

# The Impact of Wealth Inequality on Imperfect Capital Markets

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*For Patrick and my parents*



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It is hard to describe what learning to become a researcher is like. Especially in the beginning, I often had to encourage myself with Albert Einstein's (1879-1955) words: "*If we knew what we were doing, it would not be called research, would it?*" However, I am lucky to have benefited from a lot of support and inspiration along my way. This not only helped me to grow as an economist and as a researcher, but also as a character.

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## Abstract

This dissertation contributes to the analysis of the macroeconomic impact of wealth inequality on imperfect capital markets. The analysis assesses its occurrence and consequences in a general equilibrium framework with wealth heterogeneity among agents and a convex technology. The essential point is that because of the commitment value of wealth on imperfect capital markets, productive opportunities might vary along the wealth distribution via access to credit. Thus, the larger the fraction of society whose financial constraint is binding, the more the capital allocation and the production outcome deviate from their full information counterparts. In each of the three substantive Chapters, the setting is adapted to focus on a specific aspect of the role of inequality: its impact on (i) growth when explicitly allowing for autarkic production, (ii) efficiency when coupled with banking market power as an additional market friction and (iii) the beneficence of international financial integration when countries are heterogeneous.

Chapter 2 dynamizes the basic model, in order to show how productive inefficiencies affect the evolution of inequality and how they lead to a multiplicity of steady-states, which depend on initial conditions. However, taking into account the possibility of autarkic production allows identifying the existence of credit-constrained net lenders. The fact that they profit from high interest rates countervails the poor's pauperization, changes distributional dynamics and thus the convergence to the bad steady-state.

Extending the static setting, Chapter 3 challenges what hitherto models typically take for granted: perfect competition on the deposit and loan market. Among others, banking market power triggers the existence of autarkic entrepreneurs in equilibrium, but might also help to solve the imperfect information problem and consequently to restore the first-best outcome. In this context, inequality is found to be constraining the scope of banking market power.

Likewise through the imperfect capital market lens, Chapter 4 finally scrutinizes the real consequences of financial market integration. It dismantles credit rationing as a new cost of financial market integration and shows that inequality provides higher absorptive capacities for capital inflows after external financial liberalization.



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# Chapter 1

## Introduction

Economic inequality has become of prime interest in economic and policy-making circles. Still, the macroeconomic role of wealth inequality is far from being well understood. Inequality had long been considered as a final outcome rather than as a major determinant of economic performance.<sup>1</sup> No earlier than in the 1990s, a new branch of literature emerged that reversed the classical view on inequality in the spirit of Ricardo (1820). He had already called for "*an enquiry into the laws which determine the division of the produce of industry, amongst the classes who concur in its formation*". A growing number of studies has meanwhile lent theoretical and empirical support to a causal relationship running from inequality to the economic performance.<sup>2</sup> Especially three channels of influence have been put forward. A first channel evolves from political economy considerations. Alesina and Rodrik (1994), Bertola (1993) or Persson and Tabellini (1994) play on a poor median voter's inclination to opt for higher tax rates in order to promote redistribution or a high ratio of government expenditures to GDP. Yet, higher taxes incur greater distortions, which dampen aggregate output. A second channel emerges from social conflicts. While e.g. Alesina and Perotti (1996) build on reduced investment levels ensued by political instability, Fajnzylber et al. (1998) draw on the high opportunity costs caused by violence. Rodrik (1998), instead, blames the ability of political systems to efficiently respond to external shocks. At last, a third channel centers on capital market imperfections. Especially Galor and Zeira (1993) and Banerjee and Newman (1993) established the view that when financial relations are hampered by imperfect information, an agent's wealth becomes crucial for gaining access to credit because of its commitment value. That is how wealth makes productive opportunities vary along the wealth distribution.

The focus of this dissertation lies on the imperfect capital market channel. Asymmetric information still remains the main impediment to the perfect functioning of financial systems and so to an efficient allocation of financial resources. The research builds on a class of well-established, but in part diverse contributions. Irrespective of the models' particular details, however, their key feature is that agents seek access to some productive, but risky activity that

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<sup>1</sup>An exception marks Kuznets (1955) empirical generalization of inequality first increasing and then decreasing in the course of development. It is meanwhile regarded as refuted (see e.g. Deininger and Squire, 1998).

<sup>2</sup>The empirical literature is reviewed in Section 3.2.

requires them to make an initial investment (like e.g. for education or a special training, the acquisition of patents, licences, land or machinery). Those deprived of sufficient own wealth have to borrow in order to be able to start production at all, at least at an efficient scale. But problematically, the lending relation is fraught with asymmetric information: the project outcome and hence repayment crucially depend on the borrower's labor input (moral hazard; see e.g. Aghion and Bolton, 1997; Piketty, 1997; or Grüner and Schils, 2007), individual ability (adverse selection; see e.g. Jaffee and Stiglitz, 1990; or Grüner, 2003) or level of physical output (see e.g. Banjee and Newman, 1993; or Galor and Zeira, 1993). In all these models, lenders then use the commitment value of initial wealth, so that richer agents can borrow more. While collateralizing helps to mitigate information problems, it also leads to credit rationing. Wealth acts as a selective device. Those who are too poor to put up the collateral are prevented from seizing the most productive activity given their skills, that would have benefited themselves and the economy as a whole. Imperfect information and incomplete contracts cause a credit market failure. Loans that would have been profitable are not granted, which in turn causes poverty to be more persistent and the economy's production potential to not fully being exhausted. Dynamizing this setting through bequests makes economic growth appear as a process of distributional dynamics. Such inefficiencies may then also translate into the existence of multiple steady-states, each depending on initial conditions. It is this path-dependence that might drag countries into poverty traps.

This thesis centers on the examination of the role of wealth inequality on imperfect capital markets in a general equilibrium framework with wealth heterogeneity among agents and a convex technology. It confirms the implicit finding of most earlier contributions in that the scope of credit rationing decreases with a higher fraction of society complying with the wealth requirement. The lower the fraction of society that is credit-constrained, the higher credit demand and the less the equilibrium market rate of return,<sup>3</sup> allocation of credit and aggregate outcome deviate from their full information levels. Yet, this thesis goes one step further, questioning typical assumptions and applying the mechanisms to new policy issues. It particularly poses three key problems:

- (i) What is the growth-impact of inequality on imperfect capital markets when explicitly allowing for autarkic production?
- (ii) How is the relationship between wealth inequality and the economic performance affected by the variation of the competitiveness structure on both sides of the capital market?
- (iii) Given its own and its partner's aggregate wealth and wealth distribution, when is international financial integration beneficial for a country?

Each subsequent Chapter of this thesis is devoted to one of these questions. Although the Chapters are closely connected, they can be read independently of each other. All Proofs and references are collected in the Appendixes and in the Bibliography respectively.

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<sup>3</sup>A positive relation between inequality and the real interest rate has recently also empirically been confirmed by Brückner, Gerling and Grüner (2007) for three major OECD economies.

The **second Chapter** exposes how Piketty's (1997) evolutionary predictions in a Solow model with credit rationing must be modified after loosening an implicit ad-hoc assumption of his, i.e. the non-existence of autarkic production. Due to moral hazard in production, some agents are too poor to get credit for an effort investment, but too rich to only make the lower shirking investment. Contrary to Piketty (1997), the equilibrium therefore involves credit-constrained net lenders for sufficiently high interest rates. The fact that more agents profit from increasing interest rates alters the model's dynamics: interest payments release a trickle-down effect. The higher the interest rate, the higher the returns for a certain middle class. This counteracts the pauperization of credit-constrained agents and leaves the worst case steady-state substantially attenuated. This insight prepares the ground for long-term growth- and equity-enhancing policy interventions.

However, the universality of the results for the static setting might be confined, as it follows hitherto models in assuming perfect competition on the capital market for lenders and borrowers alike. In order to overcome these limitations, the **third Chapter** therefore builds on the static framework of the second Chapter, but introduces banks on both sides of the market and then varies the competitiveness specification. This allows to study how banking competition on the market for deposits and loans affects equilibrium financial contracts, credit rationing, firm sizes, agents' incomes, aggregate output and surplus. The results challenge the received view that asymmetric information and imperfect competition between banks necessarily adversely affect the economic outcome. They so give rise to many novel policy implications. As credit rationing tends to contain aggregate effort costs, it might help to allocate capital more efficiently, even under perfect competition. Banking market power might then deteriorate or further enhance efficiency. Any conclusion subtly depends on the prevailing wealth distribution - except for under double-sided discriminating monopoly. Drawing on a pecuniary externality and a capital reallocation benefit, it improves on the competitive outcome irrespective of the wealth distribution and is so the most likely banking regime to restore first-best efficiency.

Finally, the **fourth Chapter** seeks to use the insights derived before, in order to give a new perspective on international financial integration. It assesses the mechanisms through which financial market integration affects the pattern of international capital flows and the domestic economic performances when explicitly accounting for wealth inequality on imperfect capital markets. Balancing the impact of a firm size and a credit rationing effect on the net credit position and aggregate production will help predicting the distribution of gains and losses among and within countries on the basis of a country's aggregate wealth and its distribution. Altogether, the results contribute new explanations for some empirical puzzles. They also bear important implications for policy making, supranational treaty design and financial stability.

Altogether, the major message this thesis aims to deliver is the following. An explicit provision of wealth inequality not only alters some common results of the literature, but also helps to reconcile theory and evidence in the presence of an imperfect capital market. The findings improve our understanding of the functioning of capital markets and allow a substantiated

derivation of appropriate regulations and policy measures. However, more research is needed in this area. New promising lines to explore might be applying this reasoning to new fields (e.g. monetary policy) or adding new features (esp. the problem of adverse selection or a richer contractual framework allowing borrowers e.g. to resort to a variety of -also unsecured- financial products and multiple lenders).

## Chapter 2

# Growth and Inequality under Credit Rationing with Autarkic Production

### 2.1 Introduction

In a seminal contribution on the macroeconomic impact of wealth inequality, Piketty (1997) introduces credit rationing into the Solow model with a closed capital market. Consequently, growth is determined by agents' ability to borrow in order to finance an entrepreneurial investment. Piketty's main contribution is to derive the existence of multiple stationary interest rates and wealth distributions. In particular, he obtains two kinds of long-run equilibria, depending on whether credit rationing will be eliminated or persistent. The latter fundamentally relies on the fact that initially high interest rates will be self-reinforcing through higher credit rationing and lower capital accumulation.

However, as investment projects are fully divisible, this paper completes agents' option space by explicitly taking into account the possibility of self-financing a smaller than the optimal project size (henceforth called autarkic production).<sup>1</sup> This helps to reveal the existence of credit-constrained net lenders for sufficiently high interest rates. That is how it unearths the existence of credit-constrained net lenders for sufficiently high interest rates. Then, increasing interest rates do not exclusively benefit the rich and so will not univocally lead to lower wealth mobility. Dynamics change and the set of potential realizations of Piketty's low-wealth second steady-state has an upper limit. It follows that the worst case scenario for an economy may be less severe than predicted by Piketty (1997).<sup>2</sup>

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<sup>1</sup>I thank Robin Boadway, Antonio Ciccone, Hans-Peter Grüner, Andrea Prat, Elisabeth Schulte-Runne, Kenichi Ueda, Evguenia and Viktor Winschel as well as seminar participants at the University of Mannheim and the ENTER Jamboree in Brussels for valuable discussions and helpful comments.

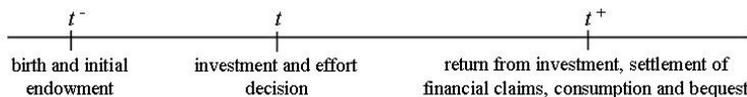
<sup>2</sup>Aghion and Bolton (1997) study a similar setting, but with a fixed investment size. Then, the poor are credit-constrained and therefore forced to become lenders. However, the uniqueness of their convergence result crucially depends on them making additional high demands on productivity and the saving rate. These assumptions help

The paper at hand is organized as follows. While Section 2.2 relaxes the implicit ad-hoc assumption in the original model, Section 2.3 scrutinizes how this impacts on the static equilibrium. Therefrom, Section 2.4 derives the evolutionary results. Section 2.5 concludes and points out some long-term growth- and equity-enhancing policy implications. All Proofs are sent to Appendix A.

## 2.2 The Model

### 2.2.1 Restating the Original Model

Piketty (1997) models a closed economy with an infinite, discrete time horizon  $t = 0, 1, \dots, \infty$ , which is populated by a continuum of dynasties of mass 1. In each period  $t$ , every dynasty is represented by an agent  $i$ , who is risk-neutral, maximizes expected current period income  $I_{it}$  and walks through the following chronology of events during his one-period life:



At the beginning of the period *at date*  $t^-$ , agent  $i$  is born and endowed with one indivisible unit of labor effort that he can only invest in his own project and initial wealth  $w_{it} \in [0, \bar{w}_t] \subseteq \mathbb{R}_+$ . The latter is the only source of heterogeneity among agents. Denote  $G_t(w)$  the cumulative wealth distribution as the measure of agents with wealth less than  $w$  at date  $t$ . Then, aggregate wealth is equal to average wealth and given by  $W_t = \int_0^{\bar{w}_t} w g_t(w) dw$ .

When agents start their risky projects *at date*  $t$ , they need to choose (i) a capital investment  $k$  and (ii) an effort level  $e \in \{0, 1\}$ . There is free access to a production technology  $f(k)$ , which is strictly increasing and concave (i.e.  $f' > 0$ ,  $f'' < 0$ ). It also satisfies the standard INADA conditions (i.e.  $f(0) = 0$ ,  $f'(0) = \infty$ ,  $f(\infty) = \infty$  and  $f'(\infty) = 0$ ). Once sunk,  $k$  cannot be recovered. Output is stochastic at the individual level: with probability  $q$ , the project succeeds and yields an outcome  $f(k)$ , whereas in case of failure, the outcome is 0. Crucially, there is moral hazard in production. By providing effort, which comes at a cost of  $e = 1$ , the entrepreneur can increase the probability of success to  $p$  with  $0 < q < p < 1$ . Given a market rate of return  $r$ , a diligent [resp. a shirking] entrepreneur's expected project profit then amounts to

$$\pi(r) = pf(k) - rk \quad [\text{resp. } \pi_0(r) = qf(k_0) - rk_0], \quad (2.1)$$

so that the optimal investment, henceforth denoted  $k(r)$  [resp.  $k_0(r)$ ], must be such that

$$pf'(k(r)) = r \quad [\text{resp. } qf'(k_0(r)) = r]. \quad (2.2)$$

It follows that  $k(r) > k_0(r)$  for all  $r$ . Let  $y(r)$  [resp.  $y_0(r)$ ] be the expected profit given the optimal investment  $k(r)$  [resp.  $k_0(r)$ ] and assume that at least for  $r = 1$ , a diligent entrepreneur

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to rule out a multiplicity of steady-state equilibria, because they ensure that capital accumulates sufficiently fast to remove any difficulties from persistent credit rationing. Contrariwise, Piketty (1997) and the study at hand also study this case, which gives rise to the second steady-state.

always generates a higher surplus than a shirking one - even if effort costs are properly taken into account.

$$\text{Assumption 1: } y(1) - 1 > y_0(1). \quad (\text{A1})$$

This way, (A1) implicitly defines a crucial  $\bar{r}(q) > 1$ , such that only for all  $r \leq \bar{r}(q)$ , it holds that  $y(r) - 1 \geq y_0(r)$ , i.e. that exerting effort ( $e = 1$ ) is preferred to shirking ( $e = 0$ ). It is the other way round for  $r > \bar{r}(q)$ .<sup>3</sup>

However, if an agent  $i$  owns less [resp. more] than the optimal investment given  $r$ , he may want to raise [resp. lend] the remainder.<sup>4</sup> Besides a costless storage option, there is a perfectly competitive capital market that enables agents to smooth their capital needs. Also considering that agents cannot end up with negative cash-holdings at the end of their lives, yields standard debt contracts in equilibrium. These are characterized by a repayment of  $t_f = 0$  in case of failure and of  $t_s = r[k(r) - w]/p$  for an effort contract [resp.  $t_{s0} = r[k_0(r) - w]/q$  for a non-effort contract] in case of success. Yet, as effort remains unobservable, the lender anticipates that after the effort contract has been signed, a borrowing entrepreneur only provides costly effort, if the incentive constraint holds:

$$(\text{IC}): p[f(k(r)) - t_s] - 1 \geq q[f(k(r)) - t_s]. \quad (2.3)$$

Substituting the repayment schedule uncovers that (2.3) is only satisfied for borrowers who are wealthy enough to make at least an initial contribution of  $\omega(r)$

$$w \geq \omega(r) := p/r(p - q) - [pf(k(r)) - rk(r)]/r. \quad (2.4)$$

This stems from the fact that the more an agent has to borrow in order to reach the efficient scale, the larger the fraction of marginal returns from effort he has to share with his lender and so the lower his incentives to provide costly effort.<sup>5</sup>

At the end of their lifetime *at date*  $t^+$ , agents realize their initiated projects' returns and settle their financial claims. Each agent gives birth to exactly one child, consumes and dies. The initial generation starts out with a scarce aggregate capital endowment of  $W_0 < k_0(1) < k(1)$ , which is distributed according to  $G_0(w)$ . Subsequent generations inherit a fixed fraction  $s$  of their parent's end-of-life wealth, i.e.  $b_{it} = w_{it+1}(w_{it}) = sw_{it}$ . Aggregate wealth in  $t+1$  therefore amounts to  $W_{t+1} = sY_t(G_t(w))$ , where  $Y_t(G_t(w))$  represents the economy's aggregate output in  $t$ .

### 2.2.2 Loosening an Implicit Ad-Hoc Assumption

With firms being technically operable at all capitalization levels  $k > 0$ , there is no reason to make production contingent on entrepreneurs' access to the capital market. That is why we now relax this implicit assumption of Piketty's (1997) and explicitly allow agent  $i$  to also open a fully self-financed firm of size  $k = w_i$ .<sup>6</sup> For this autarkic firm, incentive compatibility

$$(\text{IC}_a): y_a - 1 \geq y_{a0} \quad (2.5)$$

<sup>3</sup>As  $q$  measures the outside option of diligent agents, effort pays off the less, the higher  $q$ , so that  $d\bar{r}(q)/dq < 0$ .

<sup>4</sup>Exclude simultaneous lending and borrowing, since agents cannot gain from it anyway.

<sup>5</sup>Hence, as self-financers of  $k(r)$  stay full residual claimants on all returns from their effort,  $e = 1$  always pays off for them for any given  $r \leq \bar{r}(q)$ .

<sup>6</sup>Because of  $f$ 's concavity, agent  $i$  with  $w_i < k(r)$  will not choose to invest less than  $w_i$ .

yields that diligence only pays off, if the agent  $i$ 's wealth amounts at least to

$$w \geq \tilde{w} := f^{-1}(1/(p-q)). \quad (2.6)$$

## 2.3 Static Equilibrium with Credit Rationing

For simplicity, the analysis is first cast in terms of static considerations. After having derived optimal individual decisions, these will be aggregated to obtain the capital market equilibrium.

### 2.3.1 Optimal Individual Decisions under Moral Hazard

Given the prevailing market rate of return  $r$  and his wealth endowment  $w_i$ , each agent has to decide upon how to feasibly and income-maximizingly allocate his wealth on (i) an entrepreneurial investment (and if necessary adequate borrowing), (ii) lending and (iii) costless storage. The solution to this problem is depicted in *Figure 2.1* and requires to combine two steps of analysis. First, drawing on agents' access to credit expressed in  $\omega(r)$ , Piketty (1997) proves that there is a  $q_0$ , such that for  $0 < q < q_0$ ,  $\omega(r)$  is strictly increasing in  $r$ . Furthermore, he shows that there exists a  $\underline{r}(q) < \bar{r}(q)$  such that if  $r \leq \underline{r}(q)$ , it will be that  $\omega(r) < 0$ , so that all entrepreneurs would deliberately provide effort, irrespective of their initial wealth.<sup>7</sup> Second, we use the concavity of  $\omega(r)$ , the convexity of  $k(r)$  and  $k_0(r)$ , (A1) and  $\tilde{w}$  to determine the relative position of these functions' graphs. Although autarkic firms with varying firm sizes other than the optimal firm sizes  $k(r)$  and  $k_0(r)$  will not materialize in equilibrium, they serve as an anchor, in particular at  $r = \bar{r}(q)$  for deriving  $k_0(\bar{r}) < \omega(\bar{r}) < \tilde{w} < k(\bar{r})$  (also see the Proof of *Lemma 2.2*). That is how three crucial interest rate cut-off levels materialize:  $\underline{r}(q)$ ,  $\bar{r}(q)$  and  $\tilde{r}(q)$ . Piketty (1997) is lacking the last one, which lies at the intersection of  $k_0(r)$  and  $\omega(r)$ .

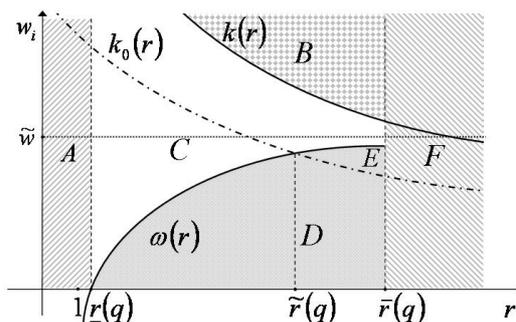


Figure 2.1: Solution to an individual agent's investment problem given  $r$

Thus, optimal individual decisions are as follows: *For any given*  $1 < r < \underline{r}(q)$ ,<sup>8</sup> (2.4)  $\omega(r) < 0$ , so that even agents with no wealth prefer to work diligently (area A). The same is true for

<sup>7</sup>Indeed, with (A1),  $pf(k(1)) - k(1) > p/(p-q)$  for sufficiently small  $q$ , so that  $\omega(r) < 0$  for sufficiently small  $r$ .

<sup>8</sup>For  $r < 1$ , agents would only invest  $k(1)$  [resp.  $k_0(1)$ ] and store any remainder.

self-financers with  $w_i \geq k(r)$  (area  $B$ ). However, if  $r(q) < r \leq \bar{r}(q)$ , the lender must require the borrower to put up  $\omega(r) > 0$  in order to break even. This stake ensures that the borrower faces the proper incentives to be diligent. If the agent has wealth  $w_i \geq \omega(r)$ , he receives the needed funds for the effort investment  $k(r)$  (white area  $C$ ). Whereas, if  $w_i < \omega(r)$ , the lender anticipates that the borrower will shirk. Then, the project becomes unsustainable at the effort scale  $k(r)$  and only guarantees the lender an expected return of  $r$  at the lower non-effort scale  $k_0(r)$ .<sup>9</sup> That is how borrowers with wealth  $w_i \in [0, \omega(r))$  find themselves credit-rationed, i.e. they are not offered effort, but only non-effort contracts (areas  $D$  and  $E$ ).<sup>10</sup> Finally, if  $r > \bar{r}(q)$ , credit repayments become so high that effort-provision does not pay off for any project anymore. No matter how rich they are, all agents invest  $k_0(r)$  and deposit any remainder (area  $F$ ).

**Lemma 2.1** [*Piketty (1997), Proposition 2*](A1) implies that there exists a  $q_0 \in (0, p)$  such that for any  $q \in (0, q_0)$ , there exists  $\underline{r}(q) \in (1, \bar{r}(q))$  such that:

(i) For  $r \leq \underline{r}(q)$ , there is no credit rationing: all agents with  $w_i < k(r)$  obtain sufficient credit to make the first-best optimum investment  $k(r)$ .

(ii) For  $\underline{r}(q) < r \leq \bar{r}(q)$ , there is some credit rationing: there exists  $\omega(r) > 0$  with  $d\omega/dr > 0$ ,  $\omega(r) \rightarrow 0^+$  as  $r \rightarrow \underline{r}(q)^+$ , such that if  $w_i \geq \omega(r)$ , agent  $i$  obtains sufficient credit to make the first-best optimum investment  $k(r)$ . Yet, if  $w_i < \omega(r)$ , agent  $i$  is credit-rationed and only invests  $k_0(r)$ .

(iii) For  $r > \bar{r}(q)$ , all agents with  $w_i < k_0(r)$  obtain sufficient credit to make the first-best optimum investment  $k_0(r)$ .

Going further than Piketty (1997) in explicitly accounting for autarkic production  $k = w_i$ , reveals that two types of credit-constrained agents can exist: net-borrowing and net-depositing ones. Whilst for  $r < \tilde{r}(q)$ , agent  $i$  with  $w_i < k_0(r) < \omega(r)$  prefers to open a shirking firm and to raise any required funds  $k_0(r) - w_i$  through a low-effort contract with  $t_{s0}$  (area  $D$ ), things change for  $\tilde{r}(q) < r \leq \bar{r}(q)$ . Then, agent  $i$  with  $k_0(r) \leq w_i < \omega(r)$  can afford to self-finance the low investment  $k_0(r)$  and prefers to deposit any remainder  $w_i - k_0(r)$  (area  $E$ ). Thus, not only the very rich with  $w > k(r)$  deposit (area  $B$ ), but also credit-constrained agents  $i$  with  $\omega(r) > w_i > k_0(r)$  (area  $E$ ). As this is fundamentally different from Piketty (1997),<sup>11</sup> Lemma 1 must be amended:

**Lemma 2.2** There exists a  $\tilde{r}(q) \in (r(q), \bar{r}(q))$  such that  $k_0(\tilde{r}) = \omega(\tilde{r})$  and  $k_0(r) < \omega(r)$  for  $r > \tilde{r}(q)$ . Then, for any given  $r \in (\tilde{r}(q), \bar{r}(q))$ , agents with  $w_i \in (k_0(r), \omega(r))$  become credit-constrained net lenders. They open autarkic shirking firms of size  $k_0(r)$  and lend  $w_i - k_0(r)$  at rate  $r$ .

<sup>9</sup>To see why (2.3) IC cannot be satisfied for any investment level  $k$  below the first-best level  $k(r)$  recall the fact that an entrepreneur's payoff in case of success is already maximal for the optimal investment. Hence, incentives to shirk increase for any suboptimal investment level.

<sup>10</sup>With first-best information, i.e. if effort was observable, effort firms would also be opened up in areas  $D$  and  $E$ . On top, credit-constrained agents always obtain a non-effort credit contract for investing  $k_0(r)$ , since they cannot further cheat on the lender than  $e = 0$ .

<sup>11</sup>Piketty (1997) explicitly states, e.g. in his footnote 19, that  $k_0(r) > \omega(r)$ .

## 2.3.2 Capital Market Equilibrium

Aggregating individual decisions allows to derive the capital market equilibrium.

**Definition 2.1** *A capital market equilibrium consists of a market rate of return  $r^*$  and individual decisions as described in Lemmata 2.1 and 3.6 such that decisions are optimal given  $r^*$  and gross capital demand  $D(r^*)$  equals supply  $S(r^*)$ .*

While  $S(r)$  is given by total aggregate wealth  $W$ ,  $D(r)$  amounts to the sum of all agents' investments intended at rate  $r$ .<sup>12</sup> On these grounds, it follows:

**Corollary 2.1** *In equilibrium, the market rate of return  $r^*$  and aggregate output  $Y^*$  are as follows:*

(i) *If  $pf'(W) \leq \underline{r}(q)$ , first-best obtains. All agents choose the high investment  $k(r^*) = W$  and exert effort.  $r^*$  and  $Y^*$  only depend on aggregate wealth  $W$ :  $r^* = pf'(W) \leq \underline{r}(q)$  and  $Y^* = pf(W)$ .*

(ii) *If  $qf'(W) > \bar{r}(q)$ , first-best obtains. All agents choose the low investment  $k_0(r^*) = W$  and shirk.  $r^*$  and  $Y^*$  only depend on aggregate wealth  $W$ :  $r^* = qf'(W) > \bar{r}(q)$  and  $Y^* = qf(W)$ .*

(iii) *Otherwise, there is credit rationing  $\omega(r^*) > 0$ . A fraction  $G(\omega(r^*))$  of the agents shirkingly invests  $k_0(r^*)$ , whereas all others diligently invest  $k(r^*)$ .  $r^*$  and  $Y^*$  depend on aggregate wealth  $W$  and its distribution  $G(w)$ :*

$$r^* \in (\underline{r}(q), \bar{r}(q)] \text{ s.t. } W = G(\omega(r^*))k_0(r^*) + [1 - G(\omega(r^*))]k(r^*) \quad (2.7)$$

$$\text{and } Y^* = G(\omega(r^*))qf(k_0(r^*)) + [1 - G(\omega(r^*))]pf(k(r^*)). \quad (2.8)$$

For  $\underline{r}(q) < r^* \leq \bar{r}(q)$ , the impact of hidden information becomes evident: The scope of credit rationing depends on the distribution of wealth among agents and restrains  $D(r^*)$ , which in turn lowers  $r^*$  as well as  $Y^*$  compared to as if there was no problem of moral hazard.<sup>13</sup> The market outcome is constrained Pareto optimal. Aiming at maximizing aggregate output and facing the same information restrictions than the agents, a social planner would also achieve  $Y^*$ .

## 2.4 Impact on the Dynamic Results

After having tracked individual transitions, the evolution of the economy and of the wealth distribution are analyzed.

### 2.4.1 Individual Transitions

In the most interesting case of an equilibrium interest rate in period  $t$  of  $r_t \in (\underline{r}(q), \bar{r}(q)]$ , transitional equations take the form of

$$w_{it+1}(w_{it}) = \begin{cases} s[\gamma + r_t \cdot \max\{0, w_{it} - k(r_t)\}] & \text{for } w_{it} \geq \omega(r_t) \\ s[\gamma_0 + r_t \cdot \max\{0, w_{it} - k_0(r_t)\}] & \text{for } w_{it} < \omega(r_t) \end{cases} \quad (2.9)$$

<sup>12</sup>With  $W < k_0(1)$ ,  $r^* > 1$ , so that the storage option becomes unattractive for all agents.

<sup>13</sup>Also,  $D'(r) = G(\omega(r))k'_0(r) - [k(r) - k_0(r)]G'(\omega(r))\omega'(r) + [1 - G(\omega(r))]k'(r) < 0$ .

with

$$\gamma = \begin{cases} f(k(r_t)) - r_t \cdot \max\{0, [k(r_t) - w_{it}]/p\} & \text{with probability } p \\ 0 & \text{with probability } (1-p) \end{cases}$$

$$\gamma_0 = \begin{cases} f(k_0(r_t)) - r_t \cdot \max\{0, [k_0(r_t) - w_{it}]/q\} & \text{with probability } q \\ 0 & \text{with probability } (1-q) \end{cases}$$

They are concave by  $f$ 's concavity.<sup>14</sup> Allowing for the emergence of credit-constrained net lenders (i.e. for  $\tilde{r} < r < \bar{r}(q)$ , agents with  $k_0(r_t) < w_i < \omega(r_t)$  in area  $E$  in *Figure 2.1*), entails that transit functions must take into account agents' safe capital incomes and therefore differ from Piketty's (1997).

### 2.4.2 Steady-States

The individual wealth trajectories (2.9) and the equilibrium interest rate schedule (2.7) establish a non-linear aggregate transition function  $G_{t+1}(G_t)$ . Kicked off by  $G_0$ , Piketty (1997) proves that the resulting infinite sequence of  $G_t$  and  $r_t$  approaches these variables' long-run steady-state values. Indeed, with (A1),  $q \in (0, q_0)$  and by  $f$ 's concavity, for any possible long-term interest rate  $r_\infty \in [1, \bar{r}(q)]$ , individual transitions (2.9)  $w_{it+1}(w_{it})$  determine a linear, globally ergodic Markov process, that converges towards a unique stationary wealth distribution  $G_{r_\infty}(w)$ .<sup>15</sup> Then,  $r_\infty$  is a long-term steady-state interest rate of the dynamic system  $(G_{t+1}(G_t), r_t = r(G_t))$ , only if it equals the equilibrium interest rate  $r(G_{r_\infty})$  coming along with its stationary distribution  $G_{r_\infty}(w)$ . Notice that with  $0 < q < p < 1$ , project outcomes' indeterminacy will prevent inequality from ever vanishing.<sup>16</sup>

On this note, two kinds of steady-state obtain: one without and one with persisting credit rationing, henceforth labelled  $(G_{r_\infty}^u(w), r_\infty^u)$  and  $(G_{r_\infty}^c(w), r_\infty^c)$  respectively. As compared to Piketty (1997), the analysis at hand does not have to alter the first in substance, whereas the contrary is true for the second.

#### 2.4.2.1 The No-Credit-Rationing Steady-State

Piketty (1997) shows that there exists an aggregate wealth saving propensity  $s_0 = s_0(q)$ , such that  $s \geq s_0$  together with (A1) and  $q \in (0, q_0)$  ensure that if  $r_0 \leq \bar{r}(q)$ ,  $r_t$  will also remain below  $\bar{r}(q)$ . Whereas if  $r_0 > \bar{r}(q)$ ,  $r_t$  will only temporarily remain above  $\bar{r}(q)$ . Consequently, wealth accumulates over periods, permanently putting downward pressure on  $r_t^*$ . This progressively shifts lending terms in favor of a smaller and smaller pool of borrowers. It also attenuates credit rationing little by little, so increasing the share of borrowers that provide effort and make the high investment  $k(r)$ . If thereby, the interest rate (transitionally) materializes above  $\tilde{r}(q)$ , credit-constrained net lenders emerge. But as it is straightforward to see from (2.9)

<sup>14</sup>For  $r_t \leq \bar{r}(q)$  [resp.  $r_t > \bar{r}(q)$ ], when all agents are diligent and invest  $k(r_t)$  [resp. shirk and invest  $k_0(r_t)$ ], the transitional function reduces to the upper [resp. lower] branch.

<sup>15</sup>If it was that  $\omega(1) > 0$ , the map from current wealth to future bequests would be non-monotone, ruling out the application of Hopenhayn and Prescott's (1992) findings.

<sup>16</sup>Yet, (2.9)  $w_{it+1}(w_{it})$ 's concavity implies that family lineages are not trapped, but can hop between any two possible wealth levels with positive probability in a finite time.

$w_{it+1}(w_{it})$ , their safe interest incomes only account for a faster realization of the steady-state, but leave Piketty's steady-state unaffected in essence. This allows to restate a combined version of Piketty's (1997) *Proposition 1* and *3*:<sup>17</sup>

**Proposition 2.1** *With (A1),  $q \in (0, q_0)$ ,  $s \geq s_0$ , but irrespective of  $G_0(w)$ , there is a unique credit-unconstrained steady-state of the dynamic system  $(G_{t+1}(G_t), r_t = r(G_t))$  with  $G_r(\omega(r_\infty^u)) = 0$  and  $r_\infty^u = r(G_{r_\infty^u}^u) \in [1, r(q)]$  associated to  $G_{r_\infty^u}^u(w)$ . More precisely,  $r_\infty^u = pf'(W_\infty^u)$ ,  $Y_\infty^u = pf(W_\infty^u)$  and  $W_\infty^u = sY_\infty^u$ .*

### 2.4.2.2 The Credit Rationing Steady-State

However, Piketty (1997) claims the coexistence of a second steady-state for the same parameter values, that may arise if the economy does not grow sufficiently quickly to drive interest rates down to  $r_\infty \in [1, r(q)]$ , where  $G_r(\omega(r_\infty)) = 0$ . He obtains that compared to this no-credit-rationing steady-state  $(G_{r_\infty}^u, r_\infty^u)$ , this second steady-state  $(G_{r_\infty}^c, r_\infty^c)$  is associated with a higher interest rate  $r_\infty^c > r(q) > r_\infty^u$ , a dominant stationary distribution  $G_{r_\infty}^c(w) > G_{r_\infty}^u(w)$  and a lower aggregate capital stock  $W_{r_\infty}^c < W_{r_\infty}^u$ . Most importantly, it involves a persistently positive fraction  $G_r(\omega(r_\infty^c))$  of credit-constrained agents and only arises if two conditions simultaneously hold:

1. The steady-state fraction of credit-constrained borrowers  $G_r(\omega(r))$  increases sufficiently in the interest rate  $r$ :  $dG_r(\omega(r))/dr > 0$ . There are two ambiguous effects at play. On the one hand, higher  $r$  lead to an *interest-income effect*: they make it more unlikely to fall in the credit rationing interval, since rich net lenders have high interest incomes even if their investment projects fail. On the other hand, higher  $r$  cause a *credit-constraint effect*: they make it more difficult to escape the credit rationing interval  $(0, \omega(r))$ , since  $\omega(r)$  is higher and since, as Piketty (1997) claims, credit-constrained agents are net borrowers, i.e.  $k_0(r) > \omega(r)$ . Therefore, the net effect of higher  $r$  on  $G_r(\omega(r))$  is only positive, if the second effect prevails, i.e. if higher  $r$  depress the rich's downward mobility less severe than the poor's upward mobility.
2. The equilibrium interest rate  $r^*$  rises in the fraction of credit-constrained agents  $G(\omega(r^*))$ :  $dr^*/dG(\omega(r^*)) > 0$ . For this to be true, an additional assumption has to be made (in the form of (A2):  $f(k)/kf'(k)$  increases with  $k$ ) and a critically low level of  $q$  has to be derived (henceforth denoted  $q_1$  with  $q_1 < q_0$ ), below which capital accumulation is sufficiently perturbed for  $r^*$  being increasing in  $G(\omega(r^*))$ .<sup>18</sup>

Taking a closer look at these two conditions in the light of *Lemma 2.2* reveals that while the fulfillment of the second remains unaffected by the emergence of credit-constrained net

<sup>17</sup>Note that this steady-state is the same than under first-best (i.e. when effort was observable, removing the need for credit collateralization).

