# Essays in Macroeconomics and Consumer Finance 

Inauguraldissertation<br>zur Erlangung des akademischen Grades eines Doktors der Wirtschaftswissenschaften der Universität Mannheim


im Herbst-/Wintersemester 2022 vorgelegt von
Tsung-Hsien Li

Abteilungssprecher:<br>Prof. Klaus Adam, Ph.D.<br>Vorsitzender der Disputation: Prof. Dr. Matthias Meier<br>Referentin:<br>Prof. Michèle Tertilt, Ph.D.<br>Koreferent:<br>Prof. Klaus Adam, Ph.D.

## Acknowledgments

First and foremost, I am incredibly grateful and indebted to my main advisor Michèle Tertilt and my co-advisors Klaus Adam and Matthias Meier. Michèle provided tremendous support in weekly meetings throughout my Ph.D. studies. I could not have finished my doctorate without her constant guidance and encouragement. Klaus offered the weekly colloquium where preliminary research ideas could be exchanged, and I benefited greatly from the discussions. I learned many things in Matthias' courses and am thankful for his useful feedback.

I would like to thank my co-author Jan Sun for the continued cooperation since our third year. In addition, my research has benefited from the valuable comments by Andreas Gulyas, Antoine Camous, David Koll, Husnu C. Dalgic, Konrad Stahl, Krzysztof Pytka, Minchul Yum, and other seminar participants at the Mannheim Macro Seminar and the discussions with other fellow students at the Center for Doctoral Studies in Economics (CDSE).

I would like to thank Alexander Matusche, Carl-Christian Groh, Federico Innocenti, Marco Reuter, Mykola Ryzhenkov, and So Jin Lee for making my Ph.D. studies a pleasant experience. In particular, the Halo night with Christian and Federico was the most relaxing moment in my second year.

I would like to thank Caroline Mohr, Corinna Jann-Grahovac, Golareh Khalilpour, Julia Potapov, Marion Lehnert, and Ulrich Kehl for their administrative support throughout my stay at Mannheim.

I am grateful for generous financial support from the foundation "Stiftung Geld und Währung" and the Graduate School of Economics and Social Sciences (GESS) at the University of Mannheim. I also acknowledge support by the state of Baden-Württemberg through bwHPC and the German Research Foundation through grant INST 35/1134-1 FUGG. I am thankful to Jeremy Tobacman for providing data relevant to my second chapter.

And finally, I want to thank my parents for their support and encouragement. It was quite a journey studying in Germany for five years that kept me distant from Taiwan and did not allow me to spend as much time with them as I wanted. Nonetheless, it came to a beautiful end full of harvest.

## Contents

Acknowledgments ..... iii
List of Figures ..... ix
List of Tables ..... xi
Preface ..... xiii
1 Credit Card and Payday Loan Borrowing ..... 1
1.1 Introduction ..... 1
1.2 Data ..... 2
1.3 Results ..... 3
1.3.1 Life Cycle ..... 3
1.3.2 Income and Wealth ..... 4
1.3.3 Education and Financial literacy ..... 6
1.4 Conclusion ..... 8
Appendices ..... 11
1.A Related Survey Questions in the SCF ..... 11
1.A. 1 Credit Card and Payday Loan Borrowers ..... 11
1.A. 2 Financial Literacy ..... 11
1.B More Results in the SCF ..... 11
1.B. 1 Credit Card Holders ..... 11
1.B. 2 Credit Card Borrowers ..... 12
1.B. 3 Payday Loan Borrowers ..... 12
1.B. 4 Net Worth Octile and Duo-Decile ..... 12
2 The Payday Loan Puzzle ..... 17
2.1 Introduction ..... 17
2.2 Related Literature ..... 21
2.3 The Model ..... 23
2.3.1 Timing ..... 25
2.3.2 Households ..... 26
2.3.3 Financial Intermediaries ..... 27
2.3.4 Evolution of the Household Distribution ..... 31
2.3.5 Equilibrium ..... 31
2.4 Calibration ..... 32
2.5 Pooling and Cross-Subsidization ..... 35
2.6 The Payday Loan Puzzle ..... 38
2.6.1 Identification of the Payday Loan Puzzle ..... 38
2.6.2 The Reputation Protection Channel ..... 41
2.6.3 Profile of Puzzling Households ..... 43
2.7 Policy Experiments ..... 44
2.7.1 Payday Loan Regulation ..... 44
2.7.2 Bankruptcy Regulation ..... 48
2.8 Conclusion ..... 50
Appendices ..... 51
2.A Assignment of Posterior Type Score ..... 51
2.B Computation ..... 51
2.B. 1 Grid Specifications ..... 51
2.B. 2 One-Loop Algorithm ..... 52
2.C Robustness Check: Same Default Costs ..... 53
2.D General Results ..... 54
3 Consumer Bankruptcy and Financial Frictions ..... 59
3.1 Introduction ..... 59
3.2 Related Literature ..... 64
3.3 The Model ..... 66
3.3.1 Timing ..... 67
3.3.2 Households ..... 68
3.3.3 Firms ..... 69
3.3.4 Banks ..... 70
3.3.5 Evolution of the Household Distribution ..... 73
3.3.6 Equilibrium ..... 73
3.4 Calibration ..... 74
3.5 Consumer Credit with Financial Frictions ..... 77
3.5.1 Benchmark v.s. Frictionless Economy ..... 77
3.5.2 Varying Degree of Financial Frictions ..... 79
3.6 Regulation of Consumer Credit Markets ..... 81
3.6.1 Welfare Measures ..... 82
3.6.2 Wage Garnishment ..... 83
3.6.3 Exclusion from Borrowing Markets ..... 87
3.6.4 Welfare Effects of Varying Financial Frictions ..... 90
3.7 Conclusion ..... 94
Appendices ..... 97
3.A Model Details ..... 97
3.A. 1 Bank Optimization ..... 97
3.A. 2 Equilibrium Conditions ..... 99
3.B Computation Details ..... 99
3.B. 1 Grid Specifications ..... 99
3.B. 2 Algorithm for Solving Stationary Equilibrium ..... 99
3.B.3 Algorithm for Solving Transition Dynamics ..... 101
3.C Transition Paths of Banking Leverage Ratio ..... 102
3.D Robustness Check: Degree of Financial Frictions ..... 102
Bibliography ..... 107
Eidesstattliche Erklärung ..... 113
Curriculum Vitae ..... 115

## List of Figures

1.1 Avg. Fraction of Borrowers by Age over the SCF 2010-2019 ..... 4
1.2 Avg. Fraction of Borrowers by Income over the SCF 2010-2019 ..... 5
1.3 Avg. Fraction of Borrowers by Net Worth over the SCF 2010-2019 ..... 6
1.4 Age of Credit Card Holders and Borrowers by Net Worth ..... 7
1.5 Avg. Fraction of Borrowers by Education over the SCF 2010-2019 ..... 8
1.B. 1 Fraction of Credit Card Holders over the SCF 2010-2019 ..... 12
1.B. 2 Fraction of Credit Card Borrowers over the SCF 2010-2019 ..... 13
1.B.3 Fraction of Payday Loan Borrowers over the SCF 2010-2019 ..... 14
1.B. 4 Fraction of Borrowers by Net Worth over the SCF 2010-2019 ..... 15
2.1 Layout of the Economy and Information Structure ..... 23
2.2 Borrowing and Default Behavior across Types ..... 35
2.3 Cross-Subsidization of Bank Loans across Types ..... 36
2.4 Formal Default Probability across Payday Loans ..... 37
2.5 Cross-Subsidization of Bank Loans across (Non-)Payday Loan Borrowers ..... 38
2.6 Identification of the Payday Loan Puzzle ..... 40
2.7 Histogram of Monetary Costs of Payday Loan Puzzle ..... 41
2.8 Reputation Protection Incentive ..... 42
2.9 Cost-Benefit Analysis among Seemingly Puzzling Households ..... 43
2.10 Earnings Distribution among Both Loan Borrowers ..... 44
2.11 Type Score Updating across Persistent Earnings ..... 45
2.12 Distribution of Payday Loan Size across Types ..... 47
2.D. 1 Default Probabilities ..... 55
2.D. 2 Pricing Schedule and Discounted Borrowing Amount ..... 56
2.D. 3 Earnings Distribution among Borrowers and Savers ..... 57
2.D. 4 Type Score Distribution ..... 57
2.D. 5 Posterior Type Score Dynamic ..... 57
2.D. 6 Avg. Interest Rates for Bank and Payday Loans across Type Scores ..... 58
3.1 Transition Paths of Banking Leverage Ratio ..... 85
3.2 Welfare for Total Households ..... 90
3.3 Welfare for Households with Good Credit History ..... 91
3.4 Welfare for Households with Bad Credit History ..... 92
3.5 Aggregate Welfare (CEV) v.s. Financial Frictions ..... 93
3.C. 1 Transition Paths of Banking Leverage Ratio ..... 102
3.C. 2 Transition Paths of Banking Leverage Ratio with $\theta^{L}$ ..... 103
3.C. 3 Transition Paths of Banking Leverage Ratio $\theta^{H}$ ..... 103

## List of Tables

1.1 Borrower Fraction over the SCF 2010-2019 ..... 3
1.2 Correctness Rates for Big 3 Questions over the SCF 2016-2019 ..... 7
2.1 Exogenously Chosen Parameters ..... 33
2.2 Internally Calibrated Parameters ..... 34
2.3 Untargeted Moments: Data v.s. Model ..... 34
2.4 Equilibrium across Types ..... 39
2.5 Policy Counterfactual: Restricting Payday Loan Size ..... 46
2.6 Policy Counterfactual: Higher Filing Costs ..... 48
2.B. 1 Grids Used for Model Computation ..... 51
2.C. 1 Counterfactual: Same Default Costs ..... 54
3.1 Exogenously Chosen Parameters ..... 75
3.2 Internally Calibrated Parameters ..... 76
3.3 Untargeted Moments: Data v.s. Model ..... 77
3.4 Effects of Financial Frictions on Equilibrium Outcomes ..... 78
3.5 Effects of Varying Degree of Financial Frictions by $\theta$ ..... 80
3.6 Effects of Varying Degree of Financial Frictions by $\psi$ ..... 81
3.7 Counterfactual of Wage Garnishment: Equilibria Comparison ..... 84
3.8 Counterfactual of Wage Garnishment: Welfare Implications ..... 86
3.9 Counterfactual of Prob. of Flag Removal: Equilibria Comparison ..... 88
3.10 Counterfactual of Prob. of Flag Removal: Welfare Implications ..... 89
3.11 Effects of Incentive and Divestment Channels v.s. Financial Frictions ..... 93
3.B. 1 Grids Used for Model Computation ..... 99
3.D. 1 Counterfactual of Wage Garnishment with $\theta^{L}$ : Equilibria Comparison ..... 104
3.D. 2 Counterfactual of Wage Garnishment with $\theta^{L}$ : Welfare Implications ..... 104
3.D. 3 Counterfactual of Wage Garnishment with $\theta^{H}$ : Equilibria Comparison ..... 105
3.D. 4 Counterfactual of Wage Garnishment with $\theta^{H}$ : Welfare Implications ..... 105

## Preface

This dissertation studies questions in macroeconomics focusing on consumer finance and default using quantitative methods. It contains three self-contained chapters.

Chapter 1 is titled "Credit Card and Payday Loan Borrowing: Evidence in the SCF 2010-2019." Each year, almost $40 \%$ of U.S. households have credit card debts and $4 \%$ borrow using a high-cost payday loan. I explore the similarities and differences between both types of borrowers. Using the Survey of Consumer Finances (SCF) from 2010 to 2019, I document that: (1) credit card borrowers are middle-aged, upper-middle-class, with some college exposure, and financially literate; (2) payday loan borrowers are young, low-income and low-wealth, less educated, and less financially literate; and (3) payday loan borrowers lack the financial knowledge of inflation and risk diversification, but not of interest compound.

Chapter 2 is titled "The Payday Loan Puzzle: A Credit Scoring Explanation" and is co-authored with Jan Sun. We propose the novel reputation protection explanation to account for the so-called Payday Loan Puzzle. A payday loan is a short-term, high-cost unsecured consumer loan popular in the U.S. In particular, these loans carry enormous interest costs corresponding to annualized rates of several hundred percent, compared to the annual interest rates for credit cards between 10 to 30 percent. Previous literature has documented that two-thirds of individuals took up a payday loan while having liquidity left on their possessed credit cards. This borrowing behavior results in significant monetary costs and has been coined the "Payday Loan Puzzle." We propose and formalize the novel explanation that households use payday loans to protect their credit scores. A credit score is a statistic computed by credit bureaus to measure a borrower's creditworthiness. These scores significantly impact U.S. households' credit access, including credit costs, mortgage terms, and even job application prospects in the future. While using credit cards affects one's credit score, using payday loans does not. Even though payday loans are much more expensive than credit cards, we hypothesize that using payday loans instead of credit cards leads to long-term reputational benefits at the short-term cost of higher interest fees.

To quantitatively investigate this hypothesis, we build a two-asset Huggett-type model
with two default options as well as hidden information and actions. Our calibrated model can account for $40 \%$ of the empirically identified payday loan borrowers with liquidity left on their credit cards. We can also match the untargeted magnitude of monetary costs due to this seeming pecuniary mistake. Payday loans are a hotly debated regulatory topic in the U.S.: critics have argued for an outright ban due to their high costs, while advocates stress their essential role in smoothing consumption. To inform the policy debate over payday lending, we assess the welfare implications of several policy counterfactuals. We find that either banning payday loans or increasing their default costs results in aggregate welfare losses.

Chapter 3, titled "Consumer Bankruptcy: the Role of Financial Frictions," studies the role of financial frictions in consumer bankruptcy. A considerable body of literature has shown that financial frictions significantly affect financial intermediation and regards them as the primary driver of the 2007-2008 Financial Crisis. Some papers have also suggested that financial frictions affect consumer credit markets. However, even though consumer credit is highly regulated through consumer bankruptcy laws, no work has been done to analyze the interaction between financial frictions and legal environments in consumer credit markets. In essence, frictional financial intermediation not only affects household borrowing and default behavior but also may influence the extent to which welfare is affected by bankruptcy strictness.

To quantitatively explore the effects of financial frictions in this regard, I develop an Aiyagari-type model with consumer default and an endogenous banking leverage constraint. Under my calibrated model, the borrowing prices of consumer loans are determined by idiosyncratic default premia and aggregate banking capitalization. Frictional financial intermediation results in higher borrowing costs, thus leading to declines in household debt and firm investment. To shed light on the role of financial frictions in consumer bankruptcy, I evaluate the welfare implications of several policy counterfactuals. I find that stricter bankruptcy regimes, through either higher wage garnishment or longer borrowing exclusion, result in aggregate welfare gains. Moreover, the sensitivity of welfare to bankruptcy strictness depends positively on the degree of financial frictions.

## Chapter 1

## Credit Card and Payday Loan Borrowing: Evidence in the SCF 2010-2019

### 1.1 Introduction

Unsecured borrowing plays an important role for consumers in smoothing consumption. There are two popular consumer loans in the United States: credit cards and payday loans. A credit card is granted with a line of credit that allows its holder to borrow liquidity repeatedly at annual interest rates between 10 to 30 percent. Exhausting the credit line and failing in repayment affect cardholders' credit scores. ${ }^{1}$ Credit cards are one ubiquitous product among the mainstream financial services (MFS): $70 \%$ of U.S. households have a credit card, and almost $40 \%$ of them borrow money using their cards.

On the other hand, a payday loan is a short-term small-amount unsecured loan with a duration of a few weeks for a typically small amount of \$300. Crucially, it carries enormous interest costs corresponding to annualized rates of several hundred percent (Stegman, 2007). Payday loans are one of the popular products among alternative financial services (AFS): there are more storefronts of payday lenders than fast-food chain restaurants (Karger, 2005) and around $4 \%$ of households take up a payday loan in the U.S. Compared to credit cards, borrowing or defaulting on payday loans usually does not affect credit scores. ${ }^{2}$

Given the similarities and differences in liquidity provision between credit cards and

[^0]payday loans, plus their popularity in the U.S., it is essential to better understand the heterogeneity across credit card and payday loan borrowers. To this end, this paper attempts to understand: (1) what type of households borrow using credit cards or payday loans? (2) in which dimensions do credit card borrowers differ from payday loan borrowers? To address these questions, I identify credit card and payday loan borrowers using the Survey of Consumer Finances (SCF) from 2010 to 2019. I then compare credit card borrowers with payday loan borrowers in terms of their life-cycle profile, income, net worth, education, and financial literacy. ${ }^{3}$

I document that: (1) most credit card borrowers are middle-aged, upper-middle-class, with some college exposure, and financially literate; (2) payday loan borrowers are often young, low-income and low-wealth, less educated, and less financially literate; and (3) although payday loan borrowers have a relatively lower degree of financial literacy than credit card borrowers, this lack of financial knowledge results from misunderstanding inflation and risk diversification, not interest compound.

The rest of the paper is organized as follows. Section 1.2 provides a brief introduction to the SCF and an overview of the average fractions of credit card and payday loan borrowers. Section 1.3 presents the evidence in the SCF 2010-2019. Section 1.4 concludes with potential avenues for further research.

### 1.2 Data

The SCF is a triennial cross-sectional survey of U.S. households. It contains information on demographic characteristics and great details on financial positions. In addition to credit card borrowing, payday loan usage has also been collected since 2010. Therefore, I use the SCF from 2010 to 2019, the latest available survey, to study the household borrowing behavior of credit cards and payday loans. To this end, I identify credit card borrowers as the survey respondents with a total balance still owed on their credit cards after the last payments. ${ }^{4}$ Payday loan borrowers refer to the households who took up a payday loan over the past 12 months proceeding to the survey. The exact survey questions used to construct the variables in the paper are summarized in Appendix 1.A. All reported statistics are weighted using the survey weights in the following discussions.

Table 1.1 reports the fraction of credit card and payday loan borrowers in the SCF from 2010 to 2019. The column "Average" shows the average fractions over 2010-2019. Overall, $36.3 \%$ of households have credit card debts, and $3.6 \%$ of households take up a payday loan in the U.S. These figures are undoubtedly high, especially given the high-

[^1]| Variable (in \%) | $\mathbf{2 0 1 0}$ | $\mathbf{2 0 1 3}$ | $\mathbf{2 0 1 6}$ | $\mathbf{2 0 1 9}$ | Average |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| All households |  |  |  |  |  |
| Credit card | 34.10 | 32.55 | 38.57 | 40.00 | 36.30 |
| Payday loan | 3.85 | 4.15 | 3.42 | 2.80 | 3.55 |
|  |  |  |  |  |  |
| Households aged 20-60 | 37.94 | 35.66 | 42.47 | 44.27 | 40.08 |
| Credit card | 4.78 | 5.08 | 4.12 | 3.64 | 4.40 |
| Payday loan |  |  |  |  |  |

Table 1.1: Borrower Fraction over the SCF 2010-2019

Notes: All statistics are weighted with the SCF survey weights. The upper panel "All households" reports the average borrower fractions among all respondents in the survey. The bottom panel "Households aged $20-60$ " shows the results conditional on the households with household heads aged between 20 and 60 . This subgroup constitutes around $70 \%$ of total respondents in each survey year.
interest costs for payday loans, which can be up to several hundred percentage points. Conditional on households aged from 20 to $60,{ }^{5}$ both fractions of credit card and payday loan borrowers increase further to $40.1 \%$ and $4.4 \%$, respectively.

### 1.3 Results

In this section, I document the properties of credit card and payday loan borrowers in terms of age, income and net worth, as well as education and financial literacy. For brevity, I present here only the unweighted average results over the SCF 2010-2019 waves and leave the results across each survey year in Appendix 1.B.

### 1.3.1 Life Cycle

Figure 1.1a and 1.1 b display the fractions of credit card and payday loan borrowers over life cycles, respectively. In particular, I divide households into the age groups of (20-34, $35-44,45-54,55-64,65-74$, and $75+$ ) and then compute the average fraction of credit card and payday loan borrowers for each age bin. Note that the SCF is a repeated cross-sectional survey. Therefore, these figures are not exactly the true life-cycle patterns. However, averaging the results using the SCF waves from 2010 to 2019 should filter out year-specific noises and thus yields a good proxy for the stationary life-cycle patterns.

Focusing first on Figure 1.1a, one can see that the life-cycle pattern of credit card borrowers is hump-shaped. This finding is consistent with Exler and Tertilt (2020). The fraction of households with any credit card debt is around $36 \%$ at the beginning of the life cycle. The fraction then increases steadily to over $45 \%$ until age 45-54. After that, it

[^2]Figure 1.1: Avg. Fraction of Borrowers by Age over the SCF 2010-2019


Notes: All statistics are weighted with the SCF survey weights. The average fraction of credit card borrowers is computed as the average of the fraction of households with a negative balance on their credit cards in each age bin over the SCF 2010-2019. The average fraction of payday loan borrowers is computed as the average of the fraction of households who took up a payday loan over the last year in each age bin over the SCF 2010-2019.
gradually decreases to around $20 \%$ after the age of 75 . On the other hand, the life-cycle pattern of payday loan borrowing in Figure 1.1b exhibits a decreasing relationship with age. The fraction of households who took up at least a payday loan remains around $5 \%$ up to age 44 and then decreases to less than $1 \%$ towards the end of the life cycle. ${ }^{6}$

The difference between the life-cycle patterns of credit card and payday loan borrowers could result from the fact that the youth often have not built up their credit scores or even processed a credit card. Refer to Appendix 1.B. 1 for the fraction of credit card holders by age group. As (partially) excluded from the mainstream financial markets, younger people turn to payday lenders to borrow against their future income to smooth consumption.

### 1.3.2 Income and Wealth

First of all, Figure 1.2 shows the average fractions of borrowers for each income decile over the SCF 2010-2019, where Figure 1.2a presents the results of credit cards and Figure 1.2 b illustrates the ones of payday loans. Income is measured as the pre-tax sum of wages, interest, dividends, realized capital gains, and miscellaneous sources of income for all household members.

In Figure 1.2a, the fraction of credit card borrowers among the lowest income decile is the lowest at around $17 \%$, compared to near $50 \%$ for the eighth decile. The fraction remains high at $45 \%$ among the ninth decile but slumps to near $30 \%$ for the richest decile. On the contrary, as seen in Figure 1.2b, the fraction of payday loan borrowers

[^3]Figure 1.2: Avg. Fraction of Borrowers by Income over the SCF 2010-2019


Notes: All statistics are weighted with the SCF survey weights. The average fraction of credit card borrowers is computed as the average of the fraction of households with a negative balance on their credit cards in each income decile over the SCF 2010-2019. The average fraction of payday loan borrowers is computed as the average of the fraction of households who took up a payday loan over the last year in each income decile over the SCF 2010-2019.
for the poorest decile is around $4 \%$ and increases to the highest at nearly $6 \%$ among the third decile. The fraction of households who took up a payday loan then monotonically decreases sixfold to less than $0.5 \%$ for the richest decile.

It is surprising to observe the hump-shaped borrowing patterns across income deciles for both credit cards and payday loans, as one may expect that the poorest are most likely to borrow. However, poor households can be excluded selectively from consumer credit markets due to relatively higher interest costs charged by lenders. For example, less than $40 \%$ of the poorest households possess a credit card, as illustrated in Appendix 1.B.1.

Second, the average fractions of credit card and payday loan borrowers for each net worth decile from the SCF 2010 to 2019 are presented in Figure 1.3a and 1.3b, respectively. Net worth denotes the net financial position of gross assets and liabilities. ${ }^{7}$ The results are robust with net worth octile or duo-decile, and see Appendix 1.B.4 for details.

Surprisingly, Figure 1.3a shows that the relationship between credit card borrowing and net worth is not monotonically decreasing. The peak of the fraction of credit card borrowers occurs in the lowest net worth decile, almost $50 \%$ of which have negative balances owed on their credit cards since the last payments. The fraction then drops dramatically to less than $20 \%$ for the second decile and then increases steadily to $45 \%$ among the fifth and sixth deciles. The fraction then decreases smoothly to below $20 \%$ for the richest decile. On the other hand, Figure 1.3b indicates that the fraction of payday loan borrowers is decreasing with net worth as expected: from the highest at $8 \%$ among the

[^4]Figure 1.3: Avg. Fraction of Borrowers by Net Worth over the SCF 2010-2019


Notes: All statistics are weighted with the SCF survey weights. The average fraction of credit card borrowers is computed as the average of the fraction of households with a negative balance on their credit cards in each net worth decile over the SCF 2010-2019. The average fraction of payday loan borrowers is computed as the average of the fraction of households who took up a payday loan over the last year in each net worth decile over the SCF 2010-2019.
first decile to the lowest of nearly $0 \%$ for the last decile.
So, why is there a slump in the fraction of credit card borrowers for the second net worth decile, i.e., the second least wealthy households? This is because many credit card borrowers among the second net worth decile are relatively older and, unlike younger households who would like to borrow much against their future income to smooth consumption, older households borrow less due to the near end of their life cycle. Figure 1.4a plots the age distribution of credit card holders for each net worth decile, and Figure 1.4 b reports the average age of credit card borrowers among each net worth decile. First, the age distribution of credit card holders in the second poorest decile is relatively more right-skewed, i.e., more middle-aged and older households, compared to the overall distributional pattern in age across net worth deciles. Second, the average age of credit card borrowers for the second net worth decile spikes to a higher level, deviating from the gradual upward age trend with net worth.

### 1.3.3 Education and Financial literacy

Figure 1.5a and 1.5b visualize the average fractions of credit card and payday loan borrowers by education groups in the SCF from 2010 to 2019, respectively. I divide households into the education groups of (no high school, high school, some college, and college degree).

Among those with no high school, the fraction of credit card borrowers is around $25 \%$. The fraction increases to over $45 \%$ for those who attended some college. The fraction then falls to around $37 \%$ for those with college degrees. On the other hand, the fraction of payday loan borrowers among those with a high school degree is around $4 \%$. The fraction

Figure 1.4: Age of Credit Card Holders and Borrowers by Net Worth


Notes: All statistics are weighted with the SCF survey weights. I decompose the credit card holders among each net worth decile into the age groups of (20-44, 45-64, 65+) and denote them as young, middle-aged, and older credit card holders, respectively. The average age of credit card borrowers for each net worth decile is computed as the average age of households with a negative balance on their credit cards in each net worth decile over the SCF 2010-2019.

| Variable (in \%) | Q1: Interest Rate | Q2: Inflation | Q3: Risk Diversification |
| :--- | :---: | :---: | :---: |
|  |  |  |  |
| Total households | 79.02 | 76.56 | 61.69 |
| Credit card borrowers | 78.95 | 77.15 | 61.02 |
| Payday loan borrowers | 75.23 | 68.11 | 47.54 |

Table 1.2: Correctness Rates for Big 3 Questions over the SCF 2016-2019
Notes: All statistics are weighted with the survey weights. The correctness rate is computed as the fraction of households correctly answering the respective question in each group.
then reaches the highest at $5.5 \%$ for those who attended some college. Lastly, the fraction for those with a college degree falls to less than $1.5 \%$. The results indicate that payday loan borrowers are less educated than credit card borrowers. ${ }^{8}$

The "Big Three" financial literacy questions were introduced to the 2016 SCF wave. These questions are meant to measure the financial knowledge regarding interest rate, inflation, and risk diversification. Refer to the Appendix 1.A for the exact survey questions. Table 1.2 reports the average correctness rates for the three questions over the SCF 2016-2019, conditional on all households, credit card borrowers, and payday loan borrowers.

In Table 1.2, one can first see that the correctness rates for all questions by credit card borrowers are not significantly lower than total households. The similar accuracy

[^5]Figure 1.5: Avg. Fraction of Borrowers by Education over the SCF 2010-2019


Notes: All statistics are weighted with the SCF survey weights. The average fraction of credit card borrowers is computed as the average of the fraction of households with a negative balance on their credit cards in each education bin over the SCF 2010-2019. The average fraction of payday loan borrowers is computed as the average of the fraction of households who took up a payday loan over the last year in each education bin over the SCF 2010-2019.
implies that households in credit card debt are not financially illiterate compared to other households. In contrast, payday loan borrowers did an excellent job answering the first question about the interest calculation, but performed poorly in answering the questions of inflation and risk diversification. Compared to credit card borrowers, the correctness rates for these two questions by payday loan borrowers fall by $9.04 \%$ and $13.48 \%$, respectively.

The results suggest that although payday loan borrowers are less financially literate, they lack financial knowledge in specific aspects: ${ }^{9}$ they do understand the concept of compound interest to the extent of the other households; however, they are deficient in the knowledge of inflation and risk diversification. This evidence contrasts with the common argument that payday loans harm consumers because most borrowers ex-ante do not understand how expensive a payday loan can escalate ex-post.

### 1.4 Conclusion

Unsecured borrowing plays an essential role for consumers in smoothing consumption. Credit cards and payday loans are two popular loan choices in the U.S.: almost $40 \%$ of them borrow on their credit cards, and around $4 \%$ of households take up a payday loan. To better understand the characteristics of credit card and payday loan borrowers, I identify them using the SCF 2010-2019 and then compare both types of borrowers in terms of life-cycle profile, income, net worth, education, and financial literacy.

[^6]The findings are threefold. First, most credit card borrowers are middle-aged, upper-middle-class, with some college exposure, and financially literate. Second, payday loan borrowers are young, low-income and low-wealth, less educated, and less financially literate. Third, although payday loan borrowers are less financially literate, they lack the financial knowledge of inflation and risk diversification, but not of interest compound.

In the future, a natural extension is to explore the properties of credit card and payday loan borrowers from other perspectives. For instance, how do the two types of borrowers differ in the search effort for liquidity? Did they confront higher expenses before the survey year than expected initially? Is the borrowing behavior of credit cards and payday loans associated with the marital status of households or other household attributes? These questions are included in the SCF and thus can be easily employed. In addition, other surveys can complement the evidence found in the SCF. For example, the National Financial Capability Study collects information on financial capability and thus could be complementary to the analysis of financial literacy in the paper.

## Appendix

## 1.A Related Survey Questions in the SCF

## 1.A. 1 Credit Card and Payday Loan Borrowers

X413: After the last payment(s) (was/were) made, what was the total balance still owed on (this account/all these accounts)?

X7973: Do you (or anyone in your family living here) have a credit card such as a Visa, MasterCard, Discover, or American Express card that allows you to carry a balance from month to month that you can pay off over time?
X7063: During the past year, have you (or anyone in your family living here) taken out a "payday loan," that is, borrowed money that was supposed to be repaid in full out of your next paycheck?

## 1.A. 2 Financial Literacy

X7558: Do you think that the following statement is true or false: buying a single company's stock usually provides a safer return than a stock mutual fund?
X7559: Suppose you had $\$ 100$ in a savings account and the interest rate was $2 \%$ per year. After 5 years, how much do you think you would have in the account if you left the money to grow: more than $\$ 102$, exactly $\$ 102$, or less than $\$ 102$ ?

X7560: Imagine that the interest rate on your savings account was $1 \%$ per year and inflation was $2 \%$ per year. After 1 year, would you be able to buy more than today, exactly the same as today, or less than today with the money in this account?

## 1.B More Results in the SCF

## 1.B. 1 Credit Card Holders

The results of the fraction of credit card holders from the SCF 2010 to 2019 by age group, education group, income decile, and net worth decile are in Figure 1.B.1.

Figure 1.B.1: Fraction of Credit Card Holders over the SCF 2010-2019


Notes: All statistics are weighted with the SCF survey weights. The fraction of credit card holders is computed as the fraction of households possessing at least a credit card in each group for each survey year.

## 1.B. 2 Credit Card Borrowers

The results of the fraction of credit card borrowers from the SCF 2010 to 2019 by age group, education group, income decile, and net worth decile are in Figure 1.B.2.

## 1.B. 3 Payday Loan Borrowers

The results of the fraction of payday loan borrowers from the SCF 2010 to 2019 by age group, education group, income decile, and net worth decile are in Figure 1.B.3.

## 1.B. 4 Net Worth Octile and Duo-Decile

The fractions of credit card and payday loan borrowers for each net worth octile and duo-decile from the SCF 2010 to 2019 are presented in Figure 1.B.4. One can observe that the patterns of credit card and payday loan borrowing by net worth decile in Figure

Figure 1.B.2: Fraction of Credit Card Borrowers over the SCF 2010-2019


Notes: All statistics are weighted with the SCF survey weights. The fraction of credit card borrowers is computed as the fraction of households with a negative balance on their credit cards in each group for each survey year.
1.3 are also preserved under the octile and duo-decile of net worth.

Figure 1.B.3: Fraction of Payday Loan Borrowers over the SCF 2010-2019


Notes: All statistics are weighted with the SCF survey weights. The fraction of payday loan borrowers is computed as the fraction of households who took up a payday loan over the last year in each group for each survey year.

Figure 1.B.4: Fraction of Borrowers by Net Worth over the SCF 2010-2019
(a) Credit Card by Net Worth Octile

(c) Payday Loan by Net Worth Octile

(b) Credit Card by Net Worth Duo-Decile

(d) Payday Loan by Net Worth Duo-Decile


Notes: All statistics are weighted with the SCF survey weights. The fraction of credit card borrowers is computed as the fraction of households with a negative balance on their credit cards in each net worth group for each survey year. The fraction of payday loan borrowers is defined as the fraction of households who took up a payday loan over the last year in each net worth group for each survey year.

## Chapter 2

## The Payday Loan Puzzle: A Credit Scoring Explanation

Joint with Jan Sun.

### 2.1 Introduction

Agarwal, Skiba, and Tobacman (2009) observe that two-thirds of individuals who use both credit cards and payday loans have at least $\$ 1,000$ of credit card liquidity left when taking out a payday loan. ${ }^{1}$ This behavior is seemingly puzzling as payday loans carry very high interest rates corresponding to annualized percentage rates of several hundred percent, compared to 10 to 30 percent on credit cards. The authors calculate that this seeming pecuniary mistake is very costly: these people could have saved on average $\$ 200$ over a year by borrowing up to their credit card limits before taking out payday loans. This phenomenon has been termed the "Payday Loan Puzzle."

Why do households take out expensive payday loans when they have far cheaper credit options available? Various behavioral explanations, such as self-control problems and financial illiteracy, have been put forward. In this paper, we propose a novel rational explanation for the payday loan puzzle, inspired by the following interview of an actual payday lender:
"Why are people taking out [payday] loans instead of using their cards?" Ranney told me, "This guy was implying that these people weren't smart enough to make the 'right' decision. I laughed in his face. 'They're protecting the card!' I told him. [...]" Whereas failure to repay a payday loan won't

[^7]
## affect a consumer's credit score, failure to repay a credit card will.

