# The Political Economy of Public Debt, Reforms, and Public Investments

Inauguraldissertation

zur Erlangung des akademischen Grades

eines Doktors der Wirtschaftswissenschaften

der Universität Mannheim

vorgelegt von

Christoph Esslinger

August 2016

| | |
|---|---|
| Abteilungssprecher: | Prof. Dr. Carsten Trenkler |
| Referent: | Prof. Dr. Eckhard Janeba |
| Korreferent: | Prof. Pierre Boyer, Ph.D. |
| Tag der mündlichen Prüfung: | 7. Oktober 2016 |

*Meinen Eltern*

# Acknowledgements

First of all, I would like to express my sincerest thanks to my supervisors Eckhard Janeba and Pierre Boyer. Both knew the right path even when times turned tricky and they always put me on track towards great research. I thank them for all their invaluable comments and discussions that made the present thesis possible. It was a privilege to learn from such great academic researchers and teachers over the years. I also truly enjoyed having Pierre Boyer as the coauthor of the second chapter of this dissertation and see his great motivation in action.

I would like to also greatly thank Cornelius Müller, the coauthor of the fourth chapter of this dissertation. We both kept each other fascinated in pursuing this research and Cornelius also became a very good friend outside the academic life.

A big thanks goes also to Hans Peter Grüner. He supported me with very helpful advice on my work and he often opened up interesting new views on aspects that I came to appreciate in a whole different way.

I would like to thank our whole public economics group, Andreas Peichl, Anna Raute, Sebastian Siegloch, and Galina Zudenkova, our chair's secretary Angelika Abuja, all further members and research assistants at the chair of Eckhard Janeba, my former chair colleagues Andreas Bernecker and Hannes Kammerer, and especially my office mate Océane Briand.

Similarly, I would like to thank all members of team C5 of the special research area SFB 884, Melissa Berger, Sebastian Blesse, Friedrich Heinemann and Christoph Schröder. I also thank the Deutsche Forschungsgemeinschaft for financial support.

I thank all my friends and colleagues who accompanied me during my time at the University of Mannheim. I would really like to list all, but this runs the risk of forgetting some. Representatively for all my friends, let me thank my oldest companions that even did their diploma in Mannheim with me: Andreas Dzemski, Timo Hoffmann, Fabienne Rasel, Henning Roth, Stefan Weiergräber, and of course my flat mate in Mannheim, Benjamin Lutz, and my best friend from my Toronto time, Ahmed Huzayyin.

My very special thanks go to my parents Ute and Werner. Their great support made the present thesis possible. They kept me going all the time and always made me believe in myself. I could count on their support whichever path I might choose, and this allowed me to follow the path that culminated in this thesis. I would therefore like to dedicate this thesis to them.

Of course, I would like to say my great thanks to my brother Patrick. Nobody could ever replace him in giving the support that only a brother can give. He was also my great example in finding the best balance between work and life. I hope I will always be able to celebrate achievements with him and his girlfriend Kim looking at the panorama of the great Alps.

My dearest thanks go to my lovely Hana. Meeting her on a public economics conference in London was the luckiest moment of my life. She has been my constant inspiration and motivation. Her love and her belief in me and my research made this thesis a wonderful experience. I hope I will always be able to give her the same wonderful support that she has given me and keeps giving me each day of my life.

# Contents

# Chapter 1

# General Introduction

It is generally acknowledged that one important driving force behind an economy's performance are decisions taken by governments today that have costs in the short-run but carry positive implications in the future. For instance, Besley and Persson (2011) show that in order to explain why some states have well-functioning systems of tax administration or legal protection, which are conducive to a better performance of the economy, we need to understand the incentives behind investing into these forms of state capacity, where the initial investments are costly. More generally, we can think of many reforms implemented by governments that have costs today but will yield benefits in the future. One example could be a reform improving the efficiency of a country's educational system. Often the positive long-run effects outweigh the short-run costs, making the implementation of such reforms the optimal decision from an efficiency viewpoint.

Such reforms or investments are usually implemented by elected politicians that follow their own agendas. Therefore, it is necessary to analyze the reform and investment decisions in a setup of political economy. This setup takes the motives of politicians explicitly into account when explaining why certain investments or reforms will or will not be implemented.

Besides reforms and public investments, another important dynamic decision of governments is the decision to raise public debt. With the recent European debt crisis, the decision on public debt has regained the interest not only of media but also of the academic debate. We have seen a revival of the political economic literature on public debt, which tries to explain the apparent tendency of the political process to raise high public debt.

However, what has been neglected in the political economy literature until now is the interaction between the reform/investment decision and the decision to raise public debt. Since the political process usually decides on both issues simultaneously, there might be important trade-offs arising between them. To investigate the interactions between reforms/public investments and public debt in a political economic setup is the general topic of the present dissertation.

The dissertation consists of three self-contained chapters, each one constituting a theoretical analysis of specific aspects of this general topic. Chapter 2, which is joint work with Pierre Boyer, investigates the interaction between public debt and growth-enhancing reforms when politicians compete for votes in an electoral campaign. The political candidates decide both on reform implementation and public debt while at the same time trying to target electoral favors to a majority of voters. We find that the growth-enhancing reform will always be implemented in the political process when enough public debt can be raised. In contrast, restrictions on the use of public debt inhibit the success of such efficient reforms in electoral campaigns. Importantly, in this chapter, high public debt does not create any costs on the economy itself but is mainly a way of shifting resources across time.

In Chapter 3, I introduce debt-related distortions into the setup of electoral competition and find that the incentivizing effect of public debt on the implementation of growth-enhancing reforms has to be weighed against the costs implied by high public debt. Such costs can for instance arise from the consequences of sovereign debt default, which occurs more likely at higher debt levels.

While such default is not modeled explicitly in chapter 3, chapter 4 contains an explicit modeling of sovereign default. This final chapter, which is joint work with Cornelius Müller, investigates the interaction between public debt and investments in capacities of a state to raise taxes and to enforce the law. It does not model electoral competition, but rather considers the incentives of rival political groups alternating in power and their incentives to invest into state capacities during their term in office. Also in this setup, we find an incentivizing effect of public debt on these investments, which is however weakened the higher are default-related costs of using public debt.

Appendices to all chapters are collected in a joint appendix and the bibliography at the end jointly lists all chapters' references.

## 1.1 Public Debt and the Political Economy of Reforms

As already pointed out, many reform decisions by governments have costs in the short run but imply higher long-run gains, making the implementation of these reforms the efficient choice. However, whether such reforms are really implemented, depends on the incentives of the politicians that have to propose them. In the second chapter of this dissertation, we look at the incentives of political candidates competing for election by targeting electoral favors to subsets of voters at the expense of others. Reforms that affect the allocation of resources across time will have an impact on the capacity to target favors to voters. Therefore, the decision to propose these reforms is influenced by the electoral incentives of political candidates. Similarly, the decision to raise public debt also has an impact on the inter-temporal allocation of resources and the capacity to target them to voters. We should hence expect interactions between the decisions for reform and public debt in such a setup of electoral competition.

We develop a two-period model of voter targeting building on Lizzeri (1999). The same two politicians compete for election in each period. They do so by redistributing available resources across voters in order to convince the beneficiaries of this process to vote for them. This tactical voter targeting does not have any efficiency implications. In contrast, the decision to implement a reform, which exists in the first period, costs resources today but will yield higher benefits in the next period. The decision to reform therefore has the positive efficiency implication described above. The second dynamic decision, which also exists in the first period, is the decision how much public debt to raise. In contrast to the third chapter, this choice on public debt does not have any efficiency implications in this chapter.

We show that the ability to raise public debt to target current voters can help sustain the efficient reform in political competition. Without public debt, politicians compete only on how to target first-period resources to voters. This implies a disadvantage for a reforming candidate, who loses targetable resources through the first-period reform costs. In contrast, the use of public debt allows politicians to also compete on targeting the pie of future resources. This gives a competitive edge to a reformer, since her advantage lies in the future. We show that putting a less restrictive limit on public debt will increase the probability of reform. The impact is especially high for growth-enhancing reforms that create more targetable resources in the future. In that case, if no exogenous limit restricts public debt, the efficient reform will always be implemented. If the reform mainly creates non-targetable public good benefits, public debt cannot ensure the implementation of the reform anymore. However, a higher ability to raise public debt still decreases the relative importance of the reform costs and still increases the probability of reform.

Our results highlight a new view on the trade-off between targeted pork-barrel spending and efficient spending. As long as the efficient policy creates benefits only in the next electoral cycle, allowing enough debt-related targeted spending might be necessary in electoral competition to incentivize spending on the efficient policy.

## 1.2 The Political Economy of Reforms with Distortionary Public Debt

The third chapter takes this result from chapter 2 as a starting point. As just described, we find that efficient policies, like growth-enhancing reforms, with costs today and benefits in the next electoral cycle might necessitate the simultaneous use of public debt in order to get them through electoral campaigns. In the analysis of chapter 2, public debt does not have any direct efficiency implications itself. That is, the use of high public debt does not impose any costs on the economy. As the recent European debt crisis made clear, excessive public debt can, however, have significant negative consequences. These negative consequences include, for instance, costs related to sovereign default, which gets more likely with high public debt. A related detrimental consequence of high public debt is less possibilities to smooth out tax-related distortions across time when the further increase of public debt gets very costly at high debt levels.

In terms of thinking about efficient policies, if a high level of public debt implies a high degree of such debt-related distortions, then investing into low public debt can be seen itself as an efficient dynamic policy with costs in the current electoral cycle and benefits in future electoral cycles. It is the only such policy which by its nature *cannot* be incentivized by the use of more public debt. In the third chapter, I therefore investigate the trade-off between the decision to implement a growth-enhancing reform and the decision to raise public debt when high public debt does impose costs on the economy. I investigate this trade-off in the two-period setup of voter targeting employed in the second chapter. However, in the third chapter, the focus is solely on growth-enhancing reforms that create potentially redistributable benefits.

The main insights from my analysis are the following. If the distortionary cost of raising high public debt is very high, then the political process eliminates high public debt on its own. In particular, if high-debt policies create a high efficiency loss, then even the majority of voters that is advantaged in the process of targeted redistribution is better off with a low-debt policy. In this case, the incentivizing effect of public debt on the pie-increasing reform, which is highlighted in the second chapter, is not present and hence the reform will not be implemented with certainty.

On the other hand, if the distortionary cost of raising high public debt is low enough compared to the net benefit of the reform, then the political process creates a bias for high public debt. Specifically, a politician that uses high public debt for targeting current voters can gain an advantage over her opponent as long as that politician can still compensate a majority of voters for the implied efficiency loss of public debt. In this case, we confirm the result from the second chapter that the growth-enhancing reform will always be implemented in electoral competition if no exogenous restrictions are placed on public debt. In particular, with some probability high public debt is raised and we do have an efficiency loss due to debt-related distortions, but this ensures efficiency on the side of implementing the pie-increasing reform. This trade-off persists once we introduce an exogenous debt limit into the analysis. Such a debt limit reduces the distortionary costs of public debt. However, at the same time, not implementing the reform gets more attractive since public debt cannot unfold its incentivizing effect on reforms as well. Hence, when setting a debt limit, more efficiency on the side of public debt has to be weighed against lower efficiency in terms of implementing the pie-increasing reform.

## 1.3 State Capacity and Public Debt: A Political Economy Analysis

The last chapter, which is joint work with Cornelius Müller, turns away from general growth-enhancing reforms and towards investments of governments into specific capacities of a state: the capacity to raise taxes and the capacity to provide a functioning legal environment. The combination of high public debt and low capacities of the state to raise taxes and to enforce the law can upset even developed economies. In light of this, it is important to understand the mechanisms underlying the combined evolution of state capacity and public debt.

In contrast to chapters 2 and 3, we turn to a different political economic setup in order to study the interaction of state capacity investments and public debt. Specifically, we build on the recent literature on state capacity, which was pioneered by Besley and Persson (2009). We integrate a baseline model of state capacity investment building on Besley and Persson (2010, 2011) with the strategic use of public debt, fluctuating incomes, and the possibility of default. In a two-period framework, an incumbent government cannot be sure to remain in power in the future. It wants to benefit its own clientele, and decides about investments in the future capacities of the state to raise taxes and to enforce the law. Besides investing, the incumbent government can spend on a common-interest public good or channel money towards its own clientele. The "cohesiveness" of institutions determines to what degree these clientele politics are possible. Specifically, low cohesiveness means it is very easy to do clientele politics. We also introduce the possibility of default on public debt, which gives us a tractable way to study the effects of increasing costs of debt financing.

We derive two main sets of results. First, in a model without default, we show that the possibility to raise debt can create an additional incentive to invest in state capacity. In particular, we find that politicians' incentives to execute investments while in power interact with public debt in a similar way to incentives of electoral candidates to propose reforms, as analyzed in chapters 2 and 3. Specifically, public debt allows to draw future tax resources to the present. In the setup of this chapter, this circumvents the problem of a use of future public funds that is not in line with the current incumbent's objective. In particular, high political instability and low cohesiveness make the first-period incumbent afraid of giving additional state capacity to the future government. By high political instability, this government is likely to be from the opposed political group, and by low cohesiveness, it can use the higher taxing power to heavily extract money from the period-1 incumbent group. Without debt, the only possibility to protect against such an adverse use of future public funds is to decrease investments in state capacity. However, if debt can be used to bring future public funds at the disposal of the first-period incumbent, then this incumbent can decide about their use. With the simultaneous use of public debt, the incumbent therefore has higher incentives to invest in state capacity.

Our second set of results shows how the debt mechanism can be weakened, thereby allowing the original low-investment mechanism to partly resurface. Specifically, with fluctuating incomes, the cost of raising additional debt depends on the possibility of default. When debt is raised to the point where default becomes possible, it becomes increasingly expensive to use the debt channel to draw future public funds to the present. To the extent that it is very costly to draw newly created future public funds to the present, the low-investment mechanism resurfaces. Specifically, it resurfaces the stronger, the higher are the income fluctuations. Besley and Persson (2011) identified so-called development clusters with weak states combining all the negative outcomes in terms of low investments in state capacity, low income and higher internal violence. We show that, under high income fluctuations, this notion of of a negative cluster can be extended to include high public debt and a high risk of default, because public debt then might not be able to overcome the low investment incentives that define these weak states.

# Chapter 2

# Public Debt and the Political Economy of Reforms[1]

## 2.1 Introduction

What determines whether efficient reforms are implemented in the political process? This question has dominated the academic and policy spheres over the last decades.[2] A key to explaining the political decision to reform is to understand under which circumstances electoral incentives can stand in the way of reforms. Electoral competition occurs to a considerable degree through targeting electoral favors to subsets of voters in order to gain their support. Since many reforms imply a shift of resources across time, the decision to reform should be influenced by this incentive to target resources to voters. The second important decision that determines the allocation of resources

[1]This chapter has been co-authored with Pierre Boyer. We thank Enriqueta Aragonès, Raicho Bojilov, Jean-Marc Bourgeon, Micael Castanheira, Davide Cantoni, Antonio Ciccone, Gregory Corcos, Eduardo Davila, Philipp Denter, Philippe De Donder, Matthias Doepke, Allan Drazen, Clemens Fuest, Vincenzo Galasso, Catarina Goulão, Renato Gomez, Olivier Gossner, Hans Peter Grüner, Matthias Hertweck, Roland Hodler, Guillaume Hollard, Eckhard Janeba, Paul Klein, Yukio Koriyama, Michel Le Breton, Aniol Llorente-Saguer, Ralf Meisenzahl, Jean-Baptiste Michau, Massimo Morelli, Salvatore Nunnari, Andreas Peichl, Eugenio Peluso, Eduardo Perez, Facundo Piguillem, Mattias Polborn, Jean-Pierre Ponssard, Niklas Potrafke, Anna Raute, Alessandro Riboni, Maik Schneider, Klaus Schmidt, Guido Tabellini, Michèle Tertilt, Karine Van Der Straeten, Ngo Van Long, and Galina Zudenkova for very helpful comments, as well as seminar and conference participants at Bocconi University, Munich University, University Carlos III Madrid, Toulouse School of Economics, Mannheim University, École Polytechnique, the 2015 Workshop "Political Economy: Theory meets Empirics" in Konstanz, LAGV 2015 in Aix-en-Provence, PET 2015 in Luxembourg, CESifo Area Conference on Public Sector Economics 2016, and the Verein für Socialpolitik in Münster. The first author gratefully acknowledges the Investissements d'Avenir (ANR-11-IDEX-0003/Labex Ecodec/ANR-11-LABX-0047) for financial support. The second author gratefully acknowledges the Collaborative Research Center 884 for financial support. The usual disclaimer applies.

[2]See for instance Williamson (1994), Tommasi and Velasco (1996), Rodrik (1996), Persson and Tabellini (2000), and Drazen (2000).

across time is the decision to raise public debt. Therefore, in order to understand incentives for reform, it is important to investigate how the decisions to reform and to raise public debt interact with political competition.

We develop a two-period model of redistributive politics that builds on Lizzeri (1999). Two politicians compete for election in each period. They do so by targeting available resources to subsets of voters at the expense of others. This tactical redistribution does not imply any efficiency gain. In the first period, politicians also choose the level of public debt and whether to implement a reform. The reform is efficient in the sense that it costs resources in the first period but yields higher benefits in the second period. The introduction of the reform decision besides the decision to raise debt is our main contribution.

A main insight from our analysis is that the electoral incentives to implement the efficient reform depend on the interaction between the reform decision and the decision to raise public debt. We show that the ability to raise public debt to target current voters can help sustain the efficient reform in political competition. The argument is the following: resources left in the future cannot be targeted to specific voters due to endogenously arising electoral uncertainty between the two periods. Without public debt, politicians compete only on how to target first-period resources to voters. This implies a disadvantage for a reforming candidate, who loses a potentially big proportion of these targetable resources through the first-period reform cost. In contrast, the use of public debt allows politicians to also compete on targeting future resources. This gives a competitive edge to a reforming candidate, since her advantage lies in the future, where the reform benefits occur. Indeed, our results show that the reform will always be implemented in political equilibrium when the use of public debt allows a reformer to make up for her loss in targeting capacity in the first period. On the other hand, the efficient reform will not be implemented for sure if the reform mainly creates non-targetable public good benefits or if the use of public debt is too heavily restricted. Both aspects hinder a reformer to compensate for her first-period targeting disadvantage. However, even if the reform corresponds to investing in a pure public good, we show that putting a more restrictive limit on public debt will still decrease the probability of reform. The driving force behind this result is the following: restricting public debt means reducing the amount of targetable resources on which electoral competition occurs. A given amount of reform costs therefore creates a relatively bigger disadvantage in terms of targeting capacity.

These results highlight a new view on the tradeoff between targeted pork-barrel spending, which does not increase aggregate welfare, and efficient policies like spending on a reform. As long as the efficient policy creates benefits only in the next electoral cycle, allowing enough debt-related targeted spending might be necessary to incentivize spending on the efficient policy.

In the following, we describe in more detail the setup we use to derive our results. To focus on the effects of electoral competition in its purest form, we consider an environment without any pre-imposed heterogeneity. There are two periods and the same two politicians run for election in each period. These politicians are purely office-motivated and voters are ex-ante homogenous with each voter having a resource endowment of one in each period. Politicians compete for voters by redistributing the available resources across voters. They are not restricted in the tools available for redistribution of resources within a given period. In the first period, politicians additionally decide on the level of public debt and on the implementation of a reform that costs some resources today and will yield benefits in the next period. All voters and politicians know the reform benefits and costs. The reform is efficient in the sense that the benefits surmount the costs. On the other hand, targeting resources to voters is a purely tactical instrument and does not create a net gain like the reform.

In our analysis the reform benefits can take the form of an increase in the endowment of the economy and/or they can have a public good nature. By way of illustration, starting from a situation with deficient enforcement of property and civil rights, consider a reform of the legal system that ensures efficient and universal enforcement of these rights. This is what is usually termed establishing the rule of law.[3] By decreasing uncertainty for investors, such a reform will lead to an increase in the economy's GDP,[4] which in our case corresponds to an increase in the endowment of the economy. Besides that there will be a general increase in well-being beyond the increase in the endowment. For instance, everybody will feel more safe in such a functioning legal

[3]See, for instance, La Porta et al. (2008), Besley and Persson (2011), and Acemoglu and Robinson (2012).

[4]Rodrik et al. (2004) and Djankov et al. (2007) provide empirical support for this claim.

environment. This second kind of benefit has the properties of a public good in the sense that it is non-rival and non-excludable.[5] When the benefits result in an increase in the endowment, which can be taxed, the benefits can potentially be redistributed to specific voters. In the case where the benefits have a public good nature, a reforming politician cannot shuffle the benefits that voters derive from the reform. In line with the political economy literature, we therefore assume that benefits that have a private good nature can be targeted to individual voters whereas targeting is precluded for the public good part of the reform.

We prove two main sets of results. In the first part, we focus on how the nature of the reform impacts on the interaction between the reform and debt decisions. Therefore, we impose no restrictions on debt except for the natural debt limit. Implementing the reform increases the natural debt limit by the induced increase in the second-period endowment.

We show that, if the proportion of reform benefits that increases the endowment is high enough, then both politicians will choose to implement the reform with probability one. The intuition behind this result is the following: due to endogenously arising electoral uncertainty, resources in the second period cannot be targeted to specific voters. This gives both candidates the incentive to transfer as many resources as possible to the first period by debt in order to target them to specific voters. Since debt repayment capacity increases by the reform-induced increase in the endowment, a reformer can raise higher debt than a non-reformer. This allows a reformer to compensate for the disadvantage of losing targetable resources through the reform costs.

In contrast, when the reform benefits are mainly of a public good nature and don't increase the endowment much, the result is overturned. In that case, a reformer cannot raise much more debt than a non-reformer. If the major part of the reform benefits has the character of a public good, then this part is non-targetable by nature and also cannot be made targetable through the use of public debt. Therefore, by saving on the costs of the reform, a non-reforming candidate has more *targetable* resources and she can use this advantage to compensate at least a majority of voters for missing out on the net gain that the reform creates. Due to this efficiency gain, the reform will

[5]Excluding some people from access to the legal system would mean a failure to establish the rule of law.

still be implemented with positive probability, but it will no longer be implemented with certainty. For reforms that create mainly public good benefits, we therefore get a failure of the political process to implement the efficient policy.

Our second set of results focuses on the availability of the debt channel. Constitutional limits on deficit and debt are a popular response to the recent debt crisis and are present in many jurisdictions: most U.S. states have a balanced-budget rule, the Stability and Growth Pact in the European Union limits gross government debt to sixty percent of the country's GDP. We first look at a reform that mainly creates benefits corresponding to an increase in the endowment. Such benefits can potentially be targeted to first-period voters through the use of public debt. We show that an exogenous restriction on public debt that prevents a reformer from raising more debt than the non-reforming candidate gives these potentially targetable benefits the character of non-targetable public good benefits. From the point of view of first-period voters, future reform benefits that cannot be transferred to the present have the character of providing a public good that promises higher utility for everyone, but whose benefits cannot be targeted to specific voters. Above, this public good character was given through the nature of the reform. Now, it is artificially created through the debt limit. We show that if the debt limit becomes too stringent, the efficient reform is no longer implemented with probability one.

However, even if the reform only creates non-targetable public good benefits, we show that putting a more restrictive exogenous limit on public debt will still decrease the probability of reform. The probability is lowest if public debt is not allowed at all. If there is no public debt, this means restricting electoral competition of targeting favors to the subset of only the present resources. A given disadvantage of having to finance the first-period reform costs is then relatively bigger. In other words, allowing public debt gives politicians the opportunity to also compete on targeting the pie of future resources. Such targeting does not create any efficiency gain itself. However, by putting a reformer in a relatively better position, it incentivizes spending on efficient policies whose benefits only occur in the next electoral cycle. This gives us a new view on the effects of targeted spending, which until now has mainly been shown to disincentive efficient spending on public goods in the same electoral cycle.

The rest of the paper is organized as follows. Section 2.2 discusses the related literature. Section 2.3 describes the formal framework. Section 2.4 solves for the political equilibrium in the last period of the game. Our main results are presented in Section 2.5, where we solve for the equilibrium in the first period, and in Section 2.6, where we study the implications of debt limits. The last section contains concluding remarks. We relegate all proofs to the Appendix.

## 2.2 Related literature

Our work builds on the game-theoretic literature on the "divide-the-dollar"-game. Following Myerson (1993), this literature features models of political competition in which a policy proposal specifies how a cake of a given size should be distributed among voters.[6] Our model differs from these models in that policy proposals affect the size of the cake that is available for redistribution.[7]

Lizzeri and Persico (2001) extend the framework of Myerson (1993) by characterizing political equilibria under the assumption that politicians face a choice between an efficient public good and pork-barrel redistribution.[8] In their static framework, they show that targeted pork-barrel spending stands against the efficient policy of providing a public good that creates a net gain in utility.[9] In contrast, we consider an efficient policy that is of a dynamic nature in the sense that its benefits only occur in the next electoral cycle. For such policies, we show that allowing more debt-related targeted spending can actually increase the probability of implementing the efficient policy.

The first extension of Myerson (1993)'s setup to a dynamic model was done by Lizzeri (1999). Lizzeri (1999) shows that in a two-period model of "divide-the-dollar" electoral competition, candidates will always raise the maximal debt, because it allows them to better target the pool of resources to voters. Our analysis builds on Lizzeri

[6]Contributions to this literature include Laslier and Picard (2002), Roberson (2006), Sahuguet and Persico (2006), Carbonell-Nicolau and Ok (2007), Kovenock and Roberson (2008), Kovenock and Roberson (2009), Crutzen and Sahuguet (2009). See Kovenock and Roberson (2012) for a review.

[7]Some related papers that endogenize the size of the redistributive pie are Ueda (1998), Bierbrauer and Boyer (2016), and Boyer and Konrad (2014), however these papers are static and do not study the interaction between debt and reforms.

[8]See also Lizzeri and Persico (2004) and Lizzeri and Persico (2005).

[9]Roberson (2008) adds the possibility to provide different public goods to different districts.

(1999) by studying the interaction between debt and reform in such a redistributive politics setup. This setup allows to distill the pure effect of electoral competition on policy outcomes, because it does not impose any exogenous heterogeneity on politicians or voters. Furthermore, it derives political turnover endogenously as the outcome of the electoral game. In contrast, the literature on strategic debt has derived the tendency of the political process to accumulate debt from partisan preferences combined with the exogenously imposed threat that a currently ruling government is replaced in the future. Alesina and Tabellini (1990) show that a currently ruling party that has different spending objectives than a potential future incumbent uses debt to tie its successor's hands.[10] Recently there has been a revival of the literature on the political economy of public debt.[11] Battaglini and Coate (2008) introduce Barro (1979)'s tax smoothing setup of public debt into an infinite horizon model of legislative bargaining. Similar to Alesina and Tabellini (1990) they show that, when an electoral district is not sure to remain in the governing coalition, the incentive of politicians to spend pork on their own district leads to the use of public debt even when this means accepting higher tax distortions in the future.[12] In that sense, investing into low public debt is an efficient dynamic policy whose benefits only occur in the future. By its very nature, it is the only such policy that cannot be incentivized by a higher use of public debt. We add an important aspect to this literature by establishing this incentivizing effect of public debt for all other efficient dynamic policies that have costs today and benefits in the future.

We also complement the existing literature on political economy of reforms. In this paper, we shut down the channels that the previous literature has identified as impediments to reform.[13] Our objective is to show how efficient reforms and public

[10] Other early papers in this line of research are Persson and Svensson (1989), Aghion and Bolton (1990) and Tabellini and Alesina (1990). See Martimort (2001) for an extension of these models to an optimal income taxation setup à la Mirrlees (1971).

[11] See Alesina and Passalacqua (forthcoming) and Battaglini (2011) for recent reviews of this literature.

[12] See Battaglini and Barseghyan (forhcoming) for a recent application of the legislative bargaining model investigating public debt in a growth setup. Further papers with different setups are Yared (2010), Drazen and Ilzetzki (2011), Song et al. (2012), Maskin and Tirole (2014), and Azzimonti et al. (2014).

[13] In contrast to Fernandez and Rodrik (1991) and Cukierman and Tommasi (1998), our analysis does not link the benefits and costs of reform to specific voters. Consequently, we also do not consider problems of asymmetric information in compensating losers of the reform as in Grüner (2002). Furthermore, we have no uncertainty regarding appropriate timing of the reform as in Laban and Sturzenegger (1994b,a) and Mondino et al. (1996). Reforms do not fail because of insufficient technical knowledge by decision makers as in Caselli and Morelli (2004) and Mattozzi and Merlo (2015). We also exclude powerful vested interest that could block reform as in Olson (1982), Benhabib and

debt interact in a setup of electoral competition, absent all the previously identified channels. The only previous papers that have looked at public debt in combination with reforms do not model electoral competition. Specifically, Beetsma and Debrun (2004, 2007) rely on the assumption of an exogenous probability of change in political power. In contrast to us, they do not consider a feedback of the decisions on debt and reform on the electoral outcome. As we show in our model, these forces of political competition are crucial to understanding the interaction between debt and reforms.[14]

## 2.3 The model

Our setup builds on Lizzeri (1999), who introduces debt in a model of redistributive politics but does not consider the decision to perform an efficient reform.

**The electorate.** There are two periods and a continuum of voters of measure one.[15] All voters are ex-ante homogenous. They are risk-neutral, live for the two periods, and have a discount factor equal to 1. There are two goods, money and a public good. Voters have linear utility over goods with the marginal utility of money normalized to one.[16] All voters in each period have one unit of money which is perfectly divisible.

**Political process.** In each period, there is an election where voters choose between two candidates. The set of candidates is the same for both dates. One candidate is denoted by $A$, the other by $B$. Each candidate $i \in \{A, B\}$ is purely office-motivated and maximizes vote-shares.

---

Rustichini (1996) and Gehlbach and Malesky (2010). There is no conflict between different groups about who will bear the costs of reform as in Alesina and Drazen (1991), Drazen and Grilli (1993) and Hsieh (2000). Finally, the success of the reform does not depend on the competence of politicians as in Prato and Wolton (2014). Inefficiencies of the political process to pursue efficient investment have been investigated in several setups, see, e.g., Besley and Coate (1998), Battaglini and Coate (2007), Azzimonti et al. (2009), Battaglini et al. (2012), and Azzimonti (2015).

[14] Ribeiro and Beetsma (2008) take a first step towards endogenizing political turnover. However, they still need to add a final period with exogenous probability of change in power. Furthermore, one politician is forced to run a reform platform and she cannot decide not to reform, while her opponent is exogenously set to run a no-reform platform. Again, this precludes to see the workings of the forces of political competition that we establish in this paper.

[15] This is a convenient assumption that is meant to be an approximation for a game with a large (but finite) number of voters.

[16] Our results extend if we introduce some curvature in the utility function on public good consumption.

**Policies.** In the first period, a strategy $p_1^i$ has three elements: the decision to enact a reform, a level of public debt, and promises of taxes and transfers to each individual voter.

*1. Reform.* The possibility to enact a reform is present only in the first period. The cost of this reform is incurred in the first period and the benefit occurs in the second period. Denote by $c$ the per capita cost and by $e$ the per capita benefit from the reform. We assume that

$$e - c > 0, \tag{2.1}$$

$$c < 1. \tag{2.2}$$

Assumption (2.1) states that the reform is beneficial and therefore should always be implemented from an efficiency perspective. Assumption (2.2) ensures that there is enough first-period endowment to finance the reform.

We allow the reform to generate benefits that have pure public good and/or private good characters. Formally, a fraction $\lambda \in [0, 1]$ of the reform benefits has a private good character. The remaining part of the benefits has a pure public good character, i.e. it is non-rival and non-excludable. The public good nature of a policy makes it impossible for politicians to affect the distribution of the utility gains derived by the voters. This is often referred to as the non-targetable part of the policy in the political economy literature. In contrast, the targetable aspect of the reform associated with its private goods aspect can be redistributed between voters in the political process. Hence, for $\lambda = 0$, we have the case of a pure public good and $e$ denotes the utility that each and every voter derives from the reform. For $\lambda = 1$ on the other hand, we have the case of a pie-increasing reform in the sense that $e$ corresponds to the increase in the per-capita endowment of the economy for the second period. Since the proportion of benefits with private good character can be potentially targeted to voters, we refer to $\lambda$ also as the targetability of reform benefits.

We also introduce the possibility that the reform is not offered by politicians for sure and denote by $\beta^i$ the probability with which candidate $i$ proposes the reform.

*2. Debt.* Government debt is financed by borrowing from abroad and there is no possibility of default. The size of the deficit in the first period is interpreted as the fraction of the average voter's second-period resources that is pledged to the repayment of the debt.[17]

The natural limit on debt corresponds to the total resources that can be mobilized to repay debt. Formally, the debt level $\delta$ belongs to $[-1, 1]$ if the reform is not implemented. The lower bound represents the case where the maximal budget *surplus* of 1 is run and transferred from the first to the second period. The upper bound represents the case where the total amount of resources available in the second period is transferred to the first period by debt. It corresponds to the natural debt limit in case of no-reform. Implementing the reform increases the natural debt limit by the induced increase in the second-period endowment. This increase corresponds to the proportion of reform benefits that has a private good character, i.e. $\lambda e$. Hence, when the reform is undertaken, the maximal amount of resources that can be transferred to the present increases by $\lambda e$. The debt level $\delta$ then belongs to $[-1 + c, 1 + \lambda e]$.

Later on in the analysis, we also introduce the possibility that the amount of debt that can be incurred is exogenously restricted. We interpret such a restriction as a constitutional limit on debt, which we denote by $\bar{\delta}$.

In total, we denote by $\delta_R^i \in [-1 + c, \min\{\bar{\delta}, 1 + e\}]$ (resp. $\delta_N^i \in [-1, \min\{\bar{\delta}, 1\}]$) the debt level proposed by candidate $i$ when the reform is (resp. is not) implemented by this candidate.

*3. Redistribution.* We formalize the transfer offers that candidates make to voters with offer distributions from which the actual transfer promises to voters are drawn. A negative transfer corresponds to taxing a voter and the lowest possible transfer offer of $-1$ corresponds to taxing away the full endowment of a voter. We denote the cumulative distribution functions from which the transfer offers are drawn as $F^i(\cdot), i \in \{A, B\}$. Specifically, we follow Myerson (1993) and assume that the favors offered to different voters are *iid* random variables with probability distribution $F$. We appeal to the law of large numbers for large economies and interpret $F(x)$ not only as the probability that any one individual receive an offer weakly smaller than $x$, but also as the population share of voters who receive such an offer. Therefore, $F$ describes the redistributive pattern resulting under the corresponding transfer scheme.

[17] We also allow for the possibility that the government runs a surplus which will, however, never occur in equilibrium.

Finally, we denote by $F^i_{1,R}$ and $F^i_{1,N}$ the first-period cumulative distribution functions proposed by candidate $i$ for the case where the reform is or is not implemented by this candidate, respectively.

In total, a first-period strategy $p^i_1$ of candidate $i$ is then given by $\{\beta_i, \delta^i_R, \delta^i_N, F^i_{1,R}, F^i_{1,N}\}$, where $\beta_i$ was the probability with which candidate $i$ proposes the reform.

A second-period strategy $p^i_2$ of candidate $i$ consists only of the choice of the second-period cumulative distribution function $F^i_2$. Denote by $\delta^*$ the debt level realized after the first period.

**Feasible policies.** Strategies are feasible if they satisfy the following budget constraints.

First-period budget constraint:

if the reform is not undertaken,

$$\int_{-1}^{+\infty} x dF^i_{1,N}(x) = \delta^i_N; \tag{2.3}$$

if the reform is undertaken,

$$\int_{-1}^{+\infty} x dF^i_{1,R}(x) = \delta^i_R - c; \tag{2.4}$$

Second-period budget constraint:

if the reform is not undertaken,

$$\int_{-1}^{+\infty} x dF^i_2(x) = -\delta^*; \tag{2.5}$$

if the reform is undertaken,

$$\int_{-1}^{+\infty} x dF^i_2(x) = \lambda e - \delta^*. \tag{2.6}$$

Thus in the first period, the *additional* resources that can on average be given to each voter are the resources transferred from the future by debt, $\delta^i$, minus the costs $c$ that have to be paid in case of reform. In the second period, the debt $\delta^*$ of the winner of the first-period election has to be repaid. However, in case of reform, the amount of resources that can be redistributed across voters increases by the amount of reform benefits with private good character, $\lambda e$.[18] Note that in the second period, in addition to the transfer offers, which are represented in the budget constraints above, each voter will receive the public good utility $(1-\lambda)e$.

**Timing.** The timing of the game is as follows:

**Period 1:**

Stage 1 Each candidate $i = \{A, B\}$ plays a strategy $p_1^i$ in order to win the election.

Stage 2 Each voter observes her draw $(x_1^A, x_1^B)$ from each candidate's distribution plan, the reform proposals, the proposals for debt, and then votes.[19] When voters are indifferent between the two candidates, they flip a coin to decide who to vote for.

At period 2 everybody observes the first-period debt level that has to be repaid and whether the reform was undertaken so that the strategies are conditioned on the first-period outcome; there are two stages:

**Period 2:**

Stage 1 Candidates choose distribution plans $F_2^i(\cdot)$, $i = A, B$.

Stage 2 Each voter observes her draw $(x_2^A, x_2^B)$ from each candidate's distribution plan and then votes.

[18]Note that since the budget constraints are formulated in terms of *transfers*, we look at changes in the existing endowment of people. That is a transfer of -1 means that the person loses its full endowment. We treat the targetable reform benefits $\lambda e$ not as an additional per-capita endowment, but as a general increase in resources available for transfers (similar to the resources that are additionally available in the first period if debt is raised). This is why the lower bound of the last integral is $-1$: the whole per capita endowment of 1 is taken away and nothing from the additional pie is given to the worst-off individual. We could also work with $\lambda e$ occurring as an additional person-specific endowment, in which case this lower bound would become $-(1+\lambda e)$ and $\lambda e$ would disappear from the right hand side of the last budget constraint.

[19]Note that at stage 1, candidates potentially play a mixed strategy against each other in terms of implementing the reform. However, at the stage when voters decide, each candidate is committed to either reform or no-reform. This approach is taken because, in terms of finding the equilibrium of the electoral game, we want to allow politicians to use mixed strategies in case there is no equilibrium in pure strategies. On the other hand, in terms of the voter decision, we do not want to introduce limited-commitment issues on the side of politicians.

**Vote-shares.** We denote by $S_t^i(p_t^i, p_t^j)$ the share of the votes of candidate $i$ in period $t \in \{1, 2\}$ if she chooses to play strategy $p_t^i$ and the other candidate $j$ chooses to play strategy $p_t^j$. Then, $S_t^j(p_t^j, p_t^i) = 1 - S_t^i(p_t^i, p_t^j)$.

The solution concept used is subgame-perfect Nash equilibrium.

## 2.4 Second-period political equilibrium

We start by presenting the second-period equilibrium offer distributions.

Every voter has access to the public good part of the reform and thus receives at least a utility of $(1-\lambda)e$. All that candidates can compete over is redistributing all available *targetable* resources. The amount of targetable resources is increased by the reform benefits with private good character in the case that the reform was implemented in the first period, and it is decreased by any debt that has to be repaid. We define by $\mu_2$ the resources available in the second period for making transfer offers. Specifically, $\mu_2$ corresponds to the right-hand side of the second-period budget constraints (2.5) or (2.6). When debt is raised, $\mu_2$ can take a negative value, which means that resources that would otherwise be available for transfers have to be taken away from the voters. The lowest value of $\mu_2$ occurs when the debt raised is so high that all future resources are necessary for debt repayment. In that case, $\mu_2$ takes a value of $-1$. The maximal amount of targetable resources in the second period is reached if the elected first-period politician ran a full surplus and has implemented the reform. Formally, $\mu_2 \in [-1, 1+\lambda e - c]$. The following proposition derives the equilibrium offer distributions for any given $\mu_2$.

**Proposition 2.4.1** *In the unique second-period equilibrium, both candidates generate offers to all voters from a uniform distribution on* $[-1, 1+2\mu_2]$.

The proof of Proposition 2.4.1 is similar to the one of Theorem 1 in Lizzeri (1999).[20] We provide a sketch of the arguments. The resources that are available in the second period for transfers are $\mu_2 = -\delta^*$ when no reform is undertaken and $\mu_2 = \lambda e - \delta^*$ when the reform is undertaken. Therefore, in the second period, we are back to a

[20]Myerson (1993) shows a uniqueness proof in the class of symmetric equilibria.

static version of the divide-the-dollar game where the average resources available for making transfer offers are given by $\mu_2$. We show that both candidates playing an offer distribution that is uniform on $[-1, 1+2\mu_2]$ indeed constitutes an equilibrium. Suppose politician $A$ plays the uniform distribution on $[-1, 1+2\mu_2]$. Then the vote share of politician $B$ playing any budget balanced distribution is given by:

$$
\begin{aligned}
S_2^B(p_2^B, p_2^A) &= \int_{-1}^{+\infty} F_2^A(x) dF_2^B(x) \\
&\leq \int_{-1}^{+\infty} \frac{x+1}{2+2\mu_2} dF_2^B(x) \\
&= \frac{\mu_2+1}{2+2\mu_2} = \frac{1}{2},
\end{aligned}
$$

where the inequality in the second line is strict if candidate B makes any offer $x > 1+2\mu_2$ with positive probability.

Notice that, if all second-period targetable resources are necessary for debt repayment, both candidates' offer distribution are degenerate on $\mu_2 = -1$.

The crucial feature of the second-period election is the uncertainty for voters regarding the outcome of the process of redistributive politics. Given a uniform distribution on $[-1, 1+2\mu_2]$, in period 2 each voter expects to get $\mu_2 = -\delta^*$ in the case of no-reform. In case of reform, each voter expects a transfer offer of $\mu_2 = \lambda e - \delta^*$, but additionally she gets the public good utility $(1-\lambda)e$. In total, each voter therefore expects a utility of $e - \delta^*$ in case of reform. For the analysis of the first period, this expectation about future utility fully captures how a voter evaluates the future effects of a proposed policy. However, the equilibrium distribution implies that some voters are treated very well and others are treated very badly. The politicians have an incentive to "cultivate favored minorities" as in Myerson (1993). This uncertainty is going to be a driving force behind the electoral incentives to do the reform and accumulate debt in the first period.

## 2.5 First-period political equilibrium

We turn to the analysis of the first-period equilibrium. We want to see how electoral incentives shape the decision to reform, in particular when we focus on the interaction of the reform decision with the decision to raise public debt. As it turns out, this interaction depends on the nature of reform benefits in the sense of them having a private or a public good character.

### 2.5.1 Private good reform

As a benchmark, we first treat a polar case where the second-period reform benefits are only of a private good nature, i.e. $\lambda = 1$. Therefore, they can be potentially redistributed across voters. We provide a complete equilibrium characterization in Corollary 2.5.2 below.

**Proposition 2.5.1** *When the reform benefits are of a private good nature, the unique equilibrium is such that both candidates always reform and announce the maximum debt level.*

In our setup, the second-period reform benefits surmount the first-period costs of the reform. Since all benefits have a private good nature, the reform will therefore lead to an increase in the size of the total endowment of the economy and this increase is potentially targetable to voters. However, resources that are left in the future cannot be targeted to voters in the first period, because the outcome of the electoral game of redistribution in the second period is uncertain. This is the reason why a no-reform candidate might have an advantage: if such a candidate has more resources available in the first period through saving on the costs of the reform, she can skew the distribution of these resources to win a majority. However, by making use of public debt, a reforming candidate can compensate for the targeting disadvantage that arises from covering the first-period costs of the reform. Debt is only restricted by its natural limit: a reformer can raise $e$ more in debt than a non-reformer. Since the benefits surmount the costs, a reformer then has more resources available for targeting voters in the first period than a non-reformer. Through this advantage a reforming candidate will always be able to convince a majority of voters to vote for her if she faces a non-reforming opponent. In equilibrium, therefore, the reform is proposed by both candidates with probability 1.

The fact that both candidates raise the maximum debt follows the political forces highlighted in Lizzeri (1999). Whatever amount of resources is left in the future is not targetable to first-period voters. A candidate that does not run the maximal debt is therefore forced to offer an egalitarian distribution for the resources that she leaves in the future. This goes against the incentive to skew the distribution of resources in order to gain the electoral support of the voters that are treated favorably in the process of redistribution. The electoral uncertainty is not an artifact of the assumption that politicians are unable to commit to second-period transfers. Lizzeri (1999) shows that allowing candidates to commit does not change the electoral incentives to run debt: a candidate who commits to future transfers can only make promises about her own future behavior, not about the transfers made by the other candidate. This implies that, if a candidate does not run the maximal deficit, there is still an element of redistributive uncertainty concerning the second-period outcome. This uncertainty is enough to make voters view the outcome of the future election as relatively egalitarian.

From Proposition 2.5.1, we can draw two implications for the interaction between public debt and the decision to reform when proposed by competing political candidates. First, the reform increases the repayment capacity for debt and this can help the reformer to compensate her targeting disadvantage from having to finance the reform costs. Second, since both candidates will go for the reform in equilibrium, this will push both candidates to the maximal possible debt level under reform. If one candidate did not reform, the other candidate would not need to go up to the maximal debt under reform in order to win the election. For instance, against a non-reformer, it is enough that the debt raised by the reformer surpass the debt under no-reform by $c$, the costs of the reform. Then a reforming candidate has at least as many resources available for targeting voters in the first period. On top of that, a reformer can offer a higher expectation of future transfers. Through this, she would still beat any non-reformer. However, in equilibrium, every candidate competes against another reforming candidate. Therefore, both are pushed to transfer the full reform benefits to the present by debt.

The following corollary provides a complete equilibrium characterization, including the equilibrium transfer distributions.

**Corollary 2.5.2** *Suppose that the reform benefits are of a private good nature, i.e.* $\lambda = 1$. *Then, in the unique equilibrium*

*(i) both candidates reform with probability 1:* $\beta^A = \beta^B = 1$*;*

*(ii) both candidates announce the maximum debt level:* $\delta^A = \delta^B = 1 + e$*;*

*(iii) first-period offers to voters are drawn from a uniform distribution on* $[-1, 3+2(e-c)]$. *That is both candidates draw first-period offers from the following distribution:*

$$F_R^*(x) = \begin{cases} 0, & \text{if } x \leq -1, \\ \frac{x+1}{2+2(1+e-c)}, & \text{if } -1 \leq x \leq 3+2(e-c), \\ 1, & \text{if } x \geq 3+2(e-c). \end{cases} \tag{2.7}$$

*Second-period offers are degenerate on* $\mu_2 = -1$.

In the electoral game considered here, any candidate has an incentive to skew the distribution of resources in order to gain the electoral support of the voters. In the first period, the total resources available from debt and first-period endowment minus the cost of the reform are allocated as in Myerson (1993)'s analysis: the incentives to "cultivate favored minorities" remains a feature of electoral competition as in the second-period equilibrium outcome.

### 2.5.2 Reform with private and public good benefits

We now consider the case where the reform benefits partly have the character of a public good, i.e. $\lambda < 1$. Since the proportion of benefits with private good character can be potentially targeted to voters, we refer to $\lambda$ also as the targetability of reform benefits. The following proposition summarizes the outcome of the electoral game for different compositions of reform benefits.

**Proposition 2.5.3** *(I.) When the reform has a large share of private good benefits* $\lambda$ *such that* $e - \lambda e - 2(c - \lambda e) > 0$, *in the unique equilibrium both candidates always reform and announce the maximal debt.*

*(II.) When the reform has a low share of private good benefits* $\lambda$ *such that* $H := 2(c-\lambda e)-(e-\lambda e) > 0$, *in the unique equilibrium both candidates reform with probability* $1 - \frac{1}{2}H < 1$ *and announce the maximal debt.*

**Decision to reform.** When only a share $\lambda$ of the reform benefits translates into an increase in the second-period endowment, then the natural debt limit under reform increases only by $\lambda e$ compared to no-reform. This means that through using public debt, the reformer can only make the part $\lambda e$ of reform benefits targetable to first-period voters. For the remaining part, she is forced through the public good nature of these benefits to offer them equally across all voters. However, Part (I.) in Proposition 2.5.3 subsumes the case where targetability $\lambda$ is so high that that $\lambda e \geq c$. In this case, the additional debt that a reformer can raise is high enough for her to completely cover the reform costs, without having less targetable resources in the first period. Furthermore, she will be able to offer the additional public good benefits created in the second period. For this subcase, any reformer will therefore beat a non-reforming opponent.

In the following, we focus on the case where $\lambda e < c$. That is, even when using her higher debt repayment capacity, the reformer cannot fully compensate for the targeting advantage of the non-reformer, who saves the costs of the reform. However, the reform is efficient in the sense that total benefits surmount total costs. Thus the reformer has more to offer in total even if she is partly forced to distribute this bigger pie in an egalitarian way. The question then is whether this efficiency gain combined with the increased debt capacity is enough to compensate the first-period cost savings of the non-reformer.

For the case $\lambda e < c$, a no-reform candidate has more resources available in the first period for targeting voters. Specifically, the additional per-capita amount available to her equals the difference between the reform costs and the part of the future benefits that can be transferred to the present through debt, $c - \lambda e$. On the other hand, in case of reform everyone expects a boost in future utility through the public good benefits of the reform. More specifically, each voter expects additional utility $e - \lambda e$ in case of reform. Since the reform is efficient in the sense that benefits $e$ are greater than costs $c$, the additional public good utility, $e - \lambda e$, surmounts the loss in targetable resources in the first period, $c - \lambda e$. However, these public good benefits cannot be targeted. Hence, the additional public good utility must be high enough so that the non-reformer cannot convince a majority to vote for her. In particular, she should not be able through her advantage in targetability to make at least half of the voters as well off as under reform. This is exactly the condition of Proposition 2.5.3: $e - \lambda e > 2(c - \lambda e)$. The factor "2" on the right hand side of this inequality is explained by the fact that, in order to win a majority through targeting, a candidate can promise very low offers to $\frac{1}{2}$ of the voters in order to offer attractive benefits to the other half. If the condition $e - \lambda e > 2(c - \lambda e)$ is fulfilled, as in Part (I.) of Proposition 2.5.3, then the efficiency gain of the reform is high enough to trump the targetability advantage of the non-reformer.[21]

In Part (II.) of Proposition 2.5.3, $H = 2(c - \lambda e) - (e - \lambda e) > 0$, and the additional public good utility under reform is not enough to compensate for the fact that a no-reform candidate has more targetable resources in the first period. We therefore interpret $H$ as the net targeting advantage of *not* doing the reform. If $H > 0$, the more in targetable resources is enough to outweigh the efficiency gains from reform and the reform cannot be offered with probability 1 in equilibrium. This means that we get a failure of the political process to deliver the efficient outcome.

Note, however, that even with a net targeting advantage of no-reform, the reform will still be played with positive probability in equilibrium as long as it is efficient, i.e. $e - c > 0$. The reason for this will be discussed after having described the full equilibrium in Corollary 2.5.4.

**Decision to raise debt.** In Proposition 2.5.3, the incentives to go for the maximal possible debt persist and the intuition is similar to the one in Proposition 2.5.1. An important insight from Proposition 2.5.3 is that the ability to raise higher debt under reform ensures the implementation of the reform with certainty only when the benefits of the reform are mainly of a private good nature. In the opposite case, when the nature of the reform is such that only a small part of the reform benefits have a private good aspect, a large share of the reform benefits are non-targetable to begin with and cannot be targeted to first-period voters through the use of debt. Therefore, we are getting into the trade-off between efficient (non-targetable) public good spending and targetable transfer spending. This trade-off is at the core of the static setup of Lizzeri and Persico (2001) and we discuss it after Corollary 2.5.4.

[21] Note that one way to fulfill this condition is the case where the reform benefits are higher than double the costs of the reform, $e > 2c$. Hence if the net benefit of the reform is high enough, the reform will always be implemented independent of the nature of the reform benefits. In that case, the efficiency gain is so high that it can compensate for the targeting disadvantage of having to finance the reform costs even if none of the reform benefits are targetable.

In the following corollary, we characterize the full equilibrium including the equilibrium transfer distributions.

**Corollary 2.5.4** *(I.) Suppose that the reform has mainly private good benefits, i.e.* $\lambda$ *is high enough such that* $e - \lambda e - 2(c - \lambda e) > 0$. *Then, in the unique equilibrium*

*(i) both candidates reform with probability* $\beta^A = \beta^B = 1$;

*(ii) announce the maximal debt:* $\delta^A = \delta^B = 1 + \lambda e$;

*(iii) first-period offers to voters are drawn from a uniform distribution on* $[-1, 3 + 2(\lambda e - c)]$. *That is both candidates draw first period offers from the following distribution:*

$$F_R^*(x) = \begin{cases} 0, & \text{if } x \leq -1, \\ \frac{x+1}{2+2(1+\lambda e - c)}, & \text{if } -1 \leq x \leq 3 + 2(\lambda e - c), \\ 1, & \text{if } x \geq 3 + 2(\lambda e - c). \end{cases} \tag{2.8}$$

*Second-period transfer offers are degenerate on* $\mu_2 = -1$. *Additionally, everybody receives utility* $(1 - \lambda)e$ *from the public good aspect of the reform. In total, using the linear utility assumption, the second-period outcome is equivalent to a transfer distribution that is degenerate on* $-1 + (1 - \lambda)e$.

*(II.) Suppose that the reform has mainly public good benefits, i.e.* $\lambda$ *is low enough such that* $H := 2(c - \lambda e) - (e - \lambda e) > 0$. *Then, in the unique equilibrium*

*(i) both candidates reform with probability* $\beta^A = \beta^B = 1 - \frac{1}{2}H < 1$;

*(ii) both candidates announce the maximal debt:* $\delta_N^i = 1$ *in case of no-reform,* $\delta_R^i = 1 + \lambda e$ *in case of reform,* $i \in \{A, B\}$.

*(iii) When candidates do not reform, they draw first-period offers from the following*

*distribution:*

$$F_N^*(x) = \begin{cases} 0, & \textit{if } x \leq -1, \\ \frac{1}{2}\left(\frac{x+1}{H}\right), & \textit{if } -1 \leq x \leq -1+H, \\ \frac{1}{2}, & \textit{if } -1+H \leq x \leq 3-H, \\ \frac{1}{2}\left(1+\frac{x-3+H}{H}\right), & \textit{if } 3-H \leq x \leq 3, \\ 1, & \textit{if } x \geq 3. \end{cases} \tag{2.9}$$

*Second-period offers are degenerate on* $\mu_2 = -1$.

*When candidates reform, they draw first-period offers from the following distribution:*

$$F_R^*(x) = \begin{cases} 0, & \textit{if } x \leq -1, \\ \frac{x+1}{4-2(c-\lambda e)}, & \textit{if } -1 \leq x \leq 3-2(c-\lambda e), \\ 1, & \textit{if } x \geq 3-2(c-\lambda e). \end{cases} \tag{2.10}$$

*Second-period transfer offers are degenerate on* $\mu_2 = -1$. *Additionally, everybody receives utility* $(1-\lambda)e$ *from the public good aspect of the reform.*

When the reform has mainly private good benefits, the candidates can target these benefits to particular voters in the first period: the reform is implemented with certainty and maximal debt is raised. Therefore, both candidates compete on redistributing the same amount of resources in the first period. The form of the transfer distribution follows again the insight of Myerson (1993).

When the reform has mainly public good benefits, the efficiency gain of doing the reform cannot compensate for the targeting disadvantage of having to cover the reform costs. Nevertheless, the reform will still be implemented with positive probability. The underlying mechanism has been analyzed by Lizzeri and Persico (2001) in a static setup. By still playing the reform option with some probability, a candidate can use the efficiency gain of the reform to force her opponent to concentrate half of her offers on relatively "expensive" voters: these voters can be convinced to vote against the reform by receiving at least a transfer that fully covers the public good utility loss from the no-reform decision and also compensates for the additional transfer offered by the reforming candidate. As can be seen from Corollary 2.5.4, the distribution offered in case of no-reform, $F_N^*(x)$, has a disconnected support with an upper and a lower part. The upper part starts where any transfer on this part will ensure the vote against the reform distribution: an offer in the upper part covers at least the utility loss $(1 - \lambda)e$ from not implementing the reform plus the best transfer offered by the reforming candidate $3 - 2(c - \lambda e)$. Due to the efficiency gain of the reform, it does not make sense, in terms of maximizing the expected vote share, to attack a reformer with offers that don't beat her best offer for sure. In that sense, part of the no-reform offers that lie above the best offer under reform are more "expensive" than if reform were played with probability zero and a no-reform candidate could play a uniform distribution with a connected support. Therefore, a candidate that reforms with positive probability gains an advantage if her opponent were to never offer the reform.

When the net targeting advantage $H$ of the non-reformer decreases, the probability of reform goes up. Ceteris paribus, $H$ decreases when the targetability $\lambda$ of reform benefits goes up. That is, the more private good aspects a reform has, the more public debt can help in overcoming the reformer's targeting disadvantage from financing the reform costs and the higher the chance of the reform to be implemented in electoral competition. On the other hand, if a reform has a high share of public good benefits, then public debt, which can only transfer the private good aspects to the present, cannot help as much in overcoming the targeting disadvantage of a reformer. For the same efficiency gain, such a reform will therefore be implemented with lower probability as an electoral outcome.

## 2.6 Constitutional limit on debt

We now study how an exogenous restriction on the amount of debt that can be incurred by politicians changes the chances of reform adoption. Limits on deficit and debt are a popular response to the recent debt crisis and are present in many jurisdictions: most U.S. states have a balanced-budget rule, the Stability and Growth Pact in the European

Union limits gross government debt to sixty percent of the country's GDP. Germany adopted in 2009 a constitutional rule referred to as the debt brake that requires the federal and state governments to run balanced budgets from 2016 and 2020 onwards respectively (see Janeba (2012) for details).

### 2.6.1 Debt limit and the nature of reform benefits

We have seen in the last section, the nature of reform benefits and the availability of public debt are the crucial determinants of the success of reforms in the political process. We now focus on the effect of public debt only and study the case where reform benefits are of a private good nature, i.e. $\lambda = 1$. That is, the nature of reform benefits will not be the restrictive factor that makes reforms fail to be implemented.

We denote by $\bar{\delta}$ the first-period debt limit. In this subsection, we assume that $\bar{\delta} \geq 1$. This implies that the debt limit is more restrictive than the natural debt limit only in case of reform. This assumption will simplify the comparison with the results in the previous section. In the following subsection, we will then consider the case $\bar{\delta} < 1$. Furthermore, we assume that $\bar{\delta} \leq 1 + e$. This is the interesting case to consider since it implies that the exogenous debt limit is lower than the natural debt limit under reform.

For the equilibrium characterization, we work with a modified definition of the debt limit. We define $\bar{\rho} = \bar{\delta} - 1$ so that $\bar{\rho} > 0$ implies a maximal debt of $\bar{\delta} = 1 + \bar{\rho}$. The term $\bar{\rho}$ captures the amount by which debt can be higher than the per-capita endowment of 1. By definition, it is possible to raise debt higher than the endowment only in case of reform. For the reform case, this measure captures how easily the reform benefits $e$ can be drawn to the first period by debt. If $\bar{\rho} > 0$, then $e - \bar{\rho}$ is the amount of the reform benefits that cannot be drawn to the first period by debt and remains available in the second period for transfers. If $\bar{\rho} = 0$, then the maximal debt $\bar{\delta}$ equals the endowment of 1. In case of reform, the full benefits $e$ then remain available in the future for transfers. The following proposition characterizes the equilibrium strategies in terms of reform and public debt decisions. Corollary A.1.1 in the Appendix presents the complete equilibrium characterization including the equilibrium offer distributions.[22]

[22] Corollary A.1.1 is analogous to Corollary 2.5.4 with the debt limit $\bar{\rho}$ taking the place of the fraction of reform benefits with private good nature $\lambda e$.

