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Abstract

This paper investigates the roles psychological biases play in empirically estimated deviations between subjective survival beliefs (SSBs) and objective survival probabilities (OSPs). We model deviations between SSBs and OSPs through age-dependent inverse S-shaped probability weighting functions (PWFs), as documented in experimental prospect theory. Our estimates suggest that the implied measures for cognitive weakness, *likelihood insensitivity*, and those for motivational biases, *relative pessimism*, increase with age. We document that direct measures of cognitive weakness and motivational attitudes share these trends. Our regression analyses confirm that these factors play strong quantitative roles in the formation of subjective survival beliefs. In particular, cognitive weakness is an increasingly important contributor to the overestimation of survival chances in old age.

JEL Classification: D12, D83, I10.

Keywords: Subjective Survival Beliefs, Probability Weighting Function, Confirmatory Bias, Cognition, Optimism

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1 Introduction

Important economic problems, such as the decision about when to retire, how much to save for retirement and whether to purchase life-insurance, depend on the formation of survival beliefs over an individual's life-cycle. A rational individual would be modeled as a statistician whose survival beliefs are given as data-based (Bayesian or frequentist) estimates. For this rational benchmark, any differences between subjective survival beliefs and their objective counterparts can only result from an insufficient amount of data, and biases will decrease when the individual collects more data with age. Empirical studies on subjective survival beliefs, however, do not support this notion of convergence of perceived survival chances to objective survival probabilities. Instead, the literature robustly documents a *flatness bias*, i.e., relatively young respondents (younger than age 70) express underestimation, whereas relatively old respondents (older than age 75) express overestimation of survival chances on average.[1] Moreover, we document that these biases are not negligible: on average, 65 year-old respondents underestimate their survival probabilities by roughly 10 percentage points, whereas 85 year-old respondents overestimate them by roughly 15 percentage points. What is driving these age-dependent patterns of survival belief biases on top of any statistical learning process?

We argue in this paper that the answer to this question is *psychological factors*. In particular, we show that cognitive weakness/strength and pessimism/optimism play important quantitative roles in the formation of age-dependent subjective survival beliefs. As the first step leading us to this finding, we provide a descriptive analysis of survival belief biases by comparing subjective survival beliefs (SSBs) to objective survival probabilities (OSPs) using data from the Health and Retirement Study (HRS). In the HRS, interviewees are asked about their beliefs on whether they will survive from the interview age to some target age that is several years ahead. To compare these individual SSBs with their objective counterparts, we estimate for each interviewee the corresponding individual-level OSP by using the information on actual HRS mortality and several conditioning

[1] Inspired by Hamermesh (1985), a growing body of economic literature documents such a *flatness bias*, cf., e.g., Elder (2013), Ludwig and Zimper (2013), Peracchi and Perotti (2014) and Groneck et al. (2016).

variables, including mortality trends. For this purpose, we adapt the methods used by Khwaja et al. (2007), Khwaja et al. (2009) and Winter and Wuppermann (2014) to estimate mortality hazard rates at the individual level. Plotting SSBs against OSPs over age, we document the *flatness bias* in the form of an average underestimation of respondents of age 70 and younger, respectively, an overestimation of respondents of age 75 and older. *Within* a given age group, we find that respondents with low OSPs express overestimation, whereas respondents with high OSPs express underestimation, resulting in a "flattening out" of SSBs compared to the 45-degree line of OSPs.

To formally describe these biases, we next adopt the concept of inverse S-shaped transformations of objective probabilities, as known from experimental prospect theory (PT) (cf. Kahneman and Tversky (1979), Tversky and Kahneman (1992), Wakker (2010)). More specifically, we assume that SSBs can be modeled as age-dependent Prelec (1998) probability weighting functions. In line with the usual interpretation of the parameters of the Prelec function (cf. Wakker (2010)), we assume that the motivational factor of *relative pessimism* is expressed through the *elevation* and that the cognitive factor of *likelihood insensitivity* corresponds to the *flatness* of the Prelec function. Likelihood insensitivity refers to a cognitive weakness according to which people cannot distinguish well among the respective likelihoods of events that are neither impossible nor certain. An extreme case of such flattening-out are *fifty-fifty* probability judgments, which are well-documented in the psychological literature (Bruine de Bruin et al. 2000).[2] Estimating age-specific Prelec probability weighting functions on our data of SSBs, we find that the *elevation* of the Prelec function decreases with age, whereas its *flatness* increases. These findings thus imply that the implicit measures of the relative pessimism of respondents and their likelihood insensitivity increase with age.[3]

Our next objective is to compare the age patterns of these implicit cognitive and motivational factors to directly observable counterparts. Because we do not have individual-level data on relative pessimism and likelihood insensitivity,

[2]Gonzalez and Wu (1999) refer to these concepts as *attractiveness* and *diminishing sensitivity*, respectively.

[3]Our finding of increasing likelihood insensitivity with age is consistent with Booij et al. (2010), who also find that the elderly are more insensitive to likelihood.

we look at proxies for these variables in the HRS. From wave 8 onward, the HRS contains measures on *dispositional optimism (pessimism)* that are derived from the same statements as in the well-known Life Orientation Test-Revised (LOT-R).[4] We find that dispositional optimism is decreasing with age, whereas dispositional pessimism is increasing, on average.[5] To obtain a good proxy for *likelihood insensitivity*, we consider HRS measures on the cognitive weakness of the respondent, which is motivated by a cognitive interpretation of likelihood insensitivity (Wakker 2010). This cognitive measure is a version of a composite score taken from RAND, and it combines the results of several cognitive tests. We find that cognitive weakness is strongly increasing with age. Thus, these age patterns are consistent with the age patterns of our implied measures of *relative pessimism* and *likelihood insensitivity* that we obtained from the age-dependent Prelec functions.

Finally, in order to estimate the quantitative impact of cognitive and motivational factors on subjective survival beliefs, we combine the Prelec transformation of OSPs with the HRS data on direct cognitive and motivational measures. Specifically, we specify both parameters of the Prelec function—*relative pessimism* and *likelihood insensitivity*—as linearly dependent on dispositional optimism (pessimism) and cognitive weakness. This linear dependence also features a constant. With our estimate of that constant, we identify a significant *base bias* in the form of a baseline inverse-S-shaped transformation of the objective survival probabilities. We interpret this base bias as capturing incomplete statistical learning of, respectively, (rational) inattention to, the OSP of the individual. Thus, the base bias reflects that individuals may only partially use their individual-level OSP in their formation of subjective beliefs. Since the shape of the inverse S-shaped transformation of OSPs attributable to this base effect is constant over age by construction, changes in differences between SSBs and OSPs attributable to the base bias reflect movements of the underlying OSPs. For example, because OSPs are relatively high at the age of 65, the base effect induces an underestima-

[4]The Life Orientation Test-Revised questionnaire (LOT-R) was developed to measure dispositional optimism, i.e., a generalized expectation of good outcomes in one's life (Scheier and Carver 1987; Scheier et al. 1994).

[5]While it may seem that optimism is just the opposite of pessimism, psychologists measure the two phenomena separately. We further explore the differences in Section 4.

tion of long-horizon survival chances of approximately 10 percentage points. At age 85, however, OSPs are relatively low and the base bias therefore induces an overestimation of approximately 7 percentage points at that age. In addition, our estimates identify a *change of cognition* over the life-cycle that describes a clockwise tilting of the probability weighting function in age so that lack of cognition increases with age (the Prelec function becomes flatter). Relative to the base bias, this leads to an *additional* underestimation at age 65 by minus 5 percentage points and to an *additional* overestimation at age 85, also by 5 percentage points. In contrast to these dynamic effects of cognition, the effects of the motivational factors pessimism and optimism are roughly constant in age. *Pessimism* leads to a downward bias by 5 percentage points, and *optimism* leads to an upward bias by 10 percentage points.

We thus find that cognitive and motivational factors are important drivers of subjective survival beliefs beyond any statistical learning processes that may take place. While the motivational bias measured as optimism and pessimism does not significantly change with age, cognitive weakness, measured as likelihood insensitivity, does. We therefore conclude that cognitive weakness is an increasingly important contributor to the overestimation of survival chances in old age.

Relation to the Literature. Our work contributes to the economics literature on subjective expectations (Manski 2004), particularly on subjective survival beliefs, which is inspired by Hamermesh (1985). On the one hand, the literature documents that SSBs are broadly consistent with OSPs and co-vary with direct measures of health, such as health behavior (e.g., smoking) or health status, in the same way as OSPs (Hurd and McGarry 1995; Gan et al. 2005) in that SSBs serve as predictors of actual mortality (Hurd and McGarry 2002; Smith et al. 2001) and that individuals revise their SSBs in response to new adverse (health) shocks (Smith et al. 2001).

On the other hand, several authors document important biases in subjective survival beliefs when comparing sample average beliefs to objective survival probabilities (Elder 2013; Ludwig and Zimper 2013; Peracchi and Perotti 2014; Groneck et al. 2016). We emphasize that motivational (optimism and pessimism) and cognitive factors are important contributors to these biases. In this respect, our work relates to medical studies examining the link between psychosocial dispositions and health shocks (Kim et al. 2011) or subjective body weight (Sutin 2013). Mirowsky and Ross (2000) and Griffin et al. (2013) study how incorporating motivational factors influences subjective life expectancy. We extend their work by controlling for OSPs. D'Uva et al. (2015) investigate the effects of cognition on the accuracy of longevity expectations. We go beyond their analyses by combining motivational and cognitive variables and by focussing on probabilities.

Manifestations of biases driven by motivational factors have also been discussed in the behavioral learning literature in the form of *confirmatory* biases (Rabin and Schrag 1999), *myside* biases (Zimper and Ludwig 2009), *partisan* biases (Jern et al. 2014; Weeks 2015), and *irrational belief persistence* (Baron 2008). Simply speaking, people who are biased by motivational factors 'only see/learn what they want to see/learn' so that any new information tends to confirm already existing beliefs and convictions. One would expect that motivational biases play an important role in the formation of survival beliefs, as most people strongly dislike to die. According to Kastenbaum (2000) "[...], most of us prefer to minimize even our cognitive encounters with death." Under the plausible assumption that "cognitive encounters with death" increase with age, elderly people might express more *optimistic* attitudes towards their likelihood of surviving. An age-increasing motivational bias in the form of optimism could then explain why elderly people increasingly overestimate their survival chances compared with younger people, for whom the prospect of death is less apparent.

Although our analysis confirms that a motivational (i.e., confirmatory) bias is important for the formation of survival beliefs at all ages, we find that it leads to a roughly constant bias in age, on average. If anything, our descriptive analysis suggests that probability weighting functions express more pessimistic rather than optimistic beliefs as individuals become older. In contrast, both our descriptive and regression analyses suggest that cognitive weakness is an increasingly important quantitative contributor to the overestimation of survival chances over an individual's life-cycle.

To model age-dependent survival beliefs, we employ a Prelec probability weighting function applied to objective survival probabilities, which is a prominent approach in *prospect theory* (PT). As a generalization of *rank-dependent utility theories*, pioneered by Quiggin (1981, 1982), modern PT has developed into a comprehensive decision theoretic framework that combines empirical insights—starting with Kahneman and Tversky (1979))—with theoretical results about integration with respect to non-additive probability measures; cf. the Choquet expected utility theories of Schmeidler (1989) and Gilboa (1987).

Of the many aspects of PT, our model of age-dependent biases in survival beliefs is thus related to the experimental PT literature, which shows that subjective probability judgments cannot be described as additive probabilities. To be precise, the experimental *two-stage* PT literature shows that in a first stage, subjective probability judgments (=beliefs) resemble an inverse S-shaped transformation of additive probabilities, while in a second stage, these transformed probability judgments themselves undergo another inverse S-shaped transformation (emphasizing pessimism) to become decision weights reflecting the decision maker's preferences; cf., e.g., Tversky and Fox (1995), Fox et al. (1996), Fox and Tversky (1998), Gonzalez and Wu (1999), Kilka and Weber (2001), and Wakker (2004, 2010). According to this two-stage approach, probability weighting is relevant for the formation of beliefs *and* decision weights.[6]

Our findings contribute to the literature on the two-stage approach within the special context of age-dependent survival beliefs. While the inverse S-shape of probability judgments has typically been documented in experimental situations, only a very few papers document evidence of inverse S-shaped probability judgments in non-experimental data.[7] In contrast to experimental situations, for

[6]The two-stage approach is not uncontroversial. For example, Barberis (2013) argues that probability weighting is exclusively a feature of preferences (recent contributions in this line of literature are Woodford (2012) and Steiner and Stewart (2016), who postulate that perception biases may arise as a second-best solution if the information processing capacity is limited or if the processing is noisy). However, this view ignores, e.g., people who judge the chances of A versus NOT A as *fifty-fifty*, even if they were told that the mutually exclusive events A, B, and C are equally likely. Arguably, many real-life people commit such cognitive errors, "reflecting insufficient sensitivity to changes in likelihood" (Wakker 2004, p. 239). While inverse S-shaped beliefs are thus prevalent even in situations under *risk*—in which individuals are told objective probabilities—they are apparently even more pronounced in situations under *uncertainty*, for which no objective probabilities are provided; cf. Wakker (2004).

[7]For example, Polkovnichenko and Zhao (2013) and Andrikogiannopoulou and Papakonstantinou (2016) find evidence for inverse S-shaped probability weighting for option prices and betting markets, respectively.

which the experimental design can clearly distinguish between risk and uncertainty, this clear-cut distinction does not apply to non-experimental HRS data on survival beliefs: Although there exist OSPs, we cannot know how much the subjects of the HRS are aware of these objective probabilities, so we look at a hybrid situation for which both aspects, risk and uncertainty, are relevant. This is reflected in our estimates of the base bias. Since it is plausible to assume that assessments of long-run survival chances involve ample uncertainty, the strong quantitative role of the base bias we uncover can be interpreted as a confirmation that inverse S-shaped probability weighting is indeed very pronounced in situations with uncertainty.

Importantly, it speaks to the robustness of the experimental PT findings on probability judgments that we can confirm the typical inverse S-shape for the survival belief data at all ages. Moreover, our regression analyses with respect to direct motivational and cognitive measures support the typical interpretations of the PT literature on probability judgments.

The remainder of this paper is organized as follows. Section 2 presents the main stylized facts on survival belief biases. Section 3 provides a structural interpretation of these biases through prospect theory. Section 4 looks at the direct psychological measures elicited in the HRS. Section 5 quantifies the role of cognitive and motivational variables for subjective survival beliefs. Finally, Section 6 concludes. Separate appendices contain further information on the data and additional results.