\author{

- Servon (2017): The Unbanking of America ${ }^{2}$
}

Our proposed "reputation protection" hypothesis is that people do not exhaust their credit card limits because they want to protect their credit scores. A credit score is a statistic computed by credit bureaus to access a person's default risk. ${ }^{3}$ Borrowing or defaulting on credit cards will affect one's credit score, while payday lenders in the U.S. usually do not report to credit bureaus (Consumer Financial Protection Bureau, 2017). ${ }^{4}$ People care about their credit scores as they influence credit access, credit costs, mortgage terms, and even job application prospects in the future. Therefore, using payday loans to protect one's credit score leads to dynamic reputational benefits at the static cost of higher interest fees.

To better understand the reasons behind the payday loan puzzle and to formalize the above hypothesis, we extend the type scoring framework of Chatterjee, Corbae, Dempsey, and Ríos-Rull (2020). The authors study a Huggett-type model with consumer default and asymmetric information. Households differ in their degrees of patience measured by discount factors (called their "types"). These factors influence their default behavior and thus their riskiness as borrowers. However, banks are unable to observe household types directly. As a result, banks resort to using "type scores" to infer the probability of each individual being patient with a high discount factor (the good type). A type score thus represents an individual's reputation in the credit markets and is analogous to a credit score in practice. ${ }^{5}$

We extend their framework by adding a second debt option (payday loans) and a second default option on only payday loans. Thus, in addition to bank loans, households in our model can also borrow using payday loans offered by the second type of financial intermediary called payday lenders. Households can default in two ways: (1) "formal default" where households default on both bank and payday loans; ${ }^{6}$ and (2) "payday default" where households default selectively only on their payday loans. Default costs include filing fees, utility loss (stigma), and temporary exclusion from the respective asset markets. In equilibrium, payday loans have higher interest rates compared to bank loans because of higher default premia and operating costs. Crucially, banks cannot observe the payday loan choices of households. Payday loans thus introduce hidden actions into

[^8]the price setting and type score updating problem of banks. To our knowledge, we are the first to explicitly model payday loans using a two-asset structure and two default options.

In our model, a dynamic trade-off emerges between the short-run costs of payday loans and the long-run reputational credit score gains. Households trade off between the marginal benefit of maintaining one's type scores versus the marginal cost of borrowing on more expensive payday loans. The intuition behind the type score protection is as follows. Banks cannot observe a household's type and its payday loan usage. If a household is hit by a low income shock and borrows using bank loans to smooth consumption, banks regard this as being indicative of impatience and thus downgrade the type score. Taking up payday loans instead helps protect against being misclassified in the current period. Moreover, it also lowers the probability of a type score downgrade due to default on bank loans in the future in case of sufficiently low future income shocks. We are the first to formally examine the reputation protection explanation for the payday loan puzzle in a theoretical model.

Limited information of banks regarding households' types and payday loan choices gives rise to cross-subsidization in the bank loan market. Conditional on the same level of bank borrowing, impatient households or payday loan borrowers are more likely to default. However, banks cannot observe either a household's type or payday loan usage. This imperfect information restricts banks from designing contracts conditioned on these two characteristics. Both impatient households and payday loan borrowers thus face cheaper borrowing rates than the actuarially fair rates when banks have full information. As a result, impatient households (payday loan borrowers) are subsidized by patient households (non-payday loan borrowers) in the bank lending market.

To understand the payday loan puzzle documented in Agarwal et al. (2009), we calibrate our model to the U.S. households in 2004. Most parameters are exogenously determined by direct empirical evidence or estimates from the literature. We internally calibrate the stigma costs of defaults to match default rates in the bank and payday markets. Our calibrated model can account for various untargeted moments, such as the fraction of payday loan borrowers and the average interest rate on payday loans.

Our calibrated model endogenously gives rise to the reputation protection channel: households invest in their type scores by paying higher interest costs on payday loans. We can quantitatively account for $40 \%$ of the empirically identified payday loan borrowers who have not exhausted their credit cards yet. We can also match the magnitude of the monetary costs. Neither of these moments was targeted in the calibration. ${ }^{7}$ In particular, the model predicts average annual monetary costs of $\$ 230$, which is similar to its empirical counterpart of $\$ 200$ as calculated by Agarwal et al. (2009). Using our calibrated model, we are the first to generate and quantitatively match the empirically identified payday

[^9]loan puzzle.
Payday loans have been a controversial subject of debate in the U.S. in recent years. Critics of payday loans have focused on the high costs of these loans and have argued for outright payday loan bans. ${ }^{8}$ However, we show that payday loans serve an essential insurance purpose even in the presence of these high costs. We are the first to inform the payday loan policy debate in a structural framework by conducting a series of counterfactual policy experiments.

First, we investigate the effects of limiting the maximum payday loan size, a quantity cap, and an outright ban of payday loans. We find that a quantity cap decreases overall welfare. However, there is heterogeneity across households: impatient households lose while patient ones gain. Impatient households are more likely to borrow larger payday loans and are thus more heavily affected by the quantity cap. In addition, the quantity cap imposes less unobservable options on payday loans. This reduction in hidden actions enables banks to better infer payday loan usage of households, thus reducing the amount of information asymmetry in the bank loan market. As a result, banks can better identify households' discount factors, leading to a decline in cross-subsidization of impatient by patient households. In contrast to the quantity cap, a full ban on payday loans is welfarereducing for both types of households. The reason for the welfare loss is the reduction in available insurance. Both impatient and patient households use payday loans to smooth idiosyncratic shocks without harming their type scores. With a full ban, the insurance loss outweighs the gains from reduced cross-subsidization for patient households. These results imply that current regulatory efforts in certain U.S. states to ban payday loans may be misguided in the sense that they end up hurting all households. ${ }^{9}$

Second, we examine the implications of increasing either the formal or payday default cost. The increase in default costs is calibrated to reflect the increase in Chapter 7 filing costs after the 2005 Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) in the U.S. ${ }^{10}$ We find that increasing formal default costs leads to a welfare gain, whereas increasing payday default costs leads to a welfare loss for both types of households. Higher default costs make it harder to smooth consumption across states by defaulting, but easier to smooth consumption over time by borrowing through lower default premia (Zame, 1993). In equilibrium, households prefer smoothing across states by defaulting on payday loans while smoothing over time by borrowing bank loans for three reasons: (1) defaulting on payday loans does not directly affect a household's type

[^10]score, whereas formally defaulting on a bank loan does; (2) interest rates for bank loans are much lower than payday loans; and (3) payday default costs are lower than formal default costs. Higher formal (payday) default costs exactly help (hamper) households in achieving smoothing over time (across states).

The rest of the paper is organized as follows. Section 2.2 gives an overview of the related literature. Section 2.3 details the model framework. Section 2.4 presents the calibration of the model. Section 2.5 illustrates the fundamental mechanism of pooling and cross-subsidization in our framework. In Section 2.6, we discuss in detail the payday loan puzzle and the reputation protection channel in our model. Section 2.7 presents the policy experiments and Section 2.8 concludes with some potential extensions.

### 2.2 Related Literature

In this section, we discuss the literature related to our paper. The consumer finance literature (both empirical and theoretical) is extensive; thus, we will only focus on the papers most directly related to our own. We start by discussing papers that we build on in terms of the underlying methodology and then briefly summarize the literature on payday loans.

Our theoretical framework is based on the type scoring framework developed by Chatterjee et al. (2020). In their paper, they build on the consumer default workhorse models developed by Chatterjee et al. (2007) and Livshits et al. (2007) in which households are allowed to default on their loans as insurance against idiosyncratic risk. ${ }^{11}$ Both Chatterjee et al. (2007) and Livshits et al. (2007) assume that lenders are fully informed about all household characteristics that affect repayment in the next period. Chatterjee et al. (2020) depart from this assumption and introduce heterogeneity across households in the form of different discount factors, which are unobservable by banks. As the patience of households affects their loan repayment probability, banks try to infer households' types by computing an individual-specific type score. This score denotes the Bayesian assessment by banks of individual type based on observable household behavior. Our paper extends this model by introducing a second asset and an additional default option. In addition, banks cannot observe payday loans and default and thus face hidden actions.

Our paper is also closely related to the empirical literature on the seeming pecuniary mistakes in using payday loans. Using matched credit card and payday loan data, Agarwal et al. (2009) document that many borrowers use payday loans when they still have sufficient credit left on their credit cards, even though payday loans carry much higher

[^11]interest rates. They compute that this behavior is very costly and leads to monetary costs of several hundred U.S. dollars over one year. They coin this finding the "Payday Loan Puzzle." Furthermore, Carter, Skiba, and Tobacman (2011) look at a dataset of credit union members and their payday loan borrowing behavior. They also find a pecuniary loss due to the usage of payday loans instead of cheaper alternatives similar to the previous paper. We contribute to this literature by generating the payday loan puzzle in a theoretical model and offering a rational explanation for part of its occurrence.

Payday loans and their effects on consumers are a hotly debated regulatory topic in the U.S. The literature on the effects of payday loans on consumers is in disagreement about its sign. Using household panel survey data, Zinman (2010) finds that restricting access to payday loans leads consumers to shift to bank overdrafts and late payments. The result is a decline in the financial health of affected households and an overall harmful effect of restricting payday loans. Similarly, Morse (2011) uses natural disasters and estimates that access to payday lenders increases welfare. Morgan, Strain, and Seblani (2012) find that the banning of payday lending leads to an increase in bounced checks and overdraft fees. Bhutta, Goldin, and Homonoff (2016) find that consumers switch to other high-cost alternatives in response to payday loan bans. These authors stress that payday loans are instrumental for households to mitigate the negative effects of transitory income or expenditure shocks, especially when access to the mainstream financial system is impaired.

On the other hand, many authors point out that using payday loans can further worsen households' financial situation. Skiba and Tobacman (2019) estimate that using payday loans significantly increases bankruptcy rates by depressing the cash flow of households. Melzer (2011) finds that access to payday loans worsens the ability of households to pay mortgages, rent, and utility bills. Carrell and Zinman (2014) use exogenous variation in payday loan access for military personnel to estimate that usage of payday loans decreases job performance, retention, and readiness. Campbell, Martínez-Jerez, and Tufano (2012) find that access to payday lending increases rates of involuntary bank account closures. We contribute to this literature by offering a theoretical framework in which we jointly model mainstream financial and payday loans as well as their interaction with credit scores. We then use our framework to conduct counterfactual policy exercises, such as banning payday loans, and investigate the resulting welfare implications for households.

Our paper is also related to Exler (2020). He examines the welfare impact of different policy alternatives to regulate small-dollar loans. He builds and calibrates a quantitative model of unsecured lending where individuals can declare bankruptcy or become delinquent. His findings suggest welfare improving changes to the legislation proposed by the Consumer Financial Protection Bureau (CFPB). In contrast to our approach, he considers only one asset and does not model credit scores. Saldain (2021) considers a model of

Figure 2.1: Layout of the Economy and Information Structure

Unobservable to Both Intermediaries
$\beta$ - type (discount factor)
$z$ - transitory earnings
Unobservable to Banks Only
$p / p^{\prime}-$ old/new payday loans
$P D$ - payday default
Observable to Both Intermediaries
$e$ - persistent earnings
$s$ - prior type score
$b / b^{\prime}$ —old/new bank assets
$\mu$ - household distribution

only payday loans with behavioral households and studies policy regulations on payday lending.

### 2.3 The Model

Time is discrete and infinite. We follow the convention of dynamic programming that the time subscript is removed, and the next-period variable is expressed with prime ${ }^{\prime}$. The market is incomplete. There is a measure one of rational households populating the economy. In addition, there exist two financial intermediaries, banks and payday lenders, which operate in perfectly competitive markets. Both offer lending services in oneperiod unsecured loans. Banks also provide saving services. The layout of the economy is illustrated in Figure 2.1.

In every period, households survive at a rate $\rho$, and those who die are replaced by newborns. Households receive persistent earnings $e$ following a stationary finite-state Markov process $Q^{e}\left(e^{\prime} \mid e\right)$ and transitory earnings $z$ determined by an i.i.d. process $Q^{z}(z)$. All in-
come realizations are independent across individuals. There are two types of households: impatient households with a low discount factor $\beta_{L}$ and patient households with a high discount factor $\beta_{H}$. A household's discount factor follows a stationary two-state Markov process $Q^{\beta}\left(\beta^{\prime} \mid \beta\right)$ and evolves independently across individuals. We call a household's discount factor her type.

Households derive utility from consumption $c$. They can either borrow or save an amount $b^{\prime}$ at the discount price $q_{b}$ with banking institutions. Furthermore, they may also take out payday loans $p^{\prime}$ at the discount price $q_{p}$. These actions are illustrated with the solid arrows in Figure 2.1. At the beginning of each period, if a household has any kind of debt, she can choose to repay $(d=R)$ or default. There are two default options available: formal default $(d=F D)$ and payday default $(d=P D)$. Formal default discharges all debts (including potential payday loans) but incurs the out-of-pocket bankruptcy costs $\kappa_{F D}$ (e.g., attorney fees) and stigma (utility) costs $\xi_{F D}$. In addition, no saving or borrowing is possible in the filing period. Alternatively, she may choose payday default to selectively discharge her payday loan only at the cost of filing fees $\kappa_{P D}$ and stigma costs $\xi_{P D}$. Compared to formal default, she becomes excluded only from the payday lending market, and potential bank loans still need to be repaid, but she retains access to the bank asset market. ${ }^{12}$

Banks can observe households' persistent earnings $e$, bank asset position $b$, bank asset choice $b^{\prime}$, formal default $F D$, and household distribution $\mu$. On the contrary, they cannot observe households' transitory earnings $z$, payday loan position $p$, payday loan choice $p^{\prime}$, payday default $(d=P D)$, and discount factors $\beta$. We denote $(e, b, s)$ as the bankobservable state $\omega_{b}$. This information structure is summarized on the left-hand side in Figure 2.1. As all unobservable variables are relevant for the repayment probability of loans in the next period, banks would like to infer them. While banks cannot infer transitory earnings $z$ as they are i.i.d. across time and households, the other variables can be.

For a household's payday loan position $p$, we assume that banks are not able to track it at an individual level, but banks know the aggregate distribution of payday loans in the population (rational expectations). As a result, banks exploit the cross-sectional distribution of households to form their expectation about a household's payday loan position. ${ }^{13}$ Banks then handle unobservable payday loan choices $p^{\prime}$ by summing them out. In addition, banks cannot observe whether payday loans are repaid. Hence, they cannot distinguish between full repayment or payday default by households. These two choices are accordingly subsumed under non-formal default $(\tilde{d}=N F D \equiv R \vee P D)$.

[^12]Households' discount factors are unobservable to financial intermediaries. Banks infer these factors using type scores $s$, which denote the probability of being patient. Past actions are informative about a household's discount factor as it follows a persistent process. The prior assessment of a household being patient at the beginning of a period is denoted as $s \equiv \mathbb{P}\left(\beta_{i}=\beta_{H}\right)$. Given bank-observable states $\omega_{b}$ and choices ( $\left.\tilde{d}, b^{\prime}\right)$, banks will update a household's type score $s$ using Bayes' rule each period. The posterior type score is denoted as $s^{\prime}=\psi_{\beta_{H}^{\prime}}^{\left(\hat{d}, b^{\prime}\right)}\left(\omega_{b}\right)$ where superscripts denote actions and variables in parentheses denote states. As the updated type score may not lie on the type score grid, it is assigned to the nearest grid points using the function $Q^{s}\left(s^{\prime} \mid \psi\right) .{ }^{14}$ The type score updating process is indicated by the dashed arrows in Figure 2.1. Thus, the bank loan pricing function $q_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)$ will be affected by an individual's observable choices and characteristics, including type scores.

Payday lenders are assumed to be more informed than banks. In addition to what banks can observe, payday lenders can certainly tell payday loan decisions. This information structure is also summarized on the left-hand side in Figure 2.1. For simplicity, we assume that payday lenders use the identical type scores as banks. ${ }^{15}$

The rest of the section is structured as follows. Section 2.3.1 summarizes the timing in each period. Section 2.3.2 details the household's maximization problem. Section 2.3.3 presents the problems of both financial intermediaries. In particular, type score updating is discussed in Section 2.3.3. Section 2.3.4 shows the evolution of the crosssectional distribution of households. In Section 2.3.5, we close the section by defining the equilibrium.

### 2.3.1 Timing

The timing in every period is summarized as follows:

1. Households begin each period with state $\left(\beta, z, \omega_{b}, p\right)$.
2. Given bank prices $q_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)$ and payday prices $q_{p}^{\left(R, b^{\prime}, p^{\prime}\right)}\left(\omega_{b}\right)$, households choose to either repay all debt $d=R$, default on the payday loan only $d=P D$, or formally default on both loans $d=F D$.

- If $d=R$, they also choose $b^{\prime}$ and $p^{\prime}$ and consume $c^{\left(R, b^{\prime}, p^{\prime}\right)}$.
- If $d=P D$, they also choose $b^{\prime}$ and $p^{\prime}=0$ and consume $c^{\left(P D, b^{\prime}, 0\right)}$.

[^13]- If $d=F D$, they consume the leftover earnings $c^{(F D, 0,0)}$.

3. Based on bank-observable states $\omega_{b}$ and choices ( $\tilde{d}, b^{\prime}$ ), banks update their type scores from prior $s$ to posterior $\psi_{\beta_{H}^{\prime}}^{\left(\tilde{d}, b^{\prime}\right)}\left(\omega_{b}\right)$.
4. $\beta^{\prime}, z^{\prime}, e^{\prime}$, and $s^{\prime}$ are drawn from $Q^{\beta}\left(\beta^{\prime} \mid \beta\right), Q^{z}\left(z^{\prime}\right), Q^{e}\left(e^{\prime} \mid e\right)$, and $Q^{s}\left(s^{\prime} \mid \psi\right)$. Newborn households begin with discount factor $\beta^{\prime}$ drawn from the initial distribution $G_{\beta}$, transitory earnings $z^{\prime}$ from $G_{z}$, persistent earnings $e^{\prime}$ from $G_{e}$, no bank or payday loan assets $\left(b^{\prime}, p^{\prime}\right)=(0,0)$, and a type score $s^{\prime}$ consistent with $G_{\beta}$.

### 2.3.2 Households

Households take as given the bank and payday loan pricing functions $q_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)$ and $q_{p}^{\left(R, b^{\prime}, p^{\prime}\right)}\left(\omega_{b}\right)$ as well as the type scoring function $\psi_{\beta_{H}^{\prime}}^{\left(\tilde{d}, b^{\prime}\right)}\left(\omega_{b}\right)$. Households can choose between repayment $(d=R)$, defaulting on payday loans only $(d=P D)$, or formally defaulting on both bank and payday loans $(d=F D)$.

Following Chatterjee et al. (2020), we introduce the action-specific utility shocks. These shocks are i.i.d. across time and households. For each action ( $d, b^{\prime}, p^{\prime}$ ) and household, an unobservable additive utility shock $\epsilon^{\left(d, b^{\prime}, p^{\prime}\right)}$ is drawn from an extreme value distribution. These shocks capture other unobservable heterogeneity that is not explicitly modeled in a reduced but tractable way. Policy functions also become probabilistic with these shocks. Without such randomness, households' actions are perfectly informative about their true types.

The value function is thus given by:

$$
\begin{equation*}
V\left(\epsilon, \beta, z, \omega_{b}, p\right)=\max _{\left(d, b^{\prime}, p^{\prime}\right)} v^{\left(d, b^{\prime}, p^{\prime}\right)}\left(\beta, z, \omega_{b}, p\right)+\epsilon^{\left(d, b^{\prime}, p^{\prime}\right)} \tag{2.1}
\end{equation*}
$$

where $\epsilon^{\left(d, b^{\prime}, p^{\prime}\right)}$ is drawn from the following extreme value distribution $E V(\epsilon)$ :

$$
\begin{equation*}
E V(\epsilon)=\exp \left\{-\exp \left(-\frac{\epsilon-\mu_{\epsilon}}{\alpha}\right)\right\} \tag{2.2}
\end{equation*}
$$

where $\alpha>0$ determines the variance of the shock and $\mu_{\epsilon}=-\alpha \gamma_{E}$ makes the shock mean zero and $\gamma_{E}$ is the Euler's constant. ${ }^{16}$

The conditional value function is given by:

$$
\begin{align*}
& v^{\left(d, b^{\prime}, p^{\prime}\right)}\left(\beta, z, \omega_{b}, p\right)=u\left(c^{\left(d, b^{\prime}, p^{\prime}\right)}\left(z, \omega_{b}, p\right)\right)-\xi_{P D} \cdot \mathbb{I}_{[d=P D]}-\xi_{F D} \cdot \mathbb{I}_{[d=F D]} \\
& \quad+\beta \rho \cdot \sum_{\left(\beta^{\prime}, z^{\prime}, e^{\prime}, s^{\prime}\right)} Q^{\beta}\left(\beta^{\prime} \mid \beta\right) \cdot Q^{z}\left(z^{\prime}\right) \cdot Q^{e}\left(e^{\prime} \mid e\right) \cdot Q^{s}\left(s^{\prime} \mid \psi\right) \cdot W\left(\beta^{\prime}, z^{\prime}, \omega_{b}^{\prime}, p^{\prime}\right), \tag{2.3}
\end{align*}
$$

[^14]where the utility function defined on consumption $u(c)$ is additively separable over time, continuous, increasing, and concave; $\xi_{P D}$ and $\xi_{F D}$ represents the stigma costs for payday and formal default; $\mathbb{I}$ denotes the indicator function equal to one if the condition in the squared parentheses is true; $W$ is the unconditional value function which will be defined below; and consumption $c^{\left(d, b^{\prime}, p^{\prime}\right)}\left(z, \omega_{b}, p\right)$ is defined as:
\[

$$
\begin{cases}e \cdot z+b+p-q_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right) \cdot b^{\prime}-q_{p}^{\left(R, b^{\prime}, p^{\prime}\right)}\left(\omega_{b}\right) \cdot p^{\prime}, & \text { if }\left(d, b^{\prime}, p^{\prime}\right)=\left(R, b^{\prime}, p^{\prime}\right)  \tag{2.4}\\ e \cdot z-\kappa_{P D}+b-q_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right) \cdot b^{\prime}, & \text { if }\left(d, b^{\prime}, p^{\prime}\right)=\left(P D, b^{\prime}, 0\right) \\ e \cdot z-\kappa_{F D}, & \text { if }\left(d, b^{\prime}, p^{\prime}\right)=(F D, 0,0)\end{cases}
$$
\]

where $\kappa_{P D}$ and $\kappa_{F D}$ denote the out-of-pocket bankruptcy costs for payday and formal default. ${ }^{17}$

Let the set of feasible actions be defined as:

$$
\begin{equation*}
\mathcal{F}\left(z, \omega_{b}, p\right)=\left\{\left(d, b^{\prime}, p^{\prime}\right) \mid c^{\left(d, b^{\prime}, p^{\prime}\right)}\left(z, \omega_{b}, p\right)>0\right\} . \tag{2.5}
\end{equation*}
$$

Under the distributional assumption on the utility shocks in Equation (2.2), the choice probabilities take the following form: ${ }^{18}$

$$
\sigma^{\left(d, b^{\prime}, p^{\prime}\right)}\left(\beta, z, \omega_{b}, p\right)=\left\{\begin{array}{ll}
\frac{\exp \left\{\frac{v^{\left(d, b^{\prime}, p^{\prime}\right)}\left(\beta, z, \omega_{b}, p\right)}{\alpha}\right\}}{\sum_{\left(d, b^{\prime}, p^{\prime}\right) \in \mathcal{F}} \exp \left\{\frac{v^{\left(\hat{d}, b^{\prime}, p^{\prime}\right)}\left(\beta, z, \omega_{b}, p\right)}{\alpha}\right\}} & \text { if }\left(d, b^{\prime}, p^{\prime}\right) \in \mathcal{F}\left(z, \omega_{b}, p\right)  \tag{2.6}\\
0 & \text { otherwise }
\end{array} .\right.
$$

The unconditional value function is then given by:

$$
\begin{align*}
W\left(\beta, z, \omega_{b}, p\right) & =\mathbb{E}_{\epsilon} V\left(\epsilon, \beta, z, \omega_{b}, p\right) \\
& =\alpha \cdot \ln \left(\sum_{\left(d, b^{\prime}, p^{\prime}\right) \in \mathcal{F}\left(\beta, z, \omega_{b}, p\right)} \exp \left\{\frac{v^{\left(d, b^{\prime}, p^{\prime}\right)}\left(\beta, z, \omega_{b}, p\right)}{\alpha}\right\}\right) . \tag{2.7}
\end{align*}
$$

We use $\mu\left(\beta, z, \omega_{b}, p\right)$ to denote the cross-sectional distribution of households.

### 2.3.3 Financial Intermediaries

In this section, we detail the financial intermediaries. Section 2.3.3 presents the banking sector and Section 2.3.3 outlines the payday lenders.

[^15]
## Banks

Banks can borrow from the international credit market at risk-free interest rate $r_{f}$. The bank's profit $\pi_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)$ for a contract $\left(N F D, b^{\prime}\right)$ is given by:

$$
\pi_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)=\left\{\begin{array}{ll}
\rho \cdot \frac{\mathbb{P}_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right) \cdot\left(-b^{\prime}\right)}{1+r_{f}}-q_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right) \cdot\left(-b^{\prime}\right) & \text { if } b^{\prime}<0  \tag{2.8}\\
q_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right) \cdot b^{\prime}-\rho \cdot \frac{b^{\prime}}{1+r_{f}} & \text { if } b^{\prime} \geq 0
\end{array},\right.
$$

where $\rho$ is the survival probability and $\mathbb{P}_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)$ denotes the repayment probability of a contract $\left(N F D, b^{\prime}\right)$ conditional on bank-observable states $\omega_{b}$. Given perfect competition, the zero-profit condition implies for each contract that:

$$
q_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)= \begin{cases}\rho \cdot \frac{\mathbb{P}_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)}{1+r_{f}} & \text { if } b^{\prime}<0  \tag{2.9}\\ \frac{\rho}{1+r_{f}} & \text { if } b^{\prime} \geq 0\end{cases}
$$

Recall that banks cannot observe discount factors $\beta$, transitory earnings $z$, payday loan holdings and choices $\left(p, p^{\prime}\right)$, as well as the exact choice of repayment or payday default $(d=P D \vee R)$. To determine the repayment probability $\mathbb{P}_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)$, banks solve an inference problem over these unobservables in three steps.

1. Filter out unobservable states and actions $\left(p, p^{\prime}, R, P D\right)$ to obtain the choice probabilities of bank-observable actions $\tilde{\sigma}_{b}^{\left(\tilde{d}, b^{\prime}\right)}\left(\beta, z, \omega_{b}\right)$.
2. Assess the probability that an individual is patient tomorrow $\beta^{\prime}$ given bank-observable state $\omega_{b}$ and choices $\left(\tilde{d}, b^{\prime}\right)$, i.e., the posterior type score $s^{\prime}=\psi_{\beta_{H}^{\prime}}^{\left(\tilde{d}^{\prime}, b^{\prime}\right)}\left(\omega_{b}\right)$.
3. Compute the individual's repayment probability given transition over $\omega_{b}$ for each possible $\beta^{\prime}$. Then, use the weighted sum over $\beta^{\prime}$ to compute $\mathbb{P}_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)$.

In the first step, banks filter out payday loan holdings $p$ using the household distribution $\mu$ and sum out payday loan choices $p^{\prime}$ as follows:

$$
\begin{equation*}
\sigma_{b}^{\left(d, b^{\prime}\right)}\left(\beta, z, \omega_{b}\right)=\sum_{p^{\prime}} \sum_{p} \sigma^{\left(d, b^{\prime}, p^{\prime}\right)}\left(\beta, z, \omega_{b}, p\right) \cdot \frac{\mu\left(\beta, z, \omega_{b}, p\right)}{\sum_{\hat{p}} \mu\left(\beta, z, \omega_{b}, \hat{p}\right)}, \tag{2.10}
\end{equation*}
$$

where the last fraction denotes the marginal distribution of $p$ conditional on $\left(\beta, z, \omega_{b}\right)$. The idea is straightforward: since banks have rational expectations, they deal with the unobservables by weighting them with the distribution of unobservables conditional on the observables. Banks then form the probability of formal default $(\tilde{d}=F D)$ versus nonformal default $(\tilde{d}=N F D \equiv R \vee P D)$ to obtain the choice probabilities of bank-observable
actions as follows:

$$
\tilde{\sigma}_{b}^{\left(\tilde{d}, b^{\prime}\right)}\left(\beta, z, \omega_{b}\right)=\left\{\begin{array}{ll}
\sigma_{b}^{\left(d, b^{\prime}\right)}\left(\beta, z, \omega_{b}\right) & \text { if } \tilde{d}=F D  \tag{2.11}\\
\sum_{d \in\{R, P D\}} \sigma_{b}^{\left(d, b^{\prime}\right)}\left(\beta, z, \omega_{b}\right) & \text { if } \tilde{d}=N F D
\end{array} .\right.
$$

Accordingly, the feasible set from the bank's perspective is defined as:

$$
\begin{equation*}
\tilde{\mathcal{F}}_{b}\left(\beta, z, \omega_{b}\right)=\left\{\left(\tilde{d}, b^{\prime}\right) \mid \tilde{\sigma}_{b}^{\left(\tilde{d}, b^{\prime}\right)}\left(\beta, z, \omega_{b}\right)>0\right\} . \tag{2.12}
\end{equation*}
$$

In the second step, an individual's type score update is computed using Bayes' rule: ${ }^{19}$

$$
\psi_{\beta_{H}^{\prime}}^{\left(\tilde{d} b^{\prime}\right)}\left(\omega_{b}\right)=\left\{\begin{array}{ll}
\sum_{z} Q^{z}(z) \cdot \sum_{\beta} Q^{\beta}\left(\beta^{\prime} \mid \beta\right) \cdot \frac{\tilde{\sigma}_{b}^{\left(\tilde{d}, b^{\prime}\right)}\left(\beta, z, \omega_{b}\right) \cdot s(\beta)}{\sum_{\hat{\beta}^{\tilde{b}}}^{\tilde{b}_{b}^{\left(\tilde{d}, b^{\prime}\right)}\left(\hat{\beta}, z, \omega_{b}\right) \cdot s(\hat{\beta})}} & \text { for }\left(\tilde{d}, b^{\prime}\right) \in \tilde{\mathcal{F}}_{b}  \tag{2.13}\\
\sum_{\beta} Q^{\beta}\left(\beta^{\prime} \mid \beta\right) \cdot s(\beta) & \text { for }\left(\tilde{d}, b^{\prime}\right) \notin \tilde{\mathcal{F}}_{b}
\end{array},\right.
$$

where $s\left(\beta_{L}\right) \equiv 1-s\left(\beta_{H}\right)$ by abuse of notation. For completeness, the second case in Equation (2.13) handles the score updating for an infeasible action. The updating process is intuitive: banks' prior belief $s$ is updated with the relative choice likelihood of observable actions across types $\left(\tilde{\sigma}_{b}^{\left(\tilde{d}, b^{\prime}\right)} / \sum_{\beta} \tilde{\sigma}_{b}^{\left(\tilde{d}, b^{\prime}\right)} \cdot s\right)$, and with the exogenous transition of discount factors $Q^{\beta}$ and transitory earnings $Q^{z}$. The posterior type score $s^{\prime}$ is denoted by $\psi_{\beta_{H}^{\prime}}^{\left(\hat{d}, b^{\prime}\right)}\left(\omega_{b}\right)$. There are two observations: (1) rebuilding type scores is costly due to priors; and (2) the updating process is dominated by priors when banks are certain about households' types. As $s^{\prime}$ may not lie on the score grid, we randomly assigned it to one of the two nearest points. This assignment is characterized by the function $Q^{s}\left(s^{\prime} \mid \psi\right)$. Refer to Appendix 2.A for details.

In the final step, the next-period repayment probability of a contract ( $N F D, b^{\prime}$ ) for banks is computed as:

$$
\begin{align*}
\mathbb{P}_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)= & \sum_{\left(\beta^{\prime}, z^{\prime}, e^{\prime}, s^{\prime}\right)} s^{\prime}\left(\beta^{\prime}\right) \cdot Q^{z}\left(z^{\prime}\right) \cdot Q^{e}\left(e^{\prime} \mid e\right) \cdot Q^{s}\left(s^{\prime}\left(\beta^{\prime}\right) \mid \psi_{\beta^{\prime}}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)\right) \\
& {\left[\mathcal{W}_{P D}^{b^{\prime}}\left(\omega_{b}\right) \cdot\left(1-\sigma^{(F D, 0,0)}\left(\beta^{\prime}, z^{\prime}, \omega_{b}^{\prime}, p^{\prime}=0\right)\right)+\right.} \\
& \left.\left(1-\mathcal{W}_{P D}^{b^{\prime}}\left(\omega_{b}\right)\right) \cdot \sum_{p^{\prime}} \mathcal{W}_{p^{\prime}}^{\left(R, b^{\prime}\right)}\left(\omega_{b}\right) \cdot\left(1-\sigma^{(F D, 0,0)}\left(\beta^{\prime}, z^{\prime}, \omega_{b}^{\prime}, p^{\prime}\right)\right)\right], \tag{2.14}
\end{align*}
$$

where the weighting factor $\mathcal{W}_{P D}^{b_{D}^{\prime}}\left(\omega_{b}\right)$ denotes the probability that a household with bankobservable states $\omega_{b}$ and bank loan choice $b^{\prime}$ chooses payday default $d=P D$ between full

[^16]repayment and payday default in the current period. It is given by:
\[

$$
\begin{equation*}
\mathcal{W}_{P D}^{b^{\prime}}\left(\omega_{b}\right)=\sum_{z} Q^{z}(z) \cdot \frac{\sum_{\beta} s(\beta) \cdot \sigma_{b}^{\left(P D, b^{\prime}\right)}\left(\beta, z, \omega_{b}\right)}{\sum_{\hat{d} \in\{P D, R\}} \sum_{\beta} s(\beta) \cdot \sigma_{b}^{\left(\hat{d}, b^{\prime}\right)}\left(\beta, z, \omega_{b}\right)} . \tag{2.15}
\end{equation*}
$$

\]

In this case, provided that an individual has chosen to default on her payday loan in the current period, the bank realizes that the only possible payday loan choice in the next period is zero $p^{\prime}=0$.

