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Optimal Investment Policies for Hybrid Pension Plans
Analyzing the Perspective of Sponsors and Members

von

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Optimal Investment Policies for Hybrid Pension Plans

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Abstract:

This paper analyzes investment strategies in the context of alternative hybrid pension plans which are optimal either from the perspective of the plan sponsor or the beneficiaries. The focus is in particular on how the introduction of minimum and maximum limits for pension benefits as well as minimum guarantees and caps on the return of the members' individual investment accounts affect the investment decision. The study finds that portfolio choice of sponsor and beneficiaries shows substantial differences depending on the exact plan design and the beneficiaries' risk aversion. The introduction of caps on investment returns emerged a possible means to reduce such differences and to share investment risks and returns more equally between sponsor and beneficiaries.

JEL Classification:

G11, G23

1. Introduction

Since being firstly introduced in the United States by the Bank of America in 1985, hybrid forms of occupational pension plans broke through the traditional dichotomy of pension plan designs (Schieber 2003). Before that, pension promises were either of pure defined benefit (DB) or pure defined contribution (DC) character.

In a DB scheme, the plan sponsor promises to the plan beneficiaries a final level of pension benefits. This level is usually defined according to a benefit formula as a function of salary trajectory and years of service. In general, benefits are paid as life annuities rather than lump sums. As Bodie et al. (1988) point out, the foremost advantage of DB plans is that they offer stable income replacement rates to the plan beneficiaries at retirement. The major drawbacks of DB schemes are the lack of benefit portability when leaving the company and the complex valuation of plan liabilities. Moreover, the plan sponsor is exposed to substantial investment and longevity risk, which could result in significant contribution expenses.

In a DC scheme, the plan sponsor commits to paying funds into the beneficiaries' individual accounts according to a specified formula, e.g. a fixed percentage of annual salary. The most prominent feature of DC schemes is their inherent flexibility. By construction, a DC plan is fully funded in individual accounts. The value of the pension benefits is simply determined as the market value of the backing assets. Therefore, the pension benefits are easily portable in case of job change. Additionally, the beneficiaries have full control over the investment strategy and is free to choose an adequate decumulation plan at retirement, i.e. either life annuities, phased withdrawal plans, lump sum payment or any combination of these. While the employer is only obliged to make regular contributions, the employee bears the risk of uncertain replacement rates, especially caused by fluctuations in the capital markets (Bodie and Merton 1992).

The term hybrid pension plan subsumes the manifold class of pension schemes that combine both, elements of defined contribution and defined benefit plans. The motivation of hybrid pension plans is to combine the best characteristics of DB and DC plans while circumventing their major disadvantages. A broad range of hybrid pension plans combine a DC type individual account, whereby minimum and/or maximum annuity benefits at retirement may be specified by a defined benefit formula. Additionally, investment returns credited to the individual accounts may be subject to return guarantees and/or return caps.

The asset allocation decision is of major importance for all types of pension plans. In a pure DC plan, the plan members have full control over the investment strategy for their accounts. This enables them to adapt the risk/return profile of the portfolio to their individual taste of risk. Since the investment risk is completely borne by the members, the plan sponsor is indifferent towards the investment policy. This is in sharp contrast to pure DB plans. As the sponsor is fully exposed to capital market risk, the asset allocation decision must lie with him to control his funding situation. In a hybrid pension plan, both parties have an interest to influence the investment policy, which may result in conflicts of interest. The extent of this possible conflict of interests for different hybrid pension plan specifications is the object of investigation of this paper.

For this, we construct a set of hypothetical hybrid pension plans, assume a fixed set of parameters and study over a given set of investment alternatives the optimal investment strategy for particular objective functions, i.e. cost minimization from the perspective of plan sponsor versus maximizing risk adjusted pension benefits from the perspective of plan members. Although neither the design of the plan nor the assumed parameter specification correspond to any actually existing pension plan, it draws on elements typically found in real world pension plans. In particular the model and results presented in this paper benefited substantially from prior work undertaken on the European Central Bank retirement plan.¹ Compared to actually existing plans, the plan developed in this paper shows a reduced complexity. Inter alia, it omits modeling any aspects related to dependant benefits and it also assumes a simplified population model.

The remainder of this study is organized as follows. In the next section, we first discuss the general plan design and the core hybrid elements studied here, such as the minimum pensions guarantee, maximum pension limits, and the return guarantee/caps. Subsequently, we focus on technical aspects of our underlying model and the decision making process assumed before analyzing the optimal investment strategy, both from the perspective of the plan sponsor as well as the plan members.

2. Designing a Hybrid Pension Plan

2.1 General Structure of the Pension Plan

The model pension plan analyzed in this study is a non-contributory funded plan consisting of two types of accounts. First, every plan member owns an individual account. All

individual accounts are endowed by the plan sponsor with a payment of 20 percent of the members' annual salary. These payments are the employer's regular contributions to the plan. In addition to that, the plan sponsor owns a separate account, the contingency reserve, which takes the role of a settlement account for transfers from or to the individual accounts. The sum of the funds in both types of accounts, individual accounts as well as contingency reserve, represents the total plan assets. All plan funds are invested into the same asset allocation and the return on this portfolio is credited pro rata to the individual accounts and the contingency reserve.

Besides the promise by the plan sponsor to fund the individual accounts with regular contributions, the plan design includes a combination of additional guarantees and/or limits. These are related either directly to the level of benefits at retirement and/or to the asset return credited to the individual accounts. Incorporating such elements has an impact on the obligations arising from the plan and may require from the sponsor further payments in addition to the regular contributions, so called supplementary contributions.

Supplementary contributions may be triggered in two cases. Firstly, when guaranteeing a minimum return on the plan assets, the plan sponsor has to cover possible shortfalls below the target return by replenishing the individual accounts through supplementary contributions. Secondly, supplementary contributions can become necessary in cases with guaranteed minimum pension benefits. If the market value of the total plan assets falls below 90 percent of the actuarial present value of the plan liabilities, i.e. the solvency ratio is below 0.9, the plan sponsor will have to endow the contingency reserve with enough funds to re-establish a solvency ratio of 1.

Furthermore, participation in this pension scheme is assumed to be mandatory and the beneficiaries cannot withdraw funds during the accumulation phase. The management of the plan does not take into account possible benefits provided by other (government-run) systems. Members leaving the plan before retirement are entitled to either leave their funds in the plan or to receive the balance of their individual account as a lump sum, limited to the actuarial value of the maximum pension where applicable. At the retirement age of 65, the available funds are converted into a life annuity. This conversion may be subject to the guaranteed minimum pension benefits and to maximum pension limits depending on the exact design of the benefit structure, which will be discussed subsequently.

