Non–Technical Summary

In empirical studies on the dynamics of firm investment behaviour it is usually assumed that prices of factor inputs, i.e. wages and prices of investment goods, are independent of the firms' level of demand, whereas prices of output goods are assumed to depend on the firms' level of supply.

In this paper the standard model usually applied for the empirical analysis of firm investment behaviour, which is known as the Euler equation model, is extended for imperfectly competitive structures on the factor markets. Therefore, prices depend on the level of factor demand. Although economically reasonable, for technical reasons the resulting investment equation cannot be econometrically estimated. However, it is shown that proceeding in the usual way, assuming wages and prices of investment goods to be given for individual firms, may end in misleading results.

Interpreting Estimation Results of Euler Equation Investment Models when Factor Markets are Imperfectly Competitive

by

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Abstract: In this paper the standard Euler equation investment model with imperfectly competitive product markets is extended for imperfectly competitive structures on the factor markets: labour markets and markets for investment goods. This extension leads to two additional explanatory variables in the Euler equation. Although economically reasonable, the resulting equation for a simple reason cannot be estimated: parts of the explanatory variables are perfectly collinear. For estimation purposes at least one of these variables has to be neglected. Neglecting one of the additional variables, the coefficients to be estimated have to be interpreted as linear combinations of the 'true' coefficients. The differences between the 'true' coefficients and the linear combinations are numerically demonstrated.

Keywords: Firm Investment Behaviour, Euler Equation Model

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1 Introduction

The Euler equation model introduced by Bond and Meghir (1994) has become a standard tool in modern empirical analysis of firm investment behaviour (see the surveys of Blundell, Bond, and Meghir, 1996 and Chirinko, 1993). Different model specifications have been more or less successfully estimated at the level of the firm, industry or economy. Most of these empirical studies assume imperfectly competitive product markets. Compared to the standard model, this assumption implies an additional explanatory variable: the firm's real output in relation to its real capital stock. However, all of the existing studies assume perfectly competitive factor markets. That means, prices of input factors, like wages or prices of investment goods, are independent of the level of input.

In this paper, we allow for imperfectly competitive structures on all markets: product markets, labour markets, and markets for investment goods. Lagrangian arguments are used to solve the intertemporal optimization problem to the Euler equation of investment behaviour, as advocated by Chow (1992, 1993). Adopting the standard procedure of Bond and Meghir (1994) to transform the stochastic Euler equation into an equation linear in observables, this results in two more additional explanatory variables: real labour costs and real user costs of capital, both in relation to the firm's real capital stock. Although economically reasonable, the resulting equation for a simple reason cannot be estimated: parts of the explanatory variables are perfectly collinear. For estimation purposes at least one explanatory variable has to be neglected. Neglecting one of the additional variables, the coefficients to be estimated have to be interpreted as linear combinations of the coefficients of the 'true' model. The differences between the 'true' coefficients and the linear combinations are numerically demonstrated.

2 The Model

The firm is choosing the inputs to the production process at any time period t so as to maximize the expected present value of future dividend flows. Since the profits can either be distributed to the share holders or be used in order to finance investments, the sum of dividends is the difference between profits and investment spending. The profits are the difference between sales, more precisely the firm's net value added, and labour costs. This leads to the maximization of the expected present value

$$V_t \;=\; \mathrm{E}_t \left\{ \sum_{s=t}^{T} (1+r)^{-(s-t)} \left[\mathrm{p}_s(Q_s) Q_s - \mathrm{w}_s(L_s) L_s - \mathrm{c}_s(I_s) I_s \right] \right\} \qquad (t = 1, \ldots, T), \tag{1}$$

where Q_s indicates the firm's output, L_s the amount of hired labour, I_s gross investment spending in fixed capital, and p_s, w_s, c_s the prices of output goods, labour, and investment goods respectively.

The output price p_s is allowed to depend on the firm's output Q_s due to imperfectly competitive product markets. For similar reasons, the price of labour w_s depends on the amount of hired labour L_s and the price of investment goods c_s on the level of gross investment spending I_s. Decisions are made conditional on information available at time t, which is denoted by the index of the expectations operator $\mathrm{E}_t\{\cdot\}$. The discount factor $(1+r)^{-1}$ is assumed to be time invariant for simplicity.