2 Age Patterns of Biases in Survival Beliefs

2.1 Data Sources

We base our estimates of subjective survival beliefs (SSBs) and of the corresponding objective survival probabilities (OSPs) on the Health and Retirement Study (HRS), which is a national representative panel study of the elderly US population. Individuals are interviewed on a biennial basis. Interviews of the first wave were conducted in 1992. The interviewees are individuals older than 50 and their spouses regardless of age. An overview of the survey, its waves and the interview cohorts is contained in Appendix A.

In our descriptive analyses, we use waves 8-11. In our regression analyses with lagged variables, we use waves $9-11$ (years $2006-2012$) because motivational variables are only available for waves 8-12, and our measure of the individual level OSP is dependent on our index of contemporaneous cognitive weakness index which is only available up to wave 11. To estimate the individual-level objective survival probabilities (OSPs), we accordingly use waves $4-12$ of the HRS as well as data from the Human Mortality Database (HMD).[8] For further details on the sample selection, see again Appendix A.

2.2 Subjective Survival Beliefs

In the HRS, an interviewee i of age h is asked about her SSB to live to at least a certain target age m, which we denote as $SSB_{i,h,m}$. We focus on individuals in the survey who are age 65 and older. This sample restriction is used because the data set does not allow us to estimate OSPs for ages less than 65, with details provided in Subsection 2.3 below. The assignment of target age $m(h)$ to interview age h for our sample is given in Table 1. Observe that the distance between interview age h and target age $m(h)$ is the same across all interview age/target age groups.

Table 1: Interview Age h and Target Age $m(h)$

Interview age h	Target Age $m(h)$
65-69	80
70-74	85
75-79	90
80-84	95
85-89	100

Source: Health and Retirement Study (HRS).

[8] We explore the index of cognitive weakness up to wave 11 and compute panel mortality between waves 11 and 12.

2.3 Objective Survival Probabilities

To study survival misconceptions at the individual level, our first objective is to assign to each individual in the sample its respective objective survival probability (OSP). Using aggregate data from (cohort) life-tables for this purpose[9] is ill-suited because individual (objective) survival rates generally deviate from sample averages. To estimate individual-level OSPs, we adapt the methods described in Khwaja et al. (2007), Khwaja et al. (2009) and Winter and Wuppermann (2014). We accordingly employ a duration model to estimate hazard rates conditional on several individual-level characteristics. Among standard variables such as age, socioeconomic status, and health behavior, the set of explanatory variables includes predicted average OSPs (AOSPs) in order to capture time trends of mortality hazards. We extract the time trend by applying the Lee and Carter (1992) procedure; see Appendix A.2.

We estimate the relationship between individual-level observable variables and the AOSPs, both collected in $\mathbf{x}_i$, and mortality using a hazard function given by

$$\lambda(t|\mathbf{x}_i') = \lambda_0(t)\exp(\mathbf{x}_i'\beta), \tag{1}$$

where time to failure t is the number of years to death. $\lambda_0(t)$ is the baseline hazard, for which we choose the specification given by the Weibull (1951) hazard model.[10] This allows us to model duration dependence, i.e., the fact that mortality rates are an increasing function of age. Accordingly, we impose the structure

$$\lambda_0(t) = \alpha t^{\alpha-1} \tag{2}$$

[9]As, e.g., in Perozek (2008), Ludwig and Zimper (2013), Peracchi and Perotti (2014), and Groneck et al. (2016).

[10]A specification of the hazard function that allows for unobserved heterogeneity may be preferable. However, when we tried to estimate the individual OSPs with a specification of the hazard function that allows for unobserved heterogeneity, we faced serious convergence problems in many of our bootstrap iterations. Thus, we compared the results of the first bootstrap of a specification while allowing for unobserved heterogeneity with our specification in the paper. The coefficient estimates and the estimate for duration dependence are very similar. Additionally, we compared the distributions of the predicted OSPs for both specifications, which are very similar as well. Hence, we are confident that our results are not affected much by ignoring unobserved heterogeneity in our specification of the hazard function.

which allows for $\alpha > 1$ (capturing positive duration dependence). $\exp(\mathbf{x}'\beta)$ is the proportional hazard. In our estimation, survivors are treated as censored, and we estimate function (1) by maximum likelihood.

The objective survival probabilities (OSPs) for all prediction horizons t and each individual i of interview age h are given by (cf., e.g., Wooldridge (2002) and Cameron and Trivedi (2005)):

$$OSP_{i,h}(t) = \exp\left[-\exp(\mathbf{x}_i'\beta)t^{\alpha}\right] \tag{3}$$

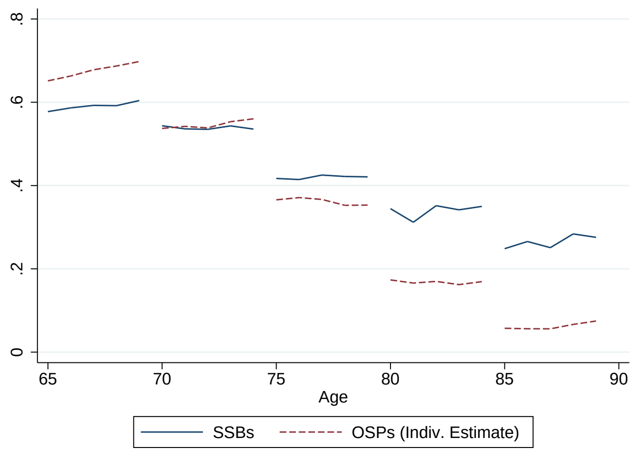
From this, we can also construct the OSP until the target age (with horizon $t = m(h) - h$), $OSP_{i,h,m(h)}$, which we assign to the respective $SSB_{i,h,m(h)}$ of individual i.

2.4 Biases

Our following descriptive analysis compares individual-specific SSBs from the survey data with our individual-specific measures of OSPs. As a first step, we replicate the results of previous literature—e.g., Hamermesh (1985), Elder (2013), Ludwig and Zimper (2013) and Peracchi and Perotti (2014)—on the age patterns of survival beliefs in Figure 1. As a crucial difference from this literature, we calculate average OSPs from our individual measures instead of using average (cohort) life-tables. The step function in the figure is due to the change in assignment of the interview and target ages, cf. Table 1. Our findings confirm the well-established *flatness bias*: At ages prior to age 70, individuals, on average, underestimate their probabilities to survive, whereas for ages above 75, they overestimate it.

Next, we take a new perspective for which individual-level data are needed. Instead of computing averages over age, we average over OSPs, i.e., for each OSP, we compute the average SSB. In the upper left panel of Figure 2, we show the corresponding results by plotting the average SSBs against the average OSPs. If the SSBs are aligned along the 45-degree line, then there is no bias. However, we observe a very systematic pattern of misconception: Individuals with low OSPs, on average, overestimate their survival chances, whereas those with high OSPs, on average, underestimate it.

Figure 1: “Flatness Effect”

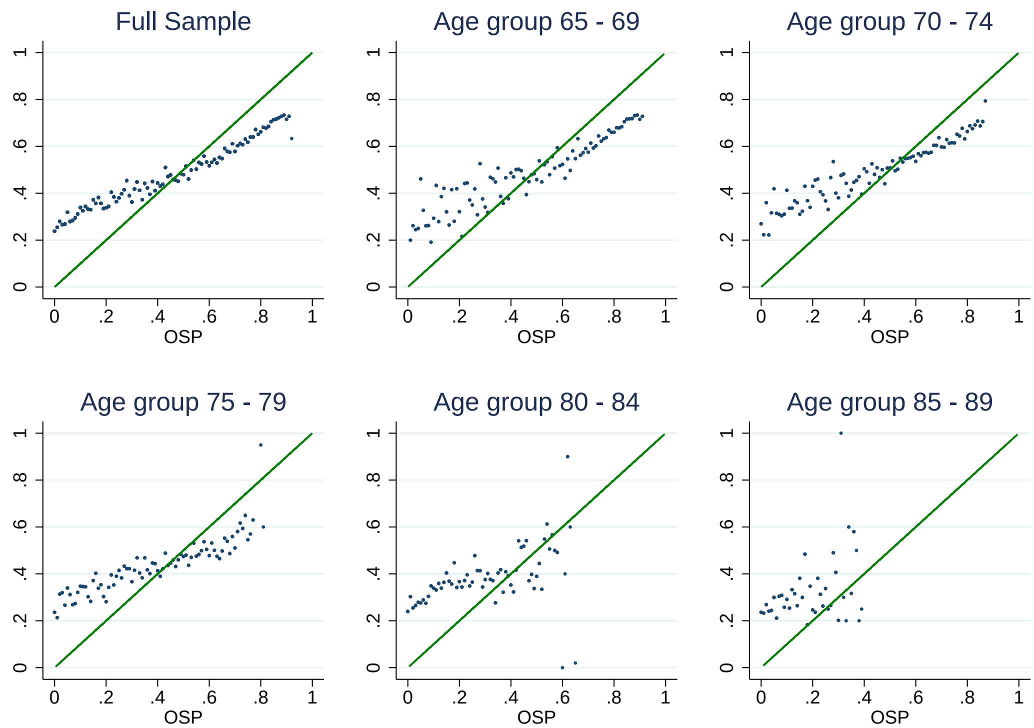


Notes: Average subjective beliefs about surviving to different target ages (SSBs, solid line) and corresponding average objective survival probabilities (OSPs, dashed line), cf. equation (3). SSBs are elicited in the HRS for a combination of the age at interview of the individual (which is shown on the abscissa) and a corresponding target age, cf. Table 1. The step function follows from changes in the interview age/target age assignment. *Source*: Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

The two perspectives on the data taken in Figure 1 and the upper left panel of Figure 2 suggest a very simple explanation for the observed biases across age. Suppose that individuals were to always resolve any uncertainty about their survival chances in a 50-50 manner (Bruine de Bruin et al. 2000), i.e., their response were a weighted average of a fifty percent chance of survival and the actual OSP. Such a 50-50 heuristic could obviously explain the pattern in the upper left panel of Figure 2. Furthermore, young respondents in our data have OSPs above 50 percent. If they were to apply such a simple heuristic, then they would, on average, underestimate their chances to survive. Old respondents, on the other hand, have long-run OSPs less than 50 percent, on average.[11] Under such a heuristic, they would accordingly overestimate their OSPs, on average. Hence, such a 50-50 bias could simultaneously explain the patterns of Figure 1 and the upper left panel of Figure 2.

[11]Recall from Table 1 that the target age is several years ahead of the interview age.

Figure 2: Objective Survival Probabilities and Subjective Survival Beliefs by Age Group



Notes: SSB over OSP by age group. The upper left panel is for all ages. The remaining age group panels focus on different target ages according to Table 1. *Source*: Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

We next argue that there is more information content in the data, giving rise to alternative interpretations.[12] For this purpose, we repeat the previous analysis for different age groups. In the remaining panels of Figure 2, we display SSBs as a function of OSPs for each target age group; cf. Table 1. The figure suggests that the flatness of SSBs against OSPs grows stronger with increasing age—compare, e.g., age group 65-69 with age group 80-84. In addition, the

[12]The general notion of more information content beyond a mere 50-50 bias is also shared in the earlier work by Hurd and McGarry (1995), Hurd et al. (1999), Smith et al. (2001), Smith et al. (2001), Hurd and McGarry (2002) and Gan et al. (2005). We add to this literature by emphasizing the roles of cognitive and motivational factors.

intersection with the 45-degree line moves downward, from approximately 50 percent for age group 65-69 to approximately 40 percent for age group 80-84. Therefore, the average tendency for underestimation increases across age groups.

Hence, it appears that, on average, pessimism and likelihood insensitivity are both increasing with age. The next section provides a structural interpretation of these biases and trends over age.

3 Modeling Subjective Survival Beliefs

3.1 The Prelec Probability Weighting Function

We interpret these biases in survival beliefs through the lens of prospect theory (PT).[13] More precisely, we take from PT the concept of inverse S-shaped probability weighting functions in order to model probability judgments in the form of SSBs. The use of inverse S-shaped probability weighting functions enables us to model the flatness of SSBs relative to the underlying OSPs shown in Figure 2. We additionally aim at capturing the dynamics of the PWFs across age, i.e., the increasing flatness of SSBs and the decreasing intersection with the 45-degree line we observe in the remaining panels of Figure 2. We approach this by a specific functional form assumption on the probability weighting function using a parsimonious parameterization, which, employing the terminology of Wakker (2010), gives rise to two psychological interpretations of these data features. First, the increasing flatness reflects, along a *cognitive* dimension, an increasing insensitivity to the objective likelihood of the decision maker (likelihood insensitivity). Second, the decreasing intersection with the 45-degree line reflects increasing pessimism, respectively, decreasing optimism, and hence a *motivational* interpretation of the data. Our aim is to estimate these implicit cognitive and motivational measures.[14]

[13]As a generalization of rank dependent utility theories (pioneered by Quiggin (1981, 1982)), modern prospect theory (PT) (Tversky and Kahneman 1992) has developed into a comprehensive decision theoretic framework that combines empirical insights (starting with Kahneman and Tversky (1979)) with theoretical results regarding integration with respect to non-additive probability measures (cf. the Choquet expected utility theories of Schmeidler (1989) and Gilboa (1987)).

[14]Our estimates may be biased by two important features of the data. First, for the oldest two age groups, we only cover part of the full support of OSPs, because the long-run objective

To simultaneously capture these *cognitive* and *motivational* dimensions, we adopt the non-linear probability weighting function (PWF) suggested by Prelec (1998). Consider the mapping of the OSP to the SSB according to

$$SSB = \left(\exp\left(-\left(-\ln\left(OSP\right)\right)^{\xi}\right)\right)^{\theta} \tag{4}$$

for parameters $\xi \geq 0, \theta \geq 0$. These two parameters control the elevation and the curvature of the function, which can be interpreted as measures of pessimism/optimism and likelihood insensitivity, respectively.

To see this, observe that for $\xi = \theta = 1$, function (4) coincides with the 45-degree line. Holding θ constant at one, an increase of ξ above one leads to an S-shaped pattern and a decrease below one leads to an inverse S-shape, whereby the intersection with the 45-degree line is at the objective probability of $OSP = \exp(-1) \approx 0.37$.[15] This dependency on ξ is illustrated in Panel (a) of Figure 3, where we decrease ξ from one to zero.[16] In the limit where $\xi = 0$, the curve is flat. Hence, ξ can be interpreted as a measure of likelihood insensitivity: lowering ξ decreases the responsiveness of the SSB in the OSP, i.e., there is a lower likelihood sensitivity, respectively, higher likelihood insensitivity. As we further illustrate in Panel (b) of Figure 3, decreasing θ leads to an upward shift of the PWF, whereas increasing it leads to a downward shift. Accordingly, θ can be interpreted as a measure of relative pessimism whereby a higher value of θ means higher pessimism. Finally, notice that unless $\theta = 1$ (or $\xi = 1$), these two properties of the PWF interact. This can be seen in Panel (b) of Figure 3, where varying the pessimism parameter θ simultaneously affects the shape (slope) of the probability weighting function.

