Essays on Financial Crises and Bank Capital Regulation

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Eidesstattliche Erklärung

Hiermit erkläre ich, dass ich die vorliegende Dissertation selbstständig angefertigt und die benutzten Hilfsmittel vollständig und deutlich angegeben habe.

Mannheim, 24. August 2018 Xue Zhang

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To my grandfather

Introduction

This thesis consists of three chapters, all of which contribute to the literature on financial crises and bank capital regulation. A common feature of the quantitative models in the three chapters is the endogenous bank run equilibrium à la Gertler and Kiyotaki (2015). Chapter 1 studies the cost and benefit of retail bank capital requirements in an economy with both retail and shadow banks, where financial crises take the form of shadow bank runs. Chapter 2 compares the effect of three macroprudential policies on financial stability in an economy with intertwined housing and banking crises. Chapter 3 analyzes the macroeconomic effects of the policy combination of bank capital requirement (an ex-ante intervention policy) and credit easing (an ex-post intervention policy). Chapter 1 is co-authored with Johannes Pöschl. Chapter 2 is a joint work with Johannes Pöschl and Marcus Mølbak Ingholt. Chapter 3 is single-authored.

Chapter 1, titled *Endogenous Shadow Banking Crises and Bank Capital Regulation*, sheds light on the optimal level and dynamic design of retail bank capital requirements in an economy with two banking sectors – retail and shadow banking. Systemic banking crises occur endogenously in the form of self-fulfilling runs on shadow banks. A negative externality exists, as banks do not internalize the effects of their leverage choices on the probability of bank runs, creating a role for government interventions, e.g., through bank capital requirements.

We show that a dynamic capital requirement, which requires retail banks to build up capital buffers during normal times and allows the buffers to be depleted during a bank run, can reduce the frequency and severity of systemic banking crises. We highlight the importance of relaxing the capital requirement in a timely manner when a bank run happens. Otherwise, the capital requirement would restrict the retail banks' ability to absorb liquidated assets of shadow banks, resulting in more frequent and more severe banking crises. Meanwhile, tightening the capital requirement leads to less financial intermediation and shifts banking activities from retail to shadow banks. Based on our calibration, we find retail bank capital requirements undesirable, as the welfare cost of less financial intermediation outweighs the benefit of fewer bank runs.

Chapter 2 is titled *Housing, Financial Crises and Macroprudential Regulation: The Case of Spain*. Based on the observation of the intertwined housing and banking crisis in Spain between 2008 and 2016, our objective is to study the effectiveness of different macroprudential regulation policies in preventing crises of this kind.

We develop a dynamic stochastic general equilibrium (DSGE) model in which we introduce a housing market and mortgage credit while keeping the mechanism of banking crises as in Chapter 1. However, instead of distinguishing retail and shadow banking, we simplify the model by including only one banking sector in the economy. The equilibrium mortgage credit allocation in the economy is determined either by a capital requirement on the banks or a loan-to-value (LTV) constraint on the borrowers. Large drops in the house price (housing crises) occur endogenously and can lead to runs on the banking sector (banking crises).

We calibrate the model to match the Spanish economy in 2007-2017. We find that all three macroprudential policies can reduce the mortgage default rate and the frequency of bank runs, but the effect is stronger with the higher capital requirement and the tighter LTV constraint. Dynamic loan loss provisioning is effective at reducing the cyclicality of the capital structure of banks, while a tighter LTV constraint amplifies the cyclicality of bank and household leverage.

Chapter 3 is titled *Capital Requirements and Credit Easing: Ex-ante vs Ex-Post Intervention Policy*. In the first two chapters, the policy focus is on ex-ante macroprudential policies, which are designed to improve financial stability and prevent potential financial crises in the future. In this chapter, I turn to ex-post intervention policies. In particular, I introduce a credit easing policy by the central bank to an economy with endogenous banking crises similar to Chapter 1 but has only one banking sector.

I show that bank capital requirements and credit easing policies exhibit very different trade-offs. In particular, tightening bank capital requirements effectively reduces bank leverage but leads to less financial intermediation. For the credit easing policy, I highlight an unintended ex-ante effect: it decreases the banks' risk premium in a financial crisis, resulting in more leverage taking of banks and a higher frequency of bank runs. Nonetheless, the credit easing policy facilitates financial intermediation in both normal and crisis periods and stabilizes asset prices during financial crises. A combination of the two policies can offset their respective negative effects, reducing the frequency and severity of financial crises while maintaining efficient financial intermediation in the economy.

The thesis is structured as follows. The quantitative model and main findings of each chapter are presented in respective Chapters 1 to 3. Data sources, full statement of the model equilibrium, and numerical solution algorithms for Chapter 1 and 2 are gathered in Appendices A and B, respectively. All the references are in Bibliography.

Chapter 1

Endogenous Shadow Banking Crises and Bank Capital Regulation

with Johannes Pöschl

1.1 Introduction

In the aftermath of the 2007-2009 Global Financial Crisis, bank capital regulation as an intervention policy for combating future crises has been subject to extensive debates among policymakers and researchers. Supporters of stringent bank capital regulations emphasize that higher capital requirements make the banking system more resilient to financial panics, therefore enhance financial stability of the economy. Opponents argue that higher capital requirements increase the financing cost of the financial institutions and diminish financial intermediation in the economy. Moreover, some worry that stringent regulations on retail banks can cause banking activities shifting to the unregulated shadow banking sector, which could undermine financial stability. This policy debate leads to our research question: How should bank regulators optimally design capital requirements and adjust them dynamically in response to changes in the state of the economy?

In this chapter, we study the macroeconomic effects of regulating retail banks in a non-linear dynamic stochastic general equilibrium (DSGE) model with both retail and shadow banks. Taking shadow banks into consideration is crucial for several reasons. First, shadow banking has grown tremendously over the last decades into an essential part of the modern financial system.[1] Second, the collapse of the shadow banking sector played an important role in the financial turmoil that ultimately turned into a global

[1]According to the Global Shadow Banking Monitoring Report 2016 by the Financial Stability Board, in 2015, shadow banking accounts for 13 percent of the total financial system in 27 jurisdictions included in the report, where the shadow banking to the GDP ratio is around 70 percent.

financial crisis. Third, it is important to take into account the unintended spillover effects on the shadow banks when analyzing the effects of retail bank regulations. In this chapter, we define shadow banks as financial institutions that (i) borrow from other financial institutions on a wholesale funding market, (ii) are highly leveraged, and (iii) are more efficient than retail banks in investments to non-financial firms.[2]

Specifically, we consider a closed economy in the spirit of Gertler, Kiyotaki, and Prestipino (2016) (henceforth GKP), which is populated with households, retail banks, shadow banks, capital producers and consumption goods producers. In equilibrium, households make deposits at retail banks, which in turn lend to shadow banks through a wholesale funding market. Both banking sectors face an endogenous leverage constraint due to a moral hazard problem. In addition, retail banks face a regulatory capital requirement. There are two production sectors: consumption goods producers, which produce consumption goods using capital and labor subject to productivity shocks, and capital producers, which transform consumption goods into capital subject to capital adjustment cost. Uncertainty arises in the form of productivity shocks and liquidation cost shocks, which affect the liquidation value of capital in bank runs. A shadow bank run equilibrium exists when retail banks foresee that a fall in the capital price in the even of a run can reduce the value of the shadow banks' assets below their liabilities, making shadow banks insolvent. We focus on the case of shadow bank runs and not runs on the retail banks, as the subprime mortgage crisis was essentially a shadow banking crisis (see, e.g., GKP).

In our model, financial regulators can address two externalities by imposing retail bank capital requirements. First, banks do not internalize the effects of their leverage decisions on the likelihood of systemic bank runs, which we call the *bank run externality*. To our best knowledge, this externality has not been addressed in an infinite horizon DSGE model on macroprudential regulations in the literature before. Second, as price-taking agents also do not internalize the effects of their decisions on asset prices, there is a *pecuniary externality*. If banks expand their balance sheets by increasing leverage, then the capital price increases, which relaxes the banks' borrowing constraints. As a result, banks borrow and invest more, and drive up the capital price even further. The capital price externality amplifies the business cycle volatility, creating welfare losses for households. This pecuniary externality has been explored theoretically by Lorenzoni (2008) and Dávila and Korinek (2017) and in a quantitative framework by Bianchi (2011).

[2] The last characteristic of shadow banks can be a result of either benefits of specialization or due to fewer regulatory restrictions for shadow banks. Examples of shadow banks by our definition include financial institutions such as finance companies, stand-alone broker-dealers, asset-backed security originators, and non-bank affiliated structured investment vehicles.

Our model captures the following trade-off of bank capital requirements. On the positive side, a higher *dynamic* capital requirement reduces the frequency and severity of banking crises. Under a higher capital requirement, retail banks build up more capital buffer during expansion periods, which can be depleted to absorb liquidated assets upon bank runs. As such, the liquidation price of capital is higher in bank runs, which in turn increases the recovery rate of the retail banks' lending to shadow banks in a run. As the likelihood of a shadow bank run is negatively related to the recovery rate, the probability of bank runs is reduced. We show that for this policy to take effect, it is crucial that the regulators relax capital requirements timely during a bank run, allowing retail banks to absorb liquidated assets of shadow banks. On the negative side, a higher capital requirement on retail banks leads to less financial intermediation and a shifting of banking activity from retail to shadow banks. The high capital requirement pushes up the financing cost for retail banks, which is further passed through wholesale funding to the shadow banks. This results in less financial intermediation and a higher required return on capital, a lower aggregate capital stock, and eventually lower output of the economy. Furthermore, the relative share of financial intermediation conducted through the shadow banking sector will increase as the capital requirement on retail banks increases, which in turn makes bank runs on the shadow banking sector more costly and therefore increases the severity of financial crises.

Three assumptions are crucial for the trade-off results above. First, there is no equity market between households and banks. As such, banks respond to a higher capital requirement by reducing assets and not expanding equity. Second, agents differ in their investment efficiencies. More precisely, households are the least efficient investors and shadow banks are the most efficient investors in the economy. Under a higher bank capital requirement, capital reallocates from the banking sector to the households. As a result, financial intermediation decreases and consequently the steady state capital stock shrinks. Third, banks can divert assets, which leads to an endogenous leverage constraint. Importantly, for shadow banks, assets funded through wholesale funding are more difficult to divert than those funded by deposits. Hence, wholesale funding allows for higher leverage, making it the preferred funding option for shadow banks. For retail banks, wholesale lending to shadow banks is more difficult to divert than lending to consumption goods producers. Thus, wholesale lending can be funded with a higher leverage. As a result, shadow banks borrow from retail banks in equilibrium. A higher cost of capital for retail banks is therefore passed through to shadow banks. In addition, in response to a higher capital requirement, retail banks reduce lending to consumption goods producers more than lending to shadow banks, which leads to the reallocation of banking activity from retail banks to shadow banks.

We solve the model non-linearly using global methods, since bank runs are catastrophic events that drive the economy far away from its steady state. To solve the model, we use a projection method and rely on a sparse grid algorithm developed by Judd, Maliar, Maliar, and Valero (2014). We calibrate our model to match stylized facts of the US economy and banking system, and the post-war financial crises in OECD countries (since financial crises are rare events in a single economy). The calibrated model generates financial crises of plausible magnitude and business cycle comovement between real and financial variables similar to the US data.

As a first numerical exercise, we compute the welfare cost of bank runs. We find that despite low risk aversion implied by log utility and that bank runs are only rare events, households are willing to pay 0.2 percent in permanent consumption equivalent units to avoid bank runs. Risk-neutral retail and shadow banks are willing to pay 1.1 and 8.5 percent in consumption equivalent units to avoid bank runs, respectively.

We then investigate the effects of static and dynamic capital requirements on financial stability and welfare. We find that increasing the bank capital requirement from 8 to 15 percent leads to a decrease in bank run frequency from 2.8 to 0.8 runs per 100 years. We highlight that this is only the case if the regulators relax bank capital requirements during a bank run. Otherwise, the frequency of bank runs would increase instead of decrease. Meanwhile, the steady state capital stock decreases by approximately 5 percent under the dynamic capital requirement policy above. Overall, based on our calibration, the steady state effect dominates. Higher retail bank capital requirements reduce welfare despite effectively eliminating banking crises.

1.1.1 Related Literature

This paper is closely related to two strands of literature. The first studies the role of financial frictions as a driving force of financial crises. Early studies in this line of literature include Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). In these papers, financial frictions are embedded in the financial structure of firms instead of financial intermediaries. After the 2007-2009 financial crisis, there is an emerging literature that links financial frictions in the banking sector to the outbreak of the worst global financial crisis since the Great Depression.[3] Gertler and Kiyotaki (2015) develop a canonical macroeconomic framework of financial crisis in the form of bank runs. Gertler, Kiyotaki, and Prestipino (2016) extend Gertler and Kiyotaki (2015) by subdividing the financial intermediation sector into a retail and a wholesale banking sector. In their model, aggregate capital stock is fixed and banks are unregulated. We build on their framework and modify it by adding capital accumulation and retail bank capital requirement to analyze the welfare and financial stabilization effect of bank capital regulation.

[3] See, for example, Gertler and Kiyotaki (2011), Gertler and Karadi (2011), Brunnermeier and Sannikov (2014), Boissay et al. (2016).

Recently, there is a growing literature on the role of shadow banking system in evaluating the bank regulation policy, such as Plantin (2015) and Huang (2014). The way they model shadow banking corresponds to the off-balance sheet shadow banking activities conducted by traditional banks. In contrast, we consider shadow banking to be an independent banking sector that conducts financial intermediation outside the regulatory framework of banks. Our notion of shadow banks corresponds to the external shadow banking sector in contrast to the off-balance sheet shadow banking activities carried out by traditional banks. We consider the internal shadow banking as a part of the traditional banking as bank capital regulations (such as Basel III) are, or at least supposed to be, implemented on a fully consolidated basis. In contrast to internal shadow banking, external shadow banking is a result of gains from specialization and vertical integration rather than a result of regulatory arbitrage as is the internal shadow banking (see, e.g., Adrian and Ashcraft, 2016).

Our modeling framework is most closely related to Gertler, Kiyotaki, and Prestipino (2017), where banking panics are introduced into an infinite horizon DSGE model. A key distinction is that we introduce endogenous capital accumulation, which allows us to study the steady state effects of bank capital regulation. Moreover, we discuss the motive (bank run and pecuniary externality) and welfare effects of bank capital regulation and compute the welfare cost of bank runs. Finally, we allow for multiple shocks, including productivity shocks, whereas uncertainty in their model arises exclusively through capital quality shocks.

Another paper that addresses a similar research question to this chapter is Begenau and Landvoigt (2017), which also studies retail bank capital requirements in an economy with an unregulated external shadow banking sector and endogenous capital accumulation. The key difference to their framework lies in the flow of funds in our economy. In our model, households have direct access to capital markets and there is a wholesale funding market that links the retail and the shadow banking sector. In their model, households hold both debt and equity of retail and shadow banks but have no access to the capital market, and there is no interbank market between the two banking sectors. Consequently, the spillover effects of regulating retail bank capital on shadow banks are small in their model. They also model bank runs, but the probability of a bank run is determined exogenously and independently of the liquidation losses.

Finally, regarding the effectiveness of regulation, the results of this chapter are related to Angeloni and Faia (2013), who study the effectiveness of dynamic capital requirements in the presence of bank runs. Our model setup differs in the following. First, we include shadow banks in our analysis. This gives us a spillover effect of regulation of retail banks on the shadow banking sector. Second, we consider runs on the interbank market as opposed to depositor runs. Financial instability in our model arises therefore for a different reason, and gives a different motive to regulate retail

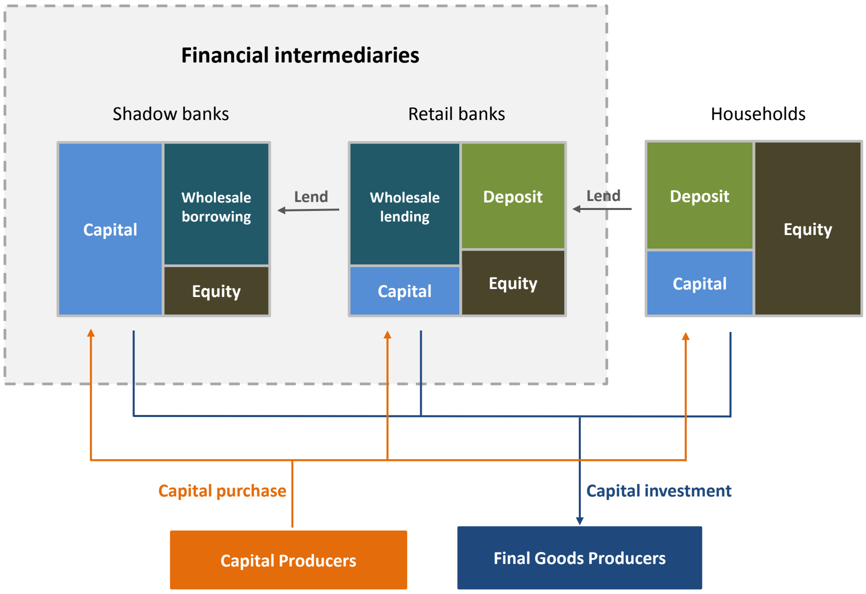


Fig. 1.1 Model overview

banks. Third, banks in our model invest into long-lived assets, which gives rise to self-fulfilling crises through changes in asset prices. Therefore, macroprudential policy can have additional effects through affecting these asset prices.

We proceed as follows. In section 1.2, we explain the model. The calibration strategy is outlined in section 1.3. We compare the model to the data in section 1.4. We discuss the main mechanism in section 1.5 and present the welfare cost of bank runs as well as the welfare effects of bank capital regulation in section 1.6. Finally, section 1.7 concludes.

1.2 Model

This section outlines a closed economy with an infinite horizon, which is populated with households, retail banks, shadow banks, capital producers and consumption goods producers. Households consume, deposit at banks, and make direct capital investments to consumption goods producers. Banks receive deposits from households, obtain funds from a wholesale funding market, and invest capital at consumption goods producers. During a bank run, the wholesale funding market breaks down. Figure 1.1 shows an overview of the flow of funds in the no-run equilibrium of the model.

1.2.1 Households

Households maximize utility from consumption. Their utility function is given by

$$\mathbb{E}_t\left[\sum_{s=t}^{\infty}\beta^{s-t}\ln(c_s^H)\right], \tag{1.2.1}$$

where β is the discount factor of the household and c_t^H denotes household consumption in period t. We follow the convention that lower case letters for variables denote variables of individual agents, while upper case letters denote aggregate variables.

Households consume, invest in capital k_{t+1}^H and make deposits d_{t+1}^H at banks. They supply labor inelastically and receive W_t in return. In addition, they own the capital producers and receive the profits Π_t^Q.[4] Deposits yield a safe gross return R_{t+1}^D in the subsequent period. Capital can be sold and purchased at price Q_t and yields an uncertain net return r_{t+1}^K in the subsequent period. Capital depreciates at rate δ. The remaining fraction of the capital stock in the next period is valued at the next period capital price, Q_{t+1}. The net worth of the household at the beginning of period t is given by

$$n_t^H = \left[r_t^K + (1-\delta)Q_t\right]k_t^H + R_t^D d_t^H + W_t + \Pi_t^Q. \tag{1.2.2}$$

In reality, households delegate credit supply to banks, because banks have a relative advantage of screening and monitoring non-financial firms.[5] To capture this advantage of banks in a simple way, we follow GKP and introduce a quadratic holding cost of new capital for households. This capital holding cost takes the form

$$\frac{\eta^H}{2}\left(\frac{k_{t+1}^H}{K_t}\right)^2 K_t.$$

Following GKP, we assume that retail banks purchase capital management services from specialized firms.[6] Retail banks pay a linear fee f_t^R to these firms for each unit of capital managed. The profit of these capital management firms is given by

$$f_t^R\tilde{K}_{t+1}^R - \frac{\eta^R}{2}\left(\frac{\tilde{K}_{t+1}^R}{K_t}\right)^2 K_t$$

[4] Profits of capita producers are zero in steady state, but may be positive or negative outside of the steady state due to a quadratic capital adjustment cost. Subsection 1.2.3 provides a detailed discussion of the profit of the capital producers.

[5] Diamond (1984) develops a model in which a monitoring advantage of banks arises through diversification.

[6] Some examples of such firms are appraisal management companies, which determine the value of a property, and credit bureaus, which determine the credit worthiness of a household.

where $\tilde{K}^R_{t+1}$ is the amount of capital managed and the second component captures the capital management cost. These firms are owned by the households. They operate in a competitive market, which means the equilibrium fee f^R_t is taken as given by households and retail banks, and is determined in equilibrium such that capital management firms are willing to manage the capital of the retail banks, i.e., $\tilde{K}^R_{t+1} = K^R_{t+1}$.[7]

The optimization problem of the household can be summarized as

$$\max_{\left\{c^H_t, k^H_{t+1}, d^H_{t+1}, \tilde{K}^R_{t+1}\right\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \ln\left(c^H_t\right) \right], \tag{1.2.3}$$

s.t.

$$c^H_t = n^H_t - Q_t k^H_{t+1} - d^H_{t+1} - \frac{\eta^H}{2}\left(\frac{k^H_{t+1}}{K_t}\right)^2 K_t + \left(f^R_t - \frac{\eta^R}{2}\frac{\tilde{K}^R_{t+1}}{K_t}\right)\tilde{K}^R_{t+1},$$

$$k^H_{t+1}, d^H_{t+1}, c^H_t \geq 0,$$

with n^H_t given by Equation (1.2.2).

1.2.2 Banks

There is a unit measure of both retail banks (R-banks) and shadow banks (S-banks) in the economy. J-banks, J $\in$ {R,S} can take deposits d^J_{t+1} from households and borrow or lend on the wholesale funding market b^J_{t+1}. In addition, they purchase capital k^J_{t+1}, which is invested into the production of consumption goods.[8]

Banks differ in their ability in investing at the consumption goods producers. In particular, retail banks pay a linear fee $f^R_t k^R_{t+1}$ for the capital management services provided by the screening firms owned by the households. There is no such fee and no capital holding cost for the shadow banks.[9]

Following GKP, we assume that banks of type J receive an exit shock each period with probability σ^J. In the case of such a shock, the banks liquidate their assets and

[7] The capital management firms assumption is important for technical reasons. It ensures that the decision problem of the retail banks is linear in their net worth. Therefore it is sufficient to characterize the decision problem of a representative retail bank.

[8] In practice, banks' lending to the non-financial sector is largely in the form of debt rather than equity. In the context of our model, banks' investment in the non-financial sector takes the form of equity investment rather than debt. This is a common assumption in the literature with financial intermediation for simplicity. Otherwise, another layer of liability of the non-financial sector has to be added.

[9] Adrian and Ashcraft (2016) discuss reasons for the existence of shadow bank credit intermediation in addition to retail bank credit intermediation. They argue that securitization allowed shadow banks to reduce informational frictions in credit markets, thereby offering loans to high-risk creditors which yield a superior return.

exit the economy. To keep the measure of banks constant over time, new banks with mass σ^J enter the economy with an exogenous endowment $\upsilon^J K_t / \sigma^J$.[10]

Both types of banks can divert a fraction of their assets after they have made their borrowing and lending decisions. How much they can divert depends both on the type of assets and the source of financing for the assets. Capital investment is easier to divert than wholesale lending, and assets financed through wholesale funding are harder to divert compare to those financed by deposits or equity.[11] In particular, a fraction ψ, $0 < \psi < 1$, of equity or deposit financed capital investment can be diverted. Only a fraction $\gamma\psi$, $0 < \gamma < 1$, of equity or deposit financed wholesale loans is divertible. And a fraction $\omega\psi$, $0 < \omega < 1$, of wholesale funding financed capital investment can be diverted. The parameter ω captures the monitoring intensity of the creditors of wholesale lending. Adrian and Ashcraft (2016) argue that due to deposit insurance, depositors have a lower incentive to monitor investments of the banks than wholesale lenders who lend against securitized assets. For the former, the implicit government guarantee is enough to ensure depositors that their lending is risk-free, whereas for the latter, the riskiness of their lending depends on the diversification of the borrowers. For $\gamma < 1$, the intuition lies in the higher standardization of wholesale lending compared to other lending activities. For example, the collateral underlying a repo contract (a typical wholesale lending instrument) is often a high quality government bond, whose market value is easy to verify for creditors. The collateral underlying a loan can be real estate, for which only a rough estimate of the market value exists. Hence, the potential for diversion is much higher for loans compared to wholesale lending.

If banks divert assets, they will not repay their liabilities. In this case, they will be forced to exit the economy by their creditors (depositors or wholesale fund lenders). Because diversion occurs at the end of the period before next period uncertainty realizes, an incentive constraint (IC) on the banks can ensure that diversion will never occur in equilibrium. This incentive constraint states that the benefit of diversion must be smaller or equal to the continuation value of the bank.

Figure 1.2 displays the timing of intra-period decisions of banks. The intra-period problem of a J-bank consists of three stages: survival, borrowing and investment decisions and diversion. After the productivity uncertainty of the consumption goods producers has realized, banks receive the exit shock. If they exit, they consume their net worth, otherwise they make their investment and borrowing decisions. New banks

[10] We scale the endowment of newly entering banks by the capital stock to ensure that the arguably stylized assumptions on entry do not affect the comparative statics through changes in the relative size of the endowment.

[11] Diversion entails the liquidation of the banks' assets and a subsequent default on creditors. One way to justify why equity financed assets cannot be diverted fully is that diversion creates a loss to the diverting bank which is equal to $1 - \psi$ times the diverted assets. Since banks utility is linear in consumption, such a cost may either be a pecuniary cost in the form of a penalty or a stigmatic utility cost.

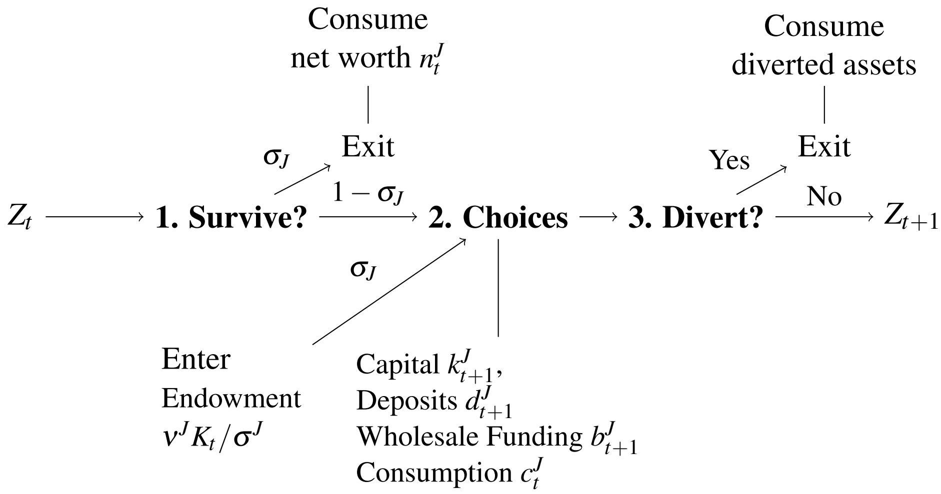


Fig. 1.2 Sequence of decisions for a bank of type J

enter the economy and also make these intertemporal decisions. Finally, after banks have decided how much to invest and how much to borrow, they can decide whether or not to divert their assets. If they divert, they consume the gain from diversion, default on their next period debt and exit the economy. Otherwise, they transition to the next period and the same procedure repeats.

Shadow Banks

If shadow banks do not exit, they consume c_t^S, borrow funds on the wholesale funding market b_{t+1}^S and invest in capital k_{t+1}^S. If they do exit, they consume their net worth n_t^S. The utility function of shadow banks is linear in consumption:

$$\mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \left\{ \left[\beta(1-\sigma^S) \right]^{s-t} \left[\sigma^S n_s^S + (1-\sigma^S) c_s^S \right] \right\} \right\}, \tag{1.2.4}$$

where β is the discount rate, σ^S is the exit probability, n^S is the net worth of the shadow bank and c^S is consumption in the case that the shadow bank does not exit. The net worth of an incumbent shadow bank in period t is given by

$$n_t^S = R_t^K k_t^S - R_t^B b_t^S. \tag{1.2.5}$$

A new shadow bank is endowed with an exogenous amount of resources when entering the economy, which equals their net worth in period t:

$$\tilde{n}_t^S = \frac{\upsilon^S K_t}{\sigma^S}. \tag{1.2.6}$$

The balance sheet constraint of shadow banks requires that assets equal liabilities plus equity:

$$\underbrace{Q_t k^S_{t+1}}_{\text{Asset}} = \underbrace{b^S_{t+1}}_{\text{Liability}} + \underbrace{n^S_t - c^S_t}_{\text{Equity}}. \tag{1.2.7}$$

Since shadow banks borrow exclusively from retail banks and lend only to consumption goods producers, their payoff from diversion is given by

$$\psi\left[(n^S_t - c^S_t) + \omega b^S_{t+1}\right].$$

The incentive constraint is given by

$$\psi\left[(n^S_t - c^S_t) + \omega b^S_{t+1}\right] \leq \beta \mathbb{E}_t\left[V^S_{t+1}\right], \tag{1.2.8}$$

where $\mathbb{E}_t\left[V^S_{t+1}\right]$ is the continuation value of the shadow bank defined below. This constraint states that the value from continuing to operate the shadow bank must be at least as high as the value of diverting assets, therefore ensures that asset diversion would never occur in equilibrium.

Continuing shadow banks reinvest their entire net worth, i.e., $c^S_t = 0$. This is an optimal choice, whenever

$$Q_t < \beta \mathbb{E}_t\left[\frac{V^S_{t+1}}{n^S_{t+1}} R^K_{t+1}\right].$$

This equation says that even if the shadow bank invests his entire net worth in capital, the benefit of investment still exceeds the cost of investment. We verify that this condition holds in our numerical solution.

The incentive constraint is always binding. In this case, the problem of a shadow bank reduces to:

$$V^S_t = \max_{\left\{k^S_{s+1}, b^S_{s+1}\right\}_{s=t}^{\infty}} \mathbb{E}_t\left[\sum_{s=t}^{\infty}\left[\beta(1-\sigma^S)\right]^{s-t}\sigma^S n^S_s\right], \tag{1.2.9}$$

s.t.

$$Q_t k^S_{t+1} = b^S_{t+1} + n^S_t, \qquad \text{(Balance Sheet Constraint)}$$

$$\psi\left(n^S_t + \omega b^S_{t+1}\right) \leq \beta \mathbb{E}_t\left[\sigma^S n^S_{t+1} + (1-\sigma^S)\psi\left(n^S_{t+1} + \omega b^S_{t+2}\right)\right], \quad \text{(IC)}$$

$$k^S_{t+1}, b^S_{t+1} \geq 0,$$

with net worth given by (1.2.5) for incumbent banks and by (1.2.6) for new shadow banks. Finally, we define leverage as the market value of assets over equity, i.e.,

$$\phi_t^S \equiv \frac{Q_t k_{t+1}^S}{n_t^S}.$$

Retail Banks

Retail banks consume c_t^R, invest in capital k_{t+1}^R, take deposits from households d_{t+1}^R, and lend to shadow banks on the wholesale funding market b_{t+1}^R. Their utility function is given by

$$\mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \left\{ \left[\beta(1-\sigma^R)\right]^{s-t} \left[\sigma^R n_s^R + (1-\sigma^R) c_s^R\right] \right\} \right\}. \tag{1.2.10}$$

In period t, incumbent retail banks obtain a gross return on capital, $R_t^K k_t^R$, and a gross return from lending to shadow banks, $R_t^B b_t^R$. They return $R_t^D d_t^R$ to households for their deposits. The retail bank's net worth in period t is given by

$$n_t^R = R_t^K k_t^R + R_t^B b_t^R - R_t^D d_t^R. \tag{1.2.11}$$

The net worth of newly entering retail banks given by

$$\tilde{n}_t^R = \frac{\upsilon^R K_t}{\sigma^R}. \tag{1.2.12}$$

The balance sheet of retail banks states that assets equal liabilities plus equity:

$$(Q_t + f_t^R) k_{t+1}^R + b_{t+1}^R = d_{t+1}^R + n_t^R - c_t^R. \tag{1.2.13}$$

We again focus on the case of zero consumption of continuing banks, which is optimal whenever

$$Q_t + f_t^R < \beta \mathbb{E}_t \left[\frac{V_{t+1}^R}{n_{t+1}^R} R_{t+1}^K \right].$$

Since retail banks lend on the wholesale funding markets and refinance themselves exclusively through deposits and net worth, their payoff from diversion is given by

$$\psi \left[(Q_t + f_t^R) k_{t+1}^R + \gamma b_{t+1}^R \right] \leq \beta \mathbb{E}_t \left[V_{t+1}^R \right]. \tag{1.2.14}$$

Further, define the leverage ratio of retail banks as

$$\phi_t^R \equiv \frac{(Q_t + f_t^R) k_{t+1}^R + \gamma b_{t+1}^R}{n_t^R}. \tag{1.2.15}$$

This leverage ratio excludes a fraction $(1-\gamma)b_{t+1}^R$ of wholesale loans, because this fraction is non-divertible and can therefore be completely financed with deposits. The fraction of equity financing used by retail banks is thus given by $\frac{1}{\phi_t^R}$. With this formulation, the incentive constraint for retail bank pins down the leverage ratio:

$$\psi \phi_t^R n_t^R \leq \beta \mathbb{E}_t \left[V_{t+1}^R \right]. \tag{1.2.16}$$

In this sense, the incentive constraint can be interpreted as a market imposed leverage constraint.

Capital Regulation

The regulator can impose a capital requirement on depository institutions, which stipulates that the bank's equity to asst ratio has to be above a certain threshold. Importantly, we assume that whether a bank is regulated depends on whether the bank accepts deposits or not and not on the type of bank per se. We make this assumption to avoid situations where households will shift deposits from regulated retail banks to unregulated shadow banks. This assumption can be justified, because the ability to issue deposits requires participation in a deposit insurance scheme, like the FDIC in the US, which is usually attached to stringent oversight requirements. In equilibrium, only retail banks accept household deposits and are thus regulated. We assume that the regulator weighs assets in the same way as the market. That is, a fraction $(1-\gamma)$ of wholesale loans can be financed completely with deposits. Accordingly, the capital requirement can be formulated as

$$\frac{1}{\phi_t^R} \geq \frac{1}{\bar{\phi}_t}, \tag{1.2.17}$$

where $\frac{1}{\bar{\phi}_t}$ is the minimum capital ratio that the regulator allows.

Recall that there are two classes of assets on the retail bank's balance sheet: retail bank's capital holding, $\left(Q_t + f_t^R\right) k_{t+1}^R$, and wholesale lending, b_{t+1}^R. The interpretation of (1.2.17) is that the retail bank can finance at most $1 - \frac{1}{\bar{\phi}_t}$ share of its capital holding by households' deposit and at most $1 - \frac{\gamma}{\bar{\phi}_t}$ share of its wholesale lending by households' deposit. In other words, with $\gamma < 1$, at least a share $\frac{1}{\bar{\phi}_t}$ of capital holding has to be financed by retail banks' own equity, but only a share $\frac{\gamma}{\bar{\phi}_t}$ of wholesale lending has to be financed by equity.

The problem of the retail bank in the presence of a regulatory capital requirement can be summarized as:

$$V_t^R = \max_{\left\{k_{s+1}^R, b_{s+1}^R, d_{s+1}^R\right\}_{s=t}^{\infty}} \mathbb{E}_t \left\{ \sigma^R \sum_{s=t}^{\infty} \left[\beta(1-\sigma^R)\right]^{s-t} n_s^R \right\}, \tag{1.2.18}$$

s.t.

$$(Q_t + f_t^R)k_{t+1}^R + b_{t+1}^R = d_{t+1}^R + n_t^R, \qquad \text{(Balance Sheet Constraint)}$$

$$(Q_t + f_t^R)k_{t+1}^R + \gamma b_{t+1}^R \leq \bar{\phi}_t n_t^R, \qquad \text{(Regulatory Capital Requirement)}$$

$$\psi\left[(Q_t + f_t^R)k_{t+1}^R + \gamma b_{t+1}^R\right] \leq \beta \mathbb{E}_t \left\{\sigma^R n_{t+1}^R + (1-\sigma^R)\psi\left[(Q_{t+1} + f_{t+1}^R)k_{t+2}^R + \gamma b_{t+2}^R\right]\right\}, \quad \text{(IC)}$$

$$k_{t+1}^R, d_{t+1}^R, b_{t+1}^R \geq 0,$$

with n_t^R given by Equation (1.2.11) for incumbent retail banks and Equation (1.2.12) for new retail banks. We refer to the economy in which the regulatory bank capital requirement is so low that retail bank leverage is always determined by the incentive constraint of the retail banks as the baseline economy.

