

# Deadlock-Detection in Component-Based Systems is NP-hard

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**Abstract.** Interaction systems are a formal model for component-based systems. Combining components via connectors to form more complex systems may give rise to deadlock situations. We present here a polynomial time reduction from 3-SAT to the question whether an interaction system contains deadlocks.

## 1 Introduction

We consider a setting where components are combined via connectors to form more complex systems, see, e.g. [GS03], [S04], [S05], [GGMS06] and [BBS06]. Each individual component  $i$  offers ports  $a_i, b_i, \dots \in A_i$  for cooperation with other components. Each port in  $A_i$  represents an action of component  $i$ . The behavior of a component can be represented via a labeled transition system with starting state, where in each state there is at least one action available. Components are glued together via connectors, where each connector connects certain ports. In the global system obtained by gluing components (local) deadlocks may arise where a group of components is engaged in a cyclic waiting and will thus no longer participate in the progress of the global system (cf. [T01]). We show here that deadlock-detection is NP-hard by encoding the classic 3-SAT problem in (deadlock detection for) interaction systems. For this we apply two ideas. First, we ensure that in all situations where a deadlock arises, a global deadlock arises.<sup>1</sup> Second, the components we introduce for a clause of a 3-KNF formula will always be able to progress while the clause evaluates to false. So, at the time a deadlock occurs no progress is possible and, that is, no clause evaluates to false.

The paper is organized as follows. Section 2 contains the basic definitions. Section 3 gives the polynomial time reduction from 3-SAT to the problem of deadlock detection in interaction systems. Section 4 contains a short conclusion and a discussion of related work.

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<sup>1</sup> As a global deadlock is a special case of a local deadlock. This also means that we reduce 3-Sat to both local and global deadlock analysis.

## 2 Components, Connectors and Interaction Systems

We consider here *interaction systems*, a model for component-based systems that was proposed and discussed in detail in [GS03], [S05], [GS05] and [BBS06]. An *interaction system* is a tuple  $Sys = (K, \{A_i\}_{i \in K}, C, \{T_i\}_{i \in K})^2$ , where  $K$  is the set of *components*. W.l.o.g. we assume  $K = \{1, \dots, n\}$ . Each component  $i \in K$  offers a finite set of *ports*  $A_i$  for cooperation with other components. The port sets  $A_i$  are pairwise disjoint. Cooperation is described by *connectors*. A *connector* is a set of actions  $c \subseteq \bigcup_{i \in K} A_i$ , where for each component  $i$  at most one action  $a_i \in A_i$  is in  $c$ . A connector set  $C$  is a set of connectors, s.t. every action of every component occurs in at least one connector of  $C$  and no connector contains any other connector.

The local behavior of each component  $i$  is described by  $T_i = (Q_i, A_i, \rightarrow_i, q_i^0)$ , where  $Q_i$  is the finite set of local states,  $\rightarrow_i \subseteq Q_i \times A_i \times Q_i$  the local transition relation and  $q_i^0 \in Q_i$  is the local starting state. Given a connector  $c \in C$  and a component  $i \in K$  we denote by  $i(c) := A_i \cap c$  the participation of  $i$  in  $c$ .

For  $q_i \in Q_i$  we define the set of *enabled actions*  $ea(q_i) := \{a_i \in A_i \mid \exists q'_i \in Q_i, \text{ s.t. } q_i \xrightarrow{a_i} q'_i\}$ . We assume that the  $T_i$ 's are non-terminating, i.e.  $\forall i \in K \forall q_i \in Q_i \ ea(q_i) \neq \emptyset$ .

The global behavior  $T_{Sys} = (Q, C, \rightarrow, q^0)$  of  $Sys$  (henceforth called *global transition system*) is obtained from the behaviors of the individual components, given by the transition systems  $T_i$ , and the connectors  $C$  in a straightforward manner:

- $Q = \prod_{i \in K} Q_i$ , the Cartesian product of the  $Q_i$ , which we consider to be order independent. We denote states by tuples  $(q_1, \dots, q_n)$  and call them global states.
- the relation  $\rightarrow \subseteq Q \times C \times Q$ , defined by
 
$$\forall c \in C \forall q, q' \in Q \ q = (q_1, \dots, q_n) \xrightarrow{c} q' = (q'_1, \dots, q'_n) \quad \text{iff}$$

$$\forall i \in K \ (q_i \xrightarrow{i(c)} q'_i \text{ if } i(c) \neq \emptyset \text{ and } q'_i = q_i \text{ otherwise}).$$
- $q^0 = (q_1^0, \dots, q_n^0)$  is the starting state for  $Sys$ .

