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Modeling Asset Returns: A Comparison of Theoretical and Empirical Models

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Non-Technical Summary

This paper presents and compares three classes of time-series models for returns of broad-based stock indices: empirical, classical and behavioral models.

Empirical models usually have no economic foundation and have been derived from a purely statistical reasoning. As a representative of this class the NGARCH (1,1)-in-mean model is used. This model assumes, that the (time-varying) volatility is the only factor that influences the stock returns.

Classical economic return models are consistent with an equilibrium with rational expectations and von Neumann-Morgenstern utility functions. We propose a time-series model which is consistent with a representative investor with a general HARA utility function. This type of utility function allows for decreasing and increasing (relative) risk aversion. Thus, for example, an investor with decreasing relative risk aversion likes to invest more in risky assets when his wealth is higher.

Finally, recent experimental studies provide evidence that people do not act rationally and their choices often do not seem consistent with von Neumann-Morgenstern utility functions. One of these behavioral phenomena is loss aversion. This means, that risk aversion of an investor is not constant but increases for negative stock returns. The classical model is augmented to account for such behavioral aspects.

All models are tested and compared in a consistent empirical framework. We find that the standard NGARCH (1,1)-in-mean model performs well. However, the augmented model derived from an equilibrium model performs better for some countries (Germany, Japan). For these two countries decreasing (relative) risk aversion has been found, whereas for the other countries (France, UK, USA) the relative risk aversion is constant.

But the behavioral components are not significant and, thus, do not improve the model performance.

Future research should be devoted to the derivation of economically founded and empirically tractable time-series models. This paper has shown, that equilibrium return models can lead to interesting time-series models which also empirically outperform ad-hoc specifications.

Modeling Asset Returns:
A Comparison of Theoretical and Empirical Models

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Abstract

This paper presents and compares several time-series models for returns of broad-based stock indices. These models nest a nonlinear asymmetric GARCH (NGARCH) model as a special case. Some of these models are empirically motivated ad-hoc specifications others are derived from a representative investor economy with HARA-utility and some are behavioral, i.e. are based on recent findings in behavioral finance. To compare these models we use the inflation adjusted MSCI total return indices of 5 large economies, USA, United Kingdom, Germany, France and Japan. The empirical results show that although the pure NGARCH model performs well, the estimation for two indices could be significantly improved by an extension which follows from the representative investor model with HARA-utility.

JEL Classification: G12, G15, C22

Key Words: asset pricing, HARA-utility function, behavioral finance, NGARCH-in-mean.

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Introduction[*]

Although over the past decades we have observed a tremendous interest in new time-series models for asset returns, the geometric Brownian motion might still be considered as the predominant model for stock prices. This is on the one hand certainly due to the fact that the Black-Scholes option pricing model is derived from the assumption that the underlying asset is governed by a geometric Brownian motion, but on the other hand the geometric Brownian motion is also empirically tractable and it has a sound economic foundation. This makes it very interesting for researchers to use. However, there is empirical evidence against the geometric Brownian motion and more flexible models perform much better. In empirical applications researchers often use "statistical models". These purely empirically motivated time-series models as (G)ARCH models or empirically motivated multi-factor models usually fit the data better. However, these stochastic processes do not only lack an equilibrium foundation but some of them have been even proved to be inconsistent with such an equilibrium. The so-called viability discussion (see, e.g. Bick, 1990 and He and Leland, 1993) has shown that widely used time-series models as the Ornstein-Uhlenbeck process and the constant elasticity of variance model, originally proposed by Cox and Ross (1976), are not viable models of the market portfolio in a representative investor economy. Other empirically motivated time-series models as the long-memory processes are in general not arbitrage-free (see, e.g. Beran, 1994, Beran, 1999, Rogers, 1997, and Beran et al., 2002). Recently, research focusing on behavioral aspects of asset pricing has been very popular. These models try to explain asset return characteristics by deviations from the classical assumption of rational expectations and von Neumann-Morgestern utility functions. Kahneman and Tversky (1979), and more recently Benartzi and Thaler (1995), Barberis and Huang (2001) and Barberis, Huang and Santos (2001) argue in favor of *loss aversion*. But also mental accounting, overconfidence and many other "behavioral explanations" for asset return characteristics are used. Thus, the current state of the literature leaves us basically with the choice between three classes of models: empirical, classical and behavioral.

This paper does not attempt to resolve this problem by providing a unifying model but we aim to bridge the gap between these classes of models. We start by deriving in a representative investor economy with rational expectations and HARA-utility an extension to the geometric Brownian motion. We compare this model to a standard

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NGARCH-in-mean model[1] and to some behavioral ad-hoc specifications. These behavioral models capture the loss aversion phenomenon by relating the risk premium, and hence the representative investor's risk aversion, to past returns, i.e. recent gains and losses.

The empirical evidence against the geometric Brownian motion as a model for index returns and returns of individual stocks is compelling. Many studies document heteroskedasticity of asset returns and the asymmetric volatility phenomenon, i.e. negative correlation between asset returns and volatility. Thus empirical studies document non constant volatility of asset returns, this indeed is inconsistent with the geometric Brownian motion as a model for asset prices.[2] Moreover, asset returns seem to be weakly predictable. Many studies document positive serial correlation of asset returns on shorter horizons and negative serial correlation on longer horizons. Related to this are the predictive power of financial ratios and the success of trading strategies as momentum strategies and the Winner-Loser Effect. Though many studies find return predictability it is still controversial whether it is economically significant or simply a statistical artifact.[3] However, there is compelling empirical evidence against the geometric Brownian motion and researchers have proposed uncountable many alternative, mainly empirically motivated, time-series models. The class of (G)ARCH models has been most successful. Especially the NGARCH model captures both aspects, the persistence in return volatility and the negative correlation with asset returns.

