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Social Learning in Dualistic Societies: Segregation, Growth and Distribution

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Abstract:

Using a simple model of social learning, we endogenize growth and distribution in a dualistic developing society. For given parameters of the learning technology, a trade-off between growth and equity results. On the other hand, more intensive social interaction between agents will raise the growth rate and lower the income differential at the same time. The economic consequences of lacking social integration are sluggish growth and high inequality.

Key Words: Social Learning, Dualistic Development, Growth, Distribution, Local Externalities

JEL Classification: O15, D83, D33, O41, O33

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Social Learning in Dualistic Societies: Segregation, Growth and Distribution

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1. Introduction

Taking the classic description of dualistic economies by Lewis as a starting point, this paper highlights three essential features of a modernization process, namely socio-cultural development, capital accumulation and sectoral reallocation of labor. In the modern sector, two different types of agents coexist: those with a traditional background and those adapted to a modern economic environment. This latter type is more productive and has a higher income. The modern sector is connected to a traditional *hinterland* via the labor market.

By observing the modern type individuals, the traditional agents may acquire the new characteristics. The joint presence of individuals with different backgrounds in the modern sector drives an acculturation process. We carefully adapt the epidemic model of diffusion to the given context. The *aggregate learning curve* for the economy is a decreasing function relating diffusion speed to the proportion of modern type individuals in the labor force.

This leads to a qualified answer to the question of whether there is a trade-off between growth and equity. In the steady state, all factor quantities are growing at the same rate. A higher equilibrium growth rate requires a faster spread of modern characteristics. For a *given* social learning technology, any such acceleration is feasible only at the expense of a higher proportion of poor individuals. An *enhanced* efficiency of social learning, however, will raise the growth rate and lower the proportion of marginalized individuals in the modern sector, thereby smoothing income differentials.

Social integration plays a prominent role in our description of dualistic development. The intensity of contact between agents of different types is crucial for their ability to learn from each other by observation. A low contact intensity implies sluggish growth, together with high income differentials and a large proportion of the modern sector population being marginalized. A better mixing of the population makes for a more efficient process of social learning, thus facilitating high growth, a flat income profile and high upward mobility. This important result

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[1] Any opinions expressed are mine and cannot be interpreted as the opinions of the Deutsche Bundesbank. This paper draws heavily on the first chapters of v. Kalckreuth (1999). I owe a great debt to my advisors, Jürgen Schröder and Martin Hellwig at the Economics Department of the University of Mannheim, for intellectual leadership, patience, and many valuable comments. Paul Romer at Stanford encouraged me to investigate the economic consequences of cultural change and he drew my attention to important new literature. Haizhu Zhang introduced me to the theory of dualistic development and Richard Klophaus did the same with regard to the diffusion of innovations. I discussed the stability properties of the dynamical equilibrium with Kerstin Hoffmann. My sister Stefanie proofread the English version. Various comments on seminars held at Mannheim and at the Passau Conference on International Economic Relations proved very helpful. The responsibility for all remaining errors and omissions rests with me.

might help to redirect the attention of development economists and theorists towards the socio-cultural aspect of transformation processes.

The paper draws on three separate strains of literature. The macroeconomic framework is derived from the well-known model of dualistic development pioneered by Lewis (1954, 1958), which triggered a lot of important research in the field, such as Ranis and Fei (1961), Todaro (1969), or Cole and Sanders (1985). Our use of the dualistic setting is closest to Findlay and Rodriguez (1981). These authors insert a classical human capital production technology into the framework of the Lewis model. Their approach, however, it is not suitable for investigating distributional issues. In their model, all agents are identical and the lifetime incomes of skilled and unskilled workers are equalized. As there are no externalities, the significance of discrimination and segregation cannot be addressed.

Second, socio-economic development is modeled using ideas on the diffusion of innovations by Rogers (1983), Griliches (1957), and Mansfield (1968); on cultural transmission by Coleman (1964), Cavalli-Sforza and Feldman (1981) and Boyd and Richerson (1985); and on social learning by Ellison and Fudenberg (1993, 1995). Finally, the tremendous potential significance of local externalities for growth and the distribution of income is highlighted by Loury (1977), Durlauf (1996), and Bénabou (1996a, 1996b). This paper owes much inspiration to the questions posed by these authors and their answers.

2. A Prototypical Model of Dualistic Development

In his seminal work, Lewis (1954, 1958) distinguishes a traditional sector and a modern, mainly urban sector. The traditional sector makes no use of capital goods, nor does it generate any savings. The income of the labor force is considered to be stationary. Lewis and Fei and Ranis (1964) maintain that the stationary wage rate is the result of traditional mechanisms of income sharing. As a consequence of overpopulation, the marginal productivity of labor is very low or even zero. A socio-cultural subsistence level is sustained by non-market institutions of income redistribution. Income is shared within larger clans or kinship groups, or a patron guarantees the subsistence income to his tenants.[2] As long as the marginal productivity of labor remains *below* the subsistence income, the nexus between marginal productivity and wage income is loose or non-existent – the (private) opportunity costs of labor in the traditional sector are constant.[3]

The output of the modern sector is a perfect substitute for the traditional sector product. For its production, capital is being used. The modern sector is open for the supply of labor from

[2] Banerjee and Newman (1996) argue that in a traditional environment, agents easily obtain insurance against idiosyncratic shocks. Monitoring cost are low because agents know a lot about each other. The modern sector is more productive, but plagued by information asymmetry.

[3] Dualistic models are surveyed by Ranis (1988). Sen (1966) investigates the surplus-labor thesis.

the traditional sector. This supply is infinitely elastic at a reservation wage $\overline{w}$ that surpasses the subsistence level in the traditional sector by a constant amount. This differential not only pays for the higher costs of living in the modern sector, but also compensates for living in an unfamiliar environment.

Using this as a basis, Lewis develops a growth model that is "almost provocatively simple".[4] Let K be the modern sector capital stock and L the aggregate input of labor. The aggregate production function, $\mathrm{F}(K, L)$, displays constant returns to scale and diminishing marginal returns. Both factors are homogeneous. The infinite elasticity of labor supply determines the equilibrium value k^* of the capital intensity k as the unique solution to the equation

$$\mathrm{f}(k) - k\,\mathrm{f}'(k) = \overline{w},$$

where $\mathrm{f}(k)$ represents the per capita production function and $\mathrm{f}'(k)$ the net marginal productivity of capital.

Lewis makes the classic assumption that only capital owners save. With a constant savings rate s, the following equation of motion for the capital stock results:

$$\dot{K} = s\,\mathrm{f}'(k^*) \cdot K.$$

The growth rates of the capital stock, the aggregate income, and the labor force employed in the modern sector are thus given by:

$$\hat{K} = \hat{L} = s\,\mathrm{f}'(k^*). \tag{1}$$

This equation makes the equilibrium growth rate equal to the product of the savings rate and the marginal productivity of capital in k^*, the profit rate. The latter will be high when the reservation wage is low.

The development process according to Lewis is dominated by the accumulation of capital. The growing stock of capital draws more and more workers into the modern sector. The economy is evolving fast if its agents are frugal, i.e., if capitalists consume little and workers are willing to accept low wages.[5] In spite of their simplicity, the equations constitute a fully developed model of endogenous growth. The unlimited supply of labor at a fixed price ensures that the marginal productivity of capital is stabilized and growth does not falter. The resulting rate of growth itself is explained by technology and preferences.[6]

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4 Ranis and Fei (1982), p. 31.

5 See Zhang (1996) for a detailed exposition.

6 As the model does not have any transitional dynamics, it is to be classified as an *AK*-model. Lewis manages to expose it without using a single equation. The formal exposition above follows Findlay (1982).

