

Essays in Financial Economics

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Chapter 1

Introduction

This thesis consists of three distinct research articles which belong to the field of financial economics. A central theme of these contributions is the interaction between publicly available information, or the lack thereof, and incentives to acquire private information, primarily in the context of securitized assets traded in financial markets. Private information is a central issue in many economic interactions as it may inhibit realization of gains from trade due to a lemons problem.²

Opacity and Liquidity Chapter 2 proposes a novel way of modelling the opacity of an asset, characterized by the degree of public knowledge about its detailed payoff characteristics, and investors' incentives to become privately informed. In this context, private information is detrimental to welfare as its sole purpose is to exploit less well informed trading partners. The article sheds light on recent pushes for increased transparency in the financial system: Policymakers typically view transparency as a one-way street to a better financial system and implicitly presume that opacity harms liquidity – it increases the scope for agents to have different information sets. With that line of reasoning, adverse selection should be more pronounced and liquidity low.

However, private information acquisition is in fact endogenous: Not only the scope for information acquisition, but also the incentives to become informed in the first place are affected by an asset's opacity. It is thus not straightforward why information should be more valuable for opaque assets, resulting in higher degrees of private information and less liquidity. In fact, many opaque assets are frequently traded and have low bid-ask spreads, signifying little worry by market makers about asymmetric information. One example is the banking industry. Banking is considered a very opaque business. Nonetheless, the major banks' stocks are heavily traded and display high liquidity.

²See [Akerlof \(1970\)](#).

The paper provides a formalization of opacity which is independent of the expected payoff and risk of an asset and provides a microfoundation for the idea that opacity may in fact deter information acquisition by limiting incentives to become privately informed. As private information acquisition is not valuable for opaque assets despite the large scope for information acquisition, these assets should be highly liquid. The same holds for transparent assets, where the scope for private information is either small or nonexistent. By contrast, assets of intermediate opacity yield both incentives and scope for private information acquisition: In equilibrium, these assets display a lack of liquidity; gains from trade can not be realized due to fear of being exploited by better informed trading partners.

It can thus be privately and socially optimal to issue opaque assets to deter information acquisition. This may explain why opacity in the financial system has remained high, despite the enormous improvements in information dissemination and analysis technologies in recent decades. Moreover, artificial opacity creation, e.g. by resorting to complex securitization structures, is not necessarily harmful to investors as often argued by policymakers. Instead, it may preserve *common ignorance* and thus liquidity.

The article contributes to the policy debate by providing a simple microfoundation for why opacity per se is not harmful to market participants. Adverse effects of legislation increasing transparency of liquidity may arise and should be taken into account by policymakers. As in many regulatory settings, subsidies are better suited to increase social welfare (by aligning private and social incentives) than mandatory disclosure levels.

Securitization Practices before and after the recent crisis Chapters 3 and 4 contribute to the theoretical analysis of securitization practices in both the run-up and aftermath of the recent financial crisis.

It has been argued that one main driving force of the recent financial crisis were lenders' incentives to *originate-to-sell*: By selling securitized loans to investors, lenders were not liable when borrowers defaulted and thus did not screen potential borrowers carefully enough (or even deliberately included bad risks in the securitized pools) – this may lead to a propagation of bad risk in the economy which, through securitization, is spread to a multitude of agents and has been perceived as one of the main causes of the collapse in 2007. The argumentation has received empirical support (e.g. by [Keys, Mukherjee, Seru, and Vig \(2010\)](#), [Keys, Seru, and Vig \(2012\)](#)) and, in light of this as

well as anecdotal evidence, motivated policymakers to act, e.g. by requiring minimum retentions.

However, the relevance of this type of moral hazard problem as a central reason of the recent crisis is not undisputed. [Gorton \(2010\)](#) argues that adverse selection concerns, instead of moral hazard, were the driving force behind the crisis. Moreover, [Bubb and Kaufman \(2014\)](#) provide empirical evidence in direct opposition to [Keys, Mukherjee, Seru, and Vig \(2010\)](#) and [Keys, Seru, and Vig \(2012\)](#). From a theoretical perspective, it is also unclear how such a lack of screening can be sustained in an equilibrium framework with rational agents: Under rationality, the lack of incentives to screen properly should be anticipated by investors.

Chapter 3 therefore analyzes a theoretical framework in which lenders may condition their screening and lending decision on a publicly observable signal of borrowers' creditworthiness such as the FICO score. As screening produces private information for the lender, this complicates realizations of potential gains from trade in the loan sale market. The model considers retention by lenders as a signalling device. A key finding is that, comparing a scenario with and without loan sale opportunities, screening intensity decreases for borrowers with an intermediate FICO score: While both equilibria with and without screening can be sustained, the lack of private information in the no-screening case allows for comparatively greater realization of gains from trade at the loan sale stage, which increases lender profitability.

The article thus provides theoretical evidence that securitization may in fact decrease screening intensity at least for some borrowers in a framework where agents are fully rational. Moreover, this may be detrimental to borrower welfare: Borrowers who obtain a loan only in the securitization regime, i.e. borrowers who are not screened but where screening would have revealed negative information, may ex post prefer never to have applied for (and received) a loan – however, as they are never screened, they can never infer their type from the lender's behavior in equilibrium. The framework is also consistent with empirically observed discontinuities in lending and default rates around a FICO score of 620. A key prediction is that ex-post retention by lenders should be differential across the range of FICO scores, which is seemingly opposed to empirical findings by [Bubb and Kaufman \(2014\)](#). However, the article argues that a true empirical assessment needs to focus not on securitization but on actual risk transfer as a measure of retention by lenders. This relates to the literature on securitization without risk transfer (see e.g. [Acharya, Schnabl, and Suarez \(2013\)](#)): Securitization often occurs via conduits where the lenders as sponsors retained the risk of the securitized loans

by guaranteeing that the conduit will not default. This implicit risk retention is in line with retention serving as a signalling device when private information by lenders is a concern. This purpose may also explain the prevalence of securitization via ABCP conduits despite its lack of profitability.

Finally, Chapter 4 focuses on a different interaction between public and private information. The article characterizes securities which are most robust to interim public information in a security design and trading framework which builds on [Dang, Gorton, and Holmström \(2011\)](#). While private information acquisition itself is not contained in the analysis, the securities most robust to public information are found to be contracts composed of debt-like tranches. These complex securities best preserve value irrespective of whether interim public information is positive or negative by putting the security's payoff primarily in those parts of the underlying payoff distribution which is as equally likely as possible to realize under different public signals. In the context of the model, robustness of this form is desirable as gains and losses from public information are realized differentially due to cash-in-the-market pricing: While losses are fully incurred, gains in value due to positive interim information can not necessarily be capitalized upon.

However, this introduces a misalignment in the securitizers' incentives when addressing public and private information concerns: Complex securities composed of debt-like tranches are most robust to public information, while standard debt contracts minimize private information acquisition incentives. This is in contrast to [Dang, Gorton, and Holmström \(2013\)](#), which restricts the type of public information considered. The prevalence of *debt-on-debt* in securitization practices before the crisis can thus be interpreted as having been potentially optimal from an ex-ante perspective (if private information concerns dominate); ex-post, however, these security structures were (too) vulnerable to interim public information, which contributed to the crisis.

To summarize, this thesis contributes to the theoretical analysis of securitization practices before and after the recent financial crisis by focussing on the interaction of public information and private information acquisition incentives in various settings. The findings are relevant for policy debates such as the push for increased transparency in the financial system and the mandatory retention by lenders, which are already implemented by legislators in several countries.

Authorship Of the three articles featured in this thesis, two are single-authored (Chapters 3 and 4). Chapter 2 builds on joint work with Wolf B. Wagner (Tilburg University).

Relationship to Previous Work This thesis consists of five chapters, and includes three distinct research papers. Chapters 1, 2, 3 and 5 were exclusively created after my Master's thesis. Chapter 4 extends my own work of [Stenzel \(2011\)](#), submitted as a Master Thesis at the University of Mannheim. The main results, however, are all novel. The previous work focused on generalizing [Dang, Gorton, and Holmström \(2011\)](#) with respect to the optimality of debt in precluding private information acquisition. The current focus is on the optimal security with respect to a generalized formulation of interim public information. [Stenzel \(2011\)](#) did contain Example 2, which served as a starting point for the present contribution.

Chapter 2

Opacity and Liquidity

Abstract

We present a model that links the opacity of an asset to its liquidity. While low opacity assets are liquid, intermediate levels of opacity provide incentives for investors to acquire private information, causing adverse selection and illiquidity. High opacity, however, benefits liquidity by reducing the value of a unit of private information to investors. The cross-section of bid-ask spreads of U.S. firms is shown to be consistent with this hump-shape relationship between opacity and illiquidity. The analysis suggests that uniform disclosure requirements may not be desirable; optimal information provision can be achieved by subsidizing information. The model also delivers predictions about when it is optimal for asset originators to sell intransparent products or pools composed of correlated assets.

2.1 Introduction

Opacity and illiquidity are two central concepts in economics. They are, however, rarely distinguished from each other. Both arise from incompleteness of information. An asset can be said to be opaque when agents generally have limited knowledge about its pay-offs. By contrast, when some agents know more than others about an asset, the asset tends to be illiquid because of adverse selection problems. Thus, opacity reflects incompleteness of public information, while illiquidity arises from asymmetric realizations of private information.

This chapter is based on joint work with Wolf B. Wagner. We thank participants of the Annual Meeting of the European Finance Association 2014 and the Econometric Society European Meeting 2014, as well as the ENTER Jamboree 2015, and seminar audiences at Cambridge, ULB-ECARES, Leicester, Rotterdam School of Management, Mannheim and Tilburg. We are grateful to Jacques Cr mer, Paolo Conteduca, Raphael Levy, Volker Nocke and Yuki Sato for valuable comments.

How can the two be related? At first, one would expect a positive link between opacity and illiquidity. When there is more opacity, there is more scope for agents having different information sets. Adverse selection should be more pronounced and liquidity be low. This reasoning is consistent with common thinking among policy makers that transparency is beneficial for the financial system: more public information should deter wasteful private acquisition of information and also reduce the potential for asymmetries among investors.

This argumentation, however, ignores the fact that private information is endogenous. Gathering it is costly; hence it has to be profitable for investors to acquire it. The relationship between opacity and liquidity will thus depend on the scope for private information as well as on the incentives to acquire such information. It is not obvious why the value of information should be higher for opaque assets. Casual observation also throws doubt on an exclusively positive link between opacity and illiquidity. Many opaque assets are frequently traded and have low bid-ask spreads. A case in point is the banking industry. Banking is considered a very opaque business. Nonetheless, the major banks are heavily traded and their stocks display high liquidity.

This paper presents a model that analyzes the link between opacity and liquidity. We consider an investor who holds an asset of given opacity, where opacity is defined as the fraction of states of the world in which it is publicly not known what the pay-offs are. The investor can decide how much he wants to learn about these states.³ Doing so incurs a fixed cost per state. Following this, the state of the world becomes known. The investor may be hit by a liquidity shock that forces him to sell the asset to the public. Illiquidity arises at this stage since market participants anticipate that the investor will sometimes trade opportunistically on his private information.⁴

For a completely transparent asset, there is no scope for private information. Such an asset trades without an adverse selection discount and hence is liquid. At the other extreme, for a very opaque asset the scope for private information is maximal. At the same time, however, the incentives to acquire information are low. The reason is that acquiring knowledge about a certain number of states is then less valuable as these states constitute a smaller share of the overall number of opaque states. For a

³Learning about a state can be thought of as understanding how the asset's pay-off depends on a certain factor, e.g., an oil price change or a recession. For more opaque assets, the set of states that would need to be analyzed is naturally larger.

⁴In this sense, information acquisition is ultimately self-defeating as it is anticipated and priced by investors in equilibrium. This is reminiscent of [Glode, Green, and Lowery \(2012\)](#), where investments in financial expertise are offset by similar investments by counterparties.

sufficiently high level of opacity, it can be shown that it is never optimal to acquire any information. Complete symmetry of information is preserved and the asset is liquid. At intermediate values of opacity, however, the investor always acquires information and there is adverse selection.⁵ A key prediction of the model is thus a hump-shape relationship between opacity and illiquidity; a prediction we find supported in the cross-section of U.S. firms.

The main analysis considers an asset of given opacity. However, since opacity affects information acquisition and liquidity, an investor's valuation of an asset will depend on its opacity. This in turn affects the incentives of originators of assets. We turn to the question of how much information an original owner of an asset wants to publicly release, prior to selling to the investor. The issuer's decision is guided by two motives. First, he wants to sell an asset that maximizes value to the investor, as this will benefit him through a higher sale price. Second, he wants to minimize costs associated with releasing information to the public (arising, for example, because third parties have to be hired to certify information).

Two conclusions can be drawn from the analysis of endogenous opacity. First, it can be (privately and socially) optimal to issue opaque assets such as to deter information acquisition. This may explain why opacity in the financial system has remained high, despite the enormous improvements in information dissemination technologies in recent decades (which should have, by themselves, led to much better public information and lower opacity). It can even be desirable to increase an asset's opacity beyond its natural level (for example, by drawing up complex securitization structures). Second, issuers may privately choose opacity levels that are higher than the ones that are desirable for the financial system. This occurs because issuers have to fully bear the cost of reducing opacity, but only partially internalize any benefits for other agents.

Our framework can be applied to understand other decisions of issuers. Consider for instance an originator who wants to sell a pool of assets. From a diversification perspective, such a pool should contain assets of different risk profiles. This is however in sharp contrast to the observed practice of pooling mostly similar assets.⁶ Our analysis

⁵The seminal paper by [Grossman and Stiglitz \(1980\)](#) considers the incentives of agents to learn about the expected pay-off of an asset. A lower quality of the signal reduces the incentives to become informed, leading to equilibrium prices reflecting fundamentals less well. While in [Grossman and Stiglitz \(1980\)](#) information incompleteness arises with respect to the expected pay-off of the asset (the “fundamentals”), in our analysis of opacity the latter is known. Instead, learning takes place about the mapping between (future) states of the world and pay-offs.

⁶[Gorton and Metrick \(2012\)](#) refer to this as one of the major puzzles of securitization.

suggests that issuing correlated assets has a benefit because it lowers information costs: correlated pools avoid duplication of information because learning about one asset is then informative about the rest of the pool. However, the incentives to acquire information are higher in correlated pools, so there is a trade-off. For certain parameter values, the model predicts that informational costs are minimized by selling a correlated pool. The model also delivers testable predictions for other characteristics of asset sales, such as the decision whether to sell assets in a pool or separately.

There are several implications for policy. Uniformly mandated increases in transparency are not desirable because of the non-monotonic nature of the relationship between opacity and liquidity (which coincides with welfare in our setting). In principle, a two-class policy where regulators distinguish between assets according to their opacity can achieve efficiency. For assets that are fairly transparent, the standard policy prescription applies that more transparency increases efficiency. However, assets that are relatively intransparent to start with should not be forced to higher levels of transparency. Such a conditional transparency regime seems, however, informationally demanding.⁷ A better approach is to provide subsidies for issuers to voluntarily increase transparency. Subsidies are efficiency-enhancing regardless of a firm's opacity level since they directly address the source of inefficient information choices of issuers (the positive externality of information for other agents in the financial system). They may, for example, take the form of governments sponsoring infrastructure for services that promote transparency, such as public information repositories.

2.1.1 Related Literature

Our setting is closely related to recent literature which has analyzed how security design affects information acquisition by investors. While the focus in the present paper is on the question of how much information should be released about an asset, the security design literature studies how an asset's pay-off streams can be separated into different parts to make information acquisition less attractive. A central theme in this literature is the optimality of debt contracts: because debt has a flat payoff for most of the domain (and otherwise its payoff is determined by limited liability), it minimizes the benefits of acquiring private information.⁸ [Dang, Gorton, and Holmström \(2013\)](#)

⁷Although differentiated disclosure policies exist in practise (for example, different standards for listed firms).

⁸The literature mostly considers situations where only one party in a potential trade can become informed, in which case information acquisition is welfare-reducing. [Farhi and Tirole \(2014\)](#) study

formally introduce the concept of the *information sensitivity of a security* and show in a model of strategic security design and multiple trading rounds that debt contracts minimize market participants' incentives to acquire information. Using a generalized information structure, [Yang \(2012\)](#) finds standard debt to be least sensitive to private information, irrespective of the composition of the underlying asset pool. [Farhi and Tirole \(2014\)](#) highlight the importance of commonality of information. They show that for an asset to be liquid it is important that information is symmetric. This can be achieved either by common knowledge or by common ignorance. In our paper, informational symmetry arises either for very transparent assets (common knowledge) or for very opaque assets (because of common ignorance). Intermediate levels of opacity, in contrast, lead to one-sided information and cause adverse selection.

There is a small but growing literature that analyzes asset opacity. [Kaplan \(2006\)](#) examines a bank's choice of whether to release information about assets at an interim stage. The paper shows that it can be efficient for the bank to commit to keep information secret, even though this forces the bank to offer non-contingent deposit contracts ex-ante. The reason is that the cost of revealing negative information at an interim stage can outweigh the benefits of positive information. [Sato \(2014\)](#) considers a setup with opacity at the fund and the asset level. He finds that opaque funds invest in opaque assets and that such funds can trade at a premium. The reason is that managers of opaque funds inflate investors' beliefs about future returns by (secretly) overinvesting in opaque assets and leveraging up.

[Pagano and Volpin \(2012\)](#) analyze a model where investors differ in their ability to process information. Releasing information about assets is subject to a trade-off. On the one hand, information decreases primary market liquidity because it induces a "winner's curse" problem for unsophisticated investors who cannot parse information. On the other hand, information increases secondary market liquidity as information not released by issuers creates scope for private information acquisition and hence leads to adverse selection. The second channel is also present in our model. While in [Pagano and Volpin \(2012\)](#) information is of an all-or-nothing nature, in our model information is continuous. This allows us to show that the value of a unit of information can vary with the asset's level of opacity, which is the source of the opacity benefit in our paper. A second, related, difference to the literature is that opacity in our framework continuously alters the degree of publicly available information while the literature

information acquisition on both sides. In this case, information acquisition can improve liquidity as it can increase symmetry across agents.

typically distinguishes between opacity and transparency in a binary form, where only for transparent assets information can be acquired at all.⁹

Carlin, Kogan, and Lowery (2013) focus on an issue similar to the differential information processing in Pagano and Volpin (2012). They consider an experimental setting in which the complexity of an asset is varied. Complexity relates to the computational difficulty required to obtain information about the asset's payoff. Carlin, Kogan, and Lowery (2013) find that when subjects are aware that other subjects are more adept at performing the required calculations, adverse selection becomes pronounced. This is consistent with agents anticipating a lower degree of common information present in markets.

While in our setting there is no social benefit to information, recent papers by Monnet and Quintin (2015) and Dang, Gorton, Holmström, and Ordoñez (2013) have shown that transparency (i.e. more information) can lead to more efficient interim decisions.¹⁰ However, there is also a cost, as investors may be forced to liquidate their positions in response to negative information. In the presence of secondary markets that are not always liquid, the benefits of good interim information cannot be fully capitalized by investors. Transparency is shown to mitigate this problem, at the cost of allocative efficiency.

The remainder of the paper is organized as follows. Section 2.2 sets up the baseline model for the analysis of the link between opacity and illiquidity. Section 2.3 examines the cross-section of U.S. firms to see whether it exhibits a hump-shape relationship. In Section 2.4 we consider the incentives of asset originators. Section 2.5 discusses some policy implications. Section 2.6 concludes.

2.2 The Model

We develop a very simple model of information acquisition that nevertheless micro-founds how the cost and value of information depends on an asset's opacity. In the model, an investor can learn about an asset of varying degrees of opacity. This learning

⁹This binary structure of asset opacity is also present in Di Maggio and Pagano (2014), who analyze it in conjunction with different degrees of market transparency. Market transparency, as opposed to asset transparency, is also the focus of e.g. Fuchs, Öry, and Skrzypacz (2015).

¹⁰Boot and Thakor (2001) provide an analysis of disclosure of various types of information that are all beneficial (as it reveals agent's types). They show that in equilibrium firms find it beneficial to disclose all types of information.

is not about the asset's expected pay-off (which is the focus of [Grossman and Stiglitz \(1980\)](#) and several other papers) but about how it pays in different states of the world. This can be likened to an investor (or the risk manager of a financial institution) exerting effort in analyzing how an asset performs under several scenarios (e.g., an oil price shock, deflation or an economic downturn). For opaque assets, this will be inherently more difficult than for transparent ones.

Take for instance the stock of Coca-Cola (or a community bank) versus the stock of JP Morgan. The business models of Coca-Cola and the community bank are simple and transparent; it is hence easy to predict how their stock will perform in a set of circumstances. By contrast, the operations of JP Morgan are extremely complex, involving a wide set of activities (such as trading in derivatives, or holdings of securitization products) which are often difficult to understand even on an individual basis. Learning about how JP Morgan's business will perform under different circumstances is hence difficult and requires substantial effort by investors. Another example is credit products. A mezzanine tranche formed from a portfolio of credits, for instance, is much more opaque than an exposure to a single name. As a result, learning about its pay-offs in different states (for instance, its dependence on a clustering of default events in the economy) is more demanding. Conglomerates can also be seen as opaque firms, as opposed to standalone firms, as it will be more challenging to understand how they are impacted by shocks.¹¹

The economy consists of an investor I and an agent M , representing the market. There are two dates, $t = 1, 2$. The preferences of both agents are linear and given as follows:

- The investor's utility depends on whether she is patient or impatient. If patient (occurring with probability $\pi \in (0, 1)$), the investor can consume at both dates: $U^I = C_1^I + C_2^I$. If impatient, the investor derives only utility from consumption at date 1: $U^I = C_1^I$. The investor privately learns her type (patient or not) at $t = 1$ and this information is not verifiable.
- The market agent consumes at both dates: $U^M = C_1^M + C_2^M$.

The endowments of the agents are as follows. At $t = 1$, the investor holds an asset which pays off at date 2. This asset returns one in a subset L (of mass $l \in (0, 1)$) of uniformly distributed states of the world $s \in S = [0, 1]$ and zero otherwise. Given the uniform distribution, the unconditional value of the asset is hence l . While the set L

¹¹Consistent with this, [Cohen and Lou \(2012\)](#) provide evidence that it takes more time for a piece of information to be incorporated in the price of a conglomerate.

is unknown, its mass l is publicly known. The market agent has a cash endowment of $w^M > 1$ at date 1.¹² The agents hold no other endowments.

Given the allocation of endowments, it is natural that gains from trade can be realized. If the investor turns out to be impatient at date 1, she can sell the asset to M . However, reaping these gains is complicated by the opportunity for the investor to acquire private information about the asset prior to trading: Acquisition of private information results in adverse selection when trading with the market. The incentives to acquire information, in turn, are affected by the asset's opacity.

Opacity is modeled as follows. There is a set of states O containing the payoff states ($L \subseteq O$). This set O is publicly known. We refer to the mass of this set, $o \in [l, 1]$, as the asset's opacity. Maximum opacity ($o = 1$) arises when there is no information about the set of payoff states. At the other extreme, if $o = l$, the precise set of payoff states is common knowledge and there is no scope for private information acquisition – the asset is transparent. For $o \in (l, 1)$, opacity is of an intermediate degree and there is incomplete knowledge about payoff states. The more transparent the asset, the smaller o and the more precise is the public information about the location of the payoff states, that is, the circumstances under which the asset pays off. Note that opacity is distinct from the asset's ex-ante return and risk: the expected payoff is l and variance of the asset is $l(1 - l)$.

At the beginning of date 1, the investor has the option to acquire private information. Specifically, she decides on an amount $a \in [0, o - l]$ of information to acquire. Following this, nature reveals a random subset of states A of mass a from O for which the asset does not pay off. Private information acquisition reduces the size of the set containing the payoff states from o to $o - a$. There are proportional costs of acquiring information $k_I \cdot a$, where $k_I > 0$. We assume that these costs take intermediate values:

Assumption 2.1

$$\frac{\pi l(1 - \pi)}{1 - \pi l} < k_I < \pi.$$

This assumption ensures that information acquisition is nontrivial.

The choice of a as well as the realization of the subset A are private to the investor and are not verifiable. Following the investor's information acquisition decision, the state

¹²We abstract from issues that can arise because endowments are constrained (as in [Dang, Gorton, Holmström, and Ordoñez \(2013\)](#)), which potentially lead to cash-in-the-market pricing.

of the world s becomes available. Subsequently, the investor can sell the asset to the market. For this we assume that the market posts a competitive price for the asset and the investor decides whether or not to sell at this price.¹³

To focus the analysis, it is convenient to rearrange the states s of the world. Specifically, we reorder states such that the payoff states are on $[0, l]$, the public set of potential payoff states is on $[0, o]$, and the set of potential payoff states privately known to the investor is $[0, o - a]$. In addition, agents no longer observe the exact state, but only the set in which the state falls. If $s > o$, both investor and the market learn that the state of the world falls outside the public set O , and hence that the asset does not pay off. If $s \in (o - a, o]$, the state of the world is in the public set of possible states of the world O , but not in the investor's private set. The investor privately learns that the asset does not pay, while the market only learns that s is within the public set of potential payoff states of mass o . If $s \leq o - a$, both investor and market have incomplete knowledge about the payoff. The investor knows that s is within the private set of payoff states, while the market only observes that the state is within the public set.

The timing of the model can be summarized as follows:

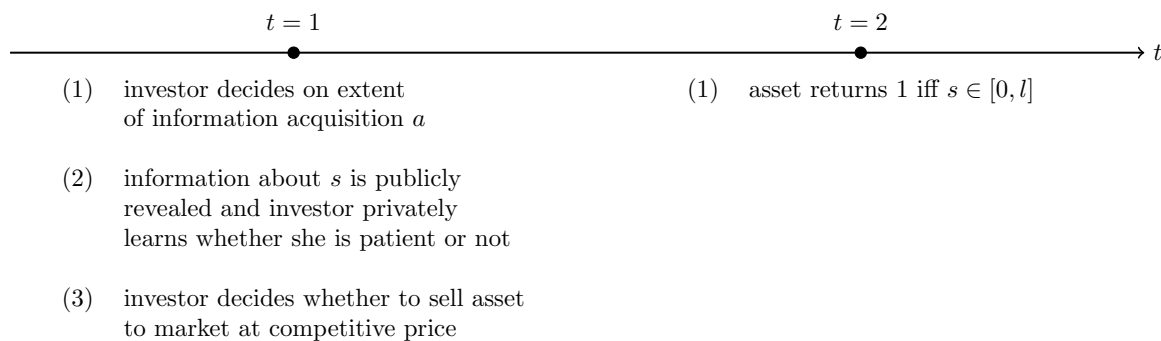


Figure 2.1: Timeline of the Baseline Model

2.2.1 Trade with the Market

To solve for an equilibrium of the game, we first analyze the final stage in which the investor has the opportunity to sell to the market. At this stage, public information about the state s has been revealed. The public set of payoff states depends on the asset's opacity level and is given by $[0, o]$. Furthermore, the investor has potentially

¹³This avoids the use of price as a signal about the asset's quality or the investor's type. A competitive price may, for example, arise if market participants compete by posting bid prices for the asset.

acquired information a ; her private set of payoff states is thus $[0, o - a]$. Denote by \tilde{a} the market's beliefs about how much information the investor has acquired, and by $p(\tilde{a}, o)$ the competitive price given these beliefs and the opacity level o .

We first analyze the investor's selling decision for a given price p . To rule out no-trade equilibria, we assume that the investor has a weak preference for selling when she is impatient, and a weak preference for not selling when she is patient. We focus on pure strategy equilibria.

The following cases arise depending on the realization of s . First, there is the trivial case of s being outside the public set ($s > o$). Both the investor and the market know that the asset does not pay off and trade is irrelevant. We can ignore this case for the analysis of the trade equilibrium as trade, if it takes place, occurs at a price of zero.

Consider next the case of s being inside the public set ($s \leq o$). If an investor is impatient, she will sell regardless of price (given her weak preference for selling) since there is zero utility from holding on to the asset. For a patient investor, the decision to sell depends on whether the signal is in the private set. If the signal is outside the private set ($s > o - a$), the investor knows that the asset is worthless. She will hence sell at any positive price. If the signal is inside the private set ($s \leq o - a$), the investor's expected utility of keeping the asset is $\frac{l}{o-a}$. Taking into account the weak preference for holding on to the asset, she will hence sell the asset if and only if the price p is larger than $\frac{l}{o-a}$. Such a price, however, is inconsistent with market rationality. To see this, note that $\frac{l}{o-a}$ is higher than the value of the asset even without adverse selection (that is, when the investor only sells when she is impatient). The market can hence never break even at this price and such a price cannot prevail in equilibrium. It follows that when $s \leq o - a$ the patient investor does not sell the asset.

We can summarize these results as follows:

Lemma 2.1 *In equilibrium, the asset is offered to the market (when $s \leq o$) iff*

- i) the investor is impatient, or*
- ii) the investor is patient and s is in her public set but outside her private set ($s \in (o - a, o]$).*

We next solve for the price at which an asset is sold (in the case of $s \leq o$). Since the price is set competitively, the market breaks even in expectation. The price hence has

to be equal to the asset's expected value (given beliefs \tilde{a}) *conditional on being sold*. According to Lemma 2.1, the asset is sold either when the investor is impatient, or when she is patient and the state is outside her private set. The first case occurs with probability $1 - \pi$ and the likelihood of the asset paying off in this case is $\frac{l}{o}$, i.e., the ratio of the size of the payoff set l to the size of the public set o . The second case, a patient investor with s outside her private set, is perceived by the market to occur with probability $\pi \cdot \frac{\tilde{a}}{o}$ ($\frac{\tilde{a}}{o}$ is the likelihood of the state being outside the private set given beliefs \tilde{a} about the extent of information acquisition). The asset is worthless in this case. The expected value of the asset (conditional on being sold) is hence $\frac{(1-\pi)\frac{l}{o}}{1-\pi+\pi\frac{\tilde{a}}{o}}$. Rearranging yields the competitive price $p(\tilde{a}, o)$ given beliefs \tilde{a} and opacity level o :

$$p(\tilde{a}, o) = \frac{1 - \pi}{o - \pi(o - \tilde{a})}l. \quad (2.1)$$

Note that for $\tilde{a} = 0$ (that is, if the market believes there is no private information) we have $p(0, o) = \frac{l}{o}$. Furthermore, $\frac{\partial p}{\partial \tilde{a}} < 0$ because if the market believes that the investor privately acquired more information, it prices in more adverse selection as it becomes more likely that a worthless asset is offered.

2.2.2 Information Acquisition

Consider a candidate for information acquisition a^* , and corresponding market beliefs \tilde{a} . For a^* to constitute an equilibrium amount of information acquisition, it has to be the case that a^* maximizes the investor's utility given that the market believes $\tilde{a} = a^*$. We thus have for a^* that

$$a^* = \operatorname{argmax}_{a \in [0, o-l]} u(a, a^*),$$

where $u(a, \tilde{a})$ denotes the investor's expected utility given that she chooses a level of information acquisition a and the market holds beliefs \tilde{a} .

We can derive $u(a, \tilde{a})$ as follows. With probability $1 - o$, the state of the world falls outside the public set ($s > o$). In this case, the investor does not derive any utility from owning the asset as it is common knowledge that the asset is worthless. With probability o , the state of the world falls inside the public set ($s \leq o$). The investor then sells whenever she is impatient or when she is patient and the state is outside her private set ($s \in (o - a, o]$). The combined probability for this is $1 - \pi + \pi \cdot \frac{a}{o}$ and she obtains $p(\tilde{a}, o)$ from selling the asset. When she is patient and the state is inside

the private set ($s \in [0, o - a]$) she holds onto the asset. This happens with probability $\pi \cdot \frac{o-a}{o}$ and she receives (in expectation) $\frac{l}{o-a}$ from the date-2 return. Together with the information costs $k_I \cdot a$, her utility is thus

$$u(a, \tilde{a}) = o \left((1 - \pi + \pi \frac{a}{o}) p(\tilde{a}, o) + \pi \frac{o-a}{o} \frac{l}{o-a} \right) - k_I \cdot a. \quad (2.2)$$

Note that when beliefs are consistent with actual information acquisition ($\tilde{a} = a$), the above simplifies to $l - k_I \cdot a$.

Differentiating with respect to a , we obtain the marginal gain from acquiring information:

$$\frac{\partial u(a, \tilde{a})}{\partial a} = \pi p(\tilde{a}, o) - k_I. \quad (2.3)$$

Equation (2.3) shows that information acquisition trades off marginal benefits $\pi p(\tilde{a}, o)$ with information acquisition costs k_I . The benefits are derived as follows: By acquiring one additional unit of information, the investor reduces her private set by one state. If this state realizes, she knows that the asset is worthless. If she turns out to be patient, she will hence sell and obtain $p(\tilde{a}, o)$, while before she would have held a worthless asset. Note that the incentives to acquire information increase in the asset's price.

The marginal benefits in (2.3) are constant as they do not depend on the amount of information acquired (a). There are hence three cases to consider. If $\pi p(\tilde{a}, o) - k_I < 0$ (or rearranging, if $p(\tilde{a}, o) < \frac{k_I}{\pi}$), the marginal benefits are always outweighed by the marginal costs. Zero information ($a^* = 0$) thus maximizes investor utility. Likewise, if $\pi p(\tilde{a}, o) - k_I > 0$ ($p(\tilde{a}, o) > \frac{k_I}{\pi}$), the marginal benefits outweigh the marginal costs and the highest possible level of information acquisition ($a^* = o - l$) maximizes utility. Finally, if $p(\tilde{a}, o) = \frac{k_I}{\pi}$, the investor is indifferent as to which level of information acquisition to choose. We can hence summarize for the investor's choice of information given beliefs \tilde{a} :

$$\operatorname{argmax}_{a \in [0, o-l]} u(a, \tilde{a}) = \begin{cases} 0 & \text{if } p(\tilde{a}, o) < \frac{k_I}{\pi} \\ [0, o-l] & \text{if } p(\tilde{a}, o) = \frac{k_I}{\pi} \\ o-l & \text{if } p(\tilde{a}, o) > \frac{k_I}{\pi}. \end{cases} \quad (2.4)$$

This allows us to solve for equilibrium information acquisition. Note that higher opacity reduces the price p for a given belief \tilde{a} (equation (2.1)) and hence the incentives to acquire information. Define \underline{o} as the critical opacity level which just leads to full information acquisition ($a^* = o - l$). Recall that in equilibrium, we have that $a^* = \tilde{a}$.

Inserting $\tilde{a} = \underline{o} - l$ into $p(\tilde{a}, \underline{o}) = \frac{k_I}{\pi}$, we obtain after rearranging: $\underline{o} = \pi l + \frac{\pi(1-\pi)l}{k_I}$. Likewise, define \bar{o} as the critical opacity which deters acquisition of any information. We obtain $\bar{o} = \frac{\pi l}{k_I}$ by rearranging $p(0, \bar{o}) = \frac{k_I}{\pi}$. For intermediate values of o , an interior equilibrium arises. By solving for \tilde{a} in the condition $p(\tilde{a}, o) = \frac{k_I}{\pi}$, we obtain for the interior equilibrium that $a^* = \tilde{a} = \frac{(1-\pi)}{\pi}(\frac{\pi l}{k_I} - o)$.

Note that Assumption 2.1 ensures $\underline{o} < \min\{\bar{o}, 1\}$, which allows to summarize

Proposition 2.1 *The equilibrium level of information acquisition a^* is*

$$a^*(o) = \begin{cases} o - l & \text{if } o \leq \underline{o} \\ \frac{(1-\pi)}{\pi}(\frac{\pi l}{k_I} - o) & \text{if } o \in (\underline{o}, \bar{o}) \\ 0 & \text{if } o \geq \bar{o} \end{cases} \quad (2.5)$$

with $\underline{o} = \pi l + \frac{(1-\pi)\pi l}{k_I}$ and $\bar{o} = \frac{\pi l}{k_I}$.

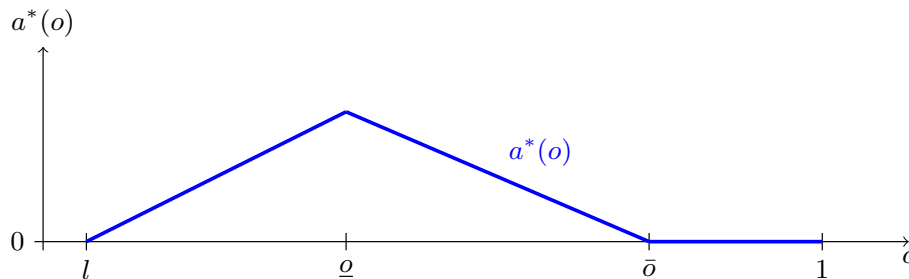


Figure 2.2: Information Acquisition as a Function of o

Figure 2.2 shows equilibrium information acquisition $a^*(o)$ as a function of an asset's opacity o . At $o = l$, the asset is fully transparent and it is not possible to acquire information ($a^* = 0$). For values of o between l and \underline{o} , the maximum feasible amount of information is acquired ($a^* = o - l$). In this range, opacity increases information acquisition, as higher opacity increases the feasible amount. Beyond \underline{o} , however, opacity reduces information acquisition. This is until \bar{o} is reached, at which point no information is acquired. Note that while in the figure we have that $\bar{o} < 1$, this is not necessarily always the case. If not, information will be acquired even at full opacity.

What is the reason why opacity can deter information acquisition? When opacity is high, the public set is large and hence a realization of s in this set becomes less informative about payoffs. This can be appreciated from the fact that p (for given \tilde{a}) is

declining in opacity (see equation (2.1)). A lower p in turn means that learning about a given number of states in the public set becomes less valuable for the investor as the investor benefits from private information by selling to the market in cases where the asset is worthless.

Note that the non-monotonic impact of opacity on information acquisition translates also into a non-monotonic impact on liquidity as well as welfare. This is, first, because information acquisition always lowers liquidity, and second, because information acquisition is the only source of welfare losses in our setting.

2.2.3 Robustness

In this section, we discuss several modifications of the model.

Random Discovery of the Payoff Interval

We have considered an information acquisition technology which is deterministic: The investor eliminates non-paying states from the set of potential payoff states with certainty. This has significantly simplified the analysis. Alternatively, the outcome of information acquisition may be random. In particular, the investor may decide to analyze a specific state and then find out whether the asset pays off in this state or not. Following this, she may decide to acquire information about more states. A consequence is that the extent to which opacity is eliminated becomes random: the investor may be lucky and discover the payoff interval early on or she may be unsuccessful and decide to stop after having acquired a certain amount of information. Another consequence is that the amount of information acquired (and hence also the deadweight loss from information acquisition) becomes random as well.

Appendix 2.A analyzes an information technology with random discovery of states. The results from the baseline model carry over in that information acquisition is first increasing and then decreasing in opacity. There can also be interior equilibrium amounts of information acquisition, where there is a threshold for information acquisition such that an investor acquires information until this threshold is reached or until the pay-off interval is discovered.

Learning about Loss States

What happens when information acquisition allows us to learn about the set of states where the asset does not pay off? Suppose that – the exact opposite of the baseline model – the asset pays off on $[l, 1]$ but not on $[0, l]$. In addition, suppose that reducing opacity and information acquisition also work in the opposite way: the owner’s opacity choice narrows down the set of loss states to $[0, o]$, while information acquisition further narrows it to $[0, o - a]$.

A difference to the baseline model is that the investor now benefits from states in which he has *positive* private information about the asset. The intuition for this observation is as follows (Appendix 2.B contains the full analysis). Suppose that selling the asset yields a given price p . Suppose a state s realizes in which the investor knows that the asset pays off but the market does not ($s \in [o - a, o]$). A patient investor will then not sell the asset and thus realize a return of 1, whereas she would have realized p without information acquisition. Suppose next that a state of the world realizes where both investor and market are uncertain about whether there is a pay-out ($s \in [l, o - a]$). Since the investor also observes that this state is not within her private set of payoff-states $[o - a, o]$, she perceives a higher chance that the asset will not pay than the market. This will cause her to sell the asset when patient. However, under symmetric information, the investor would have been indifferent between selling and not selling so that no additional gains are incurred.

Information acquisition makes it more likely that a state realizes where the investor has positive information about the asset. In such a state the investor will refrain from selling the asset, while prior to information acquisition she would have sold the asset. A consequence is that the gains from information acquisition are decreasing in the market price p , the opposite to the case in the baseline model (equation (2.3)). This eliminates the possibility for interior choices of information acquisition. However, as shown in the appendix, it is still the case that opacity lowers information acquisition at low opacity levels and that sufficiently high opacity prevents information acquisition.

State-Dependent Information Acquisition Costs

The baseline model assumes that the cost of acquiring information is proportional to the number of states which are analyzed. Implicit to this is that states have equal

information costs. Alternatively, information costs may differ across states. For example, it might be easier to ascertain the value of an asset in the case of an inflationary shock, than for instance in the event of a financial crisis. If costs are state-dependent, it becomes optimal for an investor to first analyze cheaper states, resulting in increasing marginal information acquisition costs. Intuitively, increasing costs make it more likely that we obtain an interior equilibrium. Appendix 2.C contains the analysis of increasing marginal costs, showing that the qualitative results are the same as in the baseline model. In particular, the relationship between opacity and information acquisition still follows a hump-shape.¹⁴

It is critical, however, that the *total* cost of gathering information is higher when more states are analyzed. To see this, suppose to the contrary that any level of information acquisition incurs a fixed cost, independently of how many states are analyzed. The marginal benefit from information once some information has been acquired ($a > 0$) is then strictly positive ($\frac{\partial u(a, \tilde{a})}{\partial a} = \pi p(\tilde{a}, o) > 0$ from equation (2.3) for $k_I = 0$). Hence there can no longer be an interior equilibrium. The investor will hence either acquire no information or all information. This case corresponds to the technology of information acquisition considered in [Dang, Gorton, and Holmström \(2013\)](#) and [Dang, Gorton, Holmström, and Ordoñez \(2013\)](#).

Alternative Mechanisms

In our model, opacity discourages information acquisition by reducing the price at which an asset can be sold in the case of negative private information. An opaque asset is worth less in such a sale for the following reason: when the public domain is wider, it is less likely that a state in this domain belongs to the payoff interval. The main mechanism, however, does not rely on this aspect of the model. There are various other reasons for why opaque assets generally will command lower prices. For instance, market participants may have heterogenous preferences for assets (arising, for example, because they have different endowment profiles). For transparent assets there is more knowledge about when the asset pays out, allowing market participants with the highest valuation for these states to bid for the asset. Alternatively, more transparency may allow for more efficient hedging in the transparent region, decreasing hedging costs.

¹⁴The analysis of state-dependent information acquisition costs is similar to an analysis where states of the world are not uniformly distributed: While state-dependent costs alter the cost-side of acquiring information, different distributions alter the benefits.

In appendix 2.E we develop a model of endogenous valuation of opaque assets based on inefficient liquidations. We consider a setting in which a market agent, after having bought the asset, may decide to scrap the asset before $t = 2$. Efficiency requires to scrap whenever the asset is worthless at $t = 2$. If the asset is opaque, it is more likely that the scrapping decision is inefficient as there are then less states where the market is informed about the future payout. Anticipating this, the market's valuation of an opaque asset will be lower at $t = 1$. This in turn makes information acquisition less attractive for investors. The appendix shows that in this richer setup it still holds that information is only acquired when opacity is not large. For sufficiently large opacity, there is no information acquisition.

2.3 The Cross-Section of Bid-Ask Spreads and Opacity

The model's key prediction is that opacity encourages private information only up to a point. Beyond this point, the relationship inverts, and opacity makes information acquisition less attractive (see Figure 2.2). In this section we analyze firm-level data to see whether such a pattern is consistent with the data.¹⁵

Following the theoretical contributions of [Glosten and Milgrom \(1985\)](#) and [Kyle \(1985\)](#), private information leads to higher bid-ask spreads on a firm's stock. Market makers need to be compensated for the risk of trading with an informed party, leading them to widen the bid-ask spread when they expect private information to be more prevalent. Similarly, in our model, the market bids less when information asymmetries are higher. Empirically, it is well documented that adverse selection is an important determinant of bid-ask spreads (see e.g. [Stoll \(1989\)](#), [Huang and Stoll \(1997\)](#)). We hence proxy the amount of private information on a firm using its stock's bid-ask spread.

The opacity of a firm is measured by the extent of disagreement among analysts about future earnings (following [Flannery, Kwan, and Nimalendran \(2004\)](#) and others). The idea is that opaque firms exhibit large potential for divergence among analysts, while

¹⁵[Agarwal \(2007\)](#) finds a hump-shape relationship between institutional ownership and liquidity. This is interpreted as the presence of two offsetting effects. On the one hand, higher institutional ownership leads to more informational asymmetries and hence lower liquidity. On the other hand, it leads to more competition (among institutions) which should result in pricing better reflecting information and hence higher liquidity. This exercise differs from ours in that we vary asset characteristics (opacity) rather than characteristics of the holders of the asset.

disagreement is naturally limited for transparent firms.¹⁶ The literature has suggested alternatives to the dispersion proxy, which are however less appropriate for our purpose. For example, [Morgan \(2002\)](#) uses rating splits as measure of firm opacity. While conceptually similar to analyst dispersion, rating splits are not ideal in our context because testing our theory requires an opacity measure that varies over a sufficiently large interval in order to be able to identify a non-monotonic relationship (rating splits, in their simplest form, are a binary measure). Another measure of opacity that is used in the literature is the number of analysts following a firm, see e.g. [Roulstone \(2003\)](#). This measure does not measure underlying firm opacity itself, but rather the extent to which analyst activity alleviates this opacity. We will account for this in our analysis by including the number of analysts following a stock as a control variable.

2.3.1 Data

We conduct an analysis of firms listed in the U.S. by relating their bid-ask spreads to the dispersion in analyst forecasts. In our analysis, we control for factors that may affect bid-ask spreads and which are unrelated to adverse selection.

We use the universe of firms contained in the CRSP database. Our measure of the bid-ask spread is the average of a firm's bid-ask spread in CRSP during the last three months of 2013 (October 2013-December 2013). In addition, we obtain various controls from CRSP: the (log) of the market capitalization as a measure of firm size and the stock price itself (both as of October 2013); the standard deviation of stock price returns, trading volume and the standard deviation of trading volume averaged over the two years prior to October 2013.

The dispersion measure is obtained from the monthly summary statistics of the I/B/E/S database. Specifically, we calculate dispersion as the average standard deviation of monthly one-year ahead earnings forecasts. We calculate this average over the two years prior to October 2013 (October 2011 until September 2013) and scale the standard deviation by the mean earnings forecast. We deliberately choose a long horizon to capture structural disagreement among analysts and to mitigate the impact of any short term events that may cause analysts to diverge or converge in their forecasts, e.g.

¹⁶Suppose that we enrich our model by “analysts” who all randomly learn about different parts of the set of potential pay-off states. If polled about the asset value after the state s is revealed, there is a higher chance of divergence among analysts for an opaque asset as it is then more likely that analysts have different information sets.

earnings announcements. We also obtain the average number of analysts submitting forecasts following a firm from I/B/E/S.

To be included in our final data set, we require firms to have complete information during each month over which averages are computed and to have at least two analyst forecasts in any month (results are robust to requiring a higher number of analysts). As there are outliers in both the spread and dispersion measure, we exclude observations in the 1% tail in either variable. We also drop stocks with an average price of less than \$5 (a common practice in the literature) because such stocks tend to trade infrequently. We arrive at a final sample of 2067 observations. Table 2.1 in Appendix 2.F contains the summary statistics for all variables.

2.3.2 Results

We first summarize the relationship between spreads and analyst dispersion using rolling windows which sort on dispersion. Figure 2.3 depicts the results for a window size of 500 (the first datapoint is the mean spread of the sample of firms with the 500 smallest dispersion measure, the second datapoint is the mean of the firms with a dispersion rank between 2 and 501, ...). Up to around window 800, there is a clear positive relationship between dispersion and bid-ask spreads. However, for subsequent windows, the bid-ask spread drops significantly. There is thus a non-monotonic relationship between the two variables – as predicted by theory. Interestingly, for the extreme dispersion portfolios (starting at around window 1300) the negative relationship ceases. Spreads no longer fall (and even increase somewhat). A potential explanation for this is that at this point the degree of opacity that prevents information acquisition is reached (\bar{o} in our model). We also note that the range of spreads implied by dispersion changes is large. While portfolios with intermediate dispersion have an average spread of around 0.028, spreads at the low and high end of the dispersion spectrum around are at 0.024 and 0.023 (the standard deviation over the entire sample is 0.03).

The rolling window analysis of Figure 2.3 is based on the raw data and is subject to the disadvantage that for any window information from the observations outside the window are completely ignored. This is an inefficient use of data and, among others, results in a more variable relationship in the figure. In addition, it does not allow inferences for individual specific dispersion levels, as each datapoint equally summarizes 500 data

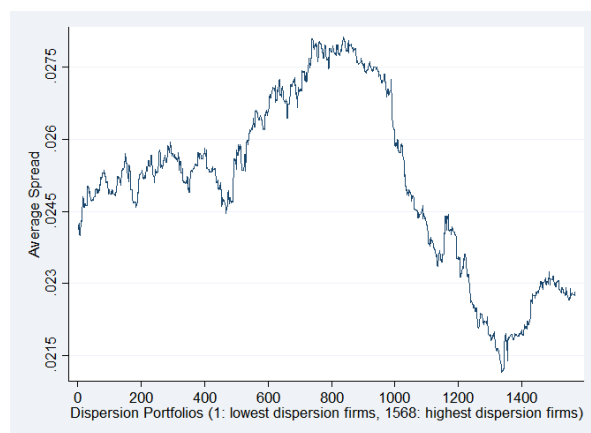


Figure 2.3: Rolling Windows Analysis of Bid-Ask Spreads

points. As an alternative, we analyze the relationship using Lowess smoothing.¹⁷ Figure 2.8 in Appendix 2.F presents the results, which confirm the rolling windows analysis. In particular, there is a monotonically increasing relationship between dispersion and spreads up to the 36th dispersion percentile of firms, after which a negative relationship obtains. Around the 86th percentile, the lowest bid-ask spread and hence highest liquidity is reached. Beyond, only smaller fluctuations in the liquidity proxy occur, consistent with opacity levels that preclude information acquisition being reached.

