.	Est./s.e	P-value
Δt_k	-0.0012	0.0010	-1.138	0.1276
Z_{k-1}	-0.1025	0.0181	-5.668	0.0000
Z_{k-2}	0.0003	0.0184	0.015	0.4940
Z_{k-3}	0.0186	0.0146	1.276	0.1009
ss_{k-1}	0.0080	0.0405	0.198	0.4214
bb_{k-1}	-0.0178	0.0402	-0.442	0.3294
α_1	-0.4427	0.0333	-13.303	0.0000
α_2	0.4073	0.0332	12.250	0.0000
Δt_k	0.0010	0.0018	0.531	0.2978
BAS_{k-1}	0.0432	0.0231	1.871	0.0306
$DVOL1_{k-1}$	-0.7546	0.0539	-13.997	0.0000

number of observations: 3,729

Table 2 presents ordered probit estimation results for three representative trading days at LIFFE. α_i $(i = 1, 2)$ denotes the partition boundaries. The coefficients below the boundaries are for variance. "P-value" denotes the significance level at which the null hypothesis would just be significant.

TABLE 3

ESTIMATION RESULTS FOR THE BASIC SPECIFICATION OF DTB

DTB, Aug. 08, 1994

Parameter	Estimates	Std. err.	Est./s.e	P-value
Δt_k	-0.0027	0.0013	-2.104	0.0177
Z_{k-1}	-0.4450	0.0494	-9.009	0.0000
Z_{k-2}	-0.1492	0.0406	-3.672	0.0001
Z_{k-3}	-0.0823	0.0375	-2.194	0.0141
$log(vol_{k-1})$	-0.0225	0.0545	-0.412	0.3401
$log(vol_{k-2})$	0.0354	0.0543	0.653	0.2570
$log(vol_{k-3})$	0.0411	0.0543	0.758	0.2244
α_1	-0.7518	0.0983	-7.649	0.0000
α_1	0.8216	0.1003	8.190	0.0000
Δt_k	0.0032	0.0010	3.289	0.0005
$log(vol_{k-1})$	-0.0103	0.0538	-0.191	0.4244

number of observations: 1,488

DTB, Aug. 11, 1994

Parameter	Estimates	Std. err.	Est./s.e	P-value
Δt_k	-0.0014	0.0017	-0.860	0.1950
Z_{k-1}	-0.2583	0.0235	-11.001	0.0000
Z_{k-2}	-0.0272	0.0190	-1.427	0.0768
Z_{k-3}	0.0258	0.0182	1.423	0.0773
$log(vol_{k-1})$	0.0949	0.0331	2.865	0.0021
$log(vol_{k-2})$	-0.0313	0.0322	-0.973	0.1653
$log(vol_{k-3})$	0.0095	0.0321	0.295	0.3838
α_1	-0.5394	0.0653	-8.258	0.0000
α_2	0.7786	0.0696	11.186	0.0000
Δt_k	0.0060	0.0016	3.777	0.0001
$log(vol_{k-1})$	-0.1127	0.0376	-3.001	0.0013

number of observations: 3,549

DTB, Aug. 18, 1994

Parameter	Estimates	Std. err.	Est./s.e	P-value
Δt_k	-0.0025	0.0023	-1.103	0.1351
Z_{k-1}	-0.1622	0.0177	-9.176	0.0000
Z_{k-2}	0.0003	0.0154	0.019	0.4925
Z_{k-3}	0.0296	0.0149	1.983	0.0237
$log(vol_{k-1})$	-0.0247	0.0316	-0.782	0.2172
$log(vol_{k-2})$	-0.0480	0.0304	-1.581	0.0569
$log(vol_{k-3})$	0.0309	0.0304	1.015	0.1551
α_1	-0.7007	0.0622	-11.263	0.0000
α_2	0.5404	0.0593	9.118	0.0000
Δt_k	0.0152	0.0022	6.953	0.0000
$log(vol_{k-1})$	-0.1698	0.0354	-4.799	0.0000

number of observations: 3,915

Table 3 presents ordered probit estimation results for three representative trading days at DTB. α_i $(i = 1, 2)$ denotes the partition boundaries. The coefficients below the boundaries are for variance. "P-value" denotes the significance level at which the null hypothesis would just be significant.

If the non-40 volumes are indeed recorded correctly (which is unclear), another way to interpret the effect of $DVOL_{k-1}$ is to regard it as a "slow trade" dummy because a proper manual recording is easier if few trades occur per minute. This is a somewhat hazardous assumption since it completely contradicts the earlier finding of the impact of Δt_k on the conditional variance of Z_k^*.[28] The assumption can be tested by including a "slow trade indicator" in the conditional variance of Z_k^*. This indicator is coded one if Δt_k is below its median and zero otherwise. The indicator is then interacted with actual time between trade.[29] Such a "small time between trade" variable specification did not have a significant impact on the conditional variance. Therefore, $DVOL_{k-1}$ does not capture information about the trading intensity.

Cuttoff points and convergence: The boundaries α_1 and α_2 are measured with high precison as indicated by high t-statistics. A comparison of trading days exhibits that the boundaries are close to each other in magnitude. Note that a comparison is, due to the scaling of the coefficients by σ_k, only possible if the estimated $\hat{\boldsymbol{\beta}}$'s are divided by an arbitrarily chosen other coefficient so that the σ_k's cancel out. Both boundaries are very close to each other in absolute magnitude.

