Discussion Paper No. 12-045

Prices, Debt and Market Structure in an Agent-Based Model of the Financial Market

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Non-technical Summary

In this paper we develop an agent-based model of the financial market. Agent-based modeling is a simulation-based technique that is gaining popularity in economics. In an agent-based model autonomously acting and interacting units (e.g. representing financial market participants) endogenously generate structures and system properties which become subject to analysis. In contrast to conventional equilibrium models, agent-based models prominently take into account mutual dependencies between agents and feedback processes.

In our model boundedly rational agents trade a financial asset. Their trading strategy thereby depends on their return forecast which is formed by either considering fundamentals or technical analysis. This setup is well established in the literature and has been exceptionally successful in reproducing several stylized facts of financial asset price time series. With our model we analyze the behavior of a large number of heterogeneous agents. The heterogeneity can predominantly be attributed to differing trading strategies and the ensuing divergence in the structure of agents' balance sheets. The balance sheet aspect of the financial markets has been largely neglected in extant literature.

Specifically, we endow each agent with a balance sheet comprising of a risky asset and cash on the asset side and equity capital and debt on the liabilities side. The risky asset is traded among agents at an endogenously set price. We assume that agents actively manage their respective balance sheet in two regards. Firstly, they choose a portfolio which optimizes the ratio between risky assets and cash conditional on their current return forecast, and secondly they aim at a fixed ratio between equity and debt (leverage ratio). Agents are constrained in their ability to acquire and dispose of debt by the credit supply of a risk managing financier and credit frictions, which hinder agents to make immediate changes to their debt levels.

We simulate our model and show that it can reproduce several empirically observable facts and relationships. Although we initially endow all agents with identical balance sheets, the size distribution of agents quickly converges to a lognormal distribution, which is typically observed for investment banks. We furthermore observe a natural tendency for inequality to increase over time. When we impose low credit frictions on the model financial market, leverage becomes procyclical, which is also typical for investment banks.

In a next step we vary central parameters of the model exogenously in order to identify their effect on financial stability. By varying the leverage target of agents, we find that an increased target goes along with increased price volatility. Furthermore, a higher leverage ratio leads to greater size-inequality among agents. Leverage thereby naturally generates agents that are "too-big-to-fail". When varying the degree of credit frictions agents are confronted with, we find that lower frictions increase price volatility. This suggests that short term debt, which is essential for achieving immediate changes to debt levels, may have detrimental effects on financial market stability. On the other hand, we find that lowering credit frictions can drastically reduce the number of bankruptcies encountered in simulations.

Nicht-technische Zusammenfassung

In dem vorliegenden Papier entwickeln wir ein agentenbasiertes Modell für den Finanzmarkt. Die agentenbasierte Modellierung stellt eine in den Wirtschaftswissenschaften an Bedeutung gewinnende Methodik dar, in der eine Vielzahl von dezentral handelnden Einheiten (bspw. Finanzmarktakteure) mit Hilfe von Computersimulationen abgebildet und analysiert werden. Anders als bei sog. Gleichgewichtsmodellen spielen in agentenbasierten Modellen wechselseitige Abhängigkeiten zwischen den Agenten und Rückkopplungseffekte eine wesentliche Rolle. Die sich aus der Interaktion heterogener Agenten herausbildenden Systemeigenschaften und Strukturen sind Untersuchungsgenstände agentenbasierter Modelle.

Die beschränkt rationalen Agenten handeln in diesem Modell ein Wertpapier aufgrund unterschiedlicher Renditeerwartungen, welche auf Basis einer Fundamentalwert- und einer Chartistenstrategie geformt werden. Dieser Modellierungsansatz ist gut in der Literatur etabliert und eignet sich hervorragend, um statistische Eigenschaften von realen Zeitreihen auf Finanzmärkten zu replizieren. Wir untersuchen eine große Anzahl von Agenten mit heterogenen Eigenschaften. Die Heterogenität in unserem Modell ist vor allem auf individuelle Bilanzstrukturen der Agenten zurückzuführen. Diesem Aspekt wurde in der bestehenden Literatur bisher nur wenig Beachtung geschenkt.

Konkret statten wir die Agenten mit einer Bilanz aus, die auf der Aktivseite aus risikofreien liquiden Mitteln und dem riskanten Vermögensgegenstand, welcher auf dem artifiziellen Markt gehandelt wird, besteht. Auf der Passivseite der Bilanz steht Eigenkapital Fremdkapital gegenüber. Wir nehmen an, dass die Agenten ihre Bilanzen aktiv managen. Zunächst werden riskante Papiere gehandelt, um ein von der Renditeerwartung abhängiges optimiertes Verhältnis zu erreichen. In einem nächsten Schritt wird aktiv Verschuldung aufgenommen oder reduziert, um ein avisiertes Verhältnis zwischen Fremd- und Eigenkapital (Leverage) zu erzielen. Hierbei ist es wichtig, dass die Verschuldungsmöglichkeiten beschränkt sind. Diese sind zum einen abhängig von dem Angebot an Fremdkapital, welches ein Finanzier auf Basis eines Risikomodells zur Verfügung stellt. Auf der anderen Seite erlauben geringe Kreditfriktionen kurzfristig finanzierten Agenten, ihre Passivseite schnell umzustrukturieren.

Monte-Carlo Simulationen des Modells zeigen, dass dieses in der Lage ist, verschiedene empirisch beobachtbare Effekte nachzubilden. Obwohl die Grundannahme ist, dass alle Agenten zunächst eine identische Bilanzstruktur haben, ergibt sich aus dem Modell, dass die Größenverteilung der Agenten durch eine Log-Normalverteilung beschrieben werden kann. Dieses Verhalten ist auch in realen Daten für Investmentbanken beobachtbar. Des Weiteren finden wir für den Fall geringer Kreditfriktionen prozyklische Verschuldung, wie sie empirisch für Investmentbanken gemessen wird.

In einem nächsten Schritt können wir einige zentrale Parameter des Modells exogen variieren und daraus Politikempfehlungen hinsichtlich des Ziels der Finanzstabilität ableiten. Hohe Verschuldungshebel verstärken die Preisvolatilität und die Größenungleichheit zwischen den unterschiedlichen Akteuren. Das Modell deutet somit darauf hin, dass eine Kontrolle des Verschuldungsgrades auch die „too-big-to-fail"-Problematik adressieren könnte. Geringe Finanzierungsfriktionen verringern die Anzahl der Insolvenzen. Auf der anderen Seite erhöhen diese jedoch die Volatilität der Märkte, so dass hier ein Zielkonflikt zwischen verschiedenen Maßzahlen der Finanzstabilität besteht.

Prices, Debt and Market Structure in an Agent-Based Model of the Financial Market*

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Abstract

We develop an agent-based model in which heterogenous and boundedly rational agents interact by trading a risky asset at an endogenously set price. Agents are endowed with balance sheets comprising the risky asset as well as cash on the asset side and equity capital as well as debt on the liabilities side. The introduction of balance sheets and debt into an agent-based setup is relatively new to the literature and allows us to tackle several research questions that are mostly inaccessible following conventional methodology, especially representative agent models. A number of findings emerge when simulating the model. We find that the empirically observable log-normal distribution of bank balance sheet size naturally emerges and that higher levels of leverage lead to a greater inequality among agents. When further analyzing the relationship between leverage and balance sheets, we observe that decreasing credit frictions result in an increasingly procyclical behavior of leverage, which is typical for investment banks. We show how decreasing credit frictions increase volatility but decrease the number of bankruptcies.

JEL classification: C63 - D53 - D84
Keywords: agent-based model - financial markets - instability - balance sheets - leverage - size distribution - credit frictions

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1 Introduction

The past few years have indicated that the understanding of the dynamics in financial markets is far from satisfactory. Economists and regulators often seem to rely on intuition rather than model-guided comprehension when pondering and designing new rules for the financial system. As a result, most of the suggested financial market regulations are impeded by controversy about their efficiency and uncertainty about their impact. One reason why financial market dynamics prove so difficult to grasp and model is that they are driven by heterogeneous market participants' actions and interactions that feed back into the financial system.

We present an uncalibrated agent-based model (ABM) that includes debt in order to facilitate an analysis of the dynamics ensued by agents' capital structure. Each agent in our model is therefore endowed with a highly stylized balance sheet containing a tradable risky asset and cash on the asset side and equity capital and debt on the liabilities side. Agents trade according to their price expectations, which they form through either fundamental value considerations (fundamentalists) or technical analysis (chartists). The price of the risky asset depends on agents' transactions and therefore evolves endogenously. Leverage can generally be managed by agents but is constrained by the debt supply of an exogenous risk managing financier. Simulations are conducted to demonstrate the general working of our model as well as some of the new possibilities of analysis provided by the model, which are unfeasible with either standard representative agent models or existing agent-based financial market models focusing predominantly on price dynamics. We can report several findings. Specifically we show how credit frictions[1] can change the relationship between leverage and assets and thereby account for the differences observed for commercial and investment banks in this context: for investment banks leverage is procyclical, while no such relation can be observed for commercial banks. By looking at the emergent market structure of the model, we find that balance sheet size is approximately lognormally distributed and that there is a natural tendency for inequality to increase over time. Higher leverage intensifies the evolution towards higher inequality between agents. Where possible, we compare the outcomes of the simulations with balance sheet data from a sample of international banks and make reference to the views expressed in the relevant literature. Policy implications, especially with regard to financial stability, are given where appropriate.

[1] We define credit frictions as the latency with which agents can acquire and dispose of debt.

The remainder of the paper is organized as follows. After reviewing the related literature and presenting a methodological motivation for our approach in the next section, Section 3 presents the model. Section 4 then provides simulation results. Here, we start by showing some basic dynamics of an exemplary simulation in Section 4.1. We proceed, in Section 4.2, by looking at the distribution of agents' balance sheet size and the effects of leverage on balance sheet evolution. The role of credit frictions in our model market is analyzed in Section 4.3. Section 5 concludes.

