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***Exclusive Dealing and Vertical Integration in  
Interlocking Relationships***

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# Exclusive Dealing and Vertical Integration in Interlocking Relationships\*

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## Abstract

We develop a model of interlocking bilateral relationships between upstream firms (manufacturers) that produce differentiated goods and downstream firms (retailers) that compete imperfectly for consumers. Contract offers and acceptance decisions are private information to the contracting parties. We show that both exclusive dealing and vertical integration between a manufacturer and a retailer lead to vertical foreclosure, to the detriment of consumers and society. Finally, we show that firms have indeed an incentive to sign such contracts or to integrate vertically.

*JEL Classification:* L13, L42, D43.

*Keywords:* vertical relations, exclusive dealing, vertical merger, foreclosure, bilateral contracting.

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# 1 Introduction

In this paper, we study interlocking relationships in vertically related oligopolies, where the same competing upstream firms deal with the same competing downstream firms. We develop a framework that allows for general contracts between upstream and downstream firms, the terms of which are private information to the contracting parties. In this framework, firms have an incentive to sign exclusive dealing provisions or, alternatively, to integrate vertically, at the expense of consumers and society. The contribution of this paper is thus two-fold: It provides a general and yet tractable framework for the analysis of interlocking relationships; and it sheds light on the long-standing policy debate on vertical foreclosure.

*Interlocking relationships.* Interlocking relationships are endemic in both consumer goods and intermediate goods industries. For example, most supermarkets carry both Coca Cola and Pepsi Cola, and competing aircraft manufacturers procure components (e.g., avionics, wheels and brakes) from the same competing suppliers (such as Honeywell and Thales). Yet, despite the prevalence of such interlocking relationships, so far the IO literature on vertically related markets has mostly focused on upstream (or downstream) monopoly, or on “competing vertical structures” where each upstream firm deals with a distinct set of downstream firms (e.g., franchise networks).<sup>1</sup> The few papers that do allow for such interlocking vertical relationships have two types of limitations: they often restrict attention to particular types of (public) contracts such as linear tariffs or two-part tariffs (see, e.g., Dobson and Waterson (2007), Rey and Vergé (2010), and Allain and Chambolle, 2011), or they assume that the upstream firms produce a homogeneous good (see, e.g., de Fontenay and Gans (2005) and Nocke and White (2007,2010)). Moreover, most papers assume that contracts are publicly observable, giving rise to strategic commitment effects which may not seem very plausible.<sup>2</sup>

By contrast, we propose a framework, based on secret contracting, which allows for general (menus of) non-linear tariffs and imperfect competition both upstream (product differentiation) and downstream (Cournot competition). We show that the framework is tractable by characterizing the complete set of equilibrium outcomes in a range of situations, in the absence or presence of (single or pairwise) exclusive dealing or vertical integration.

*Vertical foreclosure.* Whether firms can engage in exclusive dealing or merge vertically for

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<sup>1</sup>Papers featuring competing vertical structures include Bonanno and Vickers (1988), Rey and Stiglitz (1988, 1995), Gal-Or (1991), Jullien and Rey (2007), and Piccolo and Miklos-Thal (2012).

<sup>2</sup>Models of interlocking relationships with observable contracts may also run into equilibrium existence problems; see, e.g., Rey and Vergé (2010) and Schutz (2013).

purely anti-competitive purposes has been the object of a long-standing debate in policy circles as well as in the academic literature.<sup>3</sup> The so-called *Chicago critique* pointed out that the “monopoly leverage” concept originally used was based on a confusion, as there is only one source of profit in a vertically related industry. In response to this critique, Ordover, Saloner and Salop (1990)<sup>4</sup> showed that an integrated firm may stop supplying downstream rivals, in order to confer market power to the other suppliers and raise in this way downstream rivals’ costs. As pointed out by Hart and Tirole (1990) and Reiffen (1992), however, this analysis relied on the assumptions that (i) an integrated supplier could somehow pre-commit itself not to supply downstream rivals (as *ex post* it would have an incentive to supply the rival), and (ii) contracting with the other suppliers is inefficient (linear tariffs, giving rise to double marginalization). Hart and Tirole (1990), O’Brien and Shaffer (1992), and McAfee and Schwartz (1994) emphasize instead that, under secret contracting, exclusive dealing or vertical integration can help a dominant supplier exert its market power. While these papers is not subject to the same limitations as Ordover, Salop and Saloner (1990), they restrict attention to upstream monopoly or quasi-monopoly settings, which has severely limited their impact on actual policy decisions. We contribute to this debate by showing that in interlocking relationships as well, firms can have incentives to engage in vertical foreclosure in order to exert market power at the expense of consumers and society.

*This paper.* We consider a successive duopoly framework. For the sake of exposition, we will refer to the upstream firms as manufacturers and to the downstream firms as retailers; it should however be clear that the analysis can be transposed to other types of vertically related industries. Manufacturers produce differentiated goods, and retailers can choose which of the goods (if any) to stock and sell on to consumers. While we introduce mild regulatory conditions on demand, we do not impose any restriction on tariffs; finally, we assume that manufacturers’ offers as well as retailers’ acceptance decisions are private information to the contracting parties, thereby discarding strategic effects.

We first provide a complete characterization of equilibria in the absence of any exclusive dealing or vertical integration. While there exists a range of equilibria, they all yield the same retail prices and quantities, and only differ in the type of contracts being signed and on how manufacturers and retailers share profits. Manufacturers prefer the equilibrium outcome induced by two-part tariffs, which is also the unique equilibrium outcome when below-cost pricing is ruled

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<sup>3</sup>For reviews of this debate, see, e.g., Rey and Tirole (2007) and Whinston (2006).

<sup>4</sup>See also Salinger (1992).

out.

We then analyze the impact of exclusive dealing and vertical integration. Under single exclusive dealing, there is again a range of equilibria, which here only differ in how manufacturers share profits with the common retailer. Under pairwise exclusive dealing, the division of profit is also uniquely determined. Under single vertical integration, there exists an equilibrium that leads to the complete foreclosure of the independent downstream rival, and thus yields the same retail prices and quantities as under (single) exclusive dealing. Under pairwise vertical integration the unique equilibrium outcome is identical to that under pairwise exclusive dealing, as each vertically integrated firm chooses to foreclose its rival.

Finally, we show that firms have indeed an incentive to engage in exclusive dealing or to integrate vertically, to the detriment of consumers and society.

*Roadmap.* Section 2 describes our framework. Section 3 provides a complete characterization of equilibria in the absence of any exclusive dealing or vertical integration. Sections 4 and 5 respectively study exclusive dealing and vertical integration. Section 6 concludes.

## 2 The Framework

We consider a vertically related industry with two symmetrically differentiated manufacturers,  $M_A$  and  $M_B$ . Manufacturer  $M_i$ ,  $i \in \{A, B\}$ , produces good  $i$  at constant unit cost  $c > 0$ . The manufacturers distribute their goods through two perfectly substitutable retailers,  $R_1$  and  $R_2$ , each of whom faces the same constant unit cost  $\gamma$ . For notational simplicity, we henceforth set  $\gamma \equiv 0$ .

For expositional simplicity consumer demand is assumed to be symmetric: The inverse demand for good  $i = A, B$  is given by  $P(Q_i, Q_j)$ ,  $j \neq i \in \{A, B\}$ , where  $Q_i \equiv q_{i1} + q_{i2}$  denotes total consumption of good  $i$ , and  $q_{ih} \geq 0$  the quantity of good  $i$  purchased from retailer  $R_h$ ,  $h \in \{1, 2\}$ .<sup>5</sup> Throughout the paper, we further impose the following conditions:

**(A.1)**  $P(0, 0) > c$  and, for  $Q$  sufficiently large,  $P(Q, 0) < c$  and  $P(0, Q) < c$ .

**(A.2)** For any  $(Q_i, Q_j) \geq 0$ ,<sup>6</sup>

$$\partial_1 P(Q_i, Q_j) \leq \partial_2 P(Q_i, Q_j) \leq 0,$$

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<sup>5</sup>Demand symmetry implies  $\partial_2 P(Q, Q') = \partial_2 P(Q', Q)$  ( $= \partial_{12} U(Q, Q')$ , where  $U(\cdot, \cdot)$  denotes consumers' gross surplus), and thus  $\partial_{12}^2 P(Q, Q') = \partial_{22}^2 P(Q', Q)$ , for all  $Q$  and  $Q'$ .

<sup>6</sup>Throughout the paper,  $\partial_n f$  denotes the partial derivative of the function  $f$  with respect to its  $n^{\text{th}}$  argument; likewise,  $\partial_{nm}^2 f$  will denote the second-order partial derivative with respect to the  $n^{\text{th}}$  and  $m^{\text{th}}$  arguments.

with strict inequalities when  $P(Q_i, Q_j) > 0$ .

Condition (A.1) is essentially a viability assumption, whereas condition (A.2) simply asserts that goods  $A$  and  $B$  are (imperfect) substitutes.<sup>7</sup>

We confine attention to *vertical* contracts; that is, we do not allow for any kind of “horizontal” agreements such as, e.g., market-share contracts, and consider instead contracts purely based on the quantity traded: Formally, a contract between  $M_i$  and  $R_h$  is a tariff  $\tau_{ih} : \mathfrak{R}_+ \rightarrow \mathfrak{R}$ , where  $\tau_{ih}(q)$  is the payment from  $R_h$  to  $M_i$  in return for a quantity  $q$  of good  $i$ .<sup>8</sup> We do not impose any further restriction, however, and thus allow for any nonlinear tariff; special cases of interest are:

- *Two-part tariff*:  $\tau_{ih}(q) = F + wq$ , where  $F$  is the fixed (or “franchise”) fee, and  $w \geq 0$  the marginal wholesale price; we will denote such a two-part tariff by  $(w, F)$ .
- *Forcing contract*:

$$\tau_{ih}(q) = \begin{cases} \hat{T} & \text{if } q = \hat{q}, \\ \infty & \text{otherwise,} \end{cases}$$

where  $\hat{q}$  is the “forced” quantity; we will denote such a forcing contract by  $(\hat{q}, \hat{T})$ .

The contracting terms between  $M_i$  and  $R_h$  are private information to the two parties. The timing is as follows:

**Stage 1** Manufacturers simultaneously offer (secret) contracts to retailers.

**Stage 2** Retailers simultaneously (and secretly): (i) accept or reject the offers; and (ii) for each accepted contract, choose how much to put on the final market.<sup>9</sup> The resulting prices are such that markets clear.

We will also consider variants involving exclusive dealing and vertical integration. An exclusive dealing provision restricts the set of partners to whom offers can be made (in case of exclusive distribution), or from whom they can be accepted (in case of single branding). When

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<sup>7</sup>The condition on  $P(Q, 0)$  follows from the others but is mentioned explicitly for the sake of exposition.

<sup>8</sup>For the sake of exposition, we will assume that parties contract on the quantity  $q$  sold to consumers, rather than the quantity bought from the manufacturer. The distinction becomes moot when the production cost is large enough, as then a retailer will not want to buy more than it needs in any relevant scenario.

<sup>9</sup>As acceptance and output decisions are simultaneous, there is no role here for menus of contracts: Offering a menu of tariffs is *de facto* equivalent to offering the lower envelope of these tariffs.

instead  $M_i$  and  $R_h$  are vertically integrated, they are assumed to maximize their joint profits, regardless of internal transfer prices.  $M_i$  and  $R_h$  moreover “share information” in the sense that, when making its acceptance and output decisions,  $R_h$  is informed about the offer that its upstream affiliate  $M_i$  has previously made to the rival retailer  $R_k$ . By contrast, as acceptance and output decisions are made simultaneously, when making its output decisions  $R_h$  cannot be informed of whether the rival  $R_k$  accepted  $M_i$ ’s offer.<sup>10</sup>

We will study Perfect Bayesian Equilibria with *passive* beliefs, in which retailers do not revise their beliefs about the offer made to the other retailer when receiving an out-of-equilibrium offer. As retailers compete downstream in quantities, these passive beliefs coincide with the “wary beliefs” introduced by McAfee and Schwartz (1994), as the contract signed with a retailer has no impact on a manufacturer’s gains from trade with the other retailer.

### 3 Baseline Model

In this section, we characterize the equilibria of our baseline model. We first define the notion of a “cost-based” contract, in which the marginal input price coincides with the marginal cost of production, and show that unintegrated manufacturers sign cost-based contracts with every available retailer. Drawing on this insight, we then characterize the equilibria in the absence of exclusive dealing and vertical integration.

#### 3.1 Independent Manufacturers

Throughout the analysis, we will use indices  $i \neq j$  when referring to  $M_A$  and  $M_B$ , and  $h \neq k$  when referring to  $R_1$  and  $R_2$ . Let

$$\chi(q_{ik}, q_{jh}, q_{jh}) \equiv \arg \max_{q_{ih}} [P(q_{i1} + q_{i2}, q_{j1} + q_{j2}) - c] q_{ih} + P(q_{j1} + q_{j2}, q_{i1} + q_{i2}) q_{jh}$$

denote the set of bilaterally efficient values for the output  $q_{ih}$ , from the standpoint of the pair  $M_i - R_h$ , holding fixed all other outputs.<sup>11</sup> We will say that the equilibrium contract signed by  $M_i$  and  $R_h$  is “cost-based” if it induces a bilaterally efficient output, given the other equilibrium outputs:

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<sup>10</sup>This avoids having to take a stand on how the integrated retailer would interpret an unexpected acceptance or rejection of the upstream affiliate’s offer.

<sup>11</sup>Note that this set does not depend on the tariffs  $\tau_{ik}$  and  $\tau_{jh}$ .

**Definition 1** *The equilibrium contract  $\tau_{ih}(\cdot)$  between  $M_i$  and  $R_h$  is said to be cost-based if, when accepted, and given the outputs of the other channels  $(q_{ik}, q_{jh}, q_{jh})$ ,  $\tau_{ih}(\cdot)$  induces a quantity  $q_{ih} \in \chi(q_{ik}, q_{jh}, q_{jh})$ .*

The following lemma characterizes the equilibrium contracts signed by unintegrated manufacturers:

**Lemma 1** *Suppose  $M_i$  is not vertically integrated (whereas  $M_j$  may or may not be vertically integrated). Then, in any equilibrium  $M_i$  signs a cost-based contract with every retailer  $R_h$  that is available, given the exclusive dealing provisions (with the convention that they sign a “null” contract if it is bilaterally efficient not to trade).*

**Proof.** See Appendix A. ■

The intuition is simple: Under passive beliefs, each  $R_h$  expects its rival  $R_k$  to stick to the equilibrium quantities even when receiving a deviant offer from an independent  $M_i$ . Moreover, such a deviant offer does not affect the profit that  $M_i$  makes on its contract with  $R_k$ . In equilibrium, the contract between  $M_i$  and  $R_h$  must therefore maximize the joint bilateral profit of the contracting parties, assuming that  $R_k$  sticks to its equilibrium quantities, which is achieved by signing a cost-based contract.

### 3.2 Non-Exclusive Relationships and Vertical Separation

We now characterize the set of equilibria in the absence of exclusive dealing and vertical integration (which we will index by the superscript “ $\circ$ ”). From Lemma 1, we know that *each*  $M_i$  must sign a cost-based contract with *each*  $R_h$ , implying the following result:

**Proposition 1** *In the absence of exclusive dealing and vertical integration, the set of equilibrium quantities  $(q_{A1}^\circ, q_{A2}^\circ, q_{B1}^\circ, q_{B2}^\circ)$  coincides with that of a Cournot multiproduct duopoly in which the two firms (1 and 2) can produce the same two goods (A and B) at marginal cost  $c$ .*

**Proof.** This follows directly from Lemma 1. ■

For the sake of exposition, it is useful to pin down the equilibrium outcome; the following mild regularity conditions ensure that the equilibrium retail prices and quantities are uniquely defined and symmetric:

**(A.3)** For any  $(Q_i, Q_j) \geq 0$  such that  $P(Q_i, Q_j) > 0$ , we have

$$2\partial_1 P(Q_i, Q_j) + \partial_{11}^2 P(Q_i, Q_j)Q_i < \partial_2 P(Q_i, Q_j) + \partial_{12}^2 P(Q_i, Q_j)Q_i < 0.$$



(A.4) For any  $(Q_i, Q_j) \geq 0$  such that  $P(Q_i, Q_j) > 0$ , and for any  $q_i \in [0, Q_i]$  and any  $q_j \in [0, Q_j]$ , we have

$$\begin{aligned} & 2\partial_1 P(Q_i, Q_j) + \partial_{11}^2 P(Q_i, Q_j) q_i + \partial_{22}^2 P(Q_j, Q_i) q_j \\ & < \partial_2 P(Q_i, Q_j) + \partial_2 P(Q_j, Q_i) + \partial_{12}^2 P(Q_i, Q_j) q_i + \partial_{12}^2 P(Q_j, Q_i) q_j \\ & < 0. \end{aligned}$$

Condition (A.3) ensures that a Cournot duopoly game, in which one firm sells good  $A$  and the other good  $B$ , would have a unique, stable equilibrium. Condition (A.4) further ensures that profits remain concave when both firms can sell goods  $A$  and  $B$ . In the case of linear demand, (A.3) and (A.4) boil down to  $\partial_1 P < \partial_2 P < 0$ , and are thus implied by (A.2).

We have:

**Proposition 2** *In the absence of exclusive dealing and vertical integration, under Assumptions (A.3)-(A.4) the equilibrium quantities must satisfy  $q_{ih}^\circ = q^\circ$ , for  $i \in \{A, B\}$  and  $h \in \{1, 2\}$ , where  $q^\circ$  is the unique solution to:*

$$P(2q^\circ, 2q^\circ) - c + [\partial_1 P(2q^\circ, 2q^\circ) + \partial_2 P(2q^\circ, 2q^\circ)] q^\circ = 0.$$

**Proof.** See Appendix B. ■

Hence, any equilibrium must be bilaterally efficient, which under the regularity assumptions (A.3)-(A.4) implies that each “channel” must sell the same quantity  $q^\circ$ . It follows that all equilibria generate the same industry-wide aggregate profit,

$$\Pi^\circ \equiv [P(2q^\circ, 2q^\circ) - c] 4q^\circ.$$

$M_i$ 's profit is therefore of the form  $\pi_i^\circ = \pi_{i,1}^\circ + \pi_{i,2}^\circ$ , where  $\pi_{i,h}^\circ = \tau_{ih}^\circ(q^\circ) - cq^\circ$  is  $M_i$ 's profit on its contract with  $R_h$ , whereas  $R_h$ 's profit is  $\pi_h^\circ = 2P(2q^\circ, 2q^\circ)q^\circ - \tau_{Ah}^\circ(q^\circ) - \tau_{Bh}^\circ(q^\circ)$ .

To support such an equilibrium, contracts must be somewhat flexible: As shown in Appendix D, there is no equilibrium in which a manufacturer offers a single forcing contract. The intuition is as follows. In equilibrium, each retailer must be indifferent between accepting both manufacturers' offers, and only one (either one): If a retailer strictly preferred dealing with both manufacturers than with only one of them, then the rival manufacturer could profitably deviate by asking for a larger share of the profits. But if, say,  $M_i$  offers  $R_h$  a single forcing contract  $(q^\circ, T_{ih}^\circ)$ , then  $M_i$  is also indifferent between whether or not  $R_h$  also accepts upstream rival  $M_j$ 's offer; hence, the joint profit of  $M_i$  and  $R_h$  must be the same as what they would obtain if  $R_h$

were to deal exclusively with  $M_i$ . But in equilibrium, the sum of  $R_h$ 's profit and of  $M_i$ 's profit from its contract with  $R_h$  is given by

$$\begin{aligned}\pi_{i,h}^\circ + \pi_h^\circ &= [\tau_{ih}^\circ(q^\circ) - cq^\circ] + [2P(2q^\circ, 2q^\circ) - \tau_{ih}^\circ(q^\circ) - \tau_{jh}^\circ(q^\circ)] \\ &= [\tau_{ih}^\circ(q^\circ) - cq^\circ] + [P(2q^\circ, q^\circ) - \tau_{ih}^\circ(q^\circ)] \\ &= [P(2q^\circ, q^\circ) - c]q^\circ.\end{aligned}$$

Hence, under exclusivity  $M_i$  and  $R_h$  could generate more profit by replacing  $q^\circ$  with

$$\hat{q} \equiv \arg \max_q \{[P(q^\circ + q, q^\circ) - c]q\},$$

and share the profit increase through an appropriate transfer  $\hat{T}$ ; it follows that  $M_i$  could profitably deviate by offering the forcing contract  $(\hat{q}, \hat{T})$ , thereby inducing  $R_h$  to “bump” the rival manufacturer (note that this deviation does not affect the profit that  $M_i$  obtains from dealing with the other retailer,  $R_k$ ).

We now show that simple two-part tariffs suffice to support an equilibrium:

**Proposition 3** *In the absence of exclusive dealing and vertical integration, under Assumptions (A.3)-(A.4) there exists an equilibrium in which each manufacturer signs with each retailer the cost-based two-part tariff  $(w^\circ, F^\circ) = (c, \Delta^\circ)$ , where*

$$\Delta^\circ \equiv [P(2q^\circ, 2q^\circ) - c]2q^\circ - \max_q \{[P(q^\circ + q, q^\circ) - c]q\}$$

*denotes each manufacturer's contribution to the profit generated by a retailer.*

**Proof.** See Appendix C. ■

The intuition is as follows. First, cost-based two-part tariffs allow retailers to buy any marginal quantity at cost. It follows that: (i) as a manufacturer does not care about the level of trade chosen by retailers, it is indifferent as to whether retailers will deal with the rival manufacturer or not; and (ii) in both instances, the tariff is bilaterally efficient, whether or not the retailer deals with the other manufacturer. Second, the equilibrium fixed fees are such that each retailer is indifferent between dealing with both manufacturers, or with either one on an exclusivity basis. It follows that these tariffs eliminate the above “bumping” problem, as a manufacturer would have to give away some of its profit in order to convince a retailer to opt for exclusivity.

There exist many other equilibria, however; although all equilibria must rely on cost-based contracts, and thus generate the same industry profit,  $\Pi^\circ$ , they can differ in the way firms share this profit:

**Proposition 4** *In the absence of exclusive dealing and vertical integration, under Assumptions (A.3)-(A.4):*

- *Manufacturers' equilibrium profits are of the form  $(\pi_A^\circ = \pi_{A,1}^\circ + \pi_{A,2}^\circ, \pi_B^\circ = \pi_{B,1}^\circ + \pi_{B,2}^\circ)$ , where  $\pi_{i,h}^\circ \in [0, \Delta^\circ]$ .*
- *Any profile of profits in this range can be supported by an equilibrium in which each  $M_i$  offers each  $R_h$  a pair of forcing contracts; however, outcomes giving less than its contribution to a manufacturer (i.e., such that  $\pi_i^\circ < 2\Delta^\circ$  for some  $M_i$ ) can only be supported by tariffs that price some incremental quantity below cost.*

**Proof.** See Appendix D. ■

Proposition 4 shows that, without loss of generality, we can restrict attention to equilibria in which each  $M_i$  offers each  $R_h$  a menu of two forcing contracts: A cost-based contract  $(q^\circ, T_{ih}^\circ)$  “designed” for common agency, which  $R_h$  accepts along the equilibrium path, and a contract  $(\tilde{q}_{ih}^\circ, \tilde{T}_{ih}^\circ)$  “designed” for exclusivity, where  $\tilde{q}_{ih}^\circ > q^\circ$  and  $c(\tilde{q}_{ih}^\circ - q^\circ) \geq \tilde{T}_{ih}^\circ - T_{ih}^\circ > 0$ . In equilibrium, each retailer is indifferent between picking both  $(q^\circ, T_{Ah}^\circ)$  and  $(q^\circ, T_{Bh}^\circ)$ , or picking only  $(\tilde{q}_{ih}^\circ, \tilde{T}_{ih}^\circ)$ , from either manufacturer  $i = A, B$ .

Finally, the division of profit varies substantially across equilibria. The intuition is as follows. Even though they are not accepted in equilibrium, the “exclusive deal” offers determine retailers’ outside options, and thus how much profit is left for the manufacturers. In equilibrium, the two manufacturers’ exclusive deal offers must be equally “generous.” Moreover, each manufacturer must (weakly) prefer that the retailer does not choose the exclusive deal offer but rather the one designed for common agency: If a manufacturer were to prefer the retailer to accept the exclusive deal option over the common agency option, the manufacturer could profitably deviate by making the exclusive deal option slightly more attractive, thereby inducing the retailer to accept that option. This generates a multiplicity of equilibria in terms of how the profits are shared between the upstream and downstream levels: The more generous is one manufacturer, the more generous must be the other one. Only in the equilibrium with the least generous offers is each manufacturer indifferent between the retailer choosing the common agency contract (which is accepted in equilibrium) and the exclusive deal offered by the manufacturer (which is not accepted).

The analysis also identifies bounds on how profits can be shared. While Proposition 3 shows that cost-based two-part tariffs (where *any* incremental quantity is sold at cost) enables manufacturers to obtain exactly their contribution to industry profits,  $\Delta^\circ$ , Proposition 4 establishes

that manufacturers cannot obtain more, and can obtain much less; in particular, there are equilibria in which retailers appropriate all profits – as well as equilibria in which one manufacturer obtains its contribution  $\Delta^\circ$  when the other one gets nothing. Note however that all these other equilibria rely on tariffs that price the incremental quantity for exclusivity below cost: At least one tariff  $\tau_{ih}^\circ$  must be such that  $\tau_{ih}^\circ(\tilde{q}_{ih}^\circ) - \tau_{ih}^\circ(q^\circ) < c(\tilde{q}_{ih}^\circ - q^\circ)$ , where  $\tilde{q}_{ih}^\circ$  denotes the quantity that  $R_h$  would pick if it were to deviate and deal exclusively with  $M_i$ ;  $M_i$  would thus obtain less profit if  $R_h$  were to move to exclusivity – by contrast, the equilibria giving manufacturers their contribution  $\Delta^\circ$  are such that each  $M_i$  is indifferent as well between  $R_h$  buying  $q^\circ$  from both manufacturers, or  $\tilde{q}_{ih}^\circ$  exclusively from  $M_i$ .