<sup>18</sup> $d(f(k)/kf'(k))/dk > 0$  ensures that the average product of capital of unconstrained agents is higher than that of constrained:  $pf(k(r))/k(r) > qf(k_0(r))/k_0(r)$ . This in turn assures that the fact that the constrained agents accumulate less than the unconstrained (i.e.  $sqf(k_0(r)) < spf(k(r))$ ) dominates the fact that they also demand less  $k_0(r) < k(r)$ , so that after all, a higher fraction of credit-constrained agents indeed pushes up the equilibrium interest rate. Also see the first part of Piketty (1997)'s Proof of his *Proposition 4*.

lenders, that of the first troubles. The reason for this is that although  $k_0(r) > \omega(r)$  is the crucial element in Piketty's (1997) argumentation,<sup>19</sup> we find it to only be valid for  $r < \tilde{r}(q)$  and rather the contrary for  $\tilde{r}(q) < r < \bar{r}(q)$ . Consequently, Piketty's (1997) dynamics are valid up to  $r < \tilde{r}(q)$  and reverse thereafter, because then, the *interest-income effect* will dominate the *credit-constraint effect*. Intuitively and as contrasted in *Figure 2.2*, the existence of credit-constrained net lenders adds a third branch in the middle of individual trajectories. That is how the the lineages' wealth accumulation process is accelerated, which in turn reduces the number of consecutive periods until family lineages finally reach the critical threshold  $\omega(r)$ .

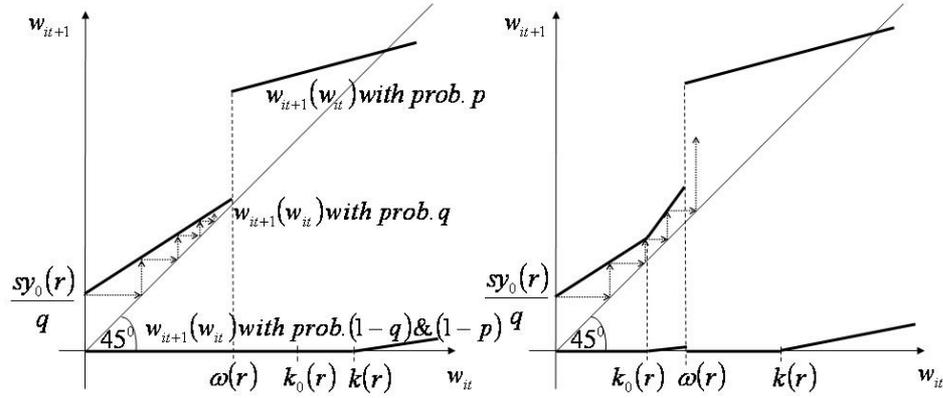


Figure 2.2: Individual transitions with credit rationing for  $\underline{r}(q) < r < \tilde{r}(q)$  (left); with credit rationing and credit-constrained net lenders for  $\tilde{r}(q) \leq r \leq \bar{r}(q)$  (right)

Against this background, we can immediately infer that the steady-state interest rate and the associated share of credit-constrained agents have an upper limit. Even if the parameter constellation was such that the economy started out at  $r^* \in (\underline{r}(q), \tilde{r}(q))$ , where  $k_0(r) > \omega(r)$ , and  $q < q_1$ , the economy would only temporarily be set on the path of increasing interest rates. As soon as  $r^* \in (\tilde{r}(q), \bar{r}(q))$ , it follows that  $k_0(r) < \omega(r)$ , triggering decreasing interest rates. Thus, the dynamic system converges to  $r_\infty^c \leq \tilde{r}(q)$  and its corresponding  $G_{r_\infty^c}^c(w)$ . This also implies that in contrast to Piketty's (1997) predictions, the steady-state fraction of credit-constrained agents  $G_r(\omega(r))$  cannot converge towards 1.<sup>20</sup> All in all:

**Proposition 2.2** *With (A1) and (A2),  $q \in (0, q_0)$ , there is a credit-constrained steady-state. It is located at  $r_\infty^c$  with  $r_\infty^c \in (\underline{r}(q), \tilde{r}(q)]$  and associated with  $G_{r_\infty^c}^c(w)$ . It involves a persistent fraction of credit-constrained agents  $G_r(\omega(r_\infty^c)) < 1$ ,  $Y_\infty^c < Y_\infty^u$  and  $W_\infty^c < W_\infty^u$ .*

<sup>19</sup>See the last but one paragraph in Piketty (1997)'s Proof of his *Proposition 5*. If it was always hurt, so that  $k_0(r) < \omega(r)$  holds, he points out himself in footnote 19 that this would destroy the second steady-state and restore uniqueness.

<sup>20</sup>Although  $k_0(r) \rightarrow 0$  for  $q \rightarrow 0$  prevents credit-constrained dynasties from accumulating much wealth from low-investment production, those with  $w_i \in (k_0(r), \omega(r))$  earn the more safe capital income from lending as soon as  $r \in (\tilde{r}, \bar{r}(q))$ , in turn accelerating the achievement of the cut-off level  $\omega(r)$ . Note that for  $w_i \in (k_0(r), \omega(r))$ , especially low  $q$  ( $d[w_i - k_0(r)]/dq < 0$ ) and high  $r$  ( $dr[w_i - k_0(r)]/dr > 0$ ) reinforce this effect, so that  $G_r(\omega(r))$  cannot converge to 1.

## 2.5 Conclusion

This paper shows that relaxing Piketty's (1997) implicit ad-hoc assumption of a non-existing autarkic production option reveals the existence of credit-constrained net lenders for sufficiently high interest rates in equilibrium. Although higher interest rates make the poor suffer from a lower project profit (due to increased credit rationing and tightened financing conditions), they also make them benefit from a higher capital income. It is this latter countervailing effect that fundamentally alters dynamics and prevents the credit rationing steady-state from becoming as severe as predicted by Piketty (1997).

Yet it only is the accurate knowledge of the evolution of aggregate wealth and its distribution that allows the derivation of policy recommendations. The most obvious starting-point thereby offers the crucial return rate  $\tilde{r}(q)$  that this paper derives as the upper market rate of return that the bad steady-state converges to. It is straightforward that policy measures that reduce the level of  $\tilde{r}(q)$  will foster long-term growth and occupational opportunities across all wealth levels. More concretely, the government could levy a profit tax on credit-constrained entrepreneurs (i.e. on all shirking and autarkic entrepreneurs) at the end of period  $t$  out of which it pays a lump sum transfer to all agents at the beginning of period  $t + 1$ . The tax would scale back the optimal non-effort investment and induce credit-constrained agents to increase their savings. These earn a save remuneration and allow credit-constrained lineages to quicker accumulate wealth over time than by risky non-effort entrepreneurship alone. That is how they can faster comply with the wealth requirement and open less risky, higher return effort firms. After all, aggregate output as well as agents' incomes are fuelled and inequality is diminished.  $\tilde{r}(q)$  will be shifted to the left, which leaves the bad steady-state substantially improved.

However, maximizing aggregate output and thus growth might not always be a desirable economic objective. As the next Chapter shows, it might come at the cost of an inefficient use of aggregate effort.

## Chapter 3

# The Impact of Banking Market Power on Credit Rationing, Efficiency and Income

### 3.1 Introduction

Many economists have tried to look into the origins and the impact of wealth and income inequalities. Although neoclassical models typically leave no room for the marginal product of an individual in an occupation to reflect any endowment effects, Frank Knight already pointed out in 1923, that *"ownership of personal or material productive capacity is based upon a complex mixture of inheritance, luck and effort, probably in that order of relative importance"*. This model features all three factors -inheritance, luck and effort- but will focus on if this causal trilogy must be amended by banking market power.<sup>1</sup>

For this purpose, this paper reverts to a closed capital market model. Agents are heterogeneous with respect to their initial wealth. They seek to raise funds in order to capitalize a firm operating with a convex technology. Yet, due to effort being an invisible, but crucial production input, credit is fraught with moral hazard. It forces lenders to offer incentive-compatible contracts. As these involve a collateral requirement, poor agents are denied full credit.<sup>2</sup> Allocative inefficiency is hampered, which in turn dampens the real interest rate, incomes, output and ultimately growth. However, this static version of Piketty's (1997) reformulation of Solow's (1956) seminal growth model will be crucially amended - above all by targeting aggregate sur-

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<sup>1</sup>I thank Thorsten Beck, Bruno Biais, Paolo Fulghieri, Hans-Peter Grüner, Martin Peitz, Elisabeth Schulte-Runne, Kenichi Ueda, Ernst-Ludwig von Thadden, Evguenia Winschel and conference participants at the Annual Congress of the European Economic Association in Budapest, the Spring Meeting of Young Economists in Hamburg, the Annual Meeting of the Scottish Economic Society in Perth, the ENTER Jamboree in Brussels and seminar participants at the University of Mannheim for valuable discussions and helpful comments.

<sup>2</sup>While it takes an inheritance of USD10.000 to double a Brit's chance to start a business (see Blanchflower and Oswald, 1998), the poor in developing countries are often entirely excluded from the credit market (see Laffont and Matoussi, 1995; Sial and Carter, 1996).

plus as the sum of agents' incomes and by a variation of the competitive structure of the jointly modelled input (deposit) and output (loan) market of financial intermediaries. That is how the asymmetry of information literature is integrated into the literature on the effect of the banking market structure on the determinants of firms' access to finance.<sup>3</sup>

Besides a comprehensive comparison of the efficiency properties of various banking regimes, this paper produces some original results. First, even under perfect information, a coordination failure among borrowers deters the market outcome from achieving first-best efficiency. Second, adding asymmetric information might then be efficiency-enhancing, as it entails credit rationing, which countervails excessive aggregate effort provision and so contains aggregate costs of the unobservable production input. Third, the interplay of banking market power and inequality matters for efficiency. Credit rationing is not a generic property of credit markets hampered by moral hazard and limited liability. Its occurrence and scope hinge upon the market structure, which determines a bank's ability to extract enough surplus from a contract to break even. Consequently, simple deposit or loan market power aggravate credit rationing, firm size heterogeneity and deadweight losses. This not only owes to higher loan costs, but also to a return rate wedge that induces the creation of suboptimally capitalized autarkic firms. Thereby, the Herfindahl index turns out to be an incomplete summary measure of bank competition. Higher inequality decimates simple market power and lets the outcome tend towards that under competition. Fourth, discriminating market power alleviates efficiency losses, since it nullifies the return rate wedge. While a discriminating monopsony restores the competitive outcome, a discriminating monopoly still keeps to higher rationing. Preserving all marginal project returns, it seeks to avoid excessive effort provision among the firms it finances and might so ameliorate the competitive outcome. Fifth, unlike all other banks, a discriminating double-sided monopoly (henceforth called monemporist<sup>4</sup>) further improves on the competitive outcome and might the likeliest restore first-best efficiency. It does so, if a capital reallocation benefit outweighs higher costs of poor borrowers' effort contracts. This benefit arises from smoothing firm sizes in the presence of a decreasing marginal product of capital. Intuitively, the result stems from the fact that competition prevents banks from apportioning contracts, from accounting for the reallocation benefit and from internalizing effort firms' pecuniary externality on firms' effort decision. Instead, banks are coerced into lending indiscriminately to all entrepreneurs above a certain wealth level. Sixth, double-sided competition might lead to monopoly outcomes at the benefit of depositors.

All in all, this paper contributes a new explanation for the predominance of uncompetitive banking markets in environments characterized by high informational frictions and low as well as unequally distributed pledgeable wealth. Think e.g. of local moneylenders in developing countries, credit unions in China and in the West at the beginning of the Industrial Revolution,<sup>5</sup> banking branch regulation in the U.S. up to the 1970s or the rather recent emergence

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<sup>3</sup>We, however, omit monitoring, screening and relationship banking as hitherto drivers of any positive relationship between banking power and access to finance.

<sup>4</sup>The term goes back to Nichol (1943).

<sup>5</sup>Hicks (1969) and North (1981) put down the start of the Industrial Revolution to the ability of a more developed financial system to finally overcome large-scale investment requirements for using technologies that had already been known for some time.

of incubators in the New Economy. As this paper studies the outcome in a variety of capital market regimes, it also implicitly reviews regulators' and policy makers' potential to mitigate the macroeconomic impact of inequality on the economic performance by their choice of an appropriate banking market regime and by the design of distributive policies.

The remainder of this paper is structured as follows. Section 3.2 reviews the literature, before Section 3.3 outlines the basic model. Against the first-best case in Section 3.4, Section 3.5 assesses the impact of informational and competitive frictions. Section 3.6 summarizes implications for banking regime design and policy making. Finally, Section 3.7 concludes. All Proofs are in the Appendix.

## 3.2 Literature Review

This paper merges two, until now separated branches of the literature.

The first deals with the **macroeconomic impact of inequality on capital markets**. It arises from two key ingredients: an up-front, sunk investment<sup>6</sup> and a problem of asymmetric information. Thereby, the project outcome and hence repayment either depend on borrowers' labor input (moral hazard; see e.g. Aghion and Bolton, 1997; Piketty, 1997; or Grüner, 2001), individual ability (adverse selection; see e.g. Jaffee and Stiglitz, 1990; or Grüner, 2003) or level of physical output (see e.g. Banjee and Newman, 1993; or Galor and Zeira, 1993). Although most of these models assume a fixed-size capital outlay and a given interest rate, this paper follows Piketty (1997) in that capitalization levels are chosen optimally with respect to an endogenously determined return rate. This allows to separate the effects of credit market imperfections from the supplementary assumption of non-convex technologies. Then, especially the poor become entrepreneurs and firms can be operated at all scales - in contrast to Piketty (1997) also with own wealth only. This can give rise to equilibria with autarkic entrepreneurs and credit-constrained net lenders. Furthermore, while most previous contributions' focus lies on aggregate output, this paper's lies on aggregate surplus. It better captures productive efficiency and agents' income situation, since it fully takes into account the costs of both, visible and invisible inputs.

Aiming at mitigating the impact of inequality on efficiency, two remedying policies have widely been put forward. One centers on political wealth redistribution. As shown by e.g. Grüner (2003) or Grüner and Schils (2007), the equalization of opportunities across households can improve both, efficiency and equity. This paper uncovers new applications for distributive measures. The other corrective policy calls on governments to foster informal financial institutions because of their ability to use monitoring and sanctions that guarantee unsecured loans; or to run NGO microfinance projects (such as the Garmeen Bank of Bangladesh and Banco Sol of Bolivia). These solve many of the informational and enforcement problems through endogenous group formation and peer monitoring (see e.g. Morduch, 1999; Besley and Coate, 1995; Ghatak, 1999; or Ghatak and Guinnane, 1999). This paper, instead, challenges a typical assumption of hitherto models: perfect competition among banks for depositors and borrowers. Banking market design then emerges as a novel policy tool.

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<sup>6</sup>Like e.g. for the acquisition of education, training, patents, licences, land or machinery.

On the empirical front, Kuznets (1955) was the first to observe that in the course of development, inequality was first rising and then falling. While there is meanwhile consensus on inequality causally impacting on the economic performance (see e.g. Deininger and Squire, 1998), a controversial debate still runs on the sign and significance of the relationship (also see the surveys by Benabou, 1996; Ferreira, 1999; and Perotti, 1996 and newer contributions such as e.g. Banerjee and Duflo, 2005 or Brückner, Gerling and Grüner, 2007).

The second strand of the literature remains undecided on the **effects of banking market power on access to credit**. Some models derive a positive or nonlinear link based on information asymmetries and agency problems. Marquez (2002) shows how efficient borrower screening vanishes with competition, since it makes borrower-specific information more dispersed. In Cetorelli and Peretto (2000), an informational externality generates a free-riding problem that leads to a trade-off: the fewer the number of banks, the greater the incentives for banks to incur screening costs and, hence, the larger the proportion of funds that is efficiently allocated to high-quality borrowers. Dinç (2000) confirms a nonlinear relationship. He analyzes how the degree of competition impacts on the bank's incentive to keep its commitment with a borrower when his credit quality worsens. Petersen and Rajan's (1995) monopolist also faces more incentives to establish long-term lending relations with young borrowers. This owes to the monopolist's ability to intertemporally share in profits and thus risk. But for all that, e.g. Pagano (1993) or Guzman (2000) still obtain that monopoly's drawbacks prevail (i.e. the reduction of quantities and appropriation of the profit). In contrast, whilst neglecting monitoring and screening, this paper shifts attention to inequality and its interplay with banking market power. In a similar, though only bilateral setting with a fixed investment size, Malavolti-Grimal (2001) receives the same level of credit rationing for a competitive and a monopoly regime. She, however, fails to model the deposit side and her monopolist takes as given a standard two-tools debt contract. Yet, when she introduces a probability of financing as an additional contract tool (which facilitates the satisfaction of the borrower's participation constraint and exacerbates that of the lender's), a monopolist might also ration credit more than a competitive bank. In this paper, such conclusions will depend on the wealth distribution, which also entails that agents get ranked according to their wealth. Although for adverse selection and an exogenous loan size, such a ranking also materializes in Besanko and Thakor (1987). Their monopoly bank does not use collateral as a signal of credit-worthiness, unless it is sufficiently valuable to make the loan riskless. It rather sorts borrowers by setting the interest rate such that the bad type leaves the market.

Also empirical evidence is ambiguous. A positive link is found by e.g. Petersen and Rajan (1995), Bonaccorsi and Dell'Ariccia (2000), Cetorelli and Gambera (2001), whereas anecdotal support comes from e.g. Gerschenkron (1965) and Mayer (1990) for big German and Japanese banks. Contrariwise, a negative link is uncovered by e.g. Black and Strahan (2002), Shaffer (1998), Berger, Kashyap, and Scalise (1995), Cetorelli and Gambera (2001) or King and Levine (1992, 1993). Beck et al. (2004) only confirm a negative link for countries with low levels of economic and institutional development. Rather complementary, e.g. Beck et al. (2000) or Rajan and Zingales (1998) find broader and deeper financial markets to be strongly associated with the economic performance. In the light of this paper's findings, however, these empirical results

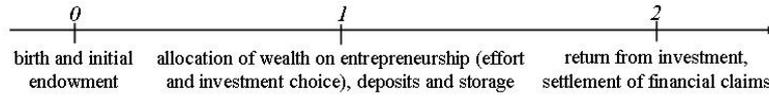
might be heavily biased. The reason is that they completely neglect inequality, discriminating market power and the allocation of bank profits.

### 3.3 The Model

Consider a closed economy populated with a continuum of agents  $i \in [0, 1]$ .

#### 3.3.1 Agents, Endowments and the Sequence of Events

Risk-neutral agents live for one period and maximize total expected income  $u_i$ .



At *date 0*, agents are born as prospective entrepreneurs with initial wealth  $w_i$ , access to the same production technology and one indivisible unit of labor effort that they can only invest in their own one-man business. At *date 1*, agents decide upon how to optimally allocate their wealth on (i) an entrepreneurial investment  $k$ , (ii) interest-bearing deposits  $w_D$  with a bank and (iii) costless storage  $w_S$ . The capital market opens, enabling them not only to deposit funds with, but also to raise further funds from the bank. Entrepreneurs start a risky and either (i) self-financed, (ii) leveraged or (iii) autarkic project, for which they have to choose a level of capitalization  $k$  and of effort  $e \in \{0, 1\}$ . At *date 2*, the returns of the initiated investment projects are realized and financial claims are settled.

#### 3.3.2 Technology and the Entrepreneur's Problem

The technology  $F(K, L)$  exhibits constant returns to scale with respect to aggregate capital and labor inputs  $K$  and  $L$ . With only potential one-man firms, production can be studied at the per-capita level:  $f(k) = F(K/L, 1)$  is strictly increasing and concave in the capital-labor ratio  $k = K/L$  (i.e.  $f' > 0$ ,  $f'' < 0$ ). It also satisfies the standard INADA conditions (i.e.  $f(0) = 0$ ,  $f'(0) = \infty$ ,  $f(\infty) = \infty$  and  $f'(\infty) = 0$ ). Once sunk,  $k$  is not recoverable anymore.

We introduce idiosyncratic risk by allowing the output to be stochastic at the individual level: with a probability of  $q$ , the project succeeds and yields an outcome  $f(k)$ ; whereas if it fails, the outcome is 0. At a cost of  $e = 1$ , effort provision increases the probability of success from  $q$  to  $p$  with  $0 < q < p < 1$ .

Before opening a firm, the entrepreneur must decide on  $k$  and  $e$ . For a start, consider a financially unconstrained and diligent [resp. shirking]. Taking the market rate of return  $r$  as given and with an expected project profit of

$$pf(k) - rk \quad [\text{resp. } qf(k_0) - rk_0], \quad (3.1)$$

he chooses the optimal investment  $k$ , henceforth labelled  $k(r)$  [resp.  $k_0(r)$ ],

$$\text{such that } pf'(k(r)) = r \quad [\text{resp. } qf'(k_0(r)) = r]. \quad (3.2)$$

Likewise,  $y(r)$  [resp.  $y_0(r)$ ] denotes the expected profit given  $k(r)$  [resp.  $k_0(r)$ ].  $k(r)$  and  $k_0(r)$  are differentiable, strictly decreasing and convex functions in  $r$ . For all  $r$ ,  $k(r) > k_0(r)$  and  $k(r)$  steeper than  $k_0(r)$ . In order to make the model work, we must assume that at least for  $r = 1$ , diligence generates a higher surplus than shirking - even if effort costs are properly taken into account:

$$\text{Assumption 1: } y(1) - 1 > y_0(1). \quad (\text{A1})$$

This way, (A1) implicitly defines a crucial  $\bar{r} > 1$ , such that  $y(r) - 1 \geq y_0(r)$  holds for all  $r \leq \bar{r}$ . Only then exerting effort ( $e = 1$ ) is individually preferable to shirking ( $e = 0$ ). It reverses for  $r > \bar{r}$ .<sup>7</sup> Hence:

**Lemma 3.1** *Given a market rate of return  $r \geq 1$ , a **financially unconstrained entrepreneur** behaves as follows:*

- (i) *If  $r \leq \bar{r}$ , he provides effort and invests  $k^* = k(r)$  s.t.  $pf'(k(r)) = r$ .*
- (ii) *If  $r > \bar{r}$ , he shirks and invests  $k^* = k_0(r)$  s.t.  $qf'(k_0(r)) = r$ .*

### 3.3.3 Aggregate Economy

Individual wealth  $w$  is the only source of heterogeneity among agents. It is continuously distributed on  $[0, \bar{w}] \subseteq \mathbb{R}_+$  according to  $g(w)$ .  $G(w)$  then represents the cumulative wealth distribution as the measure of agents with wealth less than  $w$ . And aggregate wealth  $W = \int_0^{\bar{w}} wg(w) dw$  equals average wealth. For simplicity, we assume that capital is scarce by letting  $W$  be smaller than the aggregate of entrepreneurs' intended investments if capital costs were zero:

$$\text{Assumption 2: } W < k_0(1) \leq k(1) \quad (\text{A2})$$

The economy's aggregate surplus  $U$  is the sum of agents' total expected incomes  $u_i$ , which consist of deposit remunerations and project profits, plus eventual bank profits. Thus,  $U$  also equals aggregate output  $Y$  (i.e. the sum of agents' outputs) minus aggregate effort costs.

### 3.3.4 Capital Market

There is a capital market that allows agents to smooth their capital needs given a deposit rate  $r_D$  and a loan rate  $r_L$ .

#### 3.3.4.1 Information and Financial Intermediation

While effort provision  $e$  remains an entrepreneur  $i$ 's private information, his initial wealth  $w_i$ , project input  $k_i$  and output ( $0$  or  $f(k_i)$ ) can be verified using a specific screening technology.<sup>8</sup> Access to this technology is prohibitively costly for individuals, but not for banks. That is why banks intermediate the supply of credit  $S(r_D)$  and the demand for credit  $D(r_L)$ . Banks are

<sup>7</sup>Effort pays off less, the higher the outside option of diligent agents, i.e.  $d\bar{r}/dq < 0$ .

<sup>8</sup>Wealth is likewise unobservable in e.g. Malavolti-Grimal (2001) and output costly to verify in e.g. Townsend (1979). Alternatively, banks can also arise for diversifying idiosyncratic risk on behalf of risk-averse agents (as e.g. in Townsend and Ueda, 2006).

risk-neutral and do not have funds of their own or operation costs.<sup>9</sup> They transform deposits into loans under different banking market regimes, each described in detail below.

### 3.3.4.2 Financial Contracts

On the deposit side, banks offer deposit contracts  $C_D(w) = (r_D)$  to agents with wealth  $w$ . With idiosyncratic, independent shocks cancelling out on the loan side,  $C_D(w)$  specifies a safe deposit rate  $r_D$  per unit lent and leaves depositor  $i$  with  $R(w)$ . A bank  $j$  earns an intermediation profit of  $\Pi_{D,j} = r_L - r_D$ .

On the loan side, banks offer two-tools credit contracts  $C_L(w) = (w_c, t)$  to agents with wealth  $w$ . To be financed, borrowers have to pledge an initial equity contribution  $w_c$ .<sup>10</sup> After project completion, they receive a transfer of  $t$  in case of success and nothing in case of failure.<sup>11</sup>  $t$  is a function of  $r_L$  and  $w_i$ .

A bank  $j$ , however, is only active if it makes non-negative profits with each single deposit and loan contract (which is a much stronger requirement than from across all contract partners as e.g. in Jaffe and Russel, 1976):

$$(IR_D^B): \Pi_{D,j} \geq 0 \text{ and} \quad (3.3)$$

$$(IR_L^B): \Pi_{L,ji} \geq 0 \text{ [resp. } \Pi_{L,ji0} \geq 0]. \quad (3.4)$$

Note that (3.3)  $IR_D^B$  entails  $r_D \leq r_L$ . Moreover, the deposit and the loan market are interrelated through an availability constraint, ensuring that bank  $j$  does not lend out more funds than it acquires:

$$(AC^B): S_j(r_D) \geq D_j(r_L). \quad (3.5)$$

### 3.3.4.3 Firm Types and Contractual Constraints

No agent can increase his lifetime income by simultaneously depositing with and borrowing from the bank (also see Proof of *Lemma 3.3*). We therefore emanate from all agents considering to approach one side of the capital market only.

The deposit rate  $r_D$  is the reference rate for a self-financing and an autarkic entrepreneur. In contrast to the first (whose behavior is described by *Lemma 3.1*), the second cannot afford to self-finance the optimal investment given  $r_D$ . Either refusing or being denied access to credit, he uses his wealth to open a suboptimally capitalized firm of size  $k_{a,i} \leq w_i$ . This makes him earn  $y_a - 1 = pf(k_{a,i}) - r_D k_{a,i} - 1$  with effort [resp.  $y_{a0} = qf(k_{a0,i}) - r_D k_{a0,i}$  without]. An autarkic agent  $i$  prefers to provide effort if

$$(IC_a): y_a - 1 \geq y_{a0}, \quad (3.6)$$

<sup>9</sup>As a by-product, Section 3.5.2 also implies the outcome with positive operation costs.

<sup>10</sup>Like e.g. in Holmström and Tirole (1997), collateralizable wealth is modelled as "cash", which is netted out at contract conclusion. Such an equity participation is neither asymmetrically valued by the contracting parties (as e.g. in Barro, 1976), nor a non-liquid fixed asset against which an agent borrows the entire set-up costs (as e.g. in Buraschi and Hao, 2004).

<sup>11</sup>This reward scheme represents the toughest punishment for a shirking borrower, who is protected by limited liability (i.e. who cannot end up with negative cash holdings at date 2). The course of initial output appropriation is innocuous with view to the results.

which holds if  $r_D \leq \bar{r}$  and if he owns at least  $k_{a,i} \geq \tilde{w} := f^{-1}(1/(p-q))$ . Otherwise, the net profit generated from investing  $w_i$  does not cover moral hazard costs  $1/(p-q)$ . Remark that at  $r_D = \bar{r}$ ,  $k_0(\bar{r}) < \tilde{w} < k(\bar{r})$ . Altogether:

**Lemma 3.2** *An autarkic entrepreneur is an agent  $i$  with wealth less than the optimal investment given  $r_D \geq 1$ , who behaves as follows:*

(i) *If  $r_D \leq \bar{r}$  and  $w_i \geq \tilde{w} := f^{-1}(1/(p-q))$ , he provides effort and invests his entire wealth i.e.  $k_{a,i} = w_i < k(r_D)$ .*

(ii) *Otherwise, he shirks and invests  $k_{a0,i} = \min\{w_i, k_0(r_D)\}$ .*

Given the loan rate  $r_L$ , instead, a borrowing entrepreneur  $i$  accepts bank  $j$ 's effort credit contract  $C_L(w)$  [resp. non-effort credit contract  $C_{L0}(w)$ ] in order to optimally capitalize a firm:  $k_L(r_L)$  if  $e = 1$  [resp.  $k_{L0}(r_L)$  if  $e = 0$ ]. It leaves them with expected net profits of

$$v_i = pt - r_D w_c - 1 \quad (3.7)$$

$$\text{and } \Pi_{L,j,i} = p[f(k_L(r_L)) - t] - r_D[k_L(r_L) - w_c] \quad (3.8)$$

[resp.  $v_{0i} = qt_0 - r_D w_{c0}$  and  $\Pi_{L0,j,i} = q[fk_{L0}(r_L) - t_0] - r_D[k_{L0}(r_L) - w_{c0}]$ ]. When designing financial contracts, banks have to respect three sets of constraints. First, there is incentive compatibility (IC). As borrowers can deliberately, but invisibly elevate the success probability by working hard, banks must specify loan transfer payments such that the net surplus is higher with effort than without:

$$(IC): pt - 1 \geq qt. \quad (3.9)$$

Thus, only a transfer of  $t \geq 1/(p-q)$ , covering at least moral hazard costs, induces post-contractual effort. Second, in order to make agents participate, banks must guarantee them at least the net profit of their outside option. The individual rationality constraints of the depositor ensure that depositing yields more than simple storage ( $IR_{D1}$ ) and autarkic production ( $IR_{D2}$ ). Those of the leveraged entrepreneur guarantee that borrowing is more profitable than depositing ( $IR_{L1}$ ) and autarkic production ( $IR_{L2}$ ):

$$(IR_{D1}): r_D \geq 1 \quad (3.10)$$

$$(IR_{D2}): y_a - 1 < 0 \text{ [resp. } y_{a0} < 0] \quad (3.11)$$

$$(IR_{L1}): v_i \geq 0 \text{ [resp. } v_{0i} \geq 0] \quad (3.12)$$

$$(IR_{L2}): v_i \geq y_a - 1 \text{ [resp. } v_i \geq y_{a0}, v_{0i} \geq y_{a0}, v_{0i} \geq y_a - 1] \quad (3.13)$$

Third, the bank has to respect non-negativity constraints regarding agents' deposits ( $N_D$ ), equity contribution ( $N_c$ ) and transfer ( $N_s$ ):

$$(N_D): w_i - k_i - w_{S,i} \geq w_{D,i} \geq 0 \text{ [resp. } w_i - k_{0i} - w_{S,i} \geq w_{D,i} \geq 0] \quad (3.14)$$

$$(N_c): w_c \leq w_i - w_{D,i} - w_{S,i} \text{ [resp. } w_{c0} \leq w_i - w_{D,i} - w_{S,i}] \quad (3.15)$$

$$(N_s): t \leq f(k_L(r_L)) \text{ [resp. } t_0 \leq f(k_{L0}(r_L))] \quad (3.16)$$

Balancing constraints and anticipating  $r_D \geq 1$ ,<sup>12</sup> allows to contain the decision problem and to fix if agents become financial surplus or deficit units:

**Lemma 3.3** *Given  $1 \leq r_D \leq r_L$ , agent  $i$ 's wealth  $w_i$  confines his **option space**:*

(i.) *If  $w_i$  is larger than the optimal investment given  $r_D$ , he self-finances the optimal investment given  $r_D$  and deposits any remainder at rate  $r_D$ .*

(ii.) *Otherwise, he either (ii.a) borrows any lacking funds for the optimal investment given  $r_L$  or (ii.b) simply autarkically invests  $w_i$  without approaching the capital market on either side (except for if  $r_D > \bar{r}$  or if  $w_i < \tilde{w}$ , when he only invests  $\min\{w_i, k_0(r_D)\}$  and deposits any remainder at rate  $r_D$ ).*

*Consequently, (3.15)  $N_c$  is only relevant in case (ii.a) and (3.14)  $N_D$  in case (i.) and eventually (ii.b). If  $(IR_{D2})$  is violated,  $(IR_{L2})$  implies  $(IR_{L1})$ .*

### 3.3.4.4 Capital Market Equilibrium

Regardless of the banking regime, agent  $i$  decides how much to devote to investment  $k_i$ , deposits  $w_{D,i}$  and storage  $w_{S,i}$  in view of a profile of deposit and loan contracts  $\hat{C}(w) = (C_{D,1}(w), \dots, C_{D,n}(w); C_{L,1}(w), C_{L0,1}(w), \dots, C_{L,n}(w), C_{L0,n}(w))$  offered by  $n \geq 1$  banks. The latter have the same beliefs about how agents decide when they are offered  $\hat{C}(w)$ :  $B(\hat{C}(w)) \rightarrow (k, e, w_D, w_S)$ , whose components are  $n$ -dimensional vectors. Following Grüner (2003), common beliefs must be compatible with individual maximization in the following sense:

**Definition 3.1** *The beliefs  $B(\hat{C}(w))$  satisfy the **optimality-by-agent criterion**, if they are compatible with the following behavior: (1.) Agents who decide to accept a contract, choose the one that offers them the highest expected payoff. (2.) Agents who are indifferent between the following options pick each of them with equal probability: (i) contracts offered by several banks, (ii) effort or shirking, (iii) types of entrepreneurship (self-financed, leveraged or autarkic), (iv) entrepreneurship in either form, deposits or storage.*

This being said, the symmetric subgame perfect Nash equilibrium of the economy can be formally characterized:

**Definition 3.2** *A **capital market equilibrium** consists of (1.) a deposit contract  $C_D^*(w)$  and (2.) a loan contract  $C_L^*(w)$  for each wealth class  $w$  offered by  $n \geq 1$  banks in the market, (3.) beliefs  $B(\hat{C}(w))$  satisfying the optimality-by-agent criterion and (4.) agents' equilibrium decisions  $(k, e, w_D, w_S)^*$  s.t. neither banks, nor agents have an incentive to deviate: (i) There is no excess capital demand or supply. Total capital investment  $\int_0^1 \{k_i^* + k_{a,i}^* + k_{L,i}^*\} di$  equals unstored wealth  $W - \int_0^1 w_{S,i} di$ . (ii) Actual behavior is rational and so as expected:  $(k, e, w_D, w_S)^* = B(\hat{C}(w))$ . (iii) Given beliefs  $B(\hat{C}(w))$ ,  $C_D^*(w)$  and  $C_L^*(w)$  are equilibrium contracts in the respective market form. If new banks are not hindered from entering the market, there is no incentive for them to do so.*

<sup>12</sup>Hurting  $(IR_{D1})$ ,  $r_D < 1 < \bar{r}$  is not sustainable. As  $k(1) > \tilde{w}$ , the rich with  $w_i > k(1)$  would diligently invest  $k(1)$  and store  $w_i - k(1)$ . All others would autarkically invest  $w_i$ , since for  $w_i \geq \tilde{w}$ ,  $pf'(w_i) > 1$  and for  $w_i < \tilde{w}$ ,  $qf'(w_i) > 1$ . Thus, to incite deposits,  $r_D \geq 1$ .

### 3.4 Equilibria under Perfect Information

In order to establish a benchmark, we exceptionally assume away the problem of asymmetric information: effort is observable and contractable at no cost.

#### 3.4.1 Omniscient Social Planner

A social planner aims at realizing the economy's maximal aggregate surplus  $U$ . For this purpose, given aggregate wealth  $W$  and *Lemma 3.1*, he must choose a rate of return  $r$  and a fraction of non-effort firms  $\alpha(W) \in [0, 1]$  at size  $k_0(r)$  along a fraction of effort firms  $(1 - \alpha(W))$  at size  $k(r)$ :

$$\begin{aligned} \max_r U &= \alpha(W) qf(k_0(r)) + (1 - \alpha(W)) [pf(k(r)) - 1] \\ \text{s.t. } W &= \alpha(W) k_0(r) + (1 - \alpha(W)) k(r) \end{aligned} \quad (3.17)$$

As formally derived in the Appendix, the solution depends on two critical return rates, henceforth denoted  $\hat{r}_p$  and  $\hat{r}_q$  (with  $\hat{r}_p < \hat{r}_q < \bar{r}$ ). Thereby,  $\hat{r}_p$  [resp.  $\hat{r}_q$ ] is the cut-off rate for which  $U$  is still larger for  $\alpha = 0$  than  $\alpha = \varepsilon$  [resp.  $\alpha = 1$  than  $\alpha = 1 - \varepsilon$ ] with  $\varepsilon > 0$ , but infinitesimal small. Accordingly, if  $W$  is sufficiently large [resp. low] to give rise to  $pf'(W) \leq \hat{r}_p$  [resp.  $qf'(W) \geq \hat{r}_q$ ], the planner empowers effort firms of size  $k(\hat{r}) = W$  [resp. non-effort firms of size  $k_0(\hat{r}) = W$ ] only:  $\hat{\alpha}(W) = 0$  and the optimal return rate is  $\hat{r} = pf'(W)$  [resp.  $\hat{\alpha}(W) = 1$  and  $\hat{r} = qf'(W)$ ]. Otherwise, the planner makes coexist effort and non-effort firms. Maximizing (3.17) gives

$$\hat{r} \text{ s.t. } k'(\hat{r}) / [k'(\hat{r}) - k'_0(\hat{r})] = [W - k(\hat{r})] / [k_0(\hat{r}) - k(\hat{r})], \quad (3.18)$$

so that  $0 < \hat{\alpha}(W) = [W - k(\hat{r})] / [k_0(\hat{r}) - k(\hat{r})] < 1$ . For  $r \in (\hat{r}_p, \hat{r}_q)$ ,  $\hat{\alpha}(W)$  is a linearly decreasing function in  $W$ . This, as displayed in *Figure 3.1*, leaves the average firm size  $\hat{k}(r) := \alpha(W) k_0(r) + (1 - \alpha(W)) k(r)$  stepwise convex in  $r$ .

