Human Capital Investment and Population Dynamics

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Acknowledgments

The fact that this thesis has only a single author suggests that this is the achievement of only one person. This is, however, only part of the truth. Without the help and support of many others it would have been much harder – if not impossible – to finish this dissertation. Therefore, I want to take the chance to thank all those who deserve the credit for their support through the last years.

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Contents

Chapter 1

General Introduction

This thesis consists of four self-contained papers linked by one common topic: the interaction of human capital, fertility and the macroeconomic environment. We start by looking at an economy characterized by dismal social and economic conditions: low life expectancy, high mortality, little investment into human capital, low income, and high fertility. In the first two chapters, we examine the development path from a Malthusian trap to a Solowian economy and make an attempt to provide possible explanations which forces could have potentially shaped this process. To focus only on the link between human capital, fertility and survival probabilities, we deliberately ignore the issue of physical capital accumulation which will be brought back in chapters 4 and 5. Then we move on to an economy with fundamentally different conditions. We look at the problems of an aging but developed economy. The challenges faced by these economies are radically different from those described above. Now, the problem is not that short lives and frequent diseases make investment into human capital economically unviable but rather that long living and healthy centenarians exert financial pressure on our social security systems. The disincentive to invest into human capital stems now from high taxation rather than from a short planning horizon. Capital dilution due to high fertility rates is no longer the problem but we worry that currently low fertility rates may endanger the fiscal sustainability of the pay-as-you-go pension systems and lead to distributional conflicts between old and young. No matter at which situation we look, it is evident that the answer is likely to be dominated by the interaction of these variables.

The following sections contain a short non-technical summary of each chapter. Proofs and technical details are provided directly at the end of each chapter. Due to the fact that a part of the literature is common to all chapters, references are provided in one single chapter at the end of the thesis.

1.1 From Malthus to Modern Growth: Child Labor, Schooling and Human Capital

Chapter 2 develops a dynamic general equilibrium model of fertility, human capital accumulation, child labor and uncertain child survival focusing on the qualitative and quantitative effect of declining child mortality on household decisions and economic development. Due to uncertainty about child survival, parents have a precautionary demand for children. Rising survival probability leads to falling fertility, eventually to investment into schooling and the demise of child labor. Child labor can be an obstacle to development since it lowers the incentives of parents to educate children. Furthermore, we argue that the decline of precautionary child demand as a consequence of falling mortality is not sufficient to generate a demographic transition. Falling mortality can only explain a relatively small part of the fertility decline. A sizable reduction in fertility can only be achieved by human capital investment and the induced quantity-quality trade off.

The modeling environment follows the literature by allowing families to choose their consumption level, number and quality of their children. Children's quality is measured by the level of their human capital which is the result of time investment into a human capital formation technology. At the same time, parents choose the level of child labor: children can thus contribute to their families' income. They can simultaneously work and go to school, as long as their total time budget is not exhausted. For the parents' utility function we assume that it is increasing in their own consumption, the number and quality of their surviving children but decreasing in the level of children's labor supply. Thus, parents face a trade-off between higher consumption and the disutility from child labor. Further, by having working children, they sacrifice some potential utility gain from endowing them with human capital. Since the survival of children is uncertain, parens have a precautionary demand for children. This means that in times of high child mortality, parents will have high fertility. High fertility implies that child labor can be a large source of family income whereas child education is relatively expensive. Especially, if the human capital technology to be productive requires some minimum level of investment per child, investing in children's human capital can be virtually infeasible. They simply cannot afford to endow each child with that minimum educational requirement. In times of low death probabilities, the optimal number of children is lower, potential income from child labor is also lower and parents can afford a higher level of education per child. If this level is above the minimum threshold, parents start investing into child education and decrease child labor supply. This serves as an accelerator of the development process, fueling further acquisition of human capital by future cohorts. This in turn will lead to lower mortality for the next generation and the dynamic process will lead the economy towards a steady-state with low child mortality, low fertility and high educational investment. The chapter's contribution to the literature is twofold. Firstly, it is the first paper to bring together the quantity-quality decision of children and child labor in an analytically tractable framework with uncertainty. Secondly, the paper provides quantitative evidence on the contribution of falling mortality, rising schooling and the role of child labor to the demographic transition. Using historical macro-data as benchmarks, we calibrate the model and show that the decrease of precautionary demand for children as a consequence of falling mortality is able to explain a small drop in fertility but it is unlikely to be the main driving force of the demographic transition.

1.2 Human Capital and the Demographic Transition: Why Schooling Became Optimal

As opposed to the previous chapter, here we focus on the implications of rising adult life expectancy and its role for human capital investment and fertility. Moreover, we propose an explanation how a schooling system could emerge without the intervention of a government. We show that if parents invest their own time into children's human capital, rising adult life expectancy always increases fertility. If children are educated in schools and parents pay tuition fees, fertility will fall. Furthermore, parents deciding to send their children to a school have – for any life expectancy – fewer children and invest more in their human capital. Without a schooling system, rising life expectancy therefore initially increases fertility. As life expectancy rises during the development process, a schooling system will be endogenously adopted and the relationship between fertility and longevity reversed. We argue therefore that it is important to account for the change in the nature of the costs of child education: from time costs to monetary costs.

We use a simple life cycle model in which adults differ with respect to their productivity on the labor market and decide about consumption, investment into adult and child human capital and the number of children. They also have the choice to invest some of their own time into the human capital of their children. Alternatively, they can send their offspring to a school and pay tuition fees. Agents' utility function closely follows the setup from the previous chapter. Parents like consumption over their life-cycle and a quality-quantity composite of children.

Using this setup we show that if parents increase children's human capital by using their own time, fertility will unambiguously increase. The economic explanation is that as adult life expectancy increases, this additional lifetime can be used to work and thus consume more or have more children. By concavity of utility in consumption and children, the agent will distribute the additional lifetime on both. The reaction of agents deciding to send their offspring to a school is, however, ambiguous. If parental human capital is sufficiently productive and the agents' utility in the quality-quantity composite is sufficiently concave, fertility decreases. This comes from the fact that the price of education is now fixed and does not increase with parental investment into adult human capital. In other words, if parents spend their own time to enhance children's human capital, rising life expectancy increases the price of quality and quantity. With investment into child human capital via a school, increasing adult human capital increases only the opportunity costs of quantity but leaves the price of education unchanged. Thus, if parental human capital is sufficiently productive and the marginal valuation of an additional child is rather low, the rising relative price of quantity will bias the parental decision toward more investment into quality and decrease the number of offspring. Furthermore, we show that not only the reaction of agents with respect to change in their life expectancy differs across educational systems: parents choosing the school system will – for any life expectancy – decrease fertility and increase investment into both types of human capital.

The decision which system to adopt depends on the ability level of each parent. Initially agents have low life expectancy and only high ability parents will afford education via a formal schooling system. As the economy develops and life expectancy rises, also less able agents will be able to purchase schooling. Thus, more parents will opt for the formal system. For initially low life expectancy fertility increases due to the fact that the economy is dominated by families investing own time into children's education. Later, as the share of agents participating in the schooling system reaches a threshold value, the relationship between fertility and life expectancy changes on the aggregate level. Thus, the key contribution of this paper is to provide a novel explanation for the fertility transition and the endogenous appearance of a mass schooling system in an otherwise rather standard model. The explanation is based on a change in the nature of investment in child quality from time costs to monetary costs. We thus propose a theory why a formal schooling system emerged endogenously without a state intervention on a large scale. We do not, however, make the next step and model why the society – via government and parliament – decided set up a free public schooling system financed by taxes. The extension by such a political economy element is left for future research.

1.3 Mortality, Fertility, Education and Capital Accumulation in a Simple OLG Economy

In the final two chapters of this thesis we move on to a developed economy. We reintroduce physical capital into the economy but assume now that fertility is exogenous. In chapter 4 we develop a simple two-period OLG model with exogenous fertility and mortality in the spirit of Diamond (1965) to analytically show that aging leads to increased educational efforts through a general equilibrium effect. The mechanism is that scarcity of raw labor increases the return of human capital relative to physical capital. While a reduction in the birth rate is shown to unambiguously increase educational efforts, increases in the adult survival rate have ambiguous effects. Falling birth rates also increase capital per worker but the effects of rising survival rates are again ambiguous. Therefore we argue that our model is a useful laboratory to highlight potentially offsetting effects in models with endogenous education and overlapping generations.

The model setup is fairly standard. We assume that agents live for two periods and they survive to old-age with an exogenous survival probability. In the first period ("young") agents can save, invest time into human capital accumulation and consume. Time investment in the first period increases the stock of human capital. In the second period ("old") agents work an exogenous proportion of their available time and are retired for the rest. During retirement they receive a lump-sum pension. We model a PAYG pension system in which agents contribute a share of their wage income to the pension fund. Then, by the assumption of a balanced budget and taking the population structure as given, we compute pension payments and can express the level of pensions as a share of current net wages. Using such a setup, we are able to conduct policy experiments by varying either the contribution or the replacement rate and let the other adjust. Finally, we embed the households in a general equilibrium setup and study the effect of changing survival rates and birth rates on wages and interest rates.

The key contribution of this paper is that we use a rich setup and are able to show that in general, changes in survival rates have ambiguous effects on the capital stock and education. Further, our setup enables us to analytically show where this ambiguity comes from and therefore we can conclude that it is key to consider the interactions between annuity markets, the pension system and productivity of education.

This chapter is joint work with Alexander Ludwig and has been published as "Mortality, Fertility, Education and Capital Accumulation in a Simple OLG Economy", Journal of Population Economics, 23(2): 703-735, 2010.

1.4 Demographic Change, Human Capital and Welfare

The final chapter of this thesis evaluates the role of human capital adjustments for the economic consequences of demographic change on wages, returns to capital and welfare. As opposed to the previous chapter, here we are interested how large the quantitative effects of such an adjustment are. We find that endogenous human capital formation is a quantitatively important adjustment mechanism which substantially mitigates the macroeconomic impact of population aging. On the aggregate level, the predicted decrease of the rate of return to physical capital is only one third of the predicted decrease in a standard model with a fixed human capital profile. In terms of welfare, while young agents with little assets gain up to 0.8% in consumption from increasing wages in both models, welfare losses from decreasing returns of older and asset rich households are substantial. But importantly, these losses are about 50 - 70% higher in the model without endogenous human capital formation. Ignoring this adjustment channel thus leads to quantitatively important biases of the welfare assessment of demographic change. We also document that sticking to the status quo social security system and letting contribution rates increase will largely offset any positive welfare effects for future generations. Our contribution to the literature is thus the quantification of the additional adjustment possibility of investing into human capital.

To quantify these effects, we employ a calibrated large scale overlapping generations (OLG) model with endogenous human capital formation using a Ben-Porath (1967) technology. We assume that agents life up to a maximum age of 90 years and survive to the next period (age) with an exogenous probability. Agents retire at the age of 65. During retirement they receive retirement benefits from the pension system which is financed by contribution of the working cohorts. We allow agents in each period to work, invest time into human capital or consume leisure. Investment into human capital increases, whereas depreciation decreases the stock of human capital next period. With our calibration we are able to reproduce the hump-shaped pattern of labor supply and decreasing investment into human capital as individuals proceed through their life-cycle. The role of the government in our model is limited: it manages a simple social security system with balanced budget. Taking the demographic structure and this government policy as given, we can determine equilibrium pension payments. By adjusting either contribution or replacement rates we can simulate alternative pension reform scenarios.

The underlying economic mechanism is that an aging society will accumulate more physical capital and thereby decrease the rate of return of savings. Thus, on the one hand the incentive so save decreases and on the other hand, the opportunity costs of borrowing when young decrease. Hence, young agents have a strong incentive to invest more time into human capital accumulation, borrow while spending time on the acquisition of human capital and repaying later. As summarized in the first paragraph, we find that the additional possibility to invest into human capital helps to limit welfare losses in an ageing society.

In our policy lab, we conduct two social security experiments. In our first scenario, we assume that contribution rates are freezed at current rates and let therefore the replacement rates decrease such that the budget of a pension system is balanced. In the polar scenario, we simulate a generous pension system and keep the replacement rate constant at current levels. In an aging society, this requires rising contribution rates thereby decreasing net wages of workers. We find that the distortionary effects of the rising contribution rates will largely offset potential gains from higher human capital accumulation. Thus, not reforming the social security system will involve welfare losses of future generations. Nevertheless, without a welfare criterion or knowledge about social preferences we do not make any statement about the optimality of pension policy.

This chapter is joint work with Alexander Ludwig and Thomas Schelkle. An earlier version in which we model additionally different skill types and the effect of human capital investment on the permanent growth rate in the spirit of Lucas (1988) is available as Ludwig, Schelkle, and Vogel (2007).

Chapter 2

From Malthus to Modern Growth: Child Labor, Schooling and Human Capital

2.1 Introduction

Key stylized facts characterizing the evolution of humanity from an era close to subsistence levels to today's high-tech economies are – among other facts – increasing technological progress combined with rising educational attainment and the demise of child labor on the one hand and falling mortality and fertility causing a demographic transition on the other hand. These events have occurred in today's developed countries from the onset of the industrial revolution to present times and are currently under way in developing countries. These stylized facts can therefore be observed both across time and across countries (see section 2.2). The motivation for this research is to build a model explaining these facts in a general equilibrium setup. Although there is a large body of literature on each of the mentioned items in isolation, work on the combination and interaction of these phenomena is scarce.

To replicate the historically observed sequence of these stylized facts, this paper develops a dynamic general equilibrium model of endogenous fertility, human capital investment decisions, child labor and uncertain child survival. The driving force of the model dynamics is the changing child survival rate. Parents maximize utility from own consumption, child leisure and a quantity-quality composite of children. In addition to adult labor supply, parents can choose to send children to work and thus generate additional income. Quality is measured in terms of the child's human capital whereas the quantity refers to the number of surviving children. The macroeconomic piece of the model consists of a production function with human capital and a fixed amount of land as inputs. Technological progress is initially driven by rising population and later additionally by human capital investment.

Employing this framework, the paper makes two contributions to the literature. First, it is the first paper bringing the quantity-quality decision of children on the one hand and child labor on the other hand in an analytically tractable framework with uncertainty, inspired by the seminal contribution of Kalemli-Ozcan (2003), together. Second, it provides quantitative evidence on the contribution of falling mortality, rising schooling and the role of child labor to the demographic transition. Using a calibrated version of the model it is shown that the decrease of precautionary demand for children as a consequence of falling mortality is able to explain a small drop in fertility but it is unlikely to be the main driving force of the demographic transition. The reversal of the relationship between income, mortality and population growth is ultimately triggered by the quantity-quality trade off which forces parents to curb fertility in order to endow children with schooling. We find that child labor is a potential obstacle to development in a sense that the more children can earn on the labor market, the higher is fertility and the lower is schooling. Moreover, the model is able to generate the historically observed sequencing and qualitative behavior of fertility, population growth, child labor and schooling. Initially, sending children to school is not optimal but children work and fertility is declining whereas population growth is rising. Later – with falling mortality – fertility and child labor decrease and schooling becomes optimal. This fuels technological progress which further rises survival rates, decreases fertility and child labor. Eventually, parents choose not to send their children to work but invest only into their education.

Recently, the link between child labor and human capital accumulation has shifted into the focus of growth and development economics. Basu and Van (1998) present a model in which parents are not selfishly exploiting their children but let their children work because additional income close to subsistence levels is the welfare maximizing household solution.

They also discuss the possibility of multiple equilibria. Hazan and Berdugo (2002) develop a model with child labor and schooling decision. Their central result is that technological progress increases the wage differential between children and adults leading to reduced child labor and more education. Baland and Robinson (2000) investigate the role of capital market imperfections and the role of bequests for child labor. They find that child labor is inefficient if parents can use children's income as a substitute for negative bequests or are credit constrained. Dessy (2000) argues that child labor may be the obstacle to development: if the economy is sufficiently close to a critical value of per capita human capital, the presence of child labor may pull the economy into a poverty trap which can be avoided by introducing compulsory schooling. Strulik (2004a) presents a model with child mortality and child labor and Strulik (2004b) additionally includes child health affecting child survival. Depending on the child survival rate, the economy can be stuck in a high-fertility and low growth regime with child labor or in a low fertility and perpetual growth environment. The demographic transition is generated by a quantity-quality mechanism pioneered by Becker and Lewis (1973) and Becker (1960).

The choice of human capital investment under uncertain survival has been considered in Kalemli-Ozcan (2003) and in a general equilibrium setup in Kalemli-Ozcan (2002). Parents have a "precautionary" demand for children. As a consequence of this, high mortality rates and thus high uncertainty about the survival of offspring will induce parents to have more children but endow them with little education. Lowering the risk will decrease precautionary demand and accelerate investment into schooling. In the same spirit, Tamura (2006) presents numerical evidence showing that this family of models can be used to generate realistic results for important macro- and microeconomic variables (life expectancy, fertility, population, mortality, etc.). Using a perpetual youth model Kalemli-Ozcan, Ryder, and Weil (2000) show that a reduction in the mortality rate at any age significantly increases investment into human capital. Empirical studies confirm these findings. In an econometric analysis using Swedish fertility data, Eckstein, Mira, and Wolpin (1999) find that both increases in real wages and reductions in infant and child mortality significantly contributed to the fertility decline. Most important was, however, the decline in mortality. Similar findings are confirmed for India by Ram and Schultz (1979) who argue that falling mortality was an important incentive to invest into education.

There is an enormous amount of literature offering a wide range of alternative explanations for the demographic transition and the rise of human capital investment. The seminal paper by Galor and Weil (2000) generates the transition from a Malthusian development stage to a growing economy by endogenously raising the rate of technological progress and thereby human capital investment. Hansen and Prescott (2002) examine a model with an agricultural sector with a fixed factor (land) and a modern technology with constant returns. Assuming exogenous technological progress in both sectors, the modern sector will eventually be more productive and pull the economy out of the Malthusian trap. Galor and Weil (1996) derive the fertility decline from a narrowing wage gap between men and women. By increasing the value of female labor, the costs of child rearing increase and thus the transition from a high fertility to a low fertility regime is achieved. In the model of Cervellati and Sunde (2005) the driving force of development is the rising life expectancy. Assuming that education incurs a fixed (time) cost, rising life expectancy makes education more attractive and thus agents will engage into education as their planning horizon expands. Jones (2001) is proposing a mechanism in which the introduction of property rights plays the key role in explaining growth and the demographic transition over long periods. Other explanations for the demographic transition are rooted in evolutionary economics with people having a preference for child quality eventually dominating (Galor and Moav (2002)) or changes in marriage institutions with an increasing share of women with higher human capital (Gould, Moav, and Simhon (2008)).

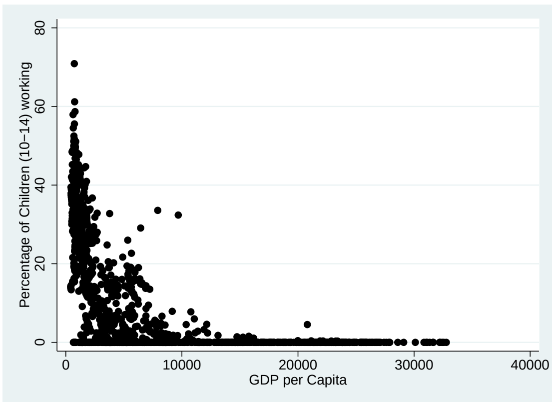
Stylized facts motivating this research are presented in section 2.2. Section 2.3 introduces the model environment and outlines the household's maximization problem. Section 2.4 describes the macroeconomic setup. Section 2.5 closes the model by establishing the links between household decisions and aggregate behavior on the one hand and the dynamic behavior of the model on the other hand. In the same section we also present the results from a calibration exercise. Section 2.6 concludes.

2.2 Stylized Facts

This section presents stylized facts motivating this research in some more detail. Note that all facts hold true for the modern world with poor and rich countries (i.e. for the cross section) and in a time series perspective using historical data for today's developed countries.

Figure 2.1 plots the percentage of working children against GDP per capita. The data refers to the period 1960-2002, revealing that child labor is still a widely spread phenomenon in today's world. In countries like Mali, Bhutan and Burundi almost 50% of the children aged 10-14 participate in the labor market and are thus an important source of family income. In the 1960s and 1970s, the share of working children was even higher. In Mali, Nepal and Burkina Faso more than half of all children had to work in order to contribute to family income. Using income per capita as a benchmark, these numbers are comparable to historical statistics. According to Lebergott (1964) at the end of the 19th century between 13 and 18 percent of all children aged 10-15 in the US were actively participating in the labor market, working even in industries like mining or manufacturing and a ten year old boy employed in agriculture had the earning capacity of about one quarter of an adult. The second empirical regularity observed in the data is the strong negative

Figure 2.1: GDP per Capita and Child Labor

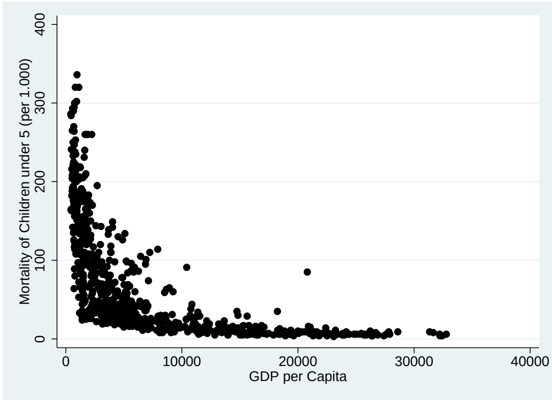


Notes: Cross sectional data 1960-2002. *Source:* The World Bank (2004).

correlation of child or infant mortality and income per capita.[1] Figure 2.2 shows that as income approaches very low (subsistence) levels, child mortality rises dramatically. Income

[1]There is a strong positive correlation (ρ >0.8, based on data from The World Bank (2004)) between adult and infant mortality suggesting that high child mortality is also a good proxy for health conditions over the entire life span.

Figure 2.2: GDP per Capita and Child Mortality

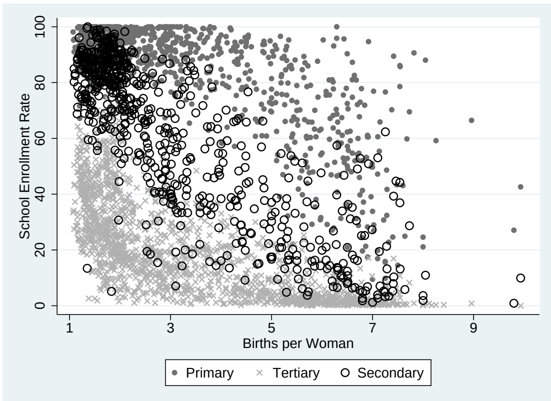


Notes: Cross sectional data 1960-2002. *Source:* The World Bank (2004).

beyond this threshold has a relatively minor influence. As a glance on on the graph reveals, there has been no large gain in child survival probabilities for some low income countries despite the huge gains in medical knowledge worldwide which suggests that income seems to be the most important factor determining child mortality. Historical statistics from Sweden (Wolpin (1997)) and England (Cutler, Deaton, and Lleras-Muney (2006)) confirm this result. Infant mortality was high and the chance to survive age 15 were as low as 60-70%. Survival probabilities conditional on having survived childhood were much higher. Due to high mortality rates earlier in life, life expectancy at birth around 1850 in England was 40 years, conditional on being 10 years old 55 years and close to 70 years at the age of 45. Thus, the dramatic increase in life expectancy came first from eliminating the risks early in life. Huge improvements in life expectancy later in life were achieved only in more recent times.

The trade off between quality and quantity is another regularity present in the data and shown in figure 2.3. There is a clear negative correlation between the enrolment rate at any schooling level and the total number of births per woman. Again, the same conclusion can be obtained from a time series perspective. French enrolment rates of children (aged 5-14) to primary school increased from 30% in 1830 to almost 90% in 1900. In England, the fraction of children with primary education was about 20% in 1860 and reached 80% in 1900. At the same time the number of birth per woman declined dramatically (Flora, Kraus, and Pfenning (1983)). The "corollary" of the higher survival rates and rising schooling is that the pattern of population growth has changed too. Initially, rising survival rates increased population growth but for rising income this relationship turned negative. As can be seen in figure 2.4, population growth for the Less Developed Countries is hump-shaped, peaked at 2.7% around 1965 and has been declining since then. The same hump-shaped pattern can be verified for today's developed countries with the peak roughly 100 years earlier.

Figure 2.3: Fertility and Enrollment Rate

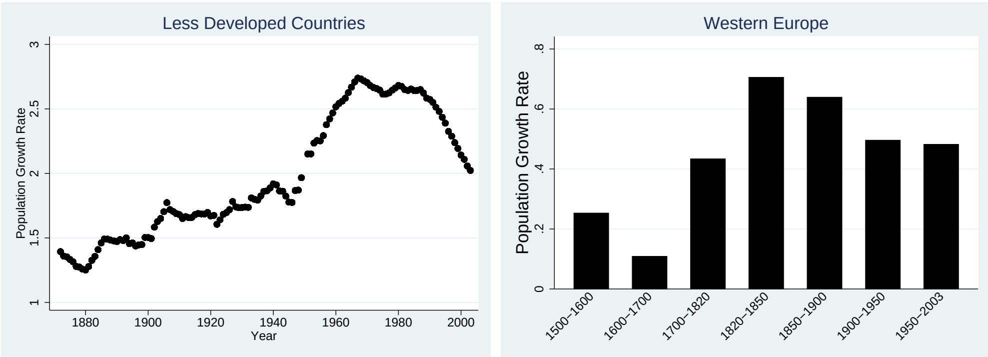


Notes: For primary and secondary enrolment rates are net enrolment rates, for tertiary education gross enrolment rates. *Source:* The World Bank (2004).

2.3 The Model

Consider an OLG economy where agents live for two periods and survival to the second period is uncertain. In the first period they are children and can work, receive some education (which enhances their adult human capital) or do both at the same time. Uncertainty concerning their survival is unraveled at the end of the first period (childhood). The earnings from their labor supply accrues to the parents. If they survive they become adults, they consume their total income and make a one-time fertility decision about the desired number of children, children's labor supply and educational attainment. Adults do not leave any bequests. Time is discrete and is extending into the infinite future. The econ-

Figure 2.4: Population Growth in LDCs and Western Europe



Notes: LDC: Africa, Latin America and Asia (without Japan), 5 year moving average. Western Europe: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, UK. *Source:* Maddison (2001)

omy produces a single consumption good using two factors of production: human capital and land.

2.3.1 Household Behavior

In this setup households choose consumption c_t, the number of newborns n_t, child labor supply ℓ_t and schooling investment s_t they give to each child. Preferences are defined over adult consumption c_t, the future earnings of the surviving children $q_t n_t h_{t+1} w_{t+1}$ where q_t is the probability to survive to adulthood, n_t is the number of children, h_{t+1} is the human capital of each child and w_{t+1} is the wage per unit of human capital. Parents also derive utility from child leisure $(1-\ell_t)$ where ℓ_t is child labor supply. For simplicity we assume that n_t is continuous, or, we deal with an average individual in the economy. The utility function of generation t can be thus written as

$$U_t = \gamma_1 \log(c_t) + \gamma_2 \{\mathbb{E}[\ln(q_t n_t h_{t+1} w_{t+1})]\} + \gamma_3 \ln(1-\ell_t) \tag{2.1}$$

where expectations are taken with respect to the child survival rate q_t. This modeling strategy is commonly used in the literature. Having children and their human capital in the utility function can be interpreted either as pure parental altruism or as an implicit old-age pension system if children are likely to support their parents.[2] In this model, the survival probability refers to the chances to survive to the age until children start making own economic decisions. Due to this simple setup it is not possible to distinguish between infant, early and late childhood mortality. Assuming that children's survival rate is binomially distributed and using the method of Kalemli-Ozcan (2003), the above expected utility maximization can be approximated[3] by

$$U_t = \gamma_1 \ln(c_t) + \gamma_2 \left[\ln(q_t n_t h_{t+1} w_{t+1}) - \frac{1-q_t}{2n_t q_t} \right] + \gamma_3 \ln(1-\ell_t). \tag{2.2}$$

The difference to a standard maximization problem without uncertain child survival is the remainder term in the parentheses. Low survival probability generates large disutility which can be minimized by having more children. The economic consequence of this additional term is that it generates a "precautionary" demand for offspring.[4] An intuitive justification why precautionary demand may be important is that replacement of children is not possible any more once mothers leave their fertile years. This problem is not present in modern economies but is certainly of importance in a high mortality environment. Note that this additional term vanishes if the survival probability approaches unity. Naturally, with $q_t = 1$ there is no more risk and we are back in the certainty case.

Human capital is produced according to

$$h_{t+1} = (\underline{s} + s_t)^{\xi} \tag{2.3}$$

where $\underline{s}$ and ξ are parameters and s_t is schooling investment into the children's human capital.[5] Investment in schooling has – from the households' point of view – only pri-

[2]The modeling alternative in which agents derive utility from the utility of their children (i.e. the dynastic approach by Becker and Barro (1988) or Barro and Becker (1989)) requires the debatable assumption that the agents know what their children will do. For models where the old-age security motive is made explicit see e.g. Boldrin and Jones (2002) and Ehrlich and Lui (1991).

[3]This is basically a third order approximation of the log-function evaluated at the mean of the distribution. See appendix 2.A for a derivation of the approximation.

[4]See also Sah (1991) for an application of a similar idea to parental welfare.

[5]There is no interaction between working and school. Some authors (e.g. Strulik (2004a)) assume that if children work, the efficiency of schooling is diminished and the accumulation process of human capital is less efficient. Although there is empirical evidence that labor has a negative effect on school achievement (Psacharopoulos (1997)), we ignore this issue here since it alters only the quantitative but not the qualitative aspects of the model.

vate benefits and households do not take possible externalities of schooling into account. Without investment into schooling, the stock of human capital is a constant scaling factor.

Adults supply labor inelastically and use a portion of their remaining time – here standardized to unity – on rearing children and (if optimal) educating them. There are no tuition fees: the cost of education are only parent's opportunity costs.[6] Each child consumes a fixed share $v \in (0, 1)$ of the parents' time which is independent of the number of children. This fixed cost per child is assumed to capture forgone wages, nutrition, clothing or other relevant expenditures. On the other hand, children can be sent to work and earn a fraction $\theta < 1$ of an adult's wage. The budget constraint is then

$$c_t = w_t h_t [1 - (v + s_t) n_t] + \theta w_t h_t \ell_t n_t. \tag{2.4}$$

Additional constraints are the (natural) "birth limit" restriction $n_t \leq 1/v$[7], non-negative consumption, non-negative schooling investment, and non-negative child labor supply. Additionally, we make the following assumptions:

Assumption 2.1. *$v > \theta$.*

Assumption 2.2. $v\xi - \underline{s} > 0$

Assumption 2.3. $v - \underline{s} - \theta > 0$

Assumption 2.1 is needed to ensure that children are always a monetary cost to parents. Assumption 2.2 guarantees that parents will invest into schooling in an environment without mortality risk. Finally, assumption 2.3 guarantees that there is always an interior solution.

[6]The introduction of tuition fees does not affect the qualitative results as long as they are proportional to income. For a model with schooling costs depending on parents' human capital see de la Croix and Doepke (2003).

[7]An alternative interpretation is that $1/v$ is a social norm for the maximum number of children.

2.3.2 Solution to the Household's Problem

Solving the household's decision problem gives the following first order conditions for the schooling decision, child labor supply and the number of offspring

$$\Delta(s_t, \lambda) \equiv \frac{\gamma_2 \xi}{\underline{s} + s_t} - \lambda w_t h_t n_t, \tag{2.5a}$$
$$= 0 \quad \text{if} \quad s_t > 0$$
$$< 0 \quad \text{if} \quad s_t = 0$$
$$\Delta(\ell_t, \lambda) \equiv \frac{\gamma_3}{1 - \ell_t} - \lambda w_t h_t n_t \theta, \tag{2.5b}$$
$$= 0 \quad \text{if} \quad \ell_t \in (0, 1)$$
$$> 0 \quad \text{if} \quad \ell_t = 0$$
$$\Delta(n_t, \lambda) \equiv \lambda w_t h_t (v + s_t - \theta \ell_t) - \gamma_2 \left[\frac{1}{n_t} + \frac{(1 - q_t)}{2 n_t^2 q_t} \right] \tag{2.5c}$$
$$= 0 \quad \text{if} \quad n_t \in (0, \frac{1}{v})$$
$$< 0 \quad \text{if} \quad n_t = \frac{1}{v},$$

where λ is the multiplier attached to the budget constraint and $\Delta(x_t, \lambda)$ is the derivative of the Lagrangian with respect to x_t and equals zero for any interior solution. Conditions (2.5a) and (2.5b) require that the marginal utility of schooling or child labor supply is larger or equal than the marginal utility of (forgone) consumption. The third equation (2.5c) requires that the marginal utility of children (quantity) is larger or equal to the lost income in terms of consumption.

Because of the various constraints these conditions need not be satisfied always with equality. In fact, some of the binding constraints and the associated corner solutions will be defining features of different stages of development. The inequality signs below the FOC's for the interior solutions provide the intuition for the corner solutions. Obviously, schooling and labor supply have always a unique solution (either interior or corner solution). The equation for the optimal number of children is nonlinear in n_t. Nevertheless, it can be shown that there is either a corner or a unique interior solution with a strictly positive number of children.[8] The intuition behind these results is that marginal utility is bounded

[8] In fact , this is a quadratic equation and in appendix 2.A it is shown that it has always a positive and

for corner solutions in (2.5a) and (2.5b) but unbounded from below for the number of newborns. Thus, parents will avoid zero children at any cost but corner solutions with zero schooling or zero child labor are possible. We will discuss the solution to the model in detail further below.

Since each adult has n_t children but only a share q_t survives to the next period, the population growth rate is given by

$$g_N = L_{t+1}/L_t - 1 = n_t q_t - 1, \tag{2.6}$$

where L_t is the size of the adult population at period t.

Equations (2.5a), (2.5b) and (2.5c) can be solved analytically to obtain closed form solutions. We do this for schooling and labor supply but show the optimal number of children only as an implicit function of the survival rate.[9] Assume that the survival rate is low and parents do not invest into human capital but have working children. Then parents only choose child labor supply and the number of children:

$$\ell_t = \frac{\gamma_1}{\gamma_1 + \gamma_3} - \frac{(1 - n_t v)\gamma_3}{n_t \theta(\gamma_1 + \gamma_3)} \tag{2.7a}$$

$$n_t \left[\frac{n_t (v - \theta)\gamma_1 + \gamma_3}{1 - n_t (v - \theta)} - \gamma_2 \right] = \gamma_2 \frac{1 - q_t}{2q_t} \tag{2.7b}$$

After some time, the survival rate may have increased sufficiently to induce parents to invest into schooling. The optimal choice of schooling, child labor and number of children is given by:

$$s_t = \frac{\xi\gamma_2}{n_t(\gamma_1 + \xi\gamma_2 + \gamma_3)} - \frac{(v - \theta)\xi\gamma_2 + \underline{s}(\gamma_1 + \gamma_3)}{(\gamma_1 + \xi\gamma_2 + \gamma_3)} \tag{2.8a}$$

$$\ell_t = \frac{\gamma_1 + \xi\gamma_2}{(\gamma_1 + \xi\gamma_2 + \gamma_3)} - \frac{(1 - n_t(v - \underline{s}))\gamma_3}{n_t\theta(\gamma_1 + \xi\gamma_2 + \gamma_3)} \tag{2.8b}$$

$$n_t \left[\frac{n_t(v - \underline{s} - \theta)\gamma_1 + \xi\gamma_2 + \gamma_3}{1 - n_t(v - \underline{s} - \theta)} - \gamma_2 \right] = \gamma_2 \frac{1 - q_t}{2q_t} \tag{2.8c}$$

If for some survival rate child labor is endogenously abandoned, parents decide about

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a negative root.

[9] Since n_t is a quadratic equation it is possible to obtain a closed form solution. However, the result is rather cumbersome and is of no use for the remainder of the paper.

optimal schooling and children according to:

$$s_t = \frac{\xi\gamma_2}{n_t(\gamma_1 + \xi\gamma_2)} - \frac{v\xi\gamma_2 + \underline{s}\gamma_1}{(\gamma_1 + \xi\gamma_2)} \tag{2.9a}$$

$$n_t \left[\frac{n_t(v - \underline{s})\gamma_1 + \xi\gamma_2}{1 - n_t(v - \underline{s})} - \gamma_2 \right] = \gamma_2 \frac{1 - q_t}{2q_t} \tag{2.9b}$$

Assume that q_t and preferences are such that neither schooling nor child labor supply are positive at the optimal solution. Then parents face only a fertility-consumption trade off. The number of children is then implicitly defined by

$$n_t \left[\frac{v\gamma_1 n_t}{1 - n_t v} - \gamma_2 \right] = \gamma_2 \frac{1 - q_t}{2q_t} \tag{2.10}$$

In principle there is also one "pathological" solution to the household problem. Assume that $\underline{s} = 0$ and that parents' valuation of child leisure γ_3 is very low. For a low survival rate it is possible that the number of children equals the maximum fertility limit $1/v$ with child labor and schooling being both positive. Put differently, due to high uncertainty parents wish to have as many children as possible but then children finance parents' consumption and their own schooling by working. We exclude this case by a restriction on the model's parameters (assumption 2.3).

The solution to the households' maximization problem reveals that the nature of household solution does not change qualitatively during the different stages of development. Schooling decreases with the number of children (quantity-quality trade off) and child labor is higher if the number of children is higher. Which regime prevails thus depends on the parameter constellation. By the choice of the parameters (mainly θ, $\underline{s}$ and γ_3) one can obtain all possible solutions ranging from no child labor and no schooling to an interior solution with simultaneous working and schooling or a realistic solution for low mortality environments without child labor but schooling investment.

The optimal reaction of the household to exogenous changes in the survival rate are summarized in the following propositions.

Proposition 2.1. *If the survival rate is increasing the number of newborns n_t is decreasing.*

Proof. See appendix 2.A. □

Proposition 2.2. *If the survival rate is increasing schooling is increasing (if positive) and child labor is decreasing (if positive).*

Proof. See appendix 2.A. □

Proposition 2.3. *There exists always a survival rate $\tilde{q}_l$ low enough such that optimal schooling is zero if $\partial h_{t+1}/\partial s_t < \infty$. If preferences and relative child productivity θ are such that child labor is optimal for $q_t < \tilde{q}_l$, then the threshold value $\tilde{q}_l$ is an increasing function of θ.*

Proof. See appendix 2.A. □

Propositions 2.1 and 2.2 are the results of the interaction of lower precautionary child demand and a quantity-quality trade off. If child survival risk is falling, the number of children will decrease – even without schooling. This is the consequence of falling uncertainty and thus falling precautionary demand for children. With a decreasing number of children, parents move out from the corner solution and will endow each offspring with education. Proposition 2.3 states that if the chance of children to survive to adulthood is low enough, parents will rather invest into quantity and will not endow their offspring with human capital. Moreover, the higher relative child labor productivity, the more likely is that parents will have many children and schooling will be delayed. The intuition is that if the child survival rate is at very low levels, parents would like to have a very high number of offspring to make sure that at least some of them survive. Since they also have a quantity-quality trade off, they will opt for zero schooling. On the other hand the more children can earn, the less costly they are. Thus, high child productivity increases the opportunity costs of schooling which explains why the number of children at the threshold $\tilde{q}_t$ is higher ($\partial \tilde{n}_t/\partial \theta > 0$) and schooling will be delayed ($\partial \tilde{q}_t/\partial \theta > 0$).

The behavior of the population growth rate g_N is a nonlinear and non-monotonic function of the survival rate which is summarized below:

Proposition 2.4. *Population growth is hump-shaped and has exactly one local maximum if there is no solution to the household problem with $\ell_t > 0$ and $s_t > 0$ simultaneously (i.e. no interior solution). If there exists an interior solution, then the population growth rate* *has the above property only if the population growth rate with child labor and schooling (i.e. interior solution) satisfies*

$$\left.\frac{\partial g_N}{\partial q}\right|_{n_t=\tilde{n}_t} \geq 0, \tag{2.11}$$

or if the regime without child labor but schooling satisfies

$$\left.\frac{\partial g_N}{\partial q}\right|_{n_t=\tilde{n}_t} \leq 0, \tag{2.12}$$

where $\tilde{n}_t = \frac{\gamma_3}{\gamma_3(v-\underline{S})+\theta(\gamma_1+\xi\gamma_2)}$ *is the number of children where child labor is endogenously abandoned. If one of the two conditions is violated, then the population growth rate has two local maxima.*

Proof. See appendix 2.A. □

For low q_t, rising survival probability dominates the drop in the number of children and population growth increases. Intuitively, for large survival risk small changes in the survival probability will not change the optimal solution for n_t much. For vanishing survival risk we have the opposite effect. If the incentive to educate children is strong enough, parents will decrease precautionary demand for children and additionally invest more time into each child. Therefore, population growth will decrease at high levels of q_t. The possible "complications" are caused by the fact that at $\tilde{n}_t$ the slope of $\partial n_t/\partial q_t$ becomes less steep (the number of children is still falling but at a lower pace). This is counterbalanced by rising survival probabilities which increases the population growth rate. Which effect will dominate in the end depends on the parameters of the model.

Lemma 2.1. *If relative child productivity increases, the number of newborns increases.*

Proof. See appendix 2.A. □

Lemma 2.2. *If relative child productivity increases child labor increases and schooling decreases.*

Proof. See appendix 2.A. □

The economic interpretation is quite intuitive. If children become relatively more productive on the labor market, the opportunity cost of child leisure goes up and thus child labor supply should rise. At the same time, the cost of children decreases. This is why the number of offspring rises. Schooling decreases because parents are engaged in a quantity-quality trade off.

2.3.3 The Steady State Solution

Assume that the economy grew out from poverty, the child survival rate approaches unity and child labor is abandoned. Then, we are back in a standard Becker-type model with a quantity-quality decision of the parents where only preferences (and some parameters from the human capital production function) determine the solution. Optimal education and number of children are then

$$n_{ss} = \frac{(1-\xi)\gamma_2}{(v-\underline{s})(\gamma_1+\gamma_2)} \tag{2.13a}$$

$$\begin{aligned} s_{ss} &= \frac{(1-n_{ss}v)\xi\gamma_2}{n_{ss}(\gamma_1+\xi\gamma_2)} - \frac{\underline{s}\gamma_1}{\gamma_1+\xi\gamma_2} \\ &= \frac{v\xi-\underline{s}}{1-\xi} \end{aligned} \tag{2.13b}$$

The results confirm the intuition behind the model. Higher fixed costs of children v, higher education productivity ξ increases education, higher fixed costs of education $\underline{s}$, decrease education. Obviously, the opposite is true for the number of children.[10]

2.4 The Macroeconomy

There is one sector producing a homogenous good used for consumption. The production technology uses human capital and an exogenously given amount of land. Output is then

[10]Note that with certain survival, mortality does not play a role for the optimal number of children and therefore population growth will monotonically increase with q_t. In such a setup, the number of children will be constant trough time and only a quantity-quality trade off is able to generate a demographic transition.

produced according to

$$Y_t = A_t H_t^{\alpha} T^{1-\alpha} \tag{2.14}$$

where H_t is human capital, T is the fixed amount of land and A_t is the level of TFP which grows over time. The fixed amount of land captures the Malthusian nature of the model. In absence of growth in H_t or A_t, a growing population will obviously drive down income per capita. Aggregate human capital is the sum of inelastic adult labor supply and the endogenously determined labor supply of children. I assume that children's and adults' labor are perfect substitutes. Aggregate human capital is then

$$H_t = h_t(L_t + L_t \theta \ell_t n_t) \tag{2.15}$$

where L_t is the number of adults at time t and $\theta \ell_t n_t$ is the effective labor supply of children. Substituting into equation (2.14) and rearranging we have

$$Y_t = A_t h_t^{\alpha} (L_t \tilde{L}_t)^{\alpha} T^{1-\alpha} \tag{2.16}$$

where $\tilde{L}_t \equiv 1 + \theta \ell_t n_t$. Thus, $L_t \tilde{L}_t$ is total labor supply in the economy. Following the literature (Galor and Weil (2000), Kögel and Prskawetz (2001)), the return to land is zero and income equals average labor productivity with

$$y_t = Y_t / (L_t \tilde{L}_t) = A_t h_t^{\alpha} (L_t \tilde{L}_t)^{\alpha - 1} T^{1-\alpha}. \tag{2.17}$$

Here, y_t is also the income of a family unit consisting of one adult and the children contributing $n_t \theta \ell_t$ to total labor supply.[11] In a developed economy without child labor, family and per capita income are identical ($\tilde{L}_t = 1$). The growth rate of efficiency wages is

$$g_t^y = g_t^A + \alpha g_t^h - (1-\alpha)(g_t^L + g_t^{\tilde{L}}). \tag{2.18}$$

Note that in steady state without child labor and stationary population the solution collapses to $g^y = g^A$ as in any growth model. During the transition, growth of human capital, changes in child labor supply and population dynamics affect the growth rate of wages.

[11] Income per capita would be smaller than y_t since θ and ℓ_t are both smaller than unity.