Evidence for heteroscedasticity: The coefficients in the variance specification are – except Δt_k at LIFFE – highly significant. Therefore, homoscedasticity can be rejected for both DTB and LIFFE.

A test for order flow dependence

So far, first ordered probit estimation results were discussed. The last lines of this section are mainly concerned with two questions: *(a)* does order-flow dependence exist and *(b)* are OOC and APT trading significantly different from each other at LIFFE.

That prices are not set independently of the entire trading history is mentioned by Easley and O'Hara (1992, p. 589) who state that "watching past market outcomes is informative". Prices hence may not satisfiy the Markov property. That is, they depend on the order flow. A sequence of price changes such as –1/–1/1 is then likely to have a quite different impact on the conditional mean of Z_k^* than a sequence of 1/–1/–1, although the total price change is the same. This implies that the coefficients of lagged price changes Z_{k-l} (where $l = 1,2,3$) are not identical. The *individual* price change and not the *sum* of price changes matters in the determination of the conditional mean. If the former is true, then the three lags of the transaction price changes should not be identical. It is straigthfoward to test this hypothesis with a Wald test since this test, alongside with the Lagrange ratio (LR) and the Lagrange mutiplier (LM) test, is based on the maximum likelihood principle and requires only the estimation of the unrestricted model.[30] Wald, LR and LM tests are intended to test hypothesis involving more than one coefficient. The null hypothesis is that the coefficients of lagged price changes are equal, that is, $H_0: \hat{\boldsymbol{\beta}}_{Z_{k-1}} = \hat{\boldsymbol{\beta}}_{Z_{k-2}} = \hat{\boldsymbol{\beta}}_{Z_{k-3}}$. A linear H_0 is yield by multiplying the vector of estimated parameters $\hat{\boldsymbol{v}} = (\hat{\boldsymbol{\beta}}_{Z_{k-1}}, \hat{\boldsymbol{\beta}}_{Z_{k-2}}, \hat{\boldsymbol{\beta}}_{Z_{k-3}})'$ by a 2×3 matrix $\boldsymbol{R}$, where

$$\boldsymbol{R} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}, \tag{13}$$

[28] No evidence was found for collinearity of Δt_k and $DVOL_{k-1}$ when the one of these variables was neglected.

[29] That is,

$$small\Delta t_k = \begin{cases} \Delta t_k & \text{if} \quad \Delta t_k < \text{median}(\Delta t_j) \\ 0 & \text{otherwise,} \end{cases}$$

where $j = 1, \ldots, K$.

[30] See, e.g., Ronning (1991, 1.3.4) for details on tests based on the maximum likelihood principle.

so that $\boldsymbol{R}\hat{\boldsymbol{v}} = 0$. The number of $\boldsymbol{R}$'s rows is given by the number of restrictions (which is two: $\hat{\boldsymbol{\beta}}_{Z_{k-1}} = \hat{\boldsymbol{\beta}}_{Z_{k-2}}$ and $\hat{\boldsymbol{\beta}}_{Z_{k-2}} = \hat{\boldsymbol{\beta}}_{Z_{k-3}}$) and the number of columns is given by the number of coefficients involved (which is three). The Wald–statistic is

$$\hat{\boldsymbol{v}}' \, \boldsymbol{R}(\hat{\boldsymbol{v}})' \left(\boldsymbol{R}(\hat{\boldsymbol{v}}) \, V[\hat{\boldsymbol{v}}] \, \boldsymbol{R}(\hat{\boldsymbol{v}})' \right)^{-1} \boldsymbol{R}(\hat{\boldsymbol{v}}) \, \hat{\boldsymbol{v}} \sim \chi^2_r, \tag{14}$$

where $V[\hat{\boldsymbol{v}}]$ denotes the estimated variance–covariance matrix. The Wald statistic is χ^2 distributed with r degrees of freedom equal to the number of restrictions. For two degrees of freedom, the critical values are 13.82, 10.60 and 4.605 at the one, five and ten % significance level.[31] The H_0 has to be rejected if the Wald statistic exceeds the critical value.

Order flow dependence can be easily rejected for both DTB and LIFFE at the usual significance levels. The test statistics for LIFFE are presented in table 4.

Inspection of the coefficients $\hat{\boldsymbol{\beta}}_{Z_{k-1}}$, $\hat{\boldsymbol{\beta}}_{Z_{k-2}}$ and $\hat{\boldsymbol{\beta}}_{Z_{k-3}}$ shows that the rejection might be due the fact that the first lag is considerably larger than the second and the third. The second and the third are quite close to each other in magnitude.

Therefore, a Wald test was applied to test if the second and the third lags of Z_k For both DTB and LIFFE, equality of the second and the third lag of Z_k can generally *not* be rejected at the common significance levels. The Wald statistics are usually higher at LIFFE than at DTB.