In addition to these empirically motivated time-series models we derive an alternative equilibrium asset price process which is consistent with HARA-utility of the representative investor. This process is interesting for several reasons. First, from a theoretical point of view, it is an interesting extension of the geometric Brownian motion since it does not imply constant elasticity of the asset specific pricing kernel but is consistent with declining, constant and increasing elasticity of the asset specific pricing kernel. To say it in other words, in a representative investor economy where the asset specific pricing kernel is equal to the standardized marginal utility of the representative investor, the geometric Brownian motion implies constant relative risk aversion of the representative investor (see Bick, 1990, Franke, Stapleton and Subrahmanyam, 1999, Camara, 2001, and Camara, 2003). The extension proposed in this paper allows also for decreasing and increasing relative risk aversion. Secondly, as will be shown, this stochastic process is more

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[1] NGARCH is the abbreviation of nonlinear asymmetric GARCH. The model was developed by Engle and Ng (1993) and is due to its flexible news impact curve a suitable model for stock markets. The NGARCH-model is also often used for option pricing models, see e.g. Duan (1995).

[2] For an overview, see for example Ghysels, Harvey and Renault (1996).

[3] For an overview, see for example Cochrane (2001).

flexible than the geometric Brownian motion since it allows for non constant volatility and non constant drift. Finally, this equilibrium based stochastic process is the same as the *displaced diffusion* which has been proposed by Rubinstein (1983). However, Rubinstein did not derive the process in an equilibrium and he proposed the displaced diffusion as a model for single stocks and not as a model for the market portfolio. Hence, the derivation in this paper provides an equilibrium justification to use the displaced diffusion as an alternative model for the market portfolio. Finally, as a third class of models we consider some ad-hoc specifications. These stochastic processes account for some sort of loss aversion. In contrast to the equilibrium based models, the risk premium of these models depends also on past returns. Hence, if the market went up, i.e. investors gained, the risk premium might be different than if investors suffered losses.

We analyze the empirical performance of the models for the inflation adjusted monthly total return MSCI indices for USA, United Kingdom, Germany, France and Japan for the time period January 1972 to March 2003. We find that the standard NGARCH-in-mean model performs quite well. However, the equilibrium based displaced diffusion with a heteroskedastic error term fits better for at least two indices. Hence, based on our results we would suggest the displaced diffusion with heteroskedastic error term to model broad-based market indices and to test for significant deviations from this model. The NGARCH-in-mean model which is nested in this model seems to be a good alternative.

The organization of the paper is as follows. The following section presents the alternative time-series models. In Section 2 the data and the methodology are presented. Empirical results are shown in Section 3. Section 4 concludes.

1 Alternative Return Models

1.1 The equilibrium return model

The predominant model for asset prices *S* is the geometric Brownian motion

$$dS_t = \mu S_t dt + \Sigma S_t dW_t, \quad 0 \le t \le T, \quad S_0 > 0 \tag{1}$$

where μ and Σ are constant parameters and W is a one-dimensional standard Brownian motion defined on a filtered probability space (Ω, F, F_t, P) where $(F_t)_{t\in[0,T]}$ is the filtration generated by W augmented by all the F-null sets, with $F = F_T$. It has been shown, that this process implies constant elasticity of the asset specific pricing kernel which equals $\frac{\mu}{\Sigma^2}$.[4] In this case, the representative agent's utility function has constant relative risk aversion, i.e.

$$RRA(x) = -\left(\frac{\frac{\partial^2 U(x)}{\partial x^2}}{\frac{\partial U(x)}{\partial x}}\right) x = \text{constant}.$$

Since the utility function with constant relative risk aversion is a special case of the HARA-class, a natural generalization would be a stochastic process which is consistent with a general HARA-utility function, i.e.

$$U(x) = \frac{1-\gamma}{\gamma}\left(\frac{x}{1-\gamma} + \theta\right)^{\gamma},\ \gamma \neq 0,\ \ \gamma \neq 1,\ \theta \in \mathbb{R}. \tag{2}$$

The HARA-class of utility functions as defined in (2) is the predominant class of utility functions in financial economics.[5] Thus, if one wants to compare popular empirical time-series models to an equilibrium founded time-series model, it is natural to consider asset price processes which are consistent with equation (2).

Note that although the HARA-class of utility functions is very widespread in financial economics and its appealing property that it captures declining, constant and increasing relative risk aversion, it has the drawback that it is only defined over the domain $\frac{x}{1-\gamma} + \theta > 0$. Moreover $\gamma > 1$ implies an upper bound for wealth. This case is not very sensible in asset pricing since asset price distributions would have to be bounded from above. Hence, for the further derivation we also restrict $\gamma < 1$. In this case, we have to impose the restriction on wealth $x > -\theta(1-\gamma)$. It follows that for $\theta > 0$ (increasing relative risk aversion) and $\theta < 0$ (declining relative risk aversion) this lower bound is negative or positive, respectively. To avoid any problems of inconsistency we will therefore derive equilibrium asset prices for an information process which generates a terminal distribution of the asset price with lower bound $-\theta(1-\gamma)$. An information process I characterizes the conditional expectations of a representative investor about the value of the shares at time T. Since this process characterizes expectations, it has to be a martingale in a model with rational expectations. Starting from an information process to derive asset

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4 See for example Bick (1990).

5 To be precise, we also exclude the case of an infinite γ. This excludes also the negative exponential utility function.

prices is a parsimonious way of modeling the information structure in the economy. It avoids modeling a complete economy with assumptions on production functions and information flow (for detailed discussions of information processes see, Brennan and Xia, 2003, Franke, Stapleton and Subrahmanyam, 1999, and Lüders and Peisl, 2001). Assuming that asset prices are governed by a geometric Brownian motion implies that the information process is also governed by a geometric Brownian motion, i.e.

$$I_t = E\left(S_T \middle| F_t\right), \quad 0 \le t \le T, \tag{3}$$

is governed by

$$dI_t = I_t \sigma dW_t, \quad 0 \le t \le T,\ I_0 > 0 \tag{4}$$

where σ is a constant parameter. This stochastic process I however allows for terminal wealth $I_T = S_T \in [0, \infty)$. To avoid terminal wealth $S_T < -\theta(1-\gamma)$ we assume that investors' expectations are governed by

$$\hat{I}_t = I_t - \theta(1-\gamma), \quad 0 \le t \le T,\ \hat{I}_T = S_T \tag{5}$$

