Public Policy in Macroeconomic Models with Incomplete Markets

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Contents

Chapter 1

General Introduction

This thesis consist of three self-contained papers linked by a common topic: positive and normative analysis of public policy when human capital investment is risky and private insurance markets against labor income risk are missing. The main reason for the incompleteness of human capital markets is an informational friction: the work effort choice of employed households and the search effort choice of unemployed households are in general publicly not observable. Hence, wages and unemployment benefit payments cannot be made contingent on the households' effort choices. The informational asymmetry induces a moral hazard friction which prevents private insurance markets against human capital risk to develop.[1] The failure of private insurance markets makes room for welfare improving public policies that provide some insurance against labor income and employment risk. In addition, as shown in the third paper (chapter 4 of the thesis), publicly provided consumption services leave income risk unaffected but reduces consumption risk. This relationship is not taken into account by the households' portfolio decisions. The presence of idiosyncratic uninsurable labor income risk makes public policies that encourage more risk taking beneficial.

All three papers build on the human capital based endogenous growth model with idiosyncratic human capital risk introduced by Krebs (2003). The key property of this framework is the analytical tractability despite the fact that idiosyncratic labor income and employ-

[1]For a more detailed argument for missing insurance markets against labor income and employment risk, see, e.g., Arrow (1963) and Rothschild and Stiglitz (1976).

ment shocks translate into household specific income histories which leads to *ex-post* heterogeneity with respect to individual wealth holdings. The analytical tractability stems from two assumptions: homothetic preferences and budget sets in which disposable income is linear homogenous in wealth. Taken together, both assumptions lead to consumption-saving decisions that are linear homogenous in wealth, which simplifies the market clearing substantially. Specifically, the equilibrium allocation can be found without solving for the complete wealth distribution, which stands in strong contrast to the standard incomplete market models discussed, e.g., in Ljungqvist and Sargent (2004).[2] The first and second paper of this thesis extend Krebs (2003) to include search model of the labor market, and the third paper introduces publicly provided consumption services. All modifications are made such that the key property, the analytical tractability of the basic framework, is preserved.

The first paper (chapter 2 of this thesis) is joint work with Tom Krebs and introduces a labor market search model in which unemployed households choose their search effort. After having established some characterization results of the competitive equilibrium, we apply a calibrated version of the model economy to evaluate employment, growth and, in particular, welfare effects of the recent major labor market reform in Germany, that was implemented in 2005 and 2006. The second paper (chapter 3 of this thesis) is single authored and builds on the model developed in the previous paper, but in contrast of making a positive statement on the effect of labor market reforms, I now take a normative approach and analyzed the optimal unemployment insurance system when the households' search effort decisions are only private information such that the government cannot make benefit payments conditional on the exerted search effort. The third paper (chapter 4 of this thesis) is joint work with Tom Krebs and introduces publicly provided consumption services that are financed through capital, labor and consumption taxes. Because there is an implicit shift from risky private consumption towards risk-free publicly provided consumption services that is not taken into account by the households' optimization decision, distortative tax systems may be welfare improving. In this paper, we take a normative approach and derive the optimal welfare improving tax system. The rest of the general introduction provides a short overview of the three papers including a preview on the

[2]See Samuelson (1969) and Merton (1969) for partial equilibrium models that exploit this property, and Stokey (2009) for a general equilibrium model.

results.

1.1 A Macroeconomic Model for the Evaluation of Labor Market Reforms

We develop a tractable macroeconomic model with employment risk and labor market search in order to evaluate the effects of labor market reform on unemployment, growth, and welfare. The model has a large number of risk-averse households who can invest in risk-free physical capital and risky human capital. Unemployed households receive unemployment benefits and decide upon the search effort.

The paper contributes to the literature with respect to two dimensions: First, we present a theoretical characterization result that facilitates the computation of equilibria substantially. In particular, we show that the the key property of the equilibrium allocation is preserved. Thus, portfolio choices, search effort decisions, and the aggregate capital-to-labor ratio, can be characterized without solving for the complete underlying wealth distribution.

Second, we calibrate the model to German data and use the calibrated model economy to simulate the macroeconomic effects of the German labor market reforms of 2005 and 2006 (*Hartz Reforms*). We find three major effects of the 2005-reform, which implemented a reduction in benefit payments to long-run unemployed households: First, the equilibrium unemployment rate has fallen by approximately 1.1 percentage points from 7.5 to 6.4 percent. Second, the reduction in unemployment has been the result of an increase in the search effort by long-term and by short-term unemployed households. The latter have contributed to almost 40 percent of the total effect on the unemployment rate. Third, employed and short-term unemployed households have experienced significant welfare gains, whereas the long-term unemployed have lost in welfare terms. The effects of the 2006-reform, which implemented a substantial reduction in the eligibility period to high unemployment benefit, are qualitatively similar, but quantitatively much smaller.

1.2 Optimal Unemployment Insurance in General Equilibrium

This paper uses the dynamic general equilibrium search model of the labor market developed in chapter 2 of this thesis to compute an optimal unemployment insurance scheme when search effort choices are only private information of the households. In contrast to the existing literature on optimal unemployment insurance, we first provide a macroeconomic (general equilibrium) perspective, second allow for precautionary saving, and third preserve the analytical tractability in the sense that the optimal unemployment insurance system can be characterized without solving for the complete underlying wealth distribution. The combination of all three properties – which have quantitatively and computationally important implications – is unique to the literature.

Households are risk-averse and decide upon consumption, saving, portfolio composition, and search effort. The latter is unobservable to the government which causes a moral hazard friction. Despite the ex-post heterogeneity due to different employment histories. We set up a mixed social planner – Ramsey problem, that preserves the wealth independence of the equilibrium allocation and the social planner solution. Specifically, the government chooses wealth independent transfer rates subject to firstly, the households' consumption-saving decision and search effort choices, which is the Ramsey part of government's optimization problem, and secondly, the portfolio allocation, which is the social planner part of the government's optimization problem.

The main results are as follows: First, conditional on being employed, the social planner provides full insurance. This is due to the fact that there are no moral hazard frictions for currently employed households. Second, the optimal unemployment benefit rate is independent of the unemployment duration. Third, while the net benefit rate for unemployed households is quite low, there are high rewards for successful job finders of 134 percent of their labor income.

1.3 Human Capital Risk, Public Consumption, and Optimal Taxation

This paper studies the optimal tax policy when, first, households make portfolio decisions between a risk-free investment opportunity (physical capital) and a risky investment opportunity (human capital) and, second, the government provides public consumption services that are beneficial to the households. More specifically, we show that in case that public consumption services are less risky than human capital and households derive utility from these public consumption services, there is less investment in the risky asset than socially optimal. Hence, there exists a welfare improving tax policy that distorts the households' portfolio decision towards the risky asset – human capital.

There is a straightforward economic intuition for the sub-optimality of the undistorted competitive equilibrium. By choosing their portfolio, households determine the mean and the volatility of their return to total investment and hence of their individual consumption growth. When making their investment decision, households take the provision of public consumption services as given. However, in aggregate more investment in the high-return risky asset also increases output and thus the provision of the publicly provided consumption good. Hence, in the undistorted competitive equilibrium households invest less in the risky asset than socially optimal.

We formalize the above intuition using a version of the endogenous growth model with incomplete markets introduced by Krebs (2003) and augment it by a government sector that provides risk-free public consumption services. The government sets linear capital, labor and consumption taxes. We show that in this framework, it is always optimal to encourage more risk taking by subsidizing the risky investment opportunity – human capital.

The quantitative analysis reveals that switching from an environment, in which the tax rates on physical and human capital are zero, to the optimal tax system leads to only small welfare gains. The reason for this is the presence of strong general equilibrium effects. Specifically, increasing the subsidy on human capital investment makes human capital more abundant and thus reduces the return to human capital. However, allowing for endogenous labor-leisure choices breaks the strong general equilibrium effect: the optimal human capital subsidy becomes substantial and induces considerable welfare gains.

Chapter 2

A Macroeconomic Model for the Evaluation of Labor Market Reforms

2.1 Introduction

There is considerable evidence that individual households face a substantial amount of labor income risk.[1] In particular, employed workers face the risk of becoming unemployed. In most countries, the government provides insurance against this type of risk through the payment of unemployment benefits. Other things being equal, the provision of unemployment insurance increases the welfare of risk-averse households. However, unemployment benefits also discourage unemployed households from exerting search effort thereby raising the overall unemployment rate. When employment drops, so does aggregate output. In designing the unemployment insurance system, governments therefore have to weigh the insurance benefits against the costs of distorted incentives.[2]

Although the incentive-insurance tradeoff is already present in a simple one-tiered unem-

[1] Using individual data on labor income dynamics, estimates for the standard deviation of labor income range from 0.15 in Hubbard, Skinner, and Zeldes (1995) over 0.19 in Meghir and Pistaferri (2004) up to 0.25 in Storesletten, Telmer, and Yaron (2004). Jacobson, LaLonde, and Sullivan (1993) focus on the specific issue of labor income dynamics after job displacement and find that long run earnings are on average 25 percent below the pre-displacement rate for long-tenured workers. For a review of the job displacement literature, see Kletzer (1998).

[2] See, e.g. Shavell and Weiss (1979), Hopenhayn and Nicolini (1997), Acemoglu and Shimer (1999) and Lentz (2009).

ployment benefit system, governments often run multi-tiered unemployment benefit systems with falling benefits schedules in order to deal with the incentive-insurance tradeoff more efficiently.[3] In 2005 and 2006, the German government implemented two major labor market reforms, the so called *Hartz Reforms*, in order to establish a more pronounced two-tiered unemployment insurance system to fight the steadily increasing unemployment rate in Germany. The 2005-reform reduced the benefit payments in the second tier, whereas the 2006-reform implemented a sharp reduction in the length of the eligibility period for high benefit payments in the first tier. Both reforms put more emphasis on the incentive effect of the unemployment system. Obviously, such reforms tend to reduce the unemployment rate, but the welfare effect is ambiguous, due to the above mentioned tradeoff. In this paper, we develop a tractable macroeconomic model, and use a calibrated version of the model to evaluate the quantitative effects of the *Hartz Reforms* on unemployment, growth, and welfare.

Our model combines the incomplete markets model developed in Krebs (2003, 2006)[4] with the labor market search model introduced by Benhabib and Bull (1983). Similar to Krebs (2003, 2006), there is a large number of risk-averse households who invest in risk-free physical capital and risky human capital. Investment in human capital is risky due to wage risk and employment risk. Following Benhabib and Bull (1983), unemployed households choose their search effort that determines their reemployment probability in the subsequent period. There is a government that provides unemployment insurance and finances these transfer payments through a consumption tax. Our main theoretical contribution is an extension of Krebs' tractability result to search models: the equilibrium allocation can be found without knowledge of the underlying wealth distribution, which facilitates the computation of equilibria substantially.

[3]Much of the theoretical literature on optimal unemployment insurance, e.g. Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997), support the idea that falling benefit schedules are optimal. However, recently Shimer and Werning (2006, 2007, 2008) challenge this result: the lower the wealth, the more easier it is to provide search incentives and the higher the benefit payments can be without distorting incentives.

[4]On the one hand, this model builds on the extensive literature of human capital based endogenous growth models, e.g. Lucas (1988) and Jones and Manuelli (1990), among many others and, on the other hand, Krebs (2003, 2006) relates to the macroeconomic incomplete markets literature, e.g. **?**, Huggett (1993) and Aiyagari (1994). In particular, Aiyagari (1994) discusses a consumption saving model with idiosyncratic shocks on labor income, that can be interpreted as employment shocks. However, he abstracts from an explicit formulation of the labor market which makes his model inappropriate to discuss different aspects of unemployment insurance systems.

Using our theoretical characterization result, we proceed with the quantitative evaluation of the labor market reforms in Germany. More specifically, we calibrate the model to match the pre-2005 German data, and then obtain the quantitative effects of the recent labor market reforms through model simulation. Our main quantitative results are as follows. First, the 2005-reform had large employment effects: the equilibrium unemployment rate has been reduced by approximately 1.1 percentage points from 7.5 to 6.4 percent. Second, the drop in unemployment has led to substantial output gains. Third, employed and short-term unemployed households have experienced a significant welfare gain. Hence, the positive incentive effect dominates the negative insurance effect. However, the long-term unemployed have lost in welfare terms. Fourth, a further decrease in the benefit rate leads only to small additional welfare gains, if at all. Finally, the effects of the 2006-reform are qualitatively similar, but quantitatively much smaller.[5]

The rest of the paper is organized as follows. After a short discussion of the related literature in section 2.2, we develop the economic model in section 2.3. Section 2.4 is devoted to the construction of a competitive equilibrium. In section 2.5, we calibrate the model to match stylized facts of the German economy and simulate the respective employment, growth, and welfare effects of the recent labor market reforms. Furthermore, we investigate the robustness of our results with respect to the critical model parameters. Section 2.6 presents our conclusions.

2.2 Related Literature

There is an extensive empirical literature on policy reform evaluation that analyzes the effect of various labor market reforms on the unemployed using micro-level data.[6] In a certain sense, papers in this field also deal with the interaction of labor market reform and labor market risk (long-term consequences of unemployment). However, work in this literature usually does not take into account any effect of labor market reform on labor demand and wages, which is arguably of first-order importance when the labor market

[5]In other words, the change in the eligibility period implemented in 2006 had only small effects on re-employment probabilities. This result is consistent with the findings of recent empirical studies (see Fitzenberger and Wilke (2007) for Germany).

[6]For a discussion of this strand of the literature, see section 2.5, and in more detail, Franz (2009).

reform affects a large number of workers. By contrast, some of the work in the applied general equilibrium literature, e.g. Boehringer, Boeters, and Feil (2005) and Immervoll, Kleven, Kreiner, and Saez (2007), explicitly deals with such labor market effects of policy reform, and some interesting applications of this approach to Germany have been done in Franz, Guertzgen, Schubert, and Clauss (2007). However, this work has neither taken into account income risk, nor considered the interaction of labor, capital, and goods markets – two issues that will take center stage in our analysis.

The present model is most closely related to Shimer and Werning (2006, 2007, 2008) who use a Bewley-type model with labor market search to analyze the optimal profile of the unemployment insurance system. Similarly to our model, they succeed in characterizing the equilibrium consumption-saving and reservation wage decision without solving for the underlying wealth distribution.[7] However, in Shimer and Werning (2006, 2007, 2008), this result relies on CARA-utility specification, whereas our approach is valid as long as preferences are homothetic and disposable income is linear homogenous in asset holdings.[8]

Our model is further related to Lentz (2009) who extends Aiyagari (1994) by a labor market module with endogenous search effort decision. In contrast to our paper, his model is not tractable in the above-mentioned sense because the consumption-saving and the search effort decision depends on the individual wealth level such that the equilibrium unemployment rate depends on the underlying wealth distribution. Thus, solving the model comes at high computational costs. There are two further differences between the above mentioned papers and our paper. First, we take a general equilibrium perspective. As Lentz (2009) showed, the relation between the time preference rate and the interest rate is crucial for the determination of welfare effects. Since the return is determined endogenously in our model, we do not suffer from this additional degree of freedom. Second, our framework is actually an endogenous growth model such that we can also analyze the long-run growth effects of the labor market reforms in question.

Our paper further relates to the extensive research of optimal unemployment insurance, starting with the influential contribution by Shavell and Weiss (1979), who explicitly ad-

[7]Specifically, the reservation wage decision is equivalent to our approach in which households choose their search effort. However, in a model where households choose their reservation wage, the quantitative results depend crucially on the non-observable wage offer distribution whereas the results in our model are quite insensitive to the parameterization of the job search technology.

[8]See, e.g., Stokey (2009).

dress the tradeoff between insurance and incentives in an asymmetric information framework. In their framework, the government is assumed to be unable to observe the households' search behavior and thus cannot condition benefit payments on search effort. In contrast, benefit duration is observable and implementing a falling benefit profile over the unemployment spell punishes in expectation the lazy job seekers more than the diligent ones. Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) show that such an unemployment insurance system is capable of dealing with insurance and incentives optimally. In contrast, allowing for precautionary savings, Shimer and Werning (2006, 2007, 2008) challenge these results. These models on the optimal unemployment insurance are, however, partial equilibrium models since they only consider the effect of the unemployment insurance schedule on a single unemployed agent abstracting from financing issues in general equilibrium.

In order to model labor markets explicitly, the literature suggests two approaches: the search theoretic approach and the matching function approach. The search theoretic approach assumes that households either receive wage offers that are randomly drawn from a pre-specified distribution, e.g. McCall (1970), Lucas and Prescott (1974) and Ljungqvist and Sargent (1998), or that households endogenously decide on their search effort which then determines their re-employment probability in the subsequent period, e.g. Benhabib and Bull (1983) and Lentz (2009). The implementation of a more generous unemployment insurance system leads either to an increase in the reservation wage or to a reduction of the search effort, which makes these models an appropriate choice for analyzing the incentive effect of unemployment benefit payments. For tractability reasons, however, most search theoretical models rely on risk-neutral agents, which makes them unsuitable for the discussion of the insurance effect.

The matching function approach, based on Phelps (1968) and in particular Pissarides (1979), exhibits in contrast to the search theoretic approach the advantage of a more detailed description of the demand side of the economy.[9] Like the search theoretic approach, these models are often based on risk neutrality such that the insurance effect of unemployment benefit payments cannot be analyzed. Merz (1995) and Andolfatto (1996) combine the matching model with the standard real business cycle model to analyze the shock

[9]For a detailed overview of the matching model approach, see Pissarides (2000).

propagation mechanisms with sluggish labor adjustments. In contrast to our model, they assume that households consist of many individual agents that are able to pool unemployment risk such that the provision of unemployment insurance is superfluous. Recently, there are some papers that take the market incompleteness and thus the idiosyncracy of employment shocks more seriously, e.g. Costain and Reiter (2005) and Nakajima (2010) and Krusell, Mukoyama, and Sahin (2009). Due to different employment histories, households are now heterogenous with respect to their asset holdings and current employment state, which forces the authors to use involved and time-consuming numerical methods in order to simulate the equilibria of the economy. The advantage of our approach, in contrast, is the simple characterization of the equilibrium which allows more theoretical insights and imposes only a low computational burden. Costain and Reiter (2005) and Nakajima (2010) focus on the insurance effect of unemployment benefit payments, but in the absence of endogenous search effort choices in their models, the provision of more unemployment insurance does not discourage households from search. In other words, in designing the unemployment insurance system, the government does not face the tradeoff between offering insurance, on one side, and providing incentives, on the other. Launov and Wälde (2010) also present a tractable macro model with income risk and search/matching, but they do not allow workers to save.

2.3 The Economy

2.3.1 Households

Consider a discrete-time, infinite-horizon, search model of the labor market with one non-perishable all-purpose good that can be either consumed or invested. There is a continuum of ex-ante identical, infinitely-lived households with unit mass. Let $S = S_1 \times S_2$ denote the space of stochastic states, where $s_{1it} \in S_1$ is the current employment state of household i, and $s_{2it} \in S_2$ denotes an i.i.d. depreciation shock to human capital.

Preferences are time-separable and each household derives utility from consumption c_{it} and disutility from search effort l_{it}. By choosing the search intensity, unemployed agents directly determine their next-period reemployment probability. The one-period utility function is separable in consumption and search. Specifically, assume

$$u(c_{it},\ l_{it}) = \ln c_{it} - \mathbf{1}_{s_t=u}\ d(l_{it})$$

where $d(l_{it})$ denotes the disutility from search, satisfying $d'(l_{it}) > 0$ and $d''(l_{it}) \geq 0$. The indicator function $\mathbf{1}_x$ equals one when statement x is true and zero otherwise. Future utility is discounted by the time discount factor β.

Let k_{it} and h_{it} denote the stocks of physical and human capital held by household i. Employed households receive capital and labor income, $r_{kt}k_{it}$ and $r_{ht}h_{it}$, with r_{kt} and r_{ht} denoting the (gross) return to physical and human capital, respectively. We assume that the income net of depreciation of unemployed households is proportional to total asset holdings. Specifically, income is given by $b_t^q(k_{it} + h_{it})$, where benefit entitlements can be either high, $q = \hbar$, or low, $q = \ell$. This assumption guarantees that the unemployed will not shift resources from human capital to physical capital as a response to a change of the benefit rate. This excludes output effects that are solely based on the unemployed's shift from unproductive human capital to productive physical capital. Furthermore, as we will show, every household chooses the same portfolio of physical and human capital in equilibrium. This excludes substantial negative human capital investment and thus allows a straightforward interpretation of the benefit rate b_t^q as unemployment benefit.[10] The households use their net income and their current wealth position to buy consumption, which is taxed at rate τ_{ct}, and next period physical and human capital stock.

Each household i chooses a complete contingent plan $\{c_{it}, k_{i,t+1}, h_{i,t+1}, l_{it}\}_{t=0}^{\infty}$ in order to maximize his lifetime utility. Specifically, the optimization problem reads

$$\max_{\{c_{it},k_{i,t+1},h_{i,t+1},l_{it}\}_{t=0}^{\infty}} \left\{ U = \mathbb{E}_S \left[\sum_{t=0}^{\infty} \beta^t \ \left(\ln c_{it} - \mathbf{1}_{s_{1it}=u}\ d(l_{it}) \right) \right] \right\}$$

[10] There is also a different interpretation of the assumption that income of unemployed households is given by $b_t^q(k_{it} + h_{it})$: The government pays unemployment benefits $\tilde{b}_t^q\ h_{it}$ and seizes a fraction ρ_t^q of the unemployed's physical capital income and uses this revenue as an additional source to finance the unemployment benefit net depreciation. The unemployed's capital income is thus taken into account when determining the unemployment compensation. Total income net of depreciation of the unemployed is given by $((1-\rho_t^q)r_{kt} - \delta_k)\ k_{it} + (\tilde{b}_t^q - \delta_h(s_{2it}))\ h_{it}$. For simplicity, we assume that the government sets $\tilde{b}_t^q$ and ρ_t^q such that $\tilde{b}_t^q - \delta_h(s_{2it}) = (1-\rho_t^q)r_{kt} - \delta_k$ and define $b_t^q = \tilde{b}_t^q - \delta_h(s_{2it})$. The unemployed's income thus simplifies to $b_t^q(k_{it} + h_{it})$.

subject to

$$(1+\tau_{ct})\ c_{it} + k_{i,t+1} + h_{i,t+1} = \begin{cases} (1+r_{kt}-\delta_k)\ k_{it} + (1+r_{ht}-\delta_h(s_{2it}))\ h_{it}, & \text{for } s_{1it}=e \\ (1+b_t^q)\ (k_{it}+h_{it}), q \in \{\hbar, \ell\} & \text{otherwise} \end{cases}$$

$$k_{i,t+1} \geq 0$$

$$h_{i,t+1} \geq 0$$

where δ_k is the depreciation rate of physical capital and $\delta_h(s_{2it})$ denotes the stochastic depreciation rate on human capital. For convenience, let the subscript of the expectation operator indicate the space with respect to which we take the expectation.