Analogously, $1-\mathcal{W}_{P D}^{b^{\prime}}\left(\omega_{b}\right)$ gives the probability of choosing full repayment $d=R$. As banks do not observe $p^{\prime}$, they must form an expectation over the individual's payday loan choice. Conditional on full repayment, $\mathcal{W}_{p^{\prime}}^{\left(R, b^{\prime}\right)}\left(\omega_{b}\right)$ denotes the probability of a household choosing a certain payday loan $p^{\prime}$ and is given by:

$$
\begin{equation*}
\mathcal{W}_{p^{\prime}}^{\left(R, b^{\prime}\right)}\left(\omega_{b}\right)=\sum_{z} Q^{z}(z) \cdot \frac{\sum_{\beta} s(\beta) \cdot \hat{\sigma}_{b}^{\left(R, b^{\prime}, p^{\prime}\right)}\left(\beta, z, \omega_{b}\right)}{\sum_{\hat{p}^{\prime}} \sum_{\beta} s(\beta) \cdot \hat{\sigma}_{b}^{\left(R, b^{\prime}, \hat{p}^{\prime}\right)}\left(\beta, z, \omega_{b}\right)} . \tag{2.16}
\end{equation*}
$$

## Payday Lenders

The payday loan pricing schedule is also endogenously determined by the zero-profit condition due to the assumption of perfect competition. ${ }^{20}$ For computational tractability, we assume payday lenders use the same type score as banks to infer a household's hidden type. ${ }^{21}$ The repayment probability of a contract $\left(R, b^{\prime}, p^{\prime}\right)$ for bank-observable states $\omega_{b}$ is thus given by:

$$
\begin{align*}
& \mathbb{P}_{p}^{\left(R, b^{\prime}, p^{\prime}\right)}\left(\omega_{b}\right)=\sum_{\left(\beta^{\prime}, z^{\prime}, e^{\prime}, s^{\prime}\right)} s\left(\beta^{\prime}\right) \cdot Q^{z}\left(z^{\prime}\right) \cdot Q^{e}\left(e^{\prime} \mid e\right) \cdot Q^{s}\left(s^{\prime}\left(\beta^{\prime}\right) \mid \psi_{\beta^{\prime}}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)\right) \\
&\left(1-\sum_{d^{\prime} \in\{F D, P D\}} \sum_{b^{\prime \prime}<0} \sigma^{\left(d^{\prime}, b^{\prime \prime}, 0\right)}\left(\beta^{\prime}, z^{\prime}, \omega_{b}^{\prime}, p^{\prime}\right)\right) . \tag{2.17}
\end{align*}
$$

Note that payday lenders have to take into account both formal default $F D$ and payday default $P D$ because payday loans can be discharged in both cases. Moreover, a payday loan can be taken only if she does not save at banks $b^{\prime \prime}<0$. The payday loan pricing function is thus given by:

$$
\begin{equation*}
q_{p}^{\left(R, b^{\prime}, p^{\prime}\right)}\left(\omega_{b}\right)=\rho \cdot \frac{\mathbb{P}_{p}^{\left(R, b^{\prime}, p^{\prime}\right)}\left(\omega_{b}\right)}{1+r_{p}} \tag{2.18}
\end{equation*}
$$

[^17]where $r_{p}$ denotes the operating costs in the payday lending industry.

### 2.3.4 Evolution of the Household Distribution

The probability for an individual to move from state $\left(\beta, z, \omega_{b}, p\right)$ to $\left(\beta^{\prime}, z^{\prime}, \omega_{b}^{\prime}, p^{\prime}\right)$ is governed by the following mapping:

$$
\begin{align*}
& T^{*}\left(\beta^{\prime}, z^{\prime}, \omega_{b}^{\prime}, p^{\prime} \mid \beta, z, \omega_{b}, p\right) \\
& \left.=\rho \cdot Q^{\beta}\left(\beta^{\prime} \mid \beta\right) \cdot Q^{z}\left(z^{\prime}\right) \cdot Q^{e}\left(e^{\prime} \mid e\right) \cdot \sigma^{\left(d, b^{\prime}, p^{\prime}\right)}\left(\beta, z, \omega_{b}, p\right) \cdot Q^{s}\left(s^{\prime}\left(\beta^{\prime}\right) \mid \psi_{\beta_{H}^{\prime}\left(\tilde{d^{\prime}}\right)}^{(\tilde{y}}\right)\left(\omega_{b}\right)\right) \\
& +(1-\rho) \cdot G_{\beta}\left(\beta^{\prime}\right) \cdot G_{z}\left(z^{\prime}\right) \cdot G_{e}\left(e^{\prime}\right) \cdot \mathbb{I}_{\left[b^{\prime}=0\right]} \cdot \mathbb{I}_{\left[s^{\prime}=G_{\beta}\right]} \cdot \mathbb{I}_{\left[p^{\prime}=0\right]} . \tag{2.19}
\end{align*}
$$

The second line describes the transition of surviving households. The third line describes the birth of newborn households. Therefore, the cross-sectional distribution of households $\mu$ evolves according to:

$$
\begin{equation*}
\mu^{\prime}\left(\beta^{\prime}, z^{\prime}, \omega_{b}^{\prime}, p^{\prime}\right)=\sum_{\left(\beta, z, \omega_{b}, p\right)} T^{*}\left(\beta^{\prime}, z^{\prime}, \omega_{b}^{\prime}, p^{\prime} \mid \beta, z, \omega_{b}, p\right) \cdot \mu\left(\beta, z, \omega_{b}, p\right) . \tag{2.20}
\end{equation*}
$$

### 2.3.5 Equilibrium

A stationary Recursive Competitive Equilibrium (RCE) is a set of (un)conditional value functions $v^{*}$ and $W^{*}$, bank loan pricing functions $q_{b}^{*}$ and repayment probability $\mathbb{P}_{b}^{*}$, payday loan pricing functions $q_{p}^{*}$ and repayment probability $\mathbb{P}_{p}^{*}$, a type scoring function $\psi^{*}$, choice probability functions $\sigma^{*}$ and $\tilde{\sigma}_{b}^{*}$, and a distribution $\bar{\mu}^{*}$ such that:

1. Household Optimality: $v^{*\left(d, b^{\prime}, p^{\prime}\right)}\left(\beta, z, \omega_{b}, p\right), \sigma^{*\left(d, b^{\prime}, p^{\prime}\right)}\left(\beta, z, \omega_{b}, p\right)$, and $W^{*}\left(\beta, z, \omega_{b}, p\right)$ satisfy Equation (2.3), (2.6), and (2.7) for all $\left(\beta, z, \omega_{b}, p\right)$, respectively.
2. Type Score Updating: $\tilde{\sigma}_{b}^{*\left(\tilde{d}, b^{\prime}\right)}\left(\beta, z, \omega_{b}\right)$ and $\psi_{\beta_{H}^{\prime}}^{*\left(\tilde{d}, b^{\prime}\right)}\left(\omega_{b}\right)$ satisfy Equation (2.11) and (2.13) for all $\left(\beta, z, \omega_{b}\right)$, respectively.
3. Zero Profits for Banks: $q_{b}^{*\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)$ and $\mathbb{P}_{b}^{*\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)$ satisfy Equation (2.9) and (2.14) for all $\omega_{b}$, respectively.
4. Zero Profits for Payday Lenders: $q_{p}^{*\left(R, b^{\prime}, p^{\prime}\right)}\left(\omega_{b}\right)$ and $\mathbb{P}_{p}^{*\left(R, b^{\prime}, p^{\prime}\right)}\left(\omega_{b}\right)$ satisfy Equation (2.18) and (2.17) for all $\omega_{b}$, respectively.
5. Stationary Distribution: $\bar{\mu}^{*}\left(\beta, z, \omega_{b}, p\right)$ solves Equation (2.20).

Note that the banking problem requires the knowledge of the cross-sectional distribution of households $\mu$. As a result, all equilibrium objects depend on the distribution, and solving the model numerically becomes a daunting task. To accelerate the computation, we implement the one-loop algorithm where value functions, the type scoring function,
pricing schedules, and the distribution are updated simultaneously in each iteration until convergence. ${ }^{22}$ Refer to Appendix 2.B for computational details.

### 2.4 Calibration

The goal of the paper is to explore to what extent the reputation protection channel can explain the payday loan puzzle documented in Agarwal et al. (2009). Given they used a payday loan dataset collected from 2000 to 2004 and to circumvent the effects of the 2005 Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA), we set the baseline calibration year to 2004. The model period is one year. We calibrate the model to the whole U.S. households. Median earnings are set to $\$ 33,176$ in 2004 from the Current Population Survey (CPS). ${ }^{23}$ Our calibration strategy is threefold: (1) standard parameters are taken from the literature; (2) parameters with a direct empirical counterpart are exogenously calibrated; and (3) the rest are internally calibrated to match targeted data moments.

The persistent and transitory earnings processes are taken from Floden and Lindé (2001). We use their process because they estimated it using wage earnings in the U.S. for the same time period considered in our paper and without life-cycle components. We assume newborn households are endowed with the lowest persistent earnings realization and with transitory earnings drawn randomly from the estimated process. These assumptions imply that newborn households start with low earnings. Following Chatterjee et al. (2020), ${ }^{24}$ we set discount factors to 0.886 and 0.915 , respectively. The turn-over rates for discount factors are $Q^{\beta}\left(\beta_{H} \mid \beta_{L}\right)=0.013$ and $Q^{\beta}\left(\beta_{L} \mid \beta_{H}\right)=0.011$. These rates imply that households change their types on average every 77 to 91 years. The share of impatient households among newborns is set to $72 \%$. This is consistent with the upward moving of credit ranking along ages observed in data. ${ }^{25}$ All are summarized in Table 2.1.

We set the CRRA parameter of the utility function to 2 , the standard value in the macro literature. The survival probability of households every period is set to 0.975 , implying an average working life span of 40 years. The risk-free rate $r_{f}$ is set to $1.4 \%$ and implies an effective interest rate of $4 \%$, consistent with the literature. According to calculations in Albanesi and Nosal (2020), the out-of-pocket filing costs for Chapter 7 before the 2005 bankruptcy reform amounted to approximately $\$ 697$, implying $\kappa_{F D}=0.02$. As

[^18]| Parameter |  | Value | Target / Source |
| :--- | :---: | :---: | :--- |
|  | $\rho_{e}$ | 0.9136 | Floden and Lindé (2001) |
| Persistence of persistent earnings | $\sigma_{e}^{2}$ | 0.0426 | Floden and Lindé (2001) |
| S.D. to persistent earnings | $\sigma_{z}^{2}$ | 0.0421 | Floden and Lindé (2001) |
| S.D. to transitory earnings | $G_{e}$ | $(1,0,0)$ | Upward earnings profile |
| Persistent earnings at birth | $G_{z}$ | $(1 / 3,1 / 3,1 / 3)$ | Upward earnings profile |
| Transitory Earnings at birth | $\beta_{L}$ | 0.886 | Chatterjee et al. (2020) |
|  | $\beta_{H}$ | 0.915 | Chatterjee et al. (2020) |
| Low discount factor | $Q^{\beta}\left(\beta_{H} \mid \beta_{L}\right)$ | 0.013 | Chatterjee et al. (2020) |
| High discount factor | $Q^{\beta}\left(\beta_{L} \mid \beta_{H}\right)$ | 0.011 | Chatterjee et al. (2020) |
| Transition from low to high | $G_{\beta}$ | $(0.72,0.28)$ | Chatterjee et al. (2020) |
| Transition from high to low |  |  |  |
| Discount factor at birth | $\rho^{2}$ | 2 | Standard |
| CRRA | $r_{f}$ | 0.975 | 40 years |
| Survival probability |  | 0.014 | Effective interest rate $=4 \%$ |
| Risk-free rate | $\kappa_{F D}$ | 0.02 | Albanesi and Nosal (2020) |
| Formal default cost | $\kappa_{P D}$ | 0.002 | Montezemolo and Wolff (2015) |
| Payday default cost | $r_{p}$ | 1.925 | Flannery and Samolyk (2005) |
| Operating cost for payday lenders |  |  |  |
| S.D. of extreme value shocks | $\alpha_{0}$ | 0.005 | $\approx$ Chatterjee et al. (2020) |

Table 2.1: Exogenously Chosen Parameters

Montezemolo and Wolff (2015) pointed out that payday defaults in practice involve two bounced checked fees (one by banks and the other by payday lenders, $\$ 35$ each), we set the out-of-pocket filing costs for payday defaults $\kappa_{P D}$ to 0.002 . According to Flannery and Samolyk (2005), the average operating costs (without default losses) per two-week payday loan of size $\$ 230$ is around $\$ 19$, thus implying the annualized operating cost for payday lenders $r_{p}$ is 1.925 . The dispersion parameter of the extreme value distribution is set to $0.005 .{ }^{26}$ Table 2.1 provides a summary.

We internally calibrate the stigma costs for formal default $\kappa_{F D}$ and for payday default $\kappa_{P D}$ jointly by matching the formal default rate and the conditional payday default rate. The conditional payday default rate refers to the write-off rate among payday loan borrowers in the year after they took out their first payday loans. Results are summarized in Table 2.2. The formal default rate in the data is computed as the total number of non-business Chapter 7 filings from American Bankruptcy Institute (ABI) normalized by the total number of U.S. households in 2004. The conditional payday default rate is taken from Skiba and Tobacman (2018) where they used the same payday loan data as in Agarwal et al. (2009). The formal and payday stigma costs are accordingly set to 0.02235 and 0.00702 , respectively. ${ }^{27}$

[^19]|  |  |  | Moment (in \%) |  |  |
| :--- | :---: | :--- | :--- | :--- | :---: |
| Parameter | Value | Target | Data | Model |  |
|  |  |  |  |  |  |
| Formal stigma cost | $\xi_{F D}$ | 0.02235 | Formal default rate | 0.99 | 0.99 |
| Payday stigma cost | $\xi_{P D}$ | 0.00702 | Payday default rate (cond.) | 29.7 | 29.7 |

Table 2.2: Internally Calibrated Parameters

| Moment (in \%) | Data | Model |
| :--- | :---: | :---: |
|  |  |  |
| Households in Debt |  |  |
| Fraction of bank loan borrowers | 20.9 | 24.26 |
| Fraction of payday loan borrowers | 5.61 | 9.46 |
| Bank debt-to-earnings (cond.) | 11.75 | 6.48 |
|  |  |  |
| Interest Rate |  |  |
| Avg. interest rate for bank loans | 9.26 | 8.56 |
| Avg. interest rate for payday loans | 447.88 | 410.85 |

Table 2.3: Untargeted Moments: Data v.s. Model

We also evaluate our model fit on a set of untargeted moments standard in the consumer finance literature. The data and model moments are summarized in Table 2.3. ${ }^{28}$ For the fraction of bank loan borrowers in the data, we use the 2004 Survey of Consumer Finances (SCF) and construct a measure of liquid net worth. ${ }^{29}$ We then compute the fraction of households with negative liquid net worth. The fraction of payday loan borrowers is computed with the 2010 SCF since information on payday loans was first collected in the 2010 wave. We also use the 2004 SCF to compute the bank debt-to-earnings ratio conditional on borrowing bank loans. ${ }^{30}$ Bank debt is measured using the same liquid net worth definition as above. Earnings is computed as wage income measured in the 2004 SCF.

The average interest rate for bank loans is computed as the average credit card interest rate among those having a positive credit card balance in the 2004 SCF, net of the oneyear ahead CPI inflation of all urban consumers from the U.S. Bureau of Labor Statistics. We use the payday loan statistics reported in Skiba and Tobacman (2018) to calculate the average interest rate for payday loans, net of the one-year ahead CPI inflation. ${ }^{31}$
average.
${ }^{28}$ Note that for all SCF-related data moments, we restrict the sample to households with household heads aged between 20 and 60 . We do this since our model does not account for retirement or childhood.
${ }^{29}$ We follow Herkenhoff (2019) in constructing this measure of liquid net worth. It is calculated as the difference between a household's liquid assets, such as checking and savings accounts, and credit card debt. We prefer this measure of net worth as we do not explicitly model illiquid assets such as housing in our framework.
${ }^{30}$ We compute the ratio of average debt to average earnings conditional on having bank debts.
${ }^{31}$ The average bi-weekly payday loan size is $\$ 317.55$ with an average interest payment of $\$ 56.4$. It implies that $\frac{56.4}{317.55} \times \frac{365}{14} \times \frac{1}{1.03388} \times 100=447.88 \%$.

Figure 2.2: Borrowing and Default Behavior across Types


Notes: Left figure: The choice likelihood ratio denotes the probability of an impatient household making a certain choice relative to a patient one. A high value for a certain choice $b^{\prime}$ implies that an impatient household is much more likely to make this choice compared to a patient one. Right figure: The solid line denotes the probability of formal default for a patient household across bank loans $b$. The dashed line denotes the same probability for an impatient household.

### 2.5 Pooling and Cross-Subsidization

In our economy, there is hidden information about a household's type in addition to hidden actions (a household's payday loan choice is unobservable to banks). Because banks cannot observe household types and payday loan choices, they cannot directly design contracts conditioned on these variables. ${ }^{32}$ As a result, this limited information structure leads to two-dimensional pooling across household types and payday loans when banks price their loans. ${ }^{33}$

We first illustrate the heterogeneity in behavior and the resulting cross-subsidization of bank loans across types. Figure 2.2 illustrates differences in borrowing and default behavior across impatient and patient households. Figure 2.2a plots the choice likelihood ratio across different bank asset choices $b^{\prime}$ conditional on a certain state. The choice likelihood ratio denotes the probability of an impatient household saving or borrowing a certain amount relative to a patient one. A high value for the ratio implies that a certain choice is more likely to be taken by an impatient household than a patient one. We can see that impatient households are much more likely to borrow and to borrow more relative to patient households. This is intuitive as households with a lower discount factor value consumption today more and will therefore tend to borrow more. Figure 2.2b illustrates how the formal default probability varies across levels of bank debt $b$. The solid

[^20]Figure 2.3: Cross-Subsidization of Bank Loans across Types


Notes: Cross-subsidization is computed as the difference between actuarially fair interest payments when banks can observe household type and actual interest payments in equilibrium.
line presents the formal default probability for a patient household, while the dashed line shows the probability for an impatient one. It can be seen that the impatient households are more likely to formally default than patient ones across most bank loan positions $b$. As a consequence, conditional on the same state (and in particular, the same bank loan size), impatient households are riskier borrowers for banks.

Since banks cannot perfectly infer a household's type, this imperfect distinction across types results in the cross-subsidization of bank loans across types. In Figure 2.3, we plot the distribution of cross-subsidization amounts in the percentage of median earnings for impatient and patient households. Such an amount denotes the extra interest payments that households face in the counterfactual when banks were able to see their types compared to the benchmark, computed as:

$$
\begin{equation*}
\left(q^{\left(N F D, b^{\prime}\right)}-q_{f a i r}^{\left(N F D, b^{\prime}\right)}(\beta)\right) \cdot b^{\prime} \times 100, \tag{2.21}
\end{equation*}
$$

where $q_{\text {fair }}^{\left(N F D, b^{\prime}\right)}(\beta)$ represents the actuarially bank loan price schedule as if banks knew household types. As shown in Figure 2.3, it is mostly impatient households who are cross-subsidized by patient households. This is due to the fact that the impatient tend to be riskier borrowers as they are more likely to default. In other words, conditional on the same level of bank borrowing, impatient households face lower interest rates on bank loans than actuarially fair rates in our economy.

Moreover, there are also differences in default behavior across payday loan borrowers. Figure 2.4 shows how the formal default probability varies across different levels of bank debt $b$ and households with extra payday debt $p=-0.15$ (dashed line) or not $p=0$ (solid line). Conditional on the same bank loan position, households with additional payday loan positions are more likely to formally default on both loans. This is straightforward

Figure 2.4: Formal Default Probability across Payday Loans


Notes: The solid line depicts the probability for a household with no payday loans to formally default. The dashed line shows the same probability for a household with a payday loan size of 0.15 .
as households with more payday loans have a higher total debt burden and are thus more likely to default. As a result, bank loan borrowers who take out extra payday loans are riskier for banks.

These differences in default behavior lead to cross-subsidization of bank loans across payday and non-payday loan borrowers. Because banks cannot observe payday loan usage by households, borrowers with extra payday loans face the same bank loan pricing schedule as borrowers who do not have payday loans. Conditional on the same level of bank loan, payday loan borrowers tend to have a higher default probability as they have more debt in total. As a result, payday (non-payday) loan borrowers pay lower (higher) rates on bank loans than actuarially fair rates. Figure 2.5 plots the distribution of the crosssubsidization amounts across payday and non-payday loan borrowers. In this case, the amount of cross-subsidization is computed as below.

$$
\begin{equation*}
\left(q^{\left(N F D, b^{\prime}\right)}-q_{f a i r}^{\left(R \vee P D, b^{\prime}, p^{\prime}\right)}\right) \cdot b^{\prime} \times 100, \tag{2.22}
\end{equation*}
$$

where $q_{f a i r}^{\left(R \vee P D, b^{\prime}, p^{\prime}\right)}$ represents the actuarially bank loan price schedule as if banks were able to observe payday loan default and choices.

Table 2.4 summarizes the main equilibrium outcomes across types. Compared to patient households, impatient households are more likely to default and borrow, and hold larger debts for both bank and payday loans. This leads to overall higher borrowing costs for the impatient even though they are partially cross-subsidized by patient households as shown in Figure 2.3.

Figure 2.5: Cross-Subsidization of Bank Loans across (Non-)Payday Loan Borrowers


Notes: Cross-subsidization is computed as the difference between actuarially fair interest payments when banks can observe payday loan usage and actual interest payments in equilibrium.

### 2.6 The Payday Loan Puzzle

In this section, we first illustrate how we identify the payday loan puzzle in our model. Then, we examine to what extent our model can account for the puzzle in the data. In addition, we quantify the type score gains and interest costs from using payday loans and investigate under what circumstances households use payday loans to protect their type scores in our model.

### 2.6.1 Identification of the Payday Loan Puzzle

In our model, we identify the households who make seeming pecuniary mistakes that are consistent with the payday loan puzzle in the following way: for each possible state $\left(\beta, z, \omega_{b}, p\right)$, we identify those feasible borrowing choices with repayment ( $R, b^{\prime}<0, p^{\prime}<$ $0) \in \mathcal{F}\left(z, \omega_{b}, p\right)$ that involve a payday loan where the same total amount of borrowing $\hat{b}^{\prime}=b^{\prime}+p^{\prime}$ could have been achieved at lower borrowing costs using bank loans only. That is:

$$
\begin{equation*}
\left|q_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right) \cdot b^{\prime}+q_{p}^{\left(R, b^{\prime}, p^{\prime}\right)}\left(\omega_{b}\right) \cdot p^{\prime}\right|<\left|q_{b}^{\left(N F D, \hat{b}^{\prime}\right)}\left(\omega_{b}\right) \cdot \hat{b}^{\prime}\right| . \tag{2.23}
\end{equation*}
$$

The borrowing choices that fulfil the above condition are the choices that we classify as the payday loan puzzle. Let the set of these choices be called $\mathcal{P}\left(\beta, z, \omega_{b}, p\right) .{ }^{34}$

To illustrate where the region with payday loan puzzle can happen, Condition (2.23)

[^21]| Moment (in \%) | Aggregate | Impatient | Patient |
| :--- | :---: | :---: | :---: |
|  |  |  |  |
| Default |  |  |  |
| Formal default rate | 0.99 | 1.27 | 0.57 |
| Payday default rate (cond.) | 29.7 | 30.6 | 27.9 |
|  |  |  |  |
| Households in debt | 24.26 | 27.5 | 19.55 |
| Fraction of bank loan borrowers | 9.46 | 10.7 | 7.65 |
| Fraction of payday loan borrowers | 8.42 | 9.54 | 6.77 |
| Fraction of both loan borrowers | 6.48 | 6.54 | 6.36 |
| Bank debt-to-earnings (cond.) | 1.91 | 2.00 | 1.73 |
| Payday debt-to-earnings (cond.) |  |  |  |
|  |  |  |  |
| Interest rate | 8.56 | 8.79 | 8.06 |
| Avg. interest rate for bank loans | 410.85 | 433.89 | 362.74 |
| Avg. interest rate for payday loans |  |  |  |

Table 2.4: Equilibrium across Types
Notes: The payday default rate and the payday debt-to-earnings ratio are conditional on having any payday loans. The bank debt-to-earnings ratio is conditional on having any bank loans.
is visualized in Figure 2.6 where we plot the discounted borrowing amounts across total borrowing conditional on a certain state. The solid line denotes the discounted borrowing amounts involving a given payday loan $p^{\prime}=-0.01$ and the dashed line denotes the discounted borrowing amounts without any payday loan $p^{\prime}=0$. The region of choices satisfying the condition is marked by asterisks and labeled as "Potential Puzzle Area."

Recall that Agarwal et al. (2009) use a matched dataset of credit cards and payday loans to identify the payday loan puzzle. We accordingly define the rate of puzzle occurrence as the fraction of households that make a choice which would be classified as the payday loan puzzle relative to all households that borrow using both loans. More specifically, the rate of puzzle occurrence in the model is calculated as follows:

$$
\begin{equation*}
\frac{\sum_{\beta, z, \omega_{b}, p} \mu\left(\beta, z, \omega_{b}, p\right) \cdot \sum_{\left(d, b^{\prime}, p^{\prime}\right) \in \mathcal{P}\left(\beta, z, \omega_{b}, p\right)} \sigma^{\left(d, b^{\prime}, p^{\prime}\right)}\left(\beta, z, \omega_{b}, p\right)}{\sum_{\beta, z, \omega_{b}, p} \mu\left(\beta, z, \omega_{b}, p\right) \cdot \sum_{\left(d, b^{\prime}, p^{\prime}\right) \in \mathcal{F}_{b o t h}\left(z, \omega_{b}, p\right)} \sigma^{\left(d, b^{\prime}, p^{\prime}\right)}\left(\beta, z, \omega_{b}, p\right)}, \tag{2.25}
\end{equation*}
$$

where the numerator represents the unconditional fraction of households making the puzzling behavior; the denominator denotes the fraction of households borrowing using both types of loans; and the feasible set of borrowings choices using both loans $\mathcal{F}_{\text {both }}\left(z, \omega_{b}, p\right)$
a certain amount is higher when using payday loans compared to only using bank loans. That is:

$$
\begin{equation*}
v^{\left(R, b^{\prime}, p^{\prime}\right)}\left(\beta, z, \omega_{b}, p\right)>v^{\left(R, \hat{b}^{\prime}, p=0\right)}\left(\beta, z, \omega_{b}, p\right) \tag{2.24}
\end{equation*}
$$

Hence, there exists the general dependency of $\mathcal{P}(\cdot)$ on $\beta$. In fact, Condition (2.24) is pretty weak as almost all borrowing choices using both loans are fulfilled.

Figure 2.6: Identification of the Payday Loan Puzzle


Notes: The discounted borrowing amount is computed as the borrowing amount multiplied by the associated discount borrowing price.
is defined as:

$$
\begin{equation*}
\mathcal{F}_{\text {both }}\left(z, \omega_{b}, p\right) \equiv\left\{\left(d, b^{\prime}, p^{\prime}\right) \mid\left(d=R, b^{\prime}<0, p^{\prime}<0\right) \in \mathcal{F}\left(z, \omega_{b}, p\right)\right\} . \tag{2.26}
\end{equation*}
$$

Our model can account for a significant fraction of the puzzling households who take out expensive payday loans with cheaper borrowing alternatives available, identified in the data. In the model, the rate of puzzle occurrence is around $26.44 \% .{ }^{35}$ Agarwal et al. (2009) empirically identify a rate of around two-thirds using a matched dataset. Thus, our model can account for around $40 \%$ of the payday loan puzzle found in the data. ${ }^{36}$

Our model can also match the magnitude of monetary costs from the payday loan puzzle. Recall that these costs denote the amounts which the puzzling payday loan borrowers could have saved if first exhausting their credit cards. Figure 2.7 shows the distribution of the corresponding annual monetary costs per household in both data (solid line) and our calibrated model (bar chart). We can see that in our model most monetary costs have the same magnitude ranging from $\$ 0$ to $\$ 500$ as in the data. ${ }^{37}$ Moreover, our calibrated model predicts average annual monetary costs of $\$ 230$, which is aligned with the average amount of around $\$ 200$ reported in Agarwal et al. (2009). Essentially, these costs represent the value of reputation protection in our model.

[^22]Figure 2.7: Histogram of Monetary Costs of Payday Loan Puzzle


Notes: The data series is from Agarwal et al. (2009). The monetary costs are the amounts which households could have saved if they first exhausted their credit cards before taking out payday loans over one year.

### 2.6.2 The Reputation Protection Channel

We now explore the reputation protection hypothesis quantitatively in our model. In our model, borrowing larger bank loans leads to a lower type score. In addition, households with lower type scores face higher bank interest rates. Hence, households have an incentive to borrow using payday loans instead of bank loans in order to avoid a negative impact on their type scores, thus giving them access to cheaper bank credit in the future.

Figures 2.2 and 2.8 illustrate how this mechanism works. Figures 2.2 a and 2.8 a show the effects of bank loan choices on type scores. In Figure 2.2a, we can see how impatient households are more likely to borrow and to borrow more relative to patient households. Figure 2.8a shows the type score updating function and depicts how a household's type score is updated conditional on different bank asset choices $b^{\prime}$. We can see that taking out a larger bank loan (or saving less) leads to a worse type score update because banks realize that the impatient are more likely to borrow larger amounts. Figures 2.2 b and 2.8b show how a lower type score leads to higher interest rates. Figure 2.2b illustrated how impatient households are more likely to formally default than patient ones across different levels of debt. Figure 2.8b illustrates the bank loan discounted price schedules for households with low (solid line), medium (dashed line), and high type scores (dashdotted line). Banks will charge households with lower type scores lower discounted prices (higher interest rates) in order to be compensated for the additional default risk.

Figure 2.9 looks at the trade-off between type score protection and monetary costs for using payday loans among the payday loan borrowers with cheaper credit available. Figure 2.9a illustrates the relative gain in posterior type scores from using payday loans compared to borrowing the same amount using only bank loans across different prior type

Figure 2.8: Reputation Protection Incentive


Notes: Left figure: The type score update is plotted across different bank asset choices $b^{\prime}$ conditional on a certain state $(e, b, s)$. A new type score of 1.0 means that a household is assessed to be patient with probability one. Right figure: The discounted price schedule for bank loans is shown across different bank loan choices $b^{\prime}$ conditional on a certain state $(e, b, s)$. The discount price is inversely related to the interest rate. The solid/dashed/dash-dotted lines denote the schedules offered to households with low/medium/high type scores.
scores. ${ }^{38}$ There exists significant prior-dependent heterogeneity. ${ }^{39}$ In particular, the gain is over $30 \%$ for those who have lower medium prior type scores. Figure 2.9 b calculates the monetary costs in U.S. dollars across prior type scores. ${ }^{40}$ These costs refer to the extra interest expenses incurred by using payday loans compared to using bank loans for the same borrowing amount. Such pecuniary costs are significant and vary across prior type scores. For example, households with the lowest possible type score are willing to pay an additional $\$ 240$ in payday loan interest fees to achieve higher type scores. On average, these puzzling households, i.e., taking out payday loans while having cheaper borrowing alternatives available, are willing to pay an additional $\$ 230$ in interest payments on payday loans for an increase in type scores by $23 \%$. On average, an $1 \%$ increase in type scores, in turn, leads to a lower borrowing interest rate by $16 \%$ in the future bank lending market. ${ }^{41}$

[^23]Figure 2.9: Cost-Benefit Analysis among Seemingly Puzzling Households
(a) Posterior Type Score Gain
(b) Monetary Costs



Notes: Left figure: The type score gain is computed by comparing the posterior type score of using payday loans relative to using only banks loans for the same borrowing amount, conditional on a prior type score, and expressed in percentage points. Right figure: The monetary costs denote the extra interest payments incurred using payday loans compared to using bank loans for the same borrowing amount across prior type scores.

### 2.6.3 Profile of Puzzling Households

In the previous subsection, we illustrated how using payday instead of bank loans can lead to significant type score gains at the cost of substantially higher interest costs in the short run. Better type scores thus lead to better access to credit markets in the long run. In this subsection, we further investigate when households engage in this seemingly puzzling behavior in our calibrated model.

Figure 2.10 plots the distribution of both loan borrowers across persistent earnings (Figure 2.10a) and transitory earnings (Figure 2.10b), conditional on whether the cheaper bank credit has been exhausted or not yet. We can see that, compared to the borrowers who have exhausted their cheaper bank credit (solid bar chart), borrowers who have not exhausted their cheaper bank credit yet (argyle bar chart) tend to have higher persistent but lower transitory earnings. In particular, households take out payday loans before exhausting cheaper bank loans when they have medium to high persistent earnings but low transitory earnings in our model. This observation indicates that these puzzling households use payday loans to smooth out the shortfall in transitory earnings without significantly damaging their type scores (such a trade-off has been explained in Figure 2.9).

However, why are the households with this earnings profile especially incentivized to borrow using payday loans instead of cheaper bank loans? Recall that banks can observe persistent earnings but not transitory earnings. Therefore, taking out bank loans to smooth out a negative transitory earnings shock while having high persistent earnings

Figure 2.10: Earnings Distribution among Both Loan Borrowers
(a) Persistent Earnings
(b) Transitory Earnings



Notes: These figures show the distribution of payday loan borrowers who have exhausted their cheaper bank loans or not across persistent (left figure) and transitory (right figure) earnings. "Cheaper bank credit available" refers to the households who borrow using both loans even though they have not exhausted cheaper bank credit (see conditions 2.23 and 2.24). " "No bank credit left" refers to the households who borrow using both loans but have exhausted cheaper bank credit.
will lead to a downgraded type score. This explanation is illustrated in Figure 2.11 which shows the type score updating across bank asset choices for different persistent earnings. Conditional on the same bank asset choice $b^{\prime}$, a household with low persistent earnings (solid line) will receive a higher type score update than a household with medium (dashed line) or high (dash-dotted line) persistent earnings. The intuition is as follows. Borrowing a larger bank loan is more indicative of impatience (low discount factor) when having high compared to low persistent earnings because banks think those with higher persistent earnings are not supposed to borrow that much. Instead, by complementing bank loans with payday loans, which are unobservable to banks, households can reduce the negative impact on their type scores while still being able to smooth out transitory earnings shocks.

### 2.7 Policy Experiments

In this section, we consider two different policy experiments that are highly relevant in the consumer credit market: policies curtailing (or outright banning) payday loans and bankruptcy law regulation.

### 2.7.1 Payday Loan Regulation

Payday loans have been a subject of intensive public debate. Opponents of payday loans have long argued that payday lenders prey on poor households and should be banned.

Figure 2.11: Type Score Updating across Persistent Earnings


Notes: This figure plots the updated type score for different bank asset choices $b^{\prime}$ across persistent earnings for a certain state. The solid/dashed/dash-dotted lines denote the type score updating function with low/medium/high persistent earnings.

Advocates emphasized the role of payday loans in smoothing consumption.
We contribute to this debate by investigating the welfare implications of limiting access to payday loans through quantity caps or an outright ban on payday loans in our model. Table 2.5 summarizes the key results of these policy counterfactuals where we report the key moments and welfare outcomes measured in consumption equivalent variation (CEV) units relative to the benchmark in percentage points. ${ }^{42}$ The column "Benchmark" describes the calibrated model as presented in the previous sections. The column "Quantity Cap" denotes the counterfactual where the possible payday loan choices are limited to a size of $\$ 300$ which is the smallest possible payday loan in the benchmark economy. ${ }^{43}$ The column "Full Ban" describes the counterfactual where payday loans become unavailable in the economy.