## 4 Exclusive Dealing

In this section, we analyze the effects of exclusive dealing provisions on the equilibrium outcome and on welfare. These provisions include *exclusive distribution* contracts, which preclude the manufacturer from selling to the rival retailer, and *single branding* contracts, which preclude the retailer from buying from the other manufacturer. We first consider the case of a single exclusive dealing provision that precludes trade between  $M_A$  and  $R_2$ , and then that of two exclusive dealing provisions that preclude trade between  $M_A$  and  $R_2$  and between  $M_B$  and  $R_1$ . Next, we make an excursion by analyzing an associated duopoly game without any vertical aspects. We then use the insights from this game to study the incentives for firms to engage in exclusive dealing. Finally, we provide a welfare analysis of the effects of exclusive dealing.

### 4.1 Equilibrium Outcomes

We begin by analyzing the equilibrium effects of a (pre-existing) exclusion dealing provision that precludes trade between manufacturer  $M_A$  and retailer  $R_2$ . Such a provision may be either an exclusive distribution contract between  $M_A$  and  $R_1$ , or a single branding contract between  $M_B$  and  $R_2$ .

The following proposition provides a characterization of the equilibrium quantities (which we will index by the superscript “\*”):

**Proposition 5** *Suppose that a single exclusive dealing provision precludes trade between  $M_A$  and  $R_2$  (i.e.,  $q_{A2}^* = 0$ ). Then, under Assumptions (A.3) and (A.4):*

- *There exists an equilibrium supported by cost-based two-part tariffs.*

- In all equilibria,  $M_A$  signs a cost-based contract with  $R_1$ , and  $M_B$  signs cost-based contracts with both  $R_1$  and  $R_2$ ; the equilibrium quantities,  $(q_{A1}^*, q_{B1}^*, q_{B2}^*)$ , are moreover positive and uniquely defined by:

$$\begin{aligned} q_{A1}^* &= \arg \max_{q_{A1}} [P(q_{A1}, q_{B1}^* + q_{B2}^*) - c] q_{A1} + P(q_{B1}^* + q_{B2}^*, q_{A1}) q_{B1}^*, \\ q_{B1}^* &= \arg \max_{q_{B1}} [P(q_{B1} + q_{B2}^*, q_{A1}^*) - c] q_{B1} + P(q_{A1}^*, q_{B1} + q_{B2}^*) q_{A1}^*, \end{aligned}$$

and

$$q_{B2}^* = \arg \max_{q_{B2}} [P(q_{B1}^* + q_{B2}, q_{A1}^*) - c] q_{B2}.$$

**Proof.** See Appendix E. ■

The market outcome is thus that of an asymmetric duopoly, in which one firm offers both goods  $A$  and  $B$ , whereas the other offers only one of these goods. Following the same steps as for Proposition 4, it can be checked that there exist multiple equilibria, which differ on how  $R_1$  shares its profit with the manufacturers. More precisely, let

$$\Pi_1^* = [P(q_{A1}^*, q_{B1}^* + q_{B2}^*) - c] q_{A1}^* + [P(q_{B1}^* + q_{B2}^*, q_{A1}^*) - c] q_{B1}^*$$

denote the profit generated by  $R_1$  in equilibrium (some of which may be captured by the two manufacturers), and

$$\begin{aligned} \Delta_{A,1}^* &= \Pi_1^* - \max_{q_{B1}} \{[P(q_{B1} + q_{B2}^*, 0) - c] q_{B1}\}, \\ \Delta_{B,1}^* &= \Pi_1^* - \max_{q_{A1}} \{[P(q_{A1}, q_{B2}^*) - c] q_{A1}\} \end{aligned}$$

$M_A$ 's and  $M_B$ 's contributions to this profit, respectively. We then have  $\Delta_{A,1}^* > \Delta_{B,1}^* > 0$ <sup>12</sup> and:

- In equilibrium,  $M_B$  now always appropriates the profit generated by  $R_2$ .
- As regards sharing the profit generated by  $R_1$ , any tuple  $(\pi_{B,1}^*, \pi_{A,1}^*)$  with  $\pi_{B,1}^* \in [0, \Delta_{B,1}^*]$  for  $M_B$  and  $\pi_{A,1}^* \in [\Delta_{A,1}^* - \Delta_{B,1}^*, \Delta_{A,1}^*]$  for  $M_A$  can be sustained as an equilibrium outcome.

---

<sup>12</sup>That  $M_B$ 's contribution is positive stems from  $q_{B1}^* > 0$ , which implies

$$\begin{aligned} \Pi_1^* &= \max_{q_{A1}, q_{B1}} \{[P(q_{A1}, q_{B1} + q_{B2}^*) - c] q_{A1} + [P(q_{B1} + q_{B2}^*, q_{A1}) - c] q_{B1}\} \\ &> \max_{q_{A1}} \{[P(q_{A1}, q_{B2}^*) - c] q_{A1}\}. \end{aligned}$$

That  $M_A$ 's contribution is larger than  $M_B$ 's follows from the fact that, from (A.2),  $P(q, q_{B2}^*) > P(q + q_{B2}^*, 0)$  for any  $q$  such that  $P(q, q_{B2}^*) > 0$ . To see this, note that  $\phi(x) = P\left(\frac{q+q_{B2}^*}{2} + x, \frac{q+q_{B2}^*}{2} - x\right)$  is decreasing in  $x$  from (A.2), and  $P(q, q_{B2}^*) = \phi\left(\frac{q-q_{B2}^*}{2}\right)$  whereas  $P(q + q_{B2}^*, 0) = \phi\left(\frac{q+q_{B2}^*}{2}\right)$ .

Suppose now that (pre-existing) pairwise exclusive dealing provisions preclude trade between  $M_A$  and  $R_2$  as well as between  $M_B$  and  $R_1$ . For example,  $M_A$  and  $R_1$  as well as  $M_B$  and  $R_2$  may have signed exclusive distribution contracts with each other, or  $M_A$  and  $R_2$  as well as  $M_B$  and  $R_1$  may have signed single branding contracts.

The following proposition characterizes the equilibrium (which we will index by the superscript “\*\*”).

**Proposition 6** *Suppose that pairwise exclusive dealing provisions preclude trade between  $M_A$  and  $R_2$  as well as between  $M_B$  and  $R_1$  (i.e.,  $q_{A2}^{**} = q_{B1}^{**} = 0$ ). Then under Assumption (A.3):*

- *There exists an equilibrium supported by cost-based two-part tariffs.*
- *In all equilibria:*
  - *$M_A$  and  $R_1$  as well as  $M_B$  and  $R_2$  sign cost-based contracts, and each manufacturer fully appropriates the profit generated by its good.*
  - *$q_{A1}^{**} = q_{B2}^{**} = Q^{**} > 0$ , where  $Q^{**}$  is the unique solution to*

$$P(Q^{**}, Q^{**}) - c + \partial_1 P(Q^{**}, Q^{**}) Q^{**} = 0.$$

**Proof.** See Appendix F. ■

The market outcome is thus that of a standard duopoly, with one firm offering good  $A$  and the other offering good  $B$ . This equilibrium outcome can, for instance, be supported by cost-based two-part tariffs. Finally, in contrast to the previous cases, equilibrium profits are here unique as well, as manufacturers appropriate all profits.

## 4.2 Excursion: Associated Duopoly Game

Consider the following hypothetical duopoly game, denoted  $\Gamma_2$ . There are two players, firms 1 and 2, and two goods,  $A$  and  $B$ . Firm 1’s strategy consists in choosing the quantity  $q_{A1} \in [0, \infty)$  of good  $A$  to sell at the same time as firm 2 chooses the quantity  $q_{B2} \in [0, \infty)$  of good  $B$ . In addition, firm 1 also sells an exogenous quantity  $\hat{q}_{B1}$  of good  $B$  and firm 2 an exogenous quantity  $\hat{q}_{A2}$  of good  $A$ , so that the profit functions of firms 1 and 2 are given by

$$\hat{\Pi}_1(q_{A1}, q_{B2}; \hat{q}_{B1}, \hat{q}_{A2}) \equiv [P(q_{A1} + \hat{q}_{A2}, \hat{q}_{B1} + q_{B2}) - c] q_{A1} + [P(\hat{q}_{B1} + q_{B2}, q_{A1} + \hat{q}_{A2}) - c] \hat{q}_{B1}$$

and

$$\hat{\Pi}_2(q_{A1}, q_{B2}; \hat{q}_{B1}, \hat{q}_{A2}) \equiv [P(\hat{q}_{B1} + q_{B2}, q_{A1} + \hat{q}_{A2}) - c] q_{B2} + [P(q_{A1} + \hat{q}_{A2}, \hat{q}_{B1} + q_{B2}) - c] \hat{q}_{A2},$$

respectively. In the special case where  $\hat{q}_{A2} = \hat{q}_{B1} = 0$ , this game simplifies to a standard differentiated goods Cournot duopoly, where each of the two goods is sold by only one firm.

For our main results, we will assume that the equilibrium of game  $\Gamma_2$  has the following properties:

**(P.1)** Game  $\Gamma_2$  has a unique Nash equilibrium  $(\tilde{q}_{A1}(\hat{q}_{B1}, \hat{q}_{A2}), \tilde{q}_{B2}(\hat{q}_{B1}, \hat{q}_{A2}))$ .

**(P.2)** In equilibrium, the aggregate profit

$$\begin{aligned} & \Pi(\tilde{q}_{A1}(\hat{q}_{B1}, \hat{q}_{A2}) + \hat{q}_{A2}, \hat{q}_{B1} + \tilde{q}_{B2}(\hat{q}_{B1}, \hat{q}_{A2})) \\ \equiv & [P(\tilde{q}_{A1}(\hat{q}_{B1}, \hat{q}_{A2}) + \hat{q}_{A2}, \hat{q}_{B1} + \tilde{q}_{B2}(\hat{q}_{B1}, \hat{q}_{A2})) - c](\tilde{q}_{A1}(\hat{q}_{B1}, \hat{q}_{A2}) + \hat{q}_{A2}) \\ & + [P(\hat{q}_{B1} + \tilde{q}_{B2}(\hat{q}_{B1}, \hat{q}_{A2}), \tilde{q}_{A1}(\hat{q}_{B1}, \hat{q}_{A2}) + \hat{q}_{A2}) - c](\hat{q}_{B1} + \tilde{q}_{B2}(\hat{q}_{B1}, \hat{q}_{A2})) \end{aligned}$$

is uniquely maximized for  $\hat{q}_{B1} = \hat{q}_{A2} = 0$ ; that is,

$$\Pi(\tilde{q}_{A1}(0, 0), \tilde{q}_{B2}(0, 0)) > \Pi(\tilde{q}_{A1}(\hat{q}_{B1}, \hat{q}_{A2}) + \hat{q}_{A2}, \hat{q}_{B1} + \tilde{q}_{B2}(\hat{q}_{B1}, \hat{q}_{A2}))$$

whenever  $\hat{q}_{B1} + \hat{q}_{A2} > 0$ .

**(P.3)** The equilibrium quantity  $\tilde{q}_{A1}(\hat{q}_{B1}, \hat{q}_{A2})$  (resp.,  $\tilde{q}_{B2}(\hat{q}_{B1}, \hat{q}_{A2})$ ) is (weakly) decreasing in  $\hat{q}_{A2}$  (resp.,  $\hat{q}_{B1}$ ).

These properties are satisfied in the case of linear demand. In Appendix G, we provide more general sufficient conditions on demand that ensure that (P.1)-(P.3) do indeed hold.

### 4.3 Incentives to Engage in Exclusive Dealing

We now study firms' incentive to engage in exclusive dealing. We first show that at least one manufacturer-retailer pair benefits from signing an exclusive distribution contract (whether or not this induces the other pair to do the same), and the other pair then benefits from doing the same. Hence, absent any rule against exclusive dealing provisions, we would expect the emergence of pairwise exclusivity.

We then consider the case of single-branding contracts. The incentives for a manufacturer-retailer pair to sign such a contract are less clear, as doing so benefits the other pair but may reduce the joint profit of the signing pair. In addition, once such a contract is in place, the other manufacturer-retailer pair does not have an incentive to adopt any exclusivity provision; the first pair, however, has an incentive to move to pairwise exclusivity, by complementing its single-branding contract with an exclusive distribution provision.

### 4.3.1 Exclusive Distribution

To analyze firms' incentives to sign exclusive distribution contracts, we proceed as follows. First, starting from an environment without exclusive dealing, we ask whether there exists a manufacturer-retailer pair  $M_i - R_h$  that could raise its joint profit by signing an exclusive distribution contract that prevents  $M_i$  from dealing with the rival retailer  $R_k$ . Second, starting from an environment in which the pair  $M_i - R_h$  has already signed such an exclusive distribution contract, we ask whether the other pair,  $M_j - R_k$ , can increase its joint profit by signing an exclusive distribution contract that prevents  $M_j$  from dealing with  $R_h$ . Despite the multiplicity of equilibria (under either no or single exclusive dealing) in terms of rent shifting between manufacturers and retailers, we obtain a strong result: Firms do have an incentive to engage in exclusive dealing, no matter how profits are shared.

**Proposition 7** *Assume (A.3)-(A.4) and (P.1)-(P.3). Then:*

- *In any equilibrium that arises in the absence of exclusivity, there exists a manufacturer-retailer pair  $M_i - R_h$  that can strictly increase its joint profit by signing an exclusive distribution contract, regardless of:*
  - *which equilibrium is selected under single exclusivity; and*
  - *whether this induces the other pair to engage in exclusive dealing as well.*
- *In any equilibrium under single exclusive distribution between  $M_i - R_h$ , the other pair  $M_j - R_k$  can strictly increase its joint profit by signing an exclusive distribution contract.*

**Proof.** See Appendix H. ■

The intuition builds upon two observations: (i) Industry profits are larger under pairwise exclusive dealing than under any other configuration; and (ii) a manufacturer-retailer pair, say  $M_A - R_1$ , obtains a larger joint profit when it is the only pair that engages in exclusive distribution, than under pairwise exclusive dealing. The first observation is intuitive and follows from (P.2).<sup>13</sup> The second observation follows from the fact, in a single exclusive dealing equilibrium,  $M_A$  and  $R_1$  must at least obtain what they could achieve by deviating to pairwise exclusivity, and moreover face a less aggressive rival than in the pairwise exclusive dealing equilibrium:  $q_{B2}^* < q_{B2}^{**}$ , from (P.3).

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<sup>13</sup>Recall that (P.2) follows in turn from the regularity assumptions provided in Appendix G.



By construction, in the absence of exclusivity at least one manufacturer-retailer pair obtains less than half of the equilibrium industry profit  $\Pi^\circ$ , which itself is (strictly) smaller than the equilibrium industry profit  $\Pi^{**}$  achieved under pairwise exclusive dealing. It follows from the above observations that this pair would benefit from signing an exclusive distribution contract – whether or not this induces the other pair to engage in exclusivity as well.

It also follows from the above observations that under single exclusive dealing, the pair that is under exclusive distribution obtains a larger joint profit than the other pair. But then, this other pair has an incentive to engage in exclusivity as well, so as to earn a bigger share (namely, one-half) of a bigger pie (as the industry profit is maximal under pairwise exclusive dealing).

### 4.3.2 Single Branding

The above analysis suggests that the incentives to adopt single branding provisions are less clear. To see this, consider an environment without exclusivity, and suppose that  $M_j$  and  $R_k$  sign a single branding contract that prevents  $R_k$  from dealing with the rival manufacturer  $M_i$ . Intuitively, this eliminates intrabrand competition for good  $A$ , and may thus increase in this way total industry profit. However, the above analysis points out that the other manufacturer-retailer pair, say  $M_i - R_h$ , gets the bigger share of that profit; hence, even if total industry profit is increased,  $M_j$  and  $R_h$  may obtain too small a share of that bigger pie, making single branding unprofitable. Indeed, in the case of the linear demand considered at the end of Section 4.4, starting from a situation without exclusivity where the manufacturer-retailer pairs  $M_i - R_h$  and  $M_j - R_k$  share the industry profit equally, then none of them can increase its joint profit by opting for single branding; in the same vein,  $M_j$  and  $R_k$  cannot benefit from signing a single branding contract if this does not allow  $M_j$  to extract more profit from the other retailer,  $R_h$  (i.e., if  $\pi_{j,h}^* \leq \pi_{j,h}^\circ$ ).

It also follows from the above analysis that, if one manufacturer-retailer pair, say  $M_j - R_k$ , opts for single branding, then the other pair,  $M_i - R_h$ , will not follow suit. Indeed, we have seen that  $M_i$  and  $R_h$ 's joint profit is larger in the single exclusive dealing situation where  $M_i$  does not deal  $R_k$ , than in case of pairwise exclusivity. However,  $M_j$  and  $R_k$  would have an incentive to complement their single branding contract with an exclusive distribution provision, in order to move towards pairwise exclusivity, so as obtain again a bigger share of a bigger pie. By the same token, starting from an environment without exclusivity, at least one pair (any pair that does not get more than half of the industry profit) would have an incentive to engage in mutual exclusivity, that is, to sign a contract involving both exclusive distribution and single branding,

in order to move towards pairwise exclusivity.

#### 4.4 Welfare Effects of Exclusive Dealing

We now turn to the welfare effects of exclusive dealing. For given quantities  $Q_A$  and  $Q_B$ , consumer surplus can be expressed as

$$S(Q_A, Q_B) \equiv \int_0^{Q_A} P(q_A, Q_B) dq_A + \int_0^{Q_B} P(q_B, 0) dq_B - P(Q_A, Q_B)Q_A - P(Q_B, Q_A)Q_B,$$

aggregate profit as

$$\Pi(Q_A, Q_B) \equiv [P(Q_A, Q_B) - c]Q_A + [P(Q_B, Q_A) - c]Q_B,$$

and social welfare as

$$W(Q_A, Q_B) \equiv \int_0^{Q_A} [P(q_A, Q_B) - c] dq_A + \int_0^{Q_B} [P(q_B, 0) - c] dq_B.$$

Let  $Q^\circ = 2q^\circ$  denote the aggregate output per good in the absence of exclusive dealing,  $Q^{**} = q^{**}$  that under pairwise exclusive dealing, and  $(Q_A^* = q_{A1}^*, Q_B^* = q_{B1}^* + q_{B2}^*)$  the aggregate outputs when a single exclusive dealing provision precludes trade between  $M_A$  and  $R_2$ .

For the second part of our first welfare result, we require the following technical assumption on demand (which holds with equality if demand is linear):

**(A.5)** For any  $(Q_i, Q_j) \geq 0$  such that  $P(Q_i, Q_j) > 0$ , we have

$$\begin{aligned} & \partial_2 P(Q_i, Q_j) [Q_i \partial_{11}^2 P(Q_i, Q_j) + Q_j \partial_{22}^2 P(Q_j, Q_i)] \\ & \geq \partial_1 P(Q_i, Q_j) [Q_i \partial_{12}^2 P(Q_i, Q_j) + Q_j \partial_{21}^2 P(Q_j, Q_i)]. \end{aligned}$$

The first welfare result says that introducing a single exclusive dealing agreement raises aggregate industry profit at the expense of consumer surplus and social welfare:

**Proposition 8** *Compared to the baseline case with no exclusive dealing, an exclusive dealing provision that precludes trade between  $M_A$  and  $R_2$ : i) under (A.4), reduces social welfare,  $W(Q^*, Q^*) < W(Q^\circ, Q^\circ)$ ; and ii) under (A.5), reduces consumer surplus,  $S(Q^*, Q^*) < S(Q^\circ, Q^\circ)$ , and increases aggregate profit,  $\Pi(Q^*, Q^*) \equiv \Pi^* > \Pi^\circ \equiv \Pi(Q^\circ, Q^\circ)$ .*

**Proof.** See Appendix I. ■

The second welfare result says that pairwise exclusive dealing increases profits at the expense of consumer surplus and social welfare:

**Proposition 9** *Compared to the baseline case with no exclusive dealing, exclusive dealing provisions that preclude trade between  $M_A$  and  $R_2$  and between  $M_B$  and  $R_1$ : i) reduce both consumer surplus and social welfare:  $S(Q^{**}, Q^{**}) < S(Q^\circ, Q^\circ)$  and  $W(Q^{**}, Q^{**}) < W(Q^\circ, Q^\circ)$ ; and ii) under (A.4), increase aggregate profit:  $\Pi(Q^{**}, Q^{**}) \equiv \Pi^{**} > \Pi^\circ \equiv \Pi(Q^\circ, Q^\circ)$ .*

**Proof.** See Appendix J. ■

The intuition is straightforward. Exclusive dealing restricts the number of retailers selling any given good, leading to less intense competition, lower outputs and higher prices. This increases firms' profits (as output levels remain above monopoly levels), but obviously harms consumers and reduces total welfare (as prices remain above marginal cost).

Intuitively, we would expect the impact of exclusivity to be more important when goods  $A$  and  $B$  are more differentiated. Indeed, exclusive dealing has no effect in the limit case of perfect substitutes (as, in that case, the retailers do not care about whether they sell one good or both of them), and enables instead firms to achieve the industry-wide monopoly outcome when goods  $A$  and  $B$  face independent demands. To illustrate this, consider the case of linear demand:<sup>14</sup>

$$P(Q_A, Q_B) = 1 - \frac{Q_A + sQ_B}{1 + s},$$

where  $s$  reflects the degree of substitution between  $A$  and  $B$ , and ranges from  $s = 0$  (independent demands) to  $s = 1$  (perfect substitutes). Normalizing the production cost to  $c \equiv 0$ , we have:

$$\begin{aligned} Q^{**} &= q^{**} = \frac{1}{2+s} < Q^\circ = 2q^\circ = \frac{2}{3}, \\ p^{**} &= P(Q^{**}, Q^{**}) = \frac{1}{2+s} > p^\circ = P(Q^\circ, Q^\circ) = \frac{1}{3}. \end{aligned}$$

Hence, pairwise exclusivity reduces output and raises prices, and all the more so as goods  $A$  and  $B$  become more differentiated:  $Q^{**}$  decreases, and  $p^{**}$  increases, as  $s$  decreases.

Intuitively, we would also expect each exclusive dealing provision to contribute to increasing profit, at the expense of consumers and allocative efficiency. This is indeed the case in the above linear model. When a single exclusive dealing provision precludes trade between  $M_A$  and  $R_2$ , the equilibrium prices and quantities are uniquely defined and given by:

$$\begin{aligned} q_{A1}^* &= \frac{1}{2} < Q^\circ = \frac{2}{3}, q_{B1}^* = \frac{2-s}{6} < q^\circ = \frac{1}{3} < q_{B2}^* = \frac{1+s}{3}, \\ p^{**} &= \frac{1}{2+s} > p_A^* = \frac{1}{2} - \frac{s}{6} > p_B^* = p^\circ = \frac{1}{3}. \end{aligned}$$

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<sup>14</sup>The demand is here normalized so as to ensure that its size remains constant (for symmetric configurations) when the degree of product differentiation varies. In particular, the benchmark monopoly quantity (summing over both goods),  $Q^M = 1/2$ , is independent of  $s$ .

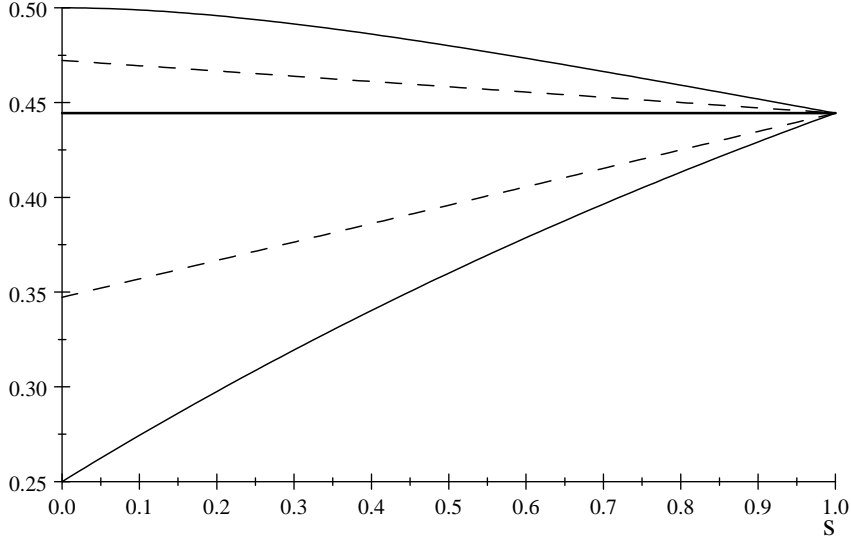


Figure 1: The figure depicts the industry profit (top curves) and consumer surplus (bottom curves) as a function of the substitution parameter  $s$ . The solid curves represent the case of pairwise exclusivity; the dashed curves the case of single exclusivity, and the bold line the non-exclusivity benchmark.

That is, starting from the baseline scenario with no exclusivity, shutting down the channel  $M_A - R_2$  induces  $R_1$  to sell more of good  $A$ , but not so much as to compensate for  $R_2$ 's lost sales of good  $A$ ; this also induces  $R_2$  to sell more of good  $B$ , a move partially offset by  $R_1$  reducing its own sales of that good (both because it faces a more aggressive rival  $R_2$  for good  $B$ , and because  $R_1$  itself sells more of the substitute good  $A$ ). As one price increases, the other one remaining constant, consumer surplus and social welfare decrease, whereas industry profit increases.

It can be checked that each exclusivity provision increases industry profit, and reduces both consumer surplus and social welfare:

$$\begin{aligned} \Pi^{**} &= 2p^{**}Q^{**} = 2\frac{1+s}{(2+s)^2} > \Pi^* = p_A^*Q_A^* + p_B^*Q_B^* = \frac{17-s}{36} > \Pi^\circ = 2p^\circ Q^\circ = \frac{4}{9}, \\ S^{**} &= S(Q^{**}, Q^{**}) = \frac{(1+s)^2}{(2+s)^2} < S^* = S(Q_B^*, Q_A^*) = \frac{25+7s}{72} < S^\circ = S(Q^\circ, Q^\circ) = \frac{4}{9}, \\ W^{**} &= S^{**} + \Pi^{**} = \frac{3+4s+s^2}{(s+2)^2} < W^* = \Pi^* + S^* = \frac{59+5s}{72} < W^\circ = S^\circ + \Pi^\circ = \frac{8}{9}. \end{aligned}$$

This is illustrated by Figure 1, which further shows that the impact of *each* exclusivity provision is also larger when the goods are more differentiated:

## 5 Vertical Integration

In this section, we analyze the positive and normative effects of vertical integration. We begin by considering the case of a single vertically integrated upstream-downstream pair, and then turn to the case of pairwise vertical integration. We show that, under single vertical integration, there exists an equilibrium in which the integrated firm forecloses its downstream rival; this equilibrium thus replicates the outcome (in terms of retail prices and quantities) under single exclusive dealing. That is, a vertical merger leads to the foreclosure of the rival retailer. We also show that pairwise vertical integration yields a unique equilibrium outcome, in which each vertically integrated firm forecloses its rival. The welfare analysis of vertical integration therefore mirrors that of exclusive dealing: vertical integration reduces both consumer surplus and social welfare.