We now discuss the space of stochastic states S (in contrast to the individual physical and human capital holdings $\{k_{it}, h_{it}\} \in \mathbb{R}_+^2$ that can be directly determined by households in the previous period) and the underlying state transition probabilities in more detail. Households are either employed, $s_{1it} = e$, or unemployed. Unemployed agents are, on the one hand, either good or bad job seekers $\{g, b\}$, and, on the other hand, either entitled to high or low unemployment benefits $\{\hbar, \ell\}$. Hence, we have to distinguish between four different unemployment states: households with good search skills that are entitled to high or low unemployment benefits, $s_{1it} = u^{g\hbar}$ and $s_{1it} = u^{g\ell}$, and households with bad job search skills who can as well be entitled to either high or low unemployment benefits, $s_{1it} = u^{b\hbar}$ and $s_{1it} = u^{b\ell}$. Taken together, the total space of employment states is given by $S_1 = \{e, u^{g\hbar}, u^{g\ell}, u^{b\hbar}, u^{b\ell}\}$.

The employment state transition is as follows: with probability σ_x, employed agents lose their job, they become unemployed and are initially both, eligible for high unemployment benefit and good job seekers. Unemployed agents exert search effort l_{it} and they find a new job in $t+1$ with probability $\pi^j(e|u^{j\hbar}; l_{it})$ respectively $\pi^j(e|u^{j\ell}; l_{it})$, for $j \in \{g, b\}$. By definition, bad job seekers that exert the same search effort as the good ones will nevertheless have a lower probability of reemployment. If, job search is not successful, they will lose, if it has not already happened before, their entitlement to high benefits with probability σ_{bt} and their good search skills with exogenously given probability σ_s. Following Shimer and Werning (2006), we interpret the shock to the search technology as depreciation of search skills. For example, search skills depreciate when households have finished searching for a job in the easily accessible proximity of their own social network and now have to consider jobs outside their network. Note that while households take σ_{bt} as exogenous to their optimization problem, the government chooses σ_{bt} as part of its labor market policy. This specification of the state transition process implies that the longer the unemployment spell, the higher the probability that households have lost their entitlement to high benefit payments and the higher the probability that households have become bad job seekers.

In addition to the employment state, there is a general independent and identical distributed depreciation shock $s_{2it} \in S_2$ on human capital. This shock is used to capture earning volatility due to e.g. promotion or changes in the working conditions and only happens to currently employed households. In contrast to the employment shock, this depreciation shock constitutes a permanent income shock. With $\bar{\delta}_h$ denoting the deterministic part of human capital depreciation, the total depreciation rate of human capital reads

$$\delta_h(s_{2it}) = \bar{\delta}_h + \mathbf{1}_{s_{1it}=e}\ s_{2it}$$

Clearly, only the current employment state has predictive power for the state in the next period: $\pi(s_{i,t+1} \mid s_{it}; l_{it}) = \pi(s_{i,t+1} \mid s_{1it}; l_{it})$.

2.3.2 Production

The production sector consists of a continuum of identical firms with neoclassical production function that uses physical and human capital to produce the *all-purpose* good that can be either consumed or invested. The production sector is competitive and can be represented by an aggregate firm whose profit function reads

$$\Pi(K_t,\ H_t^e) = F(K_t,\ H_t^e) - r_{kt}\ K_t - r_{ht}\ H_t^e$$

K_t denotes the aggregate amount of physical capital in the economy and H_t^e is the aggregate amount of human capital used in the production sector.

2.3.3 Government

The government pays out unemployment benefits $\mathbb{E}_I[b_t^q(k_{it} + h_{it}) \mid s_{1it} = u^q]$, collects consumption taxes, $\tau_{ct}\mathbb{E}_I[c_{it}]$, and seizes the unemployed's capital income, $r_{kt}\mathbb{E}_I[k_{it} \mid s_{1it} = u^q]$ for $q \in \{\mathscr{g}, \mathscr{b}\} \times \{\mathscr{h}, \mathscr{l}\}$. We assume that the government runs a balanced budget in every period. Thus, the government's budget constraint reads

$$\tau_{ct}\ \mathbb{E}_I[c_{it}] + r_{kt}\ \mathbb{E}_I[k_{it} \mid s_{1it} = u^q] = \mathbb{E}_I[b_t^q(k_{it} + h_{it}) \mid s_{1it} = u^q]$$

In addition to the consumption tax $\{\tau_{ct}\}_{t=0}^{\infty}$ and the benefit rates $\{b_t^{\mathscr{h}},\ b_t^{\mathscr{l}}\}_{t=0}^{\infty}$, the government also chooses the expected entitlement period to high benefit payments via $\{\sigma_{bt}\}_{t=0}^{\infty}$, which enters the government's budget constraint through the expectation operator. From now on, we restrict to stationary labor market policies in the sense that $\{b_t^{\mathscr{h}}, b_t^{\mathscr{l}}, \sigma_{bt}\}_{t=0}^{\infty} = (b^{\mathscr{h}}, b^{\mathscr{l}}, \sigma_b)$.

2.4 Equilibrium

In order to construct the equilibrium, we follow Krebs (2003) and transform the optimization problem into a portfolio choice problem. We also define total wealth $w_{it} \equiv k_{it} + h_{it}$ and the portfolio share of physical capital $\theta_{it} \equiv \frac{k_{it}}{k_{it}+h_{it}}$. Equipped with these definitions, the household's budget constraint and the law of motion for physical and human capital simplifies to

$$w_{i,t+1} = \begin{cases} (1 + \theta_{it}\ (r_{kt} - \delta_k) + (1 - \theta_{it})\ (r_{ht} - \delta_h(s_{2it})))\ w_{it} - (1 + \tau_{ct})\ c_{it} & \text{for } s_{1it} = e \\ (1 + b^q)\ w_{it} - (1 + \tau_{ct})\ c_{it}, q \in \{\mathscr{h}, \mathscr{l}\} & \text{otherwise} \end{cases}$$

The terms multiplying the current wealth position w_{it} are the return to total wealth and we define them as $[1 + r(\theta_{it},\ s_{it};\ r_{kt},\ r_{ht})]$ for convenience. For employed agents, this return is the portfolio weighted net return to physical and human capital. In contrast, unemployed agents just receive, at least from the perspective of households exogenous return b^q, $q \in \{\mathscr{h}, \mathscr{l}\}$. Clearly, the return to wealth for the employed households depends on the individual portfolio choice, whereas the return for unemployed agents does not.

With these definitions, the household's budget constraint simplifies further to

$$w_{i,t+1} = \left[1 + r(\theta_{it},\ s_{it};\ r_{kt},\ r_{ht})\right] w_{it} - (1+\tau_{ct})\ c_{it} \tag{2.1}$$

Instead of $\{c_{it},\ x_{kit},\ x_{hit},\ l_{it}\}_{t=0}^{\infty}$, households now directly choose $\{c_{it},\ \theta_{i,t+1}, w_{i,t+1},\ l_{it}\}_{t=0}^{\infty}$ subject to the flow budget constraint (2.1). A competitive equilibrium is defined as follows:

Definition 2.1 (Competitive Equilibrium for Given Labor Market Policy).
A competitive equilibrium for given labor market policy $(b^h,\ b^f,\ \sigma_b)$ *is*

1. *a sequence* $\{K_t,\ H_t^e\}_{t=0}^{\infty}$ *that maximizes the firm's profit for a given sequence of factor prices* $\{r_{kt},\ r_{ht}\}_{t=0}^{\infty}$*;*

2. *a sequence* $\{c_{it},\ \theta_{i,t+1},\ w_{i,t+1},\ l_{it}\}_{t=0}^{\infty}$ *that solves agent i's maximization problem for a given sequence of factor prices* $\{r_{kt},\ r_{ht}\}_{t=0}^{\infty}$*, idiosyncratic shocks* $\{s_{it}\}_{t=0}^{\infty}$ *and consumption tax rates* $\{\tau_{ct}\}_{t=0}^{\infty}$*;*

3. *a sequence* $\{r_{kt},\ r_{ht}\}_{t=0}^{\infty}$ *that satisfies market clearing on the input factor market,* $\mathbb{E}_I[k_{it}] = K_t$ *and* $\mathbb{E}_I[h_{it} \mid s_{1it} = e] = H_t^e$*; and*

4. *a sequence of consumption tax rates that satisfies the balanced budget constraint of the government* $\{\tau_{ct}\}_{t=0}^{\infty}$.

From now on, we focus on a stationary equilibrium as defined in the next proposition:

Definition 2.2 (Stationary Equilibrium).
A competitive equilibrium for given labor market policy is stationary if

1. *the returns to physical and human capital are stationary,* $r_{kt} = r_k$ *and* $r_{ht} = r_h$*,*

2. *the tax policy is stationary,* $\tau_{ct} = \tau_c$*, and*

3. *the flow into the different employment states is equal to the flow out of them.*

Let us start with the firm's optimization problem. Due to competitive markets, the usual marginal product conditions for profit maximization apply. Define the aggregate capital-to-labor ratio that is used in production $\tilde{K}_t = \frac{K_t}{H_t^e}$ and define the production technology in intensive form $f(\tilde{K}_t) = F(\tilde{K}_t, 1)$. The conditions for profit maximization in the stationary equilibrium then read

$$r_k = f'(\tilde{K}_t) \tag{2.2}$$
$$r_h = f(\tilde{K}_t) - \tilde{K}_t\ f'(\tilde{K}_t) \tag{2.3}$$

Stationarity of the factor prices immediately reveals $\tilde{K}_t = \tilde{K}$. Thus, the total investment return can be more compactly written as $r(\theta_{it},\ s_{it};\ r_k,\ r_h) = r(\theta_{it},\ s_{it};\ \tilde{K})$.

We now consider the maximization problem of the households. The Bellman equation associated with the household's optimization problem is

$$V(\theta_{it},\ w_{it},\ s_{it}) = \max_{c_{it},\theta_{i,t+1},w_{i,t+1},l_{it}} \left\{ \ln c_{it} - \mathbf{1}_{s_{1it}=u}\ d(l_{it}) + \beta\ \mathbb{E}_S\Big[V(\theta_{i,t+1},\ w_{i,t+1},\ s_{i,t+1}\Big]\right\} \tag{2.4}$$

subject to the flow budget constraint (2.1). The first-order conditions with respect to $w_{i,t+1}$, $\theta_{i,t+1}$ and l_{it} are

$$\frac{1}{c_{it}} = \beta\ \mathbb{E}_S\left[\frac{1 + r(\theta_{i,t+1},\ s_{i,t+1};\ \tilde{K})}{c_{i,t+1}}\right] \tag{2.5}$$
$$0 = \mathbb{E}_S\left[\frac{(r_k - \delta_k) - (r_h - \delta_h(s_{2i,t+1}))}{c_{i,t+1}}\right] \tag{2.6}$$
$$d'(l_{it}) = \beta\ \frac{\partial \pi^{j\prime}(e|u^{jm}; l_{it})}{\partial l_{it}}\ \mathbb{E}_{S_2}\Bigg[V(\theta_{i,t+1},\ w_{i,t+1},\ s_{1i,t+1} = e,\ s_{2i,t+1}) - \sum_{q\in\{g,b\}\times\{h,\ell\}} \pi(s_{1i,t+1} = u^q \mid s_{1it}; l_{it})\ V(\theta_{i,t+1},\ w_{i,t+1},\ s_{1i,t+1} = u^q,\ s_{2i,t+1})\Bigg], \quad j \in \{g, b\}, m \in \{h, \ell\} \tag{2.7}$$

The Euler equation (2.5) has the usual interpretation that the household's utility loss today of investing one more unit of the consumption good is equal to the utility gain tomorrow of doing so. The intra-temporal first-order condition (2.6) states that the household must be indifferent between investing one more unit into physical capital and one more unit into human capital. Finally, equation (2.7) requires that the utility loss today of searching one more unit is equal to the expected utility gain tomorrow of doing so. Any plan

$\{c_t,\ w_{t+1},\ \theta_{t+1},\ l_{t+1}\}_{t=0}^{\infty}$ that solves the system of first-order conditions equations (2.5) to (2.7), the budget constraint (2.1) and the corresponding transversality condition, is a solution to the household's constrained utility maximization problem.[11] It is easy to verify that the consumption and saving functions

$$c_{it} = \frac{1-\beta}{1+\tau_c}\ (1 + r(\theta_{it},\ s_{it};\ \tilde{K}))\ w_{it} \tag{2.8}$$

$$w_{i,t+1} = \beta\ (1 + r(\theta_{it},\ s_{it};\ \tilde{K}))\ w_{it} \tag{2.9}$$

jointly solve the budget constraint (2.1) and the Euler equation (2.5). Using these policy functions with the method of guess and verify, we can show that:

Proposition 2.1 (Lifetime Utility)**.**
The value function $V(\theta_{it},\ w_{it},\ s_{it})$ *that solves the respective Bellman equation is given by*

$$V(\theta_{it},\ w_{it},\ s_{it}) = \frac{1}{1-\beta} \ln[(1 + r(\theta_{it},\ s_{it};\ \tilde{K}))\ w_{it}] + B(s_{1it}) \tag{2.10}$$

where $B(s_{1it})$ *solves the Bellman equation in intensive form*

$$B(s_{1it}) = \max_{\theta_{i,t+1}, l_{it}} \Bigg\{ \ln \frac{1-\beta}{1+\tau_c} + \frac{\beta}{1-\beta} \ln\beta - \mathbf{1}_{s_{1it}=u}\ d(l_{it}) + \beta\ \mathbb{E}_S\left[\frac{1}{1-\beta} \ln(1 + r(\theta_{i,t+1},\ s_{i,t+1};\ \tilde{K})) + B(s_{1i,t+1})\right]\Bigg\} \tag{2.11}$$

Proof. See appendix. □

Using the consumption policy (2.8), the first-order conditions with respect to $\theta_{i,t+1}$ simplify to

$$0 = \mathbb{E}_{S_2}\left[\frac{(r_k - \delta_k) - (r_h - \delta_h(s_{2i,t+1}))}{1 + r(\theta_{i,t+1},\ s_{i,t+1};\ \tilde{K})}\right] \tag{2.12}$$

Note that since the return to wealth for unemployed households does not depend on their individual portfolio composition, the first-order condition with respect to the portfolio choice, equation (2.12), is independent of the transition probabilities and thus, independent

[11] In proposition 2.1, we solve for the value function that is associated with the plan $\{c_{it},\ w_{i,t+1},\ \theta_{it},\ l_{it}\}_{t=0}^{\infty}$. The value function is finite and thus, the transversality condition holds.

of the current employment state s_{1it} and the search effort l_{it}. Moreover, this condition is also independent of the current portfolio share and the household's current wealth, leading to the conclusion that every agent chooses the same portfolio, independent of his individual shock history and wealth. This clearly defines a policy function $\theta_{i,t+1} = \theta(\tilde{K})$.

The consumption policy (2.8) and the Bellman equation in intensive form (2.10) help to transform the first-order condition with respect to the search effort decision into

$$v'(l_{it}) = \beta \, \frac{\partial \pi^{j'}(e|u^{jm}; l_{it})}{\partial l_{it}} \, \mathbb{E}_{S_2} \Bigg[\left(\frac{\ln(1 + r(\theta_{i,t+1},\ s_{1i,t+1} = e,\ \tilde{K}))}{1-\beta} + B(s_{1i,t+1} = e) \right)$$
$$- \sum_{q \in \{\mathcal{g},\mathcal{b}\} \times \{\mathcal{h},\mathcal{l}\}} \pi(s_{1i,t+1} = u^q | s_{1it}; l_{it}) \left(\frac{\ln(1 + r(\theta_{i,t+1},\ s_{1i,t+1} = u^q,\ \tilde{K}))}{1-\beta} + B(s_{1i,t+1} = u^q) \right) \Bigg],$$
$$j \in \{\mathcal{g}, \mathcal{b}\}, m \in \{\mathcal{h}, \mathcal{l}\} \quad (2.13)$$

Note that this condition is independent of wealth, the current portfolio and the current realization of the i.i.d. depreciation shock s_{2it}. Thus, conditional on the employment status, s_{1it}, every household chooses the same search intensity. This defines a function $l_{it} = l(s_{1it} = u^q;\ \tilde{K})$, for $q = \{\mathcal{n}, \mathcal{s}\} \times \{\mathcal{h}, \mathcal{l}\}$. Our result is closely related to Shimer and Werning (2008) who consider a partial equilibrium consumption savings model in which unemployed households choose their reservation wages. Under CARA-preferences, they show that the choice of the reservation wage (which is equivalent to the search effort choice in our model) is wealth-independent with strong implications for the optimal unemployment benefit scheme. The wealth independence in our model, however, is based on the combination of more general homothetic preferences and disposable income which is linear homogenous in the agent's asset holdings. Market clearing on the input factor market requires that the households' supply of physical and human capital is consistent with the firm's demand for the two input factors. Thus, market clearing satisfies

$$\tilde{K} = \frac{\mathbb{E}_I[(1 - \theta(s_{1it};\ \tilde{K}))\ w_{it} \mid s_{1it} = e]}{\mathbb{E}_I[\theta(s_{1it};\ \tilde{K})\ w_{it}]} \quad (2.14)$$

Although in equilibrium aggregate wealth grows infinitely at a constant rate, the wealth of type s_{1t} households relative to aggregate wealth is constant. Hence, as will be shown in the appendix, the market clearing condition (2.14) depends on the wealth ratios but is independent of the absolute wealth level.

Finally, using the households' policy functions and market clearing, it is easy to verify that the government's budget constraint is independent of the absolute wealth level. Moreover, since the policy functions for saving, portfolio choices and search decisions, as well as the market clearing condition, are independent of the consumption tax rate, it is trivial to choose the consumption tax rate τ_c such that the government budget is satisfied. In particular, the choice of the consumption tax rate does not distort the equilibrium allocations.

The previous discussion is summarized in the following proposition:

Proposition 2.2 (Characterization of Competitive Equilibrium).
A stationary competitive equilibrium for given labor market policy, $(b^{h},\ b^{\ell},\ \sigma_b)$*, is characterized by*

1. *The firms' problem satisfies the usual marginal product conditions, equations* (2.2) *and* (2.3).

2. *The households' consumption and saving policies are linear homogenous in wealth and given by equations* (2.8) *and* (2.9). *Conditional on the employment state, every agent chooses the same wealth independent portfolio and search effort decision. In particular, the portfolio choice and search effort decision jointly solve equations* (2.12) *and* (2.13).

3. *Market clearing satisfies* (2.14) *and is independent of the absolute wealth level in the economy.*

4. *The consumption tax rate does not distort the above characterized equilibrium and solves the government's budget constraint.*

Observe that despite the *ex-post* heterogeneity, which makes solutions to dynamic general equilibrium models very complicated and time-consuming, we found a very simple solution within our framework, .in which the equilibrium allocation can be characterized without solving for the complete underlying wealth distribution.

2.5 Quantitative Analysis

2.5.1 Calibration

We calibrate our model economy such that the equilibrium is consistent with quarterly German data of the pre-reform period. The pre-2005 system was characterized by a rather long period of *Unemployment Benefit* entitlements and an essentially unlimited means-tested *Unemployment Assistance* after the eligibility to *Unemployment Benefit* entitlements expired. *Unemployment Benefit* was between 60 and 67 percent of the previous net income whereas *Unemployment Assistance* laylaid between 53 and 57 percent of previous net income.[12] If benefit payments were below the minimum level of subsistency, *Social Assistance* was used to meet the additional need. Taking this into account, the OECD (2006) calculates effective average net replacement rates of about 69 percent for both, *Unemployment Benefit* and *Unemployment Assistance*. Hence, the two-tiered unemployment insurance system was effectively a one-tiered system. Based on Schmitz and Steiner (2007), we calculate an average eligibility period for high benefit payments of 19.3 months which translates into $\sigma_b = 0.1554$.[13] This accomplishes the calibration of the government's pre-reform policy parameters.

Having already calibrated σ_b, we now focus on the determination of the remaining state transition rates: Since our setup abstracts from non-participation in the labor market, we have to adjust the employment-to-unemployment flows by the employment-to-non-participation flows. However, the employment-to-non-participation flows also include old households who decide to retire early, young households who return to school in order to accomplish their formal education and women who decide to take a maternity leave. These cases cannot be counted as job loss in a narrow sense, and if we would include them, our job loss rate would be upward biased. To avoid these issues, we only take the transition rates from employment to unemployment and from employment to non-participation of 25 to 55 year old males as the job loss rate. Using the calculations by Bachmann (2005) both rates add up to approximately one percent per month which yields $\sigma_x = 0.03$ per quarter. Jung and Kuhn (2010) find transition rates in the same range. For simplicity,

[12] See for example Schmitz and Steiner (2007).

[13] In particular, we assume a uniform distribution of households aged between 25 and 64 years and calculate the average (maximal) entitlement period for this age group.

Table 2.1: Calibration - Exogenous Parameters

PARAMETER	DESCRIPTION	VALUE
PREFERENCES		
A	parameter of disutility of search	25
PRODUCTION		
α	capital share	0.3600
DEPRECIATION AND DEPRECIATION SHOCKS		
δ_k	depreciation rate: physical capital	0.0150
δ_h	depreciation rate: human capital if employed	0.0150
$\mu_{s_{2t}}$	expectation of i.i.d. shock	0
$\sigma_{s_{2t}}$	standard deviation of i.i.d. shock	0.1500
LABOR MARKET AND TRANSITION RATES		
$b^{\hbar}$	unemployment benefit rate: high entitlement	0.6900
b^{ℓ}	unemployment benefit rate: low entitlement	0.6900
σ_x	job separation probability	0.0300
σ_b	probability of loosing high benefit	0.1554
σ_s	arrival rate of search technology shock	0.2500

we associate the search skill depreciation shock with long-term unemployment, which is usually defined as an unemployment spell of at least one year. This yields a probability of losing job search skills of $\sigma_s = 0.25$. For the job search technology, we follow Lentz (2009) and use an exponential specification

$$\pi^j(e \mid s_{1it}; l_{it}) = 1 - \mathrm{e}^{-\lambda^j l_{it}}, \quad \text{for } j \in \{g, b\}$$

where $\lambda^g > \lambda^b$. The search technology parameters are determined such that the equilibrium unemployment rate is 7.5 percent and the equilibrium share of long-term unemployed households to total unemployed households is 42 percent. The calibration values of the search technology parameters depend on the equilibrium search effort which in turn depends on the specification of the disutility search.