Since in this study we concentrate on broad-based total return indices we can focus on the pricing of assets without dividend payments. In this case, the price of the asset at time t is the expected future value of the asset under the equivalent martingale measure Q, discounted at the risk-free interest rate r

$$S_t = E^Q\left[\exp\left(-\int_t^{t+\tau} r ds\right) S_{t+\tau} \middle| F_t\right], \quad 0 \le t \le t+\tau \le T. \tag{6}$$

Since equation (6) is true for any $0 \le t \le t+\tau \le T$ it holds also for time T with $\hat{I}_T = S_T$,

$$S_t = E^Q\left[\exp\left(-\int_t^{T} r ds\right) \hat{I}_T \middle| F_t\right], \quad 0 \le t \le T. \tag{7}$$

Equation (7) can be rewritten in the usual pricing kernel notation. For a constant interest rate r equation (7) can then be written as

$$S_t = \exp\left(-r(T-t)\right) E\left[\hat{I}_T \Phi_{t,T} \middle| F_t\right], \quad 0 \le t \le T, \tag{8}$$

with $\Phi_{t,T} = \frac{\Phi_{0,T}}{\Phi_{0,t}}$ the forward pricing kernel and $\Phi_{0,t} = E\left(\Phi_{0,T} \middle| F_t\right)$. In a representative investor economy the pricing kernel is equal to the standardized marginal utility, i.e.

$$\Phi_{0,T} = \frac{\dfrac{\partial U(S_T)}{\partial S_T}}{E\left(\dfrac{\partial U(S_T)}{\partial S_T}\right)}. \tag{9}$$

Inserting equations (2) and (9) into equation (8) yields[6]

$$S_t = \exp\left(-r(T-t)\right)\left[Y_t \exp\left(\sigma^2 (T-t)(\gamma-1)\right) - \theta(1-\gamma)\right] \tag{10}$$

It follows from equation (10) that we have the following return-model

$$\ln\left(\frac{S_{t+1} + \left(\dfrac{\theta(1-\gamma)}{\exp\left(r\left(T-(t+1)\right)\right)}\right)}{S_t + \left(\dfrac{\theta(1-\gamma)}{\exp\left(r(T-t)\right)}\right)}\right) = r + \sigma^2(1-\gamma) - \frac{1}{2}\sigma^2 + \varepsilon_{t+1} \tag{11}$$

with $\varepsilon_{t+1} \sim N(0,1)$.

This can be rewritten as

$$\ln\left(\frac{S_{t+1} + \left(\bar{\theta}(1-\gamma)\exp(r)\right)}{S_t + \left(\bar{\theta}(1-\gamma)\right)}\right) = r + \sigma^2\left(\frac{1}{2} - \gamma\right) + \varepsilon_{t+1}$$

with $\varepsilon_{t+1} \sim N(0,1)$, (12)

and $\bar{\theta} = \dfrac{\theta}{\exp\left(r\left(T-(t+1)\right)\right)}$

Of course even this extension of the standard Black-Scholes economy to non-constant elasticity of the pricing kernel is a very simplified model. At least one would argue that the information process should have stochastic volatility as argued for example by Lüders and Peisl (2001). Introducing stochastic volatility of the

[6] A proof is given in the appendix. This result is also derived in Lüders (2002).

information process would capture the phenomenon that the information flow is more intense in certain periods than in others and hence investors' uncertainty about the fair value of the assets is higher during certain periods. A straight-forward way to account for this randomness in uncertainty would be to model the volatility term by a GARCH process. We choose the NGARCH(1,1) model since it nests the GARCH(1,1) model and it makes it possible to test for the asymmetric component i.e. the leverage effect in the volatility of stock returns. This yields the following return model

$$\ln\left(\frac{S_{t+1}+\left(\bar{\theta}(1-\gamma)\exp(r)\right)}{S_t+\left(\bar{\theta}(1-\gamma)\right)}\right)-r-\kappa_0-\sigma_{t+1}{}^2\left(\frac{1}{2}-\gamma\right)=\varepsilon_{t+1}$$

$$\text{with } \varepsilon_{t+1}\sim N(0,1) \text{ and} \tag{13}$$

$$\sigma_{t+1}{}^2=\alpha_0+\alpha_1\left(\varepsilon_t-c\sigma_t\right)^2+\beta\sigma_t{}^2$$

In the volatility equation $c \neq 0$ causes an asymmetric news impact curve.[7] In case of $c < 0$ the volatility of the stock returns exhibits a leverage effect i.e. a negative ε_t has a higher impact on σ_{t+1}^2 than a positive value. The mean equation of (13) contains, in addition, the constant term κ_0. In equation (12) the constant of the mean is represented by $\gamma\sigma^2$, but as the volatility in (13) is time-varying an additional constant term is needed.

Equations (12) and (13) are our "equilibrium return models" which we might call the HARA-processes or displaced diffusions.[8] Recent research has more and more questioned the assumptions of von Neumann-Morgenstern utility functions and rational expectations. Therefore, in the following section we present several specifications which take into account some of these behavioral patterns.

1.2 Behavioral models

Many articles have recently argued in favor of loss aversion, i.e. people behave differently if they have suffered losses recently. Hence, their risk attitude does not

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[7] See Engle and Ng (1993).

[8] Note that due to discounting, our displaced diffusion differs slightly from the version derived by Rubinstein (1983). Note also that Camara (1999) discusses option prices in a one-period model when the underlying is three-parameter lognormally distributed. Though Camara (1999) does not consider stochastic processes in this one period model, the underlying is also governed by a displaced diffusion since this generates a three-parameter lognormally distributed asset price.

only depend on their wealth but also on the change in their wealth level.[9] One way to account for such a behavior would be to let the required risk premium depend negatively on past returns. Hence, if the market has raised recently, investors (the representative investor) made profits and thus might be more willing to take further risks, i.e. the representative investor is less risk averse. Also, if investors faced losses recently, they require a higher risk premium to take risks which implies that the representative investor is more risk averse. This motivates to model the risk premium μ_t as $\mu_t = \left[\gamma_1 + \kappa_1 \ln\left(\frac{S_t}{S_{t-1}}\right) + \kappa_2 \left(\ln\left(\frac{S_t}{S_{t-1}}\right)\right)^2 \right]$.