We begin by considering the case where a single upstream-downstream pair,  $M_A - R_1$  say, is vertically integrated. Our previous analysis allows us to provide a very partial characterization of equilibrium:

**Lemma 2** *Suppose that  $M_A - R_1$  are vertically integrated whereas  $M_B$  and  $R_2$  are vertically separated. Then, in equilibrium, the unintegrated manufacturer  $M_B$  signs a cost-based contract with each retailer. The vector of equilibrium quantities,  $(q_{A1}^*, q_{A2}^*, q_{B1}^*, q_{B2}^*)$ , is thus such that*

$$q_{Bh}^* = \arg \max_{q_{Bh}} [P(q_{Bh} + q_{Bk}^*, q_{Ah}^* + q_{Ak}^*) - c] q_{Bh} + P(q_{Ah}^* + q_{Ak}^*, q_{Bh} + q_{Bk}^*) q_{Ah}^*$$

for all  $h \neq k \in \{1, 2\}$ .

**Proof.** This is an immediate implication of Lemma 1. ■

Intuitively, the integrated  $M_A$  does not need access to  $R_2$  to sell its good (any unit that  $M_A$  sells through  $R_2$  could instead be sold directly through the downstream affiliate  $R_1$ ), and it has moreover an incentive to protect its own retailer  $R_1$  from intrabrand competition. Indeed, the following proposition shows that there exists an equilibrium in which the integrated firm will not supply its downstream rival:

**Proposition 10** *Assume (P.1)-(P.3). Then, there exists an equilibrium in which:*

- $M_A$  offers  $R_2$  a quantity-forcing contract,  $(\hat{q}, \hat{T})$ , whereas  $M_B$  offers  $R_1$  a (cost-based) quantity-forcing contract,  $(q_{B1}^*, T_{B1}^*)$ , and offers  $R_2$  to supply any quantity at cost (i.e.,  $\tau_{B2}(q) = cq$ , for any  $q \geq 0$ ).

- $R_2$  is indifferent between accepting both  $M_A$ 's and  $M_B$ 's contracts, or either one of them, and rejects  $M_A$ 's offer;
- $R_1$  is indifferent between accepting and rejecting  $M_B$ 's contract, and accepts it.

**Proof.** See Appendix K. ■

The equilibrium under single vertical integration of  $M_A - R_1$ , characterized by the proposition, replicates the outcome in terms of retail prices and quantities that would obtain under an exclusive distribution contract between  $M_A$  and  $R_1$  (or under a single branding contract between  $M_B$  and  $R_2$ ). However, unlike in the case of exclusive dealing, the independent retailer  $R_2$  here extracts some rents thanks to the competition between the two manufacturers for its business. In equilibrium,  $M_A$  makes an attractive offer to  $R_2$ , the anticipation of which prompts  $M_B$  to make a generous offer to  $R_2$ , which in turn prevents  $M_A$  from winning the competition for  $R_2$ 's business.

Proposition 10 extends the analysis of Hart and Tirole (1990) to the case of oligopolistic upstream competition. As in the case of a pure upstream bottleneck, the integrated manufacturer  $M_A$  completely forecloses the downstream rival  $R_2$ , and monopolizes the distribution of its product. By contrast with the case of a pure upstream bottleneck, the competition for  $R_2$ 's business leads  $M_A$  to offer a contract guaranteeing some rents to  $R_2$ , which  $R_2$  however chooses to reject. Interestingly, this can be contrasted with another situation considered by Hart and Tirole, in which  $M_A$  would face a less efficient competitive fringe of suppliers, offering the same input but facing a higher cost than  $M_A$ . In that situation,  $M_A$  would end up supplying  $R_2$  (although on terms based on the higher cost of the competitive fringe). By contrast, here the other supplier offers a differentiated good, and each manufacturer is “more efficient” than its rival on the provision of its own input; as a result,  $M_A$  ends up *not* supplying  $R_2$ . Finally, the proposition also shows that, while the integrated manufacturer forecloses the downstream rival, the integrated retailer keeps dealing with the independent manufacturer.

We now turn to the case where there are two vertically integrated firms,  $M_A - R_1$  and  $M_B - R_2$ . Intuitively, both manufacturers have an incentive to protect their own retailers from intrabrand competition. The following proposition shows that pairwise vertical integration leads indeed to complete foreclosure of rivals, mirroring the outcome under pairwise exclusive dealing:

**Proposition 11** *Suppose  $M_A - R_1$  and  $M_B - R_2$  are vertically integrated. If Properties (P.1)-(P.3) hold, then there exists a unique equilibrium,  $(q_{A1}^{**}, q_{A2}^{**}, q_{B1}^{**}, q_{B2}^{**})$ , in which moreover there is no cross-selling:  $(q_{A1}^{**}, q_{A2}^{**}, q_{B1}^{**}, q_{B2}^{**}) = (Q^{**}, 0, 0, Q^{**})$ , where  $Q^{**} \equiv \arg \max_Q [P(Q, Q^{**}) - c] Q$ .*

**Proof.** See Appendix L. ■

The proposition shows that pairwise vertical integration leads to a strong form of foreclosure, as each integrated firm refuses to deal with the other integrated firm. In particular, combined with Lemma 2, it shows that pairwise vertical integration is “less competitive” than single vertical integration. It follows from our previous welfare analysis that vertical integration harms consumers and society. In particular, under pairwise vertical integration both prices are higher (and consumer surplus as well as social welfare are thus lower) than under vertical separation.

From our analysis of exclusive dealing, it also follows that firms have an incentive to integrate vertically: If no firm is vertically integrated, there exists a manufacturer-retailer pair, say  $M_A - R_1$ , that can increase their joint profit by merging. Moreover, if  $M_A - R_1$  are vertically integrated, the remaining manufacturer-retailer pair  $M_B - R_2$  can also increase their joint profit by merging.

We conclude this section by noting that “complete foreclosure” arises here from the fact that a single retailer suffices to serve the entire market. If it were not the case, e.g., due to downstream capacity constraints or to differentiation among the retailers, then integrated manufacturers would still wish to deal with downstream rivals in order to expand market coverage or serve customer niches; in such situations, we would thus expect vertical integration to result into partial rather than complete foreclosure. By the same token, in such situations vertical integration (and partial foreclosure) is likely to be more profitable than exclusive dealing (and thus complete foreclosure).<sup>15</sup>

## 6 Conclusion

In this paper, we develop a tractable framework for the analysis of interlocking bilateral relationships in vertically related markets. Key features of the framework are that upstream firms are horizontally differentiated, contract offers and acceptance decisions are private information to the contracting parties, and any tariff (e.g., linear, two-part, quantity forcing, and so forth) can be used. In the absence of exclusive dealing provisions and vertical integration, all channels are active and involve cost-based (nonlinear) tariffs. Under mild regularity conditions, there exists a unique equilibrium outcome in terms of retail prices and quantities; we also provide a complete characterization of the equilibrium outcomes in terms of profit sharing between manufacturers and retailers.

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<sup>15</sup>See Rey and Tirole (2007) for an analysis of the impact of downstream differentiation on the extent of foreclosure in the case of an upstream monopoly.

We use this framework to shed some light on a long-standing debate on vertical foreclosure. More specifically, we analyze the positive and normative effects of exclusive dealing and vertical integration, and show that firms have an incentive to engage in exclusive dealing or vertical integration to exert more market power, at the expense of consumers and society.

There are exciting avenues for future research. First, it would be natural to extend the model to an arbitrary number of manufacturers and retailers, and to introduce upstream and/or downstream firm heterogeneity. Second, it would be interesting to allow retailers to be horizontally differentiated. We expect that, in this case, vertical integration no longer leads to complete foreclosure of rival retailers, unlike exclusive dealing provisions. Third, it seems important to extend the analysis to downstream price competition, which is however known to raise additional issues for the treatment of out-of-equilibrium beliefs.<sup>16</sup> Finally, and perhaps most importantly, the framework developed in this paper can be used to study the positive and normative effects of other contractual arrangements, such as “fidelity rebates” based on market shares, MFN clauses, or agency contracts.

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<sup>16</sup>Passive beliefs appear to remain the most tractable approach in that case, but they become less compelling – in particular, they no longer coincide with wary beliefs – and equilibrium existence is no longer guaranteed; see Rey and Vergé (2004).



## A Proof of Lemma 1

Fix a candidate equilibrium, with associated equilibrium quantities  $(q_{ih}^e)_{i=A,B,h=1,2}$  and acceptance decisions  $(\delta_{ih}^e)_{i=A,B,h=1,2}$ , with the convention that  $\delta_{ih}^e = 1$  if  $M_i$  and  $R_h$  are vertically integrated and, when they are independent,  $\delta_{ih}^e = 1$  if the offer is accepted and  $\delta_{ih}^e = 0$  if it is not (in which case  $q_{ih}^e = 0$ ). Suppose that an unintegrated  $M_i$  deviates and offers  $R_h$  a cost-based two-part tariff  $(c, \tilde{F}_{ih})$ , where the fixed fee  $\tilde{F}_{ih}$  is as follows:

- if  $R_h$  is vertically integrated with  $M_j$ , then:

$$\begin{aligned} \tilde{F}_{ih} = \max_{q_{ih}} \{ & [P(q_{ih} + q_{ik}^e, q_{jh}^e + q_{jk}^e) - c] q_{ih} \\ & + [P(q_{jh}^e + q_{jk}^e, \tilde{q}_{ih} + q_{ik}^e) - c] q_{jh}^e + \delta_{jk}^e [\tau_{jk}^e(q_{jk}^e) - cq_{jk}^e] \} - \pi_{j-h}^e, \end{aligned} \quad (1)$$

where  $\pi_{j-h}^e$  denotes the profit of the integrated firm  $M_j - R_h$  in the candidate equilibrium. The terms in curly brackets represent the profit that the vertically integrated firm  $M_j - R_h$  would make if  $R_h$  accepted  $M_i$ 's deviant offer and maintained the equilibrium quantity  $q_{jh}^e$ , and  $R_k$  maintained the equilibrium quantities  $q_{ik}^e$  and  $q_{jk}^e$ :

- the first two terms are the profits generated by, respectively, the channels  $M_i - R_h$  and  $M_j - R_h$ ,
- whereas the third term is the profit that  $M_j$  generates in equilibrium through the sales to the other, unintegrated retailer  $R_k$ .

- if instead  $R_h$  is not vertically integrated, then:

$$\begin{aligned} \tilde{F}_{ih} = \max_{q_{ih}} \{ & [P(q_{ih} + q_{ik}^e, \delta_{jh}^e q_{jh}^e + q_{jk}^e) - c] q_{ih} \\ & + \delta_{jh}^e [P(q_{jh}^e + q_{jk}^e, q_{ih} + q_{ik}^e) q_{jh}^e - \tau_{jh}^e(q_{jh}^e)] \} - \pi_h^e, \end{aligned} \quad (2)$$

where  $\pi_h^e$  denotes the profit that the unintegrated  $R_h$  makes in equilibrium. The terms in curly brackets represent the profit that the unintegrated  $R_h$  would make if it accepted  $M_i$ 's deviant offer and maintained its acceptance decision  $\delta_{jh}^e$  vis-à-vis  $M_j$ 's contract offer as well as the equilibrium quantity  $q_{jh}^e$ , and  $R_k$  maintained the equilibrium quantities  $q_{ik}^e$  and  $q_{jk}^e$ :

- the first term is the profit generated by the channel  $M_i - R_h$ ,

– whereas the second term is the profit that  $R_h$  makes on its contract with  $M_j$ .

We first claim that  $R_h$  is willing to accept the deviant offer  $(c, \tilde{F}_{ih})$ :

1. Having passive beliefs, at the acceptance stage  $R_h$  continues to believe that its downstream rival  $R_k$  has been offered the equilibrium contracts and will sell the equilibrium quantities  $q_{ik}^e$  and  $q_{jk}^e$  in the continuation game.
2. By accepting  $M_i$ 's deviant offer,  $R_h$  can make the same profit as in the candidate equilibrium by sticking to its acceptance decision vis-à-vis  $M_j$ 's nondeviant offer and maintaining the quantity  $q_{jh}$  at its equilibrium level  $q_{jh}^e$ , and can do only better by optimizing over these decisions.
3. If instead  $R_h$  rejects  $M_i$ 's deviant offer, it obtains the same profit as in the continuation game following the rejection of  $M_i$ 's equilibrium offer. By construction, this cannot exceed  $M_i$ 's equilibrium profit: it constitutes the equilibrium profit if in equilibrium  $R_h$  rejects  $\tau_{ih}^e$ , and must be (weakly) lower otherwise.

As  $R_h$  is willing to accept this deviant offer (and can be induced to do so, if needed, by slightly reducing the fixed fee  $\tilde{F}_{ih}$ ), which gives  $M_i$  a profit equal to  $\tilde{F}_{ih}$ , this deviation is unprofitable only if  $\tilde{F}_{ih} \leq \pi_{i,h}^e$ , where

$$\pi_{i,h}^e = \delta_{ih}^e [\tau_{ih}^e (q_{ih}^e) - cq_{ih}^e]$$

denotes the equilibrium profit that  $M_i$  makes from selling through retailer  $R_h$ . But then:

- If  $R_h$  is vertically integrated with  $M_j$  (implying  $\delta_{ih}^e = 1$ ), we can rewrite  $\pi_{i,h}^e$  as follows:

$$\begin{aligned} \pi_{i,h}^e &= \{ [P(q_{ih}^e + q_{ik}^e, q_{jh}^e + q_{jk}^e) - c] q_{ih}^e \\ &\quad + [P(q_{jh}^e + q_{jk}^e, q_{ih}^e + q_{ik}^e) - c] q_{jh}^e + \delta_{jk}^e [\tau_{jk}^e (q_{jk}^e) - cq_{jk}^e] \} - \pi_{j-h}^e. \end{aligned}$$

Using (1),  $\pi_{i,h}^e \geq \tilde{F}_{ih}$  then implies  $q_{ih}^e \in \chi(q_{ik}^e, q_{jh}^e, q_{jk}^e)$ .

- If instead  $R_h$  is unintegrated, we can rewrite  $\pi_{i,h}^e$  as follows:

$$\begin{aligned} \pi_{i,h} &= \{ [P(q_{ih}^e + q_{ik}^e, \delta_{jh}^e q_{jh}^e + q_{jk}^e) - c] q_{ih}^e \\ &\quad + \delta_{jh}^e [P(q_{jh}^e + q_{jk}^e, q_{ih}^e + q_{ik}^e) q_{jh}^e - \tau_{jh}^e (q_{jh}^e)] \} - \pi_h^e. \end{aligned}$$

Using (2),  $\pi_{i,h}^e \geq \tilde{F}_{ih}$  then implies  $q_{ih}^e \in \chi(q_{ik}^e, q_{jh}^e, q_{jk}^e)$ .

## B Proof of Proposition 2

From Proposition 1, we know that the equilibrium quantities satisfy  $q_{ih}^\circ \in \chi(q_{ik}^\circ, q_{jh}^\circ, q_{jk}^\circ)$ . We first show that Assumptions (A.3)-(A.4) ensure that all quantities are positive; we then use first-order conditions to characterize the unique, symmetric equilibrium outcome.

### B.1 Interior solution

To see that all quantities are positive, suppose that  $q_{B2}^\circ$ , say, is zero.

*Step 1:*  $q_{B1}^\circ > 0$ . Suppose otherwise that  $q_{B1}^\circ = 0$ . By construction, we then have:

$$q_{Ah}^\circ = \arg \max_{q_{Ah}} [P(q_{Ah} + q_{Ak}^\circ, 0) - c] q_{Ah}.$$

Note that  $q_{A1}^\circ = 0$  would imply  $p_A^\circ = P(q_{A2}^\circ, 0) \leq c$ , and thus  $q_{A2}^\circ = 0$  as well,<sup>17</sup> but this would therefore require  $P(0, 0) \leq c$ , contradicting the viability condition (A.1). Thus, we can assume that  $q_{A1}^\circ$  is positive, and thus satisfies  $R_1$ 's first-order condition which, using

$$\pi_1 = [P(q_{A1} + q_{A2}^\circ, q_{B1}) - c] q_{A1} + [P(q_{B1}, q_{A1} + q_{A2}^\circ) - c] q_{B1},$$

and  $q_{B1}^\circ = 0$ , is given by:

$$\left. \frac{\partial \pi_1}{\partial q_{A1}} \right|_{(q_{ih})=(q_{ih}^\circ)} = P(Q_A^\circ, 0) - c + \partial_1 P(Q_A^\circ, 0) q_{A1}^\circ = 0.$$

But then, a small increase in  $q_{B1}$  would increase  $R_1$ 's profit:

$$\begin{aligned} \left. \frac{\partial \pi_1}{\partial q_{B1}} \right|_{(q_{ih})=(q_{ih}^\circ)} &= P(0, Q_A^\circ) - c + \partial_2 P(Q_A^\circ, 0) q_{A1}^\circ \\ &> P(Q_A^\circ, 0) - c + \partial_1 P(Q_A^\circ, 0) q_{A1}^\circ = 0, \end{aligned}$$

where the inequality stems from (A.2) ( $\partial_2 P > \partial_1 P$ , which also implies  $P(0, Q) > P(Q, 0)$  for any  $Q > 0$ ).

*Step 2:*  $q_{A2}^\circ > q_{A1}^\circ$ . From Step 1,  $q_{B1}^\circ$  is positive and therefore satisfies the first-order condition

$$\left. \frac{\partial \pi_1}{\partial q_{B1}} \right|_{(q_{ih})=(q_{ih}^\circ)} = P(Q_B^\circ, Q_A^\circ) - c + \partial_1 P(Q_B^\circ, Q_A^\circ) q_{B1}^\circ + \partial_2 P(Q_A^\circ, Q_B^\circ) q_{A1}^\circ = 0. \quad (3)$$

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<sup>17</sup>Otherwise, a slight reduction in  $q_{A2}$  would increase  $R_2$ 's profit  $\pi_2^\circ = (p_A^\circ - c) q_{A2}^\circ$ , since then

$$\left. \frac{\partial \pi_2}{\partial q_{A2}} \right|_{(q_{ih})=(q_{ih}^\circ)} = p_A^\circ - c + q_{A2}^\circ \partial_1 P(q_{A2}^\circ, 0) < p_A^\circ - c \leq 0.$$

From (A.2),  $\partial_1 P \leq 0$  and  $\partial_2 P \leq 0$  and thus  $P(Q_B^\circ, Q_A^\circ) \geq c > 0$ ; but (A.2) then implies  $\partial_1 P(Q_B^\circ, Q_A^\circ) < 0$ , which in turn yields  $P(Q_B^\circ, Q_A^\circ) > c$ .

Using

$$\pi_2 = [P(q_{A1}^\circ + q_{A2}, q_{B1}^\circ + q_{B2}) - c] q_{A2} + [P(q_{B1}^\circ + q_{B2}, q_{A1}^\circ + q_{A2}) - c] q_{B2},$$

the first-order condition for  $q_{B2}^\circ = 0$  yields:

$$\left. \frac{\partial \pi_2}{\partial q_{B2}} \right|_{(q_{ih})=(q_{ih}^\circ)} = P(Q_B^\circ, Q_A^\circ) - c + \partial_2 P(Q_A^\circ, Q_B^\circ) q_{A2}^\circ \leq 0. \quad (4)$$

Using (A.2) and  $P(Q_B^\circ, Q_A^\circ) > c$ , it follows that  $q_{A2}^\circ$  is positive and thus satisfies the first-order condition

$$\left. \frac{\partial \pi_2}{\partial q_{A2}} \right|_{(q_{ih})=(q_{ih}^\circ)} = P(Q_A^\circ, Q_B^\circ) - c + \partial_1 P(Q_A^\circ, Q_B^\circ) q_{A2}^\circ = 0,$$

implying  $P(Q_A^\circ, Q_B^\circ) > c$ .

Subtracting (3) from (4) yields:

$$\partial_2 P(Q_A^\circ, Q_B^\circ) (q_{A2}^\circ - q_{A1}^\circ) \leq \partial_1 P(Q_B^\circ, Q_A^\circ) q_{B1}^\circ,$$

where  $\partial_1 P_A < 0$  and  $\partial_2 P_B < 0$  (from (A.2), as both prices are positive), and  $q_{B1}^\circ > 0$  (from Step 1); therefore,  $q_{A2}^\circ > q_{A1}^\circ$ .

*Step 3:*  $q_{A1}^\circ > 0$ . Suppose otherwise that  $q_{A1}^\circ = 0$ . In that case,  $Q_A^\circ = q_{A2}^\circ$  and  $Q_B^\circ = q_{B1}^\circ$  satisfy  $Q_B^\circ = \hat{\chi}(Q_A^\circ)$  and  $Q_A^\circ = \hat{\chi}(Q_B^\circ)$ , where the best response function

$$\hat{\chi}(Q) \equiv \arg \max_Q \left\{ [P(\hat{Q}, Q) - c] \hat{Q} \right\}$$

is characterized by the first-order condition:

$$P(\hat{\chi}(Q), Q) - c + \partial_1 P(\hat{\chi}(Q), Q) \hat{\chi}(Q) = 0.$$

Assumption (A.3) ensures that this response function satisfies

$$-1 < \hat{\chi}'(Q) < 0.$$

Therefore, we must have  $Q_A^\circ = Q_B^\circ = \hat{Q}^\circ$ , where  $\hat{Q}^\circ$  is such that  $\hat{Q}^\circ = \hat{\chi}(\hat{Q}^\circ)$ , and thus satisfies:

$$P(\hat{Q}^\circ, \hat{Q}^\circ) - c + \partial_1 P(\hat{Q}^\circ, \hat{Q}^\circ) \hat{Q}^\circ = 0.$$

But then, each retailer would want to sell the other brand as well:

$$\left. \frac{\partial \pi_1}{\partial q_{A1}} \right|_{(q_{ih})=(q_{ih}^\circ)} = \left. \frac{\partial \pi_2}{\partial q_{B2}} \right|_{(q_{ih})=(q_{ih}^\circ)} = P(\hat{Q}^\circ, \hat{Q}^\circ) - c + \partial_2 P(\hat{Q}^\circ, \hat{Q}^\circ) \hat{Q}^\circ > 0$$

as  $P(\hat{Q}^\circ, \hat{Q}^\circ) > c$  from above, and thus  $\partial_1 P(\hat{Q}^\circ, \hat{Q}^\circ) < \partial_2 P(\hat{Q}^\circ, \hat{Q}^\circ) < 0$  from (A.2). Hence,  $q_{A1}^\circ > 0$ .

*Step 4.* It follows from the previous steps that  $q_{A2}^\circ$ ,  $q_{A1}^\circ$  and  $q_{B1}^\circ$  must all be positive, and thus satisfy the first-order conditions:

$$\left. \frac{\partial \pi_1}{\partial q_{A1}} \right|_{(q_{ih})=(q_{ih}^\circ)} = P(Q_A^\circ, Q_B^\circ) - c + \partial_1 P(Q_A^\circ, Q_B^\circ) q_{A1}^\circ + \partial_2 P(Q_B^\circ, Q_A^\circ) q_{B1}^\circ = 0, \quad (5)$$

$$\left. \frac{\partial \pi_1}{\partial q_{B1}} \right|_{(q_{ih})=(q_{ih}^\circ)} = P(Q_B^\circ, Q_A^\circ) - c + \partial_1 P(Q_B^\circ, Q_A^\circ) q_{B1}^\circ + \partial_2 P(Q_A^\circ, Q_B^\circ) q_{A1}^\circ = 0, \quad (6)$$

$$\left. \frac{\partial \pi_2}{\partial q_{A2}} \right|_{(q_{ih})=(q_{ih}^\circ)} = P(Q_A^\circ, Q_B^\circ) - c + \partial_1 P(Q_A^\circ, Q_B^\circ) q_{A2}^\circ = 0, \quad (7)$$

whereas the first-order condition for  $q_{B2}^\circ = 0$  yields:

$$\left. \frac{\partial \pi_2}{\partial q_{B2}} \right|_{(q_{ih})=(q_{ih}^\circ)} = P(Q_B^\circ, Q_A^\circ) - c + \partial_2 P(Q_A^\circ, Q_B^\circ) q_{A2}^\circ \leq 0. \quad (8)$$

Subtracting (7) from (5) and (6) from (8) yields:

$$\begin{aligned} -\partial_1 P(Q_A^\circ, Q_B^\circ) (q_{A2}^\circ - q_{A1}^\circ) &= -\partial_2 P(Q_B^\circ, Q_A^\circ) q_{B1}^\circ, \\ -\partial_2 P(Q_A^\circ, Q_B^\circ) (q_{A2}^\circ - q_{A1}^\circ) &\geq -\partial_1 P(Q_B^\circ, Q_A^\circ) q_{B1}^\circ. \end{aligned}$$

The first condition yields  $q_{A2}^\circ > q_{A1}^\circ$ , and thus the two conditions can be rewritten as:

$$\frac{-\partial_1 P(Q_A^\circ, Q_B^\circ)}{-\partial_2 P(Q_B^\circ, Q_A^\circ)} = \frac{q_{B1}^\circ}{q_{A2}^\circ - q_{A1}^\circ} \leq \frac{-\partial_2 P(Q_A^\circ, Q_B^\circ)}{-\partial_1 P(Q_B^\circ, Q_A^\circ)}.$$

This, in turn, implies

$$\partial_1 P(Q_A^\circ, Q_B^\circ) \partial_1 P(Q_B^\circ, Q_A^\circ) \leq \partial_2 P(Q_B^\circ, Q_A^\circ) \partial_2 P(Q_A^\circ, Q_B^\circ),$$

a contradiction as  $\partial_1 P < \partial_2 P < 0$  from (A.2). Hence, there is no equilibrium in which  $q_{B2}^\circ = 0$ .