2.2 Benefit Structure of the Pension Plan

In the course of this study, six distinct hybrid pension plan designs will be scrutinized. Every plan is characterized by a unique combination of the elements mentioned earlier. Comparing these designs aims at investigating the effects on plan costs as well as pension benefit levels that each hybrid plan element has and the implications for the optimal allocation of plan assets.

In Case I, which is the benchmark design, the pension plan consists of an individual account for every plan member that is endowed by the plan sponsor with regular contributions of 20 percent of the current salary. These funds are invested in the capital markets. Beneficiaries are protected from return shortfalls by an annual capital guarantee, i.e. a guaranteed yearly minimum return of 0 percent.² In case the funds earn less than 0 percent in any given year, the sponsor makes an additional contribution to the accounts. As these funds accumulated over a plan member's career might not be sufficient to pay for an adequate pension, this plan layout also guarantees a minimum level of pension benefits corresponding to 2 percent of the career-average salary per year of service. In addition, this plan limits the maximum level of benefits to 2 percent of the beneficiaries' final salary.³ In the event of a member either leaving the plan or retiring, any funds in the individual account that exceed the actuarial value of the maximum benefits will be transferred to the contingency reserve.

Table 1: Summary of Hybrid Pension Plan Designs

	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
Individual Account	√	√	√	√	√	√	√
Minimum Benefits	√	√	√				
Maximum Benefits	√	√		√			
Capital Guarantee	√			√	√	√	
Return Cap						√	

Source: Authors' compilation.

Notes: Minimum benefits are defined as 2 percent of career-average salary per year of service; maximum benefits are defined as 2 percent of final salary per year of service; capital guarantee refers to a guaranteed minimal return of 0 percent per year for the individual accounts; the return cap limits the annual return credited to the individual accounts to 10 percent.

The subsequent Cases II to V are constructed by eliminating certain plan elements compared to the benchmark case. Case II excludes the capital guarantee, and in Case III the maximum benefits are additionally removed. Case IV eliminates the minimum benefits from the benchmark case, while Case V only includes the annual capital guarantee. Case VI includes the annual capital guarantee and additionally a return cap of 10 percent per year but provides no further benefit elements relating to salary and years of service. If the asset return on the funds in the individual accounts in any year exceeds the 10 percent level, the excess

return will be credited to the contingency reserve. Case VII does not include any guarantees or caps and, therefore, can be interpreted as a pure defined contribution plan. Table 1 summarizes the various plan designs.

The minimum rate of return guarantee increases the complexity of the pension plan substantially. More specifically, the minimum rate of return guarantee *may* introduce an asymmetric link between assets and liabilities. Suppose the value of a given investment account corresponds to a pension payment in-between the minimum and the maximum pension limit. In this situation, a high asset return in any given year permanently increases the sponsors' liabilities for the current and future years. Negative returns in subsequent years do not decrease the liability as the minimum rate of return guarantee requires the sponsor to replenish the investment account. Thus the high asset return in the first year had a permanent effect on the liabilities.⁴ However in a situation where the investment account corresponds to a pension payment either below the minimum pension guarantee or above the maximum pension limit, asset returns do not have an immediate effect on the sponsor's liabilities.

2.3 Asset Liability Model and Decision-Making Process

This section outlines the asset liability model and decision making process employed to determine the funds' asset allocation. This first comprises the specification of assumptions about how assets and liabilities are projected forward. Secondly we specify decision rules used either by the plan sponsor or by the beneficiaries to identify the optimal asset allocation.

Independently of whether the asset allocation decision is made by the sponsor or by the beneficiaries, a two-step heuristic method is applied which is often found in practical decision making. As a first step, the set of mean variance efficient asset allocations is determined using standard Markowitz-type portfolio optimization. In the second step all portfolios from the efficient frontier are assessed against a projection of asset and liabilities over a horizon of 30 years.

In order to project the return and risk effects of a certain asset allocation over time, it is necessary to specify the stochastic processes that govern the developments of the returns of asset classes, interest rates for maturities of three months, representing money market investments, and ten years as well as inflation rates. The difference between the nominal ten year interest rate and the inflation rate, i.e. the real ten year interest rate, is used to discount future pension liabilities. The stochastic dynamics of the (uncertain) market values of the assets are modeled as geometric Brownian motion, which implies that the log return of every asset is independent and identically normally distributed. Long- and short-term interest rates as well

as the inflation rate are modeled using the multi-dimensional Ornstein/Uhlenbeck process, to cover the empirically observable mean reversion characteristics in these time series.⁵

The investment universe comprises the broad asset classes money market instruments, euro area bonds, and world-wide diversified equities as well as emerging market equities. A regime-switching model is used to derive expected returns for the fixed income asset classes (i.e. money market instruments and Eurobonds). This technique allows consistent generation of yield curve projections contingent on expectations about economic activity (Bernadell et al. 2005). In the long-term projection of the macro economic environment, we rely on the Economist Intelligence Unit (EIU) as an external provider of forecasts for the Euro area, the US and Japan.⁶ Expected returns on equity investments are approximated by add-ons to the long-term yields on government bonds. In the analysis the equity risk premium is fixed at 2.5% annually for world-wide diversified equity. Reflecting higher risk of emerging market investments we assume an equity risk premium of 4% for this asset class. All asset classes are subject to short selling constraints and, in addition, the investment in emerging market equity is restricted to a maximum of 5% of overall investments. Details of the parameter estimates for the asset, interest rate and inflation dynamics are given in the appendix.

The projection of liabilities is based on a discontinuance valuation method usually applied by plan actuaries. Discontinuance valuation relies on the assumption that service of each participant ceased on the respective valuation date. It assumes that at a given valuation date the individual investment accounts are translated into a (usually deferred) life annuity with inflation-adjusted payments, whereby the minimum and maximum pension limits laid out earlier are applied. The real discount rates used for this exercise are and the real ten-year interest rates determined by the asset model. Discontinuance valuation is performed for each year over the 30-year analysis horizon. The valuation of liabilities requires projecting population dynamics comprising the evolution of the number and composition of staff, salaries, number of retirees and dependants. For this purpose a hypothetical population comprising initially 1000 staff members is constructed. The population is evolved forward using an inhomogeneous, discrete time Markov chain. Transition probabilities are derived against assumptions made on the company's recruitment policy, promotion and turnover of staff, evolution of salaries as a function of consumer price inflation and mortality rates.

Comparing the value of liabilities with the projected value of assets at the respective valuation date allows determining the evolution of the plan's solvency ratio, supplementary contributions to be made by the sponsor and average benefits. Given the complexity of the

plan design, solutions are determined using Monte Carlo simulation over 1000 simulation runs.