Previous research has indicated that bid-ask spreads reflect other factors besides adverse selection costs. It is thus important to control for these factors in the analysis. Addressing this, we analyze bid-ask spreads which are net of these factors. For this, we first regress bid-ask spreads on a set of controls and obtain residuals from this regression. We proceed to analyze the spread residuals using rolling window portfolios and Lowess smoothing (Cleveland (1979)).

As a first control, we use size (the log of market capitalization) as larger firms are expected to have smaller spreads independent of adverse selection considerations. Second, we include the stock price, as higher price firms have a tendency to have larger spreads. We also include proxies for inventory costs, as prior literature has emphasized that such costs should result in wider spreads by market makers. A first (and inverse) proxy is trading volume (scaled by market capitalization). Higher trading volume makes it easier for market makers to adjust their inventory and should hence lead to lower spreads (see e.g. Chordia, Roll, and Subrahmanyam (2000)). A second proxy is the standard

¹⁷Lowess smoothing (Cleveland (1979)) is based on a series of local regressions which are combined using non-parametric smoothing.

deviation of the stock return. This variable captures firm risk, which has the effect of increasing the cost of holding inventory and results in larger spreads. We also include the number of analysts following a stock. This is because the presence of more analysts is considered to lead to more efficient information transmission to other market participants, effectively reducing private information (see e.g. [Roulstone \(2003\)](#)) and thus the spread. Finally, we include the standard deviation of the daily trading volume (scaled by market capitalization) as firms with volatile trading volumes require more market depth to provide smooth pricing ([Roulstone \(2003\)](#)).

Table 2.2 in Appendix 2.F summarizes the results of a regression of the spread on these controls. Several controls are significant: market capitalization, price, volatility of returns and number of analysts. In each case, the coefficient of the significant variable has the expected sign. We calculate the residuals from this regression to separate the components that do not relate to adverse selection. Figure 2.4 depicts the rolling window analysis of the residuals, using the same approach as in Figure 2.3. The pattern looks fairly similar to the previous analysis, one noticeable difference being that there is now a more pronounced peak in residual spreads (at around window 820). Figure 2.9 in Appendix 2.F presents a locally smoothed graph based on Lowess regressions which plots residual spread against dispersion rank, again showing the hump-shaped relationship.

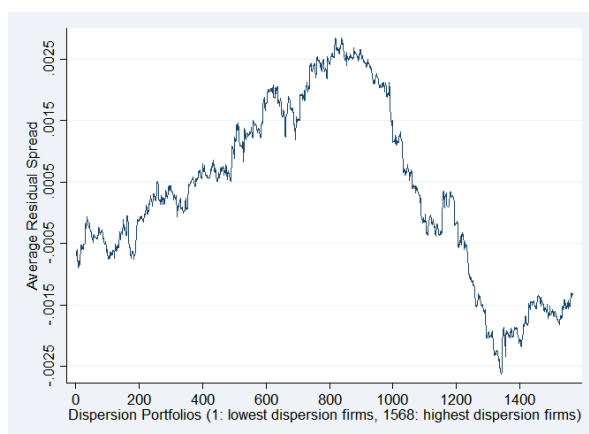


Figure 2.4: Rolling Windows Analysis of Bid-Ask Spreads Residuals

The basic result of a non-monotonic relationship between spreads and dispersion is robust to various considerations. First, the length of the rolling window can be modified within reasonable ranges without fundamentally modifying the observed pattern (similarly, results are robust to variations in the bandwidth for the Lowess regression).

Second, trading volume (which we use here as a control) arguably can be considered as a measure of liquidity itself. We hence re-run Figure 2.4 excluding trading volume in stage 1, with results unchanged. Furthermore, the results are robust to exclusion of the number of analysts as control. This is potentially important since the number of analysts may itself be a function of firm opacity, which may obscure the analysis. The analysis is also robust to requiring a high number of analysts following a firm (i.e., a minimum requirement of 5, 7 or 10 analysts following the firm). Finally, results are robust to different outlier treatments, such as including stocks with a price of less than \$5 and extending the tail cut-off for the spread and dispersion measures.

In sum, the empirical analysis provides evidence that the relationship between opacity and liquidity is not a simple negative one, as often presumed. In particular, the high liquidity of stocks with a large dispersion in analysis forecasts indicates that opacity may sustain common ignorance – even if market participants could exercise effort to acquire private information – and thus preserve liquidity.

2.4 The Incentives of Asset Originators

In this section, we endogenize several characteristics of the asset held by the investor. For this, we consider an original owner of the asset who can influence an asset's characteristics before selling it on to the investor. We first analyze the question of how much information an owner wants to release about an asset prior to the sale. Following this, we consider implications for which assets should be sold and how to sell them. Often originators (which may for instance be banks) have several assets for sale. In this case, they can decide whether to sell them together or separately, and when they sell them together, which assets to include in the bundle.

To analyze these questions, let us assume that there is an original owner of the asset, O . Prior to selling the asset to the investor, the owner can choose (some) characteristics of the asset. The choice of these characteristics affects future information acquisition by the investor, and through this, the price at which the owner can sell in the primary market.

Incorporating the owner, the economy now consists of three agents: an owner O , an investor I , and the market M . There are three dates ($t = 0, 1, 2$) of which date 1 and 2 are identical to the baseline model. The preferences of agents are as follows:

- The owner derives utility from consumption at date 0 only: $U^O = C_0^O$.
- The investor can now also consume at date 0. His utility when patient is hence $U^I = C_0^I + C_1^I + C_2^I$ and $U^I = C_0^I + C_1^I$ when impatient.
- The utility of the market is unchanged: $U^M = C_1^M + C_2^M$.

At date 0, the owner is endowed with the asset. The owner has no other endowment besides the asset. The investor has an endowment of $w^I (> l)$ at date 0; the market still has an endowment of $w^M (> 1)$ at date 1.

The owner first decides on the characteristics of the asset. Following this, he can sell the asset to the investor. For this, we assume that the owner and the investor bargain and that the owner captures a fraction $\delta \in (0, 1]$ of the investor's surplus. Following this, actions proceed as in the baseline model. Figure 2.5 depicts the timeline.

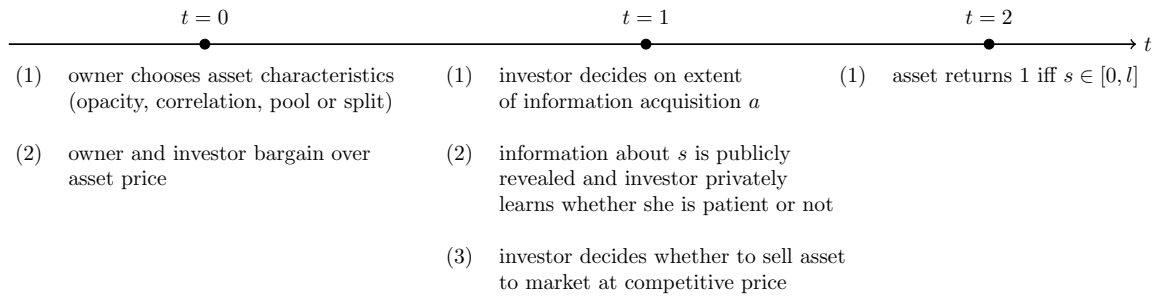


Figure 2.5: Timeline of Extended Model

2.4.1 Opacity

We first analyze the owner's choice of opacity. We assume that the owner is perfectly informed about the states in which the asset pays off. Before selling to the investor, he decides how much of this information to release. Specifically, he discloses a set of states of measure o which contain the payoff states. Releasing information comes at a cost for the owner: reducing opacity from 1 to o incurs a proportional cost of $k_O \cdot (1 - o)$ ($k_O > 0$).¹⁸ Such costs arise because it is costly to collect information about an asset and to convey it credibly to the other agents in the economy.

¹⁸A richer model could allow assets to differ with respect to *fundamental* opacity, that is, the level of opacity before any efforts by the owner to reduce opacity.

Efficient Opacity

Since the owner does not capture the full surplus whenever $\delta < 1$, his choice of opacity may differ from the welfare maximizing one. We first solve for the welfare-maximizing opacity level and subsequently contrast it with the owner's opacity choice.

From date 1 onwards, the setup is identical to the model of fixed opacity; trading and information acquisition are still characterized by Lemma 2.1 and Proposition 2.1. We now analyze the level of opacity that maximizes welfare. Given linearity of utility, (utilitarian) welfare is simply the expected sum of resources in the economy that are available for consumption. Welfare thus consists of the endowments, $w^I + w^M$, the asset's expected payoff, l , minus the cost of reducing opacity $k_O \cdot (1 - o)$, minus the cost of acquiring information $k_I \cdot a^*(o)$:

$$W(o, a^*(o)) = w^I + w^M + l - k_O \cdot (1 - o) - k_I \cdot a^*(o). \quad (2.6)$$

Welfare is hence maximized by minimizing the sum of the two costs in the economy.

The opacity choice has two effects on welfare. There is the direct cost of opacity reduction $k_O \cdot (1 - o)$ incurred by the owner. Furthermore, opacity affects date-1 information acquisition $a^*(o)$ and hence the information acquisition costs. Two cases arise. If $\bar{o} \leq 1$, information acquisition can be deterred by leaving the asset fully opaque, that is setting $o = 1$ (see Proposition 2.1). As this induces neither information acquisition (and associated costs) nor costs of opacity reduction; the first best is reached.

If $\bar{o} > 1$, this is not possible. In this case, the problem can be broken down as follows. First, choosing an opacity level that leads to partial information acquisition (that is, choosing an o on $[\underline{o}, 1)$) such that $a^*(o) \in (0, o - l)$ is never optimal. A completely opaque asset ($o = 1$) would dominate this choice as it would entail less information acquisition (recall that information acquisition is decreasing in opacity in the interior range) and also no opacity reduction cost. Second, when an opacity level of $[\underline{o}, 1)$ is chosen, all possible information is acquired ($a^*(o) = o - l$) and welfare is given by

$$W(o, a^*(o)) = w^I + w^M + l - k_O \cdot (1 - o) - k_I \cdot (o - l). \quad (2.7)$$

Equation (2.7) shows that optimal opacity depends on which cost parameter is larger. If information is more costly ($k_I > k_O$), welfare is maximized by choosing the smallest opacity in the range: $o = l$. If this is not the case ($k_I \leq k_O$), the optimal choice would

be to choose the largest opacity in the range: $o = \underline{o}$. However, as previously discussed, \underline{o} is dominated by a completely opaque asset ($o = 1$).

It follows that to find the optimal opacity level o whenever $\bar{o} > 1$, one has to compare welfare for a fully transparent and a fully opaque asset ($o = l$ versus $o = 1$). This boils down to comparing the cost of fully eliminating opacity, $k_O \cdot (1 - l)$, with the cost of investor information acquisition that arises for an entirely opaque asset, $k_I \cdot a^*(1)$.

Summarizing:

Proposition 2.2 *Selling an opaque asset ($o^* = 1$) maximizes welfare if*

(i) *this deters information acquisition ($\bar{o} \leq 1$), or*

(ii) $k_I \cdot a^*(1) < k_O \cdot (1 - l)$.

Otherwise, selling a fully transparent asset maximizes welfare ($o^ = l$).*

There are three important messages. First, it can be optimal to sell a fully opaque asset – independent of the magnitude of opacity reduction costs k_O . This is because under certain conditions, full opacity prevents any information acquisition by the investor. Second, intermediate degrees of opacity are undesirable as such opacity levels induce the investor to acquire costly information. Third, if the costs of opacity reduction are sufficiently small, it can be optimal for the owner to sell a fully transparent asset, which precludes information acquisition.

Adverse selection costs: Even though there is adverse selection at the trading stage (since a patient investor sells when he has negative private information), there are no direct welfare consequences of this in our model. This is because the impatient investor and the market have identical marginal utilities of consumption. A lower market price resulting from adverse selection thus does not affect the gains from trade (the equation for welfare does not contain the price). If an impatient investor were to have higher marginal utility than the market, this neutrality no longer obtains. Appendix 2.D analyzes this case, showing that information acquisition then has an additional, negative, effect on welfare through its effect on the equilibrium price. This, however, does not affect the key results. In particular, the hump-shaped relationship between opacity and information acquisition is still obtained.

The Owner's Choice of Opacity

The owner maximizes the price at which he can sell the asset to the investor, minus any cost incurred by him. Given that the investor's surplus is $l - k_I \cdot a^*(o)$, the owner maximizes

$$W_O(o, a^*(o)) = \delta(l - k_I \cdot a^*(o)) - k_O \cdot (1 - o). \quad (2.8)$$

The owner thus minimizes a combination of costs of opacity reduction and information acquisition costs. However, his objective function is not identical to the social one as he only internalizes a fraction δ of the investor's information acquisition costs.

Similar to the previous section, the solution can be derived as:

Proposition 2.3 *The owner sells a fully opaque asset ($o = 1$) if*

(i) *this deters information acquisition ($\bar{o} \leq 1$), or*

(ii) $\delta k_I \cdot a^*(1) < k_O \cdot (1 - l)$.

Otherwise, he sells a fully transparent asset ($o^ = l$).*

Proof. *The owner's opacity choice mirrors the one in the baseline model. If $1 > \bar{o}$, information acquisition can be deterred and the owner can avoid costs entirely by choosing full opacity ($o = 1$). If this is not the case, he chooses either full opacity or full transparency. The respective utilities from these choices are $W_O(1, a^*(1)) = \delta(l - k_I \cdot a^*(1))$ and $W_O(l, a^*(l)) = \delta o l - k_O \cdot (1 - l)$. He hence chooses full opacity iff $\delta k_I \cdot a^*(1) < k_O \cdot (1 - l)$. ■*

This yields the following corollary

Corollary 2.1 *The owner chooses an opacity level that is inefficiently high if and only if $\delta k_I \cdot a^*(1) < k_O \cdot (1 - l) < k_I \cdot a^*(1)$. Otherwise his choice of opacity is efficient.*

Proof. *Follows from comparing condition (ii) in Proposition 2.2 and 2.3. ■*

The intuition is clear. Since the owner incurs transparency costs fully but only internalizes a fraction of the investor's information acquisition costs, he has comparatively lower benefits from outcomes where transparency is high and information acquisition low. He hence may not sell a transparent asset even when transparency maximizes welfare.¹⁹

¹⁹Note that a sharing of rents between the investor and the market in the secondary trading stage does

Fundamental and effective opacity: The effective opacity of an asset (that is, the opacity of an asset when sold to the investor) will in practice consist of two factors. First it consists of the fundamental opacity of the asset, determined by its business characteristics. This was the focus of the analysis in the baseline model. For example, firms in certain industries are intrinsically more opaque. Large and complex firms will also have a fundamental tendency towards higher opacity. Second, there is the opacity choice of the owner (which we focused on in this section). This choice can be understood as efforts by the owner to reduce opacity below its fundamental opacity. In cases where such efforts are not taking place, effective opacity may approximate fundamental opacity. In addition, reaching a certain level of transparency will be more costly when initial opacity is high. In practice we would thus expect to observe a wide range of levels of effective opacity, with higher levels generally corresponding to a higher fundamental opacity.

2.4.2 Correlation

Suppose an owner wants to sell a number of assets, for instance, through a securitization. Should he include correlated or uncorrelated assets in the sale? And does this decision depend on the characteristics of the assets available?

To analyze this, we consider the following modification of the model. At $t = 0$, the owner is endowed with two pools of assets, each containing x ($x \geq 2$) assets of fixed opacity o . The assets in each pool are individually identical to the that of the baseline model: an asset pays 1 in a mass l states of the world and zero otherwise. The investor can narrow down the set of pay-off states for each individual asset by incurring cost a . The only difference between the two pools is that in the first pool, assets pay off in exactly the same states of the world. In the second pool, the payoff-states are independently distributed across assets.

At $t = 0$ the owner decides which pool of assets to sell. This choice is public information. At $t = 1$ the investor can acquire information about each asset in the pool and subsequently sell assets to the market. We assume that assets are sold individually to market participants and that each market participant cannot observe how many assets in total the investor is selling. To focus the analysis, we analyze in the following the

not lead to any bias in the owner's opacity choice as it does not create a wedge with the welfare maximizing level of opacity.

case of $\delta = 1$, in which case there is no conflict between the owner's incentives and the welfare maximizing outcome.

Suppose first that the owner chooses to sell the pool consisting of correlated assets. At the trading stage, the investor has to decide for each individual asset whether or not to sell it. The market has formed beliefs about information acquisition and since assets are identical, these beliefs boil down to a single parameter \tilde{a} about the investor's private information set $[o - \tilde{a}, o]$. The decision whether or not to sell is identical to the baseline model, but now applies to x -assets at the same time. That is, the investor will sell all assets whenever she is impatient or when she privately knows that the assets are worthless ($s \in [o - a, o]$).

At the beginning of $t = 1$, the investor decides how much information to acquire about each asset. Since assets are perfectly correlated, it is strictly optimal to acquire information about one asset only. The investor thus has a single choice a , as in the baseline model. However, acquiring information about one asset now provides additional benefits: Because of perfect correlation, the investor learns about several assets at the same time. Similar to equation (2.2), we can write the utility of the investor as

$$u(a, \tilde{a}) = x \cdot o \left((1 - \pi + \pi \frac{a}{o}) p(\tilde{a}, o) + \pi \frac{o - a}{o} \frac{l}{o - a} \right) - k_I \cdot a. \quad (2.9)$$

From this we can derive the investor's optimal information acquisition:

Proposition 2.4 *The equilibrium level of information acquisition for the correlated pool of assets is*

$$a_C^* = \begin{cases} o - l & \text{if } o \leq \underline{o}_C \\ (1 - \pi)(x \frac{l}{k_I} - \frac{o}{\pi}) & \text{if } o \in (\underline{o}_C, \bar{o}_C) \\ 0 & \text{if } o \geq \bar{o}_C \end{cases} . \quad (2.10)$$

with $\underline{o}_C = \pi l + x \frac{\pi(1-\pi)l}{k_I}$ and $\bar{o}_C = x \frac{l\pi}{k_I}$.

Proof. *Analogous to Proposition 2.1. ■*

Compared to the sale of a single asset, information acquisition now tends to be higher. First, the threshold at which the investor starts to acquire information is higher ($\bar{o}_C > \bar{o}$). Second, information acquisition in the interior cases is always higher ($a_C^* > a^*$ for

given o). The reason for this is that since information can be applied to several assets, it becomes more attractive to acquire information.

Suppose next that the owner has sold uncorrelated assets. At the trading stage the market will again have beliefs \tilde{a} about the level of private information for each asset. These beliefs will be asset-independent due to symmetry of the setup. The trading stage for each asset is hence the same as in the baseline case. Consequently, information acquisition for each individual asset is also unchanged and given by a as laid out in Proposition 2.1. Total information acquisition, however is $x \cdot a^*$.

We can now turn to the owner's choice of which assets to sell. Since the owner only consumes at $t = 0$, he does not care about the assets that are retained.²⁰ He will hence sell the pool that obtains the highest price, which will be the one with the lowest information cost. The owner's problem is thus to identify the pool that induces the lowest amount of private information. This choice will be subject to a basic trade-off. The incentives to acquire information for an individual asset are stronger in the correlated pool, as shown above. This speaks for the uncorrelated pool. However, for a given amount of information acquired about an asset, total costs are higher in the uncorrelated pool because information is then acquired about each asset individually.²¹

The consequences for the owner's decision are as follows. When information acquisition is sufficiently unattractive ($o \geq \bar{o}_C$), there will be no information acquisition for either pool and the owner is indifferent between the pools. When $\bar{o} < o < \bar{o}_C$, there will be information acquisition in the correlated pool only; hence the uncorrelated pool is preferred. For lower levels of opacity ($o < \bar{o}$), information is acquired in both pools. In this case the above trade-off comes into play. If $o > \underline{o}_C$ (that is, there is incomplete information acquisition in the correlated pool), an uncorrelated pool still maximizes welfare. This can be seen by noting that interior information acquisition in the correlated pool, $a_C^* = (1 - \pi)(x \frac{l}{k_I} - \frac{o}{\pi})$, is always higher than in the uncorrelated pool, $xa^* = x(1 - \pi)(\frac{l}{k_I} - \frac{o}{\pi})$. However, for o that is sufficiently below \underline{o}_C , information costs in the uncorrelated pool dominate (information acquisition in the correlated pool even

²⁰If the owner could also consume at $t = 2$, he would still be indifferent as to which assets are retained as both pools have the same expected payoff.

²¹Dang, Gorton, Holmström, and Ordoñez (2013) also analyze the impact of diversification on the incentives for information acquisition. They show that selling a diversified portfolio discourages private information acquisition by hiding private information. They do this in a setting where the cost of acquiring information is independent of the security design. Our model of endogenous information acquisition shows that while incentives to acquire information are indeed lower in the case of uncorrelated assets, correlated pools avoid duplicating private information production and may thus be preferred.

decline because they are then already at their maximum feasible level, $o - l$). The critical opacity level at which this happens is determined by the condition $o - l = xa^*(o)$. Rearranging yields:

$$\hat{o} = \frac{x(1 - \pi)\frac{\pi}{k_I} + 1}{x(1 - \pi) + 1}l. \quad (2.11)$$

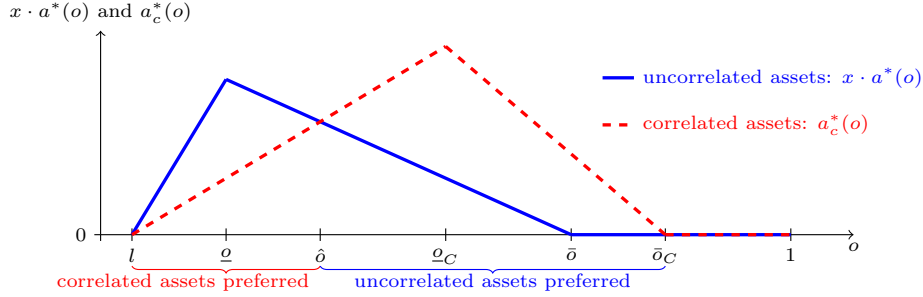


Figure 2.6: Information Acquisition as a Function of o

Figure 2.6 illustrates the different cases. We can summarize:

Proposition 2.5 *Consider the owner’s choice to sell a correlated or uncorrelated pool of assets.*

1. *If $o \leq \hat{o}$, the owner prefers to sell a correlated pool of assets.*
2. *If $o \in (\hat{o}, \bar{o}_C)$, the owner prefers to sell an uncorrelated pool of assets.*
3. *If $o \geq \bar{o}_C$, the owner is indifferent between both pools.*

In their review of securitization practices, Gorton and Metrick (2012) identify the lack of diversification as one of the main puzzles: “The choice of loans to pool and sell to the SPV also remains a puzzle. Existing theories cannot address why securitized-loan pools are homogeneous – all credit cards or all prime mortgages, for example. The existing theory suggests that credit card receivables, auto receivables, mortgages, and so on should be in the same pool – for diversification, but this never happens.” Proposition 2.5 shows that selling homogenous (or correlated assets) can be optimal. The reason is that this lowers the total cost of private information acquisition because information acquisition costs do not need to be spent on each individual asset.²²

²²On the investors’ side, [Van Nieuwerburgh and Veldkamp \(2010\)](#) consider a framework with endogenous information acquisition and investment and show that, when investors can acquire information before making their investment choice, non-diversified portfolio holdings may emerge endogenously.

2.4.3 Splitting and Pooling

Information acquisition also has consequences for whether an owner should sell cash-flows individually, or in a pool. To this end, consider that the owner has at date 0 an asset that pays x in l states and zero otherwise. The owner has the option to sell this asset in its entirety. Alternatively, he can split the asset into x smaller assets (each paying 1 in l states) and sell them to x separate investors. Assume that per-state information costs are k_I regardless of the size of the asset. In addition, assume that investors cannot credibly reveal information to each other (otherwise, one investor could obtain the information and sell them to all other investors) and that the market cannot observe how many investors are selling assets (this would reveal the private information of investors).

Consider first the sale of the asset in one piece. This case is identical to that of a correlated pool in the previous section. While for a correlated pool information acquisition for one asset applied to x assets of size one, it now applies to one asset of size x . Information acquisition is hence a_C^* as given by Proposition 2.4. Consider next the sale of split assets to different investors. Each investor is in the same situation as in the baseline model: he can decide to acquire information about an asset of size 1. Thus, the results from the baseline model apply. However, since there are now x investors in total, overall information costs are $x \cdot a^*$, identical to the case of an uncorrelated pool.

The decision whether or not to split the asset thus creates the same trade-off as the decision whether to sell a correlated pool. We can conclude:

Proposition 2.6 *Consider the owner's choice to split an asset for sale.*

1. *If $o \leq \hat{o}$, the owner prefers not to split.*
2. *If $o \in (\hat{o}, \bar{o}_C)$, the owner prefers to split.*
3. *If $o \geq \bar{o}_C$, the owner is indifferent.*

The intuition behind the trade-off is as follows. On the one hand, the incentives to acquire information for an investor who has bought the entire asset are high because private information can then be applied to an asset that pays off $x > 1$. On the other hand, when investors who have bought the split assets acquire information, information acquisition is duplicated because each individual investor will acquire information. This means that in cases where there are large incentives to acquire information, the

owner should sell the entire asset in order to avoid duplication of a large amount of information.

2.5 Public Policy

Regulation of information disclosure by firms has a long tradition and takes many forms. Examples are requirements for listed companies to publish certified accounts at specified intervals or to disclose material information in a timely fashion. Prior to the crisis of 2007-2009, disclosure policies were predominantly targeted at protecting investors in standard securities (debt and equity). Following the breakdown of trade in various classes of asset-backed securities, a new focus of regulation is on the transparency of assets issued by financial institutions. For example, the Dodd-Frank act requires disclosure of information about asset-backed securities.

Transparency policies typically take the form of minimum standards. Issuers are obliged to follow these standards, but are free to implement higher standards of transparency. The non-monotonic nature of opacity suggests that a (uniform) minimum standard is not a desirable approach to regulation. We have shown that transparency reduces adverse selection only when transparency is sufficiently large, while increasing it otherwise. Consider Figure 2.3 and 2.8, which depict the (smoothed) cross-sectional relationship between opacity and bid-ask spreads at the firm level. The turning point at which transparency reduces asset liquidity is around the 40th percentile in both figures, suggesting that a mandated increase in transparency may increase bid-ask spreads for a large share of the population of firms. Since higher transparency brings about costs for issuers, the net effect of uniformly higher transparency may hence easily be negative.²³ Note that this does not imply that transparency regulation per se is undesirable as actual opacity levels already reflect existing efforts to enhance transparency.

Nonetheless, our analysis provides a clear rationale for regulation: issuers do not internalize the full cost of opacity for other agents in the economy and may hence choose inefficiently low amounts of transparency. Firm-specific disclosure standards that take into account that optimal opacity is heterogeneous are in principle welfare-enhancing. However, as the analysis in Section 2.4.1 has shown, the extent to which transparency

²³Kurlat and Veldkamp (2015) provide an alternative reason for why disclosure can reduce welfare. The channel is based on a general equilibrium effect. Disclosure makes assets less risky. This, in turn, will result in assets commanding a lower return in equilibrium.

is optimal depends on deep parameters such as the cost of information to firms and investors. Regulation that conditions on these parameters seems practically infeasible.

A less demanding approach is to provide subsidies (implicit or explicit) to issuers for reducing transparency. From previous analysis we know that issuers sometimes choose inefficient opacity since they only take into account a fraction $\delta < 1$ of the full cost of opacity, $k_I \cdot a^*(1)$. A subsidy of $(1 - \delta)k_I \cdot a^*(1)$ for each issuer can hence implement efficiency (essentially, a negative Pigouvian tax). And when the regulator has incomplete knowledge about the size of externalities posed by individual issuers, he can still implement a welfare-improving policy through a subsidy that is equal to the minimum of $(1 - \delta)k_I \cdot a^*(1)$ across all firms (in this case, transparency will be optimally increased at some firms – without leading to any increases in transparency that are welfare-reducing at other firms).

A subsidy could, for example, take the form of a government-sponsored rating agency that allows issuers (at their discretion) to obtain free ratings. In addition, publicly-run information repositories could help reduce the costs of providing transparency to issuers. It is crucial, however, that participation is left to the discretion of the issuers – compulsory participation suffers from the same problem as mandatory disclosure requirements.

Our analysis also speaks to the current discussion on asset correlation in the financial system. There are good reasons to believe that at the system-level financial institutions inefficiently invest in assets with high correlation, due to the externalities associated from creating systemic risk (see e.g. [Acharya \(2009\)](#) or [Wagner \(2011\)](#)). Correlation is hence typically viewed with suspicion by policy-makers. On the micro-level, there is the puzzle that securitization pools predominantly consist of similar assets, which flies in the face of diversification ([Gorton and Metrick \(2012\)](#)). Our analysis suggests that there is also a benefit to issuing assets with high correlation: it minimizes information costs in the economy as information about one asset can be applied to other assets as well. Thus even though adverse selection problems for correlated assets were observed during the recent crisis, this does not necessarily indicate inefficiency, as issuing correlated assets can be optimal under certain circumstances. Similarly to asset correlation, opacity is also often associated with inefficiencies. Commentators have been especially alarmed by the opacity of complex securitization structures. Since opacity also comes with benefits – it may discourage people from obtaining private information – one has to be careful in deriving direct implications from this.

2.6 Conclusion

How does opacity affect liquidity when investors can acquire information about an asset? This paper has suggested that the link between the two is non-monotonic. Both very transparent and very opaque assets preserve commonality of information. While full transparency directly precludes information asymmetries, sufficiently large opacity deters acquisition of private information by making learning about an asset more costly. Assets with either very low or very high opacity can hence be expected to be liquid. Assets which display intermediate degrees of opacity, in contrast, are prone to information acquisition. These assets may suffer from adverse selection problems when they need to be traded. An empirical analysis of the cross-section of listed U.S. firms strongly supported this hump-shape relationship between opacity and illiquidity.

Our analysis points to a significant benefit to opacity, which may help understand the phenomenon that issuers often choose to sell surprisingly opaque assets, as for instance observed in the case of securitization products. Policy makers thus have to be careful in equating opacity with inefficiencies. The results also have implications for transparency regulation. In particular, our analysis suggests that uniform transparency requirements are not desirable. This is simply because they may increase adverse selection for the more opaque assets in the economy. Rather, a more appropriate policy is to subsidize the provision of information by issuers. This can help internalizing the externalities associated with opacity, while allowing issuers to optimally preserve heterogeneous transparency levels.

Appendix

2.A Random Discovery of the Payoff Interval

This analyzes a stochastic information acquisition technology. While in the baseline model information acquisition started at the upper end of the interval $[l, o]$, we now consider a random starting point. Specifically, we denote the *starting state* for information acquisition with y and assume that it is uniformly distributed on $[l, o]$. The distribution of the starting state is known by the investor, but not its realization.

As before, the investor learns about an interval of mass a when choosing a level of information acquisition a . For given starting state y , the investor thus learns about the interval $[y - a, y]$. If a is such that $y - a > l$, she learns that the interval $[y - a, y]$ does not contain payoff states, as in the baseline model. If a is sufficiently large such that $y - a \leq l$, she “discovers” the payoff interval. In this case, she ends up with complete knowledge about the distribution of payoff states.

We allow information acquisition to take place sequentially, that is, the investor can first decide to obtain information about a certain mass of states, and following this decide whether to analyze more states (and so on). Note that since the investor does not know the realization of y , she does not know in advance whether a certain amount of information acquisition will lead to discovery of the payoff interval.

It is easy to see that the modification in the information technology does not alter the investor’s incentives to sell to the market at date 1 (Lemma 2.1): she will offer the asset if impatient; otherwise she will offer the asset only if she knows that the asset is worthless. The price of the asset will again depend on the market’s belief about information acquisition. These beliefs, however, are no longer necessarily characterized by a single parameter since information acquisition can become stochastic (for instance, depending on y , investor may discover the payoff interval early on and stop). Let us denote the market price with \tilde{p} to indicate its dependence on beliefs.

We start with the analysis of the investor’s incentives to acquire information. When deciding about information, the investor takes as given the price \tilde{p} at which she can sell to the market. We consider information acquisition that takes place by acquiring knowledge about (small) intervals of size $b > 0$ (we later consider the limit of b tending to zero).

Consider first that the investor has already discovered the payoff interval. She then has complete information about the asset, and hence will not acquire any further information.

Consider next the decision of an investor to acquire information about an interval b given that she has already acquired an amount $a \geq 0$ of information and has not yet discovered the payoff interval. Two cases arise. First, if a is sufficiently large such that $o - a - b \leq l$, the investor knows that the payoff interval will be discovered with certainty with the next information acquisition. The discovery will benefit the investor when a state of nature s materializes that falls in the interval $[l, o - a]$ and when she is impatient. The probability of this is $(o - a - l)\pi$, in which case she is able to sell at price \tilde{p} rather than holding onto a worthless asset. Her expected gains from additional information acquisition are thus

$$u(l, \tilde{p}) - u(a, \tilde{p}) = (o - a - l)\pi\tilde{p} - bk_I. \quad (2.12)$$

These gains are identical to equation (2.3) in the baseline model - except that an interval of size $o - a - l$ is discovered by incurring costs for b ($\geq o - a - l$) states. Equation (2.12) shows that information acquisition is beneficial whenever $(o - a - l)\pi\tilde{p} > bk_I$. We can hence define the *option value* of information acquisition in this case as $\max\{(o - a - l)\pi\tilde{p} - bk_I, 0\}$.

Second, we have the case of $o - a - b > l$. In this case the investor does not know whether the next information acquisition will discover the payoff interval - it depends on the starting state y . While the realization of y is unknown to the investor, she infers from not having discovered the payoff interval up to now that $y \in [l + a, o]$. The impact of information acquisition in this case is as follows. When $y > l + a + b$, she does not discover the payoff interval. In this case, she can rule out an interval of mass b as containing payoff-states. When $y \leq l + a + b$, she discovers the payoff interval. She then rules out in total a mass of $o - a - l$ states. The likelihood of non-discovery and discovery is $1 - \frac{b}{o - a - l}$ and $\frac{b}{o - a - l}$, respectively. We hence have for the total expected mass of loss states discovered $b(1 + \frac{o - l - a - b}{o - a - l})$. Recalling that the investor benefits from knowledge about loss states when impatient, we obtain for the total expected gains from acquiring information

$$u(a + b, \tilde{p}) - u(a, \tilde{p}) = b \left(\left(1 + \frac{o - l - a - b}{o - a - l}\right)\pi\tilde{p} - k_I \right) + \left(1 - \frac{b}{o - a - l}\right) V(a + b), \quad (2.13)$$

where $V(a + b)$ is the option value from acquiring further information when the payoff interval has not been discovered.

The value of information acquisition can hence be recursively defined as

$$V(a) = \begin{cases} \max\{b \left((1 + \frac{o-l-a-b}{o-a-l})\pi\tilde{p} - k_I \right) + \left(1 - \frac{b}{o-a-l} \right) V(a+b), 0\} & \text{if } a < o-l-b \\ \max\{(o-a-l)\pi\tilde{p} - bk_I, 0\} & \text{if } a \in [o-l-b, o-l) \\ 0 & \text{if } a \geq o-l \end{cases} \quad (2.14)$$

Note that $f(a) := b \left((1 + \frac{o-l-a-b}{o-a-l})\pi\tilde{p} - k_I \right)$ is decreasing in a . This implies that the value of acquiring information about an interval of size b is declining in the amount of information already acquired. The reason is as follows. While the likelihood of discovering the payoff interval ($\frac{b}{o-a-l}$) is increasing in a , the expected gains conditional on discovery are decreasing. The latter is because the mass of states ruled out by discovery, $o-l-a-b$, falls in a . Because of this latter effect, the gains from information acquisition are ultimately decreasing.

It follows that $f(a) \leq 0$ implies $f(a+b) < 0$. In addition, we can conclude that when $a \in [o-l-b, o-l)$ (that is, when the next information acquisition discovers the payoff interval with certainty) we have $f(a-b) > (o-a-l)\pi\tilde{p} - bk_I$. From this it follows that whenever $f(a) \leq 0$, the option value of information acquisition beyond the next interval is zero ($V(a+b) = 0$). Thus, $V(a) = 0$ whenever $f(a) \leq 0$. The consequence is that an investor will acquire information as long as $f(a) > 0$, and will stop when $f(a) \leq 0$ or when the payoff interval is discovered.

An equilibrium strategy for information acquisition is hence defined by a threshold $a^* \in (0, o-l)$ such that $f(a^*) \leq 0$, but $f(a^* + b) > 0$. For arbitrarily small intervals of information acquisition ($b \rightarrow 0$), we find that $f(a) = 0$ precisely when

$$\tilde{p} = \frac{k_I}{2\pi}. \quad (2.15)$$

This condition is almost identical to the condition for an interior equilibrium in the baseline model ($\tilde{p} = \frac{k_I}{\pi}$). The difference arises because information acquisition is now more effective as it can result in the discovery of the payoff interval, in which case the entire distribution becomes known (in the baseline model, it only allowed us to proportionally narrow down the set of payoff states). In order for the gains from information acquisition to be identical to the costs k_I , the price at which the asset can be sold when information is of use to the investor hence has to be lower.

We next derive the break-even market price \tilde{p} as a function of beliefs about information acquisition. Recall that the investor's strategy can be summarized by a threshold value a^* . The market's beliefs can hence be summarized by a single parameter \tilde{a} . Note that even though information discovery is stochastic, it only has two possible outcomes: either the investor finds the payoff interval or she reaches \tilde{a} and stops. Given that the starting point y is distributed on $[l, o]$, the probability of the payoff interval being discovered is simply

$$\pi_0 = \frac{\tilde{a}}{o-l}. \quad (2.16)$$

The investor will offer the asset if either she is impatient or if she is patient and privately knows the asset will not pay out. The probability of the latter is $\frac{o-l}{o}$ when she has discovered the payoff interval and $\frac{\tilde{a}}{o}$ when she has not discovered the payoff interval. The total probability of offering is thus

$$1 - \pi + \pi \left(\pi_0 \frac{o-l}{o} + (1 - \pi_0) \frac{\tilde{a}}{o} \right). \quad (2.17)$$

An offered asset only has a positive expected value if the investor is impatient (occurring with probability $1 - \pi$), in which case the expected value to the market is $\frac{l}{o}$. We can then use (2.16) and (2.17) to express the expected value (and hence the price) of the asset conditional on being offered as

$$p(\tilde{a}, o) = \frac{1 - \pi}{(1 - \pi)o + \pi(\tilde{a}(2 - \frac{\tilde{a}}{o-l}))} l. \quad (2.18)$$

Combining (2.15) and (2.18) to eliminate $p(\tilde{a}, o)$, and solving for $a^* = \tilde{a}$ yields:

$$a^* = (o-l) - \sqrt{(o-l) \left((o-l) - (1-\pi) \left(\frac{2l}{k_I} - \frac{o}{\pi} \right) \right)}. \quad (2.19)$$

Differentiating with respect to o gives

$$\frac{\partial a^*}{\partial o} = 1 - \frac{(o-l) \left(2 - \frac{1-\pi}{\pi} \right) - (1-\pi) \left(\frac{2l}{k_I} - \frac{o}{\pi} \right)}{2\sqrt{(o-l) \left((o-l) - (1-\pi) \left(\frac{2l}{k_I} - \frac{o}{\pi} \right) \right)}} < 0. \quad (2.20)$$

Information acquisition (in an interior equilibrium) is hence declining in opacity o , as in the baseline model.

The cases of no and full information acquisition are straightforward to analyze. No information acquisition results if at $a = 0$ we have $f(a) \leq 0$. Noting that zero information acquisition implies $p = \frac{l}{o}$, we can obtain from $f(a) = 0$ a critical threshold opacity of $\bar{o} = 2\frac{l\pi}{k_I}$, such that an opacity level of $o \geq \bar{o}$ deters information acquisition. Full information acquisition arises when $f(o - l) \geq 0$ (as $b \rightarrow 0$, we can ignore the case of $a \in [o - l, o - l + b]$). Equation (2.18) yields for $a^* = o - l$ that $p = \frac{(1-\pi)l}{o-\pi l}$. Combining with $f(o - l) = 0$ and rearranging gives a critical threshold $\underline{o} = \pi l + \frac{(1-\pi)\pi l}{2k_I}$. For $o \leq \underline{o}$ we hence have a full information acquisition equilibrium.

We can summarize

Proposition 2.A.1 *The equilibrium threshold for information acquisition $a^*(o)$ is given by*

$$a^*(o) = \begin{cases} o - l & \text{if } o \leq \bar{o} \\ (o - l) - \sqrt{(o - l) \left((o - l) - (1 - \pi) \left(\frac{2l}{k_I} - \frac{o}{\pi} \right) \right)} & \text{if } o \in (\underline{o}, \bar{o}) \\ 0 & \text{if } o \geq \underline{o} \end{cases} \quad (2.21)$$

with $\underline{o} = \pi l + \frac{(1-\pi)\pi l}{2k_I}$ and $\bar{o} = 2\frac{\pi l}{k_I}$.

2.B Learning about Loss States

Assume that the asset pays 1 if nature selects a state $s \in [l, 1]$ and zero otherwise. Increasing transparency and information acquisition each narrow down the potential set of states where the asset does not pay off. In particular, for transparency choice o and information acquisition a , the public knows the set of non-paying (loss) states to be on the interval $[0, o]$, while the investor knows that the loss states are distributed on $[0, o - a]$.

For given beliefs about private information acquisition, \tilde{a} , the trading decision of the investor is as follows. When $s \geq o$, both investor and market know that the asset will certainly pay. Its price will hence be 1. An impatient investor will sell the asset, while a patient investor will not sell given the assumptions we made about the investor's actions whenever indifferent. When $s \in [o - a, o]$, the investor knows that the asset will certainly pay off but the market only has imperfect knowledge about the payoff. The investor has thus positive private information about the asset. If she is patient, she will

hence not sell. If impatient, the investor will still sell. Finally, when $s \in [0, o - a]$, both investor and market are uncertain about the payoff. However, the investor observes that s is not in her private set of payoff states $[o - a, o]$. She thus has negative private information. She will hence sell, regardless of whether she is patient or not (the expected value of the asset is $\frac{o-a-l}{o-a}$ in this case). The market price of the asset (conditional on $s < o$) can hence be derived as

$$p(\tilde{a}, o) = \frac{o - l - \tilde{a}\pi}{o - \tilde{a}\pi}. \quad (2.22)$$

Note that for $\tilde{a} = 0$, this simplifies to $p = \frac{o-l}{o}$, which is the expected value of the asset conditional on $s < o$. Note also that $\frac{\partial p(\tilde{a}, o)}{\partial \tilde{a}} < 0$, because of adverse selection.

Similar to equation (2.2) we can derive the investor's utility given market beliefs \tilde{a} :

$$u(a, \tilde{a}) = 1 - o + o \left(\frac{a}{o}(\pi + (1 - \pi)p(\tilde{a}, o)) + \frac{o - a}{o}p(\tilde{a}, o) \right) - k_I \cdot a. \quad (2.23)$$

The derivative with respect to a is

$$\frac{\partial u(a, \tilde{a})}{\partial a} = \pi(1 - p(\tilde{a}, o)) - k_I. \quad (2.24)$$

It is useful to contrast this with the marginal benefit of information acquisition in the baseline model ($\frac{\partial u(a, \tilde{a})}{\partial a} = \pi p(\tilde{a}, o) - k_I$). Private information benefits the investor whenever it causes her to modify her selling decision. In the baseline model, the investor learns that the asset will *not* pay off in certain states. A patient investor will then sell the asset if such a state materializes; and hence benefits from a higher market price. In the extension considered here, the investor learns about states in which the asset *does* pay off. She thus does not sell the asset if such a state materializes. Her gains hence decline in the market price (which she would otherwise obtain by selling the asset). This has a consequence: because more information acquisition leads to lower prices in equilibrium, the gains from information will now be increasing in the amount of information acquired (formally, we have that $\frac{\partial u(a, \tilde{a})}{\partial a} \Big|_{a=\tilde{a}}$ is increasing in a).

Two cases arise. Consider first that $\frac{\partial u(a, \tilde{a})}{\partial a} \Big|_{a=\tilde{a}} > 0$ at $a = 0$. This implies that at a conjectured equilibrium with no information acquisition, the marginal gains from information acquisition are positive. Since we know that $\frac{\partial u(a, \tilde{a})}{\partial a} \Big|_{a=\tilde{a}}$ is increasing in a , the marginal gains from information acquisition are hence also positive for any $a > 0$. The unique equilibrium is hence full information acquisition: $a^* = o - l$. Consider next

that $\left. \frac{\partial u(a, \tilde{a})}{\partial a} \right|_{a=\tilde{a}} < 0$ at $a = 0$. In this case, the gains from information acquisition at an equilibrium with no information acquisition are negative. Hence, no information acquisition is an equilibrium ($a^* = 0$). Since $\left. \frac{\partial u(a, \tilde{a})}{\partial a} \right|_{a=\tilde{a}}$ is increasing in a , there might also be a second equilibrium with positive information acquisition. However, this equilibrium would be pareto-dominated by no information acquisition (which involves no information cost) and we hence rule it out.

Whether full or no opacity is chosen thus depends on the sign of $\left. \frac{\partial u(a, \tilde{a})}{\partial a} \right|_{a=\tilde{a}}$. Using equation (2.24) one can find that this derivative is zero when $o = \frac{\pi l}{k_I}$. We can hence state

Proposition 2.B.1 *When the investor learns about loss states, the equilibrium level of information acquisition a^* is*

$$a^*(o) = \begin{cases} o - l & \text{if } o < \bar{o} \\ 0 & \text{if } o \geq \bar{o} \end{cases} \quad (2.25)$$

with $\bar{o} = \frac{\pi l}{k_I}$.

2.C Increasing Cost of Information Acquisition

To analyze increasing costs of information acquisition, let the total cost of acquiring information about a mass of a states be $K_I(a)$ with $K_I(0) = 0$, $K'_I(a) > 0$, $K''_I(a) > 0$.

Differentiating the investor's utility $u(a, \tilde{a})$ (equation (2.2), after replacing $k_I \cdot a$ with the new information cost function) with respect to a yields

$$\frac{\partial u(a, \tilde{a})}{\partial a} = \pi p(\tilde{a}, o) - K'_I(a). \quad (2.26)$$

Equation (2.26) determines a new threshold for zero information acquisition \bar{o} . Rearranging $\pi p(0, o) - K'_I(0) = \pi \frac{l}{o} - K'_I(0) = 0$ gives $\bar{o} = \frac{\pi l}{K'_I(0)}$. Since $K'_I(0) > 0$, there is hence a unique \bar{o} above which no information is acquired. Likewise, \underline{o} is uniquely pinned down by the condition: $\pi p(o - l, o) - K'_I(o - l) = \pi \frac{1-\pi}{1-\pi \frac{l}{o}} \frac{l}{o} - K'_I(o - l) = 0$. This yields $\underline{o} = \pi l + \frac{(1-\pi)\pi l}{K'_I(o-l)}$. Finally, we can write down the condition for the interior equilibrium: $\pi p(a^*, o) - K'_I(a^*) = \frac{\pi(1-\pi)l}{o-\pi(o-a^*)} - K'_I(a^*) = 0$. Totally differentiating with

respect to o and rearranging gives

$$a^{*'}(o) = -\frac{(1-\pi)K_I'(a^*(o))}{\pi K_I'(a^*(o)) + ((1-\pi)o + \pi a^*(o))K_I''(a^*(o))} < 0. \quad (2.27)$$

Thus, opacity reduces information acquisition in an interior equilibrium. We hence have the same properties as in the baseline model. For $o \leq \underline{o}$ we have full information acquisition ($a^* = o - l$). Between \underline{o} and \bar{o} there is an interior degree of information acquisition which is declining in opacity. For opacity larger than \bar{o} , no information is acquired.

2.D Adverse Selection Costs

Modify the baseline model by assuming that the utility of the impatient investor is

$$U^I = C_0^I + qC_1^I, \text{ with } q \geq 1. \quad (2.28)$$

This modification does not affect trading with the market: an impatient investor will always sell while the patient investor only sells when she know the asset is worthless. Consider next the investor's incentives to acquire information. Similar to equation (2.2), utility is now

$$u(a, \tilde{a}) = o \left(((1-\pi)q + \pi \frac{a}{o})p(\tilde{a}, o) + \pi \frac{o-a}{o} \frac{l}{o-a} \right) - k_I \cdot a. \quad (2.29)$$

The derivative with respect to a is $\pi p(\tilde{a}, o) - k_I$ – the same as in the baseline model (equation (2.3)). The incentives to acquire information are hence unchanged and Proposition 2.1 still applies. The reason is that information acquisition only benefits the investor if she turns out to be patient, thus the fact that q may be larger than one does not matter.

The expression for welfare is now as follows. Whenever the investor is impatient and sells to the market, there is an additional welfare gain of $(q-1)p(a^*(o), o)$ compared to the baseline model. We thus have for welfare that

$$W(o, a^*(o)) = w^I + w^M + l + \pi(q-1)p(a^*(o), o) - k_O \cdot (1-o) - k_I \cdot a^*(o). \quad (2.30)$$

Opacity now has a new effect, arising because it can affect the price p through a change

in information acquisition. Since p is declining in information acquisition (equation (2.1)), opacity-induced increases in information acquisition now have two effects. First, they directly lead to costs k_I . Second, they reduce the gains for the impatient investor by lowering the price at which she can sell to the market (when $q > 1$, these losses are not completely offset by gains for the market).