Equivalently, we can express the dynamics of income per family as

$$y_{t+1} = y_t(1 + g_t^A)(\underline{s} + s_t)^{\xi\alpha}(n_t q_t)^{\alpha-1}\left(\frac{1 + \theta n_{t+1}\ell_{t+1}}{1 + \theta n_t \ell_t}\right)^{\alpha-1} \tag{2.19}$$

where we have substituted h_{t+1}, population growth and $\tilde{L}_{t+1}$ out.

2.5 General Equilibrium

This section puts the household solution and the macroeconomic production side together. To close the model we develop the relationships and feedback effects between income, population, technological progress and survival rates. The last subsection presents the simulation results from a calibration exercise using realistic parameter values and data.

2.5.1 Technological Progress

The level of technology A_t is evolving according to

$$A_t = A_{t-1}(1 + g_{t-1}^A) \tag{2.20}$$

where g_{t-1}^A is the growth rate of technology. Technological progress is determined by the size of the adult population L_t and the schooling investment s_t with[12]

$$g_t^A = g(L_t, s_t) \qquad g_{t,L}^A > 0 \quad g_{t,s}^A > 0 \quad g_t^A(L_t, 0) > 0. \tag{2.21}$$

Technological progress depends here on the size of the population which introduces a strong scale effect. Although there is no clear empirical evidence for this specification in modern economies (Jones (1995)), the assumption seems to be true for a large part of human history (Kremer (1993), Galor and Weil (2000), Diamond (1998)).[13] Alternative specifications imposing some exogenous minimum g_t^A if there is no schooling investment and population is below a threshold level lead to similar results. Note that this specification

[12]One could also assume that g_A depends on the level of human capital which is the same as to assume that it depends on education since human capital is a function of education.

[13]On the other hand see Crafts and Mills (2007) for the opposite evidence.

allows for a transitory effect of a larger population on technological progress in a sense that a growing population leads to an acceleration of g^A. In the case of a stationary population technological progress can further accelerate if investment into human capital is positive.

2.5.2 Survival Law

We assume that idiosyncratic survival risk washes out and the survival rate evolves deterministically on the aggregate level . The survival rate of children $q_t \in (0, 1]$ is a function of income per capita y_{t-1} and is given by

$$q_t = q(y_{t-1}) \tag{2.22}$$

with positive first and negative second derivatives and $\lim\limits_{y_t \to \infty} q_t \to 1$. Income per capita enters the survival law in a Malthusian fashion. Falling income per capita decreases the survival probability of children which decreases population (growth) in the next period. By including y_{t-1} instead of y_t excludes contemporaneous feedback effects of the number of children on the survival law.[14]

2.5.3 The Dynamical System

The solution to the household problem is the foundation of the dynamic simulation. Given an initial child survival rate q_0 we can solve the household problem. Then, given an initial adult population L_0 and initial technological level A_0 we can feed in the households' decisions into the macroeconomic framework to calculate wages, population, technological progress and survival rates for the next period. Given these values, the entire system can be simulated. Before presenting the simulation results, we first derive some analytical results describing the dynamic behavior of the system. Using equation (2.19) and expressing all

[14] This simplifying assumption is needed because otherwise child labor, income and hence survival rates are jointly determined in general equilibrium. Although this is certainly the more realistic assumption, we abstract from this complication since it does not add any additional insights.

endogenous variables in terms of y_t and y_{t-1} we have

$$\begin{aligned} y_{t+1} &= y_t[1+g_t^A(L_t, s_t(y_{t-1}))][\underline{s}+s_t(y_{t-1})]^{\xi\alpha} \times \\ & [n_t(y_{t-1})q_t(y_{t-1})]^{\alpha-1}\left[\frac{1+\theta n_{t+1}(y_t)\ell_{t+1}(y_t)}{1+\theta n_t(y_{t-1})\ell_t(y_{t-1})}\right]^{\alpha-1} \\ L_{t+1} &= L_t n_t(y_{t-1})q_t(y_{t-1}) \end{aligned} \tag{2.23}$$

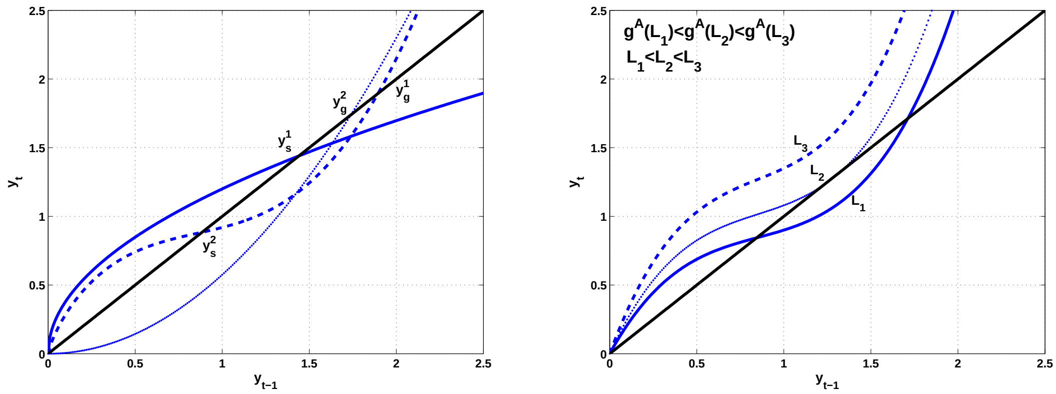
where $s_t = s_t(n_t(q_t(y_{t-1})))$, $n_t(q_t(y_{t-1}))$ and $\ell_t(n_t(q_t(y_{t-1})))$. This is a two dimensional (L, y) second order non-linear difference equation which is analytically not tractable. The "non-tractability" comes from the fact that the population growth rate is not a monotone function of y_{t-1}. It can be seen that $\partial y_{t+1}/\partial y_t > 0$, $\partial g_t^A/\partial y_{t-1} > 0$ and $\partial \ell_t/\partial y_{t-1} < 0$. This is all increasing next period's income. The fact that $\partial n_t/\partial y_{t-1} < 0$ but $\partial\ q_t/\partial\ y_{t-1}\ >\ 0$ makes a statement about the qualitative behavior of the system impossible. Assume that the population growth rate is rising as a consequence of rising survival rates which lowers income. This is counterbalanced by technological progress, rising human capital and falling child labor which pushes available resources per worker up. Thus, if the three factors contributing to rising productivity outweigh the diluting effect of population growth, income will grow, otherwise fall.

Figure 2.5(a) shows three possible functional forms for the relationship between today's and tomorrow's income per capita.[15] As can be seen, the strictly concave function has only one solution allowing only for a low income equilibrium (y_s^1). Thus, in absence of shifts in technology, income will always converge back to this stable solution. In the case of a strictly convex function, there are two solutions. If the economy starts out below the threshold y_g^2, income will converge to zero without technological progress. Otherwise, the economy will transit into a regime with endogenous growth. The S-shaped function allows also for two solutions: one stable Malthusian equilibrium (y_s^2) and a growing economy ($y_t > y_g^1$). Changes in the size of the population and the induced change in the pace of technological progress causes a shift of the curves. As shown in figure 2.5(b) a rising population shifts the curves outward. Note that due to the dependence of q on income and g^A on the population size, the economy can be in a situation with temporarily falling

[15]There is a fourth solution which is slightly less interesting. If y_{t+1} is strictly convex in y_{t-1} (and the slope is larger than unity at the origin) income is growing without bound for all initial values of income, population, and technological progress.

Figure 2.5: Solutions to the Difference Equation

(a) $g^A(L_t)$ given (b) $g^A(L_t)$ varying



income, falling survival rates but *accelerating* technological progress. In this case there is a "horse-race" between the diluting effect of population size on income on the one hand and positive effect on technological progress on the other hand. In the figure, we simultaneously move along the y-schedule to the left and shift the curve outwards due to higher technical progress. The net effect may go in either direction. Such a situation can happen if a country has an initial income per capita (and thus q_t) such that population is growing but the country is not large enough to generate a sufficiently high level of technological progress. Income and survival rates fall reducing g_L further and thus slowing down the *growth rate* of g_A. If technological progress does not catch up with population growth, the economy falls back to the Malthusian equilibrium. However, even if income temporarily falls back to the Malthusian level, the economy will not necessarily stay there forever.

Proposition 2.5. *If technological progress depends positively on the population size, the economy stays in the Malthusian equilibrium if*

$$-\frac{\partial^2 g_A(L_t,0)/\partial L_t^2}{\partial g_A(L_t,0)/\partial L_t} L_t = 1,$$

which is nothing else than requiring the elasticity of the marginal product of population with respect to population size to be unity.

Proof. See appendix 2.A □

The previous proposition states that the growth rate of TFP should not accelerate “to fast”, or the rate of increase has to decrease fast enough to just counterbalance population growth.[16] Then, income per capita will stay constant and so will the survival rate. Note that if we impose an exogenous growth rate $\bar{g}^A$ for TFP or alternatively we put an upper bound on $g_t^A(L_t, 0) = \bar{g}^A$ for all $L_t > \bar{L}$ then there is a threshold value for $\bar{g}^A$ or for the population size $\bar{L}$ which will determine whether the economy will grow or stay underdeveloped.

The fact whether child labor or schooling are optimal changes only the slope of the curves at the regime switching point. If schooling becomes optimal the slope increases, a regime change to zero child labor flattens the slope at all income levels. Shocks to income per capita (and implicitly survival rate) or relative child productivity have thus the potential to lift the economy out of a development trap.

However, if mortality rates are sufficiently high, a ban of child labor (by setting $\theta = 0$) is not a guarantee for a kickoff of the development process. As suggested in proposition 2.3 the value of q_t beyond which parents invest into education decreases with θ but if $q_t < \tilde{q}_t$ then education will be nevertheless zero.[17] Thinking in dynamic terms, eradicating child labor has a positive effect on average wages, survival rates and therefore the number of future newborns will drop which induces parents to send children to school earlier. Thus, abolishing child labor may not have immediate benefits for growth but will pay off only in the future.

2.5.4 A Calibration Exercise

This subsection contains a calibrated version of the model and discusses the dynamic development of the economy. Obviously, this highly stylized model will not be able to capture the complexity of the real world. Due to the simple structure we can only focus on a limited number of calibration targets. Therefore, the primary target of this section is to demonstrate that this type of model is able to track observed historical developments and provide some quantitative guidance. For an easier comparison with real data the model's

[16] Note that for this to hold as L_t goes to infinity, the marginal product of L_t has to go to zero.

[17] See Patrinos and Psacharopoulos (1997) on this issue who also argue that due to income effects, not working does not automatically imply that children are attending school.

predictions are transformed into annualized growth rates according to $g_r^x = (1+g_m^x)^{1/J} - 1$ where $g_{r,m}^x$ are the growth rates for real (r) and model time (m) for variable x and J is a proxy for the length of a period and chosen to be 20. Although this choice is common in the literature (Lagerlöf (2006), Boldrin and Jones (2002)) one has to keep in mind that this has a a large effect on the growth rates when transformed from the generational time dimension to yearly growth rates.

In order to simulate the entire system we need to fix parametric forms for the equations determining the evolution of the survival rate and technological progress. We choose

$$
\begin{aligned}
q_{t+1} &= 1 - \frac{\varphi_1}{y_t^{\varphi_2}} && (2.24)\\
g_t^A &= \omega_1 L_t^{\lambda_1} + \omega_2 s_t^{\lambda_2}. && (2.25)
\end{aligned}
$$

These functional forms satisfy the conditions outlined above and the parameters were calibrated to provide realistic time paths for the endogenous variables. The exact functional form is not important for the qualitative behavior of the system.

The calibration targets on the household level were the number of children (fertility) and income share generated by working children. Total fertility rate in the 19th century fluctuated around 5 in European countries (Galor (2005)) and according to Patrinos and Psacharopoulos (1997) working children in Peru contributed around 14% to family income. Historical numbers are less reliable but are in the same order of magnitude. Assuming that woman's wages are around 50% of men (Galor (2005), p. 233), children contribute in our model about 14% to family income at the beginning of the development process. The human capital production function was calibrated by choosing $\underline{s}$ such that there is a corner solution and ξ was taken from Browning, Hansen, and Heckman (1999). The weights in the utility function were calibrated such that there is a regime with child labor but no schooling, an intermediate regime with working and learning children and finally a situation without child labor. Further, we choose the parameters of the household model such that population is stationary in steady state.

On the macroeconomic level we want to generate the typical inverted U-shape for population growth, initially low growth of wages but accelerating growth rate of TFP (and equivalently of wages) as soon as schooling becomes optimal and asymptotic convergence of child mortality to zero. For growth of wages we use the numbers from Hansen and Prescott (2002) as a calibration target. The share of human capital in the production function is also taken from the same source. The remaining parameters of equation (2.25) and (2.24) have no empirical counterparts[18] but were calibrated such that the model is able to generate an economic and demographic transition within a sensible time period of about 20 generations. As can be seen in the graphs, the largest part of the decrease in mortality risk and the demographic transition is achieved within this time window. Also the adjustment of schooling, wage growth and number of children from initial to near steady state levels is completed within a reasonable period. All parameters and initial conditions are summarized in table 2.1.

Table 2.1: Summary of Parameters and Initial Conditions

Households	v	0.33	γ_1	0.30
	$\underline{s}$	0.20	γ_2	0.60
	ξ	0.80	γ_3	0.10
	θ	0.09		
Survival Rate	φ_1	0.24		
	φ_2	0.90		
TFP Growth	ω_1	0.06	ω_2	1.80
	λ_1	0.20	λ_2	1.50
Production Function	α	0.60		
Initial Conditions	A_0	1	T	40
	L_0	10	$q_0(y_0)$	0.5

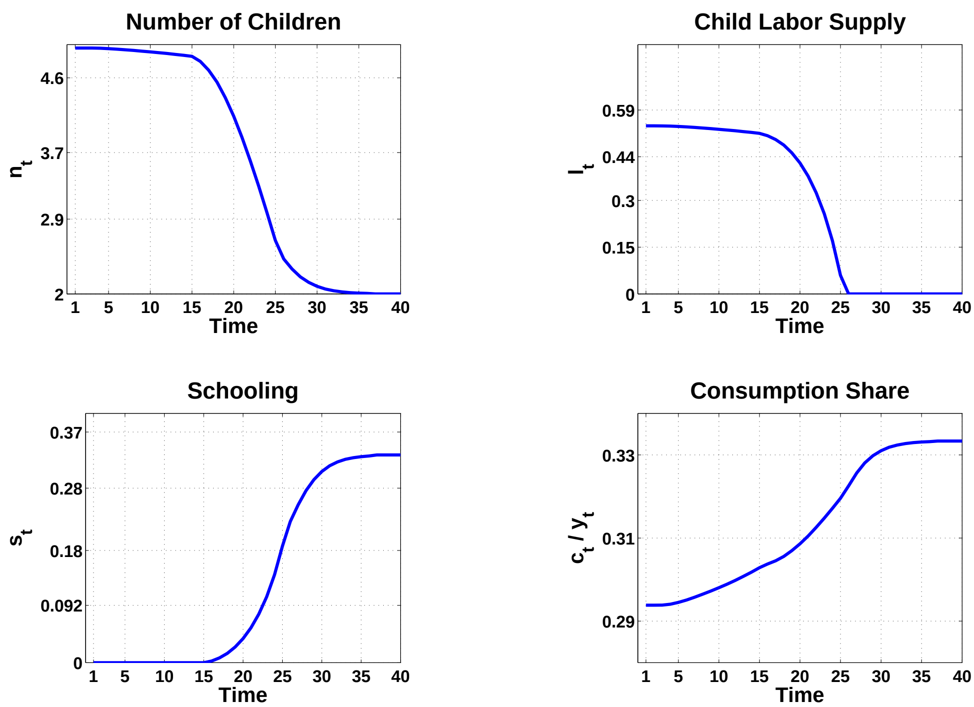
As a first step, in figure 2.6 we look at the household solutions generated by the baseline calibration from table 2.1. The number of children is high but decreasing due to falling mortality.[19] Due to rising survival rates, parents decrease their precautionary demand for children. At this stage of development this is the only reason why fertility is falling.

[18] We only know from several studies (Cutler, Deaton, and Lleras-Muney (2006), Kalemli-Ozcan (2003)) that there is a concave relationship between income and survival probabilities which dictates $\varphi_2 < 1$.

[19] In this graph the number of children is shown as *per family* which is twice the number from the model solution where each individual is allowed to have children.

Obviously, this is not the main source of the demographic transition.[20] Education is initially not zero and child labor supply is positive. Later, the survival rate approaches the critical threshold above which educational investment becomes positive. From this point in time onwards parents face also an additional quantity-quality trade off. The falling precautionary demand is now augmented by the quantity-quality trade off. Thus, the optimal number of offsprings starts to drop dramatically. Simultaneously, child labor is decreasing and later endogenously abandoned. The entire adjustment process from high fertility, high child labor and no schooling environment to a situation without child labor, low fertility and schooling investment is completed in less than 15 generations. The panel in the south-east of the graph shows the share of income spent on consumption. During the development process, parents not only decrease fertility but increase spending on the quality of children but are also able to increase own consumption.

Figure 2.6: General Equilibrium Simulation - Household



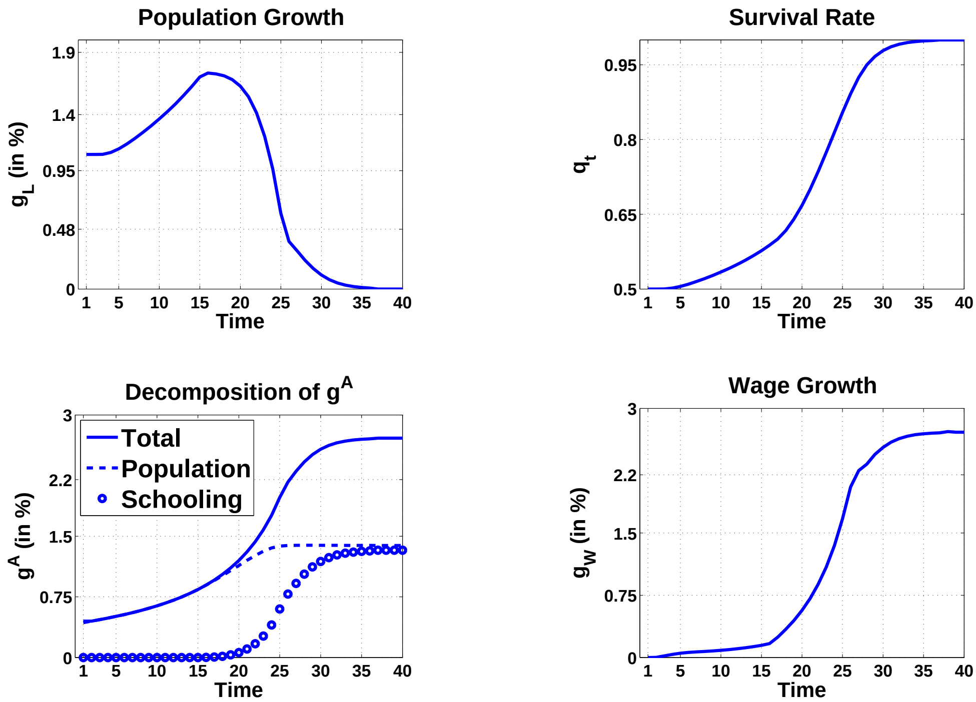
[20]This result is also confirmed by Doepke (2005) who finds that a reduction in survival risk reduces total fertility but eventually concludes that the dramatic fall in the net reproduction rate (number of surviving daughters) must have been caused by other factors than declining child or infant morality.

Figure 2.7 shows the time paths of several macroeconomic variables. Initially, the population growth rate is around 1% but starts rising as mortality (panel to the right) falls, reaches the maximum level at 1.7% and drops then monotonically to the steady state value of no population growth (by construction). The reason for the accelerating population growth is the insufficient drop in the number of children. As can be seen in the household solution the number of children is falling as the survival rate increases. However, this drop is not enough to counterbalance falling mortality. Thus, the total effect is that population growth is rising. The demographic transition starts to unfold only as parents start to invest into schooling. This can also be seen from the panel displaying the evolution of wage and TFP growth. Initially, the growth rate of TFP is just enough to counterbalance the growth of population and hence income per capita is growing only slowly.[21] This is also reflected in only small increases of the survival rate. However, as the survival rate passes the critical threshold value agents start to educate children which boosts growth of TFP and wages. This feeds back into falling mortality and falling population growth. Eventually, child mortality is almost eliminated and the model converges to its steady state solution with a constant household decisions and constant growth rates. In this calibration, the contribution of the scale effect (L_t) and of schooling to technological progress is approximately the same.

Figure 2.6 shows that the number of newborns decreases only slightly if schooling is zero. The falling fertility is only due to falling survival risk. However, it is rather obvious that this effect is not strong enough to bring fertility down to levels low enough generating a genuine demographic transition. Observe that in figure 2.7 for low values of q the population is growing even with falling fertility. This result is not driven by the choice of parameters but it is rather a general feature of the model. Figure 2.8 shows the elasticity of the number of birth with respect to the survival rate for different parameter constellations.[22] We generate three different scenarios ranging from a situation where child labor and schooling do no not overlap (labeled "no interior solution") to the situation in which children attend school and work at the same time (labeled "interior solution") and a parameter constellation without child labor but positive schooling over the entire range of q (labeled "no child labor"). To generate this solutions we have to adjust some parameters of the model but leave the

[21]Initially TFP growth is only fueled by rising population since educational investment is zero.
[22]Note that these graphs have the survival rate q on the horizontal axis.

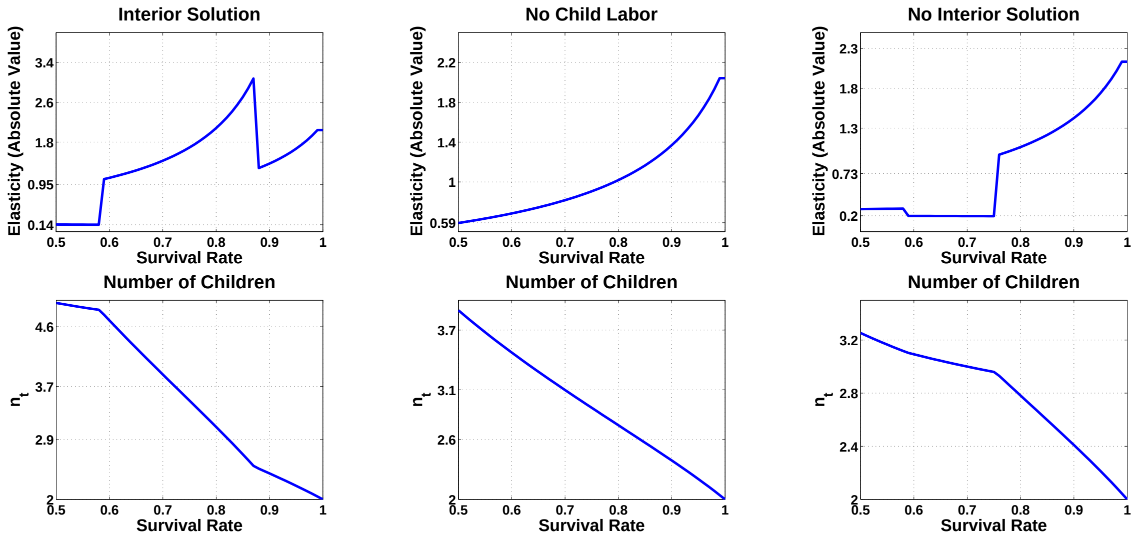
Figure 2.7: General Equilibrium Simulation - Macro



initial conditions unchanged. We set $\theta = 0$ in order to obtain the solution without child labor. The graph without an interior solution is created by setting $\underline{s} = 0.24$ and $\theta = 0.068$ and re-calibrating γ_1, γ_2, and γ_3 such that fertility for $q = 1$ is identical in all graphs. This corresponds to a situation in which child labor is abandoned early, followed by a situation without child labor (but still no schooling) and positive schooling investment only later.

It can be seen that without schooling, the elasticity is rather small (and always well below unity) without the potential to generate a demographic transition. The necessary condition is the introduction of a quantity-quality trade off via the investment into schooling. This result is rather insensitive to the parameter choice.

Figure 2.8: Elasticity of Children w.r.t. Mortality



2.6 Conclusion

This paper analyzes the dynamic co-movement of fertility, schooling and child labor assuming that child survival is uncertain. In early stages of development the economy is characterized by low income, high mortality, high fertility and child labor. Due to rising technological progress, however, income starts to grow and so will the survival probability of children. Therefore parents will start to decrease their precautionary demand for children. Because the pure effect of falling mortality is not sufficient to generate a large quantitative change in the number of children, the population growth rate is still accelerating and thus the growth rate of resources per capita is rather low. At some point, however, parents will start to invest into schooling accelerating the development process. This change will induce a sizable drop in fertility which is the trigger of a demographic transition. Eventually population growth starts to decline and the economy converges to a balanced growth equilibrium. Along the development process child labor will be abandoned as parents decide to shift more resources to child quality and the need for many offsprings (caused by high mortality) vanishes.

We show that child labor has an adverse effect on development in a sense that even if parents value child leisure, child labor will delay investment into schooling. If the survival chances of children are sufficiently low a ban of child labor, however, does not necessarily induce parents to invest into schooling. On the other hand, the model's prediction is that the effect of falling mortality alone is not sufficient to induce a large behavioral change. Thus, the demographic transition can only be explained by the rise of education and thus a quantity-quality trade off.

On the macroeconomic side we analyze the conditions for stagnation and endogenous growth. Without a link between population size and technological progress (or an upper bound on technological progress) the economy can be stuck in a Malthusian equilibrium with low income, high fertility, child labor and no human capital investment or transit into an endogenous growth regime which characterizes modern economies. The outcome depends on the parameters of the model and therefore multiple equilibria are possible. If technological progress depends on the size of the population the economy is likely to escape from the domain of attraction of the Malthusian "trap" except a knife-edge condition is satisfied.

2.A Appendix: Proofs

Proof of proposition 2.1. Rewriting the first order conditions from the text and rearranging them gives

$$
\begin{aligned}
F(n_t, q_t) &= n_t \left[\frac{n_t(v - \underline{s} - \theta)\gamma_1 + \xi\gamma_2 + \gamma_3}{1 - n_t(v - \underline{s} - \theta)} - \gamma_2 \right] - \gamma_2 \frac{1 - q_t}{2q_t} && \ell_t > 0, s_t > 0 && \text{(2.26a)} \\
F(n_t, q_t) &= n_t \left[\frac{n_t(v - \theta)\gamma_1 + \gamma_3}{1 - n_t(v - \theta)} - \gamma_2 \right] - \gamma_2 \frac{1 - q_t}{2q_t} && \ell_t > 0, s_t = 0 && \text{(2.26b)} \\
F(n_t, q_t) &= n_t \left[\frac{n_t(v - \underline{s})\gamma_1 + \xi\gamma_2}{1 - n_t(v - \underline{s})} - \gamma_2 \right] - \gamma_2 \frac{1 - q_t}{2q_t} && \ell_t = 0, s_t > 0 && \text{(2.26c)} \\
F(n_t, q_t) &= n_t \left[\frac{v\gamma_1 n_t}{1 - n_t v} - \gamma_2 \right] - \gamma_2 \frac{1 - q_t}{2q_t} && \ell_t = 0, s_t = 0 && \text{(2.26d)}
\end{aligned}
$$

Each of these equations implicitly defines $n_t = n_t(q_t)$. Note that the left part (the left hand side in the FOC) is only a function of n_t and the the right part (the right hand side in the FOC) is only a function of q_t. Thus, the effect of changing survival probabilities on the optimal number of children is given by

$$
\frac{\partial n_t}{\partial q_t} = -\frac{\partial F(\cdot)/\partial q_t}{\partial F(\cdot)/\partial n_t} \tag{2.27}
$$

Further, the behavior of the RSH for limiting cases of q_t is given by

$$
\lim_{q_t \to 1} RHS = 0 \tag{2.28a}
$$

$$
\lim_{q_t \to 0} RHS = \infty \tag{2.28b}
$$

The LHS deserves more discussion. First observe that it holds that

$$
\lim_{n_t \to 0} LHS = 0 \tag{2.29a}
$$

$$
\lim_{n_t \to 1/v} LHS \begin{cases} = \infty & \text{if } \ell_t = 0, s_t = 0 \\ < \infty & \text{else} \end{cases} \tag{2.29b}
$$

This implies that for all cases $\lim_{n_t \to 1/v} LHS < \infty$ (i.e. the birth limit is binding) there is a lower bound $\tilde{q}_t$ which satisfies $F(n_t, q_t) = 0$. For all $q_t < \tilde{q}_t$ the solution is $n_t = 1/v$.

To determine the sign of $\partial n_t/\partial q_t$ we need to evaluate the derivatives of RHS and LHS. The RHS of all cases is identical. Taking the derivative with respect to q_t gives

$$\frac{\partial F(n_t,q_t)}{\partial q_t} = \gamma_2 \frac{1}{2q_t^2} > 0 \quad \forall \quad q_t \in (0,1). \tag{2.30}$$

The derivative of the LHS depends on the scenarios and is given by the following equations

$$\begin{aligned}
\frac{\partial F(n_t,q_t)}{\partial n_t} &= \frac{n_t\tilde{v}\gamma_1(2-n_t\tilde{v})+\xi\gamma_2+\gamma_3}{(1-n_t\tilde{v})^2} - \gamma_2 && \ell_t > 0, s_t > 0 && \text{(2.31a)}\\
\tilde{v} &\equiv v - \underline{s} - \theta \\
\frac{\partial F(n_t,q_t)}{\partial n_t} &= \frac{n_t\tilde{v}\gamma_1(2-n_t\tilde{v})+\gamma_3}{(1-n_t\tilde{v})^2} - \gamma_2 && \ell_t > 0, s_t = 0 && \text{(2.31b)}\\
\tilde{v} &\equiv v - \theta \\
\frac{\partial F(n_t,q_t)}{\partial n_t} &= \frac{n_t\tilde{v}\gamma_1(2-n_t\tilde{v})+\xi\gamma_2}{(1-n_t\tilde{v})^2} - \gamma_2 && \ell_t = 0, s_t > 0 && \text{(2.31c)}\\
\tilde{v} &\equiv v - \underline{s} \\
\frac{\partial F(n_t,q_t)}{\partial n_t} &= \frac{n_t v\gamma_1(2-n_t v)}{(1-n_t v)^2} - \gamma_2 && \ell_t = 0, s_t = 0 && \text{(2.31d)}
\end{aligned}$$

We have now to evaluate each of the cases in the range of all possible solutions for n_t. For the case of interior solutions we have

$$\left.\frac{\partial F(n_t,q_t)}{\partial n_t}\right|_{n_t=0} = \gamma_3 - \gamma_2(1-\xi) \tag{2.32a}$$

$$\left.\frac{\partial F(n_t,q_t)}{\partial n_t}\right|_{n_t=\frac{1}{v}} = \frac{\left(1-\left(\frac{\alpha+\theta}{v}\right)^2\right)\gamma_1+\gamma_3}{\left(\frac{\alpha+\theta}{v}\right)^2} + \gamma_2\left(\frac{\xi}{\left(\frac{\alpha+\theta}{v}\right)^2}-1\right) \tag{2.32b}$$

For the case without schooling but child labor

$$\left.\frac{\partial F(n_t,q_t)}{\partial n_t}\right|_{n_t=0} = \gamma_3 - \gamma_2 \tag{2.32c}$$

$$\left.\frac{\partial F(n_t,q_t)}{\partial n_t}\right|_{n_t=\frac{1}{v}} = \frac{\left(1-\left(\frac{\theta}{v}\right)^2\right)\gamma_1+\gamma_3}{\left(\frac{\theta}{v}\right)^2} - \gamma_2 \tag{2.32d}$$

For the case without child labor but schooling

$$\left.\frac{\partial F(n_t, q_t)}{\partial n_t}\right|_{n_t=0} = \gamma_2(\xi - 1) \tag{2.32e}$$

$$\left.\frac{\partial F(n_t, q_t)}{\partial n_t}\right|_{n_t=\frac{1}{v}} = \frac{\left(1-\left(\frac{\alpha}{v}\right)^2\right)\gamma_1}{\left(\frac{\alpha}{v}\right)^2} + \gamma_2\left(\frac{\xi}{\left(\frac{\alpha}{v}\right)^2} - 1\right) \tag{2.32f}$$

However, if the model's parameters are such that there is a steady state with $q_t = 1$ and schooling but no child labor, then the parameter restriction $\xi > \frac{\alpha}{v}$ has to hold. Using this it is clear that the second parenthesis is positive. And for the case with zero schooling but also no child labor

$$\left.\frac{\partial F(n_t, q_t)}{\partial n_t}\right|_{n_t=0} = -\gamma_2 \tag{2.32g}$$

$$\left.\frac{\partial F(n_t, q_t)}{\partial n_t}\right|_{n_t=\frac{1}{v}} = \infty \tag{2.32h}$$

Further, it can be shown under the parameter restrictions (essentially assumptions 2.1 and 2.3) the following holds

$$\left.\frac{\partial F(n_t, q_t)}{\partial n_t}\right|_{n_t=\frac{1}{v}} > \left.\frac{\partial F(n_t, q_t)}{\partial n_t}\right|_{n_t=0} \tag{2.33}$$

$$\frac{\partial^2 F(n_t, q_t)}{\partial n_t^2} = \frac{2(v - \underline{s} - \theta)(\gamma_1 + \xi\gamma_2 + \gamma_3)}{(-1 + n(v - \underline{s} - \theta))^3} < 0 \tag{2.34}$$

This ensures monotonicity of the derivatives. If assumptions 2.1 or 2.3 are violated then $\partial F/\partial n$ may turn positive leading to two possible positive solutions. □

Proof of proposition 2.2. Use equations (2.8a) and (2.9a), take the derivative with respect to q_t and use the result $\partial n_t/\partial q_t < 0$ from proposition 2.1. Then the claim

$$\frac{\partial s_t}{\partial q_t} = \frac{\xi\gamma_2}{\gamma_1 + \xi\gamma_2 + \gamma_3}\frac{-\partial n_t/\partial q_t}{n_t^2} > 0, \tag{2.35}$$

is established where for the solution without child labor the parameter γ_3 is set to zero. To prove that child labor is decreasing if the survival probability increases use equations

(2.8b) and (2.7a) and take the derivative with respect to q_t. The result is

$$\frac{\partial \ell_t}{\partial q_t} = -\frac{\gamma_3}{\gamma_1 + \gamma_3 + \xi\gamma_2}\frac{-\partial n_t/\partial q_t}{n_t^2} < 0 \tag{2.36}$$

where we again use the result $\partial n_t/\partial q_t < 0$ from proposition 2.1. In case of a solution without schooling we have to set $\xi\gamma_2 = 0$ which does not change the sign. □

Proof of proposition 2.3. From equations (2.8a) or (2.9a) we see that schooling is a negative function of the number of children. Thus, there is threshold value $\tilde{n}_t$ such that schooling is zero. Using proposition 2.1 we can conclude that there is a survival rate low enough such that $n_t \geq \tilde{n}_t$ and thus ensuring $s_t \leq 0$. The second part can be proven by using that $\partial\tilde{n}_t/\partial\theta > 0$. The claim $\partial\tilde{q}_t/\partial\theta > 0$ follows then from the fact that the cross derivative of the left hand side of (2.8c) or (2.7b) with respect to $\{n_t, \theta\}$ $\forall n_t \in (0, 1/v]$ is negative and the derivative of the right hand side is decreasing in q_t. Thus, with rising θ and consequently rising n_t for $s_t = 0$ to hold, we need a higher survival rate. □

Proof of proposition 2.4. Rewriting equations (2.8c), (2.7b), (2.9b) and (2.10) as

$$F(n_t, q_t) \equiv n_t\left(\frac{n_t\tilde{v}\gamma_1 + \xi\gamma_2 + \gamma_3}{1 - n_t\tilde{v}} - \gamma_2\right) - \gamma_2\frac{1 - q_t}{2q_t} = 0 \tag{2.37}$$

where $\tilde{v}$, $\xi\gamma_2$ and γ_3 depend on the optimal regime (see equations in the text). Denoting the population growth factor as $L_{t+1}/L_t \equiv g_N$ and using $L_{t+1}/L_t = n_t q_t$ gives $n_t = g_N/q$. Inserting this into the above equation and rearranging gives

$$F(g_N, q_t) \equiv 2g_N^2\tilde{v}\gamma_1 + (g_N\tilde{v} - q)(1 + 2g_N - q)\gamma_2 + 2g_N q(\gamma_3 + \xi\gamma_2) \tag{2.38}$$

The change in population growth as a function of the survival rate is then

$$\frac{\partial g_N}{\partial q} = -\frac{\partial F/\partial q}{\partial F/\partial g_N} = \frac{-\gamma_2 + (2q - g_N(\tilde{v}+2))\gamma_2 + g_N 2(\gamma_3 + \xi\gamma_2)}{-\gamma_2\tilde{v}(1 + 2g_N - q) + 2(\gamma_2 q - 2g_N\tilde{v}\gamma_1) - 2q(\gamma_3 + \xi\gamma_2)}. \tag{2.39}$$

At $q = 0$ and $g_N = 0$ this gives $1/\tilde{v}$ which is always positive.[23] However, if $q = 1$ we have $g_N = \frac{(1-\xi)\gamma_2}{(v-\underline{S})(\gamma_1+\gamma_2)}$ from the steady state solution with positive schooling and no child labor.

[23]This does not imply that the population growth rate is positive. It only means that it is increasing.

Using this gives

$$\frac{\partial g_N}{\partial q} = \frac{(g_N(2+\tilde{v})-1)\gamma_2 - 2g_N\xi\gamma_2}{4g_N\tilde{v}\gamma_1 - 2(1-2\tilde{v}g_N)\gamma_2 + 2\xi\gamma_2} \tag{2.40}$$

Assuming that schooling has high returns, i.e. ξ is approaching unity, the derivative is unambiguously negative. However, for all ξ the condition ensuring declining population growth at high survival rates is

$$\frac{(1-\xi)\gamma_2}{\tilde{v}(\gamma_1+\gamma_2)}(2(1-\xi)+\tilde{v}) < 1. \tag{2.41}$$

where we have used g_N as specified as above.

The proof for the non-monotonic behavior of the shape (i.e. second derivative) involves two steps. First, observe that the population growth rate is a strictly concave function of the survival rate (and therefore for all regimes). Assume now that we have parameter values and a q_t such that we have an interior solution with positive schooling and child labor. Thus, if $\partial g_N/\partial q < 0$ evaluated at $\tilde{n}_t$[24] holds, then then g_N at this point must be below the local maximum. Further note that we can rewrite

$$\frac{\partial g_N}{\partial q_t} = \frac{\partial n_t}{\partial q_t} q_t + n_t \tag{2.42}$$

which implies that

$$\left.\frac{\partial g_N}{\partial q_t}\right|_{\ell,s>0} < \left.\frac{\partial g_N}{\partial q_t}\right|_{\ell=0,s>0}. \tag{2.43}$$

Second, it follows that if $\partial g_N/\partial q < 0$ evaluated at $\tilde{n}_t$ is negative for the solution without child labor but positive schooling (i.e. after the regime switch), then this derivative must be smaller (i.e. more negative). Due to strict concavity, g_N drops monotonically to the steady state value. If the derivative is positive, then there will be a second local maximum because the population growth rate will rise to a second local maximum.

On the other hand, if $\partial g_N/\partial q > 0$ holds for the interior solution then the population growth rate will continue to rise after the endogenous regime switch (see derivative above) and

[24] Recall that $\tilde{n}_t$ is the point where the household endogenously switches from the interior regime to the regime without child labor but schooling.

eventually start to decline after a local maximum converging monotonically to the steady state value. □

Proof of lemma 2.1. Use equations (2.8c) and (2.7b), redefine $\tilde{v} \equiv v - \alpha$ and $\varphi \equiv \xi\gamma_2 + \gamma_3$ if schooling and labor are interior and $\tilde{v} \equiv v$ and $\varphi \equiv \gamma_3$ if only child labor is optimal. Then, the first order condition for children can be written as

$$F(n_t, \theta) = n_t \left[\frac{n_t(\tilde{v} - \theta)\gamma_1 + \varphi}{1 - n_t(\tilde{v} - \theta)} - \gamma_2 \right] - \gamma_2 2 \frac{1 - q_t}{q_t} \tag{2.44}$$

The result can be established by showing that

$$\frac{\partial n_t}{\partial \theta} = - \frac{\partial F / \partial \theta}{\partial F / \partial n_t} > 0. \tag{2.45}$$

□

Proof of lemma 2.2. Use equations (2.8b) and (2.7a), redefine $\tilde{v} \equiv v - \underline{s}$ and $\varphi \equiv \xi\gamma_2$ if schooling and labor supply are interior. Otherwise $\tilde{v} \equiv v$ and $\varphi \equiv 0$. Taking then the derivative with respect to θ is

$$\frac{\partial \ell_t}{\partial \theta} = - \frac{\gamma_3}{\gamma_1 + \varphi + \gamma_3} \left(\frac{-\frac{\partial n_t}{\partial \theta}\theta - (1 - n_t\tilde{v})n_t}{(n_t\theta)^2} \right) > 0, \tag{2.46}$$

where we use the fact that $\partial n_t / \partial \theta > 0$ from proposition 2.1. To show the relationship between schooling and relative productivity use equation (2.8a) and take the derivative with respect to θ. The result is

$$\frac{\partial s_t}{\partial \theta} = \frac{\xi\gamma_2}{\gamma_1 + \xi\gamma_2 + \gamma_3} \left(1 - \frac{\partial n_t / \partial \theta}{n_t^2} \right) \gtrless 0 \tag{2.47}$$

where $\partial n_t / \partial \theta > 0$ from proposition 2.1. The result that $\frac{\partial s_t}{\partial \theta} < 0$ cannot be established analytically since it depends on the equilibrium value of n_t. However, in the numerical simulations it turns out that the equilibrium solution is always such that the claim above always holds. □

Proof that n_t has always a positive and a negative root. Using equation (2.5c) and replacing λ with γ_1/c_t gives

$$\frac{\gamma_1}{c_t} w_t h_t (v + s_t - \theta \ell_t) = \gamma_2 \left[\frac{1}{n_t} + \frac{(1-q_t)}{2n_t^2 q_t} \right]. \tag{2.48}$$

Rearrange this equation to obtain

$$-\frac{\gamma_1}{\gamma_2} \frac{w_t h_t (v + s_t - \theta \ell_t)}{c_t} n_t^2 + n_t + \frac{1-q_t}{2q_t} = 0. \tag{2.49}$$

Solving this quadratic equation for n_t proves the claim. □

Proof of proposition 2.5. Start with equation (2.18), assume that schooling is zero (which implies $g_h = 0$) and use the fact that child labor supply is constant ($g_{\tilde{L}} = 0$). Due to the Malthusian steady state assumption we have $g_y = 0$ implying $g_A(L_{t-1}(1+g_L), 0) = (1-\alpha) g_L$ where we have used $L_t = L_{t-1}(1 + g_L)$. Taking the derivative with respect to population growth gives

$$\frac{\partial g_A}{\partial L_t} L_{t-1} = 1 - \alpha.$$

With positive population growth this can only hold for all L_t if g_A is strictly concave in the population size and

$$\frac{\partial^2 g_A}{\partial L_t} L_t + \frac{\partial g_A}{\partial L_t} = 0$$

holds. Rearranging this equation proves the claim. □

Approximation of Expected Utility Maximization. The 3^{rd} order Taylor series approximation of the utility function $U(n_t q_t) = \log n_t q_t$ around the mean $\bar{n}_t$ is

$$U(n_t q_t) = \log \bar{n}_t + (\bar{n}_t - n_t q_t) \frac{1}{\bar{n}_t} - \frac{(\bar{n}_t - n_t q_t)^2}{2!} \frac{1}{\bar{n}_t^2} + \frac{(\bar{n}_t - n_t q_t)^3}{3!} \frac{2}{\bar{n}_t^3}. \tag{2.50}$$

Taking expectations gives then the result in the paper. The first term is evaluated at the mean, the second term vanishes due to $E[\bar{n}_t - n_t q_t] = 0$, the third term is just $E[(\bar{n}_t - n_t q_t)^2] = Var(n_t q_t) = n_t q_t (1 - q_t)$ and the last term is also zero because of the symmetry of the binomial distribution. Since we use a utility function which is unbounded from below and a distribution with mass on zero, we would have to include an arbitrary small constant into the utility function. However, this would not affect any of the results. $\square$

Chapter 3

Human Capital and the Demographic Transition: Why Schooling Became Optimal

3.1 Introduction and Motivation

The history of humanity was – until recently – characterized by dismal economic conditions: low income, low life expectancy, low investment into human capital and high fertility. Briefly summarized, "the life of man [used to be], solitary, poore, nasty, brutish, and short" (Hobbes (1651), p. 78). In modern Western economies we observe the opposite: high income, high life expectancy, highly educated individuals and low fertility. In this paper we develop a model which is able to rationalize the monotonic increase in human capital investment, the hump-shaped relationship between fertility and life expectancy and the endogenous appearance of a public schooling system. We argue that it is important to account for the change in the nature of the costs of child education: from time costs in an underdeveloped economy to monetary costs in a developed economy.

