

# Investment Behavior under Ambiguity: The Case of Pessimistic Decision Makers

Alexander Ludwig and Alexander Zimmer

**60-2004**

August 2004

# Investment Behavior under Ambiguity: The Case of Pessimistic Decision Makers\*

Alexander Ludwig<sup>†</sup>      Alexander Zimmer<sup>‡</sup>

August 2004

## Abstract

We define pessimistic, respectively optimistic, investors as CEU (*Choquet expected utility*) decision makers who update their pessimistic, respectively optimistic, beliefs according to a pessimistic (Dempster-Shafer), respectively optimistic, update rule. This paper then demonstrates that, in contrast to optimistic investors, pessimistic investors may strictly prefer investing in an illiquid asset to investing in a liquid asset. Key to our result is the dynamic inconsistency of CEU decision making, implying that a CEU decision maker ex ante prefers a different strategy with respect to prematurely liquidating an uncertain long-term investment project than after learning her liquidity needs. Investing in an illiquid asset then serves as a commitment device guaranteeing an ex ante favorable outcome.

*Keywords:* Ambiguity, Choquet Expected Utility Theory, Bayesian Updating, Pessimism, Optimism

*JEL Classification Numbers:* D81, G20.

---

\*We thank Jürgen Eichberger, Itzhak Gilboa, Nicolas Nalpas, and Joachim Winter for helpful suggestions. Moreover, we appreciate the feedback from participants at FUR XI, Paris. Financial support from the Deutsche Forschungsgemeinschaft, SFB 504, at the University of Mannheim and from the Gesamtverband der Deutschen Versicherungswirtschaft is gratefully acknowledged.

<sup>†</sup>Mannheim Research Institute for the Economics of Aging, University of Mannheim, L13, 17, 68131 Mannheim, Germany. Email: ludwig@mea.uni-mannheim.de

<sup>‡</sup>Corresponding author. Sonderforschungsbereich 504, University of Mannheim, L13, 15, 68131 Mannheim, Germany. Email: zimper@bigfoot.com

# 1 Introduction

This paper analyzes the investment decision of an individual who may choose between two uncertain investment projects with identical expected returns: a liquid and an illiquid investment project. Conventional wisdom suggests that the individual at least weakly prefers investing in a liquid asset to investing in an illiquid asset. As Laibson (1997) shows in his seminal contribution, this insight fails to hold for individuals with dynamically inconsistent preferences as produced with hyperbolic discount functions. Such preferences motivate individuals to constrain their future choices by investing in an illiquid asset.

We establish the same insight for a *Choquet expected utility* (CEU) decision maker. Moreover, we argue that such an investment behavior is more plausible for *pessimistic* rather than for *optimistic* CEU decision makers. The dynamic inconsistency of CEU decision making implies that a CEU decision maker ex ante prefers a different liquidation strategy with respect to prematurely liquidating a risky long-term investment project than ex post, i.e., after learning her liquidity needs. Investing in the illiquid asset then serves as commitment device guaranteeing an ex ante favorable outcome.

CEU theory (Schmeidler, 1987; Gilboa, 1988) is by now the standard generalization of expected utility theory which incorporates *ambiguity attitudes* (as first elicited by Ellsberg, 1961) by considering decision makers who maximize expected utility with respect to non-additive beliefs. Applied to situations of sequential choice, the assumption of non-additive beliefs implies that a CEU decision maker's preferences are - in general - not *dynamically consistent*, that is, preferences may change upon receiving new information. We exploit this particular dynamic inconsistency of CEU decision making in our commitment-model of investment. After investing in the project, but before its maturation, an individual learns her *liquidity type*, i.e., she learns if she either has a high or a low desire for liquidity. Using this information the individual updates her beliefs. The assumption of non-additive beliefs gives rise to several perceivable Bayesian update rules (Gilboa and Schmeidler, 1993). For two standard Bayesian update rules - the *optimistic* and the *pessimistic* update rule - we derive conditions such that realized liquidity types have strict incentives to deviate from the ex ante optimal *liquidation strategy*. In a next step, we define an optimistic, respectively pessimistic, individual as a CEU decision maker who updates her beliefs according to the *optimistic*, respectively *pessimistic*, update rule, and who additionally has so-called *optimistic*, respectively *pessimistic*, beliefs. As our main result, we show that *pessimistic* rather than *optimistic* individuals strictly prefer to invest in the illiquid asset.

The intuition behind our formal result is that sequentially rational individuals expect to become more pessimistic in the future if they update new information according to

the pessimistic update rule. As a consequence, individuals with pessimistic beliefs tend to a premature liquidation of the uncertain long-term investment projects because their decision making becomes more preoccupied with the possibility of the project's failure. The investor realizes that she will be more pessimistic in the future than from her ex ante perspective and therefore strictly prefers to sign a contract which guarantees, by assumption, that there does not occur any premature liquidation of the uncertain long-term investment project. That is, she strictly prefers to invest in an illiquid investment project. On the other hand, the future realizations of an optimistic investor shift their expectations towards the project's success, so that a sequentially rational individual could not achieve any commitment-advantage from investing in the illiquid investment project.

The decision theoretic literature on dynamic inconsistencies of Non-EU decision making is mainly motivated by the view that dynamic consistency is a desirable - if not a normatively indispensible - property of preferences, as argued by Kreps and Porteus (1979) and by Hammond (1988). Epstein and Le Breton (1993), as well as Eichberger and Kelsey (1996), consider dynamic consistency as a general property of preferences, so that an individual's preferences are dynamically consistent, if and only if, they are dynamically consistent in all possible decision-trees resulting from a given state-space.<sup>1</sup> As their main result, Epstein and Le Breton (1993) demonstrate that dynamically consistent updating of beliefs requires additive beliefs (compare also Eichberger and Kelsey (1996), who show the same result for the special case of CEU decision making). In contrast, subsequent research has explored conditions which assure dynamic consistency of Non-EU decision making when only a given single decision-tree is considered (Sarin and Wakker, 1998; Eichberger and Kelsey, 2004). For example, in the dynamic decision situation, as considered by Sarin and Wakker (1998), CEU decision making is dynamically consistent only if beliefs are additive at the first stage of the decision-tree whereas they may be non-additive at the second - and final - stage of the decision-tree.