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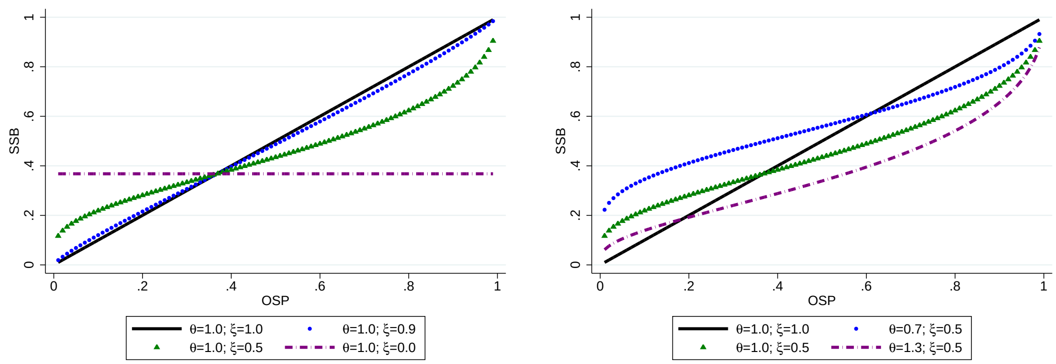
survival chances do not exceed 70%, respectively, 50%, for these two groups, cf. Figure 2. This censoring of the data may bias our estimates for these groups. Second, survival chances are naturally bounded from below by zero and from above by one so that respondents with very high (low) objective survival probabilities cannot overestimate (underestimate) their survival chances by much, whereas the respective other side is less limited. This may induce a flatness of the PWFs; see also our discussion in Section 5.

[15]For $\xi \neq 1$, $SSB = OSP$ iff $-\ln(SSB) = -\ln(OSP) = 1$; hence, $SSB = OSP = \exp(-1) \approx 0.37$.

[16]Given the data patterns shown in Figure 2, which resemble an inverse S at all interview ages, a parameterization with $\xi > 1$ is irrelevant in our context.

Figure 3: Pessimism and Likelihood Sensitivity in Stylized PWF

(a) Varying ξ (b) Varying θ



Notes: Stylized Prelec (1998) probability weighting functions. The left panel shows the impact of likelihood insensitivity, ξ, for $\theta = 1$ and $\xi \in [0, 0.5, 0.9, 1]$. The right panel shows the impact of pessimism for $\xi = 0.5$ and $\theta \in [0.7, 1, 1.3]$.

3.2 Age-Dependent Probability Weighting Functions

We next postulate that relationship (4) is an appropriate model for each individual i's subjective belief of surviving from current (interview) age h to some future age t. Accordingly, we specify for a given $OSP_{i,h,t}$ that

$$SSB_{i,h,t} = \left(\exp\left(-\left(-\ln\left(OSP_{i,h,t}\right)\right)^{\xi_h}\right)\right)^{\theta_h} + \epsilon_{i,h,t}. \tag{5}$$

The error term $\epsilon_{i,h,t}$ captures errors both in our measures of objective survival probabilities and in the round-off in answering patterns.[17] Notice that parameters θ_h, ξ_h are now (interview) age h specific. Through this, we capture the dependency of survival belief formation on the current age, as suggested by the age group-specific bias patterns displayed in Figure 2.

To estimate parameters ξ_h, θ_h, we further restrict these parameters to be the same for each interview age h assigned with a specific target age $m(h)$, i.e., we let $\xi_h = \bar{\xi}_{m(h)}$ and $\theta_h = \bar{\theta}_{m(h)}$. We identify these parameters under this constraint by minimizing the Euclidean distance between the predicted and reported

[17]One specific form of round-off is the tendency to provide focal point answers at SSBs of 0%, 50% or 100%.

subjective survival beliefs for each individual in group $m(h)$ using the data of Figure 2.

Figure 4 shows the predicted probability weighting functions with the corresponding 95% confidence intervals. Standard errors are bootstrapped and confidence intervals are computed using the percentile method.[18,19] For the fitted values of the full sample displayed in the upper left panel, we observe a quite symmetric weighting function intersecting the 45-degree line close to 0.5. As already suggested by the pattern in Figure 2, the age-specific weighting functions depicted in the other panels in Figure 4 reveal two facts: First, the functions become flatter with increasing age, and second, the intersection with the 45-degree line is at lower values for older ages—it is at approximately 55 percent for age group 65-69 and at approximately 40 percent for age group 80-84.

Figure 5 depicts the parameter estimates $\xi_h = \bar{\xi}_{m(h)}, \theta_h = \bar{\theta}_{m(h)}$, again with the bootstrapped 95% confidence intervals. The coefficient estimates $\xi_h = \bar{\xi}_{m(h)}$, shown in Panel (a) of the figure, are decreasing in h, which according to our interpretation captures increasing likelihood insensitivity. Note, however, that the differences between age groups are not always statistically significant.

Estimates $\theta_h = \bar{\theta}_{m(h)}$, shown in Panel (b), show a less clear-cut age pattern. They are increasing between interview age groups 70-74 and 85-89, but the confidence bands for the oldest group are rather large. The estimated pessimism parameters are also decreasing from age group 65-69 to age group 70-74. Overall, we can conclude that pessimism also tends to increase for ages above 70.

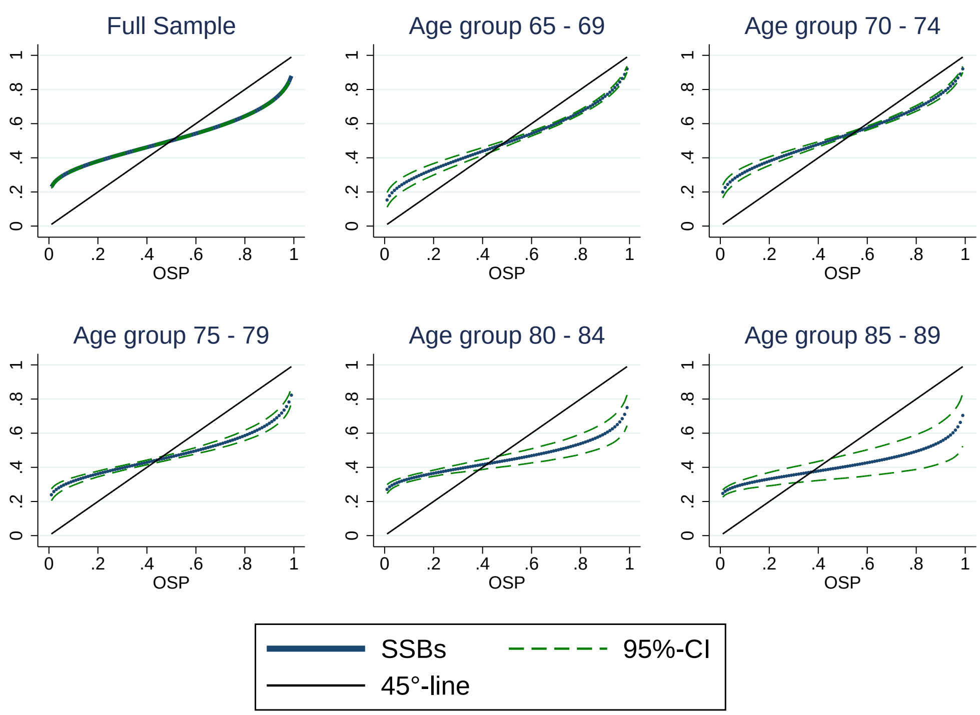
In Appendix B, we also investigate the robustness of these results by use of linear probability weighting function and thereby confirm our main findings: implied measures of likelihood sensitivity (pessimism) tend to decrease (increase) with age. In the next section, we turn to direct measures of cognitive and moti-

[18]Since our data are clustered, we perform a cluster bootstrap that samples the clusters with replacement. Thus, in each bootstrap, we solve

$$\min_{\bar{\xi}_{m(h)}, \bar{\theta}_{m(h)}} \left\{ \sum_{i=1}^{N^{m(h)}} \left[\epsilon_{i,h,m(h)}\right]^2 \right\}.$$

[19]The percentile method uses the relevant percentiles of the empirical distribution of our bootstrap estimates of the Prelec parameters.

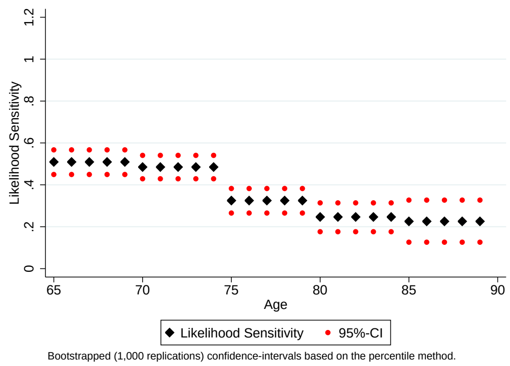
Figure 4: Estimated Non-linear Probability Weighting Functions



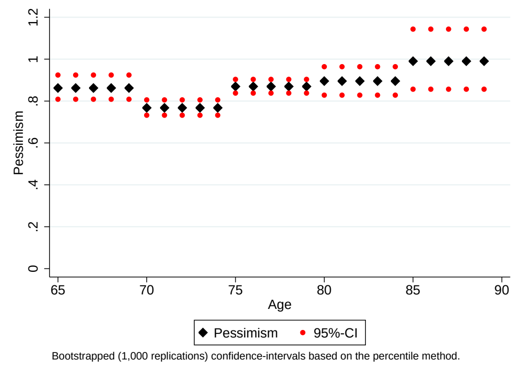
Notes: Estimated Prelec probability weighting functions for the full sample (upper left panel) and for different age groups. Parameters estimated with non-linear least squares. *Source*: Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

Figure 5: Estimated Prelec Parameters: Likelihood Sensitivity and Pessimism

(a) Likelihood Sensitivity ξ_h



(b) Pessimism θ_h



Notes: This figure shows estimates of $\xi_h = \bar{\xi}_{m(h)}$ in Panel (a), estimates of $\theta_h = \bar{\theta}_{m(h)}$ in Panel (b), and the bootstrapped (1,000 replications) 95% confidence intervals, which are based on the percentile method. *Source*: Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

vational variables to investigate whether we can observe similar age patterns.

4 Direct Psychological Measures

4.1 Measures

From wave 8 onward, the HRS contains measures on optimism and pessimism. Measures on *dispositional optimism (pessimism)* are derived from the same statements as in the well-known Life Orientation Test-Revised (LOT-R).[20] Respondents are given various statements regarding a specific latent variable. For most variables, they were asked *"please say how much you agree or disagree with the following statements"*. Each statement is rated on a scale from one (*strongly disagree*) to six (*strongly agree*). Average scores are taken to create indices for each motivational construct. Higher values for the motivational variables imply more-optimistic, respectively, more pessimistic, attitudes.[21]

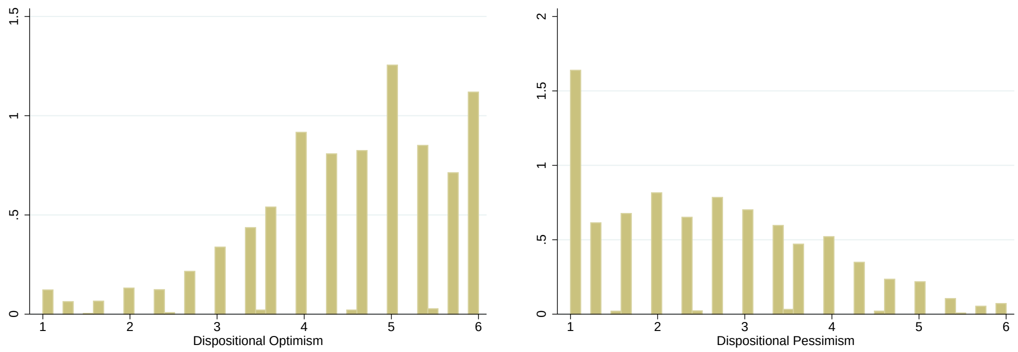
[20]Such statements are, e.g., "In uncertain times I usually expect the best".

[21]The index score is set to missing if responses on more than half of the respective statements are missing.

Figure 6: Histogram of Optimism and Pessimism

(a) Optimism

(b) Pessimism



Notes: Histograms of 'optimism' and 'pessimism' variables. Averages of answer scale, where 1 indicates 'strongly disagree' and 6 'strongly agree'. *Source*: Health and Retirement Study (HRS).

Note that *optimism* and *pessimism* are usually measured separately, i.e., respondents are asked questions with negative connotations (pessimism) or positive connotations (optimism). The reason for separate measures is that these two concepts were found to display bi-dimensionality (Herzberg et al. 2006). Figure 6, showing the histograms on both measures in our sample, underscores this aspect. Dispositional pessimism shows a clear focal point at index value 1 (="strongly disagree"), whereas dispositional optimism apparently has focal point answers at values 4, 5 and 6, and the peak is at 5. In our empirical analyses, we therefore use separate variables for each concept, although in our previous descriptive and theoretical analyses, we speak of increasing pessimism and decreasing optimism interchangeably.

For a measure corresponding to *likelihood insensitivity*, our choice of a proxy variable is motivated by our cognitive interpretation of likelihood insensitivity (Wakker 2010). Thus, we include a variable measuring the cognitive weakness of the respondent. It is a version of a composite score taken from RAND and combines the results of several cognitive tests. For instance, respondents were asked to recall a list of random words, to count backwards and to name the (Vice) President of the United States. In total, there are 35 questions and the results are summarized in an ability score. We take RAND's composite score of cognitive ability as given and create our score of cognitive weakness. For this purpose, we subtract the cognitive ability score from the maximal achievable value, i.e., our measure of cognitive weakness is 35 minus cognitive ability. A higher score indicates higher cognitive weakness.

An overview of our three measures of cognitive and motivational variables is given in Table 2.

Table 2: Cognitive and Motivational Variables

	Min	Max	Mean	SD
Cognitive Variable				
Cognitive Weakness	0	35	13.56	5.19
Motivational Variables				
Dispositional Optimism	1	6	4.51	1.15
Dispositional Pessimism	1	6	2.56	1.26

Notes: This table summarizes the sample moments our measure of cognitive weakness and the two motivational variables, dispositional optimism and pessimism. *Source:* Own calculations, Health and Retirement Study (HRS).