## B.2 The equilibrium outcome is unique and symmetric

It follows from the above analysis that all equilibrium quantities are positive and thus satisfy the first-order conditions. Adding the conditions for good  $A$ , namely:

$$\left. \frac{\partial \pi_1}{\partial q_{A1}} \right|_{(q_{ih})=(q_{ih}^\circ)} = P(Q_A^\circ, Q_B^\circ) - c + \partial_1 P(Q_A^\circ, Q_B^\circ) q_{A1}^\circ + \partial_2 P(Q_B^\circ, Q_A^\circ) q_{B1}^\circ = 0,$$

$$\left. \frac{\partial \pi_2}{\partial q_{A2}} \right|_{(q_{ih})=(q_{ih}^\circ)} = P(Q_A^\circ, Q_B^\circ) - c + \partial_1 P(Q_A^\circ, Q_B^\circ) q_{A2}^\circ + \partial_2 P(Q_B^\circ, Q_A^\circ) q_{B2}^\circ = 0,$$

implies that

$$P(Q_A^\circ, Q_B^\circ) \geq c > 0$$

and

$$Q_A^\circ = \tilde{\chi}(Q_B^\circ),$$

where  $Q_A = \tilde{\chi}(Q_B)$  denotes the “best-response” function defined by

$$\phi(Q_A, Q_B) \equiv 2[P(Q_A, Q_B) - c] + \partial_1 P(Q_A, Q_B) Q_A + \partial_2 P(Q_B, Q_A) Q_B = 0.$$

Likewise, adding the first-order conditions for good  $B$  yields  $P(Q_B^\circ, Q_A^\circ) \geq c > 0$  and  $Q_B^\circ = \tilde{\chi}(Q_A^\circ)$ . The derivatives of  $\phi$  are given by:

$$\begin{aligned} \partial_1 \phi(Q_A, Q_B) &= 3\partial_1 P(\tilde{\chi}(Q), Q) + \partial_{11} P(\tilde{\chi}(Q), Q) \tilde{\chi}(Q) + \partial_{22} P(Q, \tilde{\chi}(Q)) Q, \\ \partial_2 \phi(Q_A, Q_B) &= 2\partial_2 P(\tilde{\chi}(Q), Q) + \partial_2 P(Q, \tilde{\chi}(Q)) + \partial_{12}^2 P(\tilde{\chi}(Q), Q) \tilde{\chi}(Q) + \partial_{21} P(Q, \tilde{\chi}(Q)) Q. \end{aligned}$$

Assumptions (A.2)<sup>18</sup> and (A.4) ensure that  $\partial_1 \phi(Q_A, Q_B) < \partial_2 \phi(Q_A, Q_B) < 0$ . Hence the reaction function  $\tilde{\chi}(\cdot)$  is uniquely defined and such that

$$\tilde{\chi}'(Q) = -\frac{\partial_2 \phi(Q_A, Q_B)}{\partial_1 \phi(Q_A, Q_B)} \in (-1, 0).$$

It follows that the equilibrium is symmetric:  $Q_A^\circ = Q_B^\circ = Q^\circ$ . The first-order conditions for  $R_h$ 's quantity choices, for  $h \in \{1, 2\}$ , then yield:

$$\begin{aligned} -\partial_1 P(Q^\circ, Q^\circ) q_{Ah}^\circ - \partial_2 P(Q^\circ, Q^\circ) q_{Bh}^\circ &= P(Q^\circ, Q^\circ) - c, \\ -\partial_1 P(Q^\circ, Q^\circ) q_{Bh}^\circ - \partial_2 P(Q^\circ, Q^\circ) q_{Ah}^\circ &= P(Q^\circ, Q^\circ) - c, \end{aligned}$$

and thus

$$q_{Ah}^\circ = q_{Bh}^\circ = q^\circ \equiv -\frac{P(Q^\circ, Q^\circ) - c}{\partial_1 P(Q^\circ, Q^\circ) + \partial_2 P(Q^\circ, Q^\circ)}.$$

## C Proof of Proposition 3

Consider a candidate equilibrium in which each manufacturer offers the cost-based two-part tariffs  $(w, F) = (c, \Delta^\circ)$ , which accept it.

We first note that the continuation equilibrium is then such that both retailers sell  $(q_{Ah}, q_{Bh}) = (q^\circ, q^\circ)$ , for  $h = 1, 2$ . From the proof of Proposition 2, this constitutes the unique candidate equilibrium, and it satisfies all first-order conditions. It thus suffices to check that retailers' profit

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<sup>18</sup>Using demand symmetry (see footnote 2), (A.2) implies  $\partial_1 P(Q_A, Q_B) < \partial_2 P(Q_A, Q_B) = \partial_2 P(Q_B, Q_A)$ .

functions are concave. Anticipating that its rival will sell  $(q^\circ, q^\circ)$ , by selling  $(q_A, q_B)$  a retailer obtains a profit (gross of the fixed fees) equal to

$$\Pi_R^\circ(q_A, q_B) \equiv [P(q_A + q^\circ, q_B + q^\circ) - c]q_A + [P(q_B + q^\circ, q_A + q^\circ) - c]q_B.$$

The second-order derivatives of  $\Pi_R^\circ(\cdot, \cdot)$  are given by:

$$\begin{aligned} \partial_{11}^2 \Pi_R^\circ(q_A, q_B) &= 2\partial_1 P(q_A + q^\circ, q_B + q^\circ) + \partial_{11}^2 P(q_A + q^\circ, q_B + q^\circ)q_A + \partial_{22}^2 P(q_B + q^\circ, q_A + q^\circ)q_B, \\ \partial_{22}^2 \Pi_R^\circ(q_A, q_B) &= 2\partial_1 P(q_B + q^\circ, q_A + q^\circ) + \partial_{11}^2 P(q_B + q^\circ, q_A + q^\circ)q_B + \partial_{22}^2 P(q_A + q^\circ, q_B + q^\circ)q_A, \\ \partial_{12}^2 \Pi_R^\circ(q_A, q_B) &= \partial_2 P(q_A + q^\circ, q_B + q^\circ) + \partial_2 P(q_B + q^\circ, q_A + q^\circ) \\ &\quad + \partial_{12}^2 P(q_A + q^\circ, q_B + q^\circ)q_A + \partial_{12}^2 P(q_B + q^\circ, q_A + q^\circ)q_B. \end{aligned}$$

Assumption (A.4) then ensures that the profit function  $\Pi_R^\circ$  is strictly concave, as  $\partial_{11}^2 \Pi_R^\circ$  and  $\partial_{22}^2 \Pi_R^\circ$  are both negative, and the determinant of the Hessian is positive:  $\partial_{11}^2 \Pi_R^\circ \partial_{22}^2 \Pi_R^\circ > (\partial_{12}^2 \Pi_R^\circ)^2$ . The assumption moreover yields  $\partial_{12}^2 \Pi_R^\circ < 0$ , which will be used in the proof of 4.

Next, we note that each retailer is willing to carry both goods. Indeed, the fee  $F^\circ$  is such that, if its rival were to accept both offers and sell  $(q^\circ, q^\circ)$ , then a retailer:

- Obtains the same profit,  $\pi_R^\circ$ , by accepting both manufacturers' offers or only one of them:

$$\pi_R^\circ = 2 \{ [P(2q^\circ, 2q^\circ) - c]q^\circ - F^\circ \} = \max_q [P(q + q^\circ, q^\circ) - c]q^\circ - F^\circ;$$

- Strictly prefers securing this profit to rejecting both offers:

$$\begin{aligned} \frac{\pi_R^\circ}{2} &= [P(2q^\circ, 2q^\circ) - c]q^\circ - F^\circ \\ &= \max_q [P(q^\circ + q, q^\circ) - c]q - [P(2q^\circ, 2q^\circ) - c]q^\circ \\ &> \max_q [P(q^\circ + q, q^\circ) - c]q - [P(2q^\circ, q^\circ) - c]q^\circ \\ &\geq 0. \end{aligned}$$

Thus, if these contracts are offered, it is a continuation equilibrium for both retailers to accept both manufacturers' offers, and then to sell  $(q^\circ, q^\circ)$ . We now show that manufacturers cannot profitably deviate from this candidate equilibrium. As the profit that a manufacturer achieves with a retailer is not affected by its relation with the other retailer, without loss of generality we can restrict attention to "one-sided" deviations, in which a manufacturer offers a

deviating contract to one of the retailers. Furthermore, the above tariffs are profitable for the manufacturers:

$$\begin{aligned}
F^\circ &= [P(2q^\circ, 2q^\circ) - c] 2q^\circ - \max_q [P(q + q^\circ, q^\circ) - c] q \\
&= \max_{q_A, q_B} \{ [P(q_A + q^\circ, q_B + q^\circ) - c] q_A + [P(q_B + q^\circ, q_A + q^\circ) - c] q_B \} - \max_q [P(q + q^\circ, q^\circ) - c] q \\
&> 0,
\end{aligned}$$

where the second equality comes from the definition of  $q^\circ$  and the inequality comes from the fact that the second optimization problem is more constrained than the first one (and the optimal  $q_A$  and  $q_B$  are indeed both positive, as they are equal to  $q^\circ$ ). It follows that a deviation cannot be profitable if it is not accepted by the retailer; and since the retailer can secure its equilibrium profit  $\pi_R^\circ$  by accepting only the rival's offer, it must be the case that the deviation increases the joint profit of the manufacturer and of the retailer.

If the deviation induces the retailer to keep dealing with the other manufacturer, then the joint profit of the manufacturer and of the retailer (gross of the manufacturer's cost of supplying  $q^\circ$  to the rival retailer, which is not affected by the deviation) cannot exceed

$$\max_{q_A, q_B} \{ [P(q_A + q^\circ, q_B + q^\circ) - c] q_A + [P(q_B + q^\circ, q_A + q^\circ) - c] q_B \} = [P(2q^\circ, 2q^\circ) - c] 2q^\circ,$$

which the two parties already obtain in the candidate equilibrium. Therefore, such a deviation cannot be profitable.

If instead the deviation induces the retailer to reject the other manufacturer's offer, then the joint profit of the manufacturer and of the retailer (again gross of the manufacturer's cost of supplying  $q^\circ$  to the rival retailer) cannot exceed

$$\begin{aligned}
\max_q \{ [P(q + q^\circ, q^\circ) - c] q \} + F^\circ &= \max_q \{ [P(q + q^\circ, q^\circ) - c] q \} + [P(2q^\circ, 2q^\circ) - c] 2q^\circ \\
&\quad - \max_q [P(q + q^\circ, q^\circ) - c] q \\
&= [P(2q^\circ, 2q^\circ) - c] 2q^\circ,
\end{aligned}$$

which is again what they obtain in the candidate equilibrium. Therefore, such a deviation cannot be profitable either, which concludes the argument.

## D Proof of Proposition 4

Given the assumption on passive beliefs, we can focus on one particular retailer  $R$ , taking as given that the other retailer will sell  $(q^\circ, q^\circ)$ . Let

$$\rho_R^\circ(q_A, q_B) \equiv P(q_A + q^\circ, q_B + q^\circ) q_A + P(q_B + q^\circ, q_A + q^\circ) q_B$$



denote the total revenue generated by  $R$  selling  $(q_A, q_B)$ . As shown in the proof of Proposition 3, the associated profit

$$\Pi_R^\circ(q_A, q_B) = \rho_R^\circ(q_A, q_B) - c(q_A + q_B)$$

is strictly concave in  $(q_A, q_B)$  in the range where it is positive; this profit moreover reaches its maximum at  $(q_A, q_B) = (q^\circ, q^\circ)$ , where it is equal to half the aggregate industry profit,  $\Pi_R^\circ(q^\circ, q^\circ) = \Pi^\circ/2$ .

Let  $\tau_{iR}^\circ(q_i)$  denote the equilibrium tariff that  $M_i$  offers  $R$ ,  $\pi_{i,R}^\circ \equiv \tau_{iR}^\circ(q^\circ) - cq^\circ$  denote  $M_i$ 's equilibrium profit from supplying  $R$ , and  $\pi_R^\circ \equiv \Pi^\circ/2 - \pi_{A,R}^\circ - \pi_{B,R}^\circ$  denote  $R$ 's equilibrium profit. Also, let

$$\tilde{q}_{iR}^\circ \in \arg \max_{q_i} \rho_R^\circ(q_i, 0) - \tau_{iR}^\circ(q_i)$$

denote the output level that  $R$  would choose under exclusivity with  $M_i$ , and  $\tilde{\pi}_{i,R}^\circ \equiv \tau_{iR}^\circ(\tilde{q}_{iR}^\circ) - c\tilde{q}_{iR}^\circ$  denote  $M_i$ 's associated profit (the corresponding profit for  $R$  is thus  $\Pi(\tilde{q}_{iR}^\circ, 0) - \tilde{\pi}_i^\circ$ ). We have:

**Lemma 3** *Under Assumptions (A.2)-(A.4), the output and profit levels satisfy,  $\tilde{q}_{iR}^\circ > q^\circ$  and*

$$0 \leq \pi_{i,R}^\circ \leq \Delta^\circ \equiv \frac{\Pi^\circ}{2} - \hat{\Pi}_R^\circ \quad (9)$$

for  $i = A, B$ , where

$$\hat{\Pi}_R^\circ \equiv \max_q \Pi_R^\circ(q, 0),$$

and:

$$\pi_R^\circ = \frac{\Pi^\circ}{2} - \pi_{A,R}^\circ - \pi_{B,R}^\circ = \Pi_R^\circ(\tilde{q}_{AR}^\circ, 0) - \tilde{\pi}_{A,R}^\circ = \Pi_R^\circ(\tilde{q}_{BR}^\circ, 0) - \tilde{\pi}_{B,R}^\circ > 0. \quad (10)$$

**Proof.** We first provide bounds on equilibrium payoffs, before turning to the comparison between  $\tilde{q}_{iR}^\circ$  and  $q^\circ$ .

By construction, we have  $\pi_{i,R}^\circ \geq 0$  for  $i = A, B$ . Furthermore, if  $\pi_{i,R}^\circ > \Delta^\circ$  for some  $i \in \{A, B\}$ , then the aggregate profit of  $R$  and the other supplier  $M_j$  (gross of the profit that  $M_j$  makes with the other retailer), is such that :

$$\pi_R^\circ + \pi_{j,R}^\circ = \frac{\Pi^\circ}{2} - \pi_{i,R}^\circ < \hat{\Pi}_R^\circ.$$

But then,  $M_j$  could profitably deviate to exclusivity by offering a forcing contract of the form  $(\hat{q}, \hat{T})$ , where  $\hat{q} \equiv \arg \max_q \Pi_R^\circ(q, 0)$  denote the bilaterally efficient output under exclusivity: by accepting this offer (and only that one),  $R$  would generate a bilateral profit of  $\hat{\Pi}_R^\circ$ , which can then be shared by an appropriate  $\hat{T}$  so as to ensure that both  $M_j$  and  $R$  benefit from the

deviation. It follows that  $\pi_{A,R}^\circ, \pi_{B,R}^\circ \leq \Delta^\circ$ , which in turn implies that the retailer obtains a positive profit:

$$\pi_R^\circ = \frac{\Pi^\circ}{2} - \pi_{A,R}^\circ - \pi_{B,R}^\circ \geq \frac{\Pi^\circ}{2} - 2 \left( \frac{\Pi^\circ}{2} - \hat{\Pi}_R^\circ \right) = 2\hat{\Pi}_R^\circ - \frac{\Pi^\circ}{2} > 0,$$

where the inequality stems from the fact that, from Assumption (A.2), goods  $A$  and  $B$  are (imperfect) substitutes. Finally, (10) follows from the fact, already noted in the main text, that in equilibrium  $R$  must be indifferent between accepting both manufacturers' offers, or only one (either one).

We now establish  $\tilde{q}_{iR}^\circ > q^\circ$ . By a revealed preference argument, we have:

$$\begin{aligned} \rho_R^\circ(\tilde{q}_{iR}^\circ, 0) - \tau_{iR}^\circ(\tilde{q}_{iR}^\circ) &\geq \rho_R^\circ(q^\circ, 0) - \tau_{iR}^\circ(q^\circ), \\ \rho_R^\circ(q^\circ, q^\circ) - \tau_{iR}^\circ(q^\circ) &\geq \rho_R^\circ(\tilde{q}_{iR}^\circ, q^\circ) - \tau_{iR}^\circ(\tilde{q}_{iR}^\circ). \end{aligned}$$

Therefore:

$$\begin{aligned} &\rho_R^\circ(\tilde{q}_{iR}^\circ, 0) - \rho_R^\circ(q^\circ, 0) \geq \rho_R^\circ(\tilde{q}_{iR}^\circ, q^\circ) - \rho_R^\circ(q^\circ, q^\circ) \\ \Leftrightarrow &\int_{q^\circ}^{\tilde{q}_{iR}^\circ} \partial_1 \rho_R^\circ(x, 0) dx \geq \int_{q^\circ}^{\tilde{q}_{iR}^\circ} \partial_1 \rho_R^\circ(x, q^\circ) dx \\ \Leftrightarrow &\int_{q^\circ}^{\tilde{q}_{iR}^\circ} \int_0^{q^\circ} \partial_{12}^2 \rho_R^\circ(x, y) dx dy \leq 0. \end{aligned}$$

As  $q^\circ > 0$  and  $\partial_{12}^2 \rho_R^\circ = \partial_{12}^2 \Pi_R^\circ < 0$  from the proof of Proposition 3, it follows that  $\tilde{q}_{iR}^\circ \geq q^\circ$ .

Assume now that  $\tilde{q}_{iR}^\circ = q^\circ$ , which implies  $\tau_{iR}^\circ(q^\circ) = \tau_{iR}^\circ(\tilde{q}_{iR}^\circ)$  and thus  $\pi_{i,R}^\circ = \tilde{\pi}_{i,R}^\circ$ ; hence, from condition (10), both  $M_i$  and  $R$  are indifferent between  $R$  accepting both suppliers' equilibrium offers, or only  $M_i$ 's offer. But then,  $M_i$  could profitably deviate to exclusivity by offering a forcing contract of the form  $(\hat{q}, \hat{T})$ : by accepting this offer (and only that one),  $R$  would increase their bilateral profit from  $\Pi_R^\circ(q^\circ, 0)$  to  $\hat{\Pi}_R^\circ = \max_q \Pi_R^\circ(q, 0)$ , which can then be shared by an appropriate  $\hat{T}$  so as to ensure that both  $M_i$  and  $R$  benefit from the deviation. Therefore,  $\tilde{q}_{iR}^\circ > q^\circ$ . ■

**Corollary 1** *There is no equilibrium in which a supplier offers a single forcing contract.*

**Proof.** This follows directly from Lemma 3, which implies that equilibrium contracts must offer at least two relevant options,  $q^\circ$  and  $\tilde{q}_{iR}^\circ \neq q^\circ$ . ■

We next show that, for any equilibrium based on tariffs  $\{\tau_{AR}^\circ(\cdot), \tau_{BR}^\circ(\cdot)\}$ , there exists an equilibrium, yielding the same profits, in which each  $M_i$  offers a pair of forcing contracts:

**Lemma 4** Let  $\{\tau_{AR}^\circ(\cdot), \tau_{BR}^\circ(\cdot)\}$  denote the tariffs signed by retailer  $R$  in a given equilibrium, with associated equilibrium profits  $\pi_{A,R}^\circ$ ,  $\pi_{B,R}^\circ$  and  $\pi_R^\circ = \Pi^\circ/2 - \pi_{A,R}^\circ - \pi_{B,R}^\circ$ , and let  $\tilde{\tau}_{iR}^\circ \equiv \{(q^\circ, \tau_{iR}^\circ(q^\circ)), (\tilde{q}_{iR}^\circ, \tau_{iR}^\circ(\tilde{q}_{iR}^\circ))\}$  denote the corresponding pair of forcing contracts, respectively based on the equilibrium output level  $q^\circ$  and on the output level  $\tilde{q}_{iR}^\circ$  that  $R$  would choose under exclusivity with  $M_i$ . Then there exists an equilibrium in which each  $M_i$  offers the tariff  $\tilde{\tau}_{iR}^\circ$ , leading  $R$  to pick the forcing contract  $(q^\circ, \tau_{iR}^\circ(q^\circ))$ ; this alternative equilibrium moreover yields the same profits  $\pi_{A,R}^\circ$ ,  $\pi_{B,R}^\circ$  and  $\pi_R^\circ$ .

**Proof.** Consider an equilibrium based on tariffs  $\{\tau_{AR}^\circ(\cdot), \tau_{BR}^\circ(\cdot)\}$ , and suppose that each supplier offers instead  $\tilde{\tau}_{iR}^\circ = \{(q^\circ, \tau_{iR}^\circ(q^\circ)), (\tilde{q}_{iR}^\circ, \tau_{iR}^\circ(\tilde{q}_{iR}^\circ))\}$ . By construction,  $R$  is willing to accept both offers, in which case it is willing to choose the “option”  $(q^\circ, \tau_{iR}^\circ(q^\circ))$  from each  $\tilde{\tau}_{iR}^\circ$ ; furthermore, from Lemma 3  $R$  is indifferent between doing so and accepting only either  $M_i$ ’s offer, in which case it would choose the option  $(\tilde{q}_{iR}^\circ, \tau_{iR}^\circ(\tilde{q}_{iR}^\circ))$ . We now show that manufacturers have no incentive to deviate.

Without loss of generality, we can restrict attention to deviations in which the deviating manufacturer offers a single forcing contract. As  $\pi_{i,R}^\circ \geq 0$  from Lemma 3, to be profitable the deviant offer must be accepted, either alone or in combination with one of the two options offered by  $M_j$ ; as  $M_j$ ’s equilibrium contract offers contain, among other options, the options  $(q^\circ, \tau_{jR}^\circ(q^\circ))$  and  $(\tilde{q}_j^\circ, \tau_{jR}^\circ(\tilde{q}_j^\circ))$  this implies that the deviant offer would also be accepted in the original equilibrium, as  $R$  could then combine it with even more options. But then, as by construction there is no profitable deviation in the original equilibrium  $\{\tau_{AR}^\circ(\cdot), \tau_{BR}^\circ(\cdot)\}$ , there is no profitable deviation from  $\{\tilde{\tau}_{AR}^\circ, \tilde{\tau}_{BR}^\circ\}$  either. ■

From now on, without loss of generality we will consider equilibria in which each  $M_i$  offers two options: that is,  $\tau_{iR}^\circ = \{(q^\circ, T_{iR}^\circ), (\tilde{q}_{iR}^\circ, \tilde{T}_{iR}^\circ)\}$ .

**Lemma 5** The contracts  $(\tau_i^\circ = \{(q^\circ, T_{iR}^\circ), (\tilde{q}_{iR}^\circ, \tilde{T}_{iR}^\circ)\})_{i=A,B}$  support an equilibrium if and only if the associated profits  $(\pi_{i,R}^\circ = T_{iR}^\circ - cq^\circ, \tilde{\pi}_{i,R}^\circ = \tilde{T}_{iR}^\circ - c\tilde{q}_{iR}^\circ)_{i=A,B}$  and  $\pi_R^\circ = \Pi^\circ/2 - \pi_{A,R}^\circ - \pi_{B,R}^\circ$  satisfy (9), (10) and

$$\pi_{i,R}^\circ - \tilde{\pi}_{i,R}^\circ \leq \frac{\Pi^\circ}{2} - \Pi_R^r(\tilde{q}_{iR}^\circ), \quad (11)$$

where

$$\Pi_R^r(q_i) \equiv \max_q \Pi_R^\circ(q, q_i)$$

denotes the maximal aggregate profit that  $R$  can generate, conditional on selling  $q_i$  units of good  $i$ .

**Proof.** We first check that  $R$  is indeed willing to accept both contracts, and to pick the options  $\{(q^\circ, T_{AR}^\circ), (q^\circ, T_{BR}^\circ)\}$ :

- From (10),  $R$  is willing to accept the offers, and is indifferent between accepting  $\{(q^\circ, T_{AR}^\circ), (q^\circ, T_{BR}^\circ)\}$ ,  $\left\{\left(\tilde{q}_{AR}^\circ, \tilde{T}_{AR}^\circ\right)\right\}$ , or  $\left\{\left(\tilde{q}_{BR}^\circ, \tilde{T}_{BR}^\circ\right)\right\}$ ;
- In addition,  $R$  prefers accepting  $\{(q^\circ, T_{AR}^\circ), (q^\circ, T_{BR}^\circ)\}$ , which yield  $\pi_R^\circ$ , to accepting only  $(q^\circ, T_{iR}^\circ)$ : This amounts to

$$\frac{\Pi^\circ}{2} - \pi_{A,R}^\circ - \pi_{B,R}^\circ > \Pi_R^\circ(q^\circ, 0) - \pi_{i,R}^\circ,$$

or:

$$\pi_{j,R}^\circ < \frac{\Pi^\circ}{2} - \Pi_R^\circ(q^\circ, 0), \quad (12)$$

which follows from (9), as the RHS of (12) is strictly larger than  $\Delta^\circ$ .

- Using (10), the previous observation also implies that, if  $R$  were to accept  $M_i$ 's contract only, then it would pick the option  $\left(\tilde{q}_{iR}^\circ, \tilde{T}_{iR}^\circ\right)$  rather than  $(q^\circ, T_{iR}^\circ)$ .
- Finally,  $R$  indeed prefers accepting  $\{(q^\circ, T_{AR}^\circ), (q^\circ, T_{BR}^\circ)\}$  to accepting  $\left\{(q^\circ, T_{iR}^\circ), \left(\tilde{q}_j^\circ, \tilde{T}_{jR}^\circ\right)\right\}$ : (11) implies that the joint profit of  $M_i$  and  $R$  is larger in the equilibrium configuration; as  $M_i$  is indifferent between the two scenarios (either way, it gets  $\pi_{i,R}^\circ = T_{iR}^\circ - cq^\circ$ ),  $R$  must prefer sticking to  $\{(q^\circ, T_{AR}^\circ), (q^\circ, T_{BR}^\circ)\}$ .