In contrast to this normative approach, we treat the possible dynamic inconsistency of CEU decision making as a given fact that may bear implications for economic institutions. In particular, we consider a CEU decision maker who is ex ante aware of her dynamically inconsistent preferences, so that she may desire some device for intrapersonal commitment.

The role of economic institutions and financial products as commitment devices for individuals who recognize that their preferences are dynamically inconsistent, has already been discussed in Strotz (1956). More recently, this approach has become very popular within the context of so-called *hyperbolic discount* functions (e.g., Laibson, 1997)

---

<sup>1</sup>Also see Epstein and Schneider, 2003.

where an individual may have different ex ante preferences over saving-decisions than ex post. Our approach follows this literature on using economic institutions and financial products as ways of self-commitment by presuming a *sequentially rational* decision maker, that is, a decision maker who anticipates her future decisions and therefore chooses her optimal strategy by back-folding. Sequential rationality therefore implies that the decision maker is ex ante well aware of the occurrence of these changes, i.e., of her dynamically inconsistent behavior. What distinguishes our approach from this existing literature on the role of economic institutions as intra-personal commitment devices is the entirely different mechanism by which dynamic inconsistencies may enter our model: by decision making under ambiguity of, especially, pessimistic individuals.

There is a long list of real world examples for financial products that provide forms of commitment, such as pension plans, consumer durables and home equity, see Laibson (1997) for a more detailed discussion. Laibson points out that such “illiquid assets provide a form of commitment, though there are sometimes additional reasons that consumers might hold such assets (e.g., high expected returns and diversification).” We follow Laibson and abstract from such additional differences in financial products and economic institutions and only focus on the liquidity aspect by modelling a very stylized investment decision.

The remainder of this paper is organized as follows. Section 2 introduces our model of the liquid investment project. In section 3 the ex ante decision situation of a CEU decision maker is described who has to choose a liquidation strategy. The individual’s ex post - after learning her liquidity needs - decision problem is explored in Section 4. Section 5 introduces our notion of a stylized commitment technology provided by an illiquid investment project. In section 6 we state first results, concerning the strict preference of CEU decision makers for intra-personal commitment by investing in the illiquid asset. Our main results, referring to our definitions of optimistic, resp. pessimistic, individuals, are derived in section 7. Section 8 concludes.

## 2 Investing in a Liquid Asset

Suppose an individual invests in period 0 a fixed amount of money, denoted  $x$ , in a risky long-term project. In period 1, after learning her need for liquidity, she may either prematurely liquidate the investment project or not. Because, by assumption, the individual is uncertain in period 0 with respect to her liquidity needs in period 1, she ex ante perceives the premature liquidation as an uncertain prospect, denoted  $\Pi_1$ . If, in contrast, the long-term project is allowed to mature until period 2 the individual ex

ante expects a prospect  $\Pi_2$ , which is uncertain with respect to the project's success or failure. Thus, the individual's ex ante prospects (as perceived in period 0) with regard to the two different actions - premature *liquidation* vs. *maturity* - can be depicted as follows

	period 0	period 1	period 2
$l$	$-x$	$\Pi_1$	0
$m$	$-x$	0	$\Pi_2$

In order to capture all aspects of the individual's uncertainty we introduce as relevant state space

$$S = \{(L, f), (L, s), (H, f), (H, s)\},$$

where  $L$  ( $H$ ) indicates low (high) need for liquidity and  $s$  ( $f$ ) indicates success (failure) of the investment project. Accordingly, the events - **L**ow demand for liquidity, **H**igh demand for liquidity, **f**ailure of the project, **s**uccess of the project - are defined as follows

$$\begin{aligned} \mathbf{L} &= \{(L, f), (L, s)\} \\ \mathbf{H} &= \{(H, f), (H, s)\} \\ \mathbf{f} &= \{(L, f), (H, f)\} \\ \mathbf{s} &= \{(L, s), (H, s)\} \end{aligned}$$

Liquidation in period 1 yields utility  $b$  if the individual turns out to be a type with a high desire for liquidity, that is, if event **H** realizes (we speak then simply of a *high type* agent). In contrast, liquidation by a low type agent is evaluated by utility 0. A successfully matured project yields utility 1 whereas it only yields utility 0 in the case of failure. In what follows, we presume that  $0 < b < 1$ , i.e., a successfully matured project is perceived as best scenario whereas premature liquidation is strictly better for a high than for a low type.

