Family Policies and the Macroeconomy

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Chapter 1

Introduction

Family policies in Germany are intended to reach numerous goals at the same time - enhancing fertility, improving the employment situation of mothers, encouraging fathers to spend more time with their children, just to mention a few. However, the results are not always straightforward and clear. Based on numerous micro-econometric studies, there is empirical evidence that enhanced family policies can indeed raise the fertility rate.

Building on this finding, the first question that arises is, what the effects of family policies are in a macroeconomic general equilibrium. In a general equilibrium, the government does not only provide better support to parents but also has to finance such policies. Furthermore, the households' decisions affect prices. Thus, it is not clear *ex ante* what the overall effects of family policy are. In Chapter 2, I compare two recent German family policy reforms in order precisely to answer these questions. I find that, also in general equilibrium, enhanced family policies have a positive effect on fertility. Even more interesting is the second main result: Given the low fertility rate in Germany, one additional EUR "invested" in family policies brings the government more than one EUR in return in the long-run.

Furthermore, in Germany there are over 150 different family policy instruments which have different effects on outcomes such as fertility or labor supply. In Chapter 3 I select two of these family policies - *Kindergeld* and *öffentliche Betreuungsangebote* - and analyze their effectiveness and underlying mechanisms. Households prefer to get higher *Kindergeld* while an expansion of public child care is more beneficial for the government budget. In addition, using the additional financial resources for a reduction of labor taxes is the best government policy in terms of welfare.

In Germany, we currently experience a demographic change leading towards an older population. The demographic structure that used to be a pyramid currently transforms into a pillar. As the public pension system builds on this demographic structure, population aging has a massive negative impact on the public provision of old-age insurance. As a consequence, the only reform proposal frequently made is that the German government shall adjust the public pension system to the new demographic structure.

Contrary to this approrach, in environmental politics the reform proposals to mitigate climate change are based on two pillars: One is to “adjust the system” to adapt to climate change, and the other is to incentive people to change their behavior in order to reduce the size of climate change itself. Transposing this approach to demography politics and demographic change, only one pillar is enforced currently: Reforming the system to adapt to demographic change. However, the second pillar - incentivizing people to reduce demographic change itself - has been ignored so far. So, one can wonder if there is some scope for family policies to become this second pillar of demography politics.

In Chapter 4, I thus propose a different approach to tackle demographic change, namely to expand family policies. I simulate a negative longevity shock on the budget of the pension system and analyze whether it is preferable to respond to this shock with pension reforms or with adapted family policies. I find that in the long-run, family policy reforms are more attractive compared to reforms of the pension system.

Finally, the results of family policies depend on the underlying model setup. I therefore compare in Chapter 5 the effects of family policies in three different models, one with parental altruism, one with reversed altruism and one without any altruism. The positive effect on fertility remains in all models whereas the budgetary effect is dependent on the model and the calibration.

Chapter 2

Family Policies in General Equilibrium

2.1 Introduction

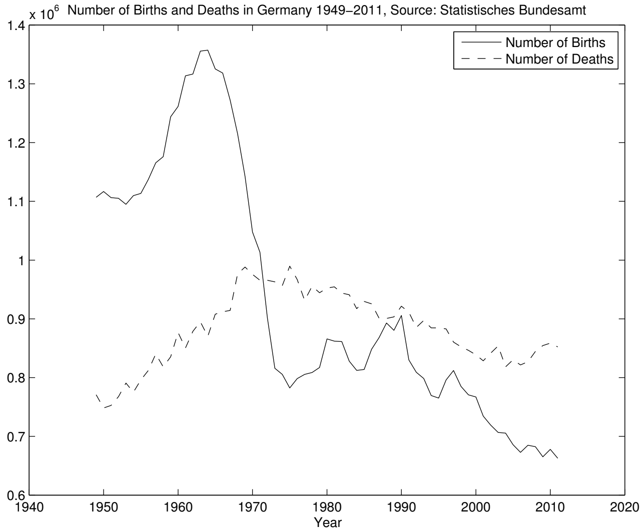
Demographic change, which is a well-known phenomenon in Germany, is driven by two factors: While mortality decreases and life expectancy increases, fertility goes down. Since 1972 the number of deaths has been exceeding the number of births per year - a situation which is assumed to get even worse in the next decades. Figure 2.1 shows this pattern for Germany between 1949 and 2011. In 2011, there were around 650,000 newborns compared to 850,000 deaths. The number of deaths has been staying rather constant even with the strong increase of life expectancy while the number of births has been steadily declining.

This change of demographic structure has implications, in particular for the economic and social life as well as for the priorities of the government. Because of the demographic change one strong priority of the German Federal government is to support families and children. One of the main goals of the German Federal government is to motivate and support couples to have more children.[1] However, do policies that aim at motivating parents to get (more) children work indeed? A micro-econometric body of literature is dealing with this question which will be discussed in Section 2.2. Overall, many studies find indeed a positive and significant effect of family policies on the fertility rate. However, the empirical micro-econometric literature does not look at the overall effect. Family subsidies such as direct monetary transfers, public child care or tax deductions for children have to be financed. What happens to the effectiveness of family policies when one takes this budgetary effect into account? Furthermore, family policies used as fertility subsidies change relative prices and therefore have an impact on other decisions

[1] See Bundesinnenministerium (2012).

of the parents such as consumption, savings or labor supply. What is the effect of those changes on aggregate prices and how do they feed back into the fertility decision of the parents?

Figure 2.1: Number of Births and Deaths in Germany between 1949 and 2011



More precisely, in this chapter I want to find out what the long-run effects of family policy in Germany on fertility, labor and savings decisions of households are in general equilibrium (GE)? I evaluate the effectiveness and analyze the outcomes of two recent reforms of German family policy. The first one consists in a reform of direct monetary benefits (the so-called *Kindergeld*) from 184 EUR to 190 EUR per child and month while the second reform corresponds to an expansion of public child care (*Kindertagesstätte* shortened as *Kita*) up to a level of 35% coverage, meaning that 35% of all children below the age of three visit a child care facility.

To analyze these reforms I use a four-period overlapping generations (OLG) model with heterogeneous households that decide about the number of children they want to have, their consumption, savings and how much they work. The households are heterogeneous with respect to their income. The model furthermore includes a government sector that collects labor income taxes and uses the tax revenues to finance family policies, a public pension system and general government spending.[2] The government offers two types of family policies, namely public child care and direct monetary benefits.[3] The model economy consists of two types of households - high and low income - to capture the effect that family policies have on parents belonging to different income groups. Moreover, households accumulate work experience when being active in the labor market which endogenously leads to a family wage gap that is actually observed:

[2]These expenditures cover sectors like military defense, education or infrastructure.

[3]There are many more instruments used for family policies in Germany, in total over 150. The German system of family policies will be discussed in Section 2.2.

Parents - mothers in particular - suffer losses of lifetime earnings because of parenthood.[4] Using the method of matching moments, the model is calibrated to Germany around the year 2010. Starting from this calibrated benchmark I introduce the two reforms and analyze their outcomes.

My findings are, first, that both family policies have a positive impact on the fertility rate taking long-run general equilibrium effects into account and, second, that the budgetary effect of the intensified family policies does not lead to a trade-off in the budget of the government. In the long-run, both reforms are more than self-financing; they actually add additional financial resources to the governmental budget. This second finding is not obvious as it implies that an investment of one Euro in family policies brings a return that is larger than one Euro to the budget. This positive return of family policy exists as long as the positive effects of family policies dominate the negative ones. The positive effects are a lower old-age dependency ratio, that is less retirees per worker, and in the case of public child care a larger accumulation of work experience because of the positive effect on the individual labor supply. The negative effects are a higher youth dependency ratio, that is more children per employee, and in the case of public child care a lower base wage rate because of the labor supply effect. I show that this result is robust across different parameter choices and for a large range of family policy levels. The 3.2% increase of the *Kindergeld* from 184 to 190 EUR leads to an increase of the fertility rate of 0.7%. The policy has a stronger effect on low income households than on high income households with an increase of the fertility rate of 0.8% and 0.4%, respectively. The expansion of child care coverage by 15 percentage points from 20 up to 35% is of much bigger magnitude and therefore its effects are quantitatively larger. The total fertility rate increases by 12.3%. The number of newborns in low income households increases by 13.6% and by 7.7% in high income households. Again, the fertility response of the lower income households is stronger. The *Kindergeld* reform creates a budgetary surplus of 1.6%. The government uses this surplus to increase the general expenditures. The corresponding increase for the child care reform is 34.1%.

There exists already a strand of literature that uses quantitative models with endogenous fertility choice to evaluate family policies. However, it mainly focuses on labor market outcomes and on female labor force participation in particular. In addition, most of these models are partial equilibrium models. Fehr and Ujhelyiova (2013) calibrate an OLG model to Germany and evaluate the effect of different public policies on fertility and employment. They find that fertility subsidies can raise the number of children while the effect on the mother's labor force participation is policy dependent. Bick (2013) calibrates an OLG model to evaluate the effect of different child care policies on fertility and female labor force participation for Germany. He

[4] See for example Beblo et al. (2009).

finds that the lack of child care is indeed responsible for a lower fertility. The size of the effect, however, is relatively small. He also finds that the timing of the family policy intervention is crucial. A different approach is taken by Erosa et al. (2010). They use a labor market model to analyze the general equilibrium effects of family leave policies. In general, there exists a positive effect of a family leave policy on fertility. However, the direction of the effect is not always clear and depends on the exact implementation of the family leave policy. Domeij and Klein (2013) investigate whether offering subsidized daily child care helps to increase female labor force participation and welfare in general. They find positive effects even in general equilibrium when child care is financed by taxing the labor income. However, there is no statement about fertility since demographics are exogenously given in their model.

My key findings contribute to this literature. First, the model creates positive fertility reactions even in general equilibrium and when the policies have to be financed. The result is even stronger: With most of the parameter constellations, family policy opens up new financial resources for the government. Furthermore, the fertility elasticities are in a reasonable range and similar to those measured in the empirical literature.

This Chapter is structured as follows: Section 2.2 summarizes the system of family policies in Germany and also discusses the empirical literature on the effectiveness of such policies on fertility. Section 2.3 describes the OLG model setup including the equilibrium definition. Section 2.4 presents the data used, the calibration strategy and the benchmark equilibrium. In Section 2.5 I analyze two family policy reforms, the 2015 *Kindergeld Reform* and a *Kita-Reform*. I check the robustness of these results in Section 2.6. Finally, Section 2.7 concludes this chapter.

2.2 System of German Family Policies and their Effectiveness to Enhance Fertility

2.2.1 German Family Policy

In 2010, there were over 150 different policies supporting families, children and married couples in Germany with an overall volume of 200 billion EUR. 75 billion EUR were devoted to the support of married couples (*ehebezogene Leistungen*) and the remaining 125 billion EUR were allocated to support families, parents and children (*familienbezogene Leistungen*).[5] This second sum can be divided into three legal or political components: Family support (*Familienförderung*) with 55 billion EUR, compensation policies for families (*Familienlastenausgleich*) with 53 billion EUR, and other expenditures to ensure a minimum income and health care level for children (*materielle*

[5]See for example Bundesfamilienministerium (2012) or Prognos (2014b).

und gesundheitliche Grundsicherung von Kindern) or for youth surveillance (*Jugendhilfe*) with 17 billion EUR. The first component is the only one that can be influenced by policy makers and reforms, the others are basically set by the Constitution.[6] From the 125 billion EUR family support benefits 46 billion EUR were tax transfers such as *Kinderfreibetrag*, 25 billion EUR were cash benefits like *Elterngeld*, 27 billion EUR were social insurance benefits and another 27 billion EUR were real transfers for instance publicly provided child care.[7]

The numerous policies can be summarized into 16 categories.[8] I want to focus on the four most prominent categories: (1) direct monetary benefits (*Kindergeld*), (2) tax exempt amount (*Kinderfreibetrag*), (3) public child care (*öffentliche Betreuungsangebote*) and (4) parental leave policies (*Elterngeld*).[9]

All parents receive *Kindergeld* for each child: 184 EUR per month and child for the first two children, 190 EUR for the third child and 215 EUR for each additional child. These benefits are independent of parental income and unconditional.[10] By law, parents perceive a yearly tax exemption of 7,008 EUR per child. Each year, the tax authorities have to check which type of family support - *Kindergeld* or *Kinderfreibetrag* - is more profitable for the parents. The more profitable policy then applies. On average, *Kindergeld* is more profitable as long as the gross labor income of the household does not exceed 63,000 EUR per year. If the household earns more the *Kinderfreibetrag* becomes more profitable. The goal of these policies is to decrease the private financial costs of parents for rearing children. Generally, these two policies have very similar effects and the tax exemption introduces an income-related benefit scheme in which households earning more than 63,000 EUR per year get slightly higher family support. In total, both policies had a volume of 41 billion EUR in 2010.

The term "subsidized public child care" describes the legal claim parents have for a child care slot for all children of age one and older.[11] The financial volume of public child care was 16 billion EUR in 2010.[12] The goal of this policy is to facilitate a higher compatibility of employment and parenthood. This policy can be seen as a time-subsidy for the parental time budget. It decreases the time needed to raise children enabling the parents to keep on working, for instance.

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[6]See Bundesfamilienministerium (2012), pp.46-47.

[7]The social insurance benefits come from the fact that children are insured via their parents without having to pay any payroll taxes. These rebates are financed by the state.

[8]See ZEW (2013), pp.146-147.

[9]For a detailed description of German family policies, see for instance Bundesfamilienministerium (2012), Prognos (2014b) or ZEW (2013).

[10]As long as the children are under the age of 18 or as long as they are in education or professional formation and under the age of 25.

[11]The legal claim to a slot for preschool children above the age of three in the *Kindergarten* exists since 1996. This was extended in 2008, such that nowadays all children of one year and older have the right to go to a *Kita*.

[12]Although public child care is actually the duty of German Counties, the Federal government heavily subsidizes this form of family policy.

An alternative viewpoint is that such a policy lessens the opportunity costs of children because foregone earnings are reduced. Foregone earnings correspond to potential labor income that cannot be realized because the parent has to stay at home to take care of the children.

Finally, *Elterngeld* is a cash benefit that a parent receives after the birth of a child as compensation for staying at home and not being able to work. It was introduced in Germany in 2007 and its benefit level depends on the previous net labor income.[13] The maximum support, which is 14 months, can only be reached when each parent stays at home for at least two months. Thus, fathers have an incentive to also stay at home for some time. In 2010, the government spent 4.5 billion EUR on this policy. The goal of this policy is also the reduction of private child rearing costs. This policy has no effect on the time budget of the parents.[14]

2.2.2 Literature on the Effectiveness of Fertility Enhancing Family Policies

The main emphasis of my analysis is the macroeconomic effect of family policies, notably through their effect on fertility but also through their effect on labor force participation. Starting with Becker (1960), economists assume children to be consumption goods. Moreover, we can distinguish between two types of costs of children: Direct costs and indirect costs. Direct costs represent all things a child needs, for instance food or clothing. Indirect costs correspond to the opportunity cost of foregone earnings for the parent who has to stay at home for child rearing and not being able to work outside the household.[15] Family policies that decrease the direct or indirect costs of children should increase the number of children.

The effects of family policies on labor force participation are less clear and the expected effects vary across different types of family policies. Policies that subsidize the direct child rearing costs should - holding everything else constant - decrease labor force participation. Indeed, assuming that children exhibit both types of costs, a decrease of direct costs increases the number of children and therefore increases the time costs so that parents have to stay at home more and have less time to work.[16] Policies that reduce the time cost of child rearing lead to an ambiguous effect: On the one hand, time costs per child are reduced but on the other hand the number of children is increased. *Ex ante*, it is not clear which effect predominates.[17]

[13] *Elterngeld* was the successor of *Erziehungsgeld* which was introduced in 1986. Thus, this family policy was not newly introduced in 2007 but significantly expanded.

[14] Actually, there is a negative effect on the working hours since the benefits are only paid when the person stays at home.

[15] See Mincer (1963). I use the terms "indirect costs" and "time costs" interchangeably.

[16] This implicitly assumes that each additional child increases the time cost of the parents.

[17] I abstract completely from two other important effects of family policies: First, the impact on the "quality" of the child and the quality-quantity trade-off as introduced by Becker and Lewis (1973); and second, the impact on intra-household bargaining and decision making process within a couple as introduced by Chiappori (1988, 1992), Manser and Brown (1980), McElroy and Horney (1981) or Lundberg and Pollak (1993).

For Germany, there are not many micro-econometric analyses on the effect of family policies, partly due to the fact that family policymakers became more active only recently. This is why I will also include some microsimulation studies and micro-econometric evidence from other countries. Moreover, most of the studies focus their analysis on the employment rather than fertility effects.

Direct Monetary Child Benefits

In 1996, the German government increased the *Kindergeld* significantly. Instead of 70 DM, parents received 200 DM per child and month. Rainer et al. (2012) investigate this expansion of monetary benefits and find a weak positive effect on fertility. The probability of getting a child increases by 27% for the group of lower educated households. The effect is mainly driven by the fertility behavior of families with no or one child before the reform. Based on the estimates they find, the authors simulate the same reform with data for Germany in 2010: In this experiment, fertility increases by 6.5%.[18] Tamm (2010) uses the same reform to identify the effect of higher *Kindergeld* on parental employment. He finds a significant negative effect on hours worked by mothers. Haan and Wrohlich (2011) estimate a structural model for Germany and then simulate the effect of an increase of monetary benefits. They find a positive and robust effect on the fertility rate and correspondingly a negative effect on mothers' labor supply. In another microsimulation of expansionary monetary benefits Wagenhals (2001) finds a negative effect on the labor supply of mothers.

Moreover, there exist a lot of studies analyzing the effects of similar family policy reforms in other countries. Milligan (2005) uses a quasi-experiment strategy to identify the effect of the introduction of direct monetary child benefits on the fertility rate in Canada. In the Canadian Province of Quebec the introduction of the Allowance for Newborn Children (ANC) represented a very strong monetary benefit for parents when getting an additional child. Depending on the number of children, the monetary benefits correspond to 1.3 to 30% of direct child-rearing costs. Milligan finds a very strong and positive effect: The fertility rate increases by 25% for the most responsive households. Duclos et al. (2001) analyze the same reform of the ANC and identify a positive effect on the probability of getting a first or an additional child. However, they cannot exclude tempo effects as a reason.[19] Azmat and Gonzalez (2010) use a tax reform in Spain in 2003 to evaluate its effect on fertility and mothers' labor force participation. The tax reform

[18] This positive fertility reaction is not significant for all education groups, in particular for higher educated households.

[19] A tempo effect means that women get their children earlier or "faster" but the fertility rate does not change in total. So, the fertility rate is not increased but the family policy pulls the fertility decision forward.

introduced a tax credit for mothers and higher tax deductibility for children. They find positive effects of this reform on both the fertility rate and labor force participation of the mother. The hours worked increase by 2% and the fertility rate by 5%. Sanchez-Mangas and Sanchez-Marcos (2008) use the same reform to identify its effect on short-run employment of mothers. They also find a positive effect.[20] Cohen et al. (2013) use several reforms of monetary child benefits in the late 1990s and beginning of the 2000s in Israel to identify the effect on fertility. An increase of child subsidies of circa 34 USD per child and month increases the probability of getting a child by 10%. This positive fertility effect is robust across several specifications. Sinclair et al. (2012) also identify an increase of the birth rate in Australia caused by the introduction of a Baby Bonus paid by the government.

Several studies investigate the fertility reaction of tax exemptions for dependents in the United States (US). Whittington et al. (1990) and Whittington (1992, 1993) find a positive effect of such implicit monetary benefit on the fertility rate. The Earned Income Tax Credit (EITC) that was introduced in the US in the 1990s resembles an implicit monetary benefit for parents. Baughman and Dickert-Conlin (2003, 2009) analyze this policy and find either weak or weakly negative effects on the fertility rate. Finally, a similar tax policy was introduced in the United Kingdom (UK) in 1999, the Working Families Tax Credit. While Francesconi and Klaauw (2007) and Francesconi et al. (2009) do not find any significant effect on the fertility rate, Brewer et al. (2012) find a significantly positive fertility reaction. To sum it up, most studies find a robust positive effect of monetary benefits on the fertility rate. However, the results seem to be less strong in the cases of tax exemptions like in the US or UK.

Public Child Care

Bauernschuster et al. (2013) investigate the recent expansion of public child care coverage in Germany. They use county variation in public child care coverage to identify the effect on fertility. An expansion of child care coverage by ten percentage points leads to an increase of the fertility rate by 3.2%. The estimate is robust to several sensitivity analyses. In their microsimulation Haan and Wrohlich (2011) reach mixed results for expanded public child care. In their projection it leads to higher female employment but the fertility rate remains unaffected. They explain this result by the two goals of public child care - employment and fertility - that cannot be reached simultaneously. Hank and Kreyenfeld (2003) use data from the Socio-Economic Panel (SOEP) for Western Germany and find that access to informal child care increases the probability of

[20] The positive employment effect of these analyses is rather surprising given what economic theory predicts. However, this result is driven by the increase of potential tax deductions when the mother keeps on working.

getting a child while access to public child care does not have a significant effect. Hank (2002) reaches the same result. Hank et al. (2004) add data for Eastern Germany in the analysis. Here, in contrast to the Western part of the country, they find a positive effect of public child care on fertility. Gathmann and Sass (2011) evaluate a family policy that was introduced in 2006 in Thuringia. This policy subsidizes parents when they do not send their child to a public child care facility. This is equivalent to the *Betreuungsgeld* on the Federal level. Gathmann and Sass (2011) find that such a policy leads to a drop in fertility by 3.3% which can be a sign of either delaying birth or reducing the number of children. So, the incentive to not send children to a public child care facility leads to a lower fertility rate. There does not exist any study on the *Betreuungsgeld* yet.

Looking at studies from other countries provide additional insights. Rindfuss et al. (2010) find very strong effects on the fertility rate for Norway. They estimate the increase in fertility to be between 0.5 and 0.7 children per woman when child care coverage for preschool children is increased from 0 to 60%. A similar study for Sweden is done by Moerk et al. (2013). They also find a positive effect of expanded public child care coverage. They estimate that a decrease of child care costs that represent 25% of average household income pushes the fertility rate by 9.5%. Kravdal (1996) looks at child care reform in the 1970s in Norway and finds very mixed results on child bearing. For Finland and Norway, Ronsen (2004) interestingly finds a negative, though partly insignificant, effect of child care on fertility. Andersson et al. (2004) do not find any effect of child care policies on the childbearing decisions of couples. There exist also empirical analyses on non-Scandinavian countries, for example Spain. Baizan (2009) finds positive effects not only on getting a first child but also on higher order births caused by higher formal child care availability. Another study by Del Boca (2002) looks at the effects in Italy. She does not only find a positive effect of child care availability on the fertility decision but also on employment. Klasen and Launov (2006) do not look specifically at a child care reform but more generally at the causes for the fertility decline in the Czech Republic. Among others, they identify the fear of insufficient child care coverage as one cause for reducing the number of children parents want to have. Heckman and Walker (1990) do not look at child care policies but directly at the opportunity cost of child rearing. They find that the number of births decreases and spacing of birth increases when female wages are higher. This provides evidence for the indirect costs of children due to foregone earnings. This implies a positive effect of fertility when indirect costs of children are reduced.

To conclude, the majority of studies find relatively strong positive effects of extended public child care coverage on the fertility rate, for example Bauernschuster et al. (2013) or Rindfuss et al. (2010). Nevertheless, some studies do not find a significant effect which is partly due to the contradicting goals of increasing employment and increasing fertility that are pursued by such child care policies.

Parental Leave Policies

Raute (2014) looks at the introduction of *Elterngeld* in Germany at the beginning of 2007. She finds significant positive effects of increased benefits, additional 1,000 EUR increase the probability of getting a child by 1.2%. She observes this positive effect for middle and high income parents in particular: As the new benefits are income-related, this result is not surprising. Lalive and Zweimüller (2009) find a positive effect of parental leave and post-birth job protection policies on childbearing for Austria. Similar studies have been done investigating a French pronatalist transfer policy called *allocation parentale d'éducation* (APE) that was expanded in the 1990s. Laroque and Salanie (2004, 2008) and Laroque and Salanie (2014) all find positive and significant effects of this expanded parental leave policy on fertility. All studies find a positive effect of parental leave policies on the fertility rate. However, it remains unclear and unanswered whether the effect measured is rather a timing or a tempo effect that might not be persistent.

Finally, there are a number of studies that look at the effect of family policies over a longer time horizon than just one particular reform. Bjoerklund (2006) measures the effect of Swedish family policy from around 1960 to 1980 and compares the observed fertility patterns in Sweden to the ones of some other European countries. He uses completed fertility rates and not total fertility rates. In line with most of the previous studies, he also concludes that higher pronatalist expenditures lead to an increased level of fertility. For Sweden, Walker (1995) computes a shadow price of childbearing that is affected by family policies. He finds a negative relationship between this price of fertility and the number of newborns. This result gives supportive evidence for a positive effect of family policies on the number of births. Manski and Mayshar (2003), who analyze the fertility patterns in Israel, also identify a positive effect of child allowances on total fertility. Ebenstein et al. (2011) use a quasi-experiment approach in collective communities in Israel to examine the effect of increased costs of raising children on fertility. They conclude that there is a significant effect: Higher costs of child rearing lead to lower fertility. However, they also emphasize that the magnitude of this effect is relatively small and should not be overstated. Finally, a broader survey is given by Gauthier (2007) or by policy reviews like ZEW (2013).

For obvious reasons, my macroeconomic model cannot exhibit all family policy instruments, so I focus on two family policies that also correspond directly with the two cost components of child rearing, direct and indirect costs. Hence, in the model the government can use monetary child benefits and public child care but no parental leave policy. As parental leave policies are the third big family policy instrument the empirical literature of its effects has been discussed anyhow.[21]

2.3 The Model

This section describes the model setup and equilibrium definition of the four-period OLG model. The three key components are (1) heterogeneous households that decide on the number of children, labor supply and savings, (2) a government which collects labor income taxes and runs the public pension system and has family policy instruments, and finally (3) a firm sector.

2.3.1 Households

Model Structure

Each individual lives for four periods, labeled by j, and there is certainty about the time of death. The four periods consist of childhood ($j = c$), young adulthood ($j = y$), middle adulthood ($j = m$) and old adulthood ($j = o$). During childhood, children do not decide anything but create costs for their parents. The period of young adulthood is the most important one: In this stage people decide on the number of children which directly affects their labor supply and budget since the children have to be taken care of. During middle adulthood children are out of the house and parents just work, consume and save. Old adulthood is the retirement phase and people live off their private savings and public pension benefits. A household in this model always consists of only one individual. Consequently, I abstract from gender and there is one parent deciding on the number of children.[22]

Heterogeneity and Intergenerational Transmission

People are heterogeneous in productivity. They can either be of high or low type, $i = \{L, H\}$. This type determines the labor productivity of a person when working in the market, z^i. The type of a child i' is influenced by the type of her parent and some random component.[23] The intergenerational type transmission follows a Markov process with the corresponding Markov transition probability matrix

[21] In addition, in the model the effects of parental leave benefits and monetary child benefits are equivalent.

[22] I use the words “parent” and “parents” interchangeably in this model setup. But in terms of the model it is always only one individual.

[23] I abstract from human capital investment and education choices.

$$\Pi(i' \mid i) = \begin{pmatrix} \pi_{HH} & \pi_{HL} \\ \pi_{LH} & \pi_{LL} \end{pmatrix}$$

where π_{ij} represents the probability of a type i parent to have a type j child. These probabilities are common knowledge to people, thus, they condition their actions on the probabilities. However, parents only learn the type of their children after they have chosen the number of children. Furthermore, a parent's utility is independent of the type of her children and therefore the probabilities do not affect the individual optimization problem. Nevertheless, aggregates and therefore prices depend on the type structure within the economy, and therefore also affect individual choices over the general equilibrium channel. Finally, I assume that all children of one parent will be of the same type. Put differently, there is no heterogeneity in types across siblings. Building on this transition probability matrix, I can define two laws of motion for the fraction of each. Define total population size as $N_t = N_t^H + N_t^L$ where N_t^i is the total amount of all people of type i. The fractions of high and low type individuals then are $h_t = \frac{N_t^H}{N_t}$ and $l_t = \frac{N_t^L}{N_t}$. The type specific fertility rate is n_t^i and the average fertility rate is $\bar{n}_t = h_t n_t^H + l_t n_t^L$. Using this definition and the transition matrix, the laws of motions for the two type shares are

$$h_{t+1} = h_t \frac{n_t^H}{\bar{n}_t} \pi_{HH} + l_t \frac{n_t^L}{\bar{n}_t} \pi_{LH}, \tag{2.1}$$

$$l_{t+1} = h_t \frac{n_t^H}{\bar{n}_t} \pi_{HL} + l_t \frac{n_t^L}{\bar{n}_t} \pi_{LL}. \tag{2.2}$$

For more details, see Appendix 2.A.1.

Child-rearing Costs

Child-bearing involves two types of costs for the parent of type i. First, child rearing is costly and parents have to pay a^i consumption goods to cover these costs. Since parents want their children to consume similarly to themselves, child expenditures are higher for high type parents compared to low type parents, $a^H > a^L$.[24] However, the costs are independent of the children's type and total costs for the parents are determined by the number of children. Second, children require b units of parents' time during which they cannot work. Parents have to spend the same amount of time with their children, independent of their types. Moreover, all children require the same amount of time, so there are no returns to scale. The childhood of the children corresponds to the young adulthood of the parent. Thus, parents face these costs during their young adulthood.

[24]For evidence on Germany, see for example Destatis (2014b), p.21: The poorest decile spends much less for children than the average.

As soon as the children enter young adulthood, they are out of the house and no further costs arise for their parent.

Both costs can be subsidized by the government. There can be a direct monetary transfer θ^a to cover parts of the goods cost. And there can be subsidized child care θ^b that reduces the time that is required to spend with children. There is no alternative to the publicly provided child care. Parents cannot hire nannies privately and when public child care is offered it is mandatory.[25] The subsidies will be discussed more extensively in Section 2.3.2.

Time Budget, Work Experience and Effective Wage Rate

A person i is endowed with one unit of time in each period t. With this time an individual can do two things: She either works, ℓ_t^i, or she raises her children, bn_t^i. Child care facilities θ^b decrease the time a parent has to spend with her children, so that the time constraint is:

$$\ell_t^i + (b - \theta^b) n_t^i \leq 1. \tag{2.3}$$

While working in period t, a person of type i and age j gains some learning-by-doing work experience $x_{j,t}^i$. Consequently, the more a person works during young adulthood the higher is the experience in the subsequent middle adulthood period.[26]

$$x_{j,t}^i = \begin{cases} \lambda_0 & \text{for } j = y \\ x_{j-1,t-1}^i + \lambda_1 + \lambda_2 \ell_{y,t-1}^i & \text{for } j = m. \end{cases} \tag{2.4}$$

λ_0 is the initial level of experience which is the same for everyone. λ_1 represents a lifecycle component, describing the wage difference between a young and a middle aged worker. λ_2 corresponds to the family wage gap between childless parents and the ones with children. This parameter captures the wage punishment for parents since they cannot work fully in the market and therefore gain less experience.

Given a person's experience $x_{j,t}^i$ and her permanent productivity type z^i, the effective wage rate $\omega_{j,t}^i$ of this person in period t with age j is

$$\omega_{j,t}^i = w_t z^i x_{j,t}^i, \tag{2.5}$$

[25] In general, this is a strong assumption. However, in my model parents neither gain any utility from spending time with their children nor can they use their time for leisure without working or child-rearing. Without these two margins, the assumption is much weaker. Nevertheless, since parents cannot choose to use public child care, the effect of a child care expansion on labor supply and fertility will be an upper bound.

[26] This experience can also be interpreted as a form of work-specific human capital.

where w_t is the wage rate per efficiency unit of labor.

Preferences and Lifetime Optimization Problem

People gain utility from their own consumption c_j^i during each period of adulthood. Furthermore, they value the number of children n^i during young adulthood.[27] The instantaneous utility function is

$$\begin{aligned} u_j^i(c_j^i, n^i) &= \frac{(c_j^i)^{1-\sigma}}{1-\sigma} + \eta \frac{(n^i)^{1-\psi}}{1-\psi} && \text{for } j = y, \\ u_j^i(c_j^i) &= \frac{(c_j^i)^{1-\sigma}}{1-\sigma} && \text{for } j \in \{m, o\}. \end{aligned}$$

The lifetime utility of a person i born in period $t-1$ then reads

$$\begin{aligned} U_{t-1}^i &= u_{y,t}^i(c_{y,t}^i, n_t^i) + \beta u_{m,t+1}^i(c_{m,t+1}^i) + \beta^2 u_{o,t+2}^i(c_{o,t+2}^i) \\ &= \frac{(c_{y,t}^i)^{1-\sigma}}{1-\sigma} + \eta \frac{(n_t^i)^{1-\psi}}{1-\psi} + \beta \frac{(c_{m,t+1}^i)^{1-\sigma}}{1-\sigma} + \beta^2 \frac{(c_{o,t+2}^i)^{1-\sigma}}{1-\sigma}, \end{aligned} \tag{2.6}$$

where σ is the CRRA coefficient for consumption, ψ is the CRRA coefficient for fertility, η is the weight on the direct utility of the number of children and β is the discount factor.

A person born in period $t-1$ faces the following optimization problem:

$$\max_{\{c_{y,t}^i, s_{y,t}^i, n_t^i, c_{m,t+1}^i, s_{m,t+1}^i, c_{o,t+2}^i\}} \quad U_{t-1}^i$$

$$s.t.$$

$$c_{y,t}^i + s_{y,t}^i + (a^i - \theta^a) n_t^i = (1-\tau)(1-(b-\theta^b)) \ell_t^i \omega_{y,t}^i \tag{2.7}$$

$$c_{m,t+1}^i + s_{m,t+1}^i = (1-\tau)\omega_{m,t+1}^i + (1+r_{t+1}) s_{y,t}^i \tag{2.8}$$

$$c_{o,t+2}^i = P_{t+2}^i + (1+r_{t+2}) s_{m,t+1}^i. \tag{2.9}$$

Here, s are savings, τ is a labor income tax rate, r is the real interest rate and P^i are the work-history dependent pension benefits from a public pay-as-you-go pension system.[28] In young and middle adulthood, a person works and earns labor income. During young adulthood, a person can use this income for either consumption, to save for future periods or to pay the costs of rearing children. Consequently, the income in middle adulthood consists of labor income and

[27] Theoretically, people could value their children in each period of their adulthood. However, this can be collapsed such that the two representations are equivalent: $\tilde{\eta}\frac{(n^i)^{1-\psi}}{1-\psi} + \beta\tilde{\eta}\frac{(n^i)^{1-\psi}}{1-\psi} + \beta^2\tilde{\eta}\frac{(n^i)^{1-\psi}}{1-\psi} = (\tilde{\eta} + \beta\tilde{\eta} + \beta^2\tilde{\eta})\frac{(n^i)^{1-\psi}}{1-\psi} = \eta\frac{(n^i)^{1-\psi}}{1-\psi}$ with $\eta = \tilde{\eta} + \beta\tilde{\eta} + \beta^2\tilde{\eta}$.

[28] The pension formula will be discussed in Section 2.3.2.

capital income from the savings during young adulthood. At this stage of life, income is used to either consume or save for retirement. There is no borrowing constraint, $s < 0$ means the household is a borrower.[29] When old, a person is retired and does not receive labor income but pension benefits. Furthermore, she has her private savings from the previous period. Since people die with certainty, they do not leave accidental bequests. As there is no incentive to leave intentional bequests, old-age income is completely spent on consumption and $s_o^i = 0$. Consequently, everyone starts with an initial asset stock of zero, i.e. $s_c^i = 0$.

