A Safety Margin Model for Revenue Management in a Make-to-Stock Production System

Working paper

1. Introduction

Although revenue management (RM) originates from service industries, its ideas are also relevant for manufacturing environment. In this paper, we consider RM approaches for demand fulfillment in a make-to-stock (MTS) production system with known exogenous replenishments and stochastic demand from multiple customer classes. We propose a safety margin model which borrows the "safety stock" idea from inventory management to account for demand uncertainty and sets up booking limits for each customer class.

In a make-to-stock system, production is forecast-driven and cannot be easily adjusted to short-term demand fluctuation. Therefore, when demand is higher than supply, it may not be possible to satisfy all incoming customer orders. The manufacturer then has to decide how to allocate the limited supply, i.e., the finished goods inventory, to his customers, since different customers may show different profitability or hold different strategic importance. This situation is similar to the traditional airline revenue management problem, where a fixed number of seats are sold to multiple fare classes. Thus, demand fulfillment in MTS system can also benefit from revenue management ideas. The only difference is that, in the MTS system, the scarce resource to allocate is the finished goods inventory rather than seats. Unlike flight seats, inventory is storable and can be replenished at certain times. Therefore, inventory holding cost and backlogging cost might be incurred, which makes profit maximization a more appropriate criterion than pure revenue maximization.

In nowadays advanced planning system (APS), the available finished goods inventory is represented by the so-called *available-to-promise* (ATP) quantities which are derived from the mid-term master planning. For demand fulfillment, APS uses a two-level planning process to answer real-time customer requests. In the first allocation planning level, customers are segmented based on their profitability and/or strategic importance and APS then allocates ATP quantities to different delivery periods and customer segments according to certain predetermined allocation rules. In the second order promising level, the allocated ATP (aATP) is consumed by the incoming orders based on simple consumption rules such as first-come-first-served. The key connection between the two planning levels is that, for each incoming order, if aATP is available for the corresponding class, ATP can be consumed and the order is quoted accordingly. Otherwise, the order promising process searches for other options to satisfy the order, e.g., by consuming aATP quantities from lower classes if nesting is applied (Kilger and Meyr 2008).

Obviously, the quality of the adopted allocation rule has a great impact on the performance of demand fulfillment. For example, when supply is scarce, if two customer segments with the same expected demand show very different profitability, it is beneficial to allocate more supply to the more profitable segment than giving both segments the same share. In the current APS practice, the ATP quantities are normally allocated according to the priority ranks of the customers, the committed forecast, or the predetermined split factors, all of which are merely simple heuristic rules and none of them is profit maximizing.

In order to achieve system optimization, researchers have developed different allocation planning approaches. One stream uses deterministic linear programming (DLP) model to maximize the expected profit (Meyr, 2009). The other stream takes a full stochastic perspective and models the problem as a dynamic program (Quante et al. 2009). Both of these approaches have limitations: The DLP model considers only expected demand and neglects demand uncertainty, therefore not all information included in the demand distribution is taken into account, which makes the solution usually suboptimal. The stochastic dynamic program, however, is computationally expensive and therefore hardly scalable.

As an alternative, this paper aims for incorporating the impact of demand uncertainty into the deterministic model. In order to do so, we develop a safety-margin model which has similar philosophy as the safety stock calculation. We consider the same problem setting as Quante et al. (2009) and Meyr (2009): A make-to-stock manufacturer is facing stochastic demand from heterogeneous customers with different unit revenues. Inventory replenishments are scheduled exogenously and deterministic. The manufacturer decides for each order whether to satisfy it from stock, backorder it at a penalty cost, or reject it, in anticipation of more profitable future orders. The objective is to maximize the expected profit over a finite planning horizon, taking into account sales revenues, inventory holding costs, and backorder penalties.

We follow the two-level planning process of APS. In the allocation planning level, we allocate the ATP quantities not only according to the expected demand, as Meyr (2009) does, but also borrowing the “safety stock” idea from inventory management to calculate “safety margins” for higher customer classes and set up corresponding booking limits for the lower classes . By doing so, we successfully take demand uncertainty into account.

For the order promising level, we quote the orders according to the predetermined booking limits. In a series of numerical simulation, we compare the performance of our safety margin model with other common fulfillment policies.

In summary, we make the following contributions to the field:

- We present a new demand fulfillment model which takes customer demand uncertainty into consideration.
- By analogizing safety margins to safety stocks, we provide insight to the relationship between the traditional inventory/supply chain management world and the relatively new and emerging RM world.
- We compare the relative performance of our safety-margin model and other fulfillment policies numerically and show that the safety-margin model improves the performance of the DLP model with even lower computational expense.

The paper is organized as follows. In § 2, we review the current literature and further motivate our research. In § 3, we explain the problem setting and the basic model formulation. The core of the paper is § 4, which derives the safety-margin model. § 5 provides a numerical study which compares the relative performance of common current fulfillment policies in MTS environment. We conclude with §6 and discuss future research potentials.

2. Literature review

In general, manufacturing systems can be divided into make-to-order (MTO) system, assemble-to-order (ATO) system and make-to-stock (MTS) system. In literature, most researches regarding RM in manufacturing focus on the MTO system. This is due to the direct analogy between the perishable production capacity in MTO and the perishable flight seats in the traditional airline RM, which makes most of the airline RM approaches directly applicable in this environment. Van Slyke and Young (2000), Defregger and Kuhn (2004, 2007), Rehkopf and Spengler (2005), Barut and Sridharan (2005), and Spengler et al. (2007) propose RM approaches for the order acceptance problem in MTO environment. Harris and Pinder (1995) apply RM to an ATO environment. Literature on RM in MTS environment is very limited and we will focus on them in what follows.

Revenue management and manufacturing have significant methodological differences. While revenue management is normally based on stochastic optimization and uses probability distributions to assess opportunity costs, manufacturing companies rely on APS which takes deterministic mathematical programming as the major tool for different planning tasks (Quante et al. 2009). Due to this methodological divide between revenue management and manufacturing, in literature there are two main streams of researches for applying revenue management to demand fulfillment in MTS manufacturing. The first stream holds the traditional APS perspective and seeks to incorporate revenue management ideas into the deterministic optimization. The second stream takes a full stochastic view and models the problem as dynamic program. In what follows, we briefly review literature from both research streams.

For the deterministic stream, Kilger and Meyr (2008) set up a two-step framework, in which demand fulfillment is accomplished through ATP allocation and ATP consumption. Ball et al. (2004) propose a similar push-pull framework for ATP models: Push-based ATP models pre-allocate available resources to different customer classes and pull-based ATP models promise the allocated resources in direct response to incoming orders. Following this framework, we first consider the allocation models.

Ball et al. (2004) develop a deterministic optimization-based model that allocates production capacity and raw materials to demand classes in order to maximize profit. They claim that the model is designed for an MTS environment, but actually it is more appropriate for an ATO environment as both capacity and materials are taken into account.

With the same problem setting as ours, Meyr (2009) proposes a deterministic linear programming model for the ATP allocation. The DLP model maximizes the overall profit and its optimal solution is used as partitioned quantity reserved for each customer class and each arrival period, based on which different consumption rules are used for order promising. A numerical study shows that, compared to the rule-based allocation methods, this model can significantly improve the performance of an APS if demand forecasting is reliable. This DLP model is computationally efficient and can therefore easily be adapted to the advanced planning system. However, the major drawback is that it utilizes only expected demand information but ignores demand uncertainty. In order to overcome this drawback, our safety-margin approach extends the DLP model by adding safety margins to expected demand to account for demand uncertainty.