A *liquidation strategy* of the individual determines for her two liquidity types whether to prematurely liquidate the project or to let it mature, i.e., a liquidation strategy is formally defined as a mapping  $\phi : \{\mathbf{L}, \mathbf{H}\} \rightarrow \{l, m\}$ . For example, the liquidation strategy  $(m, l)$  states that the low type allows the project to mature whereas the high type prematurely liquidates it. Let  $u(\phi(t))$  denote the utility of liquidation strategy  $\phi$  if state  $t \in S$  realizes and observe that there exist four different liquidation strategies in our model which give the following utilities dependent on the true state of the world

	$(L, f)$	$(L, s)$	$(H, f)$	$(H, s)$
$(m, m)$	0	1	0	1
$(m, l)$	0	1	$b$	$b$
$(l, l)$	0	0	$b$	$b$
$(l, m)$	0	0	0	1

### 3 The ex ante Decision Situation

In this section we define the individual as a CEU decision maker, that is, the individual maximizes expected utility with respect to some non-additive belief. Properties of such non-additive beliefs are used in the literature for formal definitions of, e.g., *ambiguity* and *uncertainty attitudes* (Schmeidler, 1987; Epstein, 1999; Ghirardato and Marinacchi, 2002), degrees of *confidence* in additive beliefs (Eichberger and Kelsey, 1999), *pessimism* and *optimism* (Wakker, 2001; Chateauneuf et al., 2003), as well as *sensitivity to changes in likelihood* (Wakker, 2003). Thus, besides the mere accommodation of observed choice behavior, non-additive beliefs of CEU theory also offer a wide range of psychological explanations for deviations from expected utility theory as, e.g., elicited in Allais (1954) and in Ellsberg (1961).

CEU theory was first axiomatized by Schmeidler (1986, 1989) for the framework of Anscombe and Aumann (1963) who assume the existence of random devices that generate objective probabilities. Subsequently, Gilboa (1987) as well as Sarin and Wakker (1992) have presented CEU axiomatizations for the Savage (1954) framework - where probabilities are derived from betting behavior as an exclusively personalistic concept - whereby Sarin and Wakker (1992) additionally assume the existence of *ambiguous* versus *unambiguous* events. CEU theory is equivalent to *cumulative prospect theory* (Tversky and Kahneman, 1992; Wakker and Tversky, 1993) restricted to the domain of gains (compare Tversky and Wakker, 1995). Moreover, as a representation of preferences over lotteries, CEU theory coincides with *rank dependent utility theory* as introduced by Quiggin (1981, 1982).

We proceed with a number of formal definitions. A *non-additive belief* (capacity, fuzzy measure)  $\nu$  on the state space  $S$  is a real-valued set function on the subsets of  $S$  which satisfies

- (i)  $\nu(\emptyset) = 0, \nu(S) = 1$
- (ii)  $A \subset B \Rightarrow \nu(A) \leq \nu(B)$

For  $A \subset S$  let  $u(\phi(A)) = u(\phi(t))$  if  $u(\phi(t)) = u(\phi(t'))$  for all  $t, t' \in A$ . For a given liquidation strategy  $\phi$  denote by  $E_1, \dots, E_m$  the partition of  $S$  such that  $u(\phi(E_1)) > \dots > u(\phi(E_m))$ . The Choquet expected utility of the liquidation strategy  $\phi$  with respect to  $\nu$  is

$$CEU(\phi, \nu) = \sum_{i=1}^m u(\phi(E_i)) \cdot [\nu(E_1 \cup \dots \cup E_i) - \nu(E_1 \cup \dots \cup E_{i-1})], \quad (1)$$

where we apply the convention that  $\nu(E_1 \cup \dots \cup E_0) = 0$ .

Fix some  $\nu$ , interpreted as the individual's *belief*, and apply (1) to obtain the following Choquet expected utilities for the individual's four liquidation strategies of the investment project described in the previous section.

$$CEU((m, m), \nu) = \nu(\mathbf{s}) \quad (2a)$$

$$CEU((m, l), \nu) = \nu(\{(L, s)\}) + b \cdot (\nu(\{(L, s), (H, f), (H, s)\}) - \nu(\{(L, s)\})) \quad (2b)$$

$$CEU((l, l), \nu) = b \cdot (\nu(\mathbf{H})) \quad (2c)$$

$$CEU((l, m), \nu) = \nu(\{(H, s)\}) \quad (2d)$$

Write  $\phi \succeq \psi$  if the individual prefers the liquidation strategy  $\phi$  over the liquidation strategy  $\psi$ , and define strict preference  $\succ$  and indifference  $\sim$  in the standard way. If the individual is a CEU decision maker there exists some belief  $\nu$  such that, for all liquidation strategies  $\phi, \psi$ , and some belief  $\nu$

(i)  $\phi \succ \psi$  implies  $CEU(\phi, \nu) > CEU(\psi, \nu)$

(ii)  $\phi \sim \psi$  implies  $CEU(\phi, \nu) = CEU(\psi, \nu)$

From now on we suppose the individual to be a CEU decision maker, which implies that, for some given  $\nu$ , the Choquet expected utilities in equations (2a) to (2d) represent the individuals *ex ante* preferences over liquidation strategies, that is, before the individual learns whether she has low or high desire for liquidity.

## 4 The ex post Decision Situation

Consider now the *ex post* decision problem where a CEU decision maker has already learnt her type, i.e., her need of liquidity, when she has to choose between premature



liquidation vs. *maturation*. After learning her type, either **L** or **H**, the individual then acts as if to maximize her Choquet expected utility with respect to the updated beliefs  $(\nu \mid \mathbf{L})$ , respectively  $(\nu \mid \mathbf{H})$ . In contrast to the case of additive beliefs, different rules for updating non-additive beliefs are conceivable. Since this fact will drive our results, we first sketch some seminal results of Gilboa and Schmeidler (1993).

Gilboa and Schmeidler (1993) characterize a set of *Bayesian update rules* such that any act  $f$  may define a specific *f-Bayesian update rule* if  $f$  is taken to be the "unrealized alternative". In particular, an *f-Bayesian update rule* of a CEU decision maker with preferences  $\succeq$  over acts in some given Savage-framework determines a collection of preference orderings over acts,  $\left\{ \succeq_A^f \right\}$  for all events  $A$ , such that for all acts  $g, h$

$$g \succeq_A^f h \Leftrightarrow (g, A; f, \neg A) \succeq (h, A; f, \neg A) \quad (3)$$

where  $(g, A; f, \neg A)$  denotes the act which gives consequences  $g(s)$  for all  $s \in A$  and consequences  $f(s)$  for all  $s \in \neg A$ , i.e., for all  $s$  that belong to the complement of  $A$ .