2 Literature Review

The majority of agent-based financial market models focus on price dynamics, which emerge through the interaction of heterogeneous agents. Such models have been quite successful in replicating and explaining some intriguing features of the financial market such as endogenous bubbles and crashes as well as stylized facts of return time series including fat tails and clustered volatility. Compelling reviews of the literature can e.g. be found in LeBaron (2006), Chiarella et al. (2009), Hommes and Wagener (2009), and Lux (2009). Incorporating balance sheets containing debt and equity into financial market ABMs is a sensible extension to established models and is mostly novel. Notable exceptions include Raberto et al. (2011) and Thurner et al. (2010). While the model introduced in Raberto et al. (2011) takes a macroeconomic perspective and mainly focuses on the lending channel of banks, the model presented in Thurner et al. (2010) is closer to our approach. However Thurner et al. (2010) are less interested in pure balance sheet dynamics and rather focus on the effects of leverage on returns, which they find to produce fat tails and clustered volatility. Furthermore the setups of the two approaches differ in many respects, including the portfolio choice of agents, the separation of investment and leverage strategies as well as the inclusion of both debt demand and supply.

Although the study of leverage and balance sheet dynamics is novel in the context of agent-based models, the issue has been addressed by prominent researchers in other contexts. Early work emphasizing the role of leverage and balance sheets can be found in the debt deflation theory of Fisher (1933), and in Minsky's financial instability hypothesis (see Minsky, 1986). In Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) leverage acts as a financial accelerator for non-financial borrowers. The resurfacing of research on leverage and balance sheet dynamics in the aftermath of the recent financial crisis suggests its importance for understanding the workings of the financial system and the events precipitating the crisis. Adrian et al.

(2010) argue that there is an important relation between financial intermediaries' balance sheet dynamics and real economic activity. The dynamics of market and funding liquidity, which reinforce each other and can lead to destabilizing effects on financial markets, are analyzed theoretically by Brunnermeier and Pedersen (2009), while Geanakoplos (2009) shows how changes to leverage can cause wild fluctuations in asset prices. More generally, the inclusion of the financial sector into new macroeconomic DSGE models (see e.g. Curdia and Woodford, 2009; Gertler and Kiyotaki, 2010) is a further indicator for the increasing importance of financial markets for economic theory. Conversely, the linkage between the real economy and the financial sector is also being addressed in recent agent-based research (see Lengnick and Wohltmann, 2011; Scheffknecht and Geiger, 2011; Westerhoff, 2011).

Agent-based models constitute a promising method for advancing the understanding of the financial system's underlying dynamics. In the context of our model there exists arguably a comparative advantage of the agent-based methodology over the rational representative agent paradigm.[2] In contrast to the top-down approach of representative agent models, agent-based models take a bottom-up modeling approach. Thereby they account for the *fallacy of composition* (see e.g. Caballero, 1991; Kirman, 1992), i.e. ABMs follow the assumption that the aggregate behavior of interacting agents does not have to coincide with the behavior of the individual. By substituting aggregate individual behavior with the behavior of one, mostly rational, representative agent, mainstream macroeconomic models are kept analytically tractable while satisfying the Lucas Critique which demands that models are microfounded (see Lucas, 1976). This aggregation approach seems appealing not only to economists but, as Kirman (2010) remarks, also to politicians and commentators, who, when speaking of financial markets, often refer to "the market" as if it were an individual. Such oversimplifying assumptions often suppress interesting and important details.[3] The event of a bankruptcy is e.g. not feasible within a representative agent framework. The fact that the possibility of default is mostly neglected in theoretical models (cp. Goodhart and Tsomocos, 2011) is unfortunate, not least when considering the devastating effects of the Lehman Brothers default in 2008. Agent-based models, on the other hand, have been used to explic-

[2]More profound and comprehensive criticisms of mainstream economic models can e.g. be found in Leijonhufvud (2009); Colander et al. (2009); Kirman (2010); Stiglitz (2011). Comparisons between agent-based models and DSGE models can e.g. be found in Farmer and Geanakoplos (2009); Fagiolo and Roventini (2012).

[3]There are also serious theoretical reservations that adhere to this aggregation approach. See e.g. Stoker (1993) for a thorough discussion of the issue.

itly investigate the propagation of bankruptcies in the financial system (see e.g. Battiston et al., 2009; Tedeschi et al., 2011; Lenzu and Tedeschi, 2012; Markose et al., 2012). When allowing for more than one agent, heterogeneity enters a model. Heterogeneity leads to interactions which lead to endogenous developments. Prices e.g. evolve endogenously and bubbles, crashes or return time series stylized facts emerge. Stiglitz (2011) writes:

> Standard Models focused on the wrong questions. They focused on explaining the small "normal" variations in the economy - which don't matter much - and ignored the large variations which matter a great deal. They asked how the economy responded to exogenous shocks, while some of the most important disturbances - the bubbles that periodically occur, and then break - are clearly endogenous.

When simulating our model we find that it exhibits strong path dependence. Within the same parameter constellation, repeated simulations with differing error terms (random numbers drawn from the same distribution) display highly variant outcomes ranging from relatively efficient and tranquil markets to the collapse of the system with all agents defaulting. This emergent property of agent-based models makes them seem arbitrary at times, whereas the existence of countable solutions (unique equilibrium or multiple equilibria) in most mainstream models seems to tell a clearer story. At best, however, stable and empirically testable patterns and distributions emerge in simulations of ABMs. The factors that lead to a certain pattern or distribution can then be analyzed and valuable insights about the workings of the financial system (in our case) may be disclosed. The emergent property inherent to the agent-based methodology can thus help to advance our knowledge of the dynamics in the financial system.

3 The Model

The model described in the following can be classified as a "few type" agent-based financial market model. While agents cannot produce entirely new trading strategies, as is possible through evolutionary learning algorithms in some so-called "many type" models, they can choose from a set of predefined trading rules, the rule they deem most profitable to them under the limitations imposed on their rationality. Specifically, in our model agents can select either a strategy based on fundamentals or a chartist strategy based on technical analysis. The implied assumption that real traders do choose and switch between these two strategies finds strong support in the literature (see e.g. Menkhoff and Taylor, 2007) and chartist-fundamentalist-approaches figure among the most common agent-based financial market models (see e.g. Lux and Marchesi (2000), Farmer and Joshi (2002) and Westerhoff and Dieci (2006)). Heterogeneity enters the model not only through the differing strategy types, but also through departing configurations within the strategies. Disagreement may prevail on the true fundamental value of an asset and there may be different methods and time frames considered by chartists to extrapolate future price movements from historic ones. In general, heterogeneity is key to any agent-based model. It is the emergent aggregate behavior ensuing from interacting heterogeneous agents which lies at the focus of agent-based analysis and embodies a salient distinction between ABMs and models with a representative agent. In the context of agent-based financial market models, emergent aggregate behavior e.g. encompasses stylized facts such as fat tails and clustered volatility, asset bubbles and crashes. Our model also allows for the analysis of emergent leverage and balance sheet dynamics, as well as market structure.

3.1 Model Structure

While the replication of financial market return time series stylized facts has constituted the aim of many ABMs, much less attention has been directed towards emergent behavior in the balance sheet dimension of financial markets. For this reason, we endow each agent j in our model with the following schematic balance sheet at time t:

Assets	Liabilities
$Q_{j,t}P_t$	$E_{j,t}$
$C_{j,t}$	$O_{j,t}$

The assets side of the balance sheet comprises the quantity $Q_{j,t}$ of a risky asset with price P_t as well as cash $C_{j,t}$ which can be held without risk. It will often be useful to consider logarithmic prices, which will be denoted in lower case (i.e. $p_t = \log(P_t)$)[4]. On the liabilities side, each agent is endowed

[4]In the following we will make use of lower-case letters for logarithmic values and upper-case letters for real values. The main rationale for using log prices p_t is to ensure that real prices P_t remain non-negative in the price formation process.

with equity capital[5] $E_{j,t}$ and outside capital (debt) $O_{j,t}$. The balance sheet total $B_{j,t}$ is given by:

$$B_{j,t} = Q_{j,t}P_t + C_{j,t} = E_{j,t} + O_{j,t} \tag{1}$$

From the beginning of period t to the beginning of period $t+1$ balance sheets evolve as sketched below:

Assets	Liabilities
$(Q_{j,t} + D_{j,t}) \exp(p_t + r_{t+1})$	$E_{j,t} + \Delta E_{j,t+1}$
$C_t + \Delta C_{t+1}$	$O_{j,t} + \Delta O_{j,t}$

where $D_{j,t}$ is the demand of agent j for the asset in period t and r_{t+1} is the logarithmic return, with $r_{t+1} = p_{t+1} - p_t$.[6] The debt level $O_{j,t+1}$ in period $t+1$ consists of the debt level from the start of period t, i.e. $O_{j,t}$, and a change to outside capital $\Delta O_{j,t}$, which depends on the agent's strategic demand for debt as well as the available supply of debt. As indicated by the time index, the change in outside capital $\Delta O_{j,t}$ already takes place before the end of period t, so that agents can use the newly acquired debt for trading in period t. The timing of the model is schematized in Figure 1. Each period t in the model represents a trading day in which all agents first revise and possibly change their trading strategy (see Section 3.3), forecast the return of the following period $t+1$ (see next section), make a decision on how much debt they want to hold and ultimately trade.