Quante (2008) incorporates demand uncertainty into the DLP model in another way. He adapts the randomized linear programming (RLP) idea from Talluri and van Ryzin (1999) to the MTS setting. The idea is to repetitively solve the DLP, not with the expected demand, but with a realization of the random demand with known distribution. The optimal allocation quantity is estimated by a weighted-average of the results over all repetitions. The RLP approach is appealing as it is only slightly more complicated than the DLP method but incorporates distributional information on demand. Besides, it also has the flexibility to model various possible demand distributions. However, according to the numerical study from Quante (2008), the RLP model does not show promising results and is often dominated by the DLP model.

After allocation planning, aATP quantities could be consumed in real-time mode or batch mode. Kilger and Meyr (2008) propose to use search rules for real-time order promising and suggest searching available aATP quantities along three dimensions: customer class, time and product. In order to improve the rule-based consumption methods which represent the current practice, Meyr (2009) formulates the real-time order promising problem as a linear programing (LP) model with the objective to maximize overall profits. In order to make it easy for practical implementation, he proposes several consumption rules to mimic the LP search process. For batch mode order promising, Fleischmann and Meyr (2003), Pibernik (2005, 2006) and Jung (2010) propose optimization based models.

For the stochastic stream, Quante et al. (2009) model the demand fulfillment process in MTS production as a network revenue management (NRM) problem and formulate a stochastic dynamic program. Unlike the traditional airline network revenue management problem, in the MTS setting, since products are identical, theoretically any of the available supplies can be used to satisfy any incoming order. Therefore, one has to decide not only whether or not to satisfy an order but also which supply and how many of each supply to use, as each supply alternative generates a different profit. It turns out that the optimal policy of the DP is the famous booking-limit policy which is easy to implement. Quante et al. (2009) also show that it outperforms current common fulfillment policies, such as first-come-first-served (FCFS) and the deterministic optimization model from Meyr (2009). However, because of the “curse of dimensionality”, it is computationally expensive and therefore not really applicable for real-size problem. This paper considers the same problem setting as Quante et al. (2009) and compares its performance with the proposed safety-margin model in the numerical study.

In order to deal with the computational intractability, Bertsimas and Popescu (2003) proposed a generic Approximate Dynamic Programming (ADP) algorithm, the basic idea of which is to approximate the value function of the DP by a simpler algorithm, such as linear programming (Talluri and van Ryzin 1999, Spengler et al. 2007, Erdelyi and Topaloglu 2010), affine functional approximation (Adelman, 2007) and Lagrangian relaxation approximation (Topaloglu 2009, Kunnumkal and Topaloglu 2010). Most of these researches are within the traditional airline revenue management context and we are not aware of any ADP study for the MTS environment.

In addition to the above mentioned two main streams, there is a paper from Pibernik and Yadav (2009) that is closely linked to our setting: They also consider an MTS system with stochastic demand. However, rather than pursuing the main target of revenue management: profit maximization, the authors still use the traditional service level maximization as the objective. Besides this main distinction, other differences include that the authors limit their analysis to two classes and do not allow backlogging.

3. The Demand Fulfillment Model

We consider the same demand fulfillment problem and therefore share the same problem setting with Meyr (2009) and Quante et al. (2009): We consider a MTS manufacturing system with exogenously determined replenishments and stochastic demand from heterogeneous customers. In order to maximize the expected profit, the manufacturer has to decide for each arriving order whether to satisfy it from stock, backorder it at a penalty cost, or reject it, in anticipation of more profitable future orders. The manufacturer needs to take into account not only sales revenues, but also inventory holding costs and backorder penalties.

Following the two-level framework from Kilger and Meyr (2008), we build up a demand fulfillment model which comprises an allocation planning level and an order promising level. In what follows, we summarize the modeling issues and notations for the demand fulfillment model.

We have a finite planning horizon of T, which is subdivided into discrete time periods $t = 1, \dots, T$. Customers are differentiated into K different segments, $k = 1, \dots, K$, with corresponding unit revenues of r_k $(r_1 > r_2 > \cdots > r_k)$. Orders from different segments arrive in arbitrary order and ask for a random quantity of the products. We assume that the order due dates equal to the arrival date. This assumption is legitimate for the MTS environment as customers normally expect immediate delivery. We use $D_{k\tau}$ to denote the total random demand from segment k with arrival date τ. $D_{k\tau}$ can follow any possible distribution, e.g., Poisson, Normal or Negative Binomial.

At the beginning of the planning horizon, allocation planning is conducted once for the whole planning horizon, with the following information on hand.

- Available inventory that arrives in period t which is denoted by ATP_t;
- Demand forecast: The mean and standard deviation of $D_{k\tau}$ is known.

After the allocation planning, incoming orders are processed in real time based on the allocation result. Delaying an order causes backorder cost of b per unit per period and the unit holding cost is h per period.

Table 1 Notation

Indices:	
$t = 1, \dots, T$	Periods of planning horizon
$\tau = 1, \dots, T$	Demand due date
$k = 1, \dots, K$	Customer segments
Data:	
r_k	Unit revenue from customer segment k
b	Unit backorder cost per period
h	Unit holding cost per period
ATP_t	Available ATP supply that arrives at the beginning of period t
Random variables:	
$D_{k\tau}$	Total demand from segment k with arrival date τ, follows a certain distribution with known mean $\mu_{k\tau}$ and standard deviation $\sigma_{k\tau}$

Table 1 summarizes the above notations. The profit of one unit ATP from period t which is used for customer segment k with arrival date τ can be calculated as follows

$$p_{tk\tau} = r_k - b(t - \tau)(1 - \delta_{t\tau}) - h(\tau - t)\delta_{t\tau} \tag{1}$$

where $\delta_{t\tau}$ is defined as 1 if $t \le \tau$ and 0 otherwise (Quante et al., 2009).

Quante et al. (2009) model the above demand fulfillment problem as a stochastic dynamic program and find out that the optimal policy is a generalized booking-limit policy which sets up booking limit for each segment and supply arrival. This model generates the optimal ex-ante policy and is easy to execute. The main problem is that it is computationally expensive and therefore hardly scalable.

Using the partitioned allocation of each ATP_t to segment k with arrival date τ as the decision variable, Meyr (2009) modeled the allocation planning problem as a DLP followed by a rule-based consumption process. The DLP model is efficient to solve, but as only the expected demand is taken into account, the performance is not satisfying if demand uncertainty is high. Quante et al. (2009) show in the numerical study that for low demand variability, the DLP model is competitive to the SDP model, but when demand variability increases, the performance of the DLP model deteriorates drastically.

In order to overcome the limitations of the above two models, in the next section we propose a safety margin model which can efficiently calculate the booking limits and also takes demand uncertainty into account by incorporating safety margins to more profitable customers.

4. Safety Margins

The basic idea of safety margin is analogous to the safety stock in inventory management, i.e. to reserve more stock than expected demand as "safety margin" for more profitable customers. We first consider a simple single-period, two-class case in which safety margins can be calculated by Littlewood's rule. Then we generalize the calculation to multi-period, multi-class case.

4.1 Single period, two-class case

We first consider the problem with $T = 1, K = 2$ and assume that within this single period, the lower class (Class 2) arrives before the higher class (Class 1). The problem then becomes the famous Littlewood's problem and can be solved directly using Littlewood's rule. We now illustrate how its solution can be interpreted in terms of safety margins.