1.2.3 Production

Consumption Goods Producers

The consumption goods producers hire labor from households and borrow capital from households, retail banks and shadow banks to produce consumption goods, using a Cobb-Douglas production technology:

$$Y_t = Z_t F(K_t, L_t) = Z_t K_t^{\alpha} L_t^{1-\alpha}. \tag{1.2.19}$$

The price of the consumption goods is normalized to one. Productivity Z_t follows an AR(1) process:

$$\ln(Z_t) = (1-\rho^Z)\mu^Z + \rho^Z \ln(Z_{t-1}) + \varepsilon_t, \tag{1.2.20}$$

where $|\rho^Z| < 1$ and $\varepsilon_t \sim N(0, \sigma^Z)$.

The consumption goods producers take the prices of the production input and output as given and make zero profits.[12] They maximize profits taking the aggregate wage W_t and the rental rate of capital r_t^K as given:

$$\max_{L_t, K_t} \Pi_t = Z_t K_t^{\alpha} L_t^{1-\alpha} - W_t L_t - r_t^K K_t.$$

[12]Since the consumption goods producers make zero profits, it does not matter who owns the them.

The first order conditions of the consumption goods producers' problem determine the wage and rental rate of capital in equilibrium:

$$W_t = (1-\alpha) Z_t K_t^{\alpha} L_t^{-\alpha}, \tag{1.2.21}$$

$$r_t^K = \alpha Z_t K_t^{\alpha-1} L_t^{1-\alpha}. \tag{1.2.22}$$

Capital Producers

Capital producers use a technology that transforms one unit of consumption goods into one unit of capital:

$$Y_t^K = F^K(I_t) = I_t, \tag{1.2.23}$$

where Y_t^K is the amount of capital produced in period t and I_t is the amount of consumption goods used for the production. Adjustment to the production of capital is costly. In particular, the capital adjustment cost takes the form

$$\frac{\theta}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t, \tag{1.2.24}$$

where δ is the depreciation rate of capital. This form of capital adjustment cost implies that whenever the investment rate differs from the depreciation rate, a positive proportional adjustment cost is incurred. Therefore, the relative price of capital is endogenous. The adjustment cost is scaled by the aggregate capital stock K_t, which the capital producers take as given.

As the capital adjustment cost depends on the capital stock from last period, the constant return to scale does not necessarily hold. Therefore the capital producers may earn a non-zero profit. We assume that the capital producers are owned by the households and any profits or losses are transferred to the households each period.

The capital producers' problem can be summarized as:

$$\max_{I_t} \Pi_t^Q = Q_t I_t - I_t - \frac{\theta}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t. \tag{1.2.25}$$

The first order condition of the capital producers' problem yields the following expression for the capital price:

$$Q_t = 1 + \theta\left(\frac{I_t}{K_t} - \delta\right). \tag{1.2.26}$$

1.2.4 Aggregation and Equilibrium

Aggregation

The aggregate net worth of the retail and shadow banking sector is given by the sum of the net worth of incumbent and newly entering banks:

$$
\begin{aligned}
N_t^R &= \left(R_t^K K_t^R + R_t^B B_t - R_t^D D_t\right)(1-\sigma^R) + \upsilon^R K_t, \\
N_t^S &= \left(R_t^K K_t^S - R_t^B B_t\right)(1-\sigma^S) + \upsilon^S K_t.
\end{aligned}
$$

Aggregate output is given by production net of the capital holding costs:

$$
Y_t = Z_t K_t^\alpha + \upsilon^R K_t + \upsilon^S K_t - \frac{\eta^H}{2}\left(\frac{K_{t+1}^H}{K_t}\right)^2 K_t - \frac{\eta^R}{2}\left(\frac{K_{t+1}^R}{K_t}\right)^2 K_t. \tag{1.2.27}
$$

Define I_t as net investment excluding capital adjustment costs, that is

$$
I_t = K_{t+1} - (1-\delta)K_t.
$$

We define aggregate gross investment $\tilde{I}_t$ as the total expenditure necessary to change the capital stock from K_t to K_{t+1}. Therefore, our measure of aggregate investment includes the capital adjustment costs.[13] The gross investment is given by

$$
\tilde{I}_t = I_t + \frac{\theta}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t. \tag{1.2.28}
$$

Since there is a representative household, the individual consumption and aggregate consumption are equal, $c_t^H = C_t^H$. Household consumption can be inferred from the aggregate resource constraint:

$$
C_t^H = Y_t - \tilde{I}_t - \sigma^R \frac{N_t^R - \upsilon^R K_t}{1-\sigma^R} - \sigma^S \frac{N_t^S - \upsilon^S K_t}{1-\sigma^S} \tag{1.2.29}
$$

No-Run Equilibrium

In the absence of bank runs, we use a standard sequential equilibrium definition. Crucially, in the no-run equilibrium, each retail bank expects all other retail banks to roll over shadow bank debt, such that a bank run will never arise in this equilibrium. Taking the bank capital regulation policy $\bar{\phi}_t$ as given, the no-run equilibrium is a sequence of prices

$$
\left\{Q_t, r_t^K, R_t^D, R_t^B, W_t, f_t^R\right\}_{t=0}^\infty
$$

[13] A similar distinction between gross investment and net investment is for example used in Christiano et al. (2005).

and allocations for

- households, $\{C_t^H, K_{t+1}^H, D_{t+1}^H\}_{t=0}^{\infty}$,
- retail banks, $\{C_t^R, K_{t+1}^R, D_{t+1}^R, B_{t+1}^R\}_{t=0}^{\infty}$,
- shadow banks, $\{C_t^S, K_{t+1}^S, B_{t+1}^S\}_{t=0}^{\infty}$,
- consumption goods producers, $\{K_t, L_t\}_{t=0}^{\infty}$, and
- capital producers, $\{I_t\}_{t=0}^{\infty}$,

that solve the respective optimization problems of all agents as defined above, clear the markets for

- capital $K_t = K_t^H + K_t^R + K_t^S$,
- labor $L_t = 1$,
- investment goods $I_t = K_{t+1}^H + K_{t+1}^R + K_{t+1}^S - (1-\delta)K_t$,
- deposits $D_{t+1}^R = D_{t+1}^H$,
- wholesale funding $B_{t+1}^S = B_{t+1}^R$,
- capital management services $K_{t+1}^R = \tilde{K}_{t+1}^R$,

and satisfy the aggregate resource constraint (1.2.29). In a no-run equilibrium, the bank run condition (1.2.31), which is discussed in detail in the next subsection, does not hold.

Shadow Bank Run Equilibrium

As in the model of GKP, retail banks can run on shadow banks. We consider only runs on the shadow banking sector as a whole. If such a run happens, the assets of the shadow banks are liquidated at the liquidation price Q_t^* and returned to the retail banks. Incumbent shadow banks exit once their assets are liquidated. Define x_t as the recovery rate for the retail banks through shadow bank liquidation:

$$x_t = \xi_t \frac{\left[r_t^K + (1-\delta)Q_t^*\right] K_t^S}{(1+r_t^B)B_t}, \tag{1.2.30}$$

where ξ_t is a liquidation cost shock following an iid log normal distribution with mean 0 and variance σ^{ξ}. The liquidation cost shock helps to quantitatively pin down the frequency of bank runs. One interpretation for this liquidation cost shock is that it represents a market illiquidity discount in the collateral market due to search frictions.[14] If liquidity is low, retail banks incur an additional loss on the recovery value.

Runs are persistent and continue into the next period with probability π. New shadow banks start reentering the economy at rate σ^S only once the run has ended.

Bank runs can be self-fulfilling. In that case, the market price of capital deteriorates in anticipation of a bank run. This weakens balance sheets of shadow banks so much that they cannot repay their liabilities. As a consequence, it is optimal for the retail banks to run on shadow banks. However, it will only occur if the assets of the shadow banks, valued at the liquidation price of capital, are insufficient to cover the liabilities of shadow banks, that is, if

$$x_t < 1. \tag{1.2.31}$$

We assume that once this condition is fulfilled, a bank run will be triggered. The liquidation condition in (1.2.31) states that, if a bank run happened, i.e., shadow banks' assets got liquidated, the retail banks would suffer from a loss on their wholesale lending. But if all retail banks coordinate to not run, then even if the bank run condition holds, bank runs would never take place. Our assumption eliminates this possibility by implying that retail banks can never successfully coordinate. Hence in our model, bank runs do not arise randomly as a consequence of sunspot shocks, but are closely tied to the fundamentals of the shadow banking sector. Gorton (1988) presents evidence that historically, bank runs in the United States were indeed related to an increased fundamental riskiness of deposits, that is, during times when expected losses on deposits were high. Further, the large number of retail banks in an economy and the high competition in the retail banking business reduces the ability of banks to coordinate (especially with the absence of a credible lender of last resort Rochet and Vives (2004)).

Since the recovery rate is strictly increasing in ξ_t, the probability of a bank run happening in $t+1$ can be written as a state-contingent cutoff $\bar{\xi}_t$, which is defined by

$$\bar{\xi}_t = \frac{(1+r_t^B)B_t}{\left[r_t^K + (1-\delta)Q_t^*\right] K_t^S}.$$

The probability of a bank run in t conditional on Z_t is given by

$$p_t \equiv F(\bar{\xi}_t).$$

This probability is endogenous. It depends, in particular, on the liquidation price of capital during a bank run. A lower liquidation price of capital makes a bank run

[14]For a microfoundation for time-varying liquidity discounts in collateral markets, see for example He and Milbradt (2014). In their model, collateral of defaulted bonds is sold on a search market as in Duffie et al. (2005). Liquidity is determined endogenously by the default decision of the bond issuer.

both more likely and more severe. While our liquidation shock assumption is slightly different from the sunspot shock which determines the bank run probability in GKP, it also leads to a bank run probability that is decreasing in x_t.

1.3 Calibration

In this section, we outline our calibration strategy and provide some evidence for the model fit by comparing untargeted moments from the model to the data. Of particular interest is the ability of the model to produce financial crises that are similar to financial crises observed in developed economies. We also check whether the model can generate realistic business cycle dynamics for real and financial variables.

1.3.1 Calibration Strategy

We calibrate the model using US data between 1990 and 2007 for the model economy. Since financial crises are relatively rare events and there are not enough observations for a single economy, we use financial crises data of OECD countries after WWII for the calibration of bank run parameters. The length of one period in the model is a quarter. We divide all parameters into three groups. The first group of parameters are taken from the literature. The second group are set to match steady state properties of the model to the data. The third group are calibrated to match dynamic properties of the model.

Parameters in Panel (a) of Table 1.1 are set following the literature. The capital share of consumption goods production and quarterly depreciation rate of capital are set to be 0.36 and 0.025, respectively. The banks endowments υ^R and υ^S to yield a planning horizon of shadow banks of about two years and retail banks of about five years, similar to Gertler et al. (2016).[15] These are the same targets for the banks endowment as in Gertler and Karadi (2011). We set the discount rate of households β to target at an annual steady state return on deposits of 4 percent.

We set the adjustment cost parameter θ to match an elasticity of the capital price to the investment-to-capital ratio of 0.25, which is the target of Bernanke et al. (1999). This implies a parameter of $\theta = 10$.[16] There is considerable variation in the choice of this parameter in the literature. Christiano and Fisher (2003) estimate an elasticity of 0.76, targeting asset price comovement with real GDP. Gertler et al. (2007) target an elasticity of the capital price to the investment-capital ratio of 2, which would in

[15] The planning horizons are simply $1/\sigma^R$ and $1/\sigma^S$.

[16] The elasticity of the capital price to investment is given by $\frac{\partial Q_t}{\partial I_t}\frac{I_t}{Q_t} = \theta \frac{1}{K_t}\frac{I_t}{1+\theta\left(\frac{I_t}{K_t}-\delta\right)}$. Evaluated in Steady State, this expression becomes $\theta\delta$.

(a) Parameters from the literature				
α	δ	υ^R, υ^S	β	θ
0.36	0.025	0.001	0.9902	10
			$R^D = 1.04$	$\frac{\partial \ln(Q_t)}{\partial \ln(I_t)} = 0.25$

(b) Parameters set to match steady state properties				
γ	η^H	η^R	σ^R	σ^S
0.6676 $R^{K,R} - R^B = 0.4\%$	0.0286 $R^K - R^{K,R} = 1.2\%$	0.0071 $R^{K,R} - R^D = 1.2\%$	0.0521 $K^R/K = 0.4$	0.1273 $K^S/K = 0.4$
ψ	ω			
0.2154 $\phi^R = 10$	0.5130 $\phi^S = 20$			

(c) Parameters set to match dynamic properties				
ρ^Z	σ^Z	σ^ξ	π	
0.9 $\rho(Y_t, Y_{t-1}) = 0.9$	0.01 $\sigma(Y_t) = 0.025$	0.0188 Crisis freq. of 0.68% per quarter	12/13 Runs last 3.25 yrs on avg	

Table 1.1 Calibration

our model correspond to adjustment cost parameters of 30 and 80, respectively. More recent papers in the financial accelerator literature often use investment adjustment costs, which is different from capital adjustment cost. We find this infeasible, since it would introduce another endogenous state variable, which would complicate the numerical solution of our model substantially. However, since these models typically target the steady state elasticity of the price of capital with respect to investment, we can compare their choice to our target. For example, Gertler and Kiyotaki (2011) target an elasticity of investment to the price of capital of 1.5, which would correspond to a parameter value of 60. The business cycle literature generally uses much lower parameter values for the quadratic capital adjustment costs. Begenau and Landvoigt (2017) use a parameters value of 2, Christensen and Dib (2008) of 0.5. Basu and Bundick (2017) also estimate a value of around 2 for their quadratic capital adjustment cost. By using a value of 10, we use a high value relative to the business cycle literature, but a very conservative value relative to the financial accelerator literature.

Parameters in Panel (b) of Table 1.1 are set to match steady state properties of the economy. We use the same targets for these parameters as GKP. To find data equivalents for the steady state values, GKP assume that the US economy was in steady state in the years before the financial crisis of 2007-2009. We use leverage ratios of 10 and 20 for retail banks and shadow banks, respectively, to calibrate the diversion parameters ψ and ω. We choose the remaining diversion parameter γ to match an average annualized spread between the return on retail lending and the return on wholesale funding of 0.4 percent. We set the exit shock probabilities σ^R and σ^S such that the shares of assets intermediated by retail banks and shadow banks in steady state both equal to 40 percent. These values correspond to the respective share of intermediated assets in the data between 2003 and 2007.[17] For the capital holding cost parameters η^H and η^R, we target at the spread between the return on shadow bank lending and retail bank lending and the spread between the return on retail bank lending and the deposit rate in annualized terms, respectively.

Parameters in Panel (c) of Table 1.1 are calibrated to match dynamic properties of the model. We choose ρ^Z and σ^Z to match the conditional volatility and the autocorrelation of detrended GDP for the United States. Two key parameters for the welfare cost of bank runs are the volatility of the liquidation cost shock σ^ξ and the persistence of the run π. We choose the persistence of financial crises such that the average length of a financial crisis is 3.25 years. We calibrate the volatility of the liquidation cost σ^ξ to match an annual frequency of bank runs of 2.7 percent, or one bank run every 36.76 years.

We use data of historical banking crises between 1970-2011 from Table A1 in Laeven and Valencia (2012), where the authors provide a comprehensive database on systemic banking crises during 1970-2011. They classify a time period as a systemic financial crisis if it exhibits *"[s]ignificant signs of financial distress in the banking system (as indicated by significant bank runs, losses in the banking system, and/or bank liquidations)."* and *"[s]ignificant banking policy intervention measures in response to significant losses in the banking system."* (Laeven and Valencia (2012), p. 228) This definition corresponds well to a crisis in our model, which is characterized by a shadow bank run, liquidation of the shadow banking sector and losses on both capital holdings and wholesale lending for the retail banking sector.

We calculate the frequency of a financial crises per country per quarter by dividing the number of financial crises which occurred during the period 1970-2011 in the OECD countries by the number of countries (i.e., 35) and the length of the period in quarters. We sum up the length of all financial crises which happened in the OECD countries during this time and divide the number by the number of financial crises to get the average length of financial crises.[18]

[17] According to GKP, assets intermediated by retail banks comprise equity of non-financial firms, bonds, commercial paper, household and non-financial firm loans, mortgages and consumer credit. For shadow banks, intermediated assets comprise equity of non-financial firms, mortgages and consumer credit.

[18] The banking crises started in 2008 in many countries do not have specific ending date in this table. In this case we set a uniform ending date of 2012.

	Schularick and Taylor (2012)			Model		
	0 yr	0-1yr	0-2yr	0 yr	0-1yr	0-2yr
Real GDP	-2.02%	-4.46%	-6.3%	-1.89%	-1.46%	-1.16%
Real Investment	-3.45%	-12.45%	-19.9%	-8.32%	-6.15%	-4.69%
Bank Assets	-1.89%	-6.98%	-7.7%	-3.69%	-2.89%	-2.21%

Table 1.2 Untargeted financial crisis moments

1.3.2 Numerical Solution and Simulation Procedure

We solve the model nonlinearly with a projection method on a Smolyak grid, using the toolbox of Judd et al. (2014). Details of the solution algorithm can be found in Appendix A.4.

The model has three shocks: the productivity shock ε_t, the liquidation cost shock ξ_t, and the re-entry-shock π_t. We simulate $N = 5000$ economies for $T = 1000$ periods. The first 200 periods are dropped to eliminate the effects of initial conditions. One issue in simulating the model is that the net worth N_t^R and N_t^S and the price of capital Q_t are simultaneously determined by a nonlinear equation. Therefore, we guess an initial path for $\left\{Q_t^d\right\}_{t=1}^T$, use this path to update the optimal policies and compute $\left\{N_t^{R,d+1}\right\}_{t=1}^T$ and $\left\{N_t^{S,d+1}\right\}_{t=1}^T$, and use these new sequences of net worth to compute the updated sequence $\left\{Q_t^{d+1}\right\}_{t=1}^T$. We iterate on the simulation until the distance between $\left\{Q_t^d\right\}_{t=1}^T$ and $\left\{Q_t^{d+1}\right\}_{t=1}^T$ becomes sufficiently small.

1.4 Untargeted Moments

1.4.1 Financial Crises

In this section, we show that the model is able to generate financial crises which are quantitatively similar to the crises we observed in the data. Schularick and Taylor (2012) use an event study approach to measure the cumulative change in real and financial aggregate variables relative to the pre-crisis trend. Their empirical definition of a financial crisis follows Laeven and Valencia (2012). The dataset they use covers 14 economies spanning the years 1870 to 2008. For the comparison, we use their post-WWII results, which is comparable with our calibrated model using the post-WWII US and OECD data.

Table 1.2 reports the results. Using the same data and method as in Schularick and Taylor (2012), we calculate the average cumulative percentage change in real GDP, real investment, and bank assets after a systemic banking crisis.[19] We distinguish three time intervals after the start of the crisis: the first year, the first two years, and the first three years after the breakout of the crisis. Our measure of bank assets is given by $(Q_t + f_t^R)K_{t+1}^R + Q_t K_{t+1}^S$. Real GDP is Y_t as defined in Equation (1.2.27) and real investment is $\tilde{I}_t$ as defined in Equation (1.2.28). For instance, in the case of the first three years after the systemic crisis, given that a bank run starts in period t, we compute the log change in the variables of interest between period t and period $t+11$. Using the same empirical method as in Schularick and Taylor (2012), we estimate the average change of the real and financial variables for each period between t and $t+11$ by the panel regression:

$$d.log(X_{i,t}) = \alpha_i + \sum_{s=0}^{11} \beta_{s+1} run_{i,t-s} + \varepsilon_{i,t}$$

where $X_{i,t} \in \left\{ Y_{i,t}, \tilde{I}_{i,t}, (Q_{i,t} + f_{i,t}^R)K_{i,t+1}^R + Q_{i,t}K_{i,t+1}^S \right\}$, $run_{i,t-s}$ is a dummy variable which takes the value of 1 if a run happens in period $t-s$, α_i is the unconditional growth rate of X_i in country i, and β_{s+1} is the growth rate s periods after the start of a financial crisis. We then add up the coefficients of the dummy variables to get the cumulative effect of banking crises on the variables of interest 0-2 years after the crises happened.

As shown in Table 1.2, the immediate effect of systemic banking crises on real GDP from the model matches quite closely with that in the data. In general, our model economy reacts stronger in the first year after the crisis and recovers faster afterwards from the recession compared to the data, in which a banking crisis has a more persistent negative effect on the economy.

1.4.2 Business Cycle Statistics

We compare the simulated business cycle moments of the model with the business cycles in the US to lend additional confidence to the ability of the model to account for fluctuations. For this purpose, we use data from the NIPA and the Flow of Funds between 1986Q1 and 2010Q4. For the wholesale funding rate we only have data from 2001Q1 onwards. We stop in 2010, because afterwards there has been a secular decline in wholesale funding of the shadow banking sector. We describe the data in Appendix A.1. A notable deviation from the business cycle literature is that instead of

[19] In Table 2 in Schularick and Taylor (2012), the authors report the cumulative percentage change 0-5 years after the start of the banking crises. According to our model calibration however, a systemic bank run lasts on average 3 years. Therefore we recalculate the Table 2 results for the 0-2 (instead of 0-5) year cumulative effects of banking crises using their data and method to make it comparable to the simulation results of our model. We consolidate both banking sectors for the comparison.

	$\sigma(X)/\sigma(Y)$		$\rho(X,Y)$		$\rho(X_t,X_{t-1})$	
	(1) Data	(2) Model	(3) Data	(4) Model	(5) Data	(6) Model
Output (Y_t)	0.027	0.029	1.000	1.000	0.898	0.931
Consumption (C_t^H)	0.929	0.527	0.916	0.864	0.887	0.961
Investment ($\tilde{I}_t$)	4.368	1.632	0.943	0.966	0.886	0.905
Deposits (D_{t+1})	2.449	1.657	0.759	0.895	0.891	0.946
Wholesale Lending (B_{t+1})	10.379	9.606	0.187	0.625	0.853	0.963
Deposit Rate (R_{t+1}^D)	0.648	0.104	0.431	-0.494	0.900	0.898
Wholesale Funding Rate (R_{t+1}^B)	0.532	0.136	0.249	-0.665	0.599	0.889
Return on Equity (R_t^K/Q_{t-1})	6.385	1.089	0.379	0.184	0.829	0.445

Table 1.3 Untargeted business cycle moments

the Hodrick-Prescott (HP) filter, we detrend the data using the routine proposed by Hamilton (2017), which avoids the spurious correlations that can arise with HP-filtered time series.

Columns (1) and (2) of Table 1.3 report unconditional standard deviations of variables relative to output for the data and the model.[20] We calibrate the model such that output volatility roughly matches that in the data. Consumption is less volatile than output both in the model and the data. Investment is more volatile. The volatility of deposits and the volatility of wholesale lending match the data quite well. In terms of interest rates, the volatility of the deposit interest rate, the wholesale funding rate and the return on equity is a bit low relative to the data, which is not surprising given that we use a simple log utility function.

Columns (3) and (4) report the contemporaneous correlations of all variables with output. As in the data, consumption, investment and deposits are strongly pro-cyclical. The most problematic statistic is the correlation between wholesale lending and output, which is positive in the model, but only weakly positive in the data. This seems puzzling, since one of the stylized facts of the financial crises is a contraction in wholesale lending. There is a clear trend break in the data around the year 2002 for wholesale lending, which is not properly picked up by either the HP-filter or the Hamilton filter. This trend break may be responsible for our counter-intuitive observation of acyclical wholesale lending. The deposit rate is fairly acyclical both in the model and in the data, and the return on equity is weakly pro-cyclical.

Finally, columns (5) and (6) show that the model can roughly match the autocorrelations in the data, with the exception of the return on capital being too weakly autocorrelated in the model.

[20] In the first row we report the standard deviation of output instead.

1.5 Discussion

In this section, we discuss how a regulator can improve financial stability through retail bank capital regulation. We will also discuss the steady state effects of bank capital regulation, in which capital requirements reduce financial intermediation. This leads to the trade-off between the frequency of bank runs and financial intermediation. We determine the quantitative importance of this trade-off in section 1.6.

1.5.1 How Can Bank Capital Regulation Increase Welfare?

There are two key inefficiencies in the model that retail bank capital requirements can address. First, retail banks do not internalize the effect of their leverage and asset allocation decisions on the probability of bank runs. We call this the run externality. Second, there is a feedback loop between the incentive constraints of the banks and the price of capital, which is also not internalized by the retail banks. This feedback loop increases the frequency and severity of both bank runs and business cycle fluctuations. We call this the capital price externality. Bank capital requirements can address these inefficiencies in two ways. First, bank capital requirements force retail banks to reduce leverage. Since they cannot issue equity to households, they will instead intermediate less funds to the shadow banking sector. Second, if retail banks build up higher capital buffers during normal times, they can absorb the liquidated assets of the shadow banking sector during a bank run more easily and therefore stabilize the liquidation price of capital. We will discuss in more detail about both channels in the following subsections.

The Run Externality

It might appear to be odd at first glance that retail banks do not internalize the impact of their leverage choice on bank runs. After all, it is the retail banks who initiate the run. The point here is, we consider only systemic bank runs, in which the whole retail banking sector runs on the whole shadow banking sector. The probability of such a systemic bank run depends only on aggregate equilibrium prices and quantities and not on bank-specific variables. Hence, from the perspective of an individual retail bank, the probability of bank run is exogenous. As we show below, while retail banks charge a premium on wholesale lending for the expected loss in a bank run, they do not internalize that by increasing leverage and thereby increasing lending to the shadow banking sector, they increase the probability of a bank run in the next period.

The equilibrium choices k^R_{t+1}, b^R_{t+1} and d^R_{t+1} of the retail banks are determined by either the incentive constraint (1.2.14) or the capital requirement (1.2.17), the balance sheet constraint (1.2.13) and one first order condition. Define $\Omega^R_{t+1} = V^R_{t+1}/n^R_{t+1}$. This expression corresponds to the value of an additional unit of net worth. Substituting the incentive constraint and the balance sheet constraint into Equation (1.2.18) for k^R_{t+1} and d^R_{t+1} and differentiating with respect to b^R_{t+1}, we get that

$$\mathbb{E}_t\left[(1-p_{t+1})\,\Omega^R_{t+1}\left(\frac{R^K_{t+1}}{Q_t+f^R_t}-R^D_{t+1}\right)+\int_0^{\bar{\xi}_{t+1}}\Omega^{R*}_{t+1}\left(\frac{R^{K*}_{t+1}}{Q_t+f^R_t}-R^D_{t+1}\right)dF(\xi)\right]$$
$$=\frac{1}{\gamma}\mathbb{E}_t\left[(1-p_{t+1})\,\Omega^R_{t+1}(R^B_{t+1}-R^D_{t+1})+\int_0^{\bar{\xi}_{t+1}}\Omega^{R*}_{t+1}(x_{t+1}R^B_{t+1}-R^D_{t+1})dF(\xi)\right]. \tag{1.5.1}$$

We derive this condition in Appendix A.3.2. This condition essentially states that retail banks must be indifferent between lending on the wholesale market and holding capital. The return on wholesale lending is lower than the return on capital holdings, because the retail bank can use more leverage to finance wholesale lending, i.e., since $\gamma<1$. Both the ex-post return on capital holdings and the ex-post return on wholesale lending are lower if a run occurs in the next period. The return on capital holdings is lower, because those capital holdings are valued at the liquidation price if a bank run occurs, since $R^K_{t+1}=r^K_{t+1}+(1-\delta)Q_{t+1}>r^K_{t+1}+(1-\delta)Q^*_{t+1}=R^{K*}_{t+1}$. The return on wholesale lending is lower, because retail banks will not recover the full amount when a bank run occurs, since $x_{t+1}<1$ by definition of the bank run equilibrium. Rearranging the first order condition (1.5.1) yields the following expression for the ex-ante (safe) return on retail bank lending:

$$\begin{aligned}R^B_{t+1}=\gamma\mathbb{E}_t&\left[(1-p_{t+1})\,\Omega^R_{t+1}\frac{R^K_{t+1}}{Q_t+f^R_t}+\int_0^{\bar{\xi}_{t+1}}\Omega^{R*}_{t+1}\frac{R^{K*}_{t+1}}{Q_t+f^R_t}dF(\xi)\right]\\&+(1-\gamma)R^D_{t+1}\mathbb{E}_t\left[(1-p_{t+1})\,\Omega^R_{t+1}+\int_0^{\bar{\xi}_{t+1}}\Omega^{R*}_{t+1}dF(\xi)\right]\\&\times\left(\mathbb{E}_t\left[(1-p_{t+1})\,\Omega^R_{t+1}+\int_0^{\bar{\xi}_{t+1}}\Omega^{R*}_{t+1}x_{t+1}dF(\xi)\right]\right)^{-1}.\end{aligned} \tag{1.5.2}$$

Consider first the case in which bank runs are unanticipated, i.e., $\mathbb{E}_t\left[p_{t+1}\right]=0$. In this case, the ex-post return on wholesale lending is safe. Then, Equation (1.5.2) reduces to

$$R^B_{t+1}=\gamma\frac{\mathbb{E}_t\left[\Omega^R_{t+1}\frac{R^K_{t+1}}{Q_t+f^R_t}\right]}{\mathbb{E}_t\left[\Omega^R_{t+1}\right]}+(1-\gamma)R^D_{t+1}.$$

The fraction γ of wholesale lending is financed in the same way as capital holdings and must yield a return equal to the expected return on capital. The fraction $1-\gamma$ of wholesale lending is non-divertible and can therefore be fully financed with deposits.

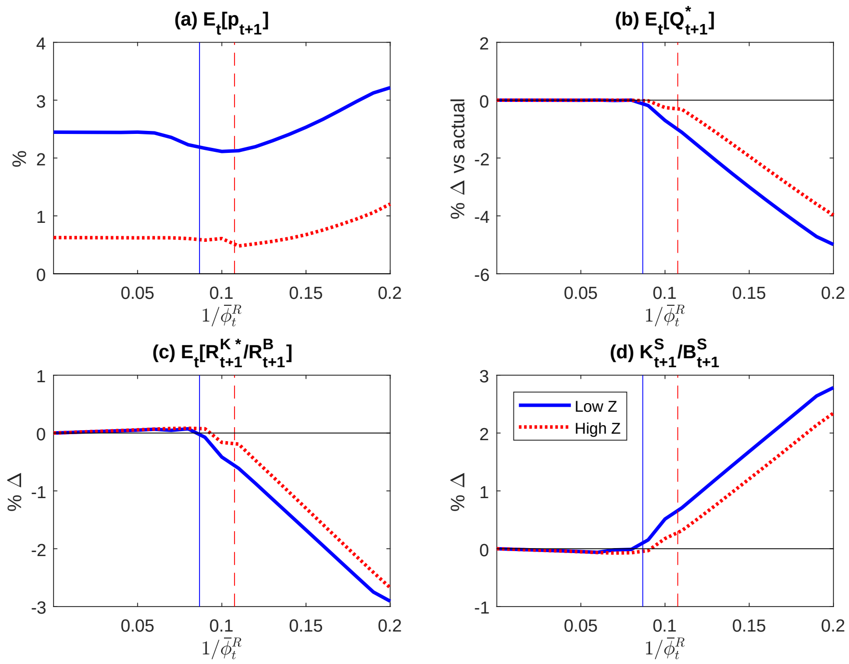


Fig. 1.3 General equilibrium effects of a constant retail bank capital requirement on the bank run probability

Since retail banks are competitive, this fraction must therefore yield a return which corresponds to the return on deposits. The full Equation (1.5.2) essentially includes an additional compensation for the loss in default in the denominator plus an adjustment to the marginal value of an additional unit of equity, Ω_{t+1}^R, which varies between bank run and no-run states. Importantly, however, agents do not incorporate how their decisions change the probability of a bank run, that is $\mathbb{E}_t\left[\partial p_{t+1}/\partial \phi_t^R\right] = 0$. Therefore, when leveraging up and expanding their balance sheet, retail banks invest too much in wholesale lending from a social welfare perspective.

Figure 1.3 shows that a constant retail bank capital requirement can substantially increase the probability of a bank run in the next period and is therefore not an appropriate policy to address the bank run externality. In this figure, we plot the next period bank run probability $\mathbb{E}_t[p_{t+1}]$ and the two components of the bank run condition (1.2.30) as a function of $\bar{\phi}_t^R$. We set K_t^H, K_t^R, K_t^S, B_t, D_t, R_t^D and R_t^B at the steady state level and report results for two Z_t, one unconditional standard deviation above and below the unconditional mean, respectively. We fix a level of $\bar{\phi}_t^R$, recompute the general equilibrium given this fixed level of $\bar{\phi}_t^R$ and then compute the statistics of interest. This figure shows the effects of a constant retail bank capital requirement on the equilibrium bank run probability.

First, in Panel (a), we report the bank run probability as a function of $\bar{\phi}_t^R$. The solid line depicts the bank run probability if productivity is one standard deviation below the mean, the dotted line if productivity is one standard deviation above the mean. We mark the policies the retail banks choose in the absence of regulation by the thin vertical solid and dashed lines, respectively. Note that the leverage ratio of retail banks ϕ_t^R is *counter*-cyclical, because during an expansion, retail banks will increase wholesale lending relative to capital holdings. Since wholesale lending enters leverage only with weight γ, this shift in the composition of assets reduces the leverage of retail banks.

As we can see in Panel (a) of Figure 1.3, imposing a higher static capital requirement can increase the future probability of a bank run. By increasing the capital requirement from 0 to about 20 percent, the probability of a bank run increases from 2.5 percent to around 3 percent per quarter in a recession, and from 0.6 percent to about 1.1 percent in an expansion. For low values of $1/\bar{\phi}_t^R$, there is no effect, since the capital requirement is never binding. There is a small effect even before the capital requirement starts binding in the current period, since it will already bind in some states of the world in the next period.

To see through which channels the capital requirements impact the run probability, we decompose the bank run condition given by Equation (1.2.30) into two components. The first component, R_{t+1}^{K*}/R_{t+1}^{B}, is the spread between asset and liability returns for shadow banks during a bank run. We show in Panel (b) that increasing the capital requirement reduces this spread, meaning that it is less likely that shadow banks can repay their liabilities. This effect is mostly due to a lower liquidation price of capital. The second component, K_{t+1}^{S}/B_{t+1}, is the assets-to-debt ratio of the shadow banks. This ratio is inversely related to the tightness of the incentive constraint of shadow banks. The higher the leverage of shadow banks, the lower the ratio of assets to liabilities is. A higher leverage ratio of retail banks hence tightens the incentive constraint of shadow banks. Panel (c) shows that the reduction in the first component less than one for one offset by an increase in the second component. Because of this under-adjustment, the probability of a bank run increases as the capital ratio of retail banks decreases. The reason for the strong increase in the bank run probability is that the regulator imposes a *less* counter-cyclical capital requirement than the market. This forces retail banks to deleverage *more* during a bank run compared to the case of no regulation, which in turn lowers the future liquidation price of capital and hence increases the bank run probability. Therefore, the regulator should optimally relax the capital requirement as much as possible ex-post during a bank run to reduce the probability of a bank run ex-ante.

Whether the bank run probability increases or decreases depends theoretically on whether K_{t+1}^{S}/B_{t+1}^{S} increases more or less than R_{t+1}^{K*}/R_{t+1}^{B} increases. This in turn

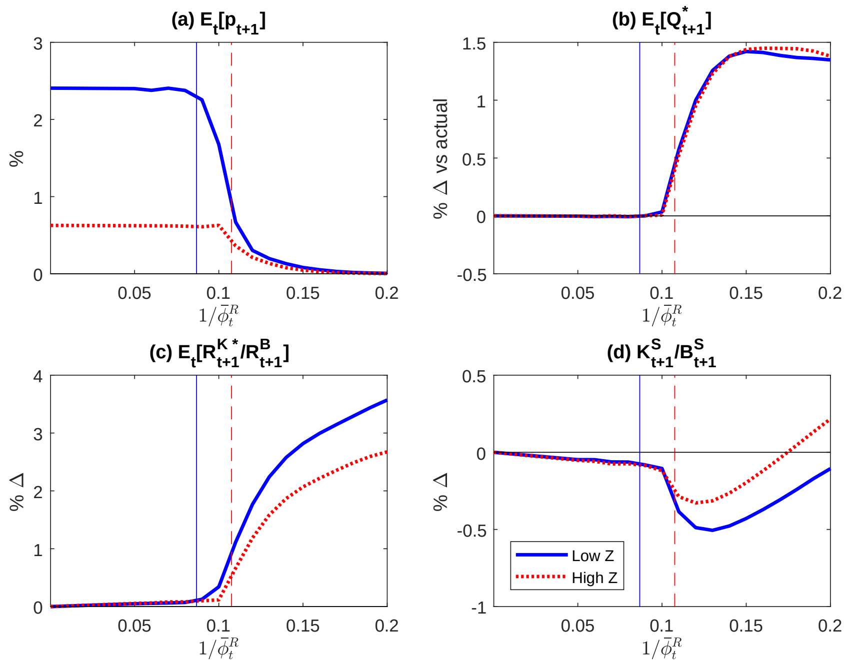


Fig. 1.4 General equilibrium effects of a run-contingent retail bank capital requirement on the bank run probability

depends on how much the incentive constraint of the shadow bank tightens in response to an reduction in the future net worth of the shadow bank.