The preference parameters are calibrated as follows: As in Hopenhayn and Nicolini (1997) or Shimer and Werning (2006, 2007, 2008), disutility of search is linear in search effort

$$d(l_{it}) = A\ l_{it} + \omega$$

where ω denotes a fixed disutility of being unemployed. In equilibrium, the parameter A is not separately identified from the parameters of the search technology λ^{g} and λ^{b}. Consequently, there is one degree of freedom such that we can set the scaling parameter to a numerically convenient value of $A = 25$. This implies $\lambda^{g} = 7.2606$ and $\lambda^{b} = 2.9999$ in order to make the equilibrium match our calibration targets. The fixed disutility of being unemployed, ω, is calibrated to match the point elasticity of the job finding rate with respect to benefit payments. Empirically, this elasticity is hard to pin down, because the data sets either do not include the required information to construct a precise measure of benefit payments (IAB data), or there are too few observations to get reliable results (GSOEP). Addison, Centeno, and Portugal (2010) use a structural search model and the European Community Household Panel (ECHP) to estimate the elasticity for several European countries, and for Germany they find values between $\eta_{\theta,b} = -1.66$ and $\eta_{\theta,b} = -1.14$. For the US, Meyer and Mok (2007) use a quasi-experimental setup in which the maximum weekly benefit payments in New York State were raised. Their approach allows for the construction of different control groups of households leading to substantial variation in the data to get reliable results.[14] They find that for the US, increasing the benefit rate by 1 percent leads to an increase in benefit duration by 0.21 percent, and translating this number into an elasticity of the reemployment probability with respect to benefit payments yields approximately $\eta_{\theta,b} = -0.2$. Clearly, the benefit level in the US is much lower than in Germany, which implies that the elasticity in Germany has to be higher in absolute terms. However, Addison, Centeno, and Portugal (2010) also estimate the respective elasticity for the UK, which has labor market institutions comparable to the US. For the UK, they find elasticities between $\eta_{\theta,b} = -0.62$ and $\eta_{\theta,b} = -0.36$, in absolute terms higher that the esti-

[14] In particular, households can be separated into three groups: i.) those who are not affected by this policy since they were not eligible to the maximum weekly benefit payments under the old regime, ii.) those who are partially affected in the sense that their new weekly benefit payments lie between the old and the new maximum weekly benefit level and iii.) those who are now eligible for the maximum benefit level under the new regime.

mates by Meyer and Mok (2007) for the US, indicating that the estimates for Germany are upward biased, in absolute terms. For this reason, we take the lower bound, $\eta_{\theta,b} = -1.14$ for the benchmark calibration, yielding $\omega = 0.2668$. Since the elasticity has a key role in determining the effect of the labor market reforms on the aggregate unemployment rate, we will run a sensitivity analysis for lower elasticities as well. Finally, the time-discount factor β is set such that the aggregate private saving rate in equilibrium is 20 percent. This yields $\beta = 0.9799$.

Table 2.2: Calibration - Endogenous Parameters

PARAMETER	DESCRIPTION	VALUE
PREFERENCES		
β	time preferences	0.9799
ω	disutility of being unemployed	0.2668
PRODUCTION		
z	productivity	0.0794
LABOR MARKET AND TRANSITION RATES		
λ^g	search technology parameter: good job seeker	7.2606
λ^b	search technology parameter: bad job seeker	2.9999
PARAMETERS ARE CHOSEN TO MATCH		
	aggregate saving rate	0.2000
	aggregate quarterly consumption growth rate	0.0051
	unemployment rate	0.0750
	share of long-term unemployment	0.42
	average benefit elasticity of reemployment probability	-1.1400

We calibrate the depreciation rates to $\delta_k = \delta_h = 0.015$, which is approximately six percent per annum. For physical capital, this value lies within the range suggested by the literature. For human capital, Browning, Hansen, and Heckman (1999) find annual depreciation rates between zero and four percent. Accounting for the infinite horizon structure in our model, we have to add an additional depreciation of two percent. Thus, a human capital depreciation rate of six percent corresponds to an upper bound of reasonable values suggested in the literature. The i.i.d. depreciation shock to human capital is normally distributed with mean zero and standard deviation $\sigma_{s_{2t}} = 0.15$, which, together with the employment shocks and loss of job specific skills, implies a standard deviation of labor income in equilibrium that is in line with micro-evidence for Germany, estimated by Krebs and Yao (2010).

Finally, the production technology is Cobb-Douglas

$$F(K_t, H_t^e) = z \left(K_t\right)^{\alpha} \left(H_t^e\right)^{1-\alpha}$$

with the capital share of output set to $\alpha = 0.36$. The scaling parameter of the production technology is chosen such that the annual equilibrium growth rate of aggregate consumption is two percent. This gives $z = 0.0794$.

2.5.2 Growth and Welfare Effect of German Labor Market Reform

The first reform, which was implemented in January 2005, replaced the *Unemployment Assistance* with *Unemployment Benefit II*, which requires tighter means tests and is independent of previous earnings.[15] Mapping this new system into our model (where unemployment benefits depend on the stock of human capital), average benefit payments in the second tier decrease substantially to about 45 percent of previous net earnings. The regulations of the second reform became binding in February 2006. The eligibility period for *Unemployment Benefit I*[16] was reduced for all unemployed households, and a particularly strong reduction was implemented for older unemployed agents. Based on Schmitz and Steiner (2007) we calculate that the average eligibility period dropped from 19.3 to 13.5 months, thus σ_b increases from 0.1554 to 0.2222.

The macroeconomic effects of both reforms are given in table 2.3. The main findings

[15]In fact, by the introduction of *Unemployment Benefit II*, *Unemployment Assistance* and *Social Assistance* were merged.

[16]The pre-reform *Unemployment Benefit* was relabeled as *Unemployment Benefit I*, in order to make the distinction between the newly introduced *Unemployment Benefit II.*

Table 2.3: Macroeconomic Effects of the Labor Market Reforms

	BENCHMARK	REFORM 1	REFORM 2
unemployment rate	7.50%	6.38%	6.25%
share of long-term unemployment	42.0%	32.8%	31.7%
annualized growth rate	2.00%	2.06%	2.08%
level effect on consumption[1]	0.00%	0.96%	1.12%
consumption tax rate	3.66%	2.69%	2.54%
capital-to-labor ratio	0.6950	0.6943	0.6927

[1] Deviation form benchmark in percent.

are as follows: first, implementing the 2005-reform leads to a substantial decrease in the equilibrium unemployment rate from 7.5 to 6.4 percent. Clearly, unemployed households in the second tier, that means those who already lost their entitlement to high benefit payments, increase their search effort in order to escape the state with low benefits more quickly. Furthermore, for unemployed households who are still eligible for high benefit payments, losing their entitlement becomes more threatening now, wherefore they increase their search effort, as well. Decomposing the contribution of both mechanisms to the decrease in the unemployment rate reveals that 57 percent of the decrease is due to the reaction of the households who directly lost their entitlement to high benefit payments. The remaining 43 percent are explained by search effort adjustments due to the increasing threat of loosing high entitlements in the near future. The adjustments of the individual search effort decisions to the labor market reforms are given in table 2.4.

When the eligibility period is reduced according to the second reform, those households who still enjoy the high benefit rate will intensify their job search to avoid losing their entitlements to high benefit payments. However, in order of magnitude, this effect is not very important, and the unemployment rate decreases by 0.13 percentage points with respect to the first reform. Put differently, the search effort decision and thus the re-employment probability is quite insensitive to the duration of high benefit entitlements. This finding is consistent with recent empirical research, e.g. by Caliendo, Tatsiramos, and Uhlendorff

(2009) and Fitzenberger and Wilke (2007). In particular, Caliendo, Tatsiramos, and Uhlendorff (2009) find that the reemployment probability peaks only for those households who are close to the exhaustion period of high benefit entitlements. Thus, if we reduce the eligibility period, only the households who become close to the new exhaustion period will raise their search effort, whereas the other households' search effort decision is almost unaffected. Our model, however, abstracts from the exhaustion period effect, since every unemployed household with high entitlements faces the same expected period of remaining entitled to high benefits, $\frac{1}{\sigma_b}$ quarters. Thus, no unemployed agent is close to the exhaustion period, making the adjustment of search effort negligible, and the equilibrium unemployment rate is hardly affected by the implementation of the second reform. Since we abstract from the exhaustion period effect, our results for the second reform have to be interpreted more cautiously as a lower bound.

Table 2.4: Household Policies

	BENCHMARK	REFORM 1	REFORM 2
θ	0.3916	0.3941	0.3943
$\pi^n(l(u^{gh}))$	0.5172	0.5379	0.5427
$\pi^n(l(u^{gl}))$	0.5172	0.5672	0.5670
$\pi^s(l(u^{bh}))$	0.1667	0.2137	0.2233
$\pi^s(l(u^{bl}))$	0.1667	0.2618	0.2616

Second, the average consumption growth rate increases by 0.06 and 0.08 percentage points on an annual basis for reform one and reform two, respectively. For the average consumption growth rate, there are two opposing forces at work. On the one hand, human capital risk increases and discourages households to accumulate human capital, which leads to a downward pressure on the aggregate consumption growth rate. On the other hand, there are more employed households in the new equilibrium who accumulate human capital at higher rates than unemployed households. This leads to an upward pressure on the aggregate consumption growth rate. In our numerical example, the second effect dominates the first one. In a similar vein, we find that the equilibrium capital-to-labor ratio is almost unaffected by the labor market reforms since there are two opposing forces at work. Discouraging human capital investment obviously raises $\tilde{K}$ while the employment effect tends to raise the absolute amount of human capital used in production such that $\tilde{K}$ decreases. Numerically, both effects almost offset each other.

Third, the increase in average consumption growth is accompanied by a considerable level effect on equilibrium consumption. In particular, the decreasing unemployment rate leads to an increase in production which finally allows an upward shift of the consumption path by 0.96 and 1.12 percent for reform 1 and 2, respectively. From a different point of view, we see that the reduction of the marginal benefit rate in the second tier, reform one, the reduction of the eligibility period to high benefit payments in the first tier, reform two, and the decrease in the total unemployment rate substantially reduce the total amount of benefit payments. Hence, the government needs less tax revenue in order to meet its balanced budget constraint, wherefore it reduces the consumption tax rate from 3.66 to 2.69 and 2.54 percent for labor market reforms one and two, respectively. Clearly, reducing the cost of consumption heaves the consumption path to a higher level.

Table 2.5: Welfare Effect of Labor Market Reforms

	REFORM 1			REFORM 2		
	WELFARE	LEVEL	INSURANCE	WELFARE	LEVEL	INSURANCE
Δ	0.41%	1.93%	−1.52%	0.45%	2.02%	−1.57%
$\Delta \mid s_{1t} = e$	0.48%	1.93%	−1.45%	0.52%	2.02%	−1.50%
$\Delta \mid s_{1t} = u^{gh}$	0.17%	1.93%	−1.76%	0.18%	2.02%	−1.84%
$\Delta \mid s_{1t} = u^{g\ell}$	−0.32%	1.93%	−2.15%	−0.27%	2.02%	−2.29%
$\Delta \mid s_{1t} = u^{bh}$	−0.53%	1.93%	−2.46%	−0.63%	2.02%	−2.65%
$\Delta \mid s_{1t} = u^{b\ell}$	−1.70%	1.93%	−3.63%	−1.65%	2.02%	−3.67%

Considering social welfare, which we define as the equally weighted average of the households' lifetime utility, there are again two opposing forces at work when we implement the labor market reforms. On the one hand, households enjoy a tax cut which allows them to consume more in each period and, thus, raises their lifetime utility. On the other hand, reducing benefit payments and shortening the entitlement period to high benefit payments increase the individual income risk which leads to losses of lifetime utility when households are risk averse. In order to quantify the welfare effects, we follow Lucas (1987) and ask the households in the pre-reform state how much additional consumption do they need in each period in order to be indifferent between implementing the reform or not. Specifically, let Δ denote the respective percentage share satisfying

$$\mathbb{E}_S \left[\sum_{t=0}^{\infty} \beta^t \ \ln((1+\Delta)\ c_t) \right] = \mathbb{E}_S \left[\sum_{t=0}^{\infty} \beta^t \ \ln c_t^{ref} \right]$$

where $\{c_t\}_{t=0}^{\infty}$ denotes the households' consumption plans without labor market reforms and $\{c_t^{ref}\}_{t=0}^{\infty}$ are the consumption plans when the reform is implemented in period 0. In table 2.5, we report the welfare effects[17] and find substantial welfare gains of 0.41 and 0.45 for reform 1 and reform 2, respectively. Hence, the welfare improving level effect of consumption dominates the welfare reducing effect from losing insurance. Clearly, the currently employed households benefit most from the labor market reforms since the loss of insurance imposes only second order risk on them in the sense that they first have to become unemployed before being directly exposed to the risk of losing the high entitlements. More surprisingly, those unemployed agents who receive the high benefit payments and are good job seekers realize a slightly positive welfare gain from the labor market reforms. Hence, for them, it still holds that the level effect of a higher consumption path dominates the loss of insurance. For the other types of unemployed agents, however, the loss of insurance dominates, leading to substantial welfare losses. Table 2.5 also reports the decomposition of the welfare effect into level and insurance effect. Further reductions in the benefit rate, however lead only to negligible additional welfare gains, if at all.

Clearly, the reaction of the equilibrium unemployment rate to the labor market reforms depends crucially on the elasticity of the job finding probability with respect to the benefit rate. The more elastic the job finding probability, the stronger the decrease in the unemployment rate which finally leads to a stronger increase in aggregate production and social welfare. In other words, the more elastic the job finding probability, the stronger the level effect of consumption on social welfare. To assess the importance of this elasticity with respect to our results, we recalibrate the model to a target elasticity of $\eta_{\pi,b} = -1.0$

[17] For the computation of the welfare effects, we take the transition phase into account. Details on the computation are deferred to the appendix.

Table 2.6: Macroeconomic Effects of the Labor Market Reform 1 - Different Target Elasticities

	BENCHMARK	$\eta_{\pi,b} = -1.14$	$\eta_{\pi,b} = -1$	$\eta_{\pi,b} = -0.5$
unemployment rate	7.50%	6.38%	6.46%	6.82%
share of long-term unemployment	42.0%	32.8%	33.5%	36.6%
annualized growth rate	2.00%	2.06%	2.05%	2.03%
level effect on consumption[1]	0.00%	0.96%	0.93%	0.83%
consumption tax rate	3.66%	2.69%	2.72%	2.81%
capital-to-labor ratio	0.6950	0.6943	0.6944	0.6946

[1] Deviation form benchmark in percent.

and $\eta_{\pi,b} = -0.5$ which is already in the range of values estimated for the United Kingdom and thus, a lower bound (in absolute terms) for our analysis. Since the second reform has only negligible effects, we focus on the first reform only. The macroeconomic effects of the re-calibrated model are given in table 2.6. As expected, the more inelastic the job finding probability with respect to benefit payments, the higher the unemployment rate: setting the elasticity to -1.0 yields an equilibrium unemployment rate of 6.5 percent and for an elasticity of -0.5, the unemployment only decreases to 6.8 percent. The smaller employment effect on aggregate output finally translates into lower welfare effects that are given in table 2.7.

2.6 Conclusions

We develop a dynamic stochastic general equilibrium model with labor market search and incomplete markets which remains despite *ex-post* heterogenous agents tractable in the sense that the equilibrium can be characterized without knowing the underlying wealth distribution. The model allows the analysis of the unemployment insurance's major tradeoff between insuring households against earning and consumption volatility and providing an incentive to exert search effort. In contrast to the existing literature, our model also

Table 2.7: Welfare Effect of Labor Market Reform 1 - Different Target Elasticities

	$\eta_{\pi,b} = -1.14$	$\eta_{\pi,b} = -1.0$	$\eta_{\pi,b} = -0.5$
Δ	0.41%	0.38%	0.23%
$\Delta \vert s_{1t} = e$	0.48%	0.45%	0.31%
$\Delta \vert s_{1t} = u^{gh}$	0.17%	0.14%	−0.03%
$\Delta \vert s_{1t} = u^{g\ell}$	−0.32%	−0.36%	−0.55%
$\Delta \vert s_{1t} = u^{bh}$	−0.53%	−0.59%	−0.84%
$\Delta \vert s_{1t} = u^{b\ell}$	−1.70%	−1.77%	−2.10%

considers long-run growth effect of the unemployment insurance system.

Applying the model to evaluate the welfare and growth effects of the recent labor market reforms in Germany, we find that society as a whole benefits from these reforms and, furthermore, even short-term unemployed benefit since the loss of insurance is dominated by the employment effect. The results remain quite robust throughout variations of the critical calibration target, the elasticity of the job finding rate with respect to the benefit level. Although this reform yields substantial welfare gains, a further decrease in the benefit rate in the second tier only causes negligible additional welfare gains. In other words, at this point, the social welfare function is already quite flat in the space of benefit rates.

Further research is devoted to the analytical derivation of optimal unemployment schedules with explicit focus on the equilibrium growth effects of those reforms. The analysis of optimal unemployment insurance is feasible due to the straightforward simple characterization of the stationary equilibrium in our model.

2.A Appendix: Proof of Proposition 2.1

In order to find the value function, we apply the method of guess and verify. For convenience, we suppress the dependency of the coefficients and the return functions on the aggregate state $\tilde{K}$. In particular, guess

$$V(\theta_{it},\ w_{it},\ s_{it}) = A(s_{1it})\ \ln[(1 + r(\theta_{it},\ s_{it}))\ w_{it}] + B(s_{1it}) \tag{2.15}$$

Together with the optimal consumption policy, the Bellman equation (2.4) reads

$$\begin{aligned} A(s_{1it})\ \ln[(1 + r(\theta_{it},\ s_{it}))\ w_{it}] + B(s_{1it}) = \max_{\theta_{i,t+1}, l_{it}} \Bigg\{ \ln \frac{1-\beta}{1+\tau_c} + \ln[(1 + r(\theta_{it},\ s_{it}))\ w_{it}] \\ - \mathbf{1}_{s_{1it}=u}\ d(l_{it}) + \beta\ \mathbb{E}_S \Big[A(s_{1i,t+1}) \ln[(1 + r(\theta_{i,t+1},\ s_{i,t+1}))\ w_{i,t+1}] + B(s_{1i,t+1}) \Big] \Bigg\} \end{aligned}$$

Combining the optimal consumption policy and the budget constraint yields $w_{i,t+1} = \beta\ (1 + r(\theta_{it},\ s_{it}))\ w_{it}$. Thus

$$\begin{aligned} & A(s_{1it})\ \ln[(1 + r(\theta_{it},\ s_{it}))\ w_{it}] + B(s_{1it}) \\ & \quad = \max_{\theta_{i,t+1}, l_{it}} \Bigg\{ (1 + \beta\ \mathbb{E}_S[A(s_{1i,t+1})])\ \ln[(1 + r(\theta_{it},\ s_{it}))\ w_{it}] + \ln \frac{1-\beta}{1+\tau_c} \\ & + \beta\, \mathbb{E}_S[A(s_{1i,t+1})]\ \ln \beta - \mathbf{1}_{s_{1it}=u}\ d(l_{it}) + \beta\, \mathbb{E}_S \Big[A(s_{1i,t+1})\ \ln(1 + r(\theta_{i,t+1},\ s_{i,t+1})) + B(s_{1i,t+1}) \Big] \Bigg\} \end{aligned}$$

Now suppose, we already found a solution for $\theta_{i,t+1}$ and l_{it} that are independent of current state variables $\theta_{it},\ w_{it}$ and s_{2it}. Then, the method of undetermined coefficients gives

$$A(s_{1it}) = \frac{1}{1-\beta}$$

and the Bellman equation then simplifies to

$$\begin{aligned} B(s_{1it}) = \max_{\theta_{i,t+1}, l_{it}} \Bigg\{ \ln \frac{1-\beta}{1+\tau_c} + \frac{\beta}{1-\beta} \ln \beta - \mathbf{1}_{s_{1it}=u}\ d(l_{it}) \\ + \beta\ \mathbb{E}_S \Big[\frac{1}{1-\beta} \ln(1 + r(\theta_{i,t+1},\ s_{i,t+1})) + B(s_{1i,t+1}) \Big] \Bigg\} \end{aligned} \tag{2.16}$$

Obviously, the optimal portfolio choice as well as the optimal search intensity that solve the Bellman equation in intensive form, equation (2.16) are independent of the current state variables θ_{it}, w_{it} and s_{2it}. To put it differently: $\theta_{i,t+1}$ and l_{it} only depend on s_{1it} and s_{3it} and the exogenous model parameters, which is consistent with our previous conjecture. The optimal policies transform the functional equation (2.16) into the respective plan equation such that we can easily solve for $B(s_1)$, thereby verifying our initial guess on the functional form of the value function, equation (2.15). This completes our proof.

2.B Appendix: Market Clearing Condition

Let $s^t = (s_t, s_{t-1}, s_{t-2}, \dots)$ denote the history of shocks, which describes the history of type-s^t-agent. Total wealth in the economy at $t+1$ is defined as

$$W_{t+1} = \sum_{s^t} w_{t+1}(s^t)\ \pi(s^t) \tag{2.17}$$

The aggregate wealth of those people with (s_t, s_{t-1}) reads

$$W_{t+1}(s_t, s_{t-1}) = \pi(s_t \mid s_{t-1}) \sum_{s^{t-2}} w_{t+1}(s^t)\ \pi(s_{t-1} \mid s^{t-2})\ \pi(s^{t-2}) \tag{2.18}$$

and their share of total wealth is defined as

$$\begin{aligned}
\rho_{t+1}(s_t, s_{t-1}) &= \frac{W_{t+1}(s_t, s_{t-1})}{W_{t+1}} = \frac{\pi(s_t \mid s_{t-1})\ \sum_{s^{t-2}} w_{t+1}(s^t)\ \pi(s_{t-1} \mid s^{t-2})\ \pi(s^{t-2})}{\sum_{s^t} w_{t+1}(s^t)\ \pi(s^t)} \\
&= \frac{\sum_{s^{t-1}} w_t(s^{t-1})\ \pi(s^{t-1})}{\sum_{s^t} w_{t+1}(s^t)\ \pi(s^{t-1})}\ \frac{\pi(s_t \mid s_{t-1})\ \sum_{s^{t-2}} w_{t+1}(s^t)\ \pi(s_{t-1} \mid s^{t-2})\ \pi(s^{t-2})}{\sum_{s^{t-1}} w_t(s^{t-1})\ \pi(s^{t-1})} \\
&= \frac{W_t}{W_{t+1}}\ \frac{\pi(s_t \mid s_{t-1})\ \sum_{s^{t-2}} w_{t+1}(s^t)\ \pi(s_{t-1} \mid s^{t-2})\ \pi(s^{t-2})}{\sum_{s^{t-1}} w_t(s^{t-1})\ \pi(s^{t-1})}
\end{aligned} \tag{2.19}$$

where we suppressed the dependency of the conditional probabilities from individual search effort decisions, for convenience.

