Asset Pricing with Imperfect Competition and Endogenous Market Liquidity

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To My Parents

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Mannheim, September 2007 *Christoph Heumann*

Chapter 1

Introduction

The equilibrium expected return, or the required return, on financial assets is a central variable in financial economics, and understanding the determinants of required asset returns is the fundamental goal of asset pricing. The basic insight of traditional neoclassical asset pricing models is that the equilibrium expected return of an asset is increasing in the systematic risk of the asset, as risk-averse investors require compensation for bearing non-diversifiable risk. Neoclassical models, however, are based on the assumption that markets for financial assets are frictionless. Accordingly, trade in these markets is regarded as costless, and the actual process of trading and price formation is, in fact, left unmodeled. The starting point of this thesis is that real-world financial markets are not frictionless and that trading of financial assets can involve considerable costs. This raises the question of how market frictions affect required asset returns.

A market friction of major importance to investors is a lack of market liquidity. Roughly stated, market liquidity refers to the ease of trading financial assets, with more liquid markets having lower trading costs. Trading costs in illiquid markets include execution costs in the form of commissions, bid-ask spreads, and price impact, and also opportunity costs in the form of delayed and uncompleted trades. The effects of such costs on investment performance are often measured by the implementation shortfall proposed by Perold (1988), and these effects can be surprisingly large. For instance, Perold (1988) shows that a hypothetical "paper" portfolio based on the weekly stock recommendations of the Value Line Investment Survey has outperformed the market by almost 20% per year over the period from 1965-1986, while the corresponding real portfolio of the Value Line Fund has outperformed the market by only 2.5% per year—the difference representing the implementation shortfall.

When trading costs are important to investors, then a lack of market liquidity may also affect required asset returns. The standard result here traces back to Amihud and Mendelson (1986), who show that trading costs in illiquid markets reduce the price of an asset and, equivalently, increase the equilibrium expected return of an asset, as investors require compensation for bearing these costs. Hence, required asset returns should reflect both a risk premium as compensation for risk and a liquidity premium as compensation for illiquidity.

This pricing effect of illiquidity has become the conventional wisdom on the role of market frictions in asset pricing. For instance, Stoll (2000, p. 1483) argues that "*frictions must be reflected in lower asset prices so that the return on an asset is sufficient to offset the real cost of trading the asset, adjusted for the holding period.*" Following this line of argument, a simple back-of-the-envelope example illustrates the effect. For stocks on the New York Stock Exchange, the average effective bid-ask spread is approximately 2.2% and the average turnover of the outstanding shares is approximately 60% on an annual basis. Ignoring trading costs other than bid-ask spreads, these numbers suggest that the average required stock return should be increased by a liquidity premium of about $0.6 \cdot 2.2\% = 1.3\%$, which in turn would have a large effect on the level of stock prices.[1]

Market liquidity is an elusive concept, however, and how exactly illiquidity affects required asset returns remains a subject of considerable controversy and debate. As O'Hara (2003) points out, the principal difficulty for understanding the precise relationship between market liquidity and required asset returns is the separation of the related fields of research into asset pricing and market microstructure. Asset pricing, on the one hand, studies the macro determinants of asset prices, but gives only a limited representation of the underlying market interactions among investors. For example, asset pricing models in the line of Amihud and Mendelson (1986) just take trading costs as given and enforce trade among investors by imposing exogenous

[1]The example is taken from Garleanu and Pedersen (2004) with numbers on NYSE stocks from Chalmers and Kadlec (1998).

life-cycle motives. Market microstructure, on the other hand, analyzes (among other things) the determinants of trading costs based on a deeper view on the trading and price formation process, but does not consider the broader level of asset prices. For example, microstructure models examine how adverse selection problems among market participants affect the size of a bid-ask spread rather than where the level of the asset price and the spread is in the first place. O'Hara (2003, p. 1335) summarizes, "*asset pricing ignores the central fact that market microstructure focuses on: asset prices evolve in markets*," and highlights the need for a linkage of asset pricing and market microstructure in order to improve the understanding of how microstructure variables such as market liquidity affect equilibrium expected asset returns.

The contribution of this thesis is to study the role of market liquidity in asset pricing within a theoretical framework that integrates the asset pricing and the market microstructure view. Our framework allows to derive market liquidity, investors' trading behavior, and required asset returns endogenously and, hence, to analyze the pricing effect of illiquidity on the basis of an explicit microstructure foundation. We employ a CARA-Gaussian asset pricing setup with a single risky asset and incorporate market liquidity by allowing for imperfect competition. Investors trade by submitting demand functions for execution against each other, and each investor recognizes that he faces trading costs in the form of price impact, arising endogenously from the demand functions of the other agents. The resulting equilibrium is thus a Nash equilibrium in demand functions as in Kyle's (1989) noisy rational expectations model on information aggregation with imperfect competition. However, we use the framework in an asset pricing context, with investors trading under symmetric information to improve risk sharing.

Our results on market liquidity are consistent with the inventory-based strand of the microstructure literature. We find that investors' risk aversion and strategic trading behavior impair market liquidity and thereby generate trading costs. Investors respond to this lack of market liquidity by restricting trading volume in order to reduce their execution costs. As a consequence, investors incur opportunity costs, as their portfolio holdings of the risky asset do not achieve Pareto-optimal risk sharing.

Our main result is, however, that illiquidity does not affect the price and the required return of the risky asset. The reason is the endogeneity of trading behavior on both sides of the market. Because of trading costs, buyers restrict their demand to buy and sellers their supply to sell, and these effects cancel each other out with respect to the price and the required return of the asset. This cancelling out of individual trading costs turns out to be valid in different versions of our framework: in a static setting, in a dynamic setting in which investors repeatedly incur trading costs, and also in a dynamic setting in which illiquidity generates additional trading risks for investors. Although investors are thus increasingly exposed to costs of illiquidity, the required asset return reflects only a standard risk premium but no additional liquidity premium in each setting.

Our results on the pricing effect of illiquidity challenge the conventional wisdom and stand in contrast to the extensive literature on the existence of liquidity premia following Amihud and Mendelson (1986). Yet, there are some models with results and intuition similar to ours. Vayanos (1998) considers a competitive model with overlapping generations of long-lived investors and exogenous bid-ask spreads. He shows that the price of an asset may increase in its trading costs. Because of trading costs, new investors are less willing to buy the asset and older investors sell back their asset holdings more slowly, and it is possible that the effect from the seller side dominates the price. Garleanu and Pedersen (2004) present a model with strategic risk-neutral investors and endogenous bid-ask spreads arising from asymmetric information. They find that trading costs do not affect the asset price, as investors' profits from information-based trading outweigh their costs from non-information-based trading. Garleanu (2006) studies a competitive model in which illiquidity originates from search frictions. He shows that, due to illiquidity, buyers reduce their demand of the asset and sellers their supply of the asset, and these effects net out with regard to the asset price. Hence, in these models illiquidity does not affect asset prices in the standard way, because they also allow for a deeper microstructure view on investors' trading behavior. Our framework is complementary to these models, as we consider strategic trading of risk-averse investors and endogenize market liquidity by using arguments from the inventory-based microstructure literature.

The outline of this thesis is as follows. Chapter 2 introduces the CARA-Gaussian setup, presents the benchmark results in frictionless markets, and describes the microstructure view of market liquidity. Chapter 3 reviews the literature on market liquidity in asset pricing. Chapter 4 presents our static asset pricing setting with imperfect competition and endogenous market liquidity, and Chapter 5 extends the analysis to settings with dynamic and stochastic trading effects. Chapter 6 contains concluding remarks.

Chapter 2

Walrasian Asset Prices and Market Microstructure

Asset pricing theory has many of its foundations in the neoclassical paradigm of general equilibrium analysis in the tradition of the early works of Walras (1874), Pareto (1909), and Fisher (1930), and their extensions to models with time and uncertainty by Arrow (1953) and Debreu (1959). The basic view of the neoclassical paradigm is that self-interested agents meet in frictionless markets where their competitive and independent actions are coordinated by the price mechanism, so that mutually beneficial trade takes place. The actual process of price formation is not made explicit, however, but is attributed to the impersonal forces of the market in the spirit of Adam Smith's (1776) "invisible hand."

The question of how prices are formed is addressed in market microstructure theory. Its focus is on the interrelations between market frictions, institutional market rules, and strategic interactions among agents. In order to analyze the effects of frictions and strategic trading on equilibrium asset returns, it is therefore promising to integrate market microstructure features into asset pricing theory.

In this chapter, we provide the building blocks for our analysis of asset pricing with imperfect competition and endogenous market liquidity. Section 2.1 reviews the assumptions that constitute the neoclassical paradigm of frictionless markets. Section 2.2 presents a standard neoclassical asset pricing model, which serves as reference point and as workhorse model for subsequent chapters. Section 2.3 discusses the neoclassical view on the price formation process; a special emphasis is given to the absence of strategic interactions among agents, which is implied by the neoclassical assumption of perfect competition. Finally, Section 2.4 describes the market microstructure concept of market liquidity in detail.

2.1 The Walrasian Paradigm

Almost all of the issues addressed in neoclassical finance can be traced back to two independent key ideas: first, the concept of no-arbitrage introduced by Keynes (1923, ch. 3) relating to the interest rate parity theory of foreign exchange rates and by Modigliani and Miller (1958) in the context of optimal corporate capital structure; and second, the theory of portfolio selection developed by Markowitz (1952, 1959) and Tobin (1958). Based upon these elementary principles, some of the most seminal contributions in the theory of asset pricing (the subfield of finance that is concerned with the valuation of financial securities and derivatives) are the static capital asset pricing model (CAPM) by Treynor (1961), Sharpe (1964), Lintner (1965), and Mossin (1966), its extensions to dynamic settings by Merton (1973a) and Breeden (1979), the arbitrage pricing theory by Ross (1976), the theory of derivatives pricing by Black and Scholes (1973) and Merton (1973b), and the connection of asset prices to the production technology in the economy by Cox, Ingersoll, and Ross (1985a, 1985b).

The features that are common to neoclassical asset pricing models and that basically constitute the paradigm of neoclassical equilibrium analysis (hereafter also referred to as "Walrasian paradigm") are the assumptions about the economy and the conditions for equilibrium. In the following, these features are described in detail. The focus thereby is on models on absolute pricing, in which all securities are valued endogenously according to investors' endowments, preferences, and information—in contrast to models on relative pricing, in which some assets are valued in terms of exogenous prices of related assets. Furthermore, the description aims at asset pricing in secondary financial markets for the case that investors trade a given stock of outstanding securities motivated by risk sharing and that asset payoffs from firms' underlying production activities are exogenous. The following combination of assumptions is characteristic for the Walrasian paradigm.

Perfect Competition: Each investor acts as price taker, i.e., each investor decides on his individual trading actions as if his actions do not affect the resulting prices. The assumption of price-taking behavior is usually motivated as a reasonable approximation to situations when there is a large number of investors in the market and each investor is small relative to the rest of the market.[1]

Perfect Capital Markets: Every asset can be traded without costs and without delay at any instant in time. There are no taxes. Assets are perfectly divisible, and short selling is allowed without restrictions. Prices are freely observable and identical for all investors.

Symmetric Information: No investor has an informational advantage over any other agent, and all investors have identical information about asset payoffs (homogenous expectations). Each investor knows his own characteristics (endowments, preferences), but does not observe the characteristics of the other agents.

Rational Investors: Each investor decides on his individual trading actions with the objective to maximize his utility, given his endowments and information. Investors derive utility from lifetime consumption of a single consumption good or, as assumed in the following, from terminal wealth. For decisions under uncertainty, each investor is risk averse and makes his decision on the basis of mean and variance of terminal wealth. Consistency of mean-variance preferences with the standard expected utility approach in the von Neumann-Morgenstern sense can be assured by assuming quadratic utility functions or by assuming normally distributed terminal wealth.[2]

Given this combination of assumptions, a neoclassical equilibrium ("Walras equilibrium") is defined by the conditions that (*i*) each investor selects an optimal portfolio of assets, and (*ii*) prices are determined such that all markets clear. As the Walrasian paradigm studies market outcomes in completely frictionless markets, it provides a useful benchmark for the analysis of asset prices under more realistic conditions where trading costs and other market frictions interfere with the actions of investors.

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[1] Aumann (1964) shows that the effect of each individual agent on prices becomes mathematically negligible when agents are represented as points in a continuum.

[2] See Ingersoll (1987, ch. 4) or Huang and Litzenberger (1988, ch. 3).

2.2 A CARA-Gaussian CAPM

The CAPM is the most prominent neoclassical asset pricing model and a cornerstone of modern finance. It provides an equilibrium analysis of the prices of risky assets and the structure of investors' portfolio holdings.[3] In this section, we present the CAPM in a static CARA-Gaussian version, which serves as a reference point and a workhorse model for the analysis in subsequent chapters. In a static model, each agent acts only once and all agents act at the same time. Thus, all investors meet in a single round of trading and thereafter simply hold their portfolios, receive terminal asset payoffs, and then consume their terminal wealth. Furthermore, the CARA-Gaussian setting places restrictions on investor preferences and asset payoffs. If w^i denotes terminal wealth to be consumed by investor i, then the following two assumptions are made.

- Each investor i has a negative exponential (CARA) utility function over terminal wealth, $u^i(w^i) = -\exp(-\rho^i w^i)$, with a constant Arrow-Pratt index of absolute risk aversion,

  $$-\frac{\partial^2 u^i / \partial (w^i)^2}{\partial u^i / \partial w^i} = \rho^i.$$

  The inverse of the absolute risk aversion measure is referred to as risk tolerance. The CARA utility function belongs to the class of utility functions for which risk tolerance is linear in wealth (also called hyperbolic absolute risk aversion (HARA) utility functions),

  $$-\frac{\partial u^i / \partial w^i}{\partial^2 u^i / \partial (w^i)^2} = a^i + b^i w^i. \tag{2.1}$$

  Clearly, CARA utility satisfies condition (2.1) with $a^i = 1/\rho^i$ and $b^i = 0$.

- The payoffs of all risky assets are jointly normally distributed (Gaussian), and terminal wealth generated by the portfolio of investor i is thus also normally distributed, $\tilde{w}^i \sim \mathcal{N}(E[\tilde{w}^i], \mathrm{Var}[\tilde{w}^i])$.

[3] The assessment of the validity of the CAPM remains controversial, however; Jagannathan and McGrattan (1995) and Fama and French (2004) summarize the debate on the CAPM.

With exponential utility and normally distributed wealth, the expected utility function over wealth becomes[4]

$$\begin{aligned} E\left[u^i\left(\tilde{w}^i\right)\right] &= \frac{1}{\sqrt{2\pi \mathrm{Var}\left[\tilde{w}^i\right]}} \int_{-\infty}^{\infty} -\exp\left[-\left(\rho^i w^i + \frac{\left(w^i - E\left[\tilde{w}^i\right]\right)^2}{2\mathrm{Var}\left[\tilde{w}^i\right]}\right)\right] dw^i \\ &= -\exp\left[-\rho^i\left(E\left[\tilde{w}^i\right] - \frac{\rho^i}{2}\mathrm{Var}\left[\tilde{w}^i\right]\right)\right]. \end{aligned}$$

Maximizing expected utility is thus equivalent to maximizing the mean-variance objective function $E\left[\tilde{w}^i\right] - \rho^i \mathrm{Var}\left[\tilde{w}^i\right]/2$.

The main motivation for the CARA-Gaussian framework is analytical tractability, which allows us to obtain closed-form solutions and to trace asset prices back to their underlying economic determinants. Furthermore, the resulting linear demand functions will be crucial for the analysis of strategic interactions among investors as carried out in later chapters of this thesis. The drawbacks of the setting are that (i) exponential utility implies that—when rates of returns on assets are given and riskless borrowing and lending is possible without restrictions—an investor's demand for risky assets does not depend on his initial wealth, and (ii) normally distributed payoffs can become negative with strictly positive probability and have unlimited downside liability. Both properties may be questioned regarding their descriptive realism.

The CARA-Gaussian version of the standard CAPM goes back to Lintner (1970). In the following, we briefly review the model and its main results; as the results are well-known, proofs are omitted.[5]

2.2.1 Description of the Economy

We consider a static framework with two risky assets (indexed by $k = 1, 2$) and a riskless asset. At the beginning of the period, assets are traded among investors with p_k denoting the price of risky asset k and p_f the price of the riskfree asset. At the end of the period, assets pay off exogenous liquidation values, and investors consume

[4]The proof can be found in Sargent (1987, pp. 154-5).

[5]Stapleton and Subrahmanyam (1978) extend the CARA-Gaussian framework to a dynamic CAPM. Lintner (1969) is an early application of the setting to a CAPM with heterogeneous expectations. Bamberg (1986) gives a detailed discussion of the economic rationale of the CARA-Gaussian approach. For presentations of standard asset pricing theory in more general settings, see e.g. Ingersoll (1987), Huang and Litzenberger (1988), LeRoy and Werner (2001), or Cochrane (2005).

their terminal wealth. The per-share payoff of each risky asset k is given by $\tilde{v}_k$, where the $\tilde{v}_k$ are normally distributed with mean $E[\tilde{v}_k]$ and covariance $\mathrm{Cov}[\tilde{v}_1, \tilde{v}_2]$. Each unit of the riskfree asset yields a payoff R. The total number of outstanding shares of risky asset k and of the riskless asset is denoted by $\bar{X}_k$ and $\bar{Y}$, respectively.

In the economy, there are I rational investors (indexed by $i = 1, 2, \ldots, I$) with homogenous expectations about asset payoffs. Each investor i is initially endowed with $\bar{y}^i$ units of the riskless asset and $\bar{x}_k^i$ units of risky asset k (where $\sum_{i=1}^{I} \bar{y}^i \equiv \bar{Y}$ and $\sum_{i=1}^{I} \bar{x}_k^i \equiv \bar{X}_k$) and hence has initial wealth

$$\bar{w}^i = p_f \bar{y}^i + p_1 \bar{x}_1^i + p_2 \bar{x}_2^i.$$

If y^i and x_k^i are the corresponding portfolio holdings after trade has taken place, then investor i's terminal wealth follows as

$$\tilde{w}^i = R y^i + \tilde{v}_1 x_1^i + \tilde{v}_2 x_2^i.$$

Each agent has a CARA utility function with a coefficient of absolute risk aversion $\rho^i > 0$. As terminal wealth is normally distributed, the objective for each investor is to choose his portfolio holdings to maximize

$$\Pi^i\left(y^i, x_1^i, x_2^i\right) := E\left[\tilde{w}^i\right] - \frac{\rho^i}{2} \mathrm{Var}\left[\tilde{w}^i\right]$$

subject to his budget constraint. Each investor takes the prices of all assets as given, and markets for all assets are perfect.

2.2.2 Walras Equilibrium

To make the concept of Walras equilibrium formal, let $\mathbb{I} = \{1, 2, \ldots, I\}$ denote the set of investors, $(\bar{y}^i, \bar{x}_1^i, \bar{x}_2^i)$ the initial endowments and Π^i the objective function for investor i, and let $\mathcal{E} = \left\{\mathbb{I}, \left\{\left(\bar{y}^i, \bar{x}_1^i, \bar{x}_2^i\right), \Pi^i\right\}_{i \in \mathbb{I}}\right\}$ denote the associated economy to be studied. Furthermore, denote by (y^i, x_1^i, x_2^i) the asset holdings of investor i after trade has taken place, by $\{(y^i, x_1^i, x_2^i)\}_{i \in \mathbb{I}}$ an allocation of assets among investors, and by (p_f, p_1, p_2) a system of prices. Then we have the following definition.

Definition 1 *A Walras equilibrium of economy $\mathcal{E}$ is an allocation $\left\{\left(y^{i,*}, x_1^{i,*}, x_2^{i,*}\right)\right\}_{i \in \mathbb{I}}$ and a price system $\left(p_f^*, p_1^*, p_2^*\right)$ such that the following conditions are satisfied:*

(*i*) *Individual Optimization: For each investor* $i \in \mathbb{I}$, *asset holdings* $\left(y^{i,*}, x_1^{i,*}, x_2^{i,*}\right)$ *solve*

$$max\ \Pi^i\left(y^i, x_1^i, x_2^i\right) \quad s.t.\ \ p_f^*\,\bar{y}^i + p_1^*\,\bar{x}_1^i + p_2^*\,\bar{x}_2^i = p_f^*\,y^i + p_1^*\,x_1^i + p_2^*\,x_2^i.$$

(*ii*) *Market Clearing:*[6] *Prices* $\left(p_f^*, p_1^*, p_2^*\right)$ *adjust to solve*

$$\sum_{i=1}^{I} y^{i,*}\left(p_f, p_1, p_2\right) = \bar{Y} \ \ and \ \ \sum_{i=1}^{I} x_k^{i,*}\left(p_f, p_1, p_2\right) = \bar{X}_k, \quad k = 1, 2.$$

Before we turn to the Walras equilibrium, we first consider the individually optimal demand function for an investor i, which specifies the optimal number of shares to hold in his portfolio for exogenous asset prices.

Optimal Asset Demand: Optimal demand for the two risky assets is given by

$$\begin{bmatrix} x_1^{i,*}\left(p_f, p_1, p_2\right) \\ x_2^{i,*}\left(p_f, p_1, p_2\right) \end{bmatrix} = \frac{1}{\rho^i} \begin{bmatrix} \mathrm{Var}\left[\tilde{v}_1\right] & \mathrm{Cov}\left[\tilde{v}_1, \tilde{v}_2\right] \\ \mathrm{Cov}\left[\tilde{v}_1, \tilde{v}_2\right] & \mathrm{Var}\left[\tilde{v}_2\right] \end{bmatrix}^{-1} \begin{bmatrix} E\left[\tilde{v}_1\right] - \dfrac{R\,p_1}{p_f} \\ E\left[\tilde{v}_2\right] - \dfrac{R\,p_2}{p_f} \end{bmatrix},$$

and optimal demand for the riskless asset, represented in terms of dollar demand, is

$$p_f\,y^{i,*}\left(p_f, p_1, p_2\right) = \bar{w}^i - p_1\,x_1^{i,*}\left(\cdot\right) - p_2\,x_2^{i,*}\left(\cdot\right).$$

For the CARA-Gaussian setting, demand for each risky asset is linear in the prices of all risky assets, but does not depend on initial wealth of investor i (or, more precisely, demand for exogenous prices does not depend on initial endowments). Accordingly, changes in initial wealth (endowments) are completely absorbed by riskless borrowing or lending. Furthermore, demand for risky assets satisfies Tobin's (1958) separation theorem, which states that, for exogenous prices, an investor's risk aversion and initial wealth are reflected only in the size of the optimal portfolio of risky assets but not in its structure, where the structure of the portfolio is specified by

$$\psi_k^{i,*}\left(p_f, p_1, p_2\right) := \frac{p_k\,x_k^{i,*}\left(\cdot\right)}{p_1\,x_1^{i,*}\left(\cdot\right) + p_2\,x_2^{i,*}\left(\cdot\right)}, \quad k = 1, 2.$$

[6] To avoid terminological ambiguity, we note that the market-clearing condition for, say, asset k can be written in two ways. The first way is $\sum_i x_k^{i,*}\left(\cdot\right) = \bar{X}_k$, which says that investors' aggregate "demand to hold shares" must be equal to the total "stock of outstanding shares" (aggregate endowment). The second way is $\sum_i \left(x_k^{i,*}\left(\cdot\right) - \bar{x}_k^i\right) = 0$, which says that investors' aggregate "demand to buy shares" (excess demand) must be equal to zero—where a negative demand to buy is a "supply to sell shares." As $\bar{X}_k \equiv \sum_i \bar{x}_k^i$, the two conditions are equivalent, of course.

Thus, the problem of optimal asset demand can be separated into a two-stage process: first, selecting the optimal structure of the portfolio of risky assets; and second, determining how much wealth to invest into risky assets overall and how much to borrow or lend at the riskless rate of interest. Under Tobin separation, only the second step of the demand decision depends on individual investor characteristics. Hence, the first step of the decision can be made independently of the specific investor characteristics and also independently of the second step.[7]

Now we turn to the Walras equilibrium of the economy. Allingham (1991) and Hens, Laitenberger, and Löffler (2002) show that both the joint assumptions of CARA utility and normally distributed payoffs (without assumption on the presence of a riskless asset) and the joint assumptions of CARA utility and presence of a riskless asset (without assumption on payoff distributions) are sufficient to guarantee a unique equilibrium in the CAPM. Hence, a unique equilibrium does also exist in the economy $\mathcal{E}$ to be studied here.

Equilibrium Prices: Equilibrium prices of economy $\mathcal{E}$ are

$$\frac{p_k^*}{p_f^*} = \frac{1}{R}\left(E\left[\tilde{v}_k\right] - \frac{\operatorname{Cov}\left[\tilde{v}_k, \tilde{v}_1\bar{X}_1 + \tilde{v}_2\bar{X}_2\right]}{\sum_{i=1}^{I}(1/\rho^i)}\right), \quad k = 1, 2. \tag{2.2}$$

The price of a risky asset is given by the expected payoff of the asset minus a risk premium, discounted at the riskfree rate of interest. The risk premium, in turn, is determined as the covariance of the per-share payoff of that asset with the total payoff of all risky assets in the economy, divided by the aggregate risk tolerance of the economy. As all investors obey their budget constraints as an accounting identity, market clearing for all but one market implies market clearing also for the remaining market (Walras' law). As a consequence, it is possible to determine prices only for all but one asset in relation to the remaining asset (numeraire asset). In the following, we choose the riskless asset as numeraire (i.e., we set $p_f = 1$) and denote the riskless rate of interest by $r := R/p_f - 1 = R - 1$.

[7] Hakansson (1969) and Cass and Stiglitz (1970) show that, in the presence of a riskless asset, Tobin separation holds (under more general payoff distributions than the Gaussian), if and only if the investor has a utility function with linear risk tolerance.

Equilibrium Allocation: The equilibrium allocation of economy $\mathcal{E}$ is

$$\begin{aligned} x_k^{i,*} &= \frac{(1/\rho^i)}{\sum_{i=1}^I (1/\rho^i)}\,\bar{X}_k \qquad \text{for all } i \text{ and all } k, \qquad (2.3)\\ y^{i,*} &= \bar{y}^i + p_1^*\,\bar{x}_1^i + p_2^*\,\bar{x}_2^i - p_1^*\,x_1^{i,*} - p_2^*\,x_2^{i,*} \quad \text{for all } i. \qquad (2.4)\end{aligned}$$

All outstanding risky assets are allocated among investors according to the proportion of each individual investor's risk tolerance to the aggregate risk tolerance of the economy. The portfolio holdings of the riskless asset simply absorb the remaining wealth not invested into risky assets; thus, we skip the closed-form representation for the riskless holdings.

Prices (2.2) and allocation (2.3) and (2.4) constitute the unique Walras equilibrium of economy $\mathcal{E}$. In the next subsection, we review the main properties of this equilibrium.

2.2.3 Equilibrium Properties

Here we describe the main properties of the Walras equilibrium of economy $\mathcal{E}$.[8] Several of these properties do not hold in more general settings of the incomplete-markets economy of the CAPM, but are known to hold for economies that are at least effectively complete. Rubinstein (1974) provides sets of sufficient conditions for effective completeness. In particular, effectively complete economies encompass economies that fulfill the following three conditions: (i) a riskless asset exists; (ii) all endowments are tradable; and (iii) all investors have utility functions with linear risk tolerance (as in (2.1)) and with identical coefficients of marginal risk tolerance ($b^i = b$ for all i). The conditions $(i) - (iii)$ are satisfied by economy $\mathcal{E}$ to be studied here.[9]

Mutual Fund Theorem: For each investor i, the structure of the risky asset portfolio is given by

$$\begin{aligned} \psi_k^i &:= \frac{p_k\,x_k^i}{p_1 x_1^i + p_2 x_2^i}\\ &= \frac{E\left[\tilde{v}_k\right]\bar{X}_k \sum_{i=1}^I (1/\rho^i) - \mathrm{Cov}\left[\tilde{v}_k\bar{X}_k, \tilde{v}_1\bar{X}_1 + \tilde{v}_2\bar{X}_2\right]}{\left(E\left[\tilde{v}_1\right]\bar{X}_1 + E\left[\tilde{v}_2\right]\bar{X}_2\right)\sum_{i=1}^I (1/\rho^i) - \mathrm{Var}\left[\tilde{v}_1\bar{X}_1 + \tilde{v}_2\bar{X}_2\right]},\ k = 1, 2. \qquad (2.5)\end{aligned}$$

[8]As all properties refer to equilibrium, we drop the stars on endogenous variables for notational simplicity.

[9]Geanakoplos and Shubik (1990) examine the CAPM in a much more general framework of a full-blown general equilibrium model with incomplete markets.

Equation (2.5) reveals that the structure of each investor's portfolio of risky assets depends on the individual risk preferences of all agents, but for all investors in the same way (on the aggregate risk tolerance of the economy). Thus, we have

$$\psi_k^i \;=\; \psi_k \qquad \text{for all } i \text{ and all } k,$$

i.e., the structure of risky asset holdings is the same for all investors, even if investors differ in individual risk aversion or initial wealth. It is as if all risky assets were arranged in a single mutual fund, each risky asset with the proportion ψ_k, and each investor holds a fraction of this fund. Rubinstein (1974) shows that conditions (i) – (iii) are sufficient for the mutual fund theorem to hold.

Market Portfolio: An important insight of the CAPM is that the pricing-relevant risk of an asset is not given by the variance of its payoff but rather by the covariance of its payoff with the payoff of a particular portfolio that contains all risky assets of the economy. The pricing-relevant risk is called systematic risk, and the particular portfolio is called market portfolio. If the structure of the market portfolio is denoted by

$$\Psi_k := \frac{p_k \bar{X}_k}{p_1 \bar{X}_1 + p_2 \bar{X}_2}, \qquad k = 1, 2,$$

then we have

$$\Psi_k = \psi_k, \qquad k = 1, 2.$$

Hence, the structure of the market portfolio coincides with the structure of the risky asset portfolio of each investor. This is the case, as all investors have the same optimal structure of risky asset holdings and all investors hold all outstanding risky assets in their portfolios altogether. The structure of the market portfolio is mean-variance efficient, i.e., risky asset portfolios with this structure (held alone or combined with riskless borrowing or lending) have maximum expected return for a given level of risk. If the payoff and the price of the market portfolio are denoted by $\tilde{V}_M := \tilde{v}_1 \bar{X}_1 + \tilde{v}_2 \bar{X}_2$ and $P_M := p_1 \bar{X}_1 + p_2 \bar{X}_2$, respectively, then

$$\Lambda := \frac{E\left[\tilde{V}_M\right] - (1+r)\, P_M}{\operatorname{Var}\left[\tilde{V}_M\right]} = \left(\sum_{i=1}^{I} (1/\rho^i)\right)^{-1} \tag{2.6}$$

is Lintner's (1970) version of the market price of (dollar) risk. The market price of risk in the CARA-Gaussian setting is thus equal to the inverse of the aggregate risk tolerance of the economy.[10]

Pareto Optimality (Pareto Efficiency): The allocation of assets among investors is Pareto optimal, i.e., it is not possible to redistribute assets to make some investor strictly better off without making some other investor strictly worse off. That the risk-tolerance-weighted allocation rule in (2.3) provides the unique Pareto-optimal allocation for the risky assets is known from the analysis of risk sharing among groups by Wilson (1968); accordingly, the Pareto-optimal allocation is also referred to as optimal (efficient) risk sharing. However, that the Pareto-optimal allocation is established in the CAPM is not self-evident, as Pareto optimality for Walras equilibria from the first welfare theorem requires a complete-markets economy. With incomplete markets, in contrast, the allocation is not Pareto optimal in general, but only in certain special cases. Rubinstein (1974) shows that conditions $(i)-(iii)$ are sufficient for Pareto optimality to be achieved in incomplete-market economies. Thus, Pareto optimality holds for the economy $\mathcal{E}$ studied here. The establishing of Pareto optimality within the single round of trading in the static CAPM implies that, even

[10]The more familiar version of the CAPM using rates of returns rather than prices can now be obtained as follows. First, let the return on an asset (or on a portfolio of assets) be denoted $\tilde{r} := \tilde{v}/p - 1$, such that

$$E\left[\tilde{v}_k\right] = p_k\left(1 + E\left[\tilde{r}_k\right]\right), \tag{2.7}$$

$$\mathrm{Cov}\left[\tilde{v}_k, \tilde{v}_j\right] = p_k p_j \mathrm{Cov}\left[\tilde{r}_k, \tilde{r}_j\right]. \tag{2.8}$$

Next, a more standard version of the market price of risk is defined by $\Lambda' := \Lambda\, P_M$ and can be derived by rewriting (2.6) and using (2.7) and (2.8) to obtain

$$\Lambda' = \frac{\left(E\left[\tilde{V}_M\right] - (1+r)P_M\right)\ P_M^2}{P_M\ \mathrm{Var}\left[\tilde{V}_M\right]} = \frac{E\left[\tilde{r}_M\right] - r}{\mathrm{Var}\left[\tilde{r}_M\right]}. \tag{2.9}$$

Finally, the standard security market line can be derived by substituting (2.7) and (2.8) into (2.2) and using (2.6) and (2.9) to obtain

$$E\left[\tilde{r}_k\right] = r + \frac{\left(E\left[\tilde{r}_M\right] - r\right)\mathrm{Cov}\left[\tilde{r}_k, \tilde{r}_M\right]}{\mathrm{Var}\left[\tilde{r}_M\right]}, \quad k = 1, 2. \tag{2.10}$$

Note that (2.9) is not an explicit (closed-form) solution for the market price of risk (as $\tilde{r}_M$ depends on the prices of all risky assets), and hence the security market line (2.10) and all other pricing relations based upon (2.9) are also merely implicit representations. For a comparative static analysis of the market price of risk for other utility functions than the exponential, see Rubinstein (1973).

if markets were re-opened at later dates for additional rounds of trading among the same investors (as in a dynamic model), no additional trades would occur, as there is no potential for additional mutually beneficial trading left.