We show the effects of a policy that imposes a capital requirement only during the no-run equilibrium in Figure 1.4. This policy is successful in eliminating bank runs. Panel (a) shows that imposing a capital requirement of 20 percent can eliminate bank runs both in recession and in expansion states. Panel (b) shows that the lower bank run probability is due to a higher spread between the returns on assets and liabilities of shadow banks during a bank run. This effect is primarily driven by a higher liquidation price of capital. As the capital requirement is relaxed during a run, retail banks can increase leverage and will hence expand their capital holdings, the only asset they have access to in the case of a bank run. This higher investment demand will increase the liquidation price of capital. From Panel (c), we can see that this policy increases shadow bank leverage initially, because a lower bank run probability relaxes the incentive constraint of shadow banks by raising their continuation value, allowing them to use more leverage and hence a lower ratio of assets to liabilities. However, leverage also decreases for capital requirements higher than 12 percent. This reversal occurs because for a small enough bank run probability, a reduction in the bank run probability only relaxes the incentive constraint little. In addition, a higher capital requirement still tightens the incentive constraint of shadow banks during normal times.

In summary, agents fail to internalize the effects of their decision on the probability of a bank run. In particular, the probability of a bank run is very sensitive to the capital ratio of retail banks, especially if the economy is in a recession. Therefore, retail bank capital requirements can be an effective way of reducing shadow bank runs, but only if the regulator relaxes them during a bank run.

The Capital Price Externality

Due to the incentive constraints (1.2.8) and (1.2.14), the extent to which banks can leverage their equity depends on the aggregate capital price. A low capital price lowers the maximum leverage ratio of shadow banks, since the value of diverting assets today decreases less than the continuation value. Therefore, banks have to deleverage, which lowers their desired level of investment. This lower level of investment in turn leads to a lower capital price. The model therefore exhibits a feedback loop, the classical financial accelerator effect, which operates through the endogenous capital price and the incentive constraints.[21]

An inefficiency arises, because banks do not internalize the effects of their current borrowing decisions on the future aggregate price of capital. By borrowing less during times of high capital prices, banks could reduce the co-movement between the tightness of the incentive constraint and the price of capital and thereby reduce the strength of this feedback loop.[22] This feedback loop is especially prominent in the economy with bank runs, because bank runs are more likely to occur and more severe if the capital price is more volatile.

A regulatory policy that is designed to restrict deposit lending during good economic conditions and ease deposit lending during bad economic conditions can increase welfare by mitigating the feedback loop and reducing the frequency and severity of bank runs. In contrast, a policy that restricts lending in all states of the world equally may actually reduce welfare, because such a policy acts as a tighter borrowing constraint and makes the feedback loop more severe.

In Figure 1.5, we illustrate that a higher constant retail bank capital requirement $1/\bar{\phi}_t^R$ reduces both the expected future capital price and the expected future liquidation price of capital. We follow the same procedure to compute these statistics as in Figure 1.3. The expected future capital price $\mathbb{E}_t\left[Q_{t+1}\right]$ in Panel (a) incorporates the probability that a bank run may occur in the next period.

[21] The financial accelerator effect was first introduced in the business cycle literature by Bernanke et al. (1999).

[22] This mechanism is well known in the literature, see for example Lorenzoni (2008) for theoretical work and Bianchi (2011) for numerical work that studies this pecuniary externality.

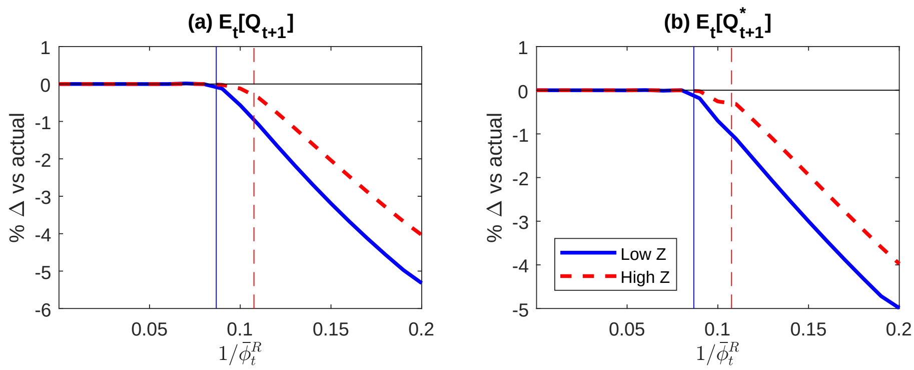


Fig. 1.5 General equilibrium effects of a constant capital requirement on the future capital price

The thick, solid line in the left panel of Figure 1.5 is the expected future capital price as a function of $\bar{\phi}_t^R$ if productivity Z_t today is one unconditional standard deviation below the mean. The thin, solid line denotes the level of ϕ_t^R that retail banks actually choose in equilibrium. We report the expected capital price relative to the expected capital price under the actually chosen policy, which in a recession implies a capital ratio of around 9 percent. Imposing a capital requirement $1/\bar{\phi}_t^R$ of 20 percent decreases the future capital price by more than 5 percent. The unconditional quarterly standard deviation of the capital price in the simulated model is about 0.6 percent, so this is a large change. The reason for this effect is that the regulator tightens the financial constraint of the retail banking sector, which forces them to deleverage, reducing both lending to consumption goods producers and shadow banks. Retail banks and shadow banks are forced to contract their balance sheets, which reduces investment and the price of capital.

The thick, dashed line in the left panel of Figure 1.5 is the same function, except when productivity is one standard deviation above the mean. In this case, retail banks choose a higher capital ratio, which is shown by the thin, dashed line. Increasing the capital ratio has a weaker effect on the future price of capital. Imposing a capital requirement of 20 percent reduces the capital price by only 4 percent. Looking at the right panel, we see that imposing a constant capital requirement has a similar effect on the future liquidation price of capital.

In Figure 1.6, we show the effects of a capital requirement that is imposed during the no-run equilibrium, but relaxed to zero during a bank run. This policy has a weaker negative effect on the expected future capital price, because it has a strong positive effect on the future liquidation price of capital. In addition, this policy reduces the frequency of bank runs, which in turn lowers the probability that assets are valued at the liquidation price of capital, increasing the capital price even further. Contrasting Panels (a) of Figures 1.5 and 1.6, we can see that by setting a capital requirement of 20

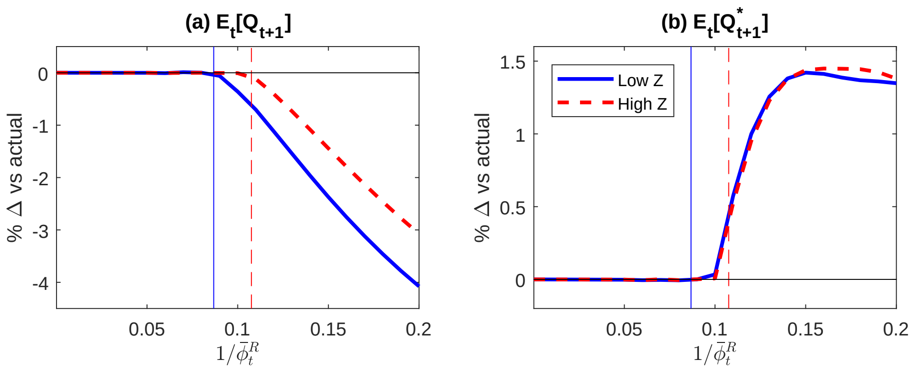


Fig. 1.6 General equilibrium effects of a run-contingent retail bank capital requirement on the future capital price

percent that is relaxed during a bank run, the regulator still reduces the capital price, but by only 4 instead of more than 5 percent if he does not relax the capital requirement during a run. Panel (b) shows that a higher capital requirement now strongly increases the liquidation price of capital, both in recessions and expansions. Imposing a capital requirement of 20 percent increases the liquidation price of capital by 1.5 percent relative to the model without regulation. This is because by restricting lending today, the regulator increases the capital buffer of retail banks during a bank run, which increases their ability to increase leverage and therefore their capital holdings during a bank run. As a consequence, the liquidation price is higher compared to the baseline model.

In summary, the retail bank capital ratio can affect future capital prices substantially, so a policy aimed at influencing this retail bank capital ratio can mitigate the financial accelerator effect. In particular, the regulator should tighten the capital requirement during expansions and relax it during recessions and bank runs to stabilize the capital price and reduce both the financial accelerator effect during normal business cycles as well as the probability of large bank runs.

1.5.2 The Steady State Effect of Retail Bank Capital Requirements

In this section, we explore the steady state implications of retail bank capital requirements. We characterize the non-stochastic steady state equilibrium absent of bank runs in Appendix A.2. In the steady state, the capital adjustment cost is zero, therefore the price of capital Q equals 1. We denote steady state variables without time subscript.

In the following subsections, we conduct comparative statics analysis of the impact of varying bank capital regulation on retail banks, i.e., the consequence of changing the policy parameter $\frac{1}{\phi_R}$ while keeping other parameters constant.

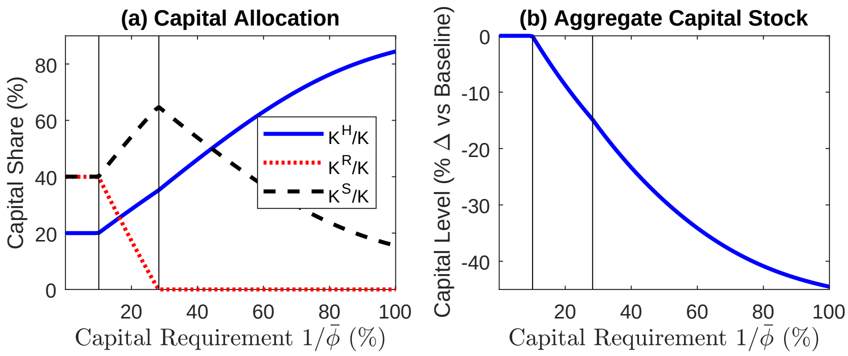


Fig. 1.7 Steady state effect of a retail bank capital requirement on the capital allocation and the aggregate capital stock

Capital Allocation and the Aggregate Capital Stock

Other things being equal, under a tighter capital requirement, retail banks are faced with a higher financing cost as they have to use more costly capital and less relatively cheap deposit from the households. This higher financing cost is further passed on by the retail banks to the shadow banks. Therefore, the required return for capital investment from the banking sector increases. On the other hand, the required return for capital investment for households remains the same. As a result, households' capital holding share increases. The first order condition regarding K^H_{t+1} in steady state is given by (B.5 in the Appendix):

$$R^K = \frac{1}{\beta}\left(1 + \eta^H \frac{K^H}{K}\right).$$

Hence the return on capital, R^K, increases as the household hold a larger share of capital of the economy. As investment in the economy becomes more costly, the aggregate capital stock and aggregate output decrease.

Figure 1.7 shows the comparative statics for the distribution of capital among households, retail banks and shadow banks in Panel (a) and the aggregate capital stock in Panel (b). We vary the minimum capital requirement between 0 and 100 percent. The solid line is the share of capital held by households, the dotted line is the share of retail bank capital holdings and the dashed line is the share of shadow bank capital holdings. For a retail bank capital requirement below 10 percent, the capital requirement is not binding and the retail bank leverage ratio is determined by its incentive constraint.

For a retail bank capital requirement between 10 and 25 percent, the capital requirement is binding and retail banks will invest in both capital and wholesale lending. As the capital requirement increases in this range, retail banks substitute away from capital lending to wholesale lending. This is because an additional unit of capital lending requires $1/\bar{\phi}$ of equity finance, while an additional unit of wholesale lending only requires $\gamma/\bar{\phi}$ of equity finance. If the regulator tightens the retail bank capital requirement, wholesale lending will therefore become relatively more attractive for retail banks. Hence, direct capital holdings by retail banks decrease, and capital holdings by shadow banks increase in this range.

For a capital requirement above 25 percent, retail banks will only invest through wholesale lending. If the regulator increases the capital requirement in this range, retail banks can no longer substitute away from capital holding and therefore can only reduce wholesale lending. Consequently, both retail and shadow banks will reduce their assets in this range.

In Panel (b), we plot the aggregate capital stock relative to the unregulated economy as a function of the capital requirement. The capital stock is very sensitive to the capital requirement: If retail banks were required to finance themselves using 100 percent equity, the capital stock would reduce by more than 40 percent. The reason for this strong effect is that banks in this economy cannot raise outside equity from households. Hence, a higher capital requirement forces retail banks to sharply cut the asset side of their balance sheet, which in turn forces the shadow banks to reduce their assets as well. If banks could issue equity to households, their required return on equity would not increase monotonically with a higher capital requirement, which would imply a lower bound on the aggregate capital stock.

Leverage and the Coverage Ratio of Shadow Banks

In Figure 1.8, we report how leverage of retail and shadow banks and the coverage ratio of shadow banks change with the capital requirement. We define the steady state coverage ratio of shadow banks as the ratio of beginning of period assets over liabilities by shadow banks, i.e.,

$$\frac{R^K K^S}{R^B B^S} = \frac{R^K}{R^B} \frac{\phi^S}{\phi^S - 1}.$$

Mechanically, an increase in the retail bank capital requirement lower the leverage ratio of retail banks. For shadow banks, there are two cases. As long as retail banks can substitute away from capital holdings towards wholesale lending, higher retail bank capital requirements increase the leverage ratio of shadow banks. When retail banks lend only on the wholesale market, regulating retail banks reduces shadow bank leverage.

The coverage ratio is an interesting statistic, because it indicates how run-prone the shadow banking sector is. While it does not exactly correspond to the recovery rate of wholesale lending by the retail banks after a bank run, a given fall in the liquidation price of capital can ceteris paribus trigger a bank run more often if the coverage

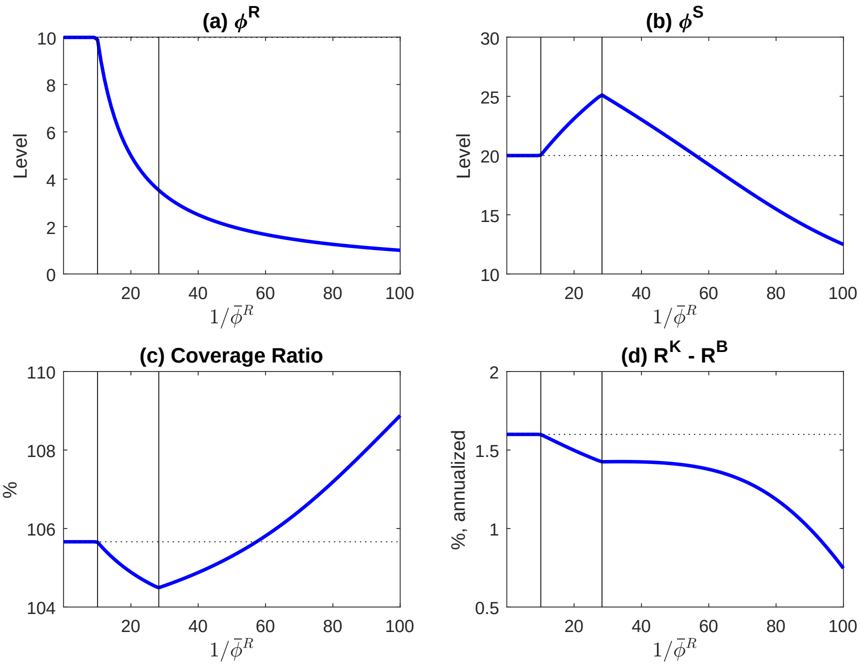


Fig. 1.8 Steady state effect of a retail bank capital requirement on leverage and the coverage ratio

ratio is low. We can see that the coverage ratio decreases as soon as the retail bank capital requirement becomes binding and increases as soon as retail banks exclusively lend on the wholesale funding market. This has a significant effect on the bank run probability. In the baseline model, the economy could sustain a drop in the liquidation price of capital of at most 5.5 percent without a bank run being triggered. With fully equity-financed retail banks, the economy could sustain a drop in the liquidation price of capital of more than 8.5 percent without a self-fulfilling bank run being triggered.[23]

The coverage ratio is decreasing in the leverage of shadow banks and increasing in the excess return $R^K - R^B$. Looking at the right upper and lower panel of Figure 1.8, we find that the increase in the coverage ratio is primarily driven by the lower leverage ratio of shadow banks, because $R^K - R^B$ is decreasing in the capital requirement throughout.

Consumption and Welfare

Figure 1.9 shows how retail bank capital requirements affect consumption and hence welfare of agents in steady state. Households consume less in a regime with a higher capital requirement. For households this is because they can save less through deposits

[23] The formula to calculate this threshold price is $Q^* = \left(\frac{R^B B}{K^S} - r^K\right)\frac{1}{1-\delta}$.

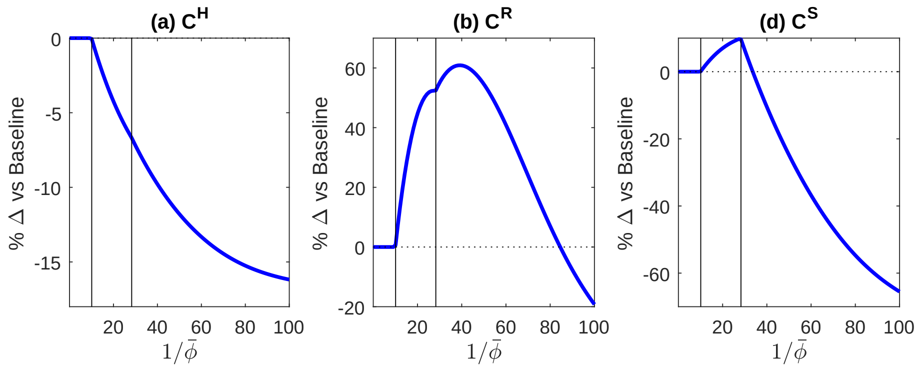


Fig. 1.9 Steady state effect of a retail bank capital requirements on consumption

and make more inefficient direct investment. Also, aggregate output decreases strongly due to the decrease in the aggregate capital stock, which lowers wages.

Retail banks and shadow banks, on the other hand, enjoy higher consumption under a higher capital requirement. This is because competitive retail banks do not internalize that by lending less to consumption goods producers they can increase the returns on their assets, $R^K/(Q+f^R)$ and R^B, relative to the return on their liabilities, R^D, and thereby increase their net worth. Shadow banks receive a consumption gain as long as they can increase leverage, and a decrease in consumption once retail banks can no longer substitute from direct lending to wholesale lending.

Overall, because in our calibration consumption of retail and shadow banks is very small relative to household consumption, higher retail bank capital requirements lead to a welfare loss. However, by reducing the coverage ratio of shadow banks, retail bank capital requirements can reduce the susceptibility of the economy to shadow bank runs.

1.6 Counterfactuals

We conduct two experiments in this section. First, we compute the welfare cost of bank runs from the perspective of households, retail banks and shadow banks. This experiment gives us an upper bound on the positive effect of a policy designed to reduce bank runs. Second, we consider different rules for capital requirements, both in an economy with and without shadow bank runs.

1.6.1 Welfare Computation

We compute welfare in consumption equivalent terms. We use the realized consumption sequences to compute welfare. For households, welfare computation is straightforward. For the banks, we include both incumbent and newly entering banks

	Baseline, No Runs	Baseline, With Runs	% Δ
C^H	0.824	0.822	0.203
C^R	0.576	0.570	1.055
C^S	0.205	0.189	8.467

Table 1.4 Agents' willingness to pay to avoid bank runs in the baseline model

into our welfare measure. Since they have linear utility, the regulator can then simply consider the welfare of each of the banking sectors as a whole. The utility of banking sector J, $J \in \{R,S\}$, as a whole if net worth is constant over time is given by

$$U^J = \sigma^J \frac{N^J - W^J K}{1-\sigma^J} + \beta(1-\sigma^J)U^J, \tag{1.6.1}$$

with consumption given by $C^J = \sigma^J \frac{N^J - W^J K}{1-\sigma^J}$. The consumption equivalent welfare of banking sector J is therefore

$$C^J_{\text{equiv}} = \left[1-\beta(1-\sigma^J)\right] U^J. \tag{1.6.2}$$

1.6.2 The Welfare Cost of Bank Runs

Before investigating the welfare effects of bank capital requirements, we want to know how costly bank runs are in our calibrated model. For this purpose, we conduct the following experiment. We first simulate the model with the liquidation cost shock. We compute the permanent consumption equivalent of welfare for each type of agent in this economy. Next, we simulate a model without bank runs by setting the liquidation cost shock to a large enough number > 1. We then calculate the permanent consumption equivalent for each type of agent in this economy without bank runs. The difference between the two consumption equivalents is the welfare gain if bank runs were completely eliminated, expressed in permanent consumption equivalent units.

Table 1.4 shows how much households, retail banks and shadow banks are willing to pay to avoid bank runs. We report the results as percentage change in the consumption equivalent of welfare from eliminating bank runs, i.e., if welfare in consumption equivalent terms is given by C^J, the percentage change in welfare for agent J is

$$\frac{C^J_{\text{No Runs}} - C^J_{\text{Runs}}}{C^J_{\text{Runs}}}.$$

Shadow banks gain the most from eliminating bank runs and would be willing accept an 8.4 percent permanent decrease in consumption to avoid bank runs. Bank runs are also very costly for retail banks, who would pay 1 percent of their permanent consumption to eliminate bank runs. Households gain about 0.2 percent in quarterly consumption equivalent terms from the elimination of bank runs.

To compare our results to the literature, Chatterjee and Corbae (2007) estimate a consumption equivalent welfare gain from eliminating the likelihood of economic crises [24] to be around 0.97 percent. Their estimated contribution of a reduction in consumption volatility to this welfare gain is around 0.196 percent in consumption equivalent terms, which is comparable to the welfare gain of households in our model. Their depression state has a similar frequency and similar output effects to a financial crisis in our model. The unconditional probability of a depression state in their model is 9.75 percent, whereas the unconditional probability of a financial crisis state in our model is around 8 percent, which is also comparable. They assume however a constant relative risk aversion of 3, which is much higher than the value of 1 that we use.

Barro (2009) estimates a welfare gain of 4 percent in output equivalent terms for a representative household with log utility from eliminating consumption disasters like World War II. In his case, disasters however have an output cost of almost 30 percent on average, which is one order of magnitude larger than the output loss from a bank run in our model. Overall, we conclude that the welfare gain from eliminating bank runs is sizable for all agents in the economy.

1.6.3 Policy Experiments

We discuss two different rules for setting the capital requirement $\frac{1}{\bar{\phi}_t}$. First, we consider the simple case of a constant capital requirement:

$$\frac{1}{\bar{\phi}_t} = \frac{1}{\bar{\phi}}. \tag{1.6.3}$$

Second, we look at the case where the regulator can condition the capital requirement on whether or not the economy is in a run equilibrium. Denote as $\mathbb{1}_t^{\text{Run}}$ an indicator variable that is 1 if the economy experiences a run in period t and 0 otherwise. Then, we can write a capital requirement that conditions on the no-run state as

$$\frac{1}{\bar{\phi}_t} = \frac{1}{\bar{\phi}}(1 - \mathbb{1}_t^{\text{Run}}). \tag{1.6.4}$$

Such a requirement has the advantage that the regulator can impose higher equity buffers of retail banks during normal times, which can be used to absorb the liquidated capital from shadow banks during a run, thereby pushing up the liquidation price of

[24] Chatterjee and Corbae (2007) define economic crises as depressions of the same magnitude of the Great Depression in terms of increase in unemployment.

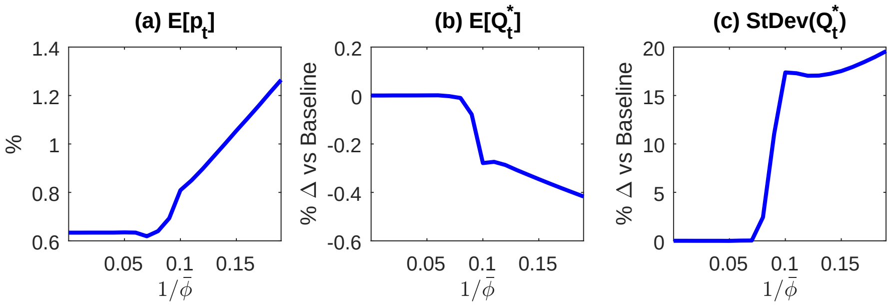


Fig. 1.10 Probability of bank runs with a constant minimum capital requirement

capital. In this sense, the more access to deposits retail banks have during a banking crisis, the higher the fire sale price of the capital will be ex-post, and the less likely bank runs would happen ex-ante. Therefore, the optimal capital requirement in face of a bank run is its lower bound, i.e., zero. In what follows we focus on this specific run-contingent capital requirement.

For each policy experiment, we are interested in two questions. i) How effective is the policy in reducing bank runs? ii) What is the welfare effect of the policy?

Constant Capital Requirements

First, we discuss the case of a constant capital requirement. The Panel (a) of Figure 1.10 shows that the frequency of bank runs increases as the minimum capital requirement increases. The reason is that the capital requirement tightens very strongly during a bank run, which lowers the ability of the retail banking sector to absorb the liquidated capital of the shadow banking sector. Therefore, this capital requirement has a negative effect on the liquidation price of capital. Since the higher constant capital requirement implies that the capital price is more volatile, bank runs become more likely.

In Figure 1.11, we show the welfare effects of a constant capital requirement. On the x-axis, we vary the capital requirement between 0 percent and 20 percent. On the y-axis, we show the percentage change in welfare relative to the capital requirement of 0 percent, which is never binding. We report the percentage change in welfare relative to the model without regulation for each type of agents as well as for the sum of utilities, which corresponds to a utilitarian welfare function. We show the results for three different variations of the model. The dashed line is the steady state equilibrium. The dotted line is the model without bank runs and the solid line is the model with bank runs. The purpose of including the steady state is to illustrate the strong steady state effect, and the purpose of the No Runs case is to illustrate the isolated effect of

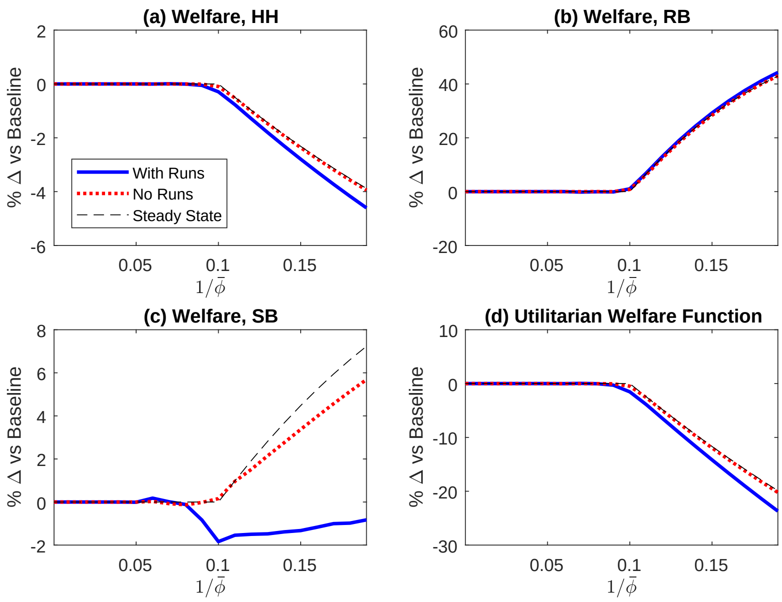


Fig. 1.11 Welfare with a constant minimum capital requirement

the capital requirement on the capital price externality that is also present in the model without runs.

First, we can see that in steady state, a higher capital requirement reduces the welfare of households, but increases the welfare of retail and shadow banks. Overall, measured in consumption equivalent units, capital requirements are welfare reducing in steady state. Retail and shadow banks gain in welfare terms, because the return on their assets increases more than the return on their liabilities, which increases the net worth of incumbent banks.

Second, we can see that a constant capital requirement reduces welfare *more* in the dynamic model without bank runs relative to the steady state. This is because this constant capital requirement amplifies the effect of the pecuniary externality. To see this, compare the incentive constraint (1.2.14) of the retail banks to the case of a constant capital requirement (1.2.17), we get:

$$\psi\left[(Q_t+f_t^R)k_{t+1}^R+\gamma b_{t+1}^R\right]\leq\beta\mathbb{E}_t\left[V_{t+1}^R\right],$$
$$\psi\left[(Q_t+f_t^R)k_{t+1}^R+\gamma b_{t+1}^R\right]\leq\bar{\phi}n_t^R.$$

While the left hand sides are identical, the right hand sides differ. The incentive constraint has the continuation value of the retail bank on the right-hand side, while the capital requirement has the current net worth times some constant on the right-hand

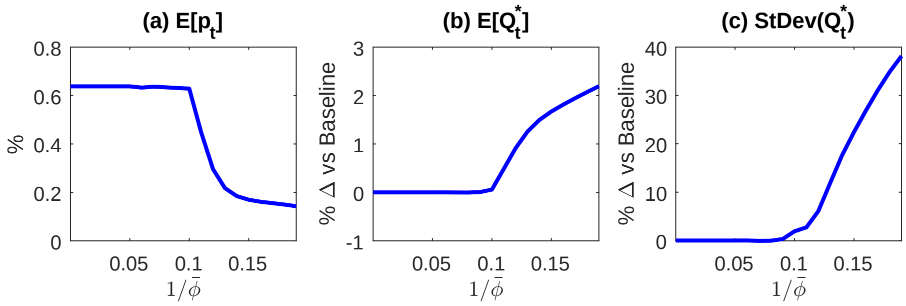


Fig. 1.12 Probability of bank runs with a run-contingent capital requirement

side. In particular, rewriting the incentive constraint slightly, we get

$$\psi\left[(Q_t + f_t^R)k_{t+1}^R + \gamma b_{t+1}^R\right] \leq \beta\mathbb{E}_t\left[\Omega_{t+1}^R \frac{n_{t+1}^R}{n_t^R}\right] n_t^R.$$

Using that both the marginal value of net worth Ω_{t+1}^R and the net worth growth rate n_{t+1}^R/n_t^R are independent from n_t^R, which we show in Appendix A.3.2, we can state that the derivative of the right hand side of the incentive constraint with respect to n_t^R is given by $\beta\mathbb{E}_t\left[\Omega_{t+1}^R \frac{n_{t+1}^R}{n_t^R}\right]$, which in our calibration is *counter*-cyclical. The derivative of the right hand side of the capital requirement with respect to n_t^R is $\bar{\phi}$, which is constant. Hence, the market imposed leverage partially offsets fluctuations in net worth of the retail bank, which reduces the pro-cyclicality of the retail bank balance sheet. The constant regulatory capital requirement does not do this.

Third, we can see that a constant capital requirement reduces welfare even more in the dynamic model with bank runs compared to the dynamic model without bank runs. The reason is that, as can be seen in Figure 1.10, in addition to amplifying the effect of the pecuniary externality, a higher capital requirement increases the frequency and severity of bank runs. To summarize the results: From a macro-prudential perspective, constant capital requirements not only distort the allocation of capital, which leads to a steady state welfare loss, but they also amplify the effects of the pecuniary externality during normal times and increase the frequency and severity of bank runs. While constant capital requirements may be beneficial at the microprudential level, our results indicate that macroprudential regulation of the retail banking sector should not use constant capital requirements.

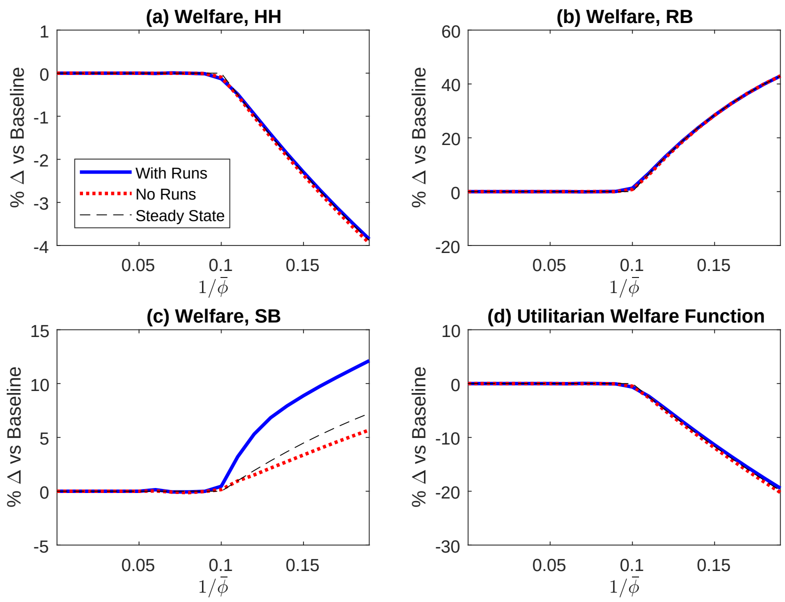


Fig. 1.13 Welfare with a run-contingent capital requirement

Run-Contingent Capital Requirements

In Figures 1.12 and 1.13, we show the effects of a capital requirement that is only imposed if the economy is in the no-run equilibrium. First, we can see in Panel (a) in Figure 1.12 that by implementing a run-contingent capital requirement, the regulator can reduce the probability of bank runs substantially. As we show in Panels (b) and (c), the main channel through which the regulator achieves this effect is through a higher liquidation price of capital: increasing the capital requirement to 20 percent increases the liquidation price of capital by more than 2 percent. While the run-contingent capital requirement also increases the volatility of the liquidation price of capital, this effect is more than offset by the higher mean of the liquidation price. Nevertheless, a policy which is designed to offset cyclical fluctuations in the price of capital may lead to superior welfare outcomes.

The welfare results for the steady state case and the case without bank runs are the same as in figure 1.11. Focusing on the welfare results for the model with bank runs, we can see that a run-contingent capital requirement can undo the negative externality of capital requirements on the probability of bank runs. In fact, welfare of shadow banks increases more in the model with runs compared to the model without runs. However, the capital requirements are still overall welfare reducing, and more so in the dynamic models than in the steady state. This is because the run-contingent capital requirement still increases the pro-cyclicality of the retail bank balance sheet constraint during normal times, which amplifies the pecuniary externality from the capital price and therefore the welfare cost of business cycles.

The intuition for the better performance of the run-contingent capital requirement relative to the constant capital requirement is as follows. A higher capital requirement increases the net worth of retail bank, which in turn increases the continuation value of the retail banks. This means that the incentive constraint and hence the market imposed borrowing constraint is relaxed. If the regulator now removes the capital requirement during a shadow bank run, the retail banks can increase leverage relative to the case without regulation. Hence, they can absorb the liquidated capital of the shadow banks more easily, which increases the liquidation price of capital. Finally, a higher liquidation price of capital reduces the ex-post cost of realized bank runs and reduces the ex-ante probability of bank runs. The success of this policy is illustrated by a relatively higher welfare gain from regulation in the model with bank runs for all agents compared to the steady state model. However, the steady state cost of bank capital regulation is still dominant, such that capital regulation overall lowers welfare. This high cost relies on the extreme assumption that banks can never raise outside equity from households, no even in the long run. Removing this constraint may yield a significantly less pessimistic cost of bank capital requirements.

1.7 Conclusion

We study the macroeconomic effects of retail bank capital regulation in a quantitative model with regulated retail banks and unregulated shadow banks. In our model, financial crises occur in the form of runs on shadow banks. A negative externality exists because banks do not internalize that their capital structure decisions affect the likelihood of financial crises, which leads to over-borrowing during normal times. This externality creates a role for bank capital regulation.

From the regulators' perspective, the trade-off that determines the optimal capital requirement is: on the one hand, higher capital requirements increase the ability of retail banks to absorb liquidation losses during a shadow bank run, thereby reducing the frequency and severity of bank runs. For this effect, it is crucial that capital requirements are relaxed during a bank run. A higher constant capital requirement induces more bank runs instead. On the other hand, tightening capital requirements reduces the steady state capital stock and output due to less financial intermediation.

We conclude that capital requirement on retail banks is an effective tool to reduce banking crises and improve financial stability. However, there is substantial costs of the policy, especially when capital accumulation is endogenous and equity issuance is very costly for banks. Therefore, the optimal capital requirement in a model with endogenous capital accumulation should be substantially lower than that in a model with exogenous capital.