In the global transition system a transition labeled  $c$  may take place when each component participating in  $c$  is ready to perform  $i(c)$ .

For an example of an interaction system see Example 1 at the end of section 3. For a global state  $q = (q_1, \dots, q_n) \in Q$  we refer to the local state  $q_j$  of component  $j \in K$  by  $q(j)$ .

Let  $q = (q_1, \dots, q_n) \in Q$  be a global state. We say that some non-empty set  $D = \{j_1, j_2, \dots, j_{|D|}\} \subseteq K$  of components is in *deadlock* in  $q$  if  $\forall i \in D \forall c \in C$ , s.t.  $c \cap ea(q_i) \neq \emptyset \exists j \in D$ , s.t.  $j(c) \not\subseteq ea(q_j)$ . We say that  $i$  waits for  $j$  then.

<sup>2</sup> The model in [GS03] is more general, introduces a notion of interaction, which is a subset of a connector and distinguishes between connectors and complete interactions. We are able to show NP-hardness for deadlock detection in interaction systems without the use of complete interactions, so we omit them for ease of notation. Note that this yields a stronger, not weaker result than using complete interactions. Readers who are familiar with interaction systems may simply assume  $Comp = \emptyset$  for  $Sys(F)$  in Section 3.

A system has a local *deadlock* in some global state  $q$  if there is  $D \subseteq K$ , that is in deadlock in  $q$ . If  $D = K$ , the system is globally deadlocked. Hence a global deadlock is a special case of a local deadlock. A system is *deadlock-free*, if there is no reachable state  $q$  and  $D \subseteq K$ , such that  $D$  is in deadlock in  $q$ .

We denote by IS the set of all interaction systems and by DLIS the set of interaction systems that contain local deadlocks:

$$DLIS := \{Sys \in IS \mid Sys \text{ contains local deadlocks}\}$$

We consider the well-studied NP-complete 3-SAT problem [GJ79] where the formula is a conjunction of clauses  $k_i$ , each of which is a disjunction of 3 literals, (i.e. possibly negated variables) and reduce it to DLIS.

### 3 Reducing 3-SAT to DLIS

Let  $F = k_1 \wedge \dots \wedge k_n$  with  $k_i = (l_{(i,1)} \vee l_{(i,2)} \vee l_{(i,3)})$  be a propositional formula in 3-KNF, where  $l_{(i,1)}, l_{(i,2)}, l_{(i,3)}$  are positive literals (i.e. variables) or negative literals (i.e. negated variables). In the following, we construct an interaction system  $Sys(F)$ , s.t.  $(F \in 3-SAT) \Leftrightarrow (Sys(F) \in DLIS)$ . We represent each clause  $k_i$  by a component  $(i, 0)$  and each literal  $l_{(i,j)}$  by a component  $(i, j)$ . By  $i + 1$  we mean  $i + 1$ , if  $1 \leq i \leq n - 1$  and 1 if  $i = n$ .

$Sys(F) = (K, \{A_{(i,j)}\}_{(i,j) \in K}, C, \{T_{(i,j)}\}_{(i,j) \in K})$ , where:

$$K = \{(i, j) \mid 1 \leq i \leq n, 0 \leq j \leq 3\}.$$

$$A_{(i,0)} = \{init_{(i,0)}, false_{(i,0)}\} \text{ for } 1 \leq i \leq n$$

$$A_{(i,j)} = \{init_{(i,j)}, set-to-1_{(i,j)}, set-to-0_{(i,j)}, true_{(i,j)}, false_{(i,j)}\} \text{ for } \\ 1 \leq i \leq n \text{ and } 1 \leq j \leq 3$$