2.3.2 Government

The government collects a flat labor income tax τ that all employees have to pay. With the tax revenues the government finances two types of family policies θ^a and θ^b, a public pay-as-you-go pension system with benefits P^i, and residual expenditures G. The budget has to be balanced in every period. Since the government finances the pension system and other policies with this tax, it should be interpreted as the overall tax burden on labor consisting of labor income tax and payroll taxes for social security contributions. The first type of family policy are direct monetary benefits, θ^a. They are paid per child and directly reduce the child rearing costs in terms of consumption goods for the parents. Thus, total expenditures are the total number of children in the economy times the level of the benefits. The second type is subsidized public child care, θ^b. For subsidized child care the government has to employ nannies to take care of the children. Each nanny can take care of Q children, simultaneously. Hence, the governmental demand for nannies is the total number of children times the time the government wants to take care of them adjusted by the "productivity" of a nanny, reflected by the children-to-nanny ratio Q:

$$L_t^N = \theta^b \frac{N_t^c}{Q}.$$

L_t^N is the number of full-time nannies the government has to employ in period t. The government offers nannies a wage of w^N such that governmental expenditures stay at a minimum but sufficient employees are attracted to work within the nanny sector.

All retirees receive pension benefits P_t^i that are determined by the replacement q and the average net labor income $\hat{\omega}^i$ a person received during her career:

$$P_t^i = q\hat{\omega}_t^i,$$

[29]For this case, I assume a perfect financial market without transaction costs so that the borrowing rate equals the lending rate.

where $\hat{\omega}_t^i$ is defined as

$$\hat{\omega}_t^i = \frac{1}{2}\left[(1-\tau)\ell_{y,t-2}^i \omega_{y,t-2}^i + (1-\tau)\omega_{m,t-1}^i\right].$$

The replacement rate q can be set by the government while the average net labor income is determined by previous labor supply decisions of the household. This pension formula creates a link between pension contributions and benefits which is also present in the German pension system and similarly done in the previous literature.[30] A more detailed discussion on the German public pension system can be found in Chapter 4. All tax revenues that the government does not use for family policies or the pension system are summarized as residual government expenditures G. Expenditures of this category do not affect agents and their decision making.[31] Putting it all together, the government budget constraint in period t is

$$\sum_{j\in\{y,m\}}\sum_{i\in\{H,L\}} \tau \ell_{j,t}^i \omega_{j,t}^i N_{j,t}^i = \theta^a N_t^c + L_t^N w^N + \sum_{i\in\{H,L\}} P_t^i N_{o,t}^i + G_t. \tag{2.10}$$

$N_{j,t}^i$ describes the number of persons of type i and age j in period t, and N_t^c stands for all the children in period t independent of their type.

2.3.3 Firms

Firms are active in a competitive market so that prices, interest rate r_t and wage rate w_t, are taken as given. Input factors are effective labor H_t^M and capital K_t, and output Y_t^M is produced.[32] A is the total factor productivity and the capital stock depreciates with rate δ in each period. The production function is of Cobb-Douglas form

$$Y_t^M = A(K_t)^\alpha (H_t^M)^{1-\alpha}.$$

Firms maximize profits. Normalizing the price of the output good to 1, the profit maximization problem has the following form:

$$\max_{\{H_t^M, K_t\}} A(K_t)^\alpha (H_t^M)^{1-\alpha} - w_t H_t^M - r_t K_t. \tag{2.11}$$

[30] See for example Ludwig and Reiter (2010). I also abstract from ceilings on social security contributions and benefits. As I only have two income types, there are no households with very high income in the model. Therefore this assumption is not critical.

[31] One can think of expenditures for military defense, infrastructure or judiciary. For sure, people benefit from better roads or a more efficient judicial system. However, this would be a level effect for all persons of the economy and therefore I abstract from this effect in this model setup.

[32] Total effective labor H_t^M corresponds to total labor L_t^M.

2.3.4 Equilibrium Definition

Given a tax rate τ, family policies θ^a and θ^b and residual government expenditures G a stationary equilibrium consists of

- sequences of choices $\{c_y^i, c_m^i, c_o^i, s_y^i, s_m^i, n^i\}_t$ for $i = H, L$,
- capital stock, aggregate labor and aggregate effective labor $\{K, L^M, L^N, H^M, H^N\}_t$,
- factor prices $\{w(K, H^M), r(K, H^M)\}_t$ and
- a replacement rate $\{q(\tau, \theta^a, \theta^b, G)\}_t$

such that

- households maximize lifetime utility given in equation (2.6),
- factor prices are consistent with the firms' problem given in equation (2.11),
- the government budget presented in equation (2.10) holds, and
- labor and asset markets clear.

  For asset market clearing, the sum of all savings in the economy has to equal the capital demand from the firm sector:

$$K_t = \sum_{j\in\{y,m\}} \sum_{i\in\{H,L\}} s_{j,t}^i N_{j,t}^i. \tag{2.12}$$

  On the labor market, people work either in the firm or in the nanny sector. Consequently, the total labor supplied by all working households has to equal the demand from public child care facilities L_t^N and the firms L_t^M:

$$L_t^M + L_t^N = \sum_{j\in\{y,m\}} \sum_{i\in\{H,L\}} \ell_{j,t}^i N_{j,t}^i. \tag{2.13}$$

  The same has to hold for the effective labor supply H_t, which is shown in Appendix 2.A.2.

- Finally, the distribution of types has to be stationary.

  Starting from equations (2.1) and (2.2), for a stationary type distribution it must hold that

$$h_{t+1} = h_t = h = h\frac{n^H}{\bar{n}}\pi_{HH} + l\frac{n^L}{\bar{n}}\pi_{LH},$$
$$l_{t+1} = l_t = l = h\frac{n^H}{\bar{n}}\pi_{HL} + l\frac{n^L}{\bar{n}}\pi_{LL}.$$

This system of equations can be reformulated to

$$h = \frac{\tilde{\pi}_{LH}}{1 - \tilde{\pi}_{HH}} l, \tag{2.14}$$

$$l = \frac{\tilde{\pi}_{HL}}{1 - \tilde{\pi}_{LL}} h, \tag{2.15}$$

where $\tilde{\pi}_{ij}$ is the probability for a parent of type i to get a child of type j adjusted by the relative type specific fertility rate of type i. Solving this system of equations leads to the stationary type distribution.[33]

2.3.5 Equilibrium Characterization

In a stationary equilibrium, all choices and prices are time-independent and therefore the time indices are dropped. The three first-order conditions are:

$$n_i: \quad \eta n_i^{-\psi} - c_{y,i}^{-\sigma}[(a - \theta^a) + (1 - \tau)(b - \theta^b)] = 0 \tag{2.16}$$

$$s_{y,i}: \quad -c_{y,i}^{-\sigma} + \beta(1 + r)c_{m,i}^{-\sigma} = 0 \tag{2.17}$$

$$s_{m,i}: \quad -c_{m,i}^{-\sigma} + \beta(1 + r)c_{o,i}^{-\sigma} = 0. \tag{2.18}$$

Hence, the six equations (2.7), (2.8), (2.9), (2.16), (2.17), (2.18) and six unknowns describe the household's optimal behavior. This is done for both types.[34]

Profit maximization implies that the marginal return of input factors is equal to their price:

$$r = \alpha A \left(\frac{H^M}{K}\right)^{1-\alpha} - \delta, \tag{2.19}$$

$$w = (1 - \alpha) A \left(\frac{K}{H^M}\right)^{\alpha}. \tag{2.20}$$

Given the equilibrium definition it is the replacement rate q that balances the governmental budget, which yields

$$q = \frac{\sum_{j\in\{y,m\}} \sum_{i\in\{H,L\}} \tau \ell_j^i \omega_j^i N_j^i - \theta^a N^c - L^N w^N - G}{\sum_{i\in\{H,L\}} \hat{\omega}^i N_o^i}. \tag{2.21}$$

In equilibrium, the government will offer exactly $w^N = \omega_y^L$, that is the nanny wage equals the wage for young low type workers. As a consequence the only group working as nannies are the

[33] Note, that $\bar{n}$ includes the fractions h and l such that it cannot be solved explicitly for the stationary distribution but numerically by iterating on these two equations. For more details, see Appendix 2.A.1.

[34] The household optimization solution algorithm is presented in Appendix 2.A.3.

young and low productivity people. Therefore, market clearing in the nanny sector leads to

$$L^N = \theta^b \frac{N^C}{Q}. \tag{2.22}$$

Given this allocation in the nanny sector, market clearing for effective labor supply in the firm sector is

$$H^M = \sum_{j \in \{y,m\}} \sum_{i \in \{H,L\}} \ell^i_j z^i x^i_j N^i_j - L^N z^L x^L_y. \tag{2.23}$$

I solve for the benchmark steady state applying the solution algorithm shown in Appendix 2.A.3. Before presenting the benchmark steady state I discuss the calibration and further parameter choices for the model.

2.4 Calibration

In this section I discuss the choice of parameter values and the calibration procedure including targets and data used. The model parameters are calibrated so that it replicates Germany in the year 2010. Seven parameters are determined by calibration while the rest is set based on external evidence. I start with discussing the latter one.

2.4.1 Externally Determined Parameters

In the model there are two different types of households and therefore the choice of the empirical counterparts to the types is important. In particular, the productivity and intergenerational transmission parameters depend on this choice. The cut-off between the two household types is based on the educational attainment: I define all people that have not finished *Abitur* as low income households and all who did pass the *Abitur* as high income households. The big advantage of this type definition is that information on educational attainment is broadly available so that intergenerational transmission probabilities can be calculated. Applying this cut-off to German data, three quarters of German employees are classified as low income types and one quarter as high income types.[35]

With this type distinction and based on estimates provided by Heineck and Riphahn (2007) I compute transition probabilities π_{HH} and π_{LL} of the following form: Given the educational background of parents what is the educational attainment of the child. The values I get are $\pi_{HH} = 0.5734$ and $\pi_{LL} = 0.7783$. This means that the probability for (high type) parents with

[35]See Destatis (2010), p.488. This appears to be rather too many people without *Abitur*, however, this results is driven by the fact that participation rates for the *Gymnasium* started to go up not so long ago and that there are a lot of employees in the labor market not having had the possibility to go to *Gymnasium*.

Abitur to get a high type child is 57 % while the probability for low type parents without *Abitur* is 78% that their children stay in the same education category.[36] For more details, see Appendix 2.A.4.

While the productivity level of low types z^L is determined by matching moments (see Section **??**), the high type productivity z^H is defined as a function of z^L. In the model, the productivity factor $\frac{z^H}{z^L}$ captures the labor income gap between young high and low types, $\frac{w_y^H}{w_y^L} = \frac{x_y^H z^H \omega}{x_y^L z^L \omega}$. Since the work experience x_y and the base wage rate ω is the same for both young types, the productivity z is the only parameter that can account for different wages. According to data from Destatis (2010) the labor income gap is 42.07% in Germany, see Appendix 2.A.4 for more details. However, observed labor income is the hourly wage times the hours worked, $\frac{\ell_y^H w_y^H}{\ell_y^L w_y^L}$. As I am interested in the productivity difference and therefore in the gap of the effective wage rate, I have to abstract from individual labor supply. Using the model assumption that households can either work or spend time with their children and the fact that the average fertility of low income households is higher than the one of high income households, I can compute the productivity factor $\frac{z^H}{z^L}$ which is 1.34. Put it differently, the high income type is 34% more productive than the low income type.[37] The complete derivation of the productivity gap is presented in Appendix 2.A.4.

The parameters in the function that describes the cumulation of experience also need to be set. Recall the functional form of this accumulation function. It consists of three components: an initial value λ_0, a life cycle component λ_1 and a family wage gap component λ_2. The initial value λ_0 is normalized to 1. For determining the other two components, λ_1 and λ_2, I solve jointly a system of two equations. The life cycle component determines the wage growth between the young and middle adulthood period of a low type person. Using estimates on rates of return on experience from Dustmann and Meghir (2005), the wage difference is 74.6%.[38] To put the number in some relation, the implied wage growth over 20 years for the US is 88.75% (by Voena (2013)), or 65.9% (by Attanasio et al. (2008)). This implies an annual wage growth of 3.4% or 2.7% respectively. Olivetti (2006) finds a range of 3% to 5%. The implied annual growth based on Dustmann and Meghir (2005) is 2.83%. The family wage gap component is chosen to match

[36] Heineck and Riphahn (2007) use SOEP data for their analysis. Given my education cut-off I pool together people with “basic and middle education”, that is *Hauptschule* and *Realschule*. This leads to a larger group size and therefore also to a higher persistence. In Section 2.6 I test for different transition probabilities using a different underlying assumption as a robustness check.

[37] Note that this productivity gap is only present when working for a firm. When working as a nanny both types have the same productivity.

[38] See Dustmann and Meghir (2005), p.92, Table 3. They estimate these rates of return for several years of experience, leading to the following computation for one period of 20 years: $\frac{w_y^L}{w_m^L} = 1 \times 1.146 \times 1.082 \times 1.064 \times 1.043 \times 1.016^{15} = 1.7460$. This calculation is based on data from unskilled workers. For skilled workers the computed wage growth is $1 \times 1.047^2 \times 1.060 \times 1.067 \times 1.021^{15} = 1.6934$, see Dustmann and Meghir (2005) p.94 Table 4.

the ratio between a middle aged low type without children and a corresponding person with one child. Beblo et al. (2009) estimate this gap to be 24% in Germany. Then, I get the following two conditions:

$$\frac{w_m^L}{w_y^L} = 1.746$$
$$\frac{w_m^{L,n=0}}{w_m^{L,n=1/2}} = 1.24.$$

Here, I can use the model definitions for wages and the normalization of λ_0 to 1 such that I get the following two equations:

$$\lambda_1 = 0.398 - 0.801\lambda_2$$
$$\lambda_2 = -2.783 - 2.783\lambda_1.$$

As a result λ_1 is -2.1384 and λ_2 is 3.1681. For further details on the computation, see Appendix 2.A.4.

An important parameter are the time costs of children b. Based on estimates for the US from Schoonbroodt (2014) the value of b is 0.31.[39] According to the estimates parents spend 15.5% of their potential working time with one child or consequently 31% for two children. As I have normalized the total time endowment to 1 and because of the fact that there is always only one parent and therefore a fertility rate of n equals 1 in the model represents two children in reality, 31% of the time endowment is 0.31.[40]

The children-to-nanny ratio Q determines the productivity of the nanny sector. Destatis (2014a) finds different child-caring ratios depending on the age of the children. I choose the ratio of the widest age-range, where one nanny can care for 5.0 children of the age between zero and eight.[41]

For the manufacturing sector, the capital share α is 0.32 which reflects the capital share of national income in Germany in the year 2010.[42] Total factor productivity A is normalized to be 3.5 and the annual depreciation rate $\hat{\delta}$ is 7.73%. This is in the range of other studies on Germany, such as Ludwig and Reiter (2010) who use 8.1% or Fehr and Ujhelyiova (2013) with

[39]Such an estimate does not exist for Germany, yet. Nevertheless, it is not implausible that, abstracting from nannies or child care facilities, children in the US require the same amount of time as in Germany.

[40]In Section 2.2 of Schoonbroodt (2014) p.8ff, she defines example 1 which is a model with labor supply and child-rearing margin only. Thus, this is the same as in my model. Bringing this model to the data, she presents some numbers in section 6.1.1 on pp.31-32. She provides numbers for mothers, fathers and parents. Since I do not have different genders I use the value for parents.

[41]This ratio is 4.6 for children between zero and three, and 9.1 for children between two and eight years old, see Destatis (2014a), p.10.

[42]The corresponding numbers are from Destatis (2014c), p.18.

5.9%. Since one model-period corresponds to 20 years, this leads to a period-wise depreciation rate δ of 80%.

Table 2.1: Externally Determined Parameters

Parameter		Value
CRRA utility	σ	1.5
discount factor	$\hat{\beta}$	0.95
time for children	b	0.31
productivity high type	z^H	$1.31z^L$
initial experience level	λ_0	1
life cycle component	λ_1	-2.1384
family wage gap	λ_2	3.1681
persistence high type	π_{HH}	0.5734
persistence low type	π_{LL}	0.7783
nanny-to-children ratio	Q	5.0
TFP	A	3.5
capital intensity	α	0.3198
depreciation rate	$\hat{\delta}$	7.73 %
labor tax rate	τ	39.6%
public child care	θ^b	0.062

Governmental parameters that are determined externally are the labor income tax τ and public child care θ^b. The parameter τ does not only represent the average labor income tax burden in Germany but also includes the social security contributions, in particular the contributions to the German public pension system. The OECD computes this number to be 39.6%.[43] For public child care I use data presented by Bauernschuster et al. (2013). They use administrative data for Western German counties and found an average for public child care coverage of 14.3% in 2009.[44] However, information on Eastern Germany is missing and it is known that coverage rates are much higher in the East compared to the West. However, there are also fewer children such that the higher coverage rate has a relatively lower weight. Thus, I assume that overall child care coverage is 20%, meaning 20% of the time cost of children b are covered by the state. Therefore, θ^b is 0.062.[45]

[43]See OECD (2014), p.292. There are 20.4% social security contributions and 19.1% is added by the labor income tax. I use the the number for "single earner's average rate of income tax and social security contribution (less cash transfers)".

[44]See Bauernschuster et al. (2013), p.9, Table 1.

[45]Actually, the coverage rate measures which share of children goes to a public child care facility. In the model, the coverage rate has a different interpretation. Here, the coverage rate stands for the percentage reduction of the time that parents have to spend with their children because the remaining time costs of children are covered by the public child care facility. In combination with the assumption that parents cannot choose whether they want to give their children to public child care, this definition of child care coverage has an important implication: All parents, independent of their type, use public child care in the same way and by definition there cannot be a heterogeneous usage of public child care.

Finally, I use a CRRA coefficient σ of 1.5 for consumption which is in the usual range.[46] The discount factor $\hat{\beta}$ is 0.95 per year. The period-wise discount factor β then is 0.3585.[47] Table 2.1 presents an overview.

2.4.2 Calibrated Parameters

There are seven parameters that are calibrated by minimizing the distance between a data target and the model counterpart:

- The weight on fertility in the utility function η,
- the CRRA coefficient for fertility ψ,
- the cost of children in terms of consumption for high income types a^H,
- and also for low income types a^L,
- the low type productivity z^L,
- the monetary child benefits θ^a,
- and the residual government expenditures G.

The resulting parameter values are shown in Table 2.2. For the method of minimizing moments I use seven moments that are closely related to the parameters. However, I cannot identify clear individual relationships between parameters and moments and therefore the set of parameters jointly minimizes a distance function. The distance function is the sum of all absolute percentage deviations of the model moments from their data counterparts. More details on the procedure and the algorithm are in Appendix 2.A.3.

The first targeted moment is the average level of fertility in Germany: The official German total fertility rate in 2012 is 1.378.[48] The corresponding model moment is $2\bar{n}$. It is twice the average fertility rate in the model because in each household there is one parent. A fertility rate of one leads to a constant population size in the model, and so does a fertility rate of two in reality.

Next, I target the fertility differential between low and high types. Using numbers provided by Destatis (2012a) the fertility differential is 0.38. This means that on average a low type mother

[46] Voena (2013) also uses 1.5 and refers to Attanasio et al. (2008). Ludwig and Reiter (2010) use log utility in their benchmark and for their sensitivity analysis the coefficient is 2. Domeij and Klein (2013) and Fehr and Ujhelyiova (2013) use $\sigma = 2$.

[47] $\hat{\beta}^{20} = 0.95^{20} = 0.3585$.

[48] See Destatis (2012b), p.15.

gets 0.38 more children than a high type mother. See Appendix 2.A.4 for further information on the calculation of the differential. Equivalent to the logic from above, that one child in the model corresponds to two children in reality, the corresponding model moment is $2(n^L - n^H)$.

To pin down child rearing costs I use information on consumption costs for children of high type parents as a share of parental consumption, a^H/c_y^H. High income parents spend 1,587 EUR per month for their children and 3,245 EUR for themselves, thus, the ratio is 48.9%.[49]

Furthermore, high type parents spend 2.5 times more for their children (per child) compared to low type parents. The corresponding model moment is computed by a^H/a^L.[50]

The level of productivity of low income households, z^L, uniquely determines a young worker's wage w_y^L. This is true because the parameter for initial experience λ_0 is normalized to one and the base wage rate ω is a) set by market clearing and b) the same for both types. The moment I target to determine the productivity level is therefore the wage earnings of young low type workers relative to GDP per capita $\ell_y^L w_y^L/y$. Data for labor income comes from Destatis (2010) and with the procedure described in Appendix 2.A.4 I compute the low type wage to be 33,574 EUR per year. GDP data is taken from Destatis (2014c) where I use GDP per capita for the year 2010 which is 31,511 EUR. Thus, the targeted ratio is 1.07. The corresponding data moment is computed in the same way. The labor income is an obvious outcome, however, the GDP is not. I define GDP as all manufacturing output plus services which in this case are all government wage payments to nannies. Thus, GDP in the model is $Y = AK^\alpha H_M^{1-\alpha} + L^N w^N$.

As targeted moment for the direct monetary child benefits θ^a, I use the *Kindergeld* as a ratio of a young low type worker's income, $\theta^a/ell_y^L w_y^L$.[51] Parents in Germany receive 184 EUR per child and month as direct monetary child benefit, which makes 2,208 EUR per child per year. The average labor income of young low type workers is 33,574 EUR per year which leads to a ratio of 0.066. Finally, I target the current average net replacement rate in the German public pension system which is 55.3% according to the OECD.[52]

Table 2.2 presents an overview over the calibrated parameter values. The weight on fertility in the utility function η is 0.36, which is about a third of the weight on consumption which is normalized to one. Thus, the households give consumption a weight of three quarters and fertility of one quarter. The CRRA coefficient on fertility ψ is 1.10 and therefore lower than the one on consumption (σ is 1.5). The cost parameters for child rearing a^H and a^L are 0.59 and 0.24. The

[49] See Appendix 2.A.4 for more information on child rearing costs. The numbers I use are from Destatis (2014b), Table 14, p.46.

[50] See also Destatis (2014b), Table 14. p.46.

[51] The absolute value of the *Kindergeld* is not a meaningful target but has to be put in relation to another model outcome.

[52] See OECD (2013), Table 4.10, p.143.

productivity level for a low type person z^L is 2.0, consequently the high type productivity level is 2.64. The monetary child benefits from the government θ^a are 0.11 and therefore close to 50% of the child rearing costs for a low type parent. Finally, the residual governmental expenditures G are 1.81.

Table 2.2: Parameters Determined by Calibration

Parameter		Value
fertility weight	η	0.355
CRRA fertility	ψ	1.104
cost of children high type	a^H	0.588
cost of children low type	a^L	0.235
productivity low type	z^L	2.018
monetary child benefit	θ^a	0.114
wasteful governmental expenditure	G	1.811

The model moments are shown in Table 2.3 which also presents the data counterparts. The model perfectly matches six of the seven moments: the average fertility rate, the differential fertility, the child cost to consumption ratio, the child cost ratio, the relative monetary child benefits and the replacement rate. The only target which cannot be reached very well is the relative labor income of a young low type person. With 0.88 the income ratio is too low compared to 1.07 in the data.

Table 2.3: Targeted Moments

Target		Data Moment	Model Moment
average fertility rate	$2\bar{n}$	1.378	1.378
differential fertility	$2(n^L - n^H)$	0.38	0.38
child cost to consumption ratio	a^H/c_y^H	0.489	0.489
child cost ratio	a^H/a^L	2.5	2.5
relative labor income	$\ell w_y^L/y$	1.07	0.88
relative monetary child benefit	$\theta^a/\ell w_y^L$	0.066	0.066
replacement rate	q	0.553	0.553

2.4.3 Benchmark Steady State

Before I turn to the analysis of the two recent family policy reforms in Germany, I want to mention some features - or “un-targeted moments” - of the benchmark steady state, that are presented in Tables 2.4 and 2.5, and in Appendix 2.A.5.

The type-specific fertility rates are 1.12 for high types and 1.50 for low types. The data counterparts are 1.22 and 1.61 and therefore 0.1 children per woman higher than the model

Table 2.4: Untargeted Moments

	Model Moment	Data Moment
fertility high type	1.12	1.22
fertility low type	1.50	1.61
share of high types	0.31	0.25
old-age dependency ratio	0.86	0.34
total dependency ratio	1.14	0.65
child costs-to-income ratio (high type)	0.26	0.18
child costs-to-income ratio (low type)	0.20	0.33

result.[53] In combination with the average fertility rate being 1.38 in both model and data, this implies that the share of low income households must be higher in the model compared to the sample used by the *Statistische Bundesamt* for the calculation of the data moment. The share of low income types in the model is 69%, and 31% for high types, respectively.[54]

The old-age dependency ratio in the model is 0.86: For every 100 employees there are 86 retirees in the economy. This ratio is much higher than the 0.34 that is measured in Germany in 2009. The discrepancy remains when looking at the total dependency ratio. The total dependency ratio is the old-age dependency ratio plus the youth-dependency ratio which is the number of people aged 0 to 20 over the number of all employees. The model produces a total dependency ratio of 1.14 while it is 0.65 in the data.[55] However, this discrepancy can be explained by the concept of a stationary equilibrium. In a steady state, fertility rates must be the same for all cohorts and therefore the fertility rate of old people in the model is significantly lower than in reality. The actual fertility rate in Germany has been steadily declining over the last decades and the former and higher fertility rates imply a lower old-age dependency relative to the model outcome.

For a closer look on child rearing costs, I compute child costs relative to net labor income of the parent. A low type parent spends 19.7% of her net labor income on her children while a high type parent spends 26.1%. The empirical counterparts are 32.9% for low types and 17.6% for high types. So, while in reality low income households (have to) spend relatively more for their children it is the other way round in the model.

In the benchmark steady state the only household type that is a borrower is the category of young adults with high income. They anticipate the higher future labor income and inter-temporally smooth consumption. Private old-age savings during middle adulthood are

[53]See Appendix 2.A.4 for more details.

[54]The study does not report type shares that could be used for a comparison. Using official data from Destatis (2010), Table 22.1, the share of low types is 75% and 25% for high types. This does not fit with the previous conclusion and this contradiction lies in different underlying data bases.

[55]See Destatis (2009), Figure 5, p.19.

substantial. Since the real interest rate in the model is quite high with 9% per year, this leads to an overall retirement wealth which relies mainly on private savings.[56] This is true for both household types: Private savings count for 70% of total retirement wealth for high types, and 73% for low types. The fact that poorer households save relatively more than richer ones seems counter-intuitive. One explanation is that public pension benefits are linked to the individual labor income. Therefore, public pension benefits for high income households are significantly higher compared to households that earn less during their work-life.[57]

Lastly, I turn to the government sector. In the benchmark steady state, the government spends 2.8% of the tax revenues for monetary child benefits and 0.7% for public child care. In total, family policy expenditures add up to 3.5% of the overall budget. With 68.9% the largest part is spent for the public pension system while the remaining 27.6% are for further expenditures like infrastructure or military defense. In 2010, the budget of the Federal German ministry for Family Affairs was 2.0% of the overall budget so the model gets close to the actual data.[58] Moreover, the government employs 0.4% of the total labor force as nannies. All nannies are young low type workers. Within this specific group 1.5% of employees are working as nannies.

Table 2.5: Benchmark Steady State: Government

tax revenues	6.56	
pension system	4.52	68.9%
family policies	0.23	3.5%
residual expenditures	1.81	27.6%
nanny sector (as % of total workforce)		0.4%
nanny sector (as % of young low types)		1.5%

2.5 Policy Reforms

In this section I use the calibrated model to evaluate the effects of two family policy reforms that were introduced in Germany in recent years:

[56]The high real interest rate has another implication: $\beta(1+r)$ is greater than one and therefore the consumption pattern for both household types is increasing.

[57]It is also counterfactual that private old-age insurance is more important for households' retirement wealth than public old-age insurance. However, this model result fits the reality much better when taking into account other old-age saving channels like housing.

[58]See Bundesfinanzministerium (2010). Comparing the other categories to their data counterparts does not make too much sense for mainly two reasons: First, I do not distinguish between the Federal budget and the public pension system and pool together social security contributions and labor taxes. Actually, these are two separated budgets. Second, I only have the labor income tax in my model and abstract from all other tax categories, for instance the value-added tax. The pension system and the model assumption on the joint financing of the pension system and the general budget are discussed in more detail in Chapter 4.

1. In 2015, the German Federal government increased the *Kindergeld* from 184 EUR to 190 EUR per child and month.
2. In 2007, the German government organized the so-called *Krippengipfel* on which they announced reforms of the public child care system. An official target for the public child care coverage rate of 35% was announced, meaning that the government wanted 35% of all children under the age of three to visit a public child care facility. As policy experiment, I assume this target to be realized such that child care coverage increases from 20% up to 35%.

I compare each reform outcome separately with the benchmark steady state and analyze and discuss the effects in particular on the individual decisions for child bearing and the aggregate effect on the governmental budget. Based on the micro-econometric evidence discussed in Section 2.2 the fertility rate is expected to increase. When computing the reform steady states, the expanded family policies have to be financed in a way that the government budget still holds. In order not to add additional distortions, the parameter that adjusts are residual expenditures G.[59]

2.5.1 The 2015 *Kindergeld* Reform

In March 2015 the German government decided to increase the *Kindergeld* by 6 EUR per child and month.[60] The new *Kindergeld* is 190 EUR per child and month and therefore 3.3% higher than before. In the model this directly translates into an increase of monetary child benefits θ^a by 3.3% from 0.113 to 0.117. The *Kindergeld* increase reduces the parental costs for rearing their children which can be seen in Tables 2.6 and 2.7. For low income households direct costs are reduced by 3.0% (per child) and for high income households costs decrease by 0.8%. As the reform does not affect the time budget of parents, time costs per child remain constant. As expected, a higher level of *Kindergeld* leads to a higher number of newborns for both household types. However, low income households react much stronger to the reform than high income households. In fact, their fertility rate increases twice as much as the one of high income households (+0.8% versus +0.4%). From the parent's point of view, children are ordinary goods and therefore the stronger price decrease for low income households leads to a stronger fertility reaction.

[59] Note for clarification that this chapter does not include a comparison of the two family policy reforms. This will be done in Chapter 3.

[60] Actually, this has been done in two steps which I merge into one: The first increase from January 1st 2015 was 4 EUR, and additional 2 EUR were added from January 1st 2016 onwards. Furthermore, the potential for tax deductions - *Kinderfreibetrag* - and further subsidies - *Kinderzuschlag* - are also increased. Since my model focuses on direct monetary benefits I abstract from the other two reform components, see www.bundesregierung.de/Content/DE/Artikel/2015/03/2015-03-24-kabinett-anhebung-grund-und-kinderfreibetrag.html.

This behavior of lower income households is in line with empirical evidence given by Rainer et al. (2012): For the *Kindergeld* reform of 1996 they find that the fertility reaction of lower educated households has been much stronger than the one of higher educated households. Fehr and Ujhelyiova (2013) find the same result in their policy simulation.

The higher number of children causes higher child rearing costs for parents such that total direct costs (costs times number of children) is only reduced by 0.3% and 2.2%. Moreover, parents have to spend more time for child rearing and suffer higher foregone earnings. Thus, after the reform total child rearing costs for both household types are the same as before even if total child rearing costs per child go down. The cost reduction induced by the reform is completely compensated by the higher number of children.

Table 2.6: *Kindergeld Reform* - Child Rearing Costs (High Type)

		benchmark	post-reform	Δ (in %)
direct costs	per 2 children	0.474	0.470	-0.8
	total	0.264	0.263	- 0.3
time costs	per 2 children	0.248	0.248	0.0
	total	0.138	0.139	0.4
foregone earnings	per 2 children	0.416	0.416	0.0
	total	0.232	0.233	0.5
total costs	per 2 children	0.890	0.887	-0.4
	total	0.496	0.496	0

Table 2.7: *Kindergeld Reform* - Child Rearing Costs (Low Type)

		benchmark	post-reform	Δ (in %)
direct costs	per 2 children	0.122	0.118	-3.0
	total	0.091	0.089	- 2.2
time costs	per 2 children	0.248	0.248	0.0
	total	0.186	0.187	0.8
foregone earnings	per 2 children	0.317	0.317	0.0
	total	0.237	0.239	0.8
total costs	per 2 children	0.439	0.435	-0.8
	total	0.328	0.328	0

The average fertility rate raises by 0.7% from 1.38 to 1.39. The corresponding fertility reaction is 0.0015, meaning that additional 10 EUR of monetary child benefits increases the fertility rate by 0.015 children per woman. This number is slightly higher but relatively close to the one that Rainer et al. (2012) find in their empirical analysis of the 1996 reform. In 1996, *Kindergeld* was increased from 36 to 102 EUR, thus, a plus of 66 EUR. In their analysis, this leads to an increase of the fertility rate from 1.39 to 1.48 and the corresponding fertility reaction is 0.0013.[61] Fehr and Ujhelyiova (2013) who also use a general equilibrium framework find a much stronger fertility reaction. In their experiment, they raise direct monetary benefits by 25%. This corresponds to an increase of *Kindergeld* by 46 EUR from 184 to 230 EUR. They find that the fertility rate increases by 0.36 from 1.46 to 1.82. The resulting fertility reaction is 0.0078 which is five times higher compared to the present analysis.[62]

As direct counterpart of the increased fertility rate, labor supply of both household types slightly decreases. This negative labor-supply effect of monetary child benefits is observed in basically all empirical studies on the topic.[63]

What happens to the government budget? Recall, that in this simulation the residual government expenditures G have to adjust to finance the reform. However, in the post-reform steady state the government has more financial resources to spend on G. The 3.2% higher monetary child benefits increase residual expenditures by 2.3%. Given that the tax and replacement rate as well as the public child care coverage remains unchanged, this leads to a strong result: In the long run the expansion of monetary child benefits is not only self-financing but actually adds financial resources to the government budget. Changes in the different components of the total budget are shown in Table 2.8.

Table 2.8: *Kindergeld Reform* - Government Budget

	benchmark		post-reform	
tax revenues	6.56		6.60	
pension system	4.52	68.9%	4.51	68.3%
monetary child benefits	0.19	2.8%	0.20	3.0%
public child care	0.04	0.7%	0.04	0.7%
residual expenditures	1.81	27.6%	1.85	28.1%

However, one has to take into account the fact that the population size is larger in the post-reform steady than it was before. A larger economy can entertain a larger governmental budget. But also when I express the budget in per capita terms, the key result holds: Residual expenditures per capita still increase by 1.6%.[64]

This main result is driven by a lower old-age dependency ratio. The higher fertility rate causes the ratio to decrease from 86% to 85%. This implies that in the post-reform steady state there are relatively more people working and paying taxes than retirees getting pension benefits.

[61]See Rainer et al. (2012), p.155. Based on the estimated elasticity they compute a counterfactual effect on the fertility rate assuming the same reform was introduced in 2010.

[62]When I simulate a 25% increase of monetary child benefits, the fertility reaction is 0.006.

[63]See for example Rainer et al. (2012) or Tamm (2010) for Germany.

[64]Table 2.20 in Appendix 2.A.6 presents the figures for the budget in per capita terms.

Thus, the additional tax revenues finance both the expanded monetary child benefits as well as the additional retirees.[65] On top of that, there are still financial resources left such that the government can increase other expenditures by 1.6% per capita.

The expenditures for both family policies increase due to the higher number of newborns. The government spends 4.6% more for direct monetary benefits per capita and 1.3% more for public child care. Finally, tax revenues and pension benefits per capita remain basically unchanged, and actually slightly decrease. The reason is the following: In the new steady state the effective wage rate for middle-aged employees ω_m slightly decreases since people gain less experience due to their lower individual labor supply.[66] Therefore, per capita tax revenues are also slightly reduced. Given that their pension benefits are linked to the labor income retirees face the same effect so pension expenditures for the government co-move with the tax revenues.

To have an impact on the other household choices the size of the reform is too small. Savings in both periods of work-life remain almost unchanged. The consumption pattern of both household types flattens a bit. Consumption during young adulthood slightly increases compared to the benchmark steady state while it is lower when being old.[67] Tables 2.21 and 2.22 in Appendix 2.A.6 present the resulting household choices and Table 2.23 shows some aggregate statistics.