The output Q_s depends on the capital stock K_s, the amount of hired labour L_s, and the current gross investment spending I_s according to a linear homogeneous production function F and a linear homogeneous convex adjustment cost function G:

$$Q_s \;=\; \mathrm{F}(K_s, L_s) - \mathrm{G}(K_s, I_s) \qquad (s = t, \ldots, T). \tag{2}$$

Following SUMMERS (1981), the adjustment cost function G is assumed to be quadratic in the rate of investment:

$$\mathrm{G}(K_s, I_s) \;=\; \frac{b}{2} \left(\frac{I_s}{K_s} - a \right)^2 K_s \qquad (s = t, \ldots, T), \tag{3}$$

where a, b are parameters.

The capital stock K_s develops according to the transition equation and the initial condition

$$K_s \;=\; (1-\delta) K_{s-1} + I_s \qquad (s = t, \ldots, T) \tag{4}$$

$$K_s \;=\; \bar{K} \qquad (s = t-1) \tag{5}$$

with δ as the rate of physical depreciation, which is again assumed to be time invariant for simplicity.

The literature presents different approaches to the mathematical solution of the optimization problem. For example BOND AND MEGHIR (1994) use Bellman's principle. In this paper we use Lagrangian arguments, as advocated by CHOW (1992, 1993). The Lagrangian for a given time period t is given by

$$\begin{aligned}\mathcal{L}_t[L_s, I_s, K_s, \lambda_s] \;&=\; \mathrm{E}_t\left\{(1+r)^{-(s-t)}\sum_{s=t}^{T}\Big(p_s(Q_s)Q_s - w_s(L_s)L_s - c_s(I_s)I_s\right.\\ &\qquad\left. +\lambda_s\Big[(1-\delta)K_{s-1} + I_s - K_s\Big]\Big)\right\}, \qquad (6)\end{aligned}$$

where Q_s stands for $\mathrm{F}(K_s, L_s) - \mathrm{G}(K_s, I_s)$ and λ_s for the Lagrangian parameter. Assuming regularity conditions that allow for the interchange of taking expectations and partial derivation, the necessary conditions for a maximum are derived from setting to zero the partial derivatives of the Lagrangian with respect to its arguments:

$$\begin{aligned}
\mathcal{L}_{t,L} &\propto \mathrm{E}_t\left\{(1+\theta_s^Q)p_s\mathrm{F}_L(K_s, L_s) - (1+\theta_s^L)w_s\right\} = 0 \quad (s = t, \ldots, T) && (7)\\
\mathcal{L}_{t,I} &\propto \mathrm{E}_t\left\{-(1+\theta_s^Q)p_s\mathrm{G}_I(K_s, I_s) - (1+\theta_s^I)c_s + \lambda_s\right\} = 0 && (8)\\
&\qquad (s = t, \ldots, T)\\
\mathcal{L}_{t,K} &\propto \mathrm{E}_t\left\{(1+\theta_s^Q)p_s[\mathrm{F}_K(K_s, L_s) - \mathrm{G}_K(K_s, I_s)] + \frac{1-\delta}{1+r}\lambda_{s+1} - \lambda_s\right\} = 0 && (9)\\
&\qquad (s = t, \ldots, T-1)\\
\mathcal{L}_{t,K} &\propto \mathrm{E}_t\left\{(1+\theta_s^Q)p_s[\mathrm{F}_K(K_s, L_s) - \mathrm{G}_K(K_s, I_s)] - \lambda_s\right\} \quad (s = T) && (10)\\
\mathcal{L}_{t,\lambda} &\propto \mathrm{E}_t\left\{(1-\delta)K_{s-1} + I_s - K_s\right\} \quad (s = t, \ldots, T). && (11)
\end{aligned}$$

Subscripts others than t and s indicate partial derivatives with respect to the subscript. $\theta_s^Q = 1/\eta_s^Q$ represents the inverse of the price elasticity of demand for output goods, $\theta_s^L = 1/\eta_s^L$ and $\theta_s^I = 1/\eta_s^I$ the inverses of the price elasticities of supply for labour and investment goods. Equation (7) is the marginal condition for the sequence of hired labour L_s, equation (8) for the sequence of gross investment I_s, equations (9) and (10) for the sequence of capital stocks K_s. Equation (11) is the marginal condition for the sequence of Lagrangian parameters λ_s.

