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DISCUSSION PAPER

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Congestion Management Games in Electricity Markets

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Abstract

This paper proposes a game-theoretic model to analyze the strategic behavior of inc-dec gaming in market-based congestion management (redispatch). We extend existing models by considering incomplete information about competitors' costs and a finite set of providers. We find that inc-dec gaming is also a rational behavior in markets with high competition and with uncertainty about network constraints. Such behavior already occurs in our setup of two regions. Comparing market-based redispatch with three theoretical benchmarks highlights a lower efficiency level of market-based redispatch and inflated redispatch payments. Finally, we study seven variations of our basic model to assess whether different market fundamentals or market design changes mitigate inc-dec gaming. None of these variations eliminate inc-dec gaming entirely.

JEL Classification: D43, D44, L13, Q41, Q48

Keywords: Energy market, Game theory, Auctions/bidding, Congestion management, Inc-dec gaming

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1 Introduction

Through much of the 20th century, the power industry was vertically integrated, mostly fossil-fuel based, and organized regionally. Market liberalization and the energy transition lead to an increased distance between generators and consumers (Hesamzadeh et al., 2021). Renewable energy plants are most productive at sites with high resource availability. The expansion of wind turbines is usually concentrated in windy areas, for example along coastlines. Both wind and solar power stations are preferably built where land prices are low. Further strain on transmission infrastructure results from a closer integration of neighboring electricity markets.

In Europe, among others, electricity markets are organized in large pricing (or "bidding") zones. Within a bidding zone, the electricity price is uniform for each time step. This implies that prices do not reflect intrazonal transmission constraints, i.e., they are operated as if the entire internal network was a copper plate. While this simplification has helped the integration of the European markets into the world's largest electricity market, it imposes new challenges in terms of increasing congestion in the network.

Various power systems worldwide addressed congestion through the introduction of locational marginal pricing, also known as nodal pricing (see for example Joskow, 2008). This market design considers transmission constraints in the spot market clearing. While nodal pricing is applied among others in all liberalized U.S. power markets, a transition to nodal pricing in continental Europe is unlikely for political reasons (Eicke & Schittekatte, 2022). Instead, a bidding zone review is conducted to better align the delineation of bidding zones. While this process might result in smaller pricing zones in Europe with presumably less congestion, the management of transmission constraints within zones remains necessary as long as the zonal market design persists.

In zonal markets, system operators such as transmission system operators and distribution system operators relieve the overload of network elements within zones through out-of-the market measures. The most important instrument is the "redispatch" of power stations: some market participants who contribute to congestion are ordered to alter their generation or consumption such that the power flow on congested lines is reduced.[1] In many European countries, the use of redispatch has increased over the years, leading to higher costs (ACER & CEER, 2021). How to procure redispatch resources is subject to an intense debate. While participating in redispatch has often been mandatory, the European Commission proposed to turn this into voluntary redispatch markets (EU, 2019). Here, system operators procure redispatch resources from market participants through auctions.

This paper proposes a game-theoretic model to study strategic behavior in market-

[1] The European Regulation 2019/943 on the internal market for electricity defines the term redispatch as both generation and consumption control measures for congestion management (EU, 2019).

based redispatch. It has been long understood that firms factor in profit opportunities from redispatch markets and change their spot market bids accordingly (Hirth & Schlecht, 2019). Such behavior is known as "inc-dec gaming". Inc-dec gaming increases the probability of congestion, and has caused severe problems in the past. Most prominent is the case of California, where strategic behavior contributed to a series of rolling blackouts in 2000/01 (Alaywan et al., 2004; Brunekreeft et al., 2005). This eventually led to the introduction of nodal pricing in California in 2009 (Cramton, 2019). Other cases where inc-dec gaming occurred are the Scottish-English border, where congestion strongly increased after the rapid expansion of wind energy in Scotland (Konstantinidis & Strbac, 2015), and the Italian market (Graf et al., 2021).

So far, only few authors have studied market-based redispatch by analytically solving for the Nash equilibrium of the game played by market participants. The main reference is Holmberg and Lazarczyk (2015). They assume a continuum of infinitesimally small generators, which have full information about all costs. The authors find that the dispatch under nodal pricing equals the final dispatch of a zonal spot market with uniform pricing followed by market-based pay-as-bid redispatch,[2] which are both an efficient allocation. Also the local market-based redispatch prices are the same as in markets with nodal pricing. A key difference in the outcomes of the two market designs is that generators' total profits are higher in zonal markets with redispatch. This is for two reasons: first, generators trade at the higher of the uniform zonal or their local redispatch price. Second, generators at export-constrained nodes can make a profit without producing electricity: they sell electricity at the zonal price and buy it back at the lower local redispatch price. Hence, one main result of Holmberg and Lazarczyk (2015) is that inc-dec gaming also occurs in a perfectly competitive market. This finding is supported by Hirth and Schlecht (2019), who point out a number of consequences of inc-dec gaming.

Another stream of literature studies inc-dec gaming by using bi-level equilibrium modeling (Sarfati et al., 2019; Sarfati & Holmberg, 2020). These papers have in common that they assume oligopolistic competition and full information about costs. Their models also provide evidence for inc-dec gaming in redispatch markets. Grimm et al. (2022) combine analytical and numerical modeling and find in a full-information model that market-based redispatch with a system operator that minimizes redispatch costs may result in inefficient (i.e., not welfare-maximizing) outcomes for more than two nodes or regions.

Our paper extends the analysis by Holmberg and Lazarczyk (2015) by relaxing two of their strong assumptions: we assume incomplete information about competitors' costs and a finite set of providers. The assumption of incomplete information of market participants about their competitors' costs is motivated by the increasing market shares of renewable

[2]The outcome is the same whether there is one bid per market stage or with a single bid for both stages.

energy sources and flexible assets such as storage and flexible consumers. The costs of both types of technologies are difficult to predict for other market participants because the availability of renewable energy resources varies over time and the costs of flexible market participants are mainly determined by opportunity costs. Second, we consider cases with a finite set of generators in a situation of strategic interaction, reflecting these aspects of the oligopoly situation in some electricity markets.

We derive four main results from our model. First, market-based redispatch leads to inefficient outcomes and high payments due to inc-dec gaming. Second, inc-dec gaming occurs even when the number of generators goes to infinity or when the uncertainty about competitors' costs vanishes. Third, the analysis of the effect of three different market fundamentals (competition and market size, different congestion levels, and the probability of congestion) shows that inc-dec gaming occurs in these setups. Fourth, this also applies to four market design changes proposed in the literature and by practitioners to mitigate inc-dec gaming: the possibility to bid differently in spot and redispatch markets, price caps, uniform pricing in the redispatch market, and a hybrid model, combining cost-based and market-based redispatch.

In the next section, we provide background information on redispatch markets and on inc-dec gaming. Section 3 introduces our basic model as well as its results, and highlights the inefficiency and high payments resulting from market-based redispatch. We analyze how gaming opportunities are affected by fundamental characteristics of power markets in Section 4 and market design changes in Section 5. Section 6 summarizes and concludes.

2 Background: Markets for redispatch

Redispatch can be organized in multiple ways. In cost-based redispatch, generators are obliged to participate, hence it is also termed mandatory redispatch. The system operator appoints assets that must provide redispatch services and compensates them for the incurred costs. The main problem of this approach is that the incurred costs vary between assets. Because the system operator has incomplete information on these costs, they need to be estimated. Such estimations are already difficult for conventional power plants, where costs depend, among others, on the asset's efficiency and fuel costs. For flexible demand and storage facilities, including electric vehicles or heat pumps, such an estimation is nearly impossible (for the system operator but also for competitors).[3] As a result, these assets are usually not considered in cost-based redispatch. The limited number of assets participating in the redispatch mechanism lowers its economic efficiency and potentially even renders the redispatch impossible if not enough steerable units are available in a deficit region. For

[3] To determine an adequate compensation for these assets, system operators would need to estimate each consumer's opportunity costs, i.e., their willingness to pay for electricity.

comparison, we use cost-based redispatch with a completely informed system operator as a theoretical benchmark in Section 3.2, which we refer to as idealized cost-based redispatch due to the assumption of complete information.

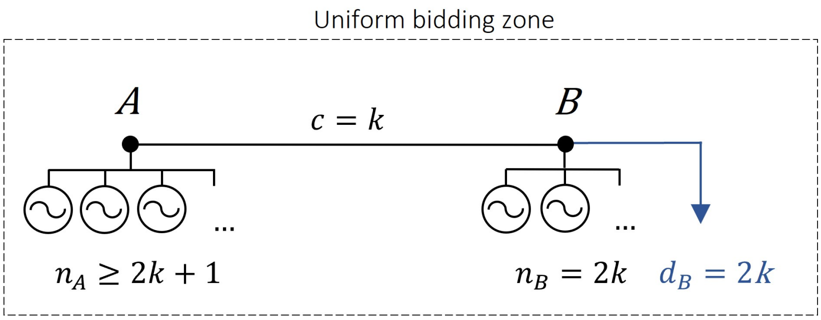
An alternative approach is voluntary market-based redispatch, which is at the core of this paper. In this auction-based mechanism, market participants can submit bids to indicate at which price they are willing to adjust their output or consumption. This facilitates the participation of flexible demand and storage facilities. Proponents of market-based redispatch argue that this mechanism would be more efficient due to the participation of flexible assets. The European Commission is among these proponents. In 2016, it stipulated market-based redispatch as the default mechanism in the Electricity Market Regulation recast, which became effective in 2019 (EU, 2019).[4] Currently, many member states apply market-based redispatch (e.g., Finland, Italy, Sweden, and the Netherlands), others make use of the exemptions mentioned in Footnote 4 and apply cost-based redispatch (e.g., Germany), and a third group applies a combination of market-based and cost-based redispatch (e.g., Denmark, France, and Ireland).

Market-based redispatch can be interpreted as a two-stage market where the first stage is a zonal short-term market (spot market) and the second stage are local redispatch markets. These two market stages open opportunities for inc-dec gaming. When congestion occurs due to the spot market allocation, generators in the importing region prefer selling on their regional redispatch market, where prices are higher than on the zonal spot market, which does not reflect local scarcity in supply. Thus, generators bid sufficiently high in the spot market not to be dispatched. Conversely, generators in the export-constraint region can profit by being downward redispatched. They bid low in the spot market, thereby ensuring being dispatched. On the redispatch market, generators with high generation costs buy the electricity back at a lower price.

Such inc-dec gaming in market-based redispatch is problematic for multiple reasons. First, it inflates the volume of redispatch: congestion increases because generators in the deficit area increase their bids in the spot market to benefit from higher redispatch prices. Second, generators that participate in gaming will be able to extract rents, i.e., gaming generates windfall profits. Third, gaming is problematic for financial markets, since the spot market, which serves as underlying for futures and forward contracts, becomes less meaningful. This reduces the possibility to hedge prices. Finally, gaming provides perverse investment incentives: it incentivizes the construction of additional generators in surplus regions. In the extreme, it leads to investments of generation assets in the oversupplied

[4] The regulation states that redispatch must be organized via a competitive mechanism, unless one of the following exemptions apply: no market-based alternative is available, all available market-based resources are exhausted, the number of available units is too low to ensure effective competition, or the current grid situation leads to congestion in such a frequent and predictable way that market-based redispatch would result in strategic bidding.

Figure 1: Setup of the model in the basic case



region with the sole purpose to engage in gaming but never to actually generate electricity.

3 Strategic behavior in market-based redispatch

In this section, we introduce our game-theoretic model to study the behavior of profit-maximizing generators in the presence of redispatch markets.

3.1 Basic model

The model is chosen such that it captures the relevant characteristics of the strategic situation of generators under market-based redispatch and at the same time is as simple as possible to enable analytical tractability and the traceability of results. We focus on the behavior of generators and assume price inelastic consumers. This reveals the fundamental incentives under market-based redispatch, which are also transferable to flexible consumers.

The setup of the basic model is depicted in Figure 1. There are $n = n_A + n_B$ single-unit-supply generators, each of whom can provide one unit of electricity at one of two regions, A and B. This setup can be interpreted as one bidding zone in which congestion occurs only on the transmission line between the regions A and B. The network capacity c for an exchange between A and B is k units, $c = k$, $k \in \{1, 2, 3, \ldots\}$.

Regions A and B differ with respect to demand and number of generators. Demand at A is $d_A = 0$ and demand at B is $d_B = 2k$. There are $n_A \geq 2k + 1$ generators in A and $n_B = 2k$ generators in B. Thus, generators in B are able to satisfy demand, but there is competition from A, which can satisfy up to half of the demand (due to the transmission capacity).[5]

The game has two stages: In the first stage, generators submit a bid in the spot market, and, depending on the spot market outcome, redispatch markets are conducted in which generators can submit a second bid.

[5] We vary demand (relative to k) and certainty of demand in sections 4.2 and 4.3. The case $n_A = 2k$ is analyzed separately in Section 4.1.

We assume that generators are profit-maximizing and sequentially rational. Each provider knows its generation costs and their relative position in the distribution of generators' costs. Generation costs are private information, i.e., providers do not know their competitors' costs.

