UNIVERSITY OF MANNHEIM

University of Mannheim / Department of Economics

Working Paper Series

***Coalition formation for unpopular reform in the presence of private reputation costs.***

Evguenia Winschel

Working Paper 13-08

2012

# Coalition formation for unpopular reform in the presence of private reputation costs.

Evguenia Winschel[1]

2012

**Abstract**

This paper studies coalition formation under asymmetric information. An outside party offers private payments in order to influence the collective decision over an unpopular reform. The willingness to accept such payments is private information. The paper demonstrates that a supermajority coalition induces truth-telling and secures the implementation of the decision for a price close to the full information minimal winning coalition price. On the contrary, if the minimal winning coalition is formed, then no revelation is possible.

Keywords: coalition formation, minimal winning coalition, supermajority coalition, private information.

JEL: D71, D72.

[1]Mannheim University, e mail - eugeniaw@rumms.uni-mannheim.de
This paper is supported by SFB 884 grant. This a revised version of my PHD thesis. I would like to thank Hans Peter Grüner, Oriana Ponta, Olga Gorelkina for useful conversation and comments.

# 1 Introduction

This paper investigates the coalition formation for an unpopular reform in the presence of private payments and private information.

I consider the following situation: a reform is introduced and needs the support of players who are going to lose from the implementation. Such players can be compensated for their support with private payments. But the compensation induces some additional costs: players bear private costs if they accept private payments in case of the reform implementation. Moreover, I assume that such costs are private information. There is an outside party which is interested in reform approval and tries to form a coalition for this purpose. I show that, in contrast to minimal winning coalition, in case the supermajority coalition is formed, the full revelation of private costs is possible and the support of cheapest players can be secured.

The labour market reform which has to be supported by at least some trade unions could be presented as an example of such a situation. The similar case occurs when the prime minister proposes a rather unpopular reform and needs the support of ministers in the government and the parties in the government coalition. The alternative interpretation can be offered by some interest group, trying to influence collective political decision.

In the government example, the payments can be governmental portfolios or the implementation of other projects in which the politicians might be interested. The interest group can use illegal bribes or legal campaign contributions. Private costs associated with private payments can be interpreted in a number of ways. Firstly, the players can bear some "reputation" costs: politicians can be considered corrupted by some voters, trade unions or some union members can loose their reputation, which can have negative influence in the future. Secondly, the costs can take a form of a private "scruple": a politician just suffers if he is associated with the undesired decision. In the lobby example there is a risk of punishment if the lobbying activity is illegal. In this case politician can be punished for obtaining illegal private payments. When private payments take the form of a campaign contribution, a realistic interpretation for costs can be the reduction in the reelection chances (Prat 2004, Prat, Puglisi and Snyder, Jr. 2005).

The existence of costs, associated with obtaining private benefits rather than with the actual vote, is the important feature of my model . If the unpopular reform or project are implemented, everyone associated with the decision has to bear costs, not only the players who voted for the implementation. Such situation indeed is possible if the supermajority coalition is formed: there is no need for all coalition members to actually support the decision during the voting stage, but they are still defined as coalition members and can bear costs if they obtained the private payments from implementation.

A good example for this cost structure is the pullout process in Israel in 2005, where some politicians did not want to be included in the coalition in order not to be associated with the disintegration process. The following citation introduces the concept of the "collective responsibility" and explains the spirit of my cost assumption:

*"Worst of all is the way the politicians – government ministers, mainly – think they can have their cake and eat it, too. They want to keep their seats in a government that voted for the pullout, but without voicing solidarity for its decisions. They serve in a government that elected to leave Gaza, but they don't understand that even if they voted "no," they are partners in the decision. Because there is a principle known as collective responsibility. Sharon's success will be their success. Yet no one has come out in defense of Sharon - or in defense of the decision reached by his government, to be more precise."* ("Silence of the lambs as Israel burns" Yoel Marcus, 01.07.2005 Haaretz).

A part of Sharon's government remained government members, receiving all office "perks", despite voting against the disintegration plan. But others considered the damage of the "collective responsibility" for their future political carrier as too high and left the government. For example, the financial minister and Sharon's main rival in the Likud party Benjamin Netanyahu, whose supporters considered the disintegration plan as the opposite to "success", left the government shortly before the disintegration plan implementation. In other words, a coalition member cannot simultaneously enjoy private payments associated with the coalition membership and the political capital associated with a "no" vote.

I investigate the closed voting procedure, advocated by Dal Bo (2007) and Grüner and Felgenhauer (2008)[2], where the actual vote of a politician cannot be observed and private payments can be conditioned only on the reform implementation. I do it for the two reasons. First, my cost assumptions becomes stronger. Without the information about an actual vote, the private payment is the only signal concerning coalition participation. Second, under open voting procedures Dal Bo (2007) and Grüner and Felgenhauer (2008) propose mechanisms which allow to influence decision at no or very small costs[3].

I consider the following mechanism which represents the simple majority and supermajority coalition formation: the group of players (politicians) are

[2]Both papers belong to the recent literature that compares open and closed voting rules. Dal Bo (2007) and Felgenhauer and Grüner (2008) found that under an open rule an outside party can manipulate the committee decision at very small costs or even at no cost at all. As a result both papers advocate for a closed voting procedure.

Dal Bo presents a complete information model where all committee members are to some extent against the approval of the project desired by an interest group.

Felgenhauer and Grüner discuss the case where committee members have private information about what might be the best policy. Moreover, they are influenced by an external interest group, which favors a certain policy.

[3]Such mechanisms obviously do not include full compensation of coalition members. Therefore, such mechanism usually would be not robust to repeated interaction or any reputational concerns of the coalition builder. If it is essential that the coalition members for whatever reasons, may be not explicitly modeled, are fully compensated for their loss, then the private information concerning their costs is crucial. In such model my results may be applied for open voting procedure as well.

supposed to decide about the reform implementation under the simple majority voting rule. The player who wants to influence the decision (the prime minister) commits to the size of the coalition. The politicians make an announcement of their type: their private costs of obtaining the private payments. The prime minister forms a coalition from the players with smallest types and in the case of reform implementation compensates them their loss from the reform itself and private costs of coalition participation. It reminds an auction where one buyer has to form at least a simple majority coalition of sellers and collects their bids, committing to a coalition size. The difference is that "sellers" do not have to commit to their bids: the politicians can always change their mind and vote against a reform, even if they are proposed a private payment according to their announcement.

The main result of my paper is that if the types of players are discrete then the truthful implementation exists under the supermajority coalition in contrast to the minimal winning coalition. If the types are continuous, then there is no truthful revelation under both types of coalition, but under the supermajority coalition it can be achieved using Crawford and Sobel (1982) partition strategy.

