Computable General Equilibrium Analysis: Opening a Black Box

Christoph Böhringer, Thomas F. Rutherford
and Wolfgang Wiegard

ZEW

Zentrum für Europäische
Wirtschaftsforschung GmbH

Centre for European
Economic Research

Discussion Paper No. 03-56

Computable General Equilibrium Analysis: Opening a Black Box

Christoph Böhringer, Thomas F. Rutherford
and Wolfgang Wiegard

Non-Technical Summary

Quantitative simulations to evaluate alternative policy measures play a key role in applied economic research. Numerical models accommodate the systematic analysis of economic problems where analytical solutions are either not available or do not provide adequate information. Compared to analytical models, the numerical approach facilitates the analysis of complex economic interactions and the impact assessment of structural policy changes.

Among numerical methods, computable general equilibrium (CGE) models are widely employed by various national and international organizations (EU Commission, IMF, World Bank, OECD, etc.) for economic policy analysis at the sector-level as well as the economy-wide level. CGE analysis constitutes a powerful scientific method for the comprehensive ex-ante simulation of adjustment effects induced by exogenous policy interference. The main virtue of the CGE approach is its comprehensive micro-consistent representation of price-dependent market interactions. However, the barriers to entering the field of CGE analysis for students and professionals are commonly perceived as rather prohibitive. Without detailed programming knowledge, the CGE approach is doomed to remain a "black box" for non-modelers.

The main objective of this article is to lower the barriers to CGE analysis for two target groups. The first group consists of economists that want to actively enter the field of CGE analysis. We lay out the most recent developments in CGE model implementation for this group: The MCP approach, combined with the calibrated share form, provides a comprehensive framework to implement equilibrium problems in a transparent and efficient way. The second group includes economists that want to refrain from programming but are interested in CGE analysis as a tool kit. For this group, we present an interactive interface that offers convenient access to CGE analysis. Without technical programming skills, they can use the intuitive and simple Windows-based interface to explore the determinants of policy effects. Interactive learning-by-doing then involves the specification of additional policy simulation, the ex-ante economic reasoning on the potential effects, and the ex-post cross-check with the results provided by the computer simulations.

.

Computable General Equilibrium Analysis:

Opening a Black Box

Christoph Böhringer

boehringer@zew.de

ZEW Mannheim

Thomas Rutherford

rutherford@colorado.edu

University of Colorado at Boulder

Wolfgang Wiegard

wolfgang.wiegard@wiwi.uni-regensburg.de

University of Regensburg

ABSTRACT

Computable general equilibrium models play an important role in applied economic research. These models accommodate the micro-consistent systematic analysis of complex economic problems where analytical solutions are either not available or do not provide adequate information. However, the computational approach to economic policy analysis also has severe shortcomings of its own. In general, scientific publications include a complete listing of neither the algebraic model underlying the numerical simulation nor the data used to calibrate model parameters. Even if the model algebra and the data were fully laid out, replication of results would still require specialized programming skills. As a consequence, CGE analyses are often perceived as a *black box* to non-expert readers. This article aims at lowering the barriers to numerical CGE analysis along two lines. First, we provide a compact introduction to recent methods of CGE model implementation that may substantially ease entry for potential modelers. Second, and equally important, we illustrate the non-technical access to CGE analysis by means of a user-friendly interface that allows non-modelers to test and develop their economic intuition on general equilibrium interactions without programming skills.

1 INTRODUCTION

Quantitative simulations to evaluate alternative policy measures play a key role in applied economic research. Numerical models accommodate the systematic analysis of economic problems where analytical solutions are either not available or do not provide adequate information. Compared to analytical models, the numerical approach facilitates the analysis of complex economic interactions and the impact assessment of structural policy changes.

Among numerical methods, computable general equilibrium (CGE) models are widely employed by various national and international organizations (EU Commission, IMF, World Bank, OECD, etc.) for economic policy analysis at the sector-level as well as the economy-wide level.[1] The main virtue of the CGE approach is its micro-consistent representation of price-dependent market interactions. The simultaneous explanation of the origin and spending of the agents' income makes it possible to address both economy-wide efficiency as well as distributional impacts of policy interference. This has made CGE models a standard tool for the quantitative analysis of policy interference in many domains including fiscal policy, trade policy, and environmental policy.[2]

The computational approach to policy analysis, however, has also shortcomings of its own. In general, scientific publications include a complete listing neither of the algebraic model underlying the numerical simulation nor the data used to calibrate model parameters. Even if the model algebra and the data were fully laid out, replication of results or sensitivity analysis would still require specialized programming skills.[3] As a consequence, CGE analyses is often perceived as a *black box* to non-expert readers. All they can do is to "... deliver summary grunts of belief or disbelief but find it difficult to articulate reasons in a disciplined way" (McCloskey 1983, p. 502).

The objective of this introductory paper is to lower the barriers of entry to CGE analysis for graduate students and professionals that are interested in this method for applied policy analysis. We provide a compact introduction to recent methods of CGE model implementation that highly facilitate the accessibility of CGE analysis for potential modelers. Furthermore, we illustrate the non-technical

---

[1] The importance of CGE modeling in academic research is reflected by the fact, that – since 1991 - the *Journal of Economic Literature* has listed a separate classification number for this type of analysis (D 58: *Computable and other Applied General Equilibrium Models*).

[2] For survey articles see e.g. Conrad (1999, 2001), Bergman (1990), Gottfried et al. (1990), Borges (1986), Kehoe and Kehoe (1994), Klepper et al. (1995), Pereira and Shoven (1988), Shoven and Whalley (1984, 1992), Fehr and Wiegard (1996), Piggot and Whalley (1985, 1991), Bhattacharyya (1996), Gunning and Keyzer (1995).

[3] Obviously, errors in the numerical model specification can never be ruled out. In fact, Dewald, Thursby and Anderson (1986) conclude from their famous *The Journal of Money, Credit and Banking Project* „that inadvertent errors in published empirical articles are a commonplace, rather than a rare, occurrence." (pp. 587).

access to CGE analysis by means of a user-friendly interface, allowing non-modelers to test and develop their economic intuition on general equilibrium interactions without programming skills.

The remainder of this paper is organized as follows. In section 2, we briefly describe the basic procedure in CGE analysis. In section 3, we lay out the implementation of the common textbook 2x2x1-model using the mixed complementarity approach in which the equilibrium problem is formulated as a system of weak inequalities and complementary slackness conditions (between equilibrium variables and equilibrium conditions). In section 4, we introduce a generic visual interface for performing quantitative equilibrium analyses without technical programming skills. In section 5, we provide policy-relevant examples of activity analysis and explicit price restrictions as well as quantity constraints to demonstrate the advantages of the mixed complementarity approach in general equilibrium analysis. In section 6, we extend the stylized 2x2x1-model to a large-scale multi-region, multi-sector model of global trade and energy use in order to investigate the economic impacts of international climate protection policies. Finally, we conclude.

2   BASIC APPROACH

CGE models build upon general equilibrium theory that combines behavioral assumptions on rational economic agents with the analysis of equilibrium conditions. Analytically, the economic implications of policy interference could be studied in a comparative-static framework: One derives the total differentials of the market equilibrium conditions and solves this system of equations for the relative changes in the variables of interest. This is the normal procedure employed in the theoretical literature. Although theoretical analysis provides important qualitative insights, its contribution to actual policy analysis remains limited. This is because theoretical models are highly stylized to keep analytical tractability. As soon as certain real-world complexities are taken into account, e.g. a more detailed production structure, analytical solutions are no longer available and numerical solution methods are required. Furthermore, analytical models can only be applied to marginal analysis (given non-linear relationships), whereas numerical models are suited for the investigation of structural changes.

Figure 1 summarizes the five main steps involved in constructing and using CGE models. Initially, the policy issue must be carefully studied to decide on the appropriate model design as well as the required data. The second step involves the use of economic theory (at best, the draft of a simple analytical maquette model) in order to lay out key economic mechanisms that drive the results in the more complex numerical model. Data work, model formulation, and implementation then delivers the framework for numerical policy analysis. This step also involves the set-up of alternative policy instruments and strategies that induce changes vis-à-vis the reference situation (scenario definition). In determining results of policy simulation, the choice and parameterization of functional forms are crucial.

Figure 1:   Steps in computable general equilibrium analysis
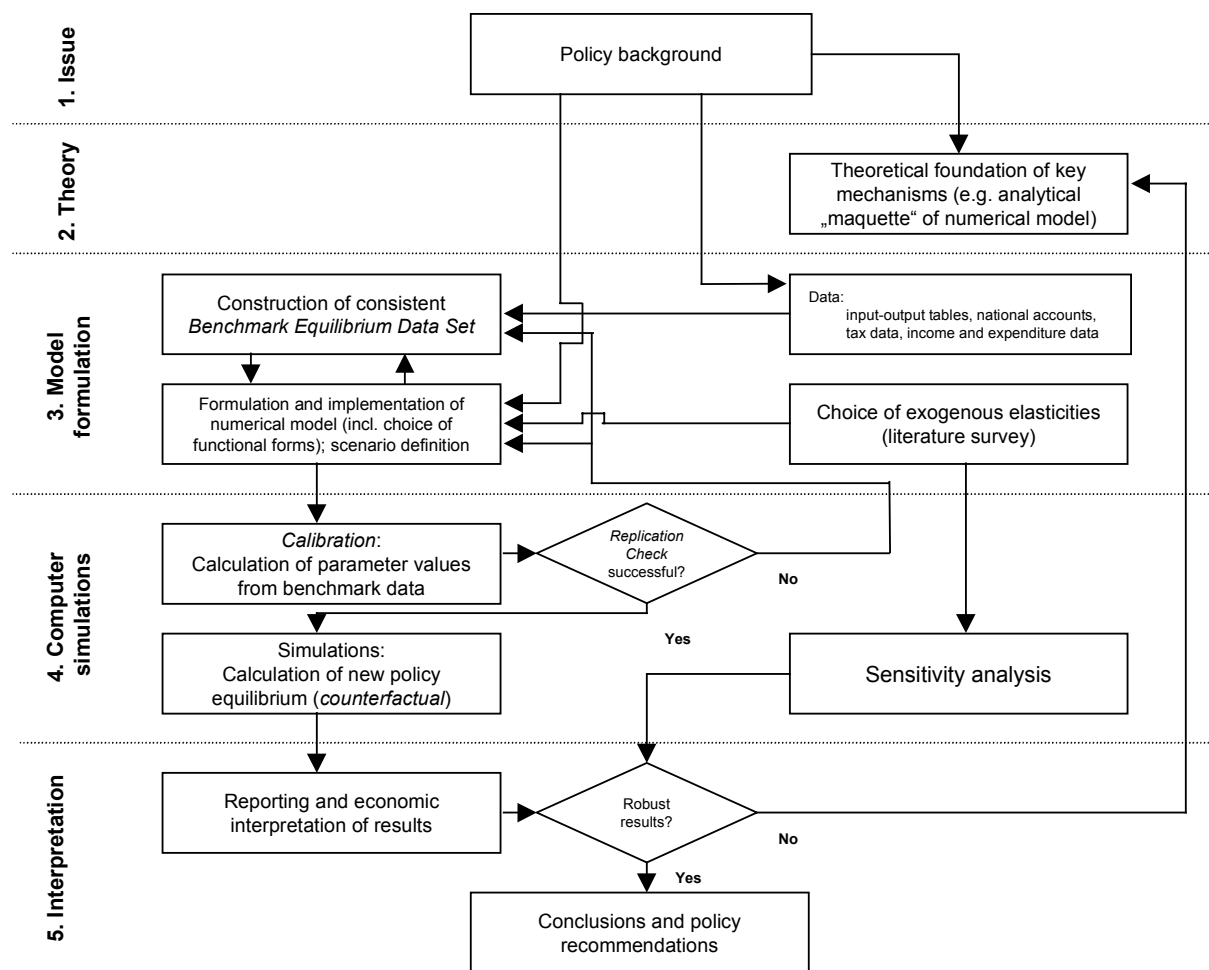
The procedure most commonly used to select parameter values is known as calibration (see Mansur and Whalley 1984). Calibration of the free parameters of functional forms requires a consistent one year's data (or a single observation represented as an average over a number of years), together with exogenous elasticities that are usually taken from literature surveys. The calibration is a deterministic procedure and does not allow for a statistical test of the model specification. The one consistency check that must necessarily hold before one can proceed with policy analysis is the replication of the initial benchmark: the calibrated model must be capable of generating the base-year (benchmark) equilibrium as a model solution *without* computational work.