TABLE 4
A WALD TEST FOR ORDER FLOW DEPENDENCE (LIFFE DATA)

date	$\hat{\beta}_{Z_{k-1}}$	$\hat{\beta}_{Z_{k-2}}$	$\hat{\beta}_{Z_{k-3}}$	Wald-stat.	P-val.
Aug 01	-0,1995	0,0047	0,0572	45,57	0,0000
Aug 02	-0,2103	-0,0427	-0,0401	40,11	0,0000
Aug 03	-0,2573	-0,0192	0,0120	71,04	0,0000
Aug 04	-0,2038	-0,0102	0,0197	0,00	0,9875
Aug 05	-0,1664	-0,0440	-0,0197	22,48	0,0000
Aug 08	-0,3508	-0,0865	0,0270	74,06	0,0000
Aug 09	-0,3060	-0,0577	-0,0163	81,62	0,0000
Aug 10	-0,3430	-0,0810	-0,0288	69,58	0,0000
Aug 11	-0,0171	0,0046	-0,0288	11,67	0,0029
Aug 12	-0,0659	-0,0474	-0,0040	10,01	0,0067
Aug 15	-0,1973	0,0099	0,0095	23,64	0,0000
Aug 16	-0,1642	0,0141	-0,0215	46,78	0,0000
Aug 17	-0,1942	-0,0152	-0,0078	55,27	0,0000
Aug 18	-0,1025	0,0003	0,0186	33,77	0,0000
Aug 19	-0,2234	-0,0423	0,0095	50,67	0,0000
Aug 22	-0,1469	0,0564	0,0337	66,10	0,0000
Aug 23	-0,1647	-0,0310	-0,0261	43,59	0,0000
Aug 24	-0,2243	-0,0208	-0,0261	60,54	0,0000
Aug 25	-0,3436	-0,0099	-0,0239	68,36	0,0000
Aug 26	-0,2203	-0,0822	-0,0455	37,24	0,0000
Aug 30	-0,1976	-0,0417	-0,0091	37,58	0,0000

Table 4 shows Wald test results of a test for order flow dependence. The first three colimns present the coefficients tested for equality. "P-value" denotes the significance level at which the null hypothesis would just be significant.

[31]The critical values for the χ^2 distribution are here and hereafter taken from Jeske (1995, p. 317).

Are APT and OOC charateristically different?
Some differences in the emprical findings of the DTB and LIFFE specification were dicussed above. A comparison of the emprirical findings for the dealership market OOC and the electronic trading system APT at LIFFE should yield further insights on how explanatory variables affect price changes under different trading systems. Note, however, that APT is primarily used to offset positions at the end of a trading day so that differences may not neccessarily due to different trading systems.
In least squares models it would be straightforward to estimate the paramameters for OOC and APT separately and then to run a Chow test. A Chow test cannot be applied here since residuals are not obtained.
This problem can be solved by splitting up the explanatory variables into two groups. That is, explanatory variables which are observed during OOC are separated from those observed during APT. Both are then included in the same specification. The vector of explanatory variables for the conditional mean of Z_k^*, $\boldsymbol{X_k}$, is then given by:

$$\boldsymbol{X_k} = \left(\Delta t_k^{OOC}, \Delta t_k^{APT}, Z_{k-l}^{OOC}, Z_{k-l}^{APT}, ss_k^{OOC}, ss_k^{APT}, bb_k^{OOC}, bb_k^{APT}\right)', \tag{15}$$

where $l = 1,2,3$. For the conditional variance, the vector of explanatory variables, $\boldsymbol{W_k}$, is given by

$$\boldsymbol{W_k} = \left(\Delta t_k{}^{OOC}, \Delta t_k{}^{APT}, BAS_{k-1}^{OOC}, BAS_{k-1}^{APT}, DVOL1_k\right)'. \tag{16}$$

The conditional mean and the conditional variance were estimated separately. A partitioning of all explanatory variables caused the maximum likelihood function not to converge.[32]
The null hypothesis of equality of the explanatory variables in the *conditional mean* of Z_k^* leads to the following null hypothesis:

$$H_0 : \Delta t_k^{OOC} = \Delta t_k^{APT}, Z_{k-l}^{OOC} = Z_{k-l}^{APT}, ss_k^{OOC} = ss_k^{APT}, bb_k^{OOC} = bb_k^{APT}, \tag{17}$$

where $l = 1,2,3$. The matrix $\boldsymbol{R}$ has then the dimension 6×12, since there are six restrictions and twelve coefficients. $\boldsymbol{R}$ is then given by:

$$\boldsymbol{R} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdot & & & & & & & & & & & \cdot \\ \cdot & & & & & & & & & & & \cdot \\ \cdot & & & & & & & & & & & \cdot \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}. \tag{18}$$

The vector of coefficients, $\hat{\boldsymbol{v}}$, consists of the coefficients that are tested for equality in the mean function: $\hat{\boldsymbol{\beta}}_{\Delta t_k^{OOC}}, \hat{\boldsymbol{\beta}}_{\Delta t_k^{APT}}, \hat{\boldsymbol{\beta}}^{OOC}_{Z_{k-l}}, \hat{\boldsymbol{\beta}}^{APT}_{Z_{k-l}}, \hat{\boldsymbol{\beta}}_{bb_k^{OOC}}, \hat{\boldsymbol{\beta}}_{bb_k^{APT}}$ ($l = 1,2,3$).
Equality of the coefficients corresponding to OOC and those corresponding to APT trading has to be rejected if the Wald statistic exceeds the critical values 22.46, 18.55 and 16.81 (one, five and ten % significance level). 13 trading days did not yield convergence.[33] The H_0 cannot be rejected ten times at the ten % significance level, one time at the five % and two times at the one % significance level. With regard to the sample size, the impact of the explanatory variables seems actually depend on the trading system. The test statistics are presented in table 5.
An inspection of the coefficients shows that the inequality might be due to the diffe-

