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QALYs versus HYEs - What's Right and What's Wrong

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1. Introduction

The last two decades have seen cost-utility analysis of health care programmes becoming increasingly popular, both at the theoretical level and in empirical applications. This certainly explains why utility measurement in health care has attained such a prominent position in the field of health economics. On the benefit side of changes in resource allocation, the twin aspects of morbidity and mortality effects are to be combined to yield an assessment in line with individual preferences. Thus, at least in theory, the utility of the people involved provides the yardstick with which to gauge health effects.

In order to obtain economically meaningful descriptions of the utility impact due to changes in individual health, two approaches currently coexist on somewhat less than peaceful terms. The older one, the quality-adjusted life years (henceforth: QALY) approach, lends itself to empirical application fairly easily since several methods have been devised in order to capture the QALY impact due to health care programmes. More recently, its position has been challenged by the healthy-years equivalent (henceforth: HYE) approach, mainly on the grounds that the QALY approach be likely to misrepresent individual preferences for health effects.

To economists outside the health economics profession, the strong emphasis on individual utility characteristic of both approaches may seem peculiar. More precisely, given that an infinite number of utility representations exists for the same individual preference relation, it is legitimate to question the use of utility analysis in an effort directed at measuring the intensity of preferences. However, as will be made clear below, both the QALY and the HYE approach introduce some sort of scaling the effect of which is to single out exactly one functional representation of individual preferences. Therefore, the non-uniqueness problem mentioned before is not really present.

From the very outset, its main proponents have argued that “unlike QALYs, HYEs fully represent patients´ (or other individuals) preferences“.[1] Since this is obviously a rather strong claim, it is not particularly surprising to find proponents of the QALY approach taking issue with that statement. In fact, a veritable dispute has developed over the comparative merits of both approaches, as witnessed, e.g., by several papers published in

[1] Mehrez and Gafni (1989), p. 143.

two more recent issues of the "Journal of Health Economics".[2] More precisely, the debate centers on the ability of either approach to capture individual preferences for health under certainty and in the presence of risk. Apart from that, considerable confusion has arisen over side issues such as the appropriateness of the two-step procedure employed by the HYE approach.

Given the prominent position of utility measurement in the economic evaluation of health care programmes and, more generally, in the assessment of health outcomes, it would certainly be helpful to know about the validity of alternative views held by the contributors to the QALYs versus HYEs debate. In principle, knowledge about the extent to which both approaches succeed in generating information on individual preferences, respectively, is available from Bleichrodt´s paper.[3] However, the theoretical part of his paper is directed at providing characterizations of the QALY and the HYE approach in terms of individual preferences. While his results no doubt are important in their own right, they offer rather little direct guidance on the issues at stake in the current debate.[4]

Therefore, the purpose of this paper is a simple one. By assessing the main views to be found in the literature from a theoretical perspective, it tries to make a contribution to the ongoing QALYs versus HYEs debate. More precisely, I shall demonstrate that the major issues can all be resolved in a surprisingly simple manner. If one is prepared to accept the multi-attribute decision theoretic framework adopted here, the validity of each argument of the debate can be determined without ambiguity. In turn, this explains the rather ambitious title chosen for this paper.

The paper is organized as follows. Section 2 presents the general intertemporal setting drawn from multi-attribute decision theory. In particular, care is taken to describe conditions which are sufficient to establish the existence of preferences for health, both under certainty and in the presence of risk. Next, section 3 describes the QALY and the HYE approach in some detail. Specifically, it includes an interpretation of both approaches in terms of either value or utility theory. This prepares the ground for section

[2] Cf. Buckingham (1993a), Culyer and Wagstaff (1993), Gafni, Birch and Mehrez (1993) as well as Loomes (1995), Johannesson (1995), Bleichrodt (1995), Culyer and Wagstaff (1995).

[3] Cf. Bleichrodt (1995).

[4] In fact, Bleichrodt´s paper contains only a brief discussion of some of the issues at stake, cf. Bleichrodt (1995), pp. 25-26.

4 which is devoted to an examination of alternative views held in the QALYs versus HYEs debate. In addition, the approximation procedure for the HYE approach proposed by Mehrez and Gafni comes under scrutiny. The final section puts forward reasons which may explain why the debate over QALYs versus HYEs contains an astoundingly large number of errors and misconceptions.

2. Existence of preferences for health

This section presents conditions which ensure that an individual does indeed have preferences for health in a sense to be defined below. Since both the QALY and the HYE approach rely on the existence of such preferences for health, an investigation of the underlying assumptions seems to represent a worthwhile undertaking. Basically, the analysis will start from a point where health constitutes but one source of individual utility. More precisely, preferences for health will be embedded in a more general setting in which the individual is taken to have preferences over intertemporal profiles of health and some set of non-health attributes, the latter referring to more common objects of choice, i.e., consumption of (other) commodity bundles.

While to some degree, of course, the framework of analysis represents an arbitrary choice, it should be emphasized from the outset that the one considered below is quite general. Given that it is truly intertemporal, the general setting captures the trade-off between health and other sources of utility at any point in time as well as the trade-off between any two attributes over time. As such, it is general enough to be compatible with the preference specification adopted in a variety of life-cycle models currently prominent in health economics, e.g., the full Grossman model or the Berger et al. model.[5]

2.1 The general setting

In the general setting, the analysis is conducted in continuous time and the basic objects of individual choice are given by flows involving health status $h(t)$ and non-health

[5] Cf. Grossman (1972) and Berger, Blomquist, Kenkel and Tolley (1986). The “full Grossman model“ refers to the model including both a consumption and an investment benefit of health.

attributes $x(t)$ where t refers to the time index. In human capital terminology, $h(t)$ represents a service produced by the stock of health capital. While, in principle, it could be a vector, $h(t)$ will be assumed scalar without loss of generality. More specifically, it is useful to interpret $h(t)$ as health status at time t. As mentioned above, the vector $x(t)$ is taken to refer to the consumption of other commodities. To be precise, $x(t)$ denotes the respective rate of consumption. These flows of $[h(t), x(t)]$ evolve over time and their occurence may either be certain or uncertain in the sense that a discrete probability distribution applies.

Consider first the case of certainty. More precisely, assume a real number T to represent a finite upper bound on the individual´s lifetime measured in years, i.e., any flow of health and non-health attributes under consideration will have the individual dying no later than after T years. Clearly, the objects of choice are then given by flows $[h(t), x(t)] = \{(h(t), x(t) | 0 \leq t \leq T)\}$ of health and non-health attributes. Furthermore, suppose the individual´s relation of weak preference ${}^{T}\succ$ over such flows to be complete, transitive and continuous. Then, applying a result due to Debreu,[6] a continuous **value function** φ can be shown to exist such that it represents the preferences of the individual.[7] In other words, one has for any two flows $[h(t), x(t)]^1$ and $[h(t), x(t)]^2$:

(1) $$[h(t), x(t)]^1 \; {}^{T}\succ [h(t), x(t)]^2 \quad \Leftrightarrow \quad \varphi\left([h(t), x(t)]^1\right) \geq \varphi\left([h(t), x(t)]^2\right)$$

Moreover, any monotonic transformation of φ will also represent the preference relation ${}^{T}\succ$. Thus, the function φ simply denotes some member of the equivalence class of all value functions capable of representing the individual´s preferences over flows of health and non-health attributes.

Let me now turn to the more general case of risk. It is customary to distinguish between a risk involving health status (a morbidity risk) and a risk with respect to (remaining) length of life (a mortality risk). In the analysis of the present paper, both types of risk may be present. As for mortality, suppose again an upper bound of T years to exist which now represents maximum possible remaining lifetime. Clearly, the objects of

[6] Cf. Debreu (1983), p. 108. Note that the variant of this theorem commonly used in consumer theory is of no use here since the objects of choice do not belong to a finite-dimensional Euclidean space.
[7] On the notion of a value function see Keeney and Raiffa (1976), p. 80ff. or French (1986), p. 74ff..

choice are then lotteries l over both health and non-health attributes, where, due to the assumption of a discrete probability distribution, the description of a lottery includes some countable set of probabilities relating to the possible states of nature and the corresponding flows of attributes:

(2) $$l = \left(\pi_1, \pi_2, \ldots; [h(t), x(t)]^1, [h(t), x(t)]^2, \ldots\right); \quad \sum_i \pi_i = 1; \quad \pi_i \rangle 0$$

To impose some structure, take the individual´s relation of weak preference ${}^T\succ^l$ over such discrete lotteries to be complete, transitive and continuous. Applying again Debreu´s theorem, this implies the existence of a continuous function Ψ which represents the individual´s preferences, i.e., one has for any two lotteries l_1 and l_2:

(3) $$l_1 \quad {}^T\succ^l \quad l_2 \qquad \Leftrightarrow \qquad \Psi(l_1) \;\geq\; \Psi(l_2)$$

In particular, any monotonic transformation of Ψ will also represent individual preferences as described by the relation ${}^T\succ^l$. Therefore, the function Ψ can be interpreted as a typical member of the equivalence class of all functions capable of representing the individual´s preferences over risky streams of health and non-health attributes.

