Essays in international trade with heterogeneous firms

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Chapter 1

Introduction

This thesis examines the effect of international trade between industrialised countries on three topics of interest from a macroeconomic perspective. In chapter 2, I investigate the effects of a reduction in the costs of international trade on wage inequality between skilled and unskilled labour. Chapter 3 analyses how trade liberalisation affects firm-level and aggregate investment in process innovation. In chapter 4, I turn to the links between trade openness and output volatility. I approach these three questions with a common methodology, which stresses the importance of accounting for firm-level heterogeneity in order to gain a better understanding of the macroeconomic consequences of international trade. I build on the insights of the heterogeneous firms literature in trade to shed a new light on the three topics outlined above.

The 1990s have seen the development of a large empirical literature pointing to the fact that firms which export part of their output to foreign countries are larger, more productive, more capital intensive and pay higher wages than non-exporters. These observations were established for U.S. firms by the seminal paper of Bernard and Jensen (1995) and have been replicated for a large number of countries[1]. These robust empirical regularities have posed a challenge to the representative firm models used in the traditional trade theories and called for a new framework of analysis.

In order to make sense of the above observations and to draw their conclusions for the analysis of international trade on welfare and productivity, a new theoretical literature, led by Melitz (2003), has developed in the last decade. This literature extends the representative firm model of Krugman (1979), which provides a rationale for intra-industry international trade between perfectly symmetric countries, to a heterogeneous firms framework. The Krugman (1979) model relies on two main features. First, it

[1] see among others Bustos (2005) for Argentina, Eaton et al. (2004) for France, Wagner (1995) for Germany or De Loecker (2007) for Slovenia.

assumes that consumers have preferences exhibiting love of variety[2], which means that for a given budget, they would like to consume as many different varieties as possible. Second, each variety is produced with increasing returns to scale by one firm at one location. As consumers in each country want to consume all varieties available in the world, they import those that are produced in the foreign country, thereby generating international trade. Krugman (1979) shows that intra-industry trade is beneficial, as it expands the set of varieties available to consumers in each country.

Melitz (2003) adds two core elements to the representative firm framework of Krugman (1979). First, he assumes that firms are heterogeneously productive for exogenous reasons. Productive firms optimally charge low prices, sell large amounts of their variety and make high profits. Second, he argues that exporting to a foreign country requires the payment of a fixed cost, which can be interpreted as the cost of establishing a distribution network in a foreign country, adapting a product to foreign regulation or learning foreign business laws and customs. If these costs are large, only highly productive firms generate enough profits on the export market to cover the fixed costs of exporting. This generates a partitioning between exporting and non-exporting firms, where exporters are more productive and larger than average. The Melitz (2003) model furthermore provides a new rationale for gains from trade. It shows that trade liberalisation leads to a reallocation of market shares from inefficient firms, which exit the market, to more efficient firms, which export. This reallocation induces aggregate productivity gains.

The work of Melitz (2003) has been a major breakthrough in the international trade literature in the last six years. It provides a tractable workhorse model on which to build for the study of a variety of problems related to international trade betweeen symmetric countries. The present thesis contributes to this literature by applying its insight to the three topics outlined above and stresses the importance of accounting for firm heterogeneity in order to draw conclusions for the macroeconomic effects of international trade.

The second chapter shows how trade liberalisation between symmetric countries affects wage inequality between skilled and unskilled labour (henceforth the 'skill premium').

Strong empirical evidence shows that the U.S. skill premium has been dramatically rising through the 1980s and 1990s. Politicians and economists were quick to point to the increasing volumes of U.S. trade with developing countries as a potential driver of this trend. Standard trade theory[3] indeed predicts that if the U.S., which has a relatively skilled labour force, trades with countries with a relatively unskilled labour force, the skill premium in the U.S. should rise. However, U.S. trade volumes with developing countries were low in the 1980s and 1990s, leading Krugman and Lawrence (1993) to

[2] This type of preferences originally comes from Dixit and Stiglitz (1977).

[3] the Stolper-Samuelson effect in a Heckscher Ohlin framework.

disqualify international trade as an explanation for the evolution of the skill premium.

I show that trade between industrialised countries, which constitutes the bulk part of U.S. and world trade, also affects the skill premium. I build on a Melitz (2003) type model in which I introduce a positive correlation between the productivity of a firm and its skill intensity, defined as the ratio of skilled employees to total employment at the firm level. This conforms to empirical evidence and replicates the fact that exporting firms are more skill intensive than average. A reduction in the costs of trade induces a reallocation of market shares from non-exporters to exporters, which produce with different skill intensities. This affects the aggregate demand for both factors, which are in fixed supply in the economy, and therefore their relative price.

I show that the effect of trade liberalisation on wage inequality depends on the type of trade costs considered. On the one hand, a reduction in the variable costs of trade (tariffs, transportation costs) unambiguously raises wage inequality. On the other hand, a drop in the fixed costs of trade has a non-monotonic effect on the skill premium. The mechanism behind these results is as follows.

A reduction in the variable costs of trade has three effects on firms in the economy. First, exporting firms can sell on the foreign market at lower costs and export more. Since these firms are more skill intensive than average, their increase in production raises the relative demand for skilled labour in the economy and therefore the skill premium. Second, the least productive firms are driven out of the market as they lose market shares. Since these firms are the least skill intensive, they release much unskilled labour, which further increases the skill premium. Third, some firms which were not exporting prior to trade liberalisation now find it profitable to enter the export market. The effect of their entry on the skill premium is ambiguous and depends on their relative skill intensity. If the initial trade costs were high, only very productive and skill intensive firms were able to export. The new entrants are therefore more skill intensive than average and the expansion of their production also drives the skill premium up. In the case of low initial trade costs however, new entrants on the export market are less skill intensitve than average and the increase in their production provides a countervailing force to the rise in the skill premium. I show that the first and second effect described above always dominate the effect of new entrants on the export market, so that a decrease in the variable costs of trade generates an unambiguous rise in the skill premium.

In the case of a reduction in the fixed costs of trade, firms already exporting are not affected at the margin and do not increase their exports. The first effect described above, which was raising the skill premium, therefore drops out. I show that the aggregate effect of a drop in the fixed costs of trade on the skill premium is inverted U-shaped, which is due to the non-monotonicity of the effect of firms entering the export market (the third effect above).

The calibration of the model to standard parameters of the U.S. economy shows that the effect of a drop in variable costs of trade between industrialised countries can explain up to a fourth of the rise in the U.S. skill premium over the last 30 years.

In the third chapter, I examine the impact of trade liberalisation on firm-level and aggregate investment in productivity improvements.

Recent empirical evidence shows that the *between* firm reallocation of productive resources pointed out by Melitz (2003) is not the only channel through which trade liberalisation influences productivity in the economy. A number of studies[4] point to a *within* firm adjustment of productivity following a change in market conditions.

The third chapter develops a Melitz (2003) type model in which firms, after observing their exogenous productivity level, can decide on a level or investment in process innovation (henceforth 'investment') to improve their productivity. This decision is continuous in the sense that each firm decides how much to spend on innovation and is not restricted to a binary decision between investment and no investment. I denote for simplicity the mapping of the investment level into the production function as the 'technology'.

The main contribution of this chapter is to develop a new analysis for the effect of trade liberalisation on firm level investment and draw its consequences for aggregate innovation spending.

I show that a reduction in the variable costs of trade, which induces a reallocation of market shares from non-exporters to exporters, impacts the incentives to invest for these two groups in opposite directions. Since exporters increase their sales, they find it profitable to cut their unit costs of production and raise their investment level. Non-exporters, on the other hand, sell less and cut down their investment.

The effect of trade liberalisation on the investment intensity of a firm, which is the ratio of investment spending to sales, depends on a simple observable property of the investment technology. If the technology is such that firm size and investment intensity are positively correlated in equilibrium, trade liberalisation raises the investment intensity of exporters and reduces that of non-exporters. I further show that the increase in investment intensity of exporters in this case dominates the reduction in investment intensity of non-exporters, so that trade liberalisation tends to raise the aggregate investment intensity in the economy. The reverse argument holds if the technology induces a negative equilibrium correlation between size and investment intensity. This result qualifies the view that the scale effect of international trade fosters innovation and provides a simple condition about the investment technology under which trade is likely to raise aggregate innovation spending.

[4]Trefler (2004) and Bernard et al. (2006) among others.

The model in chapter 3 also addresses two recent puzzles to the heterogeneous firms literature pointed out by Nocke and Yeaple (2006). First, they argue that trade liberalisation has reduced the skewness of the size distribution of firms. They find that a decrease in trade costs does not affect all firms proportionately, and that the size differential between two given firms tends to decrease, which cannot be explained by the Melitz (2003) framework. In the present model, the heterogeneous reaction of firm investment to a change in trade costs can account for this fact. Second, they point out that the relationship between the Tobins Q of a firm and its size is empirically negative, contrarily to a straightforward extension of the standard heterogeneous firms model. I show how the different investment levels across firms can explain this fact, depending on the properties of the technology.

The fourth chapter analyses the link between output volatility and openness, defined as the ratio of exports to global sales, at the firm level.

The question whether trade raises or reduces output volatility in the economy is central to the political debate and regularly gives rise to protectionist demands to shield domestic consumers and firms from foreign induced shocks. The political interest for the topic has in the last decade been matched by a growing scientific literature investigating the empirical relationship between trade and output volatility. However, virtually no study investigates the precise transmission mechanisms of shocks between markets. One reason is that the standard approach has taken a purely macroeconomic perspective, leaving aside more disaggregated levels of analysis, in particular firm behaviour. Chapter 4 examines the relationship between trade openness and output volatility at the firm level in order to provide a better understanding of the mechanisms underlying the aggregate effects.

I develop a model of trade with demand uncertainty to examine the impact of market-specific demand shocks on the production and export of different firms. It has two important features. First, firms are heterogeneous with respect to the demand parameters for their product on both markets and therefore with respect to their degree of openness, defined as the ratio of exports to global sales. This allows me to study the link between volatility and openness at the firm level. Second, firms produce with convex costs in the short run. These two core features of the model allow me to generate a number of empirical predictions that I test using the Amadeus dataset, which provides comprehensive firm-level balance sheet and export information, for French firms between 1997 and 2006.

The main conclusions of the chapter are threefold. First, exporting firms substitute sales in the short run between their domestic and export markets. For example, a firm facing a positive demand shock on its domestic market and no change in demand on the export market will reduce its exports in order to sell more on the domestic market.

I show theoretically and empirically that a higher than average sales growth in one market is associated with a lower than average growth in the other. Second, exporters with a high openness level have in equilibrium more volatile domestic sales and less volatile exports than exporters with a low openness level. It is a direct consequence of the market substitution highlighted above: demand shocks induce a sales substitution which is proportionally small for the larger market and large for the smaller market. The empirical analysis confirms the quantitative importance of this effect. Third, I show that exporting firms with an openness level above 25% are more volatile than comparable non exporters. Those with a lower openness level are on the other hand less volatile than comparable non-exporters. This last observation is in line with a standard diversification argument according to which selling to uncorrelated markets reduces volatility.

The thesis is organized in such a way that the chapters can be read independently of each other. All references are collected in the bibliography.

Chapter 2

Trade between symmetric countries and the skill premium

2.1 Introduction

The strong rise in wage inequalities has, over the last decades, been one of the most debated political issues in industrial countries. The large increase in the returns to education (college wage premium[1]) from the early 1980s to the mid 1990s is well documented, and has been of dramatic magnitude from a historical perspective: Acemoglu (2002) shows that the U.S. college premium increased from 1.4 to 1.8 between 1979 and 1996. This trend has given rise to a large literature, which has spanned different fields of economics. Though prominent in the popular debate, international trade has never been a major explanation among economists, partly due to the weakness of its quantitative effect. A reason for this is that most trade studies have addressed the question using models based on North South heterogeneity[2], thereby leaving aside the bulk part of international trade, made of exchanges between industrial countries. It is only recently that trade economists have turned to intra-industry trade models as a potential determinant of the evolution of the skill premium.

In the last years, models of monopolistic competition with heterogeneous firms have been at the core of most developments in the international trade literature. Their popularity is based on their ability to match a number of well-established stylized facts linking firm characteristics to their export behaviour. Empirical studies such as Bernard and Jensen (1995, 1997) show that exporting firms are relatively skill intensive, while firms shutting

[1]The college premium is defined as the ratio of the wage of college graduates to the wage of non-graduates. I will refer to 'college premium' or 'skill premium' indifferently.

[2]Such as Heckscher-Ohlin types of model - see for example Krugman (2000) and Leamer (2000) on their relevance, or outsourcing models à la Feenstra and Hanson (2001).

down are less skill intensive than average (Bernard and Jensen (2007)). These facts suggest that trade liberalisation, due to its heterogeneous effects on different firms, can impact the skill wage premium through a reallocation of productive resources between firms. The aim of the present model is to explore this channel.

I develop a one-sector monopolistic competition framework with heterogeneous firms, in which firms produce with two factors of production: skilled and unskilled labour. I assume that firms are heterogeneous in the relative productivity of skilled labour, in the sense that some use skills more effectively than others. This establishes a correlation between productivity, skill intensity and exports: more productive firms are relatively skill intensive and export more than other firms, as confirmed by empirical studies. Following Melitz (2003), the present model is built on two types of trade costs: variable costs, which can be interpreted as transport costs or as tariffs, and fixed export costs, usually seen as administrative costs to export in a foreign country. As is standard in this type of models, these costs generate a partitioning between exporting and non-exporting firms.

The main contribution of this approach is to show that the evolution of the skill premium following trade liberalisation depends on two factors: the kind of trade costs considered and the magnitude of trade costs before liberalisation takes place (the 'initial' trade costs). On the one hand, a reduction in the variable costs of trade unambiguously raises the skill wage premium. On the other hand, a drop in the fixed costs of trade has an ambiguous, potentially non-monotonic effect on the skill premium. Indeed, for sufficiently low initial costs of trade, a further reduction in the fixed costs of exporting decreases the skill premium. I calibrate the model to match key variables of the U.S. economy and show that it has a substantial quantitative effect. A plausible reduction in the variable costs of trade between three identical countries (a reduction of the iceberg costs of trade from 1.5 to 1.1) can account for an increase in the skill premium of more than 10 percentage points, which is roughly a fourth of the observed rise in the 1980s and 1990s.

The core mechanism driving the results is the reallocation of productive resources between firms following trade liberalisation. A reduction in the variable costs of trade makes the export activity cheaper, so that exporting firms, which are more skill intensive than average, scale up their demand for labour (the first effect). Relatively unproductive firms are, on the other hand, driven out of the market, releasing much unskilled labour (the second effect). Both effects tend to raise the skill premium. The skill intensity of firms newly entering the export market (the third effect) is however undetermined and depends on the initial costs of trade. If initial trade costs are high, only productive firms, with a higher than average skill intensity enter the export market following liberalisation. This drives the skill premium further upwards. If the costs of trade are initially low, however, the firms entering the export market are relatively unskilled intensive, and their entry provides a countervailing force to the increase in the skill premium. I show that this third effect cannot overturn the first two, so that the skill wage premium unambiguously increases. On the other hand, if the fixed costs of exporting decrease, the first effect disappears, and the skill wage premium can decrease if trade is initially cheap. The third effect, which counteracts the rise in the skill premium as trade costs decrease, provides a rationale for the observed slowdown in the growth of the skill premium from the mid 1990s onwards[3].

This paper is related to the nascent literature discussing the link between the skill premium and intra-industry trade. Epifani and Gancia (2008) assume a correlation between the scale and the skill intensity of a sector, and follow Dinopoulos et al. (2002) who assume such a correlation at the firm level. With appropriate assumptions on preferences, the increase in scale inherent to trade liberalisation therefore exerts a bias towards skill demand and raises the skill wage premium. Both models use a representative firm's framework, so that the mechanisms driving the results are different from the present paper, which concentrates on factor reallocation between heterogeneous firms. Yeaple (2005) uses a monopolistic competition model in which ex-ante homogeneous firms choose between two production technologies with different complementarities to skills. High technology firms self-select into exporting while low technology firms remain purely domestic. There is no difference between firms of the same technology type. Labour consists of a continuum of skills, with high (low) skilled labour matched to the high (low) technology. Following trade liberalisation, some firms enter the export market and switch to the high technology, driving the skill premium up no matter which type of liberalisation occurs. The present model largely differs in its construction and conclusions. The source of heterogeneity is different, since I use two types of labour and a continuum of technologies. As in the standard models in the literature, firms receive an exogenous productivity and do not switch technology while entering the export market. This feature has an important impact on the results, since it drives the countervailing effect (the third effect) of liberalisation on the skill premium. The evidence on firms switching technologies upon entering the export market is at best mixed, while there is massive evidence that new exporters differ from continuing non-exporters long before their entry in the export market[4]. These facts suggest that the present model provides a useful complementary analysis to that of Yeaple (2005), with a stronger focus on firm heterogeneity. Finally, Yavas (2006) builds on Melitz (2003) to analyse the evolution

[3]This fact has given rise to a literature contesting the standard skill-biased technological change hypothesis as an explanation for the rise in the skill premium, see Card and diNardo (2002).

[4]See Clerides et al. (1998), Bernard and Jensen (1999), Delgado et al. (2002) or Pavcnik (2002) among others. Some of these studies suggest that firms entering the export market have a high productivity growth before entry. Since this is contemporaneous with an increase in size, it cannot be however concluded that they switch technology but could be a pure size effect. Direct evidence on technology upgrading is still lacking.

of the skill premium, but does not allow for variable costs of trade, which is a major restriction considering both their empirical importance and their effect on the analysis.

The second strand of literature to which this paper relates is the rapidly expanding field of heterogeneous firm in trade[5]. The present model matches some important empirical features about exporting firms emphasised in this literature. Not surprisingly since I largely build on Melitz (2003), the fact that exporting firms are bigger and more productive is preserved, and conform to the conclusions of Bernard and Jensen (1995), Bernard et al. (2003) and many others. Additionally, the model typically generates higher average wages for exporting firms[6] due to their employing relatively more skilled labour, a feature which conforms to the results of Bernard and Jensen (1995, 1997). They provide a strong empirical case for my approach, arguing that 'the between plant movement of workers and wages, which are especially important in the increases in the aggregate wage gap, are largely determined by demand shifts across plants, and in particular by export related demand movements' (Bernard and Jensen, 1997, p.25).

The remainder of the paper is structured as follows. Section 2.2 develops the model. Section 2.3 derives the comparative statics of the skill premium following a marginal trade liberalisation. Section 2.4 provides a numerical solution to the model for illustrative and quantitative purposes. Section 2.5 concludes.

2.2 The model

2.2.1 Demand

The world consists of two identical countries. All consumers in each country are identical except for their income and share the same constant elasticity of substitution (C.E.S.) utility function over a continuum of varieties. Each consumer has utility:

$$U = \left[\int_{\omega\in\Omega} q(\omega)^{\frac{\epsilon-1}{\epsilon}} d\omega\right]^{\frac{\epsilon}{\epsilon-1}} \tag{2.1}$$

where the set Ω represents all available varieties, $q(\omega)$ stands for the consumption of variety ω by the consumer, and ϵ is the elasticity of substitution between varieties, assumed to be strictly greater than one. Consumers preferences therefore exhibit the usual love of variety property following Dixit and Stiglitz (1977). Define:

$$C \equiv \left[\int_{\omega\in\Omega} q^1(\omega)^{\frac{\epsilon-1}{\epsilon}} d\omega\right]^{\frac{\epsilon}{\epsilon-1}} \tag{2.2}$$

[5]Melitz (2003), Bernard et al. (2003), Helpman et al. (2004), Bernard et al. (2004), Chaney (2006) among others.

[6]Under the sufficient and plausible condition that there is weakly less skilled than unskilled labour in the economy and that skilled labour is at least as productive as unskilled labour.

as the optimal consumption bundle costing one unit of income. This optimal bundle is the result of the maximisation of utility with respect to the consumption of each variety and subject to the budget constraint: $PC = 1$, where P denotes the price index of the optimal bundle. This yields:

$$q^1(\omega) = \left(\frac{p(\omega)}{P}\right)^{-\epsilon} C = p(\omega)^{-\epsilon} P^{\epsilon-1} \tag{2.3}$$

where P is:

$$P = \left[\int_{\omega\in\Omega} p(\omega)^{1-\epsilon} d\omega\right]^{\frac{1}{1-\epsilon}} \tag{2.4}$$

Since preferences are homothetic, the aggregate demand for a variety in a country is given by:

$$q(\omega) = p(\omega)^{-\epsilon} P^{\epsilon-1} I \tag{2.5}$$

where I denotes the aggregate income of all consumers in a country. This income consists of the proceeds of capital, labour and of profits. From (2.5), the demand for a variety decreases in its relative price and increases in national income.

2.2.2 Production

In each country, there is a continuum of firms, each producing a different variety. Production uses two factors, skilled (s) and unskilled (u) labour, internationally immobile and in fixed aggregate supply. They are combined in a C.E.S. production function:

$$y = \left[u^{\frac{\sigma-1}{\sigma}} + z^{\frac{1}{\sigma}} s^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{2.6}$$

where $\sigma > 1$ is the elasticity of substitution between the two factors of production. This is an empirically founded assumption, as most studies estimate a parameter σ between 1 and 2 for industrial countries.[7] Firms are heterogeneous as to the productivity of skilled labour, indexed by z, which is the realisation of a random variable, drawn from an exogenously given continuous distribution with support $[\underline{z}, \infty]$, where $\underline{z} \geq 1$ in order to ensure that skilled labour is more productive than unskilled labour. Acemoglu (2002) among others uses the same form of production function to study skill-biased technological change, where $z^{\frac{1}{\sigma}}$ increases exogenously over time[8]. In order to produce, a firm needs to pay a fixed cost f, in terms of capital K, at a unit price r. This assumption largely simplifies the labour market equilibrium conditions and is neutral for the study

[7] See Acemoglu (2002) p.20

[8] Note that z enters to the power $\frac{1}{\sigma}$ only for simplicity.

of the skill premium[9]. I assume that both countries are perfectly symmetric in every respect, so that factor prices are equal across countries.

The marginal costs of production (m) are the lowest possible costs for a firm to produce a unit of its variety. This is the solution to the minimisation problem:

$$\min_{u,s}(w_u u + w_s s) \quad \text{s.t. } y \geq 1 \tag{2.7}$$

where w_u and w_s are respectively the wage of unskilled and skilled labour. The first order conditions of the minimisation problem yield:

$$s = zuw^{-\sigma} \tag{2.8}$$

where $w = \frac{w_s}{w_u}$ is the skill premium. From (2.6) and (2.8), the unit unskilled and skill requirements are given by:

$$u = y\left(1 + zw^{1-\sigma}\right)^{\frac{\sigma}{1-\sigma}} \tag{2.9}$$

$$s = yzw^{-\sigma}\left(1 + zw^{1-\sigma}\right)^{\frac{\sigma}{1-\sigma}} \tag{2.10}$$

The marginal cost of production of a firm having drawn z are therefore given by:

$$m(z) = w_u(1 + zw^{1-\sigma})^{\frac{1}{1-\sigma}} \tag{2.11}$$

The first equation states that the skill intensity of a firm increases in z and decreases in the skill premium. The second shows that marginal costs decrease with z. The following lemma follows directly:

Lemma 2.1 *Skill intensive firms have lower marginal costs of production.*

This correlation between productivity and skill intensity is well established empirically, as shown by Idson and Oi (1999), Haltinwanger et al. (1999) and Bernard and Jensen (1995).

The domestic profits of a firm z ($\pi_d(z)$) are given by:

$$\pi_d(z) = (p - m(z))q_d - fr \tag{2.12}$$

[9]Assuming that fixed costs are paid as a fraction of output as in Yeaple (2005) would here not be neutral for the skill premium since the composition of fixed costs would depend on the skill intensity of the firm. Assuming that these are paid in terms of skilled labour as in Ekholm and Midelfahrt (2005) would complicate the labour market conditions and generate an inverse relationship between size and skill intensity which runs counter to empirical evidence.

Due to the monopolistic competition structure of the model, there is no strategic interaction between firms, which maximise their profits taking the average price P as given. The optimal decision of a firm is to set the price for its variety on its domestic market equal to a constant markup over marginal costs:

$$p_d(z) = \frac{\epsilon}{\epsilon - 1} m(z) \tag{2.13}$$

Using the demand equation (2.5) and the optimal price (2.13), the quantity sold by a firm on its domestic market is:

$$q_d(z) = \left(\frac{\epsilon}{\epsilon - 1}\right)^{-\epsilon} (m(z))^{-\epsilon} P^{\epsilon-1} I \tag{2.14}$$

A firm sells more the lower its marginal cost and the higher the average price of its competitors P. More productive firms are therefore larger and, from Lemma 2.1, more skill intensive. The domestic profits realised by a firm z are therefore:

$$\pi_d(z) = A(m(z))^{1-\epsilon} P^{\epsilon-1} I - fr \tag{2.15}$$

where for simplicity: $A \equiv \frac{1}{\epsilon}\left(\frac{\epsilon}{\epsilon-1}\right)^{1-\epsilon}$. The profits of a firm are increasing in I, the level of income of the country, P and z, which indexes its productivity.

Due to the existence of fixed costs of production, not all firms find it profitable to produce. The least productive producing firm is the one having drawn a productivity $z = z^*$, at which it makes zero profits. All entrepreneurs having drawn a $z < z^*$ do not find it profitable to produce and stay out of the market. Setting $\pi(z^*) = 0$ in (2.15) and solving for the price index:

$$P^{\epsilon-1} = \frac{fr(m(z^*))^{\epsilon-1}}{AI} \tag{2.16}$$

This establishes a negative relationship between the price index P and the cutoff level z^*. If the price index is low, a firm faces very productive competitors on average, and the demand for its variety is low. It should therefore be relatively productive to be able to cover the fixed costs of production and make non-negative profits.

Using (2.9), (2.10), (2.11) and (2.14), the amount of unskilled and skilled labour employed by a firm $z \geq z^*$ for domestic production is:

$$u_d(z) = A(\epsilon - 1)\left(1 + zw^{1-\sigma}\right)^{\frac{\epsilon-\sigma}{\sigma-1}} P^{\epsilon-1} I w_u^{-\epsilon} \tag{2.17}$$

$$s_d(z) = zw^{-\sigma} A(\epsilon - 1)\left(1 + zw^{1-\sigma}\right)^{\frac{\epsilon-\sigma}{\sigma-1}} P^{\epsilon-1} I w_u^{-\epsilon} \tag{2.18}$$

Each firm also has the possibility to export to the other country if it finds it profitable. Exporting requires the payment of two additional types of costs. Iceberg costs $\tau \geq 1$ are the fraction of goods that must be produced in order for one unit of the good to arrive at destination. Shipping costs or tariffs are typical interpretations for such iceberg costs. Additionally, an exporting firm has to incur a fixed cost of exporting f_x, which reflects the additional costs of doing business abroad, of establishing a distribution network, etc. There is much empirical evidence about the importance of these fixed costs of exporting, which generate a partition of firms between non-exporting and exporting firms as long as: $\frac{f_x}{f}\tau^{\epsilon-1} > 1$. I assume that these fixed costs of exporting are paid in terms of capital, and that both types of trade costs are symmetric between the two countries.