$$C := \{ \{ \{ init_{(i+1,0)}, init_{(i,1)}, init_{(i,2)}, init_{(i,3)} \} \mid 1 \leq i \leq n \} \\ \cup \{ \{ set-to-1_{(i_1, j_1)}, set-to-1_{(i_2, j_2)}, \dots, set-to-1_{(i_a, j_a)} \} \mid \\ \exists \text{ variable } x \text{ that occurs in } l_{(i_1, j_1)}, \dots, l_{(i_a, j_a)} \text{ and only there} \} \\ \cup \{ \{ set-to-0_{(i_1, j_1)}, set-to-0_{(i_2, j_2)}, \dots, set-to-0_{(i_a, j_a)} \} \mid \\ \exists \text{ variable } x \text{ that occurs in } l_{(i_1, j_1)}, \dots, l_{(i_a, j_a)} \text{ and only there} \} \\ \cup \{ \{ false_{(i,0)}, false_{(i,1)}, false_{(i,2)}, false_{(i,3)} \} \mid 1 \leq i \leq n \} \\ \cup \{ \{ true_{(i,j)}, init_{(i+1,0)} \} \mid 1 \leq i \leq n, 1 \leq j \leq 3 \}$$

The local transition systems  $T_{(i,0)}$  for  $1 \leq i \leq n$  are given in Figure 1 (a). The local transition systems  $T_{(i,j)}$  for  $1 \leq i \leq n$ ,  $1 \leq j \leq 3$  and  $l_{(i,j)}$  is a positive (resp. negative) literal are given in Figure 1 (b) (resp. (c)).

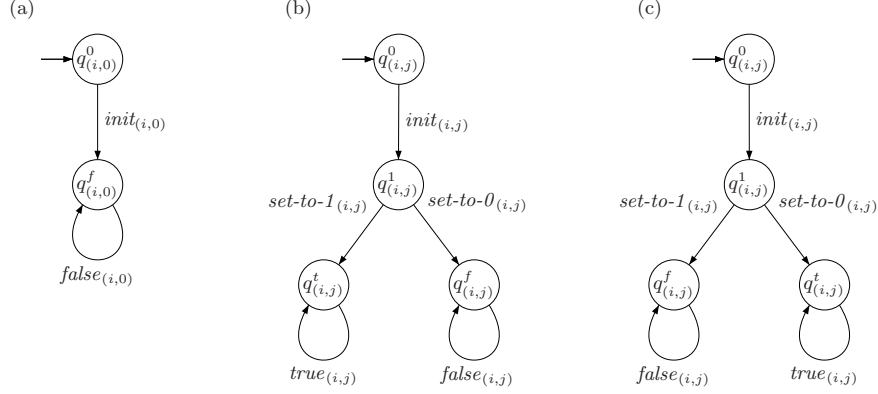
We call components  $(i, 0)$  clause-components and components  $(i, j)$  where  $1 \leq j \leq 3$  literal-components. For a component  $(i, j)$  we call the state  $q_{(i,j)}^f$  its *false-state* and, if it exists, the state  $q_{(i,j)}^t$  its *true-state*. We call both  $q_{(i,j)}^t$  and  $q_{(i,j)}^f$  *local final states*. We call a global state  $q \in Q$  *global final state*, if all components are in local final states in  $q$ .

There is a natural 1-to-1-correspondence between assignments and reachable global final states:

An assignment  $\sigma$  for  $F$  corresponds to the global final state  $q^{end} := state(\sigma)$ ,

where all clause-components are in their false-states (they have no other local final state) and any literal-component  $(i, j)$  that represents a literal of variable  $x$  with  $\sigma(x) = 1$  ( $\sigma(x) = 0$ ) is in the local final state that is reachable by the set-to-1-action (by the set-to-0-action).

A global final state  $q^{end}$  that is in fact reachable starting in  $q^0$  (i.e. all literal-components for the same variable have been set conjointly) corresponds to the assignment  $\sigma := ass(q^{end})$ , where for each variable  $x$ ,  $\sigma(x) = 1$  ( $\sigma(x) = 0$ ) if the literal-components in which  $x$  occurs are in their local final states that are reached by the set-to-1-action (by the set-to-0-action).