Furthermore, take the individual to satisfy the von Neumann-Morgenstern axioms, i.e., he/she maximizes expected utility. It is well-known that these assumptions are sufficient to imply the individual´s preferences for risky streams of the two (sets of) attributes over time to be representable by the expectation of a continuous von Neumann-Morgenstern (vNM) **utility function** ψ .[8] Hence, one has:

(4a) $$\Psi(l) = \sum_i \pi_i \psi\left([h(t), x(t)]^i\right)$$

and, thus, for any two lotteries l_1 and l_2:

(4b) $$l_1 \;{}^T\succ^l\; l_2 \qquad \Leftrightarrow \qquad \sum_i \pi_i \psi\left([h(t), x(t)]^i\right) \geq \sum_j \pi_j \psi\left([h(t), x(t)]^j\right)$$

where i (j) indexes the states of nature contained in lottery l_1 (l_2). More precisely, the function ψ maps flows $[h(t), x(t)]$ into the set of real numbers and is defined uniquely up to positive affine transformations. In other words, ψ is some member of the equivalence class of all vNM utility functions capable of representing the individual´s

[8] On the notion of a utility function see Keeney and Raiffa (1976), p. 219f. and French (1986), p. 153ff..

preferences as described by the relation ${}^{T}\succ^{l}$ iff the von Neumann-Morgenstern axioms hold.

A final remark on the utility function ψ . Since it represents the individual´s preferences over degenerate lotteries involving just one flow of health and non-health attributes, it can also be employed to describe preferences under certainty. In other words, given the validity of the von Neumann-Morgenstern axioms, a utility function ψ also represents a value function.[9] However, the converse is not true. From what has been said above it should be clear that the equivalence class of utility functions to which ψ belongs is strictly smaller than the equivalence class of value functions to which φ belongs. Therefore, a value function need not be a utility function.[10]

2.2 Additional structure

Consider again first the case of certainty. More specifically, assume the timepath of health status to be **preferentially independent** of the timepath of non-health attributes.[11] This implies the individual´s preferences for flows of health status to be independent of the particular flow of non-health attributes under consideration. Accordingly, it is then possible to speak about preferences for flows of health status.[12] Moreover, under this condition the preference relation ${}^{T}\succ$ induces a relation of weak preference ${}^{T}\succ_{h}$ on the space of health attribute flows in the following manner:

(5a) $\quad [h(t)]^{1}\ {}^{T}\succ_{h} [h(t)]^{2} \quad \Leftrightarrow \quad [h^{1}(t),\tilde{x}(t)] \succ [h^{2}(t),\tilde{x}(t)]$ for some $[\tilde{x}(t)]$.

Likewise, a value function φ induces a value function v on the same space as follows:

(5b) $\quad \varphi([h(t),x(t)]) \;=\; g\{v([h(t)]),[x(t)]\}; \qquad \dfrac{\partial g}{\partial v} \rangle\ 0\,.$

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[9] Similarly, the restriction of the relation ${}^{T}\succ^{l}$ to degenerate lotteries yielding some flow of health and non-health attributes with certainty generates the relation ${}^{T}\succ$ in a sense made precise below; cf. the end of section 2.2.

[10] Cf. Keeney and Raiffa (1976), p. 220 and French (1986), p. 150.

[11] On the notion of preferential independence of (sets of) attributes, see Keeney and Raiffa (1976) and French (1986).

[12] In other words, the assumption of preferential independence entails the existence of a marginal preference relation which is defined for flows of health status. Otherwise, individual preferences for flows of health status would depend on the flow of non-health attributes and it would not make sense to speak about preferences for (flows of) health (status); cf. French (1986), p. 108.

Any monotonic transformation of v yields another value function v' which also represents the same preferences. Hence, the function v is a member of the equivalence class which consists of all value functions representing the individual´s preferences over flows of health status as described by the relation ${}^{T}\!\succ_{h}$.

Consider now the presence of risk. In order to be able to discuss preferences for health in this case, the individual´s preferences over lotteries involving only the flow of health status - i.e., **special lotteries** l^h - must be independent from the particular flow of non-health attributes $[\tilde{x}_t]_{t=1}^{T}$ under consideration. This is achieved by assuming the flow of health states to be **utility independent** from the flow of non-health attributes.[13] This implies the existence of a relation of weak preference ${}^{T}\!\succ_{h}^{l}$ for lotteries l^h involving only the timepath of the health attribute. Correspondingly, if one looks at the vNM function ψ for a given flow of non-health attributes, any two such functions must be strategically equivalent, i.e. one can be obtained from the other through a positive affine transformation.[14] Thus, the assumption of utility independence gives rise to a vNM utility function u defined for flows of health status whose expectation represents the individual´s preferences over the set of lotteries l^h.

In order to present these results more formally, define a special lottery l^h as follows:

(6) $$l^h \quad = \quad \left(\pi_1, \pi_2, \ldots; [h(t)]^1, [h(t)]^2, \ldots\right); \quad \sum_i \pi_i = 1; \quad \pi_i \rangle\, 0\,.$$

By utility independence of the flow of health states from the flow of non-health attributes, one has:

(7a) $$\psi\left([h(t), x(t)]\right) = \alpha\, u\left([h(t)]\right) + \beta\ ; \quad \alpha \rangle\, 0; \quad u\left([h(t)]\right) = \psi\left([h(t), \tilde{x}(t)]\right)$$

where both α and β depend on the flow of non-health attributes $[x(t)]$. Furthermore:

(7b) $$l_1^h \ {}^{T}\!\succ_{h}^{l} \ l_2^h \qquad \Leftrightarrow \qquad \sum_i \pi_i u\left([h(t)]^i\right) \geq \sum_j \pi_j u\left([h(t)]^j\right),$$

where i (j) indexes the states of nature contained in l_1^h (l_2^h) and, to be sure, the comparison of the two lotteries involves the same flow of non-health attributes.

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[13] On the property of utility independence see Keeney and Raiffa (1976) and French (1986). For an assessment of this assumption see Bodily (1980).

[14] Cf. Keeney and Raiffa (1976), p. 226.

As already noted above, any positive affine transformation of u will produce another utility function u´ which may serve equally well to represent individual preferences. Thus, the function u denotes a member of the equivalence class comprising all utility functions capable of representing the relation ${}^{T}\succ_{h}^{l}$ over special lotteries.

Consider finally the restriction of ${}^{T}\succ_{h}^{l}$ to the set of degenerate special lotteries, i.e., special lotteries yielding a particular flow of health status for certain. Clearly, if one is willing to identify such lotteries with the corresponding flows $[h(t)]$, this restriction of ${}^{T}\succ_{h}^{l}$ simply generates the relation ${}^{T}\succ_{h}$. Hence, utility independence of the flow of health attributes from the flow of non-health attributes implies the property of preferential independence.

2.3 The introduction of discrete time

Up to now, the analysis has been set up in continuous time. Both the QALY and the HYE approach, however, relate to a discrete time setting - i.e., periods of time - where each period equals one year. In order to comply with this, suppose T to be a positive integer. Retaining the assumptions introduced in the previous section, attention is now confined to flows of health status which are constant in any given year but may vary between years. Hence, the objects of choice are, in fact, **sequences of health status** $\{h_t\}_{t=1}^{T}$ in the case of certainty and lotteries involving only sequences of health status under conditions of risk.

It is important to see that, in discrete time, the individual´s preferences for health can be described by means of the instruments discussed in the previous section, i.e., the preference relations and the corresponding functional representations. This holds because health status sequences are but special flows of health status. In turn, this implies the choice sets of the discrete time setting to be proper subsets of the corresponding choice sets in continuous time. More precisely, the set of health status sequences is a proper subset of the domain of ${}^{T}\succ_{h}$. Therefore, both ${}^{T}\succ_{h}$ and any value function based on it generate a ranking of health status sequences in line with individual preferences.

Likewise, the set of lotteries involving sequences of health status constitutes a proper subset of the set of lotteries defined with respect to health status flows in general. It follows that, in the discrete time setting introduced above, the relation ${}^{T}\succ_{h}^{l}$ and any corresponding vNM utility function are fully capable of representing the individual´s preferences for health in the case of risk.