Compared to the benchmark, a quantity cap leads to fewer payday loan borrowers as there are less payday loan choices available. Conditional on borrowing payday loans, payday debt-to-earnings ratio also drops. It then leads to a decrease in the (unconditional) payday default rate to $2.2 \%$ since it is less advantageous to default on smaller payday loans. ${ }^{44}$ The unconditional payday default rate also drops mechanically as there are less payday loan borrowers. In addition, the conditional effective default rate on payday loans, which is defined as the fraction of households defaulting on payday loans through either formal or payday default conditional on have any payday loans, also decreases from around $34.68 \%$ in the benchmark to $31.24 \%$. Accordingly, the average payday interest

[^24]| Variables (in \%) | Benchmark | Quantity Cap | Full Ban |
| :--- | :---: | :---: | :---: |
|  |  |  |  |
| Default | 0.99 | 0.96 | 0.89 |
| Formal default rate | 2.81 | 2.19 | - |
| Payday default rate | 34.68 | 31.24 | - |
| Eff. payday default rate (cond.) |  |  |  |
|  |  |  |  |
| Households in debt | 24.26 | 24.06 | 23.15 |
| Fraction of bank loan borrowers | 9.46 | 8.22 | - |
| Fraction of payday loan borrowers | 8.42 | 7.36 | - |
| Fraction of both loan borrowers | 6.48 | 6.61 | 6.84 |
| Bank debt-to-earnings (cond.) | 1.91 | 1.40 | - |
| Payday debt-to-earnings (cond.) |  |  |  |
|  | 8.56 | 8.53 | 8.46 |
| Interest rate | 410.85 | 341.88 | - |
| Avg. interest rate for bank loans |  |  |  |
| Avg. interest rate for payday loans | - | -0.0012 | -0.0291 |
| Welfare | - | -0.0029 | -0.0331 |
| Welfare - aggregate | - | 0.0013 | -0.0233 |
| Welfare - impatient households |  |  |  |
| Welfare - patient households |  |  |  |
| Cross-Subsidization of bank loans | - | -10.5 | -15.1 |
| Avg. cross-sub. across types | -21.0 | -100.0 |  |
| Avg. cross-sub. across payday loan borrowers | - |  |  |

Table 2.5: Policy Counterfactual: Restricting Payday Loan Size
Notes: The conditional effective payday default rate is defined as the fraction of households choosing to default on payday loans through either formal or payday default, conditional on having any payday loans. The bank debt-to-earnings ratio is conditional on having any bank loans. The payday debt-toearnings ratio is conditional on having any payday loans. Welfare is measured in CEV units relative to the benchmark in percentage points. The average cross-subsidization amount of bank loans is computed as in Section 2.5 but expressed in percentage changes relative to the benchmark.
rate decreases. The formal default rate also decreases slightly and as such there is no substitution from payday default to formal default as a consequence of the payday loan cap. This in turn gives rise to a mild decrease in average bank interest rate. Surprisingly, the extensive margin of bank loan borrowing also decreases: the fraction of bank loan borrowers drops slightly. The lack of an increase in the extensive margin of bank loan borrowers is explained by the fact that most payday loan borrowers were already borrowing bank loans in the benchmark economy. Instead, limiting the size of payday loans leads to an increase in the intensive margin of bank loan usage: conditional on borrowing, bank debt-to-earnings ratio rises. This is because borrowers now partially substitute bank loans for payday loans. In the full ban counterfactual, all of these changes are magnified.

The overall welfare effects of both policy counterfactuals are negative. ${ }^{45}$ More in-

[^25]Figure 2.12: Distribution of Payday Loan Size across Types


Notes: This figure illustrates the distribution of payday loan borrowers across different payday loan amounts for impatient households (solid line) and patient households (dashed line).
terestingly, the welfare implications of experiments are heterogenous across household types. Impatient households lose in terms of welfare whenever the payday loan market becomes more constrained. In contrast, patient households have higher welfare in the quantity cap counterfactual but lower welfare in the full ban counterfactual compared to the benchmark economy. The reasons for the declines in welfare for impatient households are intuitive. First, impatient households are more likely to borrow larger payday loans in the benchmark economy and are thus more affected by the quantity cap or ban, as shown in Figure 2.12 of the distribution of payday loan size conditional payday loan borrowers across types in the benchmark. Second, imposing a payday loan quantity cap or banning payday loans also reduces the informational asymmetry regarding payday loan usage in the bank market. In turn, this reduction allows banks to better assess a household's type and reduces pooling across types in the bank loan market. As a result, there is less cross-subsidization of impatient by patient households as we can see in Table 2.5. This decrease in cross-subsidization explains the increase in welfare for patient households but the decrease in welfare for impatient households in the quantity cap counterfactual.

So what explains the decrease in welfare for patient households when payday loans are fully banned? The answer is that there is a second factor at play apart from crosssubsidization: insurance. Constraining payday loan choices makes it harder for everyone in the economy, including patient households, to insure against idiosyncratic shocks. When payday loans are quantity capped but still available in the economy, the reduction in cross-subsidization outweighs this reduced insurance for patient households. But patient households do depend on payday loans to smooth shocks, for example in order to reduce the negative effect on type scores of a transitory earnings shock as discussed in Section 2.6. In the full ban economy, this loss of insurance outweighs the gain from reduced cross-

| Variables (in \%) | Benchmark | $\mathbf{1 . 3 5 \times \boldsymbol { \kappa } _ { \boldsymbol { F D } }}$ | $\mathbf{1 . 3 5 \times \boldsymbol { \kappa } _ { \boldsymbol { P D } }}$ |
| :--- | :---: | :---: | :---: |
|  |  |  |  |
| Default | 0.99 | 0.84 | 0.99 |
| Formal default rate | 2.81 | 3.03 | 2.60 |
| Payday default rate | 34.68 | 33.59 | 33.78 |
| Eff. payday default rate (cond.) |  |  |  |
| Households in debt | 24.26 | 26.35 | 24.21 |
| Fraction of bank loan borrowers | 9.46 | 10.11 | 9.07 |
| Fraction of payday loan borrowers | 6.48 | 7.56 | 6.48 |
| Bank debt-to-earnings (cond.) |  |  |  |
|  |  |  |  |
| Interest rate | 8.56 | 7.51 | 8.56 |
| Avg. interest rate for bank loans | 410.85 | 395.01 | 398.23 |
| Avg. interest rate for payday loans |  |  |  |
| Welfare | - | 0.1236 | -0.0032 |
| Welfare - aggregate | - | 0.1404 | -0.0036 |
| Welfare - impatient households | - | 0.0991 | -0.0026 |
| Welfare - patient households |  |  |  |

Table 2.6: Policy Counterfactual: Higher Filing Costs
Notes: The conditional effective payday default rate is defined as the fraction of households choosing to default on payday loans through either formal or payday default, conditional on having any payday loans. The bank debt-to-earnings ratio is conditional on having any bank loans. The payday debt-toearnings ratio is conditional on having any payday loans. Welfare is measured in CEV units relative to the benchmark in percentage points.
subsidization for patient households. This result implies that in our model fully banning payday loans makes both types of households worse off.

### 2.7.2 Bankruptcy Regulation

Another approach to regulation in the consumer finance market taken by policy makers is through bankruptcy laws. The most notable overhaul of bankruptcy regulation in recent years is the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) in 2005. Among other changes, this legislation increased the total out-of-pocket filing cost for Chapter 7 filings by around $35 \%$ (Albanesi and Nosal, 2020). To examine the effect of such an increase in monetary filing cost in our model, we simulate a counterfactual where the formal filing cost is increased by $35 \%\left(1.35 \times \kappa_{F D}\right)$. In addition, we also consider the policy counterfactual where the payday filing cost rises by the same magnitude $\left(1.35 \times \kappa_{P D}\right)$ to assess the implication of stricter regulation on payday lending. The key results of these policy counterfactuals are summarized in Table 2.6. The column " $1.35 \times \kappa_{F D}$ " denotes the counterfactual where the formal filing cost is increased by $35 \%$. The column " $1.35 \times \kappa_{P D}$ " describes the counterfactual where the payday filing cost is increased by $35 \%$.

Focusing first on the case where the formal filing cost is increased, we can observe that
this change leads to a significant decrease in the formal default rate. This is caused by substitution from formal default to payday default as the (unconditional) payday default rate rises. The drop in the formal default rate leads to a decrease in the average bank interest rate as banks require a lower default premium on their loans. This, in turn, makes borrowing using bank loans cheaper and increases bank loan borrowing both in terms of the extensive (fraction of loan borrowers) and intensive (debt-to-earnings) margins. Interestingly, the increase in bank loan borrowing is not accompanied by a decrease in payday loan borrowing. Rather payday loan usage also increases, leading to an overall higher level of debt in the economy. This is because the conditional effective default rate on payday loans actually drops from $34.68 \%$ in the benchmark to $33.59 \%$, thus implying cheaper borrowing costs for payday loans.

Continuing to the case where the filing cost for payday default is increased, the payday default rate drops mechanically as it becomes more expensive to default on payday loans. This is associated with a lower average payday loan interest rate. We can also see that the fraction of payday loan borrowers drops even though payday interest rates have fallen. The reason is that in our economy households often default on payday loans. The utility of payday loan borrowers decreases as the increase in payday default costs outweighs the lower payday interest costs. All bank-related variables remain roughly unchanged.

The welfare implications of increasing the filing costs for either formal or payday default are the opposite: an increase in formal default costs leads to a welfare gain for both types of households, whereas an increase in payday default costs leads to a welfare loss. On the one hand, a stricter bankruptcy regime through higher default costs leads to lower interest rates, making borrowing cheaper. On the other hand, a stricter regime makes it more costly to default in response to bad shocks. ${ }^{46}$ In our model, it is cheaper to borrow using bank loans compared to payday loans. At the same time, it is less costly to default on payday than bank loans as both the reputational and monetary filing costs are lower. Thus, households prefer to borrow using bank loans and to default on their payday loans first. ${ }^{47}$ Increased formal default costs exactly allow households to take out bank loans at even lower interest rates, which explains the welfare gain in this counterfactual. In contrast, increased payday default costs make it harder for households to default on their payday loans, which explains the welfare loss in this case.

[^26]
### 2.8 Conclusion

One puzzle in the consumer finance literature is the so-called "Payday Loan Puzzle": households use expensive payday loans even when they still have cheaper alternatives, such as credit cards. We propose a new rational explanation of this behavior: these households use payday loans to protect their credit scores since payday lenders do not report to credit bureaus. To investigate this hypothesis, we build a two-asset Huggett-type model with two types of consumer default as well as asymmetric information and hidden actions. Households can be of one of two types: patient with a high discount factor or impatient with a low discount factor. This household type is unobservable to lenders. In order to form an expectation of a household's type, lenders compute an individual-specific type score based on one's credit history. In addition, a household's payday loan choice is also not observable to banks. This information structure then endogenously creates an incentive for households to use payday loans instead of cheaper bank loans to protect their type scores.

Our model can successfully replicate the payday loan puzzle by matching both the fraction of households that show behavior consistent with the payday loan puzzle as well as the magnitude of the monetary costs. Furthermore, we illustrate how the reputation protection channel leads to the emergence of the payday loan puzzle in our framework. We then conduct a series of policy experiments. We show that restricting the size of payday loans benefits patient households at the expense of impatient ones, while a full ban on payday loans results in a welfare loss for both types of households. In addition, we also show that increasing the costs of defaulting on payday loans is welfare-reducing, whereas increasing the costs of formal default is beneficial in terms of welfare. These results imply that current regulatory efforts in the U.S. to curtail or even ban the payday loan sectors may potentially be harmful to households.

In the future, estimating the model using the simulated method of moments could make the policy conclusions more robust. However, such an estimation is often constrained by the availability of payday loan data at the individual level. In addition, we are planning to consider a case where banks can observe payday loan usage by requiring payday lenders to report. This alternative specification would allow us to more cleanly separate the effect on policy outcomes of pooling across types versus pooling across payday loan borrowers, thus guiding the regulation of the payday lending industry.

## Appendix

## 2.A Assignment of Posterior Type Score

As the updated type score $\psi$ may not lie on the original type score grid, it is randomly assigned to one of the two nearest grid points $s_{i}^{\prime}\left(\beta^{\prime}\right)$ and $s_{j}^{\prime}\left(\beta^{\prime}\right)$ for all $\beta^{\prime}$ with $s_{i}^{\prime}\left(\beta^{\prime}\right) \leq$ $\psi_{\beta_{H}^{\prime}}^{\left(\hat{d}, b^{\prime}\right)} \leq s_{j}^{\prime}\left(\beta^{\prime}\right)$, and assign probability $\chi\left(\beta^{\prime} \mid \psi\right)$ to $s_{i}^{\prime}\left(\beta^{\prime}\right)$ and $1-\chi\left(\beta^{\prime} \mid \psi\right)$ to $s_{j}^{\prime}\left(\beta^{\prime}\right)$, where

$$
\begin{equation*}
\chi\left(\beta^{\prime} \mid \psi\right)=\frac{s_{j}^{\prime}\left(\beta^{\prime}\right)-\psi_{\beta_{H}^{\prime}}^{\left(\tilde{d}, b^{\prime}\right)}\left(\omega_{b}\right)}{s_{j}^{\prime}\left(\beta^{\prime}\right)-s_{i}^{\prime}\left(\beta^{\prime}\right)}, \quad \forall \beta^{\prime} . \tag{2.27}
\end{equation*}
$$

For all $s^{\prime}$ such that $s^{\prime}\left(\beta^{\prime}\right) \in\left\{s_{i}^{\prime}\left(\beta^{\prime}\right), s_{j}^{\prime}\left(\beta^{\prime}\right)\right\}$ for all $\beta^{\prime}$, the probability of receiving score $s^{\prime}$ in the next period is thus equal to

$$
\begin{equation*}
Q^{s}\left(s^{\prime} \mid \psi\right)=\prod_{s^{\prime}\left(\beta^{\prime}\right)=s_{i}^{\prime}\left(\beta^{\prime}\right)} \chi\left(\beta^{\prime} \mid \psi\right) \cdot \prod_{s^{\prime}\left(\beta^{\prime}\right)=s_{j}^{\prime}\left(\beta^{\prime}\right)}\left(1-\chi\left(\beta^{\prime} \mid \psi\right)\right) . \tag{2.28}
\end{equation*}
$$

For all other $s^{\prime}, Q^{s}\left(s^{\prime} \mid \psi\right)=0$.

## 2.B Computation

## 2.B. 1 Grid Specifications

| Variable | Symbol | \# of Points | Value / Range |
| :--- | :---: | :---: | :---: |
|  |  |  |  |
| Persistent earnings | $e$ | 3 | $\{0.57,1.00,1.74\}$ |
| Transitory earnings | $z$ | 3 | $\{0.78,1.00,1.29\}$ |
| Bank assets | $b$ | 191 | $[-0.40,15.00]$ |
| Payday loans | $p$ | 16 | $[-0.15,0.00]$ |
| Type scores | $s$ | 8 | $[0.013,0.989]$ |

Table 2.B.1: Grids Used for Model Computation

We discretize the persistent and transitory earnings processes, each with three points, using Adda and Cooper (2003) and uniform distribution, respectively. We choose the lower
bounds for bank and payday loans to ensure that the endogenous borrowing limits are included. Check Appendix 2.D for the pricing schedules in equilibrium. We then consider an equally-spaced grid of 40 points for bank loans and an exponentially-spaced grid of 150 points for bank savings. More importantly, the grid for payday loans is designed with the same spacing as bank loans to properly compare the borrowing choices between bank and payday loans when identifying the payday loan puzzle.

## 2.B. 2 One-Loop Algorithm

1. Set parameters and tolerances for convergence.
2. Create grids for $\left(\beta, z, \omega_{b}, p\right)$ with lengths $\left(n_{\beta}, n_{z}, n_{\omega}, n_{p}\right)$ where $n_{\omega}=n_{e} \times n_{b} \times n_{s}$.
3. Initialize algorithm with starting guesses:
(a) $W(:,:,:,:, s,::)=W^{F I}$ for all $s$ where $W^{F I}$ denotes the unconditional value function under full information.
(b) $\psi_{\beta_{H}^{\prime}}^{\left(\tilde{d}, b^{\prime}\right)}\left(\omega_{b}\right)=s \cdot Q^{\beta}\left(\beta_{H} \mid \beta_{H}\right)+(1-s) \cdot Q^{\beta}\left(\beta_{H} \mid \beta_{L}\right)$ for all $\omega_{b}$ and $\left(\tilde{d}, b^{\prime}\right)$.
i. $s_{i}^{\prime}=\max \left\{s \in \mathcal{S} \mid s \leq \psi_{\beta_{H}^{\prime}}^{\left(\tilde{d}, b^{\prime}\right)}\left(\omega_{b}\right)\right\}$ and $s_{j}^{\prime}=\min \left\{s \in \mathcal{S} \mid s \geq \psi_{\beta_{H}^{\prime}}^{\left(\tilde{d}, b^{\prime}\right)}\left(\omega_{b}\right)\right\}$.
ii. $Q^{s}\left(s_{i}^{\prime}\left(\beta_{H}\right) \mid \psi_{\beta_{H}^{\prime}}^{\left(\tilde{d}, b^{\prime}\right)}\left(\omega_{b}\right)\right)=\frac{s_{j}^{\prime}-\psi_{\beta^{\prime}}^{\left(\tilde{d}, b^{\prime}\right)}\left(\omega_{b}\right)}{s_{j}^{\prime}-s_{i}^{\prime}}$ and $Q^{s}\left(s_{j}^{\prime}\left(\beta_{H}\right) \mid \psi_{\beta_{H}^{\left(d, b^{\prime}\right)}}^{(\tilde{d})}\left(\omega_{b}\right)\right)=\frac{\psi_{\beta_{H}^{\prime}}^{\left(\widetilde{d}, b^{\prime}\right)}\left(\omega_{b}\right)-s_{i}^{\prime}}{s_{j}^{\prime}-s_{i}^{\prime}}$.
(c) $q_{b}^{\left(N F D, b^{\prime}\right)}(:, b, s)=q_{b}^{F I}$ for all $b, s$ where $q_{b}^{F I}$ denotes the bank loan price function under full information.
(d) $q_{p}^{\left(R, b^{\prime}, p^{\prime}\right)}(:, b, s)=q_{p}^{F I}$ for all $b, s$ where $q_{p}^{F I}$ denotes the payday loan price function under full information.
(e) $\mu(:,:,:,:, s,:,:)=\frac{1}{n_{s}} \times \mu^{F I}$ for all $s$ where $\mu^{F I}$ denotes the cross-sectional distribution of households under full information.
4. Begin the one-loop algorithm:
(a) Solve for new $W_{1}$ taking as given $W_{0}$.
i. Find set of feasible actions ( $d, b^{\prime}, p^{\prime}$ ) using (2.4).
ii. For each $\left(\beta, z, \omega_{b}, p\right)$, compute the value $v^{\left(d, b^{\prime}, p^{\prime}\right)}\left(\beta, z, \omega_{b}, p\right)$ for each feasible action ( $d, b^{\prime}, p^{\prime}$ ) according to (2.3).
iii. Compute new $W_{1}$ using (2.7).
(b) Compute $\sigma^{\left(d, b^{\prime}, p^{\prime}\right)}\left(\beta, z, \omega_{b}, p\right)$ according to (2.6).
(c) Compute new equilibrium functions.
i. On bank side:
A. Compute $\tilde{\sigma}_{b}^{\left(\tilde{d}, b^{\prime}\right)}\left(\beta, z, \omega_{b}\right)$ using (2.10) and (2.11).
B. Then $\psi_{\beta_{H}^{\prime}}^{\left(\tilde{d},{ }^{\prime}\right)}\left(\omega_{b}\right)$ using (2.13).
C. Then $\chi\left(\beta^{\prime} \mid \psi\right)$ using (2.27) for all $\psi$ from previous step.
D. Then $Q^{s}\left(s^{\prime} \mid \psi\right)$ using (2.28) for all $\psi$ from previously.
E. Then $\mathbb{P}_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)$ using (2.14).
F. Finally $q_{b}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)$ using (2.9).
ii. On payday lender side:
A. Compute $\mathbb{P}_{p}^{\left(R, b^{\prime}, p^{\prime}\right)}\left(\omega_{b}\right)$ using (2.17).
B. Then $q_{p}^{\left(R, b^{\prime}, p^{\prime}\right)}\left(\omega_{b}\right)$ using (2.18).
(d) Compute stationary distribution $\mu_{1}$ using (2.20).
(e) Assess convergence of $W, \psi, q_{b}, q_{p}$, and $\mu$.
i. If achieved, continue to the next step.
ii. Otherwise, update the initialization of the targeted objects with relaxation and return to step (a).
5. Compute moments.

## 2.C Robustness Check: Same Default Costs

Given that payday default costs are lower than those for formal default, households might take out payday loans because of the better across-state insurance through defaulting on payday loans at lower costs. To argue that this filing channel is not the primary driver for our calibrated framework to generate the payday loan puzzle, we consider a counterfactual where we set the filing and stigma costs for formal default to those for payday default. That is, defaulting on bank loans is as cheap as on payday loans, either pecuniarily or mentally. Important moments and the rate of payday loan puzzle occurrence are reported in Table 2.C.1, along with the benchmark results.

We can see that, compared to the benchmark, households substitute formal default for payday default as it becomes cheaper to execute formal default. A higher formal default rate increases the interest costs for bank and payday loans since households can discharge both loans with formal default. Higher borrowing costs result in drops in the fractions of either loan borrowers at the extensive margin. More importantly, the rate of puzzle occurrence is almost two times larger than the one in the benchmark. The increase can be explained by the fact that payday loans are very costly in the counterfactual. As a result, Condition (2.23) is much more likely to be satisfied, conditional on borrowing using both loans. This result suggests that cheaper costs for payday default than formal default are not the main driving force for our calibrated model to generate the payday loan puzzle.

| Moment (in \%) | Benchmark | Same Default Costs |
| :--- | :---: | :---: |
|  |  |  |
| Default | 0.99 | 5.21 |
| Formal default rate | 29.7 | 22.0 |
| Payday default rate (cond.) |  |  |
|  |  | 16.40 |
| Households in debt | 24.26 | 9.24 |
| Fraction of bank loan borrowers | 9.46 | 8.42 |
| Fraction of payday loan borrowers | 8.42 |  |
| Fraction of both loan borrowers |  | 59.38 |
| Interest rate | 8.56 | 1435.12 |
| Ave. interest rate for bank loans | 410.85 |  |
| Ave. interest rate for payday loans |  | 51.38 |
| Payday loan puzzle | 26.44 |  |
| Rate of puzzle occurrence |  |  |

Table 2.C.1: Counterfactual: Same Default Costs

## 2.D General Results

Figure 2.D. 1 depicts how default probabilities vary across (persistent) earnings $e$ and types $\beta$. The left-hand side shows how the probability of a household choosing formal default increases as its debt burden grows ( $b$ becomes more negative). Households with lower earnings start to formally default at lower debt burdens compared to households with higher earnings. Furthermore, more impatient households $\left(\beta_{L}\right)$ also start to formally default at smaller debt levels. In contrast, as can be seen on the right-hand side the probability of payday default decreases as the debt burden grows. This is due to the switching from payday to formal default: As bank loans increase households switch from payday defaulting on their payday loans only to formally defaulting on all debt in order to discharge their larger bank loans. We can see in Figure 2.D.1b that this switching starts earlier at lower debt levels for households with less income (black line starts dropping at lower $b$ ) and for households that are more impatient (dashed lines drop more quickly than solid lines). This happens because low types are less concerned about the long-term reputational damage from formal default.

The pricing schedules and the risky borrowing limits of bank and payday loans across earnings in the model are depicted in Figure 2.D.2. These results are quite standard in consumer default models. The intuition is clear: On the one hand, borrowing more this period will lead to a higher default probability next period c.p. as the gain from defaulting is larger. As a result we can see in Figure 2.D.2a that borrowing more (more negative $b^{\prime}$ ) leads to lower prices/higher interest rates. Furthermore, an individual with lower persistent earnings $e$ will face lower prices compared to one with higher $e$ c.p. due to the difference in default probability in the following period. Similarly, the payday loan

Figure 2.D.1: Default Probabilities

pricing schedules and the risky borrowing limits across earnings in the model are in the bottom panel. These results are similar to those of bank loans. The significant disparity in levels across bank and payday loans results from the fact that payday lenders have higher operating costs than banks (i,e, higher lending costs).

Figure 2.D. 3 illustrates what kind of household in our economy saves or borrows. On the left, Figure 2.D.3a shows the distribution of savers and borrowers across persistent income. Unsurprisingly, savers in our economy tend to have higher (persistent) income compared to borrowers. We can also see that households who use bank loans (either only bank loans or together with payday loans) are overwhelmingly poor (the red bars). Perhaps more interestingly, payday loan borrowers, while still being poor compared to savers, tend to have higher persistent income than bank loan borrowers. On the right, Figure 2.D.3b shows the distribution of households across transitory income. Compared to Figure 2.D.3a it can be seen that payday loan borrowers tend to have lower transitory income than bank loan borrowers. These two figures suggest that the two types of loans are used to smooth different types of income shocks in our model: households use bank loans to smooth persistent income shocks whereas payday loans are used to smooth transitory shocks. This makes sense: Payday loans are more expensive than bank loans and are much more costly to smooth a persistent negative income shock. On the other hand, using payday loans does not (directly) affect your type score. As a result, it can make sense to smooth transitory income shocks using payday loans in order avoid long-term reputational damage to a household.

Figure 2.D.4a plots the type score distributions among borrowers and savers. We can see that savers in our economy tend to have higher type scores compared to either bank or payday loan borrowers. Interestingly, payday loan borrowers have slightly lower type scores compared to bank loan borrowers. Figure 2.D.4b instead depicts the type score distribution among puzzle and non-puzzle users. We can see that the prior type score distributions of both users are skewed to the right. More importantly, puzzle borrowers,

Figure 2.D.2: Pricing Schedule and Discounted Borrowing Amount

those who take out payday loans before exhausting cheaper bank credit, tend to have lower prior type scores in contrast to non-puzzle borrowers, those who take out payday loans without cheaper bank credit available. This is because the reputation gain (the interest costs) are higher (lower) for households with lower type scores (see Figure 2.9).

Figure 2.D. 5 plots the variation in updated type scores relative to priors among puzzle users in percentage (solid line) compared to the counterfactual when they were to borrow the same amount using only bank loans (dotted line). Borrowing only banks loans results in overall lower posterior type scores across all priors, compared to borrowing a mixture of bank and payday loans. This is intuitive as banks can observe only bank loans. Borrowing more bank loans thus indicates more impatience.

Figure 2.D. 6 plots the average interest rates for bank loans (Figure 2.D.6a) and payday loans (Figure 2.D.6b) across type scores. We can see that higher type scores lead to lower interest rates in both bank and payday lending markets. In particular, the difference in bank loan interest rates between households with the lowest and highest type scores is over $2 \%$. On the other hand, the interest rate difference in the payday lending market can be up to $90 \%$.

Figure 2.D.3: Earnings Distribution among Borrowers and Savers
(a) Persistent Earnings
(b) Transitory Earnings

Distribution of persistent income


Distribution of transitory income


Figure 2.D.4: Type Score Distribution


Figure 2.D.5: Posterior Type Score Dynamic


Figure 2.D.6: Avg. Interest Rates for Bank and Payday Loans across Type Scores

(a) Bank Loans
(b) Payday Loans

## Chapter 3

## Consumer Bankruptcy: the Role of Financial Frictions

### 3.1 Introduction

Consumer credit serves as an important financial instrument for households in smoothing consumption. In the U.S., more than $40 \%$ of households had credit card debts, and the outstanding revolving consumer credit was over one trillion in 2019. ${ }^{1}$ It has been widely shown in the literature that financial frictions affect financial intermediation (Bernanke and Gertler, 1989; Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011). Some papers have suggested that financial frictions influence consumer credit markets. For example, Nakajima and Ríos-Rull (2014) and Fieldhouse, Livshits, and MacGee (2016) study the business cycles of credit card debt and Chapter 7 bankruptcy. They both find that adding countercyclical intermediation costs can help account for the high volatility of consumer credit. Dempsey and Ionescu (2022) document that interest rate spreads of credit cards observed in the data cannot be explained solely by household heterogeneity and argue that banks adopt time-varying lending standards over business cycles. Lee, Luetticke, and Ravn (2020) incorporate frictional financial intermediation into a heterogeneous agent model with consumer loans and analyze the effects of the endogenous countercyclical interest spread induced by financial frictions.

Consumer bankruptcy provides essential insurance for households to mitigate the impacts of adverse financial events. Households can discharge unaffordable debts by defaulting. However, bankruptcy leniency leads to higher default risks and increased borrowing costs. The welfare implication of a bankruptcy law thus depends on evaluating these two counteracting forces. ${ }^{2}$ The trade-off between the two forces has been quantitatively

[^27]investigated in the literature under various theoretical frameworks with a focus on creditdemand factors. For example, Athreya (2002) emphasizes the role of idiosyncratic income risks in household borrowing and default behavior. Livshits et al. (2007) point out the importance of expenditure risks and life-cycle earnings profile in the welfare assessment of bankruptcy regulations. Nakajima $(2012,2017)$ argue that consumer temptation accounts for increasing overindebtedness. Exler et al. (2020) point out the consumer's over-optimism over income prospects results in over-borrowing and delayed bankruptcy filing. Herkenhoff (2019) and Herkenhoff, Phillips, and Cohen-Cole (2021) examine the effects of consumer credit access on unemployment, self-employment, and entrepreneurship. However, no work has been done to analyze the role of credit supply.

How do financial frictions affect household borrowing and default behavior? Through what channels and to what extent does frictional financial intermediation shape the welfare implications of consumer bankruptcy laws? To address these questions, I extend the workhorse model of consumer credit and default in Chatterjee et al. (2007). They study a heterogeneous agent model with consumer default. Households receive stochastic endowments of labor productivity and face preference shocks. If hit by a preference shock, a household becomes impatient with a lower discount factor. She thus takes up a larger loan than she would have taken with the baseline (higher) discount factor. Households can file for bankruptcy at default costs, including wage garnishment in the filing period and bad credit history in the subsequent periods. Households with a bad credit history are excluded from borrowing markets, but their flags could be erased with a certain probability per period. Following Chatterjee et al. (2020), I introduce extreme value shocks to default decisions to capture the effects of other unobservable heterogeneity that are not modeled under my framework. Banks have full information about households and thus charge each borrower her risk-based interest price. Crucially, there is no friction in financial intermediation, and banks can be entirely financed with external deposits.

I extend their framework by adding financial frictions. In particular, I focus on the one proposed by Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) (hereafter, the GK-type frictions). ${ }^{3}$ They assume that an agency problem exists between banks and creditors (i.e., savers) since banks may default by diverting assets if the continuation value for banks is lower than the diverting benefits. The benefits are larger if banks have more external funding via deposits. Household savers lose their savings at banks in the event of bank default. An incentive constraint thus comes into effect to limit banks' ability to manage assets and prevent banks from the diversion. Therefore, banks face an endogenous leverage constraint and must accumulate sufficient net worth to conduct lending services. The degree of financial frictions is governed by the fraction of assets that banks can divert and the exit rate of banks. A larger diverting fraction and a higher exit rate correspond

[^28]to a higher degree of financial frictions because banks are more tempted to default in both cases. Banks use deposits and net worth to issue loans to firms and households. Firms commit to repayment, but households may default. ${ }^{4}$ To my knowledge, I am the first to explicitly model consumer default and financial frictions under a heterogeneous agent framework.

In my model, borrowing prices depend on loan size, household characteristics, and aggregate banking net worth. A household's assessed default risk is high if she takes a large loan or has a bad future income prospect. As a result, banks charge her a high borrowing interest rate today to compensate for the potential default loss in the future. In addition, when banks possess little net worth and thus become highly leveraged, they have higher incentives to default. In order to prevent the deviation of banks from continuing, an extra incentive premium endogenously arises for all loans. As a result, future asset returns increase, and diverting the claims on these assets today becomes less profitable for banks. I contribute to the consumer finance literature by considering the endogenous effects of aggregate banking capitalization on individual borrowing costs. ${ }^{5}$

To understand the effects of financial frictions on consumer credit markets, I calibrate my model to the U.S. economy in 2004 to avoid the effects of the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 (BAPCPA). Most parameters are exogenously determined by direct empirical evidence or estimates from the literature. I internally calibrate the dispersion of the extreme value distribution and the probability of preference shocks to match the Chapter 7 default rate and the banking leverage ratio in the data. My calibrated model can account for several untargeted data moments, such as the average credit card interest rate and debt-to-earnings ratio.

Compared to the frictionless economy, frictional financial intermediation entails higher borrowing costs, thus leading to fewer household debt and lower production. These effects are amplified as the degree of financial frictions increases. Under the benchmark calibration, the incentive constraint binds in equilibrium, and an incentive premium emerges to compromise the incentive conflicts between banks and depositors. However, the extra premium causes borrowing prices to increase and results in a decline in household debt. I label this mechanism as the incentive channel. On the other hand, firms reduce capital investment due to higher borrowing costs. Therefore, production and wages decrease. This mechanism is denoted as the divestment channel. In addition, when a higher degree of financial frictions is confronted in the economy, a larger incentive premium must be charged to mitigate the worse agency problem. The effects of incentive and divestment channels are thus intensified. Therefore, households borrow further less from banks.

[^29]Firms reduce their investment further, thus leading to much lower production and wages. Both channels thus adversely influence households because higher borrowing prices and lower wages worsen the ability of households to smooth consumption.

Consumer credit and its effects on households have been a crucial policy subject in the U.S. For example, the most significant reform in recent years was the 2005 BAPCPA which limited the provision of personal bankruptcy via increased out-of-pocket filing costs (Albanesi and Nosal, 2020). The Consumer Financial Protection Bureau (CFPB) was established in 2011 and aims to protect consumers in consumer finance markets (Consumer Financial Protection Bureau, 2011). Many papers in the literature have evaluated the welfare effects of several policy proposals. However, I am the first to inform the effects of consumer credit regulations under a theoretical framework that features both consumer default and financial frictions. Importantly, I also consider the transition dynamics of policy changes for the welfare evaluation of households. ${ }^{6}$ Therefore, the welfare evaluation of a policy change depends on the policy per se, the transition dynamics of households to the new policy, and the degree of financial frictions.

