Rates of Return of the German PAYG System - How they can be measured and how they will develop

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ABSTRACT

With the adoption of the latest German pension reform in spring 2004 a public debate arose on whether rates of return for future pensioner cohorts were threatened to become negative as a result of the new reform. In order to make the system sustainable, the reform had restricted future rises in the contribution rate at the expense of further decreases in pension levels.

The paper contributes to this ongoing discussion by providing (1) a thorough discussion on the appropriate measurement of rates of return of the German public pension system and (2) projections of the rates of return for future pensioner cohorts based on the German public pension system after the 2004 reform. It is found that under realistic assumptions of future demographic and labour market developments, rates of return will be lower than for present retirees, but remain positive.

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By Christina Benita Wilke

1 Introduction

In summer 2004, in the aftermath of the latest German pension reform, the chair of the Federal Constitutional Court, Hans-Jürgen Papier, made an official statement that the constitutional conformity of the German public pension system would be threatened should future rates of return become negative for younger cohorts.[1] The statement referred to the previous public debate that had reflected fears that the newly introduced sustainability factor[2] would decrease pension levels to such an extent that future cohorts will receive benefits that are smaller than their contributions.

This raises two questions. The first one is obvious: How high are future rates of return essentially going to be? Calculations based on the new reform measures were first made by Ohsmann and Stolz (2004) and then also taken up by the Sozialbeirat (2004) in the context of the 2004 pension insurance report of the government and by the Sachverständigenrat (2004) in its annual report. They find that future rates of return will remain clearly positive. In contrast, Raffelhüschen (2005) as well as Ottnad and Wahl (2005) calculate (partly) negative rates of return for younger cohorts, based on very different assumptions.[3]

The second question is: What is the appropriate calculation method to obtain cohort-specific rates of return? The common approach in the German pension literature is to calculate rates of return for different demographic groups (single men, single women and married couples) and different scenarios (retirement at the statutory retirement age, retirement at earlier ages etc.). This so called "scenario-based approach" was used by all of the above named studies.

The scenario-based view, however, does not allow to adequately consider all risks that are covered by the German pension insurance. Apart from longevity risk

[1] Die Welt (2004).
[2] For a description of the factor and its effects see Börsch-Supan, Reil-Held and Wilke (2003).
[3] The studies of Raffelhüschen (2005) and Ottnad and Wahl (2005) will be briefly discussed later on where relevant.

(covered through old-age pensions) and the risk of survival (covered through survivor pensions), this is most notably the invalidity risk[4] (covered through disability pensions) which is hard to capture within a scenario-based approach.[5] In order to reflect the whole range of insurance benefits the German pension insurance provides it is crucial to account for all risks simultaneously.

This paper therefore presents a calculation method that is based on a stochastic rather than a scenario-based, deterministic approach. It computes the rate of return of the expected payment flows (where the expectation includes longevity, survival and invalidity risk as well as the time of retirement) rather than the rate of return of a specific deterministically defined scenario. It thus allows to consider the whole range of possible scenarios simultaneously.[6] In particular, the artificial and non-representative figure of the (modified) standard pensioner[7] used for the scenario-based approach as well as for most official calculations in the German public pension system is replaced by a (weighted) average of pensioners in each cohort.[8]

The paper is structured as follows. Section 2 gives a brief introduction to the issue of rates of return in the context of the German pay-as-you-go (PAYG) system. Section 3 then looks at alternative calculation concepts and presents both the deterministic and the stochastic approach in more detail. Section 4 presents the results of the two approaches. Section 5 concludes.

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[4] Papier (1998) emphasizes that potential benefits from invalidity pensions have to be considered in the calculation of the rates of return.

[5] In general, the amount of contributions paid into the system is adjusted by a correction factor that accounts for the invalidity share of annual pension expenditures. See also section 3.2.

[6] For an older calculation of the rates of return based on a stochastic approach see Schnabel (1998).

[7] The standard pensioner is a fictive person that worked for exactly 45 years, always earned the average wage and retires at the statutory retirement age of 65 years, receiving a normal old-age pension. See also section 3.2.

[8] Papier (1998) points out that only if future rates of return become negative for the "average" or "typical" pensioner would this question the constitutional conformity of the system and the property rights on pension claims that are inherent in the German pension system. See also Sozialbeirat (2004).

2 Cohort-specific rates of return of the German PAYG system

In the German PAYG system the size of the individual pension is derived from (1) the sum of earning points[9] at the time of retirement entry $EP_{i,RE}$ that the individual accumulated during the working period, (2) the adjustment factor F_{RE} that adjusts the pension size according to the time of retirement entry[10] or the pension type respectively (e.g. widow pension) and (3) the current pension value PV_t that annually readjusts the value of the pension according to the development of average wages and internal system parameters:

$$P_{i,t} = EP_{i,RE} \times F_{RE} \times PV_t \qquad (1)$$

The size of the individual pension is thus linearly linked to the amount of contribution payments during the working life. Unlike many other pension systems, the German system of old-age pension does not redistribute within cohorts ('relative equivalence'). Differences in the rate of return across individuals of the same cohort are therefore only due to differences in life expectancy and the timing of life-time contributions.

A politically very sensitive parameter is the "pension level" defined as the monthly pension of a standard pensioner divided by the average monthly wage of an insured employee.[11] In the past, the German pension insurance recorded comparably high pension levels (70% net) that could be maintained until the end of the 1990s but were paid by substantial increases in the contribution rate. As a consequence, the rates of return of today's younger pensioner cohorts are lower than those of the

[9] The amount of earning points collected in one year is derived from the ratio of the earned wage to the average wage of all employed workers of the same year. If the average wage is earned, exactly one earning point is acquired. If the wage is only 80% of the average wage, only 0.8 earning points are acquired.

[10] Pension benefits are reduced by 0.3% for each month that the individual retires earlier then the statutory retirement age 65 and are increased by 0.5% for each month the individual retires later than 65.

[11] Note that pension levels in Germany do not correspond to the economic notion of a replacement rate which relates pension benefits to the wage immediately before retirement. The standard pensioner is defined as a person with an earnings history equivalent to 45 years at the average wage and retirement at age 65, see also section 3.2.

elderly. Due to the rise in labour costs and their potentially negative effect on economic growth, the two latest pension reforms of 2001 and 2004 limited future increases of the contribution rate while allowing for a reduction in pension levels. The resulting past and future development of contribution rates and (gross!)[12] pension levels of the German PAYG system is depicted in Figure 1.[13] The rates of return of future pensioner cohorts thus will be affected both by rising contribution rates during their contribution period and declining pension levels during their pension phase.

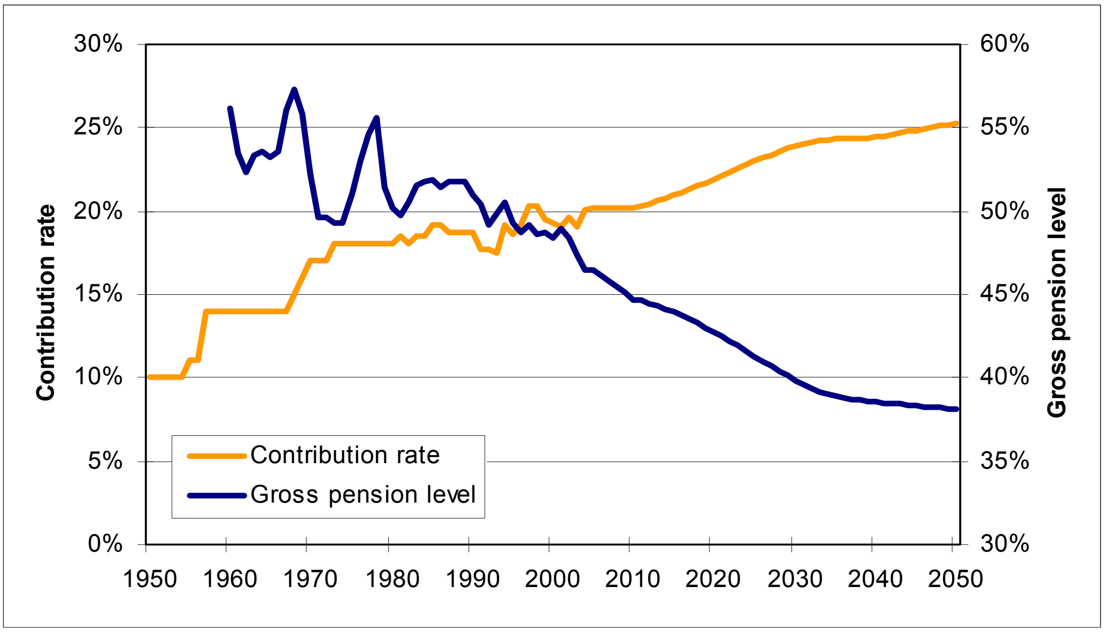

FIGURE 1: DEVELOPMENT OF CONTRIBUTION RATES AND PENSION LEVELS (1950 TO 2050)

Source: Author's calculations based on the MEA-PENSIM model.[14] Projected figures are based on the demographic and labour market forecasts used by the "Rürup Commission" (Kommission für die Nachhaltigkeit in der Finanzierung der sozialen Sicherungssysteme, 2003). Note that contribution rates turn our higher than according to the commission's report since the proposed but not yet implemented increase in the statutory retirement age is not taken into account here. In addition, simulations are run from 2005 onwards and thereby already fully incorporate the effects of the extremely low growth rates of the past two years.

[12] Note that pension levels until recently have been recorded as net pension levels in the German official statistics and are only newly recorded as gross pension levels. Since there is no data on the development of current pension values before 1960, pension levels are displayed only from 1960 on.

[13] For the rate of return calculations in the subsequent sections data on contribution rates and pension levels beyond 2050 is needed. For the purpose of this paper, it is assumed that the rates remain constant after 2050. Long-term projections – based on the underlying demographic and labour market assumptions – corroborate this trend.

[14] MEA-PENSIM is a pension simulation program that was developed at the Mannheim Research Institute for the Economics of Aging (MEA) and served among others as a cross-check for the calculations of the "Rürup Commission". For a description of the program see Wilke (2004).

3 Calculation concepts

This section starts off with the question of how rates of return of the German pension system can be measured. Thereafter, two different calculation concepts are described: the traditional, deterministic scenario-based approach and an alternative stochastic approach. Both concepts will be used for the calculation results presented in section 4.

3.1 How to measure rates of return

How can the rate of return of the German public pension system for a specific cohort be measured? In general, rates of return describe the size of gains or losses on an investment. In the context of the German PAYG system, the investment consists of the contributions paid to the system (negative payment flow) whereas the benefits received during the pension phase (positive payment flow) represent the pay-off of the investment. The rate of return can thus be measured as the proportion of the size of benefits to the size of contributions.

The internal rate of return. Since contribution payments and pension benefits occur at different points in time, the flow of positive and negative payments has to be discounted to a common date in order to make the different values of the payments comparable. In general, this is the starting point of the payment flow, which in this case is the date of entry into the labour force.[15] The rate of return can then be calculated as the so-called internal rate of return, for which the capital value of the overall payment flow is zero:

$$CV_{c,a_0} = \sum_{a=a_{RE}}^{A_c} P_{c,a} \left(\frac{1}{1+r} \right)^{a-a_0} - \sum_{a=a_0}^{a_{RE}-1} Contrib_{c,a} \left(\frac{1}{1+r} \right)^{a-a_0} = 0 \qquad (2)$$

$CV_{c,a0}$	… Capital value of the payment flow of cohort *c* at age a_0
a	… Age index
a_0	… Age of entrance into the labour force
a_{RE}	… Age at retirement entry
A_c	… Maximum age/ end of pension period of cohort *c*
$P_{c,a}$	… Pension payments to cohort *c* at age *a*
r	… Internal rate of return
$Contrib_{c,a}$	… Contribution payments by cohort *c* at age *a*

[15] For selected questions such as on early retirement incentives, it makes sense to discount the values of the payment flow to a point in time that is close to potential retirement, at which the individual can be assumed to be confronted with the decision on when exactly to retire. Such calculations can e.g. be found in Schnabel (1998). For the rate of return, however, the point in time chosen as t_0 is irrelevant as it does not change the results.