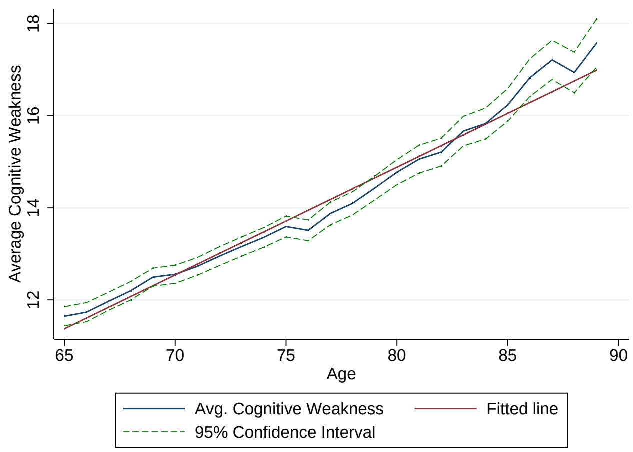
4.2 Age Patterns

Figures 7 and 8 display the age patterns of cognitive weakness and of the two motivational measures, respectively. The average index value of cognitive weakness is quite strongly and statistically significantly increasing from 11.8 to 17.9 between ages 65 and 89. Optimism decreases by 2.9% and pessimism increases by 12.2% from age 65 to 90, but both age patterns are less pronounced.[22] That pessimism increases more strongly than optimism decreases supports the notion of the bi-dimensionality of these two measures.

Hence, the age trends of the direct cognitive and motivational measures coincide with the indirect measures we derived from estimating probability weighting

[22] Regressing the average cognition, pessimism and optimism on age gives slope coefficients of 0.2341, 0.0080 and -0.0055, respectively.

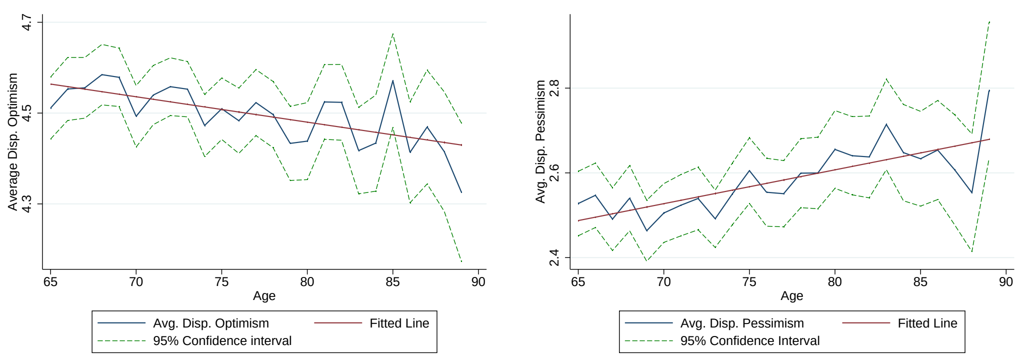
Figure 7: Average Cognitive Weakness Score over Age



Notes: Average cognitive weakness score over age with 95% confidence-interval calculated based on a normality assumption. The straight line shows the prediction of a linear regression of cognitive weakness against age. *Source*: Own calculations, Health and Retirement Study (HRS).

Figure 8: Average Optimism and Pessimism over Age

(a) Optimism (b) Pessimism



Notes: Average optimism and pessimism scores over age with 95% confidence intervals calculated based on a normality assumption. The straight line shows the prediction of a linear regression of optimism (pessimism) against age. *Source*: Own calculations, Health and Retirement Study (HRS).

functions on the data of subjective survival beliefs. These findings lead us to conjecture that cognition and motivational attitudes play a strong role in the formation of subjective survival beliefs. Our next aim is to investigate this interpretation of the data further through regression analyses.

5 Regression Analyses

5.1 Parameterized Non-linear PWFs

To investigate whether our measures of cognitive and motivational factors play a role in the formation of subjective survival beliefs and to quantify their impact, we consider a parameterized variant of the Prelec (1998) function. Specifically, we postulate that for each individual in the sample i and each age h, the implicit measures of cognition, $\xi_{i,h}$, and optimism/pessimism, $\theta_{i,h}$, from equation (5) are linearly dependent on the cognitive, respectively, motivational, variables as follows:

$$\xi_{i,h} = \xi_0 + \xi_1 c_{i,h-2} \tag{6a}$$

$$\theta_{i,h} = \theta_0 + \theta_1 p_{i,h-2} + \theta_2 o_{i,h-2} \tag{6b}$$

In the above, $c_{i,h-2}$ is the lag of our measure of cognitive weakness, and $p_{i,h-2}$ is the lag of our measure of pessimism, whereas $o_{i,h-2}$ is the lag of our measure of optimism. Using lags of these measures allows us to treat them as weakly exogenous so that we avoid spurious correlation[23]. It also allows us to interpret our findings on the relationship between cognitive and motivational measures and SSBs, at least tentatively, as causal.[24]

Replacing in (5) the age-specific parameters ξ_h and θ_h with the individual and age-specific parameters $\xi_{i,h}, \theta_{i,h}$ and using (6), our specification of survival beliefs

[23]E.g., health shocks may affect cognition and motivational attitudes directly and lead to adjustments of subjective survival beliefs.

[24]While the approach of using lags for causal identification is widespread in the social sciences, this approach is not without criticism; cf. Bellemare et al. (2015). We therefore speak of a "tentative" interpretation. Our results hold unchanged if we use contemporaneous measures of our cognitive and motivational variables (available upon request). Reporting those instead would change our wording in statements such as "a change of cognition (or pessimism) *leads to* a change of SSBs by factor x", where *"leads to"* would be replaced with *"is associated with"*.

reads as

$$SSB_{i,h,m(h)} = \left(\exp\left(-\left(-\ln\left(OSP_{i,h,m(h)}\right)\right)^{\xi_0+\xi_1 c_{i,h-2}}\right)\right)^{\theta_0+\theta_1 p_{i,h-2}+\theta_2 o_{i,h-2}}, \quad (7)$$

which we estimate on the pooled sample of HRS data.

Turning to the parameters of interest in specification (7), we refer back to our analysis of Section 3, in particular to the illustration in Figure 3. In light of our discussion there, parameters ξ_0 and θ_0 capture a *base effect* in subjective beliefs. With regard to the base effect in cognition, ξ_0, we conjecture that this base effect indeed exists in form of an inverse S, and therefore, we expect $\xi_0 \in (0, 1)$. This may reflect an average degree of cognitive weakness, incomplete statistical learning, (rational) inattention with respect to objective survival probabilities or a statistical artifact from truncation of the data.[25] On the other hand, with regard to a base effect in optimism/pessimism captured $\theta_0 \neq 1$, we do not have a specific prior expectation. Recall from our discussion in Section 3 that $\theta_0 < 1$ reflects rather optimistic beliefs, whereas $\theta_0 > 1$ reflects rather pessimistic beliefs. Depending on which of these two motivational factors dominate, we would find an average upward or downward shift of the probability weighting function. Furthermore, recalling the illustrative analysis of Figure 3, a lower likelihood sensitivity, ξ_h, leads to a flatter PWF. Therefore, if changes in cognitive weakness are relevant to the formation of subjective beliefs, we would find its coefficient to be negative, $\xi_1 < 0$. Also, since increasing relative pessimism, θ_h, leads to a lower elevation of the PWF, we expect that $\theta_1 > 0$. Likewise, since increasing relative optimism leads to a higher elevation, we expect that $\theta_2 < 0$. To summarize, our main hypotheses on the signs of the coefficients are that $\xi_0 \in (0, 1), \xi_1 < 0, \theta_1 > 0$ and $\theta_2 < 0$. We do not have a hypothesis on the sign of θ_0.

Our main results summarized in Table 3 show that there is indeed a significant average inverse-S-shaped transformation of objective survival probabilities ($\xi_0 = 0.54 < 1$). At the same time, we do not identify an additional baseline shifter of the PWF that would reflect average optimistic or pessimistic beliefs, because θ_0 is not statistically different from 1 and the point estimate is close to one.

[25] SSBs cannot be less than zero or above one. Such truncation may induce overestimation, on average, for OSPs close to zero and underestimation for OSPs close to one, which leads to a natural flatness of the PWF relative to the 45-degree line.

Thus, the base effect exclusively captures factors that lead to an inverse-S of the PWF relative to the 45-degree line, without additional shifting. Since we cannot distinguish between the alternative explanations for this effect discussed above, we subsequently speak of this average base effect simply as a *base bias*. Beyond the base bias, increasing lack of cognition leads to increasing likelihood insensitivity ($\xi_1 = -0.01$), which flattens the non-linear PWF. Furthermore, increasing pessimism leads to a significant downward shift ($\theta_1 = 0.029$) and increasing optimism to a significant upwards shift ($\theta_2 = -0.05$) of the non-linear PWF. Thus, cognitive and motivational factors do have a significant effect on the formation of subjective survival beliefs of the expected sign.

Table 3: The Effects of Cognition and Motivational Measures on Subjective Survival Beliefs

	point estimate	CI-	CI+
Cog.Weak. Intercept (ξ_0)	0.5457	0.4952	0.5955
Cog.Weak. Slope (ξ_1)	-0.0134	-0.0170	-0.0095
Psycho. Intercept (θ_0)	1.0285	0.9471	1.1160
Pessimism Slope (θ_1)	0.0295	0.0176	0.0420
Optimism Slope (θ_2)	-0.0583	-0.0722	-0.0435
OSP_0	0.3513	0.2948	0.4144
AIC	2990.0	2778.1	3190.8

Notes: Number of observations: 8858. Column 2 shows the point estimates. Columns 3 and 4 show the respective bounds of the bootstrapped 95%-confidence intervals (CI- and CI+), which are based on the percentile method (1,000 replications). AIC: Akaike (1973) information criterion. *Source:* Own calculations, Health and Retirement Study (HRS).

To quantify the impact of these factors, we next compute the predicted subjective beliefs of the probability weighting function. To isolate the effect of each factor, we decompose the probability weighting function into the following elements:

$$\text{base bias:} \quad SSB^{b}_{i,h,m(h)} = \Big(\exp\Big(-\big(-\ln\big(OSP_{i,h,m(h)}\big)\big)^{\xi_0}\Big)\Big)^{\theta_0} \tag{8a}$$

$$\text{base + cogn. weakn.:} \quad SSB^{bc}_{i,h,m(h)} = \Big(\exp\Big(-\big(-\ln\big(OSP_{i,h,m(h)}\big)\big)^{\xi_0+\xi_1 c_{i,h-2}}\Big)\Big)^{\theta_0} \tag{8b}$$

$$\text{base + pess.:} \quad SSB^{bp}_{i,h,m(h)} = \Big(\exp\Big(-\big(-\ln\big(OSP_{i,h,m(h)}\big)\big)^{\xi_0}\Big)\Big)^{\theta_0+\theta_1 p_{i,h-2}} \tag{8c}$$

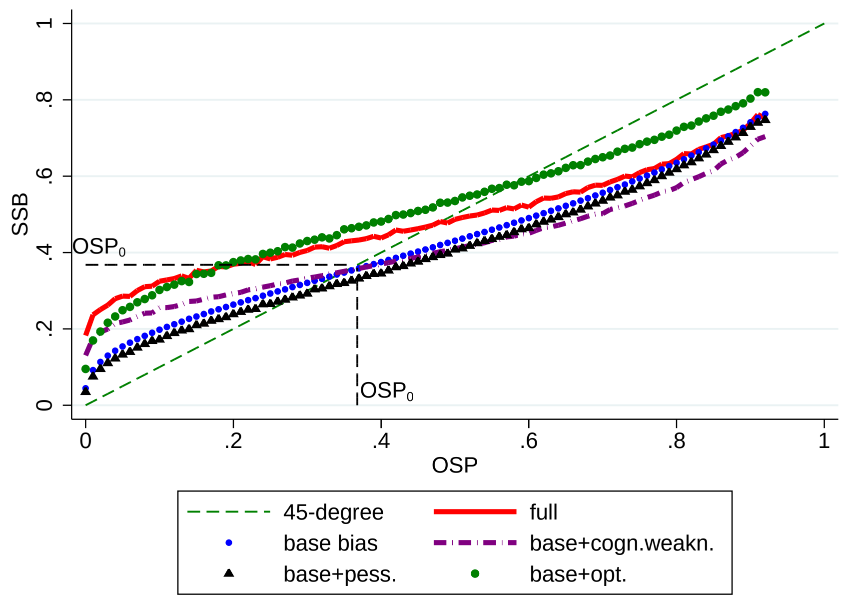
$$\text{base + opt.:} \quad SSB^{bo}_{i,h,m(h)} = \Big(\exp\Big(-\big(-\ln\big(OSP_{i,h,m(h)}\big)\big)^{\xi_0}\Big)\Big)^{\theta_0+\theta_2 o_{i,h-2}} \tag{8d}$$

To carry out this decomposition, we first predict the full model and the sub-PWFs defined in (8) for each individual and then take sample averages. In our subsequent description, we denote these respective predicted values by $\widehat{\ }$, i.e., $\widehat{SSB}$ stands for the sample average of the predicted PWFs under the "full" model, equation (7), and so forth.

The results on the predictions for the full model and its decomposition are displayed in Figure 9. The predicted base bias $\widehat{SSB}^{b}$ ("base bias") displays a pronounced inverse S and intersects with the 45-degree line at $\widehat{OSP}_0 = 0.35$. Since the estimate $\hat{\theta}_0$ is close to one, this intersection is close to the theoretical intersection at $\exp(-1) \approx 0.37$; cf. Section 3, in particular Figure 3. Predictions for the base bias plus changes in cognitive weakness $\widehat{SSB}^{bc}$ ("base + cogn. weakn.") lead to a clockwise rotation of the PWF, while the intersection with the 45-degree line stays at roughly $\widehat{OSP}_0 = 0.35$, which again is a consequence of $\hat{\theta}_0 \approx 1$. The predictions from the base model with optimism, $\widehat{SSB}^{bo}$ ("base + opt."), imply a parallel upward shift relative to the base model, which dominates the smaller downward shift of pessimism; see $\widehat{SSB}^{bp}$ ("base + pess."). As a consequence of all these effects, the PWF in the full model ("full") is both flatter and shifted upwards relative to the PWF of the base bias.