Representative Investor: As the allocation of assets is Pareto optimal, asset prices in economy $\mathcal{E}$ are the same as in an economy with a single, suitably constructed representative investor. Deriving asset prices via a representative agent economy provides a means of analytical simplification.[11] In general, however, the construction of the representative agent depends on the initial distribution of endowments among investors in the underlying economy. Hence, redistributions of endowments change the characteristics of the representative agent and also change asset prices. Once again, Rubinstein (1974) shows that conditions $(i) - (iii)$ are sufficient to have an exceptional case where representative agent and asset prices are independent of the initial distribution of endowments among investors (aggregation property). In particular, the representative investor for the CARA-Gaussian economy $\mathcal{E}$ is characterized by the risk tolerance $1/\rho^{\text{repr}} = \sum_{i=1}^{I} (1/\rho^i)$.

To conclude the discussion, we note that the sufficiency of Rubinstein's (1974) conditions $(i) - (iii)$ for several of the above equilibrium properties relies on the frictionless setting of the Walrasian paradigm. That is, mutual fund theorem, Pareto optimality, and representative investor pricing with aggregation property do not necessarily carry over to economics that satisfy $(i) - (iii)$, but exhibit trading costs or other market frictions.

2.3 The Myth of the Walrasian Auctioneer

In the Walrasian paradigm, which underpins neoclassical asset pricing and which also underpins neoclassical economics as best represented by the Arrow-Debreu model of general equilibrium (GE), prices play a crucial role: prices equate demand and supply on all markets and reflect the various marginal rates of substitution; prices are freely observable and summarize all relevant information needed by agents in order to decide what their optimal actions are; and prices are the coordinating force that aligns the

[11] Note that representative investor analysis provides a simplified pricing technique, but is not to be taken literally, of course. If there were de facto only a single investor, then there could not be trade and hence there would be no need for prices (i.e. trading ratios) in the first place.

competitive and independent actions of the different agents, so that mutually welfare-increasing trade can take place.

What is missing in the Walrasian paradigm, however, is an explicit mechanism by which these prices are established. While prices are derived analytically by solving a simultaneous equation system for the economy—and conditions under which such a solution exists have been examined with considerable mathematical effort and rigor—it remains unspecified who or what, exactly, in the economy enforces this system of prices and how neoclassical equilibrium is actually attained. Accordingly, neoclassical models point out the logical possibility of equilibrium existence, but do not show how, or if, equilibrium will actually be reached on the basis of any price formation process in the market. Yet, the question of how equilibrium is established is important in at least two respects.

First, the abstraction from an explicit price mechanism involves the implicit assumption that, whatever mechanism is actually employed, it has no effect on the market outcomes to be studied. Similarly, potential frictions associated with the actual price mechanism are also assumed to have no effects on the outcomes of the market. This aspect provides the focus for the asset pricing analysis in subsequent chapters of this thesis. The main question will be: how do frictions in the trading and price formation process affect the equilibrium expected return on risky assets, when investors interact under an explicit set of market rules?

Second, the abstraction from an explicit price mechanism is important from a methodological point of view. The ultimate purpose for studying equilibrium models is, of course, to gain understanding of causal relations between economic variables in the real world. This approach is based on the premise that the variables in the real world will ultimately reach, or at least gravitate towards, the equilibrium state of the model. As pointed out by von Hayek (1937, p. 44), *"it is only with this assertion [of a tendency to equilibrium] that economics ceases to be an exercise in pure logic and becomes an empirical science."* As long as the tendency towards equilibrium is merely an assertion, however, any insights drawn from neoclassical equilibrium models rest on a very uncertain foundation, as the adjustment of the relevant variables towards equilibrium is, a priori, not self-evident. This line of objection leads several authors to a skeptical appraisal of the neoclassical approach. Blaug (1992, p. 165), for instance, argues that *"the empirical content of GE theory is nil,"* and Schneider (2001, p. 378) deprives neoclassical economics of its economic content by paraphrasing it as *"wirtschaftliche Namen verwendende Mathematik."*[12]

The question of how an economy reaches equilibrium is well recognized in neoclassical economics; it can be traced back to Walras (1874) and his notion of a fictitious auctioneer who establishes equilibrium via a process of tâtonnement or "groping." In this Walrasian auction, all agents of the economy participate in a series of preliminary bidding rounds that is supposed to converge to equilibrium. First, an auctioneer calls out tentative prices for all markets, and agents specify their optimal demand and supply quantities at these prices. The auctioneer then aggregates the ordered quantities for each market and revises his prices according to what is dubbed the "law of demand and supply:" if there is excess demand in a market, he raises the price; if there is excess supply, he lowers the price. Subsequently, the auctioneer calls out the revised but still tentative prices, and agents have a chance to re-optimze their demand and supply orders. This trial-and-error process of price (and quantity) adjustments is continued until the auctioneer eventually finds a system of prices such that demand equals supply for each market. Only after this system of prices has been found, actual trading occurs. As the prices clear all markets simultaneously and the quantites traded at these prices are individually optimal for each agent, the conditions for neoclassical equilibrium are finally fulfilled.[13]

A characteristic feature of the Walrasian paradigm and the imaginary auction mechanism is the complete absence of market frictions: there are no costs for participating in the auction, setting prices, and processing trades; there are no informational problems regarding the traded objects or the trading counterparties; and there are no incentive conflicts to comply with the agreed conditions of the trades. Although this frictionless setting is apparently an oversimplification for most real-world markets (e.g. labor markets or credit markets), it is often argued to be a well-suited

[12]For more methodological discussion on neoclassical economics, see Blaug (1992, ch. 8) and Schneider (2001, pp. 349-78). A related discussion with a focus on equilibrium models in neoclassical finance is given by the dispute between Krahnen (1987) and Schneider (1987).

[13]To be accurate, we note that Walras' presentations of the tâtonnement process vary among the different editions of his "Elements," and that conflicting interpretations of the process can be found in the literature; see Walker (1987). An alternative mechanism for attaining neoclassical equilibrium is proposed by Edgeworth's (1881) notion of recontracting.

approximation for securities markets, in which trading is conducted similarly to a multilateral and highly organized auction mechanism. Accordingly, the use of the Walrasian framework, at least in an "as if" sense, seems a plausible and justified approach for the theory of asset pricing.

The Walrasian framework fails, however, to account for several important phenomena that can be observed in real-world securities markets. For instance, the existence of intermediaries (such as market makers or dealers), which act as middlemen in bringing together buyers and sellers and also step in as a trading counterparty on their own account, cannot be justified under frictionless market conditions. Furthermore, the existence of stock exchanges with their detailed institutional market rules (including, e.g., specifications of admissible order types and standards for clearing and settlement procedures) can similarly not be explained in a Walrasian setting. Institutions and intermediaries derive their "raison d'être" from their particular ability to overcome the adverse effects of frictions on market outcomes and, thus, from the presence of frictions in the first place (see e.g. Spulber (1996)). Consequently, the importance of stock exchanges and intermediaries in the trading of financial assets indicates significant deviations from the frictionless Walrasian framework even for real-world securities markets.

Irrespective of arguments about descriptive realism, the notion of a Walrasian auctioneer involves a more fundamental flaw: the auction mechanism for establishing neoclassical equilibrium is not part of the analytical representation of the economy, but merely provides an illustrative anecdote entirely external to the model. More concretely, when an individual agent in the model determines his optimal demand and supply, he does neither recognize how his order submitted to the fictitious auctioneer is translated into the ultimate trading conditions, nor does he recognize any interdependency of his order decision with the decisions of all other agents in the market. Thus, the individual agents are not aware of the Walrasian auction mechanism they are allegedly subject to, and any effects from the mechanism, per se, on the individual trading decisions and the resulting market outcomes are thereby implicitly assumed away from the outset.

Without an explicit price formation mechanism based on agents' individual trading decisions, the common interpretation of neoclassical equilibrium as a spontaneous order for an economy where the competitive and independent actions of different agents are coordinated by the impersonal forces of the market has no theoretical foundation. To the contrary, the coordination mechanism that de facto hides behind the analytical representation of neoclassical models is an omnipotent agent, called the auctioneer, who determines and enforces an equilibrium system of prices based on his implicit a priori knowledge of all relevant parameters of the economy. The individual agents themselves are not involved in this price formation procedure. Any strategic interactions among the agents for reaching equilibrium are thus ruled out (but, in the presence of an omnipotent auctioneer, also redundant).[14]

The absence of strategic interactions among the individual agents follows from the assumption of perfect competition, which may be the single most characteristic assumption of the Walrasian paradigm. Under perfect competition, each agent passively takes the trading prices announced by the fictitious auctioneer as given, but does not try to affect prices to his own advantage or to obtain prices particularly favorable to him—although exactly these activities are argued, according to the common interpretation, to trigger price adjustments to bring a competitive equilibrium about in the first place.[15] As forcefully stated by von Hayek (1949, p. 92):

> *"... what the theory of perfect competition discusses has little claim to be called 'competition' at all. ... The reason for this seems to me to be that this theory throughout assumes that state of affairs already to exist which, according to the truer view of the older theory, the process of competition tends to bring about (or to approximate) and that, if the state of affairs assumed by the theory of perfect competition ever existed, it would not only deprive of their scope all the activities which the verb 'to compete' describes but would make them virtually impossible."*

Consequently, perfectly competitive equilibria study market outcomes, as if the individual agents are not engaged in competitive activities at all.

[14]This point leads several authors to the conclusion that neoclassical equilibrium models provide not a depiction of a decentralized market economy, but rather of a centrally-planned economy; see e.g. De Vroey (1990).

[15]For example, no-arbitrage pricing is based on the notion that investors try to find and exploit trading opportunities in mis-priced assets and exactly thereby bring prices towards their arbitrage-free values.

A concluding appraisal of the neoclassical view on the price formation process is given by Hahn (1987, p. 137): *"The auctioneer is a co-ordinator deux ex machina and hides what is central."* This critical evaluation of the Walrasian auctioneer and his role in the neoclassical approach to economics and asset pricing leads us to the field of market microstructure theory.

Market microstructure theory is the analysis of price formation with a focus on the interrelations between frictions, institutional market rules, and strategic interactions among agents.[16] As O'Hara (1995, p. 1) notes, the study of market microstructure emerged from "*the desire to know how prices are formed in the economy.*" The major analytical issues in this subfield of finance include: short-term price behavior, order decisions of individual investors, the incorporation of private information into prices, the determinants of market liquidity, and the performance of different market organizations. Whereas market microstructure models consider these topics in their own right, the objective of this thesis is to integrate microstructure features into asset pricing theory. In particular, we are interested in the effects of market liquidity on equilibrium asset returns. As a preliminary step for this analysis, the next section describes the market microstructure view on market liquidity in detail.

2.4 Market Liquidity

In this section, we specify the term "market liquidity" as we apply it throughout this thesis. To begin with, we note that the objective of our specification is not to resolve the varying views and the definitional disputes on the exact meaning of liquidity, but rather to provide a useful definition for our subsequent analysis of market liquidity in asset pricing.

Unfortunately, liquidity has different meanings in different contexts in financial economics. In corporate finance, for example, liquidity refers to both the ability of firms to meet irrefutable payment obligations (solvency) and the ability to raise funds to cover financing needs (funding liquidity). In macroeconomics and banking,

[16] The term "market microstructure" was coined by Garman (1976). Krahnen (1993) points out that the implementation of explicit trading and price formation rules in a market dissolves the traditional dichotomy between markets (in the abstract sense of neoclassical economics) and institutions (as sets of contracting rules in the line of neo-institutional economics) as two alternative coordination mechanisms for economic activities.

liquidity describes the provision of credit facilities by banks to other sectors of the economy or within the banking sector itself (money supply). In several other contexts, liquidity relates to the potential of a real or financial asset to be converted into money (moneyness). In market microstructure, finally, the term is used to capture the ease with which financial assets can be traded among investors (tradability, market liquidity). Beyond the plurality of meanings of liquidity, the different meanings are also intertwined. For example, a firm's solvency is supported by a high tradability of financial assets that are held as a financial cushion, and a firm's funding liquidity is enhanced by a high moneyness of real assets that can be used as banking collateral. The terminological complexity notwithstanding, we use the term liquidity in the following exclusively in the market microstructure sense.

Even after liquidity is now pinned down to the ease of trading financial assets, the notion of market liquidity remains elusive. Harris (1990, p. 3) gives a definition that grasps the concept as a whole: *"A market is liquid if traders can quickly buy or sell large numbers of shares when they want and at low transaction costs."* Thus, liquidity has a price aspect ("at low costs") and a time aspect ("quickly") and, overall, refers to the absence of frictions in the trading of financial assets. The theoretical benchmark of a perfectly liquid market can therefore be associated with the frictionless Walrasian conditions, where prices and portfolios are determined as if each investor can buy and sell any arbitrary number of shares immediately and at no costs. Any friction relative to this benchmark makes the market less than perfectly liquid; we refer to a market with less than perfect liquidity as "illiquid."

In the remainder of this section, we consider market liquidity in more detail: first, we delineate the different dimensions of illiquidity that may impede an investor's trading decision; and second, we review the factors and interdependencies by which the degree of liquidity in a market is determined.

2.4.1 Dimensions of Liquidity

The concept of market liquidity is commonly subdivided into four dimensions: tightness (width), depth, immediacy, and resiliency. Neither terminology nor definitions for these dimensions are consistent in the literature, however. Harris (1990) describes the four dimensions as follows: width refers to the bid-ask spread for a given number of shares; depth refers to the number of shares that can be traded at given bid and ask quotes; immediacy refers to how quickly trades of a given size can be done at a given cost; and resiliency refers to how quickly prices revert to former levels after temporary order imbalances. Tightness (width) and depth represent the price aspect of liquidity, immediacy and resiliency the time aspect. In the following, we largely adopt the terminology of Harris (1990), but we give slightly different definitions and disregard the resiliency dimension.[17]

Tightness: A market is perfectly tight at a given point in time, if an investor can buy a given number of shares at the same price at which he can sell that number of shares. Otherwise, there exists a bid-ask spread in the market, such that the price to buy (ask price) is higher than the price to sell (bid price).

Depth: A market is perfectly deep at a given point in time, if an investor can trade different quantities of shares at the same price per share. Otherwise, the investor has price impact (market impact), such that buying a larger number of shares drives up the price to buy or selling a larger number of shares drives down the price to sell.

Immediacy: A market has perfect immediacy, if an investor can trade a given number of shares instantly at any time he wants. Otherwise, there are costs from the time delay of trading.

Figure 2.1 illustrates tightness and depth of a market for an investor i. The step function displayed in the figure represents buy and sell orders of other agents, e.g. quotes from market intermediaries or limit orders from other investors submitted to an order book, which provide liquidity to the market and against which investor i can trade. In the figure, the market is not perfectly tight, as the price p_{ask} to buy θ^i shares is higher than the price p_{bid} to sell the same number of shares $(-\theta^i)$. Furthermore, the market is not perfectly deep, as buying the larger quantity $\theta^{i'} > \hat{\theta} > \theta^i$ drives up the price to $p'_{ask} > p_{ask}$.

The exact conditions for order execution in a particular market are dictated by the institutional details of the trading and price formation rules of that market. These rules specify, for example, whether the order to buy $\theta^{i'}$ shares is executed under uniform pricing, where investor i pays the price p'_{ask} for all units of his order,

[17]For further discussion on how to define and how to measure market liquidity, see Bernstein (1987), Hasbrouck and Schwartz (1988), Amihud and Mendelson (1991), Kempf (1999), Stoll (2000), and Aitken and Comerton-Forde (2003).

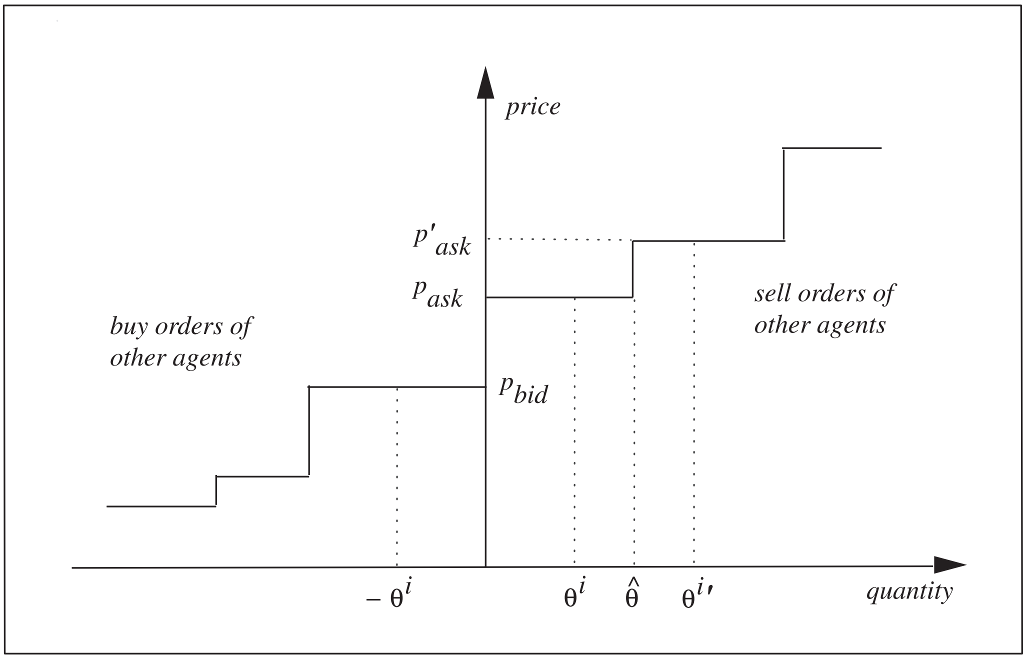

Figure 2.1: Price dimensions of market liquidity.

or whether the order "walks up the book," so that investor i pays the price p_{ask} for the first units of the order and the less favorable price p'_{ask} only for the last units of his order. Although such specifications are important features of real-world market organizations, we largely abstract from them in our subsequent analysis of market liquidity in asset pricing. Thus, we presume that although the institutional microstructure details affect the level of liquidity in a market, they are of minor relevance for the question of how illiquidity, per se, affects equilibrium expected asset returns.

In order to disentangle further facets of market liquidity, a few additional comments are useful.

- Liquidity may vary over time. For example, bid-ask spreads may be relatively high for a newly established trading venue, but decrease over time, when the market attracts additional investors and trading becomes more active.

- Liquidity may be stochastic. For example, an investor may not be able to observe the existing orders in an order book and, hence, is uncertain about the price impact of his own order, or an investor may not be able to evaluate time variations in liquidity and, thus, faces uncertainty about the bid-ask spreads for his potential future trades.

- The different dimensions of market liquidity are interdependent. For example, an investor can often improve the immediacy of trading at the cost of a higher bid-ask spread and price impact, and vice versa.

- Illiquidity causes different types of costs. First, an investor may incur costs such as commissions and trading taxes that have to be paid in addition to the pure trading price (explicit execution costs). Next, an investor may have to pay fees to an intermediary that are already incorporated in the quoted bid and ask prices, or the investor may have price impact that directly worsens his trading price (implicit execution costs). Finally, an investor may decide to trade a smaller quantity or not to trade at all in order to avoid paying execution costs, but as a consequence holds an unbalanced portfolio or misses a speculative profit and, hence, incurs an expected utility loss because of the forgone trade (opportunity costs).

In summary, market liquidity has several dimensions and facets, which ideally should be considered together in order to analyze the effects of illiquidity on equilibrium asset returns. A comprehensive analysis of market liquidity, however, requires also a reasonable view on how illiquidity arises from the trading interactions of different agents in the market microstructure process. This is what we turn to next.

2.4.2 Determinants of Liquidity

In the standard Walrasian paradigm, markets are perfectly liquid. Perfect liquidity, however, is not derived endogenously from the interactions of the different agents in the market, but is assumed from the outset by imposing the assumptions of perfect competition (no price impact) and perfect capital markets (no bid-ask spread, no time delay of trading). This subsection presents a short review of the market microstructure arguments on how the degree of liquidity in a market is determined.

The degree of market liquidity for an investor is determined by his own costs of market participation and, more importantly in the context of securities markets, by the presence and willingness of sufficiently many other agents to step in as a trading counterparty. These other agents can be market intermediaries or other investors. In a dealer market (quote-driven market), for example, intermediaries are obliged to stand ready as a trading counterparty and to quote bid and ask prices against which an investor can trade a given number of shares. In an auction market (order-driven market), alternatively, other investors submit limit orders to an order book against which an investor can trade by submitting a matching own order that is executed either immediately (continuous auction) or at the next periodically scheduled trading date (call auction, batched auction).

Irrespective of the particular market organization and trading partner, however, market microstructure theory highlights three "basic frictions" that make other agents reluctant to take the counterpart of a trade and that therefore can be regarded as the origins of illiquidity in securities markets: (*i*) asymmetric information, (*ii*) inventory problems, and (*iii*) costs for searching counterparties and processing orders. In the following, we briefly review the main arguments for why these basic frictions impair the tradability of financial assets.[18]

Asymmetric Information: The presence of asymmetric information in a market provokes an adverse selection problem and impairs market liquidity, as potential trading counterparties worry about losses from trading against investors with superior (private) information (Akerlof (1970)). In the early models of Bagehot (1971) and Copeland and Galai (1983), bid-ask spreads arise as market makers require the spread revenues from trades with uninformed investors in order to recoup their losses from trades with informed investors. In the seminal works of Glosten and Milgrom (1985) and Kyle (1985), market makers recognize that order flow from investors is a (noisy) signal of the underlying private information, and bid-ask spreads and price impact emerge in the learning process of inferring the private information from the flow of orders. For example, a market maker quotes a bid-ask spread for the next incoming order, since a buy order will indicate good news about the asset and hence will drive the market maker's valuation of the asset up to the ask, whereas a sell order will indicate bad news and drive the valuation of the asset down to the bid. In the model of Kyle (1989), price impact is generated as agents try to learn private

[18]For a detailed exposition of market microstructure models, see e.g. O'Hara (1995), Madhavan (2000), Brunnermeier (2001), or Biais, Glosten, and Spatt (2005).

information directly by conditioning their own demand schedules on prices and, for instance, interpret a higher price due to strong demand of other agents as driven by good private news about the asset.[19] In asymmetric information models, illiquidity is increasing in the relative extent of informed versus uninformed trading. The no-trade theorems of Milgrom and Stokey (1982) and Tirole (1982) show that markets are completely illiquid for information-motivated (non-Pareto-improving) trading, if all agents are rational and risk averse, if the distribution of initial endowments is already Pareto optimal, and if all of this is common knowledge.

Inventory Problems: Illiquidity arises also when potential trading counterparties are unwilling to move away from their current inventory or portfolio position or are willing to do so only at a price concession. In Garman (1976), the market maker sets bid-ask spreads for stochastic order flow as a mechanism to control the dynamics of his inventory and to avoid bankruptcy in terms of running out of shares or money. In Stoll (1978), Ho and Stoll (1981, 1983), and Grossman and Miller (1988), bid-ask spreads and price impact arise as market makers are risk averse and are willing to change their portfolios only according to their risk-return trade-offs. For example, a risk-averse market maker is willing to step in as a buyer only at a price discount of a lower (bid) price in order to be compensated by a higher expected return for his exposure to the additional risk. In risk-aversion-based inventory models, illiquidity is increasing in the risk aversion of the trading counterparties and in the risk of the asset.

Search Costs and Order Processing Costs: Search costs impede the tradability of assets in markets for which there is no central trading place and trades have to be initiated and negotiated in bilateral processes, for example in over-the-counter markets. The costs may encompass fixed and variable components for both the trades actually taking place and also for participating in the market and engaging in

[19] The early rational expectations equilibrium (REE) models on information aggregation and transmission through prices by Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980), and Diamond and Verrecchia (1981) are based on the assumption of perfect market liquidity—including, in particular, the assumption of perfect competition (price-taking behavior, no price impact). As Hellwig (1980) points out, however, price-taking behavior in a REE requires agents to behave "schizophrenically:" on the one hand, agents think they do not impact prices; but on the other hand, agents try to learn private information from prices and, thus, there must be price impact for the information to become incorporated into prices in the first place.

the search process in the first place. If the costs for direct bilateral trading among investors are too large, then trading may be economized by establishing a central trading venue operated by market intermediaries. Trading costs for investors are then determined by the fees charged by the intermediaries to cover their costs of running the market and processing orders and, in case of market power for intermediaries, by an additional profit mark-up.[20]

The focus of market microstructure models is to explain market outcomes from the interactions of different market participants under explicit trading and price formation rules. Yet, for analytical tractability, most of the models introduce agents who trade for exogenous reasons (random noise traders) or employ agents whose trading decisions are endogenous but not interdependent with the decisions of the other agents ("small" traders, competitive fringe). However, with increasing interdependency of trading decisions, the provision of market liquidity among agents tends to exhibit strategic complementarity. Strategic complementarity prevails, if each investor's incentive to act in a certain way (e.g. to participate in a market or to increase his trading aggressiveness) increases as the other agents act this way as well and, hence, if expected utility for each investor increases with the degree of coordination with the other agents (Bulow, Geanakoplos, and Klemperer (1985)). Strategic complementarity in liquidity provision arises for instance with respect to market participation, as investors are more willing to participate in a market with a high level of liquidity and, conversely, the level of liquidity is higher with a larger number of agents participating in the market. This mechanism of mutually re-inforcing liquidity provision is commonly paraphrased by the expressions that "liquidity begets liquidity" or that "liquidity feeds on itself."

Models with strategic complementarity often yield multiple (non-Walrasian) equilibria. For example, in the microstructure models on concentration versus fragmentation of trading over time or across markets by Admati and Pfleiderer (1988) and Pagano (1989a), agents tend to cluster together in a single period or market because

[20]Braun (1998) presents a microstructure analysis in which intermediaries in a first step decide whether to participate in the costly establishment of a trading venue and, if so, in a second step provide liquidity for exogenous investor orders in the presence of different frictions. The model allows to endogenously relate the number of intermediaries and their market power to the level of liquidity in the market.

of strategic complementarity in liquidity provision, and multiplicity of equilibrium arises as trading and liquidity may be concentrated in one period or market or the other. Pagano (1989b) and Dow (2004) present microstructure models with multiple equilibria in which the different equilibria entail different levels of liquidity and, in particular, investors may be trapped in a Pareto-inferior, low-liquidity equilibrium as consequence of a coordination failure.

To conculde this chapter, we note that our analysis of asset pricing with imperfect competition and endogenous market liquidity in later chapters of this thesis takes a rather puristic view on the market organization. We use a highly stylized trading and price formation mechanism, which in fact closely resembles a Walrasian auction. This might seem surprising at first, as a large part of microstructure theory centers around institutional market details and actively trading intermediaries. Yet, all that is needed for endogeneity of market liquidity is to introduce a basic friction and to allow for strategic interactions among investors under explicit, even though simple, market rules. It is in this spirit that we integrate market microstructure features into asset pricing theory.

Chapter 3

Market Liquidity and Asset Prices: Literature Review

In this chapter, we review the literature on market liquidity in asset pricing. This literature is vast, comprising a wide range of approaches to capture the multi-faceted concept of market liquidity and its implications for asset prices. The aim of our review, however, is not to provide an extensive overview of the literature, but rather to illustrate the main results of those models that are most closely related to our own analysis in the following chapters. Hence, we restrict the review to models that focus on the theoretical understanding of how illiquidity in the form of bid-ask spreads or price impact affects equilibrium expected asset returns.

Among the approaches not included in our review are pure portfolio selection models, models applying what is sometimes called "utility indifference pricing," and reduced-form models. Portfolio selection models study the optimal investment and consumption policy of an investor in the presence of trading costs, but treat asset prices as exogenous.[1] Hence, these models do not address the pricing effect of illiquidity.

Models following the utility indifference approach also start from a portfolio selection analysis for exogenous asset prices, but then argue about the pricing effect of illiquidity by comparing the expected utility of an investor under his optimal portfolio

[1] See, for instance, Davis and Norman (1990), Duffie and Sun (1990), Dumas and Luciano (1991), and Liu (2004).

policy in two separate cases: first, for trading a perfectly liquid asset; and second, for trading an otherwise identical but illiquid asset. The illiquid asset is then postulated to yield a return premium over its liquid counterpart such that the investor's expected utility is the same in both cases. For instance, Constantinides (1986) defines the liquidity premium as the increase in the expected return on the illiquid asset that makes an investor indifferent between trading the illiquid asset and trading the liquid asset.[2] However, although these models make predictions about asset returns, they do not derive asset prices endogenously in a standard way of price formation such as by imposing market-clearing conditions.

Reduced-form models, finally, employ pre-specified factor structures with one of the factors accounting for illiquidity. This approach is particularly useful as a preliminary step for empirical analyses.[3] Yet, from a theoretical point of view, reduced-form models again do not derive the pricing effect of illiquidity endogenously and, thus, do not provide an explanation for why the illiquidity factor constitutes a component of required asset returns.

In the remainder of this chapter, we review models that endogenize the effects of market liquidity on equilibrium asset returns. Section 3.1 presents asset pricing models in which investors are subject to exogenous bid-ask spreads. Section 3.2 considers models on large investors in asset pricing, in which some of the agents face illiquidity in the form of price impact. Section 3.3 studies asset pricing models in which illiquidity is integrated endogenously due to a basic friction in the market microstructure interactions among investors. Section 3.4 provides a discussion of the reviewed approaches and relates them to the analysis carried out in the following chapters of this thesis. Bühler and Sauerbier (2003) and Amihud, Mendelson, and Pedersen (2005) present comprehensive surveys of the literature on market liquidity in asset pricing, covering several of the approaches not included in this review here.