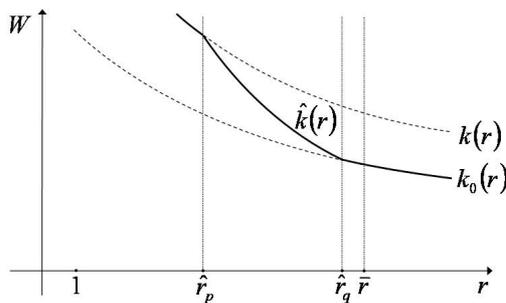


Figure 3.1: The average firm size  $\hat{k}(r)$  implemented by a social planner

With (A2) ruling out storage, we can fully characterize the Pareto-optimum:

**Proposition 3.1** *In the first-best equilibrium, the rate of return  $\hat{r}$  depends on aggregate wealth  $W$ :*

(i) *If  $pf'(W) \leq \hat{r}_p$ , all agents are made run effort firms of size  $k(\hat{r}) = W$ . Thus,  $\hat{r} = pf'(W)$  with  $\hat{r} \in (1, \hat{r}_p]$ .*

(ii) If  $qf'(W) \geq \hat{r}_q$ , all agents are made run non-effort firms of size  $k_0(\hat{r})=W$ . Thus,  $\hat{r} = qf'(W)$  with  $\hat{r} \in [\hat{r}_q, \infty)$ .

(iii) Otherwise, a share of agents  $0 < \hat{\alpha}(W) < 1$  is made run non-effort firms of size  $k_0(\hat{r})$  and  $(1-\hat{\alpha}(W))$  effort firms of size  $k(\hat{r})$ . Thus, (3.18) gives  $\hat{r}$  with  $\hat{r} \in (\hat{r}_p, \hat{r}_q)$ .

In case (i), the planner realizes an aggregate surplus of  $\hat{U} = \hat{Y} - 1$  with aggregate output  $\hat{Y} = pf(W)$ . It is lower in case (iii), when  $\hat{U} = \hat{Y} - (1 - \hat{\alpha}(W))$  with  $\hat{Y} = \hat{\alpha}(W) qf(k_0(\hat{r})) + (1 - \hat{\alpha}(W)) pf(k(\hat{r}))$  and even lower in case (ii), when  $\hat{U} = \hat{Y} = qf(W)$ .

### 3.4.2 Market Outcome

We now assess the allocation of the social planner against that of a perfectly competitive capital market under perfect information.

#### 3.4.2.1 Financial Contracts and Individual Decisions

Banks take the deposit rate  $r_D$  and hence contracts  $C_D(w) = (r_D)$  as given from a competitive loan market. In a Bertrand manner, they then compete away the intermediation margin by their simultaneous offer of loan contracts to agents of different wealth classes.<sup>13</sup>

**Lemma 3.4** *In a competitive market equilibrium, banks make zero profits.*

A positive return rate wedge can therefore not exist with zero intermediation costs.  $r := r_D = r_L$  applies to deposit and loan contracts alike, so that:

**Definition 3.3** *Given a return rate  $r^* \leq \bar{r}$ , deposit contract  $C_D^* = (r^*)$ , beliefs  $B(\hat{C}(w))$  and Lemma 3.3, a loan contract  $C_L(w) = (w_c, t)$  is a **Bertrand equilibrium effort loan contract**, if it solves the problem of an agent  $i$  with  $w_i < k(r^*)$ :*

$$\max_{t, w_c, k_L} v_i(r^*) = pt - r^*w_c - 1 \text{ s.t. } (k, e, w_D, w_S) = B(\hat{C}(w)),$$

$$(3.4) \text{ } IR_L^B: \Pi_{L,ji} = 0, (3.13) \text{ } IR_{L2} \text{ and } (3.15) \text{ } N_c.$$

Banks just break even by offering standard debt contracts. These entail a success reward that encompasses the total output minus whatever it takes to cover interest payments in expected terms:  $t = f(k_i) - r[k_i - w_c]/p$ . Borrowers then find it optimal to mimic self-financers. They choose  $k_L = k(r)$  and, with an equal return on the project and on deposits,  $w_c = w_i$ . Hence, full information loan contracts do not restrict any agent's access to credit. They enable agents of all wealth classes to behave as described in Lemma 3.1:<sup>14</sup>

**Lemma 3.5** *For a given return rate  $r \geq 1$ , the solution to the **individual contracting problem** with perfect information and loan market competition is:*

- (i) For  $r \leq \bar{r}$ , agent  $i$  with  $w < k(r)$  borrows  $k(r) - w_i$  at rate  $r/p$ .
- (ii) For  $r > \bar{r}$ , agent  $i$  with  $w_i < k_0(r)$  borrows  $k_0(r) - w_i$  at rate  $r/q$ .

<sup>13</sup>Under single-sided competition, banks can always raise enough deposits at a given rate  $r_D$  to meet loan demand. As Section 3.5.5 exhibits, this changes under double-sided competition, i.e. when banks additionally have to compete for deposits.

<sup>14</sup>No-effort loan contracts only require to rewrite Definition 3.3 for a return rate  $r^* > \bar{r}$ .

### 3.4.2.2 Capital Market Equilibrium and Aggregate Output

The equilibrium market rate of return  $r^*$  equalizes gross capital supply  $S(r^*)$  and demand  $D(r^*)$ . While  $S(r^*)$  amounts to aggregate wealth  $W$  minus the funds devoted to storage,  $D(r^*)$  adds up all agents' investments intended at rate  $r^*$ . The latter can sectionwisely be obtained from *Lemma 3.5*.

For a given  $r \leq \bar{r}$ , borrowing constraints are absent. No agent is hindered to invest  $k(r)$ . It follows that  $D(r) = k(r)$  and that  $MPC^e = pf'(k(r)) = r$ . (A2) then ensures  $r^* > 1$ , which makes costless storage unattractive:  $w_S^* = 0$  and  $S(r) = W$ . After substitution,  $S(r^*) = D(r^*)$  transforms to

$$k(r^*) = W, \text{ so that } r^* = pf'(W).$$

The dispersion of wealth levels  $G(w)$  does obviously neither affect the allocation of capital across agents, nor  $D(r)$  or  $r^*$ : every agent simply invests average wealth  $W$ . For this purpose, poor agents cover a deficit of  $k(r^*) - w_i$  by borrowing from rich agents, who deposit a surplus of  $w_{D,i} = w_i - k(r^*)$ . Due to zero bank profits, aggregate output  $Y^* = pf(W)$  is totally split among agents. Each of them ends up with a wealth-dependent gross capital income  $rw_i$  and a wealth-independent project profit  $pf(W) - rW - 1$ , so that  $U^* = Y^* - 1$ . Analogously proceeding for  $r > \bar{r}$  gives  $k_0(r^*) = W$  and  $U^* = Y^* = qf(W)$ .

For  $r = \bar{r}$ , however,  $k_0(\bar{r}) \leq W \leq k(\bar{r})$ . There is either too much or too little aggregate wealth for all agents to make the same investment. Given  $W$ , banks' supply then only contains a fraction  $\alpha^*(W) \in (0, 1)$  of non-effort and  $(1 - \alpha^*(W))$  of effort contracts such that  $W = \alpha^*(W)k_0(\bar{r}) + (1 - \alpha^*(W))k(\bar{r})$ . But as (A1)  $y(\bar{r}) - 1 = y_0(\bar{r})$  leaves agents indifferent between investing  $k_0(\bar{r})$  without effort and  $k(\bar{r})$  with effort, they pick each contract with equal probability. After all,  $Y^*(\bar{r}) = \alpha^*(W)qf(k_0(\bar{r})) + (1 - \alpha^*(W))pf(k(\bar{r}))$ , but  $U^*(\bar{r}) = Y^*(\bar{r}) - (1 - \alpha^*(W)) = pf(W) - 1 = qfW$ .

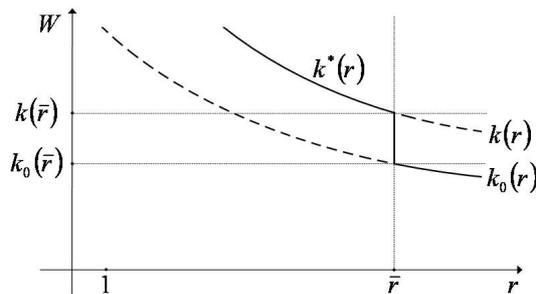


Figure 3.2: The average firm size  $k^*(r)$  with a competitive capital market

As illustrated in *Figure 3.2*, the average firm size  $k^*(r)$  will always equal  $W$ . We also see from the comparison with *Figure 3.1* that in equilibrium, the market's only resembles the social planner's capital allocation for  $r^* \notin (\hat{r}_p, \bar{r})$ . Otherwise, there is an over-provision of effort, which hampers aggregate surplus  $U$ . This is because in contrast to the planner, privately optimizing agents do not internalize the pecuniary externality that they put on each other when they open an effort firm: effort firms require higher capital inputs, which put an upwards pressure on

the equilibrium market rate of return via higher capital demand and so lower all firms' optimal sizes. Although effort firms' output is still higher than non-effort firms', the difference decreases convexly in  $r$  and finally ceases to outweigh aggregate effort costs for  $\hat{r}_q < \bar{r}$  (also see the Proof of Lemma 3.1).

That is how effort costs  $e = \{0, 1\}$  cause a coordination failure among creditors: as their impact on  $U^*$  is not captured by  $r^*$ , too many agents find it individually optimal to run effort firms when  $r^* \in (\hat{r}_p, \bar{r})$ . Compared to the socially optimal behavior, they thereby fuel  $r^*$  and  $Y^*$ , but lower  $U^*$ .

**Proposition 3.2** *In an equilibrium with perfect information and a competitive loan market, the return rate  $r^*$  depends on aggregate wealth  $W$ :*

- (i) *If  $pf'(W) \leq \bar{r}$ , all agents choose the same high investment  $k(r^*) = W$  and exert effort. Thus,  $r^* = pf'(W)$  with  $r^* \in (1, \hat{r}_p]$ .*
- (ii) *If  $qf'(W) \geq \bar{r}$ , all agents choose the same low investment  $k_0(r^*) = W$  and shirk. Thus,  $r^* = qf'(W)$  with  $r^* \in [\bar{r}, \infty)$ .*
- (iii) *Otherwise, a share of agents  $\alpha^*(W)$  opens non-effort firms of size  $k_0(\bar{r})$  and  $(1-\alpha^*(W))$  effort firms of size  $k(\bar{r})$ . Thus,  $r^* = \bar{r}$ .*

*However, the allocation only reaches first-best efficiency if  $W$  is sufficiently high or low, i.e. if  $W$  is s.t. it entails  $r^* \notin (\hat{r}_p, \bar{r})$ .*

## 3.5 Equilibria under Imperfect Information

Following e.g. Diamond (1984), Piketty (1997), Aghion and Bolton (1997) or Grüner and Schils (2007), we now reintroduce asymmetric information (Subsection 3.5.1). Later, we add banking market power and study a deposit monopsony (Subsection 3.5.2), loan monopoly (Subsection 3.5.3), double-sided monopoly (Subsection 3.5.4) and two-sided competition (Subsection 3.5.5).

### 3.5.1 Competitive Loan Market

Like in the benchmark case, we consider Bertrand competition among banks.

#### 3.5.1.1 Competitive Financial Contracts and Individual Decisions

Taking  $r_D$  and  $C_D(w) = (r_D)$  as given from a competitive deposit market, banks compete by their offer of loan contracts until in equilibrium (as stated in Lemma 3.4) they end up only trading zero-profit contracts with  $r_C := r_D = r_L$ .

**Definition 3.4** *Given a return rate  $r_C^* \leq \bar{r}$ , deposit contract  $C_D(w) = (r_C^*)$ , beliefs  $B(\hat{C}(w))$  and Lemma 3.3,  $C_L(w) = (w_c, t)$  is a **Bertrand equilibrium effort loan contract**, if it solves the problem of an agent  $i$  with  $w_i < k(r_C^*)$ :*

$$\begin{aligned} \max_{w_c, t, k_L} v_i(r^*) = pt - r^*w_c - 1 \quad \text{s.t. } (k, e, w_D, w_S) = B(\hat{C}(w)), \\ (3.4) \quad IR_L^B : \Pi_{L,ji} = 0, (3.9) \quad IC, (3.13) \quad IR_{L2} \text{ and } (3.15) \quad N_c. \end{aligned} \tag{3.19}$$

For the same reason as in the benchmark case, a standard effort debt contract with  $t = f(k_L) - r[k_L - w_c]/p$  and  $w_c = w_i$  induces participation. It enables agents to employ  $k_L = k(r)$ . However, with the effort input being invisible, the bank must ensure beforehand that contract terms also prevent borrowers from future opportunistic behavior. Rewriting (3.9) IC

$$p[f(k(r)) - r[k(r) - w_i]/p] - 1 \geq q[f(k(r)) - r[k(r) - w_i]/p]$$

reveals that effort provision only pays off for borrowers, who own at least

$$w_i \geq \omega(r) := p/r(p - q) - [pf(k(r)) - rk(r)]/r. \quad (3.20)$$

$\omega(r)$  amounts to the difference between the expected discounted costs of moral hazard and the project's net present value.<sup>15</sup> It is a strictly increasing and concave function in  $r$ . This owes to the fact that the higher  $r$ , the larger the fraction of marginal returns from effort the borrower has to share with his lender and so the lower his incentive to actually provide costly effort. On the other hand,  $\omega(r)$  becomes zero for with  $r \in (1, \hat{r}_p)$ .<sup>16</sup> The need to break even prevents banks from further improving contract terms for borrowers.

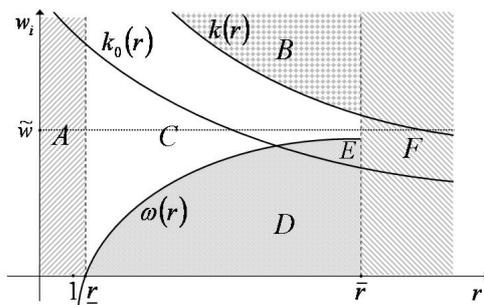


Figure 3.3: Solution to an individual agent's investment problem given  $r$

On these grounds, *Figure 3.3* summarizes agents' occupational choices.

(i) For  $1 < r \leq \bar{r}$ ,  $\omega(r) \leq 0$ , so that even have-nots with zero wealth prefer to provide effort (area A). Thus, all agents open effort firms of size  $k(r)$ .

(ii) For  $\bar{r} < r \leq \hat{r}$ , self-financers still run effort firms (area B), since they stay full residual claimants on all returns from effort. Banks, however, know that the fraction that borrowers keep from their returns on effort is only sufficient to induce effort for those who can pledge an equity contribution  $w_c \geq \omega(r)$ . Only they get effort contracts and open effort firms of size  $k(r)$  (area C). Agents with  $w_i < \omega(r)$ , instead, cannot guarantee the bank an expected return of  $r$ . They get credit-rationed, i.e. banks refuse to provide them with sufficient funds for effort

<sup>15</sup>Measuring agents' outside option,  $q$  indicates the toughness of credit rationing:  $d\omega(r)/dq > 0$ . Likewise,  $\omega(r) < 0$  for  $q \leq 0$ , eliminating the commitment issue.

<sup>16</sup>The reason is that for  $r \leq \hat{r}$ , effort firms' external effect on each other via their pressure on  $r$  is neither sufficient to affect agents' individual, nor the planner's aggregate effort decision. However, the planner on top internalizes a countervailing capital reallocation benefit. As he receives all marginal project returns, he benefits from equalizing firm sizes in the presence of a decreasing MPC. Thus,  $\hat{\alpha}(W)$  becomes zero for  $\hat{r}_p > \hat{r}$ . Also note, that  $d\hat{r}/dq < 0$ .

firms.<sup>17</sup> They are rather offered ample credit for non-effort firms of size  $k_0(r)$ , since they are unable to cheat more on the lender than  $e = 0$ . Agents with  $w_i < \omega(r)$  accept non-effort contracts  $C_{L0} = (0, t_0 = f(k_0(r)) - r(k_0(r) - w_i)/q)$  (area  $D$ ), except for if  $k_0(r) \leq w_i < \omega(r)$  (area  $E$ ). These agents can afford to self-finance the low investment  $k_0(r)$  and prefer to deposit any remainder  $w_i - k_0(r)$  at rate  $r$ . Hence, there might not only be very rich depositors with  $w_i > k(r)$ , but also some poor ones who are credit-constrained.<sup>18</sup>

(iii) For  $r > \bar{r}$ , capital costs climb so high that effort does not pay off for anyone, even not for self-financers. All agents choose  $e = 0$  and  $k_0(r)$  (area  $F$ ).

Remember from Subsection 3.4.2, that under full information, competitive banks also empower effort firms in areas  $D$  and  $E$ . Altogether:

**Lemma 3.6** *For a given return rate  $r \geq 1$ , the solution to the **individual contracting problem** with moral hazard and loan market competition is as follows:*

- (i) For  $r \leq \underline{r}$ , agent  $i$  with  $w < k(r)$  borrows  $k(r) - w_i$  at rate  $r/p$ .
- (ii) For  $\underline{r} < r \leq \bar{r}$ , agent  $i$  borrows  $k(r) - w_i$  at rate  $r/p$  if  $\omega(r) \leq w_i < k(r)$  and  $k_0(r) - w_i$  at rate  $r/q$  if  $w_i < \omega(r)$ .
- (iii) For  $r > \bar{r}$ , agent  $i$  with  $w_i < k_0(r)$  borrows  $k_0(r) - w_i$  at rate  $r/q$ .

### 3.5.1.2 Capital Market Equilibrium and Aggregate Output

As laid out in *Definition 3.2*,  $D(r_C^*) = S(r_C^*)$  gives the equilibrium return rate  $r_C^*$ . While gross capital demand  $D(r_C^*)$  equals the sum of all agents' investments intended at rate  $r_C^*$ ,  $S(r_C^*) = W$  (with  $r_C^* \geq 1$  ensued by  $\underline{r} > 1$  and (A2)).

Preconceive from *Lemma 3.6* that  $D(r_C^*)$  is unconstrained if  $W$  is sufficiently high [resp. low] to push  $r_C^*$  below  $\underline{r}$  [resp. above  $\bar{r}$ ]. All agents make the same high investment  $k(r)$  [resp. low investment  $k_0(r)$ ]. The equilibrium return rate  $r_C^*$  and aggregate surplus  $U_C^*$  therefore only depend on aggregate wealth  $W$ :  $r_C^* = pf'(W)$  and  $U_C^* = Y_C^* - 1$  with  $Y_C^* = pf(W)$  [resp.  $r_C^* = qf'(W)$  and  $U_C^* = Y_C^* = qf(W)$ ]. Yet for intermediate levels of  $W$ ,  $D(r_C^*)$  is constrained by credit rationing.<sup>19</sup> A fraction of agents  $G(\omega(r_C^*))$  is denied credit for  $k(r_C^*)$  and so left to invest  $k_0(r_C^*)$  only. This makes  $r_C^*$  and  $U_C^*$  become a function of the entire wealth distribution  $G(w)$ :

$$r_C^* = r_C^*(G(w)) \text{ s.t. } W = G(\omega(r_C^*))k_0(r_C^*) + [1 - G(\omega(r_C^*))]k(r_C^*) \quad (3.21)$$

$$U_C^*(G(w)) = G(\omega(r_C^*))qf(k_0(r_C^*)) + [1 - G(\omega(r_C^*))][pf(k(r_C^*)) - 1]. \quad (3.22)$$

Being subject to the same informational constraints as competitive banks deters a non-omniscient, non-repressive social planner from improving on the market outcome. Hence, unlike the market outcome under perfect information in Subsection 3.4.2, that under imperfect information is (constrained) Pareto-optimal:

<sup>17</sup>To see why (3.9) IC cannot be satisfied for any  $k$  below the optimal level  $k(r)$ , recall the fact that an entrepreneur's payoff in the high output state is already maximal for the  $k(r)$ . Hence, incentives to shirk increase for any suboptimal investment level.

<sup>18</sup>Not accounting for autarkic production, Piketty (1997) obtains that for  $r < \bar{r}$ , agent  $i$  with  $w_i < k(r)$  is always a net borrower.

<sup>19</sup>Notice that  $D'(r) = G(\omega(r))k_0'(r) - [k(r) - k_0(r)]G'(\omega(r))\omega'(r) + [1 - G(\omega(r))]k'(r) < 0$ .

**Proposition 3.3** *In a capital market equilibrium with moral hazard and Bertrand competition, the rate of return  $r_C^*$  is as follows:*

(i) *If either  $pf'(W) \leq \bar{r}$  or  $qf'(W) \geq \bar{r}$ , first-best efficiency obtains:  $r_C^*$  only depends on aggregate wealth  $W$  and Proposition 3.1 applies accordingly.*

(ii) *Otherwise, constrained efficiency obtains: credit rationing  $\omega(r_C^*) > 0$  makes  $r_C^*$  s.t.  $D(r_C^*) = W$  dependent on aggregate wealth  $W$  and its distribution  $G(w)$ : A share of agents  $G(\omega(r_C^*))$  shirkingly invests  $k_0(r_C^*)$ , whereas all others diligently invest  $k(r_C^*)$ . If  $G(\omega(r_C^*))$  equals the first-best share of non-effort firms  $\hat{\alpha}(W)$ , constrained and first-best efficiency coincide. If  $G(\omega(r_C^*)) > \hat{\alpha}(W)$  [resp.  $G(\omega(r_C^*)) < \hat{\alpha}(W)$ ], too many [resp. few] non-effort firms dampen [resp. fuel]  $r_C^*$  relative to the first-best return rate  $\hat{r}$ .*

Another difference to the full information market outcome is the existence of credit rationing. It unfolds a dampening effect on credit demand, the equilibrium return rate and aggregate output,<sup>20</sup> but a boosting effect on aggregate surplus. In fact, an omniscient planner and uninformed competitive banks resemble in that they avoid excessive aggregate effort provision, although for very different reasons. While the first apportions effort contracts to maximize aggregate surplus by internalizing effort firms' external effects, the latter ration effort contracts in order to break even. They also differ in that with competitive banks, the identity of effort entrepreneurs and thus agent  $i$ 's project profit becomes wealth-dependent:  $qf(k_0(r_C^*)) - r_C^*k_0(r_C^*)$  if  $w_i < \omega(r_C^*)$  and  $pf(k(r_C^*)) - r_C^*k(r_C^*) - 1$  if  $w_i \geq \omega(r_C^*)$ . With  $k(r_C^*) > k(r^*) > k_0(r_C^*)$ , credit-constrained agents realize a lower project profit. All others even earn more, if an increase in project profit outweighs a decrease in gross capital income.

Interestingly, the higher wealth inequality,<sup>21</sup> the lower the scope of credit rationing and so the higher  $r_C^*$  and  $Y_C^*$ . Although this makes the outcome converge towards the full information market outcome, it might not always be efficiency-enhancing in that it also increases  $U_C^*$ . Comparing the share of non-effort firms with the planner  $\hat{\alpha}(W)$  and with competitive banks  $G(\omega(r_C^*))$  for  $r^* \in (r, \bar{r})$ , reveals that depending on the dispersion of wealth levels  $G(w)$ , there can be more, the same or less effort firms than under first-best. But as deviations to either side incur an efficiency loss, they leave scope for efficiency-enhancing distributive policies. Lowering inequality would then help to reduce aggregate effort costs via an increased scope of rationing (and vice versa).

### 3.5.2 Monopsony Deposit Market

Suppose that the economy is split up in  $n$  identical regions.<sup>22</sup> Populated with a single bank and a continuum of agents, each region  $j \in [1, n]$  disposes of aggregate wealth  $W_j = W/n$ . Whilst taking the loan rate  $r_L$  and hence loan contracts  $C_L(w)$  as given from a national competitive loan market, each bank controls its regional deposit market. As a monopsonist, it chooses a deposit rate  $r_{D,j} := r_{D,j}(r_L)$  with  $r_{D,j} \leq r_L$  and deposit contracts  $C_D(w)$ .

<sup>20</sup>Also in Stiglitz and Weiss (1981), the competitive equilibrium involves credit rationing, if the "Walrasian return rate" is s.t. there exists a lower  $r$  for which bank profits are higher.

<sup>21</sup>In this paper, higher inequality means a larger mass of agents complying with (3.20)  $\omega(r)$ .

<sup>22</sup>Although fixed and finite, the exact number of regions does not affect the results.

### 3.5.2.1 Monopsony Financial Contracts and Individual Decisions

On the **national competitive loan market**, monopsony banks are presented with the optimization problem stated in *Definition 3.4*. Thus, *Lemma 3.6* reapplies, but for diverging deposit and loan rates:  $r_{D,j} < r_L$ . With self-financers being geared to  $r_{D,j}$  and borrowers to  $r_L$ , (A1) must also be rewritten for either rate and so entails  $\bar{r}_{D,j}$  and  $\bar{r}_L \leq \bar{r}_{D,j}$ . Accordingly, for  $r_{D,j} \leq \bar{r}_{D,j}$  [resp.  $r_L \leq \bar{r}_L$ ], self-financers' [resp. borrowers'] optimal effort investment is  $k(r_{D,j})$  s.t.  $pf'(k(r_{D,j})) = r_{D,j}$  [resp.  $k_L = k(r_L)$  s.t.  $pf'(k(r_L)) = r_L$ ]. Borrowers need to fill their financing gap through a standard debt contract with  $t = f(k(r_L)) - r_L[k(r_L) - w_i]/p$ . Qualitatively, credit rationing remains unaltered. Albeit as a function of the loan rate, wealth constraint (3.20)  $\omega(r_L)$  recurs. Owing to  $r_{D,j} \leq r_L$ , agents a fortiori prefer to pledge all they have, so that  $w_c = w_i$ .

On the **regional monopsony deposit market**, each bank then freely chooses the deposit rate  $r_{D,j}$  given  $r_L$ .  $r_L$  adjusts firms' national net capital demand  $D(r_L) = nD_j(r_L)$  and depositors' net capital supply  $nS_j(r_{D,j})$ .

**Definition 3.5** *Given a loan rate  $r_L^* \leq \bar{r}_L$ , demand  $D_j(r_L^*)$  and contracts  $C_L^*(w)$  and  $C_{L0}^*(w)$ , non-effort borrowers  $i$  with  $w_i \in [0, \min\{\omega(r_L^*), k_0(r_L^*)\}]$  and effort borrowers  $i$  with  $w_i \in [\omega(r_L^*), k(r_L^*)]$ , beliefs  $B(\hat{C}(w))$  and *Lemma 3.3*, contract  $C_D(w) = (r_{D,j}(r_L))$  is a **monopsony equilibrium deposit contract**, if it solves the following problem of bank  $j$ :*

$$\begin{aligned} \max_{r_{D,j}} \Pi_j &= [r_L^* - r_{D,j}] D_j(r_L^*) \quad \text{s.t. (3.5) } AC^B: D_j(r_L^*) \leq S_j(r_{D,j}), \\ (k, e, w_d, w_s) &= B(\hat{C}(w)), \quad (3.3) \text{ } IR_D^B, \quad (3.4) \text{ } IR_L^B, \quad (3.10) \text{ } IR_{D1}, \quad (3.11) \text{ } IR_{D2}, \quad (3.14) \text{ } N_D. \end{aligned}$$

However, the observability of agents' wealth by banks constitutes discriminating banking power and so calls for a distinction of cases.

**3.5.2.1.1 Case 1: Simple Monopsony** Forgoing discriminating power, a monopsony bank offers a constant return  $r_{D,j} < r_L$  on each unit of deposit.

Qualitatively, the return rate wedge adds a further segment of individual behavior. For  $\bar{r}_L < r_L < \bar{r}_{D,j}$ , leveraged effort firms do no longer pay off, but self-financed effort firms do. It then follows from  $r_{D,j}(\bar{r}_L) \leq \bar{r}_L$ , *Lemmata 3.1, 3.2* and *3.6* that  $k_0(\bar{r}_L) \leq k_0(r_{D,j}(\bar{r}_L)) < k(r_{D,j}(\bar{r}_L))$  and that

$$k_0(\bar{r}_L) < \omega(\bar{r}_L) < \tilde{w} < k(\bar{r}_L) < k(r_{D,j}(\bar{r}_L)). \quad (3.23)$$

Moreover, (A1),  $r_{D,j} < r_L$  and  $dy/dr < 0$  establish  $y(r_{D,j}(\bar{r}_L)) - y_0(r_{D,j}(\bar{r}_L)) > 1$  and even more notably

$$y(\bar{r}_L) - 1 < y_0(r_{D,j}(\bar{r}_L)). \quad (3.24)$$

Despite  $k_0(r_{D,j}) < k(r_L)$  and  $p/q > r_L/r_{D,j}$  for  $r_L \leq \bar{r}_L$ , the relative position of  $k_0(r_{D,j}(\bar{r}_L))$  is not universally trackable. Depending on the parameter constellation,<sup>23</sup> three possibilities remain (each represented by a dashed-dotted line and dotted areas in *Figure 3.4*):  $k_0(r_{D,j}(\bar{r}_L))$

<sup>23</sup>As derived in the Appendix, the position depends on the magnitude of the return rate spread ( $r_L - r_{D,j}$ ) and its relation with the success probability margin ( $p - q$ ).

can be (i) below  $\omega(\bar{r}_L)$ , (ii) between  $\omega(\bar{r}_L)$  and  $\tilde{w}$  or (iii) above  $\tilde{w}$  (but below  $k(\bar{r}_L)$ ). In any case and for any given  $r_L$ , there will be autarkic production (light grey-shaded areas), leveraged production (white area plus the checked grey area of credit-constrained agents) and self-financed production combined with depositing (dark grey-shaded area).

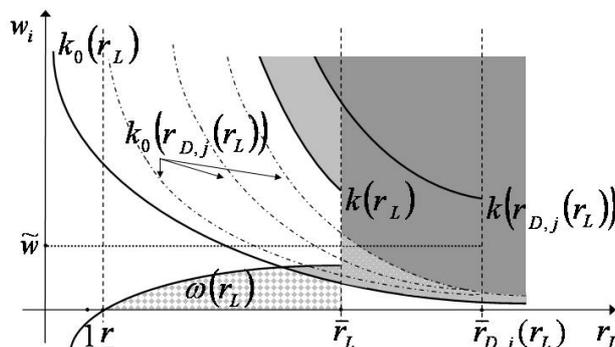


Figure 3.4: Solution to an agent's investment problem given  $r_L$

Quantitatively, the larger the return rate spread, the larger the fraction of autarkic firms and so the lower the volume of intermediated funds. In order to deduce how far to optimally cut  $r_{D,j}$  below  $r_L$ , the simple monopsonist must sectionwisely sum up agents' capital supply and demand:

(i) For  $1 \leq r_L \leq \bar{r}$ , all agents exert effort. A share of agents  $G(k(r_L))$  leverages  $k(r_L)$ . In contrast,  $[1-G(k(r_{D,j}))]$  self-finances  $k(r_{D,j})$  and deposits any remainder. All others autarkically invest  $w_i$ .

(ii) For  $\bar{r} < r_L \leq \bar{r}_L$ , credit rationing arises. A fraction of agents  $[G(k(r_L)) - G(\omega(r_L))]$  diligently leverages  $k(r_L)$ , whereas  $G(\min\{\omega(r_L), k_0(r_L)\})$  operates leveraged non-effort firms of size  $k_0(r_L)$ . Whilst depositing any remainder,  $[1-G(k(r_{D,j}))]$  opens self-financed effort firms of size  $k(r_{D,j})$  and  $[G(\omega(r_L)) - G(\min\{\omega(r_L), k_0(r_{D,j})\})]$  self-financed non-effort firms of size  $k_0(r_{D,j})$ . Finally,  $[G(\min\{\omega(r_L), k_0(r_{D,j})\}) - G(k_0(r_L))]$  runs autarkic non-effort firms and all others autarkic effort firms of size  $w_i$ .

(iii) For  $\bar{r}_L < r_L \leq \bar{r}_{D,j}$ , as opposed to self-financed and eventually some autarkic firms, leveraged firms are no longer profitable with effort. Depositing any remainder, a share of agents  $[1-G(k(r_{D,j}))]$  diligently self-finances  $k(r_{D,j})$  and  $[G(k(r_{D,j})) - G(k_0(r_{D,j}))]$  shirkingly invests  $k_0(r_{D,j})$ . Then,  $G(k_0(r_L))$  shirkingly leverages  $k_0(r_L)$ .  $[G(k_0(r_{D,j})) - G(\min\{\tilde{w}, k_0(r_{D,j})\})]$  autarkically invests  $w_i$  with effort and  $[G(\min\{\tilde{w}, k_0(r_{D,j})\}) - G(k_0(r_L))]$ .

(iv) At last, for  $r_L > \bar{r}_{D,j}$ , effort becomes unprofitable and all agents shirk. A share of agents  $G(k_0(r_L))$  leverages  $k_0(r_L)$ .  $[1-G(k_0(r_{D,j}))]$  self-finances  $k_0(r_{D,j})$  and deposits any remainder, whereas all others autarkically invest  $w_i$ .

Consequently, e.g. for  $r_L \leq \bar{r}_L$ , we obtain a deposit supply  $S_j(r_{D,j})$  of

$$\int_{\min\{k_0(r_{D,j}), \omega(r_L)\}}^{\omega(r_L)} [w - k_0(r_{D,j})] g(w) dw + \int_{k(r_{D,j})}^{\tilde{w}} [w - k(r_{D,j})] g(w) dw.$$

Substituting (3.5)  $AC^B$  into the object function in *Definition 3.5* gives

$$\Pi_j = [r_L - r_{D,j}] S_j(r_{D,j}). \quad (3.25)$$

An interior solution<sup>24</sup> satisfies  $d\Pi_j/dr_{D,j} = -S_j(r_{D,j}) + [r_L - r_{D,j}] S'_j(r_{D,j}) = 0$ . Looking at the FOC, *Figure 3.4* and borrowers' wealth levels stated in *Definition 3.5* exhibits the simple monopsonist's trade-off: increasing the volume of intermediated funds is feasible, but only at the cost of leaving more to depositors. A reformulation yields an intuitive pricing formula: given  $r_L$ , the optimal deposit rate  $r_{D,j}^*$  is determined by

$$r_{D,j}^* + S_j(r_{D,j}^*)/S'_j(r_{D,j}^*) = r_L. \quad (3.26)$$

Accordingly, the marginal total costs of attracting an additional unit of deposit (*LHS*) must equal  $r_L$ , i.e. the given revenue from lending out an additional unit of loan (*RHS*). With the supply curve  $S_j(r_{D,j})$  sloping upwards, an increase of the deposit volume calls for depositing entrepreneurs to reduce their firm sizes, which in turn requires a rising deposit rate. But as the monopsonist must pay a higher deposit rate on all units of deposit and not only on the marginally attracted one, each further unit of deposit costs more than the average. For the marginal deposit costs to be rising in  $r_{D,j}$ , it must be assumed that the deposit supply is sufficiently steep and its slope sufficiently slowly increasing in  $r_{D,j}$ :<sup>25</sup>

$$\text{Assumption 3: } d[r_{D,j} + S_j(r_{D,j})/S'_j(r_{D,j})]/dr_{D,j} > 0 \quad (A3)$$

This constitutes:

**Lemma 3.7** *For a given loan rate  $r_L \geq 1$ , the solution to the **simple monopsonist's problem** is to set a deposit rate  $r_{D,j}^* = r_L - S_j(r_{D,j}^*)/S'_j(r_{D,j}^*)$  if  $r_L - S_j(r_{D,j}^*)/S'_j(r_{D,j}^*) \geq 1$  and  $r_{D,j}^* = 1$  otherwise.*

Rearranging terms in (3.26) gives Blair and Harrison's (1993, p. 48) buyer power index. It measures the percentage markup over price, which in optimum equals the reciprocal elasticity of supply  $\varepsilon$ , i.e.

$$[r_L - r_{D,j}^*]/r_{D,j}^* = S_j(r_{D,j}^*)/r_{D,j}^* S'_j(r_{D,j}^*) = 1/\varepsilon. \quad (3.27)$$

The intermediation margin (and thus the deviation from the competitive price  $r_C^*$ ), is the higher, the lower  $\varepsilon$  - i.e. the smaller the responsiveness of the deposit supply to changes in the deposit rate (essentially, the greater  $S'_j(r_{D,j}^*)$  or the flatter the supply curve).<sup>26</sup> An upper limit of the percentage markup over price finally follows from taking advantage of (3.26) and  $p/q > r_L/r_D$  for  $r_L < \bar{r}_L$ :

$$1/\varepsilon < (p - q)/q \quad (\text{whereby } (p - q)/q > 1).$$

<sup>24</sup>Despite  $\Pi_j$ 's discontinuity, we can neglect corner solutions without loss of generality.