**Proposition 2.6.1** *Suppose that the reform benefits are of a private good nature, i.e.* $\lambda = 1$*, and that there is a constitutional debt limit* $\bar{\delta} = 1 + \bar{\rho} \geq 1$*.*

*(I.) When the debt limit* $\bar{\rho}$ *is such that* $e - \bar{\rho} - 2(c - \bar{\rho}) > 0$*, in the unique equilibrium both candidates always reform and announce the maximal debt.*

*(II.) When the debt limit* $\bar{\rho}$ *is restrictive enough such that* $\widehat{H} := 2(c - \bar{\rho}) - (e - \bar{\rho}) > 0$*, in the unique equilibrium both candidates reform with probability* $1 - \frac{1}{2}\widehat{H} < 1$ *and announce the maximal debt.*

Comparing these results to Proposition 2.5.3, we see that restricting public debt has a similar effect as increasing the proportion of reform benefits with public good nature. In particular, even if now the full reform benefits are potentially targetable, any reform benefits that have to be left in the future due to the debt limit acquire the characteristics of a non-targetable public good from the point of view of first-period voters. The amount $e - \bar{\rho}$ corresponds to the part of future reform benefits that cannot be transferred to the present. Since the outcome of future redistribution is uncertain, these resources cannot be skewed to specific voters. On the contrary, each voter expects the same amount $e - \bar{\rho}$ of additional second-period transfers under reform. The part $e - \bar{\rho}$ of the reform benefits that has to be left in the future is just like a public good whose benefits cannot be targeted. The difference to Proposition 2.5.3 is that, if more debt was allowed, this part could also be targeted to first-period voters. For the case of Proposition 2.5.3, the public good characteristic was given through the nature of the reform and could not be changed. Here, in contrast, it is created through the debt limit combined with future electoral uncertainty.

Through this analogy, we get the same case distinction as before. If the debt limit is not too restrictive such that enough future reform benefits can be targeted to first-period voters, then the cost-saving advantage of the non-reformer is overcome and reform is implemented with certainty in the political equilibrium. On the other hand, if the debt limit becomes too restrictive, the no-reformer has a net targeting advantage. Due to its efficiency gain the reform will still be implemented with some probability for the same reason as discussed for Proposition 2.5.3. However, the reform will no longer be implemented with certainty and the political process fails to deliver the efficient outcome.

Note, in particular, that the probability of reform decreases in $\bar{\rho}$. That is, the probability of reform is the lower, the more restrictive the debt limit. In this subsection, the debt limit $\bar{\delta} = 1 + \bar{\rho}$ is restricted to be greater or equal to 1, the natural debt limit under no-reform. Therefore, $\bar{\rho}$ captures how much more debt the reformer can raise compared to the non-reformer. Since the reformer creates additional resources in the amount of $e$ in the future, $\bar{\rho}$ captures how much of these resources can be made targetable to first-period voters by transferring them to the present. A more restrictive debt limit means that this transfer is inhibited and therefore the reformer has less possibility to compensate for the loss in targetable resources caused by the reform costs.

If $\bar{\rho} = 0$ or equivalently $\bar{\delta} = 1$, then both the reformer and the non-reformer can raise the same amount of debt. In that case, none of the reform benefits can be transferred to the present. If the debt limit becomes even more restrictive and drops below the natural limit under no-reform ($\bar{\delta} < 1$ or $\bar{\rho} < 0$), then both the reformer and the non-reformer can still raise the same amount of debt. One could therefore expect that the probability of reform does not change compared to the case $\bar{\delta} = 1$, when the debt limit is decreased below 1. Nevertheless, as we show in the following subsection, making the debt limit more restrictive in this sense still decreases the probability of reform.

### 2.6.2 Very restrictive debt limit

In the following, we consider the case $\bar{\delta} < 1$. That is, the exogenous debt limit is decreased below the natural debt limit under no-reform. This means that none of the reform benefits can be transferred to the present. Therefore, the nature of the reform benefits is immaterial and the results here apply both to a purely pie-increasing reform as well as to a reform that creates only public good benefits. In the previous subsection, we introduced the term $\bar{\rho} = \bar{\delta} - 1$ to capture by how much the exogenous debt limit is *higher* than the natural limit under no-reform. For the case considered here, we define $\bar{\sigma} = 1 - \bar{\delta}$ to capture by how much the exogenous debt limit is *lower* than the natural limit under no-reform.

The following proposition summarizes the equilibrium outcomes in terms of the debt and reform decisions in the case $\bar{\delta} < 1$. The full characterization of the equilibrium including the equilibrium offer distributions can be found in Corollary A.1.2 in the Appendix. The results resemble the ones in Proposition 2.5.3 and Proposition 2.6.1 but the concepts used before need to be adjusted in the way explained below. Importantly, in terms of interpretation, these adjustments will imply a second interaction between restrictions on public debt and reforms that was not present in the previous subsection.

**Proposition 2.6.2** *Suppose that the exogenous debt limit is such that $\bar{\delta} = 1 - \bar{\sigma} < 1$.*

*(I.) When the reform benefits are high enough compared to the reform costs such that $e > 2c$, in the unique equilibrium both candidates always reform and announce the maximal debt.*

*(II.) When the reform benefits are low enough compared to the reform costs such that $\widetilde{H} := 2c - e > 0$, in the unique equilibrium both candidates reform with probability $1 - \frac{1}{2-\bar{\sigma}}\widetilde{H} < 1$ and announce the maximal debt.*

Note first, that what we defined as the net targeting advantage of no-reform is now given by $\widetilde{H} = 2c - e$. Recall that this concept relates the gain in targetable resources from not doing the reform to the additional non-targetable resources that the reform creates. In the case where the exogenous debt limit is more restrictive than the natural debt limit under no-reform, the reformer cannot raise more debt than the non-reformer. Thus the reformer's loss in targetable resources in the first period corresponds to the full reform costs.[23] Furthermore, since none of the reform benefits can be transferred to the first period, the additional non-targetable resources that the reform creates are equal to the full reform benefits.

We have already discussed shortly in subsection 2.5.2 that, if the efficiency gain of the reform is high enough, in the sense of the benefits surmounting the double amount of the costs, then the reform will always be implemented. This is stated formally as Part (I.) of Proposition 2.6.2. In that case, $\widetilde{H} = 2c - e$ becomes negative, and hence the non-reformer does not gain an advantage from saving on the costs of the reform. The benefits are so high that, even though they are completely non-targetable, the reformer will still be able to offer more to a majority of voters. In this case, the implementation

[23] Recall that the factor 2 in front of the reform costs accounts for the fact that targetable resources can be shifted across voters and only 50 percent of voters must be convinced through such a shift in order to win a majority.

of the reform does not depend on issues of targetability and hence is also not affected by the exogenous debt limit.

Part (II.) of Proposition 2.6.2 is the more interesting case, where the benefits are low enough such that issues of targetability play a role for the implementation of the reform. In this case, $\widetilde{H} = 2c - e > 0$, such that there is a net targeting advantage for the non-reformer. However, this is no longer the only parameter that determines the probability of reform. This probability is now given by $1 - \frac{1}{2-\bar{\sigma}}\widetilde{H}$. As we can see, it also depends on $\bar{\sigma}$, the amount by which the exogenous debt limit is lower than the natural debt limit under no-reform. In particular, a more restrictive debt limit, which corresponds to an increase in $\bar{\sigma}$, decreases the probability of reform. The intuition is the following: the parameter $\bar{\sigma}$ captures the part of the potentially targetable second-period endowment that *both* the reformer and the non-reformer must leave in the future. Since only resources in the first period are targetable to first-period voters, this decreases the size of the targetable pie. Politicians compete on this targetable pie when implementing their pattern of redistribution. Therefore, if the total size of the targetable pie shrinks, the loss in targetable resources through financing a certain amount of costs is now relatively bigger. In other words, a given net targeting advantage of the non-reformer, $\widetilde{H} = 2c - e > 0$, puts the reformer in a relatively worse position, the lower the size of the targetable pie. By decreasing the size of this targetable pie, a stricter exogenous debt limit therefore makes reform less likely.

Here we can see a second reason why a more restrictive debt limit can be detrimental to the implementation of reforms. In the previous subsection, where the debt limit was higher than the natural debt limit under no-reform, a stricter debt limit decreased the probability of reform by hampering the transfer of reform benefits to the present. In this subsection, we have considered the case of a debt limit that is already so restrictive that no reform benefits can be transferred to the present. Therefore, a more restrictive debt limit now implies restricting the transfer of future resources that would exist even in the absence of reform. As explained, this decreases the targetable pie on which both candidates compete. The loss in targetable resources due to the reform costs then becomes relatively more important. Therefore, the probability of reform still decreases with a stricter debt limit. As noted at the beginning, the results here apply also to a reform that only creates public good benefits. Even though public debt cannot ensure the implementation of such a reform, it can still make the reform more likely.

These results point to a new view on the trade-off between targeted pork-barrel spending and efficient spending decisions, like the financing of a beneficial reform. As long as the efficient policy creates benefits only in the next electoral cycle, allowing enough debt-related targeted spending might be necessary in electoral competition to incentivize spending on the efficient policy.[24]

## 2.7 Concluding remarks

In this paper we show that the interaction between the reform decision and the decision to raise public debt is decisive in shaping electoral incentives for reform. We consider a "pure" electoral game where politicians are office-motivated and compete for election by targeting resources to subsets of voters at the expense of others. A reform that costs resources today and yields higher benefits in the future forces a politician to give up targetable resources today. Due to endogenously arising uncertainty about the outcome of future elections, future resources cannot be targeted to specific voters in the present as long as these resources remain in the future. This mechanism on its own puts a reformer at a disadvantage to a non-reformer, since targetable resources are more valuable for convincing a majority of voters today. However, public debt potentially allows politicians to transform future reform benefits into resources that can be targeted to today's voters. This can help overcome the original disadvantage that results from the dynamic distribution of reform benefits and costs.

We show that the reform is always implemented when sufficient debt can be raised. This is the case if enough reform benefits are of a private good nature in the sense of translating into an increase in the future endowment. Such reform benefits can potentially be transferred to the present by debt.

We also show that restricting the use of public debt hampers the chances of a reform to go through the political process. With a lower capacity to raise public debt, a reforming politician can only commit to a lower present-day redistribution. This makes it harder to sustain the implementation of beneficial reforms in electoral

[24] For the case of a purely pie-increasing reform, we also considered an extension to three periods and showed that our results are robust in such a multi-period setup.

competition. This result holds even for reforms whose benefits cannot be transferred to the present by their public good nature. This implies that constitutional restrictions on public debt might be a hurdle for the implementation of reforms by politicians.

Our results point towards a new evaluation of the trade-off between targeted spending and efficient spending decisions. In particular, enough debt-related targeted spending might be necessary in electoral competition to incentivize efficient spending on dynamic policies whose benefits only accrue in the next electoral cycle.

# Chapter 3

# The Political Economy of Reforms with Distortionary Public Debt[1]

## 3.1 Introduction

A recent paper by Boyer and Esslinger (2016)[2] investigates the interaction between the decision to raise public debt and the decision to implement growth-enhancing reforms in an electoral setup of targeted redistribution.[3] In this setup, politicians target available resources to subsets of voters in order to maximize their vote share. Boyer and Esslinger (2016) show that efficient dynamic policies, like growth-enhancing reforms, might only have a chance in the political process if politicians have enough access to public debt. The argument is that such reforms usually incur costs in the current electoral cycle while yielding their benefits only in future electoral cycles. Therefore, seen on their own, these reforms imply a disadvantage in terms of promising electoral favors to current voters. This is because, first, expenditures for costly reforms in the present cannot be used as electoral favors to current voters. Second, a reform-induced increase in future resources, as long as it remains in the future, cannot be promised to specific subsets of current voters due to uncertainty about the future electoral outcome. However, if politicians can use public debt to shift resources that are the results of present reform decisions from the future to the present, then this disadvantage in the

[1]I thank Pierre Boyer, Eckhard Janeba, and Andreas Peichl for very helpful comments, as well as seminar participants at Mannheim University. The author gratefully acknowledges the Collaborative Research Center 884 for financial support. The usual disclaimer applies.

[2]This citation refers to the preceding chapter of this dissertation.

[3]This setup of electoral competition was first introduced by Myerson (1993).

capacity to target electoral favors can be completely overcome. Indeed, Boyer and Esslinger (2016) show that without any restrictions on public debt, the efficient policy of implementing a growth-enhancing reform will always be carried out.[4] Furthermore, they show that introducing a limit on public debt prevents the certain implementation of the efficient reform when the debt limit becomes restrictive enough. The probability of reform then decreases continuously the more restrictive the debt limit gets.

The electoral setup of targeted redistribution creates a bias of the political process towards high public debt.[5] As already mentioned, resources that are left in the future cannot be promised as electoral favors to current voters. By shifting future resources to the present, these resources can be used to convince a majority of voters to win the election. Since voters are self-interested, they prefer to be advantaged today and secure their favors instead of leaving the distribution of future resources to a future election whose outcome in terms of allocating favors is uncertain.[6] This bias towards high debt is therefore closely related to other political economic biases that also imply the tendency for high debt, like a short-term horizon of policy makers or the common pool problem. Actually, since only a majority of voters is advantaged in terms of the distribution of future resources but everyone has to contribute to repaying the resulting debt, we do have some form of a common pool problem here. Similarly, the interest of politicians in winning the current election creates a short-term bias for them. The previous literature that has focused on these political economic biases for high debt has shown that they exist even when high public debt creates distortions and imposes an efficiency loss compared to low debt levels.[7]

In contrast, one important assumption in Boyer and Esslinger (2016) is that public debt has no direct efficiency implications itself. That is, raising high public debt does not create any distortions that impose a cost to the economy. Instead, public debt is merely a means of shifting money across time. Given the high costs that arose for many European countries recently due to a high level of public debt, it makes sense to investigate the effect of debt-related distortions in the above setup

[4] Note that in this paper, the focus is purely on growth-enhancing reforms that increase the pie of potentially redistributable resources. Boyer and Esslinger (2016) additionally consider the case where the reform can create public good aspects that cannot be redistributed.

[5] This was first shown by Lizzeri (1999).

[6] Note that this is not an argument about risk-aversion. Rather, when future resources are already allocated today, the majority of advantaged voters gets more than what they would expect to get if the distribution is left to a future election.

[7] See, for instance, Battaglini and Coate (2008) for a recent contribution to this literature.

of political competition. This is the goal of the current paper. In the case of debt-related distortions, the incentivizing effect of public debt on efficient reforms, which was highlighted in Boyer and Esslinger (2016), has to be weighed against possible negative effects of high public debt. These negative effects include, for instance, costs related to sovereign default, which gets more likely with high public debt. A related detrimental consequence of high public debt is less possibilities to smooth out tax-related distortions across time. This happens when the further increase of public debt gets more costly at high debt levels. Such a situation can prevent the use of public debt exactly then when debt would be most necessary to avoid high taxes. In terms of thinking about efficient policies, if a high level of public debt implies high distortions, then investing into low public debt can be seen itself as an efficient dynamic policy with costs in the current electoral cycle and benefits in future electoral cycles. It is the only such policy which by its nature *cannot* be incentivized by the use of more public debt.

The main insights from our analysis are the following. In a setup where high public debt creates distortions, the most efficient policy bundle is to avoid high public debt and to implement the pie-increasing reform. Together, this implies the maximization of the total resources available to the economy. If the distortionary cost of raising high public debt is very high, the political process eliminates high public debt on its own. In particular, a majority of voters will only support a high-debt policy if at least this majority can be compensated for the efficiency loss resulting from high public debt. If the efficiency loss is so high that the advantaged majority is worse off compared to what they would expect to get under a low-debt policy, then only low-debt policies will be supported in electoral campaigns. In this case, the incentivizing effect of public debt on the pie-increasing reform is not present and hence the reform will not be implemented with certainty.

On the other hand, if the distortionary cost of raising high public debt is low enough compared to the net benefit of the reform, then the reform will always be implemented in electoral competition if no restrictions are placed on public debt. In this case, the political process creates a bias for high public debt, but this debt unfolds its incentivizing effect on reform implementation. In particular, with some probability high public debt is raised and this allows the reform policy to be used in a high enough degree for targeting current voters. Due to the use of high public debt, we do have an efficiency loss due to debt-related distortions, but this ensures efficiency on the side of implementing the pie-increasing reform. This trade-off persists once we introduce an exogenous debt limit into the analysis. Such a debt limit reduces the distortionary costs of public debt. However, at the same time, not implementing the reform gets more attractive since public debt cannot unfold its incentivizing effect on reforms as well. Hence, when setting a debt limit, more efficiency on the side of public debt has to be weighed against lower efficiency in terms of implementing the pie-increasing reform.

The setup we use to derive our results builds on Boyer and Esslinger (2016) but makes the use of public debt distortionary. Specifically, we consider a model with two periods and the same two politicians run for election in each period. These politicians are purely office-motivated and voters are ex-ante homogenous with each voter having a resource endowment of one in each period. Politicians compete for voters by redistributing the available resources across voters. They are not restricted in the tools available for redistribution of resources within a given period. In the first period, politicians additionally decide whether to raise maximal public debt or no debt at all[8] and on the implementation of a reform that costs some resources today but will increase resources in the next period. Importantly, in the case of maximal public debt, there are expected costs on the economy in the second period. Voters and politicians know the reform benefits and reform costs and the costs related to a high level of public debt. The reform is efficient in the sense that its benefits surmount its costs. Raising high public debt is inefficient in the sense that the debt-related costs reduce the total size of the resource pie. Finally, targeting resources to voters is a purely tactical instrument and does not have any efficiency implication.

Our contributions are two-fold. First, on the methodological side, we develop the necessary tools to deal simultaneously with two efficiency-related policy issues - pie-increasing reform and distortionary public debt - and still keep the generality of the setup in Boyer and Esslinger (2016) in terms of modeling electoral competition. The two relevant factors determining the probabilities of different debt-reform policy bundles in the optimal electoral strategy are efficiency - in terms of maximizing the total

[8] This binary modeling allows politicians to follow the tendency for maximal debt identified in this electoral setup by Lizzeri (1999) and Boyer and Esslinger (2016) but allows at the same time an avoidance of debt-related distortions through the possibility of a no-debt policy.

size of the pie - and targetability - in terms of the capacity to transfer enough resources to a majority of voters in order to win their votes. We show how these two factors can be captured by two simple parameters of the transfer distributions that politicians offer to voters under different debt-reform policies: the maximal transfer given to a voter under the respective transfer distribution and the resource constraint of that distribution. We show that, in order to study the effect of the debt and reform policy choices on the electoral outcome, all that needs to be understood is how the combination of these policy choices determines the resource constraints and the upper bounds of the transfer distributions that can be offered to voters. Given the complexity of the constructive proofs, this insight is necessary to keep the derivations tractable.

This methodological contribution will have interesting applications beyond the scope of the specific issue studied in this paper. Specifically, it allows to develop general results for the above-employed setup of electoral competition following Myerson (1993) when other policy issues besides redistributive transfers determine the electoral outcome. That is, reform and public debt can be replaced by other policy issues with direct efficiency implications and the resulting electoral outcome can be studied. In contrast to the previous literature, our methods can deal with two additional efficiency-related policy issues besides redistributive transfers instead of only one.[9] For instance, Lizzeri and Persico (2001) analyzed the additional provision of a public good as the only policy issue besides redistributive transfers, while Lizzeri (1999) introduced public debt as the one additional policy issue besides redistributive transfers into the setup of Myerson (1993). Closely related to our study of debt-related distortions is the paper by Crutzen and Sahuguet (2009), which deals with tax-related distortions. Nevertheless, this paper also only deals with this one additional issue besides redistributive transfers.[10]

Our second contribution lies on the substantive side. In particular, we deploy the just mentioned tools to study the interaction between the two policy issues of distortionary public debt and pie-increasing reforms in the setup of targeted electoral competition. A detailed discussion of the related literatures on public debt and reforms

[9] The decisive difference to Boyer and Esslinger (2016) is that besides the reform, public debt now also has efficiency implications.

[10] In their final section, Crutzen and Sahuguet (2009) discuss the issue of two different tax instruments, but they do not solve for the equilibrium when both tax instruments are used at the same time. A more detailed discussion of the literature following Myerson (1993) can be found in Boyer and Esslinger (2016).

can be found in Boyer and Esslinger (2016). As already mentioned, when debt-related distortions are very high, the political process avoids public debt on its own. In contrast, for the case where debt-related distortions are low enough compared to the net benefits of the reform, a political economic bias for high debt exists. For this case, we show how reaching efficiency on the side of the reform has to be traded off against efficiency on the side of public debt. In particular, when studying the introduction of an exogenous limit on public debt, we identify a direct and indirect efficiency gain on the side of public debt. First, restricting the level of public debt reduces directly the distortionary costs that arise with a higher level of public debt. Second, since debt is used to target future resources to current voters, a stricter debt limit reduces the targetability advantage of debt-related policies. This implies that no-debt policies are played with higher probability in the political equilibrium. Since such no-debt policies incur no debt-related costs, this results in a further efficiency gain.

Nevertheless, these direct and indirect efficiency gains through a debt limit have to be weighed against the efficiency losses in terms of implementing the reform. Specifically, when the debt limit becomes stricter, public debt cannot exert its incentivizing effect on reform implementation as well. Specifically, it gets more difficult to target future reform benefits to present voters. Therefore, with a stricter debt limit, the pie-increasing reform will be played with lower and lower probability in the political equilibrium. For a given amount of reform costs, we find that this negative effect of a debt limit is more pronounced for reforms with a lower net benefit.

In terms of policy implications, our analysis implies that a careful evaluation and weighting of the costs related to high public debt and the potential benefits of important growth-enhancing reforms is necessary to determine the optimal level of a debt limit. Our analysis gives some first qualitative indications which factors tilt the trade-off between reform-related efficiency and debt-related efficiency in a given direction. Specifically, when expected debt-related distortions are at levels where the political process exhibits a bias towards high public debt, a debt limit can help reduce the resulting distortions from excessive public debt. When net benefits of potential growth-enhancing reforms are substantial, a stricter debt limit does not decrease the probability of reform that much. However, any given decrease in this probability is worse, because higher net reform benefits are foregone. We also discuss that it is never optimal to restrict public debt below a level where distortions are unlikely to arise. In that case, restricting public debt has no efficiency gains on the side of public debt and only implies an efficiency loss in the sense that the pie-increasing reform is implemented with lower probability.

The rest of the paper is organized as follows. Section 3.2 describes the formal framework. Section 3.3 solves for the political equilibrium in the last period of the game. Our main results are presented in Section 3.4, where we solve for the equilibrium in the first period for low debt-related distortions, in Section 3.5, where we study the implications of debt limits, and in Section 3.6, where we derive the first-period equilibrium for the case of very high debt-related distortions. The last section contains concluding remarks. We relegate all proofs to the Appendix.

## 3.2 The model

Our setup builds on Boyer and Esslinger (2016) who combine the decisions to raise public debt and to implement a growth-enhancing reform in a model of redistributive politics. Here, we add distortions resulting from high public debt.

**The electorate.** There are two periods and a continuum of voters of measure one.[11] All voters are ex-ante homogenous. They are risk-neutral, live for the two periods, and have a discount factor equal to 1. There is only one good, which for comparability to Boyer and Esslinger (2016), we refer to as money. Voters have linear utility over this good with the marginal utility normalized to one. All voters in each period have an endowment of one unit of money which is perfectly divisible.

**Political process.** In each period, there is an election where voters choose between two candidates. The set of candidates is the same for both dates. One candidate is denoted by $A$, the other by $B$. Each candidate $i \in \{A, B\}$ is purely office-motivated and maximizes vote-shares.

[11] This is a convenient assumption that is meant to be an approximation for a game with a large (but finite) number of voters.

**Policies.** In the first period, a strategy $p_1^i$ has three elements: the decision to enact a reform, a level of public debt, and promises of taxes and transfers to each individual voter. We will define $p_1^i$ formally after introducing these three elements.

*1. Reform.* The possibility to enact a reform is present only in the first period. The cost of this reform is incurred in the first period and the benefit occurs in the second period. Denote by $c$ the per capita cost and by $e$ the per capita benefit from the reform. Specifically, $e$ corresponds to the increase in the per-capita endowment of the economy for the second period. We assume that:

$$e - c > 0, \tag{3.1}$$

$$c < 1, \tag{3.2}$$

$$2c - e > 0. \tag{3.3}$$

Assumption (3.1) states that the reform is beneficial and therefore should always be implemented from an efficiency perspective. Assumption (3.2) ensures that there is always enough first-period endowment to finance the reform. Assumption (3.3) excludes the rather uninteresting case of reforms whose net benefit is so high that they would always go through the political process independently of redistributive politics considerations.

*2. Debt.* Government debt is financed by borrowing from abroad and there is no possibility of default.[12] The size of the deficit in the first period is interpreted as the fraction of the average voter's second-period resources that is pledged to the repayment of the debt. The natural limit on debt corresponds to the total resources that can be mobilized to repay debt. In Boyer and Esslinger (2016), raising public debt did not create any distortion, it was merely a way of shifting money across time. In their model, the natural debt limit in case of no-reform is equal to 1 and corresponds to the case where the total amount of resources available in the second period is transferred to the first period by debt.

Here, we want to introduce distortions that arise when raising a high level of public debt. Given the already high generality of the model setup in terms of electoral competition, we do not model these distortions explicitly but capture them in a reduced

[12] While the distortion we introduce shortly could also be related to default in a more complex setup, for reasons of tractability, we stick to the no-default assumption here.

form parameter, which we introduce shortly. Politicians have the choice between two debt levels: no debt and the maximal debt, where the latter corresponds either to the natural debt limit defined above or a stricter exogenously imposed debt limit. This binary modeling allows politicians to follow the tendency for maximal debt identified in this electoral setup by Lizzeri (1999) and Boyer and Esslinger (2016) but allows at the same time an avoidance of debt-related distortions through the possibility of a no-debt policy.

We introduce a parameter $\gamma$ that captures in a reduced form way distortions that arise in the process of raising high public debt. In particular, we assume that, when transferring 1 unit of money across time through public debt, $\gamma$ units are lost in the process, with $0 < \gamma < 1$. The natural debt limit in case of no-reform is therefore $1 - \gamma$ and so the debt level $\delta$ belongs to $\{0, 1 - \gamma\}$ if the reform is not implemented and there are no exogenous restrictions on public debt. In a more complex world with infinite horizon and distortionary taxes, $\gamma$ could be interpreted as the loss in the future productive capacity of the economy when tax distortions can no longer be smoothed out very well at high debt levels due to the inability to increase debt much further. It makes sense to assume that such distortions play a role only for fairly high debt levels. When restricting public debt through an exogenous limit, one might therefore reach debt levels for which these distortions no longer arise. We will discuss the implications of this aspect in section 3.5.

Implementing the reform increases the natural debt limit by the induced increase in the second-period endowment adjusted for the distortion. Hence, when the reform is undertaken, the maximal amount of resources that can be transferred to the present increases by $\gamma e$. The debt level $\delta$ then belongs to $\{0, (1 - \gamma)(1 + e)\}$.

We also allow for the possibility that the amount of debt that can be incurred is exogenously restricted. We interpret such a restriction as a constitutional limit on debt, which we denote by $\bar{\delta}$. To allow comparability to Boyer and Esslinger (2016), we define $\bar{\delta}$ as the level of second-period resources that is allowed to be used for the repayment of debt. This is because we want to indirectly capture through $\bar{\delta}$ how much of the second-period resources are *not* transferred to the first period through debt and remain in the future. With $\bar{\delta}$ defined like that, a distortion implies that debt in the first period is actually restricted to be below $(1 - \gamma)\bar{\delta}$. In the following, we assume that $\bar{\delta} \leq (1+e)$.[13]

Finally, we denote by $\delta^i$ the debt level that is proposed by candidate $i$. When she proposes a no-debt policy, then $\delta^i = 0$. In case she goes for the maximal debt, then this maximal debt level also depends on her decision about the reform. In case of reform, we have $\delta^i \in \{0, \min\{(1-\gamma)\bar{\delta}, (1-\gamma)(1+e)\}\}$, where the second term inside the minimum operator is the natural debt limit under reform. In case of no-reform, we have $\delta^i \in \{0, \min\{(1-\gamma)\bar{\delta}, 1-\gamma\}\}$, where the second term inside the minimum operator is the natural debt limit under no-reform.

*3. Redistribution.* We formalize the transfer offers that candidates make to voters with offer distributions from which the actual transfer promises to voters are drawn. A negative transfer corresponds to taxing a voter and the lowest possible transfer offer within one period is $-1$ and corresponds to taxing away the full endowment of a voter in a given period. We denote the cumulative distribution functions from which the transfer offers are drawn as $F^i(\cdot), i \in \{A, B\}$. Specifically, we follow Myerson (1993) and assume that the favors offered to different voters are *iid* random variables with probability distribution $F$. We appeal to the law of large numbers for large economies and interpret $F(x)$ not only as the probability that any one individual receive an offer weakly smaller than $x$, but also as the population share of voters who receive such an offer. Therefore, $F$ describes the redistributive pattern resulting under the corresponding transfer scheme.

*Definition of strategies.* Now we are in a position to formally define a first-period strategy $p_1^i$ of candidate $i$. In total, there are four possible reform-debt bundles. To simplify the reference to these four different bundles, we denote them by the following capital letter notation:

$$
\begin{aligned}
&\{R;\, D\}: && \textit{Reform} + \textit{Debt};\\
&\{R;\, ND\}: && \textit{Reform} + \textit{No Debt};\\
&\{NR;\, D\}: && \textit{No Reform} + \textit{Debt};\\
&\{NR;\, ND\}: && \textit{No Reform} + \textit{No Debt}.
\end{aligned}
$$

As we will see shortly, the choices on reform and debt determine which transfers can

[13]The case $\bar{\delta} > (1+e)$ is uninteresting because such an exogenous debt limit implies a maximal debt that is higher than the natural debt limit under reform. Hence, such a debt limit would never bind.

be offered to the voting population. Therefore, we denote by $F^i_{1,Y}$ the cumulative distribution function for transfers in the first period by candidate $i$ for the case where $i$ implements reform-debt bundle $Y$, with $Y \in \{\{R;\, D\}, \{R;\, ND\}, \{NR;\, D\}, \{NR;\, ND\}\}$. Similarly, we denote by $\beta^i_Y$ the probability with which bundle $Y$ and the associated transfer distribution is played by candidate $i$. A first-period strategy $p^i_1$ of candidate $i$ is then given by the choice of the probabilities for different reform-debt bundles and the choice of the associated transfer distributions:
$p^i_1 = (\beta^i_{\{R;\,D\}}, \beta^i_{\{R;\,ND\}}, \beta^i_{\{NR;\,D\}}, \beta^i_{\{NR;\,ND\}}, F^i_{1,\{R;\,D\}}, F^i_{1,\{R;\,ND\}}, F^i_{1,\{NR;\,D\}}, F^i_{1,\{NR;\,ND\}})$.

A second-period strategy $p^i_2$ of candidate $i$ consists only of the choice of the second-period cumulative distribution function $F^i_2$. Finally, denote by $\delta^*$ the debt level realized after the first period.

**Feasible strategies.** Strategies are feasible if the first-period probabilities of the different reform-debt bundles add up to one and if the transfer distributions satisfy the following budget constraints.

First-period budget constraint:

if $\{NR;\, D\}$ is chosen,

$$\int_{-1}^{+\infty} x dF^i_{1,\{NR;\,D\}}(x) = \delta^i = \min\{(1-\gamma)\bar{\delta}, (1-\gamma)\}; \tag{3.4}$$

if $\{NR;\, ND\}$ is chosen,

$$\int_{-1}^{+\infty} x dF^i_{1,\{NR;\,ND\}}(x) = \delta^i = 0; \tag{3.5}$$

if $\{R;\, D\}$ is chosen,

$$\int_{-1}^{+\infty} x dF^i_{1,\{R;\,D\}}(x) = \delta^i - c = \min\{(1-\gamma)\bar{\delta}, (1-\gamma)(1+e)\} - c; \tag{3.6}$$

if $\{R;\, ND\}$ is chosen,

$$\int_{-1}^{+\infty} x dF^i_{1,\{R;\,ND\}}(x) = \delta^i - c = -c; \tag{3.7}$$

Second-period budget constraint:

if the reform is not undertaken and for a debt level $\delta^* \in \{0, \min\{(1-\gamma)\bar{\delta}, 1-\gamma\}\}$,

$$\int_{-1}^{+\infty} x dF_2^i(x) = -\delta^* \cdot \frac{1}{1-\gamma}; \tag{3.8}$$

if the reform is undertaken and for a debt level $\delta^* \in \{0, \min\{(1-\gamma)\bar{\delta}, (1-\gamma)(1+e)\}\}$,

$$\int_{-1}^{+\infty} x dF_2^i(x) = e - \delta^* \cdot \frac{1}{1-\gamma}. \tag{3.9}$$

Thus, in the first period, the *additional* resources that can on average be given to each voter are the resources transferred from the future by debt, $\delta^i$, minus the costs $c$ that have to be paid in case of reform. In the second period, the debt $\delta^*$ of the winner of the first-period election has to be repaid. Given the distortion, this requires a repayment of $\delta^* \cdot \frac{1}{1-\gamma}$. However, in case of reform, the amount of resources that can be redistributed across voters increases by the amount of reform benefits $e$.[14]

**Timing.** The timing of the game is as follows:

**Period 1:**

Stage 1 Each candidate $i = \{A, B\}$ plays a strategy $p_1^i$ in order to win the election.

Stage 2 Each voter observes her draw $(x_1^A, x_1^B)$ from each candidate's distribution plan, the reform proposals, the proposals for debt, and then votes.[15] When voters are indifferent between the two candidates, they flip a coin to decide who to vote for.

At period 2, everybody observes the first-period debt level that has to be repaid and if the reform was undertaken so that the strategies are conditioned on the first-period outcome; there are two stages:

[14]Note that since the budget constraints are formulated in terms of *transfers*, we look at changes in the existing endowment of people. That is a transfer of -1 means that the person loses its full endowment. We treat the reform benefits $e$ not as an additional per-capita endowment, but as a general increase in resources available for transfers (similar to the resources that are additionally available in the first period if debt is raised). This is why the lower bound of the last integral is $-1$: the whole per capita endowment of 1 is taken away and nothing from the additional pie is given to the worst-off individual. We could also work with $e$ occurring as an additional person-specific endowment, in which case this lower bound would become $-(1+e)$ and $e$ would disappear from the right hand side of the last budget constraint.

[15]Note that at stage 1, candidates potentially play a mixed strategy against each other in terms of (not) implementing the reform and in terms of (not) raising public debt. However, at the stage when voters decide, each candidate is committed to either reform or no-reform and to either the maximal debt or no debt. This approach is taken because, in terms of finding the equilibrium of the electoral game, we want to allow politicians to use mixed strategies in case there is no equilibrium in pure strategies. On the other hand, in terms of the voter decision, we do not want to introduce limited-commitment issues on the side of politicians.

**Period 2:**

Stage 1 Candidates choose distribution plans $F_2^i(\cdot)$, $i = A, B$.

Stage 2 Each voter observes her draw $(x_2^A, x_2^B)$ from each candidate's distribution plan and then votes.

**Vote-shares.** We denote by $S_t^i(p_t^i, p_t^h)$ the share of the votes of candidate $i$ in period $t \in \{1, 2\}$ if she chooses to play strategy $p_t^i$ and the other candidate $h$ chooses to play strategy $p_t^h$. Then, $S_t^h(p_t^h, p_t^i) = 1 - S_t^i(p_t^i, p_t^h)$.

The solution concept used is subgame-perfect Nash equilibrium.

## 3.3 Second-period political equilibrium

We start by presenting the second-period equilibrium offer distributions.

In terms of the electoral game of the second period, there is no change compared to a model without debt-related distortions. In case of maximal debt all resources are necessary for debt repayment, and in case of no debt, the amount of resources available for redistribution in the second period is also the same as in a model without debt-related distortions. Following Boyer and Esslinger (2016), define by $\mu_2$ the resources beyond existing endowments available in the second period for making transfer offers. Specifically, $\mu_2$ corresponds to the right-hand side of the second-period budget constraints (3.8) or (3.9). The following proposition derives the equilibrium offer distributions for any given $\mu_2$. The arguments for its proof can be found in Boyer and Esslinger (2016).

**Proposition 3.3.1** *In the unique second-period equilibrium, both candidates generate offers to all voters from a uniform distribution on* $[-1, 1 + 2\mu_2]$.

From the point of view of the first period, voters cannot anticipate which offer they get in the second period. Therefore, they just anticipate what they get in expectation in the second period under a certain policy proposed in the first period. Specifically, if a policy implies resources beyond existing endowments of $\mu_2$ in the second period, each voter has an expected transfer offer of $\mu_2$ in the second period.

## 3.4 The interaction between public debt and reforms for low debt-related distortions

In the following, we analyze the first-period equilibrium, focusing on the interaction of the reform decision with the decision to raise public debt. In Boyer and Esslinger (2016), the only decision that has direct efficiency implications is the decision to implement the pie-increasing reform. Due to the implied increase in overall resources available to the economy, the implementation of the reform is Pareto efficient. On the other hand, by incurring costs today and creating benefits only in the future, the reform policy on its own implies a disadvantage in terms of targeting resources to first-period voters. This is because only resources that are available in the first period can be targeted to specific voters. By transferring reform-induced second-period resources to the present, public debt can remedy this problem. Indeed, Boyer and Esslinger (2016) show that for efficient policies, like the reform, that create their benefits only in the next electoral cycle while having costs today, the use of public debt can overcome this disadvantage in terms of targeting first-period voters.

Since raising public debt now creates distortions, the decision to raise debt also has direct efficiency implications. In particular, avoiding public debt is itself an efficient policy that, in terms of targeting resources to first-period voters, has costs today and implies an overall increase in available resources realizing only in the future. Importantly, running low public debt is by definition the only such policy that cannot be incentivized through a higher use of public debt. Hence, it will be interesting to see how the trade-off between the two efficient dynamic policies of implementing a pie-increasing reform and avoiding debt-related distortions plays out in political equilibrium.

**Political equilibrium: Main results.**

We first present the political equilibrium when there are no exogenous restrictions on public debt. The following proposition summarizes the main results. A complete equilibrium characterization can be found in Proposition 3.4.3 below. The proof of this latter proposition hence also proves the following results.

**Proposition 3.4.1** *When the distortion caused by raising the maximal debt is low enough compared to the net benefit of the reform, $\gamma < \frac{e-c}{1+e}$, then in equilibrium both candidates always reform. The candidates mix over playing maximal debt and no debt.*

This result shows that, as long as the debt-related distortion is not too high compared to the net benefit of the reform,[16] the reform is implemented with probability one when public debt is not exogenously restricted. The important point is that this outcome arises although we do not get maximal debt with probability one any more. Instead, as the second result shows, the candidates mix over playing maximal debt and no debt. In the setup of Boyer and Esslinger (2016) without debt-related distortions, debt was always raised to the maximum. In the case of low debt-related distortions, the incentivizing effect of public debt on reform implementation persists even when maximal debt is not always employed.

The intuition is the following. In terms of the optimal electoral strategy, what counts is which debt-reform bundle gives most in terms of efficiency and which bundle gives most in terms of targetability. Specifically, the most efficient bundle maximizes the size of the total pie and any strategy that plays this bundle with the right probability can capitalize on this efficiency advantage. On the other hand, the targetability-maximizing bundle gives the highest capacity to target resources to a subset of voters at the expense of others. Any strategy that employs this bundle with the right probability can capitalize on this capacity.[17] In total, the optimal electoral strategy combines both efficiency and targetability in the best possible manner to maximize the vote share.

As long as there is a distortion caused by raising high public debt, the debt-reform bundle that maximizes efficiency, in terms of maximizing the size of the total pie, is reform and no debt, $\{R; ND\}$. Due to the distortion, it is actually necessary to raise no debt in order to achieve the maximal possible efficiency gain.[18] Additionally, this maximal efficiency gain necessitates to implement the pie-increasing reform.

On the other hand, as will be explained in detail below, for a low enough distortion, the bundle that yields most targetable resources in the first period is reform and

[16]We will discuss the case of very high distortions in section 3.6.

[17]A more detailed discussion of the term “targetability” in the context of this paper follows shortly. Importantly, we use it in a slightly different accentuation than in Boyer and Esslinger (2016).

[18]In Boyer and Esslinger (2016), there was no need to reduce public debt to achieve maximal efficiency, since public debt created no distortions.

maximal debt, $\{R; D\}$.[19] In terms of maximizing targetability, public debt is hence necessary to make the implementation of the reform the optimal choice. Here, we can see the incentivizing effect of public debt on reform implementation described in Boyer and Esslinger (2016). We can understand this even better after looking at the full characterization of the equilibrium. This will also help us explain why the no-reform bundles are not used in the optimal electoral strategy.

**Full characterization of equilibrium.**

As a preliminary, note that in this and the following sections, when we refer to transfer offer distributions, we always refer to the distribution of *total expected offers*. When voters decide who the vote for in the first period, they vote for the candidate whose proposal gives them the highest expected transfer offer over both periods. That is, a voter evaluates candidate $i$ by adding the transfer offer she gets from $i$ in the first period to the transfer offer she expects in the second period under the debt-reform bundle proposed by $i$. This is what we call the total expected offer. That is, voters anticipate the electoral equilibrium in the second period. Specifically, as we have argued in section 3.3, the debt-reform bundle chosen in the first period determines the second-period budget constraint and thereby the expected offer in the second-period electoral campaign. Assume candidate $i$ has chosen debt-reform bundle $Y$ and an associated first-period transfer distribution $F^i_{1,Y}$. The distribution of total expected offers, $\hat{F}^i_{1,Y}$, is then obtained by adding to the first-period distribution of transfer offers, $F^i_{1,Y}$, a degenerate distribution at $\mu_{2,Y}$. Thereby, $\mu_{2,Y}$ denotes the expected transfer offer in the second-period electoral equilibrium given debt-reform bundle $Y$ was chosen in the first period. Since the total expected offer over both periods determines the voting decision in the first period, the distribution of total expected offers is the one politicians ultimately care about when trying to win the first-period election. Note also that we consider distributions of *net* transfers. That is, an offer of $-2$ means that a voter completely loses both periods' endowments, whereas an offer of $+1$ means that on top of her endowments, the voter additionally gets a transfer of one unit of resources.

[19] Even with a continuous debt choice set, as long as the distortion is not too high, the bundle of reform and maximal debt would still maximize targetability. Even with more than two possible debt levels, maximal debt should hence always be played with some probability in order to capitalize on this targetability advantage.

Any reform-debt bundle $Y$ is defined by three characteristics in terms of the distribution of total expected transfers that can be offered under this bundle. First, denote by $M_Y$ the additional resources that are available above the already existing endowments under bundle $Y$.[20] Therefore, $M_Y = 0$ means that any positive transfer to one voter has to be financed by a negative transfer of the same absolute amount to another voter. In the case of distortions, $M_Y$ can also take a negative value when debt-related costs make resources drop below the existing endowments. For efficiency - in terms of maximizing the total size of the resource pie - only the resource constraint matters. Hence, $M_Y > M_Z$ means that bundle $Y$ has a higher efficiency than bundle $Z$ in the sense that the total amount of resources is bigger. Second, denote by $l_Y$ the lowest possible offer under bundle $Y$. This will be the lower bound of the support of the transfer distribution that is played under this bundle. Third, denote by $u_Y$ the maximal offer that can be financed for 50 percent of voters given a lowest possible offer $l_Y$ and a budget beyond existing endowments of $M_Y$: $u_Y = 2M_Y - l_Y$. This will be the upper bound of the support of the distribution that is played under bundle $Y$. Since targetability refers to the capacity to benefit one subset of voters at the expense of another subset, a comparison of these maximal offers under different debt-reform bundles gives us a ranking of these bundles in terms of targetability. That is bundle $Y$ has higher targetability than bundle $Z$ if $u_Y > u_Z$. We define $u_Y - u_Z$ as the net targetability advantage of bundle $Y$ over bundle $Z$.

We summarize the above in the following definition:

**Definition 3.4.2** *Consider two debt-reform bundles $Y$ and $Z$.*

***Efficiency:*** *Define $M_j$ as the additional resources that are available above the already existing endowments under debt-reform bundle $j$. Then bundle $Y$ has a higher efficiency than bundle $Z$ if $M_Y > M_Z$. The bundle that maximizes efficiency is indexed by $E$ for efficiency.*

***Targetability:*** *Define $u_j$ as the maximal (total expected) offer that can be financed for 50 percent of voters under bundle $j$ given a lowest possible offer $l_j$ and a budget beyond existing endowments of $M_j$: $u_j = 2M_j - l_j$. Then bundle $Y$ has higher tar-*

[20] In terms of the budget constraints (3.4) to (3.9), $M_Y$ can be calculated by adding up the right-hand sides of the first- and second-period budget constraints for a given debt-reform choice $Y$.

*getability than bundle $Z$ if $u_Y > u_Z$. The bundle that maximizes targetability is indexed by $T$ for targetability.*

***Net Targetability Advantage:*** *$u_Y - u_Z$ is called the net targetability advantage of bundle $Y$ over bundle $Z$.*

The reason for the terminology *net* targetability advantage is the following. When relating targetability to the maximal total expected offer that can be financed for half of the voters, higher (net) targetability refers to the following: A policy bundle has a net targetability advantage, when the more in targetable resources of this bundle over a more efficient bundle is enough to compensate at least a majority of voters for the loss in total resources compared to the more efficient policy bundle. Seen the other way, consider a policy bundle that has more targetable resources in the first period than the efficiency-maximizing bundle $E$. At the same time, it loses so much resources in terms of the total size of the pie that the highest total expected transfer that can be offered to half of the voters under this bundle is lower than $u_E$. Such a bundle will have a net targetability *disadvantage* compared to bundle $E$. In the following, when we talk about higher targetability, we always mean the just described advantage in terms of *net* targetability.

We are now in a position to state the full characterization of the equilibrium. It can be found in the following Proposition:

**Proposition 3.4.3** *Suppose the debt-related distortion is low enough compared to the net benefit of the reform such that $\gamma < \frac{e-c}{1+e}$. Then we have:*

*(a) The bundle $\{R; ND\}$ maximizes efficiency:*[21] *$E = \{R; ND\}$. At the same time, $\{R; ND\}$ has the lowest targetability.*

*(b) The bundle $\{R; D\}$ maximizes targetability: $T = \{R; D\}$.*

*(c) Furthermore, the targetability-maximizing bundle $T = \{R; D\}$ has higher efficiency than the no-reform bundles, $\{NR; ND\}$ and $\{NR; D\}$.*

*This implies that in equilibrium:*

[21] In this and the following propositions, when we refer to efficiency or targetability, we always refer to them as defined in Definition 3.4.2

*(i) Both candidates play the targetability-maximizing bundle $T = \{R;\ D\}$ with probability $\beta_T = \frac{2H}{u_T - l_T}$, where $H = u_T - u_E$ is the net targetability advantage of the targetability-maximizing bundle $T = \{R;\ D\}$ over the efficiency-maximizing bundle $E = \{R;\ ND\}$.*

*(ii) Both candidates play the efficiency-maximizing bundle $E = \{R;\ ND\}$ with probability $\beta_E = 1 - \beta_T$.*

*(iii) Both candidates play the no-reform bundles $\{NR;\ D\}$ and $\{NR;\ ND\}$ with probability zero.*

*(iv) Both candidates play the distributions of total expected transfer offers illustrated in Figure 3.1.*

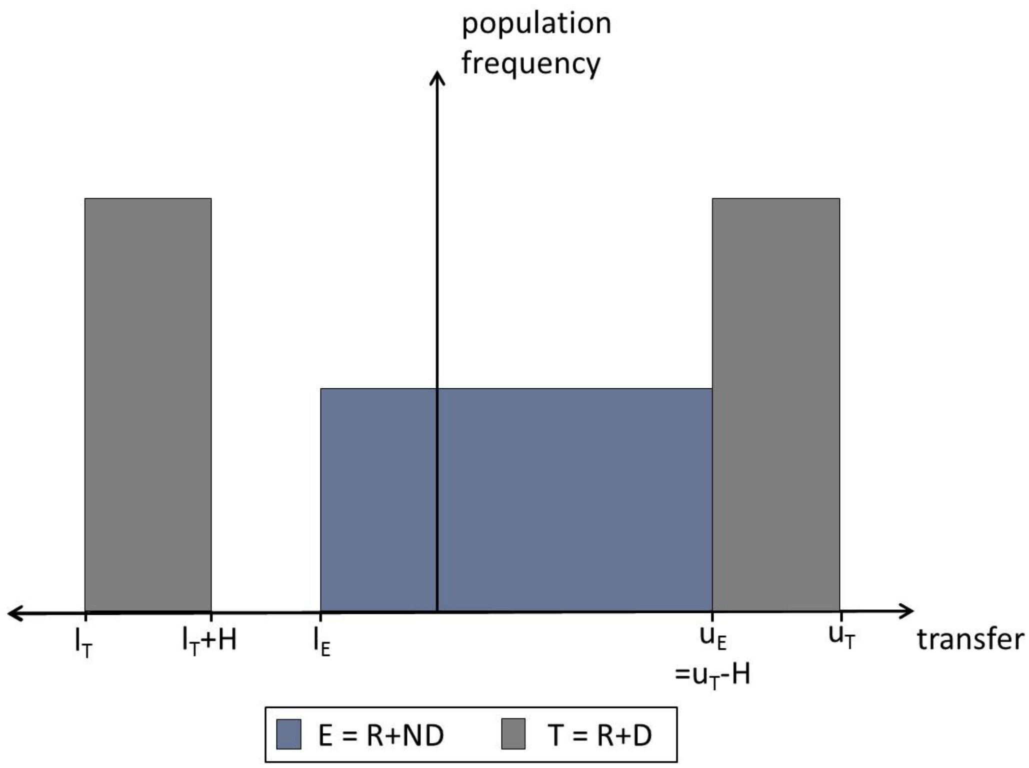


Figure 3.1: Transfer offer distributions without debt limit

The complete proof of Proposition 3.4.3 relies on applying Proposition B.2.1 in the appendix. More specifically, the above proposition follows from Corollary B.1.1 to Proposition B.2.1, which are both stated and proven in the appendix.

**Intuition and interpretation of results.**

As already mentioned in the discussion following Proposition 3.4.1, the two factors that determine the use of different debt-reform bundles in the optimal electoral strategy are efficiency and targetability. The bundle $\{R;\, ND\}$ maximizes the size of the pie by implementing the pie-increasing reform while at the same time avoiding any loss of resources by not creating any distortions through public debt. This bundle hence maximizes efficiency and is therefore indexed by $E$. The resources beyond existing endowments under this bundle, $M_E$, are higher than for any other bundle. Nevertheless, this efficiency gain comes at the expense of a loss in targetability. First, by not transferring future resources to the present, these resources cannot be targeted to current voters. Second, by implementing a reform that costs resources today, further targetable resources in the first period are lost and the reform gains in the future are again not targetable. In terms of Definition 3.4.2, bundle $\{R;\, ND\}$ has the lowest targetability in the sense that the highest possible offer $u_E$ that can be offered to half of the voters under this bundle is lower than for all other bundles. Nevertheless, in order to capitalize on the efficiency gain implied by bundle $E = \{R;\, ND\}$, the optimal electoral strategy plays this bundle with a uniform transfer distribution with support $[l_E, u_E]$, reminiscent of the uniform distribution in Myerson (1993). Due to its dominance in terms of total resources, bundle $E = \{R;\, ND\}$ cannot be attacked on this support by any other bundle.

The only chance for other bundles to be played with positive probability is to make use of a targetability advantage and offer below and above the support of bundle $E$. If the debt-related distortion is low enough compared to the net benefit of the reform, as is the case in the above proposition,[22] then the bundle $\{R;\, D\}$, which combines the reform with the maximal debt, maximizes targetability and is therefore indexed by $T$. Specifically, for low enough distortions, not too much is lost in the process of transferring money across time and so transferring the existing endowment to the present through debt implies a higher targetability than not raising debt. This means that out of the no-reform bundles, the bundle with debt, $\{NR;\, D\}$, has a higher targetability than the one without debt, $\{NR;\, ND\}$. Furthermore, when transferring the reform benefits to the present through debt, still enough resources arrive in the

[22]We will discuss the case where the distortion $\gamma$ is very high in section 3.6.

the first period to compensate for the reform costs incurred in that first period. This implies, in particular, that out of the debt bundles, the reform bundle, $\{R;\, D\}$, has higher targetability than the no-reform bundle, $\{NR;\, D\}$. In total, bundle $T = \{R;\, D\}$ therefore dominates all other bundles in terms of targetability. In terms of Definition 3.4.2, bundle $\{R;\, D\}$ has the highest targetability in the sense that the maximal total expected transfer $u_T$ that can be offered under this bundle is higher than for all other bundles.

In particular, the highest offer $u_T$ that can be financed under bundle $T$ is greater than $u_E$, and we have $H = u_T - u_E > 0$. We defined $H$ as the net targetability advantage of $T$ over $E$. In particular, bundle $T$ can offer transfers in the interval $[u_E, u_T]$ above the support of $E$ and finance these by offers in the interval $[l_T, l_T + H]$, which lies below the support of $E$. This is illustrated in Figure 1. Note that bundle $T$ can only finance offers above the highest possible offer of $E$, because it can take more away from the worst-off voters and shift these resources to the best-off voters. Hence, the higher maximal offer of $T$ comes purely from this higher ability to shift resources.[23] In terms of the optimal electoral strategy, it makes sense to make use of this net targetability advantage of bundle $T$ by playing half of the offers uniformly distributed on $[u_E, u_T]$ above the support of $E$ and finance these offers by offers uniformly distributed on the interval $[l_T, l_T + H]$, which lies below the support of $E$. The equilibrium probabilities with which the two bundles $E$ and $T$ are played are then constructed in a way such that it does not make sense to shift more than half of the offers above $u_E$ for bundle $T$.[24]

Intuitively, the use of both bundles $E = \{R;\, ND\}$ and $T = \{R;\, D\}$ in the way described in the Proposition makes optimal use of the efficiency advantage of bundle $\{R;\, ND\}$ and the net targetability advantage of bundle $\{R;\, D\}$. In particular, note that as the net targetability advantage $H$ of bundle $\{R;\, D\}$ over bundle $\{R;\, ND\}$ goes down, the probability $\beta_T$ with which the targetability maximizing bundle $T = \{R;\, D\}$ is played goes down. Intuitively, the net targetability advantage of bundle $\{R;\, D\}$ then loses significance compared to the efficiency advantage of bundle $\{R;\, ND\}$.

[23] If $T$ where restricted to offer $l_E$ as its lowest possible offer, it would only be able to finance a maximal offer below $u_E$ because of lower total resources than bundle $E$.

[24] Specifically, such a shift would win more votes when the other candidate plays bundle $E$, but would lose votes when the other candidate plays bundle $T$. With the equilibrium probabilities of playing bundles $E$ and $T$, the second effect dominates and the shift would imply a decrease in the total expected vote share.

We are now in a position to explain why only the bundles involving reform are played with positive probability, while the no-reform bundles are played with probability zero for the case in Proposition 3.4.3. Since we always have that $E = \{R;\, ND\}$ dominates all other bundles in terms of efficiency, neither of the no-reform bundles, $\{NR;\, D\}$ and $\{NR;\, ND\}$, can attack the efficiency-maximizing bundle $E = \{R;\, ND\}$ on its support $[l_E, u_E]$. The only possibility for these bundles to be played is to make use of their net targetability advantage against $E$ and play offers in the intervals $[u_E, u_{\{NR;\,D\}}]$ or $[u_E, u_{\{NR;\,ND\}}]$ above $u_E$.[25] However, these intervals lie inside the support of bundle $T = \{R;\, D\}$, since $T$ has the highest net targetability advantage over bundle $E$. If the distortion fulfills the condition in the corollary, $\gamma < \frac{e-c}{1+e}$, then the net gain from the reform is high enough compared to the distortionary costs of public debt to make $T = \{R;\, D\}$ dominate both no-reform bundles not only in terms of targetability but also in terms of efficiency. This efficiency advantage of $T$ implies that the no-reform bundles cannot attack bundle $T = \{R;\, D\}$ an its support either. Therefore, they are played with probability zero.

## 3.5 Constitutional limit on debt

We now want to see which effects the introduction of an exogenous restriction on public debt has on the outcome of the electoral game. Since public debt creates distortions in the above model, a debt limit should have a direct positive effect on efficiency by reducing the costs incurred when using high public debt. On the other hand, as was shown in Boyer and Esslinger (2016) for the case without debt distortions and as was confirmed in the previous section for the case with distortions, public debt has an indirect positive effect on efficiency. In particular, a reform policy with costs today and benefits in the future implies a targetability disadvantage when reform benefits are left in the future. Public debt, through transferring the reform benefits to the present, can turn this targetability disadvantage into a targetability advantage. As was shown in the previous section for distortions that are not too high, the unrestricted use of public debt allowed the policy bundle of reform and debt, $\{R;\, D\}$, to actually maximize targetability and therefore be played alongside the most efficient policy bundle

[25] Again, such offers would be financed by offers below $u_E$.

of reform and no debt, $\{R; ND\}$, as part of the political equilibrium. If we restrict the use of public debt, a no-reform bundle might replace the reform bundle $\{R; D\}$ as the targetability-maximizing bundle and in that sense efficiency would be indirectly decreased through a restriction on public debt. The following analysis investigates this trade-off in more detail.

Recall that we defined the debt limit $\bar{\delta}$ as the level of second-period resources that is allowed to be used for the repayment of debt. To allow comparability to Boyer and Esslinger (2016), for the equilibrium characterization, we work with a modified definition of the debt limit. We define $\bar{\rho} = \bar{\delta} - 1$ so that $\bar{\rho} > 0$ implies that maximally $\bar{\delta} = 1 + \bar{\rho}$ second-period resources can be used for the repayment of debt. The term $\bar{\rho}$ then captures the amount of resources beyond the second-period endowment of 1 that can be committed to the repayment of debt.[26] Similarly, $\bar{\rho} < 0$ implies that maximally $\bar{\delta} = 1 + \bar{\rho} < 1$ second-period resources can be used for the repayment of debt. In that case, $\bar{\rho}$ captures the amount by which the debt repayment capacity lies below the second-period endowment of 1.[27]

**Political equilibrium: Main results.**

The following proposition characterizes the equilibrium strategies in terms of reform and public debt decisions. Proposition 3.5.2 below presents the complete equilibrium characterization.

**Proposition 3.5.1** *Suppose the distortion caused by raising the maximal debt is small enough compared to the net benefit of the reform such that* $\gamma < \frac{e-c}{1+e}$*, and suppose that there is a constitutional debt limit* $\bar{\rho} \in (-1 + \frac{2c-e}{2(1-\gamma)-1}, e]$.

*(I.) When the debt limit* $\bar{\rho}$ *is not very restrictive in the sense that* $\bar{\rho} > \frac{2c-e}{2(1-\gamma)-1}$*, then in equilibrium both candidates always reform and mix over playing no debt and the maximal debt. With a stricter debt limit (i.e. when* $\bar{\rho}$ *decreases), the probability with which the efficient policy bundle* $\{R; ND\}$ *is played increases.*

*(II.) When the debt limit* $\bar{\rho}$ *is restrictive enough such that* $\bar{\rho} < \frac{2c-e}{2(1-\gamma)-1}$*, then in equi-*

[26] Of course, only in the case of reform, more resources than the second-period endowment can be committed to debt repayment. Specifically, the reform benefits provide such additional resources.

[27] Note that with a distortion, the repayment capacity does not correspond to how many resources will arrive in the first period. Recall that a distortion implies that debt in the first period is actually restricted to be below $(1-\gamma)(1+\bar{\rho})$.

*librium both candidates start playing the most inefficient policy bundle $\{NR;\ D\}$ alongside the reform bundles $\{R;\ ND\}$ and $\{R;\ D\}$. With a stricter debt limit (i.e. when $\bar{\rho}$ decreases), the probability with which the most inefficient policy bundle $\{NR;\ D\}$ is played increases, while the probability of the reform bundle $\{R;\ D\}$ decreases.*

Before turning to the interpretation of the results, note the following preliminary. By our assumptions, the cut-off $\frac{2c-e}{2(1-\gamma)-1}$ that separates non-restrictive from restrictive debt limits lies in the interval $(0, e)$. That is, above this cut-off, a debt limit only restricts the transfer of reform benefits across time. Below this cutoff, once the debt limit drops below zero, the debt limit also restricts the transfer of the second-period endowment across time.

The first part of the proposition shows that if the debt limit is not too restrictive and allows a high enough transfer of future reform benefits to the present, the reform and debt policy bundle, $\{R;\ D\}$, still maximizes targetability and is played alongside the efficiency-maximizing bundle $\{R;\ ND\}$ in the political equilibrium. Nevertheless, the net targetability advantage that the reform bundle with debt, $\{R;\ D\}$, has over the reform bundle without debt, $\{R;\ ND\}$, decreases when the use of public debt becomes more restricted. This implies that the efficiency advantage of bundle $\{R;\ ND\}$ gets a higher weight compared to the net targetability advantage of bundle $\{R;\ D\}$ in the optimal policy mix. Therefore, the probability of bundle $\{R;\ D\}$ goes down. Said differently, in the range where the debt limit is still so non-restrictive that only reform bundles are played with positive probability, a stricter debt limit increases the probability with which the most efficient policy bundle $\{R;\ ND\}$ is played. Thus, a debt limit in this range has two effects. First, there is a direct effect because allowing a lower maximal level of public debt reduces the distortionary costs that go along with this maximal debt level. Second, there is an indirect efficiency-enhancing effect by reducing the net targetability advantage of the policy bundle involving debt and increasing the probability with which the most efficient bundle involving no debt is played. The full characterization of the equilibrium for part one of the proposition can be found in Corollary B.1.2 in the Appendix. It corresponds closely to Proposition 3.4.3 presented in the previous section for the case without debt limit.