The Lagrangian method leaves us with a sequence of marginal conditions for every decision time. Since the decisions are reoptimized at each time period t, only the first elements of these sequences remain effective. Thus, the evaluation of equation (9) at $s = t$ provides a difference equation for the shadow price of the capital stock. If we further assume that all variables dated on time t are elements of the information set, we obtain:

$$\begin{aligned} &(1+\theta_t^Q)p_t\Big[\mathrm{F}_K(K_t, L_t) - \mathrm{G}_K(K_t, I_t)\Big] - \lambda_t \\ &= \; - \mathrm{E}_t\left\{\frac{1-\delta}{1+r}\lambda_{t+1}\right\} \qquad (t = 1, \ldots, T-1). \end{aligned} \tag{12}$$

A terminal condition at $t = T$ can be similarly derived from equation (10). Evaluating equation (8) at $s = t$ and $s = t + 1$ and substituting λ_{it} and the expectation of $\lambda_{i,t+1}$ in (12), we obtain the Euler equation of intertemporal investment behaviour

$$\begin{aligned} &(1+\theta_t^Q)p_t\Big[\mathrm{F}_K(K_t, L_t) - \mathrm{G}_K(K_t, I_t) - \mathrm{G}_I(K_t, I_t)\Big] - (1+\theta_t^I)c_t \\ &= \; - \mathrm{E}_t\left\{\frac{1-\delta}{1+r}\Big[(1+\theta_{t+1}^Q)p_{t+1}\mathrm{G}_I(K_{t+1}, I_{t+1}) + (1+\theta_{t+1}^I)c_{t+1}\Big]\right\} \\ &\qquad (t = 1, \ldots, T-1). \end{aligned} \tag{13}$$

To obtain an investment equation linear in observables we adopt the procedure of Bond and Meghir (1994). Assuming rational expectations, remembering the linear homogeneity properties of the production function and the adjustment cost function:

$$\begin{aligned} \mathrm{F}_K(K_t, L_t) &= \frac{\mathrm{F}(K_t, L_t) - \mathrm{F}_L(K_t, L_t)L_t}{K_t} \\ \mathrm{G}_K(K_t, I_t) &= \frac{\mathrm{G}(K_t, I_t) - \mathrm{G}_I(K_t, I_t)I_t}{K_t}, \end{aligned}$$

and substituting the marginal condition for the amount of hired labour (7), we obtain

$$\begin{aligned} &(1+\theta_t^Q)p_t\frac{Q_t}{K_t} - (1+\theta_t^L)w_t\frac{L_t}{K_t} - \left[(1+\theta_t^Q)p_t\left(1-\frac{I_t}{K_t}\right)\mathrm{G}_I(K_t, I_t) + (1+\theta_t^I)c_t\right] \\ &= \; - \frac{1-\delta}{1+r}\Big[(1+\theta_{t+1}^Q)p_{t+1}\mathrm{G}_I(K_{t+1}, I_{t+1}) + (1+\theta_{t+1}^I)c_{t+1}\Big] + \epsilon_{t+1} \\ &\qquad (t = 1, \ldots, T-1), \end{aligned} \tag{14}$$

where ϵ_{t+1} stands for the expectations error. Substituting the marginal derivation of the adjustment cost function (3), i.e. $\mathrm{G}_I(K_t, I_t) = b(I_t/K_t) - ba$, assuming time invariant price elasticities $\eta_t^Q = \eta^Q$, $\eta_t^L = \eta^L$, $\eta_t^I = \eta^I$, as well as a time invariant rate of inflation of output goods $p_{t+1}/p_t = 1+\pi$ and rearranging results in

$$\begin{aligned}
\frac{I_{t+1}}{K_{t+1}} = &\ a(1-\phi) + (1+a)\phi\frac{I_t}{K_t} - \phi\left(\frac{I_t}{K_t}\right)^2 \qquad (15)\\
&- \frac{\phi}{b}\frac{\eta^Q}{1+\eta^Q}\left(\frac{Q_t}{K_t} - \frac{(w_t/p_t)L_t}{K_t} - \left[1 - \frac{(1-\delta)c_{t+1}}{(1+r)c_t}\right]\frac{(c_t/p_t)K_t}{K_t}\right)\\
&- \frac{\phi}{b}\frac{1}{1+\eta^Q}\left(\frac{Q_t}{K_t}\right) + \frac{\phi}{b}\frac{\eta^Q/\eta^L}{1+\eta^Q}\left(\frac{(w_t/p_t)L_t}{K_t}\right)\\
&+ \frac{\phi}{b}\frac{\eta^Q/\eta^I}{1+\eta^Q}\left(\left[1 - \frac{(1-\delta)c_{t+1}}{(1+r)c_t}\right]\frac{(c_t/p_t)K_t}{K_t}\right)\\
&+ \frac{\phi}{b}\frac{\eta^Q}{1+\eta^Q}\frac{\epsilon_{t+1}}{p_t} \qquad (t = 1, \dots, T-1)
\end{aligned}$$

with ϕ representing $(1+r)/[(1-\delta)(1+\pi)]$.