Since both the QALY and the HYE approach rely on discrete time, one may question the need to discuss the continuous time case at all. When assessing individual preferences for health, however, both approaches rely on reference flows of health which can validly be constructed only in a continuous time setting.[15]

More specifically, as will become more obvious below, this statement holds true for the HYE approach in toto and for the QALY approach based on the time trade-off method. Thus, although neither the QALY nor the HYE approach go beyond the evaluation of health status sequences in the sense defined above, the respective evaluation procedures upon which the focus of the QALYs versus HYEs debate lies each require individual preferences to be defined for health status flows proper. Accordingly, it is necessary to work with the more general concepts of a preference relation for health status flows in the case of certainty and a preference relation for lotteries involving health status flows in the presence of risk.

3. The two approaches

The primary purpose of this section is to present the QALY and the HYE approach, respectively, in terms of multi-attribute decision theory. Moreover, throughout the remainder of this paper it will be assumed that no measurement errors and no strategic biases are present. Hence, individuals are taken to respond truthfully and unerringly to any question concerning their preferences for health. While this assumption may not be particularly realistic, it helps to bring out most clearly the theoretical differences between the two approaches of interest.

[15] As far as I know, this point has not received any attention in the literature to date.

3.1 The QALY approach

While the notion of quality-adjusted life years has probably been mentioned first by Zeckhauser and Shepard in their 1976 paper,[16] its origins trace back further in time. For example, earlier work by Fanshel and Bush - i.e., the concept of “function years“ - seems to be closely related in spirit,[17] and the basic idea is certainly present in the 1971 paper by Culyer et al.[18] Indeed, the notion that life years of different quality should be made comparable had already been applied before by Klarman et al. in their cost-effectiveness analysis dealing with alternative modes of treatment of chronic renal disease.[19]

In comparing different streams of health status over time, the QALY approach demands that both the quantity (i.e., length of life, mortality) and the quality (i.e., morbidity) of life should be taken into account. This is especially important for program evaluation in health care, because simply counting life years saved may fail to adequately capture the associated health benefit if the quality aspect is left out of the picture. Thus, the QALY approach explicitly recognizes that life years are no homogenous commodity.

In fact, two separate variants of the QALY approach need to be distinguished.[20] One of them, which has been introduced by Pliskin, Shepard and Weinstein in their important 1980 paper, will not be considered any further in this section. Although, in principle, this variant also relates to arbitrary health status sequences, it does so only by relying on the conceptual device of a **health status annuity** which is to be elicited at an intermediate stage. Furthermore, the assumptions imposed on the way to a simple utility specification are discussed with respect to this intermediate stage while their implications as regards the original health status sequence remains unclear. Hence, this variant of the QALY approach bears directly only on the evaluation of chronic health states. This issue will be taken up again in section 5 below.

The other variant of the QALY approach - which is essentially the one proposed by Zeckhauser and Shepard - roughly can be described as follows. In order to account for both morbidity and mortality aspects, a given sequence of health states is assessed by

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[16] Cf. Zeckhauser and Shepard (1976), p. 11.
[17] Cf. Fanshel and Bush (1970).
[18] Cf. Culyer, Lavers and Williams (1971).
[19] Cf. Klarman, Francis and Rosenthal (1968).
[20] Cf. Loomes and McKenzie (1989) and Bleichrodt (1995).

looking at the individual health states themselves. Thus, the focus is on health states rather than on sequences of health status. Several methods have been devised in order to accomplish this task. Evaluation of a sequence of health states then follows from a simple aggregation procedure. More precisely, this is done by relying on either a constant or a variable rate of discount utility model, where the former category includes the special case of no discounting.[21]

In the literature, a distinction is made between chronic and temporary health states, where the former are usually taken to last for the rest of the individual´s life while the latter cover just one period, i.e., one year. More specifically, a chronic health state is characterized by some constant health status h^c and a time of duration T´, T´ ≤ T.[22] Hence, a chronic health state represents a health status sequence $H^{c,T'}$ as follows:

(8) $$H^{c,T'} = \left\{h^c\right\}_{t=1}^{T'}$$

Note that this notation forces one to be explicit about the duration of the chronic health state under consideration. Hence, two chronic health states involving the same health status may still be different.

Consider the assessment of chronic health states by the QALY approach first.[23] The **rating scale method** asks the individual to assign a value between zero and one to the sequence $H^{c,T'}$, where the top value is reserved for a sequence offering perfect health up to period T´ while immediate death usually occupies the bottom position.[24] Accordingly, this method elicits how the individual values sequences of health status involving a chronic health state and a length of life equal to T´. More precisely, the rating scale method produces the image of some value function which represents the preference relation ${}^{T'}\!\succ_h$ and is defined for all chronic health states of duration T´.[25]

In some contrast, the **time trade-off method** asks the individual to find another health status sequence in which the state of perfect health is followed by death after some time

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[21] On the implications of these two models, both under certainty and in the presence of risk, see Bleichrodt and Gafni (1996).
[22] Note that, due to the fact that T represents maximum possible survival time, T´ may well be smaller than T.
[23] The following draws on Torrance (1986), p. 17ff., cf. also McGuire et al. (1988), p. 21ff..
[24] This is not true if health states worse than death are possible.
[25] Note that, due to the scaling, this value function is uniquely determined.

τ. More specifically, τ needs to be chosen so as to produce indifference with respect to the special sequence of health under consideration. Let denote the indifference relation derived from the relation ${}^{T'}\succ_h$. Furthermore, letting $[\tau]$ denote the largest integer not higher than τ and $\bar{h}$ ($\underline{h}$) the state of perfect health (death), one has by definition:

$$(9) \qquad \left\{h^c\right\}_{t=1}^{T'} \qquad \left(\left\{\bar{h}_{t=1}^{[\tau]}\right\}, \left(\bar{h}, \tau-[\tau], \underline{h}\right), \left\{\underline{h}\right\}_{t=[\tau]+2}^{T'}\right)',$$

where $\left(\bar{h}, \tau-[\tau], \underline{h}\right)$ describes health status in period $[\tau]+1$ which is a combination of perfect health during the first part of that period as measured by the fraction τ-$[\tau]$ and death later on.[26]

Observe that (9) implicitly defines a function from the set of chronic health states of duration T´ into the real numbers. Hence, a chronic health state H(c,T´) is mapped into another length of life τ[H(c,T´)] associated with perfect health such that the quality aspect is accounted for. More precisely, the time trade-off method assigns a value which equals the ratio of τ[H(c,T´)] and T´ to the chronic health state of interest. This implies that the "chronic states" of perfect health and immediate death obtain values of one and zero, respectively. Like the rating scale method, the time trade-off method produces the image of some value function - which, again, represents the relation ${}^{T'}\succ_h$ - applied to chronic health states lasting for T´ periods.[27]

The **standard gamble method** is different from the two methods considered above in that it forces the individual to consider the presence of risk explicitly. More specifically, the individual has to evaluate a chronic health state by means of a gamble involving perfect health up to period T´ with probability p and immediate death with probability 1-p. The probability p is fixed such that, for the individual, the chronic health state represents the certainty equivalent of the gamble described above. Thus, the standard gamble method supplies values of a utility function for chronic health states of duration T´, where this function is based on the preference relation ${}^{T'}\succ_h^l$.[28]

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[26] Note that $0 \leq \tau-[\tau] \leq 1$ must hold by definition.
[27] Since there is a scaling of the two extreme sequences, this value function is also well-defined.
[28] Note that, due to the scaling implied by the fact that p is a probability, the utility function is uniquely defined.

Consider now temporary health states, i.e., health states which last for one period only.[29] More precisely, the purpose is to assess the experience of being in some health state h_t for one period while returning to good health after that. Again, the rating scale method asks the individual to evaluate the temporary health state under consideration in relation to the best (i.e., being in perfect health) and the worst temporary health state. Thus, the method attempts to elicit a value function for temporary health states **on the proviso that such a function exists**.

Similarly, the time trade-off method looks at the event of being in the worst temporary health state for some fraction y of a period while returning to good health afterwards. Now the individual needs to choose y so that indifference prevails to being in the temporary health state under consideration for one period. Then, the valuation of this state is given by 1-y. More specifically, this generates a point on one of the individual´s value functions for temporary health states, again under the provision that any such function exists.