We are now interested in how the wealth shares $\rho_{t+1}(s_t, s_{t-1})$ evolve over time. By the optimal consumption policy (2.8), we know that the law of motion of wealth in equilibrium is governed by $w_{i,t+1} = \beta\,[1 + r(\theta_{it}(s_{i,t-1}),\ s_{it};\ \tilde{K}_t)]\ w_{it}$. Using this condition, together with our Markov process that governs the evolution of the state, we can rewrite the definition of total wealth, equation (2.17), as follows:

$$
\begin{aligned}
W_{t+1} &= \sum_{s^t} w_{t+1}(s^t)\ \pi(s^t)\ \ =\ \ \sum_{s_t}\ \sum_{s^{t-1}}\ w_{t+1}(s_t, s^{t-1})\ \pi(s_t \mid s^{t-1})\ \pi(s^{t-1}) \\
&= \sum_{s_t}\ \sum_{s^{t-1}}\ \beta\ [1 + r(\theta_t(s_{t-1}),\ s_t;\ \tilde{K}_t)]\ w_t(s^{t-1})\ \pi(s_t \mid s^{t-1})\ \pi(s^{t-1}) \\
&= \sum_{s_t}\ \sum_{s_{t-1}}\ \sum_{s_{t-2}}\ \sum_{s^{t-3}}\ \beta\ [1 + r(\theta_t(s_{t-1}),\ s_t;\ \tilde{K}_t)]\ w_t(s^{t-1}) \\
&\quad \times \pi(s_t \mid s_{t-1})\ \pi(s_{t-1} \mid s_{t-2})\ \pi(s_{t-2} \mid s_{t-3})\ \pi(s^{t-3}) \\
&= \sum_{s_t}\ \sum_{s_{t-1}}\ \beta\ [1 + r(\theta_t(s_{t-1}),\ s_t;\ \tilde{K}_t)]\ \pi(s_t \mid s_{t-1}) \\
&\quad \times \left[\sum_{s_{t-2}} \pi(s_{t-1} \mid s^{t-2})\ \sum_{s^{t-3}}\ w_t(s^{t-1})\ \pi(s_{t-2} \mid s^{t-3})\ \pi(s^{t-3})\ \right] \\
&= \sum_{s_t}\ \sum_{s_{t-1}}\ \beta\ [1 + r(\theta_t(s_{t-1}),\ s_t;\ \tilde{K}_t)]\ \pi(s_t \mid s_{t-1})\ \sum_{s_{t-2}}\ \rho(s_{t-1}, s_{t-2})\ W_t
\end{aligned}
$$

Hence,

$$
\frac{W_t}{W_{t+1}} = \frac{1}{\sum_{s_t}\ \sum_{s_{t-1}}\ \beta\ [1 + r(\theta_t(s_{t-1}),\ s_t;\ \tilde{K}_t)]\ \pi(s_t \mid s_{t-1})\ \sum_{s_{t-2}}\ \rho(s_{t-1}, s_{t-2})} \tag{2.20}
$$

In addition,

$$
\begin{aligned}
&\frac{\pi(s_t \mid s_{t-1})\ \sum_{s^{t-2}} w_{t+1}(s^t)\ \pi(s_{t-1} \mid s^{t-2})\ \pi(s^{t-2})}{\sum_{s^{t-1}} w_t(s^{t-1})\ \pi(s^{t-1})} \\
&= \pi(s_t \mid s_{t-1})\ \beta\ [1 + r(\theta_t(s_{t-1}),\ s_t;\ \tilde{K}_t)]\ \frac{\sum_{s^{t-2}} w_t(s^{t-1})\ \pi(s_{t-1} \mid s^{t-2})\ \pi(s^{t-2})}{\sum_{s^{t-1}} w_t(s^{t-1})\ \pi(s^{t-1})} \\
&= \pi(s_t \mid s_{t-1})\ \beta\ [1 + r(\theta_t(s_{t-1}),\ s_t;\ \tilde{K}_t)] \\
&\quad \times \frac{\sum_{s_{t-2}}\ \sum_{s^{t-3}}\ w_t(s^{t-1})\ \pi(s_{t-1} \mid s_{t-2})\ \pi(s_{t-2} \mid s^{t-3})\pi(s^{t-3})}{\sum_{s^{t-1}} w_t(s^{t-1})\ \pi(s^{t-1})} \\
&= \pi(s_t \mid s_{t-1})\ \beta\ [1 + r(\theta_t(s_{t-1}),\ s_t;\ \tilde{K}_t)] \\
&\quad \times \sum_{s_{t-2}} \frac{\sum_{s^{t-3}}\ \pi(s_{t-1} \mid s_{t-2})\ w_t(s^{t-1})\ \pi(s_{t-2} \mid s^{t-3})\pi(s^{t-3})}{\sum_{s^{t-1}} w_t(s^{t-1})\ \pi(s^{t-1})} \\
&= \pi(s_t \mid s_{t-1})\ \beta\ [1 + r(\theta_t(s_{t-1}),\ s_t;\ \tilde{K}_t)]\ \sum_{s_{t-2}}\ \rho_t(s_{t-1}, s_{t-2})
\end{aligned} \tag{2.21}
$$

Using (2.20) and (2.21) in (2.19) yields the following law of motion for the wealth share:

$$\rho_{t+1}(s_t, s_{t-1}) = \frac{\pi(s_t \mid s_{t-1})\ \beta\ [1 + r(\theta_t(s_{t-1}),\ s_t;\ \tilde{K}_t)]\ \sum_{s_{t-2}}\ \rho_t(s_{t-1}, s_{t-2})}{\sum_{s_t}\ \sum_{s_{t-1}}\ \beta\ [1 + r(\theta_t(s_{t-1}),\ s_t;\ \tilde{K}_t)]\ \pi(s_t \mid s_{t-1})\ \sum_{s_{t-2}}\ \rho_t(s_{t-1}, s_{t-2})} \tag{2.22}$$

Now, consider the aggregate stock of physical and human capital used in production. By definition, the stock of physical capital used in production reads

$$\begin{aligned}
K_{t+1} &= \sum_{s^t}\ k_{t+1}(s^t)\ \pi(s^t) = \sum_{s^t}\ \theta_{t+1}(s_t)\ w_{t+1}(s^t)\ \pi(s^t) = \sum_{s^t}\ \theta_{t+1}(s^t)\ w_{t+1}(s^t)\ \pi(s^t) \\
&= \sum_{s_t}\ \sum_{s_{t-1}}\ \sum_{s^{t-2}}\ \theta_{t+1}(s_t)\ w_{t+1}(s^t)\ \pi(s_t \mid s_{t-1})\ \pi(s_{t-1} \mid s^{t-2})\ \pi(s^{t-2}) \\
&= \sum_{s_t}\ \sum_{s_{t-1}}\ \theta_{t+1}(s_t)\ \pi(s_t \mid s_{t-1})\ \sum_{s^{t-2}}\ w_{t+1}(s^t)\ \pi(s_{t-1} \mid s^{t-2})\ \pi(s^{t-2}) \\
&= \sum_{s_t}\ \sum_{s_{t-1}}\ \theta_{t+1}(s_t)\ W_{t+1}(s_t, s_{t-1}) \\
&= \sum_{s_t}\ \sum_{s_{t-1}}\ \theta_{t+1}(s_t)\ \rho_{t+1}(s_t, s_{t-1})\ W_{t+1}
\end{aligned} \tag{2.23}$$

For the stock of human capital that is used in production, only those agents that are currently employed, are relevant. Hence,

$$H^e_{t+1} = \sum_{s^t}\ \pi(s_{1,t+1} = e \mid s^t)\ [1 - \theta_{t+1}(s_t)]\ w_{t+1}(s^t)\ \pi(s^t)$$

which simplifies, by the similar procedure as above, to

$$H^e_{t+1} = \sum_{s_t}\ \sum_{s_{t-1}}\ \pi(s_{1,t+1} = e \mid s_t)\ [1 - \theta_{t+1}(s_t)]\ \rho_{t+1}(s_t, s_{t-1})\ W_{t+1} \tag{2.24}$$

By (2.23) and (2.24), the capital-to-labor ratio reads

$$\tilde{K}_{t+1} = \frac{\sum_{s_t}\ \sum_{s_{t-1}}\ \theta_{t+1}(s_t)\ \rho_{t+1}(s_t, s_{t-1})}{\sum_{s_t}\ \sum_{s_{t-1}}\ \pi(s_{t+1} = e \mid s_t)\ [1 - \theta_{t+1}(s_t)]\ \rho_{t+1}(s_t, s_{t-1})} \tag{2.25}$$

Clearly, stationarity of the equilibrium implies $\rho_t = \rho,\ \forall t$. Thus, every $\tilde{K}$ that solves this condition (note that the ratios ρ defined previously, depend on the capital to labor ratio via the return functions) implicitly solves market clearing on the input factor markets. Observe, market clearing is independent of aggregate wealth.

2.C Appendix: Welfare Effects

In this appendix, we show the details on how to compute the welfare effects in our model with transition dynamics.

Step 1: Calculate the stationary equilibrium before and after the implementation of the reform. In particular, calculate the respective portfolio choices and search intensities, the aggregate capital-to-labor ratios and the wealth shares $\rho_t(s_{t-1}, s_{t-2})$. Suppose we implement the labor market reform in $t = 0$ and the new stationary equilibrium is *reached* in $t = T$. Then, the respective lifetime utilities read

$$V(\theta_{i0},\ w_{i0},\ s_{i0}) = \left[\frac{1}{1-\beta}\ln(1 + r(\theta_{i0},\ s_{i0};\ \tilde{K})) + B(s_{1i0}, s_{3i0})\right] + \frac{1}{1-\beta}\ \ln\ w_{i0}$$
$$V(\theta_{iT},\ w_{iT},\ s_{iT}) = \left[\frac{1}{1-\beta}\ln(1 + r(\theta_{iT},\ s_{iT};\ \tilde{K})) + B(s_{1iT}, s_{3iT})\right] + \frac{1}{1-\beta}\ \ln\ w_{iT}$$

Observe that the social planner cannot simply aggregate the lifetime utilities in $t = 0$ and $t = T$ and compare them, because the equilibrium distribution of agents and the wealth distribution is different in $t = 0$ and $t = T$. Nevertheless, as we will show below, it is sufficient to compute the term in square brackets, which is perfectly feasible, and for convenience, we define

$$\tilde{V}_{iT} = \frac{1}{1-\beta}\ln(1 + r(\theta_{iT},\ s_{iT};\ \tilde{K})) + B(s_{1iT}, s_{3iT})$$

such that

$$V(\theta_{iT},\ w_{iT},\ s_{iT}) = \tilde{V}_{iT} + \frac{1}{1-\beta}\ \ln\ w_{iT}$$

Step 2: Fix T. Since in our model, the transition from the old to the new stationary equilibrium is very fast, it suffices to set $T = 25$.

Step 3: Guess a sequence of wealth shares $\{\rho_t(s_{t-1}, s_{t-2})\}_{t=0}^{T}$.

Step 4: Since $t = T$ is already the new equilibrium, we know $\theta_{iT}(s_{i,T-1})$ and hence, $l_{i,T-1}$. In other words, we know the household decisions made in period $T-1$. Thus, we can immediately consider the household decision in period $T-2$. The respective Bellman equation reads

$$
\begin{aligned}
&V(\theta_{i,T-2},\ w_{i,T-2},\ s_{i,T-2}) \\
&= \max \Bigg\{ \ln c_{i,T-2} - \mathbf{1}_{s_{1i,T-2}=u}\ d(l_{i,T-2}) + \beta\, \mathbb{E}\Bigg[\ln c_{i,T-1} - \mathbf{1}_{s_{1i,T-1}=u}\ d(l_{i,T-1}) \\
&\qquad + \beta \left(\tilde{V}_{iT} + \frac{1}{1-\beta} \ln w_{iT} \right) \Bigg] \Bigg\} \\
&= \max \Bigg\{ \ln c_{i,T-2} - \mathbf{1}_{s_{1i,T-2}=u}\ d(l_{i,T-2}) \\
&\qquad + \beta\, \mathbb{E}\Bigg[\ln \frac{1-\beta}{1+\tau_{c,T-1}} [1 + r(\theta_{i,T-1},\ s_{i,T-1}; \tilde{K}_{i,T-1})] w_{i,T-1} - \mathbf{1}_{s_{1i,T-1}=u}\ d(l_{i,T-1}) \\
&\qquad + \beta \left(\mathbb{E}[\tilde{V}_{iT}] + \frac{1}{1-\beta} \ln \beta [1 + r(\theta_{i,T-1},\ s_{i,T-1}; \tilde{K}_{i,T-1})] w_{i,T-1} \right) \Bigg] \Bigg\}
\end{aligned}
$$

Where the last line follows from the optimal consumption policy and its implied law of motion for individual wealth holdings. For given wealth shares in period $T-1$, we can simultaneously solve for the optimal policies in $T-2$ and the capital-to-labor ratio in $T-1$. Note that the optimal policies are wealth independent, even during the transition phase. With these results, we compute

$$
\begin{aligned}
\tilde{V}_{T-1} = \ln \frac{1-\beta}{1+\tau_{c,T-1}} [1 + r(\theta_{i,T-1},\ s_{i,T-1}; \tilde{K}_{i,T-1})] - \mathbf{1}_{s_{1i,T-1}=u}\ d(l_{i,T-1}) \\
+ \beta \left(\mathbb{E}[\tilde{V}_{iT}] + \frac{1}{1-\beta} \ln \beta [1 + r(\theta_{i,T-1},\ s_{i,T-1}; \tilde{K}_{i,T-1})] \right)
\end{aligned}
$$

and thus

$$
V(\theta_{i,T-1},\ w_{i,T-1},\ s_{i,T-1}) = \tilde{V}_{i,T-1} + \frac{1}{1-\beta}\ \ln\ w_{i,T-1} \tag{2.26}
$$

Repeat this backward solving algorithm until $t = 0$.

Step 5: Use the optimal policies in step 4 to calculate the implied wealth shares, solving forward from $t = 0$ to $t = T$.

Step 6: If the wealth shares of the guess and the forward calculated wealth shares of step 5 are close enough, go to step 7. Otherwise, update your guess on the initial sequence of wealth shares and go back to step 4.

Step 7: In order to compute the welfare effect for households with initial state $(\theta_{i0},\ w_{i0},\ s_{i0})$, we ask the households in the pre-reform state how much additional consumption do they need in each period in order to be indifferent between implementing the reform or not. Specifically, let $\Delta(s_0)$ denote the respective percentage share. $V(\cdot)$ and $V^{ref}(\cdot)$ respectively denote the lifetime utility without and with labor market reform. Then

$$\frac{1}{1-\beta}\ \ln(1+\Delta(s_0)) + V(\theta_{i0},\ w_{i0},\ s_{i0}) = V^{ref}(\theta_{i0},\ w_{i0},\ s_{i0})$$

Solving for Δ yields the welfare effect for households with current state s_{it}:

$$\Delta(s_0) = \mathrm{e}^{(1-\beta)\ (V^{ref}(\theta_{i0},\ w_{i0},\ s_{i0}) - V(\theta_{i0},\ w_{i0},\ s_{i0}))}$$

Note that by (2.26), we know that w_{i0} cancels out such that the welfare effects are independent of the wealth level and the wealth distribution. Finally, having calculated the welfare effects $\Delta(s_0)$ for all s_0, the social welfare is the weighted average over $\Delta(s_0)$ with the shares of s_0 in the population as weights.[18]

[18] Alternatively, we could also first compute the social welfare and afterwards the welfare effects. However, since most people belong to one type, the employed, the results are fairly equivalent.

Chapter 3

Optimal Unemployment Insurance in General Equilibrium

3.1 Introduction

The empirical literature documents a substantial degree of labor income risk, with a large fraction actually caused by (un)employment risk.[1] The existing (un)employment risk points to inherently missing insurance markets due to informational frictions. Specifically, if the job seekers' effort choices are not publicly observable, benefit payments cannot be made contingent on the job search decision, and the existing moral hazard friction leads to a collapse of private unemployment insurance markets.[2] In designing the unemployment insurance system, the government has to weigh labor income insurance insurance against distorting the search incentives.[3]

In this paper, we use a dynamic general equilibrium search model of the labor market to compute an optimal unemployment insurance scheme when search effort choices are only

[1]For the United States, Hubbard, Skinner, and Zeldes (1995), Meghir and Pistaferri (2004), and Storesletten, Telmer, and Yaron (2004) estimate a standard deviation of the log of labor income for between 0.15 and 0.20. Furthermore, Jacobson, LaLonde, and Sullivan (1993) find that the long run earning loss upon job displacement is around 25 percent. Although Farber (2003) only finds losses half as large, it is undeniable, that job displacement and the associated unemployment spell substantially contribute to labor income risk. For a more detailed discussion, see Kletzer (1998).

[2]Arrow (1963) and Rothschild and Stiglitz (1976) have already made this point in different contexts.

[3]For an analysis of the trade-off between insurance and distorted incentives, see, e.g., Shavell and Weiss (1979), Hopenhayn and Nicolini (1997), Shimer and Werning (2006, 2007, 2008), and Pavoni (2007).

private information of the households. In contrast to the existing literature on optimal unemployment insurance, we first provide a macroeconomic (general equilibrium) perspective, second allow for precautionary saving, and third preserve the analytical tractability in the sense that the optimal unemployment insurance system can be characterized without solving for the complete underlying wealth distribution. The first property is motivated by Lentz (2009) who find that the welfare effects in his labor market search model substantially depend on the relation between the time discount factor and the interest rate. In general equilibrium, the interest rate is endogenously determined such that we get rid of this additional degree of freedom. The second property is motivated by Shimer and Werning (2006, 2007, 2008) who show that when households make consumption-saving decisions, the optimal benefit profile differs substantially from the case in which households do not make the decision, as, e.g., in Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997). The third property allows us to consider richer unemployment duration dependent policies as in the previous literature. The reason for this is that we do not have to compute the underlying wealth distribution as Hansen and Imrohoroglu (1992), Wang and Williamson (1996), and Young (2004). The combination of all three properties – which have quantitatively and computationally important implications – is unique to the literature.

The model we use is based on chapter 2. Households are risk-avers and make consumption-saving, portfolio composition, and search effort decisions, the latter being unobservable to the government which causes the moral hazard friction. Despite the ex-post heterogeneity due to different employment histories, the equilibrium allocation is tractable in the sense that the equilibrium allocation can be characterized without knowing the complete underlying endogenous wealth distribution. This property is preserved in the optimal unemployment insurance system that is obtained by a mixed social planner – Ramsey problem. Specifically, the government chooses wealth independent transfer rates subject to the households' consumption-saving and search effort decisions, which is the Ramsey part of government's optimization problem, and the portfolio allocation, which is the social planner part of the government's optimization problem.

The main results are as follows: First, conditional on being employed, the social planner provides full insurance. This is due to the fact that there are no moral hazard frictions for currently employed households. Second, the optimal unemployment benefit rate is independent of the unemployment duration. This result is consistent with Shimer and Werning (2006, 2007, 2008) who show that under the absence of wealth effects, when there is a consumption saving decision, the benefit profile is constant with respect to unemployment duration. In contrast, Hopenhayn and Nicolini (1997) find falling benefit profiles without an endogenous consumption-saving decision. Third, while the net benefit rate for unemployed households is quite low, there are high rewards for successful job finders of 134 percent of their labor income. This result is consistent with Wang and Williamson (1996) and Hopenhayn and Nicolini (1997), but we find even stronger effects.

The rest of this paper is organized as follows: section 3.2 presents the model economy and section 3.3 constructs the competitive equilibrium. Because this model builds on the framework developed in chapter 2, we will keep the model description and the derivation of the properties of the competitive equilibrium short. Section 3.4 sets up the restricted social planner problem. In section 3.5 based on a calibrated version of the model economy, we present some numerical results. Chapter 3.6 concludes.

3.2 Economy

The economy is populated by a continuum of *ex-ante* identical, infinitely-lived households with unit mass who derive period utility from consumption c_t and avoiding to exert job search effort l_t. For convenience we use lower case letters to denote idiosyncratic variables. The period utility function is given by $u(c_t, l_t) = \ln c_t - d(l_t)$, with $d(l_t)$ denoting the disutility from exerting job search effort. We assume that the utility cost of job search are increasing and convex in the exerted search effort, $d'(l_t) > 0$ and $d''(l_t) > 0$, and there is no search independent disutility of being unemployed, $d(0) = 0$. For a similar specification see, e.g., Lentz (2009).

Let $s_t \in S = \{e, u_1, u_2, u_3, \dots\}$ denote the current employment state of an arbitrary household. Households are either employed, $s_t = e$, or unemployed, $s_t = u_j$, where $j = 1, 2, 3, \dots$ denotes the duration of the current unemployment spell. The employment history of an arbitrary household is denoted by $s^t = (s_t, s_{t-1}, s_{t-2}, \dots)$. Let $\pi(s^t)$ denote the unconditional probability of experiencing employment history s^t. According to our normalization, the unconditional probability equals the mass of households with the respective employment history. The individual employment state follows a first-order Markov process with $\pi(s_{t+1}|s_t)$ denoting the probability of ending up in state s_{t+1} in the next period, given the household is currently in state s_t. While the probability of losing a job $\pi(u_1|e)$ is exogenously given, households determine their reemployment probability $\pi(e|u_j)$ by their search effort choice. For convenience, we leave the dependence of the state transition rate on search effort implicit. Clearly, the more search effort the household exerts, the higher the probability of finding a new job in the subsequent period, $(\partial\pi'(e|u_j))/(\partial l_t) > 0$, for $j = 1, 2, 3, \ldots$.