The second part of the proposition shows that the effects get more involved when the debt limit drops below the level $\frac{2c-e}{2(1-\gamma)-1} > 0$. In that case, the part of the reform benefits that can be transferred to the present is too low to compensate for the reform costs and to keep the policy bundle $\{R;\, D\}$ the targetability-maximizing bundle. Instead, the most inefficient bundle $\{NR;\, D\}$, which combines no reform with maximal debt becomes the bundle with the highest net targetability advantage over the most efficient bundle $\{R;\, ND\}$. This is an indirect negative effect on efficiency that was not present for the non-restrictive debt limit in part one of the proposition. It is the effect that was highlighted in Boyer and Esslinger (2016): The fact that reform costs occur in the present while reform benefits realize only in the future gives non-reform policies a targetability advantage when public debt becomes too restricted. To understand why the $\{R;\, D\}$ policy is still played with positive probability although it is now dominated in terms of targetability by $\{NR;\, D\}$, it helps to look at the full characterization of the equilibrium. This will also help us explain the comparative static results concerning a change in the debt limit.

**Restrictive debt limit: Full characterization of equilibrium.**

The full characterization of the equilibrium for the case of a restrictive debt limit is summarized in the following proposition.

**Proposition 3.5.2** *Suppose the distortion caused by raising the maximal debt is small enough compared to the net benefit of the reform such that* $\gamma < \frac{e-c}{1+e}$*, and suppose that there is a constitutional debt limit* $\bar{\rho} \in (-1 + \frac{2c-e}{2(1-\gamma)-1}, e]$.

*Suppose furthermore the debt limit* $\bar{\rho}$ *is restrictive in the sense that* $\bar{\rho} < \frac{2c-e}{2(1-\gamma)-1}$*. Then we have:*

*(a) The bundle* $\{R;\, ND\}$ *maximizes efficiency:* $E = \{R;\, ND\}$*. At the same time,* $\{R;\, ND\}$ *has the lowest targetability.*

*(b) The bundle* $\{NR;\, D\}$ *maximizes targetability:* $T = \{NR;\, D\}$*. At the same time* $\{NR;\, D\}$ *has the lowest efficiency.*

*(c) The bundle* $\{R;\, D\}$*, which is indexed by* $I$*, dominates the remaining bundle* $\{NR;\, ND\}$ *in terms of efficiency and targetability.*

*This implies that in equilibrium:*

(i) *Both candidates play the targetability-maximizing bundle* $T = \{NR;\, D\}$ *with probability* $\beta_T = \frac{2H''}{u_T - l_T}$*, where* $H'' = u_T - u_I$ *is the net targetability advantage of the targetability-maximizing bundle* $T = \{R;\, D\}$ *over bundle* $I = \{R;\, D\}$*, which has the second highest targetability. This probability increases with a stricter debt limit (with smaller* $\bar{\rho}$*).*

(ii) *Both candidates play bundle* $I = \{R;\, D\}$ *with probability* $\beta_I = \frac{2H'}{u_I - l_I} \frac{u_T - l_T - 2H''}{u_T - l_T}$ *where* $H' = u_I - u_E$ *is the net targetability advantage of bundle* $I = \{R;\, D\}$ *over the efficiency-maximizing bundle* $E = \{R;\, ND\}$*. This probability decreases with a stricter debt limit (with smaller* $\bar{\rho}$*).*

(iii) *Both candidates play the efficiency-maximizing bundle* $E = \{R;\, ND\}$ *with probability* $\beta_E = 1 - \beta_I - \beta_T$*.*

(iv) *Both candidates play the remaining bundle* $\{NR;\, ND\}$ *with probability zero.*

(v) *Both candidates play the distributions of total expected offers illustrated in Figure 3.2.*

The complete proof of Proposition 3.5.2 relies on applying Proposition B.2.2 in the appendix. More specifically, the above proposition follows from Corollary B.1.3 to Proposition B.2.2, which are both stated and proven in the appendix.

**Interpretation of results.**

As can be seen from Figure 3.2, once the debt limit $\bar{\rho}$ drops below $\frac{2c-e}{2(1-\gamma)-1}$, the *no-reform* policy bundle $\{NR;\, D\}$ achieves the highest possible offer $u_T$ and therefore becomes the bundle that maximizes targetability. Since it has lower total resources than the resource-maximizing policy bundle $E = \{R;\, ND\}$, the offer distribution associated to bundle $T = \{NR;\, D\}$ will again not be played on the support of bundle $E$.

Nevertheless, we now get the situation that bundle $I = \{R;\, D\}$, which was the targetability-maximizing policy bundle before, still dominates bundle $E$ in terms of targetability and can play offers above and below the support of $E$. Importantly, bundle $I = \{R;\, D\}$ has higher efficiency than bundle $T = \{NR;\, D\}$, because resources

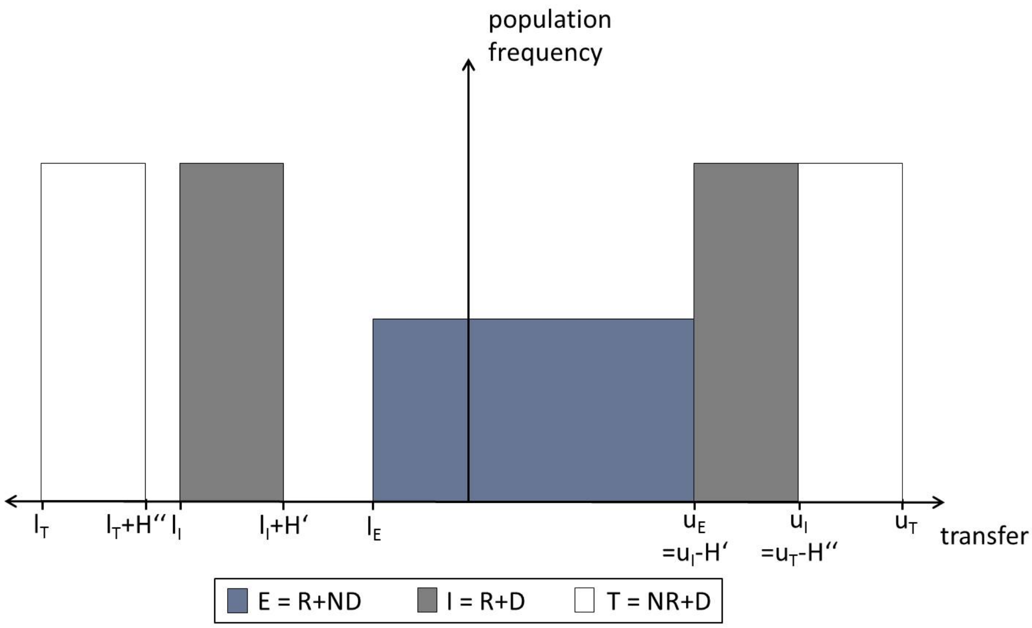


Figure 3.2: Transfer offer distributions for restrictive debt limit

are higher under reform than under no-reform, $M_I > M_T$. This means that bundle $T$ cannot attack bundle $I$ on its support. Intuitively, in order to make optimal use of the efficiency gain of bundle $I = \{R;\, D\}$ over bundle $T = \{NR;\, D\}$ and the net targetability advantage of bundle $I = \{R;\, D\}$ over bundle $E = \{R;\, ND\}$ , it makes sense to still play bundle $I$ alongside the efficiency-maximizing bundle $E$ and the targetability-maximizing bundle $T$.

The last bundle $\{NR;\, ND\}$ has lower efficiency than bundle $E = \{R;\, ND\}$ and can therefore only have offers outside the support of bundle $E$. At the same time, it has lower targetability than bundle $I = \{R;\, D\}$ in the sense that $u_{\{NR;\, ND\}} < u_I$.[28] This is because, for low enough debt-related distortions,[29] the gain in targetability from raising debt versus not raising debt is higher than the gain in targetability from not reforming versus reforming. Therefore, with $u_{\{NR;\, ND\}} < u_I$, bundle $\{NR;\, ND\}$ can only play offers inside the support of bundle $I$. However, bundle $I = \{R;\, D\}$ also dominates bundle $\{NR;\, ND\}$ in terms of efficiency. Specifically, for low enough debt-related distortions, as considered here, the efficiency gain from reforming versus not reforming is higher than the efficiency gain from not raising debt versus raising

[28]This holds if the debt limit is not extremely strict, $\bar{\rho} > -1 + \frac{2c-e}{2(1-\gamma)-1}$, as assumed in the above proposition. Otherwise, potentially all four policies are played. For this case, further formal tools beyond Propositions B.2.1 and B.2.2 need to be developed. This is left as a task for future research.

[29]Recall that, in this section, we are still considering a distortion that fulfills $\gamma < \frac{e-c}{1+e}$.

debt. With $I = \{R;\, D\}$ dominating $\{NR;\, ND\}$ in terms of efficiency, $I$ hence cannot be attacked by $\{NR;\, ND\}$ on its support. As just argued, the support of $I$ was the last remaining option for offers of bundle $\{NR;\, ND\}$. That is why bundle $\{NR;\, ND\}$ is played with probability zero.

**Comparative statics for probabilities of different policies.**

Figure 3.3 illustrates the relationship between the equilibrium probabilities with which the different bundles are played and the strictness of the debt limit for the following parameter values: $e = 0.4; c = 0.3; \gamma = 0.6\frac{e-c}{1+e}$.[30] We plotted the same graph for different parameter specifications, and we will discuss the difference across different specifications at the end. The main effects we discuss here for Figure 3.3 are present across all specifications, however the relative size of different effects varies.[31]

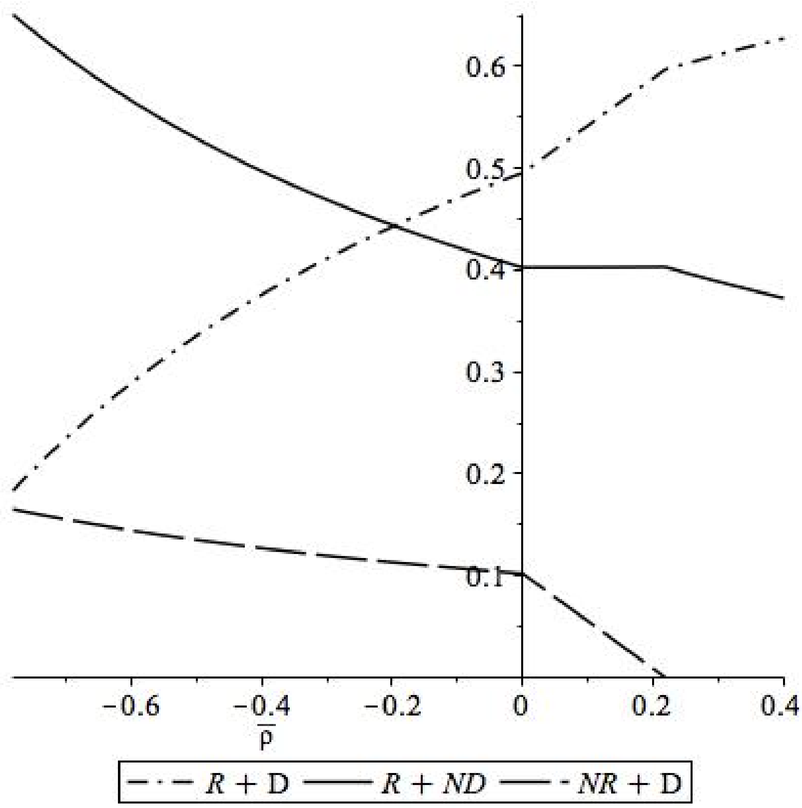


Figure 3.3: Equilibrium probabilities of policy bundles

[30] The values are chosen such that all our assumptions hold. For instance, $2c = 0.6 > 0.4 = e$, or $\gamma = 0.6\frac{e-c}{1+e} < \frac{e-c}{1+e}$.

[31] Note that the intention here is still a *qualitative* analysis. In that sense, the above parametrization should not be seen as capturing a realistic size of reform benefits compared to the endowment of an economy. For such a quantitative analysis the above model is too stylized with its two-period setup and the fact that here the full endowment is targetable across voters.

The figure has to be read starting from the right at the highest possible debt limit of $\bar{\rho} = e = 0.4$. As we can see, there is a range of debt levels for which only the reform policy bundles $\{R;\, ND\}$ and $\{R;\, D\}$ are played with positive probability. This was described in part (I.) of Proposition 3.5.1. It is the case as long as the debt limit is non-restrictive in the sense that $\bar{\rho} > \frac{2c-e}{2(1-\gamma)-1} (\approx 0.22)$. For this case, we have also explained above that with a stricter debt limit, the net targetability advantage of the policy bundle involving debt, $\{R;\, D\}$, goes down. Therefore, this bundle is played with lower probability if we move to the left. Correspondingly, the most efficient bundle, $\{R;\, ND\}$, which does not involve debt, is played with a higher probability.

When the debt limit $\bar{\rho}$ drops below the cut-off $\frac{2c-e}{2(1-\gamma)-1} (\approx 0.22)$, we enter the realm of part (II.) of Proposition 3.5.1 and the most inefficient bundle $\{NR;\, D\}$ becomes the targetability-maximizing bundle. As we can see in Figure 3.3, if the debt limit $\bar{\rho}$ is in the range $[0, \frac{2c-e}{2(1-\gamma)-1})$, then a stricter debt limit implies that policy bundle $\{NR;\, D\}$ replaces policy bundle $\{R;\, D\}$ in the sense that the probability of the latter decreases in almost the same manner as the probability of the former increases. The probability $\beta_E$ of the efficient bundle $E = \{NR;\, D\}$ stays almost constant in this range. Analytically, the reaction of $\beta_E$ to a stricter debt limit can actually go both ways.

The intuition for these results is the following. In the range $\bar{\rho} \in [0, \frac{2c-e}{2(1-\gamma)-1})$, policy bundle $\{NR;\, D\}$ has the highest maximal offer $u_T$ and hence the highest degree of targetability. In terms of Figure 3.2, this maximal offer as well as the maximal offer $u_E$ of the efficient bundle stay the same as long as $\bar{\rho} \in [0, \frac{2c-e}{2(1-\gamma)-1})$. The only effect is that with a stricter debt limit less reform benefits can be transferred to the present and so the maximal offer $u_I$ under policy $\{R;\, D\}$ decreases with a stricter debt limit. This means that $u_T - u_I$, the net targetability advantage of the debt policy involving no-reform, $T = \{NR;\, D\}$, over the other two policy bundles increases. Therefore, policy bundle $T = \{NR;\, D\}$ is played with higher probability, whereas the sum of the probabilities of the other two bundles goes down. The relative size of the probabilities of these other two bundles is then determined by the following. With less possibility to transfer the reform benefits to the present, the net targetability advantage of bundle $I = \{R;\, D\}$ over the efficient bundle $E = \{R;\, ND\}$, $u_I - u_E$, decreases. Therefore, the reform and debt bundle $I$ should be played with lower probability relative to the efficient bundle $E$. Furthermore, recall from above that the sum of the probabilities of $E$ and $I$, $\beta_E + \beta_I$, decreases. Therefore, bundle $I$ is unambiguously played with a lower probability when the debt limit becomes stricter. For bundle $E$, there are two countervailing effects. On the one hand, $\beta_E + \beta_I$ decreases, on the other hand, bundle $E$'s probability relative to bundle $I$'s probability increases. As Figure 3.3 illustrates, for the range $\bar{\rho} \in [0, \frac{2c-e}{2(1-\gamma)-1})$, the two effects basically cancel for bundle $E$ and it is played with an almost constant probability.

As soon as the debt limit $\bar{\rho}$ drops below zero, besides completely restricting the transfer of reform benefits across time, the debt limit now also restricts how much of the second-period endowment can be transferred across time.[32] This implies that for $\bar{\rho} < 0$, both debt policies $I = \{R;\, D\}$ and $T = \{NR;\, D\}$ lose targetability in the same degree when the debt limit becomes stricter. Specifically, both lose targetability in the degree that a stricter debt limit lets both of them only transfer a lower amount of the existing second-period endowment to the present. This implies that $u_T - u_I$, the net targetability advantage of the debt policy involving no-reform, $T = \{NR;\, D\}$ over the other two policy bundles stays constant when the debt limit becomes stricter. However, $u_T - l_T$, the range of the support of the targetability-maximizing transfer distribution, decreases. This range captures the maximal size of the targetable pie. When the size of the targetable pie decreases, this implies that a given net targetability advantage gains a higher relative importance. This effect has also been highlighted in Boyer and Esslinger (2016) for the case without debt distortions. Therefore, for $\bar{\rho} < 0$, the probability of the inefficient but targetability-maximizing bundle $T = \{NR;\, D\}$ still increases with a stricter debt limit compared to the other two bundles, because its targetability advantage gains relatively more weight with a lower size of the targetable pie.

For $\bar{\rho} < 0$, the net targetability advantage of the debt policy bundle $I = \{R;\, D\}$ over the efficient no-debt bundle $E = \{R;\, ND\}$ also still decreases with a stricter debt limit. Specifically, debt can be used less and less to gain a targetability advantage by transferring future resources to the present. Therefore, the probability of $I = \{R;\, D\}$ decreases unambiguously, as was the case for $\bar{\rho} \in [0, \frac{2c-e}{2(1-\gamma)-1})$. For the efficient bundle $E = \{NR;\, D\}$, there are again the two countervailing effects: its probability relative

[32] Recall from the definition of $\bar{\rho}$ that for $\bar{\rho} < 0$, $\bar{\rho}$ captures the amount by which the debt repayment capacity lies below the second-period endowment of 1.

to the probability of bundle $I$ increases, but the combined probability of bundles $E$ and $I$ decreases. Nevertheless, our numerical simulations show that for $\bar{\rho} < 0$, the first effect seems to dominate. This might have to do with the fact that the increase of the probability of the targetability-maximizing bundle is no longer based on a direct increase in targetability compared to the other two bundles. Instead, it is only based on the indirect effect of a decrease in the targetable pie, which makes a given targetability advantage more important. Hence, for $\bar{\rho} < 0$, it seems that with a stricter debt limit the most efficient policy is played with a higher probability.

**Implications of results.**

In total, when there are distortions related to raising high public debt, we have to weigh several effects against each other. First, there is a direct efficiency-enhancing effect of a debt limit, which was only shortly mentioned in the above discussion: With a stricter debt limit, any policy bundle involving public debt can only raise lower public debt and therefore will create a lower loss in resources due to distortions.[33] However, this effect has to be qualified in the following manner. For low enough levels of public debt, debt-related distortions might no longer be present. For instance, if one thinks of the distortionary costs coming from a reduced tax-smoothing capacity at high debt-levels, then such costs will not arise for a moderate debt level. Hence, the above efficiency-enhancing effect should be most prominent for still quite high debt limits. This cautionary note applies to all effects that are linked to debt-related distortions, and we will discuss it also for the following effects.[34]

Besides the just described direct efficiency-enhancing effect of a debt limit, there are indirect effects. By determining the net targetability advantage of different policies, the debt limit determines the probability with which different policy bundles are played. In particular, once the debt limit becomes restrictive in the sense that $\bar{\rho}$ drops below $\frac{2c-e}{2(1-\gamma)-1}$, the most inefficient policy of no reform and debt, $\{NR;\, D\}$, is played with higher and higher probability. As we have seen, for $\bar{\rho} < 0$ this effect might be partly compensated by the fact that the most efficient policy $\{R;\, ND\}$ is also played with

[33]However, as long as $\bar{\rho} > 0$, the $\{NR;\, D\}$ policy is not affected by a stricter debt limit in terms of the maximal amount of debt that can be raised.

[34]Note that the arguments presented in this paper still go through if, instead of no debt, a debt level below which no distortions arise can be played as the alternative to maximal debt.

higher probability. However, let us take again into account that, below a certain level of debt, no distortions should arise anymore. Then, as soon as the debt limit restricts public debt below this level, we would again be in the no-distortions case from Boyer and Esslinger (2016). That is, only the policies $\{NR;\ D\}$ and $\{R;\ D\}$ would be played. In that case, Figure 3.3 would change to Figure 3.4. Once the debt limit reaches the level below which no distortionary costs are to be expected ($\bar{\rho} = -0.3$ in Figure 3.4), policies that raise debt up to this limit have no efficiency disadvantage anymore compared to no-debt policies. In particular, $\{R;\ D\}$ would have the same efficiency as $\{R;\ ND\}$ but higher targetability and hence would always be played as the unique reform policy bundle. Similarly, $\{NR;\ D\}$ would be the unique no-reform bundle that is played. In this range, the only effect of a debt limit is hence the one described in Boyer and Esslinger (2016) that a stricter debt limit makes the more efficient reform policy bundle less likely to be implemented.

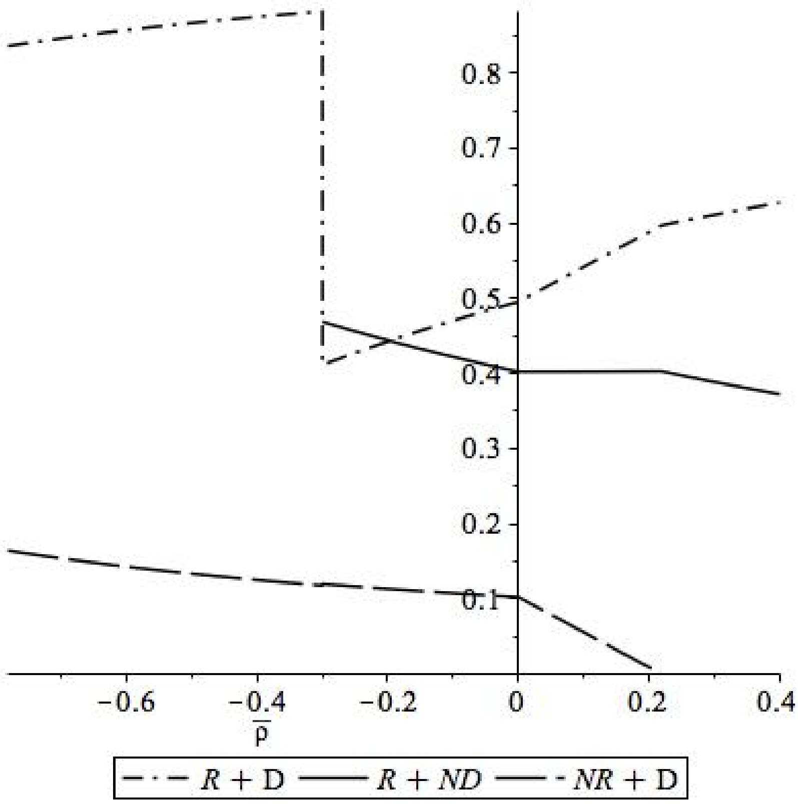


Figure 3.4: Equilibrium probabilities without distortions below a certain level of debt

In summary, we can draw the following policy recommendations from the above analysis. First, when setting a debt limit, one should be careful only to restrict public debt up to a point where debt-related distortions are no longer likely to arise. If debt is restricted below such debt levels, then there is no direct efficiency-enhancing effect of a debt limit and the indirect effect is negative in the sense that a stricter debt limit makes efficient reforms less likely to be implemented.

Second, in the range of debt levels where distortionary costs of public debt are likely to arise, a debt limit has a direct efficiency-enhancing effect by reducing these distortionary costs. Furthermore, by decreasing the targetability advantage of the reform policy involving debt over the most efficient reform policy involving no debt, there is a positive indirect effect by increasing the relative probability with which the more efficient no-debt policy bundle is played amongst the reform policy bundles. Nevertheless, these two efficiency-enhancing effects still are counterbalanced by the effect that the most inefficient policy of no-reform and debt becomes the targetability-maximizing policy and is played with higher and higher probability. In reality, these effects hence have to be weighed against each other when setting a debt limit.

At this point, it makes sense to discuss how the shape of Figure 3.3 changes for different parameter values.[35] When increasing the net reform benefits $e - c$ while keeping reform costs $c$ constant,[36] the efficiency-decreasing effect of a debt limit is lower in the sense that the inefficient $\{NR;\, D\}$ policy is played with rather low probability. Intuitively, due to the high implied efficiency gain, reforms with higher net benefits are attractive even if they imply a loss in targetable resources. In this case, the main effect of a debt limit is that among the reform bundles, the efficient no-debt policy is played with higher and higher probability. For high benefits of contemplated reforms, stricter debt limits hence do not harm as much in terms of the probability of reform implementation. However, a given decrease in this probability is worse because higher net reform benefits are foregone. Furthermore, one should still not overshoot the goal and restrict debt below levels where distortions are no longer likely to arise. In contrast, for lower reform benefits, the efficiency-decreasing effect in terms of increasing the probability of the most inefficient policy bundle $\{NR;\, D\}$ is quite pronounced. However, the forgone net reform benefits in case of not implementing the reform are

[35]See Figures B.1 and B.2 in the appendix for alternative specifications of the reform benefits $e$ given fixed reform costs $c$.

[36]For all our parametrizations, the size of the net benefit $e - c$ always respects assumption (3.3): $2c > e$. If the net reform benefits are so high that $e > 2c$, then the reform will always go through the political process, because its efficiency gain trumps any targetability considerations. In that case, it therefore does not matter for reform implementation how much debt is allowed. In reality, these highly beneficial reforms should however be rather rare.

lower. Depending on the specific situation, it can make sense to allow a debt level where distortions can still arise. These distortions are then counterbalanced by the positive effect on reform implementation. Note, however, that the above model can only give qualitative recommendations. It is not intended as a quantitative model that can put specific sizes on the different effects.

## 3.6 The interaction between public debt and reforms for high debt-related distortions

Note that the results in the previous two sections have been derived under the assumption that the debt-related distortion was not too high compared to the net benefit of the reform: $\gamma < \frac{e-c}{1+e}$. If the distortion rises above this level, possibly all four bundles are played at the same time, especially when a debt limit gets involved. The formal analysis of this case requires a further development of the tools presented in Propositions B.2.1 and B.2.2 in the appendix, and it is left for future research. Nevertheless, it is possible to analyze the case of very high distortions and no debt limit with an application of Proposition B.2.1. The results for this case are summarized in the following proposition. This proposition follows from Corollary B.1.4 to Proposition B.2.1, which are both stated and proven in the appendix.

**Proposition 3.6.1** *Suppose the debt-related distortion is so high that $\gamma > \frac{1}{2}$. Then we have:*

*(a) The bundle $\{R; ND\}$ maximizes efficiency: $E = \{R; ND\}$.*

*(b) The bundle $\{NR; ND\}$ maximizes targetability: $T = \{NR; ND\}$*

*(c) Furthermore, the targetability-maximizing bundle $T = \{NR; ND\}$ has higher efficiency than the debt bundles, $\{R; D\}$ and $\{NR; D\}$*

*This implies that in equilibrium*

*(i) Both candidates play the targetability-maximizing bundle $T = \{NR; ND\}$ with probability $\beta_T = \frac{2H}{u_T - l_T}$, where $H = u_T - u_E$ is the net targetability advantage of the targetability-maximizing bundle $T = \{NR; ND\}$ over the efficiency-maximizing bundle $E = \{R; ND\}$.*

*(ii) Both candidates play the efficiency-maximizing bundle $E = \{R;\ ND\}$ with probability $\beta_E = 1 - \beta_T$.*

*(iii) Both candidates play the debt bundles $\{NR;\ D\}$ and $\{R;\ D\}$ with probability zero.*

*(iv) both candidates play the distributions of total expected offers illustrated in Figure 3.1. However, $T$ is now the no-debt bundle $\{NR;\ ND\}$.*

As these results show, with high enough debt-related distortions, the loss in resources through the use of public debt is so high that the no-debt policy bundles are the only bundles that are played in the political equilibrium. In particular, the bundle that plays no reform and no debt, $\{NR;\ ND\}$, now dominates all other bundles in terms of net targetability. This is because so much is lost of the second-period endowment when transferring it to the first period, that the more in resources in the first period cannot compensate for the high efficiency loss. The net targetability in the sense of what can maximally be offered to 50 percent of voters is higher with $\{NR;\ ND\}$ than $\{NR;\ D\}$. Similarly, so much is lost when transferring the reform benefits to the first period that it is not enough to compensate for the loss in targetable resources due to the reform costs. In terms of net targetability, $\{NR;\ D\}$ can offer more to voters than $\{R;\ D\}$. Finally, clearly $\{NR;\ ND\}$ must dominate the most efficient bundle $\{R;\ ND\}$ in terms of net targetability. This is because, when both bundles do not raise debt, the no-reform bundle has more to target due to saving on the targetable first-period reform costs in exchange for giving up the non-targetable second-period reform benefits.

In terms of the equilibrium probabilities with which the different bundles are played, the same reasoning as for Proposition 3.4.3 now applies. The efficient bundle $E = \{R;\ ND\}$ cannot be attacked on its support by any bundle, because it dominates in terms of total available resources. The only chance for the debt bundles $\{NR;\ D\}$ and $\{R;\ D\}$ to be played is to attack the targetability-maximizing bundle $T = \{NR;\ ND\}$ on its support, which lies above and below the support of $E$. However, due to the high costs of public debt, bundle $T$ also dominates these two debt bundles in terms of efficiency. Therefore, the debt bundles $\{NR;\ D\}$ and $\{R;\ D\}$ have no chance against bundle $T$ on its support either. Therefore, the debt bundles are played with zero probability.

This implies that with high enough debt-related distortions the political process takes care of eliminating public debt itself. In particular, since voters in the first period are forward looking, they only promote a less efficient policy if the efficiency loss can still be compensated for at least a majority of voters. Recall that, only under the following condition does a less efficient policy bundle with more targetable resources today have a net targetability advantage: The more in targetable resources in the first period is enough to compensate at least 50 percent of the electorate for the efficiency loss implied by that bundle. Otherwise, the more efficient bundle also dominates in terms of net targetability. This is exactly what happens with very high debt-related distortions. Even though the use of public debt gives more targetable resources today, the implied efficiency loss is so high that in terms of net targetability a majority of voters cannot be convinced to support such high inefficiency.

This result shows that, if voters are fully informed about an impending danger for high debt-related distortions, they should support low-debt policies even in an environment where each voter tries to get out as much as possible for herself in terms of current electoral favors. In that vain, the recent European debt crisis can have a positive effect, because it clearly demonstrated such costs for several European economies. Due to the involvement of the whole Eurozone in solving the crisis, these costs also became imprinted in the awareness of European citizens whose country was not itself directly hit by a debt crisis.

In terms of intermediate debt-related distortions, which are not quite as high as discussed in this section, but higher as in the previous sections, we hypothesize a mix of the highlighted mechanisms. On the one hand, with higher debt-related distortions, the direct and indirect efficiency gains induced by a debt limit weigh stronger. On the other hand, the dis-incentivizing effect on reform implementation still remains. Additionally, the argument that a debt limit should never restrict debt below the point where distortions do no longer arise still has full validity here. In particular, it also applies when the distortions in the range of high debt levels are higher. Furthermore, as shown in this section, the higher debt-related distortions become, the more no-debt policies will become attractive on their own due to their higher efficiency and do not need to be incentivized too much through a very strict debt limit. A full formal analysis of the case of intermediate debt-related distortions is certainly an interesting task for future research.

## 3.7 Concluding remarks

This paper has analyzed the interaction between distortionary public debt and growth-enhancing reforms in a setup of electoral competition where politicians try to win votes by targeting available resources to subsets of voters. The presence of distortions caused by high debt levels gives any high-debt policy a direct negative impact on efficiency in the sense of reducing the total size of the pie. On the other hand the growth-enhancing reform has a positive effect on efficiency by increasing the total size of the pie. In an electoral setup of targeted redistribution, the factor besides efficiency that determines the electoral success of different policies is how well a given pie of resources can be targeted to specific subsets of voters. Since the reform has costs today, while its benefits only realize in the future, it gives politicians less possibility to target current voters. This is because future resources cannot be promised in the present due to electoral uncertainty between the present and the future. By allowing a transfer of future resources to the present, public debt can help overcome this targetability disadvantage of the reform. In terms of the capacity to target voters under a proposed policy bundle, the interaction between public debt and reform is therefore essential.

A recent paper by Boyer and Esslinger (2016) has analyzed this interaction between public and reforms for the case where high public debt did not create any distortions and hence did not directly effect efficiency. In that case, a limit on public debt is detrimental. This is because the only effect of public debt is the indirect efficiency-enhancing effect in terms of increasing the probability of implementing a pie-increasing reform. We have shown that in the presence of debt-related distortions, we still get the dis-incentivizing effect of a debt limit on the implementation of an efficient reform. In particular, among the policies that raise high debt, the no-reform policy is implemented with relatively higher probability when the debt limit gets stricter. This negative effect, however, now has to be weighed against positive effects of a debt limit in terms of increasing efficiency through a reduction in debt-related distortions. Besides reducing these distortions directly, we have also shown that a stricter debt limit can increase the probability with which efficient no-debt policies are implemented.

# Chapter 4

# State Capacity and Public Debt: A Political Economy Analysis[1]

## 4.1 Introduction

The combination of high public debt and low capacities of the state to raise taxes and to support markets can upset even developed economies. Countries with a low level of fiscal capacity, the institutional infrastructure necessary to collect and enforce taxes, usually exhibit a high level of shadow economic activities. Furthermore, low property rights protection indicates a low level of legal capacity, the legal infrastructure necessary to provide a secure investment climate. State capacity, the combination of legal and fiscal capacity, is a crucial determinant of a state's financial strength. A country with low state capacity that has at the same time a tendency to accumulate high public debt might run into severe problems. In light of this, it is important to understand the mechanisms underlying the combined evolution of state capacity and public debt.

However, the political economy literature of public debt[2] usually takes the institu-

[1]This chapter has been co-authored with Cornelius Müller. We are grateful to Eckhard Janeba for many insightful discussions. We thank Klaus Adam, Pierre Boyer, Océane Briand, Philippe De Donder, Georg Dürnecker, Tore Ellingsen, Hans Peter Grüner, Hubert Kempf, Anke Kessler, Heléne Lundqvist, Andreas Madestam, Torsten Persson, David Strömberg, Michele Tertilt, Johannes Voget, Galina Zudenkova, conference participants at the 2013 PET Meeting, the 2013 IIPF Congress, the 2014 Second Annual TARC workshop, the EPCS 2014 Meeting, the 2014 Annual Congress of the Verein für Socialpolitik, the 2015 EEA Congress, and seminar participants at the University of Mannheim and the University of Stockholm for helpful comments. The first author gratefully acknowledges the Collaborative Research Center 884 for financial support. The usual disclaimer applies.

[2]See Alesina and Passalacqua (forthcoming) and Battaglini (2011) for recent reviews of this literature. For a survey of the earlier contributions see Alesina and Perotti (1995).

tional infrastructure necessary to raise taxes as given and does not consider this fiscal capacity as an investment object of the government. The legal infrastructure necessary for the proper functioning of a market economy is also not modeled as an endogenous political choice in this literature. In contrast, the recent political economy literature of state capacity, pioneered by Besley and Persson (2009), endogenizes fiscal and legal capacity as investment objects of the state. However, this literature does not include public debt. Analyzing the combined evolution of state capacity and public debt necessitates an integrated analytical framework. Otherwise, important aspects of the *interaction* between these dynamic variables will remain unexplored. We provide such an integrated model, and we uncover interactions between state capacity and public debt that cannot be understood by studying the two issues separately.

We integrate a baseline model of state capacity investment building on Besley and Persson (2010, 2011) with the strategic use of public debt, fluctuating incomes, and the possibility of default. In a dynamic framework, an incumbent government cannot be sure to remain in power in the future. It wants to benefit its own clientele, and decides about investments in the future fiscal and legal capacities. The incumbent government can additionally spend on a common-interest public good or channel money towards its own clientele. The "cohesiveness" of institutions determines to what degree such clientele politics are possible. Following Besley and Persson, we say a country has low cohesiveness, when it is very easy to do clientele politics to the benefit of the own group. Importantly, we allow the incumbent to also raise public debt. Debt is restricted by future state capacity, because the latter determines the debt repayment capacity in the second period. The income level attainable for a given legal support as well as the value of public goods fluctuate over time, introducing a business cycle component into the model. The implied possibility of default gives us a tractable way to study the effects of increasing costs of debt financing.

We derive two main sets of results. First, in a simple basic model without fluctuating incomes and without default, we show that the possibility to raise debt can create an additional incentive to invest in state capacity. The intuition is that debt allows to draw future tax resources to the present. This circumvents the problem of a use of future public funds that is not in line with the current incumbent's objective. Specifically, high political instability and low cohesiveness make the first period incumbent afraid of giving additional state capacity to the future government. By high political instability, this government is likely to be from the opposed group, and by low cohesiveness, it can use the higher taxing power to heavily redistribute away from the period-1 incumbent group. In the model without debt, the only possibility to protect against such an adverse use of future public funds is to lower investments in fiscal and legal capacity. We call this the low-investment mechanism. However, if debt can be used to bring future public funds at the disposal of the first-period incumbent, then this incumbent can decide about their use. Given that there are profitable uses of tax resources in the first period, the incumbent now has higher incentives to invest in state capacities in order to increase the amount of public funds at her disposal. We call this the debt mechanism. The strength of this latter mechanism depends on how easily it can be used. For the basic model without default, there are no restrictions on using this mechanism. Therefore, it can completely cancel out the original low-investment mechanism.

Our second set of results shows how the debt mechanism can be weakened, thereby allowing the original low-investment mechanism to partly resurface. Specifically, with fluctuating incomes, the cost of raising additional debt depends on the possibility of default. When debt is raised to the point where default becomes possible, it becomes increasingly expensive to use the debt channel to draw future public funds to the present. To the extent that it is very costly to draw newly created future public funds to the present, the low-investment mechanism resurfaces. Specifically, it resurfaces the stronger, the higher are the income fluctuations. For high income fluctuations, we therefore get results close to the original no-debt model. In particular, a polarized country with low institutional protection of the non-ruling group and high political instability will invest only little in state capacities. Furthermore, this 'weak state' situation is now worsened by a built-up of high debt and a positive probability of default. Besley and Persson (2011) identified so-called development clusters with weak states combining all the negative outcomes in terms of low investments in both forms of state capacity, low income and higher internal violence. We show that, under high income fluctuations, this notion of a negative cluster can be extended to include high public debt and a high risk of default.

From these results, we can draw the following policy implications. Besides political instability, cohesiveness is identified as an important driving force behind the combined evolution of state capacity and public debt. Cohesiveness enters as an exogenous parameter in our model, but our comparative static results show which implications follow from changing it. In our model, a country with high cohesiveness will be close to the social planner optimum. Increasing cohesiveness in the real world necessitates deep reforms that go at the core of the functioning of the state. Examples of such reforms include implementing a functioning system of checks and balances, establishing an independent press that names and shames clientele politics, creating provisions in the constitution that prevent clientele politics, or strengthening the constitutional court in its power to enforce such provisions. Comprehensive reforms in these directions can prevent a country from running into a situation of high debt and low state capacity. Importantly, our results highlight that, in order to put a country on a path of investments for the future without making it run up high debt at the same time, it is necessary to first incentivize deeper institutional reforms. Otherwise political economic incentives will always push back towards using low investments or high public debt to protect against a politically adverse future.[3]

The rest of this paper is organized as follows. Section 4.2 discusses the relation of this paper to the existing literature. Section 4.3 sets out a basic model of state capacity and public debt which does not yet include fluctuating incomes or default. Building on Besley and Persson (2010, 2011), we combine a workhorse model of state capacity investment with the political economic incentive to strategically use public debt. In Section 4.4, we analyze this model. Comparing the results to the model without debt, we find that the possibility to raise debt can create a novel incentive to invest in state capacity. In Section 4.5, we introduce exogenous income fluctuations into the model to allow for sovereign default. We investigate under which circumstances the incentivizing effect of public debt on state capacity investments can fade. In that case, countries can run into the situation with low state capacity and high public debt. We also take a short look at available cross-country data to see whether they give a clear indication

[3]Note that the clientele politics here do not correspond to the targeting of electoral favors during an electoral campaign, which has been analyzed in the previous two chapters. Any democracy will be characterized by some form of such an allocation of electoral favors to the majority of voters. The clientele politics in this chapter correspond rather to a form of rent extraction for the benefit of the own clientele once politicians are in power. In particular, in this chapter, we do not consider a feedback of politicians' decisions on the probability of being in power. We do not even take a stand on whether a change in political power is determined by a democratic election. Therefore, the results here do not contradict the result from before that targeted pork-barrel spending might be necessary in electoral campaigns to secure the success of growth-enhancing reforms.

in which direction the interaction between debt and state capacity investments plays out. Section 4.6 concludes and discusses topics for future research. We relegate all proofs to the Appendix.

## 4.2 Related Literature

Analyzing the political incentives behind investing in state capacity and raising public debt, we bring together the two strands of the political economy literature that have analyzed these concepts in isolation. The concept of state capacity was brought back to the minds of economists by Timothy Besley and Torsten Persson in a series of recent papers (Besley and Persson, 2009, 2010).[4] These were condensed into their book *Pillars of Prosperity* (Besley and Persson, 2011). All of these models include two aspects of state capacity, legal and fiscal capacity, in a tractable political economy model with two periods.

Our model builds on the workhorse model in Besley and Persson (2011). This model has been extended in several directions. Besley and Persson (2009), for instance, provide a micro foundation for the growth-enhancing effect of legal support by explicitly modeling a credit market whose effectiveness depends on the level of legal support. Besley et al. (2013) drop legal capacity and extend the remaining fiscal capacity model to comprise multiple periods and to include decreasing marginal benefits of public good spending. They show that the main results from the two-period model generalize to this setup.

The main novel feature of our paper is that we consider in one model the interaction of the strategic use of public debt and the decision of an incumbent government to invest in its future powers to raise taxes and to grant legal support. With regard to the state capacity literature, we find that the possibility to raise debt can provide an additional mechanism to incentivize state capacity investments that cannot be seen in a model without debt. However, debt might be used to tie down the additional investments for uses that are not in accordance with the social planner's objective. We also derive conditions under which the link between debt and state capacity investments

[4] Early studies concerning state capacity are the ones of Cukierman et al. (1992) regarding fiscal capacity and Svensson (1998) regarding legal capacity. In recent years, Acemoglu (2005) and Acemoglu et al. (2011) made further contributions to the literature on state building.

is weak. In this case, the weak state situation of low investments in state capacity, identified by Besley and Persson, is worsened by an additional buildup of high debt.

While the state capacity literature has not included the debt channel at all, the debt literature usually includes taxes. Nevertheless, this latter literature takes fiscal capacity as given. Implicitly, the institutional capacity to raise taxes is often assumed to be maximal in this context. In particular, when labor taxes are considered, the upper bound on taxation is given by the tax rate maximizing the resulting Laffer curve. Therefore, fiscal capacity is not included as an endogenous dynamic variable in this literature.

The strand of the debt literature closest to our setup is the literature on strategic debt initiated by Persson and Svensson (1989), Alesina and Tabellini (1990) and Tabellini and Alesina (1990).[5] Persson and Svensson (1989) and Alesina and Tabellini (1990) consider debt in the setup of distortionary labor taxes. The cost of raising debt therefore involves higher tax distortions in the future. Both papers show that in political competition between parties with differing objectives, too much debt is raised compared to a normative benchmark. This happens because a currently ruling government cannot be sure to remain in power in the future and therefore uses debt to bind its successor's hands. Since our model has non-distortionary taxation, it is more closely related to the one of Tabellini and Alesina (1990). They examine a two-period model with non-distortionary taxation and a group of heterogeneous individuals with different preferences over two public goods. The social planner would run a balanced budget, but in the political equilibrium, the uncertainty regarding the future median voter leads to a positive debt level to bind the hands of the future median voter.

A similar trade-off as in these models also arises in our setup. The spending purpose on which groups have differing preferences is now redistributive transfers for the purpose of clientele politics. Also similarly, our model can produce the incentive to over-accumulate debt compared to a social planner. Specifically, clientele politics are not beneficial from a social planner's point of view. A social planner will therefore only spend money on public goods. This implies that the social planner will not raise debt when the future value of public goods is expected to be higher than in the present. At

[5] Recent models in the field of political economy of public debt with rich dynamic frameworks can be found in Battaglini and Coate (2008), Yared (2010), Song et al. (2012), and Battaglini and Barseghyan (forhcoming).

the same time, a political government might still accumulate debt in order to finance redistribution towards its own clientele in the present.

The literature of borrowing with default goes back to the seminal study of Eaton and Gersovitz (1981). Arellano (2008) extends their approach and applies it to sovereign debt default, especially in the context of developing countries. Both studies use an infinite horizon model in which the borrower can choose to default. The incentives to not default are given by an embargo on future borrowing, an additional penalty, or direct output costs. Due to the two-period setting, our model does not include the embargo on future borrowing. It is therefore somewhat in the spirit of Alesina and Tabellini (1989), who also rely an a two-period model of sovereign debt default. Furthermore, we only model ability-to-pay default and not willingness-to-pay default. The modeling of default is used here mainly to include increasing costs of debt financing in a tractable way. A more involved modeling of the default decision is left to future research.

## 4.3 Model Setup

**Political Environment.** The model set out here combines a workhorse model of state capacity with the strategic use of public debt. Building on Besley and Persson (2011), the model has two periods $s = 1, 2$ and considers a country consisting of two equally sized groups of individuals. Total population size is normalized to 1. One of the groups holds governmental power in the first period. Individuals that are a member of the *incumbent* group in a given period are superscripted by the letter $I$, whereas members of the *opposition* group are superscripted by the letter $O$. With exogenous probability $\gamma$, power is transferred to the other group after the first period. Higher $\gamma$ thus captures higher political instability from the point of view of the first period incumbent group.

**Intra-temporal Policies.** In period $s$, an individual of group $J \in \{I, O\}$ has an income of $\omega(p_s^J)$, where $\omega(\cdot)$ is an increasing and concave function of legal support $p_s^J$ granted to group $J$. More broadly, one can think of $p_s^J$ as any kind of market supporting policies that increase the private income of individuals of group $J$. Following Besley and Persson (2009, 2010), we interpret $p_s^J$ as legal enforcement which is conducive to a more efficient functioning of capital markets.[6]

The utility of an individual of group $J \in \{I, O\}$ in period $s$ is linear in private consumption $c_s^J$ and public good consumption $g_s$:[7]

$$u_s^J = \alpha_s g_s + c_s^J \tag{4.1}$$

where $\alpha_s$ parametrizes the marginal value of public good consumption relative to that of private consumption.[8] Income available for private consumption is determined by the non-distortionary tax rate $t_s$ on income $w$ and the per-capita transfers $r_s^I, r_s^O$ awarded by the government to the different groups. Therefore, individual utility in period $s$ becomes:

$$u_s^J = \alpha_s g_s + (1 - t_s)\omega(p_s^J) + r_s^J \tag{4.2}$$

Future utility is discounted with the factor $\delta \in (0, 1)$.

The value of public goods fluctuates over time. In a developing country setup, a period with a high value of public good spending can be interpreted as a situation with a high threat of an external war. For a developed country, it is harder to find a perfect real world match for this assumption. Nevertheless, we can think of certain rescue actions in times of an economic crisis whose benefit to the overall economy exceeds possible additional private benefits by far. The stabilization of the economy can then be interpreted as a *quasi*-public good, whose value is high in crisis times. One example would be the nationalization of a system-relevant bank.

To model this fluctuation in the simplest possible way, the value of public goods is drawn each period from a two-point distribution: $\alpha_s \in \{\alpha_H, \alpha_L\}$, with $\alpha_H > 2 > \alpha_L > 1$, and Prob$[\alpha_s = \alpha_H] = \phi$. As will become clear in the subsequent analysis, the high value $\alpha_H$ is chosen such that public good spending in this state of the world will be preferable to transfer spending that benefits the incumbent group. In a situation with $\alpha_L$, this is not necessarily the case. Since public goods benefit everybody the same, the desired size of fiscal infrastructure will depend on the probability $\phi$ of ending up in a situation where the state definitely spends on common-interest public goods.

[6] For a microfoundation of the above reduced form modeling, see Besley and Persson (2009).

[7] Our results are robust to an extension for quasi-linear utility where we introduce curvature in the utility function on public good consumption.

[8] Note that the whole analysis is in *per-capita* terms.

**Inter-temporal Policies.** The crucial feature of the model in Besley and Persson (2011) is that it includes two aspects of state capacity, fiscal capacity $\tau_s$ and legal capacity $\pi_s$. Existing fiscal capacity $\tau_s$ puts an upper bound on the tax rate that can be raised from income in period $s$: $t_s \leq \tau_s$. In this simple model, $(1-\tau_s)$ can be interpreted as the fraction of income that an individual can earn in an informal sector. To increase second period fiscal capacity $\tau_2$, the period-1 government can invest in the built-up of $[\tau_2 - \tau_1]$ additional units of fiscal capacity.[9] For the sake of parsimony, we assume zero depreciation of the stocks of state capacity, in contrast to Besley and Persson (2011). We require $[\tau_2 - \tau_1] \geq 0$, so disinvestment is not allowed. There is an increasing and convex cost $F(\tau_2 - \tau_1)$ of carrying out the investment, with $F(0) = F_\tau(0) = 0$. Here, as in the following, subscripts on functions denote partial derivatives, and the last equation can be interpreted as the first marginal investment having negligible costs.

Legal capacity $\pi_s$ puts an upper bound on the legal protection for both groups: $p_s^J \leq \pi_s$ for $J \in \{I, O\}$. The idea is that the existing legal infrastructure restricts the level of legal protection a government can grant to any group. The government in period one can invest in the future legal capacity that becomes available in period two via an increasing and convex cost function $L(\pi_2 - \pi_1)$ with $L(0) = L_\pi(0) = 0$. As for fiscal capacity, we require $[\pi_2 - \pi_1] \geq 0$, so disinvestment is not allowed.

Besides the dynamic decisions to invest in fiscal and legal capacities, we now add a third dynamic decision to the policy set: the possibility to raise public debt. Specifically, the country is assumed to start with a stock of debt equal to zero, $b_0 = 0$. The period-1 incumbent government can now issue one-period risk-free bonds $b_1 \geq 0$ on an international bond market.[10] These bonds have to be repaid in the second period. The interest rate which has to be paid on bonds is given by $\rho = 1/\delta - 1$ , where $\delta$ is the discount factor of the individuals. Since the bonds are supposed to be risk-free, the maximal amount of bonds is determined by the requirement that fiscal and legal

[9]There will be a technological maximum $\bar{\tau} < 1$ above which fiscal capacity cannot be expanded $(\tau_s \leq \bar{\tau})$. Here, this would be determined by the fact that some small black market jobs just cannot be detected. Besley et al. (2013) interpret $\bar{\tau}$ as the highest technologically feasible tax rate (while $\tau_s$ is the highest institutionally feasible tax rate) and argue that in a richer model with distortionary taxation, $\bar{\tau}$ could be the peak of the Laffer curve. However, in the following, we focus on a situation where the optimal level $\tau_2$ will not hit this upper bound.

[10]We make the assumption that a government cannot invest into assets and that the starting debt level is zero just to keep the analysis focused. An extension that relaxes these assumptions does not add significant insights to the analysis.

capacity in the second period must be high enough to repay the bonds: $b_1 \leq \frac{\tau_2 \omega(\pi_2)}{1+\rho}$. The right-hand side of this inequality is the discounted maximal tax revenue that can be raised in the second period. Following Besley and Persson (2011) we still assume that the citizens themselves cannot save or borrow. Firstly, due to the linearity of the utility function, this assumption does not alter the results. Secondly, for the developing world, where the issue of state building is most pertinent, there is evidence for private agents' lack of access to credit markets (see Claessens, 2006).

**Feasible Policies.** The incumbent group government is assumed to maximize its own group's utility subject to a usual budget constraint and a constraint imposed by the country's "cohesiveness" of institutions. The budget constraint requires that government revenues are enough to finance all government expenditures:[11]

$$\sum_{J \in \{I,O\}} \frac{t_s \omega(p_s^J)}{2} + b_s \geq g_s + m_s + n_s + \frac{r_s^I + r_s^O}{2} + (1+\rho) b_{s-1} \tag{4.3}$$

where $m_s$ and $n_s$ represent the investment costs in fiscal and legal capacity, which only occur in period 1, and hence are given by

$$m_s = \begin{cases} F(\tau_2 - \tau_1) & \text{if } s = 1 \\ 0 & \text{if } s = 2 \end{cases} \tag{4.4}$$

and

$$n_s = \begin{cases} L(\pi_2 - \pi_1) & \text{if } s = 1 \\ 0 & \text{if } s = 2. \end{cases} \tag{4.5}$$

Since the groups have equal size, $\frac{r_s^I + r_s^O}{2}$ is the *average* per-capita transfer that the government pays out.

The institutional constraint requires that for each unit of transfers awarded by the government to its own group it must transfer at least $\sigma \in [0,1]$ units to the other group. Besley and Persson (2011) introduce the parameter $\theta = \frac{\sigma}{1+\sigma} \in [0, \frac{1}{2}]$ to describe the "cohesiveness" of institutions. $\theta = \frac{1}{2}$ refers to completely cohesive institutions which make sure that the opposition is treated in exactly the same way as the incumbent group. For $\theta < \frac{1}{2}$, clientele politics are possible that lead to a redistribution of money

[11] Recall that total population is normalized to 1.

towards the incumbent group. Given that the incumbent government respects the institutional setting as just another constraint but ultimately is only concerned about its own group's utility, it will set transfers to the opposition group as small as possible:

$$r_s^O = \sigma r_s^I = \frac{\theta}{1-\theta} r_s^I \tag{4.6}$$

In the following, we therefore assume the government is choosing only transfers $r_s^I$ to its own group, while setting $r_s^O$ according to (4.6).

Plugging (4.6) into the budget constraint (4.3), we arrive at a modified budget constraint that already includes the constitutional constraint:

$$\sum_{J\in\{I,O\}} \frac{t_s \omega(p_s^J)}{2} + b_s \geq g_s + m_s + n_s + \frac{r_s^I}{2(1-\theta)} + (1+\rho)b_{s-1}, \tag{4.7}$$

with $b_0 = b_2 = 0$.

**Timing.** The timing of the whole two-period model is now as follows:

**Period 1:**

1. The initial stock of fiscal capacity is $\tau_1$ and group $I_1$ is in power. Nature draws the public good value $\alpha_1$.

2. The government from the currently incumbent group $I_1$ chooses the set of period-1 policies $\left\{t_1, g_1, r_1^I, r_1^O, b_1, p_1^I, p_1^O\right\}$ and by its investment decision chooses the period-2 stocks of fiscal capacity $\tau_2$ and legal capacity $\pi_2$.

**Period 2:**

1. $I_1$ remains in power with probability $1-\gamma$, and nature draws $\alpha_2$

2. The government from the future incumbent group $I_2$ chooses period-2 policies $\left\{t_2, g_2, r_2^I, r_2^O, p_2^I, p_2^O\right\}$ while honoring the debt commitments.

The applied solution concept is subgame perfect equilibrium.

## 4.4 The Incentivizing Effect of Public Debt

Public debt as well as state capacity investments generate dynamic links across periods. However, given the linear utility function, we can derive the optimal policy decision between public good spending and transfer spending for any period taking as given the levels of state capacity, state capacity investments and debt. Furthermore, the non-distortionary nature of taxes makes the level of taxes in a given period depend only on the level of fiscal capacity in that period. In a second step, the optimal debt level will be determined using the optimal policy functions on public good and transfer spending and still taking state capacity and state capacity investments as given.[12] Having derived the optimal policy decisions on spending and debt for different levels of state capacity investments, the optimal level of these investments can then be determined in a last third step.

**Intra-temporal policies**

Turning to the first step, legal protection will be set maximally for both groups: $p_s^I = p_s^O = \pi_s$. This is because, first, the incumbent group gains from an increase of its own income. Second, it also benefits from an increase of the other group's income, because the resulting higher tax revenues can be used for additional public good or transfer spending.

Taxes will be used up to the full fiscal capacity: $t_s = \tau_s$. The reason is the following: The marginal benefit of public spending is always at least as high as the opportunity cost of lost private consumption, since $max\{\alpha_s, 2(1-\theta)\} \geq 2(1-\theta) \geq 1$.

Compared to the model without debt, the introduction of debt does not change the trade-off between public goods and transfers. This trade-off depends solely on the constant marginal benefits of these two forms of spending. The only effect is on the level of spending. Specifically, the residual revenues after investment spending have to be adjusted for the net inflow of money through issuing new debt and repaying old debt, $b_s - (1+\rho)b_{s-1}$.

[12]The reason that the debt decision can be analyzed *before* the state capacity decisions has to do with the constancy of the marginal value of spending in each period.

The optimal policy function for public good spending becomes

$$G(\alpha_s, \tau_s, \pi_s, m_s, n_s, b_s, b_{s-1}) = \begin{cases} \tau_s\omega(\pi_s) - m_s - n_s + b_s - (1+\rho)b_{s-1} & \text{if } \alpha_s \geq 2(1-\theta) \\ 0 & \text{otherwise.} \end{cases} \tag{4.8}$$

That is, public goods are provided at the maximal level if the gross marginal value of public good spending, $\alpha_s$, exceeds the gross marginal value of transfers for the incumbent group, $2(1-\theta)$. At the same time, transfers and therefore redistribution towards the incumbent group are zero. If the ordering between the marginal values is the opposite way, we only get redistributive transfers and no public goods.

Using the new budget constraint (4.7) with $t_s$ set to $\tau_s$ and $p_s^J$ set to $\pi_s$, the indirect payoff function for group $J \in \{I, O\}$ in period $s$ becomes:

$$\begin{aligned} &W(\alpha_s, \tau_s, \pi_s, m_s, n_s, b_{s-1}, b_s, \beta^J) \\ &= \alpha_s G + (1-\tau_s)\omega(\pi_s) + \beta^J[\tau_s\omega(\pi_s) - G - m_s - n_s + b_s - (1+\rho)b_{s-1}] \end{aligned} \tag{4.9}$$

where $\beta^I = 2(1-\theta)$ and $\beta^O = 2\theta$ can be interpreted respectively as the gross marginal value of transfer spending for the incumbent (I) and for the opposition (O). Note that we have suppressed the arguments of the $G$ function. Furthermore, $\beta^J[\tau_s\omega(\pi_s) - G - m_s - n_s + b_s - (1+\rho)b_{s-1}] \geq 0$ are the transfers to group $J$.

The "value functions" capturing the within-period utility in the second period for a group that is the incumbent (I) or the opposition (O) become:

$$\begin{aligned} &U^I(\tau_2, \pi_2, b_1) \\ &= \phi W[\alpha_H, \tau_2, \pi_2, 0, 0, b_1, 0, 2(1-\theta)] + (1-\phi)W[\alpha_L, \tau_2, \pi_2, 0, 0, b_1, 0, 2(1-\theta)] \\ &= \phi[\alpha_H(\tau_2\omega(\pi_2) - (1+\rho)b_1) + (1-\tau_2)\omega(\pi_2)] \\ &\quad + (1-\phi)W[\alpha_L, \tau_2, \pi_2, 0, 0, b_1, 0, 2(1-\theta)] \end{aligned} \tag{4.10}$$

$$\begin{aligned} &U^O(\tau_2, \pi_2, b_1) \\ &= \phi W[\alpha_H, \tau_2, \pi_2, 0, 0, b_1, 0, 2\theta] + (1-\phi)W[\alpha_L, \tau_2, \pi_2, 0, 0, b_1, 0, 2\theta] \\ &= \phi[\alpha_H(\tau_2\omega(\pi_2) - (1+\rho)b_1) + (1-\tau_2)\omega(\pi_2)] \\ &\quad + (1-\phi)W[\alpha_L, \tau_2, \pi_2, 0, 0, b_1, 0, 2\theta] \end{aligned} \tag{4.11}$$

Note that, when the value of the public good is high, whatever is left after repaying debt, $(\tau_2\omega(\pi_2)-(1+\rho)b_1)$, will be spent on the public good. Finally, the total expected utility of the period-1 incumbent group, as seen from the first period, is:

$$W(\alpha_1,\tau_1,\pi_1,F(\tau_2-\tau_1),L(\pi_2-\pi_1),0,b_1,2(1-\theta))+\delta([1-\gamma]U^I(\tau_2,\pi_2,b_1)+\gamma U^O(\tau_2,\pi_2,b_1)) \tag{4.12}$$

### Inter-temporal policies

Having solved for the optimal intra-temporal policies, we now turn to the inter-temporal policies $b_1$, $\tau_2$ and $\pi_2$. To make the following analysis easier, we define $\lambda_1$, the gross marginal benefit of public funds in period 1, and $E(\lambda_2)$, the expected gross marginal benefit of public funds in period 2. We have

$$\lambda_1 \equiv max\{\alpha_1, 2(1-\theta)\} \tag{4.13}$$

and

$$E(\lambda_2) \equiv \phi\alpha_H + (1-\phi)\lambda_2^L \tag{4.14}$$

with

$$\lambda_2^L = \begin{cases} \alpha_L & \text{if } \alpha_L \geq 2(1-\theta) \\ (1-\gamma)2(1-\theta)+\gamma 2\theta & \text{otherwise,} \end{cases} \tag{4.15}$$

since $E(\lambda_2)$ depends on the use of public funds in the future, which is uncertain.