If κ_1 is negative, then the drift is smaller after positive past returns and higher after negative past returns. The quadratic term is included to capture potential nonlinear effects. γ_1 represents the constant part of the risk premium which is equal to $(0.5-\gamma)$ in the equations (12) and (13). We add this "behavioral component" to our two return models from the previous section, i.e. equation (12) and equation (13). This yields the following two models, where we can, in addition, test for the "behavioral components", i.e. $\kappa_1 \neq 0, \kappa_2 \neq 0$:

$$\ln\left(\frac{S_{t+1} + \left(\overline{\theta}(1-\gamma)\exp(r)\right)}{S_t + \left(\overline{\theta}(1-\gamma)\right)}\right) - r - \left[\gamma_1 + \kappa_1 \ln\left(\frac{S_t}{S_{t-1}}\right) + \kappa_2 \left(\ln\left(\frac{S_t}{S_{t-1}}\right)\right)^2\right]\sigma^2 = \varepsilon_{t+1} \qquad (14)$$

with $\varepsilon_{t+1} \sim N(0,1)$.

$$\ln\left(\frac{S_{t+1} + \left(\overline{\theta}(1-\gamma)\exp(r)\right)}{S_t + \left(\overline{\theta}(1-\gamma)\right)}\right) - r - \kappa_0 - \left[\gamma_1 + \kappa_1 \ln\left(\frac{S_t}{S_{t-1}}\right) + \kappa_2 \left(\ln\left(\frac{S_t}{S_{t-1}}\right)\right)^2\right]\sigma_{t+1}{}^2 = \varepsilon_{t+1}$$

with $\varepsilon_{t+1} \sim N(0,1)$ and (15)

$$\sigma_{t+1}{}^2 = \alpha_0 + \alpha_1\left(\varepsilon_t - c\sigma_t\right)^2 + \beta\sigma_t{}^2$$

Hence, our behavioral model (15) nests the models (12), (13) and (14). Finally, if $\overline{\theta}$ is zero, model (15) simplifies to

[9] See Barberis and Huang (2001) and Barberis, Huang and Santos (2001).

$$\ln\left(\frac{S_{t+1}}{S_t}\right)-r-\kappa_0-\left[\gamma_1+\kappa_1\ln\left(\frac{S_t}{S_{t-1}}\right)+\kappa_2\left(\ln\left(\frac{S_t}{S_{t-1}}\right)\right)^2\right]\sigma_{t+1}{}^2=\varepsilon_{t+1}$$

$$\text{with } \varepsilon_{t+1}\sim N(0,1) \text{ and} \qquad (16)$$

$$\sigma_{t+1}{}^2=\alpha_0+\alpha_1\left(\varepsilon_t-c\sigma_t\right)^2+\beta\sigma_t{}^2$$

which would be the purely "behavioral model".

1.3 Empirical models

In the last two sections we proposed several behavioral respectively equilibrium models. In empirical research on asset pricing, statistical models for return processes have been prevalent. Especially the (G)ARCH and (G)ARCH-in-mean models are very common and researchers have been quite successful in fitting these models to asset returns. As, for example, the exponential GARCH (EGARCH) of Nelson (1991), the asymmetric GARCH (AGARCH) (see Engle and Ng (1993)) and the GJR-GARCH (see Glosten, Jagannathan and Runkle (1993)) the NGARCH-model is particularly designed to model an asymmetric behavior of volatility with regard to return innovations, particularly the so called leverage effect. Therefore we also fit a pure NGARCH(1,1)-in-mean model

$$\ln\left(\frac{S_{t+1}}{S_t}\right)-r-\kappa_0-\gamma_1\sigma_{t+1}{}^2=\varepsilon_{t+1}$$

$$\text{with } \varepsilon_{t+1}\sim N(0,1) \text{ and} \qquad (17)$$

$$\sigma_{t+1}{}^2=\alpha_0+\alpha_1\left(\varepsilon_t-c\sigma_t\right)^2+\beta\sigma_t{}^2$$

which we will also compare to the models proposed in the two previous sections. In section 3 we report inter alia the results of Likelihood Ratio-tests that compare the six models (12)-(17) and show which of them has the best fit regarding the empirical behavior of the inflation adjusted stock indices.

2 Data and Methodology

For the estimations we use inflation adjusted stock indices to measure the real wealth of the representative investor.[10] An economic justification for the use of the real country stock indices as a representation of the real wealth of the investors is the

[10] Approximating representative investor's wealth by the value of a broad based stock index is very common, see e.g. also Rosenberg and Engle (2002) who use the S&P500-index. For recent critical discussions of this approximation see Camara (2001) and Camara (2003).

well-known home bias, which means that investors prefer domestic stocks to international stocks and, thus, hold most of their equity portfolio in domestic assets.[11] The stock indices are the country indices of MSCI (Morgan Stanley Capital International Inc.) for France, Germany, Japan, UK and US. Hence, we consider the five largest stock markets in the world. All indices are total return indices and, thus, include all cash flows, for example dividends, paid to the investor. The real stock index S_t is defined as $S_t = MSCI_t / CPI_t$, where *MSCI* indicates the nominal country stock index and *CPI* is the seasonally adjusted country specific consumer price index from the OECD. CPI_t is equal to 1.0 in the first period (January 1972). As risk free nominal interest rate we use the money market rate from the IMF.[12] This interest rate has been converted into real terms (r_t) using the ex-post consumer price inflation to the same month one year before. All estimations start in January 1972 and end in March 2003 which means that we use 375 months for each country.

The equations (12) to (17) are estimated using maximum likelihood (ML). As the tables in the appendix reveal the distribution of the residuals is in all countries and all models significantly different from the normal distribution.[13] Nevertheless, we assume normally distributed residuals in the ML-estimation and apply a pseudo- or quasi-ML estimation and calculate robust asymptotic covariance matrices.[14]

In (12)-(15) the mean equations cannot be expressed in the usual form $y=f(x,\beta)$, where the dependent variable y is a function of some exogenous variables x and the parameter vector β. Instead, the mean equation can only be expressed as $g(y,\omega)=f(x,\beta)$, where $g(\cdot) = \ln[S_{t+1} + \left(\overline{\theta}\left(1-\gamma\right)\exp\left(r\right)\right)]$. Therefore, to receive unbiased estimates the likelihood equation has to be augmented by the following Jacobian term J_t:[15]

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11 See e.g. Carmichael and Coen (2003) for the latest developments in this field of research.