This is modeled as follows. Generation costs are i.i.d. random variables X_i, $i \in \{1, 2, \ldots, n\}$, with the cumulative distribution function F with full support on the interval $[\underline{x}, \bar{x}]$, which is normalized to $[\underline{x}, \bar{x}] = [0, 1]$ for convenience. The probability density function is denoted by f. The realization x_i of X_i is private information of generator i (independent private values approach). We also use the conditional random variable $X_i \mid X_i < y$, whose cumulative distribution function and probability density function are given by $G(x \mid y) = F(x)/F(y)$ and $g(x \mid y) = f(x)/F(y)$. The jth order statistic of n draws is denoted by $X_{(j,n)}$ or $X_{(j,n)} \mid X_{(j,n)} < y$, and their respective cumulative distribution function and probability density function by $F_{(j,n)}$ and $f_{(j,n)}$ or $G_{(j,n)}$ and $g_{(j,n)}$.[6]

Consequently, the generators with the lowest costs can be in any of the two regions. The basic model captures both the case that generators' costs are such that cost-minimizing dispatch makes full use of the transmission capacity and the case that no congestion occurs. Regions are asymmetric in the sense that congestion of a transmission line is possible only in one direction. Thus, an efficient dispatch only sometimes involves congestion (depending on the realized costs), but if a congestion occurs, its direction is always the same.[7]

Maximum bids in the spot market and the market for upward redispatch are limited to r, where $r \geq 1$, and minimum bids in the market for downward redispatch to $r = 0$. These boundaries enable all generators to bid up to or down to their costs. On the spot market, each generator i submits a price bid b^S_{Ai} or b^S_{Bi}, where the notation indicates region A or B. Bids are sorted in ascending order and the lowest bids win. The spot market price p is determined by the lowest rejected bid. With $d_B = 2k$, the $2k$ lowest bids win, and the $(2k + 1)$-lowest bid determines the clearing price.

Market-based redispatch is conducted if the spot market allocation is not feasible due to the grid constraint. In our basic model, in which transmission capacity is limited to $c = k$, ℓ units of electricity will be redispatched when $k + \ell$ generators with a winning bid in the spot market are located in A. Then, in region A, $k + \ell$ generators with a winning bid in the spot market compete for downward redispatch of ℓ units by submitting bids b^R_{Ai}. In B, generators without a winning bid in the spot market compete for upward redispatch of ℓ units by submitting bids b^R_{Bi}. After the spot market is cleared, only the market price is published, not the individual bids of the generators. That is, participants in the redispatch market may gain only limited knowledge about their competitors' costs from the spot market

[6]Notation: $X_{(1,n)} \leq X_{(2,n)} \leq \ldots \leq X_{(n,n)}$, i.e., the first order statistic $X_{(1,n)}$ is the smallest, $X_{(1,n)} = \min\{X_1, X_2, \ldots, X_n\}$ (Ahsanullah et al., 2013).

[7]This assumption can be justified by real-world network characteristics in which congestion usually occurs in the same direction (until the network may finally be enforced) or by predictability of the direction of a congestion for specific market time units.

(in equilibrium, they will be able to infer the $(2k+1)$-lowest cost of a generator in A).

The redispatch markets are organized as auctions with pay-as-bid pricing.[8] At the redispatch stage, demand for upward and downward redispatch is known. In the auction for downward redispatch, the highest bids win and the winners pay their bid (i.e., the provider buys the electricity back); in the auction for upward redispatch, the lowest bids win and the winners are paid their bid (i.e., the provider sells the electricity). Thus, a provider that is downward redispatched has a profit (payoff) equal to the spot market price minus the provider's redispatch bid; a provider that is upward redispatched has a profit equal to the provider's redispatch bid minus generation costs.

In this game there exists a Perfect Bayesian Equilibrium in which only generators in A win in the spot market and redispatch is always necessary.[9] Since in both A and B the equilibrium bidding function is symmetric, we omit the provider index i. The proposition is proven in Appendix A.1.

Proposition 1. *The game has a Perfect Bayesian Equilibrium in which only generators in A are dispatched in the spot market, redispatch is always necessary, and the final dispatch is inefficient.*

The equilibrium bidding functions $\beta_A^S(x)$ and $\beta_B^S(x)$ in the spot market are

$$\beta_A^S(x) = \int_0^x t g_{(k,2k-1)}(t \mid x) dt = \mathbb{E}[X_{(k,2k-1)} \mid X_{(2k-1,2k-1)} < x] \quad \textit{for all } x \in (0,1], \tag{1}$$

$$\beta_B^S(x) \in [\mathbb{E}[X_{(k,2k-1)}], r] \textit{ arbitrary for all } x \in [0,1]. \tag{2}$$

The equilibrium bidding functions $\beta_A^R(x)$ and $\beta_B^R(x)$ in the redispatch markets comprise

$$\beta_A^R(x) = \frac{\int_0^x t g_{(k,2k-1)}(t \mid y) dt}{G_{(k,2k-1)}(x \mid y)} = \mathbb{E}[X_{(k,2k-1)} \mid X_{(k,2k-1)} < x, X_{(2k-1,2k-1)} < y] \tag{3}$$

$$\textit{for all } x \in (0,y], \textit{ where } y = (\beta_A^S)^{-1}(p) \textit{ and } p \textit{ is the spot market price,}$$

$$\beta_B^R(x) = \frac{\int_0^x t f_{(k,2k-1)}(t) dt}{1 - F_{(k,2k-1)}(x)} = \mathbb{E}[X_{(k,2k-1)} \mid X_{(k,2k-1)} > x] \quad \textit{for all } x \in [0,1). \tag{4}$$

Boundary bids are $\beta_A^S(0) = 0$, $\beta_A^R(0) = 0$, and $\beta_B^R(1) = 1$.[10]

In the equilibrium, all spot market bids of generators in A are lower than the bids of generators in B, $\beta_A^S(x) \leq \mathbb{E}[X_{(k,2k-1)}]$ for all x. Thus, $2k$ generators in A win in the

[8] In Section 5.3, we will show that uniform pricing leads to the same outcome.

[9] As is common in auction theory analysis, we concentrate on symmetric equilibria in pure and increasing strategies (e.g. Krishna, 2010). That is, all generators in A use the same increasing bidding function β_A, and all generators in B use the same increasing bidding function β_B. An exception is β_B^S, which is symmetric, but not necessarily increasing, since any bid in the interval $[\mathbb{E}[X_{(k,2k-1)}], r]$ can be chosen.

[10] The off-the-equilibrium-path parts of the equilibrium bidding strategies, which are complete bidding plans for each type and each contingency, are described in Appendix A.1.

spot market, and the redispatch of k units is always necessary. Note that the probability for congestion would be much lower if generators did not engage in gaming and bid their generation costs in the spot market ($\beta^S(x) = x$). For $k = 1$, redispatch would occur only with probability $\binom{n_A}{n_A-2}/\binom{n_A+2}{n_A}$, e.g., 30% if $n_A = 3$.

The equilibrium bidding functions $\beta^S_A(x)$, $\beta^R_A(x)$, and $\beta^R_B(x)$ are strictly increasing in x. Thus, the $2k$ generators with the lowest costs in A win in the spot market, and the spot market price p is determined in A by the provider with the $(2k+1)$th-lowest costs. In the redispatch markets, in A the k generators with the higher costs among the $2k$ generators that win in the spot market are downward redispatched, and in B, the k generators with the lowest costs are upward redispatched.

As $\beta^S_A(x) < x$ for all $x \in (0, 1]$, the spot market price is below the costs of the price-setting provider. Generators in A bid weakly lower in the spot market than in the redispatch market: $\beta^S_A(x) = \beta^R_A(x)$ for $x \in [0, y]$ and $k = 1$, and $\beta^S_A(x) < \beta^R_A(x)$ for $x \in (0, y)$ and $k \geq 2$. However, their bid in the redispatch market is of course lower than the spot market price, $\beta^R_A(x) \leq p$.

The intuition behind the equilibrium is as follows. First consider the second stage, the redispatch markets. On the redispatch market in region A, the $2k$ spot market winners compete for buying (back) k units and saving their costs. The bidding function $\beta^R_A(x)$ is the same as in a pay-as-bid forward auction with k units and $2k$ generators (Krishna, 2010), with costs distributed between zero and y, the costs of the price-setting provider in the spot market. That is, generators bid the (conditional) expected costs of their median competitor under the condition that these expected costs are below the own costs.[11] In the redispatch market in B, generation must be increased compared to the spot market clearing. $2k$ generators compete for selling k units and, thus, the bidding function $\beta^R_B(x)$ is the same as in a pay-as-bid reverse (procurement) auction with k units and $2k$ generators. That is, generators bid the expected costs of their median competitor under the condition that these are above their own costs. In the spot market, generators in A compete for the spot market payment, which either implies incurring their costs (if they are among the spot market winners with the lower costs) or buying back their electricity at the price $\beta^R_A(x)$ (if they are among the spot market winners with the higher costs). Generators in B abstain from competing in the spot market because there is less competition for the k units at their redispatch market than in the spot market.

A provider who wins at the worst spot market price conditional on winning, i.e., a price equal to the provider's bid, knows the price-setting provider has the same type, $y = x$. Such a provider bids in the redispatch market as in the spot market, $\beta^R_A(x) = \mathbb{E}[X_{(k,2k-1)} \mid$

[11] The order statistic $X_{(k,2k-1)}$ is the random variable of the median of the sample of $2k-1$ independent draws from F. Thus, the bids $\beta^S_A(x)$, $\beta^R_A(x)$, and $\beta^R_B(x)$ are equal to an expected value of the sample median conditional on x.

$X_{(k,2k-1)} < x, X_{(2k-1,2k-1)} < x] = \mathbb{E}[X_{(k,2k-1)} \mid X_{(2k-1,2k-1)} < x] = \beta_A^S(x)$. Then this provider will win in the redispatch market and buy back the electricity at the same price as the spot market price, so the profit is zero, as if the provider had not won in the spot market. A provider who has bid below the spot market price has a probability of less than one of winning in the redispatch market and buying back the electricity at a positive profit (as the spot market price is higher than that generator's redispatch bid), and also has a positive probability of having to produce and making a loss.

In both regions, the expected profit of generators is positive and decreases with their costs. Moreover, the expected profit of a provider in B is higher than of a provider in A with the same costs. Unlike the generators in B, the generators in A with $x > 0$ run the risk of incurring a loss because they bid below their costs in the spot market, $\beta_A^S(x) < x$ for $x \in (0, 1]$. Therefore, it is possible for a provider to win in the spot market at a price p below x, but not win in the redispatch market. This provider suffers a loss of $p - x$.

In this setting, the generators' uncertainty about the competitors' costs requires them to trade off the different potential outcomes that may result from their bid, making their decisions risky. As a result of the uncertainty, the final dispatch may not be efficient, i.e., it is not the cost-optimal subset of generators that operate. Inefficiency occurs whenever more than k of the $2k$ generators with the lowest costs are located in region B. In this case, in the efficient outcome all these lowest-costs generators in B generate electricity, but in equilibrium only k do so. Thus, inc-dec gaming in an incomplete information setup adds a source of inefficiency not present in a full information model with atomistic generators, for which Grimm et al. (2022) find an inefficient dispatch only for settings with at least three regions.

Furthermore, note that in the equilibrium outcome, the maximum volume of k units are always redispatched. This holds even for cost realizations where more than k of the $2k$ generators with the lowest costs are located in B. If costs were common knowledge in this case, there would be an equilibrium where generators bid their costs in the spot market and no congestion occurs.[12] In that sense, as cases without congestion have positive probability

[12]This is an equilibrium: No single losing or winning bidder can cause congestion by unilaterally deviating, all winning bidders prefer winning to losing and cannot lower the price, and all losing bidders prefer losing to winning and cannot win at a lower price. Note that also in the opposed case of cost realizations where k or less of the $2k$ generators with the lowest costs are in B, with complete information and a finite number of generators there are equilibria that resemble those of Holmberg and Lazarczyk, 2015 for atomistic generators. Assume for simplicity that all generators' realized costs differ, and assume for equilibrium existence that ties are broken in favor of efficiency. There is an equilibrium in which all generators bid the same in the spot market as in the redispatch market. In A, the generators with the k- to $2k+1$-lowest costs in A bid the k-lowest costs in A and all others in A bid their costs. In B, the generators with the lowest to $k+1$-lowest costs bid the $k+1$-lowest costs in B and the others bid their costs. In the outcome, $2k$ generators in A win in the spot market, k units are redispatched, the redispatch price in A is equal to the spot market price whereas the redispatch price in B is higher, and the final allocation is efficient as the k generators in A with the lowest costs deliver. There are further equilibria with the same outcome, and others with different allocation and/or prices. For example, the asymmetric Bayesian equilibria in Footnote 13 with arbitrary

(70% for $k = 1$ and $n_A = 3$, see above), incomplete information about costs increases the probability and expected volume of redispatch.

