

Discussion Paper No. 18-016

**Idiosyncratic Risk,  
Aggregate Risk, and the Welfare Effects  
of Social Security**

Daniel Harenberg and Alexander Ludwig

**ZEW**

Zentrum für Europäische  
Wirtschaftsforschung GmbH

Centre for European  
Economic Research

Discussion Paper No. 18-016

**Idiosyncratic Risk,  
Aggregate Risk, and the Welfare Effects  
of Social Security**

Daniel Harenberg and Alexander Ludwig

Download this ZEW Discussion Paper from our ftp server:

<http://ftp.zew.de/pub/zew-docs/dp/dp18016.pdf>

Die Discussion Papers dienen einer möglichst schnellen Verbreitung von neueren Forschungsarbeiten des ZEW. Die Beiträge liegen in alleiniger Verantwortung der Autoren und stellen nicht notwendigerweise die Meinung des ZEW dar.

---

Discussion Papers are intended to make results of ZEW research promptly available to other economists in order to encourage discussion and suggestions for revisions. The authors are solely responsible for the contents which do not necessarily represent the opinion of the ZEW.

# Idiosyncratic Risk, Aggregate Risk, and the Welfare Effects of Social Security\*

Daniel Harenberg<sup>†</sup>     Alexander Ludwig<sup>‡</sup>

This Version: December 1, 2017

## Abstract

We ask whether a pay-as-you-go financed social security system is welfare improving in an economy with idiosyncratic productivity and aggregate business cycle risk. We show analytically that the whole welfare benefit from joint insurance against both risks is greater than the sum of benefits from insurance against the isolated risk components. One reason is the convexity of the welfare gain in total risk. The other reason is a direct risk interaction which amplifies the utility losses from consumption risk. We proceed with a quantitative evaluation of social security's welfare effects. We find that introducing an unconditional minimum pension leads to substantial welfare gains in expectation, even net of the welfare losses from crowding out. About 60% of the welfare gains would be missing when simply summing up the isolated benefits.

*JEL classification:* C68; E27; E62; G12; H55

*Keywords:* social security; idiosyncratic risk; aggregate risk; welfare

---

\*An earlier version of this paper circulated under the title "Social Security and the Interactions Between Aggregate and Idiosyncratic Risk". We thank the editor Harold Cole and three anonymous referees as well as Klaus Adam, Alan Auerbach, Martin Barbie, Tino Berger, Antoine Bommier, Johannes Brumm, Georg Dürnecker, Ayşe Imrohoroğlu, David Jaeger, Philip Jung, Tom Krebs, Dirk Krueger, Per Krusell, Felix Kubler, Marten Hillebrand, Michèle Tertilt, Fabrizio Zilibotti, and various seminar participants at several places for helpful discussions. Daniel Harenberg gratefully acknowledges financial support by the German National Research Foundation (SFB 884), by Swiss Re Foundation, and by ETH Zürich Foundation. Alex Ludwig gratefully acknowledges financial support by the German National Research Foundation (SPP 1578) and by the Research Center SAFE, funded by the State of Hessen initiative for research LOEWE.

<sup>†</sup>Chair of Integrative Risk Management and Economics, ETH Zürich; dharenberg@ethz.ch.

<sup>‡</sup>SAFE, Goethe University Frankfurt; ludwig@safe.uni-frankfurt.de.

# 1 Introduction

Many countries operate large social security systems. One reason is that social security can provide insurance against risks for which there are no private markets. However, these systems also impose costs by distorting prices and decisions. The question arises whether the benefits of social security outweigh the costs.

We address this question in a model economy featuring two types of risk, aggregate business cycle risk in form of aggregate wage and asset return risk as well as idiosyncratic productivity risk. We follow the literature and assume that insurance markets for both types of risk are incomplete. In such a setting, social security can increase economic efficiency by providing partial insurance. However, it also distorts decisions leading to welfare losses from crowding out of capital formation. Our analysis differs from the previous literature in that prior studies characterized social security's welfare effects in models with only one type of risk. One strand of the literature examined social security when only aggregate risk is present, e.g., Krueger and Kubler (2006). In that setting, social security—by pooling aggregate wage and asset return risks across generations—can improve *intergenerational* risk sharing. The other strand only considered idiosyncratic risk, cf., e.g., İmrohoroğlu, İmrohoroğlu, and Joines (1995, 1998) and Conesa and Krueger (1999). There, social security provides *intragenerational* insurance by redistributing ex-post from high to low productivity households. Broadly speaking, both strands of this literature conclude that the costs of introducing social security outweigh the benefits.

Such a segregated view is incomplete because households face both types of risk over the life-cycle and because social security, when appropriately designed, can (partially) insure both types of risk. We also argue that simply combining the findings from previous studies leads to severe biases in the overall welfare assessment. Our theoretical contribution is to show analytically why the whole insurance benefit exceeds the sum of the benefits from insurance against isolated risk components. Our quantitative contribution is to establish that joint insurance against both types of risk leads to large net welfare gains, thereby turning previous findings in the literature upside down: social security is welfare improving from the ex-ante perspective.

We emphasize that two important biases emanate from simply combining previous findings. The first arises even when the two types of risk are statistically independent. This bias is a consequence of the *convexity of the welfare gain (CWG)* in total risk. The welfare gain is convex in the amount of total risk because the marginal utility of insurance increases disproportionately as risk increases. Joint presence of idiosyncratic productivity and aggregate business cycle risk strongly fans out the earnings and consumption distributions. If social security is designed as a Beveridgean system with flat pension benefits, it provides partial insurance against this total

life-cycle risk exposure. Because of *CWG*, the whole benefit from insurance is therefore greater than the sum of benefits from insurance against the single risk components. We show that this difference in welfare assessments, the “*CWG* bias”, increases in the total amount of risk. Since total life-cycle risk is large, we can expect this bias to be large.

The second bias stems from a direct interaction of risks in form of a *counter-cyclical cross-sectional variance (CCV)* of idiosyncratic productivity risk: the variance of persistent idiosyncratic shocks is higher in a downturn than in a boom. The *CCV* has been documented in the data (Storesletten, Telmer, and Yaron 2004b) and analyzed with respect to its asset pricing implications (Mankiw 1986; Constantinides and Duffie 1996; Storesletten, Telmer, and Yaron 2007).<sup>1</sup> It leads to a high variance of the idiosyncratic income component when the aggregate income component is low. Due to concavity of the utility function this amplifies the welfare gains from insurance against both risks.

To expose these biases we start our analysis by employing an analytically tractable two-period life-cycle model in which a household faces idiosyncratic and aggregate wage risk in the first period of life. In absence of social security, retirement consumption is financed by private savings which bear aggregate return risk. We study the welfare consequence of introducing a pay-as-you-go (PAYG) financed social security system with flat, unconditional pension payments. By pooling idiosyncratic wage risks within and aggregate risks across generations, this Beveridgean system jointly provides partial insurance against idiosyncratic and aggregate risks. We measure welfare gains by a consumption equivalent variation. Abstracting from *CCV*, we derive a term capturing the welfare difference between the whole insurance benefit and the sum of the benefits from insurance against the isolated risk components. This difference reflects the *CWG* bias. We subsequently modify the two-period model to account for the *CCV* mechanism and show how an additional welfare difference emerges.

Our arguments so far ignore behavioral reactions, i.e., the reduction of savings caused by social security. In general equilibrium, this savings reaction leads to crowding out of aggregate capital, which entails corresponding changes in relative prices. Therefore, both the sign and the size of the welfare effects of introducing social security in a model with both risks have to be determined in a quantitative general equilibrium analysis.

To conduct such a quantitative analysis we build a large-scale overlapping generations model in the tradition of Auerbach and Kotlikoff (1987), extended by idiosyncratic productivity risk and aggregate wage and asset return risk. Households can save privately by investing in a

---

<sup>1</sup>Based on Guvenen, Ozkan, and Song (2014) a recent paper by Busch and Ludwig (2017) finds that in addition to the variance the skewness of persistent idiosyncratic shocks is countercyclical. Adding such a countercyclical left-skewness would strengthen our results, because more households would find themselves in situations with high marginal utility.

risk-free bond and a risky stock. Including this portfolio choice is important. It allows us to appropriately calibrate the risk-return structure of the private savings technologies, which directly affects the value of social security. The possibility to save in two assets also implies that households have additional means of self-insurance. In our computational experiment, we consider a stylized social security reform by introducing a pure Beveridgean PAYG social security system—like in our analytic two-period model—with a contribution rate of 2%. This is the size of the U.S. system when first introduced in 1935. We hence study the welfare implications of introducing a flat rate minimum pension.<sup>2</sup>

By calibrating the model to the U.S. economy we find that such a marginal introduction of social security leads to a strong welfare gain of 2.6% in terms of a consumption equivalent variation. This welfare improvement is obtained because strong partial equilibrium insurance gains of 5.2% outweigh the substantial welfare losses from crowding out of capital of 2.6% in general equilibrium. Our key finding of net welfare gains stands in stark contrast to the previous literature. When instead replicating the earlier literature by considering economies with only one type of risk we indeed observe net welfare losses. We therefore conclude that it is of crucial quantitative importance to jointly consider both risks.

To uncover the sources of the partial equilibrium welfare gain of 5.2%, we decompose it into the components that are attributable to insurance against the isolated risks as well as the two bias terms, *CCV* and *CWG*. We find that the combined effect of the two bias terms scales up the partial equilibrium welfare gains by 60%. This strong effect reemphasizes our key finding on the quantitative importance of jointly considering both risks. Finally, we investigate how much of the general equilibrium welfare effects stem from changes in *mean* consumption and from changes in the intra- and intergenerational *distribution* of consumption.

The notion that social security can insure against aggregate risks dates back to Diamond (1977) and Merton (1983). They demonstrate how it can partially complete financial markets, thereby increasing economic efficiency. Building on these insights, Shiller (1999) and Bohn (2001, 2009) show that social security can reduce consumption risk of all generations by pooling labor income and capital income risks across generations. Gordon and Varian (1988), Matsen and Thøgersen (2004), Krueger and Kubler (2006), and Ball and Mankiw (2007) use a two-period partial equilibrium model in which households only consume in the second period of life, i.e., during retirement. For our analytical results, we extend this model by adding idiosyncratic risk. Among the few quantitative papers with aggregate risk and social security, Krueger and Kubler (2006) is the most similar to our work. They conclude that the introduction

---

<sup>2</sup>Nearly all OECD countries feature either a minimum or a basic pension that is independent of previous income, see OECD (2015).

of a small PAYG system does generally not constitute a Pareto-improvement.<sup>3</sup> The concept of a Pareto-improvement requires that they take an ex-interim welfare perspective, whereas we calculate welfare from an ex-ante perspective. Our analysis also differs substantially because we include idiosyncratic risk and analyze interactions between the risks.<sup>4</sup>

Many quantitative papers consider idiosyncratic risk and social security, e.g., Conesa and Krueger (1999), İmrohorođlu, İmrohorođlu, and Joines (1995, 1998), Huggett and Ventura (1999), and Storesletten, Telmer, and Yaron (1999). One general conclusion from this literature is that welfare in a stationary economy without social security is higher than in one with a PAYG system. More recently Nishiyama and Smetters (2007), Fehr and Habermann (2008), and Golosov, Shourideh, Troshkin, and Tsyvinski (2013) focus on modeling the institutional features of existing social security systems in detail, which we abstract from. Our results demonstrate the benefits of a flat minimum pension. Like all these papers, we conduct a limited policy design experiment by restricting attention to insurance through the social security system, taking insurance through taxes and transfers during the working period as given.<sup>5</sup>

We derive our analytical results in Section 2. Section 3 describes the quantitative model, Section 4 presents the calibration and Section 5 the main results of our quantitative analysis. We conclude in Section 6. Proofs as well as computational and calibration details are relegated to separate appendices.

## 2 A Two-Generations Model

### 2.1 Model

In each period  $t$ , a continuum of households is born who live for two periods only. A household has preferences over consumption in the second period. In the first period of life, a household experiences an idiosyncratic productivity shock, denoted by  $\eta$ . This shock induces ex-post heterogeneity and we denote ex-post different households by  $i$ . Age is indexed by  $j$ , with  $j = 1$  being working age and  $j = 2$  being retirement. Denoting by  $c_{i,2,t+1}$  consumption in retirement,

---

<sup>3</sup>The recent work by Hasanhodzic and Kotlikoff (2015) mirrors these findings.

<sup>4</sup>Other related papers are Ludwig and Reiter (2010) who assess how pension systems should optimally adjust to demographic shocks, Olovsson (2010) who contends that pension payments should be highly risky because this increases precautionary savings and thereby capital formation, Peterman and Sommer (2015, 2016) who discuss the insurance benefits of social security in the Great Depression and the Great Recession, respectively, modeling each event as a one-time macroeconomic shock, and, finally, Gomes, Michaelides, and Polkovnichenko (2012) who use a model similar to ours to study how changes in fiscal policy and government debt affect asset prices and the wealth distribution.

<sup>5</sup>Huggett and Parra (2010) point out the importance of analyzing the optimal design of social security and the income tax system jointly.

the expected utility of a household born in period  $t$  is  $E_t[u(c_{i,2,t+1})]$ . We assume a CRRA per period utility function with coefficient of relative risk aversion  $\theta$ ,  $u(c_{i,2,t+1}) = \frac{c_{i,2,t+1}^{1-\theta}}{1-\theta}$ .

Gross wage income is given by  $\eta_{i,1,t}w_t$ , where  $w_t$  is the aggregate and  $\eta_{i,1,t}$  is the idiosyncratic, stochastic wage component. Wage income is subject to social security contributions at rate  $\tau$ . During retirement, the household receives a flat pension income,  $y_{t+1}^{ss}$ . Accordingly, the budget constraints are given by

$$s_{i,2,t+1} = (1 - \tau)\eta_{i,1,t}w_t \quad \text{and} \quad c_{i,2,t+1} \leq s_{i,2,t+1}R_{t+1} + y_{t+1}^{ss}, \quad (1)$$

where  $s_{i,2,t+1}$  denotes gross savings and  $R_{t+1} = 1 + r_{t+1}$  is the risky gross interest factor. While contributions depend on the idiosyncratic shock  $\eta_{i,1,t}$ , retirees receive the same flat pension payment,  $y_{t+1}^{ss}$ . Thus, social security provides partial *intragenerational* insurance against idiosyncratic risk.

We denote by  $\zeta_t$  the shock to aggregate wages and by  $\varrho_t$  the shock to returns.<sup>6</sup> We further assume that wages grow deterministically at rate  $\lambda$ . Denoting by  $\bar{R}$  and  $\bar{w}_t$  the deterministic components of returns and wages we accordingly get:

$$w_t = \bar{w}_t\zeta_t = \bar{w}_{t-1}(1 + \lambda)\zeta_t \quad \text{and} \quad R_t = \bar{R}\varrho_t. \quad (2)$$

Abstracting from population growth,<sup>7</sup> the balanced budget of the pure PAYG system reads

$$\tau w_t = y_t^{ss}. \quad (3)$$

From equations (2) and (3) one can see that social security provides partial *intergenerational* insurance against aggregate risk if  $\zeta_t$  and  $\varrho_t$  are imperfectly correlated.

## 2.2 Analysis

**The CWG Bias.** We analyze the welfare effects of introducing a marginal social security system of size  $d\tau > 0$  under the following assumptions:

**Assumption 1.** All shocks  $\eta_{i,1,t}$ ,  $\zeta_t$ ,  $\varrho_t$ : (a) are distributed log-normal with means  $\mu_{\ln \eta}$ ,  $\mu_{\ln \zeta}$ ,  $\mu_{\ln \varrho}$  and variances  $\sigma_{\ln \eta}^2$ ,  $\sigma_{\ln \zeta}^2$ ,  $\sigma_{\ln \varrho}^2$ , (b) have a mean of one:  $E\zeta = E\varrho = E\eta = 1$ , (c) are uncorrelated over time, and (d) are statistically independent from each other.

<sup>6</sup>In this section, we limit the analysis to a partial equilibrium, and hence wages and returns are exogenous.

<sup>7</sup>Our quantitative model also features population growth.



Assumptions 1a-b are frequently employed for analytical tractability. Assumption 1c can be justified by the long periodicity of each period in a two-period overlapping generations model of approximately 30–40 years. Assumption 1d is important to illustrate the *CWG*. Below, we relax it to extend the model by the *CCV*.

To evaluate welfare, we adopt an ex-ante perspective. The social welfare function of a cohort born in period  $t$  is the unconditional expected utility of a generation,  $E[u(c_{i,2,t+1})]$ . We study the consumption equivalent variation (CEV) from a marginal introduction of social security, which is the percentage increase in consumption,  $g_c$ , required to make the household indifferent between being born into an economy without social security ( $\tau = 0$ ) and with a small social security system ( $\tau = d\tau > 0$ ). We include a superscript PE for “partial equilibrium” to remain consistent with the subsequent quantitative analysis, which considers a general equilibrium. We also index the CEV by *AR* and *IR* to indicate presence of aggregate and idiosyncratic risk, respectively. We can now state our first proposition, which we prove in Appendix A:

**Proposition 1.** *Under Assumption 1, the consumption equivalent variation from a marginal introduction of social security is given by*

$$g_c^{PE}(AR, IR) = \left( \frac{1 + \lambda}{\bar{R}} \cdot \underbrace{\exp\left(\theta \left(\sigma_{\ln AR}^2 + \sigma_{\ln \eta}^2\right)\right)}_{\equiv \Psi(\sigma_{\ln AR}^2, \sigma_{\ln \eta}^2)} - 1 \right) d\tau \quad (4)$$

where  $\sigma_{\ln AR} \equiv \sqrt{\sigma_{\ln \zeta}^2 + \sigma_{\ln \rho}^2}$ . Therefore,  $g_c^{PE}(AR, IR) \geq g_c^{PE}(AR, 0) + g_c^{PE}(0, IR)$  with the inequality being strict for  $\sigma_{\ln \eta}^2 > 0 \wedge \sigma_{\ln AR}^2 > 0$ .

To interpret this proposition, first consider a deterministic economy, where  $g_c^{PE}(0, 0) = \left(\frac{1+\lambda}{\bar{R}} - 1\right) d\tau$ . This reflects the well-known Aaron (1966) condition, i.e., social security increases welfare in a deterministic economy if (and only if) its implicit return exceeds the market rate of return,  $(1 + \lambda) > \bar{R}$ . In the non-degenerate stochastic case where  $\sigma_{\ln \eta}^2 > 0 \wedge \sigma_{\ln AR}^2 > 0$ , term  $\Psi$  captures the welfare benefits from intergenerational and intragenerational (partial) insurance provided by the system.  $\Psi$  is (i) increasing in risk aversion  $\theta$ , reflecting the standard intuition that more risk-averse households value insurance more; (ii) increasing in  $\sigma_{\ln \eta}^2$  because social security pools histories of idiosyncratic earnings risk; (iii) increasing in  $\sigma_{\rho}^2$  because pension payments are not affected by return risk; (iv) increasing in  $\sigma_{\zeta}^2$  because social security reduces exposure to the wage shock,  $\zeta$ , when young and increases it when old;<sup>8</sup> (v) convex in

<sup>8</sup>Since  $\zeta$  is uncorrelated over time, mixing  $\zeta_t$  and  $\zeta_{t+1}$  by having  $\tau \in (0, 1)$  is welfare improving.

total risk,  $\sigma_{\ln AR}^2 + \sigma_{\ln \eta}^2$ . This last finding is central to our analysis.<sup>9</sup> As a consequence of the convexity, the whole welfare gain is greater than the sum of the gains from insurance against individual risk components. We denote the welfare difference attributable to the convexity of the welfare gain by  $\Delta_{CWG}$ . To further characterize it, we provide the following formal definition:

**Definition 1** (Components of CEV). *The contributions to  $g_c^{PE}(AR, IR)$  attributable to idiosyncratic and aggregate risk are defined as*

$$\begin{aligned} dg_c^{PE}(IR) &= g_c^{PE}(0, IR) - g_c^{PE}(0, 0), \\ dg_c^{PE}(AR) &= g_c^{PE}(AR, 0) - g_c^{PE}(0, 0), \end{aligned}$$

so that the CEV can be written as

$$g_c^{PE}(AR, IR) = g_c^{PE}(0, 0) + dg_c^{PE}(AR) + dg_c^{PE}(IR) + \Delta_{CWG}.$$

Under Assumption 1 we can express  $\Psi$  in terms of variances of levels instead of variances of logs:  $\Psi(\sigma_{AR}, \sigma_{\eta}) \equiv \left(1 + \sigma_{\eta}^2 + \sigma_{AR}^2 + \sigma_{\eta}^2 \sigma_{AR}^2\right)^{\theta}$ , where  $\sigma_{AR} \equiv \sqrt{\sigma_{\zeta}^2 + \sigma_{\varrho}^2 + \sigma_{\zeta}^2 \sigma_{\varrho}^2}$ . Employing Definition 1 for logarithmic utility ( $\theta = 1$ ), the CEV writes as

$$g_c^{PE}(AR, IR)|_{\theta=1} = \underbrace{\left(\frac{1+\lambda}{\bar{R}} - 1\right) d\tau}_{\equiv g_c^{PE}(0,0)|_{\theta=1}} + \underbrace{\frac{1+\lambda}{\bar{R}} \sigma_{AR}^2 d\tau}_{\equiv dg_c^{PE}(AR)|_{\theta=1}} + \underbrace{\frac{1+\lambda}{\bar{R}} \sigma_{\eta}^2 d\tau}_{\equiv dg_c^{PE}(IR)|_{\theta=1}} + \underbrace{\frac{1+\lambda}{\bar{R}} \sigma_{AR}^2 \sigma_{\eta}^2 d\tau}_{\equiv \Delta_{CWG}|_{\theta=1}}.$$

For logarithmic utility, the  $\Delta_{CWG}$  is accordingly directly related to the product of variances of aggregate and idiosyncratic risk. By providing a flat, unconditional transfer, social security reduces the variance of retirement consumption, thereby reducing exposure to each risk component as well as their multiplicative interaction.<sup>10</sup> As we show formally in Appendix B,  $dg_c^{PE}(AR)$ ,  $dg_c^{PE}(IR)$ , and  $\Delta_{CWG}$  are increasing in risk aversion  $\theta$ , so that for  $\theta > 1$ , the contribution of each component in the equation above constitutes a lower bound on welfare gains.

**Modification: The CCV Bias.** We alter Assumption 1 by conditioning the variance of idiosyncratic productivity risk on the aggregate state of the economy while keeping its unconditional

<sup>9</sup>The finding mirrors an important result from the literature on the welfare costs of aggregate fluctuations, namely that the welfare gain of insuring against aggregate risk is a convex function of risk, cf. Lucas (1978), De Santis (2007), and Krebs (2007). Relative to this literature we study the effects of joint insurance and therefore total risk is the sum of the risk components.

<sup>10</sup>Retirement consumption in the absence of social security is given by  $\bar{w}_t \bar{R} \eta_{i,1,t} \zeta_t \varrho_{t+1}$ . Its variance is  $(\bar{w}_t \bar{R})^2 \text{var}(\eta_{i,1,t} \zeta_t \varrho_{t+1}) = (\bar{w}_t \bar{R})^2 (\sigma_{\eta}^2 + \sigma_{AR}^2 + \sigma_{\eta}^2 \sigma_{AR}^2)$ , because the shocks are independent and have a mean of one, cf. Goodman (1960).

variance equal to  $\sigma_{\ln \eta}^2$ . Focusing on logarithmic utility we extend Definition 1 by the *CCV*:

**Assumption 2.** (a) Let  $\zeta_t \in \{\zeta_-, \zeta_+\}$  for all  $t$ , with  $\zeta_{\pm} = \chi \cdot \exp(1 \pm \sigma_{\ln \zeta}) > 0$  and probabilities  $\pi(\zeta_t = \zeta_+) = \pi(\zeta_t = \zeta_-) = \frac{1}{2}$ , where  $\chi$  is a normalizing constant. Let  $\eta_{i,1,t}$  be distributed as log-normal with conditional variance  $\sigma_{\ln \eta}^2(\zeta_t = \zeta_+) = \sigma_{\ln \eta}^2 + \Delta_{\eta}$ , and  $\sigma_{\ln \eta}^2(\zeta_t = \zeta_-) = \sigma_{\ln \eta}^2 - \Delta_{\eta}$ . The rest of Assumption 1 continues to hold. (b) Utility is logarithmic, i.e.,  $\theta = 1$ .

**Definition 2** (Components of CEV with *CCV*). The contribution to the CEV with *CCV*,  $g_c^{PE}(AR, IR, CCV)$ , that is attributable to *CCV* is defined as  $\Delta_{CCV} = g_c^{PE}(AR, IR, CCV) - g_c^{PE}(AR, IR)$ . Hence, the total CEV with *CCV* can be written as  $g_c^{PE}(AR, IR, CCV) = g_c^{PE}(0, 0) + dg_c^{PE}(AR) + dg_c^{PE}(IR) + \Delta_{CWG} + \Delta_{CCV}$ .

We can now state our next result. The proof is provided in Appendix A.

**Proposition 2.** Under Assumption 2 and using Definition 2 we get

$$g_c^{PE}(AR, IR, CCV) = \left( \frac{1 + \lambda}{\bar{R}} \cdot \exp(\sigma_{\ln \varrho}^2) \left( \frac{1}{\zeta_-} \exp(\sigma_{\ln \eta_l}^2) + \frac{1}{\zeta_+} \exp(\sigma_{\ln \eta_h}^2) \right) - 1 \right) d\tau \quad (5a)$$

$$\text{and } \Delta_{CCV} = \frac{1 + \lambda}{\bar{R}} \exp(\sigma_{\ln \varrho}^2) \Delta_{\eta} \left( \frac{1}{\zeta_-} - \frac{1}{\zeta_+} \right) d\tau > 0. \quad (5b)$$

Equation (5a) is the analogue to equation (4) for discrete  $\zeta$  and including *CCV*. Equation (5b) shows the increase of welfare gains through the *CCV* mechanism. This is due to the fact that the *CCV* raises (reduces) the variance of idiosyncratic productivity risk in states where average consumption already tends to be low (high). Since utility is concave, this mechanism increases the value of social security. The amplification of welfare is stronger the larger aggregate risk ( $\sigma_{\ln \zeta}^2$  and  $\sigma_{\ln \varrho}^2$ ) and the larger the variance shifter,  $\Delta_{\eta}$ .

## 2.3 Extensions

Harenberg and Ludwig (2015) provide an extension of the simple model with utility from first period consumption in general equilibrium to analytically derive a number of additional important insights, which we summarize and extend in Appendix B.2. This shows, first, how life-cycle and precautionary savings are reduced in response to the social security reform, leading to crowding out of capital. Second, it shows that the biases in the welfare assessment

of crowding out are ambiguous. While crowding out becomes stronger with more risks, this does not only reduce the deterministic component of wages, it also reduces exposure to wage risk because wage risk positively depends on the size of the capital stock.<sup>11</sup> Third, we uncover the importance of discounting: the lower the discount rate, the more relevant are the welfare benefits from insurance against second period consumption risk and the lower are the welfare costs of crowding out. In addition to the insights we worked out in our simple two period model, these aspects will play crucial roles in our quantitative analysis to which we turn next.