3. Two Cultures and the Social Learning Curve

In order to introduce socio-cultural change, the assumption of homogeneous labor has to be abandoned. The traditional sector is home to *T-individuals*, a type of agents with cultural characteristics that have developed in a long evolutionary process under the conditions of the traditional environment. This type of labor can also be used in the modern sector. However, because of differences regarding technology and social organization, another set of characteristics is optimal here. We call *M-individuals* the carriers of these characteristics better adapted to the exigencies of a modern environment. The traditional sector supplies T-labor at a constant price. In the modern sector – and only there! – the descendants of the migrants may acquire the new characteristics by social learning.

The characteristics in question are considered as *routines* in the sense of Nelson and Winter (1983), standardized patterns of behavior designated to solve economic problems:

> The term routine connotes, deliberately, behavior that is conducted without much explicit thinking about it, as habits or customs.[7]

Routines seem appropriate to an agent in a certain situation, without being the result of explicit optimization geared to the specific circumstances.[8] In our discussion, routines are considered as given, they constitute *technologies.* No attempt is made here to explain their evolution in the social system. Their spread implies a process in which agents adopt an innovation. The literature on the diffusion of innovation emphasizes that the social environment of an individual has a crucial influence on whether a certain innovation is adopted or not. Rogers gives the following well-known definition:

> Diffusion is the process by which an innovation is communicated through certain channels over time among the members of a social system.[9]

People receive a large part of the information relevant to them from personal contacts. The users of an innovation exert an external effect on non-users. The epidemic model of diffusion was originally adopted from population biology and epidemiology.[10] Paradigmatic applications are the works of Griliches (1957) on the diffusion of hybrid corn and Mansfield (1968) on the spread of technological innovations in various sectors of the American industry.

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7 Nelson (1995), p. 68.

8 Nelson and Winter develop the concept in order to investigate the adaptation and evolution of standardized activities by *firms*. It can be applied to *individuals* just as well. The advantages of rule-based behavior for individuals are shown by Heiner (1983).

9 Rogers (1985), p. 5. He elaborates on every part of this definition.

10 See Arrow (1994), Bartholomew (1967) and the introduction in Rogers (1983). For the structure of epidemiological models the classic reference is Bailey (1957). The analogy between cultural diffusion and biological reproduction also underlies Dawkins' (1989) ideas on the dissemination of 'memes'.

Let T be the number of traditional individuals living in the modern sector and M the number of modern type individuals. It is convenient to conduct the analysis in continuous time,[11] so we assume a demographic structure in which death and replacement is distributed evenly in time. Conceptually, the life cycle of individuals consists of two periods: youth and maturity. During youth, routines are acquired from the parent individual and from the social environment. In the second period, routines are fixed and the individual works and raises own descendants. In our mathematical representation, the duration of the first period is contracted to an instant. Birth of descendants happens at the end of the life cycle.

The individuals have (a mean of) $1+\nu$ descendants. Duration of life is distributed exponentially with parameter λ. This parameter is the instantaneous replacement probability, the probability density for the event that an individual living at any given moment dies and is replaced by his or her descendants.[12] Aggregating over many individuals, by virtue of the law of large numbers the instantaneous replacement probability becomes a deterministic replacement rate. Choosing the unit of time allows us to normalize $\lambda = 1$.

Following Cavalli-Sforza and Feldman (1981), we divide the acquisition of cultural identity in two stages.[13] As a first step, the young individual adopts the cultural type of the parent. In the second stage, he or she can acquire routines from the extrafamiliar environment. If he or she adopts a new set of routines, we speak of *assimilation*. A new set of routines is either adopted completely or not at all. M-individuals invariably have a higher income in the modern sector. The descendants of M-individuals can only lose by acquiring T-attributes. We therefore assume that social learning is limited to T-individuals.

Every T-descendant maintains personal relations to a certain group of individuals, his or her *reference group*. These individuals can serve as role models for the adoption of new characteristics. The assimilation costs c of the young are continuously distributed on R_0^+ with distribution function $\mathrm{F}_{\tilde{c}}(c)$. Among other things, the position of this distribution function depends on how *similar* the two cultures are. Assimilation is cheaper when the new pattern of characteristics is not too different from the old one and only few elements have to be re-

[11] In discrete time, stocks may overshoot or undershoot their equilibrium values due to the recursive nature of social learning, and even chaos may result. In our context, this type of behavior is meaningless. For the demographic structure employed here, see Banerjee and Newman (1993), and Blanchard (1985).

[12] If $\tilde{z}$ is the exponentially distributed duration of life, the distribution function is given by $\mathrm{P}\{\tilde{z} \le \tau\} = \mathrm{F}_{\tilde{z}}(\tau) = 1 - e^{-\lambda\tau}$, leading to the probability density $\mathrm{f}_{\tilde{z}}(\tau) = \lambda e^{-\lambda\tau}$ for death and replacement. The expected lifetime for an individual *still living* after time τ is constantly equal to $1/\lambda$. This structure is sometimes called the 'model of eternal youth'.

[13] Cavalli-Sforza and Feldman (1981) speak of *vertical transmission* if cultural traits are passed from parents to children and they call *horizontal transmission* the exchange of traits between members of the same generation. Boyd and Richerson (1985) have adopted this distinction as well as the notion that culture can be represented as a set of interrelated traits that is handed down as a whole. Both monographs describe a host of possibilities of treating different aspects of cultural transmission formally.

placed.[14] Let u be the subjective yield of assimilation. The probability that a certain T-descendant will assimilate when given the chance to do so is

$$\phi = F_{\tilde{c}}(u).$$

A proportion $1-\phi$ of T-descendants will stick to the parental characteristics, no matter how the reference group is composed. A part ϕ will adopt the M-attributes if and only if there is at least one M-individual among the members of the reference group. Ellison and Fudenberg (1995) call this a *must-see-condition.* The quantity u – and therefore ϕ as well – will be treated as a parameter. It can be shown that making u functionally dependent on the income differential will not lead to different conclusions.[15] The proportion ϕ can be interpreted as measuring the T-individuals' ability to assimilate.

The members of an individual's reference group constitute a random sample of size *n*, drawn from the modern sector population. The share of M-individuals in the modern sector population is labeled $q = M/(M+T)$. The size *n* may be distributed among the individuals in an arbitrary way. In the simple case where $n=1$ holds identically, the probability of successful assimilation will be $\phi \cdot q$. Integrating over the whole modern sector, we obtain the change of *M* per unit of time as the sum of natural growth and assimilation:

$$\dot{M} = \phi q T + \nu M. \tag{2}$$

The corresponding growth rate of the M-population will be called the *diffusion rate*:

$$\hat{M} = \phi(1-q) + \nu. \tag{2'}$$

The graph of this relationship is the *aggregate learning curve* of the economy. Figure 1 displays the schedule for different sizes of the reference group. Given *q*, the rate of diffusion is higher when the reference groups are larger, because the impact of the *must-see-condition* diminishes. More importantly, the diffusion rate is a *decreasing* function of the proportion *q* of modern type individuals in the labor force. This is a consequence of modeling diffusion as an epidemic. With a high *q*, there are only few T-individuals to whom a given number of M-individuals can transmit their characteristics, and thus the cultural 'rate of reproduction' will be low. Loury (1977), Lazear (1995) and Kubin and Rosner (1996) have treated assimilation in a similar way. In the following, we will assume the linear case $n=1$. Appendix A shows that the general case $n \geq 1$ leads to more complicated expressions, without producing different results.

