Essays on Belief-Driven Stock Price Fluctuations

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Erklärung

Hiermit erkläre ich, dass ich die vorliegende Dissertation bis auf die deutlich gemachte Mitwirkung durch Koautoren selbstständig angefertigt und die benutzten Hilftsmittel vollständig und deutlich angegeben habe.

Sebastian Merkel

Contents

List of Figures

List of Tables

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Preface

This dissertation consists of two self-contained articles extending the work on belief-driven stock price fluctuations by Adam, Marcet, and Nicolini (2016) and Adam, Marcet, and Beutel (2017). In these papers, the authors develop a theory of stock price behavior that can resolve a number of empirical puzzles, most prominently that observed stock prices are "excessively" volatile if related to the volatility of dividends (LeRoy and Porter, 1981; Shiller, 1981) and that stock price dynamics seem to display boom-bust cycles that are hard to justify by shifts in fundamental values. This theory is attractive, because it makes – relative to competing ones – only minimal adjustments to a standard Lucas-type consumption-based asset pricing model (Lucas, 1978) along a dimension that has both intuitive appeal and can be supported by empirical evidence.

Specifically, Adam, Marcet, and Nicolini (2016) and Adam, Marcet, and Beutel (2017) relax the rational expectations hypothesis (REH) in favor of a subjective belief specification according to which investors make statistical inference about future stock price behavior based on past observed stock price behavior. Stocks are a very long-duration asset[1] and the dividend yield is typically only a small fraction of stock returns, whereas capital gains play a more important role.[2] These facts imply that for most investors the expected resale price at the end of the intended holding period is much more important for judging the merits of a stock market investment ex ante than is the expected stream of dividend cash flows to be received during the holding period.[3] Under rational expectations, all investors know how the stock price is determined in market equilibrium as a function of fundamentals (e.g. dividends). Consequently, forecasting future prices amounts to forecasting future fundamentals. While future resale prices are still important for investors, investors do not need to be concerned about them independently of considerations about future fundamentals, because they know that the price equals the –

[1]The quarterly price-dividend ratio in the United States has on average been around 140 over the post-war sample.

[2]The (nominal) dividend yield of the S&P 500 over the U.S. postwar sample has on average been 84 basis points, whereas (nominal) capital gains have on average amounted to 200 basis points.

[3]This is opposed to the situation for many fixed-income assets with finite maturity. For those, some investors intent to hold the asset until maturity. Given this strategy, expectations about intermediate secondary market prices are entirely irrelevant (although expectations may still be relevant to decide on such a strategy).

appropriately discounted – present value of future dividends on all future trading dates.

However, despite their name, rational expectations generally do not follow from rationality. Instead, rational expectations can be the outcome of a deductive reasoning process of a rational individual only under very strong assumptions on what investors know about the market process and other investors' behavior (see Adam and Marcet, 2011). In practice, it is unlikely that investors are endowed with a sufficient amount of knowledge to predict with certainty how fundamentals are mapped into market outcomes.[4] The REH rests thus on very restrictive assumptions and relaxing it has immediate intuitive appeal: stock investments may in practice be perceived as risky, precisely because investors are uncertain about future market prices, independently of and beyond the uncertainty about future dividends (or other fundamentals). Furthermore, if investors try to use observed market prices to draw inference about future market prices and subsequently make trading decisions based on this inference, price-belief dynamics become partially self-referential and may occasionally be delinked from movements in fundamentals. This is the explanation for high stock price volatility and the emergence of boom-bust cycles put forward by Adam, Marcet, and Nicolini (2016) and Adam, Marcet, and Beutel (2017).

This rationale is also supported by empirical evidence. First, the models presented in these papers match a set of key asset pricing facts under reasonable calibrations. This is a requirement to accept these models as quantitatively credible explanations of stock price behavior. Second, and more importantly, Adam, Marcet, and Beutel (2017) present direct evidence on investor expectations based on a number of surveys.[5] In all those surveys, investors tend to expect high returns, when prices are already high relative to dividends, and low returns, when prices are low. In contrast, actual stock returns display the opposite pattern. This inconsistency of survey responses with actual returns is statistically strong enough to lead Adam, Marcet, and Beutel (2017) to reject the REH, but at the same time it is qualitatively and quantitatively consistent with the alternative subjective belief specification these authors propose and utilize in their asset pricing model.

Chapter 1 of this dissertation studies the effects of a financial transaction tax in an extension of the Adam, Marcet, and Beutel (2017) model. As this framework features stock price fluctuations that lead in their most extreme form to large and persistent boom-bust cycles, one may be concerned that these fluctuations are inefficient and thus socially undesirable. A policy instrument that discourages investor speculation based on price beliefs could then raise welfare. In European policy debates following the financial crisis, a financial transaction tax has featured prominently with precisely the intention to discourage speculation and improve price efficiency. The chapter analyzes whether such a tax is a suitable and desirable tool to achieve this outcome in the context of the Adam, Marcet, and Beutel (2017) model. In particular, it asks the question whether a

[4]The competing theories found in the asset pricing literature suggest that at least academic economists seem to lack such knowledge.

[5]Similar evidence is also presented by Greenwood and Shleifer (2014).

transaction tax can prevent boom-bust cycles in stock prices. The chapter is the outcome of a joint project with Klaus Adam, Johannes Beutel and Albert Marcet, an earlier version of which was based on my master thesis, Merkel (2014). It has been published as Adam, Beutel, Marcet, and Merkel (2016). It also appeared as a chapter of Johannes Beutel's dissertation (Beutel, 2017, Chapter 2).

To make a financial transaction tax an interesting policy in the framework of analysis, we extend the Adam, Marcet, and Beutel (2017) model by introducing investor belief heterogeneity that leads to equilibrium trade. Such trade is important for prices to have any allocative implications, whereas in the absence of it, price fluctuations cannot have welfare effects. We motivate belief heterogeneity by a more fine-grained analysis of investor survey data and show that the extended model is quantitatively consistent with the joint behavior of stock prices, price expectations and trading volume. In this extended model, price fluctuations lead to redistribution from experienced to inexperienced investors, which the policy maker might consider undesirable.[6] We next introduce a financial transaction tax into the framework and show that such a tax does indeed discourage speculative trading. However, this comes at the cost of reduced market liquidity. Therefore, prices have to react strongly, whenever the market has to absorb small variations in stock supply. If such supply variations are present,[7] strong price reactions might in turn trigger strong belief-revisions. Occasionally, these belief revisions are strong enough to trigger a self-reinforcing price-belief feedback loop resulting in the very boom-bust cycles, the imposition of the tax seeks to prevent. As we show in our quantitative analysis, this indirect liquidity effect dominates the direct discouragement effect, thereby increasing the probability of boom-bust cycles. Based on this analysis, a financial transaction tax seems to be an undesirable instrument for improving the efficiency of stock prices.

Chapter 2 extends the framework of Adam, Marcet, and Nicolini (2016) and Adam, Marcet, and Beutel (2017) along an entirely different dimension. Their models are based on an endowment economy in which by assumption price fluctuations cannot have any (aggregate) allocation effects. Chapter 2 embeds these stock price theories into a production economy based on a real business cycle model. It is shown that the resulting model matches a standard set of moments describing key aspects of the behavior of stock prices, business cycles and their interaction. The framework thus provides a unified and quantitatively credible explanation for the joint behavior of stock prices and business cycles. The chapter is based on a paper written jointly with Klaus Adam, Adam and Merkel (2018).

Modeling the joint behavior of stock prices and business cycles has proven to be quantitatively challenging. This is the case because business cycles are relatively smooth, in

[6]Because the model does not feature any aggregate effects of belief-driven stock price fluctuations (for such a framework, see Chapter 2), any redistribution of the aggregate consumption quantity across agents is Pareto-efficient. Whether such redistribution is perceived as undesirable depends thus on the precise welfare objective.

[7]In our model this comes from the presence of a tiny amount of noise trades.

particular so is consumption, whereas stock prices are very volatile. Generating volatile prices in the presence of smooth consumption and output fluctuations is difficult in a rational expectations model, even if output and consumption are endowment processes. It becomes even more difficult, if endogenous production decisions provide agents with additional tools to smooth consumption and self-insure against risks.[8] To nevertheless generate good quantitative predictions about the joint behavior of stock prices and business cycles, the existing literature has almost exclusively relied on non-separable preferences and labor market frictions. As we argue in detail in Chapter 2, these assumptions may be undesirable. Our theory, in contrast, does not rely on such assumptions. Instead, it utilizes the subjective belief specification of Adam, Marcet, and Beutel (2017) that has proven successful in an endowment economy. In our framework, belief-driven stock price fluctuations lead to cyclical variations in investment demand; booms in stock prices are associated with booms in investment and hours worked. Consequently, in this model belief-driven price fluctuations have aggregate effects and it becomes possible to assess the welfare effects of such fluctuations. We find that, on average, hours worked are higher and consumption lower than in the absence of belief distortions, because investment choices do not fully reflect productivity developments. Instead, they are partially driven by (false) optimism and pessimism. However, the welfare gains of eliminating belief-driven price cycles turn out to be modest, comparable in magnitude to the gains of eliminating the business cycle.

[8] E.g. if agents are sufficiently averse to non-smooth consumption profiles, asset prices have to react a lot to make agents accept the non-smooth profile in an endowment economy. But if agents can in addition control their labor supply, they may well choose to eliminate consumption fluctuations by adjusting their work hours in a production economy.

Chapter 1

Can a Financial Transaction Tax Prevent Stock Price Booms?*

1.1 Introduction

Following the financial crisis, there has been a widespread desire among policymakers to introduce financial transaction taxes (FTTs). The European Commission, for example, proposed the introduction of FTTs in September 2011. Subsequently, France introduced in 2012 a 0.1% tax on stock market and related transactions and has recently increased the tax rate to 0.2%. Italy introduced a 0.1% tax on stock market transactions in 2013.[1]

One of the stated policy objectives of the European Commission is that FTTs should 'discourage financial transactions which do not contribute to the efficiency of financial markets'. The present paper seeks to analyze to what extent FTTs actually increase the efficiency of stock market transactions and stock market prices. In particular, it investigates whether FTTs can prevent boom and bust like dynamics in stock prices; over recent decades such price dynamics have become pervasive in a number of important stock markets and have contributed to the redistribution of wealth between different kinds of investors.[2] The effect that FTTs have on boom-bust like dynamics in stock markets should thus be of prime importance to policymakers.

To analyze this issue, we use a modeling framework that can generate stock price fluctuations roughly of the size observed in the data, including occasional large upswings and reversals in stock market prices. The model also quantitatively replicates important

*This chapter was published as Adam, Beutel, Marcet, and Merkel (2016). The version presented here is identical to the published one up to typesetting and the correction of minor errors.

[1]FTTs are already a widely used tax instrument in housing markets. Spain, for example, levies an 8% transaction tax on real estate transactions and Germany levies a 5% tax, both additionally levy capital gains taxes.

[2]See Brunnermeier and Nagel (2004) for evidence on how the tech stock boom and bust around the year 2000 redistributed wealth between hedge fund and other investors.

data moments characterizing the behavior of trading volume, as well as its comovement with stock prices and investor expectations. Credibly replicating the behavior of trading volume appears key for an analysis that seeks to understand the effects of taxing trading activity and is a distinguishing feature of the present analysis.[3]

Besides being quantitatively plausible, our modeling framework gives FTTs the best possible chance to generate positive welfare effects: first, we consider a framework where subjective belief components cause asset prices not to be fully efficient, so that there is - at least in principle - room for increasing the efficiency of financial market prices; second, within the presented framework, the gains from trade exist only in subjective terms, i.e., due to belief differences, so that taxing trading activity may appear desirable on a priori grounds, see Simsek (2013); third, we abstract from a number of adverse consequences likely to be associated with the introduction of FTTs, such as costly evasive behavior, which may involve redirecting orders to other exchanges, the adverse liquidity effects resulting from financial market fragmentation, or the costly creation of alternative financial instruments that are not subject to the tax.

Our main finding is that even within this very conducive setting, the introduction of FTTs fails to ‘discourage transactions which do not contribute to the efficiency of financial markets’. Indeed, we find that the introduction of FTTs increases the likelihood that the stock market embarks on a significant boom and bust cycle in valuation, and thereby increases the overall amount of wealth redistribution. The reasons for this finding are subtle, as we explain below, but show that FTTs may actually not be a suitable policy instrument for increasing the efficiency of stock markets.

The modeling framework used in the present paper builds upon prior work by Adam, Beutel, and Marcet (2015), which replicates stock price behavior within a representative agent framework with time-separable preferences. The present analysis adds (1) by introducing investor heterogeneity and thereby equilibrium trade, (2) by showing that the resulting trading patterns are empirically plausible, and (3) by studying the pricing and welfare effects of introducing FTTs.

While the equilibrium pricing patterns of the representative agent model in Adam, Beutel, and Marcet (2015) prove rather robust to introducing agent heterogeneity, i.e., stock prices continue to be very volatile and to display occasional boom-bust cycles, the addition of agent heterogeneity helps in generating auto-correlated trading volume, trading volume that correlates positively with absolute price changes, and trading volume that correlates positively with investor disagreement, in line with what is found in the data.

The presented model is one where boom-bust dynamics arise from subjective price beliefs, but in a setting where investors take fully optimal investment decisions given their beliefs, following Adam and Marcet (2011). The introduction of subjective stock price beliefs is motivated by empirical evidence presented in Adam, Beutel, and Marcet (2015),

[3]See Section 1.2 for a discussion of the related literature.

who show that the joint dynamics of realized capital gains and capital gain expectations, as observed from survey data, are strongly inconsistent with the rational expectations hypothesis. This implies – amongst other things – that rational asset price bubbles, e.g., those derived in classic work by Froot and Obstfeld (1991), are inconsistent with the joint dynamics of actual and expected capital gains in the data.

Following Adam, Beutel, and Marcet (2015), we consider investors who hold subjective stock price beliefs of a kind such that Bayesian updating causes investors to extrapolate (to different degrees) past capital gains into the future. The degree of extrapolation is thereby calibrated to the one that we document to be present in survey data. In particular, we show that less experienced stock market investors extrapolate more compared to investors with longer investment experience.

Extrapolative behavior, which gives rise to investor optimism and pessimism, potentially supports a strong argument in favor of introducing FTTs. Specifically, in our setting, price booms emerge because investors become optimistic once they see past prices going up, causing them to bid up today's prices, thereby creating additional optimism in the next period and further price increases. FTTs can prevent investors from trading on their optimistic beliefs, i.e., prevent them from bidding up prices once optimism has increased, thereby preventing the positive feedback loop between price increases and increased optimism just described.

While intuitively plausible, this argument ignores an important additional consequence of FTTs. By preventing agents from trading, even arbitrarily small exogenous shocks to stock supply can have a disproportionately large effect on realized prices. Specifically, linear transaction taxes imply that investors, whose stockholdings are close to their subjectively optimal level, do not want to trade, unless there is a significant change in the stock price.[4] As a result, FTTs can increase price volatility in normal times. With realized prices feeding into investors' beliefs, due to extrapolative behavior, this ultimately increases the likelihood that the stock market embarks on a large self-fueling boom and subsequent bust. The predicted effect of a 4% FTT is an increase by one third of the number of stock price boom episodes relative to the case without taxes.

Our quantitative analysis shows that FTTs manage to decrease the size and duration of stock price booms, including the volatility of prices during boom times. At the same time, FTTs increase price volatility during normal times.[5] The latter together with increased likelihood of (volatile) boom and bust episodes causes FTTs to increase overall stock price volatility.

Motivated by the observation that it is undesirable to levy FTTs in normal times,

[4]This is so because the gains from trade are of second order close to the optimum, while the cost of the tax are of first order.

[5]Normal times are times that are not classified as boom times. Boom times begin when the quarterly price dividend ratio exceeds a certain level and end when the PD ratio falls below a certain lower level. In our numerical application, we set the first threshold to 250 and latter to 200. Results turn out to be rather robust to the precise threshold values.

as they increase price volatility and thereby the likelihood of boom-bust cycles, we also consider the effects of state contingent taxes that are only levied once prices exceed a certain threshold. We show that such taxes give rise to non-continuous stock demand functions and thereby to problems of equilibrium multiplicity and non-existence. State-contingent transaction taxes appear problematic on these grounds.

The remainder of this paper is organized as follows. Section 1.2 discusses some of the related literature. Section 1.3 provides basic facts about the joint behavior of stock prices, trading volume and investor expectations that we seek to quantitatively match within our asset pricing framework. Section 1.4 introduces the asset pricing model. Section 1.5 shows that the model performs poorly in terms of replicating price and trading dynamics when investors hold rational price expectations. Section 1.6 evaluates the quantitative performance of the model with subjective price beliefs and in the absence of a transactions tax. In Section 1.7 we show how stock price boom and bust dynamics redistribute wealth between different investor types. Section 1.8 presents the implications of introducing linear FTTs and Section 1.9 considers the effects of state-contingent taxes. A conclusion briefly summarizes. Technical material and information about the employed data sources is summarized in Appendix A.

1.2 Related Literature

The present paper is closely connected to an extensive literature on financial transaction taxes going at least back to the well-known proposal by Tobin (1978). We provide here a selective overview of the literature, making reference to work that is most closely related to the present paper.

In a comprehensive theoretical study, Dávila (2013) determines optimal linear transaction taxes for a setting where investors hold heterogeneous beliefs. He shows that the optimal transaction tax of a social planner who maximizes social welfare under her own (possibly different) probability beliefs, depends on the cross-sectional covariance between investors' beliefs and equilibrium portfolio sensitivities.

Scheinkman and Xiong (2003) analyze how asset price bubbles and trading volume are affected by transactions taxes in a setting with risk-neutral investors who face a short-sale constraint and who hold different beliefs because they assign different information content to publicly available signals. In their setting, transaction taxes strongly affect trading volume but may have only a limited effect on the size of asset price bubbles.

The present paper adds to these contributions by considering the effects of FTTs within a quantitatively credible setting that replicates important data moments describing the joint behavior of stock prices, trading volume and investor expectations. Furthermore, by incorporating learning from market prices, investors' belief distortions depend in important ways on market outcomes. This gives rise to feedback effects that are absent in models in which agents consider market prices to offer only redundant information.

In related work, Buss, Dumas, Uppal, and Vilkov (2013) consider the effects of FTTs and other policy instruments on stock market volatility in a production economy in which some stock market participants overinterpret the information content of public signals, as in Dumas, Kurshev, and Uppal (2009). The present paper considers an endowment economy but evaluates model performance also with regard to the ability to match trading activity. Similar to our findings, Buss, Dumas, Uppal, and Vilkov (2013) show how financial transaction taxes increase the volatility of stock market returns.

With financial transaction taxes being almost equivalent to trading costs, the present paper also relates to the transaction costs literature. As in Constantinides (1986), transaction costs generate within the present setup partially flat demand curves, see also subsequent work byAiyagari and Gertler (1991) and Heaton and Lucas (1996). Different from Constantinides (1986), the asset price effects of transaction costs fail to be of second order within the present setting because we consider agents that use price realizations to update beliefs about the price process. Guasoni and Muhle-Karbe (2013) and Vayanos and Wang (2013) provide recent surveys of the transaction cost literature.

Empirical evidence on the volatility effects of financial transaction taxes is provided in Umlauf (1993), Jones and Seguin (1997) and Hau (2006). These studies tend to find that market volatility increases with the introducing of a tax, see also McCulloch and Pacillo (2011) for a recent overview of the empirical literature. Coelho (2014) and Colliard and Hoffmann (2015) analyze the recent experiences with the introduction of FTTs in France and Italy, documenting how FTTs increase price volatility and reduce market depth.

The market microstructure literature also studies financial transaction taxes, focusing on the differential impact that such taxes have on the participation of noise traders, which create exogenous market volatility or mispricing, versus the participation of informed traders who evaluate prices according to fundamentals, see for example Jeanne and Rose (2002) or Hau (1998). The general conclusion of this theoretical literature is that if financial transaction taxes cause noise traders to participate less in the market, then market volatility can fall as a result.

1.3 Stock Prices, Price Expectations and Trading Volume: Empirical Evidence

This section documents key facts about the joint behavior of U.S. stock prices, investors' price expectations and stock market trading volume that we seek to quantitatively replicate with our asset pricing model. The next section presents empirical evidence about stock price behavior, the behavior of dividends and the behavior of average stock price expectations. Section 1.3.2 complements this with key facts about the behavior of trading volume and its relation with price behavior and the behavior of price expectations. It shows – amongst other things – that trading volume correlates positively with disagreement across investors about future prices. Finally, Section 1.3.3 shows that disagreement between investors can be systematically related to investors' stock market experience.

1.3.1 Stock Prices, Dividends and Average Price Expectations

Table 1.1 presents key facts about the behavior of quarterly U.S. stock prices, dividends and stock return expectations as available from survey data.[6] The facts presented in Table 1.1 are the main data moments guiding the analysis in Adam, Beutel, and Marcet (2015) and we summarize them here for convenience.[7]

Table 1.1 shows that the average quarterly price dividend ratio ($E[PD]$) is around 140 and has a standard deviation ($std(PD)$) of approximately half its average value.[8] Stock prices are thus very volatile. The quarterly auto-correlation of the price dividend (PD) ratio ($corr(PD_t, PD_{t-1})$) is 0.98, showing that deviations of the PD ratio from its sample mean are very persistent over time. As a result, quarterly real stock returns are very volatile, with a standard deviation ($std(r^s)$) of around 8% per quarter. Real stock returns are thus much more volatile than real dividend growth, which has a standard deviation ($std(D_t/D_{t-1})$) of just 1.92%. The mean real stock return ($E[r^s]$) is 1.89% per quarter and much higher than the average growth rate of real dividends ($E[D_t/D_{t-1} - 1]$), which equals 0.48% per quarter.

Table 1.1 also documents that the average investor's expected real returns in the UBS survey correlates strongly and positively with the PD ratio ($corr(PD_t, \overline{E}_t R_{t+1})$): the correlation equals 0.79.[9] Adam, Beutel, and Marcet (2015) show that this fact is robust against using other survey data sources and against alternative ways to distill expectations from the survey data. They also show that this fact is inconsistent with investors holding rational price expectations, which is why we include the correlation between the PD ratio and expected returns in the set of data moments that we seek to match.

1.3.2 Trading Volume, Stock Prices and Disagreement

This section presents empirical facts about trading activity and its comovement with prices and price expectations. It shows that trading volume is highly persistent, that trading volume is largely uncorrelated with stock market valuation, instead correlates

[6] The data sources used in this and the subsequent sections are described in Appendix A.1.

[7] We include here all asset pricing facts considered in Adam, Beutel, and Marcet (2015), except for those involving the bond market, as the present model does not feature a bond market.

[8] The quarterly PD ratio is defined as the price over quarterly dividend payments, see Appendix A.1 for further details.

[9] The number reported in Table 1.1 uses the mean of the expected returns of the own portfolio return expectations of all investors in the UBS survey. The survey data are avaialable from 1998:Q2 to 2007:Q2 and have been transformed into real values using the median of expected inflation reported in the survey of professional forecasters.

Table 1.1
Quarterly stock prices, dividends and survey expectations

Considered moments	U.S. data, 1949:Q1-2012:Q1
Stock prices	
$E[PD]$	139.7
$std(PD)$	65.3
$corr(PD_t, PD_{t-1})$	0.98
$std(r^s)$	8.01%
$E[r^s]$	1.89%
Survey expectations	
$corr(PD_t, \overline{E}_t R_{t+1})$	0.79
Dividends	
$E[D_t/D_{t-1} - 1]$	0.48%
$std(D_t/D_{t-1})$	1.92%

positively with *absolute* price changes. Furthermore, it documents - to our knowledge for the first time - that aggregate trading volume and disagreement about future aggregate stock market returns, as measured by survey data, are positively correlated.

The finance literature studies a range of empirical measures to capture trading activity, see Lo and Wang (2009) for an overview. To account for trading in individual shares, Lo and Wang (2009) argue that 'shares traded divided by shares outstanding is a natural measure of trading activity when viewed in the context of standard portfolio theory and equilibrium asset-pricing models' (p.243). Clearly, for individual shares, this measure is identical to using the dollar volume of shares traded divided by the dollar volume of shares outstanding. Since this latter measure aggregates more naturally across different stocks and since we are interested in the aggregate stock market, we use the dollar volume of shares traded over the dollar volume of share outstanding as our preferred measure of trading volume.

We aggregate daily trading volume into a quarterly series by summing up the daily trading volumes over the quarter, following Lo and Wang (2009). While being standard, this procedure is likely going to lead to an overstatement of the model relevant trading volume, as many of the daily trades recorded in the data may be reversed with opposing trades within the same quarter. Indeed, with the advent of high frequency trading strategies, many of the recorded trades are likely to be undone within seconds, if not milliseconds. Dealing properly with this issue in the data is difficult, as it would require information about individual portfolios of all investors. We seek to account - at least partially - for the increasing share of high-frequency trades over time, therefore use de-

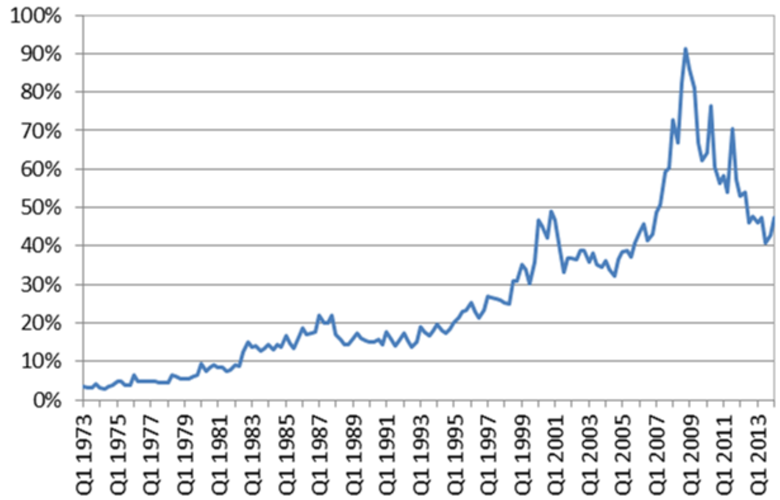

Figure 1.1. Quarterly trading volume (% of outstanding shares, undetrended)

trended data on trading volume. Since detrending can affect the cyclical properties of the trading volume series, we report below only facts that turn out to be robust to a range of plausible detrending methods.

Figure 1.1 depicts the (undetrended) quarterly trading volume of the U.S. stock market, where data is available from January 1973. Trading volume displays a clear upward trend over time. In the early 1970's trade during a quarter amounted to around 5% of the market value of outstanding shares; at the end of the sample period this number reaches close to 50%; the data also shows temporary spikes in trading volume around the 1987, 2000 and 2008 stock market busts.

Table 1.2 presents a number of facts about detrended trading volume. As a baseline, we use simple linear detrending, but the table also displays outcomes for other commonly used detrending methods. In particular, it considers linear-quadratic detrending, the outcomes obtained from HP-filtering with a smoothing parameter of 1600, as well as so-called moving average (MA) detrending, which normalizes trading volume by the average trading volume recorded in the preceding four quarters.

Table 1.2 shows that trading volume displays considerable autocorrelation across quarters. The autocorrelation is statistically significant at the 1% level for all detrending methods.[10] For higher frequencies, this is a well-known fact that has been documented in the finance literature, we show it here for the quarterly frequency at which we will evaluate our asset pricing model.

Table 1.2 also shows that there exists no statistically significant correlation between

[10]We test the null hypothesis $H_0 : corr(\cdot, \cdot) = 0$ in this and subsequent tables using robust standard errors, following Roy and Cléroux (1993), which are implemented with a Newey-West estimator with 4 leads and lags.

Table 1.2
Trading Volume and Price Behavior

Correlations	Detrending Method			
	Baseline (linear)	Linear-quadratic	HP filter	MA
$corr(TV_t, TV_{t-1})$	0.89***	0.88***	0.66***	0.43***
$corr(TV_t, PD_t)$	-0.07	0.01	-0.03	-0.06
$corr(TV_t, \|P_t/P_{t-1} - 1\|)$	0.34***	0.33***	0.33***	0.23***

*/**/*** indicate significance at the 10%/5%/1% significance level, respectively.

trading volume and the level of the PD ratio.[11] This illustrates that claims about the existence of a high positive correlation between the level of stock prices and trading volume, see for example Scheinkman and Xiong (2003) and the references cited therein, disappear once one removes the trend displayed by trading volume.[12]

The previous finding does not imply that trading volume and prices are unrelated. Indeed, as Table 1.2 documents, trading volume correlates positively and in a statistically highly significant way with normalized absolute price changes. This finding holds again for all detrending methods. It is in line with patterns documented by Karpoff (1987) and shows that periods of high volume are associated with large relative price changes.

The facts presented in Table 1.2 are fairly standard in the light of the existing finance literature studying trading volume. We complement these facts below with additional empirical evidence on the relationship between trading volume and belief disagreement. Models in which investors disagree about the future prospects from investment have a long tradition in the finance literature, see Hong and Stein (2007) for a survey. We document in Table 1.3 below that there exists a fairly robust positive correlation between aggregate trading volume and the amount of cross-sectional disagreement about future aggregate stock market returns.

Table 1.3 reports the correlation between trading volume and the cross-sectional standard deviations of real survey return expectations ($corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$), as obtained from various survey data sources.[13] The point estimate of the correlation is always positive and often statistically significant when using linear or linear-quadratic detrending or the HP filter. The evidence is less strong when detrending trading volume using the moving average approach, but is otherwise rather robust. Furthermore, to document that

[11] Table 1.2 uses the undetrended PD ratio. Detrending the PD ratio leads to very similar conclusions. For example, using instead the linearly detrended or HP filtered PD ratio, the point estimates for the correlations with turnover range between -0.27 and 0.02, depending on the way turnover is detrended.

[12] Our findings also hold true if one uses data only up to the year 2006, which shows that results are not driven by the recent financial crisis.

[13] Since the Shiller survey asks for expected capital gains, the reported correlations for this survey pertain to the cross-sectional dispersion of capital gain expectations.

Table 1.3
Correlation between trading volume and disagreement

Survey sources	Detrending Method			
	Baseline (linear)	Linear-quadratic	HP filter	MA
UBS-Gallup survey (1-year horizon)				
$corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$	0.41*	0.41**	0.43*	0.17
$corr(TV_t, IQR(\widetilde{E}_t^i R_{t+1}))$	0.36	0.50*	0.65**	0.41**
Shiller survey (3-months horizon)				
$corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$	0.37*	0.40*	0.43**	-0.06
$corr(TV_t, IQR(\widetilde{E}_t^i R_{t+1}))$	0.52**	0.54**	0.63***	0.19
Shiller survey (6-months horizon)				
$corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$	0.60***	0.60*	0.58***	0.03
$corr(TV_t, IQR(\widetilde{E}_t^i R_{t+1}))$	0.43*	0.46*	0.47**	0.09
Shiller survey (1-year horizon)				
$corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$	0.51**	0.52**	0.51***	0.18
$corr(TV_t, IQR(\widetilde{E}_t^i R_{t+1}))$	0.49**	0.55**	0.56***	0.23
CFO survey (1-year horizon)				
$corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$	0.70**	0.65**	0.64***	-0.02

*/**/*** indicate significance at the 10%/5%/1% significance level, respectively.

results are not driven by outliers in the surveys, Table 1.3 also reports the correlation between detrended trading volume and the inter-quartile range (IQR) of the cross-section of survey expectations ($corr(TV_t, IQR(\widetilde{E}_t^i R_{t+1}))$).[14] Results turn out to be robust towards using this alternative dispersion measure.

Overall, the evidence in Table 1.3 shows that trading volume and disagreement are positively correlated in the data.

1.3.3 Disagreement and Stock Market Experience

Given the evidence presented in the previous section, which shows that investor disagreement is systematically related to trading volume, this section explores potential sources of investor disagreement more closely. In particular, it shows that disagreement can be partly related to investor experience: the price expectations of investors with less stock market experience are more heavily influenced by recent stock market performance than

[14]For the CFO survey, we do not observe individual survey responses or the interquartile range, thus cannot perform this robustness check.

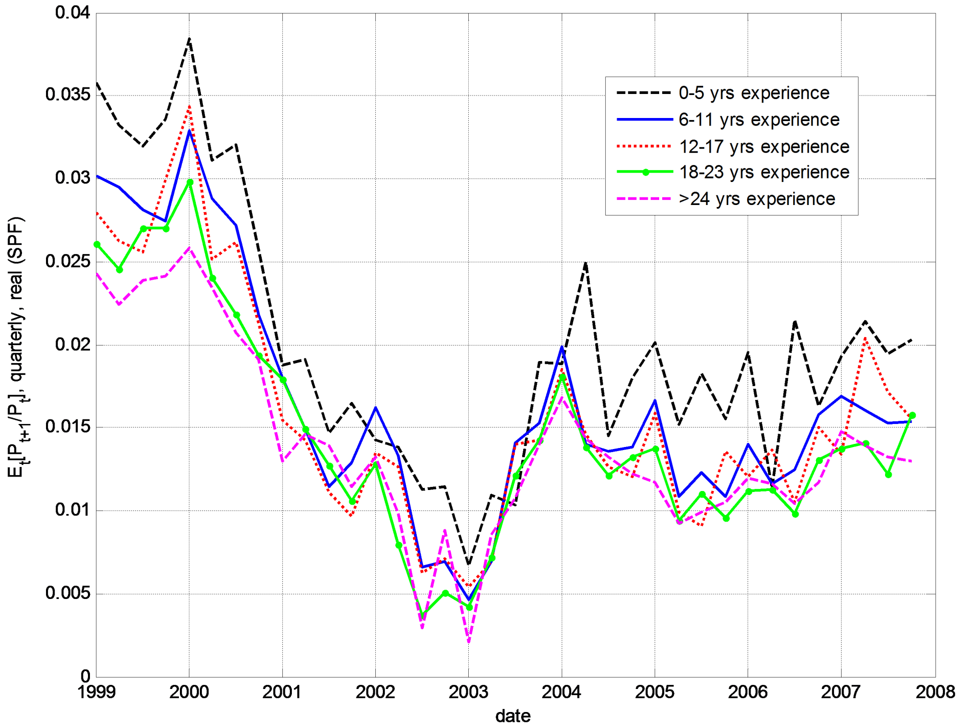

Figure 1.2. Price growth expectations by experience group (UBS survey, real, in quarterly growth rates)

those with more experience.

Adam, Beutel, and Marcet (2015) show that the empirical time series behavior of the *average* price growth expectation in the UBS survey data ($\overline{E}_t[P_{t+1}/P_t]$) can be captured very well by an extrapolative updating equation of the form

$$\overline{E}_t[P_{t+1}/P_t] = \overline{E}_{t-1}[P_t/P_{t-1}] + g\left(\frac{P_t}{P_{t-1}} - \overline{E}_{t-1}[P_t/P_{t-1}]\right), \tag{1.1}$$

which stipulates that the average investor extrapolates observed capital gains into the future. We document below that investors with different numbers of years of experience extrapolate to different degrees.

Figure 1.2 depicts the evolution of quarterly real price growth expectations held by investors with different years of stock market experience, as available from the UBS survey.[15,16] It shows that in the year 1999 and until the beginning of the year 2000, when prior stock market returns have been very high due to the preceding tech stock boom, it is the less experienced investors that tend to be most optimistic about future capital gains. Indeed, investors with 0-5 years of experience expect an average real capital gain of around 3.5% per quarter, i.e., a real gain of about 14% per year, while the most experienced group expects considerably lower capital gains (albeit still very high ones by historical standards). Following the subsequent stock market bust, belief dispersion across investor groups significantly narrows and reaches a low point during the stock market trough in the year 2003. Clearly, this happens because less experienced investors updated expectations more strongly during the market bust. Following the stock market recovery after the year 2003, belief dispersion widened again, with the least experienced investor group then holding once more the highest return expectations, while the two most experienced groups hold the lowest expectations.

Figure 1.2 suggests, in line with evidence presented in Malmendier and Nagel (2011), that the capital gain expectations of less experienced investors react more strongly to realized capital gains. We formally check this hypothesis by estimating the updating parameter g in equation (1.1) for each experience group separately, using the same approach as employed in Adam, Beutel, and Marcet (2015). Table 1.4 reports the estimation outcome and shows that the updating parameter is monotonically decreasing with experience, with the updating parameter of the most inexperienced group of investors being approximately 75% higher than that of the most experienced investor group. The estimated updating gains are all statistically significantly different from zero at the 1% level.[17] Appendix A.4 shows that the gains are significantly different from each other for sufficiently distant experience groups and that the gain of the most experienced investor group is different from those of all other groups at the 1% level.

[15]We choose experience groups with equidistant group boundaries (except for the highest group) and in a way that groups are approximately of similar size. The reported results are robust to using different numbers of groups or different group boundaries, provided one does not consider too many groups, which causes results to become more noisy.