[2] See also Longstaff (2001) and Levy and Swan (2006) for models following this line of reasoning. Longstaff (1995) and Koziol and Sauerbier (2007) use a similar argument, applying option-pricing techniques to derive the value of perfect market liquidity for an investor.

[3] See Kempf and Uhrig-Homburg (2000), Grinblatt (2001), Longstaff, Mithal, and Neis (2005), Ericsson and Renault (2006), Liu, Longstaff, and Mandell (2006), and Bühler and Trapp (2007).

3.1 Exogenous Bid-Ask Spreads

The largest group of models on frictions in asset pricing considers exogenous imperfections in the capital market such as trading costs, personal investor taxes, borrowing restrictions, or other portfolio constraints. In all other respects, however, these models still conform to the Walrasian paradigm, i.e., these models maintain the assumptions of perfect competition, symmetric information, and rational investors, and define equilibrium by the conditions of individual portfolio optimization for all investors and market clearing for all assets.[4]

In this section, we review models that address the effects of exogenous illiquidity in the form of bid-ask spreads on equilibrium expected asset return.[5] We classify these models into: (*i*) models with deterministic liquidity effects, (*ii*) models with liquidity shocks, and (*iii*) models with random time variations in liquidity. Easley and O'Hara (2003) and Cochrane (2004) survey the models on asset pricing with exogenous liquidity costs.

3.1.1 Deterministic Liquidity Effects

The beginning of the analysis of illiquidity in asset pricing is commonly attributed to the seminal model of Amihud and Mendelson (1986), in which trading costs have stochastic effects because investors are subject to liquidity shocks. It is useful, however, to first consider the pricing implications of illiquidity in more basic settings with deterministic liquidity effects.

Amihud, Mendelson, and Pedersen (2005) study a discrete-time overlapping generations (OLG) model in which a new generation of risk-neutral investors is born at each date t and leaves the economy at the next date $t + 1$. In the economy, there is a perfectly liquid riskless asset with exogenous interest rate $r > 0$ and an illiquid risky asset with a fixed number of outstanding shares. At each date t, the risky asset

[4] Early asset pricing models in this vein are, for instance, Milne and Smith (1980) on proportional trading costs, Mayshar (1979, 1981) on fixed trading costs, Mayers (1973) on nonmarketable assets, Brennan (1970), Litzenberger and Ramaswamy (1979), and Constantinides (1984) on personal investor taxes, Brennan (1971) on divergent borrowing and lending rates, Black (1972) on the absence of a riskless asset, and Levy (1978) on portfolio constraints.

[5] Kempf (1999) and Sauerbier (2006) present equilibrium asset pricing models in which illiquidity is introduced exogenously by restricting the possible trading dates for some of the assets; hence, these models focus on the time aspect of market liquidity.

pays a dividend $\tilde{d}_t$ and has an ex-dividend price p_t per share. The dividends are independently and identically distributed over time with mean $\bar{d}$. Investors can buy the risky asset at price p_t, but must sell it at $p_t - c$. Hence, the liquidity cost c is modeled as a per-share trading cost of selling the risky asset and can be interpreted as a bid-ask spread charged by a market intermediary. Because of the two-period-lived OLG stucture of the model, any portfolio position built up by an investor when born must already be liquidated at the next possible trading date before leaving the economy.

Amihud, Mendelson, and Pedersen (2005) argue that at each date t the price p_t must be determined such that the expected liquidity-adjusted (net) return of the risky asset is equal to the riskfree rate,

$$\frac{\bar{d} + p_{t+1} - c}{p_t} - 1 = r, \tag{3.1}$$

since, otherwise, investors born at t are willing to buy an arbitrary large number of shares if the price is lower or are willing to sell short an arbitrary large number of shares if the price is higher.[6]

From (3.1), a stationary equilibrium for the economy, i.e. an equilibrium with constant prices $p_t = p$ for all t, is characterized by the equilibrium price of the risky asset

$$p = \frac{\bar{d}}{r} - \frac{c}{r}. \tag{3.2}$$

Thus, the equilibrium price is equal to the present value of all expected future dividends of the asset minus a liquidity discount, and the liquidity discount, in turn, is given by the present value of all future trading costs over the lifetime of the asset. The liquidity discount reflects not only the costs incurred by the current generation of investors, but also all future costs for the repeated trading of the asset, since the current generation anticipates the liquidity discount established by future generations of investors.

[6]The argument is not exactly correct for the short-selling part, since short positions incur the selling cost c at t, not at $t+1$, and thus lose interest on the trading cost. However, this point is not relevant to equilibrium (and otherwise may be remedied by re-interpreting the cost c as a cost of unwinding a position—either by selling a long position or by buying to cover a short position).

Alternatively, the effect of trading costs can also be expressed with respect to the equilibrium expected gross return of the risky asset. From (3.2) the required gross return follows as

$$E\left[\tilde{r}\right] := \frac{\bar{d}}{p} = r + \frac{c}{p}. \tag{3.3}$$

Hence, the required gross return is equal to the riskfree rate plus a liquidity premium according to the relative per-period trading costs of the risky asset. Note that, although prices are constant and have the same liquidity discount at all dates t, investors earn positive gross returns (including the liquidity premium) because of the dividends.

Equation (3.3) represents the standard pricing effect of liquidity costs: illiquidity is priced as an additional premium on the expected return of the asset. Intuitively, as Amihud and Mendelson (1986, p. 228) put it, *"the positive association between return and [bid-ask] spread reflects the compensation required by investors for their trading costs."* In the model of Amihud, Mendelson, and Pedersen (2005), the liquidity premium exactly compensates investors for their costs of trading the illiquid asset, and the required net return on the risky asset is thus equal to the return on the perfectly liquid riskfree asset.

In a two-period-lived OLG model, investors cannot optimally choose the timing of their trades, because they have only two rounds of trading available for building up and unwinding their portfolios. Further insights into the pricing effect of illiquidity can be obtained by allowing investors to live for more than two trading dates, so that investors' trading horizons become endogenous.

Vayanos (1998) considers a continuous-time OLG model with a riskless asset and multiple risky assets. Each risky assets carries a bid-ask spread and cannot be sold short. The riskless asset is perfectly liquid and can be traded without restrictions. Investors are born with initial wealth to be invested, have exponential utility functions over their lifetime consumption, and trade the assets for life-cycle reasons.

Vayanos (1998) finds that investors buy the risky assets when born, then hold this portfolio for a short time period, and then sell back their holdings slowly over time before leaving the economy. Under these portfolio strategies, an increase in trading costs has two opposing effects on asset prices. First, new investors are less willing to buy the illiquid assets; as a consequence, prices reflect a liquidity discount according to the present value of all future trading costs of the asset (similar to the pricing effect in (3.2)). Second, older investors sell back their illiquid asset holdings more slowly. Vayanos (1998) shows that the second effect countervails the first pricing effect and that, overall, either effect can dominate. In particular, when the second effect dominates, then the price of a risky asset actually increases in its trading costs—in contrast to the standard result that trading costs reduce the price of an illiquid asset.

Vayanos and Vila (1999) provide a continuous-time OLG model with two riskless assets, one perfectly liquid and the other carrying a bid-ask spread, and with a numeraire consumption good, thereby endogenizing the prices of both the liquid and the illiquid asset. Investors face borrowing constraints, receive deterministic labor income streams to be invested, and derive utility from consumption over their lifetime.

Vayanos and Vila (1999) show that it is optimal for an investor to first buy the illiquid asset when born, next buy the liquid asset, then sell the liquid asset, and finally sell the illiquid asset. Hence, the illiquid asset has a lower turnover rate than the liquid asset, and there is a clientele effect as the illiquid asset is held by investors with a longer remaining lifetime. Furthermore, the price of the illiquid asset is reduced relative to the price of the liquid asset by the present value of all its future trading costs.

Regarding the absolute prices of the assets, Vayanos and Vila (1999) find that trading costs increase the price of the liquid asset (i.e., trading costs decrease the riskfree rate of interest), but have an ambiguous effect on the price of the illiquid asset. If the number of outstanding shares of the illiquid asset is small relative to that of the liquid asset, then the absolute price of the illiquid asset is decreasing in trading costs. However, if the number of outstanding shares of the illiquid asset is large, then trading costs may increase the absolute price of the illiquid asset and, hence, increase the absolute price of the liquid asset even more. Vayanos and Vila (1999) conclude that a change in trading costs for a significant fraction of assets may have stronger effects on the prices of the remaining assets than on those subject to the change.

3.1.2 Liquidity Shocks

In the previous subsection, trading costs and their effect on asset returns are considered in a deterministic way. One approach for capturing stochastic effects of illiquidity is to introduce liquidity shocks, so that investors' trading horizons become random. A liquidity shock occurs, for instance, when investors experience a surprise consumption need or when professional portfolio managers are subject to unforeseen fund withdrawals and thereby are forced to liquidate their asset holdings to cover the sudden need for cash. Per-period net returns of assets are then uncertain, even if trading costs and asset payoffs are deterministic, because the holding period over which the trading costs get amortized is random: net returns are low for investors who must liquidate the assets soon after buying them, but high for investors who liquidate the assets after a relatively longer holding period. Although the literature refers to the forced portfolio liquidations as liquidity shocks, the shocks apply not to market liquidity but rather to agents' wealth and funding liquidity.

Amihud, Mendelson, and Pedersen (2005) extend their basic OLG model with risk-neutral investors to the case that agents live for more than one period and thus may have multi-period holding horizons for the risky asset. In any period, however, there is some probability q that an agent receives a liquidity shock and is forced to sell his asset holdings and to leave the economy. Amihud, Mendelson, and Pedersen (2005) show that the equilibrium price of the risky asset is then given by

$$p = \frac{\bar{d}}{r} - q\frac{c}{r},$$

and, equivalently, the required gross return is equal to

$$E\left[\tilde{r}\right] = r + q\frac{c}{p}. \tag{3.4}$$

Equation (3.4) reveals that the liquidity premium is now adjusted for the probability of a liquidity shock, which can also be interpreted as the expected holding period or the average trading frequency of the asset. Hence, an asset that is traded more frequently is predicted to yield a higher liquidity premium, as investors incur the trading costs more often over the liftetime of the asset.

Amihud and Mendelson (1986) provide a model with clientele effects by allowing for investor-specific probabilities of liquidity shocks. In the model, there are I types of investors (indexed by $i = 1, 2, \ldots, I$), K risky assets ($k = 1, 2, \ldots, K$), and a riskless asset. Investors are risk neutral, arrive at the market with initial wealth used to purchase assets, and sell their positions when they receive a liquidity shock. Investor types are numbered by decreasing probability of a liquidity shock, $q^1 \geq q^2 \geq \ldots \geq q^I$, or, put differently, by increasing expected holding period. Each risky asset pays a random dividend $\tilde{d}_k$ per unit of time, can be bought at price p_k, but must be sold at $p_k - c_k$. The riskless asset yields an exogenous interest rate r and can be traded without costs. For all assets (including the riskless asset), short sales are ruled out; hence, investors are subject to borrowing constraints.

The optimal portfolio strategy for an agent is to invest his total wealth into the assets with the highest expected net returns. As the investors with the lowest q^i (type I) can amortize the trading costs over the longest expected holding period, they can outbid agents with shorter holding periods for any risky asset while still attaining a higher expected return. Because of limited initial wealth and borrowing constraints, however, it is not possible for investors of type I to buy up all of the risky assets.

Amihud and Mendelson (1986) impose market-clearing conditions for all risky assets and obtain the following equilibrium results: (*i*) assets with higher relative bid-ask spreads offer higher expected gross returns; (*ii*) assets with higher relative spreads are held by investors with longer expected holding periods (clientele effect), with investor partition into different clienteles being determined by the initial wealth of each investor type; and (*iii*) the relation between relative spreads and expected gross returns is concave, as the clientele effect moderates the spread effect on the return.

Furthermore, Amihud and Mendelson (1986) provide a numerical example which implies that the required gross return of risky asset k, if held by investors of type i, has the structure

$$E\left[\tilde{r}_k\right] = r + q^i \frac{c_k}{p_k} + \Delta_k^i,$$

with $\Delta_k^i \geq 0$. Thus, the required gross return is equal to the riskfree rate plus the relative bid-ask spread of that asset, adjusted for the average trading frequency of the specific asset holders, plus a non-negative additional term. In particular, the additional term Δ_k^i is strictly positive for all risky assets held by investors of types $i > 1$, the term is larger for assets with higher relative spreads and, hence, is larger also for assets held by investors with longer expected holding periods (because of the clientele effect). As the term Δ_k^i generates an expected net return in excess of the riskfree rate to investors with longer expected holding periods, it can be interpreted as a rent for long-term investments into illiquid assets.[7]

When agents may experience liquidity shocks, asset net returns are risky because of the random holding period over which the trading costs get amortized. In the models considered so far, the liquidity premium incorporates only the expected holding period, however, since investors were assumed to be risk neutral. If investors are risk averse, then an additional pricing effect for the illiquidity-induced risk can be derived.

Huang (2003) studies a continuous-time OLG model on liquidity shocks in which investors have utility functions with constant relative risk aversion. Each investor derives utility only from terminal consumption after experiencing a liquidity shock and selling his asset holdings. The probability for a liquidity shock is the same for all investors. In the economy, there are two riskless assets that are identical expect that one is perfectly liquid whereas the other is traded at a deterministic bid-ask spread. Both assets cannot be sold short.

Huang (2003) defines the liquidity premium as the return premium of the illiquid asset over the liquid asset and derives equilibrium for two different cases. In the first case, each investor starts with an up-front endowment of initial wealth to be invested, but does not receive additional income over time from sources other than asset payoffs; this case is similar to a situation in which agents do receive intertemporal income from other sources and can initially monetarize it by borrowing against the future income (no borrowing constraint case). In the second case, each investor starts with zero initial wealth and receives an exogenous deterministic income stream over time, but cannot borrow against his future income (borrowing constraint case). Huang (2003) finds that in both cases, the liquidity premium is higher if investors are risk averse than if investors are risk neutral. Comparison of the two cases using numerical analysis reveals that the impact of risk aversion on the liquidity premium is small

[7]Kane (1994) presents a closed-form solution for the clientele effect.

in the case without borrowing constraint, but can be very high in the case with borrowing constraint. Hence, Huang (2003) argues that illiquidity has large effects on asset returns when risk-averse investors are subject to liquidity shocks and cannot borrow.

Vayanos (2004) considers a continuous-time model in which professional portfolio managers are subject to liquidity shocks in the form of performance-based fund withdrawals. Portfolio managers can invest their funds into a perfectly liquid riskless asset and multiple illiquid risky assets. Trading costs for the risky assets are deterministic and constant over time, but differ across assets. The managers have CARA utility functions over consumption derived from their fees, which are charged as a constant fraction of the funds under management. When the performance of a portfolio manager falls below a certain threshold, his customers completely withdraw their funds and the manager has to liquidate all of his asset holdings.

The driving factor in Vayanos (2004) is the extent of uncertainty in the market, which is represented by stochastic dividend volatility common to all risky assets. In times of high uncertainty (i.e. high dividend volatility), prices of risky assets decline, as portfolio managers try to rebalance their portfolios towards the riskless asset ("flight to quality"). The decline in prices results in a low portfolio performance for the managers and, thus, increases the risk of fund withdrawals. As a consequence, portfolio managers seek to shift their portfolios towards more liquid assets in order to get rid of the illiquidity-induced risk ("flight to liquidity"). The reduced willingness of managers to hold illiquid assets leads to a widening of liquidity premia and to a stronger correlation of illiquid assets with each other and with the market during times of high uncertainty. In summary, the performance-based liquidity shocks in Vayanos (2004) provide an explanation for time variations in liquidity premia, even if trading costs per se are constant over time.[8]

[8] Other models on the pricing effect of deterministic bid-ask spreads when investors may experience liquidity shocks are Aiyagari and Gertler (1991), Heaton and Lucas (1996), and Favero, Pagano, and von Thadden (2005). Lo, Mamaysky, and Wang (2004) consider a related model in which trading involves fixed (per-trade) liquidity costs.

3.1.3 Random Time Variations in Liquidity

In the previous subsection, trading costs are treated as deterministic and the stochastic effect of illiquidity on asset returns is driven by investors' uncertainty about their trading horizons. However, market liquidity may vary over time in a random way, so that trading costs, per se, become a source of uncertainty for investors.

Acharya and Pedersen (2005) consider a discrete-time OLG model in which each generation of investors lives for two periods. Investors are born with initial wealth to be invested at date t and have CARA utility functions over their resulting terminal wealth at date $t+1$. In the economy, there are K illiquid risky assets (indexed by $k = 1, 2, \ldots, K$), each with a total of $\bar{X}_k$ outstanding shares, and a perfectly liquid riskless asset with exogenous interest rate r. At each date t, risky asset k pays a normally distributed dividend $\tilde{d}_{k,t}$, can be bought at an ex-dividend price $p_{k,t}$ but must be sold at $p_{k,t} - \tilde{c}_{k,t}$. The trading cost $\tilde{c}_{k,t}$ is also normally distributed. Short-selling is allowed for the riskless asset, but not for the risky assets.

Acharya and Pedersen (2005) analyze the equilibrium relationship between an asset's gross return,

$$\tilde{r}_{k,t} = \frac{\tilde{d}_{k,t} + p_{k,t}}{p_{k,t-1}} - 1,$$

the return on the market portfolio,

$$\tilde{r}_{M,t} = \frac{\sum_k \bar{X}_k \left(\tilde{d}_{k,t} + p_{k,t}\right)}{\sum_k \bar{X}_k \, p_{k,t-1}} - 1,$$

the relative liquidity costs of the asset,

$$\tilde{\gamma}_{k,t} = \frac{\tilde{c}_{k,t}}{p_{k,t-1}},$$

and the relative liquidity costs of the market portfolio,

$$\tilde{\gamma}_{M,t} = \frac{\sum_k \bar{X}_k \, \tilde{c}_{k,t}}{\sum_k \bar{X}_k \, p_{k,t-1}}.$$

To determine equilibrium, Acharya and Pedersen (2005) first consider an economy in which each risky asset k pays dividends $\tilde{d}_{k,t} - \tilde{c}_{k,t}$ and can be traded without costs. The equilibrium for this imagined economy satisfies the standard CAPM structure,

$$E\left[\tilde{r}_k - \tilde{\gamma}_k\right] = r + \Lambda_\gamma \operatorname{Cov}\left[\tilde{r}_k - \tilde{\gamma}_k, \tilde{r}_M - \tilde{\gamma}_M\right] \quad \text{for all } k, \tag{3.5}$$

where $\Lambda_\gamma = \left(E\left[\tilde{r}_M - \tilde{\gamma}_M\right] - r\right) / \operatorname{Var}\left[\tilde{r}_M - \tilde{\gamma}_M\right]$ is the market price of risk. Since in the imagined frictionless economy each investor holds a long position in the market portfolio which yields the same terminal wealth as in the original economy with liquidity costs, the equilibrium in the original economy is the same as in the imagined economy. Hence, (3.5) translates into a liquidity-adjusted CAPM for the original economy such that equilibrium expected gross returns are given by

$$\begin{aligned} E\left[\tilde{r}_k\right] = r + E\left[\tilde{\gamma}_k\right] + \Lambda_\gamma \operatorname{Cov}\left[\tilde{r}_k, \tilde{r}_M\right] + \Lambda_\gamma \operatorname{Cov}\left[\tilde{\gamma}_k, \tilde{\gamma}_M\right] \\ - \Lambda_\gamma \operatorname{Cov}\left[\tilde{r}_k, \tilde{\gamma}_M\right] - \Lambda_\gamma \operatorname{Cov}\left[\tilde{\gamma}_k, \tilde{r}_M\right] \quad \text{for all } k. \end{aligned} \tag{3.6}$$

Equation (3.6) states that the equilibrium expected return on each risky asset increases, as in the standard CAPM, with the riskfree rate of interest and with the systematic (payoff) risk of the asset, and, in addition, reflects four channels for a liquidity premium. First, the required return increases with the expected relative per-period trading cost of the asset; this effect is similar to the standard liquidity premium in models with deterministic liquidity effects. Second, the required return increases with the covariance between the asset's liquidity costs and the liquidity costs of the aggregate market, as investors demand a premium for investing in an asset that becomes illiquid at the same time when the market in general becomes illiquid. Third and fourth, the required return decreases with the covariance between the asset's return and the liquidity costs of the aggregate market and with the covariance between the asset's liquidity costs and the market return, since investors are willing to accept a lower return because of the diversification effects between asset payoffs and liquidity costs. In summary, the random nature of trading costs in Acharya and Pedersen (2005) generates three additional pricing factors for illiquidity due to three forms of liquidity risk.[9]

[9] Jacoby, Fowler, and Gottesman (2000) consider a static model with random trading costs and also derive a liquidity-adjusted CAPM; however, they do not impose assumptions to separate the stochastics of asset payoffs and trading costs and, hence, do not disentangle the different payoff risk and liquidity risk factors.

3.2 Large Investors

A substantial fraction of trading volume in financial markets is driven by institutional investors who trade quantities large enough to affect prices.[10] Since these large investors bear liquidity costs in the form of price impact, the Walrasian assumption of perfect competition (price-taking behavior) is not tenable when modeling their trading behavior.

Theoretical models on asset pricing with large investors usually employ a CARA-Gaussian setting with two types of agents: first, a group of small, price-taking investors ("competitive fringe"); and second, one or multiple large, price-affecting investors. Trading takes place under symmetric information to improve risk sharing. Whereas small investors submit to the market a whole demand function (which is linear in price for the CARA-Gaussian setting), large investors trade by choosing quantities against the competitive demand. Asset prices are then determined by imposing market-clearing conditions.[11] Since large investors trade against the demand schedule of the small investors, their price impact is linear in trading quantity and stems from the limited risk-bearing capacity of the competitive fringe.[12]

In the following, we classify the models on large investors in asset pricing into: (*i*) static models, (*ii*) dynamic models with a single large investor, and (*iii*) dynamic models with multiple large investors. Pritsker (2002) surveys the literature on large investors with a focus on market manipulation and market stability.

[10] Numerous empirical studies have documented a significant price impact for large institutional trades; see, for instance, Scholes (1972), Holthausen, Leftwich, and Mayers (1990), Chan and Lakonishok (1993), and Keim and Madhavan (1996).

[11] Another strand of the literature on large investors is given by the models on optimal dynamic liquidation strategies for a price-affecting agent; see Bertsimas and Lo (1998), Almgren and Chriss (2000), Subramanian and Jarrow (2001), Huberman and Stanzl (2005), and Mönch (2005). In these models, a large investor trades against an exogenous price impact function, and prices are therefore not fully endogenuous—in contrast to prices derived by imposing market-clearing conditions on the trading schedules of different optimizing agents.

[12] Linearity of price impact in trading quantity from the CARA-Gaussian setup is motivated by analytical tractability. Empirical studies on the slope of price impact functions find, in general, a non-linear relation, but disagree over the actual shape of the function (concave or convex); see, e.g., Loeb (1983), Hasbrouck (1991), Madhavan and Cheng (1997), Kempf and Korn (1999), and Griese and Kempf (2006).

3.2.1 Static Models

The early models on large investors restricts attention to a static framework.[13] Lindenberg (1979) considers a setting with multiple risky assets and a riskless assets. In the economy, there are multiple large investors, who have price impact for each risky asset, and a group of small investors. Lindenberg (1979) shows that the equilibrium expected excess return over the riskfree rate for each risky asset satisfies a two-factor structure where one factor is the systematic risk of the asset and the other factor is determined by the comovement of that asset's return with the return on the aggregate equilibrium portfolio of the large investors. After restricting the model to a single price-affecting investor, Lindenberg (1979) finds further that the large investor's equilibrium portfolio holdings for each risky asset increase linearly in the initial endowment of the investor with that asset. Hessel (1981) generalizes the result on equilibrium portfolio holdings to the case of multiple large investors.

Rudolph (1982), in a simplified version of Lindenberg (1979), presents closed-form equilibrium prices and elaborates the main intuition of the large investor models. In the economy, there is a riskless asset with exogenous interest rate r and, furthermore, a single risky asset with $\bar{X}$ outstanding shares and terminal per-share payoff $\tilde{v}$. Trading takes place between a small investor (indexed by s) and a large investor (indexed by l). Each investor $i = s, l$ has a coefficient of absolute risk aversion $\rho^i > 0$ and is initially endowed with $\bar{y}^i$ units of the riskless asset and $\bar{x}^i$ units of the risky asset (with $\bar{x}^s + \bar{x}^l \equiv \bar{X}$).

Under the CARA-Gaussian assumptions of the model, each investor i chooses his portfolio holdings x^i to maximize a mean-variance objective function

$$(1+r)\bar{y}^i + E\left[\tilde{v}\right] x^i - (1+r)\, p(x^i) \left(x^i - \bar{x}^i\right) - \frac{\rho^i}{2} \left(x^i\right)^2 \operatorname{Var}\left[\tilde{v}\right], \quad i = s, l. \tag{3.7}$$

The small investor acts as price taker (i.e., under the assumption $\partial p / \partial x^s = 0$) and thus has the following inverse demand function for the risky asset

$$p = \frac{E\left[\tilde{v}\right] - \rho^s \operatorname{Var}\left[\tilde{v}\right] x^s}{1+r}. \tag{3.8}$$

The large investor, in contrast, recognizes that he trades against the demand schedule of the small investor and, from the market-clearing condition $x^s + x^l = \bar{X}$ and from

[13] James (1976) provides a static portfolio selection model for a large investor who has price impact in the market for one risky asset, but is price taker for all other risky assets and the riskless asset.

(3.8), has linear price impact according to

$$\frac{\partial p}{\partial x^l} = \frac{\rho^s \mathrm{Var}\left[\tilde{v}\right]}{1+r} > 0. \tag{3.9}$$

The optimal portfolio holdings of the large investor can then implicitely be written as

$$x^l = \frac{E\left[\tilde{v}\right] - \left(1+r\right)p}{\rho^l \mathrm{Var}\left[\tilde{v}\right]} - \frac{\partial p}{\partial x^l}\frac{\left(1+r\right)}{\rho^l \mathrm{Var}\left[\tilde{v}\right]}\left(x^l - \bar{x}^l\right). \tag{3.10}$$

The second term on the right-hand side of (3.10) reveals that the optimal portfolio for the large investor is distorted towards his initial endowment, as the investor responds to his price impact by reducing his trading quantity $(x^l - \bar{x}^l)$, similar to a textbook monopolist or monopsonist.

After deriving the explicit solution for x^l and applying the market-clearing condition, the equilibrium price follows as

$$p = \frac{E\left[\tilde{v}\right]}{1+r} - \frac{\rho^s\left[\left(\rho^s + \rho^l\right)\bar{X} - \rho^s \bar{x}^l\right]\mathrm{Var}\left[\tilde{v}\right]}{\left(1+r\right)\left(2\rho^s + \rho^l\right)}. \tag{3.11}$$

Equation (3.11) shows that, if there is both a small investor and a large investor in the economy, the equilibrium price not only depends on the aggregate endowment of the risky asset, but in addition increases linearly in the endowment of the price-affecting agent.

Rudolph (1982) points out that the equilibrium price can be explained by the relation between the large investor's initial endowment $\bar{x}^l$ and his (risk-tolerance-weighted) holdings under Pareto-optimal risk sharing, which simplify to $x^{l,c} = \rho^s \bar{X} / \left(\rho^s + \rho^l\right)$ in the two-agent economy. If the price-affecting investor is endowed with only few shares of the risky asset ($\bar{x}^l < x^{l,c}$), he acts as buyer in equilibrium, as agents trade towards the optimal risk-sharing allocation. Since the large agent restricts his demand to buy for the risky asset in order to mitigate his price impact, the equilibrium price (3.11) turns out to be lower than in the corresponding economy under perfect competition (i.e. with only price takers). Conversely, if the large investor is endowed with many shares ($\bar{x}^l > x^{l,c}$), he restricts his supply to sell and the equilibrium price is higher than under perfect competition. In both cases, the two investors tend to hold their endowments and the equilibrium allocation is not Pareto optimal because of the reduced trading volume. Only for the special case that investors' initial endowments already correspond to the Pareto-optimal allocation, the equilibrium price is equal to the price under perfect competition and the equilibrium allocation is Pareto optimal; in this case, there is no trade, however.[14]

To sum up the static models on large investors, equilibrium asset prices are distorted towards the side of the market on which the price-affecting investor is active (buyer side or seller side of the market) and equilibrium allocations are not Pareto optimal in general, as the large investor restricts his trading volume in order to reduce his liquidity costs.

3.2.2 Dynamic Models with a Single Large Investor

Dynamic models with a single large investor focus on the time-consistency of the large agent's trading behavior and relate the analysis to arguments from durable-goods monopoly and the Coase (1972) conjecture. A durable-goods monopolist sells units of his product repeatedly over time to different customers. The monopolist's future sales therefore compete with his current sales and erode his monopoly power. When the monopolist is unable to credibly commit to a dynamic price strategy, customers with an intermediate willingness to pay postpone their purchases as they anticipate that the monopolist will reduce the price and "flood the market" in the future. As a consequence, the monopolist charges a lower first-period price and makes lower intertemporal profits than in the case with commitment. The Coase (1972) conjecture then proposes that, when the time horizon is infinite and price adjustments occur continuously over time (and commitment is not possible), the monopolist will charge the competitive price right from the beginning and intertemporal profits are driven to zero. The connection between durable-goods monopoly and dynamic securities trading with large investors is first pointed out by DeMarzo and Bizer (1993).[15]

Basak (1997) considers a general equilibrium model for a complete-markets economy without resorting to the CARA-Gaussian setting. There is a single large investor who affects the price of each Arrow-Debreu security in trading against the competi-

[14] Hirth (1997) extends the model of Rudolph (1982) to two rounds of trading and finds that price and allocation of the risky asset adjust closer, though still incompletely, towards the market outcomes under perfect competition after the additional round of trading. Spremann and Bamberg (1983) apply a variant of Rudolph's (1982) model to address the optimal financial policy for a firm that has price impact in the primary stock market.

[15] On durable-goods monopoly and for references on formal proofs of the Coase conjecture, see Tirole (1988, ch. 1).

tive fringe. Basak (1997) starts with a static model and shows that the intuition from Rudolph (1982) carries over to his more general setup. As the large investor restricts his trade size to mitigate his adverse price impact, his portfolio and consumption are distorted towards his initial endowments and the equilibrium allocation is not Pareto optimal in general. Likewise, the initial endowment of the large investor appears as an additional pricing factor, and Arrow-Debreu state prices are lower (higher) compared to a perfectly competitive economy, when the large investor acts as buyer (seller).

Basak (1997) then extends the model to a dynamic framework in continuous time. The large investor initially chooses a dynamic trading strategy to which he is able, by assumption, to credibly commit, i.e., the agent does not deviate from that strategy at later dates. In the optimal self-commitment strategy, the large investor takes into account the effects of his future trades on current asset prices. For this case, equilibrium prices and allocations of the dynamic model are affected by the large agent's endowment similarly to the static model. Basak (1997) remarks that the large investor's dynamic strategy is time-inconsistent, however. When the agent gets to later trading dates, he no longer cares about his effects on the past prices and, hence, has an incentive to deviate from the strategy chosen initially. The derivation of the time-consistent strategy is intractable in the rather general framework of the model though.

Kihlstrom (2000) analyzes the durable-goods aspect of dynamic securities trading in the tractable CARA-Gaussian framework. In the model, a single large investor initially owns all shares of the risky asset and sells part of them to small investors over two rounds of trading. Kihlstrom (2000) derives equilibrium for two scenarios: (*i*) a commitment scenario, in which the large agent can credibly commit in the first trading round to the price he will charge in the second trading round; and (*ii*) a time-consistent scenario, in which the large agent is unable to make a commitment. In the commitment scenario, the price-affecting investor chooses a path of prices above the competitive level such that he sells shares to the small investors in the first trading round, but no additional shares in the second round. As the commitment strategy evades the intertemporal competition from the second period, the large investor obtains a relatively high price by restricting his supply to sell in the first period. In the time-consistent scenario, the price-affecting investor chooses prices again above the competitive level, but lowers the price over time in order to sell additional shares to the small investors in the second period.