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14 Rogers (1993) defines *compatibility* as the degree to which an innovation is perceived as being consistent with the existing values, past experiences, and needs of potential adopters. Rogers demonstrates with numerous examples that the rate of diffusion of an innovation depends positively on its compatibility.

15 See v. Kalckreuth (1999), pp. 70-72.

4. Discrimination and Segregation

By being potential role models, M-individuals have an external effect on T-individuals. The power of this externality depends on the *contact intensity* between the two types. Assume that the costs of assimilation are infinitely high for a fraction $\mu \le 1$ of T-individuals. For these, assimilation will not be possible for any subjective yield *u*. Only a fraction $1-\mu$ of T-individuals actually participates in the diffusion process. This gives us:

$$\hat{M} = \phi(1-\mu)(1-q) + \nu. \tag{3}$$

The parameter μ can be used as a measure of *discrimination,* e.g., for ethnical or religious reasons. Equation (3) results if a fraction μ of T-individuals is not allowed to interact closely enough with individuals of the other type. Another reason for low contact intensity is *segregation,* i.e., spatial separation. Assimilation is the consequence of a local externality which is limited to the modern sector. The effect of segregation on diffusion is akin to a quarantine during an epidemic.[16]

In Appendix B we show formally that the aggregate learning process slows down when a mixed population is divided into two parts of different social composition. This is immediately clear for the case of *perfect segregation,* when the two types of individuals do not have any contact at all. In essentially the same way *local concentrations of agents,* as in squatter settlements or informal sector firms, will hamper socio-cultural development.[17] Analytically, the number $\dot{M}$ of assimilations per unit of time is a strictly concave function of the composition *q* of the modern sector labor force, see especially Figure 7 in Appendix B. If, for example, $n = 1$ holds identically and $N = M + T$, (2) gives us:

$$\dot{M} = \phi(1-q)qN + \nu qN.$$

Now consider a partition of the population in two separate communities with different social composition. Because of strict concavity in *q*, the sum of assimilations will always be smaller than in the reference case of a perfectly mixed community. The decrease of the assimilation rate will be more pronounced if the communities differ strongly in their composition, i.e. if the spatial structure of the population is polarized. This provides an analytic explanation for the *concentration effects* of ghettos, as emphasized by the sociologist Wilson (1987). Streufert (1991) gives a different but related account: He argues that the subjective valuation of characteristics like working habits, drinking habits, education, etc. might be subject to a selectivity bias if the neighborhood of adolescents consists mainly of 'losers'. The significance of convexity or concavity for the effect of segregation in communities with neighborhood externalities is

[16] Morris (1993) investigates the significance of social barriers for the spread of diseases like AIDS, that require close contact for contagion.

[17] Local concentrations in neighborhoods and informal sector firms are treated in v. Kalckreuth (1999). Chapter 2 contains an equilibrium model of sectoral dualism based on communication costs.

studied by Bénabou (1996a), and Bénabou (1996b) investigates the connection between segregation and income distribution.

The simplest case is given when a fraction μ of *both* M- and T-individuals set themselves apart in isolated communities. The composition of the (smaller) integrated rest is then unchanged and again (3) results. Below we will therefore use μ quite generally to represent a lack of social integration that results in a slower speed of diffusion.

5. Production Technology and Wages

The negative slope of the aggregate learning curve has important consequences. Consider a steady state, where the stock of M-individuals grows by the same rate as the inputs of all other factors. Given the social learning curve, a higher growth rate is only possible if the portion of traditional individuals also rises. But then, the greater scarcity of modern type individuals will generally cause a rising wage for this type of labor. This constitutes a trade-off between equity and growth which can only be surmounted by pushing the relevant frontier outside, i.e., by enhancing the efficiency of the social learning process. In the following, we will elaborate on this argument with the help of a very simple macroframework.

In the modern sector, firms use the factors capital, M-labor and T-labor. Consider

$$Y = \mathrm{F}(K, T, M)$$

being a linear homogeneous, twice continuously differentiable aggregate production function with diminishing marginal returns which are positive throughout. The cross-derivative F_{12} is positive. A higher capital input will improve the productivity of simple labor.[18] Specifically modern characteristics are not necessary for production:[19]

$$\mathrm{F}(K, T, 0) > 0 \ \forall K, T > 0.$$

By varying the quantity of capital being used, it is always possible to equalize the marginal productivity of T-labor to the exogenously given reservation wage, $\overline{w}$:

$$\mathrm{F}_2(0, T, M) < \overline{w} < \lim_{K \to \infty} \mathrm{F}_2(K, T, M) \quad \forall \ T > 0, M \geq 0. \tag{4}$$

Factor markets in the modern sector operate efficiently. The real factor returns – r for capital, w_{T} for traditional labor and w_{M} for modern labor – are given by their respective marginal productivity. Finally, the wages of M-individuals are always at least as high as the remuneration of T-labor: $w_{\mathrm{M}} \geq w_{\mathrm{T}}$. One can think of the wages for modern type labor as consisting of two parts: the price of simple labor plus a premium for specifically modern characteristics.

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[18] Behind this assumption is the idea that the services of workers can be aggregated to a composite factor: $\mathrm{F}(\cdot) = \mathrm{G}(K, L)$ with $L = \mathrm{H}(M, T)$. If G and H both are linear homogeneous, G is strictly quasiconcave and the aggregator H is weakly quasiconcave, the production function will possess the attributes stated above.

[19] This assumption allows us to depict the Lewis model as a borderline case. Apart from this, it is unessential.

6. Labor Market and Diffusion

The aggregate learning curve gives the rate of social learning as a function of the composition of the modern sector labor force. The composition itself is determined by labor market equilibrium. The supply of T-labor is perfectly elastic at wage rate $\overline{w}$. The M-individuals have adapted to the modern sector environment and do not have any property rights in the traditional sector. Their labor supply will be perfectly inelastic.

At any moment, the stocks M and K are given by the accumulation and assimilation decisions of the past. The ratio between these two quantities,

$$m = M/K,$$

shall be labeled the *stock ratio* of the system. As in the Lewis model, there will be just enough T-individuals working in the modern sector to equalize their wage rate to the reservation wage. The constant returns assumption ensures that in equilibrium a relationship between the stock ratio m and the share q of modern type individuals in the labor force will hold:

$$\mathrm{F}_2\left(\frac{K}{M},\frac{T}{M},1\right) = \mathrm{F}_2\left(\frac{1}{m},\frac{1-q}{q},1\right) = \overline{w}. \tag{5}$$

The fraction q can be considered an implicit function $\mathrm{Q}(m,\overline{w})$ of the stock ratio. Given labor market equilibrium, the M-share q will increase in m:

$$\mathrm{Q}_m(\cdot) = -\left(\frac{q}{m}\right)^2 \frac{\mathrm{F}_{12}}{\mathrm{F}_{22}} > 0.$$

Economically, this rests on the assumption that T-labor and capital are no close substitutes. Imagine a labor market equilibrium with given quantities M and K. The T-labor in the modern sector is just enough for its marginal productivity to equalize the reservation wage. Now we mentally raise the stock ratio m by doubling the quantity M of modern type labor. If the quantity of T-labor could also double, its share of modern sector labor force would be unaffected. In that case, however, there would be only half as much capital per T-individual as in the original equilibrium. The marginal productivity of T-labor must not fall below the reservation wage, so the number of T-individuals cannot grow proportionately. Their weight in the modern sector labor force will go down.