The advantage of this internal rate of return method is that there is no need to determine an appropriate discount rate that otherwise had to be specified in advance. The results are therefore independent of any reference rate.[16]

Gross versus net rates of return. Due to the gradual transition to deferred taxation which will start in 2010 and will be fully completed by 2050, net values loose their importance within the German public pension system. The rates of return calculated in this paper are therefore based on gross values where tax reductions of pension benefits and health and care insurance contributions are not accounted of. However, it should be pointed out that this transition to deferred taxation in general will have a positive effect on the size of the rates of return. Due to the German tax progression, the transition to deferred taxation will mostly lead to a larger tax relief during the working life than the additional tax burden it creates during the retirement phase as long as retirement income is lower than labour income. The resulting effect on the rates of return will therefore be positive, which means that our calculations tend to underestimate the true rates of return.

Nominal versus real rates of return. Past as well as current calculations of the rates of return for the German pension system mostly have been presented in nominal terms.[17] This is typically justified by the fact that most people are more familiar with the concept of nominal rates of return since they know it from the capital market. However, nominal rates of return are strongly biased by inflation. In Germany, inflation reached extremely high rates in the 1970s which of course deeply affects the real value of contribution payments made during this period. Figure 2 shows the development of past and future nominal and real average gross wages. If inflation is to be controlled for, real rates thus have to be looked at. In the following, nominal rates of return are therefore mainly displayed for comparison.

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[16] Note that in the calculations in this paper, neither the state subsidy to the pension system nor non-insurance benefits are considered. While they have a different impact on the rate of return for certain individuals which makes the following calculations even more complicated (e.g. mothers, and in the past low wage earners and the highly educated) the aggregate effect is neutral because the state subsidy is meant to exactly cover the non-insurance benefits. Ottnad and Wahl (2005) drop this assumption and calculate rates of return for a scenario where an increasing amount of government subsidies is used to finance ordinary insurance benefits. As expected, the resulting rates of return turn out considerably lower and partly even turn negative.

[17] An exception is Schnabel (1998, 2001) and the Sozialbeirat (2004) that presents both nominal and real rates of return.

It is important to note that future real gross wages are assumed to continue to increase at a rate of 1.5%, which means in its economic essence that annual long-run economic growth remains positive.[18] Contribution payments to the pension insurance (in real terms) thus do not remain constant but rise over a cohort's life cycle. This effect is strengthened even more by the projected increases in the contribution rate itself. Due to the compound interest effects in the internal rate of return calculations, the earlier, lower contribution payments are weighted higher which has a favourable effect on the resulting rates of return. If this effect is neglected and constant payments in real values of today are assumed over the entire contribution period, the true rate of return is thus underestimated for those cohorts with a contribution history in the past. On the other hand, real net wage growth also leads to increases in benefit payments over the retirement period and thus has a favourable effect on resulting rates of return even though later payments are again considered less. Studies that choose a simplified approach based on constant payment flows, such as the one of Raffelhüschen for the MDR Umschau (2005) should therefore be interpreted with care.

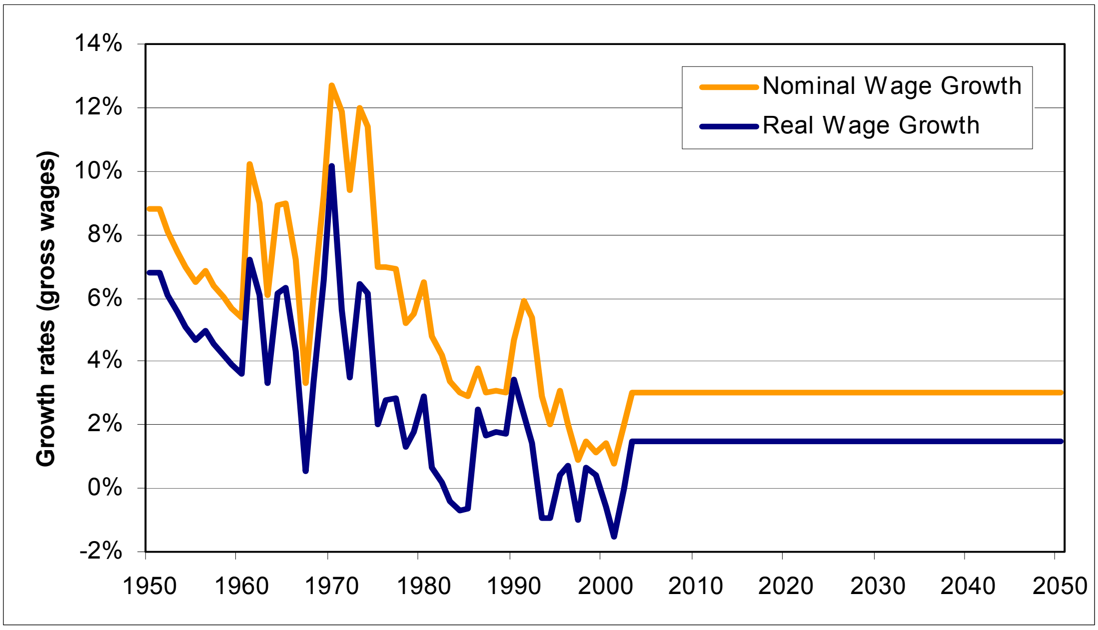

FIGURE 2: DEVELOPMENT OF NOMINAL AND REAL WAGE GROWTH (1950 TO 2050)

Source: Historical figures are taken from the National Statistical Office (www.destatis.de). The projections correspond to the assumptions by the "Rürup Commission" (Kommission für die Nachhaltigkeit in der Finanzierung der sozialen Sicherungssysteme, 2003) and assume a 3% nominal annual growth and an inflation rate of 1,5% in the long run.

[18] Even though it is unlikely that future real net growth will turn zero, some studies use this assumption as a worst case scenario, see e.g. Ottnad and Wahl (2005).

As mentioned at the outset of this paper, the annual positive and negative payment flows for the internal rate of return calculations can be computed in two fundamentally different ways. In the following, the common scenario-based approach will be explained first. Thereafter, an alternative stochastic approach will be presented. Within both approaches, rates of return will be computed for three demographic groups separately (single men, single women and married couples).

3.2 The deterministic approach

The deterministic approach takes a scenario-based view. Given a set of assumptions regarding the working life, the time of retirement entry and the applicable type(s) of pension benefits, the respective contribution and benefit history for a specific cohort is calculated (for different demographic groups) and the corresponding internal rates of return are determined. Different scenarios allow to look at different sets of assumptions. This deterministic approach clearly dominates the German pension literature[19] and was also used for the most recent calculations on the effects of the 2004 German pension reform by Ohsmann and Stolz (2004), Sozialbeirat (2004) and Sachverständigenrat (2004).

The Standard Pensioner – Male, single. The standard scenario is that of the standard pensioner. The standard pensioner is a fictive person that starts working at age 20, works for 45 years, earns in all years the average wage[20] and retires at the statutory retirement age of 65 years. Based on this fictive earnings history the standard pensioner accumulates 45 earning points. Since he retires at the statutory retirement age he is entitled to receive an old-age pension (without any reductions), the size of which is determined by the sum of earning points times the current pension value of the respective year.

[19] See e.g. Eitenmueller, S. (1996) and Eitenmüller, S. and S. Barth (1997), an earlier paper by Ohsmann and Stolz (1997) or Hain, W., Gliesmann, H. and E.-J. Horn (1998).

[20] In reality, income usually is lower than the average income of the labour force at the beginning and higher than the average wage versus the end of an individual's working life. This has an effect on the resulting rates of return, since the internal rate of return method weighs earlier contribution payments higher than later ones. In order to evaluate this effect, an age-specific earning profile was introduced that allows for a rise in wages over the working life but equally results in 45 earning points if the individual retires at the statutory retirement age. The profile was derived from the medium profile of an estimation of sector- and education-specifc wage trajectories by Fitzenberger et al. (2001) for Western Germany. The calculations show, that for a typical individual this effect is only marginal and thus can be neglected. The same applied for the stochastic approach in section 3.3.

Equation 2 can now be written as follows:

$$\sum_{a_{RE}=65}^{A_c}\left[\left(\sum_{a_0=20}^{64} EP_a\right)\times PV_{c+a}\left(\frac{1}{1+r}\right)^{a-a_0}\right]-\sum_{a_0=20}^{64} AGW_{c+a}\times\tau^{PS}{}_{c+a}\left(\frac{1}{1+r}\right)^{a-a_0}=0 \qquad (3)$$

A_c	… Now: Remaining life expectancy of cohort *c* at age 65
EP_a	… Acquired earning points at age a
PV_{c+a}	… Current pension value in year t=c+a
AGW_{c+a}	… Average gross wages in year t=c+a
$\tau^{PS}{}_{c+a}$	… Contribution rate to the pension system in year t=c+a

It is assumed that the length of the pension period corresponds to the remaining life expectancy at age 65, i.e. if the latter is 20 years, 20 years of pension benefit payment flows are considered for the calculation. Of course, this scenario of the standard pensioner can also be calculated for women. In this case the only difference lies in a higher remaining life expectancy.[21] Figure 3 shows the past and projected development of the remaining life expectancy for both men and women.

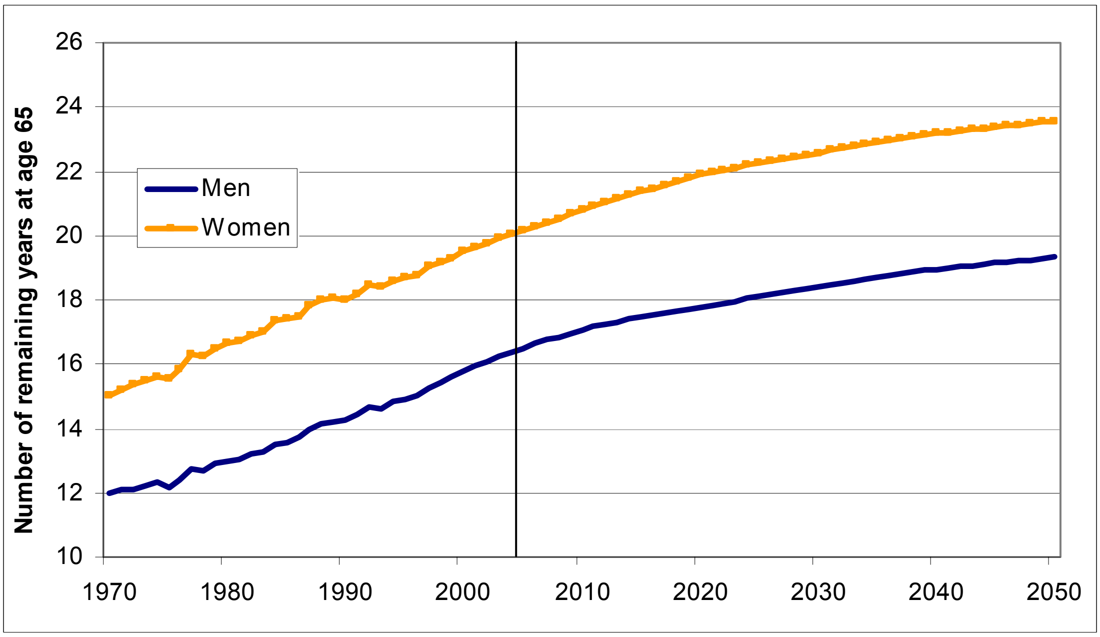

FIGURE 3: DEVELOPMENT OF THE REMAINING LIFE EXPECTANCY AT AGE 65 (1970 TO 2050)

Source: Historical figures are taken from the National Statistical Office (www.destatis.de), while future figures are consistent with the demographic forecast used by the "Rürup Commission" (Kommission für die Nachhaltigkeit in der Finanzierung der sozialen Sicherungssysteme, 2003).

[21] It could be argued that different earning points should be considered for different demographic groups (e.g. less earning points for women than for men). However, since the German pension system ensures relative equivalence (see section 2), such intra-cohort effects are not subject of this paper.