Equity capital grows with the returns R_{t+1} and R_C on the risky and risk free (cash) asset, respectively, and decreases with the interest i paid on debt. Both the risk free rate and the interest rate on debt are exogenous in our model. In a frictionless market we would assume $R_C = i$. The equity capital (equivalent to an agent's net worth) evolves endogenously:

$$\Delta E_{j,t+1} = (Q_{j,t}P_t)R_{t+1} + C_{j,t}R_C - iO_{j,t+1} \tag{2}$$

We thereby assume that new assets (i.e. $D_{j,t}$) are bought and sold at price $P_{t+1} = P_t(1 + R_{t+1})$. In a model with debt there is always the possibility of bankruptcy, i.e., the equity capital $E_{j,t}$ of an agent becomes smaller or equal to zero. This possibility needs to be taken into account by introducing a resolution procedure for bankrupt agents. We force bankrupt agents to liquidize all assets they hold on their balance sheet upon bankruptcy.[7] Bankruptcies

[5] After the initial endowment we assume that agents cannot issue new equity, for instance in the form of a seasoned equity offering. Equity capital therefore evolves as the difference between the balance sheet total and debt. Strategic changes to the liabilities side of the balance sheet can therefore only be incurred by changes to the debt level.

[6] The relation between logarithmic (r) and real (R) returns is defined as $r = \log(1+R)$.

[7] Technically the demand function from Equation (7) changes to $D_{j,t} = -Q_{j,t}$ when $E_{j,t} \leq 0$, i.e., all assets of a bankrupt agent are thrown on the market regardless of the execution price.

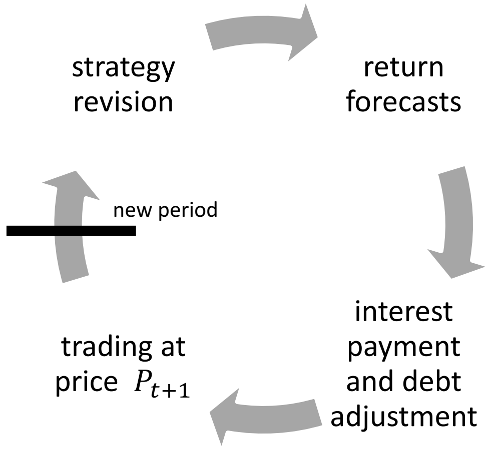


Figure 1: Timing of the model

can thereby impose a fire sale externality on the market. The bankrupt agent then disappears from the market and all losses are borne by the exogenous financier.

Using the balance sheet equality from Equation (1), the change in cash amounts to

$$\Delta C_{j,t+1} = -D_{j,t}P_{t+1} + C_{j,t}R_C - iO_{j,t+1} + \Delta O_{j,t} \tag{3}$$

We model the agent's portfolio choice (i.e. the proportion $A_{j,t+1}$ of the balance sheet he wants to hold in the risky asset in the upcoming period $t+1$) in dependence of the agent's forecast of log excess return and his confidence in this forecast, which is modeled with a measure of historic forecast errors $\sigma_{j,t}^{\mathrm{FE}}$:

$$A_{j,t+1} = \frac{\mathrm{E}_{j,t}[r_{t+1}] - r_C}{\gamma \sigma_{j,t}^{\mathrm{FE}}} \tag{4}$$

Generally we denote the forecast of agent j made in period t for the variable x in period $t+1$ as $\mathrm{E}_{j,t}[x_{t+1}]$. The parameter $\gamma > 0$ can be viewed as a risk aversion parameter. The forecast error is modeled as the square root of an exponentially weighted moving average of squared differences between return expectations and return realizations:

$$\sigma_{j,t}^{\mathrm{FE}} = \sqrt{\theta^{\mathrm{FE}}(\mathop{\mathrm{E}}_{j,t-1}[r_t] - r_t)^2 + (1-\theta^{\mathrm{FE}})(\sigma_{j,t-1}^{\mathrm{FE}})^2}, \tag{5}$$

with $0 \leq \theta^{\mathrm{FE}} \leq 1$ being a memory parameter defining how much weight should be assigned to the most recent forecast error.

Note that for Equation (4) we choose a similar structure as in classical myopic portfolio choice models with CARA utility functions or models that maximize a linear combination of return mean and variance (see e.g. Campbell and Viceira, 2002). The essential difference is that here the portfolio choice variable $A_{j,t+1}$ represents the ratio of risky assets to balance sheet total rather than the ratio of risky assets to net worth. Thus, to implement the portfolio choice from Equation (4), an agent j must act so that the following relation is satisfied in the balance sheet dimension:

$$A_{j,t+1} = \frac{\mathrm{E}_{j,t}[P_{t+1}](Q_{j,t} + D_{j,t})}{\mathrm{E}_{j,t}[B_{j,t+1}]} \tag{6}$$

The proportion of an agent's balance sheet held in the risky asset is bounded by $[-1, 0] \leq A_{j,t+1} \leq 1$. The upper bound 1 is due to an agent's budget constraint, while the lower bound can take a value between 0 and -1, depending on the constraints imposed on short selling. The closer to 0 the lower bound is set, the higher the barriers for going short. By varying the lower bound we can thus study how short selling constraints of different intensities affect the financial market.[8]

The approach detailed in Equations (4) and (6) allows us to separate an agent's leverage strategy from his portfolio choice. In classical myopic portfolio choice models leverage is linked to investment opportunities - only when large returns are expected does leverage enter the model (i.e. when $A_{j,t} > 1$). Here, on the other hand, the agent's debt choice enters the demand function, which can be obtained by rearranging Equation (6):

$$D_{j,t} = \frac{A_{j,t+1}\, \mathrm{E}_{j,t}[B_{j,t+1}]}{\mathrm{E}_{j,t}[P_{t+1}]} - Q_{j,t} \tag{7}$$

with

$$\begin{aligned} \underset{j,t}{\mathrm{E}}[B_{j,t+1}] &= \underset{j,t}{\mathrm{E}}[P_{t+1}](Q_{j,t} + D_{j,t}) + C_{j,t} + \underset{j,t}{\mathrm{E}}[\Delta C_{j,t+1}] \\ &= \underset{j,t}{\mathrm{E}}[P_{t+1}]Q_{j,t} + C_{j,t}(1 + R_C) - i\tilde{O}_{j,t+1} + \Delta\tilde{O}_{j,t} \end{aligned} \tag{8}$$

The amount of debt $\tilde{O}_{j,t+1}$ held by the agent in the upcoming period is subject to negotiations (indicated by the tilde) between agent and financier. It depends on an agent's demand for debt and the financier's willingness to supply the desired debt. If both negotiating parties do not wish to make

[8] While a lower bound of 0 implies that an agent can sell only as many assets as he owns (i.e. $D_{j,t} = -Q_{j,t}$), a lower bound of -1 implies that an agent cannot go short in more assets than he has means for repurchasing at any given point in time.

any changes to the debt level, i.e. $\Delta\tilde{O}_{j,t} = 0$, the absolute debt volume $O_{j,t}$ will need to be rolled over at interest rate i. To determine the trading price we choose a process that can be described as Walrasian tâtonnement where all agents trade at the market clearing price p_t^*, i.e. the price for which $\sum_{j=1}^{J} D_{j,t} = 0$. The demand of an agent is thereby contingent on his forecast of future returns (see Equation (7)), which is, as detailed in the next section, a function of the current price p_t. By means of numerical analysis the current price is changed until $p_t = p_t^*$ and markets clear.

3.2 Fundamental, Chartist and Debt Strategies

Agents can choose between a fundamental and a chartist strategy when forming expectations of future returns. When following a fundamental strategy, agents (i.e. $j \in \mathcal{F}$) believe that prices will revert to fundamental value. They therefore compare their perception of fundamental value $\mathrm{E}_{j,t}[f_{t+1}]$ with the current price in order to obtain a forecast of future returns:

$$\underset{j,t}{\mathrm{E}}[r_{t+1}] = \alpha^F(\underset{j,t}{\mathrm{E}}[f_{t+1}] - p_t), \quad \forall j \in \mathcal{F} \tag{9}$$

with $\alpha^F > 0$ being the speed at which the fundamentalist believes prices converge to fundamental value. The fundamentalist updates his perception of fundamental value by evaluating relevant fundamental news Δf_t, which can be modeled as an arbitrary stochastic process, and by identifying and correcting past valuation errors:

$$\underset{j,t}{\mathrm{E}}[f_{t+1}] = \underbrace{\underset{j,t-1}{\mathrm{E}}[f_t]}_{\text{past valuation}} + \underbrace{(\Delta f_t + \epsilon_{j,t})}_{\text{evaluation of news}} + \underbrace{\theta^F(f_t - \underset{j,t-1}{\mathrm{E}}[f_t])}_{\text{past error correction}} \tag{10}$$

The error term $\epsilon_{j,t} \sim \mathcal{N}(0, \sigma_f^2)$ accounts for fundamentalists' imperfect information and limited cognition and implies disagreement about the true value f_t of the risky asset. In the model we assume that disagreement on fundamental value may persist for some time, but agents will eventually become aware of erroneous evaluations and correct for them. The speed of this error correction is thereby given by $0 \le \theta^F \le 1$.

In order to obtain a forecast of future returns, chartists ($j \in \mathcal{C}$), in a first step, extrapolate a buy or sell signal. They do so by employing moving average (MA) rules, which are among the simplest and most popular with practicing technical analysts.[9] The signal is generated by comparing a short-term MA of prices to a long-term MA of prices. Specifically, the chartist

[9] Brock et al. (1992) provide evidence for the MA rule's capability to predict stock returns; in an agent-based context Chiarella et al. (2006) analyze the ensuing price dynamics when agents employ MA rules.

identifies an emerging upward trend and a buy signal ($S_{j,t} = +1$) is generated when the short-term MA is higher than the long-term MA, and vice versa for a downward trend and a sell signal ($S_{j,t} = -1$):

$$S_{j,t} = \operatorname{sgn}\left(\frac{1}{s_{j,t}}\sum_{u=0}^{s_{j,t}-1} P_{t-u} - \frac{1}{l_{j,t}}\sum_{v=0}^{l_{j,t}-1} P_{t-v}\right), \quad \forall j \in \mathcal{C} \tag{11}$$

The maximum number of lags $s_{j,t}$ and $l_{j,t}$ may differ from agent to agent as well as throughout time. Note that in order to allow for additional heterogeneity within the chartist strategy we do not specify $s_{j,t} < l_{j,t}$. A chartist j will thus follow a contrarian strategy whenever $s_{j,t} > l_{j,t}$. The forecast of future returns then depends on the direction in which the extrapolated signal is pointing, the aggressiveness of the chartist denoted by α^C and the absolute value of a random component $\rho_{j,t} \sim \mathcal{N}(0, \hat{\varsigma}_t^2)$:

$$E_{j,t}[r_{t+1}] = \alpha^C S_{j,t} \, |\rho_{j,t}| \tag{12}$$

The random component is necessary because the signal $S_{j,t}$ extrapolated by chartists does not imply a specific return expectation. We assume that while the moving average rule indicates the direction of the expected return, chartists randomly choose an absolute value of the expected return, which is scaled with the perceived price variability calculated as an exponentially weighted moving average:

$$\hat{\varsigma}_t^2 = \theta^S (r_t - r_{t-1})^2 + (1 - \theta^S)\hat{\varsigma}_{t-1}^2, \tag{13}$$

with θ^S being a memory parameter specifying how much weight is attributed to the most recent log return movement. The chartist thus adapts his return expectation to the prevailing price volatility. Chartists can therefore also be viewed as volatility traders who take strong positions in times of high volatility and vice versa.