The driving force of the model is rising adult life expectancy. We use a simple life-cycle setup in which adults differ with respect to their productivity on the labor market and decide about consumption, investment into adult and child human capital and the number of children. They also chose whether they educate their children at home or whether they endow them with human capital by sending them to a school and paying tuition fees. If parents increase children's human capital using their own time, rising adult life expectancy will unambiguously increase fertility. On the contrary, if children are educated in the public schooling system, agents' reaction to rising life expectancy is ambiguous. We show that if adult human capital is sufficiently productive and parents' preferences for children are sufficiently concave, fertility falls as parents' life expectancy rises. Furthermore, parents deciding to send their children to a school have – for any life expectancy – fewer children and invest more in their human capital.

The decision which educational system to choose depends on parents' life expectancy and ability level. Given tuition fees, more productive agents choose the public schooling system whereas less productive agents decide to spend own time on children's human capital. As life expectancy – and thus lifetime income – increases, also less productive agents will opt for the schooling system. Thus, rising adult life expectancy induces a *composition* effect (public vs. private schooling) and a *behavioral* effect (effect of life expectancy can increase or decrease fertility if children are educated in schools). For initially low life expectancy, the share of agents participating in the public schooling system is low. Higher life expectancy thereby pushes economy-wide fertility up. With rising life expectancy during the development process, public schooling becomes efficient for more and more people generating a drop in aggregate fertility and an increase in human capital investment.

Thus, the key contribution of this paper is to provide a novel explanation for the fertility transition and the endogenous appearance of a mass schooling system in an otherwise rather standard model. The explanation is based on a change in the nature of investment in child quality from time costs to monetary costs. We thus propose a theory why a public schooling system emerged endogenously *without* a state intervention on a large scale. What we do *not* explain is why eventually schooling become free, i.e. why the society – via government and parliament – decided to first heavily subsidize primary schooling and then set up a schooling system financed by taxes. This would require to develop a theory in which political decisions (tax system, educational institutions, etc.) are determined endogenously within the model. We leave this extension for further research.

This paper is not the first to provide a possible explanation for the dramatic economic and demographic change occurring in the second half of the 19th century. Possible causes for declining fertility are declining child mortality rates (Kalemli-Ozcan (2002), Kalemli-Ozcan (2003), Tamura (2006)), natural selection favoring parents with a higher preference for child quality than quantity (Galor and Moav (2002)) and the narrowing of the gender wage gap making children more expensive (Galor and Weil (1996)). Further explanations are changing marriage institutions with a rising proportion of better educated women (Gould, Moav, and Simhon (2008)), structural change and an increasing share of people investing into human capital (Doepke (2004)) or the introduction of compulsory schooling (Sugimoto and Nakagawa (2010)). In Cervellati and Sunde (2005), Cervellati and Sunde (2007), and Soares (2005) rising adult life expectancy serves as the key explanatory variable for the observed economic development.[1] Particularly, higher life expectancy induces agents to invest more into human capital and decrease fertility. The driving mechanisms are the increasing opportunity costs of fertility as adult's human capital investment rises. Empirical evidence for the differential impact of life expectancy on population growth is provided by Cervellati and Sunde (2009). They show that before the demographic transition, improvements in life expectancy primarily increased fertility. Using historical time series, Clark (2005b) agues that fertility is not monotonically related to income or life expectancy. This hypothesis is supported by Lehr (2009) based on data from contemporary developing countries.

More recently, the literature started to deal with the question why schooling systems came into existence. Galor and Moav (2006) explain the rise of a general schooling system (and thus of mass education) with a regulatory intervention by a ruling capitalist class. If skills and capital are complementary in production, diminishing marginal returns to capital accumulation can be counteracted by increasing workers' human capital. They argue, that capitalists lobbied for the introduction of compulsory schooling out of a profit maximizing rationale. In Boucekkine, de la Croix, and Peeters (2007) the appearance of a public schooling system emerges from profit maximizing behavior of municipalities. As population density increased, more and more schools were constructed decreasing the distance (transportation costs) of each agent to the next school which increased school

[1]There is an enormous amount of papers not explicitly targeting to explain the demographic transition but provide explanations for various other aspects of the development process. Prominent papers are Galor and Weil (2000), Hansen and Prescott (2002), Jones (2001), Kremer (1993), Strulik (2004a), Kögel and Prskawetz (2001), Ehrlich and Lui (1991), Ngai (2004), Lagerlöf (2003), Fernández-Villaverde (2001), Tamura (2002). An excellent overview is provided by Galor (2005).

attendance rates.[2]

Section 3.2 provides stylized facts and section 3.3 contains a detailed description of the model and the solution to the individual choice problem. The dynamic behavior of the economy with a discussion of the development path and an illustrative simulation exercise can be found in section 3.4. Section 3.5 concludes the paper. All proofs are relegated to the appendix.

3.2 Life Expectancy, Schooling and Fertility

After a stagnation of living standards over centuries, the 19th century was the starting point of an unprecedented change in almost all aspects of economic and social life.[3] GDP per capita and population entered steep growth paths (Fig. 3.1d). Simultaneously we can observe that a lengthening of life become a trend rather than an occasionally lucky event. Although increases in life expectancy at birth were initially driven by falling child mortality, survival probabilities for adults also increased substantially (Fig. 3.1a). These improvements in living conditions of daily life were initially reflected in higher fertility. Crude birth rates and net reproduction rates reached their historical peaks around 1820 and started to fall soon thereafter.[4]

At the same time, acquisition of formal human capital started to gain momentum for the first time in history. The earliest statistics indicate that in 1850 around 10% of the children of age 5-14 attended primary school. Secondary school (10-19 years) did not enter official statistics before 1900 when the demographic transition was already well under way. Human capital measured by the ability to sign marriage contracts was, however, considerably higher. In the early 19th century, around 30% of all brides and 60% of grooms signed their marriage contracts with their names instead of using an "X" (Fig. 3.1c).[5] Note that

[2]For instance, Acemoglu and Robinson (2000), Bertocchi and Spagat (2004) and Grossman and Kim (2003) argue that providing eduction to the masses decreases the potential for social conflict and civic disorder. According to these papers, the introduction of a free (and compulsory) education system was not necessarily an altruistic act but served rather the interests of the ruling class.

[3]In this paper we use data for England and Wales but the same pattern can be also observed in other countries around the same time with good data; one prominent and often studied example being Sweden.

[4]There is no data available on total fertility rates before 1850. However, the few data points available show that TFR peaked around 1870 at 5 children per woman and started to decrease afterwards.

[5]Whether literacy was a useful skill before and during the industrial revolution is hotly debated in the

the timing of the fertility reversal is closer to the introduction of primary schooling than to rising secondary school enrollment rates. It is also remarkable that primary schooling enrollment rates started to increase *before* the introduction of a compulsory schooling. The Elementary Education Act 1870 (also known as Forster's Education Act) provided only partial funding for schools in underdeveloped regions but fees were still charged. The Elementary Education Act of 1880 made schooling compulsory for children aged 5-10 (but was never aggressively enforced) and only the Free Education Act of 1891 made basic education virtually free by heavily subsidizing primary schooling.

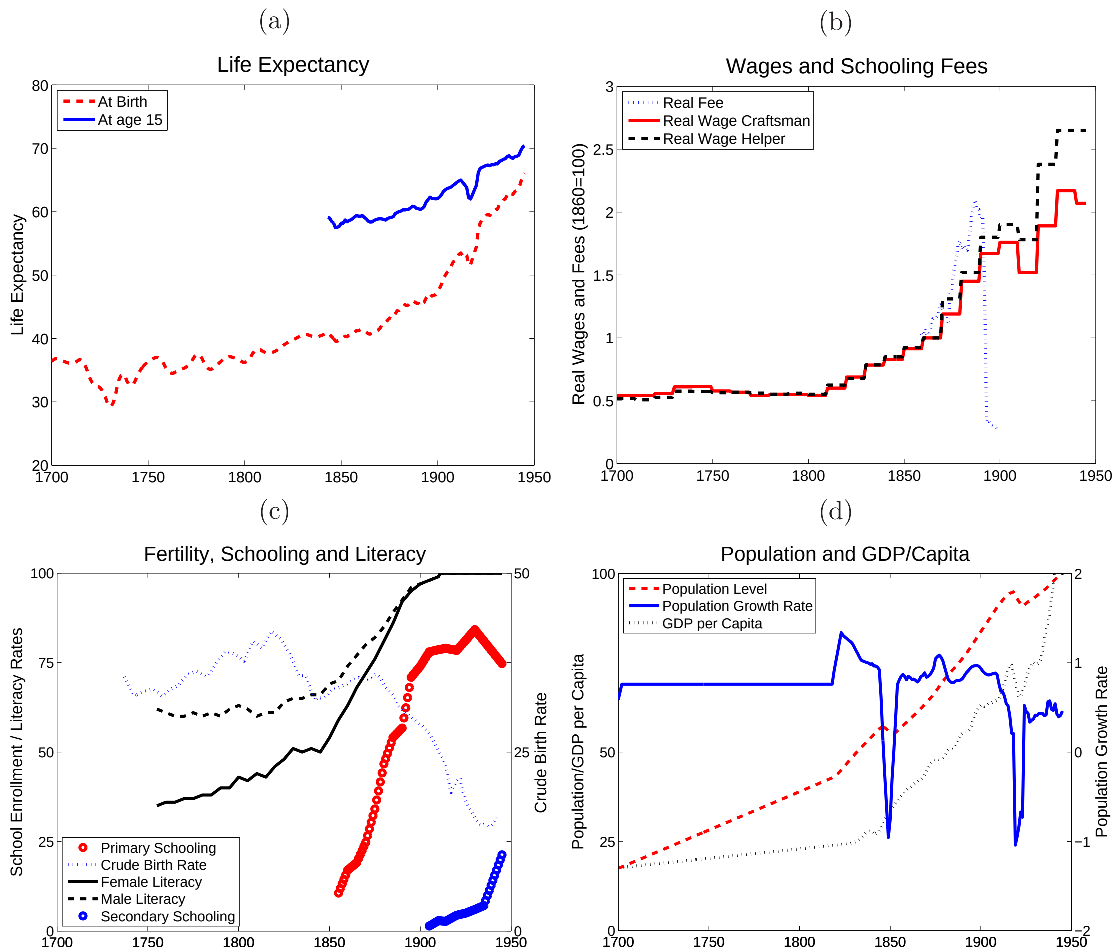
Initially, education was thus not free but financed by parents. Data for 1834 Manchester show that up to 80% of children's education was paid for only by parents (West (1970), p. 84). On the aggregate level approximately 1% of Net National Income in 1833 was spent on day-schools (West (1970), p. 87). The development of fees relative to wages is shown in figure 3.1b demonstrating that the rise of tuition fees kept pace with the general wage increase and even outpaced it shortly before the Free Education Act was enacted. This suggests that education became relatively more expensive with a, *ceteris paribus*, detrimental effect on educational investment. Nevertheless, we observe that some parents decided to send their children to costly schools. These families were most likely not member of the rich bourgeois (they could afford e.g. private tutors anyway) but rather from the lower or middle class indicating that they recognized the value of education, were able and willing to pay for it.

3.3 The Model

In this section we describe the setup and solution to the model. The first subsection deals with the timing and notational conventions, followed by the description of aggregate production, production of human capital and the pricing of public schooling. Then we move on to the households' preferences and constraints and solve the individual maximization problem. Finally, we solve for the general equilibrium and discuss the dynamic behavior

literature. After the seminal paper by Galor and Weil (2000) human capital has been accepted as *the* key ingredient of any unified growth theory. Mokyr (2004), however, claims that literacy was restricted to a small share of the population (government officials, military personnel or members of the aristocracy) and is unlikely to serve as a good explanatory variable.

Figure 3.1: Stylized Facts



Data sources: Crude Birth Rate: Chesnais (1992). Tuition Fees: Mitch (1986). School Enrolment Rates: Flora, Kraus, and Pfenning (1983). Population and GDP per Capita: Maddison (2003). Literacy: Schofield (1973). Life Expectancy: Wrigley and Schofield (1981) and Human Mortality Database (2008). Wages: Clark (2005a). The data refer to England and Wales. Enrollment data refer to the percentage of the age group (corresponding to the Net Enrollment Ratio).

of the economy.

3.3.1 Timing and Conventions

Consider an overlapping generations economy in which adults live for $T + a_B$ years. T is the life expectancy of an adult agent who enters adulthood at age a_B which may be regarded as the "economic" birth.[6] As a child, the agent may receive some education from her parents but is otherwise passive and does not make any own decisions. The agent can decide about consumption, number and education of children, and investment into adult human capital. Children are born right after parents enter adulthood at a_B. Parents can decide to educate children at home ("private system") or they can decide to send their children to school and pay tuition fees ("public system").[7] Time investment into adult human capital increases productivity on the labor market and agents differ with respect to their labor productivity.

The economy is populated by a discrete number of overlapping generations and each generation (cohort) is indexed by τ. The new household born at time t has a life expectancy of $T_t + a_B$ where the length of childhood a_B is time invariant whereas life expectancy will change during the development process. Life expectancy is identical across agents and determined by exogenous forces outside of the households' control. Population size is the number of agents (including children) at any time t. Investment into adult human capital, child human capital and the number of children are continuous variables.[8] Reproduction is asexual, one agent can be interpreted as a family making joint decisions. The notation in the paper is as follows: the subscripts τ and t denote cohort and calendar time, an individual's ability (type) is denoted by μ, a prime indicates a partial derivative, and the

[6] Some papers, e.g. de la Croix and Licandro (2009) refer to this as puberty. In the context of this paper this could also be understood as marriage, see Voigtländer and Voth (2009) on the link of marriage and fertility.

[7] We use the terminology private and public to distinguish between education at home and education in some institution not requiring parents' time but money to "buy" the time of a teacher. Thus, private and public does *not* refer to the modern notion of private and public schools. See de la Croix and Doepke (2004) who model such a framework and examine the long-run effects in the educational system on growth and inequality.

[8] See Doepke (2005) for a model with discrete and sequential fertility decision. Although in presence of uncertainty the indivisibility assumption has an effect on the fertility behavior, he states that the "quantitative predictions of the models are remarkably similar".

superscripts $j \in \{pr, pu\}$ refer to the private and respectively public schooling system. Variables with a bar (e.g. $\bar{x}$) denote averages and a tilde (e.g. $\tilde{x}$) indicates some threshold value for a variable. When no misunderstanding is expected, we omit indexes.

3.3.2 Aggregate Production

Human capital is the only productive factor in this economy and there is only one sector producing a homogeneous consumption good. We use a simple vintage model in which technological vintages are characterized by cohort specific productivity levels and each generation can operate only its cohort specific technology (each newborn generation automatically uses the new vintage).[9] Thus, agents earn over their entire working life the output (wage) of that vintage. This allows us to concentrate only on the labor market equilibrium at one point in time for one generation and avoids making assumptions about the substitutability of agents over different ages and vintages of human capital. Aggregate production for a generation τ is given by the linear technology

$$Y_\tau = A_\tau \mathcal{H}_\tau, \tag{3.1}$$

where A_τ denotes cohort specific productivity and $\mathcal{H}_\tau$ is the aggregate stock of effective labor supply, respectively. Effective labor supply is defined as $\mathcal{H}_\tau = P_\tau \mathcal{L}_\tau$ where P_τ is the number of workers, and $\mathcal{L}_\tau$ is average effective labor supply per worker. To exhaust total production, wages per unit of human capital and per capita income are

$$\omega_\tau = A_\tau, \tag{3.2}$$
$$w_\tau = \omega_\tau \mathcal{L}_\tau. \tag{3.3}$$

Hence, wages per unit of human capital increase in the general level of productivity and income per capita increases with higher individual effective labor supply. As will become clear later, nothing hinges on the *absolute* level of income per capita or wages. In order to focus on the main predictions of the model, we abstain therefore from including a

[9]See Cervellati and Sunde (2005) for similar assumptions. Chari and Hopenhayn (1991) develop a model showing that new technologies are not immediately adopted. In an empirical study Weinberg (2004) shows that older workers are more likely to operate old machines and new entrants into the labor market (young workers) will operate the most recent vintage.

Malthusian element by introducing a concave production function with a fixed factor.

3.3.3 Human Capital of Adults

Upon becoming adults, agents may decide to spend h units of time on the acquisition of adult human capital. Agents' heterogeneity translates into different productivity on the labor market where we assume that ability μ is distributed uniformly on the $[0, 1]$ interval and shifts individual productivity linearly. Then, adults' human capital is given by

$$f(h) = \mu \frac{h^\theta}{\theta}, \tag{3.4}$$

with $\theta < 1$.[10] The implicit assumption here is that human capital is embodied in people and therefore it has to be built up from zero by every new generation. Technological progress caused by human capital therefore shows up only in the level of aggregate productivity A_τ.

3.3.4 The Price of Education

We do not model a detailed education sector, the main reason being the lack of consensus in the literature how a realistic modeling environment might look like. It is, for instance, conceivable that private schools are selfish profit maximizing organizations hiring teachers on the market and selling educational services but it is also equally plausible that schools are managed by non-profit organizations or run by a government attempting to recover only their costs (i.e. operating on a zero-profit basis). To begin with, we assume that each agent working in the education sector can produce one unit of educational services by using its human capital and one unit of time. This corresponds to the assumption made for education at home: each parent has to spend one unit of time and its human capital to produce one unit of "time input" into production of human capital of children. Further, we assume that the efficiency of this unit of time spent educating children increases with the average level of human capital $\bar{f}(h)$ in the economy. Using these assumptions, we have $e^{pu}(\mu) = m(\bar{f}(h))\ell^{pu}(\mu)$ where e^{pu} and ℓ^{pu} are education services produced and time spent

[10] The uniform distribution is not crucial for the main argument of the paper. Further, the choice of this production function implies that it is never optimal for agents to choose $h = 0$. This can be relaxed at the cost of having corner solutions which would endogenously vanish once life expectancy is high enough.

teaching of an agent of ability μ. $m(\cdot)$ is an increasing function mapping average human capital into a positive externality.[11] Agents are indifferent between working in schools or in the production sector and we assume that that they are randomly drawn from the population. Then, a teacher is just a representative (average) agent. Since competition on the labor market requires wages per unit of human capital ω to be identical, the price of schooling is

$$p = \omega \frac{\bar{f}(h)}{m(\bar{f}(h))}. \tag{3.5}$$

The price of education is increasing with the wage level and the average educational attainment of the adult population but decreasing with the externality created by a better educated population. Since this element will turn out to be crucial for the development path, we will come back to this issue in section 3.4.2.

3.3.5 Household Preferences and Constraints

The agent's utility function follows rather standard assumptions. An agent likes consumption over the life-cycle and values educated children. Conditional on the decision $j \in \{pr, pu\}$ how to educate the offspring the agent from cohort τ maximizes the utility function

$$U^j = \int_0^T e^{-\rho a} da \log c(a)^j + \beta u(n^j z(e^j)) \tag{3.6}$$

where $\log c$ is the period utility obtained from consumption at age a, ρ is discounting future utility and $u(nz(e))$ is the intrinsic value of the quality-quantity composite weighted by β. The utility function u takes the number n of children times their quality which is captured by their human capital $z(e)$. This is a common assumption in the literature and can be understood as pure parental altruism or an implicit old-age pension system.[12] Human

[11] A similar assumption concerning externalities is common in endogenous growth models with knowledge externalities, see e.g. Romer (1986). The main conclusion would not change if we required that teachers posses some minimum skill level (see also de la Croix and Doepke (2003) for a setup where teachers are average agents).

[12] The alternative formulation in which agents derive utility from the utility of their children (i.e. the dynastic approach in the spirit of Becker and Barro (1988) or Barro and Becker (1989)) requires that agents know (or form expectations) what their children will do. This would render the model intractable

capital of children increases with investment e. The input into the production function is either parental time or teachers' time bought on the market for the price p per unit of time (tuition fees). Children survive with probability one until adulthood.[13] The timing of the agents' decisions is kept as simple as possible: agents first complete their schooling and fertility and start working afterwards. Working on the labor market is an absorbing state and there is no retirement.

Since this paper does not focus on life-cycle dynamics or precise life-cycle profiles but rather on the trade-off between human capital investment (child and adult) and fertility, we assume that the discount rate and the interest rate are both zero.[14] This simplifying assumption obviously eliminates the traditional life-cycle elements for consumption, saving and labor supply. However, the "qualitative" structure of the problem is not altered: rising life expectancy has exactly the same effect as in a more realistic setup. Using the assumptions from above, the problem can now be written as

$$\{j, c^j, h^j, n^j, e^j\} = \arg\max \quad T \log c^j + \beta u(n^j z(e^j)) \tag{3.7}$$

subject to the constraints

$$Tc^j \leq \omega f(h^j)(T - \phi n^j - h^j) - pe^j n^j, \qquad \text{if } j = pu. \tag{3.8}$$

in the public system where the educations costs are monetary costs. In the private system

and would probably not change the main message. For models where the old-age security motive is made explicit see e.g. Boldrin and Jones (2002) and Ehrlich and Lui (1991).

[13]We ignore uncertainty about child survival. See e.g. Kalemli-Ozcan (2003) and Strulik (2004a) on the theoretical and Eckstein, Mira, and Wolpin (1999) or Ram and Schultz (1979) empirical relationship between child survival and investment into human capital.

[14]Since marginal utility from consumption and the quantity-quality composite are independent, proceeding with $\rho = r \neq 0$ would not change the results (only the slope of the consumption profile would change), see the identical assumptions in e.g. de la Croix and Licandro (2009), Soares (2005) or Cervellati and Sunde (2005).

the constraint is[15]

$$Tc^j \leq \omega f(h^j)(T - (e^j + \phi)n^j - h^j), \qquad \text{if } j = pr. \tag{3.9}$$

In both constraints fixed time costs per child are identical and denoted by ϕ. The problem is in general analytically not tractable. Therefore we make the following assumptions about functional forms

Assumption 3.1.

$$u(n,e) = \frac{(nz(e))^{1-\sigma}}{1-\sigma} \tag{3.10}$$

$$z(e) = \frac{e^\gamma}{\gamma} \tag{3.11}$$

with $\gamma < 1$, $\sigma > 0$.[16]

3.3.6 Individual Maximization Problem

The strategy is to solve the individual maximization problem – given wages and the price of schooling – conditional on the choice of the schooling system. This should highlight the conditional dynamics of fertility, investment into child, and adult human capital as life expectancy increases. Then we will analyze the effect of rising life expectancy on the choice of the parents' utility maximizing educational system. Using this two-stage procedure we can isolate the effect of rising life expectancy on the composition of the economy, i.e. private vs. public schooling and then the change in individual behavior conditional on this choices. Finally, we will put the individuals into a general equilibrium framework to allow for feedback effects and examine the dynamics of the aggregate economy. That is,

[15] In this setup we ignore the important issue of child labor. See e.g. Basu and Van (1998), Hazan and Berdugo (2002) or Baland and Robinson (2000) for models incorporating a child labor decision into growth models. However, note that we can rewrite the budget constraint by assuming that children can earn $n\varphi\omega f(h)$ where φ represents the relative wage of child labor. Then, $p = p^g - \omega f(h)$ and $\phi = \phi^g(1-\varphi)$ where ϕ^g and p^g are *gross* and p and ϕ are *net* costs of schooling.

[16] We use the log-function for sub-utility from consumption to keep fertility independent from the *level* of ω – for a given T – as the economy is growing. A non-neutral effect of wages on fertility can be brought back into the model by choosing a utility function which does not balance income and substitution effects. See Jones, Schoonbroodt, and Tertilt (2008) for an excelling literature overview how in theoretical models income is related to fertility.

we examine the simultaneous interaction of the behavioral and compositional change.

Household Solution in the Private Education System

In this subsection we solve the household's problem assuming that agents educate their children at home. At this stage, we do not ask which schooling system is optimal for parents but examine their behavior given that they decided to stay in the private system. This is essentially an environment without public schooling where all investment into human capital is done at home by the parents. Thus, children consume a share of their parents' time which is not available for productive work.

Using λ to denote the multiplier attached to the resource constraint, the first order conditions[17] of the problem (skipping the index $j = pr$) are

$$\beta u' z - \lambda \omega f(h)(e + \phi) = 0 \tag{3.12}$$

$$\beta u' z' - \lambda \omega f(h) = 0 \tag{3.13}$$

$$f'(h)(T - (e + \phi)n - h) - f(h) = 0 \tag{3.14}$$

where a prime denotes partial derivatives. Note that the optimality condition for adult human capital h is independent of the marginal utility of consumption λ. Thus, without any further restrictions, adult human capital will just maximize lifetime income. Furthermore, combining the FOC's for fertility and child schooling capital we obtain

$$\frac{z}{z'} = \phi + e \tag{3.15}$$

implying that optimal investment into children's human capital is independent of adult life expectancy, parents' human capital (and skill level) or wages. It is a constant determined by the relative time cost of children ϕ, and the properties of the human capital production function $z(e)$. Solving the entire household problem with private education leads to the following proposition:

Proposition 3.1. *Assume that children are educated in the private system. If adult life ex-*

[17]To economize on notation we will not spell out the solution using the specified functional forms but rather use the general notation. For the proofs in the appendix we will, of course, switch to the specific functional forms whenever necessary.

pectancy increases, adult schooling and fertility will increase. Investment into child human capital is constant.

Proof. See appendix 3.A. □

The intuition behind this result is rather simple. Adult schooling is rising since the time over which the benefits of educational investment can be reaped is increasing.[18] The result is based on the trade-off between the opportunity costs of schooling today and future benefits. Child human capital is constant since the assumption made on u implies that the agent maximizes quasi-linear utility. Under this assumption marginal utility from child schooling and marginal costs are proportional in n. However, the price of one additional unit of fertility or child schooling is linearly increasing in $f(h)$ whereas returns to child schooling are concave. Thus, investment into child human capital does not change and income effects are absorbed by (rising) fertility. Fertility, on the other hand, is rising because of the intratemporal optimality between consumption and fertility. Rising life expectancy implies rising total lifetime income and the agent will – by concavity of both utility functions - distribute some of the additional "free" income to increase consumption and fertility.[19]

Household Solution in the Public Education System

Parents could also have their children educated by teachers ($j = pu$) for tuition fees p per unit of time. The crucial difference to the private system is that the opportunity costs of child human capital are not valued by forgone adult wages but purely by monetary costs.

[18] This is the Ben-Porath (1967) mechanism. See however Hazan (2009) and Kalemli-Ozcan and Weil (2002) for opposite views on the link between life expectancy, human capital investment and lifetime labor supply (including the timing of retirement).

[19] An alternative way is to consider one equilibrium allocation of consumption c^* and time $\{n^*, h^*\}$ given a life expectancy T^*. Assume now that life expectancy rises but we hold fertility and adult schooling at $\{n^*, h^*\}$ constant. Then, per period consumption will always rise since at the given equilibrium allocation, the propensity to spend out of an additional unit of income is smaller than one but holding $\{n^*, h^*\}$ constant implies that the entire additional income is available for consumption. On the other hand, rising life expectancy makes investment into education more profitable introducing an additional "multiplier" effect increasing total lifetime income even more. Thus, marginal utility of consumption will decrease even further requiring a rise in fertility to equate marginal utilities. Consequently, the agent will sacrifice some income when young and thereby equate marginal utility from consumption and fertility. This result can also be established by assuming concavity of the production function $f(h)$ and concave utility functions $u(\cdot)$ and $z(e)$ only and does not hinge on any functional form.

Formally, the first order conditions associated with the problem are

$$\beta u' z - \lambda(ep + \omega f(h)\phi) = 0 \tag{3.16}$$

$$\beta u' z' - \lambda p = 0 \tag{3.17}$$

$$f'(h)(T - h - \phi n) - f(h) = 0 \tag{3.18}$$

In contrast to the setup with private education, child schooling costs are no time costs any more. Thus, they do not enter the equation determining the optimal solution for adult human capital but are only monetary costs valued by the marginal utility of consumption. Combining again the first order conditions for quantity and quality of children we obtain

$$\frac{z}{z'} = \frac{ep + \omega f(h)\phi}{p} \tag{3.19}$$

$$e = \omega f(h)\frac{\phi}{p}\frac{\gamma}{1-\gamma} \tag{3.20}$$

where now adult human capital increases also investment in child quality. This is an income effect stemming from the fact that the scarce factor time competing for labor supply and adult human capital accumulation is freed up. Thus, the "production function" for child human capital becomes linear instead of the convex costs caused by concave utility and concave production of adult human capital. However, the fact that child schooling is now purely a monetary cost implies also that the price of fertility relative to the price of schooling is rising in adult human capital (see FOC). Thus, it is not straightforward any more how fertility is changing if life expectancy (and h) is changing. Note that the introduction of a free public schooling system is still compatible with this setup. It is reasonable to assume that even without tuition fees the costs of schooling are bounded away from zero. This ensures that the household has a well defined demand for child education.

Solving the household problem now allows us to state the following proposition

Proposition 3.2. *Assume that children are educated in the public system. If adult life expectancy increases, adult schooling and child schooling will increase. Fertility is always rising for $\sigma \leq 1$ but may fall for $\sigma > 1$.*

Proof. See appendix 3.A. □

Again, investment into adult human capital rises because of the horizon effect. Child human capital rises because the price of schooling does not increase in $f(h)$ and therefore only the positive income effect is left over. Whether fertility increases or decreases depends on the coefficient of relative risk aversion with respect to the quantity-quality composite. Intuitively, if $\sigma \leq 1$, then the income effect of higher adult human capital will dominate (i.e. marginal utility from the quality-quantity composite is "less convex"). However, if $\sigma > 1$, then the effect of higher price of fertility may dominate. Given $\sigma > 1$, the reaction of fertility with respect to change in life expectancy depends mainly on the properties of the adult human capital production function. The more adult human capital increases, the more expensive fertility becomes and therefore fertility is likely to decline.

We can draw two main conclusions from analyzing household behavior under the two schooling regimes. Firstly, if there is no public schooling system and the only input into children's human capital is parental time we will not observe a decrease in fertility if adult life expectancy is increasing. Secondly, if there is a public schooling system, a decrease in fertility is more likely but will not necessarily happen. If the agents are less risk averse with respect to fertility than with respect to consumption, fertility will not decline as life expectancy increases. Then, there is no change in behavior with respect to fertility either. If $\sigma > 1$, then fertility may decrease if $f(h)$ is increasing sufficiently as a consequence of more investment into adult human capital h. In either case, child schooling will (given p and ω) rise in the public system as parental human capital increases. Aggregate fertility may decrease if the compositional effect is strong enough, i.e. sufficiently many agents decide to switch to the public system. Being able to match the stylized facts, we proceed for the rest of the paper with

Assumption 3.2.

$$\sigma > 1 \tag{3.21}$$

While interpreting the results, we have to keep in mind that we have operated in a highly stylized environment without any frictions. Returns to education are not affected by technological progress, an assumption frequently made in the literature.[20] We also implicitly

[20]See e.g. Schultz (1964), Foster and Rosenzweig (1996) or Bartel and Sicherman (1998) on the link between technological progress and investment into human capital.

assumed that parents and teachers are equally efficient in teaching children. This may be true if we deal with educated parents (who could also be teachers) or the level of knowledge is rather low. Our assumption may be of limited use if parents are unskilled laborers or illiterate. Further, the choice of the log-utility for consumption implies that income effects – by simply raising wages (and p proportionally) – does not affect households' allocations. The purpose of these assumptions was to isolate the effect of different time allocation schemes on individual decisions. The quantitative relevance of this mechanism relative to competing explanations is ultimately an empirical question.

3.3.7 The Choice of Private vs. Public Schooling

Agents' optimal choice includes the decision in which education system their children are educated. For their decision, parents take their ability μ, wages ω, and the price of education p as given. One would expect that more able parents find it optimal to send their children to school and pay the tuition fees by spending more time on the labor market. The decision which schooling system to chose depends only on the potential income of parents. If they decide to send their children to a school, they do this because income earned on the labor market outweighs the costs of tuition fees. This is the sorting mechanism the model relies upon. The determinant is the wage-price ratio (ω/p) relative to potential earnings. Potential earnings are determined by individual ability and life expectancy. The more able agents are and the higher their life expectancy is, the cheaper and more efficient is education for their children in a public system. Note that by assuming that ability is bounded from above, there may be such a vector $\{p, \omega\}$ that even the most able agents decide not to participate in the public schooling system.[21] Then, we can state the following proposition:

Proposition 3.3. *If there is a vector $\{p, \omega\}$ such that agents are indifferent between the private and public system, agents with $\mu \geq \tilde{\mu}$ will decide to educate their children in the public system whereas agents with $\mu < \tilde{\mu}$ will decide to educate their children at home. This threshold ability level $\tilde{\mu}$ is decreasing with rising life expectancy.*

[21]The standardization with the upper bound of $\mu = 1$ does not matter. It is obvious that for any finite bounded ability, there is a sufficiently high price to deter even the most able agent from participating in the public school system.

Proof. See appendix 3.A. □

Proposition 3.3 means that for a given relative price structure, also less able agents find it optimal to switch to the public system as life expectancy is rising. The economic explanation is that lower ability is partly compensated by higher investment into human capital. In turn, rising life expectancy increases optimal investment into adult human capital thereby raising the opportunity costs of educating children at home also for less able agents. And obviously, if for an agent with ability $\mu = \tilde{\mu}$ it was optimal to join the public system given T, this will be optimal for higher life expectancy too: the price of schooling is constant and not increasing in h, thus the agent is always better off in the public system. Rising life expectancy therefore implies that the indifferent agent becomes less able as T rises. However, whether this happens on the aggregate level once we allow for feedback effects of rising adult human capital investment on the price of education is not clear (see next section). Further, switching from the private to the public education system means that households' investment into human capital and fertility will change discontinuously.

Lemma 3.1. *If agents decide to educate their children in the public system, they will increase investment in both types of human capital and decrease fertility.*

Proof. See appendix 3.A. □

This change in the optimal education system causes a change in educational attainment and fertility due to a changing composition but does not necessarily involve a behavioral change. Families choosing the public system still could increase their fertility as their life expectancy increases. Moreover, heterogeneity in ability is now also reflected in the heterogeneity of decisions.

Lemma 3.2. *If children are educated in the private system, ability does not change the solution to the households' problem. If children are educated in the public system, higher ability increases adult and child human capital investment and decreases fertility.*

Proof. See appendix 3.A. □

The negative correlation between parental education (ability) and fertility is a well documented and widely accepted fact (Skirbekk (2008)). Note that for this pattern to emerge we need that the price of children is partly decoupled from parents' own human capital. Without the adoption of a public schooling system, agents' allocations are identical despite different ability levels. Higher ability introduces an effect which proportionally raises prices of fertility and both types of human capital. Thus, more able agents enjoy only higher lifetime utility (due to higher consumption) without changing their allocation of time. Heterogenous behavior as a consequence of heterogeneity of skills requires that higher ability "buys" more time. This is, however, only the case if the price of child schooling is not perfectly linked to parents human capital.[22]

3.3.8 Aggregation

Aggregate human capital in goods' production for a given generation τ is given by

$$\mathcal{H}_\tau = P_\tau \left[\tilde{\mu}_\tau f(h_\tau^{pr}) \ell_\tau^{pr} + \int_{\tilde{\mu}_\tau}^{1} f(h_\tau^{pu}(a)) \ell_\tau^{pu}(a) da - \mathcal{E}_\tau \right] \tag{3.22}$$

where the first term measures effective labor supply of agents educating their children at home. The second term measures total labor supply of agents educating their children in the public system and the last term is the labor supply of teachers not available for producing consumption goods. Total education time purchased on the market is given by

$$\mathcal{E}_\tau = \bar{f}_\tau(h) \int_{\tilde{\mu}_\tau}^{1} n_\tau^{pu}(a) e_\tau^{pu}(a) da. \tag{3.23}$$

Using the assumption of uniformly distributed ability in the population, average human capital in the economy and fertility for any cohort τ are

$$\bar{f}_\tau(h) = \tilde{\mu}_\tau f(h_\tau^{pr}) + \int_{\tilde{\mu}_\tau}^{1} f(h_\tau^{pu}(a)) da \tag{3.24}$$

$$\bar{n}_\tau = \tilde{\mu}_\tau n_\tau^{pr} + \int_{\tilde{\mu}_\tau}^{1} n_\tau^{pu}(a) da. \tag{3.25}$$

The first term is human capital and fertility of agents educating their children at home and the second term denotes the corresponding value for the families participating in the

[22] We could also allow for both – monetary and time costs – of child human capital with obviously identical qualitatively conclusions.

public system.

3.4 The Dynamic System

The development process is shaped by the interaction of individually optimal decisions and macroeconomic externalities. Having solved the households' problem with fixed prices and for a given life expectancy, we will trace out the dynamics of simultaneous changes in prices and life expectancy. First, we study how the driving force of the model, adult life expectancy, is linked to the agents' individual decisions and how it evolves over time. Then, we will analyze the adjustment process of tuition fees when life expectancy is endogenous. Finally, we look at the dynamics of demographic variables.

3.4.1 Life Expectancy

Research has led to mainly two competing explanations why life has increased over the last centuries: improvements in nutrition and progress in medical knowledge. Whereas e.g. Fogel (1997) argues that the increases in the intake of calories is responsible for decreasing mortality, Cutler, Deaton, and Lleras-Muney (2006) object in their survey that it was mainly progress of medical knowledge.[23] Although is seems reasonable that life expectancy at some time will reach a biological upper bound, there are no signs that this will happen within the next generations. On the contrary, Oeppen and Vaupel (2002) show that record ("best-practice") life expectancy has risen for 150 years at a pace of 2.4 years per decade making it impossible to derive a sensible prediction about maximum life expectancy. Using the insights from this literature we assume that life expectancy is a positive function of the available human capital in the economy capturing both effects. Particularly, we link the next cohort's life expectancy to the average human capital of the current cohort, implicitly assuming that parents' knowledge, health behavior, etc. determines the life expectancy of

[23]Cutler, Deaton, and Lleras-Muney (2006) show that there was no health-income gradient before the Age of Enlightenment and assert that ideas like germ theory, boiling water or simply washing hands are independent of the level of income. Mokyr (1993) takes a stand between the two theories writing that "...knowledge is believed to respond to market signals, social and political pressures, changes in incentives, institutions, and so on." In other words, as people escape from a state of mere subsistence, they can afford to spend resources on the advancement of knowledge.

their children. We formalize this by writing

$$T_{\tau+1} = \Psi(\bar{f}(h(T_\tau))), \tag{3.26}$$

where Ψ is a strictly concave and non-decreasing function capturing the positive externality of average human capital on life expectancy. To escape from a trivial solution we make

Assumption 3.3.

$$T_{\tau+1} - T_\tau = \Delta(T_\tau) = \Psi(\bar{f}(h(T_\tau))) - T_\tau \geq 0 \quad \forall \ T_\tau > 0. \tag{3.27}$$

This is a simple nonlinear difference equation leading to an arbitrary high but finite life expectancy. By imposing the restriction that Ψ is non-decreasing and strictly concave we rule out possible non-monotonicities on the development path. We do this for the sake of clarity of the paper's argument: the implications of a less restrictive specification of Ψ are that we may end up with no or more than one steady-state without gaining additional insights.

3.4.2 Schooling Choice in General Equilibrium

It is clear that as life expectancy rises, agents invest more into human capital and thus p will increase. What matters, however, is the evolution of p relative to the evolution of potential income. Rising tuition fees do not matter as long as they are outweighed by sufficiently large increases in life expectancy. Put it differently: for any increase in potential income, there is a corresponding surge in the price of education such that the indifferent agent is characterized by exactly the same ability level. If prices increase by less, also less able agents will decide to join the public system and $\tilde{\mu}$ will decrease. If the price of education rises faster then some less able agents will withdraw their offspring from public schools. That is, the threshold ability level will increase undoing the positive effect of a higher potential income on children's education.

Proposition 3.4. *If life expectancy rises, the share of agents participating in the public schooling system may, depending on the strength of the externality increase or decrease.*

Without externalities, $\tilde{\mu}$ will monotonically increase and may hit the upper bound of ability for sufficiently large life expectancy.

Proof. See appendix 3.A. □

For the sake of simplicity, assume for the moment that agents are homogenous or that each agents hires a teacher from her own ability group. The equilibrium price of education is then given by $p = \omega f(h(\mu))$. Plugging this price into the households' solution for optimal education (3.19) leads to a straightforward result: educational investment in the private and public system are identical. The ratio of potential parental earnings $\omega f(h)$ and p is unity in both cases. However, the absolute price of schooling in the public system is higher since agents invest more into adult human capital (see lemma 3.1). In this case utility in the public system is always lower than in the private system: agents in the public system neglect the (negative) externality on the price of schooling they generate by investing more into human capital. Hence, $U^{pr} > U^{pu}$ holds for all T and optimizing agents will never chose the public schooling system.[24] This is the very reason why we need some positive externality of higher human capital. Intuitively, if average human capital increases but there is no externality, the average will increase faster than the human capital of the agent with $\mu = \tilde{\mu}$.[25] The agent indifferent between public and private schooling must therefore be more skilled in order to compensate the higher price of education and make her indifferent between the two options.

3.4.3 Population Dynamics

The dynamics of average (aggregate) fertility is ambiguous and depends on the strengths of the different mechanisms at work. Simplifying (3.25) leads to

$$\bar{n}_\tau = \tilde{\mu}_\tau \bar{n}_\tau^{pr} + (1 - \tilde{\mu}_\tau)\bar{n}_\tau^{pu}. \tag{3.28}$$

[24]See e.g. Bagnoud (1999) for Switzerland, Birchenough (1914) for England and Wales and Becker, Cinnirella, and Woessmann (2009) for Prussia for evidence on the reluctance of people (especially peasants and poor workers) to send their children to school despite compulsory schooling.

[25]Recall from lemma 3.2 that human capital investment is a positive function of μ. Then, human capital of agents participating in the public system will raise by more than $f(h(\tilde{\mu})^{pu})$.

We know that a rising share of parents participating in the public system decreases fertility via the composition effect. However, if the weight of these families is initially small, this effect is likely to be dominated by the agents choosing the private system. Secondly, even fertility of agents participating in the public system may rise for a while as T rises. While the composition effect is then still working towards lower aggregate fertility, fertility of each subgroup will increase as life expectancy rises. Only if the share of the public schooling is high enough and these agents also have fewer children as T goes up, will aggregate fertility of a cohort decrease unambiguously. Total population P_t and cohort size P_τ evolve according to

$$P_{t+1} = P_t + N_t(a)\,|_{a=a_B} \int_0^1 n(T_{t-a_B}, \mu) d\mu - N_t(a)\,|_{a=T_m}, \tag{3.29}$$

$$P_{\tau+1} = P_\tau(\bar{n}_\tau - 1), \tag{3.30}$$

where $T_m = T(t-T_m)$ denotes the life expectancy of the oldest agent in $t-1$ (who dies in t) born T_m years ago. $N_t(a)$ is the number of adults in t who either are of childbearing age ($a = a_B$) or die ($a = T_m$) this period.[26] The number of newborns per agent of childbearing age is determined by its life expectancy T_{t-a_B} and ability μ, and is denoted by $n(T_{t-a_B}, \mu)$. Population growth rate for any stationary life expectancy T is implicitly defined by

$$g_P = \frac{n(T)}{T} \left[\frac{(1+g_P)^T - 1}{(1+g_P)^T} \right] \tag{3.31}$$

The following corollary is rather obvious.

Corollary 3.1. *If fertility is a hump-shaped function of adult life expectancy, the population growth rate is also hump-shaped.*

Proof. See appendix 3.A. □

Population dynamics is slightly more complicated if we start from a stationary population and let life expectancy increase. Then we have initially a positive effect on the population growth rate due to higher fertility and a (delayed) positive effect due to the fact that old agents are now living longer. However, the second effect is only transitory and vanishes

[26] Alternatively, population can be also written as the integral over all living cohorts $P_t = \int_{t-T_m}^{t+a_B} N_t(a) da$.

as life expectancy settles at a constant value. Whether in the new steady state population is growing or shrinking depends on the fertility associated with the steady-state life expectancy.

3.4.4 Technological Progress

As outlined in the introduction, technological progress occurs through the invention of better and more productive machines which can be operated only by the cohort entering the labor force at the time of the introduction of the new vintage. Following the literature, we assume that higher level of human capital facilitates invention of more productive technologies[27] and model this by assuming

$$\frac{A_\tau}{A_{\tau-1}} = g(\bar{f}_{\tau-1}(h)), \tag{3.32}$$

with g increasing and concave. After having defined the static solution to the households' problem and specified how aggregate variables change over time, we are ready to define the equilibrium development path of the economy.

Definition 1 (Equilibrium)**.** *Given an initial population P_0 and initial life expectancy T_0, an equilibrium consists of a sequence of aggregate variables $\{\mathcal{H}_\tau, Y_\tau, A_\tau, T_\tau\}$, prices $\{p_\tau, \omega_\tau\}$, and individual decision rules $\{c^j_\tau, n^j_\tau, h^j_\tau, e^j_\tau, j\}$, $j \in \{pu, pr\}$, such that*

1. *households optimality conditions given by equations (3.12) and (3.16) and subject to the constraints (3.8) or (3.9) are satisfied,*

2. *aggregate variables are given by (3.1), (3.22), (3.26), and (3.32), prices by (3.3) and (3.5), and*

3. *life expectancy, total population, and cohort size evolve according to (3.26), (3.29), and (3.30).*

[27] See e.g. Lucas (1988) or the excellent literature reviews by Jones (2005) and Klenow and Rodrígues-Clare (2005).