**Fig. 1.** The  $T_{(i,j)}$ 's for clause-components (a) and literal-components (b) & (c)

**Remark 1:**

Note that there is no blow-up in notation when we go from  $F$  to  $Sys(F)$ . The four transition systems we introduce for each clause are of constant size. Also, the set-to-1- and set-to-0-connectors have an overall size which is linear in the number of literals in  $F$  and the other  $(5n)$  connectors in  $C$  are of constant size.

**Proof:** ( $F$  is satisfiable)  $\Leftrightarrow$  ( $Sys(F)$  contains a global deadlock):

$\Rightarrow$ :

Let  $F = k_1 \wedge \dots \wedge k_n$  with  $k_i = (l_{(i,1)} \vee l_{(i,2)} \vee l_{(i,3)})$  be a satisfiable 3-KNF formula and let  $\sigma(F) = 1$  for an assignment  $\sigma$ .

The starting state of  $Sys(F)$  is  $q^0 := (q_{(1,0)}^0, q_{(1,1)}^0, q_{(1,2)}^0, q_{(1,3)}^0, q_{(2,0)}^0, \dots, q_{(n,3)}^0)$ .

Let  $Sys(F)$  perform the following transitions:

- 1) For all  $1 \leq i \leq n$  perform the interactions  $\{init_{(i+1,0)}, init_{(i,1)}, init_{(i,2)}, init_{(i,3)}\}$ .

Then all clause-components  $(i, 0)$  ( $1 \leq i \leq n$ ) are in their false-states  $q_{(i,0)}^f$  and all literal-components  $(i, j) \forall 1 \leq i \leq n, 1 \leq j \leq 3$ , are in their states  $q_{(i,j)}^1$ .

- 2) Let  $x$  be a variable that occurs in  $F$  at the positions  $(i_1, j_1), (i_2, j_2), \dots, (i_a, j_a)$  (and only there), and let  $\sigma(x) = 1$  (or  $\sigma(x) = 0$ , respectively).

Then perform the interaction  $\{set\text{-}to\text{-}1(i_1, j_1), set\text{-}to\text{-}1(i_2, j_2), \dots, set\text{-}to\text{-}1(i_a, j_a)\}$  (or  $\{set\text{-}to\text{-}0(i_1, j_1), set\text{-}to\text{-}0(i_2, j_2), \dots, set\text{-}to\text{-}0(i_a, j_a)\}$ , respectively).

After having performed the corresponding interaction for each variable that occurs in  $F$  we reached the global final state  $q^{end} = state(\sigma)$  that we described above.

As  $\sigma(F) = 1$  we have  $\sigma(k_i) = 1 \vee 1 \leq i \leq n$ , i.e. in each clause there is at least one literal that evaluates to 1 under  $\sigma$ . This means there is at least one positive literal  $l_{(i,j)} = x$  with  $\sigma(x) = 1$  or a negative literal  $l_{(i,j)} = \bar{x}$  with  $\sigma(x) = 0$ . In both cases the corresponding transition system  $T_{(i,j)}$  has reached its local state  $q_{(i,j)}^t$  (cf. Figure 1, (b) and (c)).

Hence, we have  $\forall 1 \leq i \leq n \ q^{end}(i, 0) = q_{(i,0)}^f$  and  $ea(q_{(i,0)}^f) = \{false_{(i,0)}\}$ .

Furthermore,  $\forall 1 \leq i \leq n \ \exists j \in \{1, 2, 3\}$ , s.t.  $q^{end}(i, j) = q_{(i,j)}^t$  and  $ea(q_{(i,j)}^t) = \{true_{(i,j)}\}$ .

Obviously,  $Sys(F)$  is in global deadlock in  $q^{end}$  (, or in other words  $D = K$  is in deadlock in  $q^{end}$  in  $Sys(F)$ ), as every clause-component  $(i, 0)$  waits for at least one of its literal-components  $(i, 1), (i, 2), (i, 3)$ . Those literal-components in  $(i, 1), (i, 2), (i, 3)$  that are in their  $q^f$ -states, also wait for those that are in their  $q^t$ -states and those that are in their  $q^t$ -states wait for the clause-component  $(i + 1, 0)$ . Hence, we observe a cyclic waiting over all clauses (cf. Example 1, Figure 3).