By a similar argument to the one presented for the domestic case, the optimal price charged by a firm on the export market is:

$$p_x(z) = \tau p_d(z) \tag{2.19}$$

An exporting firm charges the same mark-up on both markets due to the C.E.S. preferences, but faces higher costs of selling on the export market due to the iceberg costs. Using the demand equation (2.5), this translates into the following profits on the export market:

$$\pi_x(z) = \tau^{1-\epsilon} A w_u^{1-\epsilon} \left(1 + z w^{1-\sigma}\right)^{\frac{\epsilon-1}{\sigma-1}} P^{\epsilon-1} I - f_x r \tag{2.20}$$

Setting these profits equal to zero defines the level z_x^* of the productivity parameter z that makes a firm indifferent between exporting or not. Plugging (2.16) for the price index in $\pi_x(z_x^*) = 0$ yields:

$$\left(1 + z_x^* w^{1-\sigma}\right)^{\frac{\epsilon-1}{\sigma-1}} = \frac{f_x}{f}\tau^{\epsilon-1}\left(1 + z^* w^{1-\sigma}\right)^{\frac{\epsilon-1}{\sigma-1}} \tag{2.21}$$

From the assumptions that $\frac{f_x}{f}\tau^{\epsilon-1} > 1$, it is immediate that the cutoff export level z_x^* is higher than the domestic cutoff level z^*. This generates the well-known partitioning between exporting and non-exporting firms, and by Lemma 2.1, the empirically established facts that exporting firms are more productive and more skill intensive than non-exporting firms. Furthermore, if $w > 1$[10], this ensures that exporters pay on average higher wages than non exporters, as many empirical studies confirm.

In the same way as for the domestic decision, the number of workers used for export production by firms $z \geq z_x^*$ are given by:

$$u_x(z) = \tau^{1-\epsilon} u_d(z) \tag{2.22}$$

$$s_x(z) = \tau^{1-\epsilon} s_d(z) \tag{2.23}$$

Due to the iceberg costs, firms must employ more labour to be able to sell the same amount on the export market than on the domestic market. But the higher price they

[10]Sufficient for this is that $\underline{z} > 1$, i.e. that skilled labour is always more productive than unskilled labour, and that there are weakly more unskilled than skilled workers in the economy.

charge decreases the demand for their variety on the export market, and therefore the labour they employ. This second effect dominates, so that firms employ weakly less labour for their exports than for their domestic production.

2.2.3 Equilibrium

There is an exogenous constant mass of entrepreneurs M in each country. This conforms to Chaney (2006) and differs from the original Melitz (2003) framework in that there is no free entry condition, no dynamics of entry and exit, and that there are positive aggregate profits in the economy. The amount of factors available in a country is exogenous, and denoted as K for the stock of capital, U for the mass of unskilled labour and S for the mass of skilled labour. Using (2.17), (2.18), (2.22) and (2.23), the factor market equilibrium in each country is given by:

$$U = M\left[\int_{z^*}^{\infty} u_d(z)dG(z) + \tau^{1-\epsilon}\int_{z_x^*}^{\infty} u_d(z)dG(z)\right] \quad (2.24)$$

$$S = M\left[\int_{z^*}^{\infty} s_d(z)dG(z) + \tau^{1-\epsilon}\int_{z_x^*}^{\infty} s_d(z)dG(z)\right] \quad (2.25)$$

$$K = M\left[(1-G(z^*))f + (1-G(z_x^*))f_x\right] \quad (2.26)$$

These market clearing conditions, as well as (2.21), define the equilibrium.

Since the primary interest of this paper is the evolution of the skill wage premium, it is worth noting that the equilibrium tuple (z^*, w) solves the following system of equations:

$$\frac{U}{S} = \frac{\int_{z^*}^{\infty} u_d(z)dG(z) + \tau^{1-\epsilon}\int_{z_x^*(z^*)}^{\infty} u_d(z)dG(z)}{\int_{z^*}^{\infty} s_d(z)dG(z) + \tau^{1-\epsilon}\int_{z_x^*(z^*)}^{\infty} s_d(z)dG(z)} \quad (2.27)$$

$$\frac{f_x}{f} = \left(\frac{1+z_x^*(z^*)w^{1-\sigma}}{1+z^*w^{1-\sigma}}\right)^{\frac{\epsilon-1}{\sigma-1}}\tau^{1-\epsilon} \quad (2.28)$$

The first equation is the ratio of the two labour market equilibrium conditions (2.24) and (2.25), where the market equilibrium for capital (2.26) defines the implicit function $z_x^*(z^*)$. The second equation is the indifference condition of the cutoff exporting firm (2.21). Equations (2.27) and (2.28) allow to solve for the skill wage premium independently of the interest rate r.

A number of assumptions on the parameters is needed at this stage to ensure the existence of an equilibrium:

Assumption 2.1 $\frac{K}{M} \leq f$

Assumption 2.1, which is sufficient but not necessary for the results, means that capital is scarce in the sense that the stock of capital is not sufficient for all potential entrepreneurs in a country to pay the fixed costs of production. This is due to the construction of the model, which features a fixed mass of potential entrepreneurs and in which capital is only required for the payment of fixed costs, meaning that there is a maximum amount of capital which firms can demand. Imposing Assumption 2.1 ensures both that the capital market equilibrium can hold with equality, and that the ratio $\frac{z_x^*}{z^*}$ can become arbitrarily large[11].

Assumption 2.2 $\epsilon \geq \sigma$

This assumption requires that the varieties are better substitutes in the utility functions than skilled and unskilled labour in the production function. This is a rather innocuous assumption considering the numerous empirical estimations for these parameters. Estimates of σ are usually[12] comprised between 1 and 2 while ϵ tends to be higher, between 3 and 6[13].

Proposition 2.1 *Under Assumptions 2.1 and 2.2, there exists a unique equilibrium.*

Proof. *See Appendix* ■

Under Assumptions 2.1 and 2.2, (2.27) and (2.28) respectively establish a positive and a negative relationship between w and z^*, as illustrated in figure 2.2. I show in the Appendix that the two curves defined in the (z^*, w) space intersect and that an equilibrium exists and is unique.

2.3 Trade liberalisation

The central aim of this model is to study the impact of a marginal perturbation in the costs of exporting on the wage gap between unskilled and skilled labour. I use two definitions of trade liberalisation: (i) a bilateral decrease in the iceberg costs, which can be interpreted as a reduction in tariffs or freight costs, (ii) a bilateral reduction in the fixed costs of exporting, which may occur due to the dismantling of certain types of non tariff barriers[14] or of any other measure hampering entry on the export market. Distinguishing between these two types of trade liberalisation is of importance for policy analysis. In order to derive the results, I conduct a comparative static analysis by totally differentiating (2.27) and (2.28) with respect to both trade costs measures f_x and τ.

[11]Under this assumption, if z^* is small enough, z_x^* has to go to infinity for (2.26) to hold. Even if there are only domestic firms on the market, capital is used up and has a positive price.

[12]Acemoglu (2002) p.20

[13]Bernard et al. (2003) and many others use 3.8, while a reasonable markup of 20 percent would require $\epsilon = 6$. Both σ and ϵ depend on the sector considered.

[14]Adapting a product to foreign regulation is a common example for fixed costs of exporting.

2.3.1 Trade liberalisation as a marginal decrease in the variable costs of exporting

I first consider the effect of a reduction in the iceberg costs τ on the skill premium w, and refer to the 'initial' situation for the equilibrium prevailing before any perturbation in the costs of trade.

Lemma 2.2 *$\frac{dw}{d\tau}$ has the same sign as Δ, defined as follows:*

$$\Delta \equiv \eta \int_{z_x^*}^{\infty} (1 + zw^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}} (Bzw^{-\sigma} - C) dG(z) + \xi \frac{Bz_x^* w^{-\sigma} - C}{1 + z_x^* w^{1-\sigma}} - \xi \frac{Bz^* w^{-\sigma} - C}{1 + z^* w^{1-\sigma}} \tag{2.29}$$

where $\eta, \xi < 0$ and:

$$\begin{aligned} Bz'w^{-\sigma} - C &= w^{-\sigma} \int_{z^*}^{\infty} (w^{1-\sigma} + z)^{\frac{\epsilon-\sigma}{1-\sigma}} (z' - z) dF(z) \\ &+ w^{-\sigma} \tau^{\epsilon-1} \int_{z_x^*}^{\infty} (w^{1-\sigma} + z)^{\frac{\epsilon-\sigma}{1-\sigma}} (z' - z) dF(z) \end{aligned} \tag{2.30}$$

for all $z' \geq z^*$

Proof. *See Appendix* ■

First, note that $Bz'w^{-\sigma} - C$ represents the relative skill intensity of a given firm z'. It is positive (negative) if firm z' is more (less) skill intensive than the average. The above lemma can be interpreted as follows.

For a constant w, trade liberalisation has three effects on firms, as shown by the three components of (2.29). These are essentially identical to those highlighted in Melitz (2003), and are represented in figure 2.1.

First, following a decrease in the marginal costs of exporting, the most productive firms, which are initially exporting, find it profitable to scale up their production aimed at the export market (arrow 1 in figure 2.1). Second, the most productive among the initially non-exporting firms can now make weakly positive profits on the export market, in which they enter (arrow 2). This decreases the export cutoff level z_x^*. As this last channel tends to increase the demand for capital, the cutoff level of domestic production z^* needs to rise for the capital market equilibrium to hold. The least productive producing firms in the economy therefore drop out of the market (arrow 3), which is the third effect of trade liberalisation on firms. I will denote these three effects respectively as Effect 1, 2 and 3 in accordance with the arrows of figure 2.1 and the order in which they appear in (2.29).

These three effects have different consequences for the unskilled and skilled labour markets. Two of them have a clear positive effect on the relative demand for skilled labour. First, initially exporting firms are productive and relatively skill intensive. An expansion of their production (Effect 1) thus increases the demand for skilled labour more than that of unskilled labour, as shown by the negative sign of the first term in (2.29). This tends to raise w. Second, firms dropping out of the domestic market (Effect 3) are relatively unproductive and unskilled intensive. They therefore release much unskilled labour, and their effect on (2.29), represented by the second term, is also negative. These two effects are exactly conform to the empirical evidence of Bernard and Jensen (1995, 1997, 2007).

The impact of firms newly entering the export market (Effect 2) is however undetermined. It depends on the relative unskilled intensity of the cutoff export firms, represented by the third term in (2.29). A positive (negative) sign means that these firms are relatively skilled (unskilled) intensive. From (2.30), it is immediate that this sign depends on the initial z_x^*, which is a function of the initial size of the trade costs τ and f_x by (2.28). If trade is initially expensive, only very productive firms are able to export, and the cutoff export firm is relatively skill intensive. In this case, firms newly entering the export market following a marginal trade liberalisation are also skilled intensive, and increase the relative demand for skilled labour. On the other hand, if trade is initially cheap, unskilled intensive firms can benefit from the trade liberalisation by becoming able to export. If this is the case, the entry of firms with a lower than average skill intensity on the export market provides a countervailing force to the rise in the skill wage premium.

Central for the analysis is to determine whether the third effect described above can overturn the other two and cause a decrease in the skill premium. It appears that this cannot be the case.

Proposition 2.2 *A decrease in the variable costs of trade unambiguously increases the skill wage premium.*

Proof. *See Appendix* ■

The proposition states that the effect of new firms entering the export market cannot be strong enough to reduce the skill premium. The reason is that effect 2 cannot overturn effect 3, as can be seen from (2.29). The additional effect of initially exporting firms therefore guarantees that the skill wage premium strictly increases when τ decreases.

In the description of the three effects above, it was implicitly assumed that a rise in the relative demand for skilled labour raises the skill premium. As shown in the Appendix[15], this is indeed the case. The increase in the skill premium however does not have the same effect on all firms: larger firms, which are more skill intensive, see their cost rise by a higher proportion. Smaller firms therefore benefit from this countervailing force to trade liberalisation, which may even overcompensate the direct effect of a decrease

[15] Appendix A.2., equation (2.48) and its interpretation

in trade costs and yield a decrease in the cutoff level of firm z^*. Though relevant for quantitative purposes or for the welfare analysis, this indirect effect does not affect the qualitative result that a decrease in the variable trade costs of trade raises the skill premium.

A potential concern about the specification of the model is that the assumption of iceberg transportation costs may drive the results. The assumption on trade costs is indeed not neutral for the skill premium, as it requires that they are paid in terms of exported goods. Since exporting firms are more skill intensive than average, it implies that variable trade costs are relatively skill intensive. As long as a decrease in variable trade costs raises the amount of resources used for transportation[16], this effect may tend to increase the skill premium. A simple way of showing that it does not drive the result is to assume that trade costs are paid in terms of an asset, which is in infinitely elastic supply, has a price t and is held by a third country. I moreover assume that in order to export one unit of its output, a firm z has to pay a cost $tm(z)$. This assumption has the similarity with iceberg transport costs that more productive firms pay lower transportation costs per unit shipped[17] and allows me to concentrate on the relevant aspect for the skill premium. The marginal costs of selling to a foreign consumer are therefore given by:

$$m_x(z,t) = (1+t)w_u(1+zw^{1-\sigma})^{\frac{1}{1-\sigma}} \tag{2.31}$$

Using the demand equation (2.5), the price index (2.16) and the results of the cost minimisation problem (2.8) gives the amount of labour used by a firm z for its exports:

$$u_{xt}(z) = (1+t)^{-\epsilon}\frac{r}{w_u}(\epsilon-1)(1+zw^{1-\sigma})^{\frac{\epsilon-1}{\sigma-1}}(1+z_x^*w^{1-\sigma})^{\frac{1-\epsilon}{\sigma-1}} \tag{2.32}$$

$$s_{xt}(z) = u_{xt}(z)zw^{-\sigma} \tag{2.33}$$

The only change to the equilibrium conditions is therefore that $\tau^{1-\epsilon}$ in (2.27) should be replaced by $(1+t)^{-\epsilon}$. It can be readily seen from (2.49) in Appendix that the comparative statics of the model with respect to t are qualitatively similar to those derived with respect to τ. This small extension therefore confirms that the result of Proposition 2.2 does not rely on the skill intensity of iceberg trade costs.

2.3.2 Trade liberalisation as a decrease in the fixed costs of exporting

Trade liberalisation can also be defined as a drop in the fixed costs of exporting f_x.

[16]It will be the case as long as the rise in trade implied by a decrease in costs is stronger than the gain due to the lower unit cost of trade.

[17]The assumption is plausible for insurance costs of trade or for ad valorem tariffs (more productive firms sell in this model goods of lower price), less so for freight costs.

Lemma 2.3 $\frac{dw}{df_x}$ *has the same sign as* Δ'*, defined as follows:*

$$\Delta' \equiv \theta(Bz_x^* w^{-\sigma} - C) + \kappa(Bz^* w^{-\sigma} - C) \tag{2.34}$$

where θ *and* κ *are defined in the Appendix*

Proof. *See Appendix* ■

The mechanism at stake is very similar to that highlighted in the case of variable costs of trade but the first effect in (2.29) (Effect 1) disappears, because a decrease in the fixed costs of exporting, though it raises the profits of all exporting firms, does not change their level of production and employment, which only depends on the marginal costs of exporting. Arrow 1 in figure 2.1 is therefore not relevant in the present case. Initially exporting firms therefore do not scale up their demand for labour. The only two effects remaining are the marginal effects of firms dropping out of the domestic market (Effect 3) and firms entering the export market (Effect 2), as can be seen from the two terms in (2.34).

Proposition 2.3 *A decrease in the fixed costs of exporting has an ambiguous effect on the skill wage premium. For* τ *small enough, there is a level of* f_x *below which a marginal reduction in the fixed costs of exporting decreases the skill wage premium.*

Proof. *See Appendix* ■

Comparing Propositions 2.2 and 2.3 highlights the qualitative difference between the two types of trade liberalisation for their impact on the skill premium. The proof of Proposition 2.3 requires to show that, contrary to the variable cost case, the effect on the skill premium of firms exiting the market (Effect 3) can here be overturned by the effect of new firms entering the export market (Effect 2) if these are sufficiently unskilled intensive.

As shown by (2.57) in Appendix, Δ' can be rewritten as:

$$\begin{aligned} \Delta' &= \gamma \left[\frac{Bz^* w^{-\sigma} - C}{1 + z^* w^{1-\sigma}} - \frac{Bz_x^* w^{-\sigma} - C}{1 + z_x^* w^{1-\sigma}} \right] \\ &- \xi' \left[Bz_x^* w^{-\sigma} - C + \frac{g(z^*) f}{g(z_x^*) f_x} (Bz^* w^{-\sigma} - C) \right] \end{aligned} \tag{2.35}$$

where $\gamma, \xi' > 0$.

A reduction in f_x has an impact on the cutoff levels z^* and z_x^* (effects 3 and 2) through two channels, which determine the relative sizes of both effects. First, the increase in the profits of exporting decreases z_x^* since some firms find it profitable to enter the export market. By the capital market equilibrium (2.26), z^* therefore increases. This first channel, given by the first line in (2.35), is exactly the one that prevailed in the case of a decrease in iceberg costs. From this channel, effect 2 cannot overturn effect 3, so that the skill premium tends to rise. As $z_x^* \to z^*$, the effect goes to zero. Second, the reduction in f_x has a direct effect on the capital market equilibrium (2.26). For a constant z_x^*, a marginal decrease in f_x releases capital, since exporting firms need to pay lower costs. The additional capital mitigates the increase in z^* that is needed for the capital market equilibrium to hold. The exit of unskilled intensive firms (effect 3) is therefore attenuated in comparison to the effect of firms entering the export market (effect 2). If these new exporting firms are sufficiently unskilled intensive, this second channel, which corresponds to the second line in (2.35), allows effect 2 to overturn effect 3, and therefore the skill premium to rise.

The fact that effect 2 can overturn effect 3 if f_x decreases, but cannot if τ decreases, may at first seem driven by the construction of the model. Indeed, fixed costs of exports have an additional direct effect on the demand for capital that variable costs do not have. However, allowing for such direct effects of variable trade costs on the capital market would strengthen the results. Indeed, for a constant z_x^*, a lower τ would raise the demand for productive factors[18], thereby strengthening effect 2 compared to effect 3. This difference in the relative strength of effects 2 and 3 is therefore due to a property of heterogeneous firms models, in which, for a constant z_x^*, a decrease in τ raises the demand for production factors, while a decrease in f_x reduces it. This effect has, to my knowledge, never been explicitly pointed out.

2.4 Numerical simulations

In the present section, I calibrate the model to match key variables of the U.S. economy and study the quantitative impact of trade liberalisation on the skill premium. This serves the purpose of illustrating the results and testing their quantitative importance as well as to ensure that the restrictions on the parameters imposed in the theoretical part are quantitatively sensible. I find that a plausible multilateral reduction in variable trade costs has a substantial effect on the skill premium.

2.4.1 Calibration

For all exogenous parameters of interest, I use values which are now well established in the literature. Following Bernard et al. (2003) and subsequent studies, I assume that the consumers' elasticity of substitution (ϵ) is approximately equal to 4. I set the elasticity of substitution between factors in the production function (σ) to 1.5, which is in the

[18] The reason is that exporting firms would scale up their production for the export market.

middle of the range suggested by Acemoglu (2002)[19]. I set $\frac{U}{S} = 1.4$, which was the value prevailing in the U.S. in 1995 according to Acemoglu (2002) and $f = 1$ without loss of generality.

The literature on heterogeneous firms suggests that the right tail of the distribution of firms domestic sales can be well approximated by a Pareto distribution. Axtell (2001), in a seminal contribution, estimates the parameter of this distribution using U.S. census data for 1997 to be 1.06 for large firms. In line with the literature, I assume that the productivity parameter z is Pareto distributed, i.e. that:

$$G(z) = 1 - \left(\frac{\underline{z}}{z}\right)^a \tag{2.36}$$

I further assume that $\underline{z} = 1$, which requires that skilled labour be always at least as productive as unskilled labour. The original model of Melitz (2003) has the nice property that a Pareto distribution of productivity yields a Pareto distribution for sales, a property that is not preserved in the present model. However, as argued in the appendix, for firms with large z, the distribution of sales converges to a Pareto distribution with parameter $\frac{a(\sigma-1)}{\epsilon-1}$. In order to be in line with Axtell (2001), I impose that $a = 1.06\frac{\epsilon-1}{\sigma-1} = 6.36$.

For the calibration of the model, I assume that $\tau = 1.3$ as in Ghironi and Melitz (2005), which is close to Obstfeld and Rogoff (2000). I calibrate $\frac{K}{M}$ and f_x such that: (i) $w = 1.8$, which corresponds to the estimation of the skill premium presented in Acemoglu (2002) for the U.S. in 1996. (ii) The percentage of exporting firms: $\frac{1-G(z_x^*)}{1-G(z^*)}$ is 21%, which is a common estimate for the U.S. and is close to the estimate for other industrial countries (Bernard et al. (2003), Ghironi and Melitz (2005)). In order to match these values, the fixed costs of export should be set at: $f_x = 0.916$ and the stock of capital $\frac{K}{M} = 0.765$. It is worth noting that Assumption 2.1 is then fulfilled.

This provides the benchmark case for the simulation. In the next step, I examine the impact of both types of trade costs on the skill premium.

2.4.2 Results

Figure 2.3 isolates the effect of a change in τ for a given fixed cost of exports which remain at 0.916. This confirms the result of Proposition 2.2 and shows that a marginal decrease in the *variable* costs of trade from the reference situation would increase the skill premium. Indeed, a reduction of the iceberg trade costs from 1.3 to 1.1 increases the skill premium by four percentage points, which is not negligible.

Figure 2.4 on the other hand shows the effect of a decrease in the *fixed* costs of exports on the skill wage premium for a constant $\tau = 1.3$. The fixed costs of exports are allowed to

[19] I checked the results for different values of the elasticities in the range usually estimated. It has only little influence on the quantitative results.

decrease down to a lower limit of $f_x = 0.46$, below which there would be no partitioning between exporting and non-exporting firms (the condition $\tau^{\epsilon-1}\frac{f_x}{f} > 1$ would not hold anymore). As shown in Proposition 2.3, for sufficiently low trade costs a further decrease in f_x reduces the skill premium. This effect is relatively small, which is not surprising considering that it is driven solely by the cutoff firms. At the benchmark case of $f_x \approx 0.9$, a decrease in the fixed costs of export however still raises the skill premium, albeit by a very small amount.

The quantitative effects derived here are limited magnitude partly because of the two countries assumption. A multilateral liberalisation between many symmetric countries reinforces the effects at stake, since exporting firms scale up their exports to many countries (and their demand for labour) following a decrease in variable trade costs. I conduct a straightforward extension of the model to study an economy with three symmetric countries. I recalibrate the model in the same manner as above, and obtain $K = 1.075$ and $f_x = 0.905$. As can be seen from figures 2.5 and 2.6, the effects of trade liberalisation are larger in the three country case: a decrease in τ from 1.5 to 1.1 triggers an increase in the skill premium from 1.74 to 1.85, i.e. 11 percentage points. This constitutes a substantial quantitative effect considering that the total increase in the skill premium from 1979 to 1995 in the U.S. was of around 35 percentage points [20]. For the present range of parameter values, a further increase in the number of countries only marginally strengthen the quantitative results.

2.5 Conclusion

This paper shows how the reallocation of productive resources between heterogeneous firms following trade liberalisation influences the skill premium. For this I use a model of monopolistic competition with heterogeneous firms and two factors of production: skilled and unskilled labour. I introduce a correlation between productivity and skill intensity in the production process, which generates the empirically observed link between firm size, export status, wages and skill intensity. Within this consistent framework, I analyse the different effects of two types of trade liberalisation, defined as a reduction in variable and fixed costs of trade, and stress that they lead to different effects for the skill premium. A decrease in variable costs of trade has an unambiguously positive effect on the skill premium, thereby widening inequalities, while a decrease in the fixed costs of trade mitigates inequalities if trade costs are initially low.

The core mechanism at stake in both types of liberalisation is the reallocation of labour from low productive, unskilled intensive firms to more productive firms, which are more skill intensive. The difference between the two scenarii is twofold. First, exporting firms,

[20] from 1.45 to 1.8, see Acemoglu (2002) p.15.

which are highly skill intensive, only scale up their demand for labour in the variable costs trade liberalisation. Second, more unproductive, unskilled intensive firms drop out of the market following a reduction in variable trade costs than following a decrease in fixed export costs. Both these channels account for the larger effect of a decrease in variable costs on the skill premium.

Though the model does not provide a full-flegded welfare analysis[21] this differentiation between two types of trade liberalisation may be of particular interest for policy analysis. This suggests that bilateral trade liberalisation, when concentrating on the dismantling on Non Tariff Barriers of the fixed cost nature or any measure hampering the access of exporters to a foreign market, may come at no costs in terms of inequalities between skilled and unskilled labour. This is however not the case for a decrease in tariffs.

[21] Numerical simulations suggest that, if profits and capital income accrue a third group in the population, unskilled as well as skilled workers see their welfare increase following a trade liberalisation in the range considered, albeit in different proportions.

2.6 Appendix to chapter 2

Existence and Uniqueness

Proof of Proposition 2.1

The structure of the proof is as follows. Lemmas 2.4 and 2.5 show that the two equilibrium conditions (2.27) and (2.28) respectively establish a positive and a negative relationship between w and z^*, as illustrated in figure 2.2. This ensures that if an equilibrium exists, it is unique. Lemma 2.6 completes the proof by showing that the two curves defined by the equilibrium conditions in the (z^*, w) space do cross, and therefore that an equilibrium exists.