<sup>25</sup>Note that  $d[r_{D,j} + S_j(r_{D,j})/S'_j(r_{D,j})]/dr_{D,j} = 2 - S_j(r_{D,j}) S''(r_{D,j})/[S'_j(r_{D,j})]^2$ , where  $S'_j(r_{D,j}) > 0$  and  $S''_j(r_{D,j}) > 0$ .

<sup>26</sup>The monopsony result qualitatively also holds for a Cournot **oligopsony**. Then, the *RHS* of (3.27) is multiplied with the squared sum of market shares (also called the Herfindahl concentration measure), so that frictions increase with higher concentration.

**3.5.2.1.2 Case 2: Discriminating Monopsony** Unlike a simple, a discriminating monopsonist is able to push depositors down to their reservation utility. (3.10)  $IR_{D1}$  and (3.11)  $IR_{D2}$  become binding, if each unit of deposit is just paid its simple storage return of 1 or the  $MPC^e$  it would have generated if it had been autarkically invested. Hence, in return for depositing  $w_i - k(r_{D,j})$ , agent  $i$  with wealth  $w_i > k(r_{D,j})$  claims a total deposit remuneration  $R(w)$  of

$$R(w) = p[f(k(1)) - f(k(r_{D,j}))] + \max\{0, [w - k(1)]\}. \quad (3.28)$$

Owing to  $f$ 's strict concavity and the storage option, the average remuneration per unit of deposit is non-increasing in wealth. It equals at least 1 and at the most  $r_{D,j}$  (which now denotes the deposit rate paid on the cut-off unit of attracted deposits). Irrespective of the wealth distribution  $G(w)$ , it is therefore profit-maximizing for bank  $j$  to increase the deposit volume by rising  $r_{D,j}$  until the profit margin is completely shrunk away:  $r_{D,j}^* = r_L$ . With the return rate wedge fading, the effort and non-effort investment curves in *Figure 3.4* become congruent again, so that *Figure 3.3* finally reoccurs.

**Lemma 3.8** *For a given loan rate  $r_L \geq 1$ , the solution to the **discriminating monopsonist's problem** is to fix  $r_{D,j}^* = r_L$  and:*

(i) *if  $1 \leq r_L \leq \bar{r}$ , to pay depositors (i.e. all  $i$  with  $w_i > k(r_L)$ ) a total deposit return  $R^*(w) = p[f(k(1)) - f(k(r_L))] + \max\{0, [w_i - k(1)]\}$ .*

(ii) *if  $r_L > \bar{r}$ , to pay depositors (i.e. all  $i$  with  $w_i > k_0(r_L)$ ) a total deposit return  $R^*(w) = q[f(k_0(1)) - f(k_0(r_L))] + \max\{0, [w_i - k_0(1)]\}$ .*

### 3.5.2.2 Capital Market Equilibrium and Aggregate Output

The equilibrium loan rate  $r_L^*$  clears the competitive national loan market  $D(r_L^*) = nS_j(r_{D,j}^*(r_L^*))$ . The very existence of credit rationing makes  $r_L^*$  and aggregate output dependent on  $W$  and  $G(w)$  again.

A **simple monopsony** additionally involves a wealth-distribution-dependent return rate wedge  $r_L^* - r_{D,j}^*(r_L^*) > 0$ .<sup>27</sup> It not only incites the formation of autarkic, but also of leveraged firms that are smaller in size than self-financed firms. This depresses deposit supply relatively more than loan demand. Unlike the competitive regime in Subsection 3.5.1, a simple monopsony regime is therefore associated with a higher loan rate, which in turn tightens credit rationing. As besides, the  $MPC^e$  varies across firms, aggregate output slumps. The income effect differs across agents. Borrowers earn less, whereas depositors' higher project profit might not outweigh the lower deposit yield. For autarkic agents, it depends on whether they would have been borrowers or depositors in a competitive regime. Interestingly, higher wealth inequality c.p. tends to depress the percentage deviation of the loan from the deposit rate. For this to see, imagine that wealth redistribution lowers the mass of credit-constrained agents. The ensued surge in capital demand and thus in  $r_L^*$  directly translates into an equal rise of  $r_{D,j}^*$  ( $dr_{D,j}^*/dr_L^* = 1 > 0$ ), so that the relative magnitude of the return rate spread (3.27) decreases. That is how higher inequality ameliorates output losses owing to monopsony power, which in turn makes equilibrium values tend towards those of the competitive outcome in Subsection 3.5.1. Altogether, the

<sup>27</sup>As the outcome's central driver is the return rate wedge, simple **intermediation costs** would also give rise to *Figure 3.4* for a competitive setting like in Subsection 3.5.1.

return rate spread prevents the allocation from achieving first-best and constrained efficiency. However, an exception makes a particular parameter constellation characterized below, in which the constrained efficient aggregate surplus can be reached or even surpassed. The fact that the scope of rationing accrues as the beneficial driver, again assigns a role to distributive policies.

In contrast, the **discriminating monopsony** eliminates the return rate spread. Only the impact of moral hazard remains, so that the competitive equilibrium of Subsection 3.5.1 reemerges. Autarkic firms are inexistent and the  $MPC^e$  is the same across all production units. *Figure 3.3* and *Proposition 3.3* reapply. The equilibrium return rate is given by (3.21) and aggregate surplus by (3.22). Yet, the allocation of the surplus differs. While borrowers earn the same, the bank now skims the whole net depositor rent. When there is [resp. is no] credit rationing, the outcome reaches constrained [resp. first-best] efficiency.

**Proposition 3.4** *In a capital market equilibrium with moral hazard, a monopsony deposit and competitive loan market, the deposit and loan rate  $r_{D,j}^*$  and  $r_L^*$  are as follows:*

(i) *In case of a **simple monopsonist**, the return rate wedge  $r_{D,j}^*(r_L^*) - r_L^* < 0$  and  $r_L^*$  s.t.  $D(r_L^*) = nS_j(r_{D,j}^*(r_L^*))$  depend on aggregate wealth  $W$  and its distribution  $G(w)$ . The allocation neither reaches first-best, nor constrained efficiency - except for if  $r_L^* \in (r, \bar{r}_L]$  [resp.  $r_L^* \in (\bar{r}_L, \bar{r}_{D,j}]$ ] and if there are too few non-effort firms under competition relative to first-best  $G(\omega(r_C^*)) < \hat{\alpha}(W)$ . Then, constrained-efficient aggregate surplus obtains if efficiency gains from the higher share of non-effort firms  $G(\omega(r_L^*)) > G(\omega(r_C^*))$  [resp.  $G(\tilde{w}) > G(\omega(r_C^*))$ ] outweigh efficiency losses from the return rate wedge.*

(ii) *In case of a **discriminating monopsonist**,  $r_{D,j}^* = r_L^*$ . Thus, Proposition 3.3 re-applies.*

### 3.5.3 Monopoly Loan Market

The economy is composed of  $n$  identical regions. Each region  $j \in [1, n]$  features a single bank, a continuum of agents and aggregate wealth  $W_j = W/n$ . A nationwide competitive deposit and regional monopoly loan markets coexist.

#### 3.5.3.1 Monopoly Financial Contracts and Individual Decisions

On the **national competitive deposit market**, the deposit return rate  $r_D$  arises from aligning depositors' net capital supply  $S(r_D)$  with entrepreneurs' net capital demand  $nD_j(r_{L,j})$  from the  $n$  regions:  $S(r_D) = nD_j(r_{L,j})$ . Consequently,  $r_D$  fixes deposit contracts  $C_D(w)$  and the group of depositors  $i$  with  $w_i \in [\bar{k}_1 = \min\{k_0(r_D), \omega(r_{L,j})\}, \omega(r_{L,j})] \cup [k(r_D), \bar{w}]$ . Bank  $j$  takes these as given, when it sets a loan rate  $r_{L,j} := r_{L,j}(r_D) \geq r_D$  and loan contracts  $C_L(w)$  and  $C_{L0}(w)$  on the **regional monopoly loan market**.

**Definition 3.6** *Given a deposit rate  $r_D^* \leq \bar{r}_D$ , supply  $S(r_D^*)$ , contract  $C_D^*(w) = (r_D^*)$ , beliefs  $B(\hat{C}(w))$  and Lemma 3.3,  $C_{L0}(w) = (w_c, t_0)$  and  $C_L(w) = (w_c, t)$  are **monopoly equilibrium non-effort and effort loan contracts**, if they solve the following problem of bank  $j$ : maximize total expected profits  $\Pi_j$  by assigning input levels  $k_{L0}$  and  $k_L$  and a loan rate  $r_{L,j}$  to non-effort borrowers  $i$  with  $w_i \in [0, \bar{k}_1 := \min\{k_0(r_D^*), \omega(r_{L,j})\}]$  and effort borrowers  $i$  with  $w_i \in$*

$[\omega(r_{L,j}), \bar{k}_2 := k(r_{L,j})]$ , of whom  $\pi(w) = 1$  get credit and  $\pi(w) = 0$  do not.

$$\begin{aligned} \max_{\pi(w), t, t_0, r_{L,j}, w_c, k_L, k_{L0}} \Pi_j &= \int_0^{\bar{k}_1} \pi(w) [qf(k_{L0}) - r_D^* [k_{L0} - w_c] - t_0] g(w) dw \\ &+ \int_{\omega(r_{L,j})}^{\bar{k}_2} \pi(w) [pf(k_L) - r_D^* [k_L - w_c] - t] g(w) dw \\ \text{s.t. } (k, e, w_d, w_s) &= B(\hat{C}(w)), \quad (3.4) \text{ IR}_L^B, \quad (3.5) \text{ AC}^B, \quad (3.9) \text{ IC}, (3.13) \text{ IR}_{L2}, \quad (3.15) \text{ N}_c. \end{aligned}$$

The observability of initial wealth principally prepares the ground for the exercise of discriminating market power, so again requiring a case differentiation.

**3.5.3.1.1 Case 1: Simple Monopolist** Doing without discriminating agents according to their wealth, a simple monopolist enforces standard debt contracts. While effort contracts comprise a transfer  $t = f(k_L) - r_{L,j} [k_L - w_i] / p$  and a high investment  $k_L = k(r_D)$  s.t.  $pf'(k_L(r_D)) = r_D$ , non-effort contract specify  $t_0 = f(k_L) - r_{L,j} [k_L - w_i] / q$  and  $k_{L0} = k_0(r_D)$  s.t.  $qf'(k_{L0}(r_D)) = r_D$ . (3.9) IC then makes the wealth constraint (3.20) materialize as

$$w_i < \omega(r_{L,j}) := p/r_{L,j} (p - q) - [pf(k(r_D)) - r_{L,j}k(r_D)] / r_{L,j}. \quad (3.29)$$

It is for two reasons that a simple monopolist fixes a higher rationing threshold than competitive banks. First, it sets a comparably higher loan rate and second, a higher amount of credit (as it inflicts a larger firm size on borrowers than those would have chosen on their own).<sup>28</sup>  $r_D < r_{L,j}$  not only ensures that per-contract profits are always non-negative (so that  $\pi(w) = 1$ ), but also that agents prefer to invest their entire wealth into their own project  $w_c = w_i$ .

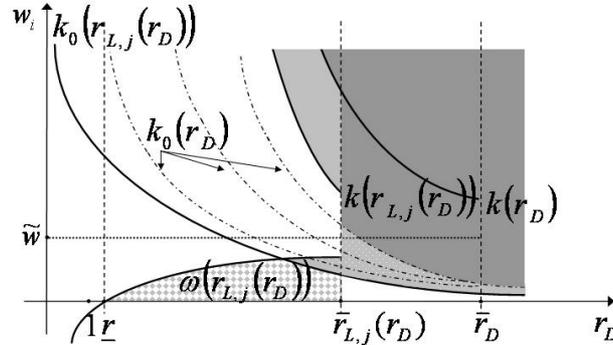


Figure 3.5: Solution to an agent's investment problem given  $r_D$

Qualitatively, the pure existence of a return rate wedge presents agents with a similar contractual decision problem under simple loan monopoly as under simple deposit monopoly. The findings of Subsection 3.5.2 therefore directly allow to construct *Figure 3.5*. Quantitatively, it then remains for the simple monopolist  $j$  to ascertain how much to lift  $r_{L,j}$  above a given  $r_D$  by determining aggregate capital needs based on agents' optimal behavior:

<sup>28</sup>If borrowers decided upon the firm size themselves, they would optimize  $k$  based on  $r_{L,j}$  and, for  $r_D \leq \bar{r}_{L,j}(r_D)$ , choose  $k(r_{L,j})$  with  $k(r_{L,j}) \leq k(r_D)$  because of  $r_{L,j} \geq r_D$ .

(i) For  $1 \leq r_D \leq \bar{r}$ , all agents exert effort. A share of agents  $G(k(r_{L,j}))$  leverages  $k(r_D)$ , whereas  $[1-G(k(r_D))]$  self-finances  $k(r_D)$  and deposits any remainder. All others autarkically invest  $w_i$ .

(ii) For  $\bar{r} < r_D \leq \bar{r}_L$ , there is some credit rationing. Therefore, a fraction of agents  $G(\min\{\omega(r_{L,j}), k_0(r_{L,j})\})$  is left to leverage non-effort firms of size  $k_0(r_D)$  only. While  $[G(\omega(r_{L,j})) - G(\min\{G(\omega(r_{L,j})), k_0(r_D)\})]$  self-finances non-effort firms of size  $k_0(r_D)$  and deposits the remainder,  $[G(\min\{\omega(r_{L,j}), k_0(r_D)\}) - G(k_0(r_{L,j}))]$  runs autarkic non-effort firms of size  $w_i$ .  $[1-G(k(r_D))]$  diligently self-finances  $k(r_D)$  and deposits the rest. Finally,  $[G(k(r_{L,j})) - G(\omega(r_{L,j}))]$  leverages  $k(r_D)$ . All others autarkically and diligently invest  $w_i$ .

(iii) For  $\bar{r}_L < r_D \leq \bar{r}_D$ , leveraged effort firms do not pay. That is why a share of agents  $G(k_0(r_{L,j}))$  shirkingly leverages  $k_0(r_D)$ .  $[G(k_0(r_D)) - G(\min\{\tilde{w}, k_0(r_D)\})]$  runs autarkic effort and  $[G(\min\{(k_0(r_D)), \tilde{w}\}) - G(k_0(r_{L,j}))]$  autarkic non-effort firms of size  $w_i$ . Depositing any remainder,  $[1-G(k(r_D))]$  diligently self-finances  $k(r_D)$  and  $[G(k(r_D)) - G(k_0(r_D))]$  shirkingly self-finances  $k_0(r_D)$ .

(iv) For  $r_D > \bar{r}_D$ , all agents shirk. A share of agents  $G(k_0(r_{L,j}))$  leverages  $k_0(r_D)$ , whereas  $[1-G(k_0(r_D))]$  self-finances  $k_0(r_D)$  and deposits any remainder. All others autarkically invest  $w_i$ .

Sectionwisely summing up loan demand  $D_j(r_{L,j})$  allows to rewrite the object function in *Definition 3.6* as

$$\max_{r_{L,j}} \Pi_j = [r_{L,j} - r_D] D_j(r_{L,j}). \quad (3.30)$$

It displays the monopolist's trade-off: increasing the loan rate  $r_{L,j}$  at the cost of not only minimizing the pool of loan applicants (think alone of  $d\bar{k}_2/dr_{L,j} < 0$ ), but also worsening the mix of loan applicants ( $\omega'(r_{L,j}) > 0$ ). Effort provision pays off for fewer agents than before, so that the monopolist diminishes their capital input by  $k(r_D) - k_0(r_D)$ .<sup>29</sup> Therefore,  $D'_j(r_{L,j}) < 0$  for all  $r_{L,j}$ . The interior solution satisfies the FOC  $d\Pi_j/dr_{L,j} = 0$ . It simplifies to an intuitive pricing formula for choosing  $r_{L,j}^*$  as a function of  $r_D$ :

$$r_{L,j}^* + D_j(r_{L,j}^*)/D'_j(r_{L,j}^*) = r_D. \quad (3.31)$$

Thus, the monopolist's loan demand soars until the marginal total return from lending out an additional unit of loan (*LHS*) just equals  $r_D$ , i.e. the given cost of raising an additional unit of deposit (*RHS*).<sup>30</sup> As  $D_j(r_{L,j}^*)/D'_j(r_{L,j}^*) < 0$ , the monopolist always realizes a positive, wealth-dependent intermediation margin - even with the marginal borrower. (3.4)  $IR_L^B$  holds, so ensuring that all borrowers get financed ( $\pi(w) = 1$ ). However, in view of the downward sloping demand curve  $D_j(r_{L,j})$ , an increase in the loan volume must be accompanied by an increase in the number or size of leveraged firms. For both,  $r_{L,j}$  must decline. Yet, as it equally applies to all units of credit, each further marginal unit of credit costs less than the average. For the marginal loan return to really be decreasing, it must be assumed that the loan demand is convex in  $r_{L,j}$ .<sup>31</sup>

$$\text{Assumption 4: } d[r_{L,j} + D_j(r_{L,j})/D'_j(r_{L,j})]/dr_{L,j} < 0 \quad (\text{A4})$$

<sup>29</sup>If  $r_{L,j} > \bar{r}_L$ , the second effect is missing, while the first is still present ( $d\bar{k}_1/dr_{L,j} < 0$ ).

<sup>30</sup>Being an atomistic price-taker on the deposit market, the bank can always satisfy (3.5) AC.

<sup>31</sup>(A4) yields  $2 - D_j(r_{L,j}) D''_j(r_{L,j}) / [D'_j(r_{L,j})]^2$  and is negative if  $D''_j(r_{L,j}) > 0$ .

**Lemma 3.9** For a given deposit rate  $r_D \geq 1$ , the solution to the **simple monopolist's problem** is to set a probability of financing  $\pi^*(w) = 1$ , a loan rate  $r_{L,j}^* = r_D - D_j(r_{L,j}^*)/D_j'(r_{L,j}^*)$ , capital input levels  $k_L^* = k(r_D)$  and  $k_{L0}^* = k_0(r_D)$ , loan contracts  $C_L(w) = (w_i, f(k(r_D)) - r_{L,j}^*[k(r_D) - w_i]/p)$  as well as  $C_{L0}(w) = (w_i, f(k(r_D)) - r_{L,j}^*[k(r_D) - w_i]/q)$  and a minimum equity contribution  $\omega(r_{L,j}^*) = p/r_{L,j}^*(p - q) - [pf(k(r_D)) - r_{L,j}^*k(r_D)]/r_{L,j}^*$ .

Furthermore, rewriting (3.31) gives the Lerner index of market power, according to which a percentage markup of price over marginal costs equals one divided by the negative elasticity of demand  $\eta$ , i.e.

$$[r_{L,j}^* - r_D]/r_{L,j}^* = D_j(r_{L,j}^*)/r_{L,j}^*D_j'(r_{L,j}^*) = -1/\eta. \quad (3.32)$$

That is why the intermediation margin, and thus the deviation from the competitive price  $r_C^*$ , increases with  $|\eta|$ , i.e. with a more inelastic and thus flatter demand schedule.<sup>32</sup> Finally, (3.26) and  $p/q > r_L/r_D$  for  $r_{L,j} < \bar{r}_L$  restrict the percentage markup over price to

$$1/\eta < -(p - q)/p \quad (\text{whereby } 0 > -(p - q)/p > -1).$$

**3.5.3.1.2 Case 2: Discriminating Monopolist** Not having to take into account the price-volume relation of the simple monopolist, a discriminating monopolist adapts the loan rate to the refinancing cost of  $r_D$  that it takes as given from the national deposit market:  $r_{L,j}^* = r_D$ . (A1) applies to self-financing and leveraged entrepreneurs alike, so that for e.g.  $r_D \leq \bar{r}$ , agent  $i$  with  $w_i < k(r_D)$  is put in the position of a potential borrower. For him, the monopolist endogenously derives contractual agreements  $C_L(w)$  and  $C_{L0}(w)$ .

Having all the bargaining power and wanting to leave as few as possible to borrowers, the discriminating lender chooses the maximal feasible initial contribution and the smallest feasible transfer. He therefore makes (3.15)  $N_c$  and (3.13)  $IR_{L2}$  just binding and so comes up with a take-it-or-leave-it offer to borrowers:  $k_L = k(r_D)$ ,  $w_c = w_i$  and  $t = f(w_i)$ . The bank appears as the residual claimant of all marginal returns from leveraged projects. But substituting this loan contract into (3.9)  $IC$  gives  $f(w_i) \geq 1/(p - q)$  or

$$w_i \geq \tilde{w} := f^{-1}(1/(p - q)). \quad (3.33)$$

Accordingly,  $t_{w_i \geq \tilde{w}} = f(w_i)$  only induces effort provision by borrowers who can put up  $w_i \geq \tilde{w}$ . All others shirk, which leaves the bank with three options for them: (i) assign  $k(r_D)$  and incite effort with  $t_{w_i < \tilde{w}} = 1/(p - q)$  in order to satisfy (3.9)  $IC$ ; (ii) assign  $k_0(r_D)$  and accept shirking with  $t_{w_i < \tilde{w}, 0} = f(w_i) < t_{w_i < \tilde{w}}$  in order to further only satisfy (3.13)  $IR_{L2}$  or (iii) deny a contract. As already  $\Pi_{L,j i 0} \geq 0$ , the third option deprives the bank of non-negative profits, which induces it to maximize the occurrence of financing (i.e.  $\pi(w) = 1$ ). Given  $r_D \leq \bar{r}$ , bank  $j$  prefers the first to the second option for agent  $i$  with  $w_i < \tilde{w}$ , if it generates more per-contract

<sup>32</sup>The monopoly result qualitatively also holds for a Cournot **oligopoly**. Then, the *RHS* of (3.32) is multiplied with the squared sum of market shares, implying that the higher the Herfindahl concentration measure, the greater deviations from the competitive outcome.

profits,<sup>33</sup> i.e. if  $\Pi_{L,ji} \geq \Pi_{L,ji0}$  or

$$\begin{aligned} & pf(k(r_D)) - r_D[k(r_D) - w_i] - p/(p-q) \\ & \geq qf(k_0(r_D)) - r_D[k_0(r_D) - w_i] - qf(w_i). \end{aligned} \quad (3.34)$$

On top exploiting that  $p/(p-q) = pf(\tilde{w}) = qf(\tilde{w}) + 1$ , yields

$$y(r_D) - y_0(r_D) \geq 1 + q[f(\tilde{w}) - f(w_i)]. \quad (3.35)$$

We see that the lower  $r_D$  and the higher  $w_i$  respectively, the rather (3.35) holds for  $w_i < \tilde{w}$ .<sup>34</sup> Rearranging (3.35) gives

$$\psi(r_D) := w_i \geq f^{-1}(f(\tilde{w}) - [y(r_D) - y_0(r_D) - 1]/q) \quad (3.36)$$

as the critical wealth level  $\psi(r_D) \leq \tilde{w}$ , above which the bank offers effort contracts to  $w_i < \tilde{w}$ .  $\psi(r_D)$  behaves like (3.20)  $\omega(r_D)$  and also cuts the abscissa for some  $r_\psi > 1$ . As however  $r_\psi < r$  (so that  $\omega(r_\psi) < 0$ ) and  $\psi(\bar{r}) = \tilde{w}$  (so that  $\psi(\bar{r}) > \omega(\bar{r})$ ), we get that  $\psi(r_D) > \omega(r_D)$  for all  $r_D \in (r_\psi, \bar{r}]$ . Overall, a discriminating monopolist exacerbates the rationing threshold relative to competitive banks. The reason is that the discriminating monopolist's return from an effort contract with an agent, who would not provide effort in an autarkic firm, is increasing in his wealth. Like the social planner, the discriminating monopolist also seeks to avoid excessive aggregate effort provision among the entrepreneurs it finances, since it keeps their marginal project returns. Yet unlike the social planner and irrespective of the wealth distribution, it rations poor borrowers already on a per-contract basis for  $r_D \in (r_\psi, \hat{r}_p]$ . This owes to the fact that it fails to internalize the capital reallocation benefit stemming from the equalization of MPCs across firms with a concave production function.

The solution to agents' contracting problem can nevertheless be read from *Figure 3.3*, but with  $\psi(r_D)$  instead of the correlative  $\omega$  curve. All in all, the bank behaves as follows:

**Lemma 3.10** *For a given deposit rate  $r_D \geq 1$ , the solution to the **discriminating monopolist's problem** is  $r_{L,j}^* = r_D$ ,  $w_c^* = w_i$ ,  $\pi^*(w) = 1$  and:*

(i) *If  $r_D \leq r_\psi$  (with  $r_\psi < r < \hat{r}_p$ ), assign borrowers  $i$  with  $w_i < k(r_D)$  effort firms of size  $k_L^* = k(r_D)$  only and pay  $t_{w_i \geq \tilde{w}}^* = f(w_i)$  to those with  $\tilde{w} \leq w_i < k(r_D)$  and  $t_{w_i < \tilde{w}}^* = 1/(p-q)$  to all others.*

(ii) *If  $r_\psi < r_D \leq \bar{r}$ , implement rationing: only borrowers  $i$  with  $w_i \geq \psi(r_D)$  are assigned effort firms of size  $k_L^* = k(r_D)$ , of whom those with  $w_i \in [\tilde{w}, k(r_D))$  get  $t_{w_i \geq \tilde{w}}^* = f(w_i)$  and all others  $t_{w_i < \tilde{w}}^* = 1/(p-q)$ . Borrowers  $i$  with  $w_i < \psi(r_D)$  are assigned non-effort firms of size  $k_{L0}^* = k(r_D)$  and get  $t_{w_i < \tilde{w}, 0}^* = f(w_i)$ .*

(iii) *If  $r_D > \bar{r}$ , assign borrowers  $i$  with  $w_i < k_0(r_D)$  non-effort firms of size  $k_{L0}^* = k_0(r_D)$  only and pay  $t_0^* = f(w_i)$ .*

<sup>33</sup>The discriminating monopolist decides on a per-contract basis. Taking  $r_D$  as given, it neither faces a procurement problem, nor does it internalize effort firms' pecuniary externality.

<sup>34</sup>It directly follows from (A1) that (3.35) holds for all  $w_i \geq \tilde{w}$ .

### 3.5.3.2 Capital Market Equilibrium and Aggregate Output

National deposit market clearing  $S(r_D^*) = nD_j(r_{L,j}^*(r_D^*))$  determines an equilibrium deposit rate  $r_D^*$ . Owing to persistent asymmetric information, there is positive credit rationing. Hence,  $r_D^*$  and aggregate output depend on aggregate wealth  $W$  and its dispersion across agents  $G(w)$ .

Beyond that, a **simple monopoly** drives a wealth-distribution-dependent wedge between the deposit and the loan rate  $r_{L,j}^*(r_D^*) - r_D^* \geq 0$ . As compared to the competitive outcome in Subsection 3.5.1, the spread induces autarkic production at inefficient scales and makes the  $MPC^e$  fluctuate across firms. The higher loan rate deteriorates credit rationing, whereas the lower deposit rate scales up leveraged and self-financed firms. All that dampens aggregate output. While the impact of banking market power on agents' incomes is similar to that under simple monopsony derived in Subsubsection 3.5.2.2, firm sizes vary less (as the bank makes leveraged firms' sizes dependent on its own refinancing costs  $r_D$ ). At last, higher wealth inequality c.p. tends to reduce the relative magnitude of the return rate spread. Distributive policies augmenting the mass of agents above the critical wealth threshold make deposits relatively scarcer and put an upward pressure on  $r_D$ . This directly leads to an equal increase in  $r_{L,j}^*$  ( $dr_{L,j}^*(r_D)/dr_D = 1 > 0$ ) and depresses (3.32). This improves the output loss resulting from monopoly power, so that the outcome tends towards the competitive result in Subsection 3.5.1. However, the return wedge impedes first-best efficiency when there is no credit rationing and constrained efficiency when there is credit rationing, except for in a unique parameter constellation. As specified below, constrained efficient aggregate surplus can then be achieved or even surpassed. There is again scope for distributive measures, as these affect the fraction of agents who cannot comply with the crucial wealth requirement and thus get rationed.

A **discriminating monopolist**, instead, involves no return rate spread and no autarkic firms, so that the  $MPC^e$  gets equalized across production units. Even though only moral hazard frictions remain, the equilibrium only qualitatively mimics the competitive equilibrium summarized in *Proposition 3.3*. In quantitative relations, the rationing threshold gets reinforced, so that  $\psi(r_D) > \omega(r_D)$  dampens the equilibrium rate of return, aggregate output and aggregate effort costs. Besides, the allocation of the output differs. While the total borrower rent accrues to the bank, depositors earn lower capital, but higher project incomes. Taken altogether, the allocation does not [resp. does] reach constrained [resp. first-best] efficiency in the presence [resp. absence] of credit rationing. Even though the appropriation of all marginal project returns makes the discriminating monopolist improve on the constrained outcome for  $r_D^* \in (r_\psi, \bar{r})$ , it still cannot always automatically achieve first-best. In fact, calculating on a per-contract basis prevents it from taking into account the wealth distribution and from internalizing effort firms' pecuniary externality on the return rate that it takes as given.

**Proposition 3.5** *In a capital market equilibrium with moral hazard, a competitive deposit and monopoly loan market, the deposit and loan rate  $r_D^*$  and  $r_{L,j}^*$  are as follows:*

(i) *In case of a simple monopolist, the return rate wedge  $r_{L,j}^*(r_D^*) - r_D^* > 0$  and  $r_D^*$  s.t.  $S(r_D^*) = nD_j(r_{L,j}^*(r_D^*))$  depend on aggregate wealth  $W$  and its distribution  $G(w)$ . The allocation neither reaches first-best, nor constrained efficiency - except for if  $r_D^* \in (r, \bar{r}_{L,j})$  [resp.  $r_D^* \in (\bar{r}_{L,j}, \bar{r}_D]$ ] and if there are too few non-effort firms under competition relative to*

first-best  $G(\omega(r_C^*)) < \hat{\alpha}(W)$ . Then, constrained-efficient aggregate surplus obtains if efficiency gains from the higher share of non-effort firms  $G(\omega(r_{L,j}^*(r_D^*))) > G(\omega(r_C^*))$  [resp.  $G(\tilde{w}) > G(\omega(r_C^*))$ ] outweigh efficiency losses from the return rate wedge.

(ii) In case of a **discriminating monopolist**, Proposition 3.3 applies, but with  $\psi(r_D) > \omega(r_D)$ : If either  $pf'(W) \leq r_\psi$  (with  $r_\psi < r < \hat{r}_p$ ) or  $qf'(W) \geq \bar{r}$ , the allocation reaches first-best efficiency. Otherwise, it improves on constrained efficiency, but only reaches first-best efficiency if  $G(\psi(r_D^*))$  equals the first-best share of non-effort firms  $\hat{\alpha}(W)$ .

### 3.5.4 Discriminating Monempory

In the light of the previous results, we now immediately confine our attention to a discriminating monemporic bank.

#### 3.5.4.1 Monempory Financial Contracts and Individual Decisions

As the monemporist is the only bank in the economy, it perfectly controls contract conclusion with agents on the deposit and loan market alike.

**Definition 3.7** Given aggregate wealth  $pf(W) \leq \bar{r}$ , self-financers' optimal investments  $k(r)$  and  $k_0(r)$ , beliefs  $B(\hat{C}(w))$  and Lemma 3.3, deposit contracts  $C_D(w) = (R(w))$  as well as effort and non-effort loan contracts  $C_L(w) = (w_c, t)$  and  $C_{L0}(w) = (w_c, t_0)$  are **discriminating monempory equilibrium contracts** if they solve the following problem of the bank: maximize total expected profits  $\Pi$  by setting a cut-off return rate  $r$  to separate depositors  $i$  with  $w_i \in (k(r), \bar{w}]$  from borrowers with  $w_i \in [0, k(r))$ . On the deposit side, choose a total depositor remuneration  $R(w)$ . On the loan side, fix a wealth constraint  $v(r)$  and contributions  $w_c$ , the identity of borrowers to be leveraged ( $\pi(w) = 1$  and 0 if not), capital effort and non effort inputs  $k_L$  and  $k_{L0}$  as well as effort and non-effort transfers  $t$  and  $t_0$ :

$$\begin{aligned} \max_{r, R(w), v(r), t, t_0, w_c, k_L, k_{L0}, \pi(w)} \Pi = & \int_0^{v(r)} \pi(w) [qf(k_{L0}) - t_0] g(w) dw \\ & + \int_{v(r)}^{k(r)} \pi(w) [pf(k_L) - t] g(w) dw - \int_{k(r)}^{\bar{w}} R(w) g(w) dw \end{aligned} \quad (3.37)$$

$$\begin{aligned} \text{s.t. } (k, e, w_d, w_s) = & B(\hat{C}(w)), (3.3) IR_D^B, (3.4) IR_L^B, (3.5) AC^B, \\ (3.9) IC, (3.10) IR_{D1}, & (3.11) IR_{D2}, (3.13) IR_{L2}, (3.14) N_D \text{ and } (3.15) N_c. \end{aligned}$$

The monemporist's calculus combines insights from Subsections 3.5.2 and 3.5.3: Being a discriminating deposit monopsonist, the monemporist leaves depositors with their outside option only. Thus, Lemma 3.8 applies w.r.t.  $R^*(w)$ . On top being a discriminating loan market monopolist, the monemporist also leaves borrowers with their outside option only. That is why Lemma 3.10 reemerges w.r.t.  $\pi^*(w) = 1$ ,  $k_L^* = k(r)$ ,  $k_{L0}^* = k_0(r)$ ,  $t_{w_i \geq \tilde{w}}^* = t_{w_i < \tilde{w}, 0}^* = t_0^* = f(w_i)$  and  $t_{w_i < \tilde{w}}^* = 1/(p-q)$ . The existence of double-sided market power, however, earns the bank an additional advantage: it can freely choose  $r$ .

Against this background, (3.37) reduces to:

$$\begin{aligned} \max_{r, v(r)} \Pi = & \int_0^{v(r)} [qf(k_0(r)) - f(w)] g(w) dw + \int_{v(r)}^{k(r)} pf(k(r)) g(w) dw \\ & - \int_{v(r)}^{\tilde{w}} 1/(p-q) g(w) dw - \int_{\tilde{w}}^{k(r)} f(w) g(w) dw - \int_{k(r)}^{\bar{w}} R(w) g(w) dw \end{aligned} \quad (3.38)$$

$$\text{s.t. (3.5) } AC^B: W = G(v(r))k_0(r) + [1 - G(v(r))]k(r)$$

Irrespective of the wealth distribution,  $t_{w_i \geq \tilde{w}}^* = t_{w_i < \tilde{w}, 0}^* = t_0^* = f(w_i)$  makes the monemporist always fix the rationing threshold  $v(r)$  at the most at  $\tilde{w}$ . But how much will the monemporist cut  $v(r)$  below  $\tilde{w}$ ? For this to see, emanate from some  $\dot{r}$  and  $v(\dot{r}) = \tilde{w}$ . Then, relax rationing by making a single borrower with  $w_i < \tilde{w}$  open an effort firm instead of a non-effort firm. On the one hand, this generates additional expected gains from effort ( $p > q$ ) and from the larger capital input ( $k > k_0$ ). On the other hand, it causes additional costs from a higher expected transfer  $t_{w_i < \tilde{w}}^* > t_{w_i < \tilde{w}, 0}^*$  as well as from the monemporist's increased financial needs ensued by this additional effort firm ( $k > k_0$ ). Yet, in order to attract more deposits, the monemporist must induce agents to scale down their firm sizes. This requires him to raise  $\dot{r}$  to  $\ddot{r}$  s.t. with  $v(\ddot{r}) > v(\dot{r})$ , it still holds that  $k(\ddot{r}) = [W - G(v(\ddot{r}))k_0(\ddot{r})] / [1 - G(v(\ddot{r}))]$ . This, in turn, sets free a capital reallocation benefit. Owing to the production function's concavity, a higher  $r$  reduces the difference in size between an effort and non-effort firm firm. What follows is a surge in aggregate output  $Y$  and so also in the total output produced from the monemporist's pool of leveraged projects. That is how the discriminating monopolist follows the social planner in internalizing effort firms' pecuniary externality on the return rate and thus on the profitability of effort provision.