Generally, we define the change in exposure to outside capital as

$$\Delta\tilde{O}_{j,t} = \mu^O \left(\tilde{O}_{j,t+1} - O_{j,t}\right), \tag{14}$$

with $\tilde{O}_{j,t+1}$ being the targeted debt volume after negotiation with the financier. Since neither the agent nor the financier can force the other party to supply or demand more debt than that party is willing to supply or demand, the debt volume will be set to the lower value of the financier's supply $O_{j,t+1}^S$ and the agent's demand $O_{j,t+1}^D$:

$$\tilde{O}_{j,t+1} := \min\{O_{j,t+1}^D, O_{j,t+1}^S\} \tag{15}$$

The parameter $0 \leq \mu^O \leq 1$ in Equation (14) introduces credit friction into the debt market. When $\mu^O < 1$ the targeted changes to debt volume take place more slowly than desired by either agent or financier. When the financier is delimitating the debt demand of the agent (i.e. $O^D_{j,t+1} > O^S_{j,t+1}$), the friction may e.g. be interpreted as credit maturity hindering the financier to withdraw his funds at once; when the financier is willing to cover the agent's full debt demand (i.e. $O^D_{j,t+1} \leq O^S_{j,t+1}$), the friction may e.g. be interpreted as delays in raising funds from different investors. Furthermore, a very low value for μ^O could be interpreted as limited institutional space to actively manage debt levels. Customer deposits held by commercial banks e.g. constitute such a limitation: while a commercial bank can invest customer deposits to a certain extent, it cannot directly increase or decrease them at will.

The structure of our model allows for the integration of arbitrary debt demand and supply functions. A simple debt strategy for an agent could be to aim for a constant leverage ratio:[10]

$$\lambda^{\text{fix}} = \frac{O^D_{j,t+1}}{\mathrm{E}_{j,t}[E_{j,t+1}]} = \frac{O^D_{j,t+1}}{\mathrm{E}_{j,t}[B_{j,t+1}] - O^D_{j,t+1}} \tag{16}$$

Note that agents are forward-looking, i.e., their desired debt level depends on their expectation of the size of their future balance sheet. Following from the previous equation, debt demand can be derived:

$$O^D_{j,t+1} = \frac{\lambda^{\text{fix}}\,\mathrm{E}_{j,t}[B_{j,t+1}]}{1 + \lambda^{\text{fix}}} \tag{17}$$

With Equation (8) it can be algebraically deduced that for the period $t+1$ agent j demands:

$$O^D_{j,t+1} = \frac{\mathrm{E}_{j,t}[P_{t+1}]Q_{j,t} + C_{j,t}(1 + R_C) - O_{j,t}}{i + \frac{1}{\lambda^{\text{fix}}}}. \tag{18}$$

We assume that financiers do not form expectations about future price movements, but rather try to assess the risk of supplying debt to individual agents. Due to the seniority of debt over equity the financier focuses on the risk that incurred losses in the subsequent periods fully deplete an agent's equity capital (i.e. the agent goes bankrupt). Specifically, the financier is willing to supply debt $O^S_{j,t+1}$ if the probability of default over the next M periods is lower than ω:

$$\Pr\left\{(E_{j,t} + O^S_{j,t+1})(1 + \mathbf{R}^B_{j,t})^M \leq O^S_{j,t+1}(1+i)^M\right\} \leq \omega \tag{19}$$

[10]We define leverage as the ratio of debt to equity capital (net worth): $\lambda = O/E$.

Since the financier does not have the expertise to assess an agent's strategy, he must solely rely on the agent's past performance (i.e debt-adjusted balance sheet growth $r^B_{j,t}$), which is for the sake of simplicity modeled as a lognormal random variable with $\log(1+\mathbf{R}^B_{j,t}) = \mathbf{r}^B_{j,t} \sim \mathcal{N}(\mu^B_{j,t}, z^2_{j,t})$. Mean and variance are estimated by the financier as exponentially weighted moving averages:

$$\begin{aligned}
\mu^B_{j,t} &= \theta^{\text{fin}} \underbrace{(\log(B_{j,t} + iO_{j,t}) - \log(B_{j,t-1} + \Delta O_{j,t}))}_{r^B_{j,t}} + (1-\theta^{\text{fin}})\mu^B_{j,t-1} \\
z^2_{j,t} &= \theta^{\text{fin}}(r^B_{j,t} - r^B_{j,t-1})^2 + (1-\theta^{\text{fin}})z^2_{j,t-1} \qquad (20)
\end{aligned}$$

θ^{fin} thereby defines how much weight is attributed to the respective last observation. With the risk constraint in Equation (19) and with $H^{-1}(\cdot)$ being the inverse cumulative distribution function of the random variable $\mathbf{r}^B_{j,t}$, the maximum amount of debt the financier is willing to supply to agent j can be derived:

$$O^S_{j,t+1} = \frac{E_{j,t} \exp\left(MH^{-1}(\omega)\right)}{(1+i)^M - \exp\left(MH^{-1}(\omega)\right)}. \qquad (21)$$

3.3 Choosing a Strategy

Agents in the model try to adapt to the prevailing situation by updating their trading strategy if it seems to be underperforming. For this purpose, each agent revises his strategy every τ_j periods. In order to avoid a synchronized change in strategy, $1 < \tau_j < n$ is a random number drawn from a discrete uniform distribution with n being the maximum number of periods before an agent revises his strategy. Formally, agent j revises his strategy at time $t \in K_j := \{t | t \bmod \tau_j = 0\}$.[11] When deciding on whether to keep or change a strategy, each agent compares a measure of the profit $\Pi_{j,t}$ his strategy has earned to a benchmark $\bar{\Pi}_t$. This comparison is modeled by a discrete choice model pioneered by Manski and McFadden (1981) and popularized in the context of agent-based models by Brock and Hommes (1998). Specifically, when agent j revises his current strategy he will stick to it with probability

$$W^F_{j,t} = \frac{\exp(\eta \Pi_{j,t})}{\exp(\eta \Pi_{j,t}) + \exp(\eta \bar{\Pi}_t)} \quad \forall t \in K_j, \qquad (22)$$

whereby $\eta > 0$ can be understood as a (bounded) rationality parameter. It limits agents' abilities to identify whether their strategies are performing

[11] The modulo operator ensures that each agent only trades in a period t which is a multiple of his trading frequency τ_j.

well or poorly in comparison to the benchmark. Low values for η imply poor identification ability and vice versa.

The profitability measure is computed as an exponentially weighted average of the most recent growth in an agent's equity capital and past equity growth:

$$\Pi_{j,t} = \begin{cases} \bar{\Pi}_t & \text{if the strategy in } t \text{ does not equal the strategy in } t-1 \\ \theta^{\Pi}(\log(E_{j,t}) - \log(E_{j,t-1})) + (1-\theta^{\Pi})\Pi_{j,t-1} & \text{else} \end{cases} \tag{23}$$

with $\theta^{\Pi} \in [0,1]$ being a memory parameter assigning how much weight is attributed to the most recent equity growth. Note from Equation (23) that the profitability measure for agent j is set to the benchmark when he changes his strategy. Thereby $\bar{\Pi}_t$ is simply the average of all agents' profitability measures, i.e.:

$$\bar{\Pi}_t = \frac{1}{J}\sum_{j=1}^{J} \Pi_{j,t} \tag{24}$$

We assume that although agents cannot directly observe the benchmark profitability, they have a notion of whether their own strategy is performing better or worse than the average strategy. The fact that this notion is not perfect is reflected by the rationality parameter η in Equation (22).

Upon opting for a chartist strategy, an agent must choose the specifications for the moving average rule, i.e. he must determine the maximum lags in Equation (11). In period $t \in K_j$ agent $j \in \mathcal{C}$ draws $s_{j,t}$ and $l_{j,t}$ randomly from a triangular distribution with the respective lower limits s^{low} and l^{low}, the respective upper limits s^{up} and l^{up} and the respective modes $c^{s}_{j,t}$ and $c^{l}_{j,t}$, with $s^{\text{low}} \leq c^{s}_{j,t} \leq s^{\text{up}}$ and $l^{\text{low}} \leq c^{l}_{j,t} \leq l^{\text{up}}$. The purpose of employing a triangular distribution with a variable mode is to ensure that chartists gravitate to the specifications of successful moving average rules. Specifically the modes are chosen so that the expected value of the triangular distribution[12] equals the expected value for the lag parameters $\hat{s}_{j,t}$ and $\hat{l}_{j,t}$ computed from a probability mass function where the respective lags for each chartist is weighted by its relative profitability:

$$\hat{s}_{j,t} = \sum_{j\in\mathcal{C}} \left(s_{j,t} \frac{\exp(\eta\Pi_{j,t})}{\sum_{j\in\mathcal{C}} \exp(\eta\Pi_{j,t})} \right) \tag{25}$$

$$\hat{l}_{j,t} = \sum_{j\in\mathcal{C}} \left(l_{j,t} \frac{\exp(\eta\Pi_{j,t})}{\sum_{j\in\mathcal{C}} \exp(\eta\Pi_{j,t})} \right) \tag{26}$$

[12] Given the lower and upper limits, the relation between the mode and the expected value of a triangular distribution amounts to $c = 3\mu - (x^{\text{up}} + x^{\text{low}})$ (Evans et al., 2000).