### 3 The Quantitative Model

#### 3.1 Time, Risk, and Demographics

Time is discrete and runs from  $t = 0, \dots, \infty$ . At the beginning of each period  $t$ , an aggregate shock  $z_t$  hits the economy. For a given initial  $z_0$ , a date-event is uniquely identified by the history of shocks  $z^t = (z_0, z_1, \dots, z_t)$  where the  $z_t$  follow a Markov chain with finite support  $\mathcal{Z}$  and nonnegative transition matrix  $\pi_z$ . Thus,  $\pi_z(z_{t+1}|z_t)$  represents the probability of  $z_{t+1}$  given  $z_t$ .

At every point in time  $t$ , the economy is populated by  $J$  overlapping generations indexed by  $j = 1, \dots, J$ . We denote the size of a generation by  $N_j(z^t)$ . Each generation consists of a continuum of households. We normalize the initial population size to unity, i.e.,  $\sum_{j=1}^J N_j(z_0) = 1$ . Population grows at the exogenous rate of  $n$ . To keep the analysis focused we abstract from survival risk.<sup>12</sup> Total population at  $t$  is therefore  $N(z^t) = (1 + n)^t$ .

Households within a cohort are ex-ante identical but receive an idiosyncratic shock  $e_j$  each period so that there is ex-post intragenerational heterogeneity. We denote by  $e^j$  the history of idiosyncratic shocks with probability  $\pi_e(e^j)$ . We assume that  $e_j$  follows a Markov chain with finite support  $\mathcal{E}$  and strictly positive transition matrix  $\pi_e$ . The transition probabilities are  $\pi_e(e_{j+1}|e_j)$ .<sup>13</sup>

---

<sup>11</sup>In general equilibrium, wages increase in capital and shocks are multiplicative in wages. Such pecuniary effects play key roles for welfare in heterogeneous agent economies, cf. Davila, Hong, Krusell, and Ríos-Rull (2012) and Krueger and Ludwig (2017). Also, Harenberg and Ludwig (2015) focus on log utility and therefore the saving rate does not react to changes of the capital stock. Furthermore, for analytical tractability, the model features a degenerate distribution of households who are all ex-ante identical. Both aspects play additional important roles for the welfare effects of crowding out, see our discussion of the quantitative results in Section 5.4.

<sup>12</sup>In presence of survival risk, social security can be beneficial if it partially substitutes for missing annuity markets. Caliendo, Guo, and Hosseini (2014) demonstrate that this may not hold because social security crowds out accidental bequests. Also, it is not straightforward to jointly model survival risk and financial risk with Epstein-Zin preferences, see Bommier, Harenberg, and Le Grand (2017).

<sup>13</sup>By a Law of Large Numbers  $\pi_e(e^j)$  represents both the individual probability for  $e^j$  and the fraction of the population with that shock history. Likewise,  $\pi_e(e_{j+1}|e_j)$  represents both the individual transition probability and its population counterpart.

### 3.2 Households

At any date-event  $z^t$ , a household is fully characterized by its age  $j$  and its history of idiosyncratic shocks  $e^j$ . Denote by  $u_j(c, e^j, z^t)$  the expected remaining life-time utility from consumption allocation  $c$  at age  $j$ , history  $e^j$ , and date-event  $z^t$ . Preferences are represented by a recursive utility function  $u_j(c, \cdot)$  of the Epstein-Zin-Weil kind (Epstein and Zin 1989, 1991; Weil 1989).<sup>14</sup>

$$u_j(c, e^j, z^t) = \left( \left[ c_j(e^j, z^t) \right]^{\frac{1-\theta}{\gamma}} + \beta \left( \sum_{z^{t+1}} \sum_{e_{j+1}} \pi_z(z_{t+1}|z_t) \pi_e(e_{j+1}|e_j) \left[ u_{j+1}(c, e^{j+1}, z^{t+1}) \right]^{1-\theta} \right)^{\frac{1}{\gamma}} \right)^{\frac{\gamma}{1-\theta}},$$

$$u_J(c, e^J, z^t) = c_J(e^J, z^t), \quad c > 0,$$

where  $\beta$  is the discount factor and  $\theta$  controls risk aversion. Parameter  $\gamma$  is defined as  $\gamma \equiv \frac{1-\theta}{1-\frac{1}{\psi}}$  with  $\psi$  denoting the intertemporal elasticity of substitution.

Households inelastically supply one unit of labor until they retire at the fixed retirement age  $j_r$ . They are endowed with a deterministic life-cycle productivity profile  $\epsilon_j$ . The idiosyncratic, stochastic component of income,  $\eta(e_j, z_t)$ , depends on the realization of idiosyncratic and aggregate shocks. The dependence of  $\eta(e_j, z_t)$  on the aggregate shock is necessary to model the *CCV*. We assume that  $E(\eta(e_j, z_t)|z_t) = 1$ . Labor income is  $y_j(e_j, z^t) = w(z^t)\epsilon_j\eta(e_j, z_t)$ , where  $w(z^t)$  is the real aggregate wage in terms of the consumption good at  $z^t$ . Insurance markets for labor income risk are closed by assumption.

Households can transfer wealth between periods by holding stocks and bonds in amounts  $s_{j+1}(e^j, z^t)$  and  $b_{j+1}(e^j, z^t)$ , respectively. The stock has a risky return  $r_s(z^{t+1})$  that depends on the realization of the aggregate shock in the following period, whereas the bond pays a one-period risk-free interest rate  $r_b(z^t)$ . The sequential budget constraint is standard:

$$c_j(e^j, z^t) + s_{j+1}(e^j, z^t) + b_{j+1}(e^j, z^t) = (1 + r_s(z^t))s_j(e^j, z^t) + (1 + r_b(z^{t-1}))b_j(e^j, z^t) + (1 - \tau)y_j(e_j, z^t)I(j) + y^{ss}(z^t)(1 - I(j)),$$

where  $\tau$  is a fixed social security contribution rate,  $y^{ss}(z^t)$  is pension income, and  $I(j)$  is an indicator function that takes the value 1 if  $j < j_r$  and 0 otherwise.<sup>15</sup> Households cannot die in

<sup>14</sup>In a slight abuse of notation, we use letter  $u$  to denote remaining lifetime utility in this recursive formulation, which was used in Section 2 to denote the per-period utility function.

<sup>15</sup>We do not consider an exogenous borrowing constraint. This may bias results in favor of social security because income (and asset) poor households can relax their budget constraint. With an exogenous borrowing

debt,  $s_{J+1}(e^J, z^t) + b_{J+1}(e^J, z^t) \geq 0$ . Since there are no bequests, households are born with zero assets, i.e.,  $s_1(e^1, z^t) = b_1(e^1, z^t) = 0$ .

### 3.3 Firms

There is a representative firm that produces output,  $Y(z^t)$ , using capital,  $K(z^t)$ , and labor,  $L(z^t)$ . The production technology is Cobb-Douglas with capital share  $\alpha$  and deterministic labor-augmenting productivity growth  $\lambda$ . At each date-event there is a multiplicative shock to total factor productivity,  $\zeta(z_t)$ , so that we have  $Y(z^t) = \zeta(z_t)K(z^t)^\alpha((1 + \lambda)^t L(z^t))^{1-\alpha}$ .

Assuming a stochastic depreciation rate  $\delta(z_t)$ ,<sup>16</sup> the capital stock evolves according to  $K(z^t) = I(z^{t-1}) + K(z^{t-1})(1 - \delta(z_{t-1}))$ . Because of perfect competition, the firm remunerates the factors of production according to their marginal productivities. Thus, the aggregate wage,  $w(z^t)$ , and the return on capital,  $r(z^t)$ , are given by

$$w(z^t) = (1 + \lambda)^t(1 - \alpha)\zeta(z_t) \left( \frac{K(z^t)}{(1 + \lambda)^t L(z^t)} \right)^\alpha, \quad (6a)$$

$$r(z^t) = \alpha\zeta(z_t) \left( \frac{(1 + \lambda)^t L(z^t)}{K(z^t)} \right)^{1-\alpha} - \delta(z_t). \quad (6b)$$

The capital stock is financed by issuing stocks and bonds in quantities  $S$  and  $B$ , so that  $K(z^t) = S(z^t) + B(z^t) = S(z^t)(1 + \bar{\kappa}_f)$ . The debt-equity ratio,  $\bar{\kappa}_f$ , is exogenous and constant. Therefore, the firm only decides on aggregate capital and not on the capital structure.<sup>17</sup> This mechanical leverage allows us to keep the depreciation shocks, which drive stock return volatility, small in the calibration. This is desirable, because large depreciation shocks imply unrealistically large fluctuations on the real side of the economy. As derived in Appendix B.5, the leveraged stock return is

$$r_s(z^t) = r(z^t) + \bar{\kappa}_f \left( r(z^t) - r_b(z^{t-1}) \right), \quad (7)$$

which shows that leverage increases mean and variance of stock returns.<sup>18</sup>

---

constraint it would be natural to modify the social security system to have a progressive contribution rate with an exemption for income poor households.

<sup>16</sup>The same assumption is employed by Storesletten, Telmer, and Yaron (2007), Gomes and Michaelides (2008), and Krueger and Kubler (2006), among others.

<sup>17</sup>Leverage is frequently modeled this way in the finance literature to increase the volatility of stock returns, cf., e.g., Boldrin, Christiano, and Fisher (1995) and Croce (2014).

<sup>18</sup>As Gomes and Michaelides (2008) point out, the empirical equity premium is for levered firms. Our model is consistent with this target, whereas standard models should rather compare to an “unlevered” empirical counterpart.

### 3.4 Social Security

Social security is organized as a PAYG system just like in the two-generations model of Section 2. Denoting by  $P(z^t)$  the number of pensioners,  $P(z^t) = \sum_{j=j_r}^J N_j(z^t)$ , the budget constraint accordingly reads as  $\tau w(z^t)L(z^t) = y^{ss}(z^t)P(z^t)$ .<sup>19</sup>

### 3.5 Equilibrium

We study a competitive general equilibrium, where households and firms maximize and all markets clear. The corresponding value function of the household is denoted  $v_j(\cdot)$ . In the computational solution, we focus on recursive Markov equilibria. We express all aggregate variables in terms of labor efficiency units, i.e., we divide aggregate variables by  $(1 + \lambda)^t L(z^t) = (1 + \lambda)^t \sum_{j=1}^{j_r-1} \epsilon_j N_j(z^t)$ . The corresponding normalized variable is written in lower case, e.g.,  $k(z^t) = \frac{K(z^t)}{(1+\lambda)^t L(z^t)}$ . Individual variables are detrended only by the level of technology, and the corresponding variables are denoted with a tilde, e.g.,  $\tilde{c}_j(\cdot) = \frac{c_j(\cdot)}{(1+\lambda)^t}$ . Accordingly, the monotone transformation of the value function is denoted by  $\tilde{v}_j(\cdot)$ . Since the model has (ex-post) heterogeneous households and aggregate uncertainty, the distribution of households becomes part of the state space. We denote by  $\Phi$  the distribution of households over age, current income state, stocks, and bonds. The corresponding equilibrium law of motion,  $\Phi' = H(\Phi, z, z')$ , is induced by household's optimal choices and the exogenous shock processes.<sup>20</sup> Every period there are five markets that clear: consumption good, capital, labor, stocks, and bonds. A precise definition of the recursive Markov equilibrium is relegated to Appendix B.

### 3.6 Computational Solution

We compute an equilibrium of our model by applying the Krusell and Smith (1998) method.<sup>21</sup> To approximate the law of motion of the distribution,  $H(\Phi, z, z')$ , we consider various forecast functions,  $\hat{H}$ , of the average capital stock and the ex-ante equity premium and select the one with the best fit. The average goodness of fit of the selected approximate law of motion is in the range of  $R^2 = 0.99$  for all of the calibrations. The state space is further reduced by one dimension by recasting the problem in terms of cash-on-hand. To speed up the solution, we

<sup>19</sup>By the balanced budget, intergenerational sharing of aggregate risk is limited to generations alive at the same point in time. It may be desirable to also share this risk with future, unborn generations. This could be achieved by adding a social security trust fund to the model.

<sup>20</sup>Next period's aggregate shock  $z'$  is an element of the law of motion because it determines the distribution of next period's idiosyncratic income states.

<sup>21</sup>Also see, e.g., Storesletten, Telmer, and Yaron (2007) and Gomes and Michaelides (2008).

employ a variant of the endogenous grid method (Carroll 2006) that allows for two continuous choices. Details of the computational solution are provided in Appendix C.

### 3.7 Welfare Criterion and Dynamic Efficiency

**Social Welfare Function.** We employ the same welfare concept as in the two-generations economy of Section 2, namely ex-ante expected utility of a household at the start of economic life. As explained in Davila, Hong, Krusell, and Ríos-Rull (2012), in an economy with ex-ante identical but ex-post heterogeneous agents, this concept represents a natural objective for a social planner who is behind the Rawlsian veil of ignorance. It is a Utilitarian welfare criterion, which weighs lifetime utilities with their respective probabilities. In our model with aggregate shocks, this criterion means that we evaluate the expected life-time utility of many different households that are randomly born into a state of an economy and then form the welfare index by taking the unconditional average of these households' expected life-time utility.

Formally, a household's ex-ante expected welfare of being born into an economy with policy A can be written as the unconditional expectation  $E[\tilde{v}_1(\tilde{c}^A, e^1, z^t)]$ , where the expectation is taken over all date-events  $z^t$ . It is an expectation over all possible equilibrium values of aggregate capital and induced prices. Analogous to Section 2, when comparing policy A to policy B we express the welfare difference of two such ex-ante welfare measures in terms of a consumption equivalent variation,  $g_c$ . As we prove in Appendix B.5, it is given by

$$g_c = \frac{E[\tilde{v}_1(\tilde{c}^B, e^1, z^t)]}{E[\tilde{v}_1(\tilde{c}^A, e^1, z^t)]} - 1. \quad (8)$$

A positive number indicates the percentage of lifetime consumption a household would be willing to give up under policy A in order to be born into an economy with policy B. We compare the long-run welfare effects of such a reform. While this does not include the transition between the two economies, it is important to understand that for the experiment described below (an introduction of social security), including the welfare effects along the transition would increase  $g_c$ . The reason is that moving from policy A to policy B implies a gradual decrease in capital. Thus, generations that live through the transition experience the full benefits from insurance but are spared some of the long-run welfare costs of crowding out. Therefore, by ignoring the transition, we calculate a lower bound on the welfare effects.

**Dynamic Efficiency.** In our economy, there are two sources for inefficiencies. One is missing insurance markets against aggregate and idiosyncratic risk, the other is the possibility of an



inefficient intergenerational allocation of mean consumption across generations even when insurance markets are complete. The latter is known as *dynamic inefficiency*, which can arise in OLG models (Samuelson 1958; Diamond 1965). In a dynamically inefficient economy an intergenerational reallocation of resources from the young to the old through a PAYG pension system can help to cure this inefficiency.

In our experiments we want to focus on dynamically efficient economies to avoid making a normative case for social security that is not based on a partial completion of missing asset markets for insuring idiosyncratic and aggregate risk. To this aim, we check the dynamic efficiency criterion of Demange (2002), Theorem 1, which applies to stochastic economies such as ours and very general notions of efficiency (e.g., ex-ante efficiency). Specifically, we compute the conditions proposed by Krueger and Kubler (2006) in Proposition 1, which are sufficient conditions for Demange's efficiency criterion. While these conditions can be conveniently evaluated numerically, they may be far from necessary conditions.<sup>22</sup> We restate them in the following definition which is adapted to our notation.

**Definition 3** (Dynamic efficiency, Krueger and Kubler (2006)). *Suppose that*

- a) *whenever the one period risk-free interest rate,  $r_b(z^t)$ , is larger than the implicit average social security return,  $(1+n)(1+\lambda) - 1$ , then there exist two next period states  $\tilde{z}_{t+1}, \tilde{\tilde{z}}_{t+1} \in \mathcal{Z}$  such that (i) next period's bond returns in the corresponding date-events are above the implicit return next period, i.e.,  $r_b(\tilde{z}^{t+1}) > (1+n)(1+\lambda) - 1$  and  $r_b(\tilde{\tilde{z}}^{t+1}) > (1+n)(1+\lambda) - 1$ , and (ii) the stock return satisfies  $r_s(\tilde{z}^{t+1}) > r_b(z^t)$  and  $r_s(\tilde{\tilde{z}}^{t+1}) < r_b(z^t)$ , and*
- b) *from any initial equilibrium state, a high interest rate  $r_b(z^t) > (1+n)(1+\lambda) - 1$  is reached in finite time.*

*If conditions a) and b) are fulfilled, the economy is dynamically efficient.*

This definition of dynamic efficiency implies that economies can be dynamically efficient even if the *average* bond return is less than the *average* implicit social social security return. It is crucial to understand that bond returns and implicit social security returns are *fluctuating* in our quantitative model. While the average bond return may be less than the average implicit social security return, condition (a) states that in equilibrium there need to exist states with high bond returns and the economy needs to stay in such a state with positive probability. Condition (b) in turn says that such a state with a high bond return must be reached in finite time.

---

<sup>22</sup>Details on the numerical implementation can be found in Appendix C.6.

### 3.8 Experiment and Decomposition Analyses

**Experiment.** In terms of the previous section, our computational experiment consists of comparing policy A, which has a social security contribution rate of  $\tau = 0\%$ , to policy B, which has  $\tau = 2\%$ . It can be interpreted as the introduction of a marginal social security system in form of a minimum pension, as in Section 2. We then compute the welfare gains from this policy reform by comparing two long-run equilibria.

If the introduction of social security leads to a welfare improvement despite the fact that the economy is dynamically efficient, then this must be a consequence of the partial completion of markets through social security. As in the simple model of Section 2, this partial completion of markets decreases the consumption variance. It also leads to behavioral adjustments through reduced savings, cf. Section 2.3, and increased stock holdings, both of which tend to increase average consumption. Disentangling these effects is crucial for understanding our quantitative results which we achieve with the decomposition analyses described next.

**Gains from Insurance and Losses from Crowding Out.** Our first decomposition of the general equilibrium welfare effects aims at disentangling the effects of welfare gains in partial equilibrium from those induced by the crowding out of capital, as in our general equilibrium extension of the simple model, cf. Section 2.3. We thereby also disentangle the long run (= general equilibrium) from the short run (= partial equilibrium) welfare effects. In our partial equilibrium experiment, the social security system changes, but prices, i.e., wages and returns, remain unaffected. Conceptually, this corresponds to a small open economy with free movement of the factors of production. To formalize this, denote by  $\mathcal{P}_A = \{\{z^t, r(z^t), r_s(z^t), r_b(z^t), \tilde{w}(z^t)\}_{t=0}^\infty | \tau = 0\%\}$  the sequence of shocks and prices obtained from the general equilibrium of the economy without a social security system, i.e., under policy A ( $\tau = 0\%$ ). Likewise, denote by  $\hat{H}_A$  the approximate law of motion of this equilibrium. We compute the partial equilibrium under the old price sequence  $\mathcal{P}_A$  and the old law of motion  $\hat{H}_A$ , but with policy B ( $\tau = 2\%$ ). The welfare gains stemming from insurance are then:<sup>23</sup>

$$g_c^{PE} = \frac{E \left[ \tilde{v}_1(\tilde{c}^B, e^1, z^t) | \mathcal{P}_A, \hat{H}_A, \tau = 2\% \right]}{E \left[ \tilde{v}_1(\tilde{c}^A, e^1, z^t) | \mathcal{P}_A, \hat{H}_A, \tau = 0\% \right]} - 1. \quad (9)$$

---

<sup>23</sup>We need to take into account the approximate law of motion  $\hat{H}_A$  in this definition because households form their expectations based on the laws of motion.

Analogously, the corresponding general equilibrium number is

$$g_c^{GE} = \frac{E \left[ \tilde{v}_1(\tilde{c}^B, e^1, z^t) | \mathcal{P}_B, \hat{H}_B, \tau = 2\% \right]}{E \left[ \tilde{v}_1(\tilde{c}^A, e^1, z^t) | \mathcal{P}_A, \hat{H}_A, \tau = 0\% \right]} - 1, \quad (10)$$

where the crucial difference is that in the new equilibrium with policy B ( $\tau = 2\%$ ), households optimize given the new general equilibrium prices and laws of motion,  $\mathcal{P}_B, \hat{H}_B$ . The welfare costs of crowding out (CO) are given by the difference  $g_c^{CO} = g_c^{GE} - g_c^{PE}$ .

To relate the costs of crowding out to our concept of dynamic efficiency in Section 3.7, notice that dynamic (in)efficiency refers to the mean allocation of consumption across generations. In our model, however, there is also a dispersion of consumption around the mean which is induced by different idiosyncratic shock histories. Therefore, from the ex-post perspective, households may gain or loose from a decrease of the capital stock because—depending on each idiosyncratic shock history and resulting asset position—either the negative aggregate wage or the positive aggregate return effect dominates, cf., e.g., Kuhle (2012). From the ex-ante perspective the question whether there is too much or too little capital in the economy then depends on the weight a household receives in the respective welfare criterion, cf. Davila, Hong, Krusell, and Ríos-Rull (2012). The reduction in the capital stock could therefore by itself lead to an increase in welfare even in a dynamically efficient economy, meaning that  $g_c^{CO} > 0$ . In our results, we never encountered this case; this is why we speak of welfare “costs” from crowding out.

**Welfare Implications of Changes in the Mean and the Distribution of Consumption.** The equivalent variations  $g_c^{GE}, g_c^{PE}, g_c^{CO}$  encompass two effects. One is the welfare implication of policy-induced changes of mean consumption allocations, the *mean* effect, the second is the welfare implication from a change in the intra- and intergenerational distribution of consumption, the *distribution* effect. We decompose the total CEV into these effects by computing the welfare change due to a change of the distribution as (see Appendix B.5):

$$\begin{aligned} g_c^{PE,distr} &= \frac{E \left[ \tilde{C}^A | \mathcal{P}_A, \hat{H}_A, \tau = 0\% \right]}{E \left[ \tilde{C}^B | \mathcal{P}_A, \hat{H}_A, \tau = 2\% \right]} \left( 1 + g_c^{PE} \right) - 1, \\ g_c^{GE,distr} &= \frac{E \left[ \tilde{C}^A | \mathcal{P}_A, \hat{H}_A, \tau = 0\% \right]}{E \left[ \tilde{C}^B | \mathcal{P}_B, \hat{H}_B, \tau = 2\% \right]} \left( 1 + g_c^{GE} \right) - 1, \end{aligned} \quad (11)$$

where  $\tilde{C}^A$  ( $\tilde{C}^B$ ) is aggregate, growth-adjusted consumption under policy regime A (B). The respective differences  $g_c^{PE,mean} = g_c^{PE} - g_c^{PE,distr}$  and  $g_c^{GE,mean} = g_c^{GE} - g_c^{GE,distr}$  are then the

equivalent variations capturing the welfare implications of changes in mean consumption.

In partial equilibrium, insurance through social security reduces both the intragenerational consumption distribution as well as the consumption growth rate over the life-cycle because precautionary savings go down (intergenerational distribution). Additional distributional changes arise in general equilibrium because the crowding out of capital causes increasing returns and decreasing wages, the welfare impact of which we capture by  $g_c^{CO,distr} = g_c^{GE,distr} - g_c^{PE,distr}$ . Likewise, the mean effect of crowding out, i.e., the change in mean consumption due to a change in equilibrium prices, is computed as  $g_c^{CO,mean} = g_c^{GE,mean} - g_c^{PE,mean}$ .

**Sources of Partial Equilibrium Welfare Gains.** Finally, we decompose  $g_c^{PE}$  into the effects attributable to insurance against aggregate risk, idiosyncratic risk, as well as the two biases,  $CWG$  and  $CCV$ , respectively, as in our simple model of Section 2. Recalling our decomposition of the CEV in Definitions 1 and 2 we have:

$$\begin{aligned}
g_c^{PE}(AR, IR, CCV) &= g_c^{PE}(0, 0) + dg_c(AR) + dg_c(IR) + \Delta_{CWG} + \Delta_{CCV} \\
g_c^{PE}(AR, IR) &= g_c^{PE}(0, 0) + dg_c(AR) + dg_c(IR) + \Delta_{CWG} \\
g_c^{PE}(0, IR) &= g_c^{PE}(0, 0) + dg_c(IR) \\
g_c^{PE}(AR, 0) &= g_c^{PE}(0, 0) + dg_c(AR).
\end{aligned}$$

The right-hand side of the first line shows all of the components. To isolate those, we compute  $g_c^{PE}(AR, 0)$  and  $g_c^{PE}(0, 0)$ , as in equation (9), but for an economy with only aggregate risk and one without risk, respectively.<sup>24</sup> With those numbers at hand, we can back out the welfare effect attributable to aggregate risk,  $dg_c(AR)$ . Likewise, we compute  $g_c^{PE}(0, IR)$  for an economy featuring only idiosyncratic risk to back out  $dg_c(IR)$ . Next, we compute  $g_c^{PE}(AR, IR)$ . As we already know  $dg_c(AR)$  and  $dg_c(IR)$ , we can back out the  $\Delta_{CWG}$ . In the same manner, we obtain  $\Delta_{CCV}$ . While we are mainly interested in the overall effect attributable to the respective risk component, we further decompose those into the respective welfare effects of changes in the mean and the distribution of consumption.

## 4 Calibration

The selection of targets and parameters to be calibrated is informed by our theoretical insights, in particular Propositions 1 and 2, as well as Section 2.3. Accordingly, the coefficient of relative

<sup>24</sup>As shown in Appendix B.4,  $g_c^{PE}(0, 0)$  can be calculated from the present discounted value of lifetime income, independent of preference parameters.

risk aversion,  $\theta$ , the variances of the shocks, the returns on savings and the discount factor are crucial in determining the value of social security. Guided by this, our baseline calibration takes a very conservative approach, in the sense that it features a low  $\theta$  and small aggregate shocks. In the sensitivity analysis of Section 5.3, we then first increase  $\theta$  to match the Sharpe ratio,  $\varsigma = \frac{E[r_{s,t} - r_{b,t}]}{\sigma[r_{s,t} - r_{b,t}]}$ , and then aggregate shocks to match the equity premium,  $\mu = E[r_{s,t} - r_{b,t}]$ . For the discount factor our target is also conservative and we report results with a less conservative target in Appendix E.3.