Within the policy simulations, single parameters or exogenous variables are changed and a new (counterfactual) equilibrium is computed. Comparison of the counterfactual and the benchmark equilibrium then provides information on the policy-induced changes of economic variables such as employment, production, consumption, relative prices, etc. Finally, the model results must be interpreted based on sound economic theory. Due to the reliance on exogenous elasticity values and a single base-year observation, comprehensive sensitivity analysis on key elasticities (and possibly

alternative assumptions on economic incentives) should be performed before concrete policy recommendations are derived.

The essence of CGE economics is the combination of general equilibrium theory with a consistent data set (most commonly from National Accounts) in order to derive policy insights.

3   MODEL IMPLEMENTATION AND ANALYSIS

At first glance, the requirements of numerical equilibrium analysis appear prohibitive. Shoven and Whalley, two prominent protagonists of the CGE approach, bemoan the necessity for modelers to be a jack-of-all-trades: „Modelers must know general equilibrium theory so that their models have a sound theoretical basis; they must know how to solve their models; they need to be able to program; they must understand the policy issues on which they work; they have to know about data sources and all their associated problems; and they have to be conversant with relevant literature, especially that on elasticities" (Shoven and Whalley 1984, p.1047). However, there has been substantial progress during the last two decades, particularly in the fields of numerical solution techniques and higher-level programming languages, which has dramatically lowered the barriers to enter numerical modeling. Compared to the "old days", there is no longer a need for modelers to program their own tailored solution algorithms, since a variety of powerful commercial solution packages for nonlinear equation systems now exist (e.g. CONOPT (Drud 1985), MINOS (Murtagh and Saunders 1995) or PATH (Dirkse and Ferris 1995). Moreover, higher-level programming languages (such as MATLAB (1997), MATHEMATICA (Wolfram 1996), AMPL (Fourer, Gay, and Kernighan, 1987) or GAMS (Brooke, Kendrick, and Meeraus 1988)) have been developed that greatly facilitate the translation of an algebraic model formulation into a computer-readable program.[4]

The traditional approach in CGE analysis is to cast equilibrium conditions as a system of equations where functional forms are calibrated to benchmark data in the so-called coefficient form. This approach, however, implies restrictions with respect to the representation of potentially important economic features and can be rather messy concerning the calibration of the free parameters of functional forms. To avoid such drawbacks, our approach employs the mixed complementarity framework to specify an economic equilibrium and the so-called calibrated share form for function calibration.

---

[4] For our computer implementation we employ GAMS (*Generalized Algebraic Modeling System*), whose fundamental strength lies in the ease with which mathematically-defined models can be formulated and solved. GAMS itself does not provide model solutions but delegates this task to subsystems, i.e. specialized algorithms for particular classes of mathematical programs.

In section 3.1, we specify an economic equilibrium as a mixed complementarity problem (MCP) where equilibrium conditions are formulated as weak inequalities and conditions of complementary slackness between variables and equilibrium conditions. Compared to the traditional equation-based format, the MCP approach easily accommodates the explicit treatment of activity analysis (i.e. the possibility of regime shifts between alternative activities) as well as discrete price and quantity constraints that may become binding in equilibrium or not. In section 3.2, we present common functional forms in CGE analysis to characterize technologies and preferences. Apart from the traditional coefficient form, we introduce the equivalent *calibrated share form* which substantially reduces the efforts as well as sources of errors in the determination of free function parameters. In section 3.3, we introduce the standard textbook 2x2x1-model and present an illustrative policy simulation.

3.1 General Equilibrium as a *Mixed Complementarity Problem* (MCP)

An economic equilibrium can be cast as a mixed complementarity problem. For illustration, we consider a standard Arrow-Debreu economy with *n* commodities (incl. factors), *m* sectors and *h* households (incl. government).[5]

The endogenous variables of the Arrow-Debreu economy can be classified into 3 categories (Mathiesen 1985):

- p:= a non-negative *n*-vector in prices for all goods and factors (I ={1,...,n}),
- y:= a non-negative *m*-vector for activity levels of CRTS-production sectors
  (J={1,...,M}), and
- M:= a non-negative *k*-vector in incomes (H ={1,...,k}).

In equilibrium the variables must fulfill three classes of conditions:

- zero profit of CRTS-producers (exhaustion-of-product constraint)

$$-\Pi_j(p) = C_j(p) - R_j(p) \geq 0 \quad \forall j$$

  where (using Hotelling's Lemma):

$\Pi_j(p)$ the unit profit function,

$$C_j(p) \equiv \min\left\{\sum_i p_i \frac{\partial \Pi_j}{\partial p_i} \Big| f_j(\cdot) = 1\right\} \qquad \text{the unit costs function, and}$$

$$R_j(p) \equiv \max\left\{\sum_i p_i \frac{\partial \Pi_j}{\partial p_i} \Big| g_j(\cdot) = 1\right\} \qquad \text{the unit revenue function.}$$

---

[5] The competitive setting can be easily adjusted with price restrictions and quantity constraints to reflect market imperfections.

The functions f_j and g_j characterize feasible input- and output-combinations of production in sector j.

- market clearance for all goods and factors:

$$\sum_j y_j \frac{\partial \Pi_j(p)}{\partial p_i} + \sum_h b_{ih} \geq \sum_h d_{ih} \quad \forall i$$

where:

b_{ih} the initial endowment of household h with commodity i, and

$$d_{ih}(p, M_h) \equiv \arg\max \left\{ U_h(x) \middle| \sum_i p_i x_i = M_h \right\}$$ the demand for good i by household h

maximizing utility.[6]

- budget constraints for households:

$$\sum_h p_i b_{ih} = M_h \geq \sum_h p_i d_{ih} \quad \forall h$$

For common utility functions (non-satiation), households are always on their budget line, i.e. $\sum_h p_i b_{ih} = M_h = \sum_h p_i d_{ih}$, and Walras' law holds.[7]

Using Walras' law, aggregation of market clearance conditions and zero profit conditions yield:

$$\sum_j y_j \Pi_j(p) = 0 \,; \quad y_j \Pi_j(p) = 0 \quad \forall j$$

and

$$p_i \left(\sum_j y_j \frac{\partial \Pi_j(p)}{\partial p_i} + \sum_h b_{ih} - \sum_h d_{ih} \right) = p_{i\xi} \xi_i = 0 \quad \forall i$$

as well as:

$$M_h \left(\sum_h p_i b_{ih} - \sum_h p_i d_{ih} \right) = 0 \quad \forall h \,.$$

Thus, economic equilibrium features complementarity between equilibrium variables and equilibrium conditions: positive market prices imply market clearance – otherwise commodities are in excess supply and the respective prices fall to zero. Activities will be operated as long as they break even - negative revenues imply the shutdown of the respective production activities.

---

[6] U_h denotes the utility function of household h.
[7] Due to linear-homogeneity of profit functions and homogeneity of demand functions of degree zero in prices, the economic equilibrium only determines relative prices.

The MCP (Cottle and Pang 1992, Rutherford 1995):

$$Given: f : R^n \to R^n$$
$$Find: z \in R^n \qquad \qquad \textbf{(MCP)}$$
$$s.t.: f(z) \geq 0, \ z \geq 0, \ z^T f(z) = 0$$

corresponds to the problem of finding an economic equilibrium for $z = [y, p, M]$ and $f(z) =$ $[\Pi_j(p), \xi_i, (\sum_h p_i b_{ih} - \sum_h p_i d_{ih})]$, thereby stating complementarity between variables and equilibrium conditions.

In this context, the term „*mixed complementarity problem*" (MCP) is straightforward: „*mixed*" indicates that the mathematical program includes equalities as well as inequalities; „*complementarity*" refers to complementary slackness between system variables and system conditions. The above formulation of an economic equilibrium is very compact: Variables such as consumption and factor demand or commodity supplies are treated implicitly, which reduces computation time significantly for higher-dimensional models.

3.2 Functional Forms in *Calibrated Share Form*

Numerical calculation of an economic equilibrium requires the choice of concrete functional forms for production possibilities and preferences. In applied modeling, combinations of Leontief-, Cobb-Douglas-, or constant-elasticity-of-substitution (CES) functions are most common. Such functions have certain mathematical properties (regularity) that ease the numerical analysis considerably, but are still flexible enough to allow for an appropriate representation of economic behavior.

Tables 1 and 2 provide an overview of standard functional forms in conventional coefficient form and (less common) calibrated share form.

The following notations are adopted with respect to functional forms characterizing production:

α_i	distribution parameter for input i,
γ	scale parameter,
ρ	substitution parameter,
σ	substitution elasticity ($\rho := \dfrac{\sigma - 1}{\sigma}$),
$\overline{x_i}$	benchmark demand for input i,
$\overline{w_i}$	benchmark price for input i,

$$\bar{y} \qquad \text{benchmark output level } \left[\bar{y}=f(\bar{x})=1\right],$$

$$\bar{p} \qquad \text{benchmark output price,}$$

$$\bar{C} \qquad \text{benchmark cost,}$$

$$\bar{c} \qquad \text{benchmark unit cost, and}$$

$$\theta_i = \frac{\bar{x}_i \cdot \bar{w}_i}{\bar{y} \cdot \bar{p}} \qquad \text{benchmark value share of input } i.$$

Equivalent notations for functional forms characterizing final demand include:

$$\bar{d}_i \qquad \text{benchmark consumption demand for good } i,$$

$$\bar{p}_i \qquad \text{benchmark consumer price for good } i,$$

$$\bar{U} \qquad \text{benchmark utility level } \left[\bar{U}=g(\bar{d})=1\right],$$

$$\bar{E} \qquad \text{benchmark expenditure,}$$

$$\bar{e} \qquad \text{benchmark unit expenditure,}$$

$$\bar{M} \qquad \text{benchmark income, and}$$

$$\theta_i = \frac{\bar{d}_i \cdot \bar{p}_i}{\bar{M}} \qquad \text{benchmark value share of consumption demand } i.$$

The so-called calibrated share form eases the calculation of free parameters of functional forms, because there is no need to invert demand functions (as opposed to the coefficient form approach). Equivalence of the coefficient form and the calibrated share form is straightforward. For example, let us consider CES production functions. Using the inverted factor demand functions in coefficient form

$$\alpha_i = \gamma^{(1-\sigma)/\sigma} \cdot \left(\frac{x_i}{y}\right)^{1/\sigma} \frac{w_i}{p},$$

we obtain:

$$\alpha_i = \gamma^{(1-\sigma)/\sigma} \cdot \theta_i \cdot \bar{x}_i^{-(1-\sigma)/\sigma} \quad .$$

Substituting this expression within the coefficient forms for production, cost and demand functions, we can derive the equivalent calibrated share forms:

- for the CES production function:

$$y = \gamma \left(\sum_i \alpha_i x_i^\rho \right)^{1/\rho} = \bar{y} \cdot \gamma \left(\sum_i \left(\gamma^{-\rho} \cdot \theta_i \cdot \bar{x}_i^{-\rho} \right) x_i^\rho \right)^{1/\rho} = \bar{y} \cdot \left[\sum_i \left(\theta_i \cdot \left(\frac{x_i}{\bar{x}_i} \right)^\rho \right) \right]^{1/\rho},$$