First, to understand the interplay between the first two components, I conduct the policy experiments of wage garnishment and borrowing exclusion while holding the degree of financial frictions fixed at the benchmark calibration. Higher garnishment and longer exclusion correspond to stricter bankruptcy regimes, whereas lower garnishment and shorter exclusion denote more lenient rules. I find that stricter (more lenient) regulations increase (decrease) overall welfare when financial frictions exist, regardless of the exact policy instruments. Higher default costs make it more difficult for households to smooth consumption across states by defaulting, while easier to smooth consumption over time by borrowing at lower interest costs due to lower default premia (Zame, 1993). In equilibrium, households prefer smoothing over time in lieu of smoothing across states for three reasons: (1) the effective disposable incomes of households are almost always positive since there are no expenditure risks in my model; $;^{7}(2)$ preference shocks cause more households to over-borrow than to default in the first place; ${ }^{8}$ and (3) the adverse effects that result from the incentive and divestment channels are attenuated under a stricter regime. Under a stricter legal regime, lower default risks give rise to lower default premia charged by banks. The over-borrowing problem triggered by preference shocks is thus mitigated because impatient households can pay fewer interest expenses for borrowing. A stricter code also decreases the borrowing prices relative to savings, thus leading to fewer

[^30]deposits in equilibrium. As a result, banks become less leveraged with external funding and thus face a milder agency tension with depositors. As the adverse effects of financial frictions are mitigated, households thus benefit from a lower incentive premium and higher wages. The quantitative results suggest that the gains from lower borrowing costs (either lower default premia or decreased incentive premium) and higher wages combined are greater than the insurance loss from higher default costs under a stricter rule, and vice versa.

However, there is heterogeneity across households under the counterfactual of longer borrowing exclusion: households with good credit history gain, while those with bad credit history lose. As discussed, households should benefit significantly from lower borrowing prices and higher wages for consumption smoothing under stricter bankruptcy law. So, why are households with bad credit history worse off under longer exclusion? This is because the reform directly impacts those households already with bankruptcy flags. Although they can benefit from lower interest costs when regaining access to borrowing markets in the future and higher wages since the onset of the new policy, they must first endure longer exclusion from borrowing markets than they would have to under the benchmark policy. For this subgroup of households, it turns out that under the counterfactual of longer borrowing exclusion, the loss of borrowing ability in the short run outweighs the benefits from lower default premia and the attenuated agency problem in the long run.

Second, I explore how and to what extent financial frictions shape the previous welfare conclusions, focusing on the interactions between financial frictions and legal changes. I begin by comparing the welfare implications of the proposed policy experiments with and without financial frictions. I find that the welfare sensitivity to bankruptcy strictness with financial frictions is larger than the one without financial frictions. This difference results from the extra adverse effects of bankruptcy rules on borrowing costs and wages through the incentive and divestment channels. Under a more lenient regime, higher default risks give rise to higher relative prices of borrowing in terms of saving. Therefore, banks receive more deposits and face a higher leverage ratio. However, when financial frictions exist, banks must charge a higher incentive premium to mitigate the increased incentive conflicts with depositors. As a result, the higher incentive premium leads to increased borrowing costs and decreased wages via the incentive and divestment channels in equilibrium. Both price changes work against household benefits and thus cause extra welfare losses. On the contrary, a stricter code yields additional welfare gains from lower borrowing costs and higher wages. These extra effects on borrowing costs and wages are absent without financial frictions. Therefore, financial frictions significantly impact to what extent welfare is affected by the strictness of bankruptcy rules through their adverse effects on borrowing costs and wages.

To further gauge the extent to which financial frictions shape the welfare assessment of a policy change, I evaluate the welfare implications of the policy proposals with different degrees of financial frictions. I find that: (1) stronger financial frictions strengthen the negative welfare effects of a more lenient rule but attenuate the positive welfare effects of a stricter code; and (2) weaker financial frictions lead to the opposite results. These findings arise because the effects of incentive and divestment channels on borrowing prices and wages are related positively to the degree of financial frictions. A higher degree of financial frictions implies a more severe agency problem. Ceteris paribus, banks have to charge a higher incentive premium to align with their incentives with depositors. Accordingly, borrowing costs increase further, and wages fall lower. Both price changes worse the ability of households to smooth consumption. As a result, these extra negative effects partially offset the welfare gains from a stricter rule and aggravate the welfare losses from a more lenient regime. In contrast, weaker financial frictions result in lower borrowing costs and higher wages in equilibrium. Both price variations are beneficial to households and lead to extra positive welfare effects. Therefore, a more lenient code becomes less welfare-reducing, and a stricter rule yields larger welfare gains.

The rest of the paper is organized as follows. I begin in section 3.2 by giving an overview of the related literature. Section 3.3 presents the theoretical framework. Section 3.4 discusses the calibration of the model. In Section 3.5, I explore the effects of financial frictions in consumer credit markets. Section 3.6 studies the role of financial frictions in the welfare evaluations of consumer bankruptcy regulations. Sections 3.7 concludes with potential avenues for further research.

### 3.2 Related Literature

In this section, I discuss the literature related to this paper. I begin with papers in the consumer finance and financial frictions literature that are close to my theoretical framework. Then, I focus on the literature about the welfare implications of consumer bankruptcy regulations.

My theoretical framework is based on the consumer default workhorse models developed by Chatterjee et al. (2007) and Livshits et al. (2007). In their papers, households are allowed to file for bankruptcy to insure themselves against idiosyncratic shocks-for instance, income and expenditure uncertainty. Both Chatterjee et al. (2007) and Livshits et al. (2007) assume that financial intermediaries are funded fully with deposits from household savers. In addition, intermediaries can fulfil any liquidity needs by household borrowers through the expansion of their balance sheets. It implies that intermediaries do not possess any internal funding and thus have an infinite leverage ratio. I depart from this assumption by introducing a more realistic modeling of financial intermediation into
a canonical model of consumer default.
My paper is also closely related to the literature on financial frictions. There are many types of financial frictions in the macro literature. The most relevant one for the paper is the one developed by Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) For example, Lee et al. (2020) study the implications of the GK-type frictions on individual's marginal propensity to consume (MPC) in a heterogeneous agent new Keynesian (HANK) model. Arslan, Guler, and Kuruscu (2020) build a mortgage default model with the GK-type frictions to study the boom and bust in housing markets. My contributions to this strand of literature include: (1) developing a heterogeneous agent framework that features both consumer default and the GK-type frictional financial intermediaries; and (2) studying the implications of personal bankruptcy regimes under the innovative framework.

The welfare effects of consumer bankruptcy laws have been studied in the literature. First, most papers focus on the role of credit-demand factors, whereas no work has been done to quantify the credit-supply effects. In addition to idiosyncratic income heterogeneity, Livshits et al. (2007) emphasize the importance of expenditure risks and life-cycle earnings profile in the welfare assessment of alternative bankruptcy rules. Nakajima (2017) study the welfare implications of the 2005 Bankruptcy Reform in a model with household temptation and self-control. Chatterjee et al. (2020) develop a consumer default model with asymmetric information between borrowers and lenders to investigate the role of borrower reputation in credit markets. Exler et al. (2020) analyze consumer credit markets with behavioral households who are over-optimistic about their income realizations. Sun (2022) study the role of intra-household insurance via spousal earnings in the welfare outcomes of consumer bankruptcy regulations. Compared to these papers, I focus on financial frictions and quantify its effects on consumer borrowing and default behavior.

Second, several papers have explored the welfare consequences of several policy proposals to regulate consumer finance markets. For example, Athreya (2002) and Li and Sarte (2006) find welfare gains from abolishing personal bankruptcy. Both Athreya (2002) and Chatterjee et al. (2007) find positive welfare effects of means-testing. Livshits et al. (2007) compare the welfare outcomes between the Chapter 7 bankruptcy code versus long-term repayment plans. Chen and Zhao (2017) and Exler (2019) study the effects of repayment plans via wage garnishment on endogenous labor supply. Chatterjee and Gordon (2012) compare the effects of bankruptcy and wage garnishment laws. Chen and Corbae (2011) investigate the welfare consequences of removing bankruptcy flags and find marginal welfare gains of erasing the flag after one year. Herkenhoff et al. (2021) also find that bankruptcy flag removal results in welfare gains for households to obtain liquidity for their businesses. Gordon (2015) studies the role of aggregate risks in the wel-
fare evaluation of bankruptcy laws. See also Exler and Tertilt (2020) for a recent survey. I contribute to the literature by exploring the welfare effects of wage garnishment and the removal of bankruptcy flags while taking into account financial frictions. Moreover, I solve the transition dynamics for each household towards the new policy equilibrium, along with the aggregate leverage adjustment by financial intermediaries. Hence, I can evaluate the welfare gain or loss from the beginning of a policy change for each household.

### 3.3 The Model

Time is discrete and infinite. I follow the convention of dynamic programming that the time subscript is removed, and the next-period variable is expressed with prime ${ }^{\prime}$. The market is incomplete. There is a unit continuum of households. In addition, there exist firms and banks. Both operate in perfectly competitive markets. Firms produce homogeneous goods using a constant returns to scale technology. Banks offer saving and lending services in one-period assets and unsecured loans, respectively.

In each period, households survive at rate $\rho$, and those who die are replaced by newborn households. Household labor productivity $e$ is composed of three components: (1) the permanent labor productivity $e_{1}$ is fixed at birth; (2) the persistent labor productivity $e_{2}$ is drawn from a stationary finite-state Markov process $Q^{e_{2}}\left(e_{2}^{\prime} \mid e_{2}\right)$; and (3) the transitory labor productivity $e_{3}$ is determined by an i.i.d. process $Q^{e_{3}}\left(e_{3}\right)$. The total household labor productivity is defined as $e=e_{1} \times e_{2} \times e_{3}$. Newborns draw their labor productivity from the initial distributions $G^{e_{1}}\left(e_{1}\right), G^{e_{2}}\left(e_{2}\right)$, and $G^{e_{3}}\left(e_{3}\right)$. All the realization of labor productivity are independent across households. For brevity, I use $Q^{e}\left(e^{\prime} \mid e\right)$ to denote the evolution of total labor productivity and $G^{e}(e)$ for the newborn distribution in the following discussions. In addition, households face i.i.d. preference shocks $\nu \sim Q^{\nu}(\nu)$ that temporarily affect households' time preference measured by discount factors $\beta$. Household credit history $h$ summarizes household payment history in financial markets.

Households are risk-averse and derive utility from consumption $c$. They supply their labor force in the efficiency unit inelastically and receive wages earnings $w \cdot \exp (e)$. Households with good credit history $h=0$ can either borrow or save an amount $a^{\prime}$ at the discount price $q$ with banks. If a household with good credit history has any debt $a<0$, she can choose to repay $d=0$ or file for bankruptcy $d=1$. If defaulting, she can discharge her debt $a=0$ but her wage earnings are subject to garnishment at rate $\eta$ and her credit history turns bad $h^{\prime}=1$. In addition, neither saving nor borrowing is allowed in the filing period. Households with bad credit history $h=1$ are excluded from the borrowing markets but can save at the risk-free rate $r_{f}$. A bankruptcy flag could be erased with probability $\mathbb{P}_{h}$. Household states are summarized as $(a, e, \nu, h)$. The cross-sectional distribution of households is denoted by $\mu(a, e, \nu, h)$.

Firms produce homogeneous goods using physical capital $K$ and aggregate labor in the efficiency unit $E \equiv \int \exp (e) d \mu$ with a standard Cobb-Douglas technology of capital share $\alpha$. Capital spending must be financed with bank loans and firms commit to full repayment. Capital depreciates at rate $\delta$.

There is a unit continuum of risk-neutral banks owned by foreign investors that are not modeled in the economy. ${ }^{9}$ Banks might exit the industry at rate $(1-\psi)$ and pay their accumulated net worth as dividends to foreign owners. Those who leave are replaced by newly entering banks with some start-up funds $\omega$ from foreign investors. The objective of banks is to maximize the sum of future dividends discounted at $r_{f}$. To this end, banks use their internally accumulated net worth $N$ and deposits externally from household savers $S^{\prime}$, to lend to firms $K^{\prime}$ and household borrowers $L^{\prime}$. Since banks have full information regarding households, banks can compute risk-based discount borrowing prices $q\left(a^{\prime}, e\right)$, conditional on loan size $a^{\prime}$ and household characteristics $e$.

Crucially, financial frictions arise endogenously because of an agency problem between banks and depositors (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011). After determining asset positions ( $K^{\prime}+L^{\prime}$ ), banks can sell the claims on these assets in secondary frictionless markets, and abscond with a fraction $\theta$ of the asset sales. To prevent banks from diverting assets, the continuation value of banks must be greater than or equal to the gain from asset diversion. This concern translates into an incentive constraint that restricts the ability of banks to asset management. The parameterized diverting fraction of assets $\theta$ thus represents the degree of financial frictions in the economy.

The rest of the section is structured as follows. Section 3.3.1 summarizes the timing in each period. Section 3.3.2 details the household problem. Section 3.3.3 sketches the standard firm problem. The problem of banks is presented in Section 3.3.4, where I introduce the set-up of financial frictions. Section 3.3.5 discusses the evolution of the cross-sectional household distribution. I close the section by defining the equilibrium in Section 3.3.6.

### 3.3.1 Timing

The timing in every period is summarized as follows:

1. Households begin each period with state $(a, e, \nu, h)$.
2. Given borrowing prices $q\left(a^{\prime}, e\right)$, households with good credit history $h=0$ choose to either repay debt $d=0$ or file for bankruptcy $d=1$.

- If $d=0$, they also choose $a^{\prime}$ and consume $c=w \cdot \exp (e)+a-q\left(a^{\prime}, e\right) \cdot a^{\prime}$.

[^31]- If $d=1$, they consume the leftover earnings $c=(1-\eta) \cdot w \cdot \exp (e)$ and their credit history turns bad $h^{\prime}=1$.

3. Households may die at a rate of $(1-\rho)$.

- Among households who survive, $e^{\prime}$ and $\nu^{\prime}$ are drawn from $Q^{e}\left(e^{\prime} \mid e\right)$ and $Q^{\nu}\left(\nu^{\prime}\right)$. Bad credit history could be removed with probability $\mathbb{P}_{h}$.
- Newborn households begin with no assets $a^{\prime}=0$, labor productivity $e^{\prime}$ drawn from $G^{e}$, no present bias $\nu^{\prime}=1$, and good credit history $h^{\prime}=0$.


### 3.3.2 Households

Households take as given the bank discount pricing function $q\left(a^{\prime}, e\right)$. At the beginning of each period, households with good credit history $h=0$ can choose between full repayment $d=0$ and filing for bankruptcy $d=1$.

Following Chatterjee et al. (2020), I introduce the action-specific utility shocks. These shocks are i.i.d. across time and households. For each household and action between repayment and default $d$, an unobservable additive utility shock $\epsilon^{d}$ is drawn from an extreme value distribution. These shocks capture other unobservable heterogeneity that affects household default decision in a reduced but tractable way. ${ }^{10}$

The value function of households with good credit history is thus given by:

$$
\begin{equation*}
V(\epsilon, a, e, \nu, h=0)=\max _{d}\left[V^{d=0}(a, e, \nu, h=0)+\epsilon^{d=0}, V^{d=1}(q, e, \nu, h=0)+\epsilon^{d=1}\right] \tag{3.1}
\end{equation*}
$$

where $\epsilon^{d}$ is drawn from the following extreme value distribution $E V\left(\epsilon^{d}\right)$ :

$$
\begin{equation*}
E V\left(\epsilon^{d}\right)=\exp \left\{-\exp \left(-\frac{\epsilon^{d}-\mu_{\epsilon}}{\zeta}\right)\right\} \tag{3.2}
\end{equation*}
$$

where $\zeta>0$ determines the variance of the shock and $\mu_{\epsilon}=-\zeta \cdot \gamma_{E}$ makes the shock mean zero and $\gamma_{E}$ is the Euler's constant.

The conditional value function of repayment is given by:

$$
\begin{align*}
V^{d=0}(a, e, \nu, h=0)=\max _{a^{\prime}}[ & u\left(w \cdot \exp (e)+a-q\left(a^{\prime}, e\right) \cdot a^{\prime}\right)  \tag{3.3}\\
& \left.+\nu \cdot \beta \cdot \rho \cdot \sum_{\left(e^{\prime}, \nu^{\prime}\right)} Q^{e}\left(e^{\prime} \mid e\right) \cdot Q^{\nu}\left(\nu^{\prime}\right) \cdot V\left(a^{\prime}, e^{\prime}, \nu^{\prime}, h^{\prime}=0\right)\right]
\end{align*}
$$

where the utility function defined on consumption $u(c)$ is additively separable over time, continuous, increasing, and concave. The conditional value function of defaulting is then

[^32]given by:
\[

$$
\begin{align*}
V^{d=1}(a, e, \nu, h=0)= & u((1-\eta) \cdot w \cdot \exp (e))  \tag{3.4}\\
& +\nu \cdot \beta \cdot \rho \cdot \sum_{\left(e^{\prime}, \nu^{\prime}\right)} Q^{e}\left(e^{\prime} \mid e\right) \cdot Q^{\nu}\left(\nu^{\prime}\right) \cdot V\left(a^{\prime}=0, e^{\prime}, \nu^{\prime}, h^{\prime}=1\right)
\end{align*}
$$
\]

where recall that $\eta$ denotes the wage garnishment rate. Moreover, I assume that default is restricted to households with debts larger than the respective default costs to avoid the incidence of default due to the utility shocks. That is, filing for bankruptcy is feasible only if $a<-\eta \cdot \exp (e)$.

Under the distributional assumption on the utility shocks in Equation 3.2, the default choice probability $g_{d}$ takes the following form:

$$
g_{d}(a, e, \nu, h=0)= \begin{cases}\frac{\exp \left\{\frac{V^{d=1}(a, e, \nu, h=0)}{\varsigma}\right\}}{\exp \left\{\frac{V^{d=0}(a, e,, \nu, h=0)}{\zeta}\right\}+\exp \left\{\frac{V^{d=1}(a, e,, \nu, h=0)}{\zeta}\right\}} & \text { if } a<-\eta \cdot \exp (e)  \tag{3.5}\\ 0 & \text { otherwise }\end{cases}
$$

The unconditional value function of households with good credit history is then given by:

$$
\begin{align*}
& V(a, e, \nu, h=0)=\mathbb{E}_{\epsilon} V(\epsilon, a, e, \nu, h=0) \\
& \quad=\zeta \cdot \ln \left(\exp \left\{\frac{V^{d=0}(a, e, \nu, h=0)}{\zeta}\right\}+\exp \left\{\frac{V^{d=1}(a, e, \nu, h=0)}{\zeta}\right\}\right) \tag{3.6}
\end{align*}
$$

The value function of households with bad credit history $h=1$ is given by:

$$
\begin{align*}
V(a, e, \nu, h=1)= & \max _{a^{\prime} \geq 0}\left[u\left(w \cdot \exp (e)+a-\bar{q} \cdot a^{\prime}\right)+\nu \cdot \beta \cdot \rho \cdot \sum_{\left(e^{\prime}, z^{\prime}, h^{\prime}\right)} Q^{e}\left(e^{\prime} \mid e\right) \cdot Q^{\nu}\left(\nu^{\prime}\right)\right. \\
& \left.\cdot\left(\mathbb{P}_{h} \cdot V\left(a^{\prime}, e^{\prime}, \nu^{\prime}, h^{\prime}=0\right)+\left(1-\mathbb{P}_{h}\right) \cdot V\left(a^{\prime}, e^{\prime}, \nu^{\prime}, h^{\prime}=1\right)\right)\right], \tag{3.7}
\end{align*}
$$

where $\bar{q} \equiv \rho /\left(1+r_{f}\right)$ denotes the discount risk-free rate and bad credit record could be removed with probability $\mathbb{P}_{h}$. I use $\mu(a, e, \nu, h)$ to denote the cross-sectional distribution of households.

### 3.3.3 Firms

Firms produce homogeneous goods $Y$ using physical capital and aggregate labor in the efficiency unit with a standard Cobb-Douglas technology:

$$
\begin{equation*}
Y=F(K, E)=K^{\alpha} E^{1-\alpha}, \tag{3.8}
\end{equation*}
$$

where $\alpha$ denotes capital share and aggregate labor in the efficiency unit is defined as:

$$
\begin{equation*}
E=\sum_{(a, e, \nu, h)} \exp (e) \cdot \mu(a, e, \nu, h) \tag{3.9}
\end{equation*}
$$

Firms finance capital expenses via bank borrowing and commit to repaying. Profit maximization implies the gross rate of return on physical capital and wages are given by:

$$
\begin{align*}
1+r_{k} & =F_{K}(K, E)+(1-\delta),  \tag{3.10}\\
w & =F_{E}(K, E), \tag{3.11}
\end{align*}
$$

where $\delta$ denotes the capital depreciation rate. Equation (3.10) and (3.11) imply that firms make zero profits in equilibrium and distribute their sales revenue net of capital depreciation to banks and workers as borrowing costs and wages, respectively.

### 3.3.4 Banks

There is a unit continuum of risk-neutral banks indexed by $j \in[0,1]$, owned by foreign investors. A bank $j$ uses its accumulated net worth $n$, deposits from household savers $s^{\prime}$ to lend to firms $k^{\prime}$ and household borrowers $l^{\prime}$. Its balance sheet constraint is given by:

$$
\begin{equation*}
k_{j}^{\prime}+l_{j}^{\prime}=n_{j}+s_{j}^{\prime}+\tau_{j}^{\prime} \tag{3.12}
\end{equation*}
$$

where $\tau^{\prime}$ denotes the amount that a bank either borrows or lends to the international markets at $r_{f}$ to balance its domestic positions.

The next-period net worth of bank $j$ is computed as the gross returns on lending to firms and households net of the principal and interest payments to savers and the international markets. That is,

$$
\begin{align*}
n_{j}^{\prime} & =\left(1+r_{k}^{\prime}\right) \cdot k_{j}^{\prime}+\left(1+r_{l}^{\prime}\right) \cdot l_{j}^{\prime}-\left(1+r_{f}\right) \cdot\left(s_{j}^{\prime}+\tau_{j}^{\prime}\right),  \tag{3.13}\\
& =\left(r_{k}^{\prime}-r_{f}\right) \cdot k_{j}^{\prime}+\left(r_{l}^{\prime}-r_{f}\right) \cdot l_{j}^{\prime}+\left(1+r_{f}\right) \cdot n_{j}, \tag{3.14}
\end{align*}
$$

where $r_{l}^{\prime}$ denotes the rate of return on household lending and the second equality results from plugging Equation (3.12).

A bank might exit the industry at rate $(1-\psi)$ and pay its accumulated net worth as dividends to foreign owners. Taking prices as given, a bank $j$ chooses $\left\{k_{j}^{\prime}, l_{j}^{\prime}, s_{j}^{\prime}\right\}$ to maximize the discounted sum of dividends paid to foreign investors. Following Gertler and Karadi (2011), I introduce an agency problem between banks and their creditors (i.e., depositors): after determining its asset portfolio, a bank $j$ can divert a fraction $\theta$ of total
assets and transfer the benefits to foreign investors. ${ }^{11}$ Therefore, creditors require that the banking continuation value must be greater than or equal to the diverting gain and $\theta$ represents the degree of financial frictions. The constrained optimization problem of bank $j$ is thus given by:

$$
\begin{align*}
W\left(n_{j}\right)= & \max _{\left\{k_{j}^{\prime}, l_{j}^{\prime}, s_{j}^{\prime}\right\}}\left(\frac{1}{1+r_{f}}\right)\left[(1-\psi) \cdot n_{j}^{\prime}+\psi \cdot W\left(n_{j}^{\prime}\right)\right]  \tag{3.15}\\
\text { s.t. } & n_{j}^{\prime}=\left(r_{k}^{\prime}-r_{f}\right) \cdot k_{j}^{\prime}+\left(r_{l}^{\prime}-r_{f}\right) \cdot l_{j}^{\prime}+\left(1+r_{f}\right) \cdot n_{j},  \tag{3.16}\\
& W\left(n_{j}\right) \geq \theta \cdot\left(k_{j}^{\prime}+l_{j}^{\prime}\right), \tag{3.17}
\end{align*}
$$

where Equation (3.17) denotes the incentive constraint. Note that both $\theta$ and $\psi$ govern the degree of financial frictions. Either a larger diverting fraction or a higher exit rate implies that banks are more tempted to default due to higher diverting gain and lower continuation value, thus corresponding to a higher degree of financial frictions.

Proposition 1. A solution to the constrained optimization problem from Equation (3.15) to (3.17) can be characterized by:

$$
\begin{align*}
W\left(n_{j}\right) & =\xi \cdot n_{j}  \tag{3.18}\\
\xi & =\frac{1-\psi+\psi \cdot \xi^{\prime}}{1-\lambda}  \tag{3.19}\\
\lambda & =\max \left\{1-\left(\frac{1-\psi+\psi \cdot \xi^{\prime}}{\theta}\right) \cdot\left(\frac{N}{K^{\prime}+L^{\prime}}\right), 0\right\}  \tag{3.20}\\
\iota & =\lambda \cdot \theta \cdot\left(\frac{1+r_{f}}{1-\psi+\psi \cdot \xi^{\prime}}\right)  \tag{3.21}\\
\iota & =r_{k}^{\prime}-r_{f}=r_{l}^{\prime}-r_{f} \geq 0 \tag{3.22}
\end{align*}
$$

where $\xi$ denotes the marginal value of banking net worth, $\lambda$ stands for the multiplier on the incentive constraint, $N, K^{\prime}$, and $L^{\prime}$ are aggregate net worth and lending to firms and households, and $\iota$ denotes the incentive premium.

Proof. See Appendix 3.A.1.
Proposition 1 is standard in the literature, e.g., see Bocola (2016). There are important observations: (1) $\xi$ is independent of bank- $j$-specific variables, implying banks are symmetric; ${ }^{12}(2)$ whether the incentive constraint binds $(\lambda>0)$ or not $(\lambda=0)$ depends on the banking leverage ratio $\left(\frac{K^{\prime}+L^{\prime}}{N}\right)$; (3) if binding, $\lambda$ decreases with $N$; (4) $\iota$ is proportional to $\lambda$ (to what extent the incentive constraint is binding) and $\theta$ (the fraction

[^33]that banks can divert) but inversely to $\left(1+r_{f}\right)^{-1}, \psi$, and $\xi^{\prime}$ (the degree of banks being forward-looking); (5) Equation (3.22) denotes the no-arbitrage conditions, i.e., the excess returns on lending to firms and households are the same equal to $\iota$.

The explanation for the extra interest wedge is straightforward. When the diverting benefit is greater than the banking continuation value (i.e., the incentive constraint becomes binding), banks are incentivized to charge the incentive premium and attached it to the asset returns for equalizing the incentive constraint. On the one hand, higher asset returns result in an increased continuation value. On the other hand, firms and households decrease their borrowings with banks because of higher borrowing costs. As a result, total assets decrease and so does the diverting gain.

Since households can discharge their debts by defaulting and banks have full information, banks provide risk-based borrowing prices conditional on loan size and household characteristics. In particular, the expected repayment for a borrowing contract of $a^{\prime}$ can be computed as:

$$
\begin{align*}
R\left(a^{\prime}, e\right)=\sum_{\left(e^{\prime}, \nu^{\prime}\right)} Q^{e}\left(e^{\prime} \mid e\right) \cdot Q^{\nu}\left(\nu^{\prime}\right) \cdot[ & \left(1-g_{d}\left(a^{\prime}, e^{\prime}, \nu^{\prime}\right)\right) \cdot\left(-a^{\prime}\right) \\
& \left.+g_{d}\left(a^{\prime}, e^{\prime}, \nu^{\prime}\right) \cdot \eta \cdot w^{\prime} \cdot \exp \left(e^{\prime}\right)\right] \tag{3.23}
\end{align*}
$$

where $g_{d}$ denotes the default choice probability defined in Equation (3.5) and credit status $h, h^{\prime}$ are ignored for brevity as only those with good credit history can borrow. The bank loan pricing function is thus given by:

$$
\begin{equation*}
q\left(a^{\prime}, e\right)=\rho \cdot \frac{R\left(a^{\prime}, e\right)}{\left(1+r_{f}+\iota\right) \cdot\left(-a^{\prime}\right)} \tag{3.24}
\end{equation*}
$$

Note that the canonical case without financial frictions, e.g., Chatterjee et al. (2007) and Livshits et al. (2007), is nested in Equation (3.24) when banks are not allowed to divert any assets, i.e., $\theta=0$. Under this case, $\iota$ equals zero by construction.

I can derive the evolution of aggregate banking net worth. It consists of the net worth of existing banks $N_{\text {existing }}^{\prime}$ and the one of newly entering banks $N_{\text {new }}^{\prime}$. Among the existing banks, their net worth can be summed up due to the symmetry property and only a fraction $\psi$ of them may stay. $N_{\text {existing }}^{\prime}$ is thus given by:

$$
\begin{equation*}
N_{\text {existing }}^{\prime}=\psi \cdot\left[\iota \cdot\left(K^{\prime}+L^{\prime}\right)+\left(1+r_{f}\right) \cdot N\right] . \tag{3.25}
\end{equation*}
$$

Each new entrant receives from foreign investors a start-up fund equal to a fraction $\left(\frac{\omega}{1-\psi}\right)$ of the total assets that banks have managed (Gertler and Karadi, 2011). The aggregate
net worth of new entrants is thus given by:

$$
\begin{equation*}
N_{n e w}^{\prime}=\omega \cdot\left(K^{\prime}+L^{\prime}\right) . \tag{3.26}
\end{equation*}
$$

Therefore, the evolution of aggregate banking net worth is defined as:

$$
\begin{equation*}
N^{\prime}=\psi \cdot\left[\iota \cdot\left(K^{\prime}+L^{\prime}\right)+\left(1+r_{f}\right) \cdot N\right]+\omega \cdot\left(K^{\prime}+L^{\prime}\right) . \tag{3.27}
\end{equation*}
$$

Note that $\omega$ can help match the targeted banking leverage ratio. Hence, it will be chosen such that the targeted ratio is supported and there is no international lending or borrowing $T=\int \tau_{j} d j$ in equilibrium.

### 3.3.5 Evolution of the Household Distribution

The probability for an individual to move from state $(a, e, \nu, h)$ to ( $a^{\prime}, e^{\prime}, \nu^{\prime}, h^{\prime}$ ) is governed by the following mapping:

$$
\begin{align*}
T\left(a^{\prime}, e^{\prime}, \nu^{\prime}, h^{\prime} \mid a, e, \nu, h\right)= & \rho \cdot \mathbb{I}_{\left[a^{\prime}=g_{a}(a, e, \nu, h)\right]} \cdot Q^{e}\left(e^{\prime} \mid e\right) \cdot Q^{\nu}\left(\nu^{\prime}\right) \cdot Q^{h}\left(h^{\prime} \mid h\right) \\
& +(1-\rho) \cdot \mathbb{I}_{\left[a^{\prime}=0\right]} \cdot G^{e}\left(e^{\prime}\right) \cdot \mathbb{I}_{\left[\nu^{\prime}=1\right]} \cdot \mathbb{I}_{\left[h^{\prime}=0\right]}, \tag{3.28}
\end{align*}
$$

where $g_{a}(a, e, \nu, h)$ denotes the policy function of households for assets and $Q^{h}\left(h^{\prime} \mid h\right)$ characterizes the evolution of credit history consistent with $g_{d}(a, e, \nu, h)$ and $\mathbb{P}_{h}$. Therefore, the cross-sectional distribution of households $\mu$ evolves according to:

$$
\begin{equation*}
\mu^{\prime}\left(a^{\prime}, e^{\prime}, \nu^{\prime}, h^{\prime}\right)=\sum_{(a, e, \nu, h)} T\left(a^{\prime}, e^{\prime}, \nu^{\prime}, h^{\prime} \mid a, e, \nu, h\right) \cdot \mu(a, e, \nu, h) . \tag{3.29}
\end{equation*}
$$

### 3.3.6 Equilibrium

A stationary Recursive Competitive Equilibrium (RCE) is a set of (un)conditional value functions $V^{*}$ and $W^{*}$, household policy functions $g_{a}^{*}$ and $g_{d}^{*}$, factor prices $r_{k}^{*}$ and $w^{*}$, bank loan pricing function $q^{*}$ and expected repayment $R^{*}$, incentive multiplier $\lambda^{*}$ and premium $\iota^{*}$, aggregate variables $N^{*}, D^{*}, L^{*}$, and $K^{*}$, and a household distribution $\mu^{*}$ such that:

1. Household Optimality: $V^{*}(a, e, \nu, h), g_{a}^{*}(a, e, \nu, h)$, and $g_{d}^{*}(a, e, \nu, h)$ satisfy Equation (3.3), (3.4), (3.5), (3.6), and (3.7) for all $(a, e, \nu, h)$.
2. Factor Prices: $r_{k}^{*}$ and $w^{*}$ satisfy Equation (3.10) and (3.11).
3. Bank Optimality: $W^{*}, \lambda^{*}, \iota^{*}, K^{*}$, and $N^{*}$ solve Equation (3.15), (3.16), (3.17), (3.21), and (3.27). $q^{*}\left(a^{\prime}, e\right)$ and $R^{*}\left(a^{\prime}, e\right)$ satisfy Equation (3.24) and (3.23) for all $\left(a^{\prime}, e\right)$, respectively.
4. Market Clearing Conditions: $L^{*}$ and $D^{*}$ are consistent with $g_{a}^{*}$ and $\mu^{*}$.
5. Stationary Distribution: $\bar{\mu}^{*}(a, e, \nu, h)$ solves Equation (3.29).

Note that the banking problem involves an occasionally binding constraint (i.e., the incentive constraint). Computing the banking leverage ratio requires the knowledge of the cross-sectional distribution of households. As a result, all equilibrium objects depend on the distribution via the incentive premium and solving the model numerically becomes a daunting task. To this end, I propose a bisection-based one-loop algorithm to solve the model. In a nutshell, I adopt a bisection procedure to deal with the occasionally binding incentive constraint. The one-loop algorithm is suggested by Hatchondo et al. (2010) to accelerate the computation for solving models with endogenous default. Refer to Appendix 3.B for computational details.

### 3.4 Calibration

The objective of this paper is to quantitatively investigate the implications of financial frictions for consumer bankruptcy. The model period is set to a year and calibrated to match the U.S. households in 2004 to circumvent the effects of the 2005 bankruptcy reform. My calibration strategy is threefold: (1) standard parameters are taken from the literature; (2) parameters with direct empirical counterparts are exogenously calibrated; and (3) the rest are internally chosen to match targeted data moments, including banking leverage ratio and Chapter 7 default rate. Table 3.1 provides an overview of the parameters with standard values and chosen exogenously. Internally calibrated parameters are presented in Table 3.2.