Finally, we make a number of specific assumptions about selection criteria used to determine the plan's optimal asset allocation. To this end, two different regimes are introduced. Under the first regime, arguably the standard for hybrid pension plans, the plan sponsor is solely responsible for the investment strategy. Correspondingly, the second regime assumes that decisions are made by the beneficiaries. In both cases investment decisions apply simultaneously to all individual investment accounts and the contingency reserve.

For the sponsor we assume the objective to minimize the costs of running the plan. More specifically the sponsor minimizes the worst-case value of discounted supplementary contributions, whereby the worst-case value is defined as the 5 percent quantile of the distribution of the sum of discounted supplementary contributions over the 30-year investment horizon. Thus decision criteria in addition to costs, such as plan solvency, are not considered explicitly. The plan funding is accounted for by the solvency rule, as specified in Section 2.1, according to which the funding ratio can not fall short of 90 percent in any single year. More formally, let SC_t be the total amount of supplementary contributions to be made by the plan sponsor in period t and r the appropriate discount rate, then the objective function is given by:

$$\min VaR_{5\%} \left[\sum_{t=1}^{30} \frac{SC_t}{(1+r)^t} \right] \quad (1)$$

Investment decisions by the beneficiaries are made collaboratively for all investment accounts. These decisions may be made in the context of an investment committee composed of staff representatives. Such a body is assumed to maximize the expected value of the constant-relative-risk-aversion (CRRA) utility function $u(PBF)$ with risk aversion parameter $\gamma > 0$.

$$\max E[u(PBF)] = \max E \left[\left(\frac{PBF^{1-\gamma}}{1-\gamma} \right) \right] \quad (2)$$

Utility is defined over the pension benefit factor PBF which refers to pension payments per year service expressed as the percentage of final salary at time of retirement. Factor PBF comprises all simulation runs and all plan members retiring over the 30-year investment horizon.

3. The Plan Sponsor's Investment Decision

3.1 Finding the Cost-Optimal Asset Allocation

In this section we take the perspective of the sponsor of a pension plan. Here we are interested in the interrelation between the asset allocation for the individual pension accounts and the resulting plan costs measured in terms of supplementary contributions by the plan sponsor. We therefore look at the different plan designs described above. Figure 1 depicts the worst case supplementary contributions for Cases I to IV for different portfolio allocations, and Figure 2 for Cases V and VI. Worst case costs are measured as the 5 percent value at risk of the supplementary contributions, i.e. the present value of contributions by the plan sponsor exceeding the regular payments of 20 percent of the salaries. The portfolio allocations are represented by the mean-variance efficient portfolio returns. The details of the cost-optimal asset allocations, i.e. the asset weights for Cash, Eurobonds, Global-, and Emerging Markets Equities, are reported in panel 1 of Table 2. Panel 2 contains the distributional characteristics of the discounted supplementary contributions for the cost-optimal asset allocations. Finally, panel 3 reports the resulting pension benefits for these allocations in terms of security equivalents. These security equivalents are calculated according to the utility function stated above and for four different parameters of relative risk aversion.

Looking at the base case, Case I, in Figure 1 it can be observed that the worst case plan costs form a U-shaped curve. With increasing expected portfolio returns the costs first decrease and then increase again, resulting in minimum supplementary contributions for an asset allocation with an expected return of 5.57 percent. This portfolio consists of about 84 percent bonds and 16 percent equities. The minimum supplementary contributions amount to 42.49 percent of the expected regular contributions. Hence, for every discounted Euro the sponsor regularly paid into the plan additional payments of 42.49 discounted Cents are required to cover the costs of the plan. The U-shape of the cost curve can be directly related to the guarantees included in Case I. Investing in portfolios mainly consisting of cash or bonds will result in assets not being able to generate enough return to cover the costs of the guaranteed minimum benefits. These costs have to be borne by the plan sponsor. As the expected return on the portfolio increases, it becomes more and more likely that the funds suffice to at least pay the minimum pension without further contributions by the plan sponsor. The rise in expected portfolio return is in turn accompanied by an increase in return volatility, which induces costs resulting from falling short of the guaranteed minimum annual asset return of 0 percent. From a certain level of volatility onwards, these newly induced costs overcompensate

sate the cost savings related to the minimum pension benefits and the overall costs increase again.

Table 2: Optimal Investment Decision - Sponsor's View

	Case I	Case II	Case III	Case IV	Case V	Case VI
<i>Panel 1: Cost-Optimal Asset Allocation</i>						
Mean Return	5.57%	5.70%	5.76%	5.63%	4.55%	5.95%
Volatility	4.68%	5.23%	5.53%	4.94%	2.17%	6.53%
Cash	0.00%	0.00%	0.00%	0.00%	71.28%	0.00%
Eurobonds	84.27%	79.16%	76.61%	81.72%	28.04%	68.95%
Global Equities	10.73%	15.84%	18.39%	13.28%	0.68%	26.05%
EM Equities	5.00%	5.00%	5.00%	5.00%	0.00%	5.00%
<i>Panel 2: Distributional Characteristics of DSC with Optimal Asset Allocation</i>						
Mean DSC	19.64%	18.44%	22.14%	9.24%	13.02%	7.00%
Std. DSC	13.03%	12.89%	12.60%	9.62%	8.01%	9.17%
5%-VaR DSC	42.49%	41.50%	43.95%	27.01%	28.04%	25.87%
25%-Q DSC	10.27%	9.04%	12.71%	0.00%	6.95%	0.00%
50%-Q DSC	17.56%	16.12%	19.87%	6.70%	12.15%	2.77%
75%-Q DSC	26.92%	25.65%	29.41%	15.17%	17.96%	11.39%
<i>Panel 3: Distributional Characteristics of PB with Optimal Asset Allocation</i>						
Mean PB	1.875%	1.872%	2.257%	1.793%	1.531%	1.939%
Std. PB	0.045%	0.050%	0.415%	0.102%	0.205%	0.307%
Security Equivalent ($\gamma = 1$)	1.874%	1.872%	2.223%	1.790%	1.517%	1.915%
Security Equivalent ($\gamma = 5$)	1.872%	1.869%	2.121%	1.776%	1.470%	1.827%
Security Equivalent ($\gamma = 10$)	1.869%	1.866%	2.040%	1.750%	1.421%	1.726%

Source: Authors' calculations.