Figure 10 provides the decomposition over age, relating us back to Figure 1. Panel (a) shows the data on SSBs and OSPs (i.e., the data points of Figure 1) and the predicted values for the full model and for the base bias. Overall, the

Figure 9: Decomposition of Non-linear PWFs

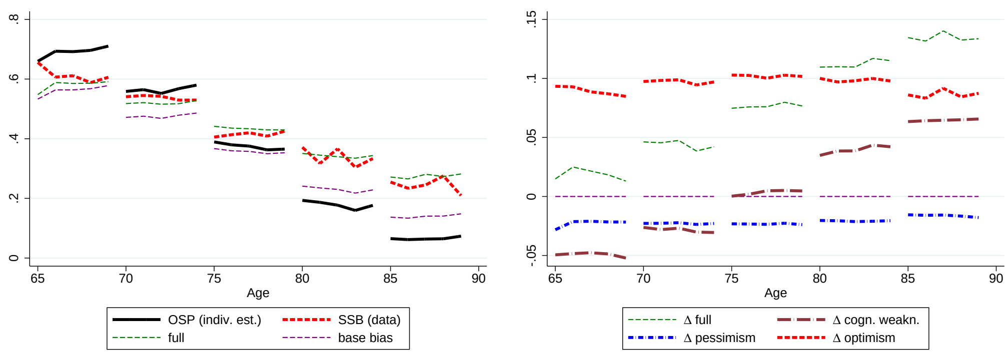


Notes: Sample averages of predicted non-linear probability weighting functions according to equations (7) and (8); "full": $\widehat{SSB}$; "base cogn. weakn.": $\widehat{SSB}^{b}$; "total cogn. weakn.": $\widehat{SSB}^{bc}$; "base+pess.": $\widehat{SSB}^{bp}$; "base+opt.": $\widehat{SSB}^{bo}$. *Source*: Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

full model displays a very close match to the average data on SSBs. Consistent with our findings in Figure 9, the base bias leads to a stronger underestimation than in the full model. Panel (b) provides the differences relative to the base bias caused by changes of cognitive weakness with age, the parallel shifts induced by pessimism and optimism, and, finally, the full model. That is, we show the differences between (8b), (8c), respectively, (8d)) to (8a), and we denote these differences by Δ. Due to the increasing cognitive weakness over age, individuals, on average, overestimate their survival chances increasingly more as they grow older: relative to the base bias, cognitive weakness initially leads to a downward bias of approximately -5%p and eventually to an overestimation by slightly more than 5%p. We also observe that pessimism leads to an underestimation of survival chances by roughly -5%p and optimism to a strong overestimation by approximately 10%p for all age groups. Overall, the effects of cognitive and motivational variables on subjective survival beliefs are quite strong, with a net effect of approximately 2%p for age group 65-69 and almost 12%p for age group 85-89. Importantly, the effects of cognitive factors are changing with age, whereas the effects of the two motivational factors optimism and pessimism are roughly constant. We can therefore conclude that lack of cognition plays an increasingly important role in the observed overestimation of survival chances in old age.

Figure 10: Non-Linear PWF: Decomposition over Age

(a) Base Cogn. Weakn. (b) Additional Effects



Notes: Sample averages of predicted subjective survival beliefs according to equations (7) and (8) by age; Panel (a): "full": $\widehat{SSB}$; "base bias": $\widehat{SSB}^b$; Panel (b): "Δ full": $\widehat{SSB} - \widehat{SSB}^b$; "Δ base+cogn. weakn.": $\widehat{SSB}^{bc} - \widehat{SSB}^b$; "Δ base+pess.": $\widehat{SSB}^{bp} - \widehat{SSB}^b$; "Δ base+opt.": $\widehat{SSB}^{bo} - \widehat{SSB}^b$. *Source*: Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

5.2 Parameterized Linear PWFs

To investigate the sensitivity of our findings with respect to the functional form of the PWF, we repeat our regressions for a neo-additive PWF (Chateauneuf et al. 2007a), which is linear for interior survival probabilities, thereby approximating the non-linear model. The main advantage of the linear model is that we can interpret coefficient estimates directly as marginal effects. Furthermore, it is straightforward to add additional control variables in the linear framework. However, one drawback is that the neo-additive PWF is only a crude approximation to answering patterns for extreme OSPs close to 0% or 100%. In addition, the structural parameters are only partially identified. In particular, without further assumptions, we cannot identify the base bias and the marginal effect of an increase of cognitive weakness; cf. Appendix B.2. To identify both, we invoke our insights from the estimates of the non-linear PWF and derive in Appendix B.2 an indirect approach to identify both effects.[26] There, we derive the following reduced form specification of the linear model:

$$
\begin{aligned}
SSB_{i,h,m(h)} = \beta_0 + \beta_1 OSP_{i,h,m(h)} + \beta_2 c_{i,h-2}\left(OSP_{i,h,m(h)} - OSP_0\right) \\
+ \gamma_1 p_{i,h-2} + \gamma_2 o_{i,h-2} + \gamma_3\left(p_{i,h-2}c_{i,h-2}\right) + \gamma_4\left(o_{i,h-2}c_{i,h-2}\right).
\end{aligned} \tag{9}
$$

The results from estimating (9) are summarized in Table 4. All coefficient estimates on the pure effects are of the expected sign and are significant, whereas the interaction terms between cognition and the motivational variables are small and insignificant. This latter finding is convenient for our decomposition analysis because it means that the marginal effects of an increase in cognitive weakness as defined earlier can be identified by setting these interactions to zero. This marginal effect is accordingly given by $\hat{\beta}_2\left(OSP_{i,h,m(h)} - \widehat{OSP}_0\right)$. Since $\hat{\beta}_2 < 0$, the effect is negative (positive) for $OSP_{i,h,m(h)} > \widehat{OSP}_0$ ($OSP_{i,h,m(h)} < \widehat{OSP}_0$), which reflects the clockwise tilting of the PWF induced by an increase of cognitive weakness, just as in the non-linear model.[27] To illustrate the effects of cognitive weakness, consider first an individual with an OSP of $OSP_{i,h,m(h)} = 0.9$. For this person, the marginal effect is $-0.0107 \cdot (0.9 - 0.3676) \cdot 100\%p = -0.6\%p$. Likewise, for a person with an OSP of $OSP_{i,h,m(h)} = 0.1$, the effect is positive at $0.3\%p$. Our estimates also suggest that a one-point increase of pessimism leads respondents to underestimate survival changes by $1.7\%p$, and a one-point increase of optimism leads respondents to overestimate them by $2.6\%p$.[28]

[26] To identify the base bias, we postulate that the base bias plus the effect of cognition, $\widehat{SSB}^{bc}$, intersects with the line of the base bias, $\widehat{SSB}^{b}$, in the same point on the 45-degree line, just as we found for the non-linear model. Accordingly, parameter OSP_0 is estimated from the intersection of $\widehat{SSB}^{bc}$ with the 45-degree line.

[27] Our estimate of OSP_0 of 0.36 is very close to the corresponding estimate for the non-linear model; cf. Table 3.

[28] Finally, comparing the AIC between Tables 3 and 4, there is no statistical difference in the goodness of fit between the non-linear and linear models.

Table 4: Linear Model: The Effects of Cognition and Motivational Measures on Subjective Survival Beliefs

	point estimate	CI-	CI+
Constant	0.0486	-0.0506	0.1451
OSP	0.6316	0.5691	0.6929
OSP × Cog. Weak.	-0.0107	-0.0155	-0.0059
Pessimism	-0.0167	-0.0307	-0.0016
Optimism	0.0261	0.0110	0.0412
Optimism × Cog. Weak.	-0.0004	-0.0015	0.0009
Pessimism × Cog. Weak.	0.0000	-0.0012	0.0010
OSP_0	0.3571	0.3019	0.4126
AIC	2946.1	2735.5	3137.8

Notes: Number of observations: 8858. Column 2 shows the point estimates. Columns 3 and 4 show the respective bounds of the 95%-confidence intervals (CI- and CI+), which are calculated with the percentile method (1,000 replications). AIC: Akaike (1973) information criterion. *Source:* Own calculations, Health and Retirement Study (HRS).

In addition to these interpretations of the marginal effects, we decompose the linear probability weighting functions analogously to equation (8) and Figures 9 and 10, with details provided in Appendix B.2. This confirms our main findings for the non-linear model: the quantitative roles of our cognitive measure and of the two motivational variables for the age-specific differences between SSBs and OSPs are very similar. Thus, our findings for the non-linear model are robust to the linear approximation.

The Linear Model and Statistical Learning. On the basis of formal models of statistical learning, individuals learn their individual OSP by obtaining more information. This suggests that they base their survival beliefs on the OSP and additional variables (e.g., those we use to predict the respective OSPs) as well as (in case of biased beliefs) cognitive and motivational factors. A *snapshot* of a reduced form learning model, as in Viscusi (1985) and Smith et al. (2001), and for biased beliefs in Ludwig and Zimper (2013) and Groneck et al. (2016), can be approximated as a linear regression by adding controls to (9) so that

$$\begin{aligned} SSB_{i,h,m(h)} = \beta_0 + \beta_1 OSP_{i,h,m(h)} + \beta_2 c_{i,h-2}\left(OSP_{i,h,m(h)} - OSP_0\right) \\ + \gamma_1 p_{i,h-2} + \gamma_2 o_{i,h-2} + \gamma_3\left(p_{i,h-2}c_{i,h-2}\right) + \gamma_4\left(o_{i,h-2}c_{i,h-2}\right) + \vec{\chi}'\vec{x}_{i,h}, \end{aligned} \tag{10}$$

where $\vec{x}_{i,h}$ is the vector of control variables.[29]

Table 5: Linear Model: The Effects of Cognition and Motivational Measures on Subjective Survival Beliefs: Adding Control Variables

	point estimate	CI-	CI+
Constant	-0.0093	-0.3379	0.3371
OSP	0.4081	0.3032	0.5067
OSP × Cog. Weak.	-0.0111	-0.0161	-0.0065
Pessimism	-0.0116	-0.0253	0.0032
Optimism	0.0211	0.0058	0.0358
Optimism × Cog. Weak.	-0.0002	-0.0013	0.0010
Pessimism × Cog. Weak.	-0.0004	-0.0015	0.0007
OSP_0	0.3729	0.2616	0.6204
AIC	2588.3	2336.0	2741.6

Notes: Number of observations: 8858. Column 2 shows the point estimates. Columns 3 and 4 show the respective bounds of the 95%-confidence intervals (CI- and CI+), which are calculated with the percentile method (1,000 replications). AIC: Akaike (1973) information criterion. *Source:* Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

The results on our main parameters of interest are reported in Table 5, and estimates for the control variables are contained in Table 8 of Appendix B.2 (which are of the expected sign and are in line with findings in the literature). Our main findings from Table 4 are unchanged. The coefficient on the constant decreases and becomes very imprecise (i.e., the constant is basically zero) and

[29] Analogously, we could replace in (7) the variable $OSP_{i,h,m(h)}$ with function $f\left(OSP_{i,h,m(h)}, \vec{x}\right)$, for $f : OSP_{i,h,m(h)}, \vec{x} \to [0,1]$. E.g., f could be a logistic function or a hazard function. We have experimented with such specifications but faced severe convergence problems. We therefore rely on the linear model to investigate the robustness of our findings w.r.t. the inclusion of control variables.

the coefficient estimate on the OSP, $\hat{\beta}_1$, decreases due to the additional explanatory power attributable to the control variables (also leading to a decrease in the AIC, indicating better fit). One interpretation of the finding that additional control variables matter for the formation of subjective survival beliefs and take on explanatory power from the objective survival probabilities is that the base bias indeed partially captures learning mechanisms in the face of uncertain survival beliefs. Otherwise, the parameter estimates are unchanged (the confidence intervals overlap). Thus, our main findings on the effects of cognitive and motivational factors for the formation of subjective survival beliefs are also robust to the inclusion of control variables in the empirical specification.

5.3 Quantile Regressions

So far, our analyses have been based on a strong structural interpretation of the data. In particular, we have postulated (and found) that cognitive weakness leads to a clockwise tilting of the PWF and that optimism induces a parallel upward shift of the PWF (respectively, pessimism leads to a parallel downward shift). We now investigate the robustness of these findings by running quantile regressions. This allows us to detect relationships that are not captured by mean effects. In our quantile regressions, we take the difference between SSBs and OSPs as a dependent variable. Additionally, we include the level of the objective survival probability in our set of explanatory variables because the interval of our dependent variable is directly linked to the level of the OSP. We analyze every decile and estimate the results for all deciles jointly. As in our previous OLS regressions, standard errors are bootstrapped. Our regression specification is

$$
\begin{aligned}
SSB_{i,h,m(h)} - OSP_{i,h,m(h)} = & \\
& \beta_0 + \beta_1 OSP_{i,h,m(h)} + \beta_2 c_{i,h} + \beta_3 p_{i,h-2} + \beta_4 o_{i,h-2} + \epsilon_{i,h,m(h)}. \quad (11)
\end{aligned}
$$

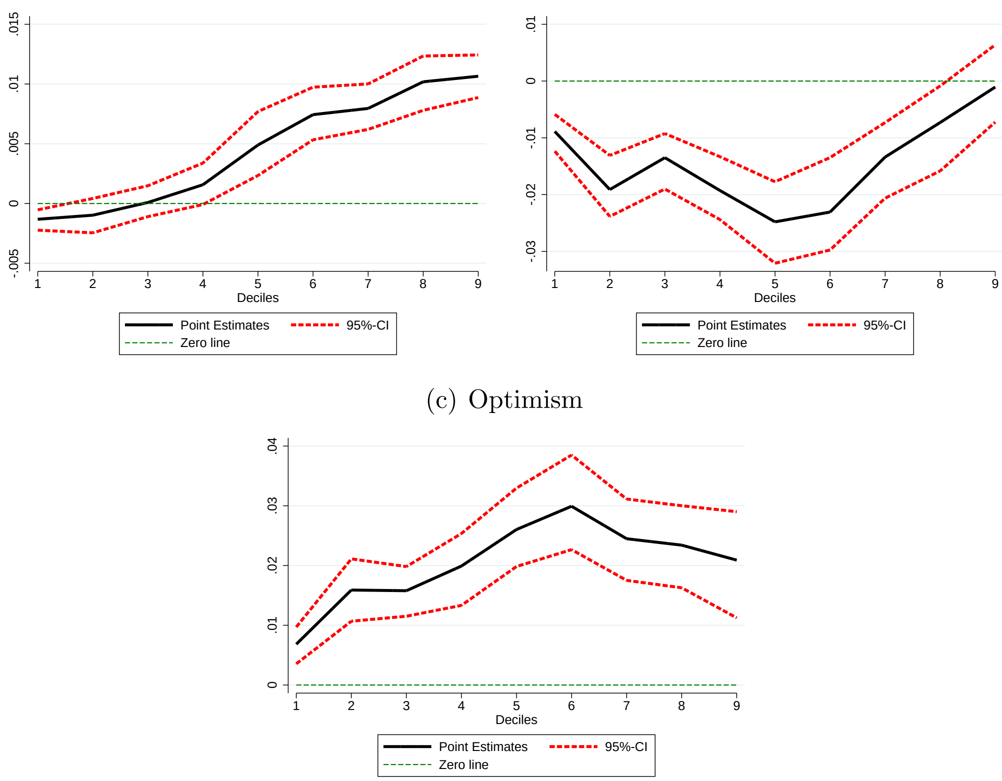
By including the OSP on the right-hand-side of the regression, we control for biases induced by truncation and censoring, as underestimators cannot report SSBs less than zero and overestimators cannot report SSBs above one. The clockwise tilting of the PWF from increasing cognitive weakness we identified earlier would be consistent with negative estimates of β_2 in lower percentiles and positive estimates in upper percentiles. This would mean that increasing cognitive weakness leads to a more pronounced underestimation for underestimators (who, on average, have high OSPs) and a more pronounced overestimation for overestimators (who, on average, have low OSPs). Irrespective of the percentiles, we also expect that $\beta_3 < 0$ and $\beta_4 > 0$.