To the best of my knowledge, supermajority coalition formation as a tool in mechanism design is a new explanation to its existence. Usually it is a minimal winning coalition that appears as a key prediction of various models of coalition formation and vote buying (Baron and Ferejohn 1989, Denzau and Munger 1986, Shepsle 1974, Riker 1962). But in reality, oversized coalitions appear at least as often as minimal winning coalitions (see discussion in Groseclose and Snyder (1996), Diermeier et all (2002, 2003)).

The direct explanation of the supermajority existence in an uncertain environment is intuitive: it serves to increase the probability of a desirable decision (Koehler 1972, 1975, Riker 1962). Another possible reason is the norm of universalism (Klingaman 1969, Weingast 1979). Groseclose and Snyder (1996) suggest that supermajority coalitions may be cheaper than minimal winning coalitions if there is a competition among vote buyers who move sequentially. Diermeier and Merlo (2000) analysis accounts for surplus and minority governments in a two periods model of the government formation and termination with a stochastic default policy. Axelrod (1970) suggests that supermajority coalitions arise when small, ideologically centrist parties are included to reduce the conflict of interest among parties in the government. Baron and Diermeier (2001) believe that supermajority coalitions form when the status quo policy is ideologically extreme. Crombez (1996) predicts that the largest party will propose a supermajority coalition when it holds few seats and is ideologically extreme. Carrubba and Volden (2000) propose that supermajority coalitions come about when policy logrolls are difficult to achieve and sustain over time. Lijphart (1984) and Sjolin (1993) argue that supermajority coalitions are formed in order to ensure control of the upper chamber in bicameral systems[4].

This paper is also related to Tsai and Yang (2010a, 2010b). In these two

[4]See Carrubba and Volden (2004) for a summary of theoretical explanations for the supermajority existence.

papers Tsai and Yang introduce asymmetric information into the coalition formation models proposed by Baron and Ferejohn (1989) and Persson, Roland and Tabellini (2000) respectively. The main result is that both supermajority coalitions and minimal winning coalitions may arise in the equilibrium. This is in contrast to the certain world in which only minimal winning coalitions appear. But in Tsai and Yang there is no possibility of information exchange between the players, so the incentives to form the supermajority coalition are quite different from the current paper.

The paper is structured in the following way: the next section describes the model. Section 3 discusses coalition formation under asymmetric information: section 3.2 presents minimal winning coalition and section 3.3 investigates the supermajority coalition. Section 4 discusses the optimal coalition size. Section 5 considers continuous types. Section 6 concludes.

# 2 The model

There is a group of $n$ politicians who decide about the implementation of some reform. Let $n$ be an odd number. The decision rule is the simple majority voting rule, so the reform is implemented if at least $\frac{n+1}{2}$ politicians support it. If the reform is not implemented, a default policy takes place.

Assume that there is some player, called the prime-minister, who is interested in the implementation of the reform and has the ability to influence the decision by proposing private payments to the politicians. I denote the vector of payments by $t$, $t_i \geq 0$, where $t_i$ is the payment to politician $i$.

## 2.1 Preferences

The politicians derive utility from either the implemented reform or the default policy as well as from private payments. I assume that all politicians obtain a utility normalized to zero, if the reform is implemented. Otherwise it is equal to some constant $p > 0$. Therefore, without additional incentives, all politicians vote against the reform.

I assume that politicians are considered to be coalition members if they obtain private payments from the prime-minister. Then they bear some positive costs if the reform is implemented. The politicians differ with respect to these costs. The cost for politician $i$ is denoted by $d_i$ and is distributed independently on the interval $[d_L, d_H]$, $0 < d_L < d_H$, according to some distribution function $D$. Moreover, I assume that $d_i$ is private information and is observed only by politician $i$. From now on, I refer to $d_i$ as to the type of politician $i$. Note, that costs are realized only if $t_i > 0$. If $t_i = 0$ then politician $i$'s cost is zero.

I consider the closed voting rule, so the payments are conditioned on the implementation of the reform and not on the individual votes. As a result, the utility obtained by politician $i$ is:

$$U_i\left(d_i, t_i\right) = \begin{cases} t_i - d_i & \text{if the reform is implemented and } t_i > 0 \\ 0 & \text{if the reform is implemented and } t_i = 0 \\ p & \text{if the reform is not implemented} \end{cases} . \qquad (1)$$

I do not explicitly model the prime-minister's utility, but I assume that it decreases in the sum of payments $\sum_i t_i$ and increases in the probability of the reform's approval. I further assume that the value of the reform for the prime-minister is relatively high, so she can credibly commit to large private payments.

## 2.2 Mechanism

There exist a number of potential options for the prime-minister, if she wants to influence the decision and to reduce the aggregate payments. In the case of open voting, the prime-minister could make proposals according to Dal Bo (2007) mechanism and trigger the reform implementation at no costs for herself. But in the case of the closed voting it would not work.

What else could the prime-minister do? First of all, the prime-minister can always propose the payment equal to $p + d_H$ to a simple majority. Such a proposal will definitely guarantee the reform implementation, but it can be quite expensive for the prime-minister. The prime-minister can also propose lower payments to some politicians and face a certain risk of the reform rejection.

In this paper I do not try to search for the optimal strategy for the prime-minister. I rather compare two popular coalition forms: a minimal winning coalition (MWC) with supermajority coalition, and show that in case the supermajority coalition is formed, the full revelation of private costs is possible and the support of cheapest players can be secured.

As a benchmark, let us consider the complete information case. The standard result for close voting models is that the prime-minister forms MWC: she invites $\frac{n+1}{2}$ politicians with the lowest types into the coalition. The payment to each coalition member in the case of the reform implementation equalizes the utility from the reform implementation and the coalition membership with the default option[5]:

$$t_i - d_i = p \qquad (2)$$

$$\Rightarrow t_i = p + d_i > 0$$

The proof follows direct from the observation that MWC minimizes the aggregate payment to the politicians, given that the reform is implemented.

However, under asymmetric information the prime-minister does not know which politicians are cheapest to buy and has to use some mechanism in order to find it out.

[5] As it is customary in the literature, I assume that if a politician is indifferent, he will vote for the reform.

I propose the following mechanism: given the vector of announced types $\widetilde{d}$, where $\widetilde{d}_i$ is the announced type of politician $i$, the prime-minister invites $c$, $\frac{n+1}{2} \leq c \leq n$, politicians with the lowest announced types into the coalition and promises to pay to coalition member $j$ the private payment $t_j$

$$t_j(\widetilde{d}_j) = p + \widetilde{d}_j \tag{3}$$

in the case of the reform implementation. All other politicians are called the opposition members and obtain zero payment. If the prime-minister is indifferent who out of several politicians is to be included into the coalition (which happens if they report the same type), she randomizes between them. If the politicians announce their true types, then such payments, first, minimize the aggregate payment to the politicians. Second, they guarantee the reform implementation.