For convenience, I denote the right hand side of (2.27) as $H(z^*, w)$ and its numerator and denominator respectively as $B(z^*, w)$ and $C(z^*, w)$:

$$\begin{aligned}
B(z^*, w) &\equiv \int_{z^*}^{\infty} \left(1 + zw^{1-\sigma}\right)^{\frac{\epsilon-\sigma}{\sigma-1}} dG(z) + \tau^{1-\epsilon} \int_{z_x^*(z^*)}^{\infty} \left(1 + zw^{1-\sigma}\right)^{\frac{\epsilon-\sigma}{\sigma-1}} dG(z) \\
C(z^*, w) &\equiv \int_{z^*}^{\infty} zw^{-\sigma} \left(1 + zw^{1-\sigma}\right)^{\frac{\epsilon-\sigma}{\sigma-1}} dG(z) + \tau^{1-\epsilon} \int_{z_x^*(z^*)}^{\infty} zw^{-\sigma} \left(1 + zw^{1-\sigma}\right)^{\frac{\epsilon-\sigma}{\sigma-1}} dG(z) \\
H(z^*, w) &\equiv \frac{B(z^*, w)}{C(z^*, w)}
\end{aligned}$$

Lemma 2.4 *Equation (2.27) establishes a continuous monotonic positive relationship between z^* and w.*

Proof. The proof uses the implicit function theorem on (2.27):

- Step 1: $\frac{\partial H(z^*,w)}{\partial z^*} < 0$

From (2.26):

$$\frac{dz_x^*}{dz^*} = -\frac{g(z^*)f}{g(z_x^*)^* f_x} \tag{2.37}$$

Using this relationship in the derivative of $H(z^*, w)$ with respect to z^* and rearranging:

$$\frac{\partial H(z^*, w)}{\partial z^*} = g(z^*)(1 + z^* w^{1-\sigma})^{\frac{\epsilon-1}{\sigma-1}} \left[\frac{Bz^* w^{-\sigma} - C}{1 + z^* w^{1-\sigma}} - \frac{Bz_x^* w^{-\sigma} - C}{1 + z_x^* w^{1-\sigma}}\right] < 0 \tag{2.38}$$

where the inequality holds when there is partitioning between exporting and non-exporting firms because $\frac{Bz^* w^{-\sigma} - C}{1+z^* w^{1-\sigma}} < \frac{Bz_x^* w^{-\sigma} - C}{1+z_x^* w^{1-\sigma}}$. Indeed, $Bz^* w^{-\sigma} - C$ is negative and smaller than $Bz_x^* w^{-\sigma} - C$, and divided by $1 + z^* w^{1-\sigma} < 1 + z_x^* w^{1-\sigma}$, which yields the above inequality.

- Step 2: $\frac{\partial H(z^*,w)}{\partial w} > 0$

$$
\begin{aligned}
\frac{\partial H}{\partial w} C^2 &= \frac{\sigma}{w} BC \\
&+ (\epsilon - \sigma) w^{-\sigma} \left(\int_{z^*}^{\infty} \kappa(z) z^2 w^{-\sigma} dz B - \int_{z^*}^{\infty} \kappa(z) z dz C \right) \\
&+ (\epsilon - \sigma) w^{-\sigma} \left(\tau^{1-\epsilon} \int_{z_x^*}^{\infty} \kappa(z) z^2 dz B - \tau^{1-\epsilon} \int_{z_x^*}^{\infty} \kappa(z) z dz C \right)
\end{aligned}
$$

where $\kappa(z) \equiv (1 + zw^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}-1} g(z)$

$\frac{\sigma}{w} BC$ is strictly positive. Since $\epsilon > \sigma$ by Assumption 2.2, it is sufficient for the above term to be positive that the sum of the second and third lines be weakly positive. Using $\kappa(z)$ to rewrite B and C and simplifying, a sufficient condition for $\frac{\partial H(z^*,w)}{\partial w}$ to be positive is:

$$
\begin{aligned}
&\int_{z^*}^{\infty} \kappa(z) z^2 dz \int_{z^*}^{\infty} \kappa(z) dz - \left(\int_{z^*}^{\infty} \kappa(z) z dz \right)^2 + \tau^{2-2\epsilon} \left(\int_{z_x^*}^{\infty} \kappa(z) z^2 dz \int_{z_x^*}^{\infty} \kappa(z) dz - \left(\int_{z_x^*}^{\infty} \kappa(z) z dz \right)^2 \right) \\
&+\tau^{1-\epsilon} \left(\int_{z^*}^{\infty} \kappa(z) z^2 dz \int_{z_x^*}^{\infty} \kappa(z) dz + \int_{z_x^*}^{\infty} \kappa(z) z^2 dz \int_{z^*}^{\infty} \kappa(z) dz - 2 \left(\int_{z^*}^{\infty} \kappa(z) z dz \int_{z_x^*}^{\infty} \kappa(z) z dz \right) \right) \geq 0 \qquad (2.39)
\end{aligned}
$$

The first line above is positive by a direct application of the Cauchy-Schwarz inequality:

$$
\int \kappa(z) z dz = \int \kappa(z)^{\frac{1}{2}} \kappa(z)^{\frac{1}{2}} z dz \leq \left(\int \kappa(z) dz \right)^{\frac{1}{2}} \left(\int \kappa(z) z^2 dz \right)^{\frac{1}{2}} \qquad (2.40)
$$

By the same reasoning, the second line would also be positive if $z^* = z_x^*$. On the other hand, as $z_x^* \to Z$, the second line would become zero. To ensure that the second line is positive for any z_x^*, I differentiate it with respect to z_x^*, which I denote ζ:

$$
\zeta \equiv 2 \left[\int_{z^*}^{\infty} \kappa(z) z dz \right] \kappa(z_x^*) z_x^* - \left[\int_{z^*}^{\infty} \kappa(z) dz \right] \kappa(z_x^*) z_x^{*2} - \left[\int_{z^*}^{\infty} \kappa(z) z^2 dz \right] \kappa(z_x^*) \qquad (2.41)
$$

Completing the square and using the Cauchy-Schwarz inequality shows that ζ is negative for all z_x^*. The second line in (2.39) is therefore weakly positive, and $\frac{\partial H(z^*,w)}{\partial w} > 0$.

- Step 3: By the implicit function theorem, steps 1 and 2 show Lemma 2.4.

Lemma 2.5 *Equation (2.28) establishes a continuous monotonic negative relationship between z^* and w.*

Proof. The proof uses the implicit function theorem on (2.28):

Define :

$$
J(z^*, w) = \left(\frac{1 + z_x^* w^{1-\sigma}}{1 + z^* w^{1-\sigma}} \right)^{\frac{\epsilon-1}{\sigma-1}} - \frac{f_x}{f} \tau^{\epsilon-1} \qquad (2.42)
$$

Using (2.37):

$$
\begin{aligned}
\frac{\partial J}{\partial z^*} &= \frac{1-\epsilon}{\sigma-1} w^{1-\sigma} \frac{(1 + z_x^* w^{1-\sigma})^{\frac{\epsilon-1}{\sigma-1}}}{(1 + z^* w^{1-\sigma})^{\frac{\epsilon-1}{\sigma-1}+1}} \left[1 + \frac{(1 + z^* w^{1-\sigma})}{(1 + z_x^* w^{1-\sigma})} \frac{g(z^*) f}{g(z_x^*) f_x} \right] < 0 \qquad (2.43) \\
\frac{\partial J}{\partial w} &= \frac{(\epsilon-1) w^{-\sigma}}{(1 + z^* w^{1-\sigma})(1 + z_x^* w^{1-\sigma})} \left(\frac{1 + z_x^* w^{1-\sigma}}{1 + z^* w^{1-\sigma}} \right)^{\frac{\epsilon-1}{\sigma-1}} (z^* - z_x^*) < 0 \qquad (2.44)
\end{aligned}
$$

where the last inequality is strict whenever there is partitioning between exporting and non-exporting firms.

The implicit function theorem immediately implies Lemma 2.5.

Lemma 2.6 *Under Assumption 2.1, the two curves defined by Lemmas 2.4 and 2.5 intersect.*

Proof.

- Limits of the curve defined by Lemma 2.4.

For $w \rightarrow 0$, $H(z^*, w) \rightarrow 0$. In order for $\frac{U}{S} = H(z^*, w)$ to hold, z^* has to decrease as much as possible. From Assumption 2.1, this is attained at a level $\tilde{z} > \underline{z}$ defined by:

$$K = (1 - G(\tilde{z}))f \tag{2.45}$$

I denote the wage attained at $\tilde{z}$ by $\tilde{w}$, below which the equality cannot hold. Similarly, if $w \rightarrow \infty$, $H(z^*, w) \rightarrow \infty$, which requires that z^* increases as much as possible. z^* is required to be smaller than the upper bound $\overline{z}$, defined as the solution to:

$$\frac{K}{M} = (f + f_x)(1 - G(\overline{z})) \tag{2.46}$$

The wage defined by (2.27) at $\overline{z}$ is denoted as $\overline{w}$.

- Limits of the curve defined by Lemma 2.5:

$$\lim_{w \rightarrow 0} J(z^*, w) = \left(\frac{z_x^*}{z^*}\right)^{\frac{\epsilon-1}{\sigma-1}} - \tau^{\epsilon-1}\frac{f_x}{f}$$

Under Assumption 2.1 and since $\tau^{\epsilon-1}\frac{f_x}{f} > 1$, the limit of $J(z^*, w)$ will be equal to zero for some $z^* \in [\tilde{z}, \overline{z}]$.

$$\lim_{w \rightarrow \infty} J(z^*, w) = 1 - \frac{f_x}{f}\tau^{\epsilon-1}$$

In order for $J(z^*, w)$ to remain equal to zero, $\frac{z_x^*}{z^*} \rightarrow \infty$. Under Assumption 2.1, this is the case for $z^* \rightarrow \tilde{z}$.

Combining Lemmas 2.4, 2.5 and 2.6 completes the proof of Proposition 2.1. A graphical representation summarising the proof is given in figure 2.2.

Comparative statics

In order to see how the skill wage premium w responds to a change in the costs of trade $T \in \{f_x, \tau\}$, I use Cramer's rule on the following system:

$$\begin{bmatrix} \frac{\partial J(z^*,w)}{\partial w} & \frac{\partial J(z^*,w)}{\partial z^*} \\ \frac{\partial H(z^*,w)}{\partial w} & \frac{\partial H(z^*,w)}{\partial z^*} \end{bmatrix} \begin{bmatrix} dw \\ dz^* \end{bmatrix} = \begin{bmatrix} -\frac{\partial J(z^*,w)}{\partial T} \\ -\frac{\partial H(z^*,w)}{\partial T} \end{bmatrix} d\tau \tag{2.47}$$

This yields:

$$\frac{dw}{dT} = \frac{\frac{\partial J(z^*,w)}{\partial z^*}\frac{\partial H(z^*,w)}{\partial T} - \frac{\partial J(z^*,w)}{\partial T}\frac{\partial H(z^*,w)}{\partial z^*}}{\frac{\partial J(z^*,w)}{\partial w}\frac{\partial H(z^*,w)}{\partial z^*} - \frac{\partial J(z^*,w)}{\partial z^*}\frac{\partial H(z^*,w)}{\partial w}} \tag{2.48}$$

From the proof of Proposition 2.1, it is immediate that the denominator of the right hand side above is positive. The sign of the effect of trade liberalisation on the skill wage premium is therefore the sign of the numerator, which depends on the trade costs considered.

Proof of Lemma 2.2

$$\frac{\partial H(z^*,w)}{\partial \tau} = \frac{\epsilon-1}{C^2}\tau^{-\epsilon}\int_{z_x^*}^{\infty}(1+zw^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}(Bw^{-\sigma}z-C)dG(z) \tag{2.49}$$

$$\frac{\partial J(z^*,w)}{\partial \tau} = (1-\epsilon)\frac{f_x}{f}\tau^{\epsilon-2} \tag{2.50}$$

Using (2.48) and the partial derivatives of $H(z^*,w)$ and $J(z^*,w)$ immediately yields Lemma 2.2 where:

$$\eta = -\frac{(\epsilon-1)^2}{\sigma-1}w^{1-\sigma}\left(\frac{1+z_x^*w^{1-\sigma}}{1+z^*w^{1-\sigma}}\right)^{\frac{\epsilon-1}{\sigma-1}}\left[(1+z^*w^{1-\sigma})^{-1}+(1+z_x^*w^{1-\sigma})^{-1}\frac{g(z^*)f}{g(z_x^*)f_x}\right]\frac{\tau^{-\epsilon}}{C^2}$$

$$\xi = (1-\epsilon)\frac{f_x}{f}\tau^{\epsilon-2}g(z^*)(1+z^*w^{1-\sigma})^{\frac{\epsilon-1}{\sigma-1}}$$

Proof of Proposition 2.2

From the first step of the proof of Lemma 2.4, it holds that: $\frac{Bz^*w^{-\sigma}-C}{1+z^*w^{1-\sigma}} < \frac{Bz_x^*w^{-\sigma}-C}{1+z_x^*w^{1-\sigma}}$. This, in combination with the definition of $\xi < 0$, shows that the second and third terms in (2.29) are, taken together, negative. Since $\eta < 0$, it remains to show that:

$$\phi \equiv \int_{z_x^*}^{\infty}(1+zw^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}(Bzw^{-\sigma}-C)dG(z) > 0 \tag{2.51}$$

From the definitions of B and C, it can be derived that:

$$\phi \propto \frac{\int_{z_x^*}^{\infty}(1+zw^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}zdG(z)}{\int_{z_x^*}^{\infty}(1+zw^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}dG(z)} - \frac{\int_{z^*}^{\infty}(1+zw^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}zdG(z)}{\int_{z^*}^{\infty}(1+zw^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}dG(z)} > 0 \tag{2.52}$$

The above term is the difference between two weighted sums of z, the first for $z \geq z_x^*$, the second for $z \geq z^*$. Since $z_x^* > z^* > 1$, the above term is positive. Differentiating the weighted sum with respect to the lower bound shows this fact formally.

Proof of Lemma 2.3

$$\frac{\partial H(z^*,w)}{\partial f_x} = \frac{1-G(z_x^*)}{f_x}\frac{\tau^{1-\epsilon}}{C^2}(1+z_x^*w^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}(Bz_x^*w^{-\sigma}-C) \tag{2.53}$$

$$\frac{\partial J(z^*,w)}{\partial f_x} = -\frac{\tau^{\epsilon-1}}{f} + \frac{\epsilon-1}{\sigma-1}\frac{w^{1-\sigma}}{1+z_x^*w^{1-\sigma}}\left(\frac{1+z_x^*w^{1-\sigma}}{1+z^*w^{1-\sigma}}\right)^{\frac{\epsilon-1}{\sigma-1}}\frac{1-G(z_x^*)}{g(z_x^*)f_x} \tag{2.54}$$

Using (2.48) and the partial derivatives of $H(z^*, w)$ and $J(z^*, w)$ immediately yields Δ' in Lemma 2.2 where:

$$\kappa = -\frac{g(z^*)(1+z^*w^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}}{C^2}\left[-\frac{\tau^{\epsilon-1}}{f} + \frac{\epsilon-1}{\sigma-1}\frac{w^{1-\sigma}}{1+z_x^*w^{1-\sigma}}\left(\frac{1+z_x^*w^{1-\sigma}}{1+z^*w^{1-\sigma}}\right)^{\frac{\epsilon-1}{\sigma-1}}\frac{1-G(z_x^*)}{g(z_x^*)f_x}\right] \quad (2.55)$$

$$\theta = -\frac{\epsilon-1}{\sigma-1}\frac{1-G(z_x^*)}{fC^2}(1+z_x^*w^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}w^{1-\sigma}\left[(1+z^*w^{1-\sigma})^{-1} + (1+z_x^*w^{1-\sigma})^{-1}\frac{fg(z^*)}{f_x g(z_x^*)}\right]$$
$$+\frac{g(z^*)}{C^2}\frac{(1+z^*w^{1-\sigma})^{\frac{\epsilon-1}{\sigma-1}}}{1+z_x^*w^{1-\sigma}}\left[-\frac{\tau^{\epsilon-1}}{f} + \frac{\epsilon-1}{\sigma-1}\frac{w^{1-\sigma}}{1+z_x^*w^{1-\sigma}}\left(\frac{1+z_x^*w^{1-\sigma}}{1+z^*w^{1-\sigma}}\right)^{\frac{\epsilon-1}{\sigma-1}}\frac{1-G(z_x^*)}{g(z_x^*)f_x}\right] \quad (2.56)$$

Proof of Proposition 2.3

Rearranging Δ' using (2.21)gives:

$$\Delta' = \frac{1-\epsilon}{\sigma-1}w^{1-\sigma}(1+z_x^*w^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}\frac{1-G(z_x^*)}{f(1+z^*w^{1-\sigma})}\left[Bz_x^*w^{-\sigma} - C + \frac{g(z^*)f}{g(z_x^*)f_x}(Bz^*w^{-\sigma} - C)\right]$$
$$+ \frac{(1+z_x^*w^{1-\sigma})^{\frac{\epsilon-1}{\sigma-1}}}{f_x}g(z^*)\left[\frac{Bz^*w^{-\sigma}-C}{1+z^*w^{1-\sigma}} - \frac{Bz_x^*w^{-\sigma}-C}{1+z_x^*w^{1-\sigma}}\right] \quad (2.57)$$

For $\tau = 1$, as $f_x \to f$, $z_x^* \to z^*$ and the second line in the above equation approaches zero, while the first line is strictly positive. For higher f_x and (or) τ, however, the first line is negative, while the second is ambiguous, so that it is not possible to draw more elaborate conclusions without further assumptions.

Calibration

As assumed for the calibration, z is drawn from a Pareto distribution with lower bound equal to one and parameter a:

$$G(z) = 1 - z^{-a} \quad (2.58)$$

I now define the cumulative distribution function of z conditional on firm z producing:

$$\Gamma(z) = 1 - z^{*a}z^{-a} \quad (2.59)$$

Domestic sales ($n(z)$) of a given z firm are given from (2.13) and (2.5) by:

$$n(z) = \lambda(1+zw^{1-\sigma})^{\frac{\epsilon-1}{\sigma-1}} \quad (2.60)$$

where $\lambda \equiv \epsilon A w_u^{1-\epsilon}P^{\epsilon-1}I$. Solving for z in (2.60) gives the productivity of a firm as a function of its domestic sales.

$$z(n) = w^{\sigma-1}\left[\left(\frac{n}{\lambda}\right)^{\frac{\sigma-1}{\epsilon-1}} - 1\right] \quad (2.61)$$

A simple transformation of variable gives the c.d.f. of firms domestic sales conditional on their producing as:

$$F(n) = \Gamma(z(n)) = 1 - z^{*a}\left(w^{\sigma-1}\left[\left(\frac{n}{\lambda}\right)^{\frac{\sigma-1}{\epsilon-1}} - 1\right]\right)^{-a} \quad (2.62)$$

For large n:

$$1 - F(n) \approx b^a w^{-a(\sigma-1)}\lambda^{\frac{a(\sigma-1)}{\epsilon-1}}n^{-a\frac{\sigma-1}{\epsilon-1}} \quad (2.63)$$

The log right tail probability of the distribution of domestic firm size (measured as sales or as employment) are therefore a straight line with coefficient: $-a\frac{\sigma-1}{\epsilon-1}$.

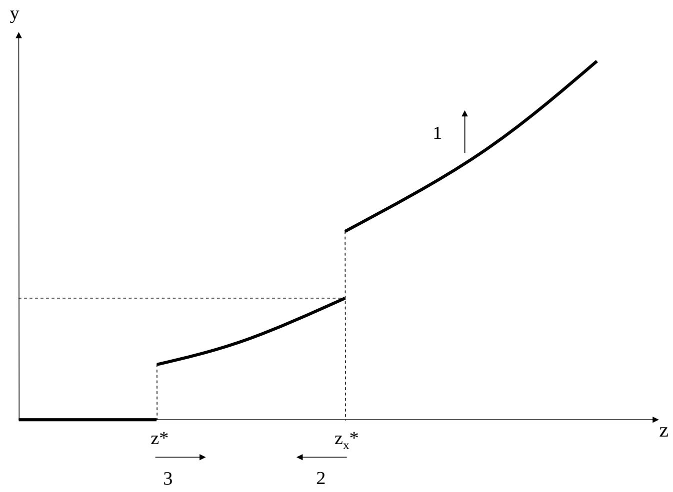

Figure 2.1: Effects of a reduction in τ on the production of different z firms for a fixed w.

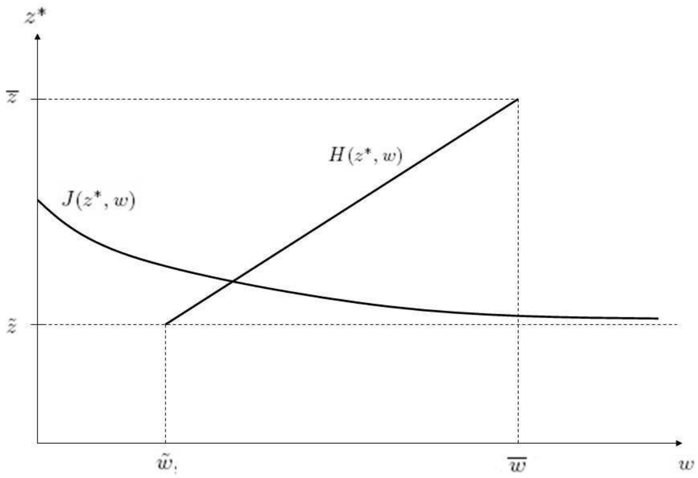

Figure 2.2: Sketch of the proof of existence and uniqueness of equilibrium. $H(z^*, w)$ stands for (2.27), $J(z^*, w)$ for (2.28).

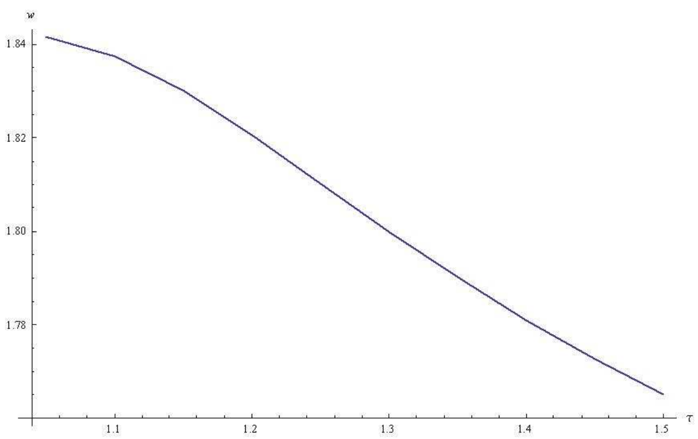

Figure 2.3: The skill premium as a function of τ for $f_x = 0.916$, 2 country case.

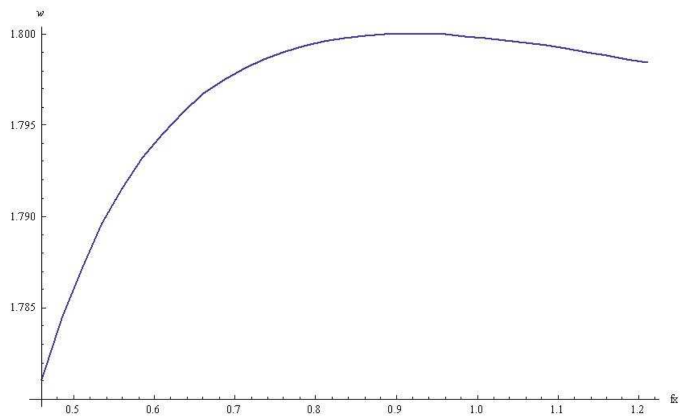

Figure 2.4: The skill premium as a function of f_x for $\tau = 1.3$, 2 country case.

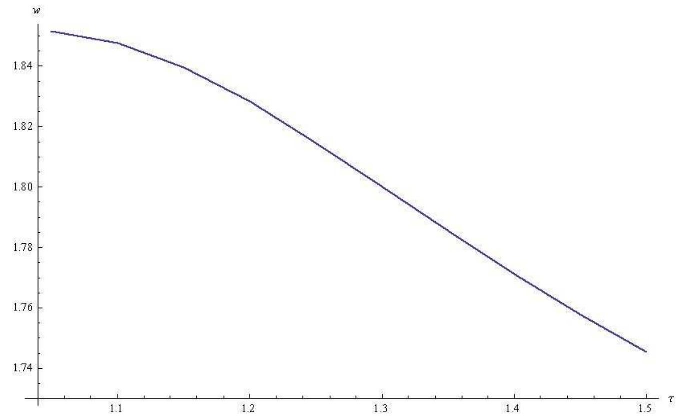

Figure 2.5: The skill premium as a function of τ for $f_x = 0.906$, 3 country case.

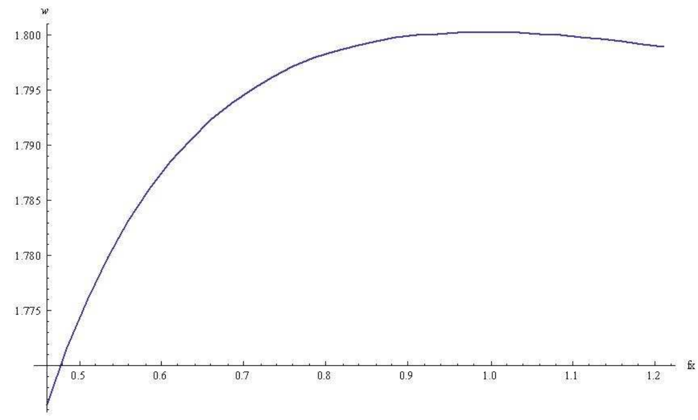

Figure 2.6: The skill premium as a function of f_x for $\tau = 1.3$, 3 country case.

Chapter 3

Trade liberalisation and investment in process innovation

3.1 Introduction

Models of heterogeneous firms have recently provided a new rationale for gains from trade. They suggest that trade liberalisation leads to a reallocation of productive factors from inefficient firms, which exit the market, to efficient firms, which export more. This source of gains from trade, which increases the productivity of the economy, has been confirmed by many studies[1]. However, empirical evidence points to another channel of productivity gains that occurs *within* firms[2] , and not only through a reallocation of factors *between* firms as shown by Melitz (2003). This suggests that firms take actions to influence their productivity.

I develop a model à la Melitz (2003) in which heterogeneously productive firms decide on a level of investment in process innovation (thereafter 'investment') in order to increase their productivity. This decision is continuous in the sense that each firm decides how much to spend and is not restricted to a binary decision between investment and no investment. I denote the mapping of the investment level into the production function as the 'investment technology'.

The main contribution of this paper is to examine the effect of trade liberalisation on firm-level and aggregate investment in productivity improvements.