[16]The figure reports the 'own portfolio' return expectations from the UBS survey, as these are available for a longer time period. Results do not depend on this choice, though. We transform nominal return expectations into real expectations using the median inflation forecast from the Survey of Professional Forecasters. To be consistent with our asset pricing model, which models capital gain expectations, we transform real return expectations into a measure of real price growth expectations using the identity $R_{t+1} = \frac{P_{t+1}}{P_t} + \frac{D_{t+1}}{P_t} = \frac{P_{t+1}}{P_t} + \beta^D \frac{D_t}{P_t}$ where β^D denotes the expected gross quarterly real growth rate of dividends that we set equal to its sample average, i.e, $\beta^D = 1.0048$, see Table 1.1. Results are very similar when using alternative plausible values for β^D. Also, since the UBS survey does not have a panel structure, the figure is based on a pseudo panel and reports at each point in time the median expectation of the considered experience group.

[17]Standard errors in Table 1.4 and the p-values reported in Appendix A.4 are computed in a standard way, exploiting the fact that the procedure used for estimating the gain is a nonlinear least squares estimation.

Table 1.4
Estimated updating parameters

Experience (yrs)	0-5	6-11	12-17	18-23	> 23
Estimated g^i	0.0316***	0.0286***	0.0264***	0.0230***	0.0180***
(std. deviation)	(0.0028)	(0.0013)	(0.0017)	(0.0013)	(0.0090)

*/**/*** indicate significance at the 10%/5%/1% significance level, respectively.

1.4 The Asset Pricing Model

This section presents the asset pricing model that we use to replicate the empirical facts documented in the previous section. We consider a model with a unit mass of atomistic investors who trade on a competitive stock market, where trade may be subject to a linear transactions tax. At the beginning of each period, stocks pay a stochastic dividend D_t per unit and investors earn an exogenous wage income W_t. Income from both sources takes the form of perishable consumption goods.

There are $I \geq 1$ types of investors in the economy and a mass $\mu^i > 0$ of each type $i \in \{0, \ldots, I\}$, where $\sum_{i=1}^{I} \mu_i = 1$. Types differ with respect to the beliefs they entertain about the behavior of future stock prices and with regard to their accumulated stockholdings. For the special case without a financial transactions tax and when there is a single investor type, the setup reduces to the one studied in Adam, Beutel, and Marcet (2015).

The Investment Problem. The representative investor of type $i \in \{1, ..., I\}$ solves

$$\begin{aligned} &\max_{\{C_t^i \geq 0, S_t^i\}_{t=0}^{\infty}} E_0^{\mathcal{P}^i} \sum_{t=0}^{\infty} \delta^t \frac{(C_t^i)^{1-\gamma}}{1-\gamma} \qquad (1.2) \\ &\text{s.t.: } S_t^i P_t + C_t^i = S_{t-1}^i (P_t + D_t) + W_t - \tau \left|(S_t^i - S_{t-1}^i) P_t\right| + T_t^i \\ &S_{-1}^i \text{ given,} \end{aligned}$$

where C^i denotes consumption, $\gamma > 1$ the coefficient of relative risk aversion, S^i the agent's stockholdings, $P \geq 0$ the (ex-dividend) price of the stock, $\tau \geq 0$ a linear financial transactions tax, which is levied on the agents' trading volume $\left|(S_t^i - S_{t-1}^i) P_t\right|$ and $T^i \geq 0$ lump sum tax rebates.

Investors' choices are contingent on the history of variables that are exogenous to their decision problem, i.e., time t choices depend on $\{P_j, D_j, W_j, T_j\}_{j=0}^{t}$ and the initial condition S_{-1}^i. $\mathcal{P}^i$ denotes a subjective probability measure, which assigns probabilities to all possible infinite histories $\{P_t, D_t, W_t, T_t\}_{t=0}^{\infty}$. The agent's subjective probabilities may or may not coincide with the objective probabilities, i.e., agents may not know the true probabilities characterizing the behavior of the variables $\{P_t, D_t, W_t, T_t\}_{t=0}^{\infty}$, which are beyond their control, but agents are 'internally rational' in the sense of Adam and Marcet (2011), i.e., behave optimally given their beliefs about external variables.

We consider linear transaction taxes because they are most easily implemented in practice. In addition, non-linear transaction taxes would create incentives to either partition trades into smaller increments or bundle trades of several investors into larger packages, so as to economize on transaction costs. The resulting tax rate would effectively be linear again. To simplify the analysis, we also assume that transaction taxes paid by investors of type i are rebated in the same period in a lump sum fashion, i.e.,

$$T_t^i = \tau \left|(S_t^i - S_{t-1}^i)P_t\right|, \tag{1.3}$$

where S_t^i and S_{t-1}^i on the r.h.s. of the previous equation denote the choices of the representative investor of type i.[18] We thereby eliminate the income effects associated with raising transaction taxes.[19] Appendix A.5 considers an alternative setup without tax rebates ($T_t^i \equiv 0$ for all t, i). It shows that the main quantitative findings are robust to assuming that taxes are not rebated to investors.

The exogenous wage and dividend processes take the form considered previously in Adam, Beutel, and Marcet (2015), with dividends evolving according to

$$\ln D_t = \ln \beta^D + \ln D_{t-1} + \ln \varepsilon_t^D, \tag{1.4}$$

where $\beta^D \geq 1$ denotes the mean growth rate of dividends and, $\ln \varepsilon_t^D$ an i.i.d. growth innovation described further below. The wage income process W_t is chosen such that the resulting aggregate consumption process $C_t = W_t + D_t$ is empirically appealing.[20] In particular, we assume

$$\ln W_t = \ln \rho + \ln D_t + \ln \varepsilon_t^W, \tag{1.5}$$

where

$$\begin{pmatrix} \ln \varepsilon_t^D \\ \ln \varepsilon_t^W \end{pmatrix} \sim iiN\left(-\frac{1}{2}\begin{pmatrix} \sigma_D^2 \\ \sigma_W^2 \end{pmatrix}, \begin{pmatrix} \sigma_D^2 & \sigma_{DW} \\ \sigma_{DW} & \sigma_W^2 \end{pmatrix}\right) \tag{1.6}$$

which implies $E\varepsilon_t^D = E\varepsilon_t^W = 1$.

Substituting the constraint into the objective function and dividing the objective function by $D_0^{1-\gamma}$, the investor's problem can be written as

$$\max_{\{S_t^i\}_{t=0}^\infty} E_0^{\mathcal{P}^i} \sum_{t=0}^\infty \delta^t \left(\frac{D_t}{D_0}\right)^{1-\gamma} \frac{\left(S_{t-1}^i\left(\frac{P_t}{D_t}+1\right) + \frac{W_t + T_t^i}{D_t} - \tau\left|\frac{(S_t^i - S_{t-1}^i)P_t}{D_t}\right| - S_t^i \frac{P_t}{D_t}\right)^{1-\gamma}}{1-\gamma}$$
$$s.t.: S_{-1}^i \text{ given} \tag{1.7}$$

[18]Agents' fully understand that what matters for tax rebates is the trading decision of the representative investor of type i and not their own decision.

[19]Alternative assumptions, e.g., a rebate that is identical across investors at each point in time, would make rebates dependent on the whole distribution of trades in equilibrium and thus on the distribution of investors' beliefs. This would add many additional state variables into investors' decision problem.

[20]For further details, we refer the reader to Adam, Beutel, and Marcet (2015), Section 4.

Due to the linear transaction cost specification, the preceding optimization problem fails to be differentiable. We explain in Section 1.4.1 how we deal with this difficulty.

Subjective Beliefs. To complete the description of the investment problem we now specify investors' subjective probability measure $\mathcal{P}^i$. We first assume that agents know the processes (1.4) and (1.5), i.e., hold rational dividend and wage expectations.[21] In a second step, we seek to specify subjective price beliefs in a way that allows us to capture the extrapolative nature of price expectations, as implied by survey data. In particular, following Adam, Beutel, and Marcet (2015), we set up a belief system for prices that leads to expectation dynamics of the kind described by equation (1.1), which captures the empirical behavior of survey expectations. To this end, we endow agents with a belief system that allows for persistent deviations of the growth rate of prices from the growth rate of dividends. Specifically, we assume that agent i's perceived law of motion of prices is given by

$$\ln P_{t+1} - \ln P_t = \ln \beta^i_{t+1} \ + \ln \varepsilon^{1,i}_{t+2} + \ln \varepsilon^{2,i}_{t+1}, \tag{1.8}$$

where $\varepsilon^{1,i}_{t+2}, \varepsilon^{2,i}_{t+1}$ denote (not directly observable) transitory shocks to price growth and β^i_{t+1} a persistent price growth component that slowly drifts over time according to

$$\ln \beta^i_{t+1} \ = \ln \beta^i_t \ + \ln \nu^i_{t+1}, \tag{1.9}$$

and where the persistent component of price growth $\ln \beta^i_{t+1}$ is also unobserved. The setup just described can capture periods with sustained increases in the price dividend ratio ($\beta^i_{t+1} > \beta^D$), as well as periods with sustained decreases ($\beta^i_{t+1} < \beta^D$). The perceived innovations $\ln \varepsilon^{1,i}_{t+2}, \ln \varepsilon^{2,i}_{t+1}$ and $\ln v^i_{t+1}$ are assumed to be jointly normally distributed according to

$$\begin{pmatrix} \ln \varepsilon^{1,i}_{t+2} \\ \ln \varepsilon^{2,i}_{t+1} \\ \ln \nu^i_{t+1} \end{pmatrix} \sim iiN \left(\begin{pmatrix} -\frac{\sigma^2_{\varepsilon,1}}{2} \\ -\frac{\sigma^2_{\varepsilon,2}}{2} \\ -\frac{(\sigma^i_v)^2}{2} \end{pmatrix}, \begin{pmatrix} \sigma^2_{\varepsilon,1} & 0 & 0 \\ 0 & \sigma^2_{\varepsilon,2} & 0 \\ 0 & 0 & \sigma^{i\,2}_\nu \end{pmatrix} \right), \tag{1.10}$$

where the variances $\sigma^2_{\varepsilon,1}$, $\sigma^2_{\varepsilon,2}$ of the transitory components are identical for all agents. We allow the perceived variance of the innovation to the persistent component ($\sigma^{i\,2}_\nu$) to differ across investors, so as to be able to capture the different responsiveness of survey expectations to realized price growth rates, as documented in Section 1.3.3.

The previous setup defines an optimal filtering problem for agents, in which they need to decompose observed price growth ($\ln P_{t+1} - \ln P_t$) into its persistent and transitory components ($\ln \beta^i_{t+1}$ and $\ln \varepsilon^{1,i}_{t+2} + \ln \varepsilon^{2,i}_{t+1}$, respectively). In the special case, that the

[21] This is motivated by the fact that within the present setting with time-separable preferences, (reasonable amounts of) extrapolation of wage and dividend beliefs would add very little to price volatility. This holds true for models with rational price expectations, as discussed in Section 2 in Adam, Beutel, and Marcet (2015), but also for models with subjective price beliefs, see for example Section V.A in Adam, Beutel, and Marcet (2015).

two transitory shock components are both unobserved and can thus be combined to $\ln \varepsilon_t^i = \ln \varepsilon_{t+1}^{1,i} + \ln \varepsilon_t^{2,i}$ with variance $\sigma_\varepsilon^2 = \sigma_{\varepsilon,1}^2 + \sigma_{\varepsilon,2}^2$, Adam, Beutel, and Marcet (2015) show, that under the assumption of a normal prior with variance equal to its Kalman filter steady state value, price growth beliefs can be summarized by a single state variable m_t^i that evolves according to

$$\begin{aligned} \ln m_t^i &= \ln m_{t-1}^i - \frac{(\sigma_v^i)^2}{2} \\ &\quad + g^i \left(\ln P_t - \ln P_{t-1} + \frac{(\sigma_\varepsilon^i)^2 + (\sigma_v^i)^2}{2} - \ln m_{t-1}^i \right) \end{aligned} \tag{1.11}$$

$$g^i = \frac{(\sigma^i)^2}{\sigma_\varepsilon^2 + (\sigma^i)^2}, \tag{1.12}$$

where

$$(\sigma^i)^2 \equiv \frac{-(\sigma_\nu^i)^2 + \sqrt{((\sigma_\nu^i)^2)^2 + 4(\sigma_\nu^i)^2\sigma_\varepsilon^2}}{2}$$

is the Kalman filter steady state variance. The state variable $\ln m_t^i$ describes the mean of $\ln \beta_t^i$ conditional on the information available at time t, i.e., $\ln \beta_t^i$ is conditionally $N(\ln m_t^i, (\sigma^i)^2)$-distributed, which implies

$$E_t^{\mathcal{P}^i}\left[\frac{P_{t+1}}{P_t}\right] = m_t^i e^{(\sigma^i)^2/2}.$$

This previous result, together with equation (1.11) shows that optimal belief updating delivers - up to a log-exponential transformation - the updating equation (1.1) considered in the empirical section. Moreover, equation (1.12) shows that the optimal updating parameter g^i is a positive function of the variance $(\sigma_\nu^i)^2$, which allows us to replicate the empirically observed heterogeneity in the belief updating equations.

To avoid simultaneity between prices and price beliefs, which may give rise to multiple market clearing price and price belief pairs, we shall rely on a slightly modified information structure, where agents observe $\ln \varepsilon_t^{1,i}$ as part of their time t information set. Adam, Beutel, and Marcet (2015) show how such a modified information structure gives rise to an updating equation of the form

$$\ln m_t^i = \ln m_{t-1}^i + g^i \left(\ln P_{t-1} - \ln P_{t-2} - \ln m_{t-1}^i \right) - g \ln \varepsilon_t^{1,i}, \tag{1.13}$$

which has lagged price growth enter.[22]

To complete the description of the belief system, we need to specify investors' beliefs about the behavior of the lump sum tax rebate T_t^i. We shall assume that agents understand that the tax rebates do not depend on their own decision, instead on the choices

[22] Price growth expectations are then given by $E_t^{\mathcal{P}^i}[P_{t+1}/P_t] = m_t^i$.

of the representative investor of the same type i. Moreover, we assume that agents know the tax rebate function (1.3).[23]

Market Clearing. The stock market clearing condition is given by

$$\sum_{i=1}^{I} S_t^i \mu_i = 1 + u_t,$$

where the left-hand side denotes total stock demand by investors of all types and the right-hand side total stock supply. We incorporate a small exogenous stochastic component u_t into stock supply, which we assume to be white noise, uniformly distributed and to have support $[-\overline{u}, \overline{u}]$ for some $\overline{u} > 0$ sufficiently close to zero. Stock supply shocks u_t may thereby capture the issuance of new stocks or stock repurchases by firms.[24] We add these shocks because linear financial transaction taxes lead to piecewise price-insensitive demand curves, which can give rise to equilibrium price indeterminacy in the absence of supply shocks. In our numerical applications, we make sure that $\overline{u}$ is sufficiently small such that it has no noticeable effects on the outcomes that emerge in the absence of a financial transaction tax. For the case with transaction taxes, the supply shock effectively only selects the equilibrium price whenever price-insenstitive demand curve may create the potential for price indeterminacy.

1.4.1 Solution Approach

This section explains how one can solve for the optimal solution of the non-differentiable problem (1.7). The approach we pursue consists of defining an alternative optimization problem with a differentiable transaction cost specification, so that a standard solution approach based on first order conditions can be applied. The alternative problem has the property that all choices that are feasible in the original problem are also feasible in the alternative problem. Therefore, if the optimal solution to the differentiable problem is a feasible choice in (1.7), then it must also solve (1.7).

The alternative problem we consider is

$$\begin{aligned} &\max_{\{S_t^i\}_{t=0}^{\infty}} E_0^{\mathcal{P}^i} \sum_{t=0}^{\infty} \delta^t \left(\frac{D_t}{D_0}\right)^{1-\gamma} \frac{\left(S_{t-1}^i\left(\frac{P_t}{D_t}+1\right) + \frac{W_t+T_t^i}{D_t} - \tau_t^i \frac{(S_t^i - S_{t-1}^i)P_t}{D_t} - S_t^i \frac{P_t}{D_t}\right)^{1-\gamma}}{1-\gamma} \\ &s.t.: S_{-1}^i \text{ given} \end{aligned} \tag{1.14}$$

[23] This assumption considerably simplifies the analysis: since in equilibrium individual actions coincide with those of the representative investor of the same type, we do not need to incorporate any additional state variables that characterize the future evolution of lump sum taxes, when writing a recursive representation of the agents' decision problem.

[24] Alternatively, they may capture changes to asset float, as discussed in Ofek and Richardson (2003) and Hong, Scheinkman, and Xiong (2006). In any case, these shocks capture (exogenous) stock demand or supply that is not coming from the consumers described around equation (1.2).

where $\tau_t^i \in [-\tau, \tau]$ denotes a state-contingent but fully linear transaction tax/subsidy and where T_t^i is given by (1.3). Problem (1.14) is differentiable and can be solved in a standard way using first-order conditions. Moreover, since $\tau_t^i \in [-\tau, \tau]$, all stockholding plans that are feasible in the original problem (1.7) continue to be feasible in the alternative problem (1.14).

Suppose that the state-contingent transactions cost function τ_t^i and the associated optimal stockholding plan $\left\{S_t^{i,opt}\right\}_{t=0}^{\infty}$ solving (1.14) jointly satisfy for all $t \geq 0$ the following property

$$\begin{array}{ll} \tau_t^i = \tau & \text{at contingencies where } S_t^{i,opt} > S_{t-1}^{i,opt} \\ \tau_t^i = -\tau & \text{at contingencies where } S_t^{i,opt} < S_{t-1}^{i,opt} \\ \tau_t^i \in [-\tau, \tau] & \text{at contingencies where } S_t^{i,opt} = S_{t-1}^{i,opt}, \end{array} \tag{1.15}$$

then $\left\{S_t^{i,opt}\right\}_{t=0}^{\infty}$ is also feasible in the original problem (1.7) and thus the solution to (1.7). The task of solving the original non-differentiable problem (1.7) is thus equivalent to finding a state contingent tax function τ_t^i such that condition (1.15) holds for the optimal solution of the alternative differentiable problem (1.14).

For a given $\{\tau_t^i\}_{t=0}^{\infty}$ the solution to (1.14) is characterized by the first order condition

$$\left(\frac{C_t}{D_t}\right)^{-\gamma} (1+\tau_t^i)\frac{P_t}{D_t} = \delta E_t^P \left(\frac{C_{t+1}}{D_{t+1}}\right)^{-\gamma} \left(\frac{D_{t+1}}{D_t}\right)^{1-\gamma} \left(\frac{P_{t+1}}{D_{t+1}}(1+\tau_{t+1}^i)+1\right) \tag{1.16}$$

As noted above, investor i's subjective beliefs can be summarized by the recursively evolving state variable m_t^i. Provided the state contingency of the tax function can be expressed in the form $\tau_t^i = \tau^i(S_{t-1}^i, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t^i)$, where the arguments in the function should be interpreted as the choices and beliefs of the representative agent of type i, the optimal stock holding policy then also has a recursive representation of the form $S_t = S^i(S_{t-1}^i, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t^i)$, by the same arguments as put forward in Adam, Beutel, and Marcet (2015).[25,26]

Our numerical solution routines, which are described in Appendix A.2 simultaneously solve for the functions $\tau^i(\cdot)$ and $S^i(\cdot)$ that jointly satisfy equations (1.15) and (1.16). Numerically solving for the optimal solution is computationally costly. Despite extensive reliance on parallelization, the numerical computation of the solution and the evaluation of the Euler errors takes around 30 hours of computing time.

[25] The fact that the transaction costs are linear and that under the stated assumptions the tax rebate T^i is a function of the same state variables is key for this result.

[26] The fact that τ_t^i depends on S_{t-1}^i is just a convenient way to summarize dependence of the tax function on past values of P_t, D_t and W_t. It does not mean that the agent thinks that τ_t^i depends on its own choices, in fact, as should be clear from the first order condition (1.16), the agent takes τ_t^i as exogenously given.

1.5 Outcomes under Objective Price Beliefs

Before presenting the model outcome under subjective price beliefs, this section briefly discusses the model predictions for the case where agents hold rational price expectations. With objective price beliefs and with investors holding identical initial stock endowments, differences between investor types disappear. The model then reduces to a representative agent rational expectations model with time-separable preferences. As shown in Adam, Beutel, and Marcet (2015), the pricing implications of the model then display a well-known set of shortcomings. The standard deviation of the price dividend ratio, for instance, is one order of magnitude below that observed in the data and displays virtually no persistence over time. The model thus fails to replicate the large and protracted run-ups and reversals that can be observed for the PD ratio in U.S. data. The model also fails to replicate the positive correlation between the PD ratio and expected returns, as evidenced in survey data. Finally, with rational price expectations, the model does not give rise to trade in equilibrium, thus cannot be related to the documented facts on trading activity. As we show in the next section, model performance strongly improves, once one incorporates the kind of extrapolative behavior documented in survey data.

1.6 Quantitative Model Performance

This section evaluates the quantitative performance of our asset pricing model in the absence of FTTs with subjective price beliefs given by equations (1.8) and (1.9). Performance is evaluated in terms of the ability to match the stylized facts presented in Section 1.3. The effects of introducing FTTs will be studied in Section 1.8.

We parameterize our model using the model parameters employed in Adam, Beutel, and Marcet (2015), which are summarized in Table 1.5. Table 1.5 also lists the value for the support of stock supply shocks, which is a new parameter and set such that the amount of trade caused by these shocks amounts to less than 0.3% of the average trading volume in a setting without FTTs. Since trading volume is only weakly affected for the considered range of FTTs, the same holds approximately true for the case with FTTs. Furthermore, we verify that in the absence of FTTs, stock supply shocks affect the model moments in almost non-noticable ways.

Motivated by the evidence in Table 1.4, we consider a model with 5 agent types, each of which has mass 1/5, and assign to them the point estimates of the updating gains from Table 1.4.[27]

Table 1.6 compares the model generated moments in the absence of FTTs to those in the data.[28] The model moments for our baseline calibration are reported in the third

[27]Recall that we chose the experience groups in Table 1.4 so as to have approximately the same number of investors in each group.

[28]All simulation results are based on 100.000 quarters of simulated data, where the first 10.000 quarters

Table 1.5
Model calibration

Parameter	Value	Calibration target
β^D	1.0048	average quarterly real dividend growth
σ_D	0.0192	std. deviation quarterly real dividend growth
ρ	22	average consumption-dividend ratio
σ_{DW}	$-3.74 \cdot 10^{-4}$	*jointly chosen* s.t. $corr_t(C_t/C_{t-1}, D_t/D_{t-1}) = 0.2$
σ_W	0.0197	and $std_t(C_t/C_{t-1}) = \frac{1}{7} std_t(D_t/D_{t-1})$
σ_ε	0.0816	std. deviation of quarterly real stock price growth
δ	0.995	average PD ratio
γ	2	– none –
$\overline{u}$	$1 \cdot 10^{-5}$	– none –

column, while the fourth column reports the associated t-ratios for each considered data moment.[29] Overall, our asset pricing model does a good job in replicating the pure stock price moments, i.e., the first five moments reported in the table. It matches particularly well the mean and autocorrelation of the PD ratio, as well as the mean of quarterly real stock returns. It produces, however, too much volatility for the PD ratio and for returns. The model also does a good job in capturing the observed high positive correlation between the PD ratio and average return expectations in the survey data ($corr(PD_t, \overline{E}_t R_{t+1})$).

Regarding the newly added moments, the model generates a high positive autocorrelation in trading volume ($corr(TV_t, TV_{t-1})$), albeit the model correlation is too high relative to the one found in the data. The model also manages to quantitatively capture the positive correlation between trading volume and (normalized) absolute price changes ($corr(TV_t, |P_t/P_{t-1} - 1|)$). When looking at the correlation between trading volume and the PD ratio ($corr(TV_t, PD_t)$), the model produces a fairly weak positive correlation, but one that is stronger than in the data. The model also generates a positive correlation between trading volume and cross sectional dispersion of return expectations ($corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$), but again overstates this correlation relative to the data.[30] The latter should not be surprising, given that in our simple model belief dispersion is the

are considered as a burn-in and discarded when calculating model moments. Also, to make results from different simulations more comparable, we use fixed sequences for the exogenous driving processes (wages, dividends, stock supply shocks).

[29] The t-ratio is based on an estimate of the standard deviation of the data moment as a measure of uncertainty. Since the data moments in Table 1.6 are not truly data moments, but functions of such moments, we estimate the standard deviation using the so-called delta method, as described in Cox (1998) or in the online appendix to Adam, Marcet, and Nicolini (2016).

[30] The data moment reported in Table 1.6 is the one pertaining to the UBS survey, which has also been used to compute $corr(PD_t, \overline{E}_t R_{t+1})$ in the data.

Table 1.6
Quantitative match of the asset pricing model

Considered moments	U.S. data	Baseline model (no tax)	t-ratio	Reduced gain (no tax)	t-ratio
$E[PD]$	139.77	135.77	0.16	117.16	0.91
$std(PD)$	65.17	122.13	-3.84	92.96	-1.88
$corr(PD_t, PD_{t-1})$	0.98	0.98	0.84	0.98	-0.13
$std(r^s)$ (%)	8.00	11.63	-9.05	8.27	-0.68
$E[r^s]$ (%)	1.89	2.11	-0.47	1.84	0.11
$corr(PD_t, \overline{E}_t R_{t+1})$	0.79	0.84	-0.78	0.84	-0.73
$corr(TV_t, TV_{t-1})$	0.89	0.97	-4.29	0.97	-4.14
$corr(TV_t, PD_t)$	-0.07	0.37	-5.79	0.47	-7.09
$corr(TV_t, \vert P_t/P_{t-1} - 1\vert)$	0.34	0.25	1.12	0.28	0.78
$corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$	0.41	0.95	-3.67	0.92	-3.50

only reason why agents want to trade.

Since the baseline model produces an 'anti-puzzle' in the form of too much stock price volatility relative to the data, we also consider a model version in which we dampen the extrapolative component in belief updating. This is motivated by the fact that the updating gains in Table 1.4 are themselves estimated with uncertainty. Specifically, we reduce the point estimates from Table 1.4 by 2.5 times the estimated standard deviation of the point estimate[31], leaving all other parameters unchanged. The resulting model moments are reported in the second to last column in Table 1.6 below, with the last column reporting the associated t-ratios. Price and return volatility are now in line with data, while all other moments remain largely unaffected.

Overall, we find that the model does a good job in quantitatively replicating the joint behavior of stock price, trading volume and price expectations.

1.7 Asset Price Booms and their Implications

This section illustrates that stock prices in the model occasionally embark on a self-sustaining asset price boom and bust cycle. Unlike in the representative agent model of Adam, Beutel, and Marcet (2015), such cycles have large welfare implications for different agent types.

To illustrate the potential of the model to generate boom-bust cycles and to compute the welfare implications of such cycles, we conduct a simple controlled experiment using

[31] We use for each gain parameter the gain specific standard deviation reported in the last row of Table 1.4.

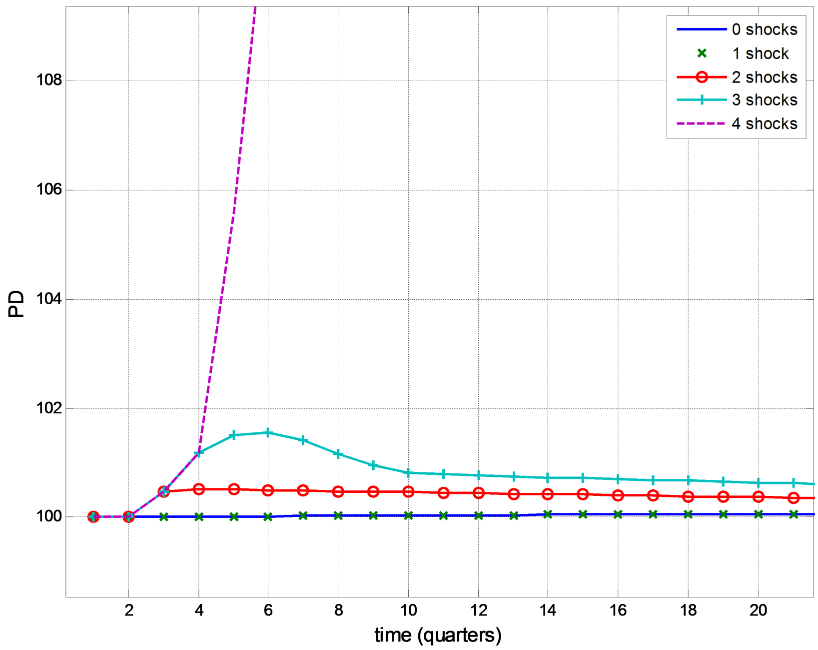

Figure 1.3. Response of the PD ratio to dividend growth shocks: initial periods

the baseline model from the previous section: we fix agents' initial stockholdings and initial beliefs at their ergodic sample means; we then shock the economy with n positive dividend growth shocks of a two standard deviation size. Such or larger positive dividend shocks occur with a probability of about 2.5% per quarter. We shut down all other shocks, including dividend growth shocks after period n. We begin the experiment with $n = 1$ and successively increase n until we obtain a stock price boom and bust cycle from period $n+1$ onwards. Figure 1.3 depicts - for different values of n - the equilibrium outcomes for the PD ratio during the initial periods. While the PD ratio reacts very little to the positive news when $n = 1$ (stock prices, however, do react to the positive dividend news), the increase in price optimism starts to increase slightly the PD ratio for $n = 2$ and $n = 3$. For $n = 4$ one suddenly obtains a very large stock price boom and a subsequent price bust, see Figure 1.4.[32] The economic forces driving the boom and bust dynamics are explained in detail in Adam, Beutel, and Marcet (2015). Here, we only note that the boom results from the fact that agents - having observed price increases - become optimistic about future price growth and eventually bid up stock prices by sufficient amounts, so that price increases and increasing optimism mutually reinforce each other. This effect is set in motion whenever a sufficient number of positive

[32]Increasing n further would lead to very similar boom-bust dynamics as for the case with $n = 4$.

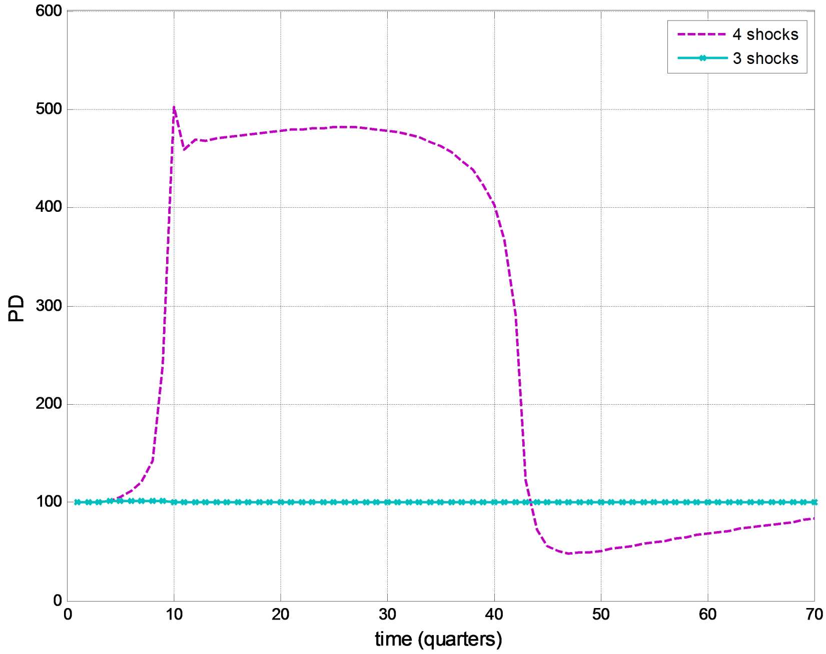

Figure 1.4. Response of the PD ratio: three versus four dividend growth shocks

fundamental shocks, e.g., dividend growth shocks, occurs. The boom comes to an end, when agents' increased wealth leads them to eventually increase consumption demand, so that stock demand ceases to increase further with increased optimism. Prices then stagnate, which means that they fail to fulfill the high growth expectations of agents. Agents then revise growth beliefs downwards and set in motion a price bust. The bust causes a temporary undershooting of the PD ratio below its ergodic mean, but prices eventually return close to their ergodic mean absent further shocks, see Figure 1.4.

Figure 1.5 depicts the PD ratio (top panel) together with agents' equilibrium trading decisions (middle panel) and return expectations (bottom panel) for the boom-bust episode triggered by four positive dividend growth shocks. To increase readability of the graph, we only report the trading patterns and return expectations of agents with the highest and lowest updating gain parameters.[33] In the UBS survey, high gains were estimated for agents with few years of stock market experience, while the most experienced group displayed a low updating gain. For this reason we refer to agents with a high (low) gain as inexperienced (experienced) agents.

Figure 1.5 shows that in the initial phase of the stock price boom, inexperienced

[33] Agents types with intermediate updating gain values take intermediate decisions that are in between those shown in the figure.

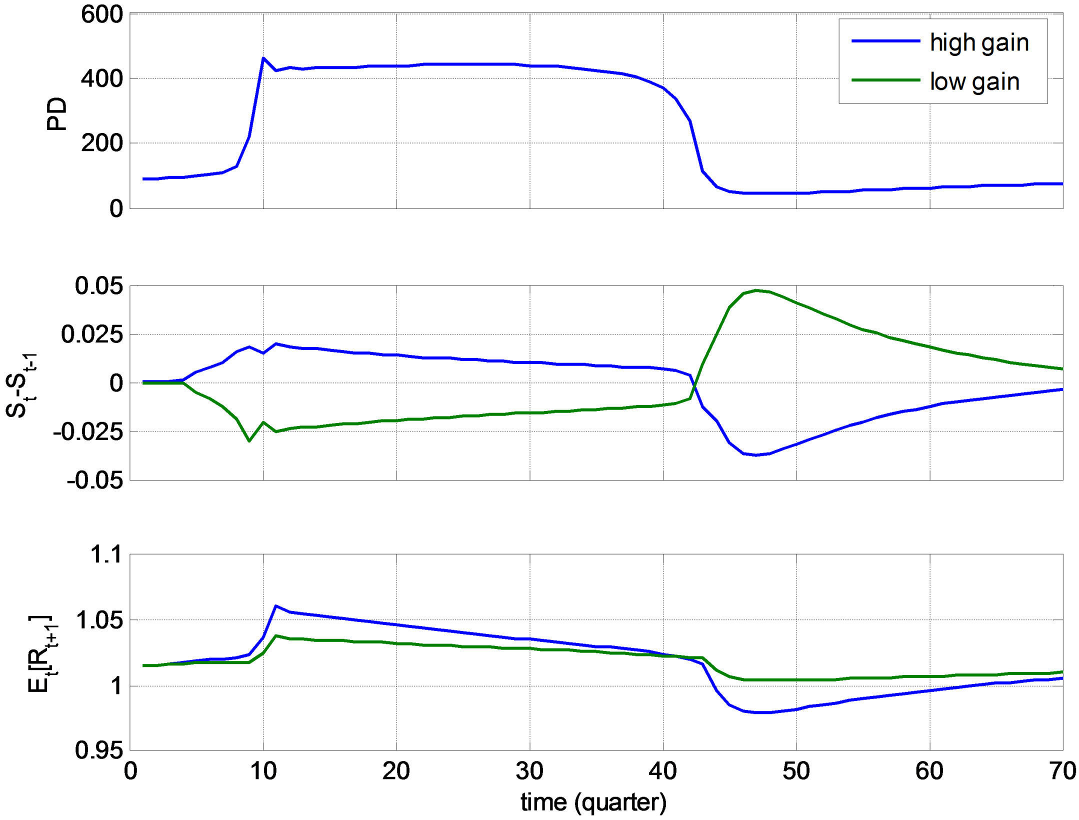

Figure 1.5. PD ratio, trading and return expectations over a boom-bust cylce (baseline model, no tax)

Table 1.7
Welfare cost of a stock price boom-bust episode

Gain	0.0316	0.0286	0.0264	0.0230	0.0180
Permanent cons. variation (%)	-7.01	-3.51	-1.27	1.73	5.24

agents do rather well. They start buying stocks early on and well before prices approach their peak value. Experienced investors sell assets during the boom phase, i.e., much too early. Yet, once the PD ratio is high, inexperienced investors are much more optimistic about future returns than experienced investors, see the bottom panel. As a result, inexperienced investors continue buying stocks from low gain types at high prices (relative to dividends). Also, inexperienced investors continue buying during much of the price bust phase and only sell in significant amounts once the PD ratio started undershooting its long-run mean. Thus, even though inexperienced investors are doing well initially, this fails to be the case over the entire boom-bust cycle.

To gauge the welfare effects of a boom-bust episode, we compare the outcome in Figure 1.5 to a situation in which the same shocks occur, but where agents hold their beliefs constant at the initial value, i.e., do not respond to the price movements triggered by the dividend growth shocks, so that there is no asset price boom. We can then compute the permanent proportional consumption variation that would make (ex-post realized) utility in the setting with constant beliefs and without an asset price boom identical to the (ex-post realized) utility in the setting with the asset price boom shown in Figure 1.5. Outcomes are reported in Table 1.7, which shows that asset price booms are extremely costly for inexperienced agents and extremely beneficial for experienced investors: the welfare equivalent consumption variations of a boom-bust episode amount to a permanent change in consumption of several percentage points.