Optimal opacity is determined analogous to the baseline model. For $1 \leq \bar{o}$, full opacity maximizes welfare. For $1 > \bar{o}$, one needs to compare welfare under full and no opacity (because of the dependence on p , welfare is now non-linear in the region $[l, \bar{o}]$, but this does not affect the optimal decision). Full opacity is optimal if and only if $W(1, a^*(1)) > W(l, 0)$, which is when $\pi(q - 1)p(a^*(1), 1) - k_I \cdot a^*(1) > \pi(q - 1)p(0, o) - k_O \cdot (1 - l)$. Otherwise full transparency should be chosen.

2.E Endogenous Discounting of Opaque Assets

We present a model in which low pricing of opaque assets does not follow from the assumed information acquisition technology (in the baseline model, a state in the public domain is less likely to pay-out when opacity is higher, resulting in a lower market price). Rather, the market values opaque assets less because it does not allow him to make efficient scrapping decisions.

We modify the baseline model as follows. First, we assume a distribution of asset returns and a type of information acquisition for which the expected value of an asset in the public domain is independent of opacity. As in the baseline model, the asset's pay-off states are randomly distributed. We assume that each state has the same likelihood of paying out, given by l . An asset is said to have opacity o when the returns on $(o, 1]$ are publicly known (there is thus a mass $(1 - o)l$ of known pay-off states and mass $(1 - o)(1 - l)$ of known non-payoff states) but the returns in the states $[0, o]$ are not. An amount of information acquisition a results in the investor learning the returns in the interval $[o - a, o]$. She will thus know about a mass $a \cdot l$ of pay-off states and a mass $a \cdot (1 - l)$ of states where the asset does not pay off.

Second, we assume that at $t = 1$, after potential information acquisition by the investor, a public signal s' about the state of the world arrives (in the baseline model, the state itself was revealed at this stage). The signal is imperfect in that only with probability σ ($\sigma \in (0, 1)$) it points to the actual state of the world ($s' = s$). With probability $1 - \sigma$, the signal is wrong, in which case the distribution of states is identical to the ex-ante

distribution (each state of the world $s \in [0, 1]$ is then equally likely). At $t = 1.5$, the state of the world is publicly revealed. Following the revelation of state, a market agent who has bought the asset can decide to scrap the asset. Scrapping the asset yields a sure pay-off of γ ($< l$). We assume in addition that $\gamma < \frac{1-\sigma}{\sigma}$.

All other actions are as in the baseline model. Figure 2.7 summarizes the timing of the modified model.

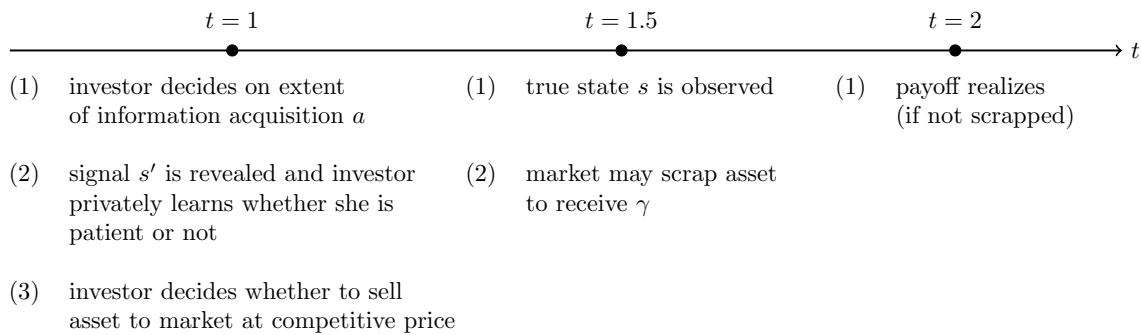


Figure 2.7: Timeline of Modified Model

The model is significantly more complicated than the baseline model in that multiple equilibria exist for a range of opacity levels and that small perturbations in the parameters can lead to discrete changes in equilibrium variables. In particular, there is no intuitive characterization of the behaviour of the equilibria with positive amounts of information acquisition. We hence focus the analysis on showing that for sufficiently high opacity, no information is acquired. By contrast, for assets of low or intermediate opacity, equilibria without private information acquisition are unsustainable.

We prove this by analyzing the investor's incentives to deviate from a conjectured equilibrium without information acquisition. We first derive the valuation of an asset sold by the investor presuming that the market believes no information has been acquired. Following this we solve for the investor's gains from deviating by acquiring information.

2.E.1 Market Price without Information Acquisition

Suppose that the market believes that no information has been acquired by the investor ($\tilde{a} = 0$). In this case information asymmetry is absent and the selling decision of the investor does not provide any signal to the market.

Consider the scrapping decision of the market at $t = 1.5$. At this stage the market knows the initial signal s' as well as the actual state s . Optimal scrapping is as follows. If s is in the transparent region ($s \in (o, 1]$), the market knows whether the asset will pay off at $t = 2$. The market will thus scrap if and only if the asset does not pay off. If s is in the opaque region ($s \in [0, o]$), the market is not certain whether the asset will pay-off at date 2. Since the likelihood of pay-off is l , it is optimal not to scrap since $\gamma < l$.

Similar to the baseline model, we derive the price conditional on the signal being in the opaque region ($s' \in [0, o]$). Given the efficient scrapping decision, the expected value of an asset to the market, and hence the price, is given by

$$p(o, \tilde{a} = 0) = \sigma l + (1 - \sigma)[ol + (1 - o)(l + (1 - l)\gamma)]. \quad (2.31)$$

This is derived as follows. With probability σ , the signal is true ($s = s'$). In this case, the actual state of the world is in the opaque region. The market then does not scrap and receives l in expectation. With probability $1 - \sigma$, the signal is not true. In this case, the state falls in the opaque region with probability o . The market then again does not scrap and receives an expected value of l . With probability $1 - o$, the state falls in the transparent region and the pay-off is known to the investor. If it is a pay-off state (occurring with probability l), the investor receives 1. If it is not a pay-off state (occurring with probability $1 - l$), the investor scraps and receives γ .

Note that $p(o, \tilde{a} = 0)$ simplifies to $l + (1 - \sigma)(1 - o)(1 - l)\gamma$. We hence have $p > l$ due to the option value from scrapping. Second, more opaque assets (high o) command lower prices. This is because for such assets there is a greater chance that a state realizes for which the investor does not know the date-2 payoff of the asset, in which case she cannot make efficient scrapping decisions.

2.E.2 Information Acquisition

We next analyze the investor's incentives to deviate by choosing a positive amount of information $a > 0$.

Consider the selling decision of an investor who has acquired information a . Information acquisition is only relevant in cases where the signal falls in the opaque region ($s' \in$

$[0, o)$), so we can restrict attention to this region. Note that if the investor sells, she obtains $p(o, \tilde{a} = 0)$ as the market believes there is no information acquisition.

If the investor is impatient, it is strictly dominating to sell to the market at $p(o, \tilde{a} = 0)$. If the investor is patient, there are three cases to consider:

1. The signal s' falls in the private domain ($s' \in [o - a, o]$) and s' is not a pay-off state. The expected pay-off for the investor is then $(1 - \sigma)l$, which is smaller than $p(o, \tilde{a} = 0)$. It is hence optimal to sell.
2. The signal s' falls in the private domain ($s' \in [o - a, o]$) and s' is a pay-off state. The expected payoff from holding on to the asset is then given by $\sigma \cdot 1 + (1 - \sigma)l$. This is less than $p(o, \tilde{a} = 0)$ by our assumption that $\gamma < \frac{1-\sigma}{\sigma}$. It is hence optimal not to sell the asset.
3. The signal s' falls outside the private domain ($s' \in [0, o - a)$). The investor then does not know whether the asset pays off if the signal turns out to be true. Her expected payoff from holding on to the asset is then simply l . Since $l < p(o, \tilde{a} = 0)$, she strictly prefers to sell.

The selling decision of the patient investor is thus similar to the baseline model: she continues to hold the asset unless she has negative private information about the realization of the asset at date 2.

It follows that the investor benefits from information acquisition *if and only if* she has negative private information (Case 1). The probability of this is $a \cdot l$. She benefits by receiving $p(o, \tilde{a} = 0)$ from selling the asset, instead of the getting $(1 - \sigma)l$ in expectation from holding onto the asset. The gains from acquiring information are thus given by $a(p(o, \tilde{a} = 0) - (1 - \sigma)l) - k_I a$. Simplifying gives:

$$u(a, \tilde{a} = 0) - u(0, \tilde{a} = 0) = a \cdot [\sigma l + (1 - \sigma)(1 - o)(1 - l)\gamma] - k_I a. \quad (2.32)$$

The marginal benefits from a unit of information are hence

$$\frac{\partial u(a, \tilde{a} = 0)}{\partial a} = \sigma l + (1 - \sigma)(1 - o)(1 - l)\gamma - k_I, \quad (2.33)$$

and are constant. It follows that an equilibrium with no information acquisition can only arise when $\frac{\partial u(a, \tilde{a}=0)}{\partial a} \leq 0$. Rearranging, this gives a critical opacity level \bar{o} :

$$\bar{o} = 1 - \frac{k_I - \sigma l}{(1 - \sigma)(1 - l)\gamma}, \quad (2.34)$$

such that for $o \geq \bar{o}$ no information is acquired. By contrast, for $o < \bar{o}$ investors always have incentives to acquire information when the market believes that no information has been acquired. In this case there is no equilibrium without information; in any equilibrium there hence has to be some level of information acquisition.

2.E.3 Alternative Mechanism

There exists a second, independent, reason for why opaque assets may lower information acquisition. It arises when the informational gain from a given amount of information depends on opacity. A unit of information is conceivably less informative if an asset is very opaque as there will then be large uncertainty even after the unit has been acquired. To demonstrate, consider a situation where an asset is valuable to an agent only if it meets a criterion in every state of the world. For instance, agents may have a subsistence requirement \bar{c} ; reaching this level of consumption gives a utility of one, if it is not reached, utility will be zero. Suppose an agent can acquire information about an asset (=project) before deciding whether to undertake it (the alternative to investment being to store funds to meet subsistence requirements). Clearly, the agent will only choose the asset if the subsistence requirement is fulfilled in every state. This requires the agent to investigate all states. Suppose that there are a discrete number o of opaque states and that in all transparent states it is known that the asset pays at least \bar{c} .²⁴ Let the probability of a pay-off in an individual state meeting the subsistence requirement be $q \in (0, 1)$ and let payoffs be independent across states. The likelihood of all states meeting the criterion is then q^o . The expected benefit from acquiring full information is given by $q^o \bar{c} \cdot 1 - k_I o$, where k_I is the per-state cost of information acquisition. Dividing by o yields a benefit $\frac{q^o}{o} \bar{c} - k_I$ of acquiring a unit of information. This value of information is decreasing in opacity for two reasons: First, for higher opacity o , the likelihood that the asset will eventually meet the criteria is lower (q^o is lower). Second, for higher o more states have to be inspected, hence the gain per state is lower.

²⁴If the latter is not the case, the asset is known to be worthless and hence there is also no incentive to acquire information about it.

2.F Empirical Analysis

Table 2.1: Summary statistics

Variable	Description	Time Window	Mean	Std. Dev.	Min.	Max.
Spread	Average spread	Oct'13-Dec'13	0.025	0.03	0.01	0.241
Market capitalization	Market capitalization (log)	Oct'13	14.656	1.494	10.67	19.876
Share price	Share price	Oct'13	41.339	40.054	5.020	672.78
Volume	Avg monthly trading volume scaled by market cap	Oct'11-Sep'13	0.116	0.212	0.002	3.83
St.Dev. Volume	St.Dev. of daily returns	Oct'11-Sep'13	0.025	0.032	0.008	0.622
Volatility	St.Dev. of scaled monthly trading volume	Oct'11-Sep'13	0.538	1.153	0.008	19.585
Number of analysts	Average number of analyst forecasts	Oct'11-Sep'13	10.994	7.229	2	53.167
Dispersion	Average standard deviation of analyst forecasts	Oct'11-Sep'13	0.117	0.217	0.007	1.651

Table 2.2: Control factors in the determination of bid-ask spreads

This table summarizes results from an ordinary least squares regression where the bid-ask spread is the dependent variable. We report coefficients for the independent variables logged market capitalization, share price, average trading volume scaled by market capitalization, the standard deviation of past daily returns, the standard deviation of scaled past trading volume, and the number of analysts giving estimates.

Coefficient	Spread
Market capitalization	-0.00961*** (0.000605)
Share price	0.000417*** (0.000026)
Volume	-0.00960 (0.00645)
St.Dev. volume	-0.00161 (0.00122)
Volatility	0.11520*** (0.04058)
Number of analysts	-0.000263*** (0.00008)
Constant	0.15023*** (0.00874)
Observations	2,067
R-squared	0.344

Robust standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

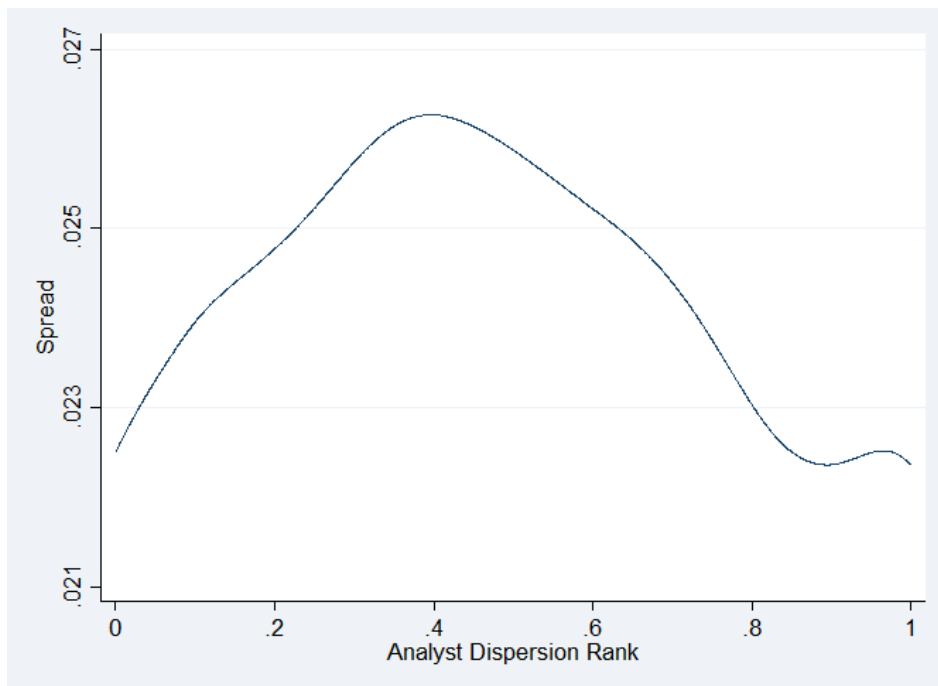


Figure 2.8: Lowess Regression for Bid-Ask Spread

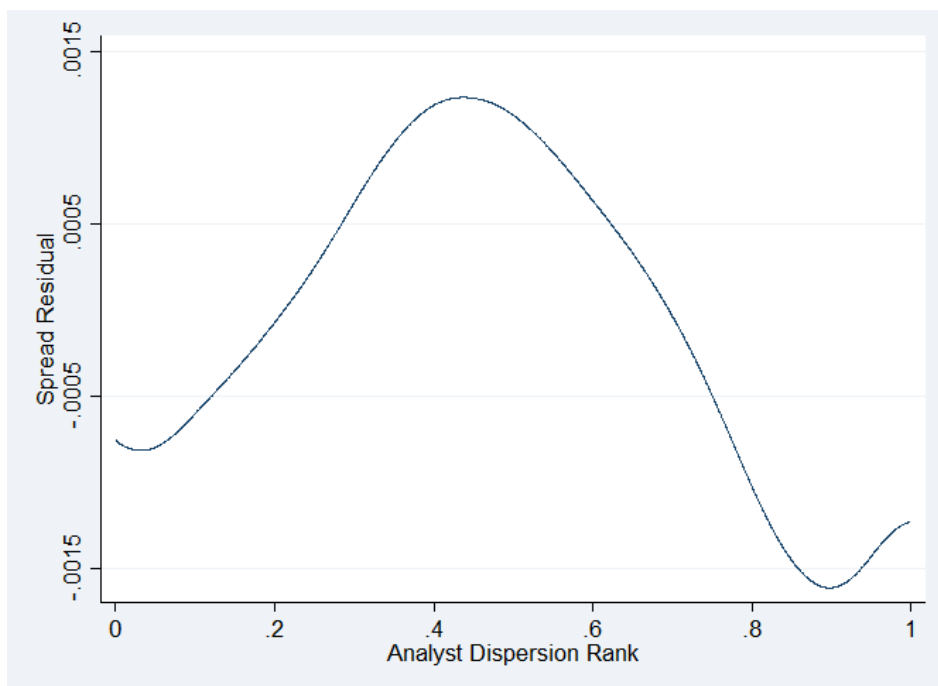


Figure 2.9: Lowess Regression of Bid-Ask Spread Residual

Chapter 3

Loan Sales and Screening with Two-Dimensional Borrower Types

Abstract

We consider a model of lending with subsequent loan sale opportunities. Market participants observe a public signal about the creditworthiness of each borrower. Lenders additionally have the opportunity to privately screen potential borrowers, at a cost. The model rationalizes empirically documented discontinuities in lending and default rates around a FICO credit rating score of 620, while providing a foundation for the endogenous emergence of a cutoff *rule-of-thumb*. We show that loan sale opportunities have a positive impact on borrowers' access to credit contingent on screening revealing positive information whenever the public information about a borrower's type is relatively bad. At the same time, average borrower quality for intermediate borrower types decreases as gains from trade via loan sales increase the relative profitability of loans to unscreened borrowers compared to loans to screened borrowers which imply significant risk retention. Loan sale opportunities can lead to adverse effects on borrower welfare while strictly increasing lender profitability.

3.1 Introduction

In the wake of the recent financial crisis, policymakers and researchers alike have increasingly focused on the role of lenders' incentives to screen potential borrowers. The analysis centers on potential incentives to *originate-to-sell*: By selling securitized loans

I am indebted to my supervisors Ernst-Ludwig von Thadden and Volker Nocke for their support and guidance. I thank seminar participants in Mannheim and in particular Raphaël Levy, Christian Michel, Nicolas Schutz, Jan-Peter Siedlarek and Emanuele Tarantino for helpful comments.

to investors, lenders were not liable when borrowers defaulted and thus may have abstained from screening borrowers before the lending decision or even deliberately included bad risks in the securitized pools.

Perceiving this issue to be at the core of the recent crisis, policymakers have acted. The European Parliament has adopted a proposal requiring originators to retain at least 5% of securitized portfolios – The hope is that the resulting risk retention and induced *skin-in-the-game* leads to better screening practices. However, the need for this type of regulation is not undisputed. For example, [Gorton \(2010\)](#) argues that concerns about adverse selection accounted for the collapse of markets for asset backed securities.

This article contributes to the debate by considering a tractable model of screening, lending and subsequent loan sales aimed at a better understanding of how screening practices were affected by securitization opportunities. We consider the following framework: A prospective borrower approaches a lender who has to decide whether to approve the loan or not. The lending decision can be based on two types of information. There is a public measure of creditworthiness, such as the FICO score, and a private signal which may be obtained by the lender, at a cost. Following the screening and lending decision, the lender has the opportunity to sell the loan to investors. For this, he decides which fraction of the loan to retain and which to sell.²⁵ Investors observe the public measure of creditworthiness and the risk retention by the lender and use this to form beliefs about the value of the offered loan. Retention thus serves as a signalling device – different levels of retention are associated with different beliefs about the lender’s private information. This signalling device is essential for sustaining equilibria with positive realization of gains from trade as the lender needs to be precluded from selectively using her private information.

Our main findings are as follows. The outcome of both the screening and lending and the loan sale stage depend on the ex-ante value of the lending transaction, that is, on the publicly observable information about the borrower. For low ex-ante values, screening occurs and loans are approved conditional on screening revealing positive information. At the same time, incentives to ‘originate-to-sell’ negative NPV-loans need to be prevented, which implies significant risk-retention by lenders in the loan sale stage. Borrowers with high ex-ante creditworthiness are approved, while the screening decision and degree of loan sales may take place at varying intensity.²⁶

²⁵We thus abstract from security design considerations and model risk retention as a vertical slice. By incorporating security design considerations, quantitative predictions would differ while the qualitative predictions should carry over, see Section 3.7.

²⁶If these borrowers are screened, they obtain a loan irrespective of the screening outcome.

Comparing a scenario in which loan sales are infeasible with the full model including loan sales, we show that the advent of securitization and thus introduction of loan sales has two effects on the screening and lending choice of lenders. In the region of the publicly observable creditworthiness such that borrowers are screened, loan sales are strictly positive, which increases lender profitability. This induces the lender to screen for lower levels of ex-ante creditworthiness; borrowers whose screening yields a positive signal represent a positive NPV lending opportunity and obtain a loan where they could not absent loan sale opportunities. This increases both lender and borrower welfare.

Conversely, the threshold above which all borrowers obtain a loan is reduced such that the average loan quality actually decreases for intermediate FICO scores. The reasoning is as follows. While equilibria with screening and partial lending, as well as equilibria without screening and full lending are sustainable, no-screening equilibria lead to a larger realization of gains from trade as private information is not present on the equilibrium path. Thus, relative profitability of no-screening equilibria for the lender is higher than for screening equilibria with partial lending. Bad borrowers are thus no longer screened out whereas they would be if loan sales were not feasible. This implies that, while lender profitability increases, loan sales can lead to adverse effects on borrower welfare if such borrowers are ex-post better off not obtaining a loan. In particular, this leads to an ambiguous overall welfare effect of the introduction of loan sales. Finally, at the margin between the screening and no-screening regions, there is an implied discontinuity in lending rates, average borrower quality and risk retention.

Empirically, discontinuities in lending and default rates around a FICO score of 620 are well-documented. Our model shows that this can be rationalized without imposing some form of irrationality either on the investors' or on the lenders' side – lending rates discontinuously increase at the screening cutoff, while average per-loan profitability decreases due to lack of screening. Average profitability in turn can be interpreted as average borrower quality, which is inverse to default rates. We show that by accounting for the interplay between screening incentives and subsequent loan sales, the discontinuities in lending and default rates arise endogenously in a model with rational agents. The model furthermore predicts that they are associated with a discontinuity in loan sale rates.

The model thus rationalizes the emergence of an underwriting *rule-of-thumb*.²⁷ To assess whether lender incentives are properly aligned, the ex-post risk retention needs to be

²⁷See [Keys, Mukherjee, Seru, and Vig \(2010\)](#), [Keys, Seru, and Vig \(2012\)](#) and [Bubb and Kaufman \(2014\)](#).

carefully assessed: The model predicts that risk-retention should be partial and in fact increasing in ex-ante creditworthiness for low FICO scores. In contrast, no risk retention is necessary for sufficiently high FICO scores. In determining the ex-post retention, it is important to distinguish between loan sales and securitization – [Acharya, Schnabl, and Suarez \(2013\)](#), among others, show that securitization often occurs without risk transfer. Lenders securitize loans via special purpose vehicles but remain liable for the risk of default. While regulatory arbitrage is typically cited as a driving force for this type of securitization, we provide an additional motif: signalling via risk-retention. Taking into account securitization without risk transfer, our results suggest that the prevailing underwriting standards are not per se indicative of misaligned lender incentives in the sense that lenders exploit irrational investors. However, as outlined above, the overall effect on loan quality and welfare may nonetheless be negative when contrasting the prevailing practice with a world absent loan sales.

The remainder of the paper is structured as follows: Section 3.1.1 discusses the institutional setting and discusses the related literature. Section 3.2 presents the model framework. Section 3.3 derives the equilibrium in a restricted model where loan sales are infeasible, before Section 3.4 derives all sustainable equilibria of the full model. Section 3.5 addresses the equilibrium multiplicity before the welfare effect of loan sales is assessed in Section 3.6. Section 3.7 discusses the robustness of the framework and discusses potential extensions. Section 3.8 concludes.

3.1.1 Institutional Setting and Related Literature

Institutional Setting Securitization of mortgage-backed loans generally proceeds in the following form. First, a lender who is approached by a borrower decides on the extent of the *review* the prospective borrower faces. Information gleaned from the review process helps the lender to form an assessment of the borrowers' creditworthiness. In determining the extent of the review process, automated underwriting systems play a crucial role. These systems use information about the borrower, such as the FICO credit rating score, loan-to-value ratio and debt-to-income-ratio to compute a recommendation whether to approve the loan or begin a manual underwriting process involving more intense scrutiny. Following the final lending decision, lenders may sell securitized loans via either government sponsored enterprises (GSEs), private-label securitizers, or instead choose to retain the loans on their balance sheet.

In the mid-1990s, the GSEs adopted guidelines for underwriting credit loans which explicitly recommended differential screening levels based on the FICO score of an individual. The FICO score is a backward-looking rating of an individual's creditworthiness. It is based on an individual's payment history, amounts owed, length of credit history, the types of credit used and recently opened credit accounts (new credit).²⁸ In securitization, particularly of mortgage-backed loans, the FICO score is instrumental in determining a prospective borrower's likelihood of default. This information is available to lenders at the initial lending stage and is reported to investors (in terms of its distribution for loan pools) during subsequently occurring securitization. For example, Freddie Mac determined in its industry letter (1995) that for 1-unit single-family dwellings, borrowers with a FICO score above 660 faced the lowest underwriting standards. By contrast, borrowers with a FICO credit rating between 620 and 660 should consider all aspects of the borrower's credit history, while applicants with a FICO score below 620 should undergo a particularly detailed review – "Unless there are extenuating circumstances, a credit score in this range should be viewed as strong indication that the borrower does not show sufficient willingness to repay as needed."²⁹ At this point, GSEs as guarantors of a significant amount of outstanding mortgages had the largest available dataset on past performance of mortgage-backed loans. It is hence unsurprising that the established cutoffs found their way into the automated underwriting systems (see [Bubb and Kaufman \(2014\)](#)). In particular, lenders were contractually obligated to follow the GSEs guidelines when selling their loans via GSEs. Nonetheless, the information gathered from the review process is not necessarily fully verifiable, resulting in private information for the lender which needs to be taken into account when assessing subsequent loan sales.

Differential, and in particular discontinuous, screening intensities manifest themselves in discontinuities in default rates. The main focus of empirical analyses which aim to assess whether lenders' screening incentives were misaligned prior to the financial crisis is on eliciting the underlying cause for observed discontinuities in default rates around a FICO score of 620. Several articles have documented that at a FICO score of 620, there is a pronounced discontinuity in default rates – observably worse loans to borrowers with a FICO score of 619- exhibit lower default rates than observably better loans to borrowers with a FICO score of 620+. This discontinuity is rationalized with increased screening incentives for loans in the 619- range, resulting in screening out 'bad' borrowers in that region, while borrowers with a FICO of 620+ face less intense

²⁸Source: www.myfico.com/crediteducation/whatsinyourscore.aspx

²⁹See Freddie [Mac \(1995\)](#).

screening.

Empirical Analyses of Rules-of-Thumb [Keys, Mukherjee, Seru, and Vig \(2010\)](#) and [Keys, Seru, and Vig \(2012\)](#) attribute the discontinuity to a rule-of-thumb induced by the prevalent GSE recommendations in the securitization stage. They argue that both the GSEs themselves and other investors such as hedge funds used the cutoff of 620, which results in a vast and discontinuous increase in demand for mortgage-backed securities (MBS) when the underlying loan pool consists of borrowers with a FICO score of at least 620. Adherence to such a threshold is further corroborated by evidence that rating agencies employed similar cutoffs when rating MBS, see [Temkin, Johnson, and Levy \(2002\)](#). Analyzing a vast sample of securitized loans, they document that at a FICO score of 620, the number of securitized loans jumps by 80%, while it is also associated with a 20% increase in defaults (see [Keys, Mukherjee, Seru, and Vig \(2010\)](#)). At the same time, the time-to-securitize is significantly lower for loans composed of FICO 620+ loans, see [Keys, Seru, and Vig \(2012\)](#). The authors interpret the difference in the number of securitized loans and the time-to-securitize as evidence for an exogenous shift in demand around a FICO of 620. Using this exogenous variation, the discontinuity in default rates can easily be rationalized by differential screening incentives – as loans with a FICO of 620+ are less likely to be retained, an originate-to-sell mentality with a lack of screening of borrowers can be sustained. By contrast, as loans to borrowers with a FICO score of 619- are more likely to remain on the balance sheet, screening incentives are stronger. At the threshold, this leads to the observed discontinuity in default rates despite underlying observables being smoothly distributed.

In contrast, [Bubb and Kaufman \(2014\)](#) argue that the rule-of-thumb is relevant already at the lending stage. Exploiting a dataset comprising both securitized and unsecuritized loans, they document that the discontinuities in the securitized number of loans and default rates can be confirmed. However, they are due to underlying discontinuities in lending rates – the number of mortgage loans already exhibits a significant discontinuity at a FICO score of 620; this discontinuity prevails for portfolio (non-securitized) loans as well as both types of securitized (private-label and GSE) loans. In particular, the securitization rate is nearly constant at 95% across all observed FICO scores. The lack of a discontinuity in the FICO score thus calls into question the validity of using only the number of securitized loans as proxy for ease of securitization (as done in [Keys, Mukherjee, Seru, and Vig \(2010\)](#)). [Bubb and Kaufman \(2014\)](#) sketch a theoretical framework which establishes that if differential screening results in private information

for the lender, risk retention should be high whenever screening occurs to properly align incentives, while this is not necessary whenever private information is a non-issue. The near-constant securitization rate is hence interpreted as evidence that lenders used the cutoff rules already in their lending decision, while the differential screening does not result in private information as otherwise risk retention should be larger. However, we argue that the lack of securitization rate discontinuities is not indicative of a lack of risk retention per se – as [Acharya, Schnabl, and Suarez \(2013\)](#), among others, have documented, securitization often occurs without risk-transfer.

We extend the theoretical analysis in [Bubb and Kaufman \(2014\)](#) by explicitly characterizing the full range of sustainable equilibria in a game where lender screening results in private information and where loan sales lead to the realization of gains from trade.³⁰ Furthermore, our model allows the lender to selectively use her private information when trading with the investor – this channel is not present in [Bubb and Kaufman \(2014\)](#). We confirm the prediction that, in equilibrium, screening requires sizeable retentions to preclude incentives to *originate-to-sell* negative NPV loans whenever screening results in private information. By itself, the result that gains from trade can be partially realized is not innocuous: [Dang \(2008\)](#) shows that bargaining with endogenous information may lead to nonexistence of equilibria with trade. The key reason why positive levels of trade can be sustained in our framework is that the trade decision is not zero/one; instead, partial retention serves as a signalling device by affecting the seller’s (lender’s) residual payoff.

Theoretical Contributions on Screening and Loan Sales The interaction between potential screening or effort and the possibility of loan sales has been analyzed in several theoretical articles. [Rajan, Seru, and Vig \(2010\)](#) present a simple framework where borrowers have binary soft and hard information types. Banks can engage in costly screening and in particular can condition the screening decision on the publicly observable hard information and the fraction of the loan which is sold subsequently – this fraction is an exogenous parameter of their model. They show that soft information is acquired whenever the hard information signal is low and the level of securitization is not too high. [Bester, Gehrig, Stenbacka, et al. \(2012\)](#) aim at determining the effects of loan sales on costly screening intensity in the lending stage and in particular focus on variations in how screening intensity affects type-I/type-II-errors respectively. They

³⁰Gains from trade are not present in [Bubb and Kaufman \(2014\)](#), which significantly simplifies the analysis and in particular implies that screening thresholds are unaffected by loan sale opportunities.

find that the impact of loan sales on screening intensity depends on the specifics of the screening technology: Loan sales soften screening whenever the benefit of screening is mainly to avoid erroneously classifying bad loans as good. Conversely, if the main benefit of higher screening intensity is to avoid misclassifying good loans as bad, loan sales induce higher screening intensity.

Fender and Mitchell (2009) and subsequent work by Kiff and Kisser (2014) assess the impact of different types of forced retention in the loan sale stage on screening prior to the lending stage in a framework which incorporates systemic risk (proxying for the economic outlook). It is established that equity tranche retention best incentivizes loan screening but at the same time has the highest costs in terms of capital requirements. This generally leads to the issuer preferring mezzanine retention (when it can choose freely) as it leads to higher profits. While we consider a fixed form of retention in the form of a *vertical slice*, we focus on the differential screening and loan sale intensity given the heterogeneity in prospective borrowers along a publicly verifiable dimension such as the FICO score.

Chemla and Hennessy (2014) consider a framework with a moral hazard problem with respect to unobservable improvement of asset quality and determine securitization (loan sale) levels and the impact of regulation. Moreover, during securitization, originators can be either transparent or opaque; in case of transparency, some market participants may exert costly effort to learn the originator's type (which is binary, as is the state of the world). The authors fully characterize security design, trading patterns and disclosure choices which allows to solve for the optimal effort level. They find that effort levels are strictly below a framework without asymmetric information and establish that the optimal regulation consists of mandating opacity and zero retention by the originator whenever effort can not be induced. Inducing separating securitization by contrast necessarily mandates junior retention by both types, whereas inducing pooling securitization mandates zero retention in the bad state. However, effort-inducing regulation is shown to not necessarily be socially desirable.

Finally, Parlour and Plantin (2008) consider a bank's incentives to monitor a firm it has lent to. In their setup, monitoring has two effects: It decreases the private benefit of shirking for the monitored firm (and thus alleviates a moral hazard problem) and produces private information about its future payoff. In a secondary stage, the bank may sell loans to investors. The selling decision may be due to either a liquidity shock or negative private information. They provide conditions under which this secondary loan market is active, which may only occur whenever banks are relatively more likely

to sell loans due to liquidity shocks as compared to private information, and contrast the secondary loan market activity with the efficient level, showing that liquidity is excessive for loans to highly rated firms and too low for loans to risky firms. While we are similarly concerned with the effects of potential private information acquisition on secondary market trade, we allow the lender to signal the quality of its loans via risk retention.

3.2 Model Setup

We consider a simple two-stage game comprising screening at the lending stage and subsequent loan sales. There is a single lender and a single investor who represents a competitive market. The lender is approached by a single borrower. The borrower type may vary along a publicly observable dimension (e.g. the FICO score) which is costlessly observable by all agents in the economy. Furthermore, it may vary along an unverifiable dimension which can be elicited via screening. In terms of interpretation, consider the following: While the FICO score yields an estimate of a prospective borrower's creditworthiness, it is by construction a backward-looking score. The rating depends on *past* behavior in the use of credit cards and other forms of borrower credit. However, other factors such as current employment and salary or the current family situation are of critical importance for forecasts of whether a loan can be repaid and hence for the profitability of the lending transaction. While these factors can be considered and analyzed through detailed processing by loan officers, the obtained information is considerably subjective and generally hard to credibly transmit to prospective investors who wish to acquire securitized loan pools.

Observing the prospective borrower's public information type, the lender decides whether to screen the borrower or not. Screening is costly and yields a signal about the borrower's soft information type, which we assume to be binary. Contingent on the available information, the lender decides whether to lend to the borrower or not. In case lending has taken place, the lender may subsequently sell parts of the loan to the investor. For this, he chooses the fraction r of the loan which is retained and offers the remaining $(1 - r)$ to the investor. The investor observes the public information about the borrower and the retention level r and uses this to infer the value of the offered loan. By assumption, the market is competitive and the investor pays this amount in

full to the lender.³¹ Formally, the model is described as follows.

Time Time is discrete and covers 3 periods, $t = 0, 1, 2$. The screening and lending decision occur at $t = 0$. The subsequent loan sale stage covers periods $t = 1$ and $t = 2$. At $t = 1$, the lender and investor may trade (a fraction) of the loan. At $t = 2$, loans mature and the profits from the lending transaction are (verifiably) realized.

Borrower Types and Profitability of the Lending Transaction At $t = 0$, a prospective borrower (“she”) arrives at a lender. For now, we treat borrowers as non-strategic: They always ask for a loan and accept it whenever they are approved. We later on endogenize borrower behavior when assessing welfare implications of loan sale opportunities. Each borrower’s type θ is two-dimensional, $\theta = (\theta^F, \theta^S)$. θ^F is public information to all agents in the economy. θ^S is assumed to be binary, $\theta^S \in \Theta^S \equiv \{\underline{\theta}^S, \bar{\theta}^S\}$. Each θ^S -type is equally likely for a given θ^F .

Given $\theta = (\theta^F, \theta^S)$, the *profitability* π of a lending transaction with a borrower of type θ is given by

$$\pi(\theta) = \pi(\theta^F, \theta^S) = \theta^F - \Delta + 2\Delta I_{\theta^S = \bar{\theta}^S} = \begin{cases} \theta^F + \Delta & \text{if } \theta^S = \bar{\theta}^S \\ \theta^F - \Delta & \text{if } \theta^S = \underline{\theta}^S \end{cases}, \quad (3.1)$$

where θ^F represents the value of a lending transaction absent screening and Δ the increase (decrease) in profitability of lending to a $(\theta^S = \bar{\theta}^S)$ -type (a $(\theta^S = \underline{\theta}^S)$ -type). By assumption, for each θ^F , half the prospective borrowers are of type $(\theta^S = \underline{\theta}^S)$ and lending to them yields a payoff of $\theta^F - \Delta$. Conversely, half are of type $(\theta^S = \bar{\theta}^S)$ and lending to them yields $\theta^F + \Delta$. If screening does not take place, we hence have

$$E[\pi(\theta^F, \theta^S) | \theta^F] = \frac{1}{2} (\theta^F - \Delta) + \frac{1}{2} (\theta^F + \Delta) = \theta^F.$$

To cover all possible cases, we make the following assumption.

Assumption 3.1 *The public type θ^F is distributed on $(-\infty, \infty)$ according to some cdf $F(\cdot)$ with strictly positive density $f(\cdot)$.*

³¹This assumption is easily microfounded, e.g. by two prospective investors engaging in Bertrand price competition.

Assumption 3.1 guarantees that θ^F may take any realization on $(-\infty, \infty)$, ensuring that all cases are covered: Some borrowers (low θ^F) will never obtain a loan as lending to them is unprofitable even contingent on screening revealing positive information. Similarly, some borrowers (high θ^F) are profitable to lend to even if screening reveals negative information.

Lender The lender L (“he”) cares about consumption at $t = 1$ and $t = 2$. Furthermore, he has a strict preference for consumption at $t = 1$ which reflects e.g. further lending opportunities for which cash is needed. His expected utility is given by

$$U^L = C_1^L + \delta \cdot C_2^L - c_s \cdot \mathbb{I}_{s=1},$$

where C_i^t denotes the consumption of agent i in period t and $\delta \in (0, 1)$ parametrizes the preference for cash at $t = 1$ (the lower δ , the higher the preference for consumption at $t = 1$). At $t = 0$, the lender has to decide whether to screen ($s = 1$) or not ($s = 0$); if screening occurs, his overall utility is lowered by c_s . The lender holds no endowments besides the rights to the profits of the lending transaction if it occurred at $t = 0$. We assume that screening is not too costly.

Assumption 3.2 *The screening cost is strictly bounded from above by*

$$c_s < \frac{1}{2} \delta \Delta.$$

Assumption 3.2 ensures that even in the absence of loan sale opportunities, screening is sufficiently cheap such that it may be sustained in equilibrium.³²

Investor The investor I cares about consumption at $t = 1$ and $t = 2$. I is indifferent between consuming at these periods, i.e.

$$U^I = C_1^I + C_2^I.$$

³²When screening and lending to $\theta^S = \bar{\theta}^S$ -types only, half of the applicants obtain a loan after screening reveals positive information. The value of the lending transaction is $\theta^F + \Delta$, that is, compared to not lending, the gain in value of an approved lending transaction is Δ , which is discounted by δ in the absence of loan sales as loans are fully retained.

The investor is assumed to be deep-pocketed in the sense that the investor's funds are sufficient for acquiring any amount of securitized assets.

Loan Sales From the setup, it is straightforward that gains from trade may be realized by letting lender and investor trade at $t = 1$. We consider the following trading protocol: If the lender owns a loan to a θ^F -type, he decides on a risk retention level $r \in [0, 1]$ and offers the remaining fraction $(1 - r)$ to the investor. The investor observes θ^F and r and uses this to update the beliefs about the θ^S -type of the borrower. I then pays this expected value in full.³³

3.3 Equilibrium without Loan Sales

Suppose first that loan sales are not feasible and hence that full retention $r(\theta^F, \theta^S) = 1 \Leftrightarrow (1 - r(\theta^F, \theta^S)) = 0$ is required for all $\theta = (\theta^F, \theta^S)$. Consider a given θ^F -type who approaches the lender. In that case, the lender has three possible strategies:

- (a) Do not lend. This directly implies not to screen to save the cost c_s . The lender's expected utility is then given by

$$E[U^L(\theta^F)] = 0.$$

- (b) Do not screen and approve all borrowers. The lender's expected utility is then given by

$$E[U^L(\theta^F)] = \delta\theta^F.$$

- (c) Screen the borrowers and approve only $\bar{\theta}^S$ -types.³⁴ The lender's expected utility is then given by

$$E[U^L(\theta^F)] = \frac{1}{2}\delta [\theta^F + \Delta] - c_s.$$

³³This implicitly assumes a perfectly competitive market for investing in securitized loans. A micro-foundation for this assumption is given by considering two (or more) investors engaging in Bertrand competition in a limit order market. As long as investors do not hold the full bargaining power, results are qualitatively unchanged.

³⁴Screening and lending to all borrowers is strictly dominated by not screening and lending to all borrowers as it saves c_s .

Let cutoffs κ_a and κ_f be characterized as follows:

$$\kappa_a = \frac{2c_s}{\delta} - \Delta \stackrel{\text{Ass. 3.2}}{<} 0$$

$$\kappa_f = \Delta - \frac{2c_s}{\delta} \stackrel{\text{Ass. 3.2}}{>} 0.$$

Given Assumption 3.1.(ii), it follows immediately from comparing the expected utilities of each action that the optimal screening and lending behavior by L is characterized as follows.

Proposition 3.1 (Equilibrium without Loan Sales) *Let κ_a and κ_f be uniquely defined as above with $\kappa_a < 0 < \kappa_f$. The lender's screening and lending decision depends on the public type θ^F and is characterized as follows:*

- (i) *If $\theta^F < \kappa_a$, the lender is not screened and lending does not take place. For $\theta^F = \kappa_a$, the lender is indifferent between screening the borrower and approving $\theta^S = \bar{\theta}^S$ -types only, and not approving the borrower without screening. His expected profit is 0.*
- (ii) *If $\theta^F \in (\kappa_a, \kappa_f)$, the lender screens the borrower and approves $\theta^S = \bar{\theta}^S$ -types only. His expected profit is $\frac{1}{2}\delta(\theta^F + \Delta) - c_s > 0$. For $\theta^F = \kappa_f$, the lender is indifferent between screening the borrower and approving $\theta^S = \bar{\theta}^S$ -types only, and approving the borrower without screening.*
- (iii) *If $\theta^F > \kappa_f$, the lender approves the borrower without screening. His expected profit is $\delta\theta^F$.*

We next assess how the equilibrium is affected whenever loan sale opportunities exist. In particular, we aim to assess whether the threshold levels κ_a for approval of some borrowers and κ_f for full lending to all borrowers are affected by the advent of securitization, that is, by loan sales.

3.4 Equilibrium with Loan Sales

Consider the full model with loan sale opportunities. The initial focus of the analysis is to characterize the maximal range of sustainable equilibrium behaviors comprising

the screening and lending decision along with the subsequent retention choice during the loan sale stage. To facilitate this, we consider the *most pessimistic* off-path beliefs. Formally, let $\mu(\theta^F, r)$ denote the investor's belief that the θ^S -type underlying the offered loan is $\bar{\theta}^S$ (for a given θ^F and observed retention level r). Given $\mu(\theta^F, r)$, the investor values the offered security characterized by the implied retention r

$$(1 - r) \cdot [\theta^F - \Delta + 2\mu(\theta^F, r)\Delta] = (1 - r) \cdot [\theta^F + \Delta(2\mu(\theta^F, r) - 1)],$$

where $(1 - r)$ is the offered fraction of the loan (i.e. what is not retained by the lender) and $\theta^F + \Delta(2\mu(\theta^F, r) - 1)$ is the expected value of the loan. The most pessimistic off-path beliefs imply that for any pair (θ^F, r) such that $r(\theta^F, \theta^S)$ is not played with positive probability by the lender for any θ^S , it holds that $\mu(\theta^F, r) = 0$, i.e. that the investor believes this loan to be to a $\theta^S = \underline{\theta}^S$ -type with probability 1.³⁵

We look for sustainability of all possible types of equilibria: Equilibria where neither screening nor lending takes place, equilibria where borrowers are screened and are approved iff screening reveals positive information, equilibria where borrowers are not screened and approved, and equilibria where borrowers are screened but obtain a loan irrespective of the information revealed by screening. Whether a given equilibrium type is sustainable depends crucially on the value of the public information θ^F .

3.4.1 No Screening, no Lending

Suppose first that the lender chooses not to screen and not to lend to any borrowers. This yields an expected payoff of 0. Proposition 3.1 implies that a profitable deviation exists iff the public type θ^F is such that $\theta^F > \kappa_a = \frac{2c_s}{\delta} - \Delta$: At this stage, even if all loans are retained, profits are strictly positive (either by screening out $\theta^S = \underline{\theta}^S$ -types or by approving all borrowers without screening). Conversely, for $\theta^F \leq \kappa_a$, not screening and not lending can be sustained given the most pessimistic off-path beliefs as any non-retained loans would yield a negative price $\theta^F - \Delta < 0$ and absent loan sales, there exists no profitable deviation.

Lemma 3.1 *If loan sales are possible, not screening and not lending can be sustained for the public type θ^F such that $\theta^F \leq \kappa_a$.*

³⁵In the interior of sustainable equilibrium ranges, there will typically less pessimistic beliefs which are also able to sustain the equilibrium. As we are interested in characterizing the maximal range of sustainable equilibria, we assume most pessimistic beliefs off-path throughout the analysis.

3.4.2 Screening and Partial Lending

Suppose next that the lender chooses to screen and lend to $\theta^S = \bar{\theta}^S$ -type borrowers only, that is, after screening has revealed positive information about the prospective borrower. Denote the equilibrium retention level by \bar{r}^* . We consider whether such an equilibrium is sustainable for a given public type θ^F . From here on out, we relegate exact derivations and proofs to the appendix and present the intuition in the main text.

For an equilibrium with screening and lending to $\theta^S = \bar{\theta}^S$ -type borrowers only, there are two constraints which arise on the equilibrium retention rate. On the one hand, the equilibrium candidate needs to be profitable, which implies an upper bound on the retention rate \bar{r}^* – the less is retained, the more gains from trade are realized, which increases profitability.

At the same time, there also is a lower bound on \bar{r}^* . This lower bound arises because incentives for the lender to deviate to not screening borrowers need to be precluded. By deviating, the lender would gain by saving the screening cost c_s and by – in expectation – selling more loans to the investor at an inflated price $\theta^F + \Delta$ instead of the actual value θ^F . However, the lender also loses because the retained fraction $(1 - \bar{r}^*)$ includes loans with a negative value $\theta^F - \Delta$. As such, a sufficiently high equilibrium retention level \bar{r}^* is necessary to preclude this deviation.

Equilibrium existence and constraints on \bar{r}^* are summarized in the following proposition. Note that in this equilibrium type, the lower bound of required retention levels $\bar{r}^*(\theta^F)$ sustainable in equilibrium increases in the public type θ^F : This is because the implied punishment by retaining a given fraction r of the loan decreases due to the increase in value. At the threshold θ^F such that $\theta^F = \kappa_f$ such that an equilibrium with screening and partial lending can be sustained, the required retention $\bar{r}^*(\kappa_f) = 1$ is full – the range of sustainable equilibrium retentions is a singleton.

Lemma 3.2 *When loan sales are possible, an equilibrium where a borrower of public type θ^F is screened and obtains a loan conditional on screening revealing $\theta^S = \bar{\theta}^S$ exists if and only if θ^F is such that $\theta^F \in [\kappa_a^{LS}, \kappa_f]$, where $\kappa_a^{LS} < \kappa_a$ is characterized by (3.3). The equilibrium retention level $\bar{r}^*(\theta^F)$ is constrained from above and below and depends on θ^F according to (3.2).*

Proof. See Appendix 3.A. ■

Remark 3.1 *Availability of loan sales decreases the threshold-level for the public type θ^F such that lending contingent on screening revealing $\theta^S = \bar{\theta}^S$ can be sustained from κ_a to κ_a^{LS} . The gains from trade which are partially realized increase lender profitability for $\theta^F \in [\kappa_a^{LS}, \kappa_a)$, rendering screening and partial lending profitable.*

Figure 3.1 illustrates the sustainable equilibrium \bar{r}^* as a function of the public type θ^F . As discussed above, the lower bound on possible equilibrium retentions (which maximizes gains from trade) is increasing in θ^F and in particular equal to 1 at $\theta^F = \kappa_f$.