[32]Especially the slope of the gradients of the time between trade variables turned out to be considerably larger than zero.
[33]It was again time between trade that caused difficulties in the numerical optimization.

rent magnitude of Δt_k in the mean function for the two trading systems. Therefore, another Wald test was conducted where all variables but Δt_k were split up into the different trading periods. The null hypothesis then is: $H_0: \ Z_{k-l}^{OOC} = Z_{k-l}^{APT}, ss_k^{OOC} = ss_k^{APT}, bb_k^{OOC} = bb_k^{APT} \ (l = 1, 2, 3)$.
The critical values for the new null hypothesis are 20.52, 16.75 and 15.09 (one, five and ten % significance level). The H_0 cannot be rejected nine times at the ten % significance level, one time at the five % and two times at the one % level. Furthermore the Wald-statistics did not alter in magnitude so that - recalling the large sample size - it actually makes a difference in the determination of the conditional mean of Z_k^* if a trade is conducted via OOC or APT. The difference is not due to time between trade.
The *conditional variance* of Z_k^* shows an interesting pattern if the explanatory variables are split up into the different trading periods. During OOC, Δt_k carries a negative sign while it is positive during APT. This finding is consistent with the results for the basic specification in section 3. At the electronic exchange DTB, Δt_k entered the conditional variance of Z_k^* positively while its impact was negative at the dealership market LIFFE. The same reversion of signs is true for the lagged bid-ask spread which is positive during OOC and negative during APT. The *absolute* impact of Δt_k and lagged bid-ask spread on the conditional variance is nevertheless almost the same. A Wald test on the equality of the coefficients for OOC and APT in the conditional variance shows that equality has to be rejected at any of the 21 trading days that yield convergence.
Due to the scaling of the coefficients by σ_k^2, it is not possible to test if DTB and LIFFE significantly differ from each other. Meaningful comparisons across exchanges can thus not be conducted. The same is true for comparisons across different trading days. This problem can be solved by applying a minimum distance estimator.

TABLE 5
A WALD TEST FOR DIFFERENCES IN OOC AND APT TRADING AT LIFFE: MEAN FUNCTION

date	Wald-stat.	P-val.
Aug 01	20,82	0,0020
Aug 02	26,97	0,0001
Aug 03	no convergence	
Aug 04	no convergence	
Aug 05	10,18	0,1172
Aug 08	2,68	0,8480
Aug 09	no convergence	
Aug 10	no convergence	
Aug 11	no convergence	
Aug 12	10,54	0,0104
Aug 15	1,07	0,9828
Aug 16	10,62	0,1008
Aug 17	no convergence	
Aug 18	7,59	0,2696
Aug 19	4,42	0,6204
Aug 22	no convergence	
Aug 23	14,8	0,0218
Aug 24	no convergence	
Aug 25	16,27	0,0124
Aug 26	5,49	0,4823
Aug 30	49,07	0,0000

Table 5 shows Wald test results of a test for order flow dependence. The first three columns present the coefficients tested for equality. "P-value" denotes the significance level at which the null hypothesis would just be significant.

4 Stylized facts reviewed: Theory and empirical evidence

Hausman et al. (1992, p. 376) claim that one of the major advantages of the ordered probit model is that it enables to determine the components of transaction price changes without putting much emphasis on market microstructure. Although indeed nonstructural, the ordered probit model in fact also allows to empirically check the validity of market microstructure theory. This literature usually seeks to explain the occurence of particular empirical findings called "stylized facts". For example, Easley and O'Hara (1987) investigate the stylized fact that "large trades (...) are made at 'worse' prices than small trades" (p. 69).[34] In turn, emprical research also tries to validate microstructure theory. The following pages review six stylized facts in market microstructure literature and their theoretical foundation.

(1) Diamond and Verrechia (1987, p. 293): "Future bid and ask prices remain unchanged when no trade is observed."

Time between trade did not play a role in earlier market microstructure approaches like Kyle's (1985) and Glosten and Milgrom's (1985). In Kyle's paper, trades are batched and all trades clear at a single price so that it does not matter *when* individual orders arrive.[35] Diamond and Verrechia's (1987) model was among the first that examined the information content of the time elapsed between two successive trades. They assume the same three types of market participants as Kyle (1985):

(1) Risk neutral[36] *insiders* who maximize expected profits. Insiders have access to private information about the true value of a risky asset.

(2) *Noise* or *liquidity traders* who act randomly. Easley and O'Hara (1987, p. 72) pose that noise traders act to gain liquidity for portfolio decisions and call such trades "exogenous demand".[37]

(3) Competitive, risk neutral *market makers*[38] who efficiently set bid and ask prices conditional on information from prior trading periods. The market maker's business is to provide the market with liquidity.

