Three Essays on Optimal Acquisition and Use of Product Value Information

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To my parents Heidi and Helmut,

my brothers Levin and Rouven,

and to Katharina

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Summary

This dissertation is concerned with the management of testing processes to reduce uncertainty about products' values and the subsequent use of the gathered information. It consists of three independent essays and is motivated by two business examples, which, while differing at first sight, exhibit several similarities with regard to the acquisition of product value information through testing, namely, new product development (NPD) and used-product acquisition management. Additionally, the investigated settings involve strategic actors and related information asymmetries. These are the issues that we analyze. The first essay investigates a situation in new product development in which a firm delegates the testing of different design alternatives to experts to subsequently select the most promising design. The central undertaking in this work comprises finding the optimal incentive schemes for delegation of testing and, based on that, characterizing the optimal testing mode and how it is driven by delegation. The key results are that delegation favors sequential testing and that the heterogeneity of testing outcomes' qualities has an impact on the optimal number of experts under sequential testing: if qualities are homogeneous, a single expert should run all tests; otherwise, each design should be tested by a different expert. The second essay investigates a setting related to used-product acquisition at firms in the recommerce business. Here, quality-dependent acquisition prices are offered on a firm's website in combination with a certain acquisition process with counteroffers in which product holders provide upfront product-quality statements. We analyze how to optimally set up this acquisition process and compare it to an alternative process with regard to profitability. The key results are that product holders have an incentive to lie about products' qualities, whereas in the alternative process, holders can be incentivized to be truthful; however, neither of the processes outperforms the other under any circumstances. The third essay investigates another setting present in the business field of recommerce. Here, a firm acquires used products through a retailer that tests products and offers quality-dependent acquisition prices. We answer how to optimally manage this acquisition channel. The key results include the characterization of the optimal contracts and acquisition prices, determination of conditions for upfront testing and quality-dependent offers being beneficial, and the observation that the investigated acquisition channel is efficient.

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Chapter I

Introduction

In several business areas, firms apply testing processes to reduce uncertainty about certain product aspects. The information gathered serves as a basis for subsequent decision making and eventually for firms' success. This dissertation is concerned with the management of such testing processes and the subsequent use of the acquired information. It is motivated by two business examples, which, while differing at first sight, exhibit several similarities with regard to acquisition of product information through testing, namely, new product development (NPD) and used-product acquisition management. In NPD, the testing of different design alternatives is necessary to separate those that exhibit promise from those that do not. Only promising designs are further developed. Regarding the second example, used products are heterogeneous with regard to quality. Therefore, testing them before acquisition enables firms to offer differentiated, quality-dependent prices. This process is standard in the recommerce business.

Both investigated business areas share a number of common characteristics. First, product values are uncertain. Second, this uncertainty can be partly resolved through testing. Testing is costly, and the reliability of the obtained results depends on the effort exerted. Third, the firms that are reliant on the testing information do not exclusively test the products by themselves but rather have independent actors execute the testing. These parties may act strategically, potentially taking advantage of the firms' ignorance regarding products, i.e., of information asymmetries.

In order to provide a clear picture of the business examples, both are presented separately and in more detail with regard to the discussed characteristics. In NPD, different designs for a future product are created. Initially, each design's technological feasibility, its durability, and attractiveness for future customers are unknown to the firm. Those aspects determine a design's value for the firm. Here, testing is used to eliminate that lack of knowledge and comprises, for instance, experiments, simulations, and/or prototype building. The testing outcomes serve as a basis for resource allocation decisions, on which the most promising designs are selected for further development, whereas others are omitted. Prior academic work regarding NPD has identified many aspects that bear on the efficacy of a firm's design testing process—including such diverse factors as test efficiency, testing costs, lead times, and learning effects—and thus on the firm's optimal testing strategy (see, e.g., Weitzman 1979, Dahan and Mendelson 2001, Loch et al. 2001, Erat and Kavadias 2008). Nevertheless, additional complexities that come into play through the presence of independent actors and information asymmetries have so far only received scant attention, despite the fact that in many industries, information asymmetries between senior management and testing experts distort the outcomes of a testing process (Sommer and Loch 2009, Mihm 2010, Schlapp et al. 2015).

In used-product acquisition, products that firms want to buy in order to process and sell them again are heterogeneous with regard to the time for which and intensity with which they were used by their owners. This results in, e.g., differing optical conditions and technical functionality, which determines how much effort to put into processing and how much to charge for sale and, hence, the products' values for the firm. The testing process comprises, e.g., optical inspection, disassembly, or testing software. The gathered product value information serves as basis for quality-dependent acquisition price offers. Research regarding quality-dependent acquisition pricing (see, e.g., Guide et al. 2003, Ray et al. 2005, Karakayali et al. 2007, Bulmuş et al. 2014), which builds on the assumption that products are tested before acquisition, has barely considered the presence of information asymmetries between independent actors. However, regarding an example from the recommerce business, in which product holders must provide product-quality statements before acquisition, Hahler and Fleischmann (2017) report that a non-negligible proportion of actual product submissions exhibit a significant mismatch between those quality statements and the true qualities determined by the firms. This discrepancy hints at inefficiencies related to the presence of multiple actors in used-product acquisition processes.

This dissertation consists of three essays. Each essay is concerned with one of the presented business examples that are characterized by a firm's uncertainty about products' values and the potential ability to resolve this uncertainty. Additionally, the natural presence of self-interested actors and the corresponding information asymmetries are taken into account.

The first essay presented in Chapter II (joint work with Jochen Schlapp) examines the presented NPD business example in which a firm must resolve uncertainties about valuation of possible product designs in order to subsequently select the most promising design for further development. This is performed via costly and time-consuming tests, which the firm delegates to experts due to their knowledge and expertise. The basic decision of how to delegate testing in the sketched situation comprises the choice of testing mode, i.e., whether to let the experts test different designs simultaneously or in sequence and the setup of contracts that are used to prevent problems resulting from misaligned incentives. Motivated by this business situation, we raise the following questions: what is the optimal testing mode and what are the corresponding optimal incentive structures to offer the experts? How does delegation drive the testing mode choice?

To answer those questions, we apply principal-agent theory in order to find the optimal incentive schemes for each testing mode that induce high-effort testing and truth telling by each expert. A comparison of the resulting profits under each testing mode leads to a characterization of when to apply which testing mode: parallel testing is only optimal if testing is sufficiently cheap and if time consumption is expensive. Otherwise, tests should be executed sequentially. In addition, we show how testing outcomes' qualities for different designs drive optimal setup of sequential testing: if outcomes' qualities are rather homogeneous across different designs, the firm should mandate a single expert to execute all tests, whereas under rather heterogeneous qualities, each test should be executed by a different expert. Furthermore, based on a profit comparison, we investigate how delegation drives optimal testing mode choice. We find that delegation is in favor of sequential testing.

The second essay (joint work with Moritz Fleischmann) presented in Chapter III considers a setting related to used product acquisition that is situated in the business field of recommerce. The examined firm's main activities can be broken down into acquisition, re-processing and re-marketing of used products (mostly small electronic devices). The most prominent means to acquire used products for such a firm is the online channel. Here, product holders interested in selling can visit the firm's website, determine their products' conditions based on various aspects, and, depending on the determined conditions, receive acquisition price offers. If satisfied with the offers, the products are sent to the firm at the firm's expense. There, actual qualities are determined, and depending on their relation to product holders' statements, counteroffers can be made. Those can in turn be accepted or rejected, where rejections result in products being sent back. Rejections thus induce losses for the firm. The setting is on the one hand characterized by the firm's uncertainty about products' values for their holders, which is crucially linked to the counteroffer decision. On the other hand, product holders are uncertain about products' values for the firm in terms of how efficiently the firm reprocesses products and what can be achieved through remarketing. Those uncertainties may cause inefficiencies in the process. Motivated by this scarcely investigated acquisition process, we address the following questions: how can we optimally set up the online used-product acquisition process? What are the optimal acquisition prices? Can the presented process be improved?

To answer the questions, we first build a sequential game-theoretical model incorporating the described information asymmetries. We find the corresponding equilibrium given the acquisition prices. We observe that product holders have an incentive to upward deviations from true quality statements. Based on the equilibrium, we build an optimization model that gives the optimal quality-dependent acquisition prices. Furthermore, we present an alternative acquisition process. For this process, we build an optimization model that gives the optimal quality-dependent acquisition prices that make product holders state their products' true qualities. Finally, both processes are compared with regard to profitability. We find that the extent of uncertainty at product holders about products' quality-dependent values for the firm is negatively correlated with firm's profit for both processes. Furthermore, the larger the gaps between quality-dependent values for the firm, the more profitable the newly presented process is compared to the current process. Nevertheless, neither of the processes outperforms the other under any circumstances.

The third essay (joint work with Moritz Fleischmann and Jochen Schlapp) presented in Chapter IV investigates another setting related to used product acquisition in the business field of recommerce. The considered firm collaborates with a retailer to achieve additional used-product supply, where the retailer is equipped with the ability to test products before acquisition. Based on the obtained testing information, the retailer makes differentiated acquisition price offers. Here, the retailer has more information than the firm with regard to the effort put into testing and testing outcomes. From the firm's perspective, this may result in an undesired testing and acquisition behavior by the retailer. It is reported that the testing outcomes of a firm and retailer sometimes differ with regard to valuation of products and hence the paid acquisition prices. This difference could suggest both inaccurate testing and misaligned incentives. We address the following related research questions: should a firm acquire used products through an intermediary with or without quality-differentiation and upfront testing? How should the collaboration be set up and the intermediary made to act in the firm's favor?

In order to answer the questions, we undertake the following steps: first, we separately investigate how to optimally set up the acquisition policies based on principal-agent models. This comprises finding the optimal contracts and quality-(in)dependent acquisition prices. Based on the resulting profits, we determine conditions under which to apply which policy. We find that differentiated acquisition is only profitable if products in different conditions significantly differ in value for the firm or for product holders, if testing is cheap, and if the likelihood for products being in different conditions is significant. Furthermore, we discuss the efficiency of the acquisition channel and find that the optimal contracts coordinate the supply chain, thus resulting in the optimal centralized supply chain profit, irrespective of the acquisition policy. Finally, we discuss under what conditions collaboration can be simplified by the application of simpler contracts. For acquisition without upfront testing and differentiated prices, close collaboration with extensive information exchange is not necessary. For differentiated acquisition, this is not always the case. Here, both parties have to be willing to truthfully share information.

Chapter II

Delegated Testing of Design Alternatives: Incentives and Testing Strategy

with Jochen Schlapp[1]

2.1. Introduction

Scholars have long acknowledged the crucial role that design testing plays in the success of any research and development (R&D) or new product development (NPD) initiative (Simon 1969, Allen 1977, Clark and Fujimoto 1989, Wheelwright and Clark 1992, Thomke 1998). This view is confirmed by the significant amount of time and resources that firms invest in activities related to testing their new products. Thus the automotive industry spent more than $100 billion (US) on R&D activities in 2015, with a large portion of this budget dedicated to design testing (Jaruzelski and Hirsh 2016); Airbus spent more than seven years evaluating different design options for its next-generation A380 aircraft before deciding on the final design (*The Economist* 2007); and the high-tech sector is expected to invest 40% of its information technology budget in new testing processes—such as "virtual" testing and robotics—by 2019 (Buenen and Muthukrishnan 2016). These numbers indicate that firms continuously seek to improve their testing processes, from both a technological and a managerial perspective, as a way of reducing costs and resource consumption yet without compromising the quality of their testing efforts.

Prior academic work has identified many aspects that bear on the efficacy of a firm's design-testing process—including such diverse factors as test efficiency, testing costs, lead times, and

[1]The research presented in this chapter is based on a paper entitled "Delegated Testing of Design Alternatives: The Role of Incentives and Testing Strategy", coauthored with Jochen Schlapp.

learning effects—and thus on the firm's optimal testing strategy (see, e.g., Weitzman 1979, Dahan and Mendelson 2001, Loch et al. 2001, Erat and Kavadias 2008). So far, however, one critical aspect has received only scant attention in the literature: *design* testing is often delegated to self-interested experts who may pursue their own respective agendas. It is an organizational reality in many industries that information asymmetry between senior management and these testing experts distorts the outcomes of a testing process (Sommer and Loch 2009, Mihm 2010, Schlapp et al. 2015). The goals of this paper are to determine (i) precisely how information asymmetry affects a firm's testing process and (ii) how the firm can mitigate the negative effects of associated agency issues by devising appropriate incentive structures and an adequate testing strategy.

One can more clearly understand the effect of information asymmetry on a firm's testing process by considering a wind turbine manufacturer that seeks to set up a new "wind farm"—a grid-connected installation of multiple wind turbines—in a pre-determined location.[2] Wind farms are built to convert the wind's kinetic energy into electricity (Krohn et al. 2009). A wind farm is a viable (i.e., an economically rational) contender in the production of electricity only if it satisfies three basic requirements: "(1) produce energy, (2) survive, and (3) be cost effective" (Manwell et al. 2009, p. 505).

In fact, the "produce energy" requirement has become a moot point. Wind turbine manufacturers can now produce a variety of wind turbine designs that have proven their technological effectiveness through standardized testing procedures and widespread application in practice. Moreover, technical developments are pushing modern wind turbines closer to the theoretical efficiency limits dictated by Betz's law[3] (Burton et al. 2011, p. 63); hence current wind turbine designs are an excellent choice also for future wind farms. Yet one crucial question remains: Which turbine design is the best choice for a given wind farm location?

The answer to this question is closely tied to evaluating Manwell et al.'s requirements (2) and (3). A wind farm can be cost effective and long-lived only if the wind turbines used are technically reliable and do not result in strong negative externalities on the environment. In other words: wind farm builders are looking for the wind turbine designs that best match site-specific wind conditions, climatic factors, and regulatory constraints yet have minimal effects on animal wildlife, emit little noise, and do not generate severe electromagnetic interference (Manwell et al. 2009, pp. 321ff). Unfortunately, many of these factors are known only

[2]The siting of wind farms usually proceeds in close collaboration with regulatory bodies. Therefore, wind turbine manufacturers can influence but not ultimately control decisions about where new wind farms will be located.

[3]According to Betz's law, no turbine can capture more than 59.3% of the wind's kinetic energy.

imperfectly ex ante, and there are no standardized testing procedures for evaluating them (in contrast to the purely mechanical testing of the energy produced by a particular wind turbine design). Instead, wind farm builders must rely on teams of experts that acquire and deliver information regarding the suitability of different wind turbine designs for a given location. Since there is no standard testing procedure, this information acquisition process—and the interpretation of the acquired information—is a process that creates *tacit* knowledge and that relies strongly on the experts' prior experience and knowledge, the quality of information sources, the synthesis of implicit information, and often also on gut feeling.

As a result, it is almost impossible for a wind farm builder to assess the quality of the information on which expert recommendations are based, let alone to verify that the experts have actually shared their knowledge (and all acquired information) with senior management. To overcome this information asymmetry, wind farm builders must adequately incentivize those experts to investigate design suitability in a thorough manner and to share the test outcomes with senior management in a truthful manner. These incentives must, of course, be aligned with the firm's overall testing strategy (e.g., parallel vs. sequential testing). Hence the questions that arise are: What is the firm's optimal testing strategy, and what are the corresponding optimal incentive structures? Our paper's main contribution is development of a game-theoretic model that delivers answers to these questions.

In particular, this study makes three main contributions to the literature. First we show that—almost regardless of the firm's testing strategy—the optimal compensation schemes that adequately incentivize the experts have a surprisingly simple two-payment structure: a success bonus; and a consolation award if an expert's design is not chosen for development. The optimal balance between these two payments depends on the informational quality of an expert's test outcomes. For rather simple designs that can be tested with high precision, the firm should place a strong emphasis on the success bonus. Yet designs that are more complex and subject to a less precise testing process demand more tolerance for failure, so in these cases the firm should offer higher consolation awards. In short, a one-size-fits-all approach to incentives is not advisable.

Second, we find that the firm's optimal testing strategy depends primarily on two parameters: the testing costs and the test efficiency. For low testing costs, the firm prefers a parallel testing strategy whereby all design alternatives are tested simultaneously; for higher testing costs, a sequential testing approach is the firm's preferred choice. We also address the question of how best to conceptualize a sequential strategy under delegation; that is, in which order should the designs be tested, and how many experts should be hired for the testing process? Our analysis reveals that the greater the heterogeneity in test efficiency across the design alternatives, the more experts the firm should hire. The reason is that employing many experts makes it more difficult for any one expert to extract high information rents—an issue that is most salient when the quality of testing is unbalanced across designs. With regard to the optimal order in which design alternatives should be tested, we challenge results in the literature that argue for the optimality of "reservation price rules" (Weitzman 1979, Adam 2001, Erat and Kavadias 2008). We establish that, when there is information asymmetry, it might be better for the firm to test less promising designs *first* in order to reduce agency costs.

Our third main contribution is to show that information asymmetry always results in a suboptimal testing process; of perhaps even more importance is our finding that the negative effect of information asymmetry is greater on parallel than on sequential testing strategies. These results indicate that, under delegation, a parallel testing approach is less suitable than promised by extant research (Dahan and Mendelson 2001, Loch et al. 2001). This finding likely also explains why so few parallel testing efforts are observed in practice despite sharply reduced testing costs in recent years: firms simply want to avoid the high agency costs associated with parallel design testing.

2.2. Related Literature

The challenges associated with managing the design process of a novel product have been a long-standing and central concern in the NPD literature (Simon 1969, Allen 1977, Clark and Fujimoto 1989, Wheelwright and Clark 1992, Thomke 1998, Loch et al. 2001, Pich et al. 2002, Erat and Kavadias 2008, Sommer et al. 2009). In his foundational work, Simon (1969, pp. 128f) describes the product design process "as involving, first, the generation of alternatives and, then, the testing of these alternatives". This view has served as the foundation of much of the subsequent academic literature, and as such it has triggered numerous extensions (see, e.g., Clark and Fujimoto 1991, Wheelwright and Clark 1992, Thomke 1998, 2003). Following the seminal classification of Simon (1969), the extant literature can be divided into two broad categories. The first group of studies focuses on the *search* dimension of product design by investigating successful strategies for finding design alternatives. In contrast, research in the second group emphasizes the *testing* dimension of product development by analyzing how best to evaluate the performance of a given design alternative.

The literature on optimal *search* dates back to the pioneering work of March and Simon

(1958) and Simon (1969), who were among the first to describe organizational problem solving as a search process. This notion of viewing the innovation process as a search over a complex design landscape inspired the subsequent proposal of different conceptual models to describe the underlying search spaces. The two most influential models of search spaces are the exploration–exploitation trade-off described in March (1991) and Manso (2011) and the NK landscape of Kauffman and Weinberger (1989) and Kauffman (1993). Building on these conceptualizations of a search space, scholars have extensively investigated how the efficiency of the search process—and thus the firm's innovation performance—changes with the complexity of the problem (Ethiraj and Levinthal 2004, Mihm et al. 2003, Billinger et al. 2014), organizational hierarchy (Rivkin and Siggelkow 2003, Siggelkow and Rivkin 2005, Mihm et al. 2010), team structure (Kavadias and Sommer 2009, Girotra et al. 2010), unforeseeable uncertainties (Sommer and Loch 2009, Sommer et al. 2009), the particular search strategy employed (Sommer and Loch 2004, Kornish and Ulrich 2011), and competition (Oraiopoulos and Kavadias 2014). More recently, Erat and Krishnan (2012), Lewis (2012), and Ulbricht (2016) have analyzed how delegation affects both the breadth and overall performance of a search process. All these cited papers focus on how to discover a set of potentially promising design alternatives, which is the quintessential first step in an innovation process. However, we are concerned with the second step in that process: determining the most reliable way to select the best alternative from among the candidates. As a consequence of that different focus, the formal model we propose differs considerably from those in the search literature.

Much closer to our work is the literature on design *testing* as initiated by Weitzman (1979). In his terminology, any design alternative can be considered a “black box”, and uncertainty about its value can be resolved only by costly testing activities. This generic model of a testing process has become a building block for almost all research on design testing, and it has proven itself flexible enough to accommodate two very different kinds of testing processes: feasibility testing and selection testing.

The primary purpose of *feasibility testing* is to discover whether (or not) a given design is technologically feasible. Answering this question requires that the design in question be repeatedly tested until there is sufficiently strong evidence either for or against its feasibility. Prime application areas for this testing procedure include the pharmaceutical industry, where a new molecule is tested and retested during clinical trials to evaluate whether it reliably produces the desired effects. Research on feasibility testing seeks to answer such questions as when in the development process to test the design as well as how many tests to pursue and at what level of fidelity to a real-world counterpart (see, e.g., Thomke and Bell 2001, Terwiesch and Loch 2004). In the economics literature, the question of how to motivate an expert's participation in this dynamic information acquisition process has recently gained traction (Gromb and Martimort 2007, Gerardi and Maestri 2012, Hörner and Samuelson 2013), leading to a theory of optimal incentives for feasibility testing. There are two ways in which our paper is connected to that stream of literature. First, from previous work on feasibility testing we borrow the insight that nearly all testing processes are imperfect and so, even with the most thorough of testing efforts, there will still be uncertainty about a design's true value. Second, we answer the question of how the firm should manage delegated testing activities when it is concerned with selection testing—the second main challenge in design testing. Thus our work complements the extant literature and broadens our knowledge about devising optimal incentives and strategies for delegated design testing.

In contrast to feasibility testing, which focuses on the technological feasibility of a single design, *selection testing* is concerned with choosing the best alternative out of a set of different candidate designs. For instance, as explained in the Introduction, wind farm builders are confronted with such a selection issue when choosing a particular wind turbine design for a new wind farm. The existing literature describes two diametrically opposed testing strategies to tackle this issue: sequential and parallel testing. Weitzman (1979) advocates the use of a sequential testing approach, in which the different designs are tested in sequence and the testing process can be stopped after each design test. In his seminal contribution, he establishes the now classical reservation price rule (a.k.a. "Pandora's rule") for determining both the order in which to test the alternatives and also when to stop testing. Adam (2001) and Erat and Kavadias (2008) study how these results are affected by the firm's ability to learn between design tests. In contrast to those papers, Dahan and Mendelson (2001) promote the use of a parallel testing approach in which all design alternatives are tested simultaneously. Loch et al. (2001) build on these results by directly comparing the performance of sequential and parallel testing strategies. They find that sequential testing is preferable when design tests are expensive and test efficiency is low whereas parallel testing is preferable when testing processes are slow and the firm needs quick results.

It is remarkable that past work on selection testing has not considered the role played by information asymmetry in the design of an optimal testing process—that is, given the ubiquity of such asymmetries in practice. Our principal contribution is to investigate how, exactly, delegation affects a firm's optimal testing strategy; we derive simple yet optimal incentive structures that counter the effects of information asymmetries. The analysis yields several new insights regarding the management of delegated testing processes. First, our derivation of the firm's optimal testing strategy reveals that information asymmetry is much more detrimental to a parallel than to a sequential testing strategy. This finding implies that, when testing is delegated, parallel strategies are probably less effective than advertised (e.g., Dahan and Mendelson 2001, Loch et al. 2001). Second, the existing literature is silent about how to conceptualize a sequential testing approach; that is, should the firm hire multiple experts to test the different design alternatives, or should it rather assign all testing activities to a single expert? We show that the multi-expert approach is preferable when test efficiency varies considerably across the design alternatives whereas the single-expert approach is preferable when test efficiency is relatively homogeneous. As a corollary we also find that, under delegation, the classical reservation price rules (promulgated in Weitzman 1979, Adam 2001, Erat and Kavadias 2008) no longer generate the optimal order for testing alternative designs. Finally, we derive the optimal incentive structures for delegated design testing, which turn out to be extremely simple irrespective of the chosen testing strategy.

2.3. Model Setup

Consider a firm engaged in NPD; it faces the challenge of selecting one out of $N \geq 1$ possible design alternatives for the new product being developed. The value that the firm receives from choosing a certain design depends on two factors: the design's technological feasibility, which is uncertain at the outset and can end up being either good or bad; and the design's inherent economic potential. The firm's goal is to choose and develop the design alternative that offers the highest value upon implementation.

At the core of our model is a testing phase in which the firm can acquire costly but imperfect information regarding each design's technological feasibility and, eventually, its value. Yet the firm cannot access this information directly. Instead, it must delegate the desired testing activities to experts who then collect information about the designs' technological feasibility through experiments, simulations, and/or prototype building. Each design test gives the corresponding expert more refined information about the evaluated design's feasibility, which enables that expert to provide a more informed recommendation vis-à-vis the firm's decision on whether or not to develop the focal design. Once the firm has collected enough recommendations, it develops the most promising design alternative; if no alternative is sufficiently convincing, then the development process may be abandoned.

In the real world, the delegated nature of this testing process gives rise to information asymmetry between the firm and the experts. Thus it is difficult if not impossible for the firm to verify the informational quality of an expert's recommendation. Two factors are responsible for this adverse situation. First, testing activities are costly for the expert, who may therefore choose to be less diligent with respect to some tests than others. However, firms cannot ascertain the diligence of experts because the interpretation of testing outcomes relies critically on an expert's gathering and synthesis of information and—most notably, owing to the tacit nature of such knowledge—prior experience. Second, experts may be reluctant to share their testing outcomes truthfully with the firm. This form of information asymmetry captures the reality that experts can use their recommendations strategically to influence the firm's design choice. So in practice, delegated design testing involves two different forms of information asymmetry: *moral hazard* during the testing phase and *adverse selection* during the recommendation phase. It follows that the firm must offer an appropriately designed compensation scheme if it hopes to incentivize experts to test the focal design(s) thoroughly and then to communicate the testing outcomes truthfully. This compensation scheme, in turn, should be carefully coordinated with the firm's second major decision: the choice of testing strategy. In particular, the firm must decide about whether the design alternatives should be tested in parallel or in sequence, how many experts to employ, how many design alternatives to test, and (if tested in sequence) the best order in which to test the different designs. In the rest of this section we provide more detail on our model setup and assumptions.

2.3.1. Delegated Design Testing

The value V_i that the firm receives from developing design $i \in \mathcal{N} = \{\infty, \ldots, \mathcal{N}\}$ depends on the design's technological feasibility $\Theta_i \in \{G, B\}$, its inherent economic potential $v_i > 0$, and the development costs $K \geq 0$. We assume more specifically that, once developed, a technologically feasible design ($\Theta_i = G$) generates a value of $v_i - K$ whereas an unfeasible design ($\Theta_i = B$) results in a loss of $-K$. Prior to development, however, each design's technological feasibility is uncertain; that is, neither the firm nor the experts know the design's true feasibility ex ante. To simplify the presentation, we assume also that each state is ex ante equally likely.[4] Experts engage in costly testing activities in their efforts to (partially) resolve this uncertainty. We represent the testing activities for design i by an expert's testing effort e_i, which can be either high ($e_i = h$) or low ($e_i = l$) and is not observable by the firm. An expert who engages in high-effort testing incurs a private cost $c > 0$, whereas the costs of low-effort testing are normalized to zero. Of course, the chosen testing effort affects the

[4]The sole purpose of this assumption is to reduce the complexity of our mathematical exposition. From a structural standpoint, all our results continue to hold for arbitrary prior probabilities.

quality of collected information. Formally, we model testing outcomes for each design i as an imperfect signal $s_i \in \{g, b\}$, which is received only by the expert testing design i and that indicates whether the design is technologically feasible ($s_i = g$) or not ($s_i = b$). We denote the precision (or quality) of this signal $q(e_i)$ because it depends on the expert's testing effort: an expert who exerts high effort receives a signal of quality $q(e_i = h) = q_i \in (1/2, 1]$; in contrast, low effort leads to an uninformative signal $q(e_i = l) = 1/2$. We assume that signals are stochastically independent across designs.

After receiving signal s_i, the expert updates—in accordance with Bayesian rationality—her belief about design i's technological feasibility. Using this refined information, the expert gives the firm an unverifiable recommendation $r_i \in \{g, b\}$, which states whether design i is considered to be technologically feasible ($r_i = g$) or unfeasible ($r_i = b$).

2.3.2. The Firm's Decisions

The ultimate goal of a firm is to maximize expected profits by selecting the most valuable design for its product while minimizing the costs of a delegated testing process. For this problem to be relevant, we assume that $q_i v_i \geq K \geq (1 - q_i)v_i$ for all $i \in \mathcal{N}$. Otherwise, the firm's decision would be a trivial one: if $q_i v_i < K$ then the firm could safely exclude design i from consideration because of the economic irrationality of developing such a design; at the other extreme, if $(1 - q_i)v_i > K$ then design i is so promising that the firm would develop it even without prior testing.

Designing an optimal testing process requires that the firm determine the testing strategy and also a scheme for compensating the experts. With regard to the former decision, we assume that the firm can select among three different testing strategies: parallel testing, multi-expert sequential testing, and single-expert sequential testing. Under a *parallel* testing strategy, the firm first decides on the number $|\mathcal{I}_P|$ and identity $\mathcal{I}_P \subseteq \mathcal{N}$ of design alternatives to test; it then assigns a separate expert to each design $i \in \mathcal{I}_P$, and all testing processes are carried out simultaneously. After reviewing the experts' recommendations, the firm decides which design (if any) to develop. Under a *multi-expert sequential* testing strategy, the firm again decides on the number (here, $|\mathcal{I}_M|$) and identity ($\mathcal{I}_M \subseteq \mathcal{N}$) of design alternatives to test—but it also determines the order in which the different designs will be tested. The firm then assigns a different expert to each design $i \in \mathcal{I}_M$ and the designs are tested, one after the other, in the order specified. After each design test, the firm can choose to stop the testing process and develop the latest design alternative; we assume that the firm always does so after receiving a good/feasible recommendation for the current design alternative. An intuitive consequence of this assumption is that, once a firm in this position continues with the testing process, it can no longer implement any previously tested design.[5] Throughout the modeled testing process, all payments are discounted at a constant rate of $\delta \in (0, 1]$ after each design test. Under a *single-expert sequential* testing strategy, the firm makes the same decisions as in the multi-expert case (viz., deciding on $|\mathcal{I}_S|$, on $\mathcal{I}_S \subseteq \mathcal{N}$, and on the order of tested designs); the only difference here is that just one expert is assigned to perform all the tests. As before, the firm can stop the testing process after each design test and develop the latest design alternative (which always occurs if the firm is given a good recommendation) yet does not have the option of developing any formerly tested design.

As regards the firm's schemes for compensating the experts, we allow the firm to offer asymmetric and nonlinear contracts that include any combination of action- and evidence-based payments.[6] An *action-based* payment is contingent on a specific action taken by the firm—for instance, developing design i.[7] In contrast, an *evidence-based* payment depends on a design's true technological feasibility Θ_i. In practice, any pay-for-performance contract must be action-based and/or evidence-based; the reason is that only such criteria are verifiable and thus enforceable by courts. In our model, then, an expert who is testing design i is eligible for the following payment types: (i) a "success bonus" u_{ig} if the firm successfully develops design i (i.e., $\Theta_i = G$); (ii) an "allowance" u_{ib} if the firm's development of design i fails (i.e., $\Theta_i = B$); (iii) a "consolation award" u_{ia} if the expert tested design i but the firm did not choose it for development; and (iv) a "termination bonus" u_{it} if none of the available design alternatives is chosen for development. We assume throughout that the firm must make all wage payments immediately when due; that is, it cannot hold back any wages. It is intuitive that the evidence-based payments u_{ig} and u_{ib} incentivize an expert to test the design thoroughly and also to recommend the design's development in the event of a favorable signal. In contrast, u_{ia} and u_{it} reflect the firm's tolerance for failure. That is to say, the firm appreciates an expert's testing efforts even in the case of negative testing outcomes; hence this payment incentivizes experts to refrain from recommending a bad design for development.

Given such compensation schemes, the utility π_i received by an expert for testing design i is

[5] As discussed in Sections 2.4.2 and 2.6, this assumption greatly reduces mathematical complexity yet has almost no bearing on the generalizability of our results. Furthermore, if the firm were unwilling to develop favorably recommended designs, then experts would have no motive to expend effort testing design alternatives.

[6] Note that, without loss of optimality, we do not need to consider fixed wages because in optimum, such a fixed wage must be zero as experts are shielded by limited liability.

[7] Payments cannot depend on the *expert*'s action because such actions cannot be verified.

the (discounted) sum of all his wage payments net of his effort costs. We follow the principal–agent literature in assuming that all experts are risk neutral and protected by limited liability; in other words, the compensation of each expert must be nonnegative at all times. The firm is risk neutral, too, and its profit Π consists of the realized value of the developed design (net of any development costs) *minus* the compensation paid to experts.

2.4. Incentives for Delegated Testing

In this section we characterize the optimal compensation schemes for the different testing strategies, given that the set of design alternatives $\mathcal{I} \subseteq \mathcal{N}$ (with $|\mathcal{I}| = n \geq 1$) is exogenously fixed.[8] In line with the revelation principle, we limit our attention to contracts that incentivize all experts to evaluate design(s) thoroughly and to reveal test outcomes truthfully. We begin by identifying the optimal compensation scheme for parallel testing (Section 2.4.1), after which we characterize the optimal contract for multi-expert sequential testing and derive the optimal testing order (Section 2.4.2). Finally, we examine the optimal incentive scheme and testing order for the single-expert sequential testing strategy (Section 2.4.3).

2.4.1. Parallel Testing

Under a parallel testing strategy, the firm assigns a different expert to each design $i \in \mathcal{I} = \{1, \ldots, n\}$, and all experts then perform their testing activities simultaneously. This setup allows the firm to receive refined information on each design's value and to use this information to select, ex post, the most promising alternative for development. The firm's overarching goal is to maximize expected profits, which amount to the expected market value of the developed design *net* of development costs and compensation paid to experts. Formally, the firm solves the following optimization problem P (the mathematical derivation

[8] We endogenize the firm's choice of design alternatives in Section 2.5.1.

and all other formal proofs, has been relegated to the Appendix):

$$P: \max_{u,y} \ \Pi := \sum_{j=1}^{n} \frac{1}{2^j} \left[\sum_{i=1}^{n} y_i^{(j)} (q_i v_i - q_i u_{ig} - (1-q_i) u_{ib} - (2^j - 1) u_{ia} - 2^{j-n} u_{it}) \right] - \sum_{j=1}^{n} \frac{1}{2^j} K \tag{2.1}$$

$$\text{s.t. } y_i^{(j)} (q_i u_{ig} + (1-q_i) u_{ib}) \geq y_i^{(j)} (u_{ia} + 2^{j-n} u_{it}) \qquad \forall i, j \in \mathcal{I} \tag{2.2}$$

$$y_i^{(j)} ((1-q_i) u_{ig} + q_i u_{ib}) \leq y_i^{(j)} (u_{ia} + 2^{j-n} u_{it}) \qquad \forall i, j \in \mathcal{I} \tag{2.3}$$

$$y_i^{(j)} (u_{ig} - u_{ib}) \geq y_i^{(j)} 2^{j+1} c / (2q_i - 1) \qquad \forall i, j \in \mathcal{I} \tag{2.4}$$

$$y_i^{(j)} [q_i (v_i - u_{ig}) - (1-q_i) u_{ib} - \sum_{k \neq i} u_{ka}] \geq y_i^{(j)} \sum_{l=1}^{n} y_l^{(j+1)} [q_l (v_l - u_{lg}) - (1-q_l) u_{lb} - \sum_{k \neq l} u_{ka}] \quad \forall i \in \mathcal{I}, j \in \mathcal{I} \backslash \{n\} \tag{2.5}$$

$$\sum_{i=1}^{n} y_i^{(j)} = 1, \ \sum_{j=1}^{n} y_i^{(j)} = 1, \ y_i^{(j)} \in \{0, 1\} \qquad \forall i, j \in \mathcal{I} \tag{2.6}$$

$$u_{ig}, u_{ib}, u_{ia}, u_{ia} + u_{it} \geq 0 \qquad \forall i \in \mathcal{I} \tag{2.7}$$

Although complex at first sight, this optimization problem P has an intuitive structure. As a starting point, note that $y_i^{(j)}$ is an indicator variable that reflects whether design i is the firm's jth most preferred alternative (constraint (2.6) ensures that this mapping is indeed one-to-one). That is, if $y_i^{(j)} = 1$ then the firm chooses design i for development only if it receives an unfavorable recommendation for all designs with a lower ranking $j' < j$. Clearly, a design's attractiveness is not exogenously given and instead depends endogenously on the offered compensation scheme. This fact is reflected by (2.5), which guarantees that the firm makes an ex post optimal selection decision. Conditions (2.2)–(2.4) represent each expert's incentive compatibility constraints, which depend on the relative attractiveness of the design she has been assigned to evaluate. Thus (2.2) and (2.3) ensure that each expert truthfully reveals, respectively, a "good" and a "bad" signal. These constraints eliminate the adverse selection problem during the recommendation phase. Condition (2.4) similarly negates the moral hazard problem during the design-testing phase because it ensures that each expert prefers high-effort to low-effort testing. Finally, experts are protected by limited liability; hence (2.7) ensures that all wage payments are nonnegative. The following proposition characterizes—under mild conditions on the properties of designs in $\mathcal{I}$—the optimal incentive structures for parallel testing and the resulting firm profits.[9]

Proposition 2.1 (Parallel Testing). *Suppose the designs in $\mathcal{I}$ can be ordered such that $q_i v_i \geq q_{i+1} v_{i+1} + 2^{i+1} c[q_i/(2q_i - 1) - 2q_{i+1}/(2q_{i+1} - 1)]^+$ for all $i \in \mathcal{I}\backslash\{n\}$, where $[x]^+ = \max\{0, x\}$. Then the following statements hold.*

(i) *Under a parallel testing strategy, the optimal contract that induces truth telling and high-effort testing for each design satisfies $u_{ig} = 2^{i+1}c/(2q_i - 1)$, $u_{ib} = 0$, $u_{ia} = 0$, and $u_{it} = 2^{n+1}(1-q_i)c/(2q_i-1)$ for all $i \in \mathcal{I}$. Moreover, $u_{it}/u_{ig} = 2^n(1-q_i)/2^i < 2^{n-i-1} \leq 2^{n-2}$ for all $i \in \mathcal{I}$.*

(ii) *Ex ante, the firm's expected profit is $\Pi_P = \sum_{i=1}^{n}((q_i v_i - K)/2^i - 2c/(2q_i - 1))$.*

Perhaps the most remarkable aspect of this proposition is the simplicity of the optimal contract's structure. For each design (and thus for each expert) $i \in \mathcal{I}$, the firm need offer only two payments: a success bonus u_{ig} if design i is successfully developed, and a termination bonus u_{it} if none of the design alternatives is chosen for development (i.e., if test outcomes indicate that all designs are technologically unfeasible). But what respective roles do these payments play in the firm's incentive system? As (2.4) reveals, the primary purpose of u_{ig} is to motivate expert i to engage in high-effort testing. Put differently, u_{ig} is a purely individual incentive that resolves each expert's moral hazard concern. In contrast, u_{it} is a common (or shared) incentive that collectively compensates the experts if all designs are considered to be technologically unfeasible. It therefore decreases the likelihood of an expert giving a positive recommendation despite receiving a negative test outcome—and thereby induces truth telling; see constraint (2.3).

It is intuitive that, when effort becomes less rewarding (i.e., effort costs c increase) and recommendations become less reliable (the signal quality q_i decreases), expert i becomes more reluctant to invest high effort and to report test outcomes truthfully. Under these circumstances, the firm must provide stronger incentives; this explains why u_{ig} and u_{it} are increasing in c and decreasing in q_i. Conversely, in the extreme case of perfect testing ($q_i = 1$), expert i has no incentive to misrepresent the test outcomes because such a false recommendation would be easily detected by the firm and so would not benefit him. It follows that if testing is perfect then the firm can forgo payment of any shared incentives ($u_{it} = 0$).

[9] The condition in Proposition 2.1 holds unless $\mathcal{I}$ contains a design for which the test is of exceptionally low efficiency.

Whereas neither u_{ig} nor u_{it} depends directly on a design's inherent economic potential v_i, these terms *are* affected by the total number n of designs to be tested and by a design's relative value, which we also index via i. In particular, u_{ig} increases with i because, with a higher index i, it becomes more likely for tests to indicate that a design with a smaller index is technologically feasible—which would render futile the expert's testing efforts. Similarly, with a higher n it becomes less likely that all n designs are technologically unfeasible; hence the termination bonus u_{it} is correspondingly less likely to be paid out. To compensate the experts for this reduced payment probability, the firm must offer a higher u_{it}.