One set of parameters, the set of first-stage parameters, is determined exogenously by either taking its value from other studies or measuring it in the data. The second set of parameters is jointly calibrated by matching the model-simulated moments to their corresponding moments in the data. Accordingly, we refer to those parameters as second-stage parameters.<sup>25</sup>

Table 1 summarizes our conservative baseline calibration, described next.<sup>26</sup> Additional information on our empirical approach to measure calibration targets and on the numerical implementation of the procedure is provided in Appendices C and D, respectively.

## 4.1 Demographics

Households begin working at the biological age of 21, which corresponds to  $j = 1$ . We set  $J = 58$ , implying a life expectancy at birth of 78 years, which is computed from the Human Mortality Database (HMD) for year 2007. We set  $j_r = 45$ , corresponding to a statutory retirement age of 65. Population grows at a rate of 1.1%.

## 4.2 Households

In our baseline calibration, we treat the coefficient of risk aversion as a first-stage parameter, setting it to 3, which is well within the standard range of  $[2, 4]$ .<sup>27</sup> The intertemporal elasticity of substitution is set to 0.5. This is at the lower end of the range of values used in the literature, as reviewed, e.g., by Bansal and Yaron (2004). A higher value of the elasticity of substitution means that households react more strongly to price changes. As a consequence, welfare losses from crowding out are lower, as shown in our sensitivity analysis in Section 5.3. In our baseline

<sup>25</sup>The second-stage parameters jointly determine all targeted moments. When we say that we calibrate a parameter to a target, we mean that it has the strongest impact on that target.

<sup>26</sup>For lack of better data on the period when the social security system was introduced in the United States in 1935 with a contribution rate of 2% (the data analogue to our thought experiment), we take averages for postwar data for calibration. In Section 5.3 we report results when the contribution rate is also set to its postwar average of 9.5%.

<sup>27</sup>Given this choice, our model produces a Sharpe ratio of  $\varsigma = 0.076$  and an equity premium of  $\mu = 0.76\%$ , well below their empirical counterparts of 0.33 and 5.60%, which we explicitly target in Section 5.3.

Table 1: Summary of the Baseline Calibration

Parameter	Value	Target (source)	Stage
<i>Demographics</i>			
Biological age at $j = 1$	21		1 <sup>st</sup>
Model age at retirement, $j_r$	45	Statutory retirement age of 65 (SSA)	1 <sup>st</sup>
Model age maximum, $J$	58	Life expectancy of 78 years (HMD)	1 <sup>st</sup>
Population growth, $n$	0.011	U.S. Social Sec. Admin. (SSA)	1 <sup>st</sup>
<i>Households</i>			
Discount factor, $\beta$	0.987	Capital output ratio, 2.65 (NIPA)	2 <sup>nd</sup>
Coefficient of relative risk aversion, $\theta$	3.0		1 <sup>st</sup>
Intertemporal elasticity of substitution, $\psi$	0.5		1 <sup>st</sup>
Age productivity, $\{\epsilon_j\}$	Cf. Appendix D	Estimates based on PSID data	1 <sup>st</sup>
Autocorrelation of log earnings, $\rho$	0.969	Estimates based on PSID data	1 <sup>st</sup>
Variance of persistent shock, $\sigma_v^2(z)$	{0.024, 0.008}	Estimates based on PSID data	1 <sup>st</sup>
Variance of transitory shock, $\sigma_\epsilon^2$	0.085	Estimates based on PSID data	1 <sup>st</sup>
<i>Firms</i>			
Capital share, $\alpha$	0.32	Wage share (NIPA)	1 <sup>st</sup>
Leverage ratio, $\bar{k}_f$	0.66	Rajan and Zingales (1995)	1 <sup>st</sup>
Technology growth, $\lambda$	0.018	TFP growth (NIPA)	1 <sup>st</sup>
Mean depreciation rate of capital, $\delta_0$	0.102	Bond return, 0.023 (Shiller)	2 <sup>nd</sup>
<i>Aggregate Risk</i>			
Standard deviation of depreciation, $\bar{\delta}$	0.080	Std. of consumption growth, 0.030 (Shiller)	2 <sup>nd</sup>
Aggregate productivity states, $1 \pm \zeta$	0.029	Std. of TFP, 0.029 (NIPA)	1 <sup>st</sup>
Transition probabilities of productivity, $\pi^\zeta$	0.941	Autocorrelation of TFP, 0.88 (NIPA)	1 <sup>st</sup>
Conditional prob. of depreciation shocks, $\pi^\delta$	0.887	Corr.(TFP, returns), 0.50 (NIPA, Shiller)	2 <sup>nd</sup>

Notes: 1<sup>st</sup> stage parameters are set exogenously, 2<sup>nd</sup> stage parameters are jointly calibrated to the targets.

calibration, the discount factor  $\beta$  is calibrated to match the capital-output ratio of 2.65, which we calculate from NIPA data, cf. Appendix D.2.<sup>28</sup> We obtain  $\beta = 0.987$ , which is a reasonable estimate for a model at an annual frequency such as ours.

The parametrization of the labor income process is based on household earnings data from the PSID applying the procedure of Busch and Ludwig (2017). Our earnings measure excludes social security contributions but includes all other taxes and transfers.<sup>29</sup> The age specific productivity profile  $\epsilon_j$  is extracted from the *deterministic* component of the earnings process displayed in Appendix D.1. Calibration of the *stochastic* component  $\eta(e_j, z_t)$  is derived from the estimates of the process

$$\log(\eta_{i,j,t}) = \xi_i + \nu_{i,j,t} + \varepsilon_{i,j,t}, \quad \varepsilon_{i,j,t} \sim \mathcal{N}(0, \sigma_\varepsilon^2), \quad (12a)$$

$$\nu_{i,j,t} = \rho \nu_{i,j-1,t-1} + v_{i,j,t}, \quad v_{i,j,t} \sim \mathcal{N}(0, \sigma_v^2(z_t)), \quad (12b)$$

where the variance of the persistent shock,  $\sigma_v^2(z_t)$ , depends explicitly on the aggregate state. The estimated value of the autocorrelation coefficient is  $\rho = 0.969$ . The estimated conditional variance of the persistent shock,  $\sigma_v^2(z_t)$ , is 0.024 in recessions and 0.008 in booms.<sup>30</sup> The estimated variance of idiosyncratic shocks is  $\sigma_\varepsilon^2 = 0.085$ . We approximate the *AR*(1) process using the Rouwenhorst method, cf. Kopecky and Suen (2010), and approximate the transitory component  $\varepsilon_{j,t}$  by Gaussian quadrature (for details see Appendix C).

### 4.3 Firms

We set the value of the capital share parameter to  $\alpha = 0.32$ . This is directly estimated from NIPA data on total compensation as a fraction of GDP. Our estimate of the deterministic trend growth rate is based on data on total factor productivity. The point estimate is  $\lambda = 0.018$ , which is in line with other studies. Leverage in the firm sector is set to  $\bar{\kappa}_f = 0.66$  (Rajan and Zingales 1995). The mean depreciation rate of capital,  $\delta_0$ , is a second-stage parameter. We calibrate it to match an average bond return of 2.3%.<sup>31</sup> In economies without aggregate risk we calibrate  $\delta_0$  to produce a risk-free return of 4.2%, corresponding to the empirical estimate of Siegel (2002).

<sup>28</sup>Our estimate is in line with the estimates of, e.g., Fernández-Villaverde and Krueger (2011).

<sup>29</sup>We thank Christopher Busch for providing us with the estimates.

<sup>30</sup>See Appendix D.1 for the identification of recessions and booms in the data and Section 4.4 for the corresponding definition in the model.

<sup>31</sup>The empirical bond return, equity premium, etc., are calculated from the data on Robert Shiller's website, see <http://aida.wss.yale.edu/~shiller/data.htm>.

## 4.4 Aggregate Risk

Aggregate risk is driven by a four-state Markov chain with support  $\mathcal{Z} = \{z_1, \dots, z_4\}$  and transition matrix  $\pi_z$ . Each aggregate state maps into a combination of a total factor productivity (TFP) shock and a depreciation shock,  $(\zeta(z), \delta(z))$ . Both shocks can take a high and a low value, given by  $\zeta(z) = 1 \pm \bar{\zeta}$  and  $\delta(z) = \delta_0 \pm \bar{\delta}$ . We define *recessions* as the low TFP states  $z \in \{z_1, z_2\}$ , where  $\zeta(z) = 1 - \bar{\zeta}$ . The transition probability of remaining in a low TFP state is  $\pi^\zeta$ . To govern the correlation between TFP and depreciation shocks, we let the probability of the high depreciation state conditional on the low TFP state be  $\pi^\delta$ . Assuming symmetry of the transition probabilities, the Markov chain of aggregate shocks is characterized by four parameters,  $(\bar{\zeta}, \bar{\delta}, \pi^\zeta, \pi^\delta)$ , see Appendix D.3 for details. We set  $\bar{\zeta}$  and  $\pi^\zeta$  to match the standard deviation and autocorrelation of TFP of 0.029 and 0.88, both estimated using NIPA data. The remaining parameters,  $\bar{\delta}$  and  $\pi^\delta$ , are calibrated as second-stage parameters to jointly match the standard deviation of aggregate consumption growth of 0.03 and the correlation of the cyclical component of TFP with risky returns of 0.5. We get  $\bar{\zeta} = 0.029$ ,  $\bar{\delta} = 0.080$ ,  $\pi^\zeta = 0.941$ ,  $\pi^\delta = 0.887$ .

# 5 Results

## 5.1 Baseline Calibration

**Dynamic Efficiency.** We first report the results of checking the two conditions for dynamic efficiency of Definition 3 before the introduction of social security.<sup>32</sup> Table 2 shows that about 40% of the 72 000 simulated periods have a high bond return (larger than the average social security return). Such a high bond state is reached from any simulated initial condition in finite time—with a maximum of 120 periods—so that condition (b) is satisfied. Conditional on being in such a high bond return state, we check condition (a) and find that it is violated about 5% of the time. Since the conditions of Definition 3 are sufficient, but not necessary, we can have *at least* 95% confidence that the baseline economy is dynamically efficient.

**Aggregate Effects and Welfare Consequences.** The effects of introducing social security at a contribution rate of 2% on capital accumulation, prices and welfare are documented in Table 3. Our experiment leads, on average, to a long-run reduction in the capital stock of 11.61%, which is accompanied by a 3.8% reduction in gross wages, an increase in the return on stocks of 0.99 percentage points, and an increase in the return on bonds of 1.01 percentage points. The average

---

<sup>32</sup>We obtain similar results for the baseline economy with social security ( $\tau = 2\%$ ), as well as for our other calibrations. Details are provided in Appendix E.1.



Table 2: Dynamic Efficiency of Baseline Economy,  $\tau = 0\%$

Condition (a)		Condition (b)		Simulated periods
High Bond Returns	Conditional violation	Max. periods	Avg. periods	
38.1%	4.7%	120	11.5	72 000

*Notes:* Dynamic efficiency conditions of Definition 3. High bond returns: fraction of high bond return states in which  $1 + r_b(z^t) > (1 + n)(1 + \lambda)$ . Conditional violation: Violation of conditions (a)(i) and (a)(ii), conditional on being in a high bond return state. Avg., resp. max., periods: average, resp. maximum, number of simulation periods to reach a high bond return state. Number of total simulated periods is after discarding a phase-in period.

return on bonds increases to a greater extent, because the insurance provided through social security leads households to rebalance their portfolios towards stocks. This reduces relative demand for bonds, decreasing their price and increasing their return.

Table 3: Aggregate Effects of The Social Security Experiment

Variable	Change
Aggregate capital, $K$	$\Delta K/K = -11.61\%$
Aggregate wage, $w$	$\Delta w/w = -3.8\%$
Stock return, $r_s$	$\Delta r_s = +0.99pp$
Bond return, $r_b$	$\Delta r_b = +1.01pp$
Consumption equivalent variation	$g_c^{GE} = +2.56\%$

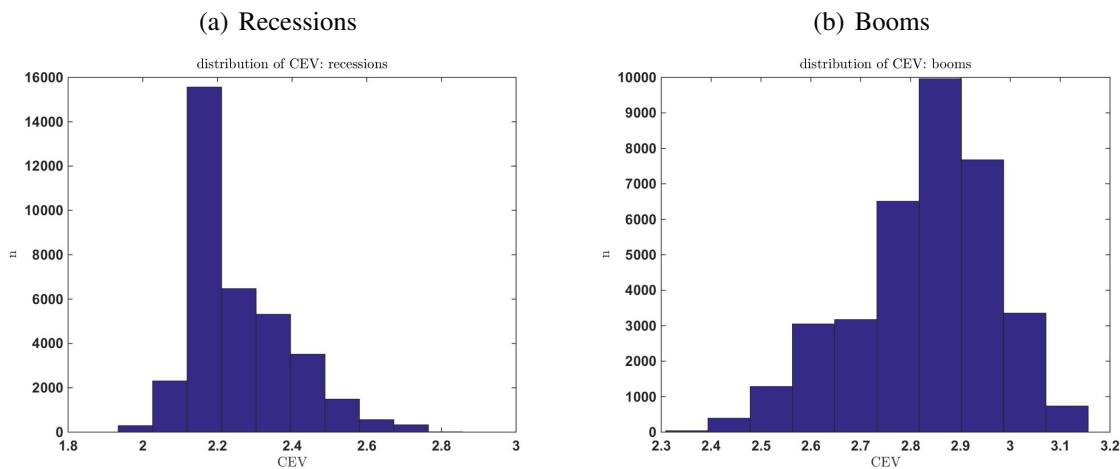
*Notes:*  $\Delta X/X$  is the expected percent change in variable  $X$  between two steady states, i.e.,  $\Delta X/X = \frac{E(X_t|\tau=2\%) - E(X_t|\tau=0\%)}{E(X_t|\tau=0\%)}$ .  $\Delta x$  is the change in variable  $x$  expressed in percentage points ( $pp$ ), i.e.,  $\Delta x = E(x_t|\tau = 2\%) - E(x_t|\tau = 0\%)$ .  $g_c^{GE}$  is the consumption equivalent variation in general equilibrium, cf. Section 3.8.

Table 3 also reports the consumption equivalent variation,  $g_c^{GE}$ , as defined in equation (10). The reform yields a CEV of 2.6% despite the sizeable crowding out of capital. This constitutes a substantial welfare gain from a minimum pension at a contribution rate of 2%.

**Conditional Distribution of Welfare Gains.** We now report the distribution of the CEV conditional on the household being born into a recession or a boom. That is, we compute the CEV for each history of aggregate shocks,  $g_{c,t}^{GE} = \frac{E[\tilde{v}_1(\tilde{c}^B, e^1, z^t)|z^t, \mathcal{P}_B, \hat{H}_B, \tau=2\%]}{E[\tilde{v}_1(\tilde{c}^A, e^1, z^t)|z^t, \mathcal{P}_A, \hat{H}_A, \tau=0\%]} - 1$ , thereby comparing a household being born into an economy with social security to a household being born into an economy without social security, before they learn their idiosyncratic shocks.

Figure 1 shows the distribution of  $g_{c,t}^{GE}$  for recessions ( $z_t \in \{z_1, z_2\}$ ) in Panel (a) and booms ( $z_t \in \{z_3, z_4\}$ ) in Panel (b). First, notice that the CEVs are always positive. Second, as contributions to social security imply higher utility costs in recessions when incomes are already low and as aggregate shocks are persistent, CEVs are on average higher in booms (with an average of 2.83%) than in recessions (with an average of 2.25%). Furthermore, the distribution of CEVs is left-skewed in recessions and right-skewed in booms.

Figure 1: Distribution of CEV: Recessions and Booms



**Benefits from Insurance versus Costs from Crowding Out.** Where do these substantial welfare gains come from? To provide an answer, we first decompose the total welfare gain into the benefits from insurance and the losses from crowding out by conducting the partial equilibrium (PE) experiment described in Section 3.8. Accordingly, the sequences of wages and returns before and after the introduction of social security are identical. As a consequence, the CEV in this experiment reflects purely the benefits from insurance. Subtracting this number from the overall welfare gain reported in Table 3 yields the losses from crowding out. As Table 4 reveals, the net welfare gains attributable to the total insurance provided by social security amount to +5.2% and the losses from crowding out stand at -2.6%.

**Welfare Implications of Changes in the Mean and the Distribution of Consumption.** Table 5 reports the results of our welfare decomposition of the numbers in Table 4 into the mean effect,  $g_c^{mean}$ , and the distribution effect,  $g_c^{distr}$ , as described in Section 3.8. Turn first to the distribution effect. The gains from a reduction of the dispersion of consumption are large,

Table 4: Benefits from Insurance versus Costs from Crowding Out

CEV	GE	PE	CO
$g_c$	+2.56%	+5.18%	-2.62%

Notes:  $g_c$  is the CEV in general equilibrium (GE) and partial equilibrium (PE). The difference of the two CEVs is the loss from crowding out (CO). See Section 3.8 for the formal definitions of these terms.

standing at 3.4% in partial and at 2.3% in general equilibrium. This is a conservative estimate of  $g_c^{distr}$ , because—as the results of our simple model show and as is confirmed in our quantitative sensitivity analysis of Section 5.3—the welfare gains of the distribution effect increase in risk aversion, the value of which is moderate in our baseline calibration. The distribution effect of crowding out stands at  $-1.0\%$ , which is due to the ex-ante welfare losses from the increased exposure to return risk (recall from Table 3 that crowding out increases returns) and the induced (ex-post) widening of the wealth distribution.

Next, observe that the mean effect in general equilibrium is 0.24%. It is small, compared to the total CEV of 2.6%; therefore the mean effect is of secondary importance for the welfare effects of social security.

But how can mean aggregate consumption increase in a dynamically efficient economy? This gain is possible because social security provides partial insurance. To understand this, it is instructive to first analyze the partial equilibrium mean effect which stands at 1.8%. This is large because the implicit average return of social security is  $(\lambda + n) \cdot 100[\%] = 2.8\%$ , which exceeds the average bond return. Social security is, therefore, an attractive implicit asset. Furthermore, the insurance provided through social security induces households to increase their financial share invested in stock in response to the policy reform, thereby increasing their mean portfolio returns and mean consumption. This portfolio reallocation is large because of the relatively low risk aversion of the baseline calibration.<sup>33</sup> To corroborate this intuition, we perform a counterfactual experiment where we hold constant portfolio allocation policy functions when social security is introduced. Then aggregate consumption falls by  $-0.62\%$ , as one would expect in a dynamically efficient economy without endogenous portfolio choice. Also, as documented in Figure 2 below, in partial equilibrium the portfolio adjustments in combination with a reduction of savings primarily increase consumption of the young. Notice that in a dynamically inefficient economy we would instead observe the opposite, namely that social security redistributes consumption from the young to the old, cf. Section 3.7.

<sup>33</sup>Qualitatively, these adjustments are the same as in Krueger and Kubler (2006).

Opposing this positive mean effect in partial equilibrium is the reduction of mean consumption induced by relative price changes, which is given by the crowding out mean effect of  $-1.58\%$ . It is smaller than the partial equilibrium mean effect, so that the net mean effect in general equilibrium is positive. We want to stress that the mean effect is positive only in our conservative baseline calibration with relatively low risk aversion and low implied risky returns. As we document in our sensitivity analysis, higher risk aversion dampens the portfolio reallocation so that the mean effect is negative in general equilibrium.

Table 5: Benefits from Changes in the Mean and the Distribution

CEV	GE	PE	CO
$g_c$	+2.56%	+5.18%	-2.62%
$g_c^{distr}$	+2.32%	+3.36%	-1.04%
$g_c^{mean}$	+0.24%	+1.82%	-1.58%

Notes: The total  $g_c$  is decomposed into the mean effect,  $g_c^{mean}$ , and the distribution effect,  $g_c^{distr}$ . See Section 3.8 for the formal definitions of these terms.

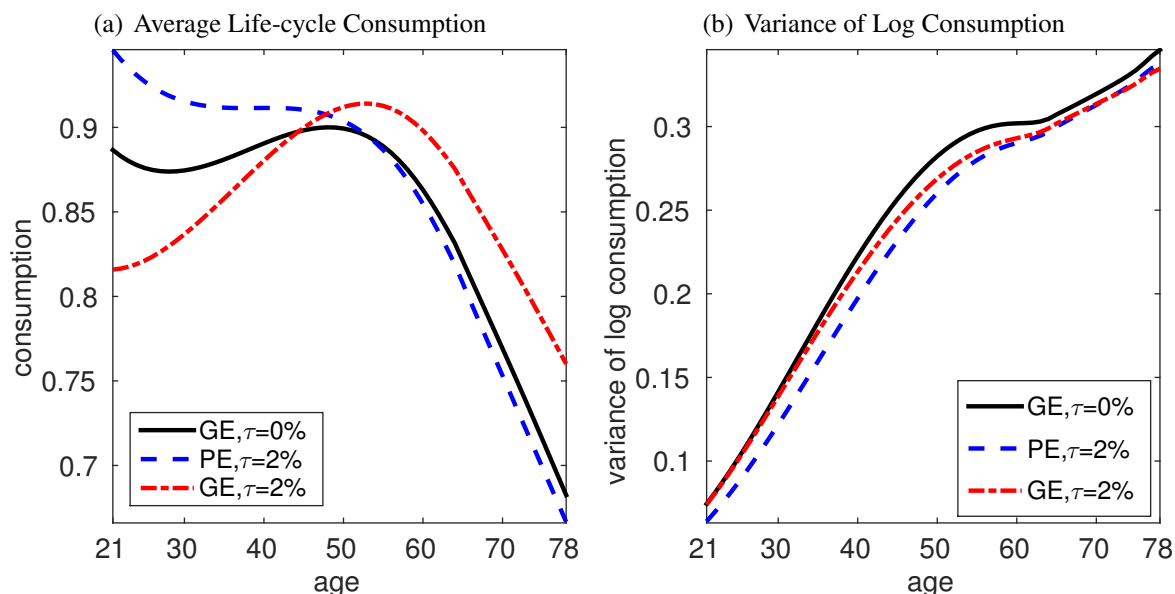
**A Closer Look at the Distributional Implications.** The previous discussion makes clear that distributional implications are important for interpreting the welfare consequences of the policy reform. To shed further light on these, Figure 2 displays average life-cycle consumption in Panel (a) and the variance of log consumption over the life-cycle in Panel (b). The increase of the variance of log consumption during retirement shown in Panel (b) is a consequence of aggregate risk in our model.<sup>34</sup> Without that risk the variance during the retirement period would be constant in the  $\tau = 0\%$  economy because all the dispersion would result from pre-retirement shock histories and because preferences are homothetic.

The introduction of social security in partial equilibrium leads to better consumption insurance as in the simple model of Section 2 and therefore reduces precautionary savings. Consequently, the consumption profile is pivoted clockwise so that households consume more

<sup>34</sup>The kink in the variance of log consumption at retirement is due the CCV mechanism. This amplification of labor income risk vanishes with retirement, which leads households to increase their exposure to aggregate risk by investing a larger financial share in stocks, as in Storesletten, Telmer, and Yaron (2007). Overall, the variance of log consumption over age is similar to the data from age 40 to age 75 (Guvenen 2007; Aguiar and Bils 2015). At younger ages, our model underestimates the variance. One explanation is the absence of initial cross-sectional heterogeneity in our model. Also notice that the variance of log consumption is flatter during retirement when we consider a more generous baseline pension system, cf. Section 5.3 below. This corresponds to Storesletten, Telmer, and Yaron (2004a) who report the variance of consumption based on CEX data for the period 1980-1990 when a generous social security system in the U.S. with its distributional components was in place (the contribution rate increased from 10.16% in 1980 to 12.4% in 1990).

on average in the early stages of the life-cycle at the expense of reduced average consumption when old. Due to discounting, the early consumption gains are weighted more strongly than the later consumption losses. Simultaneously, the variance of log consumption decreases over the life-cycle. Both effects underlie the strong partial equilibrium welfare gain.

Figure 2: Life-cycle Consumption



Notes: Average consumption in Panel (a) and variance of log consumption in Panel (b) at each age for the economy without social security (GE,  $\tau = 0\%$ ), with social security (GE,  $\tau = 2\%$ ), and the partial equilibrium with social security and old prices (PE,  $\tau = 2\%$ ).

In the post-experiment general equilibrium, the consumption profile is pivoted counter-clockwise, because crowding out of capital leads to lower wages and higher returns, cf. Table 3. In response to higher returns, households increase their life-cycle savings when young and increase consumption when old.<sup>35</sup> Consumption remains below its pre-experiment, general equilibrium level until age 44. This lower average level and high volatility of consumption when young drives the welfare losses from crowding out. On the positive side, the variance of log consumption is smaller than in the pre-experiment economy after age 40.

Finally, Table 6 reports the Gini coefficients for assets, labor earnings, and consumption. We make three observations. First, the simulated Gini coefficients for earnings and assets closely align with the data.<sup>36</sup> This is notable because they were not a target in the calibration, and it is

<sup>35</sup>In addition, households increase the share invested in stock. Note that, since there is no borrowing constraint in the model, households are leveraged in stocks at young ages despite the positive correlation of aggregate wage and return risk and the presence of idiosyncratic risk. Despite this leverage at young age, portfolio shares after age 30 are very similar to those predicted by Cocco, Gomes, and Maenhout (2005), Cocco (2005), cf. Appendix E.4.

<sup>36</sup>We take the data from Krueger, Mitman, and Perri (2016). Their estimates of the Ginis for assets and earnings

not easy to match them.<sup>37</sup> Second, the Gini coefficient for assets increases. This is so because households take on more risky portfolio compositions in response to the introduction of social security and because of higher average returns, see Table 3. Third, improved consumption insurance leads to a slightly lower degree of consumption dispersion in the economy.

Table 6: Distributional Consequences: Gini Coefficients

Variable	$\tau = 0.00$	$\tau = 0.02$	Change	Data
Assets	0.735	0.775	4.04pp	0.77
Earnings	0.456	0.456	0.00pp	0.43
Consumption	0.262	0.259	-0.32pp	0.4

Notes: *pp* stands for percentage points. Estimates in column “Data” are taken from Krueger, Mitman, and Perri (2016).

**Decomposition into Risks.** Based on our analytical results of Section 2, we investigate how much of the welfare gains in partial equilibrium of +5.2% can be attributed to insurance against aggregate and idiosyncratic risk, to the direct interaction between risks in form of the *CCV*, and to the convexity of the welfare gain, *CWG*.<sup>38</sup> Results are summarized in Table 7. The consumption equivalent variation in a deterministic environment,  $g_c^{PE}(0, 0)$ , is negative at -0.6%, because the implicit return of social security of  $(\lambda + n) \cdot 100[\%] = 2.8\%$  is below the interest rate of  $r_b = 4.2\%$ , our target in the risk-free economy.