3.4.5 An Illustrative Simulation

The goal of this paper is to demonstrate the qualitative change in the behavior of agents as they endogenously decide to invest into the human capital of their children via a formal schooling system. In this subsection we provide therefore only an illustrative simulation without any ambition to exactly match historical time series. Since the model lacks many realistic features, it would require a lot of "twisting and tweaking" of model parameters and a very lenient attitude with respect to the choice of functional forms which is of limited use as far as further insights is concerned. Especially, the model is not able to explain the drop in tuition fees caused by the introduction of the Free Education Act from 1891 see (Fig. 3.1b). We therefore restrict ourselves to the choice of rather simple functional forms and have to keep in mind that at some point of the development process, a more or less exogenous drop in p took place. Our choices for Ψ and m are

$$T_{\tau+1} = \delta T_\tau + (\bar{f}_\tau(h))^\alpha, \tag{3.33}$$

$$m = (\bar{f}_\tau(h))^\kappa. \tag{3.34}$$

The parameters of the simulated model can be found in table 3.1. To simulate a situation without positive human capital externalities on the price of schooling, we set $\kappa = 0$ and change the values for σ and δ in order to keep fertility and life expectancy within reasonable bounds.

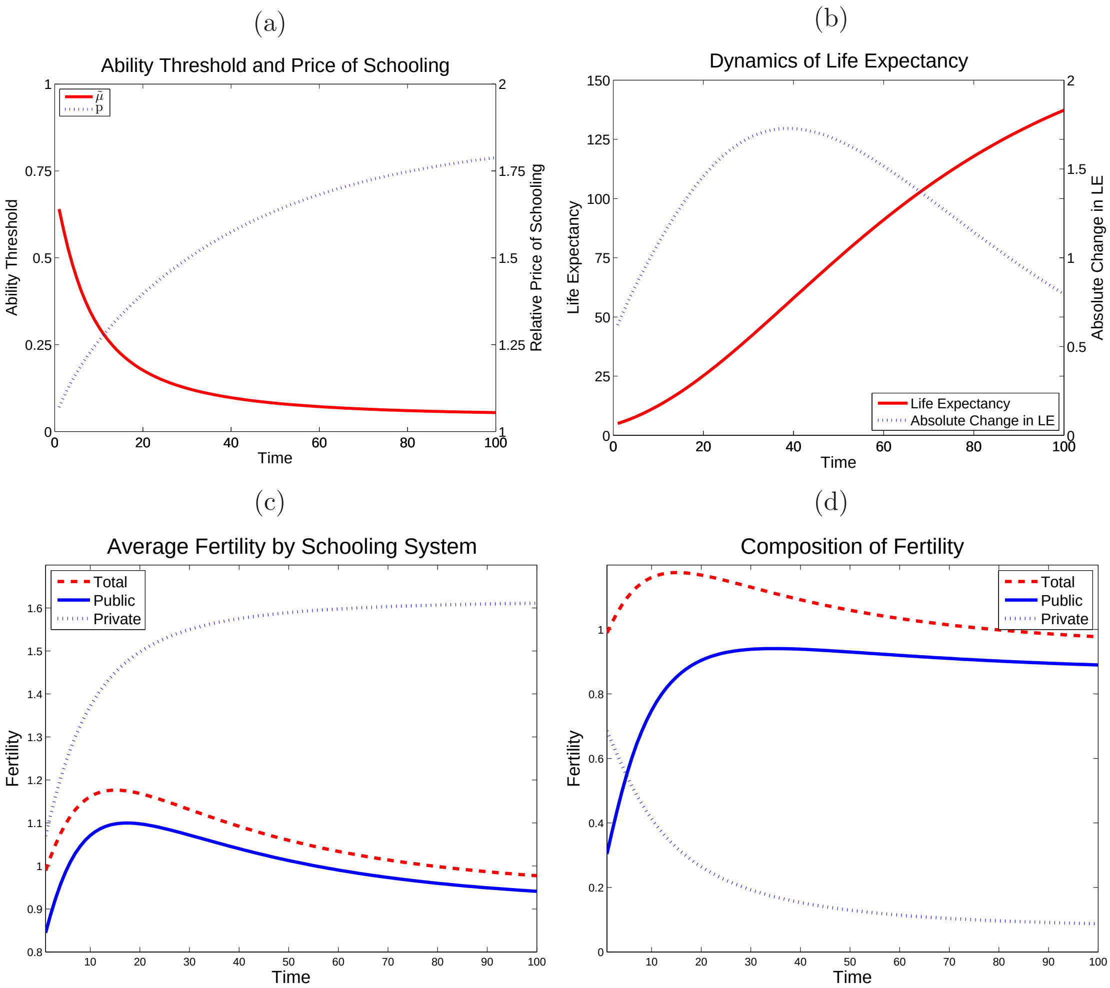
Table 3.1: Model Parameters for Simulation

	κ	β	γ	σ	ϕ	ρ	θ	κ	α	δ
With Externality	0.8	20	0.6	1.8	1.0	0.01	0.8	0.2	0.95	0.90
No Externality	0.0	20	0.6	1.2	1.0	0.01	0.8	1.0	0.95	0.85

Figure 3.2 shows the basic patterns of the development process. Initially, life expectancy is low and only the most able agents invest into human capital of their children via the public system. As life expectancy increases, despite a rising price of education the ability threshold $\tilde{\mu}$ decreases and more and more agents switch to the public schooling system. Note that life expectancy and aggregate fertility are still rising. At this stage, average fertility in both systems is still rising and the composition effect is not sufficient to bring aggregate fertility down. However, this relationship changes during the development process. Despite the

rising fertility of agents in the private system, aggregate fertility is falling: the economy is now dominated by agents choosing the public system and their fertility is falling as life expectancy keeps rising. On top of the compositional effect now also the behavioral effect works towards lower fertility.

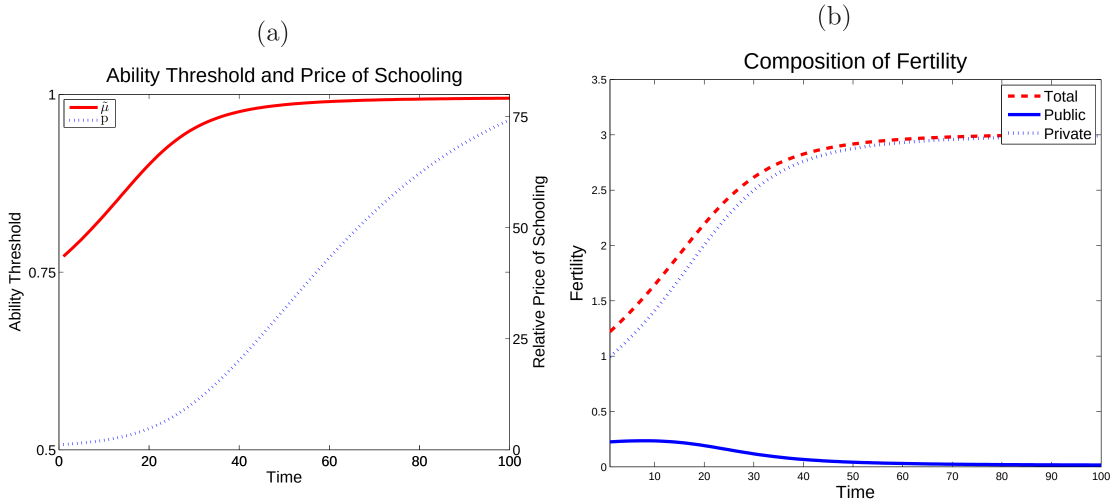
Figure 3.2: A Simulation Exercise: Human Capital Externalities



Data sources: Own simulations. See text and table 3.1 for functional forms and parameter values.

In figure 3.3 we simulate an economy without a positive human capital externality in the schooling system. The price of schooling for the indifferent agent is determined only by its human capital relative to the average. As claimed in proposition 3.4, without

Figure 3.3: A Simulation Exercise: No Human Capital Externality



Data sources: Own simulations. See text and table 3.1 for functional forms and parameter values.

externalities the ability threshold $\tilde{\mu}$ increases, and economy-wide fertility increases although life expectancy is rising.

3.5 Conclusion and Discussion

This paper proposed a simple model arguing that to understand the change in agents's behavior during the demographic transition, it is crucial to account for changing nature of the costs of child quality. We show that if the input in children's human capital production is only parental time, increasing life expectancy always increases fertility. This is because the price of child quality and quantity rise simultaneously with higher life expectancy.

A behavioral change can only occur if parents decide to educate their children in the public system. This transforms time costs into monetary costs. Then, higher lifetime income "buys" also more time. In other words, if parents spend their own time to enhance children's human capital, rising life expectancy increases the price of quality and quantity. With investment into child human capital via a school, increasing labor supply and adult human capital increases only the opportunity costs of quantity but leaves the price of education unchanged. Hence, if parental human capital is sufficiently productive and the marginal valuation of an additional child is sufficiently low, the rising relative price of quantity will bias the parental decision towards more investment into quality and lower quantity.

Since at early stages of development, the share of people deciding to educate their children at home is high, gains in adult life expectancy initially increase fertility. As life expectancy rises, more agents decide to send their children to schools, thereby strengthening the composition effect but at the same time also reinforcing the negative effect of a higher life expectancy on fertility by a potential behavioral change. Once the share of parents participating the public system is high enough, fertility will fall. Furthermore, in this paper we have proposed a theory why a formal schooling system may emerge endogenously without intervention by the state. We do not, however, make the next step and model why the society – via government and parliament – decided set up a free public schooling system financed by taxes. The extension by such a political economy element is left for future research.

3.A Appendix: Proofs

Since the problem has in general no closed form solution, we compute the comparative statics by implicitly differentiating the system of first order conditions. Then, for a change in variable X, the partial derivatives of n and h are given by

$$\begin{bmatrix} h_X \\ n_X \end{bmatrix} = - \begin{bmatrix} F_{hh} & F_{hn} \\ F_{nh} & F_{nn} \end{bmatrix}^{-1} \begin{bmatrix} F_{hX} \\ F_{nX} \end{bmatrix} = -|A|^{-1} \begin{bmatrix} F_{nn}F_{hX} & -F_{hn}F_{nX} \\ -F_{nh}F_{hX} & F_{hh}F_{nX} \end{bmatrix} \tag{3.35}$$

Proof of proposition 3.1. From (3.15) we have that e^{pr} is constant. Then we can write the household problem in terms of only n and h.

$$F_h = \frac{1}{c}\left(f(h) - \mu h^{\theta-1}\ell\right) \qquad F_n = \frac{\beta}{n}\left(nz(e)\right)^{1-\sigma} - \frac{T(e+\phi)}{\ell} \tag{3.36}$$

with $\ell \equiv T - h - n(e+\phi)$ and $c \equiv \omega f(h)\ell T^{-1}$. Partial derivatives are given by

$$\begin{bmatrix} h_T \\ n_T \end{bmatrix} = - \begin{bmatrix} -\frac{T}{h^2}\left(\theta + \frac{h^2}{\ell^2}\right) & -T\frac{e+\phi}{\ell^2} \\ -T\frac{e+\phi}{\ell^2} & -\frac{\sigma\beta}{n^2}\left(nz(e)\right)^{1-\sigma} - \frac{T(e+\phi)^2}{\ell^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\theta}{h} + \frac{T-\ell}{\ell^2} \\ \frac{(e+\phi)(T-\ell)}{\ell^2} \end{bmatrix} \tag{3.37}$$

and the determinant $|A| = F_{hh}F_{nn} - F_{hn}F_{nh} > 0$ can be shown to be positve implying that we have a maximum. Combining the elements from above establishes $n_T > 0$ and $h_T > 0$. □

Proof of proposition 3.2. The system of first order conditions is

$$F_h = \frac{1}{c}\left(f(h) - \mu h^{\theta-1}\ell\right) \qquad F_n = \frac{\beta}{n}(nz(e))^{1-\sigma} - \frac{1}{c}\left(pe + \phi\omega f(h)\right) \tag{3.38}$$

$$F_s = n\left(\beta s^{\gamma-1}(nz(e))^{-\sigma} - \frac{p}{c}\right) \tag{3.39}$$

with $\ell \equiv T - h - \phi n$ and $c \equiv (\omega f(h)\ell - npe)T^{-1}$. Combining F_s and F_n one obtains $e = f(h)\frac{\phi}{p}\frac{\gamma}{1-\gamma}$. Substituting this into the FOCs reduces the dimension of the system by one equation. We proceed by using F_h and F_n. For the comparative statics we have

$$
\begin{aligned}
\begin{bmatrix} h_T \\ n_T \end{bmatrix} &= - \begin{bmatrix} -\frac{1}{c}\left(\omega\mu h^{\theta-1}\frac{1+\theta}{\theta}\right) & -\frac{1}{c}\omega\mu h^{\theta-1}\phi \\ -\left(\frac{\omega f(h)}{c}\right)^2 \frac{\phi}{T(1-\gamma)} + (1-\sigma)\beta\gamma\theta\frac{(nz(e))^{1-\sigma}}{nh} & -\frac{\beta\sigma(nz(e))^{1-\sigma}}{n^2} + \left(\frac{\omega f(h)}{c}\frac{\phi}{1-\gamma}\right)^2\frac{1}{T} \end{bmatrix}^{-1} \\
&\times \begin{bmatrix} \frac{\omega\mu h^{\theta-1}}{c} \\ \left(\frac{\omega f(h)}{T(1-\gamma)c}\right)^2 \phi(h(1-\gamma)+\phi n) \end{bmatrix} \qquad (3.40)
\end{aligned}
$$

It can be again shown that $|A| = F_{hh}F_{nn} - F_{hn}F_{nh} > 0$. Further we have

$$
\begin{aligned}
h_T &= |A|^{-1}\left[\beta\sigma\frac{(nz(e))^{1-\sigma}}{n^2}\left(\frac{\omega f(h)\phi}{(1-\gamma)cT}\right)^2 \omega f(h)(1+\gamma\theta)\right] & (3.41) \\
n_T &= |A|^{-1}\left[\left(\frac{\phi\omega f(h)}{c(1-\gamma)T}\right)^2 \frac{n}{h}\omega f(h)(1+\gamma\theta) + \beta\gamma\theta^2(1-\sigma)\frac{\omega f(h)(nz(e))^{1-\sigma}}{nc}\right] & (3.42)
\end{aligned}
$$

where h_T is always positive. n_T is positive for $\sigma \leq 1$ but may be negative otherwise. □

Proof of Proposition 3.3. Substituting optimal choices for e and n into the utility function, utility of agents conditional on their education system choice is

$$
\begin{aligned}
U^{pr} &= T\log\left(\omega\mu\frac{h^{1+\theta}}{T\theta^2}\right) + \frac{\beta\left(nz(e)\right)^{1-\sigma}}{1-\sigma} \qquad e = \frac{\phi\gamma}{1-\gamma} & (3.43) \\
U^{pu} &= T\log\left(\omega\mu\frac{h^{\theta}(h+(h-T)\gamma\theta)}{(1-\gamma)T\theta^2}\right) + \frac{\beta\left(nz(e)\right)^{1-\sigma}}{1-\sigma} \qquad e = \frac{\omega}{p}\frac{\mu\phi\gamma h^{\theta}}{\theta(1-\gamma)}. & (3.44)
\end{aligned}
$$

and for a given vector $\{p, \omega\}$ the threshold ability level $\tilde{\mu}$ is implicitly defined by setting $U^{pr} = U^{pu}$. Since relative sub-utility from consumption is not affected directly by ω or μ (shift consumption proportionally), the decision which system to adopt depends only μ and $\{p, \omega\}$ to *via investment into education* (income effect). Note that a higher (lower) price p requires a proportionally higher (lower) ability μ to restore indifference (the allocation does not change for $j = pr$). Write the indifference condition as

$$
T\log\left(\frac{c^{pu}}{c^{pr}}\right) = \beta\left[u(n^{pr}z(e^{pr})) - u(n^{pu}z(e^{pu}))\right]. \qquad (3.45)
$$

Since the decision to join the public system is based purely on a sufficiently large income effect, it holds that $c^{pu} \geq c^{pr}$ for all solutions. For $T \to \infty$ we have

$$
\begin{aligned}
n^{pr} &= \left(\frac{\beta(1-\gamma)z(e^{pr})^{1-\sigma}}{(1+\theta)\phi} \right)^{\frac{1}{\sigma}} \\
h^{pr} &= T\frac{\theta}{1+\theta}
\end{aligned}
\tag{3.46}
$$

with e^{pr} as defined above. For indifference it must always hold that $u(n^{pr}z(e^{pr}))-u(n^{pu}z(e^{pu})) \geq 0$. Given constant $u(\cdot)^{pr}$ in the limit, we need that $u(\cdot)^{pu}$ is also constant[28] with the difference approaching zero. We also know that $\partial h^{pu}/\partial T > 0$ and hence $\partial e^{pu}/\partial T > 0$. Using $n^{pr} > n^{pu}$ and $e^{pu} > e^{pr}$ we know that it must hold that n^{pu} approaches n^{pr} from below and n^{pu} approaches e^{pr} from above. For given p and ω, this can only happen if the threshold ability level $\tilde{\mu}$ is decreasing. Since utility is non-decreasing in μ, this must hold for all μ and T. □

Proof of Lemma 3.1. Assume that we have found a vector $\{p, \omega, \tilde{\mu}\}$ such that $U^{pr} = U^{pu}$ holds. Further, we can rewrite the FOC F_n for both schooling systems to

$$
(1-\gamma)^{\sigma}\frac{T\phi\theta}{\beta} = z(e)^{1-\sigma}\frac{h(1-\gamma)}{n(h)^{\sigma}} \quad \text{if } j = pr, \tag{3.47}
$$

$$
\frac{T\phi\theta}{\beta} = z(e(h))^{1-\sigma}\frac{h+(h-T)\gamma\theta}{n(h)^{\sigma}} \quad \text{if } j = pu. \tag{3.48}
$$

The LHS if $j = pr$ is always smaller than the LHS if $j = pu$ and the same ordering must hold for the RHS in equilibrium. Since RHS is increasing in h, agents switching to the public system have lower fertility and invest more in child human capital. □

Proof of Lemma 3.2. Differentiating FOCs with respect to ability gives

$$
F_{h\mu} = 0 \qquad F_{n\mu} = 0 \qquad \text{if } j = pr \tag{3.49}
$$

$$
F_{h\mu} = 0 \qquad F_{n\mu} = \frac{\beta\gamma(1-\sigma)}{n\mu}(nz(e))^{1-\sigma} \quad \text{if } j = pu \tag{3.50}
$$

Combining this with the Hessian from above proves that ability does not change households' allocations. For $j = pu$ and assumption 3.2 more able agents invest more into adult human

[28] Obviously, e^{pu} and/or n^{pu} cannot keep growing monotonically otherwise the condition $u(\cdot)^{pr} - u(\cdot)^{pu} \geq 0$ would be violated for some T.

capital and lower fertility. Higher e follows trivially from (3.19). □

Proof of Proposition 3.4. First, use the price of education from (3.5) to express the equilibrium investment into education by parents choosing the public system. This gives

$$e = \phi \frac{\gamma}{1-\gamma} \frac{f(h(\tilde{\mu}))}{\bar{f}(h(T))} m(\bar{f}(h(T))) \tag{3.51}$$

Consider now the trivial case with $m(\bar{f}(h(T))) = \bar{f}(h(T))$. Then, the price of education relative to potential earnings is always one which brings us back to the partial equilibrium situation from proposition 3.3: $\tilde{\mu}$ decreases in T, the share of agents participating in the public system increases. Consider now the polar case without any externalities ($m(\cdot) = 1$) implying that the dynamics of p is determined by the evolution of $f(h(\tilde{\mu})^{pu})$ relative to the average $\bar{f}(h)$. Substituting this into (3.45) gives

$$T \log\left(\frac{c^{pu}}{c^{pr}}\right) = \beta \frac{z(e^{pr})^{1-\sigma}}{1-\sigma} \left[(n^{pr})^{1-\sigma} - (n^{pu} z(\Phi(\tilde{\mu}, T)))^{1-\sigma}\right] \tag{3.52}$$

$$\Phi(\tilde{\mu}, T) \equiv \frac{f(h(\tilde{\mu}, T)^{pu})}{\tilde{\mu} f(h(T)^{pr}) + \int_{\tilde{\mu}}^{1} f(h(a, T)^{pu}) da} \tag{3.53}$$

We know that the LHS is positive for all T. Since n^{pr} is converging to a constant, the same must hold for $u(n^{pu} z(e^{pu}))$. With n^{pu} approaching n^{pr} from below, $\Phi(\tilde{\mu}, T)$ must converge to a constant. Further, we have $\partial\tilde{\mu}/\partial T = -(\partial\Phi/\partial T)/(\partial\Phi/\partial\tilde{\mu}) < 0$ implying that the ability level is increasing. By monotonicity of the RHS in $\tilde{\mu}$ this is holds for all T. The result hinges on the fact that p is determined by the human capital of the indifferent agent relative to the average. As T and h rise, human capital of the $j = pu$ agent rises slower than the average (note that the agent in the numerator is the first agent in the integral in the denominator). Hence, the ability level of the indifferent agent has to rise. This increases potential earnings of agent $\mu = \tilde{\mu}$ and decreases the $\bar{f}(h)$ by shifting agents from the pr to pu (composition effect). Thus, $\tilde{\mu}$ will get arbitrarily close to 1 for large T (depends on fixed point of Ψ). □

Proof of Corollary 3.1. If fertility is a concave function of T, there are two life expectancies T_l and T_h at which fertility per family equals 2 and there must be an intermediate life expectancy T_{im} which maximizes fertility (above 2 children per couple) and population growth rate. Concavity gives us $\frac{\partial n(T_l)}{\partial T_l} > 0$ and $\frac{\partial^2 n(T_h)}{\partial T_h^2} < 0$ resulting in rising and falling population growth rate at the two stationary population levels ($g_P = 0$).

$$\left.\frac{\partial g_P}{\partial T_t}\right|_{g_P=0} = \frac{\partial n(T_t)/\partial T_t}{1+T_t} \tag{3.54}$$

Maximum population growth is defined by (3.55) and the derivative of g_P at that point is given by (3.56). Thus, g_P increases at T_l, attains a maximum at T_{im} and starts to decrease thereafter and becomes even negative at T_h.[29]

$$1 - (1+g_P)^{T_{im}} + T_{im} \log (1+g_P)^{T_{im}} = 0 \tag{3.55}$$

$$\left.\frac{\partial g_P}{\partial T_{im}}\right|_{g_P>0} = -\frac{(1+g_P)\log(1+g_P)}{T_{im}} < 0 \tag{3.56}$$

□

[29]This can be also trivially proven by applying the mean-value theorem since aggregate fertility is continuous and differentiable in T.

Chapter 4

Mortality, Fertility, Education and Capital Accumulation in a Simple OLG Economy

4.1 Introduction

Important aspects of economic history are the decline in mortality, the associated increase in life expectancy and a notable rise in investment into human capital. Life expectancy at birth in the UK was about 40 years in 1850, 65 years in 1950 and rose by 10 more years until the year 2000 (Cutler, Deaton, and Lleras-Muney (2006)). The share of children aged 10-14 attending primary schools rose from 10% in 1820 to 80% in 1930 (Flora, Kraus, and Pfenning (1983)). Universal schooling was reached soon thereafter. The same development took place for secondary and tertiary education. Net enrollment rates for secondary schooling increased from 67% in 1970 to 95% in 2000 (The World Bank (2004)). As can be seen, these processes of rising life expectancy, falling birth rates and rising investment into human capital are still going on in modern economies. The combined effect is that the population structure of developed countries is changing rapidly with a rising share of elderly people. This rise of the old-age dependency ratio and the associated rise in social security contributions have shifted the "aging problem" into the focus of the academic literature as well as public policy.

In this paper we develop an analytically tractable two generations OLG model in the spirit of Diamond (1965) in order to study the effects of demographic change on educational investment decisions and capital accumulation. We augment the simple textbook model with endogenous human capital formation. Population dynamics – the exogenous driving force of our model – are modeled by considering uncertain survival to old age and birth rates separately. Additionally, we look at the effects of changing lifetime labor supply. The strength of our setup is that we can analyze the general equilibrium effects of population dynamics using closed form solutions. The contribution of our paper is that using this rich setup, we are able to show that changes in life expectancy, population growth and lifetime labor supply have, in general, ambiguous effects on the capital stock and education. We demonstrate that it is key to consider the interactions between annuity markets, the pension system and productivity of education for understanding the qualitative and quantitative effects of variations in the population structure on changes in physical and human capital accumulation.

The relationship between mortality and investment into human capital has been investigated is a number of theoretical and empirical studies. Empirical studies find that falling mortality and the associated rise in life expectancy increase investment into human capital. Using data for post-war India, Ram and Schultz (1979) find that improvements in mortality played a major role in the rise of educational attainment. Eckstein, Mira, and Wolpin (1999) provide evidence for Sweden that the fall in child mortality was the most important factor for the demographic transition and the rising educational attainment. On the other hand, Mincer (1995) and Foster and Rosenzweig (1996) present empirical evidence that rising education premia have a positive effect on schooling.

Theoretical work dealing with the ageing-education nexus by Boucekkine, de la Croix, and Licandro (2002), de la Croix and Licandro (1999), Echevarria and Iza (2006) and Heijdra and Romp (2009) use variations of a Blanchard (1985) type of perpetual youth setup. By employing this model family the authors obtain closed form solutions and derive a number of insights by relying entirely on analytical results. These papers assume that the production processes use only labor (human capital) as an input or they consider only small open economies. Thus, the general equilibrium feedback effect of population dynamics on relative prices is ruled out by construction. A general conclusion of this literature is that increasing life expectancy increases investment into human and physical capital.

The papers by Hu (1999) and Kalemli-Ozcan, Ryder, and Weil (2000) are closest in spirit to our work. They also employ a perpetual youth setup but overcome the limitations of the above mentioned papers by developing tractable general equilibrium models. Our contributions to their work are threefold. First, we do not only study the effects of changes in mortality but also the effects of changing fertility on investment in education and human as well as physical capital accumulation. Second, we also analyze how changes in the lifetime working horizon affect educational decisions and capital accumulation. This additional channel in our model stands in for a lifelong learning motive and is increasingly important in aging societies which reform their PAYG financed pension systems by increasing retirement ages. Third, by using an OLG rather than a perpetual youth model, we reconfirm some of the findings of the above mentioned authors: Rising survival rates *may* lead to increasing educational efforts and capital accumulation. However, we emphasize that there are potentially important offsetting effects. The lower degree of analytical tractability of our OLG model – in comparison to the perpetual youth model – buys us the possibility to include and to understand several interaction effects and to show how these may change results. For example, using an equilibrium relationship of their model, Kalemli-Ozcan, Ryder, and Weil (2000) argue that the interest rate varies positively with mortality, "as would be expected from the simple intuition that shorter lives lead to lower wealth accumulation" (p. 11). We show that this positive effect is smaller when annuity markets are larger and that, by interpreting an equilibrium condition only, Kalemli-Ozcan, Ryder, and Weil (2000) ignore two important and potentially offsetting effects: increasing mortality (i) decreases the workforce and (ii) may decrease educational efforts and both effects ceteris paribus lead to a negative variation of mortality and the interest rate.

Finally, Zhang, Zhang, and Lee (2001) add to this literature by modeling endogenous fertility and child education employing a two-generations OLG setup as we do but using a dynastic framework. These differences in the two approaches makes their work less suitable as a benchmark for comparison. Furthermore, as a consequence of the endogenous nature of fertility decisions, these authors cannot study the impact of changing fertility and mortality in isolation as we do.

The remainder of this paper is structured as follows. Section 4.2 introduces the model. The results of the comparative static analysis are derived in section 4.3. In the same section we also show the results of our calibration exercise where we perform an extensive sensitivity analysis. Some concluding remarks are in section 4.4. Separate appendices contain proofs and additional results.

4.2 The Model

We develop a simple OLG model with endogenous education decisions and a PAYG financed social security system. The setup is as follows: agents live for two periods whereby survival to the second period is uncertain. In the first period agents choose time investment into education, saving and consumption. In the second period they consume their entire wealth and work only an exogenously given fraction ω of their time. The rest of their time (1 - ω) they are retired and receive a lump-sum pension, p_{t+1}. We make this assumption for analytical tractability; it allows us to analyze the effects of different social security regimes in a model of human capital accumulation à la Ben-Porath (1967) within a 2-generations model. In this setup, the parameter ω reflects a motive for life-long learning which can be affected by policy, e.g., by increasing the retirement age.

4.2.1 Demographics

Each period, there are $N_{t,0}$ young households and $N_{t,1}$ old. Let γ_t^N be the birth rate so that $N_{t,0} = \gamma_t^N N_{t-1,0}$ and s_t be the survival rate, hence $N_{t,1} = s_t N_{t-1,0}$. Using these definitions, the old-age dependency ratio ($oadr_t$) – the fraction of the old to the young – in the economy is given by

$$oadr_t = \frac{N_{t,1}}{N_{t,0}} = \frac{s_t}{\gamma_t^N}. \tag{4.1}$$

4.2.2 Markets for Annuities

We assume the existence of (imperfect) annuity markets for insurance against survival risk. Let $a_{t,0}$ be savings of the period t young. Period $t+1$ asset holdings are consequently given by

$$a_{t,0} + \lambda a_{t,0} \frac{1 - s_{t+1}}{s_{t+1}} = a_{t,0} \frac{\zeta_{t+1}}{s_{t+1}} \tag{4.2}$$

where

$$\zeta_{t+1} \equiv s_{t+1} + \lambda(1 - s_{t+1}) \tag{4.3}$$

is an annuity factor introduced here for convenience and $0 \leq \lambda \leq 1$ is the degree of annuitization, also see Hansen and Imrohoroğlu (2008). Notice that, in the case of no annuitization, we have $\lambda = 0$ and $\zeta_{t+1} = s_{t+1}$ and for complete (perfect) annuity markets we have $\lambda = 1$ and $\zeta_{t+1} = 1$. Full annuitization implies that the assets of the deceased agents are distributed uniformly among the surviving old agents which is an insurance against longevity (Yaari (1965)).

Without annuity markets there is no "insurance effect" but agents receive a lump-sum payment tr_{t+1} from the government. To keep the analysis analytically tractable we assume that in the case of incomplete annuitization the government distributes the accidental bequests to the old.[1] Accidental bequests are then redistributed to households as lump-sum transfers and given by

$$tr_{t+1} = (1 - \lambda) \frac{a_{t,0}(1 + r_{t+1})(1 - s_{t+1}) N_{t,0}}{N_{t+1,1}}. \tag{4.4}$$

and, using the fact that

$$N_{t+1,1} = N_{t,o} s_{t+1},$$

we have

$$tr_{t+1} = (1 - \lambda) \frac{a_{t,0}(1 + r_{t+1})(1 - s_{t+1})}{s_{t+1}}. \tag{4.5}$$

4.2.3 Household Optimization

Households maximize expected lifetime utility

$$\max_{c_{t,0},c_{t+1,1}} \log c_{t,0} + \beta s_{t+1} \log c_{t+1,1}, \tag{4.6}$$

subject to the constraints

$$c_{t,0} + a_{t,0} = (1-e_t)h_0 w_t (1-\tau_t) \tag{4.7a}$$

$$c_{t+1,1} = \frac{(1+r_{t+1})\zeta_{t+1}}{s_{t+1}} a_{t,0} + \omega h_{t+1,1} w_{t+1}(1-\tau_{t+1}) + (1-\omega)p_{t+1} + tr_{t+1}, \tag{4.7b}$$

where β is the raw time discount factor, e_t is investment into education when young, h_0 is the stock of human capital given at birth (taken as exogenous and constant over cohorts), w_t is the wage rate per unit of human capital, r_{t+1} is the return on financial assets, τ_t denotes the social security contribution rate, p_{t+1} are lump-sum pension payments, and tr_{t+1} are the distributed accidental bequests.

Due to the representative agent setup, two interpretations of ω are conceivable. In the first interpretation ω is the fraction of time the representative agent of age 1 works. In the second, it is the fraction in the population of age 1 that works. Either way, ω works like a policy variable and a change in ω could be interpreted, e.g., as a change in retirement legislation or labor market incentives affecting participation rates.

The present value budget constraint is accordingly given by

$$c_{t,0} + s_{t+1}\frac{c_{t+1,1}}{\zeta_{t+1}(1+r_{t+1})} = (1-e_t)h_0 w_t(1-\tau_t) + s_{t+1}\frac{\omega h_{t+1,1} w_{t+1}(1-\tau_{t+1}) + (1-\omega)p_{t+1} + tr_{t+1}}{\zeta_{t+1}(1+r_{t+1})}. \tag{4.8}$$

The education technology is

$$h_{t+1,1} = (1+g(e_t))h_0, \tag{4.9}$$

with g being a function mapping educational investment into formation of human capital. We choose g such that it is increasing, concave in e and fulfills the lower Inada condition. These are standard assumptions about the education function (see Willis (1986)).[1] Later, we specify a parametric form for $g(e_t)$ to obtain a closed form solution. Solving the maximization problem gives the Euler equation

$$c_{t+1,1} = \beta\zeta_{t+1}(1 + r_{t+1})c_{t,0}. \tag{4.10}$$

Solving for the optimal educational investment gives

$$g'(e_t) = \frac{\zeta_{t+1}(1 + r_{t+1})}{s_{t+1}} \frac{w_t(1 - \tau_t)}{\omega w_{t+1}(1 - \tau_{t+1})}. \tag{4.11}$$

This condition says that an individual invests into schooling until the marginal return of schooling equals the return on net wages relative to the effective interest rate. Following Bouzahzah, de la Croix, and Docquier (2002), we define the education function $g(e_t)$ in (4.9) as

$$g(e_t) = \xi e_t^{\psi}, \qquad \text{where } 0 < \psi < 1,\ \xi > 0. \tag{4.12}$$

Optimal education is then given by

$$e_t = \left[\omega\xi\psi \frac{w_{t+1}(1 - \tau_{t+1})}{w_t(1 - \tau_t)} \frac{s_{t+1}}{\zeta_{t+1}(1 + r_{t+1})}\right]^{\frac{1}{1-\psi}}. \tag{4.13}$$

It can be seen that educational decisions depend positively on the ratio of net wage growth to the return on capital holdings. This is the key general equilibrium effect we are interested in. The scarcity of raw labor resulting from demographic change will lead to rising wages and falling interest rates. According to equation (4.13) this will induce general equilibrium feedback effects by leading to increases in education and thereby to an increase in the second period human capital.

In addition to these general equilibrium effects, equation (4.13) shows direct effects on educational efforts through the educational productivity, ξ and ψ, the fraction of time working in the second period, ω and the probability of survival if there is some annuitization, i.e.,

[1] For analytical reasons, we assume zero depreciation of human capital and we do not make h an argument of g as in the standard Ben-Porath (1967) technology. This parametric restriction is also superimposed in some empirical studies, see the review in Browning, Hansen, and Heckman (1999).

if $\lambda > 0$. The direct effect of survival on educational decisions has in part been labeled as an effect due to an extension of the adult planning horizon, e.g., by Heijdra and Romp (2009). This is a misleading interpretation because the direct effect of survival is in fact a result of the induced adjustment of the rate of return to physical capital if there is some annuitization.[2] In the absence of annuitization, there is no adjustment of the rate of return to physical capital to the survival rate and changes in the survival rate have a direct effect only on the inter-temporal allocation of consumption (via the changing effective discount rate $s_{t+1}\beta$). In our model, the "pure" effect of extending the planing horizon is represented by an increase in ω.

Finally, households' optimal consumption follows from using (4.10) in (4.8) as

$$c_{t,0} = \frac{1}{1+\beta s_{t+1}} \left((1-e_t)h_0 w_t(1-\tau_t) + s_{t+1} \frac{\omega h_{t+1,1} w_{t+1}(1-\tau_{t+1}) + (1-\omega)p_{t+1} + tr_{t+1}}{\zeta_{t+1}(1+r_{t+1})} \right)$$

and using the above in (4.7a) gives savings as

$$a_{t,0} = \frac{1}{1+\beta s_{t+1}} \left(\beta s_{t+1}(1-e_t)h_0 w_t(1-\tau_t) - s_{t+1} \frac{\omega h_{t+1,1} w_{t+1}(1-\tau_{t+1}) + (1-\omega)p_{t+1} + tr_{t+1}}{\zeta_{t+1}(1+r_{t+1})} \right). \quad (4.14)$$

4.2.4 Firms

Firms produce output using a standard Cobb-Douglas production function

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}. \quad (4.15)$$

A_t is the firm's technology level which is determined by

$$A_{t+1} = A_t \gamma^A, \quad (4.16)$$

[2] This has already been shown by Hu (1999).

where γ^A is the exogenous gross growth rate. L_t is effective labor input which is the sum of human capital weighted labor supply of the young and of the old and accordingly given by

$$L_t \quad = \quad (1-e_t)h_0 N_{t,0} + \omega h_{t,1} N_{t,1}. \tag{4.17}$$

Competitive markets ensure that factors get paid their marginal products. We assume that capital depreciates fully after one period so that

$$1 + r_t = \alpha k_t^{\alpha-1} \tag{4.18a}$$

$$w_t = (1-\alpha) A_t k_t^{\alpha}, \tag{4.18b}$$

where $k_t \equiv \frac{K_t}{A_t L_t}$.

4.2.5 Government

The role of the government is twofold. First, the government taxes accidental bequests in the case of incomplete annuitization at a confiscatory rate and redistributes them as lump-sum payments to the old. Second, the government runs a PAYG financed social security system with a balanced budget in all periods requiring that total contributions by workers equal total pension payments.[3] By equation (4.17) we then have

$$w_t \tau_t \left((1-e_t)h_0 N_{t,0} + \omega h_{t,1} N_{t,1}\right) = (1-\omega) p_t N_{t,1}. \tag{4.19}$$

Notice that the above, using equation (4.1), implies that

$$(1-\omega) p_t = w_t \tau_t \left((1-e_t) h_0 \frac{\gamma_t^N}{s_t} + \omega h_{t,1}\right). \tag{4.20}$$

Changes in the population structure require adjustments of the social security policy. Let ϱ_t denote the replacement rate, i.e., the ratio of pension income to average net wage income.

[3] While we explicitly model this inter-generational transfer system as a pension system, it may also be interpreted as a metaphor for a more general intergenerational transfer system, e.g., a health care system.

Then pension income can be expressed as

$$p_t = \varrho_t \frac{(1-\tau_t)w_t\left((1-e_t)h_0 N_{t,0} + \omega h_{t,1} N_{t,1}\right)}{N_{t,0} + \omega N_{t,1}}.$$

Using the above definition in (4.19) and simplifying then links contribution and replacement rates by

$$\tau_t = \frac{(1-\omega)\varrho_t}{\gamma_t^N / s_t + \omega + (1-\omega)\varrho_t}. \tag{4.21}$$

It can be readily observed that τ_t increases in the fraction of pensioners, $1-\omega$, the generosity of the pension system, ϱ_t, and in the old-age dependency ratio, s_t/γ_t^N. Using this setup, fixing $\tau_t = \bar{\tau}$ corresponds to a fixed contribution rate system and holding $\varrho_t = \bar{\varrho}$ corresponds to a fixed replacement rate system.[4]

4.2.6 Equilibrium

In equilibrium all markets clear, households maximize utility and firms make zero profits. Market clearing on the capital market requires that

$$K_{t+1} = a_{t,0} N_{t,0}. \tag{4.22}$$

Using (4.1) in (4.17), aggregate labor supply can be rewritten as

$$L_t = N_{t,0} h_0 \left((1-e_t) + \omega \frac{s_t}{\gamma_t^N} (1 + g(e_{t-1})) \right). \tag{4.23}$$

Collecting elements, the following proposition gives the law of motion of the aggregate economy.

Proposition 4.1. *For given k_0 the aggregate dynamics of the economy are described by the system of first-order difference equations in $\{k_t, e_t\}$ given by*

$$k_{t+1} = \frac{\varphi_t \alpha(1-\alpha)(1-\tau_t)}{\phi_t} k_t^\alpha \tag{4.24a}$$

[4] Notice that these definitions are not the same as what is referred to as defined contribution and defined benefit systems in the literature.

$$e_t = \left(\frac{s_{t+1}}{\zeta_{t+1}}\omega\xi\psi\frac{\gamma^A(1-\tau_{t+1})k_{t+1}}{\alpha(1-\tau_t)k_t^\alpha}\right)^{\frac{1}{1-\psi}}, \tag{4.24b}$$

where

$$\begin{aligned}\phi_t \equiv \gamma^A \Bigg(\left(\alpha(2+\hat{\rho}_{t+1}) + \varphi_t \frac{(1-\alpha)\tau_{t+1}}{\zeta_{t+1}}(1+\hat{\rho}_{t+1}) \right) \frac{1-e_{t+1}}{1-e_t}\gamma_{t+1}^N \\ +\omega s_{t+1}\left(\alpha(2+\hat{\rho}_{t+1}) + \varphi_t\frac{1-\alpha}{\zeta_{t+1}}(1+\hat{\rho}_{t+1})\right)\frac{1+g(e_t)}{1-e_t}\Bigg)\end{aligned} \tag{4.25a}$$

$$\varphi_t \equiv \frac{(2+\hat{\rho}_{t+1})\zeta_{t+1}}{(2+\hat{\rho}_{t+1})\zeta_{t+1}+(1-s_{t+1})(1-\lambda))} \tag{4.25b}$$

and $\hat{\rho}_{t+1} = \frac{1}{s_{t+1}\beta} - 1$.

Proof. See appendix 4.A. □

Proposition 4.2. *If there is an equilibrium, education e_t is always interior on the interval $(0,1)$. Further, education converges always to its steady state value.*

Proof. See appendix 4.A. □

4.2.7 Steady State Analysis

Definition 2. *Along the balanced growth path (steady state) of the economy, all variables grow at constant rates so that $k = k_{t+1} = k_t$ and $e = e_{t+1} = e_t \ \forall\ t$.*

Proposition 4.3. *For $0 < \alpha < 1$ and $0 \leq \tau < 1$, the unique steady state of the economy is given by*

$$k = \left(\frac{\varphi\alpha(1-\alpha)(1-\tau)}{\phi}\right)^{\frac{1}{1-\alpha}} \tag{4.26a}$$

$$e = \left(\omega\xi\psi\frac{\gamma^A}{\alpha}\right)^{\frac{1}{1-\psi}}\left(\frac{s}{\zeta}\right)^{\frac{1}{1-\psi}} k^{\frac{1-\alpha}{1-\psi}} \tag{4.26b}$$

where

$$\phi \equiv \gamma^A \left(\left(\alpha(2+\hat{\rho}) + \varphi \frac{(1-\alpha)\tau}{\zeta}(1+\hat{\rho}) \right) \gamma^N + \right.$$
$$\left. + \omega s \left(\alpha(2+\hat{\rho}) + \varphi \frac{1-\alpha}{\zeta}(1+\hat{\rho}) \right) \frac{1+g(e)}{1-e} \right), \quad (4.27a)$$

$$\varphi \equiv \frac{(2+\hat{\rho})\zeta}{(2+\hat{\rho})\zeta + (1-s)(1-\lambda)} \quad (4.27b)$$

and $\hat{\rho} = \frac{1}{s\beta} - 1$.

Proof. See appendix 4.A. □

4.3 Comparative Statics

In this section, we use our framework to study the effects of demographic change on the economy by conducting a comparative statics analysis in steady state. In this respect our model is a useful laboratory to provide intuition for the results of much of the quantitative work, e.g., by Fougère and Mérette (1999), Sadahiro and Shimasawa (2002), Bouzahzah, de la Croix, and Docquier (2002) and Ludwig, Schelkle, and Vogel (2008). To this end, we analyze – by looking at partial derivatives – the effects of changing fertility, mortality and working time on the capital stock and education. We first do so in a social security scenario with constant contributions rates and then consider the opposite extreme by holding replacement rates constant. While we can uniquely determine the signs of many partially derivatives, we fail to do so in some cases. In these cases, our closed form solutions help us to understand the various offsetting effects at work and to detect the sources of indeterminacy. Finally, we use a calibrated version of our model to illustrate how the signs of partial derivatives depend on the parametrization of the model in the ambiguous cases.

4.3.1 Analytical Results

We drop the time indices to indicate steady state values. To begin with, we provide analytical results followed by an interpretation.