2.5.2 The *Kita-Reform* - An Increase of Subsidized Public Child Care

In 2007, the so-called *Krippengipfel* took place in Germany. The announced goal of this summit was to increase the coverage rate for public child care to 35% for children below the age of three. Furthermore, the *Kinderförderungsgesetz* from 2008 guaranteed parents a legal claim for getting a place in a public child care facility for their child from August 2013 onwards.[68] So far, the target of 35% has not been reached yet. The coverage rate in 2014 was 32.3%.[69] Nevertheless, I use the official target value for my policy experiment. In the model the increase of the coverage rate from 20 to 35% represents an increase of the child care parameter θ^b from 0.062 to 0.1085.[70]

The time cost per two children is reduced by 18.75% from 0.248 to 0.2015. As a consequence the foregone earnings (per child) also decrease by 20.1%. The decline in foregone earnings is

[65]There are also more retirees in this steady state because the higher number of children also become retirees one day.

[66]The experience channel dominates the labor supply effect. The base wage rate w is increasing because individual labor supply is reduced.

[67]As the Euler Equation has to hold and the interest rate is lower in the new equilibrium, this also implies a flatter consumption pattern.

[68]See www.bmfsfj.de/BMFSFJ/kinder-und-jugend,did=161776.html and www.bmfsfj.de/BMFSFJ/gesetze, did=133282.html from the Federal Ministry for Family Affairs.

[69]See Destatis (2015), p.8.

[70]The time cost for parents were $b = 0.31$, 20% of this is 0.062 and 35% are 0.1085, correspondingly.

stronger than the one in time cost because in the new post-reform steady state wages are lower than in the benchmark case. So, parents have to spend less time with their children and on top of that the opportunity costs of the remaining child-rearing time is also smaller. This huge drop in opportunity costs of raising children has a strong effect on the fertility rate which is also shown in Tables 2.9 and 2.10. Low income households increase their number of children by 13.6% and high income households by 7.7%. The increase of the fertility rate passes through completely to the direct child costs: Since costs per child remain unchanged the total direct costs increase by 7.8% for high income households and by 13.6% for low income households. Overall, households reduce the foregone earnings by 13.8% (high types) and 9.4% (low types). Similarly to the expansion of the *Kindergeld*, there are two effects on the household's budget: The higher availability of child care facilities decreases foregone earnings but the resulting higher fertility rate increases particularly direct child rearing costs. Here, the cost-decreasing effect of the policy reform is dominating and high income households have to spend 2.3% less on their children while these expenditures are 2.8% lower for low income households.

Table 2.9: *Kita-Reform* - Child Rearing Costs (High Type)

		benchmark	post-reform	Δ (in %)
direct costs	per 2 children	0.474	0.474	0.0
	total	0.264	0.285	7.8
time costs	per 2 children	0.248	0.202	-18.8
	total	0.138	0.121	-12.4
foregone earnings	per 2 children	0.416	0.333	-20.1
	total	0.232	0.200	-13.8
total costs	per 2 children	0.890	0.807	-9.4
	total	0.496	0.485	-2.3

Table 2.10: *Kita-Reform* - Child Rearing Costs (Low Type)

		benchmark	post-reform	Δ (in %)
direct costs	per 2 children	0.122	0.122	0.0
	total	0.091	0.103	13.6
time costs	per 2 children	0.248	0.202	-18.8
	total	0.186	0.171	-7.7
foregone earnings	per 2 children	0.317	0.254	-20.1
	total	0.237	0.212	-9.4
total costs	per 2 children	0.439	0.375	-14.5
	total	0.328	0.319	-2.8

The average fertility rate increases by 12.3%. Thus, the new overall fertility rate is 1.55, compared to 1.38 in the initial equilibrium. This means that 10 percentage points more child care coverage imply a raise of the fertility rate by 8.2%. This estimate is very close to the finding of Bauernschuster et al. (2013). Assuming a public child care coverage of 35% they estimate the

German fertility rate to be 1.53 which is very close to the 1.55 I get. Rindfuss et al. (2010) find that a 10 percentage increase of public child care leads to approximately 0.1 additional children per woman. Bauernschuster et al. (2013) estimate the fertility reaction to a 10 percentage points increase to be 3.2% which is lower than the 8.2% in the present experiment.

The result can also be compared to Fehr and Ujhelyiova (2013). Their policy experiment is a cut of time costs by 50% which leads to an increase of the fertility rate by 0.46 children per woman. Hence, a cut of 10% leads to an increase of 0.092 children per woman. The corresponding figure for my policy experiment is 0.085 and therefore lower. Equivalently to the findings of Fehr and Ujhelyiova (2013), the results show a stronger fertility reaction of low income types. While high income type parents use the higher child care coverage to earn more of the previous foregone earnings, low income type parents use the reduction in child-bearing costs to have more children.

A second choice that is highly affected by child care is parental labor supply. Generally, there are two effects present: First, more public child care reduces the time cost of children and increase parental labor supply. Recall that this model does not include leisure, hence all additionally spared time has to go into working hours.[71] Second, higher fertility induced by better child care coverage will reduce the individual labor supply since parents have to take care of more children. In this scenario, the first effect dominates and individual labor supply increases for by 2.0% for high and 1.8% for low income types.

What is the long-run budgetary effect of this *Kita* expansion? Equivalently to the *Kindergeld Reform* better child care coverage adds additional financial resources to the government budget as shown in Table 2.11. In total, the government can spend over 50% more on residual expenditures G and even per capita the government can spend 33% more. The corresponding figures can be found in Table 2.24 in Appendix 2.A.6.

Qualitatively, the result is the same as for the *Kindergeld* reform: In the long run, expansionary child care subsidies are not only self-financing but also provide additional financial resources to the government. Quantitatively, the child care reform has a much larger impact than the reform of monetary benefits: G per capita increases by one third compared to 1.6% in the first reform. However, the second reform represents a much stronger child subsidy that has a stronger impact on the fertility rate. This point will be discussed in more detail in Chapter 3 and also in Section 2.6.1.

[71] Therefore the results of this model define an upper bound of this effect. Including leisure and an additional labor-leisure margin would lead to a weakly lower effect on individual labor supply.

Per capita expenditures for public child care more than double so that the government has to hire more nannies. The total number of nannies increases by 150% and even as share of total employment the share of nannies doubles from 0.4 to 0.8%. In parallel to the effective wages of young low type workers, the nanny wage decreases by 1.6%. Monetary child benefits increase by a quarter and tax revenues increase by 3%. The cause for the higher tax revenues is the positive reaction of individual labor supply. People work more during young adulthood and have a higher labor income. In addition, their experience gain increases as well which leads to a higher effective wage rate in their middle adulthood. This in turn raises labor income and tax payments of the middle-aged employees are also higher. This effect is also present for the pension benefits. However, due to the increased fertility rate there are more non-retirees in the economy - the old-age dependency ratio drops by 15.3% - and the government has to spend less for the pension system.

Table 2.11: *Kita-Reform* - Government Budget

	benchmark		post-reform	
tax revenues	6.56		7.70	
pension system	4.52	68.9%	4.56	59.2%
monetary child benefits	0.19	2.8%	0.26	3.4%
public child care	0.04	0.7%	0.10	1.3%
residual expenditures	1.81	27.6%	2.77	36.0%

For employees, the child care reform has ambiguous effects: While the effective wage rate for young employees decreases, it increases for the middle-aged. The cause is the labor supply effect of public child care. In the post-reform steady state individual labor supply has gone up. As a consequence, the interest rate increases and the base wage rate decreases. This movement of factor prices implies that the positive labor supply effect dominates the effect on lifetime savings, which is also positive, so that the capital-to-labor ratio decreases. Young employees do not have any work experience yet what means that their effective wage rate declines together with the base wage rate. The higher individual labor supply during young adulthood leads to a bigger experience gain which makes the worker more productive in her middle adulthood period. This experience-caused productivity gain over-compensates the lower base wage and the effective wage rate in the post-reform steady state is higher than before. One consequence of these wage movements is that households earn less during young and more during middle adulthood so that both household types reduce their net savings during young adulthood and increase it one period later. All corresponding figures can be found in Tables 2.24 to 2.27 in Appendix 2.A.6.

Finally, the higher interest rate leads to a steeper consumption, meaning consumption during young adulthood decreases compared to the benchmark steady state and it increases during old adulthood. Thus, during young adulthood the price effect clearly dominates: Households shift income from consumption and savings to children which got relatively cheaper because of the child care reform.

What can be concluded from these two family policy reforms is, first, that both policies - more *Kindergeld* and more *Kitas* - have a positive effect on fertility. In both cases, the fertility reaction of low income households is stronger than of high income ones. Second, the goal of reducing total child rearing costs for the households is only reached with the child care expansion. The positive effect of more *Kindergeld* is offset by the higher number of newborns such that in total the child rearing costs remain unchanged. Expanded public child care reduces total child rearing costs for both household types. Third, higher monetary benefits go hand in hand with lower individual labor supply while better child care coverage increases individual labor supply. Fourth, both family policy reforms are more than self-financing, which means that the government does not have to cut other expenditures or raise taxes to finance the expanded family support. On the contrary, it increases the governmental revenues and therefore allows higher spendings, in this scenario on further expenditures. The main mechanism for this budgetary effect is the fact that the family policies decrease the total dependency ratio: The number of tax contributors increases relatively more strongly than the number of pure beneficiaries.

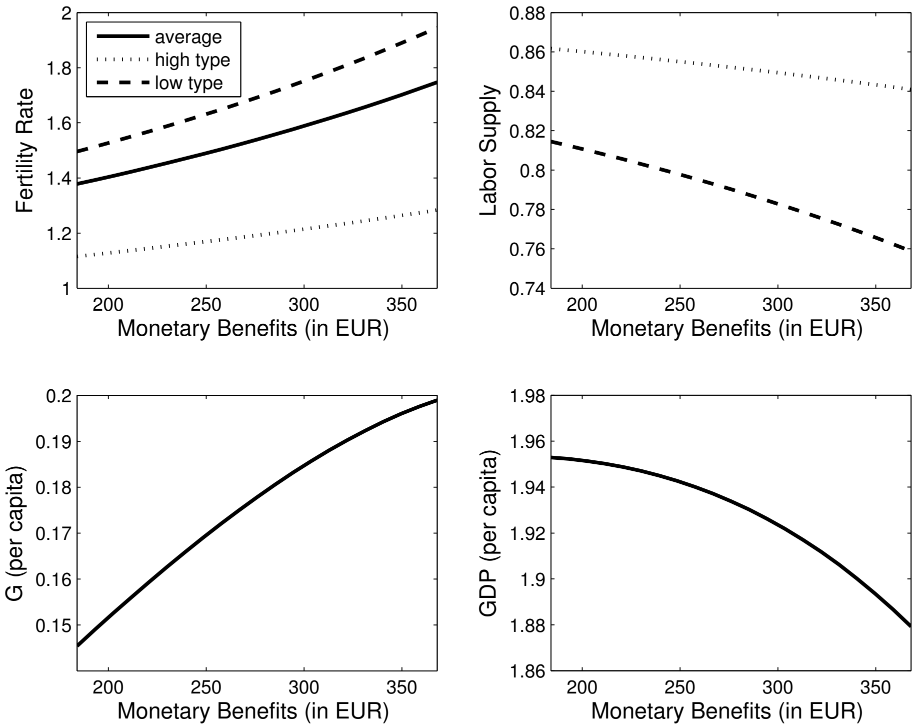
2.6 Robustness Checks

After the analysis of the two family policy reforms the question that arises is whether the main results are caused by a specific parameter constellation or rather general. In the remainder of this section, I show that the results are not based on special scenarios but are more general and for example robust to different parameter choices or different magnitudes of family policy interventions. The robustness checks that are performed are the following: First, I show a much wider range of monetary child benefits and public child care coverage rates and their corresponding effects specifically on fertility, labor supply and the government budget. With this exercise it can be shown that the positive effects of the family policies are not specific to a certain size of the reforms. Next, I provide some sensitivity analyses by using alternative assumptions on different parameters. Then, the model is re-calibrated and I simulate the two family policy reforms again. I can show that the reform results are robust across all alternative calibrations. Finally, both reforms are implemented jointly, as it basically is in reality. The results remain the same but the two effects add up linearly.

2.6.1 Broader Ranges of Family Support

As a first check, I show that the main results do not rely on the implementation of a specific reform but can be generalized to a wider range of family policy levels. To do so, I solve the model for a large range of parameter values, first, for the monetary child benefits θ^a and then for public child care θ^b. All other parameters remain unchanged compared to the calibrated benchmark. Note that the results are no comparative statics but represent a range of many different steady states. Figure 2.2 shows the main results for different levels of *Kindergeld*. The range for different benefit levels vary between 100% and 200% of the pre-reform level, thus between 184 and 368 EUR per child and month. Note that with a benefit level of 368 EUR per month and child almost 97% of the direct costs of children for low income parents are covered. In order not to create artificially negative child rearing costs I do not extend this range any further.

Figure 2.2: Range of Benefit Levels θ^a



The fertility rates are shown in the upper left Panel of Figure 2.2. For both types of households fertility increases monotonically in steady states with higher monetary child benefits. The number of children of low income households increases until almost two children per woman while high income households do not have more than 1.3 children even with the highest monetary benefits. As a consequence the average fertility also increases monotonically up to 1.8 children. The realized goal of the *Krippengipfel* for child care coverage, that is a coverage rate of 35%, brought the fertility rate increase to 1.55 children per woman. To reach the same fertility rate with higher monetary benefits, the *Kindergeld* would need to increase by 100 EUR (or 54%) from the original 184 EUR to 284 EUR per child and month.

A necessary consequence of higher monetary child benefits is decreasing individual labor supply which can be seen in the upper right Panel. One major consequence of this decrease can be observed in the lower right Panel: GDP per capita is constantly decreasing with higher monetary benefits which is mainly driven by the individual labor supply. The same argument was already made in the previous section. Not only has declining labor supply a negative impact on economic output but the negative effect is even stronger on experience accumulation which reduces the productivity of middle-aged employees. The higher the monetary child benefits are, the higher the number of children and the more prominent this mechanism is. Finally, residual government expenditures are shown in the lower left Panel. For all benefit levels, the family policy reform is more than self-financing and increases the financial resources of the government. In general, an increase of θ^a by 1% generates 0.5% more financial resources. This is line with the results for the *Kindergeld* reform where a rise of 3.2% of benefits increased G per capita by 1.6%. It also holds true for the maximum range of this policy, that is, doubling monetary benefits leads to 50% higher G per capita.

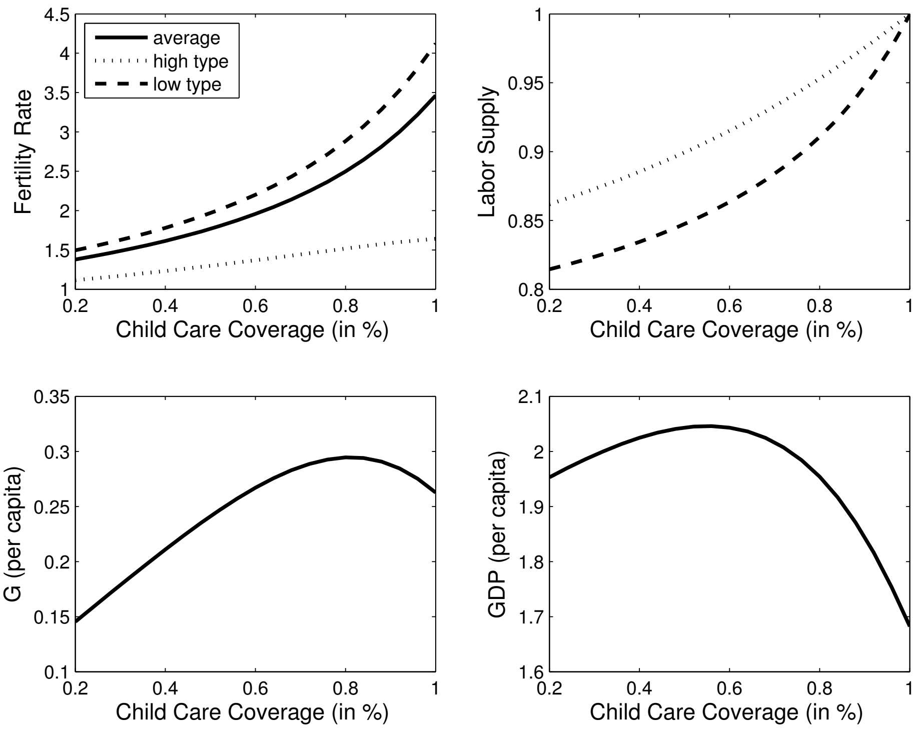
The equivalent exercise is done for public child care coverage. Figure 2.3 presents the results over a set of steady states in which child care θ^b ranges from the initial 20% up to 100% of the parental time costs.[72] Again, the fertility rates are presented in the upper left Panel. Average fertility rises to 3.5 children per woman for the case when there are essentially no time costs for parents. This effect is heavily driven by the low income households whose fertility rate exponentially increases to 4.1 children. The fertility reaction of high income households is more moderate. Even without any time costs and therefore no foregone earnings high income households only have 1.6 children. Moreover the increase is not linear as the fertility reaction becomes weaker at very high coverage rates. The cause for that diminishing effect on the high types' fertility rate is that middle life income decreases. The reason is the following: The base wage rate decreases because of the higher aggregate labor supply. Above a certain coverage rate, this negative effect on income cannot be compensated by the higher experience that results from high individual labor supply. Hence, lifetime income is lower and the negative income effect depresses fertility.

The additional provision of public child care over-compensates the fertility increase over the whole range in a way that individual labor supply monotonically increases until it is 100%

[72]Because of the numerical solution algorithm, the 100% are rather 99.99%.

when child care coverage also reaches 100%. Residual government expenditures are hump-shaped with a maximum at a coverage rate of 80%, as can be seen in the lower left Panel. With coverage rates of 80% or higher public child care becomes “too” expensive. That is, the combination of further increasing public child care coverage times a higher number of children creates higher costs than it adds financial resources to the budget. In the steady state with 100% child care coverage 6.8% of all employees are working in the child care sector.

Figure 2.3: Range of Child Care Coverage Rates θ^b



Lastly, GDP per capita is shown in the lower right Panel. It it also hump-shaped with its maximum at a coverage rate of 56%. When the coverage rate is larger than 80%, GDP per capita is even lower than in the initial steady state with 20% coverage rate. This hump-shaped pattern is mainly caused by the following trade-off: With increasing individual labor supply employees get more productive thanks to more experience which has a positive effect on GDP. However, above a certain coverage rate the total dependency ratio starts to re-increase. In other words, the amount of people who do not pay taxes grows faster than the number of tax-paying employees. Relatively more non-employed people have a negative effect on GDP. A coverage rate of 56% is the threshold from which on the increase of the dependency ratio dominates this productivity increase and thus, the GDP per capita starts to decline.[73]

[73]On top of that, with a higher coverage rate relatively more effective labor is bound in the child care sector. Effective labor in the nanny sector relative to total effective labor in the economy increases exponentially from 0.3% with a coverage rate of 20% to 4.6% with 100% child care coverage.

2.6.2 Sensitivity Analysis

The sensitivity analysis is done for three parameters by using different assumptions or parameter values. The procedure in all scenarios is, first, to re-calibrate the benchmark steady state using the alternative parameter values, and second, I re-do the two policy experiments and compare the results of the sensitivity analysis with the main results presented in Section 2.5. The analyzed parameters are:

- the transition probabilities π_{HH} and π_{LL},
- the children-to-nanny ratio Q,
- and the initial public child care coverage θ^b.

In total I analyze four scenarios since I pick two different initial coverage rates. The four parameters are the transition probabilities, the children-to-nanny ratio and twice the initial child care coverage rate.

Re-Calibrations of the Benchmark Steady States

Parameters determining the child-bearing behavior are central for the effectiveness of family policies. One might wonder whether the fertility effects of family policy reforms would be the same if the intergenerational transmission probabilities π_{HH} and π_{LL} looked differently. More precisely, a higher persistence of high types might dampen the fertility reaction as this would lead to more high types who get less children compared to low types. In the baseline calibration I divided household into two income groups using the education threshold whether a person has *Abitur* or not. The underlying assumption is that education significantly determines the income later in life.[74] As a robustness check I consider that nowadays more people reach the *Abitur* and therefore are in the high income group. I assume that 50% of people that are classified in "middle education" actually belong to the high income group.[75] The resulting intergenerational transition probability matrix looks as follows:

$$\Pi(i' \mid i) = \begin{pmatrix} 69.01\% & 30.99\% \\ 38.6\% & 61.4\% \end{pmatrix}.$$

[74] The transition probabilities in the baseline calibration are: $\pi_{HH} = 57\%$ and $\pi_{LL} = 78\%$.

[75] There are two justifications for this modification: First, there is an increasing number of pupils that continue their education after *Realschule* at the *Gymnasium* to pass the *Abitur*. Second, there is a growing share of people not having been at the *Gymnasium* but who can go to university and study, anyway. This group of people increases the share of high income earners as well. See also Appendix 2.A.4.

Now, 69% of children of high income parents will also be receivers of high income, and 61.4% of low income parents' children will remain in the low income group as their parents.

Given the new parameter values the targeted moments are not reached as well as in the benchmark calibration, see column 3 of Table 2.12. As suspected, the fertility rate is only 1.34 instead of the actual 1.38 and relative labor income is 0.82. However, the fertility rate generated by the model is still quite close to its data counterpart. All other targets are hit perfectly. Due to the modified transition probabilities the type distribution is shifted towards the high income households. Now, 52% of all people are high types compared to 31% in the benchmark calibration which is the cause for the lower average fertility. Another consequence of the shift of the type distribution is a higher GDP per capita as can be seen in Column 2 of Table 2.13. High income households are more productive and therefore the stock of effective labor in this economy is higher and GDP per capita as well. Otherwise, the transition probabilities do not affect individual choices. Before discussing the results of the family policy reforms presented in Tables 2.14 and 2.15 I discuss the re-calibrated economies of the other three sensitivity analyses.

Table 2.12: Robustness Check - Comparison of Targeted Moments

Target	Data	Benchmark	π	Q	30% θ^b	40% θ^b
average fertility rate	1.38	1.38	1.34	1.38	1.38	1.38
differential fertility	0.38	0.38	0.38	0.38	0.39	0.38
child cost to consumption ratio	0.49	0.49	0.49	0.49	0.48	0.52
child cost ratio	2.50	2.50	2.50	2.52	2.50	2.50
relative labor income	1.07	0.88	0.82	0.89	0.87	0.87
relative monetary child benefit	0.066	0.066	0.066	0.066	0.066	0.066
replacement rate	0.553	0.553	0.553	0.553	0.553	0.553

Another robustness check is done with the children-to-nanny ratio Q. The results of the policy experiments in Section 2.5 have shown strong effects of the expansion of public child care. One might argue that this is driven by the relatively high productivity of the child care sector, measured by the children-to-nanny ratio which is 5.0. As parents cannot choose between giving children to public child care or taking care of them at home, one could argue that by assumption child care must be strictly better than no child care. Therefore I set the children-to-nanny ratio to 1.0, which is the same as if one parent takes care of one child.

The targeted moments are matched as well as in the benchmark calibration, see Column 4 in Table 2.12. The child cost ratio is slightly too high but relative labor income is a bit closer to the data compared to the benchmark calibration. All other moments are matched perfectly. The initial steady state is very similar to the benchmark case with one exception: The share of people working in the child care sector is significantly higher. Instead of five children, a nanny can only care for one child and since the number of children is the same in both steady states, the government there must hire five times more nannies. In total, 1.9% of all workers are located in the child care sector. This figure and further aggregate statistics are shown in Column 3 of Table 2.13.

Table 2.13: Robustness Check - Comparison of Aggregate Statistics

	benchmark	π	Q	30% θ^b	40% θ^b
share of high types	31.0%	52.4%	31.0%	31.0%	31.0%
old-age dependency ratio	0.86	0.89	0.86	0.86	0.86
total dependency ratio	1.14	1.16	1.14	1.14	1.14
GDP per capita	1.95	2.09	1.94	2.06	1.93
interest rate (p.a.)	8.99%	9.13%	8.95%	9.05%	9.15%
wage rate rate	1.05	1.04	1.05	1.04	1.04
nanny wage rate	2.12	2.13	2.11	2.15	1.94
size of nanny sector	0.4%	0.4%	1.9%	0.6%	0.7%

Lastly, I take into account the fact that public child care is expanding very fast in Germany and that in several counties the coverage rate is higher than the initially assumed 20%. Furthermore, one might suspect that the positive effect of the public child care expansion is, at least partly, due to relatively low starting level of child care coverage. As robustness check, I therefore use 30% and 40% as child care coverage rates before the implementation of the *Kita-Reform*.

The calibration leads to very similar model moments as in the benchmark calibration. Only, the child cost-to-consumption ratio is slightly too high with an initial coverage rate of 40%. Two features of the new initial steady state differ from the benchmark case. First, the share of employees working as nannies in the new steady state is the larger, the larger the initial coverage rate is (0.6% and 0.7% compared to 0.4% in the benchmark case). Second, GDP per capita is relatively high in the case of 30% coverage and relatively low with 40% coverage. The same is true for the nanny wage rate. The reason are the calibration results for the productivity parameter z^L that can be seen in Table 2.28 in Appendix 2.A.7. The calibration with 40% child care coverage leads to the low value of z^L of 1.87 compared to a value of 2.06 in the scenario with 30% coverage. This leads to a gap in productivity and therefore to diverging stocks of human capital.

Re-Simulation of the Family Policy Reforms

The alternative results of the policy reforms are shown in Tables 2.14 and 2.15. The first column repeats the benchmark results from the main analyses, the second column shows the results with modified transition probabilities, the third one the results for the unproductive nanny sector, and the last two columns show the results for the adjusted initial child care levels.

With the modified transition probabilities the main results remain basically unchanged. This is no surprise since the probabilities do not affect individual choices. Only for the child care reform, the average fertility reaction is weaker than in the benchmark case. The cause is a type-composition effect: In the re-calibrated economy there are more high types such that their lower type-specific fertility rate has a depressing impact on the average fertility reaction to the public child care expansion.

In the scenario with the low children-to-nanny ratio, the results of the public child care expansion are very similar to the results of Section 2.5 with one exception: As nannies are less productive, with expanded child care coverage the government has to hire over-proportionally more nannies which increases the corresponding expenditures in the government budget. Therefore, the additional financial resources - measured in higher residual expenditures G per capita - that are generated by the reform are only 26% compared to 34% in the benchmark calibration. Nevertheless, even with this unproductive child care sector the financial return of expanded public child care θ^b is still significantly higher than the one of increased monetary child benefits θ^a. As already mentioned in Section 2.6.1, a 1% increase of θ^a leads to a 0.5% increase of G. For the benchmark case of public child care, a 1% increase of θ^b leads to an increase of G larger than 2%. With the low children-to-nanny ratio, the return is still above 1.5%. The results for the monetary benefits are very similar to the benchmark policy reforms.

For the sensitivity analyses with respect to the initial coverage rate of public child care I still use the increase of the coverage rate of 15 percentage points as policy experiments. So, the post-reform coverage rates are 45% and 55%, respectively.

When the initial coverage rate is 40%the effects are smaller for the child care reform compared to the baseline scenario. There are two explanations for this observation: First, the effectiveness of child care is decreasing when the initial coverage is larger. Second, it is just the way of presenting the number which is misleading. An increase from 20% to 35% coverage is an increase of 75%, and this leads to 12.3% higher fertility. The increase from 40% to 55% is a plus of 37.5% that leads to a fertility rate that is 11.2% higher. Therefore, the elasticities show an accelerating effect of child care when the initial level is higher. The second view is also in line with the results of the Section 2.6.1 where I could show that fertility increases exponentially with higher levels of public child care. Performing the policy reforms in the case of 30% coverage rate the results are very similar to the baseline scenario.

Table 2.14: Robustness Check - Comparison of Monetary Child Benefits Reform (in %)

	benchmark	π	Q	30% θ^b	40% θ^b
average fertility rate	+0.7	+0.6	+0.7	+0.7	+0.6
fertility rate high type	+0.4	+0.4	+0.4	+0.4	+0.4
fertility rate low type	+0.8	+0.7	+0.7	+0.8	+0.7
labor supply high type	-0.1	-0.1	-0.1	-0.1	-0.0
labor supply low type	-0.2	-0.2	-0.2	-0.2	-0.1
total dependency ratio	-0.4	-0.4	-0.4	-0.5	-0.4
G per capita	+1.6	+1.5	+1.6	+1.6	+1.6

Table 2.15: Robustness Check - Comparison of Child Care Reform (in %)

	benchmark	π	Q	30% θ^b	40% θ^b
average fertility rate	+12.3	+11.0	+12.5	+12.2	+11.2
fertility rate high type	+7.7	+7.7	+8.0	+7.3	+6.5
fertility rate low type	+13.6	+13.3	+13.8	+13.5	+12.5
labor supply high type	+2.0	+2.1	+2.0	+2.2	+2.3
labor supply low type	+1.8	+1.9	+1.7	+2.1	+2.5
total dependency ratio	-6.5	-6.3	-6.6	-6.5	-6.0
G per capita	+34.1	+34.9	+26.4	+34.1	+32.3

2.6.3 Joint Family Policy Reform

Both family policy reforms - the *Kindergeld Reform* and the *Kita-Reform* - have been introduced in Germany during a similar time period. Therefore, one might be interested in the joint effect of these reforms when they are introduced simultaneously. The question is whether the effects add up linearly or whether there is an additional interaction effect either amplifying or dampening the effects of a single reform. Table 2.16 compares the key results of the reforms individually and jointly. One can conclude that the two policies combined basically sum up the individual effects and neither positive nor negative interaction effects can be observed.

Table 2.16: Robustness Check - Comparison of Individual and Joint Reforms (in %)

	benefits reform	child care reform	reforms jointly
average fertility rate	+0.7	+12.3	+13.2
fertility rate high type	+0.4	+7.7	+8.2
fertility rate low type	+0.8	+13.6	+14.6
labor supply high type	-0.1	+2.0	+1.9
labor supply low type	-0.2	+1.8	+1.3
total dependency ratio	-0.4	-6.5	-6.9
G per capita	+1.6	+34.1	+35.5

2.7 Conclusion

In this chapter I have analyzed and compared the effectiveness of direct monetary benefits and public child care in a general equilibrium framework. In particular, I simulated the effects of two recent family policy reforms in Germany. First, the monetary child benefits (*Kindergeld*) were increased from 184 EUR to 190 EUR per child and month. Second, public child care was expanded to cover 35% of the child-rearing time compared to 20% before the reform.

I find that both reforms increase the fertility rate on average and for both types individually. The fertility reaction of low income households is stronger than the fertility reaction of high income households. Moreover, I can show that the fertility elasticity is in a reasonable range compared to other studies.

The main result of this chapter is that in the long-run both family policy reforms do not require higher taxes but are completely self-financing. On top of that, expanded family policies create additional financial resources in the governmental budget. Because of the reforms, the total dependency ratio declines meaning that the number of tax-paying employees increases more than the number of pure beneficiaries. This is the reason why one EUR more invested into the family policy budget leads to more than one EUR in the overall budget. This mechanism is stronger for the expansion of public child care because of additional positive labor market effects. Parents work more, hence, they gain more human capital which in turn makes them more productive. This amplifying mechanism is not present in the case of the monetary benefits reform. In contrast to the public child care reform, I find that an increase of monetary benefits reduces households' labor supply. Both results are in line with findings in the empirical literature.

The family subsidies do not generally reduce total child rearing costs. Even though the government reduces the costs per child, this effect might be offset by the increasing fertility effect of the family policies. In the case of the *Kindergeld Reform* the effects cancel out and total child rearing costs remain constant. In the case of the public child care expansion, the first effect dominates such that total child rearing costs are lower for both households.

Several robustness checks confirm that the main findings are generally. I show that the positive effects on fertility and on the government budget also hold, when I assume different intergenerational transition probabilities, a lower children-to-nanny ratio or higher initial levels of public child care coverage. I also show that the simultaneous implementation of both reforms does not lead to any positive or negative amplification. The effects are basically the sum of the effects of both individual reforms. Finally, I show that the positive effects of monetary benefits remain present for a whole range - up to doubling the amount - of the benefit level. This conclusion is only partly true for public child care reforms. Over the whole range from 0% to 100% of child care coverage rates, fertility rates increase exponentially. However, only coverage rates below 80% decrease the total dependency ratio and therefore generate a financial surplus for the government. Beyond this threshold, expanded public child care becomes the victim of its own success: When the fertility rate exceeds the replacement level of 2.0 children per woman, the total dependency ratio starts to increase. Because of the combination of higher public child care coverage and higher fertility rates, the family policy becomes over-proportionally expensive. Consequently, it cannot create a sufficient financial return and the government's financial resources decline.

2.A Appendix

2.A.1 Stationary Type Distribution

To compute a stable equilibrium, it is essential to define and find a stationary type distribution. Given the transition probabilities, type dependent fertility rates n_t^i and initial population sizes N_t^i, in a first step I set up a law of motion of the fractions of types. In a second step this law of motion is used to determine the stationary type distribution. I define total population size as $N_t = N_t^H + N_t^L$ and fractions of high and low type individuals as $h_t = \frac{N_t^H}{N_t}$ and $l_t = \frac{N_t^L}{N_t}$. Then the average fertility rate is $\bar{n}_t = h_t n_t^H + l_t n_t^L$. The fraction of high types in the next period, h_{t+1} are all the newborns of high types and low types that will become high types over next the period's total population:

$$
\begin{aligned}
h_{t+1} &= \frac{N_t^H n_t^H \pi_{HH} + N_t^L n_t^L \pi_{LH}}{N_{t+1}} \\
&= \frac{h_t N_t n_t^H \pi_{HH} + l_t N_t n_t^L \pi_{LH}}{h_t N_t n_t^H + l_t N_t n_t^L} \\
&= \frac{h_t n_t^H \pi_{HH} + l_t n_t^L \pi_{LH}}{h_t n_t^H + l_t n_t^L} \\
&= \frac{h_t n_t^H \pi_{HH} + l_t n_t^L \pi_{LH}}{\bar{n}_t} \\
&= h_t \frac{n_t^H}{\bar{n}_t} \pi_{HH} + l_t \frac{n_t^L}{\bar{n}_t} \pi_{LH}
\end{aligned}
$$

Analogously, the law of motion for the fraction of low types is:

$$
l_{t+1} = h_t \frac{n_t^H}{\bar{n}_t} \pi_{HL} + l_t \frac{n_t^L}{\bar{n}_t} \pi_{LL}.
$$

For a stationary type distribution it must hold that:

$$
\begin{aligned}
h_{t+1} = h_t = h = h \frac{n^H}{\bar{n}} \pi_{HH} + l \frac{n^L}{\bar{n}} \pi_{LH}, \\
l_{t+1} = l_t = l = h \frac{n^H}{\bar{n}} \pi_{HL} + l \frac{n^L}{\bar{n}} \pi_{LL}.
\end{aligned}
$$

This can be reformulated as

$$
[h \quad l] = [h \quad l] \times \tilde{\Pi}(i' \mid i)
$$

where $\tilde{\Pi}(i' \mid i)$ are the transition probabilities adjusted by the relative type specific fertility rates:

$$
\tilde{\Pi}(i' \mid i) = \begin{pmatrix} \frac{n^H}{\bar{n}} \pi_{HH} & \frac{n^H}{\bar{n}} \pi_{HL} \\ \frac{n^L}{\bar{n}} \pi_{LH} & \frac{n^L}{\bar{n}} \pi_{LL} \end{pmatrix}.
$$

This is a system of two equations and two unknowns which can be re-written as

$$h = \frac{\tilde{\pi}_{LH}}{1-\tilde{\pi}_{HH}} l,$$
$$l = \frac{\tilde{\pi}_{HL}}{1-\tilde{\pi}_{LL}} h.$$

Solving this system of equations leads to the stationary type distribution. Note that $\bar{n}$ includes the fractions h and l such that it cannot be solved explicitly for the stationary distribution but numerically by iterating on these two equations.