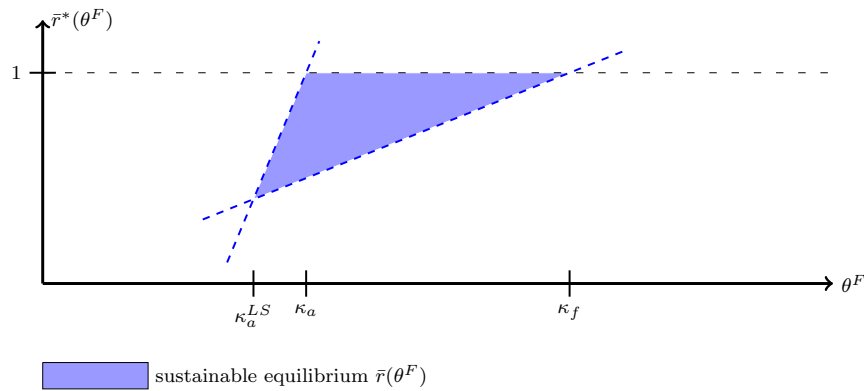


Figure 3.1: Sustainable equilibria with screening & partial lending

3.4.3 Screening and Full Lending

We next check whether screening and full lending can occur on the equilibrium path. First note that it is straightforward that this is sustainable only for public types θ^F such that $\theta^F \geq \Delta$, as profitability of lending to θ^S -types would be violated otherwise. To see this, note that separating loan sales are necessary for this type of equilibrium as otherwise not screening and offering the same retention level would save the screening cost (yielding a profitable deviation). With separating loan sales, however, profitability of lending to $\theta^S = \bar{\theta}^S$ -types is necessary and only given for θ^F sufficiently large such that $\theta^F \geq \Delta$ – otherwise, a profitable deviation would be to simply not lend to borrowers where screening yields negative information about the borrower.

Two constraints arise in that case: First note that loan sales have to be separating in that the equilibrium retention of θ^S -type loans, \underline{r}^* , is different than the retention level of $\bar{\theta}^S$ -type loans, \bar{r}^* . Otherwise, a natural deviation would be to not screen. Second,

this implies that $\underline{r}^* = 0$ as, even given the most pessimistic off-path beliefs, selling all of the $\underline{\theta}^S$ -type loans yields the fair value and thus would increase lender profitability if it were a deviation. Thus, we only need to worry about two types of deviations.

The lender may choose to not screen and mimic the $\bar{\theta}^S$ -type loans by retaining \bar{r}^* . This introduces a lower bound on \bar{r}^* – to make the deviation unattractive, a larger part of the loan needs to be retained so that the implied punishment of losing out on gains from trade is high enough. Note that this lower bound is decreasing in θ^F as a larger ex-ante value implies a larger punishment from deviating given the same level of retention. Finally, the lender may also choose to deviate by not screening and selling all loans at the (less than fair) value $\theta^F - \Delta$. This yields an upper bound on \bar{r}^* – if \bar{r}^* is too high, the lender would prefer to incur the discount on the issued loans as she saves the screening cost and receives more gains from trade. As θ^F increases, the relative attractiveness of the deviation increases as gains from trade are more relevant, and the upper bound on \bar{r}^* becomes tighter.

We show that both constraints can be satisfied for an intermediate range of θ^F -values. Given that both constraints are decreasing in the public type θ^F , the minimum retention required to sustain this equilibrium type is decreasing in θ^F .

Lemma 3.3 *When loan sales are possible, equilibria such that borrowers of a given public type θ^F are screened, but obtain a loan irrespective of θ^S , exist if and only if $\theta^F \in (\underline{\kappa}_{sf}, \bar{\kappa}_{sf}]$ with $\underline{\kappa}_{sf}, \bar{\kappa}_{sf}$ defined by (3.7) and $\bar{\kappa}_{sf} > \underline{\kappa}_{sf} > \kappa_f$.*

In that case, loans to $\theta^S = \underline{\theta}^S$ -types are fully sold, i.e. $\underline{r}^(\theta^F) = 0$, while loans to $\theta^S = \bar{\theta}^S$ -types are partially retained with the equilibrium $\bar{r}^*(\theta^F)$ constrained from above and below according to (3.8).*

Proof. See Appendix 3.B. ■

3.4.4 No Screening and Full Lending

Finally, we check for sustainability of equilibria which involve no screening and approval of all borrower types. It is clear that this is unsustainable for public types θ^F such that $\theta^F \leq 0$ due to lack of profitability (for $\theta^F = 0$, Proposition 3.1 establishes that screening and lending to $\theta^S = \bar{\theta}^S$ -types only is more profitable even if these loans are fully retained). We thus consider θ^F such that $\theta^F > 0$.

When assessing sustainability of equilibria without screening and full lending, it is important to note that most deviations can be precluded by limiting the retention level r^* in equilibrium: With high equilibrium loan sales, deviations necessarily involve retention of loans off-equilibrium and the implicit punishment through lack of realization of gains from trade is sufficiently strong. The one exception which induces a lower bound on the equilibrium retention is the lender deviation involving screening, full lending and retaining the equilibrium r^* for $\underline{\theta}^S$ -type loans while fully retaining $\bar{\theta}^S$ -type loans. In this case, the lender would profit by selling $\underline{\theta}^S$ -type loans above their value, as well as retaining $\bar{\theta}^S$ -type loans instead of selling them below value, while losing – in expectation – from holding on to $\bar{\theta}^S$ -type loans instead of realizing gains from trade, as well as from retaining a fraction $(1 - r^*)$ of $\underline{\theta}^S$ -type loans.

This potential deviation needs to be precluded unless θ^F is sufficiently large, and in particular is rendered unprofitable whenever the equilibrium retention r^* is sufficiently high. For low θ^F , we thus need to assess whether r^* exists such that this lower bound is satisfied, as well as an upper bound on r^* such that deviations to screening and partial lending are precluded – these deviations are unprofitable due to the realization of gains from trade whenever r^* is sufficiently low as the lender would lose out on lending to $\underline{\theta}^S$ -types and transferring the risk to the investor. It turns out that for sufficiently large θ^F and corresponding θ^F , r^* exists such that this is facilitated. In particular, the threshold level κ_f^{LS} such that a no-screening equilibrium can be sustained for $\theta^F \geq \kappa_f^{LS}$ (but not too large, see below) lies below the threshold κ_f in the case without loan sales.

Existence of an equilibrium without screening and full lending may nonetheless fail for some public types θ^F such that $\theta^F > \kappa_f^{LS}$. If θ^F is large, but not too large, the investor could also screen selectively, fully sell $\underline{\theta}^S$ -type loans to realize gains from trade, and retain $\bar{\theta}^S$ -type loans to gain from the value increase by Δ relative to selling these loans at value θ^F . By doing this, the investor gains from fully realizing gains from trade on $\underline{\theta}^S$ -type loans (even given pessimistic off-path beliefs, the lender obtains the fair value for these loan types), but loses from discounting the $\bar{\theta}^S$ -type loans. Sufficiently low retention r^* is able to preclude this deviation as then, realization of gains from trade in equilibrium would be large enough. It turns out that it is – depending on fundamentals – not always possible to simultaneously preclude both types of potential deviations for intermediate values of θ^F . In particular, whenever gains from trade and screening costs c_s are relatively low (δ close to 1, c_s close to 0), equilibria without screening but full lending are unsustainable for an intermediate range of θ^F .

The following Lemma summarizes the constraints on equilibrium $r^*(\theta^F)$ whenever a no-

screening equilibrium exists. Corollary 3.3 then shows that, whenever existence fails for some θ^F above κ_f^{LS} , an equilibrium with screening and full lending necessarily exists.

Lemma 3.4 *If loan sales are feasible, existence of an equilibrium where borrowers are not screened and approved depends on fundamentals and the public type θ^F . Let $\kappa_f^{LS} < \kappa_f$ be characterized by (3.10).*

- (i) *If gains from trade or the screening cost c_s are sufficiently large ($\delta \leq \frac{1}{2} + \frac{2c_s}{\Delta}$, $(2\delta - 1)\Delta \leq 6c_s$, or $\frac{(\Delta - 2c_s)^2}{4} \geq \delta\Delta[(1 - \delta)\Delta + 2c_s]$), existence is guaranteed for all $\theta^F \geq \kappa_f^{LS}$ and constraints on the equilibrium retention rate $r^*(\theta^F)$ are given in Table 3.1.*
- (ii) *Otherwise, there exists an interval $\Theta_{ne}^F \subset [\Delta + \frac{2c_s}{1-\delta}, \frac{\delta\Delta - 2c_s}{1-\delta}]$ such that a no-screening equilibrium fails to exist. For $\theta^F \geq \kappa_f^{LS}$, $\theta^F \notin \Theta_{ne}^F$, restrictions on $r^*(\theta^F)$ are given in Table 3.2.*

Proof. See Appendix 3.C. ■

Corollary 3.1 *For any public type $\theta^F \geq \kappa_f^{LS}$ such that a no-screening equilibrium exists, the equilibrium where borrowers are not screened and approved which involves the lowest equilibrium retention rate $r^*_{min}(\theta^F)$ is characterized by*

$$r^*_{min}(\theta^F) = \begin{cases} 1 - \frac{2c_s}{\delta\Delta - (1-\delta)\theta^F} & \text{if } \theta^F \leq \frac{\delta\Delta - 2c_s}{1-\delta} \\ 0 & \text{if } \theta^F > \frac{\delta\Delta - 2c_s}{1-\delta} \end{cases}.$$

Corollary 3.2 *An equilibrium with no screening and full lending exists for all $\theta^F \in [\kappa_f^{LS}, \kappa_f]$.*

Corollary 3.3 *For any $\theta^F \geq \kappa_f^{LS}$ such that no no-screening equilibrium exists, there necessarily exists an equilibrium where borrowers are screened and obtain a loan irrespective of the screening outcome.*

Proof. See Appendix 3.D. ■

Figure 3.2 depicts the two possible cases and characterizes the range of $r^*(\theta^F)$ sustainable in a no-screening equilibrium with full lending conditional on θ^F . If both gains from trade are small (δ close to 1) and the screening cost is low (c_s close to 0), the

conditions in Proposition 3.4.(i) are violated and existence of a no-screening equilibrium fails for some $\theta^F \geq \kappa_f^{LS}$ (Panel (b)). Otherwise, no-screening equilibria with full lending exist for all $\theta^F \geq \kappa_f^{LS}$ (Panel (a)).

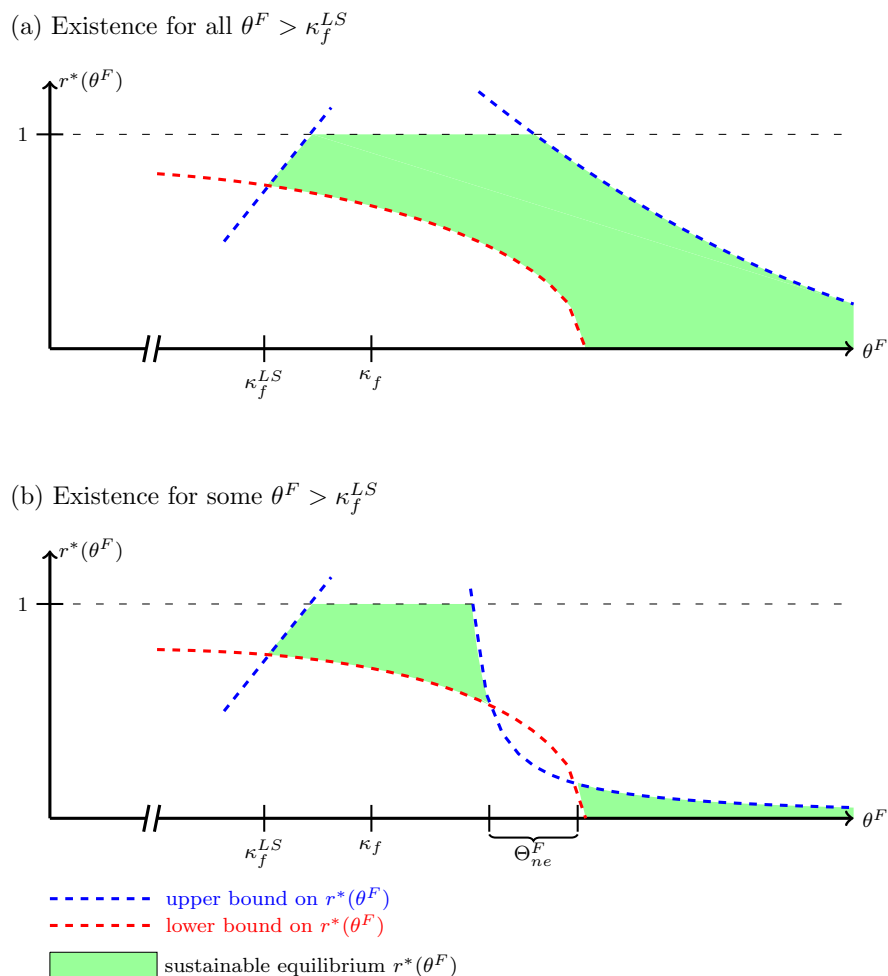


Figure 3.2: Sustainable equilibria with no screening & full lending

3.5 Equilibrium Selection

The multiplicity of sustainable equilibria, both in terms of equilibrium types and in terms of retention rates given an equilibrium type, does not allow for a straightforward welfare analysis without some form of equilibrium selection. In fact, it is straightforward from Lemmas 3.2, 3.3 and 3.4 that there exists an equilibrium of the full game such that welfare unambiguously increases (even if borrowers' utility is explicitly modelled

and taken into account): There exists an equilibrium in which (i) the lending decision is identical to the no loan sale case outlined in Proposition 3.1 and (ii) retention is bounded away from full retention for almost all θ^F , which means gains from trade are realized, increasing lender profitability.³⁶

A natural selection criterion in the present setup is to focus on the lender-preferred equilibria. These maximize welfare as they maximize gains from trade (in fact, borrowers so far are considered as non-strategic, while the lender extracts the full rent from investors). It is straightforward that for a given equilibrium type, where type refers to the screening and approval decision), the equilibrium with the lowest retention maximizes expected lender utility. Moreover, even when borrower behavior is endogenized in Section 3.6, using the lender-preferred equilibrium as a selection criterion is sensible with respect to the application: As lenders are the ones who set and adhere to lending standards and guidelines, they choose them to maximize their profits.

To find the lender-preferred equilibrium, we need to account for the multiplicity in equilibrium types given loan sale opportunities. For the public type θ^F such that $\theta^F \in [\kappa_f^{LS}, \kappa_f)$, screening equilibria with partial lending and no-screening equilibria with full lending co-exist (see Lemmas 3.2 and 3.4). Similarly, for θ^F such that a screening equilibrium with full lending exists, no-screening equilibria with full lending may exist as well (Lemmas 3.3 and 3.4). Finally, while an equilibrium without screening and lending may coexist with an equilibrium with screening and partial lending (for $\theta^F \in [\kappa_a^{LS}, \kappa_a)$), it is straightforward that the equilibrium involving actual lending is weakly preferred as profits are nonnegative.

Focus first on $\theta^F \in [\kappa_f^{LS}, \kappa_f)$. We know that a no-screening equilibrium exists in that region, where the lowest sustainable equilibrium retention rate $r^*_{min}(\theta^F)$ is given by Corollary 3.1:

$$r^*_{min}(\theta^F) = \begin{cases} 1 - \frac{2c_s}{\delta\Delta - (1-\delta)\theta^F} & \text{if } \theta^F \leq \frac{\delta\Delta - 2c_s}{1-\delta} \\ 0 & \text{if } \theta^F > \frac{\delta\Delta - 2c_s}{1-\delta} \end{cases}.$$

Consider next the equilibrium with screening and partial lending. We know that the minimum sustainable retention rate is given by

³⁶The realization of gains from trade in the screening equilibrium with full lending due to $r^*(\theta^F) = 0$ is by itself sufficient to overcome the screening cost c_s and increase profitability.

$$\bar{r}_{min}^*(\theta^F) = \frac{2c_s + \theta^F + \Delta}{(1 - \delta)\theta^F + (1 + \delta)\Delta}.$$

Note that as $\theta^F \rightarrow \kappa_f = \frac{\delta\Delta - 2c_s}{\delta}$, we have that

$$\bar{r}_{min}^*(\theta^F) \xrightarrow{\theta^F \rightarrow \kappa_f} \frac{2c_s + \frac{\delta\Delta - 2c_s}{\delta} + \Delta}{(1 - \delta)\frac{\delta\Delta - 2c_s}{\delta} + (1 + \delta)\Delta} = \frac{2\Delta - \frac{1-\delta}{\delta}2c_s}{2\Delta - \frac{1-\delta}{\delta}2c_s} = 1,$$

that is, the required retention level $\bar{r}_{min}^*(\theta^F)$ approaches full retention as θ^F goes to κ_f .

For $r_{min}^*(\theta^F)$, we have that

$$r_{min}^*(\theta^F) \xrightarrow{\theta^F \rightarrow \kappa_f} \begin{cases} 0 & \text{if } \delta < \frac{1}{2} \\ \frac{(2\delta-1)(\delta\Delta-2c_s)}{(2\delta-1)\Delta+2(1-\delta)c_s} & \text{if } \delta \geq \frac{1}{2} \end{cases},$$

where for $\delta < \frac{1}{2}$ we have that

$$\kappa_f = \frac{\delta\Delta - 2c_s}{\delta} > \frac{\delta\Delta - 2c_s}{1 - \delta} \Rightarrow r_{min}^*(\kappa_f) = 0$$

while for $\delta \geq \frac{1}{2}$ we have that

$$r_{min}^*(\kappa_f) = \frac{(2\delta - 1)(\delta\Delta - 2c_s)}{(2\delta - 1)\Delta + 2(1 - \delta)c_s} \stackrel{\text{Ass. 3.2}}{\in} (0, 1).$$

However, we know from Proposition 3.1 that at $\theta^F = \kappa_f$, a no-screening equilibrium with full retention and a screening equilibrium with partial lending and full retention lead to the same lender expected utility. It follows that at $\theta^F = \kappa_f$, the no-screening equilibrium characterized by $r_{min}^*(\kappa_f)$ yields a strictly higher expected utility to the lender than the screening equilibrium with partial lending characterized by $\bar{r}_{min}^*(\kappa_f) = 1$. Thus, we can conclude that there exists a unique $\tilde{\kappa}_f^{LS} < \kappa_f$ such that for $\theta^F \in [\tilde{\kappa}_f^{LS}, \kappa_f]$, the no-screening equilibrium with full lending and retention level $r^*(\theta^F) = r_{min}^*(\theta^F)$ is strictly preferred to any equilibrium with screening and partial lending (and in particular the screening equilibrium with partial lending and equilibrium retention $\bar{r}^*(\theta^F) = \bar{r}_{min}^*(\theta^F)$). In particular, $\tilde{\kappa}_f^{LS}$ is such that the lender is indifferent between the two most favorable equilibria, i.e. we have that

$$\begin{aligned} & \frac{1}{2} [(1 - \bar{r}_{min}^*(\tilde{\kappa}_f^{LS})) \cdot (\tilde{\kappa}_f^{LS} + \Delta) + \bar{r}_{min}^*(\tilde{\kappa}_f^{LS})\delta(\tilde{\kappa}_f^{LS} + \Delta)] - c_s \\ & = (1 - r_{min}^*(\tilde{\kappa}_f^{LS})) \cdot \tilde{\kappa}_f^{LS} + \delta r_{min}^*(\tilde{\kappa}_f^{LS})\tilde{\kappa}_f^{LS} \end{aligned}$$

where $\bar{r}_{min}^*(\tilde{\kappa}_f^{LS})$ and $r_{min}^*(\tilde{\kappa}_f^{LS})$ are as above.

The final multiplicity in equilibrium types which needs to be considered is the case where a screening equilibrium with full lending exists. Consider two possible cases. If $\delta \leq \frac{1}{2} + \frac{2c_s}{\Delta}$, we know that

$$\frac{\delta\Delta - 2c_s}{1 - \delta} \leq \Delta + \frac{2c_s}{1 - \delta}$$

and thus that whenever an equilibrium with screening and full lending exists ($\theta^F \geq \Delta + \frac{2c_s}{1-\delta}$), there also exists a no-screening equilibrium with retention $r^* = 0$. Thus, for $\delta \leq \frac{1}{2} + \frac{2c_s}{\Delta}$, the lender-preferred equilibrium in the θ^F -range where a screening equilibrium with full lending exists is necessarily the no-screening equilibrium with full loan sales: Gains from trade are maximal and the screening cost is saved.

However, it may be the case that gains from trade are small, i.e. $\delta > \frac{1}{2} + \frac{2c_s}{\Delta}$. In this case, it may be the case that a no-screening equilibrium fails to exist (see Lemma 3.4) or, if the two equilibrium types co-exist, that the equilibrium with screening, full lending and minimal retention is preferred to the equilibrium without screening and minimal retention. Irrespective of the fundamentals, however, the lender-preferred equilibrium involves approving all borrowers. We summarize these observations as follows.

Proposition 3.2 (Lender-Preferred Equilibrium) *In the lender-preferred equilibrium (which maximizes welfare given non-strategic borrowers), there exist threshold levels $\kappa_a^{LS} < \kappa_a$ and $\tilde{\kappa}_f^{LS} < \kappa_f$ such that*

- (i) *If the public type θ^F is such that $\theta^F \in [\kappa_a^{LS}, \tilde{\kappa}_f^{LS})$, borrowers are screened, obtain a loan if and only if screening reveals $\theta^S = \bar{\theta}^S$ and equilibrium retention is characterized by*

$$\bar{r}^*(\theta^F) = \frac{2c_s + \theta^F + \Delta}{(1 - \delta)\theta^F + (1 + \delta)\Delta}.$$

- (ii) *If the public type θ^F is such that $\theta^F \in [\tilde{\kappa}_f^{LS}, \Delta + \frac{2c_s}{1-\delta})$, borrowers are not screened, approved and equilibrium retention is characterized by*

$$r^*(\theta^F) = \begin{cases} 1 - \frac{2c_s}{\delta\Delta - (1-\delta)\theta^F} & \text{if } \theta^F \leq \frac{\delta\Delta - 2c_s}{1-\delta} \\ 0 & \text{if } \theta^F > \frac{\delta\Delta - 2c_s}{1-\delta} \end{cases}.$$

(iii) If the public type θ^F is such that $\theta^F \geq \Delta + \frac{2c_s}{1-\delta}$, all borrowers obtain a loan. Whether screening takes place or not, as well as the subsequent equilibrium retention level, depends on fundamentals δ, Δ and c_s .

Proposition 3.2 is important as it highlights a potentially adverse effect of the advent of securitization: By allowing lenders to realize gains from trade by selling loans to investors, the relative profitability of not screening borrowers and selling a large fraction of the raised loans increases compared to carefully screening borrowers which would imply significant risk-retention to preclude deviations. Crucially, this effect materializes when all agents are fully cognizant of the equilibrium screening and retention rates, and for intermediate values of the public type θ^F – while access to credit for very low θ^F -types, corresponding to low FICO-scores, improves, this occurs only contingent on screening revealing positive information. Screening intensity decreases, however, for intermediate FICO-scores. This is depicted graphically in Figure 3.3.

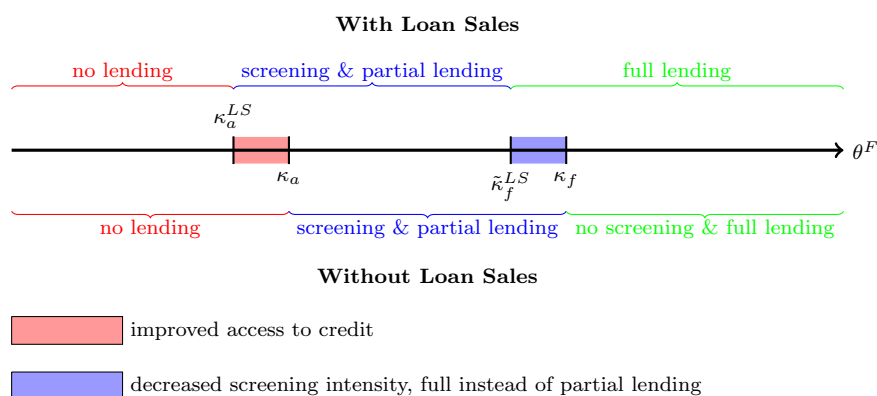


Figure 3.3: Comparison loan sales & no loan sales

To fully assess the welfare implications and to show that this may harm borrower welfare, we subsequently endogenize the so far non-strategic borrower behavior with a simple microfoundation.

3.6 Welfare Implications of Loan Sales

So far, we have not taken into account borrowers' utilities. As lender utility strictly increases with loan sales, one may think that welfare is increased. This section endogenizes borrower behavior by providing a simple microfoundation and shows that

borrower welfare may be negatively affected by the advent of securitization, implying that overall welfare effects are ambiguous.

Consider the following setup. Let borrowers obtain a private benefit B from being approved for a loan. B represents the increase in utility from owning a house in the case of financing of house-ownership via a (mortgage-backed) loan. At the same time, borrowers may face distress costs D if they are unable to repay the loan. D covers both the real financial costs of defaulting, that is the seizing of assets and loss of future access to credit, as well as the psychological cost of declaring bankruptcy. Given that θ^F and θ^S carry information about a borrower's creditworthiness, we can write borrowers' expected utility contingent on obtaining a loan as

$$U^B = B - \eta(\theta^F, \theta^S) \cdot D,$$

where $\eta(\theta^F, \theta^S)$ is the probability of default. It is natural to assume $\frac{\partial \eta(\cdot, \cdot)}{\partial \theta^F} < 0$ – the probability of default decreases in creditworthiness as measured by θ^F . Furthermore, $\eta(\theta^F, \bar{\theta}^S) < \eta(\theta^F, \underline{\theta}^S)$, i.e. $(\theta^S = \bar{\theta}^S)$ -types have a lower probability of default than $(\theta^S = \underline{\theta}^S)$ -types. In this reduced form notation, B implicitly contains the cash flow (initial loan payment and subsequent repayments) of the loan.

Suppose that borrowers are uninformed about θ^S , but only apply if their expected utility conditional on being approved is weakly larger than their outside option, which we normalize to 0.³⁷ Thus, they take into account the strategic lender behavior to infer information about θ^S from the lending decision. We will show that despite the strategic borrower behavior, loan sales can lead to decreases in borrower welfare (and potentially thus decrease overall welfare). To do this, we restrict attention to the case where absent loan sale opportunities, all borrowers who are approved in the equilibrium outlined in Proposition 3.1 ask for a loan, i.e. the expected borrower utility conditional on the screening and lending behavior is weakly positive.

In this setup, borrower welfare may be adversely affected by loan sales despite strategic behavior by borrowers. The reasoning is as follows: For the public type $\theta^F \in [\tilde{\kappa}_f^{LS}, \kappa_f)$, borrowers would be screened when loan sales are not feasible (Proposition 3.1), but would no longer be screened in the lender preferred equilibrium with loan sales (Proposition 3.2). If these borrowers (i) prefer a loan when they are uninformed about their

³⁷Qualitative results of this section carry over as long as the borrowers' information about θ^S is less precise than that obtained by the lender via screening. This is easily explained through the lenders' experience and sophistication.

θ^S -type, but (ii) prefer not to obtain a loan whenever they know that their private type θ^S is such that $\theta^S = \underline{\theta}^S$, borrower welfare decreases with the introduction of loan sales. This is because, comparing the two scenarios, only those borrower-types with $\theta^S = \underline{\theta}^S$ additionally obtain a loan, but never infer that they are of the bad private type due to lack of lender screening. This is summarized by the following Proposition.

Proposition 3.3 (Welfare Impact of Loan Sales) *Compared to the case without loan sale opportunities, lender profitability strictly increases in the case of loan sale opportunities. The change in borrower welfare depends on the ex-ante expected utility of borrowers with type θ such that $\theta^F \in [\tilde{\kappa}_f^{LS}, \kappa_f)$ and $(\theta^S = \underline{\theta}^S)$.*

- (i) *The change in borrower welfare is ambiguous and may be potentially negative if and only if some borrowers in this θ^F -region receive nonnegative expected utility from obtaining a loan without being screened, but negative expected utility conditional on knowing that $\theta^S = \underline{\theta}^S$. In this case, the change in borrower and total welfare depends on the fundamentals, in particular benefits B , distress costs D , probability of default η and the θ^F -distribution of borrowers $F(\cdot)$.*
- (ii) *In all other cases, the introduction of loan sale opportunities weakly increases borrower welfare (and thus total welfare strictly increases).*

Proof. See Appendix 3.E. ■

Proposition 3.3 highlights the potential adverse effect of loan sale opportunities on borrower welfare: It can be rational for intermediate θ^F -borrowers to apply for a loan given that they are not screened in equilibrium. At the same time, this implies that loan sales lead to lending to $(\theta^S = \underline{\theta}^S)$ -type borrowers who have a negative expected utility.

3.7 Robustness

Security Design Our framework abstracts from security design considerations by modelling lender risk retention as a vertical slice. It is well established in the economic literature that by considering alternative retention schemes such as retention of senior tranches yields a larger exposure to default risk for the issuer and best incentivizes lender effort (see e.g. [Kiff and Kisser \(2014\)](#), [Chemla and Hennessey \(2014\)](#)). More generally, [DeMarzo and Duffie \(1999\)](#) characterize the optimal security design given

that an issuer holds private information. Explicit inclusion of security design considerations could thus quantitatively alter the results by relaxing incentive constraints and in particular allow for larger realization of gains from trade in the region where screening, partial lending and sizeable risk retention prevails in equilibrium. Intuitively, the lender could sell a larger total claim on the loan and still retain the same risk exposure, which is necessary to preclude incentives to *originate-to-sell* to negative NPV borrowers. The qualitative nature of the resulting risk retention, however, is unchanged: When screening, risk retention is necessary to align incentives.

Independence Assumption We have assumed that the public (θ^F) and private (θ^S) information component of a borrower's type are independent; in particular, the two possible θ^S -types are equally likely independent of θ^F . This could be relaxed in two dimensions without altering the qualitative results. First, the assumption of equally distributed θ^S -types implies that the variance of the screening outcome is maximized compared to other possible distributions. Considering alternative distributions would decrease the benefits from screening. The relative qualitative tradeoff between screening and not screening would however be unaffected. Similarly, it could reasonably be assumed that θ^S is not independent of θ^F and in particular exhibits a variance which is decreasing in θ^F – the idea would be that borrowers with a low θ^F -type exhibit a higher ex-post heterogeneity (including the private type) than those with a high θ^F -type, where default is unlikely to occur in any case. Such a specification may for example be incorporated by letting the increase/decrease in profitability conditional on screening revealing positive/negative information, i.e. Δ , be itself a function of θ^F : $\Delta(\theta^F)$ with $\Delta'(\theta^F) < 0$. In that case, the qualitative nature of the results carries over: Incentives to screen are larger for low θ^F -types and lower for high θ^F -types. While the exact ranges of the equilibrium types would be affected, screening with partial lending and high retention would still prevail for low θ^F , while not screening, full lending and full loan sales could be sustained for high θ^F .

Endogenous Screening Intensity Another avenue to explore would be to consider a strategic choice of screening intensity as in [Bester, Gehrig, Stenbacka, et al. \(2012\)](#). This could in principle lead to a slow *phasing out* of screening in equilibrium, i.e. to screening intensity going to 0 as θ^F approaches the switching point between the equilibrium type regions, which would imply that a discontinuity in average borrower quality no longer persists. This is unlikely to occur for two reasons: On the one hand, no-screening

equilibria have been shown to be sustainable already when negative NPV borrowers are still present, i.e. whenever $\theta^F - \Delta < 0$. At the margin between the screening and no-screening equilibrium ranges, a strictly positive level of screening intensity would thus have to prevail to *screen out* bad types. Furthermore, we have established that given private information for the borrower, i.e. given screening, risk retention is necessary for the lender to preclude deviation incentives. Risk retention, however, also implies that the marginal gain from screening increases compared to a no-loan-sale framework (or equilibrium candidate). This yields a rationale for screening intensities bounded away from 0.

Continuous Soft Information Type Our model considers a binary soft information type θ^S . Alternatively, a specification with θ^S distributed continuously, e.g. $\theta^S \sim U[-\Delta, \Delta]$ could be considered. This would greatly complicate the model. Intuitively, the presented results should carry over. Conditional on screening, a cutoff type such that lending takes place would be chosen. Risk retention would in turn need to follow a continuous $q(\theta^S)$ schedule (contingent on θ^F) to preclude deviation incentives, with risk retention still necessary whenever screening takes place in equilibrium. We hope to explore this issue in detail in future research.

Retention as exclusive Signaling Device In the current framework, the lender is only able to use the retention as a signaling device for the quality of the underlying loan. This is modelled by assuming that a single borrower of known θ^F -type approaches the lender for a loan. In practice, securitization and subsequent loan sales take place not for each loan individually, but for loan pools. Thus, there is a second component which may be useful for the investor to infer the underlying quality: The size of the pool itself. If the investor has a (perfectly) accurate forecast of the number of borrowers of a given FICO score (θ^F -type in our model) who approach a particular lender, the size of the pool of loans underlying a security, as well as the implied retention level, carry information. In particular, this may lead to a higher sustainability of equilibria, mostly relevant for the sustainability of no-screening equilibria: Any deviation which implies restrictions on the equilibrium retention level necessarily involves a deviation in the size of the underlying loan pool which is sold. If the market for securities for a given lender is concentrated in that he mainly deals with a particular set of investors, these deviations may not be feasible as they would be detected by the investor upon noticing that, while the retention level is as expected, the size of the loan pool differs

sharply. Extending the model in this direction, however, complicates the analysis. It is nonetheless an interesting avenue to pursue, mainly because it may reduce the lender-preferred equilibrium to a simple cutoff rule: Screen borrowers of sufficiently low θ^F -type (low FICO score) and do not screen them whenever θ^F is high enough (high FICO score, e.g. above 620). The non-existence of no-screening equilibria for intermediate values of the lending transaction (intermediate FICO scores) should intuitively be resolved. We hope to pursue this avenue in the future.

3.7.1 Relation to Empirical Findings

Our model predicts that risk retention should be large in the region where lending rates are low as 'screening out' of bad borrowers prevails. This is seemingly at odds to the finding by [Bubb and Kaufman \(2014\)](#) that securitization rates are near constant at a level of roughly 95% across the whole range of FICO scores such that lending occurs.

There are several explanations for this. On the one hand, the model-predicted retention rates could be relaxed (i.e. decreased) by taking into account several unmodelled considerations. First of all, it is well documented that retention of senior tranches yields a larger exposure to default risk for the issuer than e.g. retention of a vertical slice or even a junior tranche (see [Kiff and Kisser \(2014\)](#), [Chemla and Hennessy \(2014\)](#)). By explicitly incorporating security design considerations, incentive constraints could hence be relaxed and larger levels of loan sales could be sustained.

In a similar fashion, reputational concerns may alleviate incentive issues: Because securitization is a repeated interaction where lenders and investors bargain repeatedly over new loan (security) issues, there is an unmodelled cost of deviating. If a particular batch of securitized loans performs badly (and differences in default rates are apparent already in the first 24 months after issuance, see [Keys, Mukherjee, Seru, and Vig \(2010\)](#)), investors will be less likely to purchase loans from a given lender in the future.

Lastly, we have assumed that screening is fully unverifiable with outcomes of the screening process being private information to the lender. In practice, this need not necessarily be the case: Screening may stochastically yield verifiable information which can in fact be credibly communicated to investors. For example, this can be seen in the different levels of documentation associated with different loan pools as in [Keys, Mukherjee,](#)

[Seru, and Vig \(2010\)](#). In particular, the standards adopted by the GSEs, such as documented in [Freddie Mac \(1995\)](#), require a certain provenance as to the screening outcome by lenders.

Nonetheless, even by incorporating the above considerations, the predicted discontinuity in loan sale rates is unlikely to disappear. Considering the empirical findings, it is however essential to distinguish between our notion of loan sales and the notion of securitization in [Bubb and Kaufman \(2014\)](#). Several articles, such as [Acharya, Schnabl, and Suarez \(2013\)](#), [Arteta, Carey, Correa, and Kotter \(2013\)](#) and [Covitz, Liang, and Suarez \(2009\)](#), have documented that securitization does not necessarily involve risk transfer. Instead, a sizeable portion of loans was securitized via asset-backed commercial paper (ABCP) conduits whereby the sponsor of the conduit retains the risk through explicit or implicit guarantees. This type of securitization would show up in [Bubb and Kaufman \(2014\)](#) as private-label securitization but is consistent with risk retention in our model.

[Acharya, Schnabl, and Suarez \(2013\)](#) argue that securitization without risk transfer was mainly due to regulatory arbitrage: Securitization via conduits allowed the sponsor of the conduit to hold less risk-capital than would be required whenever the loans would be explicitly held in the portfolio of the issuer. However, [Arteta, Carey, Correa, and Kotter \(2013\)](#) “find [...] no evidence that banks sponsored vehicles differentially in Europe and the United States due to regulatory capital requirements.” Instead, they argue that agency issues, safety nets and economies of scale and scope motivated banks to systematically take bad-tail risk in the form of ABCP vehicles.

In light of our model, a different motif for securitization via conduit arises: Despite their low profitability, conduits may serve as a signalling device. By securitizing a large fraction of subprime loans via conduits, the risk is retained by the lender/issuer which allows investors to infer that incentives not to ‘originate-to-sell’ negative NPV loans are met. Our model thus helps to explain the prevalence of securitization without risk-transfer; in particular, we offer a signalling motif that is fully independent of regulatory arbitrage considerations.

This is of particular relevance as a key assumption of the model is that retention levels are observable to the investor. Without this observability (implying that the lender could sell different parts of the loan pool to different investors without allowing for inference about the quality), positive loan sales would be a lot harder to sustain as deviations are less detectable and hence more profitable to the lender. However,

securitization via conduit solves this issue: By setting up a conduit and endowing it with guarantees, the lender signals to all investors that part of its asset are implicitly retained. Whether the remaining assets are sold to a single investor (as in the model) or separately to multiple agents is irrelevant for the signalling effect to take place.

Empirical Assessment of Moral Hazard An empirical assessment of whether moral hazard existed in the sense that lender’s incentives were distorted to deliberately *originate-to-sell* negative NPV loans thus would need to take this implicit risk retention into account. Our model suggests that absent significant risk retention for loans to borrowers whose observable creditworthiness falls below certain thresholds, a moral hazard problem would in fact prevail. We have argued above that this is not necessarily at odds with the finding of near-constant securitization rates, as documented by [Bubb and Kaufman \(2014\)](#). A rigorous analysis should take into account implicit risk retention to elicit ex-post exposure to defaults. We predict that this exposure is strictly positive and increasing in a measure of creditworthiness such as a FICO score. Subsequently, after the ‘regime-switch’ to no-screening, this exposure should ideally be zero to maximize gains from trade. Crucially, the [Bubb and Kaufman \(2014\)](#) finding does hence not rule out the presence of private information following screening by the lenders, which is a key ingredient of our model.

3.8 Conclusion

We present a model where lenders may screen potential borrowers to obtain private information about their type prior to the lending decision. We consider how the incentives to screen vary across ex-ante differences in borrower creditworthiness and analyze the interaction with subsequent loan sale opportunities. We find that in equilibrium, borrowers with sufficiently bad creditworthiness are screened and only obtain a loan upon being discovered to be of a good type. To avoid originate-to-sell incentives, loans to these borrowers come with sizeable risk retention by the lender. If the borrowers’ creditworthiness is sufficiently high, in contrast, all borrowers obtain a loan. At the margin between the two regions, there are implied discontinuities in lending, default and loan sale rates as borrowers are no longer screened, which increases both lending and default rates, while retention drops, which increases loan sale rates.

We document that the advent of loan sales improves profitability for lenders in the region where screening takes place. This allows positive NPV borrowers access to loans which they would not obtain if loan sales were disallowed. However, for borrowers with intermediate ex-ante creditworthiness, screening no longer occurs and the average loan quality worsens compared to the no-loan-sale case. This may lead to adverse welfare effects if these borrowers would ex-post prefer not to have obtained a loan. If present, these negative effects are only partially mitigated by the policy reforms mandating risk-retention for all securitized loans. Furthermore, these reforms preclude full realization of gains from trade in a large part of the no-screening region and are typically welfare-decreasing.

We relate our model to the literature on securitization without risk transfer and argue that the unprofitable securitization via conduits served not only as a means for regulatory arbitrage but also as a signalling device – investors observe that lenders retain the risk via implicit or explicit guarantees to their conduits and as such infer that the lender’s incentives preclude originating and selling of negative NPV loans.

The present article may serve as a starting point for two directions of further research: On the one hand, it informs the theoretical analysis of the interplay between securitization and lenders’ screening incentives and may be extended in several directions, most interestingly by accounting for uncertain numbers of borrowers of a given public type, i.e. a given FICO score. On the other hand, the paper argues that the empirical literature has so far not taken into account that risk retention may implicitly occur when securitizing via conduits. To properly assess ex-post retention, and hence the validity of the proposed theories, this needs be incorporated. Once this is done, the tested theories can be used to assess the welfare implications of policy interventions which have either already taken place or are currently being discussed.

Appendix

3.A Proof of Lemma 3.2

Note that given an equilibrium retention level \bar{r}^* , the expected utility of the lender can be expressed as

$$E[U^L] = \frac{1}{2} [(1 - \bar{r}^*) \cdot (\theta^F + \Delta) + \bar{r}^* \delta (\theta^F + \Delta)] - c_s$$

The first restriction on \bar{r}^* and θ^F comes from the fact that profitability is required (otherwise, not approving any borrower without screening would be a deviation). This yields

$$E[U^L] \geq 0 \Leftrightarrow \bar{r}^* \leq \frac{\theta^F + \Delta - 2c_s}{(1 - \delta)(\theta^F + \Delta)}.$$

Thus, unless $\theta^F + \Delta \geq 2c_s > 0 \Leftrightarrow \theta^F \geq 2c_s - \Delta$, the strategy is unprofitable (no $\bar{r}^* \in [0, 1]$ exists such that deviating can be precluded). For $\theta^F \in [2c_s - \Delta, \frac{2c_s}{\delta} - \Delta)$, profitability implies an upper bound on the retention level \bar{r}^* ; for $\theta^F \geq \frac{2c_s}{\delta} - \Delta$, this upper bound is irrelevant. Note that by Assumption 3.2, $\frac{2c_s}{\delta} - \Delta < 0$.

Consider next the possible deviations of the lender which involve approving at least some borrowers. For $\theta^F \geq \Delta \Leftrightarrow \theta^F - \Delta \geq 0$, a deviation necessarily exists as $\theta^S = \underline{\theta}^S$ -borrowers can be approved and profits increased. Thus, restrict attention to $\theta^F \in [2c_s - \Delta, \Delta]$. First note that any positive loan sale quantity (in any most profitable deviation) necessarily requires $r = \bar{r}^*$ as otherwise the pessimistic off-path beliefs imply a negative price. Given this, the only relevant deviation is to not screen and retain $r = \bar{r}^*$: This deviation implies that the lender gains by saving the screening cost and – in expectation – selling more loans to the investor at an inflated price, but loses by being liable for the retained fraction $(1 - r)$ of loans, which include loans with a negative value $\theta^F - \Delta$. For this deviation not to be profitable, we require

$$E[U^L] \geq (1 - \bar{r}^*)(\theta^F + \Delta) + \bar{r}^* \delta \theta^F \Leftrightarrow \bar{r}^* \geq \frac{2c_s + \theta^F + \Delta}{(1 - \delta)\theta^F + (1 + \delta)\Delta},$$

which implies a lower bound on the retention level \bar{r}^* . Note that for $\theta^F > \Delta - \frac{2c_s}{\delta}$, this lower bound becomes greater than 1 and no such equilibrium is sustainable. Whenever

both upper and lower bound are applicable, that is, for $\theta^F < \frac{2c_s}{\delta} - \Delta$, sustainability requires

$$\frac{2c_s + \theta^F + \Delta}{(1 - \delta)\theta^F + (1 + \delta)\Delta} \leq \frac{\theta^F + \Delta - 2c_s}{(1 - \delta)(\theta^F + \Delta)}$$

$$\Leftrightarrow \theta^F \geq -\Delta \frac{\delta\Delta - 2c_s}{\delta\Delta - 2(1 - \delta)c_s} \equiv \kappa_a^{LS}.$$

which gives a condition on θ^F . For θ^F sufficiently high but not too high, screening and partial lending is profitable given a sufficiently high level of loan sales ($1 - \bar{r}^*$) while \bar{r}^* is sufficiently high such that the risk retention prevents the lender from deviating (note that for θ^F too high, the lower bound on \bar{r}^* becomes too tight such that no such equilibria can be sustained). The range of equilibrium $\bar{r}^*(\theta^F)$ which can be sustained is given by

$$\bar{r}^*(\theta^F) \in \left[\frac{2c_s + \theta^F + \Delta}{(1 - \delta)\theta^F + (1 + \delta)\Delta}, \min \left\{ \frac{\theta^F + \Delta - 2c_s}{(1 - \delta)(\theta^F + \Delta)}, 1 \right\} \right]. \quad (3.2)$$

Note that

$$\kappa_a^{LS} = -\Delta \underbrace{\frac{\delta\Delta - 2c_s}{\delta\Delta - 2(1 - \delta)c_s}}_{<1} > -\Delta \quad (3.3)$$

and in particular (by Assumption 3.2)³⁸

$$\frac{2c_s}{\delta} - \Delta > \kappa_a^{LS} > 2c_s - \Delta.$$

This concludes the proof.

³⁸Note that

$$\kappa_a^{LS} > 2c_s - \Delta \Leftrightarrow \frac{(1 - \delta)}{\delta\Delta - 2(1 - \delta)c_s} > 0,$$

where both numerator and denominator are strictly positive, while

$$\kappa_a^{LS} < \frac{2c_s}{\delta} - \Delta \Leftrightarrow 2c_s < \delta\Delta.$$

3.B Proof of Lemma 3.3

Restrict attention to θ^F such that $\theta^F \geq \Delta$, let θ^F be fixed and denote by \underline{r}^* the equilibrium retention when lending to $\theta^S = \underline{\theta}^S$ -types and by \bar{r}^* the equilibrium retention when lending to $\theta^S = \bar{\theta}^S$ -types. When looking at possible deviations, we denote retention rates by \underline{r} and \bar{r} respectively. It follows immediately that to preclude deviations, $\bar{r}^* > \underline{r}^*$ is necessary, as for $\bar{r}^* = \underline{r}^*$, not screening would be profitable, while for $\bar{r}^* > \underline{r}^*$, mimicking the high-type loan when holding a low-type loan would be profitable. Subsequently, it follows that $\underline{r}^* = 0$, as otherwise, decreasing \underline{r} would increase profits (same price but less held to maturity).

Equilibrium profits can be expressed as

$$E[U^L] = \frac{1}{2} [(1 - \bar{r}^*)(\theta^F + \Delta) + \bar{r}^*\delta(\theta^F + \Delta) + (\theta^F - \Delta)] - c_s.$$

Bounds on \bar{r}^* follow from considering the full set of possible deviations:

- Don't screen, lend to both θ^S -types, $r = \bar{r}^*$. To preclude this, we need

$$\begin{aligned} E[U^L] &\geq (1 - \bar{r}^*)(\theta^F + \Delta) + \bar{r}^*\delta\theta^F \\ \Leftrightarrow \theta^F - \Delta - 2c_s &\geq (1 - \bar{r}^*)(\theta^F + \Delta) + \bar{r}^*\delta(\theta^F - \Delta) \\ \Leftrightarrow -2\Delta - 2c_s &\geq \bar{r}^* [\delta\theta^F - \delta\Delta - \theta^F - \Delta] \\ \Leftrightarrow \bar{r}^* &\geq \frac{2\Delta + 2c_s}{(1 - \delta)\theta^F + (1 + \delta)\Delta} \end{aligned}$$

This bound behaves intuitively: Gains from this type of deviation stem from (i) saving the screening cost and (ii) selling $\theta^S = \underline{\theta}^S$ -type loans at a higher value (θ^F) than they are actually worth ($\theta^F - \Delta$), while losses are incurred from holding the 'bad' loans to maturity (and discounting them) instead of fully selling them. Hence, the lower bound on \bar{r}^* is increasing in the screening cost c_s and decreasing in θ^F : A higher screening cost increases the relative profitability of the deviation, so the punishment needs to increase by increasing the retention. By contrast, a high θ^F implies that the relative cost incurred from discounting is higher and there is more leeway in terms of the equilibrium retention level \bar{r}^* which can be sustained.

- Don't screen, lend to both θ^S -types, $r = \bar{r}^* = 0$. To preclude this, we require

$$\begin{aligned}
 E[U^L] &\geq \theta^F - \Delta \\
 \Leftrightarrow \theta^F + \Delta + \theta^F - \Delta - \bar{r}^*(1 - \delta)(\theta^F + \Delta) - 2c_s &\geq 2(\theta^F - \Delta) \\
 \Leftrightarrow \bar{r}^* &\leq \frac{2\Delta - 2c_s}{(1 - \delta)(\theta^F + \Delta)}.
 \end{aligned}$$

Note that the behavior of this bound as well is intuitive: Gains from deviating stem from the fact that (i) the screening cost is saved and (ii) no loans are held to maturity (and discounted) as everything is sold, while losses are incurred due to not selling $\theta^S = \bar{\theta}^S$ -type loans at face value ($\theta^F + \Delta$) but at a discount ($\theta^F - \Delta$). The upper bound on \bar{r}^* is hence decreasing in θ^F and c_s : If θ^F is large, the gains from not holding on to the loan to maturity are large (compared to the loss from not selling at face value) and retention needs to be low for $\theta^S = \bar{\theta}^S$ -type loans. Similarly, a high screening cost increases the relative profitability of deviating. Finally, the bound is increasing in Δ as a large Δ increases the relative loss from deviating, giving more leeway in terms of retention.