Confronting the various effects gives

$$pf(k(\ddot{r})) - \ddot{r}[k(\ddot{r}) - w_i] - p/(p-q) \geq qf(k_0(\dot{r})) - r[k_0(\dot{r}) - w_i] - qf(w_i), \quad (3.39)$$

where  $\ddot{r} > \dot{r}$ . Substituting  $p/(p-q) = qf(\tilde{w}) + 1$ , simplifies (3.39) to

$$y(\ddot{r}) - y_0(\dot{r}) \geq 1 + q[f(\tilde{w}) - f(w_i)]. \quad (3.40)$$

Consequently, the monemporist applies the following strategy: It sorts agents  $i$  with  $w_i < \tilde{w}$  according to their wealth endowments in descending order and, from the top down, assigns effort contracts to agent after agent as long as (3.40) holds.<sup>35</sup> In each round, gradually increasing net effort transfers  $1/(p-q) - w_i$  and cut-off financing costs  $r$  require the bank to adapt investment levels  $k$  and  $k_0$  and to recalculate (3.40). That is how  $v^*(r) = \arg \max \Pi$  arises as the cut-off wealth level for which (3.40) just becomes binding.<sup>36</sup>  $v^*(r)$  becomes zero for  $r_v$  with  $r_v \leq \hat{r}_p$ . On these grounds, we obtain the optimal cut-off return rate  $r_{MM}^*$  as the return rate that ensures the satisfaction of the availability constraint (3.5).<sup>37</sup>

**Lemma 3.11** *The solution to the **discriminating monemporist's problem** is to behave, on the deposit side, as characterized in Lemma 3.8 and, on the loan side, as described in Lemma 3.10 - although for the following refinement: additionally choose  $v^*(r) = \arg \max \Pi$  and  $r_{MM}^*$  s.t. (3.5)  $AC^B$  holds.*

<sup>35</sup>The comparison of (3.40) with (3.35) shows that a double-sided discriminating monopolist sets a higher rationing threshold than a one-sided, because it does not take  $r$  as given.

<sup>36</sup>The respective FOC is:  $d\Pi/dv = -[pf(k(r)) - qf(k_0(r)) + f(v(r)) - f(\tilde{w})]g(v(r))v'(r) + r[k'(r) + k'_0(r)]g(w) + pf(k(r))k'(r)g(v(r)) = 0$ .

<sup>37</sup>If at the edge, there are several agents with equal wealth, but only some of them shall get effort contracts, the bank (in the sense of *Definition 3.1*) picks each agent with equal probability.

On the one side, there is an upper bound of rationing:  $v^*(r_{MM}^*) \leq \tilde{w}$ . On the other side, it follows from (3.40) that  $v^*(r_{MM}^*)$  is the lower: (i) the larger the capital reallocation benefit (i.e. the steeper  $f$ ), (ii) the lower the effort-inducing transfer  $1/(p-q)$ , (iii) the higher the wealth of borrowers  $i$  with  $w_i < \tilde{w}$  for whom the participation-inducing transfer is lower than the effort-inducing transfer (i.e.  $f(w_i) < 1/(p-q)$ ), (iv) the higher  $W$  and (v) the higher the wealth of depositors  $i$  with  $w_i > k(r)$  (as that lowers total deposit and thus refinancing costs).

### 3.5.4.2 Capital Market Equilibrium and Aggregate Output

Simultaneously controlling both sides of the capital market allows the monemporist to choose  $r_{MM}^*$  and so to equalize the capitalization of all firms. In view of the effort costs it bears via effort-inducing transfers, it chooses a positive rationing level. Depending on the parameter constellation and as compared to the competitive outcome in Subsection 3.5.1, there can be more, just the same or less credit rationing with the accordant impact on the equilibrium market rate of return and aggregate output.

Yet unlike any other bank, the monemporist is able to take advantage of the capital reallocation benefit owing to  $f$ 's concavity and of the internalization of effort firms' pecuniary externality on aggregate effort provision via their impact on the equilibrium return rate. This makes the monemporist choose a scope of credit rationing  $G(v^*(r_{MM}^*))$  that improves on the constrained efficient level  $G(\omega(r_C^*))$  w.r.t. aggregate surplus.  $G(v^*(r_{MM}^*))$  converges towards the first-best efficient level  $\hat{\alpha}(W)$  and, depending on the wealth distribution, might even attain  $\hat{\alpha}(W)$ . That is how first-best efficiency might be restored by discriminating double-sided market power despite asymmetric information and a coordination failure among borrowers. Having to respect agents' incentive and participation constraints, the monemporist essentially behaves like a discriminating non-omniscient, non-repressive social planner.

**Proposition 3.6** *In a capital market equilibrium with moral hazard and a discriminating monemporist, the return rate  $r_{MM}^*$  is as follows:*

(i) *If either  $pf'(W) \leq r_v$  (with  $r_v \leq \hat{r}_p$ ) or  $qf'(W) \geq \bar{r}$ , first-best efficiency obtains:  $r_{MM}^*$  only depends on aggregate wealth  $W$  and Proposition 3.1 reapplies accordingly.*

(ii) *Otherwise, there is credit rationing  $v^*(r_{MM}^*) > 0$ , which makes  $r_{MM}^*$  dependent on aggregate wealth  $W$  and its distribution  $G(w)$ . A share of agents  $G(v^*(r_{MM}^*))$  shirkingly invests  $k_0(r_{MM}^*)$ , whereas all others diligently invest  $k(r_{MM}^*)$ . Irrespective of  $G(w)$ , the allocation improves on constrained efficiency captured by the share of non-effort firms  $G(\omega^*(r_C^*))$ . Although contingent on  $G(w)$ , the allocation tends towards or even reaches first-best efficiency captured by the share of non-effort firms  $\hat{\alpha}(W)$ . Thus,  $|\hat{\alpha}(W) - G(v^*(r_{MM}^*))| < |\hat{\alpha}(W) - G(\omega^*(r_C^*))|$ .*

Yet, efficiency gains do not come for free, but at the cost of agents being left with their respective reservation payoffs only. Compared to all other banking regimes, monemporistic bank profits are the highest and agents' incomes the lowest. As average deposit costs are lower than  $r_{MM}^*$  and decreasing in inequality, total deposit costs are the lower, the more unequal the distribution of wealth. With total loan costs similarly declining with richer self-financers, all agents' incomes are lower than under any other discriminating regime.

### 3.5.5 Double-Sided Competition

Perceiving banks as intermediaries that, as Stewart (1770) puts it, "buy and sell again from a principle of gain", draws attention to double-sided competition.<sup>38</sup> Banks compete by simultaneously offering deposit *and* loan contracts, whilst ensuring to get *both* on board, depositors *and* borrowers. This makes banks' capacity constraint (3.5) binding and uncovers a chicken-and-egg problem: a bank cannot loan out more funds than it acquired and makes losses if it acquired more funds than it loans out. As derived by Yanelle (1989, 1997) and Stahl (1988), this induces banks to corner one market in order to gain market power on the other. Unlike with single-sided Bertrand competition, the equilibrium may then not be a Walrasian anymore.

Looking at our model through this strategic lens directly reveals that banks' lending activity is constrained by the total volume of available funds (remember (A2)  $W < k(1)$ ). Any intermediary therefore has an incentive to increase the deposit rate in order to capture the whole supply of funds, so emerging as the only supplier of credit. This allows to charge monopoly loan rates and, in turn, to pay higher deposit rates. For this reason, intermediaries overbid each other until profits from intermediation are completely eroded. Only one intermediary survives and is active in the end. Albeit making zero profits, it sets a deposit and a loan rate that both exceed the competitive rate and hence the opportunity costs of funds. This also causes autarkic production. After all, the outcome with a simple monopolist in Subsection 3.5.3 arises,<sup>39</sup> but with a different division of the surplus: bank profits are zero and the total surplus accrues to depositors at the cost of creditors.

However, there are ways to disentangle the two markets in order to restore the Walrasian outcome (and hence the outcome of Subsection 3.5.1). Ueda (2006) finds that the existence of an interbank market and the possibility of free recontracting break the crucial link between the source and the use of funds. By placing any excess deposits or raising deficit funds for credit, it allows banks to smoothly satisfy their capacity constraint. Alternatively, one could assign the task of a price-anchor to a state-owned bank. This requires endowing it with sufficient funds for preventing the  $r_D$ - $r_L$ -bidding-up-spirale from occurring. They deprive private banks of the much needed margin for being able to bid-up  $r_D$  (and in turn  $r_L$ ). In fact, the state-owned bank could still loan out funds at rate  $r_L = r^*$  and subsidize the acquisition of funds at rate  $r_D > r^*$ .

## 3.6 Comparison and Policy Implications

The analysis offers valuable insights into the functioning of capital markets, especially for  $r \in (\underline{r}, \bar{r})$ . The framework also allows to derive and evaluate policies that are of vital interest to policy makers and regulators. Yet with outcomes crucially hinging upon a subtle interplay of bank competition and wealth inequality, any policy advice can only be specific to either a particular banking regime or the prevailing wealth distribution.

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<sup>38</sup>As in Subsection 3.5.1, hitherto models typically study Bertrand competition on both markets separately, respectively taking the outcome of the other market as given.

<sup>39</sup>If banks on top perfectly price-discriminated, the monemporist's allocation in Subsection 3.5.4 would reemerge. Yet, all bank profits would accrue to depositors only.

### 3.6.1 Credit Rationing, Firm Sizes and Efficiency

In comparison to competition, simple market power -be it on the deposit or loan side- translates into higher loan rates and so further aggravates **credit rationing**. High inequality and discriminating market power, instead, may alleviate these effects. While the first cuts the scope for price setting, the second removes the return rate wedge. After having equalized the deposit and loan rate, a discriminating deposit monopsony rations credit as much as a competitive bank, whereas a discriminating loan monopolist still rations more. There are, however, parameter constellations, under which the juxtaposition of discriminative power on both sides of the market results in a first-best level of credit rationing and so implements first-best effort provision. Yet, unlike under first-best, the identity of effort entrepreneurs is wealth-dependent.

Besides that, agents' willingness and ability to borrow determine **firm sizes**.<sup>40</sup> Informational frictions cause credit rationing, which makes an effort and a non-effort firm size coexist - just as under first-best for market return rates  $r_C^* \in (r, \bar{r})$ . After having added simple market power (or intermediation costs), the dispersion of firm sizes soars. The return rate spread not only incites different capitalizations of self-financed and borrowing firms, but also encourages the formation of autarkic firms of size  $w_i$ . As discriminating market power involves no return rate spread, there are no autarkic firms. Firm size variations subside to the competitive level again.

Taken altogether, the existence of asymmetric information causes credit rationing under competition. In substance, it works on efficiency like the social planner's quotas for non-effort firms. The allocation reaches constrained, but not first-best **efficiency**.<sup>41</sup> Simple market power regimes generally fall even short of achieving constrained efficiency (with monopsony power being worse than monopoly power). Under certain parameter constellations, however, credit rationing -especially if reinforced by banking market power- helps to improve allocative efficiency since it boosts aggregate surplus. Contrariwise, a discriminating double-sided monopoly takes into account the wealth distribution and so always improves on constrained efficiency. If it fails to restore first-best efficiency, no other banking regime can do so without the correlative distributive measures.

### 3.6.2 Incomes, Equity Concerns and Distributive Policies

Although high efficiency regimes create more aggregate surplus, they counteract high and fairly distributed individual incomes. Banking market power cuts agents' **incomes**. Due to a lower outside option, it leaves the poor relatively more impoverished than the rich. That is how a role for the state arises from the necessity to accompany the assignment of market power by distributive policies of bank profits. Under certain conditions, it might pay off to establish

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<sup>40</sup>Different firm sizes are in line with a core insight from the corporate finance literature: a firm's value also depends on the way it is financed (see e.g. Harris and Raviv, 1991).

<sup>41</sup>Remark that e.g. Piketty (1997) or Aghion and Bolton (1997) target efficiency in terms of aggregate output. Then, moral-hazard-induced credit rationing and banking market power have adverse effects only as they both refrain aggregate output. This, however, owes to the neglect of effort as a costly production input, so that aggregate output indeed fails to capture productive efficiency across all production inputs.

a state-owned bank as a discriminating monempory.<sup>42</sup> Indeed, we saw that the monemporist behaves like a discriminating social planner, who has neither access to perfect information, nor the power to force agents to participate. It could not only improve on efficiency in order to enlarge the pie,<sup>43</sup> but also redistribute its profits via lump-sum transfers back to the agents. This would on top meet **equity** concerns.

Moreover, this paper's results imply that the impact of **political wealth redistribution** on efficiency depends on the banking regime. Its use and design have to be geared to stimulate aggregate surplus by making the scope of credit rationing copy the first-best quota of non-effort firms. That is how aggregate effort costs can appropriately be managed. In this context, a reduction in inequality (i.e. in the wealth held by the rich) would help to reduce aggregate effort costs via an increased scope of rationing (and vice versa). Moreover, under simple monopsony and monopoly, a reduction in inequality can be utilized to contain the bank's market power and so to lower the relative magnitude of the return rate wedge. Finally, under discriminating monempory, wealth redistribution from the rich to the very poor decreases the lender's costs of inducing effort and may strengthen the lender's effort contract supply to those otherwise credit-constrained agents.

### 3.6.3 Growth

Dynamizing the model by linking generations via bequests (as done in Chapter 2 of this thesis for a competitive capital market) emphasizes the role of the state outlined above. Credit rationing, banking market power and unredistributed bank profits alike refrain **growth**, since they dampen aggregate output. That hinders the creation and accumulation of wealth within and over generations, which makes inequality become persistent. In the extreme, this might even lead to poverty traps.<sup>44</sup> This paper, instead, qualifies this conclusion, as it shows that high aggregate output might not always be associated with high aggregate surplus. Thus, higher growth might only be achievable at the expense of agents' aggregate income. That is how the competitive banking regime tends to imply inefficient growth because of an inefficient provision of costly effort. Market power might then mitigate these adverse effects - provided that bank profits are completely redistributed to the agents via lump-sum transfers. The outcome would then also quicker attain first-best inequality.<sup>45</sup> After all, any assignment of banking market power loses its efficiency-motivated right to exist, as soon as  $W$  and  $G(w)$  are s.t. competitive banks would always implement the first-best allocation of capital and effort.

In the light of this paper's results, some growth-facilitating policies appear counterproduc-

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<sup>42</sup>Needless to say that if all commonly enumerated deficiencies of state banks were taken into account (above all e.g. an inefficient bureaucracy), these costs would have to be confronted with the benefits created in the context modelled here.

<sup>43</sup>A coexistence of a state-owned discriminating monopoly bank with competitive private banks is pointless. The first would be left with agents that got credit-constrained by private banks and could not break even from serving them. That owes to these agents' outside option: starting non-effort firms, for which they get sufficient funds from private banks. Hence, private banks would have to be legally prevented from offering non-effort contracts, so that the best outside-option of credit-constrained agents would be autarkic production.

<sup>44</sup>Unlike Piketty (1997), we find the existence of credit-constrained net lenders. As they profit of high interest rates, which countervails the poor's pauperization (see Chapter 2 of this thesis).

<sup>45</sup>Inequality will, however, never vanish, because project outcomes are stochastic.

tive. For instance, promoting savings could have fatal effects with an uncompetitive intermediary sector. Similarly, credit market liberalization, which deprives banks of discriminating power, could intensify credit rationing.

### 3.6.4 Bankruptcy Rates

Finally, in order to see that low bankruptcy rates do not necessarily go in hand with high aggregate surplus, consider the aggregate level of bankruptcy  $b$ . Its first-best level amounts to  $\hat{b} = \hat{\alpha}(W)(1 - q) + (1 - \hat{\alpha}(W))(1 - p)$ . With moral hazard in a competitive regime, its constrained-efficient level becomes

$$b_C^*(G(w)) = G(\omega(r_C^*)) (1 - q) + [1 - G(\omega(r_C^*))] (1 - p),$$

which is lower [resp. higher] than first-best for  $\hat{\alpha}(W) > G(\omega(r_C^*))$  [resp.  $\hat{\alpha}(W) < G(\omega(r_C^*))$ ]. Across all banking regimes,  $b$  is negatively correlated with a reduction in the mass of agents that comply with the wealth constraint  $\omega$  and positively with banking market power.<sup>46</sup>

## 3.7 Conclusion

Wealth requirements are often taken as a generic property of credit markets afflicted with asymmetric information. They provide the ground for wealth inequality to hamper the creation of wealth and the spread of opportunities. This paper challenges this view by adding market power as an additional source of friction and by studying their subtle interplay. Thereby, it targets efficiency in terms of aggregate surplus, since -unlike e.g. aggregate output- it properly includes visible and invisible production input costs alike. The findings highlight the sensitivity of the credit rationing literature's typical results to the targeted economic measures as well as to assumptions about the capital market's competitive structure. Consequently, Knight's (1923) causal trilogy of "*inheritance, luck and effort*" must indeed be amended by *banking market power*. Moreover, the paper implicitly explains varying firm sizes and bankruptcy rates.

The World Bank's 2006 World Development Report argues that in the long term, efficiency and equity are best seen as complements, not substitutes. Equity is identified as being "*important not just for its own sake, but also because it can enhance growth and poverty reduction*". This paper theoretically qualifies this view. It does not only help to put into perspective many commonly propagated policies, but provides a new and concrete way of mitigating the underlying problem: imposing an appropriate competitiveness structure of the capital market, whilst taking care of the allocation of eventual bank profits back to the agents. Moral-hazard-induced credit rationing thereby unfolds an efficiency-enhancing potential as it contains aggregate effort provision and thus the aggregate costs of the unobservable production input. Already under competition, the scope of credit rationing was found to be the more pronounced, the smaller aggregated wealth, the more unequal the distribution of wealth, the steeper the production function and the higher moral hazard costs. Simple banking market power tends to further

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<sup>46</sup>Also Buraschi and Hao (2004) obtain that simple monopoly leads to less lending and higher entrepreneurial bankruptcy rates than competition.

tighten rationing, but also entails a deadweight loss owing to a return rate wedge. The net effect on efficiency depends on the parameter constellation and the wealth distribution. Distributive policies might therefore help to manage aggregate effort provision and so to foster aggregate surplus. While discriminating monopsony market power restores constrained efficiency of the competitive banking regime, discriminating monopoly power improves on it. The scope of improvement, however, crucially depends on the wealth distribution. In contrast, benefitting from a capital reallocation effect and internalizing a pecuniary externality of effort firms, a monempirist is induced to fully take into account the wealth distribution. Among all banking regimes, it so improves the most on constrained efficiency for any given wealth distribution and is the most likely to attain first-best efficiency.

How to incorporate further aspects of financial intermediation into this setting -e.g. multi-tool contracts (i.e. contracts that also include e.g. project monitoring), adverse selection problems (e.g. heterogeneous, but not publicly observable production technologies), additional sources of financing (e.g. direct financing, venture capitalists or informal moneylenders), multi-stage financing or relationship lending- remains a challenge that needs to be met in future research. For instance, heterogeneous project qualities would trigger a second-degree price discrimination role for the risky debt contract. In addition to perfectly discriminating agents according to their initial wealth, the monopoly intermediary would then want to design a schedule of debt contracts that induces self-selection by entrepreneurs. Moreover, the integration into a dynamic setting could help to shed more light on the interdependence of the wealth distribution and the banking system. This would not only enhance our understanding of the relation between the real and the financial sector, but also of the evolution of financial systems and the derivation of an optimal dynamic path of regulation.

## Chapter 4

# The Real Consequences of Financial Market Integration when Countries Are Heterogeneous

### 4.1 Introduction

Retrospectively, international financial integration appears to have been the rule rather than the exception over the last centuries. Temporary interruptions mainly arose from the major wars and the Great Depression in the 1930s. While each time, the process was rather spontaneously spurred on anew by the prospect of gains from trade, financial integration has just recently gained a more powerful momentum. Ongoing regional financial integration (i.e. within the EU or ASEAN), GATS negotiations under the auspices of the WTO and the burgeoning emergence of preferential trade agreements (PTAs) including provisions on financial services trade have not only put pressure on the speed of integration, but also on the broadening of the markets involved, in terms of both, geographical scope and the number of financial assets.<sup>1</sup>

Despite this long history and growing interest among economists and policy makers alike, external financial liberalization still constitutes a controversial issue. Based on assumed differences in the marginal product of capital across countries, standard economic theory promises benefits for developing and developed countries alike. While the first use capital inflows to speed up the convergence process, the second enjoy higher returns on capital and risk reduction through enhanced portfolio diversification (see e.g. Stulz, 2005 or Eichengreen and Mussa, 1998). On the other hand, financial openness bears many risks for financial stability and must therefore be accompanied by a range of costly safeguard measures (see e.g. Schmukler, 2003 or Fischer, 1997). All the more is it a matter of dispute that many of the predicted gains in welfare

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<sup>1</sup>Also see e.g. Lothian (2001) and a survey by Chivakul, Cossé and Gerling (2007).

and growth have not always come to pass. As examined by the European Commission (2006), improvements in both, competition and efficiency have been limited despite a fully integrated EU capital market and quasi unrestricted financial services trade since 1996. Also growth (see e.g. the review by Edison et al., 2002) and the associated flows of capital from capital-abundant to capital-scarce countries have picked-up less than expected. Prasad et al. (2003, 2006) obtain that despite very few de jure restrictions to capital movements, effective external financing remains at very low levels in most African countries. With their analysis suggesting a positive correlation between a country's state of financial development and access to foreign financing, they conclude that a low financial development causes a lack of absorptive capacities for capital inflows from abroad.

The purpose of this paper is to identify the origins of lacking absorptive capacities without drawing on differences in the state of financial development or the degree of financial market competition.<sup>2</sup> Attempting to reconcile theory and evidence, it studies the impact of international financial integration coming not only from countries' capital endowments, but also from its distribution among residents in the presence of capital market imperfections. For this purpose, this paper recurs to a simple capital market model featuring a concave production technology and wealth heterogeneity among agents. These seek external financing to optimally capitalize a venture. Yet, with credit relationships being subject to a variety of agency and contractual enforcement problems, the lender can only recover a fraction of the project output if the borrower defaults. The lender therefore requires the borrower to put up a collateral. Although it ensures incentive-compatibility, it also makes the insufficiently wealthy agents credit-rationed. They are denied credit and left to open self-financed firms at suboptimal scales. This dampens aggregate capital demand and depresses the domestic equilibrium market rate of return. When two countries now get financially integrated by mutually allowing their residents to borrow and lend across their common borders without any restrictions, domestic market rates of return get equalized. The associated domestic interest rate change gives rise to either reinforcing or competing forces in the form of a firm size and a credit rationing effect. The first is negatively correlated to the rate of return, whereas the second changes sign. That is why the parameter constellation and the direction of the interest rate change matter for assessing the impact on domestic net credit positions and aggregate productions. Their sum finally gives a country's GNP. Its change serves as an overall measure of the beneficence of financial integration.

Against this background, the paper studies, in which constellations it pays off for countries to pursue financial integration. The main finding is that although it must be overall beneficial, participating countries may still be adversely affected. Consequences occur through two channels: international capital flows and, more unexpectedly, changes in the scope of domestic credit rationing. That is why not only a country's aggregate wealth, but also its distribution matters, especially in comparison to its partner country. After having identified the pattern of international capital flows and the allocation of capital, this paper shows that gains normally

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<sup>2</sup>I thank Fernando Alvarez, Hans Peter Grüner, Luigi Guiso, Hans Peter Lankes, Francesco Lippi, Fabiano Schivardi, Elisabeth Schulte-Runne and Daniele Terlizzese as well as seminar participants at the Ente "Luigi Einaudi" for valuable discussions and helpful comments. Gratefully acknowledged are also the financial support and hospitality of the Ente "Luigi Einaudi" in Rome and the hospitality of the Trade Division in the Policy Development and Review Department at the IMF in Washington D.C.

only appear in a country, if financial integration sufficiently fosters capital exports or reduces the level of efficiency-distorting credit rationing. That is how this paper also offers an explanation of why widely observed large gaps in productivity and income per capita persist across countries despite an equalization of the marginal return (see e.g. Banjee and Duffo, 2005). Moreover, this paper's results are consistent with the consensus view in the literature on growth and convergence that most of the income differences across countries can be attributed to differences in total factor productivity (also see Easterly and Levine, 2001 or Hall and Jones, 1999). In this sense, this paper's drivers are the either un- or equalizing force of the wealth-dependent borrowing constraint and the equalizing force of the diminishing returns technology.

Five policy implications deserve emphasis. Financial integration might have ambiguous welfare effects: first, across and second, within participating countries. Third, an optimal theory of financial services trade liberalization arises, underlining that countries' characteristics might require different approaches to financial integration. Fourth, in order to avoid vicious circles of beggar-thy-neighbor policies, all domestic policies affecting the level of credit rationing must be banned or harmonized in supranational treaties on financial integration. Fifth, credit rationing affects financial stability in integrated financial markets.

Altogether, this paper contributes to a growing literature on the costs and benefits of financial integration. In a calibrated neoclassical model, Gourinchas and Jeanne (2006) receive relative little welfare gains for a typical emerging market country. They conclude that large effects might occur through other channels than capital flows. Others have presented possible explanations for this phenomenon. Economic heterogeneity in the form of differing liquidity across assets is at the root of the dual-liquidity model of emerging-market crisis presented by Caballero and Krishnamurthy (2001). Emphasizing the interaction between domestic and international financial constraints, they show that entrepreneurs in less developed financial markets tend to over-borrow and to under-provision collateral. This decreases foreign lenders' incentives to enter emerging markets and exacerbates the likelihood of financial crisis. Along similar lines, Aoki, Benigno, and Kiyotaki (2006) study how production efficiency depends on the degree of capital account liberalization during the adjustment process after opening up. Whereas von Hagen and Zhang (2006) identify unequal welfare implications to different domestic agents in a small open economy. In order to smooth transition, they suggest a gradual sequencing of policy implementation. Instead, this paper presents credit rationing and its impact on productive efficiency as an additional effect of financial integration. It is therefore most closely related to Matsuyama (2005, 2007). Extending earlier work by Gertler and Rogoff (1990), Barro et al. (1995) and Boyd and Smith (1997), he was one of the first to consequently draw on capital market imperfections as an explanation of why capital may be exported from poorer countries in the South to richer ones in the North. This work however mainly differs in two respects. First, in order to separate the impact of production non-convexities and capital market imperfections, it endogenizes the project size. Second, in order to study the macroeconomic impact of wealth inequality, it allow for heterogeneous agents. This way accounting for the macroeconomic impact of wealth inequality allows to fill a gap in the hitherto literature on financial integration. Empirical support also comes from micro level studies with financial integration being found to affect entrepreneurship, firms' capital costs and financing constraints (see e.g.

Alfaro and Charlton, 2006; Chari and Hery, 2004; Harrison, Love and McMillan, 2004).

In contrast, the paper at hand abstracts from other channels that may affect the impact of financial integration. Among these is e.g. the beneficial effect of risk sharing on the overall efficiency of investment (see e.g. Obstfeld, 1994; Acemoglu and Zilibotti, 1997 or Athanasoulis and van Wincoop, 2000), capital mobility's ability to mitigate the tragedy of the commons on a common pool of resources (see Tornell and Velasco, 1992), policies enhancing openness and competition (see e.g. Detragiache and Demirgüç-Kunt, 1999 or Kaminsky and Schmukler, 2003), foreign lender's impact on the structure of lending contracts (see e.g. Alessandria and Qian, 2005) or the impact of bank specialization on systemic risk via an integrated interbank market (see e.g. Fecht, Grüner and Hartmann, 2007).<sup>3</sup>

The paper is structured as follows. Based on the model presented in Section 4.2, Section 4.3 derives the capital market equilibrium under national autarky. Against this benchmark, Section 4.4 assesses the impact of financial integration for a broad mix of country types. Section 4.5 extracts some policy implications, before Section 4.6 finally concludes. All Proofs are in Appendix C.

## 4.2 The Model

Consider an endowment economy with a single good, which is populated with a continuum of risk neutral agents  $i$  of mass one.

### 4.2.1 Agents, Endowments and Sequence of Events

The economy lasts for three dates. At *date 0*, agents are born as potential entrepreneurs, who are endowed with initial wealth  $w$  and an investment project that requires a non-fixed start-up cost  $k > 0$ . The first is the only source of heterogeneity among agents and assumed to be continuously distributed according to  $G(w)$  on  $[0, \bar{w}] \subseteq \mathbb{R}_+$ . Hence, aggregate wealth is given by  $W = \int_0^{\bar{w}} wg(w) dw$  and equal to average wealth. Aiming at maximizing their lifetime income  $I$ , at *date 1*, agents can resort to the capital market: while some seek to raise further funds for investment, others supply funds. At *date 2*, agents realize the returns of the initiated investment projects and settle financial claims.

### 4.2.2 Production

The production technology  $F(K, L)$  exhibits constant returns to scale with respect to aggregate capital  $K$  and labor  $L$ . All agents are prospective entrepreneurs. They can only work in their own firm and have access to the same technology in order to undertake a single project, so that  $F(K/L, 1) = f(k)$ . It is strictly increasing and concave in the capital-labor ratio  $k = K/L$  (i.e.  $f' > 0$ ,  $f'' < 0$ ). It also satisfies the standard INADA conditions (i.e.  $f(0) = 0$ ,  $f'(+0) = \infty$ ,  $f(\infty) = \infty$  and  $f'(\infty) = 0$ ). Once sunk,  $k$  cannot be recovered.

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<sup>3</sup>A more complete picture of the benefits and costs is e.g. provided by Agénor (2003).

### 4.2.3 Capital Market

Agents can always either remain self-financing entrepreneurs, who simply invest what they own (i.e.  $k = w_i$ ), or costlessly store wealth. A capital market allows agents to smooth their financial needs. On the one hand, there are borrowers, who are entrepreneurs that compete for others' funds in order to leverage their firm's capitalization (i.e.  $k > w_i$ ). On the other hand, there are lenders, who are agents that seek to place funds that they do not want to store or to invest in their own firm (i.e.  $k < w_i$ ).<sup>4</sup>

Given the prevailing market rate of return  $r$ , agents decide on how much to invest in the project and on if to resort to the capital market. With a project profit of  $y(k) = f(k) - rk$ , the optimal investment level, henceforth denoted  $k(r)$ , amounts to

$$k(r) \text{ such that } f'(k(r)) = r. \quad (4.1)$$

Owing to  $f$ 's functional characteristics,  $k(r)$  is strictly decreasing and convex in  $r$ . Also,  $k(r) \rightarrow 0^+$  for  $r \rightarrow \infty$ , ensuring that  $y(k(r)) \rightarrow 0^+$  for  $r \rightarrow \infty$ . Because of the storage option, agents will never invest more than  $k(1)$  or lend for less than  $r = 1$ .

Also lenders take the market rate of return as given when they perfectly compete by their offer of loan contracts. Hence, in equilibrium, only zero-profit contracts will be traded that yield the same return to lenders: from investing  $k(r)$ , a borrowing entrepreneur generates a revenue  $f(k(r))$ , out of which he must pay  $r[k(r) - w]$  to the lender. Yet, capital market efficiency is hampered by agency and enforcement problems. That is why a lender anticipates that in case of the borrower's default on his debt, he would only be able to capture a fraction  $\gamma \in [0, 1]$  of the virtual project output  $f(k)$ .  $\gamma$  can also be interpreted as the capital market's state of development. Moreover, limited liability prevents agents from ending up with negative wealth at date 2. Thus, they cannot lend or invest more than they own or borrow more than they produce.

The economy is closed, so that  $r$  arises from equalizing total capital demand  $D(r)$  and supply  $S(r)$ . Capital is scarce, i.e. aggregate wealth is not sufficient to let all agents make the optimal investment in case of zero capital costs:

$$\text{Assumption 1: } W < k(1). \quad (A1)$$

## 4.3 Equilibrium under National Autarky

Based on individual optimal decisions, the capital market equilibrium is first derived for each country under autarky. It then serves as a benchmark against which the outcome of full financial integration will be assessed.

### 4.3.1 Credit Rationing and Individual Decisions

Given diminishing returns on capital investment and  $r \geq 1$ , an agent  $i$  with wealth  $w_i$  seeks to become a borrower [resp. lender] if investing the last unit of his initial endowment would yield

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<sup>4</sup>Neglect simultaneous borrowing and lending, since no agent can win from it in equilibrium.

a higher [resp. lower] rate of return than that offered by the capital market. In view of the participation constraint of the borrower ( $PC_B$ ) [resp. the lender ( $PC_L$ )]

$$PC_B: f'(w_i) > r \quad [\text{resp. } PC_L: f'(w_i) < 1 \leq r], \quad (4.2)$$

agents  $i$  with  $w_i > k(r)$  will supply  $w_i - k(r)$  at rate  $r$  on the capital market, whereas those with  $w_i < k(r)$  will want to raise  $k(r) - w_i$ . Yet, the latter's willingness to borrow might not be sufficient to do so. Owing to capital market inefficiencies, they can only guarantee the lender the effective rate of return  $r$  if the repayment is smaller than the recoverable output. That is why a debt contract is only incentive compatible ( $IC$ ) if

$$IC: r [k(r) - w] \leq \gamma f(k(r)). \quad (4.3)$$

Solving ( $IC$ ) for the borrower's wealth, gives

$$w \geq \omega(r) := k(r) - \gamma f(k(r))/r. \quad (4.4)$$

$\omega(r)$  represents the borrower's equity participation that the lender requires to break even. It amounts to the difference between the sunk investment  $k(r)$  and the net present value ( $NPV$ ) of the pledgeable project output. As depicted below in *Figure 4.1* and as derived in the Proof of *Lemma 4.1*,  $\omega(r)$  roughly resembles a parabola that opens downwards. It has a maximum at  $r = \dot{r}$ , an inflexion point at  $r = \ddot{r}$  and approaches the abscissa for  $r \rightarrow \infty$ .

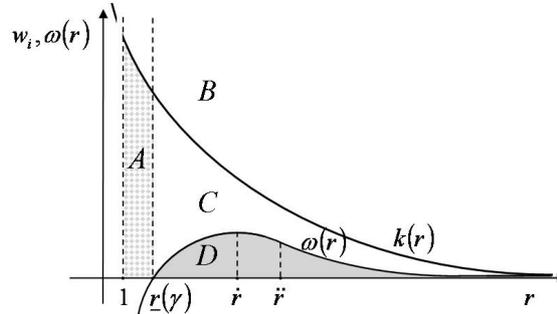


Figure 4.1: Individual investment decisions given  $r$  and  $w_i$

Intuitively,  $\omega(r)$ 's shape stems from two countervailing forces. First, as the fraction of the project return the borrower has to share with his lender is increasing in  $r$ , his incentives to repay the loan fall. This forces the lender to ask for a higher equity participation. Second, the higher  $r$ , the smaller the optimal investment  $k(r)$  the agent is striving for, so that the smaller the required external financing and thus the necessary stake of the borrower. It can be shown that the first effect prevails as long as  $\eta_{y,r} < \gamma/(1-\gamma)$  (and vice versa), where  $\eta_{y,r} > 0$  denotes the input price elasticity of output. Consequently,  $\omega(r)$  is increasing in  $r$  as long as the percentage change in output due to a percentage change in the market rate of return is sufficiently small.<sup>5</sup> Likewise,  $\omega(r)$  is found to become strictly convex as soon as  $\eta_{y,r} > 2/[(1-\gamma)\varepsilon_{k',r}/\gamma - 1]$ ,

<sup>5</sup>Remark that the better the capital market is developed (i.e. the larger  $\gamma$ ), the smaller  $\dot{r}$ .

where  $\varepsilon_{k',r} > 0$  is the factor price elasticity of the optimal investment's slope. Beyond that,  $\omega(r) \rightarrow -\infty$  for  $r \rightarrow 0$ ,  $\omega(\dot{r}) > 0$  and  $\omega$ 's continuity in  $r$  fix a  $\underline{r}(\gamma) \in (0, \dot{r})$  for any  $\gamma > 0$  with  $d\underline{r}(\gamma)/d\gamma > 0$  such that  $\omega(r) < 0$  for  $r < \underline{r}(\gamma)$ . Assume for simplicity that  $\gamma$  and the production technology are such that they ensure:<sup>6</sup>

$$\text{Assumption 2: } \underline{r}(\gamma) > 1. \quad (\text{A2})$$

With regard to  $r$  and  $w_i$ , agents decide as follows. Provided that  $r \geq 1$ , agents  $i$  with  $w_i \geq k(r)$  self-finance the optimal investment  $k(r)$  irrespective of  $r$  (area  $B$ ) and lend any remainder. Then,  $\omega(r) \leq 0$  for  $1 \leq r \leq \underline{r}(\gamma)$ , so that all agents  $i$  with  $w_i < k(r)$  are empowered to open firms at the efficient scale  $k(r)$  (area  $A$ ). Whereas if  $r > \underline{r}(\gamma)$ ,  $\omega(r) > 0$  and only agents  $i$  with  $w_i \geq \omega(r)$  get access to credit (area  $C$ ). All others are credit-rationed (area  $D$ ), i.e. they are denied to tap other agents' funds and find themselves hindered to realize the optimal capitalization level.<sup>7</sup> Note that these agents would have received sufficient credit under first-best (i.e. in the absence of capital market imperfections when  $\gamma = 1$ ). In view of (4.2)  $PC_B$ , credit-constrained agents still prefer running a self-financed firm of size  $k = w_i < k(r)$  to lending or storage. It makes them earn  $y_c(r) = f(w) - rw$ . All in all:

**Lemma 4.1** *For a given market rate of return  $r \geq 1$ , the solution to the individual financial contracting problem has the following properties:*

- (i) *For  $r \leq \underline{r}(\gamma)$ , all agents  $i$  with  $w_i < k(r)$  borrow  $k(r) - w_i$  at rate  $r$ .*
  - (ii) *Whereas for  $r > \underline{r}(\gamma)$ , those with  $\omega(r) \leq w_i < k(r)$  borrow  $k(r) - w_i$  at rate  $r$ , but those with  $w_i < \omega(r)$  are denied credit and therefore start self-financed firms of size  $w_i < k(r)$ .*
- In any case,  $k(r)$  s.t.  $f'(k(r)) = r$  and only agents  $i$  with  $w_i > k(r)$  lend  $w_i - k(r)$  at rate  $r$ .*

### 4.3.2 Capital Market Equilibrium

On these grounds, the capital market equilibrium can be derived.