The Standard Pensioner – Male, married. In contrast to singles, married couples may obtain additional benefits (survivor benefits) by the pension insurance if one partner dies and the surviving spouse does not possess considerable personal pension income. While this is still relevant for many married men, it so far plays a minor role for married women since their husbands mostly have own substantial pension income that is deducted from the survivor pension.[22] The subsequent calculations therefore focus on married men. Typically, it is assumed that the standard pensioner is married to a housewife who is 3 years younger than he is. The adjustment factor for the survivor pension F_S is 60%. For Cohorts after 1961, the factor was shortened to 55%. The rate of return r for a married man can thus be derived from equation 3:

$$\begin{aligned}&\sum_{a_{RE}=65}^{A_c} 45\times PV_{c+a}\left(\frac{1}{1+r}\right)^{a-a_0} + \sum_{a=A_c+1}^{A_{Spouse}} F_S\times 45\times PV_{c+a}\left(\frac{1}{1+r}\right)^{a-a_0}\\ &\quad -\sum_{a_{=}=20}^{64} AGW_{c+a}\times \tau^{PS}{}_{c+a}\left(\frac{1}{1+r}\right)^{a-a_0} = 0\end{aligned} \tag{4}$$

A_{Spouse} … Remaining life expectancy of the spouse

Early retirement. In the case of early retirement, e.g. at age 63, the retirement age a_{RE} in equation 3 is set to 63, the sum of earning points has to be reduced to 43 and the annual pension benefit $P_{c,a}$ is to be reduced by the appropriate adjustment factor of 7,2%[23]:

$$\sum_{a_{RE}=63}^{A_c} 43\times 0.928\times PV_{c+a}\left(\frac{1}{1+r}\right)^{a-a_0} - \sum_{a_0=20}^{62} AGW_{c+a}\times \tau^{PS}{}_{c+a}\left(\frac{1}{1+r}\right)^{a-a_0} = 0 \tag{5}$$

Limits of the deterministic scenario-based approach. There are several reasons why the scenario-based approach is less than satisfactory. First, the use of the remaining life expectancy leads two distorting effects:

1. It is assumed that the standard pensioner for whom the rate of return is determined reaches the age of 65 with a probability of 100%. In reality of course, some contributors decease earlier.

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[22] A widow pension for the man incurred for less than 25% of all pensioner couples in 2004 and less than 15% of these widow pensions were fully dispensed due to the reduction of existing pension income by the husband (Verband Deutscher Rentenversicherungsträger, 2004).

2. It is assumed that the standard pensioner lives exactly according to the cohort's average remaining life expectancy. However, the rate of return for the cohort as a whole is not equal to the rate of return for a person of this cohort with exactly the average life expectancy due to Jensen's inequality[24]: the rate of return is a highly nonlinear and concave function of its stochastic ingredients such as life length and life-time earnings. The linear approximation implicitly applied in the conventional approach therefore overestimates the true value of the rate of return.

Second, in contrast to old-age and survivor pension benefits, benefits from disability pensions are difficult to capture in this kind of scenario-based approach since there exists no 'typical' invalidity scenario. Since contribution payments comprise the coverage against invalidity risk which then cannot be reflected in the benefit flow, the standard approach in the German pension insurance literature is to include only that part of contributions in the calculations that covers old-age and survivor risks. On an annual, cross-sectional basis, this is a share of roughly 80%.[25] The remaining 20% are paid for rehabilitation and invalidity expenditures. In the calculations, annual contributions are therefore corrected by a correction factor of 80%. However, this contribution correction factor suffers from aggregation bias since the cross-sectional computation of the average budget impact does not represent the true longitudinal payment flow for a specific cohort. Moreover, the bias from Jensen's inequality, see above, equally applies since different individuals of a certain cohort face different disability risks which do not result in the same rates of return if an "average" risk is included in the calculations instead.

Finally, the scenario-based approach cannot capture the fact that the pension insurance insures against all three risks simultaneously. The calculation only captures the pay-off for those risks that apply for the chosen scenario, i.e. a typical person receiving an old-age pension, a typical disabled person, or a typical beneficiary of a survivor pension.

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[23] Theoretically, life expectancy then would also have to be adjusted to the age of 63. Due to data restrictions this is typically not done, though, and will not be done here either.

[24] According to Jensen's inequality, the expected value $E(x)$ of a non-linear function $g()$ of a random variable x is not equal to the non-linear function of the expected value of this variable: $E(g(x)) \neq g(E(x)$. For a concave function $g()$, $E(g(x)) < g(E(x))$. See also Appendix A.

[25] See Schneider (1997) for a discussion on the appropriate size of this ratio.

3.3 A stochastic approach

The stochastic approach therefore considers the whole range of possible scenarios simultaneously instead of only looking at one particular case at a time. Every possible scenario is weighted with its probability to occur. The range of possible scenarios results from the combination of the four possible events that partly already have been considered for the chosen scenarios within the deterministic approach: (1) alive and working, (2) alive and receiving an old-age pension, (3) alive and receiving a disability pension and in the case of married couples (4) death but spouse is alive, receiving a widow pension. In this section we will describe how rates of return are calculated under such a stochastic approach and how this approach differs from the deterministic approach presented above.

Looking at *expected* cohort-specific payment flows. The deterministic payment flow is turned into a stochastic payment flow by including the probabilities with which each possible event *i* might occur. It comprises the expected values of the net payments for each age *a* of the life cycle of a cohort *c*.[26] Four cases are distinguished at any age *a*: an individual may be working with a certain probability, thereby paying contributions to the system (i=1); the individual may be disabled with a certain probability, thereby receiving a disability pension (i=2); the individual may be retired, thereby receiving an old-age pension (i=3); finally, the spouse of the individual may receive a widow pension (i=4) if the individual is dead and the spouse still alive. The four cases are illustrated in Box 1.

$$
\begin{aligned}
PF_a = {} & p_a(alive) \times \begin{pmatrix} & p_a(working) & \times\, ContributionPayments_a & \ldots(i=1) \\ + & p_a(disabled) & \times\, DisabilityPensionBenefits_a & \ldots(i=2) \\ + & p_a(retired) & \times\, OldAgePensionBenefits_a & \ldots(i=3) \end{pmatrix} \\
& + p_a(dead) \times p_a(SpouseIsAlive) \times SurviorPensionBenefits_a \quad \ldots(i=4)
\end{aligned}
$$

PF_a ... Payment flow at age a
p_a ... Probability of a certain scenario to be true at age a

BOX 1: THE STOCHASTIC APPORACH IN A NUTSHELL

[26] Note that the rate of return calculated on the basis of the expected payment flow for a certain cohort is not the same as the *expected* cohort-specific rate of return. See Appendix B. Appendix B also explains why this approach was chosen instead of computing cohort-specific expected rates of return.

Note that in the stochastic approach there is no fixed retirement age a_{RE} like in the deterministic approach. Instead, contribution payments as well as pension benefits are now recorded for all years although certain event probabilities may be zero.[27]

The calculation of pension benefits under this approach has to account for path dependencies since the size of the applicable pension in a specific year depends on the age at which the individual became disabled or retired. The calculations of the cohort-specific payment flows will therefore be derived step by step as further extensions to the deterministic approach presented above.

Step 1: Introducing probabilities of survival. Under the scenario-based approach, the individual was assumed to reach each age before its assumed end of life with the probability of 1. Now it is assumed that the individual survives each year only with a certain probability $S_c(a|a=a_0)<1$. The survival rate $S_c(a|a=20)$ determines the probability for a member of the cohort c to reach age a given that it reached the conditional age a=20. This probability is age- and cohort-specific. Figure 4 depicts its development over the life cycle for both men and women and shows how the curve is further shifting outward for younger cohorts as these are projected to live longer.

Accounting for this probability of survival, the formula for the internal rate of return for a single man (or woman) who retires at the statutory retirement age and has not become disabled during his (her) working life can be extended as follows:

$$\sum_{a_{RE}=65}^{A_c} S_c\left(a|a_0=20\right)\times P_{c,a}\left(\frac{1}{1+r}\right)^{a-a_0} - \sum_{a=20}^{64} S_c\left(a|a_0=20\right)\times Contrib_{c,a}\left(\frac{1}{1+r}\right)^{a-a_0} = 0 \quad (6)$$

A_c … Now: Maximum possible age that can be achieved by cohort c

with P and *Contrib* as defined in section 3.2. In the case of married couples, the different respective survival probabilities of the husband and his (younger) wife are used.

[27] The calculations in this paper ignore the relatively rare events of retiring before age 54 or after age 71. Hence, it is assumed that the probability to receive a pension becomes positive at age 54 (namely in the form of disability pensions) while the probability to pay contributions turns zero after age 70. *A* is the maximum age that can be achieved which is assumed to be 100. These boundaries are derived from the underlying empirical data, see Berkel and Börsch-Supan (2003). The assumed time of entry into the labour force a_0 is age 20.

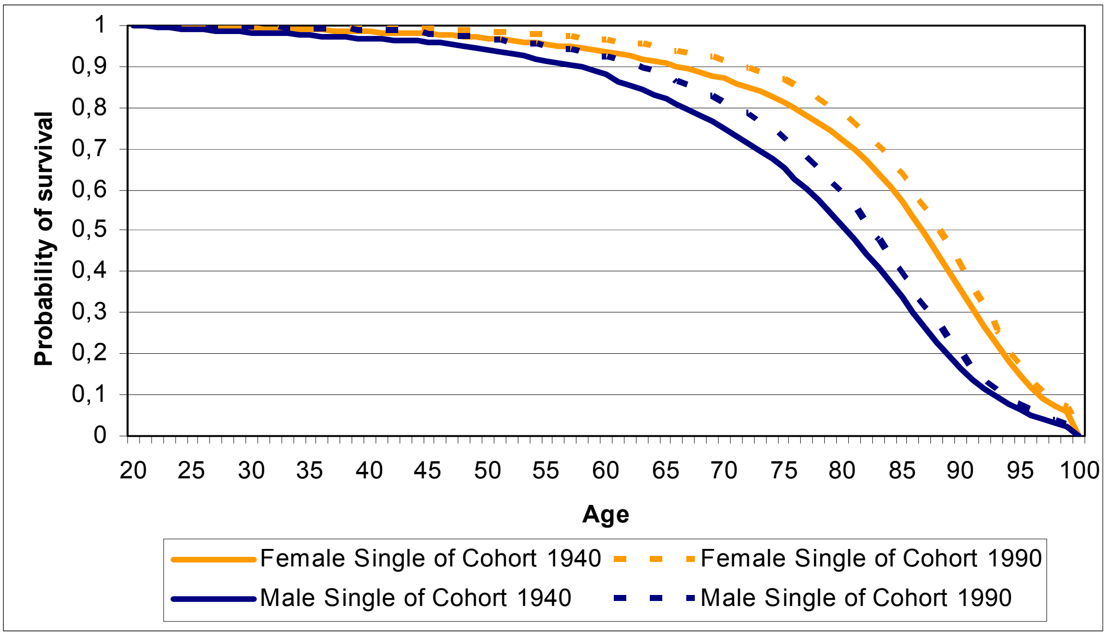


FIGURE 4: CONDITIONAL SURVIVAL RATES FOR MEN AND WOMEN (COHORTS 1940 AND 1990)

Source: Historical figures are taken from the National Statistical Office (www.destatis.de), while future figures are consistent with the demographic forecast used by the "Rürup Commission" (Kommission für die Nachhaltigkeit in der Finanzierung der sozialen Sicherungssysteme, 2003).

Step 2: Introducing probabilities of disability. Next, the invalidity risk will be introduced. Until recently, eligibility regulations for disability pensions were rather weak in Germany and thus often used as an easy early retirement option. This was changed in the context of the 2001 pension reform where these eligibility regulations were tightened considerably. Another major change was implemented with the 1992 and 1999 pension reforms which introduced adjustment factors for early retirement and increased the eligibility ages for most pension types.[28] Figure 5 on the next page depicts the phase-in of these regulations across affected cohorts for women, the disabled and the longtime insured[29].

[28] For an overview of the reform process of the German pension system during the past three decades, see Börsch-Supan and Wilke (2004).
[29] Longtime insured are those with a contribution history of at least 35 years.