8

Table 1:   Functional forms in production

		Coefficient Form	**Calibrated Share Form**
Production function	Leontief	$y = \min_i \left(\dfrac{x_i}{\alpha_i} \right)$	$y = \overline{y} \cdot \min_i \left(\dfrac{x_i}{\overline{\overline{x_i}}} \right)$
	CD	$y = \gamma \prod_i x_i^{\alpha_i}$	$y = \overline{y} \cdot \prod_i \left(\dfrac{x_i}{\overline{\overline{x_i}}} \right)^{\theta_i}$
	CES	$y = \gamma \left(\sum_i \alpha_i x_i^{\rho} \right)^{1/\rho}$	$y = \overline{y} \cdot \left[\sum_i \left(\theta_i \cdot \left(\dfrac{x_i}{\overline{\overline{x_i}}} \right)^{\rho} \right) \right]^{1/\rho}$
Cost function	Leontief	$C = \left(\sum_i \alpha_i \cdot w_i \right) \cdot y$	$C = \overline{C} \cdot \left[\sum_i \theta_i \cdot \left(\dfrac{w_i}{\overline{w_i}} \right) \right] \cdot \dfrac{y}{\overline{\overline{y}}}$
	CD	$C = \dfrac{1}{\gamma} \prod_i \left(\dfrac{w_i}{\alpha_i} \right)^{\alpha_i} \cdot y$	$C = \overline{C} \cdot \left[\prod_i \left(\dfrac{w_i}{\overline{\overline{w_i}}} \right)^{\theta_i} \right] \cdot \dfrac{y}{\overline{\overline{y}}}$
	CES	$C = \gamma^{-1/\sigma} \left[\sum_i \alpha_i^{\sigma} \cdot \gamma^{(\sigma-1) \cdot \rho} \cdot w_i^{1-\sigma} \right]^{1/(1-\sigma)} \cdot y$	$C = \overline{C} \cdot \left[\sum_i \theta_i \cdot \left(\dfrac{w_i}{\overline{w_i}} \right)^{1-\sigma} \right]^{1/(1-\sigma)} \cdot \dfrac{y}{\overline{\overline{y}}}$
Demand function	Leontief	$x_i = \alpha_i \cdot y$	$x_i = \overline{x_i} \cdot \dfrac{y}{\overline{\overline{y}}}$
	CD	$x_i = \dfrac{\alpha_i p}{w_i} \cdot y$	$x_i = \overline{x_i} \cdot \dfrac{y}{\overline{\overline{y}}} \cdot \dfrac{c}{\overline{\overline{c}}} \cdot \dfrac{\overline{w_i}}{w_i}$
	CES	$x_i = \gamma^{\sigma-1} \cdot \left(\dfrac{\alpha_i p}{w_i} \right)^{\sigma} \cdot y$	$x_i = \overline{x_i} \cdot \dfrac{y}{\overline{\overline{y}}} \cdot \left(\dfrac{c}{\overline{\overline{c}}} \cdot \dfrac{\overline{w_i}}{w_i} \right)^{\sigma}$

Table 2:   Functional forms in consumption

		Coefficient Form	**Calibrated Share Form**
Utility function	Leontief	$U = \min_i \left[\dfrac{d_i}{\alpha_i} \right]$	$U = \overline{U} \cdot \min_i \left(\dfrac{d_i}{\overline{d_i}} \right)$
	CD	$U = \prod_i d_i^{\alpha_i}$	$U = \overline{U} \cdot \prod_i \left(\dfrac{d_i}{\overline{d_i}} \right)^{\theta_i}$
	CES	$U = \left(\sum_i \alpha_i^{1/\sigma} d_i^{\rho} \right)^{1/\rho}$	$U = \overline{U} \cdot \left[\sum_i \left(\theta_i \cdot \left(\dfrac{d_i}{\overline{d_i}} \right)^{\rho} \right) \right]^{1/\rho}$
Expenditure function	Leontief	$E = \left(\sum_i \alpha_i \cdot p_i \right) \cdot U$	$E = \overline{E} \cdot \left[\sum_i \theta_i \cdot \left(\dfrac{p_i}{\overline{p_i}} \right) \right] \cdot \dfrac{U}{\overline{U}}$
	CD	$E = \prod_i \left(\dfrac{p_i}{\alpha_i} \right)^{\alpha_i} \cdot U$	$E = \overline{E} \cdot \left[\prod_i \left(\dfrac{p_i}{\overline{p_i}} \right)^{\theta_i} \right] \cdot \dfrac{U}{\overline{U}}$
	CES	$E = \left[\sum_i \alpha_i \cdot p_i^{1-\sigma} \right]^{1/(1-\sigma)} \cdot U$	$E = \overline{E} \cdot \left[\sum_i \theta_i \cdot \left(\dfrac{p_i}{\overline{p_i}} \right)^{1-\sigma} \right]^{1/(1-\sigma)} \cdot \dfrac{U}{\overline{U}}$
Demand function	Leontief	$d_i = \alpha_i \dfrac{M}{\left(\sum_i \alpha_i p_i \right)}$	$d_i = \overline{d_i} \left(\dfrac{\sum_j \theta_j \overline{p_j}}{\sum_j \theta_j p_j} \right) \left(\dfrac{M}{\overline{M}} \right)$
	CD	$d_i = \dfrac{\alpha_i M}{p_i}$	$d_i = \overline{d_i} \dfrac{M}{\overline{M}} \dfrac{\overline{p_i}}{p_i}$
	CES	$d_i = \dfrac{\alpha_i M}{e} \left(\dfrac{e}{p_i} \right)^{\sigma}$	$d_i = \overline{d_i} \dfrac{M}{\overline{M}} \left(\dfrac{e}{\overline{e}} \right)^{\sigma-1} \left(\dfrac{\overline{p_i}}{p_i} \right)^{\sigma}$

- for the CES cost function:

$$C = \gamma^{-1/\sigma} \left[\sum_i \alpha_i^\sigma \gamma^{(\sigma-1)\cdot\rho} w_i^{1-\sigma} \right]^{1/(1-\sigma)} \cdot y = \gamma^{-1/\sigma} \left[\sum_i \left(\theta_i^\sigma \cdot \overline{x}_i^{-\sigma} \gamma^{1-\sigma} \right) \gamma^{(\sigma-1)\cdot\rho} w_i^{1-\sigma} \right]^{1/(1-\sigma)} \cdot \frac{y}{\overline{\overline{y}}}$$

$$= \left[\sum_i \theta_i \cdot \left(\overline{y} \cdot \overline{p} \right)^{1-\sigma} \cdot \left(\frac{w_i}{\overline{\overline{w}}_i} \right)^{1-\sigma} \right]^{1/(1-\sigma)} \cdot \frac{y}{\overline{\overline{y}}} = \overline{C} \cdot \left[\sum_i \theta_i \cdot \left(\frac{w_i}{\overline{\overline{w}}_i} \right)^{1-\sigma} \right]^{1/(1-\sigma)} \cdot \frac{y}{\overline{\overline{y}}}$$

and

- for the CES demand function:

$$x_i = \overline{C} \cdot \left[\sum_i \theta_i \cdot \left(\frac{w_i}{\overline{\overline{w}}_i} \right)^{1-\sigma} \right]^{\sigma/(1-\sigma)} \cdot \frac{\theta_i}{\overline{\overline{w}}_i} \cdot \left(\frac{w_i}{\overline{\overline{w}}_i} \right)^{-\sigma} \cdot \frac{y}{\overline{\overline{y}}} = C^\sigma \cdot \overline{C}^{1-\sigma} \cdot \frac{\overline{x}_i}{\overline{y}\cdot\overline{p}} \left(\frac{\overline{w}_i}{w_i} \right)^\sigma \cdot \left(\frac{\overline{\overline{y}}}{y} \right)^{1-\sigma}$$

$$= \overline{x}_i \cdot \left(\frac{y}{\overline{\overline{y}}} \right)^{1-\sigma} \cdot \left(\frac{C}{\overline{\overline{C}}} \cdot \frac{\overline{w}_i}{w_i} \right)^\sigma = \overline{x}_i \cdot \frac{y}{\overline{\overline{y}}} \cdot \left(\frac{p}{\overline{p}} \cdot \frac{\overline{w}_i}{w_i} \right)^\sigma$$

3.3 Numerical Analysis for the 2x2x1-Model

We want to investigate the implications of changes in the factor endowments for a stylized equilibrium model with two factors of production, two goods (or likewise production sectors), and one household (see e.g. Boadway and Wildasin 1984 for a comprehensive analytical treatment). This simple textbook 2x2x1-model features the basic structure for all higher dimensional, more complex models that are typically used in CGE analysis of policy interference.

For numerical analysis, we must first calibrate the model to a given reference situation (the benchmark equilibrium), i.e. the economic situation *before* the variation of the factor endowment. The calibration of functional forms requires benchmark data that comply to general equilibrium conditions. As to our simple competitive 2x2x1-Arrow-Debreu economy, this means that (i) supply equals demand of goods and factors, (ii) no sector earns a positive profit, and (iii) the representative household satisfies his budget constraint.

Table 3 provides stylized data for our benchmark equilibrium in terms of a social accounting matrix (King 1985). The upper section presents the benchmark prices, the lower section reports the benchmark quantities. In general, data consistency of a social accounting matrix requires that the sums of each of the rows and columns equal zero. Whereas market equilibrium conditions are associated with the rows, the columns capture the zero-profit condition for production sectors as well as the

11

income balance for the aggregate household sector.

Table 3:   Prices and quantities in the benchmark equilibrium

Prices			
$p_1 = p_2 = r = w = 1$			
Social Accounting Matrix			
	Y1	Y2	Household	$\sum$
Y1	40	–	-40	0
Y2	–	40	-40	0
$\overline{K}$	-20	-30	50	0
$\overline{L}$	-20	-10	30	0
$\sum$	0	0	0	

Benchmark data are typically delivered in value terms, i.e. they are products of prices and quantities. In order to obtain separate price and quantity observations, the common procedure is to choose units for goods and factors so that they have a price of unity (net of all taxes and subsidies) in the benchmark equilibrium. Then, the value terms simply correspond to the physical quantities.

Benchmark prices and quantities, together with exogenous (substitution) elasticities, determine the free parameters of the functional forms laid out in Tables 1 and 2. For our illustrative 2x2x1-model, we assume Cobb-Douglas functions in production and consumption for which the elasticity of substitution is equal to one by definition. We are the now able to write down the equilibrium conditions as listed in Table 4. For our simple example, one can easily do a „replication check" of the benchmark equilibrium conditions by inserting the initial prices and quantities.

Next, we investigate the implications of a variation in factor endowments. We assume that the initial labor endowment is increased by $\Delta L = 5$ from $\overline{L} = 30$ to $\overline{L} = 35$. In many cases, the transparent interpretation of general equilibrium effects is promoted by a decomposition into partial equilibrium effects. Here, we make use of a simple decomposition to explain the outcome of the factor variation. In a first step, we keep commodity prices constant and investigate how consumption and production decisions of firms change in response to the increased factor endowment. The partial equilibrium price assumption involves goods markets that are no longer in equilibrium. We can then derive the changes in (relative) prices of goods and factors that are necessary to assure market clearance. For constant good prices, the Rybczinski theorem (Rybczinski 1955) explains the adjustment on the production side

of the economy: "If relative commodity prices are constant and if both commodities continue to be produced, an increase in the supply of a factor will lead to an increase in the output of the commodity using that factor intensively and a decrease in the output of the other commodity" (Markusen et al. 1995, p. 119). In our concrete numerical example, production of good 1 is labor intensive and, thus, we can predict an increase in the production of good 1 for increased labor supply, whereas production of good 2 should decrease. In the partial equilibrium setting, we observe an excess demand on the market for good 2 and an excess supply on the market for good 1. For general equilibrium market clearance, relative prices p_1/p_2 must to reduce production of good 1 and increase production of good 2. The associated sectoral shift in factor use leads to an excess supply of labor and an excess demand of capital. Consequently, the factor price ratio w/r declines as compared to the benchmark equilibrium.

Table 4:   MCP formulation of the 2x2x1-economy in calibrated share form

Equilibrium conditions	Variables	Complementarity conditions
Zero profit	Activity variable	
$r^{0.5} w^{0.5} \geq p_1$	$y_1 \geq 0$	$\left(r^{0.5} w^{0.5} - p_1 \right) y_1 = 0$
$r^{0.75} w^{0.25} \geq p_2$	$y_2 \geq 0$	$\left(r^{0.75} w^{0.25} - p_2 \right) y_2 = 0$
Market clearance	Price variables	
$40\, y_1 \geq 40\dfrac{M}{80}\dfrac{1}{p_1}$	$p_1 \geq 0$	$\left(40\, y_1 - 40\dfrac{M}{80}\dfrac{1}{p_1} \right) p_1 = 0$
$40\, y_2 \geq 40\dfrac{M}{80}\dfrac{1}{p_2}$	$p_2 \geq 0$	$\left(40\, y_2 - 40\dfrac{M}{80}\dfrac{1}{p_2} \right) p_2 = 0$
$30 \geq 20\, y_1 \dfrac{p_1}{w} + 10\, y_2 \dfrac{p_2}{w}$	$w \geq 0$	$\left(30 - \left(20\, y_1 \dfrac{p_1}{w} + 10\, y_2 \dfrac{p_2}{w} \right) \right) w = 0$
$50 \geq 20\, y_1 \dfrac{p_1}{r} + 30\, y_2 \dfrac{p_2}{r}$	$r \geq 0$	$\left(50 - \left(20\, y_1 \dfrac{p_1}{r} + 30\, y_2 \dfrac{p_2}{r} \right) \right) r = 0$
Budget constraint	Income variable	
$30\, w + 50\, r \geq M$	$M \geq 0$	$\left((30\, w + 50\, r) - M \right) M = 0$

Table 5 summarizes the concrete numerical results for the benchmark equilibrium (column: Benchmark), the partial equilibrium calculation with increased labor endowment holding goods prices constant at the benchmark level (column: RYBCZINSKI), and the general equilibrium *counterfactual* (column: Counterfactual) with flexible prices. In fact, the numerical results confirm the basic theoretical reasoning.