Comparing the two scenarios, Kihlstrom (2000) finds that the first-period price in the time-consistent case is indeed lower than in the commitment case. Accordingly, in the time-consistent case, trading volume is higher in both periods and investors' portfolio holdings adjust closer towards optimal risk sharing. In summary, the model of Kihlstrom (2000) illustrates that the mechanism of durable-goods monopoly applies also to dynamic securities trading with a price-affecting investor.

Vayanos (2001) presents a discrete-time, infinite-horizon model in which a risky asset is traded between a large investor, a competitive fringe of small investors and exogenous, random noise traders. The large investor receives a privately observed endowment shock in the risky asset. The endowment shock provides useful private information, because it determines the future price dynamics. For instance, if the price-affecting investor receives a positive endowment shock, he anticipates that the price will fall as soon as he will sell some of the shares to reduce his risk. The presence of noise traders generates additional price risk and precludes the small investors from inferring the private information immediately from prices. The large investor faces a trade-off between risk sharing and speculation profits: if he trades more aggressively, he adjusts his risk exposure more quickly; however, the private information is revealed also more quickly and, thus, intertemporal profits from speculation are lower.

For a time-consistent equilibrium, Vayanos (2001) shows that the large investor unwinds his endowment shock gradually over time and that the allocation of the risky asset converges to a competitive long-run limit of optimal risk sharing. The convergence of the portfolio holdings can take place in two different patterns, depending on the relation of noise trading to the risk aversion of the large investor. If noise is small and risk aversion is large, then convergence of portfolio holdings is monotonic and the trade size of the price-affecting agent decreases over time. If, in contrast, noise is large and risk aversion is small, convergence is non-monotonic. For a positive endowment shock, the large investor first sells a fraction of his endowment to establish optimal risk sharing. The competitive investors misinterpret this sale as being driven by noise traders and therefore do not adjust their price expectations sufficiently downwards. As a consequence, the large investor sells additional shares at the high price only to buy them back and to return to optimal risk sharing later at a lower price, after small investors have corrected their expectations. Thus, when the large investor trades not only for risk sharing but also based on private information, the adjustment of the market towards the competitive outcome may be affected by strategies of market manipulation.

DeMarzo and Urosevic (2006) integrate the durable-goods idea of securities trading into a corporate finance model on corporate control. In the model, risky shares of a firm are traded between a large shareholder, who can improve the payoff of the shares by engaging in costly monitoring the firm, and a fringe of small investors, who act as price takers and do not influence the firm. The large shareholder faces a trade-off between monitoring incentives and diversification in each period: if he holds a higher stake in the firm, his incentives to improve the firm's performance are enhanced; but at the same time, holding a higher stake exposes him to excessive risk. When the large shareholder decreases his stake in the firm, he adversly affects the trading price of the shares, as small investors are risk averse and, in addition, anticipate a reduction in monitoring effort and asset payoff.

In a time-consistent equilibrium with continuous trading, DeMarzo and Urosevic (2006) find that the large shareholder reduces his stake over time and ultimately trades towards the perfectly competitive allocation, even though this allocation entails inefficient monitoring. The speed of the portfolio adjustment depends on the severity of the incentive problem: if the incentive problem is weak, the large shareholder reduces his holdings immediately to the competitive allocation; if the incentive problem is large, portfolio adjustment occurs only gradually over time. Hence, the instantaneous adjustment towards the competitive outcome in line with the Coase (1972) conjecture does not hold in a corporate finance context with significant incentive problems.

3.2.3 Dynamic Models with Multiple Large Investors

Models with multiple large investors consider strategic interactions among the price-affecting agents as a Cournot game, i.e., the competitive fringe of small investors provides a downward-sloping demand schedule to the market and each large investor trades against this schedule by choosing quantities of shares, taking as given the quantity choices of all other price-affecting agents.

Pritsker (2005) studies a discrete-time CARA-Gaussian model with a representative small investor ($i = 1$) and multiple large investors ($i = 2, \ldots, I$). In the economy, there are K risky assets ($k = 1, 2, \ldots, K$), each with a total of $\bar{X}_k$ outstanding shares, and a riskless asset with exogenous interest rate r. At each date t, risky asset k pays a normally distributed dividend $\tilde{d}_{k,t}$ and has an ex-dividend price $p_{k,t}$. The dividends are independently and identically distributed over time, and the large investors have price impact for each risky asset. Although trading is limited to a finite number of periods, investors are infinitely lived and continue to receive dividend and interest payments from their final portfolios beyond the last round of trading.

Pritsker (2005) applies his model to analyze market manipulation and shock propagation in a scenario of distressed asset sales by one large investor when the other price-affecting agents anticipate that investor's need to sell.[16] In a preliminary step of the analysis (without distressed asset sales), Pritsker (2005) also derives asset pricing results. If p_t denotes the $K \times 1$ vector of risky asset prices at date t, $\tilde{d}_t$ the vector of dividends with mean $\bar{d}$ and covariance matrix Ω, $\bar{X}$ the vector of outstanding shares, and if x_t^i and $x^{i,c}$ denote the vectors of portfolio holdings after trade at date t and under optimal risk sharing, respectively, then equilibrium asset prices satisfy the following multi-factor structure,

$$p_{t+1} + \bar{d} - (1+r)p_t = \left[(1+r)\sum_{i=1}^{I}(1/\rho^i)\right]^{-1} r\,\Omega\,\bar{X} + \sum_{i=2}^{I}\delta_t^i\,\Omega\,\left(x_t^i - x^{i,c}\right), \quad (3.12)$$

where δ_t^i is a parameter that is identical for large investors with identical risk aversion. The pricing factors on the right-hand side of (3.12) are the market portfolio, as in the standard CAPM, and additional factors according to the deviation of each large investor's portfolio holdings at date t from his holdings under optimal risk sharing. Thus, the trading costs in Pritsker (2005) generate additional pricing factors for non-optimal risk sharing. For the special case that all large investors have identical risk aversion, (3.12) simplifies to a two-factor pricing relation, where the factors are the market portfolio and $\sum_{i=2}^{I}(x_t^i - x^{i,c})$. However, as investors trade towards the

[16] Attari, Mello, and Ruckes (2005) and Brunnermeier and Pedersen (2005) present other models on market manipulation against distressed investors; they assume that strategic investors are risk neutral rather than risk averse.

optimal risk-sharing allocation over time, asset prices converge to their values under perfect competition and, thus, the additional pricing factors vanish over time.[17]

Urosevic (2004) extends the corporate finance model on corporate control by DeMarzo and Urosevic (2006) to the case of multiple large shareholders. Large shareholders face a trade-off between incentives for monitoring the firm and risk sharing, and decreasing their stakes in the firm lowers the trading price as small investors anticipate a reduction in monitoring effort. With multiple large shareholders, there is an additional strategic effect: if a large agent sells more of his shares in the early periods, he decreases the willingness of other large agents to sell in later periods, as they will then receive an even lower trading price. This effect creates a "race to diversify" among large shareholders, as each of them tries to reduce his stake in the firm more quickly.

For the time-consistent equilibrium, Urosevic (2004) shows that share holdings of the large agents adjust towards the perfectly competitive allocation gradually over time and that the speed of portfolio adjustments increases in the number of large agents. Intuitively, when there are more strategic agents in the economy, trading becomes more competitive and the competitive outcome is reached more quickly.

Blonski and von Lilienfeld-Toal (2006) present a corporate finance model that relates ideas of corporate control to the concept of no-arbitrage in asset pricing. There is a single managing shareholder of the firm who can deterministically enhance the payoff of the shares by undertaking costly managing effort. Before the effort choice, shares of the firm are traded between the managing shareholder and multiple outside investors. Outside investors cannot influence the firm, but act strategically in their trading decision nonetheless (i.e., there is no competitive fringe in the model).

Blonski and von Lilienfeld-Toal (2006) show existence of an equilibrium in which a strictly positive quantity of shares is traded below their correctly anticipated value and, thus, no-arbitrage pricing does not hold. In this equilibrium, outside investors refrain from exploiting the underpricing in order to preserve incentives for the managing shareholder to maintain a high stake in the firm and to choose high effort for enhancing the payoff of the shares. However, an equilibrium in which no-arbitrage

[17] In the corresponding economy under perfect competition, optimal risk sharing is established within the first round of trading and asset prices are constant over time (Stapleton and Subrahmanyam (1978)).

pricing does hold and the managing shareholder withdraws from the firm and chooses low effort exists as well.[18]

3.3 Endogenous Market Liquidity

In this section, we consider a few recent models that examine the pricing effect of illiquidity by deriving market liquidity endogenously from the trading interactions of investors in the market microstructure process. These models, hence, trace illiquidity back to an underlying basic friction (asymmetric information, inventory problems, or search and order processing costs) and contribute to the linkage of asset pricing theory and market microstructure theory as called for by O'Hara (2003).

Garleanu and Pedersen (2004) study how current and future asymmetric information among investors generate bid-ask spreads and affect equilibrium asset returns. In the model, a finite number of risk-neutral investors trade a single risky asset and a riskless asset repeatedly over time. In each period, one randomly chosen investor receives a private signal about the next dividend of the risky asset and also a private liquidity shock. The liquidity shock can take the form of a private benefit or cost of holding the risky asset in the next period and gives the informed investor a trading motive in addition to his dividend signal. Investors can trade by submitting market or limit orders for the risky asset, and all orders are executed at a market-clearing price. The presence of liquidity shocks raises the potential for Pareto-improving trade among investors and thereby avoids a no-trade outcome, which otherwise would occur because of the adverse selection problem faced by the uninformed agents.

Garleanu and Pedersen (2004) show that in equilibrium all uninformed agents submit limit sell orders with the same (ask) price and limit buy orders with the same (bid) price, respectively, and the informed investor submits a market order or no order. In each period, a strictly positive bid-ask spread arises because of the adverse selection problem of the uninformed agents in that period. The bid-ask spread, in turn, impedes some of the Pareto-efficient trades (which would take place

[18] Other models that address the interrelation between corporate control, large shareholders and market liquidity, but do not focus on standard asset pricing issues, are Shleifer and Vishny (1986), Holmström and Tirole (1993), Admati, Pfleiderer, and Zechner (1994), Bolton and von Thadden (1998), and Maug (1998).

in the absence of asymmetric information) and thereby induces costs of allocation inefficiencies to investors.

For the asset pricing side of the model, the question is how current and future bid-ask spreads affect the price level of the risky asset. Garleanu and Pedersen (2004) find that bid-ask spreads do not directly affect the price of the risky asset, i.e., the price does not reflect a discount for the costs of trading the asset—a result that stands in contrast to models on exogenous bid-ask spreads, in which the price of an asset is reduced by the present value of all its future trading costs. The reason is that investors anticipate two effects for their future trades: first, when an investor trades due to a liquidity shock, he incurs a loss because of the trading costs; second, when an investor trades due to a dividend signal, he makes a profit because of his private information. If the different investors are equally likely to make liquidity and information trades, then these two effects offset each other with respect to the price of the risky asset. However, investors also anticipate the allocation inefficiencies caused by bid-ask spreads. As a consequence, the price of the risky asset reflects a discount according to the present value of all future allocation costs of investors. In summary, Garleanu and Pedersen (2004) find that bid-ask spreads generated by asymmetric information reduce the price of an asset indirectly through the resulting allocation costs, but not directly through the trading costs incurred by investors.[19]

A few other models, beginning with Duffie, Garleanu, and Pedersen (2005, 2006), integrate search costs into asset pricing theory and consider endogenous illiquidity in the form of bid-ask spreads or time delays of trading. Search problems are particularly relevant for markets in which trading is conducted in bilateral processes, so that it is costly and time consuming for investors to contact a potential trading counterparty and to negotiate over the terms of the trade (for instance in over-the-counter markets).[20] We do not review these search-theoretic models in more detail, however, as they are less related to our subsequent asset pricing analysis, in which the market

[19] Wang (1993), O'Hara (2003), Easley and O'Hara (2004), and Gallmeyer, Hollifield, and Seppi (2005) analyze the effects of asymmetric information on equilibrium asset returns in competitive rational expectations equilibrium models; in these models, markets are assumed to be perfectly liquid—despite the presence of asymmetric information.

[20] For models on search-theoretic aspects in asset pricing see also Garleanu (2006), Lagos and Rocheteau (2006), Lagos, Rocheteau, and Weill (2007), Vayanos and Wang (2007), and Vayanos and Weill (2007).

organization resembles more a Walrasian auction with multilateral trading (as in the other models illustrated in this chapter).

3.4 Discussion

In this chapter, we review the literature on market liquidity in asset pricing. The approaches included in the review are: asset pricing models with exogenous bid-ask spreads, asset pricing models with large investors, and asset pricing models with endogenous market liquidity. To conclude this chapter, we provide a brief discussion of these approaches and relate them to our analysis of asset pricing with imperfect competition and endogenous market liquidity as carried out in the following chapters.

The first group of reviewed models considers illiquidity in the form of exogenous bid-ask spreads. These models establish, by and large, the standard result that liquidity costs reduce the price of an asset and, equivalently, increase the expected return of an asset, as investors require compensation for bearing trading costs in illiquid markets. Though concerned with the pricing effect of illiquidity, these models treat investors' trading behavior in illiquid markets in a rather cursory way, however. This relates to two aspects.

As a first aspect, the exogenous integration of bid-ask spreads into the asset pricing analysis ignores the question of how the spread arises from investors' market microstructure interactions. An implicit assumption of these models, therefore, is that the pricing effect of illiquidity is the same, regardless of whether bid-ask spreads originate from asymmetric information, from inventory problems, or from search and order processing costs.

As a second aspect, most of the models are based on OLG structures or exogenous wealth shocks (liquidity shocks) as modeling devices to generate repeated trade among investors, since, otherwise, rational investors in a typical asset pricing setting would refuse to trade very often and, hence, would not be very concerned about incurring liquidity costs. These modeling devices provide a convenient way to reproduce the high trading volume that can be observed in real-world securities markets and, thus, to address the practical importance of market liquidity to investors. As Cochrane (2004, p. 8) argues, such devices are *"quick modeling tricks where starting* *from 'deep microfoundations' would waste too much time and space on the way to the phenomenon we want to capture."* However, an unfortunate by-product of these modeling tricks is a crude representation of investors' trading behavior as viewed from a market microstructure perspective.

For concreteness, consider models with a two-period-lived OLG structure. These models involve the assumption that, in each period, the old generation of investors sells the total stock of outstanding shares in a perfectly inelastic way to the new generation of investors. That is, the investors on the seller side of the market previously hold all outstanding risky assets in their portfolios and offer all these holdings for sale without conditioning the trade on the price to be received or on the liquidity costs to be incurred. The investors on the buyer side of the market, in contrast, have no initial holdings of risky assets and take into account the price to be paid and all potential liquidity costs to be incurred when forming their demand. The consequence of generating trade with a perfectly inelastic seller side is that in fact any arbitrary facet of illiquidity taken into account by buyers results in an additional discount on the price and, equivalently, is reflected in an additional premium on the required return of the asset. Figure 3.1 illustrates this point.

When the standard pricing effect of illiquidity is thus stripped down to its essentials, then it can be seen that the effect is basically the result of a tax in a market with inelastic supply, so that trading volume is entirely determined by sellers and price is entirely determined by buyers.[21] This representation of trading behavior is associated with several highly implausible implications for the analysis of secondary securities markets: (*i*) buyers can pass along all their liquidity costs completely to sellers, so that sellers ultimately bear all the economic burden resulting from the lack of market liquidity; (*ii*) the level of market liquidity does not affect trading volume—an implication in stark contrast to the finding of virtually every empirical study that market liquidity is positively correlated with trading volume; (*iii*) illiquidity does not induce a deadweight loss or excess burden on the economy, i.e., illiquidity does not reduce investors' expected utility by costs of allocation inefficiencies due to foregone trade (for instance in the form of inefficient risk sharing). Consequently, the modeling tricks used in most of the asset pricing models with exogenous bid-ask spreads

[21] For presentations of tax effects, see for example Varian (2003, ch. 16) or Homburg (2005).

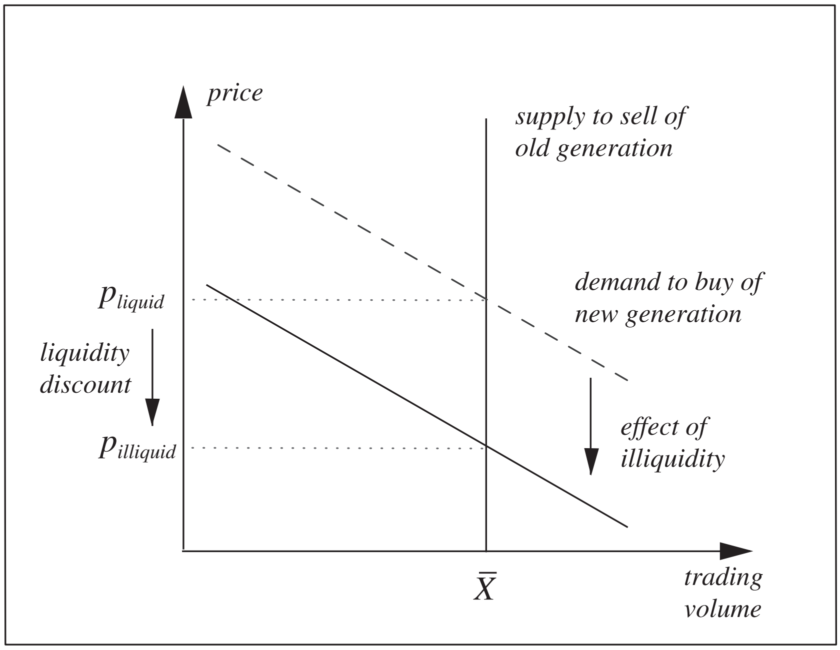

Figure 3.1: Standard pricing effect of illiquidity.

involve a crude and superficial representation of investors' trading behavior, and it is highly desirable to have a deeper market microstructure foundation for analyzing the pricing effect of illiquidity.

The second group of models reviewed in this chapter examines illiquidity in the form of price impact for large investors. In these models, a competitive fringe of small investors submits a downward-sloping demand function to the market, against which the large investors trade by choosing quantities of shares to buy or sell. The degree of market liquidity for large investors is thus endogenous according to the risk-bearing capacity of small investors as reflected in the slope of their demand function. The presence of the competitive fringe is convenient for the tractability of the models, since it allows to reduce the strategic interactions of large investors to quantity competition while still guaranteeing the existence of a unique market-clearing price. Yet, this representation of trading behavior may be questioned in two respects.

First, the classification of investors into large and small agents is exogenous, so that it remains unexplained because of which economic characteristic some agents have price impact whereas other agents trade under perfect market liquidity.[22] Second, the trading behavior of the two types of investors is counterintuitive from a market microstructure view. Large investors trade by choosing quantities ("market orders"); hence, despite their price impact, they do not control the trade for the execution price. Small investors, in contrast, trade by submitting price-contingent demand functions ("limit orders"); thus, although being small, they are the agents who provide the liquidity to the market. A more intuitive view of trading behavior and liquidity provision, however, is that those investors who are concerned about their price impact trade by using limit orders and, consequently, that it is the large investors who determine the degree of liquidity in the market.

The last group of models reviewed in this chapter studies the pricing effect of endogenous market liquidity by taking into consideration how illiquidity arises due to a basic microstructure friction in the trading interactions of investors. The few existing models in this recent strand of literature mostly treat investors as risk neutral and focus on asymmetric information and search costs in generating illiquidity. However, as standard asset pricing theories assume that investors are risk averse, it is natural to endogenize illiquidity also due to investors' risk-bearing costs in line with the inventory-based market microstructure approach.

In the next chapters, we analyze the pricing effect of illiquidity in an asset pricing framework with imperfect competition and endogenous market liquidity. This framework relates and contrasts to the approaches reviewed in this chapter as follows. As in the models with exogenous bid-ask spreads, we focus on how illiquidity affects equilibrium expected asset returns. However, in order to develop a more reasonable market microstructure representation, we treat liquidity and trading as endogenous and, thus, analyze in detail how illiquidity arises from a basic microstructure friction and how investors adjust their trading behavior in the presence of liquidity costs. As in the models with large investors, we integrate illiquidity in the form of price impact. Yet, in order to capture a more intuitive view of liquidity provision, we do not make use of a competitive fringe, but endogenize market liquidity in a fully strategic setting. Finally, as in the models with endogenous market liquidity, we ad-

[22]The common mathematical statement is that a large investor is treated as an "atom" and a small investor as a "point in a continuum," but this statement does not address the problem on an economic level.

dress the pricing effect of illiquidity within an explicit market microstructure process. However, we assume that investors are risk averse and focus on risk-bearing costs as underlying basic friction to generate illiquidity. Hence, we complement the linkage of asset pricing theory and market microstructure theory by emphasizing arguments from the inventory-based microstructure literature.

Chapter 4

Asset Prices under Imperfect Competition

In standard theories of asset pricing, the expected return of an asset is increasing in the risk of the asset, as risk-averse investors require compensation for bearing risk. Beginning with the seminal work of Amihud and Mendelson (1986), a similar argument is made regarding the pricing effect of liquidity costs: as investors are also averse to trading costs in illiquid markets and require compensation for bearing these costs, the expected return of an asset is also increasing in illiquidity. Thus, asset returns should reflect both a risk premium and a liquidity premium.

The existence of a liquidity premium is shown in several models with respect to different facets of market liquidity (see Amihud and Mendelson (1986), Vayanos and Vila (1999), Huang (2003), Vayanos (2004), Acharya and Pedersen (2005), and others). Most of these models, however, take trading costs as exogenous and generate trade among investors by imposing overlapping generations structures or exogenous wealth shocks, so that trading is partly exogenous as well. Hence, these models do not consider how illiquidity arises from an underlying market microstructure friction and give only a limited view of how investors optimally adjust their trading behavior in the presence of liquidity costs.

In this chapter, we provide an asset pricing model in which liquidity costs are endogenous due to imperfect competition among investors; this approach allows to endogenously intergrate market microstructure features of market liquidity into asset pricing theory. We study a static CARA-Gaussian setting in which investors trade a single risky asset to improve risk sharing. Investors act strategically by considering the impact of their trades on the price. In equilibrium, each investor submits a downward-sloping demand function for execution against the demand functions of the other agents. Trades have price impact, because there is a finite number of risk-averse agents in the market. In response to their price impact, investors reduce trading volume and thereby impede Pareto-optimal risk sharing. Our main result is that the expected return of the risky asset is not affected by illiquidity and, thus, reflects only a risk premium but no liquidity premium. The reason is that buyers require a price discount for their liquidity costs and sellers require a price premium, and these effects cancel each other out with respect to the market-clearing price and the expected return, respectively. This result stands in contrast to the existence of a liquidity premium in the models on exogenous trading costs cited above.

Models such as ours, in which agents act under imperfect competition and have sets of feasible trading strategies that include price-quantity combinations (demand functions), are sometimes called "demand submission games" or "limit order games." Beginning with Kyle's (1989) noisy rational expectations model on information aggregation with imperfect competition, the CARA-Gaussian setting has become a standard framework for demand submission games with risk-averse agents in market microstructure theory.[1] Demand submission games are also analyzed in three other strands of literature: the literature on share auctions, the oligopoly literature, and the literature on double auctions.[2]

In a share auction (divisible good auction, multi-unit auction), a single seller offers a divisible item of which fractional amounts are to be sold to several bidders. Bidders compete by submitting demand schedules, specifying prices for possible fractions of the supplied item, and the item is then allocated among bidders in the order of descending price until supply is exhausted. Wilson (1979) and Back and Zender (1993)

[1] See also the microstructure models by Pagano (1989a) on concentration versus fragmentation of trade across markets, Vayanos (1999) on dynamic risk sharing when agents have privately observed endowment shocks, Viswanathan and Wang (2002, 2004) on comparisons of liquidity supply between dealer markets, limit order books, and hybrid markets, and Dumitrescu (2006) on market liquidity and price informativeness when market makers have private information about the limit order book.

[2] Furthermore, there is also a literature on "strategic market games," focusing on strategic foundations for competitive general equilibrium aspects. See, for instance, Roberts and Postlewaite (1976), Shapley and Shubik (1977), Dubey and Shubik (1978), and Dubey (1982). Giraud (2003) provides an introduction to and a brief survey of this literature.

show in several settings (e.g. allowing for risk aversion, asymmetric information, or discriminatory pricing) that share auctions have equilibria with collusive bidder behavior resulting in significant underpricing of the item and low revenue for the seller. In share auction models, bidders can only buy but not sell, and strategic behavior is present only on the demand side of the market.[3]

Closely related to share auctions, from an analytical point of view, is the oligopoly literature on supply function equilibria starting with Grossman (1981) and Klemperer and Meyer (1989). In these models, firms face an exogenous, downward-sloping demand function and compete by choosing supply functions relating output quantities to market prices. Firms have thus more general strategy sets than under pure quantity competition (Cournot oligopoly) or pure price competition (Bertrand oligopoly). In oligopoly models, in turn, strategic interaction takes place only between firms on the supply side of the market.[4] The main difference of our model to the literature on oligopoly and share auctions, apart from the focus of the analysis, is that we consider investors who can both sell and buy, so that strategic interaction is present within and between both sides of the market. This setting is more appropriate to our analysis of asset prices in secondary securities markets.

In a double auction, buyers and sellers submit prices at which they are willing to trade depending on their privately know reservation values. Both sides of the market act thus strategically. Chatterjee and Samuelson (1983) initiate the analysis of the k-double auction between one buyer and one seller for a single item, and Wilson (1985) provides an early extension to the case of multiple buyers and multiple sellers, thereby endogenizing the trading volume of the auction. Most of the double auction models examine risk-neutral agents with $0-1$ demands.[5] Instead, we consider risk-averse

[3] In Back and Zender (2001) underpricing is reduced by giving the seller the option to cancel part of the supply after observing the bids, and in Kremer and Nyborg (2004) underpricing is made small when bidders have to submit finite numbers of price-quantity pairs (instead of smooth demand functions). Wilson (1979) and Wang and Zender (2002) contain settings with CARA bidders, normally distributed asset values, symmetric information, and uniform pricing, which is also the case in our model.

[4] See also Shapiro (1989, pp. 348-56) or Vives (1999, ch. 7). Supply functions include quantity and price strategies as special cases: setting a quantity is equivalent to choosing a perfectly inelastic supply function, and setting a price is equivalent to choosing a perfectly elastic supply function.

[5] See also Williams (1987) and Satterthwaite and Williams (1989) for bilateral double auctions and Gresik and Satterthwaite (1989) and Rustichini, Satterthwaite, and Williams (1994) for multilateral double auctions.

investors who each can trade multiple shares of a risky asset. Again, this setting is more appropriate to our asset pricing context.

The remainder of this chapter is organized as follows. Section 4.1 presents the setup of the model. Section 4.2 studies investors' optimization problems under imperfect competition. Section 4.3 analyzes equilibrium for the case that investors have identical risk aversion, and Section 4.4 extends the results to the case of heterogeneous risk aversion. Section 4.5 provides a concluding discussion. If the proofs of the results are short, they are stated immediately after the results; otherwise they are relegated to the Appendix of this chapter.

4.1 Description of the Economy

We consider asset pricing in a static framework. At the beginning of the period, a riskless asset and a risky asset are traded among investors. Taking the riskless asset as numeraire with perfectly elastic supply, let p be the price of the risky asset. At the end of the period, assets pay off exogenous liquidation values, and each investor consumes his terminal wealth. Each unit of the riskless asset yields a payoff $1 + r$, where r is the riskfree rate of interest, and each share of the risky asset pays a normally distributed liquidation value $\tilde{v}$. The total number of outstanding shares of the risky asset is denoted by $\bar{X}$.

In the economy, there are I rational investors (indexed by $i = 1, 2, \ldots, I$) with symmetric information regarding all relevant parameters. Each investor is initially endowed with $\bar{y}^i$ units of the riskless asset and $\bar{x}^i$ units of the risky asset (with $\sum_{i=1}^{I} \bar{x}^i \equiv \bar{X}$). If y^i and x^i are the corresponding portfolio holdings after trade has taken place, terminal wealth for investor i is given by

$$\tilde{w}^i = (1 + r)y^i + \tilde{v}x^i.$$

Each investor has a CARA utility function over terminal wealth,

$$u^i\left(w^i\right) = -\exp(-\rho^i w^i),$$

where $\rho^i > 0$ is the coefficient of absolute risk aversion (and its inverse, $1/\rho^i$, is referred to as the risk tolerance). With normally distributed terminal wealth and

CARA utility, maximizing expected utility for investor i is equivalent to maximizing the mean-variance objective function

$$\begin{aligned} \Pi^i &:= E\left[\tilde{w}^i\right] - \frac{\rho^i}{2}\mathrm{Var}\left[\tilde{w}^i\right] \\ &= (1+r)y^i + E\left[\tilde{v}\right]x^i - \frac{\rho^i}{2}\left(x^i\right)^2 \mathrm{Var}\left[\tilde{v}\right]. \end{aligned}$$

The market for the risky asset is organized as call auction (batched auction). Investors simultaneously submit orders to an auctioneer, who sets a single price to clear the market and settles all orders at this price.[6] The auctioneer himself does not trade on his own account, however. For each investor i, an order is allowed to be any non-increasing, continuously differentiable function mapping the price of the risky asset into units of the risky asset to buy: $\Theta^i : \mathbb{R} \to \mathbb{R}$, such that $\theta^i = \Theta^i(p)$. (Negative values indicate units of the risky asset to sell.) The auctioneer sets the price and allocates shares of the risky asset according to the following rules. If there exists a unique price p' that satisfies the market-clearing condition

$$\sum_{i=1}^{I} \Theta^i(p) = 0, \tag{4.1}$$

then all orders are executed at this price, and the risky asset is allocated such that $x^i = \bar{x}^i + \Theta^i(p')$ for all i. If there does not exist a unique market-clearing price, then no order is executed and each investor is forced to hold his initial endowment. The last rule is required to fully describe our trading game for all feasible order choices of investors; it does not drive any of the results in the equilibrium discussed in the next sections.

Of course, our model uses a highly stylized representation of real-world market organizations. In particular, the model abstracts from many institutional details and also from the presence of actively trading intermediaries. It will turn out, however, that our simple market rules are sufficient to endogenously integrate several microstructure properties of market liquidity into our asset pricing analysis, once we allow for strategic interactions among investors.

[6] The crucial point of simultaneous order submission is that each investor makes his trading decision without knowing the other agents' decisions. A less restrictive interpretation is therefore that each investor places his order at an arbitrary point in time within a short pre-trading phase, but without observing the other agents' orders (as in a closed order book).

Before we turn to the analysis of strategic trading and market liquidity, we briefly review the market outcomes for the corresponding economy under perfect competition (i.e. for the case that each investor acts as price taker by assumption). This case provides the benchmark of asset pricing in a perfectly liquid market.