Proposition 2.1(i) also sheds light on the severity of adverse selection—as compared with moral hazard—when the firm employs a parallel testing strategy. For designs that are extremely promising ex ante (i.e., those with a small index i), the ratio u_{it}/u_{ig} is high; the implication is that, for these designs, the firm's central concern is to incentivize truth telling. For ex ante less promising designs (those with a large index i) the ratio is low; in that event, the firm becomes relatively more concerned with incentivizing experts to exert high effort. This shifting priority of the optimal compensation scheme has an appealing explanation. From an expert's perspective, admitting that her own testing outcome is bad increases the odds of receiving *no* payments. This conclusion follows from the extreme unlikelihood of all tested designs being pronounced technologically unfeasible. The firm's concern on this point is especially strong for designs that show the most promise. At the same time, an expert assigned to a promising design has a significant intrinsic motivation to exert high effort because the potential rewards from receiving a good test outcome are high. In contrast, an expert assigned to a less promising design fears that her efforts are futile because, in all probability, a more promising design will receive a good recommendation and so the results of her testing could be irrelevant to the firm. Therefore, relatively higher individual incentives must be offered to the experts who are assigned to test less promising designs.

Finally, Proposition 2.1(ii) reveals that the firm's expected profit is decreasing in c and increasing in q_i. The reason is that, with a higher c and a lower q_i, expert i is less willing to exert high effort and to disclose the test outcomes truthfully. Hence the incentive misalignment between the firm and expert i widens, which enables the expert to extract higher information rents.

2.4.2. Multi-Expert Sequential Testing

Under a multi-expert sequential testing strategy, the firm assigns a different expert to test each design $i \in \mathcal{I} = \{1, \ldots, n\}$ and announces the testing sequence; then the experts carry out their design tests one after the other. A sequential testing approach allows the firm to stop the testing process when it receives a positive recommendation (i.e., so it can start developing that design) or to test the next viable alternative when it receives a negative recommendation.

For a given $\mathcal{I}$ and any testing order, the firm must solve the following optimization problem to derive the optimal compensation schemes. For expositional simplicity, we relabel the design alternatives such that a design's index i is identical to its position in the testing order; thus design $i = 1$ is tested first, $i = 2$ second, and so on. The optimization problem M is expressed formally as follows:

$$\begin{aligned} M: \max_{u} \ \Pi := & \sum_{i=1}^{n} \frac{\delta^{i-1}}{2^i}(q_i v_i - K) \\ & - \sum_{i=1}^{n} \frac{\delta^{i-1}}{2^i}(q_i u_{ig} + (1-q_i)u_{ib} + u_{ia} + 2^{i-n}\delta^{n-i}u_{it}) \end{aligned} \tag{2.8}$$

$$\text{s.t.} \quad q_i u_{ig} + (1-q_i)u_{ib} \geq u_{ia} + 2^{i-n}\delta^{n-i}u_{it} \qquad \forall i \in \mathcal{I} \tag{2.9}$$

$$(1-q_i)u_{ig} + q_i u_{ib} \leq u_{ia} + 2^{i-n}\delta^{n-i}u_{it} \qquad \forall i \in \mathcal{I} \tag{2.10}$$

$$u_{ig} - u_{ib} \geq \frac{4c}{2q_i - 1} \qquad \forall i \in \mathcal{I} \tag{2.11}$$

$$u_{ig}, u_{ib}, u_{ia}, u_{it} \geq 0 \qquad \forall i \in \mathcal{I} \backslash \{n\}, \qquad u_{ng}, u_{nb}, u_{na} + u_{nt} \geq 0 \tag{2.12}$$

The structure of the optimization problem M is similar to the firm's optimization problem P under a parallel testing strategy. Specifically, constraints (2.9) and (2.10) ensure that all experts truthfully reveal their testing outcomes, (2.11) guarantees that each expert engages in high-effort testing, and (2.12) accounts for the experts' limited liability. In the next proposition we derive the firm's optimal compensation schemes, describe the optimal testing order, and state the resulting firm profits for a multi-expert sequential testing strategy.

Proposition 2.2 (Multi-Expert Sequential Testing). (i) *Under a multi-expert sequential testing strategy and for any given testing order, the optimal contract that induces truth telling and high-effort testing by all experts satisfies, for all* $i \in \mathcal{I}$: $u_{ig} = 4c/(2q_i - 1)$,

$u_{ib} = 0$, $u_{ia} = 4(1 - q_i)c/(2q_i - 1)$, *and* $u_{it} = 0$. *In addition,* $u_{ia}/u_{ig} = 1 - q_i < 1/2$.

(ii) *It is optimal to test the designs in* $\mathcal{I}$ *in decreasing order of* $R_i \equiv q_i v_i - 4c/(2q_i - 1)$.

(iii) *Ex ante, the firm's expected profit is* $\Pi_M = \sum_{i=1}^{n} \delta^{i-1}(q_i v_i - K - 4c/(2q_i - 1))/2^i$.

As in the optimal contract for parallel testing, there are only two payments in the optimal compensation scheme for multi-expert sequential testing. To resolve each expert's moral hazard, the firm must again reward expert i with a success bonus u_{ig} in the event design i is developed successfully. Yet in this case the firm does not rely on shared incentives to induce truth telling; that is, $u_{it} = 0$. Instead the firm provides an individual consolation award u_{ia} to reimburse expert i for his effort costs whenever the firm dismisses the design he tested owing to the subsequent unfavorable recommendation.

This focus on purely individual incentives has two immediate consequences. First, the optimal payments depend not on a design's position in the testing order but only on its informational quality q_i: the higher the informational quality, the more aligned are the interests of firm and experts and so the lower is the compensation offered. Second, as the ratio u_{ia}/u_{ig} indicates, if testing is sequential then moral hazard is a much greater concern than adverse selection, especially when q_i is high. This finding may be better understood if one notes that, from an expert's perspective, recommending a technologically unfeasible design for development leads to zero income ($u_{ib} = 0$). So once an expert has invested high effort in testing, there is hardly any point in trying to pass off a technologically unfeasible design as a good one. Even so, motivating the expert to engage in high-effort testing at the outset requires a high effort incentive (high u_{ig}).

The optimal testing order is given in part (ii) of Proposition 2.2, the essence of which is that the firm should test designs in decreasing order of their expected net contribution R_i. This result is in the spirit of Weitzman's (1979) reservation price rule but extends it to include the costs of delegation. That is, the testing order depends not only on the designs' expected values $q_i v_i$ but also on the experts' information rents. Since these information rents are decreasing in q_i and invariant with respect to v_i, it follows that the firm—as compared to the reservation price rules advocated by Weitzman (1979), Adam (2001), and Erat and Kavadias (2008)—more strongly prefers first to test designs of high informational quality. The structure of R_i indicates that the firm's optimal testing order is myopic: a design's expected net contribution depends only on its own properties and so is independent of other design alternatives.

Recall our argument that an expert's information rents are decreasing in the quality of her information and increasing in her effort costs. From these relations it clearly follows that the firm's expected profit should be increasing in q_i and decreasing in c. Proposition 2.2(iii) confirms this intuition and also underscores how the firm's profit is adversely affected when δ, the time value of money, is low.

Finally, we emphasize that the compensation scheme and testing order presented in Proposition 2.2 remain optimal even if the firm is allowed to develop formerly tested (yet rejected) design alternatives, that is, if the firm can test "with recall". To see this, note that if the firm receives a good recommendation for some $i \in \mathcal{I}$ then it can immediately realize an expected profit of $q_i v_i - K - 4q_i c/(2q_i - 1)$ by developing design i right away. If instead the firm chooses to test the next design alternative, then the optimal ordering in Proposition 2.2(ii) implies that the firm's expected continuation profit is strictly smaller. We conclude that, as soon as the firm receives a good recommendation for a particular design, it is optimal to develop this design at once—rendering a recall option superfluous.

2.4.3. Single-Expert Sequential Testing

Under a single-expert sequential testing strategy, the firm assigns a single expert to test in sequence the design alternatives in $\mathcal{I}$. It is easy to see that—as compared with a multi-expert strategy—such reliance on the testing efforts of only a single expert will have a strong bearing on the required incentives. On the one hand, a single expert is much more inclined (than is one in a group of experts) to acknowledge an unfavorable test outcome because there is always the chance of finding a technologically feasible design later in the testing process. On the other hand, it is extremely difficult to continue incentivizing an expert to exert high testing efforts. Thus an expert's behavior during the testing process is strongly affected by his anticipation of future actions and payments.

For a given $\mathcal{I}$ and any testing order, the firm's incentive design problem is as follows. As in the preceding section, we relabel the design alternatives such that a design's index i is identical to its position in the testing order. The incentive design problem S is then

$$S: \quad \max_u \; \Pi(u) := \sum_{i=1}^{n} \frac{\delta^{i-1}}{2^i}(q_i v_i - K) - \sum_{i=1}^{n} \frac{\delta^{i-1}}{2^i}(q_i u_{ig} + (1-q_i)u_{ib} + u_{ia}) \tag{2.13}$$

$$\text{s.t.} \quad q_i u_{ig} + (1-q_i)u_{ib} \geq u_{ia} + \delta\hat{\pi}_i \qquad \forall i \in \mathcal{I} \tag{2.14}$$

$$(1-q_i)u_{ig} + q_i u_{ib} \leq u_{ia} + \delta\hat{\pi}_i \qquad \forall i \in \mathcal{I} \tag{2.15}$$

$$u_{ig} - u_{ib} \geq \frac{4c}{2q_i - 1} \qquad \forall i \in \mathcal{I} \tag{2.16}$$

$$\hat{\pi}_{i-1} = (q_i u_{ig} + (1-q_i)u_{ib} + u_{ia} - 2c + \delta\hat{\pi}_i)/2, \quad \hat{\pi}_n = 0 \qquad \forall i \in \mathcal{I} \tag{2.17}$$

$$u_{ig}, u_{ib}, u_{ia} \geq 0 \qquad \forall i \in \mathcal{I} \tag{2.18}$$

Some peculiarities of the optimization problem S warrant further discussion. First, with a single expert it is unnecessary to have an additional termination bonus u_t that compensates her when none of the design alternatives is developed. In fact, such a payment can—without loss of optimality—be folded into the expert's consolation award for testing the last design alternative (u_{na}). This follows because both payments have the same requirements and are executed simultaneously. Second, $\hat{\pi}_{i-1}$ as defined in (2.17) is the expert's expected continuation utility immediately *before* testing design i. Since the expert's decision-making process accounts for her own future utility, it is only natural for $\hat{\pi}_i$ to become an integral part of her incentive constraints; see (2.14) and (2.15). More precisely: as compared with a multi-expert sequential testing strategy, a single sequentially testing expert is more (resp. less) likely to report an unfavorable (resp. favorable) signal truthfully. Our explanation is that the expert may enjoy additional information rents by artificially keeping the testing process alive (i.e., by concealing a signal of feasibility). As shown by Proposition 2.3, that possibility has important consequences for the optimal compensation scheme.

Proposition 2.3 (Single-Expert Sequential Testing). (i) *Under a single-expert sequential testing strategy and for any given testing order, the optimal contract that induces truth telling and high-effort testing for all designs satisfies, for all $i \in \mathcal{I}$: $u_{ig} = 4c/(2q_i - 1) + [\delta\hat{\pi}_i/q_i - 4c/(2q_i - 1)]^+$, $u_{ib} = 0$, and $u_{ia} = [4(1-q_i)c/(2q_i - 1) - \delta\hat{\pi}_i]^+$. Moreover, $u_{ia}/u_{ig} \leq 1 - q_i < 1/2$.*

(ii) *If the design alternatives in $\mathcal{I}$ can be ordered such that $q_i v_i \geq q_{i+1}v_{i+1}$, $q_i \geq q_{i+1}$, and $(1-q_i)4c/(2q_i - 1) \leq \delta\hat{\pi}_i \leq 4q_i c/(2q_i - 1)$ for all $i \in \mathcal{I}\backslash\{n\}$, then it is optimal to test in* *increasing order of* i.

(iii) *Ex ante, the firm's expected profit is* $\Pi_S = \sum_{i=1}^{n} \delta^{i-1}(q_i v_i - K - \max\{4q_i c/(2q_i - 1), \delta\hat{\pi}_i, 4c/(2q_i - 1) - \delta\hat{\pi}_i\})/2^i$.

Although the optimal contract for a single-expert sequential testing strategy is structurally similar to that for a multi-expert strategy, there are some important differences. First of all, under single-expert sequential testing, the firm needs to place more emphasis on motivating high-effort testing and less on inducing experts to report truthfully. Correspondingly, the success bonus u_{ig} is higher under single-expert than multi-expert sequential testing while the consolation award u_{ia} is substantially lower. In fact, it may be optimal for the firm to offer no consolation award at all ($u_{ia} = 0$). That would be the case for sufficiently large values of $\hat{\pi}_i$, the expert's expected continuation utility. Here the single expert anticipates substantial future payments if the testing process continues; therefore, under sequential testing, that expert will always truthfully report an unfavorable signal. This dynamic has the effect of eliminating the adverse selection problem.

Second, and in contrast to the multi-expert strategy detailed previously, the optimal payments related to each design i are not myopic in the single-expert setting; instead those payments depend on the informational quality of all designs tested *after* design i. This result is a natural and direct consequence of the expert's strategic behavior, and as such it bears implications for the optimal testing order. Following the logic of Weitzman (1979) and Proposition 2.2(ii), one might well suppose that it is still optimal to test the designs in decreasing order of their expected net contribution. However, that supposition is not true in general. We can see from Proposition 2.3(ii) that such a testing order is optimal only if the expert anticipates a moderate level of continuation utilities (i.e., only if $(1 - q_i)4c/(2q_i - 1) \leq \delta\hat{\pi}_i \leq 4q_i c/(2q_i - 1)$). However, if continuing with the testing process promises continuation utilities that are exceptionally high or low, then the firm should *not* test the designs in decreasing order of attractiveness. It might rather be optimal to test the least promising designs first—with the goal of reducing the expert's strategic rent extraction.

Finally, Proposition 2.3(iii) gives the firm's expected profit under a single-expert sequential testing strategy and yields a rather surprising result. Unlike the other testing strategies, under single-expert sequential testing the firm's expected profit need not increase with quality q_i. Because of the expert's strategic behavior, a higher q_i for one design might result in the expert extracting higher information rents from the other designs being tested; this would, of course, *reduce* the firm's overall profits.

2.5. Comparison of Testing Strategies

So far, we have characterized the optimal incentive structure for the three different testing strategies given that the set of design alternatives was exogenously fixed. As a next step, we relax this assumption and determine the optimal number and identity of designs to test for each of the three testing strategies (Section 2.5.1). We then build on these results by deriving the optimal testing strategy as a function of our main contextual parameters (Section 2.5.2). Finally, we offer some insights regarding the question of how delegation, which entails information asymmetry, alters the relative ranking of testing strategies—that is, from the ranking in an otherwise identical setting but *without* information asymmetry (Section 2.5.3).

2.5.1. Optimal Set of Design Alternatives

In Section 2.4 we derived optimal compensation schemes for the different testing strategies while assuming that the firm intended to test a fixed set $\mathcal{I} \subseteq \mathcal{N}$ of design alternatives. A more realistic scenario is one in which the set of designs to test is not given exogenously but instead is chosen by the firm. In this section, then, we characterize the optimal sets of designs to be tested for the different testing strategies. Our results reveal that these sets vary considerably across those strategies.

Proposition 2.4 (Optimal Design Alternatives). (i) *Under a multi-expert sequential testing strategy, the optimal set of designs to be tested is* $\mathcal{I}_M = \{i \in \mathcal{N} \mid q_i v_i - K - 4c/(2q_i - 1) \geq 0\}$.

(ii) *Under a parallel testing strategy, the optimal set of designs to be tested satisfies* $\mathcal{I}_P \subseteq \mathcal{I}_M$.

(iii) *Let* $\mathcal{I}_S$ *be the optimal set of designs to be tested under a single-expert sequential testing strategy, and let* n *be the last design in the optimal testing order. If* $q_n v_n - K - 4c/(2q_n - 1) \geq 0$ *and if* $q_n v_n \leq q_i v_i$ *and* $q_n \leq q_i$ *for all* $i \in \mathcal{I}_S \backslash \{n\}$, *then* $\mathcal{I}_S \subseteq \mathcal{I}_M$.

Part (i) of this proposition offers a detailed characterization of the optimal identity ($i \in \mathcal{I}_M$) and number ($|\mathcal{I}_M|$) of designs to test under a multi-expert sequential testing strategy. In particular, the firm should test any design $i \in \mathcal{N}$ for which the expected value $q_i v_i$ exceeds the *sum* of (a) the expert's information rents $4c/(2q_i - 1)$ and (b) the development costs K. That is, the firm should test only those designs that promise ex ante a positive contribution margin.

A similar argument applies to the optimal set of designs to be tested under a parallel testing strategy $\mathcal{I}_P$. However, as indicated by Proposition 2.1(iii) and Proposition 2.2(iii), the experts' information rents under parallel testing are much higher than under sequential testing, which explains why the firm always tests fewer designs than under a multi-expert sequential testing strategy. There are two reasons for this difference. First, under parallel testing the firm does not have the option to stop the testing process prematurely. Second, experts testing a relatively undesirable design know that the firm will probably consider their recommendations to be irrelevant; it is therefore costly for the firm to motivate these experts to exert high testing efforts. Whereas the first dynamic has been well established by previous academic work (see, e.g., Loch et al. 2001), little attention has been paid to the second source of inefficiency.

Finally, Proposition 2.4(iii) derives some properties of the optimal set of designs to be tested under a single-expert sequential testing strategy. A few observations merit discussion here. We note that if the last design in the testing order is also the least promising alternative—yet still offers a positive contribution margin—then the firm always tests fewer designs than under a multi-expert sequential testing strategy. In other words, the expert's strategic behavior induces the firm to make reductions in the number of designs to test. Yet this generalization does admit some exceptions. In some instances, it might be profitable for the firm to include an ex ante unprofitable design in its test set $\mathcal{I}_S$ for the sole purpose of influencing the expert's continuation utility and thereby reducing his strategic behavior. In such cases, $q_n v_n - K - 4c/(2q_n - 1) < 0$ and so the firm may find it optimal to increase the number of designs to test: $\mathcal{I}_S \supset \mathcal{I}_M$.

In sum: the firm tests only ex ante profitable designs under both the parallel and multi-expert sequential testing strategy; under single-expert sequential testing, however, it may be optimal for the firm to test ex ante unprofitable designs in order to curtail rent extraction by experts.

2.5.2. Optimal Testing Strategy

Given the optimal contract structures and the optimal set of design alternatives for the different testing strategies, we can now turn to our main research question: What is the firm's optimal testing strategy under delegation? Propositions 2.1–2.3 have already indicated that the answer to this question depends mainly on two contextual parameters: the costs of effort (c) and the informational quality of test outcomes (q_i). It seems clear that these two parameters determine how much information rent the firm must sacrifice in order to align the experts' interests with the firm's agenda. The next proposition confirms this intuition.

Proposition 2.5 (Optimal Testing Strategy). *Let $\Pi_P^* = \Pi_P(\mathcal{I}_P)$, $\Pi_M^* = \Pi_M(\mathcal{I}_M)$, and $\Pi_S^* = \Pi_S(\mathcal{I}_S)$. Then the following statements hold.*

(i) *If $\delta < 1$, then there exists a $\underline{c} > 0$ such that $\Pi_P^* > \max\{\Pi_M^*, \Pi_S^*\}$ for all $c < \underline{c}$.*

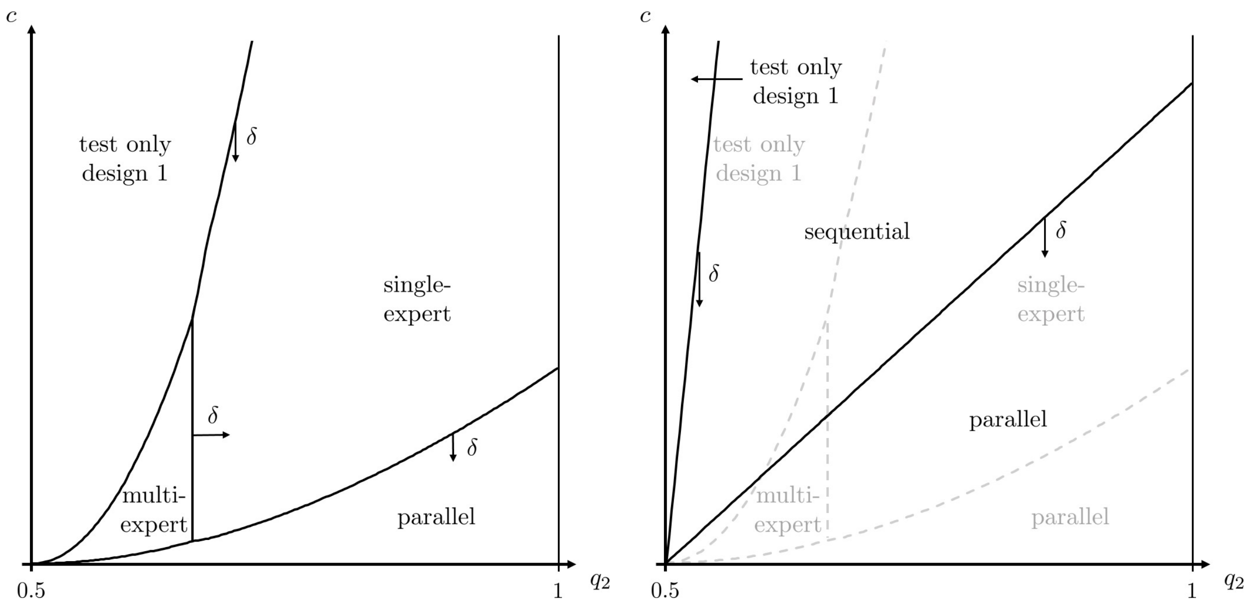
(ii) *Let $\mathcal{I}_P$ be the optimal set of designs to be tested under a parallel testing strategy. If those designs can be ordered such that $q_i v_i \geq q_{i+1} v_{i+1} + 2^{i+1} c[q_i/(2q_i - 1) - 2q_{i+1}/(2q_{i+1} - 1)]^+$ for all $i \in \mathcal{I}_P \backslash \{n\}$, then $\max\{\Pi_M^*, \Pi_S^*\} > \Pi_P^*$ provided that $c > \bar{c} \equiv \sum_{i \in \mathcal{I}_P}((1 - \delta^{i-1})(q_i v_i - K)/2^i) / \sum_{i \in \mathcal{I}_P}((2(1 - (\delta/2)^{i-1})/(2q_i - 1))$.*

(iii) *Let $\mathcal{I}_M$ be the optimal set of designs to be tested under a multi-expert sequential testing strategy, and let those designs be optimally ordered according to Proposition 2.2(ii). Then $\Pi_S^* \geq \Pi_M^*$ provided that $q_{i+1} \geq \underline{q}_i \equiv 1/2 + \delta(2q_i - 1)/(4q_i - \delta(2q_i - 1))$ for all $i \in \mathcal{I}_M \backslash \{n\}$. Moreover, $\underline{q}_i \leq \min\{q_i, 5/6\}$.*

In Figure 2.1, part (a) illustrates the key properties of the optimal testing strategy. First, a parallel testing strategy is undertaken only when the testing costs are sufficiently small ($c < \underline{c}$). Otherwise, the burden of paying all experts immediately for their testing efforts is greater than the value of information received; in that case, the firm decides to implement a sequential testing strategy. Thus design tests that are more expensive—and the resulting higher information rents—make sequential testing more economical. In this respect, parts (i) and (ii) of Proposition 2.5 extend previous findings of Loch et al. (2001) to testing processes that are prone to information asymmetry.

However, it remains an open question exactly how the firm should implement a sequential testing strategy. That is, should the firm hire multiple experts to test the different design alternatives, or should it rather assign all testing activities to a single expert? Proposition 2.5(iii) shows that this question's answer is closely tied to the test efficiency, q_i, of the different designs. Relying on a single expert is especially beneficial when the informational quality of the different design tests is relatively homogeneous (i.e., when $q_{i+1} \geq \underline{q}_i$). In contrast, if test efficiency is heterogeneous across designs then the firm is better-off assigning a different expert to each design alternative. The explanation for this finding is instructive. When the designs' test efficiencies are very different, then the firm prefers testing the designs of highest informational quality first and testing those of lowest quality last (cf. Proposition 2.3(ii)). However, the informational rents extracted from the firm by a single expert increase with any decline in the informational quality of a design test. Hence the expert tries

Figure 2.1.: The Firm's Optimal Testing Strategy



(a) Delegation (b) First-Best

The graphs plot, for an example with $N = 2$ design alternatives, the firm's optimal testing strategy under (a) delegation and (b) first-best conditions. Under a *parallel* testing strategy it is optimal to test both designs simultaneously. Under any sequential testing strategy (i.e., *multi-expert*, *single-expert*, or *first-best*) it is optimal to test design $i = 1$ first and $i = 2$ second. The other parameter values are $v_1 = 1000$, $v_2 = 400$, $q_1 = 0.9$, $K = 200$, and $\delta = 0.9$.

to keep the testing process alive as long as possible—even if that requires reporting a negative assessment of what is actually a good design. There can be no question that exposure to such strategic behavior is suboptimal for the firm, which should therefore rely instead on multi-expert sequential testing.

Figure 2.1(a) also shows the role that the discount factor δ plays in the firm's choice of an optimal testing strategy. As expected, sequential testing strategies are preferable when the time factor is less critical for the firm—that is, as δ increases. Less obvious, though, is δ's effect on the firm's preferred sequential testing strategy. We find that a higher δ facilitates the single expert's strategic extraction of rent because prolonging the testing process is then less costly for her. It follows that the size of the region in which the firm prefers multi-expert to single-expert sequential testing increases with δ (cf. the sensitivity of $\underline{q}_i$ as given in Proposition 2.5(iii)).

Finally, Propositions 2.4 and 2.5 together reveal an interesting non-monotonicity in the optimal number of designs to test. For very low testing costs c, the firm pursues a parallel testing strategy and simultaneously tests a moderate number $|\mathcal{I}_P|$ of designs. As c increases, the firm moves to a sequential testing strategy and, in so doing, increases the number of designs to test (recall that $|\mathcal{I}_M| \geq |\mathcal{I}_P|$ by Proposition 2.4(ii)). Yet when c becomes too large, design testing becomes so costly that the firm is impelled to reverse course and reduce the number of design tests. These results contradict the conventional wisdom—which is true in the *absence* of information asymmetry—that lower testing costs unequivocally lead to more design tests.

2.5.3. Costs of Delegation

Our aim in this section is to discover precisely how information asymmetries distort the firm's design-testing process. We start by describing, as a basis for comparison, the firm's first-best testing strategy: one in which both experts and firm behave as a single entity. Then, in Proposition 2.6, we compare this first-best strategy with the optimal testing strategy under delegation.

If the incentives of experts and the firm are aligned, then the latter need not pay any action- or evidence-based bonuses to motivate the former to engage in high-effort testing and to reveal their testing outcomes truthfully. So absent incentive misalignment, the firm can simply reimburse the experts for their testing efforts by paying them their effort costs c for each design test conducted. Given the resulting lack of information asymmetry, the firm's first-best (fb) expected profit under a sequential (seq) testing strategy is given by $\Pi^{\text{fb}}_{\text{seq}} = \sum_{i \in \mathcal{I}^{\text{fb}}_{\text{seq}}} \delta^{i-1}(q_i v_i - K - 2c)/2^i$; here $\mathcal{I}^{\text{fb}}_{\text{seq}} = \{i \in \mathcal{N} \mid q_i v_i - K - 2c \geq 0\}$ is the optimal set of designs to be tested, and the firm tests the designs in decreasing order of $q_i v_i$. Analogously, the firm's first-best expected profit under a parallel (par) testing strategy is $\Pi^{\text{fb}}_{\text{par}} = \sum_{i \in \mathcal{I}^{\text{fb}}_{\text{par}}}((q_i v_i - K)/2^i - c)$; here the designs in $\mathcal{I}^{\text{fb}}_{\text{par}}$ are ordered in decreasing order of $q_i v_i$, and $\mathcal{I}^{\text{fb}}_{\text{par}} \subseteq \mathcal{I}^{\text{fb}}_{\text{seq}}$. Our final proposition leverages these insights to identify how information asymmetry affects the firm's optimal testing strategy.

Proposition 2.6 (First-Best vs. Delegation)**.** (i) *Under delegated sequential testing, the optimal set of designs to be tested is a subset of the first-best set:* $\mathcal{I}_S, \mathcal{I}_M \subseteq \mathcal{I}^{\text{fb}}_{\text{seq}}$.

(ii) *Under delegated parallel testing, the firm may choose a completely different set of designs to test than under first-best conditions; thus there are cases in which* $\mathcal{I}_P \cap \mathcal{I}^{\text{fb}}_{\text{par}} = \emptyset$.

(iii) *Suppose* $q_i = q$ *for all* $i \in \mathcal{N}$. *If* $\Pi^{\text{fb}}_{\text{seq}} \geq \Pi^{\text{fb}}_{\text{par}}$, *then* $\max\{\Pi^*_M, \Pi^*_S\} \geq \Pi^*_P$; *however, the converse is not true in general.*

The main finding of Proposition 2.6 is that information asymmetry has fundamentally different effects on parallel than on sequential testing. Consider first the implications of delegation on the optimal design of a sequential testing strategy. Part (i) of the proposition shows that—as expected—the presence of information asymmetry results in a suboptimal testing process. In particular, the firm is testing too few designs and therefore stops the testing process too early; that is, $\mathcal{I}_S, \mathcal{I}_M \subseteq \mathcal{I}_{\text{seq}}^{\text{fb}}$. This result reflects that an expert's information rent makes design testing unequivocally more expensive (for the firm) than under first-best conditions.

One might suppose a similar reasoning to apply also with regard to parallel testing. In this respect, however, part (ii) of Proposition 2.6 holds a surprise. Note that even though the firm tests too few designs under a sequential testing strategy, it does still test those designs that are also the most promising ones under first-best conditions. Yet this statement does not necessarily hold for a parallel testing strategy. In fact, Proposition 2.6(ii) reveals that the optimal sets of designs to be tested with and without information asymmetry may be disjoint; under delegation, then, the firm may test an *entirely* different set of design alternatives. How can we explain this split? Recall from our discussion after Proposition 2.1 that the information rents extracted from the firm by experts are *decreasing* in the quality of those experts' information. Hence the firm never tests designs that offer relatively poor information quality—that is, with almost complete disregard for their economic potential v_i. In contrast, under first-best conditions the firm's testing costs are constant and thus do not depend on a design's informational quality; in that case, it makes sense for the firm always to test those designs promising the highest expected value $q_i v_i$. Evidently, these different priorities under first-best and delegated testing can lead to disjoint optimal test sets. This phenomenon is most likely to occur when some design alternatives are of exceptionally high economic value v_i but low test efficiency q_i.

Finally, Proposition 2.6(iii) hints at an important managerial insight: delegation favors sequential testing. This result is also clearly illustrated in part (b) of Figure 2.1, which plots the firm's optimal testing strategy vis-à-vis the first-best benchmark. Under symmetric test efficiencies ($q_i = q$ for all $i \in \mathcal{N}$), we can demonstrate formally that if the firm prefers sequential testing under first-best conditions then it does so under delegation as well. Although we are unfortunately not able to generalize this result analytically to heterogeneous test efficiencies, our numerical experiments confirm that the claim does indeed hold much more generally; see Figure 2.1(b), which allows for such heterogeneity. Our finding has implications both for the academic literature and for practice. It relativizes the claims about the effectiveness of parallel testing strategies (e.g., Dahan and Mendelson 2001, Loch et al. 2001). In the presence of information asymmetry, the benefits of such a parallel approach may be outweighed by high agency costs. This result finds further support in practice. In recent years, testing costs have declined significantly owing to technological advancements in the realms of robotics, virtualization, and computer-assisted test systems (among others). Following conventional wisdom—and previous academic insights—these developments should have led firms to focus more strongly on parallel testing strategies. However, there is no substantial evidence to date that such a general trend is underway (though the software industry is a notable exception). Our results offer a plausible and straightforward explanation for this observation: firms are reluctant to incur the high costs of delegation that come with parallel testing.

2.6. Conclusions

Design testing is an integral part of virtually any new product development initiative because it enables firms to identify the best possible designs for their new products. In reality, however, managing such testing processes is a daunting challenge. The reason is that in most cases the firm does not itself conduct the desired testing activities and so has no direct accesses to the precious information; the firm must instead rely on the recommendations of experts, who may be pursuing their own agendas. This delegated nature of the testing process gives rise to information asymmetry between the firm and the experts, which can result in a worrisome misalignment of objectives. The primary goal of this paper is to understand how the firm can set up an effective testing process that will reliably select the best design alternative—that is, notwithstanding the adverse consequences of delegation. More precisely, our main contribution is to provide insights on the questions of (i) which testing strategy the firm should choose and (ii) how the firm can optimally incentivize the experts it hires.

It is remarkable that, regardless of the chosen testing strategy, the optimal compensation scheme—one that motivates experts to test their designs with high effort and to reveal their test outcomes truthfully—always involves but two payments: a success bonus if an expert's design is developed and turns out to be technologically feasible, and a reward that reimburses an expert for his efforts in case the firm dismisses the design he is testing or terminates the testing process altogether. We show in addition that the balance between these two payments is fundamentally different for designs with different levels of test efficiency. Designs that can be tested with high precision require a strong focus on individual success bonuses, whereas designs that are evaluated with lower quality demand a stronger emphasis on consolation awards.

Our findings have immediate managerial implications. Although design testing is a complex organizational process, the structure of the optimal compensation schemes is fairly simple. As a result, the optimal contracts derived in this paper should be relatively easy to implement in practice. Regarding the relative sizes of the two payments, we emphasize that the firm must carefully adjust its contracts to reflect the quality of test outcomes. Firms that adopt a one-size-fits-all approach cannot help but sacrifice, eventually, their testing effectiveness.

As for the firm's optimal testing strategy, we find that two parameters critically determine the firm's optimal choice: testing costs and the quality of information. In line with previous research, we show that a parallel testing strategy is optimal only when testing costs are sufficiently small. In contrast, the higher the testing costs, the *more* beneficial a sequential testing strategy becomes. Yet it is an unanswered question just how the firm should set up its sequential testing strategy. Should it mandate a single expert to carry out all test activities, or should it rather assign a different expert to each design test? Our results indicate that the former approach is optimal when the informational quality of the different design tests is relatively homogeneous. When test efficiency is very heterogenous across designs, however, the firm should hire multiple experts because in that case a single expert might artificially keep the testing process alive in order to receive ongoing payments—to the firm's obvious detriment.

These results have clear consequences for practice. We identify two levers the firm can use when designing an effective testing strategy: the order of the different design tests (parallel vs. sequential), and the number of experts to employ. Whereas the former option has been extensively discussed in the academic literature, the latter option has yet to receive serious attention. It is important to recognize that the two levers address two different concerns. The choice of whether to use a parallel or sequential testing strategy depends on the testing *costs*, whereas the ideal number of experts depends on the extent to which the *efficiency* of tests (for the different designs) is heterogeneous.

Finally, we show how the presence of information asymmetry affects the various testing strategies. Overall, our results point to the same conclusion: the delegation of testing leads to a suboptimal testing process whose information asymmetries are significantly more harmful to parallel than to sequential testing strategies. In other words, our findings indicate that parallel testing may be less effective than usually claimed in the academic literature (Dahan and Mendelson 2001, Loch et al. 2001) when the testing process involves information asymmetries. This finding may also explain the practical observation that, even though technological advancements have lowered testing costs in recent years, no significant shift toward parallel testing efforts is evident.

To maintain tractability and develop a parsimonious model, we necessarily made some assumptions about the specific trade-offs inherent to a sequential testing strategy. In particular, we assumed that the design alternatives are sufficiently different that the firm cannot exploit any between-design learning effects. Also, we did not allow the firm to choose previously tested designs for development. Both assumptions clearly lead to an underestimate of the performance of sequential testing strategies, from which it follows that relaxing these assumptions could only strengthen our main message that delegation favors sequential testing. Furthermore, one can readily verify that the optimal contract structures would remain relatively intact even without these assumptions; hence our results are applicable to a wide range of practical scenarios. With regard to the firm's choice of testing strategy, we focused on "polar" cases: fully parallel versus fully sequential testing, and a single expert versus n experts. In reality, firms are free to use any mixture of parallel and sequential testing strategies, and they may also hire any arbitrary number of experts. Whether such hybrid strategies can improve the efficacy of delegated testing processes is an important question for future research. Another interesting research possibility is for empirical studies to examine the relationship between a firm's chosen testing strategy and the severity of its agency issues. Our own theoretical results lead us to conjecture that firms (and industries) with relatively strong agency problems are much more likely to use sequential than parallel testing strategies.

Chapter III

Used-Product Acquisition Processes with Quality Differentiation

with Moritz Fleischmann[1]

3.1. Introduction

Quality-dependent pricing is one means to balance quality, quantity, and timing of used-product returns (e.g., Guide et al. 2003). As a basis, firms' knowledge about products' qualities before acquisition is inevitable. This is gained through upfront grading/testing of those products. The application of a quality-dependent process for the acquisition of used products is the common approach in the recommerce business. Product holders interested in selling can submit quality statements on these firms' websites and receive quality-dependent acquisition price offers for their used devices. This process aims at shifting the quality assessment to holders and to avert those how are not willing to sell their products for the quality-specific prices. The statements, however, might not mirror the actual products' qualities due to the strategic behavior of potential sellers, which might vitiate the process' performance.

Our work is motivated by this widely applied acquisition process. On the one hand, we are interested in the dynamics in this process and, based one these dynamics, the optimal acquisition prices. On the other hand, we are interested in potential process improvements.

Among other products, recommerce providers (e.g., Gazelle, FLIP4NEW, and ReBuy) buy small used electronic devices, such as mobile phones, process them (e.g., cleaning and deletion of data), and re-sell them. For setting up their acquisition environment, those firms determine quality-specific acquisition prices for the products that they want to buy. If

[1]The research presented in this chapter is based on a paper entitled "Used-Product Acquisition Processes with Quality Differentiation under Asymmetric Information", coauthored with Moritz Fleischmann.

this has been done, the typical acquisition process comprises three stages. First, a product holder visits the website, selects his specific product, determines its quality (depending on its optical condition, technical functionality, etc.), and receives a quality-dependent, provisional price offer. If satisfied with the offer, the product is shipped to the firm. Transportation is usually paid by the firm. Second, after arrival, the product is tested, and its true quality is determined. Subsequently, the firm decides whether to accept the offer or to make a counteroffer. If accepted, the requested money is transferred to the holder, and the acquisition is completed. Making a counteroffer leads to the third and final stage, in which the holder decides to either accept or reject the counteroffer. Acceptance again leads to the conclusion of the sale. If rejected, the product is sent back to the holder at firm's expense. In this case, the counteroffer induces a loss for the firm due to already-invested money for transportation, testing, handling, etc.

This process raises incentive issues. It does not induce product holders to true quality statements, potentially forcing the firm into counteroffers. Hahler and Fleischmann (2017) mention that a non-negligible proportion of actual product submissions at FLIP4NEW, a German recommerce provider, exhibits a significant mismatch between the quality stated by product holders and the true quality determined by the recommerce provider.

Information asymmetries play a major role in this acquisition process. Naturally, firms do not know how much a product holder is willing to accept for selling his product. Therefore, making a counteroffer is risky. Furthermore, it is not obvious for product holders how efficiently the firm refurbishes used products. This can be translated into uncertainty about the margin of a product in a certain condition. The larger the margin is, the more the firm is willing to pay for a product. This uncertainty might be an incentive for product holders to not state the true quality.

Hahler and Fleischmann (2017) model and analyze the described acquisition as a sequential game with complete information. We take into account information asymmetries by modeling the acquisition process with two-sided incomplete information. First, we determine the equilibrium of the game given fixed prices. Then, based on the determined equilibrium, we build an optimization model and find the optimal acquisition prices that maximize the firm's profit. Afterward, we present a modified process by introducing bonus payments that enable the firm to incentivize product holders to true quality statements. Finally, both processes are compared with regard to their profitability.

In summary, our paper contributes to the literature as follows:

- We determine the equilibrium in the acquisition process at many recommerce providers under two-sided incomplete information.

- We determine the optimal quality-dependent acquisition prices to maximize profit based on the equilibrium.
- We propose a different acquisition process with bonus payments and determine how to maximize profit while ensuring true quality statements by product holders.
- We compare both processes with regard to profitability and provide insights about which circumstances favor one process over the other.