Table 5:   Effects of increased labor endowment ($\Delta L = 5$)

Variable	Benchmark	RYBCZINSKI	Counterfactual
Prices			
w	1.0	1.0	1.0
r	1.0	1.0	1.17
p_1	1.0	1.0	1.08
p_2	1.0	1.0	1.12
Quantities			
Y_1	40.0	55.5	43.20
X_1	40.0	42.5	43.20
Y_2	40.0	30.0	41.57
X_2	40.0	42.5	41.57
K_1	20.0	27.5	20.00
K_2	30.0	22.5	30.00
$\overline{K}$	50.0	50.0	50.00
L_1	20.0	27.5	23.33
L_2	10.0	7.5	11.67
$\overline{L}$	30.0	35.0	35.00
M	80.0	85.0	93.33

For more complex higher-dimensional settings, however, there is in many cases no clear theoretical explanation. Without numerical simulations, it would not be possible to obtain (possibly rather general) insights. On the other hand, numerical results must always be interpreted on the basis of sound theory. In this context, the reader should be aware of some fundamental restrictions with respect to numerical policy analysis. One never knows if the concrete results are correct. It must be assumed, of course, that the numerical solution algorithm works properly, but a wrong model - i.e. wrongly specified with respect to the underlying economics - may have been solved. A small programming error that can easily creep in is enough to cause such a dilemma. One may relax this problem, however, in the following way: as an initial consistency check, those policy measures should be simulated first, where the qualitative results - based on rigorous theoretical analysis - are already known (see e.g. RYBCZINSKI scenario). If the simulation model produces different results, one must assume that it is misspecified. Otherwise, one can proceed with the analysis of policy measures, where results are open in theory. Yet, one can still not be sure that the numerical solutions are "correct". Therefore, it is always important that numerical results are convincingly explained in economic terms.

4   USER INTERFACE

Despite the progress on the fields of commercial solution methods and higher-level programming language, CGE analysis still requires substantial programming skills to perform adequate quantitative

economic research. Thus, economists who are not willing to spend (sufficient) time on programming techniques may still reject CGE analysis as a black box. More recently, however, several software packages have been developed that allow modelers to set up visual interfaces quite easily in order to communicate results to non-modelers. In our view, such interfaces can greatly enhance the accessibility of CGE analysis since it permits persons who are unfamiliar with programming to perform scenario analysis, produce tables and figures, and transfer model results into spreadsheets and reports.

For demonstration purposes, we make use of the public domain GAMSsm tool (Rutherford 2001), which provides an intuitive and simple Windows-based interface to economic models (or other mathematical programs) implemented with GAMS.[8] The reader can download the tailor-made interface from ftp://ftp.zew.de/pub/zew-docs/div/M2x2x1.exe to reproduce results and study additional policy scenarios for the basic 2x2x1-economy and its variants (described in section 6). The execution of *M2x2x1.exe* installs these models in a directory of choice on the computer. When starting GAMSsm for the first time (from Windows/Start/Programs/... or via a Windows-Icon), a scenario-file (*.scn) must be loaded via the menu item FILE → OPEN (here "M2x2x1.scn").

Figure 2:   Loading of scenario file
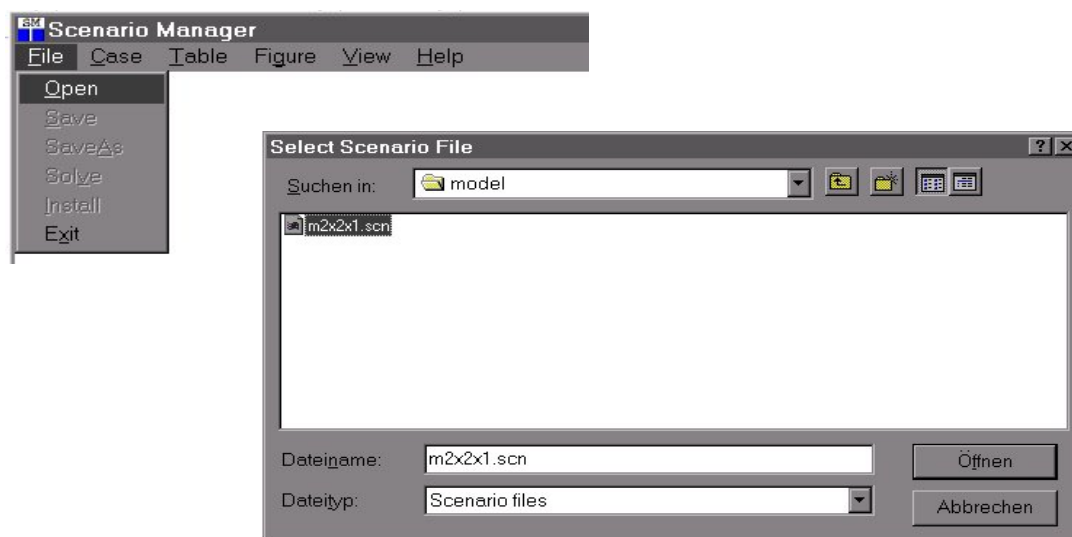
Upon invocation of the *.scn*-file, the input and output parameters that have been pre-specifed by the modeler appear on the screen. In our case, the input options include the assignment of initial factor endowments to labour and capital as well as the choice between the partial equilibrium setting (for

---

performing the Rybczinski calculation) and the general equilibrium setting (to obtain the final market equilibrium); the output is condensed in an aggregate summary report of prices and quantities.[9]

Figure 3:   Definitions of input and output parameters
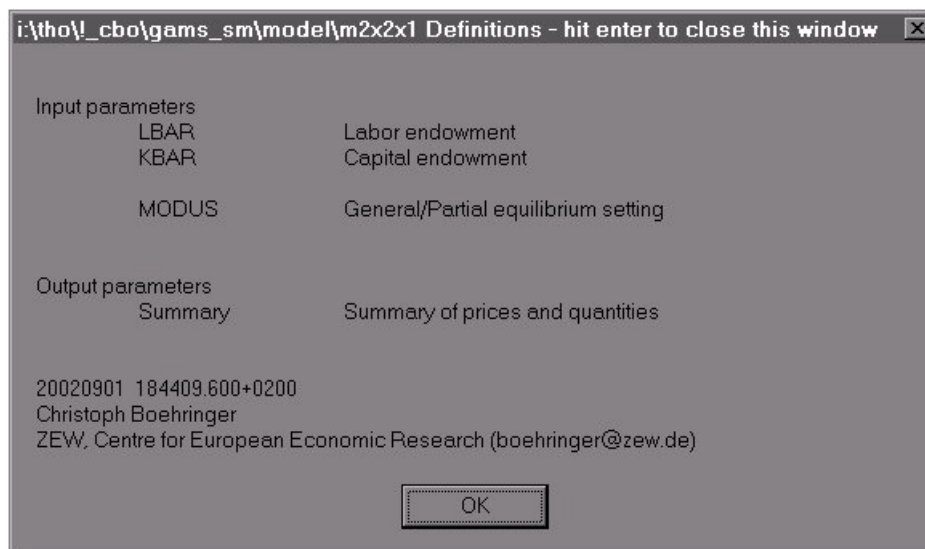
A mouse-click on „OK" opens the scenario-file showing an index of previously solved cases. In addition to the case identifiers, the „Case Index" table reports the respective scenario values of the input variables and the solution status of the case („Solved", „Unsolved", or „Approx."). In our application, the „Benchmark" scenario, which has already been pre-calculated to assure the replication check, is listed with its input specification. [10]

Figure 4:   Case Index with pre-solved scenarios

---

[9] The user can recall the model definitions of input and output parameters via VIEW → DEFINITIONS at any time.
[10] A "default" scenario which reflects the modeler's default parameterization will always be generated.

Via CASE → NEW, new scenarios can be defined. This requires a case identifier and the selection of case-specific input values. For concreteness, if we want to add the case "RYBCZINSKI", we may first adopt the initial input parameterization from the "Benchmark" scenario.

Figure 5: Definition of new case (scenario)
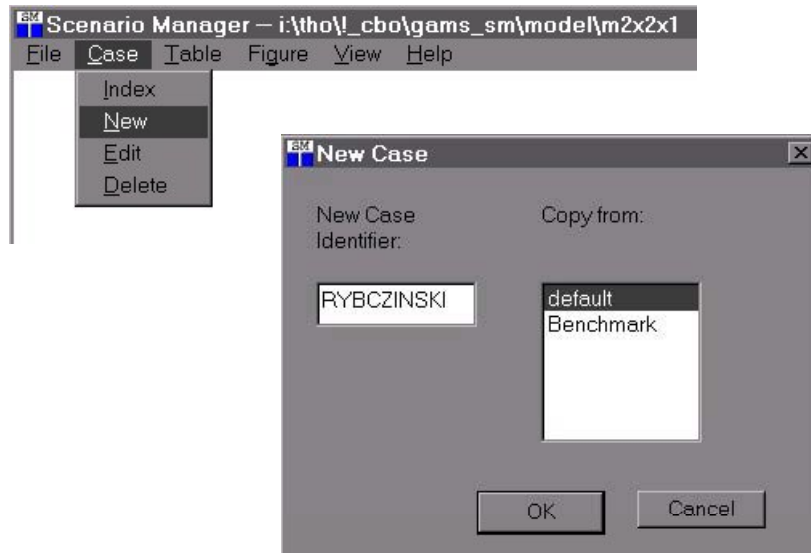
After confirming with „OK", we can then select the input parameters to be changed. For the "RYBCZINSKI" case, we set the initial labor endowment (LBAR) to 35 and choose the partial equilibrium setting with fixed output prices (MODUS „PEM").

Figure 6: Specification of input options
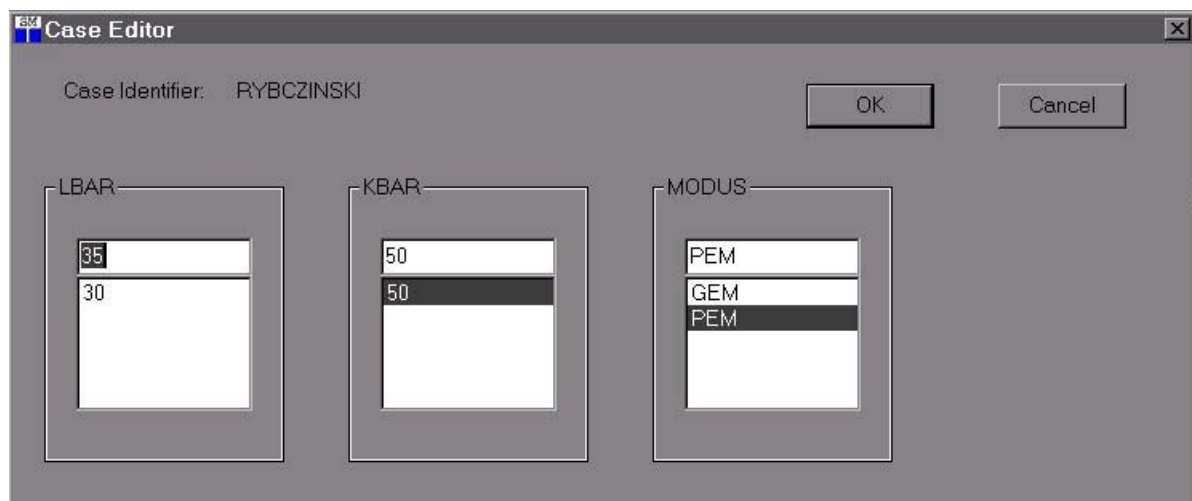
A mouse-click on "OK" delivers the newly created case as entry in the CASE Index list.

Figure 7:   Case Index (1)

Case Index

	LBAR	KBAR	MODUS	Status
Benchmark	30	50	GEM	Solved
RYBCZINSKI	35	50	PEM	Solved

Whenever numerical input parameters of new cases lie between values of formerly specified cases, GAMSsm will first provide an approximation (linear interpolation). Later on, the case can be solved via FILE → SOLVE with recourse to the associated GAMS program (here: "M2x2x1.gms"). The *real* solve requires the pre-installation of the GAMS software.

We proceed in a similar fashion with the specification of our next case „Counterfactual", which provides the full equilibrium solution (MODUS: GEM) given the increase in labor endowment.