I set the CRRA parameter of the utility function $\gamma$ to 2 , a standard value in the macro literature. Following Nakajima and Ríos-Rull (2014), the survival probability of households $\rho$ is set to 0.98 , implying an average working life span of 50 years. I set the household discount factor $\beta$ equal to 0.9592 , implying an effective discount factor of 0.94 as in Livshits et al. (2007). The capital share of the Cobb-Douglas production function $\alpha$ and capital depreciation $\delta$ are set respectively to 0.36 and 0.08 , both of which are standard values in the macro literature. The risk-free rate $r_{f}$ is set to $4 \%$, aligned with the average return on capital reported in McGrattan and Prescott (2000). The wage garnishment rate $\eta$ is set to $25 \%$ of the disposable income. The average duration of bad credit history is 10 years, consistent with the regulations in the Fair Credit Reporting Act. This implies that the probability of flag removal $\mathbb{P}_{h}$ is $1 / 10$. The bank survival rate $\psi$ is set to 0.8926 taken from Gertler and Karadi (2011), implying an average planning horizons of 10 years. The calibration for the fraction of asset diversion is suggestive. I choose $\theta=0.2918$ such that the maximum banking leverage ratio below which the incentive constraint is always

| Parameter | Value |  | Source / Target |
| :---: | :---: | :---: | :---: |
| Households |  |  |  |
| CRRA coefficient | $\gamma$ | 2 | Standard |
| Household survival rate | $\rho$ | 0.98 | Avg. working lifespan of 50 years |
| Household discount factor | $\beta$ | 0.9592 | Effective discount factor of 0.94 |
| Production |  |  |  |
| Capital share | $\alpha$ | 0.36 | Standard |
| Depreciation rate | $\delta$ | 0.08 | Standard |
| Financial market |  |  |  |
| Risk-free rate | $r_{f}$ | 0.04 | McGrattan and Prescott (2000) |
| Wage garnishment rate | $\eta$ | 0.25 | $25 \%$ of disposable income |
| Probability of flag removal | $\mathbb{P}_{h}$ | 0.10 | Avg. exclusion of 10 years |
| Bank survival rate | $\psi$ | 0.8926 | Avg. planning period of 10 years |
| Diverting fraction | $\theta$ | 0.2918 | 25\% lower than the targeted ratio |
| Transfer to newly entering banks | $\omega$ | 0.0101 | 1\% of total assets intermediated |
| Exogenous processes |  |  |  |
| S.D. of permanent labor productivity | $\sigma_{1}$ | 0.448 | Storesletten et al. (2004) |
| $\mathrm{AR}(1)$ of persistent labor productivity | $\rho_{2}$ | 0.957 | Storesletten et al. (2004) |
| S.D. of persistent labor productivity | $\sigma_{2}$ | 0.129 | Storesletten et al. (2004) |
| S.D. of transitory labor productivity | $\sigma_{3}$ | 0.351 | Storesletten et al. (2004) |
| Support of household preferences | $\left(\nu_{1}, \nu_{2}\right)$ | $(0,1)$ | Hand-to-mouth households |

Table 3.1: Exogenously Chosen Parameters
slack equals 3.43. This value is $25 \%$ lower than the targeted banking leverage ratio of 4.57. The start-up funds for new entrants to the banking industry $\omega$ are set to $1.01 \%$ of total assets that existing banks have managed in the last operational period.

The permanent, persistent, and transitory labor productivity processes are taken from Storesletten, Telmer, and Yaron (2004). I use their processes because they estimated them using labor earnings data at the household level from Panel Study of Income Dynamics (PSID) for the same time period considered in my paper. I approximate the permanent and transitory components with two-point and three-point uniform distributions, respectively. The persistent process is discretized with a three-state Markov chain using Adda and Cooper (2003). I assume that newborn households are endowed with: (1) the permanent labor productivity drawn randomly from the uniform distribution; (2) the persistent labor productivity drawn randomly according to the stationary distribution implied by the persistent process; and (3) zero transitory labor productivity. For preference shocks, I consider a two-point i.i.d. process with support $\mathcal{V}=\left\{\nu_{1}, \nu_{2}\right\}$ and probability $\mathcal{P}_{\nu}=\left\{\mathbb{P}_{\nu}, 1-\mathbb{P}_{\nu}\right\}$. For computational simplicity, $\nu_{1}$ and $\nu_{2}$ are set to zero and unity. Hence, $\nu_{1}$-type households spend all incomes on consumption (i.e., hand-to-mouth) and $\nu_{2}$-type households are forward-looking without present bias.

I then internally calibrate the probability of preference shocks $\mathbb{P}_{\nu}$ and the dispersion parameter of the extreme value distribution $\zeta$ jointly by matching the banking leverage

| Parameter |  | Value | Target | Data |
| :--- | :---: | :--- | :---: | :---: |
| Model |  |  |  |  |
|  |  |  |  |  |
| Probability of preference shocks | $\mathbb{P}_{\nu}$ | 0.01057 | Banking leverage ratio | 4.57 |
| Dispersion of E.V. shocks | $\zeta$ | 0.02150 | Chapter 7 default rate (\%) | 0.61 |

Table 3.2: Internally Calibrated Parameters
ratio and the Chapter 7 default rate. The banking leverage ratio in the data is calculated as the ratio of total assets to banking net worth among commercial banks in the U.S. over 2001-2004 using the Federal Board of Governors' seasonally adjusted H. 8 series. ${ }^{13}$ The Chapter 7 default rate in the data is computed as the total number of non-business Chapter 7 filings from American Bankruptcy Institute divided by the total number of U.S. households in 2004. The probability of preference shocks and the dispersion parameter of the extreme value distribution are accordingly set to 0.01057 and 0.02150 , respectively. The former implies that each period there are around $1 \%$ of households who are hand-to-mouth. The small latter term indicates that the equilibrium default rate is explained mostly by the structural factors in my model instead of the extreme value shocks.

In addition, I evaluate the model fit on a set of untargeted moments that are standard in the consumer finance literature. This set includes the fraction of households in debt, the debt-to-earnings ratio, and the average borrowing interest rate. The first two statistics describe household borrowings at extensive margin (whether to borrow) and intensive margin (to what extent conditional on taking up a loan), respectively. The data and model moments are summarized in Table 3.3. For the fraction of households in debt in the data, I calculate the share of households with negative net worth in the 2004 Survey of Consumer Finances (SCF). In particular, I use the SCF-calculated net worth because it is aligned with the consolidated asset position of households in my model. I consider households with heads aged between 20 to 70 to be consistent with the calibration of household life expectancy and given my model does not account for childhood and retirement. I also exclude households with negative net worth greater than $120 \%$ of total income because these debts result most likely from entrepreneurial activity following Chatterjee et al. (2007). The debt-to-earnings ratio at the aggregate level in the data is also computed using the 2004 SCF. Debts are measured using the same SCF-calculated net worth as above and earnings are computed as wage income. The average borrowing interest rates are taken from Exler and Tertilt (2020). They compute the average interest rates for two types of unsecured consumer borrowings over 1995-1999 reported in the Federal Board of Governors G. 19 series, adjusted by one-year ahead CPI inflation from the U.S. Bureau of Labor Statistics. The calibrated model does match these untargeted moments fairly well.

[^34]| Moment (in \%) | Data | Model |
| :--- | :---: | :---: |
|  |  |  |
| Fraction of households in debt | 7.05 | 8.63 |
| Debt-to-earnings ratio | 2.56 | 1.87 |
| Average borrowing interest rate | $10.93-12.84$ | 12.18 |

Table 3.3: Untargeted Moments: Data v.s. Model
Notes: The fraction of households in debt and the debt-to-earnings ratio are computed using the 2004 SCF. The average borrowing interest rate is taken from Exler and Tertilt (2020).

### 3.5 Consumer Credit with Financial Frictions

The agency problem between banks and depositors limits the ability of banks to manage assets. An incentive constraint on the banking portfolio thus endogenously emerges to regulate banks' lending behavior. As such, they cannot expand their balance sheet autonomously by issuing more loans to borrowers. Under the baseline calibration, the constraint binds in equilibrium, and financial frictions come into play. The binding economy illustrates several new insights that arise from the interplay between household finance behavior and financial frictions. For example, compared to the economy without financial frictions, the average borrowing interest rate is higher when banks are confronted with financial frictions, ceteris paribus. Household debt decreases as a result. In addition, the higher borrowing cost reduces the lending from banks to firms used for capital investment. The reduction in investment results in lower production and decreased wage earnings for all households.

The rest of the section is organized as follows. Section 3.5.1 presents the equilibrium outcomes with and without financial frictions. Section 3.5.2 explores the effects of changing the degree of financial frictions.

### 3.5.1 Benchmark v.s. Frictionless Economy

I begin by demonstrating the results with and without financial frictions to assess the importance of financial frictions in affecting the equilibrium outcomes. Table 3.4 collects the equilibrium aggregates highly related to consumer credit markets under the baseline calibrated economy and the counterfactual without financial frictions. The column "Benchmark" reports the benchmark results when financial frictions are present. The column "Frictionless" reports the results when financial frictions are deactivated artificially by setting $\theta=0$, i.e., impossible for banks to divert any assets.

The two economies exhibit distinct equilibrium outcomes as shown in Table 3.4. First of all, the average borrowing interest rate is much higher under the benchmark compared to the frictionless economy. The interest difference, $12.18 \%$ vs. $10.65 \%$, results from the extra incentive premium and increased default premium. Under the benchmark, financial

| Variable | Benchmark | Frictionless |
| :--- | :---: | :---: |
|  |  |  |
| Levels |  |  |
|  |  |  |
| Incentive premium (\%) | 0.6264 | 0.0000 |
| Avg. borrowing interest rate (\%) | 12.1829 | 10.6505 |
| Fraction of HHs in debt (\%) | 8.6335 | 9.0770 |
| Debt-to-earnings ratio (\%) | 1.8748 | 1.9551 |
| Conditional default rate (\%) | 7.0445 | 6.0182 |
|  |  |  |
| Capital | 5.1401 | 5.5655 |
| GDP | 1.8028 | 1.8552 |
| Wage | 1.1538 | 1.1873 |
| Household debt | 0.0183 | 0.0200 |
| HH debt-to-GDP ratio (\%) | 1.0169 | 1.0802 |
| \% change w.r.t. benchmark |  |  |
|  |  |  |
| Incentive premium | - | -100.0000 |
| Avg. borrowing interest rate | - | -12.5789 |
| Fraction of HHs in debt | - | 5.1374 |
| Debt-to-earnings ratio | - | 4.2824 |
| Conditional default rate | - | -14.5683 |
|  |  | 8.2751 |
| Capital | - | 2.9035 |
| GDP | - | 2.9035 |
| Wage | - | 9.3075 |
| Household debt | - | 6.2232 |
| HH debt-to-GDP ratio | - |  |

Table 3.4: Effects of Financial Frictions on Equilibrium Outcomes
Notes: The conditional default rate is defined as the fraction of households choosing to default conditional on having any loans. The upper panel "Levels" reports model moments in levels under the benchmark and the counterfactual without financial frictions. The bottom panel "\% change w.r.t. benchmark" demonstrates the percentage variations of the variables under the frictionless counterfactual compared to the benchmark.
frictions exist, and the incentive constraint binds in equilibrium. Consequently, banks are incentivized to charge an extra incentive premium uniformly for all loan contracts. ${ }^{14}$ In particular, banks charge a positive equilibrium incentive premium of $0.63 \%$ in the benchmark economy while zero in the frictionless economy. On the other hand, the conditional default rate rises from $6.02 \%$ in the frictionless case to the benchmark level at $7.04 \%$. These observations suggest that financial frictions result in a riskier composition of household borrowers. As a result of higher borrowing cost, aggregate household borrowings decrease at both extensive and intensive margins: the fraction of households in debt and the debt-to-earnings decline from $9.08 \%$ and $1.96 \%$ in the frictionless economy

[^35]to $8.63 \%$ and $1.87 \%$ in the benchmark, respectively.
Moreover, financial frictions lead to lower production and wages. Since firm investments are financed solely through bank lending, higher borrowing costs result in reduced capital, decreased production, and lower wages. In particular, capital, gross domestic product (GDP), and wages increase respectively by $8.3 \%, 2.9 \%$, and $2.9 \%$ due to the removal of financial frictions. ${ }^{15}$ In addition, household debt responds more greatly to financial frictions than GDP. The reduction in household debt outweighs the GDP decline, thus implying a decreased household debt-to-GDP ratio in the benchmark economy. Conversely, when financial frictions are gone, there is no restriction on the banking asset portfolio. Therefore, no extra incentive premium arises in equilibrium, and production reverts upward to the level implied by the risk-free rate. Households thus benefit from the more efficient allocation via higher wage earnings.

As demonstrated, frictional financial intermediation entails declined household borrowings at both intensive and extensive margins, as well as lower production and wages. In addition to the existing mechanisms in a canonical consumer default model (Chatterjee et al., 2007; Livshits et al., 2007), financial frictions bring two new mechanisms into play: incentive and divestment channels. First, the agency problem between banks and depositors limits the ability of banks to acquire external funding via deposits. In order to mitigate the agency tension, banks are incentivized to charge an extra premium attached uniformly to the returns on all assets in the next period. I call this premium the incentive premium. As such, it becomes more costly for banks to divert the claims on these assets today, and banks thus prefer continuation to collect higher returns. The extra incentive premium thus leads to increased borrowing costs for households and firms. This mechanism is labeled as the incentive channel. Second, higher borrowing costs result in firms reducing capital investment. Production and wages accordingly decrease. This mechanism is denoted as the divestment channel.

### 3.5.2 Varying Degree of Financial Frictions

The benchmark calibration of financial frictions is regarded as suggestive, given the limited data access to the direct measures of financial frictions. To understand to what extent financial frictions shape household finance behavior and the aggregate economy, I further explore the effects of varying the degree of financial frictions. In my economy, two parameters govern the degree of financial frictions: the fraction of assets banks can divert secretly $\theta$ and the probability that banks exit the industry $\psi$. In a nutshell, higher $\theta$ and $\psi$ correspond to a higher degree of financial frictions because banks either can divert more

[^36]| Variable | $\boldsymbol{\theta}=\mathbf{0 . 2 8 5 9}$ | $\boldsymbol{\theta}=\mathbf{0 . 2 8 8 8}$ | $\boldsymbol{\theta}=\mathbf{0 . 2 9 1 8}$ <br> (Benchmark) | $\boldsymbol{\theta}=\mathbf{0 . 2 9 4 7}$ | $\boldsymbol{\theta}=\mathbf{0 . 2 9 7 6}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| Consumer credit markets |  |  |  |  |  |
| Avg. borrowing interest rate (\%) | 12.0957 | 12.1411 | 12.1829 | 12.2221 | 12.2590 |
| Fraction of HHs in debt (\%) | 8.6709 | 8.6511 | 8.6335 | 8.6175 | 8.6006 |
| Debt-to-earnings ratio (\%) | 1.8852 | 1.8796 | 1.8748 | 1.8705 | 1.8667 |
| Conditional default rate (\%) | 0.6064 | 0.6073 | 0.6082 | 0.6090 | 0.6097 |
|  |  |  |  |  |  |
| Incentive \& divestment channels |  |  |  |  |  |
| Incentive premium (\%) | 0.5581 | 0.5935 | 0.6264 | 0.6570 | 0.6857 |
| Capital | 5.1839 | 5.1611 | 5.1401 | 5.1207 | 5.1026 |
| GDP | 1.8083 | 1.8055 | 1.8028 | 1.8004 | 1.7981 |
| Wage | 1.1573 | 1.1555 | 1.1538 | 1.1522 | 1.1508 |
| Household debt | 0.0185 | 0.0184 | 0.0183 | 0.0183 | 0.0182 |
| HH debt-to-GDP ratio (\%) | 1.0242 | 1.0203 | 1.0169 | 1.0139 | 1.0111 |

Table 3.5: Effects of Varying Degree of Financial Frictions by $\theta$
Notes: The conditional default rate is defined as the fraction of households choosing to default conditional on having any loans. Each column reports model moments under the given $\theta$ in the first row.
assets or are more present-biased, i.e., a lower value from continuation for banks. ${ }^{16}$
First, I simulate the counterfactuals where $\theta$ varies from values of $2 \%$ lower to $2 \%$ higher than the benchmark calibration while holding all other parameters fixed. A higher $\theta$ means banks can divert a larger fraction of assets and thus reflects a higher degree of financial frictions. The results of these experiments are reported in Table 3.5, where each table column presents the outcomes under the given $\theta$ in the first row. Since the agency problem between banks and depositors is strengthened with the degree of financial frictions, a higher incentive premium must arise to equalize the incentive conflicts between banks and depositors. As shown in Table 3.5, banks will charge a higher incentive premium if facing higher $\theta$, and vice versa. The average borrowing interest rate accordingly increases with $\theta$. Household borrowings at both margins correspondingly decrease with the degree of financial frictions, while the conditional default rate is positively related to $\theta$. Firms also reduce investment and production further in response to higher borrowing costs owing to a higher $\theta$. As a result, wages fall to a lower extent mechanically. The household debt-to-GDP ratio declines as household debt is more sensitive to the variation in $\theta$.

Second, I vary the average banking planning horizon from seven to eleven years by setting the exit rate $\psi$ to the corresponding values while all other parameters remain the same. ${ }^{17}$ A shorter average banking planning horizon implies that banks are less forwardlooking and have lower continuation values. Ceteris paribus, the myopia of banks aggra-

[^37]| Variable | $\psi=0.9091$ | $\psi=0.8926$ <br> (Benchmark) | $\psi=0.8889$ | $\psi=0.8750$ | $\psi=0.8571$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |


|  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Consumer credit markets |  |  |  |  |  |
| Avg. borrowing interest rate (\%) | 11.8810 | 12.1829 | 12.2303 | 12.4218 | 12.6524 |
| Fraction of HHs in debt (\%) | 8.8426 | 8.6335 | 8.6143 | 8.5274 | 8.4265 |
| Debt-to-earnings ratio (\%) | 1.9602 | 1.8748 | 1.8697 | 1.8514 | 1.8304 |
| Conditional default rate (\%) | 0.5969 | 0.6082 | 0.6091 | 0.6129 | 0.6149 |
| Incentive \& divestment channels |  |  |  |  |  |
| Incentive premium (\%) | 0.4670 | 0.6264 | 0.6635 | 0.8102 | 1.0141 |
| Capital | 5.2432 | 5.1401 | 5.1166 | 5.0254 | 4.9029 |
| GDP | 1.8157 | 1.8028 | 1.7998 | 1.7882 | 1.7724 |
| Wage | 1.1621 | 1.1538 | 1.1519 | 1.1445 | 1.1343 |
| Household debt | 0.0195 | 0.0183 | 0.0182 | 0.0179 | 0.0175 |
| HH debt-to-GDP ratio (\%) | 1.0740 | 1.0169 | 1.0133 | 1.0000 | 0.9850 |

Table 3.6: Effects of Varying Degree of Financial Frictions by $\psi$
Notes: The conditional default rate is defined as the fraction of households choosing to default conditional on having any loans. Each column reports model moments under the given $\psi$ in the first row.
vates the agency problem with depositors, thus resulting in a higher degree of financial frictions. Table 3.6 show the outcomes of these experiments. Similar to the conclusion drawn from the exercises of $\theta$, the incentive premium positively correlates with the degree of financial frictions (or equivalently inversely with $\psi$ ). The average borrowing rate thus increases inversely with $\psi$. Household borrowings at both margins decline as $\psi$ decreases. Production, wages, and the household debt-to-GDP ratio all increase with $\psi$.

### 3.6 Regulation of Consumer Credit Markets

Consumer credit markets are often regulated through bankruptcy laws by policymakers. However, the welfare implications of bankruptcy strictness are unclear ex-ante and depend on the canonical efficiency-insurance trade-off discussed in Zame (1993). On the one hand, households can default to insure themselves against idiosyncratic risks. In other words, default helps them smooth across states. On the other hand, bankruptcy leniency prompts banks to charge higher borrowing prices to compensate for larger default risks. Higher interest costs make it more difficult for households to smooth over time.

Since credit provision is affected by fictional financial intermediation, financial frictions play a critical role in the welfare assessment of consumer bankruptcy laws. For instance, under a lenient regime, banks charge higher default premiums to break even, and households thus face higher borrowing costs, ceteris paribus. A higher borrowing price in terms of savings results in an increased propensity to save for households. As a result, banks receive more deposits and have a greater incentive to divert assets. In
order to mitigate the agency tension, banks are incentivized to charge an extra premium for all assets. As a result, banks find it more costly to divert the claims on assets today and prefer continuation to collect higher returns. However, the increased borrowing costs make it harder for households to smooth consumption by borrowing from banks. Also, higher borrowing prices cause firms to reduce investment and thus production. Therefore, households are worse off in terms of welfare due to lower wage earnings for consumption.

To quantitatively investigate the impact of financial frictions on the welfare evaluation of consumer credit regulations, I consider two sets of bankruptcy rules highly relevant in consumer credit markets: (1) short-term monetary bankruptcy costs via wage garnishment; and (2) long-term punishment via the exclusion from borrowing markets. Generally speaking, the aggregate and welfare effects of a policy change are involved with the transition dynamics of each household to the new policy and, at the same time, dependent on the interaction with financial frictions. To better understand how each component contributes to the welfare analysis, I focus in the first step on the interplay between the legal change and the household transition dynamics by analyzing the welfare implications of these policy proposals under the benchmark calibration. These exercises are crucial for grasping how the proposed policy experiments primarily influence the aggregate economy and household welfare when financial frictions exist. In the second step to decipher the interaction between bankruptcy laws and financial frictions, I further explore how and to what extent financial frictions shape the previous benchmark welfare conclusions by changing the degree of financial frictions.

The rest of the section is structured as follows. Section 3.6.1 defines the welfare metrics that incorporate the transition dynamics of a policy change. Section 3.6.2 and 3.6.3 present the policy experiments of wage garnishment and borrowing exclusion under the benchmark calibration of financial frictions, respectively. Section 3.6.4 investigates how and the what extent financial frictions affect the welfare implications of consumer bankruptcy laws.

### 3.6.1 Welfare Measures

To evaluate the welfare effects of an unanticipated policy reform, I adopt two metrics: (1) percentage gain/loss compared to the benchmark in the consumption equivalent variation (CEV) unit; and (2) fraction of households in favor of the policy reform (i.e., majority rule). In addition, I take into account the transition dynamics of policy changes because the policy effects are heterogeneous conditional on household initial states. ${ }^{18}$ For convenience, I use superscripts old and new to denote the equilibrium objects under the old

[^38]and new policies in the following discussions.
First, I measure the lifetime percentage change in flow consumption since an unanticipated policy change. ${ }^{19}$ The welfare gain/cost $\tau(i)$ for household $i$ owning to an unanticipated new policy at $t=1$ is defined as:
\[

$$
\begin{equation*}
\mathbb{E}_{1}\left[\sum_{t=1}^{\infty} \nu_{t} \cdot(\beta \rho)^{t-1} \cdot u\left(\left(1+\frac{\tau(i)}{100}\right) \cdot c_{t}^{o l d}(i)\right)\right]=\mathbb{E}_{1}\left[\sum_{t=1}^{\infty} \nu_{t} \cdot(\beta \rho)^{t-1} u\left(c_{t}^{\text {new }}(i)\right)\right], \tag{3.30}
\end{equation*}
$$

\]

where positive $\tau(i)$ means households $i$ prefers the new policy, and vice versa. Given CRRA utility function with coefficient $\gamma, \tau(i)$ can be solved as:

$$
\begin{equation*}
\tau(i)=\left[\left(\frac{\tilde{V}_{1}(i)}{V_{\text {old }}(i)}\right)^{\frac{1}{1-\gamma}}-1\right] \times 100 \tag{3.31}
\end{equation*}
$$

where $\widetilde{V}_{1}(i)$ denotes the transition value for household $i$ at $t=1$ and $\widetilde{V}_{t}(i)$ converges to $V^{\text {new }}(i)$ when $t$ is sufficiently large.

In addition, I calculate the percentage of households in favor of the new policy as follows.

$$
\begin{equation*}
\sum_{i}\left[\mathbb{I}_{[\tau(i)>0]} \cdot \mu^{\text {old }}(i)\right] \times 100 \tag{3.32}
\end{equation*}
$$

where $\mathbb{I}$ denotes the indicator function which equals one if $\tau(i)>0$ and zero otherwise; recall that $\mu$ denotes the cross-sectional distribution of households in equilibrium. When the new policy is introduced (i.e., at the beginning of $t=1$ ), households are still distributed according to $\mu^{o l d}$. Thus, the idea is to check how many households prefer the new policy similar to majority rule. This measure can thus speak to political decision making.

### 3.6.2 Wage Garnishment

One of the bankruptcy regulation tools is the bankruptcy fees in the filing period. I model this cost using the wage garnishment rate to keep borrowers acting in good faith. To examine how wage garnishment rates affect the equilibrium outcomes with financial frictions, I simulate two counterfactuals where wage garnishment rates, relative to the benchmark value of 0.25 , are decreased by 0.05 to 0.20 and increased by 0.05 to 0.30 , respectively. The key equilibrium results of these policy experiments are summarized in Table 3.7. The column "Benchmark" reports the results in the calibrated model.

[^39]| Variable | Lower Garnishment | Benchmark | Higher Garnishment |
| :--- | :---: | :---: | :---: |
| Levels |  |  |  |
|  |  |  |  |
| Consumer credit markets |  |  |  |
| Default rate (\%) | 0.6652 | 0.6082 | 0.4322 |
| Avg. borrowing interest rate (\%) | 14.3035 | 12.1829 | 8.9899 |
| Fraction of HHs in debt (\%) | 6.7959 | 8.6335 | 11.2935 |
| Debt-to-earnings ratio (\%) | 1.2858 | 1.8748 | 2.7372 |
|  |  |  |  |
| Incentive \& divestment channels |  |  |  |
| Banking leverage ratio | 4.8967 | 4.5652 | 4.1773 |
| Incentive premium (\%) | 0.7071 | 0.6264 | 0.4893 |
| Capital | 5.0892 | 5.1401 | 5.2286 |
| GDP | 1.7964 | 1.8028 | 1.8139 |
| Wage | 1.1497 | 1.1538 | 1.1609 |
| \% change w.r.t. benchmark |  |  |  |
|  |  |  |  |
| Incentive \& divestment channels |  | - |  |
| Banking leverage ratio | 7.2613 | - | -8.4961 |
| Incentive premium | 12.8781 | - | 1.8805 |
| Capital | -0.9901 | - | 0.6160 |
| GDP | -0.3576 | 0.6160 |  |
| Wage | -0.3576 |  |  |

Table 3.7: Counterfactual of Wage Garnishment: Equilibria Comparison
Notes: The upper panel "Levels" reports model moments in levels under the benchmark and the policy experiments of wage garnishment. The bottom panel "\% change w.r.t. benchmark" shows the percentage variations of the selective moments related to the incentive and divestment channels under the policy experiments compared to the benchmark.

The column "Lower Garnishment" shows the results of the policy counterfactual where bankruptcy law becomes more lenient due to a lower wage garnishment rate of 0.20 . The column "Higher Garnishment" instead presents the results of the case where bankruptcy law becomes stricter due to a higher wage garnishment rate of 0.30

Compared to the benchmark, a lower wage garnishment rate leads to an increased default rate because of lower default costs in the filing period. As a result, the average borrowing interest rate rises from $12.18 \%$ in the benchmark to $14.30 \%$. Due to higher borrowing costs, both the fraction of households in debt (extensive margin) and the debt-to-earnings ratio (intensive margin) drop significantly. In addition, rising borrowing costs result in higher borrowing prices relative to savings, leading to fewer unsecured loans and more deposits in equilibrium. Accordingly, banks become more externally financed with deposits and have a higher leverage ratio. ${ }^{20}$ Therefore, the incentive premium increases by $12.88 \%$ and wages decrease by $0.36 \%$ through the incentive and divestment channels

[^40]Figure 3.1: Transition Paths of Banking Leverage Ratio
(a) From Benchmark to Lower Garnishment

(b) From Benchmark to Higher Garnishment


Notes: The unit of time is a year. The policy reform is unexpectedly announced at $t=1$. The banking leverage ratio remains in the old equilibrium at $t=0$ and converges to the new equilibrium at $t=80$. The left figure illustrates the transition from benchmark $(\eta=0.25)$ to lower garnishment $(\eta=0.20)$. The right figure plots the transition from benchmark $(\eta=0.25)$ to higher garnishment $(\eta=0.30)$.
mentioned previously in Section 3.5.2. In the case of a higher wage garnishment rate, all of these changes move in the opposite direction.

The converged transition paths of the banking leverage ratio for both policy counterfactuals are visualized in Figure 3.1, where Figure 3.1a plots the transition from benchmark to lower garnishment and 3.1b shows the transition from benchmark to higher garnishment. In both cases, the banking leverage ratio gradually converges to the new leverage ratios under the respective policy reforms. For example, the banking leverage ratio decreases from 4.57 to 4.18 under the policy experiment of higher garnishment. In addition, one can see that there are salient discrete jumps in banking leverage ratios in the first period. This is because more (less) households default in response to an unexpected policy change of a more lenient (stricter) bankruptcy rule. Furthermore, borrowing prices and wages vary with the transition path of the banking leverage ratio through the incentive and divestment channels. For instance, under the counterfactual of lower garnishment, the banking leverage ratio increases gradually to the higher equilibrium level. The incentive constraint thus becomes increasingly binding, and the incentive premium accordingly rises over time. As a result, households face progressively higher borrowing costs and lower wages along with the transition.

The welfare results of these policy counterfactuals under the benchmark calibration of financial frictions are summarized in Table 3.8, where I distinguish households from initial credit history, indebtedness, and the degree of patience. The column "HH Proportion" describes the initial household distribution when the policy reform is announced. The column "CEV" reports the CEV in the percentage of the policy change relative to the

| Variable (in \%) |  | Lower Garnishment |  | Higher Garnishment |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | HH Proportion | CEV | Favor Reform | CEV | Favor Reform |
| Total | 100.0000 | -0.1845 | 1.3450 | 0.2760 | 99.4748 |
| Good credit history | 94.9490 | -0.1889 | 1.4165 | 0.2810 | 99.5030 |
| Indebted | 9.0928 | -0.4267 | 15.5785 | 0.5917 | 94.5346 |
| Not indebted | 90.9072 | -0.1566 | 0.0000 | 0.2391 | 100.0000 |
| Patient | 98.9653 | -0.1868 | 1.4259 | 0.2791 | 99.5000 |
| Impatient | 1.0347 | -5.4868 | 0.5207 | 5.7604 | 99.7932 |
|  |  |  |  |  |  |
| Bad credit history | 5.0510 | -0.1062 | 0.0000 | 0.1866 | 98.9430 |

Table 3.8: Counterfactual of Wage Garnishment: Welfare Implications
Notes: All results are measured when the policy reform is announced. The column "HH Proportion" describes the initial household distribution. The column "CEV" reports the CEV in the percentage of the policy change relative to the benchmark. The column "Favor Reform" reports the fraction of households in favor of the new policy in percentage. The row "Total" shows the aggregate results. The rows "Good credit history"/"Bad credit history" illustrate the results conditional on households with good/bad credit history. The rows "Indebted"/"Not indebted" present the results among households with good credit history who have debts/no debts. The row "Impatient" shows the results conditional on households with good credit history hit by preference shocks.
benchmark. The column "Favor Reform" reports the percentage of households in favor of the new policy.

The welfare effects of decreasing or increasing wage garnishment rates are the opposite: a more lenient law through a lower wage garnishment rate is overall welfare-reducing for all households, whereas a stricter law through a higher wage garnishment rate is overall welfare-improving. The reasons are twofold. First, a stricter bankruptcy regulation via higher default costs results in lower default premia but makes bankruptcy declaration more costly in response to bad shocks. Second, the agency problem is mitigated under a stricter regime, as discussed in Table 3.7. Banks thus charge a lower incentive premium, thus leading to lower borrowing costs for firms and households. Firms thus increase capital investment, produce more, and raise wages. Hence, lower borrowing costs and higher wages allow households to better smooth consumption. In my model, the benefit from lower borrowing costs (either through lower default or incentive premium) and higher wages outweighs the loss from bankruptcy insurance through higher default costs under a stricter code. The results are the opposite under a more lenient legal environment. Therefore, a stricter (more lenient) bankruptcy regime results in a welfare gain (loss). In particular, impatient households benefit significantly from a stricter code because they can borrow at lower interest costs to mitigate the higher interest expenses due to the over-borrowing triggered by preference shocks.

However, one might find it counter-intuitive that households with bad credit history
also prefer a stricter bankruptcy regulation. Given that they have defaulted in the past with lower wage garnishment, the current imposed legal change of a higher garnishment rate does not directly impact those already with bad credit history. Although they are temporarily excluded from the borrowing markets, they can regain borrowing access in the future due to the removal of bad credit history and benefit from lower borrowing costs to smooth consumption by then. They also gain higher wages due to the reduced agency tension under a stricter legal environment. The quantitative results suggest that, for this subgroup, the gain from smoothing consumption at lower borrowing costs in the long run and higher wage earnings combined is greater than the insurance loss of higher default costs due to a stricter law.

In terms of the majority rule, almost all households prefer a higher garnishment rate, while some indebted households prefer a lower rate. Why do not indebted households support a stricter bankruptcy reform unanimously as households with good credit history but without debts do? This is because this group of households have borrowed at lower interest costs under the benchmark policy and, after the implementation of a more lenient bankruptcy law, they can thus benefit timely from discharging debts at lower default costs if hit by bad shocks in the subsequent period. Consequently, a lower wage garnishment rate is advocated by more indebted households compared to other household subgroups.

### 3.6.3 Exclusion from Borrowing Markets

Another approach to regulation in the consumer credit market is to keep track of consumer's credit history. A flag or bad record of bankruptcy filing remains on credit report for a certain period of time. During this period, consumer's borrowing ability is forbidden. In my model, this exclusion regulation is captured by the probability of flag removal $\mathbb{P}_{h}$. Recall that the benchmark calibration for $\mathbb{P}_{h}$ is set to $1 / 10$, implying an average exclusion duration of 10 years. This period length of exclusion is consistent with the Fair Credit Reporting Act. For brevity, the converged transition paths of borrowing exclusion policy experiments are reported in Appendix 3.C.

To examine the equilibrium and welfare effects of a shorter or longer duration of exclusion from borrowing markets with financial frictions, I simulate two counterfactuals where the probability of flag removal is increased to $1 / 5$ and decreased to $1 / 15$, respectively. They correspond to average exclusion duration of 5 and 15 years. The equilibrium results of these policy counterfactuals are summarized in Table 3.9 and the welfare outcomes in Table 3.10. The column "Shorter Exclusion" denotes the counterfactual where bankruptcy law becomes more lenient due to a higher probability of flag removal equal to $1 / 5$. The column "Longer Exclusion" denotes the counterfactual where bankruptcy law becomes stricter due to a lower probability of flag removal equal to $1 / 15$.