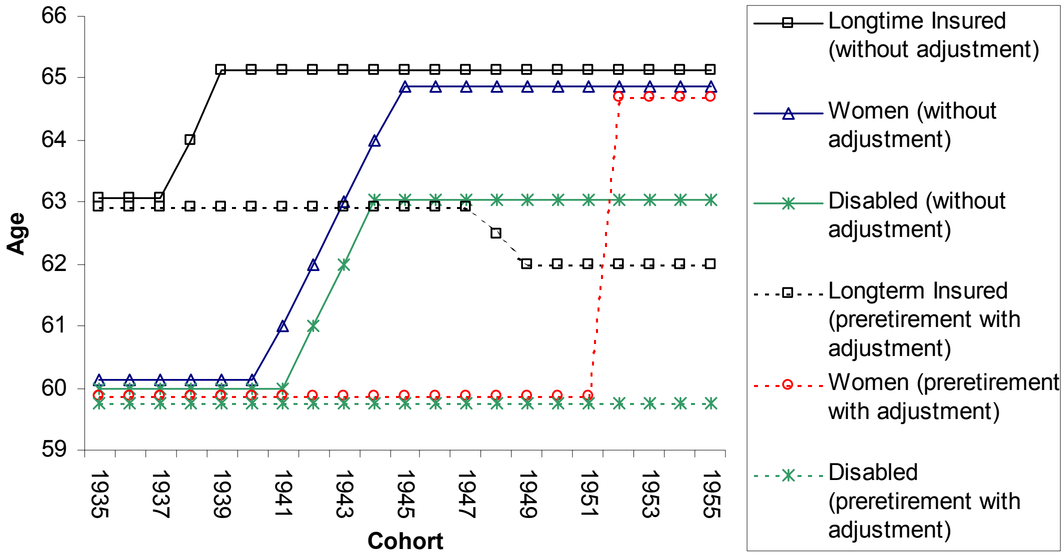


FIGURE 5: RETIREMENT AGE WITH AND WITHOUT ADJUSTMENTS (1992 AND 1999 REFORMS)

Source: Based on information provided by the Bundesversicherungsanstalt für Angestellte (BfA) (www.bfa.de).

As a result of these reform measures, probabilities of retirement due to disability differ not only by age but also across cohorts, depending on whether cohorts are already affected by the reform or not. They are displayed in Table 1 on the following page. Cohorts before 1940 are assumed to enter into disability retirement with the same probability as the 1940 cohort. The same applies to all cohorts after 1944 with respect to the 1944 cohort. It is assumed that, once a disability pension is received there is no return to the labour force.

Accounting for the probability of invalidity and subsequent disability benefits in addition to the probability of survival changes the internal rate of return equation stated in equation 6 as follows:

$$\sum_{a=65}^{A_c}\left[S_c(a|a=20)\times\left[(1-p_{c,a}(Disab))\times P_{c+a}{}^{OldAge}+p_{c,a}(Disab)\times P_{c+a}{}^{Disab}\right]\times\left(\frac{1}{1+r}\right)^{a-a_0}\right]$$
$$-\sum_{a=20}^{64}\left[S_c(a|a=20)\times\left[(1-p_{c,a}(Disab))\times Contrib_{c+a}-p_{c,a}(Disab)\times P_{c+a}{}^{Disab}\right]\times\left(\frac{1}{1+r}\right)^{a-a_0}\right]=0 \qquad (7)$$

$p_{c,a}(Disab)$ … Probability of cohort c to be disabled at age a

$P_{c+a}{}^{OldAge}$ … Old-age pension of cohort c at age a

$P_{c+a}{}^{Disab}$ … Disability pension of cohort c at age a

			Cohort		
Age	1940	1941	1942	1943	1944
			Men		
54	0,0214	0,0196	0,0178	0,0160	0,0141
55	0,0200	0,0176	0,0153	0,0129	0,0105
56	0,0230	0,0197	0,0164	0,0131	0,0097
57	0,0264	0,0224	0,0183	0,0143	0,0103
58	0,0262	0,0215	0,0169	0,0123	0,0077
59	0,0267	0,0215	0,0163	0,0111	0,0059
60	0,3152	0,2799	0,2446	0,2093	0,1739
61	0,0797	0,0885	0,0974	0,1062	0,1151
62	0,0602	0,0572	0,0543	0,0513	0,0484
			Women		
	1940	1941	1942	1943	1944
54	0,0176	0,0163	0,0151	0,0138	0,0125
55	0,0179	0,0163	0,0147	0,0131	0,0115
56	0,0192	0,0175	0,0159	0,0143	0,0126
57	0,0172	0,0155	0,0137	0,0120	0,0102
58	0,0186	0,0158	0,0129	0,0101	0,0073
59	0,0171	0,0150	0,0130	0,0110	0,0089
60	0	0,3806	0,3628	0,3450	0,3272
61	0	0	0,0795	0,0850	0,0915
62	0	0	0	0,0306	0,0304

TABLE 1: PROBABILITIES OF RETIREMENT ENTRY DUE TO DISABILITY (COHORTS 1940 TO 1944)

Source: Probabilities for the 1940 cohort (that was not affected by the reform) and for the 1944 cohort (for which the reform changes will be already fully implemented) are taken from estimations by Berkel and Börsch-Supan (2004) who explicitly model the behavioural effects of this reform. For the 1941 to 1943 cohorts that are directly affected by the phase-in of the new regulations, the respective probability values were derived by linear interpolation of the pre- and post-reform probabilities. In addition, the probabilities account for the fact that women can no longer receive their old-age pension at the age of 60 with their earliest possibility now being age 63 if they have an earnings history of at least 35 years.[30] Probabilities after age 62 are zero since it is assumed that old-age retirement turns more favourable then.

Note that introducing the probability of invalidity under the current assumptions affects both the contribution and the pension phase.[31] Also, while P_{c+a}^{OldAge} and $Contrib_{c,a}$ are here equivalent to $P_{c,a}$ and $Contrib_{c,a}$ from section 3.2, the calculation of P_{c+a}^{Disab} is more complex. For a given disability retirement age, the size of the disability pension can easily be derived from the sum of earning points that was collected over the previous working life, additional earning points that take into account the remaining years until the age of 60 and are granted in the case of

[30] In fact, individuals with an earnings history of at least 35 years of cohorts after 1950 can claim their old-age pensions already at age 62 – but only with the respective reductions of their benefits by 10.8%, which is why they are not likely to do so.

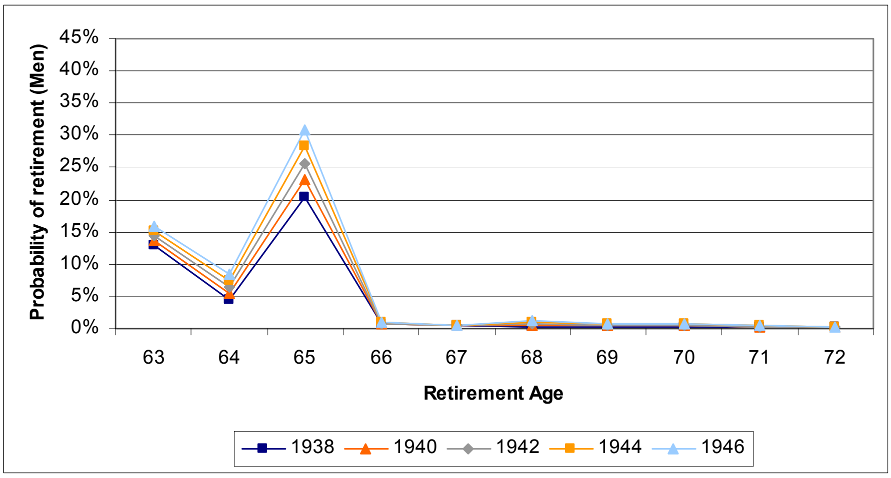
disability, an adjustment factor $F_{c+a,,Disab}$ corresponding to the one for early retirement and the current pension value. However, in this stochastic approach, each possible disability retirement age applies with a certain probability $p_{c,a}(DisabEntry)$. All potential possibilities of disability retirement entry in and before age a therefore have to be considered in the calculation of $P_{c,a}^{Disab}$:

$$
\begin{aligned}
P_{c,a}^{Disab} = {} & p_{c,a}(DisabEntry) \times \left(\sum_{k=20}^{a-1} EP_{c,k} \right) \times \left(1 + Max\left\{ \left(\frac{60-a}{(a-1)-20} \right); 0 \right\} \right) \times F_{c+a,Disab} \times PV_{c+a} \\
& + \sum_{i=54}^{a-1} \left[p_{c,i}(DisabEntry) \times \left(\sum_{k=20}^{i-1} EP_{c,k} \right) \times \left(1 + Max\left\{ \left(\frac{60-i}{(i-1)-20} \right); 0 \right\} \right) \times F_{c+i,Disab} \times PV_{c+a} \right]
\end{aligned}
\tag{8}
$$

$p_{c,a}(DisabEntry)$	… Probability to become disabled at age *a*
$F_{c+a,Disab}$	… Adjustment factor in case of disability at time $t=c+a$
$p_{c,i}(DisabEntry)$	… Probability to become disabled at any age i before age a
$F_{c+i,Disab}$	… Adjustment factor in case of disability at any age *i* before age *a*

Step 3: Introducing a stochastic retirement age. So far, it was assumed that the individual retires at the statutory retirement age of 65 unless he/she becomes disabled before that age (or chooses to retire according to the disability option). However, as Figure 5 showed, people may retire before age 65 also because of other early retirement options. Figure 6 depicts these probabilities of old-age-retirement for men and women of selected cohorts. Again, it is assumed that cohorts before 1938 and 1939 respectively retire with the same probability as the 1938 cohort. The same applies to all cohorts after 1945 and 1946 with respect to the 1945/ 1946 cohort. Probabilities turn positive at age 63 for (longtime insured) men and at age 60 for women of older cohorts. Note that the increase in the retirement age for women of younger cohorts eventually also leads to probabilities of old-age retirement of zero until age 63 as it is the case for men.

[31] Recall that for now the pension phase is still assumed to start at age 65. This assumption will be changed in the next step.



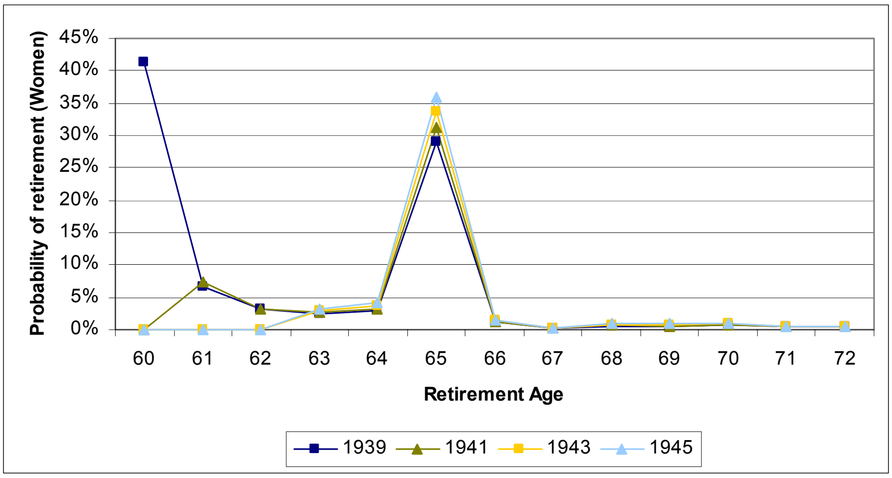


FIGURE 6: CHANGES IN OLD-AGE-RETIREMENT BEHAVIOUR (COHORT 1938 TO 1946)

Source: Probabilities for the 1938 and 1939 cohorts (that were not affected by the reforms) and for the 1945 and 1946 cohort (for which the reform changes will be fully implemented) are again taken from estimations by Berkel and Börsch-Supan (2004). For the 1940 to 1944 cohorts that are directly affected by the phase-in of the new regulations, the respective probability values were again derived by linear interpolation of the pre- and post-reform probabilities.