Figure 8:   Case Index (2)

Case Index

|	LBAR	KBAR	MODUS	Status
Benchmark	30	50	GEM	Solved
RYBCZINSKI	35	50	PEM	Solved
Counterfactual	35	50	GEM	Solved

Scenario results can be compared in tabular form through the TABLE menu under GAMSsm. A new table of results must be defined through TABLE → NEW. Here, we assign "Summary" as the table name. Since we want to compare a single report item across multiple cases, we have must "Single Item – Multiple Cases".

Figure 9:   Pull-down menu for table assignment
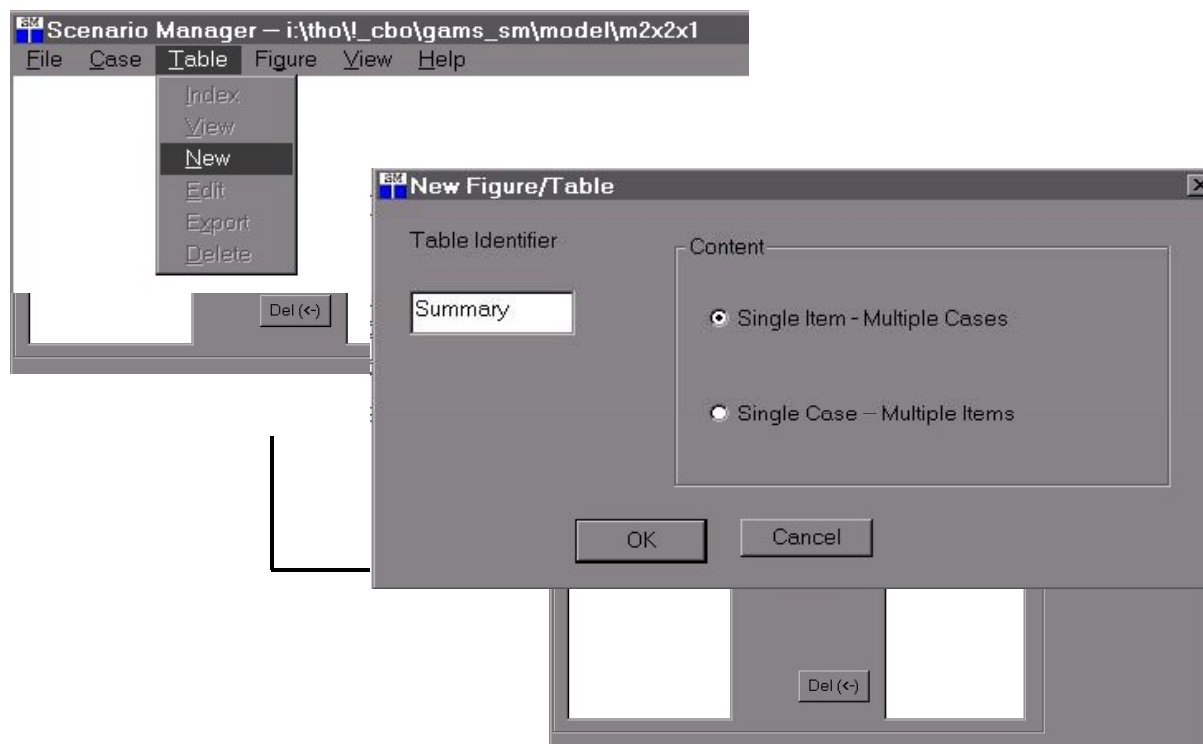
Subsequently, we must select the report item. In our example, all results are condensed in a single report item ("report").

Figure 10:   Selection of Report Item
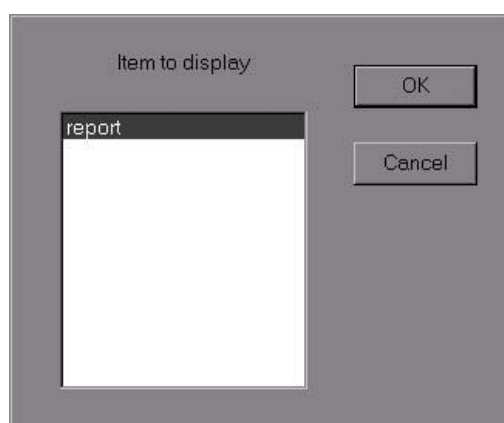
Finally, we must choose the cases across which the comparison should take place. By marking the cases of interest in the left-hand side list box and clicking on "Add", the marked cases will be transferred to the right-hand side list box and is now included in the comparison.

A mouse-click on "OK" creates the desired table of results. Note that the procedure for graphical comparison via menu item FIGURE is the same.

Figure 11:   Table of Results


```
Scenario Manager — i:\tho\l_cbo\gams_sm\model\m2x2x1            _ □ ×
File  Case  Table  Figure  View  Help

  Table: Summary( report)

                      Benchmark        RYBCZINSKI      Counterfactual
    w                      1.00              1.00              1.00
    r                      1.00              1.00              1.17
    p1                     1.00              1.00              1.08
    p2                     1.00              1.00              1.12
    Y1                    40.00             55.00             43.20
    X1                    40.00             42.50             43.20
    Y2                    40.00             30.00             41.57
    X2                    40.00             42.50             41.57
    K1                    20.00             27.50             20.00
    K2                    30.00             22.50             30.00
    KBAR                  50.00             50.00             50.00
    L1                    20.00             27.50             23.33
    L2                    10.00              7.50             11.67
    LBAR                  30.00             35.00             35.00
```

Existing tables can be modified through TABLE → EDIT to include additional report items of scenarios.

In our view, such an intuitive and simple Windows-based interface can substantially promote the use of CGE analysis in applied policy research. Users do not need to learn programming and are still able to share insights with modelers in an active way: They can test the model by specifying their own scenarios and checking the results against their economic intuition. Thus, non-modelers are no longer confronted with a black box. Moreover, the modeler himself may benefit from the interface as a convenient tool for efficient scenario management.

5   EXTENSIONS OF THE 2X2X1-MODEL

A central application of CGE models is the analysis of tax policies. The classical (welfare-)analysis of taxes within the 2x2x1-model builds on Harberger (see e.g.. Harberger 1962) and forms part of most graduate-level public finance textbooks (e.g. Boadway and Wildasin 1984). Here, we are not concerned with a detailed public finance analysis. Our primary objective is to demonstrate the advantages of the MCP implementation of equilibrium problems, i.e. activity analysis as well as the incorporation of price or quantity constraint by means of (weak) inequalities. The subsequent stylized

examples can be readily reproduced by non-modelers using the GAMSsm interface.[11]

5.1 Activity analysis: labor taxes and black market

Tax-induced crowding-out of economic activities from the *formal* sector into the *informal* black market sector constitutes an important issue in the economic policy debate. We can illustrate the basic mechanism along a simple activity analysis extension of our basic 2x2x1-model. In the benchmark, we may assume existence of an informal sector Y3 producing the same good as sector Y2.[12] However, Y3 does not break even in the benchmark and, therefore, is initially slack (inactive). Since observable market data is missing in most cases, one must revert to expert estimates or bottom-up engineering data to calibrate slack (i.e. inactive)activities. Here, we simply assume that Y3 requires λ times ($\lambda > 1$) the factor inputs to produce the same output as Y2. Table 6 captures the reference situation.

Table 6:   Initial slack activity

Prices: $p_1 = p_2 = r = w = 1$			Technology parameter for *slack*-activity Y3: $\lambda > 1$		
	Y1	Y2	Household	$\sum$	Y3
Y1	40	—	-40	0	-
Y2	—	40	-40	0	40
$\overline{K}$	-20	-30	50	0	-30 (1+λ)
$\overline{L}$	-20	-10	30	0	-10 (1+λ)
$\sum$	0	0	0	0	-40 (λ-1)

The policy maker now imposes a fiscal tax on factor use in the production of good Y2. In our simple example without an explicit public sector, tax revenues are recycled lump-sum to the representative household. The factor input tax is only applied to the formal sector, leading to a full „crowding-out" of Y2 production if the tax rate is sufficiently high. As a consequence, tax revenues drop to zero. Moreover, the tax policy induces substantial welfare losses because it forces the inefficient technology Y3 into solution.

Table 7 summarizes the algebraic formulation of the extended model. Besides the explicit zero-profit condition for the production activity Y3, and the inclusion of additional goods supply in the market clearance condition for good 2, the factor tax ty_2 is incorporated into the zero-profit condition for sector Y2, the associated factor demand functions, and the income constraint of the household.

---

[11] The respective reporting features include changes in economic variables such as tax revenues, unemployment rate, subsidy rate, and overall economic welfare.
[12] Prominent examples may include smuggling activities or illegal employment in the building industry.

Table 7: Algebraic formulation of 2x2x1-model with taxes and black market activity

Equilibrium conditions	Variables	Complementarity conditions
Zero profit	**Activity variables**	
$r^{0.5}\,w^{0.5} \geq p_1$	$y_1 \geq 0$	$\left(r^{0.5}\,w^{0.5} - p_1 \right) y_1 = 0$
$\left(r(1+ty_2) \right)^{0.75} \left(w(1+ty_2) \right)^{0.25} \geq p_2$	$y_2 \geq 0$	$\left(\left(r(1+ty_2) \right)^{0.75} \left(w(1+ty_2) \right)^{0.25} - p_2 \right) y_2 = 0$
$\lambda\, r^{0.75}\,w^{0.25} \geq p_2$	$y_3 \geq 0$	$\left(\lambda r^{0.75}\,w^{0.25} - p_2 \right) y_3 = 0$
Market clearance	**Price variables**	
$40\,y_1 \geq 40\dfrac{M}{80}\dfrac{1}{p_1}$	$p_1 \geq 0$	$\left(40\,y_1 - 40\dfrac{M}{80}\dfrac{1}{p_1} \right) p_1 = 0$
$y_3 + 40\,y_2 \geq 40\dfrac{M}{80}\dfrac{1}{p_2}$	$p_2 \geq 0$	$\left(y_3 + 40\,y_2 - 40\dfrac{M}{80}\dfrac{1}{p_2} \right) p_2 = 0$
$30 \geq 20\,y_1\dfrac{p_1}{w} + 10\,y_2\dfrac{p_2}{w(1+ty_2)} + \lambda\,0.1\,y_3\dfrac{p_2}{w}$	$w \geq 0$	$\left(30 - \left(20\,y_1\dfrac{p_1}{w} + 10\,y_2\dfrac{p_2}{w(1+ty_2)} + \lambda\,0.1\,y_3\dfrac{p_2}{w} \right) \right) w = 0$
$50 \geq 20\,y_1\dfrac{p_1}{r} + 30\,y_2\dfrac{p_2}{r(1+ty_2)} + \lambda\,0.3\,y_3\dfrac{p_2}{r}$	$r \geq 0$	$\left(50 - \left(20\,y_1\dfrac{p_1}{r} + 30\,y_2\dfrac{p_2}{r(1+ty_2)} + \lambda\,0.3\,y_3\dfrac{p_2}{r} \right) \right) r = 0$
Budget constraint	**Income variable**	
$30\,w + 50\,r + ty_2\,w10\,y_2\dfrac{p_2}{w(1+ty_2)}$ $+ ty_2 r\,30\,y_2\dfrac{p_2}{r(1+ty_2)} \geq M$	$M \geq 0$	$\big(\ 30\,w + 50\,r + ty_2\,w10\,y_2\dfrac{p_2}{w(1+ty_2)}$ $+ ty_2 r\,30\,y_2\dfrac{p_2}{r(1+ty_2)} - M\big)\,M = 0$

Figure 12 indicates tax revenues for $\lambda = 1.1$ as a function of the ad-valorem factor tax ranging from ty_2=0% (T0) to ty_2=50% (T50). We distinguish between two scenarios: one in which no black market activity is present (case: *No_Slack*), and one which includes the option of black market production of good Y2 (case: *Slack*).

The reader can reproduce our results using GAMSsm and perform additional sensitivity analysis on the choice of λ.[13]

---

[13] For this, the reader must load the scenario file "M2x2x1_act.scn" via FILE → OPEN.