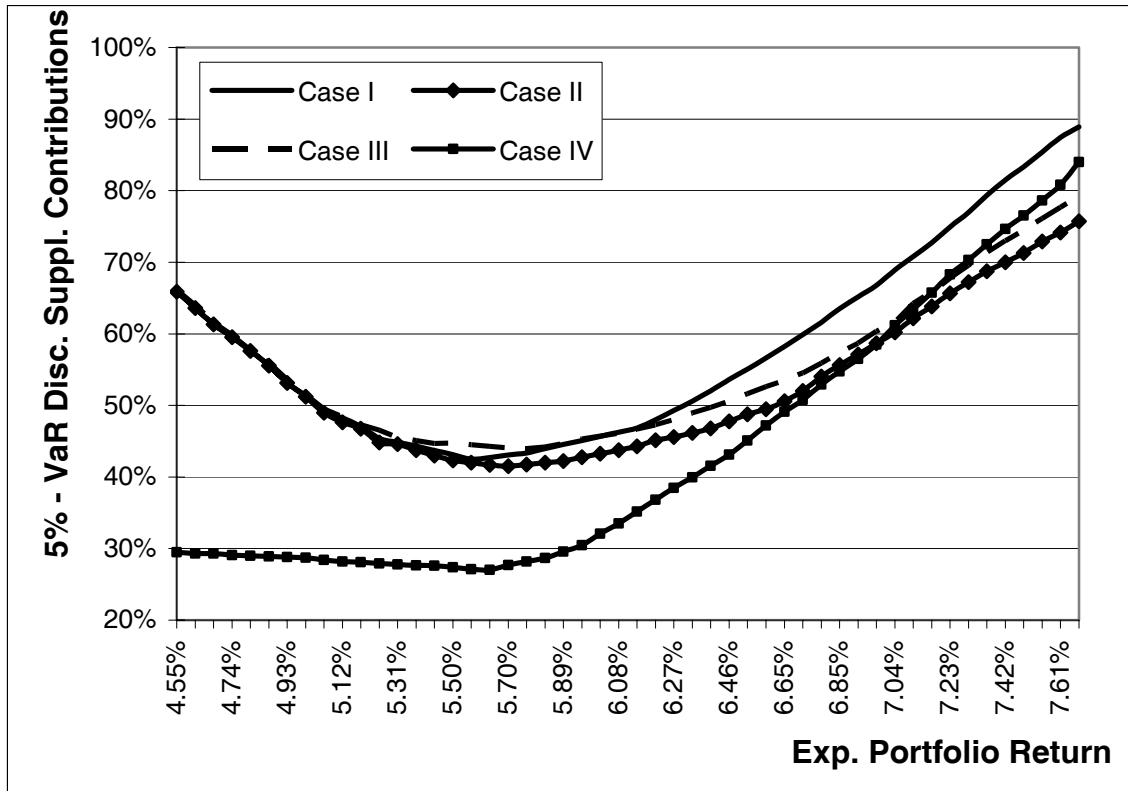
Notes: Discounted Supplementary Contributions (DSC) in percent of expected discounted regular contributions; Pension Benefits (PB) in percent of final salary per year or service; Objective function: Minimize the 5 percent VaR of DSC; Q: quantile; Case I: DC + minimum benefits + maximum benefits + capital guarantee; Case II: DC + minimum benefits + maximum benefits; Case III: DC + minimum benefits; Case IV: DC + maximum benefits + capital guarantee; Case V: DC + capital guarantee; Case VI: DC + capital guarantee + 10% cap on asset return.

Changing the structural design of the plan has the following effects. Eliminating the annual return guarantee for the individual accounts, Case II, reduces the amount of supplementary contributions, especially in the case of a more risky asset allocation. However, the asset allocation, which minimizes the costs, is only slightly different compared to Case I, i.e. about 5 percent less bonds and more equities. The total worst case costs only decrease from 42.49 percent to 41.5 percent of regular contributions. This results from the fact that for a low risk asset allocation the costs from the annual return guarantee are relatively low.

In Case III not only the capital guarantee but also the maximum pension regulation is eliminated. In this case, the plan is basically a defined contribution plan in which the beneficiaries are protected by defined minimum pension benefits against adverse capital market developments. Therefore, it is obvious that the amount of supplementary contributions increases

compared to Case II, as there are no longer funds in excess of those needed to provide maximum pension benefits, which could be credited to the plan sponsor. Looking at the cost minimizing asset allocation, the equity exposure is slightly further increased to about 13.4 percent with overall worst case supplementary contributions of 43.95 percent.

Figure 1: Worst Case Plan Costs vs. Asset Allocation



Source: Authors' calculations.

Notes: Discounted Supplementary Contributions (DSC) in percent of expected discounted regular contributions; Case I: DC + minimum benefits + maximum benefits + capital guarantee; Case II: DC + minimum benefits + maximum benefits; Case III: DC + minimum benefits; Case IV: DC + maximum benefits + capital guarantee.

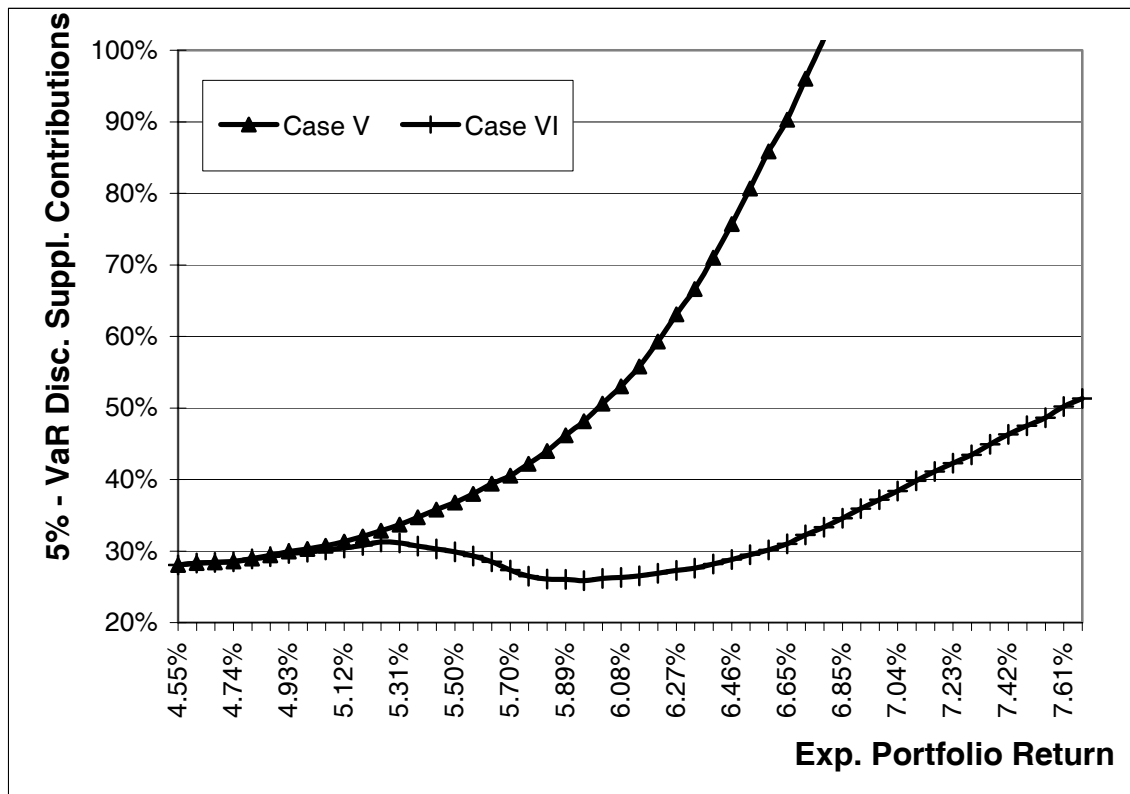
Case IV shows a quite different curve. The general U-shape is maintained showing a minimum of supplementary contributions for a portfolio return of 5.63 percent, which corresponds to an asset allocation of 81.72 percent bonds and 18.28 percent equities. While the asset allocation is comparable to the Cases I to III, the level of supplementary contributions with 27.01 percent of regular contributions is substantially lower than in the previous cases. Additionally, the left branch of the cost curve (low risk portfolios) is nearly flat while the right branch (more risky portfolios) shows a strong increase. Economically, this can be explained as follows. As discussed for Case I, the predominant source of costs, especially when investing into low risk allocations, is the minimum benefit guarantee. This guarantee is not