In this case the payment does not depend on the politician's real type but increases in the announced type $\widetilde{d}_i$. Then the utility of politician $i$ with type $d_i$ who announces type $\widetilde{d}_i$ is

$$U_i\left(d_i, \widetilde{d}_i\right) = \begin{cases} p + \widetilde{d}_i - d_i & \text{if the reform is implemented and } i \text{ is a coalition member} \\ p & \text{if the reform is not implemented} \\ 0 & \text{if the reform is implemented and } i \text{ is not a coalition member} \end{cases} \tag{4}$$

This mechanism reminds a kind of auction where the prime-minister buys the votes from the politicians. The politicians announce for how much they are ready to sell their vote and the prime-minister commits to choose $c$ cheapest ones and to pay them according to their announcements. The main difference is that, unlike in the usual auction, the politician does not have to commit to his announcement: he can always vote against the reform even if he "wins" the auction and is invited into the coalition.

## 2.3 Timing

The timing of the game is as follows:

1. The prime-minister announces the mechanism, namely, she commits to the number of coalition members $c$ and payments (3). As it is traditional in the literature, I assume that the prime-minister is able to commit to all her announcements and payments.

2. The politicians announce their types simultaneously.

3. The mechanism is implemented: the prime-minister observes the announcements and invites $c$ politicians into the coalition.

4. The voting takes place and coalition payments (3) are paid in the case of the reform implementation.

As I already mentioned, the politicians' announcements do not need to be true and the politicians cannot commit to any specific voting behavior.

I restrict the announcements to the space of feasible types. Therefore, the feasible action of politician $i$ at the second stage of the game is the type announcement $\widetilde{d}_i(d_i, c) \in [d_L, d_H]$. The feasible action of politician $i$ at the voting stage is $a_i\left(d_i, \widetilde{d}_i, t_i, c\right) \in \{accept, reject\}$. Strategy $s_i$ for player $i$ is a sequence of actions and the strategy profile $s$ is an $n$-tuple of strategies, one for each player.

The game is solved by backward induction. The solution concept is subgame perfect Bayesian Nash equilibrium. Since no player can change the outcome if a proposal is supported by more than a simple majority, I restrict attention to the equilibria in which weakly dominated strategies are eliminated.

# 3 Coalition size

In this section I obtain the main result of this paper: I investigate if the mechanism can be truthfully implemented depending on the size of coalition $c$. I start with the MWC, then I describe the case of minimal supermajority coalition with $\frac{n+3}{2}$ coalition members. Finally, I consider the general case of supermajority coalition with $c \geq \frac{n+3}{2}$.

## 3.1 Voting

In this section I introduce some lemmas which describe the voting behavior of politicians at the last stage of the game and which will be helpful throughout the paper:

**Lemma 1** *All opposition members vote against the reform.*

The proof follows directly from the assumption that $0 < p$.

From Lemma 1 I obtain the following result:

**Corollary 1** *If the coalition is a minority coalition, the reform is never implemented.*

Thus, the prime-minister never forms a minority coalition.

**Lemma 2** *If $t_i - d_i \geq p$ the coalition member votes for the reform, otherwise he votes against the reform.*

The proof follows directly from (1).

## 3.2 Minimal winning coalition

In this section I investigate what happens when the prime-minister forms the MWC. Namely, the prime-minister observes the vector of announcements $\widetilde{d}$ and proposes membership in the coalition to $\frac{n+1}{2}$ players with minimal announcements.

The first result is that if the prime-minister forms the MWC, she cannot induce the truth-telling announcement:

**Proposition 1** *Under the MWC, there are no truthful announcements.*

**Proof.** Suppose, contrary to proposition, that all politicians make truthful announcements if $c = \frac{n+1}{2}$. Consider now the politician with true type $d_H$. He either announces his true type or he can deviate and make an announcement $\widetilde{d}_i < d_H$. If he is a coalition member, his minimal utility is $p$, regardless of his announcement. This is because all coalition members are pivotal, hence, if any coalition member obtains less than $p$ as a result of the reform implementation, he can always induce the reform rejection and obtain $p$ as default payment. If he is an opposition member, his payment is 0 because the reform is always implemented. It follows that he will always announce to be $d_L$ type in order to increase his chances to be in the coalition and his announcement will never be truthful. ■

Since the MWC is a key prediction of various models for coalition formation, it may be interesting to investigate what announcements are made if the prime-minister tries to form the MWC:

**Proposition 2** *If the prime-minister forms the MWC and proposes payments according to (3), there are two possibilities:*

*(i) all players announce* $\widetilde{d}_L$. *If politicians' types are continuous, it is a unique announcement.*

*(ii) If politicians' types are discrete, then there may exist type* $d^*$ *so that all politicians with true type* $d_i < d^*$ *announce* $\widetilde{d}_i > d_i$ *and all politicians with true type* $d_i \geq d^*$ *announce* $\widetilde{d}_L$.

**Proof.** See Appendix. ■

This proposition implies an even stronger result than just lack of a truthful announcement: building MWC, the prime-minister cannot even sort politicians according to their types and induce a monotonic announcement where for each two politicians, $i$ and $j$ :

$$\widetilde{d}_i \geq \widetilde{d}_j \Leftrightarrow d_i \geq d_j \ . \tag{5}$$

### 3.3 Supermajority coalition

In this section I show that truthful implementation is possible if the prime-minister forms the supermajority coalition. In this model, the supermajority comprises $\frac{n+3}{2}$ or more politicians. I consider the case of $\frac{n+3}{2}$ coalition members, the "minimal" supermajority coalition and I discuss larger coalitions in the next section. Therefore, the prime-minister observes the vector of announcements $\widetilde{d}$ and proposes membership in the coalition to $\frac{n+3}{2}$ players with the minimal announcements. I assume, for simplicity, that there are only two possible types of politicians $\{d_L, d_H\}$ and the probability of type $d_H$ is $h$. The result is easily extended to an arbitrary number of discrete types, see remark in the end of the section. Private payments to coalition members are then as follows

$$t_i(\widetilde{d}_H) = p + d_H, \tag{6}$$

$$t_i(\widetilde{d}_L) = p + d_L. \tag{7}$$

The important feature of a supermajority coalition and the main difference from the MWC is that no coalition member is pivotal. If all but one coalition member $j$ announce their true type, then the reform is implemented regardless of the announcement, the private payment and the voting behavior of $j$. Hence, if a single player deviates from the truthful announcement strategy, his utility depends only on his announced type, true type and coalition status, but not on his voting behavior.