The strict confinement of contribution and pension phase pursued above is therefore no longer possible. Equation 3.9 demonstrates how probabilities of invalidity and retirement entry are included in our rate of return calculations:

$$\sum_{a_0=20}^{A_c} S_c\left(a|a_0=20\right)\times\begin{bmatrix} p_{c,a}\left(OldAge\right)\times P_{c,a}{}^{\text{OldAge}}+p_{c,a}\left(Disab\right)\times P_{c,a}{}^{Disab} \\ -\left(1-p_{c,a}\left(Disab\right)-p_{c,a}\left(OldAge\right)\right)\times Contrib_{c,a}\end{bmatrix}\times\left(\frac{1}{1+r}\right)^{a-a_0}=0 \quad (9)$$

$p_{c,a}\,(OldAge)$ … Probability of cohort c to be retired at age a
Note: $p_{c,72}(Disab)+p_{c,72}(OldAge)=1$

While the calculation of $P_{c,a}^{Disab}$ and $Contrib_{c,a}$ remains unchanged, $P_{c,a}^{OldAge}$ now is determined similarly to $P_{c,a}^{Disab}$:

$$P_{c,a}^{OldAge} = \sum_{i=60}^{a} \left[p_{c,i}(\text{OldAgeEntry}) \times \left(\sum_{k=20}^{i-1} EP_{c,k} \right) \times F_{i,\text{OldAge}} \right] \times PV_{c+a} \tag{10}$$

$F_{i,OldAge}$ … Adjustment factor in case of old-age retirement[32] at age i

The case of married couples. For the reason of simplicity, the above equations solely referred to the demographic group of single men or women. For married men the probability of their spouse's survival after their death has to be taken into account – as it was already shown for the deterministic approach. Adding this probability to the above equations gives us equation 11:

$$\begin{aligned} &\sum_{a_0=20}^{A_c} S_c(a|a_0=20) \times \begin{bmatrix} p_{c,a}(OldAge) \times P_{c,a}^{\text{OldAge}} + p_{c,a}(Disab) \times P_{c,a}^{Disab} \\ -(1 - p_{c,t}(Disab) - p_{c,t}(OldAge)) \times Contrib_{c,t} \end{bmatrix} \times \left(\frac{1}{1+r} \right)^{a-a_0} \\ &+ \sum_{a_0=20}^{A_{Spouse}} \left[(1 - S_c(a|a_0=20)) \times S_{c_{Spouse}}(a_{Spouse}|a_0=20) \times P_{c,a}^{Surv} \right] = 0 \end{aligned} \tag{11}$$

$S_{c,Spouse}$ … Survival probability of the spouse
$P_{c,a}^{Surv}$ … Survivor pension the spouse of cohort c receives at age a

$P_{c,a}^{Surv}$ is calculated according to the same concept applied for $P_{c,a}^{Disab}$ and $P_{c,a}^{OldAge}$ above. The size of the survivor pension is determined as 60% of the old-age pension in the case of death after old-age retirement and as 60% of the respective disability pension in the case of death before old-age retirement.

[32] Recall that the adjustment factor is equal to 1 if the individual retires at the statutory retirement age.

4 Calculation results

What are the results of the two approaches and in what respect do they differ? This question will be answered in this section. All calculations are based on the past and projected development as depicted in Figure 1 to Figure 6 above. The results of the deterministic approach will be shown first. Taking these as a starting point, the deterministic basic scenario is again extended in three consecutive steps by selected features of the stochastic approach in order to demonstrate the different impacts the two approaches have on the resulting rates of return. Finally, the overall results of the stochastic approach are compared to the deterministic outcomes from the beginning of the section.

4.1 Results from the deterministic approach

Since we used the same methodology as Ohsmann and Stolz (2004) in our realization of the deterministic approach, the results for the nominal rates of return are quite similar to those calculated by Ohsmann & Stolz (2004). Resulting real rates also roughly correspond to those calculated by Sozialbeirat (2004).

The Standard Pensioner. Table 2 summarizes the results for the standard pensioner and compares them to the results by Ohsmann and Stolz (2004) and Sozialbeirat (2004). Recall that the demographic groups here only differ with regard to their remaining life expectancies at retirement entry (age 65) and thus the length of their pension phases, while the contribution phases of all three groups are identical. Hence, single women record higher rates of return than single men (roughly 0.6 percentage points) thanks to their higher life expectancy. Likewise, married men record the highest rates of return of all three demographic groups because of their three-year younger wife (recall the assumptions) whose life expectancy obviously is higher than that of women of the same cohort as the man. Note that the three-years difference is just sufficient in order to compensate for the reduced pension benefits in form of the widow pension.

Cohort		Single, male			Single, female			Married, male		
		Ohsmann & Stolz (2004)	Sozialbeirat (2004)	Author's calculations	Ohsmann & Stolz (2004)	Sozialbeirat (2004)	Author's calculations	Ohsmann & Stolz (2004)	Sozialbeirat (2004)	Author's calculations
1939	Nom.	3,96 %	4,01 %	4,19 %	4,62 %	---	4,79 %	4,71 %	---	4,81 %
	Real	---	1,75 %	1,90 %	---	---	2,50 %	---	---	2,53 %
1940	Nom.	---	---	3,99 %	---	---	4,60 %	---	---	4,66 %
	Real	---	---	1,74 %	---	---	2,35 %	---	---	2,41 %
1944	Nom.	---	3,59 %	3,70 %	---	4,19 %	4,29 %	---	4,09 %	4,34 %
	Real	---	1,56 %	1,62 %	---	2,14 %	2,20 %	---	2,05 %	2,25 %
1975	Nom.	3,00 %	---	2,89 %	3,60 %	---	3,51 %	---	---	3,53 %
	Real	---	---	1,36 %	---	---	1,97 %	---	---	1,99 %
1980	Nom.	---	---	2,87 %	---	---	3,49 %	---	---	3,50 %
	Real	---	---	1,35 %	---	---	1,96 %	---	---	1,97 %

TABLE 2: RATES OF RETURN FOR SELECTED COHORTS ACCORDING TO THE DETERMINISTIC APPROACH

Notes: Deviations between the author's calculations and Ohsmann & Stolz (2004) and Sozialbeirat (2004) are mainly due to differences in the underlying projections. Whereas Ohsmann & Stolz (2004) refer to separate external demographic, labour market and pension system forecasts, our calculations are based on a set of consistent projections computed by the MEA pension simulation program MEA-PENSIM.[33] The underlying demographic and labour market assumptions for these projections correspond to those used by the "Rürup Commission" (Kommission für die Nachhaltigkeit in der Finanzierung der sozialen Sicherungssysteme, 2003) while assumptions about the pension system reflect the status quo. Health care contributions by the pension insurance are not included in the calculation. If they were this would lead to an increase of the rates of return by roughly 0.25 percentage points.

[33] See Wilke (2004) for a detailed description of the program.

Across cohorts, it can be seen that the rates of return for the younger 1980 cohort turn out lower than those for the 1940 cohort (about 1.1 percentage points in nominal and 0.4 percentage points in real terms). This result is not surprising taking into consideration the development of contribution rates and pension level depicted in Figure 1. Note that also the 1940 cohort is affected by this, since the projected decline in pension levels already fully affects their pension phase.

However, the decline in the rates of return cannot completely be attributed to the demographic burden and its future negative effects on contribution rates and pension levels. The rates of return for today's retiring cohorts are also higher thanks to the relatively low contribution rates of 14% until 1967[34] that lead to comparably low contribution payments during this period. The sheer development of contribution rates in the past[35] already induces a decline in the rates of return of those cohorts that have entered the labour force at later points in time. In fact, compared to today's older pensioner cohorts, already today's retiring cohorts record lower rates of return since their contribution history also comprises the past 20 years where higher contribution rates around 18% were the case.

Thus, the trend of a decline in the rates of return is not new and can already be observed today – both due to the past institutional and projected future demographic development of the system. However, while for the past the decline in the rates of return can be ascribed solely to the development of the contribution rate, future rates of return will be affected by both the development of contribution rates and that of pension levels. For the 1940 to 1980 cohorts the resulting trend is depicted in Figure 7.

[34] Only in 1973 did the contribution rate reach the 18% mark (recall FIGURE 1). Until the mid 1950s the contribution rate even was below 14%, at 10% and 11%.
[35] Note that, in contrast to the projected future rise of the contribution rate that can be attributed to demographic reasons, rises in the 1970s and 1980s mainly allowed an increasing generosity of the system.

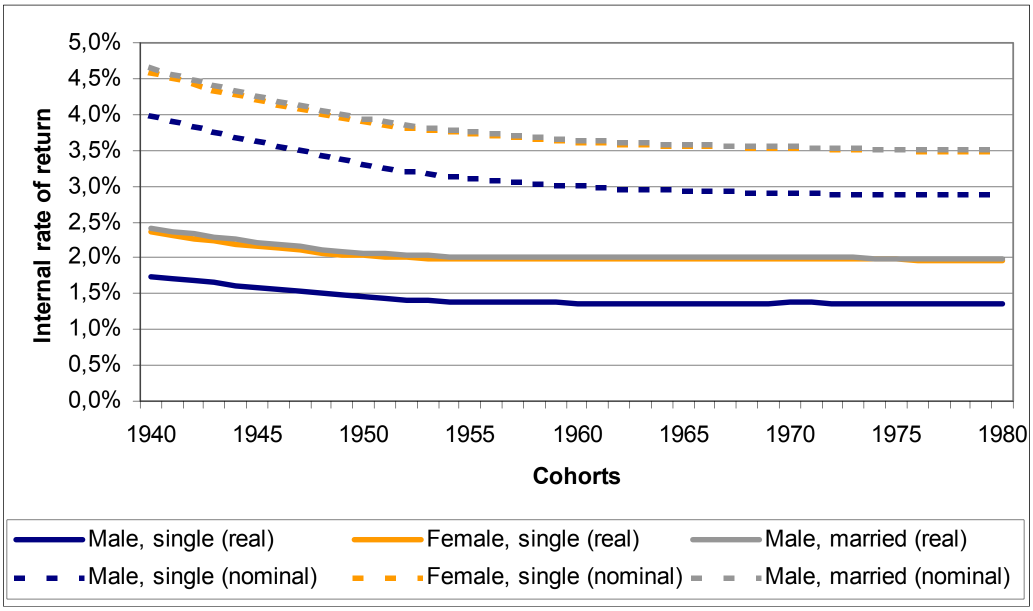


FIGURE 7: RATES OF RETURN OVER TIME ACCORDING TO THE DETERMINISTIC APPROACH

Figure 6 as well as Figure 7 also point up the difference between nominal and real rates of return. While nominal rates turn out considerably higher for the cohorts of the 1940s due to high and strongly fluctuating inflation particularly in the 1970s, their distance to the real rates reaches a stable 1.5 percentage points for cohorts from 1965 on when the largest part of the contribution phase is solely based on projected wage development. However, note that rates of return clearly remain positive under both terms.

Early retirement. In the case of early retirement, the rates of return turn out only slightly higher than for the standard pensioner which is mainly due to the adjustment factors.[36] This result corresponds to the findings by Sozialbeirat (2004) and shall not be evaluated further here. Note that it is crucial for the analysis to compare pensioners of the same cohort.[37]

[36] See Stolz and Thiede (2003) for a justification of the sizes of the adjustment factors.
[37] Ohsmann & Stolz (2004) e.g. choose to present their scenario results for different cohorts.

4.2 Results from the stochastic approach

Similar to our proceedings in section 3.3 we will analyse the results from the stochastic approach in three consecutive steps. Table 3 gives an overview of which parameters are introduced deterministically and stochastically in which steps.

	Life length	Disability	Retirement Age
Step 1	a) stochastic contributions[38] b) stochastic benefits[39] c) total effect	deterministic	deterministic
Step 2	stochastic	stochastic	deterministic
Step3	stochastic	stochastic	stochastic

TABLE 3: STEP-WISE ANALYSIS OF THE STOCHASTIC APPROACH – CONCEPTUAL OVERVIEW

Step1: Introducing probabilities of survival. In a first step, the life expectancy data used in the deterministic approach is replaced by the respective survival probabilities. In what way this affects the size of the payment flow is depicted in Figure 8 for the contribution and the pension phase respectively.[40]

For the contribution phase (Step 1a), contributions are slightly lower as compared to the deterministic approach because there is a probability that contributors die before reaching the normal retirement age. Around age 50, contribution payments are more than 5%, around age 60 already more than 10% lower than the contribution payments recorded under the deterministic approach.[41]

For the pension phase (Step 1b), Figure 8 depicts a totally different picture. Under the scenario-based approach, survival until the remaining life expectancy age 81,5 is sure and benefits increase according to real wage adjustment. After age 81,5, benefits are zero. In the stochastic approach, however, expected benefits decline steadily because the probabilities of survival decrease faster than the wage adjustment.[42]

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[38] Accounting separately for the first bias that does not account for possible death before age 65.
[39] Accounting separately for the second bias that results from neglecting Jensen's inequality.
[40] Note that the only difference between the two payment flows shown in Figure 8 lies in the use of the remaining life expectancy and survival probability respectively. In particular, this means that contribution payments are only considered with 80% according to the usual practices common for the deterministic approach. The contribution correction factor will be introduced in step 2.
[41] Figure 8 is based on real values for 2004.
[42] For data reasons, the maximum age for the calculations presented in this paper is assumed to be 100 years.