As in Melitz (2003), smaller trade costs imply a reallocation of market shares from non-exporters to exporters. Since exporters increase their global sales following a reduction in trade costs, they find it profitable to cut their unit costs of production and increase their

[1]see among others Pavcnik (2002), Bernard et al. (2006).

[2]Trefler (2004), Bernard et al. (2006), Van Biesebroeck (2005) and De Loecker (2007).

investment in productivity. Non-exporters, on the other hand, lose market shares and scale down their investment as they find it more difficult to recoup its costs. I show that the effect of trade liberalisation on firm-level investment intensity, defined as investment spending over sales, depends on a simple property of the investment technology. If the technology is such that firm size and investment intensity are positively correlated in equilibrium, trade liberalisation raises the investment intensity of exporters and reduces that of non-exporters. The reverse argument holds for a negative correlation between investment intensity and firm size.

I further show how the changes in within-firm investment induced by trade liberalisation affect the aggregate investment intensity in the economy, defined as the ratio of aggregate investment to aggregate sales. If the technology is such that larger firms have a higher investment intensity in equilibrium, the increased investment of exporters dominates the drop in investment by non-exporters, and trade liberalisation tends to raise the aggregate investment intensity. The reverse holds for a negative correlation between investment intensity and size. This result qualifies the view that the scale effect of international trade fosters innovation[3] and provides a simple, observable condition about the investment technology under which trade is likely to raise aggregate innovation.

The present paper also addresses two recent puzzles to the heterogeneous firms literature pointed out by Nocke and Yeaple (2006). First, they argue that trade liberalisation has reduced the skewness of the size distribution of firms. They find that a decrease in trade costs does not affect all firms proportionately, and that the size differential between two given firms tends to decrease, which cannot be explained by the Melitz (2003) framework. In the present model, the heterogeneous reaction of firm investment to a change in trade costs can account for this fact. Second, they show that the correlation between Tobin's Q and firm size is empirically negative in the cross section of firms, contrarily to the prediction of a straightforward extension of the standard heterogeneous firms model. I show how the different investment levels across firms can explain this correlation.

This paper relates to the rapidly growing literature on trade with heterogeneous firms, which follows the seminal contribution of Melitz (2003). A number of recent papers have introduced the possibility for firms to take an investment decision in order to improve their productivity in a Melitz (2003) type framework. Yeaple (2005), Bustos (2005) or Navas and Sala (2007) allow firms with different productivity draws to choose between two different production technologies: a low productivity, low cost technology, and a high productivity, high cost technology. This generates an equilibrium in which large exporting firms find it profitable to invest, while small unproductive firms use a low-cost technology. Their modeling strategy, which is built on a binary technology choice, however bears some limitations which this paper addresses. The continuous

[3]The view that the larger market size induced by international trade fosters innovation is widespread and often put forward in policy recommendations. See for example Onodera (2008).

investment decision used in the present paper has several advantages. First, it generates a continuous relationship between firm size and innovation spending. This is closer to empirical evidence[4] than the discrete jump in investment at a given size level generated by models with two technologies. Second, it appears that following trade liberalisation, the largest exporters continue to raise their spending in innovation[5]. This cannot be explained by binary technology models and suggests that trade liberalisation does not solely affect investment through the extensive margin of some firms switching technology, but also through changes in the investment of all firms. Third, a continuous investment technology allows to generate new predictions, such as the lower investment of non-exporters after a drop in trade costs, or the importance of the investment technology to study changes in investment at the aggregate level.

Costantini and Melitz (2007) develop a model in which firms take a discrete investment decision in a dynamic framework, and focus on the dynamic adjustment of firms to trade liberalisation, and on the causality between export and investment. Ederington and McCalman (2006) use a dynamic model of trade liberalisation with ex-ante identical firms which can choose between two technologies. The use of the high technology gradually diffuses among firms, at a speed affected by trade liberalisation. The time dimension introduces in these models some elements of continuity, though their use of a discrete investment decision bears similar problems to those mentioned above.

Van Long et al. (2007) and Atkeson and Burstein (2007) provide to my knowledge the only two models making the investment decision of firms continuous. The first is very different from the present setup, since it assumes that a firm takes its investment decision before knowing its productivity draw. The second is closer to the present model, as it builds on Melitz (2003) with a continuous investment possibility in a dynamic framework. They however make a strong assumption about the functional form of the technology, which makes the returns of process innovation proportional to firm profits. This is a special case of my - in this respect - more general formulation, which prevents them to obtain similar results on firm-level and aggregate R&D intensity or on the size distribution of firms.

In section 3.2, I present a closed economy version of the model to explain the basic mechanisms at stake. In section 3.3, I extend the model to two symmetric countries and examine the consequences of trade liberalisation for firm level and aggregate spending in process innovation. Section 3.4 shows how the model can be used to gain insight in the two aforementioned puzzles and section 3.5 concludes.

[4]see the second stylised fact in Cohen and Klepper (1996) for the U.S..

[5]Empirical results in Bustos (2007) suggests that among the group of initial exporters, the largest raise their spending in technology faster than others following trade liberalisation.

3.2 The closed economy

3.2.1 Demand

The representative consumer has a C.E.S. utility function over a continuum of varieties:

$$U = \left[\int_{\omega \in \Omega} q(\omega)^{\frac{\epsilon-1}{\epsilon}} d\omega \right]^{\frac{\epsilon}{\epsilon-1}} \tag{3.1}$$

where the measure of the set Ω is the mass of all available varieties and $q(\omega)$ stands for the consumption of variety ω. ϵ represents the elasticity of substitution between varieties. It is assumed to be strictly greater than one, which ensures that preferences exhibit the love of variety property as in Dixit and Stiglitz (1977) . The maximisation problem of the consumer yields the following demand function for a variety ω:

$$q(\omega) = \left(\frac{p(\omega)}{P} \right)^{-\epsilon} Q \tag{3.2}$$

where Q is a composite good defined as $Q \equiv U$ and P is the price of this good:

$$P = \left[\int_{\omega \in \Omega} p(\omega)^{1-\epsilon} d\omega \right]^{\frac{1}{1-\epsilon}} \tag{3.3}$$

The consumer's income consists exclusively of the proceeds of his labour, paid at a wage normalised to one. The labour supply L is inelastic, and indexes the size of the economy. The aggregate budget constraint is:

$$PQ = L \tag{3.4}$$

3.2.2 Firms

There is a continuum of firms, each producing a different variety with a production technology using exclusively labour. Firms are heterogeneous with respect to a productivity parameter z, drawn from a continuous distribution $G(z)$ with support $(0, \infty)$. Upon learning its productivity parameter, a firm decides how much to invest in process innovation (i) in order to reduce its marginal costs. The production function of a firm having drawn z is:

$$y = zlt(i)^{\frac{1}{\epsilon-1}} \tag{3.5}$$

where l is the amount of labour used for production and the function $t(i)^{\frac{1}{\epsilon-1}}$ is the investment technology, linking the amount invested in process innovation i to its impact on output[6]. I assume that the function $t(i)$, defined on the positive reals, has the following properties:

Assumption 3.1 $t'(i) > 0, \quad t''(i) < 0, \ \lim_{i\to\infty} t'(i) = 0$ *and* $\lim_{i\to 0} t'(i) = \infty$

The last part of the assumption is made to ensure that all producing firms invest a positive amount in process innovation, which simplifies the analysis[7].

Timing

The timing of the model is as follows. In a first stage, there is an unbounded mass of entrepreneurs, who decide whether to enter the market or not. As in Melitz (2003), entering the market means paying a sunk cost f_e in terms of labour in order to obtain a draw of the parameter z. In a second stage, firms decide whether to produce or not given their draw of z. If they do, they choose how much to invest in a third stage and in the fourth stage set price and quantity.

The sequentiality of the third and fourth stages appears realistic since the planning horizon of investment is usually long, and decided upon before employment. A simultaneous investment and production decision would however not affect the results.

Firms live for a single period, which is another difference to Melitz (2003), who considers the effects of exogenous death and endogenous entry of firms. The model can be straightforwardly extended to a steady state, as in Melitz (2003) but it complicates the notation without adding much insight[8].

The optimisation problem of the firm

In the fourth stage, firms set their optimal price given their investment decision. Since there is a continuum of firms, there are no strategic interactions, and each firm optimises given the market conditions summarised by the price index P. The optimal pricing decision is to set a fixed markup over marginal costs:

$$p(z) = \frac{\epsilon}{\epsilon - 1}\frac{t(i)^{\frac{1}{1-\epsilon}}}{z} \tag{3.6}$$

Using this price in the demand equation (3.2) yields the optimal quantity produced by a firm having drawn z ($q(z)$) and, from the production function (3.5), the optimal use

[6] $t(i)$ is taken to the power $\frac{1}{\epsilon-1}$ for simplicity.

[7] Having firms producing without investing would require to deal with an additional cutoff level for the investment status.

[8] Helpman et al. (2004) also use a one-period model for simplicity.

of labour for production ($l(z)$):

$$q(z) = \left(\frac{\epsilon}{\epsilon-1}\frac{t(i)^{\frac{1}{1-\epsilon}}}{z}\right)^{-\epsilon} P^{\epsilon}Q \tag{3.7}$$

$$l(z) = \left(\frac{\epsilon}{\epsilon-1}\right)^{-\epsilon} P^{\epsilon}Qt(i)z^{\epsilon-1} \tag{3.8}$$

The larger the relative productivity of a firm compared to its competitors, the lower its price and the more units it sells.

By backward induction, in stage 3, the optimal choice of investment should maximise profits which, using the optimal price, quantity and labour in (3.6), (3.7) and (3.8) are given by:

$$\pi_d(z) = p(z)q(z) - l(z) - i - f = A(zP)^{\epsilon-1}t(i)L - i - f \tag{3.9}$$

where $A \equiv \frac{1}{\epsilon}\left(\frac{\epsilon}{\epsilon-1}\right)^{1-\epsilon}$. Choosing a high level of investment allows a firm to charge a lower price and therefore to sell more, but comes at a cost (i). In order to produce, all firms must pay a fixed cost of production f. i and f are paid in labour units.

The first order condition for this problem is given by[9]:

$$A(zP)^{\epsilon-1}Lt'(i_d) - 1 = 0 \tag{3.10}$$

i_d defines the optimal level of investment of a firm as a function of its productivity parameter z. By the concavity of $t(i)$, a firm invests more the higher the price index and the higher its own productivity parameter z. For a given firm, a high z and a high price index means that it is relatively efficient in comparison to its competitors and therefore sells large quantities of its variety. The returns of investment are high for such a firm, as it reduces the costs of production of many units. This establishes the following result:

Proposition 3.1 *The optimal investment of a firm is strictly increasing in its size.*

The proof is as follows: from (3.10), the higher the z of a firm, the more it invests. It follows that $t(i_d(z))$ is increasing in z, and that, from (3.7) and (3.8), the size of a firm - whether defined as production, or labour employed - is increasing in z. ■

Using the first order condition (3.10), I rewrite the variable profits (VP_d) - given by $\pi_d(z)+i+f$ and defined as the sales minus the costs of labour used for production - as:

$$VP_d(z) = \frac{t(i_d(z))}{t'(i_d(z))} \equiv \frac{i_d(z)}{e(i_d(z))} \tag{3.11}$$

[9] Assumption 3.1 ensures the existence of a solution for any $z \in (0,\infty)$.

where $e(i) \equiv \frac{it'(i)}{t(i)}$ is the elasticity of the function $t(i)$. $VP_d(z)$ is increasing in z from the concavity of $t(i)$ and from the fact that $i_d(z)$ is increasing. It is useful at this stage to note that the optimal sales of a firm z ($s_d(z)$) are equal to:

$$s_d(z) = \epsilon VP_d(z) = \epsilon \frac{t(i_d(z))}{t'(i_d(z))} \tag{3.12}$$

I impose some additional regularity conditions on $e(i)$ to (i) make sure that very productive firms make positive profits, and (ii) to simplify the interpretation.

Assumption 3.2

(i) $e(i) < 1 - \mu$ for μ small and for all i

(ii) $e(i)$ is strictly monotonic or constant over the whole range of i and bounded away from zero.

3.2.3 The cutoff productivity level

In the second stage, each firm decides whether to produce or not given its draw of z. I define z^* as the level of z for which, given the optimal investment decision $i_d(z^*)$, a firm breaks even when it produces, i.e., it makes zero profits given its optimal investment decision. Using (3.11):

$$\frac{t(i_d(z^*))}{t'(i_d(z^*))} - i_d(z^*) - f = 0 \tag{3.13}$$

Proposition 3.2

1. *The optimal level of investment of the cutoff firm z^* is uniquely determined, and is independent of the level of z^*.*

2. *There exists a strictly positive and unique cutoff level z^* such that all firms with $z > z^*$ make strictly positive profits, and all firms with $z < z^*$ do not produce.*

Proof. *See Appendix* ■

Part 1 of the Proposition is a useful property, which allows to express $i_d(z^*)$ independently of z^*. The intuition for this result is that the sales of a firm depend on its relative productivity compared to its competitors. The least productive producing firm therefore makes its investment decision based on its rank and not on its absolute productivity level z^*. For convenience, I define: $i^* \equiv i_d(z^*)$. Part 2 of the Proposition 3.2 is similar to Melitz (2003) and gives the optimal strategy of a firm in stage 2 given its draw of z.

It is convenient to express the ideal price index P as a function of the cutoff firm z^*. From (3.10) and (3.13):

$$P^{\epsilon-1} = \frac{f + i^*}{ALt(i^*)} z^{*1-\epsilon} \tag{3.14}$$

Since i^* is constant from Proposition 3.2, the price index is inversely proportional to the cutoff productivity level z^*. This is qualitatively similar to the original Melitz (2003) model, and simplifies the analysis.

Two additional conditions are required to close the model. First, there is free entry of entrepreneurs in the first stage. In order to enter the market and obtain a draw of productivity, an entrepreneur must pay a sunk cost of f_e units of labour. He is indifferent between entering the market or not in the first stage if expected profits on the market are equal to the sunk costs of entry:

$$\int_{z^*}^{\infty} \pi_d(z) dG(z) = f_e \tag{3.15}$$

Second, the labour market must be in equilibrium:

$$L = M \left[\left(\int_{z^*}^{\infty} \left((\epsilon - 1) \frac{t(i_d(z))}{t'(i_d(z))} + i_d(z) + f \right) dG(z) \right) + f_e \right] \tag{3.16}$$

where M is the mass of entrepreneurs paying the sunk entry cost f_e in equilibrium. The right hand side aggregates the labour used for production purposes, for investment and for the payment of fixed and sunk costs.

3.3 The open economy

3.3.1 The setup

This section develops the open economy version of the model, and assumes that the world consists of two perfeclty symmetric countries, Home (H) and Foreign (F), whose economies are of the type described in the previous section. Due to the symmetry assumption, the wage in both countries is equal and normalised to one. As in Melitz (2003), exporting is associated with two kinds of additional costs: variable and fixed costs of trade. I model the variable costs as iceberg trade costs given by $\tau \geq 1$. τ states how many units of a good must be shipped for one unit to arrive at destination and reflects transportation costs or tariffs. The fixed costs of exporting f_x, paid in units of labour, can be thought of as the cost of establishing a distribution network on a foreign market, complying with foreign regulation, or learning a foreign business law. The timing is identical to the closed economy version of the model. The only difference is that in stage 2, firms decide between three strategies given their draw of z: selling on both markets (exporting firms), selling only on the domestic market (domestic firms) or not producing.

A Home firm which exports to the foreign market chooses the following optimal pricing rule:

$$p_F(z) = \frac{\epsilon}{\epsilon - 1} \frac{t(i)^{\frac{1}{1-\epsilon}}}{z} \tau = p_H(z)\tau \tag{3.17}$$

where p_F and p_H respectively denote price charged on the Foreign and on the Home market. Since the preferences are identical in both countries, an exporting firm charges the same markup on the Home and Foreign markets. Due to the variable costs of trade, however, it sets a higher price on the export market. Given the optimal price, and using the symmetry assumption between the two countries, the quantity sold by an exporter on the foreign market is given by:

$$q_F(z) = q_H(z)\tau^{-\epsilon} \tag{3.18}$$

Due to the higher price on the export market, a Home firm sells less in Foreign than at Home. If it decides to export, a firm z faces the following maximisation problem:

$$\max_i \pi_x(z) = p_H(z)q_H(z) + p_F(z)q_F(z) - \frac{t(i)^{\frac{1}{1-\epsilon}}}{z}(q_H(z) + q_F(z)) - i - f - f_x$$

Plugging the optimal prices (3.17) and quantities (3.18) in the above problem gives the global profits of an exporting firm:

$$\pi_x(z) = (1 + \tau^{1-\epsilon})A(zP)^{\epsilon-1}t(i)L - (i + f_x + f) \tag{3.19}$$

This expression is similar to the profits of a domestic firm as given by (3.9). The differences are that an exporting firm has additional revenues from the export market, weighted by $\tau^{1-\epsilon}$, and additional costs f_x.

The first order condition for optimal investment if a firm exports is given by:

$$(1 + \tau^{1-\epsilon})A(zP)^{\epsilon-1}t'(i_x)L - 1 = 0 \tag{3.20}$$

The above condition defines the optimal investment of an exporting firm ($i_x(z)$) as a function of its productivity parameter z. For a given z, a firm invests more if it exports than if it does not ($i_x(z) > i_d(z)$), which is due to the fact that it sells more when exporting. It is therefore more profitable to save on the variable costs by investing in productivity improvements. For the same reason and conditional on exporting, a firm with a higher z invests more. The present model generates a strictly monotonic relationship between the size of a firm and its investment, which fits the empirical evidence better than frameworks with binary investment[10].

From (3.19) and (3.20), I derive the global variable profits (VP_x) and sales (s_x) of an exporting firm:

$$s_x(z) = \epsilon VP_x(z) = \epsilon \frac{t(i_x(z))}{t'(i_x(z))} \tag{3.21}$$

A firm exports if it makes more profits by exporting than by producing only for its domestic market, i.e. if the difference between the right hand sides of (3.19) and (3.9) is positive:

$$A(zP)^{\epsilon-1}L\left[(1+\tau^{1-\epsilon})t(i_x(z)) - t(i_d(z))\right] - i_x(z) - f_x + i_d(z) \geq 0 \tag{3.22}$$

In contrast to Melitz (2003) and the subsequent literature, the decision to export is not taken independently of domestic considerations. The reason is that the decision to export influences the optimal level of investment, which in turn affects domestic profits. I define z_x^* as the level of z for which a firm is indifferent between exporting or not, i.e. the z for which (3.22) holds with equality:

$$A(z_x^*P)^{\epsilon-1}L\left[(1+\tau^{1-\epsilon})t(i_x(z_x^*)) - t(i_d(z_x^*))\right] - i_x(z_x^*) - f_x + i_d(z_x^*) = 0 \tag{3.23}$$

Proposition 3.3 *For f_x sufficiently high, there exists a unique cutoff firm $z_x^* > z^*$ which is indifferent between exporting and producing only for its domestic market. Firms having drawn a z above this cutoff export, while firms having drawn a lower z do not. The firm z_x^* invests discretely more than the most productive non-exporting firm.*

Proof. *See Appendix* ■

Proposition 3.3 differs in two ways from the existing literature. First, I show in the proof in Appendix that the condition for coexistence of non-exporting and of exporting firms is stronger than in the Melitz (2003) framework[11]. It is due to the fact that when deciding to export, a firm raises its investment and therefore its revenues on the domestic market at the same time. This makes it more profitable to export than in the standard Melitz framework, and requires large fixed costs of exporting in order to ensure that some firms which produce do not export. Second, there is a discrete jump in optimal investment at the cutoff export level, since the ales of the smallest exporter are discretely larger than those of the largest non-exporter.

[10] see Cohen and Klepper (1996) among others.

[11] The condition that $f_x > f(+i^*)\tau^{1-\epsilon}$, which would be the counterpart of the Melitz (2003) condition for the present model is necessary but not sufficient for partitioning.

3.3.2 Trade liberalisation

In the following, I examine the effect of a marginal decrease in the variable costs of trade on the level of investment of exporting and non-exporting firms. I use the condition that expected profits are equal to the fixed entry costs, which ensures the indifference of entrepreneurs between entering the market or not in the first stage.

$$\mathbb{E}(\pi) = \int_{z^*}^{z_x^*} \pi_d(z) dG(z) + \int_{z_x^*}^{\infty} \pi_x(z) dG(z) = f_e \tag{3.24}$$

which, using (3.9) and (3.19), is equivalent to:

$$\begin{aligned} f_e &= \int_{z^*}^{z_x^*} AP^{\epsilon-1} L z^{\epsilon-1} t(i_d(z)) - i_d(z) - f dG(z) \\ &+ \int_{z_x^*}^{\infty} (1+\tau^{1-\epsilon}) AP^{\epsilon-1} L z^{\epsilon-1} t(i_x(z)) - i_x(z) - f - f_x dG(z) \end{aligned} \tag{3.25}$$

A small drop in τ has a direct positive impact on the profit level of exporting firms, as shown by the second line of (3.25). Some endogenous variables therefore need to adapt in order to restore an equilibrium in which (3.25) holds. First, note that small changes in firm level investment have no impact on expected profits by the envelope theorem. Second, since a firm with parameter z^* makes by definition zero profit, expected profits remain unchanged if z^* changes. The same argument holds for changes in the export cutoff level z_x^*, as a firm with this parameter is indifferent between exporting or not. Since small changes in investment or in the cutoff levels have no impact on expected profits, equilibrium is restored by a decrease in P, which makes it more difficult for all firms to sell and brings expected profits back to zero.

I use the fact that trade liberalisation affects the price index to derive its effect on the cutoff level z^* and on the investment of each firm:

Proposition 3.4

A marginal decrease in variable trade costs:

1. *raises the domestic cutoff level z^*, inducing a selection effect.*
2. *decreases the optimal investment level of all firms that remain non-exporters.*
3. *raises the optimal investment of exporting firms.*

Proof. *See Appendix* ■

The first part of the proposition is a similar result to Melitz (2003) and states that following trade liberalisation, firms with low productivity drop out of the market. This is due to the decrease in the price index, which makes it more difficult for any firm to sell on the domestic market, and triggers the exit of the least efficient firms. I show in appendix that the quantitative effect of a decrease in trade costs on the domestic cutoff level z^* is strong when the fraction of exports to GDP in the economy is large. Lower trade costs have in this case a strong positive effect on average profits, as they raise the profits of exporters, which constitute a large proportion of firms. The price index must therefore decrease much for (3.25) to hold and the selection effect is strong.

Part 2 and 3 of Proposition 3.4 derive the effects of trade liberalisation for the investment of exporters and non-exporters. Two factors play a role in these results. First, the drop in the price index makes it more difficult for all firms to sell on a given market. It mechanically reduces the incentives to invest, and accounts for the effect on domestic firms. Second, exporters benefit from the reduction in the costs of exporting, which allows them to sell more on their export market. For exporters, this second effect dominates the drop in the price index, thereby increasing their global sales and raising their incentives to invest. The above proposition constitutes a major difference with the models in which the investment decision is discrete. In these frameworks, domestic firms would continue production with the low technology, while very productive firms, which already produce with the high technology would not invest more[12].

I now turn to the evolution of the cutoff export level z_x^* following a marginal change in τ, which is given by the total differentiation of (3.23). Using the envelope theorem, changes in $i_d(z_x^*)$ and $i_x(z_x^*)$ only have second order effects on the left hand side of (3.23), which is the difference between the profit levels of exporting and of non-exporting firms. A marginal drop in τ has a direct positive effect on this difference, because lower trade costs make it more profitable to export relative to not exporting. It also has an indirect negative effect on this difference through the implied decrease in the price index P, which has a larger absolute negative impact on profits for an exporting than for a non-exporting firm. As shown in the appendix, the direct effect dominates and a smaller τ raises the relative profitability of exporting. A marginal trade liberalisation therefore has the same impact on the export cutoff level as in Melitz (2003):

Proposition 3.5

A small reduction in the variable costs of trade decreases the export cutoff level z_x^*, and raises the proportion of exporting firms.

[12]This last fact especially is at odds with empirical evidence. Bustos (2007) shows that among the group of initial Argentinean exporters in 1992, spending in technology has been increased by trade liberalisation with Brazil, the more so for the largest exporters.

Proof. *See Appendix* ■

3.3.3 Investment intensity at the firm level

In this section, I examine the effects of trade liberalisation on investment intensity at the firm level, which is defined as the ratio of investment spending to sales. From the previous analysis, lower iceberg costs raise the optimal investment of exporting firms while decreasing that of non-exporting firms. This is however insufficient to draw any conclusion about the change in the investment intensity of different firms.

The investment intensity (ι) for domestic and exporting firms is:

$$\iota_k(z) \equiv \frac{i_k(z)}{s_k(z)} = \frac{1}{\epsilon}e(i_k(z)) \quad \text{for k} \in \{d, x\} \tag{3.26}$$

where the equality follows from (3.12) and (3.21). The relationship between investment intensity and size of a firm depends on the elasticity of the technology $t(i)$. If $e'(i) > 0$ (< 0), the investment intensity is increasing (decreasing) in size while it is constant for $e'(i) = 0$, which is a Cobb-Douglas case. A number of studies have examined the empirical relationship between the investment intensity of a firm (measured as R&D spending per worker) and its size or its export status. Bustos (2007) or Bernard and Jensen (1995) show that bigger, exporting firms have a higher R&D intensity, while others such as Aw et al. (2007) find the opposite result. Other works such as Cohen et al. (1996) and Cohen and Klepper (1996) point to the difficulty of establishing a clear link between R&D intensity and firm size, suggesting that this relationship may differ across industries. The question whether economies or diseconomies of scale ($e'(i) > 0$ or $e'(i) < 0$) in the production of innovation prevail is still subject to debate[13]. As the following results show, the derivative of $e(i)$ plays a central role in the analysis.

Proposition 3.6

- *if* $e'(i) = 0$, *a marginal decrease in the costs of trade has no impact on the innovation intensity at the firm level.*
- *if* $e'(i) > (<)0$, *a marginal decrease in the costs of trade raises (decreases) the skill intensity of exporting firms while decreasing (raising) that of non-exporting firms.*

Proof. The proof follows from Proposition 3.4. ■

[13] see Symeonidis (1996) for a survey.

Following trade liberalisation, exporting firms increase their global sales and therefore their investment level. If the technology t is such that larger firms have a higher investment intensity ($e'(i) > 0$), exporting firms raise their investment intensity following liberalisation. The contrary happens for non-exporting firms, which decrease their sales and therefore their investment intensity. The results are reversed if $e'(i) < 0$.

3.3.4 Aggregate investment intensity

Since the sign of the change in investment intensity differs between exporting and non-exporting firms, it is a priori unclear how the aggregate investment intensity changes.