As the planning horizon consists of only one period, we assume that there is a single inventory replenishment at the beginning of the period, namely ATP_1, and we use y_1 and y_2 to denote the allocated ATP quantities for Class 1 and Class 2 respectively. Assume the demand of Class 1 is normally distributed with mean μ_1 and standard deviation σ_1. Then, according to the Littlewoods' rule,

$$y_1^* = \Phi_1^{-1}\left(1 - \frac{r_2}{r_1}\right) = \mu_1 + z_{1-{r_2}/{r_1}} \cdot \sigma_1 \qquad (2)$$

i.e., the optimal protection level for Class 1 is y_1^* and the term $z_{1-{r_2}/{r_1}} \cdot \sigma_1$ can be considered as our safety margin for Class 1. For Class 2, the corresponding booking limit is then $\left[ATP_1 - (\mu_1 + z_{1-{r_2}/{r_1}} \cdot \sigma_1)\right]^+$.

Similar to the safety stock idea, we add a safety margin for the Class 1 customers in the allocation planning stage to better protect them.

Incorporating the safety margin of Class 1 into the DLP model from Meyr (2009), which is discussed in the previous chapter, the allocation planning problem can then be modeled as follows.

$$\max \quad r_1 y_1 + r_2 y_2 \qquad (3)$$

subject to

$$y_1 \leq \mu_1 + z_{1-r_2/r_1} \cdot \sigma_1 \qquad (4)$$

$$y_1 + y_2 \leq ATP_1 \qquad (5)$$

$$y_1, y_2 \geq 0, \text{integer} \qquad (6)$$

Constraint(4) modifies the DLP model by adding the safety margin $z_{1-r_2/r_1} \cdot \sigma_1$ in addition to the mean demand for Class 1. This simple LP forms a continuous knapsack problem whose solution is equivalent to the Littlewood's rule, i.e., by incorporating the safety margin term, we make the DLP model equivalent to the Littlewood's model which is optimal for our single-period, two-class case. This idea can be further extended to the multi-period, multi-class case.

4.2 Multi-period, multi-class case

Unlike the previous single period, two-class case, it is difficult to use the Littlewood's rule directly to calculate the safety margins for the ATP allocation problem in our MTS setting due to the following three characteristics: (1) It involves multiple customer classes instead of only two. In our MTS setting, we have multiple customer segments and in addition, orders from the same segment with different arrival dates incur different inventory holding or backlogging costs, and thus provide different profit. Therefore, these orders cannot be treated as a single class. This cost impact is a major difference between our MTS setting and the traditional airline RM, where orders from the same customer segment always generate the same profit. (2) The "low-before-high" assumption of Littlewood's rule is violated. The MTS setting involves multiple planning periods and within each period, orders from any customer segment may arrive. Therefore, orders arrive earlier may generate higher profits than orders arrive later. (3) It considers multiple replenishments, i.e., unlike the single resource case in Littlewood's model, we have multiple resources to allocate.

In order to deal with the first difficulty mentioned above, i.e., multiple customer classes, we adopt the idea of the expected marginal seat revenue (EMSR) heuristic which extends the Littlewood's rule to multi-class case (Belobaba, 1989). We consider each customer segment with a different arrival date as a different class. For a planning horizon of T periods with K customer segments, we have in total $\mathrm{N} = \mathrm{K} \cdot \mathrm{T}$ customer classes.

According to standard EMSR which also assumes low-revenue demand arrives before high-revenue demand, the profit ranking of the N classes should correspond to their arrival date, i.e., the one with the lowest profit arrives earliest and the one with the highest profit arrives latest. With this "low-before-high" assumption, EMSR ensures that the future higher classes are protected against the current lower class. However, assuming this "low-before-high" pattern is not reasonable in our MTS setting as we know that the inherent time structure of our arriving process is not the case: Each of the N classes has its specified arrival date which does not follow the "low-before-high" pattern. Therefore, the second difficulty still remains. In order to deal with it, as we know the exact arrival period of each class, we first rank them in a descending order of their arrival date. For classes with the same arrival period, we do not know their exact arrival sequence and assume that the lower classes arrive before the higher ones, i.e., they are ranked in a descending order of their unit revenue r_k. Then, the first class is the one from Segment 1 that arrives in the last period and the last class is the one from Segment K that arrives in the first period. By doing so, we ensure that by using EMSR we are indeed protecting the *future* classes against the current one. Furthermore, at each stage of the EMSR heuristic, when calculating the protection level, we only consider the future classes with higher profit than the current one. By doing so, we also achieve the goal of the standard EMSR, i.e., to protect the *future higher* classes against the current lower one.

To deal with the third difficulty, namely, the multiple resources, we consider two variants. First, we simply consider the multiple ATP supplies separately, i.e., we calculate the protection levels with respect to each ATP supply as if it is the only resource to allocate, without considering the impact of other supplies. The problem with this approach is that we are "double-counting" the demand of the higher classes when calculating protection levels – this method assumes that the future demand can only be fulfilled by a single ATP supply (the one under consideration) while actually, it has access to all ATP supplies. One may expect that this "double-counting" problem makes the safety margin model over-protect the higher classes. In order to deal with this problem, we consider another variant, i.e., to implicitly allocate the demand to individual supply: For each ATP supply, when determining the corresponding protection levels, we only take the future demand that arrive before the next supply into account. On contrary to the first case, the potential drawback of this approach is that we may not protect enough for the higher classes as we consider only a fraction of the demand when calculating the protection levels. We call the safety margin model with the first approach Safety Margin Model_Version 1(SM_1) and with the second approach Safety Margin Model_Version 2(SM_2).

4.2.1 Safety Margin Model_Version 1

Following the two-level planning procedure of APS, we first explain SM_1 in more detail with the following steps.

Allocation Planning

1. Define classes

   Rank the $N = K \cdot T$ classes in a descending order of their due date. Classes with the same due date are ranked in a descending order of their unit revenue r_k. Use a new index $j = 1, \ldots, N$ to denote customer classes and j can be considered as the customer segment/due date combination index. There is a one-to-one correspondence between each j and a combination of k, τ.

2. Calculate safety margins

   For each ATP supply t, do the following calculation:

   a. At stage $j + 1$, let $\mathfrak{I}_{tj}$ denote the set of future classes which have higher unit profit than class $j + 1$ if ATP_t is used, i.e., $\mathfrak{I}_{tj} = \{l \in \{j, j-1, \ldots, 1\}: p_{tl} > p_{t,j+1}\}$.

   b. Define the aggregated demand of set $\mathfrak{I}_{tj}$ by

$$S_{tj} = \sum_{l \in \Im_{tj}} D_l \quad (7)$$

c. Define the weighted-average profit of set $\Im_{tj}$ by

$$\bar{p}_{tj} = \frac{\sum_{l \in \Im_{tj}} p_{tl} E[D_l]}{\sum_{l \in \Im_{tj}} E[D_l]} \quad (8)$$

d. Calculate the safety margins

According to the Littlewood's rule, the protection level y_{tj}^* for set $\Im_{tj}$ is

$$y_{tj}^* = F_{\bar{tj}}^{-1}\left(1 - \frac{p_{t,j+1}}{\bar{p}_{tj}}\right) = \bar{\mu}_{tj} + \Delta_{tj} \quad (9)$$

where $\bar{\mu}_{tj} = \sum_{l \in \Im_{tj}} \mu_l$ and Δ_{tj} stands for the safety margin for set $\Im_{tj}$.