**Definition 4.1** *A capital market equilibrium consists of a rate of return  $r^*$  and individual decisions as described in Lemma 4.1 such that decisions are optimal given  $r^*$  and gross capital demand  $D(r^*)$  equals supply  $S(r^*)$ .*

While  $S(r)$  amounts to aggregate wealth  $W$  minus the funds devoted to storage,  $D(r)$  equals the sum of all agents' investments intended at rate  $r$ . But owing to (A1),  $r^* > 1$ , which makes storage unattractive and  $S(r) = W$ . Thus, if  $f'(W) \leq \underline{r}(\gamma)$ , first-best arises. All agents get sufficient credit to make the optimal investment  $k(r^*) = W$ . This entails  $r^* = f'(W)$  and aggregate output  $P^* = f(W)$ . Yet, if  $f'(W) > \underline{r}(\gamma)$ ,  $D(r)$  is dampened by credit rationing, so that  $r^*$  also becomes a function of the wealth distribution  $G(w)$ :

$$r^* = r^*(G(w)) \text{ s.t. } W = \int_0^{\omega(r^*)} wg(w) dw + [1 - G(\omega(r^*))] k(r^*). \quad (4.5)$$

<sup>6</sup>Otherwise, there would be credit rationing even if capital costs were zero:  $\omega(1) > 0$  and no area  $A$  existed in Figure 1. If the equilibrium market rate of return was then equal to 1, wealth  $U = W - \int_0^{\omega(1)} wg(w) dw + [1 - G(\omega(1))] k(1)$  would not be used, but get stored.

<sup>7</sup>As  $y(k(r))$  net of repayment is maximal for  $k(r)$ , it follows that if (4.3)  $IC$  does not hold for  $k(r)$ , it will also not hold for any  $k < k(r)$ .

That is how capital market imperfections lead to credit rationing. The rich over- and the poor underinvest. Firm sizes and hence the marginal product of capital vary over production units. This depresses aggregate output  $P^*$ , which also constitutes gross national product (GNP)  $Y^*$  under autarky:

$$P^*(G(w)) = \int_0^{\omega(r^*)} f(w)g(w)dw + [1 - G(\omega(r^*))]f(k(r^*)). \quad (4.6)$$

**Proposition 4.1** *In the autarkic capital market equilibrium, the market rate of return  $r^* \geq 1$  and GNP  $Y^*$  (which equals aggregate output  $P^*$ ) are as follows:*

(i) *If  $f'(W) \leq \underline{r}(\gamma)$ , first-best obtains. All agents make the optimal investment  $k(r^*) = W$ .  $r^*$  and  $Y^*$  only depend on aggregate wealth  $W$ :  $r^* = f'(W) \leq \underline{r}(\gamma)$  and  $Y^* = f(W)$ .*

(ii) *Otherwise, there is some credit rationing  $\omega(r^*) > 0$ . A fraction  $G(\omega(r^*))$  of the agents is credit-constrained and only invests  $w_i$ , whereas all others make the optimal investment  $k(r^*)$ .  $r^*$  and  $Y^*$  depend on aggregate wealth  $W$  and its distribution  $G(w)$ :  $\underline{r}(\gamma) < r^* < f'(W)$  and  $Y^* < f(W)$ .*

### 4.3.3 Firm Size and Capital Rationing Effect

In order to ease the subsequent analysis of financial integration, the analysis goes on with studying the main consequences of a change in  $r^*$ , which are a firm size and a credit rationing effect. This, however, prerequires an inquiry into the origins of a change in  $r^*$ , which can be classified into a net worth and a capital deepening effect.

The net worth effect captures any influence on agents' ability to comply with (4.4)  $\omega(r)$  and so to make the optimal investment  $k(r)$ . For instance, a higher  $\gamma$  increases the NPV of borrowers' projects and therefore lowers the critical threshold  $\omega(r)$ . Likewise, higher inequality boosts  $[1 - G(\omega(r))]$ , i.e. the mass of agents with  $w \geq \omega(r)$ .<sup>8</sup> Both result in an enhanced credit allocation, which in turn improves productive efficiency and so implies a higher  $r^*$ .

Per contra, although a higher  $W$  makes agents benefit of the net worth effect, it additionally releases a capital deepening effect. According to the latter, an increase in aggregate capital supply translates into a surge in investment, which results in lower  $r$  due to diminishing returns. Hence, the impact of  $\Delta W > 0$  on  $r^*$  depends on which of the two countervailing effects prevails. More directly, check if in equilibrium,  $\Delta D(r^*) > 0$  required by  $\Delta W > 0$  is achieved through a rise or fall in  $r^*$ . Differentiating the RHS of (4.5) and reformulation gives

$$dD/dr \geq 0 \quad \text{if} \quad g(\omega(r))\omega'(r) \leq -\frac{[1 - G(\omega(r))]k'(r)}{\omega(r) - k(r)}. \quad (4.7)$$

For  $r^* \leq \underline{r}(\gamma)$ , credit rationing is absent. A higher rate of return then only causes a firm size effect due to diminishing returns:  $k'(r) < 0$ . Smaller optimal firm sizes lessen gross capital demand (i.e.  $dD/dr < 0$ ). In contrast, a credit rationing effect additionally accrues for  $r^* > \underline{r}(\gamma)$ :  $\omega'(r) > 0$  if  $r < \hat{r}$  and  $\omega'(r) \leq 0$  otherwise. While for  $r < \hat{r}$ , the firm size and credit rationing effect reinforce each other, they otherwise oppose. The net effect is generally still negative (i.e.  $dD/dr < 0$ ) except for if condition (4.7) holds. Accordingly, an increase in  $D$  from lower credit rationing outweighs the decrease from smaller firm sizes if, around the turning

<sup>8</sup>Abstract from higher inequality at the lower end only, leaving  $[1 - G(\omega(r))]$  unchanged.

point at  $r = \ddot{r}$ ,  $\omega(r)$  is sufficiently steeply falling and  $g(\omega(r))$ , i.e. the mass of agents just at the rationing threshold, sufficiently large.<sup>9</sup> In order to keep things simple, assume a typical distribution of wealth:

$$\text{Assumption 3: } g(w) \text{ is a parabola that opens downwards.} \quad (\text{A3})$$

It follows that if (4.7) is binding, then a single coherent reversal area of  $dD/dr \geq 0$  exists for some  $r \in [r_{D1}, r_{D2}]$  (with  $r_{D1}, r_{D2} > \dot{r}$  such that  $dD/dr = 0$ ).<sup>10</sup> After all,  $\Delta W > 0$  is only associated with a higher  $r^*$  (implying the prevalence of the net worth effect), if (4.7) holds. We will later see, how this mechanism can redirect capital flows after integration.

Finally, a higher  $r^*$  must not necessarily go in hand with a lower  $P^*$ . Following the same argumentation as for (4.7)  $dD/dr \geq 0$  yields

$$dP/dr \geq 0 \quad \text{if} \quad g(\omega(r))\omega'(r) \leq -\frac{[1-G(\omega(r))]f'(k(r))k'(r)}{[f(\omega(r))-f(k(r))]} \quad (4.8)$$

It depicts that  $dP/dr \geq 0$  appears for some  $r \in [r_{P1}, r_{P2}]$  (with  $r_{P1}, r_{P2} > \dot{r}$  such that  $dP/dr = 0$ ) if  $g(\omega(r))\omega'(r)$  is sufficiently low. Then, efficiency gains from alleviated credit rationing temporarily dominate the loss in production from smaller firm sizes. Otherwise,  $dP/dr < 0$  prevails  $\forall r$ .

## 4.4 Equilibria under Financial Integration

From now on, the world consists of country  $A$  and several other countries  $j = \{B, C, \dots\}$  of the kind analyzed above. Countries  $l = A, j$  share the identical parameters, except for aggregate wealth  $W_l$  and its dispersion  $G_l(w)$ . While capital is perfectly mobile at no cost, agents and thus production are not. Also the sequencing remains as before, but with one exception. At *date 0*, countries can decide to become fully financially integrated by mutually allowing their residents to borrow and lend across their common borders without any restrictions. All agents will do so until the interest rates across countries are equalized, thus giving rise to a common equilibrium market rate of return  $\hat{r}^*$  and  $GNPs$   $\hat{Y}_l^*$  (instead of  $r_l^*$  and  $Y_l^*$  obtained under autarky).

Let's assume that a country bases its decision whether to financially open up to another country or not on the implied change of its  $GNP$ . As *Lemma 4.1* does not lose its validity, we can immediately turn to the capital market equilibria that arise from the various financial integration scenarios.

<sup>9</sup>With  $dD/dr$  being continuous,  $dD/dr < 0$  already for  $r < \underline{r}(\gamma)$  and  $D \rightarrow 0^+$  for  $r \rightarrow \infty$ , so that  $dD/dr \rightarrow 0^-$  for  $r \rightarrow \infty$ , (4.7) can only temporarily hold.

<sup>10</sup>Independent of the parameter constellation (that fixes the exact position of the interval):  $r_{D2} \geq \ddot{r}$  if  $g'(\omega(\ddot{r})) < 0$ . Then, an increase in  $r$  is associated with an increasing mass of agents at the rationing threshold via  $\omega'(r) < 0$ . Similarly,  $g'(\omega(\ddot{r})) > 0$  fixes  $r_{D1} \leq \ddot{r}$ . In the extreme, the reversal area might boil down to a single point. Also remark that while a uniform distribution gives rise to  $\dot{r} < r_{D1} \leq \ddot{r} \leq r_{D2}$ , more complicated shapes of  $g(w)$  could imply several  $r_{D1}, r_{D2}, \dots$ , in turn giving rise to more than one coherent reversal response area.

#### 4.4.1 Exogenous Rate of Return

As a starting point, think of country  $A$  as being small relative to the rest of the world into whose global capital market it seeks to integrate its own one. While this leaves the world unaffected,  $A$  has to adopt the still prevailing global capital market rate of return  $\hat{r}^* = r_g^*$ . For  $\hat{r}^* > \underline{r}(\gamma)$ ,  $A$  realizes a  $GNP$  of:

$$\hat{Y}_A^* = \hat{r}^* \left[ W_A - \int_0^{\omega(\hat{r}^*)} w g_A(w) dw - [1 - G_A(\omega(\hat{r}^*))] k(\hat{r}^*) \right] + \left[ \int_0^{\omega(\hat{r}^*)} f(w) g_A(w) dw + [1 - G_A(\omega(\hat{r}^*))] f(k(\hat{r}^*)) \right], \quad (4.9)$$

where the first term represents the net credit position  $\hat{X}_A^* = \hat{r}^* [W_A - D_A(\hat{r}^*)]$  and the second one aggregate output  $\hat{P}_A^*$ . If, for instance,  $r_A^* < \hat{r}^* \leq \dot{r}$ , then agents in  $A$  see a rise in the market rate of return, scale-down their optimal investments, register tighter credit rationing and start exporting capital to the world. While  $A$  loses from less domestic aggregate production  $\Delta P_A < 0$ , it wins from running a current account surplus  $\Delta X_A > 0$ .

In order to verify if integration makes  $A$  realize a higher  $GNP$ , subtract (4.6) from (4.9) to obtain  $\Delta Y_A = \hat{Y}_A^* - Y_A^* = \Delta X_A + \Delta P_A$  with  $\Delta X_A = \hat{X}_A^*$  and

$$\Delta P_A = \int_{\omega(r_A^*)}^{\omega(\hat{r}^*)} f(w) g(w) dw + [1 - G_A(\omega(\hat{r}^*))] f(k(\hat{r}^*)) - [1 - G_A(\omega(r_A^*))] f(k(r_A^*)). \quad (4.10)$$

Thereby,  $\Delta P_A$  reflects the change in production only, whereas  $\Delta X_A$  needs to be decomposed into a change in the per-unit remuneration  $\Delta r_A$  and in the quantity of traded capital  $\Delta [W_A - D_A(r)]$ . But as studied in Section 4.3.3, a firm size and a credit rationing effect influence  $\Delta P_A$  and  $\Delta D_A(r)$  (eventually even giving rise to reverse responses of  $dP_A/dr \geq 0$  and, ensued by  $dD_A/dr \geq 0$ ,  $dX_A/dr \leq 0$ ). That is why the sign of  $\Delta X_A$  and  $\Delta P_A$ , let alone the aggregate effect of  $\Delta Y_A$ , is not always immediately clear. Indeed, by the same argumentation as for (4.8)  $d\hat{P}_A^*/dr$ , we get  $d\hat{X}_A^*/dr = [W_A - D_A(r)] - rD'_A(r) > 0$  except for

$$d\hat{X}_A^*/dr \leq 0 \quad \text{if} \quad g(\omega(r))\omega'(r) \leq \frac{-[1 - G(\omega(r))]rk'(r) + [W_A - D_A(r)]}{[\omega(r) - k(r)]r}. \quad (4.11)$$

Quite alike, if (4.11) holds, it implies the existence of some  $r_{X1}, r_{X2} > \dot{r}$  s.t.  $d\hat{X}_A^*/dr = 0$  and  $d\hat{X}_A^*/dr \leq 0$  for  $r \in [r_{X1}, r_{X2}]$ . As follows from the comparison of (4.11) and (4.7), the satisfaction of the second automatically implies that of the first for  $D_A(r) > W_A$ . Starting out from the autarky allocation, the first can therefore never be fulfilled without the second. Thus, (4.11) holds when lower credit rationing exceptionally dominates lower optimal firm sizes. It fuels domestic capital demand, so that  $A$  becomes a capital importer despite  $r_A^* < \hat{r}^*$ .

Netting out  $d\hat{P}_A^*/dr$  and  $d\hat{X}_A^*/dr$  finally gives  $d\hat{Y}_A^*/dr > 0 \forall r$  but for

$$d\hat{Y}_A^*/dr \leq 0 \quad \text{if} \quad g_A(\omega(r))\omega'(r) \geq -\frac{[W_A - D_A(r)]}{[[f(\omega(r)) - r\omega'(r)] - y(r)]}, \quad (4.12)$$

which holds when  $|\Delta X_A| \leq |\Delta P_A|$  with  $d\hat{X}_A^*/dr > 0$  and  $d\hat{P}_A^*/dr < 0$  and/or when, in the light of the hitherto analysis, a reverse response area of not only  $d\hat{X}_A^*/dr \leq 0$ , but also  $d\hat{P}_A^*/dr \geq 0$  temporarily materializes for some  $r > \dot{r}$ .

**Definition 4.2** Call  $dY/dr \leq 0$  for  $r \in [r_{Y1}, r_{Y2}] = [r_{P1}, r_{P2}] \cap [r_{X1}, r_{X2}]$  with  $r_{Y1}, r_{Y2} > \dot{r}$  s.t.  $dY/dr = 0$  the  $GNP$  reversal response case ( $Y$ - $RRC$ ) and  $dY/dr > 0$  for  $\forall r \in \{\mathbb{R}^+ | r \in [r_{Y1}, r_{Y2}]\}$  the standard response case ( $Y$ - $SRC$ ).

This being said, the outcome of financial liberalization subtly depends on the parameter constellation and the direction of the interest rate change. *Table 4.1* summarizes some general results that emerge if both market rates of return fall onto the same side of  $\dot{r}$  (either  $r_A^*, \hat{r}^* \leq \dot{r}$  or  $r_A^*, \hat{r}^* > \dot{r}$ ) into the same response case (either *Y-SRC* or *Y-RRC*). Otherwise, no further refined prediction can be made but that any outcome can materialize. Nevertheless, some well-known results from standard economic theory get refuted: higher [resp. lower] interest rates make net lending [resp. net borrowing] countries not always better off.

		$\Delta r_A^* > 0: r_A^* < \hat{r}^*$	$\Delta r_A^* < 0: r_A^* > \hat{r}^*$
$r_A^*, \hat{r}^* \leq \dot{r}$ :	$r_A^*, \hat{r}^* \leq \underline{r}(\gamma)$	$\Delta Y_A > 0$	$\Delta Y_A > 0$
	otherwise	$\Delta Y_A \geq 0$ if $ \Delta P_A  \leq  \Delta X_A $	$\Delta Y_A > 0$
$r_A^*, \hat{r}^* > \dot{r}$ :	<i>Y-SRC</i>	$\Delta Y_A > 0$	$\Delta Y_A < 0$
	<i>Y-RRC</i>	$\Delta Y_A \leq 0$	$\Delta Y_A \geq 0$

Note that in all other  $r_A^*-\hat{r}^*$ -constellations (but for  $\Delta r_A = 0$  when  $\Delta Y_A = 0$ ), any result can obtain:  $\Delta Y_A = \Delta P_A + \Delta X_A \leq 0$ .

Table 4.1: Beneficence of financial integration for country *A*

Unlike with first-best credit for  $r_A^*, \hat{r}^* \leq \underline{r}(\gamma)$ , higher interest rates  $r_A^* < \hat{r}^*$  do not necessarily leave net lending countries better off with credit rationing.  $\Delta Y_A \geq 0$  when  $|\Delta P_A| \leq |\Delta X_A|$  for  $r_A^* < \hat{r}^* \leq \dot{r}$  (when  $\Delta P_A < 0$  and  $\Delta X_A > 0$ ). Only the parameter constellation decides on if the improved credit position suffices to cover the loss in domestic production incurred from tightened credit rationing and shrunken firm sizes. If so, *A* wins (even if no resident in *A* was previously credit constrained) and otherwise loses from financial integration. All the more is it remarkable that for  $\dot{r} < r_A^* < \hat{r}^*$ , diminishing credit rationing makes the generally expected result of  $\Delta Y_A > 0$  reappear for sure in the *Y-SRC*. Whereas in the *Y-RRC* (when  $\Delta P_A > 0$  and  $\Delta X_A < 0$ ),  $\Delta Y_A \leq 0$  obtains. The reason is that even though firm sizes decline, *A*'s credit rationing around  $\bar{r}$  sufficiently decreases to increase its aggregate investment. This way, *A* turns into a net borrower despite a surge in the interest rate. That is how a general result reemerges: as a net borrower, *A* loses from higher interest rates. The improved efficiency from lower credit rationing does not outweigh the losses from smaller firm sizes and from the negative net credit position.

Also the contrary, i.e. that lower interest rates  $r_A^* > \hat{r}^*$  make net borrowing countries better off, might not generally be true. But for a start, it is true with first-best for  $r_A^*, \hat{r}^* \leq \underline{r}(\gamma)$  and for  $\dot{r} \geq r_A^* > \hat{r}^*$  (when  $\Delta P_A > 0$  and  $\Delta X_A < 0$ ).  $\Delta Y_A > 0$  owes to the fact that an interest rate drop induces more net borrowing firms with non-negative profits at a higher efficient scale, which on top all generate higher profits than their credit-rationed counterparts. For  $r_A^* > \hat{r}^* > \dot{r}$ , instead, worsened credit rationing hampers efficiency so badly, that the output increase from higher optimal firm sizes cannot withal cover the negative net credit position. Hence,  $\Delta Y_A < 0$  in the *Y-SRC*. Whereas in the *Y-RRC* (when  $\Delta P_A < 0$  and  $\Delta X_A > 0$  temporarily materialize),  $\Delta Y_A \geq 0$ . Despite a drop in the per-unit capital remuneration and thus larger optimal firm sizes, *A*'s domestic gross capital demand falls, because of higher credit rationing. Yet, against conventional wisdom, becoming a net lender then even allows *A* to win from lower interest rates. Altogether:

**Proposition 4.2** *After opening up as a small country to the world, A with  $r_A^*$  and  $Y_A^*$  takes the globally prevailing rate  $r_g^* = \hat{r}^*$  as given and realizes  $\hat{Y}_A^*$ .*

(I) *If  $r_A^*, \hat{r}^* \leq \underline{r}(\gamma)$ , there neither was, nor will be credit rationing. First-best arises. As compared to  $r_A^*$ , A wins from a higher and a lower  $\hat{r}^*$  ( $Y_A^* < \hat{Y}_A^*$ ).*

(II) *If  $\hat{r}^* > \underline{r}(\gamma)$ , there will be credit rationing. (II.i) For  $r_A^*, \hat{r}^* \in (\underline{r}(\gamma), \dot{r}]$ , A may win or lose from  $r_A^* < \hat{r}^*$  ( $Y_A^* \lesseqgtr \hat{Y}_A^*$ ). Yet, A wins from  $r_A^* > \hat{r}^*$  ( $Y_A^* < \hat{Y}_A^*$ ). (II.ii) In contrast, for  $r_A^*, \hat{r}^* > \dot{r}$ , A normally wins from  $r_A^* < \hat{r}^*$  ( $Y_A^* < \hat{Y}_A^*$ ), but loses from  $r_A^* > \hat{r}^*$  ( $Y_A^* > \hat{Y}_A^*$ ). However, the opposite obtains for  $r_A^*, \hat{r}^* \in [r_{Y1}, r_{Y2}]$  if Y-RRC exists with  $r_{Y1}, r_{Y2} > \dot{r}$  such that  $dY/dr = 0$ . Y-RRC in turn materializes if  $\omega'(r) g_A(\omega(r))$  is sufficiently low.*

(III) *In any other  $r_A^*-\hat{r}^*$ -line-up (and in case II.i), the parameter constellation decides on the magnitude of  $\Delta P_A$  and  $\Delta X_A$  and thus on  $Y_A^* \lesseqgtr \hat{Y}_A^*$ .*

## 4.4.2 Endogenous Market Rate of Return

Alternatively, country A thinks of pursuing financial integration with a country  $j$  at eye height, i.e. with a partner that is not big enough to act as the world. This way, the market rate of return  $\hat{r}^*$  becomes endogenous to integration and follows from equating global capital supply  $S(\hat{r}^*)$  and demand  $D(\hat{r}^*)$ :

$$\hat{r}^* \text{ s.t. } \quad W_A + W_j = \int_0^{\omega(\hat{r}^*)} w g_A(w) dw + [1 - G_A(\omega(\hat{r}^*))] k(\hat{r}^*) + \int_0^{\omega(\hat{r}^*)} w g_j(w) dw + [1 - G_j(\omega(\hat{r}^*))] k(\hat{r}^*). \quad (4.13)$$

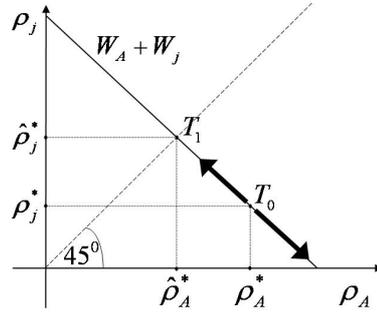
Integration entails inter-country capital flows, whose direction and magnitude are entirely driven by the differences in marginal productivity under autarky. Given e.g.  $r_A^* > r_j^*$ , A net borrows from  $j$  until the marginal productivity is equated across the two countries. This does, however, not always imply the elimination of differences in average marginal rates of productivity

$$\rho_l = \int_0^{\omega(\hat{r}^*)} f'(w) g_l(w) dw + [1 - G_l(\omega(\hat{r}^*))] f'(k(\hat{r}^*)) \quad \text{with } l = A, j. \quad (4.14)$$

If  $S(\hat{r}^*) = W_A + W_j$  is sufficiently high to ensure  $\hat{r}^* \leq \underline{r}(\gamma)$ , there is no credit rationing and the *RHS* in (4.13) reduces to  $2k(\hat{r}^*)$ . All agents make the same optimal investment  $k(\hat{r}^*)$ , so that also  $\hat{\rho}_A^* = \hat{\rho}_j^*$ . As depicted in *Figure 4.2* below, starting from  $\rho_A^* > \rho_j^*$  (e.g. ensued by  $\omega(r_A^*) > \omega(r_j^*)$  and  $k(r_A^*) < k(r_j^*)$ ) in the autarky point  $T_0$  on the resource constraint  $W_A + W_j$ , capital will flow from  $j$  to A until  $\hat{r}^* = f'(k(\hat{r}^*)) = \hat{\rho}_l^*$  in point  $T_1$ .

In contrast, if  $S(\hat{r}^*)$  is so low that  $\hat{r}^* > \underline{r}(\gamma)$ , there is credit rationing. A fraction  $[1 - G_A(\omega(\hat{r}^*))]$  of agents in country A and  $[1 - G_j(\omega(\hat{r}^*))]$  in  $j$  makes the same optimal investment  $k(\hat{r}^*)$ . As all other agents can only run self-financed sub-optimal firms, not only  $\hat{\rho}_l^* > \hat{r}^*$  persists, but also  $\hat{\rho}_A^* \neq \hat{\rho}_j^*$  if  $G_A(\omega(\hat{r}^*)) \neq G_j(\omega(\hat{r}^*))$ .<sup>11</sup> In that sense, the higher  $\hat{\rho}_l^* - \hat{r}^*$ , the larger the deviation of allocative efficiency from its first-best level. After all:

<sup>11</sup>This is consistent with the evidence reviewed in Bajeree and Duflo (2005). Also note how crucial the immobility of agents and thus of production is for the outcome. FDI and free trade of the output would eliminate any differences in  $\rho_l$  across countries. See e.g. Antràs and Caballero (2007) on the complementarity of trade and capital mobility.

Figure 4.2: Pattern of capital flows between country  $A$  and  $j$ 

**Proposition 4.3** *After mutually opening up, country  $A$  and  $j$  face a market rate of return  $\hat{r}^* \geq 1$  and GNPs  $\hat{Y}_A^*$  and  $\hat{Y}_j^*$  with the following properties:*

(i) *If  $f'([W_A + W_j]/2) \leq \underline{r}(\gamma)$ , first-best obtains. All agents make the same optimal investment  $k(\hat{r}^*) = [W_A + W_j]/2$ .  $\hat{r}^* = f'([W_A + W_j]/2) \leq \underline{r}(\gamma)$ ,  $\hat{Y}_A^*$  and  $\hat{Y}_j^*$  only depend on aggregate wealth  $W_A + W_j$ . Financial integration is beneficial for  $A$  and  $j$  alike.*

(ii) *Otherwise, there is credit rationing  $\omega(\hat{r}^*) > 0$ . A fraction  $[1 - G_A(\omega(\hat{r}^*))]$  of agents in  $A$  and  $[1 - G_j(\omega(\hat{r}^*))]$  of agents in  $j$  makes the same optimal investment  $k(\hat{r}^*)$ , whereas all others simply invest their initial endowment  $w_i$ .  $\underline{r}(\gamma) < \hat{r}^* < f'([W_A + W_j]/2)$ ,  $\hat{Y}_A^*$  and  $\hat{Y}_j^*$  depend on aggregate wealth  $W_A + W_j$  and its distribution across countries  $A$  and  $j$ . This productive inefficiency endangers the beneficence of financial integration for  $A$  and  $j$ .*

As a matter of fact, financial integration must be production-enhancing on aggregate.<sup>12</sup> However unlike for  $\hat{r}^* \leq \underline{r}(\gamma)$ , predictions about the benefits of financial integration for an individual country and the pattern of  $\rho_l$  (highlighted by the arrows in *Figure 4.2*) require to know the relation and position of all market rates of return for  $\hat{r}^* > \underline{r}(\gamma)$ . These, however, cannot be determined without taking into account the countries' characteristics with respect to their aggregate wealth  $W_l$  and its distribution  $G_l(w)$ . Recall that in comparison to a partner country, a country will be considered as richer if its aggregate wealth is higher and more unequal if a larger fraction of its residents has access to credit.

In what follows, we therefore study the five most pertinent cases:  $A$  teaming up with a (1) homogenous, (2) less unequal, (3) richer, (4) richer, less unequal as well as (5) richer, more unequal country. The last four  $W_l$ - $G_l$ -combinations are sketched in *Figure 4.3*.<sup>13</sup> In each case, equation (4.5) and *Proposition 4.1* are used to determine the relation of autarkic market rates of return  $r_l^*$ . Afterwards, equation (4.13) as well as *Propositions 4.1* and *4.3* allow to find out where the market rate of return after integration materializes. Finally, the sign of  $\Delta r_l^*$  together

<sup>12</sup>Integration goes in hand with the equalization of domestic market rates of return, so that there remains a single optimal investment level across countries (instead of two in autarky). This reduces the variation of firm scales and therewith the variation of the marginal product of capital. With a concave production function, the output then increases across countries.

<sup>13</sup>Moreover, switching indices allows  $A$  to derive the consequences of teaming up with just the opposite type of country than laid out in Cases (2) to (5).

with the location of  $r_l^*$  and  $\hat{r}^*$  (above all vis-à-vis  $\hat{r}$  and  $Y\text{-SRC}/Y\text{-RRC}$ ) will enable us to read the gains from financial integration from *Proposition 4.2*.

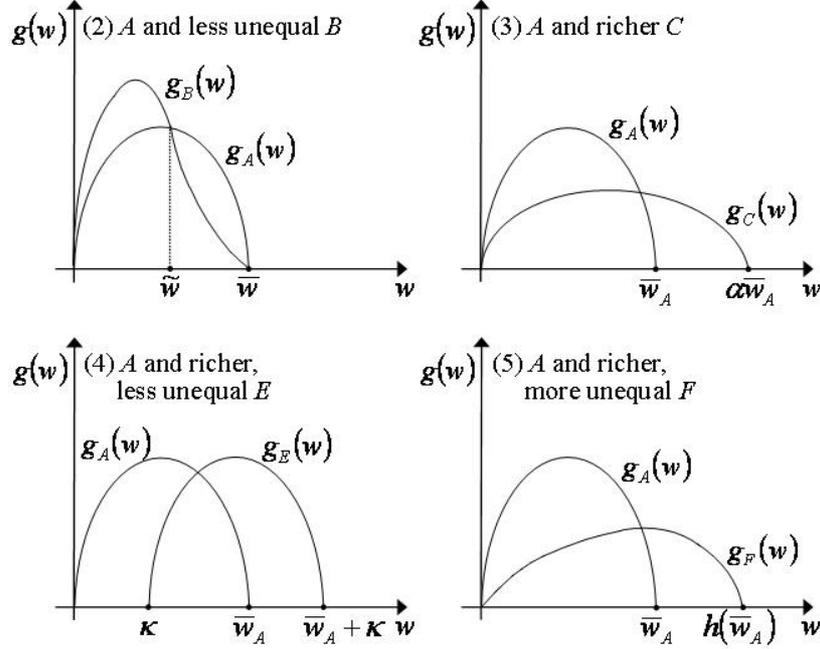


Figure 4.3: Graphical sketch of stylized  $W_l\text{-}G_l(w)$ -combinations studied

#### 4.4.2.1 Case 1: Teaming Up with a Homogenous Country

Initially, emanate from the polar case of country  $A$  pursuing financial integration with an identical country  $\tilde{A}$ :  $W_A = W_{\tilde{A}}$  and  $G_A(w) = G_{\tilde{A}}(w)$ . It is obvious that  $r_A^* = r_{\tilde{A}}^*$  under autarky and that, with integration proportionally increasing gross capital supply and demand,  $\hat{r}^* = r_A^* = r_{\tilde{A}}^*$  after integration.

**Corollary 4.1** *Financially integrating homogenous countries  $A$  and  $\tilde{A}$  with  $W_A = W_{\tilde{A}}$  and  $G_A(w) = G_{\tilde{A}}(w)$  is neutral with respect to GNP ( $\Delta Y_A^* = \Delta Y_{\tilde{A}}^* = 0$ ). The domestic autarkic equilibrium characterized in *Proposition 4.1* persists.*

#### 4.4.2.2 Case 2: Teaming Up with a Less Unequal Country

$A$  also faces the option of forming an integrated capital market with a less unequal country  $B$ . Although the two dispose of equal aggregate wealth  $W_A = W_B$ , a relatively larger part of it is in the hands of the poor in  $B$ . Given (A3),  $g_A(w) < g_B(w)$  for  $w < \tilde{w}$  (and vice versa) with  $\tilde{w} \in (0, \bar{w})$  such that  $g_A(\tilde{w}) = g_B(\tilde{w})$ . Thus, not only  $B$ 's Lorenz curve, but also  $B$ 's cumulative wealth distribution  $G_B(w)$  first-order stochastically dominate those of  $A$ .

The study of comparative statics in Section 4.3.3 showed that  $\Delta [1-G(w)]$  entails a net worth effect only. Due to  $G_A(w) < G_B(w) \forall w \in (0, \bar{w})$ , gross capital demand is always higher in  $A$

than in  $B$ . With equal gross capital supply, this puts comparatively more pressure on the equilibrium market rate of return in  $A$ , so that  $r_A^* > r_B^*$  and  $k(r_A^*) < k(r_B^*)$  under autarky. An opening up then incites  $A$  to borrow abroad. Capital flows from  $B$  to the more unequal country  $A$ , which establishes  $r_A^* > \hat{r}^* > r_B^*$ . Yet, despite a common single optimal firm size  $k(\hat{r}^*)$  and rationing threshold  $\omega(\hat{r}^*)$ ,  $\rho_A < \rho_B$  continues to hold because of  $G_A(\omega(\hat{r}^*)) < G_B(\omega(\hat{r}^*))$ . Given  $\hat{r}^*$ ,  $A$  has more absorptive capacities for productive capital than  $B$ , which would not be the case in a first-best world. Concerning the implied change in  $GNP$ , *Proposition 4.2* applies for  $\Delta r_A^* < 0$  and  $\Delta r_B^* > 0$ . For instance, if  $r_A^* \leq \hat{r}$ ,  $A$  wins, whereas this is only true for its more equal counterpart  $B$  if  $|\Delta P_B| < |\Delta X_B|$ .

**Corollary 4.2** *The beneficence of financially integrating country  $A$  and a less unequal  $B$  with  $W_A = W_B$  and  $G_A(w) < G_B(w) \forall w \in (0, \bar{w})$  can be read from *Proposition 4.2* on the basis of  $r_A^* > \hat{r}^* > r_B^*$ .*

#### 4.4.2.3 Case 3: Teaming Up with a Richer Country

Things get more complicated when country  $A$  and a richer country  $C$  mutually open up. Suppose that every agent in  $C$  owns  $\alpha$  times as much wealth as his respective counterpart in  $A$ , i.e.  $w_i^C = \alpha w_i^A$  with  $\alpha > 1$ . That is why the graph of  $g_C(w)$  appears as a horizontal dilation of  $g_A(w)$  to the right. Even though this leaves  $C$  with a higher aggregate wealth than  $A$  (i.e.  $W_C = \alpha W_A$ ), the countries' relative wealth dispersion is identical. Owing to  $G_A(w/W_A) = G_C(w/W_C)$ ,  $A$  and  $C$  share the same Lorenz curve.

As seen in Section 4.3.3,  $\Delta W$  triggers a net worth and a capital deepening effect. Hence, the impact of  $\Delta W$  crucially depends on the sign of  $dD/dr$  and we need to distinguish four interest rate scenarios. The first is the standard scenario of  $r_A^*$  and  $r_C^*$  materializing where  $dD/dr < 0$ . We know from the analysis of comparative statistics, that this yields the usually expected autarky result of  $r_A^* > r_C^*$ . But even though integration sets free capital flows from  $C$  to the poorer  $A$ , which lead to  $r_A^* > \hat{r}^* > r_C^*$ ,  $\rho_A > \rho_C$  persists. The reason is that a relatively larger fraction of agents in  $A$  still remains too poor to comply with the wealth requirement:  $G_A(\omega(\hat{r}^*)) > G_C(\omega(\hat{r}^*))$ . This lets  $A$  register less capital inflows than expected on the grounds of the differences in the optimal investments' marginal rates of productivity and the equality of the state of financial development (i.e.  $\gamma_A = \gamma_C$ ). With  $\Delta r_A^* < 0$  and  $\Delta r_C^* > 0$ , *Proposition 4.2* allows to draw the respective conclusions concerning  $\Delta Y_A$  and  $\Delta Y_C$ .

Just the contrary obtains in the reversal scenario of  $r_A^*$  and  $r_C^*$  occurring where  $dD/dr \geq 0$ . It gives  $r_A^* < r_C^*$  in autarky. Remember that this owes to the fact that here,  $\omega(r_A^*) > \omega(r_C^*)$  exceptionally outweighs  $k(r_A^*) > k(r_C^*)$  in terms of gross capital demand. Against conventional wisdom, integration then makes the relatively richer country  $C$  become a net borrower of the poorer  $A$ , so that  $r_A^* < \hat{r}^* < r_C^*$ . The redirected capital flows further widen the gap between  $\rho_A$  and  $\rho_C$  (with the first further increasing). As in the light of *Definition 4.2* and the Proof of *Proposition 4.2*, the reversal scenario of  $dD/dr \geq 0$  falls into the  $Y$ -RRC, *Proposition 4.2* offers a clear prediction for  $\Delta r_A^* > 0$  and  $\Delta r_C^* < 0$ :  $A$  always loses and its richer partner  $C$  always wins from financial integration.