1.8 The Effects of Financial Transaction Taxes

We now consider the implications of introducing linear financial transaction taxes, focusing on the implication of FTTs for the behavior of asset pricing moments, the patterns of boom-bust dynamics and trading volume.

Table 1.8 reports how the asset pricing moments from the baseline model in Table 1.6 are affected by various tax rates. The main effect of financial transaction taxes consists of increasing asset price volatility, as measured by the standard deviation of quarterly stock returns ($std(r^s)$) and the standard deviation of the PD ratio ($std(PD)$).[34] Except for the reduced correlation between trading volume and prices ($corr(TV_t, PD_t)$,

[34]For very high tax rates (10%) the volatility of the PD ratio starts to fall, while return volatility continues to increase. We discuss this issue further below.

Table 1.8
Effects of introducing financial transaction taxes

Considered moments	No tax	1% tax	2% tax	4% tax	10% tax
$E[PD]$	135.77	137.21	139.74	142.47	146.27
$std(PD)$	122.13	123.18	125.42	127.10	125.24
$corr(PD_t, PD_{t-1})$	0.98	0.98	0.98	0.98	0.98
$std(r^s)$ (%)	11.63	11.85	12.14	12.55	14.04
$E[r^s]$ (%)	2.11	2.14	2.18	2.24	2.49
$corr(PD_t, \overline{E}_t R_{t+1})$	0.84	0.85	0.86	0.87	0.89
$corr(TV_t, TV_{t-1})$	0.97	0.97	0.97	0.97	0.94
$corr(TV_t, PD_t)$	0.37	0.35	0.33	0.29	0.17
$corr(TV_t, \lvert P_t/P_{t-1} - 1\rvert)$	0.25	0.25	0.24	0.21	0.05
$corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$	0.95	0.94	0.93	0.92	0.87
# of booms per 100 yrs*	1.82	1.95	2.12	2.40	3.06
average boom length (quarters)*	32.42	31.87	31.41	30.44	27.21
average boom peak (PD)*	491.03	485.82	480.31	469.95	443.86
$E[TV]$ relative to no tax (%)	100.00	99.64	101.49	102.52	117.85

*A boom starts in the first period in which the quarterly PD ratio exceeds a value of 250 and ends once it falls below 200.

$corr(TV_t, |P_t/P_{t-1} - 1|))$, the remaining asset pricing moments from Table 1.6 prove to be rather robust towards the introduction of FTTs.

The last four rows in Table 1.8 report a number of additional statistics about asset price boom-bust episodes and trading volume. These statistics allow to asses in greater detail why asset price volatility increases with the introduction of FTTs. The fourth to last row in Table 1.8, for example, reports the number of asset price boom episodes per 100 years of simulated data, where we define the beginning of a boom as the first time in which the quarterly PD ratio exceeds a level of 250 and the end of a boom as the first time it falls below 200 thereafter.[35] The results in the table show that the number of stock price booms is monotonically increasing in the FTTs, with boom-bust episodes becoming about a third more likely relative to the case without transaction taxes when the tax rate reaches 4%.

The third and second to last rows in Table 1.8 display, respectively, information about the length of the boom episodes and the average peak value of the PD reached during these episodes. It shows that booms tend to become shorter lived and somewhat less pronounced as the tax rate rises, but these effects are not very strong for tax rates up to

[35]The reported numbers are very robust to choosing different thresholds because boom-bust episodes are periods in which prices display a clearly distinct behavior.

Table 1.9
Welfare implications of FTTs (welfare equiv. permanent cons. variations, in %)

Gain	0.0316	0.0286	0.0264	0.0230	0.0180
1% tax	-0.34	-0.14	-0.02	0.06	0.25
2% tax	-0.84	-0.36	-0.06	0.16	0.62
4% tax	-1.56	-0.64	-0.07	0.28	1.18
10% tax	-2.76	-1.00	0.10	0.39	2.15

4%. As a result, the effect of an increased number of booms dominates and the standard deviation of the PD ratio increases with the tax rate. For a 10% tax rate, the decrease in the peak level of the PD during booms and the reduced length of stock price booms start to dominate, causing the standard deviation of the PD ratio to decreases, even if the standard deviation of returns still increases.

Somewhat surprisingly, the average trading volume (relative to the case without FTTs) tends to increase with the level of FTTs. This occurs because there is more trade during booms times, as belief disagreements are then larger, and because booms become more likely with the introduction of FTTs.

Table 1.9 reports the welfare implications associated with introducing different tax rates. Starting from the ergodic mean for stock holdings and beliefs in the no-tax economy, the table reports the welfare equivalent permanent consumption variation that would make different agent types in the economy with taxes as well-off in expected terms as in the economy without taxes.[36] Table 1.9 clearly shows that agents that extrapolate more, i.e., inexperienced investors in our survey sample, tend to lose, while more experienced investors tend to win in expected terms.[37] For all agent types, except the median type, whose utility is largely unaffected by the tax rate, the gains and losses monotonically increase with the tax rate. Wealth redistribution between investors thus increases with the tax rate.

Table 1.10 provides additional insights by reporting asset price moments conditional on being in a boom period, as defined above, and conditional on being in 'normal times', i.e., periods that are not identified as boom periods. Clearly, the PD ratio is considerably higher during boom times and so is the standard deviation of the PD ratio. Mean quarterly stock returns during boom periods are considerably higher than in normal times,

[36]We use objective probabilities to compute agents' expected utility.

[37]The welfare effects in Table 1.9 are smaller than those reported in Table 1.7. The latter reports the effects of a single stock price boom episode relative to the counterfactual outcome without a boom. Since booms are (in expected terms) not likely to occur within the immediate future, when starting the simulation at the ergodic mean, the welfare effects in Table 1.9 are not as large as those reported in Table 1.7.

Table 1.10
Conditional Asset Price Moments

	No tax	1% tax	2% tax	4% tax	10% tax
Boom times*					
$E[PD]$	424.63	419.79	415.04	406.56	384.06
$std(PD)$	44.93	44.13	44.27	42.89	42.01
$corr(PD_t, PD_{t-1})$	0.48	0.48	0.47	0.49	0.53
$std(r^s)$ (%)	23.06	22.86	22.52	21.76	19.34
$E[r^s]$ (%)	3.68	3.74	3.67	3.55	3.15
$E[TV]$ rel. to no tax (%)	100.00	96.13	93.12	88.27	80.13
Normal times$^+$					
$E[PD]$	85.87	85.33	84.63	83.59	83.72
$std(PD)$	15.37	15.89	16.68	18.14	23.62
$corr(PD_t, PD_{t-1})$	0.84	0.84	0.85	0.85	0.86
$std(r^s)$ (%)	8.15	8.36	8.65	9.31%	12.26
$E[r^s]$ (%)	1.84	1.84	1.88	1.95	2.32
$E[TV]$ rel. to no tax (%)	100.00	99.60	101.43	102.69	129.90

* A boom starts in the first period in which the quarterly PD ratio exceeds a value of 250 and ends once it falls below 200.

* Normal times are all those periods not classified as boom periods.

but stock returns also display a considerably larger standard deviation. Furthermore, while the introduction of FTTs reduces the volatility of the PD ratio and returns during boom periods, FTTs increase both of these standard deviations during normal times. As we show below, it is precisely the increase in volatility during normal times coupled with extrapolative behavior which causes stock price booms to become more likely.

Table 1.10 shows that trading volume decreases with the size of the FTT during boom periods, but - somewhat paradoxically - increases during normal times. Upon closer inspection, we find that for tax rates up to 4% the increase in trading volume during normal times is purely driven by post-boom trading activity. As can be seen from Figure 1.5, trading activity stays high long after the PD ratio returned to values below 200. Once one removes these post-boom periods from the normal times, trading volume is actually decreasing with the FTTs in normal times.[38]

To illustrate further how FTTs increase the likelihood of boom-bust cycles, we now perform a similar experiment as carried out in Section 1.7 for the case without a tax.

[38]The situation is different for very high tax rates (10%). Trading activity then increases also during normal times, even when excluding post-boom periods. This occurs because the large increase in price volatility leads to an amount of belief disagreement and thus trade in normal times, which more than compensates the trade-reducing effect of the tax.

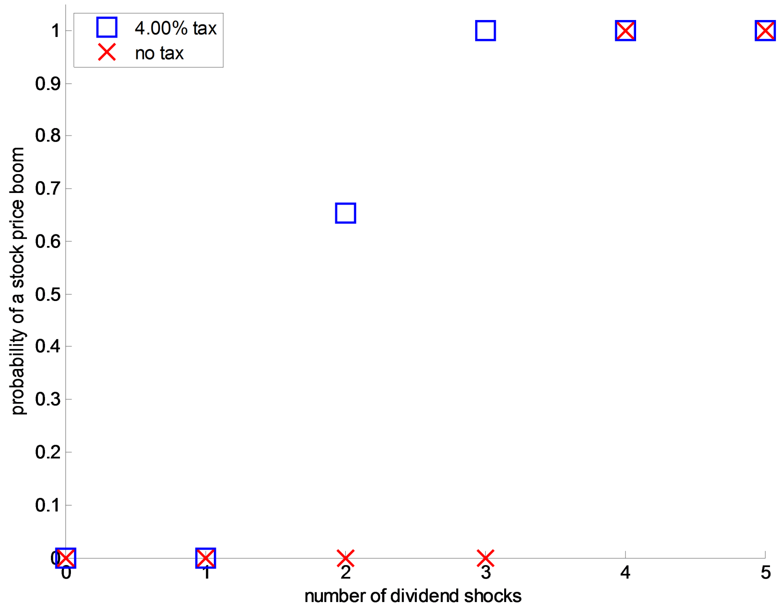

Figure 1.6. FTTs and the likelihood of stock price booms

Specifically, we consider the model with a FTT of 4% and fix initial stockholdings and initial beliefs at their ergodic sample means. We then shock the economy with $n \geq 0$ positive dividend growth shocks of two standard deviations. Yet, this time we continue to let the small exogenous stock supply shocks operate at all times. These shocks are themselves not enough to generate stock price booms, but can do so in combination with dividend shocks.

Figure 1.6 depicts the probability that the economy embarks on a stock price boom as a function of the number of dividend growth shocks, integrating over possible realizations of the stock supply shocks.[39] For the case without a FTT, booms start to emerge once n increases above 4.[40] The situation differs for the case with a 4% FTT, where fewer fundamental shocks are required to start a boom episode. For $n \leq 1$, the economy never embarks on a stock price boom, but for $n = 2$ stock price booms emerge in more than 60% of the cases and for $n \geq 3$ virtually always. This shows that booms become more

[39] As before, we define a boom as a situation where the PD ratio subsequently increases above 250 at some point. We consider up to 12 quarters after the last dividend shock. Results prove very robust to choosing different thresholds and period limits. Probabilities are computed from averaging the outcome of 500 stochastic realizations.

[40] Since we now let stock supply shocks also operate in the case without a tax, this shows that the findings of Figure 1.4 are robust to the introduction of the stock supply shock.

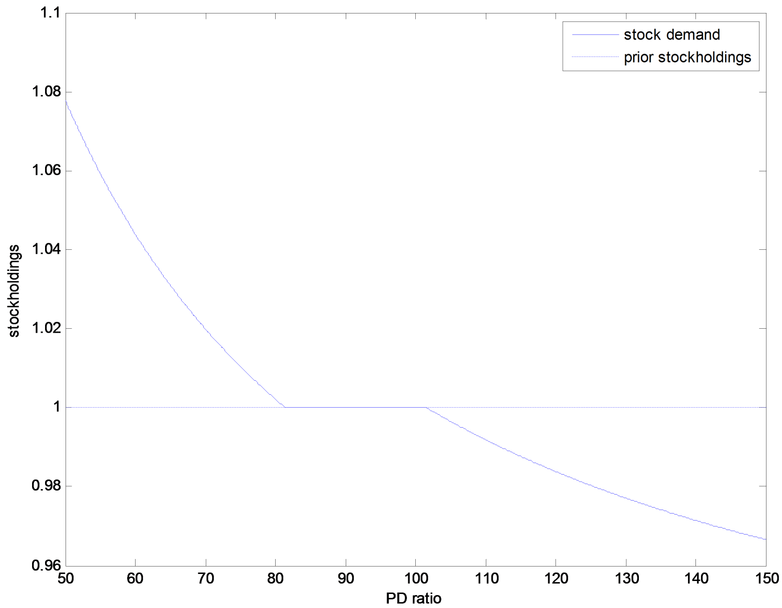

Figure 1.7. Stock demand function (4% tax)

likely in a situation with FTTs, as fewer fundamental shocks are required to set it in motion.

Figure 1.7 illustrates the driving force giving rise to this outcome. The figure depicts the stock demand function for a 4% FTT.[41] It shows that around the level of prior stockholding (assumed to be equal to one), stock demand (shown on the vertical axis) is not sensitive to the stock price (shown on the horizontal axis). This price insensitivity of stock demand covers a considerable price range and is actually increasing with the tax rate.[42] Therefore, in the presence of FTTs, even very small exogenous variations in stock supply can lead to large movements in realized prices, explaining why prices become more volatile during 'normal times'. Since agents use realized price growth to update price expectations, FTTs increase the likelihood that stock prices embark on a belief-driven stock price boom.

[41]The figure assumes $\tau = 4\%$ and the following values for the state variables: $W_t/D_t = \rho$, $m_t = \beta^D$, and $S_{-1} = 1$.

[42]Appendix A.3 explains how one can accurately determine the inaction regions.

1.9 State-Contingent Financial Transaction Taxes

Motivated by the results in the previous section, this section considers the effects of introducing state-contingent transaction taxes that are only levied once the PD ratio exceeds a certain (sufficiently high) threshold value $\overline{PD}$. The idea behind such a state-contingent tax is that it avoids the increase in price volatility during 'normal times', thereby avoiding that the stock market embarks with higher likelihood on a boom-bust cycle, while potentially limiting the duration and extent of stock price booms once they have taken hold.

Specifically, consider a setting with linear transaction taxes $\tau > 0$, which are levied only if $PD_t \geq \overline{PD}$, and zero taxes otherwise. We set the threshold value $\overline{PD}$ equal to 250, which is the value used to identify the beginning of a stock price boom episodes in previous sections. After solving for the optimal stock demand functions[43], it turns out that state-contingent taxes lead to problems of non-existence of equilibrium prices, as well as to the possibility of equilibrium multiplicities.

The non-existence problem is illustrated in Figure 1.8, which depicts the excess stock demand (on the vertical axis) as a function of the price dividend ratio (horizontal axis). The figure depicts these functions for all agent types, as well as the aggregate excess demand function.[44]

Figure 1.8 shows that once the PD ratio exceeds its critical value $\overline{PD}$, agents want to buy or sell less stocks, i.e., the excess demand functions discontinuously jump to a value closer to the no trade line (the zero line). As a result, the aggregate excess demand function also has a jump at $PD = \overline{PD}$ and for the case depicted in Figure 1.8, this leads to non-existence of an equilibrium price: the excess demand function is strictly positive for $PD < \overline{PD}$ but strictly negative for $PD \geq \overline{PD}$.

Obviously, the jump in the aggregate excess stock demand function does not necessarily have to be of the kind shown in Figure 1.8. We also encountered cases in which there was an upward jump at the critical value $\overline{PD}$. This can happen whenever agents who seek to sell stocks respond more to the tax once it is levied than agents who want

[43]The solution strategy outlined in Section 1.4.1 for the case with a non-state contingent tax can then still be applied because the tax function $\tau^i(S^i_{t-1}, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m^i_t)$ derived in Section 1.4.1 can already depend on the PD ratio. Instead of satisfying equations (1.15) and (1.16), the tax function and the stock holding policy must now jointly satisfy the first order condition (1.16) and

$$\begin{array}{ll} \tau^i_t = \tau & \text{at contingencies where } S^{i,opt}_t > S^{i,opt}_{t-1} \text{ and } PD \geq \overline{PD} \\ \tau^i_t = -\tau & \text{at contingencies where } S^{i,opt}_t < S^{i,opt}_{t-1} \text{ and } PD \geq \overline{PD} \\ \tau^i_t \in [-\tau, \tau] & \text{at contingencies where } S^{i,opt}_t = S^{i,opt}_{t-1} \text{ and } PD \geq \overline{PD} \\ \tau^i_t = 0 & \text{otherwise,} \end{array}$$

so as to be feasible in the original problem with a non-differentiable tax function (above the PD threshold).

[44]To illustrate the effects in the most transparent way, we use the setting with a 10% transaction tax, but the effects are qualitatively the same for lower tax rates.

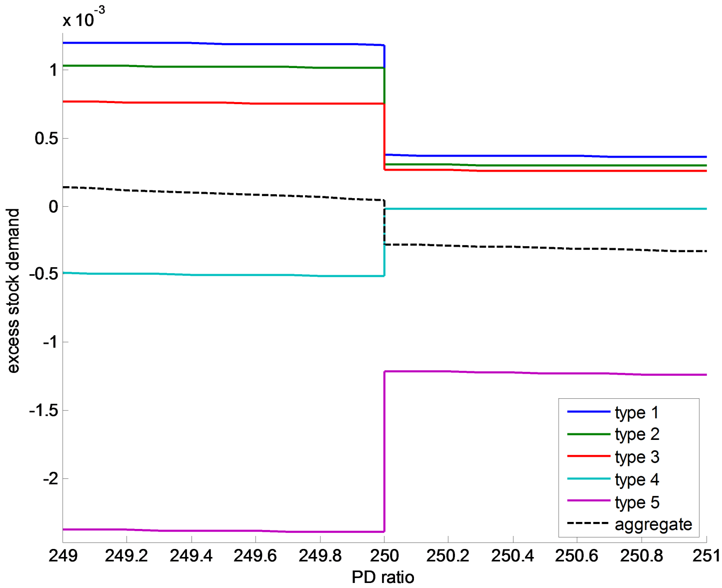

Figure 1.8. Non-existence of equilibrium with state-contingent FTTs

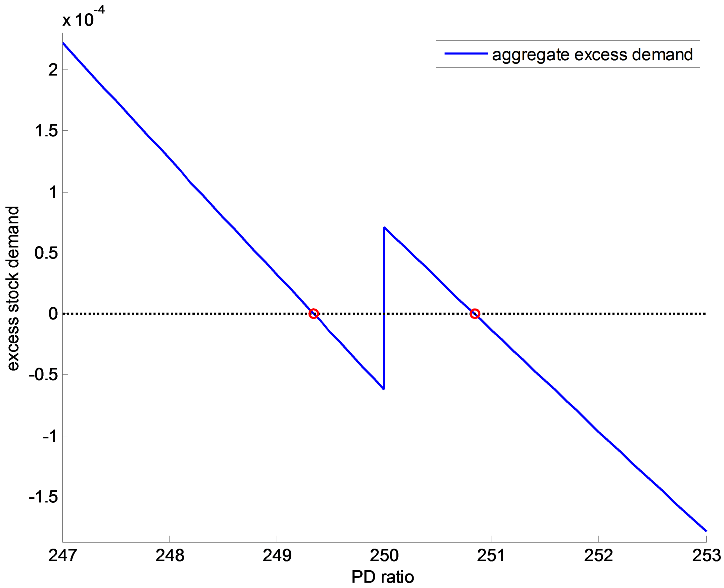

Figure 1.9. Multiple equilibrium prices with state-contingent transaction taxes

to purchase stocks. Figure 1.9 depicts an example, where the aggregate excess demand jumps upwards at $PD = \overline{PD}$. As the figure illustrates, this can give rise to multiple market clearing equilibrium prices. Since realized prices feed into agents' price beliefs, price multiplicities have the potential to significantly increase price volatility.

While the non-existence problem could possibly be overcome by introducing taxes that are a continuous function of the PD ratio, the multiplicity issue is harder to address. One would have to design state-contingent taxes in such a way that aggregate excess stock demand functions are never upward sloping in the vicinity of the zero point. It is unclear which tax design would be able to achieve this outcome.

1.10 Conclusions

We present a quantitatively credible asset pricing model in which stock prices display occasional boom and bust cycles in valuation, which redistribute large amounts of wealth between different investor types. We show how the introduction of financial transactions taxes increases price volatility during 'normal times' and thereby the likelihood that the stock market embarks on a belief-driven boom and bust cycle. State-contingent transaction taxes, which seek to avoid the increase in price volatility during normal times, generate problems via equilibrium multiplicities and non-existence. Taken together, these findings cast serious doubts on whether financial transaction taxes can fruitfully contribute towards increasing the efficiency of stock market prices and transactions.

A key insight highlighted by the present framework is that the presence of extrapolation by investors makes it an important requirement that market interventions do not increase stock price volatility during normal times, so as to avoid creating additional boom-bust episodes. Throughout the analysis, we have taken the degree of extrapolation as given. Conceivably, market interventions can also have a direct effect on the degree to which investors extrapolate past capital gains. To the extent that FTTs reduce extrapolation, FTTs can generate additional benefits that are not captured within the present analysis and may overturn our results. Obviously, if FTTs give rise to more extrapolation, they generate additional costs and strengthen the point made in the present paper. Empirically investigating the effects of FTTs on the degree of investor extrapolation thus appears to be an interesting avenue for future research.

Chapter 2

Stock Price Cycles and Business Cycles*

2.1 Introduction

We present a simple economic model that quantitatively replicates the joint behavior of business cycles and stock price cycles. The model matches standard data moments characterizing business cycle behavior, standard moments capturing stock price behavior, as well as data moments that link stock prices with business cycle variables. The model also gives rise to cycles in stock price valuation of the kind observed in the data.

Stock price cycles have – in comparison to business cycles – received relatively little attention in the literature. Somewhat surprisingly, this is the case even though stock price cycles are easily discernible in the data. Figure 2.1 depicts the S&P500 stock index from 1985 to 2014.[1] Over the considered thirty-year period, stock prices displayed three significant run-ups and two large reversals, with the price reversal amounting to a close to 50% price drop when compared to the previous peak. Similar run-ups and reversals can be observed in the stock markets of other advanced economies. We refer to this repeated boom-bust pattern in stock prices as stock price cycles.

Figure 2.2 presents an alternative approach for capturing stock price cycles: it depicts the empirical distribution of the quarterly price-dividend (PD) ratio of the S&P500.[2] While the mode of the PD ratio is slightly below 125, the PD distribution displays a long right tail with values of almost three times the mode being in the support of the distribution. The observed positive skewness and the large support of the empirical PD

*This chapter is based on and largely identical to the paper Adam and Merkel (2018).

[1]Figure 2.1 depicts the nominal value of the S&P500, but similar conclusions emerge if one deflates the nominal value by the consumer price index.

[2]The quarterly PD ratio is defined as the end-of-quarter price over a deseasonalized measure of quarterly dividend payouts, see Appendix B.1 for details.

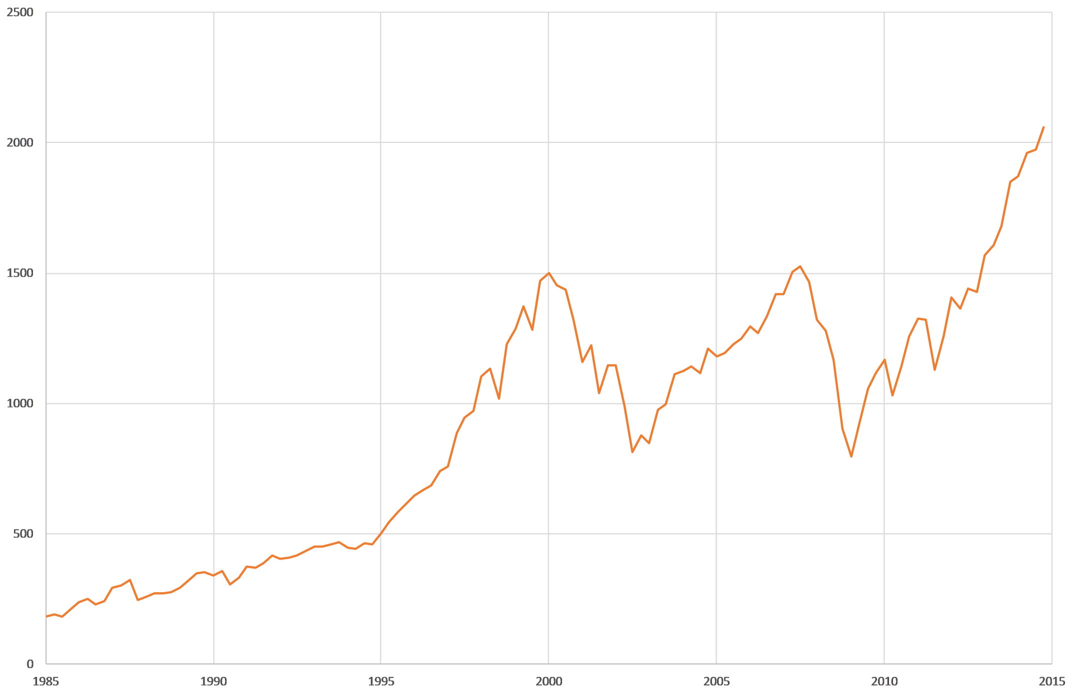

Figure 2.1. Cycles in the S&P 500: Q1:1985-Q4:2014, nominal index values

distribution are an alternative way to capture the presence of occasional stock price run-ups and reversals.

Making economic sense of these large stock price movements remains, however, challenging. This is especially true when seeking a joint explanation for stock price behavior and the behavior of the business cycle. While the latter is relatively smooth, stock prices are rather volatile. Without addressing stock price cycles, the existing literature typically relies on two approaches for reconciling the smoothness of the business cycles with the volatility of stock prices.

The first explanation combines preferences featuring a low elasticity of intertemporal substitution (EIS) with adjustment frictions.[3] A low EIS creates a strong desire for intertemporal consumption smoothing, while adjustment frictions prevent such smoothing from fully taking place. For agents to be willing to accept the observed moderate consumption fluctuations, asset prices then need to adjust strongly in equilibrium. Since one possible adjustment margin is agents' labor supply, too strong adjustments in the number of hours worked need to be prevented in such settings.[4] Therefore, EIS-based

[3]Typical examples are habit preferences (e.g. Jermann, 1998; Boldrin, Christiano, and Fisher, 2001; Uhlig, 2007; Jaccard, 2014), though sometimes also recursive preferences with a low EIS are used (e.g. Guvenen, 2009).

[4]Otherwise the high desire to smooth intertemporal fluctuations would lead to a strong adjustment in

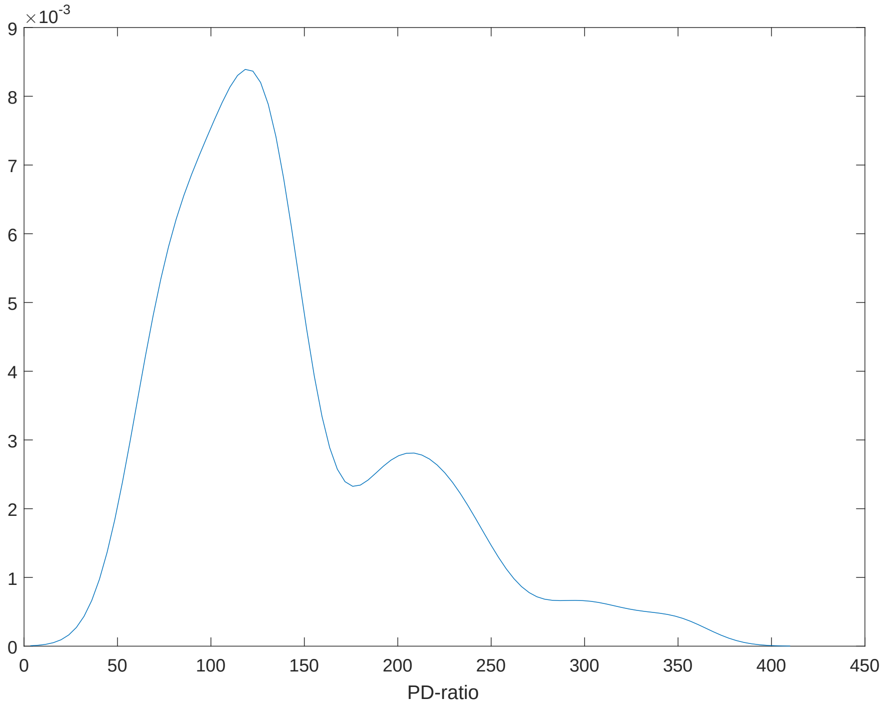

Figure 2.2. Postwar distribution of the quarterly PD ratio of the S&P 500 (kernel density estimate, Q1:1955-Q4:2014)

explanations include labor market frictions of some form (adjustment frictions, inflexible labor supply through preferences, real wage frictions), causing labor market frictions to become key for explaining asset price behavior. The centrality of labor market frictions is puzzling, in particular in light of the fact that labor market institutions differ considerably across advanced economies, while stock prices are very volatile in all these economies.

A second explanation reconciling smooth business cycles with volatile stock prices relies on specifying recursive preferences in conjunction with an additional source of exogenous uncertainty, e.g. long-run growth risk or disaster risk.[5] Such shocks have large pricing implications under recursive preferences, provided the coefficient of risk aversion is larger than the inverse EIS. These shocks then generate a large equity premium in the presence of realistic consumption dynamics. Time variation in the equity premium leads to substantial volatility in stock prices and returns, though typically less than what can be observed in the data.[6] As recently pointed out by Epstein, Farhi, and Strzalecki (2014),

hours worked and a very smooth consumption profile, which in turn would largely eliminate asset price fluctuations.

[5] See Gourio (2012), Croce (2014)

[6] Labor market frictions or the elasticity of labor supply are usually not discussed in this strand

standard calibrations of recursive preferences imply very large consumption premia for the resolution of uncertainty and the verdict on the quantitative plausibility of these premia is still outstanding.

The present paper proposes an alternative explanation for stock price and business cycle behavior. In contrast to the existing approaches, it relies on a standard separable preference specification and also features a frictionless labor market and perfectly elastic labor supply from households.[7] Despite these features, the model can jointly replicate the quantitative behavior of business cycles and stock prices using (reasonably sized) shocks to total factor productivity as its only source of exogenous random variation. The model also generates a stable risk-free interest rate and gives rise to large and persistent stock price boom-bust patterns of the kind observed in the data. The most notable dimension along which the model falls short of fully matching the data is the equity premium: it generates only about one third of the empirically observed premium. Since we consider a setting with time-separable utility and a logarithmic utility function for consumption, this still represents a respectable achievement.

Key to the empirical success of the model is a departure from the rational expectations hypothesis (REH). Following Adam, Marcet, and Nicolini (2016) and Adam, Marcet, and Beutel (2017), we consider subjectively Bayesian investors who seek to filter the long-term trend component of capital gains from observed capital gains and as a result extrapolate (to some extent) past capital gains into the future. Adam, Marcet, and Beutel (2017) show that such extrapolative behavior is in fact consistent with the available survey evidence on investors' return expectations, whereas the REH is strongly rejected by the data.[8] Adam, Marcet, and Nicolini (2016) show how subjective beliefs of such kind substantially improve the asset pricing predictions of a standard Lucas (1978) endowment economy. The present setting significantly extends their framework to one with endogenous production, featuring endogenous labor and investment choices. In doing so, we also present a conceptual approach for dealing with dynamic decision settings in which agents' expectations deviate from rational expectations along some dimension (future asset prices), but are consistent with the REH along other dimensions (all other variables beyond agents' control).

We estimate the subjective belief model using the simulated method of moments and also estimate a version with fully rational expectations. We show that the empirical improvements associated with a departure from the assumption of rational stock price

of literature. While they are certainly not as crucial as they are for the low-EIS-based explanations, flexible labor supply in a frictionless labor market is still a powerful tool for households to insure against consumption fluctuations. The fact that the asset pricing models in this second strand of literature tend to generate too little volatility of hours worked is an indication that the chosen preference specification make labor supply not flexible enough.

[7]We consider households with a linear disutility of work.

[8]Adam, Matveev, and Nagel (2018) tests to what extent survey expectations are consistent with various expectations hypotheses entertained in the asset pricing literature.

expectations is significant, both along the business cycle dimension and – even more importantly – along the stock price dimension. Under fully rational expectations, the model fails to fully replicate business cycle moments, because we do not consider investment-specific productivity shocks. We show that in a setting with subjective price beliefs such shocks are not required.

We also consider the welfare effects associated with stock price fluctuations that are driven by fluctuations in investors' subjective beliefs. Stock price movements induce volatility of investment and hours worked, but these variations are not exclusively driven by productivity developments. As a result, average consumption is higher and average hours worked lower when imposing rational stock price expectations, as investment choices then fully reflect underlying productivity. The volatility of hours and investment significantly falls under fully rational expectations and there is a dramatic drop in the volatility of stock prices. The welfare gains – measured in terms of ex-post realized utility – amount to a permanent increase in consumption of 0.29%, thus are in the order of magnitude associated with eliminating the business cycle.

The paper is structured as follows. Section 2.2 discusses the related literature. Section 2.3 presents key facts about business cycles, stock prices and their interaction in the United States. Section 2.4 describes our model with the exception of beliefs, which are discussed in detail in Section 2.5. All equilibrium conditions of the model are summarized in Section 2.6. Section 2.7 derives analytical insights and discusses the dynamics of stock prices, price beliefs and macro aggregates. Section 2.8 outlines our estimation procedure and assesses the empirical performance of our model. There, we also discuss how subjective price beliefs contribute to the empirical success of the model by contrasting our results with those of a model version where agents hold fully rational expectations. Section 2.9 compares our quantitative results to two closely related papers, Boldrin, Christiano, and Fisher (2001) and Adam, Marcet, and Beutel (2017). Section 2.10 discusses the welfare implications of belief-driven stock price cycles. Section 2.11 concludes.

2.2 Related Literature

Early attempts of jointly modeling business cycles and stock prices have relied on a combination of habit preferences and adjustment frictions to generate high stock price volatility and equity premia (Jermann, 1998; Boldrin, Christiano, and Fisher, 2001; Uhlig, 2007). Habit preferences create a low elasticity of intertemporal substitution (EIS) and thereby a strong desire to smooth consumption, which leads to volatile stock prices in a setting with empirically plausible consumption fluctuations. The intertemporal substitution channel, however, causes all asset prices to be volatile and thus generates a counterfactually high volatility of the risk-free interest rate.[9] An exception is Uhlig (2007) who

[9] The risk-free rate volatility in Jermann (1998) is twice as high as in the data; in Boldrin, Christiano, and Fisher (2001) it is almost five times as high.

considers external habits, which create strong fluctuations in risk aversion[10] and thereby allow for relatively volatile stock prices and a stable risk-free interest rate.[11] To prevent consumption smoothing, low EIS models also crucially rely on labor market frictions or inelastic labor supply: labor supply in Jermann (1998) is fully inelastic, while a timing friction forces households to choose labor supply one period in advance in Boldrin, Christiano, and Fisher (2001); Uhlig (2007) introduces a real wage rigidity that leads to labor supply rationing following negative productivity shocks. Jaccard (2014) augments the model of Jermann (1998) and allows for adjustments in labor supply, but introduces a habit specification over a composite of consumption and leisure which makes labor supply adjustments unattractive for the purpose of smoothing consumption.[12]

Models with a low EIS also typically generate the equity premium via a counterfactually high term premium. For example, the premium on long-term bonds in Jermann's (1998) model is 92% of that on stocks, compared to only 28% in the data.

Another line of literature jointly considers business cycle dynamics and stock price behavior using models with limited asset market participation.[13] In these models, a limited set of agents has access to the stock market and in addition insures the consumption of non-participating agents via other contracts. An early example is Danthine and Donaldson (2002), who consider shareholders and workers, where the latter do not participate in financial markets at all. Shareholders then offer workers a labor contract that gives rise to "operating leverage" in the sense that it causes the cash flows of shareholders to become even more volatile and procyclical. As a result, the model gives rise to an equity premium and volatile stock returns, albeit at the cost of creating too much volatility for shareholder's consumption.[14]

Guvenen (2009) considers a model with limited stock market participation in which all agents participate in the bond market. When stock market investors have a higher EIS than non-participating agents, they optimally insure the latter against income fluctuations via bond market transactions, thereby channeling most labor income risk to a small set of stock market participants. As a result, their consumption is strongly procyclical and gives

[10]See Boldrin, Christiano, and Fisher (1997) for a discussion of the differing risk aversion implications of internal and external habit specifications.

[11]It is unclear, however, whether Uhlig (2007) matches the volatility of stock returns as he only reports the sharpe ratio.

[12]As a result, the standard deviation of the growth rate of hours worked in Jaccard (2014) is only about one half of the value observed in the data.

[13]As mentioned in Guvenen (2009), stock market participation increased substantially during the 1990's. From 1989 to 2002 the number of households who owned stocks increased by 74% so that half of U.S. households had become stock owners by the year 2002.