We now turn to deviations by the manufacturers:

- $M_i$  has no incentive to deviate by making an unacceptable offer (or no offer), as  $\pi_{i,R}^\circ \geq 0$ .
- $M_i$  has no incentive to deviate to exclusivity. To see that, it suffices to note that, as  $R$  can secure its equilibrium profit by accepting  $M_j$ 's offer only, to be profitable a deviation must increase the joint profit of  $M_i$  and  $R$ ; but along the equilibrium path, this joint profit (gross of the profit that  $M_i$  makes with the other retailer) satisfies:

$$\pi_R^\circ + \pi_{i,R}^\circ = \frac{\Pi^\circ}{2} - \pi_{j,R}^\circ \geq \frac{\Pi^\circ}{2} - \Delta^\circ = \hat{\Pi}_R^\circ,$$

where  $\hat{\Pi}_R^\circ$  is the maximal profit that can be achieved under exclusivity. Note that, as  $M_i$  could induce  $R$  to switch to exclusivity by slightly reducing  $\tilde{T}_{iR}^\circ$ , we must therefore have  $\pi_{i,R}^\circ \geq \tilde{\pi}_{i,R}^\circ$ .

- $M_i$  cannot profitably deviate by inducing  $R$  to combine the deviant offer with  $M_j$ 's equilibrium option  $\left(q^\circ, T_{jR}^\circ\right)$ . As the profit generated by  $R$  is maximal along the equilibrium path (that is,  $\Pi^\circ/2 = \max_{q_A, q_B} \Pi_R^\circ(q_A, q_B)$ ), a deviation by  $M_i$  that induces  $R$  to combine the deviant offer with  $M_j$ 's equilibrium option  $\left(q^\circ, T_{jR}^\circ\right)$  cannot be profitable, as this would maintain  $M_j$ 's profit at the equilibrium level (that is,  $M_j$  would obtain a  $T_{jR}^\circ - cq^\circ = \pi_{j,R}^\circ$ ), and a deviation cannot lower  $R$ 's profit either.

• Finally,  $M_i$  cannot profitably deviate by inducing  $R$  to combine the deviant offer with  $M_j$ 's alternative option  $(\tilde{q}_j^\circ, \tilde{T}_{jR}^\circ)$ . As  $R$  can secure its equilibrium profit by accepting  $M_j$ 's offer only, such a deviation could only be profitable if it increased the joint profit of  $M_i$  and  $R$  (gross of the profit that  $M_i$  makes with the other retailer), that is, only if :

$$\Pi_R^\circ(q_i, \tilde{q}_j^\circ) - \tilde{\pi}_{j,R}^\circ > \frac{\Pi^\circ}{2} - \pi_{j,R}^\circ,$$

which is ruled out by (11) (written for  $M_j$ ). ■

We now turn to existence. We first note that relying on the bilateral efficient quantity  $\tilde{q}_{iR}^\circ = \hat{q} = \arg \max_q \{\Pi_R^\circ(q, 0)\}$  for the “exclusive deal” option restricts somewhat the range of admissible profits, as (11) is then more demanding than (9): To see this, note that (10) yields

$$\tilde{\pi}_{i,R}^\circ = \Pi_R^\circ(\hat{q}, 0) + \pi_{A,R}^\circ + \pi_{B,R}^\circ - \frac{\Pi^\circ}{2},$$

so that (11) amounts to:

$$\begin{aligned} \pi_{i,R}^\circ &\leq \pi_{A,R}^\circ + \pi_{B,R}^\circ + \Pi_R^\circ(\hat{q}, 0) - \Pi_R^r(\hat{q}) \\ \Leftrightarrow \pi_{j,R}^\circ &\geq \Pi_R^r(\hat{q}) - \Pi_R^\circ(\hat{q}, 0) = \max_q \Pi_R^\circ(\hat{q}, q) - \Pi_R^\circ(\hat{q}, 0) > 0. \end{aligned}$$

We now show that  $M_j$ 's equilibrium profit can however cover the full range  $[0, \Delta^\circ]$  by relying on a “large enough” quantity  $\tilde{q}_{iR}^\circ$  for  $M_i$ 's exclusive deal option.

**Lemma 6** For any  $\pi_{A,R}^\circ, \pi_{B,R}^\circ \in [0, \Delta^\circ]$ , there exists an equilibrium yielding profits  $\pi_{A,R}^\circ, \pi_{B,R}^\circ$  and  $\pi_R^\circ = \Pi^\circ/2 - \pi_{A,R}^\circ - \pi_{B,R}^\circ$ .

**Proof.** We first note that the expression  $\Pi_R^r(q) - \Pi_R^\circ(q, 0) = \max_{\tilde{q}} \Pi_R^\circ(q, \tilde{q}) - \Pi_R^\circ(q, 0)$  decreases as  $q$  increases. Using the envelope theorem, and letting  $q^r(q) = \arg \max_{\tilde{q}} \Pi_R^\circ(q, \tilde{q})$  denotes  $R$ 's “best response” to selling a quantity  $q$  of the other brand, we have:

$$\begin{aligned} \frac{d}{dq} [\Pi_R^r(q) - \Pi_R^\circ(q, 0)] &= \partial_1 \Pi_R^\circ(q, q^r(q)) - \partial_1 \Pi_R^\circ(q, 0) \\ &= \int_0^{q^r(q)} \partial_{12} \Pi_R^\circ(q, \tilde{q}) d\tilde{q}, \end{aligned}$$

which is negative as long as  $q^r(q) > 0$ , as  $\partial_{12} \Pi_R^\circ < 0$  from the proof of Proposition 3. From (A.1),  $q^r(q) = 0$  for  $q$  large enough; let  $\bar{q}$  denote the smallest such quantity,<sup>19</sup> which by construction is

<sup>19</sup>It is characterized by

$$P(q^\circ, q^\circ + \bar{q}) - c + \partial_2 P(q^\circ, q^\circ + \bar{q}) \bar{q} = 0.$$

also the smallest quantity satisfying

$$\Pi_R^r(\bar{q}) = \Pi_R^\circ(\bar{q}, 0).$$

For  $(\tilde{q}_{AR}^\circ, \tilde{q}_{BR}^\circ) \geq (\bar{q}, \bar{q})$ , (10) yields

$$\tilde{\pi}_{i,R}^\circ = \Pi_R^\circ(\tilde{q}_{iR}^\circ, 0) + \pi_{A,R}^\circ + \pi_{B,R}^\circ - \frac{\Pi^\circ}{2}.$$

Hence (11) becomes:

$$\begin{aligned} \pi_{i,R}^\circ - \tilde{\pi}_{i,R}^\circ &= \pi_{i,R}^\circ + \frac{\Pi^\circ}{2} - (\Pi_R^\circ(\tilde{q}_{iR}^\circ, 0) + \pi_{A,R}^\circ + \pi_{B,R}^\circ) = \frac{\Pi^\circ}{2} - \Pi_R^\circ(\tilde{q}_{iR}^\circ, 0) - \pi_{j,R}^\circ \leq \frac{\Pi^\circ}{2} - \Pi_R^r(\tilde{q}_{iR}^\circ) \\ &\iff \pi_{j,R}^\circ \geq \Pi_R^r(\tilde{q}_{iR}^\circ) - \Pi_R(\tilde{q}_{iR}^\circ, 0) = 0, \end{aligned}$$

and thus follows from (9). Therefore, from Lemma 5, contracts of the form  $(\tau_i^\circ = \{(q^\circ, T_{iR}^\circ), (\tilde{q}_{iR}^\circ, \tilde{T}_{iR}^\circ)\})_{i=A,B}$ , where  $\tilde{q}_{iR}^\circ \geq \bar{q}$ , support an equilibrium if and only if the associated profits satisfy conditions (9) and (10).

Conversely, for any  $\pi_{A,R}^\circ, \pi_{B,R}^\circ \in [0, \Delta^\circ]$ , the contracts  $(\tau_i^\circ = \{(q^\circ, T_{iR}^\circ), (\tilde{q}_{iR}^\circ, \tilde{T}_{iR}^\circ)\})_{i=A,B}$ , where

$$\begin{aligned} T_{iR}^\circ &= cq^\circ + \pi_{i,R}^\circ, \\ \tilde{q}_{iR}^\circ &\geq \bar{q}, \\ \tilde{T}_{iR}^\circ &= cq^\circ + \tilde{\pi}_{i,R}^\circ, \text{ where } \tilde{\pi}_{i,R}^\circ = \Pi_R^\circ(\tilde{q}_{iR}^\circ, 0) + \pi_{A,R}^\circ + \pi_{B,R}^\circ - \frac{\Pi^\circ}{2}, \end{aligned}$$

support an equilibrium with profits  $(\pi_{A,R}^\circ, \pi_{B,R}^\circ)$  for the manufacturers and  $\pi_R^\circ = \Pi^\circ/2 - \pi_{A,R}^\circ - \pi_{B,R}^\circ$  for  $R$ . ■

Finally, we have:

**Lemma 7** *There exist equilibria in which each  $M_i$ , too, is indifferent between  $R$  accepting  $\{(q^\circ, T_{AR}^\circ), (q^\circ, T_{BR}^\circ)\}$  or  $\{(\tilde{q}_j^\circ, \tilde{T}_{jR}^\circ)\}$ , and these equilibria yield the same equilibrium profits as the equilibrium in two-part tariffs, namely,  $M_A$  and  $M_B$  both obtain their full contribution to industry profits:  $\pi_{A,R}^\circ = \pi_{B,R}^\circ = \Delta^\circ$ .*

**Proof.** Suppose  $\pi_{i,R}^\circ = \tilde{\pi}_{i,R}^\circ$ . Together with conditions (10), this yields  $\pi_R^\circ + \pi_{i,R}^\circ = \Pi_R^\circ(\tilde{q}_{iR}^\circ, 0)$  and  $\pi_{j,R}^\circ = \Pi^\circ/2 - \Pi_R^\circ(\tilde{q}_{iR}^\circ, 0)$ . The first equality implies  $\Pi_R^\circ(\tilde{q}_{iR}^\circ, 0) = \hat{\Pi}_R$  (and thus  $\tilde{q}_{iR}^\circ = \hat{q}$ ), as otherwise  $M_i$  would have a profitable deviation to exclusivity. The second equality then implies  $\pi_{j,R}^\circ = \Pi^\circ/2 - \hat{\Pi}_R = \Delta^\circ$ .

Conversely, the contracts  $(\tau_i^\circ = \{(q^\circ, cq^\circ + \Delta^\circ), (\hat{q}, c\hat{q} + \Delta^\circ)\})_{i=A,B}$  support an equilibrium in which each  $M_i$  obtains  $\pi_{i,R}^\circ = \tilde{\pi}_{i,R}^\circ = \Delta^\circ$ . ■

## E Proof of Proposition 5

We start with the characterization of the equilibrium contracts and quantities, before establishing the existence of an equilibrium in two-part tariffs.

That contracts are cost-based follows from Lemma 1. We now show that, under Assumptions (A.1)-(A.4), the equilibrium quantities  $(q_{A1}^*, q_{B1}^*, q_{B2}^*)$  are uniquely defined and all positive. The quantities are those that would be induced in a duopoly game where firm 1's problem is given by

$$\max_{q_{A1}, q_{B1}} [P(q_{A1}, q_{B1} + q_{B2}) - c] q_{A1} + [P(q_{B1} + q_{B2}, q_{A1}) - c] q_{B1},$$

and firm 2's problem by

$$\max_{q_{B2}} [P(q_{B1} + q_{B2}, q_{A1}) - c] q_{B2}.$$

The first order conditions are given by:

$$\begin{aligned} P(Q_A^*, Q_B^*) - c + q_{A1}^* \partial_1 P(Q_A^*, Q_B^*) + q_{B1}^* \partial_2 P(Q_A^*, Q_B^*) &\leq 0, \\ P(Q_B^*, Q_A^*) - c + q_{B1}^* \partial_1 P(Q_B^*, Q_A^*) + q_{A1}^* \partial_2 P(Q_B^*, Q_A^*) &\leq 0, \\ P(Q_B^*, Q_A^*) - c + q_{B2}^* \partial_1 P(Q_B^*, Q_A^*) &\leq 0, \end{aligned}$$

where the first-order condition of quantity  $q_{ik}^*$  holds with equality if  $q_{ik}^* > 0$ , and where  $Q_A^* \equiv q_{A1}^*$  and  $Q_B^* \equiv q_{B1}^* + q_{B2}^*$ .

We first establish that all quantities are indeed positive, implying that the first-order conditions hold with equality.

(i)  $P(Q_B^*, Q_A^*) > c$ . To see this, suppose otherwise that  $P(Q_B^*, Q_A^*) \leq c$ . Then, the first-order condition of  $q_{B2}^*$  implies that  $q_{B2}^* = 0$ . Moreover, we must also have  $q_{B1}^* = 0$ ; if not, firm 1 could profitably deviate by reducing  $q_{B1}$ . Hence,  $Q_B^* = 0 \leq Q_A^*$ . By (A.2), we thus have  $P(Q_A^*, Q_B^*) \leq c$ , implying that  $q_{A1}^* = 0$  (otherwise, firm 1 could profitably deviate by reducing  $q_{A1}$ ). Hence,  $P(0, 0) \leq c$ , contradicting (A.1).

(ii)  $q_{B2}^* > 0$ . As  $P(Q_B^*, Q_A^*) > c$ , firm 2 could otherwise profitably deviate by choosing  $q_{B2} > 0$  small enough such that  $P(Q_B^* + q_{B2}, Q_A^*) > c$ .

(iii)  $P(Q_A^*, Q_B^*) > c$ . To see this, suppose otherwise that  $P(Q_A^*, Q_B^*) \leq c$ . It follows that  $q_{A1}^* = 0$ . (If not, firm 1 could profitably deviate by setting  $q_{A1} = 0$ ; if  $q_{B1}^* = 0$ , firm could combine this deviation by choosing  $q_{B1} > 0$  sufficiently small such that  $P(Q_B^* + q_{B1}, Q_A^* - q_{A1}^*) > c$ .) Hence,  $Q_A^* = 0 \leq Q_B^*$ . By (A.2), we thus have  $P(Q_A^*, Q_B^*) \geq P(Q_B^*, Q_A^*) > c$ , a contradiction.

(iv)  $q_{A1}^* > 0$ . Suppose otherwise that  $q_{A1}^* = 0$ . We have:

$$\begin{aligned} 0 &\geq P(Q_A^*, Q_B^*) - c + q_{B1}^* \partial_2 P(Q_A^*, Q_B^*) \\ &> P(Q_B^*, Q_A^*) - c + q_{B1}^* \partial_1 P(Q_B^*, Q_A^*), \end{aligned}$$

where the first inequality follows from  $q_{A1}^* = 0$ , and the second by (A.2) and  $Q_B^* > Q_A^*$ . It follows that  $q_{B1}^* = 0$ . But as  $P(Q_B^*, Q_A^*) > c$ , firm 1 could then profitably deviate by setting  $q_{B1} > 0$  sufficiently small such that  $P(Q_B^* + q_{B1}, Q_A^*) > c$ .

(v)  $q_{B1}^* > 0$ . Suppose otherwise that  $q_{B1}^* = 0$ . The induced outcome thus coincides with the equilibrium outcome in a duopoly in which firm 1 sells only good  $A$  and firm 2 sells only good  $B$ . Under Assumption (A.3), this implies that  $Q_A^* = Q_B^*$ , as shown in Proposition 6. We thus have:

$$\begin{aligned} P(Q_B^*, Q_A^*) - c + Q_A^* \partial_2 P(Q_B^*, Q_A^*) &> P(Q_A^*, Q_B^*) - c + Q_A^* \partial_1 P(Q_A^*, Q_B^*) \\ &= 0, \end{aligned}$$

where the inequality follows from (A.2) and the equality from the first-order condition of  $q_{A1}^* = Q_A^*$ . But then firm 1 could profitably deviate by slightly raising  $q_{B1}$ .

Having established that all quantities  $(q_{A1}^*, q_{B1}^*, q_{B2}^*)$  are strictly positive, implying that each of the three first-order conditions holds with equality, we now show that the quantities are unique.

Let

$$\hat{c} \equiv P(Q_A^*, Q_B^*) + q_{B2}^* \partial_1 P(Q_A^*, Q_B^*)$$

denote the marginal cost level that would induce firm 2 to produce just zero units of good  $A$  if it could produce that good at marginal cost  $\hat{c}$ , given that aggregate outputs are  $(Q_A^*, Q_B^*)$ . Equilibrium quantities are thus characterized by the following system of equations:

$$P(Q_A^*, Q_B^*) - c + q_{A1}^* \partial_1 P(Q_A^*, Q_B^*) + q_{B1}^* \partial_2 P(Q_B^*, Q_A^*) = 0, \quad (13)$$

$$P(Q_B^*, Q_A^*) - c + q_{B1}^* \partial_1 P(Q_B^*, Q_A^*) + q_{A1}^* \partial_2 P(Q_A^*, Q_B^*) = 0, \quad (14)$$

$$P(Q_B^*, Q_A^*) - c + q_{B2}^* \partial_1 P(Q_B^*, Q_A^*) = 0, \quad (15)$$

$$P(Q_A^*, Q_B^*) - \hat{c} + q_{B2}^* \partial_2 P(Q_B^*, Q_A^*) = 0. \quad (16)$$

Adding equations (14) and (15), we obtain:

$$\phi(Q_B^*, Q_A^*; c) \equiv 2[P(Q_B^*, Q_A^*) - c] + Q_B^* \partial_1 P(Q_B^*, Q_A^*) + Q_A^* \partial_2 P(Q_A^*, Q_B^*) = 0. \quad (17)$$



Similarly, adding equations (13) and (16) yields:

$$\phi(Q_A^*, Q_B^*; \hat{c}) \equiv 2P(Q_A^*, Q_B^*) - c - \hat{c} + Q_A^* \partial_1 P(Q_A^*, Q_B^*) + Q_B^* \partial_2 P(Q_B^*, Q_A^*) = 0. \quad (18)$$

Let  $R(Q_A)$  be such that  $\phi(R(Q_A), Q_A; c) = 0$ , and  $\hat{R}(Q_B, \hat{c})$  be such that  $\phi(\hat{R}(Q_B, \hat{c}), Q_B; \hat{c}) = 0$ . Hence, aggregate equilibrium outputs  $(Q_A^*, Q_B^*)$  satisfy  $Q_B^* = \hat{R}(Q_A^*, \hat{c})$  and  $Q_A^* = R(Q_B^*)$ . Using the implicit function theorem, we have for  $i \neq j \in \{A, B\}$ ,

$$R'(Q_i) = \partial_1 \hat{R}(Q_i, \hat{c}) = -\frac{2\partial_2 P(Q_j, Q_i) + Q_j \partial_{12}^2 P(Q_j, Q_i) + \partial_2 P(Q_i, Q_j) + Q_i \partial_{12}^2 P(Q_i, Q_j)}{3\partial_1 P(Q_j, Q_i) + Q_j \partial_{11}^2 P(Q_j, Q_i) + Q_i \partial_{22}^2 P(Q_i, Q_j)}.$$

(A.2) and (A.4) imply that  $R'(Q_i) = \partial_1 \hat{R}(Q_i, \hat{c}) \in (-1, 0)$ . It follows that the aggregate equilibrium outputs  $(Q_A^*, Q_B^*)$  are unique. From the first-order conditions the individual outputs are unique as well.

We now show that there exists an equilibrium in which all equilibrium contracts are cost-based two-part tariffs of the form  $(w_{i,h}^*, F_{i,h}^*) = (c, \Delta_{i,h}^*)$ , where  $\Delta_{i,h}^*$  denotes  $M_i$ 's contribution to the profit generated by  $R_h$ , namely:

$$\begin{aligned} \Delta_{A,1}^* &= (p_A^* - c) q_{A1}^* + (p_B^* - c) q_{B1}^* - \max_{q_{B1}} \{ [P(q_{B1} + q_{B2}^*, 0) - c] q_{B1} \}, \\ \Delta_{B,1}^* &= (p_A^* - c) q_{A1}^* + (p_B^* - c) q_{B1}^* - \max_{q_{A1}} \{ [P(q_{A1}, q_{B2}^*) - c] q_{A1} \}, \\ \Delta_{B,2}^* &= (p_B^* - c) q_{B2}^*. \end{aligned}$$

We first note that the continuation equilibrium is then such that  $R_1$  sells  $(q_{A1}, q_{B1}) = (q_{A1}^*, q_{B1}^*)$  and  $R_2$  sells  $q_{B2} = q_{B2}^*$ . From the first part of this proof, this constitutes the unique candidate equilibrium, and it satisfies all first-order conditions. And going through the same steps as in the proof of Proposition 3, it is easy to check that retailers' profit functions are again concave.<sup>20</sup>

Next, we note that each retailer is willing to accept all offers made. Indeed, the fees are such that each, anticipating this behavior for its rival:

- $R_1$  is indifferent between accepting both manufacturers' offers or either one (either one);
- $R_2$  is indifferent between accepting  $M_B$ 's offer or not.

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<sup>20</sup>It suffices to note that retailers' profit functions are similar to the previous function  $\Pi_R(q_A, q_B)$ , replacing the rival's equilibrium quantities  $(q^\circ, q^\circ)$  with the new equilibrium quantities  $(q_{A1}^*, q_{B1}^*)$  or  $q_{B2}^*$ .

It thus suffices to check that  $R_1$  is strictly better-off accepting the manufacturers' offers rather than rejecting both of them; indeed, we have:

$$\begin{aligned}
\pi_1^* &= (p_A^* - c) q_{A1}^* + (p_B^* - c) q_{B1}^* - \Delta_{A,1}^* - \Delta_{B,1}^* \\
&= \max_{q_{A1}} \{ [P(q_{A1}, q_{B2}^*) - c] q_{A1} \} + \max_{q_{B1}} \{ [P(q_{B1} + q_{B2}^*, 0) - c] q_{B1} \} \\
&\quad - [(p_A^* - c) q_{A1}^* + (p_B^* - c) q_{B1}^*] \\
&= \max_{q_{A1}} \{ [P(q_{A1}, q_{B2}^*) - c] q_{A1} \} - [P(q_{A1}^*, q_{B1}^* + q_{B2}^*) - c] q_{A1}^* \\
&\quad + \max_{q_{B1}} \{ [P(q_{B1} + q_{B2}^*, 0) - c] q_{B1} \} - [P(q_{B1}^* + q_{B2}^*, q_{A1}^*) - c] q_{B1}^* \\
&> \max_{q_{A1}} \{ [P(q_{A1}, q_{B2}^*) - c] q_{A1} \} - [P(q_{A1}^*, q_{B2}^*) - c] q_{A1}^* \\
&\quad + \max_{q_{B1}} \{ [P(q_{B1} + q_{B2}^*, 0) - c] q_{B1} \} - [P(q_{B1}^* + q_{B2}^*, 0) - c] q_{B1}^* \\
&\geq 0.
\end{aligned}$$

Thus, if these contracts are offered, it is a continuation equilibrium for retailers to accept all offers, and then to sell the equilibrium quantities identified above. We now show that manufacturers cannot profitably deviate from this candidate equilibrium.

We first note that the above tariffs are profitable for the manufacturers, as each manufacturer contributes positively to the profits generated by the retailers.<sup>21</sup> It follows that a deviation cannot be profitable if it is not accepted by the retailer. But then,  $M_B$  cannot profitably deviate in its offer to  $R_2$ , as it already appropriates all the profit that  $R_2$  can generate. Likewise, no  $M_i$  can profitably deviate in its dealing with  $R_1$ , as: (i)  $M_i$  and  $R_1$  cannot increase their joint profit above the equilibrium level, as  $M_j$  does not obtain more than its contribution to the profit generated by  $R_1$ ; and (ii) following a deviation by  $M_i$ ,  $R_1$  can still secure its equilibrium profit by accepting only  $M_j$ 's offer.

## F Proof of Proposition 6

We first prove the second part of the Proposition, before establishing the existence of an equilibrium based on two-part tariffs.

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<sup>21</sup>For instance,  $\Delta_{A,1}^*$  can be expressed as

$$\begin{aligned}
\Delta_{A,1}^* &= \{ [P(q_{A1}^*, q_{B1}^* + q_{B2}^*) - c] q_{A1}^* + [P(q_{B1}^* + q_{B2}^*, 0) - c] q_{B1}^* \} - \max_{q_{B1}} \{ [P(q_{B1} + q_{B2}^*, 0) - c] q_{B1} \} \\
&= \max_{q_{A1}, q_{B1}} \{ [P(q_{A1}, q_{B1} + q_{B2}^*) - c] q_{A1} + [P(q_{B1} + q_{B2}^*, 0) - c] q_{B1} \} - \max_{q_{B1}} \{ [P(q_{B1} + q_{B2}^*, 0) - c] q_{B1} \},
\end{aligned}$$

which is positive as  $q_{A1}^* > 0$ .

In the second part of the Proposition, the assertion on cost-based contracts is an immediate implication of Lemma 1. And as retailers would obtain zero profit by rejecting the offers made by the manufacturers, in equilibrium the manufacturers fully appropriate the profit generated by their goods.

To see that (A.3) implies that the equilibrium quantities are unique and symmetric, note that the first-order condition of the retailer carrying manufacturer  $M_i$ 's good is

$$\Psi(Q_i, Q_j) \equiv P(Q_i, Q_j) - c + \partial_1 P(Q_i, Q_j) Q_i = 0.$$

We have  $\Psi(0, Q_j) = P(0, Q_j) - c$ ,  $\partial_1 \Psi(Q_i, Q_j) = 2\partial_1 P(Q_i, Q_j) + Q_i \partial_{11}^2 P(Q_i, Q_j) \leq 0$  (with strict inequality if  $P(Q_i, Q_j) > 0$ ) by (A.2), and  $\Psi(Q_i, Q_j) < 0$  for  $Q_i$  sufficiently large by (A.1). Hence, the best-response to  $M_j$  selling  $Q_j$  units of output,  $\chi^E(Q_j)$ , is given by  $\chi^E(Q_j) = 0$  if  $P(0, Q_j) < c$ , and by the unique solution to  $\Psi(\chi^E(Q_j), Q_j) = 0$  otherwise. For  $Q_j$  such that  $P(0, Q_j) \geq c$ , we have

$$\frac{d\chi^E}{dQ_j}(Q_j) = -\frac{\partial_2 \Psi(Q_i, Q_j)}{\partial_1 \Psi(Q_i, Q_j)} = -\frac{\partial_2 P(Q_i, Q_j) + Q_i \partial_{12}^2 P(Q_i, Q_j)}{2\partial_1 P(Q_i, Q_j) + Q_i \partial_{11}^2 P(Q_i, Q_j)}.$$

(A.3) implies that  $-1 < d\chi^E/dQ_j < 0$ , which in turn implies that an equilibrium, if it exists, is unique and symmetric. To establish existence, it suffices to note that  $\Psi(Q, Q)$  is continuous in  $Q$ , and satisfies  $\Psi(0, 0) > 0$  and, from (A.1),  $\Psi(Q, Q) < 0$  for  $Q$  sufficiently large. Hence, there exists  $Q^{**}$  such that  $\Psi(Q^{**}, Q^{**}) = 0$ .