The remainder of the paper is organized as follows. Section 3.2 positions our work within the extant research literature. Section 3.3 models the acquisition process applied in current practice as game of incomplete information. In Section 3.4, the equilibrium of the game is presented, given fixed prices. Section 3.5 presents the optimal acquisition prices based on the equilibrium. In Section 3.6, we propose an acquisition process with bonus payments, discuss how to incentivize product holders to true quality statements by the choice of bonuses, and present the structure of the optimal bonus combination conditioned on truth-telling. Section 3.7 compares both processes. Section 3.8 concludes with some managerial implications, limitations, and suggestions for future research.

3.2. Related Literature

Our paper is situated in the research field regarding closed-loop supply chains. We refer the reader to Guide and Van Wassenhove (2009), Souza (2013) and Govindan et al. (2015) for literature reviews of this area. More specifically, we cover issues about the supply side of a reverse supply chain, i.e., used-product acquisition. Fleischmann et al. (2010) provides an overview of the literature regarding this field of research. Three specific research streams are relevant to our work: quality-dependent acquisition pricing, the value of grading information, and seller-buyer interaction in used-product acquisition.

Exploring the research field regarding product acquisition with a profit-oriented focus was initiated by Guide and Jayaraman (2000) and Guide and Van Wassenhove (2001). Earlier work mainly assumed an exogenous return process. Guide et al. (2003) introduced quality-dependent acquisition pricing in order to manage the quantity and quality of product returns. Ray et al. (2005) focus on trade-in rebates and assume a product's quality to be a continuous function of the product's age. Karakayali et al. (2007) determine optimal acquisition prices for remanufactured parts for different reverse channel structures of an OEM. Hahler and Fleischmann (2013) compare two collection system configurations. For the centralized system, one quality-independent acquisition price is offered, whereas for the decentralized system, there are finitely many quality-dependent acquisition prices. Bulmuş et al. (2014) consider simultaneous quality-dependent acquisition pricing and sales pricing for new and remanufactured products. Our paper contributes to this research stream regarding quality-dependent acquisition pricing by considering the strategic behavior of both players in the presented acquisition process for the pricing decisions.

The value of grading or yield information for subsequent processing of returned products was first considered by Souza et al. (2002) and Ketzenberg et al. (2003). Souza et al. (2002) concern themselves with the decision about remanufacturing returned products given a certain condition determined by imperfect grading. They find that reduction of grading errors leads to decreased mean flow times in a remanufacturing network. Ketzenberg et al. (2003) come to the same conclusion. Further studies analyze the value of grading information by comparing remanufacturing systems with and without upfront grading. Aras et al. (2004), Guide et al. (2005) and Zikopoulos and Tagaras (2008) contribute such studies. More recently, Mutha et al. (2016) investigate whether third-party remanufacturers should acquire used products with uncertain qualities or in sorted grades with known qualities and whether to acquire and remanufacture cores before or after demand realization. We focus on grading in a different manner. We present how grading can be shifted from firm to product holders by incentivizing them to truthfully reveal their products' qualities if interested in selling. This is valuable for the firm because grading effort can be reduced if this information is given by product holders.

Another stream of literature related to our work is on seller-buyer interaction in the acquisition of used products. One aforementioned paper is Hahler and Fleischmann (2017). They are the first to focus on the business field of recommerce and modeling this already-mentioned specific acquisition process as a sequential game. They analyze the strategic interaction under complete information and assume exogeneous acquisition prices. They determine the optimal counteroffer under incomplete information using past data and applying regression analysis. Another closely related paper is Gönsch (2014), which compares two acquisition pricing strategies: offering one fixed price versus bargaining. For bargaining, no specific negotiation process is considered, but the generalized Nash bargaining solution is used. However, used products' quality differences are not considered. We contribute to this stream of literature by analyzing the currently applied process from the recommerce business under incomplete information. Additionally, we propose an acquisition mechanism that enables the buying firm to incentivize product holders to state their products' true qualities. Finally, we introduce the quality-dependent pricing decision to this stream of research.

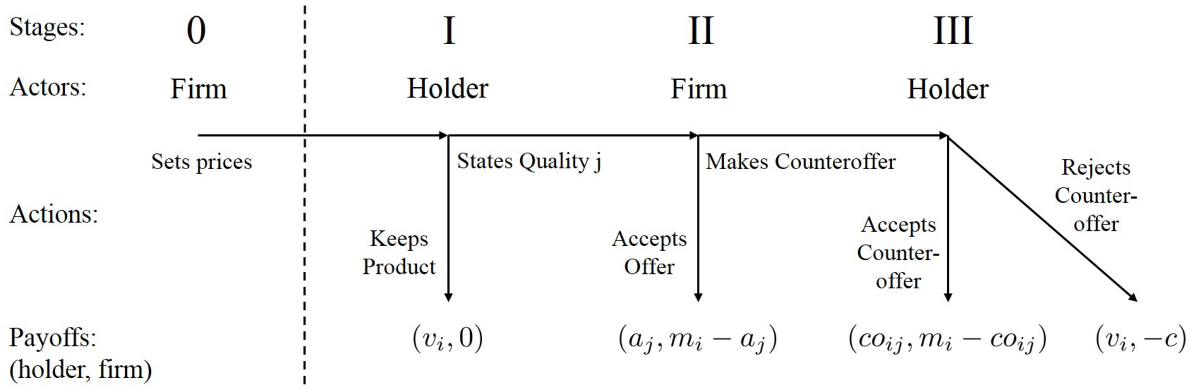
3.3. Model Setup

In this section, we formalize the initially presented common acquisition process, to which we refer as ACP. Our model consists of four stages. The initial stage comprises the quality-dependent pricing decision of the firm (she). Subsequent stages represent, first, the hand in decision of a product holder (he), second, the counteroffer decision of the firm and, finally, product holder's decision whether to accept or reject the counteroffer. See Figure 3.1 for the sequence of events.

We use the following notation and information structure: we assume the product holder and firm to be risk-neutral expected payoff maximizers. Π denotes the firm's expected profit, whereas π denotes the product holder's expected payoff. We consider one type of product, which can have finitely many qualities $i \in \{1, ..., n\}$. δ_i denotes the market share of the quality i product in the used-product market that is accessible to the firm. We normalize the whole market to 1, i.e., $\sum_{i=1}^{n} \delta_i = 1$. Every quality is characterized by the margin m_i that is achieved by the firm through reselling a product in quality i. This margin denotes the reselling price net of all investments for transportation, handling, and reprocessing the product except for the acquisition price. m_i is not known by the product holder. We assume the product holder to believe that m_i is uniformly distributed on $[\underline{m_i}, \overline{m_i}]$. A product in quality i exhibits a non-negative residual value v_i for its holder. The distribution of these values is common knowledge to the firm and product holders, and we assume the residual values to be uniformly distributed on $[\underline{v_i}, \overline{v_i}]$. The firm, however, does not know a product holder's specific v_i if a product submission takes place. We assume the quality-dependent parameters $m_i, \overline{m_i}, \underline{m_i}, \overline{v_i}, \underline{v_i}$ to be increasing in i. Furthermore, we assume $\underline{v_i} < \underline{m_i}$ for all qualities i.

We model ACP as a four-stage sequential game, with stages 0, I, II, and III. In stage 0, the firm sets the quality-dependent acquisition prices $a_1, ..., a_n$. By $\hat{a} = \max_{1 \leq i \leq n} a_i$, we denote the largest acquisition price value, which plays a special role in our analysis. In stage I, the product holder decides to either keep his product or submit it and to state a quality $j \in \{1, ..., n\}$, thereby requesting price a_j. This decision is based on the following information and beliefs that the holder has: the true quality of his used product i, the residual value v_i,

Figure 3.1.: Sequence of Events



the acquisition prices on the website corresponding to the quality levels, his prior belief about the margin m_i, and the assumption about rational behavior of the firm. If the product is kept, the game ends, the product holder has payoff v_i and the firm has payoff 0. If there is a statement j, the product is sent to the firm. Then, in stage II, the firm determines the true quality i. We assume perfect grading. Depending on the true quality i and stated quality j, the firm decides to either accept to pay a_j or to make a counteroffer $co_{ij} \geq 0$. Acceptance ends the game. The holder has payoff a_j, and the firm has payoff $m_i - a_j$. Counteroffering leads to stage III. Here, the product holder either accepts or rejects the counteroffer. Acceptance leads to payoff co_{ij} for the holder and $m_i - co_{ij}$ for the firm. Rejection leads to payoff v_i for the holder and $-c$ for the firm. Loss through rejection, c, comprises all investments made so far for acquiring the product plus the transportation costs for sending the product back to the holder.

Finally, we assume that if players are indifferent between two decisions regarding their payoffs, they choose the decision that ends the game more quickly.

3.4. Equilibrium

In this section, we present the actions of the product holder and firm in equilibrium when acquisition prices are fixed, i.e., the actions in stages I-III. Before we come to the formal characterization of equilibria, we provide some preliminary thoughts about the optimal behavior of the firm and product holder to build up intuition and derive the most important ingredients for the analysis.

First, note that the optimal action in stage III is straightforward. The product holder only has to compare counteroffer and residual value and choose the larger value by either accepting or rejecting the counteroffer.

This behavior is anticipated by the firm in stage II when deciding about the counteroffer. Furthermore, the firm knows quality statement j made in stage I and true quality i. This equips the firm with an updated belief about v_i according to Bayesian rationality, i.e., a probability of whether a certain counteroffer is accepted, $\mathbb{P}(v_i \leq co_{ij}|j)$. The firm's expected payoff for making a counteroffer, which the firm seeks to maximize, is therefore the average of payoff for accepted counteroffer and payoff for rejected counteroffer weighted by the probabilities for the corresponding outcomes:

$$\begin{aligned}\Pi(co_{ij}|j,m_i) = &\mathbb{P}(v_i \leq co_{ij}|j)(m_i - co_{ij})\\ &+ \mathbb{P}(v_i > co_{ij}|j)(-c)\end{aligned}$$

If the maximum value of this function is greater than the payoff for accepting offer a_j, $m_i - a_j$, then it is optimal for the firm to make the corresponding counteroffer.

In stage I, the product holder makes his hand in and quality statement decision based on anticipation of the optimal actions in stages II and III, his knowledge about the true quality i and residual value v_i, and his belief about margin m_i. His beliefs about the margin and about the firm's optimal behavior equip him with a probability for an offer a_j being accepted, $\mathbb{P}(\Pi(co^*_{ij}|j,m_i) \leq m_i - a_j)$, where $co^*_{ij} = \arg\max \Pi(co_{ij}|j,m_i)$. With the complementary probability, the firm makes an optimal counteroffer which, depending on its size, is accepted or rejected by the holder. Bringing this together, product holder's expected payoff for handing in and stating quality j is

$$\begin{aligned}\pi(j|i,v_i) =&\mathbb{P}(\Pi(co^*_{ij}|j,m_i) \leq m_i - a_j)a_j\\ &+\mathbb{P}(\Pi(co^*_{ij}|j,m_i) > m_i - a_j)\int_{\Pi(co^*_{ij}|j,m_i)+a_j>m_i} \max\{v_i, co^*_{ij}\}dm_i\end{aligned}$$

If the maximum value of this function is greater than the payoff for keeping the product, v_i, then it is optimal for the holder to hand in and state the corresponding quality j.

Finally, note that it cannot be optimal for the firm to make counteroffers that are greater than a requested acquisition price a_j, i.e., $co_{ij} < a_j$. She is always better off accepting the offer. It follows that it cannot be optimal for a product holder to state j if $a_j < v_i$ due to no payoff in subsequent stages being greater than v_i.

The following proposition characterizes the equilibria of stages I-III. For brevity and expositional clarity, all proofs are in the appendix.

Proposition 3.1 (Equilibrium Strategies). *The following actions characterize the equilibria for stages I-III:*

(I) *Let* $v_i^* = \min\{(\overline{m_i} + c + \underline{v_i})/2, \overline{v_i}\}$*. The product holder keeps the product if and only if* $v_i \geq \min\{v_i^*, \hat{a}\}$*. If the holder hands in and states* j*,* j *satisfies* $a_j \geq \min\{v_i^*, \hat{a}\}$*.*

(II) *Let* $co_i^* = \min\{(m_i + c + \underline{v_i})/2, \overline{v_i}\}$*. The firm makes counteroffer* co_i^* *if and only if offer* $a_j > co_i^*$*. The firm accepts offer* a_j *otherwise.*

(III) *The product holder accepts* co_i^* *if and only if* $co_i^* \geq v_i$*. The holder rejects the counteroffer otherwise.*

Since the optimal counteroffer only depends on the true quality i, not on the offer j, the index j in co_{ij}^* is dropped.

Regarding the optimal action in stage I, we observe that if there are several acquisition prices greater than v_i^*, all of them give the same (maximal) expected payoff for the holder and hence are optimal. Nevertheless, $\hat{a}$ always belongs to the set of optimal offers. Furthermore, the course and payoffs of the game are the same for every such "likewise" optimal price. Thus, $\hat{a}$ is the only driver of the firm's profit, which will be discussed in the upcoming section in more detail.

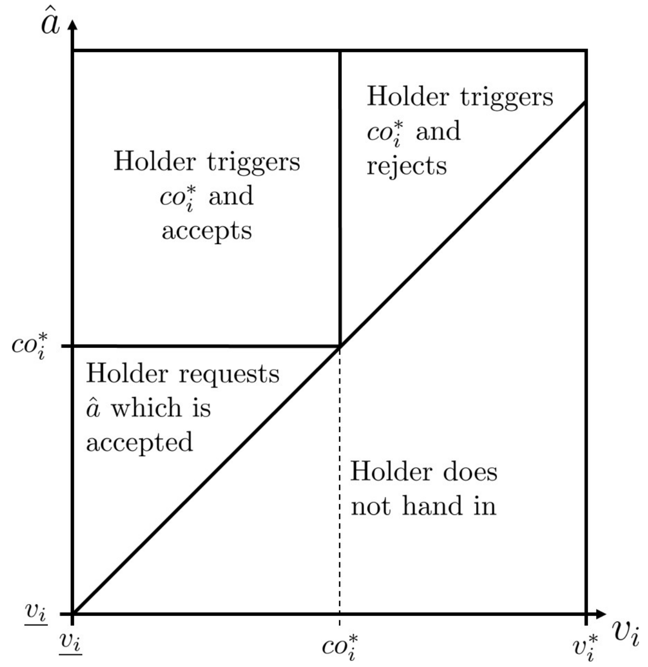
Figure 3.2 illustrates the game dynamics depending on the relation between residual value v_i of the product of quality i for the holder and the largest acquisition price $\hat{a}$. If $v_i \geq \hat{a}$, then it is not profitable for the holder to hand in (see the lower-right area). If $co_i^* > \hat{a} > v_i$, then holder hands in for $\hat{a}$ and the offer is accepted (see the lower-left area). If $\hat{a} > co_i^*$, then the holder hands in and triggers a counteroffer, which is either accepted or rejected, depending on the relation between v_i and co_i^* (see the upper areas).

There are some noteworthy observations related to the game dynamics in equilibrium. First, the firm does not learn the product's true quality through the holder's quality statement because upward deviations from true quality statements are never bad for product holders.

Second, there can be product holders that hand in for $\hat{a} > v_i$ and have v_i greater than the optimal counteroffer co_i^*. These holders cause losses to the firm due to reception of counteroffers that they do not accept. Lowering the largest acquisition price can reduce the amount of holders handing in and causing losses because handing in is only beneficial for product holders if $\hat{a} > v_i$.

Third, the two aspects of incomplete information drive the equilibria in different manners.

Figure 3.2.: Equilibrium under ACP



If the firm knew product holders' residual values, holders would not consider handing in because it would never be beneficial for them. This is due to the fact that the firm would always make a counteroffer that is less than or equal to the specific residual value, never greater. Hence, perfect knowledge about residual values leads to a situation in which in equilibrium, no acquisition occurs. If product holders knew the margins, no losses for the firm in terms of rejected counteroffers would be caused. This is due to the fact that product holders would perfectly anticipate the optimal counteroffers of the firm. Therefore, product holders would know whether it is beneficial to hand in upfront and hence would only hand in if this were the case, i.e., if they accepted the counteroffer. Hence, from the firm's perspective, knowledge about margins induces desirable behavior of product holders.

Finally, we present Corollary 3.1, which discusses a potential lever for the firm to increase profits.

Corollary 3.1 (Payment for Participation - ACP)**.** *If product holders have to pay a participation fee of $\epsilon > 0$ for a product submission, no one hands in, in equilibrium.*

That charging product holders for participation results in this undesired equilibrium outcome is basically due to the last mover advantage of the firm. It is always beneficial for the firm to reduce the requested acquisition price by the cost for product holders because product holders with a larger valuation than price net cost cannot gain anything by handing in. Hence, for this acquisition process, the firm is not able to charge money for participation. Note that this also implies that the firm cannot transfer the payment of transportation cost to product holders without deterring all of them from handing in.

3.5. Optimal Acquisition Prices

In this section, we investigate how acquisition prices have to be chosen to maximize the firm's expected profit. For this purpose, the previously determined equilibria and the probabilities for submitted products having certain qualities are taken into account.

First, consider the firm's profit in one quality class i. By Proposition 3.1 (compare also with Figure 3.2), the firm's equilibria payoffs depending on v_i are as follows: if $v_i \in [\min\{v_i^*, \hat{a}\}, \overline{v_i}]$, then the firm gains 0 because the holder does not hand in; if $v_i \in (\min\{co_i^*, \hat{a}\}, \min\{v_i^*, \hat{a}\})$, then the firm has payoff $-c$ because the holder triggers a counteroffer, which he rejects; and if $v_i \in [\underline{v_i}, \min\{co_i^*, \hat{a}\}]$, then the firm has payoff $m_i - \min\{co_i^*, \hat{a}\}$ because either the product holder is paid $\hat{a}$ or he accepts the made counteroffer co_i^*, depending on how $\hat{a}$ and co_i^* relate to one another.

Note that the specific quality-dependent acquisition prices do not play a role in equilibrium. The firm's only decision variable that impacts its profit is the value of the greatest acquisition price, $\hat{a}$. Therefore, after the optimal level of the largest price is found, one can arbitrarily set the quality-dependent prices while making sure that all of them are not greater than $\hat{a}$ and that at least one price coincides with $\hat{a}$. One convenient choice is to set $a_i = \min\{co_i^*, \hat{a}\}$. This choice also ensures that there are no counteroffers for true quality statements.

With regard to the above interval boundaries which specify conditions for product holders' behavior depending on v_i, we define the upper bound for acceptance of the counteroffer, $\hat{co}_i^*(\hat{a}) := \min\{co_i^*, \hat{a}\}$, the upper bound for handing in, $\overline{w_i}(\hat{a}) := \min\{v_i^*, \hat{a}\}$, and an auxiliary variable, $\underline{w_i}(\hat{a}) := \min\{\underline{v_i}, \hat{a}\}$ that ensures no profit or losses in quality i if $\hat{a} \leq \underline{v_i}$.

Drawing on the above observations, the firm's expected profit as a function of $\hat{a}$ considering only quality i is

$$\begin{aligned}
\Pi_{ACP}^i(\hat{a}) &= \mathbb{P}(v_i \leq \min\{co_i^*, \hat{a}\})(m_i - \min\{co_i^*, \hat{a}\}) \\
&\quad + \mathbb{P}(\min\{co_i^*, \hat{a}\} < v_i < \min\{v_i^*, \hat{a}\})(-c) + \mathbb{P}(\min\{v_i^*, \hat{a}\} \leq v_i)0 \\
&= \frac{1}{\overline{v_i} - \underline{v_i}}[(\hat{co}_i^*(\hat{a}) - \underline{w_i}(\hat{a}))(m_i - \hat{co}_i^*(\hat{a})) + (\overline{w_i}(\hat{a}) - \hat{co}_i^*(\hat{a}))(-c)]
\end{aligned}$$

Note that Π^i_{ACP} is equal to zero for $\hat{a} < \underline{v_i}$, quadratic concave in $\hat{a}$ for $\underline{v_i} < \hat{a} < co^*_i$, linearly decreasing in $\hat{a}$ for $co^*_i \leq \hat{a} < v^*_i$, and constant for $\hat{a} > v^*_i$. Furthermore, Π^i_{ACP} is differentiable except in $\underline{v_i}$ and v^*_i.

The following lemma presents the optimal largest price under negligence of all other quality classes and the corresponding profit in quality class i.

Lemma 3.1 (Optimal Acquisition Price - Single Quality)**.** *Considering only quality class i, it holds that $\hat{a}^* = \min\{(m_i + \underline{v_i})/2, \overline{v_i}\}$ maximizes Π^i_{ACP} and*

$$\Pi^i_{ACP}(\hat{a}^*) = \begin{cases} (m_i - \underline{v_i})^2/(4(\overline{v_i} - \underline{v_i})) & \text{if } (m_i + \underline{v_i})/2 \leq \overline{v_i} \\ (m_i - \overline{v_i}) & \text{otherwise} \end{cases}$$

We observe that $\hat{a}^*$ is less than co^*_i and plays the role of the hand-in cutoff level. It is always paid for a quality i product submission, which implies that there are no losses due to rejection.

In order to illustrate the single-quality case, we present Figure 3.3(a), which depicts the profit and corresponding hand-in, sales, and rejection volumes in quality class 1 depending on $\hat{a}$, given the stated parameter setting comprising three quality classes.

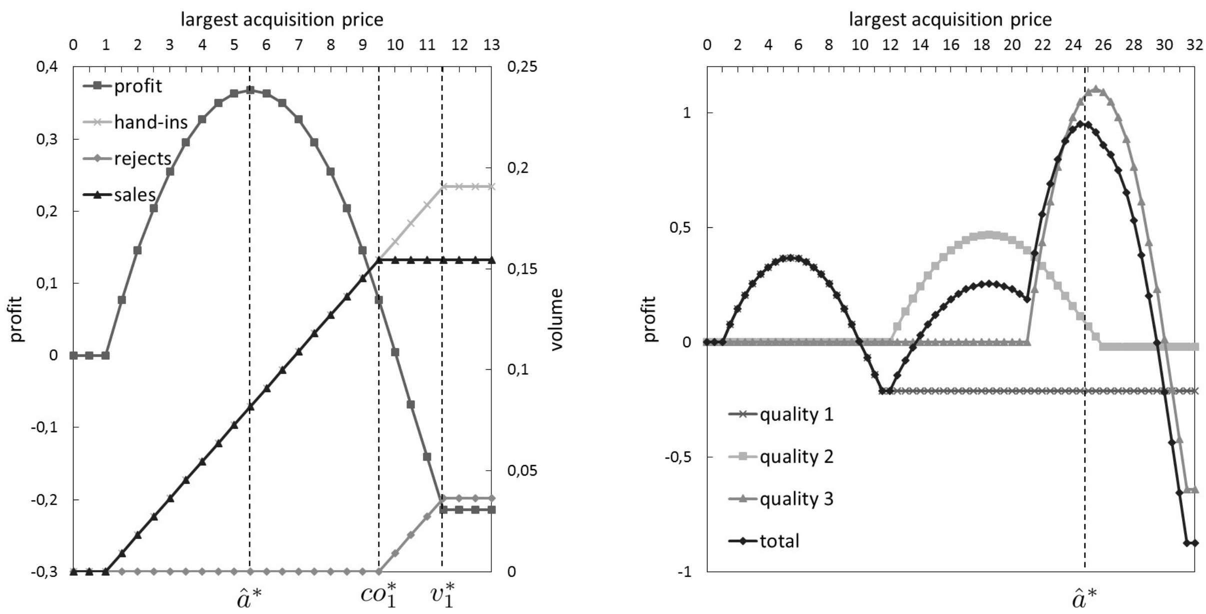
The classical price-volume trade-off reveals itself for $\hat{a}$ less than co^*_1. As the price increases, the hand-in and sales volumes increase. If $\hat{a}$ is greater than co^*_1, the sales volume stays constant, but the hand-in volume further increases. Here, product holders with residual value greater than co^*_1 are incentivized to hand in due to the high price, but they receive a counteroffer that they reject. Product holders with v_1 less than co^*_1 accept the counteroffer.

As a next step, we investigate how to choose the highest price in order to maximize the firm's overall profit considering all quality classes. It is already intuitive that the choice of $\hat{a}$ is related to the trade-off between the achievable profits in different quality classes. The firm's total expected profit is just the weighted sum of all the quality-specific profits, where the weights correspond to the market shares. The total profit is given by

$$\Pi_{ACP}(\hat{a}) = \sum_{i=1}^{n} \delta_i \Pi^i_{ACP}(\hat{a})$$

Due to the relation between Π_{ACP} and the quality-specific profits, Π^i_{ACP}, the total profit exhibits similar properties, i.e., it consists of constant, quadratic concave, and linear parts and is differentiable except at finitely many ($2n$) points.

Figure 3.3.: Firm's Profit under ACP



(a) Quality 1 Profit and Volumes

(b) Profit Π_{ACP}

Parameters: $c = 8$; for quality 1: $\delta_1 = 0.2$, $m_1 = 10$, $\overline{m_1} = 14$, $\underline{m_1} = 8$, $\overline{v_1} = 12$, $\underline{v_1} = 1$; quality 2: $\delta_2 = 0.2$, $m_2 = 25$, $\overline{m_2} = 32$, $\underline{m_2} = 20$, $\overline{v_2} = 30$, $\underline{v_2} = 12$; quality 3: $\delta_2 = 0.6$, $m_3 = 30$, $\overline{m_3} = 34$, $\underline{m_3} = 28$, $\overline{v_3} = 32$, $\underline{v_3} = 21$.

In summary, the firm's profit maximization problem is as follows:

$$P1: \max_{\hat{a}} \ \Pi_{ACP}(\hat{a})$$

$$= \sum_{i=1}^{n} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} [(\hat{co}_i^*(\hat{a}) - \underline{w_i}(\hat{a}))(m_i - \hat{co}_i^*(\hat{a})) + (\overline{w_i}(\hat{a}) - \hat{co}_i^*(\hat{a}))(-c)] \tag{3.1}$$

$$\text{s.t.} \ \hat{co}_i^*(\hat{a}) = \min\{co_i^*, \hat{a}\} \qquad \forall i \in \{1, ..., n\} \tag{3.2}$$

$$\overline{w_i}(\hat{a}) = \min\{v_i^*, \hat{a}\} \qquad \forall i \in \{1, ..., n\} \tag{3.3}$$

$$\underline{w_i}(\hat{a}) = \min\{\underline{v_i}, \hat{a}\} \qquad \forall i \in \{1, ..., n\} \tag{3.4}$$

The following proposition characterizes the highest acquisition prices $\hat{a}$ that locally solve optimization problem $P1$.

Proposition 3.2 (Locally Optimal Acquisition Prices). *$\hat{a}^*$ locally maximizes Π_{ACP} if and only if there are $0 \le j \le k < l \le n$ such that*

(i) $\hat{a}^* = (\sum_{i=j+1}^{k} \delta_i(-c)/(\overline{v_i} - \underline{v_i}) + \sum_{i=k+1}^{l} \delta_i(m_i + \underline{v_i})/(\overline{v_i} - \underline{v_i}))/(2\sum_{i=k+1}^{l} \delta_i/(\overline{v_i} - \underline{v_i}))$,

(ii) $\hat{a}^* > v_i^*$ *for all* $1 \le i \le j$,

(iii) $co_i^* \le \hat{a}^* < v_i^*$ *for all* $j < i \le k$,

(iv) $\underline{v_i} < \hat{a}^* < co_i^*$ *for all* $k < i \le l$,

(v) $\hat{a}^* < \underline{v_i}$ *for all* $l < i \le n$.

The resulting total profit is given by

$$\begin{aligned}\Pi_{ACP}(\hat{a}^*) = &\sum_{i=1}^{j} \delta_i[(co_i^* - \underline{v_i})(m_i - co_i^*) + (v_i^* - co_i^*)(-c)]/(\overline{v_i} - \underline{v_i}) \\ &+ \sum_{i=j+1}^{k} \delta_i[(co_i^* - \underline{v_i})(m_i - co_i^*) + (\hat{a}^* - co_i^*)(-c)]/(\overline{v_i} - \underline{v_i}) \\ &+ \sum_{i=k+1}^{l} \delta_i(\hat{a}^* - \underline{v_i})(m_i - \hat{a}^*)/(\overline{v_i} - \underline{v_i}).\end{aligned}$$

There are at most $3n - 2$ local maxima.

Note that j, k, l determine how $\hat{a}$ relates to co_i^*, v_i^* and $\underline{v_i}$ for all quality classes i. The profit-relevant intervals for $\hat{a}$ are $[\underline{v_i}, v_i^*]$ for all qualities i. Holders only hand in and cause either losses or profits if the residual values belong to those intervals. If some of these quality-specific intervals overlap, then the essential trade-off concerning the choice of $\hat{a}$ within such an overlap comprises balancing the profits of the corresponding quality classes.

The following corollary presents a simple procedure for determining the globally optimal largest acquisition price $\hat{a}$ by applying Proposition 3.2.

Corollary 3.2 (Finding the Global Optimum). *Due to the profit function's characteristics, there has to be at least one global optimum. Considering Proposition 3.2, for every triple (j, k, l) with $k \ne l$, determine $\hat{a}^*$ according to* (i). *Select all $\hat{a}^*$ for which conditions* (ii)-(v) *are satisfied, and determine $\Pi_{ACP}(\hat{a}^*)$. The one that yields the maximum profit is the globally optimal one.*

To illustrate Proposition 3.2, we present Figure 3.3(b), which depicts the total profit and the quality-dependent profits times their market shares depending on $\hat{a}$ for the indicated setting comprising three quality classes. The profit concerning a single quality class depends on how all corresponding parameters relate to one another. No single parameter that drives the optimal highest price can be identified. For example, there are identical market shares for classes 1 and 2, but how losses and profits through sales relate to one another is quite different when considering the shape of quality specific profit-curves. Contrarily, the market shares of qualities 1 and 3 deviate from one another, but the distances between the remaining parameters coincide. Therefore, the profits follow the same "shape", with the difference that the slope is steeper for quality 3, and therefore, the profits are magnified.

Considering the globally optimal largest price, we observe the mentioned trade-off of balancing the profits of quality classes 2 and 3. We see that the profit in class 2 has a negative slope in the optimal price, whereas the slope of the class 3 profit is positive here. Considering Proposition 3.2, the globally optimal price is equal to $\hat{a}^* = (\delta_2(-c)/(\overline{v_2} - \underline{v_2}) + \delta_3(m_3 + \underline{v_3})/(\overline{v_3} - \underline{v_3}))/(2\delta_3/(\overline{v_3} - \underline{v_3})) = 24.69$. Hence, one optimal choice for the acquisition prices is to set $a_3 = \hat{a}^*$, $a_2 = co_2^* = 22.5$, and $a_1 = co_1^* = 9.5$. By this choice, product holders with quality-1 and quality-2 products request $\hat{a}^*$, thereby triggering corresponding counteroffers that coincide with the respective quality-specific acquisition prices. Quality-3 product holders are paid acquisition price $a_3 = \hat{a}^*$. Hence, the firm incurs losses through rejection in quality class 1 caused by product holders with $v_1 \in (co_1^*, v_1^*) = (9.5, 11.5)$ and in class 2 caused by holders with $v_2 \in (co_2^*, \hat{a}^*) = (22.5, 24.69)$.

Finally, we remark that ACP leads to higher profit than the acquisition without quality differentiation. Acquisition without quality differentiation means offering only one fixed acquisition price independent of the condition of the product. To this end, assume that the optimal acquisition price, which does not depend on a product's quality, is given. Then, for ACP, $\hat{a}$ can be set equal to this price. If it is profitable to make counteroffers for submissions of products with certain qualities, the firm can do so if ACP is applied. This is not the case for having only one fixed quality-independent acquisition price. Thus, basically, the benefit is that reacting to low-quality product submissions is feasible when ACP is applied. Therefore, ACP is always preferable to the acquisition without quality differentiation.

3.6. Alternative Acquisition Process

Now that have we analyzed ACP with regard to the dynamics between the involved parties and found how to choose prices to maximize the firm's profit, we turn our attention to modifying this process. The reasons for modification are as follows: we have observed that in ACP, losses for the firm are caused due to rejections of counteroffers. Counteroffers are triggered by product holders because they have an incentive to exaggerate their products' qualities. Those exaggerations are mainly due to product holders imperfectly anticipating the firm's optimal counteroffers. Hence, if the firm wants to avoid losses, she has to incentivize holders to not trigger counteroffers. This can be achieved by two means: (i) the firm's commitment to and upfront communication of counteroffers to avoid product holders' speculation about the firm's reaction to quality statements and (ii) the correct balance of acquisition prices to signal to holders that they will not receive counteroffers for true quality statements. Additional potential benefits of having true quality statements are the following: testing effort and time could be reduced due to the firm being certain about products having the stated qualities, the cost factor comprising the organizational unit for sending products back could be reduced because holders are paid the requested money for true statements, and the firm's long-term relation to product holders could be enhanced due to product holders not receiving counteroffers being better publicity.

In order to introduce the firm's commitment to a counteroffer to the acquisition process, we modify ACP in the following manner: we give up the quality-dependent acquisition prices and instead introduce a quality-independent counteroffer as a fixed guaranteed price that is paid independently of the quality statement if the product is sold. Additionally, we introduce quality-dependent bonus payments paid for true quality statements. Then, the bonus payment plus a fixed price takes the role of the quality-dependent acquisition price of ACP, and the fixed price takes the role of any counteroffer. We denote this modified acquisition process by ABP.

Regarding Figure 3.1, the formal differences between ABP and ACP are the following: in stage 0, the firm sets the fixed price a and the quality-dependent bonus payments b_i for all $i \in \{1, ..., n\}$. Moreover, for the payoffs in subsequent stages, a_j is replaced by $a + b_j$, and co_{ij} is replaced by a.

3.6.1. Inducement to Truth-Telling

As initially discussed, incentivizing product holders to provide true quality statements and paying corresponding prices resolves the issue of rejections and has beneficial consequences. Therefore, we investigate how ABP can be used to assure true quality statements. To this end, we present conditions for the payments that induce true quality statements, i.e., conditions that cause the following equilibrium in stages I-III: for every quality i, a product holder with quality-i product either hands in, states the true quality of his product, and is paid fixed price plus bonus or else does not hand in at all. This is accomplished by fixing a and b_i in a manner such that every product holder's expected payoff for stating the true quality is not less than the payoff for stating the wrong quality. Formally, those conditions for the choice of a and b_i for all quality classes i are presented in the following proposition.

Proposition 3.3 (Incentives for True Quality Statements). *Assume that the following hold:*

(i) $b_i \geq 0$ *for all* $i \in \{1, ..., n\}$,

(ii) $0 \leq a \leq \underline{v_1}$,

(iii) $a + b_i \leq \underline{m_i} + c$ *for all* $i \in \{1, ..., n\}$,

(iv) $b_i = b_{i-1}$, *or* $a + b_i \geq \overline{m_{i-1}} + c$ *for all* $i \in \{2, ..., n\}$.

Then, in equilibrium, for all $i \in \{1, ..., n\}$, *product holder with quality-i product hands in, states the true quality and is paid* $a + b_i$ *if and only if* $v_i < a + b_i$. *Otherwise, the product is kept.*

Conditions (ii) and (iii) together signal to the product holder that if he states true quality i, he will receive $a + b_i$ with probability 1. Conditions (ii) and (iv) signal to the product holder that if a quality related to a higher bonus is stated instead of the true quality, the firm will not pay this requested higher bonus with probability 1.

Compared to ACP, we observe that here also, the extent of product holders' uncertainty about margins has an impact on the firm's acquisition performance. This dependence is due to the fact that the narrower the margin related intervals are, the more freedom for the choice of bonuses is given (see conditions (iii) and (iv)). The impact of c, the loss through rejection, on ABP is different from its impact on ACP. The larger c is, the larger is the upper bound for b_i regarding (iii), but the lower bound for b_i is larger too, regarding condition (iv). Hence, it is not obvious upfront whether an increase in the loss through rejection could even be beneficial for the firm regarding the choice of bonuses.

Finally, we present the following corollary, which, as for ACP, discusses payment for participation as another potential lever for the firm to increase profits when ABP with true quality statements is applied.

Corollary 3.3 (Payment for Participation - ABP). *Assume that product holders have to pay a participation fee of* $\epsilon > 0$ *for a product submission, conditions (i), (iii), and (iv) from Proposition 3.3 hold, and condition (ii) is replaced by* $0 \leq a \leq \underline{v_1} - \epsilon$. *Then, in equilibrium, for all i, product holders with quality i product hand in, state the true quality, pay* ϵ, *and are paid* $a + b_i$ *if and only if* $v_i < a + b_i - \epsilon$. *Otherwise, the product is kept.*

In contrast to ACP, the firm is able to charge product holders for participation without deterring all of them from handing in. This possibility entails another means for the firm to influence profit. This option is not further investigated in our analysis but is still a noteworthy benefit of ABP.

3.6.2. Optimal Bonus Payments Inducing Truth-Telling

Similarly as for ACP, we now turn our attention towards the optimal fixed price and bonus payments for ABP-inducing true quality statements. We follow the same structure as in Section 3.5 and start by considering a single quality class. Afterward, we extend the analysis to the general case comprising multiple quality classes.

If the conditions of Proposition 3.3 are satisfied (i.e., product holders are truthful and the firm pays bonuses for truthfulness), the firm's payoffs depending on v_i in one quality class i are as follows: if $v_i \in [\underline{v_i}, \max\{\min\{\overline{v_i}, a + b_i\}, \underline{v_i}\})$, then the firm has payoff $m_i - (a + b_i)$. If $v_i \in [\max\{\min\{\overline{v_i}, a + b_i\}, \underline{v_i}\}, \overline{v_i}]$, then the firm has payoff 0. Note that the firm never has payoff $m_i - a$ or $-c$ if the conditions of Proposition 3.3 hold. With regard to the above interval boundaries, we define $w_i(a, b_i) := \max\{\min\{\overline{v_i}, a + b_i\}, \underline{v_i}\}$, which is the upper bound for v_i for handing in. The firm's expected profit considering only one quality class i is given by

$$\begin{aligned}\Pi^i_{ABP}(a, b_i) &= \mathbb{P}(v_i < a + b_i)(m_i - (a + b_i)) + \mathbb{P}(v_i \geq a + b_i)0 \\ &= \frac{1}{\overline{v_i} - \underline{v_i}}(w_i(a, b_i) - \underline{v_i})(m_i - (a + b_i))\end{aligned}$$

The firm's total expected profit under consideration of all qualities is just the weighted sum of all these quality-specific profits and given by

$$\Pi_{ABP}(a, b_1, ..., b_n) = \sum_{i=1}^{n} \delta_i \Pi^i_{ABP}(a, b_i)$$

Furthermore, to ensure true quality statements, the conditions from Proposition 3.3 have to be incorporated into the model as constraints. This is straightforward except for condition (iv). We introduce quality-dependent, binary variables $y_i \in \{0, 1\}$ for all $i \in \{2, ..., n\}$ to properly capture this condition. Intuitively, $y_i = 0$ implies that b_i and b_{i-1} have to differ and both have to meet different lower bounds, whereas $y_i = 1$ forces $b_i = b_{i-1}$ sharing the same (less restrictive) lower bound. The firm's optimization problem is as follows:

$$
\begin{aligned}
P2: \quad \max_{a,b_1,\dots,b_n,y_2,\dots,y_n} \quad & \Pi_{ABP}(a,b_1,...,b_n) \\
&= \sum_{i=1}^{n} \frac{\delta_i}{\overline{v_i}-\underline{v_i}}(m_i-(a+b_i))(w_i(a,b_i)-\underline{v_i}) && (3.5)\\
\text{s.t.} \quad & w_i(a,b_i)=\min\{\max\{a+b_i,\underline{v_i}\},\overline{v_i}\} \qquad \forall i\in\{1,...,n\} && (3.6)\\
& a+b_i \le \underline{m_i}+c \qquad \forall i\in\{1,...,n\} && (3.7)\\
& a+b_i \ge (1-y_i)(\overline{m_{i-1}}+c) \qquad \forall i\in\{2,...,n\} && (3.8)\\
& y_i(b_i-b_{i-1})=0 \qquad \forall i\in\{2,...,n\} && (3.9)\\
& a\le \underline{v_1} && (3.10)\\
& a,b_i\ge 0 \qquad \forall i\in\{1,...,n\} && (3.11)\\
& y_i\in\{0,1\} \qquad \forall i\in\{2,...,n\} && (3.12)
\end{aligned}
$$

Note that if $a^*, b_1^*, ..., b_n^*$ solve the problem, then $a'^* = 0$ and $b_i'^* = a^* + b_i^*$ for all i solve the problem as well. Therefore, without loss of generality, we set $a = 0$ for the rest of the paper and do not consider a as a decision variable anymore.