For the interpretation of the results in this and the following sections, it is useful to define:

$$E(i) \equiv \frac{-t'(i)^2}{t''(i)t(i)} \tag{3.27}$$

From (3.10), $\frac{t''(i)}{t'(i)}$ is the percentage change in the marginal returns on investment for a firm which increases its investment from an initial level i. If it is very negative, investment is in this case rather insensitive to external conditions as a small change in the investment level strongly impacts its marginal return. $-\frac{t'(i)}{t''(i)}$ can therefore be interpreted as a measure of the sensitivity of optimal investment by a firm investing i. From (3.12) on the other hand, $\frac{t(i)}{t'(i)}$ is a measure of firm size. $E(i)$ therefore measures the ratio of sensitivity of investment to size. The relationship between $E(i)$ and the investment intensity of a firm is given by the following lemma:

Lemma 3.1 *Under Assumptions 3.1 and 3.2,* $e'(i)$ *and* $E'(i)$ *have the same sign.*

Proof. *See Appendix* ∎

This states that if large firms are relatively investment intensive (i.e. if the technology is such that the optimal investment intensity rises with the investment level: $e'(i) > 0$), their ratio of investment sensitivity to size is large.

Aggregate investment intensity (R) is defined as the ratio of aggregate investment to aggregate sales and is given by:

$$R \equiv \frac{\int_{z^*}^{z_x^*} i_d(z)dG(z) + \int_{z_x^*}^{\infty} i_x(z)dG(z)}{\frac{\epsilon(i^*+f)}{z^{*\epsilon-1}t(i^*)}\left(\int_{z^*}^{z_x^*} z^{\epsilon-1}t(i_d(z))dG(z) + \int_{z_x^*}^{\infty}(1+\tau^{1-\epsilon})z^{\epsilon-1}t(i_x(z))dG(z)\right)} \tag{3.28}$$

A decrease in trade costs has two types of effects on the aggregate investment intensity, which I will denote as Effects B and C. Effect B summarises the impact of firms changing their export or domestic status following a reduction in τ, i.e. firms entering the export market or stopping production. These are represented by the change in the cutoff level z^* and z_x^*. Effect B is ambiguous and depends among others on the distribution function of the productivity parameters $G(z)$. Effect C on the other hand denotes the impact of the change in the investment intensity of all other firms following a decrease in trade costs. The following Proposition summarises the impact of Effect C on the aggregate investment intensity R in the economy.

Proposition 3.7

- *if* $e'(i) = 0$*, Effect C has no impact on the aggregate investment intensity.*
- *if* $e'(i) > (<) 0$*, Effect C raises (decreases) the aggregate investment intensity.*

Proof. *See Appendix* ■

I show in the appendix that it is sufficient to examine how effect C impacts the numerator of R, which represents the aggregate investment in the economy[14], in order to know whether effect C raises or decreases the aggregate investment intensity[15]. For fixed M and cutoff levels z^* and z_x^*, aggregate investment rises following trade liberalisation if the increase in exporters' investment is stronger than the decrease in non-exporters' investment. I show that it is the case if: (i) exporters' investment is more sensitive to a change in market conditions than non-exporters' investment (ii) the share of exports in global sales is small. This second factor may at first seem counterintuitive as it states that the effect of exporters investment is stronger the smaller the weight of exports in global sales. The reason is that the higher the weight of exports, the stronger the adjustment in P following trade liberalisation and the less profitable it is for firms to invest, thereby driving aggregate investment down.

If the ratio of exporters' sensitivity of investment to size is larger than that of non-exporters, (i) and (ii) are fulfilled and effect C raises aggregate investment. This condition holds if $E'(i) > 0$, which implies by Lemma 3.1 that $e'(i) > 0$. The reverse argument holds for $e'(i) < 0$ and concludes the proof of Proposition 3.7.

Proposition 3.7 suggests a more careful interpretation of the traditional Schumpeterian argument that trade liberalisation, by raising the size of the market for exporters, raises the innovation intensity of the economy. Non-exporters, which see their scale decrease by such a drop in trade costs reduce their investment, and the properties of the investment technology, which differ between industries[16], play an important role in determining the effect of trade liberalisation on the aggregate investment intensity.

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[14] The aggregate investment in the economy is M times the numerator of R.

[15] The argument relies on showing that the effects of τ and P on aggregate sales cancel out for given M and cutoff levels. Effect C therefore influences the denominator only through changes in investment, but not as strongly as the numerator.

[16] see Acs and Audtretsch (1987) among others.

3.4 The Puzzles

Thanks to its rich structure, the model can be used to study two recent empirical puzzles on trade and heterogeneous firms as pointed out by Nocke and Yeaple (2006). These are: (i) the negative relationship between firm size and Tobin's Q (ii) the change in the skewness of the size distribution of firms when trade costs decrease.

3.4.1 Tobin's Q and firm size

Nocke and Yeaple (2006) empirically find that the Tobin's Q of a firm, defined as the ratio of market to book value, is negatively related to its size. They argue that the introduction of capital in the Melitz (2003) model would predict the opposite relationship. For this statement, they consider an extended version of the Melitz (2003) model where the fixed costs of production are paid in terms of capital, and where the production function is Cobb-Douglas with capital and labour. Without fixed costs, all firms would use the same fraction of capital in production, and the value of capital (the book value) would be a constant fraction of variable profits (the market value in a static framework) for all firms. The fixed costs paid in terms of capital however account for a higher share of the market value for small firms than for large firms, and therefore yields a Tobin's Q that is increasing with size. This, they argue, runs counter to empirical evidence.

It is straightforward to introduce capital in the present setup under the assumption that it is held by the inhabitants of a third country, and available to all firms in the economy at an exogenous price r. I assume that the fixed costs (f and f_x) and the costs of innovation (i) are paid in terms of capital so as to be in line with the interpretation of Nocke and Yeaple (2006), but abstract from the Cobb Douglas production function[17] and further assume the production function (3.5). The main difference between the present model and traditional heterogeneous firms frameworks is that larger firms invest more in productivity. This is precisely the mechanism that allows me to reverse the relationship between size and Tobin's Q as I will show next.

A firm with productivity z uses the following amounts of capital if it respectively does not and does export:

$$K_d(z) = i_d(z) + f \tag{3.29}$$

$$K_x(z) = i_x(z) + f + f_x \tag{3.30}$$

From (3.11) and (3.21), the Tobin's Q, which is equal to variable profits over capital[18]

[17]Introducing a Cobb Douglas production function would not alter the qualitative results.

[18]The Tobin's Q is equal to variable profits over capital payment: rK. Introducing capital however

is given for domestic and exporting firms respectively by:

$$T_d(z) = \frac{t(i_d(z))}{t'(i_d(z))(i_d(z)+f)} \quad (3.31)$$

$$T_x(z) = \frac{t(i_x(z))}{t'(i_x(z))(i_x(z)+f+f_x)} \quad (3.32)$$

Proposition 3.8

- *If* $e'(i) \leq 0$ *the Tobin's Q is increasing in firm size conditional on the export status.*
- *If* $e'(i) > 0$, *the Tobin's Q is increasing in size for small firms, and decreasing in size for larger firms conditional on the export status.*
- *The smallest exporting firm has a lower Tobin's Q than the largest non-exporting firm.*

Proof. *See Appendix* ■

Two factors influence the relationship between Tobin's Q and firm size in the present version of the model.

First, the fixed costs f and f_x play the same role as in Nocke and Yeaple (2006): they are paid in terms of capital and represent a larger proportion of variable profits for smaller firms. The fixed costs therefore tend to generate a positive relationship between size and Tobin's Q.

Second, the investment technology determines the investment intensity of a firm, which affects the ratio of variable profits to capital. If $e'(i) \leq 0$, the investment intensity is weakly decreasing in size from (3.26), so that large firms use proportionately little capital relative to their variable profits. The technology tends in this case to generate a weakly positive link between size and Tobin's Q. Since the effect of fixed costs and of technology go in this case in the same direction if $e'(i) \leq 0$, the relationship between size and Tobin's Q is unambiguous and given by the first part of Proposition 3.8. If $e'(i) > 0$ on the other hand, large firms use relatively much capital compared to their variable profits. The technology therefore drives the Tobin's Q of large firms down, providing a countervailing force to the effect of fixed costs. Proposition 3.8 states that the effect of fixed costs dominates for small firms while the technology effect dominates for large firms. This can account for the puzzle mentioned in Nocke and Yeaple (2006) that the Tobin's Q decreases with size.

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changes variable profits, which are equal to r times the expression in (3.11) and (3.21). The ratio of variable profits to capital payment is therefore independent of r.

Furthermore, there is a discontinuity in the relationship between size and Tobin's Q at the cutoff export level. For the firm z_x^*, exporting is associated with large fixed and investment costs but a limited increase in variable profits, so that its ratio of variable profits to capital is lower than if it were purely domestic. This provides an additional rationale for which Tobin's Q and firm size may be negatively correlated.

3.4.2 The distribution of firm size

Nocke and Yeaple (2006) empirically find that, for U.S. firms, trade liberalisation has reduced the skewness of the distribution of the logarithm of domestic sales. In other words, the relative size differential between two given firms appears to decrease following trade liberalisation.

In Melitz (2003), a decrease in trade costs induces a reduction in the price index, which has the same proportional effect on the domestic sales of all firms. In the present model however, the effect of a reduction in trade costs on the size distribution impacts the investment level and therefore domestic sales in different ways as I will show in this section.

Domestic sales of a firm with productivity z are given by:

$$s_{dd}(z) = \epsilon A(zP)^{\epsilon-1}t(i_d(z))L \quad \text{for } z < z_x^* \tag{3.33}$$

$$s_{dx}(z) = \epsilon A(zP)^{\epsilon-1}t(i_x(z))L \quad \text{for } z \geq z_x^* \tag{3.34}$$

where s_{dd} and s_{dx} respectively stand for the domestic sales of a non-exporting and of an exporting firm.

Using the equation for the price index (3.14), I rewrite these quantities as:

$$s_{dk}(z) = \epsilon(f + i^*)\frac{t(i_k(z))}{t(i^*)}\left(\frac{z}{z^*}\right)^{\epsilon-1} \quad \text{for } k \in \{d, x\} \tag{3.35}$$

The percentage change in domestic sales for non-exporting firms following a marginal trade liberalisation is given by:

$$\frac{dln(s_{dd}(z))}{d\tau} = \frac{dz^*}{d\tau}\frac{1-\epsilon}{z^*}\left[E(i_d(z)) + 1\right] \tag{3.36}$$

A decrease in τ has an impact on domestic sales which is of the opposite sign of the square bracket above. Two effects influence the domestic sales of non-exporting firms following trade liberalisation. First, there is a direct effect of the price index on sales, as shown by +1 in the square bracket. Trade liberalisation increases the average productivity of competitors, thereby decreasing the price index and the domestic sales of all non-exporting firms. Second, the reduction in sales drives the incentives to invest down, as shown by the first part of the square bracket. This further reduces domestic sales. If $e'(i) = 0$, which implies by Lemma 3.1 that $E(i)$ is constant for all i, the log of domestic sales of all non-exporting firms changes by exactly the same amount. In this case, the change in optimal investment by non-exporting firms of all sizes is such that the quantity they sell decreases by the same proportion. If $e'(i) < 0$, which implies that $E'(i) < 0$ from Lemma 3.1, the decrease in log sales following trade liberalisation is larger the smaller the non-exporter. This is due to the investment technology, which is such that small firms are proportionately more reactive in their investment decision than larger firms. Under this assumption, the size differential between non-exporting firms increases. $e'(i) > 0$ yields opposite consequences.

For exporting firms:

$$\frac{dln(s_{dx}(z))}{d\tau} = (1-\epsilon)\left[\frac{dz^*}{d\tau}\frac{1}{z^*} + \left(\frac{\tau^{-\epsilon}}{1+\tau^{1-\epsilon}} + \frac{dz^*}{d\tau}\frac{1}{z^*}\right)E(i_x(z))\right] \tag{3.37}$$

The decrease in the price index P directly affects exporting firms by making it more difficult for them to sell on the domestic market. This is captured by the first term in the square bracket in (3.37). However, exporting firms also benefit from a decrease in the transport costs, which raises their global sales, and therefore their investment and productivity from Proposition 3.4. The higher productivity in turn boosts domestic sales. Which of the two effects dominates is unclear, so that the sign of the change in domestic sales remains undetermined for exporting firms. However, the group of exporters should see its domestic sales decrease proportionately less than that of non-exporting firms due to increased investment.

The elasticity of the technology function also plays a central role in the determination of the skewness of the domestic sales distribution for exporters. A constant $e(i)$ yields the same proportional change in domestic sales by all exporting firms, although it is worth noting that this change is not equal to that of domestic firms. If $e'(i) < 0(E'(i) < 0)$, size differentials between exporting firms become smaller, since the smaller exporters raise their investment proportionally more than larger exporters. An increasing $E(i)$ has the opposite effect.

These results are summarised in the following Proposition:

Proposition 3.9 *The effect of trade liberalisation on the size distribution of firms depends on the investment technology $t(i)$.*

- *If $e'(i) = 0$, the domestic sales of all non-exporting firms decrease by the same proportion. Those of all exporting firms also change by a constant proportion, albeit different from that of non-exporting firms.*

- *If* $e'(i) > 0 (< 0)$, *the difference between the log of domestic sales of two given non-exporters decreases (increases), while that between two given exporters increases (decreases)*

As in Nocke and Yeaple (2006), the present paper can explain the fact that different firms reduce their domestic sales by different proportions following trade liberalisation. While Nocke and Yeaple (2006) predict smaller differences in the log size of two given firms after trade liberalisation, the present model suggests a non monotonic change. This does not contradict their empirical evidence as they do not allow for non-monotonicity.

3.5 Conclusion

This paper has developed a Melitz (2003) type model of trade, in which, after observing their efficiency, firms invest in productivity improvements. The investment decision is continuous, in the sense that each firm decides how much to invest. This framework preserves all main qualitative results of Melitz (2003), and provides additional insights in the investment decision of heterogeneous firms. Indeed, I am able to replicate a number of well established stylised facts, with weak assumptions for a well behaved problem.

As in Melitz (2003), smaller trade costs imply a reallocation of market shares from non-exporters to exporters. Since exporters increase their sales, they have stronger incentives to cut their unit costs of production and increase their investment in productivity. Non-exporters, on the other hand, lose market shares and scale down investment as they find it more difficult to recoup its costs. Depending on the properties of the investment technology, I show how the firm-level investment intensity, defined as investment spending over sales, changes following trade liberalisation. If the technology is such that the investment intensity of a firm is positively correlated with its size, exporters raise their investment intensity while non-exporters reduce theirs. The reverse argument holds for a negative correlation between investment intensity and size.

I further show how the changes in within-firm investment induced by trade liberalisation affect the aggregate investment intensity in the economy, defined as the ratio of aggregate investment to aggregate sales. If the investment technology implies that larger firms are more skill intensive, the increased investment of exporters dominates the drop in investment by non-exporters, and trade liberalisation raises aggregate investment intensity. The reverse holds for a negative correlation between investment intensity and size.

The present model also addresses two recent puzzles to the heterogeneous firms literature pointed out by Nocke and Yeaple (2006): (i) the negative relationship between firm size and Tobin's Q (ii) the change in the skewness of the size distribution of firms when trade costs decrease. I show that, under some assumptions on the technology function, the heterogeneous levels and sensitivities of firm investment, can explain these facts, which cannot be replicated by the existing literature.

3.6 Appendix to chapter 3

Proof of Proposition 3.2

Proof of part 1

In this section, I show that there is a unique i solving:

$$\frac{t(i)}{t'(i)} - i - f = 0 \tag{3.38}$$

For this, note that $\frac{t(0)}{t'(0)} = 0$ by Assumption 3.1. The left hand side of the above equation is therefore equal to $-f$ for $i = 0$.

Differentiating the left hand side with respect to i:

$$\frac{\partial\left(\frac{t(i)}{t'(i)} - i - f\right)}{\partial i} = -\frac{t''(i)t(i)}{t'(i)^2} > 0 \tag{3.39}$$

I now show that as i goes to infinity, the left hand side will be positive, which is to say that at least some firms will make non-negative profits in the economy.

The left hand side of (3.38) can be rewritten as:

$$\frac{t(i)}{t'(i)}\left[1 - \frac{(i+f)t'(i)}{t(i)}\right] \tag{3.40}$$

As i goes to infinity, $\frac{t(i)}{t'(i)} \to \infty$ by Assumption 3.1. Moreover, $\lim_{i\to\infty} \frac{(i+f)t'(i)}{t(i)} = lim_{i\to\infty} e(i)$, which is bounded away from 1. The square bracket above is therefore bounded, and the whole expression goes to infinity.

This completes the proof that there exists a unique i so that there is a cutoff firm that makes zero profits. This is moreover independent of z^*

Proof of part 2

For the proof of part 2, note that the function $i_d(z) : (0, \infty) \to (0, \infty)$ is bijective from (3.10), which ensures that $i'_d(z) > 0$, and from Assumption 3.1. This, combined with the result of part 1, ensures that there exists a z^* for which i^* is the optimal investment.

Proof of Proposition 3.3

For clarity of exposition, I rewrite the condition for a firm z to export (3.22):

$$\underbrace{A(zP)^{\epsilon-1}L(1+\tau^{1-\epsilon})t(i_x(z)) - i_x(z) - f - f_x}_{\pi_x(i_x(z))} \underbrace{-A(zP)^{\epsilon-1}Lt(i_d(z)) + i_d(z) + f}_{-\pi_d(i_d(z))} \geq 0 \tag{3.41}$$

which requires that the difference in profits between the two strategies (sell in both markets or sell only on the domestic market) be positive.

(i) A straightforward application of the envelope theorem shows that the derivative of the left-hand side with respect to z is positive, since the effects of z on optimal investment are of second order importance. This means that the higher the z, the larger the relative profit of the export strategy.

(ii) For $z = z^*$, $\pi_d = 0$, and $\pi_x(i_x(z^*))$ should be negative for the cutoff firm z^* not to export. This obtains if (3.41) is negative for $z = z^*$:

$$A(z^*P)^{\epsilon-1}L\left[(1+\tau^{1-\epsilon})t(i_x(z^*)) - t(i^*)\right] - i_x(z^*) + i^* - f_x < 0 \tag{3.42}$$

Using (3.14), this is equivalent to:

$$f_x > \frac{f+i^*}{t(i^*)}\left[(1+\tau^{1-\epsilon})t(i_x(z^*)) - t(i^*)\right] - i_x(z^*) + i^* \tag{3.43}$$

If the firm z^* chose the sub-optimal investment i^* when exporting, the above condition would simplify to $f_x > (f+i^*)\tau^{1-\epsilon}$, which is the equivalent to the Melitz (2003) condition in the present framework. Since firm z^* however chooses $i_x(z^*)$ when exporting, which is optimal, the right hand side of (3.43) is larger than $(f+i^*)\tau^{1-\epsilon}$ and the condition for some producing firms not to export is stronger than in the usual heterogeneous firms framework.

(iii) As $z \to \infty$, it is immediate from (3.41) that $\pi_x(i_d(z)) - \pi_d(i_d(z)) \to \infty$. Since by definition, $\pi_x(i_x(z)) > \pi_x(i_d(z))$, this implies that the difference between the profits of the export and non-export strategies goes to infinity.

Combining (i), (ii) and (iii) shows Proposition 3.3

Proof of Proposition 3.4

- Part 1: the selection effect

Plugging (3.14) into (3.25) and totally differentiating yields:

$$\frac{\partial z^*}{\partial \tau}\frac{\tau}{z^*} = -\frac{\int_{z_x^*}^{\infty}\tau^{1-\epsilon}z^{\epsilon-1}t(i_x(z))dG(z)}{\int_{z^*}^{z_x^*}z^{\epsilon-1}t(i_d(z))dG(z) + \int_{z_x^*}^{\infty}(1+\tau^{\epsilon-1})z^{\epsilon-1}t(i_x(z))dG(z)} = -\frac{X}{S} \tag{3.44}$$

Where X and S respectively denote total exports of a country, and global sales of firms from this country (which by the symmetry assumption is equivalent to total sales in a country). To derive the above expression, the envelope theorem has been used, as well as the indifference conditions of the z^* and z_x^* firms. It is negative and shows that trade liberalisation (a lower τ) yields a selection effect by increasing z^*.

- Part 2: Change in optimal investment of non-exporting firms

For non-exporting firms, the first order condition for optimal investment rearranged with (3.14) is:

$$(i^*+f)\left(\frac{z}{z^*}\right)^{\epsilon-1}\frac{t'(i_d)}{t(i^*)} = 1 \tag{3.45}$$

It is immediate that a change in τ impacts the optimal decision of non-exporting firms only through the general equilibrium effect of a drop in the price index and therefore an increase in z^* (remember that from Proposition 3.2, i^* remains constant). The rise in z^* following trade liberalisation must incur a decrease in the optimal investment of a non-exporting firm since $t(i)$ is concave. This can be immediately seen by totally differentiating (3.45):

$$di_d = (\epsilon - 1)\frac{t'(i_d)}{t''(i_d)}\frac{dz^*}{z^*} \tag{3.46}$$

Plugging in (3.44) yields:

$$di_d = (1 - \epsilon)\frac{t'(i_d)}{t''(i_d)}\frac{X}{S}\frac{d\tau}{\tau} \tag{3.47}$$

- Part 3: Change in optimal investment of exporting firms

For exporting firms, the first order condition for optimal investment rearranged with (3.14) is:

$$(1 + \tau^{1-\epsilon})\left(\frac{z}{z^*}\right)^{\epsilon-1}(i^* + f)\frac{t'(i_x)}{t(i_x)} = 1 \tag{3.48}$$

Totally differentiating the above equation and rearranging:

$$di_x = (\epsilon - 1)\left[+\frac{\tau^{1-\epsilon}}{1 + \tau^{1-\epsilon}}\frac{d\tau}{\tau} + \frac{dz^*}{z^*}\right]\frac{t'(i_x)}{t''(i_x)} \tag{3.49}$$

The square bracket shows the two effects of trade liberalisation: the first term is the direct effect that exporting firms can sell more on their export market, the second term is the indirect general equilibrium effect of a smaller price index, which makes it more difficult for these firms to sell.

Plugging in (3.44):

$$di_x = (\epsilon - 1)\frac{d\tau}{\tau}\left(-\frac{X}{S} + \frac{\tau^{1-\epsilon}}{1 + \tau^{1-\epsilon}}\right)\frac{t'(i_x)}{t''(i_x)} \tag{3.50}$$

As long as some firms do not export, $\frac{X}{S} < \frac{\tau^{1-\epsilon}}{1+\tau^{1-\epsilon}}$. This shows that the direct effect of a change in τ dominates the general equilibrium effect for the total sales of exporters (the ratio of total exports to total sales) and trade liberalisation therefore raises the investment of exporting firms.

Proof of Proposition 3.5

To highlight the general equilibrium effects of a change in trade costs on the export cutoff level, I rewrite (3.23) using (3.14):

$$\frac{(f + i^*)}{t(i^*)}\left(\frac{z_x^*}{z^*}\right)^{\epsilon-1}\left[(1 + \tau^{1-\epsilon})t(i_x(z_x^*)) - t(i_d(z_x^*))\right] - i_x(z_x^*) - f_x + i_d(z_x^*) = 0 \tag{3.51}$$

Totally differentiating the above equation and using the envelope theorem yields:

$$\frac{dz_x^*}{z_x^*} - \frac{dz^*}{z^*} - \frac{d\tau}{\tau}\frac{\tau^{1-\epsilon}t(i_x(z_x^*))}{(1 + \tau^{1-\epsilon})t(i_x(z_x^*)) - t(i_d(z_x^*))} = 0 \tag{3.52}$$

Using (3.44), this can be rewritten as:

$$\frac{dz_x^*}{z_x^*} = \frac{d\tau}{\tau}\left[\frac{\tau^{1-\epsilon}t(i_x(z_x^*))}{(1 + \tau^{1-\epsilon})t(i_x(z_x^*)) - t(i_d(z_x^*))} - \frac{X}{S}\right] \tag{3.53}$$

The first term in the square bracket above is larger than $\frac{\tau^{1-\epsilon}}{1+\tau^{1-\epsilon}}$, which is larger than $\frac{X}{S}$. The square bracket and the right hand side are therefore positive and trade liberalisation decreases the export cutoff level z_x^*.

Proof of Lemma 3.1

$e'(i)$ is equal to:

$$e'(i) = \frac{t'(i)}{t(i)} + i\frac{t''(i)t(i) - t'(i)^2}{t(i)^2} = \left(\frac{t'(i)}{t(i)}\right)^2 \left[\frac{t(i)}{t'(i)} + i\frac{t''(i)t(i)}{t'(i)^2} - i\right] \tag{3.54}$$

$e'(i)$ has therefore the sign of the square bracket on the right hand side. From Assumptions 3.1 and 3.2 (and using l'Hopital's rule to show that $\frac{t''(i)t(i)}{t'(i)^2}$ is bounded as $i \to 0$), the square bracket is equal to zero if $i \to 0$. Furthermore:

$$\frac{\partial\left[\frac{t(i)}{t'(i)} + i\frac{t''(i)t(i)}{t'(i)^2} - i\right]}{\partial i} = i\frac{\partial\frac{t''(i)t(i)}{t'(i)^2}}{\partial i} \propto iE'(i) \tag{3.55}$$

$e'(i)$ and $E'(i)$ therefore have the same sign.

Q.E.D.

Proof of Proposition 3.7

The aggregate R&D intensity in the economy is defined as the ratio of innovation spending by all firms in a sector to total sales in that sector:

$$R \equiv \frac{\int_{z^*}^{z_x^*} i_d(z)dG(z) + \int_{z_x^*}^{\infty} i_x(z)dG(z)}{\int_{z^*}^{z_x^*} \epsilon\left(\frac{z}{z^*}\right)^{\epsilon-1}\frac{t(i_d(z))}{t(i^*)}(i^*+f)dG(z) + \int_{z_x^*}^{\infty} \epsilon(1+\tau^{1-\epsilon})\left(\frac{z}{z^*}\right)^{\epsilon-1}\frac{t(i_x(z))}{t(i^*)}(i^*+f)dG(z)} \tag{3.56}$$

Totally differentiating the above equation requires to split up the problem in order to keep track of the economic intuition. In order to simplify notation, I denote the numerator of the above equation as u and the denominator as v. The total differentiation of the numerator and of the denominator can be decomposed into two parts: (i) Effect B represents the impact of the cutoff firms (z^* and z_x^*), which switch status (domestic or exporting) following trade liberalisation. They correspond to the differentiation of the boundaries of the integrals in (3.28) and their effect will be summarised by the term B_u for the numerator and B_v for the denominator. (ii) Effect C stands for the impact of the firms not switching status, of which the effect will be denoted C_u and C_v respectively for the numerator and denominator. Therefore:

$$du = B_u + C_u \tag{3.57}$$

$$dv = B_v + C_v \tag{3.58}$$

and:

$$d\left(\frac{u}{v}\right) = \frac{1}{v}\left(\underbrace{B_u - B_v\frac{u}{v}}_{\text{Effect } B} + \underbrace{C_u - C_v\frac{u}{v}}_{\text{Effect } C}\right) \tag{3.59}$$

The first part of the bracket can therefore be interpreted as the effect of the cutoff firms changing status (the extensive margin) while the second part is the change in the intensive margin of investment intensity for all non-cutoff firms.