Figure 11: Quantile Regression: Coefficient Estimates

(a) Δ Cogn. Weakn.

(b) Pessimism

(c) Optimism



Notes: Coefficient estimates of equation (11) by deciles of underestimation and the respective bounds of the 95%-confidence intervals, which are calculated with the percentile method (1,000 replications). *Source*: Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

We report our results in Figure 11, thereby confirming our hypotheses. Interestingly, we also find that the effects of optimism and pessimism are strongest for the intermediate percentiles. This is consistent with the non-linear probability weighting function: in the lowest percentiles, we have individuals with, on average, high OSPs, where the structure of the non-linear PWF forces subjective beliefs to converge to 1; cf. Figure 2. Likewise, in the highest percentile, individuals have, on average, low OSPs, which forces subjective beliefs to converge to 0. Thus, under a non-linear PWF, there is less room for motivational variables to impact the formation of SSBs at extreme OSPs of 0 and 1. This is reflected in our estimates shown in Panels (b) and (c) of Figure 11.

Overall, our results for these less-parametric quantile regressions support our structural interpretation of the data by use of inverse S-shaped probability weighting functions.

5.4 The Effects of Motivational Factors on Mortality

In our estimation of individual-level OSPs in Section 2, we include cognition as a regressor in the hazard model.[30] Therefore, our estimates in the previous subsections capture the effects of cognitive weakness on the formation of survival beliefs beyond the effects that are channeled through the objective survival probabilities. However, for sample reasons, we do not include motivational variables in this hazard model, because they are only available from wave 8 onwards. If optimists were more likely to survive and if pessimists were more likely to die, then the observed deviations from the SSBs caused by these motivational attitudes we identified would (at least partially) reflect additional information of respondents on their objective mortality risk rather than psychological biases.

Table 6: The Impact of Optimism and Pessimism on Mortality

	point estimate	CI-	CI+
Optimism	-0.0089	-0.0380	0.0256
Pessimism	0.0277	-0.0041	0.0592

Notes: Column 2 shows the point estimates. Columns 3 and 4 show the respective bounds of the 95%-confidence intervals (CI- and CI+), which are calculated with the percentile method (1,000 replications). *Source:* Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

[30] We find that the lack of cognition is positively correlated with mortality.

We can address this concern in a smaller sub-sample by re-estimating the hazard model with the inclusion of the motivational variables, using HRS data from waves 8-12. The results are shown in Table 6. We find neither optimism nor pessimism to be significant at the 5% significance level. This supports our interpretation of the effects of optimism and pessimism on SSBs as reflecting psychological biases.

6 Concluding Remarks

This paper compares subjective survival beliefs (SSBs) with objective survival probabilities (OSPs) that we estimate based on individual-level characteristics. We establish a twofold and related strong regularity of survival misperceptions. First, relatively young respondents in our sample underestimate their chances of survival, whereas relatively old respondents overestimate them. Second, respondents overestimate survival chances with low objective probabilities and underestimate chances with high objective probabilities. Based on this finding, we estimate inverse S-shaped probability weighting functions on the data and establish a strong age dependency in the shape of these functions. Our coefficient estimates suggest that implied measures of pessimism and of cognitive weaknesses are increasing with age. Direct psychological measures confirm these age patterns.

Based on these descriptive findings, we estimate reduced form variants of probability weighting functions. In particular, we find that biases induced by the motivational factor optimism leads to an overestimation of subjective survival beliefs by approximately 10 percentage points, on average, whereas the motivational factor pessimism leads to a downward bias by approximately 5 percentage points. Both biases are roughly constant in age. In contrast, increasing cognitive weakness leads to an increasing upward bias in subjective survival beliefs. By thus showing that these factors play an important role in the formation of subjective beliefs, our results support the cognitive and motivational interpretations attached to the parameters of inverse S-shaped probability weighting functions in the theoretical and experimental literature (Gonzalez and Wu 1999; Wakker 2010). Our findings also suggest that cognitive rather than motivational factors play an increasingly important quantitative role in the overestimation of survival beliefs in old age.

Our work gives rise to two related questions for future research: Can subjective beliefs with respect to other economic risks such as earnings or health risks be modeled similarly, and what is the implication of biases in subjective beliefs for economic decisions? As to the second part of this question, we employ in Groneck et al. (2016) a life-cycle model of consumption and savings to show that biased survival beliefs can contribute to resolving well-known life-cycle saving puzzles.[31] Our results in the present paper shed more light on the driving forces of such biased survival beliefs. In a standard life-cycle model of consumption and savings, they alter the *effective* discount factor, e.g., pessimism would increase discounting by decreasing survival beliefs. However, there may also be direct effects of psychological factors. For instance, according to the recent theoretical work by Gabaix and Laibson (2017), lack of cognition leads to higher *pure time* discounting, and Binswanger and Salm (2017) find that the association between subjective probabilities and decisions increases with an individual's cognitive strength, whereas lower cognitive skills are more strongly associated with heuristics. Furthermore, in extended models with multiple risks, e.g., earnings risks and health risks, psychological factors will also affect the formation of beliefs with respect these risks (Dominitz and Manski 1997; Rozsypal and Schlafmann 2017). For these reasons, the answer of how psychological factors affect economic decisions over the life-cycle can, in our view, only be provided by use of structural life-cycle models that enable researchers to explicitly take into account all these mechanisms.

[31] Direct measures of subjective beliefs are crucial in this research agenda because this approach circumvents the inherent difficulty of disentangling the effects of biased beliefs from preference parameters such as risk aversion (Wu and Gonzalez 1996; Constantinos et al. 2015).

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A Supplementary Appendix: Data

A.1 Data Sets and Samples

The main dataset used in this paper is the Health and Retirement Study (HRS) and the Human Mortality Database (HMD).

A.1.1 Health and Retirement Study (HRS)

The Health and Retirement Study (HRS) is a national representative panel study on a biennial basis, see Juster and Suzman (1995) for an overview.[32] The main purpose of the HRS is to contribute a rich panel data set to the research of retirement, health insurance, saving, and economic well-being. Since 2006 (wave 8) the HRS is complemented by a rich set of psychosocial information. These data are collected in each biennial wave from an alternating (at random) 50% of all core panel participants who were visited for an enhanced face-to-face interview (EFTF).[33] Thus, longitudinal data are available in four-year intervals and therefore the first panel with psychosocial variables is provided in 2010.

Hazard Model. We employ a hazard model to predict individual level objective survival probabilities (OSPs) based on HRS panel mortality. As the time horizons of OSPs and SSBs have to be aligned, c.f. Table 1, our sample has to cover between 11 and 15 years. In the HRS, individuals younger than 65 were asked about their subjective belief to survive another $20 - 35$ years. As the HRS data set does not yet cover this large time horizon, we cannot compute OSPs for this age group and therefore restrict our sample to individuals older than 65.

[32] The survey is administered by the Institute for Social Research (ISR) at the University of Michigan and mainly funded by the National Institute of Aging (NIA).

[33] In 2006 (wave 8) respondents were sent an additional questionnaire in case they were part of this random 50% subsample—provided they were alive and either they or a proxy completed at least part of the interview in person. In 2008 (wave 9), respondents who were not selected for the EFTF interview in 2006 were automatically selected in 2008. As in 2006 they were sent a questionnaire in case they were alive or a proxy completed at least part of the interview in person. In 2010 (wave 10) respondents who had completed the EFTF interview in 2006 were again chosen to participate in this mode of data collection. As a result the first panel is available in 2010.

Since we also do not have information on SSBs for individuals older than 89, we further restrict to individuals of age less than 90.

The sample for the hazard model includes waves 4-12 (years $1998 - 2014$). We exclude waves < 4 due to consistency problems in how some variables were measured. This concerns questions on physical health such as activities of daily living (ADL).

Cross-Sectional Analysis. The HRS contains variables about psychosocial factors from wave 8 (year 2006) onwards. In our analyses we use psychological variables in lags. Our measure of cognitive weakness is not (yet) available in wave 12 (year 2014). Since we use lags, the main cross-sectional analyses of the paper is restricted to waves $9 - 12$ (years $2008 - 2014$).

A.1.2 Human Mortality Base (HMD)

As we describe in Section A.2, next to a long list of socioeconomic and health variables, we condition OSPs also on average objective survival probabilities (AOSPs), which requires the use of (predicted) cohort life tables in order to capture the time tend of survival risk. Our out of sample predictions of cohort specific survival rates are based on a statistical model, which we estimate on the basis of period life tables taken from the Human Mortality Database (HMD) for years 1993 to 2013.[34]

A.2 Estimation of Objective Survival Probabilities (OSP)

We condition the estimation of the hazard model on average objective survival probabilities (AOSPs) and several individual level observable variables.

A.2.1 Average Objective Survival Probabilities

We construct life tables for each cohort c on the basis of a sequence of period t life tables. A period t life table contains average population mortality rates for

[34] The Human Mortality Database (HMD) is a cooperation of the Department of Demography at the University of California and the Max Planck Institute for Demographic Research in Rostock.

ages $j = 0, 1, 2, ..., J$ in year t, denoted by $\delta_{j,t}$ which is the average probability of an individual aged j born in year $c = t - j$ of dying in year t. A cohort c life table gives the average mortality rate of individuals of a given birth cohort c and in principle are obtained by simple rearrangements of period life tables. If period life tables are available from years t_{min} to t_{max}, then the cohort life table of cohort c is restricted to the age interval $\{\max(t_{min} - c; 0), t_{max} - c\}$. For our purpose, however, we require for several cohorts mortality rates that exceed the age limit $t_{max} - c$.[35] Hence, we have to predict future period life-tables from which, by re-arrangement, we can extract the corresponding cohort life tables.

To this purpose, we estimate mortality processes by adopting the Lee and Carter (1992) method and form predicted mortality rates on the basis of these estimates. We accordingly specify that the log mortality rate $\log(\delta_{j,t})$ can be decomposed into a vector of age-specific constants α_j and age specific drift terms β_j, where the drift is determined by a single index k_t according to

$$\log(\delta_{j,t}) = \alpha_j + \beta_j \cdot k_t + \varepsilon_{j,t} \tag{12}$$

where $\varepsilon_{j,t}$ is some error term that captures age and time specific random deviations from this mortality trend. The single index k_t is assumed to obey a unit-root process with drift and error term $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$:

$$k_t = \phi + k_{t-1} + \epsilon_t \tag{13}$$

We estimate these processes with data from 1950 onward (because of structural breaks in earlier periods). The estimated drift terms are $\hat{\phi} = -1.4460$ and $\hat{\phi} = -1.8114$ for men and women, respectively. Based on our estimates we predict mortality rates until 2090 by holding constant the vectors $\hat{\vec{\alpha}}$, $\hat{\vec{\beta}}$ and the drift term $\hat{\phi}$ and complete the cohort life tables on the basis of these estimates.

[35]For instance, period life tables are available from $t_{min} = 1900$ until $t_{max} = 2013$. Given a cohort $c = 1960$ (i.e., age 50 in 2010), the ($c = 1960$)-cohort life table obtained via simple rearrangement is restricted to the age interval $\{0, 53\}$ because we only have period-t life tables up to year $t = 2013$. Thus, one cannot obtain cohort $c = 1960$ mortality rates at ages above 53.

A.2.2 Individual Objective Survival Probabilities

As described in Subsection 2.3 we base our estimates of OSPs on a Weibull (1951) hazard model. We condition the baseline hazard in equation (1) on the AOSPs and on several individual level observable variables, both included in $\mathbf{x}_i$. The individual variables are summarized in Table 7.

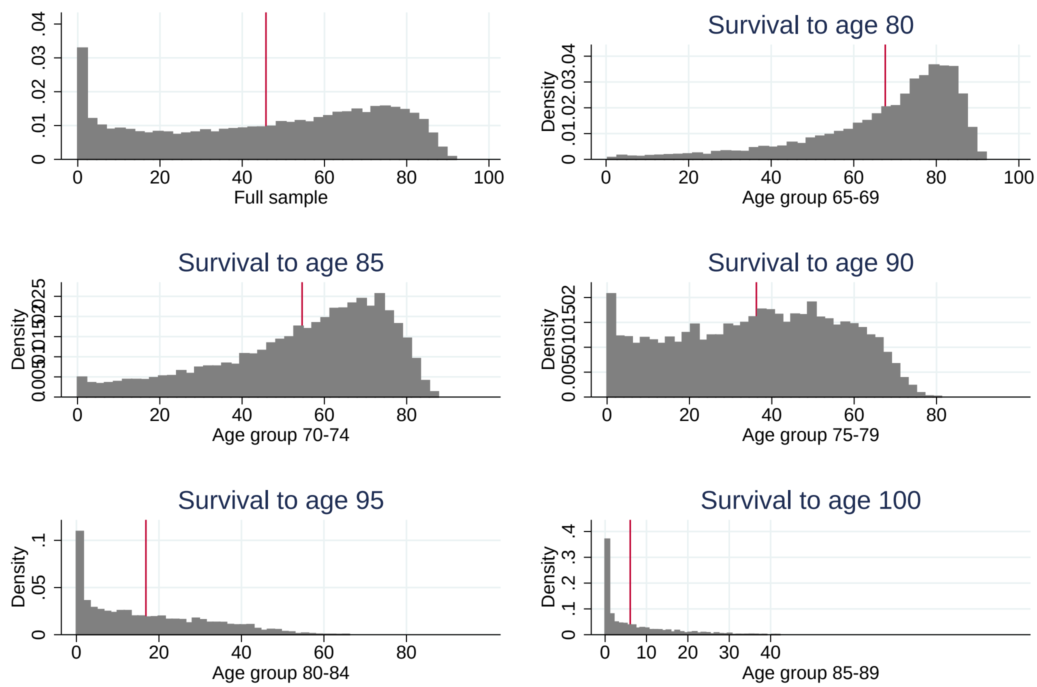
Table 7: Variables Used in Hazard Model

Variable	**Description**
Age	In years
Male	1 if male, 0 otherwise
Black	1 if black, 0 if otherwise
Married	1 if married, 0 if otherwise
Subjective Health Status (Excellent)	1 if true, 0 if otherwise
Subjective Health Status (Very Good)	1 if true, 0 if otherwise
Subjective Health Status (Good)	1 if true, 0 if otherwise
Subjective Health Status (Poor)	1 if true, 0 if otherwise
Smoke (ever)	1 if true, 0 if otherwise
Smoke (now)	1 if true, 0 if otherwise
Drink (ever)	1 if true, 0 if otherwise
ADL Index	Index between 0 and 3
Mobility Index	Index between 0 and 5
Muscle Index	Index between 0 and 4
Cognitive Weakness	Index between 0 and 35
Ever have conditions	
High blood pressure	1 if true, 0 if otherwise
Diabetes	1 if true, 0 if otherwise
Cancer	1 if true, 0 if otherwise
Lung Disease	1 if true, 0 if otherwise
Heart Diseases	1 if true, 0 if otherwise
Stroke	1 if true, 0 if otherwise
AOSP (12 years)	Avg. OSP to survive another 12 years