included in Case IV leading to substantially lower costs compared to Case I. Increasing the expected return and the volatility of the portfolio now has two opposing effects. The higher the (expected) portfolio return the more often the plan sponsor will profit from cashing in funds from the individual retirement accounts that exceed the amount necessary to cover the maximum pension benefits. Contrarily, the higher the return volatility the more often supplementary contributions will be triggered due to the annual capital guarantee. Since the former effect dominates the latter for less risky portfolios, the overall costs first decrease with increasing expected portfolio return. For more risky allocations the latter effect dominates the former, which leads to rapidly growing contributions. As the cost impact of the minimum benefit guarantee is diminishing for increasing portfolio returns, Cases I and IV hardly differ for highly risky portfolios.

We now turn to the cases with no explicit defined minimum or maximum benefits. These are depicted in Figure 2. Case V shows a plan with unlimited upside potential but with a shortfall protection resulting from the annual return guarantee. It is clear that such a plan design results in increasing supplementary contributions the higher the equity exposure. Hence, minimizing the costs in terms of supplementary contributions leads to the minimum volatility portfolio, consisting of 71.28 percent cash, 28.04 percent bonds and only 0.68 percent equities.

As in Case V, Case VI offers an annual capital guarantee and therefore protection against return shortfalls. However, the upside potential is limited due to the 10 percent return cap. This structural change has a significant impact on the shape of the cost curve. While the amount of supplementary contributions in Cases V and VI is approximately equal for low risk asset allocations, the costs in Case VI begin to decrease again for increasing portfolio return and volatility. For these allocations the increasing costs resulting from the capital guarantee are overcompensated by the profits the plan sponsor can generate through cashing in any returns that exceed the 10 percent cap. With volatility even more increasing, the costs of the capital guarantee increase disproportionately, leading to overall growing supplementary contributions. With minimum worst case costs of 25.87 percent resulting from investing in about 69 percent bonds and 21 percent equities this case proves to be the cheapest of all plan designs discussed. This holds for the minimum cost asset allocation and especially for all portfolios with high expected returns and volatilities.

Figure 2: Worst Case Plan Costs vs. Asset Allocation



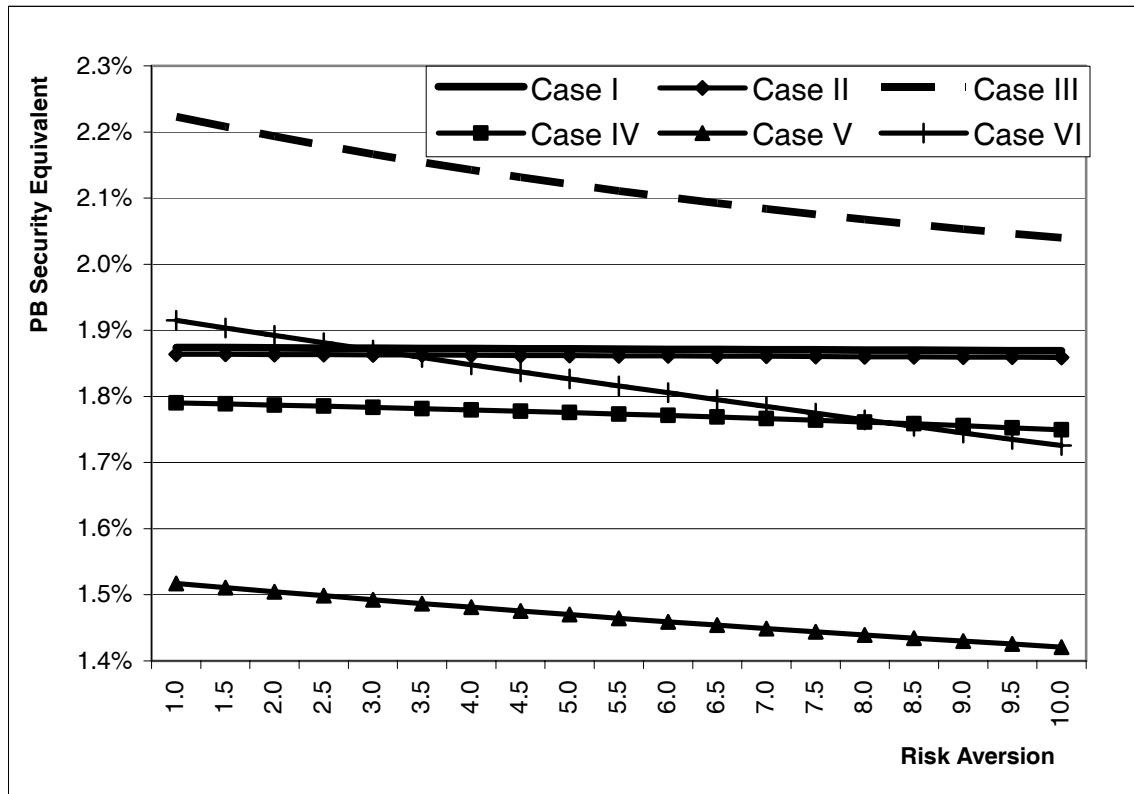
Source: Authors' calculations.