Note that the choice of memory parameter is also dependent on the rationality η of agents.

4 Simulations

In order to simulate the model described in the previous section, we first have to define parameter values and initial conditions. Quite a few parameters including rationality and memory relate to behavioral aspects of market participants and are therefore not directly observable. Since we mainly aim at deriving qualitative results and the calibration of complex agent-based models poses a considerable challenge (cp. Winker et al. (2007)), we refrain from trying to estimate the behavioral parameters for our model. The choices for parameter values are therefore often without deeper economic meaning. In the exemplary simulation presented in the following subsection, we introduce some of the dynamics the model features with the parameters and initial conditions documented in Tables 1 and 2 in the appendix. For the simulations in Sections 4.2 and 4.3 we change selected parameters in order to analyze their qualitative (ceteris paribus) effect on the model economy. As our model incorporates random terms at several instances[13], each simulation result is unique. In fact, simulation outcomes display strong path dependence. In order to ensure that the patterns emerging in our simulations are not caused by coincidence, we run - if not stated otherwise - 40 simulations[14] for each parameter value and plot the median result.

4.1 Exemplary Simulation

We define the process of fundamental value evolution as a noise process with a trend and - in order to emulate upswings and downturns - mean reversion.[15] The initial endowment of all $N = 500$ agents is the same: the balance sheet

[13]Specifically, this includes noise ϵ_t in the expectation process of the fundamental traders. The exact values for the moving average lags are randomly drawn for each agent from a specific distribution. The same holds true for the value $\rho_{j,t}$ determining the absolute value of chartists' return expectations. Moreover, when agents decide whether to change or stick to their strategy, their profitability determines the probability for a change, which of course implies some randomness. Last but not least, the frequency τ_j with which agents revise their strategy is assigned randomly at the beginning of each simulation.

[14]Simulations where all agents default are repeated. It is thus possible that the results documented in following subsections contain a sort of survivorship bias.

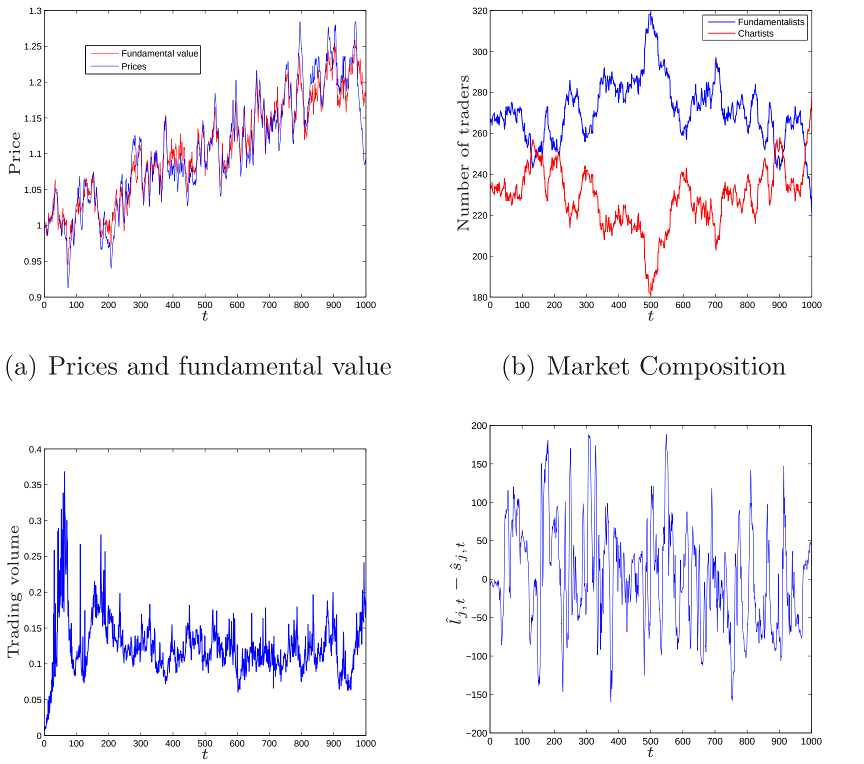
[15]Formally, this is modeled by an Ornstein-Uhlenbeck process, where the daily expected return is arbitrarily set to $\frac{0.05}{250}$ (i.e. 5% growth per trading year), volatility to $\sigma_f^2 = 0.01$, and mean reversion speed to $\theta = 0.1$. The model is initialized by setting $p_0 = f_0 = 0$.

total of each agent amounts to $B_{j,0} = 2/N$, and each agent holds the amount of risky assets that leads to an optimal portfolio when expecting the return to be equal to the trend of the fundamental value process. Agents target a leverage ratio of $\lambda = 25$, which means that agents are endowed with equity that is around 4% of total assets. At $t = 0$ the passive side of the balance sheet is constructed in order to satisfy a leverage of $\lambda = 25$. The constraints imposed by the credit supply of the financier, however, cause the agents' leverage to drop substantially in the first period. We make the simplifying assumption that $i = R_C = 0$ and thereby completely abstract from the effect of interest rates in this paper. Initially, chartists and fundamentalists each account for 50% of traders. The specific frequency τ_j with which each agent revises his strategy is initially drawn from a uniform distribution with the limits of 1 and 250, which means that agents revise their strategy at least once every trading year (one simulation period represents one trading day) and at the most every trading day. For the chartist strategy the boundaries of the moving average lags $l_{j,t}$ and $s_{j,t}$, which are initially drawn from a uniform distribution, are set to 1 and 200, which are common values in business practice (see e.g. Lo et al., 2000). In the benchmark simulation, we set the credit friction parameter to its maximum, i.e. $\mu^O = 1$, allowing agents and financiers to make immediate changes to the amount of debt they hold on their balance sheet or provide as credit.

Figure 2 shows some results of an exemplary benchmark simulation. It can be observed that the price diverges from the fundamental value on a regular basis.[16] Nevertheless, the model is stable without a single default. Periods of strong misevaluation seem to go along with a larger proportion of chartist traders in the market and, more specifically, with a larger proportion of trend-followers, which we can measure by $\hat{l}_{j,t} - \hat{s}_{j,t} > 0$. On the other hand, when contrarians ($\hat{l}_{j,t} - \hat{s}_{j,t} < 0$) dominate the population of chartists, the price is closer to the fundamental value. This is the case because a trend-following strategy amplifies the prevalent trend, while the contrarian strategy elicits a negative feedback. Trading volume fluctuates strongly in the exemplary simulation but also seems to contain a persistent component.

[16] A shortcoming of the model is the apparent smoothing of the price resulting in first-order autocorrelation. This problem could be tackled by introducing a short-term arbitrageur specialized in trading on this anomaly (LeBaron, 2010). However, since the analysis of return time series is not a focus of our model, we abstain from introducing further agent types, which would increase the complexity of the model.

Time periods of relatively high volume alternate with periods of relatively little trading.[17]



(a) Prices and fundamental value

(b) Market Composition

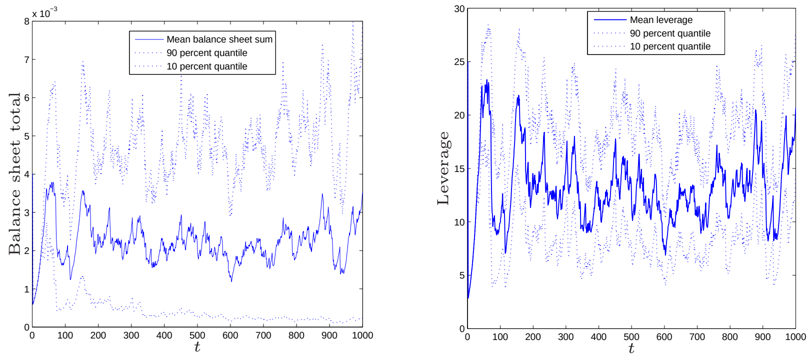
(c) Trading volume

(d) Difference between mean long moving average and short moving average

Figure 2: Dynamics in an exemplary simulation

Figure 3 shows the dynamics of the mean balance sheet total as well as mean leverage. As stated before, all agents are initially equal. However, the initial homogeneity changes quickly as simulation time progresses. As the plotted quantiles illustrate, substantial differences between agents develop. The nature of how these differences evolve in terms of balance sheet size will be addressed in the upcoming section. Noteworthy is also the apparent co-movement of mean leverage and mean balance sheet total, which will be addressed in Section 4.3.

[17] Note that the unusually high trading volume in the first 200 periods can be attributed to the fact that chartist traders do not have enough memory to correctly employ their moving average method with a maximum time horizon of 200 periods.



(a) Balance sheet total (b) Leverage

Figure 3: Mean and quantiles of agents' balance sheet total and leverage for the exemplary simulation

4.2 Distribution and Leverage

Distributions of e.g. wealth, income or output constitute an emergent property of an economy and can reveal valuable information about its state. Since redistribution is often an explicit goal of economic policy, it is important to understand the process leading to the observable distribution. Agent-based models can be helpful in this regard. In the following, we will take a look at the distribution of agents' balance sheet size, which endogenously evolves when simulating our model.