We now turn to the first part of the Proposition, and consider a candidate equilibrium in which both manufacturers offer the cost-based two-part tariff  $(w^{**}, F^{**}) = (c, \Pi_R^{**})$ , where

$$\Pi_R^{**} = [P(Q^{**}, Q^{**}) - c] Q^{**}$$

denotes the profit generated by a retailer. The retailers are willing to accept those contracts, in which case they each put  $Q^{**}$  on the market and break even. Furthermore, each manufacturer obtains all the profits generated by its good, which is moreover maximal given the output level  $Q^{**}$  of the other good; it follows that there is no profitable deviation.

## G Sufficient Conditions for Properties (P.1)-(P.3)

We provide sufficient assumptions on the inverse demand which, together with (A.1) and (A.2), yield Properties (P.1)-(P.3). In particular, throughout this section we will rely on the following assumption:

(B.1) For any  $Q_i, Q_j \geq 0$  such that  $P(Q_i, Q_j) > 0$ , and for any  $(q_i, q_j) \in [0, Q_i] \times [0, Q_j]$ , we have:

$$\begin{aligned} & 2\partial_1 P(Q_i, Q_j) + q_i \partial_{11}^2 P(Q_i, Q_j) + q_j \partial_{22}^2 P(Q_j, Q_i) \\ & < \partial_2 P(Q_i, Q_j) + q_i \partial_{12}^2 P(Q_i, Q_j) + q_j \partial_{12}^2 P(Q_j, Q_i) \\ & < 0. \end{aligned}$$

In the case of linear demand, (B.1) simplifies to  $2\partial_1 P < \partial_2 P < 0$ , and is thus implied by (A.2).

## G.1 Property (P.1)

We first show that the above assumption yields existence and uniqueness:

**Proposition 12** *Under Assumption (B.1), the game  $\Gamma_{1-1}$  has a unique Nash equilibrium  $(\tilde{q}_{A1}, \tilde{q}_{B2})$ .*

**Proof.** The derivative of firm  $i$ 's profit,  $\hat{\Pi}_i$ , with respect to  $q_{ih}$  is (for  $ih \neq jk \in \{A1, B2\}$ ):

$$\Phi(q_{ih}; q_{jk}, \hat{q}_{jh}, \hat{q}_{ik}) \equiv P(q_{ih} + \hat{q}_{ik}, \hat{q}_{jh} + q_{jk}) - c + q_{ih} \partial_1 P(q_{ih} + \hat{q}_{ik}, \hat{q}_{jh} + q_{jk}) + \hat{q}_{jh} \partial_2 P(\hat{q}_{jh} + q_{jk}, q_{ih} + \hat{q}_{ik}),$$

and is thus such that:

$$\frac{d\Phi}{dq_{ih}}(q_{ih}; q_{jk}, \hat{q}_{jh}, \hat{q}_{ik}) = 2\partial_1 P(q_{ik} + \hat{q}_{il}, \hat{q}_{jk} + q_{jl}) + q_{ik} \partial_{11}^2 P(q_{ik} + \hat{q}_{il}, \hat{q}_{jk} + q_{jl}) + \hat{q}_{jk} \partial_{22}^2 P(\hat{q}_{jk} + q_{jl}, q_{ik} + \hat{q}_{il}).$$

From (A.2),  $\Phi(q_{ih}; q_{jk}, \hat{q}_{jh}, \hat{q}_{ik}) = 0$  implies  $P(q_{ih} + \hat{q}_{ik}, \hat{q}_{jh} + q_{jk}) \geq c (> 0)$ , and thus, from (B.1),  $\frac{d\Phi}{dq_{ih}}(q_{ih}; q_{jk}, \hat{q}_{jh}, \hat{q}_{ik}) < 0$ ; it follows that firm  $i$ 's best-response

$$r(q_{jk}; \hat{q}_{B1}, \hat{q}_{A2}) = \arg \max_{q_{ih}} \hat{\Pi}_i(q_{A1}, q_{B2}; \hat{q}_{B1}, \hat{q}_{A2})$$

is single-valued. It is moreover positive whenever

$$P(\hat{q}_{ik}, \hat{q}_{jh} + q_{jk}) > c - \hat{q}_{jh} \partial_2 P(\hat{q}_{jh} + q_{jk}, \hat{q}_{ik}),$$

in which case it is characterized by the first-order condition  $\Phi(q_{ih}; q_{jk}, \hat{q}_{jh}, \hat{q}_{ik}) = 0$ . Differentiating this first-order condition with respect to  $q_{ih}$  and  $q_{jk}$  then yields:

$$\frac{dr}{dq_{jk}}(q_{jk}; \hat{q}_{jh}, \hat{q}_{ik}) = -\frac{\lambda_i}{\mu_i}, \quad (19)$$

where

$$\lambda_i \equiv \left. \begin{aligned} & 2\partial_1 P(q_{ih} + \hat{q}_{ik}, \hat{q}_{jh} + q_{jk}) \\ & + q_{ih}\partial_{11}^2 P(q_{ih} + \hat{q}_{ik}, \hat{q}_{jh} + q_{jk}) + \hat{q}_{jh}\partial_{22}^2 P(\hat{q}_{jh} + q_{jk}, q_{ih} + \hat{q}_{ik}) \end{aligned} \right|_{q_{ih}=r(q_{jk}; \hat{q}_{jh}, \hat{q}_{ik})},$$

$$\mu_i \equiv \left. \begin{aligned} & \partial_2 P(q_{ih} + \hat{q}_{ik}, \hat{q}_{jh} + q_{jk}) \\ & + q_{ih}\partial_{12}^2 P(q_{ih} + \hat{q}_{ik}, \hat{q}_{jh} + q_{jk}) + \hat{q}_{jh}\partial_{12}^2 P(\hat{q}_{jh} + q_{jk}, q_{ih} + \hat{q}_{ik}) \end{aligned} \right|_{q_{ih}=r(q_{jk}; \hat{q}_{jh}, \hat{q}_{ik})}.$$

As  $\lambda_i < \mu_i < 0$  from (B.1), (19) implies

$$-1 < \frac{dr}{dq_{jk}} < 0.$$

Hence there exists a unique Nash equilibrium, which is moreover “stable” in the usual sense. ■

## G.2 Property (P.2)

Let

$$Q^{**} \equiv \arg \max_Q [P(Q, Q^{**}) - c] Q$$

denote the “duopoly” equilibrium output per good in game  $\Gamma_{1-1}$  when  $\hat{q}_{A2} = \hat{q}_{B1} = 0$ . (From Proposition 12, we know that  $Q^{**}$  exists and is unique.)

We now provide an additional condition on demand that ensures that an increase in either  $\hat{q}_{B1}$  or  $\hat{q}_{A2}$  increases the total output of goods  $A$  and  $B$ . We then show that any such increase in aggregate output beyond that of the duopoly outcome  $(Q^{**}, Q^{**})$  reduces aggregate profit, implying (P.2).

**(B.2)** For any  $Q_i, Q_j \geq 0$  such that  $P(Q_i, Q_j) > 0$  and  $P(Q_j, Q_i) > 0$ , and for any  $q_i \in [0, Q_i]$ , we have

$$\begin{aligned} & 2\partial_1 P(Q_i, Q_j) + q_i\partial_{11}^2 P(Q_i, Q_j) + Q_j\partial_{22}^2 P(Q_j, Q_i) \\ < & \partial_2 P(Q_i, Q_j) + \partial_2 P(Q_j, Q_i) + q_i\partial_{12}^2 P(Q_i, Q_j) + Q_j\partial_{12}^2 P(Q_j, Q_i), \end{aligned} \quad (\text{B.2.a})$$

and in addition, for any  $q_j \in [0, Q_j]$ :

$$\begin{aligned} & \partial_1 P(Q_i, Q_j) \left[ \begin{array}{l} 2\partial_1 P(Q_j, Q_i) - \partial_2 P(Q_j, Q_i) \\ + q_j (\partial_{11}^2 P(Q_j, Q_i) - \partial_{12}^2 P(Q_j, Q_i)) \\ + q_i (\partial_{22}^2 P(Q_i, Q_j) - \partial_{12}^2 P(Q_i, Q_j)) \end{array} \right] \\ > & \partial_2 P(Q_i, Q_j) \left[ \begin{array}{l} 2\partial_1 P(Q_i, Q_j) - \partial_2 P(Q_i, Q_j) \\ + [Q_i - q_i] (\partial_{11}^2 P(Q_i, Q_j) - \partial_{12}^2 P(Q_i, Q_j)) \\ + [Q_j - q_j] (\partial_{22}^2 P(Q_j, Q_i) - \partial_{12}^2 P(Q_j, Q_i)) \end{array} \right]. \end{aligned} \quad (\text{B.2.b})$$

In the case of linear demand (B.2) simplifies to  $\partial_1 P < \partial_2 P$  and  $(\partial_1 P - \partial_2 P)(2\partial_1 - \partial_2 P) > 0$ , and is thus implied by (A.2). We now show that (B.1) and (B.2) together imply that any increase in  $\hat{q}_{il}$ , for  $il \in \{A2, B1\}$ , increases total output:

**Lemma 8** *Let  $\tilde{Q}_A$  (resp.,  $\tilde{Q}_B$ ) denote the total equilibrium output of good A (resp., good B) in game  $\Gamma_{1-1}$ . If (B.1) and (B.2) hold, then an increase in either  $\hat{q}_{A2}$  or  $\hat{q}_{B1}$  leads to a strict increase in the total equilibrium output  $\tilde{Q}_A + \tilde{Q}_B$ .*

**Proof.** The claim is obvious (although in a weak sense, for the “decreasing” part) when  $\tilde{q}_{A1} = \tilde{q}_{B2} = 0$ , as then  $\tilde{Q}_A = \hat{q}_{A2}$  and  $\tilde{Q}_B = \hat{q}_{B1}$ . Consider now the case where  $\tilde{q}_{ih} > 0$  whereas  $\tilde{q}_{jk} = 0$ , for  $ih \neq jk \in \{A1, B2\}$ . We then have  $\tilde{Q}_j = \hat{q}_{jh}$ , and thus  $\frac{\partial \tilde{Q}_j}{\partial \hat{q}_{ik}} = 0$ ,  $\frac{\partial \tilde{Q}_j}{\partial \hat{q}_{jh}} = 1$ . Turning to  $\tilde{Q}_i = \tilde{q}_{ih} + \hat{q}_{ik}$ , the first-order condition for  $\tilde{q}_{ih}$  is:

$$P(\tilde{q}_{ih} + \hat{q}_{ik}, \hat{q}_{jh}) - c + \tilde{q}_{ih} \partial_1 P(\tilde{q}_{ih} + \hat{q}_{ik}, \hat{q}_{jh}) + \hat{q}_{jh} \partial_2 P(\hat{q}_{jh}, \tilde{q}_{ih} + \hat{q}_{ik}) = 0,$$

or:

$$P(\tilde{Q}_i, \hat{q}_{jh}) - c + \tilde{Q}_i \partial_1 P(\tilde{Q}_i, \hat{q}_{jh}) = \hat{q}_{ik} \partial_1 P(\tilde{Q}_i, \hat{q}_{jh}) - \hat{q}_{jh} \partial_2 P(\hat{q}_{jh}, \tilde{Q}_i).$$

Differentiating this equation with respect to  $\tilde{Q}_i$  and  $\hat{q}_{jk}$  yields:

$$\hat{\lambda}_i d\tilde{Q}_i = \partial_1 P(\tilde{Q}_i, \hat{q}_{jh}) d\hat{q}_{ik} - \left[ \partial_2 P(\hat{q}_{jh}, \tilde{Q}_i) + \tilde{\mu}_i \right] d\hat{q}_{jh},$$

where

$$\begin{aligned} \hat{\lambda}_i &= 2\partial_1 P(\tilde{Q}_i, \hat{q}_{jh}) + \tilde{q}_{ih} \partial_{11}^2 P(\tilde{Q}_i, \hat{q}_{jh}) + \hat{q}_{jh} \partial_{22}^2 P(\hat{q}_{jh}, \tilde{Q}_i), \\ \tilde{\mu}_i &= \partial_2 P(\tilde{Q}_i, \hat{q}_{jh}) + \tilde{q}_{ih} \partial_{12}^2 P(\tilde{Q}_i, \hat{q}_{jh}) + \hat{q}_{jh} \partial_{21}^2 P(\hat{q}_{jh}, \tilde{Q}_i). \end{aligned}$$

Assumptions (B.1) yields  $\hat{\lambda}_i < 0$ , and thus:

$$\begin{aligned} \frac{\partial \tilde{Q}_i}{\partial \hat{q}_{ik}} &= \frac{\partial_1 P(\tilde{Q}_i, \hat{q}_{jh})}{\hat{\lambda}_i}, \\ \frac{\partial \tilde{Q}_i}{\partial \hat{q}_{jh}} &= -\frac{\partial_2 P(\hat{q}_{jh}, \tilde{Q}_i) + \tilde{\mu}_i}{\hat{\lambda}_i}. \end{aligned} \tag{20}$$

Therefore, as  $\tilde{Q}_j = \hat{q}_{jh}$ :

$$\begin{aligned} \frac{\partial (\tilde{Q}_A + \tilde{Q}_B)}{\partial \hat{q}_{ik}} &= \frac{\partial \tilde{Q}_i}{\partial \hat{q}_{ik}} = \frac{\partial_1 P(\tilde{Q}_i, \hat{q}_{jh})}{\hat{\lambda}_i} > 0, \\ \frac{\partial (\tilde{Q}_A + \tilde{Q}_B)}{\partial \hat{q}_{jh}} &= 1 + \frac{\partial \tilde{Q}_i}{\partial \hat{q}_{jh}} = 1 - \frac{\partial_2 P(\hat{q}_{jh}, \tilde{Q}_i) + \tilde{\mu}_i}{\hat{\lambda}_i} > 0, \end{aligned}$$

where the inequalities stems from  $\hat{\lambda}_i < 0$  and Assumptions (A.2) and (B.2.a), which yield  $\partial_1 P(\tilde{Q}_i, \hat{q}_{jh}) < 0$  and  $\hat{\lambda}_i < \partial_2 P(\hat{q}_{jh}, \tilde{Q}_i) + \tilde{\mu}_i$ .

Let us now consider the case where  $\tilde{q}_{A1}, \tilde{q}_{B2} > 0$ , and are thus characterized by the first-order conditions:

$$\begin{aligned} P(\tilde{q}_{A1} + \hat{q}_{A2}, \hat{q}_{B1} + \tilde{q}_{B2}) - c + \tilde{q}_{A1} \partial_1 P(\tilde{q}_{A1} + \hat{q}_{A2}, \hat{q}_{B1} + \tilde{q}_{B2}) + \hat{q}_{B1} \partial_2 P(\hat{q}_{B1} + \tilde{q}_{B2}, \tilde{q}_{A1} + \hat{q}_{A2}) &= 0, \\ P(\hat{q}_{B1} + \tilde{q}_{B2}, \tilde{q}_{A1} + \hat{q}_{A2}) - c + \tilde{q}_{B2} \partial_1 P(\hat{q}_{B1} + \tilde{q}_{B2}, \tilde{q}_{A1} + \hat{q}_{A2}) + \hat{q}_{A2} \partial_2 P(\tilde{q}_{A1} + \hat{q}_{A2}, \hat{q}_{B1} + \tilde{q}_{B2}) &= 0, \end{aligned}$$

or, in terms of total equilibrium outputs  $\tilde{Q}_A = \tilde{q}_{A1} + \hat{q}_{A2}$  and  $\tilde{Q}_B = \hat{q}_{B1} + \tilde{q}_{B2}$ :

$$\begin{aligned} P(\tilde{Q}_A, \tilde{Q}_B) - c + \tilde{Q}_A \partial_1 P(\tilde{Q}_A, \tilde{Q}_B) &= \hat{q}_{A2} \partial_1 P(\tilde{Q}_A, \tilde{Q}_B) - \hat{q}_{B1} \partial_2 P(\tilde{Q}_B, \tilde{Q}_A), \\ P(\tilde{Q}_B, \tilde{Q}_A) - c + \tilde{Q}_B \partial_1 P(\tilde{Q}_B, \tilde{Q}_A) &= \hat{q}_{B1} \partial_1 P(\tilde{Q}_B, \tilde{Q}_A) - \hat{q}_{A2} \partial_2 P(\tilde{Q}_A, \tilde{Q}_B). \end{aligned}$$

Differentiating these equations with respect to  $(\tilde{Q}_A, \tilde{Q}_B)$  and  $(\hat{q}_{A2}, \hat{q}_{B1})$  yields:

$$\begin{aligned} \tilde{\lambda}_A d\tilde{Q}_A + \tilde{\mu}_A d\tilde{Q}_B &= \partial_1 P(\tilde{Q}_A, \tilde{Q}_B) d\hat{q}_{A2} - \partial_2 P(\tilde{Q}_B, \tilde{Q}_A) d\hat{q}_{B1}, \\ \tilde{\mu}_B d\tilde{Q}_A + \tilde{\lambda}_B d\tilde{Q}_B &= -\partial_2 P(\tilde{Q}_A, \tilde{Q}_B) d\hat{q}_{A2} + \partial_1 P(\tilde{Q}_B, \tilde{Q}_A) d\hat{q}_{B1}, \end{aligned}$$

where, for  $ih \neq jk \in \{A1, B1\}$

$$\begin{aligned} \tilde{\lambda}_i &= 2\partial_1 P(\tilde{Q}_i, \tilde{Q}_j) + \tilde{q}_{ih} \partial_{11}^2 P(\tilde{Q}_i, \tilde{Q}_j) + \hat{q}_{jh} \partial_{22}^2 P(\tilde{Q}_j, \tilde{Q}_i), \\ \tilde{\mu}_i &= \partial_2 P(\tilde{Q}_i, \tilde{Q}_j) + \tilde{q}_{ih} \partial_{12}^2 P(\tilde{Q}_i, \tilde{Q}_j) + \hat{q}_{jh} \partial_{12}^2 P(\tilde{Q}_j, \tilde{Q}_i). \end{aligned}$$

Under Assumption (B.1), these coefficients satisfy  $\tilde{\lambda}_i < \tilde{\mu}_i$ ; the determinant  $D = \tilde{\lambda}_A \tilde{\lambda}_B - \tilde{\mu}_A \tilde{\mu}_B$  is therefore positive, and thus:

$$\begin{aligned} \frac{\partial \tilde{Q}_i}{\partial \hat{q}_{ik}} &= \frac{\tilde{\lambda}_j \partial_1 P(\tilde{Q}_i, \tilde{Q}_j) + \tilde{\mu}_i \partial_2 P(\tilde{Q}_i, \tilde{Q}_j)}{D}, \\ \frac{\partial \tilde{Q}_j}{\partial \hat{q}_{ik}} &= -\frac{\tilde{\mu}_j \partial_1 P(\tilde{Q}_i, \tilde{Q}_j) + \tilde{\lambda}_i \partial_2 P(\tilde{Q}_i, \tilde{Q}_j)}{D}. \end{aligned} \tag{21}$$

Therefore, we have:

$$\frac{\partial (\tilde{Q}_A + \tilde{Q}_B)}{\partial \hat{q}_{ik}} = \frac{(\tilde{\lambda}_j - \tilde{\mu}_j) \partial_1 P(\tilde{Q}_i, \tilde{Q}_j) - (\tilde{\lambda}_i - \tilde{\mu}_i) \partial_2 P(\tilde{Q}_i, \tilde{Q}_j)}{D}.$$

Assumption (B.2.b) ensures that the numerator, too, is positive, which concludes the proof. ■

Next we show that, in symmetric outcomes, increasing the output of each good beyond the “duopoly” output  $Q^{**}$  reduces aggregate profit:

**Lemma 9** Suppose that (B.1) holds, and consider a market outcome in game  $\Gamma_{1-1}$  where the total output of each good is equal to  $Q$ . Then, the aggregate profit  $\Pi(Q, Q) = 2[P(Q, Q) - c]Q$  is strictly decreasing in  $Q$  for all  $Q \geq Q^{**}$ .

**Proof.** This is obvious when  $Q$  is so large that  $P(Q, Q) = 0$ . When instead  $P(Q, Q) > 0$ , then the derivative of the aggregate profit with respect to per-good output  $Q$  is

$$\frac{d\Pi(Q, Q)}{dQ} = 2[P(Q, Q) - c + Q\partial_1 P(Q, Q) + Q\partial_2 P(Q, Q)].$$

We have:

$$\left. \frac{d\Pi(Q, Q)}{dQ} \right|_{Q=Q^{**}} = 2Q^{**}\partial_2 P(Q^{**}, Q^{**}) < 0,$$

where the inequality stems from (A.2). In addition:

$$\begin{aligned} \frac{1}{2} \frac{d^2\Pi(Q, Q)}{dQ^2} &= \frac{d}{dQ} [P(Q, Q) - c + Q\partial_1 P(Q, Q) + Q\partial_2 P(Q, Q)] \\ &= [2\partial_1 P(Q, Q) + Q\partial_{11} P(Q, Q) + Q\partial_{22} P(Q, Q)] \\ &\quad + [\partial_2 P(Q, Q) + Q(\partial_{12}^2 P(Q, Q) + \partial_{21}^2 P(Q, Q))] + \partial_2 P(Q, Q), \end{aligned}$$

where the last term is negative from (A.2) and the two terms in brackets are negative from (B.1). Hence  $d^2\Pi(Q, Q)/dQ^2 < 0$ , and thus  $d\Pi(Q, Q)/dQ < 0$  for  $Q \geq Q^{**}$ . ■

We now show that, under Assumption (B.2.a), aggregate profit decreases when total output becomes asymmetrically distributed:

**Lemma 10** Suppose (B.2.a) holds. Then, for a fixed level of aggregate output  $Q_A + Q_B = 2Q$ , the aggregate profit  $\Pi(Q_A, Q_B) = [P(Q_A, Q_B) - c]Q_A + [P(Q_B, Q_A) - c]Q_B$  is maximal for  $Q_A = Q_B = Q$ .

**Proof.** Let us fix the total output  $Q_A + Q_B = 2Q$ , and consider the impact of a variation in  $Q_i$  (thus compensated by a mirror variation in  $Q_j$ , for  $i \neq j \in \{A, B\}$ ). The aggregate profit being symmetric in  $Q_A$  and  $Q_B$ , its derivative with respect to  $Q_i$ , holding  $Q_A + Q_B$  fixed, can be expressed as

$$\left. \frac{d\Pi(Q_A, Q_B)}{dQ_i} \right|_{Q_A+Q_B=2Q} = \Psi(Q_i, Q_j) - \Psi(Q_j, Q_i), \quad (22)$$

where

$$\Psi(Q_i, Q_j) \equiv \frac{\partial \Pi(Q_i, Q_j)}{\partial Q_i} = P(Q_i, Q_j) - c + Q_i \partial_1 P(Q_i, Q_j) + Q_j \partial_2 P(Q_j, Q_i).$$



The RHS of (22) is equal to zero when  $Q_A = Q_B = Q$ ; we now show that it is negative whenever  $Q_i > Q_j$ . To see this, consider the derivative of  $\Psi$  with respect to  $Q_i$ , holding  $Q_A + Q_B$  fixed:

$$\begin{aligned} \left. \frac{d\Psi(Q_i, Q_j)}{dQ_i} \right|_{Q_A+Q_B=2Q} &= \frac{\partial\Psi(Q_i, Q_j)}{\partial Q_i} - \frac{\partial\Psi(Q_i, Q_j)}{\partial Q_j} \\ &= 2\partial_1 P(Q_i, Q_j) + Q_i \partial_{11}^2 P(Q_i, Q_j) + Q_j^2 \partial_{12}^2 P(Q_j, Q_i) \\ &\quad - [\partial_2 P(Q_i, Q_j) + \partial_2 P(Q_j, Q_i) + Q_i \partial_{12}^2 P(Q_i, Q_j) + Q_j \partial_{12}^2 P(Q_j, Q_i)] \\ &< 0, \end{aligned}$$

where the inequality follows from (B.2.a). Hence, if  $Q_i > Q = \frac{Q_A+Q_B}{2} > Q_j$ , then  $\Psi(Q_i, Q_j) < \Psi(Q, Q) < \Psi(Q_j, Q_i)$ , implying that (22) is negative; it follows that, keeping total output  $Q_A + Q_B = 2Q$  constant, the aggregate profit  $\Pi(Q_A, Q_B)$  is maximal for  $Q_A = Q_B = Q$ . ■

Combining the above three lemmas yields:

**Proposition 13** *Under Assumptions (B.1)-(B.2), the game  $\Gamma_{1-1}$  has property (P.2).*

**Proof.** Lemmas 9 and 10 together imply that  $\Pi(Q_A, Q_B) < \Pi(Q^{**}, Q^{**})$  whenever  $Q_A + Q_B > 2Q^{**}$ ; the conclusion then follows from Lemma 8. ■

### G.3 Property (P.3)

For Property (P.3), we require another condition on demand:

**(B.3)** For any  $Q_i, Q_j \geq 0$  such that  $P(Q_i, Q_j) > 0$  and  $P(Q_j, Q_i) > 0$ , and for any  $q_i \in [0, Q_i]$ , we have:

$$\partial_1 P(Q_i, Q_j) + q_i \partial_{11}^2 P(Q_i, Q_j) + Q_j \partial_{22}^2 P(Q_j, Q_i) < 0, \quad (\text{B.3.a})$$

and in addition, for any  $q_j \in [0, Q_j]$ :

$$\begin{aligned} [\partial_2 P_i + \partial_2 P_j + (Q_i - q_i) \partial_{12}^2 P_i + q_j \partial_{12}^2 P_j] &< [2\partial_1 P_i + (Q_i - q_i) \partial_{11}^2 P_i + q_j \partial_{22}^2 P_j] \\ \times [\partial_2 P_j + (Q_j - q_j) \partial_{12}^2 P_j + q_i \partial_{12}^2 P_i] &\times [\partial_1 P_j + (Q_j - q_j) \partial_{11}^2 P_j + q_i \partial_{22}^2 P_i], \end{aligned} \quad (\text{B.3.b})$$

where  $P_i \equiv P(Q_i, Q_j)$ .