- Effect C

I first turn to effect C, which is the effect this paper concentrates on. To show its effect on R, I proceed in two steps. In a first step, I show that the sign of effect C is given by the sign of a change in the numerator of (3.56). In a second step, I determine whether the numerator increases or decreases with trade liberalisation.

- First step

Total differentiation of (3.56) using (3.47) and (3.50) yields:

$$C_u = (1-\epsilon)\frac{d\tau}{\tau}\left[\int_{z^*}^{z_x^*}\frac{t'(i_d(z))}{t''(i_d(z))}dG(z)\frac{X}{S} + \int_{z_x^*}^{\infty}\frac{t'(i_x(z))}{t''(i_x(z))}dG(z)\left(\frac{X}{S} - \frac{\tau^{1-\epsilon}}{1+\tau^{1-\epsilon}}\right)\right] \tag{3.60}$$

and, using (3.10) and (3.20):

$$\begin{aligned} C_v &= \epsilon(1-\epsilon)\frac{d\tau}{\tau}\left[\int_{z^*}^{z_x^*}\frac{t'(i_d(z))}{t''(i_d(z))}dG(z)\frac{X}{S} + \int_{z_x^*}^{\infty}\frac{t'(i_x(z))}{t''(i_x(z))}dG(z)\left(\frac{X}{S} - \frac{\tau^{1-\epsilon}}{1+\tau^{1-\epsilon}}\right)\right] \\ &+ v(1-\epsilon)\left(\frac{d\tau}{\tau}\frac{X}{S} + \frac{dz^*}{z^*}\right) \end{aligned} \tag{3.61}$$

The first line on the right hand side of C_v captures the effect on aggregate sales of the change in innovation by all producing firms. The second line reflects the between firm reallocation of sales due to a drop in trade costs, which, for a constant innovation level, raises global sales of exporting firms and reduces those of purely domestic firms. The net effect of this reallocation is however zero as can be seen by setting $\frac{dz^*}{z^*}$ equal to its value in (3.44).

Effect C can therefore be written as:

$$\frac{d\tau}{\tau}(1-\epsilon)\left(1-\epsilon\frac{u}{v}\right)\left[\int_{z^*}^{z_x^*}\frac{t'(i_d(z))}{t''(i_d(z))}dG(z)\frac{X}{S} + \int_{z_x^*}^{\infty}\frac{t'(i_x(z))}{t''(i_x(z))}dG(z)\left(\frac{X}{S} - \frac{\tau^{1-\epsilon}}{1+\tau^{1-\epsilon}}\right)\right] \tag{3.62}$$

$1-\epsilon\frac{u}{v}$ is positive since all firms make non-negative profits. Indeed, $\frac{v}{\epsilon}$ is the average variable profit of a firm, which is larger than average innovation spending (u) in order to ensure that fixed costs can still be paid without making negative profits. The sign of (3.62) is therefore given by the sign of C_u, which is minus the sign of the square bracket in (3.62).

- Second step

From (3.12) and (3.21), the ratio of exports to sales can be rewritten as:

$$\frac{X}{S} = \frac{\tau^{1-\epsilon}}{1+\tau^{1-\epsilon}}\frac{\int_{z_x^*}^{Z}\frac{t(i_x(z))}{t'(i_x(z))}dG(z)}{\int_{z^*}^{z_x^*}\frac{t(i_d(z))}{t'(i_d(z))}dG(z) + \int_{z_x^*}^{Z}\frac{t(i_x(z))}{t'(i_x(z))}dG(z)} \tag{3.63}$$

Using this expression, the square bracket in (3.62) is equal to:

$$\frac{\tau^{1-\epsilon}}{1+\tau^{1-\epsilon}}\xi\left(\frac{\int_{z^*}^{z_x^*}\frac{t'(i_d(z))}{t''(i_d(z))}dG(z)}{\int_{z^*}^{z_x^*}\frac{t(i_d(z))}{t'(i_d(z))}dG(z)} - \frac{\int_{z_x^*}^{\infty}\frac{t'(i_x(z))}{t''(i_x(z))}dG(z)}{\int_{z_x^*}^{\infty}\frac{t(i_x(z))}{t'(i_x(z))}dG(z)}\right) \tag{3.64}$$

where

$$\xi = \frac{\int_{z_x^*}^{\infty} \frac{t(i_x(z))}{t'(i_x(z))} dG(z) \int_{z^*}^{z_x^*} \frac{t(i_d(z))}{t'(i_d(z))} dG(z)}{\int_{z^*}^{z_x^*} \frac{t(i_d(z))}{t'(i_d(z))} dG(z) + \int_{z_x^*}^{\infty} \frac{t(i_x(z))}{t'(i_x(z))} dG(z)} > 0 \tag{3.65}$$

Define:

$$\frac{t(i_d(z))}{t'(i_d(z))} dG(z) \equiv d\mu(z) \tag{3.66}$$

$$\frac{t(i_x(z))}{t'(i_x(z))} dG(z) \equiv d\eta(z) \tag{3.67}$$

Using the definition of $E(i)$ and changing the measure of the integral with (3.66) and (3.67):

$$\frac{\int_{z^*}^{z_x^*} \frac{t'(i_d(z))}{t''(i_d(z))} dG(z)}{\int_{z^*}^{z_x^*} \frac{t(i_d(z))}{t'(i_d(z))} dG(z)} = -\frac{\int_{z^*}^{z_x^*} E(i_d(z)) d\mu(z)}{\int_{z^*}^{z_x^*} d\mu(z)} \tag{3.68}$$

$$\frac{\int_{z_x^*}^{\infty} \frac{t'(i_x(z))}{t''(i_x(z))} dG(z)}{\int_{z_x^*}^{\infty} \frac{t(i_x(z))}{t'(i_x(z))} dG(z)} = -\frac{\int_{z_x^*}^{\infty} E(i_x(z)) d\eta(z)}{\int_{z_x^*}^{\infty} d\eta(z)} \tag{3.69}$$

Using this in (3.64) immediately shows that if $E'(i) = 0$, the whole term in (3.64) is zero and Effect C has no impact on the aggregate innovation intensity. I now expose the argument for the case: $e'(i) > 0$, which by Lemma 3.1 implies $E'(i) > 0$. In this case (remember that $i_d(z)$ and $i_x(z)$ are both strictly increasing in z):

$$\frac{\int_{z^*}^{z_x^*} E(i_d(z)) d\mu(z)}{\int_{z^*}^{z_x^*} d\mu(z)} < E(i_d(z_x^*)) \tag{3.70}$$

$$\frac{\int_{z_x^*}^{\infty} E(i_x(z)) d\eta(z)}{\int_{z_x^*}^{\infty} d\eta(z)} > E(i_x(z_x^*)) \tag{3.71}$$

Moreover, if $E'(i) > 0$, $E(i_x(z_x^*)) > E(i_d(z_x^*))$. Therefore, if $e'(i) > 0$, (3.64) is positive and Effect C raises the aggregate R&D intensity. If $e'(i) < 0$ on the other hand, all inequality signs above are reversed and the intensive margin reduces aggregate innovation intensity.

This proves Proposition 3.7.

- Effect B (cutoff firms)

For completeness, I now turn to Effect B, i.e. the impact of the cutoff domestic firms stopping production and investment, and of cutoff exporting firms starting to export and raising their innovation investment.

$$B_u = -\frac{dz^*}{z^*} i^* g(z^*) z^* + \frac{dz_x^*}{z_x^*} (i_d(z_x^*) - i_x(z_x^*)) g(z_x^*) z_x^* \tag{3.72}$$

$$\begin{aligned} B_v = & -\frac{dz_x^*}{z_x^*} \left(\frac{z_x^*}{z^*}\right)^{\epsilon-1} z_x^* g(z_x^*) \epsilon \frac{(i^* + f)}{t(i^*)} \left[(1 + \tau^{1-\epsilon}) t(i_x(z_x^*)) - t(i_d(z_x^*))\right] \\ & - \frac{dz^*}{z^*} g(z^*) z^* \epsilon (i^* + f) \end{aligned} \tag{3.73}$$

where $\frac{dz^*}{z^*}$ and $\frac{dz_x^*}{z_x^*}$ are given by (3.44) and (3.53).

The whole Effect B is therefore given by:

$$\begin{aligned} B_u - B_v \frac{u}{v} &= -\frac{dz^*}{z^*} g(z^*) z^* \left(i^* \left(1 - \epsilon \frac{u}{v}\right) - f \epsilon \frac{u}{v} \right) \\ &+ \frac{dz_x^*}{z_x^*} g(z_x^*) z_x^* \left((i_x(z_x^*) - i_d(z_x^*)) \left(\frac{u}{v}\epsilon - 1 \right) + f_x \epsilon \frac{u}{v} \right) \end{aligned} \tag{3.74}$$

The sign of the above expression is undetermined and depends among others on the density function. The first line of the right hand side represents the effect of the domestic cutoff firms dropping out of the market. They tend to decrease aggregate investment as well as aggregate sales and therefore have an ambiguous effect for the innovation intensity. The second line is the effect of new firms entering the export market. They raise their spending on innovation as well as their sales, and therefore also have an ambiguous effect on the aggregate innovation intensity.

Proof of Proposition 3.8

I first concentrate on purely domestic firms. The Tobin's Q of a domestic firm investing i is given by (3.31):

$$T_d(i) = \frac{1}{e(i) + f \frac{t'(i)}{t(i)}} \tag{3.75}$$

Therefore, $T_d'(i)$ is of the sign of:

$$T_d'(i) \propto -\left(e'(i) - f \left(\frac{t'(i)}{t(i)} \right)^2 \left(1 + \frac{1}{E(i)} \right) \right) \tag{3.76}$$

The second term in bracket is always negative so that if $e'(i) \leq 0$, $T_d'(i)$ is positive. Using that:

$$e'(i) = \frac{t'(i)}{t(i)} + i \left(\frac{t'(i)}{t(i)} \right)^2 \left(1 + \frac{1}{E(i)} \right) \tag{3.77}$$

$T_d'(i)$ can be rewritten as:

$$T_d'(i) = -\left(\frac{t'(i)}{t(i)} - (i + f) \left(\frac{t'(i)}{t(i)} \right)^2 \left(1 + \frac{1}{E(i)} \right) \right) \tag{3.78}$$

so that $\lim_{i\to\infty} e(i) = \lim_{i\to\infty} T_d'(i)$. For the case that $e'(i) > 0$, this shows that for a sufficiently high i, $T_d'(i) < 0$. The Tobin's Q then decreases with investment (and therefore size). All results up to this point can be straightforwardly extended to the case of exporting firms by replacing f by $f + f_x$. For $i = i^*$, which is the smallest producing firm, it is immediate from (3.13) that:

$$T_d'(i^*) = \frac{t'(i^*)}{t(i^*)} \frac{1}{E(i^*)} > 0 \tag{3.79}$$

The Tobin's Q is therefore increasing in i, and in size, for the smallest producing firms. From the indifference condition of the cutoff exporting firm (3.23):

$$\frac{t(i_x(z_x^*))}{t' i_x(z)} - i_x(z_x^*) - f_x - f = \frac{t(i_d(z_x^*))}{t'(i_d(z_x^*))} - f - i_d(z_x^*) \tag{3.80}$$

Using the definition of the Tobin's Q in (3.31) and (3.32), this can be rewritten as:

$$\frac{T_x(z_x^*) - 1}{T_d(z_x^*)} = \frac{i_d(z_x^*) + f}{i_x(z_x^*) + f + f_x} \tag{3.81}$$

Since the Tobin's Q is larger than one for the cutoff exporting firm, it immediately follows that $T_x(z_x^*) < T_d(z_x^*)$. This shows that the smallest exporting firm has a lower Tobin's Q than the largest non-exporting firms.

Chapter 4

Firm-level volatility and trade openness

4.1 Introduction

The opinion that trade acts as a source of economic volatility seems to gain importance in the popular debate with each recession. Politicians periodically face - or trigger - demands for increased protectionism to shield domestic consumers and firms from foreign shocks. The importance of the trade volatility relationship for the political debate has been matched in the last decade by a growing scientific interest for the question. A large literature, starting with Rodrik (1998), has examined this link empirically. Most studies find that openness to trade indeed raises macroeconomic volatility (Easterly et al. (2001), Kose et al. (2003)), though this result has been qualified by others (Bejan (2006), Cavallo (2007))[1]. However, virtually no work investigates the precise transmission mechanisms of volatility between markets, which largely remain a blackbox. One reason is that the standard approach has taken a purely macroeconomic perspective, leaving aside more disaggregated levels of analysis, in particular firm behaviour. The relationship between trade openness and volatility at the firm level remains largely unexplored, though it may provide a better understanding of the mechanisms at stake.

The main contribution of the present paper is to address these problems by taking two innovative steps.

In the first step, I build a model of trade with demand uncertainty to examine the impact of market-specific demand shocks on the production and export volumes of firms. This

[1]This literature also characterises the effect of other dimensions of globalisation, such as capital market liberalisation, on output volatility. The focus of the present paper is exclusively on trade in goods. For the effect of financial development on the volatility of firms, see for example Thesmar and Thoenig (2005).

allows me to shed light on the firm-level transmission mechanisms of volatility between markets. The model builds on a two-country monopolistic competition framework with heterogeneous firms, but departs from the literature in two important ways. First, firms are heterogeneous with respect to their openness level, defined as the ratio of exports to global sales. It is a consequence of the different and uncorrelated tastes for a firm's product on its domestic and foreign markets. This type of heterogeneity, which is empirically relevant, cannot be replicated by standard heterogeneous firms models of trade. Second, firms produce with convex costs in the short run and face idiosyncratic demand shocks on the domestic and foreign markets. These shocks are independent and induce exporting firms to substitute sales between the domestic and the export market in the short run. For example, a firm facing a positive demand shock on its export market and no change in demand on the domestic market will reduce its domestic sales in order to raise its exports. This mechanism provides a channel through which foreign shocks impact the volatility of a firm's domestic sales. These two features of the model yield a number of sharp predictions for the link between the openness level of a firm and the variance of its sales on different markets.

The second step is to test the predictions of the model empirically and to examine the link between volatility - defined as the variance of the growth rate of sales - and openness at the firm level. I use the Amadeus dataset, which provides comprehensive firm-level balance sheet and export information for French firms between 1997 and 2006.

The main conclusions of the paper are threefold. First, exporting firms substitute sales in the short run between their domestic and export markets. I show theoretically and empirically that a higher than average sales growth in one market is associated with a lower than average growth in the other. Second, exporters with a high openness level have in equilibrium more volatile domestic sales and less volatile exports than exporters with a low openness level. It is a direct consequence of the market substitution highlighted above: demand shocks induce a sales substitution which is proportionally small for the larger market and large for the smaller market. The empirical analysis confirms the quantitative importance of this effect. Third, I show that exporting firms with an openness level above 25% are more volatile than comparable non-exporters. Those with a lower openness level are on the other hand less volatile than comparable non-exporters. This last observation is in line with a standard diversification argument according to which selling to uncorrelated markets reduces volatility.

The present model is related to different strands of the literature.

From the modeling perspective, it borrows from the recent and growing literature on heterogeneous firms in trade, in the line of Melitz (2003). An important difference with most of the existing papers is that firm heterogeneity does not come from different productivity levels but from the taste parameters for varieties in the utility function, as in Crozet et al. (2007). This modeling strategy allows exporting firms to have different levels of openness, as long as they receive two uncorrelated taste parameters for their product in the domestic and export market.

The relationship between openness and volatility has been extensively studied in the last decade. Most studies investigate it at the country level using cross-section or panel data and find that trade and financial openness are positively correlated with the volatility of output growth (Rodrik (1998), Easterly et al. (2001), Kose et al. (2003)). This result has however been qualified by a number of works: Bejan (2006) shows that the correlation depends on the level of development of countries, Bekaert et al. (2006) find that financial liberalisation reduces consumption volatility, and Cavallo (2007) argues that trade openness reduces output volatility due to its dampening effect on financial volatility. di Giovanni and Levchenko (2009) interestingly go one step further and distinguish the effects of trade openness on sector level volatility, on the comovement between sectors and on country specialisation. They find that the overall effect of openness is to increase output volatility.

A number of recent works, such as Comin and Philippon (2005), Davis et al. (2006) and Buch et al. (2008), examine the evolution of firm-level output volatility over time and provide interesting insights on the factors influencing it. They however do not relate firm volatility to trade openness. Buch et al. (2006) is to my knowledge the only paper studying the link between openness and output volatility at the firm level. Using two German firm-level datasets, they point to a rather weak relationship between the two, and suggest that the sales of exporting firms may be less volatile than those of non-exporters. However, they cannot address the predictions of the present paper, since they assume that firms ship a constant exogenous share of their output to each market. Their analysis moreover suffers from data limitations, as their Germany-wide dataset does not include information on openness at the firm level.

I develop the model in section 4.2 and highlight its predictions for firm-level growth and volatility in section 4.3. I describe the data in section 4.4 and test the empirical predictions of the model in section 4.5. Section 4.6 addresses some robustness issues and section 4.7 concludes.

4.2 The Model

4.2.1 Demand

The world consists of two countries, Home (H) and Foreign (F), and many sectors. Time, indexed by t, is discrete and infinite. The analysis in the present paper concentrates on a given sector S, leaving other industries in the background. The representative consumer in country i derives a per-period subutility for sector S, which consists of a continuum of varieties, given by[2]:

$$u_{it} = \left[\int_{\omega \in \Omega} \beta_{it}(\omega)\chi_i(\omega)(q(\omega))^{\frac{\epsilon-1}{\epsilon}} d\omega \right]^{\frac{\epsilon}{\epsilon-1}} \quad \text{for } i \in \{H, F\} \tag{4.1}$$

where the measure of the set Ω is the mass of available varieties in sector S, $q(\omega)$ is the consumption level of variety ω and ϵ is the elasticity of substitution between varieties, which is assumed to be larger than one. Consumer preferences exhibit the love of variety property following Dixit and Stiglitz (1977). The taste of consumers for a given variety depends on two parameters. The first, χ_i, is time invariant and variety specific. I assume for simplicity that the distribution of χ_i across varieties has support $(0, \infty)$, and that $\chi_H(\omega)$ and $\chi_F(\omega)$ are uncorrelated. The fact that consumers in a country have a strong taste for a variety is not informative about the taste for this variety in the other country. The second parameter, β_{it}, is a time varying idiosyncratic shock to the taste for each variety, and is independent of χ_H and χ_F. It is independently drawn in each period and in each country from a country specific distribution. There is therefore no autocorrelation in the β_{it} of a given variety, and no correlation between shocks in the two countries. The distribution of β_{it}, $G_i(\beta_{it})$, is assumed constant over time with support $[\underline{\beta}_i, \overline{\beta}_i]$, mean μ_i and variance σ_i^2. The independence assumptions between the different parameters and shocks are not crucial to the qualitative results of the model, but greatly simplify the analytical results[3].

Consumers in i spend in each period a fixed amount I_i on the consumption of varieties from sector S. They maximise their per period subutility u_{it} subject to the budget constraint:

$$\int_{\omega} p_{it}(\omega)q(\omega) = I_i \tag{4.2}$$

where $p_{it}(\omega)$ is the price of a variety ω in market i and at time t. The maximisation yields the demand for a variety ω in time t and market i:

$$q_{it}(\omega) = \beta_{it}(\omega)\chi_i(\omega)(p_{it}(\omega))^{-\epsilon} P_{it}^{\epsilon-1} I_i \tag{4.3}$$

where

$$P_{it} = \left[\int_{\omega \in \Omega} (\beta_{it}(\omega)\chi_i(\omega))^{\epsilon}(p_{it}(\omega))^{1-\epsilon} d\omega \right]^{\frac{1}{1-\epsilon}} \tag{4.4}$$

is the aggregate price of a composite good defined by $Q_{it} \equiv u_{it}$. The demand for a good is an increasing function of its taste parameters and of the price index P_{it}, which summarises the prices of all available varieties in sector S at time t in country i. For simplicity, I assume at this stage that the price index is independent of time. I will show in section 4.2.3 that it is consistent with an equilibrium.

[2]Since the analysis concentrates only on sector S, I do not include a sectoral index.

[3]Most results and predictions would hold with partially correlated shocks.

I define $\eta_i \equiv P_i^{\epsilon-1} I_i$ as the common element to the demand of all varieties in sector S and market i. From (4.3), the expenditure of consumers in i at time t on variable ω is given by:

$$p_{it}(\omega)q_{it}(\omega) = (q_{it}(\omega))^{\frac{\epsilon-1}{\epsilon}}(\beta_{it}(\omega)\chi_i(\omega)\eta_i)^{\frac{1}{\epsilon}} \tag{4.5}$$

4.2.2 Firms

In each country, there is a continuum of firms in sector S, each choosing to produce a different variety ω. Production uses two factors, capital (k) and labour (l), which firms rent in each period at exogenous and constant prices r and w[4]. Firms produce with a Cobb Douglas production function combining capital and labour:

$$y = k^{\gamma} l^{1-\gamma} \tag{4.6}$$

Capital is assumed to be a fixed factor in the short run, in the sense that the stock of capital at t is decided upon at $t-1$, before the realisation of the time varying preference shocks. Labour, on the other hand, is fully flexible and can be immediately adapted.

In order to produce, firms have to pay a per-period fixed cost of production f, which generates increasing returns to scale in the long run. Whether these costs are paid in terms of labour or capital has no bearing on the results[5].

To simplify notation, define $\alpha \equiv \frac{1}{1-\gamma} > 1$. The short run cost function and the marginal costs of production are given by:

$$c(y) = rk + wy^{\alpha}k^{1-\alpha} + f \tag{4.7}$$

$$c'(y) = w\alpha y^{\alpha-1}k^{1-\alpha} \tag{4.8}$$

Due to the assumption of fixed capital in the short run, the short run costs of production are convex.

Each firm also has the possibility to export to the other country if it finds it profitable. For expositional clarity, I will from now on conduct the analysis from the point of view of the home country (H). A symmetric analysis can be performed for the foreign country. Exporting requires the payment of two types of costs. Iceberg costs τ_i are the number of goods that must be shipped in order for one unit of the good to arrive at destination in market i. It is equal to one for the domestic market ($\tau_H = 1$), reflecting that there are no costs of shipping goods within a country. Selling a good in the foreign market is on the other hand associated with costs of transportation or tariffs, which are summarised by

[4] r and w are set equal across countries for simplicity.

[5] To simplify the notation, assume that the fixed costs either require the use of $\frac{f}{r}$ units of capital or $\frac{f}{w}$ units of labour. The amount spent per period on fixed costs is therefore equal to f.

$\tau_F \equiv \tau \geq 1$. Additionally, exporting firms incur a fixed per period cost of exporting f_x, which reflects the additional costs of doing business abroad, of maintaining a distribution network, etc. I assume that the decision to pay the fixed costs f and f_x in period t has to be taken at $t-1$, before the realisation of the idiosyncratic time varying demand shocks β_{Ht} and β_{Ft}.

As usual in this type of models, I assume that firms face no financial constraints. Regardless of past profits, they can always finance capital and fixed costs as long as they expect non-negative profits for the next period[6]. The timing of the firm's problem can be summarised as follows. At time $t = 0$, firms learn their demand parameters χ_H and χ_F and the distribution of the time varying idiosyncratic shocks β_{Ht} and β_{Ft}. Firms then maximise their per period profits from period $t = 1$ onwards. The maximisation of period t profits proceeds in three steps. (i) at $t-1$, firms decide on which markets (Home and/or Foreign or none) they will be active in period t. They therefore choose at $t-1$ whether to pay f and f_x at time t. (ii) at $t-1$ and given the choice of market participation for the next period, firms choose their capital level for period t. (iii) at t, after the realisation of the time varying demand shocks, firms choose how much to produce by deciding the amount of labour to use given the capital level and the market participation strategy.

The market participation strategy of a firm at time t can be: (i) not to produce (strategy N) (ii) to sell only on the domestic market (strategy H) (iii) to sell on both markets (stategy G). For simplicity, I will denote as f_j, $j \in \{N, H, G\}$ the total fixed costs associated to a given stategy, i.e. $f_N = 0$ $f_H = f$ and $f_G = f + f_x$. Note that no firm has an interest in selling only on the export market. If it exports, a firm needs to pay both the fixed costs of production (f) and the fixed costs of export (f_x). It therefore needs to pay no additional fixed cost to sell on the Home market. As long as $\chi_H \beta_{Ht} > 0$ however, it can sell from (4.3) an infinitesimal amount on the Home market at an infinite price, and will therefore find it optimal to do so. This shows that no firm exports without selling on the domestic market. I now solve the firm's problem by backward induction.

I first derive the optimal level of sales and profits taking the capital level (k) and market participation as given. For simplicity, I define the vector $\varphi_t = (\beta_{Ht}, \beta_{Ft}, \chi_H, \chi_F)$, which summarises the four parameters governing firm heterogeneity in the model. If a firm decides to produce only for market H, it makes profits equal to the consumer's expenditure on its good in market H minus its costs of production. From (4.5), the profits of a firm depends on its draws of demand parameters χ_H and β_{Ht}, and is given by

$$\pi_{Ht}(\varphi_t, k) = q_{Ht}^{\frac{\epsilon-1}{\epsilon}} (\beta_{Ht}\chi_H\eta_H)^{\frac{1}{\epsilon}} - c(q_{Ht}) - f \tag{4.9}$$

[6]All profits of a firm are paid to the representative consumer as dividends, and all losses are covered by the consumer. Firms rent the factors of production from the representative consumer in each period and face no intertemporal constraint.