As stated, we initially assume that all agents are of equal size and thereby homogeneous. In the simulation, however, the distribution converges to a stable log-normal distribution. This result is presented in Figure 4, showing that the Jarque-Bera statistic (testing for the normality of logarithmic balance sheet size) converges to a value lower than the critical value given a 5% significance level.[18] The most convincing argument for the emergence of a log-normal distribution for balance sheet size in our model is given by *Gibrat's law*, which states that convergence to log-normality occurs when balance sheet growth is normally distributed and independent from size.[19] Figure 5(a) shows the emerging distribution for the exemplary simulation of the previous subsection. For comparison, Figure 5(b) depicts the distribution

[18]Note that we suppressed the first 200 periods due to the fact that we initially assume all agents to be equal, leading to extremely high test statistics. Furthermore, we take the median of the simulation results to control for extreme outliers.

[19]If we assume $x_t - x_{t-1} = g_t x_{t-1}$ for small values for growth rate g_t, the function converges to $\log x_t = \log x_0 + g_1 + g_2 + \cdots + g_t$, implying a log-normal distribution (Sutton, 1997).

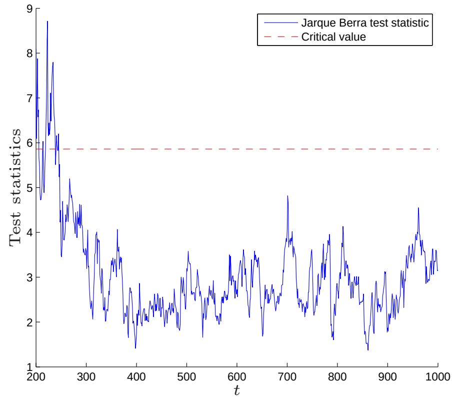
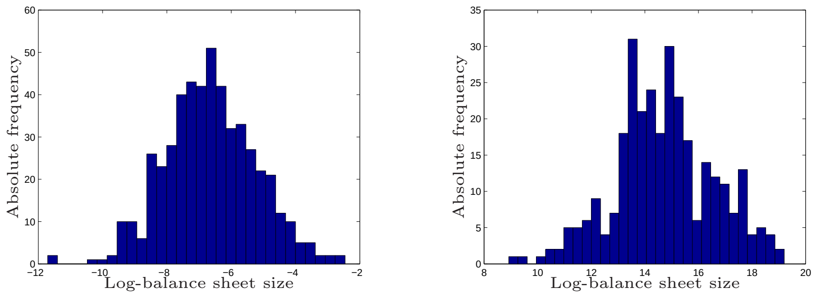


Figure 4: Jarque-Bera test statistics for the log-balance sheet size distribution (median results after 40 simulation runs)

of an international sample of investment banks.[20] The distributions qualitatively resemble each other as is also confirmed by the Jarque-Bera test for log-normality. The test statistics are provided in Table 3 in the appendix.[21]



(a) Histogram of an exemplary simulation at $t = 1000$

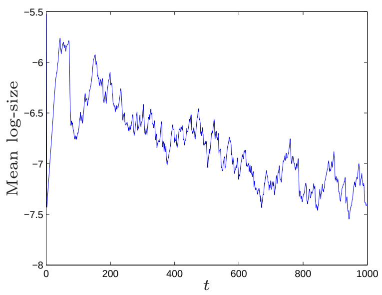
(b) Histogram of investment banks in 2009

Figure 5: Histogram for simulations and empirical data

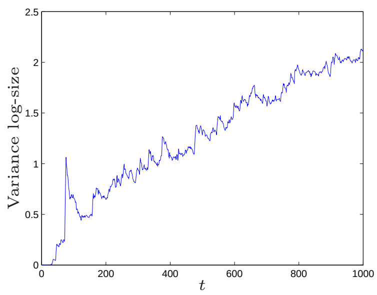
[20] Here we use annual balance sheet data of international investment banks from the Bankscope database.

[21] As presented in Janicki and Prescott (2006) this result does not hold for commercial banks, which can rather be described by a Pareto distribution. A theoretical rationale can be found in their business model and in a product differentiation argument: regional banks provide credit to regional small and medium-sized enterprises. The non-log-normal distribution of non-financial firms (cp. Axtell (2001)) therefore is also reflected in the distribution of commercial banks (Ennis, 2001).

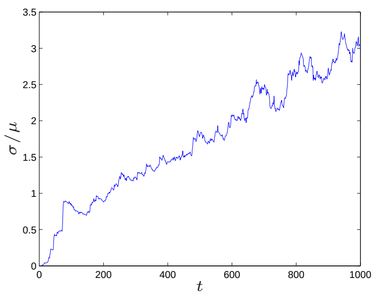
When looking at the average evolution of balance sheets throughout simulations (see Figure 6), we observe a decreasing trend of mean (log) balance sheet size while the variance steadily increases. Furthermore, the size dispersion of balance sheets, which we measure with the coefficient of variation (i.e. σ/μ), increases, which is indicative of an endogenous increase of inequality[22] with progressing simulation time. Effectively, our model suggests that the financial system naturally generates a large number of small institutions and a small number of very large institutions. There is thus a natural tendency for the system to produce institutions that are *too big to fail*.



(a) Mean log size



(b) Variance log size



(c) Coefficient of variation

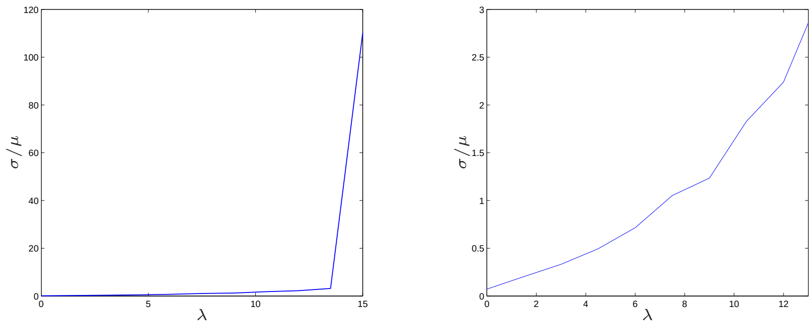
Figure 6: Endogenous average evolution of balance sheets (median results after 40 simulation runs)

Leverage seems to play an interesting role in the evolution of balance sheet distribution. In order to analyze this role we replace the debt supply function of the risk-managing financier with unlimited debt supply, while agents keep aiming for a constant leverage λ. By varying the target leverage for all agents from λ = 0-15, we can now control for the overall leverage in the model economy. Note that we only provide simulations up to a leverage

[22]The coefficient of variation provides an inequality measure insensitive to changes in the mean (Cowell, 2000).

target of $\lambda = 15$ rather than the more realistic target value of $\lambda = 25$ in the benchmark simulation. In the framework without the stabilizing financier the model market becomes highly fragile for large values of λ, with frequent breakdowns of the entire financial system.

As shown in Figure 7(a), our model displays a positive relationship between leverage and size dispersion. Theoretical studies discussing the effects



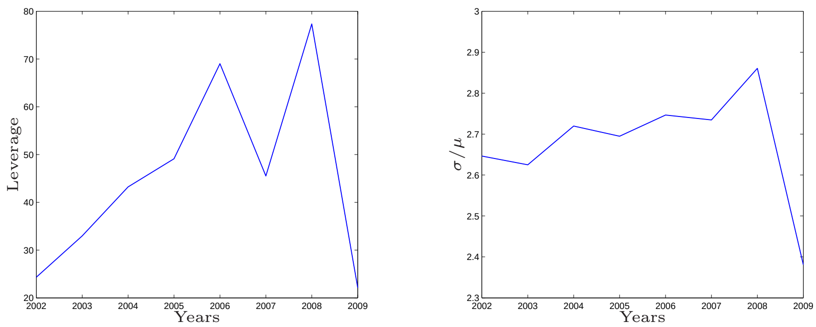
(a) Size inequality for $\lambda = 0 - 15$ (b) Size inequality for $\lambda = 0 - 13$

Figure 7: Size inequality for variation of target leverage λ in simulations (median results after 40 simulation runs)

of distribution functions frequently argue with the entry and exit mechanisms in markets. In our simple model, we do not account for entries, and exits are only possible through bankruptcies (as opposed to mergers or voluntary liquidation). In this regard, the extreme ascent of inequality observed in our model for values of $\lambda > 13$ may be attributed to the effect of bankruptcies. The linear trend for low values of λ (see Figure 7(b)) is followed by a strong non-linear behavior, especially for $\lambda > 13$. Possibly, this is brought about by defaulting agents and the associated complex market dynamics resulting from fire sales. As shown in Figure 10(c), bankruptcies strongly increase for values of $\lambda > 13$.

Despite the irregularities observed for high values of λ in the model, the quintessence of Figure 7(a) is that leverage seems to foster the natural evolution towards higher inequality described above. This conclusion may be of importance for policy makers. In this context, the introduction of a maximum leverage ratio into financial market regulation may not only help to stabilize the financial system in a more traditional sense (lower leverage decreases the probability of default), but could also decrease the speed with which inequality increases. Lower size dispersion arguably generates less institutions that classify as *too big to fail*.

A first glance at our sample of international investment banks seems to support the notion that high leverage increases inequality. In Figure 8 we plot the average leverage of investment banks from the end of the Dot-Com crisis in 2002 up to 2009. The average increase of leverage between 2002 and 2008 is accompanied by an increase in size dispersion, as predicted by our model. The significant drop in average leverage from 2008 to 2009, on the other hand, is reflected by a sharp decline in size dispersion. Note, however, that with the data available to us we cannot make inferences about the causality of the observed relationship.