12 International Financial Statistics, line 60b.

13 The tables in the appendix exhibit the results of the Jarque-Bera test applied to the standardized residuals.

14 See e.g. Greene (2000), chapter 11.5.6. According to Weiss (1986) a quasi-ML estimation leads to a consistent estimation of the parameters if the equations for the (conditional) means and variances are specified correctly. But as this estimator is inefficient in case of non-normal standardized residuals some authors choose a distribution that takes leptokurtosis explicitly into account, as e.g. the standardized multivariate t-distribution. However, when a distribution different from the normal distribution is used and this distribution is not the true distribution then the estimates are in most cases not consistent (see Newey and Steigerwald (1997)). Therefore, we prefer to apply the (conditional) normal distribution.

15 See e.g. Greene (2000), chapter 10.3.1.

$$J_t(y,\omega) = \left|\frac{\partial g(\cdot)}{\partial y}\right| = \left|\frac{\partial g(\cdot)}{\partial(\ln S_{t+1})}\right| = \left|\frac{S_{t+1}}{S_{t+1} + \left(\bar{\theta}\left(1-\gamma\right)\exp\left(r\right)\right)}\right|.$$

For the models (16) and (17) the dependent variable can be expressed directly by $\ln S_{t+1}$ and, therefore, $J(\cdot) = 1$. The likelihood equation to be maximized in the case of a time-varying conditional variance (models (13), (15)-(17)) is:

$$\ln L = \sum_{t=1}^{T} \ln L_t \ \text{ with } \ln L_t = -\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln \sigma_t^2 + \ln J_t - \frac{(\varepsilon_t)^2}{2\sigma_t^2},$$

where *T* is the total number of observations. In case of a constant variance i.e. for the models (12) and (14) σ_t^2 has to be replaced by σ^2.

3 Empirical Results

3.1 Definitions and structure of the tables in the appendix

The results of the estimations are reported in detail in the tables (1) – (5). Each of the five tables is divided into two parts. Part a) shows the parameter estimates of the models (12)–(17) and some tests regarding important characteristics of the standardized residuals, namely, tests on normality, tests on ARCH-effects and tests on autocorrelation. The estimated parameters of the mean are: κ_0, κ_1, κ_2, $\gamma_1 = (0.5-\gamma)$ and $\theta_1 = \bar{\theta}(1-\gamma)$. In equation (12) and equation (14) the term $\gamma_1\sigma^2$ represents the constant of the mean equation and therefore no additional κ_0 is estimated. γ_1 is the "classical" part of the risk premium whereas κ_1 and κ_2 are the "behavioral", autoregressive parts. θ_1 indicates the lower bound of the asset price distribution. If $\theta_1 < 0$ the lower bound is positive (see section 1.1). The sign of θ_1 gives information about the elasticity of the asset pricing kernel. For example, $\theta_1 < 0$ implies declining elasticity of the pricing kernel if $\gamma < 1$.

α_0, α_1, β and c are the parameters of the variance equation. In the models (12) and (14) only the constant variance α_0 is estimated. A significant parameter *c* indicates an asymmetric reaction of the volatility on negative and positive stock return innovations. In case of nominal stock returns the literature reports a negative *c* which measures the leverage effect i.e. a stronger impact of negative return innovations. As we estimate a model for real stock returns this need no longer be the case. *Like* gives the value of the likelihood function in the maximum. The pseudo R^2 compares the explanatory power of the models with a basic model that only consists of a constant mean and a constant variance. The pseudo R^2 is defined as $1-\frac{Like(Basis)}{Like(Model)}$ which guarantees that the value is between zero and one.

The Jarque-Bera (JB) test investigates the hypothesis that the (standardized) residuals are normally distributed. The ARCH tests use one or four lagged squared residuals and are equal to the test of Engle (1982). The null hypothesis is "no ARCH effects". In addition, the results of the Ljung-Box (LB)-Q test using one and four lagged residuals are reported. Here, the null hypothesis is "no autocorrelation". For all of these tests the table shows the p-value in percent. *Like* is the basis for the likelihood ratio (LR) tests reported in part b) of the tables. For the LR tests the following relationships between the models are used (applying the equation numbering of section 1):

$$12 \subset 14 \subset 15;\;\; 12 \subset 13 \subset 15;\;\; 17 \subset 16 \subset 15;\;\; 17 \subset 13$$

This means that, for example, the model (12) is nested in model (14) and both are nested in model (15). The other rows are to be interpreted similarly.

The test statistic of the LR-tests is calculated as $2*[Like(Model\ A)-Like(Model\ B)]$ which is χ^2_n- distributed with *n* degrees of freedom. *Model B* is nested in *Model A* and *n* is equal to the restrictions of *Model B* compared to *Model A*. Thus, the LR-tests investigate whether the restrictions of *Model B* are rejected. The null hypothesis is that both models have the same explanatory power.

3.2 Interpretation of the empirical results

Which of the proposed models performs best and what are the conclusions for future empirical research? The LR-tests reported in part b) of the tables show two different results. For US, UK and France it turns out that model (17), the pure NGARCH (1,1)-in-mean model, is the best one. Although model (13) has a slightly (but not significantly) higher explanatory power, model (17) is more parsimonious and should therefore be chosen for these countries. For Germany and Japan, in contrast, model (13) is clearly the best one and dominates all other models. The reason for this result is that the threshold parameter θ_1 is highly significant for Germany and Japan but not for the US and France. This could be interpreted as constant relative risk aversion for the two latter countries. For Germany and Japan θ_1 is significantly negative indicating a decreasing relative risk aversion. In the estimates for UK it turned out that θ_1 is significantly positive in the models (12)-(15), but in the model (16) and (17) where this parameter is not part of the model, γ_1 becomes significant.