[14]In Table 6 in Danthine and Donaldson (2002), which is the specification with the best overall empirical fit, shareholder consumption volatility is about 10 times as large as aggregate consumption volatility. Guvenen (2009) reviews the empirical evidence on stockholders' relative consumption volatility and concludes that stockholders' consumption is about 1.5-2 times as volatile as non-stockholders' and thus even less high in relative terms when compared to aggregate consumption volatility

rise to both a high equity premium and high volatility of returns. The model assumes the EIS to be low – even for shareholders – thus generates additional stock price volatility through the same channels as the habit models. The model performs quantitatively very well on the financial dimension, except for a slightly too volatile risk-free interest rate. Performance along the business cycle dimension is more mixed, as consumption is too volatile and investment and hours worked too smooth.

Tallarini (2000), Gourio (2012), Croce (2014) and Hirshleifer, Li, and Yu (2015) discuss the asset pricing predictions of the real business cycle models under Epstein-Zin preferences (Epstein and Zin, 1989), assuming that the coefficient of risk aversion is larger than the inverse EIS. Tallarini (2000) shows that increasing risk aversion while keeping the EIS fixed at one barely affects business cycle dynamics, but has substantial effects on the price of risk. While the model can give rise to a high Sharpe ratio, in line with the value observed in the data, it considerably undershoots the equity premium and the volatility of returns. The model thus falls short of reconciling the volatility of stock prices with the smoothness of business cycle dynamics. Gourio (2012) considers preferences with a larger EIS and moderate risk aversion and enriches the model by time-varying disaster risk.[15] Whereas constant disaster risk has little effect on the model dynamics, time-variation in disaster risk combined with preferences for early resolution of uncertainty generate a high equity premium and return volatility, while at the same time keeping the dynamics of macro aggregates relatively smooth, provided the disaster does not realize. While framed as a rational expectations model applied to a disaster-free data sample, one may alternatively interpret the model as a subjective belief model and the exogenous shock to the disaster probability as a shock to agents' expectations. Under this interpretation, subjective beliefs drive asset price dynamics, as is the case in the present paper. However, subjectively expected returns in his model are negatively related to the PD ratio, unlike in the data and unlike in the present model.

Another channel through which preferences for early resolution of uncertainty can generate realistic asset pricing predictions is long-run consumption growth risk as in Bansal and Yaron (2004). Croce (2014) considers a production economy with long-run productivity growth risk and shows that this translates into long-run consumption risk, thereby transferring the asset pricing predictions of the endowment economy of Bansal and Yaron (2004) to a real business cycle setting. His model can match well the equity premium and low and stable risk-free rate, but is only partially able to generate price and return volatility.

Hirshleifer, Li, and Yu (2015) also consider an economy with preferences for early resolution of uncertainty but assume inelastic labor supply. Agents know the productivity process only imperfectly and over-extrapolate recent productivity observations in their filtering problem. This mechanism endogenously generates long-run variations in

[15] A disaster is a potentially persistent event in which the economy experiences for the duration of the disaster in each period a negative productivity shock and a capital depreciation shock.

perceived technology growth and thus perceived consumption growth, despite there being only short-run technology shocks present in the data-generating process. If the extrapolation bias is small but sufficiently persistent, then the model produces a sizeable equity premium and about 50% of the observed volatility of stock returns. The model also matches a range of business cycle quantities, except for the volatility of hours worked.

In independent and complementary work, Winkler (2018) considers the asset pricing implications of a rich DSGE model featuring subjective price beliefs of the kind we also consider. His model features financial frictions, price and wage rigidities, and limited stock market participation, from which the present paper abstracts. In addition, Winkler (2018) considers a setup with "conditionally model-consistent expectations", which requires agents' beliefs to be consistent with both the subjective law of motion for prices, other agents' decision functions and a maximum number of market clearing conditions.[16] This setup allows for deviations from rational expectations also for other variables than just stock prices and differs from our setting with "partially rational expectations", which endows agents with the correct statistical model about decision-relevant variables other than prices.

Learning about stock prices in our model does not only improve stock market predictions, but helps quantitatively also along the business cycle dimension. The idea that learning can improve the business cycle predictions of the standard real business cycle model is also present in Eusepi and Preston (2011), who consider a setting where agents are learning about wages and rental rates. They show that this generates significant amplification and persistence in effects of neutral technology shocks, so that expectations ultimately drive the business cycle. In our setting, beliefs about wages and rental rates are assumed to be rational, but subjective stock price expectations do affect business cycle outcomes. Specifically, fluctuations in expectations affect capital valuations and the demand for investment goods, thereby generating fluctuations in investment and labor demand.

The paper is also related to the literature on rational stock market bubbles, as for instance derived in classic work by Froot and Obstfeld (1991). While rational bubbles provide an alternative approach for generating stock market volatility, they seem inconsistent with empirical evidence along two important dimensions. First, the assumption of rational return expectations is strongly at odds with survey measures of return expectations, which clearly favors the subjective belief specifications we consider in the present paper (compare Adam, Marcet, and Beutel, 2017). Second, Giglio, Maggiori, and Stroebel (2016) show that there is very little evidence supporting the notion that violations of the transversality condition drive asset price fluctuations, unlike suggested by the rational bubble hypothesis.

[16] Due to Walras' Law one market clearing condition can be dropped. Winkler (2018) must drop a second one, as not all markets can clear under the subjective plans.

Table 2.1
U.S. business cycle moments (quarterly real values, Q1:1955-Q4:2014)

	Symbol	Data moment	Std. dev. data moment
Std. dev. of output	$\sigma(Y)$	1.72	0.25
Relative std. dev. of consumption	$\sigma(C)/\sigma(Y)$	0.61	0.03
Relative std. dev. of investment	$\sigma(I)/\sigma(Y)$	2.90	0.35
Relative std. dev. of hours worked	$\sigma(H)/\sigma(Y)$	1.08	0.13
Correlation output and consumption	$\rho(Y, C)$	0.88	0.02
Correlation output and investment	$\rho(Y, I)$	0.86	0.03
Correlation output and hours worked	$\rho(Y, H)$	0.75	0.03

2.3 Stock Prices and Business Cycles: Key Facts

This section presents key data moments that characterize U.S. business cycles and stock price behavior and that we focus on in our empirical analysis. We consider quarterly U.S. data for the period Q1:1955-Q4:2014. The start date of the sample is determined by the availability of the aggregate hours worked series. Details of the data sources are reported in Appendix B.1.

Table 2.1 presents a standard set of business cycle moments for output (Y), consumption (C), investment (I) and hours worked (H).[17] These quantities have been divided by the working age population so as to take into account demographic changes in the U.S. population over the sample period. The second to last column in Table 2.1 reports the data moment and the last column the standard deviation of the estimated moment. We will use the latter in our simulated methods of moments estimation and for computing t-statistics.[18]

The picture that emerges from Table 2.1 is a familiar one: output fluctuations are relatively small, consumption is considerably less volatile than output, while investment is considerably more volatile; hours worked are roughly as volatile as output. Consumption, investment and hours all correlate strongly with output. A major quantitative challenge will be to simultaneously replicate the relative smoothness of the business cycle with the much larger fluctuations in stock prices to which we turn next.

[17] As is standard in the business cycle literature, we compute business cycle moments using logged and subsequently HP-filtered data with a smoothing parameter of 1600. All other data moments will rely on unfiltered (level) data. We HP filter model variables when comparing to filtered moments in the data and use unfiltered model moments otherwise.

[18] All standard deviations of moments reported in this section are computed by a procedure combining Newey-West estimators with the delta method as in Adam, Marcet, and Nicolini (2016). We refer to Appendix F of that paper for details.

Table 2.2
Key moments of stock prices, risk-free rates and dividends (U.S., quarterly real values, Q1:1955-Q4:2014)

	Symbol	Data moment	Std. dev. data moment
Average PD ratio	$E[P/D]$	152.3	25.3
Std. dev. PD ratio	$\sigma(P/D)$	63.39	12.39
Auto-correlation PD ratio	$\rho(P/D)$	0.98	0.003
Average equity return (%)	$E[r^e]$	1.87	0.45
Std. dev. equity return (%)	$\sigma(r^e)$	7.98	0.35
Average risk-free rate (%)	$E[r^f]$	0.25	0.13
Std. dev. risk-free rate (%)	$\sigma(r^f)$	0.82	0.12
Std. dev. dividend growth (%)	$\sigma(D_{t+1}/D_t)$	1.75	0.38

Table 2.2 presents a standard set of moments characterizing U.S. stock price behavior. The first three moments summarize the behavior of the PD ratio:[19] the PD ratio is rather large and implies a dividend yield of just 66 basis points per quarter. The PD ratio is also very volatile: the standard deviation of the PD ratio is more than 40% of its mean value and fluctuations in the PD ratio are very persistent, as documented by the high quarterly auto-correlation of the PD ratio. Table 2.2 also reports the average real stock return, which is high and close to 2% per quarter. Stock returns are also very volatile: the standard deviation of stock returns is about four times its mean value. This contrasts with the behavior of the short-term risk-free interest rate documented in Table 2.2. The risk-free interest rate is very low and very stable. The standard deviation of the risk-free interest rate in Table 2.2 is likely even overstated, as ex-post realized inflation rates have been used to transform nominal safe rates into a real rate. Table 2.2 also reports the standard deviation of dividend growth. Dividend growth is relatively smooth, especially when compared to the much larger fluctuation in equity returns. This fact is hard to reconcile with the observed large fluctuations in stock prices (Shiller, 1981).

Table 2.3 presents data moments that link the PD ratio to business cycle variables. It shows that stock prices are pro-cyclical: (1) the PD ratio correlates positively with hours worked; (2) stock prices also correlate positively with the investment to output ratio, but the correlation is surprisingly weak and also estimated very imprecisely. Table 2.4 below shows why this is the case: the investment to output ratio correlates positively with the PD ratio over the second half of the sample period (1985-2014), i.e., in the period with

[19] The PD ratio is defined as the end-of-quarter stock price divided by dividend payments over the quarter. Following standard practice, dividends are deseasonalized by averaging dividends over the last four quarters.

Table 2.3
Comovement of stock prices with real variables and survey return expectations (U.S., quarterly real values, Q1:1955-Q4:2014)

Correlations	Symbol	Data moment	Std. dev. data moment
Hours & PD ratio	$\rho(H, P/D)$	0.51	0.17
Investment-output & PD ratio	$\rho(I/Y, P/D)$	0.19	0.31
Survey expect. & PD ratio	$\rho(E^{\mathcal{P}}[r^e], P/D)$	0.79	0.07

large stock price cycles, but negatively in the first half of the sample period (1955-1984). Table 2.4 also shows that the overall investment to output ratio correlates much more strongly with the PD ratio if one excludes non-residential investment and investment in non-residential structure. The negative correlation in the first half of the sample period is, however, a robust feature of the data.[20]

Table 2.3 also reports the correlation of the PD ratio with the one-year-ahead expected real stock market return of private U.S. investors. It shows that investors are optimistic about future holding period returns when the PD ratio is high already. As shown in Adam, Marcet, and Beutel (2017), this feature of the data is inconsistent with the notion that investors hold rational return expectations. Since we will consider an asset pricing model with subjective return expectations, we include this data moment into our analysis.

Table 2.4
Stock prices and investment: alternative measures and sample periods (U.S., quarterly real values)

$corr(I/Y, P/D)$	1955-2014	1955-1984	1985-2014
Fixed investment	0,19	-0,64	0,40
Fixed investment, less residential inv. and nonresidential structures:	0,58	-0,66	0,77

2.4 Asset Pricing in a Production Economy

We build our analysis on a stripped-down version of the representative agent model of Boldrin, Christiano, and Fisher (2001). This model features a consumption goods producing sector and an investment goods producing sector. Both sectors produce output

[20]The correlation of the hours worked series with the PD ratio is positive in the first and second half of the sample period.

using a neoclassical production function with capital and labor as input factors. Output from the investment goods sector can be invested to increase the capital stock.

We deviate from Boldrin, Christiano, and Fisher (2001) by using a standard time-separable specification for consumption preferences instead of postulating consumption habits. In addition, we remove all labor market frictions on the firm side by making hours worked perfectly flexible.

Given a linear specification for the disutility of labor, the labor market is then perfectly flexible and competitive. While frictions are arguably present in U.S. labor markets, we prefer the fully flexible specification to illustrate that our asset pricing implications do not depend on assuming labor market frictions. This feature distinguishes the present analysis from much of the earlier work.

We furthermore simplify our setup relative to the one studied in Boldrin, Christiano, and Fisher (2001) by specifying an exogenous capital accumulation process in the investment goods sector, in line with a balanced growth path solution. This helps with analytical tractability of the model, but also insures that the supply of new capital goods is sufficiently inelastic, so that the model has a chance of replicating the large persistent swings in stock prices that can be observed in the data.[21]

2.4.1 Production Technology

There are two sectors, one producing a perishable consumption good (consumption sector), the other producing an investment good that can be used to increase the capital stock in the consumption sector (investment sector). The representative firm in each sector hires labor and rents capital, so as to produce its respective output good according to standard Cobb-Douglas production functions,

$$Y_{c,t} = K_{c,t}^{\alpha_c} \left(Z_t H_{c,t}\right)^{1-\alpha_c}, \qquad Y_{i,t} = K_{i,t}^{\alpha_i} \left(Z_t H_{i,t}\right)^{1-\alpha_i}, \tag{2.1}$$

where $K_{c,t}$, $K_{i,t}$ denote capital inputs and $H_{c,t}$, $H_{i,t}$ labor inputs in the consumption and the investment sector, respectively, and $\alpha_c \in (0,1)$ and $\alpha_i \in (0,1)$ the respective capital shares in production. Z_t is an exogenous labor-augmenting level of productivity and the only source of exogenous variation in the model. Productivity follows

$$Z_t = \gamma Z_{t-1} \varepsilon_t, \qquad \log \varepsilon_t \sim ii\mathcal{N}\left(-\frac{\sigma^2}{2}, \sigma^2\right), \tag{2.2}$$

with $\gamma \geq 1$ denoting the mean growth rate of technology and $\sigma > 0$ the standard deviation of log technology growth.

[21] In Boldrin, Christiano, and Fisher (2001), capital prices do not display large and persistent fluctuations.

Labor is perfectly flexible across sectors, but capital is sector-specific. The output of investment goods firms increases next period's capital in the consumption goods sector, so that

$$K_{c,t+1} = (1 - \delta_c)\, K_{c,t} + Y_{i,t}, \tag{2.3}$$

where $\delta_c \in (0, 1)$ denotes the depreciation rate. Capital in the investment goods sector evolves according to

$$K_{i,t+1} = (1 - \delta_i)\, K_{i,t} + X_t \tag{2.4}$$

where X_t is an exogenous endowment of new capital in the investment goods sector and $\delta_i \in (0, 1)$ the capital depreciation rate. We set X_t such that $K_{i,t+1} \propto Z_t$, which insures that the model remains consistent with balanced growth.[22] The assumed capital stock dynamics in the investment goods sector insures that capital good production that deviates from the balanced growth path is subject to decreasing returns to scale. This allows for persistent price fluctuations in the price of consumption capital around the balanced growth path.

2.4.2 Households

The representative household each period chooses consumption $C_t \geq 0$, hours worked $H_t \geq 0$, the end-of-period capital stocks $K_{c,t+1} \geq 0$ and $K_{i,t+1} \geq 0$ to maximize

$$E_0^{\mathcal{P}} \left[\sum_{t=0}^{\infty} \beta^t \left(\log C_t - H_t \right) \right], \tag{2.5}$$

where the operator $E_0^{\mathcal{P}}$ denotes the agent's expectations in some probability space $(\Omega, \mathcal{S}, \mathcal{P})$. Here, Ω is the space of realizations, $\mathcal{S}$ the corresponding σ-Algebra, and $\mathcal{P}$ a subjective probability measure over $(\Omega, \mathcal{S})$. As usual, the probability measure $\mathcal{P}$ is a model primitive and given to agents. The special case with rational expectations is nested in this specification, as explained below.

Household choices are subject to the flow budget constraint

$$\begin{aligned} C_t + K_{c,t+1} Q_{c,t} + K_{i,t+1} Q_{i,t} = W_t H_t + X_t Q_{i,t} &+ K_{c,t} \left((1 - \delta_c) Q_{c,t} + R_{c,t} \right) \\ &+ K_{i,t} \left((1 - \delta_i) Q_{i,t} + R_{i,t} \right), \end{aligned} \tag{2.6}$$

for all $t \geq 0$, where $Q_{c,t}$ and $Q_{i,t}$ denote the prices of consumption-sector and investment-sector capital, respectively, and $R_{c,t}$ and $R_{i,t}$ the rental rates earned by renting out capital to firms in the consumption and investment sector, respectively; W_t denotes the wage rate and X_t the endowment of new investment-sector capital.

[22] The model allocation is then identical to a setting in which investment is produced with labor only, i.e., $Y_{i,t} \propto Z_t \varepsilon_t^{-\alpha_i} (H_{i,t})^{1-\alpha_i}$. We prefer a specification that includes capital, capital depreciation and an exogenous investment input, as this allows us to define capital values in both sectors in a symmetric fashion.

To allow for subjective price beliefs, we shall consider an extended probability space relative to the case with rational expectations. In its most general form, households' probability space is spanned by all external processes, i.e. by all variables that are beyond their control. These are given by the process $\{Z_t, X_t, W_t, R_{c,t}, R_{i,t}, Q_{c,t}, Q_{i,t}\}_{t=0}^{\infty}$, so that the space of realizations is

$$\Omega := \Omega_Z \times \Omega_X \times \Omega_W \times \Omega_{R,c} \times \Omega_{R,i} \times \Omega_{Q,c} \times \Omega_{Q,i},$$

where $\Omega_{\mathcal{X}} = \prod_{t=0}^{\infty} \mathbb{R}$ with $\mathcal{X} \in \{Z, X, W, R_c, R_i, Q_c, Q_i\}$. Letting $\mathcal{S}$ denote the sigma-algebra of all Borel subsets of Ω, beliefs will be specified by a well-defined probability measure $\mathcal{P}$ over $(\Omega, \mathcal{S})$. Letting Ω^t denote the set of all partial histories up to period t, households' decision functions can then be written as

$$(C_t, H_t, K_{c,t+1}, K_{i,t+1}) : \Omega^t \longrightarrow \mathbb{R}^4. \tag{2.7}$$

We assume that households choose these functions, so as to maximize (2.5) subject to the constraints (2.6).

In the special case with rational expectations, $(X, W, R_c, R_i, Q_c, Q_i)$ are typically redundant elements of the probability space Ω, because households are assumed to know that these variables can at time $t \geq 0$ be expressed as *known deterministic equilibrium functions* of the history of fundamentals Z^t only.[23] Without loss of generality, one can then exclude these elements from the probability space and write:[24]

$$(C_t, H_t, K_{c,t+1}, K_{i,t+1}) : \Omega_Z^t \longrightarrow \mathbb{R}^4,$$

where $\Omega_Z^t = \prod_{s=0}^{t} \mathbb{R}$ is the space of all realizations of $Z^t = (Z_0, Z_1, ..., Z_t)$. This routinely performed simplification implies that households perfectly know how the markets determine the excluded variables as a function of the history of shocks. By introducing subjective beliefs, we will step away from this assumption.

To insure that the household's maximization problem remains well-defined in the presence of the kind of subjective price beliefs introduced below, we impose additional capital holding constraints of the form $K_{c,t+1} \leq \overline{K}_{c,t+1}$ and $K_{i,t+1} \leq \overline{K}_{i,t+1}$, for all $t \geq 0$, where the bounds $\left(\overline{K}_{c,t+1}, \overline{K}_{i,t+1}\right)$ are assumed to increase in line with the balanced growth path and are assumed sufficiently large, such that they never bind in equilibrium.[25]

[23] This assumes that there are no sunspot fluctuations in the rational expectations equilibrium.

[24] Sunspot fluctuation require either keeping some of the endogenous variables or including the sunspot variables into the probability space.

[25] These capital holding constraints are required for subjective price beliefs to be consistent with internal rationality in a setting with an effectively exogenous stochastic discount factor, see Adam and Marcet (2011) for details. While they must be chosen sufficiently large so as to never bind in equilibrium, their precise values do not matter for equilibrium determination.

2.4.3 Competitive Equilibrium

The competitive equilibrium of an economy in which households hold subjective beliefs is defined as follows:

Definition 1. For given initial conditions $(K_{c,-1}, K_{i,-1})$, a competitive equilibrium with subjective household beliefs $\mathcal{P}$ consists of allocations $\{C_t, H_t, H_{c,t}, H_{i,t}, K_{c,t+1}, K_{i,t+1}\}_{t=0}^{\infty}$ and prices $\{Q_{c,t}, Q_{i,t}, R_{c,t}, R_{i,t}, W_t\}_{t=0}^{\infty}$, all of which are measurable functions of the process $\{Z_t\}_{t=0}^{\infty}$, such that for all partial histories $Z^t = (Z_0, Z_1, ...Z_t)$ and all $t \geq 0$, prices and allocations are consistent with

1. profit maximizing choices by firms,
2. the subjective utility maximizing choices for households decision functions (2.7), and
3. market clearing for consumption goods ($C_t = Y_{c,t}$), hours worked ($H_t = H_{c,t} + H_{i,t}$), and the two capital goods (equations (2.3) and (2.4)).

The equilibrium requirements are weaker than what is required in a competitive rational expectations equilibrium, because household beliefs are not restricted to be rational. For the special case where $\mathcal{P}$ incorporates rational expectations, the previous definition defines a standard competitive rational expectations equilibrium.

2.4.4 Connecting Model Variables to Data Moments

In order to compare our model to the data, we need to define the real variables (investment, output) and stock market variables (stock prices, dividends) in our production economy. For the real variables, this is relatively straightforward. We follow Boldrin, Christiano, and Fisher (2001) and define investment as being proportional to the quantity of capital produced and use the steady state capital price Q_c^{ss} as a base price, so that fluctuations in the price of capital do not contribute to fluctuations in real investment. Investment is thus given by

$$I_t = Q_c^{ss} Y_{i,t}$$

and output correspondingly by

$$Y_t = C_t + I_t.$$

To define stock prices and dividends, we consider a setup where (investment- and consumption-sector) capital can be securitized via shares and where shares and capital can be jointly created or jointly destroyed at no cost. The absence of arbitrage opportunities then implies that the price of shares is determined by the price of the capital it securitizes. We consider a representative consumption-sector share and a representative investment-sector share. The only free parameter in this extended setup is then the dividend policy of stocks, which is indeterminate (Miller and Modigliani, 1961). To obtain a parsimonious setting, we assume a time-invariant profit payout share $p \in (0,1)$: a share p of rental income/profits per share is paid out as dividends each period and in both sectors, with the remaining share $1-p$ being reinvested in the capital stock that the share securitizes.

We now describe the setting for the consumption sector in greater detail. The setup for the investment sector is identical up to an exchange of subscripts. Let $k_{c,t}$ denote the units of (beginning-of-period t) capital held per unit of shares issued in the consumption sector. The capital is used for production and earns a rental income/profit of $k_{c,t}R_{c,t}$. Given the payout ratio $p \in (0,1)$, dividends per share are given by

$$D_{c,t} = pk_{c,t}R_{c,t}.$$

Retained profits are reinvested to purchase $(1-p)\, k_{c,t}R_{ct}/Q_{c,t}$ units of new capital per share.[26] The end-of-period capital per share then consists of the depreciated beginning-of-period capital stock and purchases of new capital from retained profits. The end-of-period share price $P_{c,t}$ is thus equal to[27]

$$P_{c,t} = (1-\delta)k_{c,t}Q_{c,t} + (1-p)k_{c,t}R_{c,t},$$

and the end-of-period PD ratio is given by

$$\frac{P_{c,t}}{D_{c,t}} = \frac{1-\delta_c}{p}\frac{Q_{c,t}}{R_{c,t}} + \frac{1-p}{p}.$$

Since the last term is small for reasonable payout ratios p, the end-of-period PD ratio is approximately proportional to the capital price over rental price ratio $(Q_{c,t}/R_{c,t})$. Moreover, as is easily verified, the equity return per unit of stock $R^e_{c,t} = (P_{c,t} + D_{c,t})\,/P_{c,t-1}$ is equal to the return per unit of capital $R^k_{c,t} = ((1-\delta_c)Q_{c,t} + R_{c,t})/Q_{c,t-1}$.

Given sectoral stock prices and PD ratios, we can define the aggregate PD ratio using a value-weighted portfolio of the sectoral investments. Let

$$w_{c,t-1} = \frac{Q_{c,t-1}K_{c,t}}{Q_{c,t-1}K_{c,t} + Q_{i,t-1}K_{i,t}}$$

denote the end-of-period $t-1$ value share of the consumption sector. The value share of the investment sector is then $1-w_{c,t-1}$. A portfolio with total value P_{t-1} at the end of period $t-1$ and value shares $w_{c,t-1}$ and $1-w_{c,t-1}$ in the consumption and investment-sector, respectively, must contain $w_{c,t-1}P_{t-1}/P_{c,t-1}$ consumption shares and

[26] In case the aggregate capital supply differs from capital demand implied by the existing number of shares, new shares are created or existing shares repurchased to equilibrate capital demand and supply.

[27] We compute end-of-period share prices, because this is the way prices have been computed in the data.

$(1 - w_{c,t-1}) P_{t-1}/P_{i,t-1}$ investment shares. The end-of-period t value of this portfolio is then given by

$$P_t = \frac{w_{c,t-1}P_{t-1}}{P_{c,t-1}} P_{c,t} + \frac{(1 - w_{c,t-1}) P_{t-1}}{P_{i,t-1}} P_{i,t} \tag{2.8}$$

and period t dividend payments for this portfolio are

$$D_t = \frac{w_{c,t-1}P_{t-1}}{P_{c,t-1}} D_{c,t} + \frac{(1 - w_{c,t-1}) P_{t-1}}{P_{i,t-1}} D_{i,t}. \tag{2.9}$$

Using the previous two equations, the aggregate PD ratio can be expressed as a weighted mean of the sectoral PD ratios where the weights are given by the share of portfolio dividends coming from each sector:

$$\frac{P_t}{D_t} = \frac{w_{c,t-1}\frac{D_{c,t}}{P_{c,t-1}}}{w_{c,t-1}\frac{D_{c,t}}{P_{c,t-1}} + (1 - w_{c,t-1}) \frac{D_{i,t}}{P_{i,t-1}}} \frac{P_{c,t}}{D_{ct}} + \frac{(1 - w_{c,t-1}) \frac{D_{i,t}}{P_{i,t-1}}}{w_{c,t-1}\frac{D_{c,t}}{P_{c,t-1}} + (1 - w_{c,t-1}) \frac{D_{i,t}}{P_{i,t-1}}} \frac{P_{i,t}}{D_{i,t}}.$$

Note that the PD ratio is independent of the initial portfolio value P_{t-1}. Aggregate dividend growth can similarly be expressed using equations (2.9) and (2.8) as a weighted average of the sectoral dividend growth rates. This completes our definition of model variables.

2.5 Price Beliefs, Probability Space and State Space

In specifying household beliefs $\mathcal{P}$, we shall consider two alternative belief specifications: (1) a standard setting in which *all* expectations are rational and (2) a setting in which households hold subjective beliefs about future capital prices $(Q_{c,t+j}, Q_{i,t+j})$, in line with the belief setup considered in Adam, Marcet, and Beutel (2017), but rational expectations about all remaining variables $(Z_{t+j}, X_{t+j}, W_{t+j}, R_{c,t+j}, R_{i,t+j})$.

We consider the setting with fully rational expectations as a point of reference. It is well known that under rational expectations the asset pricing implications of the model are strongly at odds with the data. Considering this setup, however, allows highlighting the empirical improvements achieved by introducing subjective price beliefs.

Two considerations motivate us to keep rational expectations about variables other than prices: first, we do not want to deviate from the rational expectations assumption by more than in Adam, Marcet, and Beutel (2017), so as to illustrate that the same deviation that can be used to explain stock price behavior in an endowment setting can explain stock price behavior and business cycle dynamics; second, investor expectations about future stock prices can be observed relatively easily from investor survey data, which allows disciplining the subjective belief choice. Observing beliefs about future values of $(X_{t+j}, W_{t+j}, R_{c,t+j}, R_{i,t+j})$ is a much harder task, which prompts us to keep the standard assumption of rational expectations.

Under rational expectations, decision functions in period t depend only on the history of fundamental shocks Z^t, as discussed in Section 2.4.2. Given the Markov-structure for shocks Z_t, there is furthermore a recursive time-invariant form of the decision functions, where decisions depend only on current shock Z_t and the beginning-of-period capital stocks $(Z_t, K_{c,t}, K_{i,t})$. Using this fact, we can standardly solve for the nonlinear rational expectations equilibrium using global approximation methods.[28]

We consider subjective capital price expectations of the form previously introduced in Adam, Marcet, and Beutel (2017). Specifically, we assume that agents perceive the end-of-period capital prices $Q_{s,t}$ $(s = c, i)$ to evolve as

$$\log Q_{s,t} = \log Q_{s,t-1} + \log \beta_{s,t} + \log \varepsilon_{s,t},$$

with $\varepsilon_{s,t}$ denoting a transitory shock to price growth and $\beta_{s,t}$ a persistent component, which is given by

$$\log \beta_{s,t} = \log \beta_{s,t-1} + \log \nu_{s,t}.$$

The innovations $(\varepsilon_{s,t}, \nu_{s,t})$ are independent of each other, with $\log \varepsilon_{s,t} \sim ii\mathcal{N}(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2)$ and $\log \nu_{s,t} \sim ii\mathcal{N}(-\sigma_\nu^2/2, \sigma_\nu^2)$, and also independent of all other variables.[29]

Each period t, agents only observe capital prices $\log Q_s$ up to period t and estimate the persistent component driving capital gains. Let $\log \beta_{s,t-1} \sim \mathcal{N}(\log m_{s,t-1}, \sigma^2)$ denote the period $t-1$ prior belief about the persistent component, where σ denotes the steady state Kalman filter uncertainty.[30] In period t, agents observe the new capital price $Q_{s,t}$ and update their beliefs using Bayes' law. Their period t beliefs are then given by $\log \beta_{s,t} \sim \mathcal{N}(\log m_{s,t}, \sigma^2)$ with

$$\log m_{s,t} = \log m_{s,t-1} - \frac{\sigma_v^2}{2} + g\left(\log Q_{s,t} - \log Q_{s,t-1} + \frac{\sigma_\varepsilon^2 + \sigma_v^2}{2} - \log m_{s,t-1}\right), \quad (2.10)$$

where the Kalman gain is given by

$$g = \frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2}. \quad (2.11)$$

Agents' beliefs can thus be parsimoniously summarized by the variables $(m_{c,t}, m_{i,t})$, which capture *agents' degree of optimism* about future capital gains. Equation (2.10) shows how agents' capital gain expectations are driven by observed past capital gains, whenever $g > 0$. Adam, Marcet, and Beutel (2017) show that for values of g around 2% this delivers

[28] This requires a standard transformation of variables, so as to render them stationary.

[29] The random variables $\varepsilon_{s,t+1}$, $\nu_{s,t+1}$ are not defined on the probabiltiy space Ω, but on a larger auxiliary space $\overline{\Omega}$, as they contain random variation that is independent of variation in both prices and other observables in equilibrium. Decision function can still be specified to be functions of Ω only.

[30] We have $2\sigma^2 \equiv -\sigma_\nu^2 + \sqrt{(\sigma_\nu^2)^2 + 4\sigma_\nu^2\sigma_\varepsilon^2}$.

an empirically plausible specification describing the dynamics of investors' subjective capital gain beliefs over time.

To avoid simultaneous determination of price beliefs and prices, we follow Adam, Marcet, and Beutel (2017) and use a slightly modified information setup in which agents receive at time t information about the transitory component in $t-1$, $\log \varepsilon_{s,t-1}$. The modification causes the updating equation (2.10) to contain only lagged price growth and no variance correction terms. Capital gain beliefs then evolve according to

$$\log m_{s,t} = \log m_{s,t-1} + g\left(\log Q_{s,t-1} - \log Q_{s,t-2} - \log m_{s,t-1}\right) + g \log \varepsilon^1_{s,t} \tag{2.12}$$

where $\log \varepsilon^1_{s,t} \sim ii\mathcal{N}(\frac{-\sigma^2_\varepsilon}{2}, \sigma^2_\varepsilon)$ is a time t innovation to agents' information set (unpredictable using information available to agents up to period $t-1$), which captures information that agents receive in period t about the transitory price growth component $\log \varepsilon_{s,t-1}$.[31] Agents' expectations are then given by

$$E^{\mathcal{P}}_t \left[\frac{Q_{s,t+1}}{Q_{s,t}}\right] = m_{s,t}. \tag{2.13}$$

Given the specific subjective price beliefs and the assumption of rational expectations about $(Z_{t+j,}X_{t+j}, W_{t+j}, R_{c,t+j}, R_{i,t+j})$, we can work with the probability space

$$\widetilde{\Omega} \equiv \Omega_Z \times \Omega_{Q,c} \times \Omega_{Q,i}$$

and consider decision functions of the form

$$(C_t, H_t, K_{c,t+1}, K_{i,t+1}) : \widetilde{\Omega}^t \longrightarrow R^4. \tag{2.14}$$

Moreover, the subjective belief setup allows us to summarize the history of capital prices in the two sectors using the two state variables $(m_{c,t}, m_{it}) \in \mathbb{R}^2$, so that household decision functions are time-invariant function of an extended set of state variables $(Z_t, K_{c,t}, K_{i,t}, m_{c,t}, m_{it})$. The state S_t describing the aggregate economy at time t can be summarized by the vector $S_t = (Z_t, K_{c,t}, K_{i,t}, m_{c,t}, m_{it}, Q_{c,t-1}, Q_{i,t-1})$, where the latter two variables are required to be able to describe the equilibrium dynamics of beliefs over time, as implied by equation (2.12). The economy's equilibrium dynamics can then be

[31]However, we follow the baseline specification of Adam, Marcet, and Beutel (2017) and assume that along the equilibrium path no actual information is observed, instead set $\ln \varepsilon^1_{s,t}$ to 0 each period. It is easy to see in our specific model, that – given the solution to the filtering problem – beliefs about future m values do not matter for policy functions. For this reason we work with a reduced updating equation

$$\log m_{s,t} = \log m_{s,t-1} + g\left(\log Q_{s,t-1} - \log Q_{s,t-2} - \log m_{s,t-1}\right)$$

and the reduced outcome space $\tilde{\Omega}$ below, which does not include observable innovations $\varepsilon^1_{c,t}$ and $\varepsilon^1_{i,t}$.

described by a nonlinear state transition function $G(\cdot)$ mapping current states and future technology into future states

$$S_{t+1} = G(S_t, Z_{t+1}),$$

and by an outcome function $F(\cdot)$ mapping states into economic outcomes for the remaining variables

$$(W_t, R_{c,t}, R_{i,t}, Y_t, C_t, I_t, H_t) = F(S_t).$$

We endow households with beliefs about future values $(W_{t+j}, R_{c,t+j}, R_{i,t+j})$ consistent with these equilibrium mappings.[32] Since the mappings depend themselves on the solution to the household problem and the solution to the household problem on the assumed mappings, one needs to solve for a fixed point. Because the economy features seven state variables, it is generally not feasible to solve for the fixed point of this problem at a speed that would allow estimating the subjective belief model using the simulated methods of moments. Yet, as we explain below, it becomes computationally feasible for our log-linear specification for household preferences.

2.6 Equilibrium Conditions

This section derives the equations characterizing the competitive equilibrium. These equations hold independently of the assumed belief structure. From the household's first-order conditions we have[33]

$$C_t = W_t, \tag{2.15}$$

$$Q_{c,t} = \beta E_t^{\mathcal{P}} \left[\frac{W_t}{W_{t+1}} \left((1-\delta_c) \, Q_{c,t+1} + R_{c,t+1} \right) \right], \tag{2.16}$$

$$Q_{i,t} = \beta E_t^{\mathcal{P}} \left[\frac{W_t}{W_{t+1}} \left((1-\delta_i) \, Q_{i,t+1} + R_{i,t+1} \right) \right], \tag{2.17}$$

for all $t \geq 0$. The first-order conditions of consumption-sector firms are

$$W_t = \frac{(1-\alpha_c) \, Y_{c,t}}{H_{c,t}}, \tag{2.18}$$

$$R_{c,t} = \frac{\alpha_c Y_{c,t}}{K_{c,t}}, \tag{2.19}$$

[32] That is, households beliefs $\mathcal{P}$ over the full set of exogenous variables Ω are jointly described by the equilibrium mappings F and G and a measure $\tilde{\mathcal{P}}$ on $\tilde{\Omega}$ that is consistent with the exogenous law of motion for Z and the perceived laws of motion for Q_i and Q_c implied by the learning specification above. Appendix B.3.1 provides details on how $\mathcal{P}$ can be recovered from $\tilde{\mathcal{P}}$ and the mappings F and G. Appendix B.3.2 proves that there is a unique measure $\tilde{\mathcal{P}}$ consistent with the subjective belief description given in this section.