In the Diamond and Verrecchia model, a competetive market maker[39] is assumed who rationally sets bid and ask prices to make up losses resulting from trading with insiders by gains from trading with noise traders. Diamond and Verrecchia show that with short-sale prohibition,[40] price adjustments are slower after bad news than after good

[34] Actually, there are many more papers that deal with the price/volume relationship. See Karpoff (1987) for a survey.

[35] This type of order execution is called "periodic auction" as opposed to a "continous auction" which is in place at DTB and LIFFE. See Madhavan (1992) for a discussion of the two systems.

[36] Risk neutrality is assumed in order to prevent influences of risk preferences on trading behavior.

[37] Diamond and Verrecchia (1987, p. 280) argue that "immediate consumption needs, tax planning and alternative outside investment opportunities" are the liquidity trader's reason to participate.

[38] Market maker compete through the relation of prices and quantities.

[39] Note that both DTB and LIFFE are not pure market maker markets. At DTB, quotes are not explicitely announced but limit orders exist which essentially work in the same way as quotes do. At LIFFE, there are *specialists* who differ from market makers in that they are both dealers and brokers. Other traders are allowed to announce quotes as well.

[40] "Short selling" means that the seller does not own the sold security. Rather, a security is borrowed, sold at the current price, bought back later at a (hopefully) lower price, and given back to the lender. See Chance (1995, p. 7) for further details.

news. Short selling is restricted in order to avoid boundless speculation. The reason for this lower information efficiency is that informed traders who sold short after bad news do not trade any more and thus cause time between trade to elapse. A large time between trade thus may imply bad news driving down prices in the long run.
However, there are no short sale constraints in real futures markets, which means that Diamond and Verrechia's (p. 293) initial statement applies: the absence of trade does not contain any valuable information at all and therefore leaves prices unchanged. If time between trade is large, the information it contains is simply that there are no news in the market. Hence, time between trade should not have a significant impact on bid and ask and thus on transaction prices.
Supporting evidence comes from Hausman et al. (1992) who show that time between trade has no explanatory power in the conditional mean of Z_k^* even if short sale restrictions apply. Additional - theoretical - support for Diamonds and Verrechia's thesis is provided by Easley and O'Hara (1992). In their model, no–trade outcomes are more likely if there is no information event at all. If no new information occurs, no trades take place and the market maker is forced to narrow the bid-ask spread and thus makes large price changes impossible.
How can this hypothesis be tested with the ordered probit model? A first approach was already undertaken in the basic specification, since Δt_k was included in both the conditional mean and the conditional variance of the latent variable. If time between trade does not affect future bid and ask prices, it should not have a significant impact on either the conditional mean or the conditional variance of Z_k^*. At DTB, Δt_k turned out to be significant and positive in the conditional mean at ten trading days. At LIFFE, Δt_k does not have a major impact. Time between trade is also important for the conditional variance, as will be discussed for stylized fact *(4)*. Diamond and Verecchia's thesis seems thus to be validated, at least for DTB. The reason why Δt_k is not significant in the mean at LIFFE might be simply that Δt_k is associated with both positive ***and*** negative price changes so that these effects could have offset each other.
It would be straightforward to relate time between trade with the buy/sell indicator variable or the corresponding transaction price change. Three solutions seem to apply here. The first is an interaction of time between trade and the trade sequences (buy/buy and sell/sell). This solution is chosen in order to discuss stylized fact *(2)* below. The second is to distinguish between time betwen trade associated with a fall in price and time between trade associated with a rise in price. This cannot be a serious idea since the explanatory variable would be conditioned on the endogenous which evokes a simultaneity problem. A third approach is to interact a "large time between trade"-dummy with time between trade in the following way:

$$large\Delta t_k = \begin{cases} \Delta t_k & \text{if} \qquad \Delta t_k > \text{median}(\Delta t_j) \\ 0 & \text{otherwise,} \end{cases} \tag{19}$$

where $j = 1, \ldots, K$. The large time between trade variable proved to have explanatory power in the mean function of both DTB and LIFFE. For LIFFE, the interaction variable was nine times ***positive*** significant, and for DTB seven times negative significant and ***negative.***[41] It is quite puzzling that $large\Delta t_k$ carries opposite signs in the DTB and the LIFFE specification of the conditional mean. A first suspicion is that this result might be due to correlation between trade sequences and Δt_k (at LIFFE) and Δt_k and trading volume (at DTB). This suspicion is, however, not supported, since dropping trade sequences and volume, respectively, did not change the results.
Although Δt_k carries opposite signs at DTB and LIFFE, it cannot be said that the

[41]The same model with Δt_k labeled "large" when it exceeded the mean did not alter the results.

effect is actually always inverse at any trading day. If the impact of Δt_k on the conditional mean is compared day by day, opposite signs are not found.
Δt_k does influence the conditional mean of Z_k^*, at least if it is large. It also affects the conditional variance, so that future bid and ask prices do *not* remain unchanged when no–trade is observed.

(2) Easley and O'Hara (1992, p. 578): "Two sell transactions have a very different information content if they occur continuously over time than if they occur an hour apart."