$\Leftarrow$ :

For all formulas  $F$  in 3-KNF and corresponding interaction systems  $Sys(F)$ , we show that, whenever there is  $D \subseteq K$  and a reachable global state  $q$ , such that  $D$  is in deadlock in  $q$  in  $Sys(F)$ , then  $D = K$  is in deadlock in  $q$  (or some  $q'$  reachable from  $q$ ) and  $F$  is satisfiable by some assignment  $\sigma = ass(q')$ .

**Remark 2:** ( $D$  is in deadlock in a reachable state  $q$  and  $(i, j) \in D$ )  
 $\Rightarrow ((i, j)$  is in a local final state).

Assume that  $(i, 0)$  ( $1 \leq i \leq n$ ) is part of a deadlock  $D$  and in its local non-final state  $q_{(i,0)}^0$ . Obviously in any case, the enabled  $init_{(i,0)}$ -action can be performed together with the  $init_{(i-1,j)}$ -actions of the corresponding literal-components, as those cannot have left their starting states, so  $(i, 0)$  can't be part of a deadlock. Assume that  $(i, j)$  ( $1 \leq i \leq n, 1 \leq j \leq 3$ ) is part of a deadlock  $D$  and in one of its local non-final states:

If  $(i, j)$  is in  $q_{(i,j)}^0$ , then the  $\{init_{(i+1,0)}, init_{(i,1)}, init_{(i,2)}, init_{(i,3)}\}$ -interaction can still be performed because the actions  $init_{(i,j)}$  ( $1 \leq j \leq 3$ ) occur in no other connector and the action  $init_{(i+1,0)}$  occurs in other connectors  $\{true_{(i,j)}, init_{(i+1,0)}\}$  but only together with the true-actions of the discussed components  $(i, j)$  which they do not offer until they have left their starting states which is not the case as we assumed that  $(i, j)$  is in  $q_{(i,j)}^0$ . So  $(i, j)$  can't be part of a deadlock and in particular  $(i, j)$  can still proceed to  $q_{(i,j)}^1$ .

If  $(i, j)$  is in  $q_{(i,j)}^1$ , then the  $set\text{-}to\text{-}1$ - or  $set\text{-}to\text{-}0$ -actions can still be performed in the future, because no other literal-component of the same variable can have

reached a local final state, because they can only transition conjointly (see Def. of  $C$ ). Also, any of these literal-components can proceed to  $q_{(i,j)}^1$  as explained above, if it isn't there already.

So  $(i, j)$  can still perform some action in the future and thus can't be part of a deadlock.

Now we may prove “ $\Leftarrow$ ”. Let  $q$  be a reachable state and let  $D$  be in deadlock in  $q$ . Then we show that  $D' = K$  is in deadlock in  $q$  or some global final state  $q'$  that is reachable from  $q$  and corresponds to an assignment  $\sigma = ass(q')$  with  $\sigma(F) = 1$ .

1) Let  $D \subseteq K$  be in deadlock in  $q$ . Then a literal-component  $(i, j)$  ( $1 \leq j \leq 3$ ) participates in  $D$  (because the clause-components do not communicate with each other directly).

2) Due to Remark 2,  $(i, j)$  must be in a final state. We show that at least one of the literal-components of clause  $i$  must be in its true-state: Assume that  $(i, j)$  is in  $q_{(i,j)}^f$  (else we are done). Then,  $ea(q_{(i,j)}^f) = \{false_{(i,j)}\}$ , which occurs in the connector  $\{false_{(i,0)}, false_{(i,1)}, false_{(i,2)}, false_{(i,3)}\}$ . Even if  $(i, 0) \in D$ ,  $(i, 0)$  would have to be in its local final state  $q_{(i,0)}^f$ , so  $(i, j)$  wouldn't wait for  $(i, 0)$ . Hence, one of the literal-components of clause  $i$  must participate in  $D$  and be in its true-state.