A.3 Descriptive Statistics

Figure 12 shows the distribution of OSPs for the full sample and each interview age group, cf. Table 1. Each subfigure also contains a red vertical line indicating the average objective survival probability for the respective age group. The histograms reveal that there is a significant dispersion of objective survival probabilities.[36]

Figure 12: Histograms of OSPs



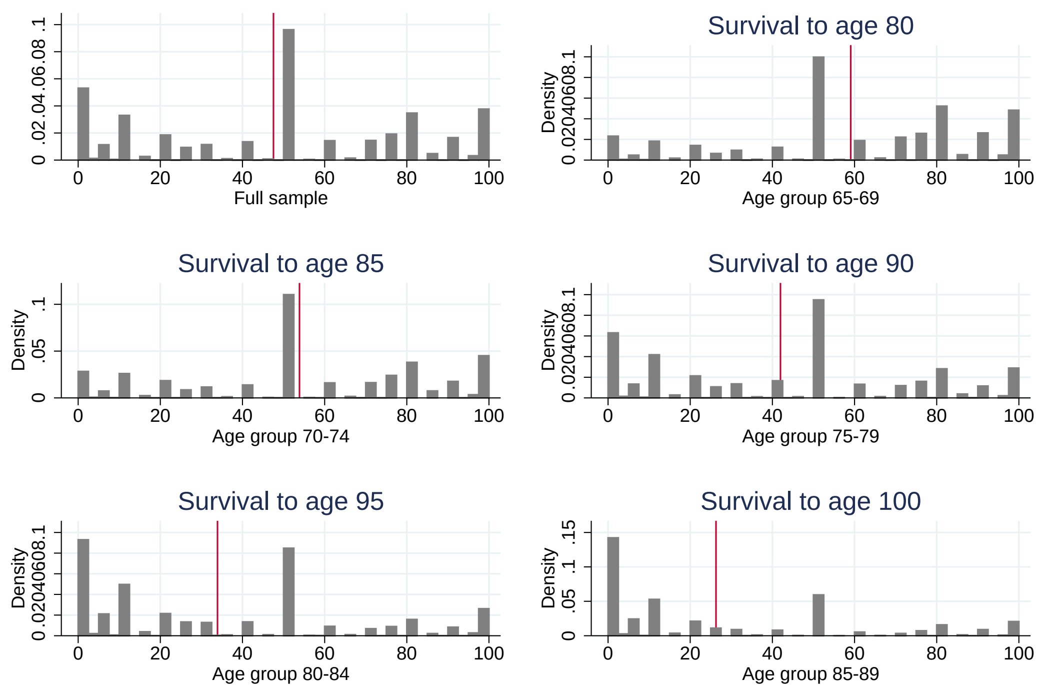
Notes: The red vertical line indicates the average objective survival probability. *Source*: Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

Figure 13 shows the corresponding distributions of SSBs. Average SSBs decrease as we move up across target age groups, as with OSPs. However, the movement is not as pronounced as for the OSPs and the difference in the averages depicted by the red lines in both figures just reflects the facts shown in Figure 1 of the main text. Second, there are focal point answers at SSBs of 0, 0.5 and 1. Observe that the fraction of individuals providing a focal point answer at 1 decreases whereas the fraction giving answer 0 increases when the target

[36]Observe that using AOSPs instead would result in only five different survival probabilities, one for each target age group.

age increases. This indicates that focal point answers do have information content that goes beyond simple heuristics that individuals may apply when being confronted with such complicated questions about survival prospects.

Figure 13: Histograms of SSBs



Notes: The red vertical line indicates the average subjective survival belief. *Source*: Own calculations, Health and Retirement Study (HRS).

A.4 Bootstrap

Standard errors of the parameters of our regressions have to be corrected in order to account for the estimation variance of OSPs. We accommodate this by implementing a two-sample bootstrap procedure with $1,000$ replications to estimate the standard errors of our coefficient estimates. In this procedure we correct for the estimation variance in objective survival probabilities as follows.[37]

[37]Note, that our two samples are both based on the HRS dataset. The first sample is based on the sample used to estimate the OSPs and the second sample is used in the overall regression

In each bootstrap replication we (i) draw a sample with replacement from the HRS sample used to estimate OSPs, (ii) estimate the OSPs, (iii) draw a sample with replacement from the cross-sectional sample used for regression analysis, (iv) perform regression analysis. Based on the resulting estimates we compute standard errors using the percentile method.

B Supplementary Appendix: The Linear Model

B.1 The Neo-Additive PWF

As an alternative to non-linear probability weighting functions, we estimate linear approximations in form of neo-additive probability capacities (Chateauneuf et al. 2007b). For $OSP_{i,h,t} \in (0,1)$[38] the neo-additive capacity is linear and writes as

$$SSB_{i,h,t} = (1-\xi_h^l)(1-\theta_h^l) + \xi_h^l OSP_{i,h,t} \tag{14}$$

where $\xi_h^l \in [0,1]$, $\theta_h^l \in [0,1]$ are parameters that are the analogues to parameters ξ_h and θ_h of the non-linear specification in (5). To see this observe that ξ_h^l controls the slope of the function, whereby for $\xi_h^l = 1$ the line in (14) corresponds with the 45-degree line. Therefore, any $\xi_h^l \in [0,1]$ can be interpreted as a measure of likelihood insensitivity. Likewise, $1-\theta_h^l \in [0,1]$ determines the intersection of (14) with the 45-degree line, whereby the intersection moves down when θ_h^l increases. Accordingly, θ_h^l can be interpreted as a measure of pessimism.

Figure 14 shows the linearly estimated probability weighting functions and Figure 15 shows the age patterns of the parameter estimates $\xi_h^l = \bar{\xi}_h^l, \theta_h^l = \bar{\theta}_{m(h)}$, again with the bootstrapped 95% confidence intervals. As for the non-linear specification in the main text, the coefficient estimates $\xi_h = \bar{\xi}_{m(h)}$, shown in Panel (a) of Figure 15, are decreasing in interview age h up to interview age group 85-89 where the estimates are very imprecise.[39] The point estimates suggest that a one percentage point increase of the OSP for age group 65-69 leads to a 0.6

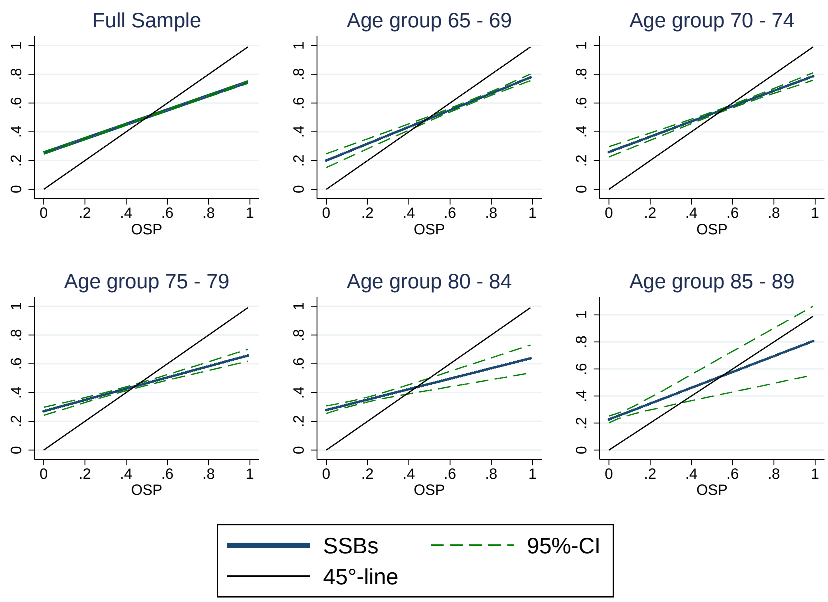
analyses.

[38] Interior OSPs follow from our specification in (3).

[39] The imprecision for this age group is much larger than for the corresponding non-linear specification, cf. Figure 5.

percentage point increase of the associated SSB, on average. At age group 75-79 the effect is only 0.4 percentage points.

Figure 14: Estimated Neo-Additive (Linear) Probability Weighting Functions



Notes: Bootstrapped (1,000 replications) 95%-confidence intervals, based on the percentile method. *Source*: Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

As with the non-linear specification, the age pattern of pessimism is less clear-cut. Pessimism increases from age group 70-74 to age group 80-84. (Estimates are very imprecise for age group 85-89.) As previously, we also observe that pessimism initially decreases from age group 65-69 to age group 70-74.

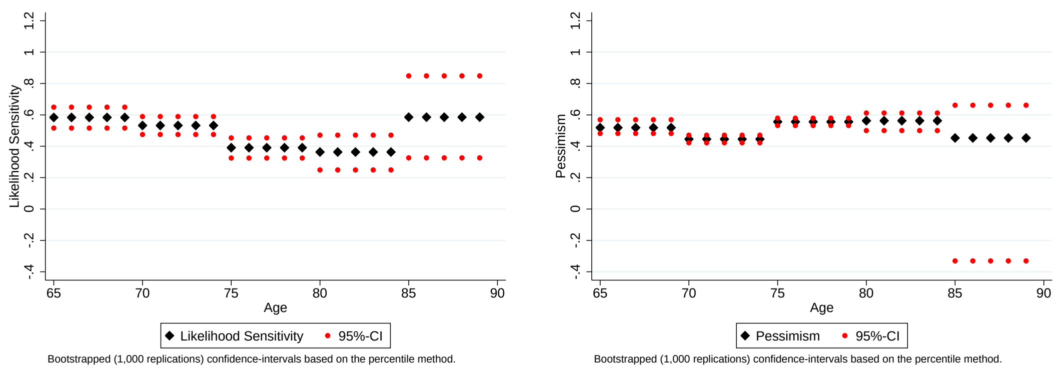
B.2 Regression Analyses

B.2.1 The Linear Regression Specification & Identification

We derive the linear specification from using (6) in (14), but also superimpose additional structure based on our insights from the non-linear model. Specifically, we assume that there is no motivational base effect, which in this linear model

Figure 15: Neo-Additive PWF: Likelihood Sensitivity and Pessimism

(a) Likelihood Sensitivity ξ_h^l (b) Pessimism θ_h^l



Notes: This figure shows estimates of $\xi_h^l = \bar{\xi}_{m(h)}^l$ of (14) in Panel (a) and estimates of $\theta_h^l = \bar{\theta}_{m(h)}^l$ in Panel (b) and the bootstrapped (1,000 replications) 95% confidence intervals, based on the percentile method. *Source*: Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

means that $\theta_0 = 0$. As in the non-linear model, this assumption implies that the base bias, $SSB_{i,h,m(h)}^b$, and the base bias plus the effect of cognition $SSB_{i,h,m(h)}^{bc}$, intersect with the 45-degree line in the same point, OSP_0.

Under these assumptions and combining (6) with (14), we get

$$SSB_{i,h,m(h)} = (OSP_0 + \theta_1 p_{i,h-2} + \theta_2 o_{i,h-2})\,(1 - (\xi_0 + \xi_1 c_{i,h-2})) + (\xi_0 + \xi_1 c_{i,h-2})\,OSP_{i,h,m(h)}. \tag{15}$$

Before turning to the reduced form of (15) and the crucial question of identification, observe that the isolated effects analogous to (8) are given by

$$\begin{aligned}
SSB^{b}_{i,h,m(h)} &= OSP_0\left(1-\xi_0\right)+\xi_0 OSP_{i,h,m(h)} && \text{(16a)}\\
SSB^{bc}_{i,h,m(h)} &= OSP_0\left(1-\left(\xi_0+\xi_1 c_{i,h-2}\right)\right)+\left(\xi_0+\xi_1 c_{i,h-2}\right)OSP_{i,h,m(h)} && \text{(16b)}\\
&= SSB^{b}_{i,h,m(h)}+\xi_1 c_{i,h-2}\left(OSP_{i,h,m(h)}-OSP_0\right),\\
SSB^{bp}_{i,h,m(h)} &= \left(OSP_0+\theta_1 p_{i,h-2}\right)\left(1-\xi_0\right)+\xi_0 OSP_{i,h,m(h)} && \text{(16c)}\\
&= SSB^{b}_{i,h,m(h)}+\theta_1\left(1-\xi_0\right)p_{i,h-2}\\
SSB^{bo}_{i,h,m(h)} &= \left(OSP_0+\theta_2 o_{i,h-2}\right)\left(1-\xi_0\right)+\xi_0 OSP_{i,h,m(h)} && \text{(16d)}\\
&= SSB^{b}_{i,h,m(h)}+\theta_2\left(1-\xi_0\right)o_{i,h-2}.
\end{aligned}$$

From the second line in (16b) we observe that the "pure" (i.e., ignoring the interactions with motivational variables) marginal effect of an increase of cognitive weakness is $\xi_1\left(OSP_{i,h,m(h)}-OSP_0\right)$. For $\xi_1<0$—i.e., increasing cognitive weakness leads to a flattening of the PWF—we find that increasing cognitive weakness gives rise to stronger underestimation for $OSP_{i,h,m(h)}>OSP_0$, and to stronger overestimation for $OSP_{i,h,m(h)}<OSP_0$, just as in the non-linear model. Likewise, we observe from (16c), that the marginal effect of an increase of pessimism is given by $\theta_1\left(1-\xi_0\right)$, respectively the effect of an increase of optimism by $\theta_2\left(1-\xi_0\right)$.