Next, we investigate the structure of the locally optimal bonus payments given fixed binary variables y_i, $i \in \{2, ..., n\}$. The reason for this choice is that if we are able to characterize the optimal solutions given fixed binary variables, we can simply go through the finitely many possibilities for the choice of values for those variables and thereby find the global optimum. Note that $y_i = 1$ has to hold if $\underline{m_i} < \overline{m_{i-1}}$. Otherwise, satisfying both constraints (3.7) and (3.8) is not feasible. Furthermore, note that having all binary variables fixed induces disjunctive index sets I_1,...,I_r, where $\bigcup_{k=1}^{r} I_k = \{1, ..., n\}$ and $r = n - \sum_{i=2}^{n} y_i$, with the following characterization: if $y_i = 1$, then $i-1$ and i belong to the same index set. If $y_i = 0$, then $i-1 \in I_k$ and $i \in I_{k+1}$ for some $k \le r-1$. Then, for indices in the same index set, we have identical corresponding bonuses, whereas for indices in different sets, the bonuses must differ. Making use of those index sets, we can define $\Pi_{ABP}^{I_k} := \sum_{i\in I_k} \delta_i \Pi_{ABP}^i$. Then, for fixed binary variables, we have $\Pi_{ABP} = \sum_{k=1}^{r} \sum_{i\in I_k} \delta_i \Pi_{ABP}^i = \sum_{k=1}^{r} \Pi_{ABP}^{I_k}$.

Now, if the binary variables are fixed, problem $P2$ can be split up into r independent subproblems. Each subproblem only contains bonuses corresponding to indices of the same index set. Hence, there is only one decision variable per subproblem due to those bonuses having to coincide by constraint (3.9).

Considering only one subproblem, we drop index $k \in \{1, ..., r\}$ and assume that $I = \{j, ..., j+u\}$, which means that $y_{j+1} = ... = y_{j+u} = 1$ and $y_j = y_{j+u+1} = 0$ by definition of such an index set. Furthermore, instead of writing $b_j, ..., b_{j+u}$, we write $\hat{b}$ because the bonuses corresponding to I have to coincide. Since all considered y_i are fixed, we insert the corresponding values into the subproblem, which is then as follows:

$$P2-S: \quad \max_{\hat{b}} \ \Pi^I_{ABP}(\hat{b}) = \sum_{i=j}^{j+u} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} (m_i - \hat{b})(w_i(\hat{b}) - \underline{v_i}) \tag{3.13}$$

$$\text{s.t. } w_i(\hat{b}) = \min\{\max\{\hat{b}, \underline{v_i}\}, \overline{v_i}\} \qquad \forall i \in I \tag{3.14}$$

$$\hat{b} \leq \underline{m_j} + c \tag{3.15}$$

$$\hat{b} \geq \overline{m_{j-1}} + c \tag{3.16}$$

$$\overline{m_0} = 0 \tag{3.17}$$

Bridging the gap with Section 3.5, we again have an optimization problem with only one decision variable, in which the objective function exhibits a similar structure as the one in problem $P1$: it consists of constant and quadratic concave parts and is differentiable except in $\underline{v_i}$ and $\overline{v_i}$. New are the truth-telling constraints, which represent lower and upper bounds on $\hat{b}$.

The following lemma characterizes all possible locally optimal solutions for $P2-S$.

Lemma 3.2 (Locally Optimal Bonus Payment - P2-S)**.** *$\hat{b}^*$ locally solves problem $P2-S$ if and only if $\overline{m_{j-1}} + c \leq \hat{b}^* \leq \underline{m_j} + c$, and there are $s, t \in I$ such that $\hat{b}^* \geq \overline{v_l}$ for $j \leq l \leq s$, $\underline{v_l} < \hat{b}^* < \overline{v_l}$ for $s < l \leq t$, $\hat{b}^* \leq \underline{v_l}$ for $t < l$, and*

(i) $\hat{b}^* = (\sum_{i=s+1}^{t} \delta_i(m_i + \underline{v_i})/(\overline{v_i} - \underline{v_i}) - \sum_{i=j}^{s} \delta_i)/(2\sum_{i=s+1}^{t} \delta_i/(\overline{v_i} - \underline{v_i}))$, *or*

(ii) $\hat{b}^* = \overline{v_p}$, *some* $p \in I$, $\lim_{x \uparrow \overline{v_p}} (\partial \Pi^I_{ABP}/\partial \hat{b})(x) \geq 0$, $\lim_{x \downarrow \overline{v_p}} (\partial \Pi^I_{ABP}/\partial \hat{b})(x) \leq 0$, *or*

(iii) $\hat{b}^* = \overline{m_{j-1}} + c$ *and* $(\partial \Pi^I_{ABP}/\partial \hat{b})(\hat{b}^*) < 0$, *or*

(iv) $\hat{b}^* = \underline{m_j} + c$ *and* $(\partial \Pi^I_{ABP}/\partial \hat{b})(\hat{b}^*) > 0$.

The resulting profit for index set I is given by

$$\Pi^I_{ABP}(\hat{b}^*) = \sum_{i=j}^{s} \delta_i(m_i - \hat{b}^*) + \sum_{i=s+1}^{t} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} (\hat{b}^* - \underline{v_i})(m_i - \hat{b}^*).$$

By using Lemma 3.2, we are now able to characterize all locally optimal solutions for problem $P2$. When the binary variables are fixed, there is no interrelation between the index sets and corresponding bonuses. All bonuses can be chosen according to the preceding lemma. Then, one only has to sum over the index sets. This is formally summarized in the following proposition.

Proposition 3.4 (Locally Optimal Bonus Payments). *Let $y_i = 1$ if $\underline{m_i} < \overline{m_{i-1}}$, and let all other y_i be arbitrarily fixed. Let $I_1, ..., I_r$ be such that $\bigcup_{k=1}^{r} I_k = \{1, ..., n\}$. If $y_i = 0$, let $i-1 \in I_k$ and $i \in I_{k+1}$ for one $k \in \{1, ..., r-1\}$, and if $y_i = 1$, let $i, i-1 \in I_k$ for one $k \in \{1, ..., r\}$. Let $a = 0$ in problem $P2$. Then, $\hat{b}_1^*, ..., \hat{b}_r^*$ locally solve $P2$ if and only if for all $k \in \{1, ..., r\}$, $\hat{b}_k^*$ locally solve $P2 - S$ for I_k in the sense of Lemma 3.2. The resulting total profit is given by*

$$\Pi_{ABP}(\hat{b}_1^*, ..., \hat{b}_r^*) = \sum_{k=1}^{r} [\sum_{i=j_k}^{s_k} \delta_i (m_i - \hat{b}_k^*) + \sum_{i=s_k+1}^{t_k} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} (\hat{b}_k^* - \underline{v_i})(m_i - \hat{b}_k^*)].$$

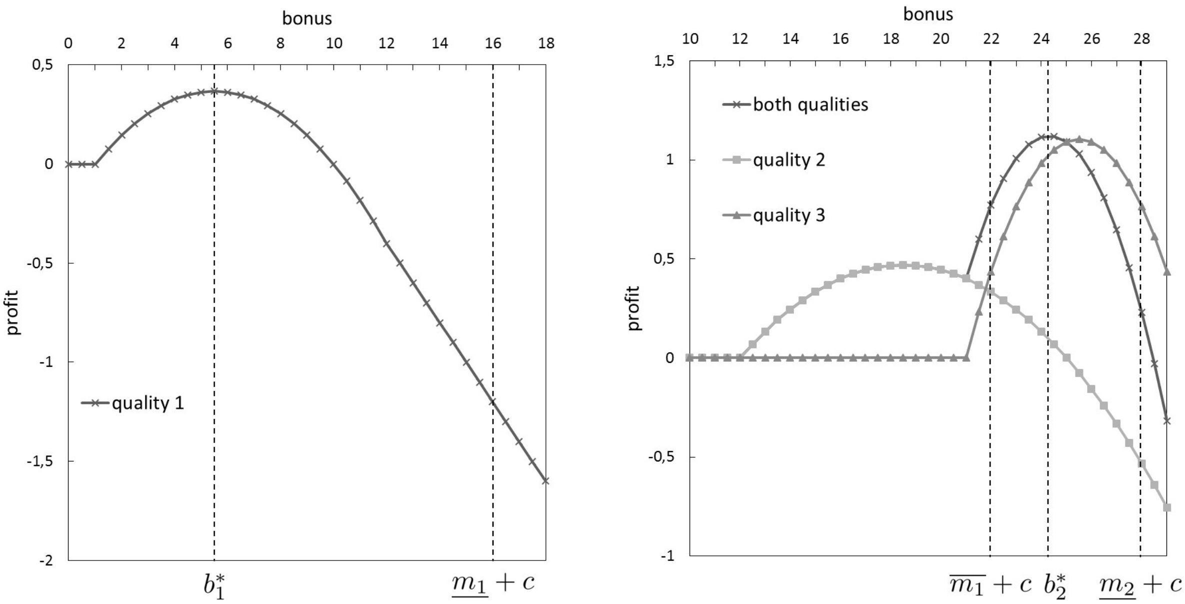
A similar procedure as for finding the globally optimal solution for $P1$ given in Corollary 3.2 can be specified for the globally optimal solution for $P2$ by use of Lemma 3.2 and Proposition 3.4. This procedure is summarized in the following corollary.

Corollary 3.4 (Finding the Global Optimum). *Due to the profit function's characteristics, there must be at least one global optimum. Considering Proposition 3.4, for every feasible value combination of binary variables y_i, do the following: for every index set $I_1, ..., I_r$, find $\hat{b}_k^*$, $k \in \{1, ..., r\}$ that globally solves $P2 - S$ for I_k by going through the different possibilities given in Lemma 3.2. The resulting bonus combination is the globally optimal solution given the fixed value combination of binary variables. The value combination of binary variables that generates the highest overall profit Π_{ABP} is the globally optimal one.*

To illustrate our findings, we again draw on the example comprising three quality classes already indicated in Figure 3.3. Furthermore, we fix the new corresponding binary variables to $y_2 = 0$ and $y_3 = 1$, thus inducing index sets $I_1 = \{1\}$ and $I_2 = \{2, 3\}$. Therefore, two corresponding bonuses $\hat{b}_1 = b_1$ and $\hat{b}_2 = b_2 = b_3$ are considered. The resulting profits are depicted in Figure 3.4.

The feasible range for bonus $\hat{b}_1$ is between 0 and $\underline{m_1} + c$. The range for bonus $\hat{b}_2$ corresponding to qualities 2 and 3 is located between $\overline{m_1} + c$ and $\underline{m_2} + c$. The lower bound ensures that product holders with a quality-1 product do not state quality 2 or 3. They know upfront that stating the incorrect quality will result in not getting paid the corresponding

Figure 3.4.: Firm's Profit under ABP



(a) Profit in $I_1 = \{1\}$ (b) Profit in $I_2 = \{2, 3\}$

Parameters: $c = 8$; for quality 1: $\delta_1 = 0.2$, $m_1 = 10$, $\overline{m_1} = 14$, $\underline{m_1} = 8$, $\overline{v_1} = 12$, $\underline{v_1} = 1$; quality 2: $\delta_2 = 0.2$, $m_2 = 25$, $\overline{m_2} = 32$, $\underline{m_2} = 20$, $\overline{v_2} = 30$, $\underline{v_2} = 12$; quality 3: $\delta_2 = 0.6$, $m_3 = 30$, $\overline{m_3} = 34$, $\underline{m_3} = 28$, $\overline{v_3} = 32$, $\underline{v_3} = 21$.

bonus. The upper bound ensures that product holders with a quality-2 or quality-3 product know that they will definitely be paid the corresponding bonus.

Deciding about bonuses within the allowed ranges comprises the natural price-volume trade-off. For I_1, it is obvious that the larger the bonus is, the more holders are willing to sell. The optimal bonus is given by $\hat{b}_1^* = (\delta_1(m_1 + \underline{v_1})/(\overline{v_1} - \underline{v_1}))/(2\delta_1/(\overline{v_1} - \underline{v_1})) = 5.5$ by Lemma 3.2. The corresponding maximum profit is $\Pi_{ABP}^{I_1}(\hat{b}_1^*) = 0.37$. The profit in I_2 has its maximum in an area where the intervals for residual values of qualities 2 and 3 overlap. By Lemma 3.2, we have $\hat{b}_2^* = (\sum_{i=2}^{3} \delta_i(m_i + \underline{v_i})/(\overline{v_i} - \underline{v_i}))/(2\sum_{i=2}^{3} \delta_i/(\overline{v_i} - \underline{v_i})) = 24.32$. The corresponding maximum profit is $\Pi_{ABP}^{I_2}(\hat{b}_2^*) = 1.12$. Hence, the total maximum profit if the binary variables are fixed such that quality classes 2 and 3 are combined is $\Pi_{ABP}(\hat{b}_1^*, \hat{b}_2^*) = 0.37 + 1.12 = 1.49$.

To find the globally optimal solution for the example, by Corollary 3.4, we have to consider every feasible combination of binary variable values. Note that $y_3 = 1$ must hold because $\overline{m_2} > \underline{m_3}$. Otherwise, satisfying (3.7) and (3.8) simultaneously is not possible. Hence, the remaining possibility is to consider $y_2 = y_3 = 1$. Solving the problem leads to $\Pi_{ABP} = 0.37$. Therefore, choosing $y_2 = 0$ and $y_3 = 1$ is optimal. This conclusion is basically due to the following: if $y_2 = y_3 = 1$, all bonuses have to coincide and respect the upper bound $\underline{m_1} + c = 16$ to signal to product holders that the bonus is paid for true quality statements. However, since $16 < \underline{v_3}$, no product holder with a quality-3 product hands in, and 60% of potential sales are lost. Additionally, in this situation, it is suboptimal to even access quality-2 product holders because the firm would pay too much for quality-1 products. If $y_2 = 0$ and $y_3 = 1$, then the firm receives the same gains in class 1 as in the other situation, but additionally, she profitably buys products from quality-2 and quality-3 product holders because the bonuses for class 1 and for classes 2 and 3 must differ.

Before proceeding to the next section, in which ACP and ABP are compared with regard to profitability, we briefly summarize the most important structural differences between the investigated acquisition processes: due to the dynamics of ACP, the firm's only lever to affect profit is the level of the highest acquisition price. Quality-dependent acquisition prices can be arbitrarily fixed as long as they are not greater than this level. Moreover, the firm can react to incorrect quality statements with quality-specific counteroffers. For ABP, the firm's decision variables are the quality-dependent bonus payments, which have to satisfy several constraints to ensure truth-telling, and binary variables, which cause combination of neighboring quality classes by offering identical or different bonuses. Here, the firm does not have any freedom in the choice of counteroffers during a single submission because they are fixed beforehand.

3.7. Process Comparison

Now that we know how the maximum profit for ACP and ABP with true quality statements can be determined with the corresponding prices/bonuses, we turn our attention to comparing both processes with regard to profitability. To this end, let Π^*_{ACP} be the profit resulting from the solution for $P1$ and Π^*_{ABP} the profit resulting from the solution for $P2$. As a first important result, note the following corollary.

Corollary 3.5 (Benchmark Profit). *If* $b_i^* = \min\{\overline{v_i}, (m_i + \underline{v_i})/2\}$ *for all* $i \in \{1, ..., n\}$ *is a feasible solution for P2, then it is optimal, and* $\Pi^*_{ABP} \geq \Pi^*_{ACP}$.

Corollary 3.5 reveals that the optimal solution for the unconstrained problem $P2$ always leads to higher profits than the optimal solution for $P1$. Intuitively, this observation tells us that the closer the optimal bonus payments solving $P2$ are to the stated solution of the unconstrained problem, the more likely ABP is to be more profitable than ACP. This means that product holders' truth-telling is desirable from firm's perspective as long as it is cheap with regard to truth-telling constraints for bonuses.

Optimization problems $P1$ and $P2$ differ structurally, and the complexity of an insightful comparison strongly increases with the number of quality classes. Therefore, for the following, we make simplifying assumptions about parameter constellations in order to allow for an analytical investigation about when which process is more profitable. Afterward, we illustrate the findings, and based on the analysis, we conclude with general conjectures.

Assumptions for Comparison:

(i) $\underline{v_1} = 0$.

(ii) *There are nonnegative parameters* K, L, R, *such that for all* $1 \leq i \leq n$, $K = m_i - \underline{v_i} = \overline{v_i} - m_i$, $L = m_i - \underline{m_i} = \overline{m_i} - m_i$ *and for all* $2 \leq i \leq n$, $R = m_i - m_{i-1}$.

(iii) $R \geq 2K$ *and* $R \geq K - L + c$.

(iv) $\delta_i/\delta_{i-1} \geq \delta_{i+1}/\delta_i$ *for all* $i \in \{1, ..., n\}$ *and* $\delta_0 = \delta_{n+1} = 0$.

Assumption (i) is just a rescaling that does not have any effect on structural properties. Assumption (ii) ensures that every quality class has the same structure, i.e., the distances between parameters within one quality class are the same for all quality classes, which supports an insightful comparison. R denotes the distance between quality-dependent margins, L captures the uncertainty about margins, and K captures the product holder heterogeneity with regard to residual values. Assumptions (i) and (ii) imply the following representation of parameters for all i: $m_i = (i-1)R + K$, $\underline{m_i} = (i-1)R + K - L$, $\overline{m_i} = (i-1)R + K + L$, $\overline{v_i} = (i-1)R + 2K$, $\underline{v_i} = (i-1)R$. Note that $L \leq K$ has to hold to satisfy $\underline{v_i} \leq \underline{m_i}$. The first inequality of assumption (iii) says that the v_i intervals do not overlap for different qualities: $\overline{v_i} \leq \underline{v_{i+1}}$ for all $i \leq n-1$. The second inequality implies that $\underline{m_i} + c \leq \underline{v_{i+1}}$ for all $i \leq n-1$. Assumption (iv) limits the possibilities for how market shares relate to one another. Still, it captures various imaginable scenarios. For instance, if $\delta_{i+1}/\delta_i \geq 1$ for all $i \in \{1, ..., n-1\}$, then δ_i is non-decreasing in i, which potentially applies to products shortly after market launch. If $\delta_{i+1}/\delta_i \leq 1$ for all $i \in \{1, ..., n-1\}$, then δ_i is non-increasing in i, which potentially applies to products that are close to the ends of their life cycles or to products that have an inherent high abrasion. Uniform distribution is also representable.

The maximal profits for the assumptions are presented in Proposition 5. The corresponding terms follow from Propositions 3.2 and 3.4.

Proposition 3.5 (Optimal Profit). *Under assumptions (i)-(iv), it holds that*

$$\Pi^*_{ACP} = \begin{cases} \delta_j \frac{K}{8} + \sum_{i=1}^{j-1} \delta_i [\frac{K}{8} - \frac{c(2L+c)}{8K}] & \text{with } j = \max_{i \leq n} i : \frac{\delta_i}{\delta_{i-1}} \geq \frac{c(2L+c)}{K^2} \quad \text{if } \frac{L+c}{2} \leq \frac{3}{2}K \\ \delta_j \frac{K}{8} + \sum_{i=1}^{j-1} \delta_i [\frac{K}{8} - \frac{c(6K-c)}{8K}] & \text{with } j = \max_{i \leq n} i : \frac{\delta_i}{\delta_{i-1}} \geq \frac{c(6K-c)}{K^2} \quad \text{if } \frac{c}{2} \leq \frac{3}{2}K \leq \frac{L+c}{2} \\ \delta_j \frac{K}{8} - \sum_{i=1}^{j-1} \delta_i K & \text{with } j = \max_{i \leq n} i : \frac{\delta_i}{\delta_{i-1}} \geq 9 \quad \text{if } \frac{3}{2}K \leq \frac{c}{2} \end{cases}$$

$$\Pi^*_{ABP} = \begin{cases} \delta_1 \frac{K}{8} & \text{if } R \leq L + c \\ \frac{K}{8} - \frac{1}{8K}(1-\delta_1)(K - 2(R-L-c))^2 & \text{if } R - \frac{K}{2} \leq L + c \leq R \\ \frac{K}{8} & \text{if } L + c \leq R - \frac{K}{2} \leq R - L + c \\ \frac{K}{8} - \frac{1}{8K}(2(L-c) - K)^2 & \text{if } R - L + c \leq R - \frac{K}{2} \end{cases}$$

The three cases for Π^*_{ACP} differ in terms of the structure of the cutoff levels for handing in, v_i^*, and the optimal counteroffers, co_i^*, mainly depending on the size of loss through rejection c. The four cases for Π^*_{ABP} differ in how the upper and lower truth-telling bounds for bonuses indicated by constraints (3.7) and (3.8) of problem $P2$ relate to the solutions for the unconstrained problem $P2$ indicated in Corollary 3.5.

Based on the assumptions and the presented terms in Proposition 3.5, the following lemma captures insights about when one of the processes is more profitable than the other.

Lemma 3.3 (Profit Observations). *Under assumptions (i)-(iv), the following hold:*

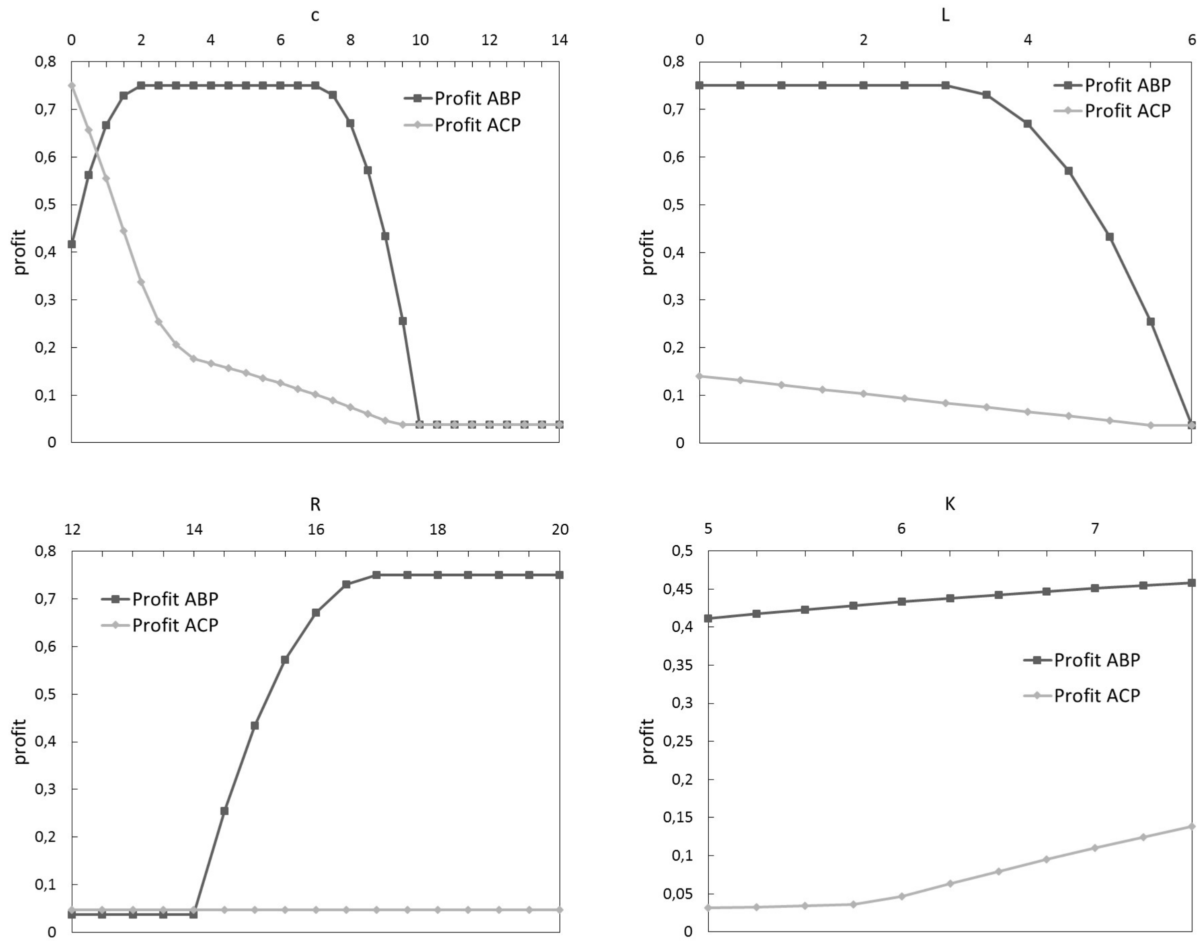
(i) *If* $L \leq c + K/2 \leq R - L$, *then* $\Pi^*_{ABP} \geq \Pi^*_{ACP}$.
(ii) *If* $L + c \geq 3K$, *and* $\delta_{i+1}/\delta_i \leq 6$ *for all* i, *then* $\Pi^*_{ABP} \geq \Pi^*_{ACP}$.
(iii) *If* $L + c \geq R$, *then* $\Pi^*_{ABP} \leq \Pi^*_{ACP}$.
(iv) *If* $c \leq K/16$, $L \geq 3K/4$, *and* $\delta_{i+1}/\delta_i \geq 1/7$ *for all* i, *then* $\Pi^*_{ABP} \leq \Pi^*_{ACP}$.
(v) Π^*_{ACP} *and* Π^*_{ABP} *are non-increasing in* L. Π^*_{ACP} *is non-increasing in* c.

The sufficient condition in observation (i) implies that the optimal solution of the unconstrained problem $P2$ is a feasible solution for the constrained problem $P2$ and hence optimal (see also Corollary 3.5). Intuitively, if the distance between margins is sufficiently large to compensate loss due to rejection and uncertainty about margins, then product holders are naturally incentivized to truth-telling if bonuses are chosen optimally without considering the truth-telling constraints. Observation (ii) states that if c is large compared to the residual value heterogeneity, K, it is beneficial to apply ABP because in ACP, it is crucial for the firm to avoid costly rejections, and therefore, it is optimal to set the highest acquisition price to be very low. This results in only quality-1 product holders handing in, whereas acquisitions of products with better quality does not occur. Observation (iii) states that if c is large compared to the distance between margins, R, ABP is less profitable than ACP. Here, having differing bonuses for different quality classes would be costly for the firm. Therefore, it is optimal to combine all quality classes by offering identical bonuses. Then, for ABP, only quality-1 product holders hand in. If the conditions of observations (ii) and (iii) apply simultaneously, it makes no difference in terms of profits whether ABP or ACP is applied. Observation (iv) states that a very small loss due to rejection in combination with a quite large uncertainty about margins favors applying ACP over ABP. Here, it is optimal to choose bonuses to be quite small to signal to product holders that they are paid the bonuses for true quality statements. This results in only few product holders handing in. For ACP, a very small loss due to rejection implies no fierce consequences of rejections. Consequently, under these circumstances, it is optimal to choose a very high highest acquisition price to access all quality classes. Finally, observation (v) shows that both profits are non-increasing in the uncertainty about margins, L, and Π^*_{ACP} is non-increasing in loss due to rejection, c. Intuitively, the higher the uncertainty about margins is, the more holders that cause losses by rejecting counteroffers are incentivized to hand in if ACP is applied. For ABP, considering one quality class, an increase in uncertainty might force a lower bonus to signal to product holders that they are paid the bonus for a true statement. Moreover, an increase in the uncertainty corresponding to a lower quality class might force a larger bonus to ensure that holders with lower-quality products are signaled that they will not be paid the bonus for the considered quality if stated.

For illustration, Figure 3.5 displays how the profits change for varying parameters c, L, R, and K based on the indicated parameter setting, which comprises five quality classes and satisfies assumptions (i)-(iv).

Not surprisingly, the loss due to rejection c always has a negative impact, if any, on ACP's profit, which is in line with Lemma 3.3 (v). For ABP, the effect varies. In the depicted situation, Π^*_{ABP} is first increasing, then constant, and then decreasing in c. For very small c and if the uncertainty about margins, L, is quite large, bonuses are chosen to be small compared to the solution of the unconstrained problem $P2$ to signal to product holders that bonuses are paid for true statements. For large c, bonuses are large. This signals to holders that they are not paid larger bonuses for upward deviations from true quality statements.

Figure 3.5.: Firm's Optimal Profit - Comparison of ACP and ABP



Parameters: $\delta_1 = 0.05$, $\delta_2 = 0.25$, $\delta_3 = 0.3$, $\delta_4 = 0.25$, $\delta_5 = 0.15$, $K = 6$, $L = 5$, $R = 15$, $c = 9$.

The uncertainty about margins, L, has a negative impact on both profits, as indicated in Lemma 3.3 (v). One can observe that for ABP, the impact is much more crucial. For ACP, the effect is constantly negative, until it no longer has an effect. The distance between margins, R, does not have any effect on ACP, which is in line with the terms in Proposition 3.5. For ABP, there is either no effect, or it is positive because that distance compensates the negative effects of margin uncertainties or loss through rejection. Finally, profits are increasing in the heterogeneity of residual values, K which is mainly due to the price-volume trade-off when deciding about payments (acquisition prices, counteroffers, or bonuses). For larger heterogeneity, smaller payments must be chosen to generate the same sales volumes.

To conclude our comparison of ABP and ACP with regard to profitability, we summarize the most significant impacts of the parameters in the following conjectures.

Conjecture 3.1. *Applying ABP with truth-telling is more profitable than applying ACP if the distance between quality-dependent margins is large.*

This implies that the acquisition mode choice is closely related to a product's architecture. For example, if a product consists of expensive components that must be replaced instead of repaired if damaged, then there might be large gaps between quality-dependent margins due to high expenses for every component that has to be replaced. For acquisition of those products, it is advisable to apply ABP with truth-telling.

Conjecture 3.2. *Applying ACP is more profitable than applying ABP with truth-telling if the uncertainty about quality-dependent margins of product holders is high.*

This implies that the trade-off for the firm between revelation of business data to convincingly inform product holders about margins and closeness about that information with regard to market competition is more crucial for ABP than for ACP.

To summarize, neither of the presented acquisition processes dominates the other in terms of profit. Firms must thoroughly analyze products' and the used-product market's characteristics to decide which process to implement. Both presented processes exhibit special benefits and drawbacks.

3.8. Conclusions

Our work is concerned with the question of how a firm should acquire used products in order to reprocess and sell them. As a starting point, we investigate an acquisition process with quality differentiation widely applied in the business field of recommerce with game-theoretical methods under incomplete information. We characterize the equilibrium, which serves as the basis for determining the optimal quality-dependent acquisition prices. The equilibrium reveals that product holders have an incentive to upward deviations from true quality statements, which is in line with existing observations. Due to this incentive issue, which induces losses for firms, we propose a modified process with bonus payments, which allows for incentivizing product holders to provide true quality statements if the bonuses satisfy certain conditions. We determine the optimal quality-dependent bonuses that induce true quality statements. After investigation of both processes in isolation, we investigate under what conditions one process is more profitable than the other one. To the best of our knowledge, we are the first to bring together quality-dependent acquisition pricing and strategic interaction between the seller and buyer under information asymmetries in reverse logistics, which is crucial for firms reprocessing and selling used products in order to secure sufficient and cheap supply.

From a managerial perspective, we find that being transparent about cost and profit structure can be beneficial for firms to reduce product holders' uncertainty about firm performance. This aspect of incomplete information of product holders has a negative impact on profit in both processes. However, firms might be reluctant to share this information due to competitive advantages.

Another important, but less surprising, observation is that reduction of the loss through rejection is beneficial for the firm if the common process is applied. Due to the fact that transportation costs constitute a large share of this parameter, firms should carefully decide about how centralized or decentralized the network should be set up. Interestingly, for the process with bonus payments, the impact of loss due to rejection on profit can, under certain circumstances, also be positive. Moreover, we identified a participation fee as another potentially profitable lever for the firm when the process with bonus payments is applied.

There are some noteworthy limiting assumptions underlying the presented models and some potential extensions that should be mentioned. That the heterogeneity of product holder's residual values and uncertainty about margins are common knowledge to firm and product holders is one sacrifice we must make to be able to analyze the process as a game with incomplete information. Furthermore, we assume that product holders can perfectly determine products' qualities. In many cases, there are helpful explanations and examples on websites that afford product holders quite precise quality assessments. Still, there might be a small amount of uncertainty left, which could be taken into account in further investigations. We account for competition only in a static manner. We assume that there is a market for used products that is segmented in a certain manner depending on quality and that is accessible by the firm depending on the pricing. Future research regarding dynamic competition in such an acquisition environment might provide interesting new insights for firms. Although we consider volume and long-term effects in our analysis, given the prevalence customer reviews on the Internet about the service of and their experience with firms, another possible direction for future research is to analyze repeated acquisition games with reputation effects. Moreover, one could also consider having a proportion of product holders stating true qualities. We suspect that this will have an interesting impact on the pricing decision and is probably more realistic than assuming that all product holders act strategically. Furthermore, empirical research regarding product holders' behavior when confronted with such acquisition processes is an interesting subject for future work. Finally, the investigation of how product holders value their products and of their beliefs about the potential margins of products for a firm, are, from our perspective, fruitful avenues for the future.

Chapter IV

Used-Product Acquisition Through an Intermediary

with Moritz Fleischmann and Jochen Schlapp[1]

4.1. Introduction

Quality-dependent pricing policies are one important lever to exploit used products' quality differences in the acquisition (Guide et al. 2003). Recommerce providers, such as Gazelle, ReBuy, and FLIP4NEW (F4N), whose success is crucially dependent on the economic supply with used products are one prime example for such policies. F4N, a German recommerce provider operating in parts of Europe, buys, re-processes, and re-sells mostly small electronic devices. Besides its prominent online acquisition channel (see, e.g., Hahler and Fleischmann 2017), F4N also runs offline channels in which it collaborates with various partner stores to reach additional, potential used-product sellers that are not accessible otherwise. This collaboration is accompanied by asymmetric information concerning the acquisition process at those partner stores, which can result in incentive conflicts.

Our work is motivated by this business situation. We address questions about how to acquire used products through an intermediary under the natural presence of products' quality heterogeneity and asymmetric information.

The detailed acquisition process in the motivating example is as follows. Before products can be acquired, F4N (from now on, "the firm") has to fix acquisition prices for products of different qualities. In practice, the prices coincide with those on the firm's website for the online acquisition. A product holder interested in selling an old product can visit the partner store (from now on, "the retailer"), where it is tested using similar but simpler criteria as

[1]The research presented in this chapter is based on a paper entitled "Used-Product Acquisition through an Intermediary under Quality Heterogeneity", coauthored with Moritz Fleischmann and Jochen Schlapp.

at the firm. Testing means determining a product's quality in terms of various aspects (e.g., optical condition, technical functionality, etc.) through searching for errors by use of, e.g., disassembly, optical inspection, and testing software. After testing, an acquisition price is offered depending on the determined quality. If accepted, the holder receives the offered price in form of a voucher from the retailer, for which the retailer is later on compensated by the firm. Acquired products are stored until they are shipped to the firm at predetermined intervals (1-2 weeks). There, the products are perfectly tested, and the actual qualities are determined, which provides the basis for the subsequent reprocessing steps. It is reported that testing results of the firm and retailer sometimes differ with regard to valuation or even determination of the correct product type.

Several characteristics of this process may cause inefficiencies. The retailer can decide about how much effort to put into testing through, e.g., training of employees, and the time, steps, and means considered for testing. More effort implies higher costs but also more reliable testing outcomes for the retailer. In the presented setting, the chosen testing effort and testing outcome are hardly observable by the firm and can be assumed to be the retailer's private information. Additionally, the vouchers that were offered are only known in hindsight by the firm and only if the offers were accepted.

Those circumstances could incentivize the retailer to not thoroughly test products and/or to not adhere to testing outcomes with offers. However, one could imagine that in-depth testing is actually not really worth the additional costs from firm's perspective. Those arguments imply that finding contracts to correctly steer the retailer is one essential part of answering our initial question.

Taking one step back, another valid question is whether upfront testing is beneficial at all. Of course, testing and quality-dependent acquisition are actually applied in our business example, which is probably due to the firm not wanting to deviate from the online acquisition policy. However, do the benefits of quality-differentiated prices really outweigh the additional complexities and efforts introduced by testing? Are there no imaginable situations in which offering just one fixed price without the whole testing procedure is more beneficial? Those questions also indicate that acquisition pricing (both quality-dependent and quality-independent) is another important means to fully answer the posed question.

In summary, our work's contribution comprises the investigation of the optimal used-product acquisition policy when acquiring through an intermediary. This is accomplished by answering the following questions. (i) Should a firm acquire used products through an intermediary with or without quality-differentiation/upfront testing? (ii) How should the additional complexities introduced by testing under quality-differentiated acquisition through an intermediary be managed? Our means for answering those questions are the optimal contracts and the optimal acquisition prices. Furthermore, we clarify the considered acquisition channel's efficiency and, additionally, discuss how and when the investigated collaboration can be simplified.

The remainder of the paper is organized as follows. Section 4.2 positions our work within the extant research literature. Section 4.3 models the acquisition process with and without quality-differentiation. In Section 4.4, we investigate the optimal undifferentiated acquisition policy. In Section 4.5, we find the optimal differentiated acquisition policies with upfront testing. Based on the findings in the previous two sections, we characterize when to apply which acquisition policy in Section 4.6. Section 4.7 discusses the efficiency of the present acquisition channel. In Section 4.8, possible simplifications of the collaboration are discussed. Section 4.9 concludes with some managerial implications, limitations, and suggestions for future research.

4.2. Related Literature

Our work builds on and contributes to research on used-product acquisition management (see Fleischmann et al. 2010, for a broad overview regarding this literature), which is a subfield of literature concerned with closed-loop supply chains (CLSCs) and reverse logistics. Various papers in the area of CLSCs consider active supply with used products under negligence of quality differences (e.g., Savaskan et al. 2004, Debo et al. 2005). These papers are concerned with long-term considerations such as which acquisition channel to use for supply for remanufacturing or whether to produce remanufacturable products. Research about used-product acquisition management, however, takes a closer look at the control of quantity/timing and quality of product returns or purchases. Due to products' qualities playing a central role, testing/grading is considered in several papers from that research field.

The substream on quality-dependent acquisition pricing, in particular, takes upfront testing into account. This stream is more related to tactical issues and was initiated by Guide et al. (2003), who investigated how to manage the quantity and quality of returns via the choice of quality-dependent acquisition and sales prices. The work discussed in the following is in line with Guide et al. (2003) and considers quality-dependent acquisition pricing. Ray et al. (2005) investigate pricing of new products and how to use trade-in rebates for the supply with remanufacturable products. Karakayali et al. (2007) investigate optimal acquisition and selling prices under different reverse channel structures and provide insights regarding under what conditions collection and/or processing operations should be outsourced. Furthermore, they show that two-part tariffs do coordinate the decentralized supply chain. Hahler and Fleischmann (2013) address the question of whether to acquire used products under a centralized system in which only one quality-independent acquisition price is offered or to acquire those under a decentralized system in which upfront testing and quality-dependent prices are offered. Bulmuş et al. (2014) consider joint quality-dependent acquisition pricing and sales pricing for new and remanufactured products. Cai et al. (2014) explore acquisition pricing and production planning in a hybrid manufacturing/remanufacturing system under two core quality levels. Wei et al. (2015) study quality-dependent refund policies for cores. Mutha et al. (2016) investigate whether third-party remanufacturers should acquire used products with uncertain qualities or in sorted grades with known qualities and whether to acquire and remanufacture cores before or after demand realization.

The presented work on quality-dependent acquisition pricing implicitly assumes perfect initial testing. Besides pricing, we do consider testing effort as a further decision that affects the reliability of testing outcomes and therefore also the optimal acquisition prices. Moreover, we introduce a not-yet-investigated setting to research about used-product acquisition management in which the buying party is not the one setting the prices but rather only decides which of the predetermined prices to offer. Furthermore, besides Karakayali et al. (2007), none of the mentioned papers also investigate the aspect of having a decentralized supply chain.

There is another literature stream with a focus on testing/grading information. On the one hand, it is concerned with the value of information about a product's condition for its processing (e.g., Souza et al. 2002, Aras et al. 2004, Zikopoulos and Tagaras 2008). On the other hand, different grading systems are compared (e.g., Tagaras and Zikopoulos 2008, Ferguson et al. 2009, Denizel et al. 2010). This stream of research, however, is not concerned with the value of testing products before acquisition and how it affects acquisition pricing, which is, among other aspects, what we investigate.