The following proposition presents the main result of the paper:

**Proposition 3** *If the prime-minister forms a supermajority coalition with* $c = \frac{n+3}{2}$*, the mechanism is truthfully implemented if*

$$\left(1 - \frac{1}{a(n,h)}\right) p \leq d_H - d_L \leq (a(n,h) - 1)\, p \tag{8}$$

*where*

$$a(n,h) \equiv \frac{1 - \sum_{k=\frac{n+3}{2}}^{n-1} \frac{(n-1)!}{(n-1-k)!k!} h^{n-k-1} (1-h)^k \left(1 - \frac{n+3}{2(k+1)}\right)}{\sum_{k=0}^{\frac{n+1}{2}} \frac{(n-1)!}{(n-1-k)!k!} h^{n-k-1} (1-h)^k \frac{n+3-2k}{2(n-k)}}.$$

**Proof.** Suppose that all politicians announce their true types. If player $i$ is of type $d_L$ and he announces his true type, then his expected utility is:

$$EU_i\left(d_L, \widetilde{d}_L\right) = p + \sum_{k=\frac{n+3}{2}}^{n-1} \frac{(n-1)!}{(n-1-k)!k!} h^{n-1-k} (1-h)^k \left(\frac{n+3}{2(k+1)} - 1\right) p, \tag{9}$$

and if he deviates and announces type $d_H$, then his expected utility is:

$$EU_i\left(d_L, \widetilde{d}_H\right) = \sum_{k=0}^{\frac{n+1}{2}} \frac{(n-1)!}{(n-1-k)!k!} h^{n-k-1} (1-h)^k \frac{n+3-2k}{2(n-k)} (p + d_H - d_L). \tag{10}$$

If player $i$ is of type $d_H$ and he announces his true type, his expected utility is

$$EU_i\left(d_H, \widetilde{d}_H\right) = \sum_{k=0}^{\frac{n+1}{2}} \frac{(n-1)!}{(n-1-k)!k!} h^{n-k-1} (1-h)^k \frac{n+3-2k}{2(n-k)} p. \tag{11}$$

If he deviates

$$EU_i\left(d_H, \widetilde{d}_L\right) = (p + d_L - d_H) + \tag{12}$$

$$\sum_{k=\frac{n+3}{2}}^{n-1} \frac{(n-1)!}{(n-1-k)!k!} h^{n-k-1} (1-h)^k \left(\frac{n+3}{2(k+1)} - 1\right) (p + d_L - d_H).$$

Then both types of politicians make a truthful announcements if

$$U_i\left(d_L,\widetilde{d}_L\right)\geq U_i\left(d_L,\widetilde{d}_H\right), \tag{13}$$

$$U_i\left(d_H,\widetilde{d}_H\right)\geq U_i\left(d_H,\widetilde{d}_L\right), \tag{14}$$

or

$$\left[\frac{1-\sum_{k=\frac{n+3}{2}}^{n-1}\dfrac{(n-1)!}{(n-1-k)!k!}h^{n-k-1}(1-h)^k\left(1-\frac{n+3}{2(k+1)}\right)}{\sum_{k=0}^{\frac{n+1}{2}}\dfrac{(n-1)!}{(n-1-k)!k!}h^{n-k-1}(1-h)^k\,\frac{n+3-2k}{2(n-k)}}-1\right]p+d_L\geq d_H, \tag{15}$$

$$\left[1-\frac{\sum_{k=0}^{\frac{n+1}{2}}\dfrac{(n-1)!}{(n-1-k)!k!}h^{n-k-1}(1-h)^k\,\frac{n+3-2k}{2(n-k)}}{1-\sum_{k=\frac{n+3}{2}}^{n-1}\dfrac{(n-1)!}{(n-1-k)!k!}h^{n-k-1}(1-h)^k\left(1-\frac{n+3}{2(k+1)}\right)}\right]p+d_L\leq d_H. \tag{16}$$

One can rewrite these conditions as

$$\left(1-\frac{1}{a(n,h)}\right)p\leq d_H-d_L\leq(a(n,h)-1)\,p \tag{17}$$

where

$$a(n,h)\equiv\frac{1-\sum_{k=\frac{n+3}{2}}^{n-1}\dfrac{(n-1)!}{(n-1-k)!k!}h^{n-k-1}(1-h)^k\left(1-\frac{n+3}{2(k+1)}\right)}{\sum_{k=0}^{\frac{n+1}{2}}\dfrac{(n-1)!}{(n-1-k)!k!}h^{n-k-1}(1-h)^k\,\frac{n+3-2k}{2(n-k)}}.$$

Moreover, for a relevant parameters range ($n>3$, $0<h<1, p>0$), it can be shown that

$$(a(n,h)-1)\,p>\left(1-\frac{1}{a(n,h)}\right)p. \tag{18}$$

Therefore, there are always $d_H$ and $d_L$ so that conditions (15) and (16) are satisfied. ■

This proposition shows that the truthful implementation exists when the difference between $d_H$ and $d_L$ is non negligible, but not extreme. The incentive to deviate for type $d_L$ is to increase the private payment in case of coalition membership, at the expense of the probability of coalition membership. If the difference between $d_H$ and $d_L$ is too large, type $d_L$ deviates. Type $d_H$ may deviate in order to increase the coalition membership probability at the expense of the private payment in case of coalition membership. He will deviate if the difference $d_H$ and $d_L$ is too small.

For example, for $n=7$, $h=0.5$ and $p=1$, both types make truthful announcements if the following conditions are satisfied:

$$.542\,14 < d_H - d_L < 1.184\,1.$$

The following results show how the change in the parameters $p$, $n$ and $h$ influences the likeliness of the implementation:

**Corollary 2** *The range of parameters $d_H$ and $d_L$, for which both types make truthful announcements, increases in $p$.*

**Proof.** From (15) and (16) it follows that the range of parameters $d_H$ and $d_L$, for which both types make truthful announcements, increases in

$$\left( (a(n,h) - 1) - \left(1 - \frac{1}{a(n,h)}\right)\right) p.$$

$(a(n,h) - 1) - \left(1 - \frac{1}{a(n,h)}\right) > 0$, therefore it increases in $p$. ■