In the case of linear demand, (B.3) simplifies to  $(\partial_2 P)^2 < (\partial_1 P)^2$ , which holds by (A.2).

**Proposition 14** *Assume (B.1) and (B.3) hold. Then, the game  $\Gamma_{1-1}$  has Property (P.3).*

**Proof.** By symmetry, it suffices to show that, say,  $\partial\tilde{Q}_B/\partial\hat{q}_{B1} \leq 1$ . This is obvious when  $\tilde{q}_{B2} = 0$ , as then  $\tilde{Q}_B = \hat{q}_{B1}$ . Consider now the case where  $\tilde{q}_{B2} > 0$ . If  $\tilde{q}_{A1} = 0$ , then from (20):

$$\frac{\partial\tilde{Q}_B}{\partial\hat{q}_{B1}} = \frac{\partial_1 P(\tilde{Q}_B, \hat{q}_{A2})}{\hat{\lambda}_B}.$$

Assumptions (A.2), (B.1) and (B.3.a) together imply  $\hat{\lambda}_B < \partial_1 P(\tilde{Q}_B, \hat{q}_{A2}) < 0$ , and thus  $\partial\tilde{Q}_B/\partial\hat{q}_{B1} < 1$ .

When instead  $\tilde{q}_{A1} > 0$ , then from (21):

$$\frac{\partial\tilde{Q}_B}{\partial\hat{q}_{B1}} = \frac{\tilde{\lambda}_A \partial_1 P(\tilde{Q}_B, \tilde{Q}_A) + \tilde{\mu}_{BA} \partial_2 P(\tilde{Q}_B, \tilde{Q}_A)}{D},$$

where  $D < 0$  under Assumption (B.1). Hence, this expression is less than one if and only if

$$\begin{aligned} & \left[ \partial_2 P_A + \partial_2 P_B + (\tilde{Q}_A - \hat{q}_{A2}) \partial_{12}^2 P_A + \hat{q}_{B1} \partial_{12}^2 P_B \right] \\ & \times \left[ \partial_2 P_B + (\tilde{Q}_B - \hat{q}_{B1}) \partial_{12}^2 P_B + \hat{q}_{A2} \partial_{12}^2 P_A \right] \\ & < \left[ 2\partial_1 P_A + (\tilde{Q}_A - \hat{q}_{A2}) \partial_{11}^2 P_A + \hat{q}_{B1} \partial_{22}^2 P_B \right] \\ & \times \left[ \partial_1 P_B + (\tilde{Q}_B - \hat{q}_{B1}) \partial_{11}^2 P_B + \hat{q}_{A2} \partial_{22}^2 P_A \right], \end{aligned}$$

which holds under Assumption (B.3.b). ■

## H Proof of Proposition 7

Let  $\Pi^\circ$ ,  $\Pi^*$ , and  $\Pi^{**}$  denote the equilibrium industry profit under no exclusive dealing, single exclusive dealing, and pairwise exclusive dealing, respectively. From (P.2), we know that  $\Pi^{**} > \Pi^*, \Pi^\circ$ . In the absence of exclusive dealing, at least one pair, say  $M_A - R_1$ , makes a weakly lower joint profit than the other pair, i.e.,  $\Pi_{M_A-R_1}^\circ \leq \Pi^\circ/2 \leq \Pi_{M_B-R_2}^\circ$ . We show below that, no matter how profits are shared, the pair  $M_A - R_1$  would benefit from  $M_A$  not dealing with  $R_2$ , and in response the other pair,  $M_B - R_2$ , would benefit from  $M_B$  not dealing with  $R_1$ ; i.e.:  $\Pi_{M_A-R_1}^* > \Pi_{M_A-R_1}^\circ$  and  $\Pi_{M_B-R_2}^{**} > \Pi_{M_B-R_2}^*$ , where  $\Pi_{M_i-R_h}^\circ$ ,  $\Pi_{M_i-R_h}^*$  and  $\Pi_{M_i-R_h}^{**}$  respectively denote the equilibrium joint profit of the pair  $M_i - R_h$  under no exclusivity, under single exclusive dealing where  $M_A$  does not deal with  $R_2$ , and under pairwise exclusive dealing.

We first note that, under single exclusivity,  $M_A$  and  $R_1$  must obtain at least what they could get by deviating to pairwise exclusivity, that is:

$$\Pi_{M_A-R_1}^* \geq \max_{q_{A1}} [P(q_{A1}, q_{B2}^*) - c] q_{A1}. \quad (23)$$

Indeed,  $M_A$  could otherwise profitably deviate by offering a forcing contract  $(\hat{q}_{A1}, \hat{T}_{A1})$ , where  $\hat{q}_{A1} \equiv \arg \max_{q_{A1}} [P(q_{A1}, q_{B2}^*) - c] q_{A1}$  and  $\hat{T}_{A1} = \pi_1^* + \varepsilon$ , where  $\varepsilon > 0$  sufficiently small. Clearly,  $R_1$  would find it profitable to accept this offer, proving the claim.<sup>22</sup>

This, in turn, implies that single exclusivity gives the pair  $M_A - R_1$  more than half of the profit under pairwise exclusivity:

$$\begin{aligned} \Pi_{M_A-R_1}^* &\geq \max_{q_{A1}} [P(q_{A1}, q_{B2}^*) - c] q_{A1} \\ &> \max_{q_{A1}} [P(q_{A1}, q_{B2}^{**}) - c] q_{A1} = \Pi_{M_A-R_1}^{**}, \end{aligned}$$

where the second inequality follows from (P.3), which implies  $q_{B2}^* < q_{B2}^{**}$ , and (A.2).

Using (P.2), we thus have:

$$\Pi_{M_A-R_1}^* > \Pi_{M_A-R_1}^{**} = \frac{\Pi^{**}}{2} > \frac{\Pi^\circ}{2} \geq \Pi_{M_A-R_1}^\circ,$$

which thus implies that, starting from no exclusivity,  $M_A$  and  $R_1$  have an incentive to engage in single exclusivity.

Furthermore, using again (P.2), we have:

$$\Pi_{M_B-R_2}^{**} = \frac{\Pi^{**}}{2} > \frac{\Pi^*}{2} > \Pi^* - \Pi_{M_A-R_1}^* = \Pi_{M_B-R_2}^*,$$

where the second inequality follows from

$$\Pi_{M_A-R_1}^* > \Pi_{M_A-R_1}^{**} = \frac{\Pi^{**}}{2} > \frac{\Pi^*}{2}.$$

It follows that, in response to  $M_A$  and  $R_1$  opting for single exclusivity,  $M_B$  and  $R_2$  have also an incentive to engage in exclusive dealing.

Note finally, by construction, the pair  $M_A - R_1$  obtains a larger joint profit under pairwise exclusivity than in the absence of any exclusivity:

$$\Pi_{M_A-R_1}^\circ \leq \frac{\Pi^\circ}{2} < \frac{\Pi^{**}}{2} = \Pi_{M_A-R_1}^{**}.$$

Therefore,  $M_A$  and  $R_1$  do have an incentive to opt for exclusivity, even if this induces  $M_B$  and  $R_2$  to engage in exclusive dealing as well.

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<sup>22</sup>  $R_1$  may find it profitable to combine  $M_A$ 's deviant offer with the equilibrium contract offered by  $M_B$ . If so, this can only increase  $R_1$ 's incentive to accept  $M_A$ 's deviant offer, without affecting  $M_A$ 's deviation profit.

## I Proof of Proposition 8

Using the same notation as in the proof of Proposition 5, consider the Cournot duopoly game where firm 1 produces both goods  $A$  and  $B$  at marginal cost  $c$  and firm 2 produces good  $A$  at marginal cost  $\hat{c}$  and good  $B$  at marginal cost  $c$ , and the two firms compete in quantities. Let  $(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c}))$  denote the solution to:

$$\hat{Q}_A(\hat{c}) = \hat{R}(\hat{Q}_B(\hat{c}), \hat{c}), \quad (24)$$

$$\hat{Q}_B(\hat{c}) = R(\hat{Q}_A(\hat{c})), \quad (25)$$

where  $\hat{R}$  and  $R$  are as defined in the proof of Proposition 5. Note that  $(Q^\circ, Q^\circ) = (\hat{Q}_A(c), \hat{Q}_B(c))$  and  $(Q_A^*, Q_B^*) = (\hat{Q}_A(\hat{c}^*), \hat{Q}_B(\hat{c}^*))$ , where

$$\hat{c}^* \equiv P(Q_A^*, Q_B^*) + q_{B2}^* \partial_1 P(Q_A^*, Q_B^*).$$

We can thus interpret the move from  $(Q^\circ, Q^\circ)$  to  $(Q_A^*, Q_B^*)$  as the evolution of the equilibrium  $(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c}))$  as  $\hat{c}$  increases from  $c$  to  $\hat{c}^*$ .

We first consider the effects on output. Differentiating (24) and (25) with respect to  $\hat{Q}_A$ ,  $\hat{Q}_B$  and  $\hat{c}$  yields:

$$\begin{aligned} d\hat{Q}_A - \frac{\partial \hat{R}}{\partial \hat{c}}(\hat{Q}_B, \hat{c}) d\hat{Q}_B &= \frac{\partial \hat{R}}{\partial \hat{c}}(\hat{Q}_B, \hat{c}), \\ d\hat{Q}_B &= R'(\hat{Q}_A) d\hat{Q}_A, \end{aligned}$$

and thus:

$$\begin{aligned} \hat{Q}'_A(\hat{c}) &= \frac{\frac{\partial \hat{R}}{\partial \hat{c}}(\hat{Q}_B(\hat{c}), \hat{c})}{1 - \frac{\partial \hat{R}}{\partial \hat{c}}(\hat{Q}_B(\hat{c}), \hat{c}) R'(\hat{Q}_A(\hat{c}))} < 0, \\ -\hat{Q}'_A(\hat{c}) &> \hat{Q}'_B(\hat{c}) = R'(\hat{Q}_A(\hat{c})) \hat{Q}'_A(\hat{c}) > 0. \end{aligned}$$

It follows that, under (A.4), introducing an exclusive dealing agreement on product  $A$  leads to a reduction in the output of  $A$  and, to a lesser extent, to an increase in the output of product  $B$ :

$$Q_A^* < \frac{Q_A^* + Q_B^*}{2} < Q^\circ < Q_B^*.$$

We now turn to the effects on social welfare. Recall that total welfare is equal to

$$W(Q_A, Q_B) = U(Q_A, Q_B) - cQ_A - cQ_B,$$

and thus

$$\frac{\partial W}{\partial Q_i}(Q_A, Q_B) = \frac{\partial U}{\partial Q_i}(Q_A, Q_B) - c = P(Q_i, Q_j) - c.$$

We now show that

$$\hat{W}(\hat{c}) \equiv W(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c}))$$

decreases as  $\hat{c}$  increases. We have:

$$\begin{aligned} \hat{W}'(\hat{c}) &= \frac{\partial W}{\partial Q_A}(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})) \hat{Q}'_A(\hat{c}) + \frac{\partial W}{\partial Q_B}(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})) \hat{Q}'_B(\hat{c}) \\ &= \left[ \frac{\partial U}{\partial Q_A}(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})) - c \right] \hat{Q}'_A(\hat{c}) + \left[ \frac{\partial U}{\partial Q_B}(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})) - c \right] \hat{Q}'_B(\hat{c}) \\ &= \left[ P(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})) - c \right] \hat{Q}'_A(\hat{c}) + \left[ P(\hat{Q}_B(\hat{c}), \hat{Q}_A(\hat{c})) - c \right] R'(\hat{Q}_A(\hat{c})) \hat{Q}'_A(\hat{c}) \\ &\leq \left[ P(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})) - P(\hat{Q}_B(\hat{c}), \hat{Q}_A(\hat{c})) \right] \hat{Q}'_A(\hat{c}), \end{aligned}$$

where the inequality uses  $\hat{P}_B > c$ ,  $\hat{Q}'_A(\hat{c}) < 0$ , and the fact that Assumptions (A.2) and (A.4) imply  $R'(Q) > -1$ . As  $\hat{Q}'_A(\hat{c}) < 0$ , to conclude the argument, it thus suffices to establish that  $\hat{P}_A < \hat{P}_B$ ; we have:

$$\begin{aligned} \frac{d}{d\hat{c}}(\hat{P}_A - \hat{P}_B) &= \left[ \partial_1 P(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})) - \partial_2 P(\hat{Q}_B(\hat{c}), \hat{Q}_A(\hat{c})) \right] \hat{Q}'_A(\hat{c}) \\ &\quad - \left[ \partial_1 P(\hat{Q}_B(\hat{c}), \hat{Q}_A(\hat{c})) - \partial_2 P(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})) \right] \hat{Q}'_B(\hat{c}) \\ &> 0, \end{aligned}$$

where the inequality stems from Assumptions (A.2) – which, using symmetry, implies  $\partial_1 P(Q_i, Q_j) < \partial_2 P(Q_j, Q_i) = \partial_2 P(Q_i, Q_j)$  (see footnote 2) – and (A.4), which imply  $\hat{Q}'_B(\hat{c}) > 0 > \hat{Q}'_A(\hat{c})$ . As  $\hat{P}_A = \hat{P}_B = P^\circ$  for  $\hat{c} = c$ , it follows that  $\hat{P}_A > \hat{P}_B$  for any  $\hat{c} > c$ .

Hence, we have that, under (A.4), introducing an exclusive dealing agreement decreases welfare.

We can use the same approach for consumer surplus. Using

$$S(Q_A, Q_B) = W(Q_A, Q_B) - \Pi(Q_A, Q_B),$$

we have:

$$\begin{aligned} \frac{\partial S}{\partial Q_i}(Q_A, Q_B) &= \frac{\partial W}{\partial Q_i}(Q_A, Q_B) - \frac{\partial \Pi}{\partial Q_i}(Q_A, Q_B) \\ &= P(Q_i, Q_j) - c - [P(Q_i, Q_j) - c + \partial_1 P(Q_i, Q_j) Q_i + \partial_2 P(Q_j, Q_i) Q_j] \\ &= -\partial_1 P(Q_i, Q_j) Q_i - \partial_2 P(Q_j, Q_i) Q_j. \end{aligned}$$

Letting

$$\hat{S}(\hat{c}) \equiv S\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right)$$

denote consumer surplus in the equilibrium of the duopoly game, we have:

$$\begin{aligned} \hat{S}'(\hat{c}) &= \frac{\partial S}{\partial Q_A}\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right) \hat{Q}'_A(\hat{c}) + \frac{\partial S}{\partial Q_B}\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right) \hat{Q}'_B(\hat{c}) \\ &= -[\partial_1 P(Q_A, Q_B) Q_A + \partial_2 P(Q_B, Q_A) Q_B] \hat{Q}'_A(\hat{c}) - [\partial_1 P(Q_B, Q_A) Q_B + \partial_2 P(Q_A, Q_B) Q_A] \hat{Q}'_B(\hat{c}) \\ &= -\left\{ \partial_1 P(Q_A, Q_B) Q_A + \partial_2 P(Q_B, Q_A) Q_B + [\partial_1 P(Q_B, Q_A) Q_B + \partial_2 P(Q_A, Q_B) Q_A] R'(\hat{Q}_A(\hat{c})) \right\} \hat{Q}'_A(\hat{c}) \\ &= -\left\{ \begin{array}{l} \partial_1 P(Q_A, Q_B) Q_A \left[ 1 + \frac{\partial_2 P(Q_A, Q_B)}{\partial_1 P(Q_A, Q_B)} R'(\hat{Q}_A(\hat{c})) \right] \\ + \partial_1 P(Q_B, Q_A) Q_B \left[ R'(\hat{Q}_A(\hat{c})) + \frac{\partial_2 P(Q_B, Q_A)}{\partial_1 P(Q_B, Q_A)} \right] \end{array} \right\} \hat{Q}'_A(\hat{c}). \end{aligned}$$

As  $\hat{Q}'_A(\hat{c}), \partial_1 P(\cdot), \partial_2 P(\cdot) < 0$  and  $R'(\cdot) > -1$ , it follows that  $\hat{S}'(\hat{c}) < 0$  if (A.5) holds.

We thus have that, under (A.4) and (A.5), introducing an exclusive dealing agreement decreases consumer surplus as well as social welfare.

We now turn to the effects on industry profit. For  $\hat{c} = \hat{c}^*$ ,  $q_{A2}^* = 0$  and thus the industry profit in the duopoly game coincides with the “true” industry profit, based on the actual cost  $c$ :

$$\Pi^* = (p_A^* - c) Q_A^* + (p_B^* - c) Q_B^*.$$

Therefore, to compare  $\Pi^*$  with  $\Pi^\circ$ , it suffices to study how the industry profit, *based on true costs*, evolves with  $\hat{c}$  in the duopoly game. Thus, let define:

$$\hat{\Pi}(\hat{c}) \equiv \left( P\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right) - c \right) \hat{Q}_A(\hat{c}) + \left( P\left(\hat{Q}_B(\hat{c}), \hat{Q}_A(\hat{c})\right) - c \right) \hat{Q}_B(\hat{c}).$$

We have:

$$\begin{aligned} \hat{\Pi}'(\hat{c}) &= \left[ P\left(\hat{Q}_A, \hat{Q}_B\right) - c + \partial_1 P\left(\hat{Q}_A, \hat{Q}_B\right) \hat{Q}_A + \partial_2 P\left(\hat{Q}_B, \hat{Q}_A\right) \hat{Q}_B \right] \hat{Q}'_A \\ &\quad + \left[ P\left(\hat{Q}_B, \hat{Q}_A\right) - c + \partial_1 P\left(\hat{Q}_B, \hat{Q}_A\right) \hat{Q}_B + \partial_2 P\left(\hat{Q}_A, \hat{Q}_B\right) \hat{Q}_A \right] \hat{Q}'_B, \end{aligned}$$

which, using the FOCs for  $R_1$ 's outputs  $\hat{q}_{A1}$  and  $\hat{q}_{B1}$ :

$$P\left(\hat{Q}_B, \hat{Q}_A\right) - c + \partial_1 P\left(\hat{Q}_B, \hat{Q}_A\right) \hat{q}_{B1} + \partial_2 P\left(\hat{Q}_A, \hat{Q}_B\right) \hat{q}_{A1} = 0,$$

$$P\left(\hat{Q}_A, \hat{Q}_B\right) - c + \partial_1 P\left(\hat{Q}_A, \hat{Q}_B\right) \hat{q}_{A1} + \partial_2 P\left(\hat{Q}_B, \hat{Q}_A\right) \hat{q}_{B1} = 0,$$

can be written as:

$$\begin{aligned} \hat{\Pi}'(\hat{c}) &= \left[ \partial_1 P\left(\hat{Q}_A, \hat{Q}_B\right) \hat{q}_{A2} + \partial_2 P\left(\hat{Q}_B, \hat{Q}_A\right) \hat{q}_{B2} \right] \hat{Q}'_A + \left[ \partial_1 P\left(\hat{Q}_B, \hat{Q}_A\right) \hat{q}_{B2} + \partial_2 P\left(\hat{Q}_A, \hat{Q}_B\right) \hat{q}_{A2} \right] R'(\hat{Q}_A) \hat{Q}'_A \\ &= \left\{ \left[ \partial_1 P\left(\hat{Q}_A, \hat{Q}_B\right) + \partial_2 P\left(\hat{Q}_A, \hat{Q}_B\right) R'(\hat{Q}_A) \right] \hat{q}_{A2} + \left[ \partial_2 P\left(\hat{Q}_B, \hat{Q}_A\right) + \partial_1 P\left(\hat{Q}_B, \hat{Q}_A\right) R'(\hat{Q}_A) \right] \hat{q}_{B2} \right\} \hat{Q}'_A. \end{aligned}$$

The first term within bracket is negative, as  $\partial_1 P(\hat{Q}_A, \hat{Q}_B) < \partial_2 P(\hat{Q}_A, \hat{Q}_B) < 0$  and  $R'(\hat{Q}_A) > -1$ . As  $\hat{Q}'_A < 0$ , it follows that  $\hat{\Pi}'(\hat{c}) > 0$  if the second term within brackets is non-positive, i.e., if:

$$\partial_2 P(\hat{Q}_B, \hat{Q}_A) + \partial_1 P(\hat{Q}_B, \hat{Q}_A) R'(\hat{Q}_A) \leq 0,$$

which amounts to Assumption (A.5). Hence, under (A.4) and (A.5), introducing an exclusive dealing agreement increases industry profit at the expense of consumer surplus and social welfare.

## J Proof of Proposition 9

We first use a revealed preference argument to show that  $Q^{**} \leq Q^\circ$ . Recall that  $Q^{**} = q^{**}$  and  $Q^\circ = 2q^\circ$  are such that

$$\begin{aligned} Q^{**} &= \arg \max_q [P(q, Q^{**}) - c] q, \\ \frac{Q^\circ}{2} &= \arg \max_q \left[ P\left(\frac{Q^\circ}{2} + q, Q^\circ\right) - c \right] q + \left[ P\left(Q^\circ, \frac{Q^\circ}{2} + q\right) - c \right] \frac{Q^\circ}{2}. \end{aligned}$$

Hence, we have

$$[P(Q^{**}, Q^{**}) - c] Q^{**} \geq [P(Q^\circ, Q^{**}) - c] Q^\circ \quad (26)$$

and

$$\begin{aligned} [P(Q^\circ, Q^\circ) - c] Q^\circ &\geq [P(Q^{**}, Q^\circ) - c] \left( Q^{**} - \frac{Q^\circ}{2} \right) + [P(Q^\circ, Q^{**}) - c] \frac{Q^\circ}{2} \\ &= [P(Q^{**}, Q^\circ) - c] Q^{**} + [P(Q^\circ, Q^{**}) - P(Q^{**}, Q^\circ)] \frac{Q^\circ}{2}. \end{aligned}$$

If  $Q^{**} > Q^\circ$ , the last term on the RHS is positive from (A.2), implying

$$[P(Q^\circ, Q^\circ) - c] Q^\circ \geq [P(Q^{**}, Q^\circ) - c] Q^{**}. \quad (27)$$

Combining (26) and (27) yields

$$[P(Q^{**}, Q^{**}) - P(Q^{**}, Q^\circ)] Q^{**} \geq [P(Q^\circ, Q^{**}) - P(Q^\circ, Q^\circ)] Q^\circ,$$

i.e.,

$$\int_{Q^\circ}^{Q^{**}} Q^{**} \partial_2 P(Q^{**}, Q) dQ \geq \int_{Q^\circ}^{Q^{**}} Q^\circ \partial_2 P(Q^\circ, Q) dQ,$$

which is equivalent to

$$\int_{Q^\circ}^{Q^{**}} \int_{Q^\circ}^{Q^{**}} \left[ \partial_2 P(\tilde{Q}, Q) + \tilde{Q} \partial_{12}^2 P(\tilde{Q}, Q) \right] d\tilde{Q} dQ \geq 0.$$

(A.2) implies that the term in brackets is strictly negative, a contradiction. Hence, we must have  $Q^{**} \leq Q^\circ$ .

Next, suppose that  $Q^{**} = Q^\circ$ . The first-order conditions of the above maximization problems (for  $q^{**} = Q^{**}$  and  $q^\circ = Q^\circ/2$ ) then yield:

$$P(Q^\circ, Q^\circ) - c = -[\partial_1 P(Q^\circ, Q^\circ) + \partial_2 P(Q^\circ, Q^\circ)] \frac{Q^\circ}{2} = -\partial_1 P(Q^\circ, Q^\circ) Q^\circ,$$

implying  $\partial_1 P(Q^\circ, Q^\circ) = \partial_2 P(Q^\circ, Q^\circ)$ , and thus contradicting (A.2). Hence, we must have  $Q^{**} < Q^\circ$ .

It follows that consumer surplus is greater in the absence of exclusive dealing, as  $S(Q, Q)$  increases with  $Q$ :

$$\frac{dS(Q, Q)}{dQ} = -2Q [\partial_1 P(Q, Q) + \partial_2 P(Q, Q)],$$

which is positive from (A.2).

Exclusive dealing also harms welfare, as  $W(Q, Q)$  increases with  $Q$  as long as  $P(Q, Q) > c$ :

$$\begin{aligned} \frac{dW(Q, Q)}{dQ} &= P(Q, Q) - c + \int_0^Q \partial_2 P(q, Q) dq + P(Q, 0) - c \\ &= P(Q, Q) - c + \int_0^Q \partial_2 P(Q, q) dq + P(Q, 0) - c \\ &= 2[P(Q, Q) - c], \end{aligned}$$

where the second equality follows from the fact that demand symmetry implies that  $\partial_2 P(q, Q) \equiv \partial_2 P(Q, q)$ . To conclude the argument, it suffices to note that  $P(Q, Q)$  is decreasing in  $Q$  from (A.2), and that the first-order condition for  $q^\circ$  yields  $P(Q^\circ, Q^\circ) > c$ :

$$P(Q^\circ, Q^\circ) - c = -[\partial_1 P(Q^\circ, Q^\circ) + \partial_2 P(Q^\circ, Q^\circ)] \frac{Q^\circ}{2}, \quad (28)$$

where (A.2) implies that the term in brackets is strictly negative, and thus the LHS is positive.

Finally, to show that  $\Pi^{**} > \Pi^\circ$ , it suffices to note that the industry-wide aggregate profit  $\Pi(Q, Q)$  is concave in  $Q$  under (A.4), and maximal to  $Q < Q^{**}$ :

$$\begin{aligned} \frac{1}{2} \frac{d^2 \Pi(Q, Q)}{dQ^2} &= [2\partial_1 P(Q, Q) + \partial_{11}^2 P(Q, Q) Q + \partial_{22}^2 P(Q, Q) Q] \\ &\quad + [2\partial_2 P(Q, Q) + 2\partial_{12}^2 P(Q, Q) Q], \end{aligned}$$

where both expressions in brackets on the RHS are negative under (A.4), and the first-order derivative, evaluated at  $Q^{**}$ , satisfies:

$$\left. \frac{d\Pi(Q, Q)}{dQ} \right|_{Q=Q^{**}} = 2Q^{**} \partial_2 P(Q^{**}, Q^{**}),$$

where the RHS is negative from (A.2).