Overall, the pure NGARCH-in-mean model (17) seems to be the best one for UK in terms of the LR-test.

The parameter γ_1 is in most cases (models and countries) not significantly different from zero. This is also true for κ_1 and κ_2. Thus, the real stock returns in our observation period are not characterized by a positive or time-varying risk premium: the risk premium is not significantly different from zero. An exception is only model (17) for the U.K. where γ_1 is significantly positive. Interestingly, the behavioral components of the model, κ_1 and κ_2, are also not significant and can therefore be eliminated from the models.

It is no surprise that for most countries the models with NGARCH dominate the models with constant variance. Only for the US no significant GARCH parameters could be found. Nevertheless, the models with GARCH also perform best for the US real stock return. In contrast to nominal stock returns the leverage parameter *c* is not negative but either insignificant or significantly positive which could be interpreted as a reverse leverage effect in the real returns: positive news have a stronger impact on volatility compared to negative news. There is only one exception: for Germany the models (16) and (17) exhibit a significantly negative *c*. But these two models are dominated by model (13) where *c* is not significant. Finally, note also that a significant parameter θ_1 also implies a correlation between volatility and asset returns which is the case for Germany and Japan. More precisely, the negative θ_1 induces a positive correlation between returns and volatility. Overall we can conclude that in contrast to nominal returns, real returns are if at all, then positively correlated with volatility.

The residual tests show that ARCH effects have to be considered in general by the models. But in all cases the residuals seem to exhibit no autocorrelation of order one or of order four. For all models the residual series show clear signs of deviations from the (standard) normal distribution which is due to a significant leptokurtosis. Another feature of the empirical results is that the models have only a very low explanatory power: the pseudo R^2 is between 1.3% and 3.2% for the best performing models of each country.

What can be learned from the estimations of the models (12) – (17)? The NGARCH (1,1)-in mean model (17) and the model (13) which also contains an NGARCH (1,1)-in-mean specification are clearly the dominant models. For both models the residuals are well-behaved with the only exception of leptokurtosis. The augmented model (13) is preferable for Germany and Japan. For both countries the economically derived threshold parameter θ_1 is significantly negative. Such a negative threshold parameter leads to relatively high volatility when asset prices are high, i.e. positive correlation between asset returns and volatility.

As a conclusion of these results the following modeling strategy can be derived: for the real stock returns first the augmented model (13) should be estimated and then a test on the parameter θ_1 can be applied which leads in some cases to a reduction of the model complexity (= model (17)).

4 Conclusion

This paper compares representatives of three model classes for asset returns: empirical, classical, behavioral. Empirical models usually have no economic foundation and have been derived from a purely statistical reasoning. As a representative of this class the NGARCH(1,1)-in-mean model is used. Classical economic return models are consistent with an equilibrium with rational expectations and von Neumann-Morgenstern utility functions. We propose a time-series model which is consistent with a representative investor with a general HARA utility function. Finally, recent experimental studies provide evidence that people do not act rationally and their choices often do not seem consistent with von Neumann-Morgenstern utility functions. One of these behavioral phenomenon is loss aversion. The classical model is augmented to account for such behavioral phenomenon. All models are tested. We find that the standard NGARCH(1,1)-in-mean model performs well. However, the augmented model which includes a threshold parameter derived from an equilibrium model performs better for some countries (Germany, Japan). The behavioral components do not improve the model performance. Hence, for future empirical studies, we suggest to estimate an NGARCH(1,1)-in-mean model which is augmented by theoretically derived parameters (see model (13)) and then test for the significance of these parameters (i.e. θ_1 and γ_1).

From a theoretical point of view, this study advocates in favor of an equilibrium model with general HARA utility function of the representative investor. However, the theoretical model should be augmented to an information flow which generates stochastic volatility of the information process, i.e. the representative investor's expectations. In this paper we accounted for the stochastic volatility of the information process heuristically by introducing an NGARCH (1,1) error term.

Future research should be devoted to the derivation of economically founded and empirically tractable time-series models. This paper has shown, that equilibrium return models can lead to interesting time-series models which also empirically outperform ad-hoc specifications.

Appendix A Proof of Proposition 1

Let $Y_t = \hat{I}_t + \theta(1-\gamma)$, then $Y_T = \hat{I}_T + \theta(1-\gamma) = S_T + \theta(1-\gamma)$. It follows from Ito's Lemma that $dY_t = Y_t \sigma dW_t$, $0 \le t \le T$. Furthermore, we have that $\hat{I}_T$ is three-parameter lognormally distributed with threshold $-\theta(1-\gamma)$ and that Y_T is two-parameter lognormally distributed. Using equations (2), (8) and (9) we get

$$S_t = \exp(-r(T-t)) \frac{E\left(S_T \frac{\partial U(S_T)}{\partial S_T} \middle| F_t \right)}{E\left(\frac{\partial U(S_T)}{\partial S_T} \middle| F_t \right)} = \exp(-r(T-t)) \left[\frac{E\left(Y_T^{\gamma} \middle| F_t\right)}{E\left(Y_T^{\gamma-1} \middle| F_t\right)} - \theta(1-\gamma) \right] \quad \text{(A.1)}$$

Since Y_T is lognormally distributed, this yields

$$\begin{aligned} S_t &= \exp(-r(T-t)) \left[\frac{Y_t^{\gamma} \exp\left(\frac{1}{2}\sigma^2 (T-t)(\gamma^2-\gamma) \right)}{Y_t^{\gamma-1} \exp\left(\frac{1}{2}\sigma^2 (T-t)\left((\gamma-1)^2-(\gamma-1)\right) \right)} - \theta(1-\gamma) \right] \\ &= \exp(-r(T-t)) \left[Y_t \exp\left(\sigma^2 (T-t)(\gamma-1)\right) - \theta(1-\gamma) \right] \end{aligned} \quad \text{(A.2)}$$

Applying Ito's Lemma yields

$$dS_t = \left\{ r + \sigma^2(1-\gamma)\left(S_t + \theta(1-\gamma)\right) \right\} dt + \sigma\left(S_t + \theta(1-\gamma)\right) dW_t, \quad 0 \le t \le T, \; S_T = \hat{I}_T. \text{(A.3)}$$