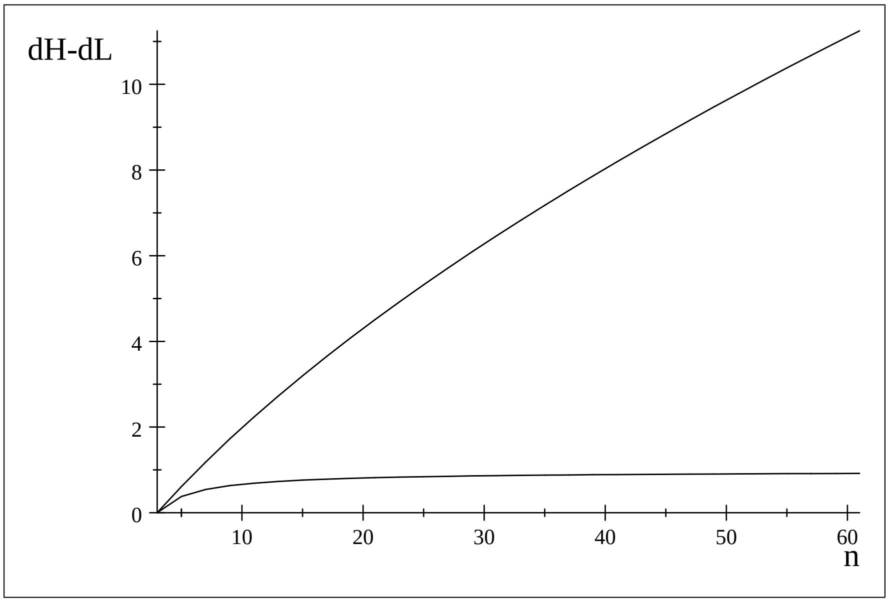
This corollary states that the higher the utility from the default policy is, the higher is the chance that the prime-minister can implement the mechanism.

The conditions under which the implementation exists are too complicated to investigate analytically, but numerical simulations provide the following results:

**Corollary 3** *The range of parameters $d_H$ and $d_L$, for which the implementation exists increases with $n$ .*

This result is represented in Figure 1. Assume that $p = 1$ and $h = 0.5$. The upper line corresponds to the LHS of condition (15) and the lower line to the LHS of condition (16). Therefore, the truth-telling equilibrium exists if the value of $d_H - d_L$ is between these two lines. With an increase in $n$ the bounds on $d_H - d_L$, implied by conditions (15) and (16), relax considerably upwards and tighten slightly downwards. The first effect is dominant, so the total effect is an increase of the parameter range for which the implementation exists.

Figure 1




This result seems counter-intuitive, because it is usually more expensive to buy the majority in a large decision body. But buying the winning coalition under truth-telling in a large decision body can still be cheaper for the prime-minister than dealing with asymmetric information in a small decision body. Furthermore, this result contradicts the famous Condorcet's jury theorem which states that the delegation of a decision to a bigger committee yields better results.

**Remark 1** *It is easy to extend this model for more than two types. Like in a two types case, the implementation exists if the distance between types is not too large and not too small. But the number of the conditions to be satisfied increases as well: for the case of $k$ types, $k(k-1)$ conditions, similar to (15) and (16) have to be satisfied.*

# 4 Supermajority size

In the previous section I analyzed the minimal supermajority: simple majority plus one politician. In this section I investigate how many coalition members the prime-minister should include into the coalition. Including an additional member into the coalition is costly for the prime-minister, and *ceteris paribus* she will prefer the "minimal" supermajority, with $\frac{n+3}{2}$ members. But one also has to consider the parameter restrictions under which the truth-telling exists. If they change, it can be the case that the supermajority with more than $\frac{n+3}{2}$ members is preferable.

Assume that prime-minister commits to include $n > c \geq \frac{n+5}{2}$ politicians into the coalition. Suppose that all politicians announce their true type.

Then the politician with type $d_L$ announces his true type if the following condition is satisfied:

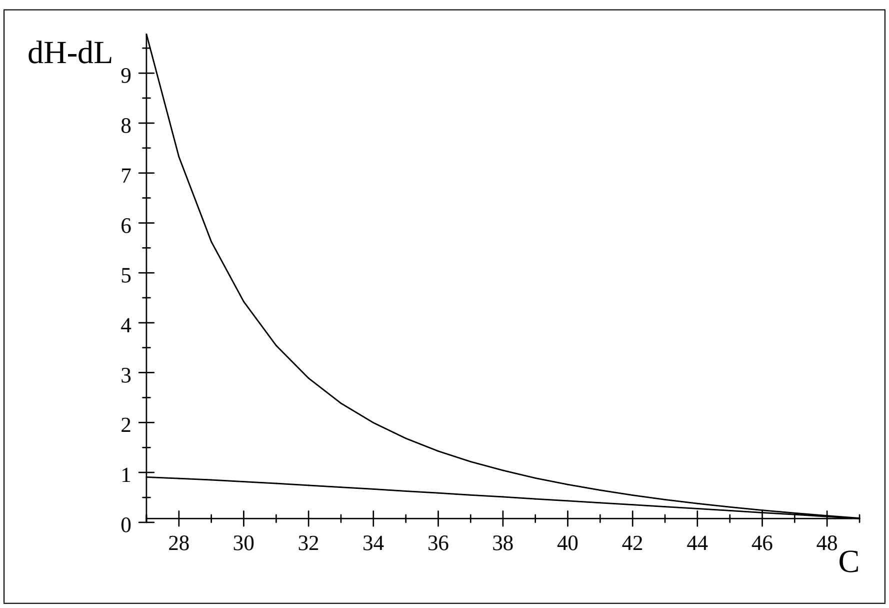
$$\left( \frac{1 + \sum_{k=c}^{n-1} \dfrac{(n-1)!}{(n-1-k)!k!} h^{n-1-k} (1-h)^k \left( \frac{c}{(k+1)} - 1 \right)}{\sum_{k=0}^{c-1} \dfrac{(n-1)!}{(n-1-k)!k!} h^{n-k-1} (1-h)^k \, \frac{c-k}{(n-k)}} - 1 \right) p \geq d_H - d_L \tag{19}$$

The politician with type $d_H$ announces his true type if the following condition is satisfied:

$$\left( 1 - \frac{\sum_{k=0}^{c-1} \dfrac{(n-1)!}{(n-1-k)!k!} h^{n-k-1} (1-h)^k \, \frac{c-k}{(n-k)}}{1 + \sum_{k=c}^{n-1} \dfrac{(n-1)!}{(n-1-k)!k!} h^{n-k-1} (1-h)^k \left( \frac{c}{(k+1)} - 1 \right)} \right) p \leq d_H - d_L \tag{20}$$

Assume that $n = 51$. Conditions (19) and (20) are represented in Figure 2. The upper line corresponds to the LHS of condition (19) and the lower line to the LHS of condition (20). Therefore, the truth-telling equilibrium exists if the value of $d_H - d_L$ is between these two lines.

Figure 2




This example shows that big supermajority can emerge. This will occur if the parameters are such that the truth-telling does not occurs for $\frac{n+3}{2}$-members coalition, but is possible for larger coalitions. In this particular example it happens if $d_H - d_L < 1$. Inviting additional players into the coalition is costly for the prime-minister, but it can still be profitable, compared to the situation when the truth-telling cannot be achieved.

Summarizing, it is important to note again that the optimal size of coalition would depend on the exact preferences of the prime-minister and other model parameters. It could be that in a case where truth-telling coalition is very expensive, which happens if it is significantly larger than a minimal one, the prime-minister may prefer some other strategy. For example she can propose $p + d_H$ to a simple majority.