Households hold physical and human capital, k_t and h_t from which they receive capital income $r_{kt}\ k_t$ and, if they are employed, labor income $r_{ht}(e)\ h_t$. There is no home production, $r_{ht}(u_j) = 0$. In addition, households receive (positive or negative) transfer payments proportional to their stock of human capital and dependent on their recent state transition. Specifically, the transfer payments are given by $Tr_t(s_t, s_{t-1})\ h_t$. Disposable income can be used for consumption and for investment into physical and human capital, x_{kt} and x_{ht}. Households maximize their lifetime utility with respect to consumption, investment in physical and human capital, and search effort. The optimization problem is given by

$$\max_{\{c_t, x_{kt}, x_{ht}, l_t\}} \left\{ U(\{c_t, x_{kt}, x_{ht}, l_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t (\ln c_t - d(l_t))\ \pi(s^t) \right\}$$

subject to

$$\begin{aligned}
c_t + x_{kt} + x_{ht} &= r_{kt}\ k_t + r_{ht}(s_t)\ h_t + Tr_t(s_t, s_{t-1})\ h_t \\
k_{t+1} &= (1-\delta_k)\ k_t + x_{kt} \\
h_{t+1} &= (1-\delta_h(s_t))\ h_t + x_{ht} \\
k_{t+1} &\geq 0 \\
h_{t+1} &\geq 0
\end{aligned}$$

with δ_k and $\delta_h(s_t)$ denoting the (potentially state dependent) depreciation rate for physical and human capital.

There is a continuum of identical firms that produce the *all-purpose* good using physical and human capital as input factors. The production technology exhibits constant returns to scale. Hence, under competitive markets, the production sector can be represented by an aggregate firm with aggregate production technology $F(K_t, H_t^e)$. K_t denotes the aggregate stock of physical capital and H_t^e denotes the aggregate stock of human capital that is used in production. The profit maximization problem of the firm reads

$$\max_{K_t, H_t^e} \{\Pi(K_t,\ H_t^e) = F(K_t,\ H_t^e) - r_{kt}\ K_t - r_{ht}(e)\ H_t^e\}$$

The government sets transfer rates $Tr_t(s_t, s_{t-1})$ conditional on the households' recent state transition, and it has to keep a balanced budget in each period. The per period government budget constraint is given by

$$\sum_{s^t} Tr_t(s_t, s_{t-1})\ h_t(s^t)\ \pi(s^t) = 0$$

3.3 Competitive Equilibrium

Following Krebs (2003), we rewrite the households' optimization problem as a portfolio choice problem. Define total (nonhuman and human) wealth $w_t \doteq k_t + h_t$ and the share of physical capital with respect to wealth as $\theta_t \doteq \frac{k_t}{w_t}$. The return to total wealth can thus be written as

$$\begin{aligned} &r_t(\theta_t, s_t, s_{t-1}; r_{kt}, r_{ht}, Tr_t(s_t, s_{t-1})) \\ &\qquad = \theta_t\ (r_{kt} - \delta_k) + (1 - \theta_t)\ (r_{ht}(s_t) - \delta_h) + (1 - \theta_t)\ Tr_t(s_t, s_{t-1}), \quad \forall s_t, s_{t-1} \end{aligned}$$

and the constraints of the households' optimization problem simplify to

$$\begin{aligned} w_{t+1} &= (1 + r_t(\theta_t, s_t, s_{t-1}; r_{kt}, r_{ht}, Tr_t(s_t, s_{t-1})))\ w_t - c_t \\ 0 &\leq \theta_{t+1} \leq 1 \end{aligned}$$

Instead of $\{c_t, x_{k,t+1}, x_{h,t+1}, l_t\}_{t=0}^{\infty}$, households now choose $\{c_t, w_{t+1}, \theta_{t+1}, l_t\}_{t=0}^{\infty}$ subject to the flow budget constraint and the portfolio share constraint (short-selling constraint).

We define a competitive equilibrium of this economy as follows:

Definition 3.1 (Competitive Equilibrium).
A competitive equilibrium is

1. *A sequence* $\{K_t,\ H_t^e\}_{t=0}^{\infty}$ *that maximizes the firm's profit for a given sequence of factor prices* $\{r_{kt},\ r_{ht}\}_{t=0}^{\infty}$*;*

2. *A sequence* $\{c_t,\ \theta_{t+1},\ w_{t+1},\ l_t\}_{t=0}^{\infty}$ *that solves the households' optimization problem for a given sequence of factor prices* $\{r_{kt},\ r_{ht}\}_{t=0}^{\infty}$*, employment shocks* $\{s_t\}_{t=0}^{\infty}$*, and transfer payments* $\{Tr_t\}_{t=0}^{\infty}$*;*

3. *A sequence* $\{r_{kt},\ r_{ht}\}_{t=0}^{\infty}$ *that satisfies market clearing on the input factor market,* $K_t = \sum_{s^t} \theta_t\ w_t(s^t)\ \pi(s^t)$ *and* $H_t^e = \sum_{s^{t-1}}(1-\theta_t)\ w_t(e, s^{t-1})\ \pi(e, s^{t-1})$*, for all t;*

4. *A sequence of transfer payments* $\{Tr_t\}_{t=0}^{\infty}$ *that satisfies the per period government budget constraint for given saving policy and portfolio choice of all households.*

For the firm's optimization problem, the usual first-order conditions apply. Define the aggregate capital-to-labor ratio $\tilde{K}_t \doteq \frac{K_t}{H_t^e}$ and the production technology in intensive form $\tilde{F}(\tilde{K}_t) \doteq \frac{F(K_t, H_t^e)}{H_t^e}$. The profit maximization conditions are

$$r_{kt} = \tilde{F}'(\tilde{K}_t)$$
$$r_{ht} = \tilde{F}(\tilde{K}_t) - \tilde{K}_t\ \tilde{F}'(\tilde{K}_t)$$

Because in equilibrium, factor prices are completely determined by the current aggregate capital-to-labor ratio, we can rewrite the returns to individual wealth as

$$r_t(\theta_t, s_t, s_{t-1}; r_{kt}, r_{ht}, Tr_t(s_t, s_{t-1})) = r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1}))$$

The individual state space of an arbitrary household consists of his capital share θ_t, current wealth w_t, and his recent state transition (s_t, s_{t-1}) which determines the transfer payments to be received. The aggregate state consists of the joint distribution of physical capital, human capital, and the employment state, on the one hand, and of the transfer system, on the other. For convenience, we leave the dependence of the value function from the aggregate state implicit. Rewriting the households' optimization problem in recursive form yields

$$\tilde{V}(\theta_t, w_t, s_t, s_{t-1}) = \max_{c_t, w_{t+1}, \theta_{t+1}, l_t} \left\{ \ln c_t - d(l_t) + \beta\ \mathbb{E}[\tilde{V}(\theta_{t+1}, w_{t+1}, s_{t+1}, s_t)] \right\}$$

subject to

$$\begin{aligned} w_{t+1} &= (1 + r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1})))\ w_t - c_t \\ 0 &\le \theta_{t+1} \le 1 \end{aligned}$$

Substituting for consumption using the flow budget constraint, the households' first-order conditions with respect to w_{t+1}, θ_{t+1}, and l_t read

$$\frac{1}{c_t} = \beta\ \mathbb{E}\left[\frac{1 + (\theta_{t+1}, s_{t+1}, s_t; \tilde{K}_{t+1}, Tr_{t+1}(s_{t+1}, s_t))}{c_{t+1}}\right] \tag{3.1}$$

$$0 = \mathbb{E}\left[\frac{(r_{k,t+1} - \delta_k) - (r_{h,t+1} + Tr_{t+1}(s_{t+1}, s_t) - \delta_h(s_{t+1}))}{c_{t+1}}\right] \tag{3.2}$$

$$\frac{\partial d(l_t)}{\partial l_t} = \beta\ \frac{\partial \pi(e|u_j)}{\partial l_t}\ \Big(\tilde{V}(\theta_{t+1}, w_{t+1}, e, u_j) - \tilde{V}(\theta_{t+1}, w_{t+1}, u_{j+1}, u_j)\Big), \quad \forall j \tag{3.3}$$

A well established result states,that under linear homogeneity of disposable income in current wealth and homothetic preferences, the consumption and saving policies are also linear homogenous in wealth.[4] Specifically, it is easy to verify that the policies

$$c_t = (1 - \beta)\ (1 + r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1})))\ w_t \tag{3.4}$$

$$w_{t+1} = \beta\ (1 + r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1})))\ w_t \tag{3.5}$$

solve the consumption-saving Euler equation (3.1) and the flow budget constraint. Inspection of (3.2) reveals that the portfolio choice does not exhibit any direct dependence on wealth if the consumption policy is linear. However, it is still possible that the portfolio choice depends indirectly on individual wealth holdings through wealth dependence of the

[4]See e.g. Krebs (2003), Stokey (2009), and the previous discussion in chapter 2.

search effort choice. By the method of guess and verify,[5] we show that

$$\tilde{V}(\theta_t, w_t, s_t, s_{t-1}) = \frac{\ln w_t}{1-\beta} + V(\theta_t, s_t, s_{t-1}) \tag{3.6}$$

where $V(\theta_t, s_t, s_{t-1})$ solves the intensive form Bellman equation

$$\begin{aligned} V(\theta_t, s_t, s_{t-1}) = \max_{l_t, \theta_{t+1}} \Bigg\{ & \ln(1-\beta) + \frac{\beta}{1-\beta}\ln\beta - d(l_t) \\ & + \frac{\ln(1 + r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1})))}{1-\beta} + \beta\mathbb{E}\left[V(\theta_{t+1}, s_{t+1}, s_t)\right] \Bigg\} \end{aligned} \tag{3.7}$$

Clearly, using (3.6), the first-order condition with respect to search effort (3.3) becomes wealth independent as long as the portfolio choice is wealth independent. Thus, portfolio choices and search effort decisions are independent of the individual wealth holdings, but dependent on the current employment state, $\theta_{t+1} = \theta_{t+1}(s_t)$.

Having solved for the firm's and the households' policies, we now close the model by analyzing market clearing on the input factor markets. Define $\rho_t(s_{t-1}) \doteq \frac{\sum_{s^{t-2}} w_t(s^{t-1})\pi(s^{t-1})}{\sum_{s^{t-1}} w_t(s^{t-1})\pi(s^{t-1})}$ as the relative wealth owned by all households whose employment state was s_{t-1} in the preceding period. Using the definition of the portfolio shares and the saving policy, the law of motion of the wealth measure is given by[6]

$$\rho_{t+1}(s_t) = \frac{\sum_{s_{t-1}} \pi(s_t|s_{t-1})(1 + r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1})))\ \rho_t(s_{t-1})}{\sum_{s_t, s_{t-1}} \pi(s_t|s_{t-1})(1 + r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1})))\ \rho_t(s_{t-1})} \tag{3.8}$$

which yields the following market clearing condition

$$\tilde{K}_{t+1} = \frac{\sum_{s_t} \theta_{t+1}(s_t)\ \rho_{t+1}(s_t)}{\sum_{s_t} (1 - \theta_{t+1}(s_t))\ \pi(e|s_t)\ \rho_{t+1}(s_t)} \tag{3.9}$$

We summarize the equilibrium characterization in the following proposition:

Proposition 3.1 (Characterization of Competitive Equilibrium).

[5]For more details on the derivation of the value function based on the method of guess and verify, see appendix 2.A.

[6]Form more details on the derivation of the law of motion of the wealth shares, see appendix 2.B.

For any transfer scheme that satisfies the per period budget constraint of the government

$$\sum_{s_t, s_{t-1}} Tr_t(s_t, s_{t-1})\ \pi(s_t|s_{t-1})\ \rho_t(s_{t-1}) = 0$$

a competitive equilibrium can be characterized as follows:

1. *The firm's demand for physical and human capital satisfies the usual first-order conditions of profit maximization under competitive markets;*

2. *The households consumption and savings policies are linear homogenous in wealth and given by* (3.4) *and* (3.5)*, the portfolio choices and search effort decisions are wealth independent and implicitly given as the solution of* (3.2) *and* (3.3) *where the intensive form value function solves the respective intensive form Bellman equation* (3.7)*;*

3. *Market clearing satisfies* (3.9) *with the evolution of wealth shares governed by* (3.8).

Observe that the equilibrium allocation is independent of the unconditional wealth distribution. Specifically, it suffices to solve for the relative wealth owned by households of type s_{t-1}. Clearly, the wealth ratios are a substantially easier mathematical object than the unconditional wealth distribution.

3.4 Optimal Unemployment Insurance

In a frictionless environment, the social planner would choose an allocation that provides full insurance against income shocks. However, for the social planner, the exerted job search effort of the individual households' is unobservable. Providing too generous insurance against unemployment restrain households from exerting sufficient search effort. Optimal unemployment insurance thus has to take the existing moral hazard friction into account.

For the analysis here, we restrict to a very specific social planner problem. The social planner's objective function, the social welfare function, is derived under two assumptions: First, the social planner weighs the lifetime utility of the individual households equally. Hence, we rule out transfer payments across types of households solely based on the fact that the social planner cares more about one type of agents than of than other. Second, although we allow the social planner to choose the consumption-saving decision of the households, we restrict to allocations that are self-enforcing in the sense that the households' consumption-saving Euler equations have to be satisfied. This assumption guarantees that the linear homogenous consumption and saving policies derived in the previous section still hold. Hence, the decomposition of the value function into an intensive form value function on the one hand, and a wealth dependent term, on the other, is still valid. The social welfare function can now be written as

$$\int_I \tilde{V}(\theta_t, w_t, s_t, s_{t-1})\mathrm{d}i = \sum_{s_t, s_{t-1}} \mu_t(s_t, s_{t-1}) V(\theta_t, s_t, s_{t-1}) + \int_I \frac{\ln w_t}{1-\beta}\mathrm{d}i$$

where $\mu(s_t, s_{t-1})$ denotes the mass of households of type (s_t, s_{t-1}). Clearly, with w_t being a state variable to the planner's problem in period t, it is sufficient for the social planner to confine attention to the social welfare function in intensive form, which is defined as the first term on the right hand side of the equation.

We restrict our analysis to the case where a social planner can fully commit to his plans. Clearly, this means that the current transfer scheme was determined in the previous period and is thus a state variable to the social planner's problem in the current period. Hence, the social planner cannot announce a policy in the current period that induces the households to exert high search effort and switch to a different high insurance policy after the search effort decision materializes into low unemployment rates in the subsequent period. This assumption has important implications on the optimal unemployment insurance system we derive. As shown by Krusell, Quadrini, and Rios-Rull (1997), Krusell (2002) for redistributive policies in general and Kankanamge and Weitzenblum (2011) for unemployment insurance specifically, implementing a time consistent insurance system comes at high welfare costs compared to the case in which the social planner can fully commit. Specifically, under limited commitment, there is more unemployment insurance at the cost of substantially higher unemployment. The optimal unemployment insurance system we derive is thus under the absence of this implementability friction.

Because search effort is unobservable for the social planner, the households' first-order conditions with respect to search effort enter as an additional constraint to the social planner's optimization problem. With individual value functions entering the constraints, standard dynamic programming techniques are not valid.[7] The standard way of dealing with this problem is to define a broader state space that also includes the current lifetime utility of the individual households. As shown by Spear and Srivastava (1987), the social planner problem is recursive on this new state space, making standard dynamic programming techniques applicable. However, this method comes at the cost that not every solution to the first-order conditions of this augmented problem is a solution to the original planner problem. Thus, we have to check whether the promised utility of the candidate solution solves the households' functional equation when the consumption and saving policies as well as the portfolio choices are given by the candidate solution.

For the incentive problem considered here, it suffices to consider the difference in lifetime utility. Thus, instead of lifetime utility, we only include the difference of lifetime utilities as state variable, reducing the dimension of the state space. Define $\Delta V_t(u_j, s_{t-1}) \doteq V(\theta_t, e, s_{t-1}) - V_t(\theta_t, u_j, s_{t-1}), j = 1, 2, 3, \dots$ as the utility difference. From the households' utility functions, we get

$$
\begin{aligned}
\Delta V_t(u_j, s_{t-1})) = d(l(u_j)) + & \frac{\ln(1 + r_t(\theta_t, e, s_{t-1}; \tilde{K}_t, Tr_t(e, s_{t-1})))}{1-\beta} \\
& - \frac{\ln(1 + r_t(\theta_t, u_j, s_{t-1}; \tilde{K}_t, Tr_t(u_j, s_{t-1})))}{1-\beta} \\
& - \beta \left[\pi(u_1|e)\Delta V_{t+1}(u_1, e) - \pi(u_{j+1}|u_j)\Delta V_{t+1}(u_{j+1}, u_j) \right] \\
& + \beta \, \frac{\ln(1 + r_{t+1}(\theta_{t+1}, e, e; \tilde{K}_{t+1}, Tr_{t+1}(e, e)))}{1-\beta} \\
& - \beta \, \frac{\ln(1 + r_{t+1}(\theta_{t+1}, e, u_j; \tilde{K}_{t+1}, Tr_{t+1}(e, u_j)))}{1-\beta}, \forall j = 1, 2,
\end{aligned}
$$

This condition is the promise keeping constraint since it requires the government to deliver the utility difference $\Delta V_t(u_j, s_{t-1})$ that was promised in the previous period.

Let $W(\theta_t, k_t, \rho_t, \mu_t, Tr_t, \Delta V_t)$ denote the social welfare. The social planner chooses the portfolio share $\theta_{t+1}(s_t)$, the search effort $l_t(s_t)$, the capital-to-labor ratio $\tilde{K}_t$, the wealth shares $\rho_{t+1}(s_t)$, the distribution of households across states $\mu_{t+1}(s_{t+1}, s_t)$, the transition dependent transfer payments $Tr_{t+1}(s_{t+1}, s_t)$, and the promised utility difference $\Delta V_{t+1}(u_{j+1}, s_t)$

[7] See, e.g. Abraham and Pavoni (2008), Mele (2010), and Marcet and Marimon (2011).

for all household types in order to maximize

$$W_t(\theta_t, \tilde{K}_t, \rho_t, \mu_t, Tr_t, \Delta V_t) = \max_{\theta_{t+1}, l_t, \tilde{K}_{t+1}, \rho_{t+1}, \mu_{t+1}, Tr_{t+1}, \Delta V_{t+1}} \Bigg\{ B - \sum_{j, s_{t-1}} \mu(u_j, s_{t-1})\ d(l)$$
$$+ \sum_{s_t, s_{t-1}} \mu(s_t, s_{t-1})\ \frac{\ln(1 + r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1})))}{1-\beta}$$
$$+ \beta\ W_{t+1}(\theta_{t+1}, \tilde{K}_{t+1}, \rho_{t+1}, \mu_{t+1}, Tr_{t+1}, \Delta V_{t+1}) \Bigg\}$$

subject to

i.) the promise keeping constraint

$$\Delta V_t(u_j, s_{t-1})) = d(l(u_j)) + \frac{\ln(1 + r_t(\theta_t, e, s_{t-1}; \tilde{K}_t, Tr_t(e, s_{t-1})))}{1-\beta}$$
$$- \frac{\ln(1 + r_t(\theta_t, u_j, s_{t-1}; \tilde{K}_t, Tr_t(u_j, s_{t-1})))}{1-\beta}$$
$$- \beta \left[\pi(u_1|e)\Delta V_{t+1}(u_1, e) - \pi(u_{j+1}|u_j)\Delta V_{t+1}(u_{j+1}, u_j) \right]$$
$$+ \beta\ \frac{\ln(1 + r_{t+1}(\theta_{t+1}, e, e; \tilde{K}_{t+1}, Tr_{t+1}(e, e)))}{1-\beta}$$
$$- \beta\ \frac{\ln(1 + r_{t+1}(\theta_{t+1}, e, u_j; \tilde{K}_{t+1}, Tr_{t+1}(e, u_j)))}{1-\beta}, \forall j = 1, 2, \ldots$$

ii.) the incentive compatibility constraint

$$\frac{\partial d(l_t)}{\partial l_t} = \beta\ \frac{\partial \pi(e|u_j)}{\partial l_t}\ \Delta V_{t+1}(u_{j+1}, u_j)$$

iii.)the evolution of wealth shares

$$\rho_{t+1}(s_t) = \frac{\sum_{s_{t-1}} \pi(s_t|s_{t-1})(1 + r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1})))\ \rho_t(s_{t-1})}{\sum_{s_t, s_{t-1}} \pi(s_t|s_{t-1})(1 + r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1})))\ \rho_t(s_{t-1})}$$

iv.)the market clearing condition

$$\tilde{K}_{t+1} = \frac{\sum_{s_t} \theta_{t+1}(s_t)\ \rho_{t+1}(s_t)}{\sum_{s_t} (1 - \theta_{t+1}(s_t))\ \pi(e|s_t)\ \rho_{t+1}(s_t)}$$

v.)the government budget constraint

$$0 = \sum_{s_{t+1}, s_t} Tr_{t+1}(s_{t+1}, s_t)\ \pi(s_{t+1}|s_t)\ \rho_{t+1}(s_t)$$

vi.)the evolution of population shares

$$\mu_{t+1}(s_{t+1}, s_t) = \sum_{s_{t-1}} \pi(s_{t+1}|s_t)\ \mu_t(s_t, s_{t-1})$$

3.5 Quantitative Analysis

3.5.1 Specification and Calibration

The quantitative analysis here uses two additional restrictions. First, we impose an *ad hoc* irreversibility constraint on human capital, which restricts the social planner in his portfolio choices and transfer policies for unemployed households. Specifically, for unemployed households the choices have to satisfy $(1 - \theta_{t+1})\ w_{t+1} \geq (1 - \delta_h(s_t))\ (1 - \theta_t)\ w_t$. With saving policies that are linear homogenous in wealth, this condition is wealth independent as well. Second, we focus on the long-run (stationary) optimal unemployment insurance system, although we are aware that the welfare gains may be offset by costly transition phases, as demonstrated by Gilles and Weitzenblum (2003).

The model is calibrated on a monthly basis to match stylized facts of the US economy. In particular, we calibrate to match the elasticity of the job finding rate with respect to benefit payments, the unemployment rate, and the monthly equilibrium growth rate. This approach is motivated by two observations: First, the elasticity takes central stage in determining the employment effects of changes in the unemployment benefit system, as shown in the sensitivity analysis of chapter 2. Second, the growth rate is key for the determination of the welfare effect. Specifically, the welfare effect is mainly determined by the consumption volatility of the employed households and its magnitude is directly linked to the the average consumption growth.