Thus, the preference ordering  $\succeq_A^f$  defines  $\neg A$  as a so-called *null-event*, that is, all consequences of acts in states of the world belonging to event  $\neg A$  are *irrelevant* to the decision maker's preferences over acts. For the decision theoretic approach, which derives any individual's likelihood considerations about events exclusively from preferences over acts, such an *irrelevance* of event  $\neg A$  is as good as if the individual perceives event  $\neg A$  as *impossible*. As a consequence, a preference ordering  $\succeq_A^f$  stands for a CEU decision maker's updated preferences who has preferences  $\succeq$  and learns that event  $A$  has occurred.

For an expected utility maximizer, the *sure-thing principle* (Postulate P2 in Savage, 1954) implies that all Bayesian update rules result in collections of preference orderings,  $\left\{ \succeq_A^f \right\}$  for all events  $A$ , which coincide for all "irrelevant" acts  $f$ . However, this is no longer the case for CEU decision makers. Gilboa and Schmeidler (1993) show that CEU preferences  $\succeq$  are updated to CEU preferences  $\left\{ \succeq_A^f \right\}$  for all events  $A$ , if and only if  $f$  is an act such that for some event  $T$

$$f = (x^*, T; x_*, \neg T),$$

that is, act  $f$  assigns to all states  $s \in T$  the best consequence  $x^*$  and to all states  $s \in \neg T$  the worst consequence  $x_*$ .

Of particular interest are the two extreme update rules for which  $f$  is either the *constant act* giving the worst consequence, i.e.,  $T = \emptyset$ , or the best consequence, i.e.,  $T = S$ . If the consequence of a *null* event is associated with the worst consequence,  $x_*$ , then Gilboa and Schmeidler (1993) speak of an "optimistic" update rule:

”... when comparing two actions given a certain event  $A$ , the decision maker implicitly assumes that had  $A$  not occurred, the worst possible outcome ... would have resulted. In other words, the behavior given  $A$  ... exhibits ”happiness” that  $A$  has occurred; the decisions are made as if we are always in ”the best of all possible worlds.””

Furthermore, they prove that the optimistically updated preferences  $\succeq_A^{x^*}$  of a CEU decision maker, who learns  $A$ , are represented by the CEU of acts with respect to conditional non-additive beliefs  $\nu(\cdot | A)$  which result from the following update rule:

### Optimistic update rule

$$\nu(B | A) = \frac{\nu(A \cap B)}{\nu(A)} \quad (4)$$

If the consequence of a *null* event is instead associated with the best consequence,  $x^*$ , i.e.,  $T = S$ , then Gilboa and Schmeidler (1993) speak of an ”pessimistic” update rule:

”... we consider a ”pessimistic” decision maker, whose choices reveal the hidden assumption that all the impossible worlds are the best conceivable ones.”

Gilboa and Schmeidler (1993) prove that pessimistically updated preferences  $\succeq_A^{x^*}$  of a CEU decision maker, who learns  $A$ , are represented by the CEU of acts with respect to conditional non-additive beliefs  $\nu(\cdot | A)$  resulting from the following update rule:

### Pessimistic (Dempster-Shafer) update rule

$$\nu(B | A) = \frac{\nu(B \cup \neg A) - \nu(\neg A)}{1 - \nu(\neg A)} \quad (5)$$

Now suppose the CEU decision maker of our model applies some Bayesian update rule after learning her liquidity needs. Given updated beliefs, e.g., liquidity agent  $\mathbf{H}$  then strictly prefers to liquidate prematurely, i.e.,  $l \succ_{\mathbf{H}} m$ , if and only if

$$CEU(l | \mathbf{H}) > CEU(m | \mathbf{H})$$

Thus, for updated belief  $(\nu | \mathbf{H})$ , respectively  $(\nu | \mathbf{L})$ , the following Choquet expected utilities (6a) - (6d) represent the liquidity agent’s *ex post* preference rankings  $\succeq_{\mathbf{H}}$ , respectively  $\succeq_{\mathbf{L}}$ , over premature liquidation vs. maturation.

$$CEU(l | \mathbf{H}) = b \tag{6a}$$

$$CEU(m | \mathbf{H}) = \nu(\mathbf{s} | \mathbf{H}) \tag{6b}$$

$$CEU(l | \mathbf{L}) = 0 \tag{6c}$$

$$CEU(m | \mathbf{L}) = \nu(\mathbf{s} | \mathbf{L}) \tag{6d}$$

**Assumption 1.** *Liquidity agent  $\mathbf{H}$  strictly prefers premature liquidation to maturation whereas liquidity agent  $\mathbf{L}$  strictly prefers maturation to premature liquidation, i.e.,*

$$l \succ_{\mathbf{H}} m \text{ and } m \succ_{\mathbf{L}} l$$

Thus, by assumption 1, liquidity agent  $\mathbf{L}$  (resp.  $\mathbf{H}$ ) has a low (resp. high) desire for liquidity, so that an individual prematurely liquidates the project after learning her liquidity needs, if and only if, she has a high desire for liquidity.

## 5 Investing in an Illiquid Asset

We now introduce an economic institution which offers in period 0, and subsequently enforces, the following contract to the individual

period 0	period 1	period 2
$-x$	0	$\Pi_2$

That is, the economic institution offers an illiquid asset by simply investing the individual's money into a risky long-term project with identical returns as the individual's liquid investment project and pays back the proceeds of the project in period 2. As the only difference to an investment in the liquid asset, investing via the financial institution here excludes the possibility of premature liquidation of the investment project. Similar to the pragmatic approach of Strotz (1956) and Laibson (1997), we simply observe that real-world financial institutions offer a great variety of illiquid assets whereas we

do not further investigate why such an institution might be able to commit itself not to prematurely liquidate the project. Furthermore, our idealized setup ignores any explicit or implicit costs of prematurely terminating the contract that might be relevant for real-world illiquid financial investments.