With this notation, the inter-temporal maximization problem of the incumbent group in period s=1 becomes:

$$\begin{aligned} max_{\tau_2,\pi_2,b_1}\ & EV^{I_1}(\tau_2,\pi_2,b_1) - \lambda_1(F(\tau_2-\tau_1)+L(\pi_2-\pi_1)-b_1) \\ s.t.\ & \tau_2 \geq \tau_1, \\ & \pi_2 \geq \pi_1, \\ & b_1 \leq \frac{\tau_2\omega(\pi_2)}{1+\rho}, \\ & \lambda_1 \equiv max\{\alpha_1, 2(1-\theta)\} \end{aligned} \tag{4.16}$$

with:

$$EV^{I_1}(\tau_2, \pi_2, b_1) = \delta((1-\gamma)U^I(\tau_2, \pi_2, b_1) + \gamma U^O(\tau_2, \pi_2, b_1)) \tag{4.17}$$

**Choice of debt**

The three dynamic variables fiscal capacity, legal capacity and debt are interlinked by the following fact. The amount of debt which can be raised is limited by the amount of future fiscal and legal capacity. The latter two determine the money the state can raise to repay debt. We now analyze the choice of debt taking the levels of fiscal and legal capacity investments as given.

Note that the investment and debt decisions determine the amount of *residual* revenue a government has at its disposal for financing public good spending or transfers after all other expenditures are covered. The linear model has the advantage that, in each period, the use of residual government revenues either on public goods or transfers is exactly determined. Furthermore, the marginal benefit of that residual use is constant in either case. This marginal benefit is what we referred to as the gross marginal benefit of public funds and denoted by $\lambda_s$.

Debt allows to make future public funds available in the present. Therefore, the optimal debt level can be found by a simple comparison of the gross marginal benefit of public funds in the two periods. Specifically, if $\lambda_1$, the gross marginal benefit in the first period, is higher than $E(\lambda_2)$, the expected gross marginal benefit in the second period, then it is optimal to raise the maximal debt that is allowed by future state capacity: $b_1 = \frac{\tau_2 \omega(\pi_2)}{1+\rho}$. If $E(\lambda_2) > \lambda_1$, then no debt is raised: $b_1 = 0$.[13,14]

Summarizing the above analysis in a policy function for the debt level chosen in period 1, we have:

$$B(\alpha_1, \tau_2, \pi_2) = \begin{cases} \frac{\tau_2 \omega(\pi_2)}{1+\rho} & \text{if } \lambda_1 > E(\lambda_2) \\ 0 & \text{otherwise.} \end{cases} \tag{4.18}$$

[13] The implicit assumption behind $b_1 \geq 0$ is that the government cannot invest in assets on the bond market.

[14] Furthermore, this result requires Assumptions (4.23) and (4.24) which will be introduced after having derived the optimal state capacity investments. We need these additional technical assumptions here because for $E(\lambda_2) > \lambda_1$, we could otherwise get that all first-period tax revenue is used for investments in future state capacity. With low enough costs of investment, it could then be beneficial to use debt for bringing future tax revenues to the present and finance even more future state capacity.

### Choice of fiscal and legal capacity

With this, we have arrived at the third step of the analysis, the decision about fiscal and legal capacity investment. We substitute the policy function (4.18) for $b_1$ in (4.12) and maximize the resulting function with respect to future fiscal capacity $\tau_2$ and legal capacity $\pi_2$, subject to the constraints that fiscal and legal capacity investments cannot be negative, $[\tau_2 - \tau_1] \geq 0$ and $[\pi_2 - \pi_1] \geq 0$, and transfers must also be weakly positive. From this, we get the following "Euler equations"

$$\begin{aligned}
&\delta([1-\gamma]\frac{dU^I[\tau_2,\pi_2,B(\alpha_1,\tau_2,\pi_2)]}{d\tau_2} + \gamma\frac{dU^O[\tau_2,\pi_2,B(\alpha_1,\tau_2,\pi_2)]}{d\tau_2}) \\
&+ W_{b_1}[\alpha_1,\tau_1,\pi_1,m_1,n_1,0,B(\alpha_1,\tau_2,\pi_2),2(1-\theta)]\frac{\partial B(\alpha_1,\tau_2,\pi_2)}{\partial \tau_2} \\
&\leq -W_m[\alpha_1,\tau_1,\pi_1,m_1,n_1,0,B(\alpha_1,\tau_2,\pi_2),2(1-\theta)]F_\tau(\tau_2-\tau_1) \\
&c.\ s.\ \tau_2-\tau_1 \geq 0
\end{aligned} \tag{4.19}$$

and

$$\begin{aligned}
&\delta([1-\gamma]\frac{dU^I[\tau_2,\pi_2,B(\alpha_1,\tau_2,\pi_2)]}{d\pi_2} + \gamma\frac{dU^O[\tau_2,\pi_2,B(\alpha_1,\tau_2,\pi_2)]}{d\pi_2}) \\
&+ W_{b_1}[\alpha_1,\tau_1,\pi_1,m_1,n_1,0,B(\alpha_1,\tau_2,\pi_2),2(1-\theta)]\frac{\partial B(\alpha_1,\tau_2,\pi_2)}{\partial \pi_2} \\
&\leq -W_n[\alpha_1,\tau_1,\pi_1,m_1,n_1,0,B(\alpha_1,\tau_2,\pi_2),2(1-\theta)]L_\pi(\pi_2-\pi_1) \\
&c.\ s.\ \pi_2-\pi_1 \geq 0,
\end{aligned} \tag{4.20}$$

where $\frac{dU^I}{d\tau_2}$, $\frac{dU^O}{d\tau_2}$, $\frac{dU^I}{d\pi_2}$ and $\frac{dU^O}{d\pi_2}$ are *total* derivatives. The trade-off is between the marginal benefit of future fiscal or legal capacity (left-hand side) against the marginal cost of financing that fiscal or legal capacity (right-hand side). Analogously to the model without debt:

$$\begin{aligned}
\lambda_1 &\equiv -W_m[\alpha_1,\tau_1,\pi_1,m_1,n_1,0,B(\alpha_1,\tau_2,\pi_2),2(1-\theta)] \\
&= -W_n[\alpha_1,\tau_1,\pi_1,m_1,n_1,0,B(\alpha_1,\tau_2,\pi_2),2(1-\theta)] = max\{\alpha_1, 2(1-\theta)\}
\end{aligned}$$

That is, the opportunity cost of using government revenues for financing investments is the gross marginal benefit of period-1 public funds.[15] It depends on the form of residual spending (public goods or transfers) in period one.

[15] This result depends again on Assumptions 4.23 and 4.24. These technical assumptions exclude the case where it is optimal (and through debt possible) that the marginal money to finance future fiscal capacity actually comes from the future.

The crucial difference to a model without debt are the left-hand sides of (4.19) and (4.20). The left-hand side of (4.19) describes the marginal benefit of additional future fiscal capacity, as seen from the first period. If no debt is raised, it is equal to $\delta\omega(\pi_2)[E(\lambda_2) - 1]$. $E(\lambda_2)$ is the expected gross marginal benefit of future public funds and is given in (4.14) and (4.15). However, when debt is raised, it is raised *maximally* and uses up all public funds in the second period. Therefore, the gross marginal value of public funds is then determined by the use of debt. Since debt is used to finance *first-period* expenditures on public goods or transfers, the gross marginal benefit of future public funds becomes $\lambda_1$, which was given in equation (4.13). The point is that debt allows to make future public funds available in the present. Therefore, the benefit of future public funds is then given by the *present* benefit of residual spending. According to this discussion, the optimality condition (4.19) can be rewritten as:

$$\delta\omega(\pi_2)\left[max\{\lambda_1, E(\lambda_2)\} - 1\right] \leq \lambda_1 F_\tau(\tau_2 - \tau_1) \tag{4.21}$$

$$c.\ s.\ \tau_2 - \tau_1 \geq 0$$

Given the assumption $F_\tau(0) = 0$, a necessary and sufficient condition for positive investments in fiscal capacity is now $max\{\lambda_1, E(\lambda_2)\} > 1$, which is always satisfied. This is a crucial difference compared to the model without debt and is discussed in more detail in Section 4.4.2.

The left-hand side of (4.20) describes the marginal benefit of additional future legal capacity, as seen from the first period. Following the same reasoning as for fiscal capacity, the optimality condition (4.20) can be rewritten as:

$$\delta\omega'(\pi_2)[1 + \tau_2[max\{\lambda_1, E(\lambda_2)\} - 1]] \leq \lambda_1 L_\pi(\pi_2 - \pi_1) \tag{4.22}$$

$$c.\ s.\ \pi_2 - \pi_1 \geq 0$$

Given the assumption $L_\pi(0) = 0$, there is always positive investment in legal capacity.

Considering the left-hand sides of equation (4.21) and equation (4.22), we notice that an investment in one of the two state capacities increases the marginal return of the other. This is because we have $max\{\lambda_1, E(\lambda_2)\} > 1$. So, fiscal and legal capacity are complements. Note that the analogous condition in Besley and Persson (2011), $E(\lambda_2) > 1$, was not guaranteed to always hold. In contrast, the introduction of debt implies that complementarity between the two forms of state capacity investment will always hold.

As in Besley and Persson (2011), this complementarity is not only an interesting fact but also allows us to apply results on monotone comparative statics. By Theorem 5 and 6 of Milgrom and Shannon (1994), any factor that increases the left-hand sides of equation (4.21) and equation (4.22) leads to an increase of both fiscal and legal capacity investments.[16] This reasoning is used to establish the comparative statics stated in the propositions in the following two subsections.

For all of the subsequent analysis, we make the following assumptions.

**Assumptions**

$$\delta\omega(\pi_2)[\alpha_H - 1] < \alpha_L F_\tau(\bar{\tau}_2 - \tau_1) \tag{4.23}$$

$$\delta\omega'(\pi_2)[1 + \tau_2[\alpha_H - 1]] < \alpha_L L_\pi(\bar{\pi}_2 - \pi_1) \tag{4.24}$$

for some $\bar{\tau}_2$, $\bar{\pi}_2$, so that $L(\bar{\pi}_2 - \pi_1) + F(\bar{\tau}_2 - \tau_1) = \tau_1\omega(\pi_1)$. That is, $\bar{\tau}_2$, $\bar{\pi}_2$ are levels of future state capacity which can be financed if the current tax revenue is only and fully used for that purpose. These assumptions mean that the curvature of the cost functions $F(\cdot)$ and $L(\cdot)$ is high enough for the marginal cost of increasing fiscal and legal capacity to surpass the marginal benefit at an interior level of investment. That is, we don't allow the marginal benefit of investment to still surpass the marginal cost at the point where all possible tax revenues are only used for investments in fiscal and legal capacity.[17] These technical assumptions are only necessary in a linear model. They can be dispensed with in a quasi-linear setup.[18]

In the following, we first analyze the normative benchmark of a social planner.

## 4.4.1 The Social Planner's Solution

The maximization problem of a Utilitarian social planner who weights the utilities of both groups equally is equivalent to the version of the model where full cohesiveness ($\theta = 1/2$) restricts the incumbent group in both periods to provide the same transfers

[16] For a more detailed formal treatment, see the proofs in the appendix.

[17] Note that the left-hand sides of (4.23) and (4.24) give the absolute maximum for the marginal benefits of fiscal and legal capacity investment. The right-hand sides give the absolute minimum for the marginal costs given investment levels $\bar{\pi}_2 - \pi_1$ and $\bar{\tau}_2 - \tau_1$ that use up all current tax revenues.

[18] The quasi-linear analysis is available from the authors upon request.

to both groups. Then, since $\alpha_L > 2(1-\theta) = 1$, the social planner always uses all residual money to provide public goods. For the basic model with debt, the results about debt and state capacity investments of a social planner are summarized in the following proposition:

**Proposition 4.4.1** *Suppose that the decisions about debt and state capacity investments are made by a Pigouvian planner with Utilitarian preferences. Then:*

1. *If $\alpha_1 = \alpha_L$:*
   - *(a) No debt is raised.*
   - *(b) No transfers are paid.*
   - *(c) There are positive investments in fiscal and legal capacity.*
   - *(d) Higher $\phi$ increases investment in fiscal and legal capacity.*
2. *If $\alpha_1 = \alpha_H$:*
   - *(a) Debt is raised maximally: $b_1 = \frac{\tau_2 \omega(\pi_2)}{1+\rho}$.*
   - *(b) No transfers are paid.*
   - *(c) There are positive investments in fiscal and legal capacity and the investments are higher than when no debt can be raised.*

For a social planner $\lambda_1 = \alpha_1$. That is, the gross marginal value of public funds in the first period corresponds to the value of public goods in the first period. Moreover, for the social planner the gross marginal value of public funds in the second period is $E(\lambda_2) = \phi\alpha_H + (1-\phi)\alpha_L > 1$.

For the first part, note that $\lambda_1 = \alpha_L < E(\lambda_2)$ implies that no debt will be raised. In a model without debt, the results of the first part are valid for both, $\alpha_1 = \alpha_L$ and $\alpha_1 = \alpha_H$ as stated in Proposition 2.1 in Besley and Persson (2011).

In the model with debt, if $\alpha_1 = \alpha_H$, we have $\lambda_1 = \alpha_H > E(\lambda_2)$ and debt will be raised maximally in order to make future public funds available in the present. Therefore, the net marginal benefit of future public funds is greater than it was without debt, which raises incentives to invest in fiscal capacity. Basically, debt allows the social planner to use the tax system of the future to finance a highly-valued public good today.

Given a high need for public funds today versus a lower need tomorrow this increases incentives to invest in fiscal capacity for the purpose of increasing spending today. By complementarity, investments in legal capacity increase as well.

We now turn to the analysis of the political equilibrium.

### 4.4.2 Three Types of States Revisited

The outcome of the political game will depend on the interplay of the parameters governing how cohesive political institutions are ($\theta$) and how stable the political system is ($\gamma$) with the public good parameters ($\phi, \alpha_H, \alpha_L$) summarizing the main features of the economic environment. In Besley and Persson (2011), the following condition ensures that political institutions are sufficiently cohesive to make the political outcome coincide with the outcome under a social planner:

***Cohesiveness***: $\alpha_L \geq 2(1-\theta)$

This condition will hold if the parameter governing the cohesiveness of political institutions, $\theta$, is close enough to $1/2$, the value it takes for a social planner. Recall that $\theta = 1/2$ ensures that both groups have to be treated equally and therefore captures perfectly cohesive political institutions.

If the cohesiveness condition fails, but the stability of the political system is high enough, Besley and Persson (2011) get a state that still has positive investments in state capacity. The corresponding stability condition is:

***Stability***: $\phi\alpha_H + (1-\phi)[(1-\gamma)2(1-\theta) + \gamma 2\theta] > 1$

This condition will hold when the probability of staying in political power, $1-\gamma$, is big enough. That is, from the point of view of the period-1 incumbent government, the political system is stable in the sense of not endangering its power. However, the condition goes further. In fact, it refers to stability in the sense of not endangering the *interests* of the period-1 government. This can also be ensured by the economic environment. For instance, if a high value of public good spending is expected with certainty ($\phi \to 1$), the stability condition will also hold. The interest of the period-1 government in high-valued future spending is then respected no matter who is in power in the future. In order to compare our results to the ones in Besley and Persson (2011), we consider the same three types of states that they derive and investigate if these types still arise after the introduction of public debt.

**Common-Interest State**

In the case where the cohesiveness condition holds, we get the following result:

**Proposition 4.4.2** *If Cohesiveness holds, then the outcome is the same as under a social planner (see Proposition 4.4.1).*

This result is analogous to the model without debt (Proposition 2.2 in Besley and Persson (2011)). The reason is that high cohesiveness makes redistribution unattractive compared to public good spending even when the latter has a low value. Therefore, by choice, each government will provide only public goods thereby behaving exactly like a social planner. The shifting of public resources over time also follows the structure of the public good values and again coincides with the social planner behavior. In line with Besley and Persson (2011), we call this state the common interest state.

**Redistributive State**

Assume that Cohesiveness fails, but Stability holds. We get the following results:

**Proposition 4.4.3** *If Cohesiveness fails and Stability holds, then:*

1. *If* $\alpha_1 = \alpha_L$ *and* $2(1-\theta) < \phi\alpha_H + (1-\phi)[(1-\gamma)2(1-\theta) + \gamma 2\theta]$*:*
   (a) *No debt is raised.*
   (b) *Residual revenues in period 1 are used to finance transfers.*
   (c) *There are positive investments in fiscal and legal capacity.*
   (d) *Higher* $\phi$ *increases investments in fiscal and legal capacity.*
   (e) *A lower value of* $\gamma$ *unambiguously raises investments, whereas an increase in* $\theta$ *raises investments if* $\gamma > 1/2$.
2. *If* $\alpha_1 = \alpha_H$ *or if* $\alpha_1 = \alpha_L$ *and* $2(1-\theta) > \phi\alpha_H + (1-\phi)[(1-\gamma)2(1-\theta) + \gamma 2\theta]$*:*
   (a) *Debt is raised maximally:* $b_1 = \frac{\tau_2 \omega(\pi_2)}{1+\rho}$.
   (b) *There are positive investments in fiscal and legal capacity and the investments are higher than when no debt can be raised.*

(c) *If $\alpha_1 = \alpha_H$, the levels of fiscal and legal capacity investments are the same as those chosen by a social planner in the same situation (see Proposition 4.4.1 Part 2).*

(d) *If $\alpha_1 = \alpha_L$, residual revenues in period 1 are used to finance transfers.*

(e) *Political instability $\gamma$ does not have an influence on the investment decisions. If $\alpha_1 = \alpha_H$, cohesiveness $\theta$ does not have an influence either.*

In an environment with a low public good value in the first period, the incumbent government prefers to spend on redistributive transfers in this first period. However, when the condition in part 1 of the proposition holds, the expected value of future public spending is still higher than the value of this first-period transfer spending. The implied preference for future spending leads to the result that no debt is raised, because this would mean taking resources from the future. Therefore, the remaining results in the first part of the proposition are analogous to the model without debt.

When one of the conditions in part 2 of the proposition holds, there is a preference for the present. Specifically, the incumbent group in period 1 can no longer be sure that spending in the second period will be in its interest in expectation. However, with debt, the period-1 government now has the possibility to bring future public funds at its disposal. Thereby, it can decide about the spending purposes which these future public funds will be used for. This will actually allow the incumbent group to solve the problem of future redistribution against itself and hence raises incentives for investing in fiscal and legal capacity.

Importantly, these bigger incentives can be driven by the desire to finance redistributive transfers in the present, which is not in the spirit of a Utilitarian social planner. Therefore, we do not only get spending on the 'wrong' intra-temporal issues, we can even get the incentive to finance more of this 'wrong' intra-temporal spending through the issuance of debt. In this case of the basic model, the additional debt-induced incentive to invest in fiscal and legal capacity therefore creates a bigger deviation of the political outcome from the social planner optimum.

**'Weak' State**

The last possibility arises when both the cohesiveness and the stability condition fail. Besley and Persson (2011) call such a state a *weak state*. In their model without debt, such a state has no incentive to invest in fiscal capacity (Proposition 2.4 in Besley and Persson (2011)).

As we have already seen, the introduction of debt can raise incentives to invest because what drives these incentives is now the use that period-2 public funds can be put to in the *first* period. This strongly suggests that the weak state situation, which is based on the fear of future public funds being used against the own group, will no longer arise in this basic model with debt. The next proposition confirms this hypothesis:

**Proposition 4.4.4** *In the basic model without the possibility of default, if Cohesiveness and Stability fail, then:*

1. *There is positive investment in fiscal capacity which is higher than the zero investment in the case without debt. Moreover, there is positive investment in legal capacity which is higher than in the case without debt.*
2. *If $\alpha_1 = \alpha_H$, the levels of investment in fiscal and legal capacity are the same as those chosen by a social planner in the same situation (see Proposition 4.4.1 Part 2).*
3. *Debt is raised maximally: $b_1 = \frac{\tau_2 \omega(\pi_2)}{1+\rho}$.*
4. *If $\alpha_1 = \alpha_L$, residual revenues are used to finance transfers.*

So, the weak state situation as in Besley and Persson (2011) does no longer arise when debt is allowed since the possibility to raise debt creates incentives for investing in state capacity. We even get the social planner's investment level if $\alpha_1 = \alpha_H$. For the case $\alpha_1 = \alpha_L$, we also get positive investments in state capacity, potentially even higher than the social planner's investments. However, from the perspective of a social planner, this case is now even worse than in the model without debt since all future tax revenue is now drawn to the present and used for transfers directed to the incumbent's clientele.

The arguments for all these results and their interpretations are the same as for Part 2 of Proposition 4.4.3, which described a redistributive state.

This result of the weak state situation no longer arising will be qualified in the following section. When we introduce the possibility of default, the costs of raising high debt enter the analysis. In cases where these costs are prominent, we can re-establish the possibility of a weak state.

## 4.5 Sovereign Default and Increasing Costs of Debt Financing

### 4.5.1 Adjustments to Model Setup

Until now, the interest rate of government bonds was constant and independent of the level of debt. Future income and tax revenues were perfectly predictable, government default was impossible, and the model had a bang-bang solution, i.e. either maximal debt or no debt at all. In this section, we extend the model by allowing income $\omega$ to be subject to shocks. As we will see below, this leads to varying interest rates, the possibility of government default, and interior solutions for the optimal level of public debt.

In the following, we allow the income $\omega_s(\pi_s)$ of the economy in period $s$ to fluctuate so that tax revenues $\tau_s\omega_s(\pi_s)$ are also uncertain. The following can thus be interpreted as a parsimonious way of including an exogenous business cycle component into the model. Specifically, there are two possible income levels for any given level of legal capacity, $\bar{\omega}(\pi_s) > \underline{\omega}(\pi_s)$. We make the assumptions that $\bar{\omega}'(\pi_s) = \underline{\omega}'(\pi_s) \equiv \omega'(\pi_s)$[19] and $\text{Prob}\left(\omega_s(\pi_s) = \bar{\omega}(\pi_s)\right) = \psi$.

In such a setting, the interest rate is endogenously determined by $R(b) = \rho + r(b)$. $r(b)$ is the risk premium, which is nonzero if $b$ exceeds a certain threshold $\bar{b}$ characterized below. So far, the interest rate $\rho = 1/\delta - 1$ was pinned down by the discount factor $\delta$ of the consumers, and therefore it was independent of the debt level

[19] Note that this is equivalent to assuming $\omega_s(\pi_s)$ to have the following form: $\bar{\omega}(\pi_s) = w(\pi_s) + \bar{v}$ and $\underline{\omega}(\pi_s) = w(\pi_s) + \underline{v}$. This means we assume the income shock (e.g. due to business cycles) to be additive and not depending on the level of legal capacity. Therefore, increasing legal capacity leads to a higher expected income (a positive growth trend), around which the actual income fluctuates with an amplitude that is constant with respect to $\pi$.

$b$. Now, the interest rate increases in the level of public debt. The higher interest rate at higher levels of debt captures the risk premium due to a higher probability of default.

Since the investment decision regarding state capacity might influence the solvency of the state and therefore the credit terms, one has to be careful regarding the timing of issuing debt and investing in state capacity. Let the timing be as described in Section 4.3. We now divide stage 2 of the timing from Section 4.3 into two stages 2a and 2b. We assume that in stage 2a the government makes the decision regarding the investments in fiscal and legal capacity and in stage 2b all other decisions including debt.[20] This allows us to capture in an easy way the fact that the investors buying government bonds condition their expectations regarding the future solvency of the state on the future levels of fiscal and legal capacity, $\tau_2$ and $\pi_2$. Besides lending money to the government, we assume that the international investors have the possibility to invest in risk-less bonds which just compensate them for their time preference. These risk-less bonds therefore have an interest rate of $\rho = 1/\delta - 1$. Since investors are assumed to be risk neutral, the risk premium has to be just high enough to make them indifferent between lending money to the model country and investing in the risk-free asset.

The threshold $\bar{b}$ is defined such that for $b \leq \bar{b}$, debt including interest can be fully paid back even for the low income realization $\underline{\omega}(\pi_2)$. In this case, there is no risk that needs compensation, so $R(b) = \rho$. The threshold $\bar{b}$ is given by:

$$\bar{b}(\tau_2, \pi_2) = \frac{\tau_2 \underline{\omega}(\pi_2)}{1 + \rho} \tag{4.25}$$

For $b > \bar{b}$, debt will be payed back fully in case of high income $\bar{\omega}(\pi_2)$ but partially else. The function for the risk premium, $r(b)$, that makes investors indifferent between lending to the country and investing in the risk-free asset is defined by

$$(1 + \rho)b_1 = \psi \cdot \underbrace{(1 + \rho + r(b_1))b_1}_{repayment\ in\ case\ of\ \bar{\omega}(\pi_2)} + (1 - \psi) \cdot \underbrace{(\tau_2 \underline{\omega}(\pi_2))}_{repayment\ in\ case\ of\ \underline{\omega}(\pi_2)} . \tag{4.26}$$

Rearranging terms leads to the expression $r(b_1) = \frac{1-\psi}{\psi}(1 + \rho - \frac{\tau_2 \underline{\omega}(\pi_2)}{b_1})$.

[20] We can also think of these actions as taking place simultaneously under the constraint that the bundle of debt, interest rate and state capacity investments is such that bond holders are indifferent between holding the country's debt and their outside option of investing into a riskless asset that compensates for the discount factor $\delta$.

It is clear that there has to be a maximum level of debt $\bar{\bar{b}}$. This is defined by the maximum debt that can be fully payed back including interest in the case of high income $\bar{\omega}(\pi_2)$. This level is given by $\bar{\bar{b}} = \frac{\tau_2 \bar{\omega}(\pi_2)}{1+\rho+r(\bar{\bar{b}})}$. Solving for $\bar{\bar{b}}$, this leads to:

$$\bar{\bar{b}}(\tau_2, \pi_2) = \frac{\tau_2(\psi\bar{\omega}(\pi_2) + (1-\psi)\underline{\omega}(\pi_2))}{1+\rho} \tag{4.27}$$

When debt is not completely paid back, which means that there is sovereign debt default, the country incurs a penalty $P$. For reasons of tractability, the penalty is assumed to reduce the after-tax income. It would certainly be more realistic to have the penalty reduce gross income as in many models of sovereign debt default (e.g. Alesina and Tabellini (1989), Arellano (2008), Eaton and Gersovitz (1981)). However, the above assumption is made to avoid technical complications. The penalty can be interpreted, for instance, as credit restrictions on retailers which make it more expensive to supply imported goods. The ensuing reduction in the purchasing power of income is captured in our simple model by the direct reduction of after-tax income through the penalty.

We assume that the penalty has the following form (where $\Delta$ is the amount not repaid):

$$P = \begin{cases} 0 & if \quad \Delta = 0 \quad (no\ default) \\ P(\Delta) & if \quad \Delta = (1+R(b_1))b_1 - (\tau_2\underline{\omega}(\pi_2)) \quad and\ \omega_2(\pi_2) = \underline{\omega}(\pi_2) \\ P_{max} & else. \end{cases} \tag{4.28}$$

This means, as long as the government shows good will, in the sense that it repays as much debt as it can, the penalty depends on the amount of debt that is not repaid. $P(\Delta)$ is assumed to be increasing and convex for $\Delta \in [0, (1+R(\bar{\bar{b}}))\bar{\bar{b}} - (\tau_2\underline{\omega}(\pi_2))] = [0, \tau_2(\bar{\omega}(\pi_2) - \underline{\omega}(\pi_2))]$ with $P(0) = 0$. If the country repays less than possible and defaults purposely, we assume the punishment to be maximal, $P_{max}$. We assume $P_{max}$ to be high enough to prevent the government from defaulting purposely. That is, we only consider *ability-to-pay* default and not *willingness-to-pay* default. This allows us to model rising costs of debt financing without having to burden the analysis with a more involved modeling of the default decision.

### 4.5.2 The Effects of Increasing Costs of Debt Financing

Concerning the intra-temporal policies, with the same reasoning as in the model without debt, fiscal and legal capacities are always fully employed. As for the policy function for public good spending, it also looks analogous to before:

$$G(\alpha_s, \tau_s, \pi_s, m_s, n_s, b_s, b_{s-1}, \hat{\omega}) = \begin{cases} \tau_s \hat{\omega}(\pi_s) - m_s - n_s + b_s & \\ -min\left\{(1+R(b_{s-1}))b_{s-1}, \tau_s \hat{\omega}(\pi_s)\right\} & \text{if } \alpha_s \geq 2(1-\theta) \\ 0 & \text{otherwise,} \end{cases} \tag{4.29}$$

where $\hat{\omega}$ can now be the high or low income realization $\overline{\omega}$ or $\underline{\omega}$. For simplifying the notation, the following contains the "expected" policy function $G = \psi G(\overline{\omega}) + (1 - \psi)G(\underline{\omega})$, where $\psi$ is the probability for the high income realization $\overline{\omega}$.

The inter-temporal maximization problem of the incumbent group in period s=1 becomes:

$$\begin{aligned} max_{\tau_2,\pi_2,b_1}\ EV^{I_1}(\tau_2, \pi_2, b_1) - \lambda_1(F(\tau_2 - \tau_1) + L(\pi_2 - \pi_1) - b_1) \\ s.t.\ \tau_2 \geq \tau_1, \\ \pi_2 \geq \pi_1, \\ b_1 \leq \overline{\overline{b}}(\tau_2), \\ \lambda_1 \equiv max\{\alpha_1, 2(1-\theta)\} \end{aligned} \tag{4.30}$$

with

$$\begin{aligned} EV^{I_1}(\tau_2, \pi_2, b_1) &= \delta((1-\gamma)U^I(\tau_2, \pi_2, b_1) + \gamma U^0(\tau_2, \pi_2, b_1)) \\ &= \delta((1-\gamma)(\phi EW(\alpha_H, \tau_2, \pi_2, 0, 0, b_1, 0, 2(1-\theta)) \\ &\quad + (1-\phi)EW(\alpha_L, \tau_2, \pi_2, 0, 0, b_1, 0, 2(1-\theta))) \\ &\quad + \gamma(\phi EW(\alpha_H, \tau_2, \pi_2, 0, 0, b_1, 0, 2\theta) \\ &\quad + (1-\phi)EW(\alpha_L, \tau_2, \pi_2, 0, 0, b_1, 0, 2\theta))) \end{aligned} \tag{4.31}$$

and

$$
\begin{aligned}
&EW(\alpha_2, \tau_2, \pi_2, m_2 = 0, n_2 = 0, b_1, b_2 = 0, \beta^J) \qquad (4.32)\\
&= \alpha_2 G + (1 - \tau_2)(\psi\bar{\omega}(\pi_2) + (1 - \psi)\underline{\omega}(\pi_2)) - (1 - \psi)P \\
&\quad + \beta^J [\tau_2(\psi\bar{\omega}(\pi_2) + (1 - \psi)\underline{\omega}(\pi_2)) - G - \psi(1 + R(b_1))b_1 \\
&\quad - (1 - \psi)min\{(1 + R(b_1))b_1, \tau_2\underline{\omega}(\pi_2)\}],
\end{aligned}
$$

which is the indirect payoff function for group $J \in I, O$ in period 2. This function is now an expected value itself, because future income $\omega_2(\pi_2)$ is uncertain. Note that for the analysis here, we assume that the fluctuations in income and in the valuation of public goods are independent. In a quasi-linear setup, we considered the other extreme, a perfect correlation between the two in the sense that public good spending has a higher value in times with low income.[21] The most realistic modeling probably lies at some intermediate level of correlation, but the extreme cases allow us to keep the analysis tractable.

Plugging (4.32) into (4.31), leads to:

$$
\begin{aligned}
EV^{I_1}(\tau_2, \pi_2, b_1) =&\delta[(\psi\bar{\omega}(\pi_2) + (1 - \psi)\underline{\omega}(\pi_2))(1 - \tau_2) - (1 - \psi)P \qquad (4.33)\\
&+ \underbrace{[\phi\alpha_H + (1 - \phi)\lambda_2^L]}_{\equiv E(\lambda_2)} \cdot [\tau_2(\psi\bar{\omega}(\pi_2) + (1 - \psi)\underline{\omega}(\pi_2)) \\
&- \psi(1 + R(b_1))b_1 - (1 - \psi)min\{(1 + R(b_1))b_1, \tau_2\underline{\omega}(\pi_2)\}]]
\end{aligned}
$$

with $\lambda_2^L$ defined by (4.15).

Regarding the solution of this optimization problem, several cases can arise:

- case a) : $b_1 = 0$[22]
- case b) : $b_1 \in (0, \bar{b})$
- case c) : $b_1 = \bar{b}$
- case d) : $b_1 \in (\bar{b}, \bar{\bar{b}})$
- case e) : $b_1 = \bar{\bar{b}}$

The first order conditions with respect to $b_1$ in case b) and d) allow us to determine when the different cases arise. In case b), the first order condition with respect to $b_1$

[21] As already noted, our results are robust to the extension with a quasi-linear utility.

[22] This is because we assume $b_0 = 0$ and that governments cannot accumulate assets. If we allowed for assets, governments would use their revenue to buy bonds in this case. If we additionally allowed $b_0 > 0$, revenues would be used to reduce debt (and possibly to buy bonds).

gives us $\lambda_1 = E(\lambda_2)$ and in case d) $\lambda_1 = E(\lambda_2) + \frac{(1-\psi)}{\psi}\frac{\partial P(\Delta)}{\partial \Delta}$. It follows that if $E(\lambda_2) < \lambda_1 < E(\lambda_2) + \frac{(1-\psi)}{\psi}\left.\frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\bar{b}}$, we are in case c), if $\lambda_1$ exceeds $E(\lambda_2) + \frac{(1-\psi)}{\psi}\left.\frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\bar{\bar{b}}}$, we are in case e) and if $\lambda_1$ is smaller than $E(\lambda_2)$, we are in case a).

Since the function $R(b_1)$ depends on the case and includes $\tau_2$ and $\pi_2$, also the first order conditions that determine $\tau_2$ and $\pi_2$ vary over the cases. The following table summarizes the expressions:[23]

| Case | Condition |
|---|---|
| a) | $\lambda_1 < E(\lambda_2)$ |
| b) | $\lambda_1 = E(\lambda_2)$ |
| c) | $E(\lambda_2) < \lambda_1 < E(\lambda_2) + \frac{(1-\psi)}{\psi}\left.\frac{\partial P(\Delta)}{\partial \Delta}\right\|_{b=\bar{b}}$ |
| d) | $\lambda_1 = E(\lambda_2) + \frac{(1-\psi)}{\psi}\frac{\partial P(\Delta)}{\partial \Delta}$ |
| e) | $\lambda_1 > E(\lambda_2) + \frac{(1-\psi)}{\psi}\left.\frac{\partial P(\Delta)}{\partial \Delta}\right\|_{b=\bar{\bar{b}}}$ |

| Case | FOC for $\tau_2$ |
|---|---|
| a) | $\delta\left\{\psi\bar{\omega}(\pi_2) + (1-\psi)\underline{\omega}(\pi_2)\right\}(E(\lambda_2) - 1) = \lambda_1 \frac{\partial F(\tau_2 - \tau_1)}{\partial \tau_2}$ |
| b) | $\delta\left\{\psi\bar{\omega}(\pi_2) + (1-\psi)\underline{\omega}(\pi_2))\right\}(E(\lambda_2) - 1) = \lambda_1 \frac{\partial F(\tau_2 - \tau_1)}{\partial \tau_2}$ |
| c) | $\delta\left\{\psi[\bar{\omega}(\pi_2) - \underline{\omega}(\pi_2)](E(\lambda_2) - 1) + \underline{\omega}(\pi_2)(\lambda_1 - 1)\right\} \leq \lambda_1 \frac{\partial F(\tau_2 - \tau_1)}{\partial \tau_2}$ |
| d) | $\delta\left\{\psi[\bar{\omega}(\pi_2) - \underline{\omega}(\pi_2)](E(\lambda_2) - 1) + \underline{\omega}(\pi_2)(\lambda_1 - 1)\right\} \leq \lambda_1 \frac{\partial F(\tau_2 - \tau_1)}{\partial \tau_2}$ |
| e) | $\delta\{(\psi\bar{\omega}(\pi_2) + (1-\psi)\underline{\omega}(\pi_2))(\lambda_1 - 1)$ $-(1-\psi)\left.\frac{\partial P(\Delta)}{\partial \Delta}\right\|_{b=\bar{\bar{b}}}(\bar{\omega}(\pi_2) - \underline{\omega}(\pi_2))\} \leq \lambda_1 \frac{\partial F(\tau_2 - \tau_1)}{\partial \tau_2}$ |

| Case | FOC for $\pi_2$ |
|---|---|
| a) | $\delta\omega'(\pi_2)[1 + \tau_2(E(\lambda_2) - 1)] = \lambda_1 \frac{\partial L(\pi_2 - \pi_1)}{\partial \pi_2}$ |
| b) | $\delta\omega'(\pi_2)[1 + \tau_2(E(\lambda_2) - 1)] = \lambda_1 \frac{\partial L(\pi_2 - \pi_1)}{\partial \pi_2}$ |
| c) | $\delta\omega'(\pi_2)[1 + \tau_2(\lambda_1 - 1)] = \lambda_1 \frac{\partial L(\pi_2 - \pi_1)}{\partial \pi_2}$ |
| d) | $\delta\omega'(\pi_2)[1 + \tau_2(\lambda_1 - 1)] = \lambda_1 \frac{\partial L(\pi_2 - \pi_1)}{\partial \pi_2}$ |
| e) | $\delta\omega'(\pi_2)[1 + \tau_2(\lambda_1 - 1)] = \lambda_1 \frac{\partial L(\pi_2 - \pi_1)}{\partial \pi_2}$ |

In the following analysis, we consider an economy that has a low value of public good spending in the first period, $\alpha_1 = \alpha_L$. In this environment, a social planner will

[23]The FOCs for $\tau_2$ and $\pi_2$ in case d) have been rearranged by using the FOC for $b_1$. The $\leq$ in the FOCs for $\tau_2$ in case c), d) and e) are due to the constraint $\tau_2 \geq \tau_1$, which might bind in these cases. Note that $\pi_2 \geq \pi_1$ does never bind.

not want to raise debt. In particular, the social planner's solution is clearly case a), since the social planner has $\lambda_1 = \alpha_1 < (1-\phi)\alpha_L + \phi\alpha_H = E(\lambda_2)$.[24] We want to see whether for such an environment, governments with a preference for the own group could exhibit a bias towards the present and hence towards excessive debt. Such a bias would make the political equilibrium differ from the social planner's solution in a significant way.

Furthermore, for the following analysis, we define 'free future revenues' as the discounted expected future tax revenues minus debt, $\delta E(\tau_2\omega_2(\pi_2)) - b_1$. Free future revenues refer to the expected future tax revenues that are not bound by debt and therefore measure the 'free' resources of the future government. We define this measure in order to capture debt in relation to a state's fiscal power, which is more informative than the absolute debt level itself.

To analyze the political equilibrium, let us distinguish between countries with high and low cohesiveness $\theta$. Assume first that cohesiveness is sufficiently high, in the sense that the cohesiveness condition of Section 4.3 holds: $\alpha_L > 2(1-\theta)$. Then the political equilibrium is case a), since we have $\lambda_1 = \alpha_1$ and $E(\lambda_2) = (1-\phi)\alpha_L + \phi\alpha_H$, as it was the case for the social planner. That is, high enough cohesiveness will make the political equilibrium coincide with the social planner outcome.

Now, consider countries with low cohesiveness, in the sense that $\theta$ is sufficiently below 1/2. These countries can end up in each of the cases a)-e), depending on the parameters $\gamma$, $\phi$ and $\alpha_H$ summarized in $E(\lambda_2)$.[25]

The following proposition summarizes the main results for the model with default. More detailed comparative static results are collected in Corollary C.1.1 in the Appendix.

**Proposition 4.5.1** *Suppose an economy in the model with sovereign default starts in the first period with $\alpha_1 = \alpha_L$. Moreover, suppose that the constraint $\tau_2 \geq \tau_1$ does not bind.*[26] *Then:*

[24] If we had $\alpha_1 = \alpha_H$, we would have $\lambda_1 > E(\lambda_2)$ for both the social planner and for a government with own group bias. Both would have a preference for the present and an incentive to raise debt, and we would end up in one of the cases c), d) or e).

[25] For the sake of completeness, further channels of influence are $\psi$, $\bar{\omega}(\cdot) - \underline{\omega}(\cdot)$ and $P(\cdot)$.

[26] It turns out that this is not a very restrictive assumption. It would only be violated if the high income realization $\overline{\omega}(\cdot)$ lied unrealistically high above the low income realization $\underline{\omega}(\cdot)$. A sufficient condition for this assumption is that $\overline{\omega}(\cdot) \leq 2\underline{\omega}(\cdot)$.

1. *In the social planner's solution as well as in the political equilibrium for* $\alpha_L \geq 2(1-\theta)$

   I. *No debt is raised. That is case a) holds.*

   II. *The corresponding comparative static results for state capacity investments are the same as in the no-default model.*

2. *In the following, consider the political equilibrium for* $\alpha_L < 2(1-\theta)$*. That is, we have* $\lambda_1 = 2(1-\theta)$ *and* $E(\lambda_2) = \phi\alpha_H + (1-\phi)[(1-\gamma)2(1-\theta) + \gamma 2\theta]$.

   (a) *If* $\lambda_1 < E(\lambda_2)$

      I. *No debt is raised. That is case a) holds.*

      II. *The corresponding comparative static results for state capacity investments are the same as in the no-default model.*

   (b) *If* $\lambda_1 = E(\lambda_2)$

      I. *The debt level is indeterminate in the range* $[0, \bar{b}]$*. That is case b) holds.*

   (c) *If* $E(\lambda_2) < \lambda_1 < E(\lambda_2) + \frac{(1-\psi)}{\psi} \left.\frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\bar{b}}$

      I. $b_1 = \bar{b}$ *is the optimal debt level. That is case c) holds.*

      II. *Debt and state capacity investments move in the same direction in response to exogenous parameter changes.*

      III. *Incentives to invest in state capacity are lower than in a no-default world with the same expected income,* $\psi\overline{\omega} + (1-\psi)\underline{\omega}$.

   (d) *If* $\lambda_1 = E(\lambda_2) + \frac{(1-\psi)}{\psi} \left.\frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\tilde{b}}$*, with* $\bar{b} < \tilde{b} < \bar{\bar{b}}$

      I. $b_1 = \tilde{b}$ *is the optimal debt level. That is case d) holds.*

      II. *There is sovereign default in the second period with probability* $1-\psi$.

      III. *Incentives to invest in state capacity are lower than in a no-default world with the same expected income,* $\psi\overline{\omega} + (1-\psi)\underline{\omega}$.

   (e) *If* $\lambda_1 > E(\lambda_2) + \frac{(1-\psi)}{\psi} \left.\frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\bar{\bar{b}}}$

      I. *Debt is raised maximally,* $b_1 = \bar{\bar{b}}$*. That is case e) holds.*

      II. *There is sovereign default in the second period with probability* $1-\psi$.

III. *Incentives to invest in state capacity are lower than in a no-default world with the same expected income, $\psi\overline{\omega} + (1-\psi)\underline{\omega}$, and lowest of all the cases considered here.*

Part 1 of the proposition establishes the social planner solution, which coincides with the political equilibrium under the cohesiveness condition, $\alpha_L \geq 2(1-\theta)$. To understand the comparative static results detailed in part 1 of Corollary C.1.1, we can look at the respective first order conditions for fiscal and legal capacity in case a). Recall that the left-hand side of these conditions gives the marginal benefit of higher investments. Since no debt is raised and future tax resources are left in the future, investment incentives are driven by the expected value of future public funds, $E(\lambda_2) = \phi\alpha_H + (1-\phi)\alpha_L$, as in a world without default. Additionally, the benefit of future fiscal capacity depends on the expected income base to which it can be applied, which is now given by $\{\psi\bar{\omega}(\pi_2) + (1-\psi)\underline{\omega}(\pi_2)\}$. Since this is increasing in $\psi$, the probability of a high income realization, investment incentives increase in $\psi$.

Part 2 of Proposition 4.5.1 illustrates the outcome of the political equilibrium when the cohesiveness condition fails. That is, cohesiveness $\theta$ is low enough such that $\alpha_L < 2(1-\theta)$. Part 2 of Corollary C.1.1 in the appendix contains the comparative static results that can be shown using the techniques of monotone comparative statics. It turns out that it is hard to get unambiguous comparative static results with respect to cohesiveness $\theta$, as soon as debt is used. Therefore, we concentrate, for the following illustration, on parameter changes which do not alter cohesiveness $\theta$. This implies that we keep the value of present public funds $\lambda_1 = 2(1-\theta)$ constant. Most of the remaining parameters enter $E(\lambda_2)$, the expected value of future public funds. Therefore, we can illustrate most comparative static results by considering the reactions to a change in $E(\lambda_2)$, keeping $\lambda_1$ constant. For a fixed and sufficiently low value of cohesiveness (and $\alpha_1 = \alpha_L$ as before), Figure 4.1 illustrates the relation between $E(\lambda_2)$, future fiscal capacity $\tau_2$, future legal capacity $\pi_2$, debt $b_1$ and free future revenues.[27]

[27] The underlying comparative statics within the different cases are established in Corollary C.1.1. So, we assume that $\tau_2 \geq \tau_1$ does not bind. However, letting it bind up to some $E(\lambda_2)$ would not change much. For the relevant values of $E(\lambda_2)$, $\tau_2$ and $\pi_2$ then would be horizontal lines from the left, the dashed line for $b_1$ in case d) would be solid and both $b_1$ and free future revenues would be constant in case c) (as long as $\tau_2 \geq \tau_1$ binds). The continuity at the border between two cases can be seen from the first order conditions. Free future revenues jump in case b) since debt jumps from $\overline{b}$ to 0. Depending on the functional forms of $F(\cdot)$, $L(\cdot)$ and $\omega(\cdot)$, the upward/downward sloping lines of the diagram are not necessarily linear.

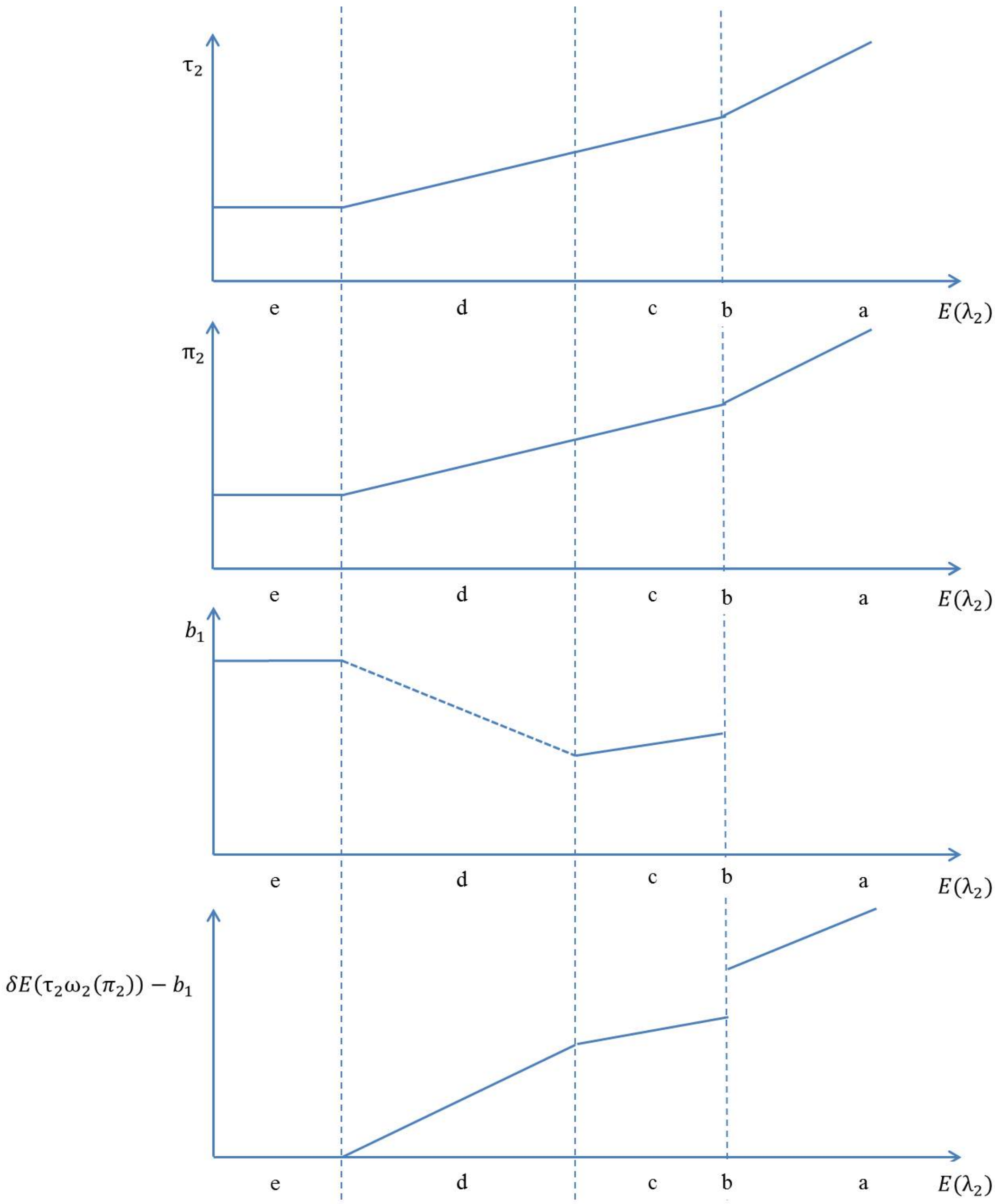


Figure 4.1: Equilibrium outcomes conditional on $E(\lambda_2)$ (holding cohesiveness fixed at a low level)

The letters below the graphs refer to the different cases we identified above. Given an expected value of future public funds, $E(\lambda_2)$, higher than the present value $\lambda_1$, we start on the right of the figure in case a). The corresponding results are summarized in Part 2.(a) of Proposition 4.5.1 and of Corollary C.1.1. No debt is raised and the comparative static results correspond to a model without debt. In particular, all parameter changes that decrease $E(\lambda_2)$ will decrease investments in fiscal and legal capacity. One important possibility to lower $E(\lambda_2)$ is an increase in political instability.

Continuing to decrease $E(\lambda_2)$, at some point, we reach the knife-edge case b) with $E(\lambda_2) = \lambda_1$.[28] The debt level in this case is indeterminate in the range $[0, \bar{b}]$.

By lowering $E(\lambda_2)$ further, we move to case c). To understand the comparative static results for case c), it is helpful to consider the respective first order condition for fiscal capacity $\tau_2$. Note that $\underline{\omega}(\pi_2)$ is the "low income part" of the expected future tax base. This part will be available for sure in the future. In contrast, $\psi[\bar{\omega}(\pi_2) - \underline{\omega}(\pi_2)]$ describes the additional expected value of the income tax base since $\bar{\omega}(\pi_2) - \underline{\omega}(\pi_2)$ will be additionally available if the high income realizes. We call $\psi[\bar{\omega}(\pi_2) - \underline{\omega}(\pi_2)]$ the "high income part".

In case c), it is optimal to exactly raise a debt level of $\bar{b}(\tau_2, \pi_2) = \frac{\tau_2 \underline{\omega}(\pi_2)}{1+\rho}$. Because of the ensuing penalty, it would be too expensive to raise more debt. This means that the low income part of the future tax base is fully drawn to the present. For this part, the marginal benefit of making more of it available to the state through fiscal capacity investments is thus proportional to $(\lambda_1 - 1)$. For case c), $E(\lambda_2) < \lambda_1$, so future public funds are more valuable when they can be used in the present through debt. Therefore, investment incentives are higher than in a world without debt through the influence of the low income part.[29]

In contrast, the high income part of expected future tax resources is not drawn to the present. For this part of the future tax base, the marginal benefit of making more of it available through fiscal capacity investments is thus proportional to $(E(\lambda_2) - 1)$. As far as this part is concerned, we therefore have the same effects as in a model without debt. For instance, given low cohesiveness, increasing political instability makes it more likely that the current government's group gets mistreated in the future by a rival government. This decreases $E(\lambda_2)$ and, through the influence of the high income part, decreases incentives to invest in fiscal capacity. By complementarity between the two forms of state capacity investment, we also get less investments in legal capacity $\pi_2$. Lower levels of fiscal and legal capacity decrease $\bar{b}(\tau_2, \pi_2) = \frac{\tau_2 \underline{\omega}(\pi_2)}{1+\rho}$. The latter is just the present value of the low income part of future public funds. Since exactly this part is drawn to the present in case c), the debt level decreases when investments in

[28] Note that, for this case, there are no comparative static results to derive.

[29] Note that the proposition does not contain results about this level comparison to a world *without debt*.

state capacity decrease. This is illustrated in the third panel of Figure 4.1. For case c), we thus have debt and state capacity moving in the same direction in response to exogenous parameter changes. Furthermore, through the influence of the high income part, we have lower incentives to invest than in a model without default that has no fluctuations in income but the same expected income, $\psi\overline{\omega}+(1-\psi)\underline{\omega}$. This is because in a world without income fluctuations, the entire income could be drawn to the present.

Continuing to decrease the value of future public funds, $E(\lambda_2)$, we enter case d). In this case, it is optimal to incur some penalty by risking default in case the low income realizes. That is default will occur with probability $(1-\psi)$. The optimality condition for debt implies that the optimal debt level lies a certain amount $db$ above $\overline{b}$. Since optimal debt $b_1$ is still lower than the maximal debt level $\overline{\overline{b}}$, there is still some proportion of expected future tax resources which is left in the future. In particular, for the *marginal* investment in fiscal capacity, the high income part of future tax resources is not drawn to the present. Therefore, as in case c), the marginal benefit of additional fiscal capacity contains a term proportional to $(E(\lambda_2)-1)$. Lowering $E(\lambda_2)$ will thus again decrease incentives to invest in fiscal and legal capacity. Since this decreases $\overline{b}(\tau_2,\pi_2)=\frac{\tau_2\underline{\omega}(\pi_2)}{1+\rho}$, we have one force that pulls debt down. Specifically, for a fixed level $db$ of debt which is raised *above* $\overline{b}$, the total amount of debt, $\overline{b}+db$, will go down. The low income part of future public funds decreases and less debt is needed to draw it to the present. This effect was the only effect at work in case c) and it was responsible for debt and state capacity moving in the same direction. We call this effect the low income effect.

In case d), we get a second effect. Specifically, from the first order condition for debt in case d) we can see the following. If $E(\lambda_2)$ decreases, $\Delta$ will adjust upwards such that the first order condition holds again with equality. That is, the amount $db$ of debt which is raised *above* $\overline{b}$ will increase and trigger a higher penalty. Given, for instance, an increasing level of political instability, the danger of getting mistreated by a rival government in the future gets bigger. Therefore, it becomes optimal to raise more debt and avoid the rival redistribution for a higher proportion of expected future tax resources. In particular, for a higher proportion of fiscal capacity, the high income part is now also drawn to the present.[30] The higher penalty implied by this is justified

[30]Note, however, that as long as we are in case d), the high income part is not drawn to the present

because the alternative of leaving money to the future also gets more expensive. We call this the high income effect.

The overall effect on the level of debt is therefore ambiguous. However, if the high income effect dominates the low income effect, we get an increase in debt if $E(\lambda_2)$ decreases, as indicated by the dashed line in the third panel of Figure 4.1. In region d), debt and state capacities can therefore start to move in opposite directions in response to exogenous parameter changes.[31]

Decreasing the value of future public funds, $E(\lambda_2)$, further, we finally enter case e). In this case, debt is raised maximally, so all future public funds are drawn to the present. However, this comes at the price of a high penalty. Despite the high penalty, these funds are drawn to the present due to the very low value of future public funds, since $E(\lambda_2) = \lambda_1 - \frac{(1-\psi)}{\psi} \frac{\partial P(\Delta)}{\partial \Delta}\Big|_{b=\bar{\bar{b}}}$ when we enter case e). If $E(\lambda_2)$ drops further, investment incentives stay constant at the level implied by $\lambda_1 - \frac{(1-\psi)}{\psi} \frac{\partial P(\Delta)}{\partial \Delta}\Big|_{b=\bar{\bar{b}}}$.

Even in case e), through the influence of the low income part, we still get higher investment incentives than in a world without debt under realistic restrictions on the income levels. However, investment incentives are lower than in a no-default world. Especially case d) and e) can reestablish a weak state situation similar to Besley and Persson (2011), where we get very low investments in fiscal and legal capacity.[32] These low investments arise from a low value of future public funds, $E(\lambda_2)$. Given the possibility to raise debt and letting $E(\lambda_2)$ be low enough that debt is chosen, $E(\lambda_2)$ influences investment incentives only through the high income part. Therefore, state capacity investments will be the lower, the higher the influence of the high income part, in the sense that the difference between the income levels, $\overline{\omega}(\cdot) - \underline{\omega}(\cdot)$, is bigger while holding the expected income, $\psi\bar{\omega}(\cdot) + (1-\psi)\underline{\omega}(\cdot)$, constant.[33]

for the *marginal* investment unit of fiscal capacity. That is why investment incentives are still driven partly by $E(\lambda_2)$.

[31]Note that in case d), the absolute level of debt could also decrease if $E(\lambda_2)$ decreases. However, this is rather an extreme scenario since this implies that $\bar{\bar{b}}$ in case e) would be lower than $\bar{b}$ at the border between case c) and d), which requires a very steep decrease of state capacities within case d). In any case, decreasing $E(\lambda_2)$ in case d) will always lead to a decrease in the level of free future revenues, that is of expected future public funds minus debt.

[32]Note that in Besley and Persson (2011), a weak state did not invest *at all* in fiscal capacity.

[33]The easiest way to see this is to look at the FOC for fiscal capacity in case e), which equals the FOC of case d) for $E(\lambda_2) = \lambda_1 - \frac{(1-\psi)}{\psi} \frac{\partial P(\Delta)}{\partial \Delta}\Big|_{b=\bar{\bar{b}}}$. If $\overline{\omega}(\cdot) - \underline{\omega}(\cdot)$ increases while $\psi\bar{\omega}(\cdot) + (1-\psi)\underline{\omega}(\cdot)$ remains constant, the LHS clearly decreases since $\Delta$ increases. So, $\tau_2$ decreases. For very high $\overline{\omega}(\cdot) - \underline{\omega}(\cdot)$, $\tau_2$ decreases until $\tau_2 = \tau_1$ is reached, which corresponds to a state which does not invest at all in fiscal capacity like a weak state in Besley and Persson (2011).

Furthermore, the situation of low investments in state capacity is now worsened by a high debt level. Debt is bad here because in the political equilibrium, $\lambda_1 = 2(1-\theta) > \alpha_L$ holds. Therefore, in case e), debt is used to fully tie down future public funds for clientele politics today. A social planner, on the other hand, would not draw future public funds to the present but would use them instead for public good expenditures in the future.

Note that the weak state situation in the preceding analysis arises for similar parameter values as in a model without debt. In particular, recall that the above analysis was done for countries with low enough cohesiveness. Furthermore, a low value of $E(\lambda_2)$ leading to case e) can be driven by high political instability $\gamma$. Low cohesiveness and high political instability constitute the parameter constellation that defines a weak state in the basic model without debt (Besley and Persson, 2011).[34] The mechanism at work has now already been highlighted several times: When the incumbent government is afraid that future public funds will be used against its interest, it will not invest in the additional creation of these public funds. We call this the low-investment-mechanism.

However, the preceding analysis also cautions that the low-investment-mechanism is only partly at work in the default setup. Specifically, we have just argued that the size of the difference between low and high income realizations plays a crucial role. Note in particular that, as this difference goes to zero, we move back to the debt model without default from Section 4.3. For this model, investment incentives were driven by the present value of public funds as soon as the future value dropped low enough. Therefore, the low-investment-mechanism was completely broken by the mechanism of using debt to bring future public funds at the disposal of the present. We call the latter the debt-mechanism.