An interesting extension of our model would be to include sticky prices and nominal debt. A bank run could then result in a Fisherian debt deflation spiral: the initial effects of the run depresses goods prices, which worsens the real debt burden of banks, which in term depresses investment, and so on. Bank runs can then lead to episodes that cause the economy to be at the lower bound of the nominal policy interest rate. In this case, the possibility of bank runs will also affect how monetary policy should be conducted.

Chapter 2

Housing, Financial Crises and Macroprudential Regulation: The Case of Spain

with Johannes Pöschl and Marcus Mølbak Ingholt

2.1 Introduction

Between 2008 and 2016, the Spanish economy witnessed a financial crisis unprecedented in its modern history. At the center of this financial crisis was a house price bust intertwined with a severe disruption in the Spanish banking sector. For this reason, macroprudential policies that focus on the prevention of such financial crises have become an important part of the agenda for the financial regulators.

Our goal in this chapter is to quantitatively assess the effectiveness of three different types of macroprudential policies in reducing the frequency and severity of financial crises resembling the 2008-2016 Spanish financial crisis. The three policies are: (i) minimum bank capital requirement, (ii) provisioning against expected credit losses[1] (also known as "dynamic loan loss provisioning" in Spain), and (iii) maximum loan-to-value (LTV) ratio restrictions.

To achieve this goal, we first build a macroeconomic model to understand the interaction among house prices, mortgage loans and bank runs in the context of the Spanish financial crisis. We calibrate the model to match dynamics of key financial and real variables during the crisis, namely house prices, total output, leverage and credit spreads of both banks and households. With the calibrated model, we conduct

[1] An accounting rule for banks, under which banks have to set aside funds to cover anticipated future losses. More details about the policy is discussed in subsection 2.3.3.

counter-factual policy experiments of the aforementioned macroprudential policies and evaluate their effects in moderating housing and banking crises.

We study a non-linear DSGE model with heterogeneous agents (patient and impatient households), a housing market and a banking sector. Our model exhibits three key features: endogenous default on mortgage loans, financially constrained borrowers and lenders in the mortgage credit market, and endogenous banking crises in the form of bank runs. Mortgage borrowers can default on their loans, in which case the banks seize the houses of the defaulted households and sell these houses in the housing market. Importantly, the mortgage market is disciplined by two constraints. On the one hand, mortgage lending is restricted by a leverage constraint faced by banks as a result of a regulatory capital requirement. In other words, given the level of bank net worth, the bank's lending ability is constrained. On the other hand, mortgage borrowers face a borrowing constraint, due to a restriction on the maximum LTV ratio they can adopt. Whether the lending constraint or the borrowing constraint binds depends on the state of the economy and therefore varies over time. Banks experience a run from the depositors whenever the liquidation value of the banks in a run is lower than the value of their outstanding debt.

There are two exogenous aggregate shocks in the model: a productivity shock and a house quality shock (a shock to the recovery values of mortgages and deposits in the event of default). The productivity shock works through the real side of the economy and affects house prices primarily through lowering the income of households and thereby lowering the demand for houses. The house quality shock works through the financial side of the economy and impacts house prices by reducing the net worth of banks and thereby lowering the supply of mortgages and demand for houses. There is a financial accelerator effect, independently of whether the lending constraint or the borrowing constraint is binding in the mortgage credit market. When the lending constraint is binding, a lower house price leads to an increase in mortgage default and a lower mortgage recovery rate, both reduce the profitability of the bank and thereby lower the bank net worth. This reduces the bank's lending ability, causing house prices to drop further. When the borrowing constraint is binding, a lower house price directly leads to lower value of the collateral for the loan, reducing the borrowers' ability to borrow, which also results in further decreases in the house price.

Bank runs occur more frequently when bank leverage is high and bank profitability is low (e.g., due to a negative shock to the recovery of defaulted mortgages through foreclosure). A bank run leads to the collapse of the mortgage market, which lowers the housing demand of the borrowers substantially. As a result, the house price decreases dramatically in the event of a bank run, lowering the liquidation value of the banks further. In that sense, the model is capable of generating intertwined housing and banking crises like the one happened in Spain.

Using the resulting model economy, we calibrate a series of productivity shocks such that the model matches the evolution of the Spanish GDP between 2003 and 2017. In the implied simulation, the model closely predicts the historical movements in untargeted variables, such as consumption, house prices, mortgage debt, and bank liabilities. This is interesting, since linear and piecewise linear DSGE models typically *also* need to rely on intertemporal preference and housing preference shocks in order to match consumption and house prices accurately (i.a., Liu et al. (2013), Guerrieri and Iacoviello (2017)). An implication of the close match is that both the consumption cycle and the housing-financial cycle can largely be accounted for by productivity shocks. Starting from 2003, positive productivity shocks initially pushed up consumption and house prices. These movements were then propagated into the banking sector via a reduction in the household credit spread and a relaxation of the collateral constraint, which lead homeowners to borrow more. From around year 2009, however, a series of negative productivity shocks overturned this expansion in output and credit, consequently causing a recession.

We find that a higher minimum capital requirement on banks can effectively eliminate bank runs in this calibrated model economy. By restricting mortgage lending of banks, a higher minimum capital requirement leads to lower aggregate household leverage. With a higher equity share in housing, the mortgage default rate declines, and the recovery rate of defaulted mortgage loans increases. All these effects contribute to less frequent and less severe bank runs. This result is in contrast with the result of a similar policy experiment in Chapter 1, where the opposite effect of the policy is found: increasing the static capital requirement would actually result in more bank runs rather than less. The reason for the different findings is the following. In the model of Chapter 1, there are two banking sectors in the economy, retail and shadow banks, and bank runs happen in the shadow banking sector instead of the regulated retail banking sector. Therefore, under a tighter static capital requirement, when a bank run happens, the retail banks' ability to absorb the liquidated assets of shadow banks is constrained, which further reduces the liquidation price of assets. As a result, bank runs are more likely to occur under higher static capital requirements. However, in the model set-up of this chapter, there is only one banking sector, which is subject to runs and is regulated. In this case, a higher capital regulation results in a stronger bank balance sheet and lower frequency of bank runs.

Provisioning against expected credit losses does not substantially affect the house prices or the consumption of households. The impact on the leverage dynamics of households and banks is also limited. It does, however, lead to a reduction in the average mortgage default rate and lower probability of bank runs, although to a much lesser extent than doubling the minimum capital requirement. Moreover, the bank leverage becomes less cyclical under the provisioning policy, suggesting that the policy does play a role in reducing the cyclicality of the capital structure of banks.

Finally, imposing a tighter LTV constraint on mortgage borrowers can also effectively reduce the bank leverage and the mortgage default rate, thereby reducing the frequency of bank runs. However, since the LTV constraint reacts more strongly on house prices (a more direct financial accelerator effect) than bank capital requirements, it amplifies the cyclicality of household and bank leverage as well as the cyclicality of the default and recovery rate of mortgage loans.

2.1.1 Related Literature

This chapter is closely related to the literature that studies financial distress and their real effects. There are two branches in this literature. The first one explores the financial accelerator effect, where weak balance sheet conditions of financial or non-financial firms undermine their access to credit, which impairs their balance sheet condition further, creating a negative feedback loop and amplifies business cycle fluctuations. This line of research is pioneered by Bernanke, Gertler, and Gilchrist (1999) and Kiyotaki and Moore (1997). Since the global financial crisis, it has been an important mechanism in many studies that try to link financial disruptions and the real effects of financial crises, such as Brunnermeier and Sannikov (2014), Gertler and Kiyotaki (2015), Boissay, Collard, and Smets (2016). The second branch studies bank run events, pioneered by Diamond and Dybvig (1983). There are two slightly different ways to model bank runs, one is as in Gertler and Kiyotaki (2015) and Gertler, Kiyotaki, and Prestipino (2016), where bank creditors suddenly stop rolling over their short-term investment in banks. The other way is to model a liquidity run due to mismatch of the liquidity from illiquid assets and liquid liabilities, such as in Martin, Skeie, and von Thadden (2014). Our model includes both a financial accelerator effect and the bank run mechanism as in Gertler and Kiyotaki (2015). One thing that this literature is silent about is the role of the housing market boom and bust in the financial crisis. This chapter adds to this literature by adding the interaction of the housing market and the financial market into a macroeconomic framework of financial crisis.

There is an extensive empirical literature on financial crises. Examples include, but are not limited to, Reinhart and Rogoff (2009), Jordà, Schularick, and Taylor (2011), Gorton and Metrick (2012), Laeven and Valencia (2012), Schularick and Taylor (2012), Mendoza and Terrones (2012), Romer and Romer (2017), Jordà, Richter, Schularick, and Taylor (2017), Krishnamurthy and Muir (2017) and Muir (2017). Relative to this literature, we document dynamics of asset prices and financial leverage during the Spanish financial crisis and study to what extent these dynamics are driven by financial and real shocks.

This chapter is also related to the literature on macroprudential regulation, which explores how regulators can ensure that the market equilibrium internalizes the pecuniary externalities that arise if there are price-sensitive borrowing constraints and endogenous capital prices. Examples include Lorenzoni (2008), Bianchi (2011), Garcia-Macia and Villacorta (2016), Farhi and Werning (2016), Korinek and Simsek (2016), Dávila and Korinek (2017) and Gersbach and Rochet (2017). We discuss different regulatory policies (dynamic provisioning and LTV constraints) and compare these policies in a concrete context, namely the Spanish housing and financial crisis.

Finally, this chapter relates to the literature that studies the interaction between housing crises and financial crises, e.g., Justiniano et al. (2015) and Guerrieri and Uhlig (2016). Most of the studies in this literature focus on the US housing crisis, whereas we study the Spanish housing crisis case.

The rest of the chapter is organized as follows. Section 2.2 documents the key dynamics of the financial crisis in Spain, which we aim to match with our quantitative model. Section 2.3 describes the model economy. Section 2.4 provides a discussion of the binding constraints in the mortgage market. Section 2.5 lays out the model calibration to the Spanish economy. Section 2.6 contains counter-factual policy experiments on the three macroprudential policies. Finally, section 2.7 concludes.

2.2 The Spanish Financial Crisis

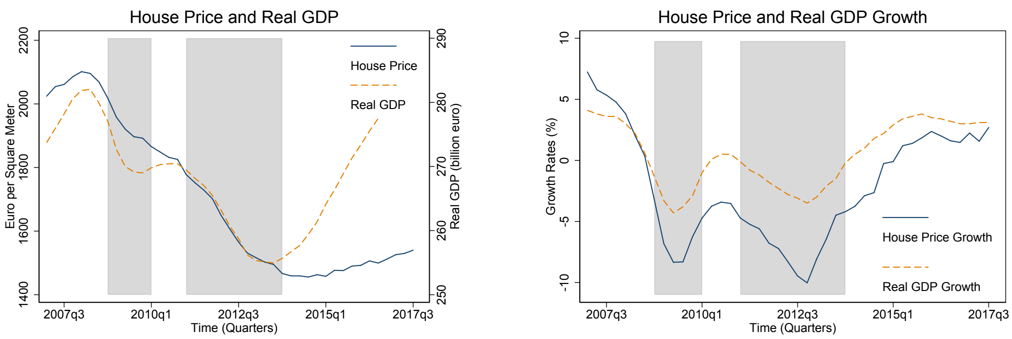
In this section, we document important dynamics of the Spanish financial crisis, which we aim to match later with our banking crisis model. We also document the pre-crisis macroprudential policy in Spain, i.e., the dynamic loan loss provisioning. We use aggregate data on GDP, house price, bank leverage, household leverage, and credit spread in Spain between 2007 and 2017. A detailed data description can be found in Appendix B.1.

2.2.1 Housing Crisis and Economic Recession

Since the mid 1990s, the real estate price in Spain embarked on an expansionary path, with nominal house prices soaring by 300 percent between 1995 and 2007.[2] In the aftermath of the global financial crisis, the Spanish real estate market collapsed. As shown in the left panel of Figure 2.1, average house price dropped from €2100 per square meter in 2008Q2 to €1450 (i.e., a 30 percent decrease) in 2015Q1, putting an end to the housing boom in Spain.

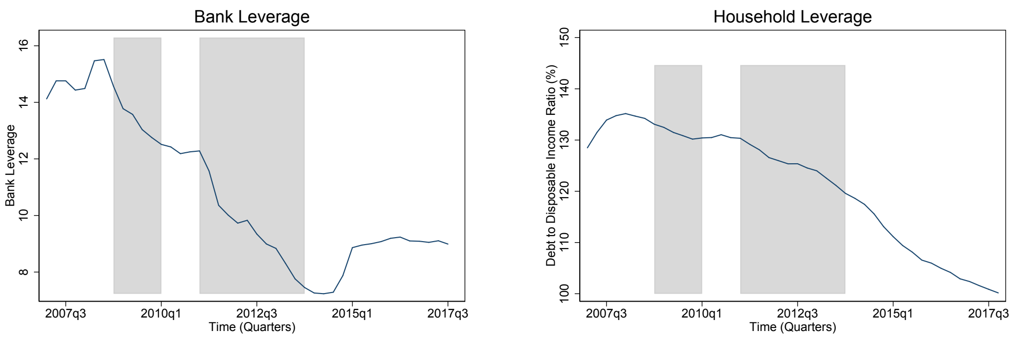
[2] See, for instance, Martín et al. (2018) for the full time trend of the Spanish house price during this period.

Fig. 2.1 House prices and real GDP in Spain, 2007-2017



Source: House price: Spain Ministry of Construction. Real GDP: Eurostat.

Fig. 2.2 Bank leverage and household leverage in Spain, 2007-2017



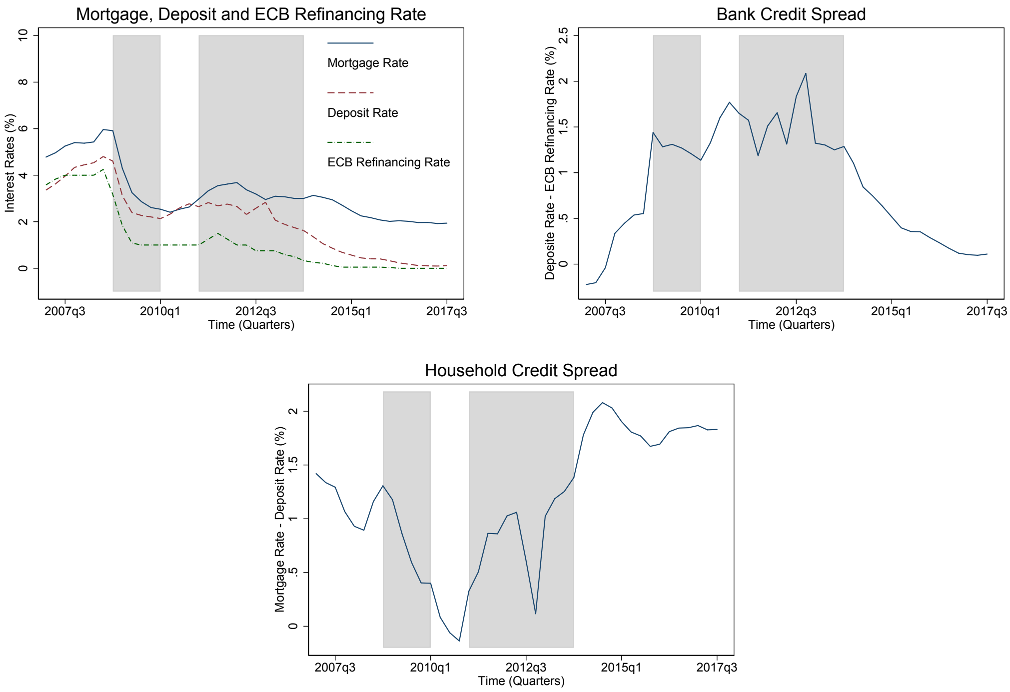
Source: Aggregate bank balance sheet data: ECB Statistical Data Warehouse. Loans granted to households as a ratio of gross disposable income: ECB Statistical Data Warehouse.

Following the conventional definition of economic recessions as two consecutive quarters of decline in real GDP, Spain experienced two economic recessions since 2000: the first one in 2008Q4–2010Q1 and the second recession in 2011Q1–2013Q4 (marked as shaded areas in the figures). The first recession corresponds with the 2007-2009 Global Financial Crisis, and the second one with the 2009-2013 European Sovereign Debt Crisis.

From the right panel of Figure 2.1, we observe an interesting pattern: the GDP growth (solid line) and house price growth (dashed line) co-move closely, suggesting a strong positive correlation between the house price and real GDP.[3] However, the decrease in house prices is much stronger than that of real GDP, with the lowest growth rate of house prices being -10 percent in 2012Q4, compared to about -5 percent for real GDP.

[3]The growth rates of GDP and the house price are calculated as the percentage change compare to the same quarter in the previous year.

Fig. 2.3 Deposit and mortgage rates and the credit spreads in Spain, 2007-2017



Source: Nominal interest rate on household deposits and mortgage loans: ECB Statistical Data Warehouse. ECB refinancing rate: ECB statistics, official interest rates. Credit Spreads: authors' own calculation based on the interest rates.

2.2.2 Banks and Households in the Financial Crisis

Leverage Over the course of the Spanish Financial Crisis, significant deleveraging took place in the banking sector. This trend can be seen in the left panel of Figure 2.2, where we define bank leverage as total asset over total equity. The average bank leverage in Spain decreased from the pre-crisis level of more than 15 in 2008 to around 7 in 2014. A simultaneous decrease in total bank asset and increase in bank equity jointly contributed to this trend.

In fact, deleveraging happened not only in the banking sector but also in the household sector during the financial crisis. Figure 2.2 shows the household debt to disposable income ratio as a measure of household leverage. However, compared to the deleveraging of banks on the left, the progress of household deleveraging is much slower yet more persistent.

Credit Spread Another important observation during the financial crisis is the credit spread dynamics, which captures the risk premium or the probability of default of the debt issuers. We calculate the bank credit spread as the difference between the annual interest rate on bank deposits and the ECB refinancing rate (interest rate on the bulk of liquidity provided to the banking system by the ECB). The household credit spread is calculated as the difference between the annual interest rate on *newly issued* mortgage loans and the annual interest rate on bank deposits. Figure 2.3 clearly shows that the credit spread of banks increased during the financial crisis, suggesting that banks had to pay higher risk premium to compensate for a higher risk of default. However, household credit spread decreased during the crisis, suggesting a lower risk premium paid by households on mortgage loans. This can be explained by a cut in bank lending to households, which forced the households to increase self-financing in house purchases. As a result, the average probability of default within the household sector decreases and the recovery of defaulted mortgage increases accordingly, reducing the credit spread.

2.2.3 Macroprudential Policy in Spain

In July 2000, Banco de España, the Spanish central bank and banking supervisor, introduced dynamic provisioning in Spain, which requires banks to provision against *expected* loan losses.[4] Under the prevailing standards, loss identification was based on "triggering events", e.g., decrease in collateral values and past-due status. The obvious drawback of the "triggering events" accounting rule is that loss recognition occurs too late, creating a procyclical effect: banks provision less (lower capital buffer) during the booming period when credit losses are low and provision more (higher capital buffer) during recessions when credit losses surge. In contrast, under the dynamic provisioning regime, banks identify potential credit losses earlier and build up buffers in good times that can be used in bad times, creating a counter-cyclical effect similar to the capital conservation buffer and counter-cyclical buffer in Basel III.

In 2014, the International Accounting Standards Board (IASB) published IFRS 9 (the accounting standard for financial instruments), which includes a new accounting standard for provisioning against expected credit losses (see, e.g., Cohen and Edwards, 2017). On that account, Spain was advanced in financial macroprudential regulation before the financial crisis hit the economy.

2.3 Model

In this section, we introduce a non-linear DSGE model with heterogeneous agents, a housing market, and a banking sector. One of the key features of the model is that both banks and households are leveraged and may default endogenously on their liabilities.

[4] See Saurina (2009) for a detailed description of the policy rule for dynamic loan loss provisioning in Spain.

2.3.1 The Model Environment

We study a closed economy with discrete time and infinite horizon. The economy is populated by patient households (fraction μ) and impatient households (fraction $1-\mu$). Households consume consumption goods and housing services, and supply labor inelastically to the production sector. They invest in housing and either borrow from or lend to banks. Banks take deposits from households and make loans to households in the form of mortgages. To keep things simple, the total housing supply is fixed to $\mathscr{H}$, so that house prices are entirely demand driven. There is no rental markets for housing, so households can only purchase houses for housing services.[5] There is a production sector that employs labor and produces consumption goods.

We denote variables related to the patient households with "P", impatient households with "I", and banks with "B".

Figure 2.4 gives an overview of how resources flow in the economy.

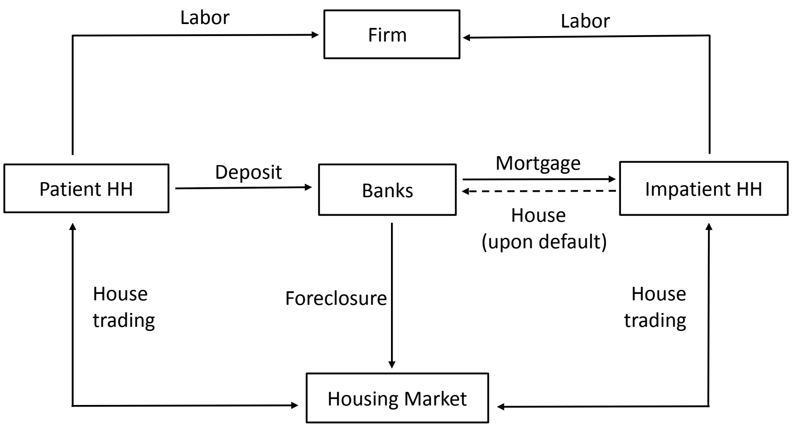


Fig. 2.4 Overview of the economy in equilibrium

2.3.2 Households

Preferences and Housing

Preferences Households of type J, $J \in \{P, I\}$, maximize their utility:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\beta^J\right)^t U^J \left(C_t^J, H_t^J\right) \right].$$

[5] Spain is a high home-ownership country with a relatively small house rental market. Between 2007 and 2017, 79 percent of the Spanish population lived in an owner-occupied dwelling, within which 32 percent had an outstanding loan or mortgage. Data source: Eurostat(ilc_lvho02).

The instantaneous utility function is given by:

$$U^J\left(C_t^J, H_t^J\right) = \chi^J \frac{(C_t^J)^{1-\sigma} - 1}{1-\sigma} + (1-\chi^J)\frac{(H_t^J)^{1-\sigma} - 1}{1-\sigma},$$

where C_t^J is consumption of household J in period t, H_t^J is housing at the beginning of period t, β^J is the discount factor, σ captures risk aversion of the households and χ^J captures the consumption weight in the utility function. Patient households and impatient households differ along two dimensions. First, impatient households discount future utility more than patient households do, i.e., $\beta^I < \beta^P$. Second, patient households have a weaker preference for housing than impatient households, i.e., $\chi^P > \chi^I$.[6]

Housing The representative household of each household type holds a portfolio of houses denoted by H_t^J, which consists of individual houses H_{it}^J:

$$H_t^J = \int_i H_{it}^J di,$$

where H_{it}^J is house i held by household J in period t and H_t^J denotes the total housing stock of households of type J at the beginning of period t. P_t is the aggregate house price expressed in units of consumption goods. The idiosyncratic price of house i, P_{it}, is the product of the aggregate house price and an idiosyncratic shock ε_{it}:

$$P_{it} = P_t \varepsilon_{it}.$$

The idiosyncratic shock ε_{it} is distributed lognormal with standard deviation σ^ε and mean $-\frac{1}{2}(\sigma^\varepsilon)^2$, such that ε_{it} has mean 1, and the expected house price is equal to the aggregate price P_t. Houses depreciate at rate δ. At the end of period t, the households sell off their remaining housing stock $(1-\delta)H_t^J$ at price P_t and make a new purchase H_{t+1}^J for the next period at the current aggregate price P_t. Therefore, the net change in housing is given by $\left[H_{t+1}^J - (1-\delta)H_t^J\right] P_t$.

[6] It is necessary that patient households and impatient households have different preferences for housing. Otherwise, impatient households would not hold sufficient housing share of the economy. As discussed in the calibration section, the impatient households' preference for housing is calibrated to match the share of owner-occupied houses with outstanding mortgage or loans.

Financial Markets

Households can lend to banks in the form of bank deposits D_t^J, and borrow from banks in the form of mortgage loans M_t^J.[7] In equilibrium, patient households are depositors and impatient households are mortgage borrowers.

Deposits If there is no bank run in period t, households receive a non-contingent gross return R_t^D in period t on the deposit they made in period $t-1$. In the event of a run, only a share X_t^D of their deposit (including due interest) can be recovered from the liquidation of the banks' assets. A detailed discussion of the recovery rate of deposits can be found in Subsection 2.3.3 about bank runs.

Mortgages A representative household borrows a portfolio of mortgages $M_t^J = \int_i M_{it}^J di$ from banks. Each individual mortgage loan M_{it}^J is secured by a corresponding house H_{it}^J. Households may default on mortgages. If a household chooses *not* to default, a gross interest rate R_t^M on the mortgage loan is paid to the bank. If the household chooses to *default*, the bank will seize the house that serves as collateral and sells it at price P_{it} to make up the losses on the loan. If the proceeds from selling the house are not sufficient to cover the losses, the bank cannot seek the deficiency balance from the borrower, i.e., the recovery is limited to the value of the house.[8] The recovery rate of the defaulted mortgage loan M_{it}^J is given by:

$$X_{it}^M = A_t \frac{(1-\delta)P_{it}H_{it}^J}{R_t^M M_{it}^J},$$

where A_t captures a multiplicative quality shock to the foreclosed houses, with

$$\begin{aligned} A_t &= \min(\hat{A}_t, 1), \\ \ln \hat{A}_t &= \rho^A \ln \hat{A}_{t-1} + \varepsilon_t^A, \text{and } \varepsilon_t^A \sim N(0, \nu^A). \end{aligned}$$

The A_t shock functions like the "capital quality shock" in Gertler, Kiyotaki, and Prestipino (2016). It provides an exogenous source of variation in the mortgage recovery rate and has a maximum value of 1. The lower the value of A_t, the less a bank recovers its defaulted mortgage from selling the house. The intuition behind this shock is that once the borrowers know they will default on the mortgage loan and give up the houses, they use the houses with less care and stop conducting proper maintenance

[7] There is no direct financial market between patient and impatient households. Therefore, in our model impatient households cannot directly borrow from patient households.

[8] In practice, mortgages are recourse loans in Spain. However, the recovery of the full value of the loan usually takes time and is not guaranteed, e.g., the mortgagor is too broke to reply. As such, we model mortgages as non-recourse loans.

of the houses. As such, there is a variation in the quality of the houses that the banks receive from defaulted borrowers.

Optimal Default Decision The repayment on a mortgage is $R_t^M M_{it}^J$. If a household chooses to default on the mortgage, they have to give up the share $A_t \in [0,1)$ of their depreciated house value $(1-\delta)P_{it}H_{it}^J$ to the bank. They keep the remaining $1-A_t$ share of the value of the house.[9] In equilibrium, it is optimal for the households to default on a mortgage if the value of the house that the household gives up is less than the outstanding liability the household owes to the bank, i.e.,

$$A_t(1-\delta)P_{it}H_{it}^J < R_t^M M_{it}^J.$$

Or equivalently, the households default on their mortgages whenever the recovery rate for the bank is less than 1, i.e., $X_{it}^M < 1$.

The aggregate mortgage default rate, Φ_t^M, can be characterized by a cutoff rule for the idiosyncratic house price shock. Define ε_t^* as the cutoff value of idiosyncratic shock, such that:

$$A_t(1-\delta)P_t H_t^J \varepsilon_t^* = R_t^M M_t^J.$$

If ε_{it}, the realized idiosyncratic shock for house i, falls below the cutoff value, it is optimal for the household to default on the mortgage. Therefore, the default rate is given by $\Phi_t^M = \Pr(\varepsilon_{it} \leq \varepsilon_t^*)$.

Borrowing Constraint The amount of the mortgage loan a representative household is able to borrow is constrained by the value of their house, i.e., the collateral of the loan:

$$M_{t+1}^I \leq \kappa P_t H_{t+1}^I,$$

where κ captures the maximum LTV ratio that a household is allowed to take. Later in the policy experiment, we will vary the value of κ to evaluate the effect of changing LTV constraints on mortgage default and bank runs.

Capital Income and Government Transfer

Patient households own the production sector and banks. Each period, they invest E^P as equity to banks and receive $\Pi_t^{B,P}$ dividends from banks and $\Pi_t^{F,P}$ profits from the firms. Capital income is taxed at rate τ.

[9] Alternatively, one could model the share $1-A_t$ as a deadweight loss of default. However, this would make the model more cumbersome to solve, since the information to compute the aggregate loss is not contained in the set of state variables we currently use.

Impatient households do not invest in bank equity and have no capital income, i.e.,$E^I = \Pi_t^{F,I} = \Pi_t^{B,I} = 0$. Instead, they receive a transfer T_t^I from the government each period. Patient households receive zero transfer, i.e.,$T_t^P = 0$.

Budget Constraint and Aggregation

Households provide labor $\bar{L^J}$ inelastically at wage W_t. The budget constraint of a representative household $J \in \{P, I\}$ is given by:

$$C_t^J + P_t \left[H_{t+1}^J - H_t^J(1-\delta)\right] + \left[1-(1-X_t^M)\Phi_t^M\right] R_t^M M_t^J + D_{t+1}^J + E^J$$
$$= W_t \bar{L^J} + M_{t+1}^J + X_t^D R_t^D D_t^J + (1-\tau)(\Pi_t^{F,J} + \Pi_t^{B,J}) + T_t^J,$$

where

$$X_t^M = \int_{X_{it}^M < 1} X_{it}^M dF(X_{it}^M)$$

is the expected recovery value of the mortgage loans in default, with $F(X)$ denoting the cumulative distribution function of X.

Full Statement of the Household's Problem In summary, the complete households' problem is given by:

$$\max_{\{C_t^J, H_{t+1}^J, M_{t+1}^J\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} (\beta^J)^t U^I\left(C_t^J, H_t^J\right)\right],$$

s.t.

$$C_t^J + P_t \left[H_{t+1}^J - H_t^J(1-\delta)\right] + \left[1-(1-X_t^M)\Phi_t^M\right] R_t^M M_t^J + D_{t+1}^J + E^J =$$
$$= W_t \bar{L^J} + M_{t+1}^J + X_t^D R_t^D D_t^J + (1-\tau)(\Pi_t^{F,J} + \Pi_t^{B,J}) + T_t^J, \qquad \text{Budget Constraint}$$
$$M_{t+1}^J \leq \kappa P_t H_{t+1}^J, \qquad \text{Borrowing Constraint}$$
$$C_t^J, H_{t+1}^J, M_{t+1}^J \geq 0.$$

2.3.3 Banks

Banks function as financial intermediaries in the economy. They take deposits from households and make mortgage loans to households. Banks are prone to runs from depositors. As such, they are subject to capital regulations.

Bank Problem

Entry and Exit Following Gertler and Kiyotaki (2015), we assume that every period η share of the banks exit the economy. This assumption makes sure that banks do not accumulate equity infinitely.[10] To keep the aggregate number of banks constant, new banks enter at the same rate as banks exit.

Objective Function Banks are owned by patient households. As such, the objective of a bank is to maximize the discounted future dividend payouts at the discount rate of the patient households:

$$\mathbb{E}_0\left\{\sum_{t=0}^{\infty}\left[\underbrace{\frac{U_c(C_t^P,H_t^P)}{U_c(C_0^P,H_0^P)}}_{\text{Stochastic Discount Factor}}\left[\beta^P(1-\eta)\right]^t\Pi_t^B\right]\right\},$$

where η is the constant bank exit rate and Π_t^B is the dividend payout in period t, which equals the net worth of the exiting banks. The net worth of an *incumbent* bank in period t is given by:

$$n_t^B = \tilde{R}_t^M M_t^B - R_t^D D_t^B,$$

where $\tilde{R}_t^M = \left[1-(1-X_t^M)\Phi_t^M\right] R_t^M$ denotes the actual return on mortgage loans, taking into account the default and recovery of the loans. The net worth of the *aggregate banking sector* (including both incumbent and newly entering banks) in period t is given by:

$$N_t^B = \eta E^P + (1-\eta)n_t^B.$$

Recall that E^P is new capital invested by patient households into the newly entering banks, which enter the economy at rate η.

Balance Sheet Banks face a balance sheet constraint, which requires that the value of mortgage loan M_{t+1}^B on the asset side of the bank must equal the sum of deposits D_{t+1}^B and bank net worth n_t^B on the liability side of the bank:

$$M_{t+1}^B = D_{t+1}^B + N_t^B.$$

Capital Requirement / Lending Constraint Banks are subject to runs from depositors, and are thus regulated. We consider bank regulation in the form of bank capital requirement. If the capital requirement is binding, it means that banks are constrained in their ability to finance lending with household deposits. Banks are required to keep

[10] An violation of such assumption would lead to banks over-accumulating equity in equilibrium, such that they no longer need to take deposits from the households, which contradicts the role of financial intermediary of the banks.

a minimum equity to asset ratio, Γ_t:

$$\Gamma_t = \underline{\Gamma} + \gamma \left\{ \mathbb{E}_t \left[\Phi_{t+1}(1 - X^M_{t+1}) \right] \right\},$$

which consists of two components. $\underline{\Gamma}$ is the static minimum capital ratio that a bank must satisfy. The second is a dynamic component that captures provisioning against expected future credit losses. The higher expected losses on mortgage loans, the higher capital ratio a bank has to keep.

Full Statement of the Bank's Problem The bank's maximization problem is given by:

$$\max_{\left\{M^B_{t+1}, D^B_{t+1}\right\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \left[\frac{U_c(C^P_t, H^P_t)}{U_c(C^P_0, H^P_0)} \left[\beta^P (1-\eta)\right]^t \Pi^B_t \right] \right\},$$

s.t.

$$M^B_{t+1} = D^B_{t+1} + N^B_t, \qquad \text{Balance Sheet Constraint}$$

$$N^B_t \geq \Gamma_t M^B_{t+1}. \qquad \text{Bank Capital Requirement}$$

Bank Runs

Existence of the Bank Run Equilibrium We model bank runs as coordination failure of the depositors, i.e.,the patient households, as in Gertler and Kiyotaki (2015). We consider the case of a bank run on the banking sector as a whole. Once a bank run happens, banks' assets get liquidated and banks can no longer take deposits or grant mortgage loans.

In a bank run, bank assets get liquidated. Patient households receive the liquidation value of the bank assets subject to the same house quality shock. The recovery rate on deposits for patient households when a bank run happens is given by:

$$X^D_t = A_t \frac{\tilde{R}^{M*}_t M^B_t}{R^D_t D_t}, \tag{2.3.1}$$

where $\tilde{R}^{M*}_t$ is the return on mortgages in a bank run, which depends on the house price in a bank run P^*_t. We use the asterisk to distinguish variables in a run state from variables in a normal state whenever the distinction matters.

Given this recovery rate of deposits in a run, a bank run equilibrium exists when patient households cannot fully recover their deposits from liquidating the assets of the bank if a run happens, i.e.,

$$X^D_t < 1. \qquad \text{(Bank-run Condition)}$$

Following Gertler and Kiyotaki (2015), a bank run occurs with probability:

$$\pi_t^{NoRun \to Run} = 1 - \min(X_t^D, 1), \tag{2.3.2}$$

The less the households recover their deposits in a run, the more likely that they would run on the banks.

Transition Matrix After the bank run, banks reenter the economy with exogenous probability $\pi^{Run \to NoRun}$, a parameter to be calibrated. The full transition matrix between the run state and the no-run state is given by:

$$\pi_t = \begin{bmatrix} 1 - \pi_t^{NoRun \to Run} & \pi_t^{NoRun \to Run} \\ \pi^{Run \to NoRun} & 1 - \pi^{Run \to NoRun} \end{bmatrix}.$$

2.3.4 Rest of the Model

Production Consumption goods producers hire labor from households to produce consumption goods. The production technology of the consumption goods producer is given by:

$$Y_t = Z_t L_t^{\alpha},$$

where log productivity follows an AR(1) process: $\ln Z_t = \rho^Z \ln Z_{t-1} + \varepsilon_t^Z$, and $\varepsilon_t^Z \sim N(0, \nu^Z)$.

Consumption goods producers maximize profits and choose the amount of labor to employ:

$$\max_{L_t} \Pi_t^F = Z_t L_t^{\alpha} - W_t L_t,$$

Rearranging the first order condition of the consumption goods producers' problem gives the equilibrium wage:

$$W_t = \alpha Z_t L_t^{\alpha - 1}. \tag{2.3.3}$$

During a bank run, a fraction of output gets lost. We model the output loss as a reduction in labor supply from $\bar{L}$ to $(1 - \xi)\bar{L}$.