To check whether an equilibrium range of \bar{r}^* exists such that both upper and lower bound are satisfied, we require

$$\begin{aligned}
 \frac{2\Delta - 2c_s}{(1 - \delta)(\theta^F + \Delta)} &\geq \frac{2\Delta + 2c_s}{(1 - \delta)\theta^F + (1 + \delta)\Delta} \\
 \Leftrightarrow \theta^F &\leq \frac{\delta\Delta^2 - c_s\Delta}{(1 - \delta)c_s} = \Delta \frac{\delta\Delta - c_s}{(1 - \delta)c_s}.
 \end{aligned} \tag{3.4}$$

Note that

$$\frac{\delta\Delta - c_s}{(1 - \delta)c_s} > 1 \Leftrightarrow \delta\Delta - c_s > c_s - \delta c_s \Leftrightarrow \delta\Delta - 2c_s > -\delta c_s$$

which is true by Assumption 3.2 as $\delta\Delta - 2c_s > 0 > -\delta c_s$. Thus, we know that (3.4) can be consistent $\theta^F \geq \Delta$. However, we also require that $\bar{r}^* \in [0, 1)$, i.e. in particular that

$$\frac{2\Delta + 2c_s}{(1 - \delta)\theta^F + (1 + \delta)\Delta} < 1 \Leftrightarrow \theta^F > \Delta + \frac{2c_s}{1 - \delta}. \tag{3.5}$$

Note that (3.4) gives

$$\theta^F \leq \Delta \frac{\delta\Delta - c_s}{(1-\delta)c_s} = \Delta + \frac{\delta\Delta - (2-\delta)c_s}{(1-\delta)c_s} \Delta.$$

Thus, for (3.4) and (3.5) to both be satisfied, we need

$$\frac{\delta\Delta - (2-\delta)c_s}{(1-\delta)c_s} \Delta \geq \frac{2c_s}{1-\delta} \Leftrightarrow \delta\Delta^2 \geq 2c_s^2 + (2-\delta)c_s\Delta. \quad (3.6)$$

Assumption 3.2 is sufficient to guarantee this, as

$$\begin{aligned} c_s < \frac{\delta\Delta}{2} &\Rightarrow 2c_s^2 + (2-\delta)c_s\Delta < \frac{\delta^2\Delta^2}{2} + (2-\delta)\frac{\delta\Delta^2}{2} \\ &\Rightarrow 2c_s^2 + (2-\delta)c_s\Delta < \delta\Delta^2. \end{aligned}$$

Finally, θ^F needs to be such that the lower bound is at least 0, which is trivially satisfied.

- The above deviations are all that need to be considered aside from profitability. The reasoning is as follows: Any non-screening deviation is dominated by one of the above deviations as (i) choosing $r = \underline{r}^* = 0$ maximizes what can be sold for any off-path $r \neq \bar{r}^*$ while prices are identical, while (ii) $r = \bar{r}^*$ is the deviation which maximizes the price which can be obtained. Finally, when screening, $\bar{r} = \bar{r}^* = \underline{r}$ or $\bar{r} = \underline{r}^* = \underline{r} = 0$ maximize the lender's utility when deviating by the same arguments; in this case, however, not screening saves c_s and we are back to the considered deviation.
- Profitability nonetheless needs to be ensured. We require

$$E[U^L] = \frac{1}{2} [(1 - \bar{r}^*)(\theta^F + \Delta) + \bar{r}^*\delta(\theta^F + \Delta) + (\theta^F - \Delta)] - c_s \geq 0.$$

As $\theta^F \geq \Delta$, we have

$$E[U^L] \geq \frac{1}{2} [(1 - \bar{r}^*)2\Delta + \bar{r}^*\delta 2\Delta] - c_s \stackrel{\bar{r}^* \leq 1}{\geq} \delta\Delta - c_s \stackrel{Ass.3.2}{>} 0.$$

Thus, as long as θ^F is such that a range of sustainable equilibrium \bar{r}^* exist which preclude deviations, profitability is ensured.

Summarizing these observations yields that an equilibrium with screening and full lending can be sustained iff

(i) $\theta^F \in (\underline{\kappa}_{sf}, \bar{\kappa}_{sf}]$ where

$$\underline{\kappa}_{sf} = \Delta + \frac{2c_s}{1-\delta} < \frac{\delta\Delta^2 - c_s\Delta}{(1-\delta)c_s} = \bar{\kappa}_{sf}. \quad (3.7)$$

(ii) $\underline{r}^*(\theta^F) = 0, \forall \theta^F \in (\underline{\kappa}_{sf}, \bar{\kappa}_{sf}]$.

(iii) $\bar{r}^*(\theta^F)$ is such that

$$\bar{r}^*(\theta^F) \in \left[\frac{2\Delta + 2c_s}{(1-\delta)\theta^F + (1+\delta)\Delta}, \min \left\{ \frac{2\Delta - 2c_s}{(1-\delta)(\theta^F + \Delta)}, 1 \right\} \right) \quad (3.8)$$

for all $\theta^F \in (\underline{\kappa}_{sf}, \bar{\kappa}_{sf}]$.

3.C Proof of Lemma 3.4

Fix $\theta^F > 0$ and denote by r^* the equilibrium retention rate. Equilibrium profits can be expressed as

$$E[U^L] = (1 - r^*)\theta^F + \delta r^*v.$$

We again need to check all possible deviations.

1. don't screen, lend to all borrowers, retain $r \neq r^*$

Note that if $r \neq r^*$, the pessimistic off-path beliefs lead to an expected utility for the lender of $(1 - r)(\theta^F - \Delta) + \delta r\theta^F$. It is thus clear that, depending on the sign of $(\theta^F - \Delta)$, $r = 0$ or $r = 1$ maximize this expression conditional on deviating.

a) $r = 0$: Expected utility upon deviating is then $\theta^F - \Delta$ and we require

$$E[U^L] = (1 - r^*)\theta^F + \delta r^*\theta^F \geq \theta^F - \Delta$$

$$\Leftrightarrow r^* \leq \frac{\Delta}{(1-\delta)\theta^F},$$

i.e. retention needs to be not too high to preclude this deviation. Note that the right-hand side is strictly below one only for large θ^F , that is, for $\theta^F > \frac{\Delta}{1-\delta}$.

- b) $r = 1$: Expected utility upon deviating is then $\delta\theta^F$. This is straightforwardly dominated by selling r^* for $r^* < 1$ and not a deviation otherwise.

2. screen, lend to all borrowers

Note that this can be the most profitable deviation iff $\theta^F - \Delta > 0$ as otherwise not lending to $\theta^S = \underline{\theta}^S$ -types is better (which will be addressed subsequently). Conditional on screening and lending, denote \underline{r} and \bar{r} the retention for $\underline{\theta}^S$ -type loans and $\bar{\theta}^S$ -type loans respectively. Clearly, only $\underline{r} = r^*$ and $\underline{r} = 0$ need to be considered: Either the loans are sold above their value as retention mimicks the presumed equilibrium play or any retention leads to a price as if the loans were to $\underline{\theta}^S$ -types. In this case, however, minimal retention ($\underline{r} = 0$) maximizes expected utility upon deviating. A similar argument applies to \bar{r} : Either $\bar{r} = r^*$ is chosen to mimick the equilibrium retention or $\bar{r} = 1$ and loans are fully retained. In principle, $\bar{r} = 0$ would also be a possible retention choice upon deviation. However, if $\bar{r} = 0$ represents the most profitable retention choice conditional on deviating, $\underline{r} = 0$ would immediately follow. In this case, however, not screening, lending to all borrowers and setting $r = 0$ for all loans would save the screening cost – this deviation has been covered above. We thus need to check three possible combinations of \bar{r}, \underline{r} upon deviating as $\bar{r} = \underline{r} = r^*$ is straightforwardly not a profitable deviation (the screening cost is incurred, otherwise expected utility is unchanged).

- a) $\underline{r} = 0, \bar{r} = r^*$. For this not to be profitable, we require

$$E[U^L] = (1 - r^*)\theta^F + \delta r^*\theta^F \geq \frac{1}{2}(\theta^F - \Delta) + \frac{1}{2}[(1 - r^*)\theta^F + \delta r^*(\theta^F + \Delta)] - c_s$$

$$\Leftrightarrow r^* \leq \frac{\Delta + 2c_s}{(1 - \delta)\theta^F + \delta\Delta}.$$

We thus obtain an upper bound on the equilibrium retention r^* which in particular is decreasing in θ^F : The more valuable the loan absent screening (large θ^F), the more profitable it is to sell off bad loans at face value to not incur the discounting.

b) $\underline{r} = 0, \bar{r} = 1$. For this not to be profitable, we require

$$E[U^L] = (1 - r^*)\theta^F + \delta r^*\theta^F \geq \frac{1}{2}(\theta^F - \Delta) + \frac{1}{2}\delta(\theta^F + \Delta) - c_s$$

$$\Leftrightarrow r^* \leq \frac{(1 - \delta)\theta^F + (1 - \delta)\Delta + 2c_s}{2\theta^F(1 - \delta)}.$$

We thus obtain a second upper bound on the equilibrium retention r^* which again is decreasing in θ^F (for similar arguments as before).

c) $\underline{r} = r^*, \bar{r} = 1$. For this not to be profitable, we require

$$E[U^L] = (1 - r^*)\theta^F + \delta r^*\theta^F \geq \frac{1}{2} [(1 - r^*)\theta^F + \delta r^*(\theta^F - \Delta) + \delta(\theta^F + \Delta)] - c_s$$

$$\Leftrightarrow r^* [(1 - \delta)\theta^F - \delta\Delta] \leq (1 - \delta)\theta^F - \delta\Delta + 2c_s.$$

It is straightforward that any $r^* \in [0, 1]$ satisfies this for $(1 - \delta)\theta^F - \delta\Delta \geq 0 \Leftrightarrow \theta^F \geq \frac{\delta\Delta}{1 - \delta}$. For $\theta^F < \frac{\delta\Delta}{1 - \delta}$, we obtain the constraint

$$r^* \geq 1 - \frac{2c_s}{\delta\Delta - (1 - \delta)\theta^F},$$

that is, a lower bound on the retention level r^* which decreases (becomes less binding) the larger θ^F . This is intuitive as the main gain from this deviation stems from the fact that $\underline{\theta}^S$ -type loans are sold above their expected value, which is relatively more appealing the lower r^* and the lower θ^F . Note that the constraint is relevant iff the right-hand side is larger than 0, that is, whenever

$$\theta^F < \frac{\delta\Delta - 2c_s}{1 - \delta}.$$

3. screen, lend to $\theta^S = \bar{\theta}^S$ -types only

Note that this can be the most profitable deviation iff $\theta^F - \Delta \leq 0$ as otherwise lending to $\theta^S = \underline{\theta}^S$ -types would be profitable. Furthermore, note that given $\theta^F - \Delta \leq 0$, we only need to consider two possible deviations: $\bar{r} = r^*$ and $\bar{r} = 1$ – any $\bar{r} \neq r^*, \bar{r} < 1$ implies that a negative per-loan price is obtained for the sold issue.

a) $\bar{r} = 1$. For this not to be profitable, we require

$$E[U^L] = (1 - r^*)\theta^F + \delta r^*\theta^F \geq \frac{1}{2}\delta(\theta^F + \Delta) - c_s$$

$$\Leftrightarrow r^* \leq \frac{(2 - \delta)\theta^F - \delta\Delta + 2c_s}{2(1 - \delta)\theta^F},$$

i.e. we obtain an upper bound on the retention r^* which is increasing in θ^F . Note that the right hand side becomes small (more strict) the lower θ^F and is relevant whenever

$$\frac{(2 - \delta)\theta^F - \delta\Delta + 2c_s}{2(1 - \delta)\theta^F} < 1 \Leftrightarrow \theta^F < \frac{\delta\Delta - 2c_s}{\delta}.$$

Finally, for $(2 - \delta)\theta^F - \delta\Delta + 2c_s \leq 0 \Leftrightarrow \theta^F < \frac{\delta\Delta - 2c_s}{2 - \delta}$, no r^* exists such that this is not a profitable deviation.

b) $\bar{r} = r^*$. For this not to be profitable, we require

$$E[U^L] = (1 - r^*)\theta^F + \delta r^*\theta^F \geq \frac{1}{2} [(1 - r^*)\theta^F + \delta r^*(\theta^F + \Delta)] - c_s$$

$$\Leftrightarrow r^* \leq \frac{\theta^F + 2c_s}{(1 - \delta)\theta^F + \delta\Delta},$$

which again is an upper bound on the retention r^* which is increasing in θ^F , relevant whenever

$$\frac{\theta^F + 2c_s}{(1 - \delta)\theta^F + \delta\Delta} < 1 \Leftrightarrow \theta^F < \frac{\delta\Delta - 2c_s}{\delta},$$

and which is not possible to satisfy for $\theta^F < -2c_s$.

Too facilitate the analysis, the following table summarizes the constraints on r^* as a function of θ^F for an equilibrium with no screening and full lending to be sustainable.

	Constraint Value	relevant iff
(C1)		$\theta^F \geq \frac{\delta\Delta - 2c_s}{2 - \delta}$
(C2)	$r^* \leq \frac{\Delta}{(1-\delta)\theta^F}$	$\theta^F > \frac{\Delta}{1-\delta}$
(C3)	$r^* \leq \frac{\Delta + 2c_s}{(1-\delta)\theta^F + \delta\Delta}$	$\theta^F > \Delta + \frac{2c_s}{1-\delta}$
(C4)	$r^* \leq \frac{(1-\delta)\theta^F + (1-\delta)\Delta + 2c_s}{2\theta^F(1-\delta)}$	$\theta^F > \Delta + \frac{2c_s}{1-\delta}$
(C5)	$r^* \leq \frac{(2-\delta)\theta^F - \delta\Delta + 2c_s}{2(1-\delta)\theta^F}$	$\theta^F < \frac{\delta\Delta - 2c_s}{\delta}$
(C6)	$r^* \leq \frac{\theta^F + 2c_s}{(1-\delta)\theta^F + \delta\Delta}$	$\theta^F < \frac{\delta\Delta - 2c_s}{\delta}$
(C7)	$r^* \geq 1 - \frac{2c_s}{\delta\Delta - (1-\delta)\theta^F}$	$\theta^F < \frac{\delta\Delta - 2c_s}{1-\delta}$

Note that (C1) stems from the fact that a certain deviation exists from constraint (C5) whenever θ^F is below this threshold. The lower bound on r^* is relevant only for $\theta^F \in \left[\frac{\delta\Delta - 2c_s}{2 - \delta}, \frac{\delta\Delta - 2c_s}{1 - \delta}\right]$. For such θ^F , constraints (C5) and (C6) potentially yield an upper bound on r^* .

Restrict attention to $\theta^F < \frac{\delta\Delta - 2c_s}{1 - \delta}$. For a no-screening equilibrium to be sustainable, we require

$$\underbrace{1 - \frac{2c_s}{\delta\Delta - (1-\delta)\theta^F}}_L \leq \min \left\{ \underbrace{\frac{\theta^F + 2c_s}{(1-\delta)\theta^F + \delta\Delta}}_{R1}, \underbrace{\frac{(2-\delta)\theta^F - \delta\Delta + 2c_s}{2(1-\delta)\theta^F}}_{R2} \right\}. \quad (3.9)$$

Note that both $R1$ and $R2$ are increasing in θ^F while L is decreasing in θ^F . At $\theta^F = \frac{\delta\Delta - 2c_s}{2 - \delta}$, we have that

$$L = \frac{\delta\Delta - 2c_s}{\delta\Delta + (1-\delta)2c_s} > 0 = R2$$

and hence that (3.9) is violated. By contrast, for $\theta^F = \frac{\delta\Delta - 2c_s}{1-\delta}$, we have that

$$L = 0 < \min \left\{ \underbrace{\frac{\delta\Delta - 2\delta c_s}{(1-\delta)(2\delta\Delta - 2c_s)}}_{R1>0}, \underbrace{\frac{1}{2(1-\delta)}}_{R2>0} \right\}$$

and hence that (3.9) is strictly satisfied. Thus, there exists a unique cutoff κ_f^{LS} characterized by

$$1 - \frac{2c_s}{\delta\Delta - (1-\delta)\kappa_f^{LS}} = \min \left\{ \frac{\kappa_f^{LS} + 2c_s}{(1-\delta)\kappa_f^{LS} + \delta\Delta}, \frac{(2-\delta)\kappa_f^{LS} - \delta\Delta + 2c_s}{2(1-\delta)\kappa_f^{LS}} \right\}. \quad (3.10)$$

For $\theta^F > \kappa_f^{LS}$, a no-screening equilibrium with approval of all θ^F -type borrowers can be supported when taking into account the lower bound on $r^*(\theta^F)$ and constraints (C5) and (C6). We still need to assess whether equilibrium existence fails whenever the lower bound constraint (C7) is relevant along with (C3) and (C4). Note that (C2) and (C7) can never both be relevant as $\frac{\Delta}{1-\delta} > \frac{\delta\Delta}{1-\delta} > \frac{\delta\Delta - 2c_s}{1-\delta}$. Suppose, however, that

$$\Delta + \frac{2c_s}{1-\delta} < \frac{\delta\Delta - 2c_s}{1-\delta} \Leftrightarrow \delta > \frac{1}{2} + \frac{2c_s}{\Delta}. \quad (3.11)$$

In that case, we have to worry that equilibrium existence might fail for $\theta^F \in \left(\Delta + \frac{2c_s}{1-\delta}, \frac{\delta\Delta - 2c_s}{1-\delta}\right)$ if (C7) and the tighter constraint between (C3) and (C4) contradict each other. To assess this potential issue, restrict attention to such θ^F and note that we can write

$$\begin{aligned} & \min \left\{ \underbrace{\frac{\Delta + 2c_s}{(1-\delta)\theta^F + \delta\Delta}}_{(C3)}, \underbrace{\frac{(1-\delta)\theta^F + (1-\delta)\Delta + 2c_s}{2v(1-\delta)}}_{(C4)} \right\} \\ &= \min \left\{ \frac{\Delta + 2c_s}{(1-\delta)\theta^F + \delta\Delta}, \frac{\Delta + 2c_s + (1-\delta)\theta^F - \delta\Delta}{2(1-\delta)\theta^F} \right\}. \end{aligned}$$

Note that

$$\begin{aligned}
 & \frac{\Delta + 2c_s}{(1 - \delta)\theta^F + \delta\Delta} > \frac{\Delta + 2c_s + (1 - \delta)\theta^F - \delta\Delta}{2(1 - \delta)\theta^F} \\
 \Leftrightarrow & 2(\Delta + 2c_s)(1 - \delta)\theta^F > (\Delta + 2c_s + (1 - \delta)\theta^F - \delta\Delta)((1 - \delta)\theta^F + \delta\Delta) \\
 \Leftrightarrow & 2(\Delta + 2c_s)(1 - \delta)\theta^F > (\Delta + 2c_s)(1 - \delta)\theta^F + (\Delta + 2c_s)\delta\Delta + (1 - \delta)^2(\theta^F)^2 - \delta^2\Delta^2 \\
 & \Leftrightarrow \delta\Delta(\delta\Delta - \Delta - 2c_s) > (1 - \delta)\theta^F [(1 - \delta)\theta^F - (\Delta + 2c_s)] \\
 & \Leftrightarrow -\delta\Delta((1 - \delta)\Delta + 2c_s) > (1 - \delta)\theta^F [(1 - \delta)\theta^F - (\Delta + 2c_s)].
 \end{aligned}$$

As we are in a range of positive θ^F , the right-hand side increases in θ^F , while the left-hand side remains constant. At $\theta^F = \frac{\delta}{1-\delta}\Delta > \frac{\delta\Delta - 2c_s}{1-\delta}$, the two sides are equal. We can thus conclude that for $\theta^F \in (\Delta + \frac{2c_s}{1-\delta}, \frac{\delta\Delta - 2c_s}{1-\delta})$, (C4) is the stricter constraint (compared to (C3)). For existence of a no-screening equilibrium in this range, we thus need to compare (C4) and (C7) and assess whether they can simultaneously be satisfied.

To establish that an r^* satisfying both (C4) and (C7) exists for $\theta^F \in (\Delta + \frac{2c_s}{1-\delta}, \frac{\delta\Delta - 2c_s}{1-\delta})$, note that we require

$$\begin{aligned}
 & \frac{(1 - \delta)\theta^F + (1 - \delta)\Delta + 2c_s}{2\theta^F(1 - \delta)} \geq 1 - \frac{2c_s}{\delta\Delta - (1 - \delta)\theta^F} \\
 \Leftrightarrow & -(1 - \delta)^2(\theta^F)^2 + (1 - \delta)\theta^F(\Delta - 2c_s) \leq \delta\Delta [(1 - \delta)\Delta + 2c_s] \quad (3.12)
 \end{aligned}$$

The right-hand side is constant in θ^F , while the left-hand side is decreasing in θ^F for $\theta^F > \frac{\Delta - 2c_s}{2(1-\delta)}$ and increasing in θ^F for $\theta^F < \frac{\Delta - 2c_s}{2(1-\delta)}$:

$$\begin{aligned}
 & \partial (-(1 - \delta)^2(\theta^F)^2 + (1 - \delta)\theta^F(\Delta - 2c_s)) / \partial \theta^F = (1 - \delta) [\Delta - 2c_s - 2(1 - \delta)\theta^F] < 0 \\
 \Leftrightarrow & \Delta - 2c_s - 2(1 - \delta)\theta^F < 0 \Leftrightarrow \theta^F > \frac{\Delta - 2c_s}{2(1 - \delta)}.
 \end{aligned}$$

Thus, the left-hand side is maximized at $\theta^F = \frac{\Delta - 2c_s}{2(1-\delta)}$. However, note the following:

- (i) It may be the case that $\frac{\Delta - 2c_s}{2(1-\delta)} > \Delta + \frac{2c_s}{1-\delta}$. In particular, this holds whenever

$$\frac{\Delta - 2c_s}{2(1 - \delta)} > \Delta + \frac{2c_s}{1 - \delta} \Leftrightarrow (2\delta - 1)\Delta > 6c_s.$$

(ii) Suppose that $(2\delta - 1)\Delta > 6c_s$. Then it follows that

$$\frac{\Delta - 2c_s}{2(1 - \delta)} < \frac{\delta\Delta - 2c_s}{1 - \delta} \Leftrightarrow 2c_s < (2\delta - 1)\Delta$$

also holds.³⁹

(iii) At $\theta^F = \frac{\Delta - 2c_s}{2(1 - \delta)}$, the inequality (3.12) reduces to

$$\frac{(\Delta - 2c_s)^2}{4} \leq \delta\Delta [(1 - \delta)\Delta + 2c_s].$$

This may be violated. To see this, note that for $c_s \rightarrow 0$, the above inequality goes to

$$\frac{\Delta^2}{4} \leq \delta(1 - \delta)\Delta^2 \Leftrightarrow \frac{1}{4} \leq \delta(1 - \delta)$$

which is clearly violated for $\delta > \frac{1}{2}$.

Note that if $(2\delta - 1)\Delta \leq 6c_s$, we have that $\frac{\Delta - 2c_s}{2(1 - \delta)} \leq \Delta + \frac{2c_s}{1 - \delta}$. As such, the left-hand side in (3.12) is decreasing for all considered θ^F . Furthermore, at $\theta^F = \Delta + \frac{2c_s}{1 - \delta}$, the inequality is satisfied irrespective of δ, Δ and c_s as

$$-(1 - \delta)^2(\theta^F)^2 + (1 - \delta)\theta^F(\Delta - 2c_s) = [(1 - \delta)\Delta + 2c_s](\delta\Delta - 4c_s) < [(1 - \delta)\Delta + 2c_s] \cdot \delta\Delta.$$

As such, we can not generically establish that a no-screening equilibrium exists for $\theta^F \in (\Delta + \frac{2c_s}{1 - \delta}, \frac{\delta\Delta - 2c_s}{1 - \delta})$. For sufficiently low screening costs, the above conditions for non-sustainability are all satisfied. In that case, there exists an interval $\Theta_{ne}^F \subset (\Delta + \frac{2c_s}{1 - \delta}, \frac{\delta\Delta - 2c_s}{1 - \delta})$ where $\frac{\Delta - 2c_s}{2(1 - \delta)} \in \Theta_{ne}^F$ where no no-screening equilibrium with full approval can be supported.

Summarizing yields the following observation:

- (i).(a) If $\delta \leq \frac{1}{2} + \frac{2c_s}{\Delta}$, an equilibrium with no screening exists for all $\theta^F \geq \kappa_f^{LS}$ and $r^*(\theta^F)$ is constrained to be

Table 3.1: Constraints on $r^*(\theta^F)$ I/II

³⁹In fact, it holds generically for $\delta > \frac{1}{2} + \frac{c_s}{\Delta}$.

θ^F	r^*
$\theta^F \in \left[\theta_{f,LS}^F, \frac{\delta\Delta - 2c_s}{1-\delta} \right]$	$r^* \in \left[1 - \frac{2c_s}{\delta\Delta - (1-\delta)\theta^F}, \min \left\{ \frac{(2-\delta)\theta^F - \delta\Delta + 2c_s}{2(1-\delta)\theta^F}, \frac{\theta^F + 2c_s}{(1-\delta)\theta^F + \delta\Delta} \right\} \right]$
$\theta^F > \frac{\delta\Delta - 2c_s}{1-\delta}$	$r^* \in \left[0, \min \left\{ \frac{(1-\delta)\theta^F + (1-\delta)\Delta + 2c_s}{2\theta^F(1-\delta)}, \frac{(2-\delta)\theta^F - \delta\Delta + 2c_s}{2(1-\delta)\theta^F}, \frac{\Delta}{(1-\delta)\theta^F} \right\} \right]$

- (i).(b) If $\delta > \frac{1}{2} + \frac{2c_s}{\Delta}$, but $(2\delta - 1)\Delta \leq 6c_s$ or $\frac{(\Delta - 2c_s)^2}{4} \geq \delta\Delta[(1-\delta)\Delta + 2c_s]$, an equilibrium with no screening exists for all $\theta^F \geq \kappa_f^{LS}$ and $r^*(\theta^F)$ is constrained to be

θ^F	r^*
$\theta^F \in \left[\theta_{f,LS}^F, \frac{\delta\Delta - 2c_s}{\delta} \right]$	$r^* \in \left[1 - \frac{2c_s}{\delta\Delta - (1-\delta)\theta^F}, \min \left\{ \frac{(2-\delta)\theta^F - \delta\Delta + 2c_s}{2(1-\delta)\theta^F}, \frac{\theta^F + 2c_s}{(1-\delta)\theta^F + \delta\Delta} \right\} \right]$
$\theta^F \in \left[\frac{\delta\Delta - 2c_s}{\delta}, \Delta + \frac{2c_s}{1-\delta} \right]$	$r^* \in \left[1 - \frac{2c_s}{\delta\Delta - (1-\delta)\theta^F}, 1 \right]$
$\theta^F \in \left[\Delta + \frac{2c_s}{1-\delta}, \frac{\delta\Delta - 2c_s}{1-\delta} \right]$	$r^* \in \left[1 - \frac{2c_s}{\delta\Delta - (1-\delta)\theta^F}, \frac{(1-\delta)\theta^F + (1-\delta)\Delta + 2c_s}{2\theta^F(1-\delta)} \right]$
$\theta^F > \frac{\delta\Delta - 2c_s}{1-\delta}$	$r^* \in \left[0, \min \left\{ \frac{(1-\delta)\theta^F + (1-\delta)\Delta + 2c_s}{2\theta^F(1-\delta)}, \frac{(2-\delta)\theta^F - \delta\Delta + 2c_s}{2(1-\delta)\theta^F}, \frac{\Delta}{(1-\delta)\theta^F} \right\} \right]$

- (ii) If $(2\delta - 1)\Delta > 6c_s$ and $\frac{(\Delta - 2c_s)^2}{4} < \delta\Delta[(1-\delta)\Delta + 2c_s]$, there exists an interval $\Theta_{ne}^F \subset \left[\Delta + \frac{2c_s}{1-\delta}, \frac{\delta\Delta - 2c_s}{1-\delta} \right]$ such that a no-screening equilibrium fails to exist. For other ranges of θ^F , restrictions on $r^*(\theta^F)$ are as follows:

θ^F	r^*
$\theta^F \in \left[\theta_{f,LS}^F, \frac{\delta\Delta - 2c_s}{\delta} \right]$	$r^* \in \left[1 - \frac{2c_s}{\delta\Delta - (1-\delta)\theta^F}, \min \left\{ \frac{(2-\delta)\theta^F - \delta\Delta + 2c_s}{2(1-\delta)\theta^F}, \frac{\theta^F + 2c_s}{(1-\delta)\theta^F + \delta\Delta} \right\} \right]$
$\theta^F \in \left[\frac{\delta\Delta - 2c_s}{\delta}, \Delta + \frac{2c_s}{1-\delta} \right]$	$r^* \in \left[1 - \frac{2c_s}{\delta\Delta - (1-\delta)\theta^F}, 1 \right]$
$\theta^F \in \left[\Delta + \frac{2c_s}{1-\delta}, \frac{\delta\Delta - 2c_s}{1-\delta} \right] \setminus \Theta_{ne}^F$	$r^* \in \left[1 - \frac{2c_s}{\delta\Delta - (1-\delta)\theta^F}, \frac{(1-\delta)\theta^F + (1-\delta)\Delta + 2c_s}{2\theta^F(1-\delta)} \right]$
$\theta^F > \frac{\delta\Delta - 2c_s}{1-\delta}$	$r^* \in \left[0, \min \left\{ \frac{(1-\delta)\theta^F + (1-\delta)\Delta + 2c_s}{2\theta^F(1-\delta)}, \frac{(2-\delta)\theta^F - \delta\Delta + 2c_s}{2(1-\delta)\theta^F}, \frac{\Delta}{(1-\delta)\theta^F} \right\} \right]$

 Table 3.2: Constraints on $r^*(\theta^F)$ II/II

3.D Proof of Corollary 3.3

The range for non-existence is characterized by $\Theta_{ne}^F \subset \left[\Delta + \frac{2c_s}{1-\delta}, \frac{\delta\Delta - 2c_s}{1-\delta} \right]$. An equilibrium with screening and full approval exists on $(\underline{\kappa}_{sf}, \bar{\kappa}_{sf}) = \left(\Delta + \frac{2c_s}{1-\delta}, \frac{\delta\Delta^2 - c_s\Delta}{(1-\delta)c_s} \right]$ by Lemma 3.3. As

$$\frac{\delta\Delta - 2c_s}{1-\delta} < \frac{\delta\Delta^2 - c_s\Delta}{(1-\delta)c_s} = \frac{\Delta}{c_s} \cdot \frac{\delta\Delta - c_s}{1-\delta},$$

which holds as $\frac{\delta\Delta - c_s}{1-\delta} > \frac{\delta\Delta - 2c_s}{1-\delta}$ and $\frac{\Delta}{c_s} > 1$ given that $2c_s < \delta\Delta \Leftrightarrow \frac{\Delta}{c_s} > \frac{2}{\delta} > 1$. Thus, if Θ_{ne}^F exists such that a no-screening equilibrium can not be supported, we necessarily have

$$\Theta_{ne}^F \subset \left(\Delta + \frac{2c_s}{1-\delta}, \frac{\delta\Delta^2 - c_s\Delta}{(1-\delta)c_s} \right] = (\underline{\kappa}_{sf}, \bar{\kappa}_{sf}].$$

3.E Proof of Proposition 3.3

Consider first borrowers with θ^F such that $\theta^F \in [\tilde{\kappa}_a^{LS}, \kappa_a)$. These borrowers could obtain a loan when loan sales are possible *conditional on screening revealing positive*

information about θ^S , but not without loan sales. For these borrowers, borrower welfare is weakly larger when loan sales are possible compared to the autarky case. To see this, first note that a borrower's expected utility conditional on being approved can be expressed as

$$E[U^B] = B - \eta(\theta^F, \bar{\theta}^S) \cdot D$$

and let κ_a and κ_a^{LS} be defined as before. Note that the expected utility is increasing in θ^F . Furthermore, by the above restriction to the case where without loan sales, lender behavior is such that all borrowers ask for a loan, we know that $B - \eta(\kappa_a, \bar{\theta}^S) \cdot D \geq 0$. The marginal borrower who obtains a loan in the no loan sale case does in fact prefer to obtain it conditional on screening revealing positive information. There are hence three cases:

(i) $B - \eta(\kappa_a, \bar{\theta}^S) \cdot D = 0$.

In this case, the marginal approved borrower with $\theta^F = \kappa_a$ in the no-loan sale case is exactly indifferent between applying (and being approved) and not applying. This implies that $B - \eta(\theta^F, \bar{\theta}^S) \cdot D < 0$ for all borrowers with $\theta^F < \kappa_a$. Even given the introduction of loan sales, those θ^F -type borrowers will not apply – they would be approved by the lender conditional on applying and screening revealing positive information, but their expected utility would be negative. Borrower welfare is thus unaffected by the introduction of loan sale opportunities.

(ii) $B - \eta(\kappa_a^{LS}, \bar{\theta}^S) \cdot D \geq 0$

In this case, the marginal borrower with $\theta^F = \tilde{\kappa}_f^{LS}$ weakly prefers to apply – the borrower obtains a loan conditional on screening revealing positive information, which yields a positive expected utility for the borrower. However, this implies that all borrowers with θ^F such that $\theta^F \in (\theta_{a,ls}^F, \kappa_a)$ strictly prefer to apply and be approved conditional on screening revealing positive information. Borrower welfare thus strictly increases for such borrowers.

(iii) $B - \eta(\kappa_a^{LS}, \bar{\theta}^S) \cdot D < 0$, but $B - \eta(\kappa_a, \bar{\theta}^S) \cdot D > 0$

In this case, by the same arguments as in (i) and (ii), there will be a unique cutoff $\widehat{\kappa}_a$ such that borrowers with $\theta^F \geq \widehat{\kappa}_a$ apply and are approved conditional on screening revealing positive information, while borrowers with $\theta^F < \widehat{\kappa}_a$ refrain from applying. Borrower welfare thus strictly increases.

Next, consider θ^F such that $\theta^F \in [\tilde{\kappa}_f^{LS}, \kappa_f)$. These borrowers may be affected as, in the lender-preferred equilibrium, they would always obtain a loan in the presence of loan sale opportunities, whereas they would receive a loan only conditional on screening revealing positive information without. We have assumed that without loan sales, the equilibrium is sustainable and borrowers ask for a loan. This implies that given $\theta^F \in [\tilde{\kappa}_f^{LS}, \kappa_f)$, we have that

$$E[U^B] = B - \eta(\theta^F, \bar{\theta}^S) \cdot D > 0.$$

For each θ^F in this region, we have the following cases.

(i) $B - \left(\frac{1}{2}\eta(\theta^F, \underline{\theta}^S) + \frac{1}{2}\eta(\theta^F, \bar{\theta}^S)\right) \cdot D < 0.$

In this case, the borrowers have negative expected utility before screening. An equilibrium with no screening is hence unsustainable as anticipating this, borrowers would not ask for a loan. However, even though no screening, and full lending is the lender-preferred equilibrium, an equilibrium with screening, partial lending and partial loan sales is also sustainable. Compared to the no-loan sale case, there would be no change in the approved borrowers and hence in borrower welfare, while lender profitability increases through partial loan sales.⁴⁰

(ii) $B - \left(\frac{1}{2}\eta(\theta^F, \underline{\theta}^S) + \frac{1}{2}\eta(\theta^F, \bar{\theta}^S)\right) \cdot D \geq 0$

In this case, borrowers accede to the lender preferred equilibrium as their expected utility is weakly positive in the absence of screening. Compared to the case without loan sale opportunities, $(\theta^S = \underline{\theta}^S)$ -type borrowers obtain a loan with loan sales, whereas they would not without. While lender profitability increases, the change in borrower surplus is ambiguous:

⁴⁰We implicitly assume that the screening decision and its outcome are observable to borrowers who can choose not to sign the loan contract if they are not screened. Otherwise, lenders would potentially have an incentive to deviate at the screening and lending stage, which could yield a breakdown of lending.

$$(ii).(a) \quad B - \eta(\theta^F, \underline{\theta}^S) \cdot D \geq 0$$

In this case, $(\theta^S = \underline{\theta}^S)$ -type borrowers have positive expected utility. Hence, borrower welfare increases through the change in lending standards from screening to not screening.

$$(ii).(b) \quad B - \eta(\theta^F, \underline{\theta}^S) \cdot D < 0$$

In this case, $(\theta^S = \underline{\theta}^S)$ -type borrowers have negative expected utility. Because the θ^S -type can never be inferred from the screening and lending decision, they obtain a loan, which decreases borrower welfare compared to the no-loan sale case.

Chapter 4

Security Design with Interim Public Information

Abstract

We analyze a strategic security design and trading game as in [Dang, Gorton, and Holmström \(2011\)](#) but with a generalized structure of public information arrival. In the absence of private information, optimal securities are those least affected by interim public information. We provide conditions such that all securities traded in equilibrium consist of multiple imperfect debt-like tranches: The tranches can not be combined into a single tranche but, conditional on specifying positive payoffs, individually share the feature of debt that payoffs are at the limited liability constraint or on a flat part of the security. Endogenous tranching obtains in the absence of private information or different risk attitudes and introduces a misalignment in the security designer's incentives: standard debt minimizes other market participants' incentives to acquire information, but debt-like tranches are most robust to public information arrival.

4.1 Introduction

The market for collateralized debt obligations (CDOs) has been at the core of the recent financial crisis. Global CDO issuance ballooned to 520.6 billion US Dollar in 2006 (up from 158 billion in 2004). While issuance decreased significantly to 4.3 billion in 2009

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in the wake of the financial crisis, it has started to pick up again, reaching 13 billion in 2011.⁴¹ From these observations and earlier episodes of financial distress it appears that debt and debt-like security structures are at the core of these periods of distress, but nonetheless prevail as one of the major forms of lending in the modern financial system.⁴² The structure of assets traded in financial markets has been a matter of interest to researchers and practitioners alike. The economic literature in the past 30 years has focused on the strategic aspects of their design process. One major finding in the literature is that standard debt contracts are optimal with respect to informational concerns in a variety of settings (see among others [Townsend \(1979\)](#), [Gale and Hellwig \(1985\)](#), [Innes \(1990\)](#), [Nachman and Noe \(1994\)](#)).

One strand of literature revolves around the information sensitivity of securities and in particular of standard debt. [Gorton and Pennacchi \(1990\)](#) point out that debt dominates equity with respect to protecting uninformed market participants from exploitation by informed ones. Hence, debt is argued to be less information sensitive than equity. [Dang, Gorton, and Holmström \(2011\)](#) fully endogenize the security design process. In a model of strategic security design where securities are traded, they find that standard debt contracts are optimal. This is due to two features: Standard debt minimizes other market participants' incentives to acquire (costly) private information and therefore mitigates adverse selection. Furthermore, it is most robust to interim public information as it minimizes resale value variance.

The present article isolates the public information problem in [Dang, Gorton, and Holmström \(2011\)](#) but generalizes its information structure. We find that the securities most robust to interim public information are composed of debt-like tranches.⁴³ This result nests the analysis with respect to public information in [Dang, Gorton, and Holmström \(2011\)](#) as a special case: Standard debt contracts (SDCs) are composed of a single standard debt tranche - and hence in the class of contracts composed of debt-like tranches - and are optimal if underlying distributions are ordered by the monotone likelihood ratio property. However, we provide conditions for the non-optimality of SDCs with respect to interim public information if the MLRP does not apply. Hence, following

⁴¹Data: Securities Industry and Financial Markets (SIFMA) press release, 2012.

⁴²See [Gorton and Metrick \(2012\)](#) for a detailed overview of the role securitization plays in modern financial systems and in particular the recent crisis.

⁴³While we provide an exact definition later, *debt-like* refers to the fact that whereas these tranches mirror the feature of debt that payoffs are either at the limited liability constraint or on a flat part of the security, this only holds *conditional on specifying positive payoffs*. Debt-like tranches may differ from debt tranches in that they do not necessarily start directly after the previous tranche and thus lack the feature of always paying a positive amount.

the analysis by [Dang, Gorton, and Holmström \(2011\)](#), there is a misalignment in the interests of investors: Standard debt contracts minimize the incentive for private information acquisition, whereas debt-like tranches - in certain cases even levered equity - are most robust to public information arrival.

We identify conditions such that the resulting security traded is composed of multiple imperfect debt-like tranches, where *imperfect* refers to the fact that combination into a single debt tranche is not possible and thus implies multiple residuals which are kept on the books of the issuer. This result is new in that it is not motivated by different risk attitudes of market participants and explicitly accounts for the issuance of multiple tranches. The literature at large typically obtains a result of tranching where a single debt tranche (or equity tranche) is split off from a cash flow and sold (or held), see for example [DeMarzo \(2005\)](#). [DeMarzo \(2005\)](#) shows that an informed investor may exploit her private information best by pooling assets it acquires from an issuer and tranching off a highly liquid standard debt tranche. This tranche can be sold due to its low information sensitivity, with the issuer retaining the resulting equity tranche. Our article, on the other hand, motivates tranching by showing that assets composed of multiple debt-like tranches are most robust to interim public information arrival. These tranches are imperfect in the sense that they can not be combined in a single debt tranche. Hence, the issuer holds on to not one but multiple residual equity tranches. These are kept on the books of the security issuing institution and correspond structurally to the risk retention by sponsors of ABCP conduits as documented by [Acharya, Schnabl, and Suarez \(2013\)](#) and [Gorton and Metrick \(2012\)](#).

From a technical perspective, the result is novel because the imperfection of the tranches implies a discontinuous optimal security structure. This is in contrast to the literature where continuity is either imposed by restricting attention to securities monotonic in both the payoff of the underlying cash flow and the residual held by the issuer, see for example [Biais and Mariotti \(2005\)](#), or where it obtains endogenously as solution to the security design problem (typically in the form of standard debt, see for example [Innes \(1990\)](#)).⁴⁴

The interaction of private and public information effects is fundamental for understanding the performance of real world financial markets, particularly the CDO market in the recent crisis. As [Dang, Gorton, and Holmström \(2011\)](#) point out, adverse information about an asset's value may render it more information sensitive and thus lead

⁴⁴It should be noted that we obtain a condition for the non-optimality of standard debt contracts even when continuity of the security design is imposed via dual monotonicity, see Section 4.6.4.

to adverse selection and a collapse of trade. The present article does not address this interaction. However, in contrast to their framework, it exhibits a basic tradeoff for the security designer, whose incentives with respect to the public and private information issues may be misaligned. Standard debt is not necessarily optimal with respect to interim public information arrival.

4.1.1 Relation to the Literature

As noted in the introduction, there are multiple channels through which information affects the security design process. Private information is a well-known issue since it may lead to losses when trading with better informed parties. Conversely, if the seller of a security is informed, she faces a lemons problem as in [Akerlof \(1970\)](#). Another important factor is public, i.e. symmetric, information arrival since gains and losses from information may be incurred differently. Specifically, it may be the case that de facto gains from positive information about an asset's value may not be fully capitalized on due to liquidity constraints of potential trading partners. If the value losses from negative information, however, are fully incurred, even symmetric information arrival during the holding period affects the security design process: There is an incentive to create securities which are robust to such public information.

Starting with the seminal papers by [Diamond and Verrecchia \(1981\)](#), [Diamond and Dybvig \(1983\)](#) and [Gorton and Pennacchi \(1990\)](#), a multitude of authors has examined the effects of asymmetric information on financial markets and connected it to the structure of traded assets. [Diamond and Verrecchia \(1981\)](#) focus on information aggregation in markets without financial intermediaries and show that price is not always fully revealing. [Diamond and Dybvig \(1983\)](#) describe how a financial intermediary can improve the situation of agents who face idiosyncratic uncertainty by providing liquidity irrespective of the state of the world.

[Gorton and Pennacchi \(1990\)](#) explicitly model both uninformed and informed traders active in a single market. They show that a financial intermediary can prevent uninformed traders' losses to insiders who hold private information by issuing (riskless) debt. The concept of liquidity in this context revolves around securities or assets which can be traded without losses to potentially better informed parties. A common feature in the literature is the existence of debt which is motivated by its feature of low information sensitivity. Low information sensitivity refers to the concept of minimizing the value of information. This concept has two main characteristics: On the one hand,

in the presence of - potentially exogenous - asymmetric information in the market, gains from exploiting the informational advantage are minimized. On the other hand, if information has to be acquired, incentives for doing so are minimized because the aforementioned exploitation yields minimal profits. This strand of literature, however, takes the existence of debt as given and thus does not explicitly show how debt arises in an endogenous fashion. While it is established that trading with debt as the financial instrument of choice is superior (in terms of preventing losses on behalf of the uninformed party) to other securities, the issue of actual optimality of debt was not studied until recently. It should be noted that in settings with costly state verification or non-verifiable returns, studied for example by [Townsend \(1979\)](#), [Gale and Hellwig \(1985\)](#) and [Aghion and Bolton \(1992\)](#), debt is shown to be optimal for issuing a security in a primary market. The asymmetric information in these settings, however, is assumed to be exogenous instead of arising endogenously following choices and actions of the involved agents.

[DeMarzo and Duffie \(1999\)](#) analyze a security designer and issuer whose private information results in illiquidity in the sense of a downward sloping demand curve. Standard debt is shown to be optimal under certain conditions, primarily the existence of a uniform worst case, because it minimizes the value of the private information the issuer holds. Furthermore, by retaining the resulting levered equity on its books, the issuer gives a signal which lessens the lemons problem. This signal is credible but costly due to the preference for cash over longterm investments. [Biais and Mariotti \(2005\)](#) consider an alternative approach to the trading game. They let the issuer commit to a price-quantity menu before private information is observed and the fraction of the security offered is chosen. Analyzing different forms of competition amongst liquidity suppliers, they find that debt is optimally issued because it minimizes the consequences of adverse selection (competitive case) and mitigates the market power of the liquidity supplier (monopolistic case).

The idea is expanded in [Dang, Gorton, and Holmström \(2011\)](#) and subsequently [Dang, Gorton, and Holmström \(2013\)](#). Building on [Dang \(2008\)](#), who analyzes bargaining with endogenous information acquisition, they show that standard debt is least information sensitive among the class of nondecreasing securities satisfying limited liability and nonnegativity constraints. In this sense, they extend the comparative result from [Gorton and Pennacchi \(1990\)](#). Furthermore, the paper proposes an explanation for the central role of debt in financial crises: The authors argue that a crisis is a collapse of trade after one-sided information acquisition has been triggered. This collapse is due to

adverse selection and hence a sharp drop in trading volume.⁴⁵ In similar fashion, [Yang \(2012\)](#) analyzes a game where information acquisition is no longer rigidly structured but flexible and arrives at the same conclusion: Standard debt contracts minimize incentives to acquire information and therefore maximize liquidity. This result is stronger than that of [Dang, Gorton, and Holmström \(2011\)](#) in that it holds irrespective of the composition of the underlying asset pool, i.e. of the number of assets and the correlation between their returns.

[Dang, Gorton, and Holmström \(2011\)](#) further show that debt is not only optimal with respect to potential private information acquisition but also most robust to public information arrival.⁴⁶ The incentives of the investor align: standard debt disincentivizes potential trading partners to acquire information and is least sensitive to public information arrival during the holding period.

A related literature considers the extent of information available to market participants and whether all such information should be disclosed. [Kaplan \(2006\)](#) shows that it can be efficient for a bank to commit to a policy keeping information about its risky assets secret despite being thus forced to offer non-contingent deposit contracts only. [Pagano and Volpin \(2012\)](#) in turn show that issuers of assets choose to publish coarse instead of precise ratings to enhance liquidity in the primary market, even though this reduces secondary market liquidity. Both articles have in common that, endogenously, not all available information is utilized. However, these articles assume the existence of debt.

Nonetheless, the optimality of debt for trading in both primary and secondary markets critically relies on the structure of the public information which becomes available between the trading periods. By altering the structure, incentives to deviate from debt and instead move towards levered securities may come into play, thus misaligning the incentives to minimize potential information production and to minimize the resale value variance.

[Farhi and Tirole \(2012\)](#) consider a security trading game with a binary state of nature. In this setup, tranching of securities is feasible only in the sense that they consist of a riskless debt and a pure equity component. They provide conditions under which the *insulation* effect, i.e. the effect that tranching off riskless debt protects this tranche from liquidity risk, outweighs the *trading adjuvant* effect of increasing the likelihood

⁴⁵This drop is due to the lemons problem.

⁴⁶Technically, this result builds on earlier work by [DeMarzo, Kremer, and Skrzypacz \(2005\)](#).

that the risky equity tranche is not sold. Furthermore, irrespective of the relative weight of the two effects, tranching always works against communality of information: Tranching deters information acquisition when it should be encouraged and encourages it when it should be deterred. Hence, even if tranching is superior because the insulation effect outweighs the trading adjuvant effect, it becomes undesirable once information acquisition is endogenized. [Farhi and Tirole \(2012\)](#) also extend their framework to a dynamic setting and show that liquidity is self-fulfilling. The expectation of liquidity in future states increases liquidity in the present.

While the conclusion that tranching has socially adverse effects is seemingly opposed to our finding that contracts composed of debt-like tranches are optimal, it is important to note the differences in the analyses: [Farhi and Tirole \(2012\)](#) consider private information and its potential acquisition whereas we are concerned with interim public information arrival. Thus, the idea that the incentives of security designers are not aligned with respect to the different types of information is in fact corroborated. Furthermore, they analyze a binary outcome space as opposed to a continuum. The richer forms of tranching securities discussed in this article are not feasible in such a setting.

The paper proceeds as follows: Section 4.2 introduces the model as well as key concepts and definitions. Section 4.3 solves the security design and trading problem after public information has arrived, and Section 4.4 addresses the security design problem at the initial trading stage. Section 4.5 further characterizes the optimal contract and provides examples. Section 4.6 discusses extensions of the model and its robustness. Section 4.7 concludes.