The schedule of this relationship is the locus of all combinations (m,q) for which the marginal productivity of T-labor is equal to the reservation wage.[20] If the stock relation exceeds a cer-

[20] Condition (4) makes sure that the function Q exists. In the arbitrage equation (5), for every $q \in \left]0,1\right]$ there is exactly one m, such that the marginal productivity F_2 assumes the value $\overline{w}$. As $\lim_{m\to 0} \mathrm{F}_2 > \overline{w}$ for every $q > 0$, for $m \to 0$ the share q will have to approach zero as well, i.e., $\mathrm{Q}(0,\overline{w}) = 0$.

tain threshold value $\overline{m}$, the T-individuals are driven out of the modern sector altogether. By assumption (4), for $m \to \infty$ the marginal productivity F_2 will fall below $\overline{w}$ even for $q = 1$.

Just as the composition q of the modern sector labor force, the diffusion rate is also a function of the stock ratio:

$$\hat{M} = Z(m,\ldots) = \begin{cases} \phi(1-\mu)(1-Q(m,\overline{w})) + \nu & \text{for } 0 < m < \overline{m} \\ \nu & \text{for } m > \overline{m} \end{cases}$$

For $m = 0$, we have $M = 0$ and a growth rate is not defined. $Z(m,\ldots)$ shall be called the *diffusion function*. We can trace the relationship between m and $\hat{M}$ in Figure 2. Part I depicts the stock ratio on the abscissa and the growth rate of M on the ordinate. The slope of the curve is determined by the equilibrium on the labor market in part II and by the aggregate learning curve in part III. With a higher stock ratio, the equilibrium percentage of migrants from the traditional sector must go down to keep the T-wage constant. Along the aggregate learning curve, this means that the diffusion rate will also have to decrease. If the threshold value $\overline{m}$ is surpassed, no T-individual can earn a living in the modern sector and the growth of M is driven solely by natural reproduction. The diffusion function is near its upper limit $\phi(1-\mu)+\nu$ in the neighborhood of $m = 0$. With m getting higher, the diffusion rate will decline until at $m = \overline{m}$ it reaches the minimum value ν.

7. Capital Market and Accumulation

Individuals forego present consumption in order to provide for old age and as a safeguard against income shocks. In a market economy, both objectives may be reached by accumulating capital goods or securities. In a traditional society, however, where no capital is used and no legal provision exists to enforce claims on others, a different kind of social technique is needed. Old age provision may be founded on a "pay as you go" scheme in small groups, i.e., socially enforced transfers from the young to the old. Risk provision may also be based on small groups, through reciprocity networks. Network members are entitled to the help of others in case of need, but they have to be permanently ready themselves to assist the other members.[21] These social techniques have in common that nothing is saved *in the aggregate*. This is quite efficient as long as the system is not expected to support accumulation of produced means of production.

As previously mentioned, in Lewis' model the workers – coming from the traditional sector – do not save. All the saving is done by the modern capitalists. Correspondingly, we will assume that only M-individuals save and furthermore that their savings rate s is constant.

[21] See Posner (1980). In her study of a Mexican shanty town, Lomnitz (1977, 1988) shows that the institution of reciprocity not only survived the transfer to the city, but is reinforced and forms the backbone of the informal sector communities.

Now the model is closed.[22] Equilibrium on the capital market implies the equality of saving and net investment:

$$\dot{K} = s(rK + w_{\mathrm{M}} M).$$

For all $m > 0$, the growth rate of the capital stock is given as a function of the stock ratio:

$$\hat{K} = \mathrm{A}(m,\dots), \qquad \text{with} \qquad \mathrm{A}(m,\dots) = s(r + w_{\mathrm{M}} m).$$

The function A shall be labeled *accumulation function*. Differentiating gives us first $\partial\hat{K}/\partial m = s\left[(\mathrm{d}\, w_{\mathrm{M}}/\mathrm{d}\, m)m + w_{\mathrm{M}} + \mathrm{d}\, r/\mathrm{d}\, m\right]$. The Euler theorem states that factor price changes, weighted by the respective quantities, invariably add up to zero:

$$\frac{\mathrm{d}r}{\mathrm{d}m}K + \frac{\mathrm{d}w_{\mathrm{T}}}{\mathrm{d}m}T + \frac{\mathrm{d}w_{\mathrm{M}}}{\mathrm{d}m}M = 0 \Leftrightarrow \qquad \frac{\mathrm{d}r}{\mathrm{d}m} + \frac{\mathrm{d}w_{\mathrm{M}}}{\mathrm{d}m}m = -\frac{\mathrm{d}w_{\mathrm{T}}}{\mathrm{d}m}\frac{T}{K}. \tag{6}$$

This gives us the following partial derivative:

$$\mathrm{A}_m(m,\dots) = s\, w_{\mathrm{M}} > 0. \tag{7}$$

In part I of Figure 2, the accumulation function is depicted alongside the diffusion function. For later use, we need to know what happens in the neighborhood of $m = 0$. For $M = 0$, the production function F is linear homogeneous in the factor inputs K and T. Assumption (4) guarantees that the equation $\mathrm{F}_2(1, T/K, 0) = \overline{w}$ has a unique solution l_0 for T/K. As the production function and its derivatives are continuous, we obtain $\lim_{m\to 0} T/K\Big|_{w_{\mathrm{T}}=\overline{w}} = l_0$ and $\lim_{m\to 0} \mathrm{A}(\cdot) = s\left[\mathrm{F}(1, l_0, 0) - \overline{w}\, l_0\right] = sr_0$, where r_0 denotes the marginal return to capital as m approaches zero and $T/K = l_0$.

8. Asymptotic Dynamics

The system dynamics are completely determined by the time path of the state variable m. The second factor ratio, $q = M/(M+T)$, is given by m via the labor market equilibrium. For positive m, both the diffusion function and the accumulation function are defined and one obtains the following equation of motion:

$$\dot{m} = m \cdot \left[\hat{M} - \hat{K}\right] = m \cdot \left[\mathrm{Z}(m,\dots) - \mathrm{A}(m,\dots)\right].$$

A steady state is a value m^* of the state variable that will not change over time. For $m > 0$ a steady state is characterized by:

$$\mathrm{Z}(m^*,\dots) - \mathrm{A}(m^*,\dots) = 0. \tag{8}$$

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[22] With only minor differences as to the assumptions on technology, the results on growth, diffusion and the distribution of labor income also follow assuming a savings function of the Solow type. The mathematical form used above, however, permits clear statements on the distribution of total income, without needing to trace the dynamics of wealth distribution: All capital income by definition belongs to the M-individuals.

The value $m = 0$ will never be reached in finite time if the starting values M_0, K_0 are positive. The graphical representation of a steady state is, as depicted in Figure 2, the intersection of the schedules for the accumulation function and the diffusion function. The dynamic behavior of the system is characterized by one of three different regimes. We first describe these regimes formally and then proceed to interpret them:

<u>Proposition 1 (Lewis regime):</u> *For*

$$\phi(1-\mu) \le sr_0, \tag{9}$$

no steady state exists. The stock ratio will converge to zero for any given starting value m_0.

<u>Proof:</u> See Figure 3a. The accumulation function is strictly increasing in *m*, whereas the diffusion function is strictly decreasing. As a necessary condition for the existence of a steady state, the highest lower bound for A, sr_0, must be smaller than the lowest upper bound for Z, $\phi(1-\mu)+\nu$. Strict monotonicity of both functions further guarantees $\dot{m}/m < \mathrm{Z}(\varepsilon,\ldots) - \mathrm{A}(\varepsilon,\ldots) < 0$ for every $m > \varepsilon$. This gives us a lower bound for the absolute value of the growth rate outside any given ε-neighborhood of zero. For every starting value $m_0 > 0$, any ε will be reached in finite time. ■

<u>Proposition 2 (Internal Dualism):</u> *If the conditions*

$$sr_0 < \phi(1-\mu)+\nu \qquad and \qquad s\,\mathrm{F}(1,\overline{m},0) > \nu$$

are fulfilled, a unique steady m^* state exists with $0 < m^* < \overline{m}$. For $t \to \infty$, m converges to m^* for any given starting value $m_0 > 0$.