The welfare gains from insurance against idiosyncratic risk,  $dg_c(IR)$ , amount to 0.7% and against aggregate risk,  $dg_c(AR)$ , to 2.0% in terms of consumption equivalent variations. Hence, the role played by aggregate risk is approximately twice as important as the role played by idiosyncratic risk. This strong contribution of aggregate relative to idiosyncratic risk may seem counterintuitive, because conventional wisdom suggests that idiosyncratic risk is higher. Based on our analysis of the simple model of Section 2 one would therefore expect that the contribution of insurance against idiosyncratic risk to the CEV is also higher. However, this intuition is misleading because the simple model misses the mean effect. A further decomposition, reported in the second and third row of Table 7, in fact shows that our results are in line with this conventional wisdom (and with our simple model) because the distribution effect of

are very similar to those of Hintermaier and Koeniger (2011).

<sup>37</sup>See, e.g., Castañeda, Díaz-Giménez, and Ríos-Rull (2003) and De Nardi (2004). The asset Gini is around 0.5 in alternative calibrations where we target the Sharpe ratio or the equity premium whereas our welfare findings are robust, cf. Section 5.3. The earnings and consumption Ginis decrease to 0.255 and 0.06 when we shut down idiosyncratic income risk, again cf. Section 5.3.

<sup>38</sup>See Section 3.8 for the decomposition procedure.

insurance against idiosyncratic risk is 0.8%, which is more than twice as large as for aggregate risk (0.3%). On the other hand, the strong positive mean effect of  $dg_c(AR)$  results—as explained above—from the endogenous portfolio reallocation, which allows households to achieve higher consumption on average, even in a dynamically efficient economy. The mean effect of  $dg_c(IR)$ , in contrast, is negative, reflecting the dynamic efficiency of the economy.

Our key finding in this decomposition analysis concerns the two bias terms. The difference in welfare attributable to the  $CCV$ , the  $\Delta_{CCV}$ , is at 1.4%. The  $\Delta_{CWG}$  is of similar size. The two welfare differences jointly account for 60% of the total insurance gains through social security, calculated as  $\frac{\Delta_{CCV} + \Delta_{CWG}}{g_c^{PE}} \cdot 100[\%]$ . Combining the findings from the previous literature—which focuses only on one risk—therefore leads to substantial quantitative biases in the welfare assessments of social security.

Table 7: Decomposition of Welfare Benefits in Partial Equilibrium

CEV, $g_c$	$g_c^{PE}$	$g_c^{PE}(0,0)$	$dg_c(IR)$	$dg_c(AR)$	$\Delta_{CWG}$	$\Delta_{CCV}$
Total	5.18% =	-0.62%	+0.67%	+2.02%	+1.68%	+1.43%
Distr.	3.36% =	0.0%	+0.77%	+0.36%	+1.16%	+1.08%
Mean	1.82% =	-0.62%	-0.09%	+1.66%	+0.52%	+0.35%

*Notes:* This table presents the decomposition of the welfare gain in partial equilibrium (PE) expressed as a consumption equivalent variation,  $g_c^{PE}$ , into various sources, cf. Section 3.8.  $IR$ : idiosyncratic risk,  $AR$ : aggregate risk,  $CWG$ : convexity of the welfare gain,  $CCV$ : counter-cyclical cross-sectional variance. The effects attributable to each risk component are further decomposed into the respective distribution effects,  $g_c^{distr}$ , and mean effects,  $g_c^{mean}$ , again see Section 3.8.

## 5.2 On the Importance of Modeling both Risks

The analysis of our baseline scenario suggests that the role played by the two biases is large. To investigate whether it is indeed the joint presence of both risks (aggregate and idiosyncratic risk) as well as their interactions that lead us to conclude that social security is beneficial in the long run, we compute the general equilibria of economies that feature only aggregate risk (*AR-only*), only idiosyncratic risk (*IR-only*), or no risk (*No-risk*). We calibrate each economy to standard targets in the literature. For the *AR-only* economy, we adopt the targets of Krueger and Kubler (2006) and match the equity premium,  $\mu = E[r_{s,t} - r_{b,t}]$ , and the volatility of stock returns. Specifically, we target an equity premium of  $\mu = 5.6\%$  and a standard deviation of stock returns of  $\sigma(r_s) = 16.8\%$ .<sup>39</sup> For the economy without aggregate risk and the deterministic economy,

<sup>39</sup>Again based on data taken from Rob Shiller’s webpage.

we target an interest rate of 4.2%, which is estimated by Siegel (2002) and which is the same rate used in our PE decomposition procedure for economies without aggregate risk. Throughout these experiments, we target a capital-output ratio of 2.65 by adjusting the discount factor,  $\beta$ . Further details of the calibration are described in Appendix D.5. All three economies are dynamically efficient (see Appendix E.1 for the *AR-only* economy).

Proposition 1 shows that welfare gains from introducing social security increase exponentially in risk aversion and the volatility of aggregate risk. With respect to these two, the *AR-only* calibration is an extreme case in that it features high aggregate risk and high risk aversion. But even with such an extreme calibration, Table 8 documents welfare losses for this case. In general equilibrium, they stand at  $-0.6\%$ , again expressed as a consumption equivalent variation. Even in the short-run, the benefits from insurance through social security do not dominate, as the welfare losses stand at  $-0.4\%$  in partial equilibrium. Most of these effects are attributable to the mean effect of  $-0.5\%$ . In contrast to our baseline results, the negative mean effect comes from the fact that households do not benefit that much from the introduction of the low yield asset social security, compared to the losses from taxation. This is so because mean stock returns now stand at 7.9%, much higher than the 3.0% of the baseline scenario. Also, the previously discussed portfolio reallocation is less strong because of the high risk aversion.

In the *IR-only* economy, we find large welfare losses in general equilibrium of  $-1.6\%$ . The mean effect again is negative at  $-0.4\%$ . As in the *AR-only* economy, this reflects that we force households to implicitly save in a low yield asset. If prices are held constant, then in partial equilibrium there is a small welfare gain, as households do value the insurance provided by social security, but the welfare cost of crowding out are much larger.

Finally, introducing social security in the no-risk economy leads to welfare losses in both general ( $-1.1\%$ ) and partial equilibrium ( $-0.6\%$ ). Here, the negative mean effect of  $-0.3\%$  is due to the downward shift of life-cycle consumption, while the negative distribution effect of  $-0.8\%$  represents the welfare impact of decreased consumption when young, cf. Figure 2.

As explained above, we re-calibrated the economies to the same targets used in the literature so as to replicate earlier findings. In Appendix E.2 we report the corresponding results without re-calibration. Qualitatively, our findings for the *AR-only* and the *No-risk* economies are unchanged. However, without re-calibration, the *IR-only* economy is dynamically inefficient and accordingly features a mild mean effect of 0.15%. The distribution effect in general equilibrium,  $g_c^{distr}$ , is also slightly positive at 0.14%. The sum of insurance gains of the *AR-only* and the *IR-only* economies continues to be negative.



Table 8: The Role of Both Risks: Benefits, Costs, and Change in Aggregates

Scenario	Consumption equivalent variation, $g_c$			$g_c^{distr}$	$g_c^{mean}$
	GE	PE	CO	GE	
<i>AR-only</i>	-0.64%	-0.40%	-0.23%	-0.13%	-0.51%
<i>IR-only</i>	-1.62%	0.08%	-1.69%	-1.22%	-0.40%
<i>No-risk</i>	-1.13%	-0.62%	-0.51%	-0.84%	-0.29%

*Notes:* GE: general equilibrium, PE: partial equilibrium, CO: crowding out; *AR-only*: economy with only aggregate risk, calibrated to match equity premium; *IR-only*: economy with only idiosyncratic risk; *No-risk*: deterministic economy. The total  $g_c$  is further decomposed into the mean effect,  $g_c^{mean}$ , and the distribution effect,  $g_c^{distr}$ , cf. Section 3.8 for formal definitions.

### 5.3 Sensitivity Analysis

**Dynamic Efficiency.** We check the sufficient conditions for dynamic efficiency of Definition 3 for all economies considered in our sensitivity analyses. The conditions are satisfied in all cases, with the statistics being very close to those of our baseline economy shown in Table 2, cf. Appendix E.1. The reason why the statistics are so similar is that we always target the same bond return and never modify the implicit return to social security, while the risky return either retains its low mean and volatility of the baseline, or has higher mean and volatility closer to the data. Therefore, the central elements of the conditions are unchanged.

**Sharpe Ratio, Equity Premium, and Intertemporal Elasticity of Substitution.** We investigate whether our key findings of long-run welfare gains and sizeable interactions are robust when we consider economies with realistic calibration targets that imply higher levels of risk. In one variant we calibrate the model to match the equity premium and the volatility of stock returns, the same targets as in the *AR – only* economy of Section 5.2. This scenario is referred to as *EP*. It implies a consumption volatility and a Sharpe ratio which are both too high relative to the data. We therefore also consider an intermediate case where we instead match the Sharpe ratio and the volatility of consumption. This scenario is referred to as *SR*. To isolate the effects of risk and risk aversion, we hold the discount factor  $\beta$  constant at its baseline value in these experiments. Therefore, a crucial preference parameter, which—as we discuss in the context of our simple model in Section 2.3—has a strong impact on welfare, remains unchanged, making the comparison and interpretation of the results much easier. The average risk-free bond return is always kept at the same level of the baseline through an appropriate calibration of  $\delta_0$ , because we know from Propositions 1 and 2 that it plays a key role for the value of social security.

We repeat this sensitivity analysis with a higher intertemporal elasticity of substitution ( $IES$ ). To this end, we first proceed as in our baseline calibration, i.e., for our choice of risk aversion of  $\theta = 3$ , we define a modified baseline ( $BL_{IES=1.5}$ ) in which we set the  $IES$  to 1.5 and recalibrate all parameters. Starting from this modified baseline, we then repeat the analogues to the  $SR$  and  $EP$  calibrations, referred to as  $SR_{IES=1.5}$  and  $EP_{IES=1.5}$ , respectively. Details on the calibration are described in Appendix D.5.

The welfare results in general equilibrium are presented in Table 9, together with the benefits from insurance and the losses from crowding out of capital formation. Our result from the baseline scenario ( $BL$ ) is confirmed: there are large welfare gains ranging from 2.5 to 5.5 percent in terms of a consumption equivalent variation when losses from crowding out are fully taken into account. In line with the prediction from our simple model of Section 2, welfare gains increase in risk aversion:  $BL$  features  $\theta = 3$ ,  $SR$  has  $\theta = 11.1$  and  $EP$  has  $\theta = 5.3$ .<sup>40</sup> Our two baseline scenarios, with an  $IES$  of 0.5 and 1.5 and a reasonable degree of risk aversion of 3, deliver the smallest welfare numbers with total welfare gains of 2.6% and 2.7%, respectively.

Table 9: Sensitivity Analysis: Benefits, Costs, Bias, and Change in Aggregates

Scenario	$g_c$			$\frac{\Delta_{CCV} + \Delta_{CWG}}{g_c^{PE}}$	$g_c^{distr}$	$g_c^{mean}$
	GE	PE	CO			
$IES = 0.5$						
$BL$	+2.56%	+5.18%	-2.62%	0.60	+2.32%	+0.24%
$SR$	+4.78%	+8.48%	-3.70%	0.66	+5.42%	-0.64%
$EP$	+3.58%	+7.40%	-3.81%	0.73	+8.72%	-5.13%
$BL_{\tau=9.5\%}$	+1.00%	+2.23%	-1.24%	0.41	+0.92%	+0.08%
$IES = 1.5$						
$BL_{IES=1.5}$	+2.69%	+3.21%	-0.53%	0.60	+2.57%	+0.12%
$SR_{IES=1.5}$	+5.62%	+8.04%	-2.42%	0.66	+5.93%	-0.31%
$EP_{IES=1.5}$	+5.56%	+7.80%	-2.23%	0.75	+7.73%	-2.16%

Notes: GE: general equilibrium, PE: partial equilibrium, CO: crowding out;  $CCV$ : counter-cyclical cross-sectional variance,  $CWG$ : convexity of the welfare gain;  $BL$ : baseline calibration with  $\theta = 3$ ;  $SR$ : scenario matching Sharpe ratio;  $EP$ : scenario matching equity premium. The total  $g_c$  is further decomposed into the mean effect,  $g_c^{mean}$ , and the distribution effect,  $g_c^{distr}$ , cf. Section 3.8 for formal definitions.

To examine the role played by the welfare biases attributable to the  $CWG$  and the  $CCV$

<sup>40</sup>Ceteris paribus, matching a higher equity premium would require a higher degree of risk aversion. However, as we simultaneously increase the variance of risky returns—by an appropriate choice of  $\bar{\delta}$ —, we also introduce more risk into the economy. As a consequence, the coefficient of risk aversion is lower in the  $EP$  than in the  $SR$ -calibration.

across scenarios, we compute the ratio  $\frac{\Delta_{CCV} + \Delta_{CWG}}{g_c^E}$ . It amounts to approximately 60 percent in our two baseline scenarios and reaches 75 percent in scenario  $EP_{IES=1.5}$ . Therefore, our finding that roughly 60% of the total welfare gains would be missed from adding up the isolated benefits is robust across calibrations.

In all scenarios, the portion of the welfare gain attributable to an increase of mean consumption,  $g_c^{mean}$ , is smaller than in our baseline. More importantly, it is negative when we match the equity premium or the Sharpe ratio (both for  $IES = 0.5$  and  $IES = 1.5$ ), while the welfare gains in those calibrations are higher than in our baseline. This is consistent with the explanation given earlier, namely that higher risk aversion mitigates the portfolio reallocation towards stocks that is induced by the introduction of social security. Also, the risky returns in those calibrations are higher, so that the implicit social security return is relatively low. At the same time, the welfare gain attributable to changes of the distribution,  $g_c^{distr}$ , increases very substantially, again due to the higher risk and risk aversion in these economies.

**Expanding the existing U.S. Social Security System.** Next, we present results on the welfare effects of expanding the existing U.S. social security system, with its average contribution rate of the postwar period of 9.5% by 2 percentage points.<sup>41</sup> With this experiment we give up some of the clarity of our thought experiment because our closed form expressions from Section 2 are derived for marginal changes around  $\tau = 0$ . We recalibrate our model to achieve the same targets as in our baseline scenario and base the calibration of the income process on estimates after all taxes and social insurance contributions, see Appendix D.5. We label this experiment as  $BL_{\tau=9.5\%}$  in Table 9. We continue to observe welfare gains in general equilibrium which are smaller than for our baseline results.<sup>42</sup> The contribution attributable to the interactions decreases to 41%. This is still sizeable.

However, a number of cautionary remarks on this experiment are in order. With this social security system we lose the interpretation of analyzing a minimum pension; instead this is a very generous flat pension. While this scenario models the average size of the actual U.S. system, the generous flat pension income does not correctly reflect the institutional feature of a mix of an earnings related and a redistributive component thereby exaggerating the insurance benefits. Finally, because the marginal excess burden is increasing in the tax level, abstracting from endogenous labor supply responses is more critical for such an experiment with an already high base distortion.

<sup>41</sup>Using the average contribution rate of 9.5% rather than the current contribution rate of 12.4% corresponds to our calibration of other moments which is based on postwar averages.

<sup>42</sup>Hence, the optimal social security contribution rate in our model is somewhere above the average rate of 9.5%.

**Other Model Parameters and Modeling Choices.** In Appendix E.3 we report and discuss further results of sensitivity analyses with respect to calibration and modeling choices. We in turn consider an exogenous decrease (increase) of risk aversion to two (to four), an increase of the targeted capital-output ratio to 3.0, a zero debt-equity ratio, a zero variance of depreciation shocks, and a zero variance of transitory income shocks. All these experiments confirm our main findings. Also, we consider a redistribution of all tax income to workers, instead of to pensioners. Welfare gains are smaller under that redistribution scheme, because it cannot insure against aggregate risk and only to a limited extent against the  $CCV$ .

## 5.4 On Biases in the Welfare Assessment of Crowding Out

One important question is whether there are also biases in the welfare assessment of crowding out of capital: Are there counterparts to  $\Delta_{CWG}$  and  $\Delta_{CCV}$  that increase the welfare losses, and what can we say about them? The answer is fundamentally difficult because crowding out is a general equilibrium phenomenon that works through price adjustments, which inhibits a decomposition of the form we perform for the insurance benefits. However, the whole point of our general equilibrium analysis is to see whether any biases in crowding out are stronger than  $\Delta_{CWG} + \Delta_{CCV}$ . Since in single-risk economies the welfare costs dominate, whereas in all our calibrations with both types of risk the benefits dominate, it must be that the biases on the insurance side are stronger than those on the crowding out side. While we cannot state this result generally, it does hold for all our carefully calibrated scenarios.

To gain additional insights on the strength of biases in the welfare cost of crowding out despite the above mentioned fundamental difficulty, we resort to a comparison of the welfare costs in our sensitivity analyses. Comparing our findings in Table 9 with those of Table 8 we indeed observe that losses from crowding out increase more strongly when both risks are modeled jointly, but this is less pronounced than for the welfare gains. Losses from crowding out in the *IR-only* economy are at  $-1.69\%$  and in the *AR-only* economy at  $-0.23\%$ , but losses in the corresponding scenario *EP* are much larger than the sum, namely at  $-3.81\%$ . This comparison is only indicative, because we compare across different equilibria and because of recalibration. Taking the non-recalibrated results from Appendix E.2, Table 13, we have analogous findings.<sup>43</sup> Hence, in our experiments, the convexity of the welfare losses from crowding out is weaker than the convexity of the welfare gains from insurance.

We now provide reasons for this finding. In Section 2.3 we already emphasize the role of

---

<sup>43</sup>Losses from crowding out in the *IR-only* economy are at  $-1.56\%$  and in the *AR-only* economy at  $-0.36\%$ ; again the sum is lower than the  $-3.81\%$  observed for scenario *EP*.

pecuniary externalities to explain why the convexity of welfare losses from crowding out may be weaker. There are two additional reasons. The first is that mean wages decrease while mean returns increase, which, in a model with heterogeneous agents, benefits some while hurting others (Davila, Hong, Krusell, and Ríos-Rull 2012). The reason is that increasing interest rates benefit the asset rich, while falling interest rates hurt the relatively asset poor. The net effect of crowding out on expected life-time utility and welfare is therefore per se ambiguous and depends on the distribution of factor incomes. As the second reason note that, in contrast to the partial equilibrium insurance gains, crowding out is a general equilibrium phenomenon. The reduction of savings leads to an important feedback in general equilibrium, because crowding out raises asset returns which induces households to save more. This substitution effect mitigates the reduction of aggregate capital and the corresponding welfare losses. Since the substitution effect becomes stronger the larger the  $IES$ , this mitigating channel can be observed by comparing the welfare losses from crowding out in the  $IES = 0.5$  and  $IES = 1.5$  scenarios of Table 9. It is indeed the case that losses from crowding out are smaller for all the  $IES = 1.5$  scenarios, which is also the main reason why the general equilibrium welfare gains are higher.<sup>44</sup>

## 6 Conclusion

This paper analyzes the welfare effects of social security by evaluating its benefits and costs when households face multiple risks in the form of idiosyncratic earnings risk and aggregate business cycle risk. We consider a pay-as-you-go (PAYG) financed social security system which partially insures both forms of risk through a minimum pension. We show that the whole gain from insurance is greater than the sum of the insurance benefits attributable to the isolated risk components. One source for this welfare difference is a direct interaction of risks, in the form of a countercyclical, cross-sectional variance of idiosyncratic income risk. The other is due to the convexity of the welfare gain in total risk.

Based on a calibrated large-scale overlapping generations model, we find that introducing a PAYG financed social security system with a contribution rate of 2% leads to long-run welfare gains of 2.6% in terms of a consumption equivalent variation despite significant crowding out of capital. Considering both risks jointly is crucial for this finding. Examining only one risk in isolation misses the two amplifying mechanisms which account for 60% of the welfare gains. In fact, when we consider only one type of risk in isolation, we find net welfare losses.

---

<sup>44</sup>Because of the substitution effect, a higher  $IES$  also leads to a smaller volatility of real aggregates. The fluctuations caused by depreciation shocks are counteracted by households' savings, so that, for higher  $IES$ , volatility of capital is smaller, leading to smaller volatility of aggregate output.

There is an interesting parallel to the literature on the welfare costs of aggregate fluctuations. In his seminal contribution, Lucas (1987) demonstrates that the costs of business cycles are negligible. However, when business cycle risk is analyzed in conjunction with idiosyncratic income risk, then welfare costs can become very large, see De Santis (2007) and Krebs (2007).

While our analysis uncovers important biases in the welfare assessment of social security and documents that they matter quantitatively, some aspects are not taken into account, two of which we emphasize here. First, we abstract from endogenous labor supply. This may bias results in favor of social security for two reasons. One is that we do not account for self-insurance against risk through endogenous labor supply adjustments. The other is that a higher contribution rate would distort labor supply decisions, crowding out aggregate labor supply if the substitution effect dominates. However, when taking labor market frictions into account and considering small policy changes, as in this paper, a calibrated model would likely only lead to small effects of endogenous labor supply reactions.

Second, we only conduct a limited policy design experiment by studying the welfare effects of improved insurance through social security, taking as given the distribution through the tax and transfer system. With this approach we follow much of the social security literature, which complements a large literature that analyzes the welfare implications of the general tax and transfer system during working life, taking as given the design of social security. There are good reasons to consider redistribution through social security beyond the insurance motives emphasized in our work. One reason is efficiency concerns based on lifetime income tax smoothing arguments put forth by Vickrey (1947), see Diamond (2003) for an analysis in the context of social security. A second is the presence of moral hazard frictions during working life that lead Michelacci and Ruffo (2015) to conclude that optimal unemployment insurance should decrease over the life-cycle thereby limiting the scope for redistribution. Since these frictions are gone in retirement, it may be optimal to redistribute less during the working period and more in retirement. In light of these aspects it is obviously important to analyze the optimal joint design of social security and progressive income taxation, respectively unemployment insurance, which we address in ongoing research.

## References

- Aaron, H. J. (1966). The Social Insurance Paradox. *Canadian Journal of Economics and Political Science* 32(3), 371–374.
- Aguiar, M. and M. Bils (2015). Has Consumption Inequality Mirrored Income Inequality? *American Economic Review* 105(9), 2725–2756.
- Algan, Y., O. Allais, W. J. Den Haan, and P. Rendahl (2014). Solving and Simulating Models with Heterogeneous Agents and Aggregate Uncertainty. In *Handbook of Computational Economics*, Volume 3, pp. 277–324. Elsevier B.V.
- Auerbach, A. J. and L. J. Kotlikoff (1987). *Dynamic Fiscal Policy*. Cambridge, MA: Cambridge University Press.
- Ball, L. and N. G. Mankiw (2007). Intergenerational Risk Sharing in the Spirit of Arrow, Debreu, and Rawls, with Applications to Social Security Design. *Journal of Political Economy* 115(4), 523–547.
- Bansal, R. and A. Yaron (2004). Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles. *The Journal of Finance* 59(4), 1481–1509.
- Bohn, H. (2001). Social Security and Demographic Uncertainty: The Risk Sharing Properties of Alternative Policies. In J. Campbell and M. Feldstein (Eds.), *Risk Aspects of Investment Based Social Security Reform*, pp. 203–241. Chicago: University of Chicago Press.
- Bohn, H. (2009). Intergenerational Risk Sharing and Fiscal Policy. *Journal of Monetary Economics* 56(6), 805–816.
- Boldrin, M., L. J. Christiano, and J. D. M. Fisher (1995). Asset Pricing Lessons for Modeling Business Cycles. *NBER Working Paper Series No. 5262*, 1–52.
- Bommier, A., D. Harenberg, and F. Le Grand (2017). Household Finance and the Value of Life. *SSRN eLibrary*, Available at SSRN: <https://ssrn.com/abstract=29157>.
- Busch, C. and A. Ludwig (2017). Higher-Order Income Risk over the Business Cycle: A Parametric Approach. Working Paper.
- Caliendo, F. N., N. L. Guo, and R. Hosseini (2014). Social Security is NOT a Substitute for Annuity Markets. *Review of Economic Dynamics* 17(4), 739–755.
- Carroll, C. D. (2006). The Method of Endogenous Gridpoints for Solving Dynamic Stochastic Optimization Problems. *Economics Letters* 91(3), 312–320.

- Castañeda, A., J. Díaz-Giménez, and J.-V. Ríos-Rull (2003). Accounting for the U.S. Earnings and Wealth Inequality. *Journal of Political Economy* 111(4), 818–857.
- Cocco, J. (2005). Portfolio Choice in the Presence Housing. *The Review of Financial Studies* 18(2), 535–567.
- Cocco, J. F., F. J. Gomes, and P. J. Maenhout (2005). Consumption and Portfolio Choice over the Life-Cycle. *The Review of Financial Studies* 18(2), 491–533.
- Conesa, J. C. and D. Krueger (1999). Social Security Reform with Heterogeneous Agents. *Review of Economic Dynamics* 2(4), 757–795.
- Constantinides, G. M. and D. Duffie (1996). Asset Pricing with Heterogeneous Consumers. *Journal of Political Economy* 104(2), 219–240.
- Croce, M. M. (2014). Long-Run Productivity Risk: A New Hope for Production-Based Asset Pricing? *Journal of Monetary Economics* 66, 13–31.
- Davila, J., J. H. Hong, P. Krusell, and J.-V. Ríos-Rull (2012). Constrained Efficiency in the Neoclassical Growth Model With Uninsurable Idiosyncratic Shocks. *Econometrica* 80(6), 2431–2467.
- De Nardi, M. (2004). Wealth Inequality and Intergenerational Links. *Review of Economic Studies* 71, 743–768.
- De Santis, M. (2007). Individual Consumption Risk and the Welfare Cost of Business Cycles. *American Economic Review* 97(4), 1488–1506.
- Demange, G. (2002). On Optimality in Intergenerational Risk Sharing. *Economic Theory* 20(1), 1–27.
- Diamond, P. (2003). *Taxation, Incomplete Markets, and Social Security*. Cambridge, MA: MIT Press.
- Diamond, P. A. (1965). National Debt in a Neoclassical Growth Model. *American Economic Review* 55(5), 1126–1150.
- Diamond, P. A. (1977). A Framework for Social Security Analysis. *Journal of Public Economics* 8(3), 275–298.
- Epstein, L. G. and S. E. Zin (1989). Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. *Econometrica* 57(4), 937–969.