If, the demand for each Class j is normally distributed with mean μ_j and variance σ_j^2, we have

$$\Delta_{tj} = z_{tj} \cdot \bar{\sigma}_{tj} \quad (10)$$

where

$$\bar{\sigma}_{tj}^2 = \sum_{l \in \Im_{tj}} \sigma_l^2 \quad (11)$$

$$z_{tj} = \Phi^{-1}\left(1 - \frac{p_{t,j+1}}{\bar{p}_{tj}}\right) \quad (12)$$

3. Incorporate safety margins to the DLP model

Adding the safety margins into the DLP model, the resulting allocation planning model is as follows

$$\max \sum_{j=1}^{N} \sum_{t=1}^{T} p_{tj} \cdot y_{tj} \quad (13)$$

subject to

$$\sum_{l \in \Im_{tj}} y_{tl} \le \bar{\mu}_{tj} + \Delta_{tj} \quad \forall t, j \quad (14)$$

$$\sum_{j=1}^{N} y_{tj} \leq ATP_t \qquad \forall t \tag{15}$$

$$y_{tj} \geq 0, \text{integer} \qquad \forall t, j \tag{16}$$

Constraint (**14**) shows that this model indeed incorporates safety margins in addition to expected demand for the higher classes.

We can use the solution of the above LP as the allocation result. Note that the above LP can actually be decomposed into single-resource problems, i.e., we can have an individual LP for each supply t. This is because in the safety margin calculation (Step 2), we explicitly consider each supply separately and determine the set of future higher classes ($\mathfrak{I}_{tj}$) with respect to the specific supply t. Therefore, the obtained safety margins in Constraint (**14**) are for each individual supply t. Besides, in the above LP, there is no constraint specifying the relation between different supplies.

However, a more convenient way is to write down the corresponding booking limits directly without solving the LP. We are able to do so because that Constraint (14) already implies a booking limit for Class $j + 1$, namely

$$b_{t,j+1} = \left[ATP_t - (\bar{\mu}_{tj} + \Delta_{tj})\right]^+ \tag{17}$$

Another advantage of using the booking limits directly is that, as we do not need to know the exact allocation to each class and the protection level term $\bar{\mu}_{tj} + \Delta_{tj}$ in (17) is independent of the real ATP consumption, in the later order processing stage, we only need to update the current ATP_t quantities before we process each incoming order. It is not necessary to repeat the allocation planning steps all over again. If we use the solution of the above LP as the allocation result, we need frequent re-solving to adapt our allocation to the real consumption.

Order Processing

In the order promise stage, we process the incoming orders in real time. The following procedure is used for processing an order from Class *j* ($j = 1, \dots, N$) with an order quantity of *d.*

1. Update the current ATP_t quantities for each supply $t = 1, \dots, T$.
2. Determine the corresponding booking limits $b_{tj}, \forall t$ using (17). Note that our way of safety margin calculation sets nested booking limits for classes with the same arrival period, i.e., within the same period, higher classes have always access to units allocated to the lower classes.
3. Search for ATP supplies to fulfill the order successively, in the order of their arrival. Let u_t denote the amount of ATP quantities from supply *t* used to satisfy the given order, we have the following steps:

   Start with $t = 1$;

   Set $u_t = \max\left(\min\left(b_{tj}, d - \sum_{k=1}^{t-1} u_k\right), 0\right)$;

   Repeat for $t + 1$.

What needs to be noticed is that the safety margins and the protection levels from (9) are independent of $\mathrm{ATP_t}$. Therefore, before each order processing, we only need to update the current $\mathrm{ATP_t}$ quantities to determine the current booking limits. It is not necessary to repeat the allocation planning steps.

In the order processing, we start our search for available ATP quantities from the earliest available ATP supply. This is because we know from Quante et al. (2009) that under certain assumptions, the optimal policy for this MTS demand fulfillment situation is also a booking-limits policy and the optimal solution is obtained through a line search, starting with the earliest available supply. Here, we are mimicking the optimal behavior in our order processing level.

4.2.2 Safety Margin Model_Version 2

For SM_2, its only difference compared to SM_1 is that when calculating the protection level with respect to each ATP supply, it only considers future demand that arrive before the next ATP supply. Therefore, it has the same procedure as SM_1 and we only need to modify set $\Im_{tj}$ (Step 2a of the allocation planning level) as follows.

For each ATP supply t, assume the next non-zero ATP replenishment arrives at the beginning of period $t+i, i \in \{1, \ldots, T-t\}$. At stage $j+1$, $\Im_{tj} = \left\{l \in \{j, j-1, \ldots, 1\}: p_{tl} > p_{t,j+1},\ \tau(l) < t+i\right\}$. Since there is a one-to-one correspondence between each class index and k, τ combination, $\tau(l)$ here denotes the arrival date of Class l.

As mentioned above, before each order processing, it is not necessary for the safety margin models to repeat the allocation planning steps due to the fact that they adopt the booking-limit policy and the safety margins we calculate are independent of the real consumption. But for the DLP model, in the allocation planning, it explicitly allocates the available ATP quantities to different classes and therefore needs frequent re-planning to adjust its allocation according to the real consumption. Otherwise, its performance might be hurt. Because of the above mentioned difference, the safety margin model we propose is computationally more efficient than the DLP model. We illustrate this further in next chapter using run-time analysis.

5. Numerical Study

In order to evaluate the performance of different demand fulfillment models, Quante et al. (2009) set up a numerical study framework, comparing their stochastic dynamic programming model (SDP) to a first-come-first served strategy as well as the deterministic linear programming model from Meyr (2009). Following the same assumptions as Quante et al. (2009), we add both versions of safety margin models to the numerical study framework.

Same as Quante et al. (2009), we consider a finite planning horizon here in order to make the models comparable to the SDP model. However, the safety margin models we propose, as well as the DLP model are also applicable in rolling-horizon planning.

Within the finite planning horizon, it is not necessary for the safety margin models or the SDP model to do any re-planning, because both methods calculate the booking limits upfront and the obtained booking limits are independent of the real ATP consumption. The DLP model, on the other hand, allocates the current ATP quantities in the allocation planning stage; therefore it is necessary to do re-planning frequently so that it has the chance to adjust its allocation according to the real consumption.

In what follows we compare the performance of the safety margin models with the following fulfillment strategies:

- First-come-first served (FCFS): A comparison with this strategy shows us the benefit of customer segmentation in the demand fulfillment process. To ensure fairness, we limit this policy to fulfill customer orders only from stock to avoid excessive backordering.

- Deterministic linear programming (DLP) model from Meyr (2009): As explained in the previous sections, this strategy allocates the ATP quantities using a DLP model, followed by a rule-based consumption process: The search starts in each incoming order's own priority class. It first looks for aATP quantities that arrives at the required due date. If the order is not fully satisfied, it searches further for aATP quantities that arrive before the due date and then after the due date. Finally, it repeats the search in lower classes. In the numerical study we recalculate the DLP model after each order processing to ensure its performance. A comparison with this strategy provides an indication of the benefit of incorporating demand uncertainty in the fulfillment process.
- Stochastic dynamic programming (SDP) model from Quante et al. (2009): This strategy models the demand fulfillment process as a stochastic dynamic program, whose optimal policy is also a booking-limits control. This strategy maximizes the expected profit and therefore generates the optimal ex-ante policy.
- Global optimum (GOP): This strategy optimally allocates ATP quantities to demand ex-post and therefore provides the highest achievable profits. In the numerical study we use it to normalize the results for comparison.

We follow the same assumptions as Quante et al. (2009) for the demand pattern: Orders of a given customer segment follow a compound Poisson process and the order processes of different segments are mutually independent. We discretize the planning horizon in such a way that at most one order could arrive in a single period and the probability of no order arrival is p_0. This single-order-arrival assumption is made for the SDP model as it is required by the Bellman equation formulation, but not necessary for the safety margin model. For each given arrival, the order size follows a negative binomial distribution (NBD). This choice allows us to analyze the effects of large demand variations. In order to make the order size strictly positive, we model it as $1 + NB(\mu - 1, \sigma)$, where μ is the mean and σ is the standard deviation. Modeling the ordering process as a compound Poisson process makes twofold variability for the customer demand, i.e., the customer demand variability depends on both the variability of the order size as well as the arrival probabilities.