<u>Proof:</u> See Figure 3b. For $m > \overline{m}$, no T-labor will be used in the modern sector at the reservation wage. Thus $\mathrm{A}(\overline{m},\ldots) = s\,\mathrm{F}(1,\overline{m},0)$ and $\mathrm{Z}(\overline{m}\ldots.) = \nu$. Under the two conditions stated, the difference $\mathrm{Z}(m,\ldots) - \mathrm{A}(m,\ldots)$ is positive for $m \to 0$ and negative for $m = \overline{m}$. Continuity and strict monotonicity guarantee the existence of a steady state between these two values. Stability follows as above. ■

<u>Proposition 3 (Solow regime):</u> *If*

$$s\,\mathrm{F}(1,\overline{m},0) \le \nu \tag{10}$$

and – provided that this limit exists –

$$\nu < \lim_{m\to\infty} s\,\mathrm{F}(1,m,0), \tag{11}$$

a unique steady state exists with $m^* \ge \overline{m}$. For $t \to \infty$, m converges to m^* for any given starting value $m_0 > 0$.

<u>Proof:</u> See Figure 3c and the proof to Proposition 2. If condition (10) holds, then $\mathrm{Z}(\overline{m},\ldots) - \mathrm{A}(\overline{m},\ldots) > 0$. Any solution to (8) will thus be greater than $\overline{m}$. Condition (11) makes sure that a solution exists. Uniqueness and stability follow as above. ■

8.1 The Lewis regime

First we consider the situation in which there is no steady state. For $m \to 0$, both T and M will asymptotically grow with rate sr_0, because assumption (4) makes sure that T/K will converge to a positive value. Under condition (9), their growth will be faster than the growth of M.

As far as production is concerned, with a vanishing m the situation is described adequately by the Lewis model. The T-individuals with their unlimited labor supply make sure that labor can grow proportionally to capital input. The M-individuals play the role of the capitalists in the Lewis model: The whole capital stock belongs to them, but the significance of their own labor input disappears. The case depicted by Proposition 1 will thus be called the *Lewis regime*. The proportion of T-individuals in the modern sector labor force will approach 1, while for any single T-individual the probability of assimilation converges to zero.[23] The individuals coming from the traditional sector keep their cultural traits, they remain *peasants in cities.*

What are the conditions for such a development to take place? On the left-hand side of (9), we find the maximum rate of diffusion, $\phi(1-\mu)+\nu$. This expression depends on the ability to assimilate, ϕ, and the parameter μ indicating the presence of social barriers. As argued above, ϕ is small when the two cultures are rather dissimilar and when T-individuals have a low preference for the lifestyle of M-individuals. The parameter μ indicates the proportion of individuals who do not take part in the diffusion process. Diffusion will be slow if the groups are poorly integrated, as it is hampered by discrimination and segregation.

The right-hand side of inequality (9) represents the asymptotic accumulation rate, $sr_0 = \lim_{m \to 0} \mathrm{A}(m,\ldots) = s\left[\mathrm{F}(1, l_0, 0) - \overline{w}\, l_0\right]$. It is equivalent to the growth rate (1) for the Lewis model. The term in brackets is the asymptotic rate of net return to capital, when all labor is provided by T-individuals. The accumulation rate sr_0 is high if the reservation wage is low,[24] and when M-individuals show a strong propensity to save. If investment is pushed in a society with extremely poor peasants, accumulation threatens to "run away" from diffusion.

8.2 Internal Dualism

The conditions in Proposition 2 assure the existence of a stable steady state that satisfies $0 < m^* < \overline{m}$. In this steady state, the inputs of all factors grow at a common rate g^* determined by the point of intersection of the schedules for A and Z. This equilibrium growth rate, has a corresponding composition $q^* < 1$ of the modern sector labor force. The coexistence of the two groups in the modern sector suggests the term *internal dualism.* The characteristics of this regime will be explored below.

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[23] Still, since we do not consider threshold effects, the process of diffusion will proceed with rate $\phi(1-\mu)+\nu$.

[24] Here, too, a factor price frontier is relevant: If $w_{\mathrm{T}} = \overline{w}$ falls, the interest rate rises.

8.3 The Solow regime

Proposition 3 depicts a configuration where for $m = \overline{m}$ the accumulation rate is still below the natural growth rate. If, however, the stock ratio surpasses $\overline{m}$, the productivity of the last T-individual falls below the reservation wage $\overline{w}$. A steady state beyond $\overline{m}$ is in every respect equivalent to the equilibrium in the growth model for the closed economy, as devised by Solow (1956). The whole income in the modern sector is earned by M-individuals, and a constant share s of total income is saved. This case shall thus be called the *Solow regime*. The second condition in Proposition 3 implies that the natural growth rate ν can be surpassed by the accumulation rate. This will be the case, if, for example, the marginal product of capital will rise without limits as $M \to \infty$ or if M-individuals are able to produce without capital.

Once again it may be asked under what circumstances a Solow regime is likely to result. The natural growth rate is situated on the right-hand side of (10). If it is high, the whole demand for M-individuals in growth equilibrium may be satisfied by natural reproduction. Actually, social learning is still involved, but solely in the form of transmission from parents to descendants. On the left-hand side of the equation, we find the accumulation rate in $\overline{m}$. It will be low when M-individuals save little or when the reservation wage is high. Furthermore, with high reservation wage, the threshold $\overline{m}$ will be low.

It is striking that the respective other extremes – high savings rate, low natural growth rate and low reservation wage – constitute a Lewis regime. Below we will show that a reservation wage rising in the course of development may lead the modern sector through all the regimes described above. The Solow regime marks the neoclassical final stage of the dualistic development process.

9. Comparative Dynamics – Growth and Distribution

Now the influence of the key parameters on the character of modernization will be explored, given a regime of internal dualism. As a measure for the economic gap between the two social groups, we will use the difference in mean income:

$$d = \mathrm{D}(m,\ldots) = \frac{Y - w_{\mathrm{T}} T}{M} - w_{\mathrm{T}} = w_{\mathrm{M}} + \frac{r}{m} - w_{\mathrm{T}}.$$

This differential will invariably be positive. Not only do M-individuals by assumption receive a premium for their specific characteristics, they also reap the whole capital income. The income differential is a *decreasing function* of the stock ratio. Differentiating and using (6) yields:

$$\mathrm{D}_m(m,\ldots) = -\frac{r}{m^2} < 0.$$

Developing societies may differ in several parameters: the propensity to save, the reservation wage, the ability to assimilate or the degree of social integration in the modern sector. Our analysis will be limited to the modern sector. Inspection of Figure 2 yields a basic insight:

Proposition 4: *Any change of parameter that leaves the character of the social learning process intact can raise the equilibrium growth rate only at the expense of increasing the proportion of marginalized individuals in the modern sector.* ■

Steady state growth implies an equal growth rate of *both* accumulated factors:

$$\hat{M} = \hat{K} = g^*.$$

If a change of parameter leaves the aggregate learning curve in place, an increase in the equilibrium growth rate of M will only be possible if the share q of modern type individuals decreases. The consequences of such a shift are problematic: The income differential between the social groups widens and the probability of assimilation shrinks. This statement has an important counterpart:

Proposition 5: *Promoting the efficiency of social learning will raise the growth rate in the modern sector and the proportion of relatively affluent individuals in the population at the same time.* ■

The aggregate learning curve describes the trade-off between the proportion of affluent individuals and the growth rate of the modern sector. Only by influencing this relationship, both variables can be increased at the same time. The investigation below will illustrate and verify both statements.