Investing in the illiquid asset yields exactly the same payoffs as the liquidation strategy  $(m, m)$ . We speak of a *sequentially rational* decision maker when this decision maker strictly prefers investing in the illiquid asset to investing in the liquid asset if and only if  $(m, m) \succ (m, l)$ . Observe that the liquidation strategy  $(m, l)$  is - by the standard backward induction (backfolding) argument - the unique solution path if the individual invests in the liquid asset. In our interpretation a sequentially rational decision maker therefore invests in the illiquid asset if she anticipates that her liquidity type  $\mathbf{H}$  - having high desire for liquidity - deviates from a supposedly ex ante favorable liquidation strategy  $(m, m)$  where no premature liquidation occurs.

**Lemma.** *Suppose assumption 1 holds and consider a CEU decision maker who updates her ex post beliefs  $\nu(\cdot | \cdot)$  according to some Bayesian update rule. If the CEU decision maker is sequentially rational, then she strictly prefers investing in the illiquid asset to investing in the liquid asset, if and only if,*

$$\frac{\nu(\mathbf{s}) - \nu(\{(L, s)\})}{\nu(\{(L, s)\} \cup \mathbf{H}) - \nu(\{(L, s)\})} > b > \nu(\mathbf{s} | \mathbf{H}) \quad (8)$$

**Proof:** To prove the lemma we have, by assumption 1, to identify conditions such that  $(m, m) \succ (m, l)$  while  $l \succ_{\mathbf{H}} m$  is also satisfied. From equations (2a)-(2d), it follows that  $(m, m) \succ (m, l)$  is equivalent to

$$\nu(\mathbf{s}) > \nu(\{(L, s)\}) + b \cdot (\nu(\{(L, s)\} \cup \mathbf{H}) - \nu(\{(L, s)\}))$$

yielding

$$\frac{\nu(\mathbf{s}) - \nu(\{(L, s)\})}{\nu(\{(L, s)\} \cup \mathbf{H}) - \nu(\{(L, s)\})} > b.$$

Moreover  $l \succ_{\mathbf{H}} m$  is equivalent to

$$b > \nu(\mathbf{s} | \mathbf{H})$$

which gives (8).□

**Remark.** Observe that, regardless which update rule is applied, for additive beliefs - i.e., where CEU theory reduces to expected utility theory - the inequalities (8) become

$$\frac{\nu(\{(H, s)\})}{\nu(\mathbf{H})} > b > \frac{\nu(\{(H, s)\})}{\nu(\mathbf{H})}$$

which is impossible to satisfy for any utility  $b$ . Thus, in our model an expected utility maximizer always prefers investing in the liquid asset to investing in the illiquid asset. This is not surprising since expected utility preferences are dynamically consistent: If the individual's preferences were dynamically consistent, then she would, by definition, also ex ante strictly prefer liquidation strategy  $(m, l)$  over  $(m, m)$ . Our results of the succeeding section are driven by the notion that this is no longer the case for a CEU decision maker with non-additive beliefs.

## 6 First Results

This section demonstrates that a sequentially rational CEU decision maker may strictly prefer investing in the illiquid asset to investing in the liquid asset, regardless whether she updates her beliefs optimistically or pessimistically. We first focus on the optimistic update rule.

**Proposition 1.** *Consider a sequentially rational CEU decision maker and suppose that assumption 1 is satisfied. If the CEU decision maker updates her beliefs according to the optimistic update rule, then there exist utility  $b \in (0, 1)$  such that she strictly prefers investing in the illiquid asset to investing in the liquid asset whenever her beliefs satisfy the following two conditions (with at least one inequality being strict)*

$$\nu(\mathbf{s}) \geq \nu(\{(H, s)\}) + \nu(\{(L, s)\}) \tag{9}$$

$$\nu(\mathbf{H}) + \nu(\{(L, s)\}) \geq \nu(\{(L, s)\} \cup \mathbf{H}). \tag{10}$$

### Proof of proposition 1.

Apply the optimistic update rule and observe that there exist some utility  $b$  satisfying equation (8), if and only if,

$$\frac{\nu(\mathbf{s}) - \nu(\{(L, s)\})}{\nu(\{(L, s)\} \cup \mathbf{H}) - \nu(\{(L, s)\})} > \frac{\nu(\{(H, s)\})}{\nu(\mathbf{H})} = \nu(\mathbf{s}|\mathbf{H}),$$

which is obviously satisfied under the conditions (9) and (10). $\square$

Now turn to the pessimistic update rule.

**Proposition 2.** *Consider a sequentially rational CEU decision maker and suppose assumption 1 is satisfied. If the CEU decision maker updates her beliefs according to the pessimistic update rule, then there exist utilities  $b \in (0, 1)$  such that she strictly prefers*

investing in the illiquid asset to investing in the liquid asset whenever her beliefs satisfy the following two conditions (with at least one inequality being strict)

$$\nu(\mathbf{s}) + \nu(\mathbf{L}) - \nu\{(L, s)\} \geq \nu(\{(H, s)\} \cup \mathbf{L}) \quad (11)$$

$$1 \geq \nu(\{(L, s)\} \cup \mathbf{H}) + \nu(\mathbf{L}) - \nu(\{(L, s)\}). \quad (12)$$

**Proof of proposition 2.**

Apply the pessimistic update rule and verify that there exist some utility  $b$  satisfying (8), if and only if,

$$\frac{\nu(\mathbf{s}) - \nu(\{(L, s)\})}{\nu(\{(L, s)\} \cup \mathbf{H}) - \nu(\{(L, s)\})} > \frac{\nu(\{(H, s)\} \cup \mathbf{L}) - \nu(\mathbf{L})}{1 - \nu(\mathbf{L})},$$

which is satisfied under conditions (11) and (12).□

## 7 Optimistic versus Pessimistic Decision Makers

So far, we considered two alternative update rules for non-additive beliefs, for which Gilboa and Schmeidler (1993) convincingly argue that they express the *implicit view of the world* of an optimistic, respectively pessimistic, CEU decision maker who receives new information. Conditions (9) and (10) (respectively (11) and (12) ) formally describe properties of non-additive beliefs such that a CEU decision maker, who updates her beliefs according to the optimistic (respectively pessimistic) update rule, strictly prefers investing in the illiquid asset to investing in the liquid asset.