The above analysis presents a first step in analyzing the relative strength of these two mechanisms. Specifically, we have highlighted how the two effects interact in a specific framework. 'Crisis countries' with very high public debt and low fiscal and legal capacity can arise. In our model, these are countries with low cohesiveness $\theta$, a low value of future public funds, $E(\lambda_2)$, and high enough income fluctuations. A low value of $E(\lambda_2)$ could be due, for instance, to high political instability $\gamma$. Since

[34]To be more precise, recall from the discussion of the stability condition in Section 4.4.2 that the probability $\phi$ for a high value of public good spending also has to be low enough.

$E(\lambda_2) < \lambda_1$ defines cases c)-e), only for sufficiently low cohesiveness we can ever end up in the high debt - low state capacity situation of case e). Therefore, the lower is cohesiveness, the more likely (i.e. for a larger range of parameter values) a country will end up in such a situation.

From this analysis, we can already draw some policy implications. A crucial factor that keeps a country from running into a debt trap, that is a situation with very high debt and low state capacity, seems to be a sufficient level of cohesiveness. High cohesiveness entails provisions in a country's constitution and other institutional features which prohibit clientele politics by the political group in power. In a politically unstable political environment, non-cohesive countries, in which it is easy to benefit your preferred clientele, will end up in the problematic situation of a debt trap. To avoid to get back in such a situation, a reform of cohesiveness seems to be beneficial. This necessitates deep reforms that go at the core of the functioning of the state. Examples of such reforms include implementing a functioning system of checks and balances, establishing an independent press that names and shames clientele politics, establishing provisions in the constitution that prevent clientele politics, or strengthening the constitutional court in its power to enforce such provisions. These reforms go deeper than the usually discussed economic reforms. Our analysis shows that in order to make such economic reform efforts sustainable, they have to be preceded by these institutional reforms. Otherwise, political incentives will constantly endanger an enduring success of economic reforms.

Note that the clientele politics here do not correspond to the targeting of electoral favors during an electoral campaign, which has been analyzed in the previous two chapters. Any democracy will be characterized by some form of such an allocation of electoral favors to the majority of voters. The clientele politics in this chapter correspond rather to a form of rent extraction for the benefit of the own clientele once politicians are in power. In particular, in this chapter, we do not consider a feedback of politicians' decisions on the probability of being in power. We do not even take a stand on whether a change in political power is determined by a democratic election. As in Besley and Persson (2011), such a change could be brought about by some kind of civil war. Therefore, the results here do not contradict the result from the previous two chapters that targeted pork-barrel spending might be necessary in electoral campaigns to secure the success of growth-enhancing reforms. Rather, the above results point to the negative effect of clientele politics that take the form of rent extraction and that are unrelated to sustaining the support of a majority of voters.

### 4.5.3 Cross-Country Correlations

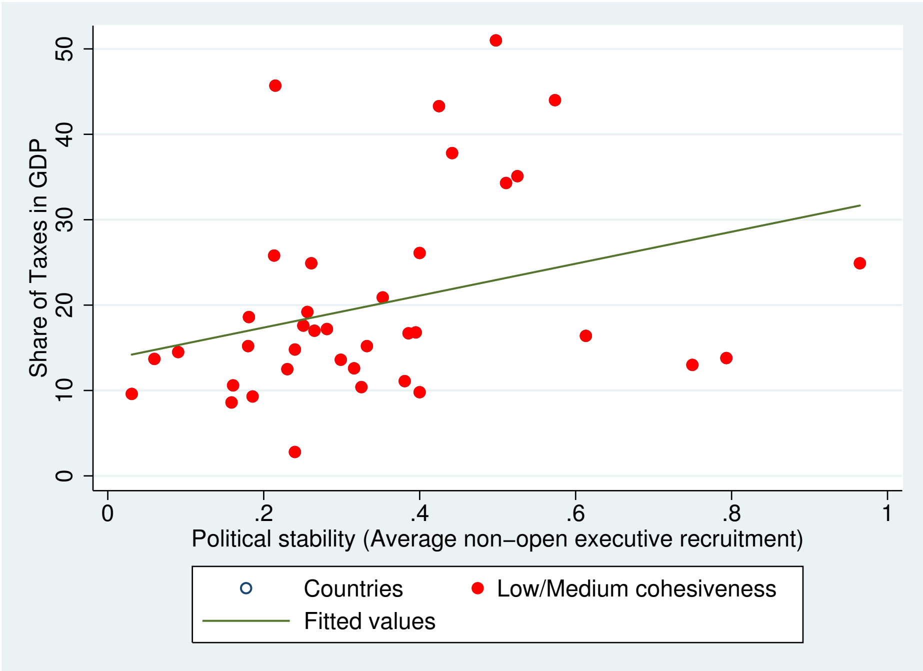


Figure 4.2

In the following, we take a short look at cross-country correlations. As in Besley and Persson (2011), this subsection is therefore not intended as a convincing test of the model's predictions. The aim is just to illustrate the theory and to see whether the presented correlations are somehow in line with results from the model. In more detail, we like to mimic Figure 4.1 with real world data. Data for state capacities and estimates for model parameters are taken from Besley and Persson (2011) and debt data is taken from Reinhart and Rogoff (2009). Joining the two datasets results in a sample of 57 countries. In Figure 4.2 we plot fiscal capacity, measured as the share of taxes in GDP in 1999 against political stability, measured as the index used by Besley and Persson (2011).[35,36] Since Figure 4.1 was derived under the assumption

[35]Detailed variable descriptions are provided in the appendix.

[36]Even though Figures 4.2, 4.3, C.1, and C.2 use only data of Besley and Persson (2011), these figures are interesting since Besley and Persson (2011) do not plot the data in this dimension, i.e. state capacities conditional on political stability. Due to the theoretical results from our model with debt as summarized by Figure 4.1, these plots are desirable here.

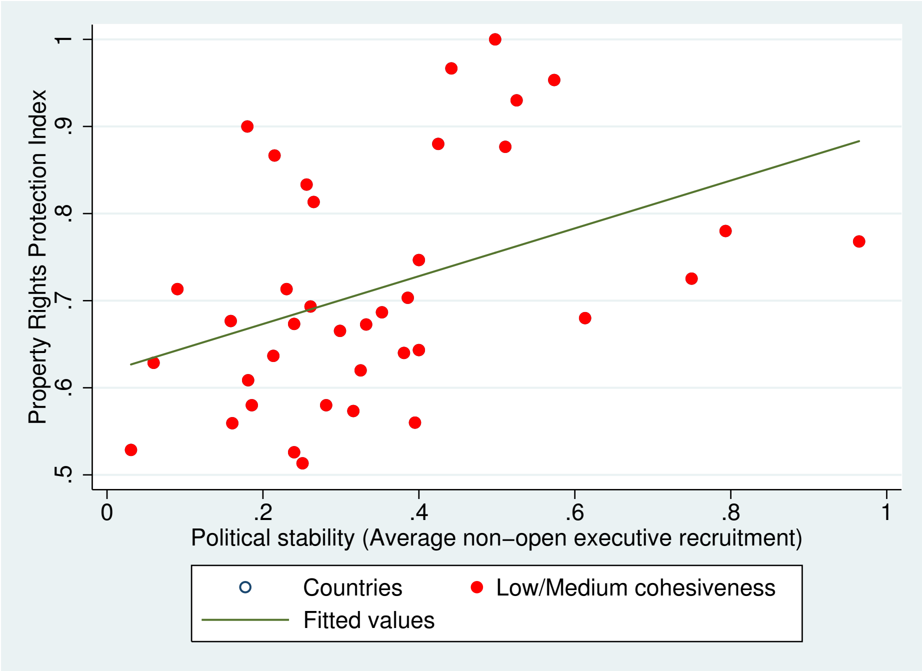


Figure 4.3

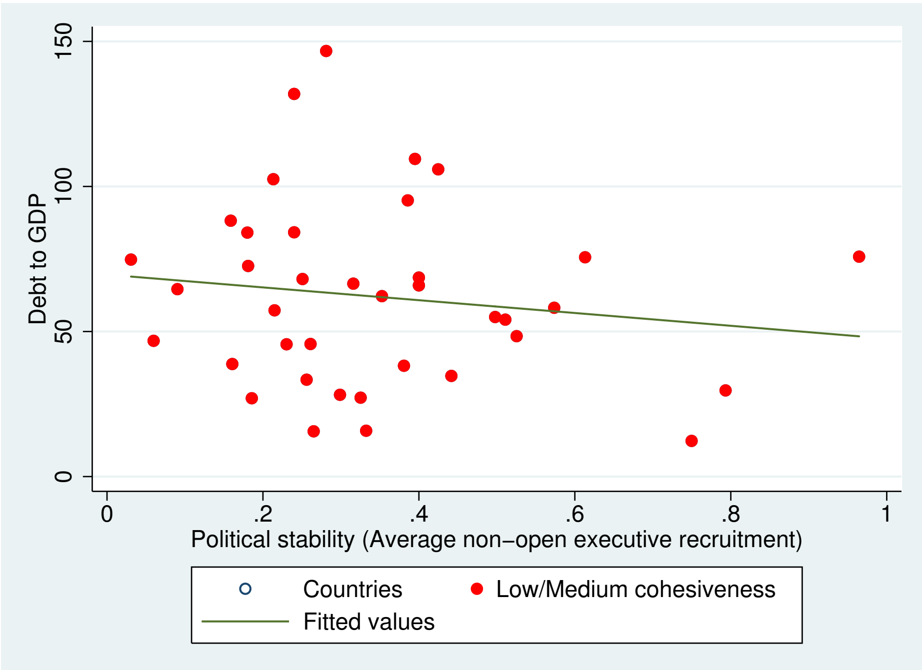


Figure 4.4

of low enough cohesiveness, Figure 4.2 focuses on countries with cohesiveness below the 66th percentile. Figure C.1 in Appendix C.3.1 presents the corresponding scatter plot including also highly cohesive countries. Cohesiveness is measured as the average executive constraints from 1800 to 2000. The theory predicts that fiscal capacity should increase with $E(\lambda_2)$. Since for non-cohesive countries, $E(\lambda_2)$ is increasing in political stability, the model predicts to observe a positive correlation between political stability and fiscal capacity for non-cohesive countries. Excluding highly cohesive countries, we find a positive correlation (+0.3200) as shown in Figure 4.2 which is statistically significant (p-value 0.0502). We also find this significant positive correlation using other measures of fiscal capacity, like the share of the non-shadow economy.

Turning to legal capacity and following the same reasoning as above, the model predicts to observe a positive correlation between political stability and legal capacity for non-cohesive countries. Figure 4.3 provides cross-country evidence, where we use the property rights protection index from Besley and Persson (2011) as a measure for legal capacity. Excluding highly cohesive countries, we find a positive correlation (+0.4115) which is statistically significant (p-value 0.0103).[37]

Finally, as the part of Figure 4.1 regarding debt illustrates, it is ultimately an empirical question whether we observe a positive or negative correlation between political stability and debt for non-cohesive countries. As Figure 4.1 is drawn now, for part $d$, debt is negatively correlated to stability. Hence, the incentive of raising more debt in order to draw more public funds to the present in the high income case dominates the effect of having to draw less public funds in the low income case. If the high income effect dominates the low income effect, we therefore expect an overall negative correlation between political stability and debt for non-cohesive countries. If the relative strengths of the two mechanisms is reversed, we expect an overall positive correlation. Figure 4.4 and Figure C.3 in Appendix C.3.1 show the corresponding cross-country data. For medium and low cohesive countries we observe a slight negative correlation (-0.1405), however, not statistically significant (p-value 0.4003). Therefore, the crude cross-country data used here are not enough to determine which of the above mentioned theoretical effects dominates in terms of the incentive to raise debt under different values of political stability.

A comprehensive empirical analysis to test the theory convincingly remains a challenging task for future research. As Besley and Persson (2011) point out, this would require data and empirical strategies that credibly isolate exogenous variations in the underlying determinants that drive the combined evolution of state capacities. What

[37] Again, the corresponding scatter plot including highly cohesive countries can be found as Figure C.2 in Appendix C.3.1.

we have shown here is which additional interactions between state capacities and debt these determinants imply. Furthermore, we have highlighted new determinants, like the size of income fluctuations that impact on these interactions.

## 4.6 Conclusion

This paper presented an integrated analytical framework for analyzing the interaction between public debt and state capacity, the power of a state to raise taxes and to provide market supporting policies. We showed that the possibility to raise debt can provide a novel incentive to invest in state capacity, because debt allows to bring future state capacity at the disposal of the current government. As long as debt can be used to protect the current government from an adverse use of future public funds, it is no longer necessary to use low investments in state capacity as a protection device.

However, we also showed how this novel mechanism can be weakened in a world with income fluctuations and the possibility of default. When high costs of raising debt make it very expensive to draw all relevant future public funds to the present, the mechanism of lowering investments resurfaces. Specifically, this mechanism is more prominent for high income fluctuations, because they increase the proportion of public funds that can only be drawn to the present at high costs. For such an environment, we get results that are closest to the original no-debt model by Besley and Persson (2011). In particular, an unstable political environment combined with insufficient institutional provisions to prevent clientele politics can then lead to a situation of low state capacity. Furthermore, this weak state situation is now worsened by a high built-up of debt, increasing the probability of sovereign default.

Our model leaves room for several generalizations which should be investigated by future research. First, to qualify the model results in light of the tax smoothing literature, it would be interesting to allow for distortionary taxation. Second, the modeling of default could be extended to a full-fledged model of willingness-to-pay default. This would certainly allow to uncover additional interesting channels that shape the interaction between debt and state capacity. Third, in light of this, it could make sense to extend the model to more than two periods.

Finally, as highlighted in the last section, a comprehensive empirical test of the model's predictions is a challenging, but promising path for future research. Such an analysis will allow to quantify the relative strengths of the mechanisms we have identified as driving forces behind the combined evolution of public debt and investment in state capacities. As our analysis shows, it is important to gauge the size of these effects correctly in order to determine whether the incentivizing effect of public debt on state building is dominant or whether a country will only add the burden of high debt to low investments in its fiscal and legal capacities.

# Appendix A

# Appendix to Chapter 2

## A.1 Constitutional limit on debt: equilibrium characterization

The following corollaries state the complete characterization of the results of Proposition 2.6.1 and Proposition 2.6.2.

**Corollary A.1.1** *Suppose the reform is purely targetable (i.e. $\lambda = 1$) and assume there is an exogenous debt limit that is higher than the natural debt limit under no-reform: $\bar{\bar{\delta}} = 1 + \bar{\rho} \geq 1$.*

***(I.) Non-restrictive debt limit:*** *When the debt limit $\bar{\rho}$ is such that $e - \bar{\rho} > 2(c - \bar{\rho})$, in the unique equilibrium*

*(i) both candidates reform with probability $\beta^A = \beta^B = 1$;*

*(ii) announce the maximal debt: $\delta^A = \delta^B = 1 + \bar{\rho}$;*

*(iii) first-period offers to voters are drawn from a uniform distribution on $[-1, 3 + 2(\bar{\rho} - c)]$. That is both candidates draw first period offers from the following distribution:*

$$F_R^*(x) = \begin{cases} 0, & \textit{if } x \leq -1, \\ \frac{x+1}{2+2(1+\bar{\rho}-c)}, & \textit{if } -1 \leq x \leq 3 + 2(\bar{\rho} - c), \\ 1, & \textit{if } x \geq 3 + 2(\bar{\rho} - c). \end{cases} \tag{A.1}$$

*Second-period transfer offers are drawn from a uniform distribution on* $[-1, -1 + 2(e - \bar{\rho})]$.

***(II.) Restrictive debt limit:*** *When the debt limit* $\bar{\rho}$ *is restrictive enough such that* $\widehat{H} = 2(c - \bar{\rho}) - (e - \bar{\rho}) > 0$, *in the unique equilibrium*

*(i) both candidates reform with probability* $\beta^A = \beta^B = 1 - \frac{1}{2}\widehat{H} < 1$;

*(ii) both candidates announce the maximal debt:* $\delta_N^i = 1$ *in case of no-reform,* $\delta_R^i = 1 + \bar{\rho}$ *in case of reform,* $i \in \{A, B\}$.

*(iii) When candidates do not reform, they draw first-period offers from the following distribution:*

$$F_N^*(x) = \begin{cases} 0, & \text{if } x \leq -1, \\ \frac{1}{2}\left(\frac{x+1}{\widehat{H}}\right), & \text{if } -1 \leq x \leq -1 + \widehat{H}, \\ \frac{1}{2}, & \text{if } -1 + \widehat{H} \leq x \leq 3 - \widehat{H}, \\ \frac{1}{2}\left(1 + \frac{x-3+\widehat{H}}{\widehat{H}}\right), & \text{if } 3 - \widehat{H} \leq x \leq 3, \\ 1, & \text{if } x \geq 3. \end{cases} \tag{A.2}$$

*Second period offers are degenerate on -1.*

*When candidates reform, they draw first-period offers from the following distribution:*

$$F_R^*(x) = \begin{cases} 0, & \text{if } x \leq -1, \\ \frac{x+1}{4-2(c-\bar{\rho})}, & \text{if } -1 \leq x \leq 3 - 2(c - \bar{\rho}), \\ 1, & \text{if } x \geq 3 - 2(c - \bar{\rho}). \end{cases} \tag{A.3}$$

*Second-period transfer offers are drawn from a uniform distribution on* $[-1, -1 + 2(e - \bar{\rho})]$.

**Corollary A.1.2** *Assume the exogenous debt limit is more restrictive than the natural debt limit under no-reform:* $\bar{\delta} = 1 - \bar{\sigma} < 1$.

***(I.) High benefit of reform:*** *When the reform benefits e are high enough compared to the reform costs c such that* $e > 2c$*, in the unique equilibrium*

*(i) both candidates reform with probability* $\beta^A = \beta^B = 1$*;*

*(ii) both candidates announce the maximal debt:* $\delta^A = \delta^B = \bar{\delta} = 1 - \bar{\sigma}$*;*

*(iii) first-period offers to voters are drawn from a uniform distribution on* $[-1, 3 - 2\bar{\sigma} - 2c]$*. That is both candidates draw first period offers from the following distribution:*

$$F_R^*(x) = \begin{cases} 0, & \text{if } x \leq -1, \\ \frac{x+1}{2(2-\bar{\sigma}-c)}, & \text{if } -1 \leq x \leq 3 - 2\bar{\sigma} - 2c, \\ 1, & \text{if } x \geq 3 - 2\bar{\sigma} - 2c. \end{cases} \tag{A.4}$$

*Second-period transfer offers are drawn from a uniform distribution on* $[-1, -1 + 2e + 2\bar{\sigma}]$.

***(II.) Low benefit of reform:*** *When the reform benefits e are low enough compared to the reform costs c such that* $e < 2c$*, in the unique equilibrium*

*(i) both candidates reform with probability* $1 - \frac{1}{2-\bar{\sigma}}\widetilde{H} < 1$*, where* $\widetilde{H} = 2c - e > 0$*;*

*(ii) both candidates announce the maximal debt:* $\delta^A = \delta^B = \bar{\delta} = 1 - \bar{\sigma}$.

*(iii) When candidates do not reform, they draw first-period offers from the following distribution:*

$$F_N^*(x) = \begin{cases} 0, & \text{if } x \leq -1, \\ \frac{1}{2}\left(\frac{x+1}{\widetilde{H}}\right), & \text{if } -1 \leq x \leq -1 + \widetilde{H}, \\ \frac{1}{2}, & \text{if } -1 + \widetilde{H} \leq x \leq 3 - 2\bar{\sigma} - \widetilde{H}, \\ \frac{1}{2}\left(1 + \frac{x-3+\widetilde{H}}{\widetilde{H}}\right), & \text{if } 3 - 2\bar{\sigma} - \widetilde{H} \leq x \leq 3 - 2\bar{\sigma}, \\ 1, & \text{if } x \geq 3 - 2\bar{\sigma}. \end{cases} \tag{A.5}$$

*Second-period transfer offers are drawn from a uniform distribution on* $[-1, -1 + 2\bar{\sigma}]$.

*When candidates reform, they draw first-period offers from the following distribution:*

$$F_R^*(x) = \begin{cases} 0, & \text{if } x \leq -1, \\ \frac{x+1}{2(2-\bar{\sigma}-c)}, & \text{if } -1 \leq x \leq 3 - 2\bar{\sigma} - 2c, \\ 1, & \text{if } x \geq 3 - 2\bar{\sigma} - 2c. \end{cases} \tag{A.6}$$

*Second-period transfer offers are drawn from a uniform distribution on* $[-1, -1 + 2e + 2\bar{\sigma}]$.

## A.2 Proofs

In the following, we prove the propositions in the paper along with the respective corollaries. It turns out that proving proposition 2.6.1 and corollary A.1.1 first is most convenient. The other results then follow through making the correct adjustments. However, in terms of notation, the proofs are easiest to carry out for proposition 2.6.1 and corollary A.1.1.

### A.2.1 Proof of Part (I.) of Proposition 2.6.1 and Corollary A.1.1

The proof consists of three steps: in Step I, we show that in any equilibrium, both candidates must reform with probability 1. In Step II, we show that in any equilibrium, both candidates must raise the maximal debt. In Step III, we characterize the equilibrium distributions.

**Step I:** First, we show that in any equilibrium, both candidates must reform with positive probability. Consider the case where candidate $A$ does not reform, raises any debt $\delta^A \leq 1$ and plays any associated distribution. We show that if candidate $B$

follows the equilibrium strategy where he is doing the reform, he defeats candidate $A$ with probability 1.

The strategy of candidate $B$ consists in doing the reform, running maximal debt, and drawing first period offers to voters from a uniform distribution on $[-1, 3+2(\bar{\rho}-c)]$, and second period offers are drawn from a uniform distribution $[-1, -1+2(e-\bar{\rho})]$.

A voter votes for the candidate that gives him the highest total expected offer. Assume candidate $i$ offers $x^i$ to the voter in the first period and proposes a debt of $\delta^i$. The resulting total expected offer to the voter is the first period offer, $x^i$, plus the amount of transfers, $\mu_2^i$, the voter expects in the second period if candidate $i$ is elected. Given a debt proposal of $\delta^i$, $\mu_2^i = -\delta^i$ in case of no-reform, and $\mu_2^i = e - \delta^i$ in case of reform. Since the outcome of the future redistribution is uncertain, today *each voter* expects $\mu_2^i$ for the second period if candidate $i$ is elected. In terms of total expected offers that a candidate proposes over both periods this means the following. If $\mu_1^i$ is defined as the mean of the first-period offer distribution $F^i(\cdot)$, candidate $i$ is effectively adding a degenerate distribution at $\mu_2^i$ to $F^i(\cdot)$ to obtain the distribution of total expected offers with mean $\mu_1^i + \mu_2^i$. Define $\hat{F}^i(\cdot)$ to be this distribution of total expected offers. A voter will vote for candidate $A$ if $A$ gives a higher total expected offer than $B$. Candidate $A$'s share of vote is equal to the probability that any random voter receives a total expected offer from candidate $B$ which is lower than the offer he receives from $A$:

$$S^A = \int_{-1-\delta^A}^{+\infty} \hat{F}^B(x) d\hat{F}^A(x).$$

Since candidate $B$ plays the equilibrium strategy, we obtain $\hat{F}^B$ by adding the equilibrium first period offer distribution (A.1) of corollary A.1.1 to the distribution degenerate at $\mu_2^B = e - \delta^B = e - (1+\bar{\rho}) = -1 + (e-\bar{\rho})$. The ex ante total expected offers that voters get from candidate $B$ are thus drawn from the following distribution:

$$\hat{F}^B(x) = \begin{cases} 0, & \text{if } x \le -2 + (e-\bar{\rho}), \\ \frac{x+2-(e-\bar{\rho})}{2+2(1+\bar{\rho}-c)}, & \text{if } -2+(e-\bar{\rho}) \le x \le 2(1+\bar{\rho}-c) + (e-\bar{\rho}), \\ 1, & \text{if } x \ge 2(1+\bar{\rho}-c) + (e-\bar{\rho}). \end{cases}$$

Since candidate $A$ does not reform, her budget constraint for the total expected offers becomes $\int_{-1-\delta^A}^{+\infty} x d\hat{F}^A(x) = 0$.

Suppose $-1-\delta^A \geq -2+(e-\bar{\rho})$, then

$$\begin{aligned}
S^A =& \int_{-1-\delta^A}^{+\infty} \hat{F}^B(x) d\hat{F}^A(x) \leq \int_{-1-\delta^A}^{+\infty} \frac{x+2-(e-\bar{\rho})}{2+2(1+\bar{\rho}-c)} d\hat{F}^A(x) \\
=& \frac{2-(e-\bar{\rho})}{2+2(1+\bar{\rho}-c)} < \frac{1}{2},
\end{aligned}$$

since $e > c$ by assumption (2.1).

Suppose $-1-\delta^A < -2+(e-\bar{\rho})$, then

$$\begin{aligned}
S^A =& \int_{-1-\delta^A}^{+\infty} \hat{F}^B(x) d\hat{F}^A(x) \leq \int_{-2+(e-\bar{\rho})}^{+\infty} \frac{x+2-(e-\bar{\rho})}{2+2(1+\bar{\rho}-c)} d\hat{F}^A(x) \\
=& \frac{1}{2+2(1+\bar{\rho}-c)} \left[ \int_{-1-\delta^A}^{+\infty} x+2-(e-\bar{\rho}) d\hat{F}^A(x) \right. \\
& \left. - \int_{-1-\delta^A}^{-2+(e-\bar{\rho})} x+2-(e-\bar{\rho}) d\hat{F}^A(x) \right] \\
=& \frac{1}{2+2(1+\bar{\rho}-c)} \left[ 2-(e-\bar{\rho}) - \int_{-1-\delta^A}^{-2+(e-\bar{\rho})} x+2-(e-\bar{\rho}) d\hat{F}^A(x) \right],
\end{aligned}$$

where $-\int_{-1-\delta^A}^{-2+(e-\bar{\rho})} x+2-(e-\bar{\rho})d\hat{F}^A(x)$ is a positive term that is maximized for $\delta^A = 1$ and by offering $-2$ to half of voters, the other half of voters getting strictly more that $-2+(e-\bar{\rho})$ so that $\hat{F}^A(-2+(e-\bar{\rho})) = \frac{1}{2}$.[1] Hence,

$$\begin{aligned}
S^A \leq& \frac{1}{2+2(1+\bar{\rho}-c)} \left[ 2-(e-\bar{\rho}) - \int_{-1-\delta^A}^{-2+(e-\bar{\rho})} x+2-(e-\bar{\rho}) d\hat{F}^A(x) \right] \\
\leq& \frac{1}{2+2(1+\bar{\rho}-c)} \left[ 2-(e-\bar{\rho}) + \frac{1}{2}[e-\bar{\rho}] \right] < \frac{1}{2},
\end{aligned}$$

The last in equality is equivalent to the condition $e-\bar{\rho} > 2(c-\bar{\rho})$, which was assumed for Part (I.) of Proposition 2.6.1 and Corollary A.1.1. Therefore, a candidate that plays reform with zero probability can be beaten for sure. In any equilibrium strategy, reform must therefore be played with positive probability.

We now show that the reform must be played with probability 1 in any equilibrium. Assume therefore that candidate $A$ reforms with positive probability $\beta^A < 1$, and for this case of reform raises any debt $\delta_R^A \leq 1+\bar{\rho}$ and plays any associated distribution. Similarly he does not reform with probability $1-\beta^A$, and for this case of no-reform raises any debt $\delta_N^A \leq 1$ and plays any associated distribution. Candidate $B$ follows the same strategy as above. Then by the above analysis candidate $B$ wins for sure if $A$ does not reform. Furthermore, it can be shown that the vote share of candidate $B$ is equal to

[1] Offering $-2$ to more than half of the voters would result in a vote share $S^A < 1/2$ anyway and is therefore excluded for the above derivation.

$\frac{1}{2}$ if $A$ reforms and plays any debt $\delta_R^A \leq 1+\bar{\rho}$ and any possible distribution. Therefore, candidate $B$'s total probability of winning is $(1-\beta)+\beta \cdot \frac{1}{2} > \frac{1}{2}$. This cannot happen in equilibrium, where both candidates should win with equal probability. Hence, in any equilibrium both candidates must reform with probability 1.

**Step II:** We follow Lizzeri (1999) and show that if both candidates reform with probability 1 and candidate $A$ does not run the maximal debt, in the sense that $\delta^A < 1+\bar{\rho}$, then candidate $A$ can be beaten for sure.[2] Define $\rho^i = \delta^i - 1$ as the amount by which the debt raised by candidate $i$ is higher than the endowment 1. Note that $\rho^A < \rho^B = \bar{\rho}$. Candidate $B$ can beat candidate $A$ for sure by choosing the maximal debt $\delta^B = 1+\bar{\rho}$ and the following first period distribution plan:

$$F^B(x) = \begin{cases} 0, & \text{if } x \leq -1, \\ \frac{1+\bar{\rho}-(1+\rho^A)}{5+3\bar{\rho}-2c-(1+\rho^A)}, & \text{if } -1 \leq x \leq -1+\bar{\rho}-\rho^A, \\ \frac{1+\bar{\rho}-(1+\rho^A)}{5+3\bar{\rho}-2c-(1+\rho^A)} & \text{if } -1+\bar{\rho}-\rho^A \leq x \\ +\frac{2(2+\bar{\rho}-c)(x+1-(\bar{\rho}-\rho^A))}{(3+\bar{\rho}-2c+(1+\rho^A))(5+3\bar{\rho}-2c-(1+\rho^A))}, & \quad \leq 3+2(\bar{\rho}-c), \\ 1, & \text{if } x \geq 3+2(\bar{\rho}-c). \end{cases} \tag{A.7}$$

Given that candidate $B$ chooses the maximal debt, $\rho^B = \bar{\rho}$, from equation (A.7) we get

$$\hat{F}^B(x) = \begin{cases} 0, & \text{if } x \leq -2+(e-\bar{\rho}), \\ \frac{1+\bar{\rho}-(1+\rho^A)}{5+3\bar{\rho}-2c-(1+\rho^A)}, & \text{if } -2+(e-\bar{\rho}) \leq x \leq -2+(e-\rho^A), \\ \frac{1+\bar{\rho}-(1+\rho^A)}{5+3\bar{\rho}-2c-(1+\rho^A)} & \text{if } -2+(e-\rho^A) \leq x \\ +\frac{2(2+\bar{\rho}-c)(x+2-(e-\rho^A))}{(3+\bar{\rho}-2c+(1+\rho^A))(5+3\bar{\rho}-2c-(1+\rho^A))}, & \quad \leq 2(1+\bar{\rho}-c)+(e-\bar{\rho}), \\ 1, & \text{if } x \geq 2(1+\bar{\rho}-c)+(e-\bar{\rho}). \end{cases} \tag{A.8}$$

Note that candidate $A$ will never offer more than the upper bound of candidate $B$'s distribution, $2(1+\bar{\rho}-c)+(e-\bar{\rho})$. Offering exactly this upper bound to a voter is enough

[2] We drop the reform subscript $R$ here, since both candidates reform with probability one.

to get the vote for sure since candidate $B$ is offering less than $2(1+\bar{\rho}-c)+(e-\bar{\rho})$ with probability 1.

The share of the votes of candidate $A$ is given by:

$$\begin{aligned}
S^A &= \int_{-2+(e-\rho^A)}^{2(1+\bar{\rho}-c)+(e-\bar{\rho})} \hat{F}^B(x) d\hat{F}^A(x) \\
&= \int_{-2+(e-\rho^A)}^{2(1+\bar{\rho}-c)+(e-\bar{\rho})} \frac{1+\bar{\rho}-(1+\rho^A)}{5+3\bar{\rho}-2c-(1+\rho^A)} \\
&\quad + \frac{2(2+\bar{\rho}-c)(x+2-(e-\rho^A))}{(3+\bar{\rho}-2c+(1+\rho^A))(5+3\bar{\rho}-2c-(1+\rho^A))} d\hat{F}^A(x) \\
&= \frac{8-(\bar{\rho}-\rho^A)^2+2(\bar{\rho}-c)(4+\bar{\rho}-c)}{(3+\bar{\rho}-2c+(1+\rho^A))(5+3\bar{\rho}-2c-(1+\rho^A))}.
\end{aligned}$$

To obtain this expression we used distribution (A.8), and the fact that, by the budget constraint for the reform option, $\int_{-2+(e-\rho^A)}^{2(1+\bar{\rho}-c)+(e-\bar{\rho})} x d\hat{F}^A(x) = e-c$.

$S^A$ achieves a maximum of $\frac{1}{2}$ for $\rho^A = \bar{\rho}$ and is strictly less than $\frac{1}{2}$ for $\rho^A < \bar{\rho}$. This can be seen by taking the derivative of $S^A$ with respect to $\rho^A$

$$\frac{\partial S^A}{\partial \rho^A} = \frac{4(2+\bar{\rho}-c)(\bar{\rho}-\rho^A)}{(3+\bar{\rho}-2c+(1+\rho^A))^2(5+3\bar{\rho}-2c-(1+\rho^A))^2}$$

the sign of which is determined by $(\bar{\rho}-\rho^A)$ since $c < 1$ by assumption.

Therefore, if candidate $A$ chooses less than the maximal debt, she is beaten for sure. This implies that in any equilibrium both candidates must run the maximal debt.

**Step III:** We have shown that in equilibrium both candidates reform and raise the maximal debt level $1+\bar{\rho}$. The latter also corresponds to the per-capita resources that are additionally available for first-period transfer offers. Therefore, we are back to a divide-the-dollar game with an exactly specified amount of resources to divide. We can therefore apply Myerson (1993) and Lizzeri (1999), and construct the first-period offer distribution analogously to the second-period offer distribution in section 2.4. Using the first-period budget constraint (2.4) for the case of reform, we can calculate the upper bound of the distribution as in section 2.4. In total we then find that both candidates will draw first period offers to voters from a uniform distribution on $[-1, 3+2(\bar{\rho}-c)]$. That this is the unique equilibrium in such a divide-the-dollar game has been established by Myerson (1993) for symmetric equilibria and by Lizzeri (1999) for the general case.

Furthermore, the amount of resources available after debt repayment in the second period is $e - \bar{\rho}$ and second period offers are drawn from a uniform distribution on $[-1, -1 + 2(e - \bar{\rho})]$. Each candidate then wins with probability $\frac{1}{2}$.

### A.2.2 Proof of Part (II.) of Proposition 2.6.1 and Corollary A.1.1: Existence of equilibrium

In the following, we show that the stated strategies indeed are an equilibrium. The proof of existence of the equilibrium follows similar steps as the proof of Theorem 5 in Lizzeri and Persico (1998), but adjusts for the dynamic setup and the use of public debt.

**Preliminaries.** For the proof we will again work with distributions $\hat{F}^*$ that add to the equilibrium first-period distributions the expected value of transfers that each voter expects in the second period if the maximal debt is raised. The resulting equilibrium distributions of total expected offers are:

$$\hat{F}_N^*(x) = \begin{cases} 0, & \text{if } x \leq -2, \\ \frac{1}{2}\left(\frac{x+2}{H}\right), & \text{if } -2 \leq x \leq -2+H, \\ \frac{1}{2}, & \text{if } -2+H \leq x \leq 2-H, \\ \frac{1}{2}\left(1+\frac{x-2+H}{H}\right), & \text{if } 2-H \leq x \leq 2, \\ 1, & \text{if } x \geq 2; \end{cases} \tag{A.9}$$

and

$$\hat{F}_R^*(x) = \begin{cases} 0, & \text{if } x \leq -2+(e-\bar{\rho}), \\ \frac{x+2-(e-\bar{\rho})}{4-2(c-\bar{\rho})}, & \text{if } -2+(e-\bar{\rho}) \leq x \leq 2-H, \\ 1, & \text{if } x \geq 2-H. \end{cases} \tag{A.10}$$

We will show that these distributions along with the equilibrium probability of reform constitute the unique equilibrium. Since the above distributions combine the equilibrium first-period distributions with running the respective maximal debt level, this proves the optimality of a maximal debt level along the way. The reason is that, without the maximal debt level, the above distributions of total expected offers are infeasible.

**Existence of equilibrium** The proof of existence has three steps: Step I shows that there is no profitable deviation from the equilibrium strategy by deviating from $\hat{F}_N^*$, step II shows that it is also not profitable to deviate on $\hat{F}_R^*$. Finally, step III argues for the optimality of the equilibrium probability of reform.

**Step I: Optimality of $\hat{F}_N^*$.** Consider candidate $A$ when he decides not to reform and assume he deviates from the equilibrium distribution under no-reform, $\hat{F}_N^*$, to another distribution $\hat{F}_N$. When this candidate $A$ meets a candidate $B$ not reforming and offering money according to $\hat{F}_N^*$, the vote share of candidate $A$ is:[3]

$$\begin{aligned} S(\hat{F}_N^*, \hat{F}_N) = & \int_{-2}^{2} \hat{F}_N^*(x) d\hat{F}_N(x) \\ = & \frac{1}{2}\left\{ \frac{M_{LN} + M_{HN}}{H} + \frac{-2+H}{H}\left[\hat{F}_N(2-H) - \hat{F}_N(-2+H)\right] \right. \\ & \left. + \left(\frac{2H-4}{H}\right)\left[1 - \hat{F}_N(2-H)\right] + \frac{2}{H} \right\}, \end{aligned} \tag{A.11}$$

where $H = 2(c - \bar{\rho}) - (e - \bar{\rho})$,

$$M_{LN} = \int_{-2}^{-2+H} x d\hat{F}_N(x), \tag{A.12}$$

and

$$M_{HN} = \int_{2-H}^{2} x d\hat{F}_N(x). \tag{A.13}$$

Equations (A.12) and (A.13) capture the money spent on transfers in the low interval $[-2, -2+H]$ and high interval $(2-H, 2]$, respectively. We choose the high interval to be open to the right, because the continuous equilibrium distribution $\hat{F}_N^*$ puts zero mass on the single offer $2-H$. In order to win a positive mass of voters in the upper interval, the respective offers must therefore be strictly higher than $2-H$.

Candidate $A$ chooses $\hat{F}_N$ under the constraint that

$$M_{LN} + M_{HN} + M_{MN} = 0, \tag{A.14}$$

[3]Without loss of generality, we exclude a mass point at the lower bound $-2$ of the support. Whenever necessary later on, we deal with this case separately.

where

$$M_{MN} = \int_{-2+H}^{2-H} x d\hat{F}_N(x) \tag{A.15}$$

is the money spent on transfer offers in the middle interval $(-2+H, 2-H]$.

In the following, we will argue that when checking for profitable deviations from the equilibrium no-reform distribution $\hat{F}_N^*$, we can concentrate on distributions $\hat{F}_N$ that have no offers in the middle interval. We will do so by showing that if $\hat{F}_N$ has a positive mass of offers in the middle interval which are not concentrated at the upper bound $2-H$ of this interval, then the vote share can be increased by shifting that mass up towards $2-H$. As a last step, it can then be shown that if a distribution only has offers in the middle interval concentrated as a mass point at $2-H$, then such a distribution can always be approximated by another distribution $\hat{F}_N'$ that has no offers in the middle interval and achieves the same vote share against the equilibrium strategy. This will establish that, when looking for deviations from the no-reform distribution $\hat{F}_N^*$, it is enough to focus on distributions $\hat{F}_N$ that have no offers in the middle interval.

Therefore, assume that $\hat{F}_N$ is a best response to the equilibrium strategy and spends a positive amount on offers in the interval $(-2+H, 2-H]$. First, it is easy to see that candidate $A$ will not make offers in the interval $(-2+H, -2+(e-\bar{\rho})]$. With such offers, candidate $A$ cannot win additional votes against $\hat{F}_N^*$, because $\hat{F}_N^*$ contains no offers in the middle interval. Furthermore, $-2+(e-\bar{\rho})$ is the lowest offer that the equilibrium distribution in case of reform, $\hat{F}_R^*$, contains. Therefore in order to win additional votes, candidate $A$ must provide definitely more than $-2+(e-\bar{\rho})$. In the following, we therefore refer to $(-2+(e-\bar{\rho}), 2-H]$ as the middle interval.

We will now argue that if $\hat{F}_N$ contains offers in the interval $(-2+(e-\bar{\rho}), 2-H)$, then $\hat{F}_N$ is actually not a best response to the equilibrium strategy. This is less straightforward to argue, because offers in this interval are made to some voters under the equilibrium reform distribution $\hat{F}_R^*$. Suppose hence that $\hat{F}_N$ is a best response to the equilibrium strategy and spends a positive amount on offers in the interval $(-2+(e-\bar{\rho}), 2-H)$. Then we can arrive at a contradiction by constructing a profitable deviation $\widetilde{F_N}$.

In particular, note that if $\hat{F}_N$ has only offers in the middle interval $(-2+(e-\bar{\rho}), 2-$

$H]$, then it will tie against the equilibrium no-reform distribution $\hat{F}_N^*$ and it will lose against the equilibrium reform distribution $\hat{F}_R^*$. Therefore, $\hat{F}_N$ must have offers in the low or high interval, $max\{|M_{LN}|, M_{HN}\} > 0$. We can then construct a deviation from $\hat{F}_N$ to $\widetilde{F_N}$ such that $\widetilde{F_N}(-2+(e-\bar{\rho})) = \hat{F}_N(-2+(e-\bar{\rho}))$ and $\widetilde{F_N}(2-H) = \hat{F}_N(2-H)$, but $\widetilde{M_{MN}} > M_{MN}$.[4] We now show that the expected vote share increases when using this deviation $\widetilde{F_N}$. When candidate $B$, who plays the equilibrium strategy, chooses not to reform, then for candidate $A$ a deviation from $\hat{F}_N$ to $\widetilde{F_N}$ is detrimental. As we can see from (A.11) combined with the budget constraint (A.14), increasing $M_{MN}$ to $\widetilde{M_{MN}}$, decreases candidate $A$'s vote share by $\frac{1}{2}\frac{\widetilde{M_{MN}}-M_{MN}}{H}$.

When candidate $B$ chooses to reform, then candidate $A$'s vote share is

$$\begin{aligned}
S(\hat{F}_R^*, \hat{F}_N) =& 1 - \hat{F}_N(2-H) + \int_{-2+(e-\bar{\rho})}^{2-H} \frac{x+2-(e-\bar{\rho})}{4-2(c-\bar{\rho})} d\hat{F}_N(x) \\
=& \frac{M_{MN}}{4-2(c-\bar{\rho})} + \frac{2-(e-\bar{\rho})}{4-2(c-\bar{\rho})}[\hat{F}_N(2-H) - \hat{F}_N(-2+(e-\bar{\rho}))] \\
&+ 1 - \hat{F}_N(2-H).
\end{aligned}$$

By the first term, a deviation of candidate $A$ from $\hat{F}_N$ to $\widetilde{F_N}$ increases his vote share by $\frac{\widetilde{M_{MN}}-M_{MN}}{4-2(c-\bar{\rho})}$.

In total, it is beneficial to increase $M_{MN}$ to $\widetilde{M_{MN}}$ if and only if

$$(1-\beta)\frac{1}{2}\frac{1}{H} < \beta\frac{1}{4-2(c-\bar{\rho})}. \tag{A.16}$$

Recall that the equilibrium probability of reform was $\beta = 1 - \frac{1}{2}H$. With this the above equation is equivalent to $e > c$, which holds by assumption (2.1).

We have therefore shown that a best response $\hat{F}_N$ to the equilibrium strategy cannot have offers in the middle interval expect at the upper bound $2-H$. It can now be shown that a distribution $\hat{F}_N$ with a mass point at $2-H$ can always be approximated by a another distribution $\hat{F}_N'$ that has no offers in the middle interval and achieves the same vote share against the equilibrium strategy.[5] Therefore, when checking for profitable deviations from the equilibrium no-reform distribution, we can concentrate

[4] That is, we shift the offers in the middle interval upwards towards $2-H$ without changing the mass of offers in the middle interval. This can be achieved because there are offers in the upper or lower interval that can be shifted downwards within the respective interval. Strictly speaking, we therefore need to exclude the case where there are offers in the lower interval only at the lower bound. The argument for this case is worked out in Esslinger (2016). This reference refers to the third chapter of this dissertation.

[5] The detailed argument can be found in Esslinger (2016).

on distributions $\hat{F}_N$ that have no offers in the middle interval and hence satisfy $\hat{F}_N(2-H) - \hat{F}_N(-2+(e-\bar{\rho})) = 0$.

For deviations that fulfill this requirement, equation (A.11) for candidate $A$'s vote share against the equilibrium no-reform distribution $\hat{F}_N^*$ becomes:

$$
\begin{aligned}
S(\hat{F}_N^*, \hat{F}_N) =& \int_{-2}^{2} \hat{F}_N^*(x) d\hat{\hat{F}}_N(x) \\
=& \frac{1}{2}\left[\frac{M_{LN}+M_{HN}}{H} + 2\left(\frac{2-H}{H}\right)\hat{F}_N(2-H)\right. \\
& \left. -2\left(\frac{2-H}{H}\right) + \frac{2}{H}\right].
\end{aligned}
$$

Candidate $A$ chooses $\hat{F}_N$ to maximize this expression under the constraint $M_{LN} + M_{HN} \leq 0$. It is clear that this constraint will not be slack, so $M_{LN} + M_{HN} = 0$ and the vote share becomes:

$$
S(\hat{F}_N^*, \hat{F}_N) = \frac{1}{2}\left[2\left(\frac{2-H}{H}\right)\hat{F}_N(2-H) - 2\left(\frac{2-H}{H}\right) + \frac{2}{H}\right].
$$

If candidate $B$, who plays the equilibrium strategy, chooses reform instead, then candidate $A$'s vote share is

$$
S(\hat{F}_R^*, \hat{F}_N) = 1 - \hat{F}_N(2-H).
$$

Candidate $B$ chooses reform with probability $\beta = \frac{1}{2}(2-H)$ and non-reform with probability $1-\beta = \frac{H}{2}$. Therefore, candidate $A$'s expected vote share when playing $\hat{F}_N$ is

$$
\begin{aligned}
& \frac{1}{2}(2-H)(1-\hat{F}_N(2-H)) \\
& + \frac{H}{2}\frac{1}{2H}[2(2-H)\hat{F}_N(2-H) + 2H - 2] \\
=& \frac{1}{2}.
\end{aligned}
$$

If candidate $B$ plays the equilibrium strategy, candidate $A$'s vote share is therefore $\frac{1}{2}$ for any distribution $\hat{F}_N$ that has no offers in the middle interval and is budget balanced. In particular, it is $\frac{1}{2}$ when playing the equilibrium no-reform distribution $\hat{F}_N^*$. This shows that candidate $A$ cannot profitably deviate from the equilibrium strategy by deviating from the equilibrium no-reform distribution $\hat{F}_N^*$.

**Step II: Optimality of $\hat{F}_R^*$.** We will now show that it is also not profitable to deviate from the equilibrium strategy by deviating from $\hat{F}_R^*$ to another reform distribution $\hat{F}_R$.

A candidate who reforms must optimally allocate the net benefits that are targetable in the first period, $\bar{\rho} - c$. In doing so, he takes into account that the debt limit implies that every voter expects to get additional resources of $e - \bar{\rho}$ in the second period which are not targetable.

Define

$$M_{MR} = \int_{-2+(e-\bar{\rho})}^{2-H} x d\hat{F}_R(x) \text{ and } M_{HR} = \int_{2-H}^{2} x d\hat{F}_R(x).$$

Then the problem of candidate $A$, if she opts for reform, is to choose $\hat{F}_R$ under the constraints $M_{MR} + M_{HR} \leq e - c$ and $\hat{F}_R(-2 + (e - \bar{\rho})) = 0$.[6]

When meeting the equilibrium reform distribution $\hat{F}_R^*$, the vote share of candidate $A$ using $\hat{F}_R$ is

$$\begin{aligned} S(\hat{F}_R^*, \hat{F}_R) =& 1 - \hat{F}_R(2-H) + \int_{-2+(e-\bar{\rho})}^{2-H} \frac{x + 2 - (e-\bar{\rho})}{4 - 2(c-\bar{\rho})} d\hat{F}_R(x) \\ =& 1 - \hat{F}_R(2-H) + \frac{e - c - M_{HR}}{4 - 2(c-\bar{\rho})} + \frac{2 - (e-\bar{\rho})}{4 - 2(c-\bar{\rho})} \hat{F}_R(2-H). \end{aligned} \tag{A.17}$$

When meeting the equilibrium no-reform distribution $\hat{F}_N^*$, $A$'s vote share is

$$\begin{aligned} S(\hat{F}_N^*, \hat{F}_R) =& \int_{-2+(e-\bar{\rho})}^{2-H} \frac{1}{2} d\hat{F}_R(x) + \int_{2-H}^{2} \frac{1}{2}\left(1 + \frac{x-2+H}{H}\right) d\hat{F}_R(x) \\ & \frac{1}{2} + \frac{1}{2}\int_{2-H}^{2} \frac{x-2+H}{H} d\hat{F}_R(x) \\ =& \frac{1}{2}\left[1 + \int_{2-H}^{2} \frac{-2+H}{H} d\hat{F}_R(x) + \frac{M_{HR}}{H}\right] \\ =& \frac{1}{2}\left[1 + \left(\frac{-2+H}{H}\right)(1 - \hat{F}_R(2-H)) + \frac{M_{HR}}{H}\right]. \end{aligned} \tag{A.18}$$

In the following we argue that, if $\hat{F}_R$ is supposed to be a best response to the equilibrium strategy, it cannot have offers in the high interval. To do so, we show that any distribution $\hat{F}_R$ that has offers in the high interval performs worse against

[6] It can be argued that any distribution $\hat{F}_R$ with a mass point at $-2 + (e - \bar{\rho})$ either looses for sure against the equilibrium strategy or can be approximated with a distribution without a mass point that wins the same expected vote share.

the equilibrium strategy than a distribution $\widetilde{F_R}$ that deviates from $\hat{F}_R$ by shifting all the offers from the high to the middle interval. This will allow us to concentrate on distributions that have no offers in the high interval when checking for profitable deviations from the equilibrium reform distribution $\hat{F}_R^*$.

Therefore, take any $\hat{F}_R$ that has $M_{HR} > 0$ and $\hat{F}_R(2-H) < 1$. We can then consider a deviation to a distribution $\widetilde{F_R}$ that shifts all the offers from the high interval to the middle interval. In terms of the above vote share formulas (A.17) and (A.18), this corresponds to decreasing $M_{HR}$ to zero and increasing $\hat{F}_R(2-H)$ by the necessary amount $\Delta$ such that it takes a value of 1.

Recall that the total expected vote share of candidate $A$ playing against the equilibrium strategy is

$$\beta S(\hat{F}_R^*, \hat{F}_R) + (1-\beta) S(\hat{F}_N^*, \hat{F}_R),$$

where $\beta = 1 - \frac{1}{2}H$ is the equilibrium probability of reform.

The above described shift in $M_{HR}$ changes this total expected vote share by

$$M_{HR}\left[\beta\frac{1}{4-2(c-\bar{\rho})} - (1-\beta)\frac{1}{2}\frac{1}{H}\right]. \tag{A.19}$$

On the other hand, the above described shift in $\hat{F}_R(2-H)$ changes this total expected vote share by

$$-\Delta(2-H)\left[\beta\frac{1}{4-2(c-\bar{\rho})} - (1-\beta)\frac{1}{2}\frac{1}{H}\right]. \tag{A.20}$$

We have already argued above that for $\beta = 1 - \frac{1}{2}H$, the term in square brackets is positive. Now note that $M_{HR} > \Delta(2-H)$, because $2-H$ is the upper bound of the middle interval and offers in the high interval must hence lie above this value. In total, we can then conclude that effect (A.19) dominates effect (A.20). Therefore, the shift towards the middle interval increases the total expected vote share and any distribution with offers in the high interval cannot be a best response to the equilibrium strategy.

Therefore, when checking for profitable deviations from the equilibrium reform distribution, we can concentrate on distributions $\hat{F}_R$ that have no offers in the high interval and hence satisfy $M_{HR} = 0$ and $\hat{F}_R(2-H) = 1$. As can be seen from (A.17) and (A.18), the vote share of any such reform distribution when meeting the equilibrium reform or no-reform distribution is $\frac{1}{2}$ in either case. In particular, also the equilibrium reform distribution $\hat{F}_R^*$ achieves the same outcome, and hence there is no profitable deviation from $\hat{F}_R^*$.

**Step III: Optimality of $\beta = 1 - \frac{1}{2}H$.** Finally, when candidate $B$ plays the equilibrium strategy, candidate $A$ is indifferent between offering reform and no-reform: In the previous two steps, we have shown that for all reform and no-reform distributions that are potential best responses to the equilibrium strategy, the total expected vote share is $\frac{1}{2}$ when playing against the equilibrium strategy. Therefore, candidate $A$ is happy to play reform with a probability of $\beta = 1 - \frac{1}{2}H$.

### A.2.3 Proof of uniqueness of equilibrium for Part (II.) of Proposition 2.6.1 and Corollary A.1.1

We prove that the equilibrium described in the above proposition and corollary is unique in the class of equilibria characterized by a probability of doing the reform $\beta^i$, debt levels in case of no-reform and reform, $\delta_N^i, \delta_R^i$, and distributions in case of no-reform and reform, $F_{1,R}^i, F_{1,N}^i$. This proof follows a similar sequence of steps as the uniqueness proof in Lizzeri and Persico (1998). However, they proved uniqueness only for a static model where the reform is a pure public good and uses up all resources in the economy. In their case, the reform distribution is therefore degenerate at the net value of the public good. In our proof, we need to account for the fact that, in case of reform, candidates can play a non-degenerate offer distribution.

Denote by $(\beta^{i*}, \delta_N^{i*}, \delta_R^{i*}, F_{1,R}^{i*}, F_{1,N}^{i*})$ an equilibrium strategy. For the following proofs, instead of working directly with the first-period distributions $F_{1,R}^{i*}, F_{1,N}^{i*}$, we work again with the distributions $\hat{F}$ that add to the above distributions the expected value of transfers that each voter expects in the second period given the respective debt level $\delta$.[7] Furthermore, recall that for the part of the above proposition and corollary that we consider here, we have $H = 2(c - \bar{\rho}) - e - \bar{\rho} > 0$.

[7]Recall from the proof of existence, that the the equilibrium distributions of total expected offers $\hat{F}_R^*$ and $\hat{F}_N^*$ combine the equilibrium first-period distributions with running the maximal debt level. Since $\hat{F}_R^*$ and $\hat{F}_N^*$ are only feasible with the maximal debt level, if we prove their uniqueness, we have also shown that in equilibrium the maximal debt must be raised.

**Lemma A.2.1** *For any feasible no-reform distribution $\hat{F}_N^i$ that is different from a uniform distribution on $[-2,2]$, there exists a feasible no-reform distribution $\hat{F}_N^j$ such that $S(\hat{F}_N^i, \hat{F}_N^j) > \frac{1}{2}$.*[8]

**Proof** See Lizzeri (1997). As for the case where the difference from a uniform distribution on $[-2,2]$ comes from the fact that the debt is less than the maximal one, the corresponding proof is analogous to Step 2 of the proof of Proposition 2.6.1 Part (I.).

**Lemma A.2.2** *The probability of reform fulfills $0 < \beta^{i*} < 1$ for $i = 1, 2$.*

**Proof**

**Step 1:** $\beta^{i*} > 0$

Suppose $\beta^{A*} = 0$, that is candidate $A$ plays no-reform for sure. Case 1: Suppose $\hat{F}_N^{A*}$ is uniform on $[-2,2]$. Then if candidate $B$ plays the equilibrium reform strategy $\hat{F}_R^*$, she gets a vote share of $\int_{-2+(e-\bar{e})}^{2-H} \hat{F}_N^{A*}(x)d\hat{F}_R^*(x) = \int_{-2+(e-\bar{e})}^{2-H} \frac{x+2}{4} d\hat{F}_R^*(x) = \frac{1}{2} + \int_{-2+(e-\bar{e})}^{2-H} x d\hat{F}_R^*(x) = \frac{1}{2} + (e-c) > \frac{1}{2}$. Case 2: Suppose $\hat{F}_N^{A*}$ is different from a uniform on $[-2,2]$. Then by Lemma A.2.1, candidate $B$ can find a no-reform distribution that beats candidate $A$ for sure and choose to play it with probability 1. Candidate $B$ would then again get a vote share greater than $\frac{1}{2}$, which cannot happen in equilibrium.

**Step 2:** $\beta^{i*} < 1$

Suppose $\beta^{A*} = 1$, that is candidate $A$ plays reform for sure. We will show that for any debt $\delta_R^A$ and any distribution $\hat{F}_R^A$ that candidate $A$ chooses, candidate $B$ can beat her for sure by reforming with probability $\beta^B = 0$, choosing the maximal debt $\delta_N^B = 1$ and playing the following distribution:

$$\hat{F}_N^B(x) = \begin{cases} 0, & \text{if } x \leq -2, \\ \frac{4-H}{8-H}, & \text{if } -2 \leq x \leq 2-H, \\ \frac{4-H}{8-H} + \frac{4}{(8-H)H}(x-2+H) & \text{if } 2-H \leq x \leq 2 \\ 1, & \text{if } x > 2. \end{cases} \tag{A.21}$$

[8] Note that for a no-reform debt less than the maximal one, $\delta_N^i < 1$, the resulting no-reform distribution $\hat{F}_N^i$ must be different from a uniform distribution on $[-2,2]$.

where $H = 2(c-\bar\rho)-(e-\bar\rho)$. Recall that $H>0$ in the case we are considering here.

Note that candidate $A$ will never offer more than the upper bound of candidate $B$'s distribution, 2. Offering exactly this upper bound to a voter is enough to get the vote for sure since candidate $B$ is offering less than 2 with probability 1.

The vote share of candidate $A$ is given by:

$$\begin{aligned}
S^A =& \int_{-2+(e-\bar\rho)}^{2} \hat F_N^B(x) d\hat F_R^A \\
=&\frac{4-H}{8-H} + \frac{4}{(8-H)H}\int_{2-H}^{2}(x-2+H)d\hat F_R^A \\
=&\frac{4-H}{8-H} + \frac{4}{(8-H)H}\left[\int_{-2+(e-\bar\rho)}^{2}(x-2+H)d\hat F_R^A - \int_{-2+(e-\bar\rho)}^{2-H}(x-2+H)d\hat F_R^A\right] \\
=&\frac{4-H}{8-H} + \frac{4}{(8-H)H}\left[e-c-2+H - \int_{-2+(e-\bar\rho)}^{2-H}(x-2+H)d\hat F_R^A\right]
\end{aligned}$$

The term $-\int_{-2+(e-\bar\rho)}^{2-H}(x-2+H)d\hat F_R^A$ is positive and can be maximized by offering $-2+(e-\bar\rho)$ to a maximal fraction $\gamma$ of voters, while offering 2 to the remaining fraction.[9] Note that in order to do this, candidate $A$ must raise the maximal debt $\bar\rho$.

When respecting the budget constraint and the constraint that no offer should be higher then 2, the maximal fraction $\gamma$ of voters to which $-2+(e-\bar\rho)$ can be offered is

$$\gamma = \frac{2-(e-c)}{4-(e-\bar\rho)}.$$

With this result we can evaluate the vote share of candidate $A$:

$$\begin{aligned}
S^A =&\frac{4-H}{8-H} + \frac{4}{(8-H)H}\left[e-c-2+H - \int_{-2+(e-\bar\rho)}^{2-H}(x-2+H)d\hat F_R^A\right] \\
\le&\frac{4-H}{8-H} + \frac{4}{(8-H)H}\left[e-c-2+H + \frac{2-(e-c)}{4-(e-\bar\rho)}(2-(e-\bar\rho)\right] \\
=&1 - \frac{8(2-(e-c))}{(8-H)H(4-(e-\bar\rho))}
\end{aligned}$$

The last expression is smaller than $\frac{1}{2}$ if and only if

$$\frac{8}{(8-H)H}\frac{(2-(e-c))}{(4-(e-\bar\rho))} > \frac{1}{2}$$

[9] Recall that candidate A will optimally never offer more than 2 to any voter.

This holds because, from our assumptions, in particular $c < 1$ and $e > c$, it follows that $H = 2(c - \bar{\rho}) - (e - \bar{\rho}) = (c - \bar{\rho}) - (e - c) < 1$ and therefore $\frac{8}{(8-H)H} > 1$. Furthermore, it can be shown that $\frac{(2-(e-c))}{(4-(e-\bar{\rho}))} > \frac{1}{2}$ whenever $H > 0$, which is the case considered here.