- Epstein, L. G. and S. E. Zin (1991). Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis. *Journal of Political Economy* 99(2), 263–286.
- Fehr, H. and C. Habermann (2008). Risk Sharing and Efficiency Implications of Progressive Pension Arrangements. *Scandinavian Journal of Economics* 110(2), 419–443.
- Fernández-Villaverde, J. and D. Krueger (2011). Consumption and Saving Over the Life Cycle: How Important Are Consumer Durables? *Macroeconomic Dynamics* 15(5), 725–770.
- Golosov, M., A. Shourideh, M. Troshkin, and A. Tsyvinski (2013). Optimal Pension Systems with Simple Instruments. *American Economic Review, Papers and Proceedings* 103(3), 502–507.
- Gomes, F., A. Michaelides, and V. Polkovnichenko (2012). Fiscal Policy and Asset Prices with Incomplete Markets. *Review of Financial Studies* 26(2), 531–566.
- Gomes, F. J. and A. Michaelides (2008). Asset Pricing with Limited Risk Sharing and Heterogeneous Agents. *The Review of Financial Studies* 21(1), 415–448.
- Goodman, L. A. (1960). On the Exact Variance of Products. *Journal of the American Statistical Association* 55(292), 708–713.
- Gordon, R. H. and H. R. Varian (1988). Intergenerational Risk Sharing. *Journal of Public Economics* 37(2), 185–202.
- Guvenen, F. (2007). Learning Your Earning: Are Labor Income Shocks Really Very Persistent. *American Economic Review* 97(3), 687–712.
- Guvenen, F., S. Ozkan, and J. Song (2014). The Nature of Countercyclical Income Risk. *Journal of Political Economy* 122(3), 1–59.
- Harenberg, D. and A. Ludwig (2015). Social Security in an Analytically Tractable Overlapping Generations Model with Aggregate and Idiosyncratic Risks. *International Tax and Public Finance* 22(4), 579–603.
- Hasanhodzic, J. and L. Kotlikoff (2015). Generational Risk—Is It a Big Deal?: Simulating an 80-Period OLG Model with Aggregate Shocks. *NBER Working Paper 19179*, 1–47.
- Hintermaier, T. and W. Koeniger (2010). The Method of Endogenous Gridpoints with Occasionally Binding Constraints Among Endogenous Variables. *Journal of Economic Dynamics and Control* 34(10), 2074–2088.

- Hintermaier, T. and W. Koeniger (2011). On the Evolution of the US Consumer Wealth Distribution. *Review of Economic Dynamics* 14(2), 317–338.
- Huggett, M. and J. C. Parra (2010). How Well Does the U.S. Social Insurance System Provide Social Insurance? *Journal of Political Economy* 118(1), 76–112.
- Huggett, M. and G. Ventura (1999). On the Distributional Effects of Social Security Reform. *Review of Economic Dynamics* 2(3), 498–531.
- İmrohoroğlu, A., S. İmrohoroğlu, and D. H. Joines (1995). A Life Cycle Analysis of Social Security. *Economic Theory* 6(1), 83–114.
- İmrohoroğlu, A., S. İmrohoroğlu, and D. H. Joines (1998). The Effect of Tax-Favored Retirement Accounts on Capital Accumulation. *American Economic Review* 88(4), 749–768.
- Kopecky, K. A. and R. M. Suen (2010). Finite State Markov-Chain Approximations to Highly Persistent Processes. *Review of Economic Dynamics* 13(3), 701–714.
- Krebs, T. (2007). Job Displacement Risk and the Cost of Business Cycles. *The American Economic Review* 97(3), 664–686.
- Krueger, D. and F. Kubler (2006). Pareto-Improving Social Security Reform when Financial Markets are Incomplete!? *American Economic Review* 96(3), 737–755.
- Krueger, D. and A. Ludwig (2007). On the Consequences of Demographic Change for Rates of Return to Capital and the Distribution of Wealth and Welfare. *Journal of Monetary Economics* 54(1), 49–87.
- Krueger, D. and A. Ludwig (2017). Precautionary Savings and Pecuniary Externalities: A Full Analytical Characterization of Optimal Capital Taxes in an OLG Economy. Working Paper.
- Krueger, D., K. Mitman, and F. Perri (2016). Macroeconomics and Household Heterogeneity. In J. B. Taylor and H. Uhlig (Eds.), *Handbook of Macroeconomics*, Chapter 11.
- Krusell, P. and J. A. A. Smith (1997). Income and Wealth Heterogeneity, Portfolio Choice, and Equilibrium Asset Returns. *Macroeconomic Dynamics* 1(2), 387–422.
- Krusell, P. and J. A. A. Smith (1998). Income and Wealth Heterogeneity in the Macroeconomy. *Journal of Political Economy* 106(5), 867–896.
- Kuhle, W. (2012). Dynamic Efficiency and the Two-Part Golden Rule with Heterogeneous Agents. *Journal of Macroeconomics* 34(4), 992–1006.

- Lucas, R. E. (1978). Asset Prices in an Exchange Economy. *Econometrica* 46(6), 1426–1445.
- Lucas, R. E. (1987). *Models of Business Cycles*. Oxford: Blackwell Publishing Ltd.
- Ludwig, A. and M. Reiter (2010). Sharing Demographic Risk - Who is Afraid of the Baby Bust? *American Economic Journal: Economic Policy* 2(4), 83–118.
- Ludwig, A. and E. Vogel (2009). Mortality, Fertility, Education and Capital Accumulation in a Simple {OLG} Economy. *Journal of Population Economics* 23(2), 703–735.
- Mankiw, N. G. (1986). The Equity Premium and the Concentration of Aggregate Shocks. *Journal of Financial Economics* 17(1), 211–219.
- Matsen, E. and O. Thøgersen (2004). Designing Social Security - A Portfolio Choice Approach. *European Economic Review* 48(4), 883–904.
- Merton, R. (1983). On the Role of Social Security as a Means for Efficient Risk Sharing in an Economy Where Human Capital is not Tradable. In Z. Bodie and J. B. Shoven (Eds.), *Financial Aspects of the United States Pension System*, pp. 325–358. Chicago: University of Chicago Press.
- Michelacci, C. and H. Ruffo (2015). Optimal Life Cycle Unemployment Insurance. *American Economic Review* 105(2), 816–859.
- Nishiyama, S. and K. Smetters (2007). Does Social Security Privatization Produce Efficiency Gains? *Quarterly Journal of Economics* 122(4), 1677–1719.
- OECD (2015). Architecture of National Pension Systems. In *Pensions at a Glance 2015: OECD and G20 Indicators*, pp. 124–126. Paris: OECD Publishing.
- Olovsson, C. (2010). Quantifying the Risk-Sharing Welfare Gains of Social Security. *Journal of Monetary Economics* 57, 364–375.
- Peterman, W. B. and K. Sommer (2015). A Historical Welfare Analysis of Social Security: Who Did the Program Benefit? *FEDS Working Paper 2015-092*.
- Peterman, W. B. and K. Sommer (2016). How Well Did Social Security Mitigate the Effects of the Great Recession? *FEDS Working Paper 2014-13*.
- Rajan, R. and L. Zingales (1995). What Do We Know About Capital Structure? Some Evidence From International Data. *The Journal of Finance* 50(5), 1421–1460.
- Ríos-Rull, J.-V. (1999). Computation of Equilibria in Heterogenous Agent Models. In R. Marimón and A. Scott (Eds.), *Computational Models for the Study of Dynamic Economies*, pp. 238–264. Oxford, N.Y.: Oxford University Press.

- Samuelson, P. A. (1958). An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money. *Journal of Political Economy* 66(6), 467–482.
- Shiller, R. J. (1999). Social Security and Institutions for Intergenerational, Intragenerational, and International Risk-Sharing. *Carnegie- Rochester Conference Series on Public Policy* 50(99), 165–204.
- Siegel, J. J. (2002). *Stocks for the Long Run : The Definitive Guide to Financial Market Returns and Long-Term Investment Strategies*. New York: McGraw-Hill.
- Storesletten, K., C. I. Telmer, and A. Yaron (1999). The Risk-Sharing Implications of Alternative Social Security Arrangements. *Carnegie-Rochester Conference Series on Public Policy* 50, 213–259.
- Storesletten, K., C. I. Telmer, and A. Yaron (2004a). Consumption and Risk Sharing Over the Life Cycle. *Journal of Monetary Economics* 51(3), 609–633.
- Storesletten, K., C. I. Telmer, and A. Yaron (2004b). Cyclical Dynamics in Idiosyncratic Labor Market Risk. *Journal of Political Economy* 112(3), 695–717.
- Storesletten, K., C. I. Telmer, and A. Yaron (2007). Asset Pricing with Idiosyncratic Risk and Overlapping Generations. *Review of Economic Dynamics* 10(4), 519–548.
- Vickrey, W. S. (1947). *Agenda for Progressive Taxation*. New York: Ronald.
- Weil, P. (1989). The Equity Premium Puzzle and the Risk-Free Rate Puzzle. *Journal of Monetary Economics* 24(3), 401–421.
- Young, E. R. (2010). Solving the Incomplete Markets Model with Aggregate Uncertainty Using the Krusell-Smith Algorithm. *Journal of Economic Dynamics and Control* 34(1), 36–41.

## A Appendix: Proofs

*Proof of Proposition 1.* The proof has various steps. Step 1 characterizes utility consequences without making the specific assumption of log-normal shocks. Step 2 modifies the respective terms for the case of log-normally distributed random variables. Finally, Step 3 derives the CEV.

### 1. Maximize

$$Eu(c_{i,2,t+1}) = \frac{1}{1-\theta} E \left( \bar{w}_t \left( \bar{R} \eta_{i,1,t} \zeta_t \varrho_{t+1} + \tau \left( (1+\lambda) \zeta_{t+1} - \bar{R} \eta_{i,1,t} \zeta_t \varrho_{t+1} \right) \right) \right)^{1-\theta}$$

which is equivalent to

$$\max \frac{1}{1-\theta} E R_{p,t,t+1}^{1-\theta},$$

where  $R_{p,t,t+1} \equiv \eta_{i,1,t} \zeta_t \bar{R} \varrho_{t+1} + \tau \left( (1+\lambda) \zeta_{t+1} - \bar{R} \eta_{i,1,t} \zeta_t \varrho_{t+1} \right)$  is a consumption (or portfolio) return. Increasing ex-ante utility for a marginal introduction of social security requires the first-order condition w.r.t.  $\tau$  to exceed zero, hence:

$$E \left[ R_{p,t,t+1}^{-\theta} \frac{\partial R_{p,t,t+1}}{\partial \tau} \right] \Bigg|_{\tau=0} > 0.$$

Taking the according partial derivatives we get, using Assumption 1b–1d, the condition

$$\frac{1+\lambda}{\bar{R}} \frac{E \left[ (\eta_{i,1,t} \zeta_t \varrho_{t+1})^{-\theta} \right]}{E \left[ (\eta_{i,1,t} \zeta_t \varrho_{t+1})^{1-\theta} \right]} - 1 > 0. \quad (13)$$

2. Define  $Z_1 \equiv (\eta_{i,1,t} \zeta_t \varrho_{t+1})^{-\theta}$  and  $Z_2 \equiv (\eta_{i,1,t} \zeta_t \varrho_{t+1})^{1-\theta}$ . By log-normality we have that  $E Z_i = \exp(E \ln Z_i + \frac{1}{2} \sigma_{\ln Z_i}^2)$ ,  $i = 1, 2$ . Turning first to  $Z_1$  observe that

$$\begin{aligned} E[Z_1] &= \exp \left( -\theta \left( E \ln \eta_{i,1,t} + \frac{\sigma_{\ln \eta}^2}{2} \right) \right) \cdot \exp \left( -\theta \left( E \ln \varrho + \frac{\sigma_{\ln \varrho}^2}{2} \right) \right) \\ &\cdot \exp \left( -\theta \left( E \ln \zeta + \frac{\sigma_{\ln \zeta}^2}{2} \right) \right) \exp \left( \frac{1}{2} \theta (1+\theta) (\sigma_{\ln \eta}^2 + \sigma_{\ln \varrho}^2 + \sigma_{\ln \zeta}^2) \right) \\ &= \exp \left( \frac{1}{2} \theta (1+\theta) (\sigma_{\ln \eta}^2 + \sigma_{\ln \varrho}^2 + \sigma_{\ln \zeta}^2) \right), \end{aligned}$$

where the second line follows from Assumption 1b. As to  $Z_2$  we get

$$\begin{aligned} E[Z_2] &= \exp\left((1-\theta)\left(E \ln \eta_{i,1,t} + \frac{\sigma_{\ln \eta}^2}{2}\right)\left(E \ln \varrho + \frac{\sigma_{\ln \varrho}^2}{2}\right)\left(E \ln \zeta + \frac{\sigma_{\ln \zeta}^2}{2}\right)\right) \\ &\quad \cdot \exp\left(\frac{1}{2}\theta(\theta-1)\left(\sigma_{\ln \eta}^2 + \sigma_{\ln \varrho}^2 + \sigma_{\ln \zeta}^2\right)\right) \\ &= \exp\left(\frac{1}{2}\theta(\theta-1)\left(\sigma_{\ln \eta}^2 + \sigma_{\ln \varrho}^2 + \sigma_{\ln \zeta}^2\right)\right) \end{aligned}$$

and hence, defining  $\sigma_{\ln AR} \equiv \sqrt{\sigma_{\ln \zeta}^2 + \sigma_{\ln \varrho}^2}$  we have

$$\frac{E[Z_1]}{E[Z_2]} = \exp\left(\theta\left(\sigma_{\ln \eta}^2 + \sigma_{\ln AR}^2\right)\right). \quad (14)$$

3. To evaluate the CEV between two scenarios, i.e., comparing  $Eu\left(c_{i,2,t+1}^{\tau>0}\right)$  with  $Eu\left(c_{i,2,t+1}^{\tau=0}\right)$ , we use that

$$Eu\left(c_{i,2,t+1}^{\tau>0}\right) = Eu\left(c_{i,2,t+1}^{\tau=0}\right) + \frac{\partial Eu\left(c_{i,2,t+1}^{\tau=0}\right)}{\partial \tau} d\tau.$$

and evaluate this expression at  $\tau = 0$ .

- (a) **Case  $\theta \neq 1$ .** We have that, evaluated at  $\tau = 0$ ,

$$\begin{aligned} \frac{\partial Eu(c_{i,2,t+1}^{\tau=0})}{\partial \tau} &= \bar{w}_t^{1-\theta} E\left[\left(\bar{R}\eta\zeta_t\varrho_{t+1}\right)^{-\theta} \cdot \left((1+\lambda)\zeta_{t+1} - \bar{R}\eta\zeta_t\varrho_{t+1}\right)\right] \\ &= \bar{w}_t^{1-\theta} \bar{R}^{1-\theta} \left(\frac{1+\lambda}{\bar{R}} EZ_1 - EZ_2\right), \end{aligned}$$

where  $Z_1, Z_2$  are defined in Step 2.

We also have that

$$\begin{aligned} Eu(c_{i,2,t+1}^{\tau=0}) &= \frac{1}{1-\theta} \bar{w}_t^{1-\theta} \bar{R}^{1-\theta} E\left(\eta_{i,1,t}\zeta_t\varrho_{t+1}\right)^{1-\theta} \\ &= \frac{1}{1-\theta} \bar{w}_t^{1-\theta} \bar{R}^{1-\theta} EZ_2. \end{aligned}$$

Therefore:

$$\begin{aligned} Eu(c_{i,2,t+1}^{\tau>0}) &= \frac{1}{1-\theta} \bar{w}_t^{1-\theta} \bar{R}^{1-\theta} EZ_2 \\ &\quad + \bar{w}_t^{1-\theta} \bar{R}^{1-\theta} \left( \frac{1+\lambda}{\bar{R}} EZ_1 - EZ_2 \right) d\tau. \end{aligned}$$

The CEV, denoted by  $g_c^{PE}$ , is defined by the relationship:

$$Eu(c_{i,2,t+1}^{\tau=0}(1+g_c^{PE})) = Eu(c_{i,2,t+1}^{\tau>0}),$$

from which, using the above formulae, we get

$$\begin{aligned} (1+g_c^{PE})^{1-\theta} &\frac{1}{1-\theta} \bar{w}_t^{1-\theta} \bar{R}^{1-\theta} EZ_2 \\ &= \frac{1}{1-\theta} \bar{w}_t^{1-\theta} \bar{R}^{1-\theta} EZ_2 + \bar{w}_t^{1-\theta} \bar{R}^{1-\theta} \left( \frac{1+\lambda}{\bar{R}} EZ_1 - EZ_2 \right) d\tau. \end{aligned}$$

Hence,

$$\begin{aligned} (1+g_c^{PE})^{1-\theta} &= 1 + \frac{\bar{w}_t^{1-\theta} \bar{R}^{1-\theta} \left( \frac{1+\lambda}{\bar{R}} EZ_1 - EZ_2 \right) d\tau}{\frac{1}{1-\theta} \bar{w}_t^{1-\theta} \bar{R}^{1-\theta} EZ_2} \\ &= 1 + (1-\theta) \left( \frac{1+\lambda}{\bar{R}} \exp\left(\theta(\sigma_{\ln \eta}^2 + \sigma_{\ln AR}^2)\right) - 1 \right) d\tau \end{aligned}$$

where the second line follows from equation (14), cf. Step 2. Hence,

$$g_c^{PE} = \left( 1 + (1-\theta) \left( \frac{1+\lambda}{\bar{R}} \exp\left(\theta(\sigma_{\ln \eta}^2 + \sigma_{\ln AR}^2)\right) - 1 \right) d\tau \right)^{\frac{1}{1-\theta}} - 1,$$

or, expressed in logs, i.e., using that  $g_c^{PE} \approx \ln(1+g_c^{PE})$  for small  $g_c^{PE}$ , we get

$$\begin{aligned} g_c^{PE} &\approx \frac{1}{1-\theta} \cdot \ln \left( 1 + (1-\theta) \left( \frac{1+\lambda}{\bar{R}} \exp\left(\theta(\sigma_{\ln \eta}^2 + \sigma_{\ln AR}^2)\right) - 1 \right) d\tau \right) \\ &\approx \left( \frac{1+\lambda}{\bar{R}} \exp\left(\theta(\sigma_{\ln \eta}^2 + \sigma_{\ln AR}^2)\right) - 1 \right) d\tau \end{aligned} \tag{15}$$

(b) **Case  $\theta = 1$ .** We have that, evaluated at  $\tau = 0$ ,

$$\begin{aligned}\frac{\partial Eu(c_{i,2,t+1}^{\tau=0})}{\partial \tau} &= E \left[ \left( \bar{R} \eta \zeta_t \varrho_{t+1} \right)^{-1} \cdot \left( (1 + \lambda) \zeta_{t+1} - \bar{R} \eta \zeta_t \varrho_{t+1} \right) \right] \\ &= \frac{1 + \lambda}{\bar{R}} E Z_1 - 1.\end{aligned}$$

We also have that

$$Eu(c_{i,2,t+1}^{\tau=0}) = \ln(\bar{w}_t \bar{R}) + E \ln(\eta_{i,1,t} \zeta_t \varrho_{t+1}).$$

Therefore:

$$Eu(c_{i,2,t+1}^{\tau>0}) = \ln(\bar{w}_t \bar{R}) + E \ln(\eta_{i,1,t} \zeta_t \varrho_{t+1}) + \left( \frac{1 + \lambda}{\bar{R}} E Z_1 - 1 \right) d\tau.$$

For  $g_c^{PE}$  we accordingly get

$$1 + g_c^{PE} = \exp \left( \left( \frac{1 + \lambda}{\bar{R}} E Z_1 - 1 \right) d\tau \right). \quad (16)$$

Approximating the above in logs we get:

$$g_c^{PE} \approx \left( \frac{1 + \lambda}{\bar{R}} \exp(\sigma_{\ln \eta}^2 + \sigma_{\ln AR}^2) - 1 \right) d\tau$$

which is the same as equation (15) for  $\theta = 1$ .

4. Finally, observe from (15) that  $g_c^{PE}(AR, 0) \approx \left( \frac{1 + \lambda}{\bar{R}} \exp(\theta \sigma_{\ln AR}^2) - 1 \right) d\tau$  and  $g_c^{PE}(0, IR) \approx \left( \frac{1 + \lambda}{\bar{R}} \exp(\theta \sigma_{\ln \eta}^2) - 1 \right) d\tau$  so that

$$g_c^{PE}(AR, 0) + g_c^{PE}(0, IR) \approx \left( \frac{1 + \lambda}{\bar{R}} \left( \exp(\theta \sigma_{\ln AR}^2) + \exp(\theta \sigma_{\ln \eta}^2) \right) - 2 \right) d\tau.$$

Hence, the inequality  $g_c^{PE}(AR, IR) \geq g_c^{PE}(AR, 0) + g_c^{PE}(0, IR)$  is equivalent to the



requirement that

$$\begin{aligned}
& \exp(\theta\sigma_{\ln AR}^2) \exp(\theta\sigma_{\ln \eta}^2) - 1 \geq \exp(\theta\sigma_{\ln AR}^2) + \exp(\theta\sigma_{\ln \eta}^2) - 2 \\
\Leftrightarrow & \exp(\theta\sigma_{\ln AR}^2) (\exp(\theta\sigma_{\ln \eta}^2) - 1) \geq \exp(\theta\sigma_{\ln \eta}^2) - 1 \\
\Leftrightarrow & \begin{cases} \exp(\theta\sigma_{\ln AR}^2) \geq 1 \text{ if } \sigma_{\ln \eta}^2 > 0 \\ \exp(\theta\sigma_{\ln \eta}^2) \geq 1 \text{ if } \sigma_{\ln AR}^2 > 0. \end{cases}
\end{aligned}$$

Hence, the inequality is strict for  $\sigma_{\ln \eta}^2 > 0 \wedge \sigma_{\ln AR}^2 > 0$ .

□

*Proof of Proposition 2.* Observe that  $E[Z_1]$  is now given by

$$E[Z_1] = \frac{1}{2} E[\varrho_{t+1}^{-1}] \left( \frac{1}{\zeta_-} E[\eta_l^{-1}] + \frac{1}{\zeta_+} E[\eta_h^{-1}] \right)$$

which, using the distributional assumptions of log-normality, can be rewritten as

$$E[Z_1] = \frac{1}{2} \exp(\sigma_{\ln \varrho}^2) \left( \frac{1}{\zeta_-} \exp(\sigma_{\ln \eta_l}^2) + \frac{1}{\zeta_+} \exp(\sigma_{\ln \eta_h}^2) \right)$$

Using this in equation (16) gives (5a). Equation (5b) then follows from applying Definition 2 to the above. □

## Supplementary Appendix (Not for Publication)

### B Supplementary Analytical Appendix

#### B.1 Two-Generations Model: Convexity of Welfare Gain

**Proposition 3.** *Applying Definition 1 to equation (4) gives the properties of the components of the CEV as follows:*

$$\frac{\partial dg_c^{PE}(i)}{\partial \theta} > 0, \text{ for } i \in \{AR, IR\}, \quad \frac{\partial \Delta_{CWG}}{\partial \theta} > 0, \quad \frac{\partial \frac{\Delta_{CWG}}{dg_c^{PE}(AR)}}{\partial \theta} > 0.$$

*Proof.* The proof consists of three steps. First, we translate the CEV as a function of variances of random variables in logs into the respective terms in levels. Second, we combine the general definition (1) with the formula for the CEV in equation (4) to write the respective terms for  $g_c(0, 0)$ ,  $dg_c(AR)$  and so forth. Third, we derive the partial derivatives.

1. By log-normality we have  $\exp\left(\theta\left(\sigma_{\ln \eta}^2 + \sigma_{\ln AR}^2\right)\right) = \left((1 + \sigma_{\eta}^2)(1 + \sigma_{AR}^2)\right)^\theta$ , where  $\sigma_{AR} \equiv \sqrt{\sigma_{\zeta}^2 + \sigma_{\varrho}^2 + \sigma_{\zeta}^2 \sigma_{\varrho}^2}$  and therefore

$$g_c^{PE} = \left(\frac{1 + \lambda}{\bar{R}} \left((1 + \sigma_{\eta}^2)(1 + \sigma_{AR}^2)\right)^\theta - 1\right) d\tau \quad (17)$$

which encompasses the case  $\theta = 1$  shown in Section 2.2.

2. Applying definition (1) to (17) readily gives

$$\begin{aligned} g_c^{PE}(AR, IR) &= \left(\frac{1 + g}{\bar{R}} \left((1 + \sigma_{\eta}^2)(1 + \sigma_{AR}^2)\right)^\theta - 1\right) d\tau \\ g_c^{PE}(0, 0) &= \left(\frac{1 + g}{\bar{R}} - 1\right) d\tau \\ g_c^{PE}(AR, 0) &= \left(\frac{1 + g}{\bar{R}} (1 + \sigma_{AR}^2)^\theta - 1\right) d\tau \Leftrightarrow dg_c(AR) = \frac{1 + g}{\bar{R}} \left((1 + \sigma_{AR}^2)^\theta - 1\right) d\tau \\ g_c^{PE}(0, IR) &= \left(\frac{1 + g}{\bar{R}} (1 + \sigma_{\eta}^2)^\theta - 1\right) d\tau \Leftrightarrow dg_c(IR) = \frac{1 + g}{\bar{R}} \left((1 + \sigma_{\eta}^2)^\theta - 1\right) d\tau \end{aligned}$$

We observe that  $dg_c(AR)$  and  $dg_c(IR)$  are both increasing in  $\theta$ . From these terms we

further get

$$\begin{aligned}\Delta_{CWG} &= g_c^{PE}(AR, IR) - \left( g_c^{PE}(0, 0) + dg_c(AR) + dg_c(IR) \right) \\ &= \frac{1+g}{\bar{R}} \left( \left( (1+\sigma_\eta^2)(1+\sigma_{AR}^2) \right)^\theta - 1 \right. \\ &\quad \left. - \left( (1+\sigma_{AR}^2)^\theta - 1 \right) - \left( (1+\sigma_\eta^2)^\theta - 1 \right) \right) d\tau \\ \Big|_{\theta=1} &= \frac{1+g}{\bar{R}} \sigma_\eta^2 \sigma_{AR}^2\end{aligned}$$

and readily observe that  $\Delta_{CWG}$  is increasing in  $\sigma_\eta$  as well as  $\sigma_{AR}$ .