Government The government runs a balanced budget every period. That is, the capital income tax from the patient households equals the transfers made to the impatient households:

$$\tau \left(\Pi_t^{B,P} + \Pi_t^{F,P} \right) = T_t^I. \tag{2.3.4}$$

2.3.5 Equilibrium

Market Clearing

There are five markets in our model economy: the consumption goods market, the labor market, the housing market, the deposit market, and the mortgage market. The deposit and mortgage markets are only active in the no-run state.

Market Clearing in the No-Run State In the case without bank runs, all five markets are active. The market clearing conditions in a no-run state are (recall that the share of patient households is μ and impatient households is $1-\mu$):

$$Y_t = \mu C_t^P + (1-\mu)C_t^I + \delta P_t \mathscr{H}, \tag{2.3.5}$$
$$L_t = \mu \bar{L^P} + (1-\mu)\bar{L^I}, \tag{2.3.6}$$
$$\mathscr{H} = \mu H_{t+1}^P + (1-\mu)H_{t+1}^I, \tag{2.3.7}$$
$$D_{t+1}^B = \mu D_{t+1}^P, \tag{2.3.8}$$
$$M_{t+1}^B = (1-\mu)M_{t+1}^I. \tag{2.3.9}$$

In Equation (2.3.5), $\delta P_t \mathscr{H}$ denotes the amount of consumption goods that is used to make up for the depreciated value of housing every period, such that housing supply in the economy is constant.

Market Clearing in the Run State In the case of a bank run, all banks get liquidated. Therefore, there are no deposit and mortgage markets in the economy. The consumption goods market and housing market clear just as in the no-run state. The labor market clearing needs to adjust for the reduction in labor supply during a financial crisis. In sum, the market clearing conditions in a bank run state are:

$$Y_t = \mu C_t^P + (1-\mu)C_t^I + \delta P_t^* \mathscr{H}, \tag{2.3.10}$$
$$L_t = (1-\xi)\left[\mu \bar{L^P} + (1-\mu)\bar{L^I}\right], \tag{2.3.11}$$
$$\mathscr{H} = \mu H_{t+1}^P + (1-\mu)H_{t+1}^I. \tag{2.3.12}$$

Competitive Equilibrium

The recursive competitive equilibrium of the model economy is given by a sequence of prices:

$$\left\{P_t, R_t^D, R_t^M, W_t\right\}_{t=0}^{\infty},$$

and allocations:

$$\{C_t^P, C_t^I, H_{t+1}^P, H_{t+1}^I, D_{t+1}^B, D_{t+1}^P, M_{t+1}^B, M_{t+1}^I, L_t\}_{t=0}^{\infty},$$

such that, if the economy is in a no-run state according to Equation (Bank-run Condition), markets clear as described in Equations (2.3.5) to (2.3.9) and agents solve their respective optimization problems described by Equations (B.3.1) to (B.3.4) in the Appendix for the impatient households, (B.3.5) to (B.3.8) for the patient households, the (Balance Sheet Constraint) and the (Bank Capital Requirement) for the banks and Equation (2.3.3) for the consumption goods producers. If the economy is in a run state according to Equation (Bank-run Condition), markets clear according to Equations (2.3.10) to (2.3.12) and agents solve the optimization problems described by Equations (B.3.1) to (B.3.8).

2.4 Financial Accelerator Effect under Different Financial Constraints

In the model, the equilibrium mortgage loan amount is determined either by the borrowing constraint faced by the households or the lending constraint faced by banks. Since house prices affect both types of constraints, they create a positive feedback loop: if house prices increase, both borrowing and lending constraints loosen. As a result, house prices increase further. Therefore, the financial accelerator effect is active in both cases, but it operates through different channels.

For the purpose of this section, it is convenient to redefine the capital requirement as a constraint on aggregate bank leverage. In other words, the capital requirement is equivalent to a constraint on the maximum level of bank leverage:

$$\frac{M_{t+1}}{N_t^B} \leq \psi_t,$$

where ψ_t is the maximum bank leverage corresponding to the minimum capital requirement faced by the bank, i.e.,$\psi_t = \frac{1}{\Gamma_t}$.

2.4.1 Case I: Binding Borrowing Constraint

If the borrowing constraint is binding, the equilibrium mortgage loan is given by

$$M_{t+1} = \kappa P_t H_{t+1}^I.$$

A marginal increase in the house price increases mortgage credit by $\kappa H^I_{t+1} + \kappa P_t \frac{\partial H^I_{t+1}}{\partial P_t}$. There is a negative income effect, a negative substitution effect and a positive wealth effect which determine the sign of $\frac{\partial H^I_{t+1}}{\partial P_t}$.

Income effect: Under a higher house price, the budget set of households shrinks, holding wealth constant. If housing is a normal good, this will reduce the demand for housing.

Substitution effect: As the relative price of housing increases, holding income and wealth constant, the demand for housing decreases.

Wealth effect: A higher house price increases the value of houses and hence assets of households. Consequently, there is a higher demand for housing.

If the income and substitution effects dominate the wealth effect, then $\frac{\partial H^I_{t+1}}{\partial P_t} < 0$, which will reduce the strength of the financial accelerator effect.

2.4.2 Case II: Binding Lending Constraint

If the lending constraint is binding, the amount of mortgage loans in equilibrium is given by the capital requirement for banks:

$$M_{t+1} = \psi_t N^B_t.$$

Constant capital requirement Consider first the case of a constant capital requirement, $\psi_t = \psi$. A marginal increase in the house price hence increases mortgage credit ceteris paribus by $\psi \frac{\partial N^B_t}{\partial P_t}$, with

$$\frac{\partial N^B_t}{\partial P_t} = (X^M_t - 1)\frac{\partial \phi^M_t}{\partial P_t} + \phi^M_t \frac{\partial X^M_t}{\partial P_t}.$$

As such, an increase in the house price has two effects on bank net worth: First, it leads to a lower mortgage default rate $\frac{\partial \phi^M_t}{\partial P_t} < 0$, which increases the net worth of the banks (since $X^M_t < 1$). Second, it increases the recovery value of banks conditional on a default, $\frac{\partial X^M_t}{\partial P_t} > 0$, which also increases the net worth of banks. Overall, $\frac{\partial N^B_t}{\partial P_t} > 0$, such that a higher house price will increase the mortgage credit. Importantly, fluctuations in bank net worth translate into fluctuations in mortgage credit by a factor of $\psi > 1$.

Dynamic Capital Requirement In the case of a dynamic capital requirement, the regulator can offset or amplify the effect of fluctuations in house prices on mortgage credit:

$$\frac{\partial M_{t+1}}{\partial P_t} = \frac{\partial \psi_t}{\partial P_t} N^B_t + \psi_t \frac{\partial N^B_t}{\partial P_t}.$$

Parameter	Description	Value	Target / Source
		Households	
β^P	Discount factor of patient households	0.9949	Annual real deposit rate = 0.3%
β^I	Discount factor of impatient households	0.9919	Annual real mortgage rate = 1.1%
χ^P	Patient HH consumption weight	0.9425	Value added of real estate activities/GDP = 5%
χ^I	Impatient HH consumption weight	0.7875	Share of houses with mortgage loans = 40%
μ	Share of patient households	0.6	Share of homeowners w/o mortgage = 60%
σ	Risk aversion	2	Kaplan, Mitman, and Violante (Kaplan et al.)
δ	Depreciation of housing stock	0.00625	Favilukis et al. (2017)
κ	Borrowing constraint	0.8	Maximum LTV ratio = 80%
ν^ε	Volatility, idiosyncratic house value shock	0.2675	Quarterly default rate = 0.5%
		Banks	
$\underline{\Gamma}$	Minimum bank capital requirement	0.08	Maximum bank leverage = 12.5
γ	Dynamic provisioning parameter	0	No dynamic provisioning in the baseline model
η	Bank exit rate	0.1	Bank asset to quarterly GDP ratio = 9.761
$\pi^{Run\to Run}$	Bank run persistence	12/13	Average run length = 3.25 yrs
		Production and Government	
α	Labor share of output	0.572	Labor share of output = 57.2%
ρ^Z	Autocorrelation, productivity	0.9704	Autocorrelation of detrended real GDP = 0.9704
ν^Z	Volatility, productivity	0.0145	Unconditional volatility of detrended real GDP = 6.13%
ρ^A	Autocorrelation, aggregate liquidation shock	0.95	Autocorrelation of bank equity = 0.9315
ν^A	Volatility, aggregate liquidation shock	0.01	Volatility of bank equity = 17.56%
τ	Capital income tax rate	0.2	Capital income tax rate = 20%
ξ	Labor supply loss in bank run	0.1	Unemployment increase during the financial crisis = 10%

Table 2.1 Parameters of the baseline model

If $\frac{\partial \psi_t}{\partial P_t} = -\frac{\psi_t}{N_t^B}\frac{\partial N_t^B}{\partial P_t}$, the regulator can offset the effect of fluctuations in house prices on mortgage credit completely.

2.4.3 Financial Constraints and Bank Runs

In general, a binding borrowing constraint implies that the lending constraint is not binding,[11] i.e.,$M_{t+1} < \psi_t N_t^B$. This means that banks have excess leverage capacity. Therefore, the probability of a bank run is low in this case.

2.5 Calibration

In this section, we describe the calibration of the model. Our goal is to characterize quantitatively the behavior of the Spanish economy in the recent financial crisis. We solve for both the no-run and run equilibrium using global nonlinear methods. A detailed description of the solution algorithm can be found in Appendix B.4.

2.5.1 Parameters

The choice of parameter values are listed in Table 2.1. Each model period corresponds to a quarter. For the baseline calibration, we use the Spanish data between 1997Q3 and 2017Q3. A detailed description of the dataset can be found in Appendix B.1.

There are in total 21 parameters in the model to be calibrated. We begin with the parameters related to the household sector. The discount factors of the households, β^P and β^I, are set to match the average real interest rate on deposit and mortgage loan,[12] respectively. The consumption weight in utility for impatient households, χ^I, is set to match the share of houses financed by mortgage loans, i.e.,share of houses owned by impatient households. The consumption weight for patient households, χ^P, is calibrated to match the value added of real estate activities as share of GDP, i.e., patient households' housing expenditure. The share of homeowners without mortgage loans in Spain is 60 percent, therefore the share of patient households μ is set to 0.6. The households risk aversion is set to $\sigma = 2$, following the convention in the macroeconomic literature. The depreciation rate of housing is set to 2.5 percent per year following Favilukis et al. (2017). The borrowing constraint parameter, κ, is set to match the maximum LTV ratio in 1997-2017, which is 80 percent. Finally the volatility of idiosyncratic house value shock is set to match an annual mortgage default rate of 2 percent.

The parameters related to banks are calibrated using the following strategy. The constant component of bank capital requirement $\underline{\Gamma}$ is set to be 8 percent, corresponding to the average bank leverage of 12.5 in 1997-2017. The bank exit rate η is set to match the average bank asset to quarterly GDP ratio of 9.761. The bank run persistence rate $\pi^{Run \to Run}$ is set to match an average length of bank runs of 3.25 years.[13]

The rest parameters are related to the production sector and the government. The production function parameter α is chosen to match the labor share of output of 57.2 percent.[14] The autocorrelation and volatility of productivity are calibrated to match the data counterpart of detrended real GDP. The autocorrelation and volatility of the aggregate liquidation shock are calibrated to match the autocorrelation and volatility of bank equity. We choose the value of capital income tax rate to match the capital income tax in Spain of 20 percent. The labor supply loss ξ is set to 10 percent to account for the increase in unemployment rate during the Spanish financial crisis.

[11] Unless both lending and borrowing constraints happen to be binding at the same time, which is unlikely.

[12] Only interest rate on newly issued mortgage loans are included in the calculation of the interest rate. Existing loans are excluded from the calculation.

[13] We use financial crises data of OECD countries after WWII to calculate the average length of bank runs in OECD countries, see Chapter 1 Section 1.3.

[14] See Estrada et al. (2014), Figure 3.

Table 2.2 lists the targeted moments where the parameter values are not set to match the data moments exactly. It can be seen that the calibrated model matches real estate share of GDP, mortgage financed house share and mortgage default rate very well. However the bank asset to GDP ratio under this calibration is rather low compare to the data.

	Model	Data
Real estate share of GDP	5.5%	5%
Houses with mortgages	41.17%	40%
Quarterly mortgage default rate	0.66%	0.5%
Bank asset to GDP ratio	1.84	9.76

Table 2.2 Model fit: targeted moments

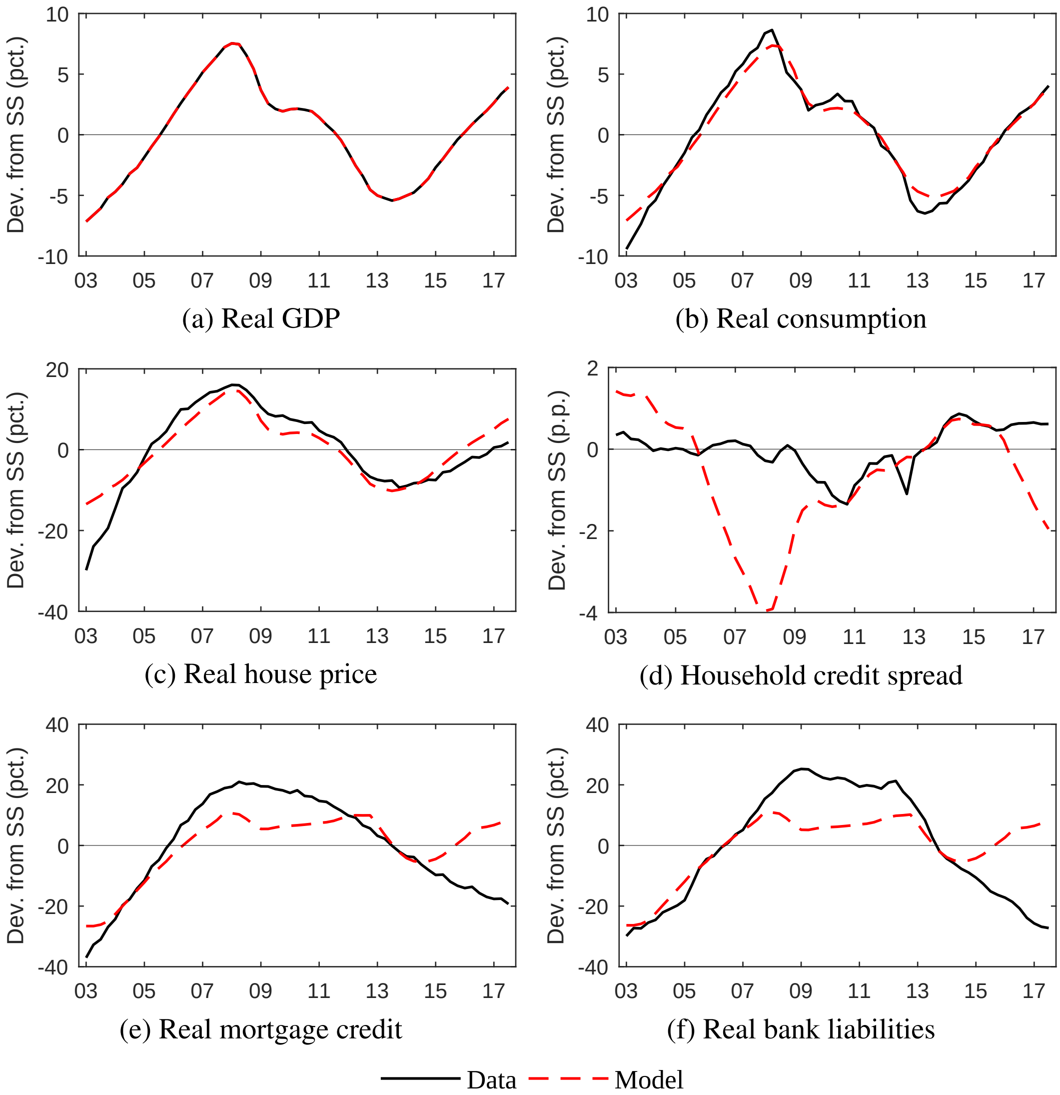
2.5.2 Matching Aggregate Dynamics

In this subsection, we calibrate a series of productivity shocks such that the model matches the historical evolution of the detrended GDP, during the years 2003-2017 in Spain. This exercise allows us to evaluate the ability of the model to predict the historical movements in key economic and financial variables, and to shed light on the contribution of productivity shocks to the consumption and housing-financial cycles.

Figure 2.5 plots the endogenous and actual movements in GDP, consumption, house prices, the household credit spread, mortgage credit, and bank liabilities, conditional on the productivity shocks. The model, by construction, perfectly matches the historical path of GDP. However, an important success of the model is that it closely predicts the empirical paths of consumption and house prices. The direct implication of this is that productivity shocks have been the principal source of variation not only in GDP, but also in house prices and consumption. This result is interesting, since linear and piecewise linear DSGE models typically *also* need to rely on intertemporal preference and housing preference shocks in order to match consumption and house prices accurately (e.g., Liu et al. (2013), Guerrieri and Iacoviello (2017)).

The model also matches the lion's share of fluctuations in mortgage credit and bank liabilities. In both the model and the data, mortgage credit and bank liabilities rise until around 2008, after which they remain roughly constant for four years, and then fall. The model captures this financial expansion through two channels. First, the initially high labor incomes induced the patient homeowners to save more, consequently forcing banks to narrow the household credit spread, so as to invest the savings. This is evident from Figure 2.5d, which shows that the theoretical credit spread fell in the boom

Fig. 2.5 Empirical and theoretical paths of aggregate variables



(a) Real GDP (b) Real consumption
(c) Real house price (d) Household credit spread
(e) Real mortgage credit (f) Real bank liabilities

Note: GDP, consumption, house prices, mortgage credit, and bank liabilities have been detrended by series-specific linear trends.

years, and rose in the bust years, roughly consistent with the data. Second, the house price appreciation relaxed the collateral constraint, concurrently allowing impatient homeowners to take on additional debt. The financial expansion was eventually undone, as a series of negative productivity shocks capped labor incomes, and caused house prices to plummet, from around 2009.

2.5.3 The Role of Financial Shocks and Real Shocks

In this subsection, we report the generalized impulse response functions (GIRF) to both the house quality shock and the productivity shock.[15] The goal is to investigate to which extent these shocks can match the following key dynamics observed during the Spanish financial crisis, as we documented in section 2:

- A strong and persistent decrease in GDP and house prices.
- Deleveraging in both the banking and the household sector.
- An increase in the credit spread for banks (i.e., the spread between deposit rates and the risk free interest rate), and a decrease in the credit spread for households (i.e., the spread between mortgage rates and deposit rates).

To compute the generalized impulse responses, we simulate one million economies for 1100 periods, discard the first 1000 periods, and hit the economy with an additional shock in period 1010. The reported impulse responses are the averages across the simulated economies.

House quality shock

Figure 2.6 shows the response of the economy to a negative five standard deviation house quality shock. In the current version of the model, this shock reduces only the value of a house that is used as collateral for a defaulted mortgage. We choose the size of the shock to ensure that the shock will induce bank runs for at least some simulated economies. A five standard-deviation house quality shock reduces the value of a house by about 7 percent.

As Figure 2.6 shows, a negative house quality shock leads to a persistent decline in output. This decline is in line with our empirical evidence. Intuitively, output declines because a house price shock can trigger a bank run by reducing the value of the assets of banks below the value of their liabilities. Houses show up on the asset side of the bank balance sheet, since banks seize the houses underlying defaulted mortgages. A bank run, in turn, causes an exogenous output loss. Quantitatively, the initial response is not very large: A decrease in the value of defaulted mortgages of 7 percent leads to a decrease in output of 0.25 percent. The response of output to a house quality shock is however very persistent, since bank runs in our model are very persistent events: Even after 20 quarters, output is on average 0.2 percent lower than before the shock.

[15] The model is non-linear. Hence, the impulse response to any given shock depends on the initial state of the economy. Therefore, we focus on generalized impulse response functions. Intuitively, these can be interpreted as the average response of the economy to a shock.

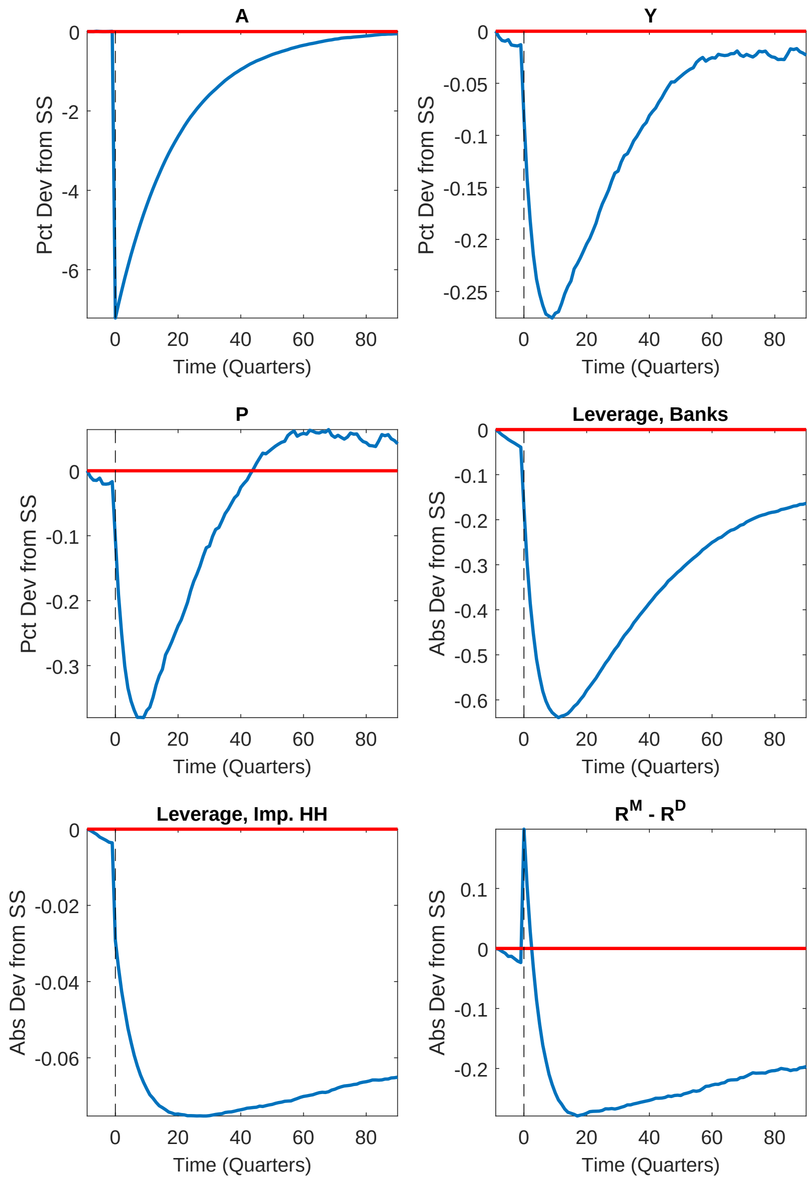


Fig. 2.6 Generalized impulse response functions to a negative 5-sd house quality shock at $t = 0$

Similarly to output, the shock leads to a persistent and endogenous decline in house prices. This is also in line with the empirical evidence. The main reason for this decline is that during a bank run, output and hence labor income of both patient and impatient households are lower, which lowers housing demand. The house price adjusts downward to equate housing demand and supply. Through a deterioration in the balance sheet of banks, this decrease on house prices endogenously amplifies the decrease in housing demand. As a result, house prices decrease by about 0.4 percent after the shock, and stay below the initial state until about 40 quarters after the shock.

As in the data, bank leverage decreases strongly in response to the house quality shock. In the current version of the model with unanticipated bank runs, this deleveraging is the result of realized bank runs, in which bank leverage decreases to 1, i.e., banks cannot use leverage. In a version of the model with anticipated bank runs, the shock would raise the borrowing costs of the banking sector even in a state without a bank run by raising the likelihood of a future bank run, and hence the bank run risk premium on deposits. This would reduce the profitability margin of banks from issuing deposits to finance mortgages, which can induce them to choose a leverage ratio which is below the regulatory leverage constraint.

Household leverage decreases, but more weakly and with a bigger delay than bank leverage. This also squares well with the empirical evidence from the Spanish financial crisis. The main reason for this decline is that banks cannot extend mortgages to households during a bank run, such that impatient households need to finance house purchases fully with equity.

The spread between deposit rates and mortgage rates increases on impact, reflecting an increase in default rates after the negative house quality shock. Subsequently, as households deleverage, the default rate of households decreases, which leads to a reduction in the spread between mortgage and deposit rates.

The spread between the deposit rate and the risk free rate is flat in the model with unanticipated bank runs.[16] In the model with anticipated bank runs, this spread would increase, reflecting both an increase in expected default losses and in the household risk premium.

All of these dynamics are consistent with the dynamics observed during the Spanish financial crisis, lending support to the importance of the house quality shock in the financial crisis. However, the shock has currently relatively weak effects, since it only affects houses serving as collateral for defaulted mortgages.

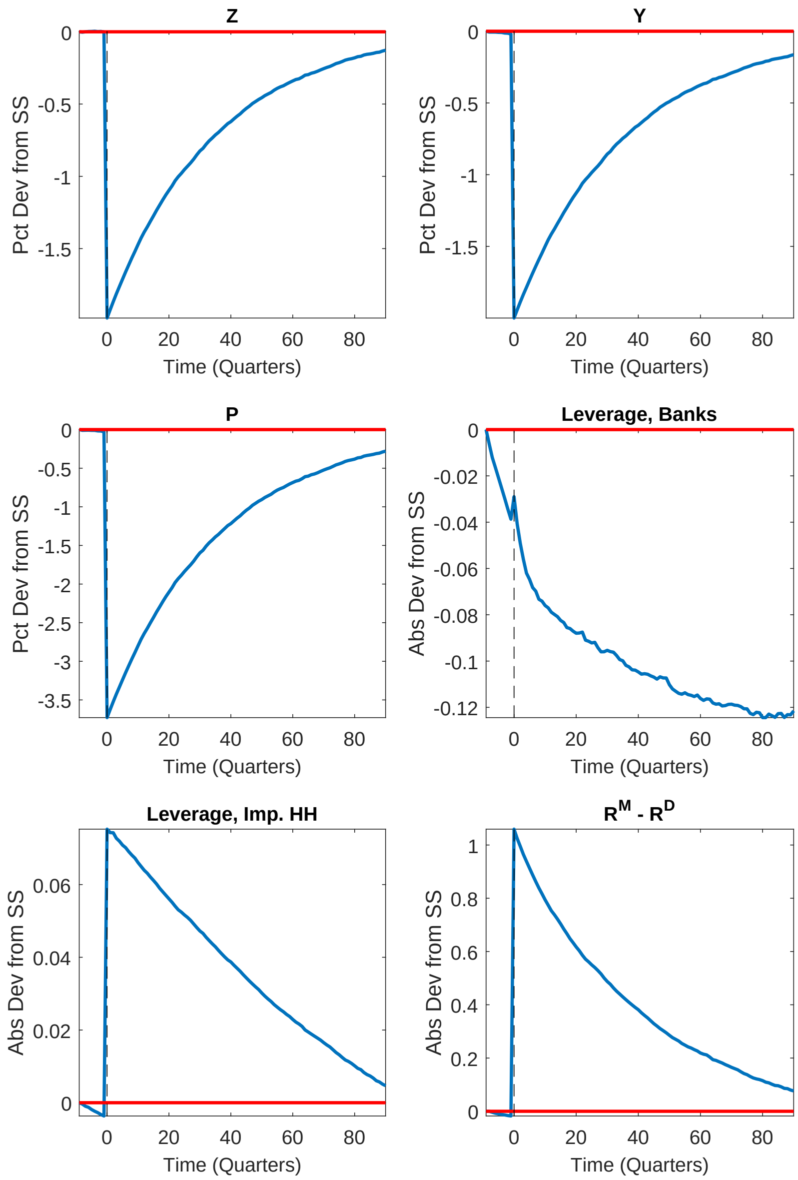


Fig. 2.7 Generalized impulse response functions to a negative 2-sd productivity shock at $t = 0$

Productivity shock

Figure 2.7 shows the result of a negative two standard deviation shock to productivity.

A negative productivity shock leads to persistent decrease in output of the same magnitude. Unless there is a bank run, output in the model moves one for one with productivity since labor supply is exogenous. In a bank run, an additional output loss can occur.

House prices also decrease, as lower household income reduces household demand. House prices are more than twice as volatile as output, but exhibit otherwise very similar dynamics.

Bank leverage decreases only very little, but very persistently in response to a productivity shock. This is because the bank profit margin from deposit financed mortgage lending, $\tilde{R}^M_{t+1} - R^D_{t+1}$, decreases. This reduction in the profit margin arises, because the consumption of patient households in response to the shock decreases more than the consumption of impatient households. Hence, the mortgage rate $\tilde{R}^M_{t+1}$, which is determined by the Euler equation of impatient households, increases less than the interest rate on deposits, R^D_{t+1}, which is determined by the Euler equation of patient households. In some cases, this induces banks to reduce their leverage below the leverage constraint.

In contrast to the data, impatient household leverage increases, as their net worth decreases more than their assets. The net worth of impatient households is given by the value of their end of period assets minus the value of their end of period liabilities. Their end of period assets are just their houses at the market value, $P_t H^I_{t+1}$. The amount of houses they own is determined by the Euler equation of the impatient household with respect to housing. Their liabilities are their mortgage debt M^I_{t+1}. The amount of mortgage debt they take up is either determined by their borrowing constraint, by the leverage constraint of the banking sector, or, if none of those constraints is binding, by the Euler equation of the impatient households with respect to mortgages. In the current calibration, it is mostly the leverage constraint of the banking sector which is binding. Hence, household mortgage borrowing is largely unresponsive to the balance sheet of the impatient households and decreases less than the value of their assets. Net worth must therefore, to satisfy the balance sheet constraint of impatient households, decrease more than the value of their assets, which pushes up the leverage of impatient households.

As a consequence of the higher household leverage, the default rate increases, which increases the default premium on mortgages. Hence, the spread between mortgage interest rates and deposits, $R^M_{t+1} - R^D_{t+1}$ increases. Note that this does not

[16] The risk-free rate is the interest rate on a non-defaultable government bond held by households that is in zero net supply.

preclude that the bank profit margin, which is given by the default risk-adjusted spread $\tilde{R}^M_{t+1} - R^M_{t+1}$, decreases.

With unanticipated shocks, the spread between the return on deposits and the risk-free interest rate is again flat. If bank runs are instead anticipated, a negative productivity shock can either increase or decrease the probability of a bank run: On the one hand, bank profit margins decrease, which increases the bank run probability. On the other hand, banks also reduce their leverage, which reduces the bank run probability. Hence, the effect of a productivity shock on the bank run premium, i.e., on the spread between the deposit rate and the risk-free rate, is theoretically ambiguous in this model.

In conclusion, while productivity shocks and house quality shocks lead to the same sign for output, house price and bank leverage dynamics, they do so through very different mechanisms. Moreover, they lead to different dynamics of household leverage and hence the spread between mortgage and deposit rates. Finally, while house quality shocks lead to a clear prediction for the spread between the deposit rate and the risk-free rate, productivity shocks do not. Therefore, given the data we have, the two shocks are in principle identified.

2.6 Macroprudential Policy

In this section, we evaluate the macroeconomic effects of three different macroprudential policies:

1. Increasing the minimum bank capital requirement
2. Imposing dynamic loan loss provisioning on banks
3. Regulating the maximum LTV ratio of mortgage borrowers

For each policy, we study their impact on the frequency of financial crisis and business cycle fluctuations, as well as their long-run effects on the households and the aggregate economy.

To do so, we simulate 1000 economies for 2000 periods and discard the first 1000 periods for each counterfactual policy. The reported results are averages across economies. The policy experiment results are reported in Table 2.3, where the baseline case is defined with $\underline{\Gamma} = 0.08$, $\gamma = 0$ and $\kappa = 0.8$.

2.6.1 Higher Capital Requirement

As the first policy experiment, we double the minimum capital requirement from 8 to 16 percent. We report the results of this exercise in column 3 of Table 2.3.

A higher capital requirement increases the average house price. It also increases the consumption of impatient households and decreases the consumption of patient households. The intuition for this result is that with a higher capital requirement, impatient households borrow less and hence accumulate more wealth. Since they value houses more than patient households, house price increases. Under a higher capital requirement, the volatility of house prices and household consumption is lower as well, which is mostly due to the lower frequency of bank runs.

Increasing the capital requirement leads to a direct decrease in bank leverage. As banks reduce mortgage lending to households, the leverage of impatient households decreases as well.

As households adopt a much lower leverage under the higher capital requirement, the mortgage default rate falls. As such, the credit spread between mortgages rates and deposit rates also decreases. The recovery rate on defaulted mortgages increases to 100 percent with the higher capital requirement, since there is almost no more mortgage default.

Finally, as a result of a combination of lower bank leverage, lower mortgage default rate and higher mortgage recovery rate, the policy eliminates bank runs from the economy completely.

2.6.2 Expected Loan Loss Provisioning

For the second policy experiment, we maintain the minimum capital requirement of 8 percent, while allowing it to increase to cover the expected losses on mortgage loans. That is, on top of the regulatory capital, banks have to hold additional capital to cover expected losses on their mortgage loans. Such a capital requirement is weakly procyclical, i.e., higher if GDP is high. It corresponds roughly to the dynamic provisioning introduced in Spain, which, despite having a counter-cyclical component, was overall procyclical. The policy experiment results are reported in column 4 of Table 2.3.

We find that the expected loan loss provisioning slightly increases the house prices. The intuition for this result is similar as before: as impatient households reduce their leverage slightly under the provisioning policy, they accumulate more wealth and demand more houses, driving up the house prices. This policy leads to a slight increase in the consumption for impatient households and has barely any effect on the patient households' consumption. Similar to the higher capital requirement policy, the expected loan loss provisioning also leads to lower volatility of house prices and household consumption.

The expected loan loss provisioning leads to a slight decrease in household leverage and a slight increase in bank leverage. Importantly, the bank leverage becomes less

	Baseline	$\underline{\Gamma} = 0.16$	$\gamma = 1$	$\kappa = 0.4$
Real Economy				
Avg House Price (Level)	7.700	7.837	7.706	7.779
Avg Consumption, Patient HH (Level)	1.112	1.108	1.111	1.112
Avg Consumption, Impatient HH (Level)	0.723	0.740	0.732	0.731
StDev(House Price) (%)	8.652	8.085	8.446	8.297
StDev(Consumption, Patient HH) (%)	5.103	4.130	4.609	4.276
StDev(Consumption, Impatient HH) (%)	11.086	3.972	6.212	5.471
Leverage				
Leverage, Impatient HH	1.811	1.231	1.803	1.608
Leverage, Banks	11.660	6.219	12.081	10.102
Corr(Household Leverage,GDP)	-0.305	-0.936	-0.585	-0.403
Corr(Bank Leverage,GDP)	0.330	0.353	0.193	0.645
Default Rates and Asset Prices				
Spread $R^M - R^D$ (% per year)	1.933	1.396	1.375	1.570
HH Default Rate (% per year)	2.905	0.040	1.497	0.223
Mortgage Recovery Rate (%)	86.263	100.000	85.317	90.674
Corr(Spread, GDP)	-0.089	-0.401	-0.215	-0.562
Corr(HH Default Rate, GDP)	-0.252	0.000	-0.677	-0.595
Corr(Recovery Rate, GDP)	0.206	0.000	0.405	0.538
Bank Runs				
Bank Runs per 100 years	1.564	0.000	0.411	0.149

Table 2.3 Policy experiment results for three different macroprudential policies: minimum capital requirement, expected loan loss provisioning, and maximum LTV constraint

cyclical under the provisioning policy, suggesting that the policy does play a role in reducing the cyclicality of the capital structure of banks.

As the households are less leveraged, the mortgage default rate decreases. As a result, the credit spread for households decreases accordingly. However, the recovery rate on mortgages decreases slightly despite the small increase in house prices.

Finally, the provisioning policy can also effectively reduce bank run frequency, but to a lesser extent than doubling the minimum capital requirement, with the frequency of bank runs decreasing from 1.6 to 0.4 runs per 100 years.

2.6.3 Tighter LTV Constraint

In the last policy experiment, we halve the maximum permissible LTV ratio of households from 80 to 40 percent while keeping the minimum bank capital requirement of 8 percent. In other words, under the new LTV constraint, households can borrow up to only 40 percent of the house value from the banks. The results of this policy experiment are reported in column 5 of Table 2.3.

Similar to the two policies discussed above, the tighter LTV constraint also leads to higher house prices. The increase in house prices under the tighter LTV constraint is weaker than the increase under the higher capital requirement but much stronger than the increase under the provisioning policy. It again leads to a tiny increase in consumption of impatient households due to lower household leverage, and has no effect on the consumption of patient households.