## K Proof of Proposition 10

### K.1 Candidate equilibrium

We first characterize some of the properties of the candidate equilibrium described in Proposition 10.

#### K.1.1 Equilibrium quantities

The equilibrium output levels satisfy:

$$\begin{aligned} q_{A1}^*, q_{B1}^* &= \arg \max_{q_{A1}, q_{B1}} \{ [P(q_{A1}, q_{B1} + q_{B2}^*) - c] q_{A1} + [P(q_{B1} + q_{B2}^*, q_{A1}) - c] q_{B1} \}, \\ q_{B2}^* &= \arg \max_{q_{B2}} \{ [P(q_{B1} + q_{B2}, q_{A1}^*) - c] q_{B2} \}. \end{aligned}$$

The equilibrium profits are thus equal to:

$$\begin{aligned} \pi_{M_A-R_1}^* &= [P(q_{A1}^*, q_{B1}^* + q_{B2}^*) - c] q_{A1}^* + P(q_{B1}^* + q_{B2}^*, q_{A1}^*) q_{B1}^* - T_{B1}^*, \\ \pi_{R_2}^* &= [P(q_{B1}^* + q_{B2}^*, q_{A1}^*) - c] q_{B2}^*, \\ \pi_{M_B}^* &= T_{B1}^* - c q_{B1}^*. \end{aligned}$$

#### K.1.2 Equilibrium fees

**Determination of  $T_{B1}^*$**  In equilibrium,  $R_1$  must be indifferent between rejecting or accepting  $M_B$ 's offer:

- $R_1$  should not benefit from rejecting the offer, otherwise it would do so;
- conversely, if  $R_1$  was strictly better off accepting the offer, then  $M_B$  could slightly increase its fee: with passive beliefs  $R_1$  would then still accept the offer, making  $M_B$ 's deviation profitable.

If  $R_1$  rejects  $M_B$ 's offer, it will sell  $\tilde{q}_{A1}$  units of good  $A$ , where:

$$\tilde{q}_{A1} \equiv \arg \max_{q_{A1}} [P(q_{A1}, q_{B2}^*) - c] q_{A1},$$

and thus obtain a profit equal to:

$$\tilde{\pi}_{M_A-R_1} \equiv [P(\tilde{q}_{A1}, q_{B2}^*) - c] \tilde{q}_{A1}.$$

Therefore, the fee  $T_{B1}^*$  should be such that  $\pi_{M_A-R_1}^* = \tilde{\pi}_{M_A-R_1}$ , or:

$$T_{B1}^* = [P(q_{A1}^*, q_{B1}^* + q_{B2}^*) - c] q_{A1}^* + P(q_{B1}^* + q_{B2}^*, q_{A1}^*) q_{B1}^* - [P(\tilde{q}_{A1}, q_{B2}^*) - c] \tilde{q}_{A1}.$$

This in particular ensures that  $M_B$ 's equilibrium profit is non-negative:

$$\begin{aligned}
\pi_{M_B}^* &= T_{B1}^* - cq_{B1}^* \\
&= [P(q_{A1}^*, q_{B1}^* + q_{B2}^*) - c] q_{A1}^* + [P(q_{B1}^* + q_{B2}^*, q_{A1}^*) - c] q_{B1}^* - [P(\tilde{q}_{A1}, q_{B2}^*) - c] \tilde{q}_{A1} \\
&= \max_{q_{A1}, q_{B1}} \{ [P(q_{A1}, q_{B1} + q_{B2}^*) - c] q_{A1} + [P(q_{B1} + q_{B2}^*, q_{A1}) - c] q_{B1} \} - \max_{q_{A1}} [P(q_{A1}, q_{B1} + q_{B2}^*) - c] q_{A1} \\
&\geq 0.
\end{aligned} \tag{29}$$

**Determination of  $(\hat{q}, \hat{T})$**  The described equilibrium is such that  $R_2$  must obtain its equilibrium profit  $\pi_{R_2}^*$  by accepting only  $M_A$ 's contract  $(\hat{q}, \hat{T})$ ; conversely,  $R_2$  should not obtain more profit by dealing with both manufacturers. As  $M_B$  offers to supply  $R_2$  at cost, this in turn implies that  $\hat{q}$  should be "large enough" to ensure that, conditional on selling  $\hat{q}$  units of good  $A$ ,  $R_2$  does not want to sell any positive quantity of good  $B$ . Assumptions (A.1) and (A.2) ensure that such large values exist for  $\hat{q}$ ; if  $R_2$ 's profit is quasi-concave in  $q_{B2}$ , then a necessary and sufficient condition is that accepting  $M_A$ 's contract lowers the marginal revenue for good  $B$  below its cost, i.e.

$$P(q_{B1}^*, q_{A1}^* + \hat{q}) + \partial_2 P(q_{A1}^* + \hat{q}, q_{B1}^*) \hat{q} \leq c. \tag{30}$$

**Remark 1** We are considering "quantity forcing" contracts where the retailer commits itself to sell the agreed quantity. If  $R_2$  only commits itself to buy the quantity  $\hat{q}$ , we would also need to check that it is willing to put all the quantity on the market; if  $R_2$ 's profit is quasi-concave in  $q_{A2}$  and  $q_{B2}$ , then a necessary and sufficient condition is:

$$P(q_{A1}^* + \hat{q}, q_{B1}^*) + \partial_1 P(q_{A1}^* + \hat{q}, q_{B1}^*) \hat{q} \geq 0. \tag{31}$$

As  $R_2$  could obtain  $P(q_{A1}^* + \hat{q}, q_{B1}^*) \hat{q} - \hat{T}$  by deviating and selecting  $M_A$ 's offer  $(\hat{q}, \hat{T})$  instead of  $M_B$ 's offer, we must have:

$$\hat{T} = P(q_{A1}^* + \hat{q}, q_{B1}^*) \hat{q} - [P(q_{B1}^* + q_{B2}^*, q_{A1}^*) - c] q_{B2}^*.$$

**Remark 2** The payment  $\hat{T}$  thus satisfies:

$$\hat{T} = \max_{q_{B2}} \{ P(q_{A1}^* + \hat{q}, q_{B1}^* + q_{B2}) \hat{q} + [P(q_{B1}^* + q_{B2}, q_{A1}^* + \hat{q}) - c] q_{B2} \} - \max_{q_{B2}} \{ [P(q_{B1}^* + q_{B2}, q_{A1}^*) - c] q_{B2} \}.$$

Intuitively, this implies that  $\hat{T} \geq 0$  if (31) holds. Indeed, when differentiating  $R_2$ ' deviation profit with respect to  $\hat{q}$ , the envelope theorem yields:

$$\frac{d\pi_{R_2}}{d\hat{q}} = P(q_{A1}^* + \hat{q}, q_{B1}^* + \hat{q}_{B2}) + \partial_1 P(q_{A1}^* + \hat{q}, q_{B1}^* + \hat{q}_{B2}) \hat{q} + \partial_2 P(q_{B1}^* + \hat{q}_{B2}, q_{A1}^* + \hat{q}) \hat{q}_{B2},$$

where  $\hat{q}_{B2}$  denotes  $R_2$ 's best response output of good  $B$  when selling  $\hat{q}$  units of good  $A$  (thus, for  $\hat{q}$  large enough,  $\hat{q}_{B2} = 0$ ). Assuming that this marginal profit decreases in  $\hat{q}$ , (31) ensures that  $\hat{T} \geq 0$ .

## K.2 Deviations

The above characterization of equilibrium quantities and fees ensures that retailers have no profitable deviation, neither at the acceptance stage nor at the product market competition stage. We thus now focus on manufacturers' deviations at the offer stage.

### K.2.1 Deviations by $M_B$

Given the passive beliefs assumption, it suffices to consider one-sided deviations. Also, by construction such a one-sided deviation cannot be profitable if it is not accepted, as in equilibrium  $M_B$  makes a non-negative profit with both  $R_1$  and  $R_2$ .

Consider first a deviant offer to  $R_1$ . Such a deviation cannot reduce  $M_A - R_1$ 's payoff, which it can secure by rejecting  $M_B$ 's offer. But it cannot increase the joint profit that  $M_B$  generates with  $M_A - R_1$  through  $R_1$ 's sales either, as the equilibrium contract offered to  $R_1$  is bilaterally efficient.

Consider now a deviant offer to  $R_2$ . Again, such a deviation cannot reduce  $R_2$ 's payoff, which it can secure by rejecting  $M_B$ 's offer and accepting instead  $M_A$ 's offer. And it cannot increase the joint profit that  $M_B$  generates with  $R_2$  either, as the equilibrium contract offered to  $R_2$  is bilaterally efficient, regardless of whether  $R_2$  accepts or rejects  $M_A$ 's offer.

### K.2.2 Deviations by $M_A - R_1$

**Consider first a deviant offer by  $M_A - R_1$  that induces  $R_2$  to reject it** Suppose first that, in the continuation equilibrium,  $R_2$  accepts  $M_B$ 's offer. If  $R_1$  also accepts  $M_B$ 's offer, then the continuation equilibrium quantities are  $(q_{A1}^*, q_{B1}^*, q_{B2}^*)$ ;  $M_A - R_1$  thus obtains its equilibrium profit, making the deviation unprofitable. If instead  $R_1$  rejects  $M_B$ 's offer then, from (P.3), in the continuation equilibrium  $M_B$  puts on the market a larger quantity  $q_{B2} > q_{B2}^*$ , and thus  $M_A - R_1$  thus obtains less than its equilibrium profit, making again the deviation unprofitable.

Therefore, if such a deviation is profitable, it must induce  $R_2$  to reject  $M_B$ 's offer. We can distinguish two cases, depending on  $R_1$ 's acceptance decision of  $M_B$ 's offer:

- If  $R_1$ , too, rejects  $M_B$ 's offer, then it will sell  $q_{A1}$  units of good  $A$ , so as to maximize

$$[P(q_{A1}, 0) - c] q_{A1},$$

and thus such that  $p_A = P(q_{A1}, 0) > c$ . But then,  $p_B = P(0, q_{A1}) > P(q_{A1}, 0) > c$  by Assumption (A.2), which in turn implies that  $R_2$  would rather accept  $M_B$ 's offer and sell a positive quantity of good  $B$ , in contradiction with  $R_2$ 's supposed rejection of  $M_B$ 's offer.

- If instead  $R_1$  accepts  $M_B$ 's offer, then it will sell  $q_{B1}^*$  units of good  $B$  and  $q_{A1}^a$  units of good  $A$ , so as to maximize

$$[P(q_{A1}, q_{B1}^*) - c] q_{A1} + P(q_{B1}^*, q_{A1}) q_{B1}^* - T_{B1}^*.$$

By revealed preference, this must exceed the profit it could achieved by rejecting  $M_B$ 's offer and selling only good  $A$ , which implies:

$$\begin{aligned} P(q_{B1}^*, q_{A1}^a) q_{B1}^* &\geq T_{B1}^* + \max_{q_{A1}} [P(q_{A1}, 0) - c] q_{A1} - [P(q_{A1}^a, q_{B1}^*) - c] q_{A1}^a \\ &> T_{B1}^* + \max_{q_{A1}} [P(q_{A1}, 0) - c] q_{A1} - [P(q_{A1}^a, 0) - c] q_{A1}^a \\ &\geq cq_{B1}^*, \end{aligned}$$

where the strict inequality follows from Assumption (A.2) and the last inequality stems from (29). Therefore,  $P(q_{B1}^*, q_{A1}^a) > c$ , which again implies that  $R_2$  would rather accept  $M_B$ 's offer and sell a positive quantity of good  $B$ , in contradiction with  $R_2$ 's supposed rejection of  $M_B$ 's offer.

**Consider now a deviant offer by  $M_A - R_1$  that is accepted by  $R_2$  together with  $M_B$ 's offer** Let  $\bar{q}$  denote the quantity of good  $A$  sold by  $R_2$  in the continuation equilibrium.

*Suppose first that, in the continuation equilibrium,  $R_1$  also keeps accepting  $M_B$ 's offer.*

In such a continuation equilibrium:

- By Property (P.2), the aggregate profit cannot exceed that achieved in the candidate equilibrium.
- $M_B$  obtains the same profit (namely,  $T_{B1}^* - cq_{B1}^*$ ) as in the candidate equilibrium;
- $R_2$  gets at least its candidate equilibrium profit  $\pi_{R2}^*$ ; if that were not the case,  $R_2$  could profitably deviate by rejecting  $M_A$ 's deviant offer and deal instead only with  $M_B$ : denoting

by  $q_{A1}^b$   $R_1$ 's output of good  $A$  in the continuation equilibrium,  $R_2$  would obtain in this way:

$$\max_{q_{B2}} \left[ P \left( q_{B1}^* + q_{B2}, q_{A1}^b \right) - c \right] q_{B2} \geq \max_{q_{B2}} \left[ P \left( q_{B1}^* + q_{B2}, q_{A1}^* \right) - c \right] q_{B2} = \pi_{R_2}^*,$$

where the inequality follows from (P.3) and the fact that  $R_2$ 's profit decreases in  $q_{A1}$ .

Thus, the deviation cannot be profitable for  $M_A - R_1$ .

**Remark 3** *Applying this reasoning to a deviant offer equal to the equilibrium shadow offer  $(\hat{q}, \hat{T})$  shows that, in equilibrium,  $M_A - R_1$  strictly prefers that  $R_2$  rejects  $M_A$ 's offer.*

**Remark 4** *The previous remark does not necessarily imply that offering  $(\hat{q}, \hat{T})$  is a dominated strategy for  $M_A - R_1$ : this will indeed not be the case if there exists a  $q_{B1}$  (together with an appropriate best response  $q_{A1}$ ) and a  $q_{B2}$  such that  $M_A - R_1$  is better off when  $R_2$  mistakenly accepts  $(\hat{q}, \hat{T})$ .*

*Suppose now that, in the continuation equilibrium,  $R_1$  rejects  $M_B$ 's offer.*

Such a deviation yields quantities  $q_{A1} = \tilde{q}_{A1}(0, \bar{q})$ ,  $q_{B1} = 0$ ,  $q_{A2} = \bar{q}$ ,  $q_{B2} = \tilde{q}_{B2}(0, \bar{q})$ , where  $(\tilde{q}_{A1}, \tilde{q}_{B2})$  denote the equilibrium outputs of the game  $\Gamma_{1-1}$  for  $\hat{q}_{B1} = 0$  and  $\hat{q}_{A2} = \bar{q}$ .

We first show that the continuation equilibrium, following  $M_A$ 's deviation (the “deviation equilibrium,” referred to below with a superscript  $c$ ), is less profitable for  $M_A - R_1$  than the equilibrium of the game  $\Gamma_{1-1}$  for  $\hat{q}_{A2} = \hat{q}_{B1} = 0$  (the “alternative equilibrium,” referred to below with a superscript  $d$ ). To see this, note that:

- $M_B$  makes the same profit in both equilibrium scenarios:  $\pi_{M_B}^d = \pi_{M_B}^c = 0$ .
- $R_2$  cannot make more profit in the alternative equilibrium than in the deviation equilibrium, i.e.:  $\pi_{R_2}^d \leq \pi_{R_2}^c$ . To show this, consider a deviation from the “deviation equilibrium,” in which  $R_2$  only accepts  $M_B$ 's offer; this deviation gives  $R_2$  a profit

$$\pi_{R_2}^e \equiv \max_{q_{B2}} \left[ P \left( q_{B2}, q_{A1}^c \right) - c \right] q_{B2},$$

which (weakly) exceeds

$$\pi_{R_2}^d = \max_{q_{B2}} \left[ P \left( q_{B2}, q_{A1}^d \right) - c \right] q_{B2},$$

as  $q_{A1}^d = \tilde{q}_{A1}(0, 0) \geq q_{A1}^c = \tilde{q}_{A1}(0, \bar{q})$ , from Property (P.3). As the “deviation from the deviation equilibrium” has to be unprofitable, we have:  $\pi_{R_2}^c \geq \pi_{R_2}^e \geq \pi_{R_2}^d$ .

- Property (P.2) ensures that the aggregate profit is larger in the alternative equilibrium than in the deviation equilibrium:  $\pi_{M_B}^d + \pi_{M_A-R_1}^d + \pi_{R_2}^d \geq \pi_{M_B}^c + \pi_{M_A-R_1}^c + \pi_{R_2}^c$ .

It follows that the integrated firm makes more profit in the alternative equilibrium than in the deviation equilibrium:

$$\pi_{M_A-R_1}^d \geq \pi_{M_A-R_1}^c.$$

But  $\pi_{M_A-R_1}^d < \tilde{\pi}_{M_A-R_1} = \max_{q_{A1}} [P(q_{A1}, q_{B2}^*) - c] q_{A1}$ , as  $R_2$  is more aggressive in the alternative equilibrium than in the “pseudo duopoly” scenario in which  $R_1$  carries  $A$  only and  $R_2$  carries  $B$  only, but  $R_2$  anticipates that  $R_1$  is also carrying  $B$ , and thus sells  $q_{B2}^* = \tilde{q}_{B2}(q_{B1}^*, 0)$  rather than  $q_{B2} = \tilde{q}_{B2}(0, 0)$ . As  $\tilde{\pi}_{M_A-R_1} = \pi_{M_A-R_1}^*$ , we have:

$$\pi_{M_A-R_1}^c \leq \pi_{M_A-R_1}^d \leq \pi_{M_A-R_1}^*.$$

That is,  $M_A - R_1$ 's deviation is not profitable.

**Finally, consider now a deviation by  $M_A - R_1$  that induces  $R_2$  to drop  $M_B$ 's offer.**

Let  $(q_{A2} = \bar{q} > 0, T_{A2} = \bar{T})$  denote the deviant offer. As  $R_2$  can costlessly accept  $M_B$ 's offer to supply at cost, and then choose  $q_{B2} = 0$ , the above reasoning (for deviations inducing  $R_2$  to accept the deviant offer by  $M_A$  as well as  $M_B$ 's offer) still applies, which concludes the proof.

## L Proof of Proposition 11

To show existence of the equilibrium, let

$$\Pi(Q_A, Q_B) \equiv [P(Q_A, Q_B) - c] Q_A + [P(Q_B, Q_A) - c] Q_B$$

denote aggregate output when  $q_{A1} + q_{A2} = Q_A$  and  $q_{B1} + q_{B2} = Q_B$ . To support the vector  $(q_{A1}^{**}, q_{A2}^{**}, q_{B1}^{**}, q_{B2}^{**}) = (Q^{**}, 0, 0, Q^{**})$  as an equilibrium outcome, we suppose that the two integrated firms do not offer contracts to each other, i.e.,  $\tau_{A2}^{**} = \emptyset$  and  $\tau_{B1}^{**} = \emptyset$ . To show that there is no profitable deviation, suppose that the integrated  $M_i - R_h$  deviates from this candidate equilibrium by offering a contract  $\tilde{\tau}_{ik}(\cdot)$  that induces a quantity  $\hat{q}_{ik}$  through the channel  $M_i - R_k$ . By assumption,  $M_j$  does not offer any contract to  $R_h$  in the candidate equilibrium, and thus we still have  $\hat{q}_{jh} = 0$ , as in the candidate equilibrium. The resulting quantities  $\tilde{q}_{ih}(\hat{q}_{ik}, 0)$  and  $\tilde{q}_{jk}(\hat{q}_{ik}, 0)$  are the equilibrium quantities in game  $\Gamma_{2-1}$  when  $\hat{q}_{jh} = 0$ :

$$\tilde{q}_{ih}(\hat{q}_{ik}, 0) = \arg \max_{q_{ih}} \Pi_h(q_{ih}, \tilde{q}_{jk}(\hat{q}_{ik}, 0); \hat{q}_{ik}, 0), \quad (32)$$

$$\tilde{q}_{jk}(\hat{q}_{ik}, 0) = \arg \max_{q_{jk}} \Pi_k(\tilde{q}_{ih}(\hat{q}_{ik}, 0), q_{jk}; \hat{q}_{ik}, 0). \quad (33)$$

Now, note that each integrated firm  $M_j - R_k$  can guarantee itself at least the candidate equilibrium profit  $\Pi^{**}/2 \equiv \Pi(Q^{**}, Q^{**})/2$  by simply rejecting  $M_i$ 's deviant offer; in this way, it would obtain:

$$\max_{q_{jk}} \Pi_k(\tilde{q}_{ih}(\hat{q}_{ik}, 0), q_{jk}; 0, 0).$$

But (P.3) implies  $\tilde{q}_{ih}(\hat{q}_{ik}, 0) \leq \tilde{q}_{ih}(0, 0) = q_{ih}^{**}$ ; as the profit of  $M_j - R_k$  decreases in  $q_{ih}$ , the above profit is at least equal to:

$$\max_{q_{jk}} \Pi_k(q_{ih}^{**}, q_{jk}; 0, 0) = \Pi^{**}/2.$$

Therefore, in order to be profitable, the deviation must increase the total profits of the two integrated firms:

$$\Pi(\tilde{q}_{ih}(\hat{q}_{ik}, 0) + \hat{q}_{ik}, \tilde{q}_{jk}(\hat{q}_{ik}, 0)) > \Pi(Q^{**}, Q^{**}).$$

But this contradicts (P.2).

To show uniqueness of equilibrium, suppose instead that there exists another equilibrium  $(q_{A1}^{**}, q_{A2}^{**}, q_{B1}^{**}, q_{B2}^{**}) \neq (Q^{**}, 0, 0, Q^{**})$ . This implies, in particular, that  $q_{A2}^{**} > 0$  or  $q_{B1}^{**} > 0$ . The induced aggregate profit is  $\Pi(Q_A^{**}, Q_B^{**})$ , where  $Q_A^{**} \equiv q_{A1}^{**} + q_{A2}^{**}$  and  $Q_B^{**} \equiv q_{B1}^{**} + q_{B2}^{**}$ . By (P.2), we have  $\Pi(Q_A^{**}, Q_B^{**}) < \Pi(Q^{**}, Q^{**})$ . The equilibrium profit of at least one of the two integrated firms, say  $M_A - R_1$ , must therefore be strictly less than  $\Pi(Q^{**}, Q^{**})/2$ . Consider the following deviation by  $M_A - R_1$ : it does not offer any contract to the rival retailer  $R_2$  nor does it accept any contract from the rival manufacturer  $M_B$ . The deviation at the offer stage induces a continuation equilibrium in which  $R_2$  expects  $R_1$  to put some quantity  $q_{B1}$  of good  $B$  on the market, and therefore chooses a quantity  $\tilde{q}_{B2}(q_{B1}, 0) \geq 0$ . Property (P.3) then implies  $\tilde{q}_{B2}(q_{B1}, 0) \leq \tilde{q}_{B2}(0, 0) = Q^{**}$ . As  $M_A - R_1$ 's deviation profit decreases with  $q_{B2}$ , it is bounded from below by  $\Pi(Q^{**}, Q^{**})/2$ , a contradiction.

## References

- [1] Allain, M.-L., and C. Chambolle (2013). “Anti-Competitive effects of Resale-Below-Cost Laws,” *International Journal of Industrial organization*, 29(4):373-385.
- [2] Bonanno, G., and J. Vickers (1988). “Vertical Separation,” *Journal of Industrial Economics*, 36(3):257-265.
- [3] Dobson, P. W., and M. Waterson (2007). “The Competition Effects of Industry-Wide Vertical Price Fixing in Bilateral Oligopoly,” *International Journal of Industrial Organization*, 25(5):935-962.
- [4] de Fontenay, C. C., and J. S. Gans (2005). “Vertical Integration in the Presence of Upstream Competition,” *RAND Journal of Economics*, 36(3):544-572.
- [5] Gal-Or, E. (1991). “Duopolistic Vertical Restraints,” *European Economic Review*, 35(6):1237-1253.
- [6] Hart, O., and J. Tirole (1990). “Vertical Integration and Market Foreclosure,” *Brookings Papers on Economic Activity. Microeconomics*, 1990, 205-276.
- [7] Jullien, B., and P. Rey (2007). “Resale Price Maintenance and Collusion,” *RAND Journal of Economics*, 38(4):983-1001.
- [8] McAfee, R. P., and M. Schwartz (1994). “Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity,” *American Economic Review*, 84(1):210-230.
- [9] Nocke, V., and L. White (2007). “Do Vertical Mergers Facilitate Upstream Collusion?” *American Economic Review*, 97(4):1321-1339.
- [10] Nocke, V., and L. White (2010). “Vertical Merger, Collusion, and Disruptive Buyers,” *International Journal of Industrial Organization*, 28(4):350-354.
- [11] O’Brien, D. P., and G. Shaffer (1992). “Vertical Control with Bilateral Contracts,” *RAND Journal of Economics*, 23(3):299-308.
- [12] Ordover, J., G. Saloner and S. C. Salop (1990). “Equilibrium Market Foreclosure,” *American Economic Review*, 80:127-142.



- [13] Piccolo, S., and J. Miklós-Thal (2012). “Colluding through Suppliers,” *RAND Journal of Economics*, 43(3):492-513.
- [14] Reiffen, D. (1992). “Equilibrium Vertical Foreclosure: Comment,” *American Economic Review*, 82(3):694-697.
- [15] Rey, P., and J. Stiglitz (1988). “Vertical Restraints and Producers’ Competition,” *European Economic Review*, 32(2-3):561-568.
- [16] Rey, P., and J. Stiglitz (1995). “The Role of Exclusive Territories in Producers’ Competition,” *RAND Journal of Economics*, 26(3):431-451.
- [17] Rey, P., and J. Tirole (2007). “A Primer on Foreclosure,” *Handbook of Industrial Organization Vol. III*, M. Armstrong and R. Porter eds, North Holland.
- [18] Rey, P., and T. Vergé (2004). “Bilateral Control with Vertical Contracts,” *RAND Journal of Economics*, 35(4):728-746.
- [19] Rey, P., and T. Vergé (2010). “Resale Price Maintenance and Interlocking Relationships,” *Journal of Industrial Economics*, 58(4):928-961.
- [20] Salinger, M. A. (1988). “Vertical Mergers and Market Foreclosure,” *The Quarterly Journal of Economics*, 77:345-356.
- [21] Whinston, Michael D. (2006). *Lectures on Antitrust Economics*, MIT Press.