3. To establish that  $\Delta_{CWG}$  is also increasing in  $\theta$ , simplify notation by defining  $\sigma_{TR} = \sqrt{\sigma_\eta^2 + \sigma_{AR}^2 + \sigma_\eta^2 \sigma_{AR}^2}$  where  $TR$  stands in for “total risk”. Using this notation, observe that

$$\begin{aligned}\frac{\partial \Delta_{CWG}}{\partial \theta} &= \frac{1+g}{\bar{R}} \left( \ln(1+\sigma_{TR}^2)(1+\sigma_{TR}^2)^\theta - \right. \\ &\quad \left. \ln(1+\sigma_{AR}^2)(1+\sigma_{AR}^2)^\theta - \ln(1+\sigma_\eta^2)(1+\sigma_\eta^2)^\theta \right)\end{aligned}$$

Evaluate this at  $\theta = 1$  to get

$$\begin{aligned}\frac{\partial \Delta_{CWG}}{\partial \theta} \Big|_{\theta=1} &= \frac{1+g}{\bar{R}} \left( \sigma_{AR}^2 \cdot \ln \left( \frac{1+\sigma_{TR}^2}{1+\sigma_{AR}^2} \right) + \sigma_\eta^2 \cdot \ln \left( \frac{1+\sigma_{TR}^2}{1+\sigma_\eta^2} \right) + \right. \\ &\quad \left. \ln(1+\sigma_{TR}^2) \cdot \sigma_{AR}^2 \cdot \sigma_\eta^2 \right) > 0.\end{aligned}$$

The general conclusion that  $\frac{\partial \Delta_{CWG}}{\partial \theta} > 0$  for all  $\theta$  then follows from continuity. Finally, we can express the contribution to the CEV of  $CWG$  relative to  $AR$  as

$$\frac{d\Delta_{CWG}}{dg_c(\sigma_{AR})} = \frac{\left( (1+\sigma_\eta^2)(1+\sigma_{AR}^2) \right)^\theta}{(1+\sigma_{AR}^2)^\theta - 1} - \frac{1}{(1+\sigma_{AR}^2)^\theta - 1} - 1 - \frac{(1+\sigma_\eta^2)^\theta - 1}{(1+\sigma_{AR}^2)^\theta - 1}$$

Take the derivative of this term w.r.t.  $\theta$  to get

$$\begin{aligned} \frac{\partial \frac{d\Delta_{CWG}}{dg_c(\sigma_{AR})}}{\partial \theta} &= \frac{1}{\left((1 + \sigma_{AR}^2)^\theta - 1\right)^2} \left( \ln \left( (1 + \sigma_\eta^2) (1 + \sigma_{AR}^2) \right) \left( (1 + \sigma_\eta^2) (1 + \sigma_{AR}^2) \right)^\theta \right. \\ &\quad \left( (1 + \sigma_{AR}^2)^\theta - 1 \right) - \left( (1 + \sigma_\eta^2) (1 + \sigma_{AR}^2) \right)^\theta \ln (1 + \sigma_{AR}^2) (1 + \sigma_{AR}^2)^\theta \\ &\quad + \ln (1 + \sigma_{AR}^2) (1 + \sigma_{AR}^2)^\theta \\ &\quad - \ln (1 + \sigma_\eta^2) (1 + \sigma_\eta^2)^\theta \left( (1 + \sigma_{AR}^2)^\theta - 1 \right) \\ &\quad \left. + \left( (1 + \sigma_\eta^2)^\theta - 1 \right) \ln (1 + \sigma_{AR}^2) (1 + \sigma_{AR}^2)^\theta \right). \end{aligned}$$

Evaluated at  $\theta = 1$  we get

$$\begin{aligned} \left. \frac{\partial \frac{d\Delta_{CWG}}{dg_c(\sigma_{AR})}}{\partial \theta} \right|_{\theta=1} &= \frac{1}{(\sigma_{AR}^2)^2} \left( \ln \left( (1 + \sigma_\eta^2) (1 + \sigma_{AR}^2) \right) (1 + \sigma_\eta^2) (1 + \sigma_{AR}^2) \sigma_{AR}^2 \right. \\ &\quad - (1 + \sigma_\eta^2) (1 + \sigma_{AR}^2) \ln (1 + \sigma_{AR}^2) (1 + \sigma_{AR}^2) + \ln (1 + \sigma_{AR}^2) (1 + \sigma_{AR}^2) \\ &\quad \left. - \ln (1 + \sigma_\eta^2) (1 + \sigma_\eta^2) \sigma_{AR}^2 + \sigma_\eta^2 \ln (1 + \sigma_{AR}^2) (1 + \sigma_{AR}^2) \right) \end{aligned}$$

Now split the numerator up as follows:

$$\begin{aligned} N &\equiv \underbrace{\ln (1 + \sigma_{TR}^2) (1 + \sigma_{TR}^2) \sigma_{AR}^2 - (1 + \sigma_{TR}^2) \ln (1 + \sigma_{AR}^2) (1 + \sigma_{AR}^2)}_{\equiv \Psi_1} \\ &\quad + \underbrace{\ln (1 + \sigma_{AR}^2) (1 + \sigma_{AR}^2) - \ln (1 + \sigma_\eta^2) (1 + \sigma_\eta^2) \sigma_{AR}^2 + \sigma_\eta^2 \ln (1 + \sigma_{AR}^2) (1 + \sigma_{AR}^2)}_{\equiv \Psi_2} \end{aligned}$$

where  $\sigma_{TR}^2$  is again the variance due to total risk. Next, notice that:

$$\begin{aligned} \Psi_1 &= (1 + \sigma_{TR}^2) \left( \sigma_{AR}^2 \ln(1 + \sigma_\eta^2) - \ln(1 + \sigma_{AR}^2) \right) \\ \Psi_2 &= (1 + \sigma_{TR}^2) \ln (1 + \sigma_{AR}^2) - \sigma_{AR}^2 (1 + \sigma_\eta^2) \ln (1 + \sigma_\eta^2) \\ \Psi_1 + \Psi_2 &= (\sigma_{AR}^2)^2 \ln(1 + \sigma_\eta^2) (1 + \sigma_\eta^2). \end{aligned}$$

Therefore  $\left. \frac{\partial \frac{d\Delta_{CWG}}{dg_c(\sigma_{AR})}}{\partial \theta} \right|_{\theta=1} > 0$  and the conclusion for general  $\theta$  again follows by continuity. □

## B.2 General Equilibrium Extension of the Simple Model

We sketch the main elements, provide and extend the key findings of Harenberg and Ludwig (2015) with the purpose (i) to show that the discount rate plays an additional decisive role for evaluating the welfare effects of social security and (ii) to expose analytically one channel for the ambiguity of risk interactions on the welfare consequences from crowding out of capital. To this end, consider a two period extension of the model of Section 2 in general equilibrium. In the first period, households are endowed with one unit of labor productivity and work full time. In the second period, they only work fraction  $\omega \in [0, 1)$  of their time and are retired with fraction  $1 - \omega$ . We also assume that the idiosyncratic shock only hits in the second period of life. The subperiod structure together with the assumption that the shock only hits in the second period enables us to model precautionary savings behavior while maintaining closed form solutions in general equilibrium when we focus on a logarithmic utility function and Cobb-Douglas production.<sup>45</sup>

### B.2.1 Modifications

The modifications of the simple model of Section 2 are threefold. First, denoting the discount factor by  $\beta$ , expected life-time utility is  $E_t [\ln(c_{1,t}) + \beta \ln(c_{2,t+1})]$ , where we restrict the analysis to log utility for analytical solutions. Second, the budget constraints in the two periods now write as

$$s_{2,t+1} + c_{1,t} = (1 - \tau_t)w_t \quad \text{and} \quad c_{i,2,t+1} \leq s_{2,t+1}R_{t+1} + \omega(1 - \tau)w_{t+1}\eta_{i,2,t+1} + (1 - \omega)y_{t+1}^{ss}.$$

Third, to close the model in general equilibrium, production takes place with a representative firm's production function  $F(K_t, \Upsilon_t L_t)$ , where  $K_t$  is aggregate capital,  $L_t = N_{t,0} + \omega N_{t,1}$  is aggregate labor, and  $\Upsilon_t$  is labor augmenting technological progress, growing at exogenous rate  $\lambda$ . Next, introduce shock  $\zeta_t$  as a standard RBC shock to output and shock  $\varrho_t$  as a shock to the user costs of capital and assume 100% depreciation (again for analytical reasons) so that profits are given by  $\Pi_t = \zeta_t F(K_t, \Upsilon_t L_t) - (1 + r_t) \varrho_t^{-1} K_t - w_t L_t$ . Denoting by  $k_t = \frac{K_t}{\Upsilon_t L_t}$  the capital stock per efficiency unit (the capital "intensity"), profit maximization of this neoclassical firm

---

<sup>45</sup>Because of the human capital wealth effect from second period income, we cannot derive closed form solutions even with logarithmic utility in partial equilibrium. Analytical tractability only arises when plugging in the general equilibrium dynamics into the first-order conditions of households, also see Krueger and Ludwig (2007), Ludwig and Vogel (2009), and Krueger and Ludwig (2017). Formally, the reason is that second period income is discounted with the market interest rate and both income and the interest rate are functions of capital in general equilibrium.

then gives the first order conditions as

$$w_t = (1 - \alpha)\Upsilon_t k_t^\alpha \zeta_t = \bar{w}_t \zeta_t \quad \text{and} \quad R_t = \alpha k_t^{\alpha-1} \zeta_t \varrho_t = \bar{R}_t \zeta_t \varrho_t.$$

which is the general equilibrium analogue to equation (2).

### B.2.2 Analysis

In general equilibrium, the law of motion of the capital intensity writes as

$$k_{t+1} = \frac{1}{(1 + \lambda)(1 + \omega)} s(\tau)(1 - \tau)(1 - \alpha)\zeta_t k_t^\alpha \quad (18)$$

$$\text{where} \quad s(\tau) = \frac{\beta\Gamma(\tau)}{1 + \beta\Gamma(\tau)} \leq \frac{\beta}{1 + \beta} \quad (19)$$

$$\text{and} \quad \Gamma(\tau) = \mathbb{E}_t \left[ \frac{1}{1 + \frac{1-\alpha}{\alpha(1+\omega)\varrho_{t+1}} (\omega\eta_{i,2,t+1} + \tau(1 + \omega(1 - \eta_{i,2,t+1})))} \right] \leq 1. \quad (20)$$

cf. Proposition 3 in Harenberg and Ludwig (2015). To interpret this, notice that in a deterministic economy (where  $\zeta_t = \varrho_t = \eta_t = 1$ ) without work effort in the second period ( $\omega = 0$ ) equation (18) is just the standard textbook variant of the law of motion of the capital intensity in a 2-period OLG economy with logarithmic utility and Cobb-Douglas production. The risk adjustment term  $\Gamma(\tau)$  in equation (19) captures (i) precautionary saving behavior w.r.t. idiosyncratic income risk— $s(\tau)$  increases in response to a mean-preserving spread of  $\eta$ ; (ii) intertemporal reallocation w.r.t. return risk— $s(\tau)$  decreases in response to a mean-preserving spread of  $\varrho$  because savings become less attractive if the return risk increases; and (iii) crowding out of savings—the saving rate decreases when  $\tau$  is increased. In the deterministic economy, life-cycle savings are reduced due to positive retirement income in the second period. In the stochastic economy, there is additional partial insurance of idiosyncratic risk through social security which decreases precautionary savings.

Based on this structure, Proposition 4 in Harenberg and Ludwig (2015) contains the main results on the welfare benefits from insurance and the welfare costs from crowding out in terms of utility units. Our next proposition extends those results to a consumption equivalent variation:

**Proposition 4.** *The CEV from a marginal introduction of social security at rate  $d\tau > 0$  in the*

stationary equilibrium is given by  $g_c^{GE} = g_c^{PE} + g_c^{CO}$ , where

$$g_c^{PE} \approx \frac{1}{1+\beta} \left( \beta \mathbb{E} \left[ \frac{\frac{1-\alpha}{\alpha} \frac{1}{\varrho_{t+1}} - \frac{1-\alpha}{\alpha} \frac{\omega}{1+\omega} \frac{\eta_{i,2,t+1}}{\varrho_{t+1}} - 1}{1 + \frac{(1-\alpha)\omega}{\alpha} \frac{\eta_{i,2,t+1}}{1+\omega} \frac{1}{\varrho_{t+1}}} \right] - 1 \right) \quad (21)$$

$$g_c^{CO} \approx -\frac{1}{1+\beta} \left( (1 - \varphi_{s,\tau}|_{\tau=0}) \left( \frac{\alpha}{1-\alpha} \frac{1+\beta}{\beta} - \Gamma|_{\tau=0} \right) \right). \quad (22)$$

$\Gamma|_{\tau=0}$  is term (20) for  $\tau = 0$  and  $\varphi_{s,\tau}|_{\tau=0}$  is the semi-elasticity of the saving rate w.r.t.  $\tau$ ,  $\varphi_{s,\tau} = \frac{\partial s/s}{\partial \tau}$ , again evaluated at  $\tau = 0$ .

*Proof of Proposition 4.* 1. Harenberg and Ludwig (2015), Proposition 4, show that social security increases ex-ante welfare in the stationary equilibrium if and only if

$$A + B > 0$$

$$\text{where } A \equiv \beta \mathbb{E} \left[ \frac{\frac{1-\alpha}{\alpha} \frac{1}{\varrho_{t+1}} - \frac{1-\alpha}{\alpha} \frac{\omega}{1+\omega} \frac{\eta_{i,2,t+1}}{\varrho_{t+1}} - 1}{1 + \frac{(1-\alpha)\omega}{\alpha} \frac{\eta_{i,2,t+1}}{1+\omega} \frac{1}{\varrho_{t+1}}} \right] - 1$$

$$\text{and } B \equiv -\beta (1 - \varphi_{s,\tau}|_{\tau=0}) \left( \frac{\alpha}{1-\alpha} \frac{1+\beta}{\beta} - \Gamma|_{\tau=0} \right).$$

2. To translate this into a CEV, observe that  $EU(C^{\tau>0}) = EU(C^{\tau=0}) + (A+B) d\tau$ , where capital letter  $U$  denotes life-time utility and capital letter  $C$  the consumption stream under the respective policy, hence  $U(C^{\tau=0}) = \ln(c_{t,1}) + \beta \ln(c_{i,t+1,2})$ . Using the definition of the CEV it is then straightforward to show that the CEV is given by  $g_c = \exp\left(\left(\frac{A+B}{1+\beta}\right) d\tau\right) - 1$ , which can be approximated as  $g_c \approx \frac{A+B}{1+\beta} d\tau$ . □

Observe that  $g_c^{PE}$  is the CEV in “partial equilibrium”, corresponding to our definition in the quantitative model of Section 3. It is the consumption equivalent variation from a marginal introduction of social security where the price sequence is held constant (i.e., does not react to social security) at the corresponding price sequence in the general equilibrium of the  $\tau = 0$  economy.  $g_c^{CO}$  is the additional effect from the crowding out of capital induced by the social security reform.<sup>46</sup>

To interpret terms  $g_c^{PE}$  and  $g_c^{CO}$  further, we first restate as observation important findings from Harenberg and Ludwig (2015) and then move on to a Corollary:

<sup>46</sup>Also notice that the aggregate productivity shock  $\zeta_t$  does not show up in (21) and (22). This is a consequence of (i)  $\zeta_t$  showing up in all sources of income—wages, interest rates, and social security—, (ii)  $\zeta_t$  showing up multiplicatively in the law of motion of the aggregate capital stock, and (iii)  $\zeta_t$  also showing up multiplicatively in the derivatives of the utility function by the assumption of log utility. All these cancel each other out.

**Observation 1.** 1. The deterministic  $\omega = 0$  economy is dynamically efficient if and only if

$$\frac{\alpha}{1 + \alpha} \frac{1 + \beta}{\beta} > 1. \quad (23)$$

2. If condition (23) holds then  $g_c^{GE} < 0$  in the corresponding stochastic economy with  $0 \leq \omega < 1$ .

3. With respect to term  $g_c^{PE}$  we find that  $\frac{\partial g_c^{PE}}{\partial \sigma_\eta^2} > 0$ ,  $\frac{\partial g_c^{PE}}{\partial \sigma_\eta^2} > 0$ ,  $\frac{\partial^2 g_c^{PE}}{\partial \sigma_\eta^2 \partial \sigma_\eta^2} > 0$ .

4. With respect to term  $g_c^{CO}$  we find that  $\frac{\partial \Gamma(\tau)|_{\tau=0}}{\partial \sigma_\eta^2} < 0$ ,  $\frac{\partial^2 \Gamma(\tau)|_{\tau=0}}{\partial \sigma_\eta^2 \partial \sigma_\eta^2} < 0$  and, under condition (23),  $\frac{\partial g_c^{CO}}{\partial \sigma_\eta^2} < 0$ ,  $\frac{\partial g_c^{CO}}{\partial \sigma_\eta^2} \leq 0$ ,  $\frac{\partial^2 g_c^{CO}}{\partial \sigma_\eta^2 \partial \sigma_\eta^2} \leq 0$ .

Observation 1.3 confirms our findings from Section 2. Importantly, Observation 1.4 states that the direction of the change of the welfare costs from crowding out when idiosyncratic risk increases (and its interaction with aggregate risk) is ambiguous. To understand this finding, suppose first that crowding out increases in risk, i.e., that  $\varphi_{s,\tau}|_{\tau=0}$  becomes more strongly negative as risk goes up.<sup>47</sup> This means that precautionary savings decrease more strongly if the amount of risk insured increases. Under this assumption, the effects of an increase of idiosyncratic risk on the costs of crowding out (as well as the cross partial w.r.t. aggregate return risk) are ambiguous. The formal reason for this finding is that  $\Gamma|_{\tau=0}$  increases in idiosyncratic risk, so that the welfare costs of crowding out are less strong. Intuitively, while crowding out reduces the mean capital stock thereby leading to welfare losses (as in a deterministic dynamically efficient economy) a lower capital stock also reduces the exposure to idiosyncratic wage risk because wages depend positively on the capital stock and are multiplicative in the shock, an effect which is (at least partially) offsetting the utility consequences of crowding out.

The next corollary on the importance of the discount factor—which is not contained in Harenberg and Ludwig (2015)—provides further guidance for the calibration of our quantitative model and the interpretation of its results:

**Corollary 1.** *The CEVs from a marginal introduction of social security in partial equilibrium,  $g_c^{PE}$ , and, under condition (23), also in general equilibrium,  $g_c^{GE}$ , increase in  $\beta$ .*

<sup>47</sup>It is not possible to show this analytically, but it is the most plausible case. It is also found to hold in the numerical analysis in Harenberg and Ludwig (2015).



*Proof of Corollary 1.* Rewrite  $g_c^{PE}$  and  $g_c^{CO}$  as

$$g_c^{PE} \approx \frac{1}{1 + \frac{1}{\beta}} \mathbb{E} \left[ \frac{\frac{1-\alpha}{\alpha} \frac{1}{\varrho_{t+1}} - \frac{1-\alpha}{\alpha} \frac{\omega}{1+\omega} \frac{\eta_{i,2,t+1}}{\varrho_{t+1}} - 1}{1 + \frac{(1-\alpha)}{\alpha} \frac{\omega}{1+\omega} \frac{\eta_{i,2,t+1}}{\varrho_{t+1}}} \right] - \frac{1}{1 + \beta}$$

$$g_c^{CO} \approx - \left( 1 - \varphi_{s,\tau} |_{\tau=0} \right) \left( \frac{\alpha}{1-\alpha} - \frac{1}{1 + \frac{1}{\beta}} \Gamma |_{\tau=0} \right)$$

and the result immediately follows.  $\square$

The intuition for the effect of discounting on the  $g_c^{PE}$  is that households value insurance of second period consumption more when  $\beta$  is increased. As to the intuition for  $g_c^{CO}$ , observe that increasing  $\beta$  lowers the welfare costs from crowding out—i.e., the distance  $\frac{\alpha}{1-\alpha} - \frac{1}{1+\frac{1}{\beta}} \Gamma |_{\tau=0} > 0$  decreases towards zero—, because increasing  $\beta$  increases life-cycle savings which moves the economy closer to the boundary of dynamic inefficiency.

### B.3 Definition of Recursive Markov Equilibrium

We here provide a formal definition of a competitive recursive Markov equilibrium, cf. Section 3.5. To this end, we define a state space that is sufficient for solving the households' maximization problem. Let  $\mathcal{E} = \{e_1, e_2, \dots, e_{max}\}$  and  $\mathcal{J} = \{1, 2, \dots, J\}$ , and let  $\mathcal{M}$  be a sigma-algebra over  $\{[\underline{s}, \bar{s}] \times [\underline{b}, \bar{b}] \times \mathcal{E} \times \mathcal{J}\}$ , where  $\underline{s}$ ,  $\bar{s}$ ,  $\underline{b}$ , and  $\bar{b}$  are the infimum and supremum on stock and bond holdings.<sup>48</sup> The measure  $\Phi$  is defined over  $\mathcal{M}$ , and the set of all such measures is denoted by  $\mathcal{Q}$ . We follow the applied literature and define the state space to consist of  $\Phi$ , the current idiosyncratic state  $(\tilde{s}, \tilde{b}, e)$ , and the current aggregate shock  $z$ . As a recursive equilibrium does not depend on the date-event, we drop time index  $t$  and use a prime for next period's variables. Finally, note that the economic dependency ratio,  $p = \frac{P(z^t)}{L(z^t)} = \frac{\sum_{j=j_r}^J (1+n)^{J-j}}{\sum_{j=1}^{j_r-1} (1+n)^{J-j} \epsilon_j}$ , and the labor-to-population ratio,  $\ell = \frac{L(z^t)}{N(z^t)} = \frac{\sum_{j=1}^{j_r-1} (1+n)^{J-j} \epsilon_j}{\sum_{j=1}^J (1+n)^{J-j}}$ , are both constant over time.

**Definition 4.** For any initial  $(z_0, \Phi_0) \in \mathcal{Z} \times \mathcal{Q}$ , a recursive competitive equilibrium consists of a measure  $\Phi$ , measurable functions for household choices  $\{\tilde{c}_j(\tilde{s}, \tilde{b}, e; \Phi, z), \tilde{s}'_j(\tilde{s}, \tilde{b}, e; \Phi, z), \tilde{b}'_j(\tilde{s}, \tilde{b}, e; \Phi, z)\}$  and an associated value function  $\tilde{v}_j(\tilde{s}, \tilde{b}, e; \Phi, z)$ , firm choices  $k(\Phi, z)$ , social security settings  $\{\tau, \tilde{y}^{ss}(\Phi, z)\}$ , factor prices  $\{\tilde{w}(\Phi, z), r(\Phi, z)\}$ , asset returns  $\{r_b(\Phi), r_s(\Phi, z)\}$ , and a law of motion  $H(\Phi, z, z')$  such that:

<sup>48</sup>For a given level of aggregate capital and a given equity premium, the infimum and supremum on bond and stock holdings are implied by the bounds on the income process and the fact that households can't hold negative positions in the asset when they die, see Section 3.2. In equilibrium, aggregate capital and the equity premium will be bounded, and the infimum and supremum can be calculated for those bounded intervals.

a) given functions for prices and returns and the law of motion, the households' policy functions  $\{\tilde{c}_j(\tilde{s}, \tilde{b}, e; \Phi, z), \tilde{s}'_j(\tilde{s}, \tilde{b}, e; \Phi, z), \tilde{b}'_j(\tilde{s}, \tilde{b}, e; \Phi, z)\}$  solve

$$\begin{aligned} & \tilde{v}_j(\tilde{s}, \tilde{b}, e; \Phi, z) \\ &= \max_{\tilde{c} > 0, \tilde{s}', \tilde{b}'} \begin{cases} \left( \tilde{c}^{\frac{1-\theta}{\gamma}} + \tilde{\beta} \left( \sum_{z'} \sum_{e'} \pi_z(z'|z) \pi_e(e'|e) \tilde{v}_{j+1}^{1-\theta}(\tilde{s}', \tilde{b}', e'; H(\Phi, z), z') \right)^{\frac{1}{\gamma}} \right)^{\frac{\gamma}{1-\theta}} \\ \tilde{c} & \text{if } j = J \end{cases} \\ & \text{s. t.} \quad \tilde{c} + \tilde{s}' + \tilde{b}' = (1 + r_s(\Phi, z))\tilde{s} + (1 + r_b(\Phi))\tilde{b} \\ & \quad \quad \quad + (1 - \tau)\tilde{y}_j(e, \Phi, z)I(j) + \tilde{y}^{ss}(\Phi, z)(1 - I(j)), \\ & \quad \quad \quad \tilde{y}_j(e, \Phi, z) = \tilde{w}(\Phi, z)\epsilon_j\eta(e, z), \\ & \quad \quad \quad \tilde{s}' + \tilde{b}' \geq 0 \quad \quad \quad \text{if } j = J, \end{aligned} \tag{24}$$

where  $\tilde{\beta} = \beta(1 + \lambda)^{\frac{1-\theta}{\gamma}}$ ,

b) functions for prices and for firm choices are related by

$$\begin{aligned} \tilde{w}(\Phi, z) &= (1 - \alpha)\zeta(z)k(\Phi, z)^\alpha, \\ r(\Phi, z) &= \alpha\zeta(z)k(\Phi, z)^{\alpha-1} - \delta(z), \end{aligned}$$

c) functions for asset returns are given by

$$\begin{aligned} r_b(\Phi) &= \frac{1}{\bar{\kappa}_f} E[r(\Phi, z)(1 + \bar{\kappa}_f) - r_s(\Phi, z)], \\ r_s(\Phi, z) &= r(\Phi, z)(1 + \bar{\kappa}_f) - \bar{\kappa}_f r_b(\Phi), \end{aligned}$$

d) the pension system budget constraint holds, i.e.,

$$\tau\tilde{w}(\Phi, z) = \tilde{y}^{ss}(\Phi, z)p, \tag{25}$$

where  $p$  is the economic dependency ratio defined above,

e) all markets clear:

$$\begin{aligned} \zeta(z)k(\Phi, z)^\alpha + (1 - \delta(z))k(\Phi, z) &= \frac{1}{\ell} \sum_{j=1}^J \sum_e \int_{\tilde{b}} \int_{\tilde{s}} \tilde{c}_j(\tilde{s}, \tilde{b}, e; \Phi, z) \Phi(\tilde{s}, \tilde{b}, e, j) d\tilde{b} d\tilde{s} \\ &\quad + k(H(\Phi, z), z')(1 + \lambda)(1 + n), \\ k(H(\Phi, z), z')(1 + \lambda)(1 + n) &= \frac{1}{\ell} \sum_{j=1}^J \sum_e \int_{\tilde{b}} \int_{\tilde{s}} (\tilde{s}'_j(\tilde{s}, \tilde{b}, e; \Phi, z) \\ &\quad + \tilde{b}'_j(\tilde{s}, \tilde{b}, e; \Phi, z)) \Phi(\tilde{s}, \tilde{b}, e, j) d\tilde{b} d\tilde{s}, \\ \frac{k(H(\Phi, z), z')(1 + \lambda)(1 + n)}{(1 + \bar{\kappa}_f)} &= \frac{1}{\ell} \sum_{j=1}^J \sum_e \int_{\tilde{b}} \int_{\tilde{s}} \tilde{s}'_j(\tilde{s}, \tilde{b}, e; \Phi, z) \Phi(\tilde{s}, \tilde{b}, e, j) d\tilde{b} d\tilde{s}, \end{aligned}$$

and by Walras' Law, the bond market also clears,

f) the law of motion  $H$  is generated by the policy functions and the Markov transition matrix  $\pi_e$ , so that

$$\Phi' = H(\Phi, z, z')$$

with the initialization at  $j = 1$  of  $\tilde{s} = \tilde{b} = 0$ .

#### B.4 Corollary: CEV in a Deterministic Economy, $g_c^{PE}(0, 0)$

For an economy with an arbitrary number of generations  $J$ , we can provide a closed-form solution for  $g_c$  for an economy without risk. Following the discussion in Section 2.2, we denote the consumption equivalent variation in an economy without risk by  $g_c^{PE}(0, 0)$ .