Finally, the standard gamble method confronts the individual with the prospect of being healthy with probability p in the period under consideration and being in the worst temporary health state with probability 1-p. In other words, the method attempts to assign a utility to temporary health states, thus clearly relying on the condition that such a utility function for health states lasting for one period exists.

Since no mention is made of the remainder of the health status sequence to which the temporary health state under consideration belongs, any of the three methods considered above will produce meaningful results only if either a value or a utility function for temporary health states can be shown to exist. Thus, as far as the evaluation of temporary health states is concerned, the QALY approach relies on the existence of a preference relation for such states. Moreover, this must be true for health status in every period since the valuations produced by either of the three methods discussed above will be used for temporary health states in any period.

[29] Of course, temporary health states may last longer than a single period. As long as no chronic health state is involved, the results discussed below apply to this case as well, with some rather obvious modifications.

Turning to the individual´s preferences, this implies the existence of a preference relation for temporary health states in every period under consideration in the case of certainty. Hence, for the time trade-off and the rating scale method to make sense, the health state attributes in any two periods must be **mutually preferentially independent**. In the presence of risk, the procedure outlined above implies the general existence of a preference relation for lotteries involving health status in only one period. Therefore, if the standard gamble method is to yield reasonable evaluations of temporary health states, the health state attributes in any two periods must be **mutually utility independent**.

It is important to observe that neither of these conditions is sufficient to gurantee that the QALY approach will succeed in correctly representing individual preferences for health either under certainty or in the case of risk.[30] However, for the purposes of the present paper it is enough to examine the relationship with the assumptions introduced in section 2.2. More specifically, it is enough to note that the assumptions required to establish the existence of preferences for health do not imply the independence assumptions needed by the QALY approach.

Looking at the case of certainty first, preferential independence of the sequence of health states from the sequence of non-health attributes has no implications whatsoever for the relation between health attributes of different periods. In particular, it fails to imply their mutual preferential independence. Likewise, the utility independence assumption of section 2.2 entails absolutely no restrictions with respect to the relation of health status between one period and another. In particular, it certainly does not imply the utility independence properties necessary for the validity of the QALY approach.

3.2 The HYE approach

In contrast to the QALY approach, the HYE approach emphasizes sequences of health status rather than individual health states[31]. More precisely, the approach attempts to evaluate sequences of health status with respect to maximum possible survival time, i.e.,

[30] In order to arrive at sufficient conditions, one must, in addition, (1) assume the value functions (in the case of risk: the utility functions) for temporary health states in different periods to be the same and (2) introduce some assumption as regards the aggregation of health state valuations across time, cf. Bleichrodt (1995).
[31] Cf. Mehrez and Gafni (1989), (1991).

T. Therefore, the domain of the approach is just given by the space of health status sequences discussed in section 2.3.

According to its main proponents, Mehrez and Gafni, the evaluation of some health status sequence $\{h_t\}_{t=1}^{T}$ should follow a two-step procedure. By means of a standard gamble, the first step fixes the corresponding utility such that perfect health in every period is assigned a utility equal to one while immediate dealth yields zero utility. In the second step, the individual is asked to find another sequence offering perfect health for some time years followed by death such that indifference obtains. Then, letting denote the indifference relation derived from the relation of weak preference $^{T}\succ_{h}^{l}$, one has for the description of the first step in formal terms:

(10a) $$\{h_t\}_{t=1}^{T} \qquad \left[p, \{\bar{h}\}_{t=1}^{T}; 1-p, \{\underline{h}\}_{t=1}^{T}\right],$$

where the gamble on the right hand side involves perfect health for maximum remaining lifetime with probability p and immediate death with probability 1-p. The second step is given by:

(10b) $$\left[p, \{\bar{h}\}_{t=1}^{T}; 1-p, \{\underline{h}\}_{t=1}^{T}\right] \qquad \left(\{\bar{h}\}_{t=1}^{[HYE]}, \left(\bar{h}, HYE-[HYE], \underline{h}\right), \{\underline{h}\}_{t=[HYE]+2}^{T}\right)$$

where health status in period $[HYE]+1$ is a combination of perfect health during the first part of that period - i.e., the fraction $HYE-[HYE]$ - and death later on.[32] This presupposes, of course, that any such number HYE exists at all, an assumption which will be used throughout this paper.

The number HYE is called the **healthy-years equivalent** of the sequence $\{h_t\}_{t=1}^{T}$ although this cannot be inferred directly from (10b). Rather, as it stands, HYE represents a certainty equivalent in terms of years in perfect health with respect to the health profile prospect included in (10b).

In order to see why the conventional interpretation of HYE is indeed true, observe that both steps of the assessment procedure outlined above involve the same indifference relation and the same health status gamble $\left[p, \{\bar{h}\}_{t=1}^{T}; 1-p, \{\underline{h}\}_{t=1}^{T}\right]$. Relying on the

[32] Note that $0 \le HYE - [HYE] \langle 1$ must hold by definition.

facts that an indifference relation is an equivalence relation and that equivalence relations are always transitive, (10a) and (10b) can be combined to imply:

$$\text{(11a)} \qquad \{h_t\}_{t=1}^{T} \qquad \left(\{\bar{h}\}_{t=1}^{[HYE]}, (\bar{h}, HYE - [HYE], \underline{h}), \{\underline{h}\}_{t=[HYE]+2}^{T} \right)$$

Moreover, since both objects of choice represent degenerate lotteries and the restriction of the preference relation ${}^{T}\succ_h^{l}$ to such lotteries coincides with the relation ${}^{T}\succ_h$ if one identifies degenerate lotteries with the respective health status sequences, (11a) further implies:

$$\text{(11b)} \qquad \{h_t\}_{t=1}^{T} \qquad \left(\{\bar{h}\}_{t=1}^{[HYE]}, (\bar{h}, HYE - [HYE], \underline{h}), \{\underline{h}\}_{t=[HYE]+2}^{T} \right)$$

Clearly, this indifference relation justifies the use of the term healthy-years equivalent since, with respect to the sequence on the left hand side of (11b), HYE does **not** constitute a certainty equivalent.

More precisely, suppose the number HYE to be well-defined, i.e., unique. Then, (11b) implicitly defines a mapping from the space of health status sequences into the set of (non-negative) real numbers which is given by $HYE\left(\{h_t\}_{t=1}^{T}\right)$. Clearly, in order to produce the required indifference for arbitrary sequences, the range of that mapping will generally not be confined to the set of integers. Indeed, the reference sequence on the right hand side of (11b) will usually represent a health status flow proper.

It is important to observe that each step of the HYE approach relies on the indifference relation which, in turn, can be derived from the relation ${}^{T}\succ_h^{l}$ introduced in section 2.2. Hence, if preferences for risky streams of health status exist, both steps outlined above are, in fact, feasible. If, in addition, the mapping $HYE\left(\{h_t\}_{t=1}^{T}\right)$ is well-defined as has been assumed above, the HYE approach can be implemented without the need to impose further assumptions.

Interestingly, the assumption that the mapping $HYE\left(\{h_t\}_{t=1}^{T}\right)$ be well-defined implies an important monotonicity property. With little additional effort, this provides for a convenient interpretation of healthy-years equivalents. In order to see this, consider health status sequences involving only the state of perfect health followed by death, where length of life is equal to some parameter A, 0 ≤ A ≤ T. In the present setting, any such sequence is given by:

$$(12) \qquad \left(\left\{ \bar{h} \right\}_{t=1}^{[A]}, \left(\bar{h}, A - [A], \underline{h} \right), \left\{ \underline{h} \right\}_{t=[A]+2}^{T} \right)$$

Clearly, these are just the reference sequences used by the HYE approach in step two.[33] Relying on the assumptions introduced in section 2.2 and on section 2.3, a continuous value function exists for such sequences. In particular, this value function is continuous with respect to the parameter A. Now the assumption that the number HYE be well-defined implies that, for any level of the value function, the corresponding value for A must be unique. Hence, there is a one-to-one correspondence between the image set of the value function and the range of the parameter A, i.e., the interval 0 ≤ A ≤ T. In other words, if the healthy-years equivalent is well-defined, the individual´s value function for health status sequences given by (12) must be monotonous with respect to the parameter A. Now consider the following, fairly innocuous assumption:

(A 1) The individual strictly prefers a life in perfect health over immediate death.