In addition to the equilibrium in Proposition 1, others may exist. However, all equilibria with a positive redispatch probability for the generators have in common that the generators in A bid below their costs in the spot market, $\beta_A^S(x) < x$, and a generator in B submits a higher bid than the same type in A, $\beta_B^S(x) > \beta_A^S(x)$ for all $x > 0$.[13] This is due to the inc-dec incentives. Without transmission constraint, it is optimal (weakly dominant strategy) for the generators to bid their costs in the spot market. With the possibility of redispatch, generators in A have an incentive to bid below their costs because they have the opportunity to make a positive profit through downward redispatch by winning in the spot market. Generators in B have an incentive to bid above the same type in A because they have the opportunity to make a positive profit through upward redispatch if they do not win in the spot market. The equilibrium in Proposition 1 is the most extreme form of this bidding difference between the generators in A and in B. Moreover, this bidding difference, combined with the different options for generators in A and B, implies that with high competition in A, an equilibrium must always have the separation of the generators in A and B as in the equilibrium in Proposition 1. In this case, even the lowest-cost generators in B have a negligible chance of winning, and the expected spot market price is very low, making redispatch the better alternative. Thus, no generator in B has an incentive to compete with the more aggressively bidding generators in A in the spot market.

3.2 Benchmarking market-based redispatch

To highlight why market-based redispatch with strategic behavior results in higher generation costs and electricity payments for consumers, we compare the approach with three theoretical benchmarks: an unconstrained grid, idealized cost-based redispatch with a completely informed system operator, and the Vickrey-Clarke-Groves (VCG) mechanism, which incents truthful bidding and results in a cost-efficient dispatch. Note that these benchmarks are no practical alternatives to market-based redispatch. Instead, the comparison with these theoretical benchmarks helps to understand where inefficiency and high payments arise in market-based redispatch and what causes them. The comparisons in this section also hold for uniform pricing in the redispatch markets and a single bid for spot and redispatch markets because these are outcome-equivalent to the basic model.

As a first benchmark, we compare market-based redispatch with the case of unconstrained grids (deviating from the basic model). Redispatch becomes dispensable when the

delivering generators and high spot market price are also equilibria in the case of complete information.

[13] An example of an asymmetric equilibrium where generators in A do not have a positive redispatch probability is given by t_A generators in A, $0 \leq t_A \leq k-1$, and $t_B = 2k - t_A$ generators in B bidding zero in the spot market, while all others bid one.

network is expanded to the extent that congestion no longer occurs. Then, generators have an incentive to bid their marginal costs in the spot market, $\beta(x) = x$. In our model, a grid capacity of $c \geq 2k$ satisfies this criterion. Whether this is economically efficient depends on the costs of network expansion, which we do not consider in this comparison. In practice, it is usually efficient to accept a certain share of congestion in order to not expand transmission infrastructure for rare events (R. Green, 2003).

The second theoretical benchmark is what we call idealized cost-based redispatch. This benchmark represents a fictitious situation in which the system operator can limit the redispatch payments to the generation costs, i.e., it can enforce price discrimination. This is only possible when the system operator has full information about the costs of all generators (deviating from the basic model). In this case, generators always have the incentive to bid their costs in the spot market, $\beta(x) = x$. There is no benefit of being upward redispatched because the resulting compensation equals the costs. Thus, a generator prefers to win in the spot market if its costs are below the spot market price. There is also no benefit from being downward redispatched because the payoff then equals the spot market price minus the costs. Hence, by bidding above costs in the spot market, the generator decreases the probability of winning and by bidding below costs, the generator increases the chance of winning only when it would be better not to win in the spot market auction because the spot market price is below costs.

The third benchmark is the Vickrey-Clarke-Groves (VCG) mechanism. This one-step mechanism determines an efficient (i.e., generation-cost-minimizing) dispatch given the transmission capacity and provides a robust incentive (a weakly dominant strategy) to bid the costs, $\beta(x) = x$ (Clarke, 1971; Groves, 1973; Krishna, 2010; Vickrey, 1961).[14] It thus provides the theoretical benchmark of an efficient allocation in our setting with incomplete information and non-atomistic generators. In the VCG mechanism, each provider is paid the difference between the sum of the other generators' (reported) costs in the cost-minimizing allocation without this provider and the sum of the (reported) costs of the other generators in the cost-minimizing allocation with this provider. Noteworthy, a result of the VCG mechanism in our setting is locationally differentiated prices depending on the relevance of the transmission constraint; thus, it is an incentive-compatible version of nodal pricing.[15] In contrast to idealized cost-based redispatch, the auctioneer is not assumed to have and use information about the generators' costs before conducting the mechanism. Thus, while the generation costs are the same under the VCG mechanism and under idealized cost-based

[14]The VCG mechanism is the unique mechanism with these properties. Only fixed transfers added to the payments resulting from a VCG mechanism, would also not affect the incentives (J. Green & Laffont, 1979; Holmstrom, 1979).

[15]In contrast to traditional locational marginal pricing, the VCG mechanism may set different local prices even if the transmission capacity is just sufficient. Therefore, generators cannot profit from inducing a congestion by misstating costs in the VCG mechanism.

Table 1: Generation costs and payments under market-based redispatch and the three theoretical benchmarks for uniformly distributed costs $X \sim U[0,1]$, $k = 1$, $n_A \geq 3$

	Scenario	Generation costs	Spot market payments	Redispatch payments	Energy payments
	Market-based redispatch	$\frac{1}{3} + \frac{1}{n_A+1}$	$2 \cdot \frac{3}{2(n_A+1)}$	$\frac{2}{3} - \frac{1}{n_A+1}$	$\frac{2}{3} + \frac{2}{n_A+1}$
Benchmarks	Unconstrained grid	$\frac{3}{n_A+3}$	$2 \cdot \frac{3}{n_A+3}$	–	$\frac{6}{n_A+3}$
	Idealized cost-based redispatch	$\frac{n_A^2+8n_A+18}{3(n_A+2)(n_A+3)}$	$2 \cdot \frac{3}{n_A+3}$	$\frac{n_A(n_A-1)}{3(n_A+2)(n_A+3)}$	$\frac{n_A^2+17n_A+36}{3(n_A+2)(n_A+3)}$
	VCG mechanism	$\frac{n_A^2+8n_A+18}{3(n_A+2)(n_A+3)}$	–	–	$\frac{2(n_A^2+8n_A+18)}{3(n_A+2)(n_A+3)}$

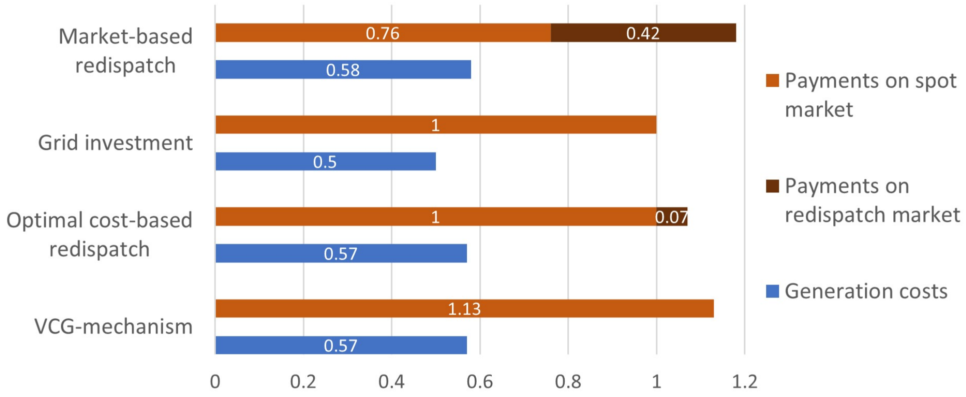
redispatch, total electricity payments with VCG are higher because generators receive an information rent.

Table 1 provides an overview of expected generation costs and electricity payments in the spot and the redispatch markets; see Appendix A.2 for the associated calculations. The electricity payments are the spot market payments (two times the spot market price) plus the redispatch payments. To simplify the comparison, we assume a uniform distribution ($X \sim U[0,1]$) and set $k = 1$ (i.e., demand in B in the basic model is $d_B = 2$). Figure 2 visualizes the costs and payments for $n_A = 3$.

Generation costs are highest with market-based redispatch due to the inefficient final dispatch in the case when the two cheapest generators are located in B. By comparison, idealized cost-based redispatch and the VCG mechanism allocate efficiently, i.e., they lead to the lowest generation costs given the transmission capacity $c = 1$, which requires generation of at least one unit in B. Generation costs are lowest under unconstrained grids because it results in an efficient dispatch of generators and does not result in congestion. The difference in generation costs between market-based redispatch and the VCG mechanism is the efficiency loss caused by the strategic behavior under market-based redispatch. As n_A increases, the generation costs of market-based redispatch, idealized cost-based redispatch, and the VCG mechanism converge because it becomes more likely that generation by only one unit in B is efficient. In the limit (n_A to infinity), generation costs are zero in A and $1/3$ in B, the spot market price and spot market payments are zero, and the redispatch payments and energy payments are $2/3$. The cost advantage of unconstrained grids increases in n_A.

The spot market price with market-based redispatch differs from the benchmark scenar-

Figure 2: Generation costs and payments under market-based redispatch and the three theoretical benchmarks for uniformly distributed costs $X \sim U[0,1]$, $k=1$, $n_A=3$



ios because it is determined by the competitive bids in region A. In the example, the spot market price is lower with market-based redispatch. As n_A goes to infinity, the spot market price in each scenario converges to zero due to intense competition.

Total energy payments with market-based redispatch are higher than in the benchmark scenarios due to the additional redispatch payments. The difference in energy payments between market-based redispatch and the VCG mechanism is the additional payment caused by the strategic behavior under market-based redispatch. As n_A increases, the difference in payments between the scenarios with $c=1$ and the scenario with unconstrained grids ($c=2$) increases. As n_A goes to infinity, redispatch payments under idealized cost-based redispatch converge to $1/3$, which equals the expected generation costs of the redispatched provider in B, and redispatch costs with market-based redispatch converge to $2/3$ (the expected generation costs of the provider that does not produce) due to the information rent to the producing provider in B. With the VCG mechanism, the information rent in the limit is also $2/3$. Even in the limit, generators in B make a profit under market-based redispatch and the VCG mechanism because the transmission constraint limits competition from region A. Total energy payments converge to zero with unconstrained grids, to $1/3$ with idealized cost-based redispatch, and to $2/3$ with market-based redispatch and with the VCG mechanism.

4 Impact of market fundamentals

This section focuses on market fundamentals that might influence the likelihood of inc-dec gaming. To this end, we model and analyze the impact of competition and market size, the probability and the level of congestion, as well as the impact of uncertainty about congestion. Formal derivations of the results are given in Appendix A.3.

4.1 Competition and market size

To assess the impact of competition and market size, we vary the number of generators n_A and investigate the limit cases of n_A or k going to infinity.

In the basic model, we have excluded the case $n_A = 2k$, which differs from the case $n_A > 2k$ because there is no competition within A for the $2k$ units. The bidding functions given in Proposition 1 form also an equilibrium in case $n_A = 2k$. However, then a bid β_B^S from market B determines the spot market price, which can therefore be very high. Furthermore, in this case, bids β_A^S just need to be sufficiently low to prevent spot market competition from region B and to assure winning $2k$ units (i.e., $\beta_A^S(x) \leq \mathbb{E}[X_{(k,2k-1)}]$ given β_B^S from (2)).

Next, consider the case of n_A growing infinitely large. The basic model comprises all $n_A \geq 2k + 1$, and the equilibrium bidding functions are independent of the number of generators. This is because competition in A boils down to competition between (the cheapest) $2k$ generators in the redispatch market to avoid producing k units (given that bidding in the spot market is sufficiently aggressive to prevent generators in B from desiring to win). If n_A goes to infinity, although bidding functions are unchanged, the spot market price and the profits of generators in A go to zero due to the selection of the generators with the lowest costs. Thus, an increase in competition lowers the effect of inc-dec gaming but does not dissolve its incentives.