Remark 1 *Under perfect competition, each investor i has the following optimal demand to buy shares of the risky asset,*

$$\Theta^{i,c}(p) = \frac{E[\tilde{v}] - (1+r)\,p}{\rho^i \mathrm{Var}[\tilde{v}]} - \bar{x}^i.$$

The resulting market-clearing price is given by

$$p^c = \frac{1}{(1+r)} \left(E[\tilde{v}] - \frac{\bar{X}\mathrm{Var}[\tilde{v}]}{\sum_{i=1}^{I}(1/\rho^i)} \right),$$

and the allocation of the risky asset among investors is Pareto optimal (optimal risk sharing) with portfolio holdings

$$x^{i,c} = \frac{(1/\rho^i)}{\sum_{i=1}^{I}(1/\rho^i)}\,\bar{X} \quad \textit{for all } i.$$

If an investor makes his trading decision under perfect competition, then his optimal demand for the risky asset is restriced by his risk aversion and the risk of the asset. Accordingly, the market-clearing price is given by the expected payoff per share minus a risk premium, both discounted at the riskfree rate of interest, and the risk premium, in turn, is increasing in the risk of the asset and decreasing in the aggregate risk tolerance of the economy. Investors' portfolio holdings conform to the risk-tolerance-weighted allocation rule of optimal risk sharing. The corresponding trading volume in the economy can be denoted

$$V^c = \frac{1}{2}\sum_{i=1}^{I}\left|x^{i,c} - \bar{x}^i\right|,$$

where the division by 2 follows the common convention to count each share traded only once (although each share bought by some investor is also sold by another investor, of course).

4.2 Trading with Imperfect Competition

In the following analysis, investors do not act as price takers, but are aware that their individual trading affects the price as consequence of the market-clearing condition; it is this awareness that makes competition imperfect and raises the question for market liquidity. More precisely, each investor chooses a trading strategy to maximize his expected utility of terminal wealth subject to his budget constraint and the market rules, taking as given the strategies of all other investors (rather than the price). This change in the trading behavior involves a change in the equilibrium concept: the perfectly competitive Walras equilibrium is replaced by the game-theoretic Nash equilibrium.

To make the concept of Nash equilibrium formal, let $\mathbb{I} = \{1, 2, \ldots, I\}$ denote the set of investors, A^i the set of feasible orders, g the consequence function—which represents the market rules and associates investors' order decisions with a trading outcome for each investor—and let Π^i denote the payoff function over the trading outcome for investor i. Then the normal form representation $\Gamma_N = \left\{\mathbb{I}, \{A^i, \Pi^i\left(g\left(\cdot\right)\right)\}_{i\in\mathbb{I}}\right\}$ captures all strategic aspects of our static asset pricing model with imperfect competition.[7] In addition, let $\Theta = \{\Theta^i\}_{i\in\mathbb{I}}$ be a profile of orders, one for each investor, and let $\Theta^{-i} = \left\{\Theta^h\right\}_{h\in\mathbb{I}\setminus i}$ be a profile of orders for all agents except for investor i. Then we have the following definition.

Definition 2 *A (pure-strategy) Nash equilibrium of trading game Γ_N is an order profile Θ^* with the property that for each investor $i \in \mathbb{I}$,*

$$\Pi^i\left(g\left(\Theta^{i,*}, \Theta^{-i,*}\right)\right) \geq \Pi^i\left(g\left(\Theta^i, \Theta^{-i,*}\right)\right)$$

for all $\Theta^i \in A^i$.

We assume that all aspects of the model are common knowledge and focus on Nash equilibria in pure strategies. Furthermore, we confine attention to equilibria in

[7] The consequence function is included to emphasize that each investor is aware of the market rules under which he makes his trading decision and how these rules affect his trading outcome. Formally, the consequence function is defined on the cross product of investors' sets of feasible orders and maps uniquely to each investor's portfolio holdings (which, in turn, determine expected utility); see Osborne and Rubinstein (1994, ch. 2).

linear strategies. Let the conjectured strategy for each investor i be denoted

$$\Theta^i(p) = \alpha^i - \beta^i p. \tag{4.2}$$

The parameters α^i and β^i will be determined in equilibrium for all i. The feasible trading strategies for each investor include market orders, limit orders, and passive investment behavior. A market order $\langle \alpha^i \neq 0, \beta^i = 0 \rangle$ specifies to buy or sell a fixed quantity of shares irrespective of the trading price. A limit order $\langle \alpha^i, \beta^i > 0 \rangle$ specifies to buy or sell different quantities of shares at different prices and, thus, conditions the trade on the price. Finally, a passive investment strategy $\langle \alpha^i = 0, \beta^i = 0 \rangle$ means that investor i does not submit any order at all, but decides to passively hold his initial endowment. As the market rules restrict the coefficient β^i to be non-negative for each investor i, the market-clearing condition (4.1) yields a unique market-clearing price,

$$p = \frac{\sum_{i=1}^{I} \alpha^i}{\sum_{i=1}^{I} \beta^i},$$

if and only if at least one investor i submits a limit order.

The first step of the analysis is to derive each investor's best-response correspondence, which specifies an individual investor's optimal order given the conjectured orders of all other agents. For this purpose, we define for each investor i a supply curve as the negative sum of the orders of all other agents $h \neq i$,

$$-\sum_{h \neq i} \Theta^h(p) \equiv -\sum_{h \neq i} \alpha^h + p \sum_{h \neq i} \beta^h. \tag{4.3}$$

The supply curve for investor i specifies the aggregate number of shares that all agents $h \neq i$ offer to sell at each possible price; it can be thought of as order book against which investor i can submit his own order to buy shares. Since each investor's individually optimal order not only depends on the supply curve offered to him, but also affects the supply curves of all other agents, the equilibrium orders for all investors will be determined interdependently in a later step of the analysis.

For the individual optimization, consider the case that at least one agent $h \neq i$ submits a limit order, so that in price-quantity space the supply curve for investor i is upward sloping; this case provides the equilibrium discussed in the next sections. If investor i trades against an upward-sloping supply curve, he can implement his

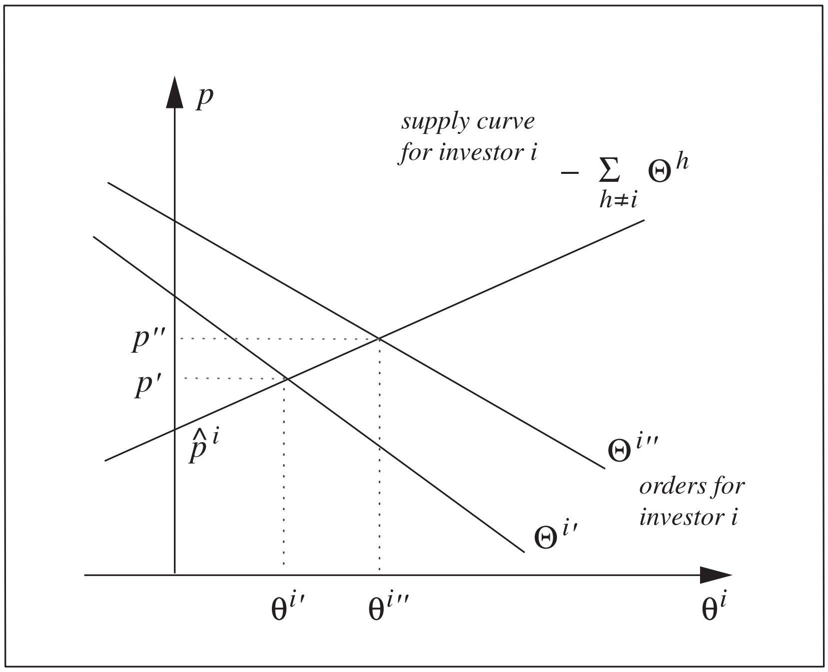

Figure 4.1: Trading against a supply curve.

optimal order by choosing quantities θ^i, since, from the market-clearing condition, each quantity is related to a unique market-clearing price according to

$$\theta^i = -\sum_{h\neq i} \Theta^h(p). \tag{4.4}$$

Consequently, from (4.3) and (4.4), it is possible to write the market-clearing price as function of the number of shares investor i demands to buy,

$$p = \hat{p}^i + \lambda^i \theta^i, \tag{4.5}$$

where $\lambda^i = \left(\sum_{h\neq i} \beta^h\right)^{-1}$ is the slope and $\hat{p}^i = \lambda^i \sum_{h\neq i} \alpha^h$ is the intercept on the price axis of the supply curve for investor i.

Figure 4.1 illustrates the role of market liquidity in the individual trading decision of investor i. The figure represents a given supply curve and two possible orders for buying two alternative quantities of the risky asset. If $\lambda^i > 0$, the market is not perfectly liquid, since investor i cannot buy and sell all he wants at a given price (as would be assumed under perfect competition). Instead, there is an implicit bid-ask spread, as the price to buy a certain number of shares is higher than the price to sell that number of shares. Furthermore, there is price impact, as buying a larger number of shares drives up the price to buy and selling a larger number of shares drives down the price to sell. The degree of market liquidity for investor i is determined by the slope of his supply curve: the steeper the slope, the less liquid the market. The inverse of the slope, $(\lambda^i)^{-1}$, is the Kyle (1985, 1989) measure of market depth; we apply this term to analyze market liquidity.[8]

The following lemma represents the best-response correspondence for each individual investor i.

Lemma 1 *Case 1: If $\beta^h > 0$ for at least one agent $h \neq i$, then the optimal order for investor i is given by*

$$\Theta^{i,*}(p) = \frac{E[\tilde{v}] - \rho^i \mathrm{Var}[\tilde{v}]\,\bar{x}^i - (1+r)\,p}{(1+r)\,\lambda^i + \rho^i \mathrm{Var}[\tilde{v}]}. \tag{4.6}$$

Case 2: If $\beta^h = 0$ for all $h \neq i$ and $\sum_{h\neq i} \alpha^h = 0$, then investor i is indifferent between all feasible trading strategies. Case 3: If $\beta^h = 0$ for all $h \neq i$ and $\sum_{h\neq i} \alpha^h > 0\,(< 0)$, then the optimal order for investor i is characterized by $\alpha^i/\beta^i = +\infty\,(-\infty)$.

Proof: The problem for investor i is to determine y^i and x^i to maximize

$$(1+r)y^i + E[\tilde{v}]\,x^i - \frac{\rho^i}{2}\left(x^i\right)^2 \mathrm{Var}[\tilde{v}] \tag{4.7}$$

subject to his budget condition $\bar{y}^i + p\bar{x}^i = y^i + px^i$ and the market rules. First, consider Case 1, where $\beta^h > 0$ for at least one agent $h \neq i$. Then, according to (4.5), there exists a unique market-clearing price $p = \hat{p}^i + \lambda^i(x^i - \bar{x}^i)$. Plugging the price into the budget condition yields the trading constraint of investor i as

$$y^i = \bar{y}^i - \hat{p}^i(x^i - \bar{x}^i) - \lambda^i\left(x^i - \bar{x}^i\right)^2. \tag{4.8}$$

Plugging (4.8) into (4.7) for y^i and differentiating for x^i gives the first-order condition

$$E[\tilde{v}] - \rho^i x^{i,*}\mathrm{Var}[\tilde{v}] = (1+r)\hat{p}^i + 2(1+r)\lambda^i(x^{i,*} - \bar{x}^i). \tag{4.9}$$

[8]Kyle's (1985) lambda emerges as risk-neutral, price-setting market makers learn private information from observing order flow, and Kyle's (1989) lambda arises as risk-averse investors infer private information by conditioning their demand schedules on prices in a call auction market. In our model, illiquidity is not driven by asymmetric information.

Using the market-clearing price to substitute for $\hat{p}^i$ on the right-hand side of (4.9) yields the optimal demand to hold shares of the risky asset

$$x^{i,*}(p) = \frac{E[\tilde{v}] + (1+r)\lambda^i \bar{x}^i - (1+r)\,p}{(1+r)\,\lambda^i + \rho^i \mathrm{Var}[\tilde{v}]}.$$

The optimal order to buy shares of the risky asset is then given by $\Theta^{i,*}(p) = x^{i,*}(p) - \bar{x}^i$. For the second-order condition

$$-2(1+r)\lambda^i - \rho^i \mathrm{Var}[\tilde{v}] < 0$$

to hold, it is sufficient that $\beta^h > 0$ for some agent $h \neq i$. Now, consider the cases where $\beta^h = 0$ for all agents $h \neq i$. Then, investor i does not trade against an upward-sloping supply curve and, thus, cannot trade different quantities of the risky asset at different prices. In Case 2, where $\sum_{h \neq i} \alpha^h = 0$, the orders of all other agents $h \neq i$ aggregate to zero at each price. Consequently, investor i cannot trade at all and is therefore indifferent between all feasible trading strategies. In Case 3, where $\sum_{h \neq i} \alpha^h > 0\ (< 0)$, the aggregate orders of all other agents $h \neq i$ constitute a (non-zero) market order. Then, investor i obtains maximum expected utility by submitting a limit order which sells (buys) against this market order and pushes the price to $+\infty$ $(-\infty)$. The Appendix to this chapter spells out the details of the last two cases, which involve the market rules for situations in which a unique market-clearing price does not exist. $\|$

Figure 4.2 depicts the effect of market liquidity on the individual trading decision of investor i for Case 1 of Lemma 1. Given the initial endowment at point A, the curve TC represents the trading constraint (4.8) for $\lambda^i > 0$, and the dashed straight line is the constraint for the limiting case of a perfectly liquid market as $\lambda^i \to 0$ (perfect competition). If $\lambda^i > 0$, investor i acts as a monopsonist over his supply curve and recognizes that the price moves against him when he trades an additional unit of the risky asset. As a consequence, his trading aggressiveness is restricted not only by his risk aversion and the risk of the asset, but also by the level of illiquidity in the market. The optimal order equates marginal expected utility of the risky asset with marginal costs, including liquidity costs (point B). The figure shows that liquidity costs from imperfect competition reduce trading volume and expected utility for investor i compared to the optimum in the perfectly competitive case (point C).

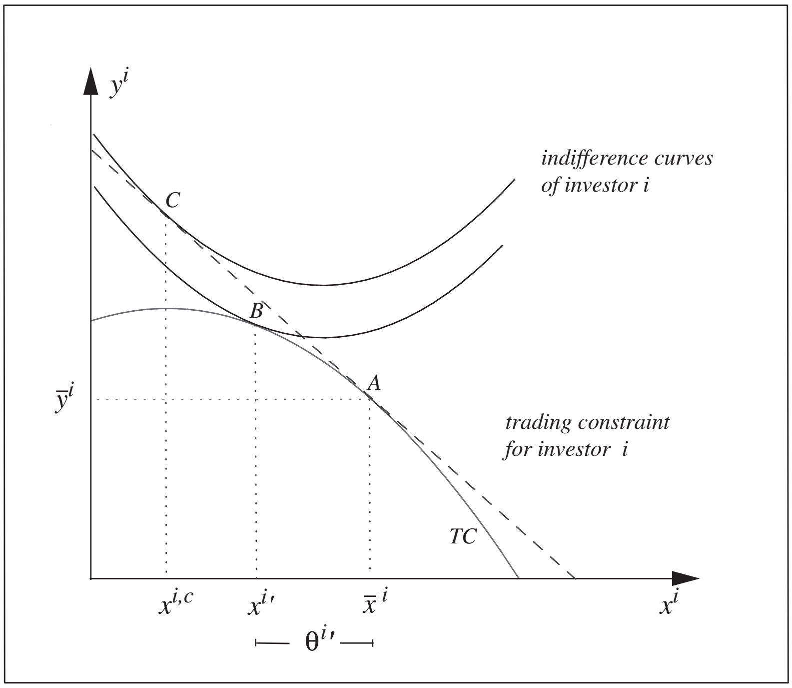

Figure 4.2: Market liquidity in the individual trading decision.

Case 1 of Lemma 1 provides the equilibrium discussed in the next sections. In this equilibrium, each investor submits a limit order and trade takes place; we refer to this equilibrium as trading equilibrium. In addition to the trading equilibrium, there exists also an equilibrium in which each investor employs the passive investment strategy. The remainder of this section argues that this equilibrium in passive strategies is of minor interest in our model.

Proposition 1 *There exists an equilibrium in which each investor plays the passive strategy. For each investor i, the passive strategy is weakly dominated by any limit order of the form*

$$\alpha^i = k^i \left(\frac{E[\tilde{v}]}{\rho^i \mathrm{Var}[\tilde{v}]} - \bar{x}_1^i \right), \quad \beta^i = k^i \frac{(1+r)}{\rho^i \mathrm{Var}[\tilde{v}]} \quad \textit{with} \quad 1 \geq k^i > 0. \tag{4.10}$$

Proof: See Appendix to this chapter. ||

The existence of the equilibrium in passive strategies is straightforward to understand: if no agent $h \neq i$ submits any order, then not-submitting an order is an optimal response for investor i, and vice versa. This trivial equilibrium with each agent doing nothing is a standard result in fully strategic trading games. As the passive strategy, however, is weakly dominated by any limit order (4.10), the trivial equilibrium is not a compelling prediction of our model (even though the choice of a weakly dominated strategy cannot be ruled out with arguments of rationality alone—in contrast to a strictly dominated strategy).

A formal way to rule out the equilibrium in passive strategies is to introduce exogenous noise traders who submit a random aggregate order $\tilde{z} \sim \mathcal{N}(0, \sigma_z^2)$, thereby provide a (non-zero) market order with probability one, and thus attract limit orders from all optimizing investors. Intuitively, when there are noise traders who have "trembling hands" in their order decisions, other investors become willing to place limit orders in the market to make profits by trading against the random market orders. However, since noise traders have no further role in our symmetric information model, we do not carry out this refinement formally.

4.3 Equilibrium with Identical Risk Aversion

In this section, we study a trading equilibrium, in which each investor submits a limit order and trade takes place. First, we establish the existence of a unique trading equilibrium. Then, we analyze the determinants of market liquidity and how illiquidity affects the market-clearing price and the allocation of the risky asset. We restrict the analysis in this section to the case that all investors have identical risk aversion and submit limit orders with identical slope, i.e., we assume $\rho^i = \rho$ for all i and $\beta^i = \beta$ for all i. This case is analytically very tractable and provides much of the intuition for our asset pricing model with imperfect competition.

The following proposition represents the trading equilibrium for the case of identical risk aversion for all investors.

Proposition 2 *Consider the case of linear orders, and assume $\rho^i = \rho$ for all i and $\beta^i = \beta$ for all i. If $I \geq 3$, then there exists a unique trading equilibrium given by*

$$\alpha^{i,*} = \frac{(I-2)}{(I-1)} \left(\frac{E[\tilde{v}]}{\rho \mathrm{Var}[\tilde{v}]} - \bar{x}^i \right), \quad \beta^{i,*} = \frac{(I-2)}{(I-1)} \frac{(1+r)}{\rho \mathrm{Var}[\tilde{v}]} \quad \text{for all } i. \tag{4.11}$$

If $I = 2$, then a trading equilibrium does not exist.

Proof: The trading equilibrium follows from Case 1 of Lemma 1, in which for each investor i, $\beta^h > 0$ for at least one agent $h \neq i$. The equilibrium is derived by equating the parameters from the conjectured order (4.2) with those from the optimal order in the best response (4.6) for each investor i, i.e., the equilibrium is the solution to the following equation system

$$\alpha^{i,*} = \frac{E[\tilde{v}] - \rho^i \mathrm{Var}[\tilde{v}]\,\bar{x}^i}{(1+r)\left(\sum_{h\neq i}\beta^{h,*}\right)^{-1} + \rho^i \mathrm{Var}[\tilde{v}]} \qquad i = 1,\dots,I, \tag{4.12}$$

$$\beta^{i,*} = \frac{(1+r)}{(1+r)\left(\sum_{h\neq i}\beta^{h,*}\right)^{-1} + \rho^i \mathrm{Var}[\tilde{v}]} \qquad i = 1,\dots,I. \tag{4.13}$$

Using the symmetry assumptions on ρ^i and β^i, (4.13) can be solved immediately. Plugging the solution for β into (4.12) yields the solution for α^i for each investor i. The second solution to (4.13), $\beta = 0$, does not yield an equilibrium from Case 1, as it violates the condition that for each investor i, $\beta^h > 0$ for at least one agent $h \neq i$. The other cases in Lemma 1 do not yield additional trading equilibria (only the passive equilibrium from Case 2). If $I = 2$, there is no relevant solution to (4.13) and, thus, a trading equilibrium does not exist—even without the symmetry restrictions on ρ^i and β^i. $\|$

In the trading equilibrium (4.11), each investor submits a limit order, a unique market-clearing price exists, and there is trade. The structure of the equilibrium orders under imperfect competition is similar to the orders under perfect competition. The only difference under imperfect competition is that each investor restricts the aggressiveness of his limit order by the coefficient $k := (I-2)/(I-1) < 1$, whereas the coefficient is equal to one when investors act under the assumption of perfect competition.

The assumption of identical β^i for all i (in addition to the assumption of identical ρ^i for all i) is used to prove that there does not exist another trading equilibrium. We conjecture that uniqueness of the trading equilibrium holds also without this assumption, but do not attempt to prove this here. A trading equilibrium does not exist, when there are only two investors. In this case, no investor is willing to submit any order, because the order would be exposed to the total market power of the other agent. The presence of a third agent, however, generates the necessary competition to make each investor willing to become active in the market and for the trading equilibrium to exist.

Now, we turn to the properties of the trading equilibrium. First, we relate an investor's price impact from imperfect competition to the notion of market liquidity as it is prevalent in market microstructure theory.

Proposition 3 *For each investor i, the degree of market liquidity is given by*

$$\lambda^{-1} = \frac{(1+r)(I-2)\rho^{-1}}{\mathrm{Var}\left[\tilde{v}\right]}. \tag{4.14}$$

Proof: The result follows immediately from the definition of λ^i and the solution to β^i. Because of the assumptions of identical ρ^i and β^i for all i, market liquidity is identical for all investors, too. ||

The rationale of this result is straightforward. If, for instance, investor i wishes to sell shares of the risky asset, he has to make other agents willing to buy these shares for the market to clear. Since the orders of the other agents are decreasing in price, the market is not perfectly liquid and investor i moves the price downward when selling his shares. The price impact (per share) is the smaller, and accordingly the degree of market liquidity is the higher, the larger the number of other agents and their risk tolerance and the smaller the risk of the asset.

The result on market liquidity corresponds qualitatively to the inventory-based microstructure approaches of Stoll (1978), Ho and Stoll (1981, 1983), and Grossman and Miller (1988). In these microstructure models, risk-averse market makers provide trading immediacy by accomodating orders from other market participants, and illiquidity (in terms of bid-ask spreads or price concessions) arises to compensate market makers for their willingness to change the risk profile of their inventory. In our model, market liquidity is not determined by the activity of a market maker, but is provided by the mutual submission of limit orders from investors themselves. Since these orders, however, reflect investors' portfolio decisions, our model captures the risk-aversion based aspect of market liquidity from the microstructure literature nevertheless.

Next, we turn to the market-clearing price and investors' portfolio holdings of the risky asset and study how these variables are affected by illiquidity. In particular, we are interested in whether the expected return of the risky asset reflects, beyond the standard risk premium, an additional liquidity premium that confirms the notion of compensation required by investors for their liquidity costs.

Proposition 4 *The market-clearing price and investors' portfolio holdings of the risky asset are given by*

$$p = \frac{1}{(1+r)} \left(E\left[\tilde{v}\right] - \frac{\bar{X}\mathrm{Var}\left[\tilde{v}\right]}{I\rho^{-1}} \right) \tag{4.15}$$

and

$$x^i = \bar{x}^i + \frac{(I-2)}{(I-1)} \left(\frac{\bar{X}}{I} - \bar{x}^i \right) \quad \textit{for all } i.$$

Proof: The market-clearing price follows from the market-clearing condition (4.1), the equilibrium orders (4.11), and from $\sum_{i=1}^{I} \bar{x}^i \equiv \bar{X}$. The number of shares bought or sold by investor i are determined by plugging the market-clearing price into the equilibrium order of the investor. Portfolio holdings are then given by $x^i = \bar{x}^i + \theta^i$. $\|$

For the case of identical risk aversion for all investors, the market-clearing price under imperfect competition is equal to the corresponding price under perfect competition: it is given by the expected payoff per share minus the standard risk premium, both discounted at the riskfree rate of interest. In particular, the market-clearing price under imperfect competition does not reflect an additional liquidity discount, and equivalently the expected return does not yield an additional liquidity premium.

The allocation of the risky asset, however, is in general different from the Pareto-optimal, risk-tolerance-weighted allocation under perfect competition, which simplifies to $x^c = \bar{X}/I$ for all i in the case of identical risk aversion. Under imperfect competition, investors trade towards the Pareto-optimal allocation, but do not achieve this allocation completely. If, for instance, investor i is endowed with only few shares of the risky asset, $\bar{x}^i < x^c$, he buys additional shares at the market-clearing price and obtains the portfolio holdings x^i with $\bar{x}^i < x^i < x^c$. Trading volume for an individual investor i is given by

$$\theta^i = \frac{(I-2)}{(I-1)} \left(x^c - \bar{x}^i \right) = \frac{(I-2)}{(I-1)} \theta^{i,c},$$

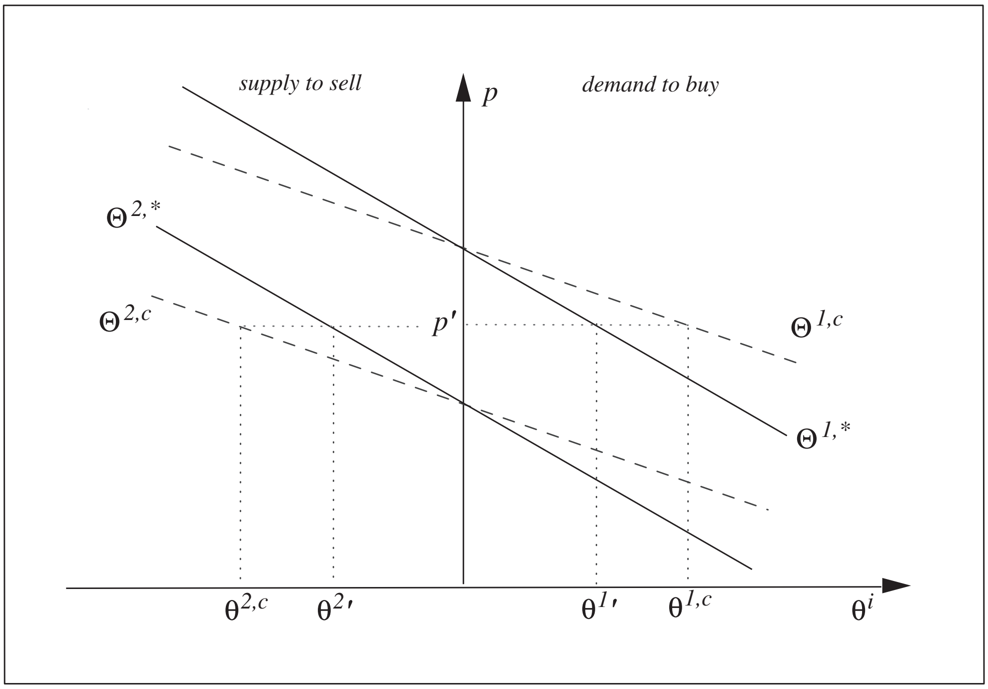

Figure 4.3: Pricing effect of illiquidity with endogenous trading.

and aggregate trading volume in the economy is given by

$$V = \frac{1}{2} \sum_{i=1}^{I} \left| \theta^i \right| = \frac{(I-2)}{(I-1)} V^c.$$

Consequently, liquidity costs from imperfect competition in general reduce trading volume and impede optimal risk sharing among investors. Only for the special case that each investor's initial endowment already corresponds to his holdings under optimal risk sharing (i.e. $\bar{x}^i = x^c$ for all i), illiquidity does not impair trading volume and optimal risk sharing. In this case, however, there is obviously no potential for mutually beneficial trade in the first place (although investors submit their limit orders nonetheless).

Figure 4.3 illustrates the results on the market-clearing price and on trading volume for an example with $I = 3$, $\bar{x}^1 < x^c < \bar{x}^2$, and $\bar{x}^3 = x^c$. In this example, trade takes place only between $i = 1$ and $i = 2$, but the presence of $i = 3$ is needed for the trading equilibrium with imperfect competition to exist. Investors' orders under imperfect competition can, from (4.11), be written

$$p = \frac{E\left[\tilde{v}\right] - \rho \mathrm{Var}\left[\tilde{v}\right] \bar{x}^i}{(1+r)} - \ k^{-1} \, \frac{\rho \mathrm{Var}\left[\tilde{v}\right]}{(1+r)} \, \theta^i, \tag{4.16}$$

with $k^{-1} = (I-1)/(I-2)$, and are represented by the solid lines in the figure.[9] It can be seen that the market clears at price p' and trades are given by $\theta^{1\prime} = -\theta^{2\prime}$. In comparison, investors' orders under perfect competition are similar to (4.16), except that the assumption of price-taking behavior implies that $k^c = 1$. Thus, each investor's order intersects the price axis at the same point as his order under imperfect competition, but is less steeply sloped, as shown by the dashed lines in the figure. Consequently, if investors act as if there are no liquidity costs from imperfect competition, the market clears at the same price p', but investors $i = 1, 2$ trade a higher volume $\theta^{1,c} = -\theta^{2,c}$.

The result on the market-clearing price is the main point of this chapter—it stands in contrast to the literature on exogenous trading costs in the line of Amihud and Mendelson (1986) and others, which finds that illiquidity reduces the price of an asset and, equivalently, increases the expected return of an asset, as investors require compensation for bearing liquidity costs. In our model, illiquidity is a cost, but it is not a priced factor. The reason is that buyers respond to liquidity costs by reducing their demand to buy and sellers by reducing their supply to sell, and these individual effects cancel each other with respect to the market-clearing price. Intuitively, as can be seen from Figure 4.3, buyers ($i = 1$) demand a price discount, and sellers ($i = 2$) demand a price premium, and thus both sides of the market would like to be compensated for their liquidity costs, but nobody is willing to grant that compensation. In the next section, we analyze this cancelling out in more detail for the case that investors have heterogeneous risk aversion.

Investors' trading decisions under imperfect competition reveal further microstructure features of market liquidity. If investor i, for instance, increases his order aggressiveness (in the sense of trading more price-elastically by choosing a higher k^i), he adversly affects his trading price, but he also generates a positive externality by improving liquidity for all other agents $h \neq i$.[10] Other agents are then willing to

[9] The order for investor $i = 3$, which is parallel to $\Theta^{1,*}$ and $\Theta^{2,*}$ and intersects the price axis at the market-clearing price, is omitted in the figure.

[10] The empirical microstructure literature on limit order books distinguishes between two concepts of order aggressiveness (say for buy orders): (i) trade aggressiveness, where the most aggressive order type is a large buy order that results in immediate execution and walks up the book above the best ask price (e.g. Biais, Hillion, and Spatt (1995)); and (ii) limit order aggressiveness, where the most aggressive type is a limit buy order that specifies a limit price above the best existing bid price

trade more aggressively themselves and thereby improve market liquidity for investor i, too. Thus, liquidity provision of each individual investor to the rest of the market is associated with a positive feedback effect of improved liquidity for that investor himself. This mutually re-inforcing liquidity provision among investors illustrates the strategic complementarity property of market liquidity and is captured in the common phrase that "liquidity begets liquidity."

In equilibrium, the price impact effect and the positive feedback effect are balanced, and no investor has an incentive to trade more aggressively, as his adverse price effect would not be offset by additional liquidity provision to him. Put differently, the submission of perfect competition orders by all investors to mutually enhance market liquidity (and trade the Pareto-optimal quantity at the same price as under imperfect competition) is not an equilibrium, since each investor would have an incentive to deviate by restricting his order aggressiveness to obtain a more favorable price.[11] Consequently, if market liquidity is endogenous, then the positive externality of liquidity provision leads to an insufficient level of liquidity in the market and thereby obstructs the processing of the Pareto-optimal trading volume.

We conclude this section with a competitive limit analysis of the imperfectly competitive trading equilibrium. It is a well acknowledged result in the economics literature that the assumption of perfect competition is problematic for markets with a finite number of participants, because price-taking behavior requires each agent to ignore the strategic influence he de facto has (Roberts and Postlewaite (1976)). Yet, the price-taking assumption is commonly motivated as a plausible approximation for trading behavior and market outcomes when there is a large number of agents in the market and each agent is small relative to the rest of the market.[12] The following proposition examines this large market justification of price-taking behavior in the context of asset pricing theory.