**Corollary 2.** Denote by  $pvi_1^A$  ( $pvi_1^B$ ) the present discounted value of lifetime income in policy  $A$  ( $B$ ). The consumption equivalent variation in the partial equilibrium of the risk-free economy is given by

$$g_c^{PE}(0, 0) = \frac{\tilde{u}_1(\tilde{c}^B)}{\tilde{u}_1(\tilde{c}^A)} - 1 = \frac{\widetilde{pvi}_1^B}{\widetilde{pvi}_1^A} - 1,$$

i.e., it is not affected by preference parameters.

*Proof.* The property follows from linearity of consumption policy functions in initial wealth which we first establish. We again simplify notation and drop the  $i$  and  $t$  indices. Recursive

substitution from  $j = J, \dots, 1$ , using that  $\tilde{u}_J = \tilde{c}_J$  gives

$$\tilde{u}_1 = \left[ \sum_{j=1}^J \tilde{\beta}^{j-1} \tilde{c}_j^{\frac{1-\theta}{\gamma}} \right]^{\frac{\gamma}{1-\theta}},$$

where  $\tilde{\beta} = \beta(1 + \lambda)^{\frac{1-\theta}{\gamma}}$ . As for the resource constraint, write

$$\sum_{j=1}^J \tilde{y}_j \left( \frac{1}{1+r} \right)^{j-1} - \sum_{j=1}^J \tilde{c}_j \left( \frac{1}{1+r} \right)^{j-1} \geq 0,$$

where, in slight abuse of notation, we use  $\tilde{y}_j$  to denote labor income during the working period and retirement income thereafter (see main text).

The Lagrangian writes as

$$\mathcal{L} = \left[ \sum_{j=1}^J \beta^{j-1} \tilde{c}_j^{\frac{1-\theta}{\gamma}} \right]^{\frac{\gamma}{1-\theta}} + \lambda \left( \tilde{a}_1 + \sum_{j=1}^J \tilde{y}_j \left( \frac{1}{1+r} \right)^{j-1} - \sum_{j=1}^J \tilde{c}_j \left( \frac{1}{1+r} \right)^{j-1} \right).$$

First-order conditions give:

$$\tilde{\beta}^{j-1} \frac{1-\theta}{\gamma} \tilde{c}_j^{\frac{1-\theta-\gamma}{\gamma}} - \tilde{\lambda} \left( \frac{1}{1+r} \right)^{j-1} = 0$$

where  $\tilde{\lambda} \equiv \lambda \left( \frac{\gamma}{1-\theta} \left[ \sum_{j=1}^J \tilde{\beta}^{j-1} \tilde{c}_j^{\frac{1-\theta}{\gamma}} \right]^{\frac{\gamma}{1-\theta}-1} \right)^{-1}$ . Using the FOC for any two ages  $j$  and  $j+1$  gives the standard Euler equation

$$\frac{\tilde{c}_{j+1}}{\tilde{c}_j} = (\tilde{\beta}(1+r))^{\frac{\gamma}{\theta+\gamma-1}} = (\tilde{\beta}(1+r))^{\psi}.$$

where  $\psi$  is the intertemporal elasticity of substitution. We consequently have

$$\frac{\tilde{c}_j}{\tilde{c}_1} = (\beta(1+r))^{\psi(j-1)}.$$

Using this in the resource constraint, which holds with equality in the optimum, and defining

by  $\widetilde{pvi}_1$  total (human) wealth of the household, we get

$$\begin{aligned}\widetilde{pvi}_1 &\equiv \sum_{j=1}^J \tilde{y}_j \left(\frac{1}{1+r}\right)^{j-1} = \tilde{c}_1 \sum_{j=1}^J \frac{\tilde{c}_j}{\tilde{c}_1} \left(\frac{1}{1+r}\right)^{j-1} \\ \Leftrightarrow \quad \widetilde{pvi}_1 &= \tilde{c}_1 \sum_{j=1}^J \left( \left( \tilde{\beta}(1+r) \right)^\psi \left( \frac{1}{1+r} \right) \right)^{j-1} = \tilde{c}_1 \sum_{j=1}^J b^{j-1} = \frac{1}{m_1} \tilde{c}_1\end{aligned}$$

where  $b \equiv \left( \tilde{\beta}(1+r) \right)^\psi \left( \frac{1}{1+r} \right)$  and  $m_1 \equiv \left( \sum_{j=1}^J b^{j-1} \right)^{-1}$  is the marginal propensity to consume out of initial wealth in period 1. We accordingly get, for any age  $j$ , that

$$\tilde{c}_j = m_j \widetilde{pvi}_1, \text{ where } m_j \equiv \tilde{\beta}(1+r)^{\psi(j-1)} m_1.$$

Using this in the utility function we get

$$\tilde{u}_1 = \left[ \sum_{j=1}^J \beta^{j-1} \left( m_j \widetilde{pvi}_1 \right)^{\frac{1-\theta}{\gamma}} \right]^{\frac{\gamma}{1-\theta}} = \left[ \sum_{j=1}^J \beta^{j-1} \left( m_j \right)^{\frac{1-\theta}{\gamma}} \right]^{\frac{\gamma}{1-\theta}} \widetilde{pvi}_1,$$

establishing linearity of the utility function in initial wealth.

Consequently, the CEV in partial equilibrium—where  $m_j$  does not change between any two policies  $A$  and  $B$  because it is only a function of the constant parameters  $r, \beta, \psi$ —is equal to the percentage change in wealth and given by

$$g_c^{PE} = \frac{\tilde{u}_1^A}{\tilde{u}_1^B} - 1 = \frac{\widetilde{pvi}_1^A}{\widetilde{pvi}_1^B} - 1.$$

□

## B.5 Additional Proofs

*Derivation of Equation (7).* Here, we derive the stock and bond return in the quantitative model. Recall that  $\bar{\kappa}_f$  is the exogenous and constant debt-equity ratio. First, we restate the relevant equation from Section 3.3,

$$K(z^t) = S(z^t) + B(z^t) = S(z^t)(1 + \bar{\kappa}_f), \quad (26)$$

where  $S$  and  $B$  denote the quantities of stock and bond, respectively. The return on capital then satisfies

$$r(z^t)K(z^t) = r(z^t)S(z^t)(1 + \bar{\kappa}_f).$$

The return on capital equals the standard first-order condition of the firm, as shown in equation (6b). Out of this total return on capital, bondholders receive

$$r_b(z^{t-1})B(z^t) = r_b(z^{t-1})\bar{\kappa}_f S(z^t),$$

where the bond return is determined one period ahead, since it is one-period risk-free. Stock holders receive the remainder, which is

$$r_s(z^t)S(z^t) = r(z^t)S(z^t)(1 + \bar{\kappa}_f) - r_b(z^{t-1})\bar{\kappa}_f S(z^t).$$

From the last equation, we immediately get (7). □

*Proof of Equation (8).* The property follows from homotheticity of Epstein-Zin preferences. To prove it we proceed by induction. We look at two alternative (expected) consumption streams  $\tilde{c}^A$  and  $\tilde{c}^B$ . One can think of them as optimal consumption under policy regime  $A$  and  $B$ . We ask how big the percentage increase of consumption stream  $\tilde{c}^A$  in each period has to be to reach the same utility level as reached for consumption stream  $\tilde{c}^B$ . For sake of simplicity we drop indices  $t$  and  $i$  and adopt the notation  $\tilde{u}_j^X \equiv \tilde{u}_j(c^X)$  for  $X \in \{A, B\}$ .

1. *Induction claim:* At each age  $j$  we have that

$$\tilde{u}_j^B = (1 + g_c)\tilde{u}_j^A.$$

2. *Induction start:* For our Epstein-Zin utility specification (cf. Section 3.2), at age  $J$  we have that

$$\tilde{u}_J^A = \tilde{c}_J^A \quad \text{and} \quad \tilde{u}_J^B = \tilde{c}_J^B.$$

Hence, by the induction claim, we get

$$\tilde{u}_J^B = (1 + g_c)\tilde{u}_J^A = (1 + g_c)\tilde{c}_J^A$$

and, correspondingly,

$$\begin{aligned}\tilde{u}_{J-1}^B &= \left[ (\tilde{c}_{J-1}^B)^{\frac{1-\theta}{\gamma}} + \beta \left( \mathbb{E}_{J-1}(\tilde{u}_J^B)^{1-\theta} \right)^{\frac{1}{\gamma}} \right]^{\frac{\gamma}{1-\theta}} \\ &= (1 + g_c) \tilde{u}_{J-1}^A.\end{aligned}$$

3. *Induction step:* Using the induction claim for any period  $j < J - 1$  we therefore have:

$$\begin{aligned}\tilde{u}_j^B &= \left[ (\tilde{c}_j^B)^{\frac{1-\theta}{\gamma}} + \beta \left( \mathbb{E}_j(\tilde{u}_{j+1}^B)^{1-\theta} \right)^{\frac{1}{\gamma}} \right]^{\frac{\gamma}{1-\theta}} \\ &= (1 + g_c) \tilde{u}_j^A.\end{aligned}$$

□

*Derivation of Equation (11).* Denote by  $\gamma_c = \frac{E[\tilde{C}^B | \tau=2\%]}{E[\tilde{C}^A | \tau=0\%]} = 1 + \Delta_c$  the consumption growth factor, where  $\Delta_c$  is the consumption growth rate. Take the definition of  $g_c$  and divide all individual consumption allocations,  $\tilde{c}^B$ , by the consumption growth factor so that mean consumption is the same in both economies. This gives the welfare benefits from changes of the distribution as

$$g_c^{distr} = \frac{E\left[\tilde{v}_1\left(\frac{1}{\gamma_c}\tilde{c}^B\right) \mid \tau = 2\%\right]}{E\left[\tilde{v}_1(\tilde{c}^A) \mid \tau = 0\%\right]} - 1 = \frac{1}{\gamma_c} \frac{E\left[\tilde{v}_1(\tilde{c}^B) \mid \tau = 2\%\right]}{E\left[\tilde{v}_1(\tilde{c}^A) \mid \tau = 0\%\right]} - 1 = \frac{1}{\gamma_c} (1 + g_c) - 1,$$

where the second equality follows from homotheticity. The  $g_c^{mean}$  is then given by

$$g_c^{mean} = g_c - g_c^{distr} = \frac{1 + g_c}{\gamma_c} \Delta_c.$$

□

## C Supplementary Computational Appendix

### C.1 Overview

The numerical solution follows Krusell and Smith (1997, 1998), Storesletten, Telmer, and Yaron (2007) and Gomes and Michaelides (2008). The algorithm consists of the following steps, details of which are given in the next subsections.

1. Choose arguments and a functional form for the approximate law of motion, and make an initial guess for its coefficients.

2. Given the approximate law of motion, solve the household's problem.
3. Simulate the economy using the obtained optimal policy functions. In every period, compute the market clearing prices.
4. Update the coefficients of the approximate law of motion by running a regression on the simulated aggregate statistics.
5. If the coefficients have converged, and the  $R^2$  of the regression is sufficiently high, stop, else go to 2.
6. Repeat steps 1 to 5 for different arguments and functional forms of the law of motion. Select the one with the highest  $R^2$ .
7. Given the functional form for the approximate law of motion that achieved the best fit, calibrate the economy to match the targets.
  - (a) Provide an initial guess for the parameters to be calibrated.
  - (b) Given the parameters, repeat steps 2 to 5.
  - (c) Calculate the target statistics from the simulations. If they are close to the targets in the data, stop, else update the guess for the parameters and go to 7b.
8. Given the calibrated parameters, increase the social security contribution rate and compute the new general equilibrium by repeating steps 2 to 5.
9. Compute the welfare gains of the experiment in general equilibrium from the simulated variables of the first and the second economy.
10. Given the approximate laws of motion and the simulated prices of the first economy, perform the risk decomposition analysis.
  - (a) Given the approximate law of motion of the first economy, solve the household's problem.
  - (b) Given the simulated prices of the first economy, simulate the economy using the obtained optimal policy functions. (Do not compute market clearing prices.)
  - (c) Increase the social security contribution rate and repeat steps 10a and 10b.
  - (d) Compute the welfare gains of the experiment in partial equilibrium (PE) from the simulated variables of the pre-experiment PE and the post-experiment PE.



- (e) If this was the no-risk, deterministic economy, stop, else turn off a risk and repeat steps 10a to 10d.

The numerical solution is implemented in Fortran and parallelized, running on 24 cores.

## C.2 Solving for the approximate law of motion

The idea behind the Krusell-Smith-method (1997, 1998) is to approximate the infinite dimensional distribution,  $\Phi$ , by a finite number of statistics. The household then uses a law of motion of these statistics,  $\hat{H}(\cdot)$ , as an approximation to the true law of motion of the distribution,  $H(\Phi, z, z')$ . The statistics have to enable the household to forecast the prices that it needs to solve its optimization problem. We follow Krusell and Smith (1997), Storesletten, Telmer, and Yaron (2007), and Gomes and Michaelides (2008) and choose mean aggregate capital,  $k$ , together with a second variable to forecast the bond return. As this second variable, we choose the expected equity premium,  $\mu = \mathbb{E}(r'_s - r'_b)$ , cf. Storesletten, Telmer, and Yaron (2007).<sup>49</sup> Thus, the approximate law of motion becomes

$$\{k'(z'), \mu'(z')\} = \hat{H}(k, \mu, z, z').$$

The functional form for  $\hat{H}$  that gives the best approximation in our baseline economy is

$$\ln k_{t+1} = \psi_{0,z}^k + \psi_{1,z}^k \ln k_t + \psi_{2,z}^k (\ln k_t)^2, \quad (27a)$$

$$\mu_{t+1,z'} = \psi_{0,z'}^\mu + \psi_{1,z'}^\mu \ln k_{t+1} + \psi_{2,z'}^\mu (\ln k_{t+1})^2, \quad (27b)$$

which is similar to the best fit regression found by Storesletten, Telmer, and Yaron (2007). Note that the forecast of capital,  $\ln k_{t+1}$ , enters as a regressor in eq. (27b). Effectively, the forecast for  $\mu_{t+1,z'}$ , which is conditional on  $z'$ , depends on  $\ln k_t$  and  $z$  through the forecast of  $\ln k_{t+1}$ . The discrete, aggregate shock,  $z$ , can take four values, so that we estimate eight equations. Therefore, we report eight coefficients of determination, which for the baseline economy are  $R_k^2 = \{0.9998, 0.9999, 0.9998, 0.9998\}$  and  $R_\mu^2 = \{0.9918, 0.9945, 0.9584, 0.9695\}$ . For the other economies, the  $R^2$ s are always higher.<sup>50</sup>

<sup>49</sup>We choose  $\mu$  instead of the bond price because this enables us to avoid  $E(r_b) > E(r_s)$  by construction. This is desirable because such a situation would never arise in equilibrium.

<sup>50</sup>For example, for the equity premium calibration with  $IES = 0.5$ , the coefficients of determination are  $R_k^2 = \{0.9999, 0.9999, 0.9999, 0.9999\}$  and  $R_\mu^2 = \{0.9961, 0.9968, 0.9944, 0.9949\}$ . This economy is the closest to Storesletten, Telmer, and Yaron (2007) and Gomes and Michaelides (2008), and the  $R^2$  are very close to the ones reported there.

To find the coefficients, we solve  $g(\Psi) = \Psi - \tilde{\Psi}(\Psi)$ , where  $\Psi$  collects all the coefficients, i.e.  $\Psi = \{\psi_{l,z}^m\}_{l=\{0,1,2\}, z=\{1,2,3,4\}, m=\{k,\mu\}}$ . To solve this nonlinear equation system, a multidimensional Broyden algorithm is used. During the solution, we normalize (and subsequently de-normalize) the coefficients around unity. For these coefficients around unity, the convergence criterion is  $\max\{|g(\Psi)|\} < 1.0^{-7}$ . The Newton-like update steps are limited to a small length, and backtracking is used to find an update, if the first step was too large.<sup>51</sup>

### C.3 Solving the household's problem

First, we rewrite the household problem in terms of cash-on-hand,  $\tilde{x}$ . This reduces the state space by one dimension, so that the idiosyncratic state consists of  $(\tilde{x}, e)$ . Second, we recast the two control variables bond,  $\tilde{b}'$ , and stock,  $\tilde{s}'$ , as total savings,  $\tilde{a}'$ , and the portfolio share invested in stock,  $\kappa$ . This enables us to employ the endogenous grid method proposed by Carroll (2006), as detailed below. And third, we replace the distribution,  $\Phi$ , by the approximation discussed in the previous section, so that the aggregate state consists of  $(k, \mu, z)$ . With a slight abuse of notation,<sup>52</sup> the optimization problem in recursive form then reads

$$\begin{aligned} & \tilde{v}_j(\tilde{x}, e; k, \mu, z) \\ &= \max_{\tilde{c} > 0, \tilde{a}', \kappa} \begin{cases} \left( \tilde{c}^{\frac{1-\theta}{\gamma}} + \tilde{\beta} \left( \sum_{z'} \sum_{e'} \pi_z(z'|z) \pi_e(e'|e) \tilde{v}_{j+1}^{1-\theta}(\tilde{x}', e'; \hat{H}(k, \mu, z, z'), z') \right)^{\frac{1}{\gamma}} \right)^{\frac{\gamma}{1-\theta}} \\ \tilde{c} & \text{if } j = J \end{cases} \end{aligned}$$

s. t.

$$\begin{aligned} \tilde{x}' &= \tilde{a}' \frac{(1 + r_b' + \kappa(r_s' - r_b'))}{1 + \lambda} + \tilde{y}'_{j+1}, \\ \tilde{a}' &\geq 0 \quad \text{if } j = J, \end{aligned}$$

<sup>51</sup>The Newton-like update step is  $\Psi_{i+1} = \Psi_i - sJ(\Psi)^{-1}g(\Psi)$ , where  $J(\Psi)$  is a finite-difference approximation to the Jacobi matrix of the system of equations and  $s$  determines the maximum step length.

<sup>52</sup>Technically, some variables would need to be renamed, e.g.  $\tilde{y}$  to  $\tilde{\tilde{y}}$ , because the state space is now different than the one in Definition 4. For sake of readability, we do not change the notation.

where  $\tilde{\beta} = \beta(1 + \lambda)^{\frac{1-\theta}{\gamma}}$ ,  $r'_s = r_s(\hat{H}(k, \mu, z, z'), z')$ ,  $r'_b = r_b(\hat{H}(k, \mu, z, z'))$ , and income in the next period is given by

$$\begin{aligned} \tilde{y}'_{j+1} &= \tilde{y}_{j+1}(e', \hat{H}(k, \mu, z, z'), z') \\ &= \begin{cases} (1 - \tau)\tilde{w}(\hat{H}(k, \mu, z, z'), z')\epsilon_{j+1}\eta(e', z') & \text{if } j + 1 < j_r, \\ \tilde{y}_{ss}(\hat{H}(k, \mu, z, z'), z') & \text{else.} \end{cases} \end{aligned}$$

The budget constraint contains a growth adjustment of  $\frac{1}{1+\lambda}$ , because  $x'$  is cash on hand at the beginning of next period, while  $a'$  is the savings choice made in the previous period. In contrast, the budget constraint in the equilibrium definition of Section 3.5 contains only contemporaneous variables, i.e., states and choices in the current period, so that no growth adjustment is needed there.

Applying the envelope theorem and simplifying we get the two first-order-conditions<sup>53</sup>

$$\mathbb{E} \left[ \tilde{v}_{j+1}(\cdot)^{\frac{(1-\theta)(\gamma-1)}{\gamma}} (\tilde{c}')^{\frac{1-\theta-\gamma}{\gamma}} (r'_s - r'_b) \right] = 0, \quad (28a)$$

$$\tilde{c} = \left( \tilde{\beta} \frac{1 + r'_b}{1 + \lambda} \left( \mathbb{E} \left[ \tilde{v}_{j+1}(\cdot)^{1-\theta} \right] \right)^{\frac{1-\gamma}{\gamma}} \mathbb{E} \left[ \tilde{v}_{j+1}(\cdot)^{\frac{(1-\theta)(\gamma-1)}{\gamma}} (\tilde{c}')^{\frac{1-\theta-\gamma}{\gamma}} \right] \right)^{\frac{\gamma}{1-\theta-\gamma}}. \quad (28b)$$

To solve for the optimal choices  $(\tilde{c}, \tilde{a}', \kappa)$ , we apply a variant of the endogenous grid method proposed first by Carroll (2006). In fact, essentially we follow a simplified version of the two-step procedure of Hintermaier and Koeniger (2010). The exogenous grid is defined on total assets in the next period,  $\tilde{a}'$ . For a given grid-point  $\tilde{a}'_i$ , we first solve eq. (28a) for the portfolio share  $\kappa$  using Brent's root-finding method. Then, given  $\tilde{a}'_i$  and the corresponding  $\kappa(\tilde{a}'_i)$ , we use eq. (28b) to get the optimal consumption,  $\tilde{c}_i(\tilde{a}'_i)$ . Finally, the budget constraint  $\tilde{x} = \tilde{c} + \tilde{a}'$  gives us the endogenous grid-point  $\tilde{x}_i$  that corresponds to the optimal choices  $(\tilde{a}'_i, \tilde{c}_i)$ .

When evaluating the expectations, we interpolate  $\tilde{v}_{j+1}$  and  $\tilde{c}'$  by multidimensional linear interpolation in the continuous states  $\tilde{x}, k, \mu$ . The aggregate shock  $z$  and the idiosyncratic shock  $\eta(e, z)$  are both discrete and follow a discrete Markov chain. As discussed in Section 4.2, our specification of the stochastic labor income process has a persistent component,  $\nu$ , and a transitory shock,  $\varepsilon$ , so that we have  $\eta(e, z) = \nu(e, z)\varepsilon(e)$ . We construct the Markov transition matrix of  $\nu(e, z)$  with the Rouwenhorst method (Kopecky and Suen (2010)), which makes it straightforward to implement the countercyclical cross-sectional variance,  $CCV$ , because the variances affect only the grid and not the transition matrix, which in turn is determined purely by  $\rho$ . We discretize the transitory shocks  $\varepsilon$  using Gauss-Hermite quadrature.

<sup>53</sup>See Weil (1989) for the envelope theorem with recursive Epstein-Zin preferences.

As is standard in life-cycle models, we iterate backwards, starting with the last generation  $J$ , for which the solution is  $\tilde{c}_J = \tilde{x}_J$ , since they do not leave bequests. In the backwards iteration, we construct age-dependent, exogenous grids  $\{\tilde{a}'_{i,j}\}_{i,j}$  to improve the approximation quality. The solution is parallelized in the dimension  $k$ , so that for each generation, the solution for all values of  $k$  is computed in parallel.

We discretize the state space in the following way. The continuous state variables cash-on-hand,  $\tilde{x}$ , aggregate capital,  $k$ , and equity premium,  $\mu$ , have 20, 18, and 10 grid-points, respectively. The discrete state variables, which are the number of generations,  $J$ , the idiosyncratic shock,  $e$ , and the aggregate shock,  $z$ , have 58, 4, and 4 grid-points, respectively, and we use 4 points for the Gauss-Hermite quadrature (for the transitory shocks). We check that this is sufficient by doubling each of the grid-points in turn and find no change to our results. The first-order-condition in eq. (28a) is solved to an accuracy of  $1.0^{-10}$ .

## C.4 Simulating the economy

We simulate the economy 24 times for 4000 periods each time and throw away the first 1000 periods, so that we are left with a total 72.000 simulation periods.<sup>54</sup> In each period, we record the aggregates, the life-cycle profiles, and the distribution. The aggregates are needed to estimate the laws of motion, and to calibrate the economy. Like in the solution of the household problem, the optimal policy functions are interpolated in the dimensions of the aggregate states  $k, \mu$  by multidimensional linear interpolation.

The distribution over households is normalized to a mass of one. We do not simulate many, discrete household units; instead we keep the continuum of households and approximate the distribution with a histogram as proposed by Young (2010). As described in Section 3.1, the Law of Large Numbers implies that  $\pi_e(e'|e)$  represents the fraction of the population moving from idiosyncratic state  $e$  to  $e'$ . Therefore, we get a nearly exact approximation in that dimension. In the cash-on-hand dimension, the distribution is discretized on a much finer grid than the policy functions obtained in the household solution, as proposed by Ríos-Rull (1999). This finer discretization improves the approximation quality substantially and helps in ensuring that no households are stuck on the bounds of the distribution. If the lowest or the highest points of the distribution have positive mass, then the cash-on-hand grid is extended and the discretization is made finer.

---

<sup>54</sup>We found that a large number of simulation periods is necessary for the distribution to converge in the sense that increasing the number of simulation periods does not change the results. In particular, we found that for less than 30.000 simulation periods, the means and standard deviations of the aggregates as well as the estimates of the laws of motion are still sensitive to the number of periods.

In each period, the beginning-of-period distribution is iterated forward by using the computed optimal policy functions and the realizations of the shock. For a given cash-on-hand at the beginning of the period, the implied cash-on-hand in the following period will almost always lie between two grid points. Since we are dealing with a continuum of households, we assign a fraction  $(1 - f)$  to the lower grid point and  $f$  to the upper grid point of the interval which contains the implied cash-on-hand, where  $f$  is the distance to the lower grid point.<sup>55</sup>

In each period  $t$ , we calculate the market-clearing prices.<sup>56</sup> The current stock return,  $r_s(\Phi_t, z_t)$  is given by the contemporaneous aggregate capital and aggregate shock. The current bond return,  $r_b(\Phi_t)$ , is determined one period before by the bond market clearing condition. We compute it with a nonlinear equation solver to an accuracy of  $1.0^{-8}$ .

We make sure that the grid for the aggregate states is large enough by checking whether the realized values lie on the bounds of the grid. If they do, the grid is increased. To get good initial guesses for the bounds of the aggregate grids and the distribution over households, we compute a degenerate equilibrium, where the realization of the aggregate shocks in the simulations is always equal to their mean. We call this a mean-shock equilibrium.

To check the accuracy of the solution, we compute in each period the 'aggregation error' and the 'income error'. The aggregation error  $e_t^{agg} = \frac{Y_t - C_t - I_t}{Y_t}$  says by how much the aggregate budget constraint is violated due to interpolation and aggregation errors, expressed in percent of output. For all economies, the maximum aggregation error is in the order of  $1.0^{-6}$  and the average is in the order of  $2.0^{-9}$ . The income error comes from Euler's formula, which says that total output must equal total factor income. Again expressed in terms of output, we find that it never exceeds  $1.0^{-14}$ .

## C.5 Calibrating the economy

The calibration procedure is cast as a system of nonlinear equations. Let  $\mathcal{T}$  denote the target statistics in the data and  $\mathcal{P}$  the model parameters to be calibrated. For given  $\mathcal{P}$ ,  $\hat{\mathcal{T}}(\mathcal{P})$  are the model-generated statistics, which we get from the simulations. Then the calibration procedure tries to find a root of  $\mathcal{T} - \hat{\mathcal{T}}(\mathcal{P}) = 0$ . We use Broyden's multidimensional secant method to solve the system to an accuracy of  $1.0^{-4}$ .