Finally, note that:

**Remark 2** *If all players are coalition members, they will announce the highest type. Therefore, there will be no truth-telling in this case.*

# 5 Continuous distribution

In this section I consider continuous types. I assume that $d_i$ is distributed independently on the interval $[d_L, d_H]$. I show that politicians do not announce their true types, even if a supermajority is formed.

The intuition for the non-existence of truthful announcements is that high type players expect that they will not be in the coalition with a high probability. Moreover, the proposal will always be accepted if all players announce their true types. Hence, the expected utility of the players with high types is close to zero. This means that it will be profitable for such players to deviate from the truthful announcement and to announce a lower type. Opposite to a discrete case when the announcement of a lower type implies a considerable change in the coalition payment (from $p$ to $p + d_L - d_H$), if the types are continuous there is always announcement $\widetilde{d} < d_H$ so that deviation is profitable. Two main differences between discrete and continuous types explain these results. If the types are discrete, first, the probability of the coalition participation does not go to zero even if the true type is high. Second, the politician cannot announce the arbitrary type $d_i - \varepsilon$, he is limited to feasible types. More formally:

**Proposition 4** *There is no truth-telling under the supermajority coalition, if the types are continuous.*

**Proof.** Assume, contrary to the proposition, that all players announce their true types and the prime-minister forms supermajority.

Consider player $i$: if he announces his true type, his expected utility is:

$$EU_i\left(d_i, d_i\right) = P_i(d_i, d_{-i}) * p + (1 - P_i(d_i, d_{-i})) * 0, \tag{21}$$

where $P_i(d_i, d_{-i})$ is the probability that player $i$ is a coalition member, which is a weakly negative function of $i$'s announcement.

If a player deviates and announces $d_i - \varepsilon$, $\varepsilon > 0$, then his expected utility is:

$$EU_i\left(d_i, \widetilde{d}_i\right) = P_i(d_i - \varepsilon, d_{-i}) * (p + (d_i - \varepsilon) - d_i) + (1 - P_i(d_i - \varepsilon, d_{-i})) * 0 \tag{22}$$

$$= P_i(d_i - \varepsilon, d_{-i}) * (p - \varepsilon).$$

If a player with type $d_H$ announces his true type, then the probability that he is in the coalition, $P_i(d_H, d_{-i})$, is equal to zero. Hence, his expected utility is zero as well. Therefore, there always exists such $\varepsilon$ that

$$P_i(d_H - \varepsilon, d_{-i}) * (p - \varepsilon) > 0. \tag{23}$$

As a result, the player with true type $d_H$ never announces his true type and truth-telling cannot be the equilibrium strategy. ■

This result follows logically from the truthful implementation existence conditions (15) and (16): if the types are continuous, they can never be satisfied and the prime-minister cannot induce the truth-telling.

In order to deal with nonexistence of the truth-telling, the prime-minister can discretize the announcements. Namely, she can adopt similar to the Crawford and Sobel (1982) the partition strategy. Then the prime-minister divides announcements space into intervals, asks politicians to announce in what intervals they are, while proposing the private payments equal to the higher endpoint of the interval. The existence of a truthful announcement in Crawford Sobel type equilibrium for this model can easily be shown if the endpoints of intervals satisfy the conditions similar to condition (17).

# 6 Conclusions

This paper presents a model of influence over the group decision under asymmetric information. I considered secret voting, so that private payments can be conditioned only on the reform implementation. The politician bears costs if he is associated with the reform via private payments, regardless of his actual vote.

In this paper I investigate the formation of the supermajority coalition under private information. By forming supermajority coalition the prime-minister can induce the politicians to reveal their true types, if the type space is discrete. This leads to guaranteed implementation of a generally undesired reform at a cost close to that of the complete information MWC.

An additional interesting result follows: with an increase in the number of politicians, truthful implementation is more likely to exist. This result is tested and found to have strong support in Carrubba and Volden (2004). Carrubba and Volden (2004) predict that coalitions are more likely to be oversized when the number and diversity of actors in a legislative chamber is greater. In order to test this predictions they use the number of actors in a legislative chamber measured by the number of seats and the number of parties. They state that coalitions will be more oversized in legislative chambers comprised of more decisive actors.

The truthful implementation exists already for the minimal supermajority: simple majority plus one politician. A bigger coalition can be formed by the prime-minister in spite of the increase in the coalition cost: introducing additional members into the coalition leads to the existence of the truth-telling equilibrium for a broader range of parameters.

If the types are continuous, then there is no truthful implementation. But this problem can be partially solved with Crawford and Sobel (1982) partition strategy.

# References


[1] Axelrod, Robert. 1970. Conflict of Interest. Chicago: Markham.

[2] Baron, David P., and Daniel Diermeier. 2001. Elections, Governments, and Parliaments in Proportional Representation Systems. Quarterly Journal of Economics 116(3):933-67.

[3] Baron, David P. and John A. Ferjohn. 1989. Bargaining in Legislatures. *American Political Science Review* 83, 1181-1206.

[4] Carrubba, Clifford J., and Craig Volden. 2000. Coalitional Politics and Logrolling in Legislative Institutions. American Journal of Political Science 44(2):261-77.

[5] Crawford, Vincent P. and Joel Sobel. 1982. "Strategic Information Transmission" *Econometrica 50 (6): 143–51.*

[6] Crombez, Christophe. 1996. Minority Governments, Minimal Winning Coalitions and Surplus Majorities in Parliamentary Systems. European Journal of Political Research 29(1):1-29.

[7] Dal Bo, Ernesto 2007. "Bribing Voters" American Journal of Political Science, 51(4).

[8] Denzau, Arthur T., and Michael C. Munger, 1986. "Legislators and Interest Groups: How Unorganized Interests Get Presented". American Political Science Review, 80, 89–106

[9] Diermeier, Daniel and Antonio M. Merlo. 2000. Governmental Turnover in Parliamentary Democracies. Journal of Economic Theory 94, 46–79.

[10] Diermeier, Daniel, Eraslan, Hülya and Antonio M. Merlo, 2002. "Coalition Governments and Comparative Constitutional Design" European Economic Review, 46, 893–907.

[11] Diermeier, Daniel, Eraslan, Hülya and Antonio M. Merlo, 2003. "A Structural Model of Government Formation" Econometrica, 71, 27–70.

[12] Felgenhauer, Mike and Hans Peter Grüner, 2008 "Committees and Special Interests." Journal of Public Economic Theory 10 (2).

[13] Groseclose, Timothy J. and James M. Snyder, 1996. "Buying Supermajorities." *American Political Science Review* 90(2): 303–15.

[14] Klingaman, David. 1969. " A Note on a Cyclical Majority Problem" *Public Choice* 6(1): 99–101.