[33] The household budget constraint will hold automatically, as we will keep all market clearing conditions in the set of equations characterizing equilibrium.

and the optimality conditions of investment-sector firms

$$W_t = (1-\alpha_i) Q_{c,t} K_{i,t}^{\alpha_i} Z_t^{1-\alpha_i} H_{i,t}^{-\alpha_i}, \tag{2.20}$$

$$R_{i,t} = \alpha_i Q_{c,t} K_{i,t}^{\alpha_i - 1} Z_t^{1-\alpha_i} H_{i,t}^{1-\alpha_i}. \tag{2.21}$$

The market clearing conditions are

$$C_t = Y_{c,t}, \tag{2.22}$$

$$H_t = H_{c,t} + H_{i,t}, \tag{2.23}$$

$$K_{c,t+1} = (1-\delta_c) K_{c,t} + Y_{i,t}, \tag{2.24}$$

$$K_{i,t+1} = (1-\delta_i) K_{i,t} + X_t. \tag{2.25}$$

For the case with subjective beliefs, we have in addition (for $s = c, i$)

$$E_t^{\mathcal{P}} [Q_{s,t+1}] = m_{s,t} Q_{s,t}, \tag{2.26}$$

where $m_{s,t}$ evolves according to (2.12) and rational expectations about all other variables. For the case with rational expectations, we have $E_t^{\mathcal{P}} [\cdot] = E_t [\cdot]$.

Equations (2.15), (2.18) and (2.22) show that labor in the consumption sector is constant and given by

$$H_{c,t} = 1 - \alpha_c. \tag{2.27}$$

Consumption variations are thus driven entirely by productivity variations and by the dynamics of capital accumulation in the consumption sector. This feature will allow the model to replicate the smoothness of the aggregate consumption series. For the investment sector, we obtain from equation (2.20) that

$$H_{i,t} = K_{i,t} Z_t^{\frac{1-\alpha_i}{\alpha_i}} \left((1-\alpha_i) \frac{Q_{c,t}}{W_t} \right)^{\frac{1}{\alpha_i}}, \tag{2.28}$$

which shows that high prices for consumption capital ($Q_{c,t}$) induce – ceteris paribus – high labor demand by firms in the investment sector and thus drive up investment in the consumption sector. This feature will allow the model to generate a positive comovement between capital prices, hours worked and investment. In addition, it follows from equation (2.20) that high prices for consumption capital ($Q_{c,t}$) and high labor input ($H_{i,t}$) induce high rental rates in the investment sector. From equation (2.17) it then follows that high prices for consumption capital ($Q_{c,t}$) transmit via this channel to higher prices for investment capital ($Q_{c,t}$), so that the asset prices in the two sectors comove with each other.

2.7 Equilibrium Dynamics under Subjective Beliefs

This section provides analytic insights into the equilibrium dynamics of the model under subjective beliefs. These analytic results show how the model can give rise to persistent swings in capital prices and thus generate volatile and persistent stock price dynamics, while simultaneously producing smooth dynamics for the remaining variables.

We start by studying the prices dynamics of consumption-sector capital. Given the assumption of rational dividend and wage expectations and given the assumed subjective price beliefs (2.26), we obtain from equation (2.16)

$$Q_{c,t} = \beta E_t \left[\frac{W_t}{W_{t+1}} \right] (1-\delta_c)\, m_{c,t} Q_{c,t} + \beta E_t \left[\frac{W_t}{W_{t+1}} R_{c,t+1} \right]. \tag{2.29}$$

Using the equilibrium relationships (2.19), (2.22) and (2.15) allows expressing the discounted expected rental rate (the last term in equation (2.29)) using period t variables:

$$E_t \left[\frac{W_t}{W_{t+1}} R_{c,t+1} \right] = \alpha_c \frac{W_t}{K_{c,t+1}} \tag{2.30}$$

Substituting into equation (2.29) and solving for $Q_{c,t}$ delivers

$$Q_{c,t} = \frac{\beta \alpha_c}{\frac{1}{W_t} - \beta (1-\delta_c) E_t \left[\frac{1}{W_{t+1}} \right] m_{c,t}} \frac{1}{K_{c,t+1}}. \tag{2.31}$$

To illustrate the asset price dynamics implied by this equation, suppose for a moment that the capital stock ($K_{c,t+1}$) and expected future wages ($E_t[1/W_{t+1}]$) are constant over time. We shall consider the additional effects associated with movements in these variables below. The asset pricing equation (2.31) then takes the particularly simple form

$$Q_{c,t} = \frac{A}{\frac{1}{W_t} - B m_{c,t}}, \tag{2.32}$$

where A and B are positive constants. From

$$W_t = C_t = Y_{c,t} = Z_t K_{c,t}^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}, \tag{2.33}$$

we get that wages are determined by productivity (Z_t) and the consumption-sector capital stock ($K_{c,t}$). This shows that that capital prices in equation (2.32) increase with productivity (Z_t), the capital stock ($K_{c,t}$) and with subjective optimism ($m_{c,t}$). Optimism is itself driven by past observed capital gains according to equation (2.12). Independent of the source of an initial increase in the capital price $Q_{c,t,}$, it follows from equation (2.32) that it will propagate over time through two self-reinforcing effects: (1) any observed capital gain today increases – via the belief updating equation (2.12) – optimism tomorrow

($m_{c,t+1}$) and thus tomorrow's capital prices; (2) an increase in the current capital price $Q_{c,t}$, increases hours worked in the investment sector (equation (2.28)), thereby tomorrow's consumption-sector capital and thus tomorrow's wages (W_{t+1}) (equation (2.33)).[34] An increased wage tomorrow raises tomorrow's capital price (equation (2.32)).

The previous arguments shows how – for given end-of-period capital stock levels ($K_{c,t+1}$) and given expectations about future wages ($E_t[1/W_{t+1}]$) – asset price increases are followed by further increases.[35] Yet, along a path where $Q_{c,t}$ rises, investment in consumption-sector capital also rises and thus the capital stock $K_{c,t+1}$ and future wages W_{t+1}. This sets in motion dampening forces that work against the capital price increase. The last term on the r.h.s of equation (2.31) shows that an increased capital stock starts to directly dampen the capital price increase. This effect emerges because an expanded capital stock leads – ceteris paribus – to falling rental rates (equation (2.30)). Similarly, since wages will increase as the capital stock expands (equation (2.33)) inverse wage expectations $E_t\left[1/W_{t+1}\right]$ in equation (2.31) start to fall. This exerts an additional dampening effect on asset prices (equation (2.31)).

It turns out that the dampening forces associated with rising capital stocks and rising expected wages are not always strong enough to terminate capital price booms. To insure that capital prices remain finite, we follow Adam, Marcet, and Nicolini (2016) and impose a projection facility that eventually bounds upward belief revisions.[36] The projection facility can be interpreted as an approximate implementation of a Bayesian updating scheme where agents have a truncated prior about the support of $\beta_{s,t}$. The details of the projection facility are spelled out in Appendix B.2.

Figure 2.3 illustrates the quantitative model dynamics by displaying the impulse responses to an exogenous one-time increase in the quarterly expected growth rate of the price of consumption sector capital ($m_{c,t}$) by 10 basis points in period zero.[37] The figure uses the estimated parameters from Section 2.8 and normalizes all variables relative to the deterministic balanced growth path values that would emerge in the absence of the shock. Figure 2.3 shows that the period zero increase in optimism and the associated

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[34]This investment increase is obvious, if it is triggered by an increase in $m_{c,t}$, as the wage W_t does then not react. If instead the initial price increase is due to an increase in productivity, we can substitute (2.32) into (2.28) to obtain $H_{i,t} = K_{i,t}Z_t^{\frac{1-\alpha_i}{\alpha_i}}\left((1-\alpha_i)\frac{A}{1-W_tBm_{c,t}}\right)^{\frac{1}{\alpha_i}}$. This shows that the joint wage and capital price increase still implies an increase in hours worked in the investment sector.

[35]Similarly, asset price decreases are followed by further decreases.

[36]The issue of bounding beliefs so as to ensure that expected utility remains finite is present in many applications of both Bayesian and adaptive learning to asset prices. The literature has typically dealt with this issue by using a projection facility, assuming that agents simply *ignore* observations that would imply updating beliefs beyond the required bound. See Timmermann (1993, 1996), Marcet and Sargent (1989), or Evans and Honkapohja (2001). This approach has two problems. First, it does not arise from Bayesian updating. Second, it introduces a discontinuity in the simulated moments and creates difficulties for our MSM estimation. To avoid this, we follow the differentiable approach to bounding beliefs used in Adam, Marcet, and Nicolini (2016).

[37]We report the impulse response to a technology shock in Section 2.8.

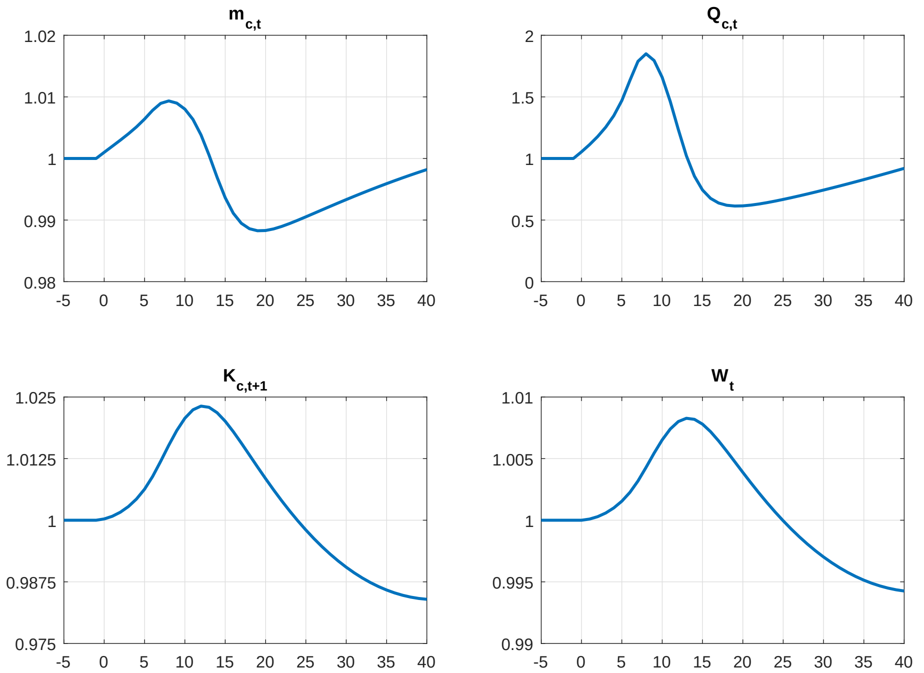

Figure 2.3. Impulse response to a +10bps shock to expected capital gains in consumption-sector capital ($m_{c,t}$)

small increase in the capital price is followed by eight quarters of further increases in optimism and capital prices. The increased capital price leads to additional investment in consumption-sector capital and also to a wage increase, as discussed before. These reactions eventually dampen the price increase and capital prices start to revert direction. Prices significantly undershoot their balanced growth path value before slowly recovering. As capital prices become depressed, investment falls and consumption-sector capital persistently undershoots its balanced growth path value. Associated with the reduction in capital is a reduction in wages.

Figure 2.4 illustrates how the dynamics in the consumption sector spill over into the investment sector. To understand the channels through which this spillover happens, note that we obtain from equation (2.17) and the assumed subjective price beliefs (2.26) the following relationship:

$$Q_{i,t} = \frac{\beta E_t \left[\frac{W_t}{W_{t+1}} R_{i,t+1} \right]}{1 - \beta \left(1 - \delta_i\right) E_t \left[\frac{W_t}{W_{t+1}} \right] m_{i,t}}. \tag{2.34}$$

Since wages increase during the price boom in consumption sector capital, see Figure 2.3,

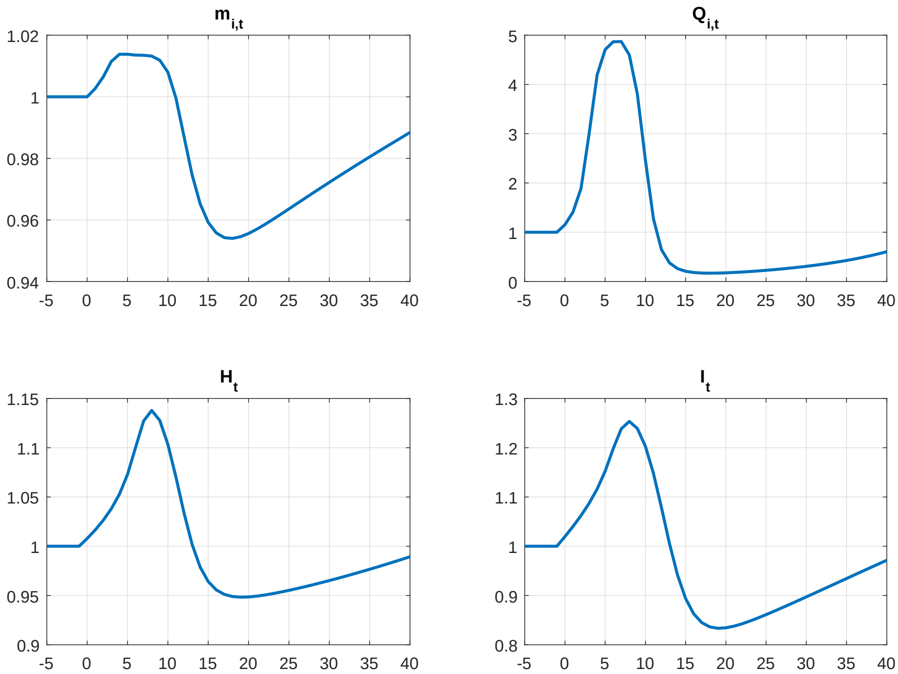

Figure 2.4. Impulse response to a +10bps shock to expected capital gains in consumption-sector capital ($m_{c,t}$)

the drop of $E\left[W_t/W_{t+1}\right]$ exerts downward pressure on the price of investment-sector capital in both the numerator and the denominator of equation (2.34). Yet, equation (2.20) shows that expected rental rates $R_{i,t+1}$ increase strongly following the increase in the price of consumption-sector capital (and also as a result of the increase in hours worked in the investment sector). Due to the strong response of $R_{i,t+1}$, the overall price effect at time zero is positive and the price boom in the consumption-sector capital spills over into an increase in the price of investment-sector capital. The initial price increase then propagates via the belief updating mechanism (2.12) and the positive dependence of capital prices on $m_{i,t}$ (equation (2.34)) into a persistent boom for the price of investment-sector capital. This shows that capital prices in the two sectors have a tendency to comove over time.

Figures 2.3 and 2.4 show that capital prices in the investment sector move considerably more than in the consumption sector. Figure 2.5 shows that this difference is far less pronounced for a standard stock market valuation measure such as the PD ratio. Since dividends in the investment sector increase strongly during the price boom (equation (2.21)) the difference in the response of the PD ratio across sectors is much more muted. Overall, the model is consistent with empirical observation that stock prices in

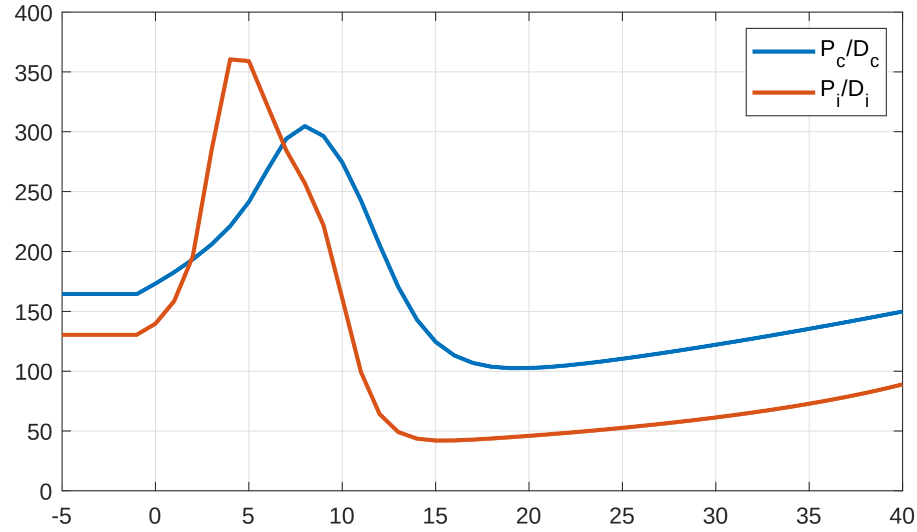

Figure 2.5. Impulse response to a +10bps shock to expected capital gains in consumption-sector capital ($m_{c,t}$)

the investment sector are more volatile than stock prices in the consumption sector.

While the PD ratios approximately double in response to the considered optimism shock, the response of real quantities is one or two orders of magnitude smaller. Consumption, for instance, increases in line with the increase in the stock of consumption sector capital and moves by less than +/- 1% over the boom-bust cycle, see Figure 2.3. Hours worked are more variable and increase by around 13% while investment increases by up to 25%. This relatively muted response of real variables allows the model to reconcile the relative smoothness of the business cycle with the observed high volatility of stock prices.

2.8 Quantitative Performance: Subjective versus Rational Price Beliefs

This section estimates the model using the simulated methods of moments (SMM). In the setting with subjective price beliefs, the model has nine parameters that need to be estimated, $(\beta, \alpha_c, \alpha_i, \delta_c, \delta_i, \gamma, \sigma, p, g)$. There is one parameter less under fully rational expectations, as the Kalman gain g then drops out.

In our baseline approach, we estimate the model using 13 target moments and a diagonal weighting matrix with the data-implied variance of the estimated data moment as diagonal entries. For the subjective belief model, these target moments include the seven business cycle moments listed in Table 2.1 and all asset pricing moments listed in Table 2.2, except for the mean and standard deviation of the risk-free interest rate. We exclude the risk-free rate moments from the set of targeted moments because the model has a hard time fully matching the equity premium. We shall nevertheless report the risk-free rate moments implied by the estimated model. For the rational expectations model, we furthermore exclude the equity return moments ($E[r^e]$, $\sigma(r^e)$) and the autocorrelation of the PD ratio ($\rho(P/D)$), as including these variables significantly deteriorates the model fit along the business cycle dimension without improving it substantially for the excluded moments.[38,39]

With fully rational expectations, equilibrium prices and equilibrium quantities are unique.[40] For the case with subjective price beliefs, this is far from obvious, but the following result shows that we have indeed uniqueness of equilibrium allocations and prices in our setting with subjective price beliefs:

Proposition 1. *There are unique functions G and F and a unique measure $\tilde{\mathcal{P}}$ on $\tilde{\Omega}$, such that*

1. *$\tilde{\mathcal{P}}$ describes the joint beliefs of households about technology and capital prices as defined in Section 2.5*
2. *G is a state transition function, $S_{t+1} = G(S_t, Z_{t+1})$, and F an outcome function, $(W_t, R_{c,t}, R_{i,t}, Y_t, C_t, I_t, H_t) = F(S_t)$, such that S_t, S_{t+1} and $(W_t, R_{c,t}, R_{i,t}, Y_t, C_t, I_t, H_t)$ are consistent with*
   - *(a) All equilibrium conditions (see Section 2.6)*
   - *(b) Households' belief updating equations (equation (2.12) for $s \in \{c, i\}$)*[41]

The proof of Proposition 1 is given in Appendix B.3.3. There, we also provide an explicit characterization of the mappings G and F based on the equations collected in Section 2.6. Essentially, this characterization determines those functions up to a static equation system (stated in Lemma 4 in the appendix). In our numerical solution procedure, we solve this equation system and construct the mapping G mirroring the construc-

[38]Excluding these moments does not conceal the true potential of the rational expectations model to match the equity return moments. Even if we give the model the best chance to match $E[r^e]$ and $\sigma(r^e)$ by making them the only estimation targets, the resulting model moments are just 0.74% and 0.50%, respectively, as opposed to the values 0.77% and 0.16% reported in Table 2.6. Relative to their empirical counterparts (1.87% and 7.98%), these numbers are small and similar in magnitude.

[39]We keep the level and standard deviation of the PD ratio as estimation targets for the RE model, because otherwise the payout ratio p would be unidentified. Excluding them has almost no impact on the remaining estimated parameters.

[40]As is well-known, the market allocation is equivalent to the solution of a social planning problem, with the latter featuring a concave objective function and a convex set of constraints.

[41]With projection adjustments as described in Appendix B.2, where necessary.

Table 2.5
Estimated parameters

Parameter	Subjective Belief Model	RE Model w/o inv. shocks	RE Model w inv. shocks
β	0.996	0.997	0.997
α_c	0.36	0.36	0.36
α_i	0.73	0.35	0.35
δ_c	0.010	0.020	0.010
δ_i	0.014	0.030	0.030
γ	1.004	1.005	1.003
σ	0.015	0.018	0.015
p	0.350	0.195	0.337
g	0.019	–	–
σ_i	–	–	0.015

tion in the proof of Proposition 1. Having recovered G and F, this essentially determines equilibrium.[42]

When estimating the learning model we impose the additional restriction that the impulse responses of capital prices to fundamental shocks display an exponential decay rate of 1.16% per quarter.[43] We do so to avoid that the estimation selects parameter values that would imply deterministic equilibrium cycles. Clearly, imposing this additional restriction can only deteriorate the fit of the targeted moments. The model moments that we present thus represent a lower bound on the fit that could be achieved in the absence of this restriction.

Table 2.5 reports the estimated parameter values and Table 2.6 target moments. We first discuss business cycle moments, which are reported in the upper half of Table 2.6. Along this dimension, our estimated subjective belief model matches the targets very well. Instead, the rational expectations model displays somewhat too much consumption volatility and considerably too little investment and hours volatility. In addition, the degree of comovement of the macro aggregates is higher in the rational expectations model than both in the data and in the subjective belief model.

The poorer performance of the rational expectations model can be fixed by introduc-

[42]Moments are computed based on a Monte Carlo simulation of model dynamics using the state transition function G. Simulated paths for all parameter combinations start in the balanced growth path and are based on the same random draw of 10,000 productivity shocks. The first 500 observations of the simulation are dropped before computing moments.

[43]Details of this restriction are provided in Appendix B.4.

Table 2.6
Model fit for targeted moments (13 / 10 targets, 9/8 model parameters for the subjective belief/RE model)

	Data (std.dev.)	Subj. Belief Model	RE Model w/o inv. shocks	RE Model w inv. shocks
	Business Cycle Moments			
$\sigma(Y)$	1.72 (0.25)	1.83	1.90	1.85
$\sigma(C)/\sigma(Y)$	0.61 (0.03)	0.67	0.75	0.66
$\sigma(I)/\sigma(Y)$	2.90 (0.35)	2.90	1.88	2.79
$\sigma(H)/\sigma(Y)$	1.08 (0.13)	1.06	0.31	0.56
$\rho(Y,C)$	0.88 (0.02)	0.84	0.98	0.86
$\rho(Y,I)$	0.86 (0.03)	0.89	0.97	0.90
$\rho(Y,H)$	0.75 (0.03)	0.70	0.89	0.80
	Financial Moments			
$E[P/D]$	152.3 (25.3)	150.0	174.6	166.0
$\sigma(P/D)$	63.39 (12.39)	44.96	7.00	8.28
$\rho(P/D)$	0.98 (0.003)	0.97	0.96	0.95
$E[r^e]$	1.87 (0.45)	1.25	0.77	0.57
$\sigma(r^e)$	7.98 (0.35)	7.07	0.16	0.16
$\sigma(D_{t+1}/D_t)$	1.75 (0.38)	2.46	1.19	1.69

ing additional investment-specific shocks.[44] While the literature has mostly focused on persistent investment-specific technology processes, we add here just a parsimonious iid component to the investment-sector production function, so as to keep the changes to the model described above minimal and to have only one additional parameter. Specifically, we replace $Y_{i,t}$ from equation (2.1) by

$$Y_{i,t} = \varepsilon_t^I K_{i,t}^{\alpha_i} \left(Z_t H_{i,t}\right)^{1-\alpha_i}$$

with $\log \varepsilon_t^I \sim \mathcal{N}(-\frac{\sigma_i^2}{2}, \sigma_i^2)$. We solve and re-estimate this augmented model under rational expectations using the same procedure and targets as for the model without investment-specific shocks. The resulting parameter estimates and model moments are reported in the third columns of Tables 2.5 and 2.6, respectively. With investment shocks, the business cycle predictions of the model are much closer to the data and overall comparable with our subjective belief model. While this approach is in principle capable of generating

[44]Investment-specific technology shocks are found to be important drivers of business cycles in estimated models (compare Justiniano, Primiceri, and Tambalotti, 2010, 2011) and thus are a natural candidate to improve business cycle predictions of the real business cycle model.

Table 2.7
Model fit for untargeted moments

Moment	Data	Subjective Belief Model	RE Model w/o inv. shocks	RE Model w inv. shocks
$E[r^f]$	0.25 (0.13)	0.78	0.77	0.57
$\sigma(r^f)$	0.82 (0.12)	0.06	0.09	0.16
$\rho(H, P/D)$	0.51 (0.17)	0.79	-0.97	-0.95
$\rho(I/Y, P/D)$	0.19 (0.31)	0.69	-0.97	-0.94
$\rho(E^{\mathcal{P}}[r^e], P/D)$	0.79 (0.07)	0.52	-0.99	-0.98

good business cycle predictions, the required volatility σ_i of the investment-specific shock is as high as the volatility σ of the neutral shock (see Table 2.5). Interestingly, this additional source of exogenous randomness is not required in our subjective belief model to generate the same amount of investment volatility (and even more hours volatility), despite σ being the same in both specifications.

We next turn to the financial target moments reported in the lower part of Table 2.6. The model with subjective beliefs is able to match the level and persistence of the PD ratio and the PD ratio in the model also displays substantial variability, although its volatility is somewhat lower than in the data. Equity returns in the model are also very volatile, generating almost 90% of the volatility observed in the data, despite a moderate – though somewhat overstated – dividend growth volatility. The model can match the level of equity returns only partially with the mean equity return in the model being only two thirds of the 1.87% per quarter earned on average on stock market investments in our sample period.

The two versions of the rational expectations model, reported in the last two columns, perform significantly worse along the financial dimension. While the level of the PD ratio (by choice of p), its persistence and the volatility of dividend growth are roughly in line with the data, the rational expectations model generates almost no price and return volatility and equity returns are considerably lower than in the subjective belief model, regardless of whether investment shocks are present or not.[45]

Table 2.7 reports all moments from Section 2.3 which where not targeted by the estimation. The first two rows show the mean and volatility of the risk-free rate. The model is consistent with a stable risk-free rate, both under subjective beliefs and rational expectations, but tends to overstate its level.[46] Yet, despite the too low mean stock return

[45] The poor ability to match $E[r^e]$ and $\sigma(r^e)$ is not because these are untargeted moments in the rational expectations estimation, compare footnote 38.

[46] The somewhat lower value for the model with investment shocks is due to the lower growth rate (γ) in that model.

and the too high mean risk-free rate, the subjective belief model generates an equity premium of 0.47%. While only around 30% of the historical equity premium, this result is still remarkable, given the low risk aversion implicit in our preference specification.[47] In comparison, there is literally no equity premium in both rational expectations versions of the model.

The subjective belief model also generates positive comovement between the business cycle and stock prices as measured by the correlations of H and I/Y with the PD ratio. In contrast, these correlations are strongly negative under rational expectations. Furthermore, the model matches the positive correlation between expected returns and the PD ratio. While quantitatively this correlation is almost four standard errors lower than in the data, it is still in the range of values that Adam, Marcet, and Beutel (2017) report for other surveys than the UBS Gallup survey, which our data moment is based on, see their Appendix A.2. We interpret this as evidence that our model generates overall realistic comovement of stock prices and expected returns. Again, rational expectations are strongly at odds with the data: because the PD ratio tends to mean-revert and expected returns must on average forecast realized returns, the correlation is strongly negative in both rational expectation specifications.

The central mechanism underlying the dynamics in the subjective belief model has already been discussed in Section 2.7. To illustrate this mechanism further, Figures 2.6 and 2.7 depict impulse response functions of the model with subjective beliefs to a technology shock (solid lines) for macro aggregates and capital prices/expectations, respectively.[48] For comparison purposes, the figures also plot impulse responses of the rational expectations model with an identical parameterization as the subjective belief model (dashed lines). We first discuss the latter as the dynamics under rational expectations are also informative for the response under subjective beliefs and the difference between the two explains well, why our subjective belief model can account better for the joint dynamics of business cycles and stock prices than the rational expectations model.

The dashed lines in Figure 2.6 show familiar adjustment dynamics present in a real business cycle model with capital adjustment frictions. On impact, the higher technology level leads to an increase in output, consumption and investment and, with one period lag,[49] also in hours worked. The increased technology increases the marginal product of

[47] Due to linear disutility of labor the effective risk aversion is even lower than what is implied by logarithmic utility in consumption: agents are *risk-neutral* with respect to all risk that is uncorrelated with wages. Thus only assets that yield low payoffs in states where wages are low carry a positive risk-premium.

[48] We start the model in a deterministic balanced growth path and plot deterministic impulse responses to a one-time one-standard-deviation positive innovation to technology, shutting down all randomness in the dynamics thereafter. For trending quantities (Y, C, I, K), the plotted dynamics are normalized by dividing by γ^t to eliminate the deterministic time trend.

[49] This lag is because of our particular timing assumption about the exogenous capital stock in the investment sector according to which the end-of-period capital stock $K_{i,t+1}$ is held proportional to Z_t. As it is not possible to reallocate capital from the consumption to the investment sector, the marginal

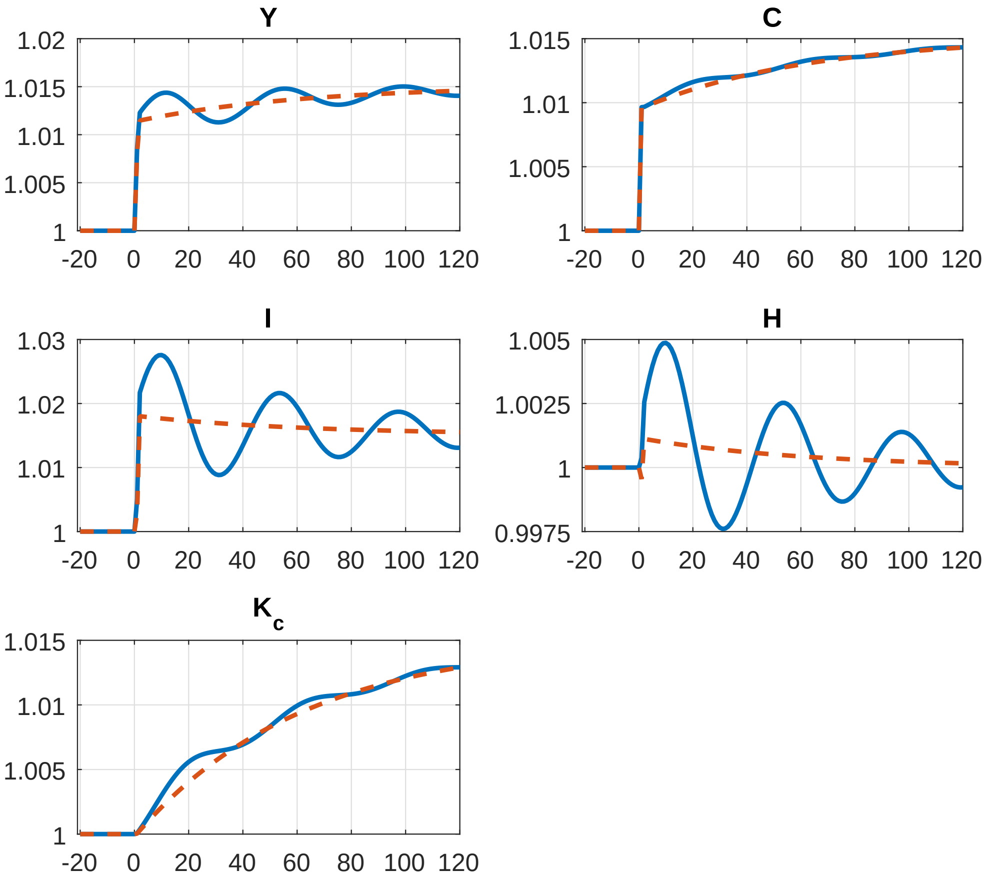

Figure 2.6. Impulse response functions of macro aggregates to a technology shock for subjective belief model (solid lines) and RE model (dashed lines)

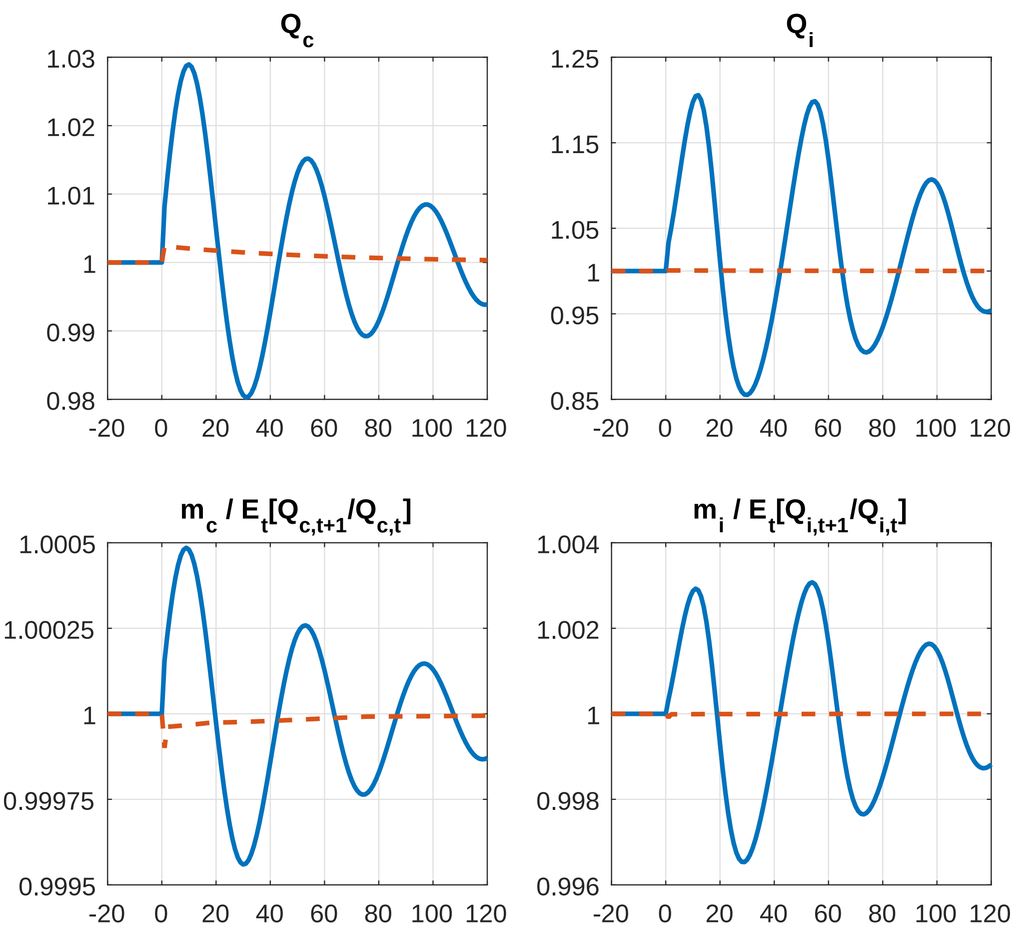

Figure 2.7. Impulse response functions of prices and beliefs to a technology shock for subjective belief model (solid lines) and RE model (dashed lines)

(consumption-sector) capital. This in turn increases capital prices in both sectors above their steady state level (see Figure 2.7),[50] which sets in motion a capital accumulation process that lasts beyond the impact period (lower panel in Figure 2.6). As capital increases over time, investment declines and consumption and output increase to their new balanced growth path, on which all three variables are permanently higher (relative to their deterministic trend) than on the pre-shock balanced growth path. As the capital stock expands, marginal products of capital and capital prices return back to steady state. The two lower plots in Figure 2.7, which show the expected capital price growth, $E_t [Q_{s,t+1}/Q_{s,t}]$ for $s \in \{c, i\}$, mirror the response of decreasing capital prices along the adjustment path.

Next, we turn to the impulse responses of the subjective belief model. Here, two channels determine the adjustment dynamics after a technology shock. First, the economy reacts on impact almost identical to the rational expectations economy[51] and, again, the shock creates a situation in which the capital stock is below its desired level, so low frequency movements of the economy display an adjustment process very close to the one observed under rational expectations. But second, as capital supply is not fully elastic, the higher capital demand on impact increases capital prices and this has substantially different implications under subjective beliefs than under rational expectations. Under rational expectations, the capital price just indicates the relative scarcity of capital and reverts back to steady state as capital is accumulated. Instead, under subjective beliefs, the initial surprise increase in prices $Q_{c,t}$, $Q_{i,t}$ triggers an upward revision in beliefs $m_{c,t+1}$, $m_{i,t+1}$ through the belief updating equation (2.12). The exogenous technology shock thus endogenously triggers a "belief shock" of the kind we have studied in Section 2.7 to illustrate the equilibrium dynamics.[52] As a result, all impulse responses cyclically fluctuate around the long-run adjustment path under rational expectations. Quantitatively, these cyclical deviations from their long-run paths are small for consumption and capital, somewhat larger for hours and output, even larger for investment and substantially larger for

product of labor in the investment sector (measured in capital units) grows on impact only due to the technology increase itself. With $\alpha_c < \alpha_i$, this increases is smaller than the increase in the wage, which even leads to a small decline of hours on impact, unless it is offset by a sufficiently strong increase in Q_c (recall, that hours in the consumption sector are constant, so all the dynamics in hours are due to adjustments in labor supply of the investment sector).