The reduced form specification follows from rewriting (15) as

$$\begin{aligned}
SSB_{i,h,m(h)} = {} & OSP_0\left(1-\xi_0\right)+\xi_0 OSP_{i,h,m(h)}+\xi_1 c_{i,h-2}\left(OSP_{i,h,m(h)}-OSP_0\right)+\\
&+\theta_1(1-\xi_0)p_{i,h-2}+\theta_2(1-\xi_0)o_{i,h-2}-\xi_1\theta_1 p_{i,h-2}c_{i,h-2}-\xi_1\theta_2 o_{i,h-2}c_{i,h-2}
\end{aligned}$$

which gives

$$\begin{aligned}
SSB_{i,h,m(h)} &= \beta_0+\beta_1 OSP_{i,h,m(h)}+\beta_2 c_{i,h-2}\left(OSP_{i,h,m(h)}-OSP_0\right)\\
&\quad+\gamma_1 p_{i,h-2}+\gamma_2 o_{i,h-2}+\gamma_3\left(p_{i,h-2}c_{i,h-2}\right)+\gamma_4\left(o_{i,h-2}c_{i,h-2}\right)\\
&= \beta_0+\beta_1 OSP_{i,h,m(h)}+\beta_2 c_{i,h-2}OSP_{i,h,m(h)}+\beta_3 c_{i,h-2}\\
&\quad+\gamma_1 p_{i,h-2}+\gamma_2 o_{i,h-2}+\gamma_3\left(p_{i,h-2}c_{i,h-2}\right)+\gamma_4\left(o_{i,h-2}c_{i,h-2}\right), \qquad (17)
\end{aligned}$$

where $\beta_3=-\beta_2 OSP_o$. The parameters in the second line can be determined by a simple linear regression, where the regression coefficients are related to the structural model parameters by

$$\beta_0 = OSP_0\,(1-\xi_0) \tag{18a}$$
$$\beta_1 = \xi_0 \tag{18b}$$
$$\beta_2 = \xi_1 \tag{18c}$$
$$\beta_3 = -\xi_1 OSP_0 \tag{18d}$$
$$\gamma_1 = \theta_1(1-\xi_0) \tag{18e}$$
$$\gamma_2 = \theta_2(1-\xi_0) \tag{18f}$$
$$\gamma_3 = \xi_1\theta_1 \tag{18g}$$
$$\gamma_4 = \xi_1\theta_2. \tag{18h}$$

In general, the reduced form does not exactly identify all parameters of the model because there are 8 parameters in the reduced form and 5 in the model. What is identified is the direct marginal effect of an increase in OSPs given by $\beta_1 = \xi_0$, and the marginal effects of pessimism is given by $\gamma_1 = \theta_1(1-\xi_0)$, respectively of optimism by $\gamma_2 = \theta_2(1-\xi_1)$. Hence, we expect that $\beta_1 \in (0,1), \gamma_1 < 0, \gamma_2 > 0$. While the marginal effect of an increase of the interaction between the OSP and cognition, $\beta_2 = \xi_1$, is identified (and while we therefore expect that $\beta_2 < 0$), the "pure" marginal effect of an increase in cognitive weakness, $\xi_1\left(OSP_{i,h,m(h)} - OSP_0\right)$, is not identified because OSP_0 is not identified. To see this, observe that using (18b) in (18a) gives $OSP_0 = \frac{\beta_0}{1-\beta_1}$, whereas using (18c) in (18d) gives $OSP_0 = -\frac{\beta_3}{\beta_2}$ and in general $\frac{\beta_0}{1-\beta_1} \neq -\frac{\beta_3}{\beta_2}$.[40]

We resolve this issue of non-identification by determining the intersection of the total effect of cognition for each individual i, $\widehat{SSB}^{bc}_{i,h}$, with the 45-degree line,

[40]To illustrate the problem of non-identification, observe that our point estimates reported in Table 4 suggest that $OSP_0 = \frac{0.0486}{1-0.6316} \approx 0.1318$ or $OSP_0 = -\frac{0.0109}{-0.0107} \approx 1.0179$ (which violates the bound constraint $OSP_0 \in [0,1]$).

i.e., we determine $OSP_{0,i}$ for each individual i from

$$\widehat{SSB}^{bc}_{i,h}(OSP_{0,i}) = \hat{\beta}_0 + \hat{\beta}_1 OSP_{0,i} + \hat{\beta}_2 c_{i,h-2} OSP_{0,i} + \hat{\beta}_3 c_{i,h-2} = OSP_{0,i}$$
$$\Leftrightarrow \qquad OSP_{0,i} = \frac{\hat{\beta}_0 + \hat{\beta}_3 c_{i,h-2}}{1 - \left(\hat{\beta}_1 + \hat{\beta}_2 c_{i,h-2}\right)}.$$

Taking the mean across all individuals gives our (mean group) estimate $\widehat{OSP}_0 = \frac{1}{n}\sum_{i=1}^{n} OSP_{0,i}$. For our decomposition analysis we further require that the base bias intersects with the 45-degree line in the same point. To this purpose we modify the base bias by an additional additive shifter $\tilde{\beta}_0$ and accordingly write[41]

$$SSB^{mb}_{i,h,m(h)} = \beta_0 + \tilde{\beta}_0 + \beta_1 OSP_{i,h,m(h)}.$$

We determine $\tilde{\beta}_{0,i}$ from

$$\widehat{SSB}^{mb}_{i}(OSP_{0,i}) = \hat{\beta}_0 + \tilde{\beta}_{0,i} + \hat{\beta}_1 OSP_{0,i} = OSP_{0,i}$$
$$\Leftrightarrow \qquad \tilde{\beta}_{0,i} = \left(1 - \hat{\beta}_1\right) OSP_{0,i} - \hat{\beta}_0.$$

Again taking the mean across all i we get the (mean group) estimate $\widehat{\tilde{\beta}}_0 = \frac{1}{n}\sum_{i=1}^{n} \tilde{\beta}_{0,i}$. To determine the confidence intervals of OSP_0 and $\tilde{\beta}_0$ we repeat these steps for all bootstrap iterations.[42]

B.2.2 Decomposition Analyses of Linear Model

The decomposition of the linear probability weighting function is presented in Figure 16. Except for the behavior in the tails, findings are very similar to those from the non-linear model. The more interesting decomposition is the one over age shown in Panels (a) and (b) of Figure 17. Again, our results are very similar

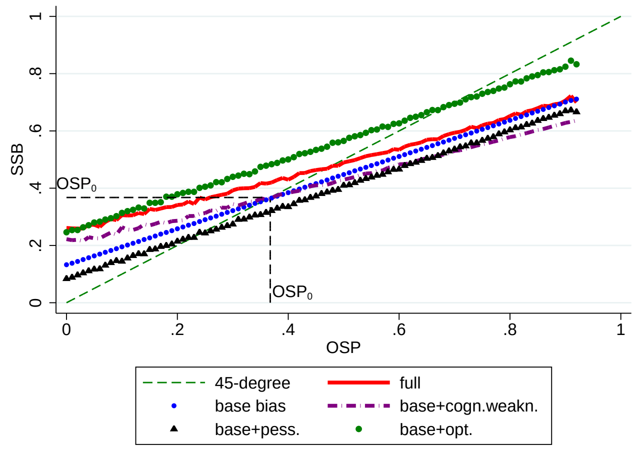
[41] This implies that the additional effects of cognition are reduced by the shifter $\tilde{\beta}_0$, hence

$$\Delta SSB^{mc}_{i,h,m(h)} = \beta_2 c_{i,h-2} OSP_{i,h,m(h)} + \beta_3 c_{i,h-2} - \tilde{\beta}_0.$$

[42] For the shifter $\tilde{\beta}_0$ we get as point estimate 0.0853 with 95% confidence interval $[0.0001, 0.1675]$.

to those from the non-linear model. The effect of cognitive weakness is basically the same, pessimism leads to a slightly stronger average underestimation and optimism to a stronger average overestimation than for the non-linear model.

Figure 16: Decomposition of Neo-Additive (Linear) PWFs



Notes: Sample averages of predicted non-linear probability weighting functions according to equations (7) and (8); "full": $\widehat{SSB}$; "base bias": $\widehat{SSB}^b$; "base+cogn. weakn.": $\widehat{SSB}^{bc}$; "base+pess.": $\widehat{SSB}^{bp}$; "base+opt.": $\widehat{SSB}^{bo}$. *Source*: Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

B.2.3 Statistical Learning and Control Variables

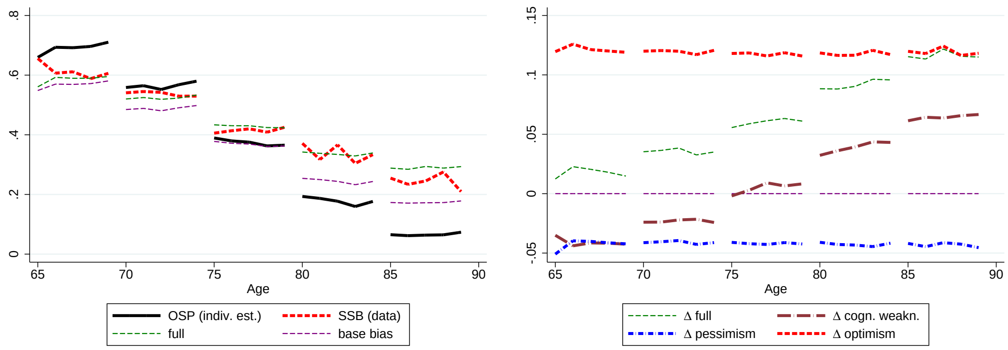
Derivation of Reduced From. To derive the linear reduced form specification, again start from (14) and replace the objective survival rate $OSP_{i,h,t}$ with $(1 - w(\vec{x}_{i,h}))\, OSP_{i,h,t}+w(\vec{x}_{i,h})\pi_0\, w(\vec{x}_{i,h})$ where π_0 is some prior belief and $w(\vec{x}_{i,h})$ is a weighting function. This linear approximation to the learning model of Groneck et al. (2016) gives rise to the following specification

$$SSB_{i,h,t} = (1 - \xi_h^l)(1 - \theta_h^l) + \xi_h^l OSP_{i,h,t} + \xi_h^l w(\vec{x}_{i,h})\,(\pi_0 - OSP_{i,h,t})\,,$$

which nests specification (14). Using (6) in the above, linearly approximating the weighting function $w(\vec{x}_{i,h})$ and ignoring all interactions between $\vec{x}_{i,h}, c_{i,h}, OSP_{i,h,t}$

Figure 17: Neo-Additive (Linear) PWF: Decomposition over Age

(a) Base Cogn. Weakn. (b) Additional Effects



Notes: Sample averages of predicted subjective survival beliefs according to equations (7) and (8) by age; Panel (a): "full": $\widehat{SSB}$; "base bias": $\widehat{SSB}^{b}$; Panel (b): "Δ full": $\widehat{SSB} - \widehat{SSB}^{b}$; "Δ base+cogn. weakn.": $\widehat{SSB}^{bc} - \widehat{SSB}^{b}$; "Δ base+pess.": $\widehat{SSB}^{bp} - \widehat{SSB}^{b}$; "Δ base+opt.": $\widehat{SSB}^{bo} - \widehat{SSB}^{b}$. *Source*: Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

for simplicity[43] results in (10).

Results on Control Variables. Table 8 shows the results of our estimation for the control variables.

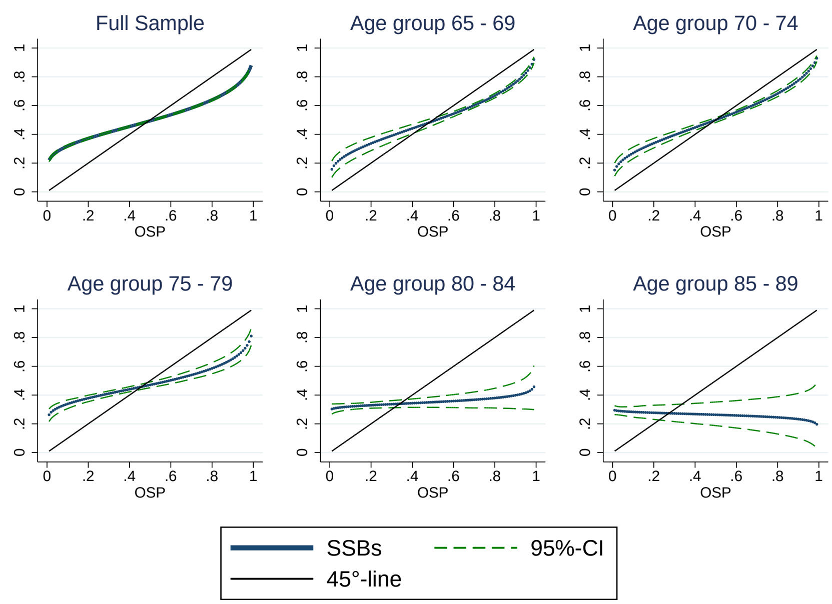
C Supplementary Appendix: Focal Point Answers

To investigate the sensitivity of our results with respect to focal point answers, we repeat the estimation of non-linear PWFs by excluding observations with focal point answers at SSBs of 0%, 50% and 100%. Results are presented in Figure 18. In contrast to the corresponding Figure 4, probability weighting functions for the highest target age group are now downward sloping. Since we regard upward sloping PWFs as plausible, this finding is another indication (beyond the

[43] In a regression where we included these interactions the AIC decreased.

histograms shown in Appendix A) that focal point answers do have information content, which justifies including all these observations in our main analyses.

Figure 18: Estimated Non-Linear Probability Weighting Functions: Excl. Focal Points



Notes: Estimated Prelec probability weighting functions for the full sample (upper left panel) and for different age-groups. Parameters estimated with non-linear least squares. *Source*: Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

Table 8: Linear Model: The Effects of Cognition and Motivational Measures on Subjective Survival Beliefs: Parameter Estimates on Control Variables

	coefficient	CI-	CI+
Constant	-0.0093	-0.3379	0.3371
Wave 9	0.0221	0.0111	0.0318
Wave 10	-0.0120	-0.0234	0.0012
TA 65	0.1649	0.0766	0.2564
TA 70	0.1469	0.0764	0.2191
TA 75	0.0817	0.0328	0.1289
TA 80	0.0665	0.0348	0.0963
Age	0.0007	-0.0034	0.0044
Male	0.0047	-0.0099	0.0180
Black	0.0987	0.0783	0.1217
Married	-0.0298	-0.0470	-0.0108
Widowed	-0.0201	-0.0402	0.0010
Mom Alive	0.0308	0.0081	0.0542
Dad Alive	0.0879	0.0441	0.1309
College	0.0127	0.0002	0.0258
Shlt	0.0785	0.0655	0.0912
ADL Index	-0.0101	-0.0224	0.0027
Mobility Index	-0.0051	-0.0116	0.0011
Muscle Index	-0.0097	-0.0151	-0.0045
Obesity	0.0059	-0.0071	0.0178
Smoke (now)	-0.0077	-0.0310	0.0162
Smoke (ever)	0.0240	0.0109	0.0360
Drink (ever)	0.0165	0.0051	0.0285
Ever have conditions			
High blood pressure	-0.0134	-0.0255	-0.0010
Diabetes	-0.0050	-0.0208	0.0104
Cancer	-0.0207	-0.0351	-0.0069
Lung Disease	0.0080	-0.0114	0.0281
Heart Disease	-0.0150	-0.0299	-0.0017
Stroke	0.0253	0.0057	0.0461
Psych. Problems	0.0171	0.0000	0.0337
Arthritis	0.0160	0.0035	0.0287

Notes: Column 2 shows the point estimates, columns 3 and 4 the respective bounds of 95%-confidence intervals (CI- and CI+), which are calculated with the percentile method (1,000 replications). *Source*: Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).