Proposition 4.4. *In the steady state of the economy we have*

1. *for $\tau = \bar{\tau}$ that*

$$\left.\frac{\partial k}{\partial \gamma^N}\right|_{\tau=\bar{\tau}} < 0 \quad \textit{and} \quad \left.\frac{\partial e}{\partial \gamma^N}\right|_{\tau=\bar{\tau}} < 0, \tag{4.28a}$$

$$\left.\frac{\partial k}{\partial s}\right|_{\tau=\bar{\tau}} \gtrless 0 \quad \textit{and} \quad \left.\frac{\partial e}{\partial s}\right|_{\tau=\bar{\tau}} \gtrless 0. \tag{4.28b}$$

$$\left.\frac{\partial k}{\partial \omega}\right|_{\tau=\bar{\tau}} < 0 \quad \textit{and} \quad \left.\frac{\partial e}{\partial \omega}\right|_{\tau=\bar{\tau}} \gtrless 0, \tag{4.28c}$$

2. *For the relationship between the cases $\tau = \bar{\tau}$ and $\varrho = \bar{\varrho}$ we have that*

$$\left.\frac{\partial k}{\partial \gamma^N}\right|_{\varrho=\bar{\varrho}} > \left.\frac{\partial k}{\partial \gamma^N}\right|_{\tau=\bar{\tau}} \quad \textit{and} \quad \left.\frac{\partial e}{\partial \gamma^N}\right|_{\varrho=\bar{\varrho}} > \left.\frac{\partial e}{\partial \gamma^N}\right|_{\tau=\bar{\tau}} \tag{4.29a}$$

$$\left.\frac{\partial k}{\partial s}\right|_{\varrho=\bar{\varrho}} < \left.\frac{\partial k}{\partial s}\right|_{\tau=\bar{\tau}} \quad \textit{and} \quad \left.\frac{\partial e}{\partial s}\right|_{\varrho=\bar{\varrho}} < \left.\frac{\partial e}{\partial s}\right|_{\tau=\bar{\tau}}, \tag{4.29b}$$

$$\left.\frac{\partial k}{\partial \omega}\right|_{\varrho=\bar{\varrho}} > \left.\frac{\partial k}{\partial \omega}\right|_{\tau=\bar{\tau}} \quad \textit{and} \quad \left.\frac{\partial e}{\partial \omega}\right|_{\varrho=\bar{\varrho}} > \left.\frac{\partial e}{\partial \omega}\right|_{\tau=\bar{\tau}}, \tag{4.29c}$$

Proof. See appendix 4.A. □

Interpretation of the partial derivatives of the capital stock and education in equation (4.28a) is rather straightforward. First, observe from (4.26b) that there is no direct effect of the birth rate, γ^N, on the education decision, e. Second, an increase of the birth rate increases the effective supply of labor in the economy which decreases k, cf. (4.26a) and (4.27a). Therefore, a change in the birth rate affects the relative prices of physical and human capital through its effect on k. An increase of k increases the wage rate, w, and decreases the return on physical capital, r. While the growth rate of wages ($\frac{w_{t+1}}{w_t}$) is unchanged in our steady state comparison, the return on physical capital decreases. Consequently, optimal education goes up, cf. (4.13).

As stated in the proposition, the signs of the partial derivatives in (4.28b) cannot be determined unambiguously. First, notice that there are various effects from increases of s on savings and thus k at work, cf. (4.27a): (i) an increase of s decreases the effective discount rate $\hat{\rho}$ which increases k. This is so because an increase of the survival rate increases savings via its effect on current period income, cf. the first term in the brackets of (4.14). (ii), however, an increase in the survival rate also increases the value of second period income as long as $\lambda > 0$ (so that $s_{t+1}/\zeta_{t+1} < 1$) which dampens the increase of savings. This dampening effect is the stronger, the larger is the size of the annuity market, i.e., the higher is λ.[5] (iii) for $\lambda > 0$, there is a direct effect of survival on education, cf. (4.26b), which varies positively with λ. This increases effective labor supply and thereby tends to decrease k. (iv) as s increases, raw labor supply increases as long as $\omega > 0$. Observe that the last two effects are stronger when the average human capital productivity is high, because ω interacts with ξ via the term $\frac{1+g(e)}{1-e}$ in (4.27a).

This discussion explains why the signs of the effects of s on k cannot be determined unambiguously. It can only be said that the capital stock is likely to increase if ω, λ and ξ are sufficiently small. For too high values of these parameters, the reaction of effective labor supply is too strong and the capital stock k_t may decrease (so that r_{t+1} increases). Second, this ambiguity with respect to the effects of s on k translates into an ambiguous effect of s on e, cf. (4.26b). However, even if k varies negatively with s, education may still increase because of the direct effect of increasing survival on the education decision in the presence of annuity markets ($\lambda > 0$). Indeed, in all of our simulations of subsection 4.3.3, schooling is found to increase if s rises, also in those cases in which k decreases when annuity markets are perfect. On the contrary, with missing annuity markets, we never find that k decreases in s so that there is also no ambiguity in the resulting educational adjustments.

The effect of a changing lifetime labor supply ω given in equation (4.28c) is unambiguously negative for the capital stock but ambiguous for the optimal education decision. First, increasing ω increases total effective labor supply and thus decreases k. Second, an increase of ω has a direct effect on education, cf. (4.13). This leads to an additional increase of effective labor which further decreases k. However, third, a decrease of k also exerts a dampening effect on education by increasing the return on physical capital. As this third effect is only a second order general equilibrium feedback effect, it cannot offset the decrease of k which explains the unambiguous sign for the partial derivative of k. However, the direct

[5] As can be immediately observed from (4.14), the overall effect of increasing survival on savings is unambiguously positive. But it is larger for $\lambda = 0$ than for $\lambda = 1$.

effect of ω on k and the resulting general equilibrium price effect could potentially be strong enough to offset the direct effect of ω on education. This explains the ambiguous sign of the partial derivative of e. While this is so analytically, we show below for a wide range of parameter constellations of our simulations that education varies positively with ω.

The effect of an adjustment of the contribution rate τ is examined in the second part of proposition 4.4. Recall that changing the contribution rate has only a direct effect on capital accumulation but does not distort education decisions in steady state. Thus, increasing the contribution rate only has an effect on steady state education to the extent that it crowds out savings in physical capital. The uniform conclusion is therefore that a rising (falling) contribution rate decreases (increases) the capital stock, thereby increases (decreases) the interest rate and thus decreases (increases) the incentives to invest into education. A brief verbal summary of the results is that the effect of falling birth rates, rising survival rates, i.e., an aging of the population, or an extension of the lifetime labor supply has a larger effect (in absolute values) on the capital stock and on education if the contribution rate τ is held constant. The results do not say, however, that the signs do not change. Since we add one layer of complexity, it is even harder to pin down the direction of change.

4.3.2 Role of Annuity Markets

This subsection discusses the role of the degree of annuitization in more detail. We show in the appendix that

Proposition 4.5. *In the steady state of the economy we have*

1. *for $\tau = \bar{\tau}$ that*

$$\left.\frac{\partial k}{\partial \lambda}\right|_{\tau=\bar{\tau}} > 0 \quad \textit{and} \quad \left.\frac{\partial e}{\partial \lambda}\right|_{\tau=\bar{\tau}} \gtrless 0, \tag{4.30a}$$

2. *For the relationship between the cases $\tau = \bar{\tau}$ and $\varrho = \bar{\varrho}$ we have that*

$$\left.\frac{\partial k}{\partial \lambda}\right|_{\varrho=\bar{\varrho}} = \left.\frac{\partial k}{\partial \lambda}\right|_{\tau=\bar{\tau}} \quad \textit{and} \quad \left.\frac{\partial e}{\partial \lambda}\right|_{\varrho=\bar{\varrho}} = \left.\frac{\partial e}{\partial \lambda}\right|_{\tau=\bar{\tau}}. \tag{4.31a}$$

Proof. See appendix 4.A. □

More complete annuity markets increase savings but have an ambiguous effect on the education decision. Again, the ambiguity comes from the fact that the direct effect of increasing annuitization on the interest rate – which reduces educational investments, c.f. equation (4.26b)– may be offset by the indirect effect of rising capital – which decreases the interest rate and thereby increases education. Furthermore, the effect of λ on capital and education is the same in both social security scenarios. This is so because the adjustment of the contribution or replacement rate does not interact with λ.

More interesting is, however, how the level of λ interacts with the derivatives of k and e with respect to s, γ^N and ω. Unfortunately, due to the algebraic complexity of the problem, it is not possible to obtain clear results for these cross-derivatives. However, as is shown in appendix 4.A, a higher λ makes it more likely that $\partial k/\partial s < 0$. Further results on the importance of annuity markets are illustrated in our numerical simulations, cf., in particular, the discussion in subsection 4.3.3.

4.3.3 Numerical Results

As stated in the previous subsection, there are cases in which the sign of the derivatives are ambiguous. For these cases we here present results from numerical simulations of our model to illustrate the sources for this ambiguity. Obviously, our stylized two period model fails to capture many relevant aspects. This exercise is therefore an illustration only and is not meant to provide exact quantitative results of population aging on the economy. We first investigate the case with perfect annuity markets and then the case without annuity markets. Furthermore, we redo the calculations for both scenarios with constant contribution and constant replacement rates.

Perfect Annuity Markets

In this subsection we focus on the case with perfect annuity markets ($\lambda = 1$) where the direct effects of changing survival rates on the education decision is strongest and consequently the effects of changing survival are likely to be ambiguous, cf. our previous discussion in subsection 4.3.1 and appendix 4.A. Furthermore, the case with perfect annuity markets, although empirically doubtful, makes our results directly comparable to the perpetual youth model of Kalemli-Ozcan, Ryder, and Weil (2000).

We take the periodicity of the model such that each generation covers a maximum of 40 years. Agents are assumed to become economically active at the actual age of 20. Correspondingly, the maximum age agents can reach is 100. Our calibration targets for some of the population parameters are for averages of the three core European countries France, Germany and Italy.[6] For the survival rate, s, we take as calibration target the remaining life expectancy at the age of 20, LE_{20}, which is currently (in 2004) 68 years. As survival in our model is certain in the first period of life, the survival rate is given by $s = LE_{20}/40 - 1.0 = 0.69$. We calibrate γ^N using the implied γ^N to match the old-age dependency ratio of 44%. Accordingly, we set $\gamma^N = 1.5574$.[7] The long-run growth rate of productivity in European countries is roughly 0.015 Barro and Sala-i-Martin (2003) annually so that $\gamma^A = 1.015^{40} = 1.81$. We set the discount factor $\beta = 0.99^{40} = 0.67$ by reference to other studies, e.g., Hurd (1990).

The most critical parameters are ω and ψ, and ξ. First, we calibrate ψ to medium value of the estimates reported in Browning, Hansen, and Heckman (1999) which is 0.6. Second, there is no direct empirical counterpart of ω because it just reflects an auxiliary variable in our model that simplifies the exposition. To calibrate this parameter we use the share of agents obtaining higher education as the calibration target.[8] Since the timing of the model is such that the first (and economically passive) period is 20 years, education can be also viewed as the share of people investing into higher education (university and post-graduate education). We construct aggregate indices using data from OECD (2008).[9] The procedure is as follows. We compute the average graduation age of a typical student for the two university (or equivalent) diploma categories (Type A and B). Then we use this number to compute how many years a person spends in tertiary education in excess of the economic starting age (which is set to 20). For example, the "average" French student (see

[6]Our population data are based on the Human Mortality Database (2008).

[7]The alternative would be to calibrate γ^N with the gross growth rate of the working age population ratio. This would require to set $\gamma^N = 1.06$. The implied *oadr* is then 0.66 and hence this alternative would overestimate the actual old-age dependency.

[8]Alternative calibration targets are e.g. the fraction of the old (age 60 and older) in the population who work which is 5.4% in the data. In our model this implies $\omega = 0.12$ and $e = 0.0077$ (0.31 years of education). The choice of this alternative measure does not change our conclusions (results available upon request).

[9]The data we use can be found in tables A1.1a, A1.3a and X1.1c. See also the same publication for more detailed information on the educational systems and definitions.

table 4.1) is obtaining a type A diploma at the age of 24.5 and a type B diploma at the age of 22. We then weight the "excess years" (4.5 and 2) by the population weights (0.11 and 0.15) to obtain years of tertiary education of a representative French agent (0.775). Then, we weight the country specific years by the population of the three countries to compute years of education for the "representative European" (0.874). As a last step, we divide this number by the duration of one period (40 years) to convert it into the model specific equivalent and use it as a calibration target. Hence, our target for e is $e = 0.01988$. Third, we calibrate ξ endogenously to match the ratio of peak life cycle wages to the wage rate at labor market entry which is 1.6 Attanasio (1999). Since we set $h_0 = 1$, this is the data equivalent to human capital holdings of the old, h_1, and our calibration target requires $\xi = 6.30$. Parameters are summarized in table 4.2.

Table 4.1: Calibration Target for Time in Tertiary Education

	Graduation Age		Share in Population		Weighted Years
Type of Diploma	B	A	B	A	B+A
France	22.0	24.5	0.11	0.15	0.775
Germany	22.0	25.5	0.09	0.14	0.950
Italy	22.5	26.0	0.01	0.12	0.601
Total					0.795

Graduation age refers to the average within the particular type of diploma. The country weights (France 0.31, Germany 0.40 and Italy 0.39) are given by the relative population size in 2006 computed from the Human Mortality Database (2008).

As our discussion of the analytical results in subsection 4.3.1 shows, the most critical parameters in the case of perfect annuitization ($\lambda = 1$) are s, ω, ξ, ψ. We therefore consider a range of alternative specifications around the benchmark specification in table 4.2 for all these parameters. The graphs have $\omega \in (0, 1)$ on the horizontal axis. The different lines are always drawn for a tuple from $\{\xi \otimes s\}$ for selected values for ξ and s where the intermediate values (solid lines) are from the benchmark calibration. Finally, in order to address the sensitivity of our results with respect to the concavity of the education technology we redo all calculations for $\psi = 0.3$.[10] We recalibrate the model when we change the value of ψ.

[10] For the sake of brevity, simulation results with varying α and β are not displayed but are available upon request.

Table 4.2: Calibration parameters

Firm sector	$\psi = 0.6$	$\psi = 0.3$
Capital share, α	0.3	0.3
Technological progress, γ^A	1.81	1.81
Household sector		
Discount factor, β	0.67	0.67
Average productivity of human capital investments, ξ	6.30	1.94
Coefficient in human capital production function, ψ	0.6	0.3
Fraction of the old working, ω	0.36	0.94
Social Security		
Replacement rate, ϱ	0.6	0.6
Demographics		
Birth rate, γ^N	1.56	1.56
Survival rate, s	0.69	0.69

The vertical black line is the calibrated value of ω. Observe that with lower concavity of the education technology (lower ψ), the calibrated value of ω is increased substantially to match the same target. Instead of reporting the rather uninformative numbers for the derivatives, in the figures we show elasticities which are better comparable across calibrations.

The effect of changing survival rates on the capital stock are displayed in figure 4.1. As claimed in proposition 4.4, the sign is ambiguous. The sign is more likely to be negative for high survival rates, high marginal productivity of education (ψ and ξ) and high labor market participation in the second period (high ω). Obviously, the higher the marginal product of education (as determined by ξ and ψ), the more will agents invest into education and the less they will work and save. The effect of ω goes into the same direction since it is reinforcing the effect of education.

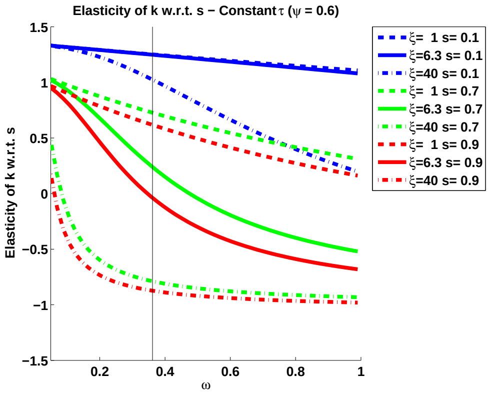
Figure 4.2 shows the elasticity of education with respect to the survival rate. Although we show in proposition 4.4 that the sign cannot be determined unambiguously, the elasticity is always positive in our simulations. Rising survival rates always increase educational attainment. The simulations also show that the elasticity is smaller for high values of ξ and higher survival rates. The curvature of the human capital production function ψ has only a minor influence.

Finally, figure 4.3 shows how education varies with the time spent on the labor market in the second period. Although the sign cannot be determined analytically, the simulations

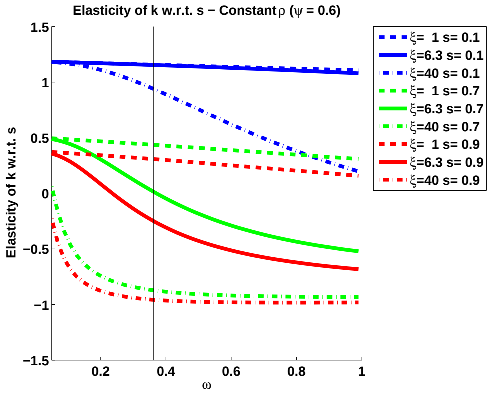
Figure 4.1: Elasticity of k with respect to s

(a) Benchmark concavity ($\psi = 0.6$)

(i) constant τ

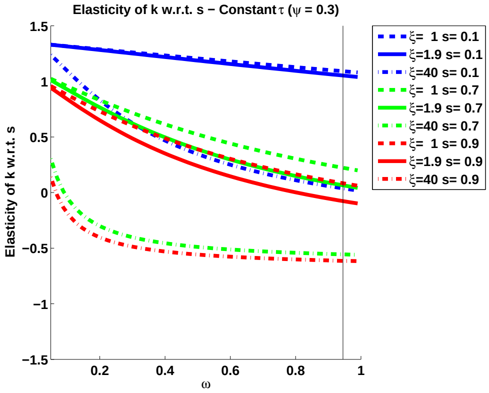


(ii) constant ρ



(b) Low concavity ($\psi = 0.3$)

(i) constant τ



(ii) constant ρ

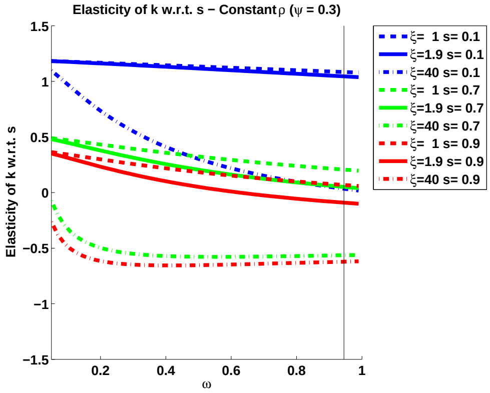
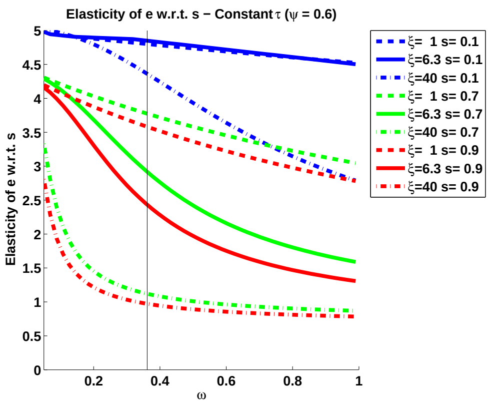


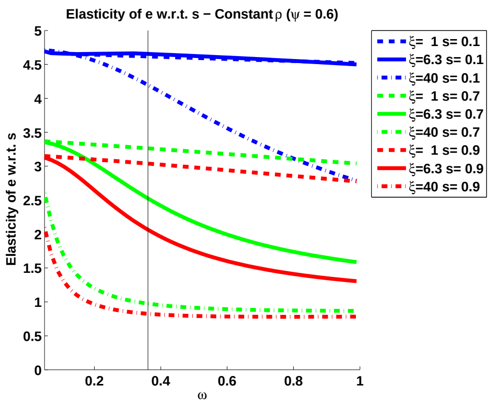
Figure 4.2: Elasticity of e with respect to s

(a) Benchmark concavity ($\psi = 0.6$)

(i) constant τ

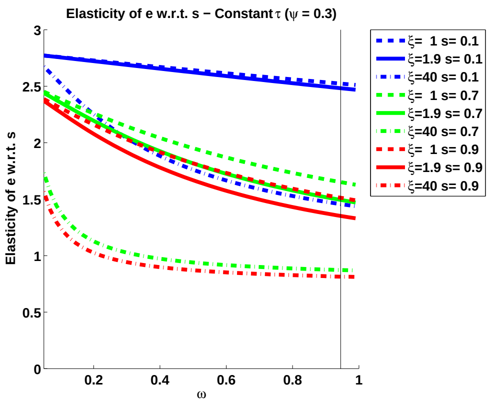


(ii) constant ρ

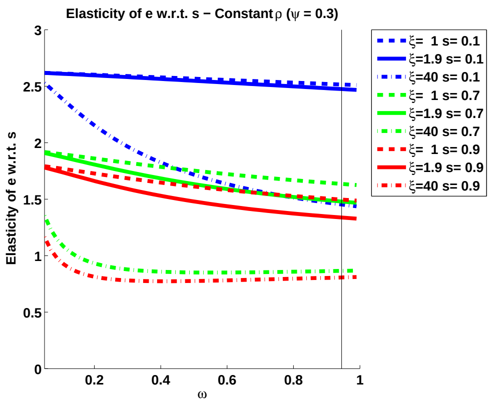


(b) Low concavity ($\psi = 0.3$)

(i) constant τ



(ii) constant ρ

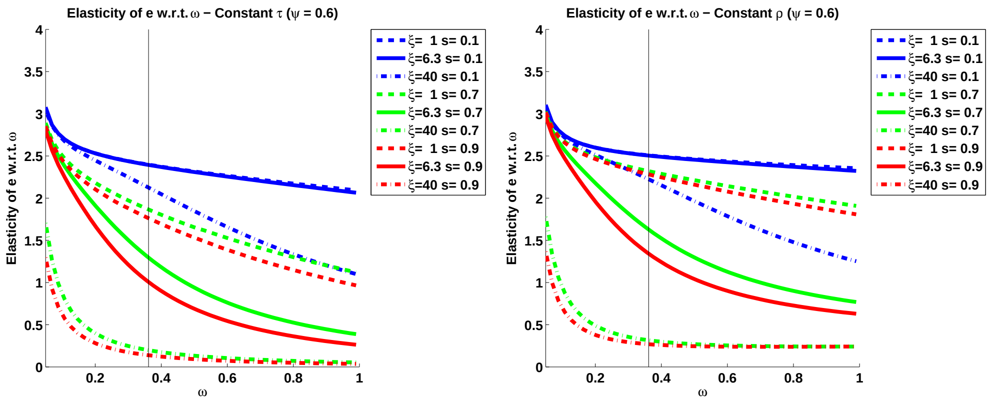


show that education always increases if ω increases. Thus, the direct effect of a rising ω is not overturned by a general equilibrium effect of rising interest rates. The factor having the largest influence is ψ which governs the shape of the marginal productivity of schooling investment and other parameters seem to have only a small effect on the behavior of the model. Not surprisingly, with more concavity of the human capital production function the (positive) effect of increasing lifetime labor supply on the education decision increases.

Figure 4.3: Elasticity of e with respect to ω

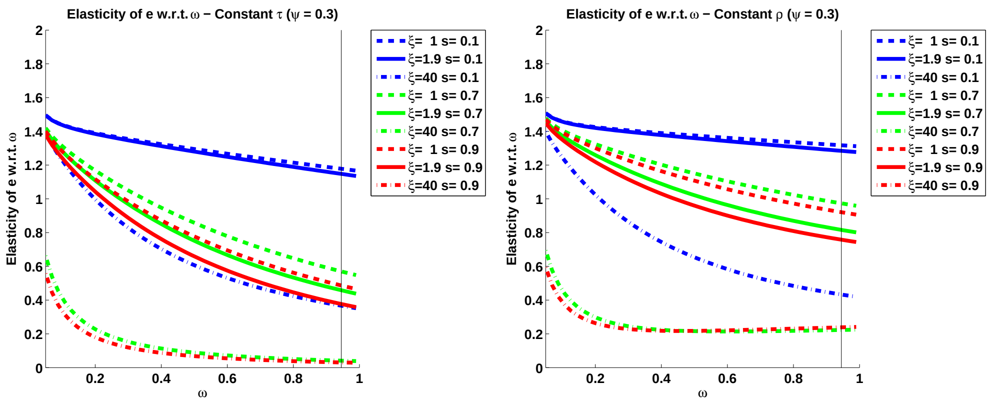
(a) Benchmark concavity ($\psi = 0.6$)

(i) constant τ (ii) constant ρ



(b) Low concavity ($\psi = 0.3$)

(i) constant τ (ii) constant ρ



No Annuity Markets

This subsection provides a sensitivity analysis with respect to changes in the degree of annuitization. We set $\lambda = 0$ (corresponding to an economy without annuity markets), recalibrate the model using the same calibration targets as above and report the new parameters in table 4.3, appendix 4.B. Since only the partial derivative $\partial k/\partial s$ changes its sign if we vary λ we here only show this result in figure 4.4. The other figures can be found in appendix 4.B.

Indeed with $\lambda = 0$ the reaction of the capital stock to changes in the survival rate is always positive, whereas for $\lambda = 1$ it may also be negative. Thus, the degree of completeness of annuity markets has an important effect on the reaction of the economy. The qualitative effects of changes in the population growth rate, γ^N, and lifetime labor supply, ω, are not affected by the choice of λ (see appendix 4.B).

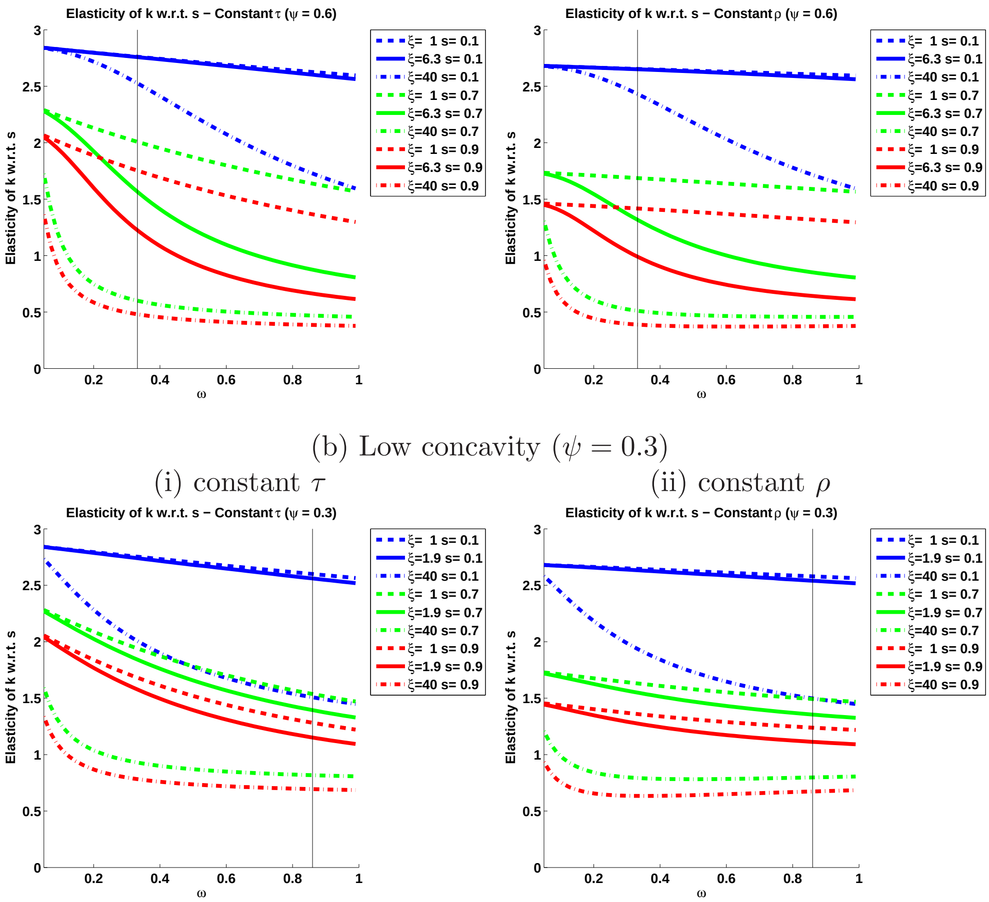
Figure 4.4: Elasticity of k with respect to s: No Annuity Markets

(a) Benchmark concavity ($\psi = 0.6$)

(i) constant τ (ii) constant ρ

(b) Low concavity ($\psi = 0.3$)

(i) constant τ (ii) constant ρ



4.4 Conclusion

This paper investigates the effects of a changing population structure on capital accumulation and educational investment in a tractable two period model in the spirit of Diamond (1965). We vary the population structure by three dimensions, first, by the fertility rate, second, by the survival rate and, third, by the degree of old-age labor supply. We show that a decrease of the fertility rate and a corresponding increase of the old-age dependency ratio unambiguously increases the capital intensity and education if contribution rates to the pension system are held constant. An increase of the survival rate, on the other hand, does not unambiguously vary with these variables. Our analytical results and our numerical illustrations shed light on the sources of this ambiguity by highlighting the various and potentially offsetting interaction effects at work. Therefore, our tractable model is a useful laboratory for understanding the magnitudes of the effects found in applied quantitative work employing models with overlapping generations.

4.A Appendix: Proofs

Proof of proposition 4.1. We have that

$$k_{t+1} = \frac{K_{t+1}}{A_{t+1}L_{t+1}}$$

which, by (4.22), can be rewritten as

$$k_{t+1}\frac{L_{t+1}}{N_{t,0}} = \frac{a_{t,0}}{A_{t+1}} \tag{4.32}$$

We first work on the LHS of (4.32). Using (4.23) we get

$$k_{t+1}\frac{L_{t+1}}{N_{t,0}} = k_{t+1}h_0\left((1-e_{t+1})\gamma^N_{t+1} + \omega s_{t+1}(1+g(e_t))\right). \tag{4.33}$$

Next, we focus on the RHS of (4.32). Using (4.5), (4.9) and (4.20) in (4.14) and bringing the terms involving $a_{t,0}$ to the LHS of the resulting expression we get

$$\begin{aligned} a_{t,0}\left(1+\frac{(1-s_{t+1})(1-\lambda)}{(1+\beta s_{t+1})\zeta_{t+1}}\right) = \frac{h_0}{1+\beta s_{t+1}}\Big(&\beta s_{t+1}(1-e_t)(1-\tau_t)w_t - \\ &\frac{w_{t+1}}{\zeta_{t+1}(1+r_{t+1})}\left(s_{t+1}\omega(1+g(e_t)) + \tau_{t+1}(1-e_{t+1})\gamma^N_{t+1}\right)\Big). \end{aligned}$$

Bringing the term postmultiplying $a_{t,0}$ to the RHS, replacing r_t and w_t by their marginal products from (4.18) and dividing by A_{t+1} gives

$$\begin{aligned} \frac{a_{t,0}}{A_{t+1}} = \varphi_t\frac{h_0}{1+\beta s_{t+1}}\Big(&\beta s_{t+1}(1-e_t)(1-\tau_t)(1-\alpha)k_t^\alpha\frac{1}{\gamma^A} \\ &-\frac{1-\alpha}{\alpha\zeta_{t+1}}k_{t+1}\left(s_{t+1}\omega(1+g(e_t)) + \tau_{t+1}(1-e_{t+1})\gamma^N_{t+1}\right)\Big). \end{aligned} \tag{4.34}$$

where

$$\varphi_t = \frac{(1+\beta s_{t+1})\zeta_{t+1}}{(1+\beta s_{t+1})\zeta_{t+1} + (1-s_{t+1})(1-\lambda)}. \tag{4.35}$$

Next, use the equation above and combine it with (4.33) to get

$$k_{t+1}\Big((1-e_{t+1})\gamma^N_{t+1}+s_{t+1}\omega(1+g(e_t))+ \varphi_t\frac{1-\alpha}{\alpha(1+\beta s_{t+1})\zeta_{t+1}}\left(s_{t+1}\omega(1+g(e_t))+\tau_{t+1}(1-e_{t+1})\gamma^N_{t+1}\right)\Big) = \varphi_t\frac{\beta s_{t+1}(1-\alpha)}{\gamma^A(1+\beta s_{t+1})}(1-e_t)(1-\tau_t)k_t^\alpha.$$

Multiply the above by $\alpha(1+\beta s_{t+1})$ and simplify to get

$$k_{t+1}\left((1-e_{t+1})\gamma^N_{t+1}\left(\alpha(1+\beta s_{t+1})+\varphi_t\frac{(1-\alpha)\tau_{t+1}}{\zeta_{t+1}}\right)+ s_{t+1}\omega(1+g(e_t))\left(\alpha(1+\beta s_{t+1})+\varphi_t\frac{1-\alpha}{\zeta_{t+1}}\right)\right) = \varphi_t\frac{\alpha(1-\alpha)\beta s_{t+1}}{\gamma^A}(1-e_t)(1-\tau_t)k_t^\alpha.$$

The expression for e_t immediately follows from replacing wages and interest rates by their respective counterparts from equations (4.18a) and (4.18b). Using $\hat{\rho}=\frac{1}{\beta s_{t+1}}-1$ proves the claim in the proposition. □

Proof of proposition 4.2. First, given that the function $g(e)$ satisfies the lower Inada condition with $\lim_{e\to 0} g'(e)\to\infty$ the solution with zero education is excluded for $\omega\in(0,1]$. Second, having full educational investment (i.e. $e=1$), labor supply and thus wage income of the young generation is zero. By the lower Inada condition of the utility function, we have that $c_{t,0}>0$ for positive wages. Consequently, savings in the first period would be negative and so will be the capital stock of the economy. Thus, if there is an equilibrium with finite and positive capital stock, education will always be lower than unity.

To show that education always converges to the steady state solution use (4.24a) in (4.24b) and rewrite the resulting expression as

$$e_t^{1-\psi}=\frac{s_{t+1}}{\zeta_{t+1}}\omega\xi\psi\frac{(1-\alpha)(1-\tau_{t+1})}{\left(\Gamma_1\frac{1-e_{t+1}}{1-e_t}+\Gamma_2\frac{1+g(e_t)}{1-e_t}\right)}$$

where

$$\begin{aligned}\Gamma_1 &\equiv \left(\alpha(2+\hat{\rho}_{t+1})+\varphi_t\frac{(1-\alpha)\tau_{t+1}}{\zeta_{t+1}}(1+\hat{\rho}_{t+1})\right)\gamma^N_{t+1}\\ \Gamma_2 &\equiv \omega s_{t+1}\left(\alpha(2+\hat{\rho}_{t+1})+\varphi_t\frac{1-\alpha}{\zeta_{t+1}}(1+\hat{\rho}_{t+1})\right)\end{aligned}$$

Defining $\Delta_t \equiv e_t - e_{t+1}(k^*)$ as measuring the distance between e_t and e_{t+1} which is ultimately a function of the steady state capital stock. Thus, Δ_t measures the change in education between t and $t+1$ outside the steady state. Rearranging gives

$$F(e_t,\Delta_t) = e_t^{1-\psi} - \frac{s_{t+1}}{\zeta_{t+1}}\omega\xi\psi\frac{(1-\alpha)(1-\tau_{t+1})}{\left(\Gamma_1\frac{1-e_t+\Delta_t}{1-e_t}+\Gamma_2\frac{1+g(e_t)}{1-e_t}\right)}. \tag{4.36}$$

Taking the derivative of e_t with respect to the distance to the steady state gives

$$\frac{\partial e_t}{\partial \Delta_t} = -\frac{\partial F/\partial \Delta_t}{\partial F/\partial e_t} < 0 \tag{4.37a}$$

$$\frac{\partial^2 e_t}{\partial \Delta_t^2} > 0. \tag{4.37b}$$

Therefore, if education is, e.g., below its new steady state level after an exogenous shock (i.e., $\Delta_t < 0$), e_t will always converge monotonically to the new steady state value.

□

Proof of proposition 4.3. Existence:
Using equation (4.22) and the assumption of constant population growth we have

$$k_{t+1} = \frac{1}{\gamma^N\gamma^A}\tilde{a}_{t,0},$$

where $\tilde{a}_{t,0}$ is equation (4.22) divided by A_t to transform $a_{t,0}$ into savings per efficient worker. Define the function

$$d(w_t, r_{t+1}) = \gamma^A\gamma^N k_{t+1} - \tilde{a}(w_t(k_t), r_{t+1}(k_{t+1}), e_t(k_{t+1})). \tag{4.38a}$$

Where $d(\cdot)$ is the change in the capital stock per effective worker. Given that we use log-utility, $e_t \in (0,1)$ and a Cobb-Douglas production function it holds that

$$0 < \tilde{a}(w_t, r_{t+1}, e_t) < \tilde{w}_t, \tag{4.38b}$$

$$0 < \frac{\tilde{a}(w_t, r_{t+1}, e_t)}{k_{t+1}} < \frac{\tilde{w}_t}{k_{t+1}}. \tag{4.38c}$$

where $\tilde{w}_t$ denotes wages scaled by the level of technology. All we have to show is that $d(\cdot)$ has opposite signs for k_{t+1} going to zero and infinity. Then by continuity of $d(\cdot)$ there is at least one capital stock satisfying $d(\cdot) = 0$. This holds since

$$\frac{d(w_t, r_{t+1})}{k_{t+1}} = \gamma^N \gamma^A - \frac{\tilde{a}(w_t, r_{t+1}, e_t)}{k_{t+1}} \tag{4.38d}$$

and taking the limits gives

$$\lim_{k_{t+1} \to \infty} \frac{d(w_t, r_{t+1})}{k_{t+1}} = \gamma^N \gamma^A > 0 \tag{4.38e}$$

$$\lim_{k_{t+1} \to 0} \frac{d(w_t, r_{t+1})}{k_{t+1}} = -\infty < 0 \tag{4.38f}$$

for sufficiently small k_{t+1}. For uniqueness it is sufficient to show that $\partial d(w_t, r_{t+1})/\partial k_{t+1} > 0$ for all k, i.e. that for a given wage rate $d(w_t, r_{t+1})$ is nondecreasing in the capital stock. Taking equation (4.25a) and recalling that $\partial e/\partial k > 0$ establishes the result. By using equation (4.26b) it is clear that a unique solution for the capital stock automatically gives a unique e. □

Proof of proposition 4.4. From (4.26) define

$$F_1(k, e; \gamma^N, s, \lambda, \omega) = \Omega(e, \gamma^N, s, \lambda, \omega)^{\frac{1}{1-\alpha}} - k = 0 \tag{4.39a}$$

$$F_2(k, e; \gamma^N, s, \lambda, \omega) = c \cdot \left(\frac{s}{\zeta}\right)^{\frac{1}{1-\psi}} \cdot k^{\frac{1-\alpha}{1-\psi}} - e = 0 \tag{4.39b}$$

where

$$\Omega(e, \gamma^N, s, \lambda, \omega) \equiv \frac{\varphi}{\phi}(1-\tau)\alpha(1-\alpha)\beta \tag{4.40}$$

$$c \equiv \left[\omega\xi\psi\frac{\gamma^A}{\alpha}\right]^{\frac{1}{1-\psi}} \tag{4.41}$$

and ϕ is as in (4.27a) and φ as in (4.27b).

1. For the case where $\tau = \bar{\tau}$, we can ignore that τ is related to γ^N and s by the steady state version of (4.21). The general problem with two implicitly defined endogenous variables can be written as

$$\begin{bmatrix} \frac{\partial k}{\partial X} \\ \frac{\partial e}{\partial X} \end{bmatrix} = - \begin{bmatrix} \frac{\partial F_1}{\partial k} & \frac{\partial F_1}{\partial e} \\ \frac{\partial F_2}{\partial k} & \frac{\partial F_2}{\partial e} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial F_1}{\partial X} \\ \frac{\partial F_2}{\partial X} \end{bmatrix} = -A^{-1} \begin{bmatrix} \frac{\partial F_1}{\partial X} \\ \frac{\partial F_2}{\partial X} \end{bmatrix} \tag{4.42}$$

where X is any variable from the vector of exogenous variables $\{\gamma_N, s, \omega,\}$ and therefore

$$\begin{bmatrix} \frac{\partial k}{\partial X} \\ \frac{\partial e}{\partial X} \end{bmatrix} = -|A|^{-1} \begin{bmatrix} \frac{\partial F_2}{\partial e} & -\frac{\partial F_1}{\partial e} \\ -\frac{\partial F_2}{\partial k} & \frac{\partial F_1}{\partial k} \end{bmatrix} \begin{bmatrix} \frac{\partial F_1}{\partial X} \\ \frac{\partial F_2}{\partial X} \end{bmatrix} \tag{4.43}$$

and rearranging gives

$$\begin{bmatrix} \frac{\partial k}{\partial X} \\ \frac{\partial e}{\partial X} \end{bmatrix} = -|A|^{-1} \begin{bmatrix} \frac{\partial F_2}{\partial e}\frac{\partial F_1}{\partial X} & -\frac{\partial F_1}{\partial e}\frac{\partial F_2}{\partial X} \\ -\frac{\partial F_2}{\partial k}\frac{\partial F_1}{\partial X} & +\frac{\partial F_1}{\partial k}\frac{\partial F_2}{\partial X} \end{bmatrix} \tag{4.44}$$

Since $\tau = \bar{\tau}$, we get

$$\frac{\partial F_1}{\partial k} = -1 < 0 \tag{4.45a}$$

$$\frac{\partial F_1}{\partial e} = \frac{1}{1-\alpha}\Omega^{1/(1-\alpha)-1}\frac{\partial \Omega}{\partial e} < 0 \tag{4.45b}$$

$$\frac{\partial F_2}{\partial k} = c\left(\frac{s}{\zeta}\right)^{\frac{1}{1-\psi}}\frac{1-\alpha}{1-\psi}k^{\frac{1-\alpha}{1-\psi}-1} > 0 \tag{4.45c}$$

$$\frac{\partial F_2}{\partial e} = -1 < 0 \tag{4.45d}$$

whereby the sign in (4.45b) follows from $\frac{\partial\Omega}{\partial e} < 0$. Consequently,

$$|A| = \underbrace{\frac{\partial F_1}{\partial k}\frac{\partial F_2}{\partial e}}_{=1} - \underbrace{\frac{\partial F_1}{\partial e}\frac{\partial F_2}{\partial k}}_{<0} > 0. \tag{4.46}$$

(a) To determine the effect of a changing population growth rate γ_N on k and e we have to replace X by γ_N in equation (4.42) which gives

$$\frac{\partial F_1}{\partial \gamma^N} = \frac{1}{1-\alpha}\Omega^{1/(1-\alpha)-1}\frac{\partial\Omega}{\partial\gamma^N} < 0 \tag{4.47a}$$
$$\frac{\partial F_2}{\partial \gamma^N} = 0, \tag{4.47b}$$

whereby (4.47a) follows from $\frac{\partial\Omega}{\partial\gamma^N} < 0$, cf. equations (4.40) and (4.27a). To get an intuitive idea what is determining the sign, note that we can write

$$\frac{\partial\Omega}{\partial\gamma^N} = \frac{\partial\varphi/\partial\gamma^N\phi - \varphi\partial\phi/\partial\gamma^N}{\phi^2} = -\frac{\varphi\partial\phi/\partial\gamma^N}{\phi^2} < 0 \tag{4.48}$$

since φ is independent of γ^N and ϕ is a positive function of γ^N, cf. equations (4.26a) and (4.27). Thus, γ^N has a direct effect on k but only an indirect effect on e via changing relative prices (this is the reason why $\partial F_2/\partial\gamma^N = 0$). Formally, we have

$$\frac{\partial k}{\partial \gamma^N} = -|A|^{-1}\left(\underbrace{\frac{\partial F_2}{\partial e}\frac{\partial F_1}{\partial \gamma^N}}_{>0} - \underbrace{\frac{\partial F_1}{\partial e}\frac{\partial F_2}{\partial \gamma^N}}_{=0}\right) < 0 \tag{4.49a}$$
$$\frac{\partial e}{\partial \gamma^N} = -|A|^{-1}\left(-\underbrace{\frac{\partial F_2}{\partial k}\frac{\partial F_1}{\partial \gamma^N}}_{<0} + \underbrace{\frac{\partial F_1}{\partial k}\frac{\partial F_2}{\partial \gamma^N}}_{=0}\right) < 0. \tag{4.49b}$$

(b) To derive the analogous steps for differentiation of (4.39) with respect to s, replace the terms in (4.47) by

$$\frac{\partial F_1}{\partial s} = \frac{1}{1-\alpha}\Omega^{1/(1-\alpha)-1}\frac{\partial \Omega}{\partial s} \gtrless 0 \tag{4.50a}$$

$$\frac{\partial F_2}{\partial s} = ck^{\frac{1-\alpha}{1-\psi}}\frac{\partial \frac{s}{\zeta}}{\partial s} \geq 0. \tag{4.50b}$$

giving

$$\frac{\partial k}{\partial s} = -|A|^{-1}\left(\underbrace{\frac{\partial F_2}{\partial e}\frac{\partial F_1}{\partial s}}_{\gtrless 0} - \underbrace{\frac{\partial F_1}{\partial e}\frac{\partial F_2}{\partial s}}_{\leq 0}\right) \gtrless 0 \tag{4.51a}$$

$$\frac{\partial e}{\partial s} = -|A|^{-1}\left(-\underbrace{\frac{\partial F_2}{\partial k}\frac{\partial F_1}{\partial s}}_{\gtrless 0} + \underbrace{\frac{\partial F_1}{\partial k}\frac{\partial F_2}{\partial s}}_{\leq 0}\right) \gtrless 0. \tag{4.51b}$$

Intuitively, the ambiguity of $\frac{\partial e}{\partial s}$ results from the fact that, holding k constant, e is increasing in s as long as $\lambda > 0$ (direct effect), but the capital stock may increase or decrease in s for given education e. As e increases in k monotonically, the ambiguity of $\frac{\partial k}{\partial s}$ translates into the ambiguity of $\frac{\partial e}{\partial s}$ (indirect effect of s on e).