---

<sup>55</sup>For details, see Young (2010).

<sup>56</sup>Algan, Allais, Den Haan, and Rendahl (2014) stress the importance of ensuring market clearing during the simulations.

## C.6 Evaluating the conditions for dynamic efficiency

After having calibrated the economy, we evaluate the sufficient conditions of Definition 3 in each period of our simulations (after discarding burn-in). Conditional on having reached a high bond return in a date-event  $z^t$ ,  $r_b(\Phi_t) > (1+n)(1+g)$ , we compute market clearing prices for all possible aggregate shocks next period,  $z_{t+1} \in \mathcal{Z}$ .<sup>57</sup> We then check whether there exist two states  $\tilde{z}_{t+1}, \tilde{\tilde{z}}_{t+1}$ , such that (i) the economy remains in a high bond return equilibrium in both corresponding date-events next period and (ii) the stock return fluctuates enough in these states relative to this period’s high bond return. If this is the case condition (a) is fulfilled.

As for condition (b), we define a counter which is initialized to zero and increased by one every period. If the economy reaches a high bond return equilibrium, the value of the counter is saved, and we start counting again from zero. If the maximum count is smaller than the number of simulated periods, then condition (b) is fulfilled. To provide more information we report in Tables 2 and 12 the maximum and the average count.

## D Supplementary Calibration Appendix

### D.1 Households

We base our estimates of the earnings process on Busch and Ludwig (2017) using PSID data of household post government earnings excluding contributions to social security. To get consistent earnings measures the sample is restricted to years from 1977 to 2012. Household pre-government earnings is the sum of labor earnings of households’ head and spouses augmented by 50% of payroll taxes excluding social security. Post-government earnings is derived from this measure by adding transfers and deducting taxes (calculated from TAXSIM), again excluding contributions to social security. A second earnings measure used for our scenario with social security ( $BL_{\tau=9.5\%}$ ) in Section 5.3 also takes contributions to social security into account.

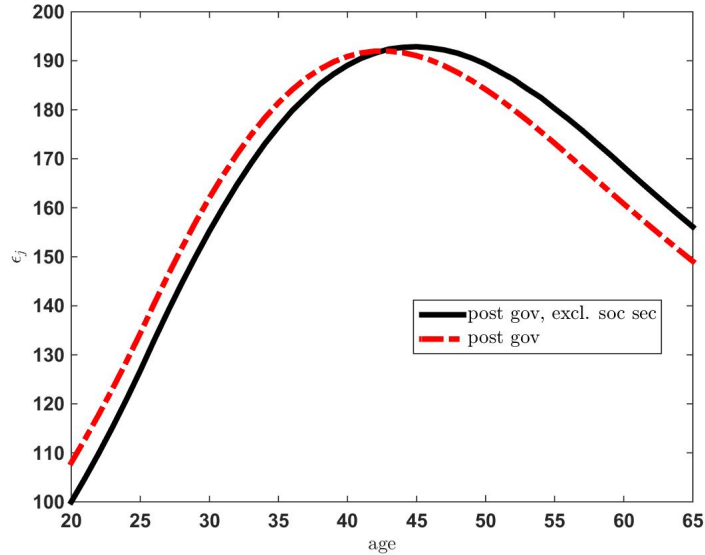
Busch and Ludwig (2017) adopt the standard strategy to first decompose a household’s log earnings into a deterministic and a stochastic component. The estimates of the age specific productivity profile  $\epsilon_j$  are taken from the deterministic component. Figure 3 displays the profile for both our earnings measures (“post gov, excl. soc sec” and “post gov”).

To estimate the stochastic earnings process, Busch and Ludwig (2017) first classify contraction years on the basis of NBER recession indicators, which, due to the sluggish adjustment of the labor market, is expanded by years of upward trending unemployment rates as in Guvenen,

---

<sup>57</sup>Since the Markov transition matrix  $\pi_z$  has non-zero entries everywhere, all  $z^{t+1}$  can be reached from  $z^t$  with positive probability.

Figure 3: Life-cycle Productivity



Notes: Age-specific productivity profile  $\epsilon_j$ . Notes: Own estimates based on Busch and Ludwig (2017).

Ozkan, and Song (2014). All years that are not accordingly classified as a recession are classified as a boom. Using their procedure and the income measures described above, our earnings process (12) is estimated with results reported in Section 4.2.<sup>58</sup>

## D.2 Firms

To estimate  $\alpha$ , we take data on total compensation of employees (NIPA Table 1.12) and deflate it with the GDP deflator (NIPA Table 1.1.4). In the numerator, we adjust GDP (NIPA Table 1.1.5), again deflated by the GDP deflator, by nonfarm proprietors' income and other factors that should not be directly related to wage income. Without these adjustments, our estimate of  $\alpha$  would be considerably higher (at  $\alpha = 0.43$ ).

To measure capital, we take the stock of fixed assets (NIPA Table 1.1), appropriately deflated. We relate this to total GDP.

We determine the growth rate of technology  $\lambda$  by estimating the Solow residual from the production function, given our estimate of  $\alpha$ , our measure for capital, and a measure of labor supply determined by multiplying all full- and part-time employees in domestic employment (NIPA Table 6.4A) with an index for aggregate hours (NIPA Table 6.4A). Notice that we ignore

<sup>58</sup>Again, we thank Christopher Busch for providing us with the estimates.

age-specific productivity which should augment our measure of employment. We then fit a linear trend specification to the Solow residual. Acknowledging the labor augmenting technological progress specification, this gives our point estimate.

### D.3 Aggregate Risk

We first provide details on how we construct the transition matrix and the values for the aggregate technology and depreciation shocks,  $(\zeta(z), \delta(z))$ . Both  $\zeta(z)$  and  $\delta(z)$  can each take a high or a low value. We let

$$\zeta(z) = \begin{cases} 1 - \bar{\zeta} & \text{for } z \in z_1, z_2 \\ 1 + \bar{\zeta} & \text{for } z \in z_3, z_4 \end{cases} \quad \text{and } \delta(z) = \begin{cases} \delta_0 + \bar{\delta} & \text{for } z \in z_1, z_3 \\ \delta_0 - \bar{\delta} & \text{for } z \in z_2, z_4. \end{cases} \quad (29)$$

Set up in this way,  $z_1$  corresponds to a low wage and a low return, while  $z_4$  corresponds to a high wage and a high return. We speak of  $z \in z_1, z_2$  as a *recessions* in the sense that these represent states in which aggregate wage shocks are low.

To calibrate the entries in the transition matrix, denote the transition probability of remaining in the low technology state by  $\pi^\zeta = \pi(\zeta' = 1 - \bar{\zeta} \mid \zeta = 1 - \bar{\zeta})$ . Assuming that the transition of technology shocks is symmetric, we then have  $\pi(\zeta' = 1 + \bar{\zeta} \mid \zeta = 1 + \bar{\zeta}) = \pi^\zeta$  and  $1 - \pi^\zeta = \pi(\zeta' = 1 - \bar{\zeta} \mid \zeta = 1 + \bar{\zeta}) = \pi(\zeta' = 1 + \bar{\zeta} \mid \zeta = 1 - \bar{\zeta})$ .

To govern the correlation between technology and depreciation shocks, let the probability of being in the high (low) depreciation state conditional on being in the low (high) technology state be  $\pi^\delta = \pi(\delta' = \delta_0 + \bar{\delta} \mid \zeta' = 1 - \bar{\zeta}) = \pi(\delta' = \delta_0 - \bar{\delta} \mid \zeta' = 1 + \bar{\zeta})$ , where the second equality follows from assuming symmetry of the matrix. We then have that the transition matrix of aggregate states follows from the corresponding assignment of states in (29) as

$$\pi_z = \begin{bmatrix} \pi^\zeta \cdot \pi^\delta & \pi^\zeta \cdot (1 - \pi^\delta) & (1 - \pi^\zeta) \cdot (1 - \pi^\delta) & (1 - \pi^\zeta) \cdot \pi^\delta \\ \pi^\zeta \cdot \pi^\delta & \pi^\zeta \cdot (1 - \pi^\delta) & (1 - \pi^\zeta) \cdot (1 - \pi^\delta) & (1 - \pi^\zeta) \cdot \pi^\delta \\ (1 - \pi^\zeta) \cdot \pi^\delta & (1 - \pi^\zeta) \cdot (1 - \pi^\delta) & \pi^\zeta \cdot (1 - \pi^\delta) & \pi^\zeta \cdot \pi^\delta \\ (1 - \pi^\zeta) \cdot \pi^\delta & (1 - \pi^\zeta) \cdot (1 - \pi^\delta) & \pi^\zeta \cdot (1 - \pi^\delta) & \pi^\zeta \cdot \pi^\delta \end{bmatrix}.$$

Now we discuss the empirical correlation of TFP and stock returns,  $\sigma(\zeta_t, r_{s,t})$ , a second-stage calibration target. Linear detrending of the data, as done, e.g., by Krueger and Kubler (2006), results in  $\sigma(\zeta_t, r_{s,t}) < 0$  as well as a negative correlation of wages and asset returns, i.e.,  $\sigma(w_t, r_{s,t}) < 0$ . Not only does this seem counter to economic intuition in an annual RBC model,



but our estimate for  $\sigma(\zeta_t, r_{s,t})$  is also statistically insignificant. Assuming instead a unit root process for the log of TFP and detrending by first differences yields a highly significant positive correlation of  $\sigma(\zeta_t, r_{s,t}) = 0.50$  (p-value 0.00).<sup>59</sup> Now also  $\sigma(w_t, r_{s,t})$  is positive and significant with  $\sigma(w_t, r_{s,t}) = 0.306$  (p-value 0.025), which coincides with our economic intuition as we would expect these variables to co-move over the cycle. Our model, however, features a linear trend, not a unit root. We therefore translate these moments to be consistent with a deterministic trend specification.

## D.4 Calibration of Single Risk Economies

Table 10 summarizes the second-stage parameters, i.e., the parameters that are jointly calibrated. The remaining first-stage parameters take the same value as in the baseline, see Table 1. Table 10 also displays the targeted moments for these economies. For comparison, the table includes the corresponding values of the baseline (*BL*).

Table 10: The Role of Both Risks: Parameters and Moments

Parameter	$\theta$	$\beta$	$\delta_0$	$\bar{\delta}$	$\pi^\delta$		
<i>BL</i>	3.00	0.987	0.102	0.080	0.887		
<i>AR-only</i>	15.10	0.994	0.076	0.111	0.833		
<i>IR-only</i>	3.00	0.974	0.079	0.000	NA		
<i>No-risk</i>	3.00	0.998	0.079	0.000	NA		
Moment	$\varsigma$	$\mu$	$E\left[\frac{K}{Y}\right]$	$E[r_b]$	$\sigma\left(\frac{\Delta C_{t+1}}{C_t}\right)$	$\sigma(r_s)$	$\sigma(\zeta, r_s)$
<i>BL</i>	0.076	0.008	2.65	0.023	0.030	0.107	0.500
<i>AR-only</i>	0.351	0.056	2.65	0.023	0.040	0.168	0.500
<i>IR-only</i>	0.000	0.000	2.65	0.042	NA	NA	NA
<i>No-risk</i>	0.000	0.000	2.65	0.042	NA	NA	NA

*Notes:* *BL*: baseline calibration with  $\theta = 3$ ; *AR-only*: economy with only aggregate risk, calibrated to match equity premium; *IR-only*: economy with only idiosyncratic risk; *No-risk*: deterministic economy.  $\varsigma = \frac{E[r_{s,t} - r_{b,t}]}{\sigma[r_{s,t} - r_{b,t}]}$ : Sharpe ratio;  $\mu = E[r_{s,t} - r_{b,t}]$ : equity premium;  $E\left[\frac{K}{Y}\right]$ : average capital-output ratio;  $E[r_b]$ : average bond return;  $\sigma\left(\frac{\Delta C_{t+1}}{C_t}\right)$ : standard deviation of aggregate consumption;  $\sigma(r_s)$ : standard deviation of stock returns;  $\sigma(\zeta, r_s)$ : correlation of TFP shocks and stock returns.

<sup>59</sup>Observe that calibrating the model to match this moment explicitly is more conservative with regard to the return implications of recessions than the assumption of Storesletten, Telmer, and Yaron (2007) who assume a perfect negative correlation of TFP and depreciation shocks.

## D.5 Calibration for Sensitivity Analyses

The calibrated parameters and targeted moments for the various scenarios we consider are summarized in Table 11. For comparison, the table includes the corresponding values of the baseline (*BL*).

For the calibration of the earnings process in the  $\tau = 9.5\%$  economy, we take the Busch and Ludwig (2017) estimates of an income process cum social security. The age productivity profile is shown in Figure 3. For the stochastic part of the income process the estimates are  $\rho = 0.966$ , and a conditional variance,  $\sigma_v^2(z_t)$ , of 0.024 in recessions and 0.01 in booms, and  $\sigma_\varepsilon^2 = 0.099$ .

Table 11: Sensitivity Analysis: Parameters and Moments

Parameter	$\theta$	$\beta$	$\delta_0$	$\bar{\delta}$	$\pi^\delta$		
<i>IES</i> = 0.5							
<i>BL</i>	3.00	0.987	0.102	0.080	0.887		
<i>SR</i>	11.10	0.987	0.020	0.045	0.829		
<i>EP</i>	5.51	0.987	0.000	0.114	0.830		
<i>BL</i> <sub><math>\tau=9.5\%</math></sub>	3.00	1.014	0.105	0.088	0.893		
<i>IES</i> = 1.5							
<i>BL</i> <sub><i>IES</i>=1.5</sub>	3.00	0.977	0.099	0.039	0.888		
<i>SR</i> <sub><i>IES</i>=1.5</sub>	12.20	0.977	0.019	0.038	0.829		
<i>EP</i> <sub><i>IES</i>=1.5</sub>	5.60	0.977	0.001	0.115	0.835		
Moment	$\varsigma$	$\mu$	$E\left[\frac{K}{Y}\right]$	$E[r_b]$	$\sigma\left(\frac{\Delta C_{t+1}}{C_t}\right)$	$\sigma(r_s)$	$\sigma(\zeta, r_s)$
<i>IES</i> = 0.5							
<i>BL</i>	0.076	0.008	2.65	0.023	0.030	0.107	0.500
<i>SR</i>	0.333	0.020	5.80	0.023	0.030	0.067	0.500
<i>EP</i>	0.357	0.056	7.67	0.023	0.066	0.168	0.500
<i>BL</i> <sub><math>\tau=9.5\%</math></sub>	0.069	0.007	2.65	0.023	0.030	0.116	0.500
<i>IES</i> = 1.5							
<i>BL</i> <sub><i>IES</i>=1.5</sub>	0.051	0.003	2.65	0.023	0.030	0.052	0.500
<i>SR</i> <sub><i>IES</i>=1.5</sub>	0.333	0.018	5.90	0.023	0.030	0.057	0.500
<i>EP</i> <sub><i>IES</i>=1.5</sub>	0.356	0.056	6.57	0.023	0.089	0.168	0.500

Notes: *BL*: baseline calibration with  $\theta = 3$ ; *SR*: scenario matching Sharpe ratio; *EP*: scenario matching equity premium.  $\varsigma = \frac{E[r_{s,t} - r_{b,t}]}{\sigma[r_{s,t} - r_{b,t}]}$ : Sharpe ratio;  $\mu = E[r_{s,t} - r_{b,t}]$ : equity premium;  $E\left[\frac{K}{Y}\right]$ : average capital-output ratio;  $E[r_b]$ : average bond return;  $\sigma\left(\frac{\Delta C_{t+1}}{C_t}\right)$ : standard deviation of aggregate consumption;  $\sigma(r_s)$ : standard deviation of stock returns;  $\sigma(\zeta, r_s)$ : correlation of TFP shocks and stock returns.

## E Supplementary Results Appendix

### E.1 Dynamic Efficiency

Let us briefly point out two misconceptions on dynamic efficiency. First, observe that the *average* bond return in our economy is 0.023 which is less than the average social security return of  $\lambda + n = 0.029$ . In our simple model of Section 2 this would indicate dynamic inefficiency. However, in our quantitative model, bond returns and implicit social security returns fluctuate, hence any conclusion based on *average* returns is misleading. Second, turn to the deterministic steady state variant of the aggregate resource constraint, cf. item (e) in Appendix B.3 and accordingly set in this equation  $\zeta(z) = 1$ ,  $\delta(z) = \delta_0 = 0.102$ , and  $k(H(\Phi, z), z') = k$ . From this we get the standard condition for dynamic efficiency that  $mpk - (\delta + n + \lambda) \geq 0$ , where  $mpk = \alpha k^{\alpha-1} = \alpha \frac{y}{k}$  is the marginal product of capital. For our calibration with  $\frac{k}{y} = 2.65$  we get  $mpk = 0.1207$  and  $\delta + n + \lambda = 0.13$ . One may therefore conclude that the economy is dynamically inefficient. This conclusion is equally misleading because there does not exist a steady state in our economy with aggregate risk, hence one cannot simply set  $\zeta(z) = 1$ ,  $\delta(z) = \delta_0$  and  $k(H(\Phi, z), z') = k$ . In fact the last equation holds in no period of our simulations.

Instead, the relevant criteria for dynamic efficiency in economies with aggregate risk, where returns fluctuate, are the Demange (2002) criteria, see the main text for an extensive discussion. Table 12 reports the results for the sufficient conditions according to Definition 3 for all model variants with aggregate risk considered in the main text. The number of simulated periods used is 72 000. Throughout, we conclude that all considered economies are dynamically efficient. The lowest confidence for this finding of at least 90.0% applies to the economy  $BLIES=1.5$ .

### E.2 On the Importance of Modeling both Risks: Results Without Re-Calibration

Complementing our analysis in Section 5.2 we summarize in Table 13 results for economies with only one form of risk, respectively for the deterministic economy, when we do not recalibrate. As in Table 8, there are welfare losses in the *AR-only* economy.

The *IR-only* economy is dynamically inefficient. The risk-free return is at 2.86%, compared to the implicit return of social security of 2.9%. That is why we find a small welfare gain in general equilibrium. Despite dynamic inefficiency, there are welfare losses from crowding out, cf. our discussion in Section 3.8 on the relationship between the costs from crowding out and dynamic efficiency in heterogeneous agent economies. Finally, the deterministic economy has

Table 12: Dynamic Efficiency in Baseline and Other Economies

	Condition (a)		Condition (b)	
	High Bond Returns	Conditional Violation	Max. Periods	Avg. Periods
<i>IES</i> = 0.5				
<i>BL</i>	38.1%	4.7%	120	11.5
<i>SR</i>	34.8%	6.0%	158	13.5
<i>EP</i>	45.1%	0.8%	134	10.8
<i>AR-only</i>	40.6%	3.9%	103	7.0
<i>BL</i> <sub><math>\tau=9.5\%</math></sub>	41.2%	3.1%	120	10.9
<i>IES</i> = 1.5				
<i>BL</i> <sub><i>IES</i>=1.5</sub>	31.8%	10.0%	120	12.1
<i>SR</i> <sub><i>IES</i>=1.5</sub>	36.0%	6.6%	124	12.0
<i>EP</i> <sub><i>IES</i>=1.5</sub>	46.3%	0.8%	120	9.9

*Notes:* Test results for dynamic efficiency conditions, cf. Definition 3. High bond returns: fraction of high bond return equilibria in which  $1 + r_b(z^t) > (1 + n)(1 + \lambda)$ . Conditional violation: Violation of conditions (a)(i) and (a)(ii), conditional on being in a high bond equilibrium. Avg., resp. max., periods: average, resp. maximum, number of simulation periods to reach high bond equilibrium. The number of simulated and tested periods is 72 000 in each scenario.

a risk-free interest rate of 5.56% and we therefore continue to find welfare losses from the introduction of social security.

### E.3 Other Sensitivity Analyses

Table 14 summarizes results for other sensitivity analyses. Throughout, we stick to the calibration strategy of the conservative baseline model—we accordingly denote all these scenarios with prefix *BL*—and vary selected parameters, respectively alter modeling choices. We accordingly recalibrate to match the calibration targets of the baseline, assuming an *IES* of 0.5.

First, we vary risk aversion in economies  $BL-\theta = 2$  and  $BL-\theta = 4$ . As expected, increasing  $\theta$  increases the overall welfare gains, increases the share of gains attributable to the insurance effect, decreases the share attributable to the mean effect and increases the share of the interactions; and vice versa for decreasing  $\theta$ .

Next, we consider a number of experiments regarding different model elements. In scenario  $BL-\sigma_\varepsilon^2 = 0$  we switch off the variance of transitory labor income shocks to show that our results are not driven by this element. Scenario  $BL-\bar{k} = 0$  sets the debt-equity-ratio to zero

Table 13: The Role of Both Risks without Re-Calibration

Scenario	Consumption equivalent variation, $g_c$			$g_c^{distr}$	$g_c^{mean}$
	GE	PE	CO	GE	
<i>AR-only</i>	-1.26%	-0.91%	-0.36%	-0.88%	-0.39%
<i>IR-only</i>	0.29%	1.84%	-1.56%	0.14%	0.15%
<i>No-risk</i>	-1.62%	-0.94%	-0.67%	-0.94%	-0.30%

*Notes:* GE: general equilibrium, PE: partial equilibrium, CO: crowding out; *AR-only*: economy with only aggregate risk, calibrated to match equity premium; *IR-only*: economy with only idiosyncratic risk; *No-risk*: deterministic economy. The total  $g_c$  is further decomposed into the mean effect,  $g_c^{mean}$ , and the distribution effect,  $g_c^{distr}$ , cf. Subsection 3.8 for formal definitions.

(so that firms are purely equity financed), thereby decreasing the equity premium from 0.76% in the baseline calibration to 0.46%, which slightly decreases the overall welfare gains from social security. Targeting a higher capital-output ratio of 3 in scenario  $BL-\frac{K}{Y} = 3$  yields higher welfare benefits. The reason is that this requires a higher discount factor (of  $\beta = 0.992$  rather than  $\beta = 0.987$ ), which increases the welfare benefits from social security in line with the predictions from the extended simple model in Appendix B.2. Finally, we consider an experiment where we shut down the depreciation shocks in the conservative baseline calibration,  $BL-\bar{\delta} = 0$ . In this experiment we recalibrate the variance of TFP shocks (which in all other experiments is taken as a first-stage parameter) to match the consumption volatility. In this experiment, the partial equilibrium gains from insurance against aggregate risks decrease from 2% to 1.4%, which leads to a decrease of the gains from insurance against the interactions of risks and to a reduction of the total insurance gains. The reason is that aggregate wage volatility increases in this calibration which makes social security less attractive.

Lastly, we model an alternative distribution scheme, labelled *BL-distr* to  $L$ . Instead of distributing the contributions of workers to pensioners each period, we redistribute lump-sum to all workers. Such a scheme does not implement a life-time income risk smoothing like social security, but rather directly insures the idiosyncratic risk each period. We find that overall welfare gains are substantially smaller than in our baseline model. To understand this, observe that, on the one hand, such a redistributive scheme insures idiosyncratic risk in each period. As a consequence, the partial equilibrium gain from insuring idiosyncratic risk increases from 0.67% in the baseline model to 1.92%. On the other hand, this scheme does not provide any insurance against aggregate risk. Also, the partial equilibrium gain from insuring the CCV decreases from a pure insurance effect of 1.08% in the baseline model to 0.47%, because this redistribution scheme can only offer very limited insurance against the countercyclical variance:

the volatility of idiosyncratic shocks is high in recessions, during which aggregate wages and the insurance payment to workers are low. This finding underscores once more the importance of risk interactions for the welfare benefits of social security and also gives support to the view that a lifetime redistributive scheme is desirable, see our concluding discussion.

Table 14: Sensitivity Analysis: Other Scenarios

Scenario	$g_c$			$\frac{\Delta_{CCV} + \Delta_{CWG}}{g_c^{PE}}$	$g_c^{distr}$	$g_c^{mean}$
	GE	PE	CO			
<i>BL</i>	+2.56%	+5.18%	-2.62%	0.60	+2.32%	+0.24%
<i>BL</i> - $\theta = 2$	+2.37%	+3.41%	-1.04%	0.53	+2.12%	+0.25%
<i>BL</i> - $\theta = 4$	+2.67%	+7.09%	-4.42%	0.65	+2.44%	+0.22%
<i>BL</i> - $\sigma_\varepsilon^2 = 0$	+2.44%	+4.30%	-1.86%	0.54	+2.22%	+0.22%
<i>BL</i> - $\bar{k} = 0$	+2.08%	+4.46%	-2.38%	0.58	+1.82%	+0.27%
<i>BL</i> - $\frac{K}{Y} = 3$	+2.97%	+4.90%	-1.93%	0.60	+2.75%	+0.23%
<i>BL</i> - $\bar{\delta} = 0$	+1.19%	+2.62%	-1.42%	0.46	+0.78%	+0.42%
<i>BL</i> -distr to <i>L</i>	+1.61%	+1.92%	-0.31%	0.28	+1.57%	+0.03%

*Notes:* GE: general equilibrium, PE: partial equilibrium, CO: crowding out; *CCV*: counter-cyclical cross-sectional variance, *CWG*: convexity of the welfare gain; *BL*: baseline calibration with  $\theta = 3$ ; *SR*: scenario matching Sharpe ratio; *EP*: scenario matching equity premium. The total  $g_c$  is further decomposed into the mean effect,  $g_c^{mean}$ , and the distribution effect,  $g_c^{distr}$ , cf. Subsection 3.8 for formal definitions.

## E.4 Life-Cycle Portfolios

Panel (a) of Figure 4 shows average shares invested in the risky asset over the life-cycle for model  $EP_{IES=0.5}$ , which comes closest to the calibration in standard life-cycle portfolio choice models like Cocco, Gomes, and Maenhout (2005). Recall from Section 3.2, in particular Footnote 15, that there is no borrowing constraint in our model. As a consequence, households are leveraged at the beginning of the life-cycle. From age 35 on, our simulated profiles are very similar to those presented in Cocco, Gomes, and Maenhout (2005).<sup>60</sup> Panel (b) shows corresponding life-cycle asset and stock holdings. Since there is no inter-generational transfer

<sup>60</sup>Unlike Storesletten, Telmer, and Yaron (2007) we do not find in this model—and in any of our other scenarios—that the CCV mechanism leads to a hump-shaped portfolio share profile. We conjecture that this is a result of the perfect positive correlation between depreciation and technology shocks in their model, which maximizes the impact of CCV, so that young households hold less assets. We instead calibrate the correlation to the data, cf. Appendix D.3.

motive, asset holdings at young ages are very low and households fully decumulate assets towards the end of their life.

Figure 4: Life-cycle Portfolios