4.2 The Model

The model is that of [Dang, Gorton, and Holmström \(2011\)](#) without private information acquisition, but with a generalized public information structure. The framework is kept deliberately simple to isolate the channel of public interim information in the security design process. There are three agents in the economy: An institution (called 'bank' or 'issuing institution') B , an investor I and a representative agent M who represents the market. In the absence of private information and associated information asymmetries, the market is composed of agents willing to transfer utility across periods. The utilities of the agents across three time periods $t = 1, t = 2$ and $t = 3$ are denoted by U^i for each agent $i \in \{B, I, M\}$. Utilities are given by

$$\begin{aligned} U^B &= C_1^B + \frac{1}{\phi}C_2^B + C_3^B \\ U^I &= C_1^I + \sigma C_2^I + C_3^I \\ U^M &= C_2^M + C_3^M. \end{aligned}$$

C_t^i denotes the consumption of agent i in period t and $\phi > 1, \sigma > 1$ are parameters reflecting the intertemporal difference in marginal utilities of consumption. The bank has a (weak) preference for consumption at $t = 1$. Think for example of a preference for undertaking outside investment options which require further cash. I prefers consumption in period 2. Consumption is assumed to occur at the end of a given period. At period 2, M represents agents willing to transfer consumption and therefore utility from the second into the third period.⁴⁷

The agents' endowments are common knowledge, nonstorable and given as follows: The bank owns a pool of assets with stochastic return X distributed on $\mathbb{I} \subseteq \mathbb{R}_+$ which is due at $t = 3$. For ease of notation, we restrict attention to an open interval of the form $(x_L, x_H) \subset \mathbb{R}_+$, which includes (x_L, ∞) .⁴⁸ The inclusion of boundary points (if an upper bound exists) would not alter the results as long as mass points are ruled out. I holds an endowment of ω^I at $t = 1$, while M holds an endowment of ω^M at $t = 2$. Formally, letting $\tilde{\omega}^i = (\omega_1^i, \omega_2^i, \omega_3^i)$ denote the endowment vector of agent i ,⁴⁹

$$\begin{aligned} \tilde{\omega}^B &= (0, 0, X) \\ \tilde{\omega}^I &= (\omega^I, 0, 0) \\ \tilde{\omega}^M &= (0, \omega^M, 0). \end{aligned}$$

X is stochastic and its payoff is publicly observable at $t = 3$. At $t = 2$, information about the distribution of X arrives: This *interim public information* is publicly observable but not verifiable in the sense that it can not be contracted upon. The endowments ω^I, ω^M are fixed with $\omega^I > x_L$.⁵⁰ The analysis performed in the following sections

⁴⁷A preference for consumption in period 3 could be introduced and would not alter the analysis performed. However, the intertemporal rate of substitution is normalized to 1 to simplify expressions. Likewise, the issuing institution is set up to be indifferent between consumption at $t = 1$ and $t = 3$ for expositional purposes.

⁴⁸Hence, canonical distributions such as exponential distributions or χ^2 -distributions are valid.

⁴⁹The remaining endowments are normalized to 0 for simplicity. Cash endowments at these points in time would lead to first trading them before securitizing the project and trading these securities. The problem would remain unchanged.

⁵⁰If $\omega^I \leq x_L$, riskless debt with a face value lower or equal to x_L can always be issued. This debt is then unaffected by public information.

remains unchanged if ω^M is stochastic, as long as it is independent of the public news. Finally, we assume $\omega^I < E_f[X]$ to ensure that the whole project cannot be acquired by I at $t = 1$. Note that since the model abstracts from private information concerns, there is no disagreement about the value of the assets involved. Thus, the problem is that of a (limited) number of agents who wish to shift a known amount of consumption intertemporally. Above some threshold value of owned assets, gains from good interim news can no longer be realized because there is no agent willing and/or liquid enough to buy the assets for their underlying value.⁵¹ The problem corresponds to that of Diamond-Dybvig-type models where a limited fraction of the population is patient and therefore willing to shift a limited amount of consumption into the last period by buying assets in the interim period, see for example [Diamond and Dybvig \(1983\)](#), [Jacklin and Bhattacharya \(1988\)](#) and [Chari and Jagannathan \(1988\)](#). This results in a form of cash-in-the-market pricing as in [Allen and Gale \(1994\)](#). The difference is that whereas in [Allen and Gale \(1994\)](#), the price of the asset adjusts so that all of it is traded in equilibrium, in our setup the asset/security is restructured to capture all the available surplus. The resulting residual is then held for future consumption.

In this setup, a public planner can realize gains from trade through a simple reallocation of endowments. For I to consume at $t = 2$, she needs to trade with B at $t = 1$ by buying (parts of) the project. She can then sell shares in the project to M at $t = 2$. When agents trade, they exchange promises contingent on the observable realization of X . These promises are called securities. Throughout this article, securities have to satisfy the following requirements:

Definition 4.1 *A security is a mapping from a domain $\mathbb{D} \subseteq \mathbb{R}_+$ into the real numbers, $s : \mathbb{D} \rightarrow \mathbb{R}$,*

that satisfies the following restrictions:

- *limited liability: $s(x) \leq x$ for all $x \in \mathbb{D}$*
- *nonnegativity: $s(x) \geq 0$ for all $x \in \mathbb{D}$*
- *nondecreasingness: $\forall x_1, x_2 \in \mathbb{D} : x_1 \geq x_2 \Rightarrow s(x_1) \geq s(x_2)$.*

The set of securities $s : \mathbb{D} \rightarrow \mathbb{R}$ satisfying these restrictions is denoted $S_{\mathbb{D}}$.

⁵¹Likewise, portfolio considerations may lead potential buyers to not wish to overinvest in the specific security class offered. Hence, they have no incentive to buy the assets at their (conditional) expected value above some threshold.

For simplicity, stochastic contracts are ruled out because they make the nondecreasingness requirement hard to evaluate. The nondecreasingness restriction in itself is a standard assumption justified by a moral hazard opportunity of the issuer, see for example [Innes \(1990\)](#). As in [Innes \(1990\)](#) and different than for example [Biais and Mariotti \(2005\)](#), we only impose nondecreasingness and not dual monotonicity, i.e. that both the security's payoff $s(x)$ and the payoff of the residual $x - s(x)$ are nondecreasing. In fact, in contrast to the standard debt contract derived as the optimal contract in [Innes \(1990\)](#) and [Dang, Gorton, and Holmström \(2011\)](#), our resulting optimal security structure explicitly violates nondecreasingness of the residual. We nonetheless feel comfortable with this assumption.⁵² The main moral hazard opportunity which impacts the security design process in our setting is the upward distortion on behalf of the issuer. Consider for example a collection of mortgages and let X be the (uncertain) aggregate repayment. It may be feasible to declare certain unpaid mortgages as repaid. While the issuer would lose out on the true repayment in that case, she might be better off were the security decreasing in the underlying aggregate return. Other moral hazard opportunities seem less plausible, however, in particular the ones which would lead to imposing dual monotonicity: Once an individual repayment has been made, the resulting paper trail makes it impossible for the issuer to hide it, i.e. to distort the true repayment downward. Likewise, an upward distortion on behalf of the security acquirer would require her to find individual mortgage holders and supply them with cash to use for mortgage repayment. Nonetheless, subsequent to our analysis, we discuss how the resulting security structure would change if dual monotonicity were imposed instead of nondecreasingness: The key result that standard debt is not necessarily optimal carries over to such a setting.⁵³

Pooling of securities based on different projects, i.e. X_1, X_2 , is not included in the model and explicitly ruled out at $t = 2$.⁵⁴ However, the random endowment X can be interpreted as a collection of different assets/securities. Similarly, the setup does not rule out tranching. Since all agents in the model have constant marginal utilities of consumption in any given period, they can be thought of as representing an arbitrarily large number of identical agents who hold an endowment with an aggregate endowment

⁵²As [DeMarzo, Kremer, and Skrzypacz \(2005\)](#) note, 'a standard motivation for dual monotonicity is that, if it did not hold, parties would "sabotage" the project and destroy output. [...] Whether revenues can be distorted in this way depends on the context.'

⁵³See Section 4.6.4.

⁵⁴Allowing pooling of securities depending on correlated underlying payoffs would affect the results. However, the main idea that individual securities should minimize resale value variance subject to the public information still factors into the security design process. See Section 4.6.2 for a more detailed discussion.

equal to what is represented in the model by ω^I (in case of I) and ω^M (in case of M) respectively. If that is the case, any tranching which overall still satisfies limited liability can also be represented by a single contract.⁵⁵ In interpreting the results, it is sometimes natural to think of the 'optimal' security as composed of different tranches which are sold separately to interchangeable market participants with identical intertemporal substitution rates.

The timing of the game is as follows: At $t = 1$, I makes a take-it-or-leave-it offer to the bank. This offer consists of a security s conditional on the return of X at $t = 3$ which she is willing to buy, and a price p which she pays in exchange. At $t = 2$, a public signal regarding the distribution of X is revealed to all agents. Then, I may make a take-it-or-leave-it offer to agent M . This offer consists of a security \hat{s} conditional on the return of s (and hence of X), and a price \hat{p} . We assume that the bargaining power lies in the hand of the investor in both stages. This, coupled with the assumption that marginal utility is constant, is made to isolate the security design process with respect to arriving public information.⁵⁶ The second trading stage is only relevant if trade occurred at $t = 1$. Furthermore, the limited liability constraint imposes that $\hat{s}(x) \leq s(x)$ for all $x \in \mathbb{I}$.

We do not allow contracts between I and M to be written *before* the public information is realized. This is due to two reasons: On the one hand, allowing such contracts would render the problem posed by the interim information irrelevant. It would be ex-ante optimal and incentive compatible to agree to always trade s in exchange for a cash payment of $E_f[s(x)]$ at $t = 2$ irrespective of the public information.⁵⁷ This would allow agents to realize the maximum gains from trade subject to participation constraints. However, this requires perfect anticipation of I being a seller at $t = 2$. To rationalize our setup where contracting is only feasible after information arrival, consider the following variation: Let I be patient with probability $\alpha \in (0, 1)$. If I is patient, she prefers consumption at $t = 3$ and will hold the security instead of trading with M . With probability $(1 - \alpha)$, our current setup arises and I is impatient. For α large enough, the 'losses' from writing an ex-ante contract and being patient outweigh the 'benefits'

⁵⁵Consider for example the issuance of two securities contingent on X , $s^1(x)$ and $s^2(x)$ with $s^1(x) + s^2(x) \leq x, \forall x \in (x_L, x_H)$, which are also nondecreasing. Due to the constant marginal utilities of consumption and the unique trading partner (see above), this is equivalent to issuing a single security $s(x) = s^1(x) + s^2(x), \forall x \in (x_L, x_H)$, which will still satisfy limited liability and nondecreasingness.

⁵⁶In this particular game, the security design problem remains identical as long as the investor has at least some bargaining power in the first trading stage.

⁵⁷This holds whenever $\omega^I = E_f[s(x)] \leq \omega^M$. Otherwise, it would be optimal to design \hat{s} satisfying limited liability with respect to s and trade \hat{s} for $E_f[\hat{s}(x)] = \omega^M$.

of having such a contract in place when impatient. Hence, only in the impatient case, a security design problem arises and in particular follows the setup of our model.

Ex ante, X is distributed randomly on \mathbb{I} with density $f(x)$, cumulative distribution function $F(x)$ and finite mean $\int_{\mathbb{I}} x dF(x) < \infty$. To model information arrival, let f be a mixture distribution, i.e. let $\lambda \in (0, 1)$ and

$$f(x) = \lambda f_1(x) + (1 - \lambda) f_2(x) \quad (4.1)$$

where f_1 and f_2 are strictly positive densities.⁵⁸ The public signal arriving at $t = 2$ reveals the true distribution, i.e. whether X is distributed according to f_1 or according to f_2 . All distributions are common knowledge, as well as λ . The public information is not verifiable, i.e. securities can not be made contingent on the realization of the public signal. [Dang, Gorton, and Holmström \(2011\)](#) order the underlying distributions by imposing the monotone likelihood ratio property, i.e. that $f_1(x)/f_2(x)$ is monotone in x . We generalize the condition by imposing ordering via first order stochastic dominance:

$$\mathbf{FOSD:} \quad \forall x \in (x_L, x_H) : F_1(x) \geq F_2(x). \quad (4.2)$$

For densities, first order stochastic dominance nests the monotone likelihood ratio property. Hence, if the monotone likelihood ratio property holds, first order stochastic dominance is also satisfied, whereas the reverse is not necessarily true. Note that for nondecreasing securities, first order stochastic dominance implies that

$$\forall s : E_{f_1}[s(x)] \leq E_f[s(x)] \leq E_{f_2}[s(x)]. \quad (4.3)$$

To sum up, this is the timeline of the game in extensive form:

$t = 1.0$: I makes a take-it-or-leave-it offer (s, p) to B

$t = 1.1$: issuer B accepts the contract (s, p) or not

$t = 2.0$: distribution F_i , $i = 1, 2$ is publicly observed

$t = 2.1$: I makes a take-it-or-leave-it offer (\hat{s}, \hat{p}) to M

⁵⁸The strict positivity facilitates but does not qualitatively change the analyses. It allows for certain existence and uniqueness statements to be made without accounting for the special case of $f(\cdot)$ being locally zero.

$t = 2.2$: agent M accepts the contract (\hat{s}, \hat{p}) or not

$t = 3.0$: x is realized and publicly observed, I is paid $s(x)$, M is paid $\hat{s}(x)$.

4.2.1 Concepts and Definitions

There are several classes of securities which play an important role in the subsequent analysis. One such class is that of *standard debt contracts*. Standard debt contracts are contracts which pay out according to the limited liability constraint up to their face value; in case the realization of the underlying cash flow exceeds this value, the payoff is capped. Formally, the following definition captures this idea.

Definition 4.2 A standard debt contract (SDC) on an interval $\mathbb{I} \subseteq \mathbb{R}_+$ is given by

$$s^{SDC}(x; D) = \min\{x, D\}$$

where $D \geq 0$ is the face value of the debt contract.

Note that if $\forall x \in \mathbb{I} : D \geq x$, s^{SDC} corresponds to an equity contract

$$s^{SDC}(x; D) = x.$$

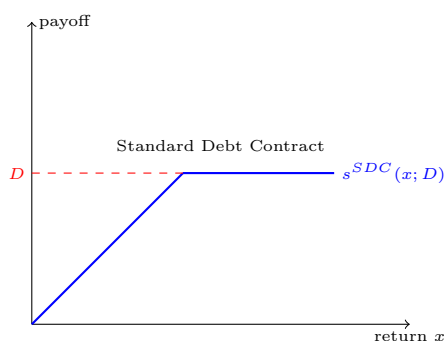


Figure 4.1: Standard Debt Contract

A second class of securities which is important for our analysis is the class of *levered equity contracts*. levered equity contracts only pay out if the payoff of the underlying cash flow exceeds a certain threshold (L), but then pay up to the limited liability constraint. Formally, this is captured by the following definition.

Definition 4.3 A levered equity contract (LE) on an interval $\mathbb{I} \subseteq \mathbb{R}_+$ is given by

$$s^{LE}(x; L) = x \cdot \mathbf{1}_{x \geq L}$$

where L is the equity cutoff.

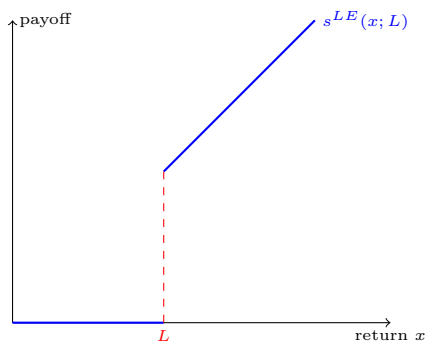


Figure 4.2: Levered Equity Contract

Given $\omega^I < E_f[x]$ and the strict positivity of densities f_1 and f_2 , there exists a unique face value D (a unique equity cutoff L) such that the induced expected value of the standard debt contract (levered equity contract) is equal to ω^I . This is captured by Lemma 4.1.

Lemma 4.1 For any given $\omega^I < E_f[X]$, there exists a unique $D(\omega^I)$ such that

$$E_f[s^{SDC}(x; D(\omega^I))] = \omega^I.$$

Likewise, there exists a unique $L(\omega^I)$ such that

$$E_f[s^{LE}(x; L(\omega^I))] = \omega^I.$$

As stated above, the proof follows immediately from the strict positivity of densities and is thus omitted.

Another important class of contracts are what we denote *contracts composed of debt-like tranches*.

Definition 4.4 A contract composed of debt-like tranches is characterized by a strictly increasing sequence $\{x_i\}_{i=1}^N \in (x_L, x_H)$ of points and a strictly increasing sequence $\{D_i\}_{i=1}^N \in \mathbb{R}_+$ of face values where

$$x_i > D_{i-1} \forall i \geq 2.$$

The contract s^{TD} is then characterized by the following payoff structure:

$$s^{TD}(x) = \begin{cases} 0 & \text{for } x < x_1 \\ \min\{x, D_i\} & \text{if } x \in (x_i, x_{i+1}), i < N \\ \min\{x, D_N\} & \text{if } x \in (x_N, x_H) \end{cases}$$

whenever N is finite, and

$$s^{TD}(x) = \begin{cases} 0 & \text{for } x < x_1 \\ \min\{x, D_i\} & \text{if } x \in (x_i, x_{i+1}), i < N \\ \min\{x, D\} & \text{if } x \geq \sup_j x_j \end{cases}$$

otherwise, where $D = \sup_j D_j$ if the supremum exists and $D = +\infty$ otherwise.⁵⁹ At all points of the sequence $\{x_i\}$, the payoff is arbitrary but has to be consistent with limited liability and nondecreasingness of s^{TD} .

Contracts composed of debt-like tranches have payoffs either on the 45 degree line $s(x) = x$, which corresponds to binding limited liability, or on a flat part of the security, up to pointwise deviations with measure 0. However, they differ from regular debt contracts in that the tranches are imperfect. Contracts which satisfy the definition are of the following form (here an example with two tranches)

$$s(x) = \begin{cases} 0 & \text{for } x \in (x_L, x_1) \\ \min\{x, D_1\} & \text{for } x \in [x_1, x_2) \\ \min\{x, D_2\} & \text{for } x \in [x_2, x_H) \end{cases}$$

where $D_2 > D_1$ and $x_2 > D_1$. This security is the sum of the two tranches:

$$s^{DT_1}(x) = \begin{cases} 0 & \text{for } x \in (x_L, x_1) \\ \min\{x, D_1\} & \text{for } x \in [x_1, x_H) \end{cases}$$

⁵⁹Hence, the last tranche is a levered equity tranche in that case.

$$s^{DT_2}(x) = \begin{cases} 0 & \text{for } x \in (x_L, x_2) \\ \min\{x, D_2\} - D_1 & \text{for } x \in [x_2, x_H]. \end{cases}$$

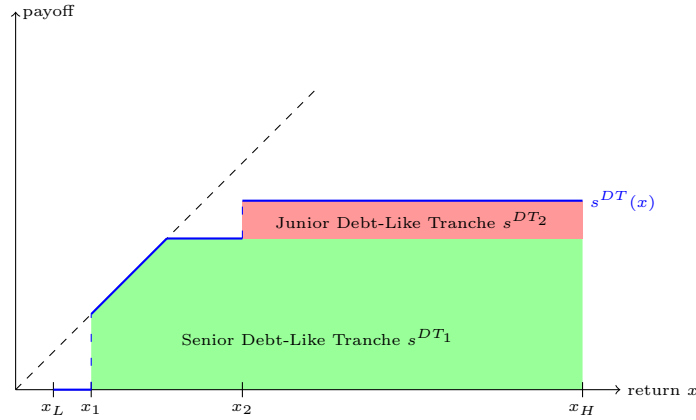


Figure 4.3: Canonical Contract Composed of Debt-Like Tranches

s^{DT_2} in the example is a junior tranche: It pays out only for high realizations of x , and thus only after s^{DT_1} , the senior tranche, has been paid in full.

From the above definitions, it is clear that any standard debt contract - as well as any levered equity contract - is also composed of debt-like tranches, but the reverse does not hold. Furthermore, the contracts differ with respect to the concept of leverage.

Definition 4.5 *A nondecreasing security s on the interval (x_L, x_H) is levered if*

$$\exists x, \hat{x} \in (x_L, x_H) \text{ such that } s(x) < x \wedge s(x) < s(\hat{x}).$$

Leverage refers to the idea of speculating on high returns. A security is levered if it does not pay up to the limited liability constraint for certain values, but then has a higher payoff for higher realizations of the underlying cash flow. By increasing the payoff where limited liability is not binding, this dependability on high returns can be mitigated.

The only non-levered contracts are standard debt contracts, whereas contracts composed of debt-like tranches which are not simultaneously standard debt are levered. As [Dang, Gorton, and Holmström \(2011\)](#) have shown, leverage leads to higher incentives

for private information acquisition.

We will show that contracts composed of debt-like tranches are optimal with respect to interim public information arrival. This introduces a tradeoff in the investor's incentives: the investor wishes to buy a standard debt contract to avoid private information acquisition by his potential trading partners and prefers debt-like tranches due to their robustness to interim public information.

4.3 Security Design and Trading after Information Arrival

To solve for an equilibrium of the game, we first determine the optimal security designed and issued after public information arrival at $t = 2$. At $t = 2$, the bank B and representative market agent M cannot profitably trade. Thus, trade may only occur if I possesses some security s acquired from B at $t = 1$. She may either sell or use this security as collateral for a new security which is offered to M at $t = 2$. Since all information is public, trade can only occur at a price equal to the common conditional expected value of the offered security.⁶⁰ Hence, the optimal strategy for I depends on the relation of the updated value of s after public information to the market endowment ω^M .

Lemma 4.2 *Suppose I holds a security s at $t = 2$ after the arrival of the public information. Any security \hat{s} which is in equilibrium traded to M satisfies:*

(i) *If $E[s(x)|f_i] \leq \omega^M$: $\hat{s}(x) = s(x)$ for all $x \in (x_L, x_H)$.*

(ii) *If $E[s(x)|f_i] > \omega^M$: $E[\hat{s}(x)|f_i] = \omega^M$ and $\hat{s}(x) \leq s(x)$ for all $x \in (x_L, x_H)$.*

\hat{s} is sold to M at its conditional expected value $\hat{p} = E[\hat{s}|f_i]$.

One particular $\hat{s}(x)$ which can implement this for $E[s(x)|f_i] > \omega^M$ is

$$\hat{s}(x) = \tau s(x) \text{ for all } x \in (x_L, x_H) \text{ where } \tau = \frac{\omega^M}{E[s(x)|f_i]}.$$

⁶⁰Recall that the bargaining power lies with I and that M is indifferent between consumption at $t = 2$ and $t = 3$.

Lemma 4.2 follows immediately from the fact that I makes a take-it-or-leave-it offer to M and has preference for consumption at $t = 2$. Intuitively, if the asset is worth weakly less than the market endowment ω^M , it is optimal for I to sell the whole security to maximize consumption at $t = 2$. If the endowment constraint binds, I sells a security that is worth strictly less than $E[s(x)|f_i]$. This new security \hat{s} must satisfy limited liability with respect to s and $E[\hat{s}|f_i] = \omega^M$ to maximize consumption at $t = 2$. In this case, I holds on to the residual security $(s - \hat{s})$ and consumes this remainder - if it is positive - at $t = 3$ after the realization of X becomes observable and s, \hat{s} pay out.

Lemma 4.2 greatly simplifies the analysis of the security design problem faced by I at $t = 1$. It allows to write the expected utility of I , given that she acquires a security s at $t = 1$, as follows:

$$\begin{aligned} EU(s) &= \omega^I - E_f[s(x)] + \lambda (\sigma \min \{E_{f_1}[s(x)], \omega^M\} + \max \{E_{f_1}[s(x)] - \omega^M, 0\}) \\ &\quad + (1 - \lambda) (\sigma \min \{E_{f_2}[s(x)], \omega^M\} + \max \{E_{f_2}[s(x)] - \omega^M, 0\}) \quad (4.4) \\ &= \omega^I + (\sigma - 1) (\lambda \min \{E_{f_1}[s(x)], \omega^M\} + (1 - \lambda) \min \{E_{f_2}[s(x)], \omega^M\}) \quad (4.5) \end{aligned}$$

where we have used $E_f[s(x)] = \lambda E_{f_1}[s(x)] + (1 - \lambda) E_{f_2}[s(x)]$ and $E_{f_1}[s(x)] \leq E_f[s(x)] \leq \omega^I$. As $\sigma > 1$, utility is weakly increasing in both $E_{f_1}[s(x)]$ and $E_{f_2}[s(x)]$. Thus, it is weakly optimal for I to exhaust her endowment at $t = 1$, i.e. to acquire a security valued $E_f[s(x)] = \omega^I$ at $t = 1$.

Furthermore, gains from trade may be limited by the friction of a limited endowment ω^M of M at $t = 2$. In (4.5), this is captured by $\min \{E_{f_1}[s(x)], \omega^M\}$ in the bad state and $\min \{E_{f_2}[s(x)], \omega^M\}$ in the good state respectively. As only nondecreasing securities are considered and since f_1 and f_2 are ordered by first order stochastic, $E_{f_1}[s(x)] \leq E_{f_2}[s(x)]$ holds for all s . Thus, a binding endowment constraint in the bad state, $\min \{E_{f_1}[s(x)], \omega^M\} = \omega^M$, implies a binding endowment constraint in the good state, i.e. $\min \{E_{f_2}[s(x)], \omega^M\} = \omega^M$. The reverse, however, is not necessarily true.

Consider two securities s_1, s_2 with the same unconditional expected value $E_f[s_1(x)] = E_f[s_2(x)] = k$, but with differing conditional expected values, i.e. $E_{f_1}[s_1(x)] \neq E_{f_1}[s_2(x)]$. Without loss of generality suppose $E_{f_1}[s_1(x)] < E_{f_1}[s_2(x)]$ and therefore $E_{f_2}[s_1(x)] > E_{f_2}[s_2(x)] \geq E_{f_1}[s_2(x)]$. Then

$$\begin{aligned}
 & EU(s_2) - EU(s_1) \\
 &= \omega^I + (\sigma - 1) (\lambda \min \{E_{f_1}[s_2(x)], \omega^M\} + (1 - \lambda) \min \{E_{f_2}[s_2(x)], \omega^M\}) \\
 &\quad - \omega^I + (\sigma - 1) (\lambda \min \{E_{f_1}[s_1(x)], \omega^M\} + (1 - \lambda) \min \{E_{f_2}[s_1(x)], \omega^M\}) \\
 &= (\sigma - 1) [(\lambda \min \{E_{f_1}[s_2(x)], \omega^M\} + (1 - \lambda) \min \{E_{f_2}[s_2(x)], \omega^M\}) \\
 &\quad - (\lambda \min \{E_{f_1}[s_1(x)], \omega^M\} + (1 - \lambda) \min \{E_{f_2}[s_1(x)], \omega^M\})]. \tag{4.6}
 \end{aligned}$$

It follows that $EU(s_1) \leq EU(s_2)$, i.e. that the security with the higher expected value under bad information leads to a weakly higher expected utility. If $\omega^M \leq E_{f_1}[s_1(x)]$, the endowment constraint binds for both securities in both states and expected utilities are equal. Likewise, if $\omega^M \geq E_{f_2}[s_1(x)]$, the constraint never binds and both securities capture the full surplus. However, if

$$E_{f_1}[s_1(x)] < \omega^M < E_{f_2}[s_1(x)], \tag{4.7}$$

it follows that $EU(s_1) < EU(s_2)$, because

Case 1: $\omega^M < E_{f_1}[s_2(x)]$

$$\begin{aligned}
 & EU(s_2) - EU(s_1) \\
 &\stackrel{(4.6),(4.7)}{=} (\sigma - 1) [\lambda \omega^M + (1 - \lambda) \omega^M - \lambda E_{f_1}[s_1(x)] - (1 - \lambda) \omega^M] \\
 &= (\sigma - 1) \lambda (\omega^M - E_{f_1}[s_1(x)]) \stackrel{(4.7)}{>} 0.
 \end{aligned}$$

Case 2: $E_{f_1}[s_2(x)] \leq \omega^M \leq E_{f_2}[s_2(x)]$

$$\begin{aligned}
 & EU(s_2) - EU(s_1) \\
 &\stackrel{(4.6),(4.7)}{=} (\sigma - 1) [\lambda E_{f_1}[s_2(x)] + (1 - \lambda) \omega^M - \lambda E_{f_1}[s_1(x)] - (1 - \lambda) \omega^M] \\
 &= (\sigma - 1) \lambda (E_{f_1}[s_2(x)] - E_{f_1}[s_1(x)]) > 0. \tag{4.8}
 \end{aligned}$$

Case 3: $E_{f_2}[s_2(x)] < \omega^M$

$$\begin{aligned} & EU(s_2) - EU(s_1) \\ & \stackrel{(4.6),(4.7)}{=} (\sigma - 1) [\lambda E_{f_1}[s_2(x)] + (1 - \lambda)E_{f_2}[s_2(x)] - \lambda E_{f_1}[s_1(x)] - (1 - \lambda)\omega^M] \\ & = (\sigma - 1) (k - \lambda E_{f_1}[s_1(x)] - (1 - \lambda)\omega^M) > 0 \end{aligned}$$

Thus, the following holds for the expected utility of the investor, $EU(s)$:

- Expected utility is increasing in the unconditional expected value $E_f[s(x)]$.
- Holding fixed the unconditional expected value, expected utility is increasing in the expected value after bad information, $E_{f_1}[s(x)]$.

Therefore, it is always optimal for I to design and acquire a security s from the bank which exhausts I 's endowment ω^I at $t = 1$ and which has maximal value after bad interim information among this set of securities. Formally, solutions to the following (equivalent) problems are maximizers of I 's expected utility.

$$(\mathbf{P1}) \max_{s \in S_{\mathbb{I}}} E_{f_1}[s(x)] \quad \text{s.t.} \quad \lambda E_{f_1}[s(x)] + (1 - \lambda)E_{f_2}[s(x)] = \omega^I$$

or equivalently

$$(\mathbf{P2}) \min_{s \in S_{\mathbb{I}}} E_{f_2}[s(x)] \quad \text{s.t.} \quad \lambda E_{f_1}[s(x)] + (1 - \lambda)E_{f_2}[s(x)] = \omega^I.$$

Denote $S_{\mathbb{I},\omega^I} \equiv \{s \in S_{\mathbb{I}} \text{ such that } E_f[s(x)] = \omega^I\}$. Then it holds that a solution to

$$(\mathbf{P1}) \max_{s \in S_{\mathbb{I},\omega^I}} E_{f_1}[s(x)]$$

exists (and hence also a solution to **(P2)**).

Proposition 4.1 *There exists a solution to **(P1)**, i.e.*

$$\exists s^* \in S_{\mathbb{I},\omega^I} : E_{f_1}[s^*(x)] \geq E_{f_1}[s(x)] \text{ for all } s \in S_{\mathbb{I},\omega^I}.$$

The proof for Proposition 4.1 is relegated to the appendix. It can be shown that **(P1)** corresponds to the maximization of a continuous mapping from the convex and closed set $S_{\mathbb{I}, \omega^I}$ into the real numbers. Hence, a maximum is attained on this set.

As noted above, incentives for I are such that acquiring a solution to **(P1)** at $t = 1$ is an equilibrium strategy. Nonetheless, it may be possible that securities which do not solve **(P1)** can be traded in equilibrium. The following proposition yields conditions under which this applies only to solutions to **(P1)**.

Proposition 4.2 *Denote $E_{f_1}[s^*(x)]$ and $E_{f_2}[s^*(x)]$ the expected value of a solution s^* to **(P1)** under bad information and good information respectively. If*

$$(i) \ E_{f_2}[s^*(x)] \geq \omega^M \geq \omega^I \text{ or}$$

$$(ii) \ E_{f_1}[s^*(x)] \leq \omega^M \leq \omega^I$$

*then only solutions to **(P1)** are traded in equilibrium at $t = 1$. Otherwise, there is multiplicity in the sense that securities with different (state-contingent) expected values may be issued at $t = 1$.*

The proof is relegated to the appendix. There are parameterizations such that the set of securities which may be traded at $t = 1$ in equilibrium consists only of solutions to **(P1)**. These parameterizations capture economically relevant problems: Unless ω^M is very high or very low, the conditions of Proposition 4.2 are satisfied.

If ω^M is high, the security design problem becomes less interesting because many securities allow a full realization of gains from trade by never inducing a binding endowment constraint. If ω^M is low, any security exhausting the constraint in both states captures the realizable surplus. The problem is again less interesting because many securities have this characteristic. Even in cases where the conditions of Proposition 4.2 do not apply, however, trading solutions to **(P1)** at $t = 1$ constitutes equilibrium behavior. Nonetheless, multiplicity not only in securities (there may be different securities solving **(P1)**) but also in state-contingent expected values arises.

For example, in the case of $E_{f_1}[s^*(x)] > \omega^M$, a security \hat{s} with $E_f[\hat{s}(x)] < \omega^I$ can be issued at $t = 1$ in equilibrium as long as $E_{f_1}[\hat{s}(x)] \geq \omega^M$, i.e. as long as it exhausts the endowment of M at $t = 2$ in both states.

Henceforth, we will focus on characterizing solutions to **(P1)** and assume that condition (i) or (ii) from Proposition 4.2 is satisfied.

To illustrate the impact of the generalized information structure in contrast to [Dang, Gorton, and Holmström \(2011\)](#), consider what happens if the two underlying distributions have an identical upper tail. This is not possible under the MLRP restriction except for the trivial case $f_1 = f_2$, i.e. the case without uncertainty about the distribution (and hence an uninformative public signal), but covered by the assumption of FOSD. If f_1 and f_2 possess a common upper tail, levered securities may be traded in equilibrium because they allow for zero value variance. Their value is not affected by the interim public revelation of the true distribution; positive payoffs occur only on the (common) upper tail of the distributions.

Proposition 4.3 *If the two distributions f_1 and f_2 have an identical upper tail, i.e. if there exists $\bar{x} \in \mathbb{I}$ such that*

$$\forall x > \bar{x} : f_1(x) = f_2(x) = f(x)$$

and if $\omega^I \leq \int_{\bar{x}}^{x_H} x dF(x)$, then any security s^* with

$$s^*(x) = 0 \text{ for } x \in (x_L, \bar{x})$$

and

$$E_f[s^*(x)] = \omega^I$$

solves **(P1)**. One such security is the levered equity contract

$$s^{LE}(x; L(\omega^I)) = x \cdot \mathbf{I}_{x \geq L(\omega^I)}.$$

The Proposition follows from the fact that specifying positive payoffs only on the common upper tail induces zero resale value variance. Due to **(FOSD)**, securities with zero resale value variance always solve **(P1)**.⁶¹ This yields leverage of the traded securities. Leverage, however, implies an incentive asymmetry for the security designer in the [Dang, Gorton, and Holmström \(2011\)](#) setting: Non-levered standard debt is optimal with respect to private information, whereas levered securities are perfectly robust to interim public information.

The following section presents the general security design problem faced by I at $t = 1$.

⁶¹Typically, such securities do not exist - otherwise, the problem would be trivial. However, they exist for the given restrictions due to the common high tail and upper bound on ω . $L(\omega^I) \geq \bar{x}$ follows from this upper bound.

4.4 The Security Design Problem

As noted before, securities are optimal and thus designed and traded at $t = 1$ if they are solutions to the problems **(P1)** and **(P2)**. In the following, we establish that there always exists a solution to **(P1)** (and thus also to **(P2)**) which is contract composed of debt-like tranches. Moreover, we provide conditions such that all solutions to **(P1)** satisfy this property. If the conditions from Proposition 4.2 are satisfied, this implies that only debt-like tranches are issued and traded in equilibrium at $t = 1$.

A key role in the analysis is played by the likelihood ratio $f_1(\cdot)/f_2(\cdot)$. The analysis in this section proceeds as follows: First, a solution to **(P1)** is established under specific global requirements on the behavior of the likelihood ratio, including the decreasing likelihood case solved in [Dang, Gorton, and Holmström \(2011\)](#). Second, it is shown that certain local variations of solutions to **(P1)** do not affect global nondecreasingness and limited liability while preserving optimality. This is utilized in the proof of the main proposition: It can be shown that there always exists a solution to **(P1)** which are composed of debt-like tranches by breaking the global problem on $(x_L, x_H) \equiv \mathbb{I}$ down into local ones where the solution is known to be a debt-like tranche. Moreover, under certain conditions on the underlying distributions f_1 and f_2 , all optimal securities, i.e. all securities solving **(P1)**, are composed of debt-like tranches.

The following lemma plays an important role in the subsequent analysis.

Lemma 4.3 *Consider two intervals $A, B \subset \mathbb{I}$ with $\int_{A \cap B} 1 dF(x) = 0$ and securities s_1, s_2 . Suppose that for all $x \in A$, $s_1(x) \geq s_2(x)$ and that for all $x \in B$, $s_1(x) \leq s_2(x)$. If $\exists k \in \mathbb{R}_+$ such that*

$$(i) \quad \int_A (s_1(x) - s_2(x)) dF_1(x) \leq k \int_A (s_1(x) - s_2(x)) dF_2(x)$$

$$(ii) \quad \int_B (s_2(x) - s_1(x)) dF_1(x) \geq k \int_B (s_2(x) - s_1(x)) dF_2(x) \text{ and}$$

$$(iii) \quad \int_{A \cup B} s_1(x) dF(x) = \int_{A \cup B} s_2(x) dF(x)$$

then

$$(iv) \int_{A \cup B} s_1(x) dF_1(x) \leq \int_{A \cup B} s_2(x) dF_1(x).$$

If (i) or (ii) holds strictly, so does (iv).

Henceforth, proofs are relegated to the appendix. Lemma 4.3 states the following: Suppose that two securities have the same expected value on the union of disjoint intervals A, B under the mixture distribution f . Furthermore, suppose that s_1 lies weakly above s_2 on A and s_2 weakly above s_1 on B . If the ratio of the expected values of the difference $s_1 - s_2$ on A under bad information (i.e. f_1 being the true distribution) to that under good information (f_2) is weakly (strictly) lower than the ratio of the expected values of the difference $s_2 - s_1$ on B under bad information to that under good information, then s_2 has a weakly (strictly) higher expected value on $A \cup B$ under bad information than s_1 . The idea is that unconditionally, decreasing s_1 to s_2 on A and simultaneously increasing s_1 to s_2 on B does not change the expected value. However, of the change in expected value on A , less is attributed to a change in the expected value under bad information than of the change on B . Therefore, s_2 yields a higher overall expected value under bad information and conversely a lower one under good information than s_1 .

The following lemma corresponds to [Dang, Gorton, and Holmström \(2011\)](#)'s proposition about the optimality of standard debt contracts whenever $f_1(x)/f_2(x)$ is weakly decreasing in x .

Lemma 4.4 *Let $f_1(x)/f_2(x)$ be weakly decreasing in x on \mathbb{I} . Then one security solving (P1) is the standard debt contract*

$$s^{SDC}(x; D(\omega^I)) = \min\{x, D(\omega^I)\}.$$

If $f_1(x)/f_2(x)$ is strictly decreasing, then the standard debt contract $s^{SDC}(x; D(\omega^I))$ is the (up to pointwise deviations) unique security solving (P1).

Intuitively, the standard debt contract $s^{SDC}(x; D(\omega^I))$ with $E_f[s^{SDC}(x; D(\omega^I))] = \omega^I$ is optimal because it puts as much of the security payoff on the lower returns of X as possible. By the MLRP, the relative likelihood of payoffs is decreasing in the realized value of the underlying cash flow. Therefore, this maximizes expected payoff under bad information, i.e. whenever f_1 is the true distribution.

As argued previously, the assumption of a decreasing monotone likelihood ratio is restrictive. For example, following Proposition 4.3, for distributions with a common upper tail levered contracts are optimal. The following lemma and its corollary capture the optimality of levered equity (debt) - in a sense the counterpart to standard debt - whenever the likelihood ratio is increasing instead. Thus, solutions to the extreme cases of global behavior of $f_1(\cdot)/f_2(\cdot)$ are obtained. These are later utilized by relating a global problem without these restrictions to multiple local problems for which the solution has been characterized.

Lemma 4.5 *Let $f_1(x)/f_2(x)$ be weakly increasing in x on \mathbb{I} . Then one security solving **(P1)** is the levered equity contract*

$$s^{LE}(x, L(\omega^I)) = x \cdot \mathbf{I}_{x \geq L(\omega^I)}.$$

*If $f_1(x)/f_2(x)$ is strictly increasing, then the levered equity contract $s^{LE}(x, L(\omega^I))$ is the (up to pointwise deviations) unique security solving **(P1)**.*

Corollary 4.1 *Let $f_1(x)/f_2(x)$ be weakly increasing in x on \mathbb{I} . Then one security solving the modified problem **(P1*)** including an upper bound u*

$$\begin{aligned} \textbf{(P1*)} \quad \max_{s \in \mathcal{S}_{\mathbb{I}}} E_{f_1}[s(x)] \quad \text{s.t.} \quad & \lambda E_{f_1}[s(x)] + (1 - \lambda) E_{f_2}[s(x)] = \omega^I \\ & s(x) \leq u \text{ for all } x \in \mathbb{I}, \end{aligned}$$

where

$$\omega^I \leq \int_{\mathbb{I}} \min[x, u] dF(x),$$

is the levered debt contract

$$s^{LD}(x; L(\omega^I), u) = \min\{x, u\} \cdot \mathbf{I}_{x \geq L(\omega^I)}$$

*where $L(\omega^I)$ is uniquely determined by Lemma 4.1. If $f_1(x)/f_2(x)$ is strictly increasing, then the levered debt contract is the (up to pointwise deviations) unique security solving **(P1*)**.*

Lemma 4.4, Lemma 4.5 and Corollary 4.1 are statements about how global behavior of $f_1(\cdot)/f_2(\cdot)$ on \mathbb{I} impacts the solution to **(P1)**. If the likelihood ratio $f_1(\cdot)/f_2(\cdot)$ is weakly decreasing, standard debt solves **(P1)**. In the case of an increasing likelihood ratio $f_1(\cdot)/f_2(\cdot)$, levered equity or, if an upper bound for the security payoffs is specified, levered debt are solutions.⁶² Note that an increasing likelihood ratio is inconsistent with $F_1(\cdot)$ being first order stochastically dominated by $F_2(\cdot)$ except for the case of $f_1 = f_2 = f$, i.e. the case where there is no uncertainty about the distribution. It nonetheless is essential for further analysis: If $f_1(\cdot)/f_2(\cdot)$ is locally increasing on some interval $(\xi_1, \xi_2) \subset \mathbb{I}$, the problem on this interval can be transformed into one where Lemma 4.5 and/or the following corollary applies. Such a transformation also applies if $f_1(\cdot)/f_2(\cdot)$ is locally decreasing. Hence, Lemma 4.4, Lemma 4.5 and Corollary 4.1 provide the foundation for the following two observations:

Lemma 4.6 *Suppose that $f_1(x)/f_2(x)$ is weakly decreasing in x on $(\underline{x}, \bar{x}) \subset \mathbb{I}$. Let s be an optimal security solving **(P1)** on $(x_L, x_H) \equiv \mathbb{I}$. Denote $e \equiv \int_{\underline{x}}^{\bar{x}} s(x)dF(x)$.*

Define

$$s^*(x) = \begin{cases} s(x) & \text{if } x \notin (\underline{x}, \bar{x}) \\ \hat{s}(x) & \text{if } x \in (\underline{x}, \bar{x}) \end{cases}$$

with

$$\hat{s}(x; D(e)) = \min\{x, D(e)\}$$

where $D(e)$ is the by Lemma 4.1 unique solution to

$$\int_{\underline{x}}^{\bar{x}} \hat{s}(x)dF(x) = e.$$

s^ is then also a solution to **(P1)**. Furthermore, s^* is globally (i.e. on (x_L, x_H)) nondecreasing and incorporates a debt-like tranche on (\underline{x}, \bar{x}) .*

Lemma 4.7 *Suppose that $f_1(x)/f_2(x)$ is weakly increasing in x on $(\underline{x}, \bar{x}) \subset \mathbb{I}$. Let s be an optimal security solving **(P1)** on $(x_L, x_H) \equiv \mathbb{I}$. Denote $e \equiv \int_{\underline{x}}^{\bar{x}} s(x)dF(x)$.*

Define

⁶²The restriction $\omega^I \leq \int_{\mathbb{I}} \min[x, u]dF(x)$ ensures that there is a security $s \in S_{\mathbb{I}}$ which satisfies $E_f[s(x)] = \omega^I$ and the constraint $s(x) \leq u$.

$$s^*(x) = \begin{cases} s(x) & \text{if } x \notin (\underline{x}, \bar{x}) \\ \hat{s}(x) & \text{if } x \in (\underline{x}, \bar{x}) \end{cases}$$

with

$$\hat{s}(x) = \begin{cases} s(\underline{x}) & \text{if } x < L \\ \min\{x, D\} & \text{if } x \geq L \end{cases}$$

where

$$D = \sup_{\xi \in (\underline{x}, \bar{x})} s(\xi)$$

and $L(e)$ is the by Lemma 4.1 unique solution to

$$\int_{\underline{x}}^{\bar{x}} \hat{s}(x) dF(x) = e.$$

s^* is then also a solution to **(P1)**. Furthermore, s^* is globally nondecreasing (i.e. on (x_L, x_H)) and incorporates a debt-like tranche on (\underline{x}, \bar{x}) .

Intuitively, Lemma 4.6 and Lemma 4.7 state that even if a security s is locally inconsistent with debt-like tranches, there exists a variation \hat{s} which is a debt-like tranche on this local interval. Furthermore, the security s^* which is equal to s everywhere but the local interval (\underline{x}, \bar{x}) , and equal to \hat{s} on that interval, is globally nondecreasing on \mathbb{I} and satisfies the limited liability and nonnegativity constraints. If s is a solution, s^* also solves **(P1)**.

We are now able to state the main Proposition of this article: Proposition 4.4 asserts that if a solution to **(P1)** exists, there also exists a solution which is composed of debt-like tranches. Furthermore, if the densities $f_1(\cdot)$ and $f_2(\cdot)$ are continuous and never proportional on any interval, any solution s to **(P1)** satisfies the property: In that case, any security designed and issued in equilibrium at $t = 1$ is composed of debt-like tranches.

Proposition 4.4 *Let s be a security solving **(P1)** on $(x_L, x_H) \equiv \mathbb{I}$. Then the following statements hold:*

- (i) *There exists a valid security s^* which is also a solution to **(P1)** on $(x_L, x_H) \equiv \mathbb{I}$ and is composed of debt-like tranches.*

(ii) If $f_1(x)$ and $f_2(x)$ are continuous and never proportional, i.e. if

$$\forall(\xi_1, \xi_2) \subseteq \mathbb{I} : \forall k \in \mathbb{R}_+ \exists x \in (\xi_1, \xi_2) : f_1 \neq kf_2$$

then s is composed of debt-like tranches.

The nonproportionality condition 4.4.(ii) requires that the likelihood ratio is never constant on any interval in \mathbb{I} . It is satisfied by most canonical distributions, including the class of exponential distributions with different rate parameters λ , the class of χ^2 -distributions with different degrees of freedom, and the class of $F(d_1, d_2)$ -distributions with fixed d_2 and varying d_1 . Furthermore, it is easy to evaluate given parameterizations of f_1 and f_2 .

The intuition for the result is as follows: Any interval where the solution s to **(P1)** is inconsistent with debt-like tranches can be decomposed into intervals where $f_1(\cdot)/f_2(\cdot)$ is weakly decreasing and weakly increasing respectively, up to points with measure zero. On these intervals, local changes preserving optimality exist by Lemmata 4.6 and 4.7. These changes yield a security s^* which is optimal and locally composed of debt-like tranches. Furthermore, if continuity and local nonproportionality of densities hold, any interval where a solution s to **(P1)** is inconsistent with debt-like tranches can be decomposed into intervals where $f_1(\cdot)/f_2(\cdot)$ is strictly decreasing and strictly increasing respectively. On these intervals, however, a local transformation exists which increases the expected payoff under bad information. This would violate optimality of s , thus implying that s must have been composed of debt-like tranches.

4.5 Characterization of Contracts Composed of Debt-Like Tranches

This section provides additional structure for the optimal contracts (under certain conditions) and further presents examples of optimal securities given specific parameterizations of f_1, f_2 .

The following Proposition provides a condition such that the contract solving **(P1)** includes a standard debt tranche.

Proposition 4.5 *Suppose that for all $x \in \mathbb{I}$ it holds that $F_1(x) > F_2(x)$. Let local nonproportionality and continuity of densities be satisfied. Then any solution s to **(P1)** satisfies*

$$\forall x \in \mathbb{I} : s(x) > x_L.$$

Thus, s includes a standard debt tranche.

$\forall x \in \mathbb{I} : s(x) > x_L$ implies inclusion of a standard debt tranche because local nonproportionality establishes that any solution is composed of debt-like tranches. The condition of strong FOSD on the interior of \mathbb{I} , $F_1(x) > F_2(x)$, is satisfied by most canonical distributions, including the aforementioned exponential distributions, χ^2 -distributions and F-distributions. Furthermore, as can be seen in Example 4.2 below, if $F_1(x) = F_2(x)$ for some $x \in \mathbb{I}$, a security perfectly robust to public information can be constructed for ω below a certain upper bound.

The following proposition yields a condition for the non-optimality of standard debt contracts.

Proposition 4.6 *Denote by $D(\omega^I)$ the face value of the standard debt contract*

$$s^{SDC}(x; D(\omega^I)), \text{ where } E_f[s^{SDC}(x; D(\omega^I))] = \omega^I.$$

Let $G(x) \equiv \frac{1-F_1(x)}{1-F_2(x)}$. Suppose that f_1, f_2 are continuous.

If

$$(i) \exists \xi \in (D(\omega^I), x_H) : G(\xi) > \inf_{x \in (x_L, D(\omega^I)]} \frac{f_1(x)}{f_2(x)}$$

*then $s^{SDC}(x; D(\omega^I))$ is not a solution to **(P1)**. Hence, the optimal contract necessarily involves leverage.*

Propositions 4.4, 4.5 and 4.6 yield the following insight: There are conditions under which the optimal security is composed of debt-like tranches, includes a standard debt tranche, but is not simultaneously a standard debt contract. Therefore, the optimal security is composed of multiple, levered tranches. In the following, Example 4.1 discusses the analysis of a case where such a structure is optimal.