Arguing formally, the ambiguity of $\frac{\partial k}{\partial s}$ comes from

$$\frac{\partial \Omega}{\partial s} = \frac{\partial(\varphi/\phi)}{\partial s} = \alpha(1-\alpha)\beta(1-\tau)\frac{\varphi'\phi - \varphi\phi'}{\phi^2} \gtrless 0 \tag{4.52}$$

where $\phi' = \partial\phi/\partial s$ and $\varphi' = \partial\varphi/\partial s$, cf. equation (4.26a). It can be shown that $\varphi' > 0$. Consequently, the sign of $\frac{\partial \phi}{\partial s}$ determines the sign of $\frac{\partial F_1}{\partial s}$(and thus $\frac{\partial \Omega}{\partial s}$) and therefore the sign of equation (4.50a) is unambiguous only if $\frac{\partial \phi}{\partial s} < 0$.

To see what determines the sign of ϕ', observe from (4.27a) that s enters in three places: (i) s pre-multiplies the term $\omega\frac{1+g(e)}{1-e}$, (ii) s decreases the effective discount rate $\hat{\rho}$, and (iii) s increases the annuity factor, ζ, as long as $\lambda < 1$. Consequently, ϕ increases in s by effect (i) whereas it decreases in s by the effects (ii) and (iii). We can therefore study an upper bound of ϕ' by setting $\lambda = 1$ so that effect (iii) is not at work.

This helps clarifying the interaction at the cost of introducing a special case. Using $\hat{\rho} = \frac{1}{\beta s} - 1$ in (4.27a) and taking the derivative of the resulting equation with respect to s, gives

$$\frac{\partial \phi}{\partial s}\frac{1}{\gamma_A} = \omega\alpha\frac{1+g(e)}{1-e} - \frac{\gamma^N}{s^2\beta}\Big[1-(1-\alpha)(1-\tau)\Big] \gtrless 0,$$

which is ambiguous.[11] The right part of this equation consists only of exogenous variables. The left part involves the endogenous education decision e for which no closed form solution is available. Thus, it is not possible to show analytically that the derivative has an unambiguous sign. However, constructing a few special cases clarifies under which conditions $\frac{\partial \phi}{\partial s} < 0$ may hold.

- For $\omega \to 0$ the left part converges to zero (e also converges to zero) and thus $\frac{\partial \phi}{\partial s} < 0$.
- For $\omega = 1$, which implies that $\tau = 0$, we have

$$\frac{\partial \phi}{\partial s}\frac{1}{\gamma_A} = \alpha\left(\frac{1+g(e)}{1-e} - \frac{\gamma^N}{s^2\beta}\right) \gtrless 0.$$

- For $\xi \to 0$ or $\psi \to 0$ we have that $e \to 0$ which means that

$$\frac{\partial \phi}{\partial s}\frac{1}{\gamma_A} = \omega\alpha - \frac{\gamma^N}{s^2\beta}\Big[1-(1-\alpha)(1-\tau)\Big] \gtrless 0.$$

Summarizing the arguments made so far, the sign of $\frac{\partial \phi}{\partial s}$ is negative (implying that k is increasing in s) if

- returns to education are low (low ξ and/or ψ)
- the horizon over which the benefits can be reaped is short (low ω)
- the discount factor β is low (i.e. high discount rate)

[11] To see what happens for $\lambda \neq 1$ define $\mu \equiv \varphi/\xi$ and $\mu' \equiv \partial\mu/\partial s$. Then the corresponding term is

$$\frac{\partial \phi}{\partial s}\frac{1}{\gamma^A} = \omega\frac{1+ge(e)}{1-e}\left(\alpha + \mu'\frac{1-\alpha}{\beta}\right) - \frac{\gamma^N}{s^2\beta}(\alpha + (1-\alpha)\mu\tau) + \mu'\frac{\gamma^N(1-\alpha)\tau}{s\beta}$$

where it is obvious that the last two terms are negative ($\mu > 0$ and $\mu' < 0$) but the sign of term in the first bracket is ambiguous again. Thus, by setting $\lambda = 1$ (perfect annuity markets) which implies $\frac{\varphi}{\zeta} = 1$ we know that $\phi'|_{0\leq\lambda<1} < \phi'|_{\lambda=1}$ holds.

- the population growth rate γ^N is high
- and the survival probability s is low.

(c) Changing the planning horizon ω gives

$$\frac{\partial F_1}{\partial \omega} = \frac{\partial \Omega}{\partial \omega} < 0 \tag{4.53a}$$

$$\frac{\partial F_2}{\partial \omega} = \frac{1}{1-\psi}\omega^{\frac{\psi}{1-\psi}} \left(\xi\psi\frac{\gamma^A}{\alpha}\right)^{\frac{1}{1-\psi}} \left(\frac{s}{\zeta}\right)^{\frac{1}{1-\psi}} k^{\frac{1-\alpha}{1-\psi}} > 0. \tag{4.53b}$$

and therefore

$$\frac{\partial k}{\partial \omega} = -|A|^{-1}\left(\underbrace{\frac{\partial F_2}{\partial e}\frac{\partial F_1}{\partial \omega}}_{>0} - \underbrace{\frac{\partial F_1}{\partial e}\frac{\partial F_2}{\partial \omega}}_{<0}\right) < 0 \tag{4.54a}$$

$$\frac{\partial e}{\partial \omega} = -|A|^{-1}\left(-\underbrace{\frac{\partial F_2}{\partial k}\frac{\partial F_1}{\partial \omega}}_{<0} + \underbrace{\frac{\partial F_1}{\partial k}\frac{\partial F_2}{\partial \omega}}_{<0}\right) \gtrless 0. \tag{4.54b}$$

Some intuition why the sign of $\partial e/\partial \omega$ is indeterminate can be gained by writing out (4.54b) and inserting the derivatives from above which gives

$$\frac{\partial e}{\partial \omega} = |A|^{-1}\left(\frac{s}{\zeta}\right)^{\frac{1}{1-\psi}} \frac{k^{\frac{1-\alpha}{1-\psi}}}{1-\psi}\left((1-\alpha)\frac{\partial \Omega}{\partial \omega}k^{-1} + \omega^{-1}\right).$$

Hence, the ambiguity is caused by the negative effect of rising labor market participation on the capital stock ($\partial\Omega/\partial\omega < 0$) and the positive counterbalancing effect of more education (ω^{-1}) due to a higher lifetime labor supply ω.

On the contrary, the reason why the sign of $\partial k/\partial \omega$ can always be determined is that the effect of ω on k and e work into the same direction. Writing out (4.54a) and simplifying yields

$$\frac{\partial k}{\partial \omega} = |A|^{-1}\left(\frac{\partial \Omega}{\partial \omega} - \frac{1}{1-\psi}\omega^{-1}e\right) < 0$$

where $\partial\Omega/\partial\omega < 0$ captures the direct effect of more labor and the second part captures the additional effect of changing education.

2. In case $\varrho = \bar{\varrho}$, there is a direct (d) and an indirect effect in the partial derivatives of Ω, $\frac{\partial \Omega}{\partial X} = (\frac{\partial \Omega}{\partial X})^d + \frac{\partial \Omega}{\partial \tau}\frac{\partial \tau}{\partial X}$. Observe from (4.21) that

$$\frac{\partial \tau}{\partial s} = \frac{\gamma^N \bar{\rho}(1-\omega)}{(s(\bar{\rho}(1-\omega)+\omega)+\gamma^N)^2} > 0 \tag{4.55}$$

$$\frac{\partial \tau}{\partial \gamma^N} = -\frac{s\bar{\rho}(1-\omega)}{(s(\bar{\rho}(1-\omega)+\omega)+\gamma^N)^2} < 0 \tag{4.56}$$

$$\frac{\partial \tau}{\partial \omega} = -\frac{s\bar{\rho}(s+\gamma^N)}{(s(\bar{\rho}(1-\omega)+\omega)+\gamma^N)^2} < 0 \tag{4.57}$$

Therefore, for given γ^N, s, and ω, the strength of the indirect effect increases in $\bar{\varrho}$. Note that changing the adjustment rule of the social security system affects only F_1 because there is no direct effect of τ on the education decision in steady state. Due to the additional indirect effect, it is not possible any more to determine the sign of the derivatives. We can only say, whether the effects become smaller or larger, compared to the $\tau = \bar{\tau}$ case.

(a) The difference between the two social security scenarios if γ^N changes and τ adjusts is given by

$$\frac{\partial F_1}{\partial \gamma^N} = \Omega^{1/(1-\alpha)-1}\alpha\beta\left(\frac{\partial \varphi/\phi}{\partial \gamma^N}(1-\tau) - \frac{\varphi}{\phi}\frac{\partial \tau}{\partial \gamma^N}\right) \tag{4.58}$$

with

$$\frac{\partial \phi}{\partial \gamma^N} = \gamma^A\left(\alpha(2+\hat{\rho}) + \varphi\frac{(1-\alpha)}{\zeta}(1+\hat{\rho})\left(\tau + \gamma^N\frac{\partial \tau}{\partial \gamma^N}\right)\right) > 0 \tag{4.59}$$

where the difference to the $\tau = \bar{\tau}$ scenario is only the term $\gamma^N\frac{\partial \tau}{\partial \gamma^N}$. Using equation (4.56) implies that

$$\left.\frac{\partial F_1}{\partial \gamma^N}\right|_{\rho=\bar{\rho}} > \left.\frac{\partial F_1}{\partial \gamma^N}\right|_{\tau=\bar{\tau}} \tag{4.60}$$

which proves that

$$\left.\frac{\partial k}{\partial \gamma^N}\right|_{\varrho=\bar{\varrho}} > \left.\frac{\partial k}{\partial \gamma^N}\right|_{\tau=\bar{\tau}} \quad \text{and} \quad \left.\frac{\partial e}{\partial \gamma^N}\right|_{\varrho=\bar{\varrho}} > \left.\frac{\partial e}{\partial \gamma^N}\right|_{\tau=\bar{\tau}}. \tag{4.61}$$

(b) To see how changes in the survival rate affect k and e with fixed replacement rate we have to evaluate

$$\frac{\partial F_1}{\partial s} = \Omega^{1/(1-\alpha)-1}\beta\alpha\left(\frac{\partial\varphi/\phi}{\partial s}(1-\tau)-\frac{\varphi}{\phi}\frac{\partial\tau}{\partial s}\right). \tag{4.62}$$

The right part in the parentheses is obviously negative. To obtain the total effect we have to evaluate $\frac{\partial(\varphi/\phi)}{\partial s}$. Since φ does not vary with τ, there is no indirect effect. Thus we again only have to evaluate the change in ϕ including now the change in the contribution rate τ. Again differentiating (4.27a) with respect to s gives

$$\frac{\partial\phi}{\partial s}\frac{1}{\gamma_A} = \omega\alpha\frac{1+g(e)}{1-e}-\frac{\gamma^N}{s^2\beta}\Big[1-(1-\alpha)(1-\tau)\Big]+(1-\alpha)\frac{\gamma^N}{s\beta}\frac{\partial\tau}{\partial s}<0$$

where we see that the derivative is identical to the case with $\tau=\bar{\tau}$ except for the last positive term. Using (4.52) and knowing that $\partial\phi/\partial s$ evaluated with the indirect effect is larger (smaller in absolute value) gives

$$\left.\frac{\partial\varphi/\phi}{\partial s}\right|_{\rho=\bar{\rho}} < \left.\frac{\partial\varphi/\phi}{\partial s}\right|_{\tau=\bar{\tau}} \quad\Rightarrow\quad \left.\frac{\partial F_1}{\partial s}\right|_{\rho=\bar{\rho}} < \left.\frac{\partial F_1}{\partial s}\right|_{\tau=\bar{\tau}} \tag{4.63}$$

which implies that

$$\left.\frac{\partial k}{\partial s}\right|_{\varrho=\bar{\varrho}} < \left.\frac{\partial k}{\partial s}\right|_{\tau=\bar{\tau}} \quad\text{and}\quad \left.\frac{\partial e}{\partial s}\right|_{\varrho=\bar{\varrho}} < \left.\frac{\partial e}{\partial s}\right|_{\tau=\bar{\tau}}. \tag{4.64}$$

(c) Differences between the two social security scenarios if ω changes are given by

$$\frac{\partial F_1}{\partial \omega} = \Omega^{1/(1-\alpha)-1}\beta\alpha\left(\frac{\partial\varphi/\phi}{\partial \omega}(1-\tau)-\frac{\varphi}{\phi}\frac{\partial\tau}{\partial \omega}\right). \tag{4.65}$$

Differentiating equation (4.27a) with respect to ω gives

$$\begin{aligned}\frac{\partial\phi}{\partial\omega} = \gamma^A\Bigg(&\varphi\frac{(1-\alpha)}{\zeta}(1+\hat{\rho})\gamma^N\frac{\partial\tau}{\partial\omega}\\ &+s\left(\alpha(2+\hat{\rho})+\varphi\frac{1-\alpha}{\zeta}(1+\hat{\rho})\right)\frac{1+g(e)}{1-e}\Bigg),\end{aligned} \tag{4.66}$$

where the difference being only the adjusting contribution rate $\frac{\partial \tau}{\partial \omega}$. Using equation (4.57) it holds that

$$\left.\frac{\partial F_1}{\partial \omega}\right|_{\rho=\bar{\rho}} > \left.\frac{\partial F_1}{\partial \omega}\right|_{\tau=\bar{\tau}} \tag{4.67}$$

proving that

$$\left.\frac{\partial k}{\partial \omega}\right|_{\varrho=\bar{\varrho}} > \left.\frac{\partial k}{\partial \omega}\right|_{\tau=\bar{\tau}} \quad \text{and} \quad \left.\frac{\partial e}{\partial \omega}\right|_{\varrho=\bar{\varrho}} > \left.\frac{\partial e}{\partial \omega}\right|_{\tau=\bar{\tau}}. \tag{4.68}$$

□

Proof of proposition 4.5. The effect of the degree of annuitization (λ) on the capital stock and education decision is given by

$$\frac{\partial F_1}{\partial \lambda} = \frac{\partial \Omega}{\partial \lambda} > 0 \tag{4.69a}$$

$$\frac{\partial F_2}{\partial \lambda} = c \cdot s^{\frac{1}{1-\psi}} k^{\frac{1-\alpha}{1-\psi}} \frac{\partial \zeta^{-1}}{\partial \lambda} < 0. \tag{4.69b}$$

Replacing the terms in (4.47) by the ones from above gives

$$\frac{\partial k}{\partial \lambda} = -|A|^{-1} \left(\underbrace{\frac{\partial F_2}{\partial e} \frac{\partial F_1}{\partial \lambda}}_{<0} - \underbrace{\frac{\partial F_1}{\partial e} \frac{\partial F_2}{\partial \lambda}}_{>0} \right) > 0 \tag{4.70a}$$

$$\frac{\partial e}{\partial \lambda} = -|A|^{-1} \left(- \underbrace{\frac{\partial F_2}{\partial k} \frac{\partial F_1}{\partial \lambda}}_{>0} + \underbrace{\frac{\partial F_1}{\partial k} \frac{\partial F_2}{\partial \lambda}}_{>0} \right) \gtrless 0 \tag{4.70b}$$

Qualitatively, changing λ has the same effects in both social security scenarios because the availability of annuity markets does not interact with the adjustment of contribution or replacement rates.

□

4.B Appendix: Numerical Results without Annuity Markets

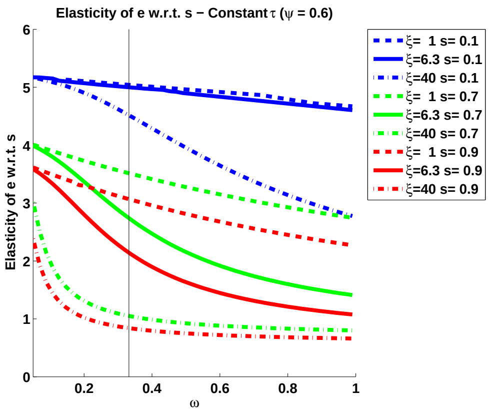
Table 4.3: Calibration parameters: No Annuity Markets

Firm sector	$\psi = 0.6$	$\psi = 0.3$
Capital share, α	0.3	0.3
Technological progress, γ^A	1.81	1.81
Household sector		
Discount factor, β	0.67	0.67
Average productivity of human capital investments, ξ	6.30	1.94
Coefficient in human capital production function, ψ	0.6	0.3
Fraction of the old working, ω	0.33	0.86
Social Security		
Replacement rate, ϱ	0.6	0.6
Demographics		
Birth rate, γ^N	1.56	1.56
Survival rate, s	0.69	0.69

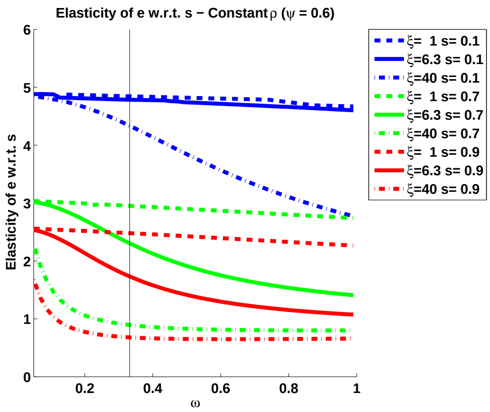
Figure 4.5: Elasticity of e with respect to s: No Annuity Markets

(a) Benchmark concavity ($\psi = 0.6$)

(i) constant τ

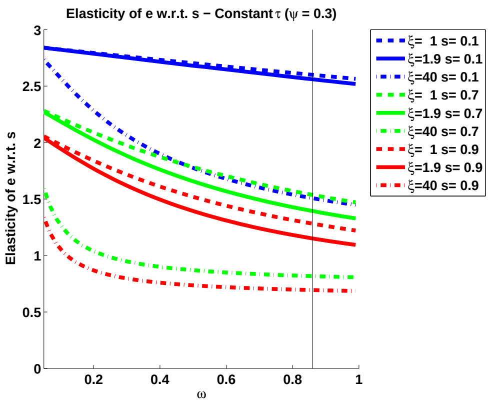


(ii) constant ρ



(b) Low concavity ($\psi = 0.3$)

(i) constant τ



(ii) constant ρ

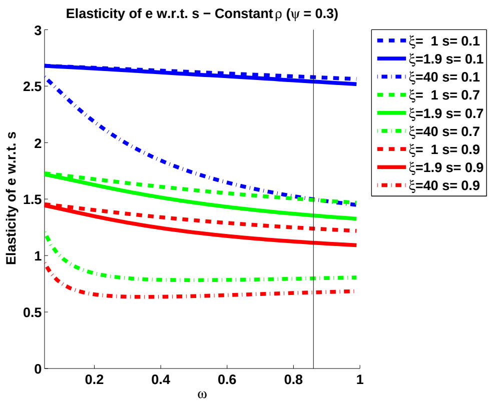
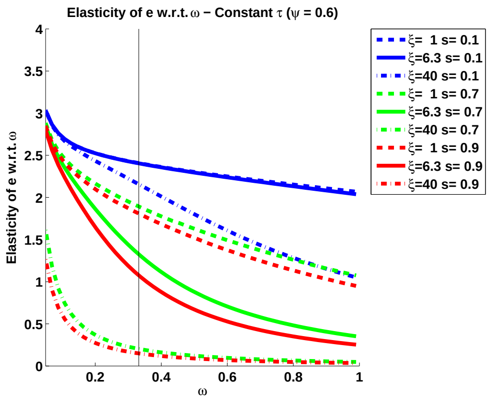
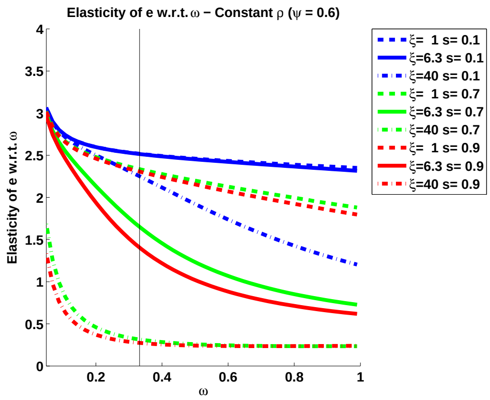


Figure 4.6: Elasticity of e with respect to ω: No Annuity Markets

(a) Benchmark concavity ($\psi = 0.6$)

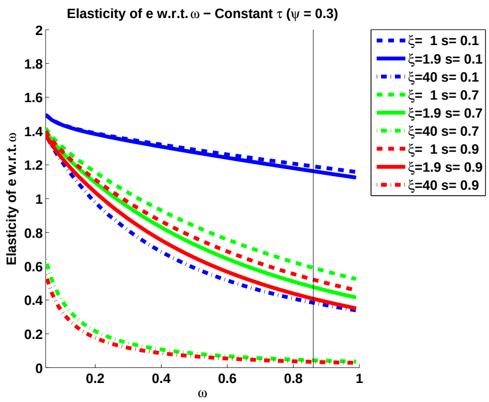
(i) constant τ (ii) constant ρ

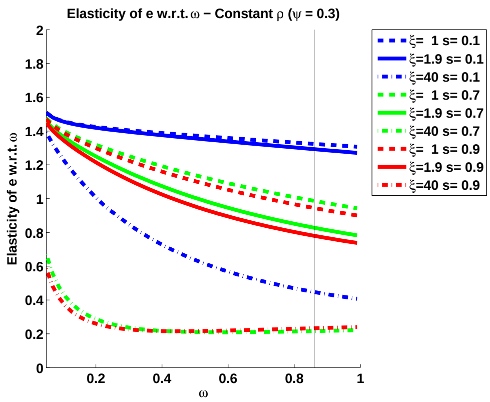




(b) Low concavity ($\psi = 0.3$)

(i) constant τ (ii) constant ρ





Chapter 5

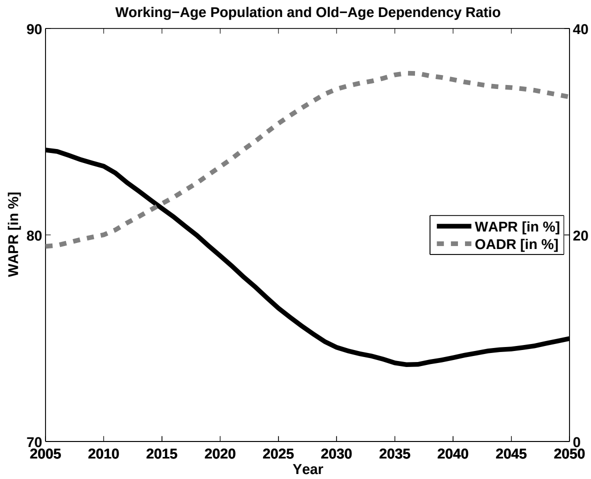
Demographic Change, Human Capital and Welfare

5.1 Introduction

As in all major industrialized countries the population of the United States is aging over time. This process is driven by increasing life-expectancy and a decline in birth rates from the peak levels of the baby boom. Consequently, the fraction of the population in working-age will decrease and the fraction of people in old-age will increase. Figure 5.1 presents two summary measures of these demographic changes: The working-age population ratio is predicted to decrease from 84% in 2005 to 75% in 2050 and the old-age dependency ratio increases from 19% in 2005 to 34% in 2050.

These projected changes in the population structure will have important macroeconomic effects on the balance between physical capital and labor. Specifically, labor is expected to be scarce, relative to physical capital, with an ensuing decline in real returns on physical capital and increases in gross wages. As shown in this paper, a strong incentive to invest in human capital emanates from the combined effects of increasing life expectancy and changes in relative prices particularly if social security systems are reformed so that contribution rates are held constant. In general equilibrium, such endogenous human capital adjustments substantially mitigate the effects of demographic change on macroeconomic aggregates and individual welfare. The key contribution of our paper is to show that the

Figure 5.1: Working age and old-age dependency ratio



Notes: Working age population ratio (WAPR, left scale): ratio of population of age $16-64$ to total adult population of age $16-90$. Old-age dependency ratio (OADR, right scale): ratio of population of age $65-90$ to working age population.
Source: Own calculations based on Human Mortality Database (2008).

human capital adjustment mechanism is quantitatively important.

We add endogenous human capital accumulation to an otherwise standard large-scale OLG model in the spirit of Auerbach and Kotlikoff (1987). The central part of our analysis is then to work out the differences between our model with endogenous human capital adjustments and endogenous labor supply and the "standard" models in the literature with a fixed (exogenous) productivity profile.

We find that as a consequence of demographic change the decrease of the return to physical capital in our model with endogenous human capital is only one third of the decrease in the standard model. Welfare consequences from increasing wages and declines in rates of return can be substantial. Newborns in 2005 experience welfare gains in the order of up to 0.8% of lifetime consumption when contribution rates to the pension system are held constant and welfare losses worth -3% of lifetime consumption when the generosity of the pension system is maintained. In contrast, asset-rich households currently alive lose from the decline in rates of return and these losses can be large depending on the future evolution of the pension system. But importantly, these losses are about $50-70\%$ higher when the human capital adjustment mechanism is shut down. Ignoring this adjustment channel thus leads to quantitatively important biases of the welfare assessment of demographic change.

Our work relates to a vast number of papers that have analyzed the economic consequences of population aging and possible adjustment mechanisms. Important examples in closed economies with a focus on social security adjustments include Huang, Imrohoroglu, and Sargent (1997), De Nardi, Imrohoroglu, and Sargent (1999) and, with respect to migration, Storesletten (2000). In open economies, Bösch-Supan, Ludwig, and Winter (2006), Attanasio, Kitao, and Violante (2007) and Krüger and Ludwig (2007), among others, investigate the role of international capital flows during the demographic transition. We add to this literature by highlighting an additional mechanism through which households can respond to demographic change.

Our paper is closely related to the theoretical work on longevity, human capital, taxation and growth[1] and to Fougère and Mérette (1999) and Sadahiro and Shimasawa (2002) who also investigate demographic change in large-scale OLG models with individual human capital decisions. In contrast to their work, we focus our analysis on the relative price changes during the demographic transition and therefore consider an exogenous growth specification.[2] We also extend their analysis along various dimensions. We use realistic demographic projections instead of stylized scenarios. More importantly, our model contains a labor supply-human capital formation-leisure trade-off. It can thus capture effects from changes in individual labor supply, i.e., human capital utilization, on the return of human capital investments. As has already been stressed by Becker (1967) and Ben-Porath (1967) it is important to model human capital and labor supply decisions jointly in a life-cycle framework. Along this line, a key feature of our quantitative investigation, is to employ a Ben-Porath (1967) human capital model and calibrate it to replicate realistic life-cycle wage profiles.[3] Furthermore, we put particular emphasis on the welfare consequences of

[1]See for example de la Croix and Licandro (1999), Boucekkine, de la Croix, and Licandro (2002), Kalemli-Ozcan, Ryder, and Weil (2000) Echevarria and Iza (2006), Heijdra and Romp (2007) and Ludwig and Vogel (2009). Our paper is also related to a literature emphasizing the role of endogenous human capital accumulation for the analysis of changes to the tax or social security system as in Lord (1989), Trostel (1993), Perroni (1995), Dupor, Lochner, Taber, and Wittekind (1996) and Lau and Poutvaara (2006), among others.

[2]Whether the trend growth rate endogenously fluctuates during the demographic transition or is held constant is of minor importance for the questions we are interested in. This is shown in our earlier unpublished working paper. Results are available upon request.

[3]The Ben-Porath (1967) model of human capital accumulation is one of the workhorses in labor economics used to understand such issues as educational attainment, on-the-job training, and wage growth over the life cycle, among others, see Browning, Hansen, and Heckman (1999) for a review. More re-

population aging for households living through the demographic transition.

The paper is organized as follows. In section 5.2 we present the formal structure of our quantitative model. Section 5.3 describes the calibration strategy and our computational solution method. Our results are presented in section 5.4. Finally, section 5.5 concludes the paper. Additional results, a description of our demographic model, and technical details can be found in appendices 5.B and 5.A.

5.2 The Model

We employ a large scale OLG model à la Auerbach and Kotlikoff (1987) with endogenous labor supply and endogenous human capital formation. The population structure is exogenously determined by time varying demographic processes for fertility and mortality, the main driving forces of our model.[4] Firms produce with a standard constant returns to scale production function in a perfectly competitive environment. We assume that the U.S. is a closed economy.[5] Agents contribute a share of their wage to the pension system and retirees receive a share of current net wages as pensions. Technological progress is exogenous.

5.2.1 Timing, Demographics and Notation

Time is discrete and one period corresponds to one calendar year t. Each year, a new generation is born. Birth in this paper refers to the first time households make own decisions and is set to real life age of 16 (model age $j = 0$). Agents retire at an exogenously given age of 65 (model age $jr = 49$). Agents live at most until age 90 (model age $j = J = 74$). At a given point in time t, individuals of age j survive to age $j + 1$ with probability

cently, extended versions of the model have been applied to study the significant changes to the U.S. wage distribution and inequality observed since the early 1970s by Heckman, Lochner, and Taber (1998) and Guvenen and Kuruscu (2009).

[4]We model neither endogenous life-expectancy or fertility, nor endogenous migration and assume that all exogenous migration is completed before agents start making economically relevant decisions (cf. appendix 5.A).

[5]For our question, the closed economy assumption is a valid approximation. As documented in Krüger and Ludwig (2007), demographically induced changes in the return to physical capital and wages from the U.S. perspective do not differ much between small and open economy scenarios. The reason is that demographic processes are correlated across countries and, in terms of speed of the aging processes, the U.S. is somewhere in the middle when looking at all OECD countries.

$\varphi_{t,j}$, where $\varphi_{t,J} = 0$. The number of agents of age j at time t is denoted by $N_{t,j}$ and $N_t = \sum_{j=0}^{J} N_{t,j}$ is total population in t.

5.2.2 Households

Each household comprises of one representative agent who decides about consumption and saving, labor supply and human capital investment. The household maximizes lifetime utility at the beginning of economic life ($j = 0$) in period t,

$$\max \sum_{j=0}^{J} \beta^j \pi_{t,j} \frac{1}{1-\sigma} \{c_{t+j,j}^{\phi}(1 - \ell_{t+j,j} - e_{t+j,j})^{1-\phi}\}^{1-\sigma}, \quad \sigma > 0, \tag{5.1}$$

where the per period utility function is a function of individual consumption c, labor supply ℓ and time investment into formation of human capital, e. The agent is endowed with one unit of time, so $1 - l - e$ is leisure time. β is the pure time discount factor, ϕ determines the weight of consumption in utility and σ is the inverse of the inter-temporal elasticity of substitution with respect to the Cobb-Douglas aggregate of consumption and leisure time. $\pi_{t,j}$ denotes the (unconditional) probability to survive until age j, $\pi_{t,j} = \prod_{i=0}^{j-1} \varphi_{t+i,i}$, for $j > 0$ and $\pi_{t,0} = 1$.

Agents earn labor income (pension income when retired) as well as interest payments on their savings and receive accidental bequests. When working they pay a fraction τ_t from their gross wages to the social security system. The net wage income in period t of an agent of age j is given by $w_{t,j}^n = \ell_{t,j} h_{t,j} w_t (1-\tau_t)$, where w_t is the (gross) wage per unit of supplied human capital at time t. There are no annuity markets and households leave accidental bequests. These are collected by the government and redistributed in a lump-sum fashion to all households. Accordingly, the dynamic budget constraint is given by

$$a_{t+1,j+1} = \begin{cases} (a_{t,j} + tr_t)(1 + r_t) + w_{t,j}^n - c_{t,j} & \text{if } j < jr \\ (a_{t,j} + tr_t)(1 + r_t) + p_{t,j} - c_{t,j} & \text{if } j \geq jr, \end{cases} \tag{5.2}$$

where $a_{t,j}$ denotes assets, tr_t are transfers from accidental bequests, r_t is the real interest rate, the rate of return to physical capital, and $p_{t,j}$ is pension income. Initial household assets are zero ($a_{t,0} = 0$) and the transversality condition is $a_{t,J+1} = 0$.

5.2.3 Formation of Human Capital

The key element of our model is endogenous formation of human capital. Households enter economic life with a predetermined and cohort invariant level of human capital $h_{t,0} = h_0$. Afterwards, they can invest a fraction of their time into acquiring additional human capital. We adopt a version of the Ben-Porath (1967) human capital technology[6] given by

$$h_{t+1,j+1} = h_{t,j}(1-\delta^h) + \xi(h_{t,j}e_{t,j})^{\psi} \quad \psi \in (0,1),\ \ \xi > 0,\ \ \delta^h \geq 0, \tag{5.3}$$

where ξ is a scaling factor, the average learning ability, ψ determines the curvature of the human capital technology, δ^h is the depreciation rate of human capital and $e_{t,j}$ is time investment into human capital formation.

The costs of investing into human capital in this model are only the opportunity costs of foregone wage income and leisure. We understand the process of accumulating human capital as a mixture of knowledge acquired by formal schooling and on the job training programs after schooling. Human capital can be accumulated until retirement age but agents' optimally chosen time investment converges to zero some time before retirement.

5.2.4 The Pension System

The pension system is a simple balanced budget pay-as-you-go system. Workers contribute a fraction τ_t of their gross wages and pensioners receive a fraction ρ_t of the current average net wages of workers.[7] The level of pensions in each period is then given by $p_{t,j} = \rho_t(1-\tau_t)w_t\bar{h}_t$, where $\bar{h}_t = \frac{\sum_{j=0}^{jr-1}\ell_{t,j}h_{t,j}N_{t,j}}{\sum_{j=0}^{jr-1}\ell_{t,j}N_{t,j}}$ denotes average human capital of workers. Using the formula for $p_{t,j}$, the budget constraint of the pension system[8] simplifies to

$$\tau_t \sum_{j=0}^{jr-1} \ell_{t,j}N_{t,j} = \rho_t(1-\tau_t)\sum_{j=jr}^{J} N_{t,j} \quad \forall t. \tag{5.4}$$

[6] This functional form is widely used in the human capital literature, cf. Browning, Hansen, and Heckman (1999) for a review.

[7] In the U.S. system, pension benefits are linked to individual monthly earnings which are indexed and averaged over the life-cycle Diamond and Gruber (1999). The replacement rate, however, is a decreasing function of monthly earnings so that the earnings related linkage is incomplete. By ignoring this earnings related linkage, we somewhat overstate the distortion of the labor-human capital formation-leisure decision induced by the pension system.

[8] The budget constraint is given by $\tau_t w_t \sum_{j=0}^{jr-1}\ell_{t,j}h_{t,j}N_{t,j} = \sum_{j=jr}^{J} p_{t,j}N_{t,j} \quad \forall t.$

Below, we consider two polar scenarios of parametric adjustment of the pension system to demographic change. In our first scenario ("const. τ"), we hold the contribution rate constant, $\tau_t = \bar{\tau}$, and endogenously adjust the replacement rate to balance the budget of the pension system. In the other extreme scenario ("const. ρ"), we hold the replacement rate constant, $\rho_t = \bar{\rho}$, and endogenously adjust the contribution rate.

5.2.5 Firms and Equilibrium

Firms operate in a perfectly competitive environment and produce one homogenous good according to the Cobb-Douglas production function

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}, \tag{5.5}$$

where α denotes the share of capital used in production. K_t, L_t and A_t are the stocks of physical capital, effective labor and the level of technology, respectively. Output can be either consumed or used as an investment good. We assume that labor inputs and human capital of different agents are perfect substitutes and effective labor input L_t is accordingly given by $L_t = \sum_{j=0}^{jr-1} \ell_{t,j} h_{t,j} N_{t,j}$. Factors of production are paid their marginal products, i.e. $w_t = (1-\alpha)\frac{Y_t}{L_t}$ and $r_t = \alpha\frac{Y_t}{K_t} - \delta_t$, where w_t is the gross wage per unit of efficient labor, r_t is the interest rate and δ_t denotes the depreciation rate of physical capital. Total factor productivity, A_t, is growing at the exogenous rate of g_t^A: $A_{t+1} = A_t(1 + g_t^A)$.
The definition of equilibrium is standard and relegated to appendix 5.A.

5.2.6 Thought Experiments

We take as exogenous driving process a time-varying demographic structure. Computations start in year 1850 ($t = 0$) assuming an artificial initial steady state. We then compute the model equilibrium from 1850 to 2400 ($t = T = 551$) – when the new steady state is assumed and verified to be reached[9] – and report simulation results for the main projection period of interest, from 2005 ($t = 156$) to 2050 ($t = 206$). We use data during our calibration period, 1960 – 2005 (from $t_0 = 111$ to $t_1 = 156$), to determine several structural model parameters (cf. section 5.3).

[9] In fact, changes in variables which are constant in steady state are numerically irrelevant already around the year 2300.

Our main interest is to compare the time paths of aggregate variables and welfare across two model variants for two social security scenarios. Our first model variant is one in which households adjust their human capital and our second variant is one in which human capital is held constant across cohorts. Therefore, our strategy is to first solve for the transitional dynamics using the model as described above. Next, we use the endogenously obtained profile of time investment into human capital formation to compute an average time investment and associated human capital profile which is then fed into our alternative model in which agents are restricted with respect to their time investment choice. We do so separately for the two polar social security scenarios described in subsection 5.2.4. The average time investment is computed as $\bar{e}_j = \frac{1}{t_1 - t_0 + 1} \sum_{t=t_0}^{t_1} e_{t,j}$ for our calibration period ($t_0 = 111$ and $t_1 = 156$). In the alternative model, we then add the constraint $e_{t,j} = \bar{e}_j$. The human capital profile is then obtained from (5.3) by iterating forward on age.[10]

5.3 Calibration and Computation

To calibrate the model, we choose model parameters such that simulated moments match selected moments in NIPA data and the endogenous wage profiles match the empirically observed wage profile in the U.S. during the calibration period $1960 - 2005$.[11] The calibrated parameters are summarized in table 5.1 below.

5.3.1 Demographics

Actual population data from $1950 - 2005$ are taken from the Human Mortality Database (2008). Our demographic projections beyond 2004 are based on a population model that

[10]By imposing the restriction of identical time investment profiles for all cohorts (instead of, e.g., imposing the restriction only on cohorts born after 2005) we shut down direct effects from changing mortality on human capital and indirect anticipation effects of changing returns. This alternative model is a "standard" model of endogenous labor supply and an exogenously given age-specific productivity profile – as used in numerous studies on the consequences of demographic change – with the only exception being that the time endowment is age-specific. By setting time endowment to $1 - \bar{e}_j$ rather than 1 we avoid re-calibration across model variants, further see below.

[11]We perform this moment matching in the endogenous human capital model and the constant contribution rate scenario. We do not re-calibrate model parameters across social security scenarios or for the alternative human capital model because simulated moments do not differ much. Furthermore, we are interested in how our welfare conclusions are affected by imposing various constraints on the model – either through our social security scenarios or by restricting human capital formation – and any parametric change in this comparison would confound our welfare analysis.

is described in detail in appendix 5.A.[12]

5.3.2 Household Behavior

The parameter σ, the inverse of the inter-temporal elasticity of substitution, is set to 2. The time discount factor β is calibrated to match the empirically observed capital-output ratio of 2.8 which requires $\beta = 0.988$. The weight of consumption in the utility function, ϕ, is calibrated such that households spend one third of their time working on average which requires $\phi = 0.411$.

5.3.3 Individual Productivity Profiles

We choose values for the parameters of the human capital production function such that average simulated wage profiles resulting from endogenous human capital formation replicate empirically observed wage profiles. Data for age specific productivity are taken from Huggett, Ventura, and Yaron (2007)[13]. We first normalize $h_0 = 1$, and then determine the values of the structural parameters $\{\xi, \psi, \delta^h\}$ by indirect inference methods (Smith (1993); Gourieroux, Monfort, and Renault (1993)). To this end we run separate regressions of the data and simulated wage profiles on a 3rd-order polynomial in age given by

$$\log w_j = \eta_0 + \eta_1 j + \eta_2 j^2 + \eta_3 j^3 + \epsilon_j. \tag{5.6}$$

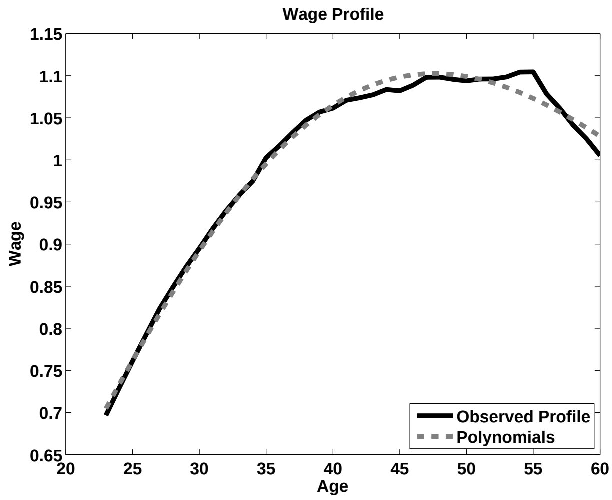
Here, w_j is the age specific productivity. Denote the coefficient vector determining the slope of the polynomial estimated from the actual wage data by $\overrightarrow{\eta} = [\eta_1, \eta_2, \eta_3]'$ and the one estimated from the simulated average human capital profile of cohorts born in $1960 - 2004$ by $\overrightarrow{\hat{\eta}} = [\hat{\eta}_1, \hat{\eta}_2, \hat{\eta}_3]'$. The latter coefficient vector is a function of the structural model parameters $\{\xi, \psi, \delta^h\}$. Finally, the values of our structural model parameters are determined by minimizing the distance $\| \overrightarrow{\eta} - \overrightarrow{\hat{\eta}} \|$, see subsection 5.3.6 for further details.

[12] The key assumptions of this model are as follows: First, the total fertility rate is constant at 2005 levels of 2.0185 until 2100 when fertility is adjusted slightly such as to keep the number of newborns constant for the remainder of the simulation period. Second, life expectancy monotonically increases from a current (2004) average life expectancy at birth of 77.06 years to 88.42 years in 2100 when it is held constant. Third, total migration is constant at the average migration for $1950 - 2005$ for the remainder of the simulation period. These assumptions imply that a stationary population is reached in about 2200.

[13] We thank Mark Huggett for sending us the data.

Figure 5.2 presents the empirically observed productivity profile and the estimated polynomials. Our coefficients[14] and the shape of the wage profile are in line with others reported in the literature, especially with those obtained by Hansen (1993) and Altig, Auerbach, Kotlikoff, Smetters, and Walliser (2001). The estimate of $\delta^h = 0.011$ is reasonable (Arrazola and de Hevia (2004), Browning, Hansen, and Heckman (1999)), and the estimate of $\psi = 0.67$ is just in the middle of the range reported in Browning, Hansen, and Heckman (1999).[15]

Figure 5.2: Wage Profiles



Notes: Observed profile: average life-cycle wage profiles taken from Huggett, Ventura, and Yaron (2007). Polynomials: predicted wage profile based on estimated polynomial coefficients of (5.6). Both profiles were normalized by their respective means.

5.3.4 Production

We calibrate the capital share in production, α, to match the income share of labor in the data which requires $\alpha = 0.33$. The average growth rate of total factor productivity, $\bar{g}^A$, is

[14]The coefficient estimates from the data are η_0: -1.6262, η_1: 0.1054, η_2: -0.0017 and η_3: 7.83e-06. We do not display the calibrated profiles in figure 5.2 because they perfectly track the polynomial obtained from the data.

[15]In a sensitivity analysis we have shown that the estimate of the average time investment productivity, $\xi = 0.16$, depends on the predetermined value of h_0, whereas the other parameters are rather insensitive to this choice. We have also found that parameterizations with a different value for h_0 yield the same results for the effects of demographic change on aggregate variables and welfare.

calibrated to match the growth rate of the Solow residual in the data. Accordingly, $\bar{g}^A = 0.018$. Finally, we calibrate $\bar{\delta}$ (and thereby scale the exogenous time path of depreciation, δ_t) such that our simulated data match an average investment output ratio of 20% which requires $\bar{\delta} = 0.039$.

5.3.5 The Pension System

In our first social security scenario ("const. τ") we fix contribution rates and adjust replacement rates of the pension system. We calculate contribution rates from NIPA data for $1960-2004$ and freeze the contribution rate at the year 2004 level for all following years. When simulating the alternative social security scenario with constant replacement rates ("const. ρ") we feed the equilibrium replacement rate obtained in the "const. τ" scenario into the model and hold it constant at the 2004 level for all the remaining years. Then, the contribution rate endogenously adjusts to balance the budget of the social security system.