Beyond the interpretation of update rules as optimistic (pessimistic) the decision-theoretic literature also describes optimistic (pessimistic) attitudes of a CEU decision maker by particular properties of her non-additive beliefs. In this section, we present two specific classes of non-additive beliefs which reflect marginal deviations from additive beliefs such that uncertainty is resolved in an optimistic (pessimistic) way. In particular, we consider so-called *optimistic* and *pessimistic beliefs*, which are special cases of the *neo-additive capacities* studied in Chateauneuf et al. (2003).<sup>2</sup>

**Definitions: Simple beliefs**

A optimistic belief  $\nu^o$  is defined as a linear combination of some additive belief  $\pi$  and a non-additive belief  $\omega^o$ , which reflects *complete confidence*, in the sense that only the null event  $\emptyset$  is considered as irrelevant. Similarly, a pessimistic belief  $\nu^p$  is defined as a linear combination of an additive belief  $\pi$  and a non-additive belief  $\omega^p$ , which reflects

---

<sup>2</sup> *Pessimistic beliefs* are discussed by Eichberger and Kelsey, 1999.

complete ambiguity or complete ignorance, in the sense that only the universal event  $S$  is considered as relevant.

- A optimistic belief,  $\nu^o$ , is defined, for some  $\lambda \in (0, 1]$ , by

$$\nu^o(E) = (1 - \lambda) \cdot \pi(E) + \lambda \cdot \omega^o(E)$$

for all  $E \subseteq S$  such that  $\pi$  is some additive probability measure and

$$\begin{aligned}\omega^o(E) &= 1 \text{ if } E \neq \emptyset \\ \omega^o(E) &= 0 \text{ if } E = \emptyset\end{aligned}$$

- A pessimistic belief,  $\nu^p$ , is defined, for some  $\gamma \in (0, 1]$ , by

$$\nu^p(E) = (1 - \gamma) \cdot \pi(E) + \gamma \cdot \omega^p(E)$$

for all  $E \subseteq S$  such that  $\pi$  is some additive probability measure and

$$\begin{aligned}\omega^p(E) &= 0 \text{ if } E \subset S \\ \omega^p(E) &= 1 \text{ if } E = S\end{aligned}$$

Notice that optimistic (pessimistic) beliefs are *concave* (*convex*) capacities. CEU decision makers with optimistic (pessimistic) beliefs are therefore *ambiguity prone* (*averse*) in the sense of Schmeidler's (1989) definition of ambiguity attitudes. To see why we speak of optimistic (pessimistic) beliefs<sup>3</sup>, observe that the CEU of an act  $\phi$  for a optimistic belief  $\nu^o$  is given by:

$$CEU(\phi, \nu^o) = (1 - \lambda) \cdot \sum_{i=1}^m u(\phi(E_i)) \cdot \pi(E_i) + \lambda \cdot \max_{t \in S} u(\phi(t)),$$

while we have for a pessimistic belief  $\nu^p$ :

$$CEU(\phi, \nu^p) = (1 - \gamma) \cdot \sum_{i=1}^m u(\phi(E_i)) \cdot \pi(E_i) + \gamma \cdot \min_{t \in S} u(\phi(t)).$$

Thus, for optimistic beliefs the CEU of an act results from a linear combination of the act's expected utility and its best outcome in some state of the world. The *degree*

---

<sup>3</sup>For a detailed discussion on this topic, we refer the reader to Chateauneuf et al. (2003) where the relation between simple beliefs and Wakker's (2001) definition of *optimism* and *pessimism*, on the one hand, and the *multiple prior approach* of Gilboa and Schmeidler (1989), on the other hand, is investigated.

of optimism  $\lambda \in (0, 1]$  then measures how much the CEU decision maker evaluates an act by the best outcome she may possibly achieve by choosing this act. Conversely, for pessimistic beliefs the CEU of an act is given by a linear combination of the act's expected utility and its worst outcome in some state of the world. How much the CEU decision maker cares about the worst outcome possible for a chosen act, is then expressed by her *degree of pessimism*  $\gamma \in (0, 1]$ .

We now combine the definition of optimistic (pessimistic) update rules on the one hand, with optimistic (pessimistic) beliefs on the other hand, in order to characterize an optimistic (pessimistic) CEU decision maker.

### Definitions.

- We define a CEU decision maker as an optimist if she has some optimistic belief which she updates according to the optimistic update rule.
- Accordingly, we define a CEU decision maker as a pessimist if she has some pessimistic belief which she updates according to the pessimistic update rule.