3) The literal-component of clause  $i$ , which is in its true-state can only wait for the clause-component  $(i + 1, 0)$ . So we have  $(i + 1, 0) \in D$  and  $(i + 1, 0)$  (due to Remark 2) has to be in its only local final state, i.e. its false-state.

4) As  $(i + 1, 0) \in D$  offers  $false_{(i+1,0)}$ , at least one of the literal-components of clause  $i + 1$  has to be in  $D$  and in its true-state. From here, we apply induction by going to 3) and conclude the same for all clauses.

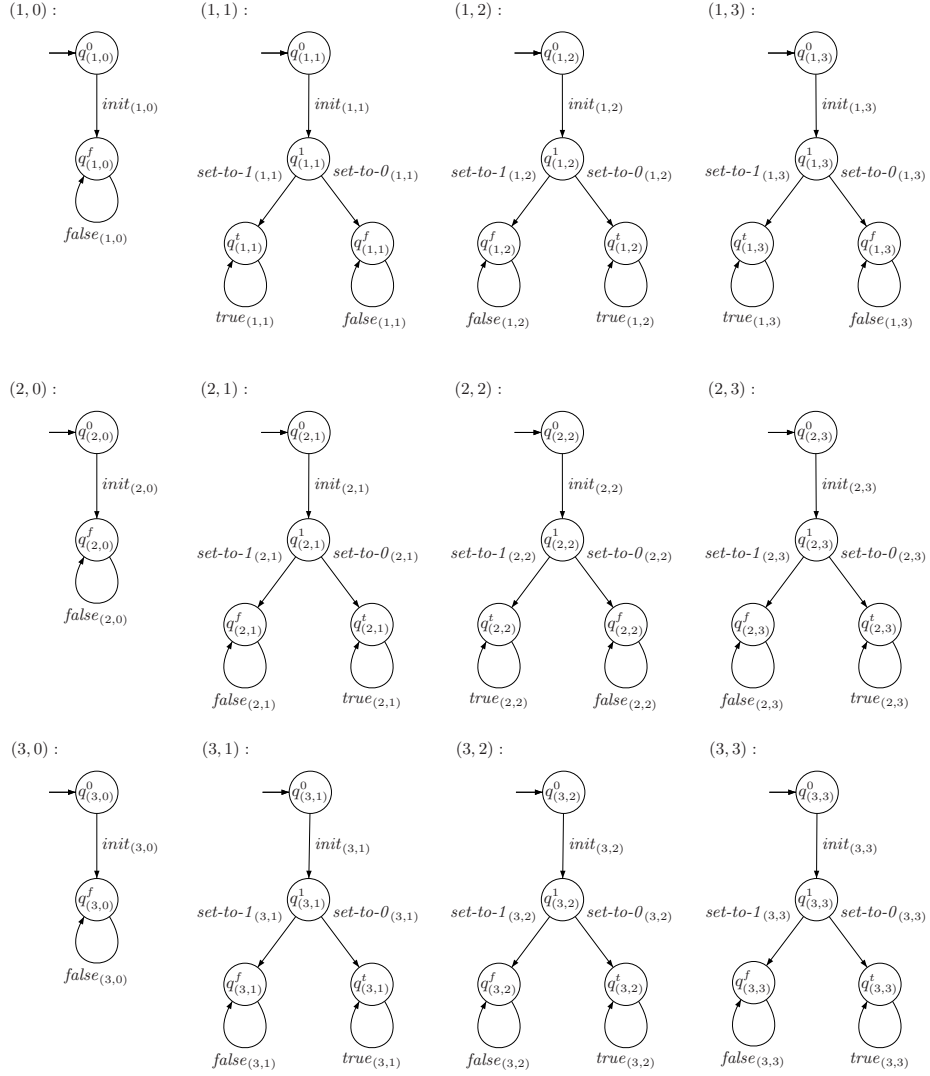
It is still possible that some variables have not yet been set to 0 or 1, i.e. the corresponding literal-components  $(\tilde{i}, \tilde{j})$  are not yet in their final states. It is however quite obvious, that we still may perform interactions such that these  $(\tilde{i}, \tilde{j})$  finally reach local final states. We call the thus reached state  $q'$  and in  $q'$ ,  $D = K$  is in global deadlock, because the  $(\tilde{i}, \tilde{j})$ , wait for components that participate in the cyclic waiting, no matter if  $q'(\tilde{i}, \tilde{j}) = q_{(\tilde{i}, \tilde{j})}^t$  or  $q'(\tilde{i}, \tilde{j}) = q_{(\tilde{i}, \tilde{j})}^f$ .