4.5.1 Examples

Example 4.1 Consider the following densities f_1, f_2 and the associated CDFs:

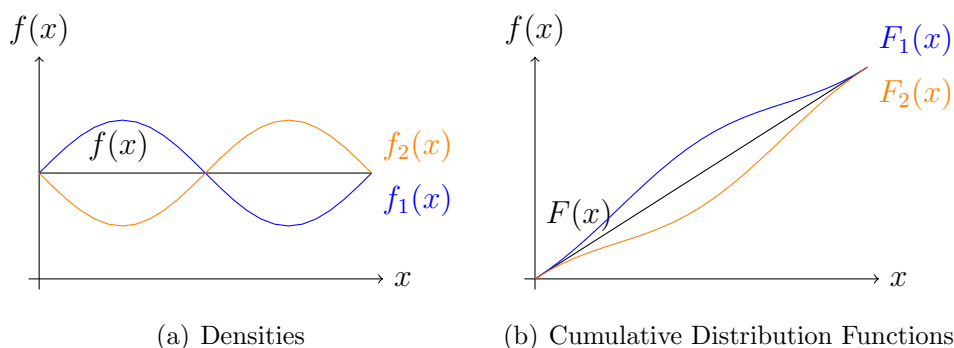


Figure 4.4: Densities and CDFs for Example 4.1

In Example 4.1, it is straightforward to see that local nonproportionality of the densities is satisfied. Hence, the optimal security is composed of debt-like tranches by Proposition 4.4. Furthermore, strict FOSD ($F_1(x) > F_2(x)$) holds for all interior x . Proposition 4.5 applies and a standard debt tranche is included in the optimal contract. Since a standard debt tranche is included, it remains to be checked whether the optimal contract is indeed a standard debt contract or whether it involves multiple tranches. Proposition 4.6 identifies a sufficient condition for non-optimality of a standard debt contract which can be evaluated using the following illustration.

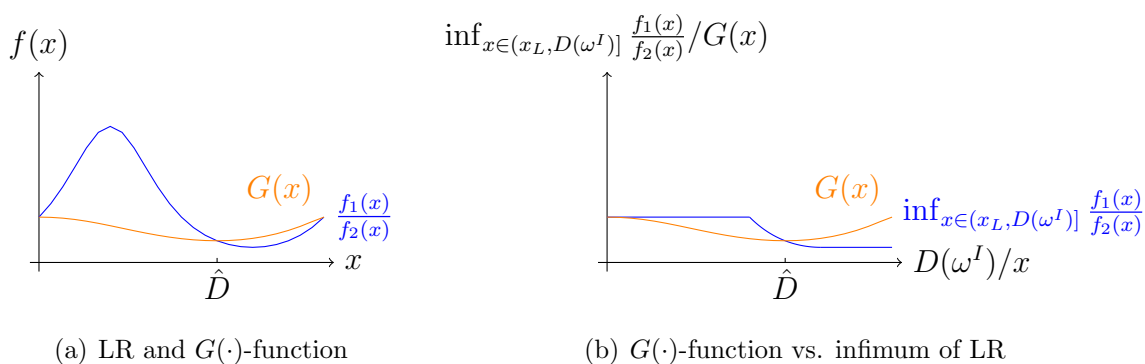


Figure 4.5: Likelihood Ratio and $G(\cdot)$ -function for Example 1

Note that $D(\omega^I)$ is strictly increasing in ω^I . Hence, for large enough ω^I and therefore large $D(\omega^I)$, condition 4.6.(i) is satisfied multiple tranches are optimal. The optimal

contract s^{TD} has a structure as indicated in Figure 5.⁶³ Note the endogenous residual equity tranche(s) which will not be traded but remain(s) on the books of the institution emitting the security.

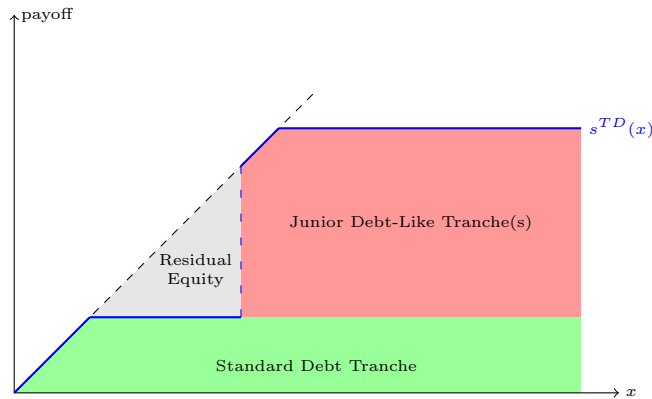


Figure 4.6: Contract Composed of Debt-Like Tranches including a Standard Debt Tranche

Example 4.2 Consider the following densities f_1, f_2 and the associated CDFs:

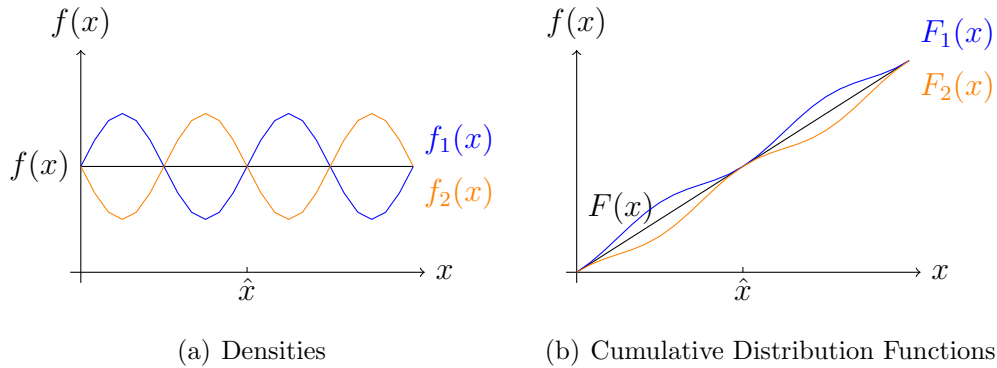


Figure 4.7: Densities and CDFs for Example 4.2

At \hat{x} , $F_1(\hat{x}) = F_2(\hat{x}) = F(\hat{x})$. Therefore, any levered debt-like tranche s^{DT} with

$$s^{DT}(x; D) = \begin{cases} 0 & \text{if } x < \hat{x} \\ D & \text{if } x \geq \hat{x}, \end{cases}$$

⁶³The number of junior debt-like tranches is not specified; at least one junior tranche is included in the optimal security by Proposition 4.6.

where $D \leq \hat{x}$, is perfectly robust to interim public information, i.e.

$$E_f[s^{DT}(x; D)] = E_{f_1}[s^{DT}(x; D)] = E_{f_2}[s^{DT}(x; D)].^{64}$$

With ω small enough, i.e. $\omega^I \leq \int_{\hat{x}}^{x_H} \hat{x} dF(x)$, there exists a unique $D(\omega^I)$ which solves $E_f[s^{DT}(x; D(\omega^I))] = \omega^I$. Because $F_1(\xi) > F_2(\xi)$ for all $\xi \in (x_L, x_H) \setminus \hat{x}$, the security $s^{DT}(x; D(\omega^I))$ is the unique security which offers perfect robustness. Hence, the optimal security is composed of a single levered debt-like tranche. Again, a residual equity tranche is kept on the books of the issuing institution (along with a levered equity tranche).

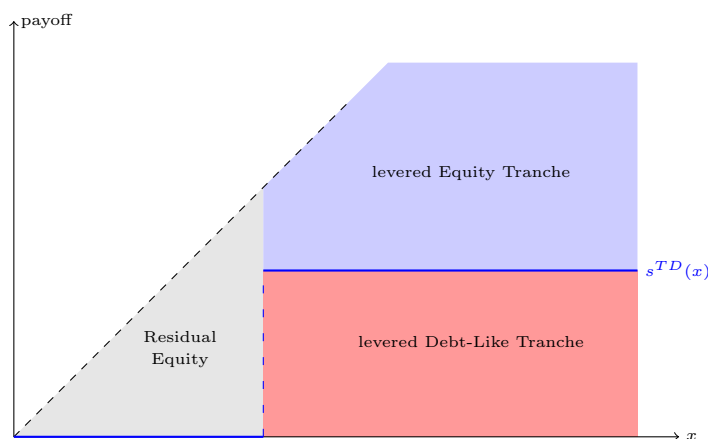


Figure 4.8: Single levered Debt-Like Tranche

4.5.2 Explaining the Residual Equity Tranches

One prediction of the model are residual equity tranches (see the above examples). The residual equity tranches are the non-traded parts of the collection of assets, X , initially owned by B and are hence kept on the books of the issuing institution. We argue that they structurally correspond to the risk retention by sponsors of ABCP conduits.

[Acharya, Schnabl, and Suarez \(2013\)](#) analyze the use of conduits, particularly asset-backed commercial paper (ABCP) conduits, in the early phase of the financial crisis of 2007-2009. They document that sponsors of conduits, especially of single seller conduits, retained significant risk when endowing conduits with assets. Extendible

⁶⁴ $E_f[s^{DT}(x; D)] = D \cdot (1 - F(\hat{x}))$, $E_{f_i}[s^{DT}(x; D)] = D \cdot (1 - F_i(\hat{x}))$

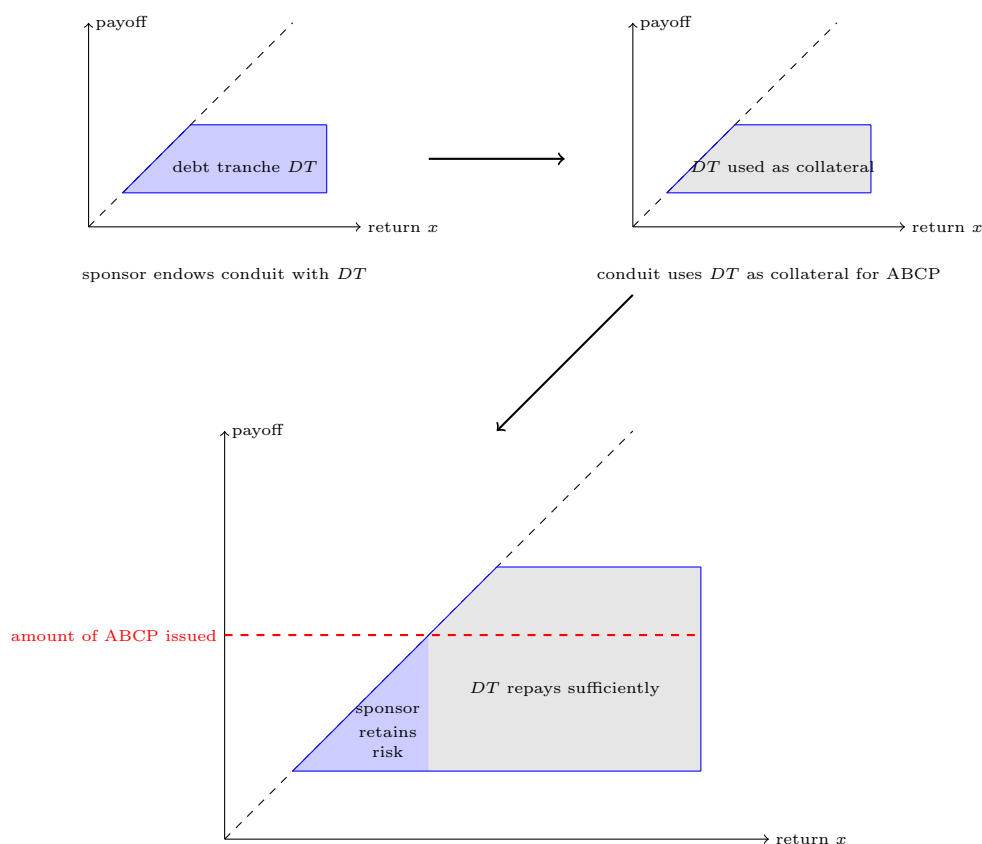


Figure 4.9: Process of Securitization via Conduit

notes guarantees and guarantees via structured investment vehicles (SIV) lead to partial insurance of the conduit's investors. Hence, the conduit's sponsor retained the risk of the conduit's assets - assets which it originally endowed the conduit with. Full credit and full liquidity guarantees went even further and in effect provided full risk insurance. This is also noted by [Gorton and Metrick \(2012\)](#) who document that 'SIVs, ABCP conduits, and credit-card securitizations were often reabsorbed [by their sponsors]'.⁶⁵

To see the correspondence to the residual equity tranches, suppose that a sponsor endows its conduit with a debt tranche (either junior or senior debt). The conduit uses this debt tranche as collateral to secure the asset-backed commercial paper it issues. If the sponsor is (partially) covering the conduit's risk, in particular the risk of a deterioration of the conduit's asset values - i.e. the value of the debt tranche the conduit was endowed with - this may give rise to the sponsoring institution being liable for parts of the residual equity tranche as depicted in Figure 4.9.

⁶⁵See page 58 of [Gorton and Metrick \(2012\)](#).

If the debt tranche is used as collateral for the asset backed commercial paper issued by the conduit, a deterioration of the conduit's asset values corresponds to a realization of states of the world where the debt tranche pays off less than the value of the issued ABCP. This issue is particularly prominent if the face value of the endowment is the base for the issued commercial paper, ignoring the 'risk' that the debt tranche itself may not pay in full. This is depicted in Figure 4.10. If the sponsor provides insurance to the conduit via guarantees, it implicitly keeps this risk on the books. This structurally corresponds to holding on to the residual equity tranche.

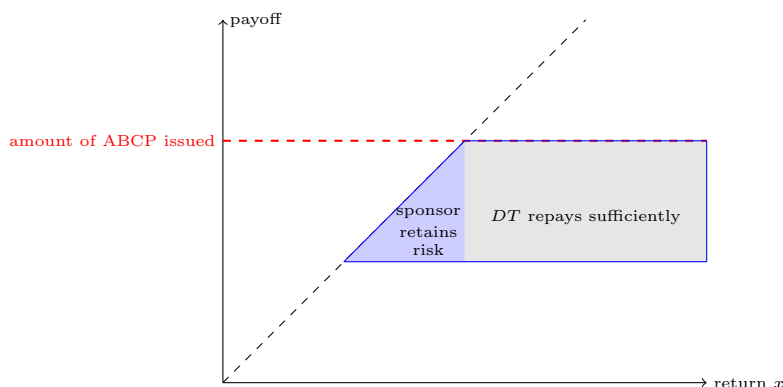


Figure 4.10: Face value of debt tranche equals amount of ABCP

4.6 Robustness and Potential Extensions

There are several ways the presented analysis can be modified and extended. Naturally, it is of interest to analyze the case of more than two underlying distributions. One way to model this goes back to [Dang, Gorton, and Holmström \(2011\)](#). There, the binary state of the world forms the baseline but the public signal does not reveal which distribution is the true one but instead the 'updated' probability of the true distribution being f_1 . Hence, the signal reveals λ . This does not affect the results about the optimality of debt-like tranches.

Furthermore, it is of interest to address pooling. Suppose that the issuing institution holds not a single collection of assets with uncertain return X but explicitly consider the case where there are multiple assets X_1, X_2, \dots, X_N which are affected by the same signal. In this case, it can again be established that for any given bundle, debt-like tranches are optimal if first order stochastic dominance is satisfied not just for the solitary assets but also for the bundle.

This leads to another avenue to explore: How are the results affected if the ordering by first order stochastic dominance does not hold and f_1, f_2 (and corresponding F_1, F_2) are arbitrary densities with full support? It turns out that there always exists an optimal security which is either composed of debt-like tranches or can be expressed as a convex combination of two such security structures. In the latter case, perfect robustness to interim public information is also implied.

In the following subsections, we address these three modifications of the model in detail. We also discuss how modifications of the nondecreasingness assumption impact the security design process in our model.

4.6.1 Multiple underlying distributions

Consider first the modification as in [Dang, Gorton, and Holmström \(2011\)](#). Let f_1, f_2 be such that $F_1(x) \geq F_2(x)$ for all $x \in \mathbb{I}$. However, let the public signal reveal λ , i.e. let $f(x) = \lambda f_1(x) + (1 - \lambda)f_2(x)$ be the distribution prior to the interim information arrival and let $f(x|\hat{\lambda}) = \hat{\lambda}f_1(x) + (1 - \hat{\lambda})f_2(x)$. Thus, $\hat{\lambda}$ is the public information arriving at the interim stage and $l(\hat{\lambda})$ the distribution of the public information with support $[0, 1]$ and $\lambda = \int_0^1 \hat{\lambda} l(\hat{\lambda}) d\hat{\lambda}$.

In that case the analysis remains unchanged and the optimal contract is composed of debt tranches. To see this, consider a solution s^* to **(P1)**, i.e. s^* solving

$$\max_{s \in S_{\mathbb{I}, \omega^I}} E_{f_1}[s(x)] \text{ where } S_{\mathbb{I}, \omega^I} \equiv \{s : \lambda E_{f_1}[s(x)] + (1 - \lambda)E_{f_2}[s(x)] = \omega^I\}$$

and any other security $s \in S_{\mathbb{I}, \omega^I}$. Note that since $E_{f_1}[s^*(x)] \geq E_{f_1}[s(x)]$ and $E_{f_2}[s^*(x)] \leq E_{f_2}[s(x)]$ by construction, it holds that

$$\begin{aligned} E[s|\lambda] &= E[s^*|\lambda] \\ \forall \hat{\lambda} \leq \lambda : E[s|\hat{\lambda}] &\leq E[s^*|\hat{\lambda}] \\ \forall \hat{\lambda} \geq \lambda : E[s|\hat{\lambda}] &\geq E[s^*|\hat{\lambda}]. \end{aligned}$$

Furthermore, define λ_{s^*} and λ_s to be the values of $\hat{\lambda}$ such that the value of the security after information arrival equals the endowment ω^M . Formally, λ_{s^*} and λ_s are the unique solutions to

$$\begin{aligned}\omega^M &= E[s|\lambda_s] \\ \omega^M &= E[s^*|\lambda_{s^*}].\end{aligned}$$

Consider now the following cases:

Case 1: $E[s|\lambda] = E[s^*|\lambda] \leq \omega^M$

In this case, it follows that $\lambda_{s^*} \geq \lambda_s$. Writing the expected utilities of acquiring s and s^* in the form

$$\begin{aligned}EU(s) &= \int_0^1 \left(\omega^I + (\sigma - 1) \left(\min\{E[s|\hat{\lambda}], \omega^M\} \right) \right) l(\hat{\lambda}) d\hat{\lambda} \\ &= \omega^I + (\sigma - 1) \int_0^1 \left(\min\{E[s|\hat{\lambda}], \omega^M\} \right) l(\hat{\lambda}) d\hat{\lambda} \\ &= \omega^I + (\sigma - 1) \int_0^1 (E[s|\hat{\lambda}] - \max\{E[s|\hat{\lambda}] - \omega^M, 0\}) l(\hat{\lambda}) d\hat{\lambda} \\ &= \omega^I + (\sigma - 1) \left[E[s|\lambda] - \int_0^1 \max\{E[s|\hat{\lambda}] - \omega^M, 0\} l(\hat{\lambda}) d\hat{\lambda} \right] \\ &= \omega^I + (\sigma - 1) \left[E[s|\lambda] - \int_{\lambda_s}^1 (E[s|\hat{\lambda}] - \omega^M) l(\hat{\lambda}) d\hat{\lambda} \right] \\ EU(s^*) &= \omega^I + (\sigma - 1) \left[E[s|\lambda] - \int_{\lambda_{s^*}}^1 (E[s|\hat{\lambda}] - \omega^M) l(\hat{\lambda}) d\hat{\lambda} \right],\end{aligned}$$

it obtains that

$$EU(s) \leq EU(s^*)$$

due to

$$\begin{aligned}\int_{\lambda_s}^1 (E[s|\hat{\lambda}] - \omega^M) l(\hat{\lambda}) d\hat{\lambda} &= \int_{\lambda_s}^{\lambda_{s^*}} (E[s|\hat{\lambda}] - \omega^M) l(\hat{\lambda}) d\hat{\lambda} + \int_{\lambda_{s^*}}^1 (E[s|\hat{\lambda}] - \omega^M) l(\hat{\lambda}) d\hat{\lambda} \\ &\geq 0 + \int_{\lambda_{s^*}}^1 (E[s|\hat{\lambda}] - \omega^M) l(\hat{\lambda}) d\hat{\lambda} \\ &\geq \int_{\lambda_{s^*}}^1 (E[s^*|\hat{\lambda}] - \omega^M) l(\hat{\lambda}) d\hat{\lambda}.\end{aligned}$$

As such, for Case 1, s^* remains optimal. The same can be established for Case 2.

Case 2: $E[s|\lambda] = E[s^*|\lambda] > \omega^M$ In this case, $\lambda_{s^*} \leq \lambda_s$ and utilities can be written as

$$\begin{aligned}
 EU(s) &= \int_0^1 \left(\omega^I + (\sigma - 1) \left(\min\{E[s|\hat{\lambda}], \omega^M\} \right) \right) l(\hat{\lambda}) d\hat{\lambda} \\
 &= \omega^I + (\sigma - 1) \int_0^1 \left(\min\{E[s|\hat{\lambda}], \omega^M\} \right) l(\hat{\lambda}) d\hat{\lambda} \\
 &= \omega^I + (\sigma - 1) \int_0^1 (\omega^M - \max\{\omega^M - E[s|\hat{\lambda}], 0\}) l(\hat{\lambda}) d\hat{\lambda} \\
 &= \omega^I + (\sigma - 1) \left[\omega^M - \int_0^1 \max\{\omega^M - E[s|\hat{\lambda}], 0\} l(\hat{\lambda}) d\hat{\lambda} \right] \\
 &= \omega^I + (\sigma - 1) \left[\omega^M - \int_0^{\lambda_s} (\omega^M - E[s|\hat{\lambda}]) l(\hat{\lambda}) d\hat{\lambda} \right] \\
 EU(s^*) &= \omega^I + (\sigma - 1) \left[\omega^M - \int_0^{\lambda_{s^*}} (\omega^M - E[s|\hat{\lambda}]) l(\hat{\lambda}) d\hat{\lambda} \right],
 \end{aligned}$$

where

$$EU(s) \leq EU(s^*)$$

follows from

$$\begin{aligned}
 \int_0^{\lambda_s} (\omega^M - E[s|\hat{\lambda}]) l(\hat{\lambda}) d\hat{\lambda} &= \int_0^{\lambda_{s^*}} (\omega^M - E[s|\hat{\lambda}]) l(\hat{\lambda}) d\hat{\lambda} + \int_{\lambda_{s^*}}^{\lambda_s} (\omega^M - E[s|\hat{\lambda}]) l(\hat{\lambda}) d\hat{\lambda} \\
 &\geq \int_0^{\lambda_{s^*}} (\omega^M - E[s|\hat{\lambda}]) l(\hat{\lambda}) d\hat{\lambda} + 0 \\
 &\geq \int_0^{\lambda_{s^*}} (\omega^M - E[s^*|\hat{\lambda}]) l(\hat{\lambda}) d\hat{\lambda}.
 \end{aligned}$$

As such, s^* remains optimal and the previous analysis holds. There is always an optimal security composed of debt-like tranches; under the conditions outlined in Proposition 4.4, this applies to all securities issued in equilibrium.

4.6.2 Pooling of multiple assets

For simplicity, consider the case where the issuing institution ex ante owns two assets with uncertain return X_1, X_2 . Further suppose that X_1 is distributed according to $f(x_1) = \lambda f_1(x_1) + (1 - \lambda) f_2(x_1)$ and X_2 according to $g(x_2) = \lambda g_1(x_2) + (1 - \lambda) g_2(x_2)$. Let the public signal reveal the true distribution in the sense that X_1 is distributed according to f_1 and X_2 according to g_1 with probability λ .⁶⁶ With probability $(1 - \lambda)$,

⁶⁶Thus, there is perfect correlation in the sense that either the true distributions are f_1, g_1 or f_2, g_2 respectively. This simplification is made for expositional purposes; pooling any two securities where

the true distribution of X_1 is f_2 and that of X_2 is g_2 . By pooling the two assets, i.e. by considering the asset $X \equiv X_1 + X_2$, this asset is distributed according to $h(x) = \lambda h_1(x) + (1 - \lambda)h_2(x)$.

If $F_1(x_1) \geq F_2(x_1)$ and $G_1(x_2) \geq G_2(x_2)$ for all x_1, x_2 in the respective support, i.e. if the ordering of the distributions by first order stochastic dominance applies in the same direction (for both assets, the signal is bad with probability λ and good with probability $(1 - \lambda)$), then it follows that $H_1(x) \geq H_2(x)$ for all $x \in \mathbb{I}$. Hence, irrespective of whether the assets are pooled or not, the analysis performed holds and the optimal contract will be a composed of debt-like tranches. It nonetheless needs to be evaluated separately whether pooling is beneficial or not; this in particular will depend on how the endowments of the liquidity shifters (agents M) covary with the pooling decision.

However, it is possible that the assets' returns are negatively correlated in the sense that good news for one asset is bad news for the other. Consider for example the case that $F_1(x_1) \geq F_2(x_1)$ and $G_1(x_2) \leq G_2(x_2)$. In this case, there is no clear ordering with respect to the distributions of the pooled asset. This directly leads to the next modification.

4.6.3 Arbitrary distributions without ordering

Suppose that f_1, f_2 are arbitrary densities and that first order stochastic dominance does not hold. Fix $\omega^M = \omega^I$ for simplicity and recall that first order stochastic dominance was only employed to reduce the game to solving the problem **(P1)**. Reconsider

$$\mathbf{(P1)} \quad \max_{s \in S_{\mathbb{I}}} E_{f_1}[s(x)] \quad \text{s.t.} \quad \lambda E_{f_1}[s(x)] + (1 - \lambda)E_{f_2}[s(x)] = \omega^I$$

and denote s a solution to **(P1)**. Note that all solutions have the same expected values in all states. Furthermore, let

$$\mathbf{(P1')} \quad \max_{s \in S_{\mathbb{I}}} E_{f_2}[s(x)] \quad \text{s.t.} \quad \lambda E_{f_1}[s(x)] + (1 - \lambda)E_{f_2}[s(x)] = \omega^I$$

and let s' be a solution to **(P1')**. By the analysis performed in this paper, it is known that there always exist s, s' which are composed of debt-like tranches. There are the

signals have an arbitrary correlation leads to a security design problem as covered in section 4.6.3 where no restriction on the underlying densities is imposed.

following cases:

	$E_{f_1}[s(x)] \geq \omega^I$	$E_{f_1}[s(x)] < \omega^I$
$E_{f_2}[s'(x)] \geq \omega^I$	Case 1	Case 2
$E_{f_2}[s'(x)] < \omega^I$	Case 3	Case 4

Case 4 is impossible as $E_{f_2}[s'(x)] < \omega^I$ implies $E_{f_1}[s(x)] \geq E_{f_1}[s'(x)] > \omega^I$. In Case 1, it follows that a convex combination s^* of s and s' can be constructed which has perfect robustness to public information, i.e. $E_f[s^*(x)] = E_{f_1}[s^*(x)] = E_{f_2}[s^*(x)] = \omega^I$. Furthermore, Case 2 corresponds to a case where the endowment constraint ω^M may only bind if the true distribution is f_2 . If that is the case, the analysis presented in the previous sections carries through and s will be an optimal security. In Case 3, the role of f_1 and f_2 is reversed: s' is optimal, f_1 can be considered good information and f_2 bad information.

This classification allows the statement that irrespective of any ordering imposed on f_1 and f_2 , there always exists an optimal security which is either composed of debt-like tranches or which is perfectly robust to interim public information and can be constructed as the convex combination of two such securities. Hence, even in the case of pooling where first order stochastic dominance no longer holds, debt-like tranches play a pivotal role in designing the optimal security with respect to the interim public information problem.

4.6.4 Modifying the Nondecreasingness Assumption

As outlined in Section 2, the nondecreasingness assumption on securities is not without question in the literature. Some studies, like [Biais and Mariotti \(2005\)](#), impose dual monotonicity, i.e. that both the security $s(x)$ and the residual $x - s(x)$ are nondecreasing in x . Others, mostly in discrete settings as [Farhi and Tirole \(2012\)](#), solve security design problems without imposing any restriction. In this section, we briefly discuss how our results are affected when moving from nondecreasingness to either dual monotonicity or no assumption on the slope of the security.

Security Design without Nondecreasingness Without nondecreasingness, the analysis of the security design problem follows that with arbitrary, non-ordered densities. First, solve for the security design s_1 that maximizes the value under f_1 and then solve for the security design s_2 that maximizes the value under f_2 . In the case of s_1 , securities which solve the problem

$$(\mathbf{P1})_{nr} \max_s E_{f_1}[s(x)] \quad \text{s.t.} \quad \lambda E_{f_1}[s(x)] + (1 - \lambda)E_{f_2}[s(x)] = \omega^I \\ \forall x \in \mathbb{I} : 0 \leq s(x) \leq x,$$

i.e. which solve the problem corresponding to $(\mathbf{P1})$ in the absence of the nondecreasingness constraint, can be characterized as follows: Define

$$\bar{r} \equiv \inf_r \left[\int_{\frac{f_1(x)}{f_2(x)} > r} x f(x) dx \leq \omega^I \right],$$

i.e. let \bar{r} be the cutoff value of r such that a security which specifies a payoff at the limited liability constraint whenever the likelihood ratio exceeds r and a payoff of 0 otherwise has an unconditional expected value of (weakly) less than ω .

It is then straightforward that any solution s_1^{nr} to $(\mathbf{P1})_{nr}$ has the property

$$s_1^{nr} = \begin{cases} x & \text{if } \frac{f_1(x)}{f_2(x)} > \bar{r} \\ 0 & \text{if } \frac{f_1(x)}{f_2(x)} < \bar{r} \end{cases} \wedge \int_{\mathbb{I}} s_1^{nr}(x) f(x) dx = \omega^I,$$

i.e. that it specifies maximal payoff ($s_1^{nr}(x) = x$) at those points where the likelihood ratio is large and minimal payoff ($s_1^{nr}(x) = 0$) when the likelihood ratio is small. It is possible that there is flexibility in the security design; in particular, whenever

$$\int_{\frac{f_1(x)}{f_2(x)} \geq \bar{r}} x f(x) dx > \omega^I,$$

the remaining payoff can be freely allocated across states where $\frac{f_1(x)}{f_2(x)} = \bar{r}$. Typically (if this multiplicity does not occur), s_1^{nr} will be of the following form:

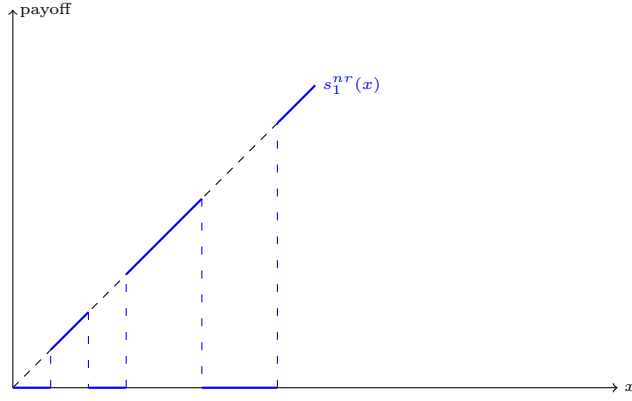


Figure 4.11: Solution to Maximization Problem without Nondecreasingness

The structure of s_2^{nr} as a solution to the corresponding maximization problem for $E_{f_2}[s(x)]$ is similar. As in the case of arbitrary f_1, f_2 without ordering, the optimal security can be characterized either as a convex combination of some such s_1^{nr}, s_2^{nr} (and offer perfect robustness to public information) or be s_1^{nr} (or s_2^{nr} respectively) whenever even s_1^{nr} (s_2^{nr}) fails to attain ω^M under f_1 (f_2). This may only occur, however, when a large part of the probability mass of $x \in \mathbb{I}$ under f_1 (f_2) lies in the region with low x where the limited liability constraint has a lot of bite.

Non-Optimality of Standard Debt under dual monotonicity In the case where dual monotonicity instead of single nondecreasingness is imposed, we do not explicitly solve the optimal control problem leading to the equilibrium security design structure. However, we obtain a condition for the non-optimality of standard debt which is similar to that derived in Proposition 4.6. In particular, consider the setting where f_1, f_2 are ordered by first order stochastic dominance. We are interested in characterizing the solution to

$$\begin{aligned}
 (\mathbf{P1})_{dm} \quad & \max_s E_{f_1}[s(x)] \quad \text{s.t.} \quad \lambda E_{f_1}[s(x)] + (1 - \lambda)E_{f_2}[s(x)] = \omega^I \\
 & \forall x \in \mathbb{I} : 0 \leq s(x) \leq x \\
 & \forall x_1, x_2 \in \mathbb{I} : x_1 > x_2 \Rightarrow s(x_1) \geq s(x_2) \wedge x_1 - s(x_1) \geq x_2 - s(x_2).
 \end{aligned}$$

The following claim states our condition for non-optimality of standard debt.

Claim 4.1 Denote $D(\omega^I)$ the face value of the standard debt contract $s^{SDC}(x; D(\omega^I))$ with

$$E_f[s^{SDC}(x; D(\omega^I))] = \omega^I.$$

Let $G(x) \equiv \frac{1-F_1(x)}{1-F_2(x)}$ and $\hat{G}(x) \equiv \frac{F_1(D(\omega^I))-F_1(x)}{F_2(D(\omega^I))-F_2(x)}$. Suppose that f_1, f_2 are continuous. If

$$(i') \exists \xi \in (D(\omega^I), x_H) : G(\xi) > \inf_{x \in (x_L, D(\omega^I)]} \hat{G}(x)$$

then $s^{SDC}(x; D(\omega^I))$ is not a solution to $(P1_{dm})$. Hence, the optimal contract necessarily involves leverage.

First note that condition (i') is stronger than condition (i) from Proposition 4.6. If (i') is satisfied, so is (i), whereas the reverse does not hold. Second, while we abstract from a detailed proof, the following figure outlines the construction of a deviation from s^{SDC} which improves E_{f_1} while adhering to dual monotonicity.

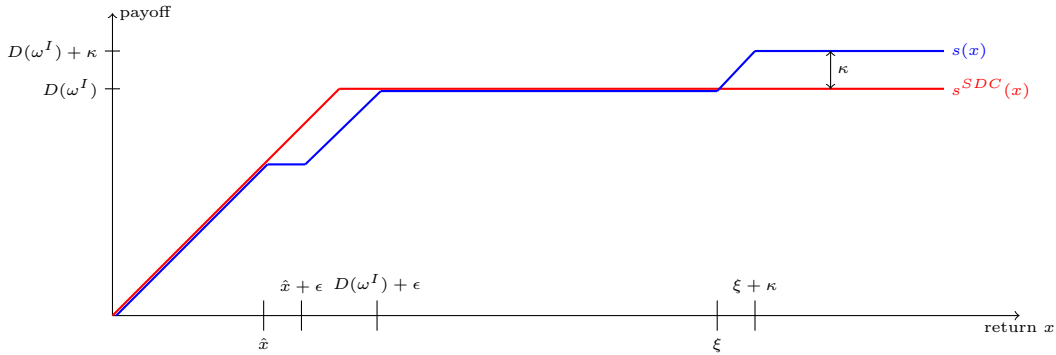


Figure 4.12: Illustration – Deviation from Standard Debt

Condition (i') implies that ϵ and κ sufficiently small can be found such that \hat{x} exists with

$$\frac{\int_{\hat{x}}^{D(\omega^I)+\epsilon} [s^{SDC}(x; D(\omega^I)) - s(x)] f_1(x) dx}{\int_{\xi}^{D(\omega^I)+\epsilon} [s^{SDC}(x; D(\omega^I)) - s(x)] f_2(x) dx} < \frac{\int_{\xi}^{x_H} [s(x) - s^{SDC}(x; D(\omega^I))] f_1(x) dx}{\int_{\hat{x}}^{x_H} [s(x) - s^{SDC}(x; D(\omega^I))] f_2(x) dx}.$$

Lemma 4.3 then ensures that s is preferred to $s^{SDC}(x; D(\omega^I))$. Intuitively, following (i'), ϵ and κ can be chosen such that $\frac{\int_{\hat{x}}^{D(\omega^I)+\epsilon} [s^{SDC}(x; D(\omega^I)) - s(x)] f_1(x) dx}{\int_{\xi}^{D(\omega^I)+\epsilon} [s^{SDC}(x; D(\omega^I)) - s(x)] f_2(x) dx}$ is arbitrarily close to $\hat{G}(\hat{x}) < G(\xi)$, with $\frac{\int_{\xi}^{x_H} [s(x) - s^{SDC}(x; D(\omega^I))] f_1(x) dx}{\int_{\hat{x}}^{x_H} [s(x) - s^{SDC}(x; D(\omega^I))] f_2(x) dx}$ arbitrarily close to $G(\xi)$.

While we abstract from further characterizing the solution to the problem $(\mathbf{P1})_{dm}$, the potential incentive asymmetry prevails: If condition (i') is satisfied, standard debt is not the optimal security with respect to interim public information, whereas it minimizes the incentives for private information acquisition of the trading parties.

4.7 Conclusion

This article has built upon the analysis by [Dang, Gorton, and Holmström \(2011\)](#) to analyze a security design problem where public information arrives between trading periods. In the absence of private information acquisition, a security is optimal if it is least sensitive to interim public information: Since 'gains' from good interim information can not be fully capitalized upon whereas 'losses' from bad information are fully incurred, an optimal security has maximal value in the bad information state and correspondingly minimal value after good information has arrived in the interim period.

If good information and bad information can be differentiated according to an ordering imposed by the monotone likelihood ratio property, standard debt is optimal as shown by [Dang, Gorton, and Holmström \(2011\)](#). However, this ordering is restrictive. Whenever the arriving information only affects e.g. the low tail of the distribution of returns of the underlying cash flow, a more general information structure seems prudent. We model such a generalization by imposing ordering by first order stochastic dominance which nests the previous analysis.

The optimal security structure is given by a composition of debt-like tranches. Contracts satisfying this structure range from standard debt to levered equity and multiple tranches of different seniorities. We identify conditions under which the optimal security includes, but is not limited to a standard debt tranche.

The results provide an explanation for the endogenous occurrence of tranches which are frequently seen in financial markets. This explanation is new in that it is independent of private information. Instead, the tranches arise because they allow to put the most weight on those returns of the underlying cash flow which are more likely to be met even after 'bad' information arrived. If the monotone likelihood ratio criterion on the underlying distributions fails to hold, it no longer is necessarily optimal to put the most weight of the payoffs on the lowest realized returns, i.e. to issue standard debt. Instead, it may even be optimal to trade levered equity. Furthermore, our notion of tranching differs from that in the literature. We provide an explanation for designing securities

composed of multiple, imperfect tranches, whereas the literature typically focuses on (pooling and) tranching off a single standard debt tranche and holding on to the residual equity. Following our results, if the initially traded contract is composed of different debt tranches, the issuer will always hold on to multiple residual equity tranches.

Technically, this is a novel result because it explicitly introduces a discontinuity in the security structure and thus differs from the literature at large which either imposes continuity by virtue of the dual monotonicity assumption or endogenously satisfies it with standard debt solving the security design problem.

The residual equity tranches are a specific prediction of the model. While they are not explicitly traded in financial markets, we argue that they arise from the use of conduits in the ABCP market as documented by [Acharya, Schnabl, and Suarez \(2013\)](#) and [Gorton and Metrick \(2012\)](#). Specifically, a sponsor who endows its conduit with a debt tranche, and who retains the risk of the conduit through guarantees, implicitly keeps the risk that the debt tranche does not pay up to face value on the books. This structurally corresponds to the residual equity tranches.

Lastly, there are implications for the relation between private and public information concerns. While private information acquisition is abstracted from in our framework, it is clear from e.g. [Dang, Gorton, and Holmström \(2011\)](#) and [Yang \(2012\)](#) that levered contracts do not minimize other market participant's incentives to acquire private information. Quite to the contrary, the possible case of levered equity maximizes these incentives. Furthermore, while tranching is beneficial with respect to the interim public information arrival, [Farhi and Tirole \(2012\)](#) show that it works against communality of information in the analysis of the private information problem. Hence, there exists a tradeoff for the security designer: With the more general structure of public information, levered tranches will be optimal in terms of creating a security robust to interim public information. Nonetheless, a standard debt contract would minimize the other market participants' desire to acquire private information. How this tradeoff plays out is an interesting avenue to explore in the future. Intuitively, for very large costs of private information acquisition, the public information issue should dominate and levered debt tranches should be issued.

Appendix

4.A Proof of Proposition 4.1

Existence is established in the following way: **(P1)** corresponds to the optimization of $E_{f_1}[s(x)]$ over the set $S_{\mathbb{I},\omega^I} \equiv \{s \in S_{\mathbb{I}} \text{ such that } E_f[s(x)] = \omega^I\}$. The objective $E_{f_1}[s(x)]$ corresponds to a mapping $h : S_{\mathbb{I},\omega^I} \rightarrow \mathbb{R}$ where $h(s) = E_{f_1}[s(x)]$. By showing that $S_{\mathbb{I},\omega^I}$ is convex and closed and h is continuous, existence of a maximum of h on the set $S_{\mathbb{I},\omega^I}$ follows.⁶⁷

To see that $S_{\mathbb{I},\omega^I}$ is convex, consider $s_1, s_2 \in S_{\mathbb{I},\omega^I}$. It has to be shown that

$$\forall \alpha \in [0, 1] : \alpha s_1 + (1 - \alpha)s_2 \in S_{\mathbb{I},\omega^I}. \quad (4.9)$$

Note that:

- nondecreasingness of s_1, s_2 implies that $\alpha s_1 + (1 - \alpha)s_2$ is nondecreasing as well
- $(\forall x \in \mathbb{I} : s_1(x) \leq x, s_2(x) \leq x) \Rightarrow \alpha s_1(x) + (1 - \alpha)s_2(x) \leq x$, for all $x \in \mathbb{I}$
hence, $\alpha s_1 + (1 - \alpha)s_2$ satisfies limited liability
- $(\forall x \in \mathbb{I} : s_1(x) \geq 0, s_2(x) \geq 0) \Rightarrow \alpha s_1(x) + (1 - \alpha)s_2(x) \geq 0$, for all $x \in \mathbb{I}$
hence, $\alpha s_1 + (1 - \alpha)s_2$ satisfies nonnegativity
- $E_f[\alpha s_1(x) + (1 - \alpha)s_2(x)] = \alpha E_f[s_1(x)] + (1 - \alpha)E_f[s_2(x)] = \omega^I$.

Thus, (4.9) follows.

To establish continuity of h and closedness of $S_{\mathbb{I},\omega^I}$, take the metric

$$d(s_1, s_2) \equiv \sup_{x \in \mathbb{I}} |s_1(x) - s_2(x)|, \quad (4.10)$$

such that

$$s_i \xrightarrow{i \rightarrow \infty} s \Leftrightarrow d(s_i, s) \xrightarrow{i \rightarrow \infty} 0. \quad (4.11)$$

⁶⁷Note that $S_{\mathbb{I},\omega^I}$ is a subset of the $S_{\mathbb{I}}$ -space and bounded there by the integrable functions $\underline{s}(x) = 0$ and $\bar{s}(x) = x$.

Closedness follows from the fact that for any converging sequence $s_i \xrightarrow{i \rightarrow \infty} s$ where $s_i \in S_{\mathbb{I}, \omega^I}$ for all i , the limit s is contained in $S_{\mathbb{I}, \omega^I}$. We will show that $s \in S_{\mathbb{I}, \omega^I}$ by establishing that it satisfies limited liability, nondecreasingness, nonnegativity and has an expected value $E_f[s(x)] = \omega^I$.

First, $E_f[s(x)] = \omega^I$ is established. Suppose $E_f[s(x)] > \omega^I$ (the contradiction for $E_f[s(x)] < \omega^I$ works analogously). Let $\delta \equiv E_f[s(x)] - \omega$. Then $s_i \xrightarrow{i \rightarrow \infty} s$ implies for any $\epsilon > 0, \epsilon < \delta$:

$$\exists N : \forall i \geq N : s_i(x) \geq s(x) - \epsilon \text{ for all } x \in \mathbb{I}. \quad (4.12)$$

Hence, for all $i \geq N$ it follows that

$$E_f[s_i(x)] \geq E_f[s(x)] - \epsilon > E_f[s(x)] - \delta = \omega^I. \quad (4.13)$$

This is a contradiction to $s_i \in S_{\mathbb{I}, \omega^I}$. Next, suppose that s does not satisfy limited liability, i.e. that $s(\xi) > \xi$ for some $\xi \in \mathbb{I}$. By $s_i \xrightarrow{i \rightarrow \infty} s$, this implies that

$$\exists N : \forall i \geq N : s_i(\xi) > \xi. \quad (4.14)$$

This contradicts $s_i \in S_{\mathbb{I}, \omega^I}$ for $i \geq N$. In the same manner, nonnegativity of s is established.

Finally, suppose that s violates nondecreasingness, i.e. that

$$\exists x_1, x_2 \in \mathbb{I} \text{ such that } x_1 < x_2 \wedge s(x_1) > s(x_2). \quad (4.15)$$

However, s_i is nondecreasing for all i . Hence, $s_i(x_1) \leq s_i(x_2)$ for all i . Since $s_i \xrightarrow{i \rightarrow \infty} s$, a contradiction again follows. $s(x_1) > s(x_2)$ requires $s_i(x_1) > s_i(x_2)$ for all $i \geq N$ for some N .

Lastly, continuity of h is established. Recall

$$h(s) = E_{f_1}[s(x)] = \int_{x_L}^{x_H} s(x) dF_1(x). \quad (4.16)$$

Now take $s_i \xrightarrow{i \rightarrow \infty} s$. It follows that

$$\begin{aligned}
 \lim_{i \rightarrow \infty} h(s_i) &= \lim_{i \rightarrow \infty} \int_{x_L}^{x_H} s_i(x) dF_1(x) \\
 &= \int_{x_L}^{x_H} \lim_{i \rightarrow \infty} s_i(x) dF_1(x) \tag{4.17}
 \end{aligned}$$

$$\begin{aligned}
 &= \int_{x_L}^{x_H} s(x) dF_1(x) \\
 &= h(s), \tag{4.18}
 \end{aligned}$$

where (4.17) follows from dominated convergence and which yields continuity of h .⁶⁸

We have thus established that $S_{\mathbb{I}, \omega^I}$ is convex and closed and that h is continuous. Hence, $h(s) = E_{f_1}[s(x)]$ attains a maximum on $S_{\mathbb{I}, \omega^I}$ and a solution to **(P1)** exists. ■

4.B Proof of Proposition 4.2

Let $E_{f_1}[s^*(x)]$ be the expected value of a solution s^* to **(P1)** after bad information. By construction of **(P1)**, all solutions have the same state-contingent expected values. Recall (4.5), i.e. the expected utility of I from acquiring a security s at $t = 1$:

$$EU(s) = \omega^I + (\sigma - 1) (\lambda \min[E_{f_1}[s(x)], \omega^M] + (1 - \lambda) \min[E_{f_2}[s(x)], \omega^M]).$$

Consider any security s which could be traded at $t = 1$, i.e. which satisfies

$$E_f[s(x)] \leq \omega^I = E_f[s^*(x)], \tag{4.19}$$

and which is not a solution to **(P1)**. It needs to hold that

$$E_{f_1}[s(x)] < E_{f_1}[s^*(x)]. \tag{4.20}$$

⁶⁸Recall that functions $s_i(\cdot)$ are bounded in the $S_{\mathbb{I}}$ -space by the integrable functions $\underline{s}(x) = 0$ and $\bar{s}(x) = x$.

If $E_{f_1}[s(x)] \geq E_{f_1}[s^*(x)]$, a contradiction would be obtained in the sense that s^* is not a solution or that s is a solution to **(P1)**. To see this, note the following: $E_{f_1}[s(x)] > E_{f_1}[s^*(x)]$ would imply that \hat{s} with $\hat{s}(x) \geq s(x)$ at all $x \in \mathbb{I}$ exists, where $E_f[\hat{s}(x)] = \omega^I$ and $E_{f_1}[\hat{s}(x)] > E_{f_1}[s^*(x)]$. This violates s^* being a solution and follows from the strict positivity of densities.

Similarly, if $E_{f_1}[s(x)] = E_{f_1}[s^*(x)]$ and $E_f[s(x)] = \omega^I$, then s would be a solution, whereas $E_{f_1}[s(x)] = E_{f_1}[s^*(x)]$ and $E_f[s(x)] < \omega^I$ together with the strict positivity of densities again would yield existence of \hat{s} with

$$\begin{aligned} & \hat{s}(x) \geq s(x) \text{ for all } x \in \mathbb{I} \\ \wedge & \exists \mathbb{A} \subseteq \mathbb{I} : \hat{s}(x) > s(x) \forall x \in \mathbb{A}, \int_{\mathbb{A}} 1 dF(x) > 0 \\ \wedge & E_f[\hat{s}(x)] = \omega^I. \end{aligned} \quad (4.21)$$

Hence, $E_{f_1}[\hat{s}(x)] > E_{f_1}[s^*(x)]$ due to $\hat{s}(x) > s(x) \forall x \in \mathbb{A}$. Thus, (4.20) needs to hold as otherwise a contradiction is obtained.