The assumption of imperfectly competitive product and factor markets leads to an investment equation, in which the rate of investment is seen as a linear function in the lagged investment rate, the lagged investment rate squared, the lagged ratio of real profits adjusted for the user costs of capital to the real capital stock, the lagged ratio of real output to the real capital stock and additionally the lagged real labour costs and the lagged real user costs of capital, both in relation to the real capital stock K_t:

$$\begin{aligned}
\frac{I_{t+1}}{K_{t+1}} = &\ \beta_0 + \beta_1\left(\frac{I_t}{K_t}\right) + \beta_2\left(\frac{I_t}{K_t}\right)^2 + \beta_3\left(\frac{Y_t}{K_t}\right)_t + \beta_4\left(\frac{Q_t}{K_t}\right) \qquad (16)\\
&+ \beta_5\left(\frac{W_t}{K_t}\right) + \beta_6\left(\frac{C_t}{K_t}\right) + u_{t+1}.
\end{aligned}$$

The variable $W_t = (w_t/p_t)L_t$ stands for real labour costs, $C_t = (1-[(1-\delta)c_{t+1}]/[(1+r)c_t])\,(c_t/p_t)K_t$ for real user costs of capital, and $Y_t = Q_t - W_t - C_t$ for real profits adjusted for the user costs of capital. u_{t+1} is the error term.

Since ϕ is the inverse of a real discount factor, the coefficient $\beta_1 = (1+a)\phi$ should be positive and near one and the coefficient $\beta_2 = -\phi$ negative and less than minus one. If the demand for output goods is elastic, i.e. $\eta^Q < -1$, $\beta_3 = -(\phi/b)\eta^Q/(1+\eta^Q)$ should be negative and the coefficient $\beta_4 = -(\phi/b)/(1+\eta^Q)$ positive with $|\beta_3| > |\beta_4|$. If the demand is inelastic, i.e. $\eta^Q > -1$, all three inequalities are inverted. For positive price elasticities of labour supply and supply of investment goods the coefficients $\beta_5 = (\phi/b)\eta^Q/[(1+\eta^Q)\eta^L]$ and $\beta_6 = (\phi/b)\eta^Q/[(1+\eta^Q)\eta^I]$ are positive in the case of elastic demand for output goods and negative in the case of inelastic demand. None of the coefficients β_3 to β_6 is defined in the limit case in which $\eta^Q = -1$. Perfectly competitive markets for investment goods are contained as a special case, since $\beta_6 \to 0$ for $\eta^I \to \infty$. The same holds for perfectly competitive labour markets in which case $\beta_5 \to 0$ for $\eta^L \to \infty$. The price elasticities can be recalculated from equation (16) by $\eta^Q = \beta_3/\beta_4$, $\eta^L = -\beta_3/\beta_5$ and $\eta^I = -\beta_3/\beta_6$.

Obviously parts of the explanatory variables in equation (16) are perfectly collinear. To avoid singularities, at least one of these variables has to be dropped for estimation purposes. Substituting the user costs term C_t/K_t, the equation to be estimated reduces to

$$\begin{aligned} \frac{I_{t+1}}{K_{t+1}} &= \beta_0 + \beta_1\left(\frac{I_t}{K_t}\right) + \beta_2\left(\frac{I_t}{K_t}\right)^2 + \tilde{\beta}_3\left(\frac{Y_t}{K_t}\right) + \tilde{\beta}_4\left(\frac{Q_t}{K_t}\right) \\ &\quad + \tilde{\beta}_5\left(\frac{W_t}{K_t}\right) + u_{t+1} \end{aligned} \tag{17}$$

with $\tilde{\beta}_3 = \beta_3 - \beta_6 < \beta_3$, $\tilde{\beta}_4 = \beta_4 + \beta_6 > \beta_4$ and $\tilde{\beta}_5 = \beta_5 - \beta_6 < \beta_5$ in the case of elastic demand for output goods and $\tilde{\beta}_3 > \beta_3$, $\tilde{\beta}_4 < \beta_4$ and $\tilde{\beta}_5 > \beta_5$ in the case of inelastic demand.