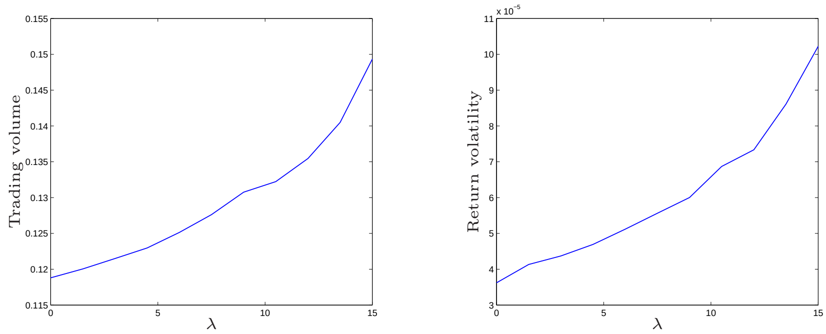
(a) Mean leverage for investment banks 2002 to 2009

(b) Coefficient of variation for investment bank size 2002 to 2009

Figure 8: Mean leverage and size inequality for international investment banks (Bankscope data)

Although there exists some empirical evidence suggesting that banks, as the agents in our model, do target a constant leverage (cp. Gropp and Heider, 2010), an unlimited supply of debt is certainly not a realistic assumption. When looking at the following effects of leverage on our model financial market, it should be kept in mind that a constrained debt supply may lead to less clear or even different results. Nevertheless, we briefly want to show some interesting patterns emerging in simulations in the context of varying leverage targets. As most of these patterns are empirically untested, further research is needed before meaningful conclusions can be reached.

Figure 9(a) shows an emerging positive relationship between leverage and trading volume. Here leverage acts as a multiplier to trades: A higher leverage target causes agents to acquire or dispose of larger sums of nominal debt in order to meet their target as the value of the risky asset on their balance sheet rises or falls, respectively. Since debt is obtained and repaid in cash, any change in agents' nominal debt also changes the composition of agents' balance sheets. The rebalancing of portfolios generates trading volume, which therefore increases as the leverage target is raised.



(a) Total trading volume (b) Return volatility

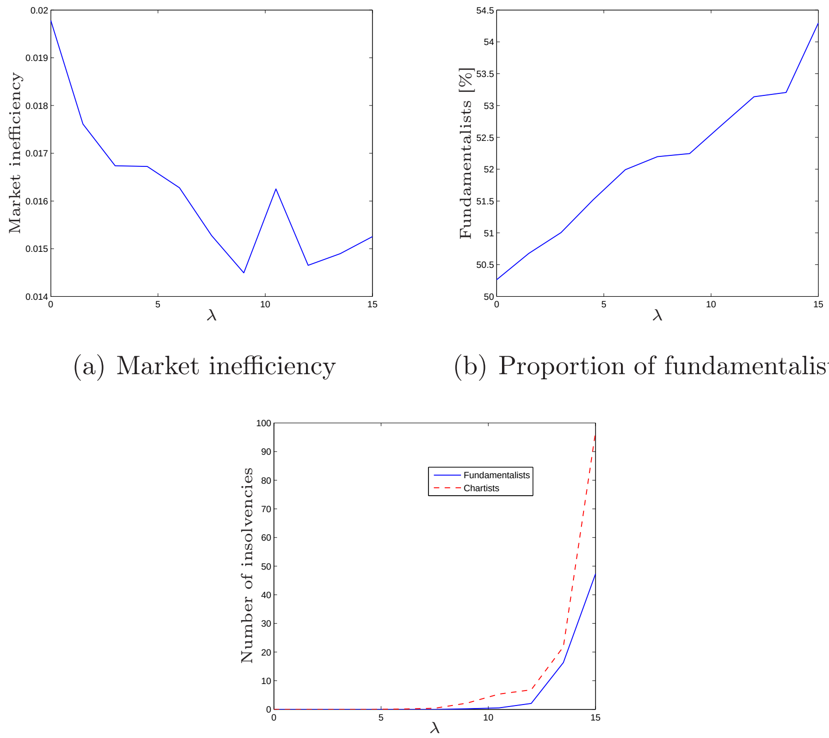
Figure 9: Trading volume and price volatility for variation of target leverage ratio λ (median results after 40 simulation runs)

Increased trading activity translates into a higher return volatility as can be observed in Figure 9(b). Somewhat of a surprise, however, is that the increased volatility goes along with higher price efficiency (Figure 10(a)), meaning that prices are more closely connected to their underlying fundamentals.[23] The reason for this counterintuitive link is depicted in Figure 10(b): higher leverage leads to a greater average proportion of fundamental traders in the model market. Higher leverage means that agents operate with less relative equity capital, which is quickly depleted in downturns. In order to survive, it becomes increasingly important for agents to anticipate price movements. Here fundamentalists are at an advantage. Figure 10(c) shows the number of bankruptcies for both fundamentalists and chartists. The number of defaulting chartists[24] is always higher than the number of defaulting fundamentalists. The losses incurred by chartists have a larger impact with increasing leverage. Leverage, in our model, may thus help to stabilize the market. This emergent behavior of the model is reminiscent of the classical argument for the existence of efficient markets. Friedman (1953) already argued that in the long run, speculative trading is not profitable and therefore eventually disappears. On the other hand, the observed efficiency

[23] As proposed in Westerhoff (2008), we measure inefficiency as the median absolute difference between log-fundamental value and price: $ME = median(|f_t - p_t|)$. In a first-order approximation, this can be interpreted as the percentage point deviation from fundamental value.

[24] More precisely, those agents who form expectations using technical analysis prior to defaulting.

gain is deceptive. Leverage strongly increases the risk of a breakdown of the entire model market. When too many agents default or experience losses at the same time, the fire sale of assets can lead to a positive feedback process triggering a debt deflation spiral as first described by Fisher (1933). In essence, falling prices call for agents to deleverage, which further suppresses prices and eventually leads to the collapse of the market. When conducting simulations we observe an increase in systemic risk with increasing leverage through the rising frequency of model breakdowns due to the default of all agents.



(a) Market inefficiency

(b) Proportion of fundamentalists

(c) Number of insolvencies for fundamental and chartist traders

Figure 10: Market efficiency, composition, and stability for variation of target leverage ratio λ (median results after 40 simulation runs)

4.3 Credit Frictions

Agents and financiers in our model actively manage their demand or supply of debt. The immediacy with which desired changes to debt can occur is constrained by the credit friction parameter μ^O in Equation (14), with a low value for μ^O implying high friction and vice versa. Frictions arise from the maturity structure of debt or from institutional characteristics of different bank types, which both restrict deliberate and immediate changes to the capital structure of agents. Credit frictions thus have the potential to affect the behavior of the financial system as a whole. To analyze the effects of credit frictions we first show how they affect the relationship between leverage and balance sheet size. Following the method of Adrian and Shin (2010), we scatter-plot the logarithmic changes of leverage against the logarithmic changes of balance sheet size.[25] Setting $\mu^O = 0$ means that agents and financiers have passive leverage strategies. The nominal debt agents are endowed with at the beginning of a simulation stays on their balance sheets while changes to the value of agents' assets lead to a negative relation between leverage and total assets.[26] This negative relationship, as plotted in Figure 11, can typically be

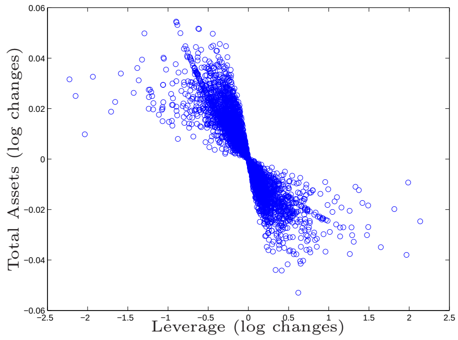


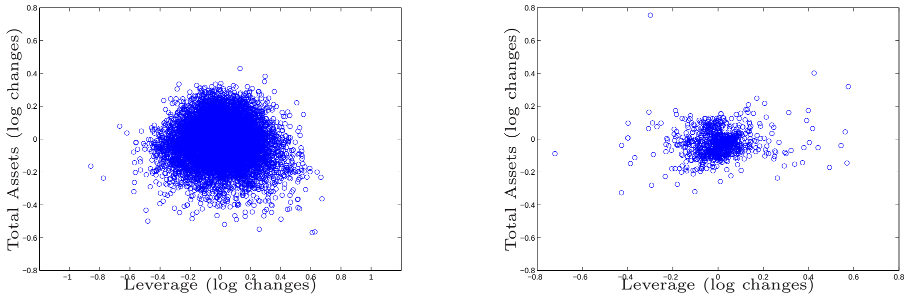
Figure 11: The relationship between balance sheet total and leverage for the passive agent

observed in household data (see Adrian and Shin, 2010). It seems however very unlikely that (professional) financial market participants would follow a completely passive leverage strategy. If we allow for slight leverage adjustment, the relationship between leverage and balance sheet size changes. A low value for μ^O implies that adjustments to agents' debt levels are constrained and take time. Commercial banks e.g. face such constraints, as customer deposits, which they cannot raise nor reduce at will, figure prominently on the liabilities side of their balance sheets. Figure 12(a) shows the

[25] More precisely, logarithmic changes of balance sheet size are changes in logarithmic balance sheet size after 50 periods, i.e. $\log(B_{j,t}) - \log(B_{j,t-50})$. The same applies for leverage.

[26] $\frac{\partial \lambda}{\partial B} = -\frac{O}{(B-O)^2} < 0$, with O being the constant nominal value of debt, B the balance sheet total and leverage being defined as $\lambda = \frac{O}{B-O}$.

leverage-balance-sheet relationship for $\mu^O = 0.01$ while Figure 12(b) shows the relationship for commercial banks in the EU27.[27] Both graphs lack a clear positive or negative relationship. On the other hand, Adrian and Shin (2010)



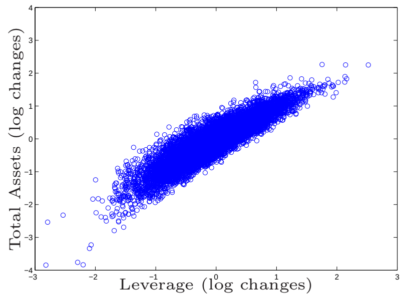
(a) Model data with $\mu^O = 0.01$ (b) Data from commercial banks

Figure 12: The relationship between balance sheet total and leverage with high credit frictions

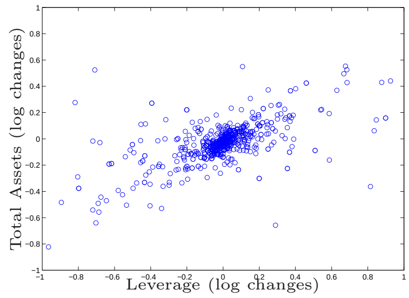
show that the relationship between leverage and balance sheet growth is positive, i.e. leverage is procyclical, for investment banks (see Figure 13(b)). Figure 13(a) exemplarily shows that our model produces a clearly positive relationship for μ^O values between approximately 0.1 and 1. In these cases there are little constraints on the adjustment of leverage. Investment banks tend to have very short-term debt (e.g. Repos) on their balance sheets, which allows them or their financiers to quickly adjust the leverage to values they deem appropriate. This characteristic is reflected in the high values for μ^O. The procyclical nature of leverage entering the model with low credit frictions can be explained by the debt supply function of the financier: a persistent positive development of an agent's balance sheet suggests to the financier that the agent is well informed, thus he perceives a lower risk level and is willing to supply more debt. On the other hand, when losses reduce the size of balance sheets, the financier will be more concerned about the safety of credit supplied to agents and will consequentially reduce his supply of debt.