Moreover, we contribute to literature regarding coordination/contracting in reverse supply chains. We refer to Guo et al. (2017) for a recent overview of research in that area. First, as already indicated, except for Karakayali et al. (2007), we do not find any literature simultaneously concerned with coordination and quality-dependent acquisition pricing.

Some studies in that field are more loosely related to our work, as they consider contracting under asymmetric information (e.g., Zheng et al. 2017, Zhao et al. 2017, Wang et al. 2017), or they investigate similar reverse supply chain settings consisting of remanufacturers and collectors/retailers (e.g. Hong et al. 2008, Zeng 2013, Gu and Tagaras 2014, Jena and Sarmah 2016). Nevertheless, the considered asymmetries and decisions/responsibilities of involved parties are very different from those in our investigated setting.

Summarizing, to the best of our knowledge, there is no paper that examines used-product acquisition through an intermediary and considers the questions of whether to have a quality-differentiated process and, if so, how to manage and exploit upfront testing. Especially, to the best of our knowledge, the setting that we investigate and the trade-off between testing cost and testing quality in used-product acquisition have not been recognized so far.

4.3. Model Setup

Consider a firm that is faced with the decision of acquiring one type of used product with initially unknown qualities through a retailer. Depending on the quality, each product represents a certain value to the firm entailing the cost of performing rework and the selling price. The firm is a price taker, which means that the values are exogenously given. The firm (perfectly) determines products' actual qualities to decide about the subsequent treatment only after the retailer has acquired and transferred them.

The retailer acquires products through offer and payment of financial incentives, which we refer to as acquisition prices. Acquiring a product does not imply that the retailer loses the whole amount given out to the product holder. For example, giving out vouchers or discounts comprising a certain value makes the retailer lose less due to not every consumer actually using vouchers and due to the mark up between the procurement/production cost and the selling prices of products in the retailer's product range, from which product holders then have to buy.

Used products do represent residual values for their holders, which are characterized by, e.g., emotional values or the prices that they would achieve through selling via eBay or other platforms. A product holder accepts an acquisition price and sells his product to the retailer only if the offered price exceeds the residual value. Naturally, neither the firm nor retailer know those residual values exactly but do have some prior beliefs about them.

There are different means for a firm to address such a market situation in terms of acquisition policy, about which the firm has to decide before collaboration with the retailer. The simplest method for the firm is to make the retailer offer a fixed, quality-independent acquisition price for a specific product. Then, upfront testing is unnecessary. The retailer just collects product holders' products from those who accept the offered price and transfers them to the firm, which compensates the retailer for the acquisition according to a predetermined contract.

Another method for the firm to acquire used products through the retailer is as presented in the business case: when products are offered for sale, the retailer can gain some (imperfect) information about products' qualities through more or less thorough upfront testing. This information and invested testing effort are the retailer's private information. Based on this information, the retailer then decides which acquisition price to offer. The quality-dependent prices have been predetermined by the firm. Hence, there are two available levers for the retailer in used-product acquisition with upfront testing to steer profit: how much effort (if any) to put into upfront testing and the acquisition price offered (if any), depending on the testing outcome.

The more effort is put into testing, the more reliable is the testing outcome, and therefore, the risk of offering the "wrong" acquisition price is reduced. Thorough testing is costly, however, due to not only time consumption but also training of employees and procurement and use of testing equipment. Therefore, there is a trade-off between information accuracy and testing cost.

As in the simpler case without testing, acquired products are transferred to the firm, which compensates the retailer according to a predetermined contract.

In the rest of this section, we provide more detail about our model setup and assumptions.

4.3.1. Setting Features

We consider one type of used product that can be of good or bad quality ($\Theta \in \{G, B\}$). We normalize the value for the firm of a good quality product to 1 and denote the value of a bad product $m \in [0, 1]$. The ex ante probability that an offered product is of good quality is $\mathbb{P}(\Theta = G) = \beta = 1 - \mathbb{P}(\Theta = B)$. We denote the retailer's acquisition price offer p. The acquisition of a product for p implies cost of δp for the retailer, where $\delta \in [0, 1]$. We denote a product's residual value for its holder, which is quality-dependent, r_Θ. It is the minimal offer value for which the holder is willing to sell. We assume r_Θ to have twice-continuously differentiable distribution functions $\mathfrak{F}_\Theta$ that are strictly increasing on support $[0, \tau]$ where $1/\delta < \tau$ and $\mathfrak{F}_\Theta(0) = 0$. The density functions are denoted $\mathfrak{f}_\Theta$. We make two additional assumptions:

Assumption 4.1. *$\mathfrak{F}_G$ and $\mathfrak{F}_B$ satisfy the monotone probability ratio order*[2]*, i.e., for all $x < y$ on $[0, \tau]$, it holds that $\mathfrak{F}_G(x)/\mathfrak{F}_B(x) \leq \mathfrak{F}_G(y)/\mathfrak{F}_B(y)$.*

This assumption is satisfied by many common families of distributions such as uniform, exponential, binomial, Poisson, and normal distributions. Therefore, Assumption 4.1 is not very restrictive. Still, it is very helpful in our analysis.

Assumption 4.2. *$\mathfrak{F}_G(p)(1 - \delta p)$ and $\mathfrak{F}_B(p)(m - \delta p)$ are concave on $[0, \tau]$.*

This assumption implies concavity of profit functions and guarantees uniqueness of the optimal prices and corresponding orderings. Nevertheless, the key results and qualitative insights remain valid without Assumption 4.2. In summary, $\mathfrak{F}_\Theta(p)$ is the probability that a product holder with quality Θ product accepts selling it for offered price p.

4.3.2. Undifferentiated Acquisition

When the firm chooses acquisition without upfront testing, the retailer makes an offer $p \in \{0, p_0\}$ when a product is put up for sale. p_0 is the acquisition price that the firm fixed beforehand, and offering 0 indicates that the retailer refrains from acquiring the product. If the offer is accepted by the holder, the retailer pays p (thereby looses δp), transfers the product to the firm, where the product's actual quality is determined, and is compensated according to a predetermined contract (to be discussed below). The firm gains 1 (m) if $\Theta = G$ ($\Theta = B$).

4.3.3. Differentiated Acquisition

When the firm decided for acquisition with upfront testing, the acquisition at the retailer proceeds as follows. When a product is offered, it is first tested by the retailer. Testing (partially) resolves uncertainty about products' qualities and depends on the retailer's testing effort e, which can be either high ($e = h$) or low ($e = l$). The corresponding testing costs are $c_h \geq c_l \geq 0$. We model the testing outcome as an imperfect signal $s \in \{g, b\}$ that indicates whether the product is of good ($s = g$) or bad ($s = b$) quality. To capture the fact that effort level has an impact on quality of testing outcomes, we set the precision of outcome $s = b$ to $\mathbb{P}(s = b \mid \Theta = B \;\&\; e) = q_e$, where $q_l = 1/2$, and $q_h = q \in (1/2, 1]$. This result implies that high-effort testing leads to more reliable outcomes than low-effort testing. For

[2]For more detailed discussion of this order see, e.g., Maskin and Riley (2000) or Hopkins et al. (2003)

outcome $s = g$, we set the precision to $\mathbb{P}(s = g \mid \Theta = G) = 1$ because testing basically means searching for errors, which is why a good product forces a good outcome, since no errors can be detected.[3] Testing effort and testing outcome are retailer's private information and not observable by the firm.

After the reception of testing outcome s, the retailer updates his belief about the product's quality in accordance with Bayesian rationality. Based on this refined information, the retailer offers $p \in \{p_g, p_b, 0\}$, where p_s for $s \in \{g, b\}$ are the quality-dependent acquisition prices that have been predetermined by the firm. The indices indicate to which testing outcomes the prices correspond. In principle, the firm wishes to have the prices for the correct, actual qualities, which is not possible if testing is imperfect. Therefore, prices have to be chosen as reactions to testing outcomes. Note that the retailer does not have to stick to outcome s with his offer ($p \neq p_s$ given outcome s is possible). Offering 0 again indicates that the retailer refrains from acquiring the product.

Through acquisition, the retailer loses δp and transfers the product. The firm determines the actual quality Θ, compensates the retailer, and gains the product's value.

4.3.4. Contracts for Collaboration

When the firm decided on an acquisition policy, a contract is offered in parallel with the announcement of acquisition prices in order to constitute the collaboration. Because policies differ structurally, the contracts do as well.

For undifferentiated acquisition, we consider the following generic contract consisting of two payments: $k = (k_\Theta)$, where $\Theta \in \{G, B\}$. k_Θ is paid to the retailer for a product transfer depending on the actual quality Θ.

For differentiated acquisition, we consider a generic contract consisting of four payments: $k = (k_{\Theta,s})$, where $\Theta \in \{G, B\}$ and $s \in \{g, b\}$. $k_{\Theta,s}$ is paid for an acquired and transferred product and depends on the product's actual quality Θ and paid acquisition price p_s.

Note that we assume that the paid acquisition price p is verifiable for the firm and that firm's testing outcome (actual quality Θ) is verifiable for the retailer. This makes the considered contracts action- and evidence-based and hence enforceable by courts: *action-based* because payments depend on retailer's offers, and *evidence-based* because payments depend on products' actual qualities.[4]

[3] All qualitative insights continue to hold for arbitrary signal precisions.

[4] Payments cannot depend on the retailer's testing effort and outcome because those are not verifiable.

To capture that the retailer only participates if it is profitable for him, we introduce value $\pi_0 \geq 0$, which denotes the minimum expected profit that the retailer requires for collaboration. The height of π_0 may also reflect the retailer's negotiating power.

Given these compensation schemes, the retailer's expected profit π consists of expected payments by the firm net testing costs, if any, and the expected cost for paying acquisition prices. We assume the retailer and the firm to be risk-neutral expected profit maximizers. The firm's expected profit Π consists of the expected product value through acquisition net expected payments to the retailer.

4.4. Acquisition without Quality Differentiation

In order to be able to decide about how to optimally acquire used products through a retailer, we first separately investigate different policies in Sections 4.4 and 4.5, i.e., acquisition without and with quality differentiation. This provides us with knowledge about how the firm can optimally steer the retailer to execute those policies. Afterward, based on that, we are able to compare profitability of those policies in Section 4.6. Following up on that, some more general discussion about supply-chain efficiency and the simplification of collaboration is provided in subsequent sections.

Now, we investigate used-product acquisition through a retailer without upfront testing and price differentiation. We refer to undifferentiated acquisition as policy N.

The firm's levers to steer policy N (which similarly applies to acquisition with quality differentiation) are the choice of acquisition price p_0 and the choice of contract payments k. Structurally, the firm's optimization problem for finding the optimal undifferentiated acquisition policy is as follows (the concrete optimization models with all formal proofs and mathematical derivations have been relegated to the Appendix.)

$$N: \quad \max_{t=(k,p_0)} \; \Pi(t|p = p_0) \tag{4.1}$$

$$\text{s.t.} \;\; \pi(p = p_0|t) \geq \pi(p = 0|t) \tag{4.2}$$

$$\pi(p = p_0|t) \geq \pi_0 \tag{4.3}$$

Constraint (4.2) ensures that it is more profitable for the retailer to offer p_0 instead of 0 if a product is put up for sale. Constraint (4.3) ensures the retailer's participation.

Determining the solution to the problem, which is also mirrored in the upcoming proposition, can be split up into two steps: first, the optimal contract k^N for a fixed, but an arbitrary price p_0 is determined. Second, based on the optimal contract (insertion into the firm's profit), the optimal acquisition price p_0^N is determined. Note that we use superscripts of the corresponding policy to indicate optimality.

The following proposition characterizes the optimal undifferentiated acquisition policy, consisting of an optimal contract and an optimal acquisition price.

Proposition 4.1 (Optimal Undifferentiated Acquisition). (i) *A contract k^N solves problem N given fixed but arbitrary p_0 if and only if*

$$k_B^N \in \mathbb{R}, \qquad k_G^N = \delta p_0 + (\pi_0 - (1-\beta)\mathfrak{F}_B(p_0)(k_B^N - \delta p_0))/(\beta \mathfrak{F}_G(p_0)).$$

(ii) *The firm's profit given price p_0 and optimal contract k^N is*

$$\Pi(k^N, p_0) = \beta \mathfrak{F}_G(p_0)(1 - \delta p_0) + (1-\beta)\mathfrak{F}_B(p_0)(m - \delta p_0) - \pi_0. \tag{4.4}$$

(iii) *Acquisition price p_0^N maximizes* (4.4) *if and only if*

$$p_0^N = \frac{\beta \mathfrak{f}_G(p_0^N)(1 - \delta \mathfrak{F}_G(p_0^N)/\mathfrak{f}_G(p_0^N)) + (1-\beta)\mathfrak{f}_B(p_0^N)(m - \delta \mathfrak{F}_B(p_0^N)/\mathfrak{f}_B(p_0^N))}{\delta(\beta \mathfrak{f}_G(p_0^N) + (1-\beta)\mathfrak{f}_B(p_0^N))}. \tag{4.5}$$

First, note that the contract characterized by $k_G = k_B = \delta p_0 + \pi_0/(\beta \mathfrak{F}_G(p_0) + (1 - \beta)\mathfrak{F}_B(p_0))$ satisfies the conditions in (i), which proves that even a fixed fee contract is optimal. As a consequence, under policy N, the sharing of information regarding a product's actual quality is not necessary, which simplifies collaboration between the firm and retailer.

Interestingly, when considering the structure of the firm's profit (4.4) under optimal contract k^N, the pricing decision is as if the firm acquires the products directly from product holders without an intermediary. This observation is discussed in more depth in Section 4.7.

With regard to the optimal acquisition price p_0^N, we make the following structural observations: the denominator of the characterizing ratio consists of the acquisition cost fraction, δ, multiplied by the change in product holder's offer acceptance probability. It is intuitive that the size of δ has a negative effect on the magnitude of the optimal acquisition price because the more expensive for the retailer (and hence, for the firm) it is to acquire products, the smaller the optimal acquisition price should be. The numerator can be interpreted as the weighted sum of revenues achieved through acquisition of a good product and revenues through acquisition of a bad product. The corresponding weights are the likelihood of the corresponding product quality times the probability of acceptance of the offered price.

Based on the optimal price's structure, several sensitivities can be observed, which are summarized in the following corollary.

Corollary 4.1 (Price Sensitivities - Undifferentiated Acquisition). *The optimal acquisition price p_0^N is unique, increases in m and β, decreases in δ, is constant in q, and satisfies $1/\delta - \mathfrak{F}_G(p_0^N)/\mathfrak{f}_G(p_0^N) \geq p_0^N \geq m/\delta - \mathfrak{F}_B(p_0^N)/\mathfrak{f}_B(p_0^N)$.*

First, uniqueness is a simple implication of the concavity of firm's profit by Assumption 4.2. The stated sensitivities are intuitive. Due to p_0^N balancing revenues through acquisition of good and bad products, depending on how numerous they are relative to one another, it is obvious that the more valuable a bad product is (m), or the larger the likelihood of a good product being offered (β), the larger is p_0^N because acquisition is basically more profitable. As already discussed, the reverse holds for δ: the more costly the acquisition is, the lower is the point (p_0^N) that optimally balances the acceptance probability and profit through acquisition trade-off. Obviously, when there is no upfront testing, the informational quality of a testing outcome, q, cannot play a role.

Finally, the bounds for the optimal price imply that it is less than $1/\delta$, i.e., it cannot be optimal for the firm to make the retailer lose more through acquisition (δp_0^N), as the firm can gain at most (value of a good product 1).

4.5. Acquisition with Quality Differentiation

After having investigated the simple case of undifferentiated acquisition, we now turn our attention to acquisition policies with upfront testing and differentiated prices.

We briefly recap the most important differences from the undifferentiated acquisition: the retailer's decisions comprise (i) the choice of testing effort, $e \in \{h, l\}$, which affects quality of testing outcome and testing cost, and (ii) based on testing outcome $s \in \{g, b\}$, the choice of acquisition price to offer, $p \in \{p_g, p_b, 0\}$. The testing effort and outcome are the retailer's private information. Again, the decisions of the firm comprise the choices of acquisition prices and contracts.

We have a typical principal-agent setting here[5]. By the revelation principle, we focus on contracts that induce truth-telling, i.e., that make the retailer offer acquisition prices corresponding to testing outcomes ($p = p_s$ for each outcome $s \in \{g, b\}$). Technically speaking,

[5] see, e.g., Fudenberg and Tirole (1991)

the contract payments have to satisfy certain constraints to cause the desired behavior from the retailer.

Besides the retailer's offer behavior, there is the effort decision, which has an impact on both the retailer's and the firm's excepted profits. Note that it is not obvious upfront whether high- or low-effort testing by the retailer is more beneficial from the firm's perspective. The firm's contract payments influence the retailer's effort decision in a manner that one choice is more profitable than the other one given that the retailer adheres with offers to testing outcomes.

As a result with regard to the retailer's effort decision, this leaves us with two different acquisition policies with quality differentiation for the firm: policy H, in which the offered contract is chosen such that it prompts the retailer to test with high effort and to make truthful offers, and policy L, in which the contract is chosen such that it prompts the retailer to engage in low-effort testing and truthful offering.

We can summarize both policies in the following optimization problem T. Offering the retailer a contract that induces $e = h$ and $e' = l$ refers to application of policy H (exertion of high testing effort is more profitable for the retailer due to constraint (4.8)). Similarly, a contract that induces $e = l$ and $e' = h$ refers to policy L.

$$T: \quad \max_{t=(k,p_g,p_b)} \Pi(t|e, p = p_s) \tag{4.6}$$

$$\text{s.t.} \quad \pi(p = p_s|s, e, t) \geq \pi(p = p'|s, e, t) \qquad \forall s \in \{g, b\}, \forall p' \neq p_s \tag{4.7}$$

$$\pi(e, p = p_s|t) \geq \pi(e', p = p_s|t) \qquad \forall e' \neq e \tag{4.8}$$

$$\pi(e, p = p_s|t) \geq \pi_0 \tag{4.9}$$

As already touched upon, constraint (4.8) is the incentive-compatibility constraint regarding effort exertion. Depending on contract k, the retailer exerts testing effort e instead of e' due to it being more profitable, given that the retailer will adhere to the testing outcomes with offers afterward. Constraint (4.7) ensures that the retailer actually follows this offer behavior. This constraint represents four different constraints: one ensures that when there is a good testing outcome, the retailer is better off offering p_g instead of p_b. The second one ensures the same with 0 instead of p_b. Then, it ensures that the retailer is better off offering p_b instead of p_g given a bad testing outcome and, finally, that offering p_b is more profitable than offering 0 given a bad testing outcome. Finally, constraint (4.9) ensures the retailer's participation if he acts according to the other constraints, i.e., if he is truthful and exerts effort according to (4.8).

To repeat and punctuate, we separately investigate policy H, which is equivalent to solving problem T for $e = h$ and $e' = l$, and policy L, in which the efforts are reversed. Thereby, we can determine the optimal contracts and acquisition prices (summarized as $t^H = (k^H, p_g^H, p_b^H)$ for policy H and the same expression but with superscript L for policy L) that maximize the firm's profits under the respective policies. Only afterward can we then determine which of the policies, H, L, or N, is the most profitable via comparison of the resulting optimal profits.

4.5.1. High-Effort Testing

In this section, we investigate policy H, in which the firm offers the retailer a contract that induces the retailer to engage in high-effort testing and truthful offers. To this end, we consider problem T with $e = h$.

The solution to the resulting problem can again be found by performing the following two steps: first, the determination of the optimal contract k^H given arbitrary but fixed acquisition prices p_g, p_b, and second, based on the resulting profit, through insertion of the optimal contract, the determination of the optimal acquisition prices p_g^H, p_b^H.

The following proposition characterizes the optimal quality-differentiated acquisition policy with high-effort testing.

Proposition 4.2 (Optimal Differentiated Acquisition - High-Effort Testing). *Let* $C \equiv 2(c_h - c_l)/((1-\beta)(2q-1))$, $\rho_{B,g} \equiv (1-\beta)(1-q)$, *and* $\rho_{B,b} \equiv (1-\beta)q$.

(i) *A contract* k^H *solves problem* T *given* $e = h$ *and fixed but arbitrary* p_g *and* p_b *if and only if for* $r_{\Theta,s} = \mathfrak{F}_\Theta(p_s)(k^H_{\Theta,s} - \delta p_s)$ *for all* Θ, s, *it holds that*

$$\begin{aligned}
&r_{B,b} \in [0, (\pi_0 + c_h)/\rho_{B,b}],\\
&r_{B,g} \leq r_{B,b} - C,\\
&r_{G,b} \leq (\pi_0 + c_h - (1-\beta)r_{B,b})/\beta,\\
&r_{G,g} = (\pi_0 + c_h - (\rho_{B,g}r_{B,g} + \rho_{B,b}r_{B,b}))/\beta.
\end{aligned}$$

(ii) *The firm's profit given prices* p_g *and* p_b *and optimal contract* k^H *is*

$$\begin{aligned}\Pi(k^H, p_g, p_b) =& \beta \mathfrak{F}_G(p_g)(1-\delta p_g) + \rho_{B,g}\mathfrak{F}_B(p_g)(m-\delta p_g) \\ &+ \rho_{B,b}\mathfrak{F}_B(p_b)(m-\delta p_b) - c_h - \pi_0.\end{aligned} \tag{4.10}$$

(iii) *Acquisition prices* p_g^H *and* p_b^H *maximize* (4.10) *if and only if*

$$p_g^H = \frac{\beta(\mathfrak{f}_G(p_g^H) - \delta\mathfrak{F}_G(p_g^H)) + \rho_{B,g}(\mathfrak{f}_B(p_g^H)m - \delta\mathfrak{F}_B(p_g^H))}{\delta(\beta\mathfrak{f}_G(p_g^H) + \rho_{B,g}\mathfrak{f}_B(p_g^H))}, \tag{4.11}$$

$$p_b^H = m/\delta - \mathfrak{F}_B(p_b^H)/\mathfrak{f}_B(p_b^H). \tag{4.12}$$

First, note that $r_{\Theta,s}$ is just the expected revenue from offering p_s for a product in quality Θ and is used to more clearly present the optimality conditions for k^H.

Considering the conditions in (i), it can be observed that any optimal contract can be generated by first fixing $k_{B,b}^H$ in the corresponding interval given in $r_{B,b}^H$'s characterization. Then, $k_{B,g}^H$ and $k_{G,b}^H$ can be chosen accordingly, satisfying the bounds depending on $k_{B,b}^H$. Finally, $k_{G,g}^H$ can be determined based on $k_{B,b}^H$ and $k_{B,g}^H$.

In particular, note the second optimality condition corresponding to $k_{B,g}^H$. It is connected to the effort constraint (4.8) and ensures that the retailer exerts high testing effort instead of low effort. This can be explained as follows: a sufficiently large gap between the payment for a bad product that was bought for p_g and the payment for a bad product that was bought for p_b induces a sufficiently large benefit of more-thorough testing to reduce the risk of accidentally paying p_g for a bad product due to a "false" testing outcome. Sufficiently large means large enough to outweigh the relative testing cost difference C.

C as a measure of how the difference between low- and high-effort testing cost, $c_h - c_l$, relates to the actual benefit of high- versus low-effort testing deserves a bit more discussion. Intuitively, testing effort is only relevant because of the existence of bad products since testing, as we defined it, means searching for errors. Errors can only be detected for bad products. Therefore, the more bad products exist (probability $1-\beta$) and the more accurate high-effort testing is compared to low-effort testing (the size of q), the larger the benefit of high-effort testing is relative to low-effort testing. Since the denominator of C decreases in β and q, C becomes larger the less beneficial high-effort testing is relative to low-effort testing given the absolute testing cost difference.

Moreover, the payment for a good product that was bought for p_b ($k_{G,b}$) can be chosen to be arbitrarily small as long as it is below a certain threshold. Then, the retailer adheres to a good testing outcome, and it does not affect the retailer's and firm's profits (adherence leads to $k_{G,b}$ never being paid because a good product can never generate a bad testing outcome, and therefore, the retailer never offers p_b for a good product).

The condition for $r_{G,g}$ is remarkable. This basically stems from making the participation constraint (4.9) binding. Intuitively, this implies that the firm can always press down the retailer's profit to a minimum to just guarantee collaboration. Hence, at optimum, the firm completely extracts the retailer's information rents.

With regard to optimal prices, both p_g^H and p_b^H have a structure similar to p_0^N. The differences in the undifferentiated price rest upon the fact that after testing, the relation between probabilities for the product being of good/bad quality is shifted. For a good outcome, it is more likely that the product is of good quality as before and hence under policy N. Before testing, it is β. Afterward, when the outcome is good, it is $\beta/(\beta+(1-\beta)(1-q))$. Therefore, the optimal price offer under a good outcome is greater than the undifferentiated price offer. For a bad outcome, the probability for the product being of bad quality even becomes one, leading to β becoming zero. Therefore, p_b^H's characterization is identical to that of p_0^N when β is set equal to zero.

The resulting sensitivities in the following corollary are easy implications of Corollary 4.1 together with the similarities between optimal prices.

Corollary 4.2 (Price Sensitivities - Differentiated Acquisition H). *The optimal acquisition prices p_g^H and p_b^H are unique. p_g^H increases in q, m and β, decreases in δ, and satisfies $1/\delta - \mathfrak{F}_G(p_g^H)/\mathfrak{f}_G(p_g^H) \geq p_g^H \geq m/\delta - \mathfrak{F}_B(p_g^H)/\mathfrak{f}_B(p_g^H)$. p_b^H increases in m, decreases in δ, and is constant in β and q. It holds that $p_g^H \geq p_0^N \geq p_b^H$.*

It is notable but not very surprising that p_g^H increases in q. Intuitively, the more accurate high-effort testing is (larger q), the higher the probability is for the product being of good quality after a good testing outcome. Therefore, the acquisition is more valuable in expectation, which makes it optimal for the firm to offer more for the product. That p_b^H is independent of β and q is easily observed by noting that after reception of a bad testing outcome, there is no remaining uncertainty about the product being of bad quality.

Finally, the presented ordering of prices can be intuitively explained by the fact that a good product is more valuable than a bad product and that all three prices balance the profits achievable through acquisition of good and bad products but with different weights/probabilities: p_g^H is offered when the likelihood for the product being of good quality is larger than before testing, whereas p_0^N is offered when the likelihood for the product being good is not updated due to no testing. p_b^H is offered when the likelihood for the product being good is zero.

4.5.2. Low-Effort Testing

Now that we have determined how to optimally manage policy H, we turn to optimally managing the acquisition policy with upfront low-effort testing, L. Therefore, we again consider optimization problem T, but with $e = l$.

The following proposition characterizes the optimal differentiated acquisition policy with low-effort testing.

Proposition 4.3 (Optimal Differentiated Acquisition - Low-Effort Testing). *Let* $C \equiv 2(c_h - c_l)/((1-\beta)(2q-1))$.

(i) *A contract* k^L *solves problem* T *given* $e = l$ *and fixed but arbitrary* p_g *and* p_b *if and only if for* $r_{\Theta,s} = \mathfrak{F}_\Theta(p_s)(k^L_{\Theta,s} - \delta p_s)$ *for all* Θ, s, *it holds that*

$$\begin{aligned} &r_{B,b} \in [0, 2(\pi_0 + c_l)/(1-\beta)], \\ &r_{B,g} \in [r_{B,b} - C, r_{B,b}], \\ &r_{G,b} \le (\pi_0 + c_l - (1-\beta) r_{B,b})/\beta, \\ &r_{G,g} = (\pi_0 + c_l - (1-\beta)(r_{B,g} + r_{B,b})/2)/\beta. \end{aligned}$$

(ii) *The firm's profit given prices* p_g *and* p_b *and optimal contract* k^L *is*

$$\begin{aligned} \Pi(k^L, p_g, p_b) =& \beta \mathfrak{F}_G(p_g)(1 - \delta p_g) + (1-\beta)\mathfrak{F}_B(p_g)(m - \delta p_g)/2 \\ &+ (1-\beta)\mathfrak{F}_B(p_b)(m - \delta p_b)/2 - c_l - \pi_0. \end{aligned} \tag{4.13}$$

(iii) *Acquisition prices* p_g^L *and* p_b^L *maximize* (4.13) *if and only if*

$$p_g^L = \frac{\beta(\mathfrak{f}_G(p_g^L) - \delta \mathfrak{F}_G(p_g^L)) + (1-\beta)(\mathfrak{f}_B(p_g^L) m - \delta \mathfrak{F}_B(p_g^L))/2}{\delta(\beta \mathfrak{f}_G(p_g^L) + (1-\beta)\mathfrak{f}_B(p_g^L)/2)}, \tag{4.14}$$

$$p_b^L = m/\delta - \mathfrak{F}_B(p_b^L)/\mathfrak{f}_B(p_b^L). \tag{4.15}$$

Again, as for policy H, we can successively determine each optimal contract by first fixing $k^L_{B,b}$, then $k^L_{B,g}$, $k^L_{G,b}$ and, finally, $k^L_{G,g}$.

Compared to the optimal contracts for policy H, the most notable difference concerns the relation between $k^L_{B,g}$ and $k^L_{B,b}$. Under policy L, it is important to have those payments close to one another such that the benefit of more precise information through high-effort testing is not worth the higher relative testing cost difference C. This incentivizes the retailer to only test with low effort and is captured by the second condition characterizing $r_{B,g}$ depending on $r_{B,b}$.

Moreover, note that also under policy L, it is optimal for the firm to press down the retailer's profit to the minimum participation profit π_0, thereby completely extracting the retailer's information rents. This is captured by $r_{G,g}$'s characterization, which is identical to constraint (4.9) of problem T given $e = l$ being binding.

Regarding the firm's profit under optimal contract k^L and the optimal prices, there is nothing new compared to policy H because pricing for policy L basically boils down to pricing for H in the special case where $q = 1/2$. Hence, it also holds that $p^L_b = p^H_b$. For completeness, the following corollary presents the remaining results with regard to optimal prices.

Corollary 4.3 (Price Sensitivities - Differentiated Acquisition L)**.** *The optimal acquisition prices p^L_g and p^L_b are unique. p^L_g increases in m and β, decreases in δ, is constant in q, and satisfies $1/\delta - \mathfrak{F}_G(p^L_g)/\mathfrak{f}_G(p^L_g) \geq p^L_g \geq m/\delta - \mathfrak{F}_B(p^L_g)/\mathfrak{f}_B(p^L_g)$. It holds that $p^H_g \geq p^L_g \geq p^N_0 \geq p^H_b = p^L_b$.*

As already mentioned, we basically observe the same price sensitivities as under policy H. The stated results are all immediate consequences of Corollary 4.2.

4.6. Optimal Acquisition Policy

Now that we investigated the firm's optimal policies separately, we want to characterize the exogeneous conditions that favor one acquisition policy over the others. For expositional clarity, we denote the maximum profit under policy $* \in \{N, H, L\}$ by Π^*. These profits result from combination of optimal contracts and optimal acquisition prices determined in the previous sections. Moreover, we use simplifying notation to shorten the terms corresponding to firm's expected revenues if a certain price is offered for a product of a certain quality. We let $\kappa^*_{G,x} \equiv \mathfrak{F}_G(p^*_x)(1 - \delta p^*_x)$ and $\kappa^*_{B,x} \equiv \mathfrak{F}_B(p^*_x)(m - \delta p^*_x)$ for $* \in \{N, H, L\}$ and $x \in \{g, b, 0\}$.

The following proposition characterizes when one acquisition policy is superior to the other ones.

Proposition 4.4 (Optimal Acquisition Policy). *Let*

$$\gamma_1 \equiv \beta[\kappa^H_{G,g} - \kappa^L_{G,g}] + (1-\beta)((1-q)[\kappa^H_{B,g} - \kappa^L_{B,g}] + (q-1/2)[\kappa^H_{B,b} - \kappa^L_{B,g}]), \quad (4.16)$$

$$\gamma_2 \equiv \beta[\kappa^H_{G,g} - \kappa^N_{G,0}] + (1-\beta)(1-q)[\kappa^H_{B,g} - \kappa^N_{B,0}] + (1-\beta)q[\kappa^H_{B,b} - \kappa^N_{B,0}], \quad (4.17)$$

$$\gamma_3 \equiv \beta[\kappa^L_{G,g} - \kappa^N_{G,0}] + (1-\beta)[\kappa^L_{B,g} - \kappa^N_{B,0}]/2 + (1-\beta)[\kappa^L_{B,b} - \kappa^N_{B,0}]/2. \quad (4.18)$$

(i) $\gamma_1, \gamma_2, \gamma_3$ *are independent of* c_h, c_l, *and* π_0. *It holds that* $\gamma_1, \gamma_2, \gamma_3 \geq 0$.

(ii) γ_1 *and* γ_2 *are increasing in* q, *whereas* γ_3 *is constant in* q.

(iii) H *is the firm's optimal acquisition policy if and only if* $\gamma_1 \geq c_h - c_l$ *and* $\gamma_2 \geq c_h$. L *is optimal if and only if* $\gamma_1 \leq c_h - c_l$ *and* $\gamma_3 \geq c_l$. N *is optimal if and only if* $\gamma_2 \leq c_h$ *and* $\gamma_3 \leq c_l$.

(iv) *If* $c_h = c_l = 0$, *then* $\Pi^H \geq \Pi^L \geq \Pi^N$.

(v) *If* $m = 1$ *and* $\mathfrak{F}_B = \mathfrak{F}_G$, *if* $\beta = 0$, *if* $\beta = 1$, *or if* $\delta = 0$, *then* $\gamma_1, \gamma_2, \gamma_3 = 0$, *i.e., the firm's optimal acquisition policy is* N.

First, note that $\gamma_1, \gamma_2, \gamma_3$ are the gains that the firm incurs through (more thorough) testing based on the three policies' optimal profits adjusted for corresponding testing cost c_h and c_l. Results (i) and (iii), which together imply (iv), support the strong intuition that if testing is not costly, it is best to test as thoroughly as possible before acquisition to be able to fully exploit quality differences based on reliable information.

Result (ii) shows that the more accurate the testing outcomes yielded by high-effort testing are, the larger the gains are compared to low-effort and no testing. This also implies that the firm is willing to invest more in making the retailer test with high effort the better his testing capabilities are.

Result (iii) gives a full characterization of when to apply which policy. It shows that if the additional gains through testing compared to undifferentiated acquisition outweigh the additional cost, then one should choose differentiated acquisition. Moreover, if the additional information accuracy through more precise testing pays off compared to the difference between high- and low-effort testing cost, then policy H should be applied.

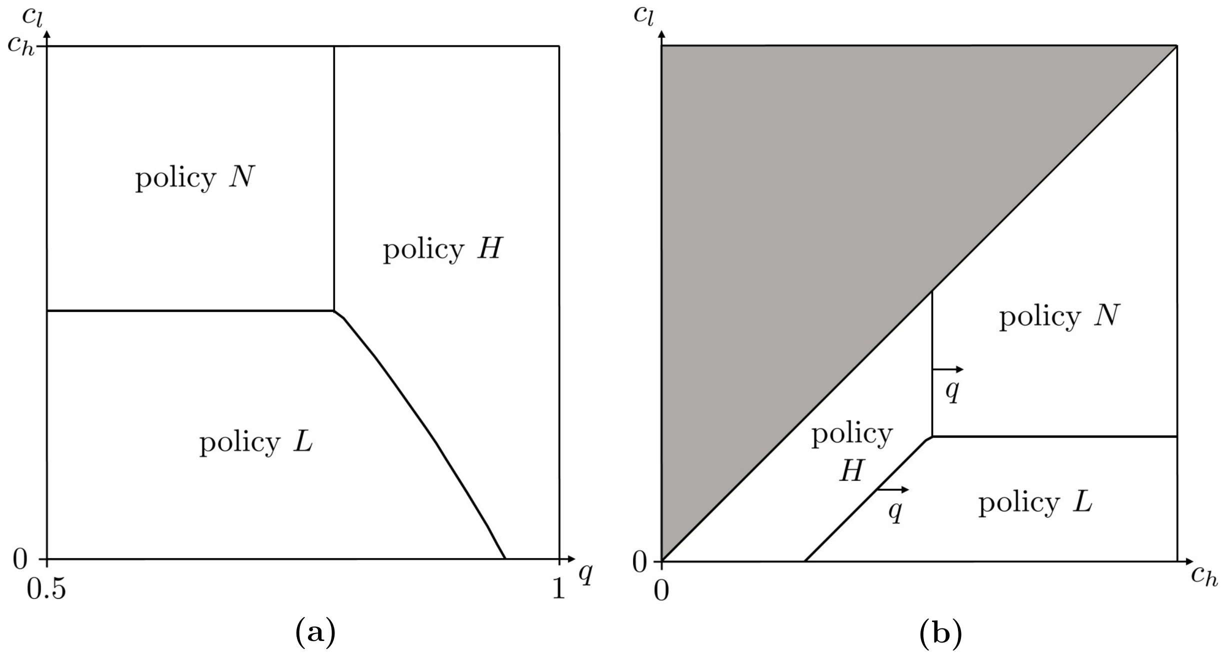
Result (v) presents conditions under which testing becomes irrelevant, i.e., the additional gains that come with (thorough) testing converge to zero. Hence, if there are no additional gains, then it cannot be profitable to invest in costly testing, and policy N is the policy of choice. Policy N is preferred if bad products exhibit the same values as good products for the firm and product holders because then, there is no benefit in testing and offering differentiated prices. In contrast, note that it suffices to have only firm or only product holders value products differently to make testing and differentiated pricing beneficial. No value differentiation may be the case for, e.g., very old mobile phones. Even if there exist phones that are well functioning and have excellent optical conditions, they may be as valuable as if they were damaged. The reason could be that the firm is only interested in the materials inside. Furthermore, if almost all products are exclusively bad (good), then testing is unnecessary because the retailer/firm does not face quality uncertainty, which has to be resolved. This might apply to very old (new) product generations. Finally, δ being close to zero implies that the fraction of acquisition prices the retailer loses through acquisition is almost negligible. Then, all acquisition prices p_g, p_b, and p_0 can be chosen to be extremely large. Thereby, all products are collected without any loss for the retailer or the firm. However, then, investment in testing is not necessary. Therefore, giving out vouchers instead of paying money for products is in favor of a no-testing policy because it is less expensive.

The reverse of (v) leads to conjectures about when differentiated acquisition is profitable or relevant: if good and bad products significantly differ in value for firm or product holders, if the likelihood for a good product submission does not deviate too much from that for a bad product submission, and if paying acquisition prices is costly for the retailer, then differentiated acquisition with upfront testing can be beneficial (depending on how expensive it is). Whether high-effort or low-effort testing should be applied depends on the information accuracy and testing cost trade-off.

Figure 4.1 illustrates how the choice of acquisition policy is driven by different parameters. For each area, the indicated policy is the most profitable one for the corresponding parameter combinations. On the left-hand side, we see how policy choice depends on low-effort testing cost c_l and quality of testing outcomes under high-effort testing q. Since both policies, N and L, are not affected by q, there is a c_l-threshold such that policy L is preferred when testing costs are smaller and that N is preferred otherwise. Since N and H are not dependent on c_l, there is a q-threshold such that H is preferred only if quality q is large enough and thereby outweighs corresponding testing cost c_h. For H and L, we have the following relation: high-quality q and high low-effort testing cost c_l are in favor of H, whereas low-quality and low low-effort testing cost are in favor of L. When one of both parameters is low and the other one is high, then both policies are likely to be comparably profitable.

On the right-hand side, we see how policy choice is driven by testing cost c_l and c_h. Due to $c_h \geq c_l$, the upper gray area is irrelevant. In the upper-right part of the relevant area, policy N is optimal. This is intuitive since here, high- and low-effort testing costs are sufficiently large that they outweigh the additional gains from differentiated acquisition. In the areas in

Figure 4.1.: The Firm's Optimal Acquisition Policy



The graphs plot an example of the firm's optimal acquisition policy depending on (a) the relation between q and c_l and (b) the relation between c_h and c_l. The remaining parameter values are the following: $\beta = 0.5$, $m = 0.4$, $\delta = 0.8$, $\pi_0 = 0$, $q = 0.8$, $c_h = 0.01$, $\mathfrak{F}_G(p) = p/2$, and $\mathfrak{F}_B(p) = 3p/4$ on $[0, 4/3]$.

which N is not optimal, we have a line characterized by the difference between c_h and c_l, which represents $\Pi^H = \Pi^L$. If c_l is below that line, the difference is larger, and hence, policy L is more profitable. H is more profitable if the difference between c_h and c_l is less than this threshold-difference because then, the additional gains from high-effort testing outweigh the additional cost. Finally, note that the impact of high-effort testing accuracy q on graph (b) is to shift the line separating H from N and the line separating H from L to the right. This is due to Propositions 4.4 (ii) and (iii), i.e., the optimal profit under H increases in q, whereas Π^N and Π^L are not affected.