2.A.2 Labor Market Clearing

Before defining labor market clearing I first define total labor supply $\tilde{L}_t$ and total effective labor supply $\tilde{H}_t$ which is the sum of all individual (effective) labor supplies.

$$\tilde{L}_t = \sum_{j\in\{y,m\}} \sum_{i\in\{H,L\}} \ell^i_{j,t} N^i_{j,t},$$
$$\tilde{H}_t = \sum_{j\in\{y,m\}} \sum_{i\in\{H,L\}} \ell^i_{j,t} z^i x^i_{j,t} N^i_{j,t}.$$

The labor market is separated into two sectors: the manufacturing sector and the public nanny sector. Total demand for nannies needed for subsidized public child care is

$$L^N_t = \theta^b \frac{N^C_t}{Q}.$$

By assumption the government can hire as many young low type workers as nannies as necessary to meet total demand for nannies. Since the government pays exactly the same wage as the young low type worker would earn in the manufacturing sector, households are indifferent whether they work in the nanny or the manufacturing sector. Moreover, experience accumulation is the same in both sectors.[76]

All people that are not employed as nannies work in the firm sector such that total labor supply in that sector is:

$$\tilde{L}^M_t = \tilde{L}_t - \tilde{L}^N_t.$$

The relevant number for the firms, however, is the effective labor supply $\tilde{H}^M_t$, which is also the difference between the total effective labor supply and the one which is bound in the nanny sector. To get an explicit number for $\tilde{H}^M_t$, I define the effective labor supply in the nanny sector,

[76] As long as the demand for nannies is lower than the total labor supply of young low type households this assumption is not critical.

$\tilde{H}_t^N$, first:

$$\tilde{H}_t^N = \sum_{j\in\{y,m\}} \sum_{i\in\{H,L\}} \ell_{j,t}^{i,N} z^i x_{j,t}^i N_{j,t}^i.$$

Consequently, the effective labor supply for the firm sector is given by:

$$\tilde{H}_t^M = \tilde{H}_t - \tilde{H}_t^N,$$
$$\tilde{H}_t^M = \sum_{j\in\{y,m\}} \sum_{i\in\{H,L\}} \ell_{j,t}^{i} z^i x_{j,t}^i N_{j,t}^i - \sum_{j\in\{y,m\}} \sum_{i\in\{H,L\}} \ell_{j,t}^{i,N} z^i x_{j,t}^i N_{j,t}^i.$$

For market clearing in this sector of the labor market it therefore has to hold that firms' demand equals the households supply of effective labor:

$$H_t^M = \tilde{H}_t^M.$$

2.A.3 Solution Algorithms

In this part of the Appendix I describe the solution algorithms for the household optimization, for the general equilibrium and for the calibration.

Household Optimization Solution Algorithm

Based on the equilibrium characterization described in Section 2.3.5 I use first-order conditions, budget constraints, time constraint and definitions for experience and effective wage rate to solve the household optimization problem:

$$\begin{aligned}
c_o &= (1+r)s_m + P,\\
c_m &= c_o(\beta(1+r))^{-1/\sigma},\\
c_y &= c_m(\beta(1+r))^{-1/\sigma},\\
n &= (\eta/\phi)^{1/\psi} c_y^{\sigma/\psi},\\
\ell &= 1-(b-\theta^b)n,\\
x_m &= \lambda_0+\lambda_1+\lambda_2\ell_y,\\
\omega_m &= wzx_m,\\
P &= \frac{1}{2}q((1-\tau)\ell_y\omega_y+(1-\tau)\omega_m),\\
s_y &= \frac{1}{1+r}(c_m+s_m-(1-\tau)\omega_m),\\
s_c &= \frac{1}{1+r}(c_y+s_y+\phi n-(1-\tau)\ell_y\omega_y).
\end{aligned}$$

The algorithm used is based on Heer and Maussner (2009) pp. 458-469.

1. Make initial guesses for s_m (for the first two iterations) and for individual pension benefits P for both household types.

2. Given s_m and P solve household problem for set of optimal choices $\mathbb{C} = (c, s_y, n, \ell)$ and experience x.

   (a) Compute c, n, ℓ, x using the above equations.

   (b) Given $\mathbb{C}$, update pension benefits P':

      i. If $\mid P - P' \mid > \epsilon$, update P and re-iterate.

      ii. If $\mid P - P' \mid < \epsilon$, go to (c).

   (c) Compute s_y and s_c:

      i. If $\mid s_c \mid > \epsilon$, update s_m and re-iterate.

      ii. If $\mid s_c \mid < \epsilon$, set of optimal household choices found, end.

For updating s_m, the Secant Method is used in the following way:

$$s_m^{i+2} = s_m^{i+1} - \frac{s_m^{i+1} - s_m^i}{s_c^{i+1} - s_c^i} \times s_c^i.$$

General Equilibrium Solution Algorithm with Stationary Type Distribution

This algorithm solves for a stationary general equilibrium and a given set of parameters.

1. Make initial guess for type distribution $\mathbb{R}$ and vector of prices $\mathbb{P} = (r, w, q)$.

2. Given $\mathbb{R}$ and $\mathbb{P}$, solve household problem for set of optimal choices $\mathbb{C} = (c, s_y, s_m, n, \ell)$:

   (a) Make initial guess for s_m.

   (b) Compute c, n, ℓ, s_y, s_c using FOCs and budget constraints.

   (c) If $\mid s_c \mid < \epsilon$, optimal choices found. Else, update s_m and re-iterate.

3. Given $\mathbb{R}$ and $\mathbb{P}$, compute aggregates $\mathbb{A} = (K, L, q)$.

4. Given $\mathbb{A}$, update prices $\mathbb{P}'$:

   (a) If $\mid \mathbb{P} - \mathbb{P}' \mid > \epsilon$, update $\mathbb{P}$ and go to 2.

   (b) If $\mid \mathbb{P} - \mathbb{P}' \mid < \epsilon$, go to 5.

5. Given $\mathbb{C}$, compute new type distribution $\mathbb{R}'$.

   (a) If $\mid \mathbb{R} - \mathbb{R}' \mid > \epsilon$, update $\mathbb{R}$ and go to 2.

   (b) If $\mid \mathbb{R} - \mathbb{R}' \mid < \epsilon$, stationary distribution and general equilibrium found, end.

Calibration Algorithm

This algorithm is used for the calibration of the parameters. It minimizes the sum of equally weighted distances between model moments and its data counterparts. Distance is defined as absolute percentage deviation.

1. Make initial guess for set of parameters $\Theta = \{\eta, \psi, a^H, a^L, z^L, \theta^a, G\}$ and compute the corresponding distance function $\mathbb{D}$.

2. Given Θ, solve for the general equilibrium with stationary type distribution to get optimal choices $\mathbb{C}$, prices $\mathbb{P}$ and type distribution $\mathbb{R}$.

3. Given $\mathbb{C}$, $\mathbb{P}$ and $\mathbb{R}$ compute model moments $\hat{M}$ and update the distance function

$$\mathbb{D}' = \sum_{\theta \in \Theta} \frac{\mid \hat{M}_\theta - M_\theta \mid}{M_\theta},$$

   where M_θ is the model moment:

   (a) If $\mid \mathbb{D} - \mathbb{D}' \mid > \epsilon$, update Θ and go to 2.

   (b) If $\mid \mathbb{D} - \mathbb{D}' \mid < \epsilon$, moments match perfectly, end.

   (c) Define $\Delta\mathbb{D}_i = \mathbb{D}_i - \mathbb{D}_{i-1}$, where i is the iteration counter.
   If $\Delta\mathbb{D}_i < \epsilon$, minimum of distance function is reached, end.

2.A.4 Calibration and Data

Intergenerational Transmission Probabilities

The calculations for the intergenerational transmission probabilities are based on Heineck and Riphahn (2007) who use SOEP data for birth cohorts from 1929 to 1978. They use three different education classes, "basic school" (*Grund- und Hauptschule*), "middle school" (*Realschule*), and "advanced school" (*Gymnasium*). Moreover, they provide the transition probabilities for sons and daughters separately, see Figure 2.4.

To make their numbers fit to my model setup I do three transformations: First, in the original tables, there are still item non-responses so that the probabilities do not sum up to one. I exclude the non-responses and re-compute the transition probabilities based on the existing observations. The adjusted probabilities can be seen in Figure 2.5

Second, and since I do not have different genders in my model, I define a child of "average-sex" that has 50% of the transition probabilities of the sons and 50% of the daughters, see Figure 2.6

Figure 2.4: Probabilities taken from Heineck and Riphahn (2007), Table 3, p.33.

		daughter		
		basic	middle	advanced
	basic	0.518	0.345	0.093
parent	middle	0.134	0.527	0.292
	advanced	0.078	0.363	0.521

		son		
		basic	middle	advanced
	basic	0.515	0,329	0.118
parent	middle	0.173	0.441	0.346
	advanced	0.094	0.285	0.581

Figure 2.5: Transition Probabilities without Missings.

		daughter		
		basic	middle	advanced
	basic	0.542	0.361	0.097
parent	middle	0.141	0.553	0.306
	advanced	0.081	0.377	0.542

		son		
		basic	middle	advanced
	basic	0.535	0.342	0.123
parent	middle	0.180	0.459	0.360
	advanced	0.098	0.297	0.605

Figure 2.6: Transition Probabilities without specific Gender.

		50% son / 50% daughter		
		basic	middle	advanced
	basic	0.539	0.351	0.110
parent	middle	0.160	0.506	0.333
	advanced	0.089	0.337	0.573

Third, I follow my education cut-off which is *Abitur* versus *no Abitur* and pool together "basic" and "middle school". The result is the following intergenerational transition probability matrix:

$$\Pi(i' \mid i) = \begin{pmatrix} 57.34\% & 42.66\% \\ 22.17\% & 77.83\% \end{pmatrix}.$$

As a robustness check in Section 2.6 I use different transition probabilities that take into account that nowadays more pupils pass the *Abitur* or go to university without *Abitur* but with a professional formation. For doing so, I spilt the education category "middle school" into two, and assume that 50% of them will end up as a high income household. The corresponding transition probability matrix is

$$\Pi(i' \mid i) = \begin{pmatrix} 69.01\% & 30.99\% \\ 38.6\% & 61.4\% \end{pmatrix}.$$

Labor Income Gap in Germany

Table 22.1 in Destatis (2010) presents the average annual gross labor income for Germany in 2010 by age and education. According to my education classification, all employees with at least *Abitur* are high income types. All below are low income types. Furthermore, I define all employees aged between 25 and 40 as "young adults" which corresponds to the first model period in adulthood, and all employees between 41 and 60 as "middle aged". I exclude all employees aged 25 and below from the young adult category because a lot of these employees are still in professional formation and therefore receive a very low labor income. Nevertheless, this leads to an upper bound for the labor income of young employees. Based on these assumptions and the original table, I compute the age and type-specific labor incomes, presented in Figure 2.7. The average labor income for young low types is 33,574 EUR while it is 47,697 EUR for high types and therefore 42% higher. This is the labor income gap.

Figure 2.7: Labor income in Germany 2010, own calculations (Source: Destatis (2010), Table 22.1).

	age groups		annual labor income (in EUR)	
type	**data**	**model**	**data age groups**	**model age groups**
low	25-30	young	30800	33574
	30-40		36347	
	40-50	middle	40318	40189
	50-60		40059	
high	25-30	young	40283	47697
	30-40		55110	
	40-50	middle	70205	69690
	50-60		69176	

Productivity gap z^H/z^L

I choose z^H such that w_y^H/w_y^L matches the labor income gap of 42% that can be computed from the data, see the section above in this appendix. Starting from the model wage-ratio between young high and low types, I first use the fact that the base wage rate resulting from profit maximization of the firm and labor market clearing is the same for both types. Experience is also the same for both types during young adulthood. Therefore both terms cancel out. Next, I use the model assumption that parents either spend their time working or with their children to substitute the type specific fertility rate for the individual labor supply. In a last step I assume the fertility rates to be the actual observed type-specific fertility rates. Then, I can solve and re-arrange the terms such that I get a value for z^H as a function of z^L.

$$\begin{aligned}
\frac{w_y^H}{w_y^L} &= \frac{x_y^H z^H \ell_y^H \omega}{x_y^L z^L \ell_y^L \omega} \\
&= \frac{\lambda_0 z^H \ell_y^H}{\lambda_0 z^L \ell_y^L} \\
&= \frac{z^H \ell_y^H}{z^L \ell_y^L} \\
&= \frac{z^H(1-(b-\theta^b)\bar{n}^H)}{z^L(1-(b-\theta^b)\bar{n}^L)} \\
\iff z^H &= \frac{1-(b-\theta^b)\bar{n}^L}{1-(b-\theta^b)\bar{n}^H} \times \frac{w_y^H}{w_y^L} z^L \\
&= 1.341550916 z^L
\end{aligned}$$

Thus, $z^H = 1.3416z^L$, or put it differently, the high types are 34.16% more productive than the low types.

Functional form of the experience accumulation function

The function form of the experience accumulation function consists of the life cycle component and the family wage gap component. The be able to compute the parameters I need to solve a system of two equations and two unknowns.

Life cycle component - λ_1

The life cycle component is such that the increase in the low type's wage $(w_m^L/w_y^L)-1$ is 74.6%. Starting from the labor income ratio of low types, the type-specific productivity z^L cancels out because it does not change over time. The base wage rate ω is the same for both as well. In the denominator I use the fact that individual labor supply is at the maximum, that is one.

The level of experience in the middle adulthood x_m^L is the only variable left, for which I plug in the functional form of the experience accumulation function (and $\lambda_0 = 1$). Next, I use the time budget to eliminate labor supply and finally, I assume the type-specific fertility rate to be the observed one. I am left with an equation relating λ_1 to λ_2:

$$\begin{aligned}
\frac{w_m^L}{w_y^L} &= \frac{x_m^L z^L \ell_m^L \omega}{x_y^L z^L \ell_y^L \omega} \\
&= \frac{x_m^L \ell_m^L}{x_y^L \ell_y^L} \\
&= \frac{\lambda_0 + \lambda_1 + \lambda_2 \ell_y^L}{\lambda_0 \ell_y^L} \\
&= \frac{1 + \lambda_1 + \lambda_2 (1 - (b - \theta^b)\bar{n}^L)}{1 \times (1 - (b - \theta^b)\bar{n}^L)} \\
\iff \lambda_1 &= (\frac{w_m^L}{w_y^L} \times (1 - (b - \theta^b)\bar{n}^L) - 1) - (1 - (b - \theta^b)\bar{n}^L) \times \lambda_2 \\
&= 0.398 - 0.801\lambda_2.
\end{aligned}$$

Family wage gap component - λ_2

The family wage gap component is chosen to match the labor income ratio between a middle-aged employee with no children and an employee with one child. Recall that one child in reality refers to $n = 0.5$ in the model as one household only consists of one adult. According to Beblo et al. (2009) the family wage gap in Germany is 24%. The labor income ratio boils down to a ratio of the two experience levels x_m since this is the only variable affected by the fertility choice. An adult without children only works during young adulthood, that is $\ell_y = 1$, while the parent with one child also has to partially stay at home. By substituting in the time budget and plugging in $n = 0.5$ and $\lambda_0 = 1$, I am left with an equation with λ_1, λ_2 and the family wage gap. I also plug in the latter so that I can re-arrange the equation such that λ_2 is a functional expression of λ_1:

$$\begin{aligned}
\frac{w_m^{L,n=0}}{w_m^{L,n=1/2}} &= \frac{x_m^L z^L \ell_m^L \omega}{x_m^L z^L \ell_m^L \omega} \\
&= \frac{x_m^L(n=0)}{x_m^L(n=0.5)} \\
&= \frac{\lambda_0 + \lambda_1 + \lambda_2}{\lambda_0 + \lambda_1 + \lambda_2 \ell_y^L} \\
&= \frac{1 + \lambda_1 + \lambda_2}{1 + \lambda_1 + \lambda_2(1 - \frac{1}{2}(b - \theta^b))} \\
\iff \lambda_2 &= \frac{1 - \frac{w_m^{L,n=0}}{w_m^{L,n=1/2}}}{\frac{w_m^{L,n=0}}{w_m^{L,n=1/2}}(1 - \frac{1}{2}(b - \theta^b)) - 1} + \frac{1 - \frac{w_m^{L,n=0}}{w_m^{L,n=1/2}}}{\frac{w_m^{L,n=0}}{w_m^{L,n=1/2}}(1 - \frac{1}{2}(b - \theta^b)) - 1} \times \lambda_1 \\
&= -2.783 - 2.783\lambda_1.
\end{aligned}$$

System of Equation

After setting $\lambda_0 = 1$ I have two parameters λ_1 and λ_2 and two equations that can be solved (that is done numerically by Matlab):

$$\begin{aligned}
\lambda_1 &= 0.398 - 0.801\lambda_2 \\
\lambda_2 &= -2.783 - 2.783\lambda_1.
\end{aligned}$$

This leads to $\lambda_1 = -2.1384$ and $\lambda_2 = 3.1681$.

Fertility differential

For the computation of the type-fertility differential I use data from Destatis (2012a), p.33. To be able to calculate type-specific fertility rates I assume all mothers saying that they have three or more children to have exactly three children. Because of this assumption I compute a lower bound of the fertility rate. However, the occurrence of more than three children is very rare in Germany. Furthermore, this assumption only has an impact on the fertility differential when the distribution of households with more than three children is very unequal. This information, however, is not provided in the data. I compute the following type-specific fertility rates: A woman of high type gets 1.35 children and her low income type counterpart 1.77. This leads to an average of 1.52 children per woman which is significantly higher than the total fertility rate of 1.38 that is reported for 2012. The 1.52 children per woman are based on data on completed fertility of women between the age of 40 and 49. The 1.38 children per woman, however, is based on the total fertility rate. Thus, the usage of different fertility concepts explains the observed difference. As a higher level of fertility potentially also leads to a higher fertility differential I do the following: I normalize the average completed fertility rate of 1.52 to the level of the total fertility rate,1.38. The compression factor is 0.91. In a next step, I use this compression factor on the type-specific fertility rates which results in 1.23 children for a high type woman and 1.61 for a low type woman. The resulting fertility differential is 0.38.

Child Rearing Costs

Table 14 on p. 46 of Destatis (2014c) reports the total consumption expenditures of a household with two children in the richest decile to be 4,832 EUR per month. This amount is split in expenditures for the children (1,587 EUR) and the residual is spent on the parents themselves (3,245 EUR). Thus, the ratio a^H/c_y^H is $1,587/3,245 = 0.489$. The total net household income is 9,007 EUR per month. Then the child cost-to-income ratio is $1,587/9,007 = 0.176$. The equivalent values for a household in the poorest decile with two children are: 1,859 EUR total consumption expenditures, 624 EUR for the children and 1,235 EUR for the parents. The child cost-to-parental consumption ratio is 0.505 and thus, very similar to the high type. Net labor income of such a household is 1,899 EUR and the child cost-to-income ratio is 0.329.

Note that according to the data none of the two household types consumes more than it earns. However, this statistics includes only consumption and excludes for examples investments in housing.

2.A.5 Benchmark Steady State

These tables present further results of the calibrated benchmark steady state.

Table 2.17: Benchmark Steady State - Choices

		High Type	Low Type
lifetime utility	U	-6.07	-6.30
fertility rate	n^i	0.558	0.748
labor supply	ℓ_y^i	0.862	0.815
experience	x_m^i	1.59	1.44
consumption	c_y^i	1.20	0.93
	c_m^i	2.13	1.65
	c_o^i	3.78	2.93
savings	s_y^i	-0.02	0.02
	s_m^i	0.401	0.323

Table 2.18: Benchmark Steady State - Income

		High Type	Low Type
effective wage rate	ω_y^i	2.78	2.12
	ω_m^i	4.42	3.05
net labor income	$j = y$	1.45	1.04
	$j = m$	2.67	1.84
total (net) income	$j = y$	1.47	1.04
	$j = m$	2.53	1.97
retirement wealth	$j = o$	3.78	2.93
- public share		30.1%	27.24%
- private share		69.9%	72.7%

Table 2.19: Benchmark Steady State - Prices and Aggregates

interest rate (p.a.)	8.99%
wage rate rate	1.05
nanny wage rate	2.12
GDP per capita	1.95

2.A.6 Further Results

The section of the appendix presents further results of the family policy reforms. Tables 2.20 to 2.23 refer to the *Kindergeld Reform*. Tables 2.24 to 2.27 refer to the *Kita Reform*.

Table 2.20: *Kindergeld Reform* - Government Budget (per capita)

	benchmark	post-reform
tax revenues	0.53	0.53
pension system	0.36	0.36
monetary child benefits	0.01	0.02
public child care	0.00	0.00
residual expenditures	0.15	0.15

Table 2.21: *Kindergeld Reform* - Choices High Type

		before reform	after reform	Δ (in %)
lifetime utility		-6.07	-6.07	0.0
fertility rate		0.558	0.560	0.4
labor supply	$j = y$	0.862	0.861	-0.1
	$j = m$	1.0	1.0	0.0
experience	$j = y$	1.0	1.0	0.0
	$j = m$	1.592	1.589	-0.1
consumption	$j = y$	1.201	1.202	0.0
	$j = m$	2.132	2.131	0.0
	$j = o$	3.783	3.778	0.0
savings	$j = y$	-0.021	-0.021	0.0
	$j = m$	0.401	0.401	0.0

Table 2.22: *Kindergeld Reform* - Choices Low Type

		before reform	after reform	Δ (in %)
lifetime utility		-6.30	-6.30	0.0
fertility rate		0.748	0.754	0.8
labor supply	$j = y$	0.815	0.813	-0.2
	$j = m$	1.0	1.0	0.0
experience	$j = y$	1.0	1.0	0.0
	$j = m$	1.442	1.438	-0.3
consumption	$j = y$	0.931	0.931	0.0
	$j = m$	1.651	1.650	-0.1
	$j = o$	2.930	2.927	-0.1
savings	$j = y$	0.20	0.21	3.2
	$j = m$	0.32	0.32	0.0

Table 2.23: *Kindergeld Reform* - Aggregate Statistics

	before reform	after reform	Δ (in %)
average fertility rate	0.689	0.694	0.7
share of high types	30.99%	30.96 %	0.0
share of low types	69.01%	69.04 %	0.0
old-age dependency ratio	0.86	0.85	-0.9
total dependency ratio	1.14	1.14	-0.4
GDP per capita	1.95	1.95	0.0
interest rate (p.a.)	8.99%	8.98	0.0
wage rate rate	1.05	1.05	0.0
nanny wage rate	2.12	2.12	0.0
size of nanny sector	0.4%	0.4%	0.0

Table 2.24: *Kita-Reform* - Government Budget (per capita)

	benchmark	post-reform
tax revenues	0.53	0.54
pension system	0.36	0.32
monetary child benefits	0.01	0.02
public child care	0.00	0.01
residual expenditures	0.15	0.20

Table 2.25: *Kita-Reform* - Choices High Type

		benchmark	post-reform	Δ (in %)
lifetime utility		-6.07	-6.05	0.4
fertility rate		0.56	0.60	7.7
labor supply	$j = y$	0.86	0.88	2.0
	$j = m$	1.0	1.0	0.0
experience	$j = y$	1.0	1.0	0.0
	$j = m$	1.59	1.65	3.4
consumption	$j = y$	1.20	1.20	-1.0
	$j = m$	2.132	2.156	1.1
	$j = o$	3.783	3.912	3.4
savings	$j = y$	-0.021		-10.8
	$j = m$	0.401	0.405	0.9

Table 2.26: *Kita-Reform* - Choices Low Type

		benchmark	post-reform	Δ (in %)
lifetime utility		-6.30	-6.26	0.7
fertility rate		0.75	0.85	13.6
labor supply	$j = y$	0.82	0.83	1.8
	$j = m$	1.0	1.0	0.0
experience	$j = y$	1.0	1.0	0.0
	$j = m$	1.44	1.49	3.1
consumption	$j = y$	0.93	0.92	-1.0
	$j = m$	1.65	1.67	1.2
	$j = o$	2.93	3.03	3.4
savings	$j = y$	0.20	0.19	- 7.0
	$j = m$	0.32	0.33	0.9

Table 2.27: *Kita-Reform* - Aggregate Statistics

	benchmark	post-reform	Δ (in %)
average fertility rate	0.69	0.77	+12.3
share of high types	31.0%	30.5%	-1.3
share of low types	69.0%	69.5 %	+0.7
old-age dependency ratio	0.86	0.73	-15.2
total dependency ratio	1.14	1.06	-6.5
GDP per capita	1.95	2.0	+2.9
interest rate (p.a.)	9.0%	9.2%	+2.2
wage rate rate	1.05	1.03	-1.6
nanny wage rate	2.12	2.08	-1.6
size of nanny sector	0.4%	0.8%	+113.5

2.A.7 Robustness Checks

Here, further results for the sensitivity analysis are presented.

Table 2.28: Robustness Check - Calibrated Parameters

Parameter		benchmark	π	Q	30% θ^b	40% θ^b
fertility weight	η	0.36	0.37	0.35	0.29	0.26
CRRA fertility	ψ	1.10	1.16	1.12	1.27	1.54
cost of children high type	a^H	0.59	0.58	0.59	0.61	0.59
cost of children low type	a^L	0.24	0.23	0.23	0.24	0.23
productivity low type	z^L	2.02	2.05	2.01	2.06	1.87
monetary child benefit	θ^a	0.11	0.11	0.11	0.12	0.11
wasteful governmental expenditure	G	1.81	1.76	1.64	1.90	1.77

Chapter 3

Comparative Analysis of Monetary Child Benefits and Public Child Care

3.1 Introduction

In a long-run general equilibrium, family policies not only increase the number of births but also open up additional financial resources for the government, as shown in Chapter 2. Moreover, these findings hold true for both types of analyzed family policies: monetary child benefits and public child care. These two German family policy reforms - the *Kindergeld-Reform* and *Kita-Reform* - were described and analyzed separately but no comparison of their effects and mechanisms was made so far. Such a comparison and analysis is the object of this chapter.

Since the size of the two reforms in Chapter 2 are very different it does not make sense to compare their quantitative impact on fertility, welfare or other household choices. Nevertheless, such a comparison could address and answer a number of questions such as: What are the overall welfare effects of family policies? Which family policy is welfare superior to the other? Are the welfare effects of the reforms different across different income types? Or else, what are the underlying mechanisms leading to these results?

Monetary child benefits are a subsidy for the direct costs of child rearing. They therefore directly affect the budget of parents. Public child care provides parents with more time so that the opportunity costs of children (indirect costs) are reduced. Hence, the price effects of both policies work through different components of child rearing costs. Moreover, public child care allows parents to increase their working hours and consequently their labor income. Thus, public child care has an income effect which is absent in the monetary child benefits scenario. Moreover, different fertility and labor supply decisions on the household level affect aggregate labor supply and can move factor prices. While it is obvious that the labor supply effect of higher monetary benefits must be negative, this effect is not clear *ex ante* for expanded public child care. A higher fertility rate decreases labor supply whereas higher public child care coverage increases it. Thus, only quantitative results can show which effect is dominant and what is the overall effect of public child care and how this compares to monetary benefits.

After implementing a family policy reform the government can choose where to use the additional financial resources. Consequently, the question arises if it matters whether the additional financial resources are used for cutting taxes or for raising benefits and if so, how. Expanded child bearing subsidies lead to more financial resources and the government can decide what to do with them. Given the model framework, there are three options:[1]

First, the government can use the money to disburden all employees by reducing labor income taxes. Retirees also benefit from this policy as their pensions depend on net wages. Second, the additional financial resources can serve to increase pension benefits. With this second policy only retirees benefit from the financial surplus. However, given that the family policies themselves support families and therefore employees, such a policy might be of interest.[2] Finally, the government can put the additional money in sectors like infrastructure or military expenditures as it has been done in Chapter 2. Given the model setup, such a policy does not affect households' utility and choices directly. Theoretically, there is a fourth option for the government: The budgetary surplus could be re-invested into family policies. However, given that one of the aims of this analysis is to see how the outcomes differ across different governmental policies, this alternative is not analyzed.

Following this structure in the analysis, I distinguish six different scenarios along the two dimensions - family policy reforms and usage of the financial surplus:

1. Family policy reforms
    (a) Reform of monetary child benefits (θ^a-reform)
    (b) Reform of public child care (θ^b-reform)
2. Usages of budgetary surplus
    (a) Increase of residual expenditures G (case 1)

[1] The three options are defined by the government budget described in Section 2.3.2 in Chapter 2. The government can use its revenues to finance either family policies, the pension system or residual expenditures such as military defense.

[2] Since I look at long-term equilibria, there is an indirect positive effect on employees: They know that they will receive higher public pension benefits during retirement and as a consequence they need to save less privately for their retirement.

(b) Cut of labor income tax τ (case 2)

(c) Increase of pension replacement rate q (case 3)

The starting point for the welfare analysis is the initial steady state of Chapter 2 that is calibrated to German data and represents roughly the year 2010. Yet, I cannot use the actual *Kindergeld Reform* of 2015 and *Kita Reformen* of the years 2007 to 2013 for the comparative analysis as both reforms were substantially different in terms of their size: The *Kindergeld* was increased by only 3.3% while public child care coverage got up by 75% from a coverage rate of 20 to 35%.

To get two comparable reforms I simulate a counterfactual reform of monetary benefits that lead to the same outcome as the public child care expansion of 75%. The outcome is measured by the change in fertility. I define the two reforms to be equivalent when the resulting average fertility rates are the same for both reforms. Monetary child benefits need to be increased from 184 EUR to 280 EUR per child and month in order to get exactly the same average fertility rate as the public child care expansion. This is a plus of 53.3%. The resulting fertility rate is 1.55 children per parent. Since I am primarily interested in the effect of family policies on the fertility behavior of households, this measure is the most appropriate. Leading to the same new fertility rate, the two different policies and their welfare effects can be compared.

The underlying assumption when determining the size of the θ^a-reform is that the government uses the financial surplus for residual expenditures (case 1). This must be the case as the results of the public child care reform also build on this assumption. Before computing cases 2 and 3 for the two family policy reforms, I take a step in-between and show a back-of-the-envelope calculation. This provides a first idea of the potentials of tax cuts and pension increases. For the cutting-tax scenario the calculation proceeds in the following way: After the reform, the residual expenditures are fixed to the pre-reform level and the surplus is added to the tax revenues. In a next step, I compute the hypothetical tax rate that would be necessary to create the same tax revenues as before the redistribution, taking into account the financial surplus from the family policy reform. The same is done for the case of increasing the replacement rate. With these numbers in mind, I compute the post-reform steady states for cases 2 and 3 for both family policy reforms. For a better understanding of the effects, I decompose the general equilibrium effects.

Generally, the public child care reform creates a larger additional financial surplus for the government compared to the monetary benefits reform. This result is not surprising since the higher child care coverage has a positive effect on labor supply and therefore increases the revenues the government collects with the labor income tax. This result holds true for all scenarios. The welfare results that are presented in Table 3.1 are far more mixed.

Table 3.1: Overview Welfare Effects

		case 1 residual expenditures	case 2 tax cut	case 3 pension increase
θ^a-reform	high type	+ 2.6%	+ 12.1%	+ 1.5%
	low type	+ 3.4%	+ 11.9%	+ 2.4 %
θ^b-reform	high type	+ 1.9%	+ 15.4%	+ 0.4%
	low type	+ 3.0%	+ 15.3%	+ 1.8%

First, both household types are better off after the reform in all scenarios. Second, welfare gains for both household types are the greatest when the government cuts the taxes. This is driven by a positive income effect of lower taxes. Furthermore, households are better off when the government uses the surplus to finance residual expenditures instead of increasing the replacement rate for pensions. This seems surprising but can be explained by a shift in factor prices. Higher pension benefits discourage people to save privately for their retirement which consequently depresses the capital-to-labor ratio and the wage rate. This creates a negative income effect which is not present when the government uses the money for residual expenditures. Third, in the cases 1 and 3 (residual expenditures and pension increase) the welfare gains are higher in case of higher monetary benefits. This can be explained by the general equilibrium effects of the public child care reform. Because of the positive effect on labor supply and consequently on aggregate labor supply higher public child care coverage leads to a lower wage rate and labor income, respectively. This negative income effect drives the welfare result. Finally, in cases 1 and 3 the welfare gain is higher for the low income households while the gain is higher for high income household in case 2. This chapter is structured as follows: Section 3.2 defines and analyzes the benchmark scenario (case 1). It also provides the back-of-the-envelope calculation for the budgetary surplus. In Section 3.3 I compute and compare the effects of the two family policies when the financial surplus is used either for tax cuts (case 2) or for a pension reform (case 3). This section also includes the decomposition of the general equilibrium effects. Finally, I summarize and conclude in Section 3.4.

3.2 Monetary Child Benefits versus Public Child Care without further Government Action

3.2.1 Case 1: Effects of Family Policies when Financing Residual Expenditures

In this section I compare the effectiveness of raising monetary child benefits (θ^a-reform) with expanding public child care coverage (θ^b-reform). To be able to do so, I need to define a θ^a-reform and a θ^b-reform so that I can compare both reforms in terms of magnitude as well. To get two comparable or equivalent reforms, some steps need to be done.

First, I define two different family policy reforms to be equivalent when the resulting average fertility rate is the same for both reforms. As the main objective of the analysis is to find out about the underlying mechanisms driving the results this definition is well suited for this purpose. With this measure the fertility effect of the reforms is normalized and the analysis can be focused on the mechanisms leading to this fertility rate and the effect on the governmental budget. As a starting point I use the initial steady state presented in Chapter 2 that is calibrated to represent Germany in 2010.

Next, I assume a θ^b-reform of 75%, meaning the child care coverage rate is increased from the initial 20 to 35%.[3] This is the same child care reform as assumed in the previous chapter. The resulting fertility rate is 1.55 children per parent. Finally, I search for the monetary benefits reform that also results in an average fertility rate of 1.55. This goal is reached when monetary benefits are increased by 53.3% from 184 EUR to 280 EUR per child and month.[4] The underlying assumption concerning the government is that the financial surplus that is created by the family policies is put into residual expenditures.

Tables 3.2 and 3.3 describe the different household reactions to both reforms. The fertility reaction of low income households is much stronger and is the main driver of overall fertility: The fertility rate of low income types increases by 13.3% while it is 7.1% for the high income types. Overall the increase is 12.3%. As the numbers are the same for both reforms, the type-specific fertility reaction is the same across the reforms. Thus, different family policy reforms do not lead to different type-specific fertility reactions.[5]

[3]This corresponds to a change of parameter θ^b from 0.062 to 0.1085.

[4]This corresponds to a change of parameter θ^a from 0.113 to 0.173.

[5]The effect of higher monetary benefits on low income households is slightly stronger than the one of public child care. However, the difference is very small and the resulting share of high types in the economy with the monetary benefits reform is 30.4% while it is 30.5% for the public child care reform as can be seen in Table 3.4.

Equivalently to the analysis in Chapter 2, labor supply decreases with higher monetary benefits and rises with expanded child care. This leads to different experience accumulations: It results in lower productivity and effective wage rates with higher child benefits and higher effective wages with expanded child care. In case of the θ^a-reform, the consumption pattern becomes flatter with higher consumption during young adulthood and less consumption in the later periods. This pattern is reversed for the θ^b-reform with better child care infrastructure. Overall, for the reform of monetary child benefits the capital-to-labor-ratio increases which leads to a lower interest rate and higher base wage rate whereas it is the other way round for the child care reform. Finally, output per capita also decreases with higher benefits and increases with higher child care coverage.

Expressed in consumption equivalence units, higher monetary child benefits increase aggregate welfare by 3.1% while the expansion of public child care leads to a rise in aggregate welfare of 2.7%.[6] In both cases, low income households benefit more from the reform than high income households. Their welfare increases by 3.4% (θ^a-reform) and 3.0% (θ^b-reform) while the corresponding numbers are 2.6% and 1.9% for high income households. The stronger fertility reaction of low income households is the reason for their higher welfare gains compared to high income households. The θ^a-reform is slightly welfare superior because of the labor supply effect of child care. There are two effects of higher labor supply on the effective wage rate: On the one hand, aggregate labor supply goes up and consequently the base wage rate decreases. On the other hand, higher labor supply increases the experience which consequently leads to higher productivity. As the first effect dominates the second, there is a negative income effect that explains the lower welfare change. This is particularly true for young low type workers as can be seen in the second-last row of Table 3.4.[7] Given the welfare results, the government should favor an increase of monetary child benefits.

Table 3.2: Case 1 - Choices High Type

	pre-reform	θ^a-reform	θ^b-reform
fertility rate	0.56	0.60	0.60
labor supply	0.86	0.85	0.88
consumption			
$j = y$	1.20	1.21	1.19
$j = m$	2.13	2.12	2.15
$j = o$	3.78	3.71	3.91
welfare change	—	0.026	0.019

[6] Aggregate welfare is measured as the the average of the two types weighted by the type fraction size.
[7] Note that the wage rate of young low type workers is the same one as nannies.