[15] Koehler, David H. 1972. "The Legislative Process and the Minimal Winning Coalition." In *Probability Models of Collective Decision Making,* ed. R.G. Niemi and H.F. Weisberg. Columbus, OH: Merrill.

[16] Koehler, David H. 1975. "Legislative Coalition Formation." *American Journal of Political Science* 29: 27–39.

[17] Lijphart, Arend. 1984. Democracies: Patterns of Majoritarian and Consensus Government in Twenty-One Countries. New Haven: Yale University Press.

[18] Persson, Torsten, Roland, Gerard, Tabellini, Guido, 2000. Comparative politics and public finance. *Journal of Political Economy* 108, 1121-1161.

[19] Prat, Andrea. "Rational Voters and Political Advertising." 2006. Oxford Handbook of Political Economy (eds. B. Weingast and D.Wittman).

[20] Prat, Andrea, Puglisi, Riccardo and James M. Snyder,. 2005 "Does Campaign Money Signal Candidate Quality?"

[21] Riker, William H. 1962. *The Theory of Political Coalitions.* New Haven: Yale University Press.

[22] Shepsle, Kenneth. 1974. "On the Size of Winning Coalitions." American Political Science Review, 86, 5005–18.

[23] Sjolin, Mats. 1993. Coalition Politics and Parliamentary Power. Lund, Sweden: Lund University Press.

[24] Tsai, Tsung-Sheng and C. C. Yang (2010), "On Majoritarian Bargaining with Incomplete Information," International Economic Review 51 (4).

[25] Tsai, Tsung-Sheng and C. C. Yang (2010), "Minimal Winning versus Oversized Coalitions in Public Finance," Social Choice and Welfare. 34 (2).

[26] Weingast, Barry A. 1979. "A Rational Choice Perspective on Congressional Norms." *American Journal of Political Science* 23: 245–62.

[27] Volden Craig and Carrubba, Clifford J.. 2004. The Formation of Oversized Coalitions in Parliamentary Democracies. American Journal of Political Science 48(3):521-37.

# 7 Appendix Proof of Proposition 2

Note that at the last stage of the game all politicians vote according to Lemmas 1 and 2. Consider politician $i$, $d_L \leq d_i \leq d_H$. If politician $i$ is an opposition member, his utility is 0 in the case of reform implementation and $p$ in the case of reform rejection, regardless of his announcement and the announcement of any other player. According to Lemma 1, he always votes against the reform.

If politician $i$ is a coalition member, his minimal utility is $p$. This is because all coalition members are pivotal, hence, if politician $i$ obtains less than $p$ as a result of the reform implementation, he can always induce the reform rejection and obtains $p$ as a default payment. Then it follows from (utility under asymmetric information), that all announcements $\widetilde{d}_i \leq d_i$ secure the same utility for coalition member $i$. Moreover, announcement $\widetilde{d}_i = \widetilde{d}_L$ weakly maximizes the chances to be in the coalition. Thus, in any equilibrium each politician $i$ announces either $\widetilde{d}_i = \widetilde{d}_L$ or $\widetilde{d}_i > d_i$. Hence, a player with true type $d_H$ always announces $\widetilde{d}_L$.

Player $i$, given the expected types and the actions of the other players, faces the following problem: if he announces $\widetilde{d}_i > d_i$ then his expected utility is

$$EU_i\left(\widetilde{d}_i, d_i, \widetilde{d}_{-i}, d_{-i}\right) = \left(1 - P_c\left(\widetilde{d}_i, \widetilde{d}_{-i}\right)\right)\left(p_o\left(\widetilde{d}_{-i}, d_{-i}\right) * 0 + \left(1 - p_o\left(\widetilde{d}_{-i}, d_{-i}\right)\right)p\right) +$$

$$P_c\left(\widetilde{d}_i, \widetilde{d}_{-i}\right)\left(p_c\left(\widetilde{d}_{-i}, d_{-i}\right)\left(p + \widetilde{d}_i - d_i\right) + \left(1 - p_c\left(\widetilde{d}_{-i}, d_{-i}\right)\right)p\right) \tag{24}$$

$$s.t.\ \widetilde{d}_i > d_i$$

where $\widetilde{d}_{-i}$ is the announcements of all other players, $P_c\left(\widetilde{d}_i, \widetilde{d}_{-i}\right)$ is the probability that player $i$ is a coalition member, $p_c\left(\widetilde{d}_{-i}, d_{-i}\right)$ is the probability of a reform's implementation, given that player $i$ is a coalition member and $p_o\left(\widetilde{d}_{-i}, d_{-i}\right)$ is the probability of a reform's implementation given that player $i$ is an opposition member. Note that if player $i$ is a coalition member, he always supports the reform and if he is an opposition member he votes against it. Hence, the probabilities of the reform's implementation $p_c\left(\widetilde{d}_{-i}, d_{-i}\right)$ and $p_o\left(\widetilde{d}_{-i}, d_{-i}\right)$ depend only on actions of other players.

If player $i$ announces $\widetilde{d}_L$, then his expected utility is as follows:

$$EU_i\left(\widetilde{d}_L, \widetilde{d}_{-i}, d_{-i}\right) = P_c\left(\widetilde{d}_L, \widetilde{d}_{-i}\right)p + \tag{25}$$

$$\left(1 - P_c\left(\widetilde{d}_L, \widetilde{d}_{-i}\right)\right)\left(p_o\left(\widetilde{d}_{-i}, d_{-i}\right) * 0 + \left(1 - p_o\left(\widetilde{d}_{-i}, d_{-i}\right)\right)p\right)$$

Note that if player $i$ is a coalition member, then the reform is not implemented, because player $i$ votes against it.

Player $i$ maximizes his expected utility over $\widetilde{d}_i$, given $d_i$ and the announcements of the other players. Note, first, that if the player announces $\widetilde{d}_L$ then his expected utility does not depend on his true type. Secondly, if the player is an opposition member, his expected utility does not depend on his announcement. Thus I can denote the opposition member's expected utility as

$$C\left(\widetilde{d}_{-i}, d_{-i}\right) \equiv p_o\left(\widetilde{d}_{-i}, d_{-i}\right) * 0 + \left(1 - p_o\left(\widetilde{d}_{-i}, d_{-i}\right)\right) p \tag{26}$$