Maximising profits with respect to q_{Ht} and rearranging using that sales are equal to expenditure for each product yields the optimal sales level:

$$s_H(\varphi_t, k) = \left(\frac{A}{w}\right)^{\frac{C}{\alpha-1}} k^C (\eta_H \chi_H \beta_{Ht})^{1-C} \equiv k^C \xi_H(\varphi_t) \tag{4.10}$$

where $A \equiv \frac{\epsilon-1}{\epsilon\alpha}$ and $0 < C \equiv \frac{(\alpha-1)(\epsilon-1)}{\alpha\epsilon-\epsilon+1} < 1$. The optimal sales of a domestic firm are increasing in the home parameter η_H, which reflects better aggregate conditions, as well as in both taste parameters β_{Ht} and χ_H. Sales are also increasing in capital. Due to the convexity of the short run cost function, optimal sales are concave in χ_H and β_{Ht}.

Plugging the optimal sales in the profits equation (4.9) yields the maximised profits for a domestic firm given capital k:

$$\pi_H(\varphi_t, k) = (1 - A)k^C \xi_H(\varphi_t) - rk - f \tag{4.11}$$

A similar analysis can be conducted for exporting firms (firms with strategy G) with global sales s_{Gt}. Let q_{GHt} and q_{GFt} denote the quantities sold by an exporting firms on its domestic and export market. Similarly, let s_{GHt} and s_{GFt} be the sales (price times quantity sold) of exporting firms on their domestic and export market. The profits of an exporting firm consists of its sales on both markets minus the costs of production and of exporting. Setting sales equal to expenditure at the variety level yields, using (4.5), the profits of a firm with strategy G:

$$\begin{aligned} \pi_{Gt}(\varphi_t, k) \quad &= \quad q_{GHt}^{\frac{\epsilon-1}{\epsilon}} (\beta_{Ht}\chi_H\eta_H)^{\frac{1}{\epsilon}} + q_{GFt}^{\frac{\epsilon-1}{\epsilon}} (\beta_{Ft}\chi_F\eta_F)^{\frac{1}{\epsilon}} \\ &\quad -c(q_{GHt} + \tau q_{GFt}) - f - f_x \end{aligned} \tag{4.12}$$

Maximising with respect to q_{GHt} and q_{GFt} and rearranging the first order conditions using (4.5) yields the firm openness level (υ), which is the optimal ratio of its exports to global sales:

$$\upsilon(\varphi_t) = \frac{s_{GF}(\varphi_t)}{s_{Gt}(\varphi_t)} = \tau^{1-\epsilon} \frac{\beta_{Ft}\chi_F\eta_F}{\beta_{Ht}\chi_H\eta_H + \tau^{1-\epsilon}\beta_{Ft}\chi_F\eta_F} \tag{4.13}$$

The openness of a firm depends both on firm specific parameters and on factors common to all firms. The common factors are: (i) the iceberg transportation costs (τ), which reduce the relative size of exports (ii) the aggregate conditions on both markets, which are summarised in η_F and η_H. The higher the price index on the export market, and the higher the aggregate income on that market, the easier it is for all firms to export and the higher their openness. The firm speficic factors, on the other hand, account for the heterogeneity in openness levels between firms. The ratio of the time invariant taste parameters ($\frac{\chi_F}{\chi_H}$) determines the long run heterogeneity in openness between firms. The ratio of the time varying idiosyncratic shocks $\frac{\beta_{Ft}}{\beta_{Ht}}$ accounts for the time varying level of openness of a given firm.

The fact that exporters differ with respect to openness is empirically well established[7]. It can however not be accounted for by traditional models of heterogeneous firms with two countries and a single source of heterogeneity[8].

Plugging (4.13) back into the first order conditions and rearranging using (4.5) yields the optimal sales on market i ($s_{Gi}(\varphi_t, k)$) of a firm facing demand parameters φ_t and having capital level k, as well as its global sales $s_G(\varphi_t, k) = s_{GH}(\varphi_t, k) + s_{GF}(\varphi_t, k)$.

$$s_{Gi}(\varphi_t, k) = k^C \left(\frac{A}{w}\right)^{\frac{C}{\alpha-1}} \tau_i^{1-\epsilon}\beta_{it}\chi_i\eta_i \left[\beta_{Ht}\chi_H\eta_H + \tau^{1-\epsilon}\beta_{Ft}\chi_F\eta_F\right]^{-C} \quad (4.14)$$

$$s_G(\varphi_t, k) = k^C \left(\frac{A}{w}\right)^{\frac{C}{\alpha-1}} \left(\eta_H\beta_{Ht}\chi_H + \tau^{1-\epsilon}\eta_F\beta_{Ft}\chi_F\right)^{1-C} \equiv \xi_G(\varphi_t)k^C \quad (4.15)$$

The interpretation is in many ways similar to the domestic case in (4.10). The main difference is that the sales of an exporting firm on a given market depend on the value of its taste parameters in both markets. This is due to the convexity of the short run cost function, which creates a link between the two markets. For instance, if a firm receives a high taste shock on its export market in a given period, the resulting increase in optimal production raises its marginal costs, and therefore reduces its sales on the domestic market.

Plugging the optimal sales from (4.14), and the implied optimal quantities, in (4.12) gives the optimised profit level of an exporter given a level of capital k:

$$\pi_G(\varphi_t, k) = (1 - A)k^C\xi_G(\varphi_t) - rk - f - f_x \quad (4.16)$$

In the second step of backward induction, I derive the optimal capital level that firms use for production. Firms choose their capital level for t at $t-1$, i.e. without knowing the realisation of the idiosyncratic demand shocks β_{Ht} and β_{Ft}. They therefore maximise their expected profits given their market participation strategy $j \in \{H, G\}$[9] and their draw of χ_H and χ_F. Expected profits for a given capital level are given by:

$$\overline{\pi}_j(\chi_H, \chi_F, k) = \int_{\underline{\beta}_H}^{\overline{\beta}_H} \int_{\underline{\beta}_F}^{\overline{\beta}_F} \pi_j(\varphi_t, k) dG_F(\beta_{Ft}) dG_H(\beta_{Ht}) \quad (4.17)$$

where the function $\pi_j(\varphi_t, k)$ is as defined in (4.11) and (4.16) for strategies H and G respectively.

Since there are no deterministic changes from one period to another in the model, a firm faces at each time t the same problem with regard to the optimal choice of its capital

[7] Eaton et al. (2008) among others.

[8] With the exception of Arkolakis (2008) who uses a marketing cost argument to show that heterogeneously productive firms choose different levels of market penetration in foreign markets.

[9] If firms decide not to produce in the next period (strategy N), they choose a zero capital level.

for the next period. The maximisation problem with respect to capital is therefore time invariant. The optimal capital level k_j^* conditional on market participation strategy j maximises the expected profits in (4.17) and is given by:

$$k_j^*(\chi_H, \chi_F) = \left(\frac{C(1-A)}{r}\right)^{\frac{1}{1-C}} \left(\overline{\xi}_j(\chi_H, \chi_F)\right)^{\frac{1}{1-C}} \tag{4.18}$$

where, for $\xi_j(\varphi_t)$ defined in (4.10) and (4.15):

$$\overline{\xi}_j(\chi_H, \chi_F) = \int_{\underline{\beta}_H}^{\overline{\beta}_H} \int_{\underline{\beta}_F}^{\overline{\beta}_F} \xi_j(\varphi_t) dG_F(\beta_{Ft}) dG_H(\beta_{Ht}) \tag{4.19}$$

$\overline{\xi}_j(\chi_H, \chi_F)$ summarises all factor influencing the expected sales of a firm except for capital. Equation (4.18) shows that the larger the expected sales of a firm, the larger its optimal capital level, due to the Cobb Douglas production function. Conditional on χ_H and χ_F, a firm producing for both markets therefore uses more capital than if it is purely domestic.

Plugging the optimal level $k_j^*(\chi_H, \chi_F)$ as defined in (4.18) into the expected profits in (4.17) yields the maximised expected profits $\overline{\pi}_j^M(\chi_H, \chi_F)$ of strategy j:

$$\overline{\pi}_j^M(\chi_H, \chi_F) = (1-C)\left[(1-A)\left(\frac{C}{r}\right)^C\right]^{\frac{1}{1-C}} \left(\overline{\xi}_j(\chi_H, \chi_F)\right)^{\frac{1}{1-C}} - f_j \tag{4.20}$$

The expected profits of a strategy j are a linear function of the optimal level of capital. The higher the expected sales of a firm, the higher its capital level and the higher its expected profits.

In a third step, I determine the market participation strategy of a firm as a function of its draws of χ_H and χ_F. Since the market participation decision for period t is taken at $t-1$, firms base their decision on the comparison of the expected profits of the different strategies. The choice of strategy between H, N of G is therefore time invariant. A firm will prefer strategy j over strategy m if:

$$\overline{\pi}_j^M(\chi_H, \chi_F) > \overline{\pi}_m^M(\chi_H, \chi_F) \tag{4.21}$$

The timing of the model is such that the market participation strategy is time invariant, meaning that it abstracts from the entry and exit of firms, both on the domestic and on the export market. Though these may play an important role for the link between trade and aggregate volatility, they would complicate the analysis without adding much insight to the channel described in the present model. I therefore leave them for future research.

Comparing the three possible strategies for firms with different (χ_H, χ_F) combinations yields the following proposition:

Proposition 4.1 *For any $f_x > 0$ and τ bounded, there is a partitioning of producing firms between non-exporters and exporters.*

Proof. *See Appendix* ■

The condition for partitioning between exporters and non-exporters is weaker than the usual condition in models of heterogeneous firms. The only requirement is that the fixed costs of exports be strictly positive and that the iceberg transportation costs are not infinite. This is due to the two dimensional heterogeneity in the present model, which allows firms to face different taste parameters in their domestic and export markets. Some firms with a very large χ_F find the export strategy profitable even for large τ, while for $\tau = 1$ and very low f_x, some firms still find it profitable not to export if their χ_F is close to zero. The partitioning is illustrated in Figure 4.1, of which I give an intuitive account in the following[10].

In a first step, I examine under which condition a firm prefers to produce for the domestic market (strategy H) than not to produce (strategy N). I show in the appendix that there exists a χ_H^* such that firms with $\chi_H > \chi_H^*$ can profitably cover the fixed costs f by their domestic sales and do not choose strategy N, regardless of χ_F. For the same reason, no firm with a $\chi_H < \chi_H^*$ would choose strategy H.

In a second step, I determine the conditions under which a firm is willing to export (strategy G). I show that for any χ_H, there will be a level of χ_F above which firms choose strategy G. Due to the link between the two markets at the firm level, this level of χ_F is a function of χ_H, which I denote as $\tilde{\tilde{\chi}}_F(\chi_H)$[11]. I show in the appendix that the higher the demand on its domestic market, the more profitable a firm finds it to export, i.e. $\tilde{\tilde{\chi}}'_F(\chi_H) < 0$. The argument for this can be separated into two parts.

First, for $\chi_H < \chi_H^*$, strategy N is preferable to H. A firm therefore exports if it finds it more profitable than not to produce. Since an exporting firm also sells on the domestic market, the larger its χ_H the lower the level of χ_F necessary to cover the fixed costs f_G and to choose strategy G. This shows that $\tilde{\tilde{\chi}}'_F(\chi_H) < 0$ for $\chi_H < \chi_H^*$.

Second, if $\chi_H \geq \chi_H^*$, a firm exports if it is more profitable than strategy H. The proof that that $\tilde{\tilde{\chi}}'_F(\chi_H) < 0$ is in this case done by showing that for a given χ_F, the profits of the G strategy increase in χ_H more rapidly than those of the H strategy. The intuition is as follows. Let two firms have identical χ_F but different χ_H. The firm with the larger χ_H employs more capital than the other as it expects to sell more. The difference in the capital levels of the two firms is proportional to the difference in their expected sales. In order to understand by how much a higher χ_H translates into higher expected sales,

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[10]The following argument would hold if the support of the distributions of χ_H and χ_F were not R^+ but a given interval in the positive reals. It would however require to make sure that the lower and upper bound of these supports are not too low or too high compared to the parameters of the model.

[11]$\tilde{\chi}_F(\chi_H)$ represents the border of the set G in figure 4.1.

it is essential to note that the sales of a firm are concave in the realisation of the shock as shown in (4.10) and (4.14). This is not driven by a risk aversion of firms but by the convex short run costs of production. If sales are very volatile, a firm with a larger χ_H therefore has only moderately higher expected sales, and invests only moderately more in capital. If sales are less volatile, on the other hand, the difference in expected sales, capital and profits between the two firms is larger. Since shocks on the Home and Foreign markets are uncorrelated, exporters benefit from a diversification effect which makes their sales relatively less volatile than those of non-exporters. Due to this volatility dampening effect , if the two firms export, the difference between their profits is larger than if the two firms do no export, so that the firm with a larger χ_H finds it relatively more profitable to export than the other firm. This concludes the proof that $\tilde{\tilde{\chi}}'_F(\chi_H) < 0$.

4.2.3 The Price index

The price index in country i builds a weighted average of the prices charged by all firms selling in this country. The price charged by a firm on its domestic and export markets can be derived by combining the optimal quantities sold on each market, the demand equation (4.3), which links quantity and price, and the optimal capital decision (4.18).

A non-exporting firm (with strategy H) charges a price:

$$p_H(\varphi_t) = \Lambda \left(\frac{\beta_{Ht}^{1-C}}{\int_{\underline{\beta}_H}^{\overline{\beta}_H} \beta^{1-C} dG_H(\beta)} \right)^{\frac{\alpha-1}{\alpha}} \tag{4.22}$$

where $\Lambda \equiv \left(\frac{A}{w}\right)^{-\frac{1}{\alpha}} \left(\frac{C(1-A)}{r}\right)^{\frac{1-\alpha}{\alpha}}$. Using the definitions of α, A and C shows that Λ is a constant markup over the long run marginal costs of production, and is the price that domestic firms would charge if there was no uncertainty or if they could adapt their capital stock in the short run. If the realisation of β_{Ht} is such that the optimal domestic sales of a firm are equal to their expected value, a firm charges a price Λ. If it has a larger than expected realisation of β_{Ht}, a firm sells more than planned, which raises its marginal costs of production and therefore its price due to the convex short run costs. The reverse holds for a lower than expected β_{Ht}.

Similarly, the price charged by an exporting firm on its domestic and export markets are:

$$p_{GH}(\varphi_t) = \Lambda \left(\frac{(\eta_H \beta_{Ht} \chi_H + \tau^{1-\epsilon} \beta_{Ft} \chi_F \eta_F)^{1-C}}{\int_{\underline{\beta}_H}^{\overline{\beta}_H} \int_{\underline{\beta}_F}^{\overline{\beta}_F} (\eta_H \beta_1 \chi_H + \tau^{1-\epsilon} \eta_F \beta_2 \chi_F)^{1-C} dG_F(\beta_2) dG_H(\beta_1)} \right)^{\frac{\alpha-1}{\alpha}} \tag{4.23}$$

$$p_{GF}(\varphi_t) = \tau p_{GH}(\varphi_t) \tag{4.24}$$

The interpretation is similar to the case of domestic firms with the main difference that shocks on both markets affect the price charged on each market. For instance, a high demand shock on the export market, by increasing the short run level of production of a firm, raises its marginal costs and therefore its prices on both markets. The price charged by a firm on its export market ($p_{GF}(\varphi_t)$) is higher than its domestic price by a factor τ, which reflects the additional costs of selling abroad.

The price index in country i builds a weighted average of the prices of all varieties available in i. These can be divided into: (i) the prices of Home firms with strategy H (ii) the prices that Home firms with strategy G charge on their domestic market (iii) the prices of varieties imported from country F. The price index integrates these prices, weighted as shown in (4.4), over β_{Ht}, β_{Ft}, χ_H and χ_F, which are the four sources of heterogeneity between firms. The integration over χ_H and χ_F accounts for the areas in figure 4.1[12] and assigns to each (χ_H, χ_F) firm its equilibrium strategy, thereby capturing which firms charge p_{Ht} and which firm p_{GHt}, as well as which foreign firms export to H.

The only time varying element in the price of each firm is the realisation of its shocks β_{Ht} and β_{Ft}. Furthermore, the market participation strategy, which defines the different areas in figure 4.1, is independent of time. Since β_{Ht} and β_{Ft} are i.i.d. and since their distribution is constant over time, the price index in each country, which builds the expectation over these shocks, is time invariant[13].

A potential issue about the definition of the price index is that the price charged by exporting firms on their domestic and export markets, ($p_{GH}(\varphi_t)$ and $p_{GF}(\varphi_t)$) themselves depend on the price indices P_H and P_F[14]. Similarly, the indifference schedules defining the optimal strategy of firms depend on both price indices[15]. The price index of country i is therefore defined as a function of the form:

$$P_i = \Phi_i(P_H, P_F) \tag{4.25}$$

I show in Appendix C that if the support of the distribution of β_{Ht} and β_{Ft} ($\underline{\beta}_H$, $\overline{\beta}_H$, $\underline{\beta}_F$ and $\overline{\beta}_F$) are bounded away from 0 and ∞, there exists at least one combination of P_H and P_F which is consistent with this definition.

4.3 Empirical Predictions

In this section, I derive three testable predictions of the above model about the link between exports and domestic sales at the firm level, knowing that the price index is

[12]These are formally defined in the Proof of Proposition 4.1 in the Appendix.
[13]Under the caveats expressed in Judd (1985) and Feldman and Gilles (1985).
[14]See (4.23) and (4.24) where $\eta_i = P_i^{\epsilon-1} I_i$.
[15]See the proof of Propostion 4.1 in Appendix.

constant as established in section 4.2.3[16]. I define the sales volatility of a firm on a given market as the variance of the growth rate of its sales on that market. I will denote the percentage change in the sales of a firm between two periods as 'growth rate' of sales, although there is no long-term growth due to the stationary structure of the model. 'Growth rate' and 'rate of change' can be used interchangeably.

4.3.1 Non-exporters

The growth rate of sales of a non exporter with successive parameters φ_{t-1} and φ_t can be directly derived from (4.10):

$$g_{Ht}(\varphi_t, \varphi_{t-1}) = \frac{s_{H(\varphi_t)}}{s_{H(\varphi_{t-1})}} - 1 = \left(\frac{k^*_{Ht}}{k^*_{Ht-1}}\right)^C \left(\frac{\beta_{Ht}}{\beta_{Ht-1}}\right)^{1-C} - 1 \quad (4.26)$$

For simplicity, I conduct the analysis using the approximation $log(1+\lambda) \approx \lambda$ for λ small:

$$g_{Ht}(\varphi_t, \varphi_{t-1}) \approx (1-C)\left(log(\beta_{Ht}) - log(\beta_{Ht-1})\right) + C(log(k^*_{Ht}) - log(k^*_{Ht-1})) \quad (4.27)$$

As shown earlier in the analysis, the optimal capital stock of a firm is constant over time, so that the second part of the right hand side term above drops out. As β_H is i.i.d. over time, I approximate the variance[17] of the growth rate of domestic sales by:

$$VAR(g_{Ht}) \approx 2(1-C)^2 \frac{\sigma_H^2}{\mu_H^2} \quad (4.29)$$

where μ_H and σ_H^2 denote the mean and the variance of the time varying idiosyncratic demand shock on the Home market β_H. The higher the variance of domestic demand shocks, the higher the sales volatility of domestic firms.

4.3.2 Exporters

For home-based exporters, it is useful to distinguish the growth rates of domestic sales $(g_{GHt}(\varphi_t, \varphi_{t-1}))$ and of exports $(g_{GFt}(\varphi_t, \varphi_{t-1}))$. From (4.14), the growth rate of sales of a home exporter in market i can be approximated as:

---

[16]Section 4.2.3 did however not establish the uniqueness of P_H and P_F. I assume from now that in the case of multiplicity, there is no jump between equilibrium price indices over time.

[17]For small shocks, the variance of a function θ of a random variable x can be approximated using a first order Taylor expansion as:

$$VAR(\theta(x)) \approx \left(\frac{1}{\theta'(x_0)}\right)^2 VAR(x) \quad (4.28)$$

where x_0 is the mean of the distribution of x.

$$
\begin{aligned}
g_{Git}(\varphi_t, \varphi_{t-1}) \approx & \quad log(\beta_{it}) - log(\beta_{it-1}) + C(log(k^*_{Gt}) - log(k^*_{Gt-1})) \\
- & \quad Clog(\tau_i^{1-\epsilon}\beta_{it}\chi_i\eta_i + \tau_n^{1-\epsilon}\beta_{nt}\chi_n\eta_n) \\
+ & \quad Clog(\tau_i^{1-\epsilon}\beta_{it-1}\chi_i\eta_i + \tau_n^{1-\epsilon}\beta_{nt-1}\chi_n\eta_n) \qquad (4.30) \\
& \qquad (4.31)
\end{aligned}
$$

where $\{i, n\} \in \{\{H, F\}, \{F, H\}\}$.

Conditional on the growth rate of capital, which is zero in the model, the growth rate of sales of an exporting firm on market i depends on the shocks that it receives in the home and in the foreign market. The growth of sales of an exporter on market i is larger (i) the stronger the increase in the realisation of the idiosyncratic demand shock (given by $log(\beta_{it}) - log(\beta_{it-1})$) (ii) the lower the rise in the weighted average of realisations of the idiosyncratic demand shocks on both markets (the second and third lines of (4.31)). The rationale behind (4.31) is that the growth of sales on a given market is large if the change in the demand shock on this market is more positive than the change on the other market. Differentiating the right hand side of (4.31) with respect to β_{it} and β_{nt} at the mean value of the random variables ($\beta_{it} = \beta_{it-1} = \mu_i$ and $\beta_{nt} = \beta_{nt-1} = \mu_n$) confirms this intuition:

$$
g_i^i(\psi_i) \equiv \left.\frac{\partial g_{Git}(\chi_i, \chi_n)}{\partial \beta_{it}}\right|_{\mu_i, \mu_n} = \frac{1}{\mu_i}[1 - C\psi_i] > 0 \qquad (4.32)
$$

$$
g_i^n(\psi_i) \equiv \left.\frac{\partial g_{Git}(\chi_i, \chi_n)}{\partial \beta_{nt}}\right|_{\mu_i, \mu_n} = -\frac{C}{\mu_n}(1 - \psi_i) < 0 \qquad (4.33)
$$

where $\psi_i(\chi_i, \chi_n) \equiv \frac{\tau_i^{1-\epsilon}\mu_i\chi_i\eta_i}{\tau_i^{1-\epsilon}\mu_i\chi_i\eta_i + \tau_n^{1-\epsilon}\mu_n\chi_n\eta_n}$ is the share of market i in the sales of a home-based exporter for the mean idiosyncratic shocks μ_n and μ_i. ψ_i is time invariant as it depends on χ_H and χ_F, which are realised at $t = 0$. ψ_F can be interpreted as the mean openness level of a firm. Since the parameters χ_F and χ_H differ across firms, exporters are heterogeneous with respect to their openness level ψ_F.

I show in the Appendix B that the first order Taylor approximation of the covariance between the growth rates of exports and domestic sales is negative. If an exporter wishes to raise its sales on a market i in the short run, it can: (i) employ more labour, which comes with convex costs (ii) reduce its production for market n. An exporter will always find it optimal to use both channels, as long as the shocks on both markets are not identical. The second channel ensures that exporters substitute production between markets in the short run in order to adapt to time varying shocks. This generates the first testable implication of the model:

Prediction 4.1 *For exporters, the within firm growth rates of domestic sales and of exports are negatively correlated.*

I now turn to the impact of the openness level of exporters on their sales volatility, which is the main objective of this paper. Using the model described above, I approach this question in two steps. In a first step, I look at the link between the openness of a firm and the volatility of its domestic sales and of its exports separately. In a second step, I concentrate on the link between openness and volatility of global sales. I conduct a comparative statics exercise for sales volatility with respect to ψ_F in order to examine the equilibrium correlation between openness and volatility in the cross-section of firms.

I first turn to a separate study of the sales volatility of exporting firms on their domestic and on their export market. As for the case of non-exporters, I use a first order Taylor expansion to approximate the volatility of sales on market i. It is a weighted sum of the variance of demand shocks in both markets:

$$VAR(g_{Git}) \approx V\tilde{A}R(g_{Git}) = 2(g_i^i(\psi_i))^2\sigma_i^2 + 2(g_i^n(\psi_i))^2\sigma_n^2 \tag{4.34}$$

Plugging (4.32) and (4.33) in (4.34) and differentating with respect to the openness level ψ_F yields:

$$\frac{\partial V\tilde{A}R(g_{GFt})}{\partial \psi_F} = -4C\left[(1-C\psi_F)\frac{\sigma_F^2}{\mu_F^2} + C(1-\psi_F)\frac{\sigma_H^2}{\mu_H^2}\right] < 0 \tag{4.35}$$

$$\frac{\partial V\tilde{A}R(g_{GHt})}{\partial \psi_F} = 4C\left[(1-C+C\psi_F)\frac{\sigma_H^2}{\mu_H^2} + C\psi_F\frac{\sigma_F^2}{\mu_F^2}\right] > 0 \tag{4.36}$$

The above derivatives suggest that, in equilibrium, firms with a larger openness have less volatile exports and more volatile domestic sales. The reason for this is the substitution between markets outlined above. Let a firm have a small openness level, a constant demand parameter on the foreign market and face a short run positive shock on the domestic market. Since such a firm exports on average a small amount of its production, it cannot increase its domestic sales much by decreasing sales on the export market, but mainly raises its use of labour in the short run. Due to the convex costs of labour however, this firm does not increase its domestic sales much, so that the volatility of domestic sales is quite low. The substitution between markets, which is small compared to the domestic sales, is however large as a proportion of exports, and ensures that the volatility of the export market is then large. This effect is summarised in the second testable prediction of the model:

Prediction 4.2 *In the cross-section of firms, the openness level of exporters is (i) positively correlated with the volatility of their domestic sales (ii) negatively correlated with the volatility of their exports.*

Differentiating (4.35) and (4.36) once more shows that the effect of openness on the volatility of both markets is non linear:

$$\frac{\partial^2 V\tilde{A}R(g_{GFt})}{\partial^2 \psi_F} = \frac{\partial^2 V\tilde{A}R(g_{GHt})}{\partial^2 \psi_F} > 0 \tag{4.37}$$

Back of the envelope calculations suggest that the channel highlighted in the derivation of Prediction 2 may be quantitatively important. Assume that the demand shocks on the home and foreign markets are identically distributed ($\sigma_H^2 = \sigma_F^2$ and $\mu_H = \mu_F$). Using the approximation of the variance in (4.34), the ratio of the volatility of exports to the volatility of domestic sales of a firm with openness ψ_F is given by:

$$\frac{VAR(g_{Ft})}{VAR(g_{Ht})} = \frac{(1 - C\psi_F)^2 + C^2(1 - \psi_F)^2}{(1 - C(1 - \psi_F))^2 + C^2\psi_F^2} \tag{4.38}$$

Using an elasticity of substitution $\sigma = 3.8$, as is standard in the literature[18], and a fraction of capital in the production function: $\gamma = 0.33$, (4.38) shows that the exports of a firm with an openness level $\psi_F = 0.15$ are 2.9 times more volatile than its domestic sales although the inherent volatility of both markets are equal.