To compare our model with a model of perfect competition and atomistic generators (Holmberg & Lazarczyk, 2015), we increase the number of generators in our game and make them smaller at the same time. To do this, we let k go to infinity, but keep the transmission capacity c and the demand d_B constant. We set $c = 1$ and $d_B = 2c = 2$ for all k. As k increases, the number of generators n_A^k and n_B^k increases, while they become smaller, each providing $1/k$ energy units. Let $n_A^1 \geq 3$ be the number of generators in A for $k = 1$, and $n_A^k = kn_A^1 \geq 3k$ for $k \geq 1$. Similarly, $n_B^1 = 2$ and $n_B^k = kn_B^1 = 2k$. So the total capacity in A is equal to n_A^1 and in B equal to n_B^1 for all k. The size and structure of the market in terms of supply, demand, and transmission capacity is therefore the same for all k. Only the size and number of generators change. To cover demand d_B, $kd_B = 2k$ generators are required, and the transmission capacity c allows $kc = k$ generators in A to deliver to B. As k goes to infinity, the non-cooperative game converges to a non-atomic limit game.[16]

With $n_A^1 \geq 3$, $d_B/n_A^1 = \kappa \in (0, 1)$ for $k \geq 1$. If k goes to infinity, according to (5),

[16]With this approach, we follow Aumann, 1964 and E. J. Green, 1984 and consider the limit of a sequence of non-cooperative games with finite players while maintaining the structure of the game.

the equilibrium bid $\beta_A^S(x)$ in (1) converges to the median of the conditional distribution $F(s \mid s \leq x)$, which is equal to $F^{-1}(F(x)/2)$. Thus, if $2k$ of n_A generators in A win in the spot market, the spot market price p converges to $F^{-1}(\kappa/2)$. According to (7), the redispatch bid $\beta_A^R(x)$ in (3) converges to x for all $x \leq p$, and converges to p for all $x > p$. According to (8), the redispatch bid $\beta_B^R(x)$ in (4) converges to the median of F, which is equal to $F^{-1}(1/2)$, for all generators with costs below the median and to x for all generators with costs higher than the median.

$$\lim_{k\to\infty} \beta_A^S(x) = F^{-1}\left(\frac{F(x)}{2}\right) \quad \text{for } x \in [0,1] \tag{5}$$

$$\lim_{k\to\infty} \beta_B^S(x) \in \left[F^{-1}\left(\tfrac{1}{2}\right), r\right] \tag{6}$$

$$\lim_{k\to\infty} \beta_A^R(x) = \begin{cases} x & \text{if } x \in \left[0, F^{-1}\left(\frac{\kappa}{2}\right)\right] \\ F^{-1}\left(\frac{\kappa}{2}\right) & \text{if } x \in \left(F^{-1}\left(\frac{\kappa}{2}\right), F^{-1}(\kappa)\right] \end{cases} \tag{7}$$

$$\lim_{k\to\infty} \beta_B^R(x) = \begin{cases} F^{-1}\left(\frac{1}{2}\right) & \text{if } x \in \left[0, F^{-1}\left(\frac{1}{2}\right)\right] \\ x & \text{if } x \in \left(F^{-1}\left(\frac{1}{2}\right), 1\right] \end{cases} \tag{8}$$

In the limit, generators in A with costs below p deliver electricity, while those with costs higher than p are redispatched or do not win in the spot market. Thus, the profits of the producing generators are positive in the limit, while the profits of the redispatched generators go to zero. A profit of zero is also made by generators whose costs are higher than $F^{-1}(\kappa/2)$ who thus are not dispatched in the spot market. Generators in B with costs below the median $F^{-1}(1/2)$ deliver electricity, while those with costs higher than the median do not. The profits of the redispatched generators are positive, while the profit of the other generators in B is zero.

When k goes to infinity, given the transmission constraint, the outcome is efficient because the marginal generator in A does not have higher costs than the marginal generator in B ($\kappa/2 \leq 1/2$).

In the limit, our model is one of full information with an infinite number of generators, and the equilibrium outcome is the same as in the equilibrium identified by Holmberg and Lazarczyk (2015). Thus, in the limit, inc-dec gaming persists, but the inefficiency vanishes, as in a setting with perfect competition.

4.2 Varying cost-related congestion

We vary cost-related congestion by varying both the probability of cost-related congestion (i.e., the probability of generation costs for which the transmission capacity prevents that the generators with the lowest costs satisfy the demand) and the cost-related congestion level (i.e., the maximum amount to be redispatched $d_B - k$).

In the basic model, the maximum amount to be redispatched is $d_B - k = 2k - k = k$ units and, whereas maximum redispatch always occurs in equilibrium, the amount of cost-related redispatch is usually less than k and the probability of cost-related congestion is less than one (see Section 3.1).

We extend our analysis by also studying scenarios where this is not the case, i.e., where congestion occurs with probability zero, or where the congestion level is higher than in the basic model. For this purpose, we vary the demand. Since we want to identify and illustrate the basic effects of varying congestion, for simplicity we set $k = 1$ and consider the two cases $d_B = 1$ and $d_B = 3$ (and keep $n_A \geq 2k + 1 = 3$ and $n_B = 2k = 2$).

If $d_B = 1$, congestion is impossible because total demand can be covered by any provider. Any spot market outcome is feasible, and the redispatch stage can be discarded. Therefore, the spot market corresponds to a second-price auction, in which bidding the costs is a weakly dominant strategy. Thus, $\beta_A^S(x) = x$ and $\beta_B^S(x) = x$ constitute an equilibrium.

If $d_B = 3$, one unit of electricity must be produced in A to cover the total demand, while the other two units of electricity must be produced in B due to the transmission capacity $c = 1$. Therefore, both generators in B will deliver electricity no matter which bids they submit. Assuming $n_A > 3$,[17] the equilibrium bidding functions are

$$\beta_A^S(x) = \int_0^x t g_{(1,2)}(t \mid x) dt \text{ for all } x \in (0, 1], \tag{9}$$

$$\beta_B^S(x) \in [\mathbb{E}[X_{(1,2)}], r] \text{ arbitrary for all } x \in [0, 1], \tag{10}$$

$$\beta_A^R(x) = \frac{\int_0^x t g_{(1,2)}(t \mid y) dt}{G_{(1,2)}(x \mid y)} \text{ for all } x \in (0, 1], \tag{11}$$

$$\beta_B^R(x) = r \text{ for all } x \in [0, 1], \tag{12}$$

where $y = (\beta_A^S)^{-1}(p) > x$ denotes the costs of the price-setting provider in the spot market. The boundary bids are $\beta_A^S(0) = 0$ and $\beta_A^R(0) = 0$.

Note that bidding in the redispatch market in region A can also be expressed as $\beta_A^R(x) = \mathbb{E}[X_{(1,2)} \mid X_{(1,2)} < x, X_{(2,2)} < y]$. Thus, generators bid the expected costs of the lower of their two competitors (whom they have to beat), given that the lower of the two competitors' costs is less than x and the higher of the two competitors' costs is less than y. Similarly, their bids in the spot market can be expressed as $\beta_A^S(x) = \mathbb{E}[X_{(1,2)} \mid X_{(2,2)} < x]$, with the intuition identical to that in the basic model (in the worst case conditional on winning, where $y = x$ and the spot market price is $\beta_A^S(x)$, the provider is just indifferent between winning and losing in the spot market). In B, the two generators face no competition in the redispatch market and thus bid the maximum bid r. On the spot market, they bid such

[17] For $n_A = 3$, the bidding functions in the redispatch markets are the same as for $n_A > 3$. Bidding in the spot market assures that generators from A win, but the spot price is determined by a bid from region B.

that they do not win, as in the basic model.

Comparing the cases $d_B \in \{1, 2, 3\}$, we find that generators in A react to higher demand (i.e., higher congestion) by bidding more aggressively (i.e., lower) in the spot market, but generators in B bid competitively in the spot market only if $d_B = 1$. Redispatch is necessary only if $d_B \in \{2, 3\}$. With higher demand, the redispatched amount is higher and therefore competition in the redispatch markets is lower. Thus, bids in the forward redispatch market in region A decrease and bids in the reverse redispatch market in region B increase. These bids feed back into the spot market. Notably, expected payoffs of generators in A do not vary with $d_B \in \{2, 3\}$ because in each case one unit is generated in A.[18]

4.3 Uncertainty about congestion

As a further variation of the model, we study the case of uncertainty about congestion in the network. One motivation for this sensitivity analysis is the claim that inc-dec gaming would only occur when congestion is well predictable.

Methodologically, we implement the uncertainty about congestion as stochastic demand, i.e., generators do not know the level of demand when submitting their bids in the spot market. This is equivalent to uncertainty about congestion because the level of demand determines whether and how much the network is congested. It is not necessary for generators to know the actual demand after the spot market has cleared because then only redispatch quantities matter. The results of the models are given in Appendix A.3.3.

For simplicity, we again set $k = 1$, and examine the case that demand d_B is stochastic in $\{2, 3\}$ each with probability 1/2 and the case that $d_B \in \{1, 2\}$ each with probability 1/2. In both cases, at the redispatch stage, demand for upward and downward redispatch is known. Therefore, the redispatch bidding functions of the basic model (3) and (4) apply when one unit of redispatch is required, and bidding functions of the case $d_B = 3$ (see (11) and (12)) apply when there is demand for two units of redispatch.

In the spot market, if demand is stochastic with $d_B \in \{2, 3\}$ and $n_A > 3$, generators in A choose bids that lie in-between those in the cases $d_B = 2$ and $d_B = 3$ (see Figure 3). Generators with low costs bid more closely to the case $d_B = 2$ whereas those with high costs bid more closely to the case $d_B = 3$. As n_A increases, the bidding function converges to that of case $d_B = 3$. Generators in B bid such that they do not win in the spot market. Depending on the realized demand, there is one or two units of redispatch. Expected payoffs of generators in A are as in the cases $d_B \in \{2, 3\}$, which are equal.

Similarly, if demand is stochastic with $d_B \in \{1, 2\}$ and n_A is sufficiently high, generators in A choose bids that lie in between those in the cases $d_B = 1$ and $d_B = 2$ and generators in

[18]The revenue equivalence theorem applies to comparing cases $d_B \in \{2, 3\}$ because in each case only the provider with the lowest costs generates and the worst-off provider type, the one with costs 1, has an expected payoff of zero.

B bid such that they do not win in the spot market. Depending on realized demand, there is either no or one unit of redispatch.

The analysis reveals that uncertainty about congestion does not necessarily reduce the occurrence of strategic behavior. A reduction in the probability of a binding transmission constraint is ineffective if it is not strong enough (a reduction by 50% in our model is ineffective) and may have unintended consequences (it may prevent the existence of pure-strategy equilibria).

5 Impact of market design

In this section, we analyze regulatory changes to the market design that have been brought forward as a means to mitigate inc-dec gaming: a single bid for spot and redispatch markets, price caps, and uniform pricing in the redispatch market (ENTSO-E, 2021).[19] We expand our model accordingly and provide formal derivations of the results in Appendix A.3. In addition, we qualitatively discuss a hybrid model combining market-based and cost-based redispatch.

5.1 Single bid for spot and redispatch market

This change relates to the argument that the possibility to adjust the bid in the spot market for the redispatch market is problematic. The rationale is that if generators could not adjust their bid, they would be unable to engage in gaming. To assess this claim, we extend the basic model accordingly: We allow only one bid per provider that is used both in the spot market and, if relevant, the respective redispatch market.

With only one bid and $k = 1$, equilibrium payoffs and final allocation are identical as in the basic model in Section 3.1. Intuitively, the bidding functions (3) and (4) form an equilibrium because bidding according to these functions both in the spot and the redispatch market is according to Proposition 1 an equilibrium even if the generators are allowed to submit different bids on the two markets. Thus, this restriction of generators' bids is inconsequential.[20] The same holds in the case of full information, as has been shown by Holmberg and Lazarczyk (2015).[21]

However, such invariance of bidding functions when only one bid is feasible does not hold in general. As shown in Section 3.1, $\beta_A^S(x) < \beta_A^R(x)$ for $k \geq 2$, and, thus, bidding

[19]For an extensive overview of mitigation measures see Klóters et al. (2022).

[20]In contrast to $k \geq 2$, market price information does not play a role in the equilibrium for $k = 1$. This is due to the fact that in this case only two generators in A compete in the redispatch stage. Since they assume in their bidding strategy (3) that the competitor's costs are lower than their own, these are also lower than the spot market price, which thus has not an explicit effect. Therefore, (1) is equal to (3). The bidding functions (2) and (4) in B are independent of the spot market price anyway.

[21]Their equilibrium is outcome equivalent to our equilibrium as k goes to infinity (see Section 4.1).

functions differ. In Appendix A.3.4, we exemplarily show for $k = 2$ the equilibrium bidding function in A for the single-bid case. We also have $\beta_A^S(x) < \beta_A^R(x)$ in the case $k = 1$ and $d_B = 3$ (see (9) and (11)).

Therefore, for these cases, the expected spot market price and the expected redispatch payments with only one bid are different from those in the corresponding case of the basic model. However, the final allocation is identical to that of the basic model, and in both A and B the expected payoff of the weakest provider with $x = 1$ is zero. If these two conditions are met, overall payoff equivalence applies (e.g., Krishna, 2010, propositions 3.1 and 5.2).

Accordingly, the one-shot game with a single bid for both markets has no advantage over the approach with two bids.

5.2 Price caps

Price caps in spot and redispatch markets are proposed to restrict profits from gaming and thus make it less attractive (Klempp et al., 2020). In the basic model, we assume price caps that do not undercut the variable costs of the most expensive generator, i.e., the cap is set at $r \geq 1$. The outcome is the same for all these price caps. The only supply reduction due to bids above costs is by generators with low costs in market B (as those with costs below $\mathbb{E}[X_{(k,2k-1)}]$ bid this amount or more), so overly high bids are not at the core of the problem.