[50] While capital in the investment sector exogenously appears as a response to the technology shock, $Q_{i,t}$ still increases due to the increased output price $Q_{c,t}$ of the investment sector. The response is, however, quantitatively small and thus hardly visible in the figure.

[51] The only difference is that under rational expectations agents expect a subsequent decline in capital prices, which dampens the initial increase. Under subjective beliefs, no such decline is expected, which leads to a larger impact response in Q_c, translating into an additional positive effect on hours worked and investment.

[52] While we have shown there a "large" increase in optimism to illustrate the emergence of a boom-bust cycle, the effect is quantitatively much smaller here. This explains the differences between the Figures 2.6 and 2.7 here and the Figures 2.3 and 2.4 there.

capital prices and price-growth expectations. This explains why the model with subjective beliefs can generate considerably more volatility in investment, hours and, particularly, stock prices, without overstating the overall volatility of consumption and output. The additional belief-driven cyclical fluctuations in the adjustment dynamics after the shock also break the almost perfect comovement of output, consumption, investment and hours under rational expectations, instead bring the correlations $\rho(Y, C)$, $\rho(Y, I)$ and $\rho(Y, H)$ closer to the data.

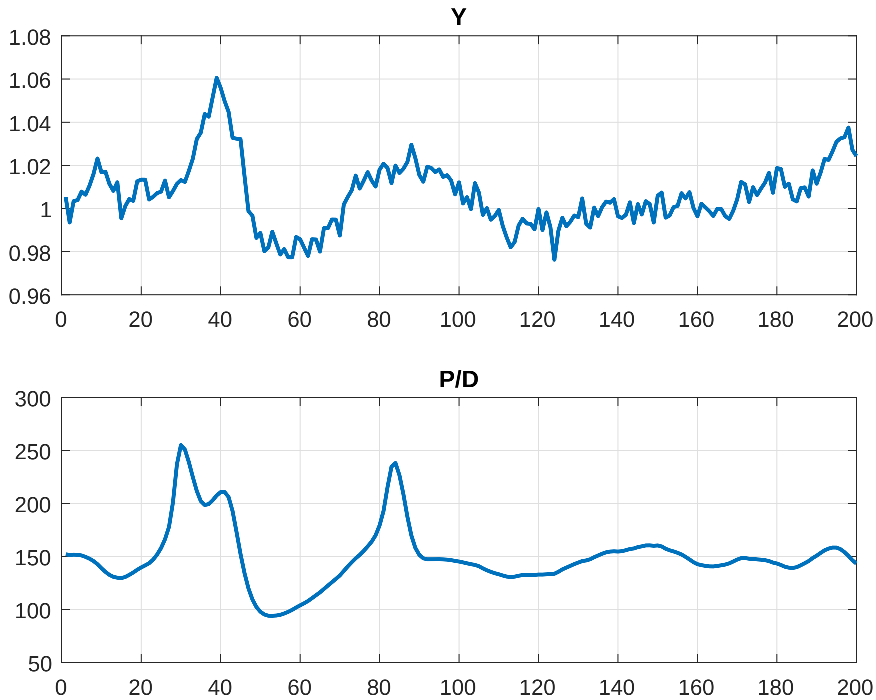

Figure 2.8. Simulated equilibrium paths of (detrended) output and the price-dividend ratio

While these impulse responses display cyclical price variations, the fluctuations in the PD ratio (not shown) implied by a single one-standard-deviation technology shock are quantitatively small. To generate sizeable fluctuations in the PD ratio as in Figure 2.5, larger amounts of capital gains optimism are required than generated by a single shock.[53] However, a sequence of positive technology shock can occasionally create a sufficient amount of optimism for dynamics to display large stock price cycles. Figure 2.8, which

[53]Note that the peak level in m_c in Figure 2.7 is just 5 basis points, whereas we have studied a one-time increase in optimism of 10 basis points in Section 2.7

plots simulated sample paths for output and the PD ratio over 200 quarters[54], shows two such price cycles in the first half of the sample (lower panel). The first, larger, price cycle is accompanied by an output boom that ends in a deep recession as prices drop sharply, mainly due to a strong decline in investment activity (not shown). The subsequent recovery is followed by a second, smaller cycle in output and prices. In the second half of the sample, prices fluctuate only little and output fluctuations are mainly driven by technology shocks and capital adjustment dynamics. From this discussion, we conclude that our model gives rise to quantitatively realistic stock price cycles, not unlike the ones depicted in Figure 2.1, and that these cycles are linked to economic activity.

The existence of stock price cycles in our model also implies that the model economy spends a considerably amount of time in states of capital gains optimism, in which PD ratios are substantially above their long-run mean. There is thus a large amount of mass in the right tail of the ergodic P/D distribution, which is illustrated in Figure 2.9. The figure plots once again the empirical kernel density of the P/D distribution in the data as in Figure 2.2, but now in addition the distribution in the model. As is visible there, the model generates a right tail that is remarkably close to the one observed in the data. However, in the model more mass is concentrated around the mode, while very small PD ratios are rare compared to the data. The reason for this asymmetry is that belief revisions have much larger effects on prices in states of optimism than in states of pessimism as is easily observable in the equations (2.31) and (2.34), such that additional negative shocks in a bust do not drive down prices as much as additional positive shocks in a boom drive them further up. The lack of mass in the left tail of the distribution also explains why we cannot match the full scale of the volatility of the PD ratio in Table 2.5 without overstating the mean.

To further illustrate by how much the shape of the P/D distribution is affected by subjective beliefs, Figure 2.10 plots the distribution of the PD ratio in the data together with the distributions implied by the two versions of the rational expectations model. In both cases, the distribution is unimodal and symmetric, with a substantial amount of mass closely centered around the mean.

[54]The figure plots output relative to its stochastic trend.

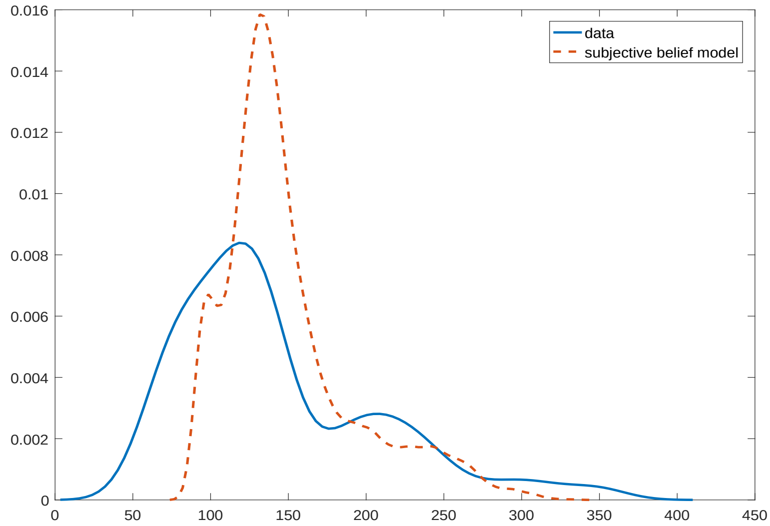

Figure 2.9. Unconditional density of PD ratio: subjective belief model versus data (not targeted in estimation, kernel estimates)

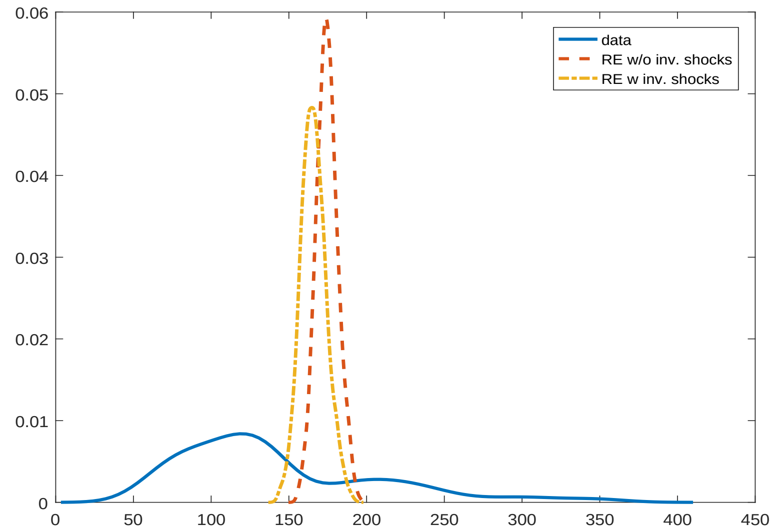

Figure 2.10. Unconditional density of PD ratio: RE models vs data (not targeted in estimation, kernel estimates)

2.9 Comparison with Boldrin, Christiano, and Fisher (2001) and Adam, Marcet, and Beutel (2017)

In this section we compare the quantitative implications of our model with Boldrin, Christiano, and Fisher (2001) and Adam, Marcet, and Beutel (2017). We choose to compare our model to Boldrin, Christiano, and Fisher (2001), because it is one of the leading joint explanations of stock prices and business cycles under rational expectations in the literature and the one closest to ours. We compare our model to Adam, Marcet, and Beutel (2017), because they study the same belief specification as we do, but in an endowment economy.

Boldrin, Christiano, and Fisher (2001) calibrate their model to a different data sample than we consider. In order to make the comparison as fair as possible, we use their reported data moments as a benchmark,[55] not the ones reported in Section 2.3, but re-estimate our model to fit those moments using the procedure outlined in Section 2.8. As estimation targets we use all moments reported in Table 2.9 with the exception of the standard deviation of the risk-free rate.[56] Boldrin, Christiano, and Fisher (2001) report annual instead of quarterly return moments despite their model being quarterly, which necessitates another change to make the two models comparable. As return autocorrelations are not fully in line with the data in either model,[57] we transform the annual return moments reported in Boldrin, Christiano, and Fisher (2001) both for the data and the model to quarterly frequency under the assumption of no return autocorrelation at that frequency.[58]

Table 2.8 shows the estimated parameters in comparison with the parameters used by Boldrin, Christiano, and Fisher (2001). Due to the similar structure of the two models, the parameters in their model have an identical interpretation as in ours. They are also quantitatively not too different. Table 2.9 reports a standard set of business cycle and financial return moments in the data and in both models. The second column reports

[55]For financial moments, they consider U.S. data covering the period 1892–1987, for business cycle moments U.S. data covering 1964:Q1–1988:Q2.

[56]The standard deviation of the risk-free rate is clearly lower in our model than in the data, particularly relative to the sample starting in 1892 used by Boldrin, Christiano, and Fisher (2001). We do not consider this as a serious shortcoming, since the data moment is likely overstated as argued in Section 2.3. For this reason, we do not attempt to match this number perfectly.

[57]In our model, returns at the quarterly frequency are positively autocorrelated. This is a known weakness of subjective price belief models of the kind studied here in the absence of transitory shocks. See Adam, Marcet, and Beutel (2017) Section VIII.A for a discussion and solution of this issue. Instead, in Boldrin, Christiano, and Fisher (2001) quarterly returns are strongly negatively correlated.

[58]This assumption is (approximately) correct for the data and it transforms – counterfactually – the good fit of the Boldrin, Christiano, and Fisher (2001) model to quarterly returns, so as to not bias the model comparison towards our model. It implies that means are divided by 4 and standard deviations are divided by 2. Similarly, we divide standard errors of means by 2 and standard errors of standard deviations by $\sqrt{2}$.

Table 2.8
Parameters for subjective belief model and model of Boldrin, Christiano, and Fisher (2001)

Parameter	Subjective Belief Model	BCF
β	0.997	0.999999
α_c	0.36	0.36
α_i	0.6	0.36
δ_c	0.15	0.021
δ_i	0.01	0.021
γ	1.004	1.004
σ	0.015	0.018
g	0.025	–
p	0.4	(0.248)

Note: BCF refers to Boldrin, Christiano, and Fisher (2001).

moments for our model, the third column moments for Boldrin, Christiano, and Fisher (2001). Overall, both models match the set of business cycle moments well. Where there are differences between the two, our model tends to do slightly better. Along the financial dimension, the model by Boldrin, Christiano, and Fisher (2001) is able to match the average levels of the risk-free rate and stock returns perfectly and generates stock return volatility close to the one observed in the data. Our model is less successful with the former two moments, but its prediction lie within a one-standard-error interval around the point estimates in the data. As Boldrin, Christiano, and Fisher (2001), we are able to generate high stock return volatility, but unlike in their paper, this is not achieved by generating counterfactually high volatility in the risk-free rate.

Table 2.10 reports additional evidence for the two models. The first four moments are statistics reported in Tables 1 and 2 of Boldrin, Christiano, and Fisher (2001) which have not been discussed in Section 2.3 of the present paper. $\rho(\Delta Y_t)$, $\rho(\Delta C_t)$ denote the autocorrelation of log output growth and log consumption growth, respectively, $\sigma(P_{hp})$ is the standard deviation of logged and HP-filtered quarterly stock prices and $\rho(Y, P_{hp})$ is the correlation of output and stock prices, both logged and HP-filtered. The two models perform similarly for the first three moments, both matching the persistence of output growth, but underpredicting the persistence of consumption growth and overstating the volatility of HP-filtered stock prices. Our model generates somewhat more comovement of stock prices with output than Boldrin, Christiano, and Fisher (2001), in line with the data.

Table 2.10 also shows statistics that relate to the behavior of dividends and the PD ratio. These statistics are not reported by Boldrin, Christiano, and Fisher (2001). We

Table 2.9
Model comparison with Boldrin, Christiano, and Fisher (2001): targeted moments

	Data (std.dev.)	Subj. Belief Model	BCF
Business Cycle Moments			
$\sigma(Y)$	1.89 (0.21)	1.83	1.97
$\sigma(C)/\sigma(Y)$	0.40 (0.04)	0.67	0.69
$\sigma(I)/\sigma(Y)$	2.39 (0.06)	2.46	1.67
$\sigma(H)/\sigma(Y)$	0.80 (0.05)	0.80	0.51
$\rho(Y,C)$	0.76 (0.05)	0.91	0.95
$\rho(Y,I)$	0.96 (0.01)	0.93	0.69
$\rho(Y,H)$	0.78 (0.05)	0.74	0.86
Financial Moments			
$E[r^f]$	0.30 (0.41)	0.68	0.30
$E[r^e - r^f]$	1.66 (0.89)	0.79	1.66
$\sigma(r^f)$	2.64 (0.52)	0.07	12.30
$\sigma(r^e)$	9.70 (1.10)	9.21	9.20

Notes: BCF refers to Boldrin, Christiano, and Fisher (2001); data and standard errors are taken from BCF and refer to their sample; moments and standard errors for financial moments are transformed to quarterly frequency by the procedure described in the main text.

Table 2.10
Model comparison with Boldrin, Christiano, and Fisher (2001): additional moments

	Data (std. dev.)	Subj. Belief Model	BCF
Additional Moments reported by BCF			
$\rho(\Delta \log Y)$	0.34 (0.07)	0.39	0.36
$\rho(\Delta \log C)$	0.24 (0.09)	-0.02	-0.05
$\sigma(P_{hp})$	8.56 (0.85)	19.7	12.1
$\rho(Y, P_{hp})$	0.30 (0.08)	0.32	0.16
Dividend and P/D Moments			
$\sigma(D_{t+1}/D_t)$	1.75 (0.38)	1.49	6.87
$E[P/D]$	152.3 (25.3)	110.5	162.4
$\sigma(P/D)$	63.39 (12.39)	32.95	13.20
$\rho(P/D)$	0.98 (0.003)	0.91	0.18
$\rho(H, P/D)$	0.51 (0.17)	0.29	-0.60
$\rho(I/Y, P/D)$	0.19 (0.31)	0.20	-0.60

Notes: BCF refers to Boldrin, Christiano, and Fisher (2001); p estimated to minimize standard-error-weighted squared distance of mean and std of P/D from data, estimate is $p = 0.24782$; data and standard errors for first four moments are taken from BCF and refer to their sample; data and standard errors for remaining moments are not reported in BCF, we therefore take the ones from Section 2.3.

therefore report our own estimates discussed in Section 2.3 in the data column and compute the respective moments in the Boldrin, Christiano, and Fisher (2001) model ourselves.[59] For the definition of dividends and prices we use the same convention as for our own model. Namely, firms pay out a fixed fraction p of capital rental income each period as dividends and reinvest the remaining fraction $1-p$ into new capital. Then, dividend growth equals the growth rate of the (sector-weighted) capital rental rates and the PD ratio is an affine linear function of the capital-price-to-rental-rate ratio, as in our model. For this reason, only the moments $E[P/D]$ and $\sigma(P/D)$ depend on the value of the parameter p, which is not present in Boldrin, Christiano, and Fisher (2001). We choose this parameter so as to minimize the sum of squared t statistics for two moments.[60] The resulting value is $p = 0.248$. In our model, dividends and the PD ratio behave qualitatively as discussed in Section 2.8, although the overall quantitative fit is somewhat worse, because the PD ratio was not targeted by the estimation. Yet, for any of the reported moments, our model clearly outperforms Boldrin, Christiano, and Fisher (2001). Dividend growth is far too volatile in their model and the PD ratio is neither as volatile nor as persistent as in the data and in our model. In addition, the PD ratio in Boldrin, Christiano, and Fisher (2001) displays negative comovement with hours worked and the investment-to-output ratio. We have encountered such negative correlations also in the rational expectations version of our model above. The reason for this negative correlation is, that capital prices are much less persistent than H and I/Y in Boldrin, Christiano, and Fisher (2001) and thus only mildly procyclical, but rental rates are similarly persistent as macro aggregates. This leads to a negative correlation of H (I/Y) and the PD ratio, despite the fact that they both move in the same direction on impact in response to a technology shock.

Next, we compare the financial moments of our model to the baseline specification of the endowment-economy model of Adam, Marcet, and Beutel (2017) reported in their Table 3 (last column, labeled “diagonal Σ matrix”). As their data sample is almost identical to ours, we consider again the estimated model from Section 2.8. Table 2.11 reports the results. Not unexpected, our production economy matches the moments less well than the endowment economy studied in Adam, Marcet, and Beutel (2017). Yet, overall the performance of our model is relatively close to that of the Adam, Marcet, and Beutel (2017) model. Given that our model has at the same time realistic business cycle implications, we consider this a substantial achievement.

[59] We solve the Boldrin, Christiano, and Fisher (2001) model by solving the associated planner problem using standard Bellman-equation-based iteration methods. We evaluate the accuracy of our solution by computing Euler errors and replicating the model moments reported in Boldrin, Christiano, and Fisher (2001). Additional moments reported by us are computed using the same method as described in footnote a in all of their tables (based on 500 simulations of sample size 200).

[60] This mirrors our estimation procedure for the subjective belief model.

Table 2.11
Model comparison with Adam, Marcet, and Beutel (2017)

Moment	Data	Model	AMB
$E[P/D]$	152.3 (25.3)	149.95	115.2
$\sigma(P/D)$	63.39 (12.39)	44.96	88.20
$\rho(P/D)$	0.98 (0.003)	0.97	0.98
$E[r^e]$	1.87 (0.45)	1.25	1.82
$\sigma(r^e)$	7.98 (0.35)	7.07	7.74
$E[r^f]$	0.25 (0.13)	0.78	0.99
$\sigma(r^f)$	0.82 (0.12)	0.06	0.83
$\sigma(D_{t+1}/D_t)$	1.75 (0.38)	2.46	1.92
$\rho(E^{\mathcal{P}}[r^e], P/D)$	0.79 (0.07)	0.52	0.79

Note: Column "Model" refers to our model, column "AMB" refers to Adam, Marcet, and Beutel (2017).

2.10 Welfare Implications of Belief-Driven Stock Price Booms

This section studies to what extent the presence of belief-driven boom and bust cycles affects allocations and household welfare. To this end, it considers the estimated subjective belief model from Section 2.8 and the allocative and welfare implications associated with imposing fully rational expectations on households. In the absence of subjective belief distortions, the competitive allocation is efficient and thus represents a natural welfare benchmark against which one can judge the costs of belief-driven boom and bust cycles. When evaluating welfare in the subjective belief model, we consider ex-post realized welfare rather than ex-ante expected welfare, as the latter would depend also on the degree of subjective optimism.

Table 2.12 reports business cycle and financial moments for a setting with and without subjective beliefs.[61] It shows that the elimination of subjective price beliefs decreases output volatility by about 13%. This is mainly driven by a substantial fall in the volatility of investment and a dramatic fall in the volatility of hours worked. These effects are partly compensated by an increase in consumption volatility, in both absolute and relative terms. The elimination of subjective beliefs has, however, its strongest effect on the volatility of financial variables. The standard deviation of the PD ratio decreases by more than three quarters and the standard deviation of stock returns falls even more dramatically. Dividend growth volatility also falls considerably.

[61] The results for both models are based on the estimated parameters for the subjective belief model in Table 2.5.

Table 2.12
The effects of shutting down subjective price beliefs

	Data	Subjective Belief Model	REE Implied by Subj. Belief Model
		Business Cycle Moments	
$\sigma(Y)$	1.72 (0.25)	1.83	1.60
$\sigma(C)/\sigma(Y)$	0.61 (0.03)	0.67	0.89
$\sigma(I)/\sigma(Y)$	2.90 (0.35)	2.90	1.59
$\sigma(H)/\sigma(Y)$	1.08 (0.13)	1.06	0.12
$\rho(Y,C)$	0.88 (0.02)	0.84	0.96
$\rho(Y,I)$	0.86 (0.03)	0.89	0.91
$\rho(Y,H)$	0.75 (0.03)	0.70	0.70
		Financial Moments	
$E[P/D]$	152.3 (25.3)	150.0	199.7
$\sigma(P/D)$	63.39 (12.39)	44.96	8.99
$\rho(P/D)$	0.98 (0.003)	0.97	0.99
$E[r^e]$	1.87 (0.45)	1.25	0.68
$\sigma(r^e)$	7.98 (0.35)	7.07	0.19
$E[r^f]$	0.25 (0.13)	0.78	0.68
$\sigma(r^f)$	0.82 (0.12)	0.06	0.06
$\sigma(D_{t+1}/D_t)$	1.75 (0.38)	2.46	0.92

The increase in consumption volatility and the fact that the volatility of hours is welfare neutral – given our linear specification for the disutility of work – suggests that the welfare implications of eliminating subjective beliefs are likely to be small. Indeed, we find that the utility gain associated with the elimination of subjective beliefs amounts to 0.29% of consumption. This is of the same order of magnitude as the welfare gains associated with the elimination of the business cycle.

The welfare gains associated with the elimination of subjective beliefs arise mainly through changes in the *mean* levels of consumption and work. Table 2.13 depicts the mean value of (detrended) consumption and labor under the two belief specifications.[62] It shows that average consumption is higher and average labor is lower in a setting with fully rational expectations. This is the case because with rational expectations asset prices correctly signal the value of investment opportunities. Instead, under subjective beliefs, investment can be triggered by a belief-driven price boom, so that high levels

[62] The utility from the common consumption trend, which is a function of the productivity process $\{Z_t\}$ only, enters utility separately.

Table 2.13
Mean values of (detrended) consumption and labor

	Subjective Belief Model	REE Implied by Subj. Belief Model
$E[C^{detr}]$	2.7719	2.7768
$E[L]$	0.7152	0.7141

of hours worked and high levels of investment might occur during times in which new capital is not particularly productive. As a result, agents can work on average more but achieve lower average consumption levels.

We conclude this section with two remarks. First, one reason for the low welfare effects of fluctuations induced by price cycles is the low curvature in assumed preferences. As curvature increases, agents would be willing to give up more consumption to eliminate fluctuations. However, the welfare gains associated with the elimination of fluctuations induced by stock price cycles are bounded above by the welfare gains associated with the elimination of all fluctuations and thus the cost of business cycles. Unless higher curvature in preferences also increases the mean effects reported in Table 2.13, we expect our result to generalize that costs of belief-driven price cycles are small and of the same order of magnitude as the costs of business cycles.

Second, the representative agent assumption is critical for the result that welfare effects of stock price cycles are small. Adam, Beutel, Marcet, and Merkel (2016) show that a belief-driven boom-bust cycle can have substantial redistributive effects in the presence of belief heterogeneity. In their model agents trade with each other at prices that are – relative to a setting without belief distortions – too high or low. In the present model, prices only affect allocations through aggregate investment. Likely, our welfare assessment would change in the presence of heterogeneity that leads to trading among agents over the course of stock price cycles, if the welfare objective was to include distributional concerns.

2.11 Conclusions

We have presented a simple theory of the joint behavior of stock prices and business cycles. This theory is quantitatively consistent with key facts about the business cycle, stock price behavior and the interaction between the two. In particular, it can reconcile smooth business cycles and volatile stock prices. Relative to the previous literature, no labor market frictions or inseparable preferences are required to achieve this. Instead, in our model stock prices are volatile because agents hold subjective stock price expectations and extrapolate observed past capital gains into the future as in Adam, Marcet, and

Beutel (2017). Extrapolation creates price-belief feedback loops that can occasionally disconnect prices from fundamentals and lead to large and persistence stock price cycles. In the model, such price cycles have allocative implications, because they lead to cyclical variation in investment demand. Our theory suggests that this variation is an important source of the business cycle fluctuations of investment and hours worked, but has only minor effects on consumption variability.

We also assess the welfare implications of belief distortions in stock markets. In the presence of such distortions, prices are only imperfect signals of the relative scarcity of capital goods. Consequently, in times of optimism there can be over-investment, in times of pessimism under-investment. Inefficient timing of investment leads to a lower average level of consumption despite a higher average amount of hours worked. However, we find the welfare costs of belief-driven stock price cycles to be relatively moderate. Several considerations absent in our model might overturn this result. First, effects of price cycles could be much larger in the presence of heterogeneity and trading in a setting where the wealth distribution matters. Second, in a richer model with a large cross-section of sectors in which sectoral price cycles only partially comove fluctuations in relative firm valuations may lead to additional misallocation, absent in our model. Third, if asset prices play an important role for the balance sheet dynamics of a financially constrained sector,[63] fluctuations in asset prices may have larger effects on allocations than in the model presented in this paper. Exploring these channels appears to be an interesting avenue for future research.

Conceptually, we show how to specify expectations that combine subjective beliefs for some model variables with rational expectations along other dimensions. The proposed expectation concept that is "partially rational" with respect to a subset of model-endogenous variables can be generalized beyond our specific setting. In particular, it may prove fruitful in other macroeconomic applications where beliefs can be disciplined empirically along some dimensions, but one wishes to maintain the rational expectations hypothesis along others.

[63] Arguably, this is less relevant for stocks than for other asset classes such as real estate. House prices also display large and persistent cycles in many advanced economies. To the extent that such cycles are also driven by the same belief dynamics analyzed here, one could easily extend the analysis to include housing.

Appendices

Appendix A

Appendix to Chapter 1

A.1 Data sources

Stock price data: Our stock price data is for the United States and has been downloaded from 'The Global Financial Database'.[1] The period covered is Q1:1949-Q1:2012. The nominal stock price series is the "SP 500 Composite Price Index (w/GFD extension)" (Global Fin code "_SPXD"). The daily series has been transformed into quarterly data by taking the index value of the last day of the considered quarter. To obtain real values, nominal variables have been deflated using the "USA BLS Consumer Price Index" (Global Fin code "CPUSAM"). The monthly price series has been transformed into a quarterly series by taking the index value of the last month of the considered quarter. Nominal dividends have been computed as follows

$$D_t = \left(\frac{I^D(t)/I^D(t-1)}{I^{ND}(t)/I^{ND}(t-1)} - 1 \right) I^{ND}(t)$$

where I^{ND} denotes the "SP 500 Composite Price Index (w/GFD extension)" described above and I^D is the "SP 500 Total Return Index (w/GFD extension)" (Global Fin code "_SPXTRD"), which contains returns from price changes and dividend payouts. In the notation of our model, $I^D(t)$ is equal to P_t and $I^{ND}(t)/I^{ND}(t-1)$ equal to $(P_t+D_t)/P_{t-1}$.We first computed monthly dividends and then quarterly dividends by adding up the monthly series. Following Campbell (2003), dividends have been deseasonalized by taking averages of the actual dividend payments over the current and preceding three quarters.

Stock market survey data: The UBS survey is the UBS Index of Investor Optimism.[2] For all our calculations we use own portfolio return expectations from 1999:Q1 to 2007:Q2. We do not use data from 1998 due to missing values. The micro dataset of the UBS survey consists of 92823 record. Data-cleaning results in the removal of 18379

[1] It is available at http://www.globalfinancialdata.com.

[2] See http://www.ropercenter.uconn.edu/data_access/data/datasets/ubs_investor.html.

this records: Following Vissing-Jorgensen (2003), we ignore survey responses with stated expected returns larger than 95% in absolute value, which results in the elimination of 16380 observations. Furthermore, we ignore records, where the difference between the respondent's age and his stated stock market experience is less than 16 years, which eliminates 2378.[3]

The Shiller survey covers individual investors and has been kindly made available to us by Robert Shiller at Yale University. The survey spans the period 1999:Q1-2012:Q4. The CFO survey is collected by Duke University and CFO magazine and collects responses from about 450 CFOs. The data span the period 2000:Q3-2012:Q4.

Inflation expectations data: The Survey of Professional Forecasters (SPF) is available from the Federal Reserve Bank of Philadelphia.

Trading volume: We have daily data from Thomson Reuters Financial Datastream from 2nd January 1973 until 31st March 2014. We look at the series "US-DS Market" (TOTMKUS), an index of 1000 U.S. stocks traded on NYSE and Nasdaq.

We compute quarterly trading volume as follows: Starting from daily trading volume (DS: VA) and daily market value (DS: MV) we compute daily trading volume (VA/MV), i.e. the share of the market that is traded on each day. We then aggregate this up, following Lo and Wang (2009), by summing the shares over all trading days in the quarter, thus arriving at the share of the market that is traded in a particular quarter up to the last trading day of the quarter (end of March, June, September, December). Thus volume is measured over the same time period where expectations are measured. Moreover, end of quarter PDs are associated with the trading volume accumulated in the preceding 3 months.

A.2 Numerical solution approach

We now describe the solution strategy for determining the functions $S^i(\cdot)$ and $\tau^i(\cdot)$ and the associated lump sum rebate $T^i(\cdot)$. To simplify notation we drop all i superscripts. Also, instead of solving for the optimal stockholding function $S(\cdot)$, we solve in our numerical approach for the optimal consumption dividend ratio $C_t/D_t = CD(S_{t-1}, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t)$. There is a one-to-one mapping between the $S(\cdot)$ policy and the $CD(\cdot)$ policy due to the flow budget constraint, which implies

$$\frac{C_t}{D_t} = S_{t-1}\left(\frac{P_t}{D_t}+1\right) + \frac{W_t+T_t}{D_t} - \tau_t(S_t - S_{t-1})\frac{P_t}{D_t} - S_t\frac{P_t}{D_t},$$

and due to the assumption that $\tau_t = \tau(S_{t-1}, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t^i)$ and $T_t/D_t = \tau\,|(S_t - S_{t-1})P_t/D_t|$.

We solve the first order condition by combining time iteration with an endogenous grid point method, thereby avoiding any root finding steps in the solution procedure.

[3]The two numbers do not add up to 18379, since some records satisfy both criteria for elimination.

This considerably speeds up the numerical solution. We now describe this procedure in detail.

We start with a guess for the future consumption policy $CD^{(j)}(\cdot)$, the transactions tax function $\tau^{(j)}(\cdot)$ and the lump sum rebate relative to dividends $TD^{(j)}(\cdot) = T^j(\cdot)/D_t$, where the superscript (j) denotes the j-th guess in the time iteration procedure and where all functions depend on the arguments $(S_{t-1}, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t)$.

Given the guesses $CD^{(j)}(\cdot)$, $\tau^{(j)}(\cdot)$ and $TD^{(j)}(\cdot)$ and given an alternative grid of current values $(S_t, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t)$ - note this alternative grid contains S_t not S_{t-1} - we can compute the updated consumption policy $\widetilde{CD}^{(j+1)}(S_t, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t)$ and the updated marginal tax function $\widetilde{\tau}^{(j+1)}$, which are both defined over the alternative grid, by iterating on the FOC (1.16). In particular, equation (1.16) implies

$$\left(\widetilde{CD}^{(j+1)}\right)^{-\gamma}(1+\widetilde{\tau}_t^{(j+1)}) = \frac{\delta E_t^{\mathcal{P}}\left(CD^{(j)}\right)^{-\gamma}\left(\frac{D_{t+1}}{D_t}\right)^{1-\gamma}\left(\frac{P_{t+1}}{D_{t+1}}(1+\tau_{t+1}^{(j)})+1\right)}{P_t/D_t} \tag{A.1}$$

Given any point $(S_t, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t)$ on the alternative grid, we can compute the distribution over future (standard) grid points $(S_t, \frac{P_{t+1}}{D_{t+1}}, \frac{W_{t+1}}{D_{t+1}}, m_{t+1})$, using the perceived evolution over prices, dividends, wages and beliefs. Together with the guesses $CD^{(j)}(\cdot)$ and $\tau^{(j)}$, this allows evaluating the r.h.s. of (A.1) using a standard numerical integration method (we use deterministic integration based on quadrature points). For future reference, let $M(S_t, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t)$ denote the value of the r.h.s. of (A.1). The l.h.s. of equation (A.1) then implies that we have also determined the value of the product $(C_t/D_t)^{1-\gamma}(1-\tau_t)$, at every alternative grid point.

It now remains to compute the updated functions $CD^{(j+1)}$, $\tau_t^{(j+1)}$ and $TD^{(j+1)}$ which are defined over the standard grid $(S_{t-1}, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t)$. We do so by fixing an arbitrary alternative grid point $(S_t^*, \left(\frac{P_t}{D_t}\right)^*, \left(\frac{W_t}{D_t}\right)^*, m_t^*)$ and by checking the range of possible situations $S_{t-1} \lesseqgtr S_t^*$.

We begin by conjecturing $S_{t-1} = S_t^*$. The flow budget constraint then determines the implied consumption dividend ratio, i.e.,

$$\frac{C_t}{D_t} = S_t^* + \left(\frac{W_t}{D_t}\right)^*. \tag{A.2}$$

We can then check whether the tax rate $\tau_t^{(j+1)}$ associated with (A.2), defined as

$$\left(\frac{C_t}{D_t}\right)^{-\gamma}(1+\tau_t^{(j+1)}) = M(S_t^*, \left(\frac{P_t}{D_t}\right)^*, \left(\frac{W_t}{D_t}\right)^*, m_t^*), \tag{A.3}$$

satisfies $\tau_t^{(j+1)} \in [-\tau, +\tau]$. If so, then we have found the optimal consumption dividend ratio $CD^{(j+1)}$ and associated shadow tax rate $\tau_t^{(j+1)}$ at the standard grid point $\left(S_{t-1} = S_t^*, \left(\frac{P_t}{D_t}\right)^*, \left(\frac{W_t}{D_t}\right)^*, m_t^*\right)$. The updated lump sum tax rebate over dividends at this gridpoint is simply $TD^{(j+1)} = 0$.

If the value of $\tau_t^{(j+1)}$ solving (A.3) satisfies $\tau_t^{(j+1)} > \tau$, then it must be that $S_t^* > S_{t-1}$.[4] We therefore set $\tau_t^{(j+1)} = \tau$ and determine the equilibrium consumption dividend ratio $CD^{(j+1)}$ from equation (A.1), which delivers

$$(CD^{(j+1)})^{-\gamma}(1+\tau) = M(S_t^*, \left(\frac{P_t}{D_t}\right)^*, \left(\frac{W_t}{D_t}\right)^*, m_t^*).$$

Finally, we use the budget constraint to compute the associated initial grid point S_{t-1}, which must solve

$$CD^{(j+1)} = S_{t-1}\left(\left(\frac{P_t}{D_t}\right)^* + 1\right) + \left(\frac{W_t}{D_t}\right)^* - S_t^*\left(\frac{P_t}{D_t}\right)^*, \tag{A.4}$$

where we used the updated lump sum rebate function $TD^{(j+1)} = \tau \cdot (S_t^* - S_{t-1})\left(\frac{P_t}{D_t}\right)^*$. We have thus determined $CD^{(j+1)}$, $\tau^{(j+1)}$ and $TD^{(j+1)}$ at the grid point $\left(S_{t-1}, \left(\frac{P_t}{D_t}\right)^*, \left(\frac{W_t}{D_t}\right)^*, m_t^*\right)$.