Based on the above assumptions, we define a numerical experiment with a test bed containing a wide range of problem instances and use simulation to evaluate the performance of the above mentioned models. In subsection 5.1 we define the test beds and in subsection 5.2 we analyze the results of the numerical study.

5.1 Test bed

We design our test bed based on a full factorial design with five design factors and six fixed parameters. We fix our planning horizon to 14 periods with two inventory replenishments in period 1 and period 8. Replenishments quantity is fixed to 50 units each time, i.e., $ATP_1 = ATP_8 = 50$. We consider 3 customer segments with different revenues. Inventory holding cost is fixed to \$1 per unit per period. We assume that the mean demand of each incoming order is constant and equal to 12 units. We summarize our choices for the design factors and fixed parameters in Table 2. This setup is similar to Quante et al. (2009), but Quante et al. (2009) consider only the first three design factors but assume equal order arrival probabilities and a fixed backlogging cost of \$10 per unit per period for all customer segments.

Table 2 Design factors and fixed parameters for the numerical study

Name	**Value**
Fixed parameters	
Planning horizon (T)	14
Arrival periods of replenishments	Period 1, Period 8
Replenishments quantity (S)	50
Number of customer segments (K)	3
Inventory holding cost (h)	1
Mean demand per order (μ)	12
Design factors	
Coefficient of variation of order size (CV)	$\left\{\frac{1}{3}, \frac{5}{6}, \frac{4}{3}, \frac{11}{6}\right\}$
Customer heterogeneity ($\boldsymbol{r}$)	$\{(100,90,80), (100,80,60), (100,70,40)\}$
Supply shortage rate (sr)	$\{40\%, 24\%, 1\%\}$
Customer arrival ratio (w)	$\{(1:2:3), (1:1:1), (3:2:1)\}$
Backlogging cost proportion (b)	$\{0.05, 0.1, 0.2\}$

The total number of all possible combinations for these design factors is $3^4 \times 4 = 324$, i.e. we have 324 scenarios. For each scenario, we generate 30 different demand profiles and run the corresponding simulations for every policy. This gives us in total $324 \times 30 = 9720$ instances for each policy in our numerical study. This scenario size ensures that both of the type I and type II error of the factorial design is limited to 5%.

We now explain the design factors in detail. The first factor in the factorial design is the coefficient of variation of order size (CV). We fix the mean of the order size to

$\mu = 12$, but the actual order size can vary from order to order and the variation is represented by the coefficient of variation of the order size $CV = {}^{\sigma}/_{\mu}$, where σ is the standard deviation of the order size. We choose the same range of CV as Quante et al. (2009) to ensure a reasonable range of variability.

The second factor of the factorial design is customer heterogeneity, which is represented by the revenue vector $\boldsymbol{r} = (r_1, r_2, r_3)$ of the customer segments. The revenue vector $(100{,}90{,}80)$ represents a low customer heterogeneity while $(100{,}70{,}40)$ represents a high customer heterogeneity. These choices are also identical to Quante et al. (2009).

The third factor of the factorial design is the supply shortage rate (sr), which reflects the degree of supply scarcity. It is defined as follows:

$$sr = 1 - \frac{\sum_{i=1}^{T} ATP_i}{(1 - p_0) \times \mu \times T}$$

Since in our case the supply quantity and the mean demand of each order are both fixed, the supply shortage rate (sr) depends solely on the no arrival probability p_0. A large p_0 corresponds to a low shortage rate while a small $p_0$leads to a high shortage rate. In our factorial design, we vary sr between 1% and 40% by varying p_0 from 0.4 to 0. We choose these levels because since we only consider situations where supply is scarce, 1% shortage rate is almost the lowest shortage rate we can use and 40% corresponds to a no arrival probability of 0 and is therefore the highest shortage rate we can use. Quante et al. (2009) use the same levels for the shortage situation, but also consider two more levels for oversupply, i.e., sr being negative.

The fourth factor of the factorial design is customer arrival ratio (w). This factor reflects the fraction of demand from each customer segment. For instance, when no arrival probability $p_0 = 0$, a customer arrival ratio $w = (1{:}\,2{:}\,3)$ corresponds with an arrival probability of $1/6$ for Segment 1, $1/3$ for Segment 2 and $1/2$ for Segment 3.

The fifth factor of the factorial design is the backlogging cost proportion (b). Quante et al. (2009) assume a fixed backlogging cost for all customer segments. We generalize this assumption to allow different backlogging cost for different customer segments, as customers from different segments pay different prices. In the numerical study, we assume that the backlogging cost for different customer segment is proportional to the corresponding revenue. When this proportion is small, e.g., $b = 0.05$, backlogging penalty is low and when this proportion is large, e.g., $b = 0.2$, backlogging cost takes 20% of the revenue, which makes the penalty high. Considering the holding cost $h = 1$, the chosen levels of backlogging cost ratio ensure that the resulting service level is within a reasonable range, e.g., if we fix the other parameters to their middle values (i.e., $CV = \frac{13}{12}, \boldsymbol{r} = (100{,}80{,}60), sr = 24\%, w = (1{:}\,1{:}\,1)$), our replenishment schedule achieves an average cycle service level between 56% and 82% for all segments if we vary b from 0.05 to 0.2.

5.2 Results Analysis

Using the test bed, we obtain the simulated profits of all the 9720 instances for each of the fulfillment strategies mentioned in the previous section. The average run-time for one simulation instance is 1774.56 seconds for the SDP model, 26.45 seconds for the DLP model, 3.63 seconds for the SM_1 model and 3.47 seconds for the SM_2 model, using a standard PC with a 2.0GHz Intel Core 2 Duo CPU and 2.00GB memory. The run-time data shows that the safety margin models are indeed much more efficient than the SDP model and even faster than the DLP model.

By comparing the simulated profits of other strategies to the simulated profits of the GOP model, we obtain the optimality gaps. We then calculate the average optimality gap for the FCFS strategy, DLP model, SDP model, and both versions of the SM model over (i) all 9720 test instances, and (ii) all subsets where one of the design factors is fixed to one of its admissible values. The results are shown in Table 3. Beside the average optimality gap (shown in bold), we also show the average backlog percentage (first value in parenthesis), the average lost sales percentage (second value in parenthesis) and the ratio between the average service levels of Segment 1 and Segment 3 (third value in parenthesis) of each strategy. As complementary data, the second and third row of Table 3 shows the average backlogging percentage and average lost sale percentage of each customer segment over all instances for each fulfillment model.

From the first row in Table 3, we see that as expected, the SDP model performs best with an average optimality gap of 3.96%, followed by SM_2 and SM_1 with an average optimality gap of 4.57% and 5.45% respectively. On average, the FCFS strategy (with an optimality gap of 7.55%) performs better than the DLP model (with an optimality gap of 8.84%).

Regarding the safety margin model, apparently both versions are considerably better than the DLP/FCFS model and perform much closer to the SDP model. As the safety margin models are developed to overcome the limitations of the DLP model and the SDP model, in what follows we will focus on comparing the safety margin models to these two models to disclose the difference.

By comparing the difference between the optimality gaps, we can see that SM_1 covers about 70% of the discrepancy between the DLP model and the optimal SDP model, and SM_2 covers 87% of the discrepancy.

As the SDP model provides the optimal solution to our problem, we compare the decisions (i.e., the backlogging, lost sale and service level behavior reflected in the bracketed value of Table 3) made by the two safety margin models and the DLP model to it to understand the profit differences.