Furthermore, a price cap in the spot market changes the outcome only if it is below the expected median costs, $r < \mathbb{E}[X_{(k,2k-1)}]$. Such a restrictive price cap in the spot market as well as price caps $r < 1$ in the redispatch market in B may prevent a feasible allocation (matching supply and demand) and thus have a negative effect on efficiency.

Price caps are hence not an adequate mitigation option for inc-dec gaming because gaming relies on bids in the range of the variable generation costs to take advantage of arbitrage opportunities.

5.3 Uniform pricing in the redispatch market

The basic model assumes pay-as-bid pricing in the redispatch markets, as it is usually applied in practice. However, changing the price rule in the redispatch markets to uniform pricing yields the same results. Under uniform pricing, the highest rejected bid determines the price in the auction for downward redispatch and the lowest rejected bid determines the price in the auction for upward redispatch. In this setting, it is weakly dominant for generators to bid their costs at their redispatch market, $\beta_A^R(x) = x$ and $\beta_B^R(x) = x$. With these bidding functions, generators' expected payments and winning probabilities in the redispatch stage are the same as in the basic model. Therefore, they bid in the spot market as in the basic model, i.e., as in (1) and (2). As a result, the expected payoffs for all generators and the dispatch are the same under uniform pricing as under pay-as-bid pricing.[22]

5.4 Hybrid model

Another suggestion to avoid inc-dec gaming is a hybrid redispatch model: a combination of cost-based redispatch for generation units and market-based redispatch for all other flexibility units. This approach would integrate all redispatch potentials in one joint merit-order. Market-based bids are used only if they are cheaper than their regulated cost-based counterparts (Cramton, 2019). To our knowledge, a hybrid approach as discussed in this subsection has not been implemented in practice as of 2023. In the following, we illustrate a case where the hybrid approach reduces the incentives for inc-dec gaming and its effects.

For the analysis of a hybrid model, we consider the basic model in Section 3.1 and assume that the costs of all $2k$ generators in B are known. On this basis, we assume cost-based redispatch for the generation units in B and market-based redispatch for the units in A. Because the profit of generators in B in the second stage is zero regardless of whether they are upward redispatched or not, it is optimal for them to bid their costs in the spot market with uniform pricing. Compared to the basic model, this increases competition for the n_A generators from A in the spot market and may result in no need for redispatch. The latter happens when k or more generators from B win in the spot market. This reduces the probability of a provider from A to make a profit with redispatch compared to the basic model where redispatch always occurs. This also reduces the effect of expected redispatch profits on the spot market bid of generators from A, and thus reduces the extent of inc-dec gaming in the form of a strategic deviation from bidding the costs in the spot market.

In conclusion, the example shows that the hybrid model reduces the incentives for inc-dec gaming compared to market-based redispatch.

6 Conclusion

We propose a game-theoretic model to study strategic behavior in redispatch markets. Our model comprises a setting with two interconnected regions within one pricing zone. We extend existing models by considering incomplete information about competitors' costs and a finite set of generators. We identify an equilibrium in which the final dispatch is inefficient due to the strategic behavior of inc-dec gaming. According to our analysis, market-based redispatch leads to inefficient outcomes and high redispatch payments. The sources and extent of inefficiency and high payments are identified by comparing market-based redispatch with different theoretical benchmarks. Moreover, we study multiple variations of our

[22]The payoff equivalence at the second stage and the overall payoff equivalence can also be derived using the payoff equivalence theorem (e.g., Krishna, 2010, propositions 3.1 and 5.2).

basic model to assess whether inc-dec gaming incentives occur only in markets with specific characteristics. Our assessment reveals that inc-dec gaming is also a rational behavior in markets with high competition and with uncertainty about congestion. Finally, we examine whether market design changes can mitigate inc-dec gaming. The analyzed market design changes comprise single bids for spot and redispatch markets, price caps, uniform pricing in the redispatch markets, and a hybrid model. None of these market design changes prevents inc-dec gaming entirely.

Our analysis is well suited to identifying and attributing the problems of market-based redispatch. However, the model has some limitations, and the analysis can be extended in future research in several ways. First, the chosen setting may seem restrictive with respect to the number of generators in B, demand, and transmission capacity. Varying these assumptions can provide additional insights. The same holds for an extension to more than two regions, meshed networks, and multi-unit generators. Besides, an attempt could be made to determine and compare other equilibria of the redispatch game. Another possible extension is the comparison of market-based redispatch and cost-based redispatch in one model framework. The model is also not suited for an analysis of alternative mitigation options in the design of redispatch markets, such as a capacity-based compensation with long-term contracts. Adjusting the model accordingly may be an interesting avenue for further research.

A Appendix

A.1 The equilibrium of the basic model

We prove Proposition 1, which considers the basic model in which $d_B = 2k$, $c = k$, $n_A \geq 2k + 1$, $n_B = 2k$, $k \in \mathbb{N}$, and we have pay-as-bid pricing in the redispatch markets.

We determine a Perfect Bayesian Equilibrium of the two-stage game (spot market followed by redispatch markets). We consider an equilibrium (see Proposition 1) in which the bidders' bidding strategies β_A^S, β_A^R, β_B^S, and β_B^R in the spot market (S) and the redispatch markets (R) are strictly increasing and in which β_A^S and β_B^S have non-overlapping image in the spot market, that is, β_A^S and β_B^S are such that bids in region A are always smaller than bids in region B. To determine the equilibrium, we apply standard techniques from auction theory (e.g., Krishna, 2010).

Our analysis begins with determining the bidding strategies for the redispatch markets on the second stage of the game. In the equilibrium in Proposition 1 that we are going to prove, $2k$ bidders from region A and none from region B win in the spot market. Thus, the amount to be downward redispatched at A is k and there are $2k$ generators that compete to be redispatched; the amount to be upward redispatched at B is k and there are $2k$

generators that compete to be redispatched.

Consider first a representative bidder with costs $x \in [0,1]$ in region A. Assume that other bidders in A in this redispatch market bid according to β_A^R. The representative bidder also uses β_A^R but pretends to have costs z. That is, the bidder submits $\beta_A^R(z)$, where z is the bidder's decision variable. The bidder's expected payoff in the redispatch market in A from either buying back the electricity or producing the electricity is

$$\pi_A^R(x,z) = -G_{(k,2k-1)}(z \mid y)\beta_A^R(z) - \big(1 - G_{(k,2k-1)}(z \mid y)\big)\, x, \tag{A.1}$$

where $G(z \mid y) = F(z)/F(y)$ and $y = (\beta_A^S)^{-1}(p)$, where y are the costs of the price-determining bidder in the spot market in A. These can be derived from the observed spot market price p, which is determined in the spot market by the representative bidder's competitor with the $2k^{th}$-lowest costs. The first-order condition (FOC) of maximizing the profit in (A.1) with respect to z is

$$\frac{\partial \pi_A^R(x,z)}{\partial z} = -\, g_{(k,2k-1)}(z \mid y)\beta_A^R(z) - G_{(k,2k-1)}(z \mid y)(\beta_A^R)'(z) + x g_{(k,2k-1)}(z \mid y) = 0.$$

To find symmetric equilibrium bidding functions for region A, we set $z = x$ and obtain the ordinary differential equation (ODE)

$$g_{(k,2k-1)}(x \mid y)\beta_A^R(x) + G_{(k,2k-1)}(x \mid y)(\beta_A^R)'(x) = x g_{(k,2k-1)}(x \mid y).$$

With the boundary condition $\beta_A^R(0) = 0$ we have

$$\beta_A^R(x) = \frac{\int_0^x t g_{(k,2k-1)}(t \mid y)dt}{G_{(k,2k-1)}(x \mid y)} = \mathbb{E}[X_{(k,2k-1)} \mid X_{(k,2k-1)} < x, X_{(2k-1,2k-1)} < y], \tag{A.2}$$

where $\beta_A^R(x) < x$. The derivative of $\beta_A^R(x)$ is

$$(\beta_A^R)'(x) = \frac{g_{(k,2k-1)}(x \mid y)[xG_{(k,2k-1)}(x \mid y) - \int_0^x t g_{(k,2k-1)}(t \mid y)dt]}{(G_{(k,2k-1)}(x \mid y))^2} > 0 \text{ for all } x \in (0, y],$$

so $\beta_A^R(x)$ is strictly increasing and the maximum is $\beta_A^R(1) = \int_0^1 t f_{(k,2k-1)}(t)dt = \mathbb{E}[X_{(k,2k-1)}]$. Thus, bidders in A submit bids between zero and $\mathbb{E}[X_{(k,2k-1)}]$. With the second derivative

$$\begin{aligned}(\beta_A^R)''(x) =& \frac{g'_{(k,2k-1)}(x \mid y)[xG_{(k,2k-1)}(x \mid y) - \int_0^x t g_{(k,2k-1)}(t \mid y)dt]}{(G_{(k,2k-1)}(x \mid y))^2} + \frac{g_{(k,2k-1)}(x \mid y)}{G_{(k,2k-1)}(x \mid y)} \\ &- \frac{2(g_{(k,2k-1)}(x \mid y))^2[xG_{(k,2k-1)}(x \mid y) - \int_0^x t g_{(k,2k-1)}(t \mid y)dt]}{(G_{(k,2k-1)}(x \mid y))^3},\end{aligned}$$

the second derivative of $\pi_A^R(x,z)$ becomes

$$\frac{\partial^2 \pi_A^R(x,z)}{\partial z^2} = g'_{(k,2k-1)}(z \mid y)(x-\beta_A^R(z)) - 2g_{(k,2k-1)}(z \mid y)(\beta_A^R)'(z) - G_{(k,2k-1)}(z \mid y)(\beta_A^R)''(z),$$

which evaluated at $z = x$ becomes $-g_{(k,2k-1)}(x \mid y) < 0$. Thus, $\beta_A^R(x)$ in (A.2) is the optimal bid in the range $[0, \mathbb{E}[X_{(k,2k-1)}]]$ if the other bidders bid according to β_A^R. Deviating to a bid above $\mathbb{E}[X_{(k,2k-1)}]$ is not profitable because compared to bidding $\mathbb{E}[X_{(k,2k-1)}]$ the winning probability does not change but the bidder's payment to the auctioneer increases. The expected equilibrium payoff of a bidder in the redispatch market in A is denoted by $\pi_A^R(x)$ and is given by

$$\pi_A^R(x) = -\int_0^x t g_{(k,2k-1)}(t \mid y) dt - \left(1 - G_{(k,2k-1)}(x \mid y)\right) x. \tag{A.3}$$

Consider a representative bidder with costs $x \in [0,1]$ in the redispatch market in B using $\beta_B^R(z)$, and the other bidders in B using β_B^R. The bidder's expected profit is

$$\pi_B^R(x,z) = (1 - F_{(k,2k-1)}(z))(\beta_B^R(z) - x). \tag{A.4}$$

The FOC of maximizing (A.4) is

$$\frac{\partial \pi_B^R(x,z)}{\partial z} = (\beta_B^R)'(z) - f_{(k,2k-1)}(z)\beta_B^R(z) - F_{(k,2k-1)}(z)(\beta_B^R)'(z) + x f_{(k,2k-1)}(z) = 0.$$

Setting $z = x$, the ODE becomes

$$(\beta_B^R)'(x) + x f_{(k,2k-1)}(x) = f_{(k,2k-1)}(x)\beta_B^R(x) + F_{(k,2k-1)}(x)(\beta_B^R)'(x). \tag{A.5}$$

Define $\beta_B^R(1) = 1$. The equilibrium bidding strategy β_B^R as a solution of (A.5) is given by

$$\beta_B^R(x) = \frac{\int_x^1 t f_{(k,2k-1)}(t) dt}{1 - F_{(k,2k-1)}(x)} = \mathbb{E}[X_{(k,2k-1)} \mid X_{(k,2k-1)} > x], \tag{A.6}$$

where $\beta_B^R(x) > x$. The derivative of β_B^R is

$$(\beta_B^R)'(x) = \frac{f_{(k,2k-1)}(x)[\int_x^1 t f_{(k,2k-1)}(t) dt - x(1 - F_{(k,2k-1)}(x))]}{(1 - F_{(k,2k-1)}(x))^2} > 0 \text{ for all } x \in [0,1),$$

so $\beta_B^R(x)$ is strictly increasing with the minimum $\beta_B^R(0) = \mathbb{E}[X_{(k,2k-1)}]$, i.e., bidders in B submit bids between $\mathbb{E}[X_{(k,2k-1)}]$ and 1. The second derivative of $\beta_B^R(x)$ in (A.6),

$$(\beta_B^R)''(x) = f'_{(k,2k-1)}(z)(x - \beta_B^R(z)) - 2f_{(k,2k-1)}(z)(\beta_B^R)'(z) + (1 - F_{(k,2k-1)}(z))(\beta_B^R)''(z),$$

becomes $-f_{(k,2k-1)}(x) < 0$ at $z = x$. Thus, $\beta_B^R(x)$ is the optimal bid in the range $[\mathbb{E}[X_{(k,2k-1)}], 1]$ if the other bidders bid according to β_B^R. Deviating to a bid below $\mathbb{E}[X_{(k,2k-1)}]$ is not profitable because compared to bidding $\mathbb{E}[X_{(k,2k-1)}]$ the winning probability does not change but the bidder gets paid less. Deviating to a bid in $(1, r]$ results in a zero payoff. The expected equilibrium payoff of a bidder in the redispatch market in B is

$$\pi_B^R(x) = (1 - F_{(k,2k-1)}(x)) \left(\frac{\int_x^1 t f_{(k,2k-1)}(t)dt}{1 - F_{(k,2k-1)}(x)} - x \right) = \int_x^1 (t - x) f_{(k,2k-1)}(t)dt. \tag{A.7}$$

Given the equilibrium bidding strategies for the redispatch markets, we now consider the spot market on the first stage of the game. Remember, in the equilibrium we prove, bidding strategies β_A^S and β_B^S have non-overlapping image in the spot market. First, we maximize the overall expected payoff of a representative bidder in A to derive the equilibrium bidding strategy β_A^S for the spot market. Assume that other bidders in A bid according to β_A^S and the considered bidder bids $\beta_A^S(z)$.