4.7. Supply-Chain Efficiency

Thus far, we have determined how to optimally apply the considered acquisition policies in the presented decentralized setting and when to apply which policy. Still, another open question we want to answer concerns the efficiency of the considered acquisition channel. To this end, we clarify how the decentralized profits relate to the centralized profits, which are the achievable profits in a situation in which the firm and retailer act as one entity.

For the sake of expositional clarity, we denote the optimal centralized supply chain profit under policy $* \in \{H, L, N\}$ Π_C^* and the optimal decentralized supply chain profit Π_D^*, which is the sum of the firm's and retailer's optimal profits under policy $*$.

The following proposition presents how centralized and decentralized profits relate to one another and how the decentralized profit is split up between firm and retailer.

Proposition 4.5 (Efficiency of Acquisition Channel)**.** *For any acquisition policy $* \in \{H, L, N\}$, the firm is able to coordinate the supply chain, i.e., the total decentralized supply chain profits are identical to the profits of a centralized supply chain ($\Pi_C^* = \Pi_D^*$). The retailer's and firm's profit shares are π_0/Π_D^* and $1 - \pi_0/\Pi_D^*$, respectively.*

Moreover, independent of policies, the retailer's share decreases in m and increases in δ. It decreases in q under policy H only.

Proposition 4.5 is in line with the profits (4.4), (4.10), and (4.13) together with the fact that in optimum the firm fully extracts the retailer's information rent and never pays the retailer more than necessary to make him collaborate, which results in $\pi = \pi_0$. Due to the magnitude of retailer's profit being independent of the firm's chosen policy, the optimal policy is the same for the centralized and decentralized systems.

To summarize, the investigated decentralized acquisition channel is efficient from the supply-chain perspective since the centralized optimal supply chain profit is obtained irrespective of chosen policy. Unfortunately, there is one drawback for firms that use this acquisition channel. Depending on how much negotiating power the retailer has, the corresponding minimal participation profit π_0 might be quite high, and therefore, even though both parties could benefit from it ($\Pi_D^* > 0$ when $*$ is optimal), it might not be profitable for the firm to collaborate ($\Pi_D^* - \pi_0 < 0$). Moreover, the retailer never receives more than the minimal participation profit π_0, of which the retailer should be aware when negotiating.

The stated sensitivities are intuitive. The retailer's share exhibits the reversed behavior as the firm's profit. Comprehensibly, the more value bad products imply for the firm or the less costly acquisition is for the retailer, the larger the firm's optimal profit and hence the smaller the retailer's share. Additionally, the more accurate high-effort testing is, the larger the firm's optimal profit is under policy H.

4.8. Contract Simplification

Thus far, we assumed that on the one hand, the actual quality, which is determined by the firm, is verifiable for the retailer. On the other hand, we assumed that the prices that the retailer paid for acquired products are verifiable for the firm. Both aspects of verifiability request close collaboration and disclosure of information on both sides, which one or the other party may not be willing to provide even though it is profitable.

Therefore, in this section, we address the question under what conditions simplification of the presented contracts does not reduce the acquisition channel's performance. First, note that this is only interesting for acquisition with quality differentiation because undifferentiated acquisition can always be set up with a fixed-fee contract (see also discussion of Proposition 4.1). Here, information transfer can be reduced to a minimum by paying for acquired products irrespective of actual quality or price paid.

Starting from our four-payment contract for differentiated acquisition, we consider (i) price-only contracts, in which payments only depend on paid acquisition prices, not on actual quality ($k_{G,s} = k_{B,s}$), (ii) quality-only contracts, in which payments are quality-dependent but not price-dependent ($k_{\Theta,g} = k_{\Theta,b}$), and (iii) fixed-fee contracts, in which there is only a single transfer payment independent of the actual qualities and prices paid. Those simplifications imply that (i) the firm does not have to report testing outcomes, and the retailer does not have to verify them, (ii) the retailer does not have to report paid acquisition prices, and the firm does not have to verify them, and (iii) both information aspects do not have to be reported or verified.

For both differentiated acquisition policies H and L, the following proposition characterizes the necessary and sufficient conditions for the presented simplified contracts being optimal.

Proposition 4.6 (Optimality of Simplified Contracts). *All ξ are positive and only depend on β, q, $\mathfrak{F}_\Theta(p_s^*)$. See the proof for the concrete ξ. Let $C \equiv 2(c_h - c_l)/((1-\beta)(2q-1))$.*

(i) *An optimal price-only contract exists under policy H if and only if $C \leq (\pi_0 + c_h)\xi_1$. It always exists under policy L.*

(ii) *An optimal quality-only contract exists under policy H if and only if $C/\mathfrak{F}_B(p_g^H) \leq \delta(p_g^H - p_b^H) \leq (\pi_0 + c_h)\xi_2$. It exists under policy L if and only if $\delta(p_g^L - p_b^L) \leq \min\{(\pi_0 + c_l)\xi_3, (\pi_0 + c_l)\xi_4 + C\xi_5\}$.*

(iii) *An optimal fixed-fee contract exists under policy H if and only if $(\pi_0 + c_h)\xi_6 + C\xi_7 \leq \delta(p_g^H - p_b^H) \leq (\pi_0 + c_h)\xi_8$. It exists under policy L if and only if $(\pi_0 + c_l)\xi_9 \leq \delta(p_g^L - p_b^L) \leq \min\{(\pi_0 + c_l)\xi_{10}, (\pi_0 + c_l)\xi_9 + C\xi_{11}\}$.*

First, note that the conditions in Proposition 4.6(iii) can define non-empty sets, i.e., there actually exist situations in which optimal fixed-fee contracts exist for policies H and L. This makes the discussion relevant. For an example of fixed-fee contract optimality under policies H and L, see the proof of Proposition 4.6.

Interestingly, for policy L, it is never necessary for the firm to verifiably share information regarding products' actual qualities. Intuitively, the firm only has to compensate the retailer for the prices paid (irrespective of actual qualities) and can thereby press down the retailer's gains to the minimum participation profit. This suffices to incentivize the retailer to make truthful offerings and test with low effort. For policy H, the relative testing cost difference C (which was discussed in Section 4.5.1) has to be sufficiently less than π_0 to make a price-only contract's optimality possible. Another supportive condition is that testing cost are both sufficiently high and sufficiently similar to result in a small relative testing cost difference. Intuitively speaking, if it is expensive to make the retailer participate or if testing costs do not differ much, then a price-only contract is likely to be sufficient to achieve optimality.

With regard to quality-only contracts, under H, structurally similar conditions as for price-only contracts have to hold. The relative testing cost difference has to be small, or making the retailer participate has to be expensive. However, additionally, the acquisition cost difference between paying the price for a bad and a good product, $\delta(p_g^H - p_b^H)$, has to be between relative testing cost difference and the term including π_0. Intuitively, this transforms the "or" condition for price-only contracts to an "and" condition under policy H: if it is expensive to make the retailer participate and if testing costs do not differ much, then a quality-only contract is likely to be sufficient to achieve optimality. For policy L, the acquisition cost difference $\delta(p_g^L - p_b^L)$ has to be sufficiently less than retailer's participation profit and relative testing cost difference to allow for quality-only contract optimality under policy L. This implies that as long as acquisition is inexpensive (small δ) or acquisition prices are close to one another, an optimal quality-only contract under L is likely to exist.

Finally, the most restrictive conditions—more restrictive than just taking those in (i) and (ii) together—are those for optimality of fixed-fee contracts, which is intuitive: if there is a fixed-fee contract, then this is also a price-only and quality-only contract. However, if there exists a price-only contract and a quality-only contract, those do not have to coincide. Under an H policy, structurally, the conditions are similar to those of a quality-only contract, with the difference that π_0 is also a part of the lower bound for the acquisition cost difference $\delta(p_g^H - p_b^H)$. For L, it is similar: compared to a quality-only contract, the conditions for optimality of a fixed-fee contract are similar, with the difference that now π_0 (and c_l) build a lower bound for $\delta(p_g^L - p_b^L)$. To conclude, irrespective of policy, for the existence of an optimal fixed-fee contract, in addition to the already-discussed conditions for the more complicated contracts, the minimum profit that guarantees the retailer's participation must not be too large.

In summary, there are situations in which collaboration between firm and retailer can be simplified by reduction of information exchange without losing contract optimality. In particular, for policy L, there always exists an optimal price-only contract.

4.9. Conclusions

Some firms from the business field of recommerce collaborate with retailers in order to generate additional supply with used products besides running their online acquisition channels. Due to the quality heterogeneity of used products, the acquisition is executed with differentiated prices, which are set by the firm. To be able to apply price differentiation, the retailer has to test products before acquisition in order to gain (imperfect) knowledge about their qualities. This testing procedure as source of information asymmetries may lead to conflicting incentives between the firm and the retailer. Therefore, the firm has to carefully decide which contract to offer. Additionally, the question of whether upfront testing and differentiated acquisition is beneficial arises.

Our work's contribution lies in the investigation of this not-yet-examined acquisition channel, which exhibits the following features: quality heterogeneity of used products, setting of acquisition prices and contracts by the firm, and testing and price offers at the retailer, where testing effort and testing outcomes are private information. Moreover, to the best our knowledge, the presented cost-accuracy trade-off in testing has not yet been considered in research regarding used-product acquisition management.

We clarify how to optimally manage this acquisition channel by contracting and price setting for different acquisition policies: undifferentiated acquisition without upfront testing and differentiated acquisition with high-/low-effort testing. Afterward, we characterize which circumstances favor one policy over the others in terms of profitability. For example, if the amount of products of one quality strongly dominates those of other qualities, then upfront testing is not valuable. If testing is very inexpensive, then one is always better off testing.

We furthermore find that the presented acquisition channel is efficient in the sense that no matter which policy the firm chooses, optimal centralized supply chain profit is reached. How this profit is split up depends on the negotiating power of the parties involved. Finally, we discuss how the restriction of information exchange drives optimality of the resulting simpler contracts. We find that undifferentiated acquisition can always be managed by a fixed-fee contract. Differentiated acquisition with low-effort testing can always be set up optimally with a price-only contract. Hence, if the firm wants to apply that policy, she can always do so without having to communicate the products' actual qualities. As a last result, we find that there actually exist situations in which for both differentiated acquisition policies, optimal fixed-fee contracts do exist.

Our findings have several managerial implications: first, to the best of our knowledge, the presented acquisition channel is not very common. Based on our findings, collaboration between firms and retailers in the presented manner can be beneficial for both sides and could be exploited a lot more. Furthermore, what we found in practice is that the same acquisition prices as for the online channel are used. However, as proven in this work, depending on how accurately products' qualities can be determined, prices should be chosen differently. Another reason for this conclusion is that the ratio between good and bad products may differ between online sellers and retailer's customers. Finally, even though almost all recommerce providers apply quality-differentiated online acquisition, those firms should thoroughly decide about whether to do this in the considered setting because the success strongly depends on the retailer's capabilities and cost structure. Therefore, for some products, undifferentiated acquisition through retailers might be better.

Our model has some limitations that should be mentioned. To maintain tractability and present a comprehensible model, we made some simplifying assumptions. Products' possible qualities and the retailer's testing effort are binary. It would be more realistic to have multiple qualities and effort levels. Nevertheless, it suffices our purposes to determine the differences between differentiated and undifferentiated acquisition and the differences between different testing effort levels. Moreover, we assume the firm to be a price-taker when selling the acquired products by assuming that products represent certain quality-dependent values. An interesting avenue would be to decide about selling and acquisition prices in parallel in the considered setting. Furthermore, the setting that we investigate is static. It would be interesting to also consider dynamic effects and investigate how to choose contracts and prices when beliefs about the distribution of products' qualities and/or values change over time.

Previous research has not investigated the cost-accuracy trade-off in upfront testing before acquisition, which builds the basis for quality-differentiated pricing. Additionally, the management of such a decentralized system is important since various firms, as in the motivating example, are not able to obtain access to retailers' customers otherwise. The aim and contribution of this paper is to present this new method of managing used-product acquisition through intermediaries and hopefully inspire firms to build fruitful collaborations in order to secure additional used-product supply and thereby reduce waste and sustain the environment.

Appendix A

Proofs of Chapter II

Proof of Proposition 2.1. To prove the claim, we first derive the firm's optimization problem P and then solve P to determine the optimal compensation scheme and the firm's expected profits.

The Optimization problem: By the revelation principle, we restrict attention to the optimal contract that induces high-effort testing and truth telling by all experts. This, however, requires several incentive constraints to be satisfied. To derive those, we need to ensure that high-effort testing and truth telling is indeed optimal for each expert $i \in \mathcal{I}$, given the assumptions that all other experts exert high effort and report truthfully, and that the firm chooses the ex post optimal design alternative.

After having received all recommendations, the firm chooses the design alternative for development that offers the highest ex post expected net contribution and for which there is a good recommendation (in case there is no good recommendation at all, the firm develops none of the designs). Given $r_i = g$, design i's ex post expected net contribution is $q_i(v_i - u_{ig}) - (1 - q_i)u_{ib} - \sum_{k \neq i} u_{ka}$. Constraint (2.5) orders the designs according to their maximum ex post expected net contribution and thus ranks them according to their relative attractiveness to the firm; represented by the index j in $y_i^{(j)}$.

We now derive the incentive compatibility constraints for design i that is the jth most attractive alternative. Given $e_i = h$ and upon receiving a good signal ($s_i = g$), expert i receives an expected utility of $\pi_{ij}^{gg} = (q_i u_{ig} + (1 - q_i)u_{ib})/2^{j-1} + (1 - 1/2^{j-1})u_{ia}$ when making a good recommendation ($r_i = g$), and $\pi_{ij}^{bg} = u_{ia} + u_{it}/2^{n-1}$ when making a bad recommendation ($r_i = b$). Similarly, given $e_i = h$ and upon receiving a bad signal ($s_i = b$), expert i receives an expected utility of $\pi_{ij}^{gb} = ((1 - q_i)u_{ig} + q_i u_{ib})/2^{j-1} + (1 - 1/2^{j-1})u_{ia}$ when making a good recommendation ($r_i = g$), and $\pi_{ij}^{bb} = u_{ia} + u_{it}/2^{n-1}$ when making a bad recommendation ($r_i = b$). Also, given truth telling, expert i's expected utility from exerting high effort is $\pi_{ij}(h) = (q_i u_{ig} + (1 - q_i)u_{ib})/2^j + (u_{ia} + (1/2)^{n-j}u_{it})/2^j + (1 - 1/2^{j-1})u_{ia} - c$, and $\pi_{ij}(l) = (u_{ig}/2 + u_{ib}/2)/2^j + (u_{ia} + (1/2)^{n-j}u_{it})/2^j + (1 - 1/2^{j-1})u_{ia}$ from exerting low effort. The incentive compatibility constraints follow from setting $\pi_{ij}^{gg} \geq \pi_{ij}^{bg}$, $\pi_{ij}^{bb} \geq \pi_{ij}^{gb}$, and $\pi_{ij}(h) \geq \pi_{ij}(l)$ for all $i, j \in \mathcal{I}$, and multiplying these inequalities with $y_i^{(j)}$. Finally, because wages must be non-negative, we require $u_{ig}, u_{ib}, u_{ia}, u_{ia} + u_{it} \geq 0$, and (2.6) follows from noting that each design i must be assigned to exactly one attractiveness rank, and each rank j is hold by exactly one design.

The firm's expected profit consists of the expected market value of the chosen design net of development costs and the experts' expected wages. Given $y_i^{(j)} = 1$, the firm develops design i with probability $1/2^j$ and receives an expected value of $q_i v_i - K$. The expected wage payments to expert i are u_{ig} with probability $q_i/2^j$, u_{ib} with probability $(1-q_i)/2^j$, u_{ia} with probability $1 - 1/2^j$, and u_{it} with probability $1/2^n$. Summing over $i, j \in \mathcal{I}$ gives the firm's expected profit Π.

(i) Suppose the designs in $\mathcal{I}$ can be ordered such that $q_i v_i \geq q_{i+1} v_{i+1} + 2^{i+1} c[q_i/(2q_i - 1) - 2q_{i+1}/(2q_{i+1} - 1)]^+$ for all $i \in \mathcal{I} \backslash \{n\}$. To solve the optimization problem P, we first derive the solution of a relaxed variant of P by dropping constraints (2.5), and then show that this solution is also feasible—and thus optimal—in P.

Given the structure of P without (2.5), maximizing the firm's expected profit is equivalent to separately minimizing the wage payments associated with each design $i \in \mathcal{I}$ whenever $y_i^{(j)} = 1$. Obviously, (2.4) implies that $u_{ig} > u_{ib}$, which allows us to rewrite (2.2) and (2.3) as $q_i u_{ig} + (1 - q_i) u_{ib} \geq u_{ia} + 2^{j-n} u_{it} \geq (1 - q_i) u_{ig} + q_i u_{ib}$. It follows that wage payments for design i with relative attractiveness j are minimized when $u_{ia} + 2^{j-n} u_{it} = (1 - q_i) u_{ig} + q_i u_{ib}$, and u_{ig} and u_{ib} are chosen as low as possible. By (2.4) and (2.7), these minimal payments are $u_{ig} = 2^{j+1} c/(2q_i - 1)$ and $u_{ib} = 0$. Moreover, (2.1) reveals that the firm prefers paying u_{it} over u_{ia}; therefore $u_{ia} = 0$ and $u_{it} = 2^{n+1}(1 - q_i) c/(2q_i - 1)$. Inserting these payments into (2.1) and using (2.6) gives $\Pi_P = \sum_{j=1}^n \sum_{i=1}^n y_i^{(j)} (q_i v_i / 2^j) - \sum_{i=1}^n 2c/(2q_i - 1) - \sum_{j=1}^n K/2^j$. By the assumed ordering, we have $q_i v_i \geq q_{i+1} v_{i+1}$, and it follows that in optimum $y_i^{(i)} = 1$ for all $i \in \mathcal{I}$, and $y_i^{(j)} = 0$ for all $i \neq j$. Moreover, this candidate optimal solution satisfies (2.6) and is thus feasible.

It remains to show that the solution also satisfies (2.5). However, this is obvious because we can rewrite this condition by $q_i v_i - (2^{i+1} q_i c/(2q_i - 1)) \geq q_{i+1} v_{i+1} - (2^{i+2} q_{i+1} c/(2q_{i+1} - 1))$, which is true by assumption.

(ii) This result follows directly from (i). □

Proof of Proposition 2.2. The optimization problem M can be derived in a similar way to the proof of Proposition 2.1. In particular, for each $i \in \mathcal{I}$, $\pi_i^{gg} = q_i u_{ig} + (1 - q_i) u_{ib}$, $\pi_i^{bg} = \pi_i^{bb} = u_{ia} + \mathbb{P}(s_j = b \; \forall j > i) \delta^{n-i} u_{it}$, $\pi_i^{gb} = (1 - q_i) u_{ig} + q_i u_{ib}$, $\pi_i(h) = (q_i u_{ig} + (1 - q_i) u_{ib})/2 + (u_{ia} + \mathbb{P}(s_j = b \; \forall j > i) \delta^{n-i} u_{it})/2 - c$, and $\pi_i(l) = (u_{ig}/2 + u_{ib}/2)/2 + (u_{ia} + \mathbb{P}(s_j = b \; \forall j >$

$i)\delta^{n-i}u_{it})/2$. Since for design $i = n$, u_{na} and u_{nt} are paid simultaneously, we only require $u_{na} + u_{nt} \geq 0$ to ensure non-negative wages.

As for the firm's profits, the firm develops design i with probability $\mathbb{P}(r_i = g,\ r_j = b\,\forall j < i)$ and receives a discounted expected value of $\delta^{i-1}(q_i v_i - K)$. The expected wage payments to expert i are $\delta^{i-1}u_{ig}$ with probability $\mathbb{P}(\Theta_i = G|s_i = g)\mathbb{P}(s_i = g,\ s_j = b\ \forall j < i)$, $\delta^{i-1}u_{ib}$ with probability $\mathbb{P}(\Theta_i = B|s_i = g)\mathbb{P}(s_i = g,\ s_j = b\ \forall j < i)$, $\delta^{i-1}u_{ia}$ with probability $\mathbb{P}(s_i = b,\ s_j = b\ \forall j < i)$, and $\delta^{n-1}u_{it}$ with probability $\mathbb{P}(s_i = b,\ s_j = b\ \forall j \neq i)$. Summing over $i \in \mathcal{I}$ gives the firm's expected profit Π.

(i) Given the structure of M, maximizing the firm's expected profit is equivalent to separately minimizing the wage payments associated with each design i. Note that (2.11) implies that $u_{ig} > u_{ib}$, which allows us to rewrite (2.9) and (2.10) as $q_i u_{ig} + (1 - q_i)u_{ib} \geq u_{ia} + 2^{i-n}\delta^{n-i}u_{it} \geq (1-q_i)u_{ig} + q_i u_{ib}$. It follows readily that $u_{ia} + 2^{i-n}\delta^{n-i}u_{it} = (1-q_i)u_{ig} + q_i u_{ib}$, and u_{ig} and u_{ib} should be chosen as low as possible. By (2.11) and (2.12), these minimal payments are $u_{ig} = 4c/(2q_i - 1)$ and $u_{ib} = 0$. Moreover, the firm is indifferent between paying u_{ia} or u_{it}, so without loss of optimality we can choose $u_{ia} = 4(1 - q_i)c/(2q_i - 1)$ and $u_{it} = 0$. Finally, $u_{ia}/u_{ig} = 1 - q_i < 1/2$ because $q_i > 1/2$.

(ii)-(iii) Given the optimal contract, we can rewrite the firm's expected profit as $\Pi_M = \sum_{i=1}^{n}(\delta^{i-1}/2^i)(q_i v_i - K - 4c/(2q_i - 1))$. Since $(\delta^{i-1}/2^i)$ is decreasing in i, the firm maximizes Π_M by testing the designs in decreasing order of $q_i v_i - 4c/(2q_i - 1)$. □

Proof of Proposition 2.3. Define the expert's expected continuation utility before testing design $i \in \mathcal{I}$ by $\hat{\pi}_{i-1} = (q_i u_{ig} + (1-q_i)u_{ib} + u_{ia} - 2c + \delta\hat{\pi}_i)/2$, with $\hat{\pi}_n = 0$. With this definition, the derivation of S is identical to that of M as given in the proof of Proposition 2.2. In particular, for each $i \in \mathcal{I}$, $\pi_i^{gg} = q_i u_{ig} + (1-q_i)u_{ib}$, $\pi_i^{bg} = \pi_i^{bb} = u_{ia} + \delta\hat{\pi}_i$, $\pi_i^{gb} = (1-q_i)u_{ig} + q_i u_{ib}$, $\pi_i(h) = (q_i u_{ig} + (1-q_i)u_{ib})/2 + (u_{ia} + \delta\hat{\pi}_i)/2 - c$, and $\pi_i(l) = (u_{ig}/2 + u_{ib}/2)/2 + (u_{ia} + \delta\hat{\pi}_i)/2$, and limited liability enforces non-negative wage payments.

As for the firm's profits, the firm develops design i with probability $\mathbb{P}(r_i = g,\ r_j = b\,\forall j < i)$ and receives a discounted expected value of $\delta^{i-1}(q_i v_i - K)$. The expected wage payments to expert i are $\delta^{i-1}u_{ig}$ with probability $\mathbb{P}(\Theta_i = G|s_i = g)\mathbb{P}(s_i = g,\ s_j = b\ \forall j < i)$, $\delta^{i-1}u_{ib}$ with probability $\mathbb{P}(\Theta_i = B|s_i = g)\mathbb{P}(s_i = g,\ s_j = b\ \forall j < i)$, and $\delta^{i-1}u_{ia}$ with probability $\mathbb{P}(s_i = b,\ s_j = b\ \forall j < i)$. Summing over $i \in \mathcal{I}$ gives the firm's expected profit Π.

(i) Using the definition of $\hat{\pi}_0$, we can rewrite the firm's expected profit as $\Pi_S = \sum_{i=1}^{n}(\delta^{i-1}/2^i)$ $(q_i v_i - K - 2c) - \hat{\pi}_0$. Thus, maximizing Π_S is equivalent to minimizing $\hat{\pi}_0$, which we do in the following. As a first step, we derive the minimum feasible $\hat{\pi}_{i-1}$ for given fixed $\hat{\pi}_i$.

Case (a): $\delta\hat{\pi}_i < 4(1-q_i)c/(2q_i-1)$. The optimal payments are $u_{ig} = 4c/(2q_i-1)$, $u_{ib} = 0$, and $u_{ia} = 4(1-q_i)c/(2q_i-1) - \delta\hat{\pi}_i$, and it follows that $\hat{\pi}_{i-1} = (3-2q_i)c/(2q_i-1)$.

Case (b): $4(1-q_i)c/(2q_i-1) \leq \delta\hat{\pi}_i \leq 4q_ic/(2q_i-1)$. The optimal payments are $u_{ig} = 4c/(2q_i-1)$, $u_{ib} = 0$, and $u_{ia} = 0$, and it follows that $\hat{\pi}_{i-1} = c/(2q_i-1) + \delta\hat{\pi}_i/2$.

Case (c): $\delta\hat{\pi}_i > 4q_ic/(2q_i-1)$. The optimal payments are $u_{ig} = \delta\hat{\pi}_i/q_i$, $u_{ib} = 0$, and $u_{ia} = 0$, and it follows that $\hat{\pi}_{i-1} = \delta\hat{\pi}_i - c$

Taken together, Cases (a)-(c) imply that $\hat{\pi}_{i-1}$ is non-decreasing in $\hat{\pi}_i$ for all $i \in \mathcal{I}$. As such, minimizing $\hat{\pi}_0$ is equivalent to separately minimizing $\hat{\pi}_i$ for each $i \in \mathcal{I}$, starting with $\hat{\pi}_n = 0$ and using Cases (a)-(c) for backwards induction. Thus, the optimal contract satisfies $u_{ig} = 4c/(2q_i-1) + [\delta\hat{\pi}_i/q_i - 4c/(2q_i-1)]^+$, $u_{ib} = 0$, and $u_{ia} = [4(1-q_i)c/(2q_i-1) - \delta\hat{\pi}_i]^+$ for all $i \in \mathcal{I}$.

(ii) If the designs in $\mathcal{I}$ can be ordered such that $q_iv_i \geq q_{i+1}v_{i+1}$, $q_i \geq q_{i+1}$ and $(1-q_i)4c/(2q_i-1) \leq \delta\hat{\pi}_i \leq 4q_ic/(2q_i-1)$ for all $i \in \mathcal{I}\backslash\{n\}$, then Cases (a) and (b) imply that $q_iu_{ig} + (1-q_i)u_{ib} + u_{ia} < q_{i+1}u_{i+1g} + (1-q_{i+1})u_{i+1b} + u_{i+1a}$ for all $i \in \mathcal{I}\backslash\{n\}$. By (2.13) and the assumption that $q_iv_i \geq q_{i+1}v_{i+1}$ it follows readily that it is optimal to test the designs in increasing order of i.

(iii) This result follows directly from inserting Proposition 2.3(i) in (2.13) and rearranging terms. □

Proof of Proposition 2.4. (i) By Proposition 2.2(iii), $\Pi_M = \sum_{i=1}^{n}(\delta^{i-1}/2^i)(q_iv_i - K - 4c/(2q_i-1))$, which reveals that the sign of the net profit contribution of each design $i \in \mathcal{N}$ is independent of the number and identity of the other designs to be tested. As a result, the firm finds it optimal to include all designs $i \in \mathcal{N}$ into the testing set $\mathcal{I}_M$ for which $q_iv_i - K - 4c/(2q_i-1) \geq 0$.

(ii) Consider the optimization problem P. By (2.2)-(2.4), we have $u_{ig} \geq 2^{j+1}c/(2q_i-1)$ and $u_{ia} + 2^{j-n}u_{it} \geq (1-q_i)u_{ig}$ for all $i, j \in \mathcal{I}$ such that $y_i^{(j)} = 1$. Hence, the profit contribution of design i with relative attractiveness j is $\Pi_i^{(j)} \leq (q_iv_i - K - q_iu_{ig} - u_{ia} - 2^{j-n}u_{it})/2^j \leq (q_iv_i - K - u_{ig})/2^j \leq (q_iv_i - K)/2^j - 2c/(2q_i-1)$. A necessary condition for $i \in \mathcal{I}_P$ is that $(q_iv_i - K)/2^j - 2c/(2q_i-1) \geq 0$ for some $j \in \mathcal{I}_P$. However, this can only be true if $q_iv_i - K - 4c/(2q_i-1) \geq 0$. Comparing this condition with $\mathcal{I}_M$ completes the proof.

(iii) By Proposition 2.2 (ii) and (iii), it is optimal to have design n in the optimal set of designs for testing since it is profitable to test and eventually develop this design due to the stated condition $q_nv_n - K - 4c/(2q_n-1) > 0$. Furthermore, all designs j with $q_j \geq q_n$ and $v_j \geq v_n$ belong to this optimal set because then, we also have $q_jv_j - K - 4c/(2q_j-1) > 0$. □

Proof of Proposition 2.5. (i) Suppose $\delta < 1$. For $c = 0$, we have $\mathcal{I}_P = \mathcal{I}_M = \mathcal{I}_S = \mathcal{N}$ and designs are tested in decreasing order of $q_i v_i$. By Propositions 2.1-2.3, it follows readily that $\Pi_P^* > \Pi_M^* = \Pi_S^*$. Thus, by continuity of the expected profits in c, there exists $\underline{c} > 0$ such that $\Pi_P^* > \max\{\Pi_M^*, \Pi_S^*\}$ for all $c < \underline{c}$.

(ii) Let $\mathcal{I}_P$ be the optimal set of designs to be tested under a parallel testing strategy, and assume that the designs in $\mathcal{I}_P$ can be ordered such that $q_i v_i \geq q_{i+1} v_{i+1} + 2^{i+1} c[q_i/(2q_i - 1) - 2q_{i+1}/(2q_{i+1} - 1)]^+$ for all $i \in \mathcal{I}_P \backslash \{n\}$. Then, by Proposition 2.1(iii), $\Pi_P^* = \sum_{i \in \mathcal{I}_P}((q_i v_i - K)/2^i - 2c/(2q_i - 1))$. Now, if the firm fixes the identity and ordering of designs, but instead uses a multi-expert sequential testing strategy, then $\Pi_M(\mathcal{I}_P) = \sum_{i \in \mathcal{I}_P} \delta^{i-1}(q_i v_i - K - 4c/(2q_i - 1))/2^i$. By comparing the different profits, we have $\Pi_M(\mathcal{I}_P) > \Pi_P^*$ if $c > \overline{c} \equiv \sum_{i \in \mathcal{I}_P}((1 - \delta^{i-1})(q_i v_i - K)/2^i)/\sum_{i \in \mathcal{I}_P}((2(1 - (\delta/2)^{i-1})/(2q_i - 1))$. Moreover, since $\mathcal{I}_P$ need not be optimal under a multi-expert sequential testing strategy, it follows that if $c > \overline{c}$, then $\Pi_P^* < \Pi_M(\mathcal{I}_P) \leq \Pi_M^* \leq \max\{\Pi_M^*, \Pi_S^*\}$.

(iii) Let $\mathcal{I}_M$ be the optimal set of designs to be tested under a multi-expert sequential testing strategy, with $\mathcal{I}_M$ optimally ordered according to Proposition 2.2(ii). By Proposition 2.2(iii) and 2.3(iii), we have $\Pi_M^* = \sum_{i \in \mathcal{I}_M}(\delta^{i-1}/2^i)(q_i v_i - K - 4c/(2q_i - 1))$ and $\Pi_S(\mathcal{I}_M) = \sum_{i \in \mathcal{I}_M} \delta^{i-1}(q_i v_i - K - \max\{4q_i c/(2q_i - 1), \delta\hat{\pi}_i, 4c/(2q_i - 1) - \delta\hat{\pi}_i\})/2^i \leq \Pi_S^*$. Clearly, a sufficient condition for $\Pi_S(\mathcal{I}_M) \geq \Pi_M^*$ is that $\delta\hat{\pi}_i \leq 4q_i c/(2q_i - 1)$ for all $i \in \mathcal{I}_M$.

By (2.17), we have $\delta\hat{\pi}_n = 0$ and $\delta\hat{\pi}_{i-1}$ increases in $\delta\hat{\pi}_i$ for all $i \in \mathcal{I}_M$. Moreover, if $\delta\hat{\pi}_i = 4q_i c/(2q_i - 1)$, then $\delta\hat{\pi}_{i-1} = \delta(4q_i c/(2q_i - 1) - c)$. Thus, by induction, if $\delta\hat{\pi}_i \leq 4q_i c/(2q_i - 1)$, then $\delta\hat{\pi}_{i-1} \leq \delta(4q_i c/(2q_i - 1) - c)$, and $\delta(4q_i c/(2q_i - 1) - c) \leq 4q_{i-1}c/(2q_{i-1} - 1)$ if $q_i \geq \underline{q}_{i-1}$. Finally, it is easy to show that $\underline{q}_i \leq q_i$ and $\underline{q}_i \leq 5/6$. □

Proof of Proposition 2.6. (i) Note that $\mathcal{I}_{\text{seq}}^{\text{fb}} = \{i \in \mathcal{N} \mid q_i v_i - K - 2c \geq 0\}$. Comparing this with $\mathcal{I}_M$ as given in Proposition 2.4(i) immediately yields $\mathcal{I}_M \subseteq \mathcal{I}_{\text{seq}}^{\text{fb}}$. We next show that for any $i \in \mathcal{I}_S$, $q_i v_i - K - 4q_i c/(2q_i - 1) \geq 0$; implying that $\mathcal{I}_S \subseteq \mathcal{I}_{\text{seq}}^{\text{fb}}$. Consider an arbitrary design $i \in \mathcal{I}_S$. By Proposition 2.3(i), if the firm receives a good recommendation for this design, the expected value of developing it is given by $q_i v_i - K - q_i u_{ig} - (1 - q_i) u_{ib} \leq q_i v_i - K - 4q_i c/(2q_i - 1)$. Obviously, the firm only develops design i if the development generates a nonnegative expected value; i.e., $q_i v_i - K - 4q_i c/(2q_i - 1) \geq q_i v_i - K - q_i u_{ig} - (1 - q_i) u_{ib} \geq 0$. Suppose to the contrary that there exists a design $i \in \mathcal{I}_S$ such that $q_i v_i - K - q_i u_{ig} - (1 - q_i) u_{ib} < 0$. Obviously, the firm would never develop this design as the firm's outside option has zero, and thus greater value. In equilibrium, the expert anticipates the firm's development decision, and as a result, it is impossible for the firm to motivate the expert to exert high testing efforts. Hence, since design i will never be tested anyways, it is optimal for the firm to erase it from the set of designs to be tested.

(ii) We prove the claim by example. Consider a setting with three design alternatives and the following parameters: $q_1 = 0.55$, $q_2 = 0.64$, $q_3 = 1$, $v_1 = 100$, $v_2 = 85$, $v_3 = 53$, $K = 50$, and $c = 0.6$. In this case, the optimal set of designs to be tested under first-best conditions is $\mathcal{I}^{\text{fb}}_{\text{par}} = \{1, 2\}$, leading to an expected profit of $\Pi^{\text{fb}}_{\text{par}} = 2.4$. In contrast, under delegation, we have $\mathcal{I}_P = \{3\}$ with an expected profit of $\Pi^*_P = 0.3$. It follows that $\mathcal{I}_P \cap \mathcal{I}^{\text{fb}}_{\text{par}} = \emptyset$.

(iii) For brevity, let $|\mathcal{I}_M| = n_M$, $|\mathcal{I}_P| = n_P$, $|\mathcal{I}^{\text{fb}}_{\text{seq}}| = n_{\text{seq}}$, and $|\mathcal{I}^{\text{fb}}_{\text{par}}| = n_{\text{par}}$. With symmetric test efficiencies (i.e., $q_i = q$ for all $i \in \mathcal{N}$), under any sequential testing strategy it is always optimal to test designs in decreasing order of v_i, and under any parallel testing strategy the designs attractiveness decreases in v_i. It follows from Propositions 2.1-2.4 that $\mathcal{I}_P \subseteq \mathcal{I}^{\text{fb}}_{\text{par}}, \mathcal{I}_M \subseteq \mathcal{I}^{\text{fb}}_{\text{seq}}$, and consequently, $n_P \leq n_{\text{par}}, n_M \leq n_{\text{seq}}$. Without loss of generality, we relabel the designs such that $v_i \geq v_{i+1}$ for all $i \in \mathcal{N}$. Given these preliminaries, we prove the result by showing that for any $n_P \geq 0$, $\Pi^{\text{fb}}_{\text{seq}} \geq \Pi^{\text{fb}}_{\text{par}}$ implies $\Pi^*_M \geq \Pi^*_P$.

Case (a): $n_P = 0$. Since it always holds that $\Pi^*_P = 0 \leq \Pi^*_M$, the claim is trivially satisfied.

Case (b): $n_P = 1$. By Proposition 2.1(iii) and 2.2(iii), we have $\Pi^*_P = (qv_1 - K)/2 - 2c/(2q - 1) = \Pi_M(n = 1) \leq \Pi_M(n_M) = \Pi^*_M$, where the inequality follows from the optimality of n_M.

Case (c): $n_P \geq 2$. Define $\Delta\Pi(x, y) = \Pi(x) - \Pi(y)$. With this notation, we can rewrite the firm's first-best expected profits as $\Pi^{\text{fb}}_{\text{seq}}(n_{\text{seq}}) = \Pi_M(n_M) + \Delta\Pi_M(n_{\text{seq}}, n_M) + \sum_{i=1}^{n_{\text{seq}}}(c(\delta/2)^{i-1}(3-2q)/(2q-1))$, and $\Pi^{\text{fb}}_{\text{par}}(n_{\text{par}}) = \Pi_P(n_P) + \Delta\Pi^{\text{fb}}_{\text{par}}(n_{\text{par}}, n_P) + \sum_{i=1}^{n_p}(c(3-2q)/(2q-1))$. Hence $\Pi^{\text{fb}}_{\text{seq}} \geq \Pi^{\text{fb}}_{\text{par}}$ is equivalent to $\Pi_M(n_M) \geq \Pi_P(n_P) + \Delta\Pi^{\text{fb}}_{\text{par}}(n_{\text{par}}, n_P) - \Delta\Pi_M(n_{\text{seq}}, n_M) + c(3-2q)/(2q-1)(n_P - (1 - (\delta/2)^{n_{\text{seq}}})/(1 - (\delta/2)))$. The right-hand side of this inequality is larger than $\Pi_P(n_P)$, which proves the claim. To see this, note that by optimality of n_{par} and n_M, we have $\Delta\Pi^{\text{fb}}_{\text{par}}(n_{\text{par}}, n_P) \geq 0$ and $\Delta\Pi_M(n_{\text{seq}}, n_M) \leq 0$, and finally, $n_P - (1 - (\delta/2)^{n_{\text{seq}}})/(1 - (\delta/2)) \geq n_P - 2 \geq 0$ because $n_P \geq 2$ by assumption.

Last, we prove that the converse statement is not always true. We do this by example. Consider a scenario with the following parameters: $N = 4$, $v_1 = 10$, $v_2 = 8$, $v_3 = 6$, $v_4 = 4$, $q = 1$, $\delta = 0.8$, $K = 2$, $c = 0.2$. Then $\Pi^*_P = 3.6 \leq \Pi^*_M = 3.82$, but $\Pi^{\text{fb}}_{\text{par}} = 4.2 \geq \Pi^{\text{fb}}_{\text{seq}} = 4.14$. $\square$

Appendix B

Proofs of Chapter III

Proof of Proposition 3.1. First, note the optimal decision in stage (III). Additionally, $co_{ij}^* < a_j$ always holds. Hence, product holder never states j if $v_i \geq a_j$. For $v_i < a_j$, it always holds that $v_i \leq \pi(j|i, v_i) \leq a_j$, where $\pi(j|i, v_i)$ is the holder's expected payoff for handing in and stating j. This holds because the product holder can at most gain a_j, and if there is a counteroffer that is less than v_i, the product is sent back.

The remaining proof consists of three steps: (a) we show that if it is optimal for a product holder with $v_i \in [\underline{v_i}, \overline{v_i}]$ to hand in and state j, then this is also optimal for any product holder with a smaller residual value; (b) we show that the stated action in (II) in Proposition 3.1 is the optimal response to players acting in the manner determined in the first step; and (c) we show that the stated actions in (I) of Proposition 3.1 are optimal.