Regarding lost sales, the SDP model has an average lost sale of 24.39%. Considering different customer segments, it has the highest lost sale rate for Segment 3 and the lowest rate for Segment 1. If we further consider its backlogging behavior we can see that it backlogs much more for Segment 1 and 2 than for Segment 3. Based on this observation we may conclude that compared to the other methods, the SDP model achieves a relatively high service level for the more profitable customers by increasing backlogging.

Compared to the SDP model, the DLP model has a higher average lost sale (28.11%). However, for Segment 1, its lost sale rate is even lower than SDP, but it loses much more customers from Segment 2 and 3. Regarding backlogging, the DLP model backlogs less in average and does not show a clear differentiation between segments. The backlogging rate for both Segment 1 and 2 are lower than SDP, i.e., the DLP model achieves a higher service level for Segment 1 with even less backlogging, but at the cost of losing much more customers from Segment 2 and 3. This provides clear evidence that the DLP model tends to over-protect high profit customers. This "over-protection" problem of DLP has also been identified by previous researches (De Boer, Freling, & Piersma, 2002).

The SM_1 model results in a lower lost sale rate (26.61%) than the DLP model. For Segment 1 and 2, its performance is very close to the SDP model, but for Segment 3, it has the highest lost sale rate among all methods. This means that our SM_1 model has also the “over-protection” problem, presumably due to the “double-counting” effect we have discussed in the previous chapter. Regarding backlogging behavior, SM_1 has a higher backlogging percentage than DLP, especially for Segment 1 and 2. Based on the behavior pattern of SDP we know that this backlogging behavior is actually favorable and might be the reason that SM_1 has less lost sale compared to DLP, which ultimately results in a higher average profit.

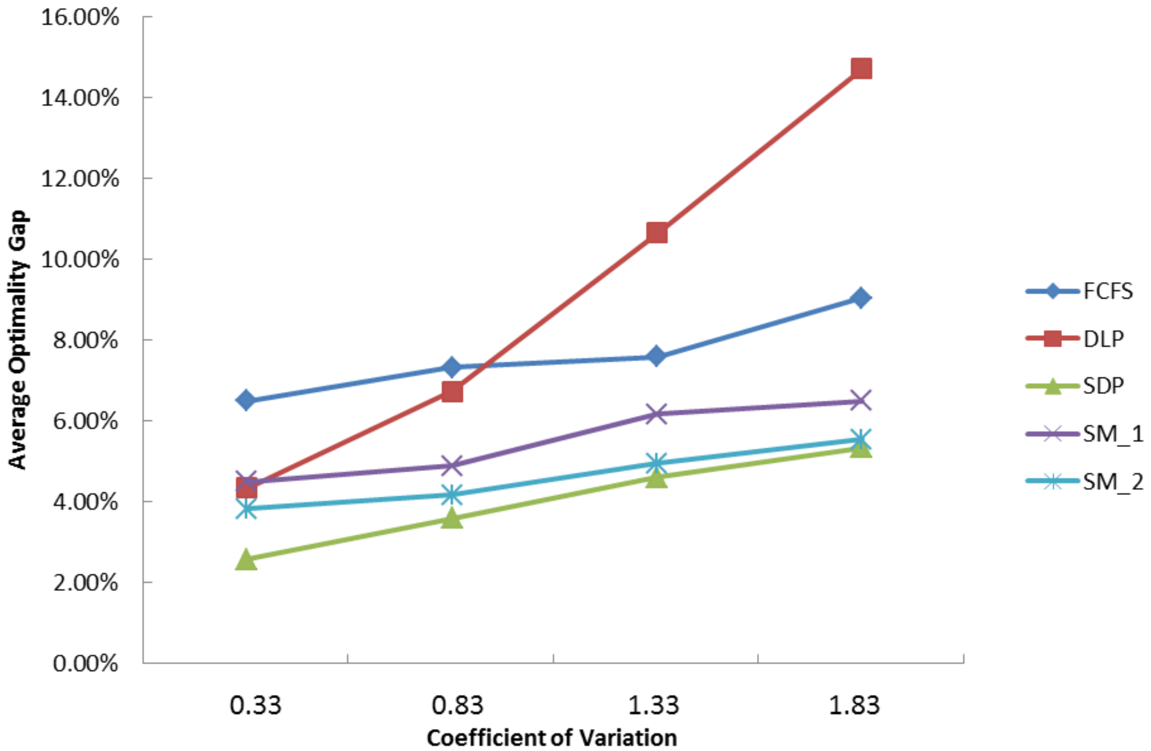
Considering our SM_2 model which is proposed to deal with the “double-counting” effect, from Table 3, we can see that is has the lowest lost sale rate (24.33%), even lower than the SDP model. This might be because that it loses more Segment 1 orders than the other strategies but much fewer Segment 3 orders and therefore indeed releases the “over-protection” problem. Considering the backlogging behavior, we can identify that it has the same pattern as the SDP model – increasing backlogging for more profitable customers to achieve better service level. From Table 3 we can see that SM_2 backlogs even more than the SDP model and this might explain why the average profit of SM_2 is still lower than SDP although it has the lowest lost sale rate.

The following part of Table 3 provides valuable information on the impact of different design factors on the performance of each fulfillment model. The customer arrival ratio (w) and the backlogging cost proportion (b) have little impact on the performance of the models, as for different levels of these two design factors the resulting optimality gaps of each fulfillment model are nearly the same. For the coefficient of variation of order size (CV), customer heterogeneity ($\boldsymbol{r}$), and supply shortage rate (sr), we find out that they have greater impact on the resulting optimality gap of each model and we analyze the impact in what follows.

Coefficient of variation of order size (CV)

From Table 3 and the following Figure 1, we can see the clear dependency between the optimality gaps and the CV values.

Figure 1 Average optimality gap for different CV values



We observe the following: (1) In general, as the CV value increases, all strategies show an increasing trend in their average optimality gaps. (2) For small CV values (i.e., low demand variability), the performance of DLP and the safety margin models are close to each other. However, as the demand variability increases the performance of the DLP model drops drastically. On the other hand, the performances of the two safety margin models are always very close to the SDP and evidently better than the DLP model for larger CV values. As the CV value increases, the gap between SM_2 and SDP is even getting closer.

For the first observation, the potential explanation is that the increasing demand variability leads to an increasing forecast error, which hurts the performance of every strategy.

In order to explain the rest observations regarding the individual performance of each model, we first summarize the response of SDP, as it provides the “right” response to parameter changes. Then we compare the decisions made by the other strategies to it.

As the CV value increases, SDP is able to keep the average lost sale rate almost constant. The backlogging percentage increases and the ratio between the average service levels of Segment 1 and 3 decreases. Based on these observations we may conclude that, as demand uncertainty increases, SDP reduces the differentiation between segments and backlogs more to retain the average service level.

Regarding backlogging, SM_1 responses the same as SDP - it increases the backlogging percentage to cope with the increasing demand uncertainty. It also reduces the differentiation between segments. However, the reduction extent is not sufficient, as its ratios between the average service levels of Segment 1 and 3 are always higher than that of the SDP model. The above reactions enable SM_1 to keep the lost sale rate at an almost constant but higher level.

For SM_2, it does not change its backlogging behavior too much as the CV value increases and the backlogging percentage is kept at a relatively high level. Similar as SDP, it also decreases the segment differentiation. Its ratios between the average service levels of Segment 1 and 3 are even lower than SDP. The high backlogging percentage as well as the low segment differentiation makes SM_2 able to keep the lost sale rate as low as the SDP model, which ultimately reflects in the very close average profits.