Due to the one-to-one correspondence of literal-components to literals and the fact that all occurrences of a variable  $x$  are consistently set to a value  $\in \{0, 1\}$  and the fact that in each clause, at least one literal evaluates to “true” we may conclude that  $\sigma(F) = 1$ .

**Example 1:** Let  $F = (x_1 \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3}) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_3})$ .

Then,  $F$  is satisfiable, namely  $\sigma(F) = 1$  for  $\sigma(x_1) = 1, \sigma(x_2) = 1, \sigma(x_3) = 0$ .

Consider the corresponding interaction system  $Sys(F) = (K, \{A_i\}_{i \in K}, C, \{T_i\}_{i \in K})$ , where  $K = \{(1, 0), (1, 1), (1, 2), (1, 3), (2, 0), \dots, (3, 3)\}$  and the port sets  $\{A_i\}_{i \in K}$  as well as the local transition systems  $\{T_i\}_{i \in K}$  can be seen from Figure 2.



**Fig. 2.** The local transition systems  $\{T_{(i,j)}\}_{(i,j) \in K}$  for Example 1

$$\begin{aligned}
C := & \{ \{ \text{init}_{(2,0)}, \text{init}_{(1,1)}, \text{init}_{(1,2)}, \text{init}_{(1,3)} \}, \{ \text{init}_{(3,0)}, \text{init}_{(2,1)}, \text{init}_{(2,2)}, \text{init}_{(2,3)} \}, \\
& \{ \text{init}_{(1,0)}, \text{init}_{(3,1)}, \text{init}_{(3,2)}, \text{init}_{(3,3)} \} \} \\
\cup & \{ \{ \text{set-to-1}_{(1,1)}, \text{set-to-1}_{(2,1)}, \text{set-to-1}_{(3,1)} \}, \\
& \{ \text{set-to-1}_{(1,2)}, \text{set-to-1}_{(2,2)}, \text{set-to-1}_{(3,2)} \}, \\
& \{ \text{set-to-1}_{(1,3)}, \text{set-to-1}_{(2,3)}, \text{set-to-1}_{(3,3)} \} \} \\
\cup & \{ \{ \text{set-to-0}_{(1,1)}, \text{set-to-0}_{(2,1)}, \text{set-to-0}_{(3,1)} \}, \\
& \{ \text{set-to-0}_{(1,2)}, \text{set-to-0}_{(2,2)}, \text{set-to-0}_{(3,2)} \}, \\
& \{ \text{set-to-0}_{(1,3)}, \text{set-to-0}_{(2,3)}, \text{set-to-0}_{(3,3)} \} \}
\end{aligned}$$

$$\begin{aligned}
& \{set\text{-to-}0_{(1,3)}, set\text{-to-}0_{(2,3)}, set\text{-to-}0_{(3,3)}\} \\
\cup & \{ \{false_{(1,0)}, false_{(1,1)}, false_{(1,2)}, false_{(1,3)}\}, \\
& \{false_{(2,0)}, false_{(2,1)}, false_{(2,2)}, false_{(2,3)}\}, \\
& \{false_{(3,0)}, false_{(3,1)}, false_{(3,2)}, false_{(3,3)}\} \} \\
\cup & \{ \{true_{(1,1)}, init_{(2,0)}\}, \{true_{(1,2)}, init_{(2,0)}\}, \{true_{(1,3)}, init_{(2,0)}\}, \\
& \{true_{(2,1)}, init_{(3,0)}\}, \{true_{(2,2)}, init_{(3,0)}\}, \{true_{(2,3)}, init_{(3,0)}\}, \\
& \{true_{(3,1)}, init_{(1,0)}\}, \{true_{(3,2)}, init_{(1,0)}\}, \{true_{(3,3)}, init_{(1,0)}\} \}
\end{aligned}$$

$$q^0 = (q_{(1,0)}^0, q_{(1,1)}^0, q_{(1,2)}^0, q_{(1,3)}^0, q_{(2,0)}^0, q_{(2,1)}^0, q_{(2,2)}^0, q_{(2,3)}^0, q_{(3,0)}^0, q_{(3,1)}^0, q_{(3,2)}^0, q_{(3,3)}^0)$$