Figure 12: Tax revenues (Laffer curve) and black market activities
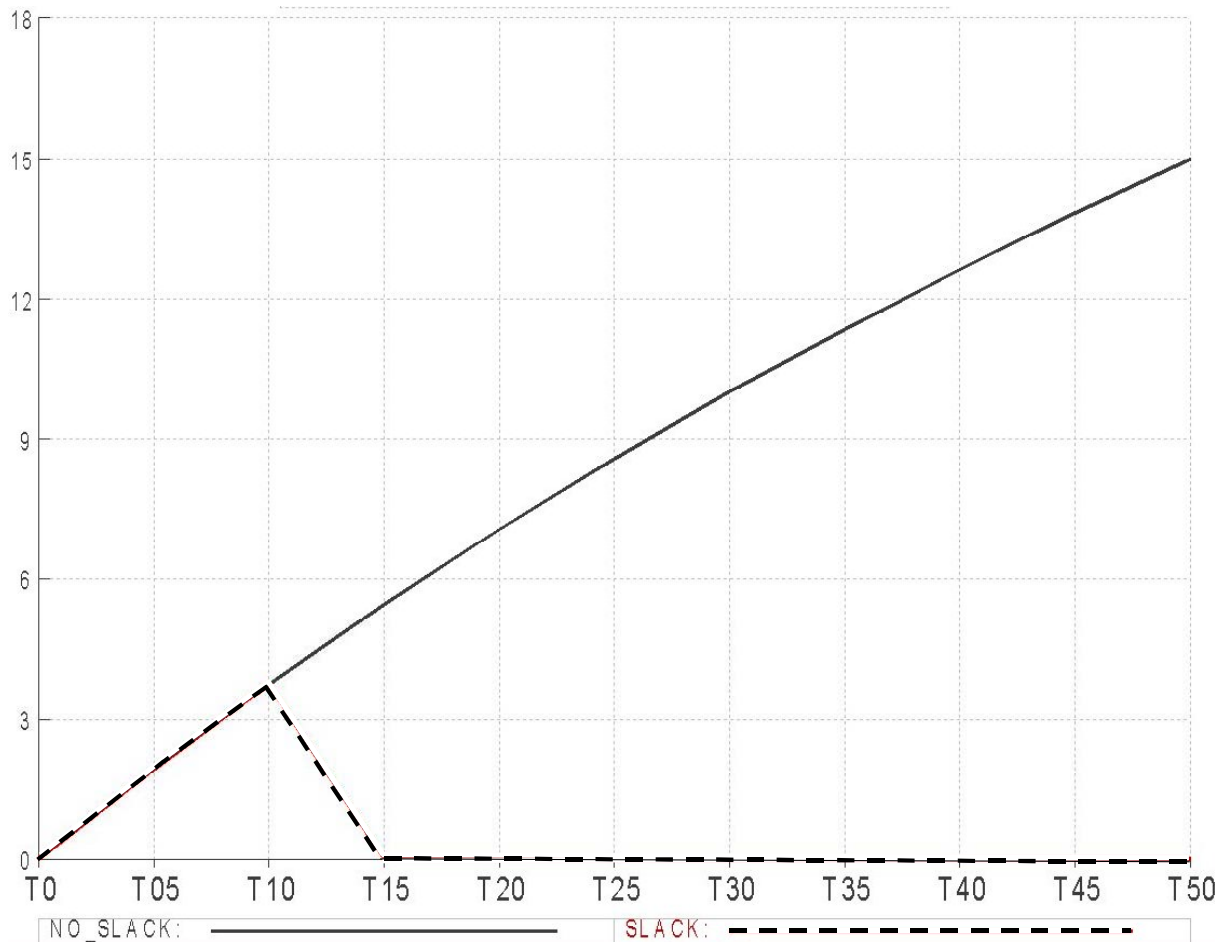
5.2 Price restrictions: minimum wages

In the labor market debate, high taxes on labor inputs, together with real wage rigidities, are often held responsible for high unemployment rates. Figure 13 illustrates the basic reasoning for the case of inelastic labor supply. In the benchmark situation without labor taxes, the market clearing real wage $(w/p)^0$ is above some potential fixed minimum wage $\bar{w}$. As a first-round effect, a labor tax makes the use of this factor more expensive, which reduces labor demand. This leads to an excess supply of labor which - in turn - forces the real wage to decline until the labor market will be cleared. For inelastic labor supply, the labor tax will then be fully borne by the real wage $(w/p)^1$. After taxes all price ratios will remain the same as in the benchmark. Without a (binding) minimum wage constraint, the labor tax simply works as a lump-sum tax and does not affect the initial allocation of resources.

Whenever the tax can not fully be shifted to the real wage because of a binding minimum wage restriction $\bar{w}$, the remaining part of the tax wedge falls on the producer wage. Labor becomes more expensive, requiring an increase in its marginal productivity. Compared to the benchmark equilibrium,

less labor will be demanded. With the wage restriction binding, labor supply will be rationed to warrant market clearance. As depicted in Figure 13, the rationed market equilibrium implies the emergence of involuntary unemployment.

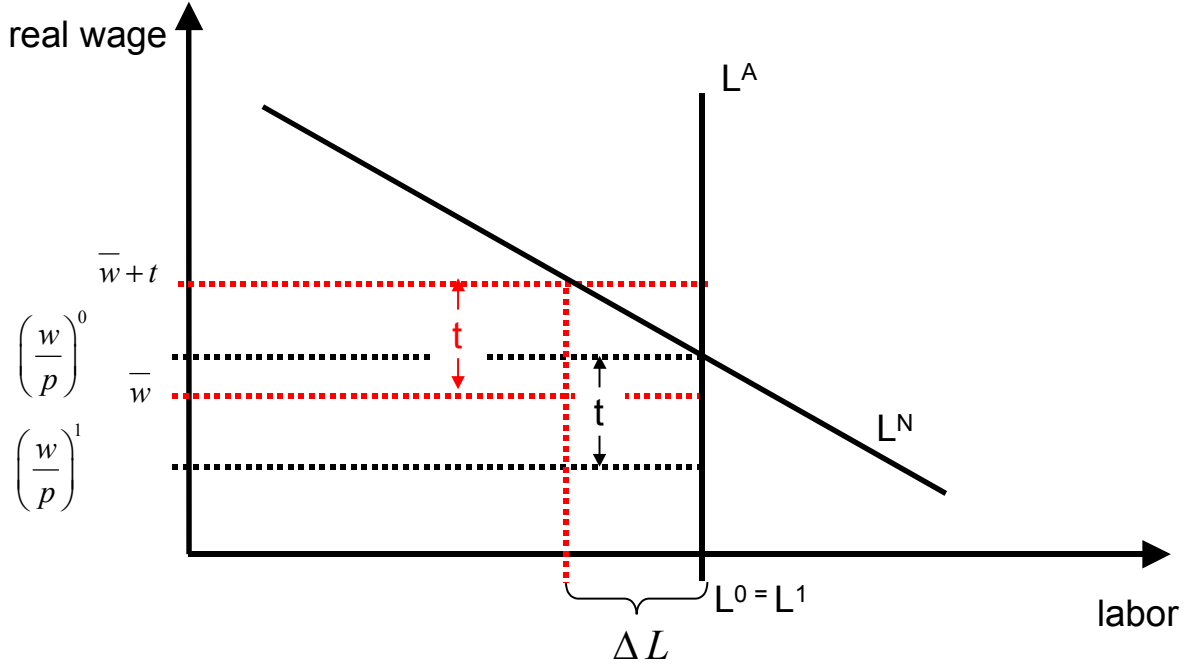Figure 13:   Unemployment due to minimum wages

Table 8 provides the algebraic formulation for the basic 2x2x1-model, extended by a minimum wage constraint. Complementary to the constraint is the variable μ that scales labor supply to effect the (rationed) labor market equilibrium. Furthermore, the tax *tl* on labor must be entered into the zero-profit conditions, the market clearance conditions and the income constraint.

Figure 14 illustrates the implication of labor taxes (ranging from 0% (T0) to 50% (T50)) for the unemployment rate μ (Y-axis), assuming that the minimum real wage is set at 95% of the benchmark equilibrium wage ($\overline{w}=0.95$). The reader can reproduce the results and do sensitivity analysis by loading the scenario file "M2x2x1_ur.scn" under GAMSsm. [14]

---

[14] Again, in order to solve additional scenarios via FILE → SOLVE, the respective GAMS model file "M2x2x1_ur.gms" must be activated.

Table 8:   Algebraic formulation of the 2x2x1-model with a minimum wage constraint and taxes

Equilibrium conditions	Variables	Complementarity conditions
Zero profit	**Activity variables**	
$r^{0.5}\left(w(1+tl)\right)^{0.5} \geq p_1$	$y_1 \geq 0$	$\left(r^{0.5}\left(w(1+tl)\right)^{0.5} - p_1\right)y_1 = 0$
$r^{0.75}\left(w(1+tl)\right)^{0.25} \geq p_2$	$y_2 \geq 0$	$\left(r^{0.75}\left(w(1+tl)\right)^{0.25} - p_2\right)y_2 = 0$
Market clearance	**Price variables**	
$40\,y_1 \geq 40\dfrac{M}{80}\dfrac{1}{p_1}$	$p_1 \geq 0$	$\left(40\,y_1 - 40\dfrac{M}{80}\dfrac{1}{p_1}\right)p_1 = 0$
$40\,y_2 \geq 40\dfrac{M}{80}\dfrac{1}{p_2}$	$p_2 \geq 0$	$\left(40\,y_2 - 40\dfrac{M}{80}\dfrac{1}{p_2}\right)p_2 = 0$
$30(1-\mu) \geq 20\,y_1\dfrac{p_1}{w(1+tl)} + 10\,y_2\dfrac{p_2}{w(1+tl)}$	$w \geq 0$	$\left(30 - \left(20\,y_1\dfrac{p_1}{w(1+tl)} + 10\,y_2\dfrac{p_2}{w(1+tl)}\right)\right)w = 0$
$50 \geq 20\,y_1\dfrac{p_1}{r} + 30\,y_2\dfrac{p_2}{r}$	$r \geq 0$	$\left(50 - \left(20\,y_1\dfrac{p_1}{r} + 30\,y_2\dfrac{p_2}{r}\right)\right)r = 0$
Budget restriction	**Income variable**	
$30\,w + 50\,r + \left(tl\,w\,30(1-\mu)\right) \geq M$	$M \geq 0$	$\left(\left(30\,w + 50\,r + \left(tl\,w\,30\right)\right) - M\right)M = 0$
Wage restriction	**Rationing variable**	
$\dfrac{w}{p_1^{0.5}p_2^{0.5}} \geq \overline{w}$	$\mu \geq 0$	$\left(\dfrac{w}{p_1^{0.5}p_2^{0.5}} - \overline{w}\right)\mu = 0$

Figure 14:   Unemployment due to minimum wage constraint
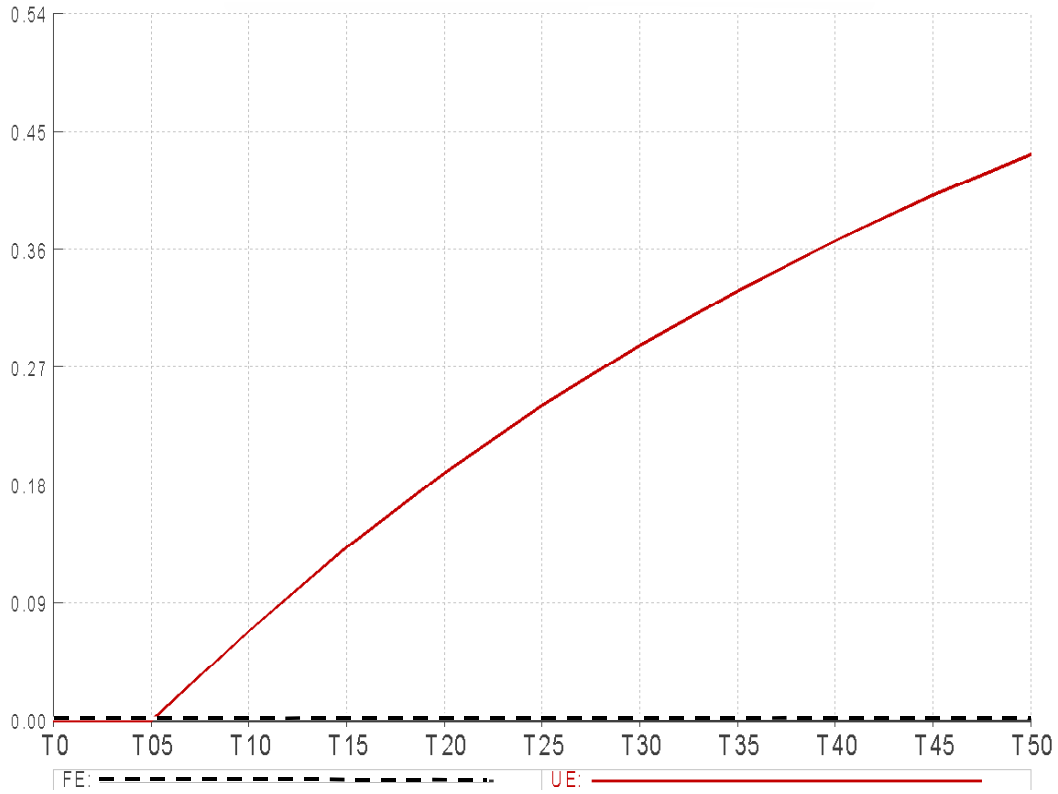
5.3   Quantity restrictions: sectoral employment targets