(a) Consider a holder's expected payoff for handing in and stating j given v_i:

$$\pi(j|i, v_i) = \mathbb{P}(\Pi(co_{ij}^*(m_i)|j, m_i) \leq m_i - a_j) \cdot a_j$$
$$+\mathbb{P}(\Pi(co_{ij}^*(m_i)|j, m_i) > m_i - a_j) \cdot \int_{\Pi(co_{ij}^*(m_i)|j,m_i)+a_j>m_i} \max\{v_i, co_{ij}^*(m_i)\} dm_i$$

Note that the optimal response of the firm is not yet specified and depends on the updated belief about v_i if a certain quality is stated. $\pi(j|i, v_i)$ exhibits the following properties:

(1) For $\epsilon > 0$, we have $\pi(j|i, v_i) > v_i \Rightarrow \pi(j|i, v_i - \epsilon) > v_i - \epsilon$. Hence, if handing in is profitable for v_i, it is also profitable for all smaller residual values.

(2) $\pi(j|i, v_i)$ is continuous in v_i.

Now, assume an arbitrary but rationally updated belief of the firm about v_i if statement j is observed, i.e., every v_i is assigned to keeping the product or to handing in and stating a certain quality (including j). The firm determines her optimal action in stage (II) based on that belief. By (1) and given the firm's updated belief, there is $\overline{w_i} \in [\underline{v_i}, \overline{v_i}]$, s.t. for all $v_i \geq \overline{w_i}$, handing in is not profitable, and for all $v_i < \overline{w_i}$, handing in is profitable. Furthermore, there are l and $v_i' < \overline{w_i}$, s.t. $\pi(l|i, v_i) \geq \pi(k|i, v_i)$ for all $k \neq l$ and for all v_i with $v_i' < v_i \leq \overline{w_i}$.

First, consider k and l, s.t. $\pi(l|i, v_i) > \pi(k|i, v_i)$ for all v_i with $v_i' < v_i < \overline{w_i}$. Assume that there is a smallest $v_i' \geq \underline{v_i}$ for which the conditions apply. Denote it by v_i''. Then, due to (2), v_i'' has to be an intersection point; hence, $\pi(l|i, v_i'') = \pi(k|i, v_i'')$. Then, for all v_i with $\overline{w_i} > v_i > v_i''$, it is more profitable to hand in and state l than to state k. This is anticipated by the firm; hence, it is rational to pay at most v_i'' if k is stated. However, then, $v_i'' = \pi(k|i, v_i'') = \pi(l|i, v_i'')$. Therefore, it is not profitable for v_i'' to hand in and state l, which contradicts (1) because it is profitable to hand in and state l for all v_i with $v_i'' < v_i < \overline{w_i}$. Hence, there is no smallest v_i' for which the stated conditions apply, and there cannot be any intersection between $\pi(l|i, v_i)$ and $\pi(k|i, v_i)$ for $v_i < \overline{w_i}$. Therefore, l has to coincide with the observed statement j, and it is optimal for all $v_i < \overline{w_i}$ to hand in and state j, whereas $v_i \geq \overline{w_i}$ do not hand in.

Now, assume there are l and $k \neq l$ s.t. $\pi(l|i, v_i) = \pi(k|i, v_i)$ for all v_i with $v_i' < v_i < \overline{w_i}$. Assume again that there is a smallest $v_i' \geq \underline{v_i}$ for which the conditions apply. Denote it again by v_i''. Then, $\pi(l|i, v_i'') > \pi(k|i, v_i'')$ would have to hold. Hence, for v_i'', it is more profitable to state l than to state k. Assume that l was stated. Then, the firm cannot differentiate whether $v_i \leq v_i''$, or $v_i'' < v_i < \overline{w_i}$ applies. In this case, all $v_i < \overline{w_i}$ act the same by handing in and stating l. Now, assume indifferent holders with v_i in $(v_i'', \overline{w_i})$ would state k. Then, the firm would know that $\pi(l|i, v_i) > \pi(k|i, v_i)$ cannot apply for v_i. Therefore, stating l for $v_i \leq v_i''$ would lead to the firm paying at most v_i''. Hence, for v_i'', it holds that $v_i'' = \pi(l|i, v_i'') < \pi(k|i, v_i'')$ by (1). This contradicts $\pi(l|i, v_i'') > \pi(k|i, v_i'')$. Therefore, if product holder with v_i is indifferent between stating k and l, it is optimal for holders with smaller residual values to follow the action of the product holder with higher residual value if handing in is profitable for him.

(b) Now, we show that the stated action in (II) of Proposition 3.1 is the optimal response to product holders' behavior. First, note that the updated belief about v_i being below a counteroffer co_{ij} if j was stated (which is important for anticipating the optimal action in stage (III)) is $\mathbb{P}(v_i \leq co_{ij}|j) = (co_{ij} - v_i)/(\overline{w_i} - \underline{v_i})$ if $\underline{v_i} \leq co_{ij} \leq \overline{w_i}$ and $\mathbb{P}(v_i \leq co_{ij}|j) = 1$ if $co_{ij} \geq \overline{w_i}$. $\overline{w_i}$ denotes the cut-off level for v_i for handing in and stating j, i.e., $v_i \in [\underline{v_i}, \overline{w_i})$ hand in and state j and $v_i \in [\overline{w_i}, \overline{v_i}]$ do not hand in. Note that $\overline{w_i} \leq \min\{a_j, \overline{v_i}\}$ has to hold. Assume that $\overline{w_i} > \underline{v_i}$; otherwise, no product holder with quality i hands in and states j. The expected payoff for making a counteroffer is as follows:

$$\Pi(co_{ij}|j, m_i) = \begin{cases} \frac{co_{ij} - \underline{v_i}}{\overline{w_i} - \underline{v_i}}(m_i + c - co_{ij}) - c & \text{if } \overline{w_i} \geq co_{ij} \geq \underline{v_i} \\ m_i - co_{ij} & \text{if } co_{ij} \geq \overline{w_i} \end{cases}$$

Differentiating $\Pi(co_{ij}|j, m_i)$ with respect to co_{ij}, setting equal to zero and rearranging with respect to co_{ij} leads to $co^*_{ij} = \min\{(m_i + c + \underline{v_i})/2, \overline{w_i}\}$. Therefore, we have $co^*_{ij} \geq \underline{v_i}$.

Now, we compare the payoff for making the optimal counteroffer to the payoff for accepting the offer a_j:

$$\Pi(co^*_{ij}|j, m_i) - (m_i - a_j) = \begin{cases} \frac{(\overline{w_i} - co^*_{ij})^2 + (a_j - \overline{w_i})(\overline{w_i} - \underline{v_i})}{\overline{w_i} - \underline{v_i}} & \textit{if } co^*_{ij} = \frac{m_i + c + \underline{v_i}}{2} \\ a_j - \overline{w_i} & \textit{if } co^*_{ij} = \overline{w_i} \end{cases}$$

It always holds that $a_j \geq \overline{w_i} \geq co^*_{ij}$. Hence, in both cases, the terms are greater than 0 if and only if $a_j > co^*_{ij}$. The terms are equal to 0 if and only if $a_j = co^*_{ij}$, which means that counteroffering does not yield any benefit. Therefore, in this case, the firm ends the game by accepting the offer. This gives us that the stated action for (II) is indeed the optimal response to product holders acting, as determined in the first step.

(c) Now, we can insert the optimal response in stage (II) into the product holder's expected payoff for handing in and stating j, giving us

$$\begin{aligned} \pi(j|i, v_i) =& \frac{1}{\overline{m_i} - \underline{m_i}} \int_{\underline{m_i}}^{\overline{m_i}} \max\{v_i, \min\{\overline{w_i}, co^*_{ij}(m_i)\}\} dm_i \\ \Leftrightarrow \quad \pi(j|i, v_i) =& \mathbb{P}(\overline{w_i} \leq \frac{m_i + \underline{v_i} + c}{2}) \cdot \overline{w_i} \\ &+ \mathbb{P}(\overline{w_i} > \frac{m_i + \underline{v_i} + c}{2} \geq v_i) \cdot \int_{\overline{w_i} > \frac{m_i + \underline{v_i} + c}{2} \geq v_i} \frac{m_i + \underline{v_i} + c}{2} dm_i \\ &+ \mathbb{P}(\frac{m_i + \underline{v_i} + c}{2} < v_i) \cdot v_i \\ =& \mathbb{P}(2\overline{w_i} - \underline{v_i} - c \leq m_i) \cdot \overline{w_i} \\ &+ \mathbb{P}(2\overline{w_i} - \underline{v_i} - c > m_i \geq 2v_i - \underline{v_i} - c) \\ &\cdot \int_{2\overline{w_i} - \underline{v_i} - c > m_i \geq 2v_i - \underline{v_i} - c} \frac{m_i + \underline{v_i} + c}{2} dm_i \\ &+ \mathbb{P}(m_i < 2v_i - \underline{v_i} - c) \cdot v_i \end{aligned}$$

It is obvious that from product holder's perspective, the firm is at most paying $\min\{a_j, (\overline{m_i} + \underline{v_i} + c)/2, \overline{v_i}\}$. That means that if v_i is less than this term, there is a positive probability that they gain more than v_i from handing in and stating j, resulting in $\pi(j|i, v_i) > v_i$. If $v_i \geq a_j$, j is not stated because it is not profitable. There exists no $v_i > \overline{v_i}$. If $v_i \geq (\overline{m_i} + \underline{v_i} + c)/2$, there is also no chance of getting paid more than $(\overline{m_i} + \underline{v_i} + c)/2$ because this is the largest imaginable optimal counteroffer if smaller than a_j and $\overline{v_i}$. Hence, $\overline{w_i} = \min\{a_j, (\overline{m_i} + \underline{v_i} + c)/2, \overline{v_i}\}$. Now, it is also obvious that if $a_k < \min\{(\overline{m_i} + \underline{v_i} + c)/2, \overline{v_i}\}$, then for $a_j > a_k$, it holds that $\pi(j|i, v_i) > \pi(k|i, v_i)$. For $a_k \geq \min\{(\overline{m_i} + \underline{v_i} + c)/2, \overline{v_i}\}$ and $a_j > a_k$, it holds that $\pi(j|i, v_i) = \pi(k|i, v_i)$. Hence, the optimal actions in stage (I) and the corresponding optimal response in stage (II) hold. Due to the fact that the optimal counteroffer co^*_{ij} has the same structure for all optimal quality statements, the index j referring to the quality statement is dropped. □

Proof of Corollary 3.1. Assume that there is a participation fee of ϵ that is subtracted from every payoff for a holder in stages (II) and (III) and added to every payoff for the firm in (II) and (III). Then, for an arbitrary cut-off level $\overline{w_i}'$, where $\underline{v_i} \leq \overline{w_i}' \leq \overline{v_i}$, the firm's optimal counteroffer solves

$$\begin{aligned}\Pi'(co^*_i|j, m_i) =& \left(\frac{co^*_i - (\underline{v_i} - \epsilon)}{\overline{w_i}' - (\underline{v_i} - \epsilon)}(m_i + \epsilon + c - \epsilon - co^*_i) - c + \epsilon\right)' \\ =& \frac{m_i + c - co^*_i - co^*_i + (\underline{v_i} - \epsilon)}{\overline{w_i}' - (\underline{v_i} - \epsilon)} = 0\end{aligned}$$

if $co^*_i \leq \overline{w_i}'$, resulting in $co^*_i = (m_i + \underline{v_i} + c - \epsilon)/2$. If $co^*_i \geq \overline{w_i}'$, it is optimal to offer $\overline{w_i}'$.

The cutoff level for handing in and stating j, $\overline{w_i}'$, is at most equal to $\overline{w_i} - \epsilon$ in the former proof because the new expected payoff for handing in is at most equal to the expected payoff without effort minus ϵ. This is due to the fact that the optimal counteroffer is lower than in the case without effort. Therefore, the firm will at most pay $\overline{w_i} - \epsilon$. This is anticipated by the product holder leading to a cutoff level that is at most equal to $\overline{w_i} - 2\epsilon$. This can again be anticipated by the firm, and so on. In the end, there is k such that $\overline{w_i} - k\epsilon \leq \underline{v_i}$, leading to the stated equilibrium at which no one hands in. □

Proof of Lemma 3.1. The statement follows immediately by differentiating Π^i_{ACP} with respect to $\hat{a}$, setting it equal to zero and rearranging. Inserting the optimal term for $\hat{a}$ into the profit gives the presented terms. □

Proof of Proposition 3.2. We have

$$\begin{aligned}\Pi_{ACP}(\hat{a}) =& \sum_{i=1}^{n} \frac{\delta_i}{\overline{v_i} - \underline{v_i}}[(m_i - \hat{co}^*_i(\hat{a}))(\hat{co}^*_i(\hat{a}) - \underline{w_i}(\hat{a})) + (\overline{w_i}(\hat{a}) - \hat{co}^*_i(\hat{a})) \cdot (-c)] \\ =:& \sum_{i=1}^{n} \Pi^i_{ACP}(\hat{a})\end{aligned}$$

and

$$\Pi'_{ACP}(\hat{a}) = \sum_{i=1}^{n} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} [-2\hat{co}_i^*(\hat{a}) \cdot a_i^{*\prime}(\hat{a}) + a_i^{*\prime}(\hat{a}) \cdot (\underline{w_i}(\hat{a}) + m_i + c)$$
$$+\hat{co}_i^*(\hat{a}) \cdot \underline{w_i}'(\hat{a}) - \underline{w_i}'(\hat{a}) \cdot m_i - \overline{w_i}'(\hat{a}) \cdot c].$$

$\hat{a}^* \neq \underline{v_i}, v_i^*$ for all i locally maximizes Π_{ACP} if and only if $\Pi'_{ACP}(\hat{a}^*) = 0$ and $\Pi''_{ACP}(\hat{a}^*) > 0$. For any $\hat{a} \neq \underline{v_i}, v_i^*$ for all i, there are indices j, k, l, where $0 \leq j \leq k \leq l \leq n$ such that $\hat{a} > v_i^*$ for all $1 \leq i \leq j$, $co_i^* < \hat{a} < v_i^*$ for all $j < i \leq k$, $\underline{v_i} < \hat{a} \leq co_i^*$ for all $k < i \leq l$, and $\hat{a} < \underline{v_i}$ for all $l < i \leq n$. This is due to the fact that $\underline{v_i} \leq co_i^* \leq v_i^*$ for all i, and these values are increasing in their quality indices. Hence, the first and second derivative in $\hat{a}$ have the following structures:

$$\Pi'_{ACP}(\hat{a}) = \sum_{i=j+1}^{k} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} \cdot (-c) + \sum_{i=k+1}^{l} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} \cdot (-2\hat{a} + \underline{v_i} + m_i)$$

$$\Pi''_{ACP}(\hat{a}) = \sum_{i=k+1}^{l} -\frac{2 \cdot \delta_i}{\overline{v_i} - \underline{v_i}}$$

We immediately see that $l \neq k$ has to hold for a proper maximum. Setting the first derivative equal to zero and rearranging with respect to $\hat{a}$ leads to

$$\hat{a}^* = \frac{\sum_{i=j+1}^{k} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} \cdot (-c) + \sum_{i=k+1}^{l} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} \cdot (m_i + \underline{v_i})}{2 \sum_{i=k+1}^{l} \frac{\delta_i}{\overline{v_i} - \underline{v_i}}}$$

, which is the term stated in the proposition. Note that for $co_i^* \neq \underline{v_i}, v_i^*$, the left-sided and right-sided derivatives exist and coincide. Therefore, the analysis also holds for $\hat{a} = co_i^*$ if the conditions are satisfied.

Now, we want to investigate the $\hat{a}$ values for which Π_{ACP} is not differentiable, which are $\underline{v_i}$ and v_i^* for all i. If Π_{ACP} was maximized by such a value, it would have to hold that $\Pi'_{ACP}(x) > 0$ for x smaller and very close to that value and $\Pi'_{ACP}(y) < 0$ for y larger and very close to that value.

First, consider v_q^* for some quality class q. For $x < v_q^*$ but very close to v_q^*, we have

$$\Pi'_{ACP}(x) = \sum_{i=q}^{k} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} \cdot (-c) + \sum_{i=k+1}^{l} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} \cdot (-2x + \underline{v_i} + m_i)$$

For $y > v_q^*$ but very close to v_q^*, we have

$$\Pi'_{ACP}(y) = \sum_{i=q+1}^{k} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} \cdot (-c) + \sum_{i=k+1}^{l} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} \cdot (-2y + \underline{v_i} + m_i)$$

It is obvious that for x, y very close to v_q^*, we can never have $\Pi'_{ACP}(x) > 0 > \Pi'_{ACP}(y)$ due to $\lim_{x \uparrow v_q^*} \Pi'_{ACP}(x) = \lim_{y \downarrow v_q^*} \delta_q(-c)/(\overline{v_q} - \underline{v_q}) + \Pi'_{ACP}(y)$.

Now, consider $\underline{v_q}$ for some quality class q. For $x < \underline{v_q}$ but very close to $\underline{v_q}$, we have

$$\Pi'_{ACP}(x) = \sum_{i=j+1}^{k} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} \cdot (-c) + \sum_{i=k+1}^{q-1} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} \cdot (-2x + \underline{v_i} + m_i)$$

For $y > \underline{v_q}$ but very close to $\underline{v_q}$, we have

$$\Pi'_{ACP}(y) = \sum_{i=j+1}^{k} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} \cdot (-c) + \sum_{i=k+1}^{q} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} \cdot (-2y + \underline{v_i} + m_i)$$

It is obvious that for x, y very close to $\underline{v_q}$, we can never have $\Pi'_{ACP}(x) > 0 > \Pi'_{ACP}(y)$ due to $\lim_{x \uparrow \underline{v_q}} \Pi'_{ACP}(x) = \lim_{y \downarrow \underline{v_q}} \Pi'_{ACP}(y) - \delta_q(-2y + \underline{v_q} + m_q)/(\overline{v_q} - \underline{v_q})$.

The number of possible maxima only depends on the amount of values of the form v_i^*, $\underline{v_i}$ and co_i^*. When increasing $\hat{a}$ starting with $\hat{a} = 0$, we pass at most $3n$ such values. We know that there are no maxima for $\hat{a} \leq \underline{v_1}$ because there, the profit is constant and equal to zero. We know that there are no maxima for $\hat{a} \geq co_n^*$ because there, the profit is linearly decreasing in $\hat{a}$ or constant. Due to the structure of the locally optimal $\hat{a}$, we know that every time we pass such a value, we have a potential locally optimal candidate until we pass the next value given by the term for the optimal price. Because there are at most $3n - 2$ such intervals (note that $v_n^* \geq co_n^*$) with at most one local maximum each, this is the maximum number of local maxima that the profit function can have. □

Proof of Proposition 3.3. Choosing the price and bonuses with respect to conditions (i)-(iv) signals the product holders that if they state the true quality i of their products, they receive

$a + b_i$ with probability one. If they state a worse quality $< i$, they will get paid less anyway, and if they state a better quality $> i$, they will either not get paid the corresponding bonus with probability one or they are paid the same bonus as for stating the true quality i.

Condition (i) is natural because a negative bonus does not incentivize a product holder to make a true quality statement. Condition (ii) leads to $\mathbb{P}(v_i < a|j) = 0$ for all i, j because $\underline{v_1}$ is the smallest residual value. Condition (iii) signals to the product holder that he is paid the bonus for stating the true quality. For deciding about paying the bonus for a true quality statement, the firm has to compare $\Pi_{ABP}(\text{no}\, b_i|i, m_i)$ and $m_i - a - b_i$ in (II). Because of (ii), $\Pi_{ABP} = -c$ holds. The choice of b_i as a signal for the product holders has to satisfy $-c \leq m_i - a - b_i$ for all $m_i \in [\underline{m_i}, \overline{m_i}]$, which is captured by condition (iii). Condition (iv) makes sure that product holders either receive the signal that they do not get paid a bonus if they state a higher quality than the actual one with probability one or do not benefit from stating the next higher quality because the bonus is the same as for stating the true quality. The condition that $b_i \geq \overline{m_{i-1}} + c - a$ tells every product holder with quality $i - 1$ that $\Pi_{ABP}(\text{no}\, b_i|i, m_{i-1}) = -c \geq m_{i-1} - a - b_i$ for all $m_{i-1} \in [\underline{m_{i-1}}, \overline{m_{i-1}}]$. Due to infeasibility through $\overline{m_{i-1}} > \underline{m_i}$ and for the sake of more possible choices for bonuses, $b_i = b_{i-1}$ allows for not satisfying the former condition. This condition makes a product holder indifferent between stating $i - 1$ and i and therefore yields no incentive for stating the wrong quality. In particular, the former condition then does not have to be satisfied.

In the end, these conditions lead to a situation in which the expected payoff for stating j if the true quality is i is equal to $a + b_j$ for $j \leq i$ and $b_j \leq b_i$. If $j > i$ the expected payoff for stating j is $a + b_i$ if $b_i = b_j$ or a if $b_j > b_i$. Therefore, product holders will state the true quality if acquisition price and bonuses satisfy the conditions. □

Proof of Corollary 3.3. The proof is identical to the proof of Proposition 3.3, with the difference that m_i, $\overline{m_i}$, $\underline{m_i}$ and c are replaced by $m_i + \epsilon$, $\overline{m_i} + \epsilon$, $\underline{m_i} + \epsilon$ and $c - \epsilon$ and every payoff for the product holder after handing in is reduced by ϵ. □

Proof of Lemma 3.2. Consider problem $P2 - S$. For fixed $\hat{b} \neq \underline{v_l}, \overline{v_l}$ for all $l \in I$ with $\overline{m_{j-1}} + c \leq \hat{b} \leq \underline{m_j} + c$, we have indices $s, t \in I$, such that $j \leq s \leq t$ and $\hat{b} > \overline{v_l}$ for $j \leq l \leq s$, $\underline{v_l} < \hat{b} < \overline{v_l}$ for $s < l \leq t$ and $\hat{b} < \underline{v_l}$ for $t < l$. We have

$$\Pi^{I\prime}_{ABP}(\hat{b}) = \sum_{i=j}^{s} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} \cdot (-1) \cdot (\overline{v_i} - \underline{v_i}) + \sum_{i=s+1}^{t} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} \cdot ((-1) \cdot (\hat{b} - \underline{v_i}) + (m_i - \hat{b}))$$

$$\Pi^{I''}_{ABP}(\hat{b}) = \sum_{i=s+1}^{t} \frac{-2\delta_i}{\overline{v_i} - \underline{v_i}}$$

Due to the second derivative, $s \neq t$ has to hold for a proper maximum. Setting the first derivative equal to zero and rearranging with respect to $\hat{b}$ gives us the term for $\hat{b}^*$ in Lemma 3.2.

In $\hat{b} = \overline{v_p}$, for some $p \in I$, we have a maximum if the stated conditions in Lemma 3.2 hold.

Next, we consider the bounds for $\hat{b}$. If $\Pi^{I'}_{ABP}(\overline{m_{j-1}} + c) < 0$, or if $\Pi^{I'}_{ABP}(\underline{m_j} + c) > 0$, then Π^{I}_{ABP} is locally maximized by these bounds. Since Π_{ABP} is just the sum over these Π^{I}_{ABP} and these are locally maximized, then the same holds for the objective function.

Finally, we cannot have maxima in any $\underline{v_p}$ for all $p \in I$. To this end, consider

$$\lim_{x \uparrow \underline{v_p}} \Pi^{I'}_{ABP}(x) = \lim_{x \uparrow \underline{v_p}} \left(\sum_{i=j}^{s} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} \cdot (-1) \cdot (\overline{v_i} - \underline{v_i}) + \sum_{i=s+1}^{p-1} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} \cdot (-2x + \underline{v_i} + m_i) \right)$$

Furthermore, we have

$$\lim_{x \downarrow \underline{v_p}} \Pi^{I'}_{ABP}(x) = \lim_{x \downarrow \underline{v_p}} \left(\sum_{i=j}^{s} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} \cdot (-1) \cdot (\overline{v_i} - \underline{v_i}) + \sum_{i=s+1}^{p} \frac{\delta_i}{\overline{v_i} - \underline{v_i}} \cdot (-2x + \underline{v_i} + m_i) \right)$$

We always have $\lim_{x \uparrow \underline{v_p}} \Pi^{I'}_{ABP}(x) \leq \lim_{x \downarrow \underline{v_p}} \Pi^{I'}_{ABP}(x)$, but for a maximum, we would need $\lim_{x \uparrow \underline{v_p}} \Pi^{I'}_{ABP}(x) > 0 > \lim_{x \downarrow \underline{v_p}} \Pi^{I'}_{ABP}(x)$

Inserting the optimal bonuses into the objective function under consideration of s and t gives the presented profit. □

Proof of Proposition 3.4. This is an immediate consequence of Lemma 3.2, where instead of considering just one index set and one corresponding bonus, multiple are considered. Still, no additional complexities come into play. □

Proof of Corollary 3.5. Consider one quality class i. It is easily seen that $b_i^* = \min\{\overline{v_i}, (m_i + \underline{v_i})/2\}$ maximizes

$$\Pi^{i}_{ABP}(b_i) = \frac{\delta_i}{\overline{v_i} - \underline{v_i}} (m_i - b_i)(\min\{\max\{\underline{v_i}, b_i\}, \overline{v_i}\} - \underline{v_i})$$

Hence, choosing b_i^* as presented for all i solves the unconstrained problem $P2$ and therefore solves $P2$ if it satisfies the constraints. Then, for $b_i^* = \min\{\overline{v_i}, (m_i + \underline{v_i})/2\}$, we have

$$\Pi^i_{ABP}(b_i^*) = \begin{cases} \frac{\delta_i}{\overline{v_i}-\underline{v_i}}(\frac{m_i-\underline{v_i}}{2})^2 & if\ \frac{m_i+\underline{v_i}}{2} \leq \overline{v_i} \\ \delta_i(m_i - \overline{v_i}) & if\ \frac{m_i-\underline{v_i}}{2} \geq \overline{v_i} \end{cases}$$

For $P1$, we consider three cases: for $\hat{a} \geq v_i^*$, we have

$$\Pi^i_{ACP}(\hat{a}) = \begin{cases} \frac{\delta_i}{\overline{v_i}-\underline{v_i}}((\frac{m_i-\underline{v_i}}{2})^2 - \frac{c^2}{4} - c(v_i^* - \frac{m_i+\underline{v_i}+c}{2})) & if\ \frac{m_i-\underline{v_i}}{2} \leq \overline{v_i} \\ \delta_i(m_i - \overline{v_i}) & if\ \frac{m_i+\underline{v_i}}{2} \geq \overline{v_i} \end{cases}$$

For $co_i^* \leq \hat{a} < v_i^*$ (note that $co_i^* < v_i^*$ implies $co_i^* = (m_i + \underline{v_i} + c)/2$), we have

$$\Pi^i_{ACP}(\hat{a}) = \frac{\delta_i}{\overline{v_i} - \underline{v_i}}((\frac{m_i - \underline{v_i}}{2})^2 - \frac{c^2}{4} - c(\hat{a} - \frac{m_i + \underline{v_i} + c}{2}))$$

For $\underline{v_i} \leq \hat{a} \leq co_i^*$, we have

$$\Pi^i_{ACP}(\hat{a}) = \frac{\delta_i}{\overline{v_i} - \underline{v_i}}(m_i - \hat{a})(\hat{a} - \underline{v_i})$$

It always holds that $\Pi^i_{ABP} \geq \Pi^i_{ACP}$. This holds for every quality class and completes the proof. □

Proof of Proposition 3.5. First, we show that Π^*_{ACP} is equal to the three different terms under the indicated conditions by making use of Proposition 3.2. Then, we show that Π^*_{ABP} is equal to the four different terms by making use of Lemma 3.2.

(1) By Proposition 3.2, $k \neq l$ is necessary for a proper maximum. If $l = i$ for some $i \in \{1, ..., n\}$, then $k = j = i - 1$ has to hold due to the fact that $\overline{v_{i-1}} \leq \underline{v_i}$ for all $i \geq 2$ by assumptions (i)-(iv). Therefore, there are n local maxima: $\hat{a} = (m_i + \underline{v_i})/2$ for all $1 \leq i \leq n$. Due to assumption (iv), the global one can be found via a marginal analysis. The structure of the optimal profit depends on how co_r^* and v_r^* for $r < i$ look like. There are three different cases (see Proposition 3.1): $co_r^* = (m_r+\underline{v_r}+c)/2$ and $v_r^* = (\overline{m_r}+\underline{v_r}+c)/2$, $co_r^* = (m_r+\underline{v_r}+c)/2$ and $v_r^* = \overline{v_r}$, and $co_r^* = v_r^* = \overline{v_r}$.

(a) $(K + L + c)/2 \leq 2K \Leftrightarrow (\overline{m_i} + \underline{v_i} + c)/2 \leq \overline{v_i}$ for all i. Therefore, $\hat{co}_i^* = \min\{(m_i + \underline{v_i} + c)/2, \hat{a}\} = \min\{(i-1)R + (K + c)/2, \hat{a}\}$ and $v_i^* = \min\{(\overline{m_i} + \underline{v_i} + c)/2, \hat{a}\} = \min\{(i-1)R +$

$(K+Lc)/2, \hat{a}\}$. Due to the fact that $\hat{a} = (m_j + \underline{v_j})/2$ for some j, the profit is then given by

$$\Pi_{ACP}(\frac{m_j+\underline{v_j}}{2}) = [\sum_{i=1}^{j-1} \frac{\delta_i}{\overline{v_i}-\underline{v_i}}((m_i - \frac{m_i+\underline{v_i}+c}{2})(\frac{m_i+\underline{v_i}+c}{2} - \underline{v_i})$$
$$-c(\frac{\overline{m_i}+\underline{v_i}+c}{2} - \frac{m_i+\underline{v_i}+c}{2}))] + \frac{\delta_j}{\overline{v_j}-\underline{v_j}}(m_j - \frac{m_j+\underline{v_j}}{2})(\frac{m_j+\underline{v_j}}{2} - \underline{v_j})$$
$$=[\sum_{i=1}^{j-1} \frac{\delta_i}{2K}(\frac{(K-c)^2}{4} - c\frac{L}{2})] + \delta_j\frac{K}{8} = [\sum_{i=1}^{j-1} \delta_i(\frac{K}{8} - \frac{c(2L+c)}{8K})] + \delta_j\frac{K}{8}$$

Now, we compare profits $\Pi_{ACP}((m_j+\underline{v_j})/2) := \Pi_j$ and $\Pi_{ACP}((m_{j+1}+\underline{v_{j+1}})/2) := \Pi_{j+1}$ for an arbitrary but fixed $j \leq n-1$: $\Pi_j \geq \Pi_{j+1} \Leftrightarrow [\sum_{i=1}^{j-1}\delta_i(K/8-(c(2L+c))/(8K))] + \delta_j K/8 - [\sum_{i=1}^{j}\delta_i(K/8-(c(2L+c))/(8K))] - \delta_{j+1}K/8 \geq 0 \Leftrightarrow c(2L+c)/(K^2) \geq \delta_{j+1}/\delta_j$. Due to assumption (iv), δ_{j+1}/δ_j is decreasing in i. Therefore, for the optimal j, it has to hold that $\delta_j/\delta_{j-1} \geq c(2L+c)/(K^2) \geq \delta_{j+1}/\delta_j$.

(b) $(K+c)/2 \leq 2K \leq (K+L+c)/2 \Leftrightarrow (m_i+\underline{v_i}+c)/2 \leq \overline{v_i} \leq (\overline{m_i}+\underline{v_i}+c)/2$ for all i. Therefore, $\hat{co}_i^* = \min\{(m_i+\underline{v_i}+c)/2, \hat{a}\} = \min\{(i-1)R+(K+c)/2, \hat{a}\}$ and $v_i^* = \min\{\overline{v_i}, \hat{a}\} = \min\{(i-1)R+2K, \hat{a}\}$. Again, because $\hat{a}^* = (m_j+\underline{v_j})/2$ has to hold for one j, the profit is then given by

$$\Pi_{ACP}(\frac{m_j+\underline{v_j}}{2}) = [\sum_{i=1}^{j-1} \frac{\delta_i}{\overline{v_i}-\underline{v_i}}((m_i - \frac{m_i+\underline{v_i}+c}{2})(\frac{m_i+\underline{v_i}+c}{2} - \underline{v_i})$$
$$-c(\overline{v_i} - \frac{m_i+\underline{v_i}+c}{2}))] + \frac{\delta_j}{\overline{v_j}-\underline{v_j}}(m_j - \frac{m_j+\underline{v_j}}{2})(\frac{m_j+\underline{v_j}}{2} - \underline{v_j})$$
$$=[\sum_{i=1}^{j-1} \frac{\delta_i}{2K}(\frac{(K-c)^2}{4} - c(2K - \frac{K+c}{2})] + \delta_j\frac{K}{8} = [\sum_{i=1}^{j-1} \delta_i(\frac{K}{8} - \frac{c(6K-c)}{8K})] + \delta_j\frac{K}{8}$$

Comparing profits $\Pi_{ACP}((m_j+\underline{v_j})/2) := \Pi_j$ and $\Pi_{ACP}((m_{j+1}+\underline{v_{j+1}})/2) := \Pi_{j+1}$ for an arbitrary but fixed $j \leq n-1$ leads to $\Pi_j \geq \Pi_{j+1} \Leftrightarrow [\sum_{i=1}^{j-1}\delta_i(K/8-c(6K-c)/(8K))] + \delta_j K/8 - [\sum_{i=1}^{j}\delta_i(K/8-c(6K-c)/(8K))] - \delta_{j+1}K/8 \geq 0 \Leftrightarrow c(6K-c)/(K^2) \geq \delta_{j+1}/\delta_j$. Due to assumption (iv), δ_{j+1}/δ_j is decreasing in i. Therefore, for the optimal j, it has to hold that $\delta_j/\delta_{j-1} \geq c(6K-c)/(K^2) \geq \delta_{j+1}/\delta_j$.

(c) $2K \leq (K+c)/2 \Leftrightarrow \overline{v_i} \leq (m_i+\underline{v_i}+c)/2$ for all i. Therefore, $\hat{co}_i^* = v_i^* = \min\{\overline{v_i}, \hat{a}\} = \min\{(i-1)R+2K, \hat{a}\}$ holds for all i. Again, $\hat{a}^* = (m_j+\underline{v_j})/2$ has to hold for one j. The profit is then given by

$$\begin{aligned}\Pi_{ACP}(\frac{m_j+\underline{v_j}}{2}) =&[\sum_{i=1}^{j-1}\frac{\delta_i}{\overline{v_i}-\underline{v_i}}((m_i-\overline{v_i})(\overline{v_i}-\underline{v_i})-c(\overline{v_i}-\overline{v_i}))]\\ &+\frac{\delta_j}{\overline{v_j}-\underline{v_j}}(m_j-\frac{m_j+\underline{v_j}}{2})(\frac{m_j+\underline{v_j}}{2}-\underline{v_j})\\ =&[\sum_{i=1}^{j-1}\frac{\delta_i}{2K}(K-2K)(2K)]+\delta_j\frac{K}{8}=[\sum_{i=1}^{j-1}\delta_i(-K)]+\delta_j\frac{K}{8}\end{aligned}$$

Comparing profits $\Pi_{ACP}((m_j+\underline{v_j})/2) := \Pi_j$ and $\Pi_{ACP}((m_{j+1}+\underline{v_{j+1}})/2) := \Pi_{j+1}$ for an arbitrary but fixed $j \leq n-1$ leads to $\Pi_j \geq \Pi_{j+1} \Leftrightarrow [\sum_{i=1}^{j-1}\delta_i(-K)]+\delta_j K/8-[\sum_{i=1}^{j}\delta_i(-K)]-\delta_{j+1}K/8 \geq 0 \Leftrightarrow 9 \geq \delta_{j+1}/\delta_j$. Therefore, for the optimal j, it has to hold that $\delta_j/\delta_{j-1} \geq 9 \geq \delta_{j+1}/\delta_j$.

(2) Note that assumptions (i)-(iv) ensure that $\overline{m_{i-1}}+c \leq \underline{m_i}+c$, $\underline{m_{i-1}}+c \leq \underline{v_i}$ and $\overline{v_{i-1}} \leq \underline{v_i}$ for all $i \geq 2$. This has several implications for the bonuses solving $P2$. If $\overline{m_{i-1}}+c \leq b_i \leq \underline{m_i}+c$ induces $\underline{v_i} \leq b_i \leq m_i$, then $y_i = 0$ for all $i \geq 2$ in $P2$ because forcing $b_i = b_{i-1}$ cannot generate profit in quality class i due to $b_i \leq \underline{m_{i-1}}+c \leq \underline{v_i}$ for all $i \geq 2$. If it is the case that b_i has to be chosen larger than m_i due to $\overline{m_{i-1}}+c \geq m_i$ for all $i \geq 2$, then $y_i = 1$ for all $i \geq 2$ is optimal, preventing losses in every quality class $i \geq 2$. Here, $b_1 = b_2 = ... = b_n \leq \underline{m_1}+c$ has to hold. The solution for the unconstrained $P2$ is $(m_i+\underline{v_i})/2$ for all $1 \leq i \leq n$ under assumptions (i)-(iv). Hence, how this term relates to $\overline{m_{i-1}}+c$, and $\underline{m_i}+c$ determines the optimal b_i.

(a) $R+K \leq K+L+c \Leftrightarrow \overline{m_{i-1}}+c \geq m_i\ \forall i \geq 2$. Therefore, as mentioned above, $y_i = 0$ for all $i \geq 2$ is optimal, forcing $b_1 = ... = b_n = (m_1+\underline{v_1})/2 = K/2$. Note that this is feasible due to $R+K \leq K+L+c \Rightarrow c \geq L \Rightarrow c+\frac{K}{2} \geq L \Leftrightarrow (m_i+\underline{v_i})/2 \leq \underline{m_i}+c$ with regard to constraint (3.7). Inserting into the objective function gives

$$\Pi_{ABP}(\frac{K}{2},...,\frac{K}{2}) = \frac{\delta_1}{\overline{v_1}-\underline{v_1}}\cdot(m_1-\frac{K}{2})\cdot(\frac{K}{2}-\underline{v_1}) = \frac{\delta_1}{2K}\cdot(K-\frac{K}{2})\cdot(\frac{K}{2}-0) = \delta_1\frac{K}{8}$$

(b) $R+K/2 \leq K+L+c \leq R+K \Leftrightarrow (m_i+\underline{v_i})/2 \leq \overline{m_{i-1}}+c \leq m_i$ for all $i \geq 2$ due to the assumptions. Therefore, by the previous discussion $y_i = 1$ for all $i \geq 2$ and $b_i = \overline{m_{i-1}}+c = (i-2)R+K+L+c$ for $i \geq 2$ and $b_1 = (m_1+\underline{v_1})/2 = K/2$ is optimal. The profit is

$$\begin{aligned}\Pi^*_{ABP} =&\frac{\delta_1}{2K}(k-\frac{K}{2})(\frac{K}{2}-0)\\&+\sum_{i=2}^{n}\frac{\delta_i((i-1)R+K-((i-2)R+K+L+c))((i-2)R+K+L+c-(i-1)R)}{2K}\\=&(1-\delta_1)\frac{(R-L-c)(K-(R-L-c))}{2K}+\delta_1\frac{K}{8}\\=&\frac{K}{8}-\frac{1}{8K}(1-\delta_1)(K-2(R-L-c))^2\end{aligned}$$

(c) $K+L+c \le R+K/2 \le R+K-L+c \Leftrightarrow \overline{m_{i-1}}+c \le (m_i+\underline{v_i})/2 \le \underline{m_i}+c\ \forall i \ge 2$. Therefore, the optimal solution for the unconstrained bonuses optimization is the optimal solution for $P2$. That means that $b_i = (m_i+\underline{v_i})/2 = (i-1)R+K/2$ for all $i \in \{1,...,n\}$ is optimal. Inserting into the objective function leads to

$$\begin{aligned}\Pi^*_{ABP} =&\sum_{i=1}^{n}\frac{\delta_i}{2K}((i-1)R+K-((i-1)R+\frac{K}{2}))((i-1)R+\frac{K}{2}-(i-1)R)\\=&\frac{1}{2K}\frac{K^2}{4}=\frac{K}{8}\end{aligned}$$

(d) $R+K-L+c \le R+K/2 \Leftrightarrow \underline{m_i}+c \le (m_i+\underline{v_i})/2\ \forall i$. That implies that $b_i = \underline{m_i}+c = (i-1)R+K-L+c$ for all i solves $P2$. The resulting profit is

$$\begin{aligned}\Pi^*_{ABP} =&\sum_{i=1}^{n}\frac{\delta_i((i-1)R+K-((i-1)R+K-L+c))((i-1)R+K-L+c-(i-1)R)}{2K}\\=&\frac{K}{8}-\frac{1}{8K}(2(L-c)-K)^2\end{aligned}$$

□

Proof of Lemma 3.3. (i) $L \le c+K/2 \le R-L \Leftrightarrow K+L+c \le R+K/2 \le R+K-L+c$ ensures that $\Pi^*_{ABP} = K/8$. This term is larger than every possible optimal value for Π^*_{ACP} in Proposition 3.5.