**Proposition 3.** *Suppose assumption 1 is satisfied. If a sequentially rational CEU decision maker is an optimist, then, for all utilities  $b \in (0, 1)$ , she strictly prefers investing in the liquid asset to investing in the illiquid asset.*

### Proof of proposition 3.

Apply the optimistic update rule and substitute optimistic beliefs  $\nu^o$  for  $\nu$  in (8). After some rearrangement, we obtain that there exists some utility  $b$  satisfying (8), if and only if,

$$\pi(\mathbf{s} \mid \mathbf{H}) = \frac{\pi(\{(H, s)\})}{\pi(\mathbf{H})} > \frac{(1 - \lambda) \cdot \pi(\{(H, s)\}) + \lambda}{(1 - \lambda) \cdot \pi(\mathbf{H}) + \lambda} = \nu^o(\mathbf{s} \mid \mathbf{H}),$$

which is impossible for all  $\lambda \in (0, 1]$ .  $\square$

**Proposition 4.** *Suppose assumption 1 is satisfied. If a sequentially rational CEU decision maker is a pessimist, then there exists some utility  $b \in (0, 1)$  such that she strictly prefers investing in the illiquid asset to investing in the liquid asset.*

### Proof of proposition 4.



Apply the pessimistic update rule and substitute pessimistic beliefs  $\nu^p$  for  $\nu$  in (8). Rearrangement yields that there exists some utility  $b$  satisfying (8), if and only if,

$$\pi(\mathbf{s} \mid \mathbf{H}) = \frac{\pi(\{(H, s)\})}{\pi(\mathbf{H})} > \frac{(1 - \gamma) \cdot \pi(\{(H, s)\})}{(1 - \gamma) \cdot \pi(\mathbf{H}) + \gamma} = \nu^p(\mathbf{s} \mid \mathbf{H}),$$

which is satisfied for all  $\gamma \in (0, 1]$ .  $\square$

For optimistic (pessimistic) CEU decision makers the proofs of proposition 3 and 4 reveal that the question whether there exists some utility  $b$  satisfying the inequality (8) of the Lemma becomes equivalent to the question whether the conditional additive part of the belief,  $\pi(\mathbf{s} \mid \mathbf{H})$ , is greater than the conditional non-additive belief  $\nu^o(\mathbf{s} \mid \mathbf{H})$  ( $\nu^p(\mathbf{s} \mid \mathbf{H})$ ). The results of propositions 3 and 4 are then immediately implied by the fact that for optimistic (pessimistic) decision makers

$$\pi(\mathbf{s} \mid \mathbf{H}) < \nu^o(\mathbf{s} \mid \mathbf{H})$$

and

$$\pi(\mathbf{s} \mid \mathbf{H}) > \nu^p(\mathbf{s} \mid \mathbf{H}).$$

That is, when an optimistic investor learns that she has a high desire for liquidity, she acts as if the project's success has now become more likely than suggested by the additive part of her belief. As a consequence, the high liquidity type of an optimistic investor would never want to prematurely liquidate the project if such a premature liquidation is not already optimal from an ex ante perspective. In contrast, after learning her high desire for liquidity, a pessimistic investor regards the project's success as less likely than suggested by the additive part of her belief. Thus, there might exist some utility  $b$  such that

$$\pi(\mathbf{s} \mid \mathbf{H}) > b > \nu^p(\mathbf{s} \mid \mathbf{H})$$

implying that the high liquidity type of a pessimistic investor has a strict incentive for premature liquidation while maturation of the project would be optimal from an ex ante perspective.

## 8 Concluding Remarks

This paper analyzes the decision problem of a CEU decision maker who has either to invest in a liquid or an illiquid uncertain investment project. The illiquid investment project of our model is simply characterized by an economic institution with the ability to invest the decision maker's money in the long-term project while excluding the possibility that the project may be prematurely liquidated by the individual. For two standard

Bayesian update rules - the optimistic and the pessimistic update rule - we then show that a sequentially rational CEU decision maker may strictly prefer investing in the illiquid project since it serves as an intra-personal commitment device which guarantees an ex ante favorable outcome.

We further argue that such an investment behavior is particularly plausible for pessimistic decision makers, which we define as CEU decision makers who apply the pessimistic update rule and who have pessimistic beliefs. The formal argument, by which we derive our result, can be intuitively interpreted as follows:

Suppose an individual expects to develop a more and more pessimistic view of the world when learning new information, so that she would liquidate an uncertain investment project because her pessimistic future-self comes to believe in the ultimate failure of the project. Whenever such an individual's view of the world is not as pessimistic yet, she may seek for an intra-personal commitment device in order to realize the uncertain long-term investment project without premature liquidation.

By exploiting the dynamic inconsistency of CEU decision making, we present a new rationale for investing in illiquid assets as a commitment device as an alternative explanation to hyperbolic discounting, see, e.g. Laibson (1997).

## 9 References

- Allais, M. (1953), "Le Comportement de l'Homme Rationnel devant le Risque: Critique des postulats et axiomes de l'École Americaine", *Econometrica* **21**, 503-546. Reprinted in English as part II in Allais, M., and O. Hagen [eds.] (1979), *Expected Utility Hypotheses and the Allais Paradox*, D. Reidel: Dordrecht etc.
- Anscombe, F.J, and R.J. Aumann (1963), "A Definition of Subjective Probability", *Annals of American Statistics* **34**, 199-205.
- Chateauneuf, A., Eichberger, J., and S. Grant (2003), "Choice under Uncertainty with the Best and Worst in Mind: Neo-additive Capacities", *mimeo*
- Eichberger, J., and D. Kelsey (1996), "Uncertainty Aversion and Dynamic Consistency", *International Economic Review* **37**, 625-640.
- Eichberger, J., and D. Kelsey (1999), "E-Capacities and the Ellsberg Paradox", *Theory and Decision* **46**, 107-140.