As said above,  $F$  is satisfiable by  $\sigma$ , so we will show that  $Sys(F)$  can reach the global final state  $state(\sigma)$ , where  $D = K$  is in deadlock:

We subsequently perform the interactions  $\{init_{(i+1,0)}, init_{(i,1)}, init_{(i,2)}, init_{(i,3)}\}$  for all  $1 \leq i \leq 3$ .

Then, the clause-components  $(i, 0)$  are in their states  $q_{(i,0)}^f$  and the literal-components  $(i, j)$  are in their states  $q_{(i,j)}^1$ .

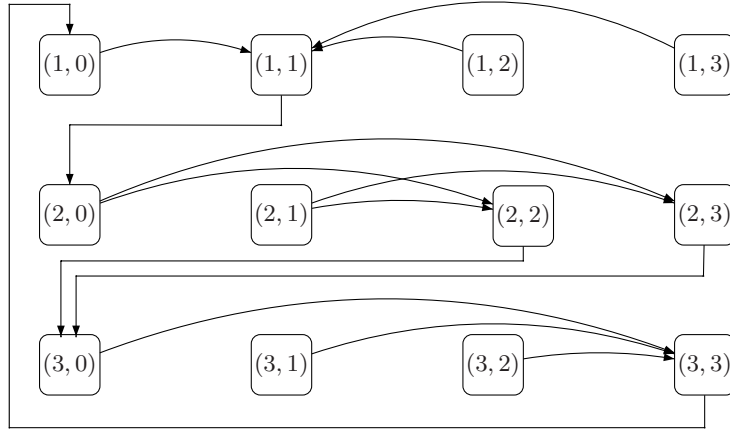
Now, we perform :

$\{set\text{-to-}1_{(1,1)}, set\text{-to-}1_{(2,1)}, set\text{-to-}1_{(3,1)}\}, \{set\text{-to-}1_{(1,2)}, set\text{-to-}1_{(2,2)}, set\text{-to-}1_{(3,2)}\}$   
and  $\{set\text{-to-}0_{(1,3)}, set\text{-to-}0_{(2,3)}, set\text{-to-}0_{(3,3)}\}$ .

Then,  $D = K$  is in deadlock in the global state

$$q^{end} = (q_{(1,0)}^f, q_{(1,1)}^t, q_{(1,2)}^f, q_{(1,3)}^f, q_{(2,0)}^f, q_{(2,1)}^f, q_{(2,2)}^t, q_{(2,3)}^t, q_{(3,0)}^f, q_{(3,1)}^f, q_{(3,2)}^f, q_{(3,3)}^t)$$

The global deadlock situation is displayed in Figure 3, where the nodes  $(i, j)$  represent the components (not their local states) and an edge from node  $(i_1, j_1)$  to  $(i_2, j_2)$  means that  $(i_1, j_1)$  waits for  $(i_2, j_2)$ .



**Fig. 3.** A graphical representation of the global deadlock in  $q^{end}$  in Example 1



## 4 Conclusion & Related Work

We showed that the questions of local and global deadlock are NP-hard for interaction systems, even without the use of complete interactions. This yields a motivation for deadlock-detection methods that work in polynomial-time such as the polynomial-time checkable sufficient condition for deadlock-freedom in interaction systems presented in [MMM07]. For the related model of Parallel Processes, Ladkin and Simons showed in [LS92] that deadlock-detection is NP-hard. A sufficient condition for liveness in interaction systems is given in [GGMMS07a]. A different approach to ensure deadlock-freedom and progress is given in [GGMMS07b].

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