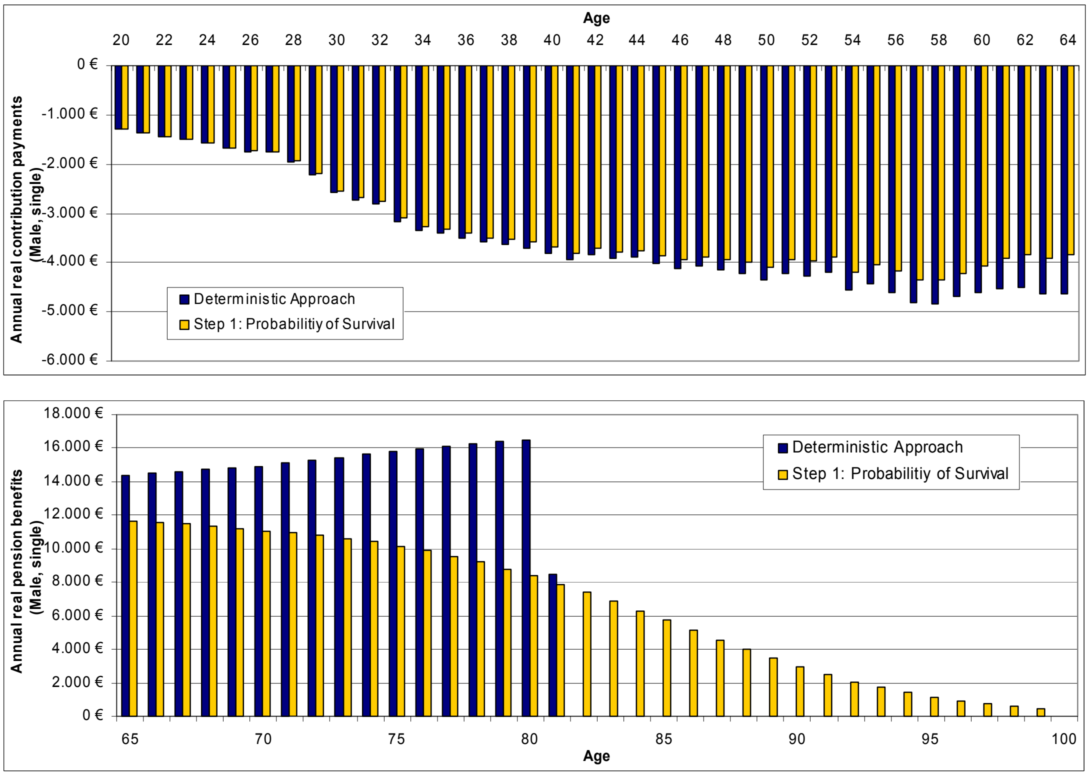

FIGURE 8: INTRODUCING PROBABILITIES OF SURVIVAL – CONTRIBUTION AND PENSION PHASE OF A SINGLE MAN OF THE 1940 COHORT

Which impact do these two effects now have on the resulting rates of return? Figure 9 shows the resulting real internal rates of return for the 1940 cohort for all three demographic groups. The effects on the contribution phase (Step 1a) and on the pension phase (Step 1b) are illustrated separately before the combined effect of both effects is shown as Step 1c.

It can be seen that the internal rates of return turn out higher if the probability of death before retirement is taken into account (Step 1a) since expected contribution payments are lower. On the other hand, internal rates of return are lower if remaining life expectancies are replaced by survival probabilities (Step 1b) due to the concave survival probability function and Jensen's inequality as it was explained at the end of section 3.2. The overall effect (Step 1c) is a decrease in the rates of return. For women, whose survival probabilities are clearly higher than those of men (recall Figure 4) the effects are considerably smaller than for single men.

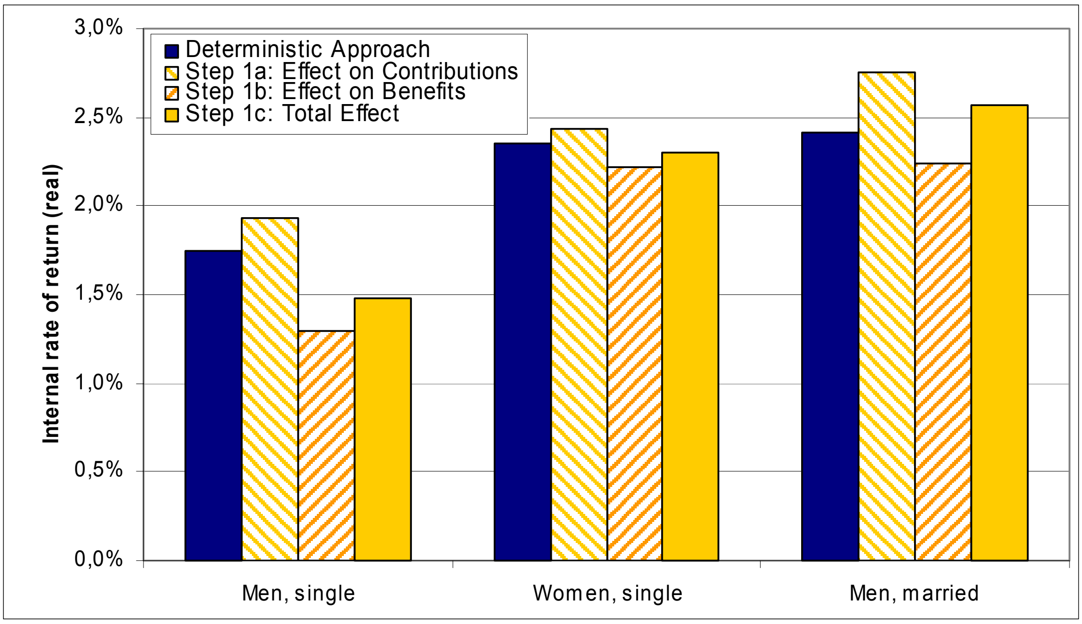

FIGURE 9: INTRODUCING PROBABILITIES OF SURVIVAL – RATES OF RETURN FOR THE 1940 COHORT

Married men are the only group recording higher rates of return. This is due to two effects. On the one hand, there is already a positive probability of death and survival of the spouse before retirement (recall section 3.3) which is not taken into account under the deterministic approach and which has a positive effect on the rate of return. On the other hand, this probability rises with age which means that according to Jensen's inequality the true rate of return is underestimated in the case the remaining life expectancy and a subsequent survivor pension are assumed as it is done under the deterministic approach. Figure 10 illustrates these effects on the overall payment flow.

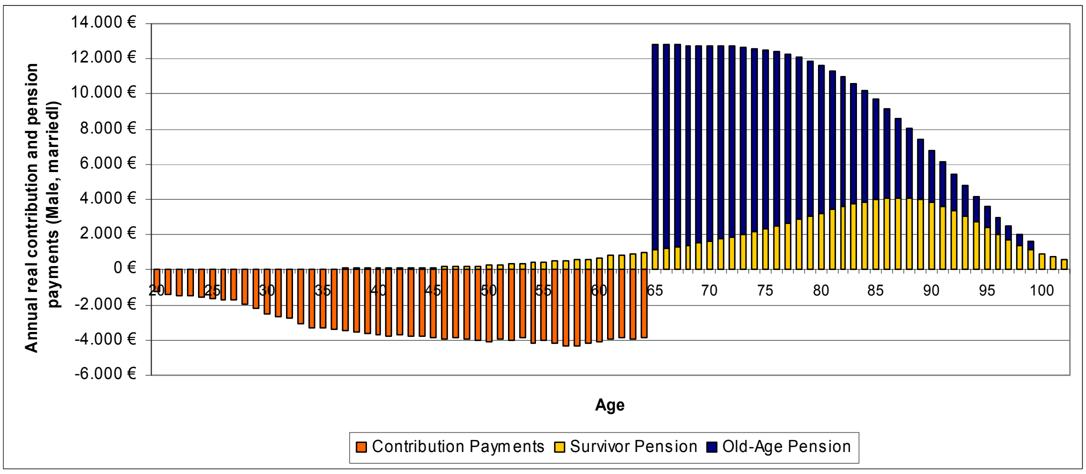

FIGURE 10: INTRODUCING PROBABILITIES OF SURVIVAL – CONTRIBUTION AND PENSION PHASE OF A MARRIED MAN OF THE 1940 COHORT

Step 2: Introducing probabilities of disability. As it was explained in section 3.2, the deterministic approach does not include benefits from disability pensions but instead uses a contribution correction factor that reduces the applicable contribution payment size by the percentage that is assumed to finance the invalidity risk.

In Step 2 we now explicitly consider disability risk under the stochastic approach. We abandon the contribution correction factor and introduce probabilities of disability instead. Since the pool of pensioners for a specific cohort remains unchanged, this means that less people retire at the statutory retirement age of 65 but retire earlier due to disability. They thus receive lower pensions, but for a longer time period.[43] The net effect is negative. As Figure 11 shows, resulting rates of return for Step 2 are lower than for Step 1.

However, a second effect counteracts this first one. The higher the probability of disability for younger ages near age 54, the higher the weight of these potential pension benefits for the remaining payment flow calculation, since for each subsequent age the potential benefit flow from an earlier claimed disability pension is considered. For this reason, the decline in the resulting rates of return turns out highest for single women. As could be seen from Table 1 and Figure 6, women of the 1940 cohort have a much lower probability to retire before age 60 than men.

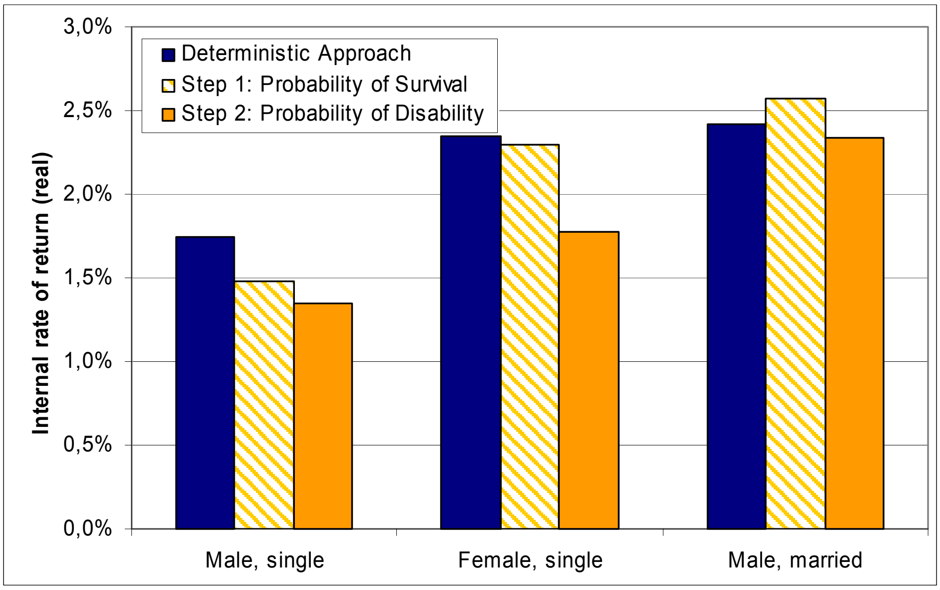

FIGURE 11: INTRODUCING PROBABILITIES OF DISABILITY – RATES OF RETURN FOR THE 1940 COHORT

[43] It is likely that those people receiving a disability pension in general have a lower life expectancy than healthy people that work until the statutory retirement age. Since data on this feedback effects is not readily available, this aspect is neglected here.

The effect of the change in retirement behaviour can also be shown nicely for single men. Figure 12 compares the effects for the 1940 and 1980 cohort. It can be seen that the size of potential disability pension flows clearly diminishes from about two thirds for the 1940 to one third of the overall pension flow for the 1980 cohort, as was to be expected from the assumptions made above.