If condition (i) or (ii) is satisfied, $E_{f_2}[s^*(x)] \geq \omega^M$.⁶⁹ Therefore, using (4.20),

$$\begin{aligned} EU(s) &= \omega^I + (\sigma - 1) [\lambda \min[E_{f_1}[s(x)], \omega^M] + (1 - \lambda) \min[E_{f_2}[s(x)], \omega^M]] \\ &\leq \omega^I + (\sigma - 1) [\lambda \min[E_{f_1}[s(x)], \omega^M] + (1 - \lambda) \omega^M] \\ &= \omega^I + (\sigma - 1) [\lambda E_{f_1}[s(x)] + (1 - \lambda) \omega^M] \\ &< \omega^I + (\sigma - 1) [\lambda E_{f_1}[s^*(x)] + (1 - \lambda) \omega^M] \\ &= EU(s^*). \end{aligned} \quad (4.22)$$

Hence we have established that only solutions to **(P1)** are traded in equilibrium at $t = 1$ if (i) or (ii) holds. ■

If $\omega^M < \omega^I$ and $E_{f_1}[s^*(x)] > \omega^M$, then multiplicity arises in the sense that securities which may be traded in equilibrium at $t = 1$ differ in their expected values across states. Specifically, any security s which satisfies $E_{f_1}[s] \geq \omega^M$ (and hence also $E_{f_2}[s] \geq E_{f_1}[s] \geq$

⁶⁹For condition (ii) this follows from $E_{f_2}[s^*(x)] \geq \omega^I$ and $\omega^I \geq \omega^M$.

ω^M by first order stochastic dominance) along with $E_f[s(x)] \leq \omega^I$ may be traded in equilibrium at $t = 1$. These securities have in common that they fully exhaust the trading capacity which is limited by the endowment constraint ω^M .

If $\omega^I \leq E_{f_2}[s^*(x)] < \omega^M$, any security s with $E_f[s(x)] = \omega^I$ and $E_{f_2}[s(x)] \leq \omega^M$ may also be issued. Even though those securities have a higher value after good interim information than solutions to **(P1)**, they still do not induce a binding endowment constraint ω^M . Gains from trade are fully realized, i.e. the security acquired at $t = 1$ is fully sold irrespective of the interim public information.

4.C Proof of Lemma 4.3

The proof is straightforward and done here for (i) or (ii) holding strictly. If both hold weakly, the same steps yield the result with the weak inequality for (iv). Fix k and suppose without loss of generality that (i) holds strictly, i.e.

$$\int_A (s_1(x) - s_2(x))dF_1(x) < k \int_A (s_1(x) - s_2(x))dF_2(x). \quad (4.23)$$

Recall (4.1), i.e. $f(x) = \lambda f_1(x) + (1 - \lambda)f_2(x)$. It holds that

$$\begin{aligned} \int_B (s_2(x) - s_1(x))dF_1(x) &\geq k \int_B (s_2(x) - s_1(x))dF_2(x) \\ \wedge \int_A (s_1(x) - s_2(x))dF_1(x) &< k \int_A (s_1(x) - s_2(x))dF_2(x) \end{aligned} \quad (4.24)$$

It follows that

$$\begin{aligned} &(\lambda + \frac{1}{k}(1 - \lambda)) \int_A (s_1(x) - s_2(x))dF_1(x) \\ &= \lambda \int_A (s_1(x) - s_2(x))dF_1(x) + \frac{1}{k}(1 - \lambda) \int_A (s_1(x) - s_2(x))dF_1(x) \\ &< \lambda \int_A (s_1(x) - s_2(x))dF_1(x) + (1 - \lambda) \int_A (s_1(x) - s_2(x))dF_2(x) \end{aligned} \quad (4.25)$$

by (4.24) and furthermore

$$\begin{aligned}
 & \lambda \int_A (s_1(x) - s_2(x)) dF_1(x) + (1 - \lambda) \int_A (s_1(x) - s_2(x)) dF_2(x) \\
 = & \lambda \int_B (s_2(x) - s_1(x)) dF_1(x) + (1 - \lambda) \int_B (s_2(x) - s_1(x)) dF_2(x) \tag{4.26}
 \end{aligned}$$

by (iii).

Since

$$\begin{aligned}
 & \lambda \int_B (s_2(x) - s_1(x)) dF_1(x) + (1 - \lambda) \int_B (s_2(x) - s_1(x)) dF_2(x) \\
 \stackrel{(4.24)}{\leq} & \lambda \int_B (s_2(x) - s_1(x)) dF_1(x) + \frac{1}{k}(1 - \lambda) \int_B (s_2(x) - s_1(x)) dF_1(x) \\
 = & (\lambda + \frac{1}{k}(1 - \lambda)) \int_B (s_2(x) - s_1(x)) dF_1(x)
 \end{aligned}$$

it holds that

$$\begin{aligned}
 (\lambda + \frac{1}{k}(1 - \lambda)) \int_A (s_1(x) - s_2(x)) dF_1(x) & < (\lambda + \frac{1}{k}(1 - \lambda)) \int_B (s_2(x) - s_1(x)) dF_1(x) \\
 \Leftrightarrow \int_A (s_1(x) - s_2(x)) dF_1(x) & < \int_B (s_2(x) - s_1(x)) dF_1(x) \tag{4.27}
 \end{aligned}$$

and therefore

$$\begin{aligned}
 & \int_{A \cup B} (s_1(x) - s_2(x)) dF_1(x) < 0 \\
 \Leftrightarrow \int_{A \cup B} s_1(x) dF_1(x) & < \int_{A \cup B} s_2(x) dF_1(x). \tag{4.28}
 \end{aligned}$$

This concludes the proof. ■

4.D Proof of Lemma 4.4

First note that $D(\omega^I)$ is unique by Lemma 4.1. Consider the standard debt contract $s^{SDC}(x; D(\omega^I))$ and any contract s with $E_f[s(x)] = \omega^I$ and $s \neq s^{SDC}(x; D(\omega^I))$ in the

sense that they differ on a subset of \mathbb{I} with positive measure. Formally, consider s such that, letting

$$\Delta \equiv \{x \in \mathbb{I} \text{ such that } s(x) \neq s^{SDC}(x; D(\omega^I))\}, \text{ it holds that } \int_{\Delta} 1dF(x) > 0. \quad (4.29)$$

To show that the standard debt contract solves **(P1)**, it is sufficient to show that for all such s , $E_{f_1}[s(x)] \leq E_{f_1}[s^{SDC}(x; D(\omega^I))]$ and equivalently $E_{f_2}[s(x)] \geq E_{f_2}[s^{SDC}(x; D(\omega^I))]$. Since $s^{SDC}(x; D(\omega^I))$ is a standard debt contract and $E_f[s(x)] = E_f[s^{SDC}(x; D(\omega^I))]$, it holds that

$$\begin{aligned} \exists \hat{x} \in \mathbb{I} \text{ s.t. } \quad & s^{SDC}(x; D(\omega^I)) \geq s(x) \quad \text{if } x < \hat{x} \\ & s^{SDC}(x; D(\omega^I)) \leq s(x) \quad \text{if } x > \hat{x}. \end{aligned} \quad (4.30)$$

There exists a point \hat{x} such that $s^{SDC}(x; D(\omega^I))$ lies weakly above s when $x < \hat{x}$, with the roles reversed for $x > \hat{x}$.

Furthermore,

$$\begin{aligned} & \int_{x_L}^{\hat{x}} (s^{SDC}(x; D(\omega^I)) - s(x))dF(x) = \int_{\hat{x}}^{x_H} (s(x) - s^{SDC}(x; D(\omega^I)))dF(x) \quad (4.31) \\ \Leftrightarrow & \lambda \int_{x_L}^{\hat{x}} (s^{SDC}(x; D(\omega^I)) - s(x))dF_1(x) + (1 - \lambda) \int_{x_L}^{\hat{x}} (s^{SDC}(x; D(\omega^I)) - s(x))dF_2(x) \\ = & \lambda \int_{\hat{x}}^{x_H} (s(x) - s^{SDC}(x; D(\omega^I)))dF_1(x) + (1 - \lambda) \int_{\hat{x}}^{x_H} (s(x) - s^{SDC}(x; D(\omega^I)))dF_2(x). \end{aligned} \quad (4.32)$$

Let $f_1(\hat{x})/f_2(\hat{x}) \equiv k$ and recall that $f_1(\cdot)/f_2(\cdot)$ is weakly decreasing. It follows that $\forall \xi \in \mathbb{I}, \xi \leq \hat{x}$

$$\begin{aligned} & \frac{f_1(\xi)}{f_2(\xi)} \geq k \\ \Leftrightarrow & f_1(\xi) \geq kf_2(\xi) \\ \Rightarrow & [s^{SDC}(\xi; D(\omega^I)) - s(\xi)]f_1(\xi) \geq k[s^{SDC}(\xi; D(\omega^I)) - s(\xi)]f_2(\xi). \end{aligned} \quad (4.33)$$

Hence,

$$\int_{x_L}^{\hat{x}} [s^{SDC}(x; D(\omega^I)) - s(x)] dF_1(x) \geq k \int_{x_L}^{\hat{x}} [s^{SDC}(x; D(\omega^I)) - s(x)] dF_2(x). \quad (4.34)$$

By the same argument,

$$\int_{\hat{x}}^{x_H} [s(x) - s^{SDC}(x; D(\omega^I))] dF_1(x) \leq k \int_{\hat{x}}^{x_H} [s(x) - s^{SDC}(x; D(\omega^I))] dF_2(x). \quad (4.35)$$

By Lemma 4.3, it thus holds that

$$\int_{x_L}^{x_H} s^{SDC}(x; D(\omega^I)) dF_1(x) \geq \int_{x_L}^{x_H} s(x) dF_1(x). \quad (4.36)$$

This implies that $s^{SDC}(x; D(\omega^I))$ is indeed a solution to **(P1)** if $f_1(\cdot)/f_2(\cdot)$ is nonincreasing.

The proof for unique optimality of $s^{SDC}(x; D(\omega^I))$ whenever $f_1(\cdot)/f_2(\cdot)$ is strictly decreasing follows the same approach. It needs to be noted that if $s \neq s^{SDC}(x; D(\omega^I))$ in the above sense, there exists $\epsilon > 0$ such that, letting $\mathbb{A} \equiv \{x \in \mathbb{I} \setminus U_\epsilon(\hat{x}) \text{ such that } s(x) \neq s^{SDC}(x; D(\omega^I))\}$,

$$\int_{\mathbb{A}} 1 dF(x) > 0. \quad (4.37)$$

s and $s^{SDC}(x; D(\omega^I))$ need to be different with positive measure outside of an ϵ -neighborhood around \hat{x} . If this were not the case, s would be equal to \hat{s} almost everywhere. The same construction as in the previous approach can be utilized, but with

$$k_1 \equiv \frac{f_1(\hat{x} + \epsilon)}{f_2(\hat{x} + \epsilon)} < k = \frac{f_1(\hat{x})}{f_2(\hat{x})} < \frac{f_1(\hat{x} - \epsilon)}{f_2(\hat{x} - \epsilon)} \equiv k_2 \quad (4.38)$$

as reference points. On any interval $\Gamma \subseteq \mathbb{I}$ where $\forall x \in \Gamma : f_1(x)/f_2(x) \geq k$, and for any $\gamma(x) : \Gamma \rightarrow \mathbb{R}$, we know that

$$\begin{aligned}
 \int_{\Gamma} \gamma(x) dF_1(x) &\geq k \int_{\Gamma} \gamma(x) dF_2(x) & (4.39) \\
 \Rightarrow \lambda \int_{\Gamma} \gamma(x) dF_1(x) + (1-\lambda) \int_{\Gamma} \gamma(x) dF_2(x) &\geq \lambda k \int_{\Gamma} \gamma(x) dF_2(x) + (1-\lambda) \int_{\Gamma} \gamma(x) dF_2(x) \\
 &\Leftrightarrow \int_{\Gamma} \gamma(x) dF(x) \geq [\lambda k + (1-\lambda)] \int_{\Gamma} \gamma(x) dF_2(x) \\
 &\Leftrightarrow \frac{1}{\lambda k + (1-\lambda)} \int_{\Gamma} \gamma(x) dF(x) \geq \int_{\Gamma} \gamma(x) dF_2(x). & (4.40)
 \end{aligned}$$

If $\forall x \in \Gamma : f_1(x)/f_2(x) \leq k$, then

$$\frac{1}{\lambda k + (1-\lambda)} \int_{\Gamma} \gamma(x) dF(x) \leq \int_{\Gamma} \gamma(x) dF_2(x). \quad (4.41)$$

Applying this on $(x_L, \hat{x} - \epsilon)$ and $(\hat{x} - \epsilon, \hat{x})$ (with corresponding k_2 and k respectively), coupled with $\int_{\mathbb{A}} 1 dF(x) > 0$, yields that

$$\begin{aligned}
 \int_{x_L}^{\hat{x}} (s^{SDC}(x; D(\omega^I)) - s(x)) dF_2(x) &\leq \frac{1}{\lambda k_2 + (1-\lambda)} \int_{x_L}^{\hat{x}-\epsilon} (s^{SDC}(x; D(\omega^I)) - s(x)) dF(x) \\
 &+ \frac{1}{\lambda k + (1-\lambda)} \int_{\hat{x}-\epsilon}^{\hat{x}} (s^{SDC}(x; D(\omega^I)) - s(x)) dF(x) \\
 &\leq \frac{1}{\lambda k + (1-\lambda)} \int_{x_L}^{\hat{x}} (s^{SDC}(x; D(\omega^I)) - s(x)) dF(x). & (4.42)
 \end{aligned}$$

Likewise, applying this on $(\hat{x}, \hat{x} + \epsilon)$ and $(\hat{x} + \epsilon, x_H)$ (with corresponding k and k_1 respectively)

$$\begin{aligned}
 \int_{\hat{x}}^{x_H} (s(x) - s^{SDC}(x; D(\omega^I))) dF_2(x) &\geq \frac{1}{\lambda k + (1-\lambda)} \int_{\hat{x}}^{\hat{x}+\epsilon} (s(x) - s^{SDC}(x; D(\omega^I))) dF(x) \\
 &+ \frac{1}{\lambda k_1 + (1-\lambda)} \int_{\hat{x}+\epsilon}^{x_H} (s(x) - s^{SDC}(x; D(\omega^I))) dF(x) \\
 &\geq \frac{1}{\lambda k + (1-\lambda)} \int_{\hat{x}}^{x_H} (s(x) - s^{SDC}(x; D(\omega^I))) dF(x) & (4.43)
 \end{aligned}$$

need to hold. Finally, by (4.37),

$$\int_{\hat{x}+\epsilon}^{x_H} (s(x) - s^{SDC}(x; D(\omega^I)))dF(x) > 0 \vee \int_{x_L}^{\hat{x}-\epsilon} (s^{SDC}(x; D(\omega^I)) - s(x))dF(x) > 0. \quad (4.44)$$

Hence, (4.42) or (4.43) needs to hold strictly, i.e.

$$\begin{aligned} \int_{x_L}^{\hat{x}} (s^{SDC}(x; D(\omega^I)) - s(x))dF_2(x) &< \frac{1}{\lambda k + (1 - \lambda)} \int_{x_L}^{\hat{x}} (s^{SDC}(x; D(\omega^I)) - s(x))dF(x) \\ \wedge \int_{\hat{x}}^{x_H} (s(x) - s^{SDC}(x; D(\omega^I)))dF_2(x) &\geq \frac{1}{\lambda k + (1 - \lambda)} \int_{\hat{x}}^{x_H} (s(x) - s^{SDC}(x; D(\omega^I)))dF(x) \\ &\vee \end{aligned} \quad (4.45)$$

$$\begin{aligned} \int_{x_L}^{\hat{x}} (s^{SDC}(x; D(\omega^I)) - s(x))dF_2(x) &\leq \frac{1}{\lambda k + (1 - \lambda)} \int_{x_L}^{\hat{x}} (s^{SDC}(x; D(\omega^I)) - s(x))dF(x) \\ \wedge \int_{\hat{x}}^{x_H} (s(x) - s^{SDC}(x; D(\omega^I)))dF_2(x) &> \frac{1}{\lambda k + (1 - \lambda)} \int_{\hat{x}}^{x_H} (s(x) - s^{SDC}(x; D(\omega^I)))dF(x) \\ & \end{aligned} \quad (4.46)$$

Thus, by Lemma 4.3,

$$E_{f_2}[s^{SDC}(x; D(\omega^I))] < E_{f_2}[s(x)] \Leftrightarrow E_{f_1}[s^{SDC}(x; D(\omega^I))] > E_{f_1}[s(x)] \quad (4.47)$$

needs to hold whenever $f_1(\cdot)/f_2(\cdot)$ is strictly decreasing. $s^{SDC}(x; D(\omega^I))$ is therefore the unique solution to **(P1)** up to pointwise deviations. ■

4.E Proof of Lemma 4.5

This proof is almost identical to the one of Lemma 4.4. $L(\omega^I)$ is uniquely determined by Lemma 4.1. Take any security $s \neq s^{LE}(x; L(\omega^I))$, in the sense that they are different on a subset of \mathbb{I} with positive measure, with $E[s(x)] = \omega^I = E[s^{LE}(x; L(\omega^I))]$. Note that by construction of $s^{LE}(x; L(\omega^I))$,

$$\begin{aligned} \exists \hat{x} \in \mathbb{I} \text{ s.t. } \quad & s^{LE}(x; L(\omega^I)) \leq s(x) \quad \text{if } x \leq \hat{x} \\ & s^{LE}(x; L(\omega^I)) \geq s(x) \quad \text{if } x \geq \hat{x}. \end{aligned} \quad (4.48)$$

By repeating the previous analysis, but with an increasing likelihood ratio, it holds that for $k \equiv f_1(\hat{x})/f_2(\hat{x})$:

$$\int_{x_L}^{\hat{x}} [s(x) - s^{LE}(x; L(\omega^I))] dF_1(x) \leq k \int_{x_L}^{\hat{x}} [s(x) - s^{LE}(x; L(\omega^I))] dF_2(x) \quad (4.49)$$

$$\int_{\hat{x}}^{x_H} [s^{LE}(x; L(\omega^I)) - s(x)] dF_1(x) \geq k \int_{\hat{x}}^{x_H} [s^{LE}(x; L(\omega^I)) - s(x)] dF_1(x). \quad (4.50)$$

Recalling $E_f[s(x)] = E_f[s^{LE}(x; L(\omega^I))]$, applying Lemma 4.3 yields

$$\int_{x_L}^{x_H} s(x) dF_1(x) \leq \int_{x_L}^{x_H} s^{LE}(x; L(\omega^I)) dF_1(x). \quad (4.51)$$

This yields the desired result that $s^{LE}(x; L(\omega^I))$ solves **(P1)**. Furthermore, as in Lemma 4.4, this inequality can be established to hold strictly whenever $f_1(\cdot)/f_2(\cdot)$ is strictly increasing. Thus, $s^{LE}(x; L(\omega^I))$ is the unique solution to **(P1)** (up to pointwise deviations). The corollary follows because the observation about the intermediate point \hat{x} extends to s^{LD} in the modified problem **(P1*)**.

$$\begin{aligned} \exists \hat{x} \in \mathbb{I} \text{ s.t. } \quad & s^{DE}(x) \leq s(x) \quad \text{if } x \leq \hat{x} \\ & s^{DE}(x) \geq s(x) \quad \text{if } x \geq \hat{x}. \end{aligned} \quad (4.52)$$

Repeating the same steps as in the proof of the Proposition yields the corollary; $\omega^I \leq \int_{\mathbb{I}} \min[x, u] dF(x)$ ensures that a security subject to the restrictions exists. ■

4.F Proof of Lemma 4.6

First, global nondecreasingness is established. Since $D(e)$ solves

$$\int_{\underline{x}}^{\bar{x}} \hat{s}(x) dF(x) = e, \quad (4.53)$$

$D(e) \geq \inf_{\xi \in (\underline{x}, \bar{x})} s(\xi)$ has to hold as otherwise

$$\int_{\underline{x}}^{\bar{x}} \hat{s}(x) dF(x) < \int_{\underline{x}}^{\bar{x}} s(x) dF(x) = e. \quad (4.54)$$

However, by nondecreasingness of s this implies $D(e) \geq s(\underline{x})$. Likewise, $D(e) \leq \sup_{\xi \in (\underline{x}, \bar{x})} s(\xi)$ has to hold as otherwise

$$\int_{\underline{x}}^{\bar{x}} \hat{s}(x) dF(x) > \int_{\underline{x}}^{\bar{x}} s(x) dF(x) = e. \quad (4.55)$$

Thus, $D(e) \leq s(\bar{x})$, which with $D(e) \geq s(\underline{x})$ and the nondecreasingness of \hat{s} by construction yields global nondecreasingness of s^* . Next, it needs to be shown that s^* solves **(P1)** on \mathbb{I} .

First note that s and s^* have the same expected value outside of (\underline{x}, \bar{x}) , i.e.

$$\int_{\xi \notin (\underline{x}, \bar{x})} s(\xi) dF_1(\xi) = \int_{\xi \notin (\underline{x}, \bar{x})} s^*(\xi) dF_1(\xi), \quad (4.56)$$

since s^* and s are equal at all those points. Consider the following modified problem on (\underline{x}, \bar{x}) :

$$\begin{aligned} \text{(P1mod)} \quad \max_{t \in \mathcal{S}(\underline{x}, \bar{x})} E_{g_1}[t(x)] \quad \text{s.t.} \quad & \lambda E_{g_1}[t(x)] + (1 - \lambda) E_{g_2}[t(x)] = e \\ & 0 \leq t(x) \leq x \text{ for all } x \end{aligned}$$

where

$$g_1(x) = f_1(x) \cdot \frac{1}{F_1(\bar{x}) - F_1(\underline{x})} \quad (4.57)$$

$$g_2(x) = f_2(x) \cdot \frac{1}{F_2(\bar{x}) - F_2(\underline{x})} \quad (4.58)$$

with associated cdf on (\underline{x}, \bar{x}) :

$$G_1(x) = [F_1(x) - F_1(\underline{x})] \cdot \frac{1}{F_1(\bar{x}) - F_1(\underline{x})} \quad (4.59)$$

$$G_2(x) = [F_2(x) - F_2(\underline{x})] \cdot \frac{1}{F_2(\bar{x}) - F_2(\underline{x})}. \quad (4.60)$$

In this formulation, since $f_1(\cdot)/f_2(\cdot)$ is weakly decreasing on (\underline{x}, \bar{x}) , so is $g_1(\cdot)/g_2(\cdot) = f_1(\cdot)/f_2(\cdot) \cdot \frac{F_2(\bar{x}) - F_2(\underline{x})}{F_1(\bar{x}) - F_1(\underline{x})}$. This implies that **(P1mod)** corresponds to a problem where Lemma 4.4 applies. Hence, \hat{s} solves **(P1mod)**. This in turn implies that

$$\begin{aligned} \int_{\underline{x}}^{\bar{x}} \hat{s}(x) dG_1(x) &\geq \int_{\underline{x}}^{\bar{x}} s(x) dG_1(x) \\ \Rightarrow \frac{1}{F_1(\bar{x}) - F_1(\underline{x})} \int_{\underline{x}}^{\bar{x}} \hat{s}(x) dG_1(x) &\geq \frac{1}{F_1(\bar{x}) - F_1(\underline{x})} \int_{\underline{x}}^{\bar{x}} s(x) dG_1(x) \\ \Rightarrow \int_{\underline{x}}^{\bar{x}} \hat{s}(x) dF_1(x) &\geq \int_{\underline{x}}^{\bar{x}} s(x) dF_1(x). \end{aligned} \quad (4.61)$$

Thus, if s is a solution to **(P1)** on \mathbb{I} , \hat{s} has to be a solution as well. With (4.56) it follows that

$$\int_{x_L}^{x_H} s^*(x) dF_1(x) \geq \int_{x_L}^{x_H} s(x) dF_1(x). \blacksquare \quad (4.62)$$

4.G Proof of Lemma 4.7

First, nondecreasingness of s^* on (x_L, x_H) is established. On $(x_L, \underline{x}]$ and $[\bar{x}, x_H)$, s^* is nondecreasing by virtue of being equal to the nondecreasing s . On (\underline{x}, \bar{x}) , \hat{s} is nondecreasing by construction. Finally $s(\underline{x}) \leq s(x) \leq s(\bar{x})$ for all $x \in (\underline{x}, \bar{x})$ as $D = \sup_{\xi \in (\underline{x}, \bar{x})} s(\xi) \leq s(\bar{x})$.

Hence, s^* is globally nondecreasing. Since

$$\int_{\xi \notin (\underline{x}, \bar{x})} s(\xi) dF_1(\xi) = \int_{\xi \notin (\underline{x}, \bar{x})} s^*(\xi) dF_1(\xi) \quad (4.63)$$

as s^* and s coincide at all those points, it is sufficient to show

$$\int_{\xi \in (\underline{x}, \bar{x})} s(\xi) dF_1(\xi) \leq \int_{\xi \in (\underline{x}, \bar{x})} s^*(\xi) dF_1(\xi) \quad (4.64)$$

to establish s^* as a solution to **(P1)**. This follows from the corollary to Lemma 4.5. Note that $e \equiv \int_{\underline{x}}^{\bar{x}} s(x) dF(x) \leq \int_{(\underline{x}, \bar{x})} \min\{x, D\}$ by construction of D . The corollary thus establishes that \hat{s} solves **(P1mod*)** on (\underline{x}, \bar{x}) , where

$$\begin{aligned} \text{(P1mod*) } \max_{t \in S_{(\underline{x}, \bar{x})}} E_{g_1}[t(x)] \quad \text{s.t.} \quad & \lambda E_{g_1}[t(x)] + (1 - \lambda) E_{g_2}[t(x)] = e \\ & t(x) \leq D \text{ for all } x \in (\underline{x}, \bar{x}), \end{aligned}$$

and

$$g_1(x) = f_1(x) \cdot \frac{1}{F_1(\bar{x}) - F_1(\underline{x})} \quad (4.65)$$

$$g_2(x) = f_2(x) \cdot \frac{1}{F_2(\bar{x}) - F_2(\underline{x})}. \quad (4.66)$$

Thus, s^* solves **(P1)** on \mathbb{I} and is indeed nondecreasing, as well as compliant with the limited liability constraint. ■

4.H Proof of Proposition 4.4

Before turning to the proofs of 4(i) and 4(ii), introduce the following notation: Let TD be the set of largest disjoint intervals where s is consistent with debt-like tranches. Further, let NTD be the set of largest disjoint intervals where s is inconsistent with this property. Denote NTD_i the set of largest disjoint intervals in NTD where $f_1(\cdot)/f_2(\cdot)$ is weakly increasing and let NTD_d be the set of largest disjoint intervals where $f_1(\cdot)/f_2(\cdot)$ is weakly decreasing within the complementary set to NTD_i with respect to NTD . Lastly, let P denote the set of points not in TD and NTD , but in \mathbb{I} .

Formally,

$$\begin{aligned}
TD &\equiv \{(\xi_1, \xi_2) \subset \mathbb{I} \mid s \text{ is composed of debt-like tranches on } (\xi_1, \xi_2) \\
&\quad \wedge \exists \epsilon > 0 : s \text{ is not consistent with debt-like tranches on } (\xi_1 - \epsilon, \xi_1) \text{ or } (\xi_2, \xi_2 + \epsilon)\} \\
NTD &\equiv \{(\xi_1, \xi_2) \subset \mathbb{I} \mid s \text{ is not consistent with debt-like tranches on } (\xi_1, \xi_2) \\
&\quad \wedge \exists \epsilon > 0 : s \text{ is composed of debt-like tranches on } (\xi_1 - \epsilon, \xi_1) \text{ or } (\xi_2, \xi_2 + \epsilon)\} \\
NTD_i &\equiv \{(\xi_1, \xi_2) \subset NTD \mid \frac{f_1(\cdot)}{f_2(\cdot)} \text{ is weakly increasing in } \xi \text{ on } (\xi_1, \xi_2) \\
&\quad \wedge \exists \epsilon > 0 : \frac{f_1(\cdot)}{f_2(\cdot)} \text{ is not weakly increasing in } \xi \text{ on } (\xi_1 - \epsilon, \xi_2) \text{ and } (\xi_1, \xi_2 + \epsilon)\} \\
NTD_d &\equiv \{(\xi_1, \xi_2) \subset NTD \setminus NTD_i \mid \frac{f_1(\cdot)}{f_2(\cdot)} \text{ is weakly decreasing in } \xi \text{ on } (\xi_1, \xi_2) \\
&\quad \wedge \exists \epsilon > 0 : \frac{f_1(\cdot)}{f_2(\cdot)} \text{ is not weakly decreasing in } \xi \text{ on } (\xi_1 - \epsilon, \xi_2) \text{ and } (\xi_1, \xi_2 + \epsilon)\} \\
NTD_p &\equiv NTD \setminus (NTD_i \cup NTD_d) \\
P &\equiv \mathbb{I} \setminus (TD \cup NTD).
\end{aligned}$$

By construction,

$$TD \cup NTD_i \cup NTD_d \cup NTD_p \cup P = \mathbb{I} \quad (4.67)$$

and

$$\int_P 1dF(x) = 0 = \int_{NTD_p} 1dF(x). \quad (4.68)$$

Consider 4(i). Take any interval $(\xi_1, \xi_2) \in NTD_i$. Consider the security

$$\hat{s}(x) = \begin{cases} s(x) & \text{if } x \notin (\xi_1, \xi_2) \\ s(\xi_1) & \text{if } x \in (\xi_1, L] \\ \min\{x, \sup_{\xi \in (\xi_1, \xi_2)} s(\xi)\} & \text{if } x \in (L, \xi_2) \end{cases} \quad (4.69)$$

where L is the by Lemma 4.1 unique solution to

$$\int_{\xi_1}^{\xi_2} s(\xi) dF(\xi) = \int_{\xi_1}^{\xi_2} \hat{s}(\xi) dF(\xi). \quad (4.70)$$

By Lemma 4.7, $\hat{s}(x)$ is composed of debt-like tranches on (ξ_1, ξ_2) and $s^* = s(x) + (\hat{s}(x) - s(x))\mathbf{I}_{x \in (\xi_1, \xi_2)}$ also solves **(P1)**. Likewise, for any $(\psi_1, \psi_2) \in NTD_d$, the security

$$\hat{s}(x) = \begin{cases} s(x) & \text{if } x \notin (\psi_1, \psi_2) \\ \min\{x, D\} & \text{otherwise} \end{cases} \quad (4.71)$$

where D is the by Lemma 4.1 unique solution to

$$\int_{\psi_1}^{\psi_2} s(\psi) dF(\psi) = \int_{\psi_1}^{\psi_2} \hat{s}(\psi) dF(\psi) \quad (4.72)$$

is composed of debt-like tranches on (ψ_1, ψ_2) and $s^* = s(x) + (\hat{s}(x) - s(x))\mathbf{I}_{x \in (\xi_1, \xi_2)}$ also solves **(P1)** by Lemma 4.6. Since the above statements hold for any $(\xi_1, \xi_2) \in NTD_i$ and for any $(\psi_1, \psi_2) \in NTD_d$, repeated use of the above local modification on all intervals in NTD_i and NTD_d yields a security s^* which is composed of debt-like tranches on

$$NTD_i \cup NTD_d \cup TD.$$

With (4.67) and (4.68) this implies that s^* is consistent with being composed of debt-like tranches on \mathbb{I} and solves **(P1)**. This concludes the proof for 4(i).

For 4(ii), suppose that s is not composed of debt-like tranches on \mathbb{I} . This implies that

$$\int_{NTD_i \cup NTD_d} 1 dF(x) > 0. \quad (4.73)$$

Suppose for the remainder of the proof that $\int_{NTD_i} 1 dF(x) > 0$.⁷⁰ By the continuity of f_1, f_2 and local nonproportionality, i.e.

$$\forall (\xi_1, \xi_2) \subseteq \mathbb{I} : \forall k \in \mathbb{R}_+ \exists x \in (\xi_1, \xi_2) : f_1 \neq k f_2, \quad (4.74)$$

⁷⁰The proof for $\int_{NTD_d} 1 dF(x) > 0$ is similar and utilizes a contradiction built with the use of Lemma 4.4.

it follows that $f_1(\cdot)/f_2(\cdot)$ is strictly increasing on any $(\xi_1, \xi_2) \in NTD_i$.⁷¹ Take any $(\xi_1, \xi_2) \in NTD_i$. Consider the following modified problem on (ξ_1, ξ_2) :

$$\begin{aligned} (\mathbf{Pmod}) \quad \max_{t \in S(\xi_1, \xi_2)} E_{g_1}[t(x)] \quad \text{s.t.} \quad & \lambda E_{g_1}[t(x)] + (1 - \lambda) E_{g_2}[t(x)] = \int_{\xi_1}^{\xi_2} s(x) dG(x) \\ & t(x) \leq \sup_{\xi \in (\xi_1, \xi_2)} s(\xi) \end{aligned}$$

where

$$g_1(x) = f_1(x) \cdot \frac{1}{F_1(\xi_2) - F_1(\xi_1)} \quad (4.75)$$

$$g_2(x) = f_2(x) \cdot \frac{1}{F_2(\xi_2) - F_2(\xi_1)}. \quad (4.76)$$

By Corollary 4.1 to Lemma 4.5, the levered debt security

$$s^{LD}(x) = \min\{x, \sup_{\xi \in (\xi_1, \xi_2)} s(\xi)\} \cdot \mathbf{1}_{x \in [L, \xi_2]} \quad (4.77)$$

with L chosen such that

$$\int_{\xi_1}^{\xi_2} s^{LD}(x) dG(x) = \int_{\xi_1}^{\xi_2} s(x) dG(x) \quad (4.78)$$

$$\Leftrightarrow \int_{\xi_1}^{\xi_2} s^{LD}(x) dF(x) = \int_{\xi_1}^{\xi_2} s(x) dF(x) \quad (4.79)$$

is the unique (up to pointwise deviations) solution to **(Pmod)** on (ξ_1, ξ_2) . Hence, since s is not composed of debt-like tranches on (ξ_1, ξ_2) and can thus not be levered debt there,

$$E_{g_1}[s^{LD}(x)] = \int_{\xi_1}^{\xi_2} s^{LD}(x) dG_1(x) > \int_{\xi_1}^{\xi_2} s(x) dG_1(x) = E_{g_1}[s(x)]. \quad (4.80)$$

Therefore,

⁷¹Weakly increasing and not strictly is ruled out by the nonproportionality, which would be necessary for a locally constant $f_1(\cdot)/f_2(\cdot)$.

$$\int_{\xi_1}^{\xi_2} s^{LD}(x) dF_1(x) > \int_{\xi_1}^{\xi_2} s(x) dF_1(x) \quad (4.81)$$

and thus

$$\int_{\mathbb{I}} \hat{s}(x) dF_1(x) > \int_{\mathbb{I}} s(x) dF_1(x) \quad (4.82)$$

where

$$\hat{s}(x) = \begin{cases} s(x) & \text{if } x \notin (\xi_1, \xi_2) \\ s^{LD}(x) & \text{otherwise.} \end{cases} \quad (4.83)$$

However, by Lemma 4.7 this $\hat{s}(x)$ is a valid security, i.e. \hat{s} satisfies limited liability and nondecreasingness. By construction

$$\int_{\mathbb{I}} \hat{s}(x) dF(x) = \int_{\mathbb{I}} s(x) dF(x) = \omega^I. \quad (4.84)$$

Hence, optimality of s is violated. Thus, the assumption that s is not composed of debt-like tranches has to be false. This concludes the proof. ■

4.1 Proof of Proposition 4.5

The proof is a proof by contradiction. Note first that since local nonproportionality holds, any solution to **(P1)** is composed of debt-like tranches. Consider some such security s with $E_f[s(x)] = \omega^I$ which does not satisfy $\forall x \in \mathbb{I} : s(x) > x_L$. As such,

$$\exists \hat{x} \in (x_L, x_H) : s(x) > x_L \Leftrightarrow x \geq \hat{x}. \quad (4.85)$$

We need to show that s cannot be a solution to **(P1)**.

First note that for s to be a solution to **(P1)**, $s(x) \geq x_L$ for all $x \in (x_L, x_H)$ is necessary. This corresponds to Lemma 4.4 of [Biais and Mariotti \(2005\)](#). To see this, suppose otherwise. Increasing the payoff to x_L at all values x increases the payoff in both states by the same value ($x \geq x_L$ with probability 1). Furthermore, any security s with $E_f[s(x)] = \omega^I > x_L$ has $E_{f_1}[s(x)] < E_{f_2}[s(x)]$ by $F_1(x) > F_2(x)$ for all $x \in (x_L, x_H)$. Hence, increasing the payoff at all $\xi \in (x_L, x_H)$ where $s(x) < x_L$ and

decreasing the payoff proportionally at all other points such that the expected payoff E_f remains unchanged increases the expected payoff under bad information. Thus, s cannot solve **(P1)**.

Having established that $s(x) \geq x_L$ for all x , denote $\{x_1, D_1\}$ the pair characterizing starting point and face value of the first tranche with a face value larger than x_L , $D_1 > x_L$. Since s specifies a payoff larger than x_L whenever $x \geq \hat{x} > x_L$ for some $\hat{x} > x_L$, it has to be the case that $x_1 > x_L$. Suppose that s consists of a finite number N of tranches and denote x_N, D_N the starting point and face value of the last debt-like tranche. Denote

$$\frac{1 - F_1(x_N)}{1 - F_2(x_N)} \equiv k < 1 \tag{4.86}$$

$$F_1(x_1) - F_2(x_1) \equiv c > 0 \tag{4.87}$$

$$D_N - D_{N-1} \equiv d > 0 \tag{4.88}$$

Since $F_1(x_L) = F_2(x_L) = 0$, it has to hold that

$$\exists \epsilon > x_L \text{ such that } \forall x_L < \delta < \epsilon : \frac{\int_{x_L}^{x_H} \min\{x, \delta\} dF_1(x)}{\int_{x_L}^{x_H} \min\{x, \delta\} dF_2(x)} > k, \tag{4.89}$$

i.e. that for sufficiently small face values δ , the corresponding standard debt tranche pays out at least k times as much (in expectation) after bad information as after good information. Existence of $\epsilon > x_L$ is due to the fact that the k is bounded away from 1, whereas the initial tranche can be constructed with a ratio arbitrarily close to 1 due to strict positivity of densities and $F_1(x_L) = F_2(x_L) = 0$. Take some such ϵ and consider $D_1 > \gamma > x_L$ where $\gamma < \epsilon$ and $\kappa \equiv \int_{x_L}^{x_1} \min\{x, \gamma\} dF(x) < d[1 - F(x_N)]$. The contract s^* with

$$s^*(x) = \begin{cases} \min\{x, \gamma\} & \text{for } x < x_1 \\ \min\{x, D_N\} - \frac{\kappa}{1 - F(x_N)} & \text{for } x \in (x_N, x_H) \\ s(x) & \text{otherwise} \end{cases} \tag{4.90}$$

can be shown to be nondecreasing, to satisfy limited liability, to have an expected value $E_f[s^*(x)] = \omega^I$ and to satisfy $E_{f_1}[s^*(x)] > E_{f_1}[s(x)]$, thus violating the supposed optimality of s . First, $E_f[s^*(x)] = \omega^I$ follows from the definition of s^* and

$\kappa \equiv \int_{x_L}^{x_1} \min\{x, \gamma\} dF(x)$. Next, nondecreasingness stems from $\kappa < d[1 - F(x_N)]$ and $\gamma < D_1$. Finally,

$$\frac{\int_{x_L}^{x_H} \min\{x, \gamma\} dF_1(x)}{\int_{x_L}^{x_H} \min\{x, \gamma\} dF_2(x)} > k \quad (4.91)$$

$$\Rightarrow \frac{\int_{x_L}^{x_H} \min\{x, \gamma\} dF_1(x) - \gamma[1 - F_2(x_1) - c]}{\int_{x_L}^{x_H} \min\{x, \gamma\} dF_2(x) - \gamma[1 - F_2(x_1)]} > k = \frac{1 - F_1(x_N)}{1 - F_2(x_N)} \quad (4.92)$$

$$\Rightarrow \int_{x_L}^{x_H} \min\{x, \gamma\} dF_1(x) - \gamma[1 - F_2(x_1) - c] - \frac{\kappa}{1 - F(x_N)}[1 - F_1(x_N)] > 0 \quad (4.93)$$

$$\Rightarrow \int_{x_L}^{x_H} s^*(x) dF_1(x) > \int_{x_L}^{x_H} s(x) dF_1(x). \quad (4.94)$$

To illustrate this construction, consider the following graph:

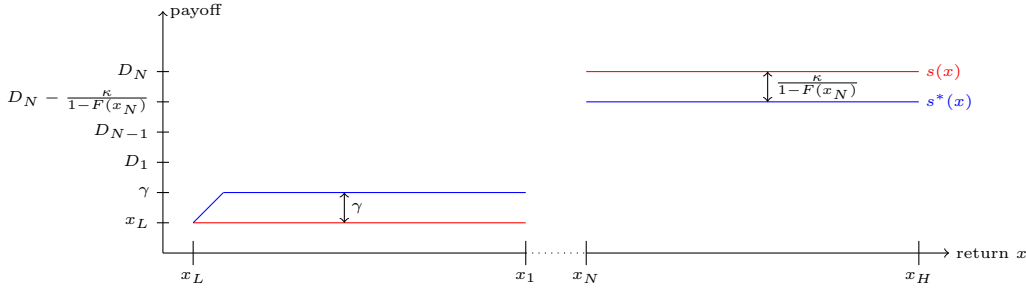


Figure 4.13: Illustration – Inclusion of Standard Debt Tranche

s^* is different from s in that it includes a standard debt tranche with face value γ and correspondingly decreases the payoff of the last tranche by $\frac{\kappa}{1-F(x_N)}$, thus ensuring that the expected values $E_f[s^*(x)] = E_f[s(x)] = \omega^I$ remain unchanged. The decrease of the face value of the most junior tranche characterized by $\{x_N, D_N\}$ affects the expected values of the security in the bad and good state with ratio k . By construction, the inclusion of the standard debt tranche with face value γ increases the expected values in the good and bad state with a larger proportion. Since the unconditional expected values of s and s^* are identical, this implies that $E_{f_1}[s^*(x)] > E_{f_1}[s(x)]$ and thus that s cannot be optimal.

If s consists of an infinite number of debt-like tranches, there has to exist at least one tranche where the ratio is bounded away from 1, i.e. where (letting j denote the

tranche) $\frac{F_1(x_{j+1})-F_1(x_j)}{F_2(x_{j+1})-F_2(x_j)} \leq k < 1$. If no such tranche existed, first order stochastic dominance would be violated. The remaining construction is then as above for the last tranche. This concludes the proof for (4.85). Coupled with the observation that any solution to **(P1)** is composed of debt-like tranches, the only securities satisfying the requirement are composed of debt-like tranches which include a standard debt tranche. ■

4.J Proof of Proposition 4.6

It can be established that a contract s with $E_f[s(x)] = E_f[s^{SDC}(x; D(\omega^I))] = \omega^I$ and $E_{f_1}[s(x)] > E_{f_1}[s^{SDC}(x; D(\omega^I))]$ exists whenever (i) holds. Consider some such ξ . By continuity of f_1, f_2 , if $G(\xi) > \inf_{x \in (x_L, D(\omega^I))} f_1(x)/f_2(x)$, there has to exist an ϵ -neighborhood around some $\hat{x} \in (x_L, D(\omega^I))$ such that

$$\forall x \in U_\epsilon(\hat{x}) : \frac{f_1(x)}{f_2(x)} < G(\xi) \quad (4.95)$$

where $x_L < \hat{x} - \epsilon < \hat{x} < \hat{x} + \epsilon < D(\omega^I)$.

Consider the following security:

$$s(x) = \begin{cases} x & \text{for } x \in (x_L, \hat{x} - \epsilon) \\ \hat{x} - \epsilon & \text{for } x \in [\hat{x} - \epsilon, \hat{x} + \epsilon] \\ \min\{x, D(\omega^I)\} & \text{for } x \in (\hat{x} + \epsilon, \xi) \\ D(\omega^I) + \kappa & \text{for } x \in (\xi, x_H) \end{cases} \quad (4.96)$$

where $\kappa(1 - F(\xi)) = \int_{U_\epsilon(\hat{x})} [x - (\hat{x} - \epsilon)] dF(x)$ and ϵ is chosen sufficiently small such that $\kappa \leq \xi - D$.

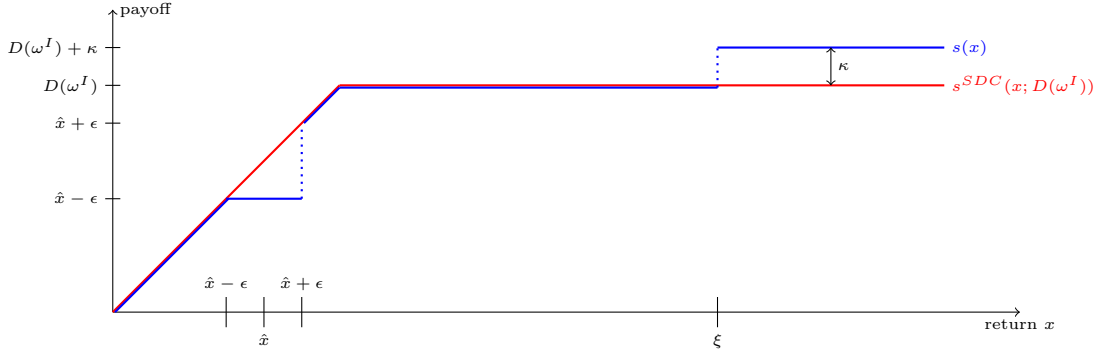


Figure 4.14: Illustration – Inclusion of Junior Debt Tranche

By construction, it holds that $E_f[s(x)] = E_f[s^{SDC}(x; D(\omega^I))]$. Thus, since $f_1(x)/f_2(x) < G(\xi)$ for all $x \in U_\epsilon(\hat{x})$,

$$\begin{aligned} \frac{\int_{U_\epsilon(x)} (s^{SDC}(x; D(\omega^I)) - s(x)) dF_1(x)}{\int_{U_\epsilon(x)} (s^{SDC}(x; D(\omega^I)) - s(x)) dF_2(x)} &< \kappa G(\xi) = \kappa \frac{1 - F_1(\xi)}{1 - F_2(\xi)} \quad (4.97) \\ \Rightarrow \frac{\int_{U_\epsilon(x)} (s^{SDC}(x; D(\omega^I)) - s(x)) dF_1(x)}{\int_{U_\epsilon(x)} (s^{SDC}(x; D(\omega^I)) - s(x)) dF_2(x)} &< \frac{\int_\xi^{x_H} (s(x) - s^{SDC}(x; D(\omega^I))) dF_1(x)}{\int_\xi^{x_H} (s(x) - s^{SDC}(x; D(\omega^I))) dF_2(x)} \\ &\Rightarrow \int_{x_L}^{x_H} s^{SDC}(x; D(\omega^I)) dF_1(x) < \int_{x_L}^{x_H} s(x) dF_1(x) \quad (4.98) \end{aligned}$$

where the last implication follows from Lemma 4.3 and the fact that s and $s^{SDC}(x; D(\omega^I))$ are identical outside of $U_\epsilon(\hat{x})$ and (ξ, x_H) . $s^{SDC}(x; D(\omega^I))$ cannot be optimal. Intuitively, the above construction does not affect the unconditional expected value. However, the decrease at all points in $U_\epsilon(\hat{x})$ yields a decrease in both the expected value under bad and good information. Of the overall change in expected value, less is attributed to the expected value under bad information (as compared to that under good information) than in the increase of the payoff by κ at all points in (ξ, x_H) . Thus, since the unconditional expected value changes exactly offset each other, s has a higher expected value under bad information than $s^{SDC}(x; D(\omega^I))$. Hence, $s^{SDC}(x; D(\omega^I))$ cannot solve **(P1)**. ■

Chapter 5

Discussion

In their review of securitization practices in the recent financial crisis, [Gorton and Metrick \(2012\)](#) write that “despite the quantitative and theoretical importance of securitization, there is relatively little research on the subject. In addition, the recent financial crisis centered on securitization, so the imperative to understand it is paramount.”

In recent years, researchers have produced both theoretical and empirical contributions which shed light on agents’ incentives in prevalent securitization practices and assess the impact of securitization on market participants’ behavior and outcomes. However, there is still much that is not fully understood.

This thesis contributes to the literature by focusing on the interaction between public information and private information acquisition incentives – the latter of which are problematic as they may imply illiquidity stemming from asymmetric information.

Chapter 2 explicitly considers the interaction between the opacity of an asset and its liquidity in a stylized framework and shows that opacity may protect liquidity by avoiding costly private information acquisition. The implicit viewpoint by policymakers that transparency is a one-way street towards a better financial system thus not necessarily comes without exemptions – liquidity may be adversely affected by transparency regulation, which should be taken into account.

Chapter 3 assesses lenders’ screening incentives in a setting where loan sales may yield gains from trade, but where private information may be selectively used during securitization. It is shown that the advent of securitization may in fact lead to less screening in equilibrium, at least for some prospective borrowers, and that this can have adverse effects on (borrower) welfare.

Finally, Chapter 4 derives optimal securities with respect to being robust to interim public information and shows that robustness in this sense is achieved by securities which do not necessarily minimize incentives to acquire private information.

Chapters 2 and 3 in particular also pave the way for future empirical analyses, either by developing testable implications and proposing a basic empirical approach which can be expanded upon (Chapter 2) or by elaborating on how the model's key assumptions can be assessed (Chapter 3). Aside from further theoretical considerations or extensions outlined in the respective Chapters, this is an avenue which seems promising and fruitful for future research.

It is my hope that the economic analysis of securitization continues to improve our understanding of why agents in the financial system behaved in the way they did prior to the recent crisis. Only then can prudential policy be designed – if necessary – to avoid future collapses which exert significant negative externalities on all agents in the economy.

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Eidesstattliche Erklärung

Hiermit erkläre ich, die vorliegende Dissertation selbständig angefertigt und mich keiner anderen als der in ihr angegebenen Hilfsmittel bedient zu haben. Insbesondere sind sämtliche Zitate aus anderen Quellen als solche gekennzeichnet und mit Quellenangaben versehen.

Außerdem erkläre ich mich damit einverstanden, dass die Universität meine Dissertation zum Zwecke des Plagiatsabgleichs in elektronischer Form speichert, an Dritte versendet, und Dritte die Dissertation zu diesem Zwecke verarbeiten.

Mannheim, 01.07.2015

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