Therefore, under our assumptions, we have $S^A < \frac{1}{2}$, and so indeed candidate $A$ can be beaten for sure if he plays reform with probability one. This completes the proof that $\beta^{i*} < 1$ for $i = A, B$

**Lemma A.2.3** *Denote by $W_N^{i*}$ the upper bound of the support of $\hat{F}_N^{i*}$ and by $W_R^{i*}$ the upper bound of the support of $\hat{F}_R^{i*}$. For all i, (1) $\hat{F}_N^{i*}(2-H) = \frac{1}{2}$, and (2) $W_R^{i*} = 2-H$, (3) $W_N^{i*} = 2$.*

**Proof** The proof proceeds in 5 steps whose sequence is determined by the requirement to use results from the earlier steps in subsequent steps.

**Step 1: $W_R^{i*} \leq 2 - H$:**

Assume candidate $A$ plays as part of his equilibrium strategy a reform distribution with $W_R^{A*} > 2 - H$ and candidate $B$ plays the equilibrium strategy. We have shown in the existence proof that, when playing against the equilibrium strategy, as long as money for the reform distribution is spent in $[2 - H, 2]$, then the vote share can be increased by shifting that money down to the interval $[-2 + (e - \bar{\rho}), 2 - H]$. But if A's reform distribution only has offers in the interval $[-2 + (e - \bar{\rho}), 2 - H]$, then A's expected vote share from playing the reform distribution is $\frac{1}{2}$. Therefore, any reform distribution with $W_R^{i*} > 2 - H$ will win a vote share of less than 50%.

Furthermore, we know that any no-reform distribution that $A$ plays cannot win more than 50% of votes because otherwise the equilibrium strategy could be beaten by playing this no-reform distribution with probability one.

Hence we can conclude that a strategy containing a reform distribution with $W_R^{i*} > 2 - H$ can be beaten by the equilibrium strategy and therefore cannot be part of an equilibrium. Hence, any reform distribution that is played as part of an equilibrium strategy has $W_R^{i*} \leq 2 - H$.

**Step 2: $W_N^{i*} \geq 2$:**

Assume candidate $A$ plays as part of his equilibrium strategy a no-reform distribution with $W_N^{A*} < 2$. Then candidate $B$ can choose to play with probability one a no-reform distribution that offers slightly more than $W_N^{A*}$ to more than 50% of voters. With this strategy he wins more than $\frac{1}{2}$ of votes against A's no-reform distribution. Since A's strategy is supposed to be an equilibrium strategy, A's reform distribution has $W_R^{i*} \leq 2 - H$ (see step 1), and so B's strategy also beats A's reform distribution. In total, any strategy containing a no-reform distribution with $W_N^{i*} < 2$ can be beaten and therefore cannot be part of an equilibrium.

**Step 3: $\hat{F}_N^{i*}(2-H) = \frac{1}{2}$:**

In can be shown that playing reform must exactly tie against playing no-reform for $0 < \beta^{i*} < 1$ to be part of an equilibrium. Given this, the proof now proceeds in two substeps. In Step (3a), we prove that any no-reform distribution with $\hat{F}_N^{i*}(2-H) < \frac{1}{2}$ will not tie against any reform distribution that would be played in an equilibrium. Similarly, in step (3b), we show that any strategy with $\hat{F}_N^{i*}(2-H) > \frac{1}{2}$ will not tie against any reform distribution that would be played in an equilibrium.

(3a) Assume that $\hat{F}_N^{A*}(2-H) < \frac{1}{2}$, that is more than 50 percent of the mass of voters gets offers above $2-H$. In part (1) we have shown that $W_R^{i*} \leq 2-H$ for all $i$. Hence any reform distribution that is part of an equilibrium strategy will lose against such a no-reform distribution that has 50 percent of offers above $2-H$. As has been pointed out at the outset of step 3), this cannot happen in equilibrium.

(3b) Assume that $\hat{F}_N^{A*}(2-H) > \frac{1}{2}$. We know from step (2) that $\hat{F}_N^{A*}$ must have offers in the interval $[2-H, 2]$ or above that interval. But then we also know from the existence proof that $\hat{F}_N^{A*}$ cannot have offers in the middle interval $[-2+H, 2-H]$ except for a mass point at $2-H$. Additionally, in the uniqueness proof here, we are only considering potential equilibrium distributions. Therefore, we can apply Myerson (1993)'s argument that a potential equilibrium distribution in this setup of electoral competition cannot have a mass point.[10] Together, this implies that more than 50% of offers lie below $-2+H$ when $\hat{F}_N^{A*}(2-H) > \frac{1}{2}$. It follows that any equilibrium reform distribution, which has a support contained in the interval $[-2+H, 2-H]$, will beat this $\hat{F}_N^{A*}$, instead of tying against it.

[10]See the proof of Theorem 2 in Myerson (1993).

**Step 4:** $W_R^{i*} \geq 2 - H$:

Assume $W_R^{A*} < 2 - H$. Then candidate B can play a budget-balanced reform distribution which is 2-part and offers uniform on $[-2 + (e - \bar{\rho}), -2 + (e - \bar{\rho}), +\Delta V]$ and uniform on $[W_R^{A*}, W_R^{A*} + \Delta V]$, where $\Delta V < (2 - H) - W_R^{A*}$.

The assumption on $\Delta V$ together with the assumption of budget balance imply that more than 50% of the mass of this reform distribution is located above $W_R^{A*}$ and so it beats A's reform distribution (whose support has upper bound $W_R^{A*}$) for sure.

Assume that, other than that, $B$ plays the equilibrium strategy. That is he plays the equilibrium probability of reform and the equilibrium no-reform distribution. Since $A$'s strategy is supposed to be an equilibrium strategy, we know from the existence proof combined with Myerson (1993)'s no-mass-point argument for equilibrium distributions that the no-reform distribution $\hat{F}_N^{A*}$ played as part of this strategy has no offers in the middle interval $[-2 + (e - \bar{\rho}), 2 - H]$. Since A's no-reform distribution $\hat{F}_N^{A*}$ is also supposed to be a best-response to B's strategy, it will not have any offers above 2. Any such best-response no-reform distribution of A that fulfills the requirements of no offers in the middle interval $[-2 + (e - \bar{\rho}), 2 - H]$ will tie against B's no-reform distribution. A's reform distribution will tie against B's no-reform distribution. A's no-reform distribution will tie against B's reform distribution, using the result in step 3) that any equilibrium no-reform distribution must have $\hat{F}_N^{i*}(2 - H) = \frac{1}{2}$. Finally, as argued above, A's reform distribution will lose to B's reform distribution. Given that in any equilibrium, reform must be played with positive probability, this means that on total A would lose to B's strategy if he played a reform distribution with $W_R^{A*} < 2 - H$. Therefore such a distribution cannot be part of an equilibrium strategy.

**Step 5:** $W_N^{i*} \leq 2$:

Assume candidate $A$ plays as part of his equilibrium strategy a no-reform distribution with $W_N^{A*} > 2$. Then candidate B can beat candidate A by playing the equilibrium no-reform distribution with probability one. We know that for $A$ to play an equilibrium strategy his reform distribution must have $W_R^{i*} \leq 2 - H$ (see step 1). The equilibrium no-reform distribution ties against such a reform distribution. Therefore, it remains to be shown that the equilibrium no-reform distribution will beat a no-reform distribution with $W_N^{A*} > 2$.

For calculating the vote share of candidate $A$ playing such a no-reform distribution, we can safely assume that $\hat{F}_N^{A*}$ has no offers in the middle interval $[-2+H, 2-H]$, because when playing against the equilibrium no-reform distribution, a distribution with offers in this interval does worse than one that would downgrade all these offers down to $-2+H$ in order to increase offers in the interval above $2-H$. With this, candidate $A$'s vote share becomes:

$$
\begin{aligned}
S^A(\hat{F}_N^*, \hat{F}_N^{A*}) &= \int_{-2}^{W_N^{A*}} \hat{F}_N^* d\hat{F}_N^{A*} \\
&= \int_{-2}^{-2+H} \frac{1}{2}\frac{x+2}{H} d\hat{F}_N^{A*} + \int_{2-H}^{2} \frac{1}{2}\left(1 + \frac{x-2+H}{H}\right) d\hat{F}_N^{A*} + \int_{2}^{W_N^{A*}} d\hat{F}_N^{A*} \\
&< \frac{1}{2}\left[\frac{1}{H}\int_{-2}^{-2+H}(x+2)d\hat{F}_N^{A*} + \int_{2-H}^{W_N^{A*}} d\hat{F}_N^{A*} + \frac{1}{H}\int_{2-H}^{W_N^{A*}}(x-2+H)d\hat{F}_N^{A*}\right] \\
&= \frac{1}{2}\frac{1}{H}\left[\int_{-2}^{-2+H}(x+2)d\hat{F}_N^{A*} + \int_{2-H}^{W_N^{A*}} x d\hat{F}_N^{A*} + (H-2+H)\int_{2-H}^{W_N^{A*}} d\hat{F}_N^{A*}\right] \\
&= \frac{1}{2}\frac{1}{H}\left[\underbrace{\int_{-2}^{-2+H} x d\hat{F}_N^{A*} + \int_{2-H}^{W_N^{A*}} x d\hat{F}_N^{A*}}_{=0}\right. \\
&\quad \left. + 2\left[\hat{F}_N^{A*}(-2+H) - \hat{F}_N^{A*}(-2)\right] + (2H-2)\left[1 - \hat{F}_N^{A*}(2-H)\right]\right] \\
&= \frac{1}{2}\frac{1}{H}\left[H - \underbrace{\hat{F}_N^{A*}(-2)}_{=0}\right] \\
&= \frac{1}{2}
\end{aligned}
$$

The last step has used the fact proven in step 3) that any equilibrium distribution must have $\hat{F}_N^{i*}(2-H) = \frac{1}{2}$. Together with the fact that $\hat{F}_N^{A*}$ has no offers in the middle interval $[-2+H, 2-H]$, this implies that $\hat{F}_N^{A*}(-2+H) = \hat{F}_N^{A*}(2-H) = \frac{1}{2}$. $\hat{F}_N^{A*}(-2) = 0$ follows again from Myerson (1993)'s no-mass-point argument for equilibrium distributions.

As intended, we have thus shown that the equilibrium no-reform distribution will beat a no-reform distribution with $W_N^{A*} > 2$.

**Lemma A.2.4** *For all $i$, $\hat{F}_N^{i*}$ is strictly increasing and continuous on* $[2-H, 2]$

**Proof** A similar result is proven in Lizerri and Persico (1998) for the case of a binary public good. The question is whether the setup here is analogous to the binary public good case. In the binary public good case, when the public good is provided, every voter gets a utility increase (above the consumption value of his one unit of money endowment) of $G-1$. Furthermore, 50 % of the transfer distribution lies above $G-1$. Specifically, the transfer distribution has 50 % of offers in the interval $[G-1, 1]$.

In our case, when the reform is implemented, we don't have a degenerate distribution. Instead, we have a real distribution. However, as has been established in the previous lemma, the support of this distribution has upper bound $W_R^{i*} = 2 - H$. Furthermore, $\hat{F}_N^{i*}(2-H) = \frac{1}{2}$, that is 50 % of the mass of the no-reform offers lie above this upper bound of the reform distribution. More specifically, since $W_N^{i*} = 2$, these 50 % lie in the interval $[2-H, 2]$

If we interpret $G-1$ in the binary public good case as the upper bound of the support of the degenerate"public good distribution", then our setup here is exactly analogous to the binary public good case. To proof the form of the no-reform distribution on the interval $[2-H, 2]$, we can therefore refer to the analogous proof in the binary public good case which proved the form of the transfer distribution on the interval $[G-1, 1]$.

**Lemma A.2.5** *For all $i$, $\hat{F}_N^{i*}$ satisfies equation* (A.9) *on* $[2-H, 2]$.

**Proof** The proof is still about proving the exact form of the no-reform distribution on the interval $[2-H, 2]$. Therefore, what has been argued in the previous lemma still applies here. Insofar as it is about proving the exact form of the no-reform distribution on the interval $[2-H, 2]$, our setup is exactly analogous to the binary public good case, where an analogous proof was done to proof the exact form of the transfer distribution on the interval $[G-1, 1]$. Therefore, the proof of the above result is analogous to the proof of Lemma 10 in Lizerri and Persico (1998).

**Lemma A.2.6** *For all $i$, $\hat{F}_N^{i*}$ satisfies equation* (A.9) *on* $[-2, -2+H]$.

**Proof** Interpreting this time $G-1$ in the binary public good case as the lower bound of the support of the degenerate"public good distribution", it is possible to use the result in Lizzeri (1997) to show that $\hat{F}_N^{i*}$ must be uniform on $[-2, q]$ for some $q \leq -2+(e-\bar{\rho})$, where $-2+(e-\bar{\rho})$ is the lower bound of the support of the reform distribution. Given this, the proof of the above result can proceed analogously to the proof of Lemma 11 in Lizerri and Persico (1998).

**Corollary A.2.7** *For all $i$, $\hat{F}_N^{i*}$ is the distribution described in equation* (A.9)*.*

**Lemma A.2.8** *For all $i$, $\hat{F}_R^{i*}$ satisfies equation* (A.10) *on $[-2+(e-\bar{\rho}), 2-H]$.*

**Proof** Given the previously established results, we know that the equilibrium reform distribution $\hat{F}_R^{i*}$ has support contained in $[-2+(e-\bar{\rho}), 2-H]$. Given the form of the equilibrium no-reform distribution established in the previous proofs, we know that no matter what its exact form, any reform distribution on this support will tie against the equilibrium no-reform distribution. All that determines the exact form of the equilibrium reform distribution is then the requirement that there should be no profitable deviation when both candidates play reform. Indeed, when both candidates play a reform distribution with support $[-2+(e-\bar{\rho}), 2-H]$, then the unique equilibrium is that both play the uniform distribution described in equation (A.10). But the proof of this result is analogous to the proof of Theorem 1 in Lizerri (1999).

**Lemma A.2.9** *In equilibrium the probability of doing the reform is $1-\frac{1}{2}H$.*

**Proof** In the previous lemmas up to now, we have established that the unique equilibrium no-reform distribution and the unique equilibrium reform distribution are the ones described in equations (A.9) and (A.10) and that both are played with positive probability in equilibrium. Therefore, to pin down the exact value $\beta^{i*}$ with which the equilibrium reform distribution is played, we can use the fact that for this $\beta^{i*}$, it should not be possible to reach a vote share greater than $\frac{1}{2}$ by choosing another no-reform distribution than $\hat{F}_N^*$. In particular, it should not be possible to increase the vote share by altering the equilibrium no-reform distribution through changing $\hat{F}_N^*(2-H)$ away from $\frac{1}{2}$. Therefore assume that candidate $A$ plays a no-reform distribution $\hat{F}_N^A$ with the same support as $\hat{F}_N^*$ but considers changing the distribution of mass across this support through a change in $\hat{F}_N^A(2-H)$. We have shown before that for such a distribution $\hat{F}_N^A$, candidate A's vote share when meeting the equilibrium no-reform distribution is $S(\hat{F}_N^*, \hat{F}_N^A) = \frac{1}{2H}[2(2-H)\hat{F}_N^A(2-H)+2(H-1)]$, and his vote share when meeting the equilibrium reform distribution is $S(\hat{F}_R^*, \hat{F}_N^A) = 1-\hat{F}_N^A(2-H)$. Given a probability of reform $\beta$ his total expected vote share when playing $\hat{F}_N^A$ is:

$$
\begin{aligned}
&(1-\beta)S(\hat{F}_N^*, \hat{F}_N^A) + \beta S(\hat{F}_R^*, \hat{F}_N^A) \\
=&(1-\beta)\frac{1}{2H}[2(2-H)\hat{F}_N^A(2-H) + 2(H-1)] + \beta[1-\hat{F}_N^A(2-H)]
\end{aligned}
$$

We need this expected vote share to be independent of $\hat{F}_N^A(2-H)$. Otherwise, it would pay to either increase or decrease $\hat{F}_N^A(2-H)$ as much as possible and we could not have $\hat{F}_N^A(2-H) = \frac{1}{2}$, as must be the case in equilibrium. Therefore we need:

$$
\begin{aligned}
&(1-\beta)\frac{1}{2H}2(2-H)\hat{F}_N^A(2-H) - \beta\hat{F}_N^A(2-H) = 0 \\
\Leftrightarrow&(1-\beta)\frac{1}{2H}2(2-H) - \beta = 0 \\
\Leftrightarrow&\beta = 1 - \frac{1}{2}H.
\end{aligned}
$$

This concludes the proof that the equilibrium probability of reform is $1 - \frac{1}{2}H$.

From all this, we can conclude that equilibrium is unique and is the one described in Proposition 2.6.1 and Corollary A.1.1.

### A.2.4 Proof of Proposition 2.5.1 and Corollary 2.5.2

Recall that in Proposition 2.5.1, we did not impose any exogenous debt limit. We can also think of this situation as an exogenous debt limit that is equal to the natural debt limit under reform, $\bar{\delta} = 1 + e$. Due to the definition of $\bar{\rho} = 1 - \bar{\delta}$, this means $\bar{\rho} = e$. Replacing $\bar{\rho}$ with $e$ in the proof of Part (I.) of Proposition 2.6.1 and Corollary A.1.1 the results follow.

### A.2.5 Proof of Proposition 2.5.3 and Corollary 2.5.4

Replacing $\bar{\rho}$ with $\lambda e$ in the proof of Proposition 2.6.1 and Corollary A.1.1 the results follow.

### A.2.6 Proof of Proposition 2.6.2 and Corollary A.1.2

This proof follows again the same steps as the proof of Proposition 2.6.1 and Corollary A.1.1. However, besides redefining $\bar{\rho}$ and $H$ an appropriate adjustment in the support of the equilibrium distributions must be made.

# Appendix B

# Appendix to Chapter 3

Recall that any reform-debt bundle $Y$ is defined by 3 characteristics in terms of the distribution of total expected transfers that can be offered under this bundle. First, $l_Y$ is the lowest possible offer under bundle $Y$. Second, $M_Y$ are the additional resources that are available above the already existing endowments under bundle $Y$. Third, $u_Y$ is the maximal offer that can be financed for 50 percent of voters given a lowest offer $l_Y$ and a budget beyond existing endowments of $M_Y$: $u_Y = 2M_Y - l_Y$. Recall that bundle $Y$ has higher (net) targetability than bundle $Z$ if $u_Y > u_Z$. Furthermore, $M_Y > M_Z$ means that bundle $Y$ has a higher efficiency than bundle $Z$ in the sense that the total amount of resources is bigger.[1]

## B.1 Equilibrium characterization for special cases

**Corollary B.1.1** *Suppose the debt-related distortion is small enough such that $\gamma < \frac{e-c}{1+e}$. Then Proposition B.2.1 applies with $\{R; ND\}$ the efficiency-maximizing bundle $E$, $\{R; D\}$ the targetability-maximizing bundle $T$, $\{NR; ND\}$ taking the place of bundle $Q$, and $\{NR; D\}$ taking the place of bundle $I$.*

*Specifically, the lowest possible offer $l$, the maximal financeable offer for half of the voters, $u$, as well as the resource constraint $M$ under the different bundles are:*

[1] See Definition 3.4.2 and the corresponding discussion in chapter 3.

$\mathbf{E} = \{\mathbf{R};\ \mathbf{ND}\}$:

$$l_E = -1 + e$$
$$M_E = e - c$$
$$u_E = 1 + e - 2c$$

$\mathbf{T} = \{\mathbf{R};\ \mathbf{D}\}$:

$$l_T = -2$$
$$M_T = (1-\gamma)e - c - \gamma$$
$$u_T = 2(1-\gamma)(1+e) - 2c$$

$\mathbf{I} = \{\mathbf{NR};\ \mathbf{D}\}$:

$$l_I = -2$$
$$M_I = -\gamma$$
$$u_I = 2(1-\gamma)$$

$\mathbf{Q} = \{\mathbf{NR};\ \mathbf{ND}\}$:

$$l_Q = -1$$
$$M_Q = 0$$
$$u_Q = 1$$

*This implies that in equilibrium*

*(i) both candidates play bundle $E = \{R;\ ND\}$ with probability $\beta_E = 1 - \frac{2H}{u_T - l_T}$, where $H = u_T - u_E$; they play bundle $T = \{R;\ D\}$ with probability $\beta_T = 1 - \beta_E$;*

*(ii) both candidates play bundles $I = \{NR;\ D\}$ and $Q = \{NR;\ ND\}$ with probability zero: $\beta_I = \beta_Q = 0$*

*(iii) both candidates play the distributions of total expected transfer offers illustrated in Figure 3.1.*

**Corollary B.1.2** *Suppose the distortion caused by raising the maximal debt is small enough compared to the net benefit of the reform such that* $\gamma < \frac{e-c}{1+e}$*, and suppose that there is a constitutional debt limit* $\bar{\rho} \in (-1 + \frac{2c-e}{2(1-\gamma)-1}, e]$.

***Non-restrictive debt limit****: Suppose the debt limit* $\bar{\rho}$ *is not very restrictive in the sense that* $\bar{\rho} > \frac{2c-e}{2(1-\gamma)-1} > 0$.[2] *Then we are still in the case of Corollary B.1.1. The only things that change are* $l_T, u_T, M_T$.

$\mathbf{T} = \{\mathbf{R};\ \mathbf{D}\}$*:*

$$l_T = -2 + (e - \bar{\rho})$$

$$M_T = e - c - (1 + \bar{\rho})\gamma$$

$$u_T = 2(1 - \gamma)(1 + \bar{\rho}) + (e - \bar{\rho}) - 2c$$

*This still implies that in equilibrium:*

*(i) both candidates play bundle* $E = \{R;\ ND\}$ *with probability* $\beta_E = 1 - \frac{2H}{u_T - l_T}$*, where* $H = u_T - u_E$*; they play bundle* $T = \{R;\ D\}$ *with probability* $\beta_T = 1 - \beta_E$*;*

*(ii) the probability* $\beta_E$ *with which the most efficient bundle* $\{R;\ ND\}$ *is played increases when the debt limit gets stricter (when* $\bar{\rho}$ *falls);*

*(iii) both candidates play bundles* $I = \{NR;\ D\}$ *and* $Q = \{NR;\ ND\}$ *with probability zero:* $\beta_I = \beta_Q = 0$*;*

*(iv) both candidates play the distributions of total expected offers illustrated in Figure 3.1.*

**Corollary B.1.3** *Suppose the distortion caused by raising the maximal debt is small enough compared to the net benefit of the reform such that* $\gamma < \frac{e-c}{1+e}$*, and suppose that there is a constitutional debt limit* $\bar{\rho} \in 1(-1 + \frac{2c-e}{2(1-\gamma)-1}, e]$.

***Restrictive debt limit:*** *Suppose the debt limit* $\bar{\rho}$ *is restrictive in the sense that* $\bar{\rho} < \frac{2c-e}{2(1-\gamma)-1}$*. Then Proposition B.2.2 applies with* $\{R;\ ND\}$ *the efficiency-maximizing*

[2] The fact that this cut-off level for the debt limit is positive is implied by the assumption made at the beginning of the corollary, $\gamma < \frac{e-c}{1+e}$ and by Assumption (3.3), $2c - e > 0$.

*bundle $E$, $\{NR;\, D\}$ the targetability-maximizing bundle $T$, $\{R;\, D\}$ taking the place of bundle $I$, and $\{NR;\, ND\}$ taking the place of bundle $Q$.*

*Specifically, the lowest possible offer $l$, the maximal financeable offer for half of the voters, $u$, as well as the resource constraint $M$ under the different bundles are:*

$\mathbf{E} = \{\mathbf{R};\, \mathbf{ND}\}$*:*

$$l_E = -1 + e$$
$$M_E = e - c$$
$$u_E = 1 + e - 2c$$

$\mathbf{T} = \{\mathbf{NR};\, \mathbf{D}\}$*:*

$$\begin{array}{llll} \textit{for}\, \bar\rho \geq 0: & l_T = -2 & \textit{for}\, \bar\rho < 0: & l_T = -2 - \bar\rho \\ & M_T = -\gamma & & M_T = -(1+\bar\rho)\gamma \\ & u_T = 2(1-\gamma) & & u_T = 2(1-\gamma)(1+\bar\rho) - \bar\rho \end{array}$$

$\mathbf{I} = \{\mathbf{R};\, \mathbf{D}\}$*:*

$$l_I = -2 + (e - \bar\rho)$$
$$M_I = e - c - (1+\bar\rho)\gamma$$
$$u_I = 2(1-\gamma)(1+\bar\rho) + (e-\bar\rho) - 2c$$

$\mathbf{Q} = \{\mathbf{NR};\, \mathbf{ND}\}$*:*

$$l_Q = -1$$
$$M_Q = 0$$
$$u_Q = 1$$

*This implies that in equilibrium*

*(i) both candidates play bundle $T = \{NR;\, D\}$ with probability $\beta_T = \frac{2H''}{u_T - l_T}$, where $H'' = u_T - u_I$; this probability increases with a stricter debt limit (with smaller $\bar\rho$);*

*(ii)* *both candidates play bundle* $I = \{R;\, D\}$ *with probability* $\beta_I = \frac{2H'}{u_I - l_I}\frac{u_T - l_T - 2H''}{u_T - l_T}$, *where* $H' = u_I - u_E$*; this probability decreases with a stricter debt limit (with smaller* $\bar{\rho}$*);*

*(iiii)* *both candidates play bundle* $E = \{R;\, ND\}$ *with probability* $\beta_E = 1 - \beta_T - \beta_I = \frac{u_I - l_I - 2H'}{u_I - l_I}\frac{u_T - l_T - 2H''}{u_T - l_T}$*;*

*(iv)* *both candidates play bundle* $Q = \{NR;\, ND\}$ *with probability zero:* $\beta_Q = 0$*;*

*(v)* *both candidates play the distributions of total expected offers illustrated in Figure 3.2.*

**Corollary B.1.4** *Suppose the debt-related distortion is so high that* $\gamma > \frac{1}{2}$*. Then Proposition B.2.1 in the appendix applies with* $\{R;\, ND\}$ *the efficiency-maximizing bundle* $E$*,* $\{NR;\, ND\}$ *the targetability-maximizing bundle* $T$*,* $\{R;\, D\}$ *taking the place of bundle* $Q$*, and* $\{NR;\, D\}$ *taking the place of bundle* $I$*. In particular, the ranking in terms of resource constraints is the second alternative stated in Proposition B.2.1:* $M_E > M_T > M_I > M_Q$*. The ranking in terms of maximal offers is the last alternative stated in Proposition B.2.1:* $u_T > max\{u_I, u_Q, u_E\}$*,* $l_I \leq l_T$*, and* $l_Q \leq l_T$*.*

*Specifically, the lowest possible offer* $l$*, the maximal financeable offer for half of the voters,* $u$*, as well as the resource constraint* $M$ *under the different bundles are:*

$\mathbf{E} = \{\mathbf{R};\, \mathbf{ND}\}$*:*

$$l_E = -1 + e$$
$$M_E = e - c$$
$$u_E = 1 + e - 2c$$

$\mathbf{T} = \{\mathbf{NR};\, \mathbf{ND}\}$*:*

$$l_T = -1$$
$$M_T = 0$$
$$u_T = 1$$

$\mathbf{I} = \{\mathbf{NR};\ \mathbf{D}\}$*:*

$l_I = -2$

$M_I = -\gamma$

$u_I = 2(1-\gamma)$

$\mathbf{Q} = \{\mathbf{R};\ \mathbf{D}\}$*:*

$l_Q = -2$

$M_Q = (1-\gamma)e - c - \gamma$

$u_Q = 2(1-\gamma)(1+e) - 2c$

*This implies that in equilibrium*

*(i) both candidates play bundle* $E = \{R;\ ND\}$ *with probability* $\beta_E = 1 - \frac{2H}{u_T - l_T}$*, where* $H = u_T - u_E$*; they play bundle* $T = \{NR;\ ND\}$ *with probability* $\beta_T = 1 - \beta_E$*;*

*(ii) both candidates play bundles* $I = \{NR;\ D\}$ *and* $Q = R+N$ *with probability zero:* $\beta_I = \beta_Q = 0$

*(iii) both candidates play the distributions of total expected offers illustrated in Figure 3.1. However, bundle* $T$ *is now* $\{NR;\ ND\}$*.*

## B.2 Equilibrium characterization in general form

The following propositions state the complete characterization of the equilibrium in a general form.

**Proposition B.2.1** *Suppose the 4 reform-debt bundles* $T, E, I, Q$ *fulfill the following criteria:*

1. $u_T > u_I > u_Q > u_E$*, i.e. the ranking in terms of targetability is* $T, I, Q, E$ *(the ranking between* $u_I$ *and* $u_Q$ *can also be flipped).*
   *Alternatively, suppose* $u_T > max\{u_I, u_Q, u_E\}$*,* $l_I \leq l_T$*, and* $l_Q \leq l_T$*.*

2. $M_E > M_T > M_Q > M_I$*, i.e. the ranking in terms of efficiency is* $E, T, Q, I$ *(the ranking between* $M_I$ *and* $M_Q$ *can also be flipped).*

*Hence, $T$ is the targetability-maximizing bundle and $E$ is the efficiency-maximizing bundle. Additionally, $T$ dominates the other bundles $I$ and $Q$ in terms of efficiency.*

*Then, in equilibrium:*

*(i) both candidates play bundle $E$ with probability $\beta_E = 1 - \frac{2H}{u_T - l_T}$, where $H = u_T - u_E = l_E - l_T - 2(M_E - M_T)$; they play bundle $T$ with probability $\beta_T = 1 - \beta_E = \frac{2H}{u_T - l_T}$;*

*(ii) both candidates play bundles $I$ and $Q$ with probability zero: $\beta_I = \beta_Q = 0$;*

*(iii) When candidates play bundle $T$, they draw total expected offers from the following distribution:*

$$\hat{F}_T^*(x) = \begin{cases} 0, & \text{if } x \leq l_T, \\ \frac{1}{2}\left(\frac{x - l_T}{H}\right), & \text{if } l_T \leq x \leq l_T + H, \\ \frac{1}{2}, & \text{if } l_T + H \leq x \leq u_T - H, \\ \frac{1}{2}\left(1 + \frac{x - u_T + H}{H}\right), & \text{if } u_T - H \leq x \leq u_T, \\ 1, & \text{if } x \geq u_T. \end{cases} \tag{B.1}$$

*When candidates play bundle $E$, they draw total expected offers from the following distribution:*

$$\hat{F}_E^*(x) = \begin{cases} 0, & \text{if } x \leq l_E, \\ \frac{x - l_E}{u_E - l_E}, & \text{if } l_E \leq x \leq u_E = u_T - H, \\ 1, & \text{if } x \geq u_E = u_T - H. \end{cases} \tag{B.2}$$

**Proposition B.2.2** *Suppose the 4 reform-debt bundles $T, E, I, Q$ fulfill the following criteria:*

1. *$u_T > u_I > u_Q > u_E$, i.e. the ranking in terms of targetability is $T, I, Q, E$.*

2. *$M_E > M_I > M_Q > M_T$, i.e. the ranking in terms of efficiency is $E, I, Q, T$.*

*Hence, $T$ is the targetability-maximizing bundle and $E$ is the efficiency-maximizing bundle. Additionally, $T$ is dominated by all other bundles in terms of efficiency, and bundle $I$ dominates bundle $Q$ in terms of efficiency and targetability.*

*Then, in equilibrium:*

*(i)* *both candidates play bundle $T$ with probability $\beta_T = \frac{2H''}{u_T - l_T}$, where $H'' = u_T - u_I = l_I - l_T - 2(M_I - M_T)$;*

*(ii)* *both candidates play bundle $E$ with probability $\beta_E = \frac{u_I - l_I - 2H'}{u_I - l_I} \frac{u_T - l_T - 2H''}{u_T - l_T}$ where $H' = u_I - u_E = l_E - l_I - 2(M_E - M_I)$;*

*(iii)* *both candidates play bundle $I$ with probability $\beta_I = 1 - \beta_T - \beta_E = \frac{2H'}{u_I - l_I} \frac{u_T - l_T - 2H''}{u_T - l_T}$;*

*(iv)* *both candidates play bundle $Q$ with probability zero: $\beta_Q = 0$;*

*(v)* *When candidates play bundle $T$, they draw total expected offers from the following distribution:*

$$\hat{F}_T^*(x) = \begin{cases} 0, & \text{if } x \leq l_T, \\ \frac{1}{2}\left(\frac{x - l_T}{H''}\right), & \text{if } l_T \leq x \leq l_T + H'', \\ \frac{1}{2}, & \text{if } l_T + H'' \leq x \leq u_T - H'', \\ \frac{1}{2}\left(1 + \frac{x - u_T + H''}{H''}\right), & \text{if } u_T - H'' \leq x \leq u_T, \\ 1, & \text{if } x \geq u_T. \end{cases} \tag{B.3}$$

*When candidates play bundle $I$, they draw total expected offers from the following distribution:*

$$\hat{F}_I^*(x) = \begin{cases} 0, & \text{if } x \leq l_I, \\ \frac{1}{2}\left(\frac{x - l_I}{H'}\right), & \text{if } l_I \leq x \leq l_I + H', \\ \frac{1}{2}, & \text{if } l_I + H' \leq x \leq u_I - H', \\ \frac{1}{2}\left(1 + \frac{x - u_I + H'}{H'}\right), & \text{if } u_I - H' \leq x \leq u_I, \\ 1, & \text{if } x \geq u_I. \end{cases} \tag{B.4}$$

*When candidates play bundle $E$, they draw total expected offers from the following distribution:*

$$\hat{F}_E^*(x) = \begin{cases} 0, & \textit{if } x \leq l_E, \\ \frac{x-l_E}{u_E-l_E}, & \textit{if } l_E \leq x \leq u_E = u_I - H, \\ 1, & \textit{if } x \geq u_E = u_I - H. \end{cases} \tag{B.5}$$

## B.3 Proofs

### B.3.1 Proof of Corollary B.1.1

In order to be able to apply Proposition B.2.1, we just have to show that the rankings of the resource constraints $M$ and the maximal offers $u$ for the specific case of Corollary B.1.1 are as stated in Proposition B.2.1.

Therefore, we first have to argue for the correctness of the parameters $l_Y$, $u_Y$, and $M_Y$ for any possible debt-reform bundle $Y$. We always have the relation: $u = 2M - l$. It can be verified that for all possible debt-reform bundles, this relation holds in Corollary B.1.1. For instance, $u_E = 1 + e - 2c = 2(e - c) - (-1 + e) = 2M_E - l_E$. Hence, we just have to argue for the correctness of $l_Y$ and $M_Y$ for a given debt-reform bundle $Y$.

Recall that $l_Y$ gives the lowest possible offer that can be offered to a voter under bundle $Y$. In the case of the bundle combining reform and no-debt, $E = \{R; ND\}$, since there is no debt, second period resources cannot be targeted. Given that the reform is implemented, any voter expects a transfer of $e$ in the second period, which cannot be taken from her in terms of expected offers from the point of view of the first period. Additionally, in the first period, maximally her endowment of 1 can be taken away, leading to a lowest total expected offer of $l_E = -1 + e$. Since the reform is implemented, the resources available over both periods beyond existing endowments correspond to the reform-induced net increase in the pie, $M_E = e - c$.

For the bundle where reform is combined with the maximal debt, $T = \{R; D\}$, the costs of the debt-related distortions reduce these resources. In particular only a fraction $(1 - \gamma)$ of the gross reform benefit remain when they are transferred across time. Additionally, a fraction $\gamma$ is lost when transferring the second-period endowment of 1 across time. Hence the total resources beyond existing endowments shrink to $M_T = (1-\gamma)e - c - \gamma$. However, with maximal debt all resources become targetable in the following sense: There are no resources left in the second period that cannot be redistributed across voters from the point of view of the first period. This implies that the worst-off voter can lose her complete endowments: $l_T = -2$.

The latter fact does not change for bundle $I = \{NR;\ D\}$, which still raises maximal debt but does no longer implement the reform. The only effect is on the resource constraint, which no longer includes any reform-related resource gains. Instead, there is only a total loss $\gamma$ in resources, which corresponds to the cost of transferring the existing second-period endowment across time. Finally, in the case of bundle $Q = \{NR;\ ND\}$, which combines no reform with no debt, there are no resource losses through debt and no gains through reform, so that there are no resources available for transfers beyond existing endowments, $M_Q = 0$. The lowest possible offer is then only the first period endowment that can be taken from the worst-off voter, $l_Q = -1$.

Turning to the ranking in terms of the maximal offers, we have:

$$\begin{aligned}
&u_T > u_I \\
\Leftrightarrow\ & 2(1-\gamma)(1+e) - 2c > 2(1-\gamma) \\
\Leftrightarrow\ & (1-\gamma)e > c \\
\Leftrightarrow\ & \gamma < \frac{e-c}{e}.
\end{aligned}$$

The last line is implied by the assumption in Corollary B.1.1 that $\gamma < \frac{e-c}{1+e}$.

Note that Assumption (3.3), $2c - e > 0$, is equivalent to $\frac{e-c}{e} < \frac{1}{2}$. Together with the just mentioned assumption from Corollary B.1.1, we therefore have: $\gamma < \frac{e-c}{1+e} < \frac{e-c}{e} < \frac{1}{2}$. Hence in the case considered, less than 1/2 is lost in transferring resources across time through public debt. Now, $\gamma < \frac{1}{2}$ implies the ranking $u_I = 2(1-\gamma) > 1 = u_Q$. Finally, $u_Q = 1 > 1 + e - 2c = u_E$ is again implied by Assumption (3.3), $2c - e > 0$.

In terms of the resource constraints, we have: $M_E = e - c > (1-\gamma)e - c - \gamma = M_T$ as soon as there exists a distortion, $\gamma > 0$, as is the case in this paper. Furthermore, we have:

$$\begin{aligned}
&M_T > M_Q \\
\Leftrightarrow\, &(1-\gamma)e - c - \gamma > 0 \\
\Leftrightarrow\, &\gamma < \frac{e-c}{1+e},
\end{aligned}$$

where the last line is exactly the assumption made in Corollary B.1.1. Finally, $M_Q = 0 > -\gamma = M_I$ again follows immediately since there is a distortion, $\gamma > 0$.

### B.3.2 Proof of Corollary B.1.2

Since nothing changes compared to Corollary B.1.1 except for $u_T$ and $M_T$, we only have to make sure that all relationships including bundle $T$ still fulfill the respective rankings compared to the other bundles. As before, let us first argue for the correctness of the specific values of $l_T$ and $M_T$ (it can be seen immediately that the relationship $u_T = 2M_T - l_T$ holds). With a debt limit $\bar{\rho}$, an amount $e-\bar{\rho}$ of resources remains in the second period and hence cannot be taken from any voter from the point of view of the first period. Therefore he lowest possible offer increases from $-2$, the full loss of both periods' endowments, to $l_T = -2 + (e - \bar{\rho})$. In terms of the resources beyond existing endowments, the reform implemented implies additional resources of $e - c$. However, with maximal debt, which under the debt limit is $(1 + \bar{\rho})$, there are distortionary costs of $(1 + \bar{\rho})\gamma$. Therefore, $M_T = e - c - (1 + \bar{\rho})\gamma$.

In terms of the rankings, we have:

$$\begin{aligned}
&u_T > u_I \\
\Leftrightarrow\, &2(1-\gamma)(1+\bar{\rho}) + (e-\bar{\rho}) - 2c > 2(1-\gamma) \\
\Leftrightarrow\, &\bar{\rho} > \frac{2c-e}{2(1-\gamma)-1},
\end{aligned}$$

where the last line is exactly the assumption made about the debt limit in this corollary. Furthermore, since a distortion exists in the sense that $\gamma > 0$, we still have $M_E = e-c > e - c - (1 - \bar{\rho})\gamma = M_T$. Finally, under the assumption on the size of the distortion made, $\gamma < \frac{e-c}{1+e}$, we have:

$$\begin{aligned} M_T &= e - c - (1-\bar{\rho})\gamma \\ &> e - c - (1-\bar{\rho})\frac{e-c}{1+e} \\ &= \frac{(e-\bar{\rho})(e-c)}{1+e} \geq 0 = M_Q, \end{aligned}$$

where the last (weak) inequality holds due to $\bar{\rho} \leq e$ as also specified in the corollary.

The additional fact that this corollary proves compared to Corollary B.1.1 is the comparative static result concerning the effect of a change in the debt limit on the probability with which the most efficient policy bundle is played. We have:

$$\begin{aligned} \beta_E &= 1 - \beta_T \\ &= 1 - \frac{2(u_T - u_E)}{u_T - l_T} \\ &= 1 - \frac{[2(1-\gamma)-1](1+\bar{\rho})}{(1-\gamma)(1-\bar{\rho}) + 1 - c}. \end{aligned}$$

Therefore, we get:

$$\begin{aligned} \frac{\partial \beta_E}{\partial \bar{\rho}} &= -\frac{\partial \beta_T}{\partial \bar{\rho}} \\ &= -\frac{[2(1-\gamma)-1](1-c)}{[(1-\gamma)(1-\bar{\rho}) + 1 - c]^2} < 0. \end{aligned}$$

The negative sign of the last expression follows from the fact $\gamma < 1/2$ derived in the proof of Corollary B.1.1 and assumption (3.2), $c < 1$. This proves that with a fall in $\bar{\rho}$ (a stricter debt limit) the probability $\beta_E$ with which the most efficient bundle is played increases. This concludes the proof of Corollary B.1.2.

### B.3.3 Proof of Corollary B.1.3

In the same vain as in the previous proofs, we show that the rankings of the maximal offers and resource constraints hold as described in Proposition B.2.2. Before turning to these rankings, let us quickly argue once more for the correctness of $l_Y$, $u_Y$, and $M_Y$ for any bundle $Y$. The bundles $E = \{R; ND\}$ and $Q = \{NR; ND\}$, which raise no debt, are not affected by a debt limit. Therefore, nothing changes for these bundles compared to the case without a debt limit in terms of the parameters that define their transfer distributions. This case was handled in Corollary B.1.1 above.

Similarly, the bundle that plays no reform and debt, $T = \{NR;\ D\}$,[3] is only affected in terms of its transfer distribution once $\bar{\rho}$ drops below zero. This is because for $\bar{\rho} \geq 0$, debt is only restricted by an amount higher than the natural debt limit under no reform and hence the debt limit does not bind. Recall that for $\bar{\rho} < 0$, $\bar{\rho}$ captures how much resources less than the second-period endowment of 1 can be used for debt repayment. These resources are left in the future and therefore cannot be taken from any voter from the point of view of the first period. That is why, for $\bar{\rho} < 0$, the lowest possible offer, $l_T$, rises from $-2$ to $-2 - \bar{\rho}$. Similarly, the amount of second-period resources which are affected by the debt-related distortion decrease from the full endowment of 1 to only $1 + \bar{\rho} < 1$. Therefore, the total loss in resources below existing endowments, which corresponds to the resource constraint for bundle $T = \{NR;\ D\}$, becomes $M_T = -(1 + \bar{\rho})\gamma$. The maximal offer is then calculated as before: $u_T = 2M_T - l_T$. The arguments for bundle $I = \{R;\ D\}$ can be found in the proof of Corollary B.1.2. Note that in this latter corollary $\{R;\ D\}$ took the role of the targetability-maximizing bundle $T$.

Turning to the ranking in terms of maximal offers we have for $\bar{\rho} \geq 0$:

$$
\begin{aligned}
& u_T > u_I \\
\Leftrightarrow\ & 2(1-\gamma) > 2(1-\gamma)(1+\bar{\rho}) + (e - \bar{\rho}) - 2c \\
\Leftrightarrow\ & \bar{\rho} < \frac{2c - e}{2(1-\gamma) - 1},
\end{aligned}
$$

where the last line is exactly the assumption made about the debt limit in this corollary. Similarly, we have for $\bar{\rho} < 0$:

$$
\begin{aligned}
& u_T > u_I \\
\Leftrightarrow\ & 2(1-\gamma)(1+\bar{\rho}) - \bar{\rho} > 2(1-\gamma)(1+\bar{\rho}) + (e - \bar{\rho}) - 2c \\
\Leftrightarrow\ & 2c > e,
\end{aligned}
$$

where the last line is exactly Assumption (3.3).

[3]Note, that compared to Corollary B.1.1, the roles of different bundles in terms of which bundle maximizes targetability are changed.

Furthermore, we have:

$$
\begin{aligned}
&u_I > u_Q \\
\Leftrightarrow\, &2(1-\gamma)(1+\bar{\rho}) + (e-\bar{\rho}) - 2c > 1 \\
\Leftrightarrow\, &\bar{\rho} > -1 + \frac{2c-e}{2(1-\gamma)-1},
\end{aligned}
$$

where the last line is exactly the lower bound imposed on the debt limit in this corollary. Finally, by Assumption (3.3), we still have $u_Q = 1 > 1 + e - 2c = u_E$.

In terms of the resource constraints, note that we have $M_E = e - c > e - c - (1-\bar{\rho})\gamma = M_I$, as long as, first, a distortion exists in the sense that $\gamma > 0$, and as long as, second, the debt limit $\bar{\rho}$ does not prohibit debt completely in the sense that $\bar{\rho} > -1$. Both these restrictions are fulfilled by assumption. Under these same two restrictions, for $\bar{\rho} > 0$, we also have $M_Q = 0 > -\gamma = M_T$. The same holds then for $\bar{\rho} < 0$: $M_Q = 0 > -(1-\bar{\rho})\gamma = M_T$. Finally, analogous to Corollary B.1.2, under the assumption on the size of the distortion made, $\gamma < \frac{e-c}{1+e}$, we have:

$$
\begin{aligned}
M_I &= e - c - (1-\bar{\rho})\gamma \\
&> e - c - (1-\bar{\rho})\frac{e-c}{1+e} \\
&= \frac{(e-\bar{\rho})(e-c)}{1+e} \geq 0 = M_Q,
\end{aligned}
$$

where the last (weak) inequality holds due to $\bar{\rho} \leq e$ as also specified in the corollary.

The final claims that need to be proven are the comparative static results concerning the effect of a change in the debt limit on the probability with which the different policy bundles are played. We have for $\bar{\rho} \geq 0$:

$$
\begin{aligned}
\beta_T &= \frac{2(u_T - u_I)}{u_T - l_T} \\
&= \frac{2c - e - [2(1-\gamma)-1]\bar{\rho}}{2-\gamma}.
\end{aligned}
$$

Therefore, we get:

$$
\frac{\partial \beta_T}{\partial \bar{\rho}} = -\frac{[2(1-\gamma)-1]}{2-\gamma} < 0.
$$

The negative sign of the last expression follows from the fact $\gamma < 1/2$ derived in the proof of Corollary B.1.1. Similarly, for $\bar{\rho} < 0$, we have:

$$
\beta_T = \frac{2(u_T - u_I)}{u_T - l_T} = \frac{2c-e}{(1-\gamma)(1+\bar{\rho})+1}.
$$

Hence, we get:

$$\frac{\partial \beta_T}{\partial \bar{\rho}} = -\frac{(1-\gamma)(2c-e)}{[(1-\gamma)(1+\bar{\rho})+1]^2} < 0,$$

where the negative sign follows again from $\gamma < 1/2$ and Assumption (3.3), $2c > e$. This proves that with a fall in $\bar{\rho}$ (a stricter debt limit) the probability $\beta_T$ with which the targetability-maximizing bundle is played increases.

Note that $\frac{u_T - l_T - 2H''}{u_T - l_T} = \beta_E + \beta_I = 1 - \beta_T$. The above result on the comparative statics of $\beta_T$ then implies that $\frac{u_T - l_T - 2H''}{u_T - l_T}$ decreases with a stricter debt limit. Hence, one factor in the terms of the probabilities of $\beta_E$ and $\beta_I$ decreases with a stricter debt limit. The comparative statics of these probabilities are hence determined by the first term in their formulas. For $\beta_I$ this first term is:

$$\frac{2(u_I - u_E)}{u_I - l_I} = -\frac{[2(1-\gamma)-1](1+\bar{\rho})}{(1-\gamma)(1-\bar{\rho})+1-c}.$$

Therefore, we get:

$$\frac{\partial \beta_I}{\partial \bar{\rho}} = \frac{[2(1-\gamma)-1](1-c)}{[(1-\gamma)(1-\bar{\rho})+1-c]^2} > 0.$$

The positive sign of the last expression follows from the fact $\gamma < 1/2$ derived in the proof of Corollary B.1.1 and assumption (3.2), $c < 1$. This proves that with a fall in $\bar{\rho}$ (a stricter debt limit) the probability $\beta_I$ decreases since both terms in its formula decrease. On the other hand, the first term in the formula for $\beta_E$ is just $1 - \frac{2(u_I - u_E)}{u_I - l_I}$ and by the above result decreases with a stricter debt limit. Therefore, the comparative statics for $\beta_E$ are indeterminate. This concludes the proof of Corollary B.1.3.

### B.3.4 Proof of Corollary B.1.4

We have to show that the rankings of resource constraints and maximal offers hold as stated in the corollary, along with the two assumptions $l_I \leq l_T$ and $l_Q \leq l_T$. the formulas for the parameter values $l_Y$, $u_Y$, and $M_Y$ of a given bundle $Y$ correspond exactly to the ones in Corollary B.1.1.

Let us start with the two assumptions on the lowest possible offers: We have $l_I = l_Q = -2 < -1 = l_T$.

Turning now to the ranking in terms of maximal offers, we need to show that bundle

$T$ has the highest maximal offer. We have:

$$\begin{aligned}
& u_T > u_I \\
\Leftrightarrow\, & 1 > 2(1-\gamma) \\
\Leftrightarrow\, & \gamma > \frac{1}{2},
\end{aligned}$$

which is just the defining assumption of this corollary. Furthermore, we have:

$$\begin{aligned}
& u_I > u_Q \\
\Leftrightarrow\, & 2(1-\gamma) > 2(1-\gamma)(1+e) - 2c \\
\Leftrightarrow\, & (1-\gamma)e < c \\
\Leftrightarrow\, & \gamma > \frac{e-c}{e}.
\end{aligned}$$

Note that Assumption (3.3), $2c - e > 0$, is equivalent to $\frac{e-c}{e} < \frac{1}{2}$. Together with the above mentioned defining assumption of this corollary, we hence have $\gamma > \frac{1}{2} > \frac{e-c}{e}$, and hence indeed $u_I > u_Q$. But combining the two above rankings we have: $u_T > u_I > u_Q$. It just remains to show that $u_T = 1 > 1 + e - 2c = u_E$, which is again implied by Assumption (3.3), $2c - e > 0$.

In terms of the resource constraints, we have $M_E = e - c > 0 = M_T$, which holds by assumption (3.1). $M_T = 0 > -\gamma = M_I$ again follows immediately since there is a distortion, $\gamma > 0$. Finally, we have:

$$\begin{aligned}
& M_I > M_Q \\
\Leftrightarrow\, & -\gamma > (1-\gamma)e - c - \gamma \\
\Leftrightarrow\, & (1-\gamma)e < c.
\end{aligned}$$

Now for $\gamma > \frac{1}{2}$, $(1-\gamma)e < \frac{1}{2}e < c$, where the last inequality holds due to Assumption (3.3), $2c - e > 0$. This concludes the proof of Corollary B.1.4.

In the following, we prove the two propositions that give a general equilibrium characterization. These propositions underly all other propositions and corollaries in this paper.

### B.3.5 Proof of Proposition B.2.1

In the following, we show that the stated strategies indeed are an equilibrium. The proof of existence of the equilibrium generalizes the corresponding proof in Boyer and Esslinger (2016), and adjusts for the fact that the debt choice now also has direct efficiency implications.

The proof of existence has four steps: Step I shows that there is no profitable deviation from the equilibrium strategy by deviating from $\hat{F}_T^*$, step II shows that it is also not profitable to deviate on $\hat{F}_E^*$. Step III shows that bundles $Q$ and $I$ are played with probability zero. Then, step IV argues for the optimality of the equilibrium probabilities with which the targetability-maximizing debt-reform bundle $T$ and the efficiency-maximizing bundle $E$ are played.

**Step I: Optimality of $\hat{F}_T^*$.** Consider candidate $A$ when he decides to play the debt-reform bundle $T$ and assume he deviates from the equilibrium distribution under this bundle, $\hat{F}_T^*$, to another distribution $F_T$. When this candidate $A$ meets a candidate $B$ that chooses also the bundle $T$ and that offers money according to $\hat{F}_T^*$, the vote share of candidate $A$ is:[4]

$$\begin{aligned} S(\hat{F}_T^*, \hat{F}_T) = & \int_{l_T}^{u_T} \hat{F}_T^*(x) d\hat{F}_T(x) \\ = & \frac{1}{2}\left\{ \frac{M_{LT} + M_{HT}}{H} - \frac{l_T + H}{H}\hat{F}_T(l_T + H) \right. \\ & \left. + \frac{u_T - H}{H}\hat{F}_T(u_T - H) + \frac{2H - u_T}{H} \right\}, \end{aligned} \tag{B.6}$$

where

$$M_{LT} = \int_{l_T}^{l_T+H} x d\hat{F}_T(x), \tag{B.7}$$

and

$$M_{HT} = \int_{u_T-H}^{u_T} x d\hat{F}_T(x). \tag{B.8}$$

Equations (B.7) and (B.8) capture the money spent on transfers in the low interval $[l_T, l_T + H]$ and the high interval $(u_T - H, u_T]$, respectively. We choose the high interval

[4] Without loss of generality, we exclude a mass point at the lower bound $l_T$ of the support. Whenever necessary later on, we deal with this case separately. We follow this approach for the remainder of this appendix.

to be open to the right, because the continuous equilibrium distribution $\hat{F}_T^*$ puts zero mass on the single offer $u_T - H$. In order to win a positive mass of voters in the upper interval, the respective offers must therefore be strictly higher than $u_T - H$.

Candidate $A$ chooses $\hat{F}_T$ under the constraint that

$$M_{LT} + M_{HT} + M_{MT} = M_T, \tag{B.9}$$

where

$$M_{MT} = \int_{l_T+H}^{u_T-H} x d\hat{F}_T(x) \tag{B.10}$$

is the money spent on transfer offers in the middle interval $(l_T + H, u_T - H]$. $M_T$ denotes the resources beyond existing endowments under bundle $T$.

In the following, we will argue that, when checking for profitable deviations from the equilibrium distribution $\hat{F}_T^*$ that is played in case of bundle $T$, we can concentrate on distributions $\hat{F}_T$ that have no offers in the middle interval. We will do so by showing that if $\hat{F}_T$ has a positive mass of offers in the middle interval which are not concentrated at the upper bound $u_T - H$ of this interval, then the vote share can be increased by shifting that mass up towards $u_T - H$. As a last step, it can then be shown that, if a distribution only has offers in the middle interval concentrated as a mass point at $u_T - H$, then such a distribution can always be approximated by another distribution $\hat{F}_T'$ that has no offers in the middle interval and achieves the same vote share against the equilibrium strategy. This will establish that, when looking for deviations from the equilibrium distribution $\hat{F}_T^*$, it is enough to focus on distributions $\hat{F}_T$ that have no offers in the middle interval.

Therefore, assume that $\hat{F}_T$ is a best response to the equilibrium strategy and spends a positive amount on offers in the middle interval $(l_T + H, u_T - H]$. First, it is easy to see that candidate $A$ will not make offers in the interval $(l_T + H, l_E]$.[5] With such offers, candidate $A$ cannot win additional votes against $\hat{F}_T^*$, because $\hat{F}_T^*$ contains no offers in the middle interval. Furthermore, $l_E$ is the lowest offer that the equilibrium distribution $\hat{F}_E^*$ in case of the efficiency-maximizing debt-reform bundle $E$ contains. Therefore in order to win additional votes, candidate $A$ must provide definitely more than $l_E$.

[5]Note that given the definition of $H$ and the relationship $l = 2M - u$, we have $l_E - (l_T + H) = 2(M_E - M_T) > 0$.

We will now argue that, if $\hat{F}_T$ contains offers in the interval $(l_E, u_T - H) = (l_E, u_E)$, then $\hat{F}_T$ is actually not a best response to the equilibrium strategy. This is less straightforward to argue, because offers in this interval are made to some voters under the equilibrium distribution $\hat{F}_E^*$ associated to bundle $E$. Suppose hence that $\hat{F}_T$ is a best response to the equilibrium strategy and spends a positive amount on offers in the interval $(l_E, u_T - H)$. Then we can arrive at a contradiction by constructing a profitable deviation $\widetilde{F_T}$.

In particular, note that if $\hat{F}_T$ has only offers in the middle interval $(l_E, u_T - H]$,[6] then it will tie against the equilibrium distribution in case of maximizing targetability, $\hat{F}_T^*$, and it will lose against the equilibrium distribution in case of maximizing efficiency, $\hat{F}_E^*$.

The latter is a more general claim which states that for any bundle $T$ with $M_T < M_E$, a distribution $\hat{F}_T$ with a support contained in $[l_E, u_T - H]$, the support of $\hat{F}_E^*$, will lose against $\hat{F}_E^*$:

$$
\begin{aligned}
S(\hat{F}_E^*, \hat{F}_T) = &\int_{l_E}^{u_T - H} \frac{x - l_E}{u_E - l_E} d\hat{F}_T(x) \\
= &\frac{M_T}{u_E - l_E} - \frac{l_E}{u_E - l_E} \\
< &\frac{M_E - l_E}{u_E - l_E} \\
= &\frac{1/2(u_E + l_E) - l_E}{u_E - l_E} = \frac{1}{2}.
\end{aligned}
$$

The inequality follows from $M_T < M_E$.

Therefore, any $\hat{F}_T$ that is supposed to be a best response to the equilibrium strategy cannot only have offers in the middle interval $(l_E, u_T - H]$ and must have a positive amount of offers in the low or high part of $T$'s equilibrium support, $max\{|M_{LT}|, |M_{HT}|\} > 0$. We can then construct a deviation from $\hat{F}_T$ to $\widetilde{F_T}$ such that $\widetilde{F_T}(l_E) = \hat{F}_T(l_E)$ and $\widetilde{F_T}(u_T - H) = \hat{F}_T(u_T - H)$, but $\widetilde{M_{MT}} > M_{MT}$. That is, we shift the offers in the middle interval $(l_E, u_T - H]$ up towards $u_T - H$ without changing the mass of offers in this or any other interval. This can be achieved, because there are offers in the upper or lower part of $T$'s equilibrium support that can be shifted downwards within the respective interval. Strictly speaking, we therefore need to exclude the case where

[6] Since we have argued that there will be no offer in $(l_T + H, l_E]$, we will henceforth refer to $(l_E, u_T - H]$ as the middle interval.

there are offers in the low interval $[l_T, l_T + H]$ only at the lower bound. We consider this case at the end.

We now show that the expected vote share increases when using the described deviation $\widetilde{F_T}$. When candidate $B$, who plays the equilibrium strategy, chooses to play bundle $T$ and the associated distribution $\hat{F}_T^*$, then for candidate $A$ a deviation from $\hat{F}_T$ to $\widetilde{F_T}$ is detrimental. As we can see from (B.6) combined with the budget constraint (B.9), increasing $M_{MT}$ to $\widetilde{M_{MT}}$, decreases candidate $A$'s vote share by $\frac{1}{2}\frac{\widetilde{M_{MT}}-M_{MT}}{H}$.

When candidate $B$ chooses to play bundle $E$ with distribution $\hat{F}_E^*$, then candidate $A$'s vote share is:

$$\begin{aligned}
S(\hat{F}_E^*, \hat{F}_T) =& 1 - \hat{F}_T(u_T - H) + \int_{l_E}^{u_T-H} \frac{x - l_E}{u_E - l_E} d\hat{F}_T(x) \\
=& \frac{M_{MT}}{u_E - l_E} + \frac{(-l_E)}{u_E - l_E}[\hat{F}_T(u_T - H) - \hat{F}_T(l_E)] \\
&+ 1 - \hat{F}_T(u_T - H).
\end{aligned}$$

By the first term, a deviation of candidate $A$ from $\hat{F}_T$ to $\widetilde{F_T}$ increases his vote share by $\frac{\widetilde{M_{MT}}-M_{MT}}{u_E-l_E}$.

In total, it is beneficial to increase $M_{MT}$ to $\widetilde{M_{MT}}$ if and only if

$$(1 - \beta_E)\frac{1}{2}\frac{1}{H} < \beta_E \frac{1}{u_E - l_E}.$$

Recall that the equilibrium probability of bundle $E$ was $\beta_E = 1 - \frac{2H}{u_T - l_T}$. With this the above equation is equivalent to

$$\begin{aligned}
&\frac{2H}{u_T - l_T}\frac{1}{2H} < \frac{u_T - l_T - 2H}{u_T - l_T}\frac{1}{u_E - l_E} \\
\Leftrightarrow& u_E - l_E = u_T - H - l_E < u_T - l_T - 2H \\
\Leftrightarrow& l_E - (l_T + H) = 2(M_E - M_T) > 0,
\end{aligned}$$

where the last inequality holds by assumption.