Notes: Discounted Supplementary Contributions (DSC) in percent of expected discounted regular contributions; Case V: DC + capital guarantee; Case VI: DC + capital guarantee + 10% cap on asset return.

3.2 Utility-Implications for the Beneficiaries

The implications investing in the cost-minimizing portfolios will have for the expected pension benefits the members will receive can be derived from panel 3 in Table 2 and from Figure 3. Panel 3 of Table 2 summarizes expected pension benefit factors and their standard deviation, expressing the pension benefits in percent of final salary per year of service. In order to relate the whole probability distribution of the pension benefit factors to the risk aversion of a representative beneficiary the pension factor security equivalents are calculated for a range of risk aversion parameters using the standard CRRA utility function. This allows a direct evaluation of the cost-optimal asset allocation for the various plan designs, Case I to VI, from the perspective of plan members with different levels of risk aversion. Figure 3 depicts the security equivalents for all parameters of risk aversion from one to ten in steps of 0.5. Additionally, numerical results for selected levels of risk aversion ($\gamma = 1, 5, \text{ and } 10$) are presented in panel 3 of Table 2.

Figure 3: Security Equivalents with Sponsor's Optimal Asset Allocation



Source: Authors' calculations.

Notes: Case I: DC + minimum benefits + maximum benefits + capital guarantee; Case II: DC + minimum benefits + maximum benefits; Case III: DC + minimum benefits; Case IV: DC + maximum benefits + capital guarantee; Case V: DC + capital guarantee; Case VI: DC + capital guarantee + 10% cap on asset return.

Attending to Figure 3, it can be easily observed that Case III results in the highest pension benefit factors for all levels of risk aversion under scrutiny, with the mean benefit factor being 2.257 percent (see Table 2, panel 3). At the same time, Case V always produces the lowest factors, on average 1.531 percent. This is an interesting result, as both cases show structural similarities. Case III and V both offer downside protection to the beneficiaries, Case III by means of guaranteed minimum pension benefits, Case V with the annual capital guarantee for the individual accounts. Neither case limits the upside potential. An explanation for this can be found when looking at the different cost-minimal asset allocations. Optimizing the amount of supplementary contributions in Case V, the plan sponsor will only invest into cash and bonds, resulting in the lowest risk exposure with respect to the capital guarantee. The highly conservative risk and return profile of the assets simultaneously leads to low expected pension benefits. Such an asset allocation, however, is not appropriate in Case III, since its return expectations are insufficient to cover the costs resulting from the guaranteed minimum benefits, i.e. 2 percent of the career-average salary per year of service. It is rather

necessary to implement a portfolio strategy that offers higher mean returns, coming at the cost of higher volatility. This, in turn, leads to substantially higher supplementary contributions, since the plan sponsor fully bears the downside volatility while only the beneficiaries profit from the upside volatility.

Implementing a maximum benefit cap (2 percent of final salary per year of service) results in considerably reduced volatilities of the pension benefit factors, i.e. 0.045 percent for Case I, 0.05 percent for Case II, and 0.102 percent for Case IV (see Table 2, panel 3). Consequently, the security equivalents of the pension benefit factors are nearly constant for the various risk aversion coefficients reported in Figure 3. Among these cases, Case I offers the highest pension benefits but is also the most costly design. Case II only offers slightly lower benefits combined with slightly lower costs.

In general it can be concluded that the plans, which offer the highest expected pension benefits, tend to cause the highest amount of supplementary contributions. Yet there are two exemptions: Case V offers by far the lowest pension benefits, however, even for the optimal asset allocation the costs are not minimal. In contrast to that, Case VI causing the lowest supplementary contribution will lead to expected pensions benefits that exceed all but one case. The rather high volatility of the pension benefit factor, however, causes the security equivalents to drop below those of most other cases for higher levels of risk aversion.

4. The Beneficiaries' Investment Decision

In this section we assume that the asset allocation decision is made by the beneficiaries of the pension plan. In this context, the plan members' objective function is to maximize the expected utility of pension benefits by choosing the appropriate asset allocation. This is done for all Cases I to VI and for the purpose of comparison also for Case VII, a pure defined contribution plan. Our interest in this section is to look at the resulting pension benefits for the plan member with different risk aversion as well as the composition of the optimal asset allocation. As in Section 3 the portfolio allocations are represented by the mean-variance efficient portfolio returns. The details of the benefit-optimal investment weights, i.e. the mean and volatility of asset returns, mean and security equivalents of pension benefit factors for plan members as well the resulting costs in terms of supplementary contribution for the plan sponsor are reported Table 3. Panel 1 contains the results for a representative plan member

with a low ($\gamma = 1$), Panel 2 with a medium ($\gamma = 5$), and Panel 3 with a high ($\gamma = 10$) coefficient of risk aversion. Details of the investment weights in Cash, Euro-bonds, Global-, and Emerging Markets Equities, are reported in Table 4.

Table 3: Optimal Investment Decision - Members' View

	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
<i>Panel 1: Low Level of Risk Aversion (Gamma = 1)</i>							
Mean Return	7.68%	7.68%	7.68%	7.68%	7.68%	7.68%	7.68%
Volatility	17.27%	17.27%	17.27%	17.27%	17.27%	17.27%	17.27%
Mean DSC	30.91%	24.60%	29.80%	26.58%	126.85%	14.29%	0.00%
5%-VaR DSC	87.93%	75.71%	79.28%	84.52%	246.62%	51.33%	0.00%
Mean PB	1.974%	1.915%	3.937%	1.949%	6.890%	1.948%	3.793%
Security Equivalent	1.974%	1.914%	3.355%	1.948%	6.014%	1.918%	3.093%
<i>Panel 2: Medium Level of Risk Aversion (Gamma = 5)</i>							
Mean Return	7.68%	7.61%	7.68%	7.68%	7.68%	5.89%	6.33%
Volatility	17.27%	16.83%	17.27%	17.27%	17.27%	6.18%	8.74%
Mean DSC	30.91%	24.14%	29.80%	26.58%	126.85%	7.35%	0.00%
5%-VaR DSC	87.93%	74.14%	79.28%	84.52%	246.62%	26.06%	0.00%
Mean PB	1.974%	1.915%	3.937%	1.949%	6.890%	1.937%	2.543%
Security Equivalent	1.973%	1.909%	2.449%	1.947%	4.011%	1.827%	2.007%
<i>Panel 3: High Level of Risk Aversion (Gamma = 10)</i>							
Mean Return	7.68%	7.61%	7.61%	7.68%	7.68%	5.76%	5.76%
Volatility	17.27%	16.83%	16.83%	17.27%	17.27%	5.53%	5.53%
Mean DSC	30.91%	24.14%	29.22%	26.58%	126.85%	7.98%	0.00%
5%-VaR DSC	87.93%	74.14%	77.68%	84.52%	246.62%	26.31%	0.00%
Mean PB	1.974%	1.915%	3.847%	1.949%	6.890%	1.930%	2.155%
Security Equivalent	1.973%	1.902%	2.160%	1.944%	2.990%	1.729%	1.760%

Source: Authors' calculations.