If the value of $\tau_t^{(j+1)}$ solving (A.3) satisfies $\tau_t^{(j+1)} < -\tau$, then we must assume $S_t^* < S_{t-1}$ and thus set $\tau_t^{(j+1)} = -\tau$. Using (A.1) we can determine the equilibrium consumption dividend ratio $CD^{(j+1)}$

$$u'(CD^{(j+1)})(1-\tau) = M(S_t^*, \left(\frac{P_t}{D_t}\right)^*, \left(\frac{W_t}{D_t}\right)^*, m_t^*).$$

Again, we use the budget constraint to compute the associated grid point S_{t-1}, which must solve

$$CD^{(j+1)} = S_{t-1}\left(\left(\frac{P_t}{D_t}\right)^* + 1\right) + \left(\frac{W_t}{D_t}\right)^* - S_t^*\left(\frac{P_t}{D_t}\right)^*, \tag{A.5}$$

where we use the updated lump sum rebate function $TD^{(j+1)} = -\tau \cdot (S_t^* - S_{t-1})\left(\frac{P_t}{D_t}\right)^*$

We perform the iterations described above until convergence of the functions $CD^{(j)}(\cdot)$, $\tau^{(j)}(\cdot)$ and $TD^{(j)}$.

---

[4]Reducing τ_t so that it satisfies $\tau_t \leq \tau$ requires that $(C_t/D_t)^{-\gamma}$ increases, see the l.h.s. of equation (A.1). From the flow budget constraint follows that this can only happen if S_{t-1} decreases below S_t^*, given the values for $(P_t/D_t)^*$ and $(W_t/D_t)^*$.

A.3 Inaction Regions and Adaptive Grid Point Choice

A transaction tax leads to partially flat stock demand curves (inaction regions) and thereby introduces a high degree of nonlinearity - non-differentiabilities in the $\frac{P_t}{D_t}$-dimension - into the consumption policy function $CD^{(j)}(\cdot)$ and the associated shadow tax $\tau^{(j)}(\cdot)$. While linear interpolation between two grid points yields very accurate approximations of these functions for most $\frac{P_t}{D_t}$ values, this is generally not true close to the boundaries of the inaction regions, if these boundaries are not elements of our discretized state space.

Including the $\frac{P_t}{D_t}$ boundaries of the inaction region into the discretized state space poses two challenges: First, the exact locations of these boundaries are not known a priori, but depend on the optimal solution. Therefore, the $\frac{P_t}{D_t}$ grid is required to change in every iteration. We describe in the sequel how we use an adaptive grid point choice to ensure that our best guess for the inaction region boundaries is always part of the $\frac{P_t}{D_t}$ grid. Second, these boundaries are not independent of other states, but vary with $(S_{t-1}, \frac{W_t}{D_t}, m_t)$. Hence, the $\frac{P_t}{D_t}$ grid is not only required to change in every iteration of the algorithm, but also to be dependent on other state variables.[5] We clarify below how we interpolate our policy to states not contained in the discretized state space.

Adaptive grid points: Since the non-differentiability problem only occurs in the $\frac{P_t}{D_t}$-dimension, we fix a vector $(S_{t-1}, \frac{W_t}{D_t}, m_t)$ in the sequel. First, we observe, that the interior of the inaction region in the $\frac{P_t}{D_t}$-dimension can be identified by the shadow tax function $\tau(\cdot)$: The optimal consumption (or, equivalently, stock holding) policy does not change in a neighborhood of the current value of $\frac{P_t}{D_t}$, if and only if $\tau\left(S_{t-1}, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t\right) \in (-\tau, \tau)$. Since in such cases $S_{t-1} = S_t$, the same relationship must hold for the function $\tilde{\tau}$ defined on the alternative "state space" $(S_t, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t)$. In our solution algorithm, we solve for this function $\tilde{\tau}$ by solving equation (A.3) under the assumption that consumption satisfies the no trade relationship (A.2) and set it to τ, whenever its value exceeds τ and to $-\tau$, whenever its value is less than $-\tau$. The boundaries of the inaction region are therefore given for those values of $\frac{P_t}{D_t}$, for which no trade consumption defined by (A.2) and $\tau_t^{(j+1)} \in \{-\tau, \tau\}$ solve equation (A.3). This yields two equations

$$\left(S_t^* + \left(\frac{W_t}{D_t}\right)^*\right)^{-\gamma}(1 \pm \tau) = M(S_t^*, \left(\frac{P}{D}\right)_\pm, \left(\frac{W_t}{D_t}\right)^*, m_t^*)$$

which we solve for the adapted grid points $\left(\frac{P}{D}\right)_\pm$ in each iteration of the above algorithm.[6] We make sure, that in our algorithm not only the functions $CD^{(j)}(\cdot)$, $\tau^{(j)}(\cdot)$ and $TD^{(j)}$,

[5] Including all inaction boundaries for any combination of $(S_{t-1}, \frac{W_t}{D_t}, m_t)$ into a common $\frac{P_t}{D_t}$ grid creates a computationally prohibitively large number of discretization points.

[6] Note, that $\left(\frac{P}{D}\right)_+$ and $\left(\frac{P}{D}\right)_-$ are functions of $((S_t^*, \left(\frac{W_t}{D_t}\right)^*, m_t^*)$, although this is suppressed in our notation.

but also these adapted grid points converge. The present approach is similar to the approach proposed in Brumm and Grill (2014). The latter cover the discretized state space with simplices and look for 'just binding' constraints on each edge of these simplices. We only look at edges that are orthogonal to the $(S_{t-1}, \frac{W_t}{D_t}, m_t)$-hyperplane, which is computationally more efficient within the present setup.

Interpolation: We fix the set of initial grid points G_S, G_{WD}, G_{PD}, G_m for the state space. Our discretized state space is, however, not given by the product $G_S \times G_{WD} \times G_{PD} \times G_m$, but instead by

$$\begin{aligned}
&G_S \times G_{WD} \times G_{PD} \times G_m \\
&\cup \{(S, WD, PD_+(S, WD, m), m) \mid (S, WD, m) \in G_S \times G_{WD} \times G_m\} \\
&\cup \{(S, WD, PD_-(S, WD, m), m) \mid (S, WD, m) \in G_S \times G_{WD} \times G_m\}
\end{aligned}$$

The standard linear interpolation method on a Cartesian product of one-dimensional grids is therefore augmented as follows: for a given query point (S_q, WD_q, PD_q, m_q), we first search for indices i, j, k, such that $S_q \in [S_i, S_{i+1}]$, $WD_q \in [WD_j, WD_{j+1}]$ and $m_q \in [m_k, m_{k+1}]$ and then linearly interpolate the policy in the PD-dimension for each combination $(S, WD, m) \in \{S_i, S_{i+1}\} \times \{WD_j, WD_{j+1}\} \times \{m_k, m_{k+1}\}$ using as a PD grid the intersection of the discretized state space with the line parallel to the PD-axis that crosses (S, WD, m). This yields eight interpolated policy values $CD_{u,v,w}$ with $(u, v, w) \in \{i, i+1\} \times \{j, j+1\} \times \{k, k+1\}$ of the function

$$(S, WD, m) \mapsto CD(S, WD, PD_q, m)$$

at the chosen closest (S, WD, m)-grid points. We then use ordinary three-dimensional linear interpolation to obtain the interpolated policy value for $CD(S_q, WD_q, PD_q, m_q)$, i.e.

$$\begin{aligned}
&CD^{interp}(S_q, WD_q, PD_q, m_q) \\
&= \sum_{u=i,i+1} \sum_{v=j,j+1} \sum_{w=k,k+1} \frac{|S_q - S_u||WD_q - WD_v||m_q - m_w|}{(S_{i+1} - S_i)(WD_{j+1} - WD_j)(m_{k+1} - m_k)} CD_{u,v,w}
\end{aligned}$$

We proceed analogously for linear extrapolation.

Table A.1
p-values for equality of gain estimates

Experience groups	6-11	12-17	18-23	> 23
0-5	0.33	0.11	0.01	0.00
6-11	-	0.30	0.00	0.00
12-17	-	-	0.11	0.00
18-23	-	-	-	0.00

A.4 Testing for Equality of Gain Estimates in Table 1.4

Table A.1 reports the p-values for the null hypothesis $H_0 : g^i = g^j$ for $i \neq j$.

A.5 No Tax Rebates

Table A.2 reports the outcomes shown in Table 1.8 in the main text for the case where tax revenue is not rebated to investors ($T_t^i = 0$ for all t,i). It shows that findings are robust to making this alternative assumption on tax rebates.

Table A.2
Effects of introducing financial transaction taxes (no tax rebate)

	No tax	1% tax	2% tax	4% tax	10% tax
$E[PD]$	135.77	137.11	140.18	143.99	152.01
$std(PD)$	122.13	122.89	125.54	128.29	131.48
$corr(PD_t, PD_{t-1})$	0.98	0.98	0.98	0.98	0.98
$std(r^s)$ (%)	11.63	11.72	11.97	12.26	13.78
$E[r^s]$ (%)	2.11	2.12	2.15	2.19	2.41
$corr(PD_t, \overline{E}_t R_{t+1})$	0.84	0.85	0.86	0.87	0.89
$corr(TV_t, TV_{t-1})$	0.97	0.97	0.97	0.97	0.94
$corr(TV_t, PD_t)$	0.37	0.35	0.33	0.29	0.16
$corr(TV_t, \lvert P_t/P_{t-1} - 1\rvert)$	0.25	0.25	0.24	0.22	0.04
$corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$	0.95	0.94	0.93	0.91	0.87
# of booms per 100 yrs	1.82	1.92	2.08	2.32	2.88
average boom length (quarters)	32.42	31.97	31.44	30.72	28.24
average boom peak (PD)	491.01	487.57	484.54	478.82	468.96
$E[TV]$ relative to no tax (%)	100.00	97.33	97.96	96.69	105.66

Appendix B

Appendix to Chapter 2

B.1 Data Sources

Data on Macro Aggregates Data series on macro aggregates and related variables have all been downloaded from the Federal Reserve Economic Data (FRED) database maintained by the federal reserve bank of St. Louis (https://fred.stlouisfed.org). All time series refer to the United States, are at quarterly frequency and cover the sample period Q1:1955 to Q4:2014. Data on output, consumption and investment is from the National Income and Product Accounts of the BEA. We measure nominal output by gross domestic product (FRED Code "GDP"), nominal consumption by personal consumption expenditures in nondurable goods ("PCND") and services ("PCESV") and nominal investment by fixed private investment ("FPI"). Nominal values of investment subcomponents for Table 2.4 are private residential fixed investment ("PRFI") and private nonresidential fixed investment in structures ("B009RC1Q027SBEA"), equipment ("Y033RC1Q027SBEA") and intellectual property products ("Y001RC1Q027SBEA"). These nominal series have been deflated by the consumer price index for all urban consumers ("CPIAUCSL") from the BLS, which is consistent with the deflating procedure used for stock prices and interest rates by Adam, Marcet, and Beutel (2017) and us. Hours worked are based on an index of nonfarm business sector hours ("HOANBS") published by the BLS. Working age population is based on data published by the OECD ("LFWA64TTUSQ647N").

Stock Prices, Interest Rates and Investor Expectations We use identical data sources as in Adam, Marcet, and Beutel (2017) and refer to their data appendix for details. They use 'The Global Financial Database' to obtain data on stock prices and interest rates until Q1:2012. We extend their stock price and interest rate data to Q4:2014 using identical data sources as they do.

We compute dividends based on price and total return index data of the SP 500 index by the procedure outlines there, resulting in a dividend series $\{D_t\}$, where t runs through all quarters from Q1:1955 to Q4:2014. Given the price series from the price index $\{P_t\}$, where P_t is the closing price of the last trading day in quarter t, we define the price-dividend ratio as the ratio P_t/D_t of the end-of-quarter closing price P_t and the within quarter dividend D_t .

B.2 Details of the Projection Facility

Following Adam, Marcet, and Nicolini (2016), we modify the belief updating equation (2.12) to

$$\ln m_{s,t} = w_{s,t}\left(\ln m_{s,t-1} + g\left(\ln Q_{s,t-1} - \ln Q_{s,t} - \ln m_{s,t-1}\right) + g \ln \varepsilon^1_{s,t}\right)$$

where $w_{s,t}(\cdot)$ is a differentiable function satisfying $w_{s,t}(x) = x$ for $x \leq \underline{m}_{s,t}$ and $w_{s,t}(x) \leq \overline{m}_{s,t}$ for all x, with $\overline{m}_{s,t} > \underline{m}_{s,t}$. Beliefs are thus bounded below $\overline{m}_{s,t}$, but evolve as described in the main text as long as they remain below $\underline{m}_{s,t}$. Following Adam, Marcet, and Nicolini (2016) we consider the function

$$w_{s,t}(x) = \begin{cases} x & \text{if } x \leq \underline{m}_{s,t} \\ \underline{m}_{s,t} + \frac{x-\underline{m}_{s,t}}{x+\overline{m}_{s,t}-2\underline{m}_{s,t}}\left(\overline{m}_{s,t} - \underline{m}_{s,t}\right) & \text{if } \underline{m}_{s,t} < x. \end{cases} \tag{B.1}$$

and calibrate the critical values $\left(\underline{m}_{s,t}, \overline{m}_{s,t}\right)$ in both sectors $s = c, i$ such that $\underline{m}_{s,t}$ is the degree of optimism that implies a quarterly PD ratio of 250 and $\overline{m}_{s,t}$ is the degree of optimism implying a PD ratio of 500. The critical PD values of 250 and 500 are taken from Adam, Marcet, and Nicolini (2016).

We now explain how these critical values can be computed. The Euler equation for capital in sector s is

$$\begin{aligned} Q_{s,t} &= \beta\left(1-\delta_s\right) E^{\mathcal{P}}_t\left[\frac{W_t}{W_{t+1}} Q_{s,t+1}\right] + \beta E^{\mathcal{P}}_t\left[\frac{W_t}{W_{t+1}} R_{s,t+1}\right] \\ &= \beta\left(1-\delta_s\right) E^{\mathcal{P}}_t\left[\frac{W_t}{W_{t+1}}\right] m_{s,t} Q_{s,t} + \beta E^{\mathcal{P}}_t\left[\frac{W_t}{W_{t+1}} R_{s,t+1}\right] \end{aligned}$$

implying

$$Q_{s,t} = \frac{\beta E^{\mathcal{P}}_t\left[\frac{W_t}{W_{t+1}} R_{s,t+1}\right]}{1-\beta\left(1-\delta_s\right) E^{\mathcal{P}}_t\left[\frac{W_t}{W_{t+1}}\right] m_{s,t}}$$

and thus the price-dividend ratio is

$$\frac{P_{s,t}}{D_{s,t}} = \frac{1-\delta}{p}\frac{Q_{s,t}}{R_{s,t}} + \frac{1-p}{p} = \frac{1}{p}\frac{\left(1-\delta\right)\beta E^{\mathcal{P}}_t\left[\frac{W_t}{W_{t+1}}\frac{R_{s,t+1}}{R_{s,t}}\right]}{1-\beta\left(1-\delta\right) E^{\mathcal{P}}_t\left[\frac{W_t}{W_{t+1}}\right] m_{s,t}} + \frac{1-p}{p}. \tag{B.2}$$

The value $\underline{m}_{s,t}$ is the value for $m_{s,t}$ in the preceding equation that causes the PD to be equal to 250; likewise, $\overline{m}_{s,t}$ is the value that causes the PD ratio to be equal to 500.[1] Since the expectations of W_{t+1} and $R_{s,t+1}$ both depend on $K_{c,t+1}$ (this follows from equations (2.18) and (2.19) and the fact that $Y_{c,t+1} = K_{c,t+1}^{\alpha_c} \left(Z_{t+1}(1-\alpha)\right)^{1-\alpha_c}$), solving for the values $\left(\underline{m}_{c,t}, \overline{m}_{c,t}\right)$ thus requires simultaneously solving for the optimal investment $Y_{i,t}$ in period t. Since the capital stock dynamics in the investment sector are exogenous, $\underline{m}_{i,t}, \overline{m}_{i,t}$ can be computed once the new belief $m_{c,t}$ that incorporates the projection facility has been determined.

B.3 Details on the Representation of Beliefs and Equilibrium Existence

B.3.1 Belief Representations on Ω and $\tilde{\Omega}$

We provide here details on the remarks made in footnote 32. To ease the discussion, we first introduce some notation that will also be used in Sections B.3.2 and B.3.3.

Notation For each random (upper case) symbol A, denote elements of Ω_A by a (these are real sequences) and their t-th component by a_t (these are real numbers).[2] For each random sequence $\{A_t\}_{t=0}^{\infty}$, write shorter just A. On the domain $\tilde{\Omega} = \Omega_Z \times \Omega_{Q,c} \times \Omega_{Q,i}$ we define the random variables (sequences) $\tilde{Z}$, $\tilde{Q}_c$ and $\tilde{Q}_i$ as projections on the first, second and third component, respectively,

$$\tilde{Z}(z, q_c, q_i) = z, \qquad \tilde{Q}_c(z, q_c, q_i) = q_c, \qquad \tilde{Q}_i(z, q_c, q_i) = q_i.$$

Similarly, we define the random variables Z, X, W, R_c, R_i, Q_c, Q_i as projections from the domain $\Omega = \Omega_Z \times \Omega_X \times \Omega_W \times \Omega_{R,c} \times \Omega_{R,i} \times \Omega_{Q,c} \times \Omega_{Q,i}$ to the respective factor.[3] We make the difference between $\tilde{Z}$ (defined on $\tilde{\Omega}$ consisting of typical elements $\tilde{\omega} = (z, q_c, q_i)$) and Z (defined on Ω consisting of typical elements $\omega = (z, x, w, r_c, r_i, q_c, q_i)$) explicit to avoid any ambiguity and confusion arising in the arguments below. In the main text, we regularly do not make these distinctions and use the same symbols, whenever a variable has the same interpretation, no matter on which space it is defined and whether it is a random variable or a realization.

[1]In addition, we do not allow $\overline{m}_{c,t}$ to exceed the theoretical upper bound on beliefs for which uniqueness has been proven in Appendix B.3.3 (see Lemma 4), irrespective of the implied PD ratio. This is done to insure equilibrium uniqueness, but is a purely theoretical concern. In practice, we encountered not a single case in which this upper bound was binding in our numerical simulations.

[2]Because the variables m_c and m_i are lower case in the model, those symbols can denote both random variables and realizations. This ambiguity should not lead to any confusion below.

[3]E.g. $Z(z, x, w, r_c, r_i, q_c, q_i) = z$.

We regularly have to work with the big vector $(Z, X, W, R_c, R_i, Q_c, Q_i)$ of random sequences on Ω.[4] For space reasons, we denote this vector just by O ("observables vector").

Furthermore, we use the following two conventions for the outcome mapping $F : s_t \mapsto (w_t, r_{c,t}, r_{i,t}, y_t, c_t, i_t, h_t)$ (where $s_t = (z_t, k_{c,t}, k_{i,t}, m_{c,t}, m_{i,t}, q_{c,t-1}, q_{i,t-1})$) introduced at the end of Section 2.5

- We denote by F_W, $F_{R,c}$, $F_{R,i}$ etc. the components of that function
- By a slight abuse of notation, we denote by F also the mapping from the full state sequence $s = \{s_t\}_{t=0}^{\infty}$ into the full outcome sequence $(w, r_c, r_i, y, c, i, h) = \{(w_t, r_{c,t}, r_{i,t}, y_t, c_t, i_t, h_t)\}_{t=0}^{\infty}$, similarly we use the component functions F_W, $F_{R,c}$, $R_{R,i}$ etc.

Mapping a Recursive Evolution into a Sequence Representation Consider a fixed initial state $s_0 = (z_0, k_{c,0}, k_{i,0}, m_{c,0}, m_{i,0}, q_{c,-1}, q_{i,-1})$ and a measurable function G describing a recursive state evolution as in the main text.[5] For any sequence $z \in \Omega_Z$ such that z_0 is consistent with s_0, the recursion

$$s_{t+1} = G(s_t, z_{t+1}) \qquad t = 0, 1, 2, \ldots$$

defines then a unique sequence $s = \{s_t\}_{t=0}^{\infty}$ in $\Sigma := \prod_{t=0}^{\infty} \mathbb{R}^7$ (Σ is the space of all state sequences). The procedure just outlined defines therefore a function $H : \Omega_Z \to \Sigma$ which maps technology sequences z into state sequences s.[6] Obviously, if G is measurable and Σ is endowed with the usual product σ-field, then H is also a measurable function. Call H the sequence representation associated with G (and the initial state s_0[7]).

Consistency of Beliefs with an Evolution The following definition clarifies the notion in the main text that beliefs about wages and rental rates be consistent with the equilibrium mappings G and F. The definition is formulated for arbitrary mappings G and F that do not necessarily need to correspond to the equilibrium mappings of the model.

Definition 2. For a given measurable state evolution G and a given measurable outcome function F, we say that a measure $\mathcal{P}$ on Ω implies beliefs about (W, R_c, R_i) consistent

[4] As a mapping, this vector just agrees with the identity id_Ω, but this notation conceals the economic interpretation of the random variables.

[5] The function G is allowed to be arbitrary here, but should have the same domain and codomain as in Section 2.5. There is no need for it to conform with any notion of equilibrium or model consistency.

[6] Strictly speaking, the mapping is only partially defined due to the requirement that z_0 has to equal the first component of s_0. This is not important for anything to follow and we will ignore it from now on (alternatively, one could just keep the last 6 components of s_0 fixed and specify that H always assigns the value z_0 to the first component of s_0).

[7] From now on we consider a fixed initial state throughout without explicitly mentioning this anymore.

with mappings G and F, if

$$W = F_W \circ H \circ Z, \qquad R_c = F_{R,i} \circ H \circ Z, \qquad R_i = F_{R,i} \circ H \circ Z$$

$\mathcal{P}$-a.s. Here, H is the sequence representation associated with G.

Constructing Consistent Beliefs from G, F and $\tilde{\mathcal{P}}$ The following lemma is the main result of this section and provides the justification why it is sufficient to work with the smaller probability space $\tilde{\Omega}$ instead of Ω.

Lemma 1. *For any given measure $\tilde{\mathcal{P}}$ on $\tilde{\Omega}$ and measurable G and F, there is a unique measure $\mathcal{P}$ on Ω with the following properties:*

1. *The distribution of (Z, Q_c, Q_i) under $\mathcal{P}$ equals $\tilde{\mathcal{P}}$;*

2. *The joint distribution of (Z, X) under $\mathcal{P}$ is consistent with the exogenous relationship between Z and X;*

3. *$\mathcal{P}$ implies beliefs about (W, R_c, R_i) that are consistent with the mappings G and F.*

Proof. We give an explicit construction of the measure $\mathcal{P}$ as the distribution of a set of suitable random variables on $\tilde{\Omega}$ under $\tilde{\mathcal{P}}$. First, the capital accumulation equation (2.25) for investment-sector capital and the assumption $K_{i,t+1} \propto Z_t$ imply

$$\bar{k}_i Z_t = (1-\delta)\,\bar{k}_i Z_{t-1} + X_t \Rightarrow X_t = \bar{k}_i\,(Z_t - (1-\delta_i)\,Z_{t-1})\,, \tag{B.3}$$

where $\bar{k}_i$ is the (fixed) proportionality constant.[8] The second requirement that the joint distribution of (Z, X) under $\mathcal{P}$ be consistent with the exogenous relationship between those variables means that equation (B.3) has to hold $\mathcal{P}$-a.s. for all t. We thus define a random variable $\tilde{X} : \tilde{\Omega} \to \Omega_X$ in a way that is consistent with the analog equation (B.3) on the $\tilde{\Omega}$ domain:[9]

$$\tilde{X}_t := \bar{k}_i \left(\tilde{Z}_t - (1-\delta_i)\,\tilde{Z}_{t-1}\right).$$

Next, let the function $H : \Omega_Z \to \Sigma$ be the sequence representation associated with G. As for X, we simply define random variables for wages, $\tilde{W} : \tilde{\Omega} \to \Omega_W$, and rental rates, $\tilde{R}_c : \tilde{\Omega} \to \Omega_{R,c}$, $\tilde{R}_i : \tilde{\Omega} \to \Omega_{R,i}$, on the domain $\tilde{\Omega}$ in a way that they satisfy the analog of the consistency condition for the probability space $(\tilde{\Omega}, \tilde{\mathcal{S}}, \tilde{\mathcal{P}})$, namely

$$\tilde{W} = F_W \circ H \circ \tilde{Z}, \qquad \tilde{R}_c = F_{R,i} \circ H \circ \tilde{Z}, \qquad \tilde{R}_i = F_{R,i} \circ H \circ \tilde{Z}.$$

[8] Strictly speaking, $\bar{k}_i$ is a model parameter. However, the choice of this parameter is not discussed in Section 2.8, because its value does not matter for any results reported in the main text. Intuitively, changing $\bar{k}_i$ just scales up or down the production of consumption-sector capital and thereby changes the units in which this capital is measured, which has no economic relevance.

[9] For $t = 0$ one must back out $\tilde{Z}_{t-1} = z_{-1}$ from the entry $k_{i,0}$ of the initial state: $z_{-1} = \frac{k_{i,0}}{\bar{k}_i}$.

With these definitions the random (observables) vector

$$\tilde{O} := (\tilde{Z}, \tilde{X}, \tilde{W}, \tilde{R}_c, \tilde{R}_i, \tilde{Q}_c, \tilde{Q}_i)$$

is a measurable mapping from $\tilde{\Omega}$ to Ω and thus its distribution defines a measure $\mathcal{P}$ on Ω. We claim that this measure satisfies the three conditions in the assertion and is the only measure to do so:

1. Z is the projection defined by $Z(z, x, w, r_c, r_i, q_c, q_i) = z$, so $Z(\tilde{O}) = \tilde{Z}$ and a similar argument shows $Q_c(\tilde{O}) = \tilde{Q}_c$ and $Q_i(\tilde{O}) = \tilde{Q}_i$. So we get

$$(Z, Q_c, Q_i)(\tilde{O}) = \left(\tilde{Z}, \tilde{Q}_c, \tilde{Q}_i\right) = \mathrm{id}_{\tilde{\Omega}}.$$

   Because $\mathcal{P}$ is the distribution of $\tilde{O}$ under $\tilde{\mathcal{P}}$, this equation implies that the distribution of (Z, Q_c, Q_i) under $\mathcal{P}$ and the distribution of $\mathrm{id}_{\tilde{\Omega}}$ under $\tilde{\mathcal{P}}$ must be identical. As the latter distribution is $\tilde{\mathcal{P}}$ itself, this proves the first property.

2. The following equation holds by definition of the random variables X, Z and $\tilde{X}$ (for all $\tilde{\omega} \in \tilde{\Omega}$)

$$\begin{aligned} X_t(\tilde{O}) = \tilde{X}_t &= \bar{k}_i \left(\tilde{Z}_t - (1 - \delta_i)\, \tilde{Z}_{t-1}\right) \\ &= \bar{k}_i \left(Z_t(\tilde{O}) - (1 - \delta_i)\, Z_{t-1}(\tilde{O})\right). \\ &= \left[\bar{k}_i \left(Z_t - (1 - \delta_i)\, Z_{t-1}\right)\right](\tilde{O}) \end{aligned}$$

   As this equation holds on $\tilde{\Omega}$, it must in particular hold $\tilde{\mathcal{P}}$-a.s., so X_t and $\bar{k}_i \left(Z_t - (1 - \delta_i)\, Z_{t-1}\right)$ must coincide a.s. with respect to the distribution of $\tilde{O}$ under $\tilde{\mathcal{P}}$, which is exactly $\mathcal{P}$. Hence, equation (B.3) holds $\mathcal{P}$-a.s.

3. The consistency proof works along the same lines as the proof that (B.3) has to hold $\mathcal{P}$-a.s. by reducing it to the analogous consistency condition in the tilde space for the tilde variables. The argument is omitted for this reason.

For uniqueness, suppose that $\mathcal{P}'$ is another (arbitrary) measure on Ω such that properties 1-3 are satisfied. Then in particular equation (B.3) holds $\mathcal{P}'$-a.s. for all t and by the definition of $\tilde{X}$ and $\tilde{Z}$ we obtain for all t

$$\begin{aligned} \tilde{X}_t \left(Z, Q_c, Q_i\right) &= \bar{k}_i \left(\tilde{Z}_t - (1 - \delta_i)\, \tilde{Z}_{t-1}\right) \left(Z, Q_c, Q_i\right) \\ &= \bar{k}_i \left(Z_t - (1 - \delta_i)\, Z_{t-1}\right) \\ &= X_t \qquad\qquad \mathcal{P}'\text{-a.s.} \end{aligned}$$

(here, all equalities except for the last hold even ω-by-ω and the last is just equation (B.3)). Similarly, from $W = F_W \circ H \circ Z$ $\mathcal{P}'$-a.s. (the consistency condition for wage beliefs under $\mathcal{P}'$) and the definition of $\tilde{W}$ and $\tilde{Z}$ we can conclude

$$\tilde{W}(Z, Q_c, Q_i) = F_W \circ H \circ \tilde{Z}(Z, Q_c, Q_i) = F_W \circ H \circ Z = W \qquad \mathcal{P}'\text{-a.s.}$$

Identical arguments also yield

$$\begin{aligned} \tilde{R}_c(Z, Q_c, Q_i) &= R_c \qquad \mathcal{P}'\text{-a.s.} \\ \tilde{R}_i(Z, Q_c, Q_i) &= R_i \qquad \mathcal{P}'\text{-a.s.} \end{aligned}$$

Combining those results with the obvious equations $\tilde{Z}(Z, Q_c, Q_i) = Z$, $\tilde{Q}_c(Z, Q_c, Q_i) = Q_c$, $\tilde{Q}_i(Z, Q_c, Q_i) = Q_i$ implies

$$\mathrm{id}_\Omega = O = \tilde{O}(Z, Q_c, Q_i) \qquad \mathcal{P}'\text{-a.s.}$$

But by property 1, the distribution of (Z, Q_c, Q_i) under $\mathcal{P}'$ must be $\tilde{\mathcal{P}}$ and thus the equation shows that the distribution of id_Ω under $\mathcal{P}'$ must be equal to the distribution of $\tilde{O}$ under $\tilde{\mathcal{P}}$, which is by definition $\mathcal{P}$. Hence, $\mathcal{P}' = \mathcal{P}$. □

B.3.2 Construction and Uniqueness of $\tilde{\mathcal{P}}$

We first construct a measure and show that it has all the properties that any candidate for $\tilde{\mathcal{P}}$ has to have. We then argue why it is the only such measure. First, the following two auxiliary constructions are required. As always, we assume implicitly, that an initial state $s_0 = (z_0, k_{c,0}, k_{i,0}, m_{c,0}, m_{i,0}, q_{c,-1}, q_{i,-1})$ is fixed.

1. Let $\mathcal{P}_Z$ be a measure on Ω_Z that describes the exogenous evolution of Z, i.e. under $\mathcal{P}_Z$ for all $t \geq 1$
$$\log \varepsilon_t := \log\left(\frac{Z_t}{\gamma Z_{t-1}}\right)$$
is iid normal with mean $-\frac{\sigma^2}{2}$ and variance σ^2 and $Z_0 = z_0$ $\mathcal{P}_Z$-a.s. Clearly, a unique measure with this property exists.

2. For $s \in \{c, i\}$ let $\mathcal{P}_{Q,s}$ be a measure on $\Omega_{Q,s}$ that describes the subjective evolution of Q_s under learning, i.e. under $\mathcal{P}_{Q,s}$ for all $t \geq 1$
$$\log \varepsilon_t^{Q,s} := \log\left(\frac{Q_{s,t}}{m_{s,t-1} Q_{s,t-1}}\right)$$

is iid normal with mean $-\frac{\sigma_Q^2}{2}$ and variance σ_Q^2 and $\sigma_Q^2 = \sigma_\varepsilon^2$. Here $m_{s,t}$ is recursively defined by equation (2.12), for $t \geq 1$[10]

$$m_{s,t} = m_{s,t-1}\left(\frac{Q_{s,t-1}}{m_{s,t-1}Q_{s,t-2}}\right)^g.$$

In addition, $Q_{s,-1} = q_{s,-1}$ $\mathcal{P}_{Q,s}$-a.s. Also, the measure $\mathcal{P}_{Q,c}$ is uniquely defined by these properties.

It is obvious, that $\mathcal{P}_Z$ indeed describes the exogenous evolution of Z as defined in equation (2.2) and that $\mathcal{P}_{Q,c}$ and $\mathcal{P}_{Q,i}$ represent the marginal distribution of Q_c and Q_i, respectively, under agents' beliefs, if they are to be consistent with the Bayesian learning formulation in Section 2.5. So any measure $\tilde{\mathcal{P}}$ on $\tilde{\Omega}$ that correctly represents subjective beliefs as defined in Section 2.5 must imply the marginal measures $\mathcal{P}_Z$, $\mathcal{P}_{Q,c}$ and $\mathcal{P}_{Q,i}$ on Ω_Z, $\Omega_{Q,c}$ and $\Omega_{Q,i}$, respectively. The only issue left to discuss is thus which assumptions about the dependence structure of the processes Z, Q_c and Q_i lead to a valid belief measure $\tilde{\mathcal{P}}$ in line with the assumptions made in Section 2.5. We claim that only independence does and thus $\tilde{\mathcal{P}}$ must be given by $\tilde{\mathcal{P}} = \mathcal{P}_Z \otimes \mathcal{P}_{Q,c} \otimes \mathcal{P}_{Q,i}$.

To see this, note that on the extended probability space that includes latent variables in households' filtering problem, agents must think that the four equations

$$\begin{aligned}
\log Q_{c,t} &= \log Q_{c,t-1} + \log \beta_{c,t} + \log \varepsilon_{c,t} \\
\log \beta_{c,t} &= \log \beta_{c,t-1} + \log \nu_{c,t} \\
\log Q_{i,t} &= \log Q_{i,t-1} + \log \beta_{i,t} + \log \varepsilon_{i,t} \\
\log \beta_{i,t} &= \log \beta_{i,t-1} + \log \nu_{i,t}
\end{aligned}$$

hold with probability 1 for all $t \geq 1$. In addition, as stated in Section 2.5, $\{\varepsilon_{c,t}\}_{t=1}^\infty$, $\{\nu_{c,t}\}_{t=1}^\infty$, $\{\varepsilon_{i,t}\}_{t=1}^\infty$, $\{\nu_{i,t}\}_{t=1}^\infty$ are independent stochastic processes and independent of all other model variables, including the process Z. But those two facts can only be simultaneously true, if the three processes Z, Q_c and Q_i are independent.[11]

B.3.3 Existence and Uniqueness of G and F

The goal of this section is to prove Proposition 1. We start with a result that collects important equations and gives an explicit characterization of the function F – up to the presence of the argument $Q_{c,t}$ which is not part of the state S_t.

[10] Note that we here ignore the additional observable innovation present in that equation in line with footnote 31.

[11] Formally, this would require a simple induction proof over time, which is not explicitly spelled out here.

Lemma 2. *In any equilibrium, irrespective of beliefs, the following equations have to hold*

$$
\begin{aligned}
W_t &= K_{c,t}^{\alpha_c} Z_t^{1-\alpha_c}(1-\alpha_c)^{1-\alpha_c} \\
R_{c,t} &= \alpha_c K_{c,t}^{\alpha_c-1} Z_t^{1-\alpha_c}(1-\alpha_c)^{1-\alpha_c} \\
R_{i,t} &= \alpha_i \left(\frac{1-\alpha_i}{W_t}\right)^{\frac{1-\alpha_i}{\alpha_i}} Z_t^{\frac{1-\alpha_i}{\alpha_i}} Q_{c,t}^{\frac{1}{\alpha_i}} \\
Y_t &= K_{c,t}^{\alpha_c} Z_t^{1-\alpha_c}(1-\alpha_c)^{1-\alpha_c} + Q_c^{ss} K_{i,t} Z_t^{\frac{1}{\alpha_i}} \left(\frac{(1-\alpha_i)\, Q_{c,t}}{W_t}\right)^{\frac{1-\alpha_i}{\alpha_i}} \\
C_t &= K_{c,t}^{\alpha_c} Z_t^{1-\alpha_c}(1-\alpha_c)^{1-\alpha_c} \\
I_t &= Q_c^{ss} K_{i,t} Z_t^{\frac{1}{\alpha_i}} \left(\frac{(1-\alpha_i)\, Q_{c,t}}{W_t}\right)^{\frac{1-\alpha_i}{\alpha_i}} \\
H_t &= 1-\alpha_c + K_{i,t} Z_t^{\frac{1-\alpha_i}{\alpha_i}} \left((1-\alpha_i)\,\frac{Q_{c,t}}{W_t}\right)^{\frac{1}{\alpha_i}}
\end{aligned}
$$

for all $t \geq 0$. In particular, $(W_t, R_{c,t}, R_{i,t}, Y_t, C_t, I_t, H_t)$ is a deterministic function of $(Z_t, K_{c,t}, K_{i,t}, Q_{c,t})$.

Conversely, if these equations hold, and $H_{c,t}$, $H_{i,t}$ are given by equations (2.27) and (2.28), then allocations $(C, H, H_c, H_i, K_c, K_i)$ and prices (Q_c, Q_i, R_c, R_i, W) are consistent with all equilibrium conditions (equations (2.15)-(2.25)), except for the two Euler equations (equations (2.16) and (2.17)) and the two capital accumulation equations (equations (2.24) and (2.25)).