5.3.6 Computational Method

For a given set of structural model parameters, solution of the model is by outer and inner loop iterations. On the aggregate level (outer loop), the model is solved by guessing initial time paths of four variables: the capital intensity, the ratio of bequests to wages, the replacement rate (or contribution rate) of the pension system and the average human capital stock for all periods from $t = 0$ until T. On the individual level (inner loop), we start each iteration by guessing the terminal values for consumption and human capital. Then we proceed by backward induction and iterate over these terminal values until convergence of the inner loop iterations.[16] In each outer loop, disaggregated variables are aggregated each period. We then update aggregate variables until convergence using the Gauss-Seidel-Quasi-Newton algorithm suggested in Ludwig (2007).

To calibrate the model in the "const. τ" scenario, we consider additional "outer outer" loops to determine structural model parameters by minimizing the distance between the simulated average values and their respective calibration targets for the calibration period

[16] As described in appendix 5.B, we implement a check for uniqueness of the solution at each age when computing optimal human capital decisions.

$1960 - 2004$. To summarize the description above, parameter values determined in this way are β, ϕ, δ, ξ, ψ and δ^h.

Table 5.1: Model Parameters

Preferences	σ	Inverse of Inter-Temporal Elasticity of Substitution	2.00
	β	Pure Time Discount Factor	0.988
	ϕ	Weight of Consumption	0.411
Human Capital	ξ	Scaling Factor	0.16
	ψ	Curvature Parameter	0.67
	δ^h	Depreciation Rate of Human Capital	1.1%
	h_0	Initial Human Capital Endowment	1.00
Production	α	Share of Physical Capital in Production	0.33
	$\bar{\delta}$	Depreciation Rate of Physical Capital	3.9%
	$\bar{g}^A$	Exogenous Growth Rate	1.8%

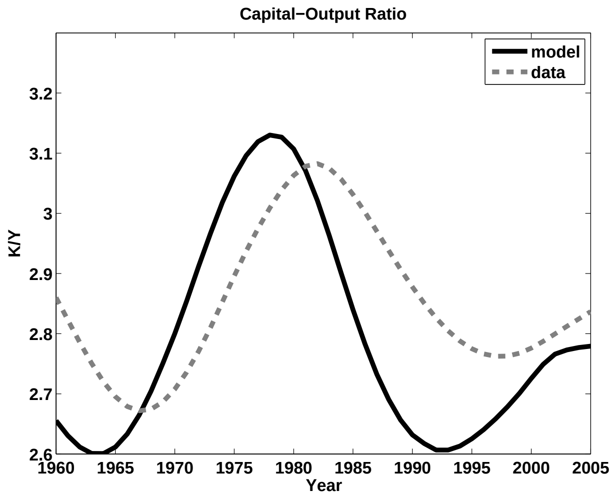
5.4 Results

Before using our model to investigate the effects of future demographic change, we show how well it can replicate observed aggregate variables and individual life-cycle profiles in the past. Next, we turn to the analysis of the transitional dynamics for the period 2005 to 2050 whereby we focus especially on the developments of major macroeconomic variables and the welfare effects of demographic change.

5.4.1 Backfitting

In order to backfit our model we do the following. First, we estimate series of TFP and actual depreciation using NIPA data. We HP filter these data series and then feed them into the model for the period 1950 to 2005. Thereafter, both parameters, g and δ are held constant at their respective means, see table 5.1. A key variable that determines paths of the rate of return to physical capital and wages is the capital output ratio. Figure 5.3 shows actual and fitted values for the period 1960-2005. Evidently, the fit of our model is quite remarkable along this key dimension of the data. Our model tracks the observed long-run swings in the data. The predicted amplitudes are slightly bigger in the model than they are in the data.

Figure 5.3: Capital Output Ratio

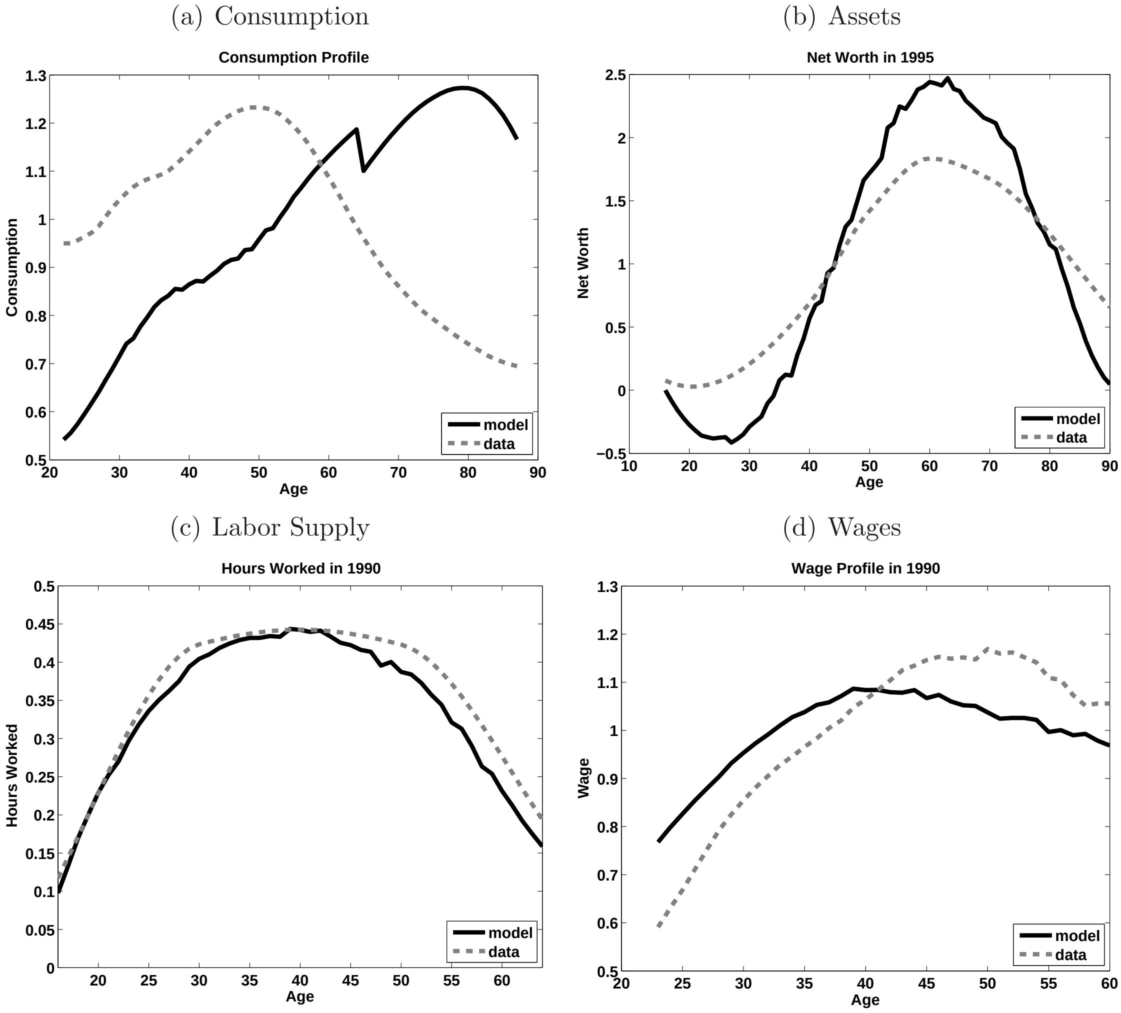


Notes: Capital-output ratio in the model and in aggregate data. The data is HP filtered.
Data Source: National income and product accounts (NIPA).

Turning to the individual level, we recognize that our model fails to replicate the empirically observed life-cycle consumption profile, cf. figure 5.4(a). The increase of consumption over the life cycle is too steep and the peak is too late compared to the data. Since in a model without idiosyncratic risk the decrease of consumption after the peak is solely caused by falling survival rates, we cannot expect to match this dimension of the data (cf. Hansen and Imrohoroğlu (2008), Fernández-Villaverde and Krüger (2007)). As shown in Ludwig, Krüger, and Börsch-Supan (2007), omitting idiosyncratic risk has only a negligible effect on welfare calculations. This is because welfare calculations are based on differences in consumption profiles and the exact shape of the consumption profile is therefore less important.

We next look at asset profiles. Figure 5.4(b) shows household net worth data from the Survey of Consumer Finances for a cross-section in 1995 obtained from Bucks, Kennickell, and Moore (2006) and the corresponding cross-sectional asset profile in the model. Our model matches the broad pattern in the data. Observed discrepancies are threefold: First, as borrowing constraints are absent from our model, initial assets are negative whereas they are positive in the data. Second, the run-up of wealth until retirement age is stronger

Figure 5.4: Life-Cycle and Cross-Sectional Profiles



Notes: Model and data profiles for consumption, assets, labor supply and wages. The model consumption profile is the life-cycle consumption profile for the cohort born in 1960. The other profiles are cross-sectional profiles in 1990 and 1995. Consumption, asset and wage profiles are normalized by their respective mean. Hours data is normalized by 76 total hours per week.
Data Sources: Based on consumption profile estimated by Fernández-Villaverde and Krüger (2007), SCF net worth data obtained from Bucks, Kennickell, and Moore (2006), hours worked data from McGrattan and Rogerson (2004) and PSID wage data.

in our model than it is in the data. Third, decumulation of assets is stronger as well. This last fact is due to the fact that our model neither has health risks as in De Nardi, French, and Jones (2009) nor explicit bequest motives.

Our model does a good job in matching the cross-sectional hours profile observed in 1990 Census data taken from McGrattan and Rogerson (2004), see figure 5.4(c).[17] We relegate a discussion of Frisch labor supply elasticities to appendix 5.A.

Figure 5.4(d) shows the cross-sectional wage profile observed in PSID data in 1990.[18] Although our model matches the broad pattern observed in the data, the fit is much better in 1970 and 1980, cf. appendix 5.A.[19]

5.4.2 Transitional Dynamics

We divide our analysis of the transitional dynamics into two parts. First, we analyze the behavior of several important aggregate variables from 2005 to 2050. Second, we investigate the welfare consequences of demographic change for generations already alive in 2005 and for households born in the future. Throughout, we demonstrate how the design of the social security system affects our results.

Aggregate Variables

The evolution of the policy variables in the two social security scenarios are presented in figure 5.5.[20] In the "const. τ" scenario pensions become less generous over time represented by a decrease in the replacement rate from around 24% in 2005 to 14% in 2050. In contrast, in the "const. ρ" scenario the generosity of the pension system remains at the 2005 level implying that contribution rates have to increase from around 12% in 2005 to 19% in 2050.

Figure 5.6 reports the dynamics of four major macroeconomic variables for the two model variants – with endogenous and exogenous human capital – in the "const. τ" social security scenario and figure 5.7 does so in the "const. ρ" scenario.

In figures 5.6(a) and 5.7(a) we show the evolution of the rate of return to physical capital

[17]The hours data is normalized with total hours per week equal to 76. This might appear to be a low number for total available hours. But such a magnitude is needed to make the McGrattan and Rogerson (2004) hours data broadly consistent with the common belief that agents spend about one third of their time working and standard practice of macroeconomists to calibrate their models (which we have followed).

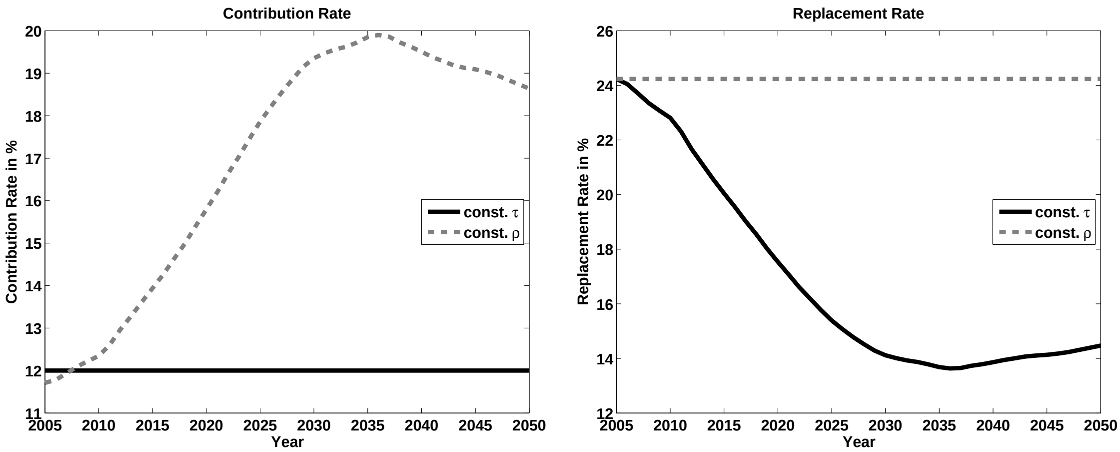
[18]In order to smooth the data we show a centered average of five subsequent PSID samples.

[19]Part of this is due to the fortunate cohorts born after the war (see Ehrlich (2007) for a discussion).

[20]Figure drawn for the endogenous human capital model. The policy variables in the exogenous human capital model are similar.

Figure 5.5: Evolution of Policy Variables

(a) Contribution Rate (b) Replacement Rate



Notes: Pension system contribution and replacement rate for the two social security scenarios. "const. τ": constant contribution rate scenario. "const. ρ": constant replcement rate scenario.

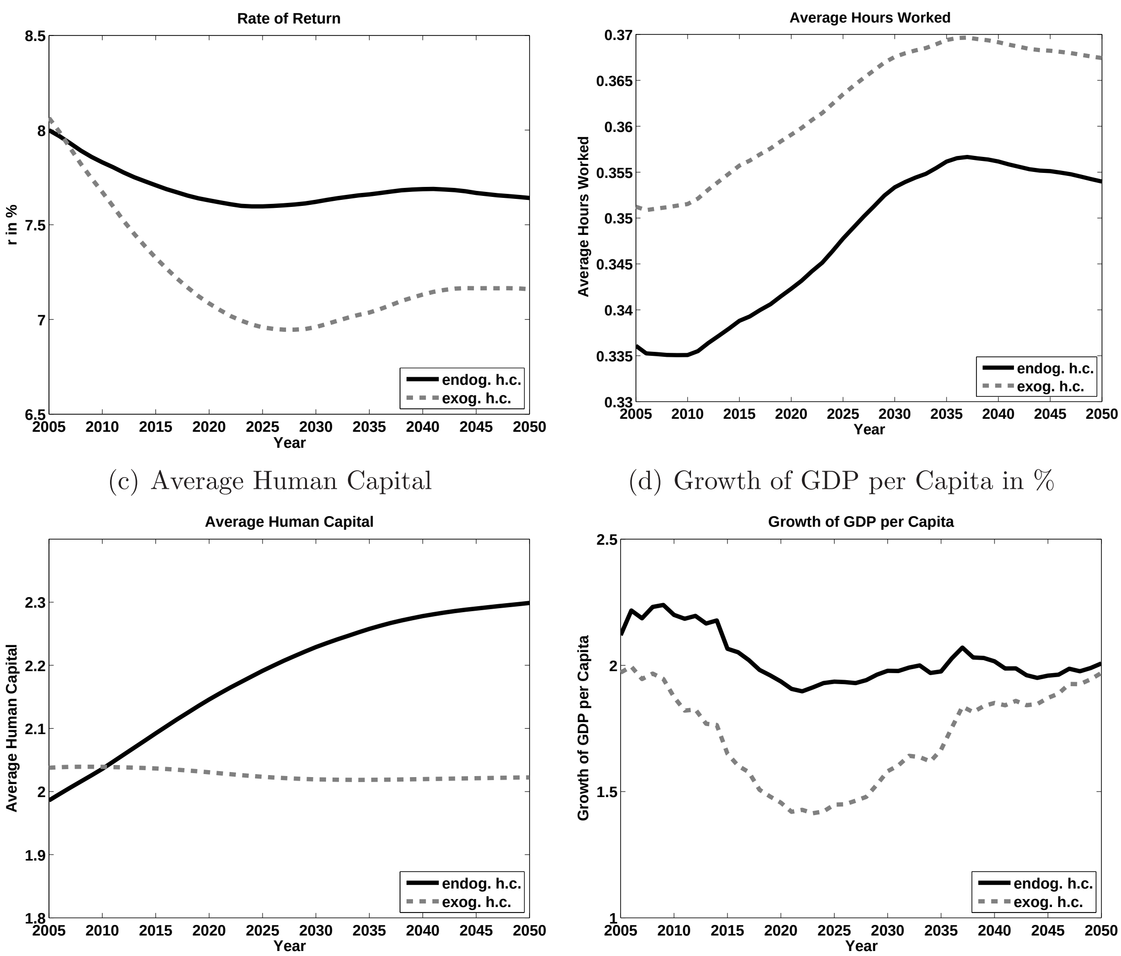
for the different models.[21] In the "standard" models with endogenous labor supply only, the rate of return decreases from an initial level of around 8% in 2005 to 7.1% in the "const. τ" scenario and to 7.5% in the "const. ρ" scenario in 2050.[22] This magnitude is in line with results reported elsewhere in the literature, cf., e.g., Bösch-Supan, Ludwig, and Winter (2006) and Krüger and Ludwig (2007) whereas Attanasio, Kitao, and Violante (2007) find slightly bigger effects. On the contrary, in the two models with endogenous human capital adjustment, the rate of return is expected to fall by only 0.3 (0.1) percentage points in the "const. τ" ("const. ρ") scenario. This difference in the decrease of the rate of return between the exogenous and the endogenous human capital models is large, at a factor of about 3 (4.5).

In figure 5.6(b) and 5.7(b) we depict the evolution of average hours worked by all working age individuals. Average hours worked increase both for the endogenous and exogenous human capital models. Observe that there are level differences between the two model

[21]There are two reasons for the small level differences in 2005 across the various scenarios. First, our calibration targets are averages for the period 1960 − 2004. Second, as already discussed in section 5.3, we do not recalibrate across scenarios. Such level differences in initial values can be observed in all of the following figures.

[22]The high initial level of the rate of return is caused by the baby boom in the past which increases the labor force and hence decreases capital intensity.

Figure 5.6: Aggregate Variables for Constant Contribution Rate Scenario



(a) Rate of Return to Physical Capital

(b) Average Hours Worked

(c) Average Human Capital

(d) Growth of GDP per Capita in %

Notes: Rate of return to physical capital, average hours worked of the working age population, average human capital per working hour and growth of GDP per capita in the constant contribution rate social security scenario for two model variants. "endog. h.c.": endogenous human capital model. "exog. h.c.": exogenous human capital model.

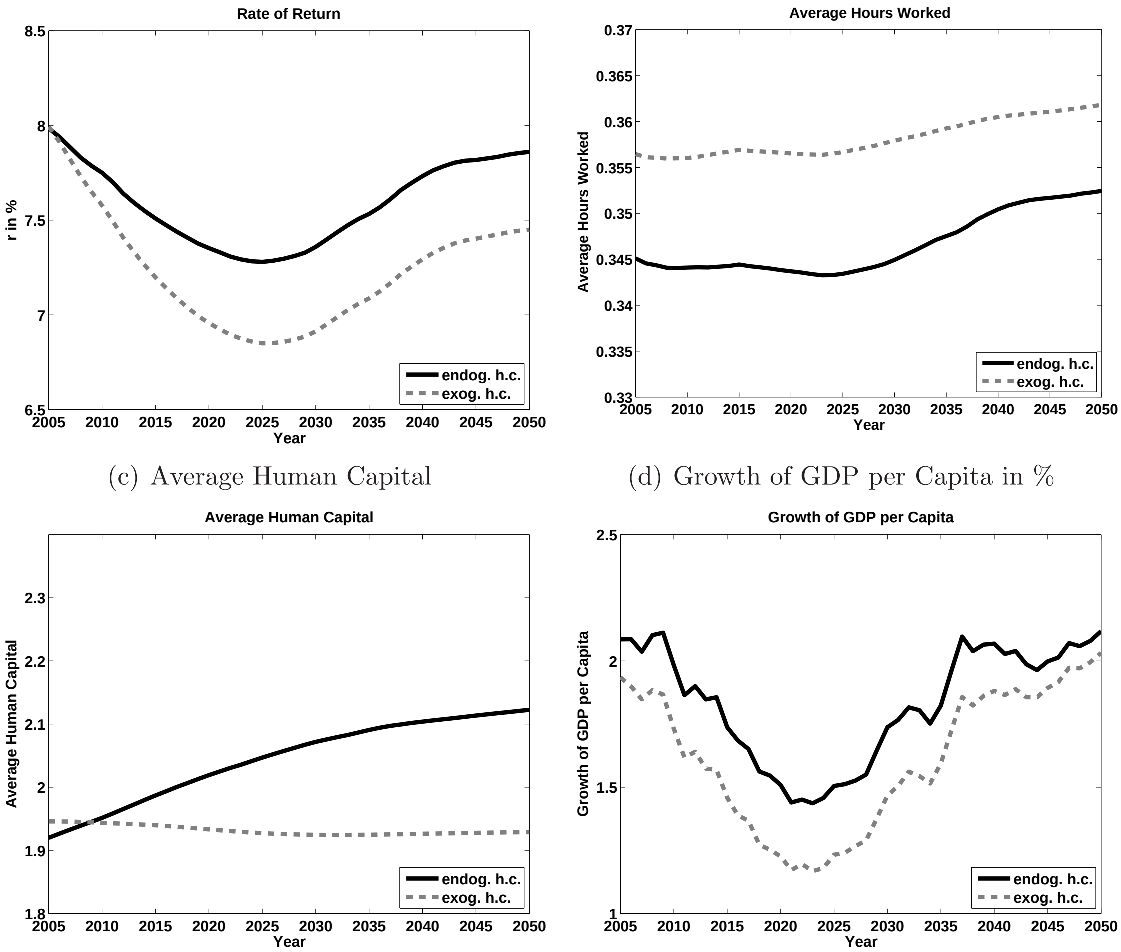
variants. This is mainly caused by differences in time investment into human capital formation.

Figures 5.6(c) and 5.7(c) show that time investment into human capital formation increases when agents are allowed to adjust their human capital. Specifically, with endogenous human capital in the "const. τ" ("const. ρ") scenario average human capital per working hour increases by around 15% (10%) until 2050.

Figure 5.7: Aggregate Variables for Constant Replacement Rate Scenario

(a) Rate of Return to Physical Capital (b) Average Hours Worked

(c) Average Human Capital (d) Growth of GDP per Capita in %



Notes: Rate of return to physical capital, average hours worked of the working age population, average human capital per working hour and growth of GDP per capita in the constant replacement rate social security scenario for two model variants. “endog. h.c.”: endogenous human capital model. “exog. h.c.”: exogenous human capital model.

Finally, we focus on the evolution of the growth rate of GDP per capita as shown in figures 5.6(d) and 5.7(d). When the U.S. aging process peaks in 2025 (cf. figure 5.1), the growth rate of per capita GDP falls in all scenarios to its lowest level. The drop is least pronounced for the endogenous human capital model with a fixed contribution rate. There, the growth rate gradually declines from 2.2% in 2005 to 1.9% in 2025.[23] Comparing the

[23]The high initial growth rate is a consequence of the past baby boom, cf. footnote 22.

two "const. τ" scenarios, it can be seen that not adjusting the human capital profile entails a big drop in the growth rate. The maximum difference in about 2025 is 0.5 percentage points. Although the difference across human capital models is only 0.3 percentage points in case the replacement rate is held constant ("const. ρ" scenarios), the same conclusion applies. The ageing process induces relative price changes so that agents increase their time investment into human capital formation and thereby cushion the negative effects of demographic change on growth.[24]

Welfare Effects

In our model, a household's welfare is affected by two consequences of demographic change. First, her lifetime utility changes because her own survival probabilities increase. Second, households face a path of declining interest rates, increasing gross wages and decreasing replacement rates (increasing contribution rates), relative to the situation without a demographic transition.

We want to isolate the welfare consequences of the second effect. To this end, we compare for an agent born at time t and of current age j her lifetime utility when she faces equilibrium factor prices, transfers and contribution (replacement) rates as documented in the previous section, with her lifetime utility when she instead faces prices, transfers and contribution (replacement) rates that are held constant at their 2005 value. For both of these scenarios we fix the households' individual survival probabilities at their 2005 values.[25] Following Attanasio, Kitao, and Violante (2007) and Krüger and Ludwig (2007), we then compute the consumption equivalent variation $g_{t,j}$, i.e. the percentage increase in consumption that needs to be given to an agent with characteristics t, j at each date in her remaining lifetime at fixed prices to make her as well off as under the situation with changing prices.[26] Positive numbers of $g_{t,j}$ thus indicate that households obtain welfare

[24]In appendix 5.A we show that the cumulative effect of these growth rate differences between the endogenous and exogenous human capital model on the level of GDP per capita are large. With human capital adjustments the detrended level of GDP per capita will increase by around 15% (10%) more until 2050 in the "const. τ" ("const. ρ") scenario than without these adjustments.

[25]Of course, they fully retain their age-dependency. We show in appendix 5.A that varying the survival probabilities according to the underlying demographic projections leaves our conclusions on welfare in the comparison across the two models essentially unchanged.

[26]With our assumptions on preferences, $g_{t,j}$ can be calculated as $g_{t,j} = \left(\frac{\bar{V}_{t,j}}{\bar{V}_j^{2005}}\right)^{\frac{1}{\phi(1-\sigma)}} - 1$, where $\bar{V}_{t,j}$

gains from the general equilibrium effects of demographic change, negative numbers are welfare losses.

Welfare of Generations Alive in 2005

Of particular interest is how the welfare of all generations already alive in 2005 will be affected by demographic change. This analysis allows for an inter-generational welfare comparison of the consequences of demographic change in terms of wellbeing that is not possible using aggregated figures such as per capita GDP. Newborns and young generations benefit from increasing wages as well as decreasing returns if they borrow to finance their human capital formation. However, older – and thus asset-rich – generations are expected to lose lifetime utility: First, they benefit less from increasing wages because they do not significantly adjust their human capital and because their remaining working period is short, second, falling returns diminish their capital income and, third, retirement income decreases in our scenario with constant contribution rates.

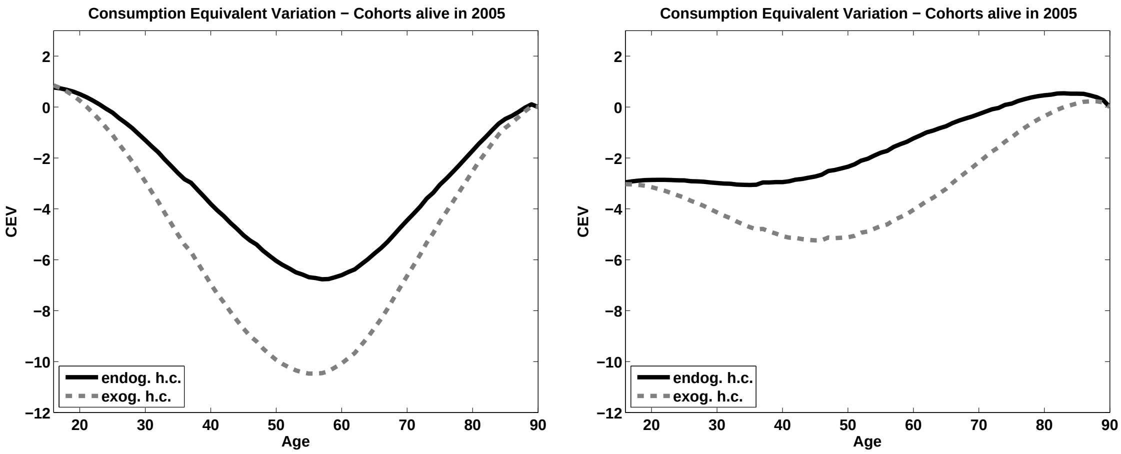
Results, shown in figure 5.8, can be summarized as follows: First, newborn agents experience welfare gains in the "const. τ" scenarios of roughly 1% of life-time consumption and welfare losses of roughly 3% in the "const. ρ" scenarios. As explained appendix 5.A, the fact that these gains (and losses) are almost identical in the two human capital models is due to a complex interaction between the value of human capital adjustments which is positive and differential general equilibrium effects which partially offset this. Second, middle-aged agents incur the highest losses in the "const. τ" scenarios: the maximum loss of agents is much larger compared to a scenario with fixed replacement rates. Clearly, constant replacement rates decrease net wages of the young but keep pensions more generous. This decreases lifetime utility of the young but narrows the loss of utility of the old (compared to a situation with falling replacement rates). The redistribution through the pension system shifts the balance somewhat in favor of the old. This also explains why the maximum of the losses occurs at a much higher age in the "const. τ" scenario in which agents close to retirement lose interest income *and* receive lower pensions. Third, independent of future pension policy, agents lose relatively less in the endogenous human capital model. Younger agents can adjust their human capital in response to higher wages

denotes lifetime utility at changing prices and $\bar{V}_j^{2005}$ at fixed 2005 prices.

whereas older (asset-rich) households benefit from a smaller drop in the interest rate (cf. figures 5.6(a) and 5.7(a)) and higher pension payments.[27]

Figure 5.8: Consumption Equivalent Variation of Agents alive in 2005

(a) Constant Contribution Rate Scenario (b) Constant Replacement Rate Scenario



Notes: Consumption equivalent variation (CEV) in the two social security scenarios.

Table 5.2 finally provides numbers on the maximum welfare loss displayed in figure 5.8 as a summary statistic. It is important to emphasize that, in the exogenous human capital model, the maximum loss is about 3.7 (2.1) percentage points or 55% (71%) higher in the "const. τ" ("const. ρ") scenario than in the endogenous human capital model. This exemplifies that ignoring the adjustment channel through human capital formation leads to quantitatively important biases of the welfare assessments of demographic change.

Table 5.2: Maximum Utility Loss for Generations alive in 2005

	Human Capital	
	Endogenous	Exogenous
Const. τ ($\tau_t = \bar{\tau}$)	-6.8%	-10.5%
Const. ρ ($\rho_t = \bar{\rho}$)	-3.1%	-5.2%

[27]In appendix 5.A, we decompose the welfare differences between the two models into effects stemming from differential changes in factor prices and the relative rise in social security benefits which is caused by additional human capital formation.

Welfare of Future Generations

We next look at the welfare consequences for all future newborns. Due to increasing wages, agents born into a "const τ"-world with endogenous human capital experience gains of lifetime utility throughout the entire projection window. Agents with exogenous human capital born after 2035 incur utility losses of up to 1% of lifetime consumption. However, welfare losses for future generations can be quite large despite the human capital channel if the social security system is not reformed ("const ρ"). Despite of human capital adjustments, they are at about -7% of lifetime consumption for cohorts born around and after 2030 (-8% for exogenous human capital).[28]

5.5 Conclusion

This paper finds that increased investments in human capital may substantially mitigate the macroeconomic impact of demographic change with profound implications for individual welfare. As labor will be relatively scarce and capital will be relatively abundant in an aging society, interest rates will fall. As we emphasize, these effects will be much smaller once we account for changes in human capital formation. For the U.S., our simulations predict that if contribution rates (replacement rates) are kept constant then the rate of return will fall by only 0.4 (0.7) percentage points until 2025 with endogenous human capital, compared to 1.1 (1.1) percentage points in the standard model with a fixed human capital profile.

We also document that the welfare consequences from the increase in wages and declines in rates of return can be substantial, in the order of up to 0.8% (-3%) with constant contribution (replacement) rates in lifetime consumption for newborns in 2005. Thus, welfare gains for newborns only come along if social security contribution rates are held constant at current levels. Households that have already accumulated assets, on the other hand, lose from the decline in rates of return. Importantly, we find that our model with exogenous human capital overstates these losses by $50 - 70\%$.

However we have operated in a frictionless environment where all endogenous human capital adjustments are driven by relative price changes. If instead human capital formation

[28]See graphs in appendix 5.A for more details.

is characterized by substantial market failures then these automatic adjustments will be inhibited. In this case appropriate education and training policies in aging societies are an important topic for future research and the policy agenda.

5.A Appendix: General Remarks

This appendix contains additional that could is not included in the main text due to space limitations. The appendix is organized as follows. Section 5.A.1 contains the formal equilibrium definition, sections 5.A.2 to 5.A.5 provide more results on the fit of our model to observed life-cycle profiles of hours and wages, the implied labor supply elasticities of our model, additional results on predicted aggregate variables during the demographic transition as well as the associated welfare effects and our model to predict the future population structure.

5.A.1 Equilibrium

Denoting current period/age variables by x and next period/age variables by x', a household of age j solves, at the beginning of period t, the maximization problem

$$V(a,h,t,j) = \max_{c,\ell,e,a',h'} \{u(c, 1-\ell-e) + \varphi\beta V(a',h',t+1,j+1)\} \tag{5.7}$$

subject to $w^n_{t,j} = \ell_{t,j} h_{t,j} w_t (1-\tau_t)$, (5.2), (5.3) and the constraints $\ell \in [0,1)$, $e \in [0,1)$.

Definition 3. *Given the exogenous population distribution and survival rates in all periods $\{\{N_{t,j}, \varphi_{t,j}\}_{j=0}^J\}_{t=0}^T$, an initial physical capital stock and an initial level of average human capital $\{K_0, \bar{h}_0\}$, and an initial distribution of assets and human capital $\{a_{t,0}, h_{t,0}\}_{j=0}^J$, a competitive equilibrium are sequences of individual variables $\{\{c_{t,j}, \ell_{t,j}, e_{t,j}, a_{t+1,j+1}, h_{t+1,j+1}\}_{j=0}^J\}_{t=0}^T$, sequences of aggregate variables $\{L_t, K_{t+1}, Y_t\}_{t=0}^T$, government policies $\{\rho_t, \tau_t\}_{t=0}^T$, prices $\{w_t, r_t\}_{t=0}^T$, and transfers $\{tr_t\}_{t=0}^T$ such that*

1. *given prices, bequests and initial conditions, households solve their maximization problem as described above,*
2. *interest rates and wages are paid their marginal products, i.e. $w_t = (1-\alpha)\frac{Y_t}{L_t}$ and $r_t = \alpha\frac{Y_t}{K_t} - \delta$,*
3. *per capita transfers are determined by*

$$tr_t = \frac{\sum_{j=0}^J a_{t,j}(1-\varphi_{t-1,j-1})N_{t-1,j-1}}{\sum_{j=0}^J N_{t,j}}, \tag{5.8}$$

4. *government policies are such that the budget of the social security system is balanced every period, i.e. equation (5.4) holds $\forall t$, and household pension income is given by $p_{t,j} = \rho_t(1-\tau_t)w_t\bar{h}_t$,*

5. *markets clear every period:*

$$L_t = \sum_{j=0}^{jr-1} \ell_{t,j} h_{t,j} N_{t,j} \tag{5.9a}$$

$$K_{t+1} = \sum_{j=0}^{J} a_{t+1,j+1} N_{t,j} \tag{5.9b}$$

$$Y_t = \sum_{j=0}^{J} c_{t,j} N_{t,j} + K_{t+1} - (1-\delta) K_t. \tag{5.9c}$$

Definition 4. *A stationary equilibrium is a competitive equilibrium in which per capita variables grow at the constant rate $1 + \bar{g}^A$ and aggregate variables grow at the constant rate $(1 + \bar{g}^A)(1 + n)$.*

5.A.2 Backfitting

Figure 5.9 presents the fit of our model to cross-sectional hours data from McGrattan and Rogerson (2004) for the years 1970, 1980, 1990 and 2000. We observe that our model does a very good job in matching the data along this dimension from 1980 onwards.

A comparison between wage profiles observed in PSID data and the model is shown in Figure 5.10. The fit of our model is very good in 1970 and 1980 and still broadly consistent with the data in 1990 and 2000.

5.A.3 Labor Supply Elasticities

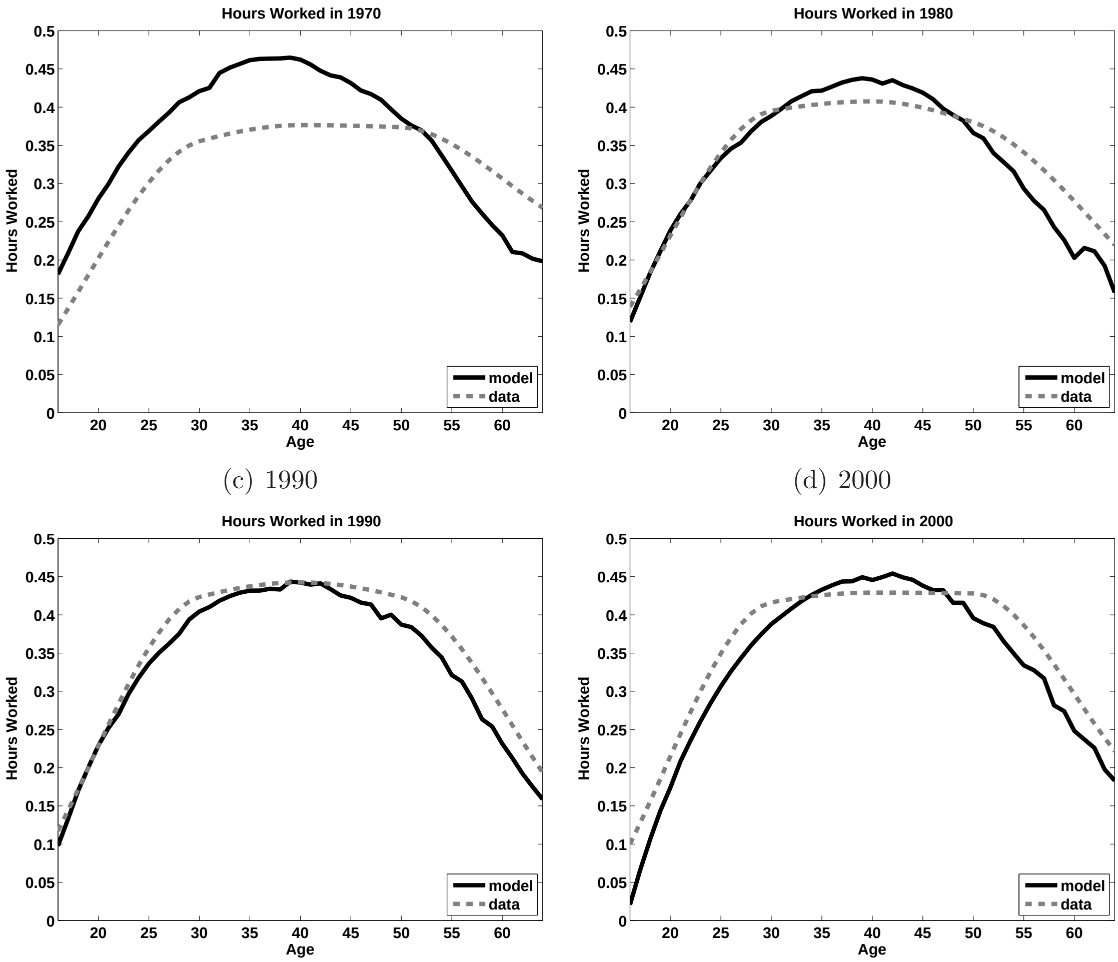
Since agents' human capital investments do not only depend on changes in relative returns but also on the extent of labor supply adjustments, realistic labor supply elasticities are key for our analysis. First, we compute the Frisch (or λ-constant) elasticity of labor supply that holds the marginal utility of wealth constant. We do so using the standard formula. In the context of our model this means holding time investment into human capital formation constant. It is then given by

$$\epsilon^j_{\ell,w} = \frac{1 - \phi(1-\sigma)}{\sigma} \frac{1 - \ell_j - e_j}{\ell_j}, \tag{5.10}$$

see Browning, Hansen, and Heckman (1999) for a derivation. In our model the Frisch elasticity depends on the amount of leisure and labor supply and therefore is age-dependent.

Figure 5.9: Labor Supply

(a) 1970 (b) 1980 (c) 1990 (d) 2000



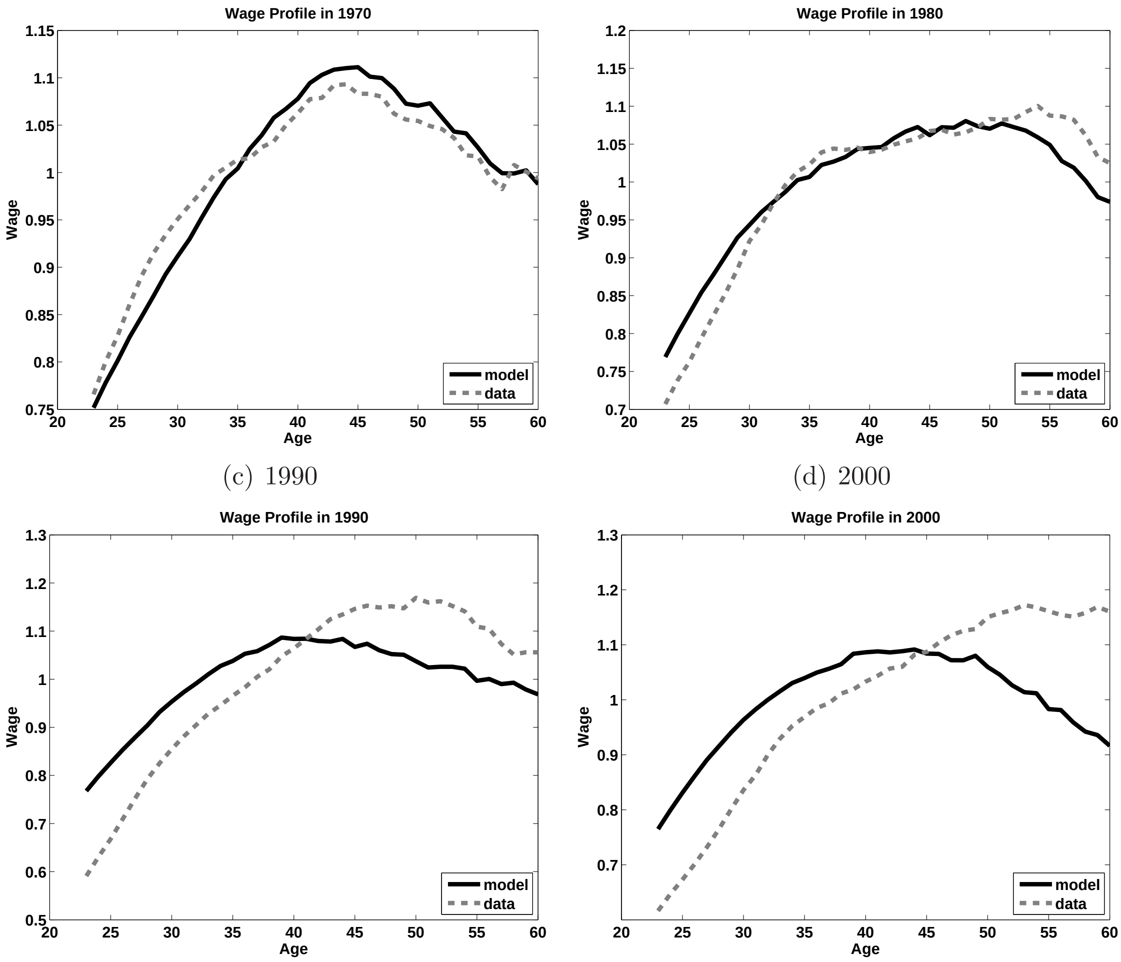
Notes: Model and data profiles for labor supply. Hours data is normalized by 76 total hours. *Data Sources:* Based on hours worked data from the Decennial Censuses obtained from McGrattan and Rogerson (2004).

As a consequence of the hump-shaped labor supply, the Frisch labor supply elasticity is u-shaped over the life-cycle. During the years 1960-2000 we find that agents between age 25 and 55 have a labor supply elasticity between 0.7 and 1.0, while it is higher for younger and older agents. For agents of age 30-50 (20-60) the average Frisch elasticity is around 0.86 (1.10), while across all agents the average is around 1.36. If we aggregate the u-shaped micro Frisch elasticities to a macro Frisch elasticity that takes the differing initial labor supply at different ages of life into account then this yields a number around 1.17 for the

Figure 5.10: Wages

(a) 1970 (b) 1980

(c) 1990 (d) 2000



Notes: Model and data profiles for wages. The data is a centered average of five subsequent PSID samples.
Data Sources: Based on PSID wage data.

macro elasticity.

We also report a Frisch labor supply elasticity that allows time investment into human capital formation to vary. In the spirit of the Frisch elasticity concept we hold the marginal utility of human capital constant in addition to the marginal utility of wealth. This Frisch elasticity is then given by

$$\widetilde{\epsilon}^{j}_{\ell,w} = \frac{1-\phi(1-\sigma)}{\sigma}\frac{1-\ell_j-e_j}{\ell_j} + \frac{1}{1-\psi}\frac{e_j}{\ell_j}. \tag{5.11}$$

As usual, an interior solution is assumed here. If we use this concept then the labor supply elasticity is higher because the second term is positive, i.e. agents invest less into human capital formation when they face a higher wage today and the marginal utility of human capital remains unchanged. Due to decreasing time investment into human capital formation, the second term decreases over the life-cycle. The resulting labor supply elasticity is still u-shaped over the life-cycle. Accordingly, during 1960-2000 for agents of age 30-50 (20-60) the resulting average Frisch elasticity with varying time investment is around 1.26 (1.79), while across all agents the average is around 2.47. Here, the macro Frisch elasticity is around 1.97 when accounting for the differing initial labor supply across agents of different age.