(ii) $L+c \ge 3K$ is equivalent to being in the second or third cases for Π^*_{ACP} in Proposition 3.5. Therefore, $c \ge 2K$ has to hold since $L \le K$. For the second case, $(K+c)/2 \le 2K \Leftrightarrow c \le 3K$ has to hold. Therefore, $c(6K-c)/(K^2) \ge 2K(6K-3K)/(K^2) = 6$ with regard to the second case. Due to the condition $\delta_{i+1}/\delta_i \le 6$ in Lemma 3.3 (ii), the optimal $\hat{a}^* = (m_j+\underline{v_j})/2 = (m_1+\underline{v_1})/2 = K/2$. Therefore, $\Pi^*_{ACP} = \delta_1 K/8$. If $c \ge 3K$, the third case for Π^*_{ACP} from Proposition 3.5 applies. In this case, since $9 > 6$, again, $\hat{a}^* = (m_j + \underline{v_j})/2 = (m_1 + \underline{v_1})/2 = K/2$ is optimal, and the optimal profit is equal to $\delta_1 K/8$. Comparing this term to the optimal profits Π^*_{ABP} shows that Π^*_{ABP} can never be smaller than $\Pi^*_{ACP} = \delta_1 K/8$.

(iii) $L + c \geq R$ implies that the first case of Π^*_{ABP} in Proposition 3.5 applies. Comparing to Π^*_{ACP} in Proposition 3.5 shows that Π^*_{ACP} can never be smaller than Π^*_{ABP} in this case.

(iv) $c \leq K/16$ and $L \geq 3K/4$ imply $L \geq K/2 + c \Leftrightarrow R + K - L + c \leq R + K/2$, meaning for Π^*_{ABP}, the fourth case in Proposition 3.5 applies. Furthermore, $c \leq K/16$ induces that the first case for Π^*_{ACP} applies. With regard to Π^*_{ACP}, it holds that $c(2L + c)/2 \leq (K/16 \cdot (2K + K/16))/(K^2) = 33/256 \leq 1/7$. Because $\delta_{i+1}/\delta_i \geq 1/7$ by assumption, the optimal j for $\hat{a}^* = (m_j + \underline{v_j})/2$ is equal to n. That leads to $\Pi^*_{ACP} = K/8 - (1 - \delta_n)c(2L + c)/(8K)$. If a lower bound for Π^*_{ACP} that is larger than a upper bound for Π^*_{ABP} is found, then the proof is completed. Due to the fact that $0 \leq c \leq K/16$ and $3K/4 \leq L \leq K$, we have

$$\begin{aligned}
\Pi^*_{ACP} =& \frac{K}{8} - (1 - \delta_n)\frac{c(2L + c)}{8K} \\
\geq& \frac{K}{8} - (1 - \delta_n)\frac{K/16 \cdot (2K + K/16)}{8K} = \frac{K}{8}(\frac{256 - 33 + 33\delta_n}{256}) = \frac{K}{8}(\frac{223 + 33\delta_n}{256}) \\
\geq& \frac{K}{8}\frac{223}{256}
\end{aligned}$$

and

$$\Pi^*_{ABP} = \frac{1}{8K}(L - c)(K - L + c) \leq K(K - \frac{12K}{16} + \frac{k}{16}) = \frac{K}{8}\frac{5}{16}$$

Since $223/256 \geq 5/16$, the proof is completed.

(v) It can easily be seen that the different cases for Π^*_{ABP} and Π^*_{ACP} are either not dependent on L or decreasing in L for $0 \leq L \leq K$. The same holds for the relation between Π^*_{ACP} and c. □

Appendix C

Proofs of Chapter IV

Proof of Proposition 4.1. (i) We write $\mathfrak{F}_\Theta^0$ instead of $\mathfrak{F}_\Theta(p_0)$. Problem N dependent on k having price p_0 fixed is as follows:

$$N: \quad \max_k \ \Pi(k|p = p_0)|_{p_0} = \beta\mathfrak{F}_G^0(1 - k_G) + (1 - \beta)\mathfrak{F}_B^0(m - k_B) \tag{C.1}$$

$$\text{s.t.} \ \beta\mathfrak{F}_G^0(k_G - \delta p_0) + (1 - \beta)\mathfrak{F}_B^0(k_B - \delta p_0) \geq 0 \tag{C.2}$$

$$\beta\mathfrak{F}_G^0(k_G - \delta p_0) + (1 - \beta)\mathfrak{F}_B^0(k_B - \delta p_0) \geq \pi_0 \tag{C.3}$$

It is easily seen that the stated conditions for k^N are optimality conditions (binds constraint (C.3), which is parallel to the objective). The insertion of such a contract into the firm's profit function (which builds the basis for (iii) and proves (ii)) results in

$$\Pi(k^N, p_0) = \beta\mathfrak{F}_G^0(1 - \delta p_0) + (1 - \beta)\mathfrak{F}_B^0(m - \delta p_0) - \pi_0. \tag{C.4}$$

(iii) For Π and p, instead of $\Pi_N(k^N, p_0)$ and p_0, the first-order condition for p^N, which is necessary and sufficient for optimality due to Assumption 4.2, is

$$\partial\Pi/\partial p|_{p^N} = \beta[\mathfrak{f}_G(p^N)(1 - \delta p^N) - \delta\mathfrak{F}_G(p^N)] + (1 - \beta)[\mathfrak{f}_B(p^N)(m - \delta p^N) - \delta\mathfrak{F}_B(p^N)] = 0. \tag{C.5}$$

(C.5) is only necessary for optimality without Assumption 4.2. Note that interior solutions ($0 < p^N < 1/\delta$) are guaranteed since $\partial\Pi/\partial p|_{p=0} > 0$ and $\partial\Pi/\partial p|_{p>1/\delta} < 0$. □

Proof of Corollary 4.1. Uniqueness is easily derived from the concavity of $\Pi(k^N, p_0)$ due to Assumption 4.2. We use Π and p instead of $\Pi(k^N, p_0)$ and p_0.

In order to determine the sensitivities, by the implicit function theorem, we use the identity $\partial p/\partial x = -(\partial^2\Pi/(\partial p\partial x))/(\partial^2\Pi/\partial p^2)$ and the fact that $\partial^2\Pi/\partial p^2 < 0$ in optimum.

By (C.5), we have $\partial^2\Pi/(\partial p\partial\delta) \leq 0$, $\partial^2\Pi/(\partial p\partial m) \geq 0$, and $\partial^2\Pi/(\partial p\partial q) = 0$, resulting in $\partial p/\partial\delta \leq 0$, $\partial p/\partial m \geq 0$, and $\partial p/\partial q = 0$, the stated sensitivities.

For $\partial p/\partial\beta$, we employ the following observation: the monotone probability ratio order (Assumption 4.1) implies $\mathfrak{f}_G(p)/\mathfrak{F}_G(p) \geq \mathfrak{f}_B(p)/\mathfrak{F}_B(p)$ for all $p \in [0, \tau]$ (see, e.g., Hopkins et al. 2003). This implies $(\mathfrak{f}_G(p)/\mathfrak{F}_G(p))(1-\delta p) - \delta \geq (\mathfrak{f}_B(p)/\mathfrak{F}_B(p))(m-\delta p) - \delta$ for $p \leq 1/\delta$. Therefore, with regard to (C.5), it has to hold that $(\mathfrak{f}_G(p^N)/\mathfrak{F}_G(p^N))(1-\delta p^N) - \delta > 0 > (\mathfrak{f}_B(p^N)/\mathfrak{F}_B(p^N))(m-\delta p^N) - \delta$, which gives us the stated bounds for p^N.

Finally, using this information together with $\partial^2\Pi/(\partial p\partial\beta)|_{p^N} = [\mathfrak{f}_G(p^N)(1-\delta p^N) - \delta\mathfrak{F}_G(p^N)] - [\mathfrak{f}_B(p^N)(m-\delta p^N) - \delta\mathfrak{F}_B(p^N)]$ results in $\partial^2\Pi/(\partial p\partial\beta)|_{p^N} \geq 0$, which proves $\partial p/\partial\beta \geq 0$. □

Derivation of Problem T. For simplicity, we write $\mathfrak{F}^s_\Theta$ instead of $\mathfrak{F}_\Theta(p_s)$. The concrete optimization problem T for general e and e' is

$$
\begin{aligned}
T:\ \max_{k,p_g,p_b}\ &\Pi(k, p_g, p_b|e, p = p_s) \\
&= \beta\mathfrak{F}^g_G(1-k_{G,g}) + (1-\beta)((1-q_e)\mathfrak{F}^g_B(m-k_{B,g}) + q_e\mathfrak{F}^b_B(m-k_{B,b}))
\end{aligned}
\tag{C.6}
$$

$$
\begin{aligned}
\text{s.t. }\ &\beta\mathfrak{F}^g_G(k_{G,g}-\delta p_g) + (1-\beta)(1-q_e)\mathfrak{F}^g_B(k_{B,g}-\delta p_g) \\
&-(\beta\mathfrak{F}^b_G(k_{G,b}-\delta p_b) + (1-\beta)(1-q_e)\mathfrak{F}^b_B(k_{B,b}-\delta p_b)) \geq 0
\end{aligned}
\tag{C.7}
$$

$$
\beta\mathfrak{F}^g_G(k_{G,g}-\delta p_g) + (1-\beta)(1-q_e)\mathfrak{F}^g_B(k_{B,g}-\delta p_g) \geq 0 \tag{C.8}
$$

$$
\mathfrak{F}^b_B(k_{B,b}-\delta p_b) - \mathfrak{F}^g_B(k_{B,g}-\delta p_g) \geq 0 \tag{C.9}
$$

$$
\mathfrak{F}^b_B(k_{B,b}-\delta p_b) \geq 0 \tag{C.10}
$$

$$
(q_e - q_{e'})(\mathfrak{F}^b_B(k_{B,b}-\delta p_b) - \mathfrak{F}^g_B(k_{B,g}-\delta p_g)) \geq (c_e - c_{e'})/(1-\beta) \tag{C.11}
$$

$$
\begin{aligned}
&\beta\mathfrak{F}^g_G(k_{G,g}-\delta p_g) + (1-\beta)(1-q_e)\mathfrak{F}^g_B(k_{B,g}-\delta p_g) \\
&+(1-\beta)q_e\mathfrak{F}^b_B(k_{B,b}-\delta p_b) - (\pi_0 + c_e) \geq 0
\end{aligned}
\tag{C.12}
$$

The reasoning is as follows: the firm's expected profit if the retailer participates, makes offers corresponding to testing outcomes, and exerts effort e is $\Pi = \mathbb{P}(\Theta = G)(\mathbb{P}(s = g|\Theta = G \ \&\ e)\mathfrak{F}^g_G(1-k_{G,g}) + \mathbb{P}(s = b|\Theta = G \ \&\ e)\mathfrak{F}^b_G(1-k_{G,b})) + \mathbb{P}(\Theta = B)(\mathbb{P}(s = g|\Theta = B \ \&\ e)\mathfrak{F}^g_B(m-k_{B,g}) + \mathbb{P}(s = b|\Theta = B \ \&\ e)\mathfrak{F}^b_B(m-k_{B,b})) = \beta\mathfrak{F}^g_G(1-k_{G,g}) + (1-\beta)((1-q_e)\mathfrak{F}^g_B(m-k_{B,g}) + q_e\mathfrak{F}^b_B(m-k_{B,b}))$. To make the retailer participate, the firm has to ensure that the retailer's payoff, π, is at least π_0, which is obviously captured by constraint (C.12) if the retailer exerts effort e and makes truthful offers. Now, we turn to the incentive-compatibility constraints (C.7)-(C.10), which ensure the retailer's adherence to testing outcomes. If the retailer receives a good testing outcome and exerted effort e, the retailer's payoffs for offering p_g, p_b, and 0 are $\pi(p = p_g|s = g \,\&\, e) = (\beta\mathfrak{F}_G^g(k_{G,g} - \delta p_g) + (1-\beta)(1-q_e)\mathfrak{F}_B^g(k_{B,g} - \delta p_g))/(\beta + (1-\beta)(1-q_e))$, $\pi(p = p_b|s = g \,\&\, e) = (\beta\mathfrak{F}_G^b(k_{G,b} - \delta p_b) + (1-\beta)(1-q_e)\mathfrak{F}_B^b(k_{B,b} - \delta p_b))/(\beta + (1-\beta)(1-q_e))$, and $\pi(p = 0|s = g \,\&\, e) = 0$, respectively. To ensure $p = p_g$ if $s = g$, it has to hold that $\pi(p = p_g|s = g \,\&\, e) \geq \max\{\pi(p = p_b|s = g \,\&\, e), \pi(p = 0|s = g \,\&\, e)\}$, which is captured by constraints (C.7) and (C.8). Correspondingly, if there is a bad testing outcome, the retailer's payoffs for offering p_g, p_b, and 0 are $\pi(p = p_g|s = b \,\&\, e) = \mathfrak{F}_B^g(k_{B,g} - \delta p_g)$, $\pi(p = p_b|s = b \,\&\, e) = \mathfrak{F}_B^b(k_{B,b} - \delta p_b)$, and $\pi(p = 0|s = b \,\&\, e) = 0$, respectively. To ensure $p = p_b$, it has to hold that $\pi(p = p_b|s = b \,\&\, e) \geq \max\{\pi(p = p_g|s = b \,\&\, e), \pi(p = 0|s = b \,\&\, e)\}$, which is ensured by constraints (C.9) and (C.10). Finally, the exertion of testing effort e is more profitable than exertion of effort e' if, afterward, the retailer adheres with offers to testing outcomes if $\pi(e, p = p_s) \geq \pi(e', p = p_s)$. It holds that $\pi(e, p = p_s) = \beta\mathfrak{F}_G^g(k_{G,g} - \delta p_g) + (1-\beta)((1-q_e)\mathfrak{F}_B^g(k_{B,g} - \delta p_g) + q_e\mathfrak{F}_B^b(k_{B,b} - \delta p_b)) - c_e$. Correspondingly, we have $\pi(e', p = p_s)$ and hence constraint (C.11). □

Proof of Proposition 4.2. (i) Consider the concrete problem T under $e = h$ and $e' = l$. Let $r_{\Theta,s} \equiv \mathfrak{F}_\Theta^s(k_{\Theta,s} - \delta p_s)$, $\rho_{B,g} \equiv (1-\beta)(1-q)$, $\rho_{B,b} \equiv (1-\beta)q$, and $C \equiv 2(c_h - c_l)/((1-\beta)(2q-1))$. For fixed p_g and p_b, problem T is then

$$\max_k \; \Pi(k|e = h, p = p_s)|_{(p_g,p_b)} = \beta\mathfrak{F}_G^g(1 - k_{G,g}) + \rho_{B,g}\mathfrak{F}_B^g(m - k_{B,g}) + \rho_{B,b}\mathfrak{F}_B^b(m - k_{B,b}) \tag{C.13}$$

$$\text{s.t. } \beta r_{G,g} + \rho_{B,g} r_{B,g} - (\beta r_{G,b} + \rho_{B,g} r_{B,b}) \geq 0 \tag{C.14}$$

$$\beta r_{G,g} + \rho_{B,g} r_{B,g} \geq 0 \tag{C.15}$$

$$r_{B,b} - r_{B,g} \geq 0 \tag{C.16}$$

$$r_{B,b} \geq 0 \tag{C.17}$$

$$r_{B,b} - r_{B,g} - C \geq 0 \tag{C.18}$$

$$\beta r_{G,g} + \rho_{B,g} r_{B,g} + \rho_{B,b} r_{B,b} - (\pi_0 + c_h) \geq 0 \tag{C.19}$$

First, note that constraint (C.19) is parallel to the firm's profit. Hence, if it is binding, the contract is optimal.

Assume that (C.19) is binding. This leads to $r_{G,g} = (\pi_0 + c_h - (\rho_{B,g} r_{B,g} + \rho_{B,b} r_{B,b}))/\beta$. This, taken together with constraints (C.15) and (C.17), leads to $r_{B,b}$'s characterization. (C.19) being binding, together with (C.14), leads to $r_{G,b}$'s characterization. (C.18) gives $r_{B,g}$'s characterization. Finally, (C.16) is dominated by (C.18), which is why it is not considered. The conditions can always be satisfied. Hence, (C.19) is indeed binding in optimum.

The insertion of an optimal contract into the firm's profit (which builds the basis for (iii) and proves (ii)) gives

$$\Pi(k^H, p_g, p_b) = \beta \mathfrak{F}_G^g (1 - \delta p_g) + \rho_{B,g} \mathfrak{F}_B^g (m - \delta p_g) + \rho_{B,b} \mathfrak{F}_B^b (m - \delta p_b) - c_h - \pi_0. \quad \text{(C.20)}$$

(iii) For Π instead of $\Pi(k^H, p_g, p_b)$, the first-order conditions for p_g^H and p_b^H, which are necessary and sufficient for optimality due to Assumption 4.2, are the following:

$$\partial\Pi/\partial p_g|_{p_g^H} = \beta[\mathfrak{f}_G(p_g^H)(1 - \delta p_g^H) - \delta \mathfrak{F}_G(p_g^H)] + \rho_{B,g}[\mathfrak{f}_B(p_g^H)(m - \delta p_g^H) - \delta \mathfrak{F}_B(p_g^H)] = 0 \quad \text{(C.21)}$$

$$\partial\Pi/\partial p_b|_{p_b^H} = (1 - \beta) q [\mathfrak{f}_B(p_b^H)(m - \delta p_b^H) - \delta \mathfrak{F}_B(p_b^H)] = 0 \quad \text{(C.22)}$$

The condition is only necessary for optimality without Assumption 4.2. Note that interior solutions are guaranteed due to $\partial\Pi/\partial p|_{p=0} > 0$ and $\partial\Pi/\partial p|_{p>1/\delta} < 0$ by (C.21) and (C.22). □

Proof of Corollary 4.2. Uniqueness is easily derived from the concavity of $\Pi(k^H, p_g, p_b)$. We use Π instead of $\Pi(k^H, p_g, p_b)$.

For sensitivities, we again use the identity $\partial p/\partial x = -(\partial^2\Pi/(\partial p \partial x))/(\partial^2\Pi/\partial p^2)$ and the fact that $\partial^2\Pi/\partial p^2 < 0$ in optimum. This holds for both p_g and p_b.

By (C.22), we have $\partial^2\Pi/(\partial p_b \partial\beta) = \partial^2\Pi/(\partial p_b \partial q) = 0$, resulting in $\partial p_b/\partial\beta = \partial p_b/\partial q = 0$, $\partial^2\Pi/(\partial p_b \partial m) \geq 0$, resulting in $\partial p_b/\partial m \geq 0$, and $\partial^2\Pi/(\partial p_b \partial\delta) \leq 0$, resulting in $\partial p_b/\partial\delta \leq 0$.

By the same reasoning, (C.21) results in $\partial p_g/\partial\delta \leq 0$ and $\partial p_g/\partial m \geq 0$. For $\partial p_g/\partial\beta$ and $\partial p_g/\partial q$, we again use the monotone probability ratio order (Assumption 4.1): $\mathfrak{f}_G(p)/\mathfrak{F}_G(p) \geq \mathfrak{f}_B(p)/\mathfrak{F}_B(p)$ for all $p \in [0, \tau]$. This implies $(\mathfrak{f}_G(p)/\mathfrak{F}_G(p))(1 - \delta p) - \delta \geq (\mathfrak{f}_B(p)/\mathfrak{F}_B(p))(m - \delta p) - \delta$ for $p \leq 1/\delta$. Therefore, with regard to (C.21), in optimum, it has to hold that $(\mathfrak{f}_G(p_g^H)/\mathfrak{F}_G(p_g^H))(1 - \delta p_g^H) - \delta > 0 > (\mathfrak{f}_B(p_g^H)/\mathfrak{F}_B(p_g^H))(m - \delta p_g^H) - \delta$, which again gives the stated bounds for p_g^H.

Finally, this information, together with $\partial^2\Pi/(\partial p_g \partial\beta)|_{p_g^H} = [\mathfrak{f}_G(p_g^H)(1 - \delta p_g^H) - \delta \mathfrak{F}_G(p_g^H)] - (1 - q)[\mathfrak{f}_B(p_g^H)(m - \delta p_g^H) - \delta \mathfrak{F}_B(p_g^H)]$ and $\partial^2\Pi/(\partial p_g \partial q)|_{p_g^H} = -(1 - \beta)[\mathfrak{f}_B(p_g^H)(m - \delta p_g^H) -$

$\delta\mathfrak{F}_B(p_g^H)]$, results in $\partial^2\Pi/(\partial p_g\partial\beta)|_{p_g^H}, \partial^2\Pi/(\partial p_g\partial q)|_{p_g^H} \geq 0$, which proves the remaining sensitivities. □

Proof of Proposition 4.3. (i) We consider problem T given fixed prices and $e = l$. We again use the notation $r_{\Theta,s} \equiv \mathfrak{F}_\Theta^s(k_{\Theta,s} - \delta p_s)$ and $C = 2(c_h - c_l)/((1-\beta)(2q-1))$. Problem T is then

$$\begin{aligned}
&\max_k \ \Pi(k|e = l, p = p_s)|_{(p_g,p_b)} \\
&\qquad = \beta\mathfrak{F}_G^g(1 - k_{G,g}) + (1-\beta)(\mathfrak{F}_B^g(m - k_{B,g}) + \mathfrak{F}_B^b(m - k_{B,b}))/2
\end{aligned} \tag{C.23}$$

$$\text{s.t.} \ \beta r_{G,g} + (1-\beta)r_{B,g}/2 - (\beta r_{G,b} + (1-\beta)r_{B,b}/2) \geq 0 \tag{C.24}$$

$$\beta r_{G,g} + (1-\beta)r_{B,g}/2 \geq 0 \tag{C.25}$$

$$r_{B,b} - r_{B,g} \geq 0 \tag{C.26}$$

$$r_{B,b} \geq 0 \tag{C.27}$$

$$r_{B,b} - r_{B,g} - C \leq 0 \tag{C.28}$$

$$\beta r_{G,g} + (1-\beta)(r_{B,g} + r_{B,b})/2 - (\pi_0 + c_l) \geq 0 \tag{C.29}$$

Again, constraint (C.29) is parallel to the firm's profit. Hence, if it is binding, the contract is optimal.

We assume that (C.29) is binding, which leads to $r_{G,g} = (\pi_0 + c_l - (1-\beta)(r_{B,g} + r_{B,b})/2)/\beta$. This, taken together with constraints (C.25) and (C.27), leads to $r_{B,b}$'s characterization. (C.29) being binding, together with (C.24), leads to $r_{G,b}$'s characterization. (C.26) and (C.28) leads to $r_{B,g}$'s characterization. The conditions can always be satisfied. Hence, (C.29) is indeed binding in optimum.

The insertion of an optimal contract into the firm's profit (which builds the basis for (iii) and proves (ii)) gives

$$\Pi(k^L, p_g, p_b) = \beta\mathfrak{F}_G^g(1 - \delta p_g) + (1-\beta)(\mathfrak{F}_B^g(m - \delta p_g^L) + \mathfrak{F}_B^b(m - \delta p_b))/2 - c_l - \pi_0. \tag{C.30}$$

(iii) Here, the same reasoning applies as for policy H with $q = 1/2$. □

Proof of Corollary 4.3. This corollary easily follows from Corollary 4.2 and the fact that $p_g^L = p_g^H$ for $q = 1/2$ and $p_b^H = p_b^L$. □

Proof of Proposition 4.4. It is easily seen that $\gamma_1 = \Pi^H + c_h - (\Pi^L + c_l)$, $\gamma_2 = \Pi^H + c_h - \Pi^N$, $\gamma_3 = \Pi^L + c_l - \Pi^N$ by the optimal profits in Propositions 4.1-4.3.

(i) Independence of c_h, c_l, π_0 is clear by definition. It is easily seen that $\gamma_1, \gamma_2, \gamma_3 \geq 0$ by noting that p_g^H maximizes $\beta\mathfrak{F}_G(p)(1-\delta p) + (1-\beta)(1-q)\mathfrak{F}_B(p)(m-\delta p)$, $p_b^H = p_b^L$ maximizes $\mathfrak{F}_B(p)(m-\delta p)$, and p_g^L maximizes $\beta\mathfrak{F}_G(p)(1-\delta p) + (1-\beta)\mathfrak{F}_B(p)(m-\delta p)/2$.

(ii) It is easily seen that Π^H increases in q, whereas Π^N and Π^L are constant in q. Therefore, we have the stated sensitivities (see also the proof of Proposition 4.5).

(iii) Immediately follows from the initially stated identities.

(iv) Immediately follows from (i) and (iii).

(v) If $m = 1$ and $\mathfrak{F}_B = \mathfrak{F}_G$, then $p_g^H = p_g^L = p_0^N = p_b^H = p_b^L = m/\delta + \mathfrak{F}_B(p_b^L)/\mathfrak{f}_B(p_b^L)$ by Propositions 4.1-4.3. Furthermore, $\Pi^N = \mathfrak{F}_G(p_b^L)(1-\delta p_b^L) - \pi_0 = \Pi^H + c_h = \Pi^L + c_l$. The same reasoning applies for $\beta = 0$.

If $\beta = 1$, then $p_g^H = p_g^L = p_0^N = 1/\delta - \mathfrak{F}_G(p_0^N)/\mathfrak{f}_G(p_0^N)$ by Propositions 4.1-4.3. Additionally, $\Pi^N = \mathfrak{F}_G(p_0^N)(1-\delta p_0^N) - \pi_0 = \Pi^H + c_h = \Pi^L + c_l$.

If $\delta = 0$, then $p_g^H = p_g^L = p_0^N = p_b^H = p_b^L = \infty$. This results in $\Pi^N, \Pi^H + c_h, \Pi^L + c_l = \beta + (1-\beta)m - \pi_0$. □

Proof of Proposition 4.5. We use $\rho_{B,g} \equiv (1-\beta)(1-q)$, $\rho_{B,b} \equiv (1-\beta)q$. Adding π_0, which is the retailer's profit in optimum, to the firm's optimal profit results in

$$\Pi_D^N = \Pi^N + \pi_0 = \beta\mathfrak{F}_G^0(1-\delta p_0^N) + (1-\beta)\mathfrak{F}_B^0(m-\delta p_0^N) \tag{C.31}$$

$$\Pi_D^H = \Pi^H + \pi_0 = \beta\mathfrak{F}_G^g(1-\delta p_g^H) + \rho_{B,g}\mathfrak{F}_B^g(m-\delta p_g^H) + \rho_{B,b}\mathfrak{F}_B^b(m-\delta p_b^H) - c_h \tag{C.32}$$

$$\Pi_D^L = \Pi^L + \pi_0 = \beta\mathfrak{F}_G^g(1-\delta p_g^L) + (1-\beta)(\mathfrak{F}_B^g(m-\delta p_g^L) + \mathfrak{F}_B^b(m-\delta p_b^L))/2 - c_l \tag{C.33}$$

It is easily seen that these terms have to be the optimal profit-terms in a centralized scenario, depending on the chosen policy, N, H, or L.

To show the sensitivities, it suffices to show that $\partial\Pi^H/(\partial m)$, $\partial\Pi^L/(\partial m)$, $\partial\Pi^N/(\partial m) \geq 0$, $\partial\Pi^H/(\partial\delta)$, $\partial\Pi^L/(\partial\delta)$, $\partial\Pi^N/(\partial\delta) \leq 0$, and $\partial\Pi^H/(\partial q) \geq 0$.

For m and by using $\partial\Pi(p_g, p_b, k^H)/(\partial p_g)|_{p_g^H} = \partial\Pi(p_g, p_b, k^H)/(\partial p_b)|_{p_b^H} = 0$, we have $\partial\Pi^H/(\partial m) = \partial\Pi(p_g^H(m), p_b^H(m), k^H)/(\partial m) = (1-\beta)((1-q)\mathfrak{F}_B(p_g^H(m)) + q\mathfrak{F}_B(p_b^H(m))) \geq 0$. For L, setting $q = 1/2$ yields the same result. For N, the same reasoning applies: $\partial\Pi^N/(\partial m) = \partial\Pi(p_0^N(m), k^N)/(\partial m) = (1-\beta)\mathfrak{F}_B(p_0^N(m)) \geq 0$.

For δ and by using $\partial\Pi(p_g, p_b, k^H)/(\partial p_g)|_{p_g^H} = \partial\Pi(p_g, p_b, k^H)/(\partial p_b)|_{p_b^H} = 0$, we have $\partial\Pi^H/(\partial\delta) = \partial\Pi(p_g^H(\delta), p_b^H(\delta), k^H)/(\partial\delta) = -p_g^H(\delta)(\beta\mathfrak{F}_G(p_g^H(\delta)) + (1-\beta)(1-q)\mathfrak{F}_B(p_g^H(\delta))) -$

$p_b^H(\delta)(1-\beta)q\mathfrak{F}_B(p_b^H(\delta)) \leq 0$. Again, the same reasoning leads to $\partial\Pi^L/(\partial\delta) \leq 0$. For N, we have $\partial\Pi^N/(\partial\delta) = -p_g^H(\delta)(\beta\mathfrak{F}_G(p_g^H(\delta)) + (1-\beta)\mathfrak{F}_B(p_g^H(\delta))) \leq 0$.

Finally, we have $\partial\Pi^H/(\partial q) = (1-\beta)(\mathfrak{F}_B(p_b^H(q))(m-\delta p_b^H(q)) - \mathfrak{F}_B(p_g^H(q))(m-\delta p_g^H(q))) \geq 0$. □

Proof of Proposition 4.6. For simplicity, we write p_g and p_b instead of p_g^* and p_b^* for $* \in \{H, L\}$ and $\mathfrak{F}_\Theta^s$ instead of $\mathfrak{F}_\Theta(p_s)$. Recall the optimality conditions of the H and L contracts: for H, consider the conditions in Proposition 4.2, which are equivalent to $k_{B,b} \in [\delta p_b, \delta p_b + (\pi_0 + c_h)/((1-\beta)q\mathfrak{F}_B^b)]$, $k_{G,b} \leq \delta p_b + (\pi_0 + c_h - (1-\beta)\mathfrak{F}_B^b(k_{B,b} - \delta p_b))/(\beta\mathfrak{F}_G^b)$, $k_{B,g} \leq \delta p_g + (\mathfrak{F}_B^b(k_{B,b} - \delta p_b) - C)/\mathfrak{F}_B^g$, $k_{G,g} = \delta p_g + (\pi_0 + c_h - (1-\beta)(1-q)\mathfrak{F}_B^g(k_{B,g} - \delta p_g) - (1-\beta)q\mathfrak{F}_B^b(k_{B,b} - \delta p_b))/(\beta\mathfrak{F}_G^g)$.

For L, the conditions in Proposition 4.3 are equivalent to $k_{B,b} \in [\delta p_b, \delta p_b + 2(\pi_0 + c_l)/((1-\beta)\mathfrak{F}_B^b)]$, $k_{G,b} \leq \delta p_b + (\pi_0 + c_l - (1-\beta)\mathfrak{F}_B^b(k_{B,b} - \delta p_b))/(\beta\mathfrak{F}_G^b)$, $k_{B,g} \in [\delta p_g + (\mathfrak{F}_B^b(k_{B,b} - \delta p_b) - C)/\mathfrak{F}_B^g, \delta p_g + \mathfrak{F}_B^b(k_{B,b} - \delta p_b)/\mathfrak{F}_B^g]$, $k_{G,g} = \delta p_g + (\pi_0 + c_l - (1-\beta)(\mathfrak{F}_B^g(k_{B,g} - \delta p_g) + \mathfrak{F}_B^b(k_{B,b} - \delta p_b))/2)/(\beta\mathfrak{F}_G^g)$.

(i) A price-only contract (it holds that $k_g = k_{B,g} = k_{G,g}$ and $k_b = k_{B,b} = k_{G,b}$) is optimal under H (L) if it satisfies the corresponding optimality conditions. To find the conditions under which a price-only contract is optimal, we first insert k_g and k_b for the corresponding payments. Then, by use of the equality condition for $k_{G,g}$, we replace k_g in all inequalities through the corresponding term depending on k_b. As a result, we have inequalities only depending on one variable, k_b. Then, we rearrange with respect to k_b, which gives us several lower and upper bounds for the choice of k_b. If those bounds define a non-empty set, then there exists an optimal price-only contract. Therefore, in a last step, we compare the upper and lower bounds for k_b and thereby find the conditions under which the upper bounds are above the lower bounds and hence the conditions for the existence of an optimal price-only contract. By this procedure, we find for H that $C \leq (\pi_0 + c_h)\beta(\mathfrak{F}_G^g\mathfrak{F}_B^b - \mathfrak{F}_G^b\mathfrak{F}_B^g)/((\beta\mathfrak{F}_G^g + (1-\beta)(1-q)\mathfrak{F}_B^g)(\beta\mathfrak{F}_G^b + (1-\beta)\mathfrak{F}_B^b)) \equiv (\pi_0 + c_h)\xi_1$ is the existence condition. For L, we find that there always exists an optimal price-only contract (e.g., $k_b = \delta p_b + (\pi_0 + c_l)/(\beta\mathfrak{F}_G^b + (1-\beta)\mathfrak{F}_B^b)$, and $k_g = \delta p_G + (\pi_0 + c_l)(2\beta\mathfrak{F}_G^b + (1-\beta)\mathfrak{F}_B^b)/((2\beta\mathfrak{F}_G^g + (1-\beta)\mathfrak{F}_B^g)(\beta\mathfrak{F}_G^b + (1-\beta)\mathfrak{F}_B^b)))$.

(ii) For quality-only contracts ($k_G = k_{G,g} = k_{G,b}$ and $k_B = k_{B,g} = k_{B,b}$), a similar procedure can be applied as for price-only contracts: the use of the equality condition to generate inequalities with only one variable, rearrangement with respect to that variable and derivation of conditions for upper and lower bounds implying non-empty sets for that variable. This procedure results in the following existence conditions under H: an optimal quality-only contract exists if and only if $C/\mathfrak{F}_B^g \leq \delta(p_g - p_b) \leq (\pi_0 + c_h)(\mathfrak{F}_G^g - \mathfrak{F}_G^b)/(\mathfrak{F}_G^b(\beta\mathfrak{F}_G^g + (1-\beta)(1-q)\mathfrak{F}_B^g)) \equiv (\pi_0 + c_h)\xi_2$. For L, we have existence of an optimal quality-only contract if and only if $\delta(p_g - p_b) \leq (\pi_0 + c_l)(\mathfrak{F}_G^g - \mathfrak{F}_G^b)/(\mathfrak{F}_G^b(\beta\mathfrak{F}_G^g + (1-\beta)\mathfrak{F}_B^g/2)) \equiv (\pi_0 + c_l)\xi_3$ and $\delta(p_g - p_b) \leq (\pi_0 + c_l)(\mathfrak{F}_B^g - \mathfrak{F}_B^b)(\mathfrak{F}_G^g - \mathfrak{F}_G^b)/(\beta\mathfrak{F}_G^g\mathfrak{F}_G^b(\mathfrak{F}_B^g - \mathfrak{F}_B^b) + (1-\beta)\mathfrak{F}_B^g\mathfrak{F}_B^b(\mathfrak{F}_g^g - \mathfrak{F}_G^b)) + C(1-\beta)(\mathfrak{F}_G^g\mathfrak{F}_B^b - \mathfrak{F}_G^b(\mathfrak{F}_B^g - \mathfrak{F}_B^b)/2)/(\beta\mathfrak{F}_G^g\mathfrak{F}_G^b(\mathfrak{F}_B^g - \mathfrak{F}_B^b) + (1-\beta)\mathfrak{F}_B^g\mathfrak{F}_B^b(\mathfrak{F}_g^g - \mathfrak{F}_G^b)) \equiv (\pi_0 + c_l)\xi_4 + C\xi_5$.

(iii) For fixed-fee contracts ($k = k_{G,g} = k_{G,b} = k_{B,g} = k_{B,b}$), again, a similar procedure can be applied. We only have to insert k for all payments. Then, we can replace k in each inequality by the corresponding term from the identity condition. The remaining payment-independent inequalities then have to be investigated with regard to feasibility. This results in the existence of an optimal fixed-fee contract under H if and only if $\delta(p_g - p_b) \geq (\pi_0 + c_h)(\mathfrak{F}_B^g - \mathfrak{F}_B^b)/(\mathfrak{F}_B^b(\beta\mathfrak{F}_G^g + (1-\beta)\mathfrak{F}_B^g)) + C(\beta\mathfrak{F}_G^g + (1-\beta)((1-q)\mathfrak{F}_B^g + q\mathfrak{F}_B^b))/(\mathfrak{F}_B^b(\beta\mathfrak{F}_G^g + (1-\beta)\mathfrak{F}_B^g)) \equiv (\pi_0 + c_h)\xi_6 + C\xi_7$ and $\delta(p_g - p_b) \leq (\pi_0 + c_h)(\beta(\mathfrak{F}_G^g - \mathfrak{F}_G^b) + (1-\beta)(1-q)(\mathfrak{F}_B^g - \mathfrak{F}_B^b))/((\beta\mathfrak{F}_G^g + (1-\beta)(1-q)\mathfrak{F}_B^g)(\beta\mathfrak{F}_G^b + (1-\beta)\mathfrak{F}_B^b)) \equiv (\pi_0 + c_h)\xi_8$. For L, there exists an optimal fixed-fee contract if and only if $\delta(p_g - p_b) \geq (\pi_0 + c_l)(\mathfrak{F}_B^g - \mathfrak{F}_B^b)/(\mathfrak{F}_B^b(\beta\mathfrak{F}_G^g + (1-\beta)\mathfrak{F}_B^g)) \equiv (\pi_0 + c_l)\xi_9$ and $\delta(p_g - p_b) \leq (\pi_0 + c_l)\beta(\mathfrak{F}_G^g - \mathfrak{F}_G^b) + (1-\beta)(\mathfrak{F}_B^g - \mathfrak{F}_B^b)/2)/((\beta\mathfrak{F}_G^g + (1-\beta)\mathfrak{F}_B^g/2)(\beta\mathfrak{F}_G^b + (1-\beta)\mathfrak{F}_B^b)) \equiv (\pi_0 + c_l)\xi_{10}$ and $\delta(p_g - p_b) \leq (\pi_0 + c_l)(\mathfrak{F}_B^g - \mathfrak{F}_B^b)/(\mathfrak{F}_B^b(\beta\mathfrak{F}_G^g + (1-\beta)\mathfrak{F}_B^g)) + C(\beta\mathfrak{F}_G^g + (1-\beta)(\mathfrak{F}_B^g + \mathfrak{F}_B^b)/2)/(\mathfrak{F}_B^b(\beta\mathfrak{F}_G^g + (1-\beta)\mathfrak{F}_B^g)) \equiv (\pi_0 + c_l)\xi_9 + C\xi_{11}$.

Finally, there exist situations in which there exist optimal fixed-fee (and hence, optimal price-/quality-only) contracts due to the following example. Consider the following parameter setting: $\beta = 0.5$, $m = 0.5$, $q = 0.8$, $c_h = 0.006$, $c_l = 0.001$, $\delta = 0.5$, and $\pi_0 = 0.3$. Furthermore, let $\mathfrak{F}_G(p) = [1 - 0.1/p]^+$ and $\mathfrak{F}_B(p) = [1 - 0.05/p]^+$ defined on $[0, \infty)$.

Then, for H, the optimal prices are $p_g^H = 0.41833001$ and $p_b^H = 0.2236068$. The optimal fixed-fee contract for H is then $k^H = 0.56312466$. The optimal profit is $\Pi^H = 0.14105927$.

For L, the optimal prices are $p_g^L = 0.38729833$ and $p_b^H = 0.2236068$. The optimal fixed-fee contract for L is then $k^L = 0.55790769$. The optimal profit is $\Pi^L = 0.14012455$. □

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