DLP fails to retain a constant lost sale rate. As the CV value increases, its lost sale rate also increases. Regarding segment differentiation, it responses in the right direction – to reduce the differentiation. But the same as SM_1, the extent of reduction is not sufficient, i.e., it keeps over-protecting the more profitable customers. DLP also makes mistakes on the backlogging behavior: Instead of backlogging more to compensate for the uncertainty increase, it even reduces the backlogging percentage as CV increases from 1/3 to 4/3. These mistakes can be attributed to the ignorance of demand uncertainty of DLP, which makes its performance drops drastically as demand variability increases.

Based on the above analysis we can conclude that while the DLP model fails to provide a satisfactory solution to our problem when demand uncertainty is high, the performance of the safety margin models we propose is promising.

Table 3 Simulation results

Test bed subset	N	Average optimality gap (%)				
		FCFS	DLP	SDP	SM_1	SM_2
All instances	9720	**7.55**(0.00, 25.39, 1.01)	**8.84**(3.49, 28.11, 1.60)	**3.96**(4.34, 24.39, 1.45)	**5.45**(4.50, 26.61, 1.65)	**4.57**(5.37, 24.33, 1.32)
Avg. backlogging (Seg.1, Seg.2, Seg.3)		*(0.00, 0.00, 0.00)*	*(3.09, 3.38, 2.23)*	*(6.07, 4.19, 1.52)*	*(4.76, 4.43, 2.52)*	*(7.52, 5.48, 1.66)*
Avg. lost sale (Seg.1, Seg.2, Seg.3)		*(0.23, 0.24, 0.24)*	*(0.09, 0.23, 0.43)*	*(0.12, 0.19, 0.39)*	*(0.12, 0.19, 0.47)*	*(0.15, 0.19, 0.36)*
CV = 1/3	2430	**6.49**(0.00, 24.73, 1.02)	**4.33**(4.48, 25.59, 1.96)	**2.57**(3.18, 24.58, 1.82)	**4.49**(4.01, 26.18, 1.89)	**3.82**(5.43, 24.22, 1.43)
CV = 5/6	2430	**7.32**(0.00, 25.30, 1.02)	**6.73**(3.58, 27.05, 1.74)	**3.58**(4.22, 24.66, 1.57)	**4.89**(4.44, 26.54, 1.79)	**4.16**(5.60, 24.31, 1.38)
CV = 4/3	2430	**7.58**(0.00, 25.18, 1.03)	**10.64**(2.61, 28.85, 1.51)	**4.60**(4.36, 24.20, 1.33)	**6.15**(4.41, 26.79, 1.55)	**4.95**(4.98, 24.23, 1.28)
CV = 11/6	2430	**9.04**(0.00, 26.37, 0.98)	**14.70**(3.29, 30.93, 1.31)	**5.34**(5.59, 24.12, 1.19)	**6.48**(5.13, 26.94, 1.44)	**5.53**(5.47, 24.57, 1.19)
r = (100,90,80)	3240	**4.48**(0.00, 25.09, 1.02)	**7.70**(3.36, 27.54, 1.60)	**2.32**(4.43, 23.53, 1.28)	**2.81**(5.58, 23.59, 1.21)	**2.86**(6.00, 23.38, 1.11)
r = (100,80,60)	3240	**7.35**(0.00, 25.58, 1.02)	**8.86**(3.52, 28.34, 1.59)	**4.22**(4.37, 24.54, 1.44)	**5.83**(4.30, 26.60, 1.73)	**4.99**(5.56, 24.31, 1.29)
r = (100,70,40)	3240	**11.44**(0.00, 25.52, 1.00)	**10.19**(3.60, 28.44, 1.61)	**5.63**(4.21, 25.10, 1.66)	**8.20**(3.62, 29.65, 2.34)	**6.16**(4.55, 25.31, 1.63)
sr = 1%	3240	**6.26**(0.00, 13.98, 1.00)	**8.03**(3.16, 15.58, 1.17)	**3.35**(4.73, 11.84, 1.09)	**5.06**(4.43, 14.83, 1.28)	**3.45**(4.50, 12.13, 1.10)
sr = 24%	3240	**7.33**(0.00, 24.61, 1.01)	**9.98**(3.91, 28.27, 1.61)	**4.24**(5.13, 23.61, 1.41)	**5.82**(4.87, 26.26, 1.67)	**4.53**(5.96, 23.48, 1.31)
sr = 40%	3240	**8.75**(0.00, 37.59, 1.04)	**8.42**(3.40, 40.46, 2.36)	**4.16**(3.15, 37.72, 2.31)	**5.40**(4.20, 38.76, 2.39)	**5.47**(5.64, 37.38, 1.74)
w = (1:2:3)	3240	**7.77**(0.00, 25.74, 1.06)	**8.69**(3.85, 27.51, 1.46)	**4.21**(4.36, 24.53, 1.38)	**5.82**(4.37, 26.93, 1.49)	**4.79**(5.35, 24.41, 1.26)
w = (1:1:1)	3240	**7.68**(0.00, 25.00, 1.00)	**8.83**(3.37, 27.74, 1.61)	**4.12**(3.94, 24.32, 1.47)	**5.87**(4.08, 26.82, 1.70)	**4.83**(4.92, 24.31, 1.31)
w = (3:2:1)	3240	**7.25**(0.00, 25.46, 0.97)	**8.99**(3.25, 29.08, 1.78)	**3.60**(4.70, 24.32, 1.50)	**4.76**(5.04, 26.10, 1.79)	**4.15**(5.83, 24.29, 1.38)
b = 0.05	3240	**8.11**(0.00, 25.39, 1.01)	**8.58**(3.71, 27.93, 1.60)	**3.62**(5.84, 23.98, 1.47)	**5.14**(6.45, 26.08, 1.67)	**4.23**(7.39, 23.95, 1.35)
b = 0.1	3240	**7.62**(0.00, 25.39, 1.01)	**8.93**(3.55, 28.10, 1.60)	**4.00**(4.47, 24.31, 1.45)	**5.50**(4.57, 26.55, 1.66)	**4.62**(5.53, 24.25, 1.33)
b = 0.2	3240	**6.92**(0.00, 25.39, 1.01)	**9.03**(3.21, 28.29, 1.60)	**4.25**(2.70, 24.87, 1.42)	**5.71**(2.47, 27.22, 1.63)	**4.86**(3.19, 24.80, 1.28)

Customer Heterogeneity (r)

There is also a clear dependency between the resulting average optimality gap and customer heterogeneity. From Table 3 and the following Figure 2, we observe: (1) In general, as the scale of customer heterogeneity increases, the performance of all strategies decreases. (2) Although all strategies show the same increasing pattern as the scale of customer heterogeneity increases, the performance difference between strategies is still evident. FCFS is most affected by increasing heterogeneity, followed by SM_1. On the other hand, the differences between DLP, SM_2 and SDP are rather constant as heterogeneity increases.