Due to the monotonicity property, (A 1) is sufficient to imply the value function for health status sequences (12) to be strictly increasing in A, i.e., the number of years in perfect health. Then, it is straightforward to demonstrate that the healthy-years equivalent as defined by (11b) provides a value for an arbitrary health sequence. In other words, one has:[34]

$$(13) \qquad \left\{ h_t^1 \right\}_{t=1}^{T} \;\; {}^{T}\!\succ_h \;\; \left\{ h_t^2 \right\}_{t=1}^{T} \quad \Rightarrow \quad HYE\left(\left\{ h_t^1 \right\}_{t=1}^{T} \right) \;\geq\; HYE\left(\left\{ h_t^2 \right\}_{t=1}^{T} \right)$$

Thus, for a given health status sequence, the HYE approach ends up with two numbers, each relating to the individual´s preferences in a specific sense. More precisely, the utility elicited in the first step can be used to represent preferences in the presence of risk given

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[33] This terminology is not quite accurate since, for $a \neq [a]$, these reference sequences are, in fact, health status flows proper.

[34] In fact, if the mapping $HYE\left(\left\{ h_t \right\}_{t=1}^{T} \right)$ is well-defined, it is only necessary to consider sequences between which the individual is not indifferent in order to prove (13). The monotonicity property based on (A 1) leads to a strictly preferred health status sequence receiving a higher healthy-years equivalent.

the assumptions of section 2.2.[35] In some contrast, the value supplied by the second step is of immediate relevance only to the case of certainty because $HYE\left(\{h_t\}_{t=1}^{T}\right)$ will, in general, only be a value function.[36]

3.3 The generalized time trade-off method

As has been outlined above, the time trade-off method constitutes one of several methods available for the calculation of quality-adjusted life years in order to represent individual preferences for health outcomes. Moreover, it is the method which represents the QALY approach in the debate over QALYs versus HYEs. For the sake of easy reference in the discussion below, it is useful to define a **generalized time trade-off method** which can be applied to sequences of health states in general.[37] Consider any such sequence $\{h_t\}_{t=1}^{T}$. The generalized time trade-off method looks for another sequence involving only the state of perfect health followed by death such that indifference between the two sequences prevails. In other words, the reference sequences used by the generalized time trade-off coincide with the reference sequences of the HYE approach, i.e., health status sequences given by (12). Thus, letting $[x]$ denote the largest integer not higher than x, one has for the generalized time trade-off assessment of a health status sequence:

$$(14) \qquad \{h_t\}_{t=1}^{T} \qquad \left(\{\bar{h}\}_{t=1}^{[x]}, \left(\bar{h}, x-[x], \underline{h}\right), \{\underline{h}\}_{t=[x]+2}^{T}\right)$$

Comparing (14) with (11b), it is evident that the generalized time trade-off method produces the healthy-years equivalent associated with $\{h_t\}_{t=1}^{T}$. Hence, under the assumptions stated in the previous section, (14) implicitly defines a value function for arbitrary health status sequences which coincides with the mapping $HYE\left(\{h_t\}_{t=1}^{T}\right)$.

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[35] Due to its implicit scaling requirement, step one identifies utility as generated by a unique member of the equivalence class of utility functions capable of representing the individual´s preferences over risky streams of health status.

[36] Again, the effect of the scaling requirement implicit in step two is to identify value as produced by a unique member of the equivalence class of all value functions capable of representing the individual´s preferences over sequences of health status.

[37] The notion of a generalized or general time trade-off method has been used implicitly in several papers, cf., e.g., Johannesson, Pliskin and Weinstein (1993), p. 283.

4. QALYs versus HYEs - an assessment of alternative views

This section presents a discussion of several views which can be found in the more recent literature. Although the central theme is the comparison of the QALY and the HYE approach in terms of their relative merits, the emphasis will lie on the latter approach. And quite understandably so since the QALY approach is rather well-known. In particular, its implications with respect to individual preferences are now fairly well understood.[38] By way of contrast, the HYE approach has been introduced only recently and, thus, is more prone to misunderstanding. Indeed, the discussion below will reveal a considerable number of misconceptions relating primarily to the HYE approach.

- The two-step procedure used to elicit the number of healthy-years equivalent is unnecessary: the first step can be skipped without doing any harm

This view has been put forward by several authors, and at times quite forcefully.[39] It is based on (11a) or, for that matter, on (11b), i.e., on the observation that the healthy-years equivalent associated with some health status sequence could be elicited validly in a single step.[40] Given that the focus of the approach, or so the argument goes, is on the healthy-years equivalent, the first step can be deleted without any loss of generality.

While the above observation is quite true, the conclusion derived from it nevertheless does not follow. The main reason is that the first step in the assessment procedure proposed by Mehrez and Gafni elicits a utility which, for an expected utility maximizer, can be used profitably in the analysis of risk. Collapsing the two steps into a single step, however, eliminates the information on the vNM utility of the health status sequence under consideration. By way of contrast, the healthy-years equivalent generally represents but a value which is of no use whenever health outcomes are stochastic.

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[38] Cf. Pliskin, Shepard and Weinstein (1980), Loomes and McKenzie (1989), Bleichrodt (1995) as well as Bleichrodt and Gafni (1996).

[39] Cf. Buckingham (1993a), Buckingham (1993b), Culyer and Wagstaff (1993), Johannesson, Pliskin and Weinstein (1993), Loomes (1995), Culyer and Wagstaff (1995).

[40] Indeed, several authors actually prove what is but a narrow variant of (11b) based on chronic health states lasting for T periods, cf. Buckingham (1993a), p. 304 and (1993b), p. 167 - beware of his notation, though, in either paper! -, Culyer and Wagstaff (1993), p. 317, Loomes (1995), pp. 1-2.

More specifically, as already pointed out in section 2.1, any utility function also constitutes a value function. This implies that the utility function generated by step one can, in fact, be employed in the evaluation of health status sequences under certainty as well. In this sense, the first part of the above assertion is indeed true, i.e., the two-step procedure is unnecessary for the representation of individual preferences over health sequences. However, contrary to the views expressed in the literature, it is, in fact, the second step that can safely be done away with without any loss of generality!

So what is the use of the second step? Basically, as Mehrez and Gafni have been quite aware of,[41] it helps to provide an intuitively accessible interpretation of individual preferences for health.[42] Without the second step, an individual´s evaluation of some health status sequence would have to be expressed in terms of "(vNM) utility scaled such that being in perfect health for maximum possible remaining lifetime receives a utility of one“. Clearly, the healthy-years equivalent represents a less cumbersome way to describe preferences.[43]

In addition, it is also important to see that the healthy-years equivalent maps the quality (i.e., morbidity) and the quantity (i.e., mortality) aspect of a health status sequence into some length of life of a given quality, i.e., perfect health. Of course, this has always been the goal of the QALY approach. Therefore, by adding a second step directed at eliciting the healthy-years equivalent associated with a health status sequence, the stage has been set for a comparison of the QALY and the HYE approach.[44]

- The number of healthy-years equivalent simply follows from an application of the time trade-off method as employed by the QALY approach[45]

This equivalence claim refers to the case of certainty since, as should be clear from section 3, both the healthy-years equivalent and the result of the time trade-off method relate to value functions. Consider first the case of chronic health states as defined in

[41] See Mehrez and Gafni (1989), pp. 142-143.
[42] In addition, without step two, the name of the HYE approach would no longer be appropriate.
[43] However, as will become clear below, this improvement as regards interpretation comes at a price which relates to the fact that HYEs represent values but not utilities.
[44] Cf. Mehrez and Gafni (1989), p. 143, who state that the HYE "...preserves the intuitively appealing meaning“ of the QALY approach.
[45] Cf. Buckingham (1993a), Buckingham (1993b), Culyer and Wagstaff (1993), (1995), and Loomes (1995).

section 3.1. Although this case is really rather special, the bulk of the literature to date has focussed exclusively on it. Let H(c,T´,T) denote the health status sequence involving the chronic health state H(c,T´) up to period T´ followed by death:

$$(15) \qquad H(c,T',T) \;=\; \left(\left\{h^c\right\}_{t=1}^{T'}, \left\{\underline{h}\right\}_{t=T'+1}^{T}\right)$$

Thus, for T´ lower than T, a chronic health state H(c,T´) represents but part of the corresponding health status sequence H(c,T´,T).

Furthermore, let $HYE\left[H\left(c,T',T\right)\right]$ denote the healthy-years equivalent associated with a sequence involving the chronic health state H(c,T´). Comparing (9) with (11b), one immediately obtains:[46]

$$(16) \qquad \tau\left[H(c,T)\right] \;=\; HYE\left[H(c,T)\right]$$

That is, if the chronic health state under consideration lasts for maximum remaining lifetime, the outcome of the time trade-off method coincides with the healthy-years equivalent. Hence, in this rather special case, the equivalence between the healthy-years equivalent and the time trade-off method as employed by the QALY approach does indeed hold.[47]

Surprisingly, the main proponents of the HYE approach have refused to accept this result on several occasions.[48] In their 1993 paper, Mehrez and Gafni present the example of a chronic health state lasting for 15 years whose healthy-years equivalent as defined by the two-step procedure is taken to be equal to 8 years.[49] In contrast, the time trade-off method is supposed to yield indifference at 11 years in perfect health. In view of the equivalence result established above, the example implies the individual to be indifferent between 8 and 11 years in perfect health, each followed by death! Hence, if the example is to make sense, the healthy-years equivalent of the chronic health state cannot be well-defined.