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and is stored in the book (Harris and Hasbrouck (1996)). The notion of aggressiveness in our call auction market corresponds to the second concept.

[11] This can be seen analytically from an out-of-equilibrium study in the example underlying Figure 4.3. Assume, investors $i = 1, 3$ submit perfect competition orders, i.e. orders (4.16) with $k^1 = k^3 = 1$. Then the optimal response for investor $i = 2$ is (from (4.6)) to submit an order (4.16) with $k^2 = 2/3 < 1$ so as to obtain a higher price $p'' > p'$ for his sale. (Perfect competition orders are not optimal for $i = 1, 3$ then either.)

[12] For further references and a short summary of the arguments on price-taking behavior, see the introduction in Hellwig (2005).

Proposition 5 *If the number of investors goes to infinity, then the market becomes perfectly liquid, equilibrium orders converge to perfect competition orders, and the expected return of the risky asset decreases to the riskfree rate of interest.*

Proof: The results follow immediately from taking the limits in (4.11), (4.14), and (4.15) for $I \to \infty$. $\|$

The results on market liquidity and equilibrium orders are in line with the common justification of price-taking behavior as an approximation for large markets: if the number of investors is large, then each investor faces a flat supply curve and thus can trade different quantities of shares at a given price. At the same time, however, the expected return of the risky asset reduces to the riskfree rate and, equivalently, the risk premium goes to zero. Although this concurrent vanishing of price impact and risk premium in our model is driven by the "risk elimination" property of CARA investors (Lintner (1970)), the intuition applies more generally: for each investor to be small relative to the rest of the market and to have zero price impact, the market must be so large that the price of risk is zero as well.

Consequently, our analysis of strategic trading interactions casts an ambiguous light on the assumption of price-taking behavior in asset pricing theory. On the one hand, the competitive limit results show that under conditions that justify price-taking behavior the analysis of risk-return relations becomes pointless, since the market, in aggregate, must be risk neutral. On the other hand, the result on the market-clearing price suggests that prices may be similar to those predicted under the assumption of price-taking behavior, even if investors have (and recognize) price impact, i.e., even if the market is small.

4.4 Heterogeneity in Risk Aversion

In this section, we study the trading equilibrium for the case that investors have heterogeneous risk aversion. The structure of the analysis is similar to the case of identical risk aversion, but the model becomes much less tractable. We thus restrict the model to the simplest possible case of heterogeneity with only three investors and two different degrees of risk aversion, i.e., we assume $I = 3$ and $\rho^1 \neq \rho^2 =$

ρ^3. Variables that are identical for investors $i = 2, 3$ are sometimes indicated by a superscript n instead of the general index i (e.g. we often use $\rho^n := \rho^2 = \rho^3$).

The following proposition establishes the trading equilibrium for the case of heterogeneity in investors' risk aversion.

Proposition 6 *Consider the case of linear orders, and assume $I = 3$, $\rho^n := \rho^2 = \rho^3$, and $\beta^n := \beta^2 = \beta^3$. Then there exists a unique trading equilibrium given by*

$$\alpha^{1,*} = k^1 \left(\frac{E[\tilde{v}]}{\rho^1 \mathrm{Var}[\tilde{v}]} - \bar{x}^1 \right), \quad \beta^{1,*} = k^1 \frac{(1+r)}{\rho^1 \mathrm{Var}[\tilde{v}]}, \tag{4.17}$$

$$\alpha^{i,*} = k^n \left(\frac{E[\tilde{v}]}{\rho^n \mathrm{Var}[\tilde{v}]} - \bar{x}^i \right), \quad \beta^{i,*} = k^n \frac{(1+r)}{\rho^n \mathrm{Var}[\tilde{v}]}, \quad i = 2, 3, \tag{4.18}$$

with

$$k^1 \equiv \frac{\sqrt{\rho^n (16\rho^1 + 9\rho^n)} - 3\rho^n}{\sqrt{\rho^n (16\rho^1 + 9\rho^n)} - \rho^n} < 1, \tag{4.19}$$

$$k^n \equiv \frac{4\rho^n}{\sqrt{\rho^n (16\rho^1 + 9\rho^n)} + 3\rho^n} < 1. \tag{4.20}$$

Heterogeneity in risk aversion affects the coefficients of order aggressiveness, k^i, according to

$$\rho^1 \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \rho^n \Rightarrow k^1 \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} k^n. \tag{4.21}$$

Proof: See Appendix to this chapter. $\|$

In the trading equilibrium with heterogeneous risk aversion, each investor's order is again given by his perfect competition order multiplied by a coefficient $k^i < 1$, which restricts the aggressiveness of the limit order. However, the coefficients of order aggressiveness are now different for investors with different risk aversion: an investor with higher risk aversion has a higher coefficient k^i, i.e., a more risk-averse investor restricts his order less strongly than a less risk-averse agent (compared to the corresponding order under perfect competition).

Now, we turn to the properties of the trading equilibrium with heterogeneous risk aversion. First, we analyze market liquidity.

Proposition 7 *Market liquidity for the different investors is given by*

$$\left(\lambda^1\right)^{-1} = \frac{8(1+r)}{\left[\sqrt{\rho^n\left(16\rho^1+9\rho^n\right)}+3\rho^n\right]\operatorname{Var}[\tilde{v}]} \tag{4.22}$$

and

$$\left(\lambda^i\right)^{-1} = \frac{4(1+r)}{\left[\sqrt{\rho^n\left(16\rho^1+9\rho^n\right)}-\rho^n\right]\operatorname{Var}[\tilde{v}]}, \quad i=2,3. \tag{4.23}$$

Proof: See Appendix to this chapter. ‖

Market liquidity for each investor is again determined by the risk-bearing willingness of the other agents in the market and, hence, is decreasing in the risk aversion of the other agents and in the risk of the asset. Heterogeneity in risk aversion reveals additional aspects of the strategic interdependency in the model, however. First, market liquidity for an investor can be related to his own risk aversion. From (4.22) it can be seen for investor $i = 1$ that

$$\frac{\partial(\lambda^1)^{-1}}{\partial\rho^1} < 0,$$

i.e., market liquidity for the investor is decreasing in his own risk aversion. This comparative static result reflects the feedback effect of liquidity provision: when the risk aversion of an investor increases, he reduces his liquidity provision to other agents by trading less aggressively and, because of strategic complementarity in order aggressiveness, experiences a decline in liquidity provision by other agents to himself in return.[13]

Second, heterogeneity in risk aversion leads to cross-sectional differences in market liquidity. A comparison of (4.22) and (4.23) shows that

$$\rho^1\left\{\begin{array}{c}>\\=\\<\end{array}\right\}\rho^n \;\Rightarrow\; (\lambda^1)^{-1}\left\{\begin{array}{c}>\\=\\<\end{array}\right\}(\lambda^n)^{-1},$$

i.e., the market is more liquid for investors with higher risk aversion. Put differently, an investor is "large" in terms of his price impact (per share), if he has a large

[13]For investors $i = 2,3$ the effect cannot be shown analytically, since a change in ρ^n cannot be related uniquely to either of the two.

risk tolerance compared to other agents in the market. The cross-sectional result is in line with the intuitive notion that large investors are characterized by high risk-bearing capacity and that large investors are major providers of market liquidity by submitting aggressive limit orders, but face illiquid markets for their own trading needs.

Next, we consider how differences in market liquidity among investors affects the market-clearing price of the risky asset.

Proposition 8 *The market-clearing price of the risky asset is given by*

$$p = \frac{1}{(1+r)}\left(E\left[\tilde{v}\right] - \frac{\bar{X}\mathrm{Var}\left[\tilde{v}\right]}{1/\rho^1 + 2/\rho^n}\right) - Z,$$

where

$$Z \equiv \sum_{i=1}^{3} \frac{1/\rho^i}{1/\rho^1 + 2/\rho^n}\,\lambda^i\left(x^i - \bar{x}^i\right). \tag{4.24}$$

Proof: See Appendix to this chapter. $\|$

For the case of heterogeneous risk aversion, the market-clearing price under imperfect competition is equal to the corresponding price under perfect competition minus an additional term Z. Thus, illiquidity from imperfect competition has a pricing effect in general. The additional pricing term Z has a nice interpretation relating to the idea that investors require compensation for bearing liquidity costs. Equation (4.24) reveals that each investor i demands a risk-tolerance-weighted price concession for his liquidity costs, which is given by his price impact per share, λ^i, times his trading volume, $x^i - \bar{x}^i$. Consequently, the pricing effect of illiquidity Z can indeed be traced back to the notion of compensation for liquidity costs on the individual investor level.

On the aggregate level, however, the individual effects again counterbalance each other, as the required price concessions are a price discount for buyers and a price premium for sellers.[14] In particular, it can be seen from (4.24) that the cancelling out of individual effects is exact and hence the perfect competition price arises ($Z = 0$),

[14]Note that, say, the price discount required by a buyer is his required price concession relative to the perfect competition price. For the overall price, in contrast, a buyer bids the price up, not down.

if there are no trades between investors with different risk aversion.[15] This condition includes, but is not limited to, two special cases: (*i*) all investors have identical risk aversion—then the perfect competition price arises independently of investors' initial endowments and the corresponding trades (as already shown in the previous section); and (*ii*) each investor's initial endowment corresponds to his holdings under optimal risk sharing—then the perfect competition price arises independently of investors' risk aversion as there is no basis for trade in the first place.

If, in contrast, the cancelling out between the individually required price concessions is not exact, then using the symmetry assumptions for investors $i = 2, 3$ to rewrite (4.24) as

$$Z \;=\; \frac{1}{1/\rho^1 + 2/\rho^n}\left[\left(\frac{\lambda^1}{\rho^1} - \frac{\lambda^n}{\rho^n}\right)\left(x^1 - \bar{x}^1\right)\right]$$

shows that the price is distorted towards the side of the market on which the larger investor (characterized by high risk tolerance and price impact) is trading. That is, the pricing effect of illiquidity is a discount compared to the price under perfect competition ($Z > 0$), if the larger investor is a buyer, and the pricing effect is a premium ($Z < 0$), if the larger investor is a seller.[16] Intuitively, the larger investor requires a higher price concession for his liquidity costs and, thus, dominates the aggregate effect on the price.

In summary, the pricing effect of illiquidity Z is different from the standard liquidity premium derived in asset pricing models on exogenous trading costs, which find that illiquidity is priced as an additional premium on the expected return of an asset and that the liquidity premium, in turn, is increasing in investors' liquidity costs. In our model, in contrast, the effects of investors' liquidity costs on the price and on the expected return "net out" to a greater or lesser extent, and the aggregate pricing effect of illiquidity is merely a residual term or price distortion, which can be positive or negative and which is not directly related to the degree of illiquidity in the market.

[15] In our economy with only two different degrees of risk aversion, this condition is also necessary. We conjecture that the necessity part of the condition does not hold with more heterogeneity in risk aversion.

[16] Since trades between agents with identical risk aversion offset exactly with respect to the price, for investors $i = 2, 3$ the aggregate trade $(x^2 - \bar{x}^2) + (x^3 - \bar{x}^3) \equiv -(x^1 - \bar{x}^1)$ is relevant for determining their side of the market.

Finally, we study the effects of heterogeneity in risk aversion on investors' portfolio holdings of the risky asset.

Proposition 9 *Investors' portfolio holdings of the risky asset are given by*

$$x^1 = \bar{x}^1 + \phi\left(x^{1,c} - \bar{x}^1\right), \tag{4.25}$$

$$x^i = \bar{x}^i + \phi\left(x^{n,c} - \bar{x}^i\right) + \xi\left(\bar{x}^i - \bar{x}^h\right) \quad i,h = 2,3,\ i \neq h, \tag{4.26}$$

where

$$\phi \equiv \frac{k^1 k^n \left(2\rho^1 + \rho^n\right)}{2k^n\rho^1 + k^1\rho^n} < 1 \quad \textit{and} \quad \xi \equiv \frac{\left(k^1 - k^n\right) k^n \rho^1}{2k^n\rho^1 + k^1\rho^n}. \tag{4.27}$$

Proof: See Appendix to this chapter. ||

The allocation of the risky asset is again governed by the improvement of risk sharing under imperfect market liquidity. If the initial endowment of investor i deviates from his holdings under optimal risk sharing, then the trade of the investor has a standard risk sharing effect $\phi\left(x^{i,c} - \bar{x}^i\right)$, adjusting the portfolio holdings towards optimal risk sharing but only incompletely because of illiquidity ($\phi < 1$). The trades of investors $i = 2,3$, however, exhibit an additional "heterogeneity effect" $\xi\left(\bar{x}^i - \bar{x}^h\right)$. The heterogeneity effect either re-inforces or countervails the risk sharing effect, depending on the relation between the risk aversions ρ^1 and ρ^n (which from (4.21) and (4.27) determines the sign of ξ) and depending on the relation between the endowments $\bar{x}^2$ and $\bar{x}^3$.

To illustrate the interplay between the risk sharing effect and the heterogeneity effect, we consider an example with $\bar{x}^1 < x^{1,c}$ and $\bar{x}^2 = x^{n,c} < \bar{x}^3$. In this example, the initial endowment of investor 2 already corresponds to his holdings under optimal risk sharing, and the trade to establish the Pareto-optimal allocation of the risky asset reduces to $x^{1,c} - \bar{x}^1 = -\left(x^{n,c} - \bar{x}^3\right) > 0$ between investor 1 and investor 3. Under perfect competition, this Pareto-optimal trade between $i = 1$ and $i = 3$ takes place, and trading volume in the economy is thus given by $V^c = \left(x^{1,c} - \bar{x}^1\right)$. Under imperfect competition, in contrast, investor 2 also trades a non-zero number of shares due to the heterogeneity effect and thereby trades away from his holdings under optimal risk

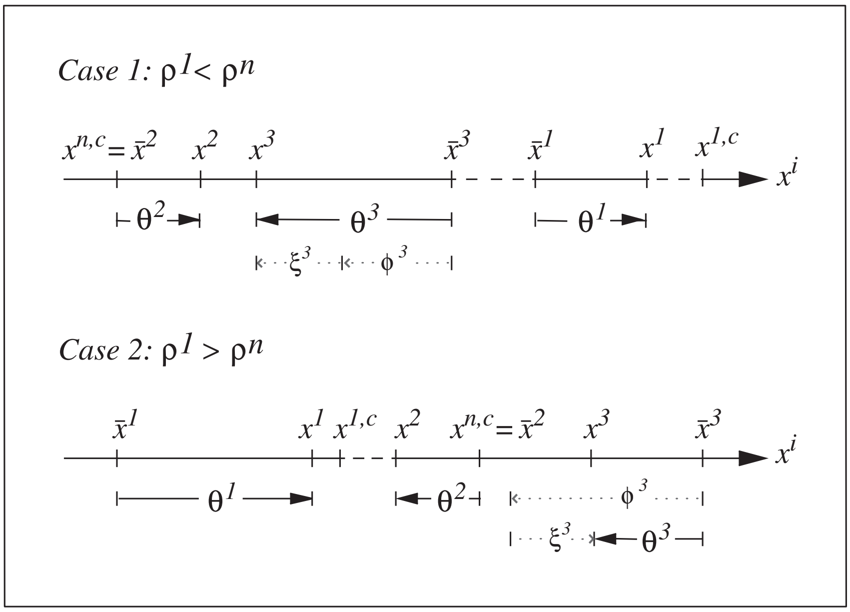

Figure 4.4: Trading volume with heterogeneity effect.

sharing. Using $\bar{x}^2 = x^{n,c}$, investors' portfolio holdings in the example follow as

$$\begin{aligned} x^1 &= \bar{x}^1 + \phi\left(x^{1,c} - \bar{x}^1\right) < x^{1,c}, \\ x^2 &= x^{n,c} + \xi\left(x^{n,c} - \bar{x}^3\right), \\ x^3 &= \bar{x}^3 + (\phi - \xi)\left(x^{n,c} - \bar{x}^3\right) > x^{n,c}, \end{aligned}$$

with

$$\phi - \xi = \frac{\left(k^1 + k^n\right) k^n \rho^1 + k^n k^1 \rho^n}{2k^n \rho^1 + k^1 \rho^n} < 1.$$

Whether investor 2 acts as buyer ($x^2 > x^{n,c}$) or as seller ($x^2 < x^{n,c}$) depends on the relation between ρ^1 and ρ^n and, hence, on who is the larger investor in the economy. In the following, we examine the two possible cases of heterogeneity for the example in detail; a graphical illustration is given in Figure 4.4.

Case 1: First, consider the case that $\rho^1 < \rho^n$ ($\Rightarrow \xi < 0$), so that investor 1 is the larger investor in the economy. As $i = 1$ acts as buyer in the example, the pricing effect of illiquidity is a discount relative to the price under perfect competition ($Z > 0$). Because of the price discount, in turn, investor 2 is willing to buy shares of the risky asset via the heterogeneity effect ($\xi\left(\bar{x}^2 - \bar{x}^3\right) > 0$) and obtains the portfolio holdings $x^2 > x^{n,c}$. Conversely, investor 3 sells shares by the heterogeneity effect

($\xi\left(\bar{x}^3 - \bar{x}^2\right) < 0$) and thereby re-inforces his selling from the risk sharing effect. Yet, despite the re-inforcing trading from the heterogeneity effect, the portfolio of $i = 3$ is still distorted towards his initial endowment ($\phi - \xi < 1$). Similarly, aggregate trading volume in the economy,

$$V = (\phi - \xi)\left(x^{1,c} - \bar{x}^1\right) = (\phi - \xi)\, V^c,$$

is increased by the heterogeneity effect beyond the trading from the risk sharing effect ($\xi < 0$), but is still lower than under perfect competition.

Case 2: Now, consider the case that $\rho^1 > \rho^n$ ($\Rightarrow \xi > 0$), so that investor 3 is the larger investor in the economy. As $i = 3$ acts as seller, illiquidity generates a premium on the price ($Z < 0$). At this higher price, investor 2 is willing to sell shares via the heterogeneity effect ($\xi\left(\bar{x}^2 - \bar{x}^3\right) < 0$) and obtains the portfolio holdings $x^2 < x^{n,c}$. Conversely, investor 3 buys shares from the heterogeneity effect ($\xi\left(\bar{x}^3 - \bar{x}^2\right) > 0$), which coutervails his selling from the risk sharing effect. Hence, the trade of $i = 3$ towards his portfolio under optimal risk sharing is partly crowded out by the heterogeneity effect. Likewise, aggregate trading volume in the economy,

$$V = \phi\left(x^{1,c} - \bar{x}^1\right) = \phi V^c,$$

is not increased beyond the trading from the risk sharing effect.

Figure 4.4 depicts the individual trading volumes for the two cases of the example. For each case, trading volume of investor 3 is composed of a risk sharing part, indicated by $\phi^3 := \phi\left(x^{n,c} - \bar{x}^3\right)$, and a heterogeneity part, $\xi^3 := \xi\left(\bar{x}^3 - \bar{x}^2\right)$. Note that in both cases of the example, investor 2 trades via the heterogeneity effect on the same side of the market as the larger investor in the economy. That is, investor 2 does not step in as a counterparty in order to alleviate the price impact of the large investor's risk-sharing trade, but instead trades in the same direction and thereby even re-inforces the large investor's price impact. Consequently, if an investor trades exclusively because of the heterogeneity effect, his activity is reminiscent of market manipulation against the trading need of a large investor in an illiquid market.[17]

[17]However, models on market manipulation against large investors are usually based on asymmetric information or exogenous endowment shocks (see, e.g., Vayanos (2001), Brunnermeier and Pedersen (2005), and Pritsker (2005)), which is not the case in our model.

4.5 Discussion

In this chapter, we study a static asset pricing model with imperfect competition and endogenous market liquidity. We find that liquidity costs reduce trading volume and impede Pareto-optimal risk sharing among investors. The expected return of the risky asset, however, is not systematically affected by liquidity costs and, thus, reflects only a risk premium but no liquidity premium—contrary to the prediction of liquidity premia in the literature on exogenous trading costs cited in the introduction to this chapter. This result deserves further discussion.

A possible objection to the absence of a liquidity premium in our model is that market liquidity, although it exhibits several typical microstructure properties, is captured incompletely. First, illiquidity originates from investors' risk-bearing costs only. Other microstructure frictions, e.g. asymmetric information or search costs, are not considered. Next, our model of one-shot trading precludes any effects of future liquidity costs on current asset prices, for instance as in a dynamic model in which investors trade repeatedly over time to further improve risk sharing or because of exogenous wealth shocks. Finally, liquidity costs are not stochastic. Even though under simultaneous order submission, an investor cannot observe his supply curve before making his own trading decision, the common knowledge assumption of the model allows each investor to infer the degree of market liquidity in equilibrium with certainty. The argument that our setting captures only a part of the multi-faceted concept of market liquidity is thus well justified. However, this argument is not crucial for the absence of a liquidity premium.

The crucial point for the absence of a liquidity premium is the endogeneity of trading on both sides of the market. It is for this reason that the price discount required as compensation for liquidity costs by buyers is offset by a price premium required by sellers. In particular, if sellers were instead exogenously forced to liquidate their portfolios, then their offsetting effect on the price would be eliminated and, consequently, a liquidity premium on the expected return would arise in our model.

A closely related result is found by Vayanos (1998) in a competitive overlapping generations model with exogenous bid-ask spreads and endogenous trading horizons. In his model, an increase in liquidity costs has two opposing effects, either of which can dominate asset prices: on the one hand, new investors are less willing to buy the illiquid asset, but on the other hand, older investors sell back their asset holdings more slowly. Vayanos (1998) shows that the second effect always offsets the standard liquidity premium to some extent. Moreover, when the second effect dominates, then asset prices are actually increasing in liquidity costs.

In conclusion, the absence of a liquidity premium in the model presented in this chapter is not primarily driven by an incomplete view of market liquidity, but rather by the endogeneity of trading on both sides of the market. However, it is nevertheless promising to consider the asset pricing implications of illiquidity in a richer setting allowing for both dynamic and stochastic trading effects. These issues are addressed in the next chapter.

Appendix: Proofs to Chapter 4

Proof of Lemma 1 (Continuation)

Here, we consider in more detail the last two cases of Lemma 1, where investor i trades against a vertical supply curve, which offers the same quantity of shares at each possible price.

In Case 2, the supply curve for investor i is given by $\beta^h = 0$ for all $h \neq i$ and $\sum_{h \neq i} \alpha^h = 0$. If i submits a limit order $\langle \alpha^i, \beta^i > 0 \rangle$, then there exists a unique market-clearing price $p' = \alpha^i / \beta^i$. At this price, $\Theta^i(p') = 0$ applies, i.e., trading volume for i is zero. (There is trade between other agents, however, if $\alpha^h \neq 0$ for at least two $h \neq i$.) If i submits a market order $\langle \alpha^i \neq 0, \beta^i = 0 \rangle$, then there does not exist a market-clearing price, and if i employs the passive strategy $\langle \alpha^i = 0, \beta^i = 0 \rangle$, then every price clears the market. Since the market rules exclude trade when there does not exist a unique market-clearing price, trading volume for i again is zero. Consequently, in Case 2, investor i does not trade, but remains with his initial endowments for each possible order choice. Thus, investor i is indifferent between all of his feasible trading strategies.

In Case 3, the supply curve for investor i is given by $\beta^h = 0$ for all $h \neq i$ and $\sum_{h \neq i} \alpha^h > 0$ (< 0). If i submits a market order $\langle \alpha^i \neq 0, \beta^i = 0 \rangle$ or employs the passive strategy $\langle \alpha^i = 0, \beta^i = 0 \rangle$, then either no price clears the market (if $\alpha^i \neq$

$-\sum_{h\neq i}\alpha^h$), or every price clears the market (if $\alpha^i = -\sum_{h\neq i}\alpha^h$); in both cases, no trade takes place, and i remains with his initial endowments. If i submits a limit order $\langle\alpha^i, \beta^i > 0\rangle$, then there exists a unique market-clearing price, and i sells (buys) $\left|\sum_{h\neq i}\alpha^h\right|$ shares. For trading against a vertical supply curve, investor i can force the price for a sale (purchase) to $+\infty$ $(-\infty)$ by setting $\alpha^i/\beta^i = \infty$ $(-\infty)$, which gives higher expected utility than holding his initial endowment. Since the corresponding trade would not be optimal for at least one other agent $h \neq i$, however, Case 3 does obviously not yield an equilibrium. Therefore, we skip the exact specification of the best response for this case, which is a purely technical problem of imposing restrictions on the parameters α^i and β^i. ‖

Proof of Proposition 1

The equilibrium in passive strategies follows from Case 2 of Lemma 1. In this case we have, for each investor i, $\beta^h = 0$ for all $h \neq i$ and $\sum_{h\neq i}\alpha^h = 0$. Then, each investor i is indifferent between all feasible trading strategies, including the passive strategy. This gives the equilibrium.

To prove the weak domination part of the proposition, we have to show that for investor i, any limit order

$$\alpha^i = k^i\left(\frac{E\left[\tilde{v}\right]}{\rho^i \text{Var}\left[\tilde{v}\right]} - \bar{x}_1^i\right), \quad \beta^i = k^i\frac{(1+r)}{\rho^i \text{Var}\left[\tilde{v}\right]} \quad \text{with} \quad 1 \geq k^i > 0 \tag{4.28}$$

does at least as well as passive investment for all feasible trading strategies of the other agents $h \neq i$ and does strictly better than passive investment for some strategies of other agents. First, substitute the budget constraint into the payoff function to write the objective function of investor i as

$$(1+r)\left[\bar{y}^i - p(x^i - \bar{x}^i)\right] + E\left[\tilde{v}\right]x^i - \frac{\rho^i}{2}\left(x^i\right)^2 \text{Var}\left[\tilde{v}\right]. \tag{4.29}$$

If i employs the passive investment strategy, then he does not trade and his expected utility is determined by

$$(1+r)\bar{y}^i + E\left[\tilde{v}\right]\bar{x}^i - \frac{\rho^i}{2}\left(\bar{x}^i\right)^2 \text{Var}\left[\tilde{v}\right], \tag{4.30}$$

irrespective of the strategies chosen by all other agents $h \neq i$. If i submits a limit order (4.28), then a unique market-clearing price

$$p' = \frac{k^i E[\tilde{v}] + \rho^i \mathrm{Var}[\tilde{v}] \left(\sum_{h\neq i} \alpha^h - k^i \bar{x}^i\right)}{k^i(1+r) + \rho^i \mathrm{Var}[\tilde{v}] \sum_{h\neq i} \beta^h} \tag{4.31}$$

exists, and, by plugging (4.31) into the order specified by (4.28), trading volume of i is given by

$$\theta^{i'} = \frac{k^i \left[\left(E[\tilde{v}] - \rho^i \mathrm{Var}[\tilde{v}]\, \bar{x}^i\right) \sum_{h\neq i} \beta^h - (1+r) \sum_{h\neq i} \alpha^h\right]}{k^i(1+r) + \rho^i \mathrm{Var}[\tilde{v}] \sum_{h\neq i} \beta^h}. \tag{4.32}$$

Next, comparing (4.29) with (4.30) shows that a limit order weakly dominates the passive investment strategy for investor i, if the condition

$$\theta^i \left[E[\tilde{v}] - (1+r)p - \frac{\rho^i}{2}\left(\theta^i + 2\bar{x}^i\right) \mathrm{Var}[\tilde{v}]\right] \geq 0 \tag{4.33}$$

holds for all feasible trading strategies of all other agents $h \neq i$ and holds with strict inequality for some strategies of other agents. Finally, plugging (4.31) and (4.32) into (4.33) and simplifying yields

$$\frac{(2-k^i)k^i\rho^i \mathrm{Var}[\tilde{v}] \left[\left(E[\tilde{v}] - \rho^i \mathrm{Var}[\tilde{v}]\, \bar{x}^i\right) \sum_{h\neq i} \beta^h - (1+r) \sum_{h\neq i} \alpha^h\right]^2}{2\left[k^i(1+r) + \rho^i \mathrm{Var}[\tilde{v}] \sum_{h\neq i} \beta^h\right]^2} \geq 0,$$

which, under the sufficient restriction $1 \geq k^i > 0$, completes the proof. $\|$

Proof of Proposition 6

The trading equilibrium follows again from Case 1 of Lemma 1. Using the assumptions on the number of investors and on heterogeneity in risk aversion, the equilibrium is the solution to the equation system

$$\alpha^{1,*} = \frac{E[\tilde{v}] - \rho^1 \mathrm{Var}[\tilde{v}]\, \bar{x}^1}{(1+r)/\left(2\beta^{n,*}\right) + \rho^1 \mathrm{Var}[\tilde{v}]}, \tag{4.34}$$

$$\beta^{1,*} = \frac{(1+r)}{(1+r)/\left(2\beta^{n,*}\right) + \rho^1 \mathrm{Var}[\tilde{v}]}, \tag{4.35}$$

$$\alpha^{i,*} = \frac{E[\tilde{v}] - \rho^n \mathrm{Var}[\tilde{v}]\, \bar{x}^i}{(1+r)/\left(\beta^{1,*} + \beta^{n,*}\right) + \rho^n \mathrm{Var}[\tilde{v}]} \qquad i = 2, 3, \tag{4.36}$$

$$\beta^{n,*} = \frac{(1+r)}{(1+r)/\left(\beta^{1,*} + \beta^{n,*}\right) + \rho^n \mathrm{Var}[\tilde{v}]}. \tag{4.37}$$

First, rearranging (4.37) yields

$$\beta^{n,*}\left(\beta^{1,*}+\beta^{n,*}\right)\rho^n \mathrm{Var}\left[\tilde{v}\right]-\beta^{1,*}(1+r)=0. \tag{4.38}$$

Then, writing (4.35) as

$$\beta^{1,*}=\frac{2\beta^{n,*}(1+r)}{(1+r)+2\beta^{n,*}\rho^1 \mathrm{Var}\left[\tilde{v}\right]} \tag{4.39}$$

and using (4.39) to eliminate $\beta^{1,*}$ from (4.38) gives

$$\beta^{n,*}\left[\left(\beta^{n,*}\right)^2 2\rho^1\rho^n\left(\mathrm{Var}\left[\tilde{v}\right]\right)^2+\beta^{n,*}3(1+r)\rho^n \mathrm{Var}\left[\tilde{v}\right]-2\left(1+r\right)^2\right]=0. \tag{4.40}$$

The relevant solution to (4.40), which does not violate the trading rules and the conditions in Case 1 of Lemma 1, is given by

$$\begin{aligned}\beta^{n,*} &= -\frac{3(1+r)}{4\rho^1 \mathrm{Var}\left[\tilde{v}\right]}+\sqrt{\frac{9\left(1+r\right)^2}{\left(4\rho^1 \mathrm{Var}\left[\tilde{v}\right]\right)^2}+\frac{\left(1+r\right)^2}{\rho^1\rho^n\left(\mathrm{Var}\left[\tilde{v}\right]\right)^2}} \\ &= \frac{\left[\sqrt{\rho^n\left(16\rho^1+9\rho^n\right)}-3\rho^n\right](1+r)}{4\rho^1\rho^n \mathrm{Var}\left[\tilde{v}\right]}>0. \end{aligned} \tag{4.41}$$

By multiplying numerator and denominator by $\sqrt{\rho^n\left(16\rho^1+9\rho^n\right)}+3\rho^n$, (4.41) can be rewritten as

$$\beta^{n,*}=\frac{4\rho^n}{\left[\sqrt{\rho^n\left(16\rho^1+9\rho^n\right)}+3\rho^n\right]}\frac{(1+r)}{\rho^n \mathrm{Var}\left[\tilde{v}\right]}. \tag{4.42}$$

Next, substituting (4.41) into (4.39) yields

$$\begin{aligned}\beta^{1,*} &= \frac{\left[\sqrt{\rho^n\left(16\rho^1+9\rho^n\right)}-3\rho^n\right]2(1+r)^2}{(1+r)4\rho^1\rho^n \mathrm{Var}\left[\tilde{v}\right]+\left[\sqrt{\rho^n\left(16\rho^1+9\rho^n\right)}-3\rho^n\right](1+r)\,2\rho^1 \mathrm{Var}\left[\tilde{v}\right]} \\ &= \frac{\left[\sqrt{\rho^n\left(16\rho^1+9\rho^n\right)}-3\rho^n\right]}{\left[\sqrt{\rho^n\left(16\rho^1+9\rho^n\right)}-\rho^n\right]}\frac{(1+r)}{\rho^1 \mathrm{Var}\left[\tilde{v}\right]}. \end{aligned} \tag{4.43}$$

Furthermore, using (4.43) and (4.41) to obtain

$$\beta^{1,*}+\beta^{n,*}=\frac{4(1+r)}{\left[\sqrt{\rho^n\left(16\rho^1+9\rho^n\right)}-\rho^n\right]\mathrm{Var}\left[\tilde{v}\right]} \tag{4.44}$$

and plugging (4.44) into (4.36) yields

$$\alpha^{i,*}=\frac{4\rho^n}{\sqrt{\rho^n\left(16\rho^1+9\rho^n\right)}+3\rho^n}\left(\frac{E\left[\tilde{v}\right]}{\rho^n \mathrm{Var}\left[\tilde{v}\right]}-\bar{x}^i\right)\qquad i=2,3. \tag{4.45}$$

Finally, substituting (4.41) into (4.34) gives

$$\alpha^{1,*} = \frac{\sqrt{\rho^n \left(16\rho^1 + 9\rho^n\right)} - 3\rho^n}{\sqrt{\rho^n \left(16\rho^1 + 9\rho^n\right)} - \rho^n} \left(\frac{E\left[\tilde{v}\right]}{\rho^1 \mathrm{Var}\left[\tilde{v}\right]} - \bar{x}^1\right). \tag{4.46}$$

In summary, the trading equilibrium in Proposition 6 is given by (4.46), (4.43), (4.45), and (4.42).