The tighter LTV constraint leads to lower leverage of both households and banks, but the effect is weaker than the higher capital requirement policy. Moreover, household leverage becomes more countercyclical and bank leverage more procyclical. This is because mortgage credit under a tight LTV constraint is more often determined by the LTV constraint instead of the capital requirement of the bank, and the LTV constraint depends more strongly on the very volatile house prices.

The mortgage default rate decreases as households use less leverage in their house purchases. Banks' recovery rate on mortgage loans also increases accordingly. Therefore, the credit spread on mortgage loan decreases with a tighter LTV constraint.

Finally, tightening the LTV constraint reduces the frequency of bank runs from 1.6 to 0.15 runs per 100 years, since it reduces the bank leverage and the mortgage default rate simultaneously.

2.7 Conclusion

We build a tractable macroeconomic model to study a joint housing and financial crisis, as was experienced in Spain in 2008-2016. We empirically document key features of this financial crisis, namely a fall in house prices, a reduction in bank and household leverage, and a rise in credit spreads for banks as well as borrowers. The model aims to capture these key features in a parsimonious way. Key features of the model are that both borrowers and banks use non-contingent debt. Borrowers may endogenously default on their mortgage loans, and banks are subject to bank runs by depositors. Dynamics in the model are driven by both real and financial shocks.

We find that all three macroprudential policies can reduce the mortgage default rate and the frequency of bank runs, but the effect is stronger with the higher capital requirement and the tighter LTV constraint. Dynamic loan loss provisioning is effective at reducing the cyclicality of the capital structure of banks, while a tighter LTV constraint amplifies the cyclicality of bank and household leverage.

In future work, we plan to use the model as a framework to disentangle to what extent financial risk and productivity risk contributed to the housing crisis in Spain.

Furthermore, we want to characterize the optimal policy mix between rule-based capital requirements as well as LTV constraints from a welfare point of view.

Chapter 3

Capital Requirements and Credit Easing: Ex-ante vs Ex-Post Intervention Policy

3.1 Introduction

In the first two chapters, the policy focus is on ex-ante intervention policies for combating financial crises, i.e., bank capital requirement in Chapter 1 and Chapter 2, and dynamic provisioning and loan-to-value (LTV) constraints in Chapter 2. These policies are introduced to improve the financial stability of the economy and prevent future financial crises from an ex-ante perspective. So far, another class of policies has been ignored: the ex-post intervention policies, such as credit easing and bank bailouts upon the onset of a financial crisis. These policies are often introduced upon the break out of a crisis to mitigate the collapse of the financial system and expedite the recovery of the economy.

The 2007-2009 global financial crisis has spurred a rapidly growing literature on policy interventions for financial crises.[1] There is extensive policy debate among policymakers and macroeconomists on both ex-ante and ex-post policies. A discussion on the policy debate over bank capital requirement as an ex-ante intervention policy is provided in Chapter 1. Regarding the ex-post intervention policies, supporters argue that they are important in increasing market liquidity and stabilizing the economy during the crises, whereas the opponents emphasize that such policies are costly for

[1] See, for instance, Gertler and Karadi (2011), Farhi and Tirole (2012), Keister (2015), Gertler et al. (2016), Bianchi (2016), Garcia-Macia and Villacorta (2016), Farhi and Werning (2016), Korinek and Simsek (2016), Dávila and Korinek (2017), Gersbach and Rochet (2017), and Begenau and Landvoigt (2017).

taxpayers and can cause moral hazard problems if financial institutions anticipate to have an easy way out in crisis times.

In this chapter, I introduce credit easing as an ex-post intervention policy and compare its macroeconomic effects with the effects of bank capital requirement, an ex-ante policy. In particular, the research questions I address in this chapter are the following: How does each policy impact the frequency and severity of financial crises? What are the effects of these policies on financial intermediation? What are their welfare implications? Are the two policies supplementaries or complementaries?

To address these questions, I introduce a credit easing policy à la Gertler and Karadi (2011) into a simplified version of the model in Chapter 1 by having only one banking sector instead of both retail and shadow banking. The credit easing policy corresponds to the United States Emergency Economic Stabilization Act of 2008, in which the US Secretary of the Treasury was authorized to spend up to 700 billion dollar to purchase distressed assets, including mortgage-backed securities and corporate bonds, to provide liquidity and increase availability of credit to the private sector.[2] Upon reasonable parameterization, I conduct quantitative policy experiments of the two intervention policies and compare their macroeconomic, financial stability and welfare implications.

I show that these two intervention policies exhibit very different trade-offs. By limiting the leverage and risk exposure of banks in normal times, the ex-ante bank capital requirement reduces the frequency and severity of financial crises, at the cost of reducing financial intermediation in the economy, which is in line with our findings in the first two chapters. To implement a credit easing policy, the central bank expands its balance sheet and purchases private-sector assets at the onset of a financial crisis to stabilize the asset prices in fire sales and reduce price volatility during a financial crisis. However, aside from the ex-post stabilizing effect, the credit easing policy also causes an *unintended ex-ante effect*: as the policy stabilizes asset prices during crisis periods, the higher fire sale prices reduce the banks' risk premium when a run hits, making it more profitable for banks to use credit. Hence, banks take on higher leverage and the financial system becomes more vulnerable to negative shocks and is subject to more frequent runs. Nonetheless, the credit easing policy facilitates financial intermediation in normal times and reduces the severity of a financial crisis by mitigating asset price drops during crises. I find the two policies complement each other well. The combination of the two policies reduces the frequency and severity of financial crisis while maintaining the efficiency of the financial system, achieving higher welfare of the agents.

[2]"US backs away from plan to buy bad assets" Reuters. Retrieved November 15, 2008. Link: https://www.reuters.com/article/us-financial-paulson/u-s-backs-away-from-plan-to-buy-bad-assets-idUSTRE4AB7P820081112.

3.1.1 Related Literature

This chapter is closely related to the fast growing literature on macroprudential policies and ex-post intervention policies for combating financial crises. It contributes to the literature that analyzes the effects of prudential policies and ex-post intervention policies on financial stability and financial intermediation. For example, Gertler and Karadi (2011) evaluate the effects of unconventional monetary policy on financial stability and illustrate the moderate effect of central bank credit interventions on economic downturns (but not bank runs). Keister (2015) argues that eliminating bailouts of financial intermediaries makes them too cautious from a social point of view, which can result in under-provision of financial services. Bianchi (2016) studies the credit bailout policy for non-financial firms and points out that the anticipation of bailout policies leads to an increase in risk-taking, making the economy more vulnerable to financial crises. Begenau and Landvoigt (2017) show that tightening bank capital requirement leads to a more financially stable retail banking system but more intermediation activity by the shadow banking sector. Among this line of literature, little has been done in comparing the effects of ex-ante and ex-post intervention policies or studying the policy implication of combined policy intervention. This chapter fills this gap in the literature.

The remainder of the chapter is structured as follows. Section 3.2 describes the model economy. Section 3.3 lays out the parameterization of the model. Section 3.4 presents the policy experiment results and provides a discussion of the findings. Section 3.5 concludes.

3.2 Model

The modeling framework is a simplified version of the two-banking sector model in Chapter 1. I merge the two banking sectors into one banking sector and add a central bank to the model. The model economy is populated by households, banks, capital producers, consumption goods producers and a central bank.

Households are the ultimate investors in the economy. There are two ways for households to make investments. One is to invest directly to consumption goods producers (or “firms" for short henceforth). The other is to make deposits at banks. Banks intermediate investments between households and firms. As financial intermediaries, banks have a relative advantage in their expertise in screening and monitoring investments comparing to the households. However, financial market frictions (asset diversion) restrict banks’ ability to borrow from households. As a result, households have to make a certain amount of direct investment. Capital producers produce capital using consumption goods subject to capital adjustment costs. Consumption goods producers hire labor and rent capital from households and banks to produce consumption goods. The central bank's role is to facilitate financial intermediation in the event of a break down of the financial system, i.e., a bank run.

Figure 3.1 illustrates the resource flows in the model economy. The solid line between households and consumption goods producers means this flow of capital happens in every state of the economy. The dotted lines jointing the banking sector denote resource flows that only exist in normal times and not in bank run periods, since all banks collapse in a bank run. The dashed lines jointing the central bank are just the opposite – the resource flows exist only in the state of bank runs (and for certain periods after the runs).

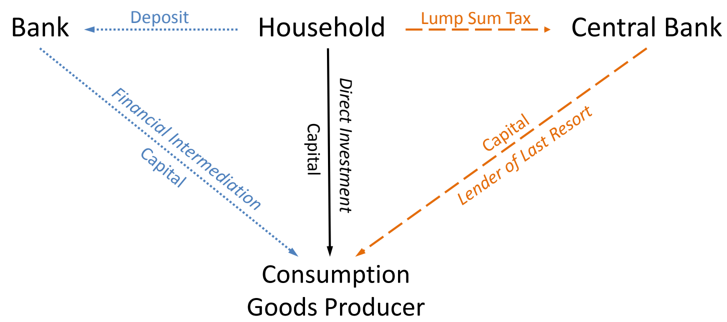


Fig. 3.1 Overview of the economy in equilibrium

3.2.1 Households

Households consume consumption goods and provide one unit of labor inelastically to receive wage W_t. They invest K^H_{t+1} directly at firms and deposit D^H_{t+1} at banks in period t. The price of capital relative to consumption goods is denoted by Q_t. In the next period, provided that a bank run does not happen, the deposits at banks yield a gross return R^D_{t+1}, and investment at firms generate a gross return R^K_{t+1}, which is given by:

$$R^K_{t+1} = r^K_t + (1-\delta)Q_t,$$

where r^K_t is the net return on capital investment and δ is the capital depreciation rate. In the case of a bank run, households recover only x_t ($x_t < 1$) share of their deposit.[3] Therefore, the return on households' deposit is given by:

$$\tilde{R^D_t} = \begin{cases} R^D_t \,, & \text{if no bank run,} \\ x_t R^D_t \,, & \text{otherwise.} \end{cases}$$

[3] A detailed discussion of the recovery rate of deposit in a bank run is provided in subsection 3.2.2.

Following Gertler and Kiyotaki (2015) and Gertler et al. (2016), I assume that households are relatively less efficient than banks in terms of screening and monitoring their investments at the firms. This inefficiency is captured by a quadratic capital investment cost for the households:

$$\frac{\eta^H}{2}\left(\frac{K_{t+1}^H}{K_t}\right)^2 K_t.$$

Households own the banks and production sectors. As such, the profits or losses generated each period are transferred to households. In addition, households make an equity injection, ωK_t, to the newly entering banks in the banking sector.

During a bank run, if the central bank conducts credit policy, a lump sum tax T_t is levied from the households to purchase liquidated assets from banks. After the run, as banks reenter the economy, the central bank reduces its capital holding and return it to the households in the form of subsidy, denoted as Π_t^G.

The optimization problem of households can be summarized as:

$$\max_{\{C_t^H, K_{t+1}^H, D_{t+1}^H\}_{t=0}^{\infty}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \ln\left(C_t^H\right)\right], \tag{3.2.1}$$

s.t.

$$C_t^H + Q_t K_{t+1}^H + \frac{\eta^H}{2}\left(\frac{K_{t+1}^H}{K_t}\right)^2 K_t + D_{t+1}^H + \omega K_t + T_t = W_t + R_t^K K_t^H + \tilde{R_t^D} D_t^H + O_t,$$

$$K_{t+1}^H, D_{t+1}^H, C_t^H \geq 0,$$

where O_t captures the sum of net profits from the capital production sector (Π_t^K), consumption good production sector (Π_t^C), the banking sector (Π_t^B) and the subsidy from the central bank after the run (Π_t^G), i.e., $O_t = \Pi_t^K + \Pi_t^C + \Pi_t^B + \Pi_t^G$.

3.2.2 Banks

Banks intermediate funds between households and consumption goods producers and have relative advantage over households in screening and monitoring investments. In period t, banks take deposits D_{t+1}^B from households and lend K_{t+1}^B capital to consumption goods producers.

Following Gertler and Kiyotaki (2015), each period, banks exit with probability σ. The net worth of the exiting banks is paid out to households in the form of dividends. Without such an assumption, banks would accumulate net worth infinitely and could save their way out of any leverage constraints. New banks enter at the same rate to keep the number of banks constant. Since banks are owned by households, the banks' objective is to maximize the discounted future payouts to households, which is equal to the net worth of the exiting banks:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \left[\left[\beta^t (1-\sigma)^{t-1} \right] \frac{U(C_t)}{U(C_0)} \Pi_t^B \right] \right\}.$$

where Π_t^B is the dividend payout to households. The net worth of newly entering banks is given by ωK_t and the net worth of incumbent banks is given by:

$$N_t^{B,I} = R_t^K K_t^B - R_t^D D_t^B.$$

Therefore, $\Pi_t^B = \sigma N_t^{B,I}$. The net worth of the aggregate banking sector is thus given by:

$$N_t^B = (1-\sigma) N_t^{B,I} + \frac{\omega K_t}{\sigma}.$$

The balance sheet constraint of the banks states that the value of bank assets must equal the sum of deposits (liability) and bank net worth (equity):

$$Q_t K_{t+1}^B = D_{t+1}^B + N_t^B,$$

where $Q_t K_{t+1}^B$ is the value of the banks' assets.

Financial Friction To introduce financial friction, I follow Gertler et al. (2016) by assuming an asset diversion problem of the banks. In particular, banks can divert a share of their asset each period and exit the economy. As a result, banks face an incentive constraint imposed by depositors such that assets are never diverted:

$$\psi Q_t K_{t+1}^B \leq \beta \mathbb{E}_t \left[V_{t+1}^B \right].$$

where ψ ($0 < \psi < 1$) is the share of asset that a bank can divert and V_{t+1}^B is the continuation value of the bank. Therefore, the incentive constraint states that the value of asset diversion must be no more than the continuation value of the bank.

Bank Capital Requirement Banks have to comply with a minimum capital requirement, such that the net worth to asset ratio has to be larger or equal to the minimum regulatory level, Γ:

$$\frac{N_t^B}{Q_t K_{t+1}^B} \geq \Gamma.$$

The banks' optimization problem can be summarized as:

$$V_t^B = \max_{\{K_{t+1}^B, D_{t+1}^B\}_{t=0}^{\infty}} \mathbb{E}_t \left[\left[\beta^t (1-\sigma)^{t-1}\right] \frac{U(C_t)}{U(C_0)} \Pi_t^B \right], \tag{3.2.2}$$

s.t.

$$Q_t K_{t+1}^B = D_{t+1}^B + N_t^B, \qquad \text{Balance Sheet Constraint}$$

$$\psi Q_t K_{t+1}^B \leq \beta \mathbb{E}_t \left[V_{t+1}^B\right], \qquad \text{Incentive Constraint}$$

$$N_t^B \geq \Gamma Q_t K_{t+1}^B, \qquad \text{Bank Capital Requirement}$$

$$K_{t+1}^B, D_{t+1}^B \geq 0.$$

Bank Runs

As in Gertler and Kiyotaki (2015), depositors (households) have an incentive to run on banks whenever they cannot fully recover their deposits, including interest, from the liquidation assets of banks when a bank run happens. Only full bank runs are considered, which means all banks get liquidated in a run. The recovery rate on deposit in a bank run is given by:

$$x_t = \xi_t \frac{\left[r_t^K + (1-\delta) Q_t^*\right] K_t^B}{(1+r_t^D) D_t}, \tag{3.2.3}$$

where ξ_t captures a multiplicative liquidation cost shock to the liquidated value of the bank assets, which follows an i.i.d. lognormal distribution with mean 0 and variance σ^{ξ}. The liquidation cost shock generates an exogenous variation in the liquidation value of the banks, which helps to better account for the probability of bank runs.

I assume that households always fail to coordinate when the recovery rate is less than 1. That is, whenever $x_t < 1$, a bank run happens.

In the period of a bank run, all bank assets get liquidated at the liquidation price Q_t^*. With probability π, the bank run persists to the next period. In other words, after the initial bank run period, new banks enter the economy at rate σ with probability $1-\pi$. Therefore, there is a run persistence shock that follows the following Bernoulli distribution:

$$\pi_t \sim B(\pi).$$

3.2.3 The Central Bank

In this subsection, I introduce a central bank. As this is new to the existing framework in the first chapter, I give more details about the assumptions made about the central bank activities.

The central bank plays the role of the lender of last resort and conducts ex-post credit easing policies in the event of a bank run. In particular, when a bank run happens, the central bank intermediates capital from households to the consumption goods producers. The central bank levies a lump sum tax T_t from the households, which is used to purchase capital. Notice that this lump sum tax is collected ($T_t > 0$) only in the period when a bank run happens. During normal times, $T_t = 0$. Therefore, the net worth of the central bank in the period of bank run is given by:

$$N_t^G = T_t.$$

The central bank is more efficient than households in making investments but less efficient than banks, i.e., $0 < \eta^G < \eta^H$. Similar to the investment cost for households, the central bank's cost for investing K_{t+1}^G to the firm in period t is given by:

$$\frac{\eta^G}{2}\left[\frac{K_{t+1}^G}{K_t}\right]^2 K_t.$$

Credit Policy Following Gertler and Karadi (2011), I assume the amount of credit that the central bank intermediates depends on the expected credit spread between the return on capital investment (R_{t+1}^K) and the deposit rate (R_{t+1}^D). This assumption is motivated by the observation that credit easing typically follows a sharp increase in spread due to higher risk premium in the credit markets, i.e., when it is costly for non-financial firms to obtain credit. In particular, the central bank intermediates Φ share of the total capital stock in the period when the bank run happens according to the following policy rule:

$$K_t^G = \Phi K_{t+1} = \phi\left[ln(R_{t+1}^K) - ln(R_{t+1}^D)\right] K_{t+1},$$

where K_t^G is central bank capital holding and ϕ is the policy parameter determining the amount of capital intermediated by the central bank. Under such policy rule, at the onset of a bank run in period t, the central bank intermediates ΦK_{t+1} amount of capital, and the households invest the rest $(1-\Phi)K_{t+1}$ directly to the consumption goods producers. The central bank's balance sheet constraint is given by:

$$Q_t K_t^G = N_t^G.$$

After the initial bank run period, banks start to re-enter with probability $1-\pi$. As banks are more efficient than central banks, they will gradually take over the capital invested by the central bank. Over time, the central bank's capital holding reduces to zero as banks accumulate enough equity to absorb all the capital held by the central bank. The central bank transfers its profit from liquidating its capital to the households in the form of subsidies:

$$\Pi_t^G = Q_t \left[(1-\delta)K_t^G - K_{t+1}^G \right] - \frac{\eta^G}{2} \left[\frac{K_{t+1}^G}{K_t} \right]^2 K_t.$$

3.2.4 Capital Producers

Capital Producers transform consumption goods into capital according to the following production function:

$$Y_t^K = I_t,$$

where Y_t^K is the amount of capital produced, and I_t is the amount of consumption goods invested into capital production. In addition, it is costly for capital producers to adjust the amount of capital produced. In particular, the the capital adjustment cost takes the form:

$$\frac{\theta}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t,$$

where δ is capital depreciation rate.

The capital producers' profit maximization problem can be summarized as:

$$\max_{I_t} \Pi_t^K = Q_t I_t - I_t - \frac{\theta}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t. \tag{3.2.4}$$

The equilibrium price of capital is given by the first order condition of the capital producer:

$$Q_t = 1 + \theta \left(\frac{I_t}{K_t} - \delta \right).$$

3.2.5 Consumption Goods Producers

Consumption goods producers hire labor and capital to produce consumption goods according to a Cobb-Douglas production technology:

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}.$$

The log productivity follows an AR(1) process, i.e., $\ln(Z_t) = (1-\rho^Z)\mu^Z + \rho^Z \ln(Z_{t-1}) + \varepsilon_t$, where $|\rho^Z| < 1$ and $\varepsilon_t \sim N(0, \sigma^Z)$.

The standard profit maximization problem of the consumption goods producers can be summarized as:

$$\max_{K_t, L_t} \Pi_t^C = Z_t K_t^{\alpha} L_t^{1-\alpha} - r_t^K K_t - W_t L_t.$$

The rental price of capital and wage are given by the standard first order conditions:

$$r_t^K = \alpha Z_t K_t^{\alpha-1} L_t^{1-\alpha}, \tag{3.2.5}$$

$$W_t = (1-\alpha) Z_t K_t^{\alpha} L_t^{-\alpha}. \tag{3.2.6}$$

3.2.6 Equilibrium

The model economy has five markets in the no-run state: a consumption good market, a labor market, a rental capital market, a capital market, and a deposit market. In a run state, since all banks get liquidated, there is no deposit market.

A competitive equilibrium is a sequence of prices $\{Q_t, R_t^K, R_t^D, W_t\}_{t=0}^{\infty}$ and allocations $\{C_t^H, D_{t+1}^H, K_{t+1}^H, D_{t+1}^B, K_{t+1}^B, K_t, Y_t, I_t, Y_t^K, K_{t+1}^G\}_{t=0}^{\infty}$, such that the following markets clear:

- Labor: $L_t = 1$,
- Rental capital: $K_t = K_t^H + K_t^B + K_t^G$,
- Capital: $Y_t^K = K_{t+1} - (1-\delta)K_t$,
- Deposits: $D_{t+1}^H = D_{t+1}^B$.

3.3 Parameterization

In this section, I provide a numerical example of the model with parameterization. The purpose of the parameterization is not to match certain moments of the data, but to create a reasonable example of the model for the numerical exercise of policy experiments, which are detailed in the next section.

The model has 15 parameters in total. One model period corresponds to a quarter. The choice of parameter values for the baseline model is listed in Table 3.1, and the corresponding steady state value of the endogenous variables are listed in Table 3.2.

I use the same parameter values for $\beta, \sigma, \psi, \eta^H, \rho^Z$ and ω as in Gertler and Kiyotaki (2015). The production function parameter α and the depreciation rate δ are set to conventional values. The bank run persistence parameter π, capital adjustment cost parameter θ, and serial correlation of productivity shocks σ^Z are set to the same values as in Chapter 1.

In the baseline parameterization, the policy parameters Γ and ϕ are set to 0.08 and 0 respectively, corresponding to an 8 percent minimum bank capital requirement and no ex-post credit intervention policy. In the subsequent policy experiment section, I vary the values of Γ and ϕ and explore the impact of different policy combinations.

Parameter	Description	Value
	Households	
β	Household discount factor	0.99
η^H	Household capital holding cost	0.008
	Banks	
σ	Exit/entry probability	0.05
ψ	Divertable asset share	0.19
ω	Exogenous bank equity endowment	0.0011
Γ	Minimum capital requirement	0.08
	Bank Run	
σ^ξ	Volatility of liquidation cost shocks	0.05
π	Persistence of bank run	$12/13$
	Central Bank	
η^G	Central bank capital holding cost	0.002
ϕ	Credit easing policy rule	0
	Production	
α	Production function curvature	0.36
ρ^Z	Serial correlation of productivity shocks	0.9
σ^Z	Volatility of productivity shocks	0.01
θ	Capital adjustment cost	10
δ	Capital depreciation rate	0.025

Table 3.1 Baseline parameterization

3.4 Policy Experiments

In this section, I conduct counter-factual policy experiments by varying the policy parameters of the bank capital requirement and the central bank credit easing policy using the parameterization detailed in Section 3.3. In particular, I pick two values[4] for each of the two policy parameters ϕ and Γ, and compare the model simulation outcome of the four policy combinations:

[4]The low level of bank capital requirement is 8 percent as in the baseline case, which is the minimum bank capital requirement from the Basel II Accord that was introduced before the 2007-2009 financial crisis, and the high level is set to 15 percent, a level that is slightly higher than the minimum bank capital requirement from Basel III (13.5 percent) including the capital conservation buffer and counter-cyclical buffer. The low level of credit easing policy ($\phi = 0$) corresponds to the lower bound of the policy - no credit intermediation by the central bank, and the high level ($\phi = 8$) is the highest credit easing policy that the current model solution can solve.

Variable	Description	Value
Q	Price of capital	1
R^D	Gross return on deposit	1.0101
R^K	Gross return on capital investment	1.0126
K^H	Capital holding of household	3.4394
K^B	Capital holding of banks	7.8669
K^G	Capital holding of central bank	0
K	Total capital stock	11.306
D	Household deposit	7.1044
W	Household endowment	0.7549
N^B	Net worth of banks	0.7625
C^H	Consumption of households	0.8657
Y	Total output	1.1879

Table 3.2 Steady state under baseline parameterization

1. **No intervention (baseline case)**: no credit intermediation by central bank after bank run ($\phi = 0$) and low bank capital requirement ($\Gamma = 8\%$).

2. **Ex-ante intervention only (tightening capital requirement)**: no credit intermediation by central bank after bank run ($\phi = 0$) and *high* bank capital requirement ($\Gamma = 15\%$).

3. **Ex-post intervention only (central bank credit easing policy)**: central bank *provide credit* to consumption goods producers ($\phi = 8$) and low bank capital requirement ($\Gamma = 8\%$).

4. **Combination of two intervention policies**: central bank *provide credit* to consumption goods producers ($\phi = 8$) and *high* bank capital requirement ($\Gamma = 15\%$).

The model has three shocks, the productivity shock ε_t, the liquidation cost shock ξ_t, and the bank run persistence shock π_t. For each policy combination, I simulate 1000 economies, with 1500 periods for each economy, where the first 500 periods are discarded to eliminate the effects of initial conditions. Therefore, eventually I have simulation results of 1000 economies for 1000 periods. Then I take the average for each variable across all 1000 economies for each period. Finally, the average values across all 1000 periods are calculated. The permanent consumption equivalent is a welfare measure that is calculated as the permanent consumption level that generates exactly the same utility as the realized utility from simulated consumption sequences of the households.

Table 3.3 lists the simulation results of endogenous variables related to bank run frequency, financial intermediation and welfare for each policy experiment.

	(1) No intervention	(2) Ex-ante Intv.	(3) Ex-post Intv.	(4) Combination
Bank Leverage	7.2898	6.5625	7.3373	6.5637
Capital Price	0.9999	0.9996	0.9999	0.9996
Capital Liquidation Price	0.9688	0.9733	0.9728	0.9768
Price Change in Run (%)	-3.1103	-2.6311	-2.7103	-2.2810
Deposit Recovery Rate	0.9821	0.9851	0.9832	0.9863
Annual Bank Run Probability (%)	3.7296	0.5692	4.2524	0.5720
Deposit Interest Rate (%)	1.0500	0.9200	0.9300	0.8800
Central Bank Capital Holding in Run (%)	0.0000	0.0000	2.8932	2.0624
Bank Capital Holding (%)	46.986	45.059	47.032	46.729
Household Capital Holding (%)	53.014	54.914	52.252	52.915
Capital Stock	10.442	10.229	10.458	10.451
Consumption Equivalent (percent Dev from Baseline)	0	0.4843	0.0709	0.7088
Consumption Volatility	0.0200	0.0182	0.0203	0.0190

Table 3.3 Policy experiment results : simulation outcomes of different intervention policy combinations

3.4.1 Discussion

In this subsection, I provide a discussion of the effects of the policy on the bank run probability, financial intermediation, and welfare implications for each policy combination.

Ex-ante Intervention

Automatically, a higher capital requirement implies lower leverage of the banks. Comparing column (1) and (2), it is obvious that increasing bank capital requirement from 8 percent to 15 percent has a significant direct negative effect on bank leverage, which decreases from 7.29 to 6.56. As banks are less leveraged, the price drop in a bank run becomes less severe. Accordingly, the capital liquidation price increases. The lower bank leverage and higher liquidation price both contribute to a significantly lower bank run probability, which drops from 3.7 percent to 0.6 percent per year.

However, increasing capital requirement restricts the banks' ability to borrow, forcing banks to increase costly equity financing, and pushes up the financing cost of banks. As a result, there is less intermediation through banks, and households hold a higher portion of the capital stock, from 53 percent to 55 percent. Since households are less efficient in managing investment, this implies less efficiency of capital investment in the economy. Aggregate capital stock also decreases, from 10.44 to 10.23, accordingly.

In terms of welfare, as bank runs become significantly less frequent under a higher capital requirement, the permanent consumption equivalent increases by 0.48 percent. For the same reason, consumption volatility decreases from 0.02 to 0.018. In sum, a higher capital requirement leads to higher and less volatile consumption.

Ex-post Intervention

Under an ex-post credit easing policy, the central bank purchases capital and lends directly to consumption goods producers in the event of a bank run. Since the central bank is more efficient than households in making capital investments, the credit easing policy increases the liquidation price of capital in the crisis. Comparing column (1) and (3), one can see that increasing the credit policy parameter ϕ from 0 to 8 leads to an increase in the capital liquidation price from 0.9688 to 0.9728, and the central bank holds on average around 3 percent of the total capital stock right after the run happens. The higher liquidation price implies a higher recovery rate for households' deposits, which drives down the risk premium faced by the banks. Therefore, the deposit rate decreases. The lower financing cost increases the banks' profitability as well as the continuation value of the bank, which relaxes the incentive constraint and results in higher bank leverage. From (1) to (3), bank leverage increases from 7.29 to 7.34. The increase in bank leverage results in a higher probability of bank run.

Under the ex-post intervention policy, banks opt for higher leverage. Banks increase borrowing, and hold a larger share of the capital in the economy. Therefore, in contrast to the capital requirement policy, the ex-post credit easing policy increases financial intermediation and reduces inefficient investment of the households. As such, the aggregate capital stock also increases accordingly.

The ex-post intervention leads to higher permanent consumption equivalent due to an increase in the capital stock and therefore output. However, consumption fluctuates more than the baseline case. This is because the policy leads to a higher frequency of bank runs, in which consumption is more volatile than in normal times. As a result, consumption volatility increases.

Combination of Two Intervention Policies

Now comparing column (1) and (4), when the capital requirement is increased to 15 percent and the central bank conducts credit easing policy in the event of a bank run, the bank leverage decreases to 6.53, indicating the effect of capital requirement policy on bank leverage is more dominant in this case. The capital liquidation price increases as both policies contribute to higher liquidation prices. The probability of bank runs decreases from 3.73 percent to 0.57 percent, which is only marginally higher than the case of only the ex-ante capital requirement policy.

In terms of capital allocation, under the policy combination, the capital holding shares of banks and households are both slightly lower than the baseline case. This is because after the run, it takes time for banks to rebuild their net worth to absorb all the capital from the central bank. Therefore, the central bank holds a small share of capital for some periods after the run, although this share decreases to 0 over time. The aggregate capital stock also increases slightly from 10.442 to 10.451.

Due to both a lower frequency of bank runs and more financial intermediation, the permanent consumption equivalent increases by 0.71 percent from the baseline case, the highest among all policy combinations. Meanwhile, less frequent bank runs also result in lower consumption volatility. Therefore, the two policies complement each other.

3.5 Conclusion

In this chapter, I introduce credit easing as an ex-post intervention policy for financial crisis into a simplified version of the bank run model in Chapter 1. The purpose of this chapter is to conduct a quantitative exercise of comparing the two policies and study their effects on financial stability, financial intermediation and welfare.

I show that despite positive welfare effects of both intervention policies, they exhibit different trade-offs and therefore complement each other well. In particular, tightening bank capital requirements effectively reduce bank leverage but leads to under-provisioning of financial intermediation. The experiment on credit easing policy highlights the ex-ante effect of the policy: it decreases the banks' risk premium in a financial crisis, resulting in more leverage taking of banks and a higher frequency of bank runs. However, the credit easing policy facilitates financial intermediation in both normal and crisis periods and stabilizes price volatility during bank runs. A combination of the two policies can offset their respective negative effects, reducing the frequency and severity of financial crises while maintaining efficient financial intermediation in the economy.

In future work, it will be useful to conduct a careful calibration of the model to specific economies. With the calibrated model, quantitative analysis of more policy combinations of ex-ante and ex-post intervention policies can be conducted to shed light on the optimal policy combination.

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Appendix A

Appendix to Chapter 1

A.1 Data

We measure output Y using real GDP, investment $\tilde{I}$ using real gross private domestic investment and household consumption C^H using personal consumption expenditures from the US National Income and Product Accounts.

For the deposit rate R^D, we use the real effective federal funds rate, provided by the Federal Reserve Board, minus a four quarter moving average of the annualized inflation rate. We use the US GDP deflator to construct the inflation rate. For the wholesale funding market rate R^B, we follow GKP, who use the 90 day asset backed commercial paper rate, which is also provided by the Federal Reserve Board, minus a four quarter moving average of the annualized inflation rate. For the return on capital R_t^K/Q_{t-1}, we use the Wilshire 5000 index, a return index, which we also deflate with the GDP deflator.

For the construction of wholesale lending B and deposit lending D, we follow the procedure in GKP. We take the data from the financial accounts of the U.S., provided by the Federal Reserve Board. We calculate deposits as the sum of the asset holdings of households and nonfinancial business of

1. checkable deposits and currency,
2. total time and savings deposits,
3. money market mutual fund shares, and
4. mutual fund shares.

The shadow banking sector comprises the following groups:

1. GSEs and federally related mortgage pools
2. Funding corporations

3. Finance companies

4. Security brokers and dealers

5. Issuers of asset-backed securities

6. Holding companies

We compute B as the net liability position of these groups in the following short-term asset classes:

1. Commercial Paper

2. Security repurchase agreements

We deflate the resulting time series using the GDP deflator. We detrend the time series for Y, $\tilde{I}$, C^H, D and B with the filter proposed in Hamilton (2017). We do not use the Hodrick-Prescott filter, since Hamilton (2017) reports that the Hodrick-Prescott filter can lead to spurious correlations and distorts the properties of the filtered series at the beginning and the end of the series. We plot the detrended time series in Figure A.1. We detrend quantities in logs and interest rates in levels. The Hamilton filter leads to substantially different detrended time series compared to the Hodrick-Prescott filter. In particular, the Hamilton-filtered time series displays a much stronger recession in 2007-2009 in terms of output, investment and consumption.