Proof. In any competitive equilibrium with subjective beliefs, the equilibrium equations (2.15)-(2.25) stated in Section 2.6 have to hold. Based on these equations and definitions in the model description, the expressions in the assertion can be computed. The wage is given by

$$W_t \overset{(2.15)}{=} C_t \overset{(2.22)}{=} Y_{c,t} \overset{(2.1)}{=} K_{c,t}^{\alpha_c} Z_t^{1-\alpha_c} H_{c,t}^{1-\alpha_c} \overset{(2.27)}{=} (1-\alpha_c)^{1-\alpha_c} K_{c,t}^{\alpha_c} Z_t^{1-\alpha_c}.$$

Similarly, the rental rate in the consumption sector is

$$R_{c,t} \overset{(2.19)}{=} \frac{\alpha_c Y_{c,t}}{K_{c,t}} \overset{(2.1),(2.27)}{=} \alpha_c (1-\alpha_c)^{1-\alpha_c} K_{c,t}^{\alpha_c-1} Z_t^{1-\alpha_c}.$$

Substituting $H_{i,t}$ as given by equation (2.28) into the capital first-order condition of investment firms (2.21) yields for the rental rate in the investment sector

$$
\begin{aligned}
R_{i,t} &= \alpha_i Q_{c,t} K_{i,t}^{\alpha_i-1} Z_t^{1-\alpha_i} \left(K_{i,t} Z_t^{\frac{1-\alpha_i}{\alpha_i}} \left((1-\alpha_i)\,\frac{Q_{c,t}}{W_t}\right)^{\frac{1}{\alpha_i}}\right)^{1-\alpha_i} \\
&= \alpha_i \left(\frac{1-\alpha_i}{W_t}\right)^{\frac{1-\alpha_i}{\alpha_i}} Z_t^{\frac{1-\alpha_i}{\alpha_i}} Q_{c,t}^{\frac{1}{\alpha_i}}.
\end{aligned}
$$

Output is defined as $Y_t = C_t + I_t$, so the asserted output equation follows from the equations for C_t and I_t. The equation for C_t follows from $C_t = W_t$ (2.15) and the equation for W_t already proven. The equation for $I_t = Q_c^{ss} Y_{i,t}$ is obtained by substituting $H_{i,t}$ stated in (2.28) into the investment-sector production function (2.1). Finally, the expression for H_t just combines hours in the two sectors (given by (2.27) and (2.28)) with labor market clearing (2.23).

For the second part of the lemma, we just remark that all the equations (2.15), (2.18), (2.19), (2.20), (2.21), (2.22) and (2.23) have been used in deriving the equations in the first part of the lemma and inverting the arguments used there shows that these equations also necessarily need to hold, if the equations in the lemma and (2.27) and (2.28) hold.

□

The functional relationships in Lemma 2 contain $Q_{c,t}$ as the sole argument that is not contained in the state of time t. $Q_{c,t}$ and $K_{c,t+1}$ are simultaneously determined by the consumption-sector Euler equation (2.16) and the accumulation equation of consumption-sector capital (2.24). It has been shown in the main text how the former equation can be solved for $Q_{c,t}$ under the assumption of subjective price beliefs, compare equation (2.31). In this equation, still a conditional expectation $E_t\left[\frac{1}{W_{t+1}}\right]$ appears. However, given the representation of W in Lemma 2, this conditional expectations can be easily solved explicitly

$$
\begin{aligned}
E_t\left[\frac{1}{W_{t+1}}\right] &= E_t\left[\frac{1}{K_{c,t+1}^{\alpha_c} Z_{t+1}^{1-\alpha_c}(1-\alpha_c)^{1-\alpha_c}}\right] \\
&= \frac{1}{(1-\alpha_c)^{1-\alpha_c} K_{c,t+1}^{\alpha_c}} E_t\left[(\gamma Z_t \varepsilon_{t+1})^{\alpha_c - 1}\right] \\
&= \frac{1}{(1-\alpha_c)^{1-\alpha_c} K_{c,t+1}^{\alpha_c}} \frac{E\left[\varepsilon_{t+1}^{\alpha_c-1}\right]}{\gamma^{1-\alpha_c} Z_t^{1-\alpha_c}},
\end{aligned}
$$

where it has been used that $K_{c,t+1}$ is known at the end of period t and $Z_{t+1} = \gamma Z_t \varepsilon_{t+1}$. The expectation $E\left[\varepsilon_{t+1}^{\alpha_c-1}\right]$ of the log-normal variable $\varepsilon_{t+1}^{\alpha_c-1}$ is simply given by $e^{(\alpha_c-1)(\alpha_c-2)\frac{\sigma^2}{2}}$. Combining this result with equation (2.31), the price of consumption capital must be given by equation (B.4) in the next lemma, which is key for the argument to follow. We first formulate a version under the assumption of predetermined beliefs $m_{c,t}$ (no projection facility applied).[12] As in Appendix B.3.1, we use in the following lower case letters to denote realizations of random variables.

[12]Note also, that the second equation (B.5) is just a combination of the capital accumulation equation (2.24) with $Y_{c,t} = \frac{I_t}{Q_c}$ and the expression for I_t from Lemma 2, so this equation has to hold along any equilibrium path as well.

Lemma 3. *For any given $(z_t, k_{c,t}, k_{i,t}, m_{c,t})$ with $z_t, k_{c,t}, k_{i,t}, m_{c,t} > 0$, the equations*

$$q_{c,t} = \frac{\alpha_c \beta}{1 - \beta\left(1-\delta_c\right) \frac{w_t}{k_{c,t+1}^{\alpha_c}(1-\alpha_c)^{1-\alpha_c}} \frac{e^{(\alpha_c-1)(\alpha_c-2)\frac{\sigma^2}{2}}}{\gamma^{1-\alpha_c} z_t^{1-\alpha_c}} m_{c,t}} \frac{w_t}{k_{c,t+1}} \tag{B.4}$$

$$k_{c,t+1} = \left(1-\delta_c\right) k_{c,t} + z_t^{\frac{1}{\alpha_i}} k_{i,t} \left(\frac{\left(1-\alpha_i\right) q_{c,t}}{w_t} \right)^{\frac{1-\alpha_i}{\alpha_i}} \tag{B.5}$$

have a unique solution $(q_{c,t}, k_{c,t+1})$ with $q_{c,t}, k_{c,t+1} > 0$. Here, $w_t = k_{c,t}^{\alpha}\left(1-\alpha\right)^{1-\alpha} z_t^{1-\alpha}$ is a function of $k_{c,t}$ and z_t.

Proof. Equation (B.4) expresses $q_{c,t}$ as a function of $k_{c,t+1}$, $q_{c,t} = f(k_{c,t+1})$, equation (B.5) expresses $k_{c,t+1}$ as a function of $q_{c,t}$, $k_{c,t+1} = g(q_{c,t})$. Clearly, due to $w_t, z_t > 0$ and $\alpha_i < 1$ the function g is strictly increasing on the domain $(0, \infty)$. Furthermore, as long as the denominator on the left of equation (B.4) is positive, i.e. for $k_{c,t+1} \in (\underline{K}, \infty)$ with

$$\underline{K} := \left(\beta\left(1-\delta_c\right) \frac{w_t}{\left(1-\alpha_c\right)^{1-\alpha_c}} \frac{e^{(\alpha_c-1)(\alpha_c-2)\frac{\sigma^2}{2}}}{\gamma^{1-\alpha_c} z_t^{1-\alpha_c}} m_{c,t} \right)^{\frac{1}{\alpha_c}},$$

$f(k_{c,t+1})$ is strictly decreasing in $k_{c,t+1}$.[13] Hence, also the function

$$h : (\underline{K}, \infty) \to \left(\left(1-\delta_c\right) k_{c,t}, \infty\right), k_{c,t+1} \mapsto g(f(k_{c,t+1}))$$

must be strictly decreasing and thus there is at most one fixed point $k_{c,t+1}^*$ (satisfying $k_{c,t+1}^* = h(k_{c,t+1}^*)$). As any solution $(q_{c,t}, k_{c,t+1})$ to (B.4) and (B.5) must satisfy $q_{c,t} = f(k_{c,t+1})$ and $k_{c,t+1} = g(q_{c,t})$, any such $k_{c,t+1}$ must necessarily be a fixed point of h. Thus, there can be at most one (positive) solution to (B.4) and (B.5).

Conversely, for any fixed point $k_{c,t+1}^*$ of h, the pair $(q_{c,t}^*, k_{c,t+1}^*) = (f(k_{c,t+1}^*), k_{c,t+1}^*)$ is obviously a (positive) solution to (B.4) and (B.5). It is thus left to show that a fixed point always exists. The function h is continuous and the following limit considerations show the existence of a fixed point by the intermediate value theorem:

- $f(k) \to 0$ as $k \to \infty$, so $h(k) \to g(0) = \left(1-\delta_c\right) k_{c,t}$ as $k \to \infty$, hence for large k $h(k) - k$ is negative
- $f(k) \to \infty$ as $k \searrow \underline{K}$ and $g(q) \to \infty$ as $q \to \infty$, so $h(k) \to \infty$ as $k \searrow \underline{K}$, hence for small k close to $\underline{K}$, $h(k) - k$ is positive

□

[13] When looking for a positive solution $(q_{c,t}, k_{c,t+1})$ to (B.4) and (B.5), we can restrict attention to $k_{c,t+1} > \underline{K}$, because otherwise $q_{c,t} = f(k_{c,t+1})$ becomes negative.

Unfortunately, the above result treats $m_{c,t}$ as fixed, but due to the projection facility described in Appendix B.2, $m_{c,t}$ is not fully predetermined in period t, instead might be projected downward, if it implies a too high PD ratio. Hence, the simple result above cannot always be applied. The following technical lemma deals with the more general case.[14]

Lemma 4. *For any given positive values of z_t, $k_{c,t}$, $k_{i,t}$, $m_{c,t}$ consider the three equations (again, $w_t = (1-\alpha_c)^{1-\alpha_c} k_{c,t}^{\alpha_c} z_t^{1-\alpha_c}$)*

$$q_{c,t} = \frac{\alpha_c \beta}{1 - \beta(1-\delta_c) \frac{w_t}{k_{c,t+1}^{\alpha_c}(1-\alpha_c)^{1-\alpha_c}} \frac{e^{(\alpha_c-1)(\alpha_c-2)\frac{\sigma^2}{2}}}{\gamma^{1-\alpha_c} z_t^{1-\alpha_c}} m_{c,t}^p} \frac{w_t}{k_{c,t+1}} \tag{B.6}$$

$$k_{c,t+1} = (1-\delta_c) k_{c,t} + z_t^{\frac{1}{\alpha_i}} k_{i,t} \left(\frac{(1-\alpha_i) q_{c,t}}{w_t} \right)^{\frac{1-\alpha_i}{\alpha_i}} \tag{B.7}$$

$$m_{c,t}^p = \begin{cases} m_{c,t}, & m_{c,t} \le \underline{m}(k_{c,t+1}) \\ \underline{m}(k_{c,t+1}) + \frac{(\overline{m}-\underline{m})(k_{c,t+1})}{m_{c,t} + (\overline{m}-2\underline{m})(k_{c,t+1})} (m_{c,t} - \underline{m}(k_{c,t+1})), & m_{c,t} \ge \underline{m}(k_{c,t+1}) \end{cases} \tag{B.8}$$

where the functions $\underline{m}$, $\overline{m}$ are the projection thresholds for m_c as defined in Appendix B.2.[15] *If*

$$m_{c,t} < \frac{1}{\beta(1-\alpha_c)} \left(\frac{\gamma}{(1-\delta_c) e^{\frac{(2-\alpha_c)\sigma^2}{2}}} \right)^{1-\alpha_c}, \tag{B.9}$$

then the equation system has a unique solution $(q_{c,t}, k_{c,t+1}, m_{c,t}^p)$.

Proof. Ignore the capital accumulation equation (B.7) and first consider equations (B.6) and (B.8). To transfer the proof from Lemma 3, we need to show that after substituting $m_{c,t}^p$ into the first equation, this still defines a decreasing relationship between $k_{c,t+1}$ and $q_{c,t}$. As $\underline{m}(k_{c,t+1})$ is strictly increasing in $k_{c,t+1}$ (and approaching $-\infty$ as $k_{c,t+1} \to 0$), there is some threshold $\hat{k}$, such that $\underline{m}(k_{c,t+1}) \le m_{c,t}$ for $k_{c,t+1} \le \hat{k}$ and $\underline{m}(k_{c,t+1}) \ge m_{c,t}$ for $k_{c,t+1} \ge \hat{k}$. We consider the two cases separately:

1. If $k_{c,t+1} \le \hat{k}$, then the projection is actually used, so we have

$$m_{c,t}^p = \underline{m} + \frac{\overline{m} - \underline{m}}{m_{c,t} + \overline{m} - 2\underline{m}} (m_{c,t} - \underline{m})$$

[14]The additional upper bound on beliefs stated in equation (B.9) is of little practical relevance. In all calibrations we consider, the upper bound is approximately equal to $\frac{1}{1-\alpha_c} \approx 1.5$, which implies an expected appreciation in the capital price of 50% within the next quarter and is much larger than any value m_c ever attained in our numerical simulations. We nevertheless tighten the projection upper bound $\overline{m}_{c,t}$ to be always consistent with equation (B.9), see footnote 1. This additional modification does not invalidate the proof of the lemma.

[15]$\underline{m}$, $\overline{m}$ also depend on z_t, w_t and $k_{c,t}$, which is suppressed in the notation.

from the third equation. For notational convenience define the constants

$$
\begin{aligned}
A &= \frac{1}{\beta(1-\delta_c)\frac{w_t \exp\left((1-\alpha_c)(2-\alpha_c)\frac{\sigma^2}{2}\right)}{\gamma^{1-\alpha_c}(1-\alpha_c)^{1-\alpha_c} z_t^{1-\alpha_c}}} \\
\underline{B} &= \frac{\beta\left(1-\delta_c\right) k_{c,t}}{\left(p\left(1+\underline{PD}\right)-1\right)\beta\left(1-\delta_c\right)\frac{w_t \exp\left((1-\alpha_c)(2-\alpha_c)\frac{\sigma^2}{2}\right)}{\gamma^{1-\alpha_c}(1-\alpha_c)^{1-\alpha_c} z_t^{1-\alpha_c}}} = \frac{\beta\left(1-\delta_c\right) k_{c,t}}{p\left(1+\underline{PD}\right)-1} A \\
\overline{B} &= \frac{\beta\left(1-\delta_c\right) k_{c,t}}{\left(p\left(1+\overline{PD}\right)-1\right)\beta\left(1-\delta_c\right)\frac{w_t \exp\left((1-\alpha_c)(2-\alpha_c)\frac{\sigma^2}{2}\right)}{\gamma^{1-\alpha_c}(1-\alpha_c)^{1-\alpha_c} z_t^{1-\alpha_c}}} = \frac{\beta\left(1-\delta_c\right) k_{c,t}}{p\left(1+\overline{PD}\right)-1} A \\
C &= \beta(1-\delta_c)\frac{w_t \exp\left((1-\alpha_c)(2-\alpha_c)\frac{\sigma^2}{2}\right)}{\gamma^{1-\alpha_c}(1-\alpha_c)^{1-\alpha_c} z_t^{1-\alpha_c}} = A^{-1}
\end{aligned}
$$

and drop all subscripts for the following argument (k refers to $k_{c,t+1}$, q to $q_{c,t}$ and m to $m_{c,t}$).

Then we have (this follows from equations (B.1) and (B.2) for $s = c$)

$$\underline{m} = Ak^{\alpha_c} - \underline{B}k^{\alpha_c - 1}, \qquad \overline{m} = Ak^{\alpha_c} - \overline{B}k^{\alpha_c - 1}$$

and (from equation (B.6))

$$q = \frac{\text{const}}{1 - Ck^{\alpha_c} m^p} \frac{1}{k}.$$

Using $m^p = \underline{m} + \frac{\overline{m}-\underline{m}}{m+\overline{m}-2\underline{m}}\left(m - \underline{m}\right)$, we obtain

$$
\begin{aligned}
\frac{\text{const}}{q} &= k - Ck^{1-\alpha_c} m^p \\
&= k - Ck^{1-\alpha_c}\left(\underline{m} + \frac{\overline{m}-\underline{m}}{m+\overline{m}-2\underline{m}}\left(m-\underline{m}\right)\right) \\
&= k - Ck^{1-\alpha_c}\Bigg(Ak^{\alpha_c} - \underline{B}k^{\alpha_c-1} \\
&\qquad + \frac{Ak^{\alpha_c} - \overline{B}k^{\alpha_c-1} - Ak^{\alpha_c} + \underline{B}k^{\alpha_c-1}}{m + Ak^{\alpha_c} - \overline{B}k^{\alpha_c-1} - 2Ak^{\alpha_c} + 2\underline{B}k^{\alpha_c-1}}\left(m - Ak^{\alpha_c} + \underline{B}k^{\alpha_c-1}\right)\Bigg) \\
&= C\underline{B} - C\left(\underline{B} - \overline{B}\right)\frac{m - Ak^{\alpha_c} + \underline{B}k^{\alpha_c-1}}{m - Ak^{\alpha_c} + \left(2\underline{B} - \overline{B}\right)k^{\alpha_c-1}}
\end{aligned}
$$

$C\underline{B}$ and $C\left(\underline{B} - \overline{B}\right)$ are positive constants, so q is decreasing in k, if and only if the expression $\frac{m-Ak^{\alpha_c}+\underline{B}k^{\alpha_c-1}}{m-Ak^{\alpha_c}+\left(2\underline{B}-\overline{B}\right)k^{\alpha_c-1}}$ is. Using that the derivative of $x \mapsto \frac{u(x)}{u(x)+v(x)}$ is given by $\frac{u'(x)v(x)-v'(x)u(x)}{(u(x)+v(x))^2}$, we find that $\frac{m-Ak^{\alpha_c}+\underline{B}k^{\alpha_c-1}}{m-Ak^{\alpha_c}+\left(2\underline{B}-\overline{B}\right)k^{\alpha_c-1}}$ is (strictly) decreasing in k, if and only if[16]

$$\begin{aligned}&\left(-A\alpha_c k^{\alpha_c-1}+(\alpha_c-1)\,\underline{B}k^{\alpha_c-2}\right)\left(\underline{B}-\overline{B}\right)k^{\alpha_c-1}\\ &<(\alpha_c-1)\left(\underline{B}-\overline{B}\right)k^{\alpha_c-2}\left(m-Ak^{\alpha_c}+\underline{B}k^{\alpha_c-1}\right)\end{aligned}$$

After expanding the products on both sides and canceling common terms, this inequality simplifies to

$$\begin{aligned}0 &< (\alpha_c-1)\left(\underline{B}-\overline{B}\right)mk^{\alpha_c-2}+A\left(\underline{B}-\overline{B}\right)k^{2\alpha_c-2}\\ \Leftrightarrow\ m &< \frac{Ak^{\alpha_c}}{1-\alpha_c}.\end{aligned}$$

Finally, using the definition of A, we obtain the condition (from now on subscripts are added back again for clarity about the timing of variables)

$$\begin{aligned}m_{c,t} &< \frac{\gamma^{1-\alpha_c}(1-\alpha_c)^{1-\alpha_c}z_t^{1-\alpha_c}}{\beta(1-\delta_c)w_t\exp\left((1-\alpha_c)(2-\alpha_c)\frac{\sigma^2}{2}\right)}\frac{k_{c,t+1}^{\alpha_c}}{1-\alpha_c}\\ &\overset{(2.33)}{=}\frac{\gamma^{1-\alpha_c}(1-\alpha_c)^{1-\alpha_c}z_t^{1-\alpha_c}}{\beta(1-\delta_c)(1-\alpha_c)^{1-\alpha_c}k_{c,t}^{\alpha_c}z_t^{1-\alpha_c}\exp\left((1-\alpha_c)(2-\alpha_c)\frac{\sigma^2}{2}\right)}\frac{k_{c,t+1}^{\alpha_c}}{1-\alpha_c}\\ &=\frac{\gamma^{1-\alpha_c}}{\beta\left(1-\delta_c\right)\left(1-\alpha_c\right)\exp\left((1-\alpha_c)(2-\alpha_c)\frac{\sigma^2}{2}\right)}\left(\frac{k_{c,t+1}}{k_{c,t}}\right)^{\alpha_c}\end{aligned}$$

This condition is tighter, the smaller is $k_{c,t+1}$. The smallest possible value of $k_{c,t+1}$ given $k_{c,t}$ is $k_{c,t+1}=(1-\delta_c)k_{c,t}$ (otherwise equation (B.7) is inconsistent with a positive $q_{c,t}$), so the above condition on $m_{c,t}$ is certainly satisfied, if

$$m_{c,t}<\frac{1}{\beta\left(1-\alpha_c\right)}\left(\frac{\gamma}{\left(1-\delta_c\right)e^{\frac{(2-\alpha_c)\sigma^2}{2}}}\right)^{1-\alpha_c},$$

which is exactly the condition required in the assertion. So as long as $k_{c,t+1}\leq\hat{k}$, the third and first equation define a strictly decreasing relationship between $k_{c,t+1}$ and $q_{c,t}$.

2. If $k_{c,t+1}\geq\hat{k}$, then $m_{c,t}^p=m_{c,t}$ does not depend on the level on $k_{c,t+1}$ anymore and thus the third and first equation define a strictly decreasing relationship between $k_{c,t+1}$ and $q_{c,t}$ by arguments made in the proof of Lemma 3.

[16] Here, $u(x)=m-Ax^{\alpha_c}+\underline{B}x^{\alpha_c-1}\Rightarrow u'(x)=-A\alpha_c x^{\alpha_c-1}+(\alpha_c-1)\,\underline{B}x^{\alpha_c-2}$ and $v(x)=\left(\underline{B}-\overline{B}\right)x^{\alpha_c-1}\Rightarrow v'(x)=(\alpha_c-1)\left(\underline{B}-\overline{B}\right)x^{\alpha_c-2}$

After substituting the third equation into the first, we have as before two functional relationships $q_{c,t} = f(k_{c,t+1})$ and $k_{c,t+1} = g(q_{c,t})$ with f strictly decreasing and g strictly increasing. The same arguments made in the proof of Lemma 3 guarantee a unique solution. The associated level of $m^p_{c,t}$ can then be computed from the third equation.

□

We have now all the tools available to prove Proposition 1.

Proof of Proposition 1. Existence and uniqueness of the measure $\tilde{\mathcal{P}}$ has already been discussed in Section B.3.2 of this appendix. Here, it is left to construct the mappings G and F. Throughout the construction only necessary equilibrium conditions are used and no construction step admits several choices. For this reason, the construction will also yield uniqueness of the mappings G and F.

Suppose $S_t = s_t$ for an arbitrary (fixed) state $s_t = (z_t, k_{c,t}, k_{i,t}, m_{c,t}, m_{i,t}, q_{c,t-1}, q_{i,t-1})$ such that $m_{c,t}$ respects the upper bound of Lemma 4. For the following argument it is also useful to define the "reduced state" $\hat{s}_t = (z_t, k_{c,t}, k_{i,t}, m_{c,t}, q_{c,t-1})$ that does not include $m_{i,t}$ and $q_{i,t-1}$. We first show the existence of a unique state transition function $\hat{G}$ for this reduced state. In any equilibrium, the wage is a function of $K_{c,t}$ and Z_t only, compare Lemma 2. Hence, conditional on $\hat{S}_t = \hat{s}_t$, $W_t = \tilde{F}_W(\hat{s}_t)$ with some deterministic function $\tilde{F}_W$ (whose explicit form is given in Lemma 2). Furthermore, in any equilibrium (with beliefs as specified in Section 2.5) the equations (B.6), (B.7), (B.8) have to hold along any equilibrium path. Lemma 4 thus implies the existence of a unique vector $(q_{c,t}, k_{c,t+1}, m^p_{c,t})$, given z_t, $k_{c,t}$, $k_{i,t}$, $m_{c,t}$ and $w_t = \tilde{F}_W(\hat{s}_t)$, which are all uniquely determined by the reduced state $\hat{s}_t$, such that all three equations hold. The equations thus implicitly define three functions

$$\tilde{G}_{Q,c}(\hat{s}_t) = q_{c,t}, \qquad \tilde{G}_{K,c}(\hat{s}_t) = k_{c,t+1}, \qquad \tilde{G}_{m^p,c}(\hat{s}_t) = m^p_{c,t}$$

and the argument given so far implies that along any equilibrium path $q_{c,t}$, $k_{c,t+1}$ and $m^p_{c,t}$ (the value of $m_{c,t}$ after projection) must necessarily be related to $\hat{s}_t$ as described by these three equations.

Next, $m_{c,t+1}$ must satisfy the belief updating equation (compare (2.12) for the consumption sector[17])

$$\ln m_{c,t+1} = \ln m^p_{c,t} + g\left(\ln q_{c,t} - \ln q_{c,t-1} - \ln m^p_{c,t}\right) =: \ln \tilde{G}_{m,c}(\hat{s}_t),$$

where the right-hand side is a function of $\hat{s}_t$, because $q_{c,t} = \tilde{G}_{Q,c}(\hat{s}_t)$ and $m^p_{c,t} = \tilde{G}_{m^p,c}(\hat{s}_t)$ are and $q_{c,t-1}$ is a component of $\hat{s}_t$.

In addition, $K_{i,t+1} \propto Z_t$ by definition. Let $\bar{k}_i$ be the proportionality constant, i.e. $K_{i,t+1} = \bar{k}_i Z_t$. This implies for realizations conditional on $\hat{S}_t = \hat{s}_t$ that $k_{i,t+1} = \bar{k}_i z_t =: \tilde{G}_{K,i}(\hat{s}_t)$.

[17] As remarked in footnote 31, we set the additional shock to agents' information set to 0 in all periods.

In total, we obtain from the discussion so far that $\hat{s}_{t+1} = (z_{t+1}, k_{c,t+1}, k_{i,t+1}, m_{c,t+1}, q_{c,t})$ must necessarily satisfy

$$\hat{s}_{t+1} = (z_{t+1}, \tilde{G}_{K,c}(\hat{s}_t), \tilde{G}_{K,i}(\hat{s}_t), \tilde{G}_{m,c}(\hat{s}_t), \tilde{G}_{Q,c}(\hat{s}_t)) =: \hat{G}\left(\hat{s}_t, z_{t+1}\right).$$

Thus, in any equilibrium, the evolution of the reduced state $\hat{s}_{t+1}$ must be governed by the transition function $\hat{G}$. From our derivation it is also clear that the evolution of $\hat{G}$ is consistent with the equations (B.6), (B.7), (B.8), the belief updating equation (2.12) in the consumption sector and the exogenous evolution of K_i. In particular, $\hat{G}$ is then consistent with the consumption-sector Euler equation (2.16) and with the capital accumulation equations (2.24), (2.25) in both sectors.

Next, we let $\hat{F}$ be the mapping from reduced states $\hat{s}_t$ to outcomes $(w_t, r_{c,t}, r_{i,t}, y_t, c_t, i_t, h_t)$ defined by the formulas given in Lemma 2, if $q_{c,t}$ is everywhere replaced by $\tilde{G}_{Q,c}(\hat{s}_t)$.[18] Combining this with the obvious state reduction mapping $s_t \mapsto \hat{s}_t$, we can define $F(s_t) := \hat{F}(\hat{s}_t)$.[19] Lemma 2 tells us then that this choice of F is the only possible choice consistent with equilibrium and in turn this F is consistent with all equilibrium equations other than the two Euler equations and the two capital accumulation equations. Consequently, F and $\hat{G}$ together are consistent with all the equilibrium conditions (2.15)-(2.25) except for the investment-sector Euler equation (2.17). In addition, $\hat{G}$ is also consistent with the belief updating equation for consumption-sector capital prices.

To complete the existence proof, it is left to show that $\hat{G}$ can be extended to a full state transition mapping G that is in addition consistent with the investment-sector Euler equation and the belief updating equation for investment-sector capital prices. First, consider the conditional expectations in equation (2.34), which is a partially solved version of the investment-sector Euler equation (2.17) from the main text. These conditional

[18]This means,

$$\begin{aligned}
\hat{F}_W(\hat{s}_t) &= k_{c,t}^{\alpha_c} z_t^{1-\alpha_c} (1-\alpha_c)^{1-\alpha_c} \\
\hat{F}_{R,c}(\hat{s}_t) &= \alpha_c k_{c,t}^{\alpha_c - 1} z_t^{1-\alpha_c} (1-\alpha_c)^{1-\alpha_c} \\
\hat{F}_{R,i}(\hat{s}_t) &= \alpha_i \left(\frac{1-\alpha_i}{\hat{F}_W(\hat{s}_t)} \right)^{\frac{1-\alpha_i}{\alpha_i}} z_t^{\frac{1-\alpha_i}{\alpha_i}} \left(\tilde{G}_{Q,c}(\hat{s}_t) \right)^{\frac{1}{\alpha_i}} \\
&\vdots
\end{aligned}$$

[19]We keep the separate mapping $\hat{F}$ as an auxiliary device for the extension of $\hat{G}$ to G below.

expectations can be written as

$$\begin{aligned} E_t\left[\frac{W_t}{W_{t+1}}R_{i,t+1}\right] &= E\left[\frac{\hat{F}_W(\hat{S}_t)}{\hat{F}_W(\hat{S}_{t+1})}\hat{F}_{R,i}(\hat{S}_{t+1})\Big|S_t, S_{t-1},\dots\right] \\ &= E\left[\frac{\hat{F}_W(\hat{S}_t)}{\hat{F}_W\left(\hat{G}\left(\hat{S}_t, Z_{t+1}\right)\right)}\hat{F}_{R,i}\left(\hat{G}\left(\hat{S}_t, Z_{t+1}\right)\right)\Big|\hat{S}_t\right], \end{aligned}$$

$$\begin{aligned} E_t\left[\frac{W_t}{W_{t+1}}\right] &= E\left[\frac{\tilde{F}_W(\hat{S}_t)}{\tilde{F}_W(\hat{S}_{t+1})} \mid S_t, S_{t-1},\dots\right] \\ &= E\left[\frac{\tilde{F}_W(\hat{S}_t)}{\tilde{F}_W\left(\hat{G}\left(\hat{S}_t, Z_{t+1}\right)\right)} \mid \hat{S}_t\right], \end{aligned}$$

where in each case the second equality follows from the fact that all information in the history S^t not already contained in $\hat{S}_t$ is redundant for predicting Z_{t+1} and $\hat{S}_t$. Hence, both conditional expectations are deterministic functions of the current reduced state $\hat{S}_t$. As these two conditional expectations and $R_{i,t} = \hat{F}_{R,i}(\hat{S}_t)$ are the only relevant variables to compute the projection bounds in the investment sector, compare Appendix B.2, conditional on $\hat{S}_t = \hat{s}_t$, the projected belief in the investment sector is given by $m^p_{i,t} = \tilde{G}_{m^p,i}(\hat{s}_t, m_{i,t})$ with some deterministic function $\tilde{G}_{m^p,i}$. By equation (2.34), $Q_{i,t}$ must then assume in equilibrium the value (conditional on $\hat{S}_t = \hat{s}_t$ and $m_{i,t}$)

$$q_{i,t} = \frac{\beta E\left[\frac{W_t}{W_{t+1}}R_{i,t+1} \mid \hat{S}_t = \hat{s}_t\right]}{1-\beta E_t\left[\frac{W_t}{W_{t+1}} \mid \hat{S}_t = \hat{s}_t\right](1-\delta)\,\tilde{G}_{m^p,i}(\hat{s}_t, m_{i,t})}.$$

The right-hand side is a function of $\hat{s}_t$ and $m_{i,t}$ and therefore of the full state s_t. Denote it by $\tilde{G}_{Q,i}(s_t)$. Finally, the belief updating equation for the investment sector defines $m_{i,t+1}$ as a function of the current state,

$$m_{i,t+1} = \tilde{G}_{m,i}(s_t) := \tilde{G}_{m^p,i}(\hat{s}_t, m_{i,t})\left(\frac{\tilde{G}_{Q,i}(s_t)}{\tilde{G}_{m^p,i}(\hat{s}_t, m_{i,t})q_{i,t-1}}\right)^g.$$

Define thus

$$\begin{aligned} G(s_t, z_{t+1}) := \Big(&\hat{G}_Z(\hat{s}_t, z_{t+1}), \hat{G}_{K,c}(\hat{s}_t, z_{t+1}), \hat{G}_{K,i}(\hat{s}_t, z_{t+1}), \hat{G}_{m,c}(\hat{s}_t, z_{t+1}), \\ &\tilde{G}_{m,i}(s_t), \hat{G}_{Q,c}(\hat{s}_t, z_{t+1}), \tilde{G}_{Q,i}(s_t)\Big). \end{aligned}$$

By construction, G is then (together with F) consistent with all equations that $\hat{G}$ is and in addition with the investment-sector Euler equation (2.17) and with the belief updating equation for m_i. This finishes the proof of the proposition. $\square$

B.4 Details on the IRF Decay Restrictions in Estimation

Capital prices in the subjective belief model show a cyclical pattern in response to a technology shock: as prices fall from a boom to steady state, agents may be already slightly pessimistic and prices undershoot, as prices recover from a bust back to steady state, agents may already be slightly optimistic, triggering another boom. If these dynamics are too strong, deterministic cycles can exist or the cyclical dynamics can even be self-amplifying (small shocks lead to a sequence of cycles of increasing magnitude). We impose a decay restriction in the estimation to rule out such dynamics. The IRF decay restriction does not impose a certain speed of decay within one cycle, but rather requires the decay of subsequent cycle peaks over time to be sufficiently fast.

Specifically, for each sector $s \in \{c, i\}$ we consider a long deterministic impulse response path (400 quarters) for the capital price Q_s to a one-standard-deviation technology shock starting in the steady state. We identify all peaks (local maxima) of the resulting path $\{Q_{s,t}\}_{t=0}^{400}$, where at t there is a "peak", if $Q_{s,t} > Q_{s,t-1}, Q_{s,t+1}$. Let T_p be the set of all peak times of the impulse response path. If $|T_p| \leq 1$, we set the peak decay rate to ∞, thereby always admitting such a parameter combination. If $|T_p| \geq 2$, we fit an exponential function through the points $\{Q_{s,t}\}_{t \in T_p}$, specifically we estimate the least-squares regression

$$\log\left(\frac{Q_{s,t}}{Q_s^{ss}} - 1\right) = a + bt + \varepsilon_t, \qquad t \in T_p,$$

where Q_s^{ss} is the steady-state value of Q_s. We call $-b$ the peak decay rate of the impulse response. Figure B.1 illustrates the procedure graphically: the blue solid line is the impulse response path, the red circles mark the peaks, i.e. the points $(t, Q_{s,t}/Q_s^{ss})$ for $t \in T_p$, and the yellow dashed line represents the fitted exponential.

On the b parameter we impose the restriction $-b \geq 1.16\%$. This implies a half-life of at most 60 quarters or – given a typical distance of approximately 40 quarters between two peaks – a size reduction of at least one third from one peak to the next.

Parameter combinations are admitted in the estimation, if they satisfy this condition for both Q_c and Q_i.

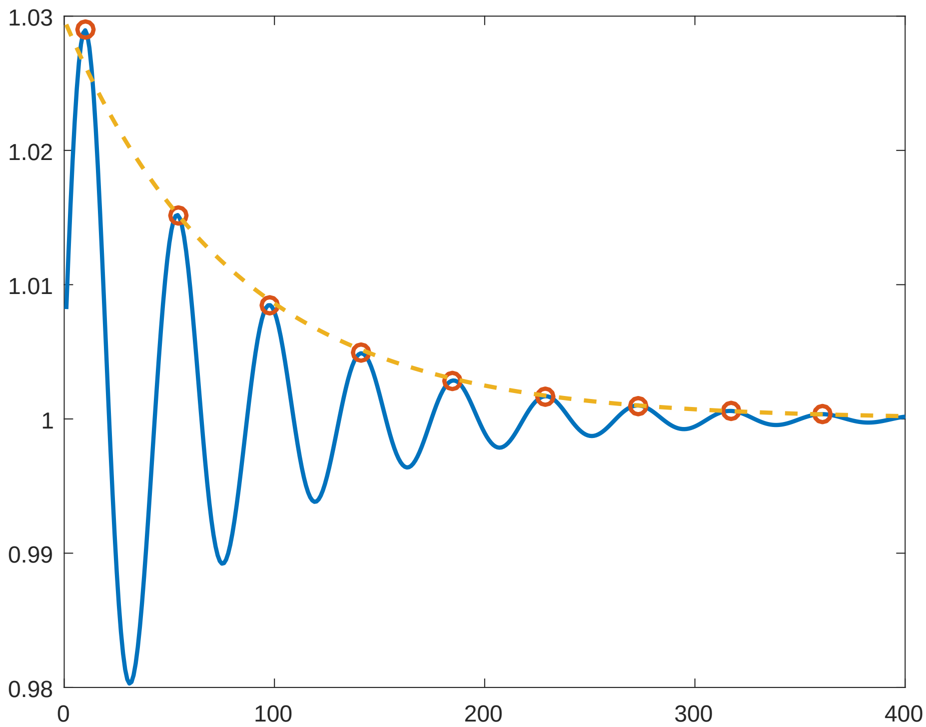

Figure B.1. Illustration of the decay rate definition

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