To derive this expected payoff, note that a deviation to a bid below $\beta_A^S(x)$, i.e., choosing $z < x$, can change the information the bidder has in the redispatch market as compared to the equilibrium beliefs used in the above calculations of the best response in the redispatch market. If the bidder wins in the spot market with such a deviation, the bidder knows that the opponents in the redispatch market have costs lower than those of the price-determining bidder (with costs y) in the spot market. Therefore, any bid above $\beta_A^R(y)$ does not change the winning probability but only increases the bidder's payment to the auctioneer. Because $\partial \pi_A^R(x,z)/\partial z = (x-z) g_{(k,2k-1)}(z \mid y)$ for all $z \in [0, y]$, the expected payoff $\pi_A^R(x, z)$ increases in z for all $z \leq y < x$, and the bidder chooses the highest bid in this range, which is $\beta_A^R(y)$. Using this, we can write the expected payoff of the bidder in A as

$$\int_z^1 \left[\beta_A^S(y) - \int_0^{\min\{x,y\}} t g_{(k,2k-1)}(t \mid y)dt - (1 - G_{(k,2k-1)}(\min\{x, y\} \mid y))x \right] f_{(2k,n_A-1)}(y)dy.$$

The FOC is

$$\left(-\beta_A^S(z) + \int_0^{\min\{x,z\}} t g_{(k,2k-1)}(t \mid z)dt + (1 - G_{(k,2k-1)}(\min\{x, z\} \mid z))x \right) f_{(2k,n_A-1)}(z) = 0.$$

Setting $z = x$, we have

$$\beta_A^S(x) = \int_0^x t g_{(k,2k-1)}(t \mid x)dt = \mathbb{E}[X_{(k,2k-1)} \mid X_{(2k-1,2k-1)} < x], \tag{A.8}$$

which is the expected sample median of a sample of size $2k - 1$ drawn from the distribution with support $[0, x]$. Since in the equilibrium $x \leq y$, (A.8) is smaller than (A.2) and therefore

$\beta_A^S(x) \leq \beta_A^R(x)$. Note that $(\beta_A^S)'(x) = xg_{(k,2k-1)}(x \mid x) > 0$ for all $x \in (0, \mathbb{E}[X_{(k,2k-1)}]$, so $\beta_A^S(x)$ is strictly increasing between 0 and $\mathbb{E}[X_{(k,2k-1)}]$. Thus, the costs of a participant in the redispatch market are lower than y. The individual rationality constraint is satisfied because the expected total payoff of a bidder in A in equilibrium, denoted by $\pi_A(x)$, is

$$\begin{aligned}\pi_A(x) &= \int_x^1 \left[\beta_A^S(y) - \int_0^x t g_{(k,2k-1)}(t \mid y)dt - (1 - G_{(k,2k-1)}(x \mid y))x\right] f_{(2k,n_A-1)}(y)dy \\ &= \int_x^1 \int_x^y (t-x) g_{(k,2k-1)}(t \mid y)dt f_{(2k,n_A-1)}(y)dy \geq 0.\end{aligned}$$

where $g_{(k,2k-1)}(t \mid y) = 0$ for all $t > y$. Deviating to a bid lower than $\beta_A^S(x)$ does not change the expected payoff. Assume that the bidder pretends to be of type $z = x - a < x$, $a \in (0, x]$. If $y < z < x$ or $z < x < y$, the result does not change. In case $z \leq y \leq x$, the bidder receives with z further payments from the spot market but pays the spot market price back in the redispatch market. That is, the payoff is changed by

$$\int_{x-a}^x \left[\beta_A^S(y) - \int_0^y t g_{(k,2k-1)}(t \mid y)dt - (1 - G_{(k,2k-1)}(y \mid y))x\right] f_{(2k,n_A-1)}(y)dy = 0. \quad \text{(A.9)}$$

Deviating to a bid larger than $\beta_A^S(x)$ and lower than $\mathbb{E}[X_{(k,2k-1)}]$ is not profitable. If $x < z < y$, the payoff does not change. If $x < y < z$ or $y < x < z$, the bidder's payoff is zero. Deviating to a bid greater than $\mathbb{E}[X_{(k,2k-1)}]$ results in either the same payoff or a payoff of zero and is therefore also not profitable.

Now consider region B in the spot market. Given that all bidders in A choose β_A^S as well as β_A^R, bidders in B can bid arbitrarily between $\mathbb{E}[X_{(k,2k-1)}]$ and r in the spot market, since their bids influence neither the spot market price nor the results of the redispatch market. The expected total payoff of a bidder in B in equilibrium is $\pi_B(x) = \pi_B^R(x) = \int_x^1 (t-x) f_{(k,2k-1)}(t)dt \geq 0$. If a bidder deviates to a bid z lower than $\mathbb{E}[X_{(k,2k-1)}]$, then either the bidder does not win in the spot market and expects the same non-negative payoff in the redispatch market, or the bidder wins in the spot market and expects a lower payoff $p - x \leq \int_0^1 t f_{(k,2k-1)}(t)dt - x \leq \pi_B(x)$.

Since a bidding strategy is a complete bidding plan for the entire game, we must also consider off-the-equilibrium-path behavior in the redispatch markets (e.g., Krishna, 2010). This comprises the following cases in which the bidder can detect a deviation from equilibrium in the spot market (because the bidder has deviated in the spot market or because spot market price or number of participants in the spot market are impossible in equilibrium) and the bidder can participate in redispatch.

- The bidder in A with costs x bids $\beta_A^S(z)$ where $z < y < x$: This case is already considered in connection with (A.9).

- At A, either k units of electricity are redispatched among $2k$ bidders and the spot market price is $p > \mathbb{E}[X_{(k,2k-1)}]$, or ℓ $(1 \leq \ell < k)$ units of electricity are redispatched among $k+\ell$ bidders: The bidder who has won in the spot market bids as in (A.2) assuming $y = 1$ and, if there are ℓ units demanded in the redispatch market, adjusting the order statistics to the observed number of goods ℓ and opponents $k+\ell-1$. The bidder's beliefs are that there are $2k$ or $k+\ell-1$ opponents with costs drawn from F.
- At B, ℓ, $1 \leq \ell < k$, units of electricity are redispatched among $k+\ell$ bidders. The bidder who has not won in the spot market bids as in (A.6), adjusting the order statistics to the observed number of goods ℓ and opponents $k+\ell-1$. His beliefs are that his opponents are $k+\ell-1$ bidders with costs drawn from F.

With this off-the-equilibrium-path behavior and the on-the equilibrium-path bidding strategies β_A^S, β_B^S, β_A^R, and β_B^R, individual deviations are not profitable and we have thus proven the equilibrium. In this equilibrium, the k bidders with the lowest costs from A and the k bidders with the lowest costs from B deliver electricity. Because there may be more than k generators from B among the $2k$ generators with the lowest costs among all generators, the final dispatch may be inefficient. This proves Proposition 1.

Further illustrations of the results are as follows. Compare the payoffs $\pi_A(x)$ and $\pi_B(x)$ for $x \in [0,1)$ to see that a bidder in B expects higher payoffs than a bidder with the same costs in A:

$$\begin{aligned}
\pi_A(x) &= \int_x^1 \int_x^y (t-x) g_{(k,2k-1)}(t \mid y) dt f_{(2,n_A-1)}(y) dy \\
&= \int_x^1 (1 - F_{(k,2k-1)}(t)) dt - \int_x^1 \frac{\partial \int_x^y (1 - G_{(k,2k-1)}(t \mid y)) dt}{\partial y} F_{(2k,n_A-1)}(y) dy \\
&= \pi_B(x) - \int_x^1 \frac{\partial \int_x^y (1 - G_{(k,2k-1)}(t \mid y)) dt}{\partial y} F_{(2k,n_A-1)}(y) dy < \pi_B(x)
\end{aligned}$$

The equilibrium bidding functions and payoffs in the special case $k = 1$ are:

$$\beta_A^S(x) = \frac{\int_0^x t f(t) dt}{F(x)} \text{ and } \beta_A^S(0) = 0, \quad \beta_A^R(x) = \frac{\int_0^x t f(t) dt}{F(x)} \text{ and } \beta_A^R(0) = 0 \tag{A.10}$$

$$\beta_B^S(x) \in [\mathbb{E}[X], 1] \text{ arbitrary}, \qquad \beta_B^R(x) = \frac{\int_x^1 t f(t) dt}{1 - F(x)} \text{ and } \beta_B^R(1) = 1 \tag{A.11}$$

$$\pi_A(x) = \int_x^1 (1 - F(t))^{n_A - 1} dt \tag{A.12}$$

$$\pi_B(x) = \pi_B^R(x) = \int_x^1 (t-x) f(t) dt \geq 0 \tag{A.13}$$

A.2 Benchmarking market-based redispatch

The generation costs and payments in Table 1 under market-based redispatch and the three theoretical benchmarks for uniformly distributed costs $X \sim U[0,1]$, $k = 1$, and $n_A \geq 3$ are calculated as follows.

Market-based redispatch Energy generation costs: $\mathbb{E}[X_{(1,2)}] + \mathbb{E}[X_{(1,n_A)}] = \frac{1}{3} + \frac{1}{n_A+1}$
Spot market payments: $2 \cdot \beta_A^S(\mathbb{E}[X_{(3,n_A)}]) = 2 \cdot \beta_A^S(\frac{3}{n_A+1}) = 2 \cdot \frac{3}{2(n_A+1)}$
Redispatch payments: $\beta_B^R(\mathbb{E}[X_{(1,2)}]) - \beta_A^R(\mathbb{E}[X_{(2,n_A)}]) = \beta_B^R(\frac{1}{3}) - \beta_A^R(\frac{2}{n_A+1}) = \frac{2}{3} - \frac{1}{n_A+1}$

Unconstrained grid Energy generation costs: $\mathbb{E}[X_{(1,n_A+2)}] + \mathbb{E}[X_{(2,n_A+2)}] = \frac{3}{n_A+3}$
Spot market payments: $2 \cdot \beta(\mathbb{E}[X_{(3,n_A+2)}]) = 2 \cdot \mathbb{E}[X_{(3,n_A+2)}] = 2 \cdot \frac{3}{n_A+3}$

Idealized cost-based redispatch Energy generation costs:

$$\frac{(2n_A+1) \cdot (\mathbb{E}[X_{(1,n_A+2)}] + \mathbb{E}[X_{(2,n_A+2)}]) + \sum_{i=1}^{n_A-1} i \cdot (\mathbb{E}[X_{(1,n_A+2)}] + \mathbb{E}[X_{(n_A+2-i,n_A+2)}])}{\frac{(n_A+1)(n_A+2)}{2}}$$
$$= \frac{2[(2n_A+1) \cdot \frac{3}{n_A+3} + \sum_{i=1}^{n_A-1} i \cdot \frac{n_A+3-i}{n_A+3}]}{(n_A+1)(n_A+2)} = \frac{n_A^2 + 8n_A + 18}{3(n_A+2)(n_A+3)}$$

Spot market payments: $2 \cdot \beta(\mathbb{E}[X_{(3,n_A+2)}]) = 2 \cdot \mathbb{E}[X_{(3,n_A+2)}] = 2 \cdot \frac{3}{n_A+3}$, as with unconstrained grid.
Redispatch payments:

$$\frac{\sum_{i=1}^{n_A-1} i \cdot \mathbb{E}[X_{(n_A+2-i,n_A+2)}] - \frac{n_A(n_A-1)}{2} \cdot \mathbb{E}[X_{(2,n_A+2)}]}{\frac{(n_A+1)(n_A+2)}{2}}$$
$$= \frac{\sum_{i=1}^{n_A-1} i \cdot \frac{n_A+2-i}{n_A+3} - \frac{n_A(n_A-1)}{2} \cdot \frac{2}{n_A+3}}{\frac{(n_A+1)(n_A+2)}{2}} = \frac{n_A(n_A-1)}{3(n_A+2)(n_A+3)}$$

VCG mechanism Energy generation costs: $\frac{n_A^2+8n_A+18}{3(n_A+2)(n_A+3)}$, as with idealized cost-based redispatch.