Player $i$ announces $\widetilde{d}_i > d_i$ if and only if

$$EU_i\left(\widetilde{d}_i, d_i, \widetilde{d}_{-i}, d_{-i}\right) > EU_i\left(\widetilde{d}_L, \widetilde{d}_{-i}, d_{-i}\right) \Leftrightarrow \tag{27}$$

$$P_c\left(\widetilde{d}_i, \widetilde{d}_{-i}\right)\left(p_c\left(\widetilde{d}_{-i}, d_{-i}\right)\left(p + \widetilde{d}_i - d_i\right) + \left(1 - p_c\left(\widetilde{d}_{-i}, d_{-i}\right)\right) p\right) - P_c\left(d_L, \widetilde{d}_{-i}\right) p +$$

$$+\left(P_c\left(d_L, \widetilde{d}_{-i}\right) - P_c\left(\widetilde{d}_i, \widetilde{d}_{-i}\right)\right) C\left(\widetilde{d}_{-i}, d_{-i}\right) > 0 \tag{28}$$

The probability to be a coalition member weakly decreases in the announcement. Hence $P_c\left(d_L, \widetilde{d}_{-i}\right) > P_c\left(\widetilde{d}_i, \widetilde{d}_{-i}\right)$, and the third term in inequality (28) is positive. Therefore, it is enough to prove that the first two terms are positive in order to show that politician $i$ announces $\widetilde{d}_i > d_i$:

$$P_c\left(\widetilde{d}_i, \widetilde{d}_{-i}\right)\left(p_c\left(\widetilde{d}_{-i}, d_{-i}\right)\left(p + \widetilde{d}_i - d_i\right) + \left(1 - p_c\left(\widetilde{d}_{-i}, d_{-i}\right)\right) p\right) - P_c\left(d_L, \widetilde{d}_{-i}\right) p > 0 \tag{29}$$

$$\Rightarrow EU_i\left(\widetilde{d}_i, d_i, \widetilde{d}_{-i}, d_{-i}\right) > EU_i\left(\widetilde{d}_L, \widetilde{d}_{-i}, d_{-i}\right). \tag{30}$$

Consider two types $d_{i1} > d_{i2}$. Suppose that for $d_{i1}$ the optimal announcement is $\widetilde{d}_i > d_{i1}$:

$$EU_i\left(\widetilde{d}_i, d_{i1}, \widetilde{d}_{-i}, d_{-i}\right) > EU_i\left(\widetilde{d}_L, \widetilde{d}_{-i}, d_{-i}\right). \tag{31}$$

Then, $\widetilde{d}_i > d_{i1} > d_{i2}$ and

$$P_c\left(\widetilde{d}_i, \widetilde{d}_{-i}\right) p_c\left(\widetilde{d}_{-i}, d_{-i}\right)\left(p + \widetilde{d}_i - d_{i1}\right) < P_c\left(\widetilde{d}_i, \widetilde{d}_{-i}\right) p_c\left(\widetilde{d}_{-i}, d_{-i}\right)\left(p + \widetilde{d}_i - d_{i2}\right). \tag{32}$$

Together with (11) it implies

$$EU_i\left(\widetilde{d}_i, d_{i1}, \widetilde{d}_{-i}, d_{-i}\right) < EU_i\left(\widetilde{d}_i, d_{i2}, \widetilde{d}_{-i}, d_{-i}\right) \tag{33}$$

$$\Rightarrow EU_i\left(\widetilde{d}_i, d_{i2}\right) > EU_i\left(\widetilde{d}_L, \widetilde{d}_{-i}, d_{-i}\right)$$

Hence, if for true type $d_{i2}$, $d_{i2} > d_{i1}$, the player announces $\widetilde{d}_i > d_{i2}$, then for true type $d_{i1}$ the announcement is also higher than the true type: $\widetilde{d}_i > d_{i1}$. Taking into account that a player with true type $d_H$ always announces $\widetilde{d}_L$, I obtain that there are only two possible equilibrium announcement strategies for player $i$:

(a) player $i$ always announces $\widetilde{d}_L$ regardless of his true type.

(b) there is type $d^*$ such that player $i$ announces $\widetilde{d}_i > d_i$ $\forall d_i < d^*$ and $\widetilde{d}_L$ otherwise.

The next step is to show that there is an equilibrium where all politicians follow the strategy (a) and announce $\widetilde{d}_L$. If the reform is rejected each player obtains $p$. If the reform is accepted, all coalition members obtain $p$ and the opposition members zero. If one player deviates and announces $\widetilde{d}_i > d_L$, he stays in opposition. As a result, he obtains a lower payment in the case of reform acceptance and is indifferent in the case of reform rejection. Hence, all players will announce $\widetilde{d}_L$.

Now I shall argue for discrete types, for some type distributions, there is an equilibrium where all players follow strategy (b). it is enough to show one example where it is true. Assume that there are only two types, $d_H$ and $d_L$, and three players. The probability for type $d_H$ is 0.5. Suppose that $0.5p + d_L < d_H$. Then there is an equilibrium where the players of type $d_L$ announce $\widetilde{d}_H$ and all players of type $d_H$ announce $\widetilde{d}_L$.

The last step is to show that if the type/announcement space is continues, then there is no equilibrium where all players follow strategy (b).

Assume that all players follow strategy (b) and announce $\widetilde{d}_i > d_i$ $\forall d_i < d^*$. Consider the incentives of player $j$ with true type very close to $d^*$. According to strategy (b) he announces $\widetilde{d}_j > d_j$. Such player expects that his announcement is a maximal among all other announcements. Then the expected probability that he is a coalition member is very close to zero. If politician $j$ is an opposition member, his utility is 0 in the case of reform implementation and $p$ in the case of reform rejection. If politician $j$ is a coalition member, his minimal utility is $p$. Then, for all $\widetilde{d}_j > d_j$, there always exists $\varepsilon$ such that

$$\left(p_o\left(\widetilde{d}_{-i}, d_{-i}\right) * 0 + \left(1 - p_o\left(\widetilde{d}_{-i}, d_{-i}\right)\right) p\right) < \left(1 - P_c\left(\widetilde{d}_i - \varepsilon, \widetilde{d}_{-i}\right)\right)\left(1 - p_o\left(\widetilde{d}_{-i}, d_{-i}\right)\right) p +$$

$$+P_c\left(\widetilde{d}_i - \varepsilon, \widetilde{d}_{-i}\right)\left(p_c\left(\widetilde{d}_{-i}, d_{-i}\right)\left(p + \widetilde{d}_i - \varepsilon - d_i\right) + \left(1 - p_c\left(\widetilde{d}_{-i}, d_{-i}\right)\right) p\right)$$

$$\Rightarrow EU_i\left(\widetilde{d}_j, d_j \lesssim d^*, \widetilde{d}_{-i}, d_{-i}\right) < EU_i\left(\widetilde{d}_j - \varepsilon, d_j, d_i, \widetilde{d}_{-i}, d_{-i}\right).$$

Therefore, player $j$ deviates and announces $\widetilde{d}_j - \varepsilon$ in order to improve his chances to be in the coalition. As a result, strategy (b) cannot be the equilibrium one. Hence, if the type/announcement space is continuous, the unique equilibrium is that all politicians announce $\widetilde{d}_L$.