9.1 An Increase in the Savings Rate

First of all, we consider an increase in the propensity to save. Here and below, all endogenous variables assume their equilibrium values. The condition for a steady state yields:

$$\frac{\mathrm{d}m^*}{\mathrm{d}s} = \frac{\mathrm{A}_s(\cdot)}{\mathrm{Z}_m(\cdot) - \mathrm{A}_m(\cdot)} = -\frac{r + w_\mathrm{M} m}{\phi(1-\mu)\mathrm{Q}_m(\cdot) + s\, w_\mathrm{M}} < 0,$$

The equilibrium stock ratio diminishes. As a consequence, the equilibrium growth rate increases, the proportion of modern type individuals in the population is reduced and the income differential widens:

$$\frac{\mathrm{d}g^*}{\mathrm{d}s} = \mathrm{Z}_m(\cdot)\frac{\mathrm{d}m^*}{\mathrm{d}s} > 0; \qquad \frac{\mathrm{d}q^*}{\mathrm{d}s} = \mathrm{Q}_m(\cdot)\frac{\mathrm{d}m^*}{\mathrm{d}s} < 0; \qquad \frac{\mathrm{d}d^*}{\mathrm{d}s} = \mathrm{D}_m(\cdot)\frac{\mathrm{d}m^*}{\mathrm{d}s} > 0.$$

A high aggregate savings rate will spur modern sector growth, just as in the Lewis model. But there are additional effects: With internal dualism, the social composition of the modern sector is adversely affected. Given the original stock ratio, an increase of the savings rate means first of all that the capital stock K will grow faster than the stock M of modern type individuals. Thus m begins to decrease. The infinitely elastic labor supply from the traditional sector pre

vents the wage w_{T} from rising, and the proportion of traditional type workers increases. Along the aggregate learning curve, this allows a faster growth of M. The decrease of m comes to a halt when this higher growth rate has reached the level $\hat{K}$. In the new equilibrium, the growth rates of both accumulated factors have risen and the lower stock ratio makes for a higher income of the M-agents. These results can easily be visualized by considering an upward shift of the accumulation function in Figure 4 and tracing the effects on the endogenous variables through the various parts of the diagram.

9.2 Costs of Assimilation and Social Integration

Given the composition of the modern sector labor force, the speed of diffusion will be high if the ability ϕ to assimilate is elevated. This parameter will be high if the M-type is attractive or if the assimilation costs are low. The effects of low frictional losses due to discrimination or segregation are identical. We will investigate the importance of social learning by inspecting the effect of a lower level of social friction, μ.

The condition (8) for a steady state gives us first:

$$-\frac{\mathrm{d}m^*}{\mathrm{d}\mu} = -\frac{\mathrm{Z}_\mu(\cdot)}{\mathrm{A}_m(\cdot)-\mathrm{Z}_m(\cdot)} = \frac{\phi(1-q)}{\phi(1-\mu)\mathrm{Q}_m(\cdot)+s\,w_{\mathrm{M}}} > 0\,.$$

The stock ratio rises. By (7), this means a higher equilibrium growth rate. The share of M-individuals also increases and the income differential becomes smaller:

$$-\frac{\mathrm{d}g^*}{\mathrm{d}\mu} = -\mathrm{A}_m(\cdot)\frac{\mathrm{d}m^*}{\mathrm{d}\mu} > 0; \quad -\frac{\mathrm{d}q^*}{\mathrm{d}\mu} = -\mathrm{Q}_m(\cdot)\frac{\mathrm{d}m^*}{\mathrm{d}\mu} > 0; \quad -\frac{\mathrm{d}d^*}{\mathrm{d}\mu} = -\mathrm{D}_m(m)\frac{\mathrm{d}m^*}{\mathrm{d}\mu} < 0\,.$$

The lower social friction will first of all raise the growth rate $\hat{M}$ above the former equilibrium value of $\hat{K}$. As a consequence, the stock ratio starts to increase. This leads to an increase in the proportion q of modern type individuals. Along the aggregate learning curve, the growth rate $\hat{M}$ decreases, and the M-individuals' total income (per unit of capital) grows. The accumulation rate goes up.

In the new equilibrium, growth rate, stock ratio and the proportion of modern type individuals have all risen. The income distribution is smoother. In Figure 5, the improvement of social integration shifts the aggregate learning curve and the graph of the diffusion function upwards.

10. An Idealized Modernization Process

By assuming that the traditional sector can supply an unlimited amount of labor at the reservation wage, we have ignored the repercussions of modern sector growth on the traditional part of the economy. The assumption of a constant reservation wage is analytically justified if

the traditional sector is large and social institutions stabilize individual consumption. The fixed wage allows us to derive a steady state growth rate for the modern sector. Interpreting this "steady state", we should keep in mind that the reservation wage does not necessarily keep this constant value throughout the entire development process. If labor transfer in the modern sector overcompensates natural growth of the traditional sector labor force, labor finally becomes scarcer and more valuable.

The model leads us to expect an S-shaped course of modernization. The basic reason for this, however, is not the mechanics of the diffusion process, but rather a slow increase in the reservation wage. As long as the reservation wage stays the same, growth rates in dynamic equilibrium will not change either.

The effects of an increasing reservation wage on the growth rates of M and K originate in the labor market. In order to satisfy the arbitrage condition, for any given M/K the ratio T/M must go down. Therefore, the labor market equilibrium will feature a higher share q of modern type individuals. As the reservation wage goes up, the traditional type individuals draw back from the modern sector. For any $q < 1$, we obtain through differentiation:

$$\mathrm{Q}_{\overline{w}}(\cdot) = -\frac{q^2}{\mathrm{F}_{22}} > 0.$$

On the aggregate learning curve, the decreasing proportion of T-individuals in the labor force dampens the diffusion rate. From (3) one obtains:

$$\mathrm{Z}_{\overline{w}}(\cdot) = -\phi(1-\mu)\mathrm{Q}_{\overline{w}} < 0.$$

Accumulation will also decrease for any given m. With higher wages for T-workers, the income of M-individuals necessarily shrinks. From $\mathrm{A}(\cdot) = s\left[\mathrm{F}\left(1, \frac{T}{K}, \frac{M}{K}\right) - \overline{w}\frac{T}{K}\right]$, we derive

$$\mathrm{A}_{\overline{w}}(\cdot) = s\left[\mathrm{F}_2 \frac{\mathrm{d}\, T/K}{\mathrm{d}\,\overline{w}} - \frac{T}{K} - \overline{w}\frac{\mathrm{d}\, T/K}{\mathrm{d}\,\overline{w}}\right] = -s\frac{T}{K} < 0.$$

In response to an increase in the reservation wage, both accumulation and diffusion function shift downwards, as is shown in Figure 6. This may lead either to a higher or to a lower value for the equilibrium stock ratio, depending on which function is depressed more strongly. The equilibrium growth rate will decrease at any rate. Differentiation of the condition $g^* = \mathrm{Z}(m^*, \overline{w}, \ldots) = \mathrm{A}(m^*, \overline{w}, \ldots)$ yields:

$$\frac{\mathrm{d}\, g^*}{\mathrm{d}\,\overline{w}} = \frac{\mathrm{Z}_{\overline{w}}(\cdot)\mathrm{A}_m(\cdot) - \mathrm{A}_{\overline{w}}(\cdot)\mathrm{Z}_m(\cdot)}{\mathrm{A}_m(\cdot) - \mathrm{Z}_m(\cdot)} = s\phi(1-\mu)\frac{(T/K)\mathrm{Q}_m(\cdot) - \mathrm{Q}_{\overline{w}}(\cdot)w_{\mathrm{M}}}{\phi(1-\mu)\mathrm{Q}_m(\cdot) + s w_{\mathrm{M}}} < 0.$$

The proportion of modern type individuals must rise, otherwise the diffusion rate could not go down. In equilibrium we have $g^* = \mathrm{Z}(m^*, \overline{w}) = \phi(1-\mu)(1-q^*)$, and therefore:

$$\frac{\mathrm{d}\, q^*}{\mathrm{d}\,\overline{w}} = -\frac{1}{\phi(1-\mu)}\frac{\mathrm{d}\, g^*}{\mathrm{d}\,\overline{w}} > 0.$$

.