- Eichberger, J., and D. Kelsey (2004), "CEU Preferences and Dynamic Consistency", *mimeo*
- Ellsberg, D. (1961), "Risk, Ambiguity and the Savage Axioms", *Quarterly Journal of Economics* **75**, 643-669.
- Epstein, L.G. (1999), "A Definition of Uncertainty Aversion", *The Review of Economic Studies* **66**, 579-608.
- Epstein, L.G., and M. Le Breton (1993), "Dynamically Consistent Beliefs must be Bayesian", *Journal of Economic Theory* **61**, 1-22.
- Epstein, L.G., and M. Schneider (2003), "Recursive Multiple Priors", *Journal of Economic Theory* **13**, 32-50.
- Ghirardato, P. and M. Marinacci (2002), "Ambiguity Made Precise: A Comparative Foundation", *Journal of Economic Theory* **102**, 251-289
- Gilboa, I. (1987), "Expected Utility with Purely Subjective Non-Additive Probabilities", *Journal of Mathematical Economics* **16**, 65-88.
- Gilboa, I., and D. Schmeidler (1989), "Maxmin Expected Utility with Non-Unique Priors", *Journal of Mathematical Economics* **18**, 141-153.
- Gilboa, I., and D. Schmeidler (1993), "Updating Ambiguous Beliefs", *Journal of Economic Theory* **59**, 33-49.
- Hammond, P.J (1988), "Consequentialist Foundations for Expected Utility", *Theory and Decision* **25**, 25-78.
- Jacklin, C.J. (1987), "Demand Deposits, Trading Restrictions, and Risk Sharing", chap. II, p. 26-47 in Prescott, E.C., and N. Wallace [eds.] (1987), *Contractual Arrangements for Intertemporal Trade*, University of Minnesota Press: Minneapolis.
- Kilka, M., and M. Weber (2001), "What determines the Shape of the Probability Weighting Function under Uncertainty", *Management Science* **47**, 1712-1726.
- Kreps, D.M., and E. Porteus (1979), "Temporal Von Neumann-Morgenstern and Induced Preferences", *Journal of Economic Theory* **20**, 81-109.
- Laibson, D. (1997), "Golden Eggs and Hyperbolic Discounting", *Quarterly Journal of Economics* **112**, 443-477.

- Quiggin, J.P. (1981), "Risk Perception and Risk Aversion among Australian Farmers", *Australian Journal of Agricultural Economics* **25**, 160-169.
- Quiggin, J.P. (1982), "A Theory of Anticipated Utility", *Journal of Economic Behavior and Organization* **3**, 323-343.
- Sarin, R., and P.P. Wakker (1992), "A Simple Axiomatization of Nonadditive Expected Utility", *Econometrica* **60**, 1255-1272.
- Sarin, R., and P.P. Wakker (1998), "Dynamic Choice and NonExpected Utility", *Journal of Risk and Uncertainty* **17**, 87-119.
- Savage, L.J. (1954). *The Foundations of Statistics*, John Wiley and Sons, Inc.: New York, London, Sydney.
- Schmeidler, D. (1986), "Integral Representation without Additivity", *Proceedings of the American Mathematical Society* **97**, 255-261.
- Schmeidler, D. (1989), "Subjective Probability and Expected Utility without Additivity", *Econometrica* **57**, 571-587.
- Strotz, R.H. (1956), "Myopia and Inconsistency in Dynamic Utility Maximization", *The Review of Economic Studies* **23**, 165-180.
- Tversky, A., and D. Kahneman (1992), "Advances in Prospect Theory: Cumulative Representations of Uncertainty", *Journal of Risk and Uncertainty* **5**, 297-323.
- Tversky, A., and P.P. Wakker (1995), "Risk Attitudes and Decision Weights", *Econometrica* **63**, 1255-1280.
- Wakker, P.P. (2001). "Testing and Characterizing Properties of Nonadditive Measures through Violations of the Sure-Thing Principle", *Econometrica* **69**, 1039-1059.
- Wakker, P.P., and A. Tversky (1993) "An Axiomatization of Cumulative Prospect Theory", *Journal of Risk and Uncertainty* **7**, 147-176.

## Discussion Paper Series

Mannheim Research Institute for the Economics of Aging Universität Mannheim

To order copies, please direct your request to the author of the title in question.

Nr.	Autoren	Titel	Jahr
47-04	Karsten Hank	Effects of Early Life Family on Women's Late Life Labour Market Behaviour: An Analysis of the Relationship between Childbearing and Retirement in Western Germany	04
48-04	Christina Benita Wilke	Ein Simulationsmodell des Rentenversicherungssystems: Konzeption und ausgewählte Anwendungen von MEA-PENSIM	04
49-04	Axel Börsch-Supan, Alexander Ludwig und Anette Reil-Held	Hochrechnungsmethoden und Szenarien für gesetzliche und private Renteninformationen	04
50-04	Axel Börsch-Supan	From Public Pensions to Private Savings: The Current Pension Reform Process in Europe	04
51-04	Axel Börsch-Supan	Gesamtwirtschaftliche Folgen des demographischen Wandels	04
52-04	Axel Börsch-Supan	Mind the Gap: The Effectiveness of Incentives to Boost Retirement Saving in Europe	04
53-04	Joachim Winter		04
54-04	Axel Börsch-Supan	Aus der Not eine Tugend – Zukunftsperspektiven einer alternden Gesellschaft	04
55-04	Axel Börsch-Supan	Global Aging – Issues, Answers, More Questions	04
56-04	Axel Börsch-Supan	Was bedeutet der demographische Wandel für die Wirtschaft Baden-Württembergs?	04
57-04	Hendrik Jürges	Self-assessed health, reference levels, and mortality	04
58-04	Alexander Ludwig	Improving Tatonnement Methods for Solving Heterogeneous Agent Models	04
59-04	Frank Betz Oliver Lipps	Stochastic Population Projection for Germany	04
60-04	Alexander Ludwig Alexander Zimmer	Investment Behavior under Ambiguity: The Case of Pessimistic Decision Makers	04