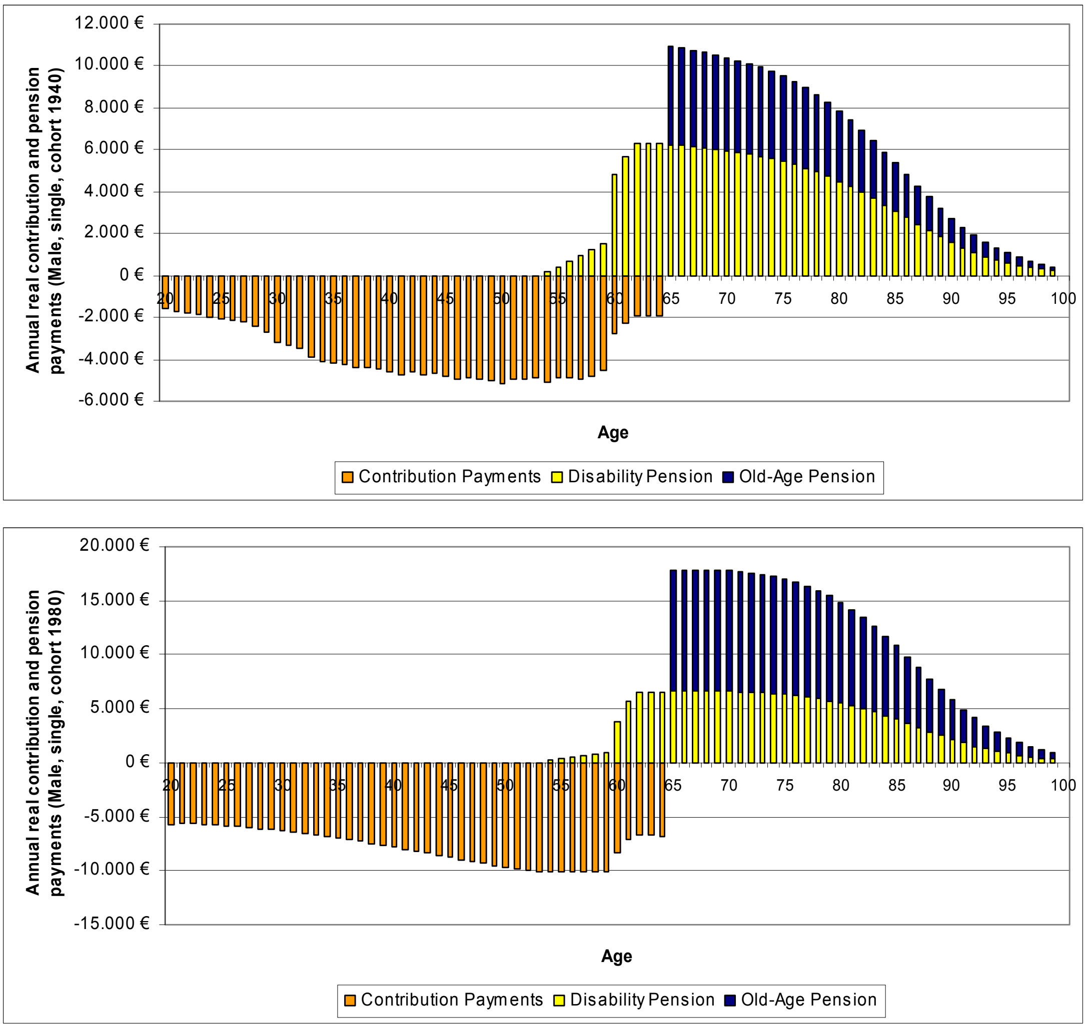

FIGURE 12: INTRODUCING PROBABILITIES OF DISABILITY – CONTRIBUTION AND PENSION PHASE OF A SINGLE MALE FOR THE 1940 AND 1980 COHORTS

Step 3: Introducing a stochastic retirement age. In contrast to the deterministic approach, the stochastic approach allows taking into account all potentially possible early, normal and late retirement scenarios. Step 3 replaces the statutory, fixed retirement age by a flexible one based on the old-age retirement probabilities displayed in Figure 6. The results are shown in Figure 13. Note that a positive old-age pension flow for ages below the statutory retirement age of 65 is now recorded.

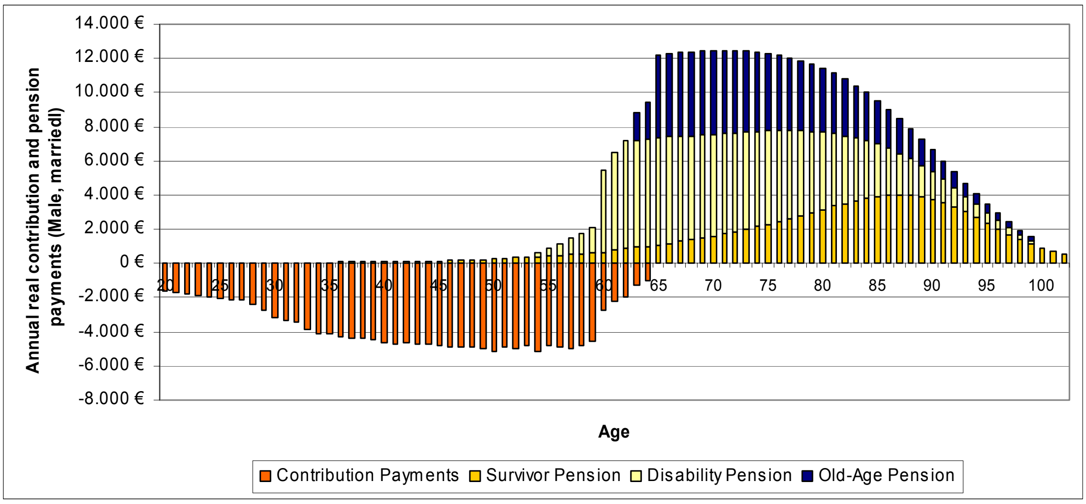

FIGURE 13: INTRODUCING A STOCHASTIC RETIREMENT AGE – CONTRIBUTION AND PENSION PHASE FOR A MARRIED MAN OF THE 1940 COHORT

The effect of the introduction of the stochastic retirement age on the resulting rates of return is shown in Figure 14 for the 1940 and 1980 cohort. While the effect is positive for the 1940 cohort across all demographic groups, the results from Step 2 remain almost unchanged for the 1980 cohort. This result nicely demonstrates the effects of the introduction of adjustment factors for early retirement with the 1992 and 1999 German pension reforms and also shows that these adjustment factors are indeed roughly actuarially neutral. The slight decrease in the rates of return (compared to Step 2) for single men and women of the 1980 cohort indicates that the concept of adjustment factors probably also goes back to a deterministic approach based on remaining life expectancy figures and that once survival probabilities are applied this actuarial fairness no longer fully holds. As could be seen from Step1, the introduction of survival probabilities leads to a negative effect on the rates of return.

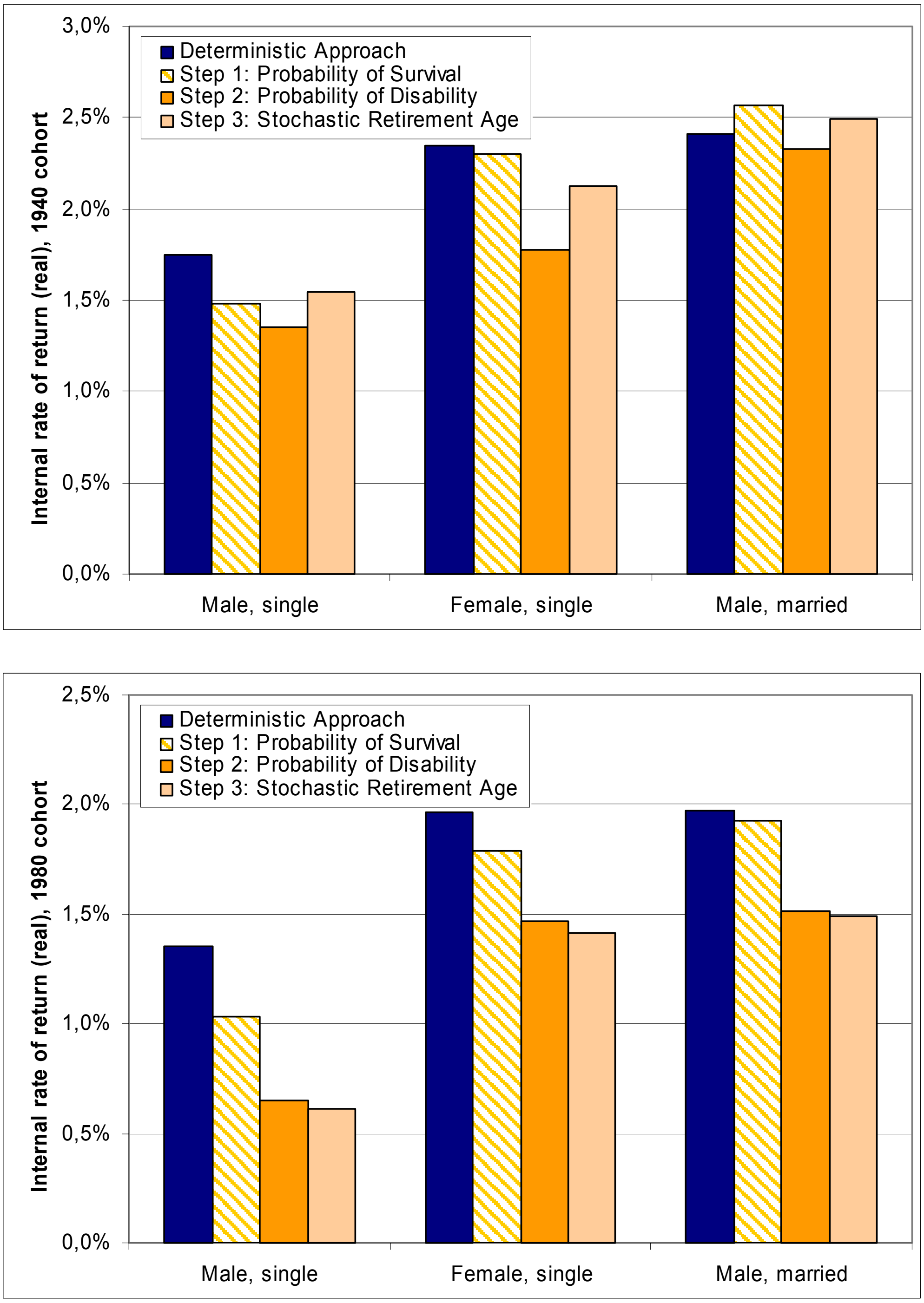

FIGURE 14: INTRODUCING A STOCHASTIC RETIREMENT AGE – RATES OF RETURN FOR THE 1940 AND 1980 COHORT

4.3 The stochastic versus the deterministic approach

The previous three steps transformed the deterministic approach into a stochastic one. A summary of the results of the two approaches for both the 1940 and 1980 cohort is given in Table 4.[44] Across demographic groups, it can be seen that both for single men and for single women rates of return turn out lower under the stochastic approach. In contrast, rates of return for married men turn out slightly higher for the 1940 and slightly lower for the 1980 cohort than under the deterministic approach. Across cohorts, both approaches deliver very similar results for the 1940 cohort that hardly differ by more than 0.2 percentage points. However, projections for younger cohorts like the 1980 cohort turn out considerably different under the two approaches. The results here differ by 0.5 to about 0.7 percentage points.

	Cohort 1940				**Cohort 1980**			
	Nominal		**Real**		**Nominal**		**Real**	
	Det.	**Stoch.**	**Det.**	**Stoch.**	**Det.**	**Stoch.**	**Det.**	**Stoch.**
Single, male	3,99%	3,83%	1,74%	1,54%	2,87%	2,12%	1,35%	0,61%
Single, female	4,60%	4,40%	2,35%	2,12%	3,49%	2,95%	1,96%	1,41%
Married, male	4,66%	4,79%	2,41%	2,49%	3,50%	2,91%	1,97%	1,47%

TABLE 4: RATES OF RETURN FOR THE 1940 AND 1980 COHORT ACCORDING TO THE DETERMINISTIC AND THE STOCHASTIC APPROACH

This outcome shows that for today's retiring cohorts the differences between the two approaches in the end are surprisingly small, given the partly significant differences that could be seen during the step-wise transformation above. However, the fact that the results vary significantly for younger cohorts calls into mind that both approaches are based on very different concepts and assumptions that react very differently to longer life spans and different retirement behaviour of future cohorts. Still, rates of return remain positive under both approaches. Calculations for even younger cohorts show no further considerable changes in the rates of return if the demographic and labour market develops according to the projections.

[44] The highlighted columns 5 and 9 in TABLE 4 are identical to the fourth bar ("Step 3") in FIGURE 14.

5 Conclusions and outlook

The calculations presented in this paper show that under realistic assumptions of future demographic and labour market development, rates of return of the German public pension system will indeed decline in the future but they will remain positive.