In the previous section, we illustrated the MCP implementation of price constraints along the example of minimum wages. In more general terms, the price constraint requires a rationing variable to ensure the (rationed) equilibrium as soon as the price constraint becomes binding. In the same manner, we can specify explicit quantity restrictions. For example, we may assume that sectoral employment policies aim at increasing the benchmark employment level in sector Y1 of our 2x2x1-model by λ %. In order to meet this quantity constraint, a complementary endogenous subsidy (tax) variable is necessary that renders labor use in sector Y1 sufficiently cheap. In the algebraic formulation, the specification of the minimum employment level in sector Y1 is linked to a partial wage subsidy on labor use in this sector. Compared to alternative policy instruments – such as an output subsidy on Y1 - the partial wage subsidy promotes the objective of sectoral production in the most direct way and is therefore most efficient in achieving a specific employment target.
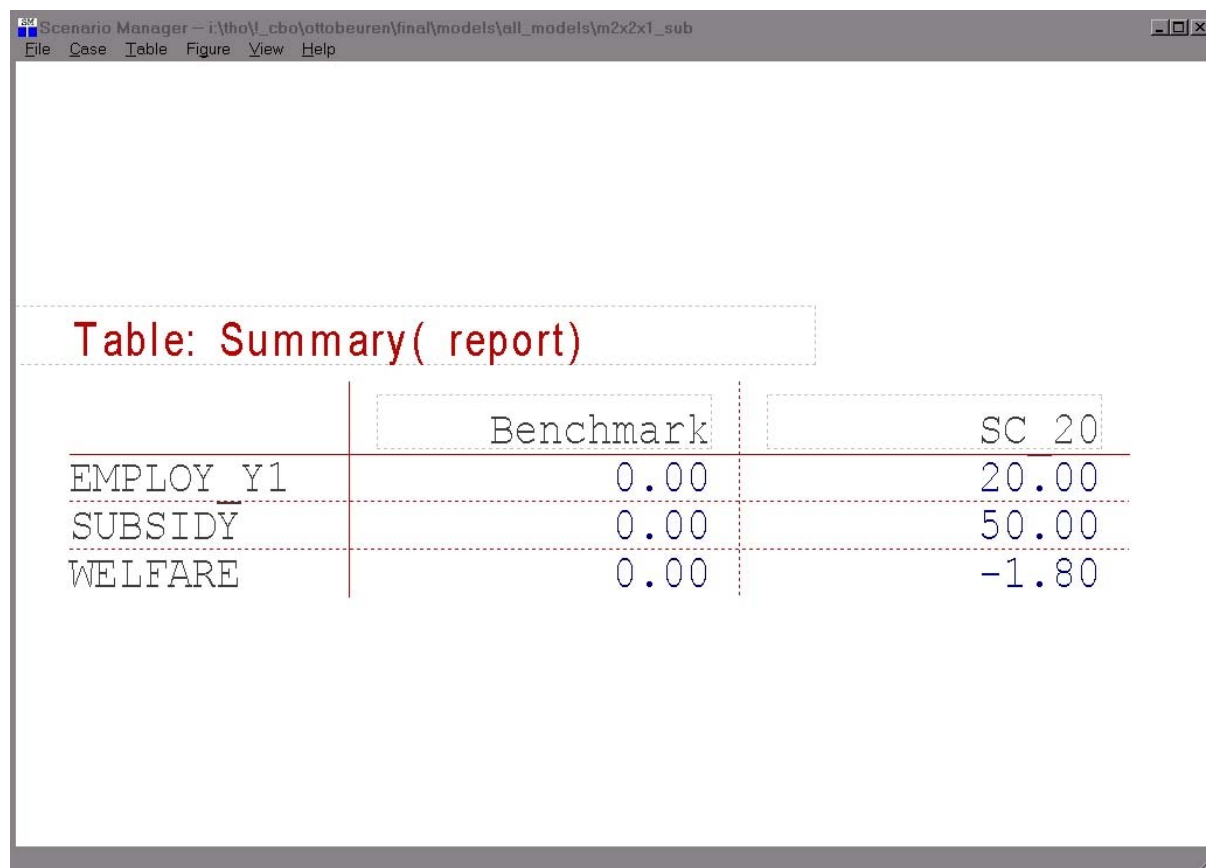
Table 9 provides the algebraic summary of the basic 2x2x1-model extension for the quantity restriction. Complementary to the quantity constraint is the endogenous subsidy variable τ, which reduces the user costs of labor in the sectoral production of good Y1. The variable τ enters the zero-profit condition for sector Y1, the sectoral labor demand function, and the budget constraint of the representative household, which must finance the subsidy policy.

Table 9:   Algebraic formulation of the model with sectoral employment targets and wage subsidies

Equilibrium conditions	Variables	Complementary conditions
Zero profit	**Activity variables**	
$r^{0.5}\left(w(1-\tau)\right)^{0.5} \geq p_1$	$y_1 \geq 0$	$\left(r^{0.5}\left(w(1-\tau)\right)^{0.5} - p_1\right)y_1 = 0$
$r^{0.75}\,w^{0.25} \geq p_2$	$y_2 \geq 0$	$\left(r^{0.75}\,w^{0.25} - p_2\right)y_2 = 0$
Market clearance	**Price variables**	
$40\,y_1 \geq 40\dfrac{M}{80}\dfrac{1}{p_1}$	$p_1 \geq 0$	$\left(40\,y_1 - 40\dfrac{M}{80}\dfrac{1}{p_1}\right)p_1 = 0$
$40\,y_2 \geq 40\dfrac{M}{80}\dfrac{1}{p_2}$	$p_2 \geq 0$	$\left(40\,y_2 - 40\dfrac{M}{80}\dfrac{1}{p_2}\right)p_2 = 0$
$30 \geq 20\,y_1\dfrac{p_1}{w(1-\tau)} + 10\,y_2\dfrac{p_2}{w}$	$w \geq 0$	$\left(30 - \left(20\,y_1\dfrac{p_1}{w(1-\tau)} + 10\,y_2\dfrac{p_2}{w}\right)\right)w = 0$
$50 \geq 20\,y_1\dfrac{p_1}{r} + 30\,y_2\dfrac{p_2}{r}$	$r \geq 0$	$\left(50 - \left(20\,y_1\dfrac{p_1}{r} + 30\,y_2\dfrac{p_2}{r}\right)\right)r = 0$
Budget constraint	**Income variable**	
$30w+50r - \left(20\,y_1\dfrac{p_1}{w(1-\tau)}\right)\tau w \geq M$	$M \geq 0$	$\left(\left(30w+50r - \left(20\,y_1\dfrac{p_1}{w(1-\tau)}\right)\tau w\right) - M\right)M = 0$
Quantity constraint	**Subsidy variable**	
$\left(20\,y_1\dfrac{p_1}{w(1-\tau)}\right) \geq (1+\lambda)\,20$	$\tau \geq 0$	$\left(\left(20\,y_1\dfrac{p_1}{w(1-\tau)}\right) - (1+\lambda)\,20\right)\tau = 0$

Figure 15 displays the sectoral (ad-valorem) wage subsidies and the welfare costs associated with an increase in sectoral production of good Y1 by λ=20% vis-à-vis the benchmark equilibrium. The reader can access the simulation within the scenario "M2x2x1_sub.scn" under GAMSsm.

Figure 15:   Sectoral employment targets

# 6   LARGE-SCALE APPLICATION: THE ECONOMIC COSTS OF KYOTO

Finally, we want to demonstrate the use of CGE models for *real-world* policy analysis. Our application refers to the Kyoto Protocol, an international environmental agreement that requires industrialized countries (as listed in its Annex B) to limit their emissions of greenhouse gases, most notably CO_2 from fossil fuel combustion. The reduction commitments amount to roughly 5 % on average as compared to the 1990 emission levels and must be achieved during the commitment period from 2008 to 2012. Here, we use a multi-sector, multi-region extension of our 2x2x1-model to investigate the economic costs of emission reduction under the Kyoto Protocol. The numerical simulations refer to a situation in which industrialized countries apply domestic carbon taxes to meet their Kyoto emission targets.

To provide a transparent interpretation of general equilibrium effects triggered by carbon constraints, we apply a decomposition method that resembles the procedure used in section 3.3 for explaining the effects of a factor variation. The effects of (climate) policy intervention in large open economies are broken down into a *domestic market effect*, assuming that international prices remain constant, and an *international market effect*, which results from the changes in international prices. The key idea with

respect to applied model analysis is that each region of a multi-region trade model (MRT) can be represented as a small open economy (SOE) in order to separate the domestic policy effect under fixed terms of trade. Policy-induced changes in international prices from the multi-region model can then be imposed parametrically on the small open economy to measure the international market effect commodity by commodity (Böhringer and Rutherford 2002).

Figure 16 illustrates the steps involved in the decomposition procedure.

Figure 16:   The MRT-SOE decomposition
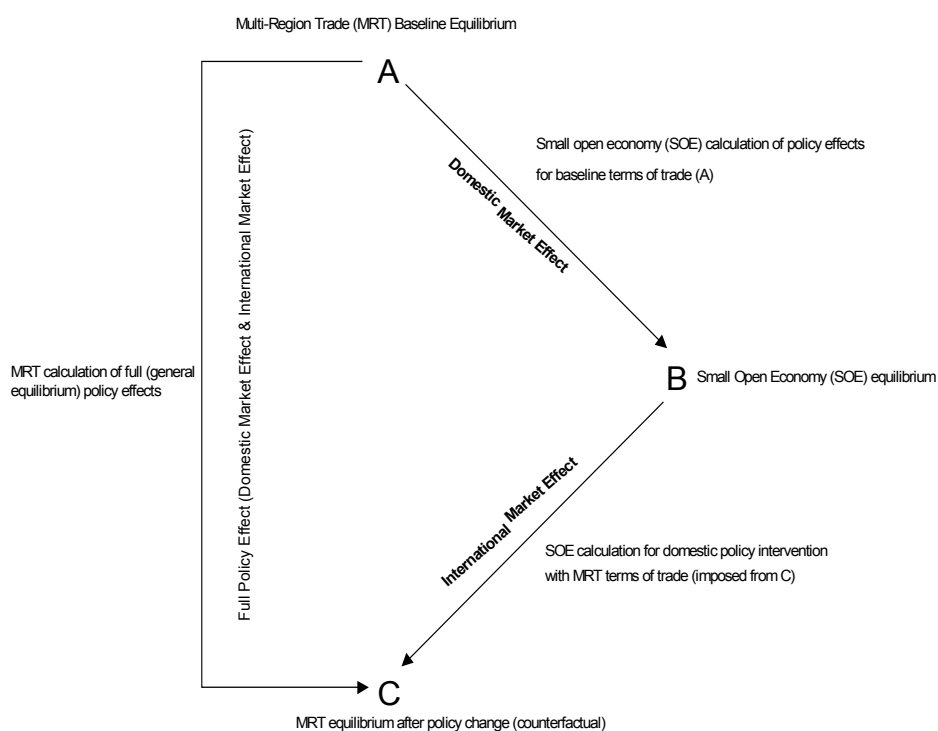
Computation of the domestic market effect simply requires the maintenance of international prices at the benchmark (reference) level and the imposition of the domestic policy change on the specific country. Hence, for the intermediate SOE equilibrium calculation (A→B), changes on the domestic market have no effect on international prices. The spillover effect for any economic or environmental

activity of a specific country is then simply the residual between the SOE equilibrium solution at benchmark terms-of-trade and the full MRT solution for the specific country (C).[15]

Table 10 summarizes results for the decomposition of economic effects induced by carbon abatement policies of industrialized countries under the Kyoto Protocol. The column *Total Policy Effect* reports aggregate consumption changes in % vis-à-vis the business-as-usual (BaU) that emerge from carbon taxes of industrialized countries to comply with their Kyoto emission reduction targets in 2010.