Similarly to the Diamond and Verrechia (1987) model, time between trade affects the probability - now conditioned on both trading volume and time between trade - that the market maker is trading with an insider. In the Easley and O'Hara (1992) model, no–trade outcomes are more likely if actually no information event occured. No trade can thus be regarded as a signal for no new information.
If time between trade is small, the likelihood that an information event occured increases. Hence, small time between trade conveys other information than a large time betwen trade.
The assumption that time between trade is informative does not necessarily mean that it directly affects transaction price changes. But if Δt_k is valuable information, then the explanatory power of a trade sequence which Hence, t–values of the interacted coefficients should be larger than those of the non–interacted ones.
Table 6 shows the estimation results. Since time between trade is zero 244 (DTB) and 274 (LIFFE) times on the average of the 21 trading days, these Δt_k's were replaced by 0.1 in order to loose as few observations as possible.
It is noted that Easley and O'Hara's (1992, pp. 582) economic model is specified in *clocktime* rather than in *transaction time* like this ordered probit approach. In their model, a trading day begins and ends with the occurence of new information. The market participants do not learn directly about an information event, but learn from the history of trading until they know almost for sure that an information event actually occured. Although in the present setting the definition of a trading day is quite different, traders are allowed to learn from observing the proxy variables. Therefore, the different definition of a trading day should not matter much in the interpretation. However, the t–values of the $ss\_\Delta t_k$ and $bb\_\Delta t_k$ variables improved for seven trading days compared with the t–values of the bb_k and ss_k variables without interaction. At eight trading days either the interacted sell/sell or the buy/buy sequences improved compared to the case of no interaction. It is quite striking that not only the trade sequences which were interacted with Δt_k improved their t–values, but so did almost all other coefficients for all trading days as well. An interaction of Δt_k and the trade sequences also improves the significance of the other regressors.[42]
Thus, the results broadly support Easley and O'Hara's thesis.[43]

(3) Hasbrouck (1988, p. 250): "The coefficients of both the indicator and the size are negative, indicating some reversal of the initial impact..."

Hasbrouck's (1988) statement concretely means that empirical evidence supports the existence of price reversals. Price reversals are due to transaction price movements going from bid to ask or from ask to bid. If price reversals are present, two successive buys are likely to be followed by a sell. This pattern was present in the basic specification of section 3.

[42]The log–likelihood function of 17 trading days converged. Ten regressors were included so that a total of 170 coefficients was estimated. An interaction of the trade sequences and time between trade improves the t–values for 81.8 % of the coefficients that were overall estimated.

[43]Despite of what was noted above, Easley and O'Hara (1992) hint that time between trade and transaction price changes could be simultaneously determined. This point is discussed in section 5.

There are two explanations for the existence of price reversals. The first are bid-ask errors. Bid and ask prices bounce around the asset's true value. Bid and ask prices are erroneous in the sense that they never reflect the asset's true value.
The second explanation is market overreaction. If, say, a high signal occured, bid and ask prices move upward to adjust to the new valuation of the asset. An overshooting occurs if traders do not notice when bid and ask prices eventually have settled around the new equilibrium. Empirical evidence is given by Kaul and Nimalendran (1990) who conclude that price reversals are mainly due to bid/ask errors.
Initially, Hasbrouck's paper tries to bring two different approaches - inventory control and asymmetric information - to market microstructure together. He empirically tests the validity of both approaches and yields evidence in favour of both microstructure models.[44]
Hasbrouck's findings are statistically weak. The buy/sell indicator IBS_k that he uses turned out to be negative, but insignificant. In the ordered probit analysis of Hausman et al. (1992), the buy/sell indicator was both highly significant and negative.
The ordered probit model of this paper strongly supports the existence of price reversals. Buy/buy sequences are significantly negative, while sell/sell sequences carry the opposite sign. For example, considering LIFFE trading at Aug. 11 (see table 2), a sell/sell sequence rises the conditional mean of Z_k^* by +0.141 while a buy/buy sequence decreases it by -0.113.
Do these findings still hold if trades occur in very short time intervals? Intuition suggests that market participants will have stronger believes that prices will rise if two successive buys occur in a very short time interval than in a large one. This thesis was tested by an inclusion of a "fast trade sequence" variable that took on the value one if the sequence was buy/buy (sell/sell) *and* time between trade was less than its median at the corresponding trading day.[45] The inclusion of the fast trade sequences did not alter the signs so that price reversals are present even if trades occur in short time intervals.

(4) Easley and O'Hara (1992, p. 598): "Variance at time $t+1$ is less than the variance at t if there is no trade at t"

Easley and O'Hara (1992) argue that the market maker narrows the bid-ask spread if she observes no-trade outcomes in order to maintain an orderly market. By reducing the bid-ask spread, she prevents large price changes and thus makes prices less volatile.
Time between trade was encountered in each of the variance-specifications. As can be seen from table 3, Δt turned out to be both *positive* and highly significant in the conditional variance for all trading days at DTB. At LIFFE - see tables 2 and 6 -, time between trade carries the opposite sign but is in the variance significant for only four trading days. Hausman et al. (1992) find a highly significant and *positive* impact of Δt on the conditional variance of Z_k^*.
The positive sign of Δt_k at DTB contradicts the thesis that the market maker narrows the bid-ask spread in care of no-trade outcomes and hence prevents large price changes. An alternative explanation has already been given in section 3: between two transactions new information could have occured causing prices to alter.