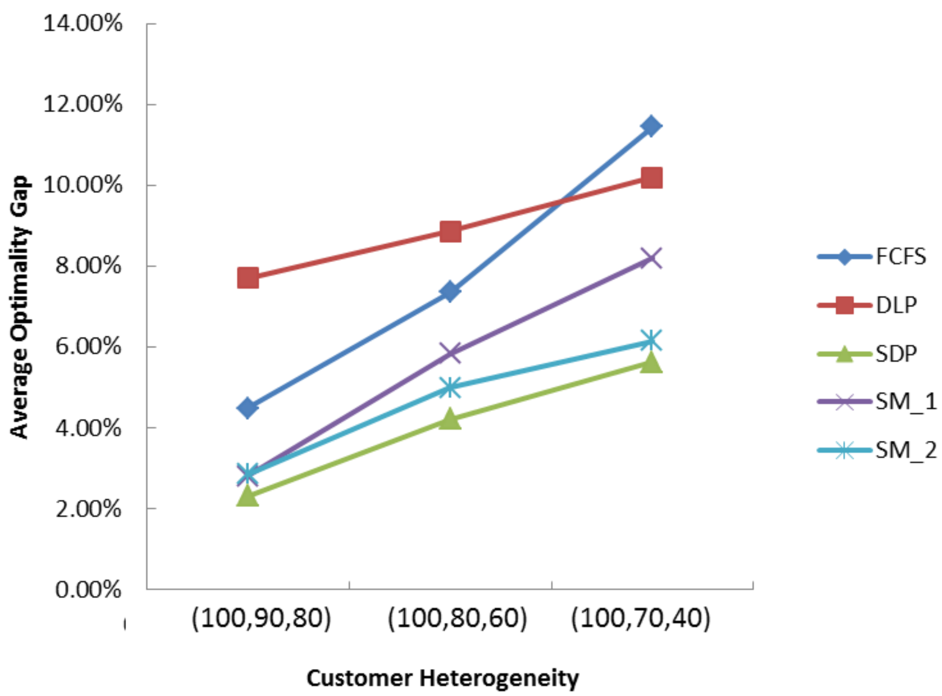
The potential explanation for the first observation might be: When the scale of customer heterogeneity is small, there is no big difference between customer segments. Therefore, the cost of “making mistakes” is low. As the scale of customer heterogeneity increases, the cost of “making mistakes” also increases, which makes larger optimality gaps.

SDP’s main reaction to the increase of customer heterogeneity is to increase the segment differentiation, which is reflected in the increasing value of the ratio between the average service levels of Segment 1 and Segment 3 (third value in parenthesis). This reaction is reasonable because it is more beneficial to better serve the more profitable customers when heterogeneity is high. As segment differentiation increases, SDP backlogs less. This is intuitive: From the average backlogging percentage of each segment in Table 3 we know that SDP does most of the backlogging for Segment 1 and 2, because it is only cost-effective to backlog the more profitable customers. As segment differentiation increases, the more profitable customers are better protected. Therefore, the necessity for backlogging decreases. The increasing segment differentiation and the decreasing backlogging percentage lead to the increase of the lost sale rate.

Both safety margin models react in the same pattern as the SDP model. However, the SM_1 model tends to overreact to the heterogeneity increase – when heterogeneity is low, its ratio between the average service levels of Segment 1 and Segment 3 is actually small, but the increase of the ratio is much higher than the SDP. This might explains why its performance deteriorates when heterogeneity is high.

In contrast, DLP has a constant average service level ratio, which means it does not react to different heterogeneity levels at all.

Figure 2 Average optimality gap for different customer heterogeneity



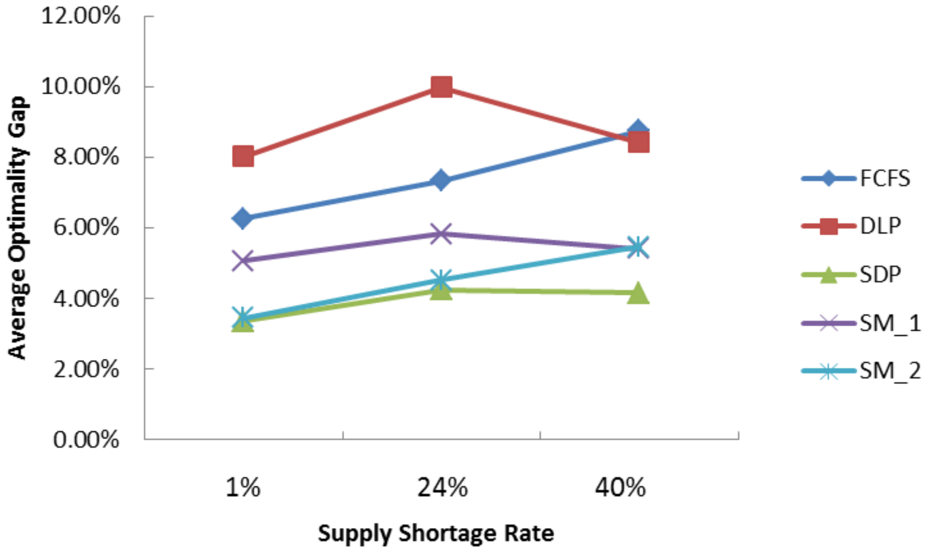
<u>Supply shortage rate (sr)</u>

Finally, we consider the impact of the degree of supply scarcity. From Table 3 and the following Figure 3 we observe: (1) The performance of DLP, SM_1 and SDP shows the same pattern, and it is not monotonic in the shortage rate (sr). All strategies perform worst for an intermediate shortage rate of 24%. (2) The performance of SM_2 shows a decreasing pattern as the shortage increases.

In response to the increasing shortage, SDP increases its segment differentiation. This makes sense as it is beneficial to better protect the more profitable customers when supply is getting scarce. Its backlogging behavior is in line with the average optimality gap, which is not monotonic in the shortage rate either, and SDP backlogs most when the shortage rate is 24%. One reasonable explanation is that for an intermediate shortage rate, the trade-off between selling a unit of supply for current low revenues versus reserving it for future higher revenues is the most difficult. If shortage is very low, the solution is clear and simple: to satisfy all the demand from all segments. If shortage rate is very high, the solution is also obvious: to reserve enough for the more profitable customers.

The other strategies react in the same way as SDP. However, for SM_2, although it also increases segment differentiation as shortage rate increases, the extent is not sufficient. When $sr = 1\%$, SM_2 has nearly the same ratio between the average service levels of Segment 1 and Segment 3 with SDP. But as the shortage increases, the difference between the ratios gets larger and larger. When $sr = 40\%$, the average service level ratio of SM_2 is much lower than SDP. This might explain why the performance of SM_2 is keeping decreasing when the shortage increases.

Figure 3 Average optimality gap for different supply scarcity



6. Conclusion and outlook

In this paper, we consider the demand fulfillment problem in make-to-stock manufacturing where customers are differentiated into different segments based on their profitability. We follow the two-level planning process of APS and develop two versions of safety margin model to allocate the pre-determined ATP quantities to different customer segments with different due date requirement, taking explicitly the demand uncertainty into account by adding safety margins to the relatively more profitable customers.

The model is motivated by the observation that the all existing approaches to the above mentioned demand fulfillment problem have their limitations and could be further improved. Based on the DLP model from Meyr (2009), we borrow the safety stock idea from inventory management to account for demand uncertainty and utilize EMSR to implement it to multi-class case. By doing so, we successfully link the traditional inventory/supply chain management world to the emerging revenue management world.

The numerical study shows that by incorporating demand uncertainty the safety margin models do improve the performance of the pure DLP model and provide a close and efficient approximation to the SDP model which is the optimal ex-ante policy but computational very expensive. Therefore, we could conclude that our results highlight the substantial opportunities for improving the demand fulfillment process in make-to-stock manufacturing and could be easily adapted to the current APS practice.

The main limitation of the safety margin models is that in the allocation stage we consider the different supplies separately, which results in the over-protection problem for SM_1 and excessive backlogging for SM_2. Besides, there could be other methods to calculate safety margins, which might improve performance even further. For the numerical study, a comparison using empirical data instead of theoretical distributions could provide us further insight into the relative performance of the different policies.