Unfortunately, in the two mixed scenarios, when  $r_A^* > r_C^*$  with either only  $r_A^*$  or  $r_C^*$  being located where  $dD/dr \geq 0$ , no general prediction is possible. The parameter constellation alone

will determine if  $\hat{r}^*$  emerges where  $dD/dr \geq 0$  or  $dD/dr < 0$ . However, this anchor is needed for deriving the direction of capital flows as well as  $\Delta Y_A$  and  $\Delta Y_C$ . Still, we can conclude, that principally and in contrast to standard economic theory, any outcome is possible here.

**Corollary 4.3** *The beneficence of financially integrating country A and a richer C with  $W_C = \alpha W_A$  ( $\alpha > 1$ ) and  $G_A(w/W_A) = G_C(w/W_C)$  can be read from Proposition 4.2 on the basis of  $r_A^* > \hat{r}^* > r_C^* \forall r$ . There is only one exception: if  $dD/dr \geq 0$  exists and  $r_A^*, r_C^* \in [r_{D1}, r_{D2}]$ , then  $r_A^* < \hat{r}^* < r_C^*$ .*

#### 4.4.2.4 Case 4: Teaming Up with a Richer, Less Unequal Country

Furthermore, A could choose a richer, less unequal partner E. For this purpose, suppose that every agent in E owns  $\kappa > 0$  units of wealth more than his counterpart in country A. The graph of  $g_E(w)$  follows from a simple horizontal shift of  $g_A(w)$  to the right by  $\kappa$ ,  $W_E = W_A + \kappa$ . Yet,  $G_A(w/W_A) < G_E(w/W_E)$  obtains, letting the Lorenz curve of E stochastically dominate that of A. The reason is that adding  $\kappa$  has a relatively larger impact on the wealth of the poor than on that of the rich and so reduces inequality in E.

As this setting appears as a combination of the two cases studied before, the analysis is straightforward. Starting out again with the standard scenario of  $r_A^*$  and  $r_E^*$  arising where  $dD/dr < 0$ , yields  $r_A^* > r_E^*$ . In fact, compared to  $r_E^*$ , supply and demand side forces reinforce each other and drive up  $r_A^*$ . A has an absolutely lower gross capital supply, since its aggregate wealth is lower. Besides that, it registers a relatively higher gross capital demand, since a higher fraction of its residents has access to credit. After opening up, E therefore exports capital to the poorer, more unequal A and  $r_A^* > \hat{r}^* > r_E^*$ . Nevertheless, capital flows remain lower than under first-best and  $\rho_A > \rho_E$ . This stems from the result that because of their lower personal wealth, relatively more agents remain credit constrained in A (i.e.  $G_A(\omega(\hat{r}^*)) > G_E(\omega(\hat{r}^*))$ ). Given  $\Delta r_A^* < 0$  and  $\Delta r_E^* > 0$ , Proposition 4.2 predicts the sign of  $\Delta Y_A$  and  $\Delta Y_E$ .

On the other hand, in the reversal scenario of  $r_A^*$  and  $r_E^*$  materializing where  $dD/dr \geq 0$ ,  $r_A^* > r_E^*$  reemerges in autarky. In fact, the credit rationing effect ( $\omega'(r) < 0$ ) temporarily dominates the firm size effect (i.e.  $k'(r) < 0$ ). This makes gross capital demand even higher than in the respective standard scenario case before. Given the still lower gross capital supply, this additionally puts relatively more pressure on  $r_A^*$ , so that  $r_A^* > r_E^*$  again. Yet, when the two countries now open their borders to capital flows, they see  $r_A^* > \hat{r}^* > r_E^*$  and E starts net borrowing from the poorer, less unequal A.  $\rho_A$  and  $\rho_E$  further depart. With, as laid out above, the reversal area of  $dD/dr \geq 0$  falling into the Y-RRC, Proposition 4.2 shows that in view of  $\Delta r_A^* < 0$  and  $\Delta r_E^* > 0$ , A's GNP clearly rises, whereas E's falls.

As for the two mixed scenarios, the findings of Case (3) apply.

**Corollary 4.4** *The beneficence of financially integrating country A and a richer, less unequal E with  $W_E = W_A + \kappa$  ( $\kappa > 0$ ) and  $G_A(w/W_A) < G_E(w/W_E)$  can be read from Proposition 4.2 on the basis of  $r_A^* > \hat{r}^* > r_E^* \forall r$ .*

#### 4.4.2.5 Case 5: Teaming Up with a Richer, More Unequal Country

At last, country  $A$  may choose to get together with a richer, more unequal country  $F$ . This situation occurs e.g. when every agent's wealth in  $F$  is a positive, increasing and convex mapping of its counterpart's wealth in  $A$ :  $w_i^F = h(w_i^A)$  with  $h'(w_i^A) > 0$  and  $h''(w_i^A) > 0$ . The graph of  $g_F(w)$  looks like a dilation (in the presence of a fixed term also going in hand with a parallel shift) of  $g_A(w)$  to the right. Hence,  $F$  exhibits a higher aggregate wealth and a more unequal wealth dispersion than  $A$ :  $W_A < W_F$  and  $G_A(w/W_A) > G_F(w/W_F)$ .

Again, the countries differ along both dimensions. Yet, this time, they differently affect (4.5)  $r^*$ . That is why already in the standard scenario of  $r_A^*$  and  $r_F^*$  emerging where  $dD/dr < 0$ , no general result obtains:  $r_A^* \gtrless r_F^*$  in autarky. Even though  $A$  indeed has a lower absolute gross capital supply, it also has a relatively lower gross capital demand than  $F$ . For any  $r$ , a larger fraction of its residents is credit constrained. It can therefore not generally be predicted, if this on net translates into a higher or lower equilibrium market rate of return in  $A$  than in  $F$ . Thus, capital flows can principally go either way and depend on the specific parameter constellation. Also,  $\Delta r_A^* \gtrless 0$  and  $\Delta r_F^* \gtrless 0$ .

Similarly, no clear picture arises in the reversal scenario of  $r_A^*$  and  $r_F^*$  materializing where  $dD/dr \geq 0$ : again,  $r_A^* \gtrless r_F^*$  in autarky. Yet, contrary to the standard scenario before, the supply and demand side forces on the equilibrium market rate of return might realign again. Whilst  $A$ 's gross capital supply still remains lower, its gross capital demand picks up. Recall that the latter originates from the credit rationing effect ( $\omega'(r) < 0$ ) temporarily dominating the firm size effect (i.e.  $k'(r) < 0$ ). Consequently, there are parameter constellations, in which this jump in  $A$ 's gross capital demand even leads to  $A$  having a relatively higher gross capital demand than  $E$ . With the supply and demand side forces reinforcing each other, we would get  $r_A^* > r_F^*$  with the consequences described in the reversal scenario in Case (4). However, if this happens depends on the magnitude of the effects, precluding any general forecasts. Thus,  $\Delta r_A^* \gtrless 0$  and  $\Delta r_F^* \gtrless 0$ .

Once more, as for the two mixed scenarios, the argumentation laid out in Case (3) reapplies correspondingly.

**Corollary 4.5** *The beneficence of financially integrating country  $A$  and a richer, more unequal  $F$  with  $W_A < W_F$  and  $G_A(w/W_A) > G_F(w/W_F)$  cannot be read from Proposition 4.2 without taking into account the exact parameter constellation. Only that can decide on not only  $r_A^* \gtrless r_F^*$ , but also  $\Delta r_A^* \gtrless 0$  and  $\Delta r_F^* \gtrless 0$ .*

## 4.5 Policy Implications

The results imply manifold recommendations for all those, who attempt to progress with financial integration in the hope of tapping its benefits.

### 4.5.1 Winners and Losers

We have seen that there is no  $W$ - $G(w)$ -combination that would principally disqualify a country for financial integration - neither as an active proponent, nor as a potential team mate. Finan-

cial integration rather appears as a menu of choices, whose benefits and costs across partner countries depends on the country's own characteristics and its counterpart's. Beyond that, there is no team constellation that does not seem appealing in at least one interest rate scenario. The variation stems from the sign and relative magnitude of the credit rationing and the firm size effect in either the *Y-SRC* or the *Y-RRC*. These also govern the pattern of capital flows. And it is obvious that when capital flows remain below their first-best levels, the same will be true for the benefits from financial integration. However, apart from the first-best case for  $r_A^*, r_B^* \leq \underline{r}(\gamma)$ , there are only two further interest rate scenarios that have the potential to be beneficial for both countries at once: first, for  $r_A^*, r_B^* < \dot{r}$  (if  $\Delta P + \Delta X \geq 0$  also for the country for which  $\Delta r^* > 0$ ) and eventually second, in some parameter constellations in all  $r_A^* - \hat{r}^*$ -constellations that cannot be generally studied (and that are therefore only mentioned in the footer of *Table 4.1*). This being said, international financial integration should only be observable in these three cases. Otherwise, other motivations must have played a role.

Indeed, unrational decisions of countries can most obviously be explained on the basis of the result that financial opening bears opposite distributional implications for domestic agents. This owes to credit rationing operating like an entry barrier. It constrains gross capital demand and depresses the equilibrium market rate of return. This is good for borrowers and bad for lenders. For instance, in the standard case for  $r \in (\underline{r}(\gamma), \dot{r}]$ , an interest rate drop makes the middle class gain (because credit rationing decreases, in turn enabling them to reach the optimal capitalization level), while the rich lose (because of the decreased remuneration of capital) and the poor are equally off (because they anyway do not take part in either side of the capital market). That is how residents are divided into supporters and opponents of financial integration. Political economy considerations can then be put forward to explain the two sides of the coin. First, why countries still pursue financial integration, even if they know the country as a whole will lose from it. And second, why they do not do so, even if they know the country as a whole would win from it. In the end, it might all depend on what group holds the political power in its hands, so that it is able to convince the government to follow its best interests and not the country's as a whole.<sup>14</sup>

## 4.5.2 Optimal Financial Services Trade Liberalization

Altogether, the results lend novel theoretical support to the observation of why some countries prefer a stepwise à la carte approach to financial integration in the framework of preferential trade agreements (PTAs) rather than within the multilateral trading system of the WTO (GATS).

First, in view of the results in Sections 4.4.1 as well as 4.4.2 and given its own characteristics, a country might be better off by hand-picking the most suitable partner than by opening up to the whole world. In that sense, this paper offers a first attempt of a theory of optimal financial services liberalization (FSTL). Even though there are only few interest rate scenarios that simultaneously increase *GNP* in both countries, political economy considerations can explain why in all other scenarios, losing countries might still agree to pursue financial integration with a

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<sup>14</sup>Alternative argumentations could be derived from dynamizing the model or extending it in the directions pointed out in the Conclusion.

winning country. Moreover, a winning country could promise to compensate the losing country in order to win its consent to a mutual opening. Second, especially if seen in a more dynamic perspective, a country's needs evolve on its path of development with capital accumulation and with the evolution of domestic inequality. In fact, the identity of winners and losers determines the development of inequality across and within countries. Now losing countries might anticipate that they will win in later periods and therefore find it optimal to open up now on the basis of the sum of discounted profits. Likewise, teaming up later with the then most appropriate partner through a PTA also appears as an appealing flexible policy tool for counteracting unpleasant influences from previous integrations, current partners' integrations with third parties or adverse domestic interest rate shocks. However remark that once the country has opened up and accumulated a non-zero current account position, the satisfaction of (4.7)  $dD/dr \geq 0$  must no longer necessarily imply that of (4.11)  $dX/dr \leq 0$ .<sup>15</sup> Thus, this paper's predictions have to be slightly adapted when applied to already open countries that look out for further partners. Third, a PTA allows countries to pursue an asymmetric à la carte approach (e.g. opening up the borrowing, but restricting the lending side and vice versa). It leaves the countries prepared to prevent a detrimental reversal of capital flows in case the *Y-RRC* occurs (e.g. after an interest rate shock or spill-overs from the other country teaming up with a third party).

### 4.5.3 Strategic Use of Domestic Policies

Furthermore, the results offer important insights for supranational treaty design. These originate from the fact that at *date*  $t$ , after the contract parties have resolved upon financial integration, each member country still has the full range of domestic economic policies at its disposal. These can be used to improve the country's economic conditions with view to the credit constraint (4.4)  $\omega(r)$ . In fact, each country can increase the share of its population being eligible for a credit through e.g. political wealth redistribution, inflation or a capital market reform that increases the state of financial development  $\gamma$ .

The mechanisms are already evident in the simplest case of two homogenous countries with  $r_A, r_{\bar{A}} \in (r(\gamma), \hat{r}]$  in Case (1) in Section 4.4.2.<sup>16</sup> The commonly received result that e.g. inflation does not affect the real economy also holds under autarky in the model at hand based on *Proposition 4.1*. This owes to inflation proportionally increasing demand and supply and so leaving the equilibrium market rate of return unaffected. Whereas after integration, *Corollary 4.1* implies that the same is only true for an equal rate of inflation  $\pi_A = \pi_{\bar{A}}$ . For e.g.  $\pi_A > \pi_{\bar{A}}$ , instead, residents in  $A$  become relative wealthier than their counterparts in  $\bar{A}$ . This enables

<sup>15</sup>If it does, mind the following: think e.g. of the *Y-RRC* in Case (3) in Section 4.4.2, where  $r_A^* > \hat{r}_0^* > r_C^*$  made  $A$  the net borrower of the poorer  $C$ . An interest rate drop to  $\hat{r}_1^*$  with  $r_A^* > \hat{r}_0^* > \hat{r}_1^*$  (ensued by e.g.  $A$  additionally teaming up with a slightly richer  $C'$  than  $C$ ) would induce  $A$  to reduce its capital imports, but not necessarily to directly turn into a capital exporter (as under autarky). Contrariwise, if it does not, (4.11) might hold without (4.7) for net borrowers with  $D > W$ . That is why in the *Y-RRC*, the country may not immediately see a total reversal of its flows, just a diminishment of its *NCP* (because lower capital imports would not be sufficient to cover a higher per-unit remuneration of capital). Whereas for net lenders with  $D < W$ , (4.11) even implies (4.7), whilst the second can again also hold without the first.

<sup>16</sup>It is straightforward how to adapt the analysis to all other settings in Section 4.4.2.

more agents in  $A$  to put up the required collateral and increases gross capital demand. The equilibrium market rate of return has to adjust upwards and induces a rise in the rationing threshold. Still, more agents in  $A$  can overcome the rationing threshold than in  $\tilde{A}$ , so that the first crowd out the latter and use them as lenders. Consequently,  $A$ 's  $GNP$  rises and  $\tilde{A}$ 's falls. As the benefit  $A$  attains from manipulating the domestic scope of credit rationing comes at the expense of  $\tilde{A}$ , the latter is forced to retaliate with similar measures.

In order to avoid such vicious circles of beggar-thy-neighbor policies, treaties on financial integration should include a harmonization or ban of all domestic policies affecting the domestic level of credit rationing.<sup>17</sup>

#### 4.5.4 Resilience to Macroeconomic Shocks

Without drawing on risk diversification per se, this paper reveals credit rationing as a shock-absorber or -amplifier. While credit rationing fosters financial stability in integrated financial markets in the standard case, it triggers contagion in the  $Y$ - $RRC$ .

For this to see, think again in terms of the simplest context of two homogenous countries and the equilibrium characterized in *Corollary 4.1* for  $r_A, r_{\tilde{A}} \in (r(\gamma), \hat{r}]$ . After integration, a negative macroeconomic shock occurs in  $A$  in the form of e.g. a sudden decline in the business climate. It makes all projects' probability of success drop from 1 to  $p < 1$ . This entails a decrease of the optimal firm size in  $A$  to  $k_A^\circ(\hat{r}^*)$  s.t.  $pf'(k_A^\circ(\hat{r}^*)) = \hat{r}^*$ . Hence, credit rationing worsens at all levels of  $r$  with  $\omega_A^\circ(\hat{r}^*) := k_A^\circ(\hat{r}^*) - \gamma pf(k_A^\circ(\hat{r}^*)) / \hat{r}^*$ . *Lemma 4.1* prevails in  $A$  but with  $\omega_A^\circ(\hat{r}^*) > \omega_A(\hat{r}^*)$ . As the business climate remains unaffected in the other country  $\tilde{A}$ , firm sizes and the scope of credit rationing stay at their previous levels, so that  $k_A^\circ(\hat{r}^*) < k_{\tilde{A}}(\hat{r}^*)$  and  $\omega_A^\circ(\hat{r}^*) > \omega_{\tilde{A}}(\hat{r}^*)$ . It follows that gross capital demand in  $A$  decreases, whereas it remains stable in  $\tilde{A}$ . The common interest rate has to fall and  $A$  starts exporting capital to  $\tilde{A}$ . While  $A$ 's  $GNP$  drops because of the firm size and the efficiency effect, it still drops less than under autarky because of  $A$ 's capital account surplus. Likewise,  $\tilde{A}$ 's  $GNP$  increases. Remark that for  $r_A, r_{\tilde{A}} > \hat{r}$ ,  $A$  still loses less in the  $Y$ - $SRC$  than under autarky and  $\tilde{A}$  wins.

Hence, integrated capital markets might indeed provide a shock absorbing capacity via the impact of credit rationing in the two countries. Any macroeconomic shock ensuing a drop in net capital demand in one country will be attenuated by the other country's intact capital demand. That is why the rate of return and the  $GNP$  under integration are still higher in the country that experiences the shock than they would have been under national autarky. The argumentation reverses in the  $Y$ - $RRC$ .

## 4.6 Conclusion

This paper complements existing theories of financial integration with the impact of wealth inequality in the presence of capital market imperfections. It draws on production inefficiencies

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<sup>17</sup>It is obvious that no vicious circle can arise if  $A$  opened up to the world, because irrespective of the domestic policy implemented,  $A$  cannot change the global market rate of return. Thus, no one would be affected and thus incited to retaliate.

due to credit rationing as a new cost, whose magnitude depends on a country's aggregate wealth endowment and wealth distribution relative to the country it opens up to.

On these grounds, a novel explanation arises for unconventional patterns of international capital flows as well as for why financial integration might turn out welfare-enhancing in some countries and welfare-deteriorating in others. These insights also allow to uncover winners and losers within a country, so paving the way for political economy considerations. Finally, the paper points out the implications of credit rationing for policy making, esp. for supranational treaty design, optimal financial services trade liberalization and financial stability.

However, the paper only makes a start and so offers various roads for future research. First of all, attention should be given to balancing out the effects of credit rationing induced by wealth inequality against the typical costs and benefits of financial integration outlined in the introduction. The paper also suggests various extensions. Countervailing effects to the impact of wealth dispersion could e.g. be based on differing states of financial development  $\gamma$ ,<sup>18</sup> increased domestic capital market competition ensued by the entry of foreign lenders or economies of scale in banking. Following Iacoviello and Minetti (2006), domestic lenders could also be assumed to be better at recovering value from a borrower's default than foreign lenders, so requiring less collateral. Above that, different technologies  $f$  can be expected to trigger a specialization effect, and hidden heterogenous project qualities (asymmetric information) to give rise to an additional quality effect in the pool of loans.<sup>19</sup> Going one step further would also require to study dynamic effects on capital accumulation and future equilibrium market rates of return<sup>20</sup> as well as to derive an optimal dynamic path of successive integrations.

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<sup>18</sup>With otherwise identical countries, the scope of credit rationing in the country with the higher  $\gamma$  would decrease ( $d\omega/d\gamma < 0$ ), so that it would attract capital flows from the other country. Additionally,  $\gamma$  could be modelled to be driven e.g. by the scope of diversification opportunities (as e.g. in Acemoglu and Zilibotti, 1997 or Martin and Rey, 2004), the presence of foreign banks with superior efficiency (as e.g. in Levine, 1996 or Rajan and Zingales, 2003) or taxed-financed financial infrastructure investments (as e.g. in Ando and Yanagawa, 2002).

<sup>19</sup>Dynamically put, progress could also be endogenized by making the technology switch dependent on surpassing financing hurdles (as in Horii, Yamamoto and Ohdoi, 2005).

<sup>20</sup>While Piketty (1997) or Chapter 2 of this thesis provide a dynamization with variable firm sizes, Matsuyama (2004) studies the impact of financial integration on the set of steady states.



# Appendix A

## Appendix to Chapter 2

### A.1 Proofs for Section 2.2

- **Proof of Lemma 2.1:**

See Piketty (1997).

- **Proof of Lemma 2.2:**

Given Lemma 2.1, two things remain to be proven.

(1) Checking the behavior and interplay of all relevant functions.

(1.1) To show:  $k(r)$  and  $k_0(r)$  are differentiable, strictly decreasing and convex functions in  $r$ . Remember that  $k(r)$  is such that  $pf'(k(r)) = r$  and that  $y(r)$  is strictly concave in  $k$ . Then,  $k'(r) < 0$  because  $f'(k(r))$  is continuous and monotonously decreasing in  $k(r)$ . On top using that  $k'(r) < 0$ ,  $k \rightarrow 0$  for  $r \rightarrow \infty$  and  $k \rightarrow \infty$  for  $r \rightarrow 0$  gives  $k''(r) > 0$ . The same reasoning applies to  $k_0(r)$ . Finally,  $k(r) > k_0(r)$  follows from  $f'(k)$ 's behavior depicted above,  $p > q$  and the fact that in optimum  $pf'(k(r)) = qf'(k_0(r)) = r$ .

(1.2) To show:  $\omega(r)$  is a strictly increasing and concave function in  $r$  for  $r < \bar{r}(q)$ . From (2.4)  $\omega(r)$ , we get

$$d\omega/dr = p[(p-q)f(k(r)) - 1] / (p-q)r^2 > 0 \text{ for } r < \bar{r}(q),$$

since  $(p-q)f(k(r)) - 1 > 0$  by (A1) for  $r < \bar{r}(q)$ . This, in turn, is derived from  $[pf(k(r)) - rk(r)] - [qf(k(r)) - rk(r)] > y(r) - y_0(r)$  (as  $y_0(r)$  is maximal for  $k_0(r)$  and hence smaller for any other  $k(r)$ ), and  $y(r) - y_0(r) > 1$  for  $r < \bar{r}(q)$ . By the same argument and with  $k'(r) < 0$ , the second derivative indeed gives

$$d^2\omega/dr^2 = [(p-q)k'(r)r^2 - 2p[(p-q)f(k(r)) - 1]] / r^3 (p-q) < 0.$$

(1.3) To show:  $k_0(\bar{r}) < \tilde{w} < k(\bar{r})$ . On the one hand, as  $y(r)$  is maximal for  $k(r)$  and hence smaller for any other  $k$ , it must be true for  $r \leq \bar{r}(q)$  that

$$[pf(k(r)) - rk(r)] - [qf(k(r)) - rk(r)] > y(r) - y_0(r). \quad (\text{A.1})$$

Now focus on  $r = \bar{r}(q)$ . Taking advantage of (A1) gives  $y(\bar{r}) - y_0(\bar{r}) = 1$ , which helps to reduce (A.1) to  $f(k(\bar{r})) > 1/(p-q)$ . Then, it is assured by  $f$ 's continuity as well as monotonicity and by  $1/(p-q) = f(\tilde{w})$  that  $k(\bar{r}) > \tilde{w}$ . On the other hand equally proceeding for  $k_0(r)$ , gives

$$[pf(k_0(\bar{r})) - \bar{r}k_0(\bar{r})] - [qf(k_0(\bar{r})) - \bar{r}k_0(\bar{r})] < y(\bar{r}) - y_0(\bar{r}) = 1, \quad (\text{A.2})$$

resulting in  $f(k_0(\bar{r})) < 1/(p-q)$  and thus  $k_0(\bar{r}) < \tilde{w}$ . Consequently,  $k_0(\bar{r}) < \tilde{w} < k(\bar{r})$ . Then, given  $k(r)$ 's shape derived in step (i) and as  $k'(r) < 0$ ,  $k(0) = \infty > \tilde{w}$ ,  $k(\infty) = 0 < \tilde{w}$ ,  $\tilde{w} > 0$  and  $d\tilde{w}/dr = 0$ , it is that  $k(\dot{r}) = \tilde{w}$  for some  $\dot{r} > \bar{r}(q)$ . Likewise, it follows that  $k_0(\ddot{r}) = \tilde{w}$  for some  $\ddot{r} < \bar{r}(q)$ .

(1.4) To show:  $k_0(\bar{r}) < \omega(\bar{r}) < \tilde{w} < k(\bar{r})$ . First, in order to verify  $\omega(\bar{r}) < \tilde{w}$ , substitute (2.4)  $\omega(r)$  and reduce it to  $pf(k(\bar{r})) - \bar{r}k(\bar{r}) > p/(p-q) - \bar{r}\tilde{w}$ . Using (A1) to replace  $y_0(\bar{r}) + 1$  for  $y(\bar{r})$  on the LHS and inserting  $f(\tilde{w}) = 1/(p-q)$  on the RHS, gives  $y_0(\bar{r}) > qf(\tilde{w}) - \bar{r}\tilde{w}$ . It is satisfied, because  $y_0$  is maximal for  $k_0(\bar{r})$  and, due to its strict concavity, less for any other capitalization (thus also less for  $\tilde{w} > k_0(\bar{r})$ ). Second, check  $k_0(\bar{r}) < \omega(\bar{r})$ . Proceeding as for  $\omega(\bar{r}) < \tilde{w}$  before, yields  $qf(k_0(\bar{r})) < qf(\tilde{w})$ . This is true by  $f$ 's positive monotonicity, since we already proved that  $k_0(\bar{r}) < \tilde{w}$ . Third, in order to prove that  $\omega(\bar{r}) < k(\bar{r})$ , again substitute (2.4)  $\omega(r)$  to obtain  $qf(\tilde{w}) - rk(\bar{r}) < y_0(\bar{r})$ . Then, use the definition of  $\tilde{w}$  to insert  $pf(\tilde{w}) - 1$  for  $qf(\tilde{w})$  on the LHS, which yields  $pf(\tilde{w}) - rk(\bar{r}) - 1 < y_0(\bar{r})$ . This holds, since the LHS is smaller than  $y(\bar{r}) - 1 = y_0(\bar{r})$  and since  $f(\tilde{w}) < f(k(\bar{r}))$ .

(1.5) To show: Relative curve positions for  $r < \bar{r}(q)$ . First,  $\omega(r) < \tilde{w}$ , because of  $\tilde{w} > 0$ ,  $\omega(\underline{r}(q)) = 0$ ,  $\omega(\bar{r}) < \tilde{w}$  and  $\omega(r)$ 's strict monotonicity and concavity for  $r < \bar{r}(q)$ . Besides, as  $k_0'(r) < 0$ ,  $k_0(0) = \infty > \omega(0)$ ,  $k_0(\infty) = 0 < \omega(\infty)$  and  $k_0(\bar{r}) < \omega(\bar{r})$ , it follows that  $k_0(\tilde{r}) = \omega(\tilde{r})$  for some  $\underline{r}(q) < \tilde{r} < \bar{r}(q)$  and  $k_0(r) > \omega(r)$  for  $r < \tilde{r}$  [resp.  $k_0(r) < \omega(r)$  for  $r > \tilde{r}$ ]. By the same argument, it becomes obvious that  $k(r) > \tilde{w}$  (and that  $k(r) < \tilde{w}$  only for some  $r > \bar{r}(q)$ ).

(2) Deriving the occupational choice of agents who own less capital than necessary for the optimal investment given  $r$ :

(2.1) For a start, emanate from an agent  $i$ , who does not want or get access to credit. In order to determine his effort, investment and deposit level, three cases have to be distinguished: (i) If  $r > \bar{r}(q)$ , step (1.1) constituted that effort does not pay off for any investment level, even not for the optimal level  $k_0(r)$ . Agent  $i$  therefore chooses to shirk too. Owing to  $qf'(w_i) > r$  for  $w_i < k_0(r)$  (remember that the profit function is concave and that  $f'(0) = \infty$ ), he invests  $w_i$  and does not deposit. He then earns  $qf(w_i) - rw_i > 0$ . (ii) On the other hand, even if  $r \leq \bar{r}(q)$ , effort provision might not always be lucrative. For similar arguments as for  $\bar{r}(q)$  from (A1) in step (1.1) above, (2.5)  $IC_a$  implies a  $\tilde{w}$  such that if  $w_i < \tilde{w} := f^{-1}(1/(p-q))$ , effort does not pay off (and vice versa). Thus, if  $w_i < \tilde{w}$ ,  $f(w_i) < 1/(p-q)$  makes the agent prefer shirking and invest his entire wealth  $w_i$  - except for if  $k_0(r) \leq w_i < \tilde{w}$ , when it yields more to invest  $k_0(r)$  s.t.  $qf'(k_0(r)) = r$  and to deposit any remainder  $w_i - k_0(r)$ . (iii) Finally, if  $r \leq \bar{r}(q)$  as well as  $w_i \geq \tilde{w}$ , agent  $i$  will provide effort. With  $pf'(w_i) > r$  for  $w_i \leq k(r)$ , he also invest all he owns.

(2.2) Now, we additionally allow agent  $i$  to ask for credit. This would make him earn  $V = p[f(k(r)) - t_s] - 1$  [resp.  $V_0 = q[f(k_0(r)) - t_{s0}]$ ]. If  $r \leq \bar{r}(q)$  and  $w_i < k(r)$ , a further refinement of  $w_i$  entails three effort cases: (i) If  $\omega(r) < \tilde{w} \leq w_i$ ,  $i$  accepts an effort contract:  $V \geq y_a - 1$  follows from the fact that the profit is maximal for  $k(r)$  and less  $\forall w_i \neq k(r)$ . (ii) Also if  $\omega(r) \leq w_i < \tilde{w}$ ,  $i$  leverages his firm:  $V \geq y_{a0}$  holds, since  $y(r) - 1 \geq y_0(r)$  is satisfied by (A1). The simplification is valid, since on the RHS,  $y_{a0} \leq y_0(r)$  and on the LHS, after substituting  $t_s$ ,  $V$  reduces to  $y(r) - 1$ . (iii) If  $w_i < \omega(r) < \tilde{w}$ ,  $i$  is credit constrained and it matters if  $w_i \geq k_0(r)$ . For  $w_i \geq k_0(r)$ , we already derived in step (2.1) that  $i$  self-finances  $k_0(r)$  and deposits any remainder. Whereas if  $w_i < k_0(r)$ , the agent asks for a non-effort credit, because then,  $V_0 \geq y_{a0}$ . After having substituted  $t_{s0}$ , this reduces to  $y_0(r) \geq y_{a0}$ . This is satisfied, since the profit is maximal for  $k_0(r)$  and less  $\forall w_i \neq k_0(r)$ . (iv) On the other hand, if  $r > \bar{r}(q)$ , shirking dominates. For  $i$  with  $w_i < k_0(r)$ , a leveraged firm size of  $k_0(r)$  is best:  $V_0 > y_{a0}$ , since the profit is maximal for  $k_0(r)$  and strictly less  $\forall w_i \neq k_0(r)$ .

## A.2 Proofs for Section 2.4

- **Proof of Proposition 2.1:**

The Proof follows from the construction of (2.9) and Piketty (1997).

- **Proof of Proposition 2.2:**

For  $r < \tilde{r}(q)$ ,  $k_0(r) > \omega(r)$ , so that there are no credit-constrained net lenders. Thus, Piketty's (1997) convergence result towards the second, i.e. the credit-constrained steady-state remains intact. In contrast, Piketty (1997) himself points out in footnote 19 that with  $k_0(r) < \omega(r)$  the first condition stated above would not hold, in turn restoring uniqueness, i.e. convergence towards the credit-unconstrained steady-state. As Lemma 2.2 constitutes that  $k_0(r) > \omega(r)$  for  $r < \tilde{r}(q)$ , it immediately follows that the steady-state interest rate is bounded above by  $\tilde{r}(q)$  and hence  $r_\infty^c \in (r(q), \tilde{r}(q)]$ .



# Appendix B

## Appendix to Chapter 3

### B.1 Proofs for Section 3.3

- **Proof of Lemma 3.1:**

As in optimum, where  $pf'(k(r)) = qf'(k_0(r)) = r$  and  $dy/dr = -k(r)$  [resp.  $dy_0/dr = -k_0(r)$ ], it also is that  $dy/dr < dy_0/dr$ . With  $y(r) \rightarrow 0$  [resp.  $y_0(r) \rightarrow 0$ ] for  $r \rightarrow \infty$ , (A1)  $y(1) - y_0(1) > 1$  implies an  $\bar{r} > 1$  s.t. for all  $r \leq \bar{r}$ , it holds that  $y(r) - 1 \geq y_0(r)$ . Thus, high effort ( $e=1$ ) is individually optimal. Likewise, shirking ( $e=0$ ) is individually optimal for  $r > \bar{r}$ .

In order to check  $k(r)$ 's shape, recall that  $k(r)$  is s.t.  $pf'(k(r)) = r$  and that  $y(r)$  is strictly concave in  $k$ . Then,  $k'(r) < 0$ , because  $f'(k(r))$  is continuous and monotonously decreasing in  $k(r)$ . Using that  $k'(r) < 0$ ,  $k \rightarrow 0$  for  $r \rightarrow \infty$  and  $k \rightarrow \infty$  for  $r \rightarrow 0$ , gives  $k''(r) > 0$ . The same reasoning applies to  $k_0(r)$ .  $k(r) > k_0(r)$  finally follows from  $f'(k)$ 's behavior depicted above,  $p > q$  and the fact that in optimum  $pf'(k(r)) = qf'(k_0(r)) = r$ . Consequently,  $k(r)$  is steeper than  $k_0(r)$ .

- **Proof of Lemma 3.2:**

We study the calculus of an agent  $i$  with  $w_i < k(r_D)$  for  $r_D \leq \bar{r}$  [resp.  $w_i < k_0(r_D)$  for  $r_D > \bar{r}$ ]. He cannot self-finance the optimal investment given  $r_D$ , but also either does not want or get credit to make the optimal investment given  $r_D$ . In order to determine the size of his necessarily suboptimally capitalized autarkic firm, deposits and effort, three cases have to be distinguished: (i) If  $r_D > \bar{r}$ , Lemma 3.1 constitutes that effort does not pay off for any investment level, even not for the optimal level  $k_0(r_D)$ . Agent  $i$  therefore chooses to shirk too. Owing to  $qf'(w_i) > r_D$  for  $w_i < k_0(r_D)$  (remember that the profit function is concave and that  $f'(0) = \infty$ ), he invests  $w_i$  and does not deposit. He then earns  $qf(w_i) - r_D w_i > 0$ . (ii) Even though  $r_D \leq \bar{r}$ , effort provision might still not be lucrative. For similar arguments as for  $\bar{r}$  from (A1) in the Proof of Lemma 3.1, (3.6)  $IC_a$  implies a  $\tilde{w}$  s.t. if  $w_i < \tilde{w} := f^{-1}(1/(p-q))$ , effort does not pay off (and vice versa). Thus, if  $w_i < \tilde{w}$ ,  $f(w_i) < 1/(p-q)$  makes the agent prefer shirking and to invest his entire wealth  $w_i$  - except for if  $k_0(r_D) \leq w_i < \tilde{w}$ , when it yields more to invest  $k_0(r_D)$

s.t.  $qf'(k_0(r_D)) = r_D$  and to deposit any remainder  $w_i - k_0(r_D)$ . (iii) If, finally,  $r_D \leq \bar{r}$  as well as  $w_i \geq \tilde{w}$ , agent  $i$  will provide effort. With  $pf'(w_i) > r_D$  for  $w_i \leq k(r_D)$ , he also invests all he owns.

It remains to relate  $k_0(r_D)$ ,  $k(r_D)$  and  $\tilde{w}$ . On the one hand, as  $y(r_D)$  is maximal for  $k(r_D)$  and smaller for any other  $k$ , we know that for  $r_D < \bar{r}$ :

$$[pf(k(r_D)) - r_D k(r_D)] - [qf(k(r_D)) - r_D k(r_D)] > y(r_D) - y_0(r_D).$$

Taking advantage of (A1)  $y(\bar{r}) - y_0(\bar{r}) = 1$  for  $r_D = \bar{r}$ , helps to reduce the equation to  $f(k(\bar{r})) > 1/(p-q)$ . Replace  $1/(p-q) = f(\tilde{w})$  and use  $f$ 's continuity and monotony to get  $k(\bar{r}) > \tilde{w}$ . On the other hand, likewise proceeding for  $k_0(r_D)$ , makes

$$[pf(k_0(\bar{r})) - \bar{r}k_0(\bar{r})] - [qf(k_0(\bar{r})) - \bar{r}k_0(\bar{r})] < y(\bar{r}) - y_0(\bar{r}) = 1$$

reduce to  $f(k_0(\bar{r})) < 1/(p-q)$ . Thus,  $k_0(\bar{r}) < \tilde{w}$ . Taking altogether and by Lemma 3.1, we finally obtain  $k_0(\bar{r}) < \tilde{w} < k(\bar{r})$ .

- **Proof of Lemma 3.3:**

First, that no agent can win from simultaneously borrowing and lending is obvious for  $r_D < r_L$ . But in order to see this for  $r_D = r_L = r$ , think e.g. of an agent who borrows more than he actually needs to invest  $k(r)$ , say  $k(r) - w_i + \check{w}$  with  $\check{w} > 0$ . Let the new corresponding transfers be  $\check{t}_f$  in case of failure and  $\check{t}_s$  in case of success. The agent now deposits  $\check{w}$  and earns  $r\check{w}$ . Using that as equity contribution,  $\check{t}_f \geq -r\check{w}$  and  $\check{t}_s \leq f(k) - r\check{w}$ . Yet, instead of concluding separate deposit and loan contracts, pool them to get one contract with net transfers  $0 - r\check{w}$  in case of failure and  $\check{t}_s - r\check{w}$  in case of success. With the non-negativity restrictions having to hold, these net transfers satisfy the same conditions as the initial borrowing contract.