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Table 1: USA

a) Model Comparison: Parameter Estimates and Model Characteristics

	Equation No.					
	(12)	(13)	(14)	(15)	(16)	(17)
κ_0	--	0.0046	--	-0.0039	-0.005	0.0043
κ_1	--	--	9.20	24.48	27.76	--
κ_2	--	--	-257.0	-15.43	-6.17	--
γ_1	1.096	-1.005	1.60	3.29	3.90	-0.961
θ_1	5.78	-3.70	5.71	-3.90	--	--
α_0	0.00196***	0.00128***	0.00195***	0.00135***	0.0013***	0.0012***
α_1	--	0.029	--	0.012	-0.0035	0.0197
β	--	0.09	--	0.098	0.095	0.088
c	--	1.839	--	1.70	1.72	1.85
Like	963.4	976.19	963.96	976.41	976.32	976.11
R^2 (in %)	0.028	1.34	0.086	1.36	1.35	1.33
JB test	0.00	0.00	0.00	0.00	0.00	0.00
ARCH (1)	2.05	55.3	13.4	43.1	46.9	58.5
ARCH (4)	13.4	89.6	29.1	84.1	85.0	89.8
LB-Q (1)	60.5	75.3	91.3	76.7	74.5	74.6
LB-Q (4)	81.9	92.7	82.8	89.8	88.1	91.5

Notes: Significance level: *, **, *** = 10%, 5%, 1%, respectively. Like = maximum value of the likelihood function. JB test = Jarque-Bera test, ARCH = test on ARCH effects for one or four lagged squared residuals, LB-Q = Ljung-Box Q test on autocorrelation for one or four lagged residuals. The results for the tests report the p-value in %.

b) Model Comparison: Likelihood-Ratio Tests of bilateral Relationships: Is Model (B) equal to Model (A)?

Test: (A) vs. (B)	Test Statistic	Degrees of Freedom
(13) vs. (12)	25.59***	4
(14) vs. (12)	1.12	2
(15) vs. (12)	26.02***	6
(15) vs. (14)	24.90***	4
(15) vs. (13)	0.43	2
(15) vs. (17)	0.61	3
(15) vs. (16)	0.19	1
(16) vs. (17)	0.42	2
(13) vs. (17)	0.18	1

Notes: Significance level: *, **, *** = 10%, 5%, 1%, respectively. Likelihood-Ratio tests for the null hypothesis model (A) = model (B), where model (B) is nested in (A).

Table 2: Japan

a) Model Comparison: Parameter Estimates and Model Characteristics

	Equation No.					
	(12)	(13)	(14)	(15)	(16)	(17)
κ_0	--	0.037***	--	0.07***	0.053***	0.053***
κ_1	--	--	21.35	-28.4	-7.44	--
κ_2	--	--	62.70	160.1	264.41**	--
γ_1	0.50	-8.23**	0.26	-17.16***	-19.15***	-18.12***
θ_1	-40.69***	-47.41***	-40.88***	-47.87***	--	--
α_0	0.0041***	0.0014**	0.004***	0.0013**	0.00038**	0.00034***
α_1	--	0.44**	--	0.46**	0.78***	0.80***
β	--	0.05	--	0.05*	0.034*	0.023
c	--	1.95	--	1.93*	1.30	1.52
Like	906.17	917.54	907.21	919.6	914.14	912.52
R^2 (in %)	0.59	1.82	0.70	2.05	1.46	1.28
JB test	0.00	0.52	0.00	0.22	0.00	0.00
ARCH (1)	0.22	99.8	0.13	55.7	75.0	54.6
ARCH (4)	1.80	75.2	0.76	69.9	68.1	45.1
LB-Q (1)	19.5	91.1	88.1	54.9	47.6	80.9
LB-Q (4)	52.0	99.8	86.7	69.2	72.6	84.8

Notes: Significance level: *, **, *** = 10%, 5%, 1%, respectively. Like = maximum value of the likelihood function. JB test = Jarque-Bera test, ARCH = test on ARCH effects for one or four lagged squared residuals, LB-Q = Ljung-Box Q test on autocorrelation for one or four lagged residuals. The results for the tests report the p-value in %.

b) Model Comparison: Likelihood-Ratio Tests of bilateral Relationships:
Is Model (B) equal to Model (A)?

Test: (A) vs. (B)	Test Statistic	Degrees of Freedom
(13) vs. (12)	22.74***	4
(14) vs. (12)	2.08	2
(15) vs. (12)	26.94***	6
(15) vs. (14)	24.85***	4
(15) vs. (13)	4.20	2
(15) vs. (17)	14.23***	3
(15) vs. (16)	11.0***	1
(16) vs. (17)	3.24	2
(13) vs. (17)	10.0***	1

Notes: Significance level: *, **, *** = 10%, 5%, 1%, respectively. Likelihood-Ratio tests for the null hypothesis model (A) = model (B), where model (B) is nested in (A).

Table 3: UK

a) Model Comparison: Parameter Estimates and Model Characteristics

	Equation No.					
	(12)	(13)	(14)	(15)	(16)	(17)
κ_0	--	-0.004	--	-0.003	-0.014**	-0.0086
κ_1	--	--	17.62	9.58	10.63	--
κ_2	--	--	184.83**	38.15	-22.08	--
γ_1	1.67	3.07	0.92	2.56	5.61**	3.69**
θ_1	59.84***	26.60**	58.6***	30.84**	--	--
α_0	0.0018***	0.0006***	0.0018***	0.0005**	0.0008***	0.0008***
α_1	--	0.62***	--	0.63***	0.54***	0.58***
β	--	0.10*	--	0.084	0.13**	0.12**
c	--	0.79*	--	0.87	0.78**	0.84**
Like	885.34	892.16	887.33	892.57	891.19	890.82
R^2 (in %)	3.06	3.80	3.28	3.85	3.70	3.66
JB test	0.00	0.00	0.00	0.00	0.00	0.00
ARCH (1)	2.69	93.3	6.8	84.6	52.7	80.0
ARCH (4)	22.8	98.6	35.0	98.4	94.3	98.0
LB-Q (1)	26.4	35.1	72.4	69.8	46.0	23.8
LB-Q (4)	32.9	49.7	29.9	55.1	32.7	33.6

Notes: Significance level: *, **, *** = 10%, 5%, 1%, respectively. Like = maximum value of the likelihood function. JB test = Jarque-Bera test, ARCH = test on ARCH effects for one or four lagged squared residuals, LB-Q = Ljung-Box Q test on autocorrelation for one or four lagged residuals. The results for the tests report the p-value in %.

b) Model Comparison: Likelihood-Ratio Tests of bilateral Relationships: Is Model (B) equal to Model (A)?