Energy payments:

$$\frac{2}{(n_A+1)(n_A+2)}\Bigg[2\mathbb{E}[X_{(3,n_A+2)}]+2\sum_{i=1}^{n_A}(\mathbb{E}[X_{(3,n_A+2)}]+\mathbb{E}[X_{(i+2,n_A+2)}])$$
$$+\sum_{i=3}^{n_A+1}\sum_{j=i}^{n_A+1}(\mathbb{E}[X_{(2,n_A+2)}]+\mathbb{E}[X_{(j+1,n_A+2)}])\Bigg]$$
$$=\frac{2\left[\frac{6}{n_A+3}+\frac{n_A(n_A+11)}{n_A+3}+\sum_{i=3}^{n_A+1}\sum_{j=i}^{n_A+1}\frac{j+3}{n_A+3}\right]}{(n_A+1)(n_A+2)}=\frac{2(n_A^2+8n_A+18)}{3(n_A+2)(n_A+3)}$$

A.3 Model extensions

A.3.1 Basic model with two bidders in region A (varying competition)

Consider the basic model with $k=1$ but with only two bidders in region A, $n_A=2$. As there are exactly as many bidders in A as there is demand, the difference between this model and the basic model in Appendix A.1 is that all bidders in A win in the spot market. Thus, the spot market price is determined in B rather than in A.

Consider a representative bidder i with costs x in A in the redispatch market. Given that the other bidder uses β_A^R, bidder i's expected payoff in the redispatch market in A by bidding $\beta_A^R(z)$ is $\pi_A^R(x,z)=-F(z)\beta_A^R(z)-(1-F(z))x$. By setting $z=x$ in the FOC and defining $\beta_A^R(0)=0$, the equilibrium bidding function is

$$\beta_A^R(x)=\frac{\int_0^x tf(t)dt}{F(x)}=\mathbb{E}[X\mid X<x], \tag{A.14}$$

which corresponds to (A.10) in the basic model with $k=1$. Thus, $\beta_A^R(x)$ in (A.14) is strictly increasing from zero to $\mathbb{E}[X]$ and deviating to another bid is not profitable.

As the spot market is not competitive for the bidders in A, they can bid arbitrarily in the range $[0,\mathbb{E}[X]]$. Deviating to a bid greater than $\mathbb{E}[X]$ either does not change the result or leads to a losing bid with zero payoff. The expected total payoff of bidders in A, $\pi_A(x)=\mathbb{E}[P^S]-\int_0^x tf(t)dt-(1-F(x))x>0$, is positive and depends on the expected spot market price $\mathbb{E}[P^S]$, where extreme outcomes like spot market prices $p^S=r$ are possible.

For bidders in B, both $\beta_B^R(x)$ and $\beta_B^S(x)$ are the same as in (A.11), and so is the expected payoff $\pi_B(x)$.

A.3.2 Model with variation of demand (varying cost-related congestion)

Consider the basic model with $k=1$ but with demand $d_B=1$. Since the total demand is equal to the transmission capacity for an electricity exchange between A and B, there is no congestion. That is, A and B can be considered as one perfectly interlinked region, any spot market outcome is feasible, and no redispatch will take place. The spot market corresponds to a second-price auction, in which bidding the costs is a weakly dominant strategy. Thus, $\beta_A(x) = x$ and $\beta_B(x) = x$ constitute an equilibrium. The expected payoffs in equilibrium are $\pi(x) = \int_x^1 (t - x) f_{(1,n_A+1)}(t)dt$.

Consider the basic model with $k = 1$ but with demand $d_B = 3$. In this case, in order to cover the total demand, one unit of electricity must be produced in region A, while the other two units of electricity must be produced in region B. As a result, both bidders in B will deliver electricity no matter which bids they submit. Consider first the redispatch market in B. As both bidders in B will win, it is optimal to bid the highest possible bid, r. Thus, $\pi_B^R(x) = r - x$.

Consider now the redispatch market in A. In case $d_B = 3$ and bidding strategies with non-overlapping image in the spot market, three bidders with the lowest costs in A win in the spot market and participate in the redispatch market. Assuming the other bidders in A use β_A^R, the expected payoff of a bidder in A with costs x who bids $\beta_A^R(z)$ is $\pi_A^R(x, z) = -G_{(1,2)}(z \mid y)\beta_A^R(z) - (1 - G_{(1,2)}(z \mid y))x$. Analogous to the basic model,

$$\beta_A^R(x) = \frac{\int_0^x t g_{(1,2)}(t \mid y)dt}{G_{(1,2)}(x \mid y)} = \mathbb{E}[X_{(1,2)} \mid X_{(1,2)} < x, X_{(2,2)} < y], \tag{A.15}$$

which is strictly increasing between zero and $\mathbb{E}[X_{(1,2)}]$. No bidder has an incentive to deviate from this bidding function and the expected payoff of a bidder in the redispatch market in A is $\pi_A^R(x) = -\int_0^x t g_{(1,2)}(t \mid y)dt - (1 - G_{(1,2)}(x \mid y))x$.

Now, consider the equilibrium bidding strategies in the spot market. As shown in Appendix A.1, choosing $z < x$ can change the information in the redispatch market, where the bidder will submit $\beta_A^R(x)$ if $x \leq y$ and $\beta_A^R(y)$ if $x > y$. Thus, the expected total payoff of the bidder in A is

$$\int_z^1 \left[\beta_A^S(y) - \int_0^{\min\{x,y\}} t g_{(1,2)}(t \mid y)dt - (1 - G_{(1,2)}(\min\{x, y\} \mid y))x \right] f_{(3,n_A-1)}(y)dy,$$

which leads to the bidding function

$$\beta_A^S(x) = \int_0^x t g_{(1,2)}(t \mid x)dt = \mathbb{E}[X_{(1,2)} \mid X_{(2,2)} < x] < \beta_A^R(x).$$

β_A^S is strictly increasing between zero and $\mathbb{E}[X_{(1,2)}]$. With the same arguments as in the basic model we can show that deviating to a bid lower than $\beta_A^S(x)$ does not change the expected payoff, while deviating to a larger bid decreases the expected payoff.

The expected total payoff of a bidder in A in equilibrium is

$$\pi_A(x) = \int_x^1 \left[\int_0^y t g_{(1,2)}(t \mid y) dt - \int_0^x (t g_{(1,2)}(t \mid y) dt - (1 - G_{(1,2)}(x \mid y)) x) \right] f_{(3, n_A - 1)}(y) dy$$
$$= \int_x^1 (1 - F(y))^{n_A - 1} dy \geq 0. \tag{A.16}$$

Now, consider spot market bidding in B. Given $\beta_A^S(x)$, $\beta_A^R(x)$, and $\beta_B^R(x)$, both bidders in B can bid arbitrarily between $\mathbb{E}[X_{(1,2)}]$ and r in the spot market and expect a payoff of $r - x$ from the upward redispatch market. They already achieve the maximum possible expected payoff under the current situation and have thus no incentive to deviate.[23]

Note that $\beta_A^S(x) < \beta_A^R(x)$, and both functions are smaller than the respective bidding function in the basic model. That is, bidders in A bid more aggressively (lower) in the spot market and less aggressively (lower) in the redispatch market when the demand increases. Since only the bidder with the lowest bid will deliver electricity, while the other two winning bidders expect a positive payoff through downward redispatch, the bidders in A bid less to win in the spot market.

A.3.3 Model with stochastic demand (uncertainty about congestion)

Consider the basic model with $k = 1$ but with stochastic demand $d_B = \{50\% : 2, 50\% : 3\}$. At the redispatch stage, spot market demand has realized and bidders participating in the redispatch markets know the demand for redispatch. Thus, they choose the bidding strategy as derived in the basic model for $d_B = 2$ and as derived in the model in Appendix A.3.2 for $d_B = 3$ which in what follows are labeled by $d = 2$ and $d = 3$ in the superscript of the bidding strategy, respectively. That is, in case $d_B = 2$ (cp. (A.10) and (A.11)), we have

$$\beta_A^{R,d=2}(x) = \frac{\int_0^x t f(t) dt}{F(x)} \quad \text{and} \quad \beta_B^{R,d=2}(x) = \frac{\int_x^1 t f(t) dt}{1 - F(x)},$$

and in case $d_B = 3$ (cp. (A.15)),

$$\beta_A^{R,d=3}(x) = \frac{\int_0^x t g_{(1,2)}(t \mid y) dt}{G_{(1,2)}(x \mid y)} \quad \text{and} \quad \beta_B^{R,d=3}(x) = r.$$

Given the bidding strategies in the redispatch markets, we now maximize the overall expected payoff of a representative bidder in A to derive the equilibrium bidding function in the spot market. We solve for equilibrium bidding strategies with non-overlapping image in the spot market ($\beta_A^S(1) \leq \beta_B^S(0)$). As shown in appendices A.1 and A.3.2, if the bidder

[23] If $n_A = 3$, analogous to the case $n_A = 2$ in Appendix A.3.1, the bids in B in the spot market determine the spot market price, which can take values up to r. Thus, $\beta_A^S(x) \in [0, \mathbb{E}[X_{(1,2)}]]$ arbitrary. Note that the interval bounds can also be $[0, \mathbb{E}[X]]$ and $[\mathbb{E}[X], r]$, as long as $\beta_A^S < \beta_B^S$.

deviates in the spot market to $z \leq y < x$, then $\beta_A^R(y)$ will be submitted in the redispatch market in A. The total expected payoff is

$$\pi_A(x,z) = \frac{1}{2}\left[\int_z^1 \left(\beta_A^S(y) - \frac{\int_0^{\min\{x,y\}} tf(t)dt + (F(y) - F(\min\{x,y\}))x}{F(y)}\right) f_{(2,n_A-1)}(y)dy\right]$$
$$+\frac{1}{2}\left[\int_z^1 \left(\beta_A^S(y) - \int_0^{\min\{x,y\}} tg_{(1,2)}(t \mid y)dt - (1 - G_{(1,2)}(\min\{x,y\} \mid y))x\right) f_{(3,n_A-1)}(y)dy\right].$$

FOC:

$$\frac{1}{2}\left(-\beta_A^S(z) + \frac{\int_0^{\min\{x,z\}} tf(t)dt + (F(z) - F(\min\{x,z\}))x}{F(z)}\right) f_{(2,n_A-1)}(z)$$
$$+\frac{1}{2}\left[-\beta_A^S(z) + \int_0^{\min\{x,z\}} tg_{(1,2)}(t \mid z)dt + (1 - G_{(1,2)}(\min\{x,z\} \mid z))x\right] f_{(3,n_A-1)}(z) = 0$$

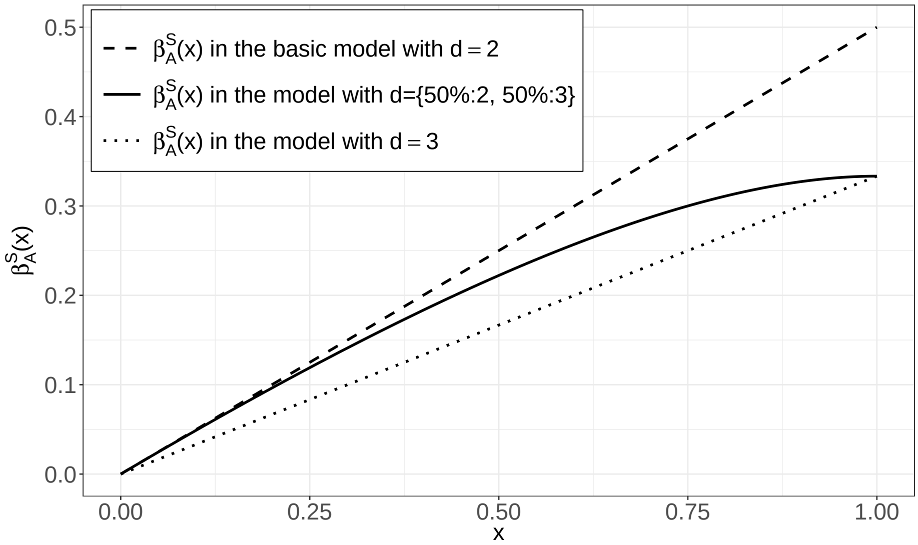
Setting $z = x$ and simplifying the FOC, we have

$$\begin{aligned}
\beta_A^S(x) &= \frac{(1-F(x))\beta_A^{S,d=2}(x) + \frac{1}{2}(n_A-3)F(x)\beta_A^{S,d=3}(x)}{(1-F(x)) + \frac{1}{2}(n_A-3)F(x)} \\
&= \beta_A^{S,d=2}(x) - \frac{\frac{1}{2}(n_A-3)F(x)(\beta_A^{S,d=2}(x) - \beta_A^{S,d=3}(x))}{(1-F(x)) + \frac{1}{2}(n_A-3)F(x)} \\
&= \beta_A^{S,d=3}(x) + \frac{(1-F(x))(\beta_A^{S,d=2}(x) - \beta_A^{S,d=3}(x))}{(1-F(x)) + \frac{1}{2}(n_A-3)F(x)}
\end{aligned}$$

Since $\beta_A^{S,d=3}(x) \leq \beta_A^{S,d=2}(x)$, it holds that $\beta_A^{S,d=3}(x) \leq \beta_A^S(x) \leq \beta_A^{S,d=2}(x)$, that is, the bidding function in case of stochastic demand lies between the bidding functions in cases $d_B = 2$ and $d_B = 3$. Moreover, $\beta_A^S(x)$ is strictly increasing between zero and $\mathbb{E}[X_{(1,2)}]$. Note that the upper limit corresponds to the upper limit of $\beta_A^{S,d=3}(x)$. This means that with higher costs (close to 1), bids are closer to those in case $d_B = 3$, while bidders with lower costs bid closely to $\beta_A^{S,d=2}(x)$. Furthermore, $\beta_A^S(x)$ converges to $\beta_A^{S,d=3}(x)$ if n_A converges to infinity. This indicates that the bidder submits a relatively low bid (closer to the case $d_B = 3$ than to $d_B = 2$) if the bidder's costs are unlikely among the lowest costs (as the bidder's costs are high or there are many bidders) because if the bidder wins it is unlikely that case $d_B = 2$ has realized. To the contrary, the bidder submits a relatively high bid (closer to $d_B = 2$ than to $d_B = 3$) if the bidder's costs are likely among the lowest costs. Figure 3 illustrates these characteristics of the bidding functions, using a uniform distribution ($F(x) = x$) and $n_A = 4$.