Notes: Discounted Supplementary Contributions (DSC) in percent of expected discounted regular contributions; Pension Benefits (PB) in percent of final salary per year of service; Objective function: Maximize the expected utility of pension benefits using a CRRA utility function defined over the pension benefits in percent of final salary per year of service; Case I: DC + minimum benefits + maximum benefits + capital guarantee; Case II: DC + minimum benefits + maximum benefits; Case III: DC + minimum benefits; Case IV: DC + maximum benefits + capital guarantee; Case V: DC + capital guarantee; Case VI: DC + capital guarantee + 10% cap on asset return; Case VII: DC.

Independent of the level of risk aversion, for Cases I to V the plan beneficiaries would opt to invest into the asset allocation that offers the highest or almost the highest expected return and the highest or almost the highest volatility. As shown in Table 4, the optimal asset allocation consists of 100% stocks. In all these cases the beneficiaries are protected against downside volatility of the international equity markets either by guaranteed minimum pension benefits and/or by the annual capital guarantee. The value of this downside protection *ceteris paribus* increases with the volatility. In analogy to the foundations of option pricing theory, the minimum pension benefits and the capital guarantee can be interpreted as a call option, for which the value is also positively related to the volatility of the underlying. Looking at the level of supplementary contributions associated with these asset allocations, it is

obvious that the cost for the plan sponsor are prohibitively high. This especially holds for Case V, in which the members' individual accounts are protected against negative fluctuations in the capital markets while at the same time offering full participation in positive returns. Here, the security equivalents of the pension benefit factors vary between 6.014 percent for a low risk aversion of $\gamma = 1$ and 2.99 percent for a relatively high coefficient of relative risk aversion of $\gamma = 10$. These high pension benefits are associated with worst case (mean) supplementary contributions of 246.62 percent (126.85 percent). It is clear that these costs are unacceptable from the perspective of the plan sponsor.

Table 4: Optimal Asset Allocation - Members' View

	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
<i>Panel 1: Low Level of Risk Aversion (Gamma = 1)</i>							
Cash	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Eurobonds	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Global Equities	95.00%	95.00%	95.00%	95.00%	95.00%	95.00%	95.00%
EM Equities	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%
<i>Panel 2: Medium Level of Risk Aversion (Gamma = 5)</i>							
Cash	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Eurobonds	0.00%	2.55%	0.00%	0.00%	0.00%	71.50%	53.63%
Global Equities	95.00%	92.45%	95.00%	95.00%	95.00%	23.50%	41.37%
EM Equities	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%
<i>Panel 3: High Level of Risk Aversion (Gamma = 10)</i>							
Cash	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Eurobonds	0.00%	2.55%	2.55%	0.00%	0.00%	76.61%	76.61%
Global Equities	95.00%	92.45%	92.45%	95.00%	95.00%	18.39%	18.39%
EM Equities	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%

Source: Authors' calculations.

Notes: Objective function: Maximize the expected utility of pension benefits using a CRRA utility function defined over the pension benefits in percent of final salary per year of service; Case I: DC + minimum benefits + maximum benefits + capital guarantee; Case II: DC + minimum benefits + maximum benefits; Case III: DC + minimum benefits; Case IV: DC + maximum benefits + capital guarantee; Case V: DC + capital guarantee; Case VI: DC + capital guarantee + 10% cap on asset return; Case VII: DC.

Cases I, II, and IV limit the upside potential available to the beneficiaries by incorporating the maximum pension benefit restriction. This results in lower benefits and lower costs compared to Case V, yet the plan members still have the incentive to choose portfolios with very high volatility. Even though the costs are substantially reduced, they are still intolerably high. For example in Case IV, i.e. Case V with incorporated maximum benefit limit, the worst case (expected) supplementary contributions amount to 84.52 percent (26.58 percent), being about three times as high as in case the plan sponsor chooses the asset allocation.

Turning to Case VI results in a different picture. This case is characterized by the annual capital guarantee and a 10 percent cap on the maximum annual asset return credited to

the beneficiaries' individual accounts. While beneficiaries with a low level of risk aversion, i.e. $\gamma = 1$, will still invest in the maximum expected return/maximum volatility portfolio, more risk averse plan members will choose an asset allocation with substantially reduced exposure to capital market risk. For a medium (high) level of risk aversion, i.e. $\gamma = 5$ (10), the allocation to bonds will increase from 0 percent, as for $\gamma = 1$, to 71.5 (78.61) percent (see Table 4). An interesting observation follows when comparing this benefit optimal asset allocation from the members' perspective (with moderate to high risk aversion) to the cost optimal asset allocation from the sponsor's perspective as given in panel 1 of Table 2. From both perspectives, the optimal investment strategy is nearly identical, i.e. including a high exposure to bonds and a low exposure to equities. This results in quite similar cost implication in terms of supplementary contributions. If the plan sponsor set the asset allocation, the 5%-Value-at-Risk of supplementary contributions would be 25.87 percent (panel 2 of Table 2), while in case a representative member with a risk aversion of $\gamma = 5$ ($\gamma = 10$) chose the (benefit) optimal asset allocation this would result in supplementary contributions for the plan sponsor of 26.06% (26.31%). Hence, if the benefit structure of the pension plan is designed according to Case VI this will lead to "harmony" of the members' and sponsor's interests, at least with respect to the asset allocation decision for a given plan design.