For the case where not all offers outside the middle interval $(l_E, u_T - H]$ are concentrated at the lower bound $l_T$ of the low part of $T$'s equilibrium support, we have therefore shown that a best response $\hat{F}_T$ to the equilibrium strategy cannot have offers in the middle interval $(l_E, u_T - H]$ expect at the upper bound $u_T - H$.

In the following, we show that, even if all offers outside the middle interval $(l_E, u_T - H)$ are concentrated as a mass point of mass $\hat{F}_T(l_T)$ at $l_T$, the vote share can still be increased. In this case, the vote share formulas become:

$$S(\hat{F}_T^*, \hat{F}_T) = \frac{1}{2} - \frac{1}{2}\hat{F}_T(l_T),$$

$$\begin{aligned} S(\hat{F}_E^*, \hat{F}_T) =& \frac{M_T - l_T\hat{F}_T(l_T)}{u_E - l_E} - \frac{l_E}{u_E - l_E}\left[1 - \hat{F}_T(l_T)\right] \\ =& \frac{1}{2} - \frac{M_E - M_T}{u_E - l_E} + \frac{l_E - l_T}{u_E - l_E}\hat{F}_T(l_T). \end{aligned}$$

Plugging in the equilibrium values for $\beta_T$ and $\beta_E$, the total expected vote share becomes:

$$\begin{aligned} &\beta_T S(\hat{F}_T^*, \hat{F}_T) + \beta_E S(\hat{F}_E^*, \hat{F}_T) \\ =& \frac{1}{2} + \left[-\frac{H}{u_T - l_T} + \frac{u_T - l_T - 2H}{u_T - l_T}\frac{l_E - l_T}{u_E - l_E}\right]\hat{F}_T(l_T) - \frac{u_T - l_T - 2H}{u_T - l_T}\frac{M_E - M_T}{u_E - l_E} \\ =& \frac{1}{2} + \frac{1}{u_T - l_T}\left[\frac{u_E - (l_T + H)}{u_E - l_E}(l_E - l_T) - H\right]\hat{F}_T(l_T) - \frac{u_T - l_T - 2H}{u_T - l_T}\frac{M_E - M_T}{u_E - l_E}. \end{aligned}$$

Now we have already shown that $l_T + H < l_E$, which implies $u_E - (l_T + H) > u_E - l_E$ and $l_E - l_T > l_T + H - l_T = H$. Therefore, the term in square brackets is positive, and the vote share can be increased by increasing $\hat{F}_T(l_T)$. This can be done until $\hat{F}_T(l_T)$ reaches its highest value for which the budget constraint still holds, which is the case when all other offers in the middle interval are at its upper bound $u_E$. Hence, we have shown that even when all offers on the equilibrium support of $T$ are concentrated as a mass point at $l_T$, the vote share can still be increased until all offers in the middle interval are concentrated at the upper bound $u_E$.

This concludes the proof that a best response $\hat{F}_T$ to the equilibrium strategy cannot have offers in the middle interval $(l_E, u_T - H]$ expect at the upper bound $u_T - H$.

It can now be shown that a distribution $\hat{F}_T$ with a mass point at $u_T - H = u_E$ and no other offers in the middle interval $(l_E, u_T - H] = (l_E, u_E]$ can always be approximated by another distribution $\hat{F}_T'$ that has no offers at all in the middle interval $(l_E, u_E]$ and achieves the same vote share against the equilibrium strategy. Hence consider a distribution $\hat{F}_T$ with a mass point of mass $\Delta$ at $u_T - H = u_E$ and no other offers in the middle interval $(l_E, u_E]$. For such a distribution we have $M_{MT} = \Delta u_E$, $\hat{F}_T(u_T - H) =$

$\hat{F}_T(u_E) = \hat{F}_T(l_E) + \Delta$, and $M_{LT} + M_{HT} = M_T - \Delta u_E$. The vote share against $\hat{F}_E^*$ becomes:

$$
\begin{aligned}
S(\hat{F}_E^*, \hat{F}_T) =& \frac{\Delta u_E}{u_E - l_E} - \frac{l_E}{u_E - l_E}\Delta + 1 - [\hat{F}_T(l_E) + \Delta] \\
=& 1 - \hat{F}_T(l_E).
\end{aligned}
$$

Therefore, the vote share against $\hat{F}_E^*$ stays the same if we shift the mass from $u_E$ to above $u_E$ by decreasing some offers inside the upper part of $T$'s equilibrium support $(u_E, u_T]$. If there are no offers in $(u_E, u_T]$, then we must decrease some offers from $u_E$ down to $l_T + H$ in order to shift the remaining offers at $u_E$ above $u_E$. In this case $\hat{F}_T(l_E)$ and therefore $S(\hat{F}_E^*, \hat{F}_T)$ changes. However, as we will argue shortly, the total expected vote share will not change through such a shift in mass.

With a mass point at $u_E$, the vote share against $\hat{F}_T^*$ becomes:

$$
\begin{aligned}
S(\hat{F}_T^*, \hat{F}_T) =& \frac{1}{2}\left\{\frac{M_T - \Delta u_E}{H} - \frac{l_T + H}{H}\hat{F}_T(l_T + H)\right. \\
&\left. + \frac{u_E}{H}[\hat{F}_T(l_T + H) + \Delta] + \frac{2H - u_T}{H}\right\} \\
=& \frac{1}{2}\left\{\frac{M_T}{H} + \frac{u_T + l_T - 2H}{H}\hat{F}_T(l_T + H) + \frac{2H - u_T}{H}\right\}.
\end{aligned}
$$

Similar to the vote share against $\hat{F}_E^*$, this vote share stays the same when the mass from $u_E$ can be completely shifted into the upper part of $T$'s equilibrium support $(u_E, u_T]$ and $\hat{F}_T(l_T + H) = \hat{F}_T(l_E)$[7] does not have to be changed. On the other hand, if $\hat{F}_T(l_T + H) = \hat{F}_T(l_E)$ needs to be decreased in order to get rid of the mass point at $u_E$, then there is still no change in the total expected vote share, because the equilibrium probabilities are constructed in a way that the total expected vote share is independent of the terms involving $\hat{F}_T(l_T + H) = \hat{F}_T(l_E)$. With probabilities $\beta_E = 1 - \frac{2H}{u_T - l_T}$ and $\beta_T = 1 - \beta_E = \frac{2H}{u_T - l_T}$, we get for the total expected vote share:

[7] Recall that we started from a distribution that had no offers in the middle interval expect at its upper bound $u_E$.

$$
\begin{aligned}
&\frac{u_T - l_T - 2H}{u_T - l_T}(1 - \hat{F}_T(l_T + H)) \\
&+ \frac{2H}{u_T - l_T}\frac{1}{2}\left[\frac{M_T}{H} + \frac{u_T - l_T - 2H}{H}\hat{F}_T(l_T + H) + \frac{2H - u_T}{H}\right] \\
=&\frac{M_T - l_T}{u_T - l_T} \\
=&\frac{M_T - l_T}{2(M_T - l_T)} = \frac{1}{2}.
\end{aligned}
$$

In total, we have shown that any distribution $\hat{F}_T$ that is supposed to be a best response to the equilibrium strategy cannot have offers in the middle interval $(l_E, u_E]$ expect at $u_E$ and that for any distribution that only has offers in the middle interval at $u_E$, there is a distribution without any offers in the middle interval that achieves the same vote share.

Therefore, when checking for profitable deviations from the equilibrium distribution under bundle $T$, we can concentrate on distributions $\hat{F}_T$ that have no offers in the middle interval $(l_E, u_E]$ and hence satisfy $\hat{F}_T(u_T - H) - \hat{F}_T(l_E) = 0$.

For deviations that fulfill this requirement, equation (B.6) for candidate $A$'s vote share against the equilibrium distribution $\hat{F}_T^*$ under bundle $T$ becomes:

$$
\begin{aligned}
S(\hat{F}_T^*, \hat{F}_T) =& \int_{l_T}^{u_T} \hat{F}_T^*(x) d\hat{\hat{F}}_N(x) \\
=&\frac{1}{2}\left[\frac{M_{LT} + M_{HT}}{H} + \frac{u_T - l_T - 2H}{H}\hat{F}_T(u_T - H) + \frac{2H - u_T}{H}\right].
\end{aligned}
$$

Candidate $A$ chooses $\hat{F}_T$ to maximize this expression under the constraint $M_{LT} + M_{HT} \leq M_T$. It is clear that this constraint will not be slack, so $M_{LT} + M_{HT} = M_T$ and the vote share becomes:

$$
S(\hat{F}_T^*, \hat{F}_T) = \frac{1}{2}\left[\frac{M_T}{H} + \frac{u_T - l_T - 2H}{H}\hat{F}_T(u_T - H) + \frac{2H - u_T}{H}\right].
$$

Candidate $A$'s vote share from playing $\hat{F}_T$ against the equilibrium distribution $\hat{F}_E^*$ under bundle $E$ becomes:

$$
S(\hat{F}_E^*, \hat{F}_T) = 1 - \hat{F}_T(u_T - H).
$$

Candidate $B$ plays $\hat{F}_E^*$ with probability $\beta_E = 1 - \frac{2H}{u_T - l_T}$ and $\hat{F}_T^*$ with probability $1 - \beta_E = \frac{2H}{u_T - l_T}$. Therefore, candidate $A$'s expected vote share when playing $\hat{F}_T$ is:

$$
\begin{aligned}
&\frac{u_T - l_T - 2H}{u_T - l_T}(1 - \hat{F}_T(u_T - H)) \\
&\quad + \frac{2H}{u_T - l_T}\frac{1}{2}\left[\frac{M_T}{H} + \frac{u_T - l_T - 2H}{H}\hat{F}_T(u_T - H) + \frac{2H - u_T}{H}\right] \\
=&\frac{M_T - l_T}{u_T - l_T} \\
=&\frac{M_T - l_T}{2(M_T - l_T)} = \frac{1}{2}.
\end{aligned}
$$

If candidate $B$ plays the equilibrium strategy, candidate $A$'s vote share is therefore $\frac{1}{2}$ for any distribution $\hat{F}_T$ that has no offers in the middle interval and is budget-balanced. In particular, it is $\frac{1}{2}$ when playing the equilibrium distribution $\hat{F}_T^*$. This shows that candidate $A$ cannot profitably deviate from the equilibrium strategy by deviating from the equilibrium distribution under bundle $T$, $\hat{F}_T^*$.

**Step II: Optimality of $\hat{F}_E^*$.** We will now show that it is also not profitable to deviate from the equilibrium strategy by deviating from $\hat{F}_E^*$ to another distribution $\hat{F}_E$ under bundle $E$.

Define

$$
M_{ME} = \int_{l_E}^{u_T - H} x d\hat{F}_E(x) \text{ and } M_{HE} = \int_{u_T - H}^{u_T} x d\hat{F}_E(x),
$$

where, as before, the upper interval $(u_T - H, u_H]$ is open to the left.

Recall that $u_T - H = u_E$, so $M_{ME}$ captures the money spent on the support of the equilibrium distribution $\hat{F}_E^*$ under bundle $E$. Then the problem of candidate $A$, if she opts for reform, is to choose $\hat{F}_E$ under the constraints $M_{ME} + M_{HE} = M_E$ and $\hat{F}_E(l_E) = 0$.[8].

When meeting the equilibrium distribution under bundle $E$, $\hat{F}_E^*$, the vote share of candidate $A$ using $\hat{F}_E$ is:

[8]It can be argued that any distribution $\hat{F}_E$ with a mass point at $l_E$ either looses for sure against the equilibrium strategy or can be approximated with a distribution without a mass point that wins the same expected vote share.

$$\begin{aligned} S(\hat{F}_E^*, \hat{F}_E) =& 1 - \hat{F}_E(u_T - H) + \int_{l_E}^{u_T-H} \frac{x - l_E}{u_E - l_E} d\hat{F}_E(x) \\ =& 1 - \hat{F}_E(u_T - H) + \frac{M_E - M_{HE}}{u_E - l_E} + \frac{-l_E}{u_E - l_E}\hat{F}_E(u_T - H) \\ =& 1 + \frac{M_E - M_{HE}}{u_E - l_E} - \frac{u_T - H}{u_E - l_E}\hat{F}_E(u_T - H). \end{aligned} \tag{B.11}$$

When meeting the equilibrium distribution under bundle $T$, $\hat{F}_T^*$, $A$'s vote share using $\hat{F}_E$ is:

$$\begin{aligned} S(\hat{F}_T^*, \hat{F}_E) =& \int_{l_E}^{u_T-H} \frac{1}{2} d\hat{F}_E(x) + \int_{u_T-H}^{u_T} \frac{1}{2}\left(1 + \frac{x - u_T + H}{H}\right) d\hat{F}_E(x) \\ & \frac{1}{2} + \frac{1}{2}\int_{u_T-H}^{u_T} \frac{x - u_T + H}{H} d\hat{F}_E(x) \\ =& \frac{1}{2}\left[1 + \int_{u_T-H}^{u_T} \frac{-u_T + H}{H} d\hat{F}_E(x) + \frac{M_{HE}}{H}\right] \\ =& \frac{1}{2}\left[1 + \left(\frac{-u_T + H}{H}\right)(1 - \hat{F}_E(u_T - H)) + \frac{M_{HE}}{H}\right]. \end{aligned} \tag{B.12}$$

In the following we argue that, if $\hat{F}_E$ is supposed to be a best response to the equilibrium strategy, it cannot have offers in the high interval $(u_T - H, u_T]$. To do so, we show that any distribution $\hat{F}_E$ that has offers in the high interval performs worse against the equilibrium strategy than a distribution $\widetilde{F_E}$ that deviates from $\hat{F}_E$ by shifting all the offers from the high to the middle interval $[l_E, u_E] = [l_E, u_T - H]$. This will allow us to concentrate on distributions that have no offers in the high interval when checking for profitable deviations from the equilibrium distribution under bundle $E$, $\hat{F}_E^*$.

Therefore, take any $\hat{F}_E$ that has $M_{HE} > 0$ and $\hat{F}_E(u_T - H) < 1$. We can then consider a deviation to a distribution $\widetilde{F_E}$ that shifts all the offers from the high interval to the middle interval. In terms of the above vote share formulas (B.11) and (B.12), this corresponds to decreasing $M_{HE}$ to zero and increasing $\hat{F}_E(u_T - H)$ by the necessary amount $\Delta$ such that it takes a value of 1.

Recall that the total expected vote share of candidate $A$ playing against the equilibrium strategy is:

$$\beta_E S(\hat{F}_E^*, \hat{F}_E) + (1 - \beta_E) S(\hat{F}_T^*, \hat{F}_E),$$

where $\beta_E = 1 - \frac{2H}{u_T - l_T}$ is the equilibrium probability to play bundle $E$.

The above described shift in $M_{HE}$ changes this total expected vote share by

$$M_{HE}\left[\beta_E \frac{1}{u_E - l_E} - (1-\beta_E)\frac{1}{2}\frac{1}{H}\right]. \tag{B.13}$$

On the other hand, the above described shift in $\hat{F}_E(u_T - H)$ changes this total expected vote share by

$$-\Delta(u_T - H)\left[\beta_E \frac{1}{u_E - l_E} - (1-\beta_E)\frac{1}{2}\frac{1}{H}\right]. \tag{B.14}$$

We have already argued above that for $\beta_E = 1 - \frac{2H}{u_T - l_T}$, the term in square brackets is positive. Recall that $\Delta$ is the mass of offers in the high interval and $M_{HE}$ is the total amount of money spent on all offers in the high interval. Hence, we have $M_{HE} > \Delta(u_T - H)$, because $u_T - H = u_E$ is the upper bound of the middle interval and offers in the high interval must hence lie above this value. In total, we can then conclude that effect (B.13) dominates effect (B.14). Therefore, the shift towards the middle interval increases the total expected vote share and any distribution with offers in the high interval cannot be a best response to the equilibrium strategy.

Therefore, when checking for profitable deviations from the equilibrium distribution under bundle $E$, we can concentrate on distributions $\hat{F}_E$ that have no offers in the high interval and hence satisfy $M_{HE} = 0$ and $\hat{F}_E(u_T - H) = 1$. As can be seen from (B.11) and (B.12), the vote share of any such reform distribution when meeting the equilibrium reform or no-reform distribution is $\frac{1}{2}$ in either case. In particular, also the equilibrium reform distribution $\hat{F}_E^*$ achieves the same outcome, and hence there is no profitable deviation from $\hat{F}_E^*$.

**Step III: Optimality of $\beta_I = \beta_Q = 0$.** Before arguing for the optimality of the exact values of the probabilities for bundles $E$ and $T$, we must still argue that bundles $I$ and $Q$ are played with zero probability in equilibrium.

To do so, we show that for $I$ and $Q$ any possible associated distribution is beaten by the equilibrium strategy. Together with the results established in steps I and II, this will imply that any strategy that plays $I$ or $Q$ with positive probability will be beaten for sure by the equilibrium strategy.

Both bundles $I$ and $Q$ are dominated in efficiency by bundles $E$ and $T$: $M_E > M_T > M_Q > M_I$. Furthermore, both $I$ and $Q$ are dominated by $T$ in terms of targetability: $u_T > u_I > u_Q$.

In step (a) of the following argument, we will use the facts $M_Q < M_E$ and $l_Q < l_T + H$ to prove that any distribution $\hat{F}_Q$ associated to bundle $Q$ that is supposed to be a best response to the equilibrium strategy cannot have offers in the middle interval $[l_E, u_E]$, the support of $\hat{F}_E^*$. This will establish that $\hat{F}_Q$ as a best response to the equilibrium strategy should only have offers on the support of $\hat{F}_T^*$. But then, in step (b), we can use the fact $M_Q < M_T$ to show that any $\hat{F}_Q$ will lose against the equilibrium strategy.

To see $l_T + H - l_Q > 0$ note that:

$$\begin{aligned}
& l_T + H - l_Q \\
=& l_T + l_E - l_T - 2(M_E - M_T) - l_Q \\
=& l_E - l_Q - 2(M_E - M_T) \\
>& l_E - l_Q - 2(M_E - M_Q) \\
&= u_Q - u_E > 0.
\end{aligned}$$

The first inequality used the fact that $M_T > M_Q$ and the last inequality holds by the assumption on the targetability ranking of the different bundles. In the alternative specification for the targetability rankings, the assumption $l_Q \leq l_T$ is included and so $l_T + H - l_Q > 0$ holds by assumption.

*Step (a):* To show that any $\hat{F}_Q$ that is supposed to be a best response to the equilibrium strategy should only have offers on the support of $\hat{F}_T^*$, we argue that the reasoning in step (I), which proved the same for any distribution $\hat{F}_T$ associated to bundle $T$, still goes through. When considering the performance against the equilibrium strategy, the difference between a distribution $\hat{F}_T$ associated to bundle $T$, as done in step (I), and a distribution $\hat{F}_Q$ associated to bundle $Q$ lies in the budget parameter $M_Q < M_T$ and the lower bound $l_Q$, which is potentially different from $l_T$. Since $l_Q < l_T + H$, when competing against the equilibrium strategy, $\hat{F}_Q$ will not offer in the interval $(l_T + H, l_E]$, as was the case for $\hat{F}_T$.

When $l_Q \geq l_T$, then nothing changes in the vote share formulas from step (I) except for $M_T$ being replaced by $M_Q$.[9] In the arguments of step (I) to establish that a best response $\hat{F}_T$ to the equilibrium strategy cannot have offers on the support of $\hat{F}^*_E$, the only time when $M_T$ played a role was when establishing that a distribution that played offers only on the support of $\hat{F}^*_E$ would lose for sure. To show this, the fact $M_T < M_E$ was used. Since $M_Q < M_E$, we can make the same argument for bundle $Q$.

In the case of $l_Q < l_T$, there is a change in the formula for $S(\hat{F}^*_T, \hat{F}_Q)$ and in the budget constraint, because the possibility for offers below $l_T$, the lowest possible offer under $\hat{F}^*_T$, arises. Since $\hat{F}^*_T$ and $\hat{F}^*_E$ only offer above $l_T$, any offer below $l_T$ will lose for sure and only makes sense in order to finance an increase in the offers above $l_T$. Therefore, all offers below $l_T$ should be chosen as a mass point at the lowest possible offer under bundle $Q$, $l_Q$, in order to finance the the highest possible offers above $l_T$. The budget constraint under which $\hat{F}_Q$ is chosen becomes:

$$M_{LQ} + M_{HQ} + M_{MQ} + l_Q \hat{F}_Q(l_Q) = M_Q, \tag{B.15}$$

where the $M$ terms are defined analogously to step (I). The vote share formula against $\hat{F}^*_T$ becomes:[10]

$$\begin{aligned} S(\hat{F}^*_T, \hat{F}_Q) =& \int_{l_Q}^{u_T} \hat{F}^*_T(x) d\hat{F}_Q(x) \\ =& \frac{1}{2} \left\{ \frac{M_{LQ} + M_{HQ}}{H} + \frac{l_T}{H} \hat{F}_Q(l_T) - \frac{l_T + H}{H} \hat{F}_Q(l_T + H) \right. \\ & \left. + \frac{u_T - H}{H} \hat{F}_Q(u_T - H) + \frac{2H - u_T}{H} \right\}. \end{aligned} \tag{B.16}$$

Recall that the goal is to construct a profitable deviation analogous to step (I) whenever $\hat{F}_Q$ has offers in the middle interval $(l_E, u_E]$ below the upper bound $u_E$. We can again show by using $M_Q < M_E$ that $\hat{F}_Q$ must have some offers outside the interval $(l_E, u_E]$. That is there must be a positive mass of offers either somewhere on $T's$ equilibrium support, $[l_T, l_T + H]$ or $(u_E, u_T]$, or as a mass point at $l_Q$.

Now the first possibility is that there are enough offers on $[l_T, l_T + H]$ or $(u_E, u_T]$ that can be decreased inside these intervals such that all offers in the middle interval

[9]For $l_Q > l_T$, there is a change in the formulas for the special case dealing with a mass point at $l_T$, which is now at $l_Q > l_T$. However, all the arguments for this special case still go through.

[10]In step (I), the analogous term to $\frac{l_T}{H}\hat{F}_Q(l_T)$, did not appear in the vote share formula, because the only mass smaller or equal to $l_T$ would be concentrated as a mass point at $l_T$. It can be shown that, as long as there are offers somewhere on the support of $T$ beside $l_T$, the vote share stays the same by shifting this mass from $l_T$ to slightly above $l_T$, while decreasing some other offers on the support of $T$. When necessary, the case where the only offers on the support of $T$ are at $l_T$, have been dealt with in the previous proofs. Now however, all mass smaller or equal to $l_T$ is concentrated as a mass point at $l_Q < l_T$, and shifting this mass up to above $l_T$ is no longer necessarily neutral on the vote share.

$(l_E, u_E]$ can be shifted up to $u_E$. In this case, $M_{LQ}+M_{HQ}$ decreases just by the amount by which $M_{MQ}$ increases but the masses in the different intervals are not changed. In that case the new terms in the formulas compared to step (I) are not affected and the argument from step (I) can be applied.

The second possibility is that, even when shifting all the offers in $[l_T, l_T + H]$ down to $l_T$ and all the offers in $(u_E, u_T]$ down to $u_E$,[11] there are still some offers in the middle interval whose shift up to $u_E$ could not yet be financed. We have to show that even in this situation there is still a possible profitable deviation. However, this deviation is easy to see, because the offers at $l_T$ can be decreased down to $l_Q$ in order to gain additional money that can be spent on shifting offers in the middle interval up. Since the offers at $l_T$, the lower bound of $T$'s equilibrium support, were also not winning before the shift down to $l_Q$, such a deviation definitely increases the vote share.

The last possibility is that we have no offers in the upper part of $T's$ equilibrium support, $(u_E, u_T]$, and all the offers from the low part $[l_T, l_T + H]$ have been shifted down to $l_Q$ and there are still some offers in the middle interval $(l_E, u_E]$ below the upper bound $u_E$. In this case, we have to redo the arguments that were done in step (I) for the special case where there were no offers in $(u_E, u_T]$ and all offers in $[l_T, l_T+H]$ were concentrated at $l_T$. These arguments still go through.

In total, we have therefore shown that for all possible constellations, the arguments from step (I) still apply and there is always a profitable deviation once there are offers in the middle interval that are not concentrated at its upper bound $u_E$.

For any $l_Q < l_T + H$, it can also still be shown in the same manner as before that any distribution $\hat{F}_Q$ that has some offers concentrated at $u_E$ can be approximated by a distribution $\hat{F}'_Q$ that has no offers in the middle interval $(l_E, u_E]$ and achieves the same vote share against the equilibrium strategy. Therefore, there is always a distribution $\hat{F}_Q$ that has no offers in the middle interval and that performs at least as good as any strategy that has some offers in the middle interval. This establishes that when considering possible best responses $\hat{F}_Q$ to the equilibrium strategy, we can focus on distributions that only play offers on the support of $\hat{F}^*_T$ (and potentially at $l_Q$ if $l_Q < l_T$).

[11] Note that this later change implies a change in $\hat{F}_Q(u_E)$ besides the shift in the $M$ terms. However, using an argument similar to the one in step (II) above, we can show that a shift from offers above $u_E$ to $u_E$ in order to finance higher offers in the middle interval still increases the total vote share.

*Step (b):* In step (I), after having established that a best response $\hat{F}_T$ would only play offers on the support of $\hat{F}_T^*$, the expected vote share using $\hat{F}_T$ was calculated to be:

$$\frac{M_T - l_T}{u_T - l_T},$$

which was shown to be equal to 1/2 using the definition of $u_T$. If $l_Q \geq l_T$, we can proceed analogously. In particular, having established in step (a) above that a best response $\hat{F}_Q$ under bundle $Q$ would only play offers on the support of $\hat{F}_T^*$, the expected vote share of playing any best-response distribution $\hat{F}_Q$ against the equilibrium strategy will now be:

$$\begin{aligned} &\frac{M_Q - l_T}{u_T - l_T} \\ <&\frac{M_T - l_T}{u_T - l_T} = \frac{1}{2}. \end{aligned}$$

The last step used the fact $M_Q < M_T$.

Similarly, if $l_Q < l_T$ then the expected vote share of playing any best-response distribution $\hat{F}_Q$ against the equilibrium strategy is:

$$\begin{aligned} &\frac{M_Q - l_T}{u_T - l_T} + \frac{(l_T - l_Q)}{u_T - l_T}\hat{F}_Q(l_Q) \\ \leq&\frac{M_Q - l_T}{u_T - l_T} + \frac{(l_T - l_Q)/2}{u_T - l_T} \\ =&\frac{(u_Q - l_T)/2}{u_T - l_T} \\ <&\frac{u_T - l_T}{u_T - l_T} = \frac{1}{2}, \end{aligned}$$

where the first weak inequality follows from $\hat{F}_Q(l_Q) \leq 1/2$. If $\hat{F}_Q$ were to put more than 1/2 of offers at $l_Q < l_T$, then it would lose for sure against the equilibrium strategy. The last step holds due to the targetability ranking $u_T > u_Q$.

This shows that any $\hat{F}_Q$ that is a best response to the equilibrium strategy will lose against the equilibrium strategy and therefore bundle $Q$ should be played with zero probability.

Analogous steps can be used for proving that bundle $I$ will not be played in equilibrium, since the same facts that were used to prove the case of $Q$ also apply to bundle $I$.

**Step IV: Optimality of $\beta_E = 1 - \frac{2H}{u_T - l_T}$ and $\beta_T = 1 - \beta_E$.** Having established that only bundles $E$ and $T$ will be played with positive probability in equilibrium we can now argue for the optimality of the respective equilibrium probabilities $\beta_E = 1 - \frac{2H}{u_T - l_T}$ and $\beta_T = 1 - \beta_E$. When candidate $B$ plays the equilibrium strategy, candidate $A$ is indifferent between playing bundle $E$ and bundle $T$: In steps (I) and (II), we have shown that for all distributions $\hat{F}_E$ and $\hat{F}_T$ played under these two bundles that are potential best responses to the equilibrium strategy, the total expected vote share is $\frac{1}{2}$ when playing against the equilibrium strategy. Therefore, candidate $A$ is happy to play bundle $E$ with a probability of $\beta_E = 1 - \frac{2H}{u_T - l_T}$ and bundle $T$ with probability $\beta_T = 1 - \beta_E$.

### B.3.6 Proof of Proposition B.2.2:

The proof of existence has five steps: Step I shows that there is no profitable deviation from the equilibrium strategy by deviating from $\hat{F}_T^*$, step II shows that it is not profitable either to deviate on $\hat{F}_I^*$, step III shows that it is not profitable to deviate on $\hat{F}_E^*$, and step IV shows that bundle $Q$ will be played with probability zero. Finally, step V argues for the optimality of the equilibrium probabilities with which bundles $T$, $I$, and $E$ are played.

**Step I: Optimality of $\hat{F}_T^*$.** Consider candidate $A$ when he decides to play the debt-reform bundle $T$ and assume he deviates from the equilibrium distribution under this bundle, $\hat{F}_T^*$, to another distribution $\hat{F}_T$. When this candidate $A$ meets a candidate $B$ that chooses also the bundle $T$ and that offers money according to $\hat{F}_T^*$, the vote share of candidate $A$ is:

$$\begin{aligned} S(\hat{F}_T^*, \hat{F}_T) = &\int_{l_T}^{u_T} \hat{F}_T^*(x) d\hat{F}_T(x) \\ =&\frac{1}{2}\left\{\frac{M_{L_T T} + M_{H_T T}}{H''} - \frac{l_T + H''}{H''}\hat{F}_T(l_T + H'')\right. \\ &\left. + \frac{u_T - H''}{H''}\hat{F}_T(u_T - H'') + \frac{2H'' - u_T}{H''}\right\}, \end{aligned} \tag{B.17}$$

where

$$M_{L_T T} = \int_{l_T}^{l_T + H''} x d\hat{F}_T(x), \tag{B.18}$$

and

$$M_{H_T T} = \int_{u_T - H''}^{u_T} x d\hat{F}_T(x). \tag{B.19}$$

Equations (B.18) and (B.19) capture the money spent under distribution $\hat{F}_T$ on transfers in the low interval $[l_T, l_T + H'']$ and high interval $(u_T - H'', u_T]$, respectively. These two intervals make up the support of the equilibrium distribution $\hat{F}_T^*$ under bundle $T$.[12] In the following, we will refer to this support as the equilibrium support of $T$. Similarly, we will refer to $[l_I, l_I + H']$ and $(u_I - H', u_I]$, the support of $\hat{F}_I^*$, as the equilibrium support of $I$. Finally, we refer to $[l_E, u_E]$, the support of $\hat{F}_E^*$, as the equilibrium support of $E$.[13]

Candidate $A$ chooses $\hat{F}_T$ under the constraint:

$$M_{L_T T} + M_{H_T T} + M_{MT} = M_T, \tag{B.20}$$

where

$$M_{MT} = \int_{l_T + H''}^{u_T - H''} x d\hat{F}_T(x) \tag{B.21}$$

is the money spent on transfer offers in the middle interval $(l_T + H'', u_T - H'']$. This middle interval is made up by the equilibrium supports of $I$ and $E$, except for the intervals $(l_T + H'', l_I)$ and $(l_I + H', l_E)$. A best response to the equilibrium strategy will never offer in these latter intervals though. This is because, for instance, offers in the interval $(l_I + H', l_E)$ are more costly than offering $l_I + H'$, but do not win more votes than offers at $l_I + H'$.[14] As before, $M_T$ denotes the resources beyond existing endowments under bundle $T$.

In the following, we will argue that, when checking for profitable deviations from the equilibrium distribution $\hat{F}_T^*$ that is played in case of bundle $T$, we can concentrate on distributions $\hat{F}_T$ that have no offers in the middle interval $(l_T + H'', u_T - H'']$.

[12] This is why the subscripts $L$ and $H$ in the above formulas, which stand for the low and high part of a given support, are themselves indexed by the letter $T$.

[13] We choose the intervals of the upper parts of the equilibrium supports of $T$ and $I$ to be open to the left, because the lower bounds of these intervals are at the same time the upper bounds of the intervals below, which belong to a different support. To have the intervals open is without loss of generality because the continuous equilibrium distributions $\hat{F}_T^*$ and $\hat{F}_I^*$ put zero mass on the single offers at these lower bounds of the respective parts in their supports. In order to win a positive mass of voters in the respective parts of their supports, the respective offers must therefore be strictly higher than these lower bounds.

[14] Since we are considering a distribution $\hat{F}_T$, whose lowest bound $l_T$ lies below the middle interval $(l_T + H'', u_T - H'']$, we can also exclude offers at $l_E$ and $l_I$, the lower bounds of $E$'s and $I$'s equilibrium supports.

Said differently, $\hat{F}_T$ should not contain offers on the equilibrium supports of $I$ and $E$. Therefore, assume that $\hat{F}_T$ has a positive mass of offers on the equilibrium supports of $I$ and $E$.

*Step (A): If $\hat{F}_T$ only has offers on the equilibrium supports of $I$ and $E$, then it will be beaten by the equilibrium strategy.*

Once $\hat{F}_T$ is restricted to only operate on the equilibrium supports of $I$ and $E$, the facts $M_T < M_I$ and $M_T < M_E$ can be used to show that $\hat{F}_T$ will lose against the equilibrium strategy.

*Step (A1): $\hat{F}_T$ will lose against the equilibrium strategy when it only offers on the equilibrium support of $E$.*

Since 50 percent of the offers of $\hat{F}_T^*$ and $\hat{F}_I^*$ lie above (respectively below) the equilibrium support of $E$, a distribution $\hat{F}_T$ that only offers on the equilibrium support of $E$ will tie against $\hat{F}_T^*$ and $\hat{F}_I^*$. Applying the fact $M_T < M_E$, it will lose against $\hat{F}_E^*$ (see step (I) in the proof of Proposition B.2.1). Since bundle $E$ is played with positive probability by the equilibrium strategy, this means that $\hat{F}_T$ will lose against the equilibrium strategy.

*Step (A2): When $\hat{F}_T$ is restricted to operate on the equilibrium supports of $E$ and $I$, then a best response to the equilibrium strategy should only offer on the equilibrium support of $I$.*

Using step (A1) and following analogous arguments as in step (I) of the proof of Proposition B.2.1, we can show this in the following way. When $\hat{F}_T$ offers a positive amount on the equilibrium support of $E$, then there is a profitable deviation from $\hat{F}_T$ given that the following condition holds:

$$
\begin{aligned}
&\beta_I \frac{1}{2H'} < \beta_E \frac{1}{u_E - l_E} \\
\Leftrightarrow &\frac{\beta_I}{\beta_E} \frac{1}{2H'} < \frac{1}{u_E - l_E}
\end{aligned}
$$

Plugging in the equilibrium values of $\beta_I$ and $\beta_E$, the above becomes:

$$\begin{aligned}
&\frac{2H'}{u_I - l_I - 2H'}\frac{1}{2H'} < \frac{1}{u_E - l_E}\\
\Leftrightarrow & l_I + H' < l_E\\
\Leftrightarrow & 2M_I - u_I + (u_I - u_E) < 2M_E - u_E\\
\Leftrightarrow & M_E > M_I.
\end{aligned}$$

The last inequality holds by the assumptions on the efficiency ranking. Hence it follows that when restricted to only offering on the equilibrium supports of $E$ and $I$, $\hat{F}_T$ should only offer on the equilibrium support of $I$.

*Step (A3): When $\hat{F}_T$ offers only on the equilibrium support of $I$, then it loses to the equilibrium strategy.*

When $\hat{F}_T$ offers only on the equilibrium support of $I$, then the vote shares against the different equilibrium distributions are:

$$\begin{aligned}
S(\hat{F}_I^*, \hat{F}_T) &= \frac{1}{2}\left[\frac{M_T}{H'} + \frac{u_I - l_I - 2H'}{H'}\hat{F}_T(u_I - H') + \frac{2H' - u_I}{H'}\right],\\
S(\hat{F}_E^*, \hat{F}_T) &= 1 - \hat{F}_T(u_I - H'),\\
S(\hat{F}_T^*, \hat{F}_T) &= 1/2.
\end{aligned}$$

Hence, the total expected vote share from playing such a distribution $\hat{F}_T$ is:

$$\begin{aligned}
&\beta_I\frac{1}{2}\left[\frac{M_T}{H'} + \frac{u_I - l_I - 2H'}{H'}\hat{F}_T(u_I - H') + \frac{2H' - u_I}{H'}\right]\\
&+ \beta_E\left[1 - \hat{F}_T(u_I - H')\right] + \beta_T\frac{1}{2}\\
=&\beta_I\frac{1}{2}\left[\frac{M_T}{H'} + \frac{2H' - u_I}{H'}\right] + \beta_E + \beta_T\frac{1}{2}\\
<&\beta_I\frac{1}{2}\left[\frac{M_I}{H'} + \frac{2H' - u_I}{H'}\right] + \beta_E + \beta_T\frac{1}{2}\\
=&\frac{u_T - l_T - 2H''}{u_T - l_T}\frac{M_I - l_I}{u_I - l_I} + \frac{2H''}{u_T - l_T}\frac{1}{2}\\
=&\frac{u_T - l_T - 2H''}{u_T - l_T}\frac{1}{2} + \frac{2H''}{u_T - l_T}\frac{1}{2}\\
=&\frac{1}{2}.
\end{aligned}$$

The inequality just used the fact that $M_T < M_I$. Hence, when $\hat{F}_T$ offers only on the equilibrium support of $I$, then it loses to the equilibrium strategy.

Combining steps (A1)-(A3), we have therefore shown the claim raised at the beginning of step (A) that if $\hat{F}_T$ only has offers on the equilibrium supports of $I$ and $E$, then it will be beaten by the equilibrium strategy.

*Step (B): If $\hat{F}_T$ spends a positive amount of offers, but not all of its offers (see step A), on the equilibrium supports of $I$ and $E$ below the upper bound $u_I$ of $I$'s equilibrium support, then we can construct a profitable deviation along the lines of step (I) in the proof of Proposition B.2.1.*

*Step (B1): If $\hat{F}_T$ spends a positive amount of offers on the equilibrium support of $I$ below $u_I$, then we can construct a profitable deviation.*

Under the conditions of step (B1), we can construct a deviation that leaves the mass of offers in all intervals the same, but increases the amount of money spent on the equilibrium support of $I$,[15] while decreasing the amount of money spent on the equilibrium support of $T$.[16] Using arguments analogous to step (I) in the proof of Proposition B.2.1, such a deviation is profitable if

$$\begin{aligned} &\beta_T \frac{1}{2H''} < \beta_I \frac{1}{2H'} \\ \Leftrightarrow &\frac{\beta_T}{\beta_I} < \frac{H''}{H'}. \end{aligned}$$

[15]That is, the offers inside this support are shifted upwards. For the case where all offers in $I$ are concentrated at $l_I + H$, the upper bound of the lower part of $I$'s equilibrium support, offers must be shifted to the high part of $I$'s equilibrium support and so masses change. However, the equilibrium probabilities are constructed in a manner that this mass shift does not change the total expected vote share and so the only effect on the vote share is still the change in the amount of money spent on the equilibrium support of $I$.

[16]That is, the offers inside this support are shifted downwards. The case where all offers on the equilibrium support of $T$ are concentrated at $l_T$ and cannot be decreased can be handled in the same manner as in in the proof of Proposition B.2.1.

Plugging in the equilibrium values of $\beta_T$ and $\beta_I$, the above becomes:

$$\begin{aligned}
&\frac{H''}{H'}\frac{u_I - l_I}{u_I - (l_T + H'')} < \frac{H''}{H'} \\
\Leftrightarrow & l_T + H'' < l_I \\
\Leftrightarrow & 2M_T - u_T + (u_T - u_I) < 2M_I - u_I \\
\Leftrightarrow & M_I > M_T.
\end{aligned}$$

The last inequality holds again by the assumptions on the efficiency ranking.

*Step (B2): If $\hat{F}_T$ spends a positive amount of offers on the equilibrium support of $E$, then we can construct a profitable deviation.*

The first possibility is that there are some offers on the equilibrium support of $E$ below $u_E$. Then we can construct a deviation that leaves the mass of offers in all intervals the same, but increases the amount of money spent on the equilibrium support of $E$, while decreasing the amount of money spent on the equilibrium support of $T$.[17] Such a deviation is profitable if

$$\beta_T \frac{1}{2H''} < \beta_E \frac{1}{u_E - l_E}.$$

But since we have already established $\beta_T \frac{1}{2H''} < \beta_I \frac{1}{2H'}$ in step (B1) and $\beta_I \frac{1}{2H'} < \beta_E \frac{1}{u_E - l_E}$ in step (A2), this follows immediately.

The other possibility is that all offers on the equilibrium support of $E$ are concentrated at the upper bound $u_E$. Using a similar argument as in the proof of Proposition B.2.1, where it was argued that any distribution with a mass point at $u_E$ achieves the same vote share as a distribution that shifts this mass into the equilibrium support of $T$, it can then be shown that in the case here, a distribution with a mass point at $u_E$ achieves the same vote share as one that shifts all the mass from $u_E$ to the equilibrium support of $I$. But this case was already handled in step (B1).

Together, steps (B1) and (B2) can be used to argue that when there are offers on the equilibrium supports of $I$ and $E$ below the upper bound $u_I$ of $I$'s equilibrium support, then we can always construct profitable deviations until all these offers are concentrated at $u_I$. At that point, applying one last time the argument that such a

[17] The case where all offers on the equilibrium support of $T$ are concentrated at $l_T$ and cannot be decreased can again be handled in the same manner as in the proof of Proposition B.2.1.

distribution achieves the same vote share as one that shifts this mass into the equilibrium support of $T$, completes the proof of step (B).

In total, combining steps (A) and (B), we have therefore shown that any distribution $\hat{F}_T$ played under bundle $T$ that is supposed to be a best response to the equilibrium strategy should have no offers on the equilibrium supports of $E$ and $I$. Said differently, we can concentrate on distributions $\hat{F}_T$ that have only offers on the equilibrium support of $T$.

For such distributions $\hat{F}_T$, the total expected vote share against the equilibrium strategy is:

$$\begin{aligned}
&\beta_T S(\hat{F}_T^*, \hat{F}_T) + (1-\beta_T)(1-\hat{F}_T(u_T - H''))\\
=&\beta_T \frac{M_T + 2H'' - u_T}{2H''} + (1-\beta_T).
\end{aligned}$$

Plugging in the equilibrium value for $\beta_T$, this becomes:

$$\begin{aligned}
&\frac{M_T - l_T}{u_T - l_T}\\
=&\frac{(u_T - l_T)/2}{u_T - l_T} = \frac{1}{2}.
\end{aligned}$$

In particular, note that $\hat{F}_T^*$ by definition also fulfills the above requirement that it only offers on the equilibrium support of $T$. Hence, there is no profitable deviation from the equilibrium strategy by deviating from the equilibrium distribution $\hat{F}_T^*$, which is played under bundle $T$.

**Step II: Optimality of $\hat{F}_I^*$.** Similar to step (I), we will argue that when checking for profitable deviations from the equilibrium distribution $\hat{F}_I^*$, which is played in case of bundle $I$, we can concentrate on distributions $\hat{F}_I$ that have no offers on the equilibrium supports of $T$ and $E$. Assume therefore that $\hat{F}_I$ spends a positive mass of offers on the equilibrium supports of $T$ and $E$.

*Step (A): $\hat{F}_I$ will lose against the equilibrium strategy when it only offers on the equilibrium support of $E$.*

Since 50 percent of the offers of $\hat{F}_T^*$ and $\hat{F}_I^*$ lie above (respectively below) the equilibrium support of $E$, a distribution $\hat{F}_I$ that only offers on the equilibrium support of $E$ will tie against $\hat{F}_T^*$ and $\hat{F}_I^*$. Applying the fact $M_I < M_E$, it will lose against $\hat{F}_E^*$ (see step (I) in the proof of Proposition B.2.1). Since bundle $E$ is played with positive probability by the equilibrium strategy, this means that $\hat{F}_I$ will lose against the equilibrium strategy.

From step (A) we can conclude that any $\hat{F}_I$ that is supposed to be a best response to the equilibrium strategy must have some offers on the equilibrium supports of $I$ or $T$. In the following step, we will show that a best response $\hat{F}_I$ should have no offers on the equilibrium support of $T$. Before we turn to this next step, note the following. In step (B1) of the proof of the optimality of $\hat{F}_T^*$, we have already shown that $l_T + H'' < l_I$. Since $l_I$ is the lowest offer that $\hat{F}_I$ can make, $\hat{F}_I$ can have no offers on the lower part of the equilibrium support of $T$, $[l_T, l_T + H'']$. Hence, we only have to exclude offers on the high part of $T$'s equilibrium support, $(u_T - H'', u_T]$, in order to exclude offers on the total equilibrium support of $T$. Since $u_T - H'' = u_I$ and $M_I = \frac{u_I + l_I}{2}$ with $l_I < u_I$, $\hat{F}_I$ cannot only have offers on the high part of $T$'s equilibrium support, $(u_T - H'', u_T]$, without violating the budget constraint. Hence, in order to exclude offers on the equilibrium support of $T$, it is sufficient to show the following:

*Step (B): When $\hat{F}_I$ has some offers on the high part of $T$'s equilibrium support, $(u_T - H'', u_T]$, and has the remaining offers on the equilibrium supports of $E$ or $I$, then the vote share can be increased by shifting the offers from $(u_T - H'', u_T]$ down towards the lower intervals.*

*Step (B1): The claim in step (B) is true when some of the remaining offers besides the offers on $(u_T - H'', u_T]$ are on the equilibrium support of $I$.*

Note that this includes the case where all the remaining offers are on the equilibrium support of $I$. What remains to be shown in step (B2) is then the case where all the remaining offers are on the equilibrium support of $E$.

To show step (B1) consider a distribution $\hat{F}_I$ that has some offers on $(u_T - H'', u_T]$

and some of the remaining offers are on the equilibrium support of $I$. The vote share formulas in this case become:

$$
\begin{aligned}
S(\hat{F}_I^*, \hat{F}_I) =& 1 - \hat{F}_I(u_I) + \frac{1}{2}\left[\frac{M_{L_I I} + M_{H_I I}}{H'} + \frac{2H' - u_I}{H'}\hat{F}_I(u_I)\right. \\
&\left. - \frac{l_I + H'}{H'}\hat{F}_I(l_I + H') + \frac{u_I - H'}{H'}\hat{F}_I(u_E)\right],
\end{aligned}
$$

$$
S(\hat{F}_E^*, \hat{F}_I) = 1 - \hat{F}_I(u_E) + \frac{M_{MI}}{u_E - l_E} - \frac{l_E}{u_E - l_E}\left[\hat{F}_I(u_E) - \hat{F}_I(l_E)\right],
$$

$$
S(\hat{F}_T^*, \hat{F}_I) = \frac{1}{2}\left[1 - \frac{u_T - H''}{H''}\left(1 - \hat{F}_I(u_I)\right) + \frac{M_{H_T I}}{H''}\right].
$$

where

$$
M_{MI} = \int_{l_E}^{u_E} x d\hat{F}_E(x)
$$

is the money that $\hat{F}_I$ spends in the middle interval, the equilibrium support of $E$, $[l_E, u_E]$. Similarly, $M_{L_I I}$ is the money spent on the lower part of the equilibrium support of $I$, $[l_I, l_I + H']$, while $M_{H_I I}$ is the money spent on the higher part of the equilibrium support of $I$, $(u_I - H'', u_I] = (u_E, u_I]$. Finally, $M_{H_T I}$ is the money spent on the higher part of the equilibrium support of $T$, $(u_T - H'', u_T] = (u_I, u_T]$.

In the following, we show that any distribution $\hat{F}_I$ that has some offers on the higher part of the equilibrium support of $T$, $(u_I, u_T]$, performs worse against the equilibrium strategy than a deviation $\widetilde{F_I}$ that shifts all these offers down to the high part of the equilibrium support of $I$, $(u_E, u_I]$. Hence, consider a distribution $\hat{F}_I$ that has $M_{H_T I} > 0$ and $\hat{F}_I(u_I) < 1$. If we decrease all the offers that are greater than $u_I$ down to $u_I$, then $\hat{F}_I(u_I)$ increases by $\Delta = 1 - \hat{F}_I(u_I)$. Furthermore, the money $M_{H_I I}$ spent in the high part of $I$'s equilibrium support increases by $\Delta \cdot u_I$. Since all these offers were previously greater then $u_I$, there is a positive amount of money $M_A = M_{H_T I} - \Delta \cdot u_I > 0$ additionally available for spending. Without this additional money the total expected vote share stays constant:

Through increasing $M_{H_I I}$ by $\Delta \cdot u_I$ while decreasing $M_{H_T I}$ by the same amount, the total expected vote share changes by:

$$\Delta \cdot u_I \left[\beta_I \frac{1}{2H'} - \beta_T \frac{1}{2H''}\right].$$

Through shifting $\hat{F}_I(u_I)$ up by $\Delta$ to reach the value one, the total expected vote share changes by:

$$\Delta \left[\beta_I \left(-1 + \frac{2H' - u_I}{2H'}\right) + \beta_T \frac{u_I}{2H''}\right]$$
$$= - \Delta \cdot u_I \left[\beta_I \frac{1}{2H'} - \beta_T \frac{1}{2H''}\right].$$

In total, the two changes cancel out. Hence, the change in the vote share depends on what happens with the additional amount of money $M_A > 0$.

*Case (i): There are enough offers in the low or the high part of I's equilibrium support, that can be shifted upwards to use up the additional money without changing the mass or money spent in any other interval.*

In this case the only additional change in the vote share formulas is that $M_{L_I I} + M_{H_I I}$ increases by $M_A$ while $M_{H_T I}$ decreases by $M_A$. The change in the expected vote share from this is:

$$M_A \left[\beta_I \frac{1}{2H'} - \beta_T \frac{1}{2H''}\right].$$

In step (B1) of the $\hat{F}_T^*$-optimality proof, it was shown that the term in square brackets is positive. Therefore, the total expected vote share increases.

*Case (ii): There are enough offers in the equilibrium support of E, that can be shifted upwards to use up the additional money without changing the mass or money spent in any other interval.*

In this case the only additional change in the vote share formulas is that $M_{MI}$ increases by $M_A$ while $M_{H_T I}$ decreases by $M_A$. The change in the expected vote share from this is:

$$M_A\left[\beta_E\frac{1}{u_E-l_E}-\beta_T\frac{1}{2H''}\right]$$

In step (B2) of the $\hat{F}_T^*$-optimality proof it was shown that the term in square brackets is positive. Therefore the total expected vote share increases.

Note that the arguments in case (i) and (ii) also imply that the vote share increases when the amounts of money spent on the equilibrium supports of $E$ and $I$ are increased simultaneously but the masses in all intervals stay the same.

*Case (iii): All the offers in the low and high part of $I$'s equilibrium support and in the equilibrium support of $E$ are concentrated at their upper bounds.*

In this case, the additional money $M_A$ can only be spent by shifting mass between different intervals. Note that the offers at $l_I+H'$, the upper bound of the low part of $I$'s equilibrium support, must counterbalance all the other offers for budget balance to hold.[18] This means that there are always enough offers at $l_I+H'$ so that we can shift enough mass from $l_I+H'$ to the upper part of $I$'s equilibrium support, $(u_E,u_I]$, in order to spent all the additional money $M_A$ on the equilibrium support of $I$. Such a shift of mass is captured in the vote share formulas by a decrease in $\hat{F}_I(u_E)$, $\hat{F}_I(l_I+H')$, and $\hat{F}_I(l_E)$ by the same amount $\mu$.

Hence, consider decreasing $\hat{F}_I(u_E)$, $\hat{F}_I(l_I+H')$, and $\hat{F}_I(l_E)$ by the necessary amount $\mu$ such that $M_{L_I I}+M_{H_I I}$ can be increased by $M_A$. At the same time $M_{H_T I}$ is decreased by $M_A$. The resulting change in the total expected vote share from the change in $\hat{F}_I(u_E)$, $\hat{F}_I(l_I+H')$, and $\hat{F}_I(l_E)$ is:

$$\begin{aligned}&\beta_E\mu+\beta_I\left[-\frac{u_I-l_I-2H'}{2H'}\right]\mu\\=&\mu\frac{u_T-l_T-2H''}{u_T-l_T}\left[\frac{u_I-l_I-2H'}{u_I-l_I}-\frac{2H'}{u_I-l_I}\frac{u_I-l_I-2H'}{2H'}\right]\\=&0.\end{aligned}$$

[18] For instance, if all offers were concentrated at the upper bound $u_E$ of $E$'s equilibrium support, the total money spent is $1\cdot u_E$. However, the available money by the resource constraint is only $M_I=\frac{u_I+l_I}{2}=\frac{u_E+(l_I+H')}{2}<u_E$ since $l_I+H'<l_E<u_E$.

That is the equilibrium probabilities are constructed in a way that the above described shift in mass does not alter the vote share. The change in the total expected vote share is therefore completely determined by the increase of $M_{L_I I} + M_{H_I I}$ by $M_A$ and the corresponding decrease of $M_{H_T I}$. The resulting change in the total expected vote share is

$$M_A \left[ \beta_I \frac{1}{2H'} - \beta_T \frac{1}{2H''} \right],$$

which is positive. We have therefore shown that also in case (iii) the total expected vote share increases by shifting all the offers down from the high part of $T$'s equilibrium support.

In total, the arguments for cases (i) to (iii) imply that we can always spent the additional money $M_A$ by first shifting up the offers in the low and high part of $I$'s equilibrium support and in $E$'s equilibrium support up to their respective upper bounds.[19] Any money that then remains can be used for a mass shift as described in case (iii). In total all such uses of $M_A$ increase the total expected vote share.

We have therefore shown the claim of step (B1): When $\hat{F}_I$ has some offers on the high part of $T$'s equilibrium support, $(u_T - H'', u_T]$, then the vote share can be increased by shifting the offers from $(u_T - H'', u_T]$ downwards for the case where the remaining offers before the shift are at least partly on the equilibrium support of $I$.

It remains to show that the same holds when the remaining offers before the shift are exclusively concentrated on the equilibrium support of $E$.

*Step (B2): When $\hat{F}_I$ has some offers on the high part of $T$'s equilibrium support, $(u_T - H'', u_T]$, and has the remaining offers only on the equilibrium support of $E$, then the vote share can be increased by shifting the offers from $(u_T - H'', u_T]$ down towards the lower intervals.*

We can still work with the same vote share formulas as in step (B1). Since there are no offers on the equilibrium support of $I$, this just means that for instance $M_{L_I I}$

[19] Note that this also includes the case where there are no offers on the equilibrium support of $E$ to begin with and hence no offers can be shifted upwards there. Hence, the arguments here also cover the case where there are only offers on the equilibrium support of $I$ before we do the proposed shift of offers down from $(u_T - H'', u_T]$.

and $M_{H_I I}$ take the value zero. Similar to step (B1), we can shift all the mass from the high part of $T$'s equilibrium support down to $u_I$ and have a positive amount $M_A$ left. As argued above, the change in the vote share is determined completely by the use of this additional money $M_A$.

Now since for step (B2) all the offers that are not on the high part of $T$'s equilibrium support are concentrated on the equilibrium support of $E$, we can always spent $M_A$ by shifting offers inside $E$'s equilibrium support upward without changing the mass of offers there.[20] This means that the expected vote share changes by $M_A \left[\beta_E \frac{1}{u_E - l_E} - \beta_T \frac{1}{2H''}\right] > 0$, as shown in case (ii) of step (B1).

In total, step (B) has therefore shown that a distribution $\hat{F}_I$ that is supposed to be a best response to the equilibrium strategy should have no offers on the equilibrium support of $T$. Since by step (A) a best response $\hat{F}_I$ cannot have only offers on the equilibrium support of $E$ either, two options remain: First, $\hat{F}_I$ only has offers on the equilibrium support of $I$, second, $\hat{F}_I$ has offers on both the equilibrium supports of $I$ and $E$. In order to exclude the second option we proceed to show the final step.

*Step (C): When $\hat{F}_I$ is restricted to operate on the equilibrium supports of $E$ and $I$, then a best response to the equilibrium strategy should only offer on the equilibrium support of $I$.*

This is analogous to step (A2) in the optimality proof of $\hat{F}_T^*$.

In total, we can therefore conclude that when checking for profitable deviations from the equilibrium distribution under bundle $I$, we can concentrate on distributions $\hat{F}_I$ that only have offers on the equilibrium support of $I$. Applying the arguments from step (A3) in the optimality proof of $\hat{F}_T^*$, it can be shown that any such distribution $\hat{F}_I$ achieves a total expected vote share of 1/2 against the equilibrium strategy. In particular, $\hat{F}_I^*$ is also such a distribution and therefore there is no profitable deviation from the equilibrium strategy by deviating from the distribution $\hat{F}_I^*$ played in case of bundle $I$.

[20] Such a shift would only be prevented if all offers were concentrated at the upper bound $u_E$ of that support, but then budget balance would be violated.

**Step III: Optimality of $\hat{F}_E^*$.** Similar to the previous steps, we will argue that, when checking for profitable deviations from the equilibrium distribution $\hat{F}_E^*$, which is played in case of bundle $E$, we can concentrate on distributions $\hat{F}_E$ that have no offers on the equilibrium supports of $T$ and $I$. In the proof of the optimality of $\hat{F}_T^*$, we have shown in step (B1) that $l_T + H'' < l_I$ and in step (A2) that $l_I + H' < l_E$. Since $l_E$ is the lowest offer that $\hat{F}_E$ can make, $\hat{F}_E$ can neither have offers on the lower part of the equilibrium support of $T$, $[l_T, l_T + H'']$, nor on the lower part of $I$'s equilibrium support, $[l_I, l_I + H']$. Hence, we only have to exclude offers on the high part of $I$'s and $T$'s equilibrium supports, $(u_I - H', u_I] = (u_E, u_I]$ and $(u_T - H'', u_T] = (u_I, u_T]$, in order to exclude offers on the total equilibrium supports of $I$ and $T$. Since $M_E = \frac{u_E + l_E}{2}$ with $l_E < u_E$, $\hat{F}_E$ cannot only have offers on the high part of $I$'s or $T$'s equilibrium supports, which both lie above $u_E$, without violating the budget constraint.

In order to exclude offers on the equilibrium support of $T$, it is therefore sufficient to show the following:

*Step (A): When $\hat{F}_E$ has some offers on the high part of $T$'s equilibrium support, $(u_T - H'', u_T]$, and has the remaining offers on the equilibrium supports of $E$ or $I$, then the vote share can be increased by shifting the offers from $(u_T - H'', u_T]$ down towards the lower intervals.*

This corresponds exactly to step (B) in the optimality proof for $\hat{F}_I^*$. In contrast to the proof there, the vote share formulas need to be adjusted by canceling the terms related to the lower part of $I$' equilibrium support, which now lies below the lowest possible offer $l_E$. The formulas become:

$$S(\hat{F}_I^*, \hat{F}_E) = 1 - \hat{F}_I(u_I) + \frac{1}{2}\left[\frac{M_{H_I E}}{H'} + \frac{2H' - u_I}{H'}\hat{F}_E(u_I) + \frac{u_I - H'}{H'}\hat{F}_E(u_E)\right]$$

$$S(\hat{F}_E^*, \hat{F}_E) = 1 - \hat{F}_E(u_E) + \frac{M_{ME}}{u_E - l_E} - \frac{l_E}{u_E - l_E}\hat{F}_E(u_E)$$

$$S(\hat{F}_T^*, \hat{F}_E) = \frac{1}{2}\left[1 - \frac{u_T - H''}{H''}\Big(1 - \hat{F}_E(u_I)\Big) + \frac{M_{H_T E}}{H''}\right]$$

where all the $M$ terms are defined analogously to before. The arguments from step (B) in the optimality proof for $\hat{F}_I^*$ can now be repeated analogously to prove the above claim. It is now even easier to do the proof because case (iii) from before is now no longer possible without violating the budget constraint.

Therefore, with step (A), we have shown that any distribution $\hat{F}_E$ that is supposed to be a best response to the equilibrium strategy should have no offers on the equilibrium support of $T$. Said differently, $\hat{F}_E$ is restricted to operate on the equilibrium support of $E$ and the high part of $I$'s equilibrium support. In the last step it remains to exclude offers on the equilibrium support of $I$.

*Step (B): When $\hat{F}_E$ is restricted to operate on the equilibrium supports of $E$ and $I$, then a best response to the equilibrium strategy should only offer on the equilibrium support of $E$.*

This is analogous to the corresponding claim in step II in the optimality proof of $\hat{F}_E^*$ in Proposition B.2.1. The same arguments can be applied here replacing, for instance, $u_T$ by $u_I$ and $u_T - H$ by $u_I - H'$.