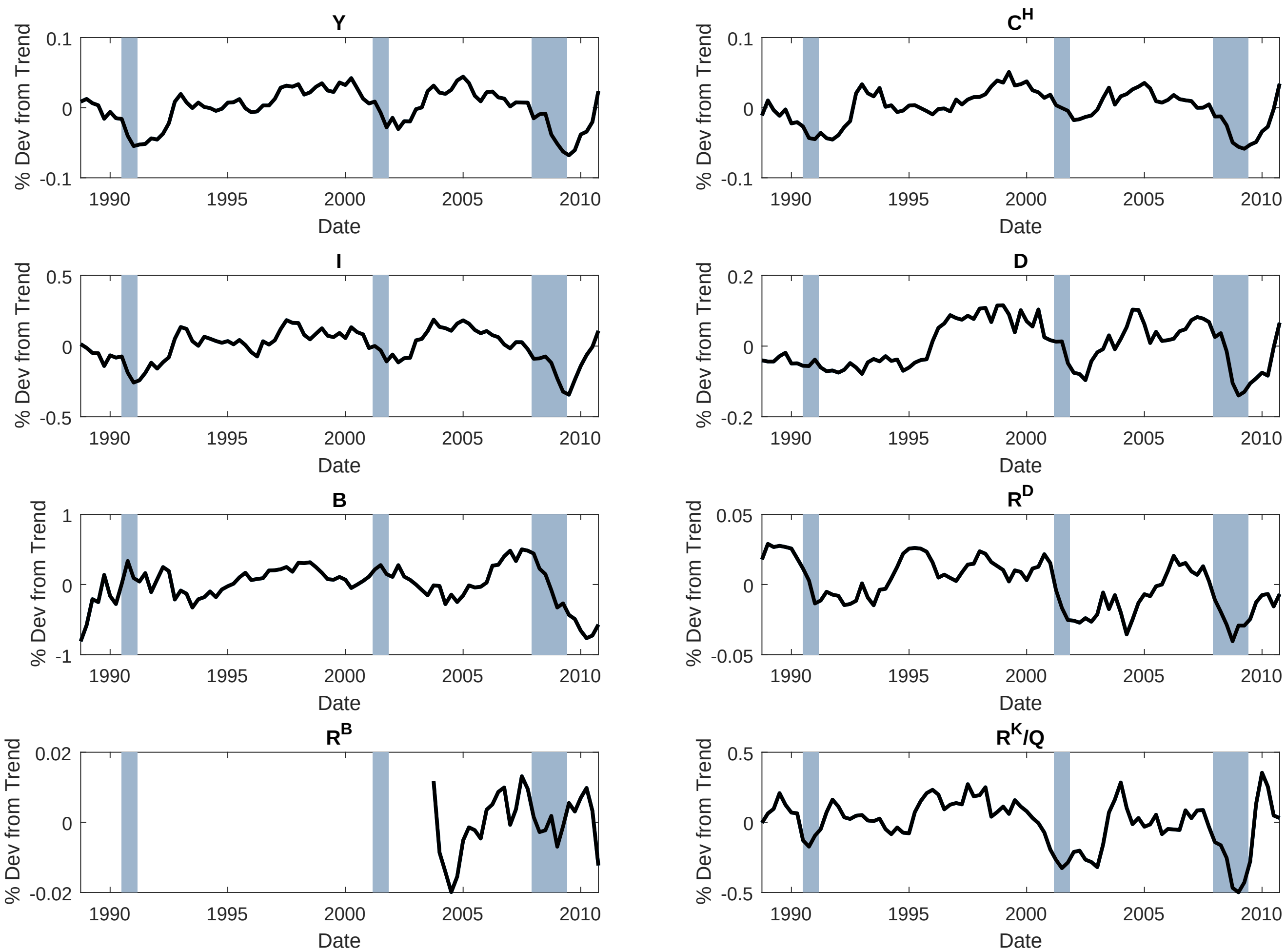


Fig. A.1 Detrended data

A.2 Steady State

We focus on the case of the model under a binding capital requirement. Our approach is to first characterize the steady state allocation of capital for a given aggregate capital stock K. We then explain how the aggregate capital stock is determined. Given K, we can compute the gross return on capital and the wage as

$$R^K_{SS} = \alpha K^{\alpha-1}_{SS} + 1 - \delta \tag{A.2.1}$$

$$W_{SS} = (1-\alpha) Z K^{\alpha}_{SS} \tag{A.2.2}$$

Steady state interest rates are determined by the first order conditions with respect to D^H_{t+1}, and B^R_{t+1}:

$$R^D_{SS} = \frac{1}{\beta} \tag{A.2.3}$$

$$R^B_{SS} = \gamma \frac{R^K_{SS}}{1+f^R_{SS}} + (1-\gamma) R^D_S S \tag{A.2.4}$$

Given R^K, K^H is determined by the euler equation of the household with respect to K^H

$$R^K_{SS} = \frac{1}{\beta}\left(1 + \eta^H \frac{K^H_{SS}}{K_{SS}}\right) \tag{A.2.5}$$

$$\frac{K^H}{K} = \frac{1}{\eta^H}\left(\beta R^K - 1\right) \tag{A.2.6}$$

We can now characterize the steady state allocation for the shadow banks: First, from the balance sheet constraint of shadow banks follows

$$B_{SS} = K^S_{SS} - N^S_{SS} \tag{A.2.7}$$

Plugging this into the law of motion for aggregate net worth, we can write net worth as

$$N^S = \frac{\upsilon^S K_{SS}}{1 - \left[(R^K - R^B)\frac{K^S}{N^S} + R^B\right](1-\sigma^S)}. \tag{A.2.8}$$

From the incentive constraint, we then get a quadratic condition for K^S/N^S:

$$\begin{aligned}
\psi\left[\omega\frac{K^S}{N^S} + (1-\omega)\right] &= \beta\left[\sigma^S + (1-\sigma^S)\psi\left[\omega\frac{K^S}{N^S} + (1-\omega)\right]\right]\frac{1}{1-\sigma^S}\left(1 - \upsilon^S\frac{K}{N^S}\right) \\
&= \beta\left[\sigma^S + (1-\sigma^S)\psi\left[\omega\frac{K^S}{N^S} + (1-\omega)\right]\right]\left(\left(R^K - R^B\right)\frac{K^S}{N^S} + R^B\right)
\end{aligned} \tag{A.2.9}$$

We then can infer $K^S = K^S/N^S N^S$. The fraction of capital holdings of retail banks are given by the market clearing condition for capital goods:

$$\frac{K^R_{SS}}{K_{SS}} = 1 - \frac{K^H_{SS}}{K_{SS}} - \frac{K^S_{SS}}{K_{SS}}. \tag{A.2.10}$$

From the balance sheet of the retail banking sector, we get

$$D = \left(1 + \eta^R \frac{K^R}{K}\right) K^R + B - N^R. \tag{A.2.11}$$

This allows us to substitute out D in the law of motion for aggregate net worth:

$$\begin{aligned}
N^R &= (R^K K^R + R^B B - R^D D)(1-\sigma^R) + \upsilon^R K \\
&= \left(R^K K^R + R^B B - R^D \left(\left(1 + \eta^R \frac{K^R}{K}\right) K^R + B - N^R\right)\right)(1-\sigma^R) + \upsilon^R K \\
&= \left(\left(R^K - \left(1 + \eta^R \frac{K^R}{K}\right) R^D\right) \frac{K^R}{N^R} + \left(R^B - R^D\right) \frac{B}{N^R} + R^D\right) N^R (1-\sigma^R) + \upsilon^R K
\end{aligned}$$

Hence,

$$N^R = \frac{\upsilon^R K}{1 - (1-\sigma^R)\left(\left(R^K - \left(1 + \eta^R \frac{K^R}{K}\right) R^D\right) \frac{K^R}{N^R} + \left(R^B - R^D\right) \frac{B}{N^R} + R^D\right)}. \tag{A.2.12}$$

Finally, from the capital requirement, we get

$$\left(1 + \eta^R \frac{K^R}{K}\right) K^R + \gamma B = \bar{\phi} \frac{\upsilon^R K}{1 - (1-\sigma^R)\left(\left(R^K - \left(1 + \eta^R \frac{K^R}{K}\right) R^D\right) \frac{K^R}{N^R} + \left(R^B - R^D\right) \frac{B}{N^R} + R^D\right)} \tag{A.2.13}$$

Substituting in the solutions for B from equation A.2.7, N^R from equation A.2.12 and K^R/K, from equation A.2.10 this is a complicated nonlinear equation in K only.

Some additional variables of interest can be calculated residually. Total output is given by:

$$Y_{SS} = Z K^{\alpha}_{SS} + \upsilon^R K + \upsilon^S K - \eta^H \left(\frac{K^H}{K}\right)^2 K - \eta^R \left(\frac{K^R}{K}\right)^2 K. \tag{A.2.14}$$

Then, household consumption is characterized by the aggregate budget constraint:

$$C^H_{SS} = Y - \delta K - \sigma^R \frac{N^R - \upsilon^R K}{1 - \sigma^R} - \sigma^S \frac{N^S - \upsilon^S K}{1 - \sigma^S}. \tag{A.2.15}$$

A.3 Equilibrium Conditions in the Dynamic Model

A.3.1 Households

The first-order conditions of the households' problem with respect to capital holding K^H_{t+1} and deposit D^H_{t+1} are given by:

$$FOC(K^H_{t+1}): \quad \frac{1}{C^H_t}(Q_t + \eta^H \frac{K^H_{t+1}}{K_t}) = \beta \mathbb{E}_t \left(\frac{1}{C^H_{t+1}} R^K_{t+1} \right) \tag{A.3.1}$$

$$FOC(D^H_{t+1}): \quad \frac{1}{C^H_t} = \beta \mathbb{E}_t \left(\frac{1}{C^H_{t+1}} R^D_{t+1} \right) \tag{A.3.2}$$

The interpretation of these first-order conditions is standard. In the first expression, the left-hand side and the right-hand side are the marginal cost and marginal benefit of capital holding, respectively. The marginal cost of capital holding has two components. One is the price the households have to pay for purchasing the capital goods, and the second is the capital holding cost due to households' low investment skills.

In addition, the households decide how much capital to hold through the retail banking sector. The first order condition with respect to K^R_{t+1} yields a first order condition which pins down f^R_t:

$$f^R_t = \eta^R \left(\frac{K^R_{t+1}}{K_t} \right).$$

Aggregate consumption of the household sector can be inferred from the resource constraint of the economy. Therefore, we do not have to track the net worth of households as a state variable.

$$C^H_t = Z_t K^\alpha_t + \upsilon^R K_t + \upsilon^S K_t - \frac{\eta^H}{2} \left(\frac{K^H_{t+1}}{K_t} \right)^2 K_t - \frac{\eta^R}{2} \left(\frac{K^R_{t+1}}{K_t} \right)^2 K_t$$
$$- I_t - \frac{\theta}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t - \sigma^R \frac{N^R_t - \upsilon^R K_t}{1-\sigma^R} - \sigma^S \frac{N^S_t - \upsilon^S K_t}{1-\sigma^S}$$

A.3.2 Banks

Shadow banks

The incentive constraint of the shadow bank is given by

$$\psi(n^S_t + \omega b^S_{t+1}) = \beta \mathbb{E}_t \left[V^S_{t+1} \right]. \tag{A.3.3}$$

The balance sheet constraint of the shadow bank reads

$$Q_t k_{t+1}^S = n_t^S + b_{t+1}^S \tag{A.3.4}$$

The net worth of an incumbent shadow bank is

$$n_t^S = R_t^K k_t^S - R_t^B b_t^S \tag{A.3.5}$$

The value of the shadow bank before the realization of the exit shock is given by

$$\begin{aligned} V_t^S &= \sigma^S n_t^S + (1-\sigma^S)\beta \mathbb{E}_t \left[V_{t+1}^S \right] \\ &= \sigma^S n_t^S + (1-\sigma^S)\psi(n_t^S + \omega b_{t+1}^S), \end{aligned}$$

where the second line uses the binding incentive constraint to substitute out the continuation value. Plugging this expression into A.3.3 yields the following characterization for the shadow banks choices for k_{t+1}^S and b_{t+1}^S:

$$\psi(n_t^S + \omega b_{t+1}^S) = \beta \mathbb{E}_t \left[(\sigma^S + (1-\sigma^S)\psi) n_{t+1}^S + \psi\omega(1-\sigma^S) b_{t+2}^S \right] \tag{A.3.6}$$

$$Q_t k_{t+1}^S = n_t^S + b_{t+1}^S \tag{A.3.7}$$

$$n_t^S = R_t^K k_t^S - R_t^B b_t^S \tag{A.3.8}$$

We now conjecture and verify that the policy functions for b_{t+1}^S and k_{t+1}^S are linear in net worth, such that it is sufficient to characterize the optimal choices of the shadow banking sector as a whole in equilibrium.

Theorem A.3.1 (Linearity of Policy Functions)**.** *The policy functions for b_{t+1}^S and k_{t+1}^S which solve the problem of the shadow bank given by equations A.3.6 to A.3.8 are linear in net worth.*

Proof. Suppose that the policy functions are given by $b_{t+1}^S = A_b^S n_t^S$ and $k_{t+1}^S = A_k^S n_t^S$, respectively. Then, it follows from equation A.3.8 that

$$\begin{aligned} n_{t+1}^S &= R_{t+1}^K k_{t+1}^S - R_{t+1}^B b_{t+1}^S \\ &= (R_{t+1}^K A_k^S - R_{t+1}^B A_b^S) n_t^S \\ &= A_n^S n_t^S. \end{aligned}$$

From equation A.3.7 follows that

$$\begin{aligned} Q_t A_k^S n_t^S &= n_t^S + A_b^S n_t^S \\ A_k^S &= \frac{1 + A_b^S}{Q_t}. \end{aligned}$$

Finally, from A.3.6 follows that

$$\begin{aligned} \psi(1+\omega A_b^S) n_t^S &= \beta \mathbb{E}_t \left[(\sigma^S + (1-\sigma^S)\psi(1+\omega A_b^S)) n_{t+1}^S \right] \\ &= \beta \mathbb{E}_t \left[(\sigma^S + (1-\sigma^S)\psi(1+\omega A_b^S)) A_n^S \right] n_t^S \\ &= \beta \mathbb{E}_t \left[(\sigma^S + (1-\sigma^S)\psi(1+\omega A_b^S)) (R_{t+1}^K \frac{1+A_b^S}{Q_t} - R_{t+1}^B A_b^S) \right] n_t^S \end{aligned}$$

This equation yields a solution for A_b^S that is independent of n_t^S.[1] Consequently, A_k^S and A_n^S are also independent of n_t^S. □

Given the linearity of policy functions, it is sufficient to characterize the policies K_{t+1}^S and B_{t+1}^S of the aggregate shadow banking sector. These choices are the solutions to

$$\begin{aligned} \psi(N_t^S + \omega B_{t+1}^S) &= \beta \mathbb{E}_t \left[(\sigma^S + (1-\sigma^S)\psi) \frac{N_{t+1}^S - \upsilon^S K_{t+1}^S}{1-\sigma^S} + \psi\omega(1-\sigma^S) B_{t+2}^S \right] \\ Q_t K_{t+1}^S &= N_t^S + B_{t+1}^S \\ N_t^S &= (R_t^K K_t^S - R_t^B B_t^S)(1-\sigma^S) + \upsilon^R K_t^S. \end{aligned}$$

Retail banks, No Regulation

We characterize the problem of a retail banks under a non-binding and a binding capital requirement. First, we consider the problem of a retail bank where the incentive constraint is binding. The incentive constraint is given by

$$\psi((Q_t + f_t^R) k_{t+1}^R + \gamma b_{t+1}^R) = \beta \mathbb{E}_t \left[V_{t+1}^R \right]. \tag{A.3.9}$$

[1] Specifically, the solution is given by $A_b^S = -p + \sqrt{p^2+q}$, with $p = -\frac{1}{2} \frac{(1/\beta - \mathbb{E}_t[R_{t+1}^K/Q_t]) - \mathbb{E}_t[R_{t+1}^K/Q_t - R_{t+1}^B]}{\omega \mathbb{E}_t[R_{t+1}^K/Q_t - R_{t+1}^B]}$ and $q = \frac{1/\beta - \mathbb{E}_t[(\sigma^S + (1-\sigma^S)\psi) R_{t+1}^K/Q_t]}{\omega \mathbb{E}_t[R_{t+1}^K/Q_t - R_{t+1}^B]}$. When $\mathbb{E}_t\left[R_{t+1}^K/Q_t - R_{t+1}^B\right] > 0$ and $\mathbb{E}_t\left[R_{t+1}^B - 1/\beta\right] > 0$, this solution is unique.

The balance sheet constraint reads:

$$(Q_t + f_t^R)k_{t+1}^R + b_{t+1}^R = n_t^R + d_t^R. \tag{A.3.10}$$

Net worth is determined according to

$$n_t^R = R_t^K k_t^R + R_t^B b_t^R - R_t^D d_t^R. \tag{A.3.11}$$

These three equations pin down k_{t+1}^R, d_{t+1}^R and n_t^R. b_{t+1}^R is determined by a first order condition of the retail banks problem:

$$\begin{aligned}
&\max_{\left\{k_{s+1}^R, b_{s+1}^R, d_{s+1}^R\right\}_{s=t}^{\infty}} \beta \mathbb{E}_t \left[V_{t+1}^R\right] \\
&\text{s.t.} \\
&V_t^R = \sigma^R n_t^R + (1-\sigma^R)\psi((Q_t + f_t^R)k_{t+1}^R + \gamma b_{t+1}^R) \\
&n_t^R + d_{t+1}^R = (Q_t + f_t^R)k_{t+1}^R + b_{t+1}^R \\
&\psi((Q_t + f_t^R)k_{t+1}^R + \gamma b_{t+1}^R) = \beta \mathbb{E}_t \left[V_{t+1}^R\right] \\
&n_t^R = R_t^K k_t^R + R_t^B b_t^R - R_t^D d_t^R \\
&k_{t+1}^R, d_{t+1}^R, b_{t+1}^R \geq 0.
\end{aligned}$$

We conjecture, as in the shadow banking problem, that the policy functions for k_{t+1}^R, b_{t+1}^R and d_{t+1}^R are linear in net worth n_t^R:

$$\begin{aligned}
k_{t+1}^R &= A_k^R n_t^R \\
b_{t+1}^R &= A_b^R n_t^R \\
d_{t+1}^R &= A_d^R n_t^R
\end{aligned}$$

Plugging in the conjectured policy functions, we can rewrite the maximization problem as

$$\begin{aligned}
&\max_{\left\{k_{s+1}^R, b_{s+1}^R\right\}_{s=t}^{\infty}} \beta \mathbb{E}_t \left[V_{t+1}^R\right] \\
&\text{s.t.} \\
&V_{t+1}^R = (\sigma^R + (1-\sigma^R)\psi((Q_{t+1} + f_{t+1}^R)A_k^R + \gamma A_b^R))n_{t+1}^R \\
&\psi((Q_t + f_t^R)k_{t+1}^R + \gamma b_{t+1}^R) = \beta \mathbb{E}_t \left[V_{t+1}^R\right] \\
&n_{t+1}^R = R_{t+1}^K k_{t+1}^R + R_{t+1}^B b_{t+1}^R - R_{t+1}^D((Q_t + f_t^R)k_{t+1}^R + b_{t+1}^R - n_t^R) \\
&k_{t+1}^R, d_{t+1}^R, b_{t+1}^R \geq 0.
\end{aligned}$$

Defining $\Omega^R_{t+1} \equiv V^R_{t+1}/n^R_{t+1} = (\sigma^R + (1-\sigma^R)\psi((Q_{t+1} + f^R_{t+1})A^R_k + \gamma A^R_b))$, the Lagrangian for this problem is given by

$$\begin{aligned}
\mathscr{L} = \beta \mathbb{E}_t & \left[\Omega^R_{t+1}(R^K_{t+1}k^R_{t+1} + R^B_{t+1}b^R_{t+1} - R^D_{t+1}((Q_t + f^R_t)k^R_{t+1} + b^R_{t+1} - n^R_t))\right] \\
& + \lambda \left[\psi((Q_t + f^R_t)k^R_{t+1} + \gamma b^R_{t+1}) \right. \\
& \left. -\beta \mathbb{E}_t \left[\Omega^R_{t+1}(R^K_{t+1}k^R_{t+1} + R^B_{t+1}b^R_{t+1} - R^D_{t+1}((Q_t + f^R_t)k^R_{t+1} + b^R_{t+1} - n^R_t))\right]\right]
\end{aligned}$$

This yields the following first order conditions:

$$\begin{aligned}
\frac{\partial \mathscr{L}}{\partial k^R_{t+1}} &= \beta \mathbb{E}_t \left[\Omega^R_{t+1}(R^K_{t+1} - R^D_{t+1}(Q_t + f^R_t))\right](1-\lambda) + \lambda \psi (Q_t + f^R_t) = 0 \\
\frac{\partial \mathscr{L}}{\partial b^R_{t+1}} &= \beta \mathbb{E}_t \left[\Omega^R_{t+1}(R^B_{t+1} - R^D_{t+1})\right](1-\lambda) + \lambda \psi \gamma = 0
\end{aligned}$$

Combining these two equations and rearranging, we arrive at the condition

$$\mathbb{E}_t \left[\Omega^R_{t+1}\left(\frac{R^K_{t+1}}{Q_t + f^R_t} - R^D_{t+1}\right)\right] = \frac{1}{\gamma}\mathbb{E}_t \left[\Omega^R_{t+1}(R^B_{t+1} - R^D_{t+1})\right].$$

This is basically a condition that ensures that the retail bank is indifferent between lending a marginal unit of funds to consumption goods producers or on the wholesale funding market.

Showing the linearity of policy functions works in the same way as in the shadow bank problem. Then, in equilibrium, it is sufficient to characterize the choices K^R_{t+1}, B^R_{t+1} and D^R_{t+1} of the retail banking sector as a whole. These choices are characterized by the following system of equations:

$$\begin{aligned}
\psi((Q_t + f^R_t)K^R_{t+1} + \gamma B^R_{t+1}) &= \beta \mathbb{E}_t \left[\Omega^R_{t+1}\frac{N^R_{t+1} - \upsilon^R K^R_{t+1}}{1-\sigma^R}\right] \\
(Q_t + f^R_t)K^R_{t+1} + B^R_{t+1} &= N^R_t + D^R_t \\
\mathbb{E}_t \left[\Omega^R_{t+1}\left(\frac{R^K_{t+1}}{Q_t + f^R_t} - R^D_{t+1}\right)\right] &= \frac{1}{\gamma}\mathbb{E}_t \left[\Omega^R_{t+1}(R^B_{t+1} - R^D_{t+1})\right] \\
N^R_t &= (R^K_t K^R_t + R^B_t B^R_t - R^D_t D^R_t)(1-\sigma^R) + \upsilon^R K^R_t
\end{aligned}$$

Retail banks, With Regulation

Under a binding regulatory capital requirement, the incentive constraint of the retail bank is replaced by the capital requirement:

$$\bar{\phi}_t n^R_t = (Q_t + f^R_t)k^R_t + \gamma b^R_{t+1}$$

Otherwise, the model is unchanged. In particular, the linearity of policy functions is preserved. Therefore, the choices of the aggregate retail banking sector are given by

$$\begin{aligned}
\bar{\phi}_t N_t^R &= (Q_t + f_t^R) K_t^R + \gamma B_{t+1}^R \\
(Q_t + f_t^R) K_{t+1}^R + B_{t+1}^R &= N_t^R + D_t^R \\
\mathbb{E}_t \left[\Omega_{t+1}^R \left(\frac{R_{t+1}^K}{Q_t + f_t^R} - R_{t+1}^D \right) \right] &= \frac{1}{\gamma} \mathbb{E}_t \left[\Omega_{t+1}^R (R_{t+1}^B - R_{t+1}^D) \right] \\
N_t^R &= (R_t^K K_t^R + R_t^B B_t^R - R_t^D D_t^R)(1 - \sigma^R) + \upsilon^R K_t^R
\end{aligned}$$

A.3.3 Production Sectors

From the problem of the capital producer follows

$$Q_t = 1 + \theta \left(\frac{I_t}{K_t} - \delta \right).$$

The first order conditions of the consumption goods producer yield

$$\begin{aligned}
r_t^K &= \alpha Z_t K_t^{\alpha - 1} \\
W_t &= (1 - \alpha) Z_t K_t^{\alpha}.
\end{aligned}$$

A.3.4 Full Statement of the Equilibrium Conditions

No Run Equilibrium

- Household:

$$\begin{aligned}
\frac{1}{C_t^H} \left(Q_t + \eta^H \frac{K_{t+1}^H}{K_t} \right) &= \beta \mathbb{E}_t \left(\frac{1}{C_{t+1}^H} R_{t+1}^K \right) \\
\frac{1}{C_t^H} &= \beta \mathbb{E}_t \left(\frac{1}{C_{t+1}^H} R_{t+1}^D \right) \\
f_t^R &= \eta^R \frac{K_{t+1}^R}{K_t} \\
C_t^H &= Z_t K_t^{\alpha} + \upsilon^R K_t + \upsilon^S K_t - \frac{\eta^H}{2} \left(\frac{K_{t+1}^H}{K_t} \right)^2 K_t - \frac{\eta^R}{2} \left(\frac{K_{t+1}^R}{K_t} \right)^2 K_t \\
&\quad - I_t - \frac{\theta}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t - \sigma^R \frac{N_t^R - \upsilon^R K_t}{1 - \sigma^R} - \sigma^S \frac{N_t^S - \upsilon^S K_t}{1 - \sigma^S}
\end{aligned}$$

- Shadow Bank:

$$\psi(N_t^S + \omega B_{t+1}^S) = \beta \mathbb{E}_t \left[(\sigma^S + (1-\sigma^S)\psi) \frac{N_{t+1}^S - \upsilon^S K_{t+1}^S}{1-\sigma^S} + \psi\omega(1-\sigma^S) B_{t+2}^S \right]$$

$$Q_t K_{t+1}^S = N_t^S + B_{t+1}^S$$

$$N_t^S = (R_t^K K_t^S - R_t^B B_t^S)(1-\sigma^S) + \upsilon^R K_t^S.$$

- Retail Bank:

$$\bar{\phi}_t N_t^R = (Q_t + f_t^R) K_{t+1}^R + \gamma B_{t+1}^R$$

$$(Q_t + f_t^R) K_{t+1}^R + B_{t+1}^R = N_t^R + D_t^R$$

$$\mathbb{E}_t \left[\Omega_{t+1}^R \left(\frac{R_{t+1}^K}{Q_t + f_t^R} - R_{t+1}^D \right) \right] = \frac{1}{\gamma} \mathbb{E}_t \left[\Omega_{t+1}^R (R_{t+1}^B - R_{t+1}^D) \right]$$

$$\Omega_t^R = \left(\sigma^R + (1-\sigma^R)\psi \left((Q_t + f_t^R) \frac{K_{t+1}^R}{\tilde{N}_t^R} + \frac{B_{t+1}^R}{\tilde{N}_t^R} \right) \right)$$

$$N_t^R = (R_t^K K_t^R + R_t^B B_t^R - R_t^D D_t^R)(1-\sigma^R) + \upsilon^R K_t^R$$

$$\tilde{N}_t^R = \frac{N_t^R - \upsilon^R K_t^R}{1-\sigma^R}$$

- Firms:

$$Q_t = 1 + \theta \left(\frac{I_t}{K_t} - \delta \right)$$

$$R_t^K = \alpha Z_t K_t^{\alpha-1} + (1-\delta) Q_t$$

$$W_t = (1-\alpha) Z_t K_t^{\alpha}$$

- Laws of Motion:

$$K_{t+1}^H + K_{t+1}^R + K_{t+1}^S = (1-\delta) K_t + I_t$$

$$\ln(Z_t) = (1-\rho^Z)\mu^Z + \rho_Z \ln(Z_{t-1}) + \varepsilon_t$$

Run Equilibrium

- Household:

$$\frac{1}{C_t^{H,*}}\left(Q_t^*+\eta^H\frac{K_{t+1}^{H,*}}{K_t}\right)=\beta\mathbb{E}_t\left[(1-\pi)\frac{1}{C_{t+1}^H}R_{t+1}^K+\pi\frac{1}{C_{t+1}^{H,*}}R_{t+1}^{K,*}\right]$$

$$\frac{1}{C_t^{H,*}}=\beta\mathbb{E}_t\left[(1-\pi)\frac{1}{C_{t+1}^H}R_{t+1}^D+\pi\frac{1}{C_{t+1}^{H,*}}R_{t+1}^D\right]$$

$$f_t^{R,*}=\eta^R\frac{K_{t+1}^{R,*}}{K_t}$$

$$C_t^{H,*}=Z_tK_t^\alpha+\upsilon^RK_t-\frac{\eta^H}{2}\left(\frac{K_{t+1}^{H,*}}{K_t}\right)^2K_t-\frac{\eta^R}{2}\left(\frac{K_{t+1}^{R,*}}{K_t}\right)^2K_t$$
$$-I_t^*-\frac{\theta}{2}\left(\frac{I_t^*}{K_t}-\delta\right)^2K_t-\sigma^R\frac{N_t^{R,*}-\upsilon^RK_t}{1-\sigma^R}$$

- Shadow Bank:

$$C_t^{S,*}=0$$
$$N_t^{S,*}=0$$
$$B_{t+1}^*=0$$
$$K_{t+1}^{S,*}=0$$

- Retail Bank:

$$\bar{\phi}_tN_t^{R,*}=(Q_t+f_t^R)K_{t+1}^{R,*}$$
$$(Q_t^*+f_t^{R,*})K_{t+1}^{R,*}=N_t^{R,*}+D_{t+1}^{R,*}$$
$$\Omega_t^R=\left(\sigma^R+(1-\sigma^R)\psi\left((Q_t^*+f_t^{R,*})\frac{K_{t+1}^{R,*}}{\tilde{N}_t^{R,*}}\right)\right)$$
$$N_t^{R,*}=(R_t^{K,*}K_t^R+\xi_tR_t^{K,*}K_t^S-R_t^DD_t^R)(1-\sigma^R)+\upsilon^RK_t^R$$
$$\tilde{N}_t^{R,*}=\frac{N_t^{R,*}-\upsilon^RK_t}{1-\sigma^R}$$

- Firms:

$$Q_t^*=1+\theta\left(\frac{I_t^*}{K_t}-\delta\right)$$
$$R_t^{K,*}=\alpha Z_tK_t^{\alpha-1}+(1-\delta)Q_t^*$$
$$W_t=(1-\alpha)Z_tK_t^\alpha$$

- Laws of Motion:

$$K^{H,*}_{t+1} + K^{R,*}_{t+1} = (1-\delta)K_t + I^*_t$$
$$\ln(Z_t) = (1-\rho^Z)\mu^Z + \rho_Z \ln(Z_{t-1}) + \varepsilon_t$$

A.4 Computation

We solve the model nonlinearly using a time iteration algorithm. Solving the model nonlinearly is important, because bank runs can lead to large deviations from steady state, where perturbation algorithms are inaccurate.

The state space of the model is $\mathscr{S} = (N^R, N^S, K, Z)$ in the *"no bank run"* equilibrium and $\mathscr{S}^* = (N^{R,*}, K, Z)$ in the *"bank run"* equilibrium. We approximate the consumption policy functions $C^H(\mathscr{S}), V^R(\mathscr{S}), V^S(\mathscr{S}), C^{H,*}(\mathscr{S}^*)$ and $V^{R,*}(\mathscr{S}^*)$ and the capital prices $Q(\mathscr{S})$ and $Q^*(\mathscr{S}^*)$ using fourth order polynomials. We compute the polynomial coefficients by imposing that the polynomial approximations must be equal to the original functions on the grid. Specifically, denoting the polynomial coefficients by α and the polynomials by $\Pi(\mathscr{S})$, we impose for example for the consumption of households

$$\Pi(\mathscr{S}_i)\alpha_{C^H} = C^H(\mathscr{S}_i) \quad i = 1, \ldots, N. \tag{A.4.1}$$

for all N grid points. We use a Smolyak grid with order $\mu = 4$ for the endogenous states and $\mu = 3$ for the exogenous states. We compute the Smolyak grid and polynomials using the toolbox by Judd, Maliar, Maliar, and Valero (2014).

One slight complication of the model is that the future net worth values N_R and N_S, depends on $Q(\mathscr{S})$. This implies that, for example, the household net worth for a given function $Q(.)$ must be computed as a solution to the nonlinear function[2]

$$N^{R,'} = \left[(r^K + (1-\delta)Q(N^{R,'}, N^{S,'}, K', Z'))K^{R,'} + R^{B,'}B' - R^{D,'}D'\right](1-\sigma^R) + \upsilon^R K. \tag{A.4.2}$$

With this in mind, we will now outline our solution algorithm for the *"no bank run"* equilibrium. Suppose we are in iteration k and have initial guesses for the no-run consumption policy functions $C^H_{(k)}(\mathscr{S}), V^R_{(k)}(\mathscr{S})$, and $V^S_{(k)}(\mathscr{S})$ and the capital price function $Q_{(k)}(\mathscr{S})$ as well as values for the future net worth $N^{R,'}_{(k)}$ and $N^{S,'}_{(k)}$.

[2] In principle, Π^Q is also a function of the states. We ignore this here for the sake of exposition. We do however account for this correctly in the code.

1. Given the value functions and the future net worth, compute the future value functions and capital prices as

$$C^{H'}_{(k)} = C^{H}_{(k)}(\mathscr{S}'_{(k)}),$$
$$Q'_{(k)} = Q_{(k)}(\mathscr{S}'_{(k)}),$$

and so on.

2. Compute the new values for $(K^{H,'}, K^{R,'}, K^{S,'}, D', B', R^{D,'}, R^{B,'}, Q)$ for all grid points $i = 1, \dots, N$ using the first order conditions A.3.1, A.3.2, A.3.9, A.3.10, A.3.3, 1.2.26 and the leverage constraints 1.2.14 and 1.2.8. Compute the future net worth where necessary according to

$$\tilde{N}^{R,'}_{(k+1)} = \left[(r^{K,'} + (1-\delta)Q_{(k)}(N^{R,'}_{(k)}, N^{S,'}_{(k)}, K', Z'))K^{H,'} + R^{B,'}B' - R^{D,'}D'\right](1-\sigma^R) \tag{A.4.3}$$
$$+ \upsilon^R K'.$$

We compute expectations using Gauss-Hermite quadrature. Note that for each quadrature node Z', a different value of $\tilde{N}^{R,'}_{(k+1)}$ must be computed.

3. Using the new policies and prices, update the consumption function of the household using equation 1.2.29, and the value functions for the retail and shadow banks using equations 1.2.9 and 1.2.18.

4. Update the next period net worth values using A.4.3, with some attenuation: $N^{R,'}_{(k+1)} = (1-\iota)N^{R,'}_{(k)} + \iota \tilde{N}^{R,'}_{(k+1)}$, with $\iota = 0.5$.

5. Repeat until the errors in the consumption, capital price and net worth values on the grid are small. We iterate until the maximum error in consumption is smaller than 1e-5 and the maximum error in the net worth is smaller than 1e-5.

If bank runs are unanticipated, we can first solve for the *"no bank run"* equilibrium and then afterwards for the *"bank run"* equilibrium. Importantly, expectations during a bank run are taken over the future *"bank run"* and *"no bank run"* states. It is therefore necessary to keep track of two sets of net worth values, $N^{R,'}_{(k)}$ and $N^{R,',*}_{(k)}$. Otherwise, the algorithm works in the same way as for the *"no bank run"* equilibrium. For the anticipated run case, we use the unanticipated run case as initial guess and solve jointly for the *"no bank run"* and *"bank run"* policy functions.

Appendix B

Appendix to Chapter 2

B.1 Data

Table B.1 Variable definitions and data sources

Variables	Description	Unit	Frequency	Source
		Banks		
Bank Assets	Total bank assets	Million Euro	Monthly	ECB Statistical Data Warehouse: Money, credit and banking: MFI aggregated balance sheet: Total Assets. Series Key: BSI.M.ES.N.A.T00.A.1.Z5.0000.Z01.E
Bank Liabilities	Total bank liabilities	Million Euro	Monthly	ECB Statistical Data Warehouse: Money, credit and banking: MFI aggregated balance sheet: Liabilities. Sum of deposits, debt securities, MMF shares, external liabilities, remaining liabilities.
Deposit Rate	Nominal annualized interest rate on bank deposits	Per cent	Monthly	ECB Statistical Data Warehouse: Financial markets and interest rates: Interest rates: Deposits. Series Key: MIR.M.ES.B.L22.F.R.A.2250.EUR.N
Mortgage Rate	Nominal annualized interest rate on mortgage loans	Per cent	Monthly	ECB Statistical Data Warehouse: Financial markets and interest rates: Interest rates: Loans. Series Key: MIR.M.ES.B.A22.A.R.A.2250.EUR.O
Risk Free Rate	ECB refinancing rate	Per cent	Monthly	ECB Statistics: ECB/Eurosystem policy and exchange rates: Official interest rates: Main refinancing operations: Fixed rate
Share of NPL	Ratio of bank nonperforming loans to total gross loans	Per cent	Annual	World Bank: Bank nonperforming loan, Spain
		Housing		
House Price	Average price per square meter of housing	Euro	Quarterly	Spanish Ministry of Construction, Ministerio de Fomento: Informacion para el ciudadano: Informacion estadistica: Vivenda y actuaciones urbanas: Estadisticas: Valor tasado de la Viviend: 1.Valor tasado de la Vivienda libre
Home Ownership	Distribution of population by tenure status	Per cent	Monthly	Eurostat (ilc_lvho02): Distribution of population by tenure status, type of household and income group - EU-SILC survey
		Households		
Household Leverage	Loans granted to households as a ratio of gross disposable income	Per cent	Quarterly	ECB Statistical Data Warehouse: Macroeconomic and sectoral statistics: Sector accounts: Sectors: Households(and NPISH) . Series Key: QSA.Q.N.ES.W0.S1M.S1.N.L.LE.F4.T._Z.XDC_R_B6G_CY._T.S.V.N._T
Unemployment Rate	Unemployment rate of active population	Per cent	Quarterly	Eurostat: Population and social conditions: Labour market: Employment and unemployment: Unemployment-LFS adjusted series: Unemployment by sex and age-quarterly average(une_rt_a)
		Economy		
Real GDP	Chain linked volumes 2010, seasonally and calendar adjusted	Million Euro	Monthly	Eurostat: Database: Economy and finance: National accounts: Quarterly national accounts: Main GDP aggregates: GDP and main components (namq_10_gdp)
GDP Growth	Chain linked volumes, percentage change compared to same period in previous year	Per cent	Monthly	Eurostat: Database: Economy and finance: National accounts: Quarterly national accounts: Main GDP aggregates: GDP and main components (namq_10_gdp)
CPI	Harmonised index of consumer prices (HICP)	2015=100	Monthly	Eurostat: Database: Economy and finance: Prices: Harmonised index of consumer prices (HICP) (prc_hicp)

Note: All data are collected for Spain for the longest possible period available.