Interpreting equation (17) as an investment equation for a model with competitive markets for investment goods, the recalculated price elasticity of demand for output goods will be too large in the case of elastic demand, since $\tilde{\eta}^Q = \tilde{\beta}_3/\tilde{\beta}_4 > \beta_3/\beta_4 = \eta^Q$ if $\eta^Q < -1$, and too small in the case of inelastic demand, since $\tilde{\beta}_3/\tilde{\beta}_4 < \beta_3/\beta_4$ if $\eta^Q > -1$. The size of the effect depends on the true values of η^Q and η^I.

Table 1 contains the calculated effects for different values of η^Q and η^I. The values for the model parameters b and ϕ are fixed to 4.0 and 1.05 without any loss of generality. The coefficients of the profit and of the output term will quite easily be more than double as large as the "true" coefficients in absolute terms. The recalculated price elasticity is numerically less effected, but still systematically wrong. The recalculation will be biased towards -1, the case in which the coefficients are not defined. The effect on the recalculated elasticity of labour supply depends on the relationship between β_5 and β_6 and could be analyzed in a similar way.

Table 1: Effects on Model Coefficients and Recalculated Price Elasticities

	η^Q	η^I	β_3	β_4	β_6	$\tilde{\beta}_3$	$\tilde{\beta}_4$	$\tilde{\eta}^Q$
	-0.90	0.50	2.3625	-2.6250	-4.7250	7.0875	-7.3500	-0.9643
inelastic	-0.90	0.75	2.3625	-2.6250	-3.1500	5.5125	-5.7750	-0.9545
demand	-0.90	1.00	2.3625	-2.6250	-2.3625	4.7250	-4.9875	-0.9474
	-0.90	1.25	2.3625	-2.6250	-1.8900	4.2525	-4.5150	-0.9419
	-0.90	1.50	2.3625	-2.6250	-1.5750	3.9375	-4.2000	-0.9375
	-1.10	0.50	-2.8875	2.6250	5.7750	-8.6625	8.4000	-1.0313
elastic	-1.10	0.75	-2.8875	2.6250	3.8500	-6.7375	6.4750	-1.0405
demand	-1.10	1.00	-2.8875	2.6250	2.8875	-5.7750	5.5125	-1.0476
	-1.10	1.25	-2.8875	2.6250	2.3100	-5.1975	4.9350	-1.0532
	-1.10	1.50	-2.8875	2.6250	1.9250	-4.8125	4.5500	-1.0577

In panel data studies on firm investment behaviour, the user costs of capital in relation to the capital stock are quite often approximated by time dummies since the relative user costs are quite constant across firms (see e.g. BOND AND MEGHIR, 1994). Replacing the user costs term in equation (17) by time dummies λ_t, another variable has to be dropped, if unit user costs of capital are exactly identical across firms. Substituting the labour costs variable, the equation to be estimated reduces to

$$\begin{aligned}\frac{I_{t+1}}{K_{t+1}} &= \beta_0 + \beta_1\left(\frac{I_t}{K_t}\right) + \beta_2\left(\frac{I_t}{K_t}\right)^2 + \breve{\beta}_3\left(\frac{Y_t}{K_t}\right) + \breve{\beta}_4\left(\frac{Q_t}{K_t}\right) \\ &\quad + \lambda_t + u_{t+1}\end{aligned} \tag{18}$$

with $\breve{\beta}_3 = \beta_3 - \beta_5 < \beta_3$ and $\breve{\beta}_4 = \beta_4 + \beta_5 > \beta_5$ in the case of elastic demand and $\breve{\beta}_3 > \beta_3$ and $\breve{\beta}_4 < \beta_5$ in the case of inelastic demand. Interpreting equation (18) as an investment equation for a model with competitive markets for labour the same arguments hold as in the case of equation (17), since $\breve{\beta}_3/\breve{\beta}_4 > \beta_3/\beta_4$ if

$\eta^Q < -1$ and $\breve{\beta}_3/\breve{\beta}_4 < \beta_3/\beta_4$ if $\eta^Q > -1$. Again the recalculated price elasticity will be biased towards -1, the case in which the coefficients are not defined. The effects will be the same as demonstrated in table 1 if η^I is replaced by η^L.

3 Conclusion

In consequence, when factor markets are imperfectly competitive and an Euler equation model for perfectly competitive factor markets is applied, the coefficients for the profit term and the output term will systematically differ from the 'true' coefficients and quite easily be more than double as large in absolute terms. The recalculated price elasticity of demand will be less effected but systematically larger than the true price elasticity if the latter is elastic and systematically lower if it is inelastic. The recalculation will be biased towards the limit case of -1 in which the coefficients of the investment equation are not defined.

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