Table 3.1: Calibration - Exogenous Parameters

PARAMETER	DESCRIPTION	VALUE
PREFERENCES		
A	parameter of disutility of search	1.0000
β	time preference rate	0.9950
PRODUCTION		
α	capital share	0.3600
DEPRECIATION RATES		
δ_k	depreciation rate: physical capital	0.0050
$\delta_h(s_t)$	depreciation rate: human capital if employed	0.0050
LABOR MARKET AND TRANSITION RATES		
$\pi(u_1\|e)$	monthly job destruction rate	0.0300

The functional forms are specified as follows: the production technology is of the Cobb-Douglas type, $F(K_t, H_t^e) = z\ K_t^\alpha\ H_t^{e(1-\alpha)}$, the disutility of search is a power function $d(l_t) = A\ l_t^\zeta$, and the job search technology is an exponential function $\pi(e|s_t) = 1 - \mathrm{e}^{-\lambda l_t}$.[8] We set the capital share in production α to 0.36, and the depreciation rates of physical and human capital δ_k and $\delta_h(s_t)$ to 0.0050 which amounts to six percent per annum. The depreciation rate of physical capital is within the range suggested by the literature. In contrast, the individual depreciation rate of human capital is estimated to be between zero and four percent per annum.[9] However, our infinite horizon model has to account for the additional mortality based human capital depreciation, which is not included in the estimates. Assuming a working life span of 50 years, we have to add an additional depreciation of 2 percent per annum, which makes our chosen value for the depreciation rate to be at the upper end of the range. Following Shimer (2005), the monthly job separation

[8]For the specification of the disutility function and the job search technology, see e.g. Lentz (2009).
[9]See Browning, Hansen, and Heckman (1999).

rate $\pi(u_1|e)$ is three percent. The time preference factor β is set to 0.9950 which is 0.94 on an annual basis. Moreover, the scaling factor of disutility of search effort A is set to one. Actually this value is no restriction since it cannot be identified independently from the search technology parameter λ that will be used to match the unemployment rate of 8 percent. The curvature of the disutility function is set to five, which implies an equilibrium reemployment elasticity of -0.25, as found by Meyer and Mok (2007).[10] Finally, setting the scaling parameter of the production technology to 0.0155 yields a monthly consumption growth rate of 0.2 percent, which amounts to approximately three percent per annum.

Table 3.2: Calibration - Endogenous Parameters

PARAMETER	DESCRIPTION	VALUE
PREFERENCES		
ζ	curvature of disutility of search	5.0000
PRODUCTION		
z	productivity	0.0155
LABOR MARKET AND TRANSITION RATES		
λ	search technology parameter:	3.0000
PARAMETERS ARE CHOSEN TO MATCH		
	aggregate monthly consumption growth rate	0.0025
	unemployment rate	0.0750
	average benefit elasticity of reemployment probability	-0.2500

3.5.2 Results

There are three main results: First, in the absence of any moral hazard friction for employed households, the government provides full insurance conditional on being employed. Specifically, the consumption growth rate is 0.27 percent, for sure. If households remain employed until the next period, the transfer payments are negative and can be interpreted

[10]For a more detailed discussion how this parameter is calibrated, see chapter 2.

as a labor income tax of 7.2 percent. In contrast, when becoming unemployed, households receive transfer payments that amount to 92.8 percent of their previous gross income.

Second, transfer rates to unemployed households that are unemployed for at least 2 periods are independent of the duration of the current unemployment spell. This result is due to the absence of wealth effects when preferences are homothetic and disposable income is linear homogenous in wealth. Our result is similar to Shimer and Werning (2006, 2007, 2008) who get rid of the wealth effects by using a CARA utility specification. However, the optimal detrended consumption profile for unemployed households is nevertheless decreasing, because the unemployed's wealth position grows at a lower rate than the economy wide average wealth positions.

Third, for unemployed households, benefit payments are quite low at 5.5 percent of the gross wage (and 12.7 percent of net wages compared to households that are employed for at least two periods). However, if unemployed households are successful job seekers, they receive a reward as high as 134 percent of their gross wage. The government uses the spread between the low benefit rate for unemployed and the high reward for successful job seekers to provide sufficient incentives for the households to exert search effort. Although this result is not new to the literature, see e.g. Wang and Williamson (1996) and Hopenhayn and Nicolini (1997), we find a substantial higher spread, again mainly due to the absence of wealth effects.

Table 3.3: Results

EMPLOYMENT STATE	CONSUMPTION GROWTH RATE		IMPLIED BENEFIT RATE	
	$s_{t+1} = e$	$s_{t+1} = u_{j+1}$	$s_{t+1} = e$	$s_{t+1} = u_{j+1}$
$s_t = e$	+0.27%	+0.27%	−7.2%	+92.8%
$s_{t+1} = u_j$	+1.0%	−0.20%	+134%	+5.42%

3.6 Conclusions

We constructed a specific social planner problem that allowed us to compute an optimal unemployment insurance system in a dynamic general equilibrium model with publicly unobservable job search effort by the households without solving for the within type wealth distribution. Our quantitative results are derived under the assumption that the government can fully commit to its policy plans. The main results are as follows: first, there is full insurance for currently employed households, second, the optimal profile of the benefit rate is independent of the duration of the current unemployment spell, and third, benefits are low for unemployed households but the reward for successful job search is substantial. The results are consistent with the existing literature on optimal unemployment insurance.

There are two directions for future research building on the model presented here. First, we plan to use the model to derive more theoretical results on optimal unemployment insurance, in particular concerning the importance of the general equilibrium effect. The wealth independence of the optimal unemployment insurance system may will simplify the analysis substantially. We will analyze the general equilibrium effect in a calibrated version of the model economy quantitatively. Second, we plan to extend the analysis to limited commitment of the government in the sense that it can deviate from its previously announced policies. Clearly, since the current transfer scheme has no impact on the households' allocation decisions, the government is tempted to provide more insurance in the current period. As argued by Kankanamge and Weitzenblum (2011), the time-consistent unemployment insurance system under limited commitment can be fundamentally different from the optimal unemployment insurance system under full commitment. This analysis will shed light on the question on how quantitatively important this additional friction is.

Chapter 4

Human Capital Risk, Public Consumption, and Optimal Taxation

4.1 Introduction

This paper is motivated by two empirical observations. First, there is strong evidence that human capital investment is risky, but complete insurance against this risk is lacking. More precisely, a significant fraction of labor income is the return to human capital investment, and a voluminous empirical literature has shown that individual households face large and highly persistent labor income shocks that have strong effects on individual consumption.[1] Second, in most developed countries, governments spend a significant amount of total output and a large fraction of this spending is used to provide public services to all households regardless of their economic status. In this paper, we show that both observations taken together imply that, in equilibrium, households always invest less in risky human capital than socially optimal. In other words, a subsidy to human capital investment financed through an incentive-neutral tax will improve social welfare. Moreover, we calibrate the model to US data and show that the growth and welfare gains from implementing the optimal policy are large when the labor-leisure choice is endogenous.

[1]For the estimation of income risk, see, for example, MaCurdy (1982), Carroll and Samwick (1997), Meghir and Pistaferri (2004), and Storesletten, Telmer, and Yaron (2004). For the consumption response, see, for example, Cochrane (1991), Flavin (1981), Townsend (1995), and Blundell, Pistaferri, and Preston (2008).

There is a straightforward economic intuition for the sub-optimality of the equilibrium allocation without taxes and subsidies. In addition to the consumption-saving decision, households have to allocate their investment between a low-return, risk-free asset (physical capital in our model) and a high return, risky asset (human capital in our model), which in turn determines the mean and volatility of individual consumption growth. When making their portfolio decisions, individual households take aggregate variables, and in particular the level of output and public consumption services, as given. Thus, they do not take into account that more investment in the high-return asset, human capital, will increase aggregate output and, as shown in this paper, the optimal level of publicly provided consumption services. If private consumption and public consumption had the same degree of riskiness, then this would not pose a problem for the optimality of the market outcome. However, if public consumption is less risky than private consumption, then in equilibrium the private risk-return trade-off differs from the social risk-return trade-off, and it becomes socially optimal to provide additional incentives for risk-taking.[2]

In this paper, we formalize the above intuition using a tractable endogenous growth model with incomplete markets. In our framework, households have the opportunity to invest in physical capital and human capital. While investment in physical capital is risk-free, human capital is subject to idiosyncratic depreciation shocks that are uninsurable and directly translate into permanent earning shocks. The government provides public services that are independent of idiosyncratic human capital shocks, has access to a linear system of taxes and subsidies, and runs a balanced budget. For given government policy, our model is tractable in the sense that the equilibrium allocation can be characterized and computed without solving for the underlying wealth distribution. Using this tractability result, we characterize the optimal government policy and show theoretically the optimality of a human capital subsidy.

For the quantitative analysis, we calibrate the model to match a number of stylized facts for the US economy. We find that in the baseline model with fixed labor-leisure choice, the optimal human capital subsidy is substantial, but switching from a zero-tax environment[3]

[2]From a different point of view, the presence of risk-free publicly provided consumption services works like an insurance system since the government applies a transfer scheme that shifts resources from risky private consumption to risk-free publicly provided consumption.

[3]In the optimal capital taxation literature that builds upon Chamley (1986) and **?**, finds that in the long-run, optimal tax rates on accumulable factors are zero. We use this result as benchmark of the

to the optimal system generates only small growth and welfare gains because of strong general equilibrium effects. More precisely, for given returns (partial equilibrium), an increase in human capital increases economic growth since human capital is the high-risk, high-return investment opportunity. In general equilibrium, this positive growth effect is dwarfed by a reduction in human capital returns since more human capital reduces the marginal product of labor, and our quantitative analysis reveals that the general equilibrium effect is quite strong. However, once we allow for an endogenous labor-leisure choice, a corresponding increase in labor-time results in only small changes in the marginal product of labor despite a large increase in human capital, and the growth and welfare effects of switching to the optimal government policy therefore become large.

This paper is related to the extensive literature on optimal income taxation analyzing the so-called Ramsey problem (Judd (1985) and Chamley (1986)).[4] Most papers in this literature assume a representative household, but Aiyagari (1995), Davila, Hong, Krusell, and Rios-Rull (2005), Conesa, Kitao, and Krueger (2009), and Imrohoroglu (1998), have also considered the effect of uninsurable idiosyncratic risk and / or binding borrowing constraints. However, none of these papers allows for a risky investment opportunity. Recently, Panousi (2007) has analyzed optimal income taxation in a model with entrepreneurial risk, but she does not consider the endogenous choice of government spending. To the best of our knowledge, this is the first paper to study optimal taxation and government spending in an economy with idiosyncratic investment risk.[5]

There is also a theoretical literature that uses two-period models to study the welfare effects of income taxation when human capital investment is risky. In particular, Eaton and Rosen (1980) argue that linear labor income taxes may reduce human capital investment risk, so that it becomes optimal to tax labor income and simultaneously subsidize human capital investment to compensate for the des-incentive effect of the labor income tax. Clearly, in this paper we emphasize a very different economic mechanism.

The rest of this paper is organized as follows. Section 4.2 presents the economic environ-

underlying quantitative analysis.

[4]There is also an important strand of the literature analyzing optimal taxation in asymmetric-information economies (Mirrlees (2005) and Golosov, Kocherlakota, and Tsyvinski (2003); for a recent review, see Kocherlakota (2005)).

[5]Acemoglu and Zilibotti (1997) also consider a setting in which it is socially optimal to encourage risk taking, but their argument is very different from ours.

ment and constructs a competitive equilibrium of the model economy. In section 4.3, we derive the main theoretical results for the baseline model, and in section 4.4, we use calibrated versions of the model economy for a quantitative analysis of the benchmark model as well as further specifications. Finally, section 4.5 concludes.

4.2 The Model

This section develops the model that underlies the theoretical and quantitative analysis conducted in the subsequent sections. As in Krebs (2003), there is a competitive production sector using a production function that displays constant returns to scale with respect to the two input factors, physical capital and efficient labor. Households are ex-ante identical, infinitely-lived and have the opportunity to invest in physical and human capital. Investment in physical capital is risk-free, but investment in human capital is subject to idiosyncratic depreciation shocks. The government provides consumption services that enter directly the households' utility function, and levies linear taxes on capital, labor and consumption in order to satisfy a balanced budget constraint.

4.2.1 The Economy

Consider a discrete-time, infinite-horizon economy with one non-perishable good that can be either consumed or invested. Competition on the input factor markets and the neoclassical production technology allows the representation of the production sector by an aggregate firm that takes factor prices as given. The aggregate firm uses physical capital K_t and efficiency units of labor L_tH_t to produce the *all-purpose* good. The production technology is given by $Y_t = F(K_t, L_tH_t)$, where L_t denotes hours worked, H_t human capital and thus, L_tH_t denotes efficiency units of hours worked. The rental rate of physical capital is r_{kt} and the rental rate of efficiency units of hours worked is r_{ht}. In each period, the firm hires capital and labor up to the point where current profits are maximized. Hence, the firm solves the following static maximization problem:

$$\max_{K_t, L_tH_t} \{F(K_t, L_tH_t) - r_{kt}K_t - r_{ht}L_tH_t\} \tag{4.1}$$

There are many ex-ante identical, infinitely-lived households with total mass of one. Households have identical preferences over private consumption plans $\{c_t\}_{t=0}^{\infty}$, private hours worked choices $\{l_t\}_{t=0}^{\infty}$ and the sequence of publicly provided consumption services $\{G_t\}_{t=0}^{\infty}$. For convenience, let lower-case letters denote individual-specific variables and upper-case letters denote aggregate variables. The specification of the utility function closely follows Barro (1990) and Guo and Lansing (1999). The one period utility function is logarithmic, and with β denoting the time preference rate, expected lifetime utility is given by

$$U(\{c_t, l_t, G_t\}_{t=0}^{\infty}) = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t (\ln c_t + \nu_l \ln(1 - l_t) + \nu_g \ln G_t)\right] \tag{4.2}$$

where ν_l and ν_g are utility parameters that measure how the household values labor and publicly provided consumption services.

Let k_t and h_t stand for the stock of physical and human capital owned by an individual household, and x_{kt} and x_{ht} denote the corresponding investment in physical and human capital. The fraction ϕ of human capital investment is bought with foregone earnings, whereas the fraction $(1 - \phi)$ is directly bought by spending wealth. Physical capital investments are, as usual, completely bought with wealth. Capital and labor markets are perfectly competitive and the government taxes (or subsidizes) capital and labor income at the flat rate τ_{kt} and τ_{ht}. In addition, the government can tax consumption at rate τ_{ct}. For convenience, we define $\tau_t = (\tau_{kt},\ \tau_{ht},\ \tau_{ct})$. The sequential budget constraint reads

$$\begin{aligned} (1 + \tau_{ct})\ c_t + x_{kt} + (1 - \phi)\ x_{ht} &= (1 - \tau_{kt})\ r_{kt}\ k_t + (1 - \tau_{ht})(l_t\ r_{ht}\ h_t - \phi\ x_{ht}) \\ k_{t+1} &= (1 - \delta_k)\ k_t + x_{kt}, \quad k_t \geq 0 \\ h_{t+1} &= (1 - \delta_h + \eta_t)\ h_t + x_{ht}, \quad h_t \geq 0 \\ &(k_0, h_0, \eta_0) \text{ given.} \end{aligned} \tag{4.3}$$

with δ_k and δ_h denoting the average depreciation rate of physical capital and human capital. The term η_t is a household-specific shock to human capital. We assume that these idiosyncratic shocks are identically and independently distributed across households and across time.[6] The random variable η_t represents uninsurable idiosyncratic labor income

[6]The budget constraint (4.3) makes two implicit assumptions about the accumulation of human capital. First, it lumps together general human capital (education and health) and specific human capital (on-the-

risk. A negative human capital shock, $\eta_t < 0$, can occur when a worker loses firm- or sector-specific human capital subsequent to job termination. In order to preserve the tractability of the model, the budget constraint rules out extended periods of unemployment because it assumes that wages are received in each period. Thus, the emphasis is on earnings uncertainty, not employment uncertainty. A decline in health provides a second example for a negative human capital shock. In this case, general and specific human capital might be lost. Internal promotions and upward movement in the labor market provide two examples of positive human capital shocks ($\eta_t > 0$). It is natural to assume that human capital shocks can never lead to the total destruction of the existing human capital stock, thus restricting the domain of the shock distribution to $\eta_t \in (-(1-\delta_h), \infty)$.

Constraint (4.3) permits households to save and dissave at the going interest rate, but does not allow for the negative financial wealth ($k_t \geq 0$ and $h_t \geq 0$). Thus, one might conjecture that the equilibrium will change once households are allowed to accumulate debt. However, this is not the case for the model analyzed here, because income shocks are permanent and not transitory and as shown in Kuhn (2008), non-negativity constraints will never bind in such an environment. More precisely, the introduction of a risk-free bond does not change the equilibrium allocation as long as the bond interest rate, r_{bt}, is given by $r_{bt} = r_{kt} - \delta_k$.

For given initial state (k_0, h_0, η_0) and given fiscal policy, $\{\tau_t,\ G_t\}_{t=0}^{\infty}$, an individual household chooses a plan, $\{c_t,\ k_{t+1},\ h_{t+1},\ l_t\}_{t=0}^{\infty}$, that maximizes his expected lifetime utility (4.2) subject to the budget constraint (4.3). Clearly, in each period, the choice $(c_t,\ k_{t+1},\ h_{t+1},\ l_t)$ is a function of the history of idiosyncratic shocks, $\eta^t = (\eta_0,\ \ldots,\ \eta_t)$.[7]

The budget constraint (4.3) can be rewritten in a way that shows that the households' optimization problem is a standard portfolio choice problem. To see this, define total wealth of an individual household as $w_t \doteq k_t + h_t$ and the fraction of total wealth invested in physical capital and human capital as $\theta_t \doteq k_t/w_t$ and $(1-\theta_t) \doteq h_t/w_t$, respectively.

job training). Second, (4.3) does not impose a non-negativity constraint on human capital investment ($x_{ht} \geq 0$).

[7] Note that the tax system τ_t may depend on t, but not on idiosyncratic shocks η_t. In this sense, the tax system does not provide insurance against idiosyncratic human capitals shocks.

Using this notation, the budget constraint simplifies to

$$w_{t+1} = \frac{(1+r_t)\ w_t - (1+\tau_{ct})\ c_t}{\theta_{t+1} + (1-\phi\ \tau_{ht})\ (1-\theta_{t+1})} \tag{4.4}$$

$$w_t \geq 0,\ \ 0 \leq \theta_t \leq 1$$

$$(k_0, h_0, \eta_0) \text{ given.}$$

with the total investment return defined as

$$r_t \doteq \theta_t\ [(1-\tau_{kt})\ r_{kt} + (1-\delta_k)] \\ + (1-\theta_t)\ [(1-\tau_{ht})\ l_t\ r_{ht} + (1-\delta_h+\eta_t)\ (1-\phi\ \tau_{ht})] - 1 \tag{4.5}$$

Equation (4.5) defines the return to investment function $r_t = r(\theta_t,\ l_t,\ \eta_t;\ r_{kt},\ r_{ht},\ \tau_{kt},\ \tau_{ht})$. Clearly, maximizing (4.2) with respect to $\{c_t,\ k_{t+1},\ h_{t+1},\ l_t\}_{t=0}^{\infty}$ subject to the budget constraint (4.3) is equivalent to maximizing (4.2) with respect to $\{c_t,\ w_{t+1},\ l_t,\ \theta_{t+1}\}_{t=0}^{\infty}$ subject to the budget constraint (4.4).

Finally, we assume that the government runs a balanced budget in each period which rules out extended periods of government debt. Thus, the government budget constraint reads

$$\tau_{ht}\ \Big(r_{ht}\ \mathbb{E}[l_t(1-\theta_t)w_t] - \phi\ \mathbb{E}[(1-\theta_{t+1})w_{t+1} - (1-\delta_h+\eta_t)(1-\theta_t)w_t]\Big) \\ + \tau_{kt}\ r_{kt}\ \mathbb{E}[\theta_t w_t] + \tau_{ct}\ \mathbb{E}[c_t] = G_t \tag{4.6}$$

From now on, we restrict the government to provide publicly consumption services proportional to the size of the economy. In particular, $G_t = \mu_t\ \mathbb{E}[c_t]$.[8]

4.2.2 Equilibrium

A competitive equilibrium of our model economy is defined as follows:

Definition 4.1 (Competitive Equilibrium).

For any given initial distribution $(w_0,\ \theta_0,\ l_0)$*, a competitive equilibrium is*

[8] Taking BEA-data from 1970 to 2009, the government-to-household consumption ratio decreased from 29 percent in 1970 to 21 percent in 2000 and started to rise again to approximately 24 percent. In the long-run a constant ratio seems to be a good approximation.

1. *A sequence of* $\{K_t, L_tH_t\}_{t=0}^{\infty}$ *that solves the firm's maximization problem* (4.1) *for given factor prices* $\{r_{kt},\ r_{ht}\}_{t=0}^{\infty}$*;*

2. *A sequence of* $\{c_t,\ w_{t+1},\ l_t,\ \theta_{t+1}\}_{t=0}^{\infty}$ *that solves the household's optimization problem* (4.2) *subject to* (4.4) *for a given sequence of factor prices* $\{r_{kt},\ r_{ht}\}_{t=0}^{\infty}$*, idiosyncratic shocks* $\{\eta_t\}_{t=0}^{\infty}$ *and fiscal policy* $\{\tau_t,\ \mu_t\}_{t=0}^{\infty}$*, for all households;*

3. *A sequence of factor prices* $\{r_{kt},\ r_{ht}\}_{t=0}^{\infty}$ *that is consistent with market clearing on the input factor markets,* $K_t = \mathbb{E}[\theta_t w_t]$ *and* $L_tH_t = \mathbb{E}[l_t(1-\theta_t)w_t]$*;*

4. *A sequence of fiscal policy* $\{\tau_t,\ \mu_t\}_{t=0}^{\infty}$ *that satisfies the government's balanced budget constraint* (4.6) *for given factor prices* $\{r_{kt},\ r_{ht}\}_{t=0}^{\infty}$ *and household policy* $\{c_t,\ w_{t+1},\ l_t,\ \theta_{t+1}\}_{t=0}^{\infty}$*, for all households.*

Introduce the aggregate capital to labor ratio $\tilde{K}_t \doteq \frac{K_t}{L_tH_t}$ and the production function in intensive form $f(\tilde{K}_t) \doteq F(\tilde{K}_t, 1)$. Using this notation, the first-order conditions associated with the firm's static profit maximization problem (4.1) are

$$r_{kt} = f'(\tilde{K}_t)$$
$$r_{ht} = f(\tilde{K}_t) - \tilde{K}_t\ f'(\tilde{K}_t)$$

Thus, $r_{kt} = r_k(\tilde{K}_t)$, $r_{ht} = r_h(\tilde{K}_t)$. Note, in equilibrium, any sequence of factor prices is completely determined by a corresponding sequence of capital-to-labor ratios $\{\tilde{K}_t\}_{t=0}^{\infty}$. For convenience, we write $r(\theta_t,\ l_t,\ \eta_t;\ r_k(\tilde{K}_t),\ r_h(\tilde{K}_t),\ \tau_{kt},\ \tau_{ht}) = r(\theta_t,\ l_t,\ \eta_t;\ \tilde{K}_t,\ \tau_{kt},\ \tau_{ht})$.