B.2 Steady State of the Model

B.2.1 Steady State Conditions

Prices:

$$R^D = \frac{1}{\beta^P} \tag{B.2.1}$$

$$W = \alpha Z L^{\alpha-1} \tag{B.2.2}$$

Patient households (P):

$$P = \beta^P \left(P + \frac{1-\chi}{\chi} \frac{C^P}{H^P} \right) \tag{B.2.3}$$

$$C^P = W\bar{L^P} + (R^D - 1)D^P + (1-\tau)\frac{O}{\mu} \tag{B.2.4}$$

Impatient households (I):

$$R^M = \max\left(\frac{1 - \beta^I \frac{PH^I}{M}\Phi}{\beta^I(1-\Phi)}, \frac{1}{\beta^I} \right) \tag{B.2.5}$$

$$X^M = \min\left(\frac{PH^I}{R^M M^I}, 1 \right) \tag{B.2.6}$$

$$P = \beta^I \left(P + \frac{1-\chi}{\chi} \frac{C^I}{H^I} \right) \tag{B.2.7}$$

$$C^I + \left[\left(1 - (1 - X^M)\Phi\right) R^M - 1 \right] M^I = W\bar{L^I} + \tau \frac{O}{(1-\mu)} \tag{B.2.8}$$

$$R^M M^I \leq \kappa P H^I \tag{B.2.9}$$

Banks (B):

$$N^B + D^B = M^B \tag{B.2.10}$$

$$n^B = \left[1 - (1 - X^M)\Phi \right] R^M M^B - R^D D^B \tag{B.2.11}$$

$$N^B = n^B(1-\eta) + E\eta \tag{B.2.12}$$

$$N^B \geq \Gamma M^B \tag{B.2.13}$$

$$\left[\left[1 - (1 - X^M)\Phi \right] R^M - R^D \right] D \geq 0 \tag{B.2.14}$$

Firms:

$$Y = Z L^\alpha \tag{B.2.15}$$

Market clearing:

$$Y = \mu C^P + (1-\mu)C^I \tag{B.2.16}$$

$$L = \mu \bar{L^P} + (1-\mu)\bar{L^I} \tag{B.2.17}$$

$$1 = \mu H^P + (1-\mu)H^I \tag{B.2.18}$$

$$\mu D^P = D^B \tag{B.2.19}$$

$$(1-\mu)M^I = M^B \tag{B.2.20}$$

Steady state variables to be determined:
Patient households: C^P, H^P, D^P;
Impatient households: C^I, H^I, M^I;
Banks: N^B, D^B, M^B;
Aggregate: H.

B.2.2 Comparative Statics in Steady State

In Figures B.1 to B.4, we show comparative statics for the leverage constraint Γ, the bank exit rate η, the discount factor of impatient households β^I and steady state productivity μ^Z. Γ is the main policy parameter we are interested in. We mark the baseline parametrization of the model with a red vertical line.

Varying Γ

Consider first the effects of raising the leverage constraint Γ displayed in Figure B.1. A higher leverage allows banks to use more deposits to finance a given amount of lending. As long as the interest rate differential $\left[1-(1-X^M)\Phi\right]R^M - R^D$ is positive, taking on more leverage is profitable and increases the net worth of the bank. To see this, we substitute B.2.10 and B.2.13 into B.2.11:

$$n^B = \left[\left[\left[1-(1-X^M)\Phi\right]R^M - R^D\right]\Gamma + R^D\right]N^B.$$

A higher bank net worth in turn implies more deposits and more mortgages, which raises the consumption of both consumption goods and housing of patient households, since they will overall save more. Similarly, it lowers consumption of consumption goods and housing of impatient households, since they save less. Moreover, a higher mortgage coupled with less housing of impatient households means that mortgages become more risky, since their recovery rate decreases and the mortgage default rate increases. This leads to a higher required return on mortgages. Finally, since a larger share of housing is now held by patient households who value housing less than impatient households, the house price index decreases.

Above a certain threshold, it is no longer the leverage constraint of banks, but the borrowing constraint of impatient households which will constrain the amount of mortgages in the economy. Beyond that threshold, raising the leverage constraint no longer has any effect on the economy.

Fig. B.1 Comparative statics with respect to the leverage constraint ψ

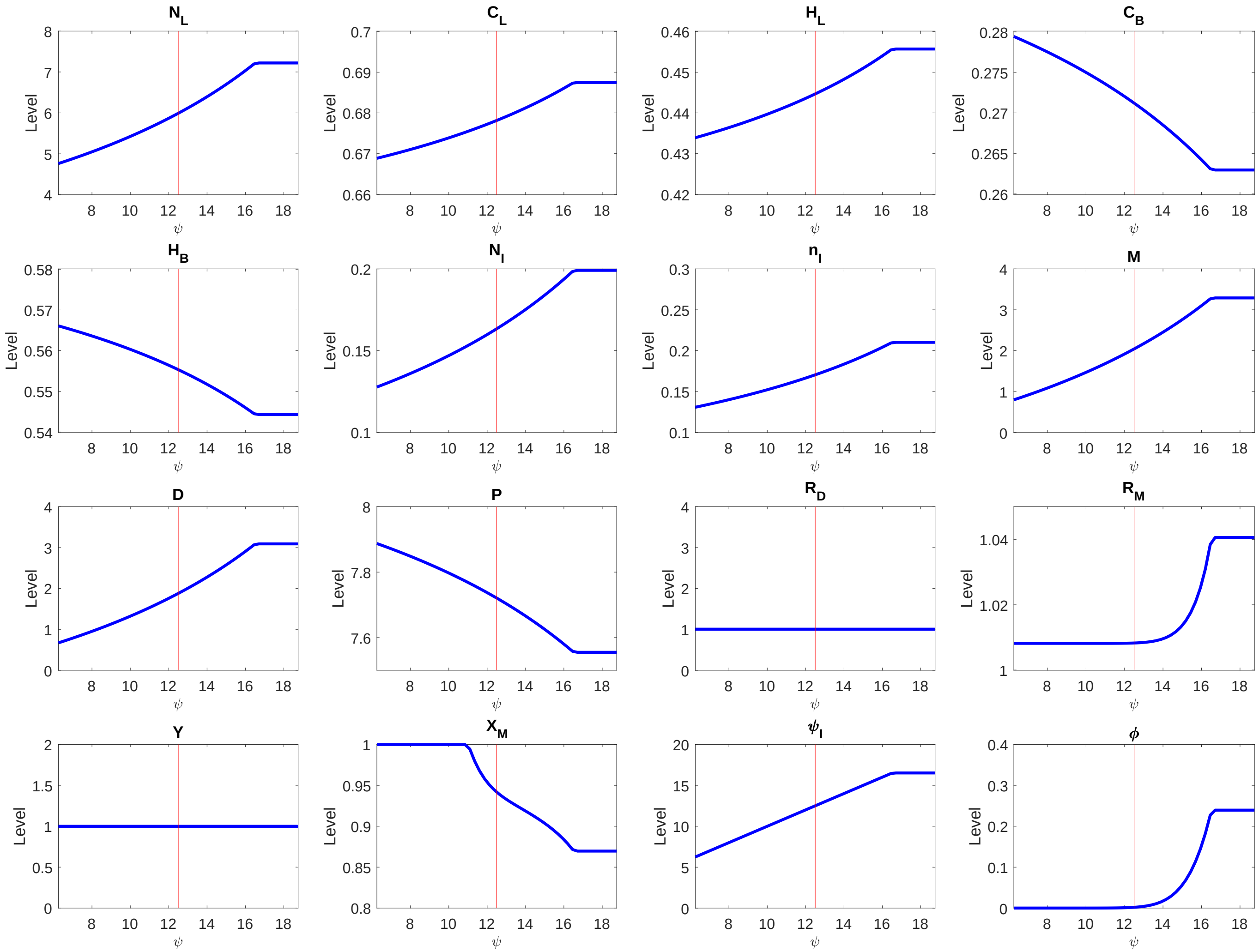


Fig. B.2 Comparative statics with respect to the exit rate of banks η

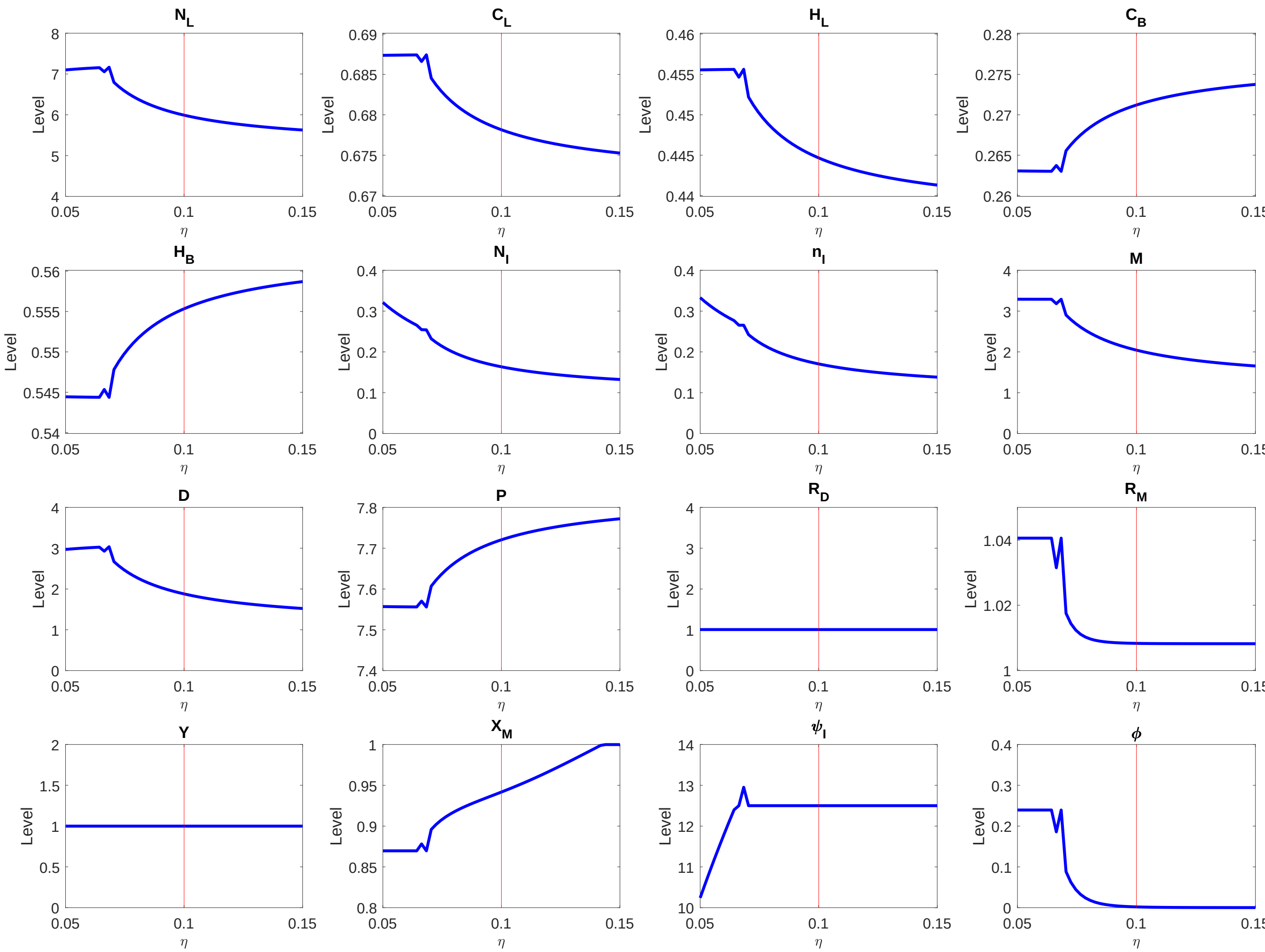


Fig. B.3 Comparative statics with respect to the discount factor of impatient households β^I

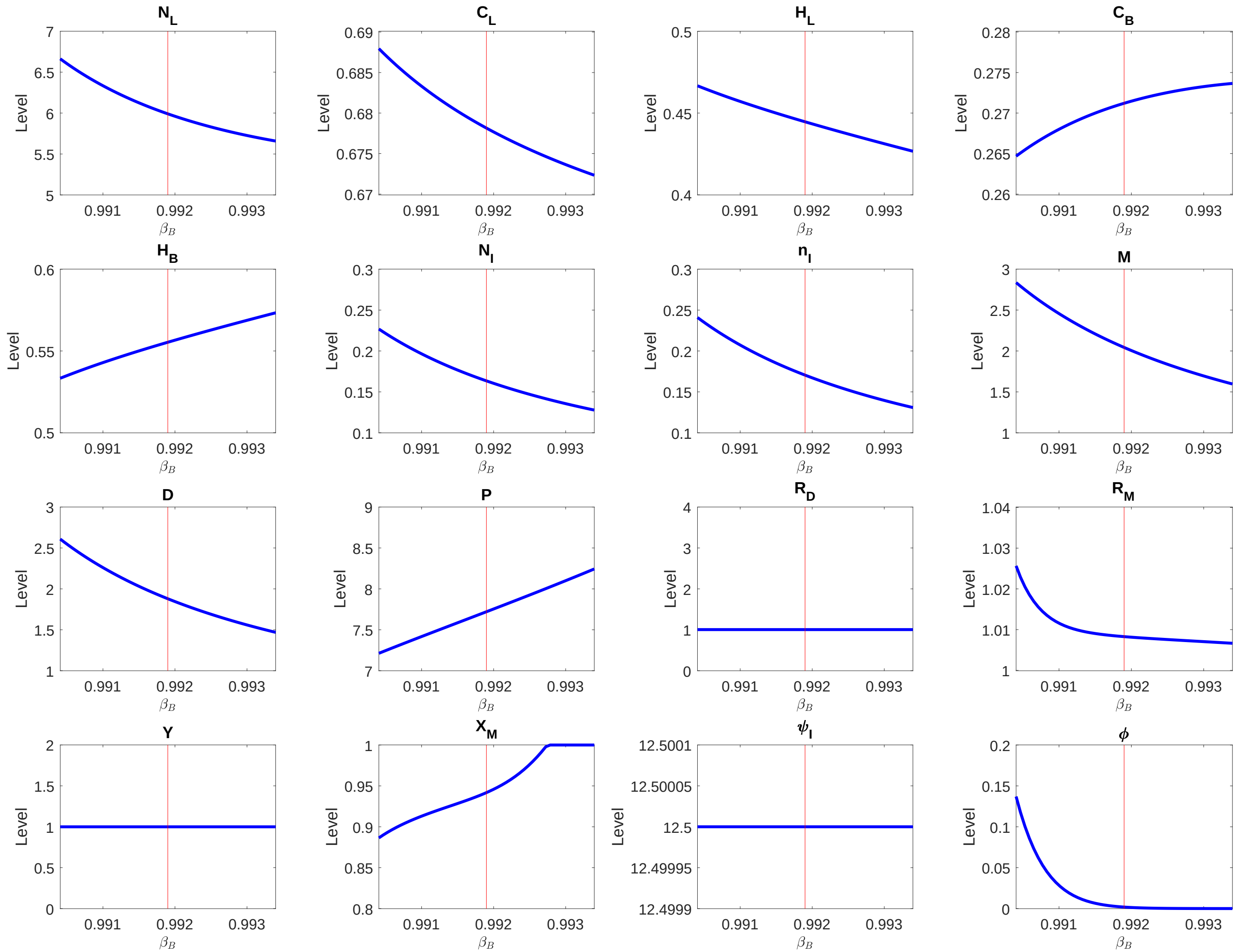
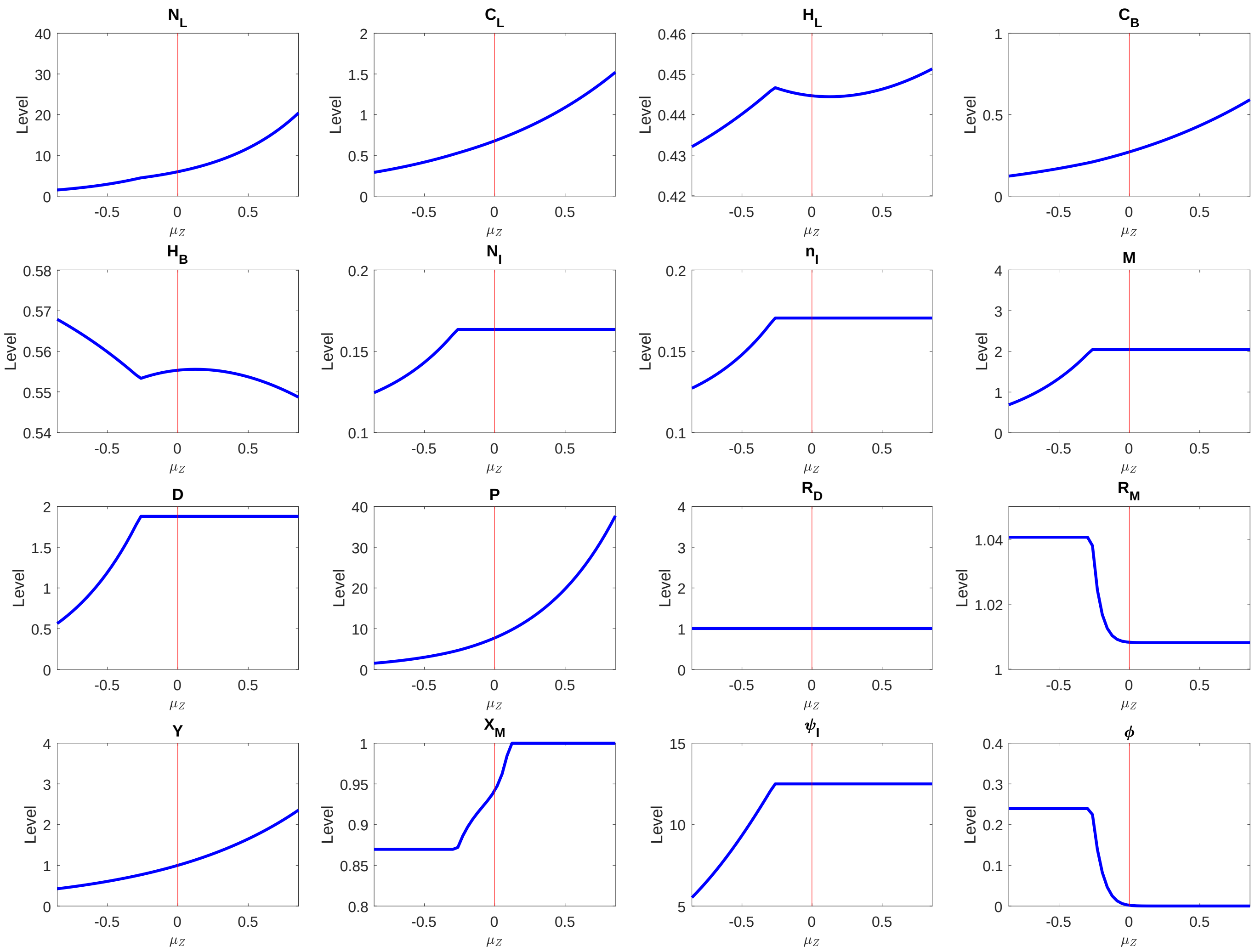


Fig. B.4 Comparative statics with respect to the steady state productivity μ^Z



B.3 Complete Statement of the Model

B.3.1 Impatient Households

The impatient households' problem can be summarized as:

$$\max_{\left\{C_t^I, H_{t+1}^I, M_{t+1}^I\right\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\beta^I\right)^t U^I \left(C_t^I, H_t^I\right) \right],$$

s.t.

$$C_t^I + P_t(H_{t+1}^I - (1-\delta)H_t^I) + \left[1 - (1 - X_t^M)\Phi_t\right] R_t^M M_t^I = W_t \bar{L}_t^I + M_{t+1}^I + T_t,$$

$$M_{t+1}^I \leq \kappa P_t H_{t+1}^I,$$

$$C_t^I, H_{t+1}^I, M_{t+1}^I \geq 0.$$

The FONC of the impatient households' problem are:

$$U_1(C_t^I, H_t^I) = \lambda_t^I, \tag{B.3.1}$$

$$\lambda_t^I P_t = \beta^I \mathbb{E}_t \left[\lambda_{t+1}^I P_{t+1} + \mathbf{U}_2(\mathbf{C}_{t+1}^I, H_{t+1}^I)\right], \tag{B.3.2}$$

$$\lambda_t^I = \beta^I \mathbb{E}_t \left[\lambda_{t+1}^I [1 - (1 - X_{t+1}^M)\Phi_{t+1}] R_{t+1}^M\right], \tag{B.3.3}$$

$$C_t^I + P_t(H_{t+1}^I - (1-\delta)H_t^I) + \left[1 - (1 - X_t^M)\Phi_t\right] R_t^M M_t^I = W_t \bar{L}_t^I + M_{t+1}^I + T_t. \tag{B.3.4}$$

B.3.2 Patient Households

The patient households face a standard consumption-and-saving problem with housing:

$$\max_{\left\{C_t^P, H_{t+1}^P, D_{t+1}^P\right\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\beta^P\right)^t U \left(C_t^P, H_t^P\right) \right],$$

s.t.

$$C_t^P + P_t(H_{t+1}^P - (1-\delta)H_t^P) + D_{t+1}^P = W_t \bar{L}_t^P + X_t^D \left(R_t^D D_t^P\right) + (1-\tau)O_t,$$

$$C_t^P, H_{t+1}^P, D_{t+1}^P \geq 0,$$

The FONC of the patient households' problem are:

$$U_1(C_t^P, H_t^P) = \lambda_t^P, \tag{B.3.5}$$

$$\lambda_t^P P_t = \beta^P \mathbb{E}_t \left[\lambda_{t+1}^P P_{t+1} + \mathbf{U}_2(\mathbf{C}_{t+1}^P, H_{t+1}^P)\right], \tag{B.3.6}$$

$$\lambda_t^P = \beta^P \mathbb{E}_t \left[\lambda_{t+1}^P R_{t+1}^D\right], \tag{B.3.7}$$

$$C_t^P + P_t(H_{t+1}^P - (1-\delta)H_t^P) + D_{t+1}^P = W_t \bar{L}_t^P + R_t^D D_t^P + (1-\tau)O_t. \tag{B.3.8}$$

B.3.3 Banks

Full statement of the bank's problem The surviving bank's maximization problem is:

$$\max_{\{M_{t+1}^B, D_{t+1}^B\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \left[\frac{U_c(C_t^P, H_t^P)}{U_c(C_0^P, H_0^P)} \left[\beta^P (1-\eta)\right]^t \eta n_t^B \right] \right\},$$

s.t.

$$M_{t+1}^B = D_{t+1}^B + n_t^B, \qquad \text{Balance Sheet Constraint}$$

$$N_t^B \geq \Gamma_t M_{t+1}^B, \qquad \text{Bank Capital Requirement}$$

where net worth of the surviving banks n_t^B is given by:

$$n_t^B = \left[1 - (1 - X_t^M)\Phi_t\right] R_t^M M_t^B - R_t^D D_t^B. \tag{B.3.9}$$

Conditional on the leverage constraint being binding, the banker's problem is fully described by the leverage constraint, the balance sheet constraint and the non-negative profit margin constraint.

B.3.4 Bank's Problem: Non-binding Optimality Conditions

Consider the case of a bank that faces a non-binding capital requirement today. Rewriting the bank's problem in recursive form yields

$$V_t = \max_{M_{t+1}^B, D_{t+1}^B} \beta^P \mathbb{E}_t \left[\frac{U_c^P(C_{t+1}^P, H_{t+1}^P)}{U_c^P(C_t^P, H_t^P)} \bar{V}_{t+1} \right]$$

s.t.

$$M_{t+1}^B = D_{t+1}^B + n_t^B$$

where

$$\bar{V}_t = \eta n_t^B + (1-\eta) V_t,$$

and

$$n_t^B = \tilde{R}_t^M M_t^B - R_t^D D_t^B.$$

Define $\Omega_t \equiv \frac{\bar{V}_t}{n_t}$. In general, Ω_t depends only on aggregate variables and is hence exogenous from the point of view of an individual bank.[1] Rewriting the bank's problem yields:

$$
\begin{aligned}
V_t &= \max_{M^B_{t+1}, D^B_{t+1}} \beta^P \mathbb{E}_t \left[\frac{U^P_c(C^P_{t+1}, H^P_{t+1})}{U^P_c(C^P_t, H^P_t)} \Omega_{t+1} n_{t+1} \right] \\
\bar{V}_t &= \eta n^B_t + (1-\eta) V_t \\
&\quad \text{s.t.} \\
M^B_{t+1} &= D^B_{t+1} + n^B_t \\
n^B_t &= \tilde{R}^M_t M^B_t - R^D_t D^B_t
\end{aligned}
$$

Optimality of Deposit Funding Consider a bank that uses deposit financing. In this case, the problem of the bank can be simplified to

$$
V_t = \max_{D^B_{t+1}} \beta^P \mathbb{E}_t \left[\frac{U^P_c(C^P_{t+1}, H^P_{t+1})}{U^P_c(C^P_t, H^P_t)} \Omega_{t+1} (\tilde{R}^M_{t+1}(D^B_{t+1} + n^B_t) - R^D_{t+1} D^B_{t+1}) \right]
$$

A bank will use some deposit funding in addition to lending out its equity whenever

$$
\frac{\partial V_t}{\partial D^B_{t+1}} = \beta^P \mathbb{E}_t \left[\frac{U^P_c(C^P_{t+1}, H^P_{t+1})}{U^P_c(C^P_t, H^P_t)} \Omega_{t+1} (\tilde{R}^M_{t+1} - R^D_{t+1}) \right] \geq 0,
$$

i.e. whenever the net benefit of raising an additional unit of deposits and lending it out in the form of mortgages is positive.

Optimality of Mortgage Lending A bank will lend out its net worth in the form of mortgages whenever

$$
\frac{\partial V_t}{\partial n^B_t} = \beta^P \mathbb{E}_t \left[\frac{U^P_c(C^P_{t+1}, H^P_{t+1})}{U^P_c(C^P_t, H^P_t)} \Omega_{t+1} \tilde{R}^M_{t+1} \right] \geq 1,
$$

i.e. when the benefit of reinvesting an additional dollar of net worth is higher than the benefit of simply paying it out to the households.

[1] This is straightforward to prove in the case of an always binding lending constraint. Noting that

$$
\begin{aligned}
M^B_{t+1} &= 1/\Gamma_t n^B_t \\
D^B_{t+1} &= (1/\Gamma_t - 1) n^B_t \\
n^B_t &= \tilde{R}^M_t 1/\Gamma_t n^B_{t-1} - R^D_t (1/\Gamma_t - 1) n^B_{t-1},
\end{aligned}
$$

we see that n^B_t / n^B_{t-1} depends only on aggregate returns. Plugging this into the bank's value function and iterating forward gives the result.

B.4 Numerical Solution Algorithm

Collect the exogenous states in $\mathscr{Y} = (A, Z)$. Collect the endogenous states in $\mathscr{S} = (H^P, N^P, N^I)$. The state space of the model without wage rigidity is characterized by $(\mathscr{S}, \mathscr{Y})$. Adding wage rigidity adds the lagged wage as a state variable. The state space in a bank run is characterized by $\left(H^P, \mathscr{Y}\right)$.

We need to find four unknown non-linear policy functions, namely $c^P_{NoRun}(\mathscr{S}, \mathscr{Y})$, $c^I_{NoRun}(\mathscr{S}, \mathscr{Y})$, $c^P_{Run}(H^P, \mathscr{Y})$ and $c^I_{Run}(H^P, \mathscr{Y})$ and laws of motion for net worth $N^{P'}_{NoRunToNoRun}$, $N^{P'}_{NoRunToRun}$, $N^{P'}_{RunToNoRun}$, $N^{P'}_{RunToRun}$ and $N^{I'}_{NoRunToNoRun}$ as functions of the endogenous and exogenous states.

The general idea is to approximate the unknown functions on a sparse state grid and then solve the model by backward iteration. The outline of the algorithm is as follows:

1. Find the steady state of the model. See above for details.
2. Set up grid: Adaptive sparse grid as in Brumm and Scheidegger (2017).
   (a) Bounds:
      - Exogenous processes: +- 4 unconditional standard deviations.
      - Endogenous processes: around steady state.

   (b) Grid level: 6, meaning that we use up to six nested sets of basis functions.
3. Initial guess for the unknown functions and the laws of motion for net worth
4. Compute expectations: Mixture of Gauss-Hermite and Gauss-Legendre quadrature. 11 quadrature nodes on each shock. Gauss-Hermite quadrature is standard to approximate the expectation over normally distributed variables. Gauss-Legendre quadrature is useful, since it allows simple integration over a bounded interval, which is what we want to do to work with the exact bank run cutoff.
5. Find new policy functions: Solving a system of non-linear equations. Need to find $H^{B'}, H^{L'}, P$ for both the run- and no-run equilibrium.
6. Update unknown functions and net worth laws of motion.
7. Check convergence: specify tolerances.

We provide details on the critical steps below.

B.4.1 Grid

The policy functions in the no-run case are approximated on a five-dimensional grid, the laws of motion for net worth in the no-run case on a seven-dimensional grid. The functions in the run case do not have N^B as a state variable.

B.4.2 Expectations

We want to numerically approximate expectations of the kind

$$\begin{aligned}&\mathbb{E}\left[f(H^{P'},N^{P'},N^{I'},A',Z')|A,Z\right] = \\ &= \int_0^\infty \left(\int_0^{A^*} f^{Run}(H^{L'},A',Z')dG(A'|A) + \right.\\ &\left.\int_{A^*}^\infty f^{NoRun}(H^{P'},N^{P'},N^{I'},A',Z')dG(A'|A)\right) dF(Z'|Z),\end{aligned}$$

where Z denotes productivity and A is the house quality shock. For productivity, we simply use Gauss-Hermite quadrature with integration nodes x^a and integration weights w^a. Since we want to compute the expectations using the exact thresholds for the house quality shock, we use Gauss-Legendre quadrature, using integration nodes $x^z \in [-1,1]$ and integration weights w^z, with $\sum_z w^z = 2$. We assume that $\ln Z$ is bounded between $\ln \underline{Z}$ and $\ln \overline{Z}$. This procedure essentially follows Hatchondo et al. (2016). The integration consists of four steps:

1. Find the exact bank run cutoffs $\varepsilon^{Z*}(H^P, R^D D, R^M M, A, Z_{-1})$. The cutoffs can be found by solving the non-linear equation

$$\begin{aligned}&Z_{-1}^{\rho^A} \exp \varepsilon^{Z*} \frac{(1-\pi(P^*))R^M M + \pi(P^*)P^* H^I}{R^D D} = 1, \\ &P^* = p^{Run}(H^P, A, Z_{-1}, \varepsilon^Z).\end{aligned}$$

2. Determine the integration nodes for Z. We distinguish two cases:

   (a) $Z^* \leq \underline{Z}$ or $Z^* \geq \overline{Z}$. In this case, there is no interior bank run cutoff. We compute the integration nodes to be equally spaced in probability: Define

$$\overline{\varepsilon} = \frac{\ln \overline{Z} - \rho^A \ln Z}{\eta^A},$$

   and

$$\underline{\varepsilon} = \frac{\ln \underline{Z} - \rho^A \ln Z}{\eta^A}.$$

Then, we compute the adjusted integration nodes ε^z as

$$cdf(\varepsilon^z) = cdf(\underline{\varepsilon}) + \frac{1+x^z}{2}(cdf(\overline{\varepsilon}) - cdf(\underline{\varepsilon})).$$

$cdf(.)$ is the cumulative distribution function of a standardized normal distribution.

(b) $\underline{Z} < Z^* < \overline{Z}$. In this case, there is an interior bank run cutoff. Hence, we compute two sets of adjusted integration nodes. Define

$$\varepsilon^* = \frac{\ln Z^* - \rho^A \ln Z}{\eta^A}.$$

Then, the first set of integration nodes is given by

$$cdf(\varepsilon^z_{Run}) = cdf(\underline{\varepsilon}) + \frac{1+x^z}{2}(cdf(\varepsilon^*) - cdf(\underline{\varepsilon})),$$

and the second set by

$$cdf(\varepsilon^z_{NoRun}) = cdf(\varepsilon^*) + \frac{1+x^z}{2}(cdf(\overline{\varepsilon}) - cdf(\varepsilon^*)).$$

3. Integrate piecewise in the No-Run and Run-region of the state space. Define

$$Z^z_{Run} = \exp(\rho^A \ln Z + \eta^A \varepsilon^z_{Run}).$$

and

$$A^a = \exp(\rho^Z \ln A + \eta^Z x^a).$$

Then, the run expectation can be approximated as

$$\begin{aligned}\mathbb{E}\left[f^{Run}|A,Z\right] &= \int_0^\infty \int_0^{Z^*} f^{Run}(H^{L'},A',Z,\varepsilon^Z)dG(\varepsilon^Z)dF(A'|A) \\ &\approx \sum_a \sum_z w^a \frac{w^z}{2} f^{Run}(H^{L'},A^a,Z,\varepsilon^z_{Run})\end{aligned}$$

The same applies to the no-run expectation $\mathbb{E}\left[f^{NoRun}|A,Z\right]$ and the expectation in the case of no interior cutoff, in which we can directly compute $\mathbb{E}[f|A,Z]$, subject to the adjustment below.

4. Sum up over the piecewise integrals. Note that since the probability mass between $\underline{Z}$ and $\overline{Z}$ is not one, we need to adjust the expectation:

$$\mathbb{E}[f|A,Z] = \frac{\mathbb{E}\left[f^{NoRun}|A,Z\right] + \mathbb{E}\left[f^{Run}|A,Z\right]}{cdf(\overline{\varepsilon}) - cdf(\underline{\varepsilon})}.$$

B.4.3 Nonlinear Equation Systems

We need to solve for the no run and the run equilibrium.

No Run System

Take the laws of motion of consumption at iteration $i-1$ as given. In iteration i, we compute the following expectations:

$$C^{P'} = c^{P}_{(i-1)}\left(h^{P'}(\mathscr{S},\mathscr{Y}), d^{'}(\mathscr{S},\mathscr{Y}), m^{'}(\mathscr{S},\mathscr{Y}), \mathscr{Y}'\right)$$

takes the law of motion of aggregate net worth as given.

With these expressions for the expectations, it's straightforward to compute the solution to the first-order conditions:

Patient Household:

$$\begin{aligned} U_1(C^P, H^P)P &= \beta^P \mathbb{E}\left[U_1(C^{L'}, H^{L'})P' + U_2(C^{L'}, H^{L'})|A, Z\right] \\ U_1(C^P, H^P) &= \beta^P R^{D'} \mathbb{E}\left[U_1(C^{L'}, H^{L'})|A, Z\right] \\ C^P + PH^{L'} + D^{'} &= WL^P + PH^L + R^D D + \Pi + \eta \frac{N^B - \eta E}{1-\eta} - \eta E \end{aligned}$$

Bank:

$$\begin{aligned} N^B + D^{'} &= M^{I'} \\ M^{'} &= \psi N^B \\ N^B &= (R^M M(1-\phi(P)) + P(\mathscr{H} - H^P)\phi(P) - R^D D)(1-\eta) + E\eta \end{aligned}$$

Impatient Household:

$$\begin{aligned} U_1(C^I, \mathscr{H} - H^P)P &= \beta^I \mathbb{E}\left[U_1(C^{B'}, H^{B'})P' + U_2(C^{B'}, H^{B'})|A, Z\right] \\ U_1(C^I, \mathscr{H} - H^P) &= \beta^I R^{M'} \mathbb{E}\left[U_1(C^{B'}, H^{B'})|A, Z\right] \\ C^I &= AL^\alpha - C^P \end{aligned}$$

where C^I comes from the aggregate resource constraint.

Firms:

$$
\begin{aligned}
A\alpha L^{\alpha-1} &= \frac{W}{1+\xi(R^K-1)} \\
\Pi &= AL^{\alpha} - WL
\end{aligned}
$$

Market Clearing:

$$
\mathscr{H} = H^{L'} + H^{B'}
$$

Effectively, this system of equations can be boiled down to solving a simple nonlinear system of equations in two variables, $H^{L'}, P$, which we do using MATLAB's fzero routine. All other variables can either be calculated explicitly or determined residually.

Run System

Patient Household:

$$
\begin{aligned}
U_1(C^P, H^P) &= \beta^P V_H^P(\mathscr{S}, A, Z) \\
C^P + PH^{L'} &= WL^P + XR^D D + \Pi + \eta\frac{N^B - \eta E}{1-\eta} - \eta E + (ZR^K - 1)K \\
X &= Z\frac{R^M M(1-\phi(P)) + P(\mathscr{H} - H^P)\phi(P)}{R^D D}
\end{aligned}
$$

Impatient Household:

$$
\begin{aligned}
U_1(C^I, \mathscr{H} - H^P) &= \beta^I V_H^I(\mathscr{S}, A, Z) \\
C^I &= AL^{\alpha} - C^P
\end{aligned}
$$

where C^I comes from the aggregate resource constraint.

Firms:

$$
\begin{aligned}
A\alpha L^{\alpha-1} &= \frac{W}{1+\xi(R^K-1)} \\
\Pi &= AL^{\alpha} - WL(1+\xi(R^K-1)) \\
K &= \xi WL
\end{aligned}
$$

Market Clearing:

$$\mathscr{H} = H^{L'} + H^{B'}$$

Curriculum Vitae

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