Table 10:   Decomposition of total consumption changes for carbon tax case (*NTR*)

	Domestic Market Effect (in % of BaU consumption)	Fossil Fuel Market Effect[**] (in % of BaU consumption)	Total Policy Effect (in % of BaU consumption)	International Spillovers[*] (in % of Total Policy Effect)	Carbon Tax (in USD$_{95}$ / ton C)	Emission Reduction (in % vs. BaU)
CAN	-0.69	-0.86	-0.88	21	230	28
CEA	0.00	0.26	0.29	100	1	-6
EUR	-0.14	-0.06	-0.06	- 116	107	14
FSU	0.00	-0.43	-1.03	100	-	-48
JPN	-0.44	-0.38	-0.30	-47	300	26
OOE	-0.13	-0.47	-0.65	81	76	16
USA	-0.36	-0.38	-0.40	9	160	27
ASI	0.00	0.26	0.14	100	-	-
BRA	0.00	0.08	0.09	100	-	-
CHN	0.00	0.26	0.20	100	-	-
IND	0.00	0.32	0.27	100	-	-
MPC	0.00	-0.77	-0.99	100	-	-
ROW	0.00	-0.05	-0.08	100	-	-

Key: CAN - Canada, CEA - Central European Associates, EUR - Europe (EU15 and EFTA), FSU - Former Soviet Union (Russian Federation and Ukraine), JPN - Japan, OOE - Other OECD (Australia and New Zealand), USA - United States, ASI - Other Asia (except for China and India), BRA - Brazil, CHN - China (incl. Hong Kong and Taiwan), IND - India, MPC - Mexico and OPEC, ROW - Rest of World

[*]   Calculated as: 100* [(Total Policy Effect) - (Domestic Market Effect)] / (Total Policy Effect)

[**] Accounting for domestic market effect and international price changes for crude oil and coal

Tax-induced reallocation of resources due to emission constraints (e.g. fuel shifting or energy savings) causes substantial adjustment costs for OECD countries. Furthermore, there are considerable international spillover effects from abating industrialized countries to non-abating countries:

---

[15] Imposing the changes in international prices which are delivered by the MRT solution (A→C), one should be able to reproduce exactly the MRT solution from the SOE perspective (B→C). This can be used as a simple consistency check.

Adjustments on international markets induce welfare losses for FSU as well as MPC and, to a much smaller extent, for ROW. All other non-abating countries benefit to varying degrees from the changes in international prices associated with emission abatement in OECD countries.

Application of the decomposition method allows insights into the different sources of welfare changes across regions. Table 10 lays out how the economic impact of carbon taxes turns from the domestic market effect into the total policy effect as changes in international prices are successively imposed upon the SOE sub-models. The column *International Spillovers* indicates the magnitude of international spillovers measured in percent of the total policy effect. Obviously, the international spillovers are identical to the total policy effect for those countries which do not undertake domestic abatement, i.e. countries whose domestic market effect is zero. As to abating countries, the decomposition provides information on the sign and relative magnitude of the primary domestic and the secondary international impacts. International spillovers are negative for USA, CAN, and OOE, whereas CEA, EUR, and JPN benefit from the adjustments on international markets.

Regarding international spillovers, most important are the adjustments on international coal and crude oil markets (see column *Fossil Fuel Market Effect*). The cutback in demands for fossil fuels from abating OECD countries depresses the international prices for oil and coal. As a consequence, countries which are net importers of coal and crude oil gain in welfare, whereas net exporting countries lose. For CAN, MPC, and ROW, which are net exporters of both coal and crude oil, the aggregate welfare effect is unambiguously negative. Similarly, net importers EUR, JPN, CHN, IND, BRA and ASI experience welfare gains. For countries which are net importer of one fossil fuel *and* net exporter of the other, the aggregate effect depends on export and import quantities as well as the relative changes in international coal and crude oil prices.

The next step of decomposition accounts for international price changes in non-energy markets where traded goods are differentiated by region of origin. On these markets, developing countries typically face adverse spillover effects. Apart from higher export prices of developed countries, developing countries suffer from a scale effect as economic activity and hence import demand by developed countries decline. On the other hand, this effect can be (partially) offset by an opposite substitution effect. Developing countries gain market shares in Annex B countries because their exports become more competitive. The same mechanisms apply to trade between abating countries with large differences in imposed carbon taxes. As an example, OOE, which has low carbon taxes, suffers from increased export prices of trading partners with high carbon taxes, such as Japan.

To sum up: Application of our decomposition method to emission regulation under the Kyoto Protocol

reveals that among signatory countries, Australia, Canada, New Zealand, and the U.S. bear a secondary burden through changes in international terms of trade, whereas Europe and Japan experience secondary benefits. Most developing countries gain a comparative advantage due to abatement in Annex B regions, but fossil fuel exporters such as Mexico and OPEC are seriously hurt. A major determinant for the differences in sign and magnitude of spillovers is the trade position of countries on international coal and crude oil markets: Depressed international prices for fossil fuels, which are due to the cutback in global fossil energy demand, provide gains for fossil fuel importers and losses for fossil fuel exporters.

7 CONCLUSIONS

In our view, CGE analysis constitutes a powerful scientific method for the comprehensive ex-ante simulation of adjustment effects induced by exogenous policy interference. The role of CGE analysis is reflected by the wide-spread use of CGE models in nearly all fields of (macro-)economic research with a steadily increasing importance for the policy debate. However, the barriers to entering the field of CGE analysis for students and professionals are commonly perceived as rather prohibitive. Without detailed programming knowledge, the CGE approach is doomed to remain a "black box" for non-modelers.

The main objective of our introductory article has been to lower the barriers to CGE analysis for two target groups. The first group consists of economists that want to actively enter the field of CGE analysis. We have laid out the most recent developments in CGE model implementation for this group. The MCP approach, combined with the calibrated share form, provides a comprehensive framework to implement equilibrium problems in a transparent and efficient way. The second group includes economists that want to refrain from programming but are interested in CGE analysis as a tool kit. For this group, the interactive interface offers convenient access to CGE analysis. Without technical programming skills, they can use the intuitive and simple Windows-based interface to explore the determinants of policy effects. Interactive learning-by-doing then involves the specification of additional policy simulation, the ex-ante economic reasoning on the potential effects, and the ex-post cross-check with the results provided by the computer simulations.

In more general terms, we hope that our combined approach stimulates students or professionals to enter the field of CGE analysis. Having gained some experience in the CGE analysis of policy interference at the non-technical level, the interested reader may be better prepared and motivated to extend our generic 2x2x1 model framework to its own needs for policy analysis.

REFERENCES

Bergman, L. (1990), The Development of Computable General Equilibrim Modeling, in: L. Bergman, D.W. Jorgenson and E. Zalai (eds.): General Equilibrium Modeling and Economic Policy Analysis, Cambridge, 3-30.

Bhattacharyya, S.C. (1996), Applied General Equilibrium Models for Energy Studies; a Survey, Energy Economics (18), 145-164.

Boadway, R.W. and D.E. Wildasin (1984), Public Sector Economics, 2nd edition, Little, Brown and Company, Boston.

Böhringer, C. and T.F. Rutherford (2002), Carbon Abatement and International Spillovers, Environmental and Resource Economics 22(3), 391-417.

Borges, A. (1986), Applied General Equilibrium Models: An Assessment of their Usefulness for Policy Analysis, OECD Economic Studies 7, 7-43.

Brooke, A., D. Kendrick and A. Meeraus (1988), GAMS - A USER'S GUIDE, The Scientific Press, San Francisco.

Conrad, K. (1999), Computable General Equilibrium Models for Environmental Economics and Policy Analysis, in: J.C.J.M. van den Bergh (ed.): Handbook of Environmental and Resource Economics, Cheltenham, 1061-1087.

Conrad, K. (2001), Computable General Equilibrium Models in Environmental and Resource Economics, in: T. Tietenberg and H. Folmer (eds.): The International Yearbook of Environmental and Resource Economics 2002/2003, 66-114.

Cottle, R.W and J.S. Pang (1992), The Linear Complementarity Problem, Academic Press.

Dewald, W. G., J. G. Thursby and R. G. Anderson (1986), Replication in Empirical Economics: The Journal of Money, Credit and Banking Project, American Economic Review 76, 587-603.

Dirkse, S. and M. Ferris (1995), The Path Solver: A Non-monotone Stabilization Scheme for Mixed Complementarity Problems, Optimization Methods and Software (5), 123-156.

Drud, A. (1985), CONOPT: a GRG code for large sparse dynamic nonlinear optimization problems. Mathematical Programming 31, 153-191.

Fehr, H. and W. Wiegard (1996), Numerische Gleichgewichtsmodelle: Grundstruktur, Anwendungen und Erkenntnisgehalt, in: Experimente in der Ökonomie, Ökonomie und Gesellschaft, Jahrbuch 13, Campus-Verlag, Frankfurt, 296-339.

Fourer, R., D. M. Gay, and B. W. Kernighan (1987), AMPL: A mathematical programming language, Computer science technical report, AT&T Bell Laboratories, Murray Hill.

Gottfried, P.E. Stöß and W. Wiegard (1990), Applied General Equilibrium Tax Models: Prosepcts, Examples, Limits, in: H.-G. Petersen und J.K. Brunner (Hrsg.): Simulation Models in Tax and Transfer Policy, Frankfurt, 205-344.

Gunning, J.W. and M.A. Keyzer (1995), Applied General Equilibrium Models for Policy Analysis, in: Handbook of Development Economics, Vol. 3A, 2025-2107.

Harberger, A.C. (1962), The incidence of the corporation income tax, Journal of Political Economy (70), 215-240.

Kehoe, P.J. and T. J. Kehoe (1994), A Primer on Static Applied General Equilibrium Models, Quarterly Review, Spring 1994, Federal Reserve Bank of Minneapolis.

King, B. (1985): What is a SAM? in: Pyatt, G. and J. I. Round (eds.), Social Accounting Matrices: A Basis for Planning. Washington D. C.: The World Bank.

Klepper, G. J.-O. Lorz, F. Stähler, R. Thiele and M. Wiebelt (1994), Empirische allgemeine Gleichgewichtsmodelle - Struktur und Anwendungsmöglichkeiten, Jahrbücher für Nationalökonomie und Statistik 213, 513-544.

Mansur, A. and J. Whalley (1984), Numerical Specification of Applied General Equilibrium Models: Estimation, Calibration, and Data. in: H. E. Scarf. and J. B. Shoven (eds..): Applied General Equilibrium Analysis, Cambridge, 69-127.

Markusen, J.R., J.R Melvin, W.H. Kaempfer and K.E Maskus (1995), International Trade – Theory and Evidence, McGraw-Hill, New York.

McCloskey, D.(1983), The Rhetoric of Economics, Journal of Economic Literature, 482-517.

Wolfram, S. (1996), MATHEMATICA- The Mathematica Book (3rd ed.), Wolfram Media/Cambridge University Press.

Mathiesen, L. (1985), Computation of Economic Equilibrium by a Sequence of Linear Complementarity Problems, in: A. Manne (ed.): Economic Equilibrium - Model Formulation and Solution, Mathematical Programming Study 23, 144-162.

MATLAB (1997), Matlab 5.1 New Features Guide The Mathworks, Inc., Natick, MA 01760 USA.

Murtagh. B.A. and M.A. Saunders (1995), MINOS 5.4 User`s Guide, Report SOL 83-20R, Department of Operations Research, Stanford University , Revised February 1995.

Peireira, A.M and J.B. Shoven (1988), Survey of Dynamic Computational General Equilibrium Models for Tax Policy Evaluation, Journal of Policy Modeling 10, 401-426.

Piggot, J. and J. Whalley (1985), New Developments in Applied General Equilibrium Analysis, Cambridge.

Piggot, J. and J. Whalley (1991), Applied General Equilibrium, Springer-Verlag, Berlin.

Robinson, S. (1999), From Stylized to Applied Models: Building Multisector CGE Models for Policy Analysis, North American Journal of Economics and Finance, 5-38.

Rutherford, T.F. (1995), Extensions of GAMS for complementarity problems arising in applied economics, *Journal of Economic Dynamics and Control* 19, 1299-1324.

Rybczinski, T.N. (1955), Factor Endowments and Relative Commodity Prices, Economica 22, 336-341.

Shoven, J.B. and J. Whalley (1984), Applied General Equilibrium Models of Taxation and International Trade, Journal of Economic Literature 22, 1007-1051.

Shoven, J.B. and J. Whalley (1992), Applying General Equilibrium, Cambridge University Press.