We now discuss the households' optimization problem. The first-order conditions with respect to w_{t+1}, θ_{t+1} and l_t read

$$\frac{\theta_{t+1} + (1-\phi\ \tau_{ht})(1-\theta_{t+1})}{(1+\tau_{ct})\ c_t} = \beta\mathbb{E}\left[\frac{1 + r(\theta_{t+1},\ l_{t+1},\ \eta_{t+1};\ \tilde{K}_{t+1},\ \tau_{k,t+1},\ \tau_{h,t+1})}{(1+\tau_{c,t+1})\ c_{t+1}}\right] \tag{4.7}$$

$$\frac{\nu_l}{1-l_t} = (1-\theta_t)\ \frac{(1-\tau_{ht})\ r_h(\tilde{K}_t)}{(1+\tau_{ct})}\ \frac{w_t}{c_t} \tag{4.8}$$

$$\frac{\phi\ \tau_{ht}}{(1+\tau_{ct})\ c_t} = \beta\mathbb{E}\left[\frac{\hat{r}_k(\tilde{K}_{t+1},\ \tau_{k,t+1}) - \hat{r}_h(l_{t+1},\ \eta_{t+1};\ \tilde{K}_{t+1},\ \tau_{h,t+1})}{(1+\tau_{c,t+1})\ c_{t+1}}\right] \tag{4.9}$$

where

$$\hat{r}_{kt}(\tilde{K}_t,\ \tau_{kt}) = (1-\tau_{kt})\ r_k(\tilde{K}_t) + (1-\delta_k)$$
$$\hat{r}_{ht}(l_t,\ \eta_t;\ \tilde{K}_t,\ \tau_{ht}) = (1-\tau_{ht})\ l_t\ r_h(\tilde{K}_t) + (1-\delta_h+\eta_t)\ (1-\phi\ \tau_{ht})$$

denote the current returns to physical and human capital net of depreciation and taxes / subsidies. The consumption-saving Euler equation (4.7) requires that the utility cost of saving one more unit of the *all-purpose* good must be equal to the expected discounted utility gain. The intratemporal first-order-condition (4.8) equates the marginal utility gain from leisure against the marginal benefit of working. Finally, the intertemporal first-order-condition (4.9) states that in the optimum, households are indifferent between investing one more unit into physical capital and one more unit into human capital. Because of the assumption that idiosyncratic shocks are independently distributed over time, it suffices to take the unconditional expectation with respect to η_{t+1}.

Any plan $\{c_t,\ w_{t+1},\ l_t,\ \theta_{t+1}\}_{t=0}^{\infty}$ that is a solution to the first-order conditions (4.7), (4.8), (4.9) and the budget constraint (4.4) and satisfies a corresponding transversality condition is also a solution to the utility maximization problem. Direct calculation shows that the consumption and saving policies

$$c_t = \frac{1-\beta}{1+\tau_{ct}}\ (1+r(\theta_t,\ l_t,\ \eta_t;\ \tilde{K}_t,\ \tau_{kt},\ \tau_{ht}))\ w_t \tag{4.10}$$

$$w_{t+1} = \frac{\beta}{(\theta_{t+1}+(1-\phi\ \tau_{ht})\ (1-\theta_{t+1}))}\ (1+r(\theta_t,\ l_t,\ \eta_t;\ \tilde{K}_t,\ \tau_{kt},\ \tau_{ht}))\ w_t \tag{4.11}$$

satisfy the household's budget constraint (4.4) and solve the consumption-saving Euler equation. Plugging the consumption and saving policies in the first-order-conditions with respect to hours worked (4.8), and solving for l_t finally yields the policy function for hours worked

$$l_t = \frac{1}{(1+(1-\beta)\ \nu_l)} - \nu_l\ (1-\beta)\ \frac{\theta_t\ ((1-\tau_{kt})\ r_k(\tilde{K}_t)+(1-\delta_k))+(1-\theta_t)\ (1-\delta_h+\eta_t)\ (1-\phi\ \tau_{ht})}{(1-\theta_t)\ (1-\tau_{ht})\ r_h(\tilde{K}_t)\ (1+(1-\beta)\ \nu_l)} \tag{4.12}$$

Note that (4.12) defines a function $l_t = l(\theta_t,\ \eta_t;\ \tilde{K}_t,\ \tau_{kt},\ \tau_{ht})$. In particular, by the linearity of l_t in the idiosyncratic shock η_t, we find $\mathbb{E}[l(\theta_t,\ \eta_t;\ \tilde{K}_t,\ \tau_{kt},\ \tau_{ht})] = l(\theta_t,\ \mathbb{E}[\eta_t];\ \tilde{K}_t,\ \tau_{kt},\ \tau_{ht})$. Using the policy functions (4.10), (4.11) and (4.12), the first-order condition with respect to the portfolio share θ_{t+1} simplifies to

$$\frac{\phi\ \tau_{ht}}{(\theta_{t+1} + (1-\phi\ \tau_{ht})\ (1-\theta_{t+1}))} = \mathbb{E}\left[\frac{\hat{r}_k(\tilde{K}_{t+1},\ \tau_{k,t+1}) - \hat{r}_h(\theta_{t+1},\ \eta_{t+1};\ \tilde{K}_{t+1},\ \tau_{k,t+1},\ \tau_{h,t+1})}{1 + r(\theta_{t+1},\ \eta_{t+1};\ \tilde{K}_{t+1},\ \tau_{k,t+1},\ \tau_{h,t+1})}\right] \tag{4.13}$$

Since the idiosyncratic shock η_{t+1} integrates out, the portfolio choice θ_{t+1} only depends on aggregate variables. Therefore, each household chooses the same capital share in his portfolio. The previously characterized plan $\{c_t,\ w_{t+1},\ l_t,\ \theta_{t+1}\}_{t=0}^{\infty}$ solves the set of the households' first-order conditions and it is straightforward to show that it also satisfies the associated transversality condition and, thus, is a solution to the utility maximization of an individual household. We summarize this result in the following proposition:

Proposition 4.1 (Solution to the Household's Optimization Problem)**.**
Given the initial distribution over $(w_0,\ \theta_0,\ l_0)$*. For any given sequence of tax rates* $\{\tau_{ct},\ \tau_{kt},\ \tau_{ht}\}_{t=0}^{\infty}$ *and for any given sequence of capital-to-labor ratios* $\{\tilde{K}_t\}_{t=0}^{\infty}$*, the solution to the household's optimization problem is characterized as follows:*

1. *The optimal consumption and saving policies are linear homogenous in current wealth and explicitly given by* (4.10) *and* (4.11)*;*

2. *The optimal labor-leisure choice is independent of the households' wealth but depends on the current portfolio and current realization of the idiosyncratic shock; the policy function is explicitly given by* (4.12)*;*

3. *The optimal portfolio choice is independent of the household's wealth and realization of the idiosyncratic shock; thus, every household chooses the same portfolio and* θ_{t+1} *is implicitly given as the solution to* (4.13)*.*

Having characterized the households' decision problem, we now discuss the market clearing condition. In equilibrium, the aggregate capital-to-labor ratio has to be consistent with the investment choices of the households. By definition, $k_t = \theta_t w_t$ and $h_t = (1-\theta_t)w_t$ and since every agent chooses the same θ_{t+1} for $t \geq 0$, market clearing is given by

$$\tilde{K}_t^* = \begin{cases} \dfrac{\mathbb{E}[\theta_t\ w_t]}{\mathbb{E}[l(\theta_t,\ \eta_t;\ \tilde{K}_t^*,\ \tau_{kt},\ \tau_{ht})\ (1-\theta_t)\ w_t]} & \text{for } t = 0 \\ \dfrac{\theta_t}{l(\theta_t,\ \mathbb{E}[\eta_t];\ \tilde{K}_t^*,\ \tau_{kt},\ \tau_{ht})\ (1-\theta_t)} & \text{for } t > 0 \end{cases} \tag{4.14}$$

Since we allow for arbitrary initial distributions of wealth, portfolios and idiosyncratic shocks, we cannot guarantee mutual independence of $w_0,\ \theta_0$ and η_0. However, for $t > 0$, we know that θ_t and l_t are independent of wealth, and, moreover, every household chooses the same capital share in his portfolio. This allows us to simplify the market clearing condition for $t > 0$ further, as we already did in (4.14). The equilibrium capital-to-labor ratio $\tilde{K}_t^*$ is a fixed point to (4.14) for given portfolio choices and tax rates. The equilibrium path of the capital-to-labor ratio, $\{\tilde{K}_t^*\}_{t=0}^{\infty}$, is completely determined by the initial distribution over $(w_0,\ \theta_0,\ \eta_0)$, the complete sequence of capital and labor income taxes $\{\tau_{kt},\ \tau_{ht}\}_{t=0}^{\infty}$ and by the corresponding sequence of capital shares $\{\theta_{t+1}\}_{t=0}^{\infty}$ that solve the respective first-order condition of the household

$$\frac{\phi\ \tau_{ht}}{(\theta_{t+1} + (1-\phi\ \tau_{ht})\ (1-\theta_{t+1}))} = \mathbb{E}\left[\frac{\hat{r}_k(\tilde{K}_{t+1}^*,\ \tau_{k,t+1}) - \hat{r}_h(\theta_{t+1},\ \eta_{t+1};\ \tilde{K}_{t+1}^*,\ \tau_{k,t+1},\ \tau_{h,t+1})}{1 + r(\theta_{t+1},\ \eta_{t+1};\ \tilde{K}_{t+1}^*,\ \tau_{k,t+1},\ \tau_{h,t+1})}\right]$$

More compactly, we write

$$IC(\theta_{t+1},\ \tilde{K}_{t+1}^*,\ \tau_{ht},\ \tau_{k,t+1},\ \tau_{h,t+1}) = 0 \tag{4.15}$$

Note that this condition already uses the optimal policy functions for consumption, saving and leisure and thus satisfies the respective first-order condition and the household's budget constraint, by construction. We denote the first-order condition mnemonically by IC, since it will be the implementability for the social planner below.

Using the households' plans as characterized in proposition 4.1 as well as the capital-to-labor ratio that satisfies market clearing, $\tilde{K}_t^*$, we find that the government budget constraint is independent of the current size of the economy. Again, using a more compact formulation, the government budget constraint reads

$$GC(\theta_t,\ \theta_{t+1},\ \tilde{K}_t^*,\ \tau_t,\ \mu_t) = 0 \tag{4.16}$$

For any initial distribution over $(w_0,\ \theta_0,\ \eta_0)$, and for any given sequence $\{\theta_{t+1},\ \tau_{kt},\ \tau_{ht}\}_{t=0}^{\infty}$, the government can freely choose a sequence of consumption taxes and publicly provided consumption services $\{\tau_{ct},\ \mu_t\}_{t=0}^{\infty}$ that balances its budget out without distorting the equilibrium decisions of the firm and the households.

The following proposition summarizes our previous discussion and characterizes the set of competitive equilibria.

Proposition 4.2 (Competitive General Equilibrium).
For any given initial distribution over $(w_0,\ \theta_0)$, the set of equilibria EQ is defined as

$$EQ = \Big\{ \{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty} \mid \{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty} \textit{ satisfies}$$
$$\{IC(\theta_{t+1},\ \tilde{K}_{t+1}^{*},\ \tau_{ht},\ \tau_{k,t+1},\ \tau_{h,t+1})\}_{t=0}^{\infty} \textit{ and } \{GC(\theta_t,\ \theta_{t+1},\ \tilde{K}_t^{*},\ \tau_t,\ \mu_t)\}_{t=0}^{\infty} \Big\} \quad (4.17)$$

In short, we can think of an equilibrium as a joint sequences $\{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty}$ that satisfies the implementability constraint, (4.15), and the government's budget constraint, (4.16). Importantly, the equilibrium is independent of the actual wealth distribution.

4.3 Optimal Taxation: Theoretical Results

For the derivation of the theoretical results, we use a simplified version of our model, which is specified in the following assumption:

Assumption 4.1.

1. *There is no disutility of work, $\nu_l = 0$, and households supply a fixed amount of hours worked that we conveniently normalize to unity, $l_t = 1,\ \forall t$.*
2. *Human capital is solely bought by spending the all-purpose good, $\phi = 0$.*

The second part of the assumption, $\phi = 0$, implies that the implementability and the government budget constraint simplify to

$$IC(\theta_{t+1},\ \tilde{K}^*_{t+1},\ \tau_{ht},\ \tau_{k,t+1},\ \tau_{h,t+1}) = IC(\theta_{t+1},\ \tilde{K}^*_{t+1},\ \tau_{k,t+1},\ \tau_{h,t+1})$$
$$GC(\theta_t,\ \theta_{t+1},\ \tilde{K}^*_t,\ \tau_t,\ \mu_t) = GC(\theta_t,\ \tilde{K}^*_t,\ \tau_t,\ \mu_t)$$

For any given sequence of capital and labor income tax rates, $\{\tau_{kt},\ \tau_{ht}\}_{t=0}^{\infty}$, proposition 4.1 specifies the equilibrium plan of consumption and wealth chosen by individual households. The particular representation of the equilibrium plan allows us to characterize the Pareto-optimal equilibrium in a simple and transparent manner. Using the households' policy functions for a given fiscal policy $\{\tau_t,\ \mu_t\}_{t=0}^{\infty}$, expected lifetime utility from private consumption can be calculated as

$$\begin{aligned} &\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \ \ln c_t\right] \\ &= \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \ \ln\left(\frac{1-\beta}{1+\tau_{ct}}\ \beta^t \ \prod_{n=0}^{t}(1+r(\theta_n,\ \eta_n;\ \tilde{K}^*_n,\ \tau_{kn},\ \tau_{hn}))\ w_0\right)\right] \\ &= h(w_0) - \sum_{t=0}^{\infty} \beta^t \ \left(\ln(1+\tau_{ct}) + \sum_{n=0}^{t} \mathbb{E}\left[\ln(1+r(\theta_n,\ \eta_n;\ \tilde{K}^*_n,\ \tau_{kn},\ \tau_{hn}))\right]\right) \end{aligned}$$

where $h(w_0)$ is a function of the underlying model parameters and the initial wealth distribution. The welfare effect of any fiscal policy $\{\tau_t,\ \mu_t\}_{t=1}^{\infty}$ is independent of the initial wealth and asset distribution $(w_0,\ \theta_0)$. Similarly, the portfolio choices $\{\theta_{t+1}\}_{t=0}^{\infty}$ are independent of $(w_0,\ \theta_0)$, as well. Thus, any Pareto-optimal equilibrium is preferred by all types of households, $(w_0,\ \theta_0)$, over all alternative equilibria, such that the set of Pareto-optimal equilibria is independent of the initial distribution over household types. More precisely, any Pareto-optimal equilibrium is the solution to the following constrained social planner problem[9]

$$\max_{\{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty}} V\left(\{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty}\right) \tag{4.18}$$

[9]Here, we use the terminology of constrained and unconstrained problems in the pure mathematical sense and do not associate them with the information structure in the economy, as it is often done in the literature.

subject to

$$\{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty} \in \Big\{ \{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty} \mid \{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty} \text{ satisfies}$$
$$\{IC(\theta_{t+1},\ \tilde{K}_{t+1}^*,\ \tau_{k,t+1},\ \tau_{h,t+1})\}_{t=0}^{\infty} \text{ and } \{GC(\theta_t,\ \tilde{K}_t^*,\ \tau_t,\ \mu_t)\}_{t=0}^{\infty} \Big\} \tag{4.19}$$

where the objective function in (4.18) is defined as

$$\begin{aligned} V\left(\{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty}\right) = (1+\nu_g)\ & \left(h(w_0) - \sum_{t=0}^{\infty}\ \beta^t\ \ln(1+\tau_{ct}) \right) + \nu_g\ \sum_{t=0}^{\infty}\ \beta^t\ \ln \mu_t \\ & + \sum_{t=0}^{\infty}\ \beta^t\ \sum_{n=0}^{t}\ \mathbb{E}\left[\ln(1+r(\theta_n,\ \eta_n;\ \tilde{K}_n^*,\ \tau_{kn},\ \tau_{hn})) \right] \\ & + \nu_g\ \sum_{t=0}^{\infty}\ \beta^t\ \sum_{n=0}^{t}\ \ln(1+r(\theta_n,\ \mathbb{E}[\eta_n];\ \tilde{K}_n^*,\ \tau_{kn},\ \tau_{hn})) \end{aligned}$$

The constrained social planner problem can be transformed into an unconstrained social planner problem as follows. Define

$$T_t = \theta_t\ \tau_{kt}\ r_{kt}(\tilde{K}_t^*) + (1-\theta_t)\ \tau_{ht}\ r_{ht}(\tilde{K}_t^*)$$

which measures to what extend total investment is taxed ($T_t > 0$), respectively subsidized ($T_t < 0$). Using the new notation, the government budget constraint (4.6) can be written as

$$\tau_{ct} = \frac{\mu_t\ (1-\beta)\ (1+r(\theta_t,\ \mathbb{E}[\eta_t];\ \tilde{K}_t^*,\ 0,\ 0) - T_t) - T_t}{(1-\beta)\ (1+r(\theta_t,\ \mathbb{E}[\eta_t];\ \tilde{K}_t^*,\ 0,\ 0) - T_t) + T_t} \tag{4.20}$$

This defines a function $\tau_{ct} = \tau_c(\theta_t,\ \mu_t,\ T_t)$. For any choice $(\theta_t,\ \mu_t,\ T_t)$, the government budget constraint can be satisfied by choosing τ_{ct} according to $\tau_c(\theta_t,\ \mu_t,\ T_t)$. Similarly, direct calculation shows that for any choice of $(\theta_t,\ \mu_t,\ T_t)$, the implementability constraint

(4.15) will hold if capital and labor income taxes are

$$\tau_{h,t+1} = -\left(\mathbb{E}\left[\frac{r_h(\tilde{K}^*_{t+1})}{1 + r(\theta_{t+1},\ \eta_{t+1};\ \tilde{K}^*_{t+1},\ 0,\ 0) - T_{t+1}}\right]\right)^{-1}$$
$$\times\ \mathbb{E}\left[\frac{\theta_{t+1}\ ((r_k(\tilde{K}^*_{t+1}) - \delta_k) - (r_h(\tilde{K}^*_{t+1}) - \delta_h + \eta_{t+1})) - T_{t+1}}{1 + r(\theta_{t+1},\ \eta_{t+1};\ \tilde{K}^*_{t+1},\ 0,\ 0) - T_{t+1}}\right] \tag{4.21}$$

$$\tau_{k,t+1} = \frac{T_{t+1} - (1 - \theta_{t+1})\ r_k(\tilde{K}^*_{t+1})\ \tau_{h,t+1}}{\theta_{t+1}\ r_k(\tilde{K}^*_{t+1})} \tag{4.22}$$

This defines functions $\tau_{ht} = \tau_h(\theta_t,\ \mu_t,\ T_t)$ and $\tau_{kt} = \tau_k(\theta_t,\ \mu_t,\ T_t)$. The constrained social planner problem (4.18) subject to (4.19) is equivalent to the unconstrained social planner problem

$$\max_{\{\theta_{t+1},\ \mu_t,\ T_t\}_{t=0}^{\infty}} \tilde{V}\left(\{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty}\right) \tag{4.23}$$

where

$$\begin{aligned}\tilde{V}\left(\{\theta_{t+1},\ \mu_t,\ T_t\}_{t=0}^{\infty}\right) = (1+\nu_g)\ &\left(h(w_0) - \sum_{t=0}^{\infty}\ \beta^t\ \ln(1 + \tau_c(\theta_t,\ T_t,\ \mu_t))\right)\\ &+ \nu_g \sum_{t=0}^{\infty}\ \beta^t\ \ln \mu_t\\ &+ \sum_{t=0}^{\infty}\ \beta^t\ \sum_{n=0}^{t}\ \mathbb{E}\left[\ln(1 + r(\theta_n,\ \eta_n;\ \tilde{K}^*_n,\ 0,\ 0) - T_n)\right] +\\ &+ \nu_g \sum_{t=0}^{\infty}\ \beta^t\ \sum_{n=0}^{t}\ \ln(1 + r(\theta_n,\ \mathbb{E}[\eta_n];\ \tilde{K}^*_n,\ 0,\ 0) - T_n)\end{aligned}$$

The discussion above is summarized in the following proposition:

Proposition 4.3 (Equivalence of Constrained and Unconstrained Planner Problem). *Any Pareto-optimal equilibrium allocation can be found by solving either the constrained social planner problem,* (4.18) *subject to* (4.19)*, or the unconstrained social planner problem,* (4.23).

Straightforward but tedious calculations reveal that the objective function in (4.23) is strictly concave. Since there is a convex choice set, the social planner problem (4.23) has at most one solution. In the appendix we show that the maximization problem has indeed a solution. Thus, there is a unique solution to the social planner problem (4.23), and therefore a unique Pareto-optimal equilibrium. As proofed in the appendix, the solution to the social planner's problem has the following properties:

Proposition 4.4 (Optimal Taxes and Public Services).
Let $\{\theta_t,\ \mu_t,\ T_t\}_{t=0}^{\infty}$*, respectively* $\{\theta_t,\ \mu_t,\ \tau_{ct},\ \tau_{kt},\ \tau_{ht}\}_{t=0}^{\infty}$*, be the solution to the social planner problem. If* $\sigma_\eta > 0$ *and* $\nu_g > 0$*, the solution to the social planner problem is characterized by:*

(i.) Optimality of a stationary fiscal policy

$$(\theta_t,\ \mu_t,\ T_t) = (\theta,\ \mu,\ T)$$

and in particular

$$(\theta_t,\ \mu_t,\ \tau_{ct},\ \tau_{kt},\ \tau_{ht}) = (\theta,\ \mu_g,\ \tau_c,\ \tau_k,\ \tau_h)$$

(ii.) Optimality level of government spending

$$\mu = \nu_g$$

(iii.) Optimality of subsidizing total investment

$$T < 0$$

(iv.) Optimality of subsidizing human capital

$$\tau_h < 0$$

Proof. The proof is deferred to the appendix. □

4.4 Optimal Taxation: Quantitative Analysis

4.4.1 Calibration

For our quantitative analysis, we use three different specifications of our model economy. We calibrate the models such that the stationary equilibrium is consistent with stylized annual facts of the US economy. In order to preserve the comparability of our results, we recalibrate the model for each specification. The specifications are as follows: First, households do not value leisure, $\nu_l = 0$, and we normalize their labor supply to unity, $l_{it} = 1$. Furthermore, investment is solely bought by spending wealth, meaning $\phi = 0$. This is basically the model setup for which we derived the theoretical results in the previous section. In the second specification, households value leisure, but there is still no human capital investment through forgone earnings. Third, we impose the restriction that a fraction $\phi = 0.25$ of human capital investment is bought through foregone earnings. This parameter value is consistent with the range of values found and applied by Trostel (1993).