Now we can sketch an idealized modernization process. If the reservation wage is very low at the onset of modernization – the traditional sector being very poor – the economy is situated in the Lewis regime. The accumulation rate is high, higher indeed than the diffusion rate. The modern sector is growing fast and the proportion of modern type individuals will be shrinking. If the reservation wage begins to rise gradually, or if it was not so low at the beginning, the regime of internal dualism is reached. There is a steady state with $0 < m^* < \overline{m}$.

If the reservation wage continues to rise, the modern sector increasingly loses its dualistic character. M-labor will be less scarce, both with regard to capital and compared to T-labor. The absorption of labor from the traditional sector, accompanied by a slow increase in the reservation wage, may continue until the labor surplus in the traditional sector is exhausted and the labor market in the traditional sector becomes efficient, or else until the modern sector reaches an equilibrium in the Solow regime. Then, no traditional type labor can be efficiently employed in the modern sector any more. Both situations are possible final stages of the dualistic scenario and the process of cultural diffusion in the modern sector.

Appendix A: The aggregate learning curve for reference groups of arbitrary size

Now we investigate the general case of a reference group with size $n \geq 1$. The reference group is a random sample from the modern sector population. Its size is countable, whereas the modern sector population forms a continuous space. The probability for the $i+1$'st member of the reference group to be of type M is independent of the type of the i'th member. The relevant probabilities are calculated as in the urn experiment with replacement. We denote the number of M-individuals in a reference group with size n as $\tilde{y}_n$. This variable follows a binomial distribution.[25] For assimilation, it is necessary that at least one M-individual be among the members of the reference group. If a portion $x = 1 - q$ of the population is of the traditional type, the probability of this event is given by $\mathrm{P}\{\tilde{y}_n \geq 1\} = 1 - \mathrm{P}\{\tilde{y}_n = 0\} = 1 - x^n$. For the individual probability of assimilation we therefore obtain $\phi\left(1 - x^n\right)$.

Obviously, this probability converges to ϕ for $n \to \infty$. With a higher number of close contacts, the *must see condition* is ever less of an impediment to assimilation, provided that there are any M-individuals in the modern sector at all. For $n \to \infty$, diffusion will follow the pattern of an *external influence model*[26] with constant transmission probability ϕ. In this borderline case, only the coexistence of different agents *per se* matters, not the details of the labor force composition.

If we aggregate over the entire population and leave aside natural growth, we obtain:

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25 Boyd and Richerson (1985), p. 139, also generalize the simple epidemic model to account for more than one role model. They do not posit a *must see condition*, instead they suppose that different cultural variants will be role models with different probabilities. Then the probability of transmission can be calculated as a weighted average. Bailey (1967) briefly discusses transmission equations based on the binomial distribution in epidemiological models.

26 See Lekvall and Wahlbin (1973).

$$\dot{M} = \phi\left(1-x^n\right)T \qquad \text{and} \qquad \hat{M} = \phi\left(1-x^n\right)\frac{T}{M} = \phi\frac{1-x^n}{1-x}(1-q). \tag{12}$$

Comparing this to the result for $n=1$ in (2)', the two differential equations only differ in a factor that depends on the size of the reference group and on the composition of the population. This factor can be written as a power series:

$$\frac{1-x^n}{1-x} = 1 + x + x^2 + \ldots + x^{n-1} = \mathrm{B}(x,n).$$

The diffusion rate will be high when the reference groups are large. Furthermore, for reference groups of any (finite) size, it will be a decreasing function of the share q of M-individuals:

$$\frac{\partial\hat{M}}{\partial q} = -\phi\left(\mathrm{B}_x(\cdot)\,x + \mathrm{B}(\cdot)\right) < 0.$$

Now consider the size of the reference group being the outcome of a discrete random variable $\tilde{n} \in \{1,2,\ldots\bar{n}\}$, distributed among the T-individuals with density $\mathrm{f}_{\tilde{n}}(n)$. The growth of M is formally calculated as a mathematical expectation. From $\dot{M} = \sum_{n=1}^{\bar{n}} \phi\left(1-x^n\right)\mathrm{f}_{\tilde{n}}(n)T$ we derive the equation of motion $\hat{M} = \phi(1-q)\sum_{n=1}^{\bar{n}} \mathrm{B}(x,n)\,\mathrm{f}_{\tilde{n}}(n) = \phi(1-q)\,\mathrm{E}\left[\mathrm{B}(x,\tilde{n})\right]$. As the expectation operator is linear, the properties discussed above are still valid.

Appendix B: Segregation and Social Learning

We denote as *segregation* the splitting up of a population into two or more subpopulations of different composition and separated from each other. First consider a single mixed population of size *N*, consisting of *M* modern type individuals and *T* traditional type individuals. With the traditional type having a share $\bar{x}$ of the population, the normalized number of M-individuals will grow according to eq. (12) as follows:

$$\frac{\dot{M}}{N} = \phi\left(1-\bar{x}^n\right)\bar{x} := \mathrm{V}(\bar{x}).$$

As can easily be seen, V is strictly concave. Now divide the population into two sectors of size N_1 and N_2, where T-individuals have shares x_1 and x_2. Without loss of generality, we assume $0 \le x_1 < \bar{x} < x_2 \le 1$. Both subpopulations are perfectly mixed, but there is no contact between them.[27] The population weight N_1/N of the first sector will be denoted by a. The total normalized number of assimilations is a weighted average of the values realized in the subsectors:

$$\left.\frac{\dot{M}}{N}\right|_{x_1,\,x_2} = a\,\mathrm{V}(x_1) + (1-a)\,\mathrm{V}(x_2) < \mathrm{V}(\bar{x}). \tag{13}$$

Because of strict concavity, this number will fall short of the reference value for the integrated case. Figure 7 gives a graphical representation of this argument. Thus, dividing an integrated population into subpopulations of different composition will invariably slow down the aggregate learning process. Furthermore, we show that the shortfall is greater when the

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[27] For Coleman (1964) this would be an example of diffusion in an 'incomplete social structure'.

compositions of the subpopulations differ strongly, i.e., if the sectors are polarized. Consider a reallocation of the population in the two sectors that leaves the T-share constant in sector 1 and increases it to $x_2{}'$ in the second sector. Because of the identity $ax_1 + (1-a)x_2 \equiv \bar{x}$, the weight of the first sector will thereby have to increase. We can interpret this as the formation of an inner city slum. If we denote by a' the new, higher weight of sector 1, the normalized number of assimilations will be:

$$\left.\frac{\dot{M}}{N}\right|_{x_1, x_2{}'} = a' \mathrm{V}(x_1) + (1-a') \mathrm{V}(x_2{}') = a \mathrm{V}(x_1) + (1-a) \left[\frac{a'-a}{1-a} \mathrm{V}(x_1) + \frac{1-a'}{1-a} \mathrm{V}(x_2{}') \right] .$$

In order to compare this with our result in (13), we must evaluate the expression in square brackets. As V is concave, we have:

$$\frac{a'-a}{1-a} \mathrm{V}(x_1) + \frac{1-a'}{1-a} \mathrm{V}(x_2{}') < \mathrm{V}\left(\frac{a'-a}{1-a} x_1 + \frac{1-a'}{1-a} x_2{}' \right).$$

Because of the identity $ax_1 + (1-a)x_2 = a' x_1 = (1-a')x_2{}'$, the right-hand side is equal to $\mathrm{V}(x_2)$. Thus, the number of assimilations is smaller in the polarized case. This argument is also represented in Figure 7.