To prove the effects of differences in risk aversion on the coefficients of order aggressiveness, k^i, substitute

$$k^1 := \frac{\sqrt{\rho^n \left(16\rho^1 + 9\rho^n\right)} - 3\rho^n}{\sqrt{\rho^n \left(16\rho^1 + 9\rho^n\right)} - \rho^n} \quad \text{and} \quad k^n := \frac{4\rho^n}{\sqrt{\rho^n \left(16\rho^1 + 9\rho^n\right)} + 3\rho^n}$$

into

$$k^1 \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} k^n$$

and simplify to obtain

$$\rho^1 \left(2\frac{\rho^1}{\rho^n} - 1\right) \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \rho^n. \tag{4.47}$$

Condition (4.47) holds if

$$\rho^1 \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \rho^n.$$

This is the desired result. $\|$

Proof of Proposition 7

Substitute (4.42) into $\left(\lambda^1\right)^{-1} = 2\beta^n$, and substitute (4.44) into $\left(\lambda^n\right)^{-1} = \beta^1 + \beta^n$. $\|$

Proof of Propositions 8 and 9

As a preliminary step for proving the market-clearing price and investors' portfolio holdings, apply the market-clearing condition to the equilibrium orders specified by (4.17) and (4.18) to obtain

$$p = \frac{1}{(1+r)} \left(E\left[\tilde{v}\right] - \frac{\left[k^1 \bar{x}^1 + k^n \left(\bar{x}^2 + \bar{x}^3\right)\right] \mathrm{Var}\left[\tilde{v}\right]}{k^1/\rho^1 + 2k^n/\rho^n}\right). \tag{4.48}$$

Equation (4.48) is used first for deriving investors' portfolio holdings in Proposition 9 and subsequently for obtaining the market-clearing price in Proposition 8 (which is an implicit representation based on the solution for the portfolio holdings).

The derivation of the portfolio holdings for each investor i in Proposition 9 is carried out by first plugging the market-clearing price (4.48) into $x^i = \bar{x}^i + \Theta^i(p)$, then substituting $\bar{X}$ into that equation, and rearranging. For investor 1, the first step gives

$$x^1 = \bar{x}^1 + \frac{k^1 \rho^n \left[k^1 \bar{x}^1 + k^n \left(\bar{x}^2 + \bar{x}^3\right)\right]}{2k^n \rho^1 + k^1 \rho^n} - k^1 \bar{x}^1. \tag{4.49}$$

By substituting $\bar{x}^2 + \bar{x}^3 = \bar{X} - \bar{x}^1$ into (4.49) and regrouping terms, portfolio holdings are

$$x^1 = \bar{x}^1 + \frac{k^1 k^n \left(2\rho^1 + \rho^n\right)}{\left(2k^n \rho^1 + k^1 \rho^m\right)} \frac{\rho^n \bar{X}}{\left(2\rho^1 + \rho^n\right)} - \frac{k^1 k^n \left(2\rho^1 + \rho^n\right)}{2k^n \rho^1 + k^1 \rho^n} \bar{x}^1,$$

where

$$\frac{\rho^n \bar{X}}{2\rho^1 + \rho^n} = x^{1,c} \tag{4.50}$$

are the Pareto-optimal holdings for investor 1. Similarly for investor 2, we first have

$$x^2 = \bar{x}^2 + \frac{k^n \rho^1 \left[k^1 \bar{x}^1 + k^n \left(\bar{x}^2 + \bar{x}^3\right)\right]}{2k^n \rho^1 + k^1 \rho^n} - k^n \bar{x}^2. \tag{4.51}$$

By plugging in $\bar{x}^1 = \bar{X} - \left(\bar{x}^2 + \bar{x}^3\right)$ and rearranging, (4.51) can be rewritten as

$$\begin{aligned} x^2 &= \bar{x}^2 + \frac{k^1 k^n \left(2\rho^1 + \rho^n\right)}{\left(2k^n \rho^1 + k^1 \rho^n\right)} \frac{\rho^1 \bar{X}}{\left(2\rho^1 + \rho^n\right)} \\ &\quad + \frac{k^n \left[-k^1 \rho^1 \left(2\bar{x}^2 - \left(\bar{x}^2 - \bar{x}^3\right)\right) + k^n \rho^1 \left(\bar{x}^2 + \bar{x}^3\right) - \left(2k^n \rho^1 + k^1 \rho^n\right) \bar{x}^2\right]}{\left(2k^n \rho^1 + k^1 \rho^n\right)} \\ &= \bar{x}^2 + \frac{k^1 k^n \left(2\rho^1 + \rho^n\right)}{\left(2k^n \rho^1 + k^1 \rho^n\right)} \frac{\rho^1 \bar{X}}{\left(2\rho^1 + \rho^n\right)} \\ &\quad - \frac{k^1 k^n \left(2\rho^1 + \rho^n\right)}{\left(2k^n \rho^1 + k^1 \rho^n\right)} \bar{x}^2 + \frac{k^n \rho^1 \left(k^1 - k^n\right)}{\left(2k^n \rho^1 + k^1 \rho^n\right)} \left(\bar{x}^2 - \bar{x}^3\right), \end{aligned}$$

where

$$\frac{\rho^1 \bar{X}}{2\rho^1 + \rho^n} = x^{2,c} \tag{4.52}$$

are the Pareto-optimal holdings for investor 2. Portfolio holdings for investor 3 are derived analogously to those for investor 2.

The representation of the market-clearing price in Proposition 8,

$$p = \frac{1}{(1+r)}\left(E[\tilde{v}] - \frac{\bar{X}\mathrm{Var}[\tilde{v}]}{1/\rho^1 + 2/\rho^n}\right) - Z$$

with

$$Z \equiv \sum_{i=1}^{3} \frac{1/\rho^i}{1/\rho^1 + 2/\rho^n}\, \lambda^i \left(x^i - \bar{x}^i\right), \tag{4.53}$$

is obtained by rewriting (4.48) as

$$p = \frac{1}{(1+r)}\left(E[\tilde{v}] - \frac{\bar{X}\mathrm{Var}[\tilde{v}]}{1/\rho^1 + 2/\rho^n}\right) - Z'$$

with

$$Z' := \frac{(k^1 - k^n)\,\left[2\rho^1\bar{x}^1 - \rho^n\left(\bar{x}^2 + \bar{x}^3\right)\right]\,\mathrm{Var}[\tilde{v}]}{(1+r)\left(2k^n\rho^1 + k^1\rho^n\right)\,\left(1/\rho^1 + 2/\rho^n\right)}, \tag{4.54}$$

and showing that $Z = Z'$. Substituting the portfolio holdings (4.25) and (4.26), the Pareto-optimal holdings (4.50) and (4.52), and $\bar{X} \equiv \bar{x}^1 + \bar{x}^2 + \bar{x}^3$ into (4.53) and simplifying gives

$$Z = \frac{\phi\left(\lambda^n/\rho^n - \lambda^1/\rho^1\right)\,\left[2\rho^1\bar{x}^1 - \rho^n\left(\bar{x}^2 + \bar{x}^3\right)\right]}{\left(1/\rho^1 + 2/\rho^n\right)\left(2\rho^1 + \rho^n\right)} \tag{4.55}$$

where

$$\phi = \frac{k^1k^n\left(2\rho^1 + \rho^n\right)}{2k^n\rho^1 + k^1\rho^n}. \tag{4.56}$$

Next, a comparison of (4.55) and (4.56) with (4.54) indicates that showing equality between Z and Z' reduces to showing equality between the terms

$$A := k^1k^n\left(\lambda^n/\rho^n - \lambda^1/\rho^1\right) \tag{4.57}$$

and

$$A' := \frac{(k^1 - k^n)\,\mathrm{Var}[\tilde{v}]}{(1+r)}. \tag{4.58}$$

After plugging (4.19), (4.20), (4.22), and (4.23) into (4.57) and plugging (4.19) and (4.20) into (4.58), tedious simplification yields

$$A = A' = \frac{\left[8\rho^1 + 2\rho^n - 2\sqrt{\rho^n\left(16\rho^1 + 9\rho^n\right)}\right]\mathrm{Var}[\tilde{v}]}{(1+r)\left[8\rho^1 + 3\rho^n + \sqrt{\rho^n\left(16\rho^1 + 9\rho^n\right)}\right]}.$$

This completes the proof. ‖

Chapter 5

Asset Prices with Time-Varying Market Liquidity

An important feature of financial markets is that assets are traded repeatedly over time under varying conditions of market liquidity. In their current trading decisions, investors therefore take into account the costs and risks of rebalancing their portfolios in the future. In asset pricing models on exogenous trading costs, current asset prices are reduced by such future illiquidity concerns, as investors require compensation for holding assets that are costly and risky to trade in the future. For example, in Amihud and Mendelson (1986) and Vayanos and Vila (1999), asset prices reflect a liquidity discount according to the present value of all future trading costs over the lifetime of the asset. Furthermore, in Huang (2003), Vayanos (2004), and Acharya and Pedersen (2005), asset prices reflect additional liquidity discounts, as illiquidity causes stochastic trading effects and investors are risk averse. As argued before, however, the models on exogenous trading costs are based on a cursory representation of trading behavior in illiquid markets, since they take market liquidity as exogenous and enforce repeated trade among investors by imposing overlapping generations structures or exogenous wealth shocks.

In this chapter, we extend our CARA-Gaussian asset pricing model with imperfect competition and endogenous market liquidity to a setting with dynamic and stochastic trading effects. Investors repeatedly trade a single risky asset by submitting downward-sloping demand functions for execution against each other. Market liquidity is time varying, as the possibilities and risks of re-trading the asset change over time. Because of illiquidity, investors split their orders over the trading horizon and, as a consequence, bear additional costs and risks for the repeated trading in the future.

Our main result is that the current price of the risky asset is not affected by illiquidity—neither by the future trading costs nor by the illiquidity-induced future trading risk. This result is in contrast to the liquidity discounts on asset prices found in the models on exogenous trading costs cited above. The reason for the absence of a pricing effect of illiquidity in our model is, again, the endogeneity of trading on both sides of the market, so that the liquidity costs for buyers and sellers offset each other with respect to the price of the risky asset. Hence, the cancelling out of liquidity costs from the static setting of the previous chapter carries over to a setting with dynamic and stochastic trading effects.

The remainder of this chapter is organized as follows. Section 5.1 presents the setup of the model. Section 5.2 derives the equilibrium. Section 5.3 studies the equilibrium properties under deterministic trading conditions, and Section 5.4 extends the results by allowing for stochastic trading effects. Section 5.5 provides a concluding discussion. All proofs are in the Appendix of this chapter.

5.1 Description of the Economy

We consider asset pricing in a dynamic framework with two rounds of trading at dates $t = 1, 2$. In the economy, there is a riskless asset and a single risky asset. Taking the riskless asset as numeraire with perfectly elastic supply, let p_t be the price of the risky asset at date t. After the second round of trading, investors hold their portfolios until assets pay off exogenous liquidation values, and then each investor consumes his terminal wealth.[1] Each unit of the riskless asset yields a liquidation value of 1 (i.e., the riskfree rate of interest is zero), and each unit of the risky asset pays a normally distributed liquidation value $\tilde{v}$. The total number of outstanding shares of the risky asset is denoted by $\bar{X}$.

We analyze two separate scenarios with respect to the information structure in

[1] As there are no asset payoffs and consumption decisions at the trading dates $t = 1, 2$, the time horizon covering the two rounds of trading may be interpreted as a short-term trading period before investors enter a long-term holding period for their portfolios.

the economy and the resulting risks faced by investors: a "basic scenario" and a "signal scenario." In both scenarios, information is symmetric throughout the model, and investors start with normally distributed prior beliefs about the payoff of the risky asset, $\tilde{v} \sim \mathcal{N}(E[\tilde{v}], \mathrm{Var}[\tilde{v}])$. In the basic scenario, investors do not receive additional information about $\tilde{v}$ during the two rounds of trading, so that their beliefs remain constant over time. The only risk faced by investors in this scenario relates to the terminal payoff $\tilde{v}$.

In the signal scenario, in contrast, investors receive a public signal $\tilde{s}$ about the payoff $\tilde{v}$ before the second round of trading. We assume that $\tilde{v} = \tilde{s} + \tilde{\varepsilon}$, where $\tilde{s}$ and $\tilde{\varepsilon}$ are independent. The signal $\tilde{s}$ is normally distributed with mean $\bar{v}$ and variance σ_s^2, and $\tilde{\varepsilon}$ is normal with zero mean and variance σ_ε^2. Hence, investors' beliefs about $\tilde{v}$ are normally distributed in the first round of trading with $E_1[\tilde{v}] = \bar{v}$ and $\mathrm{Var}_1[\tilde{v}] = \sigma_s^2 + \sigma_\varepsilon^2$ and remain normal in the second round of trading with $E_2[\tilde{v}|\tilde{s} = s] = s$ and $\mathrm{Var}_2[\tilde{v}|\tilde{s} = s] = \sigma_\varepsilon^2$.[2] The signal generates uncertainty about the asset price $\tilde{p}_2$ as viewed from date $t = 1$, as $\tilde{p}_2$ will depend on the conditional mean $E_2[\tilde{v}|\tilde{s}]$. The risks faced by investors in the signal scenario are thus payoff risk regarding $\tilde{v}$ and price risk regarding $\tilde{p}_2$. Note that the price risk stems only from the dependence of $\tilde{p}_2$ on the conditional mean of $\tilde{v}$ but not from the conditional variance of $\tilde{v}$, as the conditional variance is deterministic.

In the economy, there are $I \geq 3$ rational investors (indexed by $i = 1, 2, \ldots, I$). Each investor is initially endowed with $\bar{y}^i$ units of the riskless asset and $\bar{x}^i$ units of the risky asset (with $\sum_{i=1}^{I} \bar{x}^i \equiv \bar{X}$). If y_t^i and x_t^i are the corresponding portfolio holdings after trading at date t, then the budget condition for investor i is $\bar{y}^i + p_1 \bar{x}^i = y_1^i + p_1 x_1^i$ at $t = 1$ and $y_1^i + \tilde{p}_2 x_1^i = y_2^i + \tilde{p}_2 x_2^i$ at $t = 2$. Furthermore, terminal wealth for investor i is given by $\tilde{w}^i = y_2^i + \tilde{v} x_2^i$. Substituting from the budget conditions for y_2^i and y_1^i, investor i's terminal wealth can be rewritten as

$$\tilde{w}^i = \bar{y}^i - p_1\left(x_1^i - \bar{x}^i\right) - \tilde{p}_2\left(x_2^i - x_1^i\right) + \tilde{v} x_2^i. \tag{5.1}$$

Since $\tilde{p}_2$ and $\tilde{v}$ are normal, terminal wealth (5.1) is also normal for both the signal scenario and the basic scenario.

[2]The updating problem for investors is very simple, because we assume $\tilde{v} = \tilde{s} + \tilde{\varepsilon}$, with $\tilde{s}$ independent of $\tilde{\varepsilon}$ (as in Biais, Martimort, and Rochet (2000)), and not the more common version $\tilde{s} = \tilde{v} + \tilde{\varepsilon}$, with $\tilde{v}$ independent of $\tilde{\varepsilon}$ (e.g. Grossman and Stiglitz (1980) and Kyle (1989)).

The investors all have CARA utility functions over terminal wealth,

$$u^i\left(w^i\right) = -\exp(-\rho\, w^i),$$

with an identical coefficient of absolute risk aversion $\rho > 0$. With normally distributed terminal wealth and CARA utility, maximizing expected utility for investor i is equivalent to maximizing the mean-variance objective function

$$\Pi_t^i := E_t\left[\tilde{w}^i\right] - \frac{\rho}{2}\mathrm{Var}_t\left[\tilde{w}^i\right] \tag{5.2}$$

conditional on information at date t.

The market for the risky asset is organized as call auction. In each round of trading, investors simultaneously submit orders to an auctioneer, who sets a single price to clear the market and settles all orders at this price. For each investor i and each round of trading t, an order $\Theta_t^i : \mathbb{R} \to \mathbb{R}$ has to be a strictly decreasing, continuously differentiable function mapping the price of the risky asset into units of the risky asset to buy, such that $\theta_t^i = \Theta_t^i(p_t)$. We sometimes refer to these price-contingent orders as "limit orders." The auctioneer batches all orders, sets a unique price p_t to satisfy the market-clearing condition

$$\sum_{i=1}^{I} \Theta_t^i(p_t) = 0,$$

and allocates shares of the risky asset such that $x_1^i = \bar{x}^i + \Theta_1^i(p_1)$ at $t = 1$ and $x_2^i = x_1^i + \Theta_2^i(p_2)$ at $t = 2$ for each investor i.

As in the previous chapter, we introduce market liquidity into the model by allowing for imperfect competition among investors. That is, investors do not act as price takers, but instead act strategically by considering the impact of their trading decisions on the market-clearing price. We confine our analysis to linear orders of the form

$$\Theta_t^i(p_t) = \alpha_t^i - \beta_t^i\, p_t,$$

where the conjectured parameters α_t^i and $\beta_t^i > 0$ will be determined in equilibrium for all i and all t. Furthermore, we assume that investors' orders are symmetric in their slope at each date t, i.e., we assume $\beta_t^i = \beta_t$ for all i and all t.

For the individual optimization problem of investor i at date t, we define a supply curve as the negative sum of the conjectured orders of all other agents $h \neq i$,

$$-\sum_{h \neq i} \Theta_t^h(p) \equiv -\sum_{h \neq i} \alpha_t^h + (I-1)\,\beta_t\, p_t. \tag{5.3}$$

If investor i decides to trade the quantity θ_t^i against his supply curve (5.3), then the market-clearing condition $\theta_t^i = -\sum_{h \neq i} \Theta_t^h(p_t)$ yields the price

$$p_t = \hat{p}_t^i + \lambda_t\, \theta_t^i,$$

where $\lambda_t = [(I-1)\,\beta_t]^{-1} > 0$ is the slope and $\hat{p}_t^i = \lambda_t \sum_{h \neq i} \alpha_t^h$ is the intercept of the supply curve. Hence, investor i acts as a monopsonist over his supply curve and recognizes that the price moves against him when he trades an additional unit of the risky asset. The degree of market liquidity for investor i is determined by the slope of his supply curve: the steeper the slope, the less liquid the market. The inverse of the slope, $1/\lambda_t$, is the Kyle (1985, 1989) measure of market depth; we apply this term to analyze market liquidity.

5.2 Derivation of Equilibrium Orders

In this section, we derive the equilibrium orders of our dynamic asset pricing model with imperfect competition. We assume that all aspects of the model are common knowledge and focus on subgame-perfect Nash equilibria in pure strategies. We derive the equilibrium orders by backwards induction.

Each investor i enters the second round of trading with portfolio holdings y_1^i and x_1^i and, furthermore, has observed a signal $\tilde{s}$ and updated his beliefs about the payoff $\tilde{v}$ in the signal scenario or has beliefs about $\tilde{v}$ equal to his prior beliefs in the basic scenario, respectively. Given these conditions, the equilibrium orders in the second round of trading coincide with the equilibrium of the static model analyzed in Section 4.3. The following remark briefly reviews this equilibrium and its properties. For notational convenience, we do not yet integrate investors' beliefs about $\tilde{v}$ in explicit form, but denote them simply by $E_2[\tilde{v}]$ and $\mathrm{Var}_2[\tilde{v}]$ with the meaning that $E_2[\tilde{v}] = \tilde{s}$ and $\mathrm{Var}_2[\tilde{v}] = \sigma_\varepsilon^2$ in the signal scenario and $E_2[\tilde{v}] = E[\tilde{v}]$ and $\mathrm{Var}_2[\tilde{v}] = \mathrm{Var}[\tilde{v}]$ in the basic scenario.

Remark 2 *Equilibrium orders in the second round of trading are*

$$\Theta_2^{i,*}(p_2) = \frac{(I-2)}{(I-1)} \left(\frac{E_2[\tilde{v}] - p_2}{\rho \mathrm{Var}_2[\tilde{v}]} - x_1^i \right) \quad \textit{for all } i.$$

Market liquidity is given by

$$\lambda_2^{-1} = \frac{(I-2)\rho^{-1}}{\mathrm{Var}_2[\tilde{v}]} \quad \textit{for all } i,$$

the market-clearing price of the risky asset is

$$p_2 = E_2[\tilde{v}] - \frac{\bar{X}\mathrm{Var}_2[\tilde{v}]}{I\rho^{-1}}, \tag{5.4}$$

and investors' portfolio holdings of the risky asset are

$$x_2^i = x_1^i + \frac{(I-2)}{(I-1)} \left(\frac{\bar{X}}{I} - x_1^i \right) \quad \textit{for all } i. \tag{5.5}$$

Now, we turn to the derivation of the equilibrium orders for the first round of trading. Again we do not yet integrate investors' beliefs about the payoff $\tilde{v}$ in explicit form, but use the simplified notation $E_1[\tilde{v}]$ and $\mathrm{Var}_1[\tilde{v}]$ with $E_1[\tilde{v}] = \bar{v}$ and $\mathrm{Var}_1[\tilde{v}] = \sigma_s^2 + \sigma_\varepsilon^2$ in the signal scenario and $E_1[\tilde{v}] = E[\tilde{v}]$ and $\mathrm{Var}_1[\tilde{v}] = \mathrm{Var}[\tilde{v}]$ in the basic scenario. Similarly, for the price risk faced by investors in the signal scenario, we denote the resulting variance and covariance of $\tilde{p}_2$ simply by $\mathrm{Var}_1[\tilde{p}_2]$ and $\mathrm{Cov}_1[\tilde{p}_2, \tilde{v}]$. This allows us to carry out the derivation for the more general case of the signal scenario, including the basic scenario as a special case with $\mathrm{Var}_1[\tilde{p}_2] = \mathrm{Cov}_1[\tilde{p}_2, \tilde{v}] = 0$.

The next step of the analysis is to consider each investor's individually optimal order given the conjectured orders of all other agents in the first round of trading. The problem for investor i is to choose his limit order to maximize the objective function

$$\begin{aligned}
\Pi_1^i &= E_1\left[\tilde{w}^i\right] - \frac{\rho}{2}\mathrm{Var}_1\left[\tilde{w}^i\right] \\
&= \bar{y}^i + E_1[\tilde{v}]\, x_2^i - E_1[\tilde{p}_2]\left(x_2^i - x_1^i\right) - p_1\left(x_1^i - \bar{x}^i\right) \\
&\quad - \frac{\rho}{2}\left[\left(x_2^i\right)^2 \mathrm{Var}_1[\tilde{v}] + \left(x_2^i - x_1^i\right)^2 \mathrm{Var}_1[\tilde{p}_2] - 2x_2^i\left(x_2^i - x_1^i\right)\mathrm{Cov}_1[\tilde{p}_2, \tilde{v}]\right]
\end{aligned}$$

subject to his price impact, $p_1 = \hat{p}_1^i + \lambda_1\left(x_1^i - \bar{x}^i\right)$, and his anticipation of the market outcomes in the second round of trading, (5.4) and (5.5). The following lemma states the optimal order for each investor i.

Lemma 2 *The optimal order for investor i in the first round of trading is given by*

$$\begin{aligned}\Theta_1^{i,*}(p_1) &= \frac{(I-1)^2 E_1[\tilde{v}] - (I-1)^2 p_1 - \bar{x}^i \rho \{\mathrm{Var}_1[\tilde{v}] + \Phi\}}{(I-1)^2 \lambda_1 + \rho \{\mathrm{Var}_1[\tilde{v}] + \Phi\}} \\ &\quad - \frac{\bar{X} I^{-1} \rho \{(I-2)(I-1)\mathrm{Var}_2[\tilde{v}] + (I-2)\mathrm{Var}_1[\tilde{v}] - \Psi\}}{(I-1)^2 \lambda_1 + \rho \{\mathrm{Var}_1[\tilde{v}] + \Phi\}}, \qquad (5.6)\end{aligned}$$

with

$$\begin{aligned}\Phi &= (I-2)^2 \mathrm{Var}_1[\tilde{p}_2] + 2(I-2)\mathrm{Cov}_1[\tilde{p}_2, \tilde{v}], \\ \Psi &= (I-2)^2 \mathrm{Var}_1[\tilde{p}_2] - (I-2)(I-3)\mathrm{Cov}_1[\tilde{p}_2, \tilde{v}].\end{aligned}$$

The optimal order in the first round of trading is difficult to interpret. However, the denominator of (5.6) reveals the standard effect of liquidity costs on the individual trading decision: the investor restricts his trading aggressiveness not only by his risk aversion and the risk of the asset, but also by the degree of illiquidity, λ_1, in order to reduce the liquidity costs from his adverse price impact. For the signal scenario, the risk of the asset is composed of payoff risk regarding $\tilde{v}$, price risk regarding $\tilde{p}_2$, and the comovement between $\tilde{v}$ and $\tilde{p}_2$. For the basic scenario, the risk of the asset reduces to the payoff risk (so that $\Phi = \Psi = 0$). The term $\mathrm{Var}_2[\tilde{v}]$ appears in the numerator of (5.6) through the future (payoff) risk premium reflected in the price expectations $E_1[\tilde{p}_2]$. This future risk premium can be perfectly anticipated by investors—in the signal scenario, because the conditional variance of $\tilde{v}$ is deterministic; and in the basic scenario, because beliefs about $\tilde{v}$ remain constant over time.

The final step of the derivation is to use the individually optimal order (5.6) for each investor i to establish the equilibrium orders in the first round of trading.

Proposition 10 *Equilibrium orders in the first round of trading are given by*

$$\begin{aligned}\alpha_1^{i,*} &= \frac{(I-2)(I-1)E_1[\tilde{v}]}{\rho\{\mathrm{Var}_1[\tilde{v}] + \Phi\}} - \frac{(I-2)}{(I-1)}\bar{x}^i \\ &\quad - \frac{(I-2)}{(I-1)}\frac{\bar{X}}{I}\frac{\{(I-2)(I-1)\mathrm{Var}_2[\tilde{v}] + (I-2)\mathrm{Var}_1[\tilde{v}] - \Psi\}}{\{\mathrm{Var}_1[\tilde{v}] + \Phi\}} \quad \textit{for all } i, \\ \beta_1^* &= \frac{(I-2)(I-1)}{\rho\{\mathrm{Var}_1[\tilde{v}] + \Phi\}} \quad \textit{for all } i.\end{aligned}$$

The representation of the equilibrium orders in Proposition 10 refers again to the more general signal scenario, but includes the basic scenario as a special case with

$\Phi = \Psi = 0$. In the following sections, we discuss the properties of the equilibrium determined by Proposition 10 and Remark 2. We begin with the basic scenario and thereafter turn to the signal scenario.

5.3 Equilibrium Properties in the Basic Scenario

In this section, we study the equilibrium properties in the basic scenario. In this scenario, investors do not face price risk over the two rounds of trading, but only payoff risk at the end of the holding period. The properties of the equilibrium to be examined are the behavior of market liquidity and the effects of illiquidity on trading volume, portfolio holdings, and the market-clearing price of the risky asset.[3] First, we analyze market liquidity.

Proposition 11 *For each investor i, market liquidity is given by*

$$\begin{aligned} \lambda_1^{-1} &= \frac{(I-1)^2(I-2)\rho^{-1}}{\mathrm{Var}\left[\tilde{v}\right]}, \\ \lambda_2^{-1} &= \frac{(I-2)\rho^{-1}}{\mathrm{Var}\left[\tilde{v}\right]}. \end{aligned}$$

In line with the inventory-based microstructure literature, market liquidity for each investor is determined by the risk-bearing willingness of the other agents in the market. Hence, market liquidity is increasing in the number of agents and their risk-tolerance and is decreasing in the risk of the asset. For the dynamic behavior of liquidity, two specific results stand out: first, market liquidity declines over time; and second, the market is highly liquid in the first round of trading due to the factor $(I-1)^2$, but is yet not perfectly (or infinitely) liquid. Both results follow from the possibility for investors to repeatedly trade the risky asset over time without facing price risk.

In the first round of trading, the future asset price p_2 can be perfectly anticipated. Investors are thus willing to take large positions of the risky asset at small deviations from p_2, aiming at a riskless capital gain to be realized by re-trading the positions in

[3] Equilibrium orders for the basic scenario can be derived in explicit form from Proposition 10 and Remark 2 with $E_t\left[\tilde{v}\right] = E\left[\tilde{v}\right]$ and $\mathrm{Var}_t\left[\tilde{v}\right] = \mathrm{Var}\left[\tilde{v}\right]$ for $t = 1, 2$ and with $\Phi = \Psi = 0$.

the future. Accordingly, investors submit aggressive limit orders and thereby provide a high level of liquidity to the market.

In the second round of trading, in contrast, investors recognize that there is no further possibility to re-trade the asset in the future. Hence, the resulting portfolio positions are fixed until the end of the holding period and are subject to bearing payoff risk. As a consequence, investors reduce the aggressiveness of their limit orders and, in turn, provide less liquidity to the market. Market liquidity therefore declines over time, as the possibility for investors to re-trade the risky asset disappears.

Although there is a high level of liquidity in the first round of trading, the market is yet not perfectly liquid. The reason why investors do not submit even more aggressive limit orders when aiming at a short-term riskless capital gain is that taking a larger position in the first round of trading involves higher liquidity costs for unwinding the position in the illiquid second round of trading. Hence, it is the anticipation of future illiquidity that restricts investors' current trading aggressiveness and thereby generates current illiquidity in the market.

Next, we consider trading volume and portfolio holdings of the risky asset.