However, while Case VI seems to be most suitable among the hybrid pension plans, combining acceptable cost consequences for the sponsor and attractive pension benefit factors for the plan members, a superior plan design exists. Case VII is a pure defined contribution plan with no capital guarantee and no return cap. By construction, this plan causes no additional costs in term of supplementary contributions for the sponsor. Additionally, as shown in Table 3, a pure DC plan provides higher mean pension benefits as well as security equivalents for all levels of risk aversion. This result is due to the specification of the floor/cap structure, i.e. a minimum rate of return of 0 percent per year and a return maximum of 10 percent per year. When setting the cap to an annual return of 12.5 percent and leaving the floor constant at 0 percent leads to the following results: For members with a low level of risk aversion, i.e. $\gamma = 1$, the security equivalent for the benefit factor of the hybrid plan is still lower than in the case of a pure DC plan (2.283 percent compared to 3.093 percent). For members with medium to high levels of risk aversion, i.e. $\gamma = 5$ (10), the hybrid plan is more attractive than the pure defined contribution plan. The security equivalents are 2.100 percent compared to 2.007 percent for $\gamma = 5$, and 1.913 percent compared to 1.760 percent for $\gamma = 10$. Yet increasing the cap to 12.5 percent, the plan members will again choose the maximum volatility portfolio, i.e. 100 percent equities, independent of their

level of risk aversion. Unfortunately this will cause unacceptable costs in terms of supplementary contributions for the plan sponsor.

5. Conclusions

In this paper we analyzed the properties of a set of hypothetical hybrid pension plans by subsequently augmenting a defined contribution scheme by minimum and maximum limits for pension benefits as a function of average and final salary, as well as minimum guarantees and caps on the return of the members' individual investment accounts. For the resulting plans, we focused in particular on the optimal investment strategies from the perspectives of both, plan sponsor and beneficiaries.

The introduction of defined benefit elements increases the overall costs of running the pension plan substantially. These additional costs show a strong dependence on the chosen investment strategy.

How the usual expected return/volatility trade-off of investment returns is translated into additional costs is determined by the specifics of the defined benefit elements. For example in case of minimum pension benefits, the implied additional costs (expected and worst case values) are U-shaped as a function of expected investment returns. Therefore, assuming the objective to minimize the worst case value of additional costs, the sponsor will opt for asset allocations which deviate from the minimum risk allocation. Opposed to this, in case of minimum rate of return guarantees, additional costs increase monotonically as a function of higher expected asset return and volatility. However, the U-shape function is restored when minimum rate of return guarantees are combined with minimum or maximum pension benefits or when a cap on the return of the members' individual investment accounts is introduced. In particular the latter element emerged in the simulation study as a powerful means to limit additional costs and hence allow the sponsor to choose asset allocations with a comparable high share of equities.

From the perspective of beneficiaries the paper first focused on the utility implications of alternative defined benefit elements and the sponsors' asset allocation decisions. To this end security equivalents of pension benefits are calculated for a range of risk aversion parameters using the standard CRRA utility function. While generally higher additional costs imply higher expected pension benefits, the introduction of caps on credited asset returns al-

lows costs reduction at only slightly lower security equivalents. Secondly, the optimal investment choice from the beneficiaries' perspective was analyzed. Almost independent of the respective degree of risk aversion, plan members chose the maximum return, maximum risk asset allocation in cases the plan guarantees minimum rate of return or minimum pension benefits. Only in case of minimum rate of return guarantees in combination with a cap on the maximum return credited to the individual accounts, medium and highly risk averse members opt for less risky asset allocations.

The analysis presented in this study may also be useful to make observations about the design of hybrid pension plans. Some plan designs appear to be Pareto-inefficient (e.g. minimum and maximum pension benefits in combination with minimum rate of return guarantee) as they are dominated by others which imply lower additional costs and higher expected utility for plan members. Furthermore, in case plan sponsors and beneficiaries are jointly responsible for investment decision, caps on investment returns may reduce conflicts of interests as asset allocations respectively optimal for the parties, diverge to a lesser extent.

Appendix: Specification of Asset Dynamics, Interest, and Inflation Rates

Table A1: Parameters of Asset Returns

	Mean	Volatility	Correlations		
			Eurobonds	Global Equities	EM Equities
Eurobonds	5.1	3.7	1		
Global Equities	7.6	17.9	0.21	1	
EM Equities	9.1	27.5	0.1	0.73	1

Source: Authors' calculations.

Table A2: Parameters of Interest Rates and Inflation Dynamics

	θ	κ	σ	Correlations of Innovations		
				3m Interest Rate	10y Interest Rate	Inflation Rate
3m Interest Rate	0.043	0.114	0.012	1		
10y Interest Rate	0.05	0.075	0.01	0.8461	1	
Inflation Rate	0.02	0.286	0.011	0.7757	0.8103	1
Eurobonds				0.0683	0.1396	0.0740
Global Equities				-0.0100	0.0000	0.0100
EM Equities				-0.0100	0.0000	0.0100

Source: Authors' calculations.

Notes: The process is specified by $dX_t = \kappa(\theta - X_t)dt + \sigma dW_t$, where X_t is the value of the Ornstein/Uhlenbeck process in t , kappa (κ) is the speed of mean-reversion, theta (θ) is the long-run mean, and sigma (σ) is the volatility of changes of the process. dW_t is the increment of a standard Wiener process.

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Endnotes

¹ The ECB operates a defined contribution pension scheme. The assets of the plan, which exist solely for the purpose of providing benefits for members of the plan and their dependants, are included in the other assets of the ECB. The benefits payable, resulting from the contributions of the ECB, have minimum guarantees underpinning the defined contribution benefits.

² Alternatively to above focus on absolute return, a minimum fixed rate of return guarantee may also be applied to a relative rate of return. For example Chile's private pension funds are required to earn an annual real rate of return that is a function of the average annual real rate of return earned by all of Chile's private pension funds (see Pennacchi 1999, p. 225). Furthermore the guarantee may be applied to the account balance at the time of retirement, instead of the assumed annual basis.

³ As typical for public employees, the wage path until retirement is not-decreasing and, therefore, the guaranteed minimum pension benefits will always be lower than the maximum pension benefit limit.

⁴ This link between assets and liabilities is in contrast to the analogy developed by Bodie and Davis (2000, p.6) comparing a pension plan to an equipment trust such as those set up by an airline to finance the purchase of airplanes. Here the equipment serves as specific collateral for the associated debt obligation. The borrowing firm's liability is not affected by the value of the collateral. So, for instance, if the market value of the equipment were to double, this would greatly increase the security of the promised payments, but it would not increase their size. Opposed to this, in the scheme developed in this paper, the value of the assets may well affect the liabilities as a high return in a given year may increase the value of the liabilities as outlined above.

⁵ A drawback of the Ornstein/Uhlenbeck process is the theoretically positive probability of negative nominal interest rates. This drawback is eliminated in the simulation procedure by cutting-off the negative nominal interest rates.

⁶ The EIU forecasts are constructed with the aid of an econometric world model, maintained by the UK based Oxford Economic Forecasting.