Literature

Arrow, Kenneth J. (1994)
Methodological Individualism and Social Knowledge. Richard T. Ely Lecture.
American Economic Review 84 (Papers and Proceedings), pp. 1-9

Bailey, Norman T.J. (1957)
The Mathematical Theory of Epidemics.
1. ed.; London: Charles Griffin & Company Ltd. (2. ed. 1975)

Banerjee, Abhijit V. and Andrew F. Newman (1996)
A Dual-Economy Model of Modernization and Development.
Harvard Institute for International Development, Development Discussion Paper No. 531

Banerjee, Abhijit V. and Andrew F. Newman (1993)
Occupational Choice and the Process of Development.
Journal of Political Economy 101, pp. 274-298

Bartholomew, David (1967)
Stochastic Models for Social Processes.
London and New York: Wiley

Bénabou, Roland (1996a)
Heterogeneity, Stratification and Growth: Macroeconomic Implications of Community Structures and School Finance.
American Economic Review 86, pp. 584-609

Bénabou, Roland (1996b)
Equity and Efficiency in Human Capital Investment: The Local Connection.
Review of Economic Studies 63, pp. 237-264

Blanchard, Olivier J. (1985)
Debts, Deficits and Finite Horizons.
Journal of Political Economy 93, pp. 223-247

Boyd, Robert and Peter J. Richerson (1985)
Culture and the Evolutionary Process.
Chicago, London: University of Chicago Press

Cavalli-Sforza, Luigi L. and Feldman, Marcus W. (1981)
Cultural Transmission and Evolution: a Quantitative Approach.
Princeton: Princeton University Press

Cole, William E. and Richard D. Sanders (1985)
Internal Migration and Urban Unemployment in the Third World.
American Economic Review 75, pp. 481-494

Coleman, James S. (1964)
Introduction to Mathematical Sociology.
Toronto, Ontario: Collier-McMillan Canada

Dawkins, Richard (1994)
The Selfish Gene
2. ed.; New York, Oxford, London: Oxford University Press

Durlauf, Steven N. (1996)
A Theory of Persistent Income Inequality.
Journal of Economic Growth 1, pp. 75-93

Ellison, Glenn and Drew Fudenberg (1993)
Rules of Thumb for Social Learning.
Journal of Political Economy 101, pp. 612-643

Ellison, Glenn and Drew Fudenberg (1995)
Word-of-Mouth Communication and Social Learning.
Quarterly Journal of Economics 110, pp. 93-125
Findlay, Ronald (1982)
Protection and Growth in a Dual Economy.
in: Mark Gersovitz, Carlos F. Diaz-Alejandro, Gustav Ranis und Mark R. Rosenzweig (ed.), The Theory and Experience of Economic Development. Essays in Honor of Sir W. Arthur Lewis. London: Allen & Unwin, pp. 173-193
Findlay, Ronald and Carlos A. Rodriguez (1981)
A Model of Economic Growth with Investment in Human Capital.
Research in Human Capital and Development 2, pp. 57-72
Griliches, Zvi (1957)
Hybrid Corn: An Exploration in the Economics of Technological Change.
Econometrica 25, pp. 501-522
Heiner, Ronald A. (1983)
The Origin of Predictable Behavior.
American Economic Review 73, pp. 560-595
v. Kalckreuth, Ulf (1999) *(in German)*
Wachstum und soziale Integration in dualistischen Volkswirtschaften.
Frankfurt, Berlin, Bern, etc.: Lang
Kubin, Ingrid and Peter Rosner (1996)
Immigration, Growth and Distribution.
Department of Economics, University of Vienna, Working Paper No. 9606
Lazear, Edward P. (1995)
Culture and Language.
NBER Working Paper No. 5249
Lekvall, Per and Clas Wahlbin (1973)
A Study of Some Assumptions Underlying Innovation Diffusion Functions.
Swedish Journal of Economics, pp. 362-377
Lewis, W. Arthur (1954)
Economic Development with Unlimited Supplies of Labour.
Manchester School of Economic and Social Studies 22, pp. 139-191
Lewis, W. Arthur (1958)
Unlimited Labour: Further Notes.
Manchester School of Economic and Social Studies 26, pp. 1-32
Lomnitz, Larissa (1977)
Networks and Marginality.
Life in an Mexican Shantytown.
New York, San Francisco, London: Academic Press
Lomnitz, Larissa (1988)
The Social and Economic Organization of a Mexican Shanty Town.
in: Joseph Gugler (ed.), The Urbanization of the Third World. Oxford, New York, Toronto, etc.: Oxford University Press, pp 242-263.
Loury, Glenn C. (1977)
A Dynamic Theory of Racial Income Differences.
in: Phyllis A. Wallace und Anette A. Le Mond (ed.), Women, Minorities and Employment Discrimination. Lexington, Mass.: Lexington Books, pp. 153-186

Mansfield, Edwin (1968)
Industrial Research and Technological Innovation: An Econometric Analysis.
New York: Norton
Morris, Martina (1993)
Epidemiology and Social Networks: Modelling Structured Diffusion.
Sociological Methods and Research 22, pp. 99-126
Nelson, Richard R. (1995)
Recent Evolutionary Theorizing about Economic Change.
Journal of Economic Literature 33, pp. 48-90
Nelson, Richard R. and Sidney G. Winter (1982)
An Evolutionary Theory of Economic Change.
Cambridge, Mass., London: The Belknap Press of Harvard University Press
Posner, Richard A. (1980)
A Theory of Primitive Society, with Special Reference to Primitive Law.
Journal of Law and Economics 23, pp. 1-53
Ranis, Gustav (1988)
Analytics of Development: Dualism.
in: Hollis Chenery und T.N. Srinivasan (ed.), Handbook of Development Economics, Vol. I. Amsterdam, New York etc.: North Holland, pp. 73-92
Ranis, Gustav und John C.H Fei (1961)
A Theory of Economic Development.
American Economic Review 51, pp. 533-565
Rogers, Everett M. (1983)
Diffusion of Innovations.
3. ed.; New York: Free Press
Sen, Amartya K. (1966)
Peasants and Dualism with or without Surplus Labour.
Journal of Political Economy 74, pp. 425-450
Solow, Robert M. (1956)
A Contribution to the Theory of Economic Growth.
Quarterly Journal of Economics 70, pp. 65-94
Streufert, Peter A. (1991)
The Effect of Underclass Social Isolation on Schooling Choice.
University of Wisconsin-Madison, Institute for Research on Poverty, Discussion Paper No. 954-91
Todaro, Michael P. (1969)
A Model of Labor Migration and Urban Development in Less Developed Countries.
American Economic Review 59, pp. 138-148
Wilson, William Julius (1987)
The Truly Disadvantaged. The Inner City, the Underclass and Public Policy.
Chicago and London: The University of Chicago Press
Zhang, Haizhu (1996) *(in German)*
Wirtschaftliche Entwicklung einer offenen Dualwirtschaft.
Eine theoriegeleitete Studie, dargelegt am Beispiel der Volksrepublik China.
Frankfurt a.M., Berlin, Basel etc.: Lang