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[46] Clearly, for T´ equal to T, a chronic health state represents a health status sequence proper since the equality H(c,T´=T,T) = H(c,T) must hold.

[47] However, it should be noted that quite a few authors have taken this to imply the much more general equivalence expressed in the view above, cf. Buckingham (1993a), p. 305, Buckingham (1993b), p. 167, Culyer and Wagstaff (1993), p. 317, Loomes (1995), pp. 1-2, Culyer and Wagstaff (1995), p. 42.

[48] Cf. Mehrez and Gafni (1991), p. 145, Mehrez and Gafni (1993), p. 288, Gafni and Mehrez (1993), p. 168, Gafni, Birch and Mehrez (1993), p. 327.

[49] Cf. Mehrez and Gafni (1993), p. 288. See also Gafni, Birch and Mehrez (1993), p. 328.

Observe that for chronic health states lasting shorter than maximum remaining lifetime - i.e., T´ is strictly lower than T -, the healthy-years equivalent $HYE\left[H(c,T',T)\right]$ need not be equal to the time trade-off evaluation τ[H(c,T´)]. Now consider the following "neutrality" assumption:

(A 2) The healthy-years equivalent as defined by the indifference

(17a) $\left(\{h^c\}_{t=1}^{T'}, \{\underline{h}\}_{t=T'+1}^{T}\right) \qquad \left(\{\overline{h}\}_{t=1}^{[HYE]}, \left(\overline{h}, HYE-[HYE], \underline{h}\right), \{\underline{h}\}_{t=[HYE]+2}^{T}\right)$

is independent of maximum remaining lifetime T, T ≥ T´.

Under (A 2), the individual´s valuation of a chronic health state of duration T´ does not depend on the time horizon as given by T.[50] Thus, no matter how many periods involving the state of death follow upon the chronic health state lasting for a total of T´ periods, the number of years in perfect health necessary to achieve indifference remains constant.

Arguably, (A 2) provides the natural way to link preferences for health status sequences involving some specific length T - i.e., ${}^{T}\succ_h$ - to preferences defined for sequences of a different length T´, i.e., ${}^{T'}\succ_h$. More precisely, the restriction of ${}^{T}\succ_h$ to health status sequences involving death no later than after T´ periods generates the ordering induced by the relation ${}^{T'}\succ_h$ if one employs an obvious identification procedure.[51] In this sense, under (A 2) the individual´s preference relations for health status sequences of different length will be consistent with each other. Conversely, the assumption provides a justification for translating healthy-years equivalents obtained for one time horizon to any longer one in a particularly simple manner.

Given (A 2), it is straightforward to prove the more general result that, for the evaluation of health status sequences involving a chronic health state, the time trade-off method simply produces the healthy-years equivalent. More precisely, (17a) obviously is equivalent to:

(17b) $$HYE\left[H(c,T',T)\right] = HYE\left[H(c,T')\right] \quad \forall\, T \geq T'$$

[50] Note that, for the "chronic state" of perfect health, (A 2) is satisfied in any case.
[51] To be precise, any sequence of length T is taken to be identical with another sequence of length T´ iff they coincide for the first T´ periods. Since the attention is confined to health status sequences of length T involving death in periods T´+1,...,T, this yields a one-to-one correspondence.

Making use of (16), one arrives at:

(18) $$HYE\left[H(c,T',T)\right] \;=\; \tau\left[H(c,T')\right] \qquad \forall\; T \geq T'$$

Since $HYE\left[H\left(c,T',T\right)\right]$ exhibits no dependence on the time horizon T, it will be equal to the corresponding healthy-years equivalent obtained for the time horizon T´ which, as (16) confirms, is given by t´[H(c,T´)] as outlined above.

However, it must be kept in mind that health status sequences involving a chronic health state are but a special case. Consider now sequences which contain at least two different health states before death occurs. In fact, it is not difficult to show that the equivalence asserted above fails to hold in this more general case. Relying on (11b), the healthy-years equivalent for an arbitrary health status sequence is defined by means of the preference relation ${}^{T}\!\succ_h$. Moreover, it represents a value which can be used in the representation of individual preferences for health status sequences under certainty.

Therefore, in order to establish the equivalence asserted in the view above, one has to show that the time trade-off method as employed by the QALY approach yields a value for health status sequences involving temporary health states. Recalling the analysis of section 3.1, however, this will generally not be the case. Within the QALY approach, the time trade-off method attempts to construct an overall assessment by first evaluating the temporary health states belonging to a sequence. Unless additional assumptions are imposed, these evaluations will not represent values in the sense of multi-attribute decision theory. It follows that the alleged equivalence between the time trade-off method as used by the QALY approach and healthy-years equivalents fails to hold in general.

In order to understand the scope of this argument, it is useful to take a closer look. As pointed out in section 3.1, the existence of value functions for temporary health states in any period constitutes a necessary condition for the QALY approach to represent individual preferences for health status sequences under certainty. In particular, this is true if the approach is based on the time trade-off method. In turn, this implies the non-existence of such value functions to be sufficient for the QALY approach not to represent those preferences.

More precisely, if the individual´s preferences for health status sequences are such that the time trade-off based assessment by the QALY approach succeeds in supplying a value, this must coincide with the values provided by the generalized time trade-off due to identical scaling requirements for the extreme sequences offering perfect health until T and immediate death, respectively. Clearly, in this case the equivalence proposition will hold in general, i.e., regardless of the health status sequence under consideration.

Accordingly, the scope of the equivalence claim is governed by the level of generality of the analysis in terms of individual preferences for health: if the latter satisfy additional restrictions sufficient to make the QALY approach based on the time trade-off method work in the sense that it correctly represents these preferences, the equivalence holds for arbitrary health status sequences. Conversely, if one merely imposes the restrictions of section 2 - i.e., which only ensure the existence of preferences for health status sequences -, the equivalence proposition is confined to sequences involving a chronic health state.

It is interesting to note that, while Mehrez and Gafni are quite right in rejecting the equivalence claim contained in the view under discussion[52] since their analysis only presupposes the existence of preferences for health status sequences, they do so for the wrong reason. In several papers, they attribute the failure of the equivalence proposition to the difference between the standard gamble and the time trade-off method.[53] While, true enough, the standard gamble is used in every step of the HYE assessment procedure proposed by Mehrez and Gafni, it nonetheless has absolutely no bearing on the healthy-years equivalent of a health status sequence or, for that matter, on the mapping $HYE\left(\{h_t\}_{t=1}^T\right)$. This is expressed most clearly by (11b) which follows from the two-step procedure without further assumptions and does not involve the standard gamble method at all.

As should be obvious from the discussion above, the true reason why the equivalence claim breaks down for health status sequences in general lies entirely with the way the QALY approach uses the time trade-off method.[54] While the generalized time trade-off

[52] Although, as pointed out above, they go too far in denying the equivalence claim even for chronic health states of duration T.
[53] Cf. Mehrez and Gafni (1991), p. 145, Mehrez and Gafni (1993), p. 288. See also Gafni, Birch and Mehrez (1993), pp. 326-327.
[54] Cf. also Bleichrodt (1995), p. 26, who uses the same argument with respect to the QALY approach as introduced by Pliskin, Shepard and Weinstein (1980).

method does indeed provide the healthy-years equivalent associated with a health status sequence, the time trade-off **as employed by the QALY approach** fails to produce a value and, thus, a healthy-years equivalent for such sequences in the absence of additional assumptions.

- Healthy-years equivalents do not incorporate individual time preference[55]

This claim is somewhat surprising given that the HYE approach forces individuals to evaluate streams of health status evolving over time. Accordingly, one would expect any intertemporally relevant aspect to be taken into account. In particular, this should be true for the intertemporal trade-off between health status accruing in different periods as governed by individual time preference.[56]

As pointed out in section 3.2, if the individual´s preference relation satisfies a mild additional assumption - i.e., (A 1) -, healthy-years equivalents can be interpreted as values. This implies that healthy-years equivalents are fully capable of representing preferences over health status sequences under certainty. In particular, individual time preference for health status is accounted for. Thus, if one is prepared to accept (A 1), there is no way of denying that healthy-years equivalents do indeed incorporate individual time preference.