Proposition 12 *For each investor i, trading volume of the risky asset is given by*

$$\begin{aligned} \theta_1^i &= \frac{I-2}{I-1}\left(\frac{\bar{X}}{I}-\bar{x}^i\right), \\ \theta_2^i &= \frac{I-2}{(I-1)^2}\left(\frac{\bar{X}}{I}-\bar{x}^i\right), \end{aligned}$$

and the resulting portfolio holdings after the second round of trading are

$$x_2^i = \bar{x}^i + \frac{I\,(I-2)}{(I-1)^2}\left(\frac{\bar{X}}{I}-\bar{x}^i\right).$$

Trading of the risky asset is governed by the improvement of risk sharing under imperfect market liquidity. If, for instance, investor i is initially endowed with less shares of the risky asset than under Pareto-optimal risk sharing ($\bar{x}^i < \bar{X}/I$), he buys a positive number of shares in each round of trading, but reduces his trade size over time in response to the decline in market liquidity ($\theta_1^i > \theta_2^i > 0$). Furthermore, investor i restricts his trade size within each round of trading in order to mitigate his liquidity costs. As a consequence, the resulting portfolio holdings after the second round of trading are still slightly distorted towards the initial endowment of the investor ($x_2^i < \bar{X}/I$). In summary, investors break up, or split, their total portfolio adjustments into a sequence of smaller trades, and the allocation of the risky asset converges towards optimal risk sharing over time, but only gradually and incompletely.

The result of order splitting over time is useful in two respects. First, it provides a plausible representation of trading behavior from a market microstructure view, since order splitting by price-affecting investors is a typical feature in microstructure models (e.g. Kyle (1985), Vayanos (2001), and Huberman and Stanzl (2005)) and is also documented in empirical studies (e.g. Chan and Lakonishok (1995) and Chordia and Subrahmanyam (2004)). Second, for the asset pricing side of our model, this result generates a situation in which investors anticipate in the first round of trading that they will re-trade the risky asset in the second round when the market is much more illiquid. In order to analyze how future illiquidity affects current asset prices it is therefore not necessary to enforce repeated trade among investors exogenously (for instance by imposing an overlapping generations structure), but this situation arises fully endogenously. Accordingly, we turn now to the market-clearing price of the risky asset.

Proposition 13 *The market-clearing price of the risky asset is constant over time with*

$$p_1 = p_2 = E[\tilde{v}] - \frac{\bar{X}\mathrm{Var}[\tilde{v}]}{I\rho^{-1}}.$$

In each round of trading, the market-clearing price of the risky asset is given by the expected payoff of the asset minus the standard risk premium, but does not reflect an additional liquidity discount. The absence of a liquidity discount in the second round of trading follows from the cancelling out of concurrent liquidity costs as known from the static setting in the previous chapter.

The main point of this section is the absence of a liquidity discount in the first round of trading for the illiquidity in the second round. This result contradicts the findings of models on exogenous trading costs, in which the price of an asset is reduced by the present value of all its future trading costs. In our model, future illiquidity is a cost to investors, but it does not affect the current asset price. The reason for

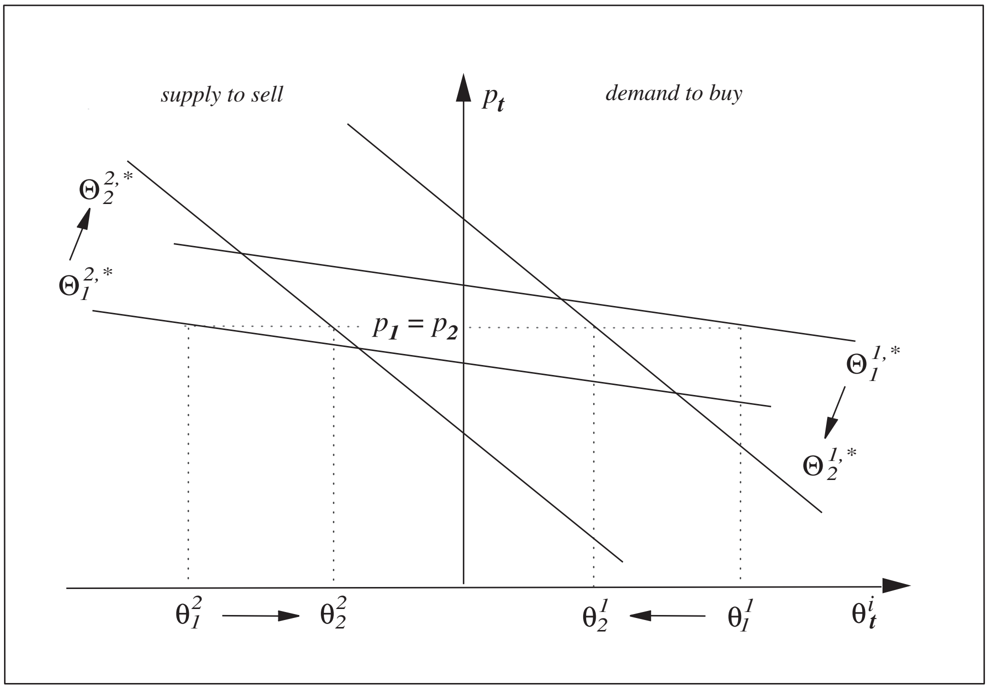

Figure 5.1: Pricing effect of future illiquidity.

the absence of a liquidity discount is, again, the endogenous trading behavior of both buyers and sellers: in anticipation of future illiquidity, buyers restrict their current demand to buy and sellers their current supply to sell, and these effects offset each other with respect to the current asset price.

Figure 5.1 summarizes the equilibrium properties in the basic scenario. The figure is based on an example with $I = 3$, $\bar{x}^1 < \bar{X}/I < \bar{x}^2$, and $\bar{x}^3 = \bar{X}/I$ and depicts the equilibrium orders for investors $i = 1, 2$ over the two rounds of trading.[4]

In the basic scenario, the time variation in liquidity is exclusively driven by an intertemporal effect: market liquidity declines over time, as the possibility to re-trade the risky asset disappears. This intertemporal effect is sufficient to raise the question of how future illiquidity affects current asset prices. Furthermore, from a market microstructure view, the effect may also capture the dynamic behavior of liquidity for assets with a short remaining lifetime and a low trading frequency. For assets with a long or infinite lifetime and a high trading frequency, however, it seems more plausible that time variations in liquidity are driven by uncertainties within the

[4]The orders for investor $i = 3$, who trades zero shares in each round of trading, are omitted in the figure.

trading process, for instance due to price risk in the short-term trading conditions. This issue is addressed in the next section.

5.4 Equilibrium Properties in the Signal Scenario

In this section, we study the equilibrium properties in the signal scenario. In this scenario, investors receive a public signal $\tilde{s}$ about the payoff $\tilde{v} = \tilde{s} + \tilde{\varepsilon}$ before the second round of trading. The signal generates uncertainty about the asset price $\tilde{p}_2$. Hence, investors face short-term price risk regarding $\tilde{p}_2$ and long-term payoff risk regarding $\tilde{v}$. Before we turn to the properties of the equilibrium under imperfect competition, we briefly state the market outcomes for the benchmark case of perfect competition (i.e. perfect market liquidity).[5]

Remark 3 *Under perfect competition, the market-clearing price of the risky asset is given by*

$$\begin{aligned} p_1^c &= \bar{v} - \frac{\bar{X}\left(\sigma_s^2 + \sigma_\varepsilon^2\right)}{I\rho^{-1}}, \\ \tilde{p}_2^c &= \tilde{s} - \frac{\bar{X}\,\sigma_\varepsilon^2}{I\rho^{-1}}, \end{aligned}$$

and the allocation of the risky asset achieves Pareto-optimal risk sharing within the first round of trading such that investors' portfolio holdings are given by

$$x_1^{i,c} = x_2^{i,c} = \bar{X}/I \quad \text{for all } i.$$

Under perfect competition, the price of the risky asset is given by the expected payoff of the asset minus the standard (payoff) risk premium. However, the price changes over time, as investors update their beliefs about $\tilde{v}$: beliefs are specified by $E_1\left[\tilde{v}\right] = \bar{v}$ and $\mathrm{Var}_1\left[\tilde{v}\right] = \sigma_s^2 + \sigma_\varepsilon^2$ at date $t = 1$ and by $E_2\left[\tilde{v}|\tilde{s}\right] = \tilde{s}$ and $\mathrm{Var}_2\left[\tilde{v}|\tilde{s}\right] = \sigma_\varepsilon^2$ at date $t = 2$. Hence, the price changes because of a random adjustment in the expected payoff and a deterministic decline in the risk premium. Furthermore, investors establish Pareto-optimal risk sharing within the first round of trading. Trading volume in the second round is thus zero.

[5]Under perfect competition, the optimal demand for the risky asset for each investor i is $\Theta_1^{i,c}(p_1) = (\bar{v} - p_1) / \left(\rho\,\sigma_s^2\right) - \sigma_\varepsilon^2\,\bar{X} / \left(\sigma_s^2\,I\right) - \bar{x}^i$ and $\Theta_2^{i,c}(\tilde{p}_2) = (\tilde{s} - \tilde{p}_2) / \left(\rho\,\sigma_\varepsilon^2\right) - x_1^i$.

Now, we turn to the properties of the equilibrium under imperfect competition. Again, we examine the behavior of market liquidity and the effects of illiquidity on trading volume, portfolio holdings, and the market-clearing price.[6] First, we study market liquidity.

Proposition 14 *For each investor i, market liquidity is given by*

$$\begin{aligned}\lambda_1^{-1} &= \frac{(I-1)^2(I-2)\rho^{-1}}{\sigma_\varepsilon^2+(I-1)^2\sigma_s^2},\\ \lambda_2^{-1} &= \frac{(I-2)\rho^{-1}}{\sigma_\varepsilon^2}.\end{aligned}$$

The dynamic behavior of market liquidity is composed of two effects. First, as in the basic scenario, there is a purely intertemporal effect. Market liquidity is high in the first round of trading due to the factor $(I-1)^2$, but declines in the second round, since investors are less willing to provide liquidity when they have no further possibility to re-trade the asset. Second, there is a signal effect which causes short-term price risk and makes re-trading the asset risky. Trading aggressiveness and market liquidity in the first round of trading are thus impaired by the uncertainty about the signal σ_s^2, but increase in the second round after the signal has been released.

To summarize the results on market liquidity, it is useful to represent the time variation with respect to the illiquidity measure λ_t and to separate the intertemporal effect and the signal effect according to

$$\lambda_2-\lambda_1 = \underbrace{\frac{\sigma_\varepsilon^2\, I}{(I-1)^2\rho^{-1}}}_{\textit{intertemporal effect}} - \underbrace{\frac{\sigma_s^2}{(I-2)\rho^{-1}}}_{\textit{signal effect}}. \tag{5.7}$$

Since it is known from the basic scenario that the purely intertemporal effect does not affect the asset price, we focus in this section on the signal effect. This effect may also be the more relevant determinant of illiquidity dynamics for most financial assets.

Next, we analyze trading volume and portfolio holdings.

[6] Equilibrium orders for the signal scenario can be derived in explicit form from Proposition 10 with $E_1[\tilde{v}]=\bar{v}$, $\mathrm{Var}_1[\tilde{v}]=\sigma_s^2+\sigma_\varepsilon^2$, and $\mathrm{Var}_1[\tilde{p}_2]=\mathrm{Cov}_1[\tilde{p}_2,\tilde{v}]=\sigma_s^2$ (as shown in the proof to this section) and from Remark 2 with $E_2[\tilde{v}]=\tilde{s}$ and $\mathrm{Var}_2[\tilde{v}]=\sigma_\varepsilon^2$.

Proposition 15 *For each investor i and each round of trading t, trading volume and portfolio holdings of the risky asset are the same as in the basic scenario.*

Trading volume and portfolio holdings are identical to the basic scenario. Hence, investors split their orders into a sequence of smaller trades, and the allocation of the risky asset converges towards optimal risk sharing over time, but only gradually and incompletely. As the speed and extent of this convergence depends solely on the number of investors and their initial endowments, the introduction of the signal does not change these results.

The important aspect of the result on trading volume is that, only because of illiquidity, investors split their orders over time and trade a non-zero number of shares in the second round of trading. (Under perfect competition, trading volume is zero in the second round.) As the price $\tilde{p}_2$ is random, illiquidity thereby generates additional trading risk for investors. Accordingly, we turn now the question of how this illiquidity-induced trading risk affects the current asset price.

Proposition 16 *The market-clearing price of the risky asset is given by*

$$\begin{aligned} p_1 &= \bar{v} - \frac{\bar{X}\,(\sigma_s^2 + \sigma_\varepsilon^2)}{I\rho^{-1}}, \\ \tilde{p}_2 &= \tilde{s} - \frac{\bar{X}\,\sigma_\varepsilon^2}{I\rho^{-1}}. \end{aligned}$$

In each round of trading, the market-clearing price of the risky asset is identical to the corresponding price under perfect competition: it is given by the expected payoff of the asset minus the standard (payoff) risk premium. The point of this result is that the asset price in the first round of trading is not affected by the illiquidity-induced trading risk for the second round. Hence, the additional trading risk makes all investors worse off, but, with endogenous trading behavior on both sides of the market, cancels out with respect to the price of the asset. This result calls into question the standard argument supported by Huang (2003), Vayanos (2004), and Acharya and Pedersen (2005) that stochastic liquidity effects should reduce the price of the asset, as risk-averse investors require compensation for holding assets that are risky to trade in the future. However, the illiquidity-induced risk in our model is different from the stochastic liquidity effects in these models. We shall return to this difference in the discussion of this chapter.

In summary, our asset pricing results challenge the conventional wisdom that illiquidity is an additional pricing factor for expected asset returns. Yet, illiquidity and expected return are nevertheless positively related in our model, as both variables are similarly determined by the risk characteristics of the asset and the risk-bearing capacity of investors. This relation holds also for the dynamic context. Using the asset prices from Proposition 16 to denote the variation in expected return as

$$\frac{\tilde{s}}{\tilde{p}_2} - \frac{\bar{v}}{p_1} = -\underbrace{\frac{\bar{X}\sigma_s^2}{I\rho^{-1}}}_{\textit{signal effect}} \tag{5.8}$$

and comparing (5.8) with the variation in illiquidity (5.7) reveals that both variables decrease over time after the uncertainty about the signal is resolved (ignoring the intertemporal effect on illiquidity). Hence, illiquidity comoves with expected return. However, this comovement is not driven by causality between the two variables, say by the presence of a liquidity premium that declines when market liquidity improves, but follows solely from the common determinants of illiquidity and the standard risk premium. From an empirical point of view, illiquidity may thus appear to be a pricing factor, even if it is not, simply because illiquidity variables are likely to proxy for mis-specifications of the standard risk premium.[7]

5.5 Discussion

In this chapter, we study a dynamic asset pricing model with imperfect competition and endogenous time variations in market liquidity. Because of illiquidity, investors incur additional costs and risks for the repeated trading of the asset in the future. However, the current asset price is not affected by illiquidity, as trading behavior is endogenous on both sides of the market and the individual effects of illiquidity therefore cancel out. This result stands in contrast to the literature on exogenous trading costs, which finds that current asset prices reflect a liquidity discount to compensate investors for holding assets that are costly and risky to trade in the future.

[7] This point is also made by Novy-Marx (2006).

A point that deserves further discussion is the nature of the illiquidity-induced risk in our model. The additional trading risk arises, as investors split their orders over time and, thus, trade also in the future when the asset price is random. This risk is different from the stochastic liquidity effects in the models on exogenous trading costs. In Huang (2003) and Vayanos (2004), investors have random trading needs because of exogenous wealth shocks, whereas trading costs are deterministic; in Acharya and Pedersen (2005), trading costs, per se, are random, while investors have deterministic trading needs.

Although these three types of illiquidity-induced risks are different with respect to their origins, they have similar effects for an individual investor: the investor incurs additional risk that would not arise, if the market were perfectly liquid or if the investor would not process the future trade. This similarity suggests that the different types of risk should have similar pricing implications in otherwise identical trading situations. Put differently, the crucial aspect for the absence of a pricing effect in our model seems not to be the specific origin of the trading risk, but indeed the endogenous trading behavior on both sides of the market.

Appendix: Proofs to Chapter 5

Proof of Lemma 2

In the first round of trading, the problem for investor i is to determine x_1^i to maximize

$$\begin{aligned} & E_1\left[\tilde{w}^i\right] - \frac{\rho}{2}\operatorname{Var}_1\left[\tilde{w}^i\right] \\ & \quad = \bar{y}^i + E_1\left[\tilde{v}\right] x_2^i - E_1\left[\tilde{p}_2\right]\left(x_2^i - x_1^i\right) - p_1\left(x_1^i - \bar{x}^i\right) \\ & \qquad - \frac{\rho}{2}\left[\left(x_2^i\right)^2 \operatorname{Var}_1\left[\tilde{v}\right] + \left(x_2^i - x_1^i\right)^2 \operatorname{Var}_1\left[\tilde{p}_2\right] - 2x_2^i\left(x_2^i - x_1^i\right)\operatorname{Cov}_1\left[\tilde{p}_2, \tilde{v}\right]\right] \end{aligned} \tag{5.9}$$

subject to his price impact,

$$p_1 = \hat{p}_1^i + \lambda_1\left(x_1^i - \bar{x}^i\right),$$

and his anticipation of the market outcomes in the second round of trading,

$$E_1\left[\tilde{p}_2\right] = E_1\left[\tilde{v}\right] - \frac{\bar{X}\operatorname{Var}_2\left[\tilde{v}\right]}{I\rho^{-1}}, \tag{5.10}$$

$$x_2^i = x_1^i + \frac{(I-2)}{(I-1)}\left(\frac{\bar{X}}{I} - x_1^i\right). \tag{5.11}$$

Differentiating the objective (5.9) for x_1^i gives the first-order condition

$$\begin{aligned} & E_1[\tilde{v}]\frac{\partial x_2^i}{\partial x_1^i} - E_1[\tilde{p}_2]\left(\frac{\partial x_2^i}{\partial x_1^i}-1\right) - p_1 - \frac{\partial p_1}{\partial x_1^i}\left(x_1^i - \bar{x}^i\right) \\ & \quad - \frac{\rho}{2}\left\{2x_2^i\frac{\partial x_2^i}{\partial x_1^i}\operatorname{Var}_1[\tilde{v}] + 2\left(x_2^i - x_1^i\right)\left(\frac{\partial x_2^i}{\partial x_1^i}-1\right)\operatorname{Var}_1[\tilde{p}_2]\right. \\ & \quad\quad \left. -2\left[\frac{\partial x_2^i}{\partial x_1^i}\left(x_2^i - x_1^i\right) + x_2^i\left(\frac{\partial x_2^i}{\partial x_1^i}-1\right)\right]\operatorname{Cov}_1[\tilde{p}_2,\tilde{v}]\right\} = 0. \quad (5.12) \end{aligned}$$

Plugging the price expectations (5.10), the portfolio holdings (5.11), as well as the derivatives $\partial x_2^i / \partial x_1^i = 1/(I-1)$ and $\partial p_2 / \partial x_1^i = \lambda_1$ into (5.12) and simplifying yields

$$\begin{aligned} & E_1[\tilde{v}] - \frac{(I-2)}{(I-1)}\frac{\bar{X}\operatorname{Var}_2[\tilde{v}]}{I\rho^{-1}} - p_1 - \lambda_1 x_1^i + \lambda_1 \bar{x}^i \\ & - \frac{\rho}{(I-1)^2 I}\left\{ \left[(I-2)\bar{X} + Ix_1^i\right]\operatorname{Var}_1[\tilde{v}] - (I-2)^2\left(\bar{X} - Ix_1^i\right)\operatorname{Var}_1[\tilde{p}_2]\right. \\ & \quad \left. + (I-2)\left[(I-3)\bar{X} + 2Ix_1^i\right]\operatorname{Cov}_1[\tilde{p}_2,\tilde{v}] \right\} = 0. \end{aligned}$$

Rearranging terms and solving for x_1^i gives the optimal demand to hold shares of the risky asset,

$$\begin{aligned} x_1^{i,*}(p_1) &= \Big[(I-1)^2 E_1[\tilde{v}] - (I-1)^2 p_1 + (I-1)^2 \lambda_1 \bar{x}^i \\ & \quad - \bar{X}I^{-1}\rho\left\{(I-2)(I-1)\operatorname{Var}_2[\tilde{v}] + (I-2)\operatorname{Var}_1[\tilde{v}] - \Psi\right\} \Big] \\ & \quad \cdot \Big[(I-1)^2\lambda_1 + \rho\left\{\operatorname{Var}_1[\tilde{v}] + \Phi\right\}\Big]^{-1} \end{aligned}$$

with

$$\begin{aligned} \Psi &\equiv (I-2)^2\operatorname{Var}_1[\tilde{p}_2] - (I-2)(I-3)\operatorname{Cov}_1[\tilde{p}_2,\tilde{v}], \\ \Phi &\equiv (I-2)^2\operatorname{Var}_1[\tilde{p}_2] + 2(I-2)\operatorname{Cov}_1[\tilde{p}_2,\tilde{v}]. \end{aligned}$$

The optimal order to buy shares, $\Theta_1^{i,*}(p_1) = x_1^{i,*}(p_1) - \bar{x}^i$, is then given by

$$\begin{aligned} \Theta_1^{i,*}(p_1) &= \Big[(I-1)^2 E_1[\tilde{v}] - (I-1)^2 p_1 - \bar{x}^i\rho\left\{\operatorname{Var}_1[\tilde{v}] + \Phi\right\} \\ & \quad - \bar{X}I^{-1}\rho\left\{(I-2)(I-1)\operatorname{Var}_2[\tilde{v}] + (I-2)\operatorname{Var}_1[\tilde{v}] - \Psi\right\} \Big] \\ & \quad \cdot \Big[(I-1)^2\lambda_1 + \rho\left\{\operatorname{Var}_1[\tilde{v}] + \Phi\right\}\Big]^{-1}. \end{aligned}$$

For the second-order condition

$$-2\lambda_1 - \rho(I-1)^{-2}\left\{\operatorname{Var}_1[\tilde{v}] + (I-2)^2\operatorname{Var}_1[\tilde{p}_2] + 2(I-2)\operatorname{Cov}_1[\tilde{p}_2,\tilde{v}]\right\} < 0$$

to hold, it is sufficient that $\operatorname{Cov}_1[\tilde{p}_2,\tilde{v}] \geq 0$. This is confirmed in (5.22). $\|$

Proof of Proposition 10

Equilibrium orders in the first round of trading follow from the individually optimal orders for all investors given in Lemma 2. By plugging $\lambda_1 = [(I-1)\,\beta_1]^{-1}$ into the individually optimal orders, the equilibrium orders are specified by the parameters $\alpha_1^{i,*}$ and β_1^* for all i that solve the equation system

$$\begin{aligned} \alpha_1^{i,*} &= \frac{(I-1)^2\,E_1[\tilde{v}] - \bar{x}^i \rho\,\{\mathrm{Var}_1[\tilde{v}] + \Phi\}}{(I-1)^2\,[(I-1)\,\beta_1^*]^{-1} + \rho\,\{\mathrm{Var}_1[\tilde{v}] + \Phi\}} \\ &\quad - \frac{\bar{X} I^{-1} \rho\,\{(I-2)\,(I-1)\,\mathrm{Var}_2[\tilde{v}] + (I-2)\,\mathrm{Var}_1[\tilde{v}] - \Psi\}}{(I-1)^2\,[(I-1)\,\beta_1^*]^{-1} + \rho\,\{\mathrm{Var}_1[\tilde{v}] + \Phi\}} \quad \text{for all } i, \quad (5.13) \\ \beta_1^{i,*} &= \frac{(I-1)^2}{(I-1)^2\,[(I-1)\,\beta_1^*]^{-1} + \rho\,\{\mathrm{Var}_1[\tilde{v}] + \Phi\}} \quad \text{for all } i. \quad (5.14) \end{aligned}$$

First, using the symmetry assumption on β_1^i to rearrange (5.14) yields

$$\beta_1^*\,[\beta_1^* \rho\,\{\mathrm{Var}_1[\tilde{v}] + \Phi\} - (I-2)\,(I-1)] = 0. \quad (5.15)$$

As the market rules restrict the parameter β_t to be strictly positive, the only relevant solution to (5.15) is

$$\beta_1^* = \frac{(I-2)\,(I-1)}{\rho\,\{\mathrm{Var}_1[\tilde{v}] + \Phi\}} > 0 \quad \text{for all } i. \quad (5.16)$$

Next, substituting (5.16) into (5.13) and simplifying gives

$$\begin{aligned} \alpha_1^{i,*} &= \frac{(I-2)\,(I-1)\,E_1[\tilde{v}]}{\rho\,\{\mathrm{Var}_1[\tilde{v}] + \Phi\}} - \frac{(I-2)}{(I-1)}\,\bar{x}^i \\ &\quad - \frac{(I-2)}{(I-1)}\frac{\bar{X}}{I}\frac{\{(I-2)\,(I-1)\,\mathrm{Var}_2[\tilde{v}] + (I-2)\,\mathrm{Var}_1[\tilde{v}] - \Psi\}}{\{\mathrm{Var}_1[\tilde{v}] + \Phi\}} \quad \text{for all } i. \quad (5.17) \end{aligned}$$

Equilibrium orders in the first trading round are thus specified by (5.17) and (5.16). ‖

Proof of Propositions 11, 12, and 13

Here, we prove the equilibrium properties of the basic scenario. The equilibrium orders in the first round of trading in this scenario are specified by (5.17) and (5.16) with investors' beliefs about the payoff $\tilde{v}$ equal to their prior beliefs at each date t and with a deterministic price p_2 such that $\Phi = \Psi = 0$. Hence, we have

$$\begin{aligned} \alpha_1^{i,*} &= \frac{(I-2)\,(I-1)\,E[\tilde{v}]}{\rho \mathrm{Var}[\tilde{v}]} - \frac{(I-2)}{(I-1)}\left[(I-2)\,\bar{X} + \bar{x}^i\right] \quad \text{for all } i, \quad (5.18) \\ \beta_1^{i,*} &= \frac{(I-2)\,(I-1)}{\rho \mathrm{Var}[\tilde{v}]} \quad \text{for all } i. \quad (5.19) \end{aligned}$$

First, we consider the degree of illiquidity in Proposition 11. The result for the first trading round follows immediately from the definition $\lambda_1 = [(I-1)\,\beta_1]^{-1}$ and (5.19). The result for the second trading round is already known from the static model as summarized in Remark 2.

Next, we turn to the market-clearing prices in Proposition 13. The price in the first trading round is obtained by applying the market-clearing condition to (5.18) and (5.19) and making use of $\sum_{i=1}^{I} \bar{x}^i \equiv \bar{X}$. The price in the second trading round is again known from Remark 2.

Finally, we derive investors' trading volumes and portfolio holdings in Proposition 12. Trading volume of investor i in the first trading round is determined by substituting the market-clearing price p_1 into the equilibrium order $\Theta_1^i(p_1)$. Trading volume in the second trading round is given in Remark 2 as

$$\theta_2^i = \frac{(I-2)}{(I-1)} \left(\frac{\bar{X}}{I} - x_1^i \right). \tag{5.20}$$

The closed-form solution for θ_2^i is obtained by plugging $x_1^i = \bar{x}^i + \theta_1^i$ into (5.20). The resulting portfolios after the second trading round are given by $x_2^i = \bar{x}^i + \theta_1^i + \theta_2^i$. $\|$

Proof of Propositions 14, 15, and 16

Here, we prove the equilibrium properties of the signal scenario. As a preliminary step, we first state investors' beliefs regarding the price risk in this scenario. From Remark 2, the market-clearing price of the risky asset in the second round of trading is given by

$$p_2 \quad = \quad E_2\,[\tilde{v}] - \frac{\bar{X}\mathrm{Var}_2\,[\tilde{v}]}{I\rho^{-1}}, \tag{5.21}$$

where $E_2\,[\tilde{v}] = \tilde{s}$ and $\mathrm{Var}_2\,[\tilde{v}] = \sigma_\varepsilon^2$ in the signal scenario. The uncertainty about (5.21) as viewed from date $t=1$ stems only from the dependence of the conditional mean of the payoff $\tilde{v} = \tilde{s} + \tilde{\varepsilon}$ on the signal $\tilde{s}$. Hence, we have

$$\begin{aligned} \mathrm{Var}_1\,[\tilde{p}_2] &= \sigma_s^2, \\ \mathrm{Cov}_1\,[\tilde{p}_2, \tilde{v}] &= \mathrm{Cov}_1\,[\tilde{s}, \tilde{s} + \tilde{\varepsilon}] = \sigma_s^2, \end{aligned} \tag{5.22}$$

where the derivation of the covariance (5.22) makes use of the assumption that $\tilde{s}$ and $\tilde{\varepsilon}$ are independent.

The equilibrium orders in the first round of trading are now specified by (5.17) and (5.16) with $E_1[\tilde{v}] = \bar{v}$, $\mathrm{Var}_1[\tilde{v}] = \sigma_s^2 + \sigma_\varepsilon^2$, and $\mathrm{Var}_1[\tilde{p}_2] = \mathrm{Cov}_1[\tilde{p}_2, \tilde{v}] = \sigma_s^2$. Hence, we have

$$\begin{aligned}
\alpha_1^{i,*} &= \frac{(I-2)(I-1)\bar{v}}{\rho\left[\sigma_\varepsilon^2 + (I-1)^2\sigma_s^2\right]} - \frac{(I-2)}{(I-1)}\left[\frac{(I-2)\sigma_\varepsilon^2 \bar{X}}{\sigma_\varepsilon^2 + (I-1)^2\sigma_s^2} + \bar{x}^i\right] \quad \text{for all } i, \\
\beta_1^{i,*} &= \frac{(I-2)(I-1)}{\rho\left[\sigma_\varepsilon^2 + (I-1)^2\sigma_s^2\right]} \quad \text{for all } i.
\end{aligned}$$

Given these conditions, the equilibrium properties of the signal scenario are derived analogously to those of the basic scenario. ‖

Chapter 6

Concluding Remarks

In this thesis, we study the role of market liquidity in asset pricing within a theoretical framework that integrates the asset pricing and the market microstructure view by deriving market liquidity, investors' trading behavior, and required asset returns endogenously. Our main result is, contrary to conventional wisdom, that illiquidity does not affect the required return of the risky asset. The reason is the endogeneity of trading on both sides of the market, so that the liquidity costs for buyers and sellers cancel each other out with respect to the price and the required return of the risky asset.

In arguing against the standard pricing effect of illiquidity, we expect to meet with criticism from skeptical readers. We thus conclude with some additional remarks. First, our assumptions of CARA utility and normally distributed asset payoffs are admittedly strong. However, this setup is also used in some of the models on exogenous trading costs, which find the existence of liquidity premia, for instance in Acharya and Pedersen (2005). Hence, these assumptions are not crucial for the absence of a liquidity premium in our model.

Next, we consider illiquidity only in the form of price impact arising from investors' risk-bearing costs. Other facets of illiquidity are not addressed. However, offsetting effects of individual liquidity costs with respect to asset prices are also found for other facets of illiquidity—in Vayanos (1998) for exogenous bid-ask spreads, in Garleanu and Pedersen (2004) for endogenous bid-ask spreads arising from asymmetric information, and in Garleanu (2006) for time delays of trading due to search frictions.

Finally, we point out that the cancelling out of liquidity costs in our model is very natural, as it arises from the endogenous trading behavior of all investors. In fact, the representation of trading behavior in our model is more plausible than in most of the models on exogenous trading costs, which treat the seller side of the market as inelastic, so that prices are entirely determined by buyers and a liquidity discount on asset prices is a direct result.

In conclusion, for the study of market liquidity in asset pricing it is important to capture a reasonable representation of investors' trading behavior. After all, if illiquidity might affect asset prices, then it is through its effects on the buying and selling decisions of investors in the underlying price formation process. An integration of asset pricing and market microstructure is thus a promising area for future research, and this thesis makes a step into this direction.

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