The procyclicality of leverage induced by low credit frictions and the financier's risk management affects the price behavior of the risky asset. Figure 14(a) shows a negative relationship between credit frictions and average return volatility. An increase in friction reduces the procyclicality of leverage and yields a lower average volatility. Empirical evidence supports this relationship: Adrian and Shin (2010) find that the growth rate of short-term debt (Repos) on dealers' balance sheets significantly forecasts changes in

[27] We use quarterly data between 1996 and 2009 from the Bankscope database.



(a) Model data with $\mu^O = 1$



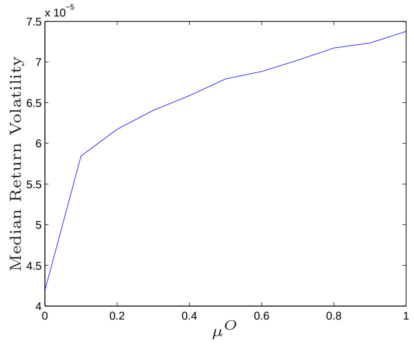
(b) Data from investment banks

Figure 13: The relationship between balance sheet total and leverage with low credit frictions

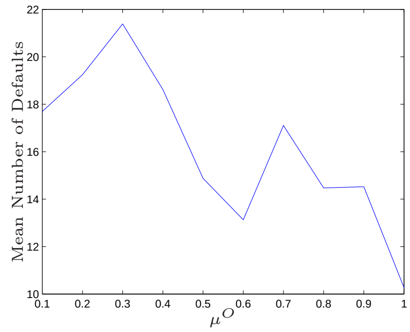
the Chicago Board Options Exchange Market Volatility Index (VIX). Our model suggests that return volatility could be reduced through an increase of credit frictions e.g. by curtailing the short-term debt supply to agents. Short-term borrowing has often been cited as a key contributor to financial instability and its curtailment has been called for on various occasions (cp. Diamond and Rajan, 2001). However, as the next graphs indicate, such regulation could turn out to be counterproductive. Figure 14(b) plots the average number of bankruptcies occurring over the 1000 periods of a simulation run for different values for the credit friction parameter μ^O. The trend indicates that the average number of bankruptcies increases with increasing credit frictions. [28] Note that the non-monotonic relationship between credit frictions and the average number of defaults in Figure 14(b) is due to the relative rarity of (many) bankruptcy events which leads to an increased impact of outliers on the average number of bankruptcies.[29] Figure 14(c) shows that the probability of a simulation run exhibiting at least a given number of bankruptcies decays exponentially: While a few defaults are relatively probable, many defaults are rare. Figure 14(d), on the other hand, shows that the probability that a given number of agents will default decreases with decreasing credit frictions. Thus, the active management of debt levels, which can best be accomplished in an environment with few credit frictions, results to be essential for the stability of our model financial market.

[28] To show a clearer graph, the first observation for $\mu^O = 0$ has been omitted. At $\mu^O = 0$ the average number of bankruptcies amounts to 278 agents, which is more than half of all agents.

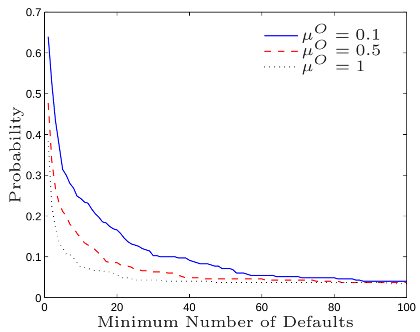
[29] We increase the number of simulation runs from 40 to 350 to somewhat dampen the effect of outliers.



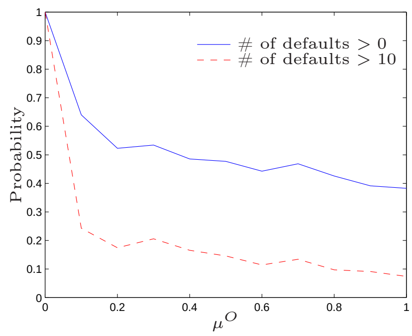
(a) Credit friction and return volatility



(b) Credit friction and bankruptcies



(c) Probability of defaulting agents



(d) Credit friction and default probability

Figure 14: The effects of credit frictions on stability (results from 350 simulation runs)

We conclude that through changes to the credit friction parameter μ^O, our model is able to reproduce the empirically observed relationship between leverage and balance sheet size of different bank types. Similar to the behavior observed for investment banks, when credit frictions in our model are small, leverage becomes procyclical. Supported by empirical evidence, procyclical leverage increases the return volatility of the model's risky asset. When judging the stability of the model's financial system through return volatility, credit frictions are beneficial to stability, whereas the opposite must be concluded when model bankruptcies are observed.

5 Conclusion

In this paper we develop an agent-based model of the financial market where agents are endowed with balance sheets that contain equity capital as well as debt. By conducting simulations we are able to analyze several aspects of the financial system that are mostly inaccessible with conventional economic models. Several results are reported: We show that the distribution of agents' balance sheet size evolves endogenously to an approximately log-normal distribution, which is typically observed for banks and especially for investment banks. With progressing simulation time, our model inherently produces higher inequality, i.e., an increasing proportion of agents are becoming smaller, while a small number of agents exhibit extraordinarily large balance sheets. Leverage is found to have a substantial effect on balance sheet evolution, with high leverage leading to high inequality and vice versa. The relationship between leverage and balance sheet size can exert a significant influence on the dynamics of financial markets. In our model the nature of this relationship depends on credit frictions, which we define as the latency with which agents can acquire and dispose of debt. When agents follow a passive debt strategy the relationship between leverage and balance sheets is negative, as observed for households. On the other hand, low credit frictions cause leverage to display the procyclical behavior observed for investment banks. In accordance with empirical findings, our model shows that procyclical leverage increases return volatility. At first glance, our model therefore suggests that higher credit frictions could help tame financial markets. However, the average number of defaults increases with higher frictions. More frequent defaults increase the danger of a fire sale externality which is detrimental to stability.

The investigations conducted in this paper represent only a small subset of feasible investigations. The effects of agents' memory or the impact of short selling constraints are among the aspects that have not been considered. The propagation of external shocks, investigations of trading volume, rationality and disagreement are other issues that could be assessed with the model framework presented in this paper. However, the potential usefulness of the model is constrained most notably by a lacking calibration and insufficient validation. While a more thorough validation of the model's qualitative predictions would clarify where the model can reveal valuable insights, a calibration of the model would make it more attractive for policy considerations.

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Appendix

	Symbol	Description	Value
Portfolio Composition	γ	Risk aversion	4
	θ^{FE}	Memory for forecast error	0.1
Fundamental Trading	$\varepsilon_{j,t}$	Error term in trading	$\varepsilon \sim \mathcal{N}(0, \sigma_f^2)$
	θ^{F}	Error correction term	0.03
	α^{F}	Aggressiveness of fundamental traders	1
Chartist trading	α^{C}	Aggressiveness of chartists	5
	θ^{S}	Memory for price variance estimator	0.1
Leverage	λ	Target leverage	25
	μ^{O}	Inverse debt debt friction	1
Financier	ω	Maximum accepted default probability	0.1%
	M	Maturity (in days)	10
	θ^{fin}	Memory of the financier for the forecast process	0.1
Switching mechanism	η	Rationality	100
	θ^{Π}	Memory for strategy comparison	0.1
	τ_j	Frequency of strategy change	Drawn from uniform distribution with the limits 1 and 250

Table 1: Benchmark simulation parameters

	Symbol	Description	Value
Financier	$\mu^B_{j,0}$	Initial estimator for adjusted balance sheet growth	$r^B_{j,t}$
	$z^2_{j,t}$	Initial estimator for volatility of adjusted balance sheet growth	$(\sigma^{FE}_{j,0})^2 = 0.05$
Portfolio composition	$(\sigma^{FE}_{j,0})^2$	Initial forecast error for all agents	$0.05\ (\triangleq 5 \cdot \sigma^2_f \triangleq 5 \cdot Var(\Delta f))$
Chartist trading	$\hat{\varsigma}^2_{j,0}$	Price volatility estimator of chartists	$0.01 \triangleq \sigma^2_f$
	$s_{j,0}/l_{j,0}$	Length of short and long moving average	Drawn from uniform distribution with the limits of 1 and 200

Table 2: Benchmark simulation initial conditions

Year	**Jarque-Bera test statistics**	**Critical value**
1996	5.06	11.53
1997	9.99	11.16
1998	4.74	10.90
1999	5.66	10.85
2000	5.84	10.86
2001	4.45	10.85
2002	2.44	10.99
2003	2.85	11.05
2004	2.34	11.10
2005	0.74	11.17
2006	0.27	11.20
2007	0.03	11.27
2008	0.79	11.42
2009	0.14	11.35

Table 3: Jarque-Bera test statistics computed for the log-balance sheet size of OECD investment banks (Bankscope data).