Conversely, if assumption (A 1) fails to hold, healthy-years equivalents will not succeed in representing an individual´s preferences for health. It would be wrong, however, to attribute this to a misrepresentation of individual time preference. Rather, the reason is that the individual prefers immediate death to living in perfect health for the next T periods. And no more can be said in this case.

By way of contrast, the QALY approach has to make some assumption concerning time preference for health because of the need to aggregate the evaluations of health states

[55] Cf. Culyer and Wagstaff (1993), p. 314, Culyer and Wagstaff (1995), p. 41.
[56] This is the argument presented by Gafni, Birch and Mehrez (1993), p. 330.
Note that, at the margin, the trade-off between health status of different periods will, in general, be governed by the rate of discount for health increments; on the difference between the rate of discount and the rate of time preference see Olson and Bailey (1981). As observed already by Zeckhauser and Shepard, however, the assumptions underlying the QALY approach - i.e., linear aggregation of period specific preference indicators - imply both rates to be equal, cf. Zeckhauser and Shepard (1976), p. 12.

pertaining to different periods. Clearly, regardless of whether a constant or a variable rate of discount model is taken to hold, such an assumption adds another restriction which may or may not be satisfied in practice. Therefore, given the general setting described in section 2.2, the QALY approach may well fail to capture individual time preference for health.

- Healthy-years equivalents represent individual preferences for health in the presence of risk

This view has been advanced repeatedly by Mehrez and Gafni, essentially on the grounds that since it is defined with the help of the individual´s utility function, the healthy-years equivalent fully incorporates preferences for health, both under certainty and under conditions of risk.[57] Given that the certainty case has already been dealt with above, it remains to investigate the case of risk. Although it is true that the two-step HYE assessment procedure does elicit the utility associated with a health status sequence, the corresponding healthy-years equivalent is but part of a reference sequence as described by (12). In turn, these reference sequences belong to the domain of a vNM utility function. Hence, as pointed out earlier by other authors, the healthy-years equivalent represents an argument of the individual´s utility function for health status sequences given by (12).[58]

In effect, the view expressed above amounts to claiming that healthy-years equivalents represent utilities. As is well-known, however, an argument of a vNM utility function will itself be a utility function only under special circumstances. Unless one imposes additional restrictions in order to make sure that these circumstances actually hold, healthy-years equivalents will not represent utilities. In fact, this has already been argued in section 3.2 where the mapping $HYE\left[\{h_t\}_{t=1}^{T}\right]$ has been shown to satisfy only the weaker requirements characterizing a value function. Thus, in the presence of risk, the expected number of healthy-years equivalent is likely to misrepresent individual preferences over lotteries involving health status sequences.

[57] Cf. Mehrez and Gafni (1989), p. 143, Mehrez and Gafni (1991), p. 141 and p. 145, Mehrez and Gafni (1993), p. 289.
[58] Cf. Johannesson, Pliskin and Weinstein (1993), p. 286 and Bleichrodt (1995), p. 26.

It is of some interest to inquire precisely when healthy-years equivalents can be relied upon in the analysis of risky health outcomes. In view of what has been said above, this will be the case iff the individual is risk-neutral with respect to (additional) life years in perfect health. Consider the healthy-years equivalent of any health status sequence given by (12). Obviously, this is simply equal to length of life A which is, in particular, linear with respect to length of life. Hence, if healthy-years equivalents can be interpreted as utilities, the utility function for reference sequences as described by (12) must be linear with respect to A.[59]

In fact, in this case more information is available on the utility function relied upon in step one. If healthy-years equivalents are to represent utilities, the corresponding function must be strategically equivalent to the von Neumann-Morgenstern utility function u defined on the set of health status sequences which is scaled such that a sequence of perfect health is assigned a utility of one while immediate death receives zero utility. Given that the corresponding healthy-years equivalents are equal to T and 0, respectively, this information is sufficient to determine the coefficients of the transformation:

(18) $$HYE\left[\{h_t\}_{t=1}^{T}\right] \;=\; a\,u\left(\{h_t\}_{t=1}^{T}\right) + b; \qquad a = T, \quad b = 0$$

Hence, the vNM utility of a health status sequence as elicited in step one simply equals the corresponding healthy-years equivalent divided by T.

- The HYE approach represents individual preferences for health in the presence of risk[60]

Given the assumptions introduced in section 2, it can be demonstrated that this claim is indeed true. If the individual behaves as an expected utility maximizer and a utility function for health trajectories exists, then the latter is sufficient for the correct representation of individual preferences under conditions of risk.[61] As can be seen from (10a), the purpose of the first step of the HYE assessment procedure is precisely to

[59] Loosely speaking - since it is true only for health status sequences given by (12) -, the individual´s utility function for life years in perfect health must be linear, cf. Johannesson, Pliskin and Weinstein (1993), p. 284, Johannesson (1995), p. 11.
[60] Cf. Gafni, Birch and Mehrez (1993).
[61] Therefore, the claim by Johannesson, Pliskin and Weinstein that, given expected utility maximization, the two-step procedure does not incorporate attitudes toward risk, is plainly wrong; cf. Johannesson, Pliskin and Weinstein (1993), p. 284.

measure the utilities corresponding to the health status sequences of interest. Clearly, these utilities can be used for the evaluation of stochastic health streams over time without the need to introduce further assumptions.

A slight complication arises, however, if one attempts to evaluate lotteries involving health status by means of healthy-years equivalents which would seem to be more in line with the spirit of the HYE approach. As pointed out above, healthy-years equivalents generally cannot be interpreted in terms of (von Neumann-Morgenstern) utilities. Hence, the information provided by the second step as described by (10b) is not of much help here. As suggested by Johannesson et al., a representation of individual preferences over prospects of intertemporal health streams relying on the concept of healthy-years equivalent has to focus on the corresponding certainty equivalents.[62]

Accordingly, the two-step assessment procedure of the HYE approach needs to be modified. Given a lottery l^h over health status sequences, a third step must be added in order to elicit the corresponding certainty equivalent in terms of healthy-years equivalent as follows:

$$(19) \qquad l^h \qquad \left(\{\bar{h}\}_{t=1}^{[\overline{HYE}]}, \left(\bar{h}, \overline{HYE} - [\overline{HYE}], \underline{h}\right), \{\underline{h}\}_{t=[\overline{HYE}]+2}^{T} \right),$$

where, of course, both l^h and the health status sequence on the right hand side are evaluated with respect to the same sequence of non-health attributes. $\overline{HYE(l^h)}$ denotes the healthy-years equivalent certainty equivalent of the intertemporal health status prospect offered by the lottery l^h.

With this modification, the HYE approach supplies a correct representation of individual preferences over risky health streams in terms of (certainty equivalents of) healthy-years equivalents for an expected utility maximizer. However, it should be noted that the improvement in interpretation comes at a considerable price as regards assessment effort. For any change in the health status lottery l^h, the third step must be carried out again. In particular, this is true for shifts in the probability distribution of the health status sequences contained in l^h. Conversely, if one were content with evaluating risky health streams in terms of von Neumann-Morgenstern utility as obtained from step one, no new

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[62] Cf. Johannesson, Pliskin and Weinstein (1993).

assessment would be necessary. Rather, a simple recalculation of expected utility would do the job.

- The approximation procedure suggested by Mehrez and Gafni[63]

It is not difficult to see that an empirical implementation of the HYE approach requires considerably more assessment effort than the QALY approach. In order to deal with this problem, Mehrez and Gafni proposed a three-step approximation procedure. The first step attempts to approximate the individual´s utility function for life years in perfect health by eliciting several values of this function from the individual. In terms of the notation employed in this paper, the first step yields an approximation of the utility function for reference sequences of health status as given by (12). The second step is used to determine the utilities associated with all health status sequences of interest. Finally, the third step involves the calculation of the corresponding healthy-years equivalents by relying on the approximation obtained in the first step. While the individual whose preferences are to be elicited is involved at stages one and two, the third stage can be carried out by the researcher on his/her own.

If applied correctly, the approximation procedure can be used profitably in the representation of individual preferences for health both under certainty and in the presence of risk.[64] Consider first the case of certainty. By locating the utility level due to some health status sequence on the utility function for reference sequences, one obtains the corresponding healthy-years equivalent, possibly subject to an error of approximation. In other words, step three of the above approximation procedure implements step two of the HYE assessment procedure as described by (10b).