Test: (A) vs. (B)	Test Statistic	Degrees of Freedom
(13) vs. (12)	13.63***	4
(14) vs. (12)	3.97	2
(15) vs. (12)	14.46**	6
(15) vs. (14)	10.49**	4
(15) vs. (13)	0.82	2
(15) vs. (17)	3.5	3
(15) vs. (16)	2.77*	1
(16) vs. (17)	0.73	2
(13) vs. (17)	2.68	1

Notes: Significance level: *, **, *** = 10%, 5%, 1%, respectively. Likelihood-Ratio tests for the null hypothesis model (A) = model (B), where model (B) is nested in (A).

Table 4: Germany

a) Model Comparison: Parameter Estimates and Model Characteristics

	Equation No.					
	(12)	(13)	(14)	(15)	(16)	(17)
κ_0	--	0.0159	--	0.002	0.008	0.009
κ_1	--	--	6.12	0.63	3.24	--
κ_2	--	--	-159.8*	-79.8	-37.12	--
γ_1	0.55	-2.35	1.07	0.68	-1.78	-2.3
θ_1	-36.23***	-35.70***	-35.8***	-36.26***	--	--
α_0	0.0055***	0.02***	0.0054***	0.002***	0.0004***	0.0004***
α_1	--	0.483***	--	0.471***	0.72***	0.72***
β	--	0.15**	--	0.162**	0.17***	0.17***
c	--	0.041	--	0.002	-0.27*	-0.25**
Like	895.28	903.29	896.89	903.66	896.5	896.28
R^2 (in %)	2.02	2.89	2.19	2.93	2.15	2.13
JB test	0.00	0.00	0.00	0.00	0.00	0.00
ARCH (1)	0.05	73.7	0.4	99.9	57.6	45.0
ARCH (4)	0.37	89.1	1.2	87.9	93.3	88.1
LB-Q (1)	39.6	44.8	93.1	64.2	22.6	10.3
LB-Q (4)	45.2	53.1	60.3	50.6	50.1	31.7

Notes: Significance level: *, **, *** = 10%, 5%, 1%, respectively. Like = maximum value of the likelihood function. JB test = Jarque-Bera test, ARCH = test on ARCH effects for one or four lagged squared residuals, LB-Q = Ljung-Box Q test on autocorrelation for one or four lagged residuals. The results for the tests report the p-value in %.

b) Model Comparison: Likelihood-Ratio Tests of bilateral Relationships: Is Model (B) equal to Model (A)?

Test: (A) vs. (B)	Test Statistic	Degrees of Freedom
(13) vs. (12)	16.03***	4
(14) vs. (12)	3.22	2
(15) vs. (12)	16.77**	6
(15) vs. (14)	13.55***	4
(15) vs. (13)	0.74	2
(15) vs. (17)	14.76***	3
(15) vs. (16)	14.32***	1
(16) vs. (17)	0.44	2
(13) vs. (17)	14.02***	1

Notes: Significance level: *, **, *** = 10%, 5%, 1%, respectively. Likelihood-Ratio tests for the null hypothesis model (A) = model (B), where model (B) is nested in (A).

Table 5: France

a) Model Comparison: Parameter Estimates and Model Characteristics

	Equation No.					
	(12)	(13)	(14)	(15)	(16)	(17)
κ_0	--	0.014	--	0.037	0.048*	0.0164*
κ_1	--	--	25.85*	-27.2	-34.84	--
κ_2	--	--	-76.38	16.3	1.85	--
γ_1	0.70	-3.4	0.88	-10.61	-12.32	-3.47
θ_1	9.49	11.01	9.39	8.89	--	--
α_0	0.0034***	0.0022***	0.0034***	0.0026***	0.003***	0.0027***
α_1	--	0.0073	--	-0.097	-0.131	-0.041
β	--	0.158*	--	0.14*	0.131**	0.146*
c	--	1.11**	--	1.26**	1.34**	1.24**
Like	850.73	862.91	852.65	863.11	862.44	862.01
R^2 (in %)	0.11	1.52	0.33	1.53	1.46	1.41
JB test	0.00	11.4	0.00	15.6	12.6	9.4
ARCH (1)	6.7	72.2	13.7	98.2	93.6	71.6
ARCH (4)	0.7	32.2	0.48	20.1	18.3	24.0
LB-Q (1)	6.3	44.9	94.0	63.0	72.2	58.9
LB-Q (4)	22.7	43.9	64.1	51.4	55.0	49.4

Notes: Significance level: *, **, *** = 10%, 5%, 1%, respectively. Like = maximum value of the likelihood function. JB test = Jarque-Bera test, ARCH = test on ARCH effects for one or four lagged squared residuals, LB-Q = Ljung-Box Q test on autocorrelation for one or four lagged residuals. The results for the tests report the p-value in %.

b) Model Comparison: Likelihood-Ratio Tests of bilateral Relationships: Is Model (B) equal to Model (A)?

Test: (A) vs. (B)	Test Statistic	Degrees of Freedom
(13) vs. (12)	24.34***	4
(14) vs. (12)	3.82	2
(15) vs. (12)	24.73***	6
(15) vs. (14)	20.90***	4
(15) vs. (13)	0.384	2
(15) vs. (17)	2.17	3
(15) vs. (16)	1.31	1
(16) vs. (17)	0.86	2
(13) vs. (17)	1.79	1

Notes: Significance level: *, **, *** = 10%, 5%, 1%, respectively. Likelihood-Ratio tests for the null hypothesis model (A) = model (B), where model (B) is nested in (A).