In contrast to the deterministic scenario-based approach typically applied in the German pension literature, this paper proposes a stochastic approach that allows to consider the entire range of possible scenarios simultaneously instead of restricting the analysis to one selected scenario at a time. Hence, all risks covered by the German pension insurance – including the invalidity risk – can be adequately captured in the rate of return calculations, which is not possible using the deterministic, scenario-based approach.

Furthermore, the stochastic approach corrects for a serious mistake of the scenario-based approach. In taking the remaining life expectancy at age 65 as the determinant of the length of the pension period, the deterministic approach makes two implicit assumptions: (1) that the respective individual with certainty reaches age 65 and (2) that he/she, again with certainty, continues to live according to the respective average remaining life expectancy of the cohort. It was shown that these two assumptions create a considerable bias, overestimating the rates of return. If age- and cohort-specific survival probabilities are taken into account in the stochastic approach, rates of return turn out substantially lower. The effect is weaker for women than for men since women have considerably higher survival probabilities for ages over 65 and therefore can better compensate the first effect of a possible death before retirement.

Future nominal rates of return are of course much higher than the real rates. The results show that they will clearly stay above 2% for all demographic groups. Since the juridical debate refers to nominal rates of return, the future constitutionality of the German public pension system seems to be warranted as long as no further demographic or labour market shocks occur.

While the stochastic approach presented in this paper allows for a more precise calculation of the size of future rates of returns, its application requires appropriate data on the respective survival and retirement probabilities. This is not so much a problem concerning the age- and cohort-specific survival probabilities as concerning the probabilities of retirement entry. For the calculations in this paper, this data was available thanks to the estimation of these probabilities by Berkel and Börsch-Supan (2003) that adequately reflects projected future changes in response to the recent reform measures in this area. If such data is not available, a stochastic computation that adequately considers all risks is no longer possible. However, as a first step towards a more proper computation of cohort-specific rates of return, the introduction of age- and cohort-specific survival probabilities into an otherwise scenario-based approach could at least correct for the two mistakes named above and is thus strongly recommended by the author.

Appendix

A. Some notes on life expectancy, survival rates and rates of return

It was pointed out in section 3.2 that under the scenario-based approach the use of the remaining life expectancy in order to determine the relevant retirement period leads to two severe biases: (1) the possible event of death before retirement is neglected and (2) the remaining life expectancy of the cohort is assumed to adequately reflect the “typical, average” pensioner of that cohort. In the following, it is explained in more detail why resulting rates of return turn out higher if these two biases persist.

Figure 15 shows how the contribution and pension phase look like if remaining life expectancies or survival probabilities are applied (recall Figure 8). The question is why rates of return turn out differently under these two approaches.

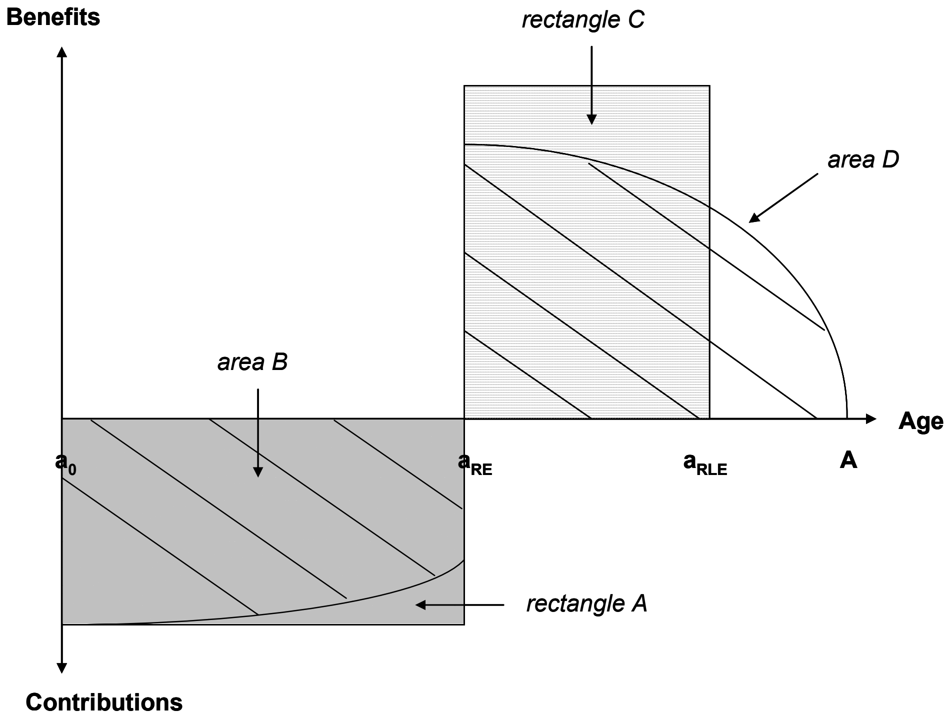

FIGURE 15: CONTRIBUTION AND PENSION PHASE FOR CALCULATIONS BASED ON THE REMAINING LIFE EXPECTANCY AND CALCULATIONS BASED ON SURVIVAL PROBABILITIES

The rates of return are derived from the proportion of the size of benefits to the size of contributions as represented by the two rectangles and areas in Figure 15. However, due to the compound interest effects, this relationship is not linear. Even if the rectangles and areas in Figure 15 were of equivalent size, the resulting rates of return in all but one case would still differ.

The following stylized example shows in what respect the rates of return calculations differ for the two approaches. Assume an annual net payment flow of -1 and plus 1 for the contribution and pension phase respectively. Using the remaining life expectancy, the rate of return *r* is derived by solving equation A1:

$$\sum_{a=a_0}^{a_{RE}-1}(-1)\times\left(\frac{1}{1+r}\right)^{a-a_0}+\sum_{a=a_{RE}}^{a_{RLE}}(1)\times\left(\frac{1}{1+r}\right)^{a-a_{RE}}=0 \qquad \text{(A1)}$$

The age of the remaining life expectancy a_{RLE} is thereby determined as follows:

$$a_{RLE}=\sum_{a=a_{RE}+1}^{A}S(a|a_{RE}=65)+1, \qquad \text{(A2)}$$

where $S(a|a_{RE}=65)=S(a-1|a_{RE}=65)\times S(a)$

and $S(a)=1-M(a)$.

The survival rate $S(a|a_{RE}=65)$ determines the probability to reach age a given that the age a_{RE} =65 was reached. S(a) represents the conditional survival rate to survive at a certain age while M(a) represents the mortality rate to die from one year to the next.

In contrast, when survival probabilities are used, the rate of return *r* is to be derived from equation A3. Note that the probability p_a is equivalent to the survival rate $S(a|a_0=20)$ and is now placed in the sum instead of determining the final sum index.

$$\sum_{a=a_0}^{a_{RE}-1}(-1)\times p_a\times\left(\frac{1}{1+r}\right)^{a-a_0}+\sum_{a=a_{RE}}^{A}(1)\times p_a\times\left(\frac{1}{1+r}\right)^{a-a_{RE}}=0, \qquad \text{(A3)}$$

where $p_a=S(a|a_0=20)=S(a-1|a_0=20)\times S(a)$

and $S(a)=1-M(a)$.

Apart from the fact that A1 relies on *S(a|a_{RE}=65)* whereas A3 relies on *S(a|a_0=20)* which alone obviously would lead to higher rates of return for the first approach, the introduction of the survival rates once as the final running index of the sum and once as a part of the sum will in general not lead to identical results.

B. Some notes on rates of return based on expected payments flows and expected rates of return

Section 3.3 presented a stochastic approach where the rate of return is calculated on the basis of the *expected payment flow* in order to consider all risks that are covered by the German pension insurance simultaneously. Alternatively, one might want to calculate the *expected rate of return E(r)* as it is known from the finance literature. In this case, the rate of return r_n is computed for each possible scenario n of a cohort c:

$$\sum_{a=a_{RE,n}}^{A_{c,n}} P_{c,a,n}\left(\frac{1}{1+r_n}\right)^{a-a_0} - \sum_{a=a_0}^{a_{RE,n}-1} Contrib_{c,a,n}\left(\frac{1}{1+r_n}\right)^{a-a_0} = 0 \qquad \text{(B1)}$$

n	… Scenario index with N=maximum number of possible scenarios
$a_{RE,n}$	… Age of retirement entry in scenario n
$A_{c,n}$	… Maximum age/ end of pension period in scenario n
$P_{c,a,n}$	… Pension payments to cohort c at age a in scenario n
r_n	… Internal rate of return for scenario n
$Contrib_{c,a,n}$	… Contribution payments by cohort c at age a in scenario n

These scenario-specific rates of return r_n are then weighted according to their probability p_n to occur:

$$E(r) = \sum_{n=1}^{N} r_n \times p_n \qquad \text{with} \qquad p_n = \prod_{a=a_0}^{A_{c,n}} p_{a,i} \quad \text{and} \quad \sum_{i=1}^{I} p_{a,i} = 1 \qquad \text{(B2)}$$

p_n	… Probability of scenario n to occur
$p_{a,i}$	… Probability of the event i to occur at age a as assumed in scenario n (e.g. old-age retirement at age 65)[45]
i	… Index of possible events (working, receiving a certain pension type, death, survivor pensions) with I=maximum number of possible events at age a

This expected rate of return method, however, has a drawback. For each case where a person dies before retirement the respective scenario-specific rate of return amounts to -100%, since this person receives no benefits at all. This amount enters into the calculation weighted with the respective probability of death at that age and has a large negative impact on the overall expected rate of return. Figure 16 illustrates this point. It depicts the scenario-specific rates of return for the case of a

[45] Note that the respective probabilities will be explained in more detail later in this section.

standard pensioner who – if surviving – retires at age 65. Figure 16 displays the rate of return by time of death.

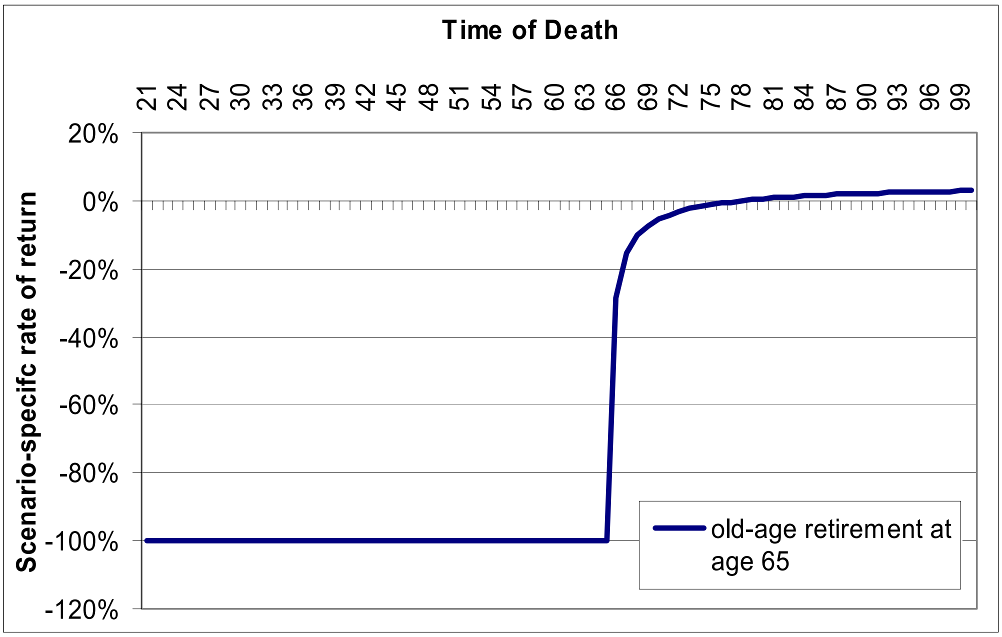

FIGURE 16: SCENARIO-SPECIFIC RATES OF RETURN FOR RETIREMENT AT AGE 65[46]

Although the scenario-specific rates of return eventually turn positive for ages above 75, they are not sufficiently large in order to make up for the extremely negative rates until age 65. Weighted with their probabilities to occur and accounting for all possible scenarios including disability, early retirement and survivor pensions, the scenario-specific rates of return lead to a highly negative expected rate of return E(r). Calculations show that the latter amounts to about –28% for the 1940 and about –22% for the 1980 cohort.[47]

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[46] Figures are based on calculations for the 1940 cohort.
[47] Note that the expected rate of return for the 1980 cohort is lower since it records considerably lower probabilities of death for younger ages (recall Figure 4).

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