In Table 3.9, one can see that longer (shorter) exclusion results in lower (higher) default

| Variable | Shorter Exclusion | Benchmark | Longer Exclusion |
| :--- | :---: | :---: | :---: |
| Levels |  |  |  |
|  |  |  |  |
| Consumer credit markets |  |  |  |
| Default rate (\%) | 0.6480 | 0.6082 | 0.5753 |
| Avg. borrowing interest rate (\%) | 12.3257 | 12.1829 | 11.9688 |
| Fraction of HHs in debt (\%) | 8.6259 | 8.6335 | 8.6814 |
| Debt-to-earnings ratio (\%) | 1.8252 | 1.8748 | 1.9334 |
|  |  |  |  |
| Incentive \& divestment channels |  |  |  |
| Banking leverage ratio | 4.5832 | 4.5652 | 4.5443 |
| Incentive premium (\%) | 0.6315 | 0.6264 | 0.6203 |
| Capital | 5.1368 | 5.1401 | 5.1440 |
| GDP | 1.8024 | 1.8028 | 1.8033 |
| Wage | 1.1535 | 1.1538 | 1.1541 |
| \% change w.r.t. benchmark |  |  |  |
|  |  |  |  |
| Incentive \& divestment channels |  | - | -0.4580 |
| Banking leverage ratio | 0.3960 | - | -0.9696 |
| Incentive premium | 0.8223 | - | 0.0752 |
| Capital | -0.0637 | - | 0.0271 |
| GDP | -0.0229 | 0.0271 |  |
| Wage | -0.029 |  |  |

Table 3.9: Counterfactual of Prob. of Flag Removal: Equilibria Comparison
Notes: The upper panel "Levels" reports model moments in levels under the benchmark and the policy experiments of borrowing exclusion. The bottom panel "\% change w.r.t. benchmark" shows the percentage variations of the selective moments related to the incentive and divestment channels under the policy experiments compared to the benchmark.
risks and thus lower (higher) borrowing interest rates. As a result, borrowings at extensive and intensive margins both rise (drop). In addition, banks become less (more) leveraged via less (more) deposits. A higher (lower) banking leverage ratio leads to higher (lower) incentive premium and lower (higher) wages. These results are qualitatively analogous to the findings of wage garnishment rates in Table 3.7. This similarity is not surprising because both a lower wage garnishment rate and a decreased probability of flag removal represent stricter bankruptcy laws, and vice versa. The major difference between these two policy tools is the timing: wage earnings are garnished only in the filing period, whereas households with bad credit history are excluded from the borrowing markets until their records are erased at the probability of flag removal.

Regarding the welfare implications, the predictions of borrowing exclusion are qualitatively similar to the one of wage garnishment. A stricter code is welfare-improving in aggregate, while a more lenient one is overall welfare-reducing. In terms of the effects of financial frictions, a stricter (more lenient) regime results in eased (greater) incentive conflicts, thus leading to a lower (higher) inventive premium and higher (lower) wages.

| Variable (in \%) |  | Shorter Exclusion |  | Longer Exclusion |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | HH Proportion | CEV | Favor Reform | CEV | Favor Reform |
| Total | 100.0000 | -0.0106 | 4.9921 | 0.0092 | 94.9358 |
| Good credit history | 94.9490 | -0.0164 | 0.8210 | 0.0127 | 99.3963 |
| Indebted | 9.0928 | -0.0429 | 9.0294 | 0.0331 | 93.3606 |
| Not indebted | 90.9072 | -0.0128 | 0.0000 | 0.0099 | 100.0000 |
| Patient | 98.9653 | -0.0162 | 0.8296 | 0.0126 | 99.3904 |
| Impatient | 1.0347 | -0.4652 | 0.0000 | 0.1608 | 99.9551 |
|  |  |  |  |  |  |
| Bad credit history | 5.0510 | 0.0925 | 83.4012 | -0.0519 | 11.0879 |

Table 3.10: Counterfactual of Prob. of Flag Removal: Welfare Implications
Notes: All results are measured when the policy reform is announced. The column "HH Proportion" describes the initial household distribution. The column "CEV" reports the CEV in the percentage of the policy change relative to the benchmark. The column "Favor Reform" reports the fraction of households in favor of the new policy in percentage. The row "Total" shows the aggregate results. The rows "Good credit history"/"Bad credit history" illustrate the results conditional on households with good/bad credit history. The rows "Indebted"/"Not indebted" present the results among households with good credit history who have debts/no debts. The row "Impatient" shows the results conditional on households with good credit history hit by preference shocks.

However, the welfare implications of borrowing exclusion are heterogeneous across household types of credit history and level of indebtedness.

Focusing first on the case of shorter exclusion in Table 3.10, one can see that the households with good credit history have lower welfare, whereas households with bad credit history have higher welfare. Moreover, this policy proposal is advocated by $80 \%$ of households with bad credit history, while by less than $1 \%$ of households with good credit history. The reasons for these differences are intuitive. First, for households with a good credit record, the loss of lower borrowing costs and higher wages outweighs the gain from better bankruptcy insurance through a shorter exclusion from borrowing markets. In contrast, this proposal helps households get rid of the bad record on their credit reports faster than in the benchmark, thus resulting in a direct positive welfare impact on those already with bad credit history. Second, among households with good credit history, $9 \%$ of indebted households favor a more lenient bankruptcy regime, while not a single household without debt appreciates bankruptcy leniency. This is because the proposed policy provides higher insurance value for indebted households by defaulting: they can discharge their debts at lower default costs in the shortfalls as they could regain access to the borrowing markets within a shorter period. In the case of longer exclusion, these welfare conclusions shift in the opposite direction.

Figure 3.2: Welfare for Total Households


Notes: These figures show the aggregate welfare results of wage garnishment and borrowing exclusion counterfactuals with and without financial frictions. Welfare in measured in CEV units relative to the benchmark policy in percentage points. The solid and dashed lines denote the welfare results with and without financial frictions, respectively.

### 3.6.4 Welfare Effects of Varying Financial Frictions

To understand how and to what extent financial frictions affect the welfare implications of policy experiments, I first iterate the simulations of the previous policy counterfactuals without financial frictions and compare this set of results with the previous welfare outcomes with financial frictions. This comparison is presented in Figure 3.2, where Figure 3.2a plots the aggregate welfare results of wage garnishment rates and Figure 3.2b displays the ones of borrowing exclusion from consumer credit markets. The solid line denotes the welfare outcomes in the CEV unit relative to the benchmark when financial frictions exist. The dashed line depicts similar welfare results but without financial frictions.

Under both policy experiments, one can see in Figure 3.2 that the aggregate welfare effects of a stricter (more lenient) bankruptcy regime are positive (negative) both with and without financial frictions, regardless of policy instruments. More interestingly, the magnitudes of welfare variations are relatively larger when financial frictions exist. So, why is the welfare sensitivity to bankruptcy strictness with financial frictions larger than those without financial frictions? This is because there are extra effects triggered by the incentive and divestment channels that come along with financial frictions. As shown in Table 3.7 and 3.9, bankruptcy leniency leads to higher default risks and higher borrowing interest costs. As a result, the relative price of borrowing in terms of saving rises, given the constant risk-free saving rate. Accordingly, banks receive more deposits and become more leveraged with external funding. A higher banking leverage ratio thus causes the incentive premium and wages to increase and decrease via the investment and divestment channels, respectively. A higher incentive premium makes borrowing more expensive, and lower

Figure 3.3: Welfare for Households with Good Credit History
(a) Wage Garnishment
(b) Borrowing Exclusion



Notes: These figures show the welfare results of wage garnishment and borrowing exclusion counterfactuals for households with good credit history with and without financial frictions. Welfare in measured in CEV units relative to the benchmark policy in percentage points. The solid and dashed lines denote the welfare results with and without financial frictions, respectively.
wages lead households to less consumption. These extra negative effects do not exist if there are no financial frictions as illustrated in Table 3.4. On the contrary, under a stricter code, the borrowing price relative to saving falls. Banks thus receive fewer deposits, implying a lower leverage ratio. As a result, the incentive premium decreases while wages increase. Hence, households benefit additionally from lower borrowing costs and higher consumption. This result implies that varying the degree of bankruptcy strictness results in relatively more considerable welfare effects with financial frictions. ${ }^{21}$

In addition, the same set of results conditional on households with either good or bad credit history are shown in Figure 3.3 and 3.4, respectively. The conclusion drawn above holds across almost all household subgroups and policy experiments, except for households with bad credit history under the borrowing exclusion counterfactual in Figure 3.4b. Recall in Section 3.6.3 that shortening the exclusion duration yields welfare gains for households with bad credit history because they can access consumer credit markets faster than in the benchmark. The extra negative effects caused by the investment and divestment channels offset the welfare gains from the shorter exclusion. In contrast, longer exclusion results in welfare losses for households with bad credit history since they remain excluded from the borrowing markets for longer than the benchmark. The extra positive effects from the investment and divestment channels thus mitigate the welfare losses in this case. As a result, the magnitudes of welfare gains (losses) are relatively larger without financial frictions.

To further explore the relationship between the welfare sensitivity to bankruptcy strict-

[^41]Figure 3.4: Welfare for Households with Bad Credit History


Notes: These figures show the welfare results of wage garnishment and borrowing exclusion counterfactuals for households with bad credit history with and without financial frictions. Welfare in measured in CEV units relative to the benchmark policy in percentage points. The solid and dashed lines denote the welfare results with and without financial frictions, respectively.
ness and the degree of financial frictions, I redo the simulations of the wage garnishment counterfactual. However, I assume these policy changes now co-occur with different degrees of financial frictions by changing the diverting fraction $\theta .{ }^{22}$ In particular, I consider two cases: (1) banks can divert a larger fraction $\theta^{H}$ of total assets by $1 \%$ compared to the benchmark calibration $\theta^{B}$, i.e., $\theta^{H}=1.01 \times \theta^{B}$; and (2) banks can instead divert a lower fraction $\theta^{L}$ of total assets by $1 \%$ than they can in the benchmark, i.e., $\theta^{L}=0.99 \times \theta^{B}$. I then compare the new welfare results with the benchmark results. The comparison of aggregate welfare is visualized in Figure 3.5, where the solid line shows the benchmark outcomes $\theta^{B}$, the dashed line presents the ones under weaker financial frictions $\theta^{L}$, and the dash-dotted line denotes the case of stronger financial frictions $\theta^{H}$. Refer to Appendix 3.C for the converged transition paths under these policy counterfactuals and Appendix 3.D for the equilibrium and welfare outcomes with $\theta^{L}$ and $\theta^{H}$ in details.

In Figure 3.5, one can see that under weaker financial frictions, a higher wage garnishment rate results in larger welfare gains, whereas a lower rate leads to less welfare losses compared to the benchmark results. In contrast, stronger financial frictions yield less welfare gains from a higher rate while greater welfare losses from a lower rate. These results are not surprising because the effects of incentive and divestment channels are dampened and strengthened under weaker and stronger financial frictions, respectively. This idea is presented in Table 3.11, where I compute the percentage variations in the incentive premium and wages compared to the benchmark under all cases. The column " $\Delta \iota$ " reports

[^42]Figure 3.5: Aggregate Welfare (CEV) v.s. Financial Frictions


Notes: This figure plots the aggregate welfare results of wage garnishment counterfactuals with benchmark/weaker/stronger financial frictions. Welfare in measured in CEV units relative to the benchmark policy in percentage points. The solid/dashed/dash-dotted lines denote the welfare results with benchmark/weaker/stronger financial frictions, respectively.

| Variable (in \%) | Lower Garnishment |  | Higher Garnishment |  |
| :--- | :---: | :---: | :---: | :---: |
|  | $\boldsymbol{\Delta} \boldsymbol{\iota}$ | $\boldsymbol{\Delta} \boldsymbol{w}$ | $\boldsymbol{\Delta} \boldsymbol{\iota}$ | $\boldsymbol{\Delta} \boldsymbol{w}$ |
|  |  |  |  |  |
| Benchmark | 12.8781 | -0.3576 | -21.8868 | 0.6160 |
| Weaker financial frictions | 8.7732 | -0.2440 | -29.3179 | 0.8276 |
| Stronger financial frictions | 16.7534 | -0.4645 | -15.0701 | 0.4230 |

Table 3.11: Effects of Incentive and Divestment Channels v.s. Financial Frictions
Notes: This table reports the variations in incentive premium and wages relative to the benchmark policy in percentage points under the wage garnishment experiment across benchmark/lower/higher degrees of financial frictions. The row "Benchmark"/"Weaker financial frictions"/"Stronger financial frictions" denotes the results with benchmark/lower/higher degrees of financial frictions, respectively.
the percentage variation in the incentive premium compared to the benchmark. The column " $\Delta w$ " shows the percentage variation in wages relative to the benchmark. ${ }^{23}$ Recall that: (1) a stricter rule results in a lower banking leverage ratio, and vice versa; (2) the higher the banking leverage ratio, the larger the distorted effects via the incentive and divestment channels in financial markets; and (3) under benchmark calibration, households prefer a stricter regime for smoothing consumption.

Under weaker financial frictions, the distorted effects are mitigated. For example, a stricter rule gives rise to a larger drop in the incentive premium by $29.32 \%$ and a larger increase in wages by $0.83 \%$ under weaker financial frictions compared to $21.89 \%$ and

[^43]$0.62 \%$ in the benchmark, respectively. On the other hand, a more lenient code yields a smaller increase in the incentive premium by $8.77 \%$ (a smaller decrease in wages by $0.24 \%$ ) compared to $12.88 \%$ ( $0.36 \%$ ) in the benchmark. These price changes in both policy experiments work in favor of households. As a result, weaker financial frictions result in larger positive welfare effects of a stricter rule and smaller negative effects of a more lenient code compared to the benchmark. Analogously, stronger financial frictions aggravate the distorted effects. Therefore, under stronger financial frictions, a stricter rule yields smaller welfare gains, and a more lenient code leads to larger welfare losses relative to the benchmark.

### 3.7 Conclusion

What are the effects of financial frictions under a heterogeneous agent framework with consumer default? To what extent are the welfare implications of consumer bankruptcy laws affected by frictional financial intermediation? To this end, I build an Aiyagari-type model of consumer default and financial frictions. Households can file for bankruptcy to insure themselves against labor productivity and preference risks. Default costs include short-term wage garnishment and long-term exclusion from borrowing markets. Firms borrow from banks to finance capital spending. Banks use net worth and deposits from household savers to lend to firms and household borrowers. However, banks are tempted to divert the claims on total assets if highly leveraged with deposits. In equilibrium, banks are thus incentivized to have skin in the game by charging an incentive premium on the asset returns. Compared to a canonical consumer default model, household borrowing prices under my framework depend on idiosyncratic default risks and aggregate banking net worth.

Under the benchmark calibration, the incentive and divestment channels emerge endogenously due to financial frictions. The incentive channel captures the direct positive effects of the incentive premium on borrowing prices. The divestment channel refers to the indirect negative effects on the wage earnings of households. Compared to the economy without financial frictions, frictional financial intermediation results in higher borrowing interest rates, leading to declines in household debt and firm investment. Production and wages accordingly decrease. All these effects are amplified as the degree of financial frictions increases.

The welfare evaluation of a policy change depends on the policy per se, the transition dynamics of households to the new policy, and the degree of financial frictions. I conduct a series of policy experiments and explore the role of financial frictions to understand the role of each component. The quantitative results indicate that Stricter bankruptcy rules are welfare-improving, whereas more lenient ones result in welfare losses, regardless
of the exact policy tools. However, the welfare implications are heterogeneous across household types. For example, impatient households favor bankruptcy strictness because they can benefit significantly from the lower borrowing costs in smoothing consumption. On the other hand, households with bad credit history find longer borrowing exclusion significantly welfare-reducing. More importantly, financial frictions affect the welfare sensitivity to bankruptcy strictness. A higher degree of financial frictions results in greater distorted effects on borrowing prices and wages through the incentive and divestment channels. These adverse effects thus dampen the welfare gains or aggravate the welfare losses from a proposed policy. The results suggest that ignoring financial frictions could lead to biased policy conclusions in consumer credit markets.

In the future, a natural extension is to introduce the general equilibrium (GE) effects into the current framework by solving the endogenous saving rate under which financial markets clear. The interaction between the GE effects and financial frictions could lead to distinct welfare implications of personal bankruptcy provision. In addition, estimating the model using the simulated method of moments could make the conclusions more robust, especially given that the current calibration of financial frictions is somewhat suggestive. However, this extension will be computationally intensive due to the occasionally binding incentive constraint. Another exciting avenue for future research is to incorporate aggregate uncertainty into my framework to study the business cycles of consumer credit and bankruptcy because my model features the interaction between consumer default and an endogenous banking leverage constraint.

## Appendix

## 3.A Model Details

## 3.A. 1 Bank Optimization

Aggregate variables are defined as:

$$
\begin{align*}
L^{\prime} & =\sum_{\left(a^{\prime}<0, a, e, \nu\right)} q\left(a^{\prime}, e\right) \cdot\left(-a^{\prime}\right) \cdot \mathbb{I}_{\left[a^{\prime}=g_{a}(a, e, \nu, h=0)\right]} \cdot \mu(a, e, \nu, h=0),  \tag{3.33}\\
D^{\prime} & =\sum_{\left(a^{\prime}>0, a, e, h\right)} q\left(a^{\prime}, e\right) \cdot a^{\prime} \cdot \mathbb{I}_{\left[a^{\prime}=g_{a}(a, e, \nu=1, h)\right]} \cdot \mu(a, e, \nu=1, h),  \tag{3.34}\\
K^{\prime} & =N+D^{\prime}-L^{\prime}, \tag{3.35}
\end{align*}
$$

where note that only households with good credit history can borrow and impatient households do not save. Bank $j$ 's optimization problem is given by:

$$
\begin{align*}
W\left(n_{j}\right)= & \max _{k_{j}^{\prime}, l_{j}^{\prime}, s_{j}^{\prime}}\left(\frac{1}{1+r_{f}}\right) \cdot\left[(1-\psi) \cdot n_{j}^{\prime}+\psi \cdot W\left(n_{j}^{\prime}\right)\right]  \tag{3.36}\\
\text { s.t. } \quad & k_{j}^{\prime}+l_{j}^{\prime}=n_{j}+s_{j}^{\prime}+\tau_{j},  \tag{3.37}\\
& n_{j}^{\prime}=\left(1+r_{k}^{\prime}\right) \cdot k_{j}^{\prime}+\left(1+r_{l}^{\prime}\right) \cdot l_{j}^{\prime}-\left(1+r_{f}\right) \cdot\left(s_{j}^{\prime}+\tau_{j}\right),  \tag{3.38}\\
& W\left(n_{j}\right) \geq \theta \cdot\left(k_{j}^{\prime}+l_{j}^{\prime}\right), \tag{3.39}
\end{align*}
$$

where the aggregate return on lending to households is defined as:

$$
\begin{equation*}
1+r_{l}^{\prime} \equiv \frac{\rho \cdot \sum_{\left(a^{\prime}<0, e, \nu\right)} R\left(a^{\prime}, e\right) \cdot \mathbb{I}_{\left[a^{\prime}=g_{a}(a, e, \nu, h=0)\right]} \cdot \mu(a, e, \nu, h=0)}{L^{\prime}} . \tag{3.40}
\end{equation*}
$$

Conjecture $W\left(n_{j}\right)=\xi \cdot n_{j}$ which will be verified shortly. With the conjecture, the above optimization problem can be rewritten as:

$$
\begin{align*}
W\left(n_{j}\right) & =\max _{k_{j}^{\prime}, l_{j}^{\prime}} \Lambda^{\prime}\left[\left(r_{k}^{\prime}-r_{f}\right) \cdot k_{j}^{\prime}+\left(r_{l}^{\prime}-r_{f}\right) \cdot l_{j}^{\prime}+\left(1+r_{f}\right) \cdot n_{j}\right]  \tag{3.41}\\
\text { s.t. } & \xi \cdot n_{j} \geq \theta \cdot\left(k_{j}^{\prime}+l_{j}^{\prime}\right) \tag{3.42}
\end{align*}
$$

where $\Lambda^{\prime}=\frac{1-\psi+\psi \cdot \xi^{\prime}}{1+r_{f}}$ denotes the bank adjusted discount factor. The first-order conditions with respect to $k_{j}^{\prime}, l_{j}^{\prime}$ and the Kuhn-Tucker condition are given by:

$$
\begin{align*}
\Lambda^{\prime} \cdot\left(r_{k}^{\prime}-r_{f}\right) & =\lambda \cdot \theta,  \tag{3.43}\\
\Lambda^{\prime} \cdot\left(r_{l}^{\prime}-r_{f}\right) & =\lambda \cdot \theta,  \tag{3.44}\\
\lambda \cdot\left(\xi \cdot n_{j}-\theta \cdot\left(k_{j}^{\prime}+l_{j}^{\prime}\right)\right) & =0, \tag{3.45}
\end{align*}
$$

where $\lambda$ denote the multiplier on the incentive constraint. It entails the following nonarbitrage conditions:

$$
\begin{equation*}
r_{k}^{\prime}-r_{f}=r_{l}^{\prime}-r_{f}=\frac{\lambda \cdot \theta}{\Lambda^{\prime}}=\lambda \cdot \theta \cdot\left(\frac{1+r_{f}}{1-\psi+\psi \cdot \xi^{\prime}}\right) \equiv \iota \geq 0 \tag{3.46}
\end{equation*}
$$

where $\iota$ denote the incentive premium. Plugging the conjecture of bank value function and first-order conditions to the objective function yields:

$$
\begin{equation*}
\xi \cdot n_{j}=\lambda \cdot \xi \cdot n_{j}+\Lambda^{\prime} \cdot\left(1+r_{f}\right) \cdot n_{j} . \tag{3.47}
\end{equation*}
$$

It follows that:

$$
\begin{equation*}
\xi=\frac{\Lambda^{\prime} \cdot\left(1+r_{f}\right)}{1-\lambda}=\frac{1-\psi+\psi \cdot \xi^{\prime}}{1-\lambda} \tag{3.48}
\end{equation*}
$$

It confirms our conjecture and indicates that banking leverage ratio dose not depend on bank-specific elements. As a results, banks are symmetric and all subscripts $j$ can be disregarded. If the incentive constraint is binding $(\lambda>0)$, then the banking leverage ratio $L R$ can be derived as:

$$
\begin{equation*}
L R \equiv \frac{\xi}{\theta}=\frac{k_{j}^{\prime}+l_{j}^{\prime}}{n_{j}}=\frac{K^{\prime}+L^{\prime}}{N}, \tag{3.49}
\end{equation*}
$$

where the capital letters denote the aggregate variables of their idiosyncratic counterparts, and the second equality results from the symmetry property. Plugging Equation (3.49) into (3.48) yields:

$$
\begin{equation*}
\lambda=\max \left\{1-\left(\frac{1-\psi+\psi \cdot \xi^{\prime}}{\theta}\right) \cdot\left(\frac{N}{K^{\prime}+L^{\prime}}\right), 0\right\} \tag{3.50}
\end{equation*}
$$

Thus, Proposition 1 has been proved.

## 3.A. 2 Equilibrium Conditions

Given $\lambda^{*}$ and $E^{*}=1$, the equilibrium conditions for aggregate variables are given by:

$$
\begin{align*}
\xi^{*} & =\frac{1-\psi}{1-\lambda^{*}-\psi},  \tag{3.51}\\
\Lambda^{*} & =\frac{1-\psi+\psi \cdot \xi^{*}}{1+r_{f}},  \tag{3.52}\\
L R^{*} & =\frac{\xi^{*}}{\theta},  \tag{3.53}\\
\iota^{*} & =\frac{\lambda^{*} \cdot \theta}{\Lambda^{*}}=r_{k}^{*}-r_{f}=r_{l}^{*}-r_{f},  \tag{3.54}\\
K^{*} & =\left(\frac{\alpha}{r_{k}^{*}+\delta}\right)^{\frac{1}{1-\alpha}} E^{*}=\left(\frac{\alpha}{r_{k}^{*}+\delta}\right)^{\frac{1}{1-\alpha}},  \tag{3.55}\\
w^{*} & =(1-\alpha)\left(\frac{K^{*}}{E^{*}}\right)^{\alpha}=(1-\alpha)\left(K^{*}\right)^{\alpha} . \tag{3.56}
\end{align*}
$$

## 3.B Computation Details

## 3.B. 1 Grid Specifications

| Variable | Symbol | \# of Points | Value / Range |
| :--- | :---: | :---: | :---: |
|  |  |  |  |
| Borrowing | $a<0$ | 101 | $[-6.0,0.0]$ |
| Saving | $a>0$ | 101 | $[0.0,400.0]$ |
| Permanent labor productivity | $e_{1}$ | 2 | $\{-0.448,0.448\}$ |
| Persistent labor productivity | $e_{2}$ | 3 | $\{-0.4851,0.0,0.4851\}$ |
| Transitory labor productivity | $e_{3}$ | 3 | $\{-0.4299,0.0,0.4299\}$ |
| Preference | $\nu$ | 2 | $\{0.0,1.0\}$ |
| Credit history | $h$ | 2 | $\{0.0,1.0\}$ |

Table 3.B.1: Grids Used for Model Computation

I choose the upper and lower bounds for bank assets to ensure that the optimal choices for all states are included. I consider an equally-spaced grid for borrowing of 101 points from -6.0 to 0.0 and an exponentially-spaced grid for saving of 101 points from 0.0 to 400.0. The permanent and transitory components are appoximated with two-point and three-point uniform distributions, respectively. The persistent process is discretized with a three-state Markov chain using Adda and Cooper (2003).

## 3.B. 2 Algorithm for Solving Stationary Equilibrium

1. Set parameters and tolerances for convergence $\varepsilon$.
2. Create grids for ( $a, e_{1}, e_{2}, e_{3}, \nu, h$ ) with lengths $\left(n_{a}, n_{e_{1}}, n_{e_{2}}, n_{e_{3}}, n_{\nu}, n_{h}\right)$.
3. Initializations:
(a) $V^{0}\left(a, e_{1}, e_{2}, e_{3}, \nu, h\right)=0, V^{d=0,0}\left(a, e_{1}, e_{2}, e_{3}, \nu\right)=0$, and $V^{d=1,0}\left(a, e_{1}, e_{2}, e_{3}, \nu\right)=$ 0 for all ( $a, e_{1}, e_{2}, e_{3}, \nu, h$ ). Note that both $V^{d=0,0}$ and $V^{d=1,0}$ do not depend on credit history $h$ as only households with good credit history can default.
(b) $g_{d}^{0}\left(a, e_{1}, e_{2}, e_{3}, \nu\right)=0$ for all $\left(a, e_{1}, e_{2}, e_{3}, \nu\right)$. This implies that zero default premia for all loans, i.e., household borrowers do not default at all.
(c) $R^{0}\left(a^{\prime}, e_{1}, e_{2}\right)=-a^{\prime}$ for all $\left(a^{\prime}, e_{1}, e_{2}\right)$ as households do not default.
(d) $q^{0}\left(a^{\prime}, e_{1}, e_{2}\right)=\frac{\rho}{1+r_{f}}$ for all ( $a^{\prime}, e_{1}, e_{2}$ ). That is, the borrowing prices equal the inverse of the constant risk-free rate, aligned with the no default initialization.
(e) $\mu^{0}\left(a, e_{1}, e_{2}, e_{3}, \nu, h\right)=\frac{1}{n}$ for all ( $\left.a, e_{1}, e_{2}, e_{3}, \nu, h\right)$, where $n \equiv n_{a} \times n_{e_{1}} \times n_{e_{2}} \times$ $n_{e_{3}} \times n_{\nu} \times n_{h}$.
(f) $\lambda^{\min }=0$ and $\lambda^{\max }=1-\sqrt{\psi}$. The latter denotes the upper bound of the incentive multiplier such that the associated incentive premium is positive in equilibrium.
4. Set up the one-loop algorithm for given $\lambda^{*}$ :
(a) Solve for the implied $L R^{*}, \iota^{*}$, and $w^{*}$ according to (3.53), (3.54), and (3.56).
(b) Solve for $V^{1}$ and $g_{d}^{1}$ taking $V^{0}, q^{0}$, and $w^{*}$ as given.
i. Compute $V^{d=0,1}\left(a, e_{1}, e_{2}, e_{3}, \nu\right)$ and $V^{d=1,1}\left(a, e_{1}, e_{2}, e_{3}, \nu\right)$ according to (3.3) and (3.4) for each ( $\left.a, e_{1}, e_{2}, e_{3}, \nu\right)$.
ii. Compute $g_{d}^{1}\left(a, e_{1}, e_{2}, e_{3}, \nu\right)$ according to (3.5) for each ( $a, e_{1}, e_{2}, e_{3}, \nu$ ).
iii. Compute $V^{1}\left(a, e_{1}, e_{2}, e_{3}, \nu, h=0\right)$ according to (3.6) for each ( $a, e_{1}, e_{2}, e_{3}, \nu$ ).
iv. Compute $V^{1}\left(a, e_{1}, e_{2}, e_{3}, \nu, h=1\right)$ according to (3.7) for each $\left(a, e_{1}, e_{2}, e_{3}, \nu\right)$.
(c) Solve for $q^{1}$ taking $V^{1}, g_{d}^{1}$, and $\iota^{*}$ as given.
i. Compute $R^{1}\left(a^{\prime}, e_{1}, e_{2}\right)$ according to (3.23) for each $\left(a^{\prime}, e_{1}, e_{2}\right)$.
ii. Compute $q^{1}\left(a^{\prime}, e_{1}, e_{2}\right)$ according to (3.24) for each ( $a^{\prime}, e_{1}, e_{2}$ ).
(d) Assess convergence of $V$ and $q$.
i. If $\left\|V^{1}-V^{0}\right\|<\varepsilon$ and $\left\|q^{1}-q^{0}\right\|<\varepsilon$, let $V^{*}=V^{1}$ and $q^{*}=q^{1}$ and continue to the next step.
ii. Otherwise, update the initial values for $V$ and $q$ with relaxation and return to step (4b).
(e) Solve for $\mu^{*}$ according to (3.29).
(f) Solve for aggregate variables $E^{*}, K^{*}, L^{*}, D^{*}$, and $N^{*}$.
(g) Compute $\mathcal{E}\left(\lambda^{*}\right)=L R^{*}-\frac{K^{*}+L^{*}}{N^{*}}$.
5. Stationary equilibrium with the occasionally binding incentive constraint:
(a) $\mathcal{E}\left(\lambda^{\mathrm{min}}\right)>0$ implies the incentive constraint is slack and stop.
(b) $\mathcal{E}\left(\lambda^{\max }\right)<0$ implies the incentive constraint cannot be satisfied and stop.
(c) Otherwise, set $\lambda^{L}=\lambda^{\min }$ and $\lambda^{U}=\lambda^{\max }$. Using the standard bisection routine to find $\lambda^{s s} \in\left[\lambda^{L}, \lambda^{U}\right]$ such that $\left|\mathcal{E}\left(\lambda^{s s}\right)\right|<\varepsilon$.
6. Compute aggregate variables of interest.

## 3.B.3 Algorithm for Solving Transition Dynamics

1. Set parameters and tolerances for convergence $\varepsilon$.
2. Compute the initial equilibrium under the old policy $E^{\text {old }}$ and the final equilibrium under the new policy $E^{\text {new }}$.
3. Set $T$ to a sufficiently large number.
4. Initializations:
(a) A bold variable $\boldsymbol{X}$ denote a $T \times 1$ vector and $\boldsymbol{X}_{\boldsymbol{t}}$ refers to the $t$-th element.
(b) $L R^{\mathbf{0}}=\left\{L R^{\text {old }}+t \cdot \frac{L R^{\text {new }}-L R^{\text {old }}}{T}\right\}_{t=1}^{T}$, implying $L R_{\boldsymbol{T}}^{\mathbf{0}}=L R^{\text {new }}$.
(c) $\boldsymbol{V}^{\mathbf{0}}=\left(0, \ldots, 0, V^{\text {new }}\right)$.
(d) $\boldsymbol{q}^{\mathbf{0}}=\left(0, \ldots, q^{\text {new }}, q^{\text {new }}\right)$.
(e) $\boldsymbol{\mu}^{\mathbf{0}}=\left(\mu^{\text {old }}, 0, \ldots, 0, \mu^{\text {new }}\right)$.
5. Given $\boldsymbol{L} \boldsymbol{R}^{\mathbf{0}}$, compute $\boldsymbol{\lambda}^{\mathbf{0}}, \boldsymbol{\iota}^{\mathbf{0}}$, and $\boldsymbol{w}^{\mathbf{0}}$ according to (3.50), (3.46), and (3.11).
6. Given $\boldsymbol{w}^{\mathbf{0}}, \boldsymbol{V}^{\mathbf{0}}$, and $\boldsymbol{q}^{\mathbf{0}}$, solve the household problem backward from $t=T$ to $t=1$ using the one-loop algorithm in Appendix 3.B. 2 to obtain $\boldsymbol{V}^{\mathbf{1}}$ and $\boldsymbol{q}^{\mathbf{1}}$.
7. With the decision rules implied by $V^{1}$, simulate the economy forward from $t=1$ to $t=T$ to obtain $\boldsymbol{\mu}^{\mathbf{1}}$ and compute $\boldsymbol{L} \boldsymbol{R}^{\mathbf{1}}$.
8. If $\left\|L R^{\mathbf{1}}-\boldsymbol{L} \boldsymbol{R}^{\mathbf{0}}\right\|<\varepsilon$, set $\boldsymbol{L} \boldsymbol{R}^{*}=\boldsymbol{L} \boldsymbol{R}^{\mathbf{1}}$ and stop. Otherwise, update the initial values for $\boldsymbol{L} \boldsymbol{R}$ with relaxation and return to step (5).
9. Compute the transition path for each aggregate variable of interest.

## 3.C Transition Paths of Banking Leverage Ratio

All transition paths of banking leverage ratio for the policy counterfactuals considered in the paper are collectively visualized here. The unit of time is a year. Conceptually, when the policy is unanticipated implemented at the beginning of $t=1$, more (less) households unexpectedly file for bankruptcy under a more lenient (stricter) bankruptcy code. This results in a sharp decrease (increase) in banking net worth, thus leading to a salient discrete increased (decreased) banking leverage ratio. Afterwards, banks adjust their portfolios to gradually achieve the new equilibrium. Recall that lower garnishment and shorter exclusion both denote a more lenient bankruptcy regime, while higher garnishment and longer exclusion both denote a stricter rule. Figure 3.1a, 3.C.1a, 3.C.2a, and 3.C.3a show the results for more lenient regimes. Figure 3.1b, 3.C.1b, 3.C.2b, and 3.C.3b instead present the results for stricter regimes.