I now turn to the volatility of the global sales of exporters. From (4.15), the growth rate of the world sales of an exporting firm with parameters φ_t and φ_{t-1} can be approximated as:

$$\begin{aligned} g_{Gt}(\varphi_t, \varphi_{t-1}) \approx & \quad (1 - C)log(\eta_H \chi_H \beta_{Ht} + \tau^{1-\epsilon} \eta_F \chi_F \beta_{Ft}) \\ & - \quad (1 - C)log(\eta_H \chi_H \beta_{Ht-1} + \tau^{1-\epsilon} \eta_F \chi_F \beta_{Ft-1}) \end{aligned} \tag{4.39}$$

and its variance as:

$$VAR(g_{Gt}) \approx V\tilde{A}R(g_{Gt}) = 2(1 - C)^2 \left[(1 - \psi_F)^2 \frac{\sigma_H^2}{\mu_H^2} + (\psi_F)^2 \frac{\sigma_F^2}{\mu_F^2} \right] \tag{4.40}$$

The variance of the global sales of an exporter is the weighted sum of the variances of the idiosyncratic taste shock parameters β_{Ht} and β_{Ft} where the weights depend on the importance of each market for the firm. Since the two shocks are not correlated[19], firms which are present in both markets benefit from a diversification effect. Differentiating the above expression with respect to ψ_F shows that openness has a U-shaped effect on volatility. The more volatile the foreign market, the lower the level of openness which minimises sales volatility. Comparing (4.40) with (4.29) establishes the following prediction:

[18] Bernard et al. (2003), Ghironi and Melitz (2005).

[19] The same would of course hold for a partial correlation.

Prediction 4.3 *If the foreign market is more volatile than the domestic market, the global sales of exporters with a small (large) openness are less (more) volatile than the sales of comparable domestic firms.*

4.4 The Data

I use the Amadeus dataset provided by Bureau van Dijk to test the predictions of the model. This pan-european dataset provides extensive balance sheet data on more than five million European businesses. I use the data for France since they include information on exports at the firm level and cover a very large fraction of enterprises. The data stem from the compulsory yearly reports of balance sheet information to the Tribunal de Commerce, which almost all firms have to comply with[20]. The Amadeus data covers approximately 90% of these firms. The large coverage of the Amadeus dataset is an important advantage, which has made it ever more popular in work on trade at the firm level[21].

In this study, I concentrate on firms from the manufacturing sector between 1997 and 2006, as this ten year period has a very good coverage. I deflate sales using the 2-digit sector specific output deflator provided by the KLEMS dataset with 2000 as reference year. Data on consolidated results, which include the balance sheets and results of French or foreign subsidiaries are dropped out of the sample in order to make sure that the results are not driven by sales of subsidiaries on foreign markets. I further drop firms with less than 5 years data on their growth rate, as well as those with an unweighted mean growth rate of sales in the top and bottom percentile[22]. This leaves me with a sample of 74750 firms.

Some descriptive statistics for these firms are shown in Table 4.1[23]. There is a large proportion of small firms in the data: the median sales are 669.000 euros and the median number of employees is 9. The size distribution of firms, whether defined as average sales or employment is as expected very skewed. The median growth rate of sales is 4.7% over the ten year period. This growth is not captured by the stationary model in sections 4.2 and 4.3, and may be due to trend increases in the stock of capital, in real aggregate income or in the development of better technologies.

I divide firms according to their export status into three categories: continuous ex-

[20] Some legal forms such as the Société en nom commun (SNC) are not subject to this obligation, but these are very rare and special organisations.

[21] Among others Helpman et al. (2004), Konings and Vandenbussche (2008), Del Gatto et al. (2007).

[22] To be kept in the sample, firms should have an average growth rate of sales per year between -0.18% and 244% over the ten years.

[23] For 'average' variables I report the median and the mean of the firm-level average of the variable over time.

	Nb Firms	Mean	Median
Average Sales (in thousand euros)	74750	8510	669
Average number of employees	70951	37.5	7.75
Number of years in sample	74750	8.74	9
Average growth rate of sales ($\overline{g}_{it}$)	74750	9.6%	4.7%
Log Variance of sales	74750	-3.5	-3.7

Table 4.1: Descriptive statistics of the dataset

porters (ce), continuous non exporters (cne), and others. Continuous exporters and non-exporters are those firms reporting respectively strictly positive exports and zero exports for all non missing observations. Other firms include those which become exporters, which stop exporting or which switch between exporting and non exporting over the time period, as well as firms always reporting missing observations. 17% of all firms are continuous exporters, while 43% are continuous non exporters. In a given year, on average 35% of firms do export. Table 4.2 summarises the main characteristics of the different groups.

	ce	cne	others
Number of firms	12907	32145	29698
Average Sales (in thousand euros)	20304	1781	10668
Average number of employees	87.6	11.9	41.4
Number of years in sample	8.86	8.60	8.84
Average growth rate of sales ($\overline{g}_{it}$)	7.2%	9.6%	10.8%
Log Variance of sales	-3.76	-3.62	-3.33

Table 4.2: Descriptive statistics of firms according to export status

As is well-known from the seminal paper of Bernard and Jensen (1995) and the large subsequent literature, exporting firms are very different from non exporters. They are much larger in terms of sales and employees, as well as in sales per employee. Their average growth rate and the variance of their sales are smaller than for non exporters, which may be due to the fact that exporters are usually older and more mature.

In order to shed more light on the characteristics of continuous exporters, I separate their sales between exports and domestic sales, which are computed as total sales minus exports. In order to avoid that the results be driven by outliers, I drop firms having a mean growth of exports, domestic sales or capital in the top or bottom percentiles. Average sales on the domestic market are 12.8 million euros and average exports are 7.5 millions euros. I compute an average openness measure for each continuous exporter as:

$$op = \frac{1}{\sum_{t=1997}^{t=2006} \mathbf{1}(s_{GFt} \neq ., s_{Gt} \neq .)} \sum_{t=1997}^{t=2006} \frac{s_{GFt}}{s_{Gt}} \tag{4.41}$$

where $\mathbf{1}(s_{GFt} \neq ., s_{Gt} \neq .)$ is the indicator function indicating that data on exports and global sales are non-missing. The distribution of *op*, which is an approximation of ψ_F in the model, is shown in Figure 4.2. It has a peak at 5% and a median of 15%. Most firms export only a small fraction of their output each year. Computing the distribution of openness separately for each year yields almost the same distribution, which has not changed over the ten year period. The median growth rate of domestic sales for continuous exporters is 4.4%, while it is 15.3% for exports. Moreover, exports appear much more volatile than domestic sales. The mean logarithm of volatility is -3.36 for domestic sales and -1.10 for exports, which is an order of magnitude higher[24]. Note that the channel highlighted in the derivation of Prediction 2, which suggests that the substitution of sales between markets in the short run magnifies the sales volatility on the smaller market, can account for a part of this difference. As shown in the discussion of (4.38), when the shocks in both markets are identically distributed, the model predicts that the exports volatility of a firm with the median openness level of 15% is approximately three times higher than the volatility of its domestic sales.

The test of Prediction 1 relies on the panel structure of the data and not on averages over the period. In order to make sure that outliers for firm-year observations do not drive the result, I drop observations in the top and bottom 1% of the growth rates of domestic sales, exports and capital[25]. This leaves me with a sample of 91324 firm-year observations for continuous exporters over 9 years (all observations for the first year in the sample are dropped due to the computation of growth rates).

4.5 Results

In this section, I test the empirical predictions derived in the model.

4.5.1 Prediction 1: the short run substitution between markets

The first prediction established that, conditional on the growth of capital, the within firm growth rate of domestic sales and of exports are negatively correlated.

In order to test this relationship, I regress the growth rate of exports on the growth rate of domestic sales using the sample of continuous exporters. I first use Pooled OLS, controlling for a number of additional regressors. ω and s are respectively a firm and a sector indicator. The estimating equation is given by:

[24] Export volatility is 10 times higher than the volatility of domestic sales.

[25] In order to be kept in the sample, the growth rate of domestic sales must be between -63% and 199%, the growth rate of exports between -92% and 1347% and the growth rate of capital between -72% and 4.40%.

$$g_{GF\omega st} = \rho_0 + \rho_1 g_{GH\omega st} + \rho_2 X_{\omega s} + \rho_3 X_{\omega st} + d_t + d_s + u_{\omega st} \tag{4.42}$$

where d_t and d_s are time and 4-digit NACE sector dummies. ρ_1 is the parameter of interest, which is the residual correlation of the growth rates of domestic sales and exports. $X_{\omega st}$ contains a number of firm specific, time varying, characteristics such as the growth rate of capital, present in all specifications, or the growth rate of employment which is taken into account in specifications 2 and 3. $X_{\omega s}$ is a vector of firm specific characteristics, which are constant over time. This includes for all specifications different measures of size, such as the average global sales, the average tangible fixed assets and the average openness over the period. Additionally, specificaton 3 includes the firm level average rate of growth of exports and of domestic sales over the period, to avoid that the relationship be due to long term factors. It controls for example for the fact that a firm develops aggressively on the export market over the period while neglecting its domestic market. Since the errors are likely to be heteroskedastic and autocorrelated, I use robust cluster standard errors for inference. The results are shown in Table 4.3. All specifications show that ρ_1 is negative and significant, i.e. that the growth rate of sales in the domestic and in the export market are negatively correlated. An alternative explanation for this negative correlation could be that shocks on the domestic and export markets are negatively correlated. It however seems implausible for French firms, which mainly export to European countries[26] with which shocks are likely to be positively correlated[27].

I then conduct a fixed effect regression of the estimating equation (4.42), where time invariant elements are dropped out. A fixed effect methodology has the clear advantage to isolate the within firm residual correlation, which is what Prediction 1 is about. The regression results are presented in Table 4.4. The estimates of ρ_1 are again negative and significant, and are between two and three times stronger than in the Pooled OLS case. Fixed effects can better capture the predicted negative correlation since the mechanism behind it occurs at the firm level.

Both POLS and fixed effect estimations however have a quite low R^2, which may be due to the fact that short-run shocks to the firm-level growth rate of sales are mostly driven by other unexplained idiosyncratic factors.

[26] 64% of French exports were directed towards its 24 partners in the European Union in 2006 according to the OECD STAN database.

[27] see Bordo and Helbling (2003) for the correlation of average demand shocks between European countries.

4.5.2 Prediction 2: Market share and volatility

The second prediction establishes that the larger the openness level of a firm (i) the larger the volatility of its domestic sales and (ii) the lower the volatility of its exports. These effects should be convex in the openness level from (4.37).

To test the first part of the prediction, I regress the log variance of the growth rate of domestic sales on the average openness of firms using OLS on a cross section of continuous exporting firms. There is one observation per firm since the time dimension has been used to compute volatility. Denoting $VARD \equiv VAR(g_{GHt})$ as the variance of the growth rate of domestic sales, the estimating equation is:

$$Log(VARD_{\omega s}) = \nu_0 + \nu_1 op_{\omega s} + \nu_2 op_{\omega s}^2 + \nu_3 X_{\omega s} + d_s + u_{\omega s} \qquad (4.43)$$

$X_{\omega s}$ controls for firm size using the log average sales of a firm over the period[28]. Size is known to have a dampening effect on volatility from other studies[29]. The reasons for this are not captured by the present model but can be among others the multiproduct characteristics of larger firms, which makes them more diversified along the product dimension, or their higher age, which is usually a sign of a more stable company. Results are reported in Table 4.5. Consistently with previous studies, size is found to have a negative effect on volatility. I also control for the average capital level of firms, as measured by their average tangible fixed assets, which appears to reduce volatility in all specifications. $X_{\omega s}$ includes in specifications 2 and 3 additional firm-level factors which have been found to impact volatility in the literature. First, I include a measure of firm leverage[30], which proxies for the financial constraint of a firm. Leverage increases firm volatility, which is in line with the discussion in Braun and Larrain (2005). Second, I add the mean rate of growth of capital over the period, which, as in Buch et al. (2006), raises volatility. In specifications 1 and 2, the coefficients ν_1 and ν_2 are positive and significant, as expected from Prediction 2. This result confirms that the more open a firm is, the more volatility it brings in its domestic market and that the effect is quantitatively large. Consider an exporter with an openness level close to zero and a variance of domestic sales at the 25th percentile of the volatility distribution for exporting firms. If this same exporter were to export almost its whole output, the variance of its domestic sales would be multiplied by seven and correspond to the 85th percentile of the volatility distribution.

The positive link between the volatility of the domestic market and the openness level could come from the fact that open firms invest in high technology, with a higher but

[28]Using the log average employment does not affect the results.

[29]Buch et al. (2006) among others.

[30]Firm leverage is here computed as a non-weighted average over time of the ratio of (non current liabilities + loans) over shareholders funds.

less certain return. Comin and Philippon (2005) point out that volatility indeed depends on R&D intensity. In order to control for such a channel, I proxy for the technology intensity by using the mean intangible fixed assets as an additional control. The value of intangible assets covers the value of patents, copyrights, etc. Specification 3 shows that firms with higher intangible assets are indeed more volatile, but this does not affect the coefficients on openness ν_1 and ν_2.

In order to test part (ii) of the prediction, I conduct a similar analysis with the log variance of export growth ($VARX \equiv VAR(g_{GFt})$) as dependent variable. The estimating equation is:

$$Log(VARX_{\omega s}) = \nu_0' + \nu_1' op_{\omega s} + \nu_2' op_{\omega s}^2 + \nu_3' X_{\omega s} + d_s + u_{\omega s} \tag{4.44}$$

Results are reported in Table 4.6. In all three specifications, the coefficients ν_1' and ν_2' are significant and, as expected, respectively negative and positive. This confirms that more open firms have a lower volatility on their export market.

The negative link between volatility of exports and openness found in Table 4.6 could however be due to the fact that more open firms export to more markets, with imperfectly correlated shocks, thereby benefiting from a diversification effect reducing the volatility of their exports. In terms of the model, this would translate into a correlation between the openness level of a firm and the variance of its export market. Equations (4.35) and (4.36) would be modified by adding a term including $\frac{\partial \sigma_F^2}{\partial \psi_F}$, which is negative. Not controlling for this effect would therefore bias the effect of openness on $VARD$ and on $VARX$ downward.

Though I cannot refute this alternative explanation due to lack of data on export destination, it should be noted that most French exporters export to only a few markets. Using data for 1986, Eaton et al. (2004) show that 35% of French manufacturing exporters export to only one market, while only 20% export to more than 10 markets. The diversification effect between export markets is therefore likely to be limited. Eaton et al. (2008) further show that the average domestic sales of French exporters are strongly positively correlated with the number of markets to which they export. This observation is consistent with models in which firms need to pay fixed costs for each market to which they export. The volumes of sales to these markets therefore has to be substantial, so that only large and productive firms are able to enter many markets. For a given level of openness, larger firms are therefore likely to export to more markets. To capture this correlation, I introduce an interaction term between the mean openness of a firm and its size, and use it as a proxy of the number of markets to which a firm exports. The coefficient on this interaction term should therefore be negative for both the $VARD$ and $VARX$ equations as shown above. The results of this extension are presented in Table 4.7 and show that the interaction term is negative in both cases but only significant for $VARD$ as a dependent variable. It increases substantially the estimates of the effect of openness on domestic volatility, while leaving those on the export volatility almost unchanged. This result should be interpreted with caution, as the interaction term is not an ideal proxy for the number of markets to which a firm exports. It may also capture a variety of other effects linked to size, which are not considered in the model, but its inclusion does not affect the test of prediction 2.

4.5.3 Prediction 3: Volatility of exporters and diversification effect

Prediction 3 highlights that exporters with small openness levels should be less volatile than comparable firms which produce only for the domestic market. This conclusion is independent of the assumptions made about the variance of both markets. Furthermore, if foreign markets are more volatile than the French market, openness should be positively correlated with volatility.

In order to test this prediction, I conduct an OLS regression on the cross section of continuous exporters and non-exporters. The estimating equation is:

$$Log(VAR_{\omega s}) = \nu_0'' + \nu_1'' op_{\omega s} + \nu_2'' op_{\omega s}^2 + \nu_3'' X_{\omega s} + \nu_4'' d_{ce} + d_s + u_{\omega s} \tag{4.45}$$

The above specification is very similar to the one used for prediction 2 except for the additional dummy variable d_{ce} which takes value 1 if a firm is a continuous exporter and 0 otherwise. Since all firms which switch status between exporters and non exporters in the period have been dropped out of the sample, the reference group is exclusively constituted of continuous non-exporters. Results are presented in Table 4.8.

All elements of the $X_{\omega s}$ vector have the same sign and interpretation as in the regressions for Prediction 2. The effect of openness on volatility is summarised by the coefficients ν_1'', ν_2'' and ν_4''. First, the continuous exporter dummy is in each specification negative and significant, a result which is consistent with Buch et al. (2006). Exporters with a small openness are less volatile than non-exporters, as expected from Prediction 3. Volatility of global sales is, on the other hand, increasing in the openness level of a firm, suggesting that foreign markets are more volatile than the French market, and that they tend to cause additional volatility of global sales. The coefficients found in specification 2 suggest that firms with an openness level of less (more) than 25% tend to be less (more) volatile than comparable non-exporters.

As in the discussion of Prediction 2, firms which export to many markets should benefit from an additional diversification effect on their export market. I take the same approach as above to proxy for this effect and include an interaction term between openness and size. As expected, the coefficient on the interaction term in specification 3 is indeed negative, but does not affect the main conclusions above. An increase in openness for large firms seems less detrimental to volatility than a similar increase for small firms.

4.6 Robustness

In this section, I conduct three main robustness checks to the results obtained in the previous section: (i) I examine the link between openness and the variance of the growth rate of employment and value added at the firm level (ii) I control for inventories in the test of prediction 3 (iii) I control for the multinational character of some firms.

4.6.1 Volatility of employment and value added

I have conducted the analysis up to now using a definition of firm volatility based on the growth rate of sales. It is the main measure used in the literature and allows to analyse the domestic and export markets separately as in predictions 1 and 2[31]. As a robustness check, I test whether the results of prediction 3 apply to other measures of volatility. Two of them are of particular interest. First, the growth rate of value added provides an interesting robustness check of the result, as it makes sure that these are not driven by the use of intermediate goods. Second, it can be proven that the model generates a similar result as prediction 3 for the volatility of the growth rate of employment. Employment volatility at the firm level may have the most important welfare consequences for individual households facing the risk of losing their job.

Table 4.9 presents the results of specification 2 of (4.45) for alternative dependent variable. For ease of comparison, the first column replicates the results of the middle column of Table 4.8, where the dependent variable is the variance of the growth rate of sales. The second and third columns present the results respectively for the variance of the growth rate of value added and the variance of the growth rate of employment as dependent variables. The main qualitative conclusions hold for all three definitions of volatility considered.

4.6.2 Inventories

Variations in inventories may be an important concern as changes in output do not in reality map one to one to changes in sales. Buch et al. (2008) for example posit that better inventory technology could influence firm volatility. As a robustness check to Prediction 3, I run the same specifications where I replace the sales of a firm by the

[31] It is impossible from the data to separate the employment or the value added of an exporter between a part assigned to export and a part assigned to the domestic market.

sum of its sales and variations in inventories. This should help to capture the quantities produced in a more realistic way. Unreported results show that the results are barely affected by this change in definition.

4.6.3 Controlling for multinationals

The analysis has up to now distinguished exporters from non exporters while leaving aside another important dimension of the internationalisation of a firm: its multinational character. These firms, which I define as those having at least one foreign subsidy or a foreign parent, are particular in many dimensions. They may for example be integrated in a global chain of production, affected by shocks in many countries, but which is not captured by the present model. I use the ownership database of Amadeus in order to determine which firms in the sample are multinationals. A substantial drawback of the data is that they only include the ownership structure of each firm at one point in time, between 2004 and 2007. Though the information on the international character of firms is only partial, it provides suggestive evidence as to whether the results may be driven by multinationals.

Of the 74750 firms in the sample: 4.07% have at least partial ownhership of one foreign subsidy or have one foreign parent. 0.3% of continuous non-exporters and 13.9% of continuous exporters are multinationals in this sense. As a robustness check, I run the whole analysis of section 5 again on the subsample of firms which are not part of multinationals. Unreported results show no major change to the results of section 5.

4.7 Conclusion

This paper looks at the relationship between trade openness and volatility at the firm level, both from a theoretical and empirical point of view.

The analysis builds on a two-country monopolisitic competition model of trade in which firms face demand uncertainty on their domestic and export markets. Due to long term differences in demand parameters, exporters have heterogeneous openness levels, defined as the ratio of exports to global sales. In the short run, firms face convex costs of production. For this reason, an increase in the production for one market affects marginal costs, and therefore sales on both markets. I show how the production and export decision of firms in this setup provide a channel through which foreign shocks affect domestic sales and their volatility. I generate a number of predictions for the link between trade openness and volatility which I test using French firm-level data from the Amadeus dataset between 1997 and 2006.

The paper has three main conclusions: (i) exporting firms substitute sales in the short run between their domestic and export markets. This effect is empirically robust to a variety of controls. (ii) the higher the openness level of an exporter, the higher the volatility of its domestic sales and the lower the volatility of its exports. It is a direct consequence of the market substitution highlighted above: demand shocks induce a sales substitution which is proportionally small for the larger market and large for the smaller market. The empirical analysis confirms the quantitative importance of this effect, which suggests to interpret simple comparative statics on the volatility of exports with caution. (iii) exporting firms with an openness level above 25% are more volatile than comparable non exporters, suggesting that foreign markets are inherently more volatile than the French market. Firms with an openness level below 25% are, on the other hand, less volatile than comparable non-exporters. This last observation is in line with a standard diversification argument according to which selling to uncorrelated markets reduces volatility.

I show that the results are robust to controling for the multinational ownership structure of firms, to changes in inventories as well as to using alternative measures of volatility based on value added and employment.

4.8 Appendix to chapter 4

Appendix A: Proof of Proposition 4.1

A firm has three possible strategies to choose from: produce only for the domestic market (strategy H), produce for both markets (strategy G) or not produce (strategy N). The proof relies on comparing all three strategies with one another. It is illustrated in figure 4.1. The argument implicitly uses the assumption that the support of the distribution of χ_H and χ_F is the positive reals, which makes sure that there exist firms to the right and to the left of each indifference schedule. A smaller support would not change the essence of the argument, but would require to impose more restrictions on the parameters.

To simplify the notation, I use in Appendix A and B the expectation operator E to denote the expectations over β_{Ht} and β_{Ft}.

- Indifference between H and N

A firm prefers domestic production to no production as long as:

$$\Delta \left(E \left[(\eta_H \beta_{Ht} \chi_H)^{1-C} \right] \right)^{\frac{1}{1-C}} - f \geq 0 \tag{4.46}$$

where $\Delta \equiv (1-A)^{\frac{1}{1-C}} r^{\frac{C}{C-1}} \left(C^{\frac{C}{1-C}} - C^{\frac{1}{1-C}} \right) \left(\frac{A}{w} \right)^{\frac{\epsilon-1}{\alpha}}$

Define χ_H^* as the level of χ_H for which a firm is indifferent between strategies H and N. χ_H^* is given by:

$$\chi_H^* = \frac{f}{\Delta \eta_H} \left(E \left(\beta_{Ht}^{1-C} \right) \right)^{\frac{1}{C-1}} \tag{4.47}$$

Regardless of their taste parameter on the foreign market, all firms having drawn a $\chi_H > \chi_H^*$ prefer strategy H to N.

- Indifference between G and N

A firm prefers to produce for both market than not to produce as long as:

$$\Delta \left(E \left[(\eta_H \beta_{Ht} \chi_H + \eta_F \beta_{Ft} \chi_F \tau^{1-\epsilon})^{1-C} \right] \right)^{\frac{1}{1-C}} - f_x - f \geq 0 \tag{4.48}$$

When the above expression holds with equality, it defines by the implicit function theorem a function $\hat{\chi}_H(\chi_F)$ with $\hat{\chi}_H'(\chi_F) < 0$. The proof only requires to show that the derivatives of the left hand side with respect to χ_H and χ_F are positive. Further note that $\breve{\chi}_H \equiv \hat{\chi}_H(0) > \chi_H^*$, which simply states that if there is no demand for its product on the export market, the demand on the domestic market of a firm which pays both types of fixed costs have to be large enough to cover these fixed costs. The demand level must therefore be higher than χ_H^*, which is necessary to cover the sole fixed costs of production. On the other hand, there exists a $\breve{\chi}_F$ such that $\hat{\chi}_H(\breve{\chi}_F) = 0$. $\breve{\chi}_F$ is defined as:

$$\breve{\chi}_F = \frac{(f_x + f)\tau^{\epsilon-1}}{\Delta \eta_F} \left(E(\beta_{Ft}^{1-C}) \right)^{\frac{1}{C-1}} \tag{4.49}$$

- Indifference between G and H

A firm prefers to sell in both markets than only on its domestic market if:

$$\Delta\left[\left(E\left[(\eta_H\beta_{Ht}\chi_H+\eta_F\beta_{Ft}\tau^{1-\epsilon}\chi_F)^{1-C}\right]\right)^{\frac{1}{1-C}}-\left(E\left[(\eta_H\beta_{Ht}\chi_H)^{1-C}\right]\right)^{\frac{1}{1-C}}\right]-f_x\geq 0 \tag{4.50}$$

Define the left hand side of the above inequality as Γ. For any given χ_H and τ bounded, it is immediate that: $\frac{\partial\Gamma}{\partial\chi_F}>0$, that $\lim_{\chi_F\to 0}\Gamma=-f_x$ and that $\lim_{\chi_F\to\infty}\Gamma=\infty$. This ensures that for all $\chi_H>\chi_H^*$, there exist firms with a χ_F sufficiently large that they choose strategy G and firms with a χ_F sufficiently low that they choose strategy H.

Though the above argument is sufficient to prove Proposition 4.1, it is of interest to define the whole indifference schedule between strategies G and H, i.e. the combinations of χ_H and χ_F which make a firm indifferent between exporting and producing for the domestic market. For this purpose, I show in the following