The numbers we find in our model are higher than the standard estimates reported in the literature which are about 0.5, see e.g. Domeij and Flodén (2006), or even lower, see table 3.3 in Browning, Hansen, and Heckman (1999). However, the data used by the empirical literature usually refers to prime-age, full-time employed, male workers and therefore captures mainly the intensive margin. As reported above, if we restrict attention to a subset of agents in the model, e.g. those of age 30-50, that is most comparable to the data set of a typical empirical study then the estimates are quite close. Furthermore, the fact that the empirical literature focusses mostly on the intensive margin and neglects much of the extensive margin suggests that the empirical estimates are a lower bound on the true labor supply elasticity. Browning, Hansen, and Heckman (1999) also report that empirical estimates for females can be much higher than for males. The u-shape of labor supply elasticities in our model can be regarded as a good property because the extensive margin is probably most relevant towards the beginning and the end of the life-cycle.

Another potential source of downward bias of the empirical literature results from not considering endogenous human capital accumulation explicitly and thereby not correctly accounting for the true opportunity cost of time. This was shown by Imai and Keane (2004) in the context of a learning-by-doing model, so it is not directly applicable to our model. But similar biases might also be present here. With regard to the Frisch elasticity with varying time investment reported above we are unaware of any attempt to estimate the Frisch elasticity empirically in this model framework, which would mean to include the marginal utility of human capital in the set of conditioning variables.

Lastly, as shown in figures 5.9 and 5.10 our model does a good job in replicating observed life-cycle profiles of hours and wages. This is probably a more meaningful test of the ability of our model to explain the relation between hours worked and wages than comparing a single number that is very hard to identify empirically.

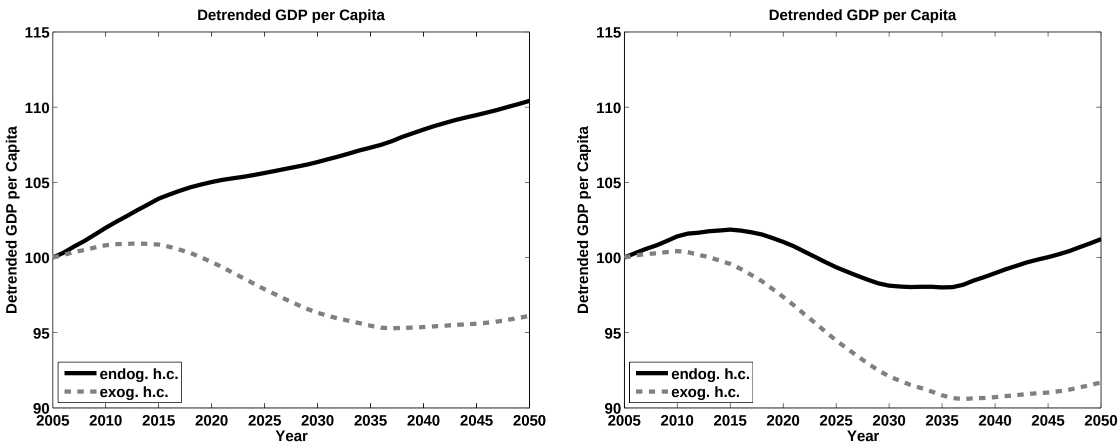
5.A.4 Transitional Dynamics

Aggregate Variables

The cumulative effect of the differences in growth rates on GDP per capita are displayed in figure 5.11. In the endogenous human capital model with constant contribution (replacement) rates, GDP per capita will increase by about 15% (10%) more until the year 2050 than without human capital adjustments.

Figure 5.11: Detrended GDP per Capita [Index, 2005=100]

(a) Constant Contribution Rate Scenario (b) Constant Replacement Rate Scenario



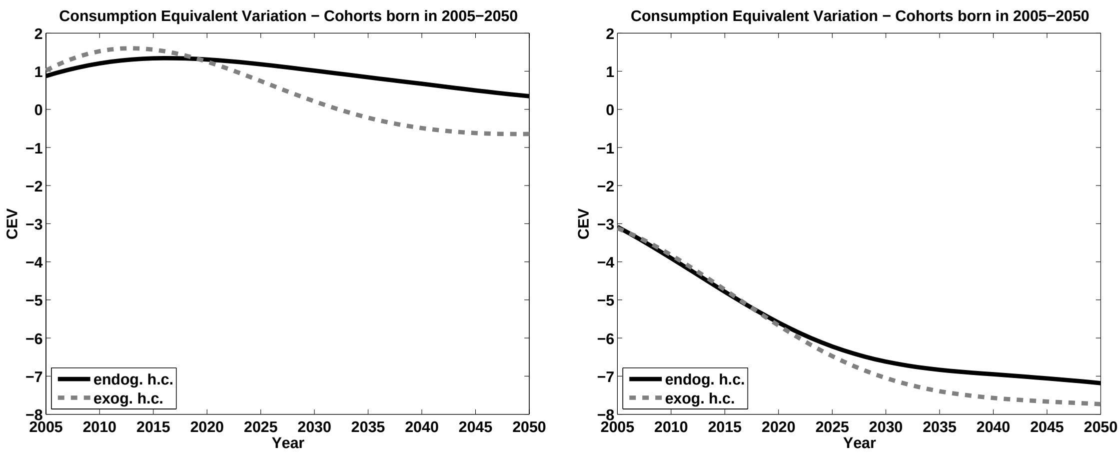
Welfare Effects

Welfare of Future Generations

In our main text, we mostly analyze the welfare consequences for agents alive in 2005 and only briefly glance at the consequences for future generations. We here look at those. Figure 5.12 shows the consumption equivalent variation for the two models and the two social security scenarios. Agents born into a world with endogenous human capital and constant contribution rates experience gains of lifetime utility throughout the entire projection window. Even if agents are allowed to invest into human capital, welfare losses of future generation can be quite large if the contribution rates rise ("const ρ"). Notice again that, in our comparison across models, differences are not large because the positive value of human capital adjustments is offset by the more beneficial general equilibrium effects in the exogenous human capital model. For this reason welfare gains for some cohorts may even be slightly higher in the exogenous human capital model when the contribution rate is held constant.

Figure 5.12: Consumption Equivalent Variation of Agents born in 2005-2050

(a) Constant Contribution Rate Scenario (b) Constant Replacement Rate Scenario



Notes: Consumption equivalent variation (CEV) in the two social security scenarios.

The Value of Human Capital Adjustments

From figure 5.8 of our main text, we observe that welfare gains (and losses) for newborns are almost identical in the endogenous and exogenous human capital models. Detailed numbers are provided in table 5.3. The explanation for these similar welfare consequences is as follows: While the value of human capital adjustments is positive (see below), the increase of wages and the associated decrease of interest rates is much stronger in the exogenous human capital model. As newborn households generally benefit from the combined effects of increasing wages and decreasing returns, welfare gains from these general equilibrium effects are higher in the exogenous human capital model. This explains why the overall welfare consequences for newborns across models do not differ much despite the fact that the value of human capital adjustments is positive.

Table 5.3: CEV for Generation Born in 2005 [in %]

	Human Capital	
	Endogenous	Exogenous
Const. τ ($\tau_t = \bar{\tau}$)	0.8%	0.9%
Const. ρ ($\rho_t = \bar{\rho}$)	-3.0%	-3.0%

Notes: CEV: consumption equivalent variation.

Our comparison across models does not tell us anything about the value of a flexible adjustment of human capital investments from the individual perspective, that is, about the value of human capital adjustments within the endogenous human capital model. To accomplish this, we store from our computation of $\bar{V}_j^{2005}$ (see above) the associated endogenous time investment profile, $\{e_j^{2005}\}_{j=0}^J$. Next, we compute $\bar{V}_{t,j}^{CE}$ as the lifetime utility of agents born at time t, age j facing constant 2005 survival rates, a sequence of equilibrium prices, transfers and contribution (replacement) rates as documented for the endogenous human capital model in the previous section, but keep the time investment profile fixed at $\{e_j^{2005}\}_{j=0}^J$. In correspondence to what we did before, we then compute

$$g_{t,j}^{CE} = \left(\frac{\bar{V}_{t,j}^{CE}}{\bar{V}_j^{2005}} \right)^{\frac{1}{\phi(1-\sigma)}} - 1, \tag{5.12}$$

as the consumption equivalent variation with constant time investment decisions. The difference $g_{t,j} - g_{t,j}^{CE}$ is then our measure of the value of endogenous human capital (where $g_{t,j}$ is the consumption equivalent variation with flexible time investments as computed above).[29]

The value of human capital adjustments is obviously positive and more or less monotonically decreasing with age (because of decreasing time investments over the life-cycle). Furthermore, for all future generations, the value of human capital adjustments can be expected to increase slightly because of the increasing rate of return to human capital formation. For sake of brevity, we do not report these results and confine ourselves to a comparison of the value of human capital adjustments of newborns in 2005, that is $g_{156,0} - g_{156,0}^{CE}$ across social security scenarios. As reported in table 5.4, the value of human capital adjustments in the "const. τ" scenario is 0.35% compared to only 0.07% in the "const. ρ" scenario and thereby around 5 times higher.

Table 5.4: The Value of Human Capital Adjustments in 2005

Const. τ ($\tau_t = \bar{\tau}$)	0.35%
Const. ρ ($\rho_t = \bar{\rho}$)	0.07%

Notes: The value of human capital adjustments is computed as $g_{t,j} - g_{t,j}^{CE}$.

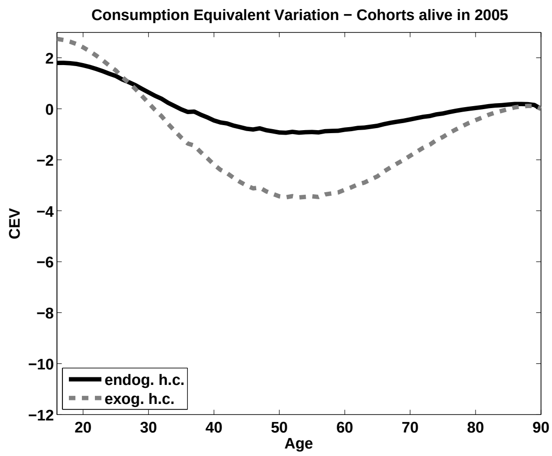
Role of the Pension System: Agents alive in 2005

We here provide a decomposition of our welfare results into the effects stemming from changes in relative factor prices and transfers and those of changing pension payments. To this end, figure 5.13 shows the welfare consequences of demographic change for agents alive in 2005 from changing factor prices alone, keeping pension payments constant. We here look only at our scenario with constant contribution rates. Table 5.5 presents the maximum utility loss for agents alive in 2005 with constant pension payments. In the exogenous human capital model, the maximum loss is about 2.6 percentage points (or 270%) higher than in the endogenous human capital model. Observe from table 5.2 of

[29] To see this more clearly, rewrite the welfare difference as $g_{t,j} - g_{t,j}^{CE} = (\bar{V}_j^{2005})^{-\frac{1}{\phi(1-\sigma)}} \left(\bar{V}_{t,j}^{\frac{1}{\phi(1-\sigma)}} - \bar{V}_{t,j}^{CE\,\frac{1}{\phi(1-\sigma)}} \right)$. The difference between the terms in the brackets is only due to the fact that agents are (or are not) allowed to adjust their human capital.

our main text that, in terms of the percentage point difference, this gain relative to the exogenous human capital model is roughly 3.7 percentage points when pension payments adjust. From comparing these numbers we can therefore conclude that roughly two thirds of the overall gain of 3.7 percentage points can be attributed to differential changes in interest rates, wages and accidental bequests and one third to the relative rise in social security benefits which is caused by the additional human capital formation and the accompanying increase of average wages.

Figure 5.13: CEV of Agents alive in 2005 with constant pensions: Constant Contribution Rates



Notes: Consumption equivalent variation (CEV) in the constant contribution rate scenario with constant pension payments. “endog. h.c.”: endogenous human capital model with constant pensions. “exog. h.c.”: exogenous human capital model with constant pensions.

Table 5.5: Maximum Utility Loss for Generations alive in 2005 with Constant Pensions

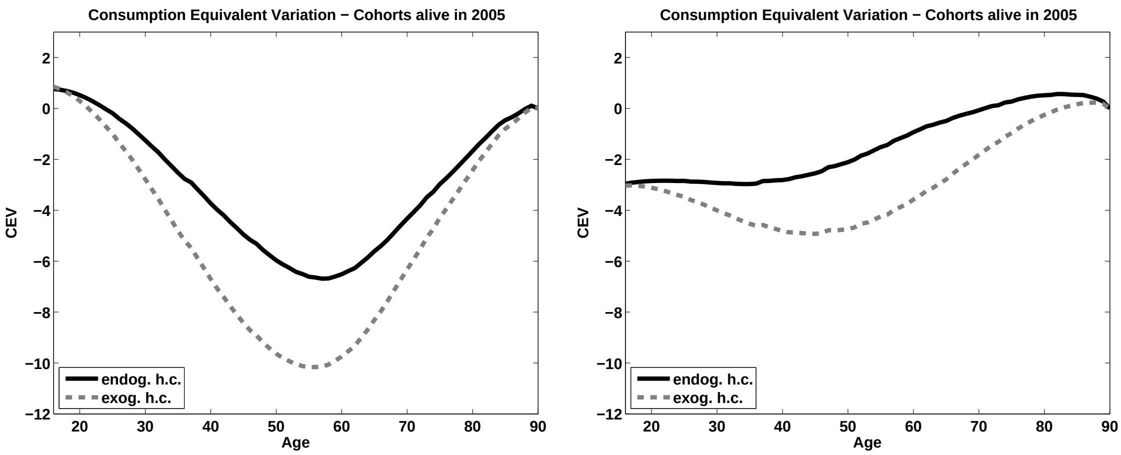
	Human Capital	
	Endogenous	Exogenous
Const. τ ($\tau_t = \bar{\tau}$)	-0.94%	-3.47%

Role of Survival Rates for Welfare Calculations

So far, we computed the welfare effects of demographic change by holding survival rates constant. We here present welfare results for varying survival rates. Figures 5.14 and 5.15 present the results of these calculations. Table 5.6 presents the maximum utility loss for agents alive in 2005 with changing survival rates. Comparing these results to those of figures 5.8 and table 5.2 of our main text and those of figure 5.12, we can conclude that holding survival rates constant or varying them according to the underlying demographic projections does not affect our conclusions about the welfare consequences of demographic change in our comparisons across various scenarios.

Figure 5.14: CEV of Agents alive in 2005 with changing Survival Rates

(a) Constant Contribution Rate Scenario (b) Constant Replacement Rate Scenario



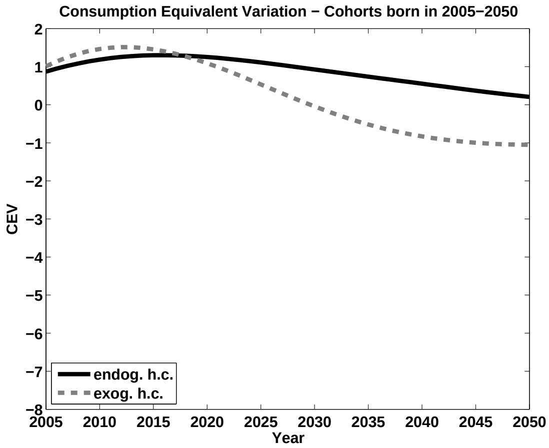
Notes: Consumption equivalent variation (CEV) calculated with changing survival rates in the two social security scenarios.

Table 5.6: Maximum Utility Loss for Generations alive in 2005 with changing Survival Rates

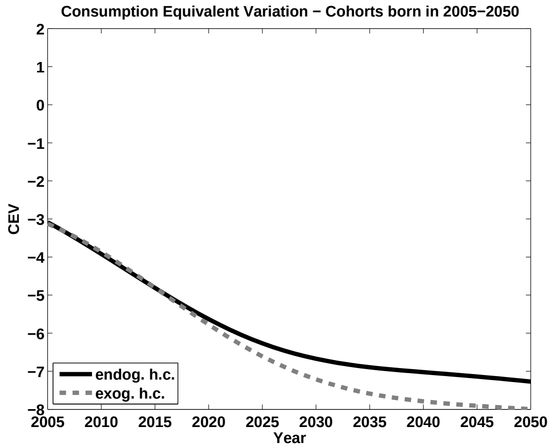
	Human Capital	
	Endogenous	Exogenous
Const. τ ($\tau_t = \bar{\tau}$)	-6.7%	-10.2%
Const. ρ ($\rho_t = \bar{\rho}$)	-3.0%	-4.9%

Figure 5.15: CEV of Agents born in 2005-2050 with changing Survival Rates

(a) Constant Contribution Rate Scenario



(b) Constant Replacement Rate Scenario



Notes: Consumption equivalent variation (CEV) calculated with changing survival rates in the two social security scenarios.

5.A.5 Demographic Data

Our demographic data are based on the Human Mortality Database (2008). Population of age j in year t is determined by four factors: (i) an initial population distribution in year 0, (ii) age and time specific mortality rates, (iii) age and time specific fertility rates and (iv) age and time specific migration rates. We describe here how we model all of these elements and then briefly compare results of our demographic predictions with those of United Nations (2007).

Initial Population Distribution

We take as data the age and time specific population for the periods $1950 - 2004$.

Mortality Rates

Our mortality model is based on sex, age and time specific mortality rates. To simplify notation, we suppress a separate index for sex. Using data from $1950 - 2004$, we apply the procedure developed by Lee and Carter (1992) to decompose mortality rates as

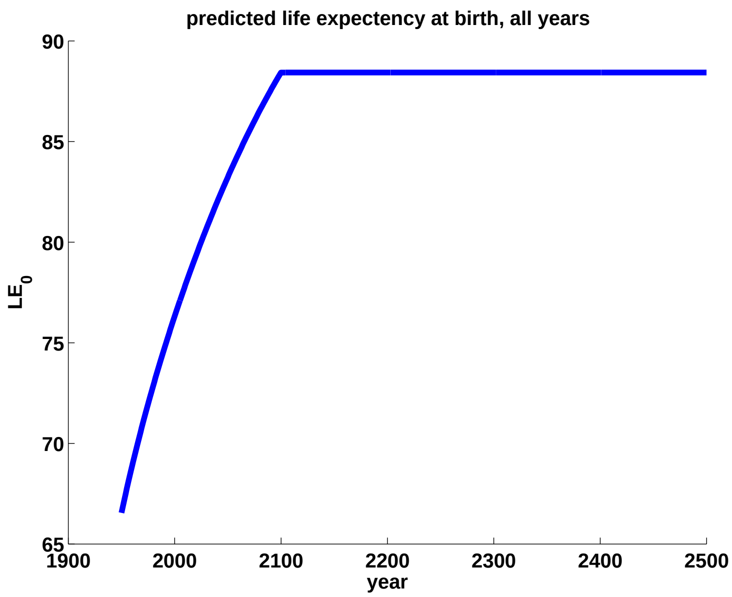
$$\ln(1 - \varphi_{t,j}) = a_j + b_j d_t, \tag{5.13}$$

where a_j and b_j are vectors of age-specific constants and d_t is a time-specific index that equally affects all age groups. We assume that the time-specific index, d_t, evolves according to a unit-root process with drift,

$$d_t = \chi + d_{t-1} + \epsilon_t. \tag{5.14}$$

The estimate of the drift term is $\hat{\chi} = -1.2891$. We then predict mortality rates into the future (until 2100) by holding $\hat{a}_j$, $\hat{b}_j$ and $\hat{\chi}$ constant and setting $\epsilon_t = 0$ for all t. For all years beyond 2100 we hold survival rates constant at their respective year 2100 values. Figure 5.16 shows the corresponding path of life expectancy at birth.

Figure 5.16: Life Expectancy at Birth



Notes: Own predictions of life-expectancy at birth based on Human Mortality Database (2008).

Fertility Rates

Fertility in our model is age and time specific. For our predictions, we assume that age-specific fertility rates are constant at their respective year 2004 values for all periods $2005, \ldots, 2100$. For periods after 2100 we assume that the number of newborns is constant. Since the U.S. reproduction rate is slightly above replacement levels this implies that the total fertility rate is slightly decreasing each year from 2100 onwards until about year 2200 when the population converges to a stationary distribution.

Population Dynamics

We use the estimated fertility and mortality data to forecast the future population dynamics. The transition of the population is accordingly given by

$$N_{t,j} = \begin{cases} N_{t-1,j-1}\varphi_{t-1,j-1} & \text{for } j > 0 \\ \sum_{i=0}^{J} f_{t-1,i} N_{t-1,i} & \text{for } j = 0, \end{cases} \tag{5.15}$$

where $f_{t,j}$ denotes age and time specific fertility rates. Population growth is then given by $n_t = \frac{N_{t+1}}{N_t} - 1$, where $N_t = \sum_{j=0}^{J} N_{t,j}$ is total population in t.

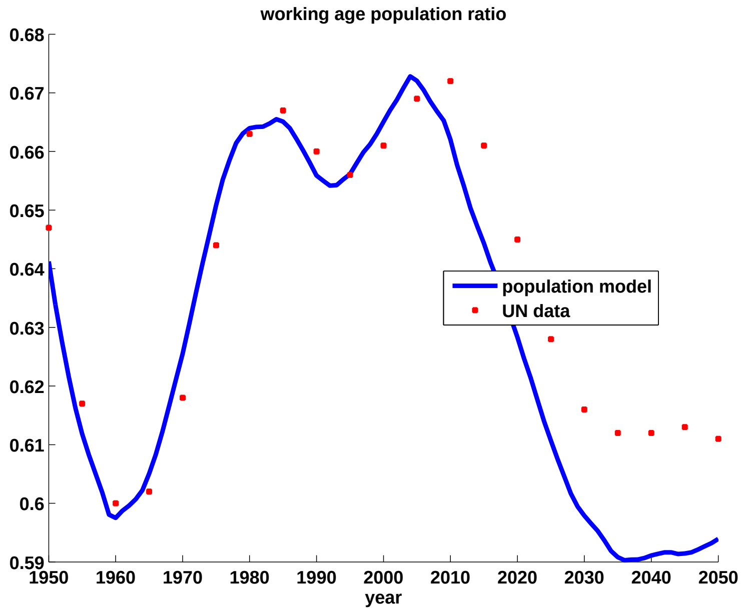
Migration

Migration is exogenous in our economic model. Setting migration to zero would lead us to overestimate future decreases in the working age population ratio and to overstate the increases in old-age dependency. We therefore restrict migration to ages $j \leq 15$ so that migration plays a similar role as fertility in our economic model. This simplifying assumption allows us to treat newborns and immigrants alike. We compute aggregate migration from United Nations (2007) and distribute age-specific migrants in each year equally across all ages $0, \ldots, 15$.

Evaluation

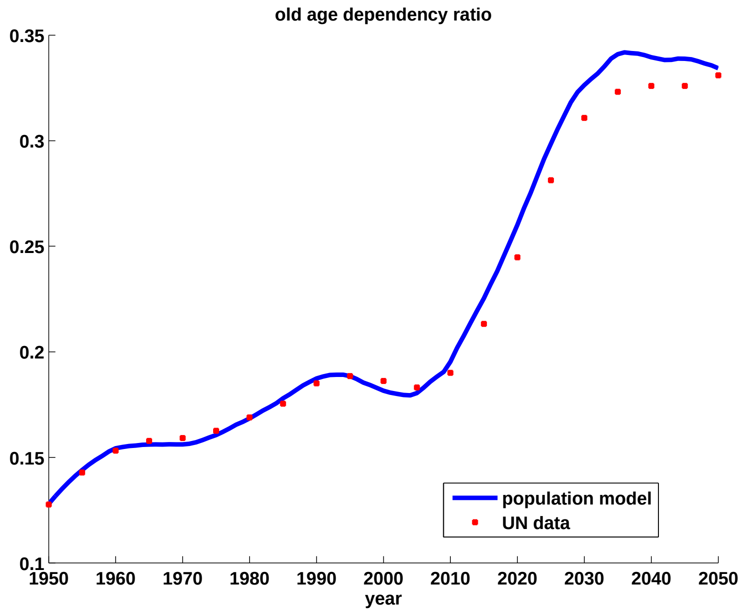
Figures 5.17-5.18 display the predicted working age population and old-age dependency ratios, according to our population model and according to United Nations (2007). Compared to this benchmark, our population model is close to the UN but predicts a slightly stronger decrease of the working age population ratio and a correspondingly stronger increase of the old-age dependency ratio until 2050.

Figure 5.17: Working Age Population Ratio



Notes: Population model: own predictions of the working age population ratio based on Human Mortality Database (2008). UN data: working age population ratio according to United Nations (2007).

Figure 5.18: Old Age Dependency Ratio



Notes: Population model: own predictions of the old-age dependency ratio based on Human Mortality Database (2008). UN data: old-age dependency ratio according to United Nations (2007).

5.B Appendix: Computational Details

5.B.1 Household Problem

To simplify the description of the solution of the household model for given prices (wage and interest rate), transfers and social security payments, we focus on steady states and therefore drop the time index t. Furthermore, we focus on a de-trended version of the household problem in which all variables x are transformed to $\tilde{x} = \frac{x}{A}$ where A is the technology level growing at the exogenous rate g. To simplify notation, we do not denote variables by the symbol $\tilde{\cdot}$ but assume that the transformation is understood. The de-trended version of the household problem is then given by

$$
\begin{aligned}
V(a,h,j) = \max_{c,\ell,e,a',h'} & \left\{u(c, 1-\ell-e) + \beta s(1+g)^{\phi(1-\sigma)} V(a',h',j+1)\right\} \\
\text{s.t.} & \\
a' &= \frac{1}{1+g}\left((a+tr)(1+r)+y-c\right) \\
y &= \begin{cases} \ell h w(1-\tau) & \text{if } j < jr \\ p & \text{if } j \geq jr \end{cases} \\
h' &= g(h,e) \\
\ell &\in [0,1], \quad e \in [0,1].
\end{aligned} \tag{5.16}
$$

Here, $g(h,e)$ is the human capital technology.

Let $\tilde{\beta} = \beta s(1+g)^{\phi(1-\sigma)}$ be the transformation of the discount factor. Using the budget constraints, now rewrite the above as

$$
\begin{aligned}
V(a,h,j) = \max_{c,\ell,e,a',h'} & \left\{u(c,1-\ell-e) + \tilde{\beta} V\left(\frac{1}{1+g}\left((a+tr)(1+r)+y-c\right), g(h,e), j+1\right)\right\} \\
\text{s.t.} & \\
\ell &\geq 0.
\end{aligned}
$$

where we have also replaced the bounded support of time investment and leisure with a one-side constraint on ℓ because the upper constraints, $\ell = 1$, respectively $e = 1$, and the lower constraint, $e = 0$, are never binding due to Inada conditions on the utility function and the functional form of the human capital technology (see below). Denoting by μ_ℓ the Lagrange multiplier on the inequality constraint for ℓ, we can write the first-order conditions as

$$c: \quad u_c - \tilde{\beta}\frac{1}{1+g}V_{a'}(a', h'; j+1) = 0 \tag{5.17a}$$

$$\ell: \quad -u_{1-\ell-e} + \tilde{\beta}hw(1-\tau)\frac{1}{1+g}V_{a'}(a', h', j+1) + \mu_\ell = 0 \tag{5.17b}$$

$$e: \quad -u_{1-\ell-e} + \tilde{\beta}g_e V_{h'}(a', h', j+1) = 0 \tag{5.17c}$$

and the envelope conditions read as

$$a: \quad V_a(a, h, j) = \tilde{\beta}\frac{1+r}{1+g}V_{a'}(a', h', j+1) \tag{5.18a}$$

$$h: \quad V_h(a, h, j) = \tilde{\beta}\left(\ell w(1-\tau)\frac{1}{1+g}V_{a'}(a', h', j+1) + g_h V_{h'}(a', h', j+1)\right). \tag{5.18b}$$

Note that for the retirement period, i.e. for $j \geq jr$, equations (5.17b) and (5.17c) are irrelevant and equation (5.18b) has to be replaced by

$$V_h(a, h, j) = \tilde{\beta}g_h V_{h'}(a', h', j+1).$$

From (5.17a) and (5.18a) we get

$$V_a = (1+r)u_c \tag{5.19}$$

and, using the above in (5.17a), the familiar inter-temporal Euler equation for consumption follows as

$$u_c = \tilde{\beta}\frac{1+r}{1+g}u_{c'}. \tag{5.20}$$

From (5.17a) and (5.17b) we get the familiar intra-temporal Euler equation for leisure,

$$u_{1-\ell-e} = hw(1-\tau)u_c + \mu_\ell. \tag{5.21}$$

From the human capital technology (5.3) we further have

$$g_e = \xi\psi(eh)^{\psi-1}h \tag{5.22a}$$

$$g_h = (1-\delta^h) + \xi\psi(eh)^{\psi-1}e. \tag{5.22b}$$

We loop backwards in j from $j = J-1,\dots,0$ by taking an initial guess of $[c_J, h_J]$ as given and by initializing $V_{a'}(\cdot, J) = V_{h'}(\cdot, J) = 0$. During retirement, that is for all ages $j \geq jr$, our solution procedure is by standard backward shooting using the first-order conditions. However, during the period of human capital formation, that is for all ages $j < jr$, the first order conditions would not be sufficient if the problem is not a convex-programming problem. And thus, our backward shooting algorithm will not necessarily find the true solution. In fact this may be the case in human capital models such as ours because the effective wage rate is endogenous (it depends on the human capital investment decision). For a given initial guess $[c_J, h_J]$ we therefore first compute a solution via first-order conditions and then, for each age $j < jr$, we check whether this is the unique solution. As an additional check, we consider variations of initial guesses of $[c_J, h_J]$ on a large grid. In all of our scenarios we never found any multiplicities.

The details of our steps are as follows:

1. In each j, $h_{j+1}, V_{a'}(\cdot, j+1), V_{h'}(\cdot, j+1)$ are known.

2. Compute u_c from (5.17a).

3. For $j \geq jr$, compute h_j from (5.3) by setting $e_j = \ell_j = 0$ and by taking h_{j+1} as given and compute c_j directly from equation (5.26) below.

4. For $j < jr$:

   (a) Assume $\ell \in [0, 1)$ so that $\mu_\ell = 0$.

   (b) Combine (5.3), (5.17b), (5.17c) and (5.22a) to compute h_j as

$$h_j = \frac{1}{1-\delta^h}\left(h_{j+1} - \xi\left(\frac{\xi\psi\frac{1}{1+g}V_{h'}(\cdot, j+1)}{\omega(1-\tau)V_{a'}(\cdot, j+1)}\right)^{\frac{\psi}{1-\psi}}\right) \tag{5.23}$$

   (c) Compute e from (5.3) as

$$e_j = \frac{1}{h_j}\left(\frac{h_{j+1} - h_j(1-\delta^h)}{\xi}\right)^{\frac{1}{\psi}}. \tag{5.24}$$

   (d) Calculate $lcr_j = \frac{1-e_j-\ell_j}{c_j}$, the leisure to consumption ratio from (5.21) as follows:

From our functional form assumption on utility marginal utilities are given by

$$\begin{aligned}
u_c &= \left(c^{\phi}(1-\ell-e)^{1-\phi}\right)^{-\sigma} \phi c^{\phi-1}(1-\ell-e)^{1-\phi} \\
u_{1-\ell-e} &= \left(c^{\phi}(1-\ell-e)^{1-\phi}\right)^{-\sigma} (1-\phi)c^{\phi}(1-\ell-e)^{-\phi}
\end{aligned}$$

hence we get from (5.21) the familiar equation:

$$\frac{u_{1-\ell-e}}{u_c} = hw(1-\tau) = \frac{1-\phi}{\phi}\frac{c}{1-\ell-e},$$

and therefore:

$$lcr_j = \frac{1-e_j-\ell_j}{c_j} = \frac{1-\phi}{\phi}\frac{1}{hw(1-\tau)}. \tag{5.25}$$

(e) Next compute c_j as follows. Notice first that one may also write marginal utility from consumption as

$$u_c = \phi c^{\phi(1-\sigma)-1}(1-\ell-e)^{(1-\sigma)(1-\phi)}. \tag{5.26}$$

Using (5.25) in (5.26) we then get

$$\begin{aligned}
u_c &= \phi c^{\phi(1-\sigma)-1}(lcr \cdot c)^{(1-\sigma)(1-\phi)} \\
&= \phi c^{-\sigma} \cdot lcr^{(1-\sigma)(1-\phi)}. \qquad (5.27)
\end{aligned}$$

Since u_c is given from (5.17a), we can now compute c as

$$c_j = \left(\frac{u_{c_j}}{\phi \cdot lcr_j^{(1-\sigma)(1-\phi)}}\right)^{-\frac{1}{\sigma}}. \tag{5.28}$$

(f) Given c_j, e_j compute labor, ℓ_j, as

$$\ell_j = 1 - lcr_j \cdot c_j - e_j.$$

(g) If $\ell_j < 0$, set $\ell_j = 0$ and iterate on h_j as follows:

i. Guess h_j

ii. Compute e as in step 4c.

iii. Noticing that $\ell_j = 0$, update c_j from (5.26) as

$$c = \left(\frac{u_c}{\phi(1-e)^{(1-\sigma)(1-\phi)}} \right)^{\frac{1}{\phi(1-\sigma)-1}} .$$

iv. Compute μ_ℓ from (5.17b) as

$$\mu_\ell = u_{1-\ell-e} - \tilde{\beta} h w (1-\tau) V_{a'}(\cdot, j+1)$$

v. Finally, combining equations (5.17b), (5.17c) and (5.22a) gives the following distance function f

$$f = e - \left(\frac{\tilde{\beta} \xi \psi h^{\psi} \frac{1}{1+g} V_{h'}(\cdot)}{\tilde{\beta} \omega h (1-\tau) V_{a'}(\cdot) + \mu_\ell} \right)^{\frac{1}{1-\psi}} , \tag{5.29}$$

where e is given from step 4(g)ii. We solve for the root of f to get h_j by a non-linear solver iterating on steps 4(g)ii through 4(g)v until convergence.

(h) *Check for uniqueness I*: What is computed above is a candidate solution under the assumption that the first-order conditions are necessary and sufficient. As a consequence of potential non-convexities of our programming problem first-order conditions may however not be sufficient and our procedure may therefore not give the unique global optimum. To address this, we next compute solutions on a grid and check if the previously computed candidate solution is indeed the only solution to our system of equations. We do so as follows: For a grid of $e_j \in [\underline{e} = 0.0001, \bar{e} = 0.9999]$, denote the equally spaced grid points by $e_{j,i}, i = 1, \ldots, ne$ and:

i. For each $e_{j,i}$, compute the corresponding $h_{j,i}$ from (5.3).

ii. Compute the corresponding $c_{j,i}, \ell_{j,i}$ by the analogous steps as described above, again taking the case distinction for binding labor into account.

iii. Compute the corresponding value of the distance function in (5.29), $f_{j,i}$.[30]

If for all $e_{j,i}, i = 1, \ldots, ne$ the value of the distance function, $f_{j,i}$, changes signs only once, then our previously computed candidate solution is indeed the unique optimum. If it would change signs more than once, then there would be mul-

[30] Notice that if $\ell_{j,i} > 0$, then we know from equation (5.23) that $x_{j,i} = \{c_{j,i}, \ell_{j,i}, e_{j,i}, h_{j,i}\}$ cannot be a solution but we still proceed by computing $f_{j,i}$.

tiplicities and our first-order conditions would accordingly not be sufficient. Setting $ne = 200$ we never found this to be the case in any of our scenarios.

5. Update as follows:

   (a) Update V_a using either (5.18a) or (5.19).

   (b) Update V_h using (5.18b).

Next, loop forward on the human capital technology (5.3) for given h_0 and $\{e_j\}_{j=0}^{J}$ to compute an update of h_J denoted by h_J^n. Compute the present discounted value of consumption, PVC, and, using the already computed values $\{h_j^n\}_{j=0}^{J}$, compute the present discounted value of income, PVI. Use the relationship

$$c_0^n = c_0 \cdot \frac{PVI}{PVC} \tag{5.30}$$

to form an update of initial consumption, c_0^n, and next use the Euler equations for consumption to form an update of c_J, denoted as c_J^n. Define the distance functions

$$g_1(c_J, h_J) = c_J - c_J^n \tag{5.31a}$$

$$g_2(c_J, h_J) = h_J - h_J^n. \tag{5.31b}$$

In our search for general equilibrium prices, constraints of the household model are occasionally binding. Therefore, solution of the system of equations in (5.31) using Newton based methods, e.g., Broyden's method, is instable. We solve this problem by a nested Brent algorithm, that is, we solve two nested univariate problems, an outer one for c_J and an inner one for h_J.

Check for uniqueness II: Observe that our nested Brent algorithm assumes that the functions in (5.31) exhibit a unique root. As we adjust starting values $[c_J, h_J]$ with each outer loop iteration we thereby consider different points in a variable box of $[c_J, h_J]$ as starting values. For all of these combinations our procedure always converged. To systematically check whether we also always converge to the unique optimum, we fix, after convergence of the household problem, a large box around the previously computed $[c_J, h_J]$. Precisely, we choose as boundaries for this box $\pm 50\%$ of the solutions in the respective dimensions. For these alternative starting values we then check whether there is an additional solution to the system of equations (5.31). We never detected any such multiplicities.

5.B.2 The Aggregate Model

For a given $r \times 1$ vector $\vec{\Psi}$ of structural model parameters, we first solve for an "artificial" initial steady state in period $t = 0$ which gives initial distributions of assets and human capital. We thereby presume that households assume prices to remain constant for all periods $t \in \{0, \dots, T\}$ and are then surprised by the actual price changes induced by the transitional dynamics. Next, we solve for the final steady state of our model which is reached in period T and supported by our demographic projections, see appendix 5.A.5. For both steady states, we solve for the equilibrium of the aggregate model by iterating on the $m \times 1$ steady state vector $\vec{P}_{ss} = [p_1, \dots, p_m]'$. p_1 is the capital intensity, p_2 are transfers (as a fraction of wages), p_3 are social security contribution (or replacement) rates and p_4 is the average human capital stock. Notice that all elements of $\vec{P}_{ss}$ are constant in the steady state.

Solution for the steady states of the model involves the following steps:

1. In iteration q for a guess of $\vec{P}^q_{ss}$ solve the household problem.
2. Update variables in $\vec{P}_{ss}$ as follows:
   (a) Aggregate across households to get aggregate assets and aggregate labor supply to form an update of the capital intensity, p^n_1.
   (b) Calculate an update of bequests to get p^n_2.
   (c) Using the update of labor supply, update social security contribution (or replacement) rates to get p^n_3.
   (d) Use labor supply and human capital decisions to form an update of the average human capital stock, p^n_4.
3. Collect the updated variables in $\vec{P}^n_{ss}$ and notice that $\vec{P}^n_{ss} = H(\vec{P}_{ss})$ where H is a vector-valued non-linear function.
4. Define the root-finding problem $G(\vec{P}_{ss}) = \vec{P}_{ss} - H(\vec{P}_{ss})$ and iterate on $\vec{P}_{ss}$ until convergence. We use Broyden's method to solve the problem and denote the final approximate Jacobi matrix by B_{ss}.

Next, we solve for the transitional dynamics by the following steps:

1. Use the steady state solutions to form a linear interpolation to get the starting values for the $m(T-2) \times 1$ vector of equilibrium prices, $\vec{P} = [\vec{p}_1', \ldots, \vec{p}_m']'$, where $p_i, i = 1, \ldots, m$ are vectors of length $(T-2) \times 1$.

2. In iteration q for guess $\vec{P}^q$ solve the household problem. We do so by iterating backwards in time for $t = T-1, \ldots, 2$ to get the decision rules and forward for $t = 2, \ldots, T-1$ for aggregation.

3. Update variables as in the steady state solutions and denote by $\tilde{\vec{P}} = H(\vec{P})$ the $m(T-2) \times 1$ vector of updated variables.

4. Define the root-finding problem as $G(\vec{P}) = \vec{P} - H(\vec{P})$. Since T is large, this problem is substantially larger than the steady state root-finding problem and we use the Gauss-Seidel-Quasi-Newton algorithm suggested in Ludwig (2007) to form and update guesses of an approximate Jacobi matrix of the system of $m(T-2)$ non-linear equations. We initialize these loops by using a scaled up version of B_{ss}.

5.B.3 Calibration of Structural Model Parameters

We split the $r \times 1$ vector of structural model parameters, $\vec{\Psi}$, as $\vec{\Psi} = \left[(\vec{\Psi}^e)', (\vec{\Psi}^f)'\right]'$. $\vec{\Psi}^f$ is a vector of predetermined (fixed) parameters, whereas the $e \times 1$ vector $\vec{\Psi}^e$ is estimated by minimum distance (unconditional matching of moments using e moment conditions). Denote by

$$u_t(\vec{\Psi}^e) = y_t - f(\vec{\Psi}^e) \text{ for } t = 0, \ldots, T_0 \tag{5.32}$$

the GMM error as the distance between data, y_t, and model simulated (predicted) values, $f(\vec{\Psi}^e)$.

Under the assumption that the model is correctly specified, the restrictions on the GMM error can be written as

$$E[u_t(\vec{\Psi}_0^e)] = 0, \tag{5.33}$$

where $\tilde{\Psi}_0^e$ denotes the vector of true values. Denote sample averages of u_t as

$$g_{T_0}(\vec{\Psi}^e) \equiv \frac{1}{T_0+1}\sum_{t=0}^{T_0} u_t(\vec{\Psi}^e). \tag{5.34}$$

We estimate the elements of $\vec{\Psi}^e$ by setting these sample averages to zero (up to some tolerance level).

In our economic model, only two parameters are pre-determined and we therefore have that

$$\vec{\Psi}^f = [\sigma, h_0]'. \tag{5.35}$$

The vector $\vec{\Psi}^e$ is given by

$$\vec{\Psi}^e = \left[g, \alpha, \delta, \beta, \phi, \psi, \xi, \delta^h\right]'. \tag{5.36}$$

We estimate the structural model parameters using data from various sources for the period 1960, ..., 2004, hence $T_0 = 44$. The parameters $\vec{\Psi}_1^e = [g, \alpha]'$ are directly determined using NIPA data on GDP, fixed assets, wages and labor supply. The remaining structural model parameters, $\vec{\Psi}_2^e = [\delta, \beta, \phi, \psi, \xi, \delta^h]'$ are estimated by simulation. Our calibration targets are summarized in table 5.7.

Table 5.7: Calibration Targets

Parameter	Target	Moment
$\vec{\Psi}^f$		
σ	predetermined parameter	
h_0	predetermined parameter	
$\vec{\Psi}_1^e$		
g^A	growth rate of Solow residual	0.018
α	share of wage income	0.33
$\vec{\Psi}_2^e$		
δ	investment output ratio	0.2
β	capital output ratio	2.8
ϕ	average hours worked	0.33
ψ, ξ, δ^h	coefficients of wage polynomial (from PSID)	

Determining the subset of parameters $\vec{\Psi}_2^e$ along the transition is a computationally complex problem that we translate into an equivalent simple problem. Point of departure of our procedure is the insight that calibrating the model for a steady state is easy and fast. However, simulated steady state moments may differ quite substantially from simulated averages along the transition even when the steady state is chosen to lie in the middle of the calibration period, in our case year 1980. We therefore proceed as follows:

1. Initialization: Choose a vector of scaling factors, $\vec{sf}$, of length e_2 that appropriately scales the steady state calibration targets (see below).

2. Calibrate the model in some steady state year, e.g., 1980, by solving the system of equations
$$\frac{\bar{y}^e_{2,i}}{sf_i} - f^{e,ss}_{2,i}(\vec{\Psi}) \tag{5.37}$$
for all $i = 1, \ldots, e_2$ to get $\hat{\vec{\Psi}}^e_2$. Here, $\bar{y}^e_{2,i}$ is the average of moment i in the data for the calibration period (1960-2004), e.g., the investment-output ratio for $i = 1$.

3. For the estimated parameter vector, $\hat{\vec{\Psi}}^e_2$, solve the model along the transition.

4. Compute the relevant simulated moments for the transition, $f^e_2(\vec{\Psi})$.

5. Update the vector of scaling vectors as
$$sf_i = \frac{f^e_{2,i}(\vec{\Psi})}{f^{e,ss}_{2,i}(\vec{\Psi})} \tag{5.38}$$
for all $i = 1, \ldots, e_2$.

6. Continue with step 2 until convergence on scaling factors (fixed point problem).

We thereby translate a complex root-finding problem into a combination of a simple root-finding problem (steady state calibration) and a fixed point iteration on scaling factors. Since scaling factors are relatively insensitive to Ψ^e_2, convergence is fast and robust. The resulting scaling factors range from 0.94 to 1.29 which means that differences between simulated moments in the artificial steady state year (1980) and averages during the transition are large (up to 30%). This also implies that calibrating the model in some artificial steady year only would lead to significantly biased estimates of structural model parameters.

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Eidesstattliche Erklärung

Hiermit erkläre ich, die vorliegende Dissertation selbständig angefertigt und mich keiner anderen als der in ihr angegebenen Hilfsmittel bedient zu haben. Insbesondere sind sämtliche Zitate aus anderen Quellen als solche gekennzeichnet und mit Quellenangaben versehen.

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