Next, turn to the case of risk. Clearly, the information gathered by the second step is sufficient to calculate the expected utility associated with some intertemporal health status prospect. By locating this utility level on the utility function for reference sequences of health status, one now obtains the corresponding certainty equivalent in terms of healthy-years equivalents, again possibly subject to an error of approximation. Hence, this simply implements stage two of the modified HYE assessment procedure

[63] Cf. Mehrez and Gafni (1991), pp. 142/143.
[64] Mehrez and Gafni confined their discussion to the case of certainty, cf. Mehrez and Gafni, op. cit.

described above by (19). Since it takes much less effort to implement while being not quite as accurate, however, it seems that this variant of the approximation procedure strikes a better compromise between the counteracting demands of easy interpretation on the one hand and low assessment effort on the other.

Obviously, the usefulness of the approximation procedure for the representation of individual preferences for health critically hinges on the extent of approximation error introduced by step one. This calls for a very careful examination of the individual´s utility function for reference sequences of health status. In particular, it would seem reasonable to elicit more points on this function than originally suggested by Mehrez and Gafni.[65]

5. Discussion

Based on a fairly general setting which explicitly recognizes the intertemporal aspect of preferences for health, the analysis undertaken in this paper has demonstrated that some of the major views expressed in the ongoing debate over QALYs versus HYEs are either partially or even outright wrong. Rather than repeating the results derived in the previous section, I would like to take the opportunity to discuss possible reasons for this. While, of course, this must to some extent remain a purely speculative exercise, I am nevertheless confident that it does help to shed light on the debate from a slightly different perspective.

First of all, I think the present paper contains a more basic message concerning the debate over QALYs versus HYEs: if one takes care to define precisely the time trade-off based QALY and the HYE approach in a multi-attribute decision theoretic setting incorporating preferences for health in a general sense, the issues at stake can be decided unambiguously with little additional effort. In particular, no complicated proofs are involved but, rather, the truth or falsity of the views under scrutiny follows, with one minor exception, almost by definition. In turn, this suggests that much of the QALYs versus HYEs debate has been based on misconceptions, primarily due to a lack of precise formulation of the concepts involved.

[65] Cf. Mehrez and Gafni (1991), p. 143.

Consider, for example, the alleged equivalence between the results of the time trade-off method as employed by the QALY approach and healthy-years equivalents. Most of the literature has concentrated on the discussion of chronic health states where, under a mild additional assumption - i.e., (A 2) -, this equivalence indeed holds.[66] As emphasized earlier, however, chronic health states represent but a special case which fails to exhaust the full range of health status sequences by a wide margin. Therefore, the focus on sequences involving a single chronic health state seems quite unwarranted.

Possibly, chronic health states have received so much attention because of the important 1980 paper by Pliskin et al. where the authors also appear to deal exclusively with such states although this is, strictly speaking, not the case.[67] In fact, the paper provides an axiomatic foundation for a von Neumann-Morgenstern utility function defined for general health status sequences. While this closely resembles the approach adopted in the present paper, Pliskin et al. introduce the conceptual device of a **health status annuity** in order to be able to separate quality (i.e., the morbidity aspect) of life from quantity (i.e., the mortality aspect) of life in a simple manner. More precisely, an arbitrary health stream is transformed into a constant health status sequence offering the same length of life - the annuity - such that the individual is indifferent between both sequences. In this manner, it becomes possible to express quality of life provided by any health status sequence through a scalar quantity, namely, the quality of life implied by the corresponding health status annuity.

Unfortunately, by concentrating on the analysis of chronic health states, the more recent literature on the QALY approach has tended to overlook the role of the health status annuity introduced by Pliskin et al..[68] In fact, to some extent the authors themselves may be blamed for this because both their axioms[69] and their discussion relates to those annuities rather than to the original health status sequences.

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[66] In particular, this is true for Buckingham (1993), Culyer and Wagstaff (1993), (1995) and Loomes (1995).

[67] Cf. Pliskin, Shepard and Weinstein (1980).

[68] Cf. Loomes and McKenzie (1989), p. 299, Broome (1993), p. 165, Culyer and Wagstaff (1993), p. 312, Johannesson, Pliskin and Weinstein (1993), p. 282, Culyer and Wagstaff (1995), p. 40.
A notable exception is Bleichrodt (1995), p. 22.

[69] More recently, it has been shown that the utility function derived by Pliskin et al. can be characterized essentially by means of risk neutrality with respect to life years, cf. Bleichrodt, Wakker and Johannesson (1996).

It is of some interest to compare the conceptual device of a health status annuity with the concept of a healthy-years equivalent. Clearly, both concepts rely on the individual´s indifference relation for health status sequences under certainty. However, while the former transforms some arbitrary health status sequence into a variable quality of life annuity of equal length, the latter attempts to find a perfect health annuity of variable length of life which generates indifference. In this sense, one concept can be obtained from the other by simply reversing the roles of quantity and quality of life. Incidentally, the quality of life implied by the health status annuity concept of Pliskin et al. could, in principle, also be used as a value function in the evaluation of health status sequences if one adopts a condition similar to (A 1).

To its credit, the HYE approach has redirected attention away from the assessment of health states and back to the evaluation of health status sequences proper. The importance of this shift in emphasis cannot be overestimated. For resource allocation decisions in health care, the intertemporal evolution of health benefits due to additional programs will usually matter.[70] More precisely, if sequencing effects are as important for the evaluation of health streams over time as recent research seems to suggest,[71] consideration of health status sequences represents the appropriate way to deal with these effects.

Moreover, by measuring preferences for health in terms of quantity of life spent in perfect health, the HYE approach has chosen an avenue which is more fruitful than the annuity approach adopted by Pliskin et al.. This holds because the length of a life in perfect health - i.e., healthy-years equivalents - represents a much simpler concept than the variable quality of life health status annuity. In particular, the latter would certainly be more difficult to explain to potential respondents.

On the other hand, the HYE approach is not without ambiguity either. Primarily, this ambiguity arises from the fact that the approach has been designed so as to pursue two different aims at the same time. First, it intends to represent individual preferences for health status sequences both under certainty and in the case of risk. Second, and just as important, the approach is to provide an intuitively accessible description of these

[70] Or, for that matter, it may also be the intertemporal spread of health costs due to cuts in health care spending.
[71] See Bleichrodt and Gafni (1996), p. 130f..

preferences. Thus, as demonstrated in more detail in the previous section, the approach comes up with two concepts each of which is capable of representing the same individual preferences. In the certainty case, the conventional two-step procedure yields both a utility and a value - i.e., the healthy-years equivalent - for a given health status sequence. Under conditions of risk, the modified procedure outlined above produces the expected utility as well as a healthy-years equivalent certainty equivalent due to some intertemporal health status prospect.

Clearly, if applied in this manner, the HYE approach includes some unnecessary duplication. In turn, as a source of ambiguity over which concept to use in order to represent individual preferences, this redundancy may have been responsible for the existing confusion as regards the appropriateness of the two-step procedure proposed by Mehrez and Gafni. More precisely, under certainty, the procedure allows to describe preferences for health by means of healthy-years equivalents whereas, in the case of risk, stochastic health streams are evaluated in terms of expected utility. In this sense, the two-step procedure falls some way short of implementing the concept of healthy-years equivalent in full.

Based on these considerations, I would recommend the following double-edged strategy for the HYE approach which attempts to meet the different demands of theoretical precision and empirical practicability. For the representation of individual preferences over health status sequences, employ the concept of healthy-years equivalent in a manner appropriate to the decision context. More precisely, rely on healthy-years equivalents as defined by (11b) under certainty and on the corresponding certainty equivalents as given by (19) under conditions of risk. In effect, this eliminates the need to calculate von Neumann-Morgenstern utilities.[72]

For the purpose of empirical implementation, certainty equivalents of healthy-years equivalents seem too cumbersome to be of practical use. As mentioned earlier, unlike von Neumann-Morgenstern utilities, these have to be elicited anew whenever the terms of an intertemporal health status prospect change. Therefore, I would favour the

[72] Note that this does not support the view criticized in the previous section, i.e., that the first part of the two-step procedure proposed by Mehrez and Gafni is superfluous. If only this procedure is available, there is no way of evaluating risky health streams in terms of healthy-years equivalents. Then, in order to be able to deal with situations involving risk, knowledge of the utilities associated with health status sequences is indispensable.

approximation procedure suggested by Mehrez and Gafni, albeit putting more emphasis on step one - i.e., the approximation of the individual´s utility function for sequences offering perfect health up to some time followed by death - in order to keep the approximation error within reasonable bounds.

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