Because the main contribution of our paper is to provide an argument that the optimal tax rates are distortionary to the portfolio decision of the households, we choose an economic environment without distortionary taxes as benchmark to which we calibrate the relevant model parameters.[10] Thus, $\tau_k = \tau_h = 0$.

We now calibrate the preference parameters. Since we are not interested in the welfare effect of changes in the public good provision, we assume that the government already provides the optimal amount of public consumption services. The optimality condition with respect to public service provision is $\nu_g = \mu$.[11] Since $\mu = \frac{g_t}{\mathbb{E}[c_t]}$, we simply set ν_g to the average government consumption to private consumption ratio found in US time series: $\nu_g = 0.255$. In order to satisfy the government budget constraint, we moreover get $\tau_c = 0.255$.[12] As noted previously, the first model specification sets $\nu_l = 0$. For the second and third model specification, we choose ν_l such that the average labor supply in equilibrium amounts to

[10]This non-distortative tax rate on accumulable assets is actually the optimal tax system derived in a Chamley-Judd economy.

[11]Actually, in section 4.3, we have shown that this condition holds for the first model specification. However, it is straightforward to prove that this condition also extends to the more elaborated specifications two and three.

[12]The government and private consumption time series includes years 1970 to 2009 and is taken from the BEA NIPA tables.

one third of the households' time endowment. For both specifications, we find $\nu_l = 2.5568$. Finally, the time preference rate is set such that the equilibrium saving rate is 20 percent. For the first specification, we get $\beta = 0.9247$ while specification two and three require $\beta = 0.9250$.

The depreciation rates are set to $\delta_k = \delta_h = 0.06$. For physical capital, this value lies within the range suggested by the literature, e.g. Cooley and Prescott (1995). For human capital, Browning, Hansen, and Heckman (1999) find annual depreciation rates between 0 and 4 percent. Accounting for the infinite horizon structure in our model, we have to add an additional depreciation of 2 percent to capture full depreciation of human capital after 50 years of work life. Thus, $\delta_h = 0.06$ is at the upper bound of reasonable values suggested in the literature. For the i.i.d. depreciation shock to human capital, we assume that $\eta \sim N(\mu,\ \sigma,\ \underline{a},\ \overline{a})$ with $\underline{a}$ and $\overline{a}$ denoting the lower and upper truncation point of the distribution. We set $\underline{a} = -0.75$ and $\overline{a} = 0.75$ which guarantees the minimum requirement on the domain of the η-distribution, $\eta > -(1 - \delta_h)$ and $\mathbb{E}[\eta] = 0$. Clearly, η is a permanent human capital shock which translates into a permanent labor income shock for the household. The evolution of the logarithm of labor income is governed by

$$\begin{aligned}
\log\ln h_{i,t+1} &= \log\ln(\beta\ [1 + r(\theta,\ \eta_t;\ \tilde{K}^*,\ \tau_k,\ \tau_h)\ h_t]) \\
&\approx \log\ln\beta + \log\ln h_{it} + r(\theta,\ \eta_t;\ \tilde{K}^*,\ \tau_k,\ \tau_h) \\
&= \omega + \frac{(1-\theta)\ (1-\phi\ \tau_h)}{1 + (1-\beta)\ \nu_l}\ \eta_t
\end{aligned}$$

with some constant ω which contains all terms that do not include the idiosyncratic human capital depreciation shock. The logarithm of labor income follows approximately a random walk with drift, and the mean and the standard deviation of the permanent component of the logarithm of labor income are $\mu_{yh} = 0$ and $\sigma_{yh} = \frac{(1-\theta)\ (1-\phi\ \tau_h)}{1+(1-\beta)\ \nu_l}\ \sigma_\eta$. In the empirical literature, the random walk specification is often used to model the permanent component of labor income risk. For example, Carroll and Samwick (1997) find a standard deviation of 0.147 whereas Meghir and Pistaferri (2004) estimate a value of 0.182. Storesletten, Telmer, and Yaron (2004) additionally condition labor income risk on the business cycle and find that labor income risk varies between 0.12 and 0.21. We calibrate σ_η such that in equilibrium, permanent labor income risk exhibits a standard deviation of 0.15. This yields

$\sigma_\eta = 0.2532$ in the first specification and $\sigma_\eta = 0.3013$ in the second and third specification.

The aggregate production technology is Cobb-Douglas with intensive form representation $f(\tilde{K}) = z\tilde{K}^\alpha$. We set $\alpha = 0.36$ to match the capital share of income according to the values suggested in the literature. The technology parameter z is chosen such that in equilibrium, aggregate consumption grows at two percent per annum. For the first specification, this yields $z = 0.3184$ and for the second and third specification, we get $z = 0.6386$. The calibration values are given in table 4.1, and the associated equilibrium allocations are provided in table 4.2.

Table 4.1: Calibration

	DESCRIPTION	(1)	(2)	(3)	MATCHES
α	technology parameter	0.3600	0.3600	0.3600	capital share
δ_k, δ_h	depreciation rates	0.0600	0.0600	0.0600	
ϕ	forgone earnings	0	0	0.2500	
ν_g	utility parameter	0.2550	0.2550	0.2550	government expenditure to private consumption ratio
z	technology parameter	0.3192	0.6442	0.6442	consumption growth rate 2%
β	time preference rate	0.9230	0.9231	0.9231	saving rate 20%
σ	sd depreciation shock	0.2602	0.3269	0.3269	labor income risk 0.15
ν_l	utility parameter	0	2.4851	2.4851	labor supply $\mathbb{E}[l_i] = 1/3$

4.4.2 Results

The optimal tax policy and its welfare and growth implications are given in table 4.2. Welfare effects ΔW are computed according to Lucas (1987) in consumption equivalent units. In the first model specification, the optimal capital and labor income tax rates are 2.0 and -2.5 percent, respectively. Due to a very strong general equilibrium effect, the possibility to encourage more risk taking is limited. Specifically, reducing the labor income tax leads to a portfolio shift from physical to human capital. Consequently, the equilibrium capital-to-labor ratio decreases thereby pushing the equilibrium interest rate upwards and the equilibrium wage rate downwards and thus discourages human capital investment. Taken together, the reduction of the tax rate on labor income induces a decrease in the wage rate such that the effect of the tax policy on the net return to human capital is almost offset. Clearly, following this line of argument, there is only a small spread between the optimal tax rates leading to minor welfare gains of about 0.06 percent from implementing the optimal tax system. Of course, in a small open economy where prices are exogenously fixed by the international financial market, the general equilibrium effect would be absent leading to more substantial welfare effects. In this sense, our framework establishes a lower bound of the welfare effects.

Adding an endogenous labor-leisure choice helps to break the strong general equilibrium effect. Reducing the labor income tax rate encourages both, investment into human capital and labor supply. Clearly, investing into human capital becomes more profitable if the household simultaneously increases his labor supply. The opposite holds as well: increasing labor supply makes investment into human capital more profitable. This reinforcing effect helps to overrule the general equilibrium effect more easily and for the calibrated model economy, the optimal labor income tax drops to -9.5 percent whereas the optimal capital income tax rises to 5.4 percent. The established spread between both tax rates of 14.9 percentage points leads to substantial welfare gains of 1.49 percent and the annual growth rate rises by substantial 0.67 percentage points.

Imposing that 25 percent of human capital investment are payed by foregone earnings leads to a stronger reduction of the optimal labor income tax compared to the previous result. This result is due to the fact that the fraction of labor income that is invested through foregone earnings is exempted from the labor income tax. Thus, subsidizing human capital needs to take into account that a fraction of the subsidy is basically not payed out and therefore raises labor income by a lower amount as in specification two. In other words, the government has to rise labor income subsidy beyond the previous result in order to encourage sufficient risk taking. The spread between capital and labor income tax increases to 17 percent, the welfare gain, however, is slightly lower as before (because the policy is not as effective as previously). The same holds for the growth effect.

Table 4.2: Results

	benchmark	(1)	(2)	(3a)	(3b)
OPTIMAL ALLOCATION					
θ	0.4127 (0.4124)	0.4024	0.3751	0.3781	0.3600
$E[l_i]$	1 (1/3)	1	0.3645	0.3657	0.3451
OPTIMAL POLICY					
τ_k	0	0.0198	0.0540	0.0510	0.0695
τ_h	0	-0.0249	-0.0950	-0.1188	0.1024
τ_c	0.2550	0.2768	0.3702	0.3869	0.2704
σ_h	0	0	0	0	0.2465
WELFARE/ GROWTH EFFECTS (IN PERCENT/ PERCENTAGE POINTS)					
$\triangle W$		0.06	1.49	1.39	1.91
$\triangle\gamma_c^*$		0.14	0.67	0.62	0.80

Finally, we allow the government to encourage human capital investment more directly by subsidizing the investment into human capital. Clearly, the direct policy dominates the indirect policy. It even happens, that it is now optimal to tax labor income in order to subsidize investment into human capital. The net effect on human capital accumulation, however, remains positive, as can be seen from the decrease in the equilibrium portfolio choice from 0.4124 to 0.3600. Thus, the government still wants to encourage more risk taking by the private households. The optimal capital income tax and the optimal labor income tax are 7.0 and 10.2 percent, respectively, and human capital investment is now subsidized by 24.7 percent. Of course, giving the government one more instrument, it cannot do worse. The welfare effect rises to substantial 1.9 percent while the annual growth rate increases by substantial 0.8 percentage points.

4.5 Conclusions

We have shown that first, there is a substantial amount of idiosyncratic risk and second, governments provide a significant amount of non-wasteful consumption services, there is too less investment into the risky asset, human capital,in the competitive equilibrium. The social planner can thus implement a welfare improving tax policy that encourages more risk taking (investment in human capital) by the households. However, there are strong general equilibrium effects that may offset almost all benefits of subsidizing the risky asset, leading to very small welfare and growth effects of the optimal policy. In our model with human capital as the risky asset, introducing an endogenous labor leisure choice substantially breaks the strong general equilibrium effect.

4.A Appendix: Proof of Proposition 4.4

For convenience, we rewrite

$$
\begin{aligned}
r_t &= \theta_t\ [(1-\tau_{kt})\ r_{kt} - \delta_k] + (1-\theta_t)\ [(1-\tau_{ht})\ l_t\ r_{ht} - \delta_h + \eta_t] \\
&= \theta_t\ [r_{kt} - \delta_k] + (1-\theta_t)\ [l_t\ r_{ht} - \delta_h] - \theta_t\ \tau_{kt}\ r_{kt} - (1-\theta_t)\ \tau_{ht}\ r_{ht} + (1-\theta_t)\ \eta_t \\
&= \Big\{\theta_t\ [r_{kt} - \delta_k] + (1-\theta_t)\ [l_t\ r_{ht} - \delta_h + \eta_t]\Big\} - T_t + (1-\theta_t)\ \eta_t
\end{aligned}
$$

and define the term in curly brackets as $\bar{r}(\theta_t, \cdot)$. Thus,

$$
r_t = \bar{r}(\theta_t, \cdot) - T_t + (1-\theta_t)\ \eta_t
$$

With this notation, we now proceed with the proof of the proposition.

Step 1: the social planner's first-order condition

The first order conditions of the social planner problem read

$$
\frac{\partial \tilde{V}(\{\theta_t,\ \mu_t,\ T_t\}_{t=0}^{\infty})}{\partial \mu_t} = -\frac{1+\nu_g}{1+\mu_t} + \frac{\nu_g}{\mu_t} = 0 \tag{4.24}
$$

$$
\begin{aligned}
\frac{\partial \tilde{V}(\{\theta_t,\ \mu_t,\ T_t\}_{t=0}^{\infty})}{\partial T_t} &= \mathbb{E}\left[\frac{1}{1+\bar{r}(\theta_t,\ \cdot) - T_t + (1-\theta_t)\eta_t}\right] \\
&\quad - \frac{(1+\nu_g)\ (1-\beta)\ \beta}{(1-\beta)(1+\bar{r}(\theta_t,\ \cdot) - T_t) + T_t} - \frac{(1-\beta-\nu_g\beta)}{1+\bar{r}(\theta_t,\ \cdot) - T_t} = 0
\end{aligned} \tag{4.25}
$$

$$
\begin{aligned}
\frac{\partial \tilde{V}(\{\theta_t, \mu_t,\ T_t\}_{t=0}^{\infty})}{\partial \theta_{t+1}} &= \mathbb{E}\left[\frac{(r_k(\theta_{t+1}) - \delta_k) - (r_h(\theta_{t+1}) - \delta_h + \eta_{t+1})}{1+\bar{r}(\theta_{t+1},\ \cdot) - T_{t+1} + (1-\theta_{t+1})\eta_{t+1}}\right] \\
&\quad + \frac{(1+\nu_g)\ (1-\beta)\ (1-\beta)\ ((r_k(\theta_{t+1}) - \delta_k) - (r_h(\theta_{t+1}) - \delta_h))}{(1-\beta)(1+\bar{r}(\theta_{t+1},\ \cdot) - T_{t+1}) + T_{t+1}} \\
&\quad - \frac{(1-\beta-\nu_g\beta)(r_k(\theta_{t+1}) - \delta_k) - (r_h(\theta_{t+1}) - \delta_h)}{1+\bar{r}(\theta_{t+1},\ \cdot) - T_{t+1}} = 0
\end{aligned} \tag{4.26}
$$

Step 2: proof of part (i.) and (ii.)

Updating equation (4.25) and combining it with equation (4.26) reveals, that the allocation (θ_{t+1}, T_{t+1}) only depends on period $t+1$ variables and thus, $(\theta_{t+1},\ T_{t+1}) = (\theta,\ T)$, $\forall t$. Moreover, condition (4.24) immediately yields $\mu_t = \nu_g$, $\forall t$. Taken together, this proofs part (i.) of the proposition, $(\theta_t,\ T_t) = (\theta,\ T)$ and $(\theta_t,\ \mu_t,\ \tau_{ct},\ \tau_{kt},\ \tau_{ht}) = (\theta,\ \mu_g,\ \tau_c,\ \tau_k,\ \tau_h)$, respectively, and part (ii.) of the proposition, $\mu_t = \nu_g$.

Step 3: proof of part (iii.)

Updating (4.25) and combining it with (4.26) yields

$$\mathbb{E}\left[\frac{\eta}{1+\bar{r}(\theta,\ \cdot) - T + (1-\theta)\eta}\right] = (1+\nu_g)(1-\beta)\frac{(r_k(\theta)-\delta_k)-(r_h(\theta)-\delta_h)}{(1-\beta)(1+\bar{r}(\theta,\ \cdot)-T)+T} \tag{4.27}$$

where we dropped the time indices for convenience. It is easier to work with (4.27) instead of (4.26). For simplicity, define

$$\Omega_T(\theta,\ T) = \mathbb{E}\left[\frac{1}{1+r(\theta,\ \cdot) - T + (1-\theta)\eta}\right] - \frac{(1+\nu_g)\ (1-\beta)\ \beta}{(1-\beta)(1+\bar{r}(\theta,\ \cdot)-T)+T} - \frac{1-\beta-\nu_g\beta}{1+\bar{r}(\theta,\ \cdot)-T} \tag{4.28}$$

$$\Omega_\theta(\theta,\ T) = \mathbb{E}\left[\frac{\eta}{1+\bar{r}(\theta,\ \cdot) - T + (1-\theta)\eta}\right] - (1+\nu_g)(1-\beta)\frac{(r_k(\theta)-\delta_k)-(r_h(\theta)-\delta_h)}{(1-\beta)(1+\bar{r}(\theta,\ \cdot)-T)+T} \tag{4.29}$$

We now have to show that there exists a (θ^*, T^*) that solves $\Omega_T(\theta^*,\ T^*) = 0$ and $\Omega_\theta(\theta^*,\ T^*) = 0$.

First, we show that for any $\theta \in [0,1]$, there exists a $T \in [\underline{T}, 0]$ that solves equation (4.28). Fix $\theta = \bar{\theta}$ and $\underline{T} = -\frac{1-\beta}{\beta}(1+\bar{r}(\bar{\theta},\ \cdot))$. Since $\bar{r}(\bar{\theta},\ \cdot)$ is finite for any $\bar{\theta}$, the lower bound $\underline{T}$ is well defined. We now apply the intermediate value theorem. On the one side, taking the limit $\lim_{T\searrow\underline{T}} \Omega_T(\bar{\theta},\ T)$, we find that the first and the third term are finite while the second term goes to infinity. Thus, $\lim_{T\searrow\underline{T}} \Omega_T(\bar{\theta},\ T) = -\infty < 0$. On the other sides, we evaluate $\Omega_T(\bar{\theta}, T)$ at $t = 0$ which yields

$$\Omega_T(\bar{\theta},\ 0) = \mathbb{E}\left[\frac{1}{1+\bar{r}(\bar{\theta}) + (1-\theta)\eta}\right] - \frac{1}{1+\bar{r}(\theta,\ \cdot)}$$

Due to strict convexity of $\frac{1}{1+r(\bar{\theta})+(1-\theta)\eta}$ in η, Jensen's inequality leads to

$$\Omega_T(\bar{\theta},\ 0) \geq \frac{1}{1+\bar{r}(\theta,\ \cdot)} - \frac{1}{1+\bar{r}(\theta,\ \cdot)} = 0$$

Thus, $\Omega_T(\bar{\theta},\ 0) \geq 0$. Clearly, $\Omega_T(\bar{\theta},\ T)$ is continuous on the domain of T. Applying the intermediate value theorem yields that for any $\theta \in [0,1], \exists\ T \in [-\frac{1-\beta}{\beta}(1+\max \bar{r}(\theta,\ \cdot)), 0]$ that solves the first order condition with respect to T. Moreover, the map $T = T(\theta)$ is continuous.

Next, we show that for the continuous map $T = T(\theta)$ defined previously, there exists a solution θ that solves equation (4.27). By construction of $T = T(\theta)$, we know that $(1-\beta)(1+\bar{r}(\theta,\ \cdot) - T(\theta)) + T(\theta) > 0$. In addition, using the natural restriction $\eta \in [-(1-\delta_h), \infty]$ together with $T(\theta) < 0$ ensures $1 + \bar{r}(\theta,\ \cdot) - T + (1-\theta)\eta > 0$. Thus, the Inada conditions imply

$$\lim_{\theta \nearrow 1} \Omega_\theta(\theta,\ T(\theta)) = -\infty$$
$$\lim_{\theta \searrow 0} \Omega_\theta(\theta,\ T(\theta)) = +\infty$$

Therefore, given $T = T(\theta)$, we know that there exists a $\theta \in [0,1]$ that solves (4.27), establishing the existence of an equilibrium $(\theta^*,\ T^*)$ with $T < 0$.

Finally, by the *Principle of Optimality* we can establish uniqueness of the equilibrium straightforwardly.

Step 4: proof of part (iv.)

We now show that it is always optimal to subsidize the risky asset, $\tau_h < 0$. Strict concavity of $\frac{\eta}{1+\bar{r}(\theta^*,\ \cdot)-T^*+(1-\theta*)\eta}$ in η and Jensen's inequality imply

$$\mathbb{E}\left[\frac{\eta}{1+\bar{r}(\theta^*,\ \cdot) - T^* + (1-\theta^*)\eta}\right] < \frac{\mathbb{E}[\eta]}{1+\bar{r}(\theta^*,\ \cdot) - T^* + (1-\theta^*)\mathbb{E}[\eta]} = 0$$

By the upper and lower bounds on T, we know that $(1-\beta)(1+\bar{r}(\theta^*,\ \cdot) - T^*) + T^* > 0$. Hence, equation (4.27) implies $(r_k(\theta^*) - \delta_k) - (r_h(\theta^*) - \delta_h) \leq 0$. Now, we focus on equation

(4.26): The second and third term can be rewritten as

$$((r_k(\theta^*) - \delta_k) - (r_h(\theta^*) - \delta_h)) \; \frac{\nu(1-\beta)(1+\bar{r}(\theta^*,\ \cdot)) - (1-\beta-2\beta\nu+\nu)T^*}{((1-\beta)(1+\bar{r}(\theta^*,\ \cdot)-T^*)+T^*)(1+\bar{r}(\theta^*,\ \cdot)-T^*)}$$

Knowing that $T^* \in [-\frac{1-\beta}{\beta}(1+\bar{r}(\theta^*,\ \cdot)), 0]$, we can show that that $\nu(1-\beta)\ (1+\bar{r}(\theta^*,\ \cdot)) - (1-\beta-2\beta\nu+\nu)T^* > 0$. Therefore, the complete expression is negative such that equation (4.26) implies

$$\mathbb{E}\left[\frac{(r_k(\theta^*) - \delta_k) - (r_h(\theta^*) - \delta_h + \eta)}{1+\bar{r}(\theta^*,\ \cdot) - T^* + (1-\theta^*)\eta}\right] > 0 \tag{4.30}$$

The optimal income tax rates (τ_k, τ_h) provide the incentive such that in competitive equilibrium, it is optimal for each household to choose θ^*. Hence

$$\begin{aligned} 0 &= \mathbb{E}\left[\frac{((1-\tau_k)r_k(\theta^*) - \delta_k) - ((1-\tau_h)r_h(\theta^*) - \delta_h + \eta)}{1+\bar{r}(\theta^*,\ \cdot) - T^* + (1-\theta^*)\eta}\right] \\ &= \mathbb{E}\left[\frac{(r_k(\theta^*) - \delta_k) - (r_h(\theta^*) - \delta_h + \eta) + (\tau_h r_h(\theta^*) - T^*)\frac{1}{\theta}}{1+\bar{r}(\theta^*,\ \cdot) - T^* + (1-\theta^*)\eta}\right] \end{aligned}$$

Solving for τ_h yields

$$\tau_h = -\frac{\mathbb{E}\left[\frac{\theta^*((r_k(\theta^*)-\delta_k)-(r_h(\theta^*)-\delta_h+\eta))-T^*}{1+\bar{r}(\theta^*,\ \cdot)-T^*+(1-\theta^*)\eta}\right]}{\mathbb{E}\left[\frac{r_h(\theta^*)}{1+\bar{r}(\theta^*,\ \cdot)-T^*+(1-\theta^*)\eta}\right]}$$

By (4.30) and $T^* < 0$, we conclude that the numerator is positive. Multiplying with (-1) and dividing by a positive number, we arrive at $\tau_h < 0$, which finally proofs part (iv.) of the proposition.

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Eidesstattliche Erklärung

Hiermit erkläre ich, die vorliegende Dissertation selbständig angefertigt und mich keiner anderen als der in ihr angegebenen Hilfsmittel bedient zu haben. Insbesondere sind sämtliche Zitate aus anderen Quellen als solche gekennzeichnet und mit Quellenangaben versehen.

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