[44]Glosten and Harris (1988) also provide empirical support for Hasbrouck's result.
[45]In order to save as many of such trade sequences, Δt_k was again replaced by 0.1 if time between trade was zero.

TABLE 6

A COMPARISON OF ORDERED PROBIT ESTIMATION RESULTS WITH AND WITHOUT INTERACTION OF ss_k AND bb_k WITH TIME BETWEEN TRADE

LIFFE, Aug. 08, 1994

	Estimates		Std. err		Est./se.	
Parameters	w/ inter.	w/o inter.	w/ inter.	w/o inter.	w/ inter.	w/o inter.
Z_{k-1}	-0.3598	-0.3504	0.0357	0.0370	-10.069	-9.481
Z_{k-2}	-0.1028	-0.0866	0.0362	0.0389	-2.838	-2.226
Z_{k-3}	0.0210	0.0269	0.0334	0.0341	0.630	0.790
ss_k	0.0021	0.1360	0.0019	0.0630	1.113	2.157
bb_k	-0.0011	0.0135	0.0016	0.0614	-0.724	0.221
α_1	-0.6983	-0.6628	0.0400	0.0476	-17.466	-13.919
α_2	0.7894	0.8295	0.0443	0.0528	17.826	15.725
Δt_k	0.0037	0.0037	0.0010	0.0010	3.742	3.640
BAS_{k-1}	0.0170	0.0172	0.0222	0.0217	0.766	0.793
$DVOL1_k$	-0.5866	-0.5804	0.0561	0.0561	-10.458	-10.338

number of observations: 1,819

LIFFE, Aug. 12, 1994

	Estimates		Std. err		Est./se.	
Parameters	w/ inter.	w/o inter.	w/ inter.	w/o inter.	w/ inter.	w/o inter.
Z_{k-1}	-0.0659	-0.0659	0.0153	0.0159	-4.293	-4.149
Z_{k-2}	-0.0055	-0.0049	0.0140	0.015	-0.393	-0.325
Z_{k-3}	-0.0167	-0.0175	0.0148	0.0149	-1.131	-1.172
ss_k	-0.0002	-0.0408	0.0012	0.0447	-0.183	-0.911
bb_k	-0.0017	-0.0632	0.0016	0.0425	-1.099	-1.486
α_1	-0.4167	-0.4405	0.0282	0.0336	-14.793	-13.09
α_2	0.3742	0.3523	0.0255	0.0308	14.701	11.434
Δt_k	0.0003	0.0004	0.0014	0.0013	0.185	0.295
BAS_{k-1}	-0.0115	-0.0120	0.0345	0.0339	-0.334	-0.354
$DVOL1_k$	-0.5564	-0.5511	0.0666	0.0662	-8.349	-8.324

number of observations: 3,198

LIFFE, Aug. 18, 1994

	Estimates		Std. err		Est./se.	
Parameters	w/ inter.	w/o inter.	w/ inter.	w/o inter.	w/ inter.	w/o inter.
Z_{k-1}	-0.1017	-0.1028	0.0175	0.0182	-5.815	-5.655
Z_{k-2}	0.0006	-0.0006	0.0172	0.0185	0.036	-0.034
Z_{k-3}	0.0188	0.0182	0.0144	0.0146	1.312	1.241
ss_k	0.0006	0.0095	0.0020	0.0407	0.276	0.234
bb_k	-0.0027	-0.0154	0.0017	0.0404	-1.584	-0.382
α_1	-0.4342	-0.4314	0.0254	0.0314	-17.083	-13.737
α_2	0.4163	0.4227	0.0244	0.0308	17.038	13.742
Δt_k	0.0012	0.0015	0.0017	0.0016	0.707	0.894
BAS_{k-1}	0.0426	0.043	0.0230	0.0231	1.850	1.866
$DVOL1_k$	-0.7592	-0.7544	0.0538	0.0538	-14.122	-14.02

number of observations: 3,729

Table 6 gives a comparison of estimation results yield with the basic specification and a specification where trade sequences and time elapsed between two successive trades are interacted. α_i ($i = 1, 2$) denotes the partition boundaries. The coefficients below the boundaries are for variance. "P–value" denotes the significance level at which the null hypothesis would just be significant.

(5) Grammatikos and Saunders (1986, p. 319): "The correlation between price variability and volume should be positive"

Franses et al. (1994, p. 19), using vector autoregressive processes, support Grammatikos and Saunders by stating that "volatility is best explained by volume".

The underlying theoretical framework of Grammatikos and Saunders (1986) empirical investigation of the price/volume relationship is the "Mixture of Distribution Hypothesis" (MDH). The MDH is discussed in Tauchen and Pitts (1983).[46] Grammatikos and Saunders test the validity of the MDH by calculation of correlation coefficients and running Granger-Sims causality tests.[47] However, strong positive correlation of price variability and volume are consistent with the MDH. Thus, an inclusion of volume in the conditional variance of Z_k^* should turn out to be both highly significant and *positive*. Large volume then increases the variance of Z_k^*.

Grammatikos and Saunders' statement is clearly invalidated by the present empirical investigation. The first lag of trading volume (in natural logarithms) is significantly different from zero and *negative* in the conditional variance at almost any of the trading days in the DTB model. Thus, the correlation between price variablity and volume is *negative*.

(6) Hasbrouck (1991, p. 179): "Trades occurring in the face of wide spreads have larger price impacts."