On these grounds, the containment of the group of depositors and borrowers follows straightforward from the participation constraints (3.10) to (3.13) as well as Lemmata 3.1 and 3.2. This, in turn, makes some constraints redundant. Provided that (3.4)  $IR_L^B$  holds, (3.16)  $N_s$  holds too. The same applies to the non-effort forms. These two constraints can thus always be omitted. From the violation of ( $IR_{D2}$ ) finally follows that if (3.13)  $IR_{L2}$  holds, (3.12)  $IR_{L1}$  holds too.

## B.2 Proofs for Section 3.4

- **Proof of Lemma 3.4:**

As established by Bertrand, bank competition removes all intermediation profits. It implies that there cannot be any equilibrium bank behavior but  $r^* = r_D = r_L$ . Applying a loan rate  $r_L < r^*$  would attract all credit demand, but cause the bank's bankruptcy: with  $r_L < r^*$ , but a deposit rate  $r^*$ , it follows from (3.3)  $IR_D^B$  that the bank would make a loss on each unit of capital intermediated. With  $r_L > r^*$ , the bank could not operate, since it would not attract any credit demand at all.

- **Proof of Proposition 3.1:**

Two things have to be derived: (i)  $\hat{r}_p$  and  $\hat{r}_q$  as well as (ii) (3.18)  $\hat{r}$ .

(i)  $\hat{r}_p$  [resp.  $\hat{r}_q$ ] is s.t. (3.18) just holds for  $k(r) = W$  [resp.  $k_0(r) = W$ ]. For expositional purposes, assume that the economy consists of two agents only.

First, we focus on  $\hat{r}_q$ : Single out the aggregate wealth level  $\bar{W}$  that -optimally distribution an effort firm and a non-effort firm- would generate a return of  $pf'(k(\bar{r})) = qf'(k_0(\bar{r})) = \bar{r}$ . Yet, we know by (A1) that at  $\bar{r}$ , the surplus from equalizing firm sizes to  $k_0 = \bar{W}/2$  is larger than from running an effort firm of size  $k(\bar{r})$  and a non-effort firm of size  $k_0(\bar{r})$ :

$$2qf(\bar{W}/2) > [pf(k(\bar{r})) - 1] + qf(k_0(\bar{r})) \quad (\text{B.1})$$

The reason is that compared to the LHS, the increase in expected output on the RHS does not cover effort costs of  $e = 1$ : by (A1), the RHS reduces to  $2qf(\bar{W}/2) > 2qf(k_0(\bar{r}))$ . This, in turn, gives  $f(\bar{W}/2) > f(k_0(\bar{r}))$ , which must hold because of  $f$ 's monotony and  $\bar{W}/2 > k_0(\bar{r})$ . That is also why  $2qf(\bar{W}/2) > 2[pf(k(\bar{r})) - 1]$ . Although only running equally large non-effort firms maximizes the economy's aggregate output for  $W = \bar{W}$ , it lowers its return rate:  $qf'(\bar{W}/2) < \bar{r}$ . (A1) and  $f$ 's concavity further ensure that (B.1) must be true for all  $W < \bar{W}$ , when the RHS declines. Hence, (B.1) implicitly defines a  $\hat{r}_q < \bar{r}$ , s.t. if  $W$  is so small that  $qf'(W/2) \geq \hat{r}_q$ , only running non-effort firms of equal size is optimal.

Second, we turn to  $\hat{r}_p$ :  $\bar{W} = 2k(\bar{r}) = 2k_0(\hat{r}_p)$  implicitly defines a  $\hat{r}_p$  with  $\hat{r}_p < \hat{r}_q$ . If  $W$  is then so high that  $pf'(W/2) \leq \hat{r}_p$ , running only equally large effort firms is optimal, i.e.:

$$2[pf(W/2) - 1] \geq [pf(k(\hat{r}_p)) - 1] + qf(k_0(\hat{r}_p)). \quad (\text{B.2})$$

(ii) Inserting  $\alpha(W) = [W - k(r)] / [k_0(r) - k(r)]$  into

$$U' = \alpha(W) qf'(k_0(r)) k_0'(r) + (1 - \alpha(W)) [pf'(k(r)) k'(r)]$$

and rearranging terms gives (3.18)  $\hat{r}$ .

- **Proof of Lemma 3.5:**

The Proof is straightforward from Definition 3.3, Lemmata 3.3 and 3.4.

- **Proof of Proposition 3.2:**

The Proof directly follows from the construction of  $D(r)$  and  $S(r)$ , the equilibrium Definition 3.2, (A2),  $\hat{r}_p > 1$  and Lemma 3.3.

## B.3 Proofs for Section 3.5

- **Proof of Lemma 3.6:**

On the grounds of Definition 3.4, Lemmata 3.3 and 3.4, the Proof follows from  $\omega(r)$  and the definition of  $\underline{r}$  and  $\bar{r}$ , whereby:

(1<sup>st</sup> step) Checking that  $\omega(r)$  is a strictly increasing and concave function in  $r$  for  $r < \bar{r}$ : From (3.20)  $\omega(r)$ , we get

$$d\omega/dr = p[(p-q)f(k(r)) - 1]/(p-q)r^2 > 0 \text{ for } r < \bar{r},$$

since  $(p-q)f(k(r)) - 1 > 0$  by (A1) for  $r < \bar{r}$ . Thus, in turn, is directly derived from  $[pf(k(r)) - rk(r)] - [qf(k(r)) - rk(r)] > y(r) - y_0(r)$  (as  $y_0(r)$  is maximal for  $k_0(r)$  and hence smaller for any other  $k(r)$ ), and  $y(r) - y_0(r) > 1$  for  $r < \bar{r}$ . By the same argument and with  $k'(r) < 0$ , the second derivative indeed gives

$$d^2\omega/dr^2 = [(p-q)k'(r)r^2 - 2p[(p-q)f(k(r)) - 1]]/r^3(p-q) < 0.$$

(2<sup>nd</sup> step) Assessing the impact of  $q$  on  $\omega(r)$ : On the one hand, there is a  $q_{01} > 0$  s.t.  $\omega(1) < 0$  for  $q > q_{01}$ .  $q_{01}$  is derived from equalizing the net present value and moral hazard costs for  $r = 1$ , i.e.  $[pf(k(1)) - k(1)] = p/(p - q_{01})$ , and strictly positive by (A1); otherwise there would be no commitment issue. On the other hand, there is a  $q_{02} > 0$  s.t.  $\omega(\bar{r}) > 0$  for  $q < q_{02}$ . First, for  $r = \bar{r}$ ,

$$(3.20) \quad \omega(\bar{r}) = p/\bar{r}(p-q) - pf(k(\bar{r}))/\bar{r} + k(\bar{r})$$

is a continuous function of  $q$  with  $\omega(\bar{r}(0)) = 0$ .

$$\begin{aligned} d\omega(\bar{r})/dq &= -p[(p-q)\bar{r}' - \bar{r}]/(p-q)^2\bar{r}^2 + k'(\bar{r})\bar{r}' \\ &\quad - [pf'(k(\bar{r}))k'(\bar{r})\bar{r}'\bar{r} - pf(k(\bar{r}))\bar{r}']/\bar{r}^2 \\ &= \omega'(\bar{r})\bar{r}' + p/(p-q)^2\bar{r}, \end{aligned}$$

On top,  $\bar{r} = [pf(k(\bar{r})) - qf(k_0(\bar{r})) - 1]/[k(\bar{r}) - k_0(\bar{r})]$  from (A1), so that  $d\bar{r}/dq = -f(k_0(\bar{r}))/[k(\bar{r}) - k_0(\bar{r})]$ , as well as  $k_0 = 0$  (and thus  $f(0) = 0$ ) for  $q = 0$ . This being said, finally allows to state that

$$d\omega(\bar{r})/dq|_{q=0} = 1/p\bar{r}(0) > 0.$$

Mind that it follows from (A1) that for  $q \rightarrow 0^+$ ,  $\bar{r} \rightarrow \infty$ . Hence, there indeed is a  $q_{02} > 0$  s.t. for  $q < q_{02}$  it is that  $\omega(\bar{r}) > 0$ . With  $\omega(r)$  being a continuous function of  $r$ , it follows that for  $q < q_{02}$  there exists  $r < \bar{r}$ , s.t.  $\omega(r) = 0$  and  $\omega(r) > 0$  for  $r \in (r, \bar{r})$ . Consequently, (i) and (ii) of Lemma 3.6 hold for  $r > 1$ , i.e. for  $q \in (0, q_0)$  with  $q_0 = \min\{q_{01}, q_{02}\}$ .

(3<sup>rd</sup> step) Scrutinizing the relative position of curves, especially  $k_0(\bar{r}) < \omega(\bar{r}) < \tilde{w} < k(\bar{r})$  for  $r = \bar{r}$ : First remember that we know from Lemma 3.2 that for  $r = \bar{r}$ ,  $k_0(\bar{r}) < \tilde{w} < k(\bar{r})$ . Then: (i) Show that  $\omega(\bar{r}) < \tilde{w}$ . For this to see, substitute (3.20)  $\omega(r)$ , which reduces to  $pf(k(\bar{r})) - \bar{r}k(\bar{r}) > p/(p-q) - \bar{r}\tilde{w}$ . Using (A1) to substitute  $y_0(\bar{r}) + 1$  for  $y(\bar{r})$  on the LHS and inserting  $f(\tilde{w}) = 1/(p-q)$  on the RHS, gives  $y_0(\bar{r}) > qf(\tilde{w}) - \bar{r}\tilde{w}$ , which holds as we know from Lemma 3.1 that the non-effort project's benefit is maximal for  $k_0(\bar{r})$  and, due to its strict concavity, less for  $\tilde{w} > k_0(\bar{r})$ . (ii) Check  $k_0(\bar{r}) < \omega(\bar{r})$ . Proceeding as for  $\omega(\bar{r}) < \tilde{w}$  before, yields  $qf(k_0(\bar{r})) < qf(\tilde{w})$ , which is true as we derived  $k_0(\bar{r}) < \tilde{w}$  in Lemma 3.2. (iii) Verify that  $\omega(\bar{r}) < k(\bar{r})$ , again by proceeding as with  $k_0(\bar{r}) < \omega(\bar{r})$

until we get  $qf(\tilde{w}) - rk(\bar{r}) < y_0(\bar{r})$ . Use the definition of  $\tilde{w}$  to substitute  $pf(\tilde{w}) - 1$  for  $qf(\tilde{w})$  on the LHS and get  $pf(\tilde{w}) - rk(\bar{r}) - 1 < y_0(\bar{r})$ . This is true, since the LHS is smaller than  $y(\bar{r}) - 1 = y_0(\bar{r})$  and since we know from Lemma 3.2 that  $\tilde{w} < k(\bar{r})$ . (iv) Study the curves' behavior for  $r < \bar{r}$ . As  $\tilde{w} > 0$ ,  $\omega(r)$  is strictly increasing, concave and  $\omega(r) = 0$  as well as  $\omega(\bar{r}) < \tilde{w}$ , it is assured that  $\omega(r) < \tilde{w}$  for  $r < \bar{r}$ . This being said, as  $k'_0(r) < 0$ ,  $k_0(0) = \infty > \omega(0)$ ,  $k_0(\infty) = 0 < \omega(\infty)$ , it follows that  $k_0(r) > \omega(r)$  for  $r < \tilde{r}_D < \bar{r}$  and  $k_0(r) < \omega(r)$  for  $r > \tilde{r}_D$ . By the same argument, it becomes obvious that  $k(r) > \tilde{w}$  and only falls below for some  $r > \bar{r}$ .

(4<sup>th</sup> step) Studying (3.13)  $IR_{L2}$  in its respective form: Knowing that for  $r \leq \bar{r}$ , it holds that  $\omega(r) < \tilde{w} < k(r)$ ,  $IR_{L2}$  must be checked for three cases: (i) If  $\omega(r) < \tilde{w} \leq w_i$ , the agent either self-finances or is eligible for an effort credit and  $v \geq y_a - 1$  follows from the fact that for  $r \leq \bar{r}$ , the net profit is maximal for  $k(r)$  and less for  $\forall w_i \neq k(r)$ . (ii) If  $\omega(r) \leq w_i < \tilde{w}$ , the agent is again eligible for an effort credit. Then,  $v \geq y_{a0}$  holds if  $y(r) - 1 \geq y_0(r)$  is satisfied (which is valid to do, since on the RHS,  $y_{a0} \leq y_0(r)$  and on the LHS, after substituting  $t$  and  $w_c$ ,  $v$  reduces to  $y(r) - 1$ ). In fact,  $y(r) - 1 \geq y_0(r)$  holds, because of (A1). (iii) If  $w_i < \omega(r) < \tilde{w}$ , the agent is credit constrained and it matters if  $w_i \geq k_0(r)$ . For  $w_i \geq k_0(r)$ , we already derived in Lemma 3.2 that the agent prefers to self-finance  $k_0(r)$  and to deposit the remainder. Whereas if  $w_i < k_0(r)$ , the agent asks for a non-effort credit, because then  $v_0 \geq y_{a0}$ . After having substituted  $t_0$  and  $w_c$ , this reduces to  $y_0(r) \geq y_{a0}$ , which is satisfied, as the net profit is maximal for  $k_0(r)$  and less for  $\forall w_i \neq k_0(r)$ . At last,  $IR_{L2}$  must be assessed for  $r > \bar{r}$ : If  $k_0(r) > w_i$ , it must be that  $v_0 > y_{a0}$ , which holds, since the net profit is maximal for  $k_0(r)$  and strictly less for  $\forall w_i \neq k_0(r)$ .

- **Proof of Proposition 3.3:**

The Proof is similar to that of Proposition 3.1, but additionally requires the inclusion of Lemma 3.6.

- **Proof of Lemma 3.7:**

Five things remain to be shown.

(1<sup>st</sup> step) Checking  $k_0(r_{D,j}(r_L))$ 's location: Recall that for  $r_L > \bar{r}_L$ ,

$$y(r_L) - 1 < y_0(r_L) < y_0(r_{D,j}(r_L)), \quad (\text{B.3})$$

so that (3.24)  $y(r_L) - 1 \geq y_0(r_{D,j}(r_L))$  could only hold for some  $r_L \leq \bar{r}_L$ . Then,  $k_0(r_{D,j}(r_L))$  and  $k(r_L)$  would intersect if  $f'^{-1}(r_{D,j}/q) = f'^{-1}(r_L/p) \iff r_L/r_{D,j} = p/q$ . Substituting  $k(r_L)$  for  $k_0(r_{D,j}(r_L))$  into (3.24) gives

$$qf(k(r_L)) - r_{D,j}k(r_L) \geq pf(k(r_L)) - r_Lk(r_L) - 1. \quad (\text{B.4})$$

Using the fact that  $r_L > r_{D,j}$ , allows to reduce the inequality to  $f(\tilde{w}) \geq f(k(r_L))$ . Thus, if  $k_0(r_{D,j}(r_L))$  and  $k(r_L)$  cross, it must be where  $\tilde{w} = k(r_L)$ . Yet,  $k(r_L)$ 's shape (derived in Lemma 3.6) and (3.23) entails that this can only be the case for  $r_L > \bar{r}_L$ .

That is how it is ensured that for  $r_L \leq \bar{r}_L$ , (3.23) holds,  $k_0(r_{D,j}(r_L)) < k(r_L)$  and so that  $r_L/r_{D,j} < p/q$ , irrespective of an intersection.

(2<sup>nd</sup> step) Refining  $k_0(r_{D,j}(\bar{r}_L))$ 's position: On the one hand, it follows from (B.4) that if  $k_0(r_{D,j}(r_L)) = k(r_L)$ , then both also equal  $\tilde{w}$  and  $k_0(r_{D,j}(\bar{r}_L)) > \tilde{w}$ . On the other hand, if there is no intersection and  $k_0(r_{D,j}(r_L)) < k(r_L) \forall r_L$ , (3.24) always holds and  $k_0(r_{D,j}(\bar{r}_L))$ 's position cannot further be contained, neither relative to  $\tilde{w}$ , nor to  $\omega(\bar{r}_L)$ : As we know that (3.24)  $y(\bar{r}_L) - y_0(r_{D,j}(\bar{r}_L)) < 1$ , it must all the more be with  $k_0(r_{D,j}(\bar{r}_L)) < k(\bar{r}_L)$  that

$$\begin{aligned} & [pf(k_0(r_{D,j}(\bar{r}_L))) - \bar{r}_L k_0(r_{D,j}(\bar{r}_L))] \\ & - [qf(k_0(r_{D,j}(\bar{r}_L))) - r_{D,j}(\bar{r}_L) k_0(r_{D,j}(\bar{r}_L))] < 1 \\ \Leftrightarrow & f(k_0(r_{D,j}(\bar{r}_L))) < f(\tilde{w}) + [\bar{r}_L - r_{D,j}(\bar{r}_L)] k_0(r_{D,j}(\bar{r}_L)) / [p - q]. \end{aligned}$$

Hence,  $k_0(r_{D,j}(\bar{r}_L)) \leq \tilde{w}$ . As the second term on the RHS is positive, the position of  $k_0(r_{D,j}(\bar{r}_L))$  relative to  $\tilde{w}$  depends on the magnitude of the interest rate spread set by the monopsonist (and thus ultimately also on  $G(w)$ ),  $p$ ,  $q$  and the steepness of the production function. On top making use of the refined pricing formula (3.26) (i.e. of  $r_{D,j}/p > S_j(r_{D,j}^*) / (p - q) S_j'(r_{D,j}^*)$ ) yields

$$pf(k_0(r_{D,j}(\bar{r}_L))) < pf(\tilde{w}) + r_{D,j} k_0(r_{D,j}(\bar{r}_L)),$$

but still does not help to generally decide  $k_0(r_{D,j}(\bar{r}_L)) \leq \tilde{w}$ . A similar consideration applies to  $k_0(r_{D,j}(\bar{r}_L))$  and  $\omega(\bar{r}_L)$ . Suppose  $k_0(r_{D,j}(\bar{r}_L)) > \omega(\bar{r}_L)$ , which reduces (after taking advantage of (A1)) to  $qf(\tilde{w}) - \bar{r}_L k_0(r_{D,j}(\bar{r}_L)) < qf(k_0(\bar{r}_L)) - \bar{r}_L k_0(\bar{r}_L)$ . This holds for  $k_0(r_{D,j}(\bar{r}_L)) > \tilde{w}$  (whereas the opposite claim would only hold for  $k_0(r_{D,j}(\bar{r}_L))$  being sufficiently smaller than  $\tilde{w}$ ).

(3<sup>rd</sup> step) Optimal behavior: With the calculus resembling the (4<sup>th</sup> step) of Lemma 3.6's Proof, we restrict ourselves to the non-obvious cases for  $r_L \leq \bar{r}_L$ , where some particularities are ensued from the two interest rates being at interplay with initial wealth.

(i) Look at  $i$  with  $w_i \in [k_0(r_{D,j}), k(r_L)]$  who prefers a leveraged effort firm not only to pure depositing (because of  $r_L > r_{D,j}$ ), but also to a self-financed non-effort firm with depositing, since

$$pf(k(r_L)) - r_L [k(r_L) - w_i] - 1 \geq qf(k_0(r_{D,j})) + r_{D,j} [w_i - k_0(r_{D,j})]. \quad (\text{B.5})$$

Indeed, this reduces to  $\vartheta(r_L) := [y_0(r_{D,j}) - y(r_L) + 1] / [r_L - r_{D,j}]$  as the critical wealth level  $\vartheta(r_L)$  above which (3.24) reverses. Substituting  $w_i = k_0(r_{D,j})$  into (B.5) gives  $y_{a0}(r_L) < y(r_L) - 1$ . It holds for  $w_i < \tilde{w}$  because of the production function's concavity and  $y_{a0}(r_L) \leq y_a(r_L)$ . Thus,  $\vartheta(r_L) \leq k_0(r_{D,j})$  and for  $w_i \geq k_0(r_{D,j})$ , (3.24) reverses for sure.

(ii)  $\vartheta(r_L)$ 's magnitude does not need to be further refined, because already for  $w_i \in [\omega(r_L), k_0(r_{D,j})]$ , self-financed non-effort production is unattainable. Thus, only leveraged effort or autarkic production remain to those agents. As for  $w_i \geq \tilde{w}$ ,  $pf(w_i) - 1 < pf(k(r_L)) - r_L [k(r_L) - w_i] - 1$  must hold by  $y$ 's concavity again, the same must

be true for  $w_i < \tilde{w}$ , because  $y_{a0}(r_L) \leq y_a(r_L)$  on the LHS. (iii) At last, consider  $w_i \in [k(r_L), k(r_{D,j})]$ . As capital units  $\tilde{w} = k(r_{D,j}) - k(r_L)$  yield  $r_{D,j}$  on the capital market, but  $r_L \geq r_{D,j}$  if invested in the project, it must be that  $pf(w_i) - 1 > pf(k(r_L)) + r_{D,j}[w_i - k(r_L)] - 1$ .

(4<sup>th</sup> step) The SOC is negative: (3.26)  $[r_L - r_{D,j}] = S_j(r_{D,j})/S'_j(r_{D,j}) > 0$  and (A3) give rise to  $d^2\Pi_j/dr_{D,j}^2 = -2S'(r_{D,j}) + [r_L - r_{D,j}]S''(r_{D,j}) < 0$ .

(5<sup>th</sup> step) All relevant side-conditions in Definition 3.5 are also satisfied, including (3.10)  $IR_{D1}$ : this is the case, since  $r_L \geq r_{D,j}^* \geq 1$  if the interior solution yields a  $r_{D,j}^* = r_L - S_j(r_{D,j}^*)/S'_j(r_{D,j}^*) \geq 1$  and  $r_L \geq r_{D,j}^* = 1$  otherwise. In both case, (3.4)  $IR_L^B$  holds, because  $r_L \geq r_{D,j}^*$  causes that the monopsonist realizes an intermediation margin that is always positive and so that he even earns a non-negative profit on the marginal contract.

- **Proof of Lemma 3.8:**

Based on Lemma 3.7, the satisfaction of all side-conditions in Definition 3.5 is straightforward.

- **Proof of Proposition 3.4:**

The Proof directly follows from Lemmata 3.7 and 3.8, Propositions 3.1 and 3.3 as well as the analysis above, but for the following:  $r_{D,j}^* \geq 1$  is ensured thanks to Lemma 3.7 and with  $r_{D,j}^* = r_C^*$ .  $r_{D,j}^* \geq 1$  holds because of Proposition 3.3 and because autarkic production reduces gross capital demand and supply by the same magnitude.

- **Proof of Lemma 3.9:**

Against the background of the monopsony results in Subsection 3.5.3, optimality follows from the calculus above, since also the *SOC* is negative. This obtains, because in optimum  $[r_{L,j} - r_D] = -D_j(r_{L,j})/D'_j(r_{L,j}) \leq 0$  and (A4) ensure that  $d^2\Pi_j/dr_{L,j}^2 = 2D'_j(r_{L,j}) + [r_{L,j} - r_D]D''_j(r_{L,j}) < 0$ .

- **Proof of Lemma 3.10:**

Two things are left to be verified. First, check that  $\Pi_{L,ji0} \geq 0$  and the bank's participation constraint: With  $t_0 = f(w_i)$  for all  $w_i < k_{L0}$  and  $k_{L0} = k_0(r_D)$ , it must be that  $qf(k_0(r_D)) - r_D[k_0(r_D) - w_i] - qf(w_i) \geq 0$ . Rearranging terms gives  $y_0(r_D) \geq y_{a0}(r_D)$ , which holds by the profit function's positive monotony for  $k_0(r_D) \geq w_i$ . Hence, already  $\Pi_{L,ji0} \geq 0$  satisfies (3.4)  $IR_L^B$ , so that if  $\Pi_{L,ji} \geq \Pi_{L,ji0}$ ,  $\Pi_{L,ji}$  would do it the more.

Second, compare (3.36)  $\psi(r_D)$  to (3.20)  $\omega(r_D)$ : With (A1)  $y(\bar{r}) - y_0(\bar{r}) = 1$  and  $d(y(r_D) - y_0(r_D))/dr_D < 0$  (see Proof of Proposition 3.1), (3.35)'s satisfaction depends on the size of agent's initial wealth  $w_i$ . Like  $\omega(r_D)$ ,  $\psi(r_D)$  is continuous,  $d\psi(r_D)/dr_D > 0$  and  $d^2\psi(r_D)/dr_D^2 < 0$ , but  $\psi(\bar{r}) = \tilde{w}$ . Thanks to the Proof of Lemma 3.6, we can thus constitute that  $\psi(\bar{r}) > \omega(\bar{r})$ . Moreover, with  $r_\psi$  being defined s.t.  $\psi(r_\psi) = 0$ , rewriting (3.36) gives  $y(r_\psi) - y_0(r_\psi) = 1 + qf(\tilde{w})$  or  $y(r_\psi) = pf(\tilde{w}) + y_0(r_\psi)$ . As  $\omega(r) = 0$  for  $y(r) = pf(\tilde{w})$ ,  $y(r_\psi) > y(r)$  and it immediately follows from  $y$ 's monotonous decline in  $r$  that  $r_\psi < r < \hat{r}_p$  and that  $\omega(r_\psi) < 0$ . Thus  $\psi(r_D) > \omega(r_D)$  for all  $r_D$ .

- **Proof of Proposition 3.5:**

The Proof directly follows from Lemmata 3.9 and 3.10, Propositions 3.1 and 3.3 as well as the analysis above.

- **Proof of Lemma 3.11:**

Based on Definition 3.7, Lemmata 3.8 and 3.10 as well as Propositions 3.1 and 3.3, the Proof is straightforward from (3.38) and the derivation of (3.40).

- **Proof of Proposition 3.6:**

In the light of Propositions 3.4 and 3.5, Lemmata 3.8 and 3.10, the Proof follows from the analysis above, Lemma 3.11 and Definition 3.2.

# Appendix C

## Appendix to Chapter 4

### C.1 Proofs for Section 4.3

- **Proof of Lemma 4.1:**

The Lemma directly follows from (A2), the derivation of  $k(r)$ ,  $\omega(r)$  and  $\mathfrak{r}(\gamma)$ . It remains to assess the functional form of (4.4)  $\omega(r)$ , which is continuous and differentiable in  $r$ . Using (4.1)  $f'(k(r)) = r$  gives:

$$\omega'(r) = [1 - \gamma] k'(r) + \gamma f(k(r)) / r^2. \quad (\text{C.1})$$

As the first term is monotonously increasing in  $r$ , whereas the second is monotonously decreasing in  $r$ , there is a single  $r = \dot{r}$ , such that  $\omega'(\dot{r}) = 0$ . Then,  $\omega'(r) > 0$  for  $r < \dot{r}$  and  $\omega'(r) < 0$  for  $r > \dot{r}$ . This stems from the fact that  $r \rightarrow 0$ ,  $k'(r) \rightarrow 0^-$  and  $f(k(r)) / r^2 \rightarrow \infty$ . Whereas for  $r \rightarrow \infty$ ,  $k'(r) \rightarrow -\infty$  and  $f(k(r)) / r^2 \rightarrow 0^+$ . Using (4.1), simple algebra yields that (C.1)  $\omega'(r) \geq 0$  as long as

$$\eta_{y,r} := -r f'(k(r)) k'(r) / f(k(r)) \leq \gamma / [1 - \gamma],$$

where  $\eta_{y,r} > 0$  is the input price elasticity of output. Likewise,

$$\omega''(r) = [1 - \gamma] k''(r) + [\gamma / r] [k'(r) - 2f(k(r)) / r^2]. \quad (\text{C.2})$$

Against the background of  $\omega(r)$ 's monotony and with the first term being positive, whilst the second one being negative for all  $r$ , there must be a single  $r = \ddot{r}$ , such that  $\omega''(\ddot{r}) = 0$ . Then,  $\omega''(r) < 0$  for  $r < \ddot{r}$  and  $\omega''(r) > 0$  for  $r > \ddot{r}$ , since  $\omega(r) \rightarrow -\infty$  for  $r \rightarrow 0$ , but  $\omega(r) \rightarrow 0$  for  $r \rightarrow \infty$ . By comparison of the terms in  $\omega'(r)$  and  $\omega''(r)$ , it must be that  $\dot{r} < \ddot{r}$ . Similarly, one can show that (C.2)  $\omega''(r) \leq 0$  for  $\eta_{y,r} \leq 2 / [(1 - \gamma) \varepsilon_{k',r} / \gamma - 1]$ , where  $\varepsilon_{k',r} > 0$  is the factor price elasticity of the optimal investment's slope. Then, it follows from the fact that  $\omega(r)$  is decreasing and concave for  $\gamma / [1 - \gamma] \leq \eta_{y,r} \leq 2 / [(1 - \gamma) \varepsilon_{k',r} / \gamma - 1]$  that  $\varepsilon_{k',r} \leq 2 + \frac{\gamma}{(1 - \gamma)}$ .

- **Proof of Proposition 4.1:**

The Proof immediately follows from (A1) to (A3), Definition 4.1, Lemma 4.1 and the construction of the demand correspondence.

## C.2 Proofs for Section 4.4

- **Proof of Proposition 4.2:**

Given (A1) to (A3), Lemma 4.1 and Proposition 4.1, first decompose the impact of  $\Delta r$  on  $A$ 's  $GNP$  into its impact on  $P_A$  and  $X_A$ :

$$d\hat{P}_A^*/dr = [f(\omega(r)) - f(k(r))] \omega'(r) g(\omega(r)) + [1 - G(\omega(r))] f'(k(r)) k'(r) \quad (C.3)$$

There are two effects at play: a credit rationing and a firm size effect. While the second is captured by the second term and always negative, the first is captured by the first term, which is negative for  $r < \hat{r}$  and positive thereafter - owing to a change in sign of  $\omega'(r)$ . As  $\hat{P}_A^*$  is decreasing in  $r$  if  $1 \leq r \leq \underline{r}(\gamma)$ , when credit rationing is absent (so that only the firm size effect is effective),  $d\hat{P}_A^*/dr < 0$  for  $r < \hat{r}$ . Yet, this standard response might not persist throughout  $r > \hat{r}$ , since the firm size and credit rationing effect then go into opposite directions w.r.t. aggregate investment. Instead, as derived in (4.8), the opposite response arises if, around the point of inflexion at  $r = \hat{r}$ , a sufficiently small  $g(\omega(r)) \omega'(r)$  entails an investment-enhancing credit rationing effect that outweighs the investment-depressing firm size effect. Besides:

$$d\hat{X}_A^*/dr = [W_A - D_A(r)] + r[k(r) - \omega(r)] \omega'(r) g(\omega(r)) - r[1 - G(\omega(r))] k'(r) \quad (C.4)$$

Starting out from autarky, the first term in (C.4) is positive if we are in the interest rate region where  $D'_A(r) < 0$  (and negative if condition (4.7) holds). The second term is positive if  $r < \hat{r}$  (and negative if not) and the third negative  $\forall r$ . For  $r \leq \underline{r}(\gamma)$ , the second as well as third term vanish and  $D'_A(r) < 0$ , so that  $d\hat{X}_A^*/dr > 0$ . Otherwise, the interest rate range and the magnitude of the terms matter. By the same analysis as for  $d\hat{P}_A^*/dr$ ,  $d\hat{X}_A^*/dr > 0 \forall r$  except for if condition (4.11) holds. Given (A3), a market rate of return span  $[r_{X1}, r_{X2}]$  then materializes for which  $d\hat{X}_A^*/dr \leq 0$ . Beyond that, it follows from the comparison of (4.11) and (4.7), that the  $RHS$  of the first is larger than that of the second for  $D_A(\hat{r}^*) > W_A$ . As both  $RHS$  are negative, the satisfaction of the first is therefore automatically implied by that of the second as long as  $D_A(r) > W_A$ . The contrary, instead, would require  $D_A(r) < W_A$ . Yet, this can never be true, since coming from an autarkic equilibrium as characterized in Proposition 4.1 makes the satisfaction of (4.7) equivalent to  $D_A(\hat{r}^*) > W_A$ . Thus, (4.11)  $d\hat{X}_A^*/dr \leq 0$  cannot be fulfilled without (4.7)  $dD_A/dr \geq 0$  holding.

Now, sum up (C.3)  $d\hat{P}_A^*/dr$  and (C.4)  $d\hat{X}_A^*/dr$  to get the derivative of (4.9)  $\hat{Y}_A^*$  w.r.t.  $r$ . Using (4.1)  $f'(k(r)) = r$ ,  $d\hat{Y}_A^*/dr = [W_A - D_A] - rD' + P'$  reduces to

$$d\hat{Y}_A^*/dr = [W_A - D_A(r)] + [[f(\omega(r)) - r\omega(r)] - y(r)] \omega'(r) g_A(\omega(r)). \quad (C.5)$$

Again, the first term is positive (except for if condition (4.7) holds) and approaches  $W_A$  for  $r \rightarrow \infty$ . In the second term,  $[f(\omega(r)) - r\omega(r)] - y(r) < 0$  (since the profit is maximal for  $k(r)$  and smaller for any other  $k \neq k(r)$ ), so that the sign of  $\omega'(r)$  depicted in Lemma 4.1 becomes crucial once more.

Thus, for  $r \leq \hat{r}$ , the signs of the two terms in (C.5) oppose. It is immediately clear that for  $r \leq \underline{r}(\gamma)$ , when there is no credit rationing,  $d\hat{Y}_A^*/dr > 0$ . Whereas for  $\underline{r}(\gamma) < r_A^* < \hat{r}^* \leq \hat{r}$ ,  $d\hat{Y}_A^*/dr \gtrless 0$ , depending on  $|\Delta P_A| \gtrless |\Delta X_A|$  with  $\Delta P_A < 0$  and  $\Delta X_A > 0$ . In contrast,  $d\hat{Y}_A^*/d\hat{r}^* > 0$  for  $r > \hat{r}$  when both terms in (C.5) share the positive sign. This is given in the *Y-SRC* characterized in Definition 4.2, when any losses from smaller firm sizes become less burdensome relative to the gains in the net credit position and the relaxed credit rationing. Whereas in the *Y-RRC*, the fulfillment of (4.7) makes  $A$  a capital importer, so that the first term in (C.5) becomes negative. Then, (4.12) assures that any dominance of the credit rationing over the firm size effect, which makes  $\Delta P_A$  and  $\Delta X_A$  temporarily change sign, translates into  $\Delta P_A \geq 0$  remaining dominated by  $\Delta X_A \leq 0$ . Thus,  $d\hat{Y}_A^*/d\hat{r}^* \leq 0$  in the *Y-RRC*.

At last, because of  $[W_A - D_A(r)]$ , there are scenarios, in which the direction of the interest rate change becomes decisive. If  $A$  sees an interest rate decline (i.e.  $\hat{r} \geq r_A^* > \hat{r}^*$ ), we get  $\Delta Y_A > 0$ , because (4.12) cannot hold. This owes to the fact that  $\Delta r_A < 0$  induces more net borrowing firms with non-negative profits at a higher optimal scale, each generating higher profits than their credit-rationed counterparts ( $y'(r) < 0$ ). That is how in the aggregate, the first term in (C.5) gets dominated by the second. As soon as  $r_A^* > \hat{r}^* > \hat{r}$ , instead, also the first term in (C.5) turns negative owing to tightened rationing. Hence,  $\Delta Y_A < 0$  in the *Y-SRC*. Contrariwise, (4.12) implies that in the *Y-RRC*,  $\Delta Y_A \geq 0$ , because the effects just reverse. As (4.7) holds,  $A$  turns into a capital exporter, making the first term positive. Then, (4.12) ensures again that the first term dominates the second, so that the positive effect prevails.

- **Proof of Proposition 4.3:**

The Proof follows from (A1) to (A3), Definition 4.1, Lemma 4.1, Propositions 4.1 and 4.2 as well as the construction  $S(r)$  and  $D(r)$ .



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1999-2000 Visiting scholar at the Institut d'Études Politiques de Paris, France  
1996-1999 Undergraduate studies in Economics and Business Administration,  
University of Mannheim, Germany  
1987-1996 Lina-Hilger-Gymnasium, Bad Kreuznach, Germany

## Professional Experience

2003-2007 Research and teaching assistant, Chair for Economic Policy,  
University of Mannheim, Germany  
2007 Visiting research fellow, Ente per gli Studi, Monetari, Bancari e  
Finanziari "Luigi Einaudi", Rome, Italy  
2006 Summer intern, International Monetary Fund, Washington D.C., USA  
2001 Intern, German Embassy, Sofia, Bulgaria  
2000 Intern, Deutsche Bundesbank, Frankfurt am Main, Germany  
2000 Intern, European Bank for Reconstruction and Development, London, UK  
1998 Intern, German-Italian Chamber of Commerce, Milan, Italy  
1996 Intern, DZ Bank, Frankfurt am Main, Germany  
1998-2002 Student teaching and research assistant, University of Mannheim

## Honors and Grants

2007 Research Fellowship, Ente "Luigi Einaudi", Rome, Italy  
1997-2002 Studienstiftung des deutschen Volkes  
2002 Landesstiftung Baden-Württemberg  
1999-2000 German Academic Exchange Service (DAAD)