Figure 3.C.1: Transition Paths of Banking Leverage Ratio


Notes: The unit of time is a year. The policy reform is unexpectedly announced at $t=1$. The banking leverage ratio remains in the old equilibrium at $t=0$ and converges to the new equilibrium at $t=80$. The left figure illustrates the transition from benchmark $\left(\mathbb{P}_{h}=1 / 10\right)$ to shorter exclusion $\left(\mathbb{P}_{h}=1 / 5\right)$. The right figure plots the transition from benchmark $\left(\mathbb{P}_{h}=1 / 10\right)$ to longer exclusion $\left(\mathbb{P}_{h}=1 / 15\right)$.

## 3.D Robustness Check: Degree of Financial Frictions

In the section, I report the results of the wage garnishment counterfactual with different degrees of financial frictions in Section 3.6.4. To be specific, I consider two cases where the fraction $\theta$ of total assets that banks can divert either decreases or increases by $1 \%$ compared to the benchmark calibration. That is, $\theta^{L}=0.99 \times \theta^{B}$ and $\theta^{H}=1.01 \times \theta^{B}$. The equilibrium and welfare results for $\theta^{L}$ are summarized in Table 3.D. 1 and 3.D.2, respectively. The ones for $\theta^{H}$ are presented in Table 3.D. 3 and 3.D.4, respectively.

Figure 3.C.2: Transition Paths of Banking Leverage Ratio with $\theta^{L}$
(a) From Benchmark to Lower Garnishment

(b) From Benchmark to Higher Garnishment


Notes: The unit of time is a year. The policy reform is unexpectedly announced at $t=1$. The banking leverage ratio remains in the old equilibrium at $t=0$ and converges to the new equilibrium at $t=80$. The left figure illustrates the transition from benchmark $(\eta=0.25)$ to lower garnishment $(\eta=0.20)$ with a lower degree of financial frictions $\left(\theta^{L}=0.99 \times \theta^{B}\right)$. The right figure plots the transition from benchmark $(\eta=0.25)$ to higher garnishment $(\eta=0.30)$ with a lower degree of financial frictions $\left(\theta^{L}=0.99 \times \theta^{B}\right)$.

Figure 3.C.3: Transition Paths of Banking Leverage Ratio $\theta^{H}$


Notes: The unit of time is a year. The policy reform is unexpectedly announced at $t=1$. The banking leverage ratio remains in the old equilibrium at $t=0$ and converges to the new equilibrium at $t=80$. The left figure illustrates the transition from benchmark $(\eta=0.25)$ to higher garnishment $(\eta=0.20)$ with a lower degree of financial frictions $\left(\theta^{H}=1.01 \times \theta^{B}\right)$. The right figure plots the transition from benchmark $(\eta=0.25)$ to higher garnishment $(\eta=0.30)$ with a higher degree of financial frictions $\left(\theta^{H}=1.01 \times \theta^{B}\right)$.

| Variable | Lower Garnishment | Benchmark | Higher Garnishment |
| :--- | :---: | :---: | :---: |
| Levels |  |  |  |
|  |  |  |  |
| Consumer credit markets |  |  |  |
| Default rate (\%) | 0.6577 | 0.6082 | 0.4318 |
| Avg. borrowing interest rate (\%) | 14.2709 | 12.1829 | 8.9363 |
| Fraction of HHs in debt (\%) | 6.8215 | 8.6335 | 11.3245 |
| Debt-to-earnings ratio (\%) | 1.2888 | 1.8748 | 2.7475 |
|  |  |  |  |
| Incentive \& divestment channels |  |  |  |
| Banking leverage ratio | 4.8570 | 4.5652 | 4.1251 |
| Incentive premium (\%) | 0.6814 | 0.6264 | 0.4428 |
| Wage | 1.1510 | 1.1538 | 1.1633 |
|  |  |  |  |
| \% change w.r.t. benchmark |  |  |  |
| Incentive \& divestment channels |  | - | -9.6396 |
| Banking leverage ratio | 6.3925 | - | -29.3179 |
| Incentive premium | 8.7732 | 0.8276 |  |
| Wage | -0.2440 |  |  |

Table 3.D.1: Counterfactual of Wage Garnishment with $\theta^{L}$ : Equilibria Comparison
Notes: The upper panel "Levels" reports model moments in levels under the benchmark and the policy experiments of wage garnishment with a lower degree of financial frictions $\left(\theta^{L}=0.99 \times \theta^{B}\right)$. The bottom panel "\% change w.r.t. benchmark" shows the percentage variations of the selective moments related to the incentive and divestment channels under the policy experiments compared to the benchmark.

| Variable (in \%) |  | Lower Garnishment |  | Higher Garnishment |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | HH <br> Proportion | CEV | Favor Reform | CEV | Favor Reform |
| Total | 100.0000 | -0.1238 | 27.3511 | 0.3815 | 99.4962 |
| Good credit history | 94.9490 | -0.1283 | 26.9135 | 0.3865 | 99.5257 |
| Indebted | 9.0928 | -0.3638 | 16.2729 | 0.7035 | 94.7835 |
| Not indebted | 90.9072 | -0.0963 | 27.9777 | 0.3437 | 100.0000 |
| Patient | 98.9653 | -0.1262 | 27.1894 | 0.3846 | 99.5229 |
| Impatient | 1.0347 | -5.4828 | 0.5207 | 5.7872 | 99.7932 |
| Bad credit history | 5.0510 | -0.0438 | 35.5772 | 0.2932 | 98.9430 |

Table 3.D.2: Counterfactual of Wage Garnishment with $\theta^{L}$ : Welfare Implications
Notes: All results are measured when the policy reform is announced. The column "HH Proportion" describes the initial household distribution. The column "CEV" reports the CEV in the percentage of the policy change relative to the benchmark. The column "Favor Reform" reports the fraction of households in favor of the new policy in percentage. The row "Total" shows the aggregate results. The rows "Good credit history"/"Bad credit history" illustrate the results conditional on households with good/bad credit history. The rows "Indebted"/"Not indebted" present the results among households with good credit history who have debts/no debts. The row "Impatient" shows the results conditional on households with good credit history hit by preference shocks.

| Variable | Lower Garnishment | Benchmark | Higher Garnishment |
| :--- | :---: | :---: | :---: |
| Levels |  |  |  |
|  |  |  |  |
| Consumer credit markets |  |  |  |
| Default rate (\%) | 0.6653 | 0.6082 | 0.4320 |
| Avg. borrowing interest rate (\%) | 14.3251 | 12.1829 | 9.0285 |
| Fraction of HHs in debt (\%) | 6.7815 | 8.6335 | 11.2654 |
| Debt-to-earnings ratio (\%) | 1.2832 | 1.8748 | 2.7261 |
|  |  |  |  |
| Incentive \& divestment channels |  |  |  |
| Banking leverage ratio | 4.9349 | 4.5652 | 4.2265 |
| Incentive premium (\%) | 0.7313 | 0.6264 | 0.5320 |
| Wage | 1.1484 | 1.1538 | 1.1587 |
|  |  |  |  |
| \% change w.r.t. benchmark |  |  |  |
| Incentive \& divestment channels |  | - | -7.4182 |
| Banking leverage ratio | 8.0981 | - | -15.0701 |
| Incentive premium | 16.7534 | -.4645 |  |
| Wage |  |  | 0.4230 |

Table 3.D.3: Counterfactual of Wage Garnishment with $\theta^{H}$ : Equilibria Comparison
Notes: The upper panel "Levels" reports model moments in levels under the benchmark and the policy experiments of wage garnishment with a higher degree of financial frictions $\left(\theta^{H}=1.01 \times \theta^{B}\right)$. The bottom panel "\% change w.r.t. benchmark" shows the percentage variations of the selective moments related to the incentive and divestment channels under the policy experiments compared to the benchmark.

| Variable (in \%) |  | Lower Garnishment |  | Higher Garnishment |  |
| ---: | :---: | :---: | :---: | :---: | :---: |
|  | HH Proportion | CEV | Favor Reform | CEV | Favor Reform |
| Total | 100.0000 | -0.2421 | 0.8999 | 0.1790 | 99.4477 |
| Good credit history | 94.9490 | -0.2464 | 0.9478 | 0.1841 | 99.4745 |
| Indebted | 9.0928 | -0.4863 | 10.4233 | 0.4887 | 94.2212 |
| Not indebted | 90.9072 | -0.2138 | 0.0000 | 0.1429 | 100.0000 |
| Patient | 98.9653 | -0.2443 | 0.9522 | 0.1821 | 99.4712 |
| Impatient | 1.0347 | -5.4907 | 0.5207 | 5.7323 | 99.7932 |
|  |  |  |  |  |  |
| Bad credit history | 5.0510 | -0.1654 | 0.0000 | 0.0886 | 98.9430 |

Table 3.D.4: Counterfactual of Wage Garnishment with $\theta^{H}$ : Welfare Implications
Notes: All results are measured when the policy reform is announced. The column "HH Proportion" describes the initial household distribution. The column "CEV" reports the CEV in the percentage of the policy change relative to the benchmark. The column "Favor Reform" reports the fraction of households in favor of the new policy in percentage. The row "Total" shows the aggregate results. The rows "Good credit history"/"Bad credit history" illustrate the results conditional on households with good/bad credit history. The rows "Indebted"/"Not indebted" present the results among households with good credit history who have debts/no debts. The row "Impatient" shows the results conditional on households with good credit history hit by preference shocks.

## Bibliography

Adda, J. and R. W. Cooper (2003): Dynamic Economics: Quantitative Methods and Applications, Cambridge: MIT Press.

Agarwal, S., P. M. Skiba, and J. Tobacman (2009): "Payday Loans and Credit Cards: New Liquidity and Credit Scoring Puzzles?" American Economic Review, 99, 412-17.

Albanesi, S. and J. Nosal (2020): "Insolvency after the 2005 Bankruptcy Reform," NBER Working Paper No. 24934.

Arslan, Y., B. Guler, and B. Kuruscu (2020): "Credit Supply Driven Boom-Bust Cycles," BIS Working Papers No, 885.

Athreya, K. (2002): "Welfare implications of the Bankruptcy Reform Act of 1999," Journal of Monetary Economics, 49, 1567-1595.

Bernanke, B. and M. Gertler (1989): "Agency Costs, Net Worth, and Business Fluctuations," The American Economic Review, 79, 14-31.

Bhutta, N., J. Goldin, and T. Homonoff (2016): "Consumer Borrowing after Payday Loan Bans," Journal of Law and Economics, 59, 225-259.

Bhutta, N., P. M. Skiba, and J. Tobacman (2015): "Payday Loan Choices and Consequences," Journal of Money, Credit and Banking, 47, 223-260.

Bocola, L. (2016): "The Pass-Through of Sovereign Risk," Journal of Political Economy, 124, 879-926.

Campbell, D., F. Martínez-Jerez, and P. Tufano (2012): "Bouncing out of the banking system: An empirical analysis of involuntary bank account closures," Journal of Banking and Finance, 36(4), 1224-1235.

Carrell, S. and J. Zinman (2014): "In Harm's Way? Payday Loan Access and Military Personnel Performance," Review of Financial Studies, 27, 2805-2840.

Carter, S. P., P. M. Skiba, and J. Tobacman (2011): Chapter 8: Pecuniary Mistakes? Payday Borrowing by Credit Union Members, Financial Literacy: Implications for Retirement Security and the Financial Marketplace, United Kingdom: Oxford University Press.

Chatterjee, S., D. Corbae, K. P. Dempsey, and J.-V. Ríos-Rull (2020): "A Quantitative Theory of the Credit Score," NBER Working Paper No. 27671.

Chatterjee, S., D. Corbae, M. Nakajima, and J.-V. Ríos-Rull (2007): "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default," Econometrica, 75, 1525-1589.

Chatterjee, S. and G. Gordon (2012): "Dealing with consumer default: Bankruptcy vs garnishment," Journal of Monetary Economics, 59, S1-S16.

Chen, D. and D. Corbae (2011): "On the welfare implications of restricting bankruptcy information," Journal of Macroeconomics, 33, 4-13.

Chen, D. and J. Zhao (2017): "The impact of personal bankruptcy on labor supply decisions," Review of Economic Dynamics, 26, 40-61.

Consumer Federation of America (2021): "Legal Status of Payday Loans by State," Available at: https://paydayloaninfo.org/state-information/ (Accessed: 04.11.2021).

Consumer Financial Protection Bureau (2011): "Building the CFPB: a progress report," Available at: https://www.consumerfinance.gov/wp-content/uploads/2 011/07/Report_BuildingTheCfpb1.pdf (Accessed: 24.07.2022).

- (2017): "I heard that taking out a payday loan can help rebuild my credit or improve my credit score. Is this true?" Available at: https://www.consumerfinanc e.gov/ask-cfpb/i-heard-that-taking-out-a-payday-loan-can-help-rebuil d-my-credit-or-improve-my-credit-score-is-this-true-en-1611/ (Accessed: 04.11.2021).

Dempsey, K. and F. Ionescu (2022): "Lending Standards and Borrowing Premia in Unsecured Credit Markets," Unpublished Manuscript, Ohio State University.

Elliehausen, G. and E. C. Lawrence (2001): "Payday Advance Credit in America: An Analysis of Customer Demand," Unpublished Manuscript, Georgetown University.

Exler, F. (2019): "Personal Bankruptcy and Wage Garnishment," Unpublished Manuscript, University of Vienna.

- (2020): "Regulating Small Dollar Loans: The Role of Delinquency," Unpublished Manuscript, University of Vienna.

Exler, F., I. Livshits, J. MacGee, and M. Tertilt (2020): "Consumer Credit with Over-Optimistic Borrowers," CEPR Discussion Paper No. DP15570.

Exler, F. and M. Tertilt (2020): "Consumer Debt and Default: A Macro Perspective," Oxford Research Encyclopedia of Economics and Finance.

Fieldhouse, D., I. Livshits, and J. MacGee (2016): "Aggregate Fluctuations, Consumer Credit and Bankruptcy," Unpublished Manuscript, Western University.

Flannery, M. and K. Samolyk (2005): "Payday Lending: Do the Costs Justify the Price?" FDIC Center for Financial Research Working Paper No. 2005/09.

Floden, M. and J. Lindé (2001): "Idiosyncratic Risk in the United States and Sweden: Is There a Role for Government Insurance?" Review of Economic Dynamics, 4(2), 406437.

Gertler, M. and P. Karadi (2011): "A model of unconventional monetary policy," Journal of Monetary Economics, 58, 17-34.

Gertler, M. and N. Kiyotaki (2010): "Chapter 11 - Financial Intermediation and Credit Policy in Business Cycle Analysis," in Handbook of Monetary Economics, ed. by B. M. Friedman and M. Woodford, Elsevier, vol. 3, 547-599.

Gordon, G. (2015): "Evaluating default policy: The business cycle matters," Quantitative Economics, 6, 795-823.

Hatchondo, J. C., L. Martinez, and H. Sapriza (2010): "Quantitative properties of sovereign default models: Solution methods matter," Review of Economic Dynamics, 13, 919-933.

Herkenhoff, K. F. (2019): "The Impact of Consumer Credit Access on Unemployment," The Review of Economic Studies, 86, 2605-2642.

Herkenhoff, K. F., G. M. Phillips, and E. Cohen-Cole (2021): "The impact of consumer credit access on self-employment and entrepreneurship," Journal of Financial Economics, 141, 345-371.

Iskhakov, F., T. H. Jørgensen, J. Rust, and B. Schjerning (2017): "The endogenous grid method for discrete-continuous dynamic choice models with (or without) taste shocks," Quantitative Economics, 8, 317-365.

Karger, H. (2005): Shortchanged: Life and Debt in the Fringe Economy, San Francisco: Berrett-Koehler.

Kim, K. T. and J. Lee (2018): "Financial literacy and use of payday loans in the United States," Applied Economics Letters, 25, 781-784.

Lee, S., R. Luetticke, and M. Ravn (2020): "Financial Frictions: Macro vs Micro Volatility," CEPR Discussion Paper No. DP15133.

Li, W. and P.-D. Sarte (2006): "U.S. consumer bankruptcy choice: The importance of general equilibrium effects," Journal of Monetary Economics, 53, 613-631.

Livshits, I. (2015): "Recent Developments in Consumer Credit and Default Literature," Journal of Economic Surveys, 29, 594-613.

Livshits, I., J. MacGee, and M. Tertilt (2007): "Consumer Bankruptcy: A Fresh Start," American Economic Review, 97, 402-418.

Lucas, R. E. (1987): Models of business cycles, vol. 26, Oxford: Basil Blackwell.
Melzer, B. T. (2011): "The Real Costs of Credit Access: Evidence from the Payday Lending Market," Quarterly Journal of Economics, 126(1), 517-555.

Montezemolo, S. and S. Wolff (2015): "Payday Mayday: Visible and Invisible Payday Lending Defaults," Research Report, Center for Responsible Lending.

Morgan, D. P., M. R. Strain, and I. Seblani (2012): "How Payday Credit Access Affects Overdrafts and Other Outcomes," Journal of Money, Credit and Banking, 44(23), 519-531.

Morse, A. (2011): "Payday lenders: Heroes or villains?" Journal of Financial Economics, 102, 28-44.

Mukoyama, T. (2010): "Welfare effects of unanticipated policy changes with complete asset markets," Economics Letters, 109, 134-138.

Nakajima, M. (2012): "Rising indebtedness and temptation: A welfare analysis," Quantitative Economics, 3, 257-288.

- (2017): "Assessing bankruptcy reform in a model with temptation and equilibrium default," Journal of Public Economics, 145, 42-64.

Nakajima, M. and J.-V. Ríos-Rull (2014): "Credit, Bankruptcy, and Aggregate Fluctuations," NBER Working Paper No. 20617.

Rust, J. (1987): "Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher," Econometrica, 55, 999-1033.

Saldain, J. (2021): "A Quantitative Model of High-Cost Consumer Credit," Unpublished Manuscript, University of Virginia.

Servon, L. (2017): The Unbanking of America: How the New Middle Class Survives, New York: Mariner Books.

Skiba, P. M. and J. Tobacman (2018): "Payday Loans, Uncertainty, and Discounting: Explaining Patterns of Borrowing, Repayment, and Default," Unpublished Manuscript, University of Delaware.

- (2019): "Do Payday Loans Cause Bankruptcy?" Journal of Law and Economics, 62(3), 485-519.

Stegman, M. A. (2007): "Payday Lending," Journal of Economic Perspectives, 21, 169-190.

Storesletten, K., C. Telmer, and A. Yaron (2004): "Cyclical Dynamics in Idiosyncratic Labor Market Risk," Journal of Political Economy, 112, 695-717.

Sun, J. (2022): "The Role of Marital Status for the Evaluation of Bankruptcy Regimes," Unpublished Manuscript, University of Mannheim.

Zame, W. R. (1993): "Efficiency and the Role of Default When Security Markets are Incomplete," American Economic Review, 83, 1142-1164.

Zinman, J. (2010): "Restricting Consumer Credit Access: Household Survey Evidence on Effects around the Oregon Rate Cap," Journal of Banking and Finance, 34(3), 546--556 .

## Eidesstattliche Erklärung

Hiermit erkläre ich, die vorliegende Dissertation selbstständig angefertigt und mich keiner anderen als der in ihr angegebenen Hilfsmittel bedient zu haben. Insbesondere sind sämtliche Zitate aus anderen Quellen als solche gekennzeichnet und mit Quellenangaben versehen.

Florence (Italy), 02.11.2022

Tsung-Hsien Li

## Curriculum Vitae

2017-2022 University of Mannheim (Germany)<br>Ph.D. in Economics<br>Spring 2020 Yale University (USA)<br>Visiting Ph.D. Student<br>2012-2015 National Taiwan University (Taiwan)<br>M.A. in Economics<br>2007-2012 National Taiwan University (Taiwan)<br>B.A. in Economics<br>B.S.S. in Bio-Industry Communication and Development


[^0]:    ${ }^{1}$ A credit score is a statistic computed by credit bureaus to measure a person's creditworthiness or default risk. In the U.S., people do care about their scores because they affect their credit access, mortgage rates, and even job application prospects.
    ${ }^{2}$ For example, Consumer Financial Protection Bureau (2017) states that payday lenders in the U.S. usually do not report to credit bureaus. Bhutta, Skiba, and Tobacman (2015) also empirically show that payday loan borrowing has no impact on credit scores.

[^1]:    ${ }^{3}$ The focus of the paper is narrowly the comparison between credit card and payday loan borrowers. See, for example, Livshits (2015) and Exler and Tertilt (2020) for surveys on the general topics.
    ${ }^{4}$ I focus on credit cards issued by banks only and exclude those cards for specific purposes, such as store and fuel cards.

[^2]:    ${ }^{5}$ The age restriction excludes retirement and childhood for the comparison purpose since many papers in the consumer finance literature focus on the working life of households, e.g., Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) and Livshits, MacGee, and Tertilt (2007).

[^3]:    ${ }^{6}$ The average ages of credit card and payday loan borrowers are 49 and 45 , respectively.

[^4]:    ${ }^{7}$ Total assets include financial assets (e.g., liquid assets, certificates of deposit, saving bonds) and nonfinancial assets (such as vehicles). Total liabilities contain mortgages, home equity loans, credit card debts, and other debts. Refer to the SCF Bulletin for details.

[^5]:    8 The other approach is to check the education distribution for each type of borrower, averaged across all survey years. For credit card borrowers, (no high school, high school, some college, and college degree) $=(7.63 \%, 27.23 \%, 31.40 \%, 33.74 \%)$. For payday loan borrowers, (no high school, high school, some college, and college degree $)=(14.60 \%, 31.92 \%, 40.04 \%, 13.44 \%)$.

[^6]:    9 The result is aligned with Kim and Lee (2018). They use the National Financial Capability Study to explore the relationship between payday loan usage and financial literacy and find that they are negatively associated.

[^7]:    ${ }^{1}$ A payday loan is a short-term unsecured loan with a duration of a few weeks for a typically small amount of around $\$ 300$. In the SCF 2010, around $5 \%$ of households used payday loans in the previous year. About $60 \%$ of payday loan borrowers possess credit cards. See, for example, Elliehausen and Lawrence (2001).

[^8]:    ${ }^{2}$ Servon was interviewing Tim Ranney, a payday lender, and Ranny was sharing a conversation he had with a risk manager at one of the largest credit card issuers in the U.S.
    ${ }^{3}$ The most well-known credit score in the U.S. is the FICO score, $35 \%$ of which is determined by the payment history and $30 \%$ by the debt burden.
    ${ }^{4}$ In line with our hypothesis, Bhutta et al. (2015) empirically document that payday borrowings has no impact on credit scores.
    ${ }^{5}$ Chatterjee et al. (2020) show that there exists a mapping from type scoring economy to credit scoring economy under some sufficient conditions.
    ${ }^{6}$ This is modeled in line with Chapter 7 bankruptcy in the U.S. which entails the liquidation of nonexempt assets in return for debt dischargement.

[^9]:    7 As mentioned previously, the unaccounted $60 \%$ of the puzzle occurrence could be potentially explained by other behavioral explanations.

[^10]:    ${ }^{8}$ For example, 16 states and the District of Columbia in the U.S. either prohibit payday loans or impose limits, while 23 states allow payday lending (Consumer Federation of America, 2021).
    ${ }^{9}$ For example, Morse (2011) uses natural disasters to identify a causal, positive relationship between welfare and access to payday loans. In other words, banning payday loans results in a welfare loss.
    ${ }^{10}$ The 2005 BAPCPA was the most significant reform of bankruptcy law in recent years. Among other changes, it significantly increased the total out-of-pocket filing costs. See also Albanesi and Nosal (2020).

[^11]:    ${ }^{11}$ Some papers extend the standard consumer default framework by incorporating behavioral components. For example, Nakajima (2017) considers households with temptation and analyzes the welfare implications of the 2005 BAPCPA. Exler, Livshits, MacGee, and Tertilt (2020) introduce over-optimism of households about future income. See also Exler and Tertilt (2020) for a complete survey.

[^12]:    ${ }^{12}$ Note that, compared to most papers in the consumer finance literature, there is no long-term exogenous exclusion imposed in our model.
    ${ }^{13}$ In principle, it is also possible to assume that banks form a joint score over type and payday loan choices $s(\beta, p)$ for each household.

[^13]:    ${ }^{14}$ To be precise, $s^{\prime}$ will be randomly assigned to one of the two nearest points between which $s^{\prime}$ lies, with probabilities inversely proportional to the relative distance of $s^{\prime}$ to the respective grid points. This assignment is captured by the function $Q^{s}\left(s^{\prime} \mid \psi\right)$.
    ${ }^{15}$ In principle, payday lenders can form another "type score" using their richer information set compared to banks. This simplifying assumption is meant to keep computation numerically tractable. Nonetheless, payday lenders can still better predict the repayment probability than banks in our economy.

[^14]:    ${ }^{16}$ Note that the noise of extreme value shocks is not the reason why our model is able to generate the payday loan puzzle. In fact, we control for it while identifying the puzzle. Refer to Section 2.6.1 for details.

[^15]:    ${ }^{17}$ There are two technical assumptions. First, we assume for computational reasons that households can only take out payday loans if they also borrow in the banking sector. Second, we assume that default is restricted to households who have debts larger than the respective monetary bankruptcy costs. For example, formal default is feasible only if $b+p<-\kappa_{F D}$.
    ${ }^{18}$ See, for example, Rust (1987).

[^16]:     for all $\omega_{b}$ and ( $\left.\tilde{d}, b^{\prime}\right)$.

[^17]:    ${ }^{20}$ This assumption can be justified by: (1) there are more payday loan storefronts than McDonald's and Starbucks combined in the U.S (Karger, 2005); (2) Flannery and Samolyk (2005) find that the annual interest rates of payday loans can be accounted by significant fixed operating costs and higher default premia.
    ${ }^{21}$ One possible justification is that developing a separate type score technology is too expensive for payday lenders.

[^18]:    ${ }^{22}$ A similar algorithm is implemented by Hatchondo, Martinez, and Sapriza (2010).
    ${ }^{23} \$ 638$ earnings per week $\times 52$ weeks $=\$ 33,176$.
    ${ }^{24}$ To determine discount factors, Chatterjee et al. (2020) use an affine approximation using the modelgenerated data to match the means and standard deviations of credit rankings across ages. Our calibrated model can match these moments fairly well.
    ${ }^{25} \mu_{H}$ denote the share of patient households. Solving $\mu_{H}=$ $\rho\left[\left(1-Q^{\beta}\left(\beta_{L} \mid \beta_{H}\right)\right) \mu_{H}+Q^{\beta}\left(\beta_{H} \mid \beta_{L}\right)\left(1-\mu_{H}\right)\right]+(1-\rho) G_{\beta_{H}}$ yields that there are $41 \%$ of patient and $59 \%$ of impatient households in equilibrium.

[^19]:    ${ }^{26}$ This value is comparable the those used in Chatterjee et al. (2020). To rule out the contribution of extreme value shocks to the payday loan puzzle, we check whether households are making such a seeming pecuniary mistake with higher values. See Section 2.6.1.
    ${ }^{27}$ The values for formal and payday stigma costs correspond to $2.18 \%$ and $0.7 \%$ of consumption loss on

[^20]:    ${ }^{32}$ As we discussed in Section 2.3.3, banks will instead use type scores and the conditional distribution of payday loans given observed variables.
    ${ }^{33}$ There is only pooling across types for payday lenders since they can observe a household's payday loan choice. In this section, we will focus on pooling and cross-subsidization in the bank lending market.

[^21]:    ${ }^{34}$ Recall that, in a model with utility shocks, any feasible action will be chosen with positive probability (not just the choice with the highest value). As a result, households might take up payday loans because mainly of such shocks. To control for this nuisance, we additionally check whether households are conscious of making this decision with higher values. To be specific, for each state $\left(\beta, z, \omega_{b}, p\right)$, the feasible borrowing choices with repayment $\left(R, b^{\prime}<0, p^{\prime}<0\right) \in \mathcal{F}\left(z, \omega_{b}, p\right)$ where the value of borrowing

[^22]:    ${ }^{35}$ The rate of puzzle occurrence among impatient households is $25.55 \%$ and among patient ones is $28.31 \%$. The unconditional fraction of puzzling households is $2.28 \%$ in aggregate, $1.5 \%$ among impatient households, and $0.78 \%$ among patient ones.
    ${ }^{36}$ Note that cheaper costs for payday default than formal default are not the main factor with which our calibrated model can generate the payday loan puzzle. Refer to Appendix 2.C for details.
    ${ }^{37}$ We can even match the distribution of these costs rather well, apart from the bins of $\$ 201-\$ 300$ and \$300-\$500.

[^23]:    ${ }^{38}$ To be precise, the relative gain in posteriors for given bank-observable states $\omega_{b}$ is computed as: $\left(\psi_{\beta_{H}^{\prime}}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)-\psi_{\beta_{H}^{\prime}}^{\left(N F D, \hat{b}^{\prime}\right)}\left(\omega_{b}\right)\right) / \psi_{\beta_{H}^{\prime}}^{\left(N F D, \hat{b}^{\prime}\right)}\left(\omega_{b}\right) \times 100$ where $\psi_{\beta_{H}^{\prime}}^{\left(N F D, b^{\prime}\right)}\left(\omega_{b}\right)$ and $\psi_{\beta_{H}^{\prime}}^{\left(N F D, \hat{b}^{\prime}\right)}$ denote the updated type scores for borrowing a bank loan of $b^{\prime}$ and for borrowing a mixture of bank and payday loans $\hat{b}^{\prime}=b^{\prime}+p^{\prime}$.
    ${ }^{39}$ The hump shape results from the fact that prior dominates in the type score updating at both ends (i.e., when banks believe a household to be a certain type).
    ${ }^{40}$ If we express these monetary costs in percentage points relative to the counterfactual, the resulting plot also exhibits a hump-shaped pattern.
    ${ }^{41}$ See Appendix 2.D for more general results.

[^24]:    ${ }^{42}$ Note that households barely change their types even though types are assumed to be stochastic for the technical reason. Given our calibration, the average life expectancy of 40 years is two times smaller than the average type-switching period of around 80 years. Refer to Section 2.4 for details.
    ${ }^{43} \$ 300$ is the average payday loan size in the data.
    ${ }^{44}$ The monetary filing cost stays the same as in the benchmark economy.

[^25]:    ${ }^{45}$ Note that our framework measures the lower bound of the welfare effects of type scores since, in practice, individuals with higher credit scores have better mortgage terms and labor market outcomes, both of which are not considered in our model.

[^26]:    ${ }^{46}$ This explanation refers to the insurance-efficiency trade-off of a bankruptcy regime between smoothing over time and smoothing across states (Zame, 1993).
    ${ }^{47}$ This argument is also valid across types. As shown in Table 2.4, the average payday interest rates are far higher than the ones for bank loans for both types.

[^27]:    ${ }^{1}$ The Survey of Consumer Finances (SCF) and the Federal Board of Governors G. 19 series in 2019.
    ${ }^{2}$ The trade-off between the two forces for a bankruptcy regime has been coined the insurance-efficiency trade-off in the default literature (Zame, 1993).

[^28]:    ${ }^{3}$ In the following, I will use the terms financial frictions and the GK-type frictions interchangeably.

[^29]:    ${ }^{4}$ The assumption that firms cannot default is meant to keep the model tractable and focus on consumer default. In practice, firms can default under Chapter 11, for example.
    ${ }^{5}$ Lee et al. (2020) also introduce the GK-type frictions into a heterogeneous agent model. However, endogenous consumer default risk is absent under their framework.

[^30]:    ${ }^{6}$ This consideration is important because households are infinite-lived and have different initial states when confronting the policy reform. As a result, the welfare effects are often heterogeneous across households. Refer to Section 3.6 for details.
    ${ }^{7}$ To be specific, the effective disposable income is defined as the sum of wage earnings and either savings revenues or loan payments. Under a model where households face significant expenditure risks, a more lenient bankruptcy rule is beneficial in terms of welfare, e.g., see Livshits et al. (2007).
    ${ }^{8}$ Preference shocks are i.i.d. and they are $8.6 \%$ of households who are indebted in equilibrium.

[^31]:    ${ }^{9}$ If necessary, banks can either borrow or save at $r_{f}$ in the international financial markets to balance their domestic positions.

[^32]:    ${ }^{10}$ The extreme value shocks can help with numerical convergence when there are discrete choice variables. See, for example, Iskhakov, Jørgensen, Rust, and Schjerning (2017).

[^33]:    ${ }^{11}$ In particular, banks can sell their claims on firm and household lending in international secondary frictionless markets. Creditor can then recover a fraction $(1-\theta)$ of total assets through a judicial process.
    ${ }^{12}$ Symmetry means that all banks choose the same leverage ratio and, as a result, their asset positions are proportional to their accumulated net worth.

[^34]:    ${ }^{13}$ To be specific, banking net worth is defined as the difference between total assets and liabilities.

[^35]:    ${ }^{14}$ This results from the optimal banking behavior because the expected returns on either asset in equilibrium must be identical; otherwise, banks can make profits by shifting funding to the asset with a higher rate of return, i.e., the no-arbitrage conditions.

[^36]:    ${ }^{15} \mathrm{GDP}$ moves in lockstep with wages because of the assumptions of homothetic production technology and inelastic labor supply.

[^37]:    ${ }^{16}$ The goal of these exercises is to explore the marginal effects of financial frictions, and such effects emerge only with a binding incentive constraint. Therefore, I focus on small variations in the two parameters where the constraint binds in equilibrium.
    ${ }^{17}$ To be specific, $\psi=1-\frac{1}{\text { average banking planning horizon }}$.

[^38]:    ${ }^{18}$ Solving the transition dynamics is not trivial in my model with financial frictions because a policy change prompts banks to adjust their leverage ratios over time to the new equilibrium level. This process takes time and affects aggregate prices, including the incentive premium and wages.

[^39]:    ${ }^{19}$ This consumption-based welfare measure is standard in the literature of business cycles dating back to Lucas (1987). See, for example, Mukoyama (2010) for applications under heterogeneous agent frameworks.

[^40]:    ${ }^{20}$ Recall that the banking leverage ratio is computed as the ratio of total assets to banking net worth. Therefore, a higher leverage ratio means that banks are more leveraged with external funding, i.e., deposits from household savers, and not financed by their internally accumulated net worth.

[^41]:    ${ }^{21}$ To be precise, the welfare effects refer to the welfare variations under policy counterfactuals relative to the respective benchmark, either with or without financial frictions.

[^42]:    ${ }^{22}$ The policy experiment of borrowing exclusion is omitted here because it generates the similar qualitative results as wage garnishment, e.g., see Section 3.6.2 and 3.6.3. $\theta$ and $\psi$ also deliver qualitatively comparable results as displayed in Section 3.5.2, so the latter is omitted here.

[^43]:    ${ }^{23}$ The divestment channel refers to firms reducing investment because of higher borrowing costs. Lower investments lead to less production and wages. The reason wages are emphasized here is because the focus is on understanding the effects of financial frictions on household welfare. From the perspective of households, they care about only their consumption which is determined by their wage earnings and borrowing capacity from banks. As a result, they would prefer higher wages and lower borrowing costs.