Hasbrouck's paper can be traced back to Easley and O'Hara's (1987) approach that was discussed earlier. The bid/ask-spread is a proxy variable for information flow. In the Easley and O'Hara (1987) framework, the market maker widens the spread if the probability that an information event occured increases. She does so in order to avoid losses from trading with insiders. Hence, bid-ask spreads are large if the market maker believes that she is trading with an insider. The bid-ask spread signals that new information could have entered the market and causes an alteration of prices.

Now, the reasoning is quite close to what was taken down for the impact of trading volume on transaction prices. Since bid-ask spreads can be associated with price changes in both directions, an inclusion of bid-ask spreads in the mean function should not – and actually does not – turn out to be significant. The same result emerged from an inclusion of a "large bid-ask spread" variable.[48]

Anyway, Hasbrouck's paper gives an good explanation why the bid-ask spread should be included in the conditional variance. And in fact, the first lag of the bid-ask spread is significantly positive for almost all trading days. Therefore, the bid-ask spread is important in the conditional variance of Z_k^* and thus actually has a price impact. An explanation of why the effect of BAS_{k-1} could be negative in the conditional variance was given in section 3.

[46] Tauchen and Pitts (1983) themselves do not label their approach Mixture of Distribution Hypothesis.

[47] The causality tests check whether volume "causes" price variability and vice versa.

[48] A bid-ask spread was labeled "large" if it exceeded its median.

5 Conclusions and suggestions for further research

The ordered probit model is a valuable tool in the investigation of transaction price changes. It takes into account the irregular spacing of transactions, the discreteness and the conditionality of the dependent variable and allows to simultaneously determine the conditional mean and the volatility of transaction price changes. Although it is a nonstructural approach, it enables to empirically test the validity of market microstrucure theory. Differences and similarities in the determination of BUND-future transaction price changes can be pointed out with respect to the competing exchanges DTB and LIFFE.
The estimation results of this paper have frequently found conflicting evidence to microstructure theory and can be summarized as follows:

(1) Time elapsed between successive trades does not leave prices unchanged as microstructure literature suggests. The effect of Δt_k on the mean and the variance differs for DTB and LIFFE. At DTB, the impact on volatility is highly significant and positive while at LIFFE it is less often significant and negative. The conditional mean of Z_k^* is negatively affected by time between trade at DTB, while Δt_k does not have a significant impact on the mean function at LIFFE.

(2) The explanatory power of trade sequences interacted with time between trade is larger than the explanatory power of trade sequences without the additional information. Thus, both the direction and the timing of transactions matter in the determination of future price changes.

(3) A tendency towards price reversals is strongly supported. Successive sells have a positive impact on transaction price changes. The reverse is true for successive buys. The result even holds if transactions occur in short time intervals.

(4) Volatility is mainly determined by time between trade, lagged bid-ask spreads and trading volume. The larger the bid-ask spread and the larger the volume, the smaller is the variance.

Order flow dependence can be rejected at the usual significance levels. That is, successive transaction price changes of +1/-1/+1 ticks have a different impact on future price changes than a -1/+1/+1 sequence, although the total price change is the same in both cases. The reason for the rejection of order flow dependence is that the coefficient of the first lag of the transaction price change is considerably larger than the last two lags. The second and third lags are almost equal in magnitude.
Open Outcry and Automated Pit Trading significantly differ from each other. The explanatory variables enter both the conditional mean and the conditional variance in significantly different ways during APT and OOC trading.

It would of course be desirable to apply some diagnostic tests on the specifications that were chosen. Unfortunately, diagnostic plots and tests based on residuals are not suitable here. Z_k^* is not observable and residuals are thus not obtained. Gourieroux, Monfort and Trognon (1985) solve this problem by introducing the concept of *generalized residuals*. The application of generalized residuals is beyond the scope of this paper so that this topic is left for further research.
Another improvement of the ordered probit model of this paper would be to run in- and out of sample forecasts. This technique is, for the same reason as the generalized residuals, also not applied here.

As it was mentioned in section 4, Easley and O'Hara (1992) give reason to assume that Δt_k and Z_k could be simultaneously determined. Simultaneity generally leads to inconsistent parameter estimates. A trader who is privately informed has an incentive to hide her information advantage. She is willing to buy (sell) a large amount of shares if she hears good (bad) news. Large shares are usually executed at worse prices so that she is tempted to split her desired trading volume up into several smaller amounts. She thus simultaneously determines transaction price changes and the time elapsing between trades. Hausman et al. (1992, pp. 348) give similar arguments for a simultaneity of Δt_k and the buy/sell indicator IBS_k. Other variables that were included in the models discussed above can also not be deemed a priori exogenous. For example, large volume is usually associated with large bid-ask spreads so that these two variables seem to be also simultaneously determined. Transaction prices changes thus may be more appropriately modeled by a compounded system. An instrument variables regression can generally solve the simultaneity problem. Hausman et al. (1992, p. 351) apply such a technique and find that estimates do not change considerably after correction for simultaneity.

Another topic that has to be left for further research is that meaningful comparisons across both exchanges and different trading days are not possible due to the scaling of the coefficients by σ_k. An application of a minimum distance estimator would make such comparisons feasible.

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