In total, we can therefore conclude that when checking for profitable deviations from the equilibrium distribution under bundle $E$, we can concentrate on distributions $\hat{F}_E$ that only have offers on the equilibrium support of $E$. For such $\hat{F}_E$, the vote share formulas become:

$$\begin{aligned}
S(\hat{F}_I^*, \hat{F}_E) &= \frac{1}{2} \\
S(\hat{F}_T^*, \hat{F}_E) &= \frac{1}{2} \\
S(\hat{F}_E^*, \hat{F}_E) &= \frac{M_{ME}}{u_E - l_E} - \frac{l_E}{u_E - l_E} \\
&= \frac{(u_E + l_E)/2 - l_E}{u_E - l_E} = \frac{1}{2}
\end{aligned}$$

Therefore any such distribution $\hat{F}_E$, which only has offers on the equilibrium support of $E$, achieves a total expected vote share of 1/2 against the equilibrium strategy. In particular, $\hat{F}_E^*$ is also such a distribution and therefore there is no profitable deviation from the equilibrium strategy by deviating from the distribution $\hat{F}_E^*$ played in case of bundle $E$.

**Step IV: Optimality of $\beta_Q = 0$.** Before arguing for the optimality of the exact values of the probabilities for bundles $T$, $I$, and $E$, we must still argue that bundle $Q$ is played with zero probability in equilibrium.

To do so, we show that for bundle $Q$ any possible associated distribution is beaten by the equilibrium strategy. Together with the results established in steps I to III, this will imply that any strategy that plays $Q$ with positive probability will be beaten for sure by the equilibrium strategy.

To start, we note some preliminaries. First, we can apply an analogous argument to step (III) in the proof of Proposition B.2.1 to show that $l_I + H' > l_Q$. In particular, we have:

$$\begin{aligned}
&l_I + H' - l_Q \\
=&l_I + l_E - l_I - 2(M_E - M_I) - l_Q \\
=&l_E - l_Q - 2(M_E - M_I) \\
>&l_E - l_Q - 2(M_E - M_Q) \\
&= u_Q - u_E > 0.
\end{aligned}$$

The first inequality used the fact that $M_I > M_Q$ and the last inequality holds by the assumption on the targetability ranking of the different bundles.

In the following, let us first consider the case $l_Q \geq l_I$.

*Step (a):* Since $l_Q \in [l_I, l_I + H']$, the only thing that changes in the vote share formulas from step (II), which proved the optimality of $\hat{F}_I^*$, is that $M_I$ is replaced by $M_Q$. In can be verified that that all the substeps of step (II) showing that a best response $\hat{F}_I$ to the equilibrium strategy should only have offers on the equilibrium support of $I$ still apply for bundle $Q$. Therefore, any best response to the equilibrium strategy played in case of bundle $Q$, $\hat{F}_Q$, should only have offers on equilibrium support of $I$. For instance, in the arguments of substep (A) to establish that a best response $\hat{F}_I$ to the equilibrium strategy cannot have offers on the support of $\hat{F}_E^*$, the fact $M_I < M_E$ was used. Since $M_Q < M_E$, we can make the same argument for bundle $Q$.

*Step (b):* Having established that a best response $\hat{F}_Q$ would only play offers on the support of $\hat{F}_I^*$, we can apply the arguments from substep (A3) of the optimality of $\hat{F}_T^*$ proof. This step showed that a distribution $\hat{F}_T$ that offers only on the equilibrium support of $I$ will lose against the equilibrium strategy due to $M_T < M_I$. Since, we have $M_Q < M_I$, the same arguments go through for bundle $Q$. Hence any possible best response $\hat{F}_Q$ would lose for sure against the equilibrium strategy and hence should be played with probability zero in equilibrium.

The same can be established for $l_Q < l_I$,[21] using additionally the corresponding arguments from step (III) in the proof of Proposition B.2.1.

**Step V: Optimality of $\beta_E = \frac{u_I - l_I - 2H'}{u_I - l_I} \frac{u_T - l_T - 2H''}{u_T - l_T}$, $\beta_T = \frac{2H''}{u_T - l_T}$, and $\beta_I = 1 - \beta_T - \beta_E = \frac{2H'}{u_I - l_I} \frac{u_T - l_T - 2H''}{u_T - l_T}$.** Having established that only bundles $E$, $T$, and $I$ will be played with positive probability in equilibrium, we can now argue for the optimality of the respective equilibrium probabilities $\beta_E = \frac{u_I - l_I - 2H'}{u_I - l_I} \frac{u_T - l_T - 2H''}{u_T - l_T}$, $\beta_T = \frac{2H''}{u_T - l_T}$, and $\beta_I = 1 - \beta_T - \beta_E = \frac{2H'}{u_I - l_I} \frac{u_T - l_T - 2H''}{u_T - l_T}$. When candidate $B$ plays the equilibrium strategy, candidate $A$ is indifferent between playing bundles $E$, $T$, and $I$: In steps (I), (II), and (III) above we have shown that for all distributions $\hat{F}_E$, $\hat{F}_T$, and $\hat{F}_I$ played under these three bundles that are potential best responses to the equilibrium strategy the total expected vote share is $\frac{1}{2}$ when playing against the equilibrium strategy. In particular, for all equilibrium distributions $\hat{F}_E^*$, $\hat{F}_T^*$, and $\hat{F}_I^*$, the total expected vote share is 1/2. Therefore, given that candidate $B$ plays the equilibrium strategy, candidate $A$ is happy to mix over playing these equilibrium distributions with the above probabilities.

[21] Note that we always have $l_Q > l_T + H''$.

## B.4 Comparative statics for probabilities of different policies under alternative parameter values

The following two figures depict the probabilities of the different policy bundles against a change in the debt limit $\bar{\rho}$ for alternative values of reform benefits $e$ given a fixed level of reform costs $c$. The relative size of the distortion compared to the size of the maximal value $\frac{e-c}{1+e}$ that it can take is kept constant at 0.6. However, the results are virtually unaffected when we keep the absolute value of the distortion constant across all figures. Note that due to $\bar{\rho} \leq e$, the upper bound of the depicted range of debt limits is different across the figures. Similarly, the lower bound of the debt limit, $-1 + \frac{2c-e}{2(1-\gamma)-1}$, also depends on the parameter value $e$. Therefore, the lower bound of the depicted range of debt limits also changes across the figures.

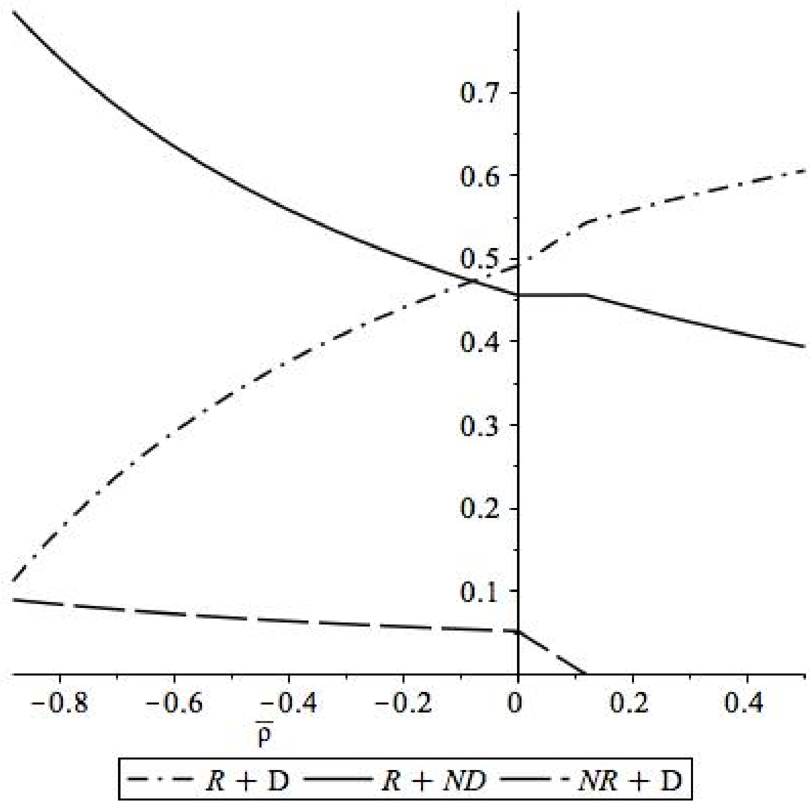


Figure B.1: Equilibrium probabilities for $e = 0.5$, $c = 0.3$, $\gamma = 0.6\frac{e-c}{1+e}$

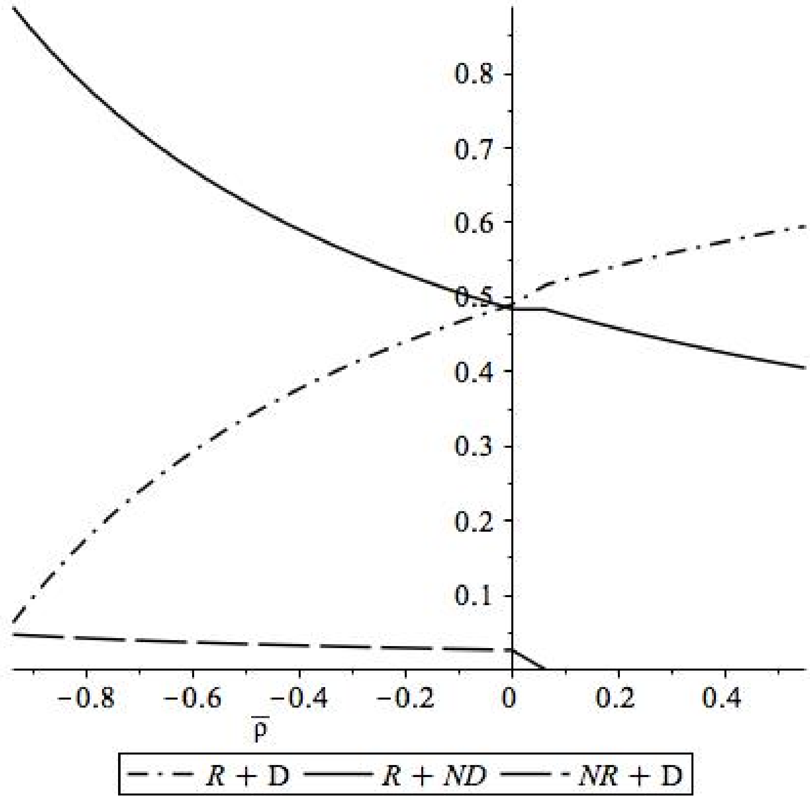


Figure B.2: Equilibrium probabilities for $e = 0.55$, $c = 0.3$, $\gamma = 0.6\frac{e-c}{1+e}$

# Appendix C

# Appendix to Chapter 4

## C.1 Comparative Statics for Default Model

**Corollary C.1.1** *Suppose an economy in the model with sovereign default starts in the first period with $\alpha_1 = \alpha_L$. Moreover, suppose that the constraint $\tau_2 \geq \tau_1$ does not bind.*[1] *Then:*

1. *In the social planner's solution as well as in the political equilibrium, if $\alpha_L \geq 2(1-\theta)$*

   I. *No debt is raised. That is case a) holds.*

   II. *No transfers are paid.*

   III. *There are positive investments in fiscal and legal capacity.*

   IV. *Higher $\phi$ increases investment in fiscal and legal capacity.*

   V. *Higher $\psi$ increases investment in fiscal and legal capacity.*

   VI. *Neither $\gamma$ nor $\theta$ have an influence on the investment decisions.*

   VII. *Free future revenues are increasing in $\phi$ and $\psi$.*

2. *In the following, consider the political equilibrium for $\alpha_L < 2(1-\theta)$. That is, we have $\lambda_1 = 2(1-\theta)$ and $E(\lambda_2) = \phi\alpha_H + (1-\phi)[(1-\gamma)2(1-\theta) + \gamma 2\theta]$.*

   (a) *If $\lambda_1 < E(\lambda_2)$*

[1] It turns out that this is not a very restrictive assumption. It would only be violated if the high income realization $\overline{\omega}(\cdot)$ lied unrealistically high above the low income realization $\underline{\omega}(\cdot)$. A sufficient condition for this assumption is that $\overline{\omega}(\cdot) \leq 2\underline{\omega}(\cdot)$.

I. *No debt is raised (case a) holds).*

II. *There are positive investments in fiscal and legal capacity.*

III. *Higher $\phi$ increases investment in fiscal and legal capacity.*

IV. *Higher $\psi$ increases investment in fiscal and legal capacity.*

V. *A lower value of $\gamma$ unambiguously raises investments, whereas an increase in $\theta$ raises investments if $\gamma$ is above 1/2.*

VI. *Free future revenues are increasing in $\theta$ if $\gamma > 1/2$, and are increasing in $\phi$, $\psi$ and $\gamma$.*

(b) *If $\lambda_1 = E(\lambda_2)$*

I. *The debt level is indeterminate in the range $[0, \bar{b}]$. That is case b) holds.*

(c) *If $E(\lambda_2) < \lambda_1 < E(\lambda_2) + \frac{(1-\psi)}{\psi} \left.\frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\bar{b}}$*

I. *$b_1 = \bar{b}$ is the optimal debt level. That is case c) holds.*

II. *Higher $\phi$ as well as lower $\gamma$ lead to an increase of both $b_1$ and free future revenues.*

III. *Higher $\psi$ leads to an increase of both $b_1$ and free future revenues if $E(\lambda_2) > 1$.*

IV. *Residual revenues are used to finance transfers.*

V. *There are positive investments in fiscal and legal capacity.*

VI. *Higher $\phi$ as well as lower $\gamma$ increase investments in fiscal and legal capacity.*

VII. *Higher $\psi$ increases investment in fiscal and legal capacity if $E(\lambda_2) > 1$.*

(d) *If $\lambda_1 = E(\lambda_2) + \frac{(1-\psi)}{\psi} \left.\frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\tilde{b}}$, with $\bar{b} < \tilde{b} < \bar{\bar{b}}$*

I. *$b_1 = \tilde{b}$ is the optimal debt level. That is case d) holds.*

II. *Higher $\psi$ leads to an increase of $b_1$ if $E(\lambda_2) > 1$.*

III. *Higher $\phi$ as well as lower $\gamma$ lead to an increase of free future revenues.*

IV. *Residual revenues are used to finance transfers.*

V. *There are positive investments in fiscal and legal capacity.*

VI. *Higher $\phi$ as well as lower $\gamma$ increase investments in fiscal and legal capacity.*

VII. *Higher $\psi$ increases investment in fiscal and legal capacity if $E(\lambda_2) > 1$.*

(e) *If $\lambda_1 > E(\lambda_2) + \frac{(1-\psi)}{\psi} \left.\frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\bar{\bar{b}}}$*

I. *Debt is raised maximally, $b_1 = \bar{\bar{b}}$ (case e) holds).*

II. *Neither $\phi$ nor $\gamma$ have an influence on the debt level, but it is increasing in $\psi$.*

III. *Free future revenues are 0 and therefore they are constant with respect to $\phi$, $\gamma$, $\psi$ and $\theta$.*

IV. *Residual revenues are used to finance transfers.*

V. *There are positive investments in fiscal and legal capacity.*

VI. *Neither $\phi$ nor $\gamma$ have an influence on the investment decision.*

VII. *Higher $\psi$ increases investment in fiscal and legal capacity.*

## C.2 Proofs of Propositions and Corollaries

### Proof of proposition 4.4.1:

**Part 1:** a) follows from (4.18) and part b) follows from (4.8). Part c) follows from the FOC for fiscal and legal capacity (i.e. (4.21) and (4.22)).

Part d): We first want to show that fiscal and legal capacity are both monotone nondecreasing in $E(\lambda_2)$. Since $\phi$ enters the model only through $E(\lambda_2)$ and $E(\lambda_2)$ is increasing in $\phi$, this suffices. Let us rewrite equation (4.16):

$$f(b_1, \tau_2, \pi_2) = EV^{I_1}(\tau_2, \pi_2, b_1) - \lambda_1(F(\tau_2 - \tau_1) + L(\pi_2 - \pi_1) - b_1) \tag{C.1}$$

As stated in Part a), $b_1 = 0$ is the debt level that maximizes (C.1). Define $g(\tau_2, \pi_2) \equiv f(0, \tau_2, \pi_2)$. Following Corollary 3 of Milgrom and Shannon (1994), it remains to show that $g(\tau_2, \pi_2)$ is quasisupermodular in $(\tau_2, \pi_2)$ and satisfies the single crossing property in $(\tau_2, \pi_2, E(\lambda_2))$. We have:

$$\frac{\partial^2 g(\tau_2, \pi_2)}{\partial \pi_2 \partial \tau_2} = \delta\omega'(\pi_2)(E(\lambda_2) - 1) > 0 \tag{C.2}$$

$$\frac{\partial^2 g(\tau_2, \pi_2)}{\partial \pi_2 \partial E(\lambda_2)} = \delta\omega'(\pi_2)\tau_2 > 0 \tag{C.3}$$

$$\frac{\partial^2 g(\tau_2,\pi_2)}{\partial\tau_2\partial E(\lambda_2)} = \delta\omega(\pi_2) > 0 \tag{C.4}$$

By Theorem 6 of Milgrom and Shannon (1994), $g(\tau_2,\pi_2)$ has increasing differences in $(\tau_2,\pi_2,E(\lambda_2))$ and is supermodular in $(\tau_2,\pi_2)$. It follows that $g(\tau_2,\pi_2)$ satisfies the single crossing property in $(\tau_2,\pi_2,E(\lambda_2))$ and is quasisupermodular in $(\tau_2,\pi_2)$. So, fiscal and legal capacity are both monotone nondecreasing in $E(\lambda_2)$.

It remains to show that fiscal and legal capacity are both strictly increasing in $E(\lambda_2)$. Since we have shown that both are monotone nondecreasing, there are three other potential possibilities: fiscal capacity is strictly increasing while legal capacity remains constant, legal capacity is strictly increasing while fiscal capacity remains constant, both are constant. However, all of them lead to a contradiction regarding the FOC (4.21) and (4.22), so they can be ruled out.

**Part 2:** a) follows from (4.18) and b) follows from (4.8). Part c): The positive investments follow from the FOC for fiscal and legal capacity (i.e. (4.21) and (4.22)). The comparison to the model without debt is due to the fact that when no debt can be raised, $max\{\lambda_1, E(\lambda_2)\}$ has to be replaced by $E(\lambda_2)$ which is smaller (the comparison can be shown rigorously using monotone comparative statics similar to the proof of part 1 d) of proposition 4.4.1).

## Proof of proposition 4.4.2:

If the cohesiveness condition holds, $\lambda_1 = \alpha_1$ and $E(\lambda_2) = \phi\alpha_H + (1-\phi)\alpha_L$. These are the same terms as for a social planner. Therefore, the results of proposition 4.4.1 hold.

## Proof of proposition 4.4.3:

**Part 1:** a) follows from (4.18) and part b) follows from (4.8). Part c) follows from the FOC for fiscal and legal capacity (i.e. (4.21) and (4.22)).

For parts d) and e), we apply again monotone comparative statics as in part 1 d) of proposition 4.4.1. As stated in part a), $b_1 = 0$ is the debt level that maximizes (C.1). Define $g(\tau_2,\pi_2) \equiv f(0,\tau_2,\pi_2)$, with $f(b_1,\tau_2,\pi_2)$ denoting the objective function given in (C.1). We have:

$$\frac{\partial^2 g(\tau_2, \pi_2)}{\partial \pi_2 \partial \tau_2} = \delta\omega'(\pi_2)(E(\lambda_2) - 1) > 0 \tag{C.5}$$

since the stability condition holds.

$$\frac{\partial^2 g(\tau_2, \pi_2)}{\partial \pi_2 \partial E(\lambda_2)} = \delta\omega'(\pi_2)\tau_2 > 0 \tag{C.6}$$

$$\frac{\partial^2 g(\tau_2, \pi_2)}{\partial \tau_2 \partial E(\lambda_2)} = \delta\omega(\pi_2) > 0 \tag{C.7}$$

$$\frac{\partial^2 g(\tau_2, \pi_2)}{\partial \pi_2 \partial \theta} = \delta\omega'(\pi_2)\tau_2(1-\phi)2(2\gamma - 1) + 2\frac{\partial L(\cdot)}{\partial \pi_2} > 0 \quad \text{for } \gamma > 1/2 \tag{C.8}$$

$$\frac{\partial^2 g(\tau_2, \pi_2)}{\partial \tau_2 \partial \theta} = \delta\omega(\pi_2)(1-\phi)2(2\gamma - 1) + 2\frac{\partial F(\cdot)}{\partial \tau_2} > 0 \quad \text{for } \gamma > 1/2 \tag{C.9}$$

Following the same reasoning as in the proof of part 1 d) of proposition 4.4.1, we are done.

**Part 2:** a) follows from (4.18). Part b): The positive investments follow from the FOC for fiscal and legal capacity (i.e. (4.21) and (4.22)). The comparison to the model without debt is due to the fact that when no debt can be raised, $max\{\lambda_1, E(\lambda_2)\}$ has to be replaced by $E(\lambda_2)$ which is smaller (the comparison can be shown rigorously using monotone comparative statics similar to the proof of part 1 d) of proposition 4.4.1). Parts c) and e) follow from the FOC for fiscal and legal capacity (i.e. (4.21) and (4.22)) and the definitions of $\lambda_1$ and $E(\lambda_2)$. Part d) follows from (4.8).

## Proof of proposition 4.4.4:

Part 1: The positive investments follow from the FOC for fiscal and legal capacity (i.e. (4.21) and (4.22)). The comparison to the model without debt is due to the fact that when no debt can be raised, $max\{\lambda_1, E(\lambda_2)\}$ has to be replaced by $E(\lambda_2)$ which is smaller (the comparison can be shown rigorously using monotone comparative statics similar to the proof of part 1 d) of proposition 4.4.1). Part 2 follows from the FOC for fiscal and legal capacity (i.e. (4.21) and (4.22)) and the definitions of $\lambda_1$ and $E(\lambda_2)$. Part 3 follows from (4.18) and part 4 follows from (4.8).

## Proof of Proposition 4.5.1 and Corollary C.1.1

The following proves Corollary C.1.1. This also proves Proposition 4.5.1, which presents the results of Corollary C.1.1 in a more compact way, except for the results on default and the comparison of investment incentives in a world with and without default. The argument for these latter results can be found in the main text.

**Part 1:**

Part I) follows from $\lambda_1 < E(\lambda_2)$ and part II) follows from $\alpha_L \geq 2(1-\theta)$. Part III) follows from the FOC for fiscal and legal capacity.

Parts IV) and V): We first want to show that fiscal and legal capacity are both monotone nondecreasing in $\psi$ and $E(\lambda_2)$. Since $\phi$ enters the model only through $E(\lambda_2)$ and $E(\lambda_2)$ is increasing in $\phi$, this suffices.

Let us rewrite equation (4.30):

$$f(b_1, \tau_2, \pi_2) = EV^{I_1}(\tau_2, \pi_2, b_1) - \lambda_1(F(\tau_2 - \tau_1) + L(\pi_2 - \pi_1) - b_1) \tag{C.10}$$

As stated in part I), $b_1 = 0$ is the debt level that maximizes (C.10). Define $g(\tau_2, \pi_2) \equiv f(0, \tau_2, \pi_2)$. Following Corollary 3 of Milgrom and Shannon (1994), it remains to show that $g(\tau_2, \pi_2)$ is quasisupermodular in $(\tau_2, \pi_2)$ and satisfies the single crossing property in $(\tau_2, \pi_2, E(\lambda_2), \psi)$. We have:

$$\frac{\partial^2 g(\tau_2, \pi_2)}{\partial \pi_2 \partial \tau_2} = \delta\omega'(\pi_2)(E(\lambda_2) - 1) > 0 \tag{C.11}$$

$$\frac{\partial^2 g(\tau_2, \pi_2)}{\partial \pi_2 \partial E(\lambda_2)} = \delta\omega'(\pi_2)\tau_2 > 0 \tag{C.12}$$

$$\frac{\partial^2 g(\tau_2, \pi_2)}{\partial \pi_2 \partial \psi} = 0 \tag{C.13}$$

$$\frac{\partial^2 g(\tau_2, \pi_2)}{\partial \tau_2 \partial E(\lambda_2)} = \delta(\psi\bar{\omega}(\pi_2) + (1-\psi)\underline{\omega}(\pi_2)) > 0 \tag{C.14}$$

$$\frac{\partial^2 g(\tau_2, \pi_2)}{\partial \tau_2 \partial \psi} = \delta(\bar{\omega}(\pi_2) - \underline{\omega}(\pi_2))(E(\lambda_2) - 1) > 0 \text{ since we have } E(\lambda_2) > \lambda_1 > 1 \tag{C.15}$$

By Theorem 6 of Milgrom and Shannon (1994), $g(\tau_2, \pi_2)$ has increasing differences in $(\tau_2, \pi_2, E(\lambda_2), \psi)$ and is supermodular in $(\tau_2, \pi_2)$. It follows that $g(\tau_2, \pi_2)$ satisfies the single crossing property in $(\tau_2, \pi_2, E(\lambda_2), \psi)$ and is quasisupermodular in $(\tau_2, \pi_2)$. So, fiscal and legal capacity are both monotone nondecreasing in $E(\lambda_2)$ and $\psi$.

It remains to show that fiscal and legal capacity are both strictly increasing in $E(\lambda_2)$ and $\psi$. Since we have shown that both are monotone nondecreasing, there are three other possibilities which we have to check: fiscal capacity is strictly increasing while legal capacity remains constant, legal capacity is strictly increasing while fiscal capacity remains constant, both are constant. All of these can be shown to lead to a contradiction as demonstrated for the following case. Consider an increase in $E(\lambda_2)$. Assume fiscal capacity is strictly increasing while legal capacity remains constant. From (C.11) and (C.12) it follows that the LHS of the FOC for $\pi$ is increasing, so the RHS has to increase as well. Since $L(\cdot)$ is convex, $\pi$ has to increase. This is a contradiction. In an analogous way, all other cases can be handled. The same can be done for an increase in $\psi$. In total, we conclude that fiscal and legal capacity must be strictly increasing in $E(\lambda_2)$ as well as $\psi$.

Part VI) follows from the FOC for fiscal and legal capacity and the definitions of $\lambda_1$ and $E(\lambda_2)$. Part VII) follows from the definition of free future revenues and the comparative statics for $\tau_2$ and $\pi_2$.

**Part 2.(a):**

Part I) follows from $\lambda_1 < E(\lambda_2)$ and part II) follows from the FOC for fiscal and legal capacity.

Parts III)-V): We want to show that fiscal and legal capacity are both strictly increasing in $\psi$, $\theta$ (for $\gamma > 1/2$) and $E(\lambda_2)$. Since $\phi$ and $\gamma$ enter the model only through $E(\lambda_2)$ and $E(\lambda_2)$ is increasing in $\phi$ and decreasing in $\gamma$, this suffices. For $\psi$ and $E(\lambda_2)$, the proof is exactly as in Part 1. It remains to show that fiscal and legal capacity are both increasing in $\theta$ (for $\gamma > 1/2$).

As stated in part I), $b_1 = 0$ is the debt level that maximizes (C.10). Define $g(\tau_2, \pi_2) \equiv f(0, \tau_2, \pi_2)$, with $f(b_1, \tau_2, \pi_2)$ denoting the objective function in (C.10). We have:

$$\frac{\partial^2 g(\tau_2, \pi_2)}{\partial \pi_2 \partial \tau_2} = \delta \omega'(\pi_2)(E(\lambda_2) - 1) > 0 \tag{C.16}$$

$$\frac{\partial^2 g(\tau_2, \pi_2)}{\partial \pi_2 \partial \theta} = \delta \omega'(\pi_2)\tau_2 2(1-\phi)(2\gamma - 1) + 2\frac{\partial L(\pi_2 - \pi_1)}{\partial \pi_2} > 0 \quad if\ \gamma > \frac{1}{2} \tag{C.17}$$

$$\frac{\partial^2 g(\tau_2,\pi_2)}{\partial\tau_2\partial\theta} = \delta(\psi\bar{\omega}(\pi_2)+(1-\psi)\underline{\omega}(\pi_2))2(1-\phi)(2\gamma-1)+2\frac{\partial F(\tau_2-\tau_1)}{\partial\tau_2} > 0 \quad if\ \gamma > \frac{1}{2} \tag{C.18}$$

Following the same reasoning as in the proof of part 1, we are done.

Part VI) follows from the definition of free future revenues and the comparative statics for $\tau_2$ and $\pi_2$.

**Part 2.(b)**:

The result follows from the FOC for debt, as argued at the beginning of Section 4.5.2

**Part 2.(c)**:

Part I) follows from the FOC for debt, as argued at the beginning of Section 4.5.2. For parts II) and III), the comparative statics for $b_1$ follow from the definition of $b_1 = \bar{b}$ and the comparative statics for $\tau_2$ and $\pi_2$ in the following parts VI) and VII). As for the comparative statics for free future revenues, note that the formula for free future revenues becomes $\delta\tau_2\psi(\overline{v}-\underline{v})$, since $\bar{\omega}(\cdot)$ and $\underline{\omega}(\cdot)$ can be written as $\bar{\omega}(\cdot) = w(\cdot)+\overline{v}$ and $\underline{\omega}(\cdot) = w(\cdot)+\underline{v}$ as argued in Section 4.5. This term is increasing in $\tau_2$. Therefore, the comparative statics for free future revenues w.r.t $\phi$ and $\gamma$ follow immediately from the comparative statics for $\tau_2$ stated in in the following parts VI) and VII). The comparative statics for free future revenues w.r.t $\psi$ follow from $\frac{\partial\delta\tau_2\psi(\overline{v}-\underline{v})}{\partial\psi} = \delta\tau_2(\overline{v}-\underline{v})+\delta\psi(\overline{v}-\underline{v})\frac{\partial\tau}{\partial\psi}$ and the comparative statics for $\tau_2$ stated in the following part VII).

Part IV) is due to $\alpha_L < 2(1-\theta)$. Part V) follows from the FOCs for $\tau_2$ and $\pi_2$.

For parts VI) and VII) we apply again monotone comparative statics. We want to show that fiscal and legal capacity are both strictly increasing in $\psi$ and $E(\lambda_2)$ (since $\phi$ and $\gamma$ enter the model only through $E(\lambda_2)$ and $E(\lambda_2)$ is increasing in $\phi$ and decreasing in $\gamma$, this suffices).

As stated in part I), $b_1 = \bar{b}$ is the debt level that maximizes (C.10). Define $g(\tau_2,\pi_2) \equiv f(\bar{b},\tau_2,\pi_2)$, with $f(b_1,\tau_2,\pi_2)$ denoting the objective function of (C.10). We have:

$$\frac{\partial^2 g(\tau_2,\pi_2)}{\partial\pi_2\partial\tau_2} = \delta(\omega'(\pi_2)(\lambda_1-1)) > 0 \tag{C.19}$$

$$\frac{\partial^2 g(\tau_2,\pi_2)}{\partial\pi_2 \partial E(\lambda_2)} = 0 \tag{C.20}$$

$$\frac{\partial^2 g(\tau_2,\pi_2)}{\partial\tau_2 \partial E(\lambda_2)} = \delta\psi(\bar{\omega}(\pi_2) - \underline{\omega}(\pi_2)) > 0 \tag{C.21}$$

$$\frac{\partial^2 g(\tau_2,\pi_2)}{\partial\pi_2 \partial\psi} = 0 \tag{C.22}$$

$$\frac{\partial^2 g(\tau_2,\pi_2)}{\partial\tau_2 \partial\psi} = \delta(\bar{\omega}(\pi_2) - \underline{\omega}(\pi_2))(E(\lambda_2) - 1) > 0 \text{ if } E(\lambda_2) > 1 \tag{C.23}$$

Following the same reasoning as in the proof of part 1, we are done.

**Part 2.(d)**:

Part I) follows from the FOC for debt, as argued at the beginning of Section 4.5.2. Part II): If $\psi$ increases, the FOC for debt says that $\Delta$ has to increase. Since $\tau_2$ and $\pi_2$ increase as stated in 2.(d) VII), $b_1$ has to increase.

Part III): Since $\phi$ and $\gamma$ enter the model only through $E(\lambda_2)$ and $E(\lambda_2)$ is increasing in $\phi$ and decreasing in $\gamma$, we have to show that free future revenues ($FR$) are increasing in $E(\lambda_2)$.

We have $\frac{\partial FR}{\partial E(\lambda_2)} = \frac{\partial \delta\tau_2(\psi\bar{\omega}(\pi_2)+(1-\psi)\underline{\omega}(\pi_2))}{\partial E(\lambda_2)} - \frac{\partial b_1}{\partial E(\lambda_2)}$ and $\frac{\partial \delta\tau_2(\psi\bar{\omega}(\pi_2)+(1-\psi)\underline{\omega}(\pi_2))}{\partial E(\lambda_2)} = \delta(\psi\bar{\omega}(\pi_2) + (1-\psi)\underline{\omega}(\pi_2))\frac{\partial\tau_2}{\partial E(\lambda_2)} + \delta\tau_2\omega'(\pi_2)\frac{\partial\pi_2}{\partial E(\lambda_2)}$. We cannot solve for $b_1$ explicitly, still we can say something about $\frac{\partial b_1}{\partial E(\lambda_2)}$. Taking the total differential w.r.t. $E(\lambda_2)$ of the FOC for $b_1$ we obtain $0 = 1 + \frac{(1-\psi)}{\psi}\frac{\partial^2 P(\Delta)}{\partial\Delta^2}\frac{\partial\Delta}{\partial E(\lambda_2)}$.

Since $P(\Delta)$ is convex, $\frac{\partial\Delta}{\partial E(\lambda_2)} < 0$. Since we are in case d), $\Delta = (1 + R(b))b - (\tau_2\underline{\omega}(\pi_2)) = \frac{1}{\psi}((1+\rho)b_1 - \tau_2\underline{\omega}(\pi_2))$ so $\frac{\partial\Delta}{\partial E(\lambda_2)} = \frac{1}{\psi}((1+\rho)\frac{\partial b_1}{\partial E(\lambda_2)} - \underline{\omega}(\pi_2)\frac{\partial\tau_2}{\partial E(\lambda_2)} - \tau_2\omega'(\pi_2)\frac{\partial\pi_2}{\partial E(\lambda_2)})$. This leads to $\frac{\partial b_1}{\partial E(\lambda_2)} < \delta(\underline{\omega}(\pi_2)\frac{\partial\tau_2}{\partial E(\lambda_2)} + \tau_2\omega'(\pi_2)\frac{\partial\pi_2}{\partial E(\lambda_2)})$.

Therefore, $\frac{\partial \delta\tau_2(\psi\bar{\omega}(\pi_2)+(1-\psi)\underline{\omega}(\pi_2))}{\partial E(\lambda_2)} > \frac{\partial b_1}{\partial E(\lambda_2)}$. So, free future revenues are increasing in $E(\lambda_2)$.

Parts IV) and V) follow analogously as parts IV) and V) in 2.(c)

Parts VI) and VII): In case d), the debt level $b^* \in (\bar{b}, \bar{\bar{b}})$ that maximizes (C.10) is implicitly defined by the FOC of $b_1$, $\lambda_1 = E(\lambda_2) + \frac{(1-\psi)}{\psi}\frac{\partial P(\Delta)}{\partial\Delta}$. Define $g(\tau_2,\pi_2) \equiv f(b^*,\tau_2,\pi_2)$. By the Envelope Theorem and plugging in the first order condition for debt, we obtain the same derivatives and cross-derivatives as in 2.(c). Following the same reasoning as for parts VI) and VII) in 2.(c), we are done.

**Part 2.(e):**

Part I) follows from the FOC for debt, as argued at the beginning of Section 4.5.2. Part II) follows from the definition of $\bar{\bar{b}}$ and part III) from the definitions of $\bar{\bar{b}}$ and free future revenues. Part IV) is due to $\alpha_L < 2(1-\theta)$. Part V) and VI) follow from the FOC for $\tau_2$ and $\pi_2$ in case e). For VII) we apply again monotone comparative statics.

As stated in part I), $b_1 = \bar{\bar{b}}$ is the debt level that maximizes (C.10). Define $g(\tau_2, \pi_2) \equiv f(\bar{\bar{b}}, \tau_2, \pi_2)$, with $f(b_1, \tau_2, \pi_2)$ denoting the objective function in (C.10). We have:

$$\frac{\partial^2 g(\tau_2, \pi_2)}{\partial \pi_2 \partial \tau_2} = \delta \omega'(\pi_2)(\lambda_1 - 1) > 0 \tag{C.24}$$

$$\frac{\partial^2 g(\tau_2, \pi_2)}{\partial \pi_2 \partial \psi} = 0 \tag{C.25}$$

$$\frac{\partial^2 g(\tau_2, \pi_2)}{\partial \tau_2 \partial \psi} = \delta(\bar{\omega}(\pi_2) - \underline{\omega}(\pi_2))(\lambda_1 - 1) + \psi \left.\frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\bar{\bar{b}}} (\bar{\omega}(\pi_2) - \underline{\omega}(\pi_2)) > 0 \tag{C.26}$$

Following the same reasoning as in the proof of part 1, we are done.

Q.E.D.

## C.3 Cross-country Correlations

### C.3.1 Scatter plots including highly cohesive countries

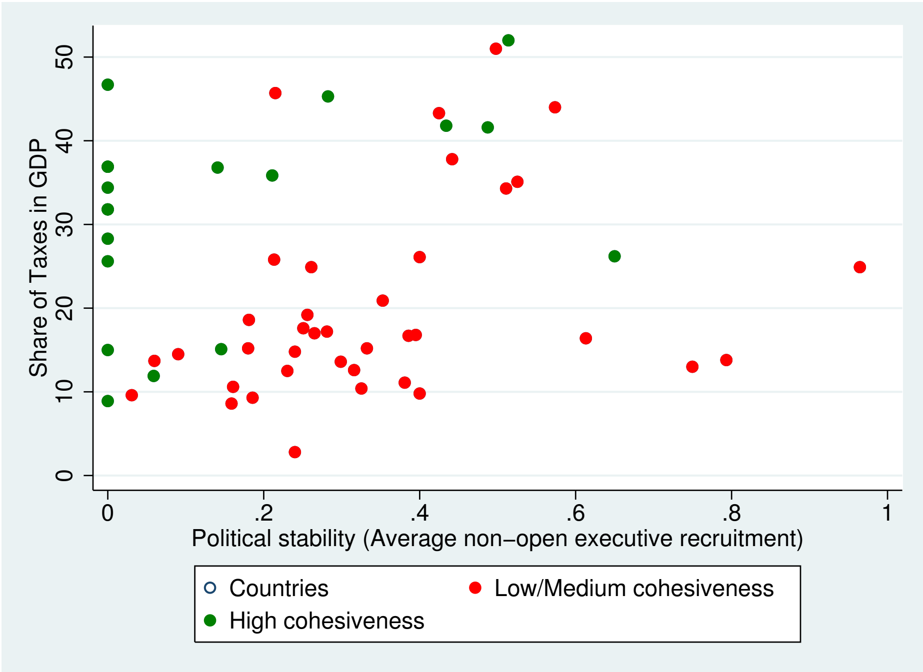


Figure C.1

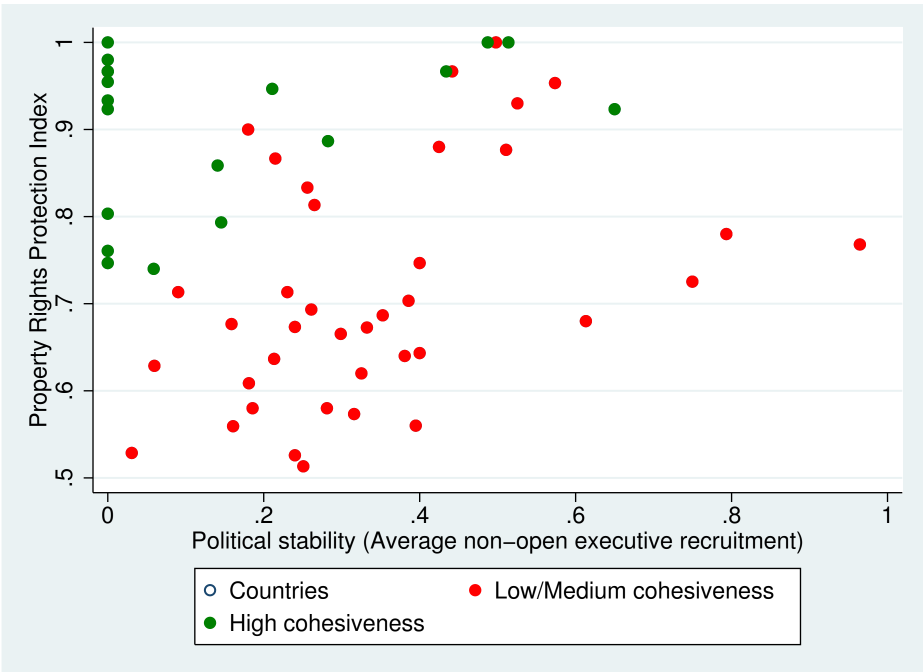


Figure C.2

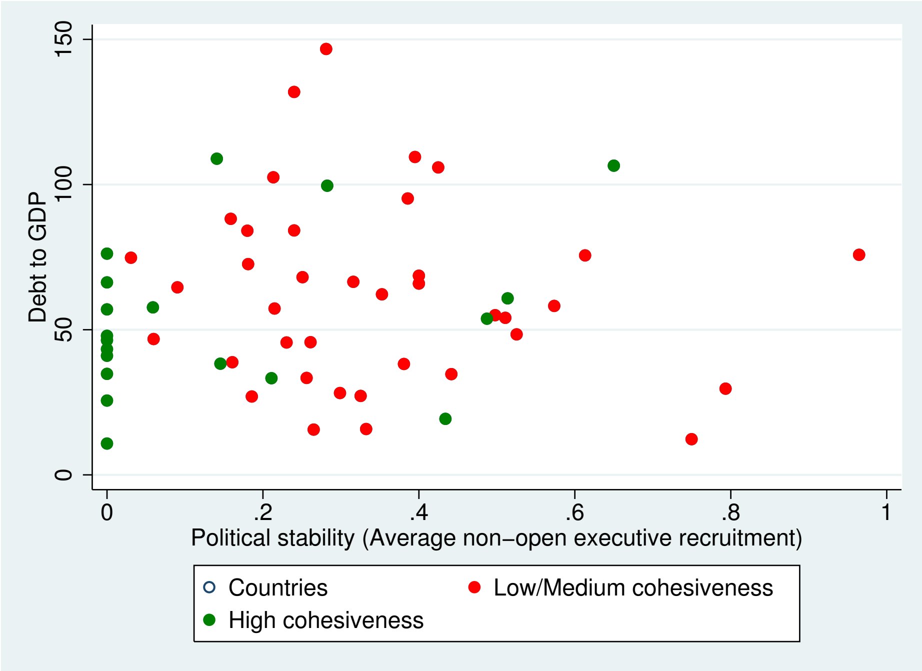


Figure C.3

### C.3.2 Variable descriptions

**Share of taxes in GDP** is the variable "taxrevenuegdp99" of Besley and Persson (2011): "The ratio of total tax revenue to GDP in 1999. This is directly taken from Baunsgaard and Keen (2005)."[2]

**Shadow economy** is the variabel "minform" of Besley and Persson (2011): "This is the original variable (Informal Economy in % of GNP 1999/2000) from Schneider (2002)."[3]

**Property Rights Protection Index** is the variable "mgadp97" of Besley and Persson (2011): "This variable tries to measure the extent of government antidiversion policies. It is calculated as an average of indexes of "law and order", "bureaucratic quality", "corruption", "risk of expropriation" and "government repudiation of contracts" from ICRG dataset in 1997 (International Country Risk Guide, The PRS Group, 1980-present) ."[4]

**Cohesiveness** is the variable "mxconst00" of Besley and Persson (2011): "Average

[2]Variable description is taken from the codebook of Besley and Persson (2011). The codebook and the data are available at `http://www-2.iies.su.se/pop/Data_1.html`.

[3]Variable description is taken from the codebook of Besley and Persson (2011). The codebook and the data are available at `http://www-2.iies.su.se/pop/Data_1.html`.

[4]Variable description is taken from the codebook of Besley and Persson (2011). The codebook and the data are available at `http://www-2.iies.su.se/pop/Data_1.html`.

executive constraints up to 2000. This measures the average value of the variable xconst (from Polity IV dataset (Marshall and Jaggers, 2010)) from 1800 (or independence date if later) up to 2000. The average is taken over non missing values of xconst (values outside $[1, 7]$ are treated as missing). This variable is normalized so that each country's scores lie between 0 and 1 (subtract 1 and divide by 6 as the possible range for the average score is from 1 to 7). This variable captures the parameter $\theta$ in the model."[5]

**Political Stability** is the variable "mgamma00" of Besley and Persson (2011): "Average non-open executive recruitment up to 2000. This measures average values of the sum of xropen and xrcomp variables in Polity IV dataset (Marshall and Jaggers, 2010) from 1800 (or independence if later) to 2000. Note that the average is taken when both xropen and xrcomp are not missing (we treat xropen and xrcomp as missing if they are less than one). The sum of xropen and xrcomp takes values between 2 and 7 in any given year so in order to normalize the average we subtract 2 and divide by 5. To get a measure of political stability this average is inverted (multiplied by minus one and add with one). This variable corresponds to the parameter $1 - \gamma$ in the model."[6]

**Debt to GDP** is the debt-to-GDP ratio in 2000, taken from Reinhart and Rogoff (2009). It measures the central government debt-to-GDP ratio. For 8 countries, this measure is not available and we use the general government debt-to-GDP ratio instead.[7]

[5] Variable description is taken from the codebook of Besley and Persson (2011). The codebook and the data are available at `http://www-2.iies.su.se/pop/Data_1.html`.

[6] Variable description is taken from the codebook of Besley and Persson (2011). The codebook and the data are available at `http://www-2.iies.su.se/pop/Data_1.html`.

[7] The data is available at `http://www.reinhartandrogoff.com/data/browse-by-topic/topics/9/`.

# Bibliography

**Acemoglu, Daron**, "Politics and economics in weak and strong states," *Journal of Monetary Economics*, 2005, *52* (7), 1199 – 1226.

_ **and James Robinson**, *Why Nations Fail: The Origins of Power, Prosperity, and Poverty*, Crown Business, 2012.

_ **, Davide Ticchi, and Andrea Vindigni**, "Emergence and persistence of inefficient states," *Journal of the European Economic Association*, 2011, *9* (2), 177–208.

**Aghion, Philippe and Patrick Bolton**, *Government Domestic Debt and the Risk of Default: A Political- Economic Model of the Strategic Role of Debt*, Cambridge, MA: MIT Press, 1990.

**Alesina, Alberto and Allan Drazen**, "Why are Stabilizations Delayed?," *American Economic Review*, 1991, *81* (5), pp. 1170–1188.

_ **and Andrea Passalacqua**, "The Political Economy of Government Debt," *Handbook of Macroeconomics*, forthcoming.

_ **and Guido Tabellini**, "External debt, capital flight and political risk," *Journal of International Economics*, 1989, *27* (3-4), 199 – 220.

_ **and** _ , "A Positive Theory of Fiscal Deficits and Government Debt," *Review of Economic Studies*, 1990, *57* (3), pp. 403–414.

_ **and Roberto Perotti**, "The Political Economy of Budget Deficits," *Staff Papers - International Monetary Fund*, 1995, *42* (1), pp. 1–31.

**Arellano, Cristina**, "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, 2008, *98* (3), pp. 690–712.

**Azzimonti, Marina**, "The dynamics of public investment under persistent electoral advantage," *Review of Economic Dynamics*, 2015, *18* (3), 653 – 678.

_ **, Eva de Francisco, and Vincenzo Quadrini**, "Financial Globalization, Inequality, and the Rising Public Debt," *American Economic Review*, 2014, *104* (8), 2267–2302.

_ **, Pierre-Daniel Sarte, and Jorge Soares**, "Distortionary taxes and public investment when government promises are not enforceable," *Journal of Economic Dynamics and Control*, 2009, *33* (9), 1662 – 1681.

**Barro, Robert J.**, "On the Determination of the Public Debt," *Journal of Political Economy*, 1979, *87* (5), pp. 940–971.

**Battaglini, Marco**, "The political economy of public debt," *Annual Review of Economics*, 2011, *3* (1), 161–189.

_ **and Levon Barseghyan**, "Political Economy of Debt and Growth," *Journal of Monetary Economics*, forhcoming.

_ **and Stephen Coate**, "Inefficiency in Legislative Policymaking: A Dynamic Analysis," *American Economic Review*, 2007, *97* (1), pp. 118–149.

_ **and** _ , "A Dynamic Theory of Public Spending, Taxation, and Debt," *American Economic Review*, 2008, *98* (1), pp. 201–236.

_ **, Salvatore Nunnari, and Thomas R. Palfrey**, "Legislative Bargaining and the Dynamics of Public Investment," *American Political Science Review*, 5 2012, *106*, 407–429.

**Baunsgaard, Thomas and Michael Keen**, "Tax revenue and (or?) trade liberalization," *IMF Working Paper*, 2005.

**Beetsma, Roel M.W.J. and Xavier Debrun**, "Reconciling Stability and Growth: Smart Pacts and Structural Reforms," *IMF Staff Papers*, 2004, *51* (3).

_ **and** _ , "The new stability and growth pact: A first assessment," *European Economic Review*, 2007, *51* (2), 453 – 477.

**Benhabib, Jess and Aldo Rustichini**, "Social conflict and growth," *Journal of Economic Growth*, 1996, *1* (1), 125–142.

**Besley, Timothy and Stephen Coate**, "Sources of Inefficiency in a Representative Democracy: A Dynamic Analysis," *American Economic Review*, 1998, *88*, 139–156.

**_ and Torsten Persson**, "The Origins of State Capacity: Property Rights, Taxation, and Politics.," *American Economic Review*, 2009, *99* (4), 1218 – 1244.

**_ and _** , "State Capacity, Conflict, and Development.," *Econometrica*, 2010, *78* (1), 1 – 34.

**_ and _** , *Pillars of Prosperity: The Political Economics of Development Clusters*, Princeton University Press, 2011.

**_ , Ethan Ilzetzki, and Torsten Persson**, "Weak States and Steady States: The Dynamics of Fiscal Capacity," *American Economic Journal: Macroeconomics*, 2013, *5(4)*, 205–235.

**Bierbrauer, Felix J. and Pierre C. Boyer**, "Efficiency, Welfare, and Political Competition," *Quarterly Journal of Economics*, 2016, *131* (1), 461 – 518.

**Boyer, Pierre C. and Christoph Esslinger**, "Public Debt and the Political Economy of Reforms," mimeo 2016.

**_ and Kai A. Konrad**, "Targeted campaign competition, loyal voters, and supermajorities," Working Paper 14, Max Planck Institute for Tax Law and Public Finance 2014.

**Carbonell-Nicolau, Oriol and Efe A. Ok**, "Voting over income taxation," *Journal of Economic Theory*, 2007, *134*, 249–286.

**Caselli, Francesco and Massimo Morelli**, "Bad politicians," *Journal of Public Economics*, 2004, *88* (3-4), 759 – 782.

**Claessens, Stijn**, "Access to Financial Services: A Review of the Issues and Public Policy Objectives," *The World Bank Research Observer*, 2006, *21* (2), 207–240.

**Crutzen, Benoît and Nicolas Sahuguet**, "Redistributive politics with distortionary taxation," *Journal of Economic Theory*, 2009, *144*, 264–279.

**Cukierman, Alex and Mariano Tommasi**, "When Does It Take a Nixon to Go to China?," *American Economic Review*, 1998, *88* (1), pp. 180–197.

_ **, Sebastian Edwards, and Guido Tabellini**, "Seigniorage and Political Instability," *American Economic Review*, 1992, *82* (3), pp. 537–555.

**Djankov, Simeon, Caralee McLiesh, and Andrei Shleifer**, "Private credit in 129 countries," *Journal of Financial Economics*, 2007, *84* (2), 299–329.

**Drazen, Allan**, *Political Economy in Macroeconomics*, Princeton, New Jersey: Princeton University Press, 2000.

_ **and Ethan Ilzetzki**, "Kosher pork," *NBER Working Paper*, 2011, *No. 16667.*

_ **and Vittorio Grilli**, "The Benefit of Crises for Economic Reforms," *American Economic Review*, 1993, *83* (3), pp. 598–607.

**Eaton, Jonathan and Mark Gersovitz**, "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, 1981, *48* (2), pp. 289–309.

**Esslinger, Christoph**, "The Political Economy of Reforms with Distortionary Public Debt," mimeo 2016.

**Fernandez, Raquel and Dani Rodrik**, "Resistance to Reform: Status Quo Bias in the Presence of Individual- Specific Uncertainty," *American Economic Review*, 1991, *81* (5), pp. 1146–1155.

**Gehlbach, Scott and Edmund J. Malesky**, "The Contribution of Veto Players to Economic Reform," *Journal of Politics*, 10 2010, *72*, 957–975.

**Grüner, Hans Peter**, "Unemployment and Labor Market Reform: A Contract Theoretic Approach," *Scandinavian Journal of Economics*, 2002, *104* (4), 641–656.

**Hsieh, Chang-Tai**, "Bargaining over reform," *European Economic Review*, 2000, *44* (9), 1659 – 1676.

**Janeba, Eckhard**, "Germany's New Debt Brake: A Blueprint for Europe?," *FinanzArchiv: Public Finance Analysis*, 2012, *68* (4), 383–405.

**Kovenock, Dan and Brian Roberson**, "Electoral Poaching and Party Identification," *Journal of Theoretical Politics*, 2008, *20* (3), 275–302.

_ **and** _ , "Inefficient redistribution and inefficient redistributive politics," *Public Choice*, 2009, *139*, 263–272.

_ **and** _ , "Conflicts with multiple battlefields," in Michelle R. Garfinkel and Stergios Skaperdas, eds., *Oxford Handbook of the Economics of Peace and Conflict*, Oxford University Press, 2012, pp. 503–531.

**La Porta, Rafael, Florencio Lopez de Silanes, and Andrei Shleifer**, "The Economic Consequences of Legal Origins," *Journal of Economic Literature*, 2008, *46* (2), 285–332.

**Laban, Raul and Federico Sturzenegger**, "Distributional conflict, financial adaptation and delayed stabilizations," *Economics & Politics*, 1994, *6* (3), 257–276.

_ **and** _ , "Fiscal conservatism as a response to the debt crisis," *Journal of Development Economics*, 1994, *45* (2), 305 – 324.

**Laslier, Jean-François and Nathalie Picard**, "Distributive Politics and electoral competition," *Journal of Economic Theory*, 2002, *103*, 106–130.

**Lizzeri, Alessandro**, "Budget Deficits and Redistributive Politics," Mimeo, Princeton University 1997.

_ , "Budget Deficits and Redistributive Politics," *Review of Economic Studies*, 1999, *66* (4), 909–928.

_ **and Nicola Persico**, "The Provision of Public Goods Under Alternative Electoral Incentives," CARESS Working Paper 98-08, University of Pennsylvania 1998.

_ **and** _ , "The Provision of Public Goods under Alternative Electoral Incentives," *American Economic Review*, 2001, *91* (1), 225–239.

— **and** — , "Why Did the Elites Extend the Suffrage? Democracy and the Scope of Government, with an Application to Britain's Age of Reform," *Quarterly Journal of Economics*, 2004, *119* (2), 707–765.

— **and** — , "A Drawback of Electoral Competition," *Journal of the European Economic Association*, 2005, *3* (6), 1318–1348.

**Marshall, Monty G. and Keith Jaggers**, "Polity IV Project: Political Regime Characteristics and Transitions, 1800-2010," 2010.

**Martimort, David**, "Optimal Taxation and Strategic Budget Deficit under Political Regime Switching," *Review of Economic Studies*, 2001, *68*, 573–592.

**Maskin, Eric and Jean Tirole**, "Pandering and Pork-Barrel Politics," *Working paper*, 2014.

**Mattozzi, Andrea and Antonio Merlo**, "Mediocracy," *Journal of Public Economics*, 2015, *130*, 32–44.

**Milgrom, Paul and Chris Shannon**, "Monotone Comparative Statics," *Econometrica*, January 1994, *62* (1), 157–80.

**Mirrlees, James A.**, "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies*, 1971, *38*, 175–208.

**Mondino, Guillermo, Federico Sturzenegger, and Mariano Tommasi**, "Recurrent High Inflation and Stabilization: A Dynamic Game," *International Economic Review*, 1996, *37* (4), pp. 981–996.

**Myerson, Roger**, "Incentives to Cultivate Favored Minorities Under Alternative Electoral Systems," *American Political Science Review*, 1993, *87* (4), 856–869.

**Olson, Mancur**, *The rise and decline of nations: Economic growth, stagflation, and economic rigidities*, New Haven: Yale University Press, 1982.

**Persson, Torsten and Guido Tabellini**, *Political Economics: Explaining Economic Policy*, Cambridge Massachusetts, 2000.

_ **and Lars E. O. Svensson**, "Why a Stubborn Conservative would Run a Deficit: Policy with Time-Inconsistent Preferences," *Quarterly Journal of Economics*, 1989, *104* (2), 325–345.

**Prato, Carlo and Stephane Wolton**, "Crises and The Puzzle of Reforms," Working Paper, Mimeo May 2014.

**Reinhart, Carmen M and Kenneth Rogoff**, *This time is different: Eight centuries of financial folly*, Princeton University Press, 2009.

**Ribeiro, Marcos Poplawski and Roel Beetsma**, "The political economy of structural reforms under a deficit restriction," *Journal of Macroeconomics*, 2008, *30* (1), 179 – 198.

**Roberson, Brian**, "The Colonel Blotto game," *Economic Theory*, 2006, *29*, 1–24.

_ , "Pork-Barrel Politics, Tagetable Policies, and Fiscal Federalism," *Journal of the European Economic Association*, 2008, *6* (4), 819–844.

**Rodrik, Dani**, "Understanding economic policy reform," *Journal of Economic Literature*, March 1996, *34*, 9–41.

_ **, Arvind Subramanian, and Francesco Trebbi**, "Institutions rule: the primacy of institutions over geography and integration in economic development," *Journal of Economic Growth*, 2004, *9* (2), 131–165.

**Sahuguet, Nicolas and Nicola Persico**, "Campaign spending regulation in a model of redistributive politics," *Economic Theory*, 2006, *28*, 95–124.

**Schneider, Friedrich**, "Size and Measurement of the Informal Economy in 110 Countries Around the World," *mimeo*, 2002.

**Song, Zheng, Kjetil Storesletten, and Fabrizio Zilibotti**, "Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt," *Econometrica*, 2012, *80* (6), 2785–2803.

**Svensson, Jakob**, "Investment, property rights and political instability: Theory and evidence," *European Economic Review*, 1998, *42* (7), 1317 – 1341.

**Tabellini, Guido and Alberto Alesina**, "Voting on the Budget Deficit," *American Economic Review*, 1990, *80* (1), pp. 37–49.

**The PRS Group**, "International Country Risk Guide," 1980-present.

**Tommasi, Mariano and Andrés Velasco**, "Where are we in the political economy of reform?," *Journal of Policy Reform*, 1996, *1* (2), 187–238.

**Ueda, Kaoru**, "On the Incentives to Cultivate Favored Minorities: A Note," *Games and Economic Behavior*, 1998, *25*, 132–143.

**Williamson, Jeffrey**, *The Political Economy of Economic Reform*, Washington: Institute for International Economics, 1994.

**Yared, Pierre**, "Politicians, Taxes and Debt," *Review of Economic Studies*, 2010, *77* (2), 806–840.

# Eidesstattliche Erklärung

Hiermit erkläre ich, die vorliegende Dissertation selbständig angefertigt und mich keiner anderen als der in ihr angegebenen Hilfsmittel bedient zu haben. Insbesondere sind sämtliche Zitate aus anderen Quellen als solche gekennzeichnet und mit Quellenangaben versehen.

Mannheim, den 08. August 2016

# Lebenslauf

| | |
|---|---|
| 2005 | Abitur, Margarete-Steiff-Gymnasium<br>Giengen an der Brenz |
| 2005-2010 | Studium der Volkswirtschaftslehre (Diplom)<br>Universität Mannheim |
| 2008-2009 | Auslandsstudium<br>University of Toronto, Toronto, Canada |
| 2010-2016 | Promotionsstudium am Center for Doctoral<br>Studies in Economics (CDSE)<br>Universität Mannheim |