Table 3.3: Case 1 - Choices Low Type

	pre-reform	θ^a-reform	θ^b-reform
fertility rate	0.75	0.85	0.85
labor supply	0.82	0.79	0.83
consumption			
$j = y$	0.93	0.93	0.92
$j = m$	1.65	1.64	1.67
$j = o$	2.93	2.87	3.03
welfare change	—	0.034	0.030

And what happens to the governmental budget? With the reform of monetary child benefits, aggregate residual government expenditures increases by 40% and the increase per capita is 23.8%. The numbers for the child care expansion are 52.7% and 34.1%. Thus, even though the two different family policies lead to the same fertility rate, they have significantly different effects on the government budget. Expanded child care opens up 40% more financial resources compared to monetary child benefits. This result is driven by the positive labor supply effect that comes along with expanded public child care: Since individual labor supply is higher, labor income is accordingly higher and the tax revenues as well. Therefore, from a budgetary point of view, the government should clearly opt for expanded public child care and not for more monetary child benefits.

Table 3.4: Case 1 - Aggregate Statistics

	pre-reform	θ^a-reform	θ^b-reform
average fertility rate	0.69	0.77	0.77
share of high types	31.0%	30.4 %	30.5%
share of low types	69.0%	69.6 %	69.5 %
GDP per capita	1.95	1.93	2.01
interest rate (p.a.)	8.99%	8.87	9.20%
wage rate	1.05	1.06	1.03
nanny wage rate	2.12	2.14	2.08
size of nanny sector	0.4%	0.5%	0.8%

3.2.2 Back-of-the-Envelope Calculation

So far, the financial surplus created by either higher monetary child benefits or expanded public child care was only used by the government for residual expenditures. Consequently, the budgetary effect leads to a new higher level of residual government expenditure that does not enter the optimization procedure of the households. Before I turn to the alternative cases, in which the government can use the financial surplus either to cut taxes (case 2) or to increase the replacement for the pension system (case 3), I provide a back-of-the-envelope calculation to answer the following question: By how much can the government reduce the labor income tax rate when the financial surplus is not used for residual expenditures but added to the tax revenues?

In case of the θ^a-reform, the pre-reform steady state value of residual expenditures is 1.8 and after the reforms it is 2.5 as can be seen in Table 3.5. Thus, the budgetary surplus created by the higher monetary child benefits is 0.7. Next, I subtract this surplus from the residual expenditures such that they are back at their pre-reform level and shift the surplus to the tax revenues which amount to 7.4 after the reform. Now, the tax revenues only need to be 6.7 to balance the government budget. Holding everything else constant, I calculate which tax rate is required such that the government collects tax revenues of 6.7. The resulting tax rate gives a rough idea of the potential effect when the government uses the family policies to cut taxes. Obviously, this calculation ignores the fact that households behave differently when they face a different tax rate. This is explicitly taken into account in Section 3.3, assuming that the government announces *ex ante* what it will do with the surplus.

Table 3.5: Back-of-the-Envelope-Calculation - Monetary Child Benefits (θ^a)-Reform

	case 1	back-of-the-envelope calculations	
	residual expenditures	tax cut	pension increase
tax revenues	7.4	6.7	7.4
pension system	4.4	4.4	5.1
family policies	0.5	0.5	0.5
residual expenditures	2.5	1.8	1.8
tax rate	0.396	0.357	0.396
replacement rate	0.553	0.553	0.645

Table 3.6: Back-of-the-Envelope-Calculation - Public Child Care (θ^b)-Reform

	case 1	back-of-the-envelope calculations	
	residual expenditures	tax cut	pension increase
tax revenues	7.7	6.8	7.7
pension system	4.6	4.6	5.5
family policies	0.4	0.4	0.4
residual expenditures	2.7	1.8	1.8
tax rate	0.396	0.347	0.396
replacement rate	0.553	0.553	0.669

The same calculation logic applies to the case of increasing the replacement rate: The surplus is added to the pension component in the budget and the new replacement rate is calculated, holding everything else constant. The same is done for the public child care reform. The results can be seen in Table 3.6.

With the monetary benefits reform, the government is able to cut taxes by 10% from 39.6% to 35.7%. With the public child care reform, labor taxes can be decreased to 34.7%.

The second possibility is to increase the replacement rate for public pensions. While it was 55.3% in the benchmark case, the replacement rate can be increased to 64.5% with the monetary child benefits reform and even to 66.9% with the public child care reform. Since the child care reform creates a higher surplus of 0.9 versus 0.7 for the monetary benefits reform, the potential tax cut or pension increase must be larger.

Based on these calculations, I conjecture that the welfare gains are much higher compared to case 1 discussed in this section. Both policy adjustments should have a strong and positive income effect on households that was not present in the case of financing more residual expenditures and this effect should persist also in the new long-run equilibria.

3.3 Monetary Child Benefits versus Public Child Care with further Government Action

3.3.1 Case 2: Effects of Family Policies when Cutting Taxes

In this scenario, the government announces that the additional financial resources from intensified family policies will be given back to the households in form of lower taxes. The results for the two household types can be found in column 2 of Tables 3.7 and 3.8.[8]

In the case of the 53%-increase of monetary benefits the government is capable to cut taxes down from 39.6 to 35.5%, see column 3 of Table 3.9. The cut is stronger than what was calculated in the back-of-the-envelope calculation. There, the tax rate could be reduced to 35.7% as can be seen in the same table in column 2.

The overall fertility reaction is also stronger. The average fertility rate is 1.60 compared to 1.55 when financial resources are not used to reduce taxes. As lower taxes increase net labor income they also lead to higher foregone earnings of child bearing which decreases the incentives of getting more children. However, the this effect is dominated by the income effect so that the fertility reaction is positive. The strong positive income effect can also be seen by looking at the consumption. For all periods, the consumption level is higher compared to the scenario when financial surplus is put into residual expenditures. Taking the two positive effects on fertility and consumption together, aggregate welfare increases by 11.9%. The large difference to the 3.1%

[8]Tables 3.7 and 3.8 show the household choices for all six scenarios. The first three columns present the results for the θ^a-reform and columns 4 to 6 the ones for the θ^b-reform. In columns 1 and 4 the results of case 1, when the surplus goes into the residual expenditures, is shown. The results of case 2, the tax cut scenario, are shown in columns 2 and 5. Finally, the results for case 3, the pension increase, can be found in columns 3 and 6.

welfare increase in case 1 is the result of the re-optimizing households. As already mentioned, net labor income is significantly higher which pushes consumption and fertility and therefore utility. The utility increase for low income households is the same as the average while the increase for high income households is even higher with 12.1%. Thus, using increased monetary child benefits to cut taxes is relatively more beneficial for high income households than for low income households. This is not surprising as the model exhibits a flat tax rate and high income households are therefore left with more after-tax income.

The public child care reform exhibits similar results and mechanisms. The results for the two household types are shown in column 5 of Tables 3.7 and 3.8 and for the government in column 3 of Table 3.10.

With expanded public child care the government can reduce the tax rate to 34.0%. Similarly to the reform of monetary child benefits, the tax cut is stronger compared to the back-of-the-envelope calculation where the potential tax rate was 34.7%. Moreover, the scope for a tax reduction is larger compared to the monetary child benefits reform. The explanation is based on a recurring mechanism: Better child care coverage has a positive effect on labor supply that in turn expands the tax base for the government. The government is not the only one benefiting from a very strong income effect. The combination of more public chid care and lower taxes leads to a significantly higher net labor income for all households. This income effect increases the average fertility rate to 1.66 where the type-specific fertility rates are 1.82 (low income types) and 1.32 (high income types). So, the two family policies lead to different fertility rates. As one can see in the first row of Tables 3.7 and 3.8 the combination of public child care and tax cut leads to the strongest fertility reaction across all scenarios. The consumption pattern experiences a strong upward level shift and as a result average welfare increases by 15.3%. Equivalently to the monetary benefits reform, the low income households' welfare change corresponds to the average value while it is slightly higher for high income households (+15.4%).

3.3.2 Case 3: Effects of Family Policies when Increasing Pensions

Using the financial surplus to enlarge the pension system is the third option available to the government. The latter can increase the pensions by increasing the replacement rate. I use the same approach as before and discuss the monetary child benefits reform before dealing with the public child care expansion. The corresponding numbers are presented in column 3 of Tables 3.7 and 3.8 and for the government in column 5 of Table 3.9.

The increase of child benefits allows the government to increase the replacement rate from 55.3% to 63.4%. In contrast to the tax cut scenario, the general equilibrium outcome is not

Table 3.7: Overview - Choices High Type

	θ^a-reform			θ^b-reform		
	case 1	case 2	case 3	case 1	case 2	case 3
fertility rate	0.60	0.64	0.59	0.60	0.66	0.59
labor supply	0.85	0.84	0.85	0.88	0.87	0.88
consumption						
$j = y$	1.21	1.30	1.20	1.19	1.31	1.17
$j = m$	2.12	2.24	2.13	2.16	2.32	2.17
$j = o$	3.72	3.84	3.81	3.91	4.10	4.02
savings						
$j = y$	-0.02	-0.02	-0.02	-0.02	-0.03	-0.02
$j = m$	0.40	0.42	0.38	0.40	0.43	0.38
welfare change	0.026	0.121	0.015	0.019	0.154	0.004

Table 3.8: Overview - Choices Low Type

	θ^a-reform			θ^b-reform		
	case 1	case 2	case 3	case 1	case 2	case 3
fertility rate	0.85	0.88	0.84	0.85	0.91	0.84
labor supply	0.79	0.78	0.79	0.83	0.82	0.83
consumption						
$j = y$	0.93	1.00	0.92	0.92	1.02	0.91
$j = m$	1.64	1.72	1.64	1.67	1.80	1.68
$j = o$	2.87	2.96	2.94	3.03	3.17	3.11
savings						
$j = y$	0.03	0.03	0.03	0.02	0.02	0.02
$j = m$	0.32	0.34	0.31	0.33	0.35	0.31
welfare change	0.034	0.119	0.024	0.030	0.153	0.018

stronger than the back-of-the-envelope-calculation which indicated a new higher replacement rate of 64.5%. The fertility rate is 1.53 children per parent. This is lower compared to the tax cut scenario but also lower than the fertility rate of case 1 in which residual expenditures were financed.

Because of higher pensions, consumption during old adulthood is higher in this case compared to case 1. Interestingly, old-age consumption is however lower compared to the tax cut scenario (case 2). So, the expansion of the public pension system leads to less retirement wealth for retirees compared to a reduction of the labor income tax.[9] There are two reasons for this result: First, a higher replacement rate implies that public pension benefits become more important for retirees, both, absolutely but also relatively to their private old-age savings. Moreover, the rate-of-return of the private savings is the interest rate which in this economy is

[9] In the last period, households consume their entire wealth. Thus, old-age consumption corresponds perfectly with retirement wealth.

relatively high. For the public pension system, the rate-of-return is basically the fertility rate which is much lower than the interest rate.[10] Thus, the pension reform pushes the households into a less efficient old-age security system while the tax cut scenario allows households to save more privately because after-tax income has increased. The second reason is that there is a general equilibrium effect leading to a negative income effect. As a result of the shift towards the public pension system, people save less and consequently the wage rate, and therefore labor income, decreases.

Putting together the relatively weak fertility effect and the less efficient composition of old-age insurance instruments, the overall welfare effect of the increase of the replacement rate is still positive but far below the tax cut scenario: Welfare of low (high) income households is 2.4% (1.5%) higher compared to the pre-reform situation. Since these negative effects are not present when the government finances residual expenditures, the welfare gains are larger in case 1. Hence, the conclusion is quite strong: Instead of increasing the replacement rate of a relatively inefficient pension system, it is welfare superior to put the money aside and not to do anything with it.

The results do not improve when the government introduces the public child care reform instead. The same mechanisms apply as for the monetary child benefits reform. In this case, the fertility rate is the lowest across all scenarios and the increase of public pension benefits makes the old-age insurance less efficient. However, there is one difference to the case of monetary benefits and that is the labor supply effect of public child care which has been already discussed. Higher child care coverage has a positive effect on the labor supply. Therefore, labor supply is higher than in the child benefits scenario. Savings are however the same as with the θ^a-reform. Therefore the capital-to-labor-ratio and consequently the wage rate are lower. As a result, aggregate welfare only increases by 1.4%. The positive number is mainly driven by low income households that face a welfare plus of 1.8% while high income households' welfare is only 0.4% higher compared to the pre-reform situation.

3.3.3 Decomposition of General Equilibrium Effects

To get a better idea about the results of the previous section, I decompose the general equilibrium effects to see and quantify which prices and parameters are the key drivers of the results. I do this for cases 2 and 3, thus, a tax cut financed by each type of family policy and the same for

[10] A fertility rate below the replacement level implies a negative rate of return of the public pension system. This mechanism will be discussed more extensively in Chapter 4.

Table 3.9: Government Budget - Monetary Benefits (θ^a)-Reform

	residual expenditures	tax cut		pension increase	
	case 1	*b-o-t-e-c*	case 2	*b-o-t-e-c*	case 3
tax revenues	7.4	6.7	7.0	7.4	7.2
pension system	4.4	4.4	4.7	5.1	5.0
family policies	0.5	0.5	0.5	0.5	0.4
residual expenditures	2.5	1.8	1.8	1.8	1.8
tax rate	0.396	0.357	0.355	0.396	0.396
replacement rate	0.553	0.553	0.553	0.645	0.634

Table 3.10: Government Budget - Public Child Care (θ^b)-Reform

	residual expenditures	tax cut		pension increase	
	case 1	*b-o-t-e-c*	case 2	*b-o-t-e-c*	case 3
tax revenues	7.7	6.8	7.3	7.7	7.5
pension system	4.6	4.6	5.0	5.5	5.3
family policies	0.4	0.4	0.5	0.4	0.4
residual expenditures	2.7	1.8	1.8	1.8	1.8
tax rate	0.396	0.347	0.340	0.396	0.396
replacement rate	0.553	0.553	0.553	0.669	0.652

the pension increase. The results of the effect decomposition are shown in the Tables 3.11, 3.12, 3.13 and 3.14.

The procedure is the following: I start in the pre-reform steady state (first column, labelled "old ss") which is the initially calibrated economy without any family policy reform. Then, I introduce the family policy (second column, either $\Delta\theta^a$ or $\Delta\theta^b$), while all other policy parameters and factor prices remain at the initial level. In a third step, I balance the governmental budget by adjusting either the tax or the replacement rate (third column, $\Delta\tau$ or Δq). In a fourth step, I add the new interest rate (fourth column, Δr). In the final step, I introduce the new wage rate. With all new policy parameters and prices this is also the new steady state (fifth column, Δw / "new ss"). Following these steps, I first discuss the reaction of the average fertility rate and then the one of welfare. The first row shows the average fertility rate given the corresponding prices and parameters.[11] The second row shows the change in fertility that is caused by "switching on" the corresponding price channel. In the same fashion, the third and fourth row show the changes in welfare.

For the tax cut scenarios, the introduction of intensified family policies explains roughly two thirds of the total increase of fertility. A higher child subsidy lowers the price of children relative to the consumption good. Therefore, in the case of lowering taxes the price effect is

[11] Here is a concrete example to be clear: The fertility rate in the third column - Δr - is the fertility rate with the new family policy, the new tax rate, the new interest rate but old wage rate.

responsible for two thirds of the fertility reaction. With expanded child care the last third is caused by the reduced tax rate and the resulting income effect.[12] With higher monetary benefits the last third of the fertility reaction is split equally between the income effect of the lower tax rate and the effects of lower interest and higher wage rate.[13]

The result is different when the government increases pension benefits. When this is done with higher monetary child benefits, fertility only reacts to the higher child subsidy. In general, the increase of pension benefits has a positive effect on income. However, since the effect is only realized in the last period it does not affect the income and therefore decisions during young adulthood. The same line of argument holds true for the scenario with expanded public child care but the fertility effect is even stronger. The partial effect of higher child care coverage on fertility is larger than the effect on fertility in general equilibrium. This is because the higher interest rate and lower wage rate cause a drop in fertility. The negative effect of factor prices is caused by the strong increase of aggregate labor supply. First, the higher fertility rate leads to a larger overall population and therefore also to a higher labor force. In addition, thanks to the better child care coverage the individual labor supply of each young adult is increased. This large supply of labor combined with reduced savings leads to lower wages and higher interest rate.[14]

Now, I turn to the effects on aggregate welfare. A similar pattern can be observed for welfare. The third row shows the total change in welfare dependent on factor price changes. The fourth row shows the change of welfare that is caused by "switching on" the price channel of the corresponding column. In general, higher family policies increase welfare. The same is true for lower taxes and higher replacement rates. Higher interest rates increase welfare whereas lower interest rates have a negative effect. The same logic holds true for wages.

For the tax cut scenarios the tax reduction affects welfare most strongly. The lower tax rate roughly counts for two thirds of the welfare gain. The last third comes from the higher child subsidy. The impact of factor prices is small. This has another implication: The welfare gains of family policies do not only come from the simple fact that higher child subsidies lead to more children which has a positive effect on parents' utility. It is actually more important to see what

[12]This is not entirely correct as a decrease in the tax rate also has a negative price effect because foregone earnings go up. Moreover, an expansion of public child care is not a pure price effect as this family policy also leads to higher labor supply and therefore with constant prices to an income effect.

[13]The higher wage rate leads as well to a positive income effect while the lower interest rate can be interpreted as a lower price of all expenditures - consumption and child rearing - during young adulthood relative to the later periods.

[14]Note that this effect is also qualitatively present with monetary benefits. However, the quantitative effect is smaller.

effect a family policy has on the households' income and what this means for the consumption pattern.

In the third case with increasing pension benefits, the welfare effect looks similar to the fertility reactions. For both family policies, the positive welfare effect of the family policy is larger than the overall welfare effect. The cause for the welfare dampening effect is once more the decreasing wage rate. Since this is a steady state comparison and pension benefits depend on former wage income, a lower equilibrium wage rate also depresses pension benefits and therefore reduces consumption possibilities of all cohorts. For public child care, this effect is so strong that in the end a strong increase of the retirement rate financed by an expansion of public child care has the smallest effect on welfare of all scenarios.

Table 3.11: Effect Decomposition - Monetary Benefits and Tax Cut

	old ss	$\Delta\theta^a$	$\Delta\tau$	Δr	Δw / new ss
fertility rate (average)	0.69	0.77	0.79	0.80	0.81
Δ fertility		+0.08	+0.02	+0.01	+0.01
welfare (CEV)		+0.023	+0.095	+0.091	+0.119
Δ welfare (Δ CEV)		+0.023	+0.072	-0.004	+0.028

Table 3.12: Effect Decomposition - Child Care and Tax Cut

	old ss	$\Delta\theta^b$	$\Delta\tau$	Δr	Δw / new ss
fertility rate (average)	0.69	0.79	0.83	0.83	0.83
Δ fertility		+0.10	+0.04	+0.00	+0.00
welfare (CEV)		+0.043	+0.149	+0.148	+0.153
Δ welfare (Δ CEV)		+0.043	+0.106	-0.001	+0.005

Table 3.13: Effect Decomposition - Monetary Benefits and Pension Increase

	old ss	$\Delta\theta^a$	Δq	Δr	Δw / new ss
fertility rate (average)	0.69	0.77	0.77	0.77	0.77
Δ fertility		+0.08	+0.00	-0.00	-0.00
welfare (CEV)		+0.023	+ 0.025	+ 0.026	+0.021
Δ welfare (Δ CEV)		+0.023	+0.002	+0.001	-0.005

Table 3.14: Effect Decomposition - Child Care and Pension Increase

	old ss	$\Delta\theta^b$	Δq	Δr	Δw / new ss
fertility rate (average)	0.69	0.79	0.79	0.78	0.77
Δ fertility		+0.10	+0.00	-0.01	-0.01
welfare (CEV)		+0.043	+0.046	+0.049	+0.014
Δ welfare (Δ CEV)		+0.043	+ 0.003	+0.003	-0.035

3.4 Conclusion

In this chapter I compared the effects of a reform of monetary child benefits and of public child care. Moreover, the government can choose between three options to use the financial surplus created by the family policy reform: It can first put the additional money into residual government expenditures. In this case, households' choices are not affected directly. Alternatively, the government can use the surplus to either reduce the labor tax rate or increase the replacement rate of the public pension system. In the two last cases, the policy parameters enter the optimization process of the household. In total, six different scenarios were analyzed.

The first observation is that in all scenarios, households are better off after the family policy reform, be it expanded child care or higher monetary child benefits. The improved situation for the households is independent from the way the government uses the financial surplus. Thus, the steady state in which Germany currently is, can be labelled as inefficient. Since fertility subsidies lead to welfare increases, the fertility rate in the initial benchmark economy must be distorted in some way.

A public child care expansion creates a larger additional financial surplus for the government than the monetary child benefits reform. This result remains the same across all scenarios. The result is driven by the higher individual labor supply that is created by higher child care coverage. On the aggregate level this increases the tax base of the government. Hence, supposing the government wants to maximize its revenues it should always go for more public child care compared to monetary child benefits.

Welfare gains are the highest for both household types in the tax cut scenario. The welfare gains are in a range of 12% to 15% measured in consumption equivalent units. In the case of the public child care reform, the actual child care improvement accounts for two third of the welfare gains while the income effect of the lower tax rate accounts for the other third. In the case of the monetary child benefits reform, the policy itself also accounts for two thirds. The last third can be explained by lower taxes and changes in factor prices with equal weight. Moreover, households are better off when the government uses the surplus to finance residual expenditures instead of increasing the replacement rate for pensions. This appears surprising but can be explained by a shift in factor prices and the inefficiency of the public pension system. Higher pension benefits discourage people to save privately for their retirement which consequently depresses the capital-to-labor ratio and the wage rate. This creates a negative income effect which is not present when the government does not promote its pension system and uses the money for residual expenditures instead. Welfare gains vary between 0.5% and 3%. In the case of higher pensions, the partial welfare effect of the two individual family policies is larger than the total welfare effect, demonstrating the importance of the general equilibrium effects. Thus, when the government does not want to maximize its revenues but rather the utility of the households, it should combine intensified family policies with a tax reduction. Furthermore, it should not force households to rely even more on public pension benefits as long as the pension system is that ineffective.

A monetary child benefits reform is welfare superior to a public child care expansion as long as the government uses the financial surplus for either residual expenditures or the pension reform. This is the other way round for the tax cut scenario. So when the government knows already *ex ante* how it will use the additional financial resources and it wants to implement a family policy reform, the welfare maximizing option is dependent on the government's choice concerning what to do with the surplus.

Finally, the reform scenarios also have different effects on the two household types. When the government invests in residual expenditures or increases pension benefits, welfare gains are larger for low income households than for high income households. It is the other way round in case of a tax reduction. This finding is independent of the type of family policy.

3.A Appendix

3.A.1 Additional Comparative Results

The following table presents aggregate statistics for the different reform scenarios (cases 2 and 3).

Table 3.15: Aggregate Statistics

	θ^a-reform		θ^b-reform	
	case 2	case 3	case 2	case 3
average fertility rate	0.81	0.77	0.83	0.77
share of high types	30.7%	30.4%	30.7%	30.5%
share of low types	69.3%	69.6%	69.3%	69.5%
GDP per capita	1.94	1.91	2.02	1.98
interest rate (p.a.)	8.66%	9.05%	8.94%	9.42%
wage rate rate	1.08	1.04	1.05	1.02
nanny wage rate	2.17	2.11	2.12	2.05
size of nanny sector	0.5%	0.5%	0.9%	0.8%

Chapter 4

Demographic Change - Family Policies as Alternative to Pension Reforms

4.1 Introduction

Demographic change in Germany has two components. First, fertility rates are below the replacement rate level which leads to fewer young people. Second, longevity increases constantly such that the old become both more numerous and older. According to Destatis (2009), in 2060 average life expectancy at birth will be 85 years for men and 89.2 for women. Compared to 2008 this means almost eight and seven additional years. At the age of 65 - and therefore at the beginning of retirement - men are expected to have 22.3 more years to live and women 25.5. On average, German retirees will live and receive pension benefits for more than five additional years.

Simultaneously, demographers assume the fertility rate to remain constant at 1.4 children per woman.[1] As a consequence the old-age dependency ratio is expected to increase significantly. This number will go up until 100 employees have to finance 62 retirees in the year 2040 or even 67 in 2060.[2] This change of the demographic structure will have a tremendous effect on old-age security in Germany. The German Federal government expects the size of the public pension system to increase by 60% from 270 billion EUR up to 430 billion EUR already by 2028. Longer projections do not exist. At the same time, the contribution rate is expected to increase from

[1] This is the main assumption of the *Statistische Bundesamt* underlying the population projection until the year 2060. Alternative scenarios assume a higher or lower fertility rate and a stronger increase of life expectancy. For more details, see Destatis (2009).

[2] See Destatis (2009), pp.19-21.

today's 18.7% to 21.4% while the replacement rate will drop from 50% to 44.4%. Moreover, the annual financial transfers from the Federal government budget to the budget of the public pension system is expected to rise from currently 60 billion EUR to 100 billion EUR per year.[3] Given the fact that the German public pension system should actually finance itself via the contribution rate, the conclusion of all these numbers can only be that demographic change is a huge financial burden for the German public pension system which cannot be solved by the public pension system alone.

This evolution is not new and there have already been many reform proposals to circumvent - or at least reduce - the financial burden for future generations. As the problem materializes in the public pay-as-you-go pension system the most obvious reform proposals apply to the pension system itself. Generally, there are three reforms possible: (1) contribution rates can be raised, (2) replacement rates can be lowered, and (3) the retirement age can be increased. Concerning the three alternatives, in a multi-country framework, Krueger and Ludwig (2007) find that raising the retirement age is the best reaction to demographic change, followed by fixing contribution rates (and reducing the replacement rate) and as last option fixing pension benefits (and increasing pension contributions). Kotlikoff et al. (2007) find in a simulation for the US that increasing the replacement rate induces lower benefits for young and lower costs for the old compared to a cut of pension benefits. According to an analysis by Ludwig and Reiter (2010), the German public pension system is close to a second-best solution. However, they do not derive any reform implications from that.[4] Berkel et al. (2004) analyze whether a higher fertility rate can circumvent the negative effects of population aging on the pay-as-you-go pension system. They find that a higher fertility rate does not solve the problem but higher human capital can be a solution. However, they assume fertility to be exogenous and not an endogenous choice. Therefore, they neglect important effects on households' decisions and general equilibrium prices.

Some of the proposals have already been implemented in Germany during the last years. The most prominent example is the introduction of the retirement age at 67 in 2007.[5] However, there are several voices claiming that the efforts so far have not been enough and the retirement age should be further increased, see for example Bundesbank (2009), Sachverständigenrat (2013)

[3]See Bundesarbeitsministerium (2014), pp.37-41. See also Holthausen et al. (2012) for a simulation of pension system adjustments that have to be implemented.

[4]There are several other publications dealing with demographic change and public pay-as-you-go pension systems like Auerbach and Kotlikoff (1987), DeNardi et al. (1999), Börsch-Supan et al. (2006) or Attanasio et al. (2007). However, none of them incorporates an endogenous fertility choice and family policies as governmental instruments.

[5]This reform is currently eroded by the retirement age of 63 years that was introduced in 2014. As long as the person has contributed to the pension system for 45 years, she can enter retirement at the age of 63 without any discount.

or Prognos (2014a). Raising contribution rates or decreasing replacement rates are no real reforms since there is some scope for policy makers such that these two parameters can be adjusted relatively easily. Following an adjustment rule the parameters are frequently determined to fit the current economic situation, see Bundesarbeitsministerium (2014). There also exist alternative reform ideas that do not apply directly to the pension system but more generally to the labor market. The idea is to increase labor force participation in order to improve the old-age-dependency ratio. The most prominent ways for the realization of this goal are more immigration, a higher female labor force participation or lower labor-market entry age.[6]

Having all those proposals in mind, I take a different viewpoint in this chapter. I propose to use family policies as an alternative to a reforms of the pension system. Based on the result that family policy reforms can generate a financial surplus for the government, I confront the effects of intensified family policies with pension reforms. I use the same model setup for this analysis as in Chapters 2 and 3.

The increase of life expectancy leads to an exogenous increase of pension expenditures for the government by roughly 30%. This financial gap has to be closed by using one of the four following policy instruments: (a) higher contribution rate, (b) lower replacement rate, (c) higher monetary child benefits or (d) expanded public child care. Which of the policy instruments is capable to re-balance the budget of the government is the first question to answer. In a second step, I evaluate which of the policies is the most beneficial from a welfare point of view. Moreover, I analyze how the different reforms affect the level and components of the retirement wealth and what role factor prices play.

Table 4.1: Main Results - Pension Reforms versus Family Policies

		$\Delta\tau$	Δq	$\Delta\theta^a$	$\Delta\theta^b$
high type	welfare change	-27.4%	+2.0%	+4.6%	+2.4%
	retirement wealth	3.37	3.63	3.67	3.95
	public pension share	26.4%	24.5%	30.6%	29.3%
low type	welfare change	-25.4%	+1.7%	+5.8%	+3.9%
	retirement wealth	2.61	2.81	2.81	3.06
	public pension share	24.0%	22.1%	26.4%	26.4%

I find that all four reforms are capable to finance the longevity shock. In case of a reform of the contribution rate, the joined tax rate needs to increase by 32.3% to 52.4%. This sharp increase of the tax rate has strong negative effects on fertility, household income and welfare. On average welfare declines by 27.4%.

[6] See for example Storesletten (2000) who quantifies the migration inflows needed to resolve financial problems created by population aging.

For the reform of the replacement rate, the German government has to lower the rate from 55.3 to 42.5%. The welfare effect of this policy is positive since it forces households to shift their old-age insurance from the relatively inefficient public pension system towards more private old-age savings. Interestingly, this reform establishes a negative relationship between fertility and the size of the pay-as-you-go pension system.

To compensate the financial longevity burden the *Kindergeld* increases to 342 EUR per month and child. With a welfare increase of 4.6% this alternative is welfare superior to all other reforms. Fertility reacts strongly positive to this policy while the retirement wealth remains relatively stable.

Finally, a child care coverage rate of 39% also closes the financial gap in the governmental budget. The combination of a positive labor supply and fertility effect leads to a very good financial situation of the government, such that public pension benefits increase compared to the situation before the longevity shock. Concerning welfare, the change is positive but smaller than with higher monetary benefits. The reason for this is the lower equilibrium wage rate caused by the higher aggregate labor supply.

An expanded child care coverage and a higher contribution rate are thus the two policies that create a balanced budget along the transition path.

One general comment on this policy comparison should be made. Even if the comparison appears to be relatively simple - which policy can fill a financial gap in the most efficient way -, it is not. While a reform of the pension system itself tries to adjust the current pension system to the new demographic structure, a family policy reform sets long-run incentives so that the demographic structure re-changes and as a consequence the pension system fits again the demographic structure. For sure, one cannot completely change the demographic structure with family policy. However, it should be clear that both approaches are fundamentally different.

The remainder of this chapter is structured as follows: Section 4.2 characterizes the pay-as-you-go pension system. Section 4.3 describes the experiment and the simulation results. The effects on the government budget, retirement wealth and factor prices are discussed in Section 4.4. Section 4.5 concludes.

4.2 Government Budget and the Public Pay-As-You-Go Pension System

In Germany, the government budget and the budget of the public pension system are legally distinct. Pension benefits are financed by a contribution rate and the government finances its expenditures by its taxes. In my model I take a more consolidated approach. There is only one labor tax rate τ that finances all expenditures. In a stylized version, government expenditures are the pension benefits P, family policies θ and residual expenditures G. L represents the financial burden arising from increased longevity. So, the budget is

$$\tau N_\tau = PN_P + \theta N_\theta + G + L,$$

where N_i are the number of people that pay the taxes or receive benefits i. So, in the present model the government has a freedom of choice that appears not to exist in reality. However, this is not true. As mentioned above, even when the budgets for the pension system and the government are separate, the government makes annual transfers from its general budget to the pension system. And these transfers, which are currently around 60 billion EUR, are expected to increase up to 300 billion EUR in the next 15 years. Furthermore, both the labor tax and the pension contribution rate are deducted from labor income, so pooling them in one combined tax rate does not affect underlying mechanisms or results. Taking the two arguments on joined budget and single tax rate together the assumption is not critical.

Theoretically, a pay-as-you-go pension system is a so-called "defined contribution" system in which individual benefits are not related to individual contributions. This leads to a redistributive scheme with potential incentives to work less. However, the actual pension system in Germany deviates from this concept.[7] Via a "point system" individual contributions are linked with individual benefits, such that the pension system becomes a "notional defined benefits" system. In a simple way, this mechanism works as follows: The more an employee earns during work-life, the more points she gets, and the more points she collects, the higher are the pension benefits. Such a scheme does not introduce a disincentive to work. Here, I abstract from the points and directly define pension benefits as earnings-dependent. More precisely and as already described in Section 2.3.2, for a person i pension benefits P_i are the product of the replacement rate q and the average net labor income $\hat{\omega}^i$ a person received during her career:

$$P_t^i = q\hat{\omega}_t^i$$

where her average net labor income is defined as:

$$\hat{\omega}_t^i = \frac{1}{2}\left[(1-\tau)\ell_{y,t-2}^i\omega_{y,t-2}^i + (1-\tau)\omega_{m,t-1}^i\right].$$

[7] For a good description of the German public pension system see for example Börsch-Supan and Wilke (2003) or Börsch-Supan et al. (2007).

With such a pension formula the individual link between individual contributions and benefits is established and the key feature of the German pension system present in the model.[8]

4.3 Increase in Longevity and Policy Reactions

4.3.1 Modeling the Increase in Longevity

Longevity increases steadily in Germany. In 2060, people in Germany will on average get seven years older compared to 2008. Most of the increased life expectancy is added at the end of life such that time in retirement is prolongated by around five years. Obviously, this is a gradual change that takes place over decades. However, for the experiment I assume this structural transition to be a one-shot change. This structural change is modeled as one huge sudden increase of longevity expenditures, L, that the government has to finance. So the government has to put more money into the pension system but individual households do not receive higher pension benefits.[9]

Given a balanced government budget in the initial steady state a sudden jump of L leads to a financial imbalance. To determine the size of the financial imbalance I use information from the German public pension insurance. On average, retirees currently receive pension benefits for 19 years.[10] Until 2060, life expectancy at the age of 65 is expected to increase by 5.2 years, and therefore, German retirees will receive pension benefits for an average duration of 24.2 years. This is an increase of 27.4% and as a consequence, the expenditures for the public pension system go up by the same amount. So, the size of the longevity shock is 27.4% of the pension expenditures in the initial steady state. These expenditures have been 4.5 initially, so that the longevity shock is 1.3. Thus, the government has to raise more taxes or cut expenditures to be able to pay the additional longevity expenditures. For doing so it can adjust one of the four policy parameters: raising the contribution rate τ, cutting the replacement rate q, increasing monetary child benefits θ^a or expanding public child care θ^b.

As a very first result, all four policy parameters individually are capable to satisfy the additional financial needs. Table 4.2 shows the effect of the longevity shock on the four policy parameters. The first column ($\Delta\tau$) shows the results for the contribution rate, the second column

[8] I also abstract from ceilings on social security contributions and benefits. As I have only two income types, there are no households with very high income in the model. Therefore this assumption is not critical. Moreover, I assume that households do not take into account the effect of the labor decision on future pension benefits.

[9] In reality, when households live longer they indeed receive more benefits which is distributed over a larger number of periods in retirement. However, as the number of periods is fixed in the model, distributing the additional government expenditures among households would be an increase of pension benefits. This is not at all what happens and therefore no modeling choice.

[10] See Rentenversicherung (2014).

(Δq) for the replacement rate, the third ($\Delta\theta^a$) for direct monetary child benefits, and the fourth ($\Delta\theta^b$) for public child care. The contribution rate needs to increase by 12.8 percentage points, such that the combined tax and contribution rate increases by a third from 39.6 to 52.8%. In the case when the government uses the replacement rate as policy adjustment, the replacement rate needs to be cut from 55.3 to 42.5%. This implies a reduction of pension benefits by almost a quarter. Using monetary child benefits to fill its budget gap, the government needs to raise benefits by 86%. The resulting *Kindergeld* is 342 EUR per month and child.[11] Public child care can also be used to re-balance the government budget. In this case, almost a doubling of the child care coverage rate is needed, from 20 to 39%.[12]

Table 4.2: Effect of Longevity Increase on Policy Parameters

	$\Delta\tau$	Δq	$\Delta\theta^a$	$\Delta\theta^b$
benchmark	0.396	0.553	0.113	0.062
higher life expectancy	0.524	0.425	0.210	0.120
absolute change	+0.128	-0.128	+0.097	+0.058
relative change	+32.3%	-23.1%	+85.8%	+93.5%

In the following sub-sections, I investigate in which way different policy adjustments affect households' behavior. I will start with the two reforms of the pension system itself and then turn to the alternative reforms of family policies. All results for the high income households are in Table 4.3, for the low income households in Table 4.4 and aggregate statistics are presented in Table 4.5. Finally, the effects on old-age insurance are presented in Tables 4.6 and 4.7. The structure is the same for all tables: The results for the reform of the contribution rate are in the first column, the second column shows the results of the reform of the replacement rate, the third presents the monetary child benefits reform, and the fourth column displays the public child care expansion.

Table 4.3: Choices High Type

	benchmark	$\Delta\tau$	Δq	$\Delta\theta^a$	$\Delta\theta^b$
fertility rate	0.56	0.43	0.57	0.63	0.61
labor supply	0.86	0.89	0.86	0.84	0.88
consumption					
$j = y$	1.20	0.91	1.23	1.22	1.18
$j = m$	2.13	1.75	2.11	2.12	2.16
$j = o$	3.78	3.37	3.63	3.67	3.95
welfare change	—	-0.274	+0.020	+0.046	+0.024

[11] The level of *Kindergeld* in the benchmark case was 184 EUR per month and child.
[12] The benchmark coverage rate was 20%. The new coverage rate is 38.7% which corresponds to an increase of 93.5%.

Table 4.4: Choices Low Type

	benchmark	$\Delta\tau$	Δq	$\Delta\theta^a$	$\Delta\theta^b$
fertility rate	0.75	0.62	0.76	0.93	0.88
labor supply	0.82	0.85	0.81	0.77	0.83
consumption					
$j = y$	0.93	0.71	0.95	0.94	0.92
$j = m$	1.65	1.36	1.64	1.62	1.68
$j = o$	2.93	2.61	2.81	2.81	3.06
welfare change	—	-0.254	+0.017	+0.058	+0.039

4.3.2 Scenario 1: Increase of Contribution Rate

The new tax rate of 52.4% reduces net labor income for all households significantly. Because of this income effect both household types have to reduce their consumption by over 15% and the consumption profile gets steeper. As a consequence, the increase of the contribution rate depresses consumption during young adulthood by 25%. Average fertility also drops from initially 1.38 to 1.12 children per household. While the number of children for low income households decreases to 1.24, there are only 0.86 children in a high income household. The lower number of children, however, results in higher labor force participation rates. Moreover, households have to lower their private old-age savings by 17% (high income households) and 16% (low income households). Higher labor supply and lower savings lead to a lower capital-to-labor ratio and consequently to lower wages and a higher interest rate. This change in factor prices amplifies the negative income effect. As a result the welfare loss of this policy reaction is huge: Low income households are worse off by 25.4% and high income households by 27.4%, measured in consumption equivalence units.

Total retirement wealth decreases by 11% for both households. Private savings for retirement drop by 18% (16%) for high (low) income households as a consequence of this reform. But since the interest rate increases private pension benefits only decrease by 6% (7%).[13] Public pension benefits fall by 22% (21%). The drop of the wage rate and the fact that pension benefits depend on the net labor income are the reasons for the reduction. As a consequence the importance of private pension benefits increase relative to public pensions: In the initial steady

[13] "Private old-age savings" describe the amount of money a household puts aside during middle adulthood and save for retirement, s_m^i. "Private pension benefits" or "private retirement wealth", $(1+r)s_m^i$, is the amount of money retirees receive as capital income.

state, 30.1% (27.2%) of the retirement wealth was provided by the public pension system, this number goes down to 26.4% (24%) after the reform.[14]

Finally, something can be said about the profitability of the two old-age insurance instruments. The profitability of private old-age savings is measured by the interest. For the public pension system, I define an implicit rate of return which compares the contribution an employee has paid into the pension system with the benefits she receives from it. Are contributions and benefits the same, the implicit rate of return is zero. In the initial steady state a household receives 38.7% less benefits than what she paid into the pension system.[15] Since a person works for 40 years, the annual implicit rate of return is -1.22%. With the increase of the tax rate the return of the public pension system is -1.83%, so more negative than in the initial steady state. In general, as the wage rate is constant in the steady state the implicit rate of return is a monotonic function of the fertility rate: The higher the fertility rate, the higher the rate of return of the pay-as-you-go pension system. The rate of return for the private savings is the interest rate which was 9.0% per year in the initial steady state and increased to 9.8%. So, the profitability gap between the private and public old-age insurance instrument gets larger when the government's reaction to increased longevity is an increase of the contribution rate.

This scenario can thus be summarized as follows: The negative income effect dominates everything else and the change of factor prices makes the situation even worse. Old-age insurance shifts more towards private savings and the rate of return of the pension system becomes more negative.

4.3.3 Scenario 2: Decrease of Replacement Rate

The second possibility for the government to adjust the pension system is to cut the replacement rate to 42.5%. The main impact of this reform is observed in the retirement period. Public pension benefits decrease by 22% for high income households and 23% for low income ones. Households react to this cut in public pensions by saving more privately. Savings during middle adulthood go up by 10% (9%) for high (low) income households. As the equilibrium interest rate decreases, private pensions only increase by 4% (3%). In total, retirement wealth and therefore consumption during old-age goes down by 4%. This is remarkable compared to the first reform scenario in which the replacement is kept constant but public pensions drop by over 20% and

[14]Note that this means that 70% of total retirement wealth is generated by private old-age savings. This does not reflect the data and is a consequence of the very high interest rate in my model. Moreover, note that high income households rely relatively more on the public pension system which is also counterfactual. This is a result of the perfect link between labor income and pension benefits in the pension formula.

[15]To compute how much a household paid into the pension system, I use the pension system expenditure share of the government. In the initial steady state, the government spends 69% of the total budget for the pension system. Then I assume that the same fraction of the individual tax payments goes into the pension system.

Table 4.5: Aggregate Statistics

	benchmark	$\Delta\tau$	Δq	$\Delta\theta^a$	$\Delta\theta^b$
average fertility rate	0.69	0.56	0.70	0.84	0.80
share of high types	31.0%	30.3%	31.1%	30.0%	30.4%
share of low types	69.0%	69.7%	68.9%	70%	69.6%
old-age-dependency ratio	0.86	1.14	0.84	0.65	0.70
young-age-dependency ratio	0.28	0.20	0.29	0.38	0.35
total dependency ratio	1.14	1.34	1.13	1.03	1.05
GDP per capita	1.95	1.83	2.00	1.90	2.02
welfare change	—	-0.260	+0.018	+0.055	+0.034
interest rate (p.a.)	8.99%	9.76%	8.68%	8.76%	9.26%
wage rate	1.05	0.99	1.07	1.07	1.03
nanny wage rate	2.12	2.00	2.17	2.15	2.07
size of nanny sector	0.4%	0.3%	0.4%	0.5%	0.9%

total retirement wealth drops by 11%. So, reducing the replacement rate leads to relatively higher public pension benefits than holding the replacement rate constant.

Concerning overall old-age insurance, this reform leads to a shift towards private old-age savings as one could expect. In the new steady state, private pension benefits contribute to 75.5% of high income households' retirement wealth, and 77.9% for low income households, respectively. The implicit rate of return increases slightly, along with the fertility rate, to -1.16%. The new interest rate for private old-age savings is 8.7%.

While the interest goes down in this scenario, the wage rate mildly increases. This creates an income effect for young adults so that they get more children. However, this effect is relatively small and with 1.40 children per woman, the new average fertility rate increases only little. Labor supply and consumption during young and middle adulthood stay basically constant. So, as fertility goes up, the model generates a negative relationship between the size of the pay-as-you-go pension system and the fertility rate. This fits into the literature that argues that larger public pensions depress fertility rates since children are no longer needed as old-age insurance instrument.[16] However, this effect exists in the present model without any altruistic link between children and parents. The model does allow children to be used as old-age insurance mechanism by assumption. As the decomposition of general equilibrium effect in Section 4.4.2 will show, the reason for this negative relation are changes in factor prices and not the enlarged pension system directly.

Welfare increases by 2% for high income households and 1.7% for low income ones. So, there is a positive effect of shrinking the size of the public pension system and this effect is

[16]See for example Nishimura and Zhang (1992), Cigno (1993), Cigno et al. (2002), Manuelli and Seshadri (2009) or Bohacek and Belush (2011).

sufficiently large to over-compensate the negative longevity shock. Furthermore, one can observe that, again, a reform that reduces the size of the government sector is more beneficial for high income households. A smaller government sector allows households to choose more freely what they want to do with their income. As the income of high type households is larger, the re-allocation of income is also larger and so is the positive welfare effect.[17]

To summarize this experiment briefly: The incentive to save more privately dominates. Old-age security shifts towards the private sector. Higher savings induce a higher wage rate which leads to a slightly higher fertility rate. This in turn leads to a better implicit rate of return in the public pension sector. Moreover, a negative relationship between the pay-as-you-go system and fertility is generated by the model.

4.3.4 Scenario 3: Increase of Monetary Child Benefits

I now turn to the family policy reforms and start with the reform with which the government increases direct monetary child benefits. The primary impact of this reform is on the costs of rearing children and therefore on the fertility rate. The average fertility rate after the reform is 1.68 and therefore 0.3 children per woman higher than before. Low income households have 1.86 children in the new steady state while for high income households the fertility rate is 1.26. With more children the labor supply decreases, the wage rate increases and the interest rate goes down to 8.8%. Old-age savings remain basically unchanged and therefore private pension benefits decrease by 4% (3%) for high (low) income households. The lower labor supply dominates the higher wage rate so that average work-life income and consequently public pension benefits decrease. High income households receive 2% less from the public pension system, while it is 8% for low income households. Total retirement wealth decreases by 3% for high and 4 % for low types.

The strong positive fertility reaction has a positive impact on welfare. High income households' welfare increases by 4.6%. With 5.8% the increase is even stronger for low income households. Hence, in terms of welfare the reform of monetary benefits is significantly better than both reforms of the pension system. Contrary to the reform of the replacement rate, higher monetary child benefits are relatively more beneficial for low income households. As the decomposition of the general equilibrium effects in Section 4.4.2 will show, the main driver for this result is the fertility reaction. However, even when ignoring this fertility effect on welfare the overall effect remains positive and for low income households even higher than with the cut of the replacement rate.

[17]The same holds true for the results in Chapter 3 for the tax cut scenario.

The results for the composition of retirement wealth are mixed. For high income households this is the only policy reform with which the share of public pensions increase (from 30.1 to 30.6%). For low income households there is no reform that leads to a higher public pension share but the monetary benefits reform is the one reducing this fraction the least (from 27.2 to 26.4%). And since fertility is the highest of all four reform scenarios, the implicit rate of return also is relatively the best with -0.58%.

This policy can be summed up as follows: The fertility subsidy leads to significantly higher fertility and also welfare. Both, private and public retirement wealth are reduced only mildly. And thanks to the high fertility rate it is the reform leading to the most efficient pay-as-you-go system

Table 4.6: Old Age Savings and Pension Benefits - High Types

	benchmark	$\Delta\tau$	Δq	$\Delta\theta^a$	$\Delta\theta^b$
replacement rate	0.553	0.553	0.425	0.553	0.553
public pension benefits	1.14	0.89	0.89	1.12	1.16
old-age savings	0.40	0.33	0.44	0.40	0.41
private pension benefits	2.65	2.48	2.75	2.54	2.79
retirement wealth	3.78	3.37	3.63	3.67	3.95
public pension share	30.1%	26.4%	24.5%	30.6%	29.3%
private pension share	69.9%	73.6%	75.5%	69.4%	70.7%
public pensions / contributions	0.613	0.477	0.626	0.791	0.742
annual implicit rate of return	-1.22%	-1.83%	-1.16%	-0.58%	-0.74%

Table 4.7: Old Age Savings and Pension Benefits - Low Types

	benchmark	$\Delta\tau$	Δq	$\Delta\theta^a$	$\Delta\theta^b$
replacement rate	0.553	0.553	0.425	0.553	0.553
pension benefits	0.80	0.63	0.62	0.74	0.81
old-age savings	0.32	0.27	0.35	0.33	0.33
private pension benefits	2.13	1.99	2.19	2.07	2.25
retirement wealth	2.93	2.61	2.81	2.81	3.06
public pension share	27.2%	24.0%	22.1%	26.4%	26.4%
private pension share	72.8%	76.0%	77.9%	73.6%	73.6%
public pensions / contributions	0.613	0.477	0.626	0.791	0.742
annual implicit rate of return	-1.22%	-1.83%	-1.16%	-0.58%	-0.74%

4.3.5 Scenario 4: Increase of Public Child Care

The last scenario simulates an expansion of public child care as response to the longevity shock. The primary effect is the same as with higher monetary benefits: Expanded child care coverage increases the fertility rate to 1.60. As always, the type-specific fertility rate of low income households reacts stronger than the one of high income households (1.76 versus 1.22). Beside the effect on fertility, more public child care generates a positive labor supply effect. As result of the child care expansion both households have more children and higher individual labor supplies. So, even with more children the households have more time to be out of the house and work. The joint increase of both fertility and labor supply leads to a strong increase of aggregate labor supply. As a consequence, the wage rate drops by 8% which reduces the net labor income significantly. Public pension benefits are 2% (1%) higher for high (low) types in the new steady state, indicating that the labor supply effect over-compensates the negative income effect. The implicit rate of return is -0.74%, which is clearly better compared to the pension reforms but worse than the monetary child benefits reform.

As always, the lower wage rate goes hand in hand with a higher interest rate. This motivates people to save more privately such that private pensions increase by 5% and 6% for high and low income households. In total, the public child care expansion is the only scenario that even leads to a higher retirement wealth (of 4% for both household types). The public component of retirement wealth is 29.3 and 26.4% for high and low income types and therefore lower as in the initial steady state but significantly higher compared to the pension reforms.

Thus, public child care is not only able to fill the financial gap created by a higher longevity but also improves the situation for retirees. This is a quite strong result that is partly reflected in the welfare results: High types are better off by 2.4% and low types by 3.9%. Hence, in the welfare ranking this reform is placed above the two pension reforms but below the monetary child benefits reform. The reason for this is the same as in the previous chapter: The strong and positive labor supply effect worsens the situation for households in general equilibrium because of the significantly lower wage rate and therefore lower income.

To put it in a nutshell: Higher child care coverage results in higher fertility and higher labor supply leading to a drop of the wage rate. Welfare is higher compared to the pension reforms but lower compared to the monetary benefits reform due to the wage drop. Both components of old-age insurance become larger and therefore retirement wealth overall increases.

4.4 Comparative Analysis

4.4.1 Government Budget

The overall budgetary effects of the four reform scenarios can be seen in the upper part of Table 4.8 and the corresponding policy parameter values in the lower part of the table. The composition of the government budget is shown in Table 4.9.

Compared to the benchmark case, total tax revenues increase in all four scenarios. While the higher contribution rate is the cause for this increase in the first reform scenario, it cannot be for the other three since it remains unchanged. However, in this first scenario of rising contribution rates, tax revenues are the lowest compared to the family policies and the decrease of the replacement rate. This shows that the tax rate is only one out of three factors that determine tax revenues. Gross labor income and the number of taxpayers are the other two factors. Since both family policy reforms have a positive impact on both gross labor income and the fertility rate, the budget of the government is 20% larger with these reforms compared to the other two. Hence, it is important for the government to take into account the behavioral and price adjustments when considering and evaluating which reform shall be implemented.

After the longevity shock and the subsequent reform, total government expenditures for the pension system increase independently of the implemented reform.[18] The government has to spend between 70% and 74% of the whole budget on the pension system. Interestingly, the highest and lowest shares are generated by the family policy reforms. As the relative size of the longevity shock is the same for both policies, 16.3%, the difference must come from the actual pension benefits. Indeed, the government can spend almost 4 percentage points more for retirees with expanded public child care compared to the monetary child benefits reform. The reason is that public child care is the most efficient family policy.[19] So, the government only spends 5.0% of its budget for family policies while it is 8.8% with monetary child benefits. As a consequence, when implementing the public child care reform the government can put more money into the pension system.

Table 4.8: Government Budget and Parameters

	benchmark	$\Delta\tau$	Δq	$\Delta\theta^a$	$\Delta\theta^b$
tax revenues	6.6	6.7	6.8	8.0	8.0
pension system	4.5	3.5	3.5	4.3	4.6
family policies	0.2	0.1	0.2	0.7	0.4
residual expenditures	1.8	1.8	1.8	1.8	1.8
longevity shock	—	1.3	1.3	1.3	1.3
tax rate	0.396	0.524	0.396	0.396	0.396
replacement rate	0.553	0.553	0.425	0.553	0.553
monetary benefits	0.113	0.113	0.113	0.210	0.113
public child care	0.062	0.062	0.062	0.062	0.120

So far, the analysis of the government budget and the conclusion that all four reform options are capable to finance the longevity shock relied on a steady state comparison. However, starting from the old steady state it takes years until the new steady state is reached. Hence,

[18] Total pension expenditures are the sum of expenditures for the pension system and the expenditures to finance the longevity shock.

[19] Efficiency is measured as "number of additional children when putting one additional EUR into the family policy".

Table 4.9: Composition of Government Budget

	benchmark	$\Delta\tau$	Δq	$\Delta\theta^a$	$\Delta\theta^b$
tax revenues	100%	100%	100%	100%	100%
pension system	68.2%	52.2%	51.5%	53.8%	57.5%
family policies	3.0%	1.5%	2.9%	8.8%	5.0%
residual expenditures	27.3%	26.9%	26.5%	22.5%	22.5%
longevity shock	—	19.4%	19.1%	16.3%	16.3%
total pension expenditures	68.2%	71.6%	70.6%	70.0%	73.8%

one can wonder what happens with the government budget during the transition phase and which reforms are capable to ensure a balanced budget along the transition path. To answer this question I compute the budget along a transition path of 30 periods.[20] In the first period, the longevity shock hits the budget and the government implements one of the four reforms.[21] Households adjust their behavior from the next period on.

The results are shown in Figure 4.1. I define the budgetary surplus in per capita terms so that the effects can be compared across the reforms. The dashed line at zero represents a balanced budget. Moreover, in period 0 the budget is already hit by the longevity shock, therefore the budgetary surplus is negative in the beginning. The results of the four reform options can be divided into two groups: On the one hand, the increase of the contribution rate and the public child care expansion are able to create a surplus along the transition. On the other hand, the reduction of the replacement rate and the increase of monetary benefits do not lead to a balanced budget throughout the entire transition path. Only in the new steady state do they reach a balanced budget. With the higher contribution rate, the government generates a budgetary surplus from the very first period that becomes smaller afterwards. Expanded public child care creates a surplus from the third period onwards.

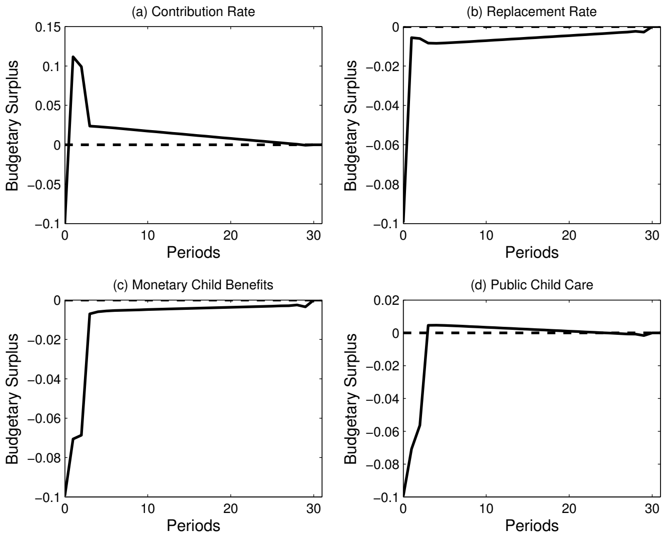
Looking at the two family policies, it is the welfare inferior one that generates budgetary surpluses along the transition. The same holds true for the pension reforms. This is no surprise as the four reforms can be divided into two further categories: A higher replacement rate or higher monetary child benefits only affect the expenditure side of the government budget.[22] A change of the tax rate affects only the revenues and public child care has an impact on both revenues and expenditures.

[20]The results are the same for a transition phase of 200 periods.

[21]Implementing a reform means that the government sets the corresponding policy parameter to the new steady state value.

[22]This is true when looking at the direct effects of the reforms. Indirectly, on the aggregate level, the total number of people in the economy changes because of the reform and therefore the tax base changes as well. This in turn affects the revenue side of the government budget.

Figure 4.1: Government Budget on the Transition Path



Thus, for a balanced budget along the transition path, it is important that the reform has an impact on the revenues of the government and not only on the expenditures. The public child care expansion is the only reform that creates a sustainable budget and also leads to a welfare increase for all households.

4.4.2 Decomposition of General Equilibrium Effects

In this section I decompose the general equilibrium effects of the four reform scenarios to get a better understanding of how factor prices, in particular, drive the results. When the government decides to increase the contribution rate as a reaction to the longevity increase, a sharp drop in fertility and welfare is the consequence. The higher tax counts for one third of the drop in fertility, see Table 4.10. The negative income effect caused by the lower wage rate accounts for the other two thirds. The resulting fertility rate would be even lower but the higher interest rate dampens the negative effect. Actually, the interest rate effect overcompensates the tax effect.[23] The negative wage effect is significantly higher for low income households. For high income households both effects, tax and wage effects, causes half of the drop in fertility.[24] In contrast to the fertility rate, welfare is mainly driven by the tax effect. It counts for almost three quarters of the welfare loss, while the last quarter is caused by the lower wage rate. Again, the interest rate has a positive effect.

Finally, it is remarkable that the lower wage rate leads to higher private and public pension benefits.

[23] The reason is the following: The higher interest rate leads to a stepper consumption pattern with lower consumption during young adulthood such that households shift income from consumption to child rearing.

[24] All type-specific results can be found in Appendix 4.A.1.

Table 4.10: Effect Decomposition - Contribution Rate - Average

	old ss	$\Delta\tau$	Δr	Δw / new ss
fertility rate (n)	0.69	0.59	0.74	0.56
Δ n		-0.10	+0.15	-0.18
private pension benefits (oas)	2.29	1.57	1.96	2.14
Δ oas		-0.72	+0.39	+0.18
public pension benefits (P)	0.90	0.74	0.70	0.71
Δ P		-0.16	-0.04	+0.01
welfare		-0.220	-0.179	-0.260
Δ welfare		-0.220	+0.041	-0.081

As already discussed in Section 4.3 the reform of the replacement rate has a small but negative effect on fertility. As one can see in Table 4.11, it is not the reduction of the replacement rate that leads to a higher fertility rate but the change in factor prices. Overall, the lower replacement rate has a negative effect on savings as well. Basically, all positive effects of this reform are driven by the wage rate, in particular, the increase of private pension benefits and welfare.[25]

Table 4.11: Effect Decomposition - Replacement Rate - Average

	old ss	Δ q	Δr	Δw / new ss
fertility rate (n)	0.69	0.69	0.69	0.70
Δ n		-0.00	+0.00	+0.01
private pension benefits (oas)	2.29	2.12	1.95	2.36
Δ oas		-0.17	-0.17	+0.41
public pension benefits (P)	0.90	0.69	0.69	0.71
Δ P		-0.21	-0.00	+0.02
welfare		-0.003	-0.006	+0.018
Δ welfare		-0.003	-0.003	+0.024

The analysis of the effect decomposition for the monetary benefits reform is straightforward. For this reform most of the effects are caused by the lower price of child-bearing thanks to the higher fertility subsidy. The fertility reaction is completely caused by this effect and it also accounts for two thirds of the welfare gain. The last third of the welfare gain is caused by the higher wage rate. Lower child-rearing costs motivate households to shift income from consumption and savings to children and as a consequence they save less. The wage effect overcompensates this negative effect on old-age savings and, in total, private pension benefits are higher than before the reform. Moreover, the effect decomposition nicely proves the statement made in Section 4.3 that the welfare result of this reform remains positive even without the positive effect of fertility on welfare.

[25]The positive effect of the higher wage rate on public pension benefits nicely shows the link between labor income and pension benefits.

Table 4.12: Effect Decomposition - Monetary Benefits - Average

	old ss	$\Delta\theta^a$	Δr	Δw / new ss
fertility rate (n)	0.69	0.83	0.83	0.84
Δ n		+0.14	+0.00	+0.01
private pension benefits (oas)	2.29	1.96	1.84	2.21
Δ oas		-0.33	-0.12	+0.37
public pension benefits (P)	0.90	0.85	0.85	0.86
Δ P		-0.05	-0.00	+0.01
welfare		+0.039	+0.036	+0.055
Δ welfare		+0.039	-0.003	+0.019

With the public child care expansion the fertility reaction is entirely caused by the change of the family policy itself. Contrary to the monetary child benefits reform, factor price changes have a mildly negative effect on fertility. In addition, the sole expansion of public child care leads to lower private pension benefits. Similarly to the monetary child benefits reform the adjustment of factor prices results in higher private pensions. However, note that factor prices move in the opposite direction compared to before. This means that households have higher private pensions when wages are lower.

Expanded public child care is the only reform that leads to higher public pension benefits. The reason for this increase is the positive labor supply effect of better child care coverage. Finally, the last row of Table 4.13 shows what has been already discussed in Section 4.3: The reform becomes partly a victim of its own success as the combination of higher labor supply and a positive fertility reaction to more child care, though very positive in terms of welfare, leads to a significantly lower equilibrium wage rate which in turn "re-reduces" the welfare gain.

Table 4.13: Effect Decomposition - Public Child Care - Average

	old ss	$\Delta\theta^b$	Δr	Δw / new ss
fertility rate (n)	0.69	0.81	0.81	0.80
Δ n		+0.12	-0.00	-0.01
private pension benefits (oas)	2.29	1.94	2.10	2.42
Δ oas		-0.33	+0.16	+0.32
public pension benefits (P)	0.90	0.93	0.93	0.91
Δ P		+0.03	+0.00	-0.02
welfare		+0.055	+0.058	+0.034
Δ welfare		+0.055	+0.003	-0.024

4.5 Conclusion

In this chapter I analyzed whether family polices are an alternative for the German government in light of the current aging of the economy. The usual policy reaction is to reform the pension system itself. I take a different point of view and propose family policies as alternative policy reaction to demographic change and the consequent financial burden for the public pension system. The idea to use higher monetary child benefits or expanded public child care to mitigate, or to reduce, the financial problems of the pension system is based on my previous findings that intensified family policies create additional financial resources for the government.

Assuming an exogenous longevity shock that creates a financial gap for the government, I find that all four reform proposals - increasing the contribution rate, cutting the replacement rate, increasing monetary child benefits and expanding public child care - are capable to fill this gap. In terms of welfare, the two family policy reforms outperform the reforms of the pension system. As in the previous chapter, higher monetary benefits lead to larger welfare gains compared to the public child care reform. The main reason is the positive labor supply effect of public child care that leads to a lower equilibrium wage rate and lowers the households' labor income. Increasing the contribution rate leads to huge welfare losses because it significantly reduces net labor income for all households. Cutting the replacement rate also leads to a welfare gain which is smaller compared to the family policy reforms.

Concerning the two pension reforms the results are in line with the literature which finds that cutting the replacement rate is relatively more beneficial compared to adjusting the replacement rate. However, the result that more children can resolve the difficult situation of the German pension system is in contrast to Berkel et al. (2004). They find that more children cannot mitigate the effects of population aging.

A really remarkable result is that three out of the four reforms actually lead to a welfare increase. The longevity shock forces the government to re-allocate its financial resources in a way that households receive less benefits from the government. And yet, welfare still increases. This indicates that the current situation in Germany is partly inefficient in two aspects: First, the public pension system exhibits a negative rate of return. However, as long as the government forces people to participate in this system they cannot shift their old-age insurance towards private old-age savings which would create a much larger return. Therefore, cutting the replacement rate allows employees to increase their private savings which leads to this positive effect. Second, the fertility rate is relatively low in Germany. This leads to a high old-age dependency ratio. In this situation, intensifying family policies reduces this ratio and generates higher revenues for the government. Moreover, and independently from what the government does with the additional financial resources there remains the positive welfare effect of higher fertility rate. Finally, since any family policy reform leads to welfare gains, the allocation within households between consumption and children has not been optimal in the initial steady state.

There are two further results that can be observed: Only policies that create more revenues can reach a balanced government budget along the transition path. Furthermore, the model result shows a negative relationship between the size of the pay-as-you-go pension system and the fertility rate which is in line with the literature.

In general, the results of this chapter shall not be interpreted in a way that intensified family policies can mitigate all problems that come along with population aging. Instead, long-run family policies aiming at a higher fertility rate - possibly in combination with further policy adjustments - can help to stabilize the financial situation of the pension system.

4.A Appendix

4.A.1 Type-specific Results of the Decomposition of General Equilibrium Effects

In this appendix, all type-specific results for the decomposition of general equilibrium effects can be found: First, for the increase of the contribution rate, then for the tax cut scenario, third for the increase of monetary child benefits and finally for the public child care expansion.

Table 4.14: Effect Decomposition - Contribution Rate - High Types

	old ss	$\Delta\tau$	Δr	Δw / new ss
fertility rate (n)	0.56	0.46	0.51	0.43
Δ n		-0.10	+0.05	-0.08
private pension benefits (oas)	2.65	1.81	2.27	2.48
Δ oas		-0.84	+0.46	+0.21
public pension benefits (P)	1.14	0.93	0.92	0.89
Δ P		-0.21	-0.01	-0.03
welfare		-0.230	-0.204	-0.274
Δ welfare		-0.230	+0.026	-0.070

Table 4.15: Effect Decomposition - Contribution Rate - Low Types

	old ss	$\Delta\tau$	Δr	Δw / new ss
fertility rate (n)	0.75	0.65	0.83	0.62
Δ n		-0.10	+0.18	-0.21
private pension benefits (oas)	2.13	1.46	1.83	1.99
Δ oas		-0.67	+0.37	+0.16
public pension benefits (P)	0.80	0.66	0.60	0.63
Δ P		-0.14	-0.06	+0.03
welfare		-0.215	-0.168	-0.254
Δ welfare		-0.215	+0.037	-0.086

Table 4.16: Effect Decomposition - Replacement Rate - High Types

	old ss	Δ q	Δr	Δw / new ss
fertility rate (n)	0.56	0.56	0.56	0.57
Δ n		-0.00	+0.00	+0.01
private pension benefits (oas)	2.65	2.46	2.26	2.75
Δ oas		-0.19	-0.20	+0.49
public pension benefits (P)	1.14	0.87	0.87	0.89
Δ P		-0.27	-0.00	+0.02
welfare		-0.004	-0.005	+0.020
Δ welfare		-0.004	-0.001	+0.025

Table 4.17: Effect Decomposition - Replacement Rate - Low Types

	old ss	Δ q	Δr	Δw / new ss
fertility rate (n)	0.75	0.75	0.75	0.76
Δ n		-0.00	+0.00	+0.01
private pension benefits (oas)	2.13	1.96	1.80	2.19
Δ oas		-0.17	-0.16	+0.39
public pension benefits (P)	0.80	0.61	0.61	0.62
Δ P		-0.19	-0.00	+0.02
welfare		-0.003	-0.007	+0.017
Δ welfare		-0.003	-0.004	+0.024

Table 4.18: Effect Decomposition - Monetary Benefits - High Types

	old ss	$\Delta\theta^a$	Δr	Δw / new ss
fertility rate (n)	0.56	0.62	0.62	0.63
Δ n		+0.06	+0.00	+0.01
private pension benefits (oas)	2.65	2.26	2.11	2.54
Δ oas		-0.39	-0.15	+0.43
public pension benefits (P)	1.14	1.11	1.11	1.12
Δ P		-0.03	-0.00	+0.01
welfare		+0.028	+0.027	+0.046
Δ welfare		+0.028	-0.001	+0.019

Table 4.19: Effect Decomposition - Monetary Benefits - Low Types

	old ss	$\Delta\theta^a$	Δr	Δw / new ss
fertility rate (n)	0.75	0.92	0.93	0.93
Δ n		+0.17	+0.01	+0.00
private pension benefits (oas)	2.13	1.83	1.72	2.07
Δ oas		-0.30	-0.11	+0.35
public pension benefits (P)	0.80	0.73	0.73	0.74
Δ P		-0.07	-0.00	+0.01
welfare		+0.044	+0.040	+0.058
Δ welfare		+0.044	-0.004	+0.018

Table 4.20: Effect Decomposition - Public Child Care - High Types

	old ss	$\Delta\theta^b$	Δr	Δw / new ss
fertility rate (n)	0.56	0.63	0.62	0.61
Δ n		+0.07	-0.01	-0.01
private pension benefits (oas)	2.65	2.24	2.43	2.79
Δ oas		-0.41	+0.19	+0.36
public pension benefits (P)	1.14	1.18	1.18	1.16
Δ P		+0.04	+0.00	-0.02
welfare		0.047	+0.049	+0.024
Δ welfare		+0.047	+0.002	-0.025

Table 4.21: Effect Decomposition - Public Child Care - Low Types

	old ss	$\Delta\theta^b$	Δr	Δw / new ss
fertility rate (n)	0.75	0.90	0.89	0.88
Δ n		+0.15	-0.01	-0.01
private pension benefits (oas)	2.13	1.81	1.96	2.25
Δ oas		-0.32	+0.15	+0.29
public pension benefits (P)	0.80	0.82	0.82	0.81
Δ P		+0.02	+0.00	-0.01
welfare		+0.059	+0.062	+0.039
Δ welfare		+0.059	+0.003	-0.023

Chapter 5

Family Policies and Altruism

5.1 Introduction

In Germany, intensified family policies increase the fertility rate, enlarge financial resources of the government and therefore help to mitigate problems of the public pay-as-you-go pension system that arise from demographic change. Moreover, intensified family policies are welfare improving. These are the main conclusions of the previous chapters.

These results are based on a macroeconomic model in which households see children as consumption goods and different generations are not connected by any form of altruism. This does not have to be the case. In general, children can be seen as consumption or investment goods and there are two forms of altruism. The first form of altruism is parental altruism where parents internalize (parts of) the utility of their children. In this case, one can say parents care about their children. Parents still see their children as consumption goods and potentially transfer money to them as outcome of their utility maximization.[1]

The second form is reversed altruism where children internalize the utility of their parents. Now, children care about their parents and the children' utility maximization might result in transfers from children to parents. As parents anticipate the potential transfers and choose the number of children accordingly, in this case children can be described as investment goods: Parents "invest" in children and the return is a form of old-age insurance when children transfer some of their income to their parents.[2]

Given the various possibilities to model endogenous fertility choices one might wonder how the model choice can affect the results of the family policy reforms in Chapters 2 to 4: Are

[1] See for example the work by Razin and Ben-Zion (1975) and Barro and Becker (1989) who paved the way for a large body of literature using models with parental altruism.

[2] See for example Carmichael (1982) who introduces reversed altruism, or Boldrin and Jones (2002). Nugent (1985) specifies the old-age insurance motive for getting children.

the fertility reactions similar across model setups? How do optimal choices of households differ given different transfer possibilities? How is the impact on the government budget and on overall welfare?

The aim of this chapter is to answer these questions. Then, the results of this chapter can be seen as some form of robustness check to the previous chapters. I set up three three-period OLG models that are identical, only the household sector looks different in each model: In the first model, the household exhibits parental altruism (PA), in the second it has reversed altruism (RA), and in the third model there is no altruism (NA). I assume an imperfect altruism, that is, people do not internalize the complete utility of their relatives but the consumption level of the period when both generations are alive simultaneously.

All three models are calibrated to reflect some key aspects of a developed country like Germany. After the calibration three policy reforms are simulated: A 40%-increase of monetary child benefits, a doubling of the public child care coverage rate and an increase of the tax rate by 10%.

A first finding is the calibration itself: Without intra-family transfers being a target, there are positive transfers from parents to children, that is bequests, in the calibrated economy with parental altruism. In the case of reversed altruism, the calibrated economy reaches the corner solution with zero transfers. Without the non-negativity constraint transfer would become negative. These results are in line with empirical evidence for Germany that finds positive bequests.[3] Since the model with parental altruism fits the targets best and also generates positive bequests, the analysis of the policy experiments emphasizes this model.

Concerning the model mechanisms, three are three main findings: (1) Independent of the model setup, family policies do not necessarily lead to more financial resources for the government, even when the fertility rate is as low as in Germany and family policies are quite effective.

(2) In the model without altruism a reform of monetary benefits leads to better welfare results compared to a public child care expansion. In the models with altruism it is the other way round. One cause for this is that public child care has a more positive impact on consumption. Models with altruism weight consumption relatively more compared to the model without altruism and therefore welfare is higher.

Finally (3), in the model with parental altruism the household has an additional choice variable, the transfers. By lowering the level of transfers the household can for instance dampen

[3] See for example Leopold and Schneider (2009) who use SOEP data and find evidence for positive bequests in Germany.

the negative effect of a reduction of pension benefits on consumption. Alternatively, an increase of the tax and replacement rate does not affect any choice of any generation as long as the redistribution of the government is anticipated by a corresponding adjustment of bequests.

The remainder of this chapter is organized as follows: Section 5.2 discusses the three model setups, presents the equilibrium definitions and describes the different equilibrium characterizations. The calibration is presented in Section 5.3. The results of the policy experiments are analyzed in Section 5.4 and Section 5.5 concludes.

5.2 The Models

This section describes the model setups, equilibrium definitions and equilibrium characterizations of the different three-period OLG models - one with parental altruism, one with reversed altruism and one without altruism. Most of the model setup applies to all three models, the key differences are the set of choice variables and the budget constraints: In the models with altruism one generation can choose to transfer resources to the other which also affects the budget constraints. There is a homogeneous household that decides on consumption, fertility, savings and intra-family transfers when generations are linked by altruism. Firm sector and government are identical across all three model setups.

5.2.1 Households

Each individual lives for three periods, labeled by j, and there is certainty about the time of death. The three periods consist of childhood ($j = c$), young adulthood ($j = y$) and old adulthood ($j = o$). Children do not decide anything but create costs for their parents. In period t, young adults decide on consumption c_t^y, savings s_t and the number of children n_t which directly affects their labor supply ℓ_t and budget since the children have to be taken care of. In the case of reversed altruism, young-aged adults ("children") also decide on the level of transfers t_t they want to give to the old-aged adults ("parent"). Old adulthood is the retirement phase, children are out of the house and people live off their private savings and public pension benefits. With reversed-altruism old-age households have an additional source of retirement wealth: the transfers from their children. In the case of parental altruism, old-aged parents split their retirement wealth between own consumption and inheritance to their (young-aged) children, t_{t+1}.[4]

[4]Equivalently to the model setup in the previous chapters, a household consists of one parent and there is no gender.

Child-rearing Costs and Time Budget

Child-rearing costs are defined in the same way as in the model of Chapters 2 to 4 and consist of two components: First, parents have to pay a consumption goods to cover the direct costs of their children. And second, children require b units of parents' time during which parents cannot work. Parents face these costs in their young adulthood, as long as their children are not parents themselves.

There are two potential fertility subsidies provided by the government: There can be a direct monetary transfer, θ^a, that covers parts of the costs in terms of goods. And there can be public child care, θ^b, that reduces the time that parents are required to spend with their children.[5]

Each person is endowed with one unit of time in each period t. With this time she can do two things: She either works, ℓ_t, or she raises her children, bn_t. Child care facilities θ^b will decrease the time a parent has to spend with her children, so that the time constraint is:

$$\ell_t + (b - \theta^b)n_t \leq 1. \tag{5.1}$$

Preferences, Budget and Lifetime Optimization Problem

A person born in period $t-1$ has the following life time utility:

$$U_{t-1} = \frac{c_{y,t}^{1-\sigma}}{1-\sigma} + \eta \frac{n_t^{1-\psi}}{1-\psi} + \gamma \frac{c_{o,t}^{1-\sigma}}{1-\sigma} + \beta \left[\frac{c_{o,t+1}^{1-\sigma}}{1-\sigma} + \rho \frac{c_{y,t+1}^{1-\sigma}}{1-\sigma} \right] \tag{5.2}$$

c_t^y is own consumption during young adulthood, c_t^o is consumption of the old-aged parents when the person born in $t-1$ is still young, c_{t+1}^o is own consumption during old adulthood and c_{t+1}^y is the consumption of the children when they are in young adulthood and the person born in $t-1$ is in old adulthood. n_t is the fertility rate. σ is the CRRA coefficient for consumption that is constant across all generations, η is the utility weight of fertility, ψ is the slope coefficient for fertility, β is the discount factor, γ is the parameter for reversed altruism and ρ the parameter for parental altruism.

In the case of parental altruism, that is, parents care for their children and internalize their consumption, the parameter ρ is positive and γ is zero. In the case of reversed altruism, children' s utility includes the consumption level of their parents and therefore γ is positive and ρ is zero. In the case of no altruism both parameters are zero.

[5] As the assumptions concerning child care are the same as in Chapters 2 to 4, the same limitations apply: Parents cannot hire nannies privately and when public child care is offered it is mandatory.

Note that there is no "full altruism" in this model. In this case, family members would internalize the utility of their parents or children. This would lead to a dynastic utility function as for example in Barro and Becker (1989) or De La Croix and Doepke (2003). In the present model, I assume that family members do not internalize the utility but the consumption of their relatives, as done by Boldrin and Jones (2002). This cuts the potential dynastic link and people only care for the direct relatives in the period both generations are alive simultaneously, that are, parents in case of reversed altruism or children with parental altruism.

Parental Altruism

In the case of parental altruism, the lifetime optimization problem of a person born in $t-1$ is

$$\max_{\{c_{y,t},c_{o,t+1},n_t,\ell_t,s_t,t_{t+1}\}} U_{t-1}$$

$$s.t.$$

$$c_t^m + s_t + (a - \theta^a)n_t = (1-\tau)\ell_t w_t + t_t \tag{5.3}$$

$$c_{t+1}^o + n_t t_{t+1} = (1 + r_{t+1})s_t + P_{t+1} \tag{5.4}$$

$$c_{t+1}^m + s_{t+1} + (a - \theta^a)n_{t+1} = (1-\tau)\ell_{t+1} w_{t+1} + t_{t+1} \tag{5.5}$$

$$t_{t+1} \geq 0. \tag{5.6}$$

τ is a labor income tax rate, r is the real interest rate, w is the wage rate, P are pension benefits from a public pay-as-you-go pension system and t_{t+1} are transfers - or "bequests" - from old to young households. Note that t_t is no choice variable in this case but from the perspective of the person born in $t-1$ this is an exogenous source of income. During young adulthood, a person can use her income for either consumption, private old-age savings or for rearing children. On the income side a young adult has her net labor income and bequests from her parents.[6] When an old person is retired she receives pension benefits as well as her private old-age savings. This income can be either consumed or inherited to her children. As an old-aged parent chooses the optimal level of bequests and these bequests affect the budget of the young-aged child, she takes into account the budget of her child. Finally, bequests cannot be negative, or put it differently, parents cannot take money from their children.

Reversed Altruism

[6]Note that in general s_t can be negative. However, then there would be a negative capital stock. Thus, the equilibrium interest rate will always be such that $s_t > 0$. This is true for all three model setups.

In the case of reversed altruism, the lifetime optimization problem of a person born in $t-1$ is

$$\max_{c_{y,t},c_{o,t+1},n_t,\ell_t,s_t,t_t} U_{t-1}$$

$$s.t.$$

$$c_t^m + s_t + (a-\theta^a)n_t + t_t = (1-\tau)\ell_t w_t \tag{5.7}$$

$$c_{t+1}^o = (1+r_{t+1})s_t + P_{t+1} + n_t t_{t+1} \tag{5.8}$$

$$c_t^o = (1+r_t)s_{t-1} + P_t + n_{t-1}t_t \tag{5.9}$$

$$t_t \geq 0. \tag{5.10}$$

The budget constraints look very similar to the case of parental altruism, however, there is one big difference. Now, t_t are transfers from young-aged children to their old-age parent. Consequently, the only source of income during young adulthood is labor income. This income has to be split between savings, consumption, child rearing and giving transfers to the parent.[7] In the period of old adulthood, there are three sources of income, public pension benefits, private old-age savings and transfer from the children. All income is entirely consumed.

Note that in this case parents are like a public good for children and it is not clear how much transfers shall be provided. Thus, there is scope for strategic interaction between siblings regarding how much transfer each one provides to the parent. As simplification for this intra-generational strategic interaction I assume that siblings behave cooperatively, meaning that everyone gives the same transfers once the generation of children has decided about the level.

Moreover, there is an inter-generational strategic interaction. This interaction arises from the following: Parents know that their fertility and saving decisions when young affect their consumption possibilities when old. However, the consumption possibilities will affect the transfer decision of their children one period later. The number of children determines the total transfers a parent receives from all of her children. Thus, children adjust their individual transfers accordingly. Savings affect private old-age retirement wealth and therefore the consumption level of parents. As this is also included in the children's utility function, the children will adjust their transfer decision accordingly. Parents know and internalize these mechanisms such that, in turn, they adjust their savings and fertility decision accordingly. For this strategic interaction I assume that parents and children play a Nash game and take best responses of the other as given. This setup is very similar to Boldrin and Jones (2002) and therefore the equilibrium characterization

[7] In this model t_{t+1} is exogenous for the person born in $t-1$.

in Section 5.2.4 will follow their approach.

No Altruism

In the case of no altruism, the lifetime optimization problem of a person born in $t-1$ is

$$\max_{\{c_{y,t},c_{t+1,o},n_t,\ell_t,s_t\}} U_{t-1}$$

$$s.t.$$

$$c_{y,t}+s_t+(a-\theta^a)n_t=(1-\tau)\ell_t w_t \tag{5.11}$$

$$c_{o,t+1}=(1+r_{t+1})s_t+P_{t+1}. \tag{5.12}$$

In this model setup there are no intra-family transfers, people earn labor income during young adulthood and can spend it for consumption, savings or child-rearing. Public pension benefits and private old-age savings are entirely consumed during old adulthood.

5.2.2 Government

Before discussing the budget of the government, I normalize the size of the young cohort N_t^y to 1. Then, the size of the cohort of children N_t^c is n_t. The cohort of old aged people N_t^o has the mass $1/n_{t-1}$.

The government collects a flat labor income tax τ that all employees have to pay. With the tax revenues the government finances two types of family subsidies θ^a and θ^b, a public pay-as-you-go pension system with benefits P, and residual expenditures G. The budget has to be balanced in every period.

The first type of family policy are direct monetary benefits, θ^a. They are paid per child and reduce the direct child rearing costs. The second type of family policies is subsidized public child care, θ^b, which reduces the opportunity costs of foregone earnings. For subsidized child care the government has to employ nannies so that they can take care of the children. Each nanny can take care of Q children. Hence, the government demand for nannies is determined by the number of children n_t and the children-to-nanny ratio Q:

$$L_t^N=\theta^b\frac{n_t}{Q}.$$

The government offers nannies a wage of w^N such that government expenditures stay at a minimum but sufficient employees are attracted to work within the nanny sector.

All retirees receive pension benefits P_t. As benefits a person receives a fraction q of her former net labor income:

$$P_t = q(1-\tau)\ell_{t-1}w_{t-1}.$$

All tax revenues that the government does not use for family policies or the pension system are summarized as residual government expenditures G. Expenditures of this category do not affect the agents' decision making. The government budget then looks as follows:

$$\tau \ell_t w_t = \theta^a n_t + L_t^N w_t^N + q(1-\tau)\ell_{t-1}w_{t-1}\frac{1}{n_{t-1}} + G_t. \tag{5.13}$$

5.2.3 Firms

Firms are active in a competitive market so that prices, interest rate r_t and wage rate w_t, are taken as given. Input factors are labor L_t^M and capital K_t, and output Y_t^M is produced. A is the total factor productivity and the capital stock depreciates with rate δ in each period. The production function is of Cobb-Douglas form

$$Y_t^M = A(K_t)^\alpha (L_t^M)^{1-\alpha}.$$

Firms maximize profits. Normalizing the price of the output good to 1, the profit maximization problem has the following form:

$$\max_{\{L_t^M, K_t\}} A(K_t)^\alpha (L_t^M)^{1-\alpha} - w_t L_t^M - r_t K_t. \tag{5.14}$$

5.2.4 Equilibrium Definition and Equilibrium Characterization

Parental Altruism

Given a tax rate τ and family policies θ^a and θ^b a stationary equilibrium consists of

- sequences of choices $\{c_y, c_o, s, n, t\}_t$,
- factor prices $w(K_t, L_t^M)$ and $r(K_t, L_t^M)$ and
- a replacement rate $q(\tau, \theta^a, \theta^b, G)$

such that

- households maximize lifetime utility given in equation (5.2) subject to the constraints (5.3), (5.4), (5.5) and (5.6),
- factor prices are consistent with the firms' problem given in equation (5.14),

- the government budget presented in equation (5.13) holds, and

- labor and asset markets clear.

  For asset market clearing, the sum of all savings in the economy has to equal the capital demand from the firm sector. Since the size of households that save is normalized to one, capital demand equals the savings rate:

$$K_t = s_t. \tag{5.15}$$

  On the labor market, people work either in the firm or nanny sector. Consequently, the total labor supplied by the working household has to equal the demand from public child care facilities L_t^N and the firms L_t^M:

$$L_t^M + L_t^N = \ell_t. \tag{5.16}$$

Before I start with the equilibrium characterization, I use the time constraint $\ell_t = 1-(b-\theta^b)n_t$ and put together the two components of child rearing costs, such that I get the expression for total child rearing costs $\phi_t = (a-\theta^a) + (b-\theta^b)(1-\tau)w_t$.

Then, the three first-order conditions are:

$$s_t: \qquad c_{y,t}^{-\sigma} = \beta(1+r_{t+1})c_{o,t+1}^{-\sigma} \tag{5.17}$$

$$n_t: \qquad \phi_t c_{y,t}^{-\sigma} + \beta t_{t+1} c_{o,t+1}^{-\sigma} = \eta n_t^{-\psi} \tag{5.18}$$

$$t_{t+1}: \qquad n_t c_{o,t+1}^{-\sigma} = \rho c_{y,t+1}^{-\sigma}. \tag{5.19}$$

Hence, the six equations (5.3), (5.4), (5.5), (5.17), (5.18), (5.19) and six unknowns describe the household's optimal behavior.

Profit maximization implies that the marginal return of input factors is equal to their price:

$$r = \alpha A \left(\frac{\ell}{s}\right)^{1-\alpha} - \delta, \tag{5.20}$$

$$w = (1-\alpha) A \left(\frac{s}{\ell}\right)^{\alpha}. \tag{5.21}$$

Given the equilibrium definition, the replacement rate q balances the government budget:

$$q = \frac{n_{t-1}}{(1-\tau)\ell w}\left[\tau \ell w - \theta^a n_t - L_t^N w_t^N - G\right]. \tag{5.22}$$

In equilibrium, the government offers exactly $w^N = w$, that is, the nanny wage equals the wage for workers.[8]

Reversed Altruism

Given a tax rate τ and family policies θ^a and θ^b a stationary equilibrium consists of

- sequences of choices $\{c_y, c_o, s, n, t\}_t$,
- factor prices $w(K_t, L_t^M)$ and $r(K_t, L_t^M)$ and
- a replacement rate $q(\tau, \theta^a, \theta^b, G)$

such that

- households maximize lifetime utility given in equation (5.2) subject to the constraints (5.7), (5.8), (5.9) and (5.10),
- factor prices are consistent with the firms' problem given in equation (5.14),
- the government budget presented in equation (5.13) holds, and
- labor and asset markets clear equivalently to equations (5.15) and (5.16).

For the equilibrium characterization, the three first-order conditions are:

$$s_t : \qquad c_{m,t}^{-\sigma} = \beta c_{o,t+1}^{-\sigma} \frac{\partial c_{o,t+1}}{\partial s_t} \tag{5.23}$$

$$n_t : \qquad \phi_t c_{m,t}^{-\sigma} = \eta n_t^{-\psi} + \beta c_{o,t+1}^{-\sigma} \frac{\partial c_{o,t+1}}{\partial n_t} \tag{5.24}$$

$$t_t : \qquad c_{m,t}^{-\sigma} = \gamma n_{t-1} c_{o,t}^{-\sigma} \tag{5.25}$$

In the first-order conditions in equations (5.23) and (5.24) the partial derivates $\frac{\partial c_{o,t+1}}{\partial s_t}$ and $\frac{\partial c_{o,t+1}}{\partial n_t}$ show up. This is because of the inter-generational interaction already mentioned. Parents decide about savings and number of children before their children decide about the transfer to their parents. Thus, parents internalize the potential transfer for their savings and fertility decision. However, the children's transfer choice also depends on the savings and fertility choice of the parents. Thus, there is a Nash game in which both generations take the best response of the other as given. With this assumption on the strategic interaction I can solve for the first-order conditions.[9] The detailed calculations are shown in Appendix 5.A.1.

[8] This requires that the demand for nannies is strictly smaller than the labor supply, otherwise there would be no production and consequently no output in the economy. This also holds true for the other two model setups and their equilibrium characterizations.

[9] This approach follows the one of Boldrin and Jones (2002).

The final first-order conditions are:

$$s_t: \qquad c_{m,t}^{-\sigma} = \beta c_{o,t+1}^{-\sigma}(1+r_{t+1}) \times \frac{1}{\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}} + 1} \tag{5.26}$$

$$n_t: \qquad \phi_t c_{m,t}^{-\sigma} = \eta n_t^{-\psi} + \beta c_{o,t+1}^{-\sigma} \frac{\Gamma \times [\frac{1}{\sigma}(\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}}) + 1] - \Phi \times \frac{\sigma-1}{\sigma}(\gamma^{-\frac{1}{\sigma}} n_t^{-\frac{1}{\sigma}})}{(\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}} + 1)^2} \tag{5.27}$$

$$t_t: \qquad c_{m,t}^{-\sigma} = \gamma n_{t-1} c_{o,t}^{-\sigma} \tag{5.28}$$

where

$$\Gamma = (1-\tau)w_{t+1} - s_{t+1} - \phi_{t+1} n_{t+1},$$
$$\Phi = (1+r_{t+1})s_t + P_{t+1}.$$

Factor prices and the replacement rate are determined equivalently to the previous equilibrium characterization, so they are

$$r = \alpha A \left(\frac{\ell}{s}\right)^{1-\alpha} - \delta,$$
$$w = (1-\alpha) A \left(\frac{s}{\ell}\right)^{\alpha},$$

and

$$q = \frac{n_{t-1}}{(1-\tau)\ell w}\left[\tau \ell w - \theta^a n_t - L_t^N w_t^N - G\right]. \tag{5.29}$$

The government again chooses $w^N = w$.

No Altruism

Given a tax rate τ and family policies θ^a and θ^b a stationary equilibrium consists of

- sequences of choices $\{c_y, c_o, s, n\}_t$,
- factor prices $w(K_t, L_t^M)$ and $r(K_t, L_t^M)$ and
- a replacement rate $q(\tau, \theta^a, \theta^b, G)$

such that

- households maximize lifetime utility given in equation (5.2) subject to the constraints (5.11), (5.12),
- factor prices are consistent with the firms' problem given in equation (5.14),

- the government budget presented in equation (5.13) holds, and
- labor and asset markets clear equivalently to equations (5.15) and (5.16).

For characterizing the equilibrium, the two first-order conditions are:

$$s_t: \quad c_{y,t}^{-\sigma} = \beta(1 + r_{t+1})c_{o,t+1}^{-\sigma} \tag{5.30}$$

$$n_t: \quad \phi_t c_{y,t}^{-\sigma} = \eta n_t^{-\psi}. \tag{5.31}$$

Hence, the four equations (5.11), (5.12), (5.30), (5.31), and the time budget (5.1), and five unknowns describe the household's optimal behavior.

Factor prices and the replacement rate are determined equivalently to the previous equilibrium characterizations, so they are

$$r = \alpha A \left(\frac{\ell}{s}\right)^{1-\alpha} - \delta,$$

$$w = (1-\alpha)A\left(\frac{s}{\ell}\right)^{\alpha},$$

and

$$q = \frac{n_{t-1}}{(1-\tau)\ell w}\left[\tau \ell w - \theta^a n_t - L_t^N w_t^N - G\right]. \tag{5.32}$$

Again, the government chooses $w^N = w$.

All models are solved by solving simultaneously the system of equations that characterize the equilibrium.

5.3 Calibration and Benchmark Economies

The primary interest of this chapter is to see what happens to the effects of family policies in a country like Germany when the model setup also exhibits altruism.

Furthermore, when individuals are altruistic they might want to give transfers to their relatives. In the case of parental altruism these transfers are bequests from parents to children. In the model with reversed altruism the transfer go the other way round, from children to parents. Since empirical evidence for developed countries shows the existence of positive bequests the focus lies on the model that can generate this pattern, that is the model setup with parental altruism.[10]

[10] For empirical evidence for positive bequests in Germany, see for example Leopold and Schneider (2009).

Thus, parameters and targets are chosen such that the model with parental altruism produces plausible results that can be connected to a developed country. Nevertheless, the model setups with reversed altruism and no altruism are calibrated to the same moments and exhibit the same externally set parameters. Given the targeted moments are hit sufficiently well, this procedure allows to compare family policy reforms across different model environments.[11]

In total, there are 16 parameters with the model setups with altruism and 15 without.[12] Two of these parameters are targets, nine are set externally and five - or four for the case of no altruism - are calibrated.

5.3.1 Parameters and Moments

In general, the parameters are chosen such that the resulting benchmark economy replicates an economy with low fertility rate, sufficiently large savings rate and large government sector, that is large tax and replacement rate. This corresponds to a developed economy like the one of Germany.

So, there are four targeted moments while the rest is determined externally. The four targeted moments are the fertility, the replacement and the savings rate and the residual government expenditure. To hit the four targets there are either five or four parameters. As a consequence, the externally determined parameters are the same across all three model setups while the calibrated parameters differ.

Before discussing the calibrated parameters I start with the externally determined parameters.

Externally Determined Parameters

Even if the model does not mimic Germany as in the previous chapters, I basically use the same parameter values as before. In this way, I ensure that the model corresponds to a developed economy.

Therefore, I set the parameter that describes the time cost of children b to 0.31, the public child care coverage θ^b to 0.062, that is 20% of b, and the children-to-nanny ratio Q to 5. These values are based on Schoonbroodt (2014), Bauernschuster et al. (2013) and Destatis (2014a).[13]

[11]In this way, all three models describe the same benchmark situation which is necessary for a comparison. Parts of the underlying parameters, however, are different. This indicates that the underlying mechanisms that lead to the same benchmark steady state differ across the models. In the end, those different mechanisms determine the differences in the effects of family policy interventions in different models.

[12]In case of "no altruism" there is no coefficient for altruism in the utility function.

[13]A more detailed description and discussion can be found in Chapter 2.

Table 5.1: Externally Set Parameters

Parameter		
CRRA utility	σ	1.5
discount factor	β	0.21
time for children	b	0.31
nanny-to-children ratio	Q	5.0
TFP	A	3.5
capital intensity	α	0.32
depreciation rate	δ	1
labor tax rate	τ	0.396
public child care	θ^b	0.062

For the manufacturing sector, capital intensity α is 0.32, total factor productivity A is normalized to 3.5 and the period-wise depreciation rate δ is set to be 100%. The combined tax and contribution rate τ is 39.6%. And finally, the CRRA coefficient for consumption σ is 1.5 and the discount factor β is 0.21.[14]

All parameters can also be found in Table 5.1.

Calibrated Parameters

Across all model setups, the parameters that are determined by calibration are

- the weight on fertility in the utility function η,
- the CRRA coefficient for fertility ψ,
- the cost of children in terms of consumption a,
- the monetary child benefits θ^a,
- the coefficient for parental altruism ρ,
- and the coefficient for reversed altruism γ.

In the case of parental altruism, the first five parameters are calibrated, in case of reversed altruism the first four parameters and the last, and in the case of no altruism the first four parameters. Table 5.2 gives an overview about which model exhibits which parameter.

To pin down the parameters, I have four targets that are hit jointly. The calibration procedure is equivalent to the one described in Section 2.4 of Chapter 2: The parameters are chosen to jointly minimize a distance function which is the sum of all absolute percentage deviations of the model moments from their data counterparts. Therefore the calibration

[14] The annual discount factor $\hat{\beta}$ is 0.95. In a three-period model, one periods corresponds to roughly 30 years, so the period-wise discount factor β is $\hat{\beta}^{30} = 0.95^{30} = 0.2146 = \beta$.

Table 5.2: Calibrated Parameters in Different Model Setups

Parameter		PA	RA	NA
fertility weight	η	✓	✓	✓
CRRA coefficient fertility	ψ	✓	✓	✓
cost of children	a	✓	✓	✓
direct monetary child benefit	θ^a	✓	✓	✓
parental altruism	ρ	✓	no	no
reversed altruism	γ	no	✓	no

algorithm is the same as described in Appendix 2.A.3 of Chapter 2 but there is no stationary distribution that has to be found.

As already mentioned I want the benchmark economy to mimic an economy with low fertility and high replacement rate. To reach this I use German data targets. The average fertility rate and therefore the data target is 1.38. The corresponding model counterpart is twice the model fertility rate n. This is because n is the number of children of one parent while in the data, fertility is related to two parents. The replacement rate in Germany is 55.3%. As data target I use half of the replacement, 27.7%, for the following reason: In reality, retirees receive pension benefits for a period that is approximately half as long as their working life. In the model, people receive half of the pension benefits for a period as long as their working life. Both expressions lead to the same result and since the number of periods is fixed, I have to choose the second expression.

Furthermore, the data target for the savings rate is 9.4% which corresponds to the savings rate in Germany in 2014.[15]

The final target is that residual government expenditures G are zero. Obviously, this is not taken from empirical data. Instead, this condition supports the comparison of the three model setups. Why is that? With this target the government cannot re-allocate financial resources from or to a component that does not affect households' decision directly. This makes the resulting benchmark economies more comparable.

Before discussing the calibration results, one side-comment concerning the data targets should be made: One natural moment in models with altruism are transfers. However, in this comparative setup this is not possible. Parental altruism leads to transfers going from parents to children, reversed altruism leads to transfers the other way round, and no altruism results in no transfer. By defining an explicit transfer target, two out of three models will never be able to reach this target and a comparison would be impossible. Moreover, I do not want to calibrate the models to reproduce a particular economy with a specific transfer pattern, so, it is

[15]See Bundesbank (2015), Table XI.8.

reasonable not to choose transfers as a target but leave it unrestrained and see which transfer pattern is generated by the calibrations.

The data targets and their model counterparts can be seen in Table 5.3. The model with parental altruism (PA) hits all targets while the model with reversed altruism (RA) exhibits a savings rate that is 30% too low. For the model without altruism (NA) the replacement rate is 27.2% and therefore slightly lower than the target. All other moments are hit perfectly.

Table 5.3: Targeted Moments

Targeted Moment		Data Target	PA	RA	NA
fertility rate	$2n$	1.38	1.38	1.38	1.38
replacement rate	q	0.28	0.28	0.28	0.27
savings rate	s	0.09	0.09	0.07	0.09
residual government expenditures	G	0	0.0	0.0	0.0

The resulting parameters are presented in Table 5.4. As already mentioned, the coefficients for the altruism in the utility function are not relevant for all models, and the zeros in the corresponding lines are not a result of the calibration procedure but set before the calibration.

Three parameters values attract some attention. First, the fertility weight η is only 0.04 in the NA-model. This means that the weight of fertility in the utility function is very low relative to consumption.[16] Second, the child costs a are only 0.07 and therefore very low for the NA-model. This makes children very cheap compared to consumption goods whose prices is normalized to one. Actually, monetary child benefits θ^a are higher than the direct child costs a. However, children are still costly for parents since the time costs b of children are relatively higher. Nevertheless, this creates a very high fertility elasticity with respect to price changes and can lead to strong reactions of the fertility rate in case of policy changes.

Third, the slope coefficient for fertility ψ is relatively low (0.3) for the RA-model. In particular, ψ is significantly lower compared to the PA-model which means that the marginal utility of fertility is lower in case of reversed altruism.

Table 5.4: Calibrated Parameters

Parameter		PA	RA	NA
fertility weight	η	0.36	0.93	0.04
CRRA coefficient fertility	ψ	2.53	0.32	1.82
cost of children	a	0.35	0.53	0.07
parental altruism	ρ	0.66	0	0
reversed altruism	γ	0	0.97	0
direct monetary child benefit	θ^a	0.20	0.19	0.21

[16]The consumption weight is normalized to one.

5.3.2 Benchmark Economies

Table 5.5 shows the three benchmark economies for the three different models. A comparison of the different benchmark economies generated by different model setups does not make much sense as one would compare three different models but nevertheless, two points are worth mentioning.

One can directly see the effect of a developed economy with a relatively large public pension sector: In the model with reversed altruism transfers from children to parents are zero, thus, households do not use the intra-family old-age insurance channel.[17] In line with this observation, the bequests are positive in the model with parental altruism: Old-age parents bequeathe approximately one fifth of their retirement wealth to their children.

Since the model with parental altruism and the one without altruism hit the targets very well, the fertility and savings rate are the same and as a consequence factor prices are the same as well.[18] Given that the model with reversed altruism generates a lower savings rate, the wage rate (interest rate) is relatively lower (higher).

Table 5.5: Choices and Prices

Parameter		PA	RA	NA
consumption young	c_y	0.52	0.46	0.60
consumption old	c_o	0.54	0.35	0.62
fertility rate	n	0.69	0.69	0.69
savings rate	s	0.09	0.06	0.09
labor supply	ℓ	0.83	0.83	0.83
transfers	t	0.12	0	—
interest rate	r	3.89	5.29	3.89
wage rate	w	1.19	1.05	1.19

5.4 Policy Experiments

To see how the different models affect the effectiveness of family policies, I introduce the following three reforms, two related to family policies and one to the tax rate:

1. An increase of monetary benefits, θ^a, by 40%.

2. An increase of public child care, θ^b, from 20% coverage to 40%.

3. An increase of the tax rate, τ, by 10%.

[17] One could wonder why the model with reversed altruism with zero transfers does not result in the same allocation as the model with no altruism. However, the altruism in the utility function is independent of the transfers and therefore children' utility still internalizes the consumption level of the elderly which affects the allocation.

[18] Via the time budget, identical fertility rates lead to identical labor supplies. And because of the normalized population size, identical labor supply and savings rate lead to the same capital-to-labor-ratio.

Every reform is simulated in all three model setups. In the discussion of the results the emphasis lies on the model with parental altruism as, given the parameter environment, this model fits best.

5.4.1 Increase of Monetary Benefits

The first reform which is analyzed is an increase of direct monetary child benefits θ^a by 40%.[19] First of all, one can see in Table 5.6 that in the model framework with parental altruism higher monetary benefits do not lead to additional financial resources for the government: The replacement rate of the public pension system has to be cut by 32% in order to finance the additional family subsidy. For the household, the price effect of lower child-rearing costs dominates. During young adulthood consumption and savings are reduced by over 10% such that the household can increase the number of children by 9%. And since savings drop stronger than labor supply, the wage rate decreases which in turn reduces labor income.

Table 5.6: 40%-increase of monetary benefits

		PA	RA	NA
consumption young	c_y	- 10.2%	- 10.9%	+ 26.9%
consumption old	c_o	-5.0%	-3.9%	-18.1%
fertility rate	n	+8.7%	+12.4%	+94%
savings rate	s	-12.8%	-16.9%	+122%
labor supply	ℓ	-2.4%	-2.4%	-19.3%
transfers	t	-66.7%	0	—
welfare change		-7.1%	-4.6%	+18.8%
interest rate	r	+10.8%	+12.7%	-60.7%
wage rate	w	-3.6%	-3.8%	+36.1%
replacement rate	q	-32.1%	-35.7%	-92.9%

Finally, bequests are reduced tremendously by 67%. This is a direct consequence of the more expensive family policy for the government. Since public pensions are cut by almost a third and private savings also decrease, old households decide to give less transfers to their children. With this balancing mechanism, on the one hand, the household can keep old-age consumption relatively constant and has to reduce it by only 5%. On the other hand, it further decreases income for young adults.

Overall, the household faces a welfare loss of 7.1% which is mainly driven by the drop of consumption during young adulthood.

For the model with reversed altruism, the results are very similar to the model with parental altruism. Fertility increases while savings and consumption in young adulthood go

[19] In Germany, this would correspond to an increase of *Kindergeld* from 184 to 258 EUR per month and child.

down. The government has to cut pensions by over a third and the capital-to-labor ratio becomes smaller. As a consequence consumption during old-age decreases. Transfers from children to parents remain zero, so, the public and private old-age insurance remain "too large to trigger" this third channel of old-age insurance.[20] In total, welfare decreases by 4.6%.

Due to the very low total child rearing costs in the model without altruism, the fertility rate almost doubles as a result of the higher child benefits. This huge increase of fertility maps into a huge decline of labor supply. This leads to a wage rate that is 36% higher than before. Thus, labor income also increases significantly which has a strong positive effect on consumption during young adulthood. For the government, the family policies become so expensive that the replacement rate of the pension system has to be reduced by 93%.[21] Higher fertility and consumption during young adulthood lead to a welfare increase of 18.8%.

5.4.2 Public Child Care Expansion

In the second experiment, the government doubles public child care coverage. The effects of this child care expansion are very similar to those of the benefits reform, as the results in Table 5.7 show. With parental altruism the households shift resources from savings (-5%) and consumption (-2%) to child rearing such that the fertility rate increases by 6%. Moreover, the positive effect of child care on labor supply dominates such that individual labor supply increases as well. For the government this means that the tax base gets larger and the necessity to cut the replacement rate in the pension system is relatively mild. Additionally, for the government public child care is the less costly or more efficient of the two family policies.[22] Therefore, the government has to re-allocate less money from the pension system to the family policies to finance the child care reform and has more financial resources that can finance pension benefits. Because of this effect the replacement rate is only 4% lower compared to before the reform.

As a consequence of less public and private old-age savings, old households decide to inherit 17% less to their children which allows the old households to consume more during retirement

[20] For example, when the tax rate is sufficiently low such that the pension system is sufficiently small, the transfers become positive. When the governmental sector shrinks by more than 90%, transfers from children to parents starts to be positive. An economy with such a small government sector would rather correspond to a developing country. Moreover, for China for example, Cai et al. (2006) or Ebenstein and Leung (2010) find positive transfers from children to parents that are for old-age insurance. So, one could conjecture that models with parental altruism fit better to developed countries while models with reversed altruism are better suited for developing countries.

[21] The government also needs to higher twice as much nannies as before, so the budgetary effect is larger than the direct effect of the increase of monetary child benefits.

[22] Given the calibration of the model, public child care is the more efficient family policy as long as the children-to-nanny ratio Q is larger than roughly 1.5. The actual value used is 5.0. Here, efficiency is measured in additional fertility when putting one additional EUR into one of the two family policies.

Table 5.7: Doubling of Public Child Care

		PA	RA	NA
consumption young	c_y	-1.7%	-2.3%	+14.9%
consumption old	c_o	+2.0%	+1.9%	-8.0%
fertility rate	n	+5.6%	+6.6%	+98%
savings rate	s	-5.3%	-6.2%	+44%
labor supply	ℓ	+4.2%	+3.6%	-9.6%
transfers	t	-16.7%	0	—
welfare change		+0.6%	+1.5%	+12.4%
interest rate	r	+6.9%	+7.6%	-35.7%
wage rate	w	-2.5%	-1.9%	+16.8%
replacement rate	q	-3.6%	-3.6%	-39.3%

compared to before. Overall welfare increases by 0.6% which is mainly driven by the higher fertility rate which overcompensates the loss of consumption during young adulthood.

In the case of reversed altruism, the household reactions to higher child care coverage is very much the same as with parental altruism. Based on the same argument for the monetary benefits reform, transfers from children to parents remain zero. Welfare increases by 1.5% which is caused by two effects: On the one hand, fertility increases by 7%, and on the other hand consumption of old households increases by 2%. Since in the model with reversed altruism old-age consumption of parents also enters the lifetime utility of a young household, this increase drives the positive welfare result.

In the model without altruism, the effects are very strong again. Fertility almost doubles and all the other choices react similarly to the monetary benefits reform but less strongly. Welfare increases by 12.4%.

Thus, independent from the exact size of the effects one observation can be made by comparing the two reforms and relating the results to the previous chapters: The welfare ranking of family policy reforms depend on the underlying model. In the model setup with no altruism, an increase of monetary child benefits leads to higher welfare gains compared to the public child care expansion. This holds true for the results in all chapters. In the models with parental or reversed altruism it is the other way round: The expansion of public child care leads to better welfare outcomes.

One potential explanation is the form of warm-glow altruism I assume. Consumption determines a larger share of lifetime utility compared to the model without altruism. Moreover, public child care has a positive effect on labor supply and therefore labor income.This positive income effect leads to a positive effect on consumption. These two arguments taken together might be one explanation for the observed welfare changes.

5.4.3 Increase of the Tax Rate

The last reform that is analyzed is not a family policy reform but an increase of the tax rate by 10%. In all three models the tax increase leads to higher replacement rates, so there is no massively negative impact of higher taxes as observed in Chapter 4.

The results for the model with parental altruism are very interesting. Basically, the household does not change any of its choices but one: Bequests increase by 33%.[23] The higher tax rate leads to higher pension benefits. This surplus for retirees is transmitted from the old-aged parents directly to their children. Hence, the consumption of old households does not change because of the reform. The level of bequests the children receive is (almost) exactly the amount they have to pay for the higher tax rate. Consequently, young households do not change their behavior but use the additional bequests to pay the additional taxes.

In the end nothing has changed besides the fact that the government shifts money from the young to the old households. Indeed, since the households are connected altruistically they shift the same amount back from the old to the young.[24]

For the other two model setups, there is a negative income effect and therefore consumption and overall welfare decrease. Interestingly, for the fertility choice the price effect of higher taxes dominates the income effect and fertility goes up. The price effect consists of lower foregone earnings of the household since the net labor income is lower because of higher taxes.

Table 5.8: 10%-Increase of the Tax Rate

		PA	RA	NA
consumption young	c_y	0.0	-4.1%	-7.1%
consumption old	c_o	0.0	-2.8%	-4.4%
fertility rate	n	0.0	+2.1%	+30%
savings rate	s	0.0	-6.2%	-11.1%
labor supply	ℓ	0.0	0.0	-6.0%
transfers	t	+33.3%	0	—
welfare change		-0.2%	-2.9%	-5.9%
interest rate	r	0.0	+4.5%	+5.4%
wage rate	w	0.0	-1.0%	-1.7%
replacement rate	q	+24.6%	+24.2%	+24.5%

[23]The results of this reform can be seen in Table 5.8. Note that 0.0 means that the change is not exactly zero but is smaller than 0.005%.

[24]This result is in line with the finding of Barro (1974) that the way a government finances its expenditures does not affect household's consumption decisions as long as generations within the household are altruistically connected.

5.5 Conclusion

Putting together the results of the three policy experiments which have been simulated in the three different model setups and relating them to the results of the previous Chapters, several conclusions can be drawn:

For a developed country like Germany where positive transfers from parents to children can be observed, the model with parental altruism might fit best because this is the only model that can generate this transfer pattern. For developing countries or countries where the public pension system is very small, for example in rural China, children are a factor for the old-age insurance of their parents which is why the model with reversed altruism might be better suited.[25] Furthermore, in a model with reversed altruism that mimics a country like Germany, the reductions of public pensions must be very huge until children start to transfer money to their parents. However, thinking of the whole life-cycle it might be possible that the direction of transfers change depending on the life stage families are in.[26]

Family policies do not necessarily lead to more financial resources for the government and higher welfare as it has been the case in the previous chapters. So, even when the fertility rate is far below the generational replacement rate of two children per woman, intensifying fertility subsidies is not *per se* welfare enhancing. It depends on the model setup.

Furthermore, in a model with no altruism an increase of monetary child benefits leads to better welfare results than an expansion of public child care. This order is flipped around in models with parental or reversed altruism. One reason might be the positive income effect of child care combined with the fact that, in models with altruism, consumption can be relatively more important for lifetime utility than in a model without altruism.

Moreover, in a model with parental altruism households have transfers as an additional choice variable, so that a decrease of transfers can reduce potentially negative effects from a shrinking public pension system. Since family policies were welfare superior to pension reforms in Chapter 4 the welfare difference might be smaller when taking into account this additional choice variable.

Finally, in a model with parental altruism, a pension reform in which higher taxes finance higher replacement rates have no effect on households. This is because households adjust their transfers in a way that old-age parents inherit the additional pension benefits to their children who use those bequests to pay the increased tax burden.

[25]See for example Cai et al. (2006) or Ebenstein and Leung (2010) who provide empirical evidence of the old-age insurance motive of parents in rural China.

[26]As proposed for example by Choukmane et al. (2013).

5.A Appendix

5.A.1 Derivation of First-order Conditions in the Case of Reversed Altruism

The FOC for transfers t(5.25) can be transformed to

$$c_{m,t} = (\gamma n_{t-1})^{-\frac{1}{\sigma}} c_{o,t}$$

Plugging in the budgets for young and old in period t, (5.7) and (5.8), leads to

$$(1-\tau)w_t - s_t - \phi_t n_t - t_t = (\gamma n_{t-1})^{-\frac{1}{\sigma}}[(1+r_t)s_{t-1} + P_t + n_{t-1}t_t]$$

This equation can be re-arranged to back out the optimal transfers t_t:

$$\begin{aligned}
(\gamma n_{t-1})^{-\frac{1}{\sigma}} n_{t-1} t_t + t_t &= (1-\tau)w_t - s_t - \phi_t n_t - (\gamma n_{t-1})^{-\frac{1}{\sigma}}[(1+r_t)s_{t-1} + P_t] \\
\iff [\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}} + 1] t_t &= (1-\tau)w_t - s_t - \phi_t n_t - (\gamma n_{t-1})^{-\frac{1}{\sigma}}[(1+r_t)s_{t-1} + P_t] \\
\iff t_t &= \frac{1}{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}} + 1}[(1-\tau)w_t - s_t - \phi_t n_t - (\gamma n_{t-1})^{-\frac{1}{\sigma}}[(1+r_t)s_{t-1} + P_t]]
\end{aligned} \tag{5.33}$$

(5.33) describes the optimal transfer decision (given all the rest).
t_t can also be derived from the budget of the current old, see equation (5.8):

$$\begin{aligned}
c_{o,t} &= n_{t-1}t_t + (1+r_t)s_{t-1} + P_t \\
t_t &= \frac{c_{o,t}}{n_{t-1}} - \frac{(1+r_t)s_{t-1} + P_t}{n_{t-1}}
\end{aligned} \tag{5.34}$$

The optimal transfer choice for sure has to fulfill the budget constraint, hence, (5.33) = (5.34) has to hold. Thus, I plug in the two equations and solve for the variable of interest, $c_{o,t}$:

$$\begin{aligned}
\frac{c_{o,t}}{n_{t-1}} - \frac{(1+r_t)s_{t-1} + P_t}{n_{t-1}} &= \frac{1}{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}} + 1}[(1-\tau)w_t - s_t - \phi_t n_t - (\gamma n_{t-1})^{-\frac{1}{\sigma}}[(1+r_t)s_{t-1} + P_t]] \\
c_{o,t} &= \frac{n_{t-1}}{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}} + 1}[(1-\tau)w_t - s_t - \phi_t n_t - (\gamma n_{t-1})^{-\frac{1}{\sigma}}[(1+r_t)s_{t-1} + P_t]] + [(1+r_t)s_{t-1} + P_t] \\
c_{o,t} &= \frac{n_{t-1}}{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}} + 1}[(1-\tau)w_t - s_t - \phi_t n_t] - \frac{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}}}{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}} + 1}[(1+r_t)s_{t-1} + P_t]] + [(1+r_t)s_{t-1} + P_t] \\
c_{o,t} &= \frac{n_{t-1}}{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}} + 1}[(1-\tau)w_t - s_t - \phi_t n_t] + \frac{1}{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}} + 1}[(1+r_t)s_{t-1} + P_t]].
\end{aligned} \tag{5.35}$$

To be a bit more extensive, the first transformation (row 2 to 3) of the second factor comes from:

$$\frac{n_{t-1}}{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}} + 1} \times (\gamma n_{t-1})^{-\frac{1}{\sigma}} = \frac{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{1-\frac{1}{\sigma}}}{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}} + 1}.$$

The second transformation (row 3 to 4) of the second factor is:

$$1 - \frac{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}}}{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}} + 1} = \frac{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}} + 1}{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}} + 1} - \frac{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}}}{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}} + 1} = \frac{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}} + 1 - \gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}}}{\gamma^{-\frac{1}{\sigma}} n_{t-1}^{\frac{\sigma-1}{\sigma}} + 1}.$$

(5.35) describes the consumption of the current old-aged as a function of their previous decisions s_{t-1} and n_{t-1} given the optimal transfer decision (see equation (5.33) since derived from FOC).

To get the equation and relation of interest (i.e. $c_{o,t+1}$ as function of s_t and n_t, I shift equation (5.35) one period forward.

$$c_{o,t+1} = \frac{n_t}{\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}} + 1}[(1-\tau)w_{t+1} - s_{t+1} - \phi_{t+1}n_{t+1}] + \frac{1}{\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}} + 1}[(1+r_{t+1})s_t + P_{t+1}] \quad (5.36)$$

Now, I can take the relevant derivatives to complete the FOCs, starting with the easy one for s_t:

$$\frac{\partial c^o_{t+1}}{\partial s_t} = (1 + r_{t+1}) \times \frac{1}{\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}} + 1} \quad (5.37)$$

The derivative with respect to n_t looks slightly messier. I define

$$\Gamma = (1-\tau)w_{t+1} - s_{t+1} - \phi_{t+1}n_{t+1},$$
$$\Phi = (1 + r_{t+1})s_t + P_{t+1}.$$

So, equation (5.36) becomes

$$c_{o,t+1} = \frac{n_t}{\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}} + 1}\Gamma + \frac{1}{\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}} + 1}\Phi$$

And the derivative is

$$\frac{\partial c_{o,t+1}}{\partial n_t} = \frac{\Gamma \times (\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}} + 1) - n_t \frac{\sigma-1}{\sigma}(\gamma^{-\frac{1}{\sigma}} n_t^{-\frac{1}{\sigma}}) \times \Gamma}{(\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}} + 1)^2} - \frac{\frac{\sigma-1}{\sigma}(\gamma^{-\frac{1}{\sigma}} n_t^{-\frac{1}{\sigma}}) \times \Phi}{(\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}} + 1)^2}. \quad (5.38)$$

In the first part of equation (5.38), one can multiply in the n_t and put the A together such that the nominator of this first part (FP) becomes

$$FP = \Gamma \times [(\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}} + 1) - \frac{\sigma - 1}{\sigma}(\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}})] \tag{5.39}$$

$$= \Gamma \times [(1 - \frac{\sigma - 1}{\sigma})(\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}}) + 1] \tag{5.40}$$

$$= \Gamma \times [\frac{1}{\sigma}(\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}}) + 1]. \tag{5.41}$$

Finally, the derivative is the following:

$$\frac{\partial c_{o,t+1}}{\partial n_t} = \frac{\Gamma \times [\frac{1}{\sigma}(\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}}) + 1] - \Phi \times \frac{\sigma-1}{\sigma}(\gamma^{-\frac{1}{\sigma}} n_t^{-\frac{1}{\sigma}})}{(\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}} + 1)^2}. \tag{5.42}$$

The two derivatives now can plugged in, such that I get the following three FOCs:

$$s_t: \qquad c_{m,t}^{-\sigma} = \beta c_{o,t+1}^{-\sigma}(1 + r_{t+1}) \times \frac{1}{\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}} + 1} \tag{5.43}$$

$$n_t: \qquad \phi_t c_{m,t}^{-\sigma} = \eta n_t^{-\psi} + \beta c_{o,t+1}^{-\sigma} \frac{\Gamma \times [\frac{1}{\sigma}(\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}}) + 1] - \Phi \times \frac{\sigma-1}{\sigma}(\gamma^{-\frac{1}{\sigma}} n_t^{-\frac{1}{\sigma}})}{(\gamma^{-\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}} + 1)^2} \tag{5.44}$$

$$t_t: \qquad c_{m,t}^{-\sigma} = \gamma n_{t-1} c_{o,t}^{-\sigma}. \tag{5.45}$$

5.A.2 Overview Parameters in Different Model Setups

Table 5.9: Overview Parameters in the Different Model Setups

Parameter		parental altruism	reversed altruism	no altruism
CRRA utility	σ	✓	✓	✓
discount factor	β	✓	✓	✓
time for children	b	✓	✓	✓
nanny-to-children ratio	Q	✓	✓	✓
TFP	A	✓	✓	✓
capital intensity	α	✓	✓	✓
depreciation rate	δ	✓	✓	✓
labor tax rate	τ	✓	✓	✓
public child care	θ^b	✓	✓	✓
parental altruism	ρ	✓	no	no
reversed altruism	γ	no	✓	no
fertility weight	η	✓	✓	✓
CRRA coefficient fertility	ψ	✓	✓	✓
cost of children	a	✓	✓	✓
direct monetary child benefit	θ^a	✓	✓	✓
wasteful government expenditures	G	✓	✓	✓

5.A.3 Further Results of Policy Reforms

In this appendix are the more detailed results for each model setup and each reform individually. The first three tables present the results of an increase of θ^b and the next three tables provide the results for the increase of θ^a. Finally, the last three tables show the results of the increase of τ.

Table 5.10: Increase of θ^b -Parental Altruism

		20% coverage	40% coverage
consumption young	c_y	0.52	0.52
consumption old	c_o	0.54	0.55
fertility rate	n	0.69	0.73
savings rate	s	0.094	0.089
labor supply	ℓ	0.83	0.865
transfers	t	0.12	0.10
interest rate	r	3.89	4.16
wage rate	w	1.19	1.16
replacement rate	q	0.28	0.27

Table 5.11: Increase of θ^b - Reversed Altruism

		20% coverage	40% coverage
consumption young	c_y	0.46	0.45
consumption old	c_o	0.35	0.36
fertility rate	n	0.69	0.73
savings rate	s	0.065	0.061
labor supply	ℓ	0.83	0.86
transfers	t	0	0
interest rate	r	5.29	5.69
wage rate	w	1.05	1.03
replacement rate	q	0.28	0.27

Table 5.12: Increase of θ^b - No Altruism

		20% coverage	40% coverage
consumption young	c_y	0.60	0.69
consumption old	c_o	0.62	0.57
fertility rate	n	0.69	1.37
savings rate	s	0.09	0.13
labor supply	ℓ	0.83	0.75
transfers	t	0	0
interest rate	r	3.89	2.50
wage rate	w	1.19	1.39
replacement rate	q	0.28	0.17

Table 5.13: Increase of θ^a - Parental Altruism

		benchmark	+40%
consumption young	c_y	0.52	0.47
consumption old	c_o	0.54	0.51
fertility rate	n	0.69	0.75
savings rate	s	0.094	0.082
labor supply	ℓ	0.83	0.81
transfers	t	0.12	0.04
interest rate	r	3.89	4.31
wage rate	w	1.19	1.15
replacement rate	q	0.28	0.19

Table 5.14: Increase of θ^a - Reversed Altruism

		benchmark	+40%
consumption young	c_y	0.46	0.41
consumption old	c_o	0.35	0.34
fertility rate	n	0.69	0.77
savings rate	s	0.065	0.054
labor supply	ℓ	0.83	0.81
transfers	t	0	0
interest rate	r	5.29	5.96
wage rate	w	1.05	1.01
replacement rate	q	0.28	0.18

Table 5.15: Increase of θ^b - No Altruism

		benchmark	+40%
consumption young	c_y	0.60	0.76
consumption old	c_o	0.62	0.51
fertility rate	n	0.69	1.33
savings rate	s	0.09	0.20
labor supply	ℓ	0.83	0.67
transfers	t	0	0
interest rate	r	3.89	1.53
wage rate	w	1.19	1.62
replacement rate	q	0.28	0.02

Table 5.16: Increase of τ - Parental Altruism

		benchmark	+10%
consumption young	c_y	0.52	0.52
consumption old	c_o	0.54	0.54
fertility rate	n	0.69	0.69
savings rate	s	0.094	0.094
labor supply	ℓ	0.83	0.83
transfers	t	0.12	0.16
interest rate	r	3.89	3.9
wage rate	w	1.19	1.19
replacement rate	q	0.28	0.34

Table 5.17: Increase of τ - Reversed Altruism

		benchmark	+10%
consumption young	c_y	0.46	0.44
consumption old	c_o	0.35	0.34
fertility rate	n	0.69	0.70
savings rate	s	0.065	0.061
labor supply	ℓ	0.83	0.83
transfers	t	0	0
interest rate	r	5.29	5.53
wage rate	w	1.05	1.04
replacement rate	q	0.28	0.34

Table 5.18: Increase of τ - No Altruism

		benchmark	+10%
consumption young	c_y	0.60	0.56
consumption old	c_o	0.62	0.59
fertility rate	n	0.69	0.89
savings rate	s	0.09	0.08
labor supply	ℓ	0.83	0.78
transfers	t	0	0
interest rate	r	3.89	4.10
wage rate	w	1.19	1.17
replacement rate	q	0.28	0.34

Chapter 6

General Conclusion

What can we conclude from the analysis of the macroeconomic general equilibrium effects of family polices?

Family policies have a positive effect on the fertility rate. This result is robust across all calibrations and assumptions made on altruism in the utility function. One major consequence is that the old-age dependency ratio decreases and the internal rate of return of a public pay-as-you-go pension system improves. Whether this effect leads to an expansion of the financial resources of the government in the long run, that is, whether family policies are more than self-financing, depends on the model structure and calibration. The results indicate, however, that a more realistic modeling, of the life cycle and of the labor income process in particular, tends to generate more positive results for the government budget.

Moreover, whether monetary child benefits or public child care are more favorable for parents also depends on the model setup.

Finally, and especially the welfare results of all chapters should be interpreted with caution. The welfare analyses are based on steady state comparisons which is a strong assumption since such a comparison basically compares two different economies with two different capital stocks. More robust results could be drawn from an analysis of the transition path. However, it seems possible that the positive results of an expansionary family policy could remain along the transition path, in particular in comparison to reforms of the pension system. This conjecture is based on the observation that the positive welfare results are not driven by shifting old-age insurance from the public to the private sector but rather by improving the rate of return of the public pension system.

In this way, positive effects of family policies do not only remain in general equilibrium but could also be used to support current reforms of public pension systems.

Bibliography

Andersson, G., A.-Z. Duvander, and K. Hank (2004): "Do child-care characteristics influence continued child bearing in Sweden? An investigation of the quantity, quality, and price dimension," *Journal of European Social Policy*, 14, 407–418.

Attanasio, O., S. Kitao, and G. L. Violante (2007): "Global demographic trends and social security reform," *Journal of Monetary Economics*, 54, 144–198.

Attanasio, O., H. Low, and V. Sánchez-Marcos (2008): "Explaining Changes in Female Labor Supply in a Life-Cycle Model," *American Economic Review*, 98, 1517–1552.

Auerbach, A. and L. Kotlikoff (1987): *Dynamic Fiscal Policy*, Cambridge University Press.

Azmat, G. and L. Gonzalez (2010): "Targeting fertility and female participation through the income tax," *Labour Economics*, 17, 487 – 502.

Baizan, P. (2009): "Regional child care availability and fertility decisions in Spain," *DemoSoc Working Paper No. 2009-31.*

Barro, R. J. (1974): "Are government bonds net wealth?" *Journal of political economy*, 82, 1095–1117.

Barro, R. J. and G. S. Becker (1989): "Fertility Choice in a Model of Economic Growth," *Econometrica*, 57, 481–501.

Bauernschuster, S., T. Hener, and H. Rainer (2013): "Does the Expansion of Public Child Care Increase Birth Rates? Evidence from a Low-Fertility Country," *ifo Working Paper*, No. 158, 1–38.

Baughman, R. and S. Dickert-Conlin (2003): "Did expanding the EITC promote motherhood?" *American Economic Review*, 93, 247–251.

——— (2009): "The earned income tax credit and fertility," *Journal of Population Economics*, 22, 537–563.

Beblo, M., S. Bender, and E. Wolf (2009): "Establishment-level wage effects of entering motherhood," *Oxford Economic Papers*, 61, 11–34.

Becker, G. S. (1960): "An economic analysis of fertility," in *Demographic and economic change in developed countries*, Columbia University Press, 209–240.

Becker, G. S. and H. G. Lewis (1973): "On the Interaction between the Quantity and Quality of Children," *Journal of Political Economy*, 81, 279–288.

Berkel, B., A. Börsch-Supan, A. Ludwig, and J. Winter (2004): "Sind die Probleme der Bevölkerungsalterung durch eine höhere Geburtenrate lösbar?" *Perspektiven der Wirtschaftspolitik*, 5, 71–90.

Bick, A. (2013): "The Quantitative Role of Child Care for Female Labor Force Participation and Fertility," *mimeo, University of Frankfurt.*

Bjoerklund, A. (2006): "Does Family Policy Affect Fertility? Lessons from Sweden," *Journal of Population Economics*, 19, 3–24.

Bohacek, R. and V. Belush (2011): "Endogenous Fertility and Social Security Reform in a Dynastic Life-Cycle Model," *mimeo, CERGE-EI Prag.*

Boldrin, M. and L. E. Jones (2002): "Mortality, Fertility, and Saving in a Malthusian Economy," *Review of Economic Dynamics*, 5, 775–814.

Brewer, M., A. Ratcliffe, and S. dSmith (2012): "Does welfare reform affect fertility? Evidence from the UK," *Journal of Population Economics*, 25, 245–266.

Bundesarbeitsministerium (2014): *Rentenversicherungsbericht 2014.*

Bundesbank (2009): "Monatsbericht Juli 2009," Jahrgang 61, Nr. 7.

——— (2015): "Monatsbericht März 2015," Jahrgang 67, Nr.3.

Bundesfamilienministerium (2012): *Familienreport 2012.*

Bundesfinanzministerium (2010): *Bundeshaushaltsplan 2010.*

Bundesinnenministerium (2012): *Jedes Alter zählt - Demografiestrategie der Bundesregierung.*

Börsch-Supan, A., A. Ludwig, and J. Winter (2006): "Ageing, Pension Reform and Capital Flows: A Multi-Country Simulation Model," *Economica*, 73, 625–658.

Börsch-Supan, A., A. Reil-Held, and C. B. Wilke (2007): "How an unfunded pension system looks like defined benefits but works like defined contributions: the German pension reform," *MEA Discussion Papers.*

Börsch-Supan, A. and C. Wilke (2003): "The German Social Security System: How it Was and How it Will Be," *NBER Working Paper No*, 10525.

Cai, F., J. Giles, and X. Meng (2006): "How well do children insure parents against low retirement income? An analysis using survey data from urban China," *Journal of public Economics*, 90, 2229–2255.

Carmichael, J. (1982): "On Barro's Theorem of Debt Neutrality: The Irrelevance of Net Wealth," *American Economic Review*, 72, 202–213.

Chiappori, P.-A. (1988): "Rational Household Labor Supply," *Econometrica*, 56, 63–90.

——— (1992): "Collective Labor Supply and Welfare," *Journal of Political Economy*, 100, 437–467.

Choukmane, T., N. Coeurdacier, and K. Jin (2013): "The One-Child Policy and Household Savings in China," *unpublished*.

Cigno, A. (1993): "Intergenerational transfers without altruism: Family, market and state," *European Journal of Political Economy*, 9, 505 – 518.

Cigno, A., L. Casolaro, and F. C. Rosati (2002): "The Impact of Social Security on Saving and Fertility in Germany," *FinanzArchiv: Public Finance Analysis*, 59, 189–211.

Cohen, A., R. Dehejia, and D. Romanov (2013): "Financial incentives and fertility," *Review of Economics and Statistics*, 95, 1–20.

De La Croix, D. and M. Doepke (2003): "Inequality and growth: why differential fertility matters," *American Economic Review*, 93, 1091–1113.

Del Boca, D. (2002): "The effect of child care and part time opportunities on participation and fertility decisions in Italy," *Journal of Population Economics*, 15, 549–573.

DeNardi, M., S. Imrohoroglu, and T. J. Sargent (1999): "Projected U.S. Demographics and Social Security," *Review of Economic Dynamics*, 2, 575 – 615.

Destatis (2009): *12. koordinierte Bevölkerungsvorausberechnung - Begleitmaterial zur Pressekonferenz am 18.November 2009 in Berlin*, Wiesbaden.

——— (2010): "Verdienste und Arbeitskosten - Verdienststrukturen," *Fachserie*, 16, 1–618.

——— (2012a): *Geburten in Deutschland - Ausgabe 2012*, Wiesbaden.

——— (2012b): *Geburtentrends und Familiensituation in Deutschland*.

——— (2014a): *Der Personalschlüssel in Kindertageseinrichtungen - Methodische Grundlagen und aktuelle Ergebnisse 2013*, Wiesbaden.

——— (2014b): *Konsumausgaben von Familien für Kinder - Berechnungen auf der Grundlage der Einkommens- und Verbraucherstichprobe 2008*, Wiesbaden.

——— (2014c): "Volkswirtschaftliche Gesamtrechnungen 2013 - Inlandsproduktberechnung Lange Reihen ab 1970," *Fachserie*, 18, 1–150.

——— (2015): *Kindertagesbetreuung regional 2014*.

Domeij, D. and P. Klein (2013): "Should daycare be subsidized?" *Review of Economic Studies*, 80, 568–595.

Duclos, E., P. Lefebvre, and P. Merrigan (2001): "A 'Natural Experiment' on the Economics of storks: Evidence on the Impact of Differential Family Policy on Fertility Rates in Canada," *Université du Quebec a Montreal Working Paper*, No. 136, 1–49.

Dustmann, C. and C. Meghir (2005): "Wages, Experience and Seniority," *Review of Economic Studies*, 72, pp. 77–108.

Ebenstein, A., M. Hazan, and A. Simhon (2011): "Raising the Financial Costs of Children and Fertility Responses: Evidence from the Kibbutz," *mimeo, The Hebrew University of Jerusalem.*

Ebenstein, A. and S. Leung (2010): "Son preference and access to social insurance: evidence from China's rural pension program," *Population and Development Review*, 36, 47–70.

Erosa, A., L. Fuster, and D. Restuccia (2010): "A general equilibrium analysis of parental leave policies," *Review of Economic Dynamics*, 13, 742 – 758.

Fehr, H. and D. Ujhelyiova (2013): "Fertility, Female Labor Supply, and Family Policy," *German Economic Review*, 14, 138–165.

Francesconi, M. and W. v. d. Klaauw (2007): "The Socioeconomic Consequences of "In-Work" Benefit Reform for British Lone Mothers," *Journal of Human Resources*, 42, 1–31.

Francesconi, M., H. Rainer, and W. Van Der Klaauw (2009): "The Effects of In-Work Benefit Reform in Britain on Couples: Theory and Evidence," *Economic Journal*, 119, 66–100.

Gathmann, C. and B. Sass (2011): "Female Labor Supply and Childcare: An Evaluation of Germany's New Childcare Subsidy," *mimeo, University of Mannheim.*

Gauthier, A. (2007): "The impact of family policies on fertility in industrialized countries: a review of the literature," *Population Research and Policy Review*, 26, 323–346.

Haan, P. and K. Wrohlich (2011): "Can child care policy encourage employment and fertility?: Evidence from a structural model," *Labour Economics*, 18, 498–512.

Hank, K. (2002): "Regional social contexts and individual fertility decisions: A multilevel analysis of first and second births in Western Germany," *European Journal of Population*, 18, 281–299.

Hank, K. and M. Kreyenfeld (2003): "A multilevel analysis of child care and women's fertility decisions in Western Germany," *Journal of Marriage and Family*, 65, 584–596.

Hank, K., M. Kreyenfeld, and C. K. Spiess (2004): "Kinderbetreuung und Fertilitaet in Deutschland/Child Care and Fertility in Germany," *Zeitschrift für Soziologie*, 228–244.

Heckman, J. J. and J. R. Walker (1990): "The Relationship Between Wages and Income and the Timing and Spacing of Births: Evidence from Swedish Longitudinal Data," *Econometrica*, 58, 1411–1441.

Heer, B. and A. Maussner (2009): *Dynamic General Equilibrium Modeling - Computational Methods and Applications*, Springer-Verlag Berlin Heidelberg.

Heineck, G. and R. Riphahn (2007): "Intergenerational transmission of educational attainment in Germany: the last five decades," *DIW Discussion Paper*, 738, 1–39.

Holthausen, A., J. Rausch, and C. Wilke (2012): "MEA-PENSIM 2.0: Weiterentwicklung eines Rentensimulationsmodells, Konzeption und ausgewählte Anwendungen," *MEA Discussion Papers*, 254-2012.

Klasen, S. and A. Launov (2006): "Analysis of the determinants of fertility decline in the Czech Republic," *Journal of Population Economics*, 19, 25–54.

Kotlikoff, L. J., K. Smetters, and J. Walliser (2007): "Mitigating America's demographic dilemma by pre-funding social security," *Journal of Monetary Economics*, 54, 247 – 266.

Kravdal, O. (1996): "How the local supply of day-care centers influences fertility in Norway: A parity-specific approach," *Population Research and Policy Review*, 15, 201–218.

Krueger, D. and A. Ludwig (2007): "On the consequences of demographic change for rates of returns to capital, and the distribution of wealth and welfare," *Journal of Monetary Economics*, 54, 49 – 87.

Lalive, R. and J. Zweimüller (2009): "How does parental leave affect fertility and return to work? Evidence from two natural experiments," *Quarterly Journal of Economics*, 124, 1363–1402.

Laroque, G. and B. Salanie (2004): "Fertility and financial incentives in France," *CESifo Economic Studies*, 50, 423–450.

——— (2008): "Does Fertility Respond to Financial Incentives?" *CESifo Working Paper*, No.2339, 1–49.

——— (2014): "Identifying the response of fertility to financial incentives," *Journal of Applied Econometrics*, 29, 314–332.

Leopold, T. and T. Schneider (2009): "Schenkungen und Erbschaften im Lebenslauf: vergleichende Längsschnittanalysen zu intergenerationalen Transfers," *SOEPpapers on Multidisciplinary Panel Data Research*.

Ludwig, A. and M. Reiter (2010): "Sharing Demographic Risk–Who Is Afraid of the Baby Bust?" *American Economic Journal: Economic Policy*, 2, 83–118.

Lundberg, S. and R. A. Pollak (1993): "Separate Spheres Bargaining and the Marriage Market," *Journal of Political Economy*, 101, 988–1010.

Manser, M. and M. Brown (1980): "Marriage and Household Decision-Making: A Bargaining Analysis," *International Economic Review*, 21, 31–44.

Manski, C. F. and J. Mayshar (2003): "Private Incentives and Social Interactions: Fertility Puzzles in Israel," *Journal of the European Economic Association*, 1, 181–211.

Manuelli, R. E. and A. Seshadri (2009): "Explaining International Fertility Differences," *Quarterly Journal of Economics*, 124, 771–807.

McElroy, M. B. and M. J. Horney (1981): "Nash-Bargained Household Decisions: Toward a Generalization of the Theory of Demand," *International Economic Review*, 22, 333–349.

Milligan, K. (2005): "Subsidizing the Stork: New Evidence on Tax Incentives and Fertility," *Review of Economics and Statistics*, 87, 539–555.

Mincer, J. (1963): "Market prices, opportunity costs, and income effects," *Measurement in Economics: Studies in mathematical economics and econometrics in memory of Yehuda Grunfeld*, 67–82.

Moerk, E., A. Sjoegren, and H. Svaleryd (2013): "Childcare costs and the demand for children - evidence from a nationwide reform," *Journal of Population Economics*, 26, 33–65.

Nishimura, K. and J. Zhang (1992): "Pay-as-you-go public pensions with endogenous fertility," *Journal of Public Economics*, 48, 239 – 258.

Nugent, J. B. (1985): "The Old-Age Security Motive for Fertility," *Population and Development Review*, 11, 75–97.

OECD (2013): *Pensions at Glance 2013*, OECD Publishing.

——— (2014): *Taxing Wages 2014*, OECD Publishing.

Olivetti, C. (2006): "Changes in women's hours of market work: The role of returns to experience," *Review of Economic Dynamics*, 9, 557 – 587.

Prognos (2014a): *Die Zukunft der Altersvorsorge - Vor dem Hintergrund von Bevölkerungsalterung und Kapitalmarktentwicklungen*, Handelsblatt Research Institute.

——— (2014b): *Gesamtevaluation der ehe- und familienbezogenen Maßnahmen und Leistungen in Deutschland - Endbericht.*

Rainer, H., S. Bauernschuster, N. Danzer, T. Hener, C. Holzner, and J. Reinkowski (2012): "Kindergeld," *ifo Forschungsbericht Nr. 60.*

Raute, A. (2014): "Do financial incentives affect fertility - Evidence from a reform in maternity leave benefits," *mimeo, University of Mannheim.*

Razin, A. and U. Ben-Zion (1975): "An Intergenerational Model of Population Growth," *American Economic Review*, 65, 923–933.

Rentenversicherung (2014): *Rentenversicherung in Zahlen*, Deutscher Rentenversicherungsbund.

Rindfuss, R. R., D. K. Guilkey, S. P. Morgan, and Ø. Kravdal (2010): "Child-care availability and fertility in Norway," *Population and development review*, 36, 725–748.

Ronsen, M. (2004): "Fertility and public policies - evidence from Norway and Finland," *Demographic Research*, 10, 143–170.

Sachverständigenrat (2013): *Gegen eine rückwärtsgewandte Wirtschaftspolitik - Jahresgutachten 2013/14.*

Sanchez-Mangas, R. and V. Sanchez-Marcos (2008): "Balancing family and work: The effect of cash benefits for working mothers," *Labour Economics*, 15, 1127 – 1142.

Schoonbroodt, A. (2014): "Costs of Children: Mapping Models and PSID-CDS Data," *unpublished*, 1–47.

Sinclair, S., J. Boymal, and A. De Silva (2012): "A Re-Appraisal of the Fertility Response to the Australian Baby Bonus," *Economic Record*, 88, 78–87.

Storesletten, K. (2000): "Sustaining Fiscal Policy through Immigration," *Journal of Political Economy*, 108, 300–323.

Tamm, M. (2010): "Child Benefit Reform and Labor Market Participation." *Jahrbücher für Nationalökonomie & Statistik*, 230.

Voena, A. (2013): "Yours, Mine and Ours: Do Divorce Laws Affect the Intertemporal Behavior of Married Couples?" *mimeo, University of Chicago*, 1–55.

Wagenhals, G. (2001): "Incentive and Redistribution Effects of the" Karlsruher Entwurf zur Reform des Einkommenssteuergesetzes," *Schmollers Jahrbuch*, 121, 625–638.

Walker, J. (1995): "The effect of public policies on recent Swedish fertility behavior," *Journal of Population Economics*, 8, 223–251.

Whittington, L. (1992): "Taxes and the Family: The impact of the tax exemption for dependents on marital fertility," *Demography*, 29, 215–226.

Whittington, L. A. (1993): "State Income Tax Policy and Family Size: Fertility and the Dependency Exemption," *Public Finance Review*, 21, 378–398.

Whittington, L. A., J. Alm, and H. E. Peters (1990): "Fertility and the Personal Exemption: Implicit Pronatalist Policy in the United States," *American Economic Review*, 80, 545–556.

ZEW (2013): *Geburten und Kinderwünsche in Deutschland: Bestandsaufnahme, Einflussfaktoren und Datenquellen.*

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