As shown in appendices A.1 and A.3.2, both if $d_B = 2$ and if $d_B = 3$, deviating to a lower bid does not change the expected payoff, while a bid larger than $\beta_A^S(x)$ decreases the

Figure 3: Spot market bidding functions for stochastic and fixed demand



expected payoff. The expected total payoff of a bidder in A in equilibrium is

$$\begin{aligned}
\pi_A(x) =& \frac{1}{2}\left[\int_x^1 (\beta_A^S(y) - \int_x^1 \frac{\int_0^x tf(t)dt + (F(y)-F(x))x}{F(y)}) f_{(2,n_A-1)}(y)dy\right] \\
&+ \frac{1}{2}\left[\int_x^1 (\beta_A^S(y) - \int_0^x tg_{(1,2)}(t \mid y)dt - (1-G_{(1,2)}(x \mid y))x) f_{(3,n_A-1)}(y)dy\right] \\
=& \int_x^1 (1-F(y))^{n_A-1}dy \geq 0.
\end{aligned}$$

This expected total payoff is equal to (A.12) in the basic model with $k = 1$ and to (A.16) in the model with $d_B = 3$, which corresponds to the revenue equivalence theorem. That is, the amount of d_B does not have an effect on the expected total payoff of a bidder in A, since only one bidder will supply the electricity and the bidder with costs $x = 1$ has the same expected payoff, zero.

Given the bidding strategies of bidders in A, bidders in B can bid arbitrarily between $\mathbb{E}[X_{(1,2)}]$ and one in the spot market. As a result, they do not win in the spot market and both bidders participate in the redispatch market. As stated before, their bids in the spot market neither have an effect on the spot market price nor on the redispatch price. The expected total payoff is $\pi_B(x) = \frac{1}{2}\int_x^1 (t-x)f(t)dt + \frac{1}{2}(r-x) \geq 0$. Deviating to a lower bid z in $[0, \mathbb{E}[X_{(1,2)}]]$ is not profitable: either the bidder does not win in the spot market and the expected payoff does not change, or the bidder wins and expects a lower payoff $p - x \leq \mathbb{E}[X_{(1,2)}] - x \leq \pi_B(x)$.

Consider the basic model with $k = 1$ but with stochastic demand $d_B = \{50\% : 1, 50\% : 2\}$. In case $d_B = 1$, no redispatch is needed. In case $d_B = 2$, the bidders will choose the same bidding functions for the redispatch market as in the basic model, see (A.10) and (A.11).

Given the bidding strategies in the redispatch market in A, we now maximize the overall expected payoff of a representative bidder in A to derive the equilibrium bidding function in the spot market. We solve for an equilibrium with bidding strategies with non-overlapping image in the spot market ($\beta_A^S(1) \leq \beta_B^S(0)$). As shown in Appendix A.1, if the bidder deviates in the spot market to $z \leq y < x$, then $\beta_A^R(y)$ will be submitted in the redispatch market in A. The total expected payoff is

$$\pi(x,z) = \frac{1}{2}\left[\int_z^1 \beta_A^S(y) f_{(1,n_A-1)}(y)dy - (1 - F_{(1,n_A-1)}(z)))x\right] \\ + \frac{1}{2}\left[\int_z^1 (\beta_A^S(y) - \frac{\int_0^{\min\{x,y\}} tf(t)dt + (F(y) - F(\min\{x,y\}))x}{F(y)}) f_{(2,n_A-1)}(y)dy\right].$$

FOC:

$$\frac{1}{2}(x - \beta_A^S(z)) f_{(1,n_A-1)}(z) \\ + \frac{1}{2}\left(- \beta_A^S(z) + \frac{\int_0^{\min\{x,z\}} tf(t)dt + (F(z) - F(\min\{x,z\}))x}{F(z)}\right) f_{(2,n_A-1)}(z) = 0.$$

Setting $z = x$ and simplifying the FOC, we have

$$\beta_A^S(x) = \frac{(1 - F(x))x + (n_A - 2)\int_0^x tf(t)dt}{(1 - F(x)) + (n_A - 2)F(x)}.$$

It is easy to show that $\beta_A^{S,d=2}(x) = \frac{\int_0^x tf(t)dt}{F(x)} \leq \beta_A^S(x) \leq x = \beta_A^{S,d=1}(x)$ and $\beta_A^S(1) = \mathbb{E}[X]$. Furthermore, $\beta_A^S(x)$ is strictly increasing.

For bidders in B we have $\beta_B^S(x) \in [\mathbb{E}[X], 1]$ and

$$\pi_B(x) = \frac{1}{2}(1 - F(x))(\beta_B^R(x) - x). \tag{A.17}$$

Note that depending on n_A and F, bidders in B may have an incentive to deviate to a bid less than $\mathbb{E}[X]$ and, thus, existence of an equilibrium with bidding strategies with non-overlapping image in the spot market may require that n_A is sufficiently large in this setting with $d_B = \{50\% : 1, 50\% : 2\}$. For example, in case of $n_A = 3$ and a uniform distribution F, if the bidder in B with $x = 0$ bids 0, the expected payoff (from selling in the spot market instead of the redispatch market) is $\frac{3}{8} > \frac{1}{4} = \pi_B(0)$. However, if n_A is sufficiently large, such an equilibrium with $\beta_B^S(x) \in [\mathbb{E}[X], 1]$ exists. Note that $\beta_A^S(x)$ is strictly decreasing in

n_A, as

$$\frac{\partial \beta_A^S(x)}{\partial n_A} = \frac{(1-F(x))(\int_0^x tf(t)dt - xF(x))}{((1-F(x)) + (n_A-2)F(x))^2} < 0 \text{ for all } x \in (0,1).$$

This means that if n_A converges to infinity, the expected spot market price converges to 0. As a result, the potential profitability of a deviation by a bidder in B to a bid less than $\mathbb{E}[X]$ vanishes as n_A increases, because the expected payoff in (A.17) is independent of n_A and is strictly positive for all $x < 1$. Therefore, there exists a finite $\underline{n}$ (whose minimum size depends on the distribution) so that for all $n_A > \underline{n}$, the bidding functions above constitute an equilibrium.

A.3.4 Model with a single bid for spot market and redispatch market

Consider the basic model with $k = 1$, but assume that each bidder can submit only a single bid, which applies to the spot market and the redispatch market (one-shot game), denoted by β_A and β_B for the two regions.

We again solve for an equilibrium with bidding strategies with non-overlapping image in the spot market, that is, β_A and β_B are such that bids in A are always smaller than bids in B. Assume that in both regions the bidding strategies β_A and β_B are strictly increasing. Consider first a representative bidder with costs $x \in [0,1]$ in A. Assume that the other bidders in A use β_A and all bidders in B use β_B. The bidder also uses β_A but pretends to have costs z. That is, the bidder submits $\beta_A(z)$, where z is the bidder's decision variable. The bidder's expected profit is

$$\pi_A(x,z) = \int_z^1 \beta_A(t) f_{(2,n_A-1)}(t)dt - \big(1 - F_{(1,n_A-1)}(z)\big)\, x - \big(F_{(1,n_A-1)}(z) - F_{(2,n_A-1)}(z)\big)\, \beta_A(z).$$

The FOC is $-f_{(1,n_A-1)}(z)\beta_A(z) - F_{(1,n_A-1)}(z)\beta_A'(z) + F_{(2,n_A-1)}(z)\beta_A'(z) + f_{(1,n_A-1)}(z)x = 0$.

Setting $z = x$ and $\beta_A(0) = 0$, the ODE is solved by $\beta_A(x) = \int_0^x tf(t)dt/F(x)$, which corresponds to (A.10) in the basic model, and is thus strictly increasing from zero to $\mathbb{E}[X]$. The bidder expects the same payoff $\pi_A(x)$ as in (A.12).

Note that $\beta_A(x) = \beta_A^S(x) = \beta_A^R(x)$ is a special case that occurs with $k = 1$. For $k \geq 2$, the equality relations do not apply. For instance, assume a uniform distribution F, $k = 2$, and $n_A = 6$. In the basic model, $\beta_A^S(x) = x/2$ while $\beta_A^R(x) = (1 - e^{2x} + 2x + 2x^2 + 4x^3)/(4x^2)$, which is strictly increasing and overlaps with $\beta_A^S(x)$. Compared to the basic model, $\beta_A(x)$ leads to a lower expected spot market price $5(163 - 21e^2)/(112) < 5/7$. Note that in both models, the dispatch is the same as the equilibrium bidding functions are strictly increasing, and the expected payoff for the type $x = 1$ is 0. According to the revenue equivalence theorem (e.g., Krishna, 2010), the expected payoffs in the equilibrium must be the same. Thus, expected redispatch prices are lower in this model than in the basic model.

Now consider bidders in region B. The equilibrium bidding strategies and the expected payoffs are the same as in the basic model, see (A.11) and (A.13).

A.3.5 Uniform pricing in the redispatch market

Consider the basic model with $k = 1$ but instead of pay-as-bid pricing consider uniform pricing (UP) in the redispatch market, where the price is determined by the highest (lowest) rejected bid in the downward (upward) redispatch market.

In general, the expected payoff of each bidder and of the seller in the equilibria of two auctions are the same if the dispatch is the same and the expected payoff for the weakest type is the same (revenue equivalence theorem, e.g., Krishna (2010), Proposition 5.2). Thus, if the dispatch under UP and pay-as-bid is the same and the weakest type ($x = 1$) earns zero under UP as under pay-as-bid, the auctions are revenue equivalent. In what follows, we prove explicitly that the model with UP is payoff-equivalent with our basic model.

If the UP rule is used in the redispatch markets, all bidders have a weakly dominant strategy to bid their costs, that is, by bidding $\beta_A^R(x) = x$ the bidder is weakly better off than with any other bid independent of the bids of the opponents. To prove this, we consider first a representative bidder with costs $x \in [0, 1]$ in A. If the bidder bids x, the bidder may either win or not win in the redispatch market. If the bidder bids x and wins, the payoff is $p - p^R$, where p is the spot market price and p^R, $p^R \leq x$, is the uniform redispatch price set by a different bidder. If the bidder deviated to a different bid $\geq p^R$, the payoff would not change. If the bidder deviated to a different bid $< p^R$ the payoff would become $p - x$, which is weakly less than $p - p^R$. If the bidder bids x and does not win, the payoff is $p - x$ and we have $x \leq p^R$. If the bidder deviated to a different bid $\leq p^R$ the payoff would not change. If the bidder deviated to a different bid $> p^R$, the bidder would win at a price $\geq p^R$, so the payoff would become weakly less than $p - p^R$, which is weakly less than $p - x$. Thus, with any deviation from $\beta_A^R(x) = x$ the bidder is weakly and sometimes strictly worse off.

The bidder's expected payoff in the redispatch market in A if all bidders bid their costs is $\pi_A^R(x) = -\int_0^x tg(t \mid y)dt - (1 - G(x \mid y))x$, which is identical to that in the basic model (A.3).

Now consider a representative bidder with costs $x \in [0, 1]$ in B. The argument to show that bidding the costs is weakly dominant strategy is essentially the same as for region A. The expected profit of a bidder in the redispatch market in B if all bidders bid their costs is $\pi_B^R(x) = \int_x^1 (t - x)f(t)dt$, which is identical to that in the basic model (A.7).

Given these identical expected payoffs in the redispatch markets as in the basic model, the equilibrium bidding strategies in the spot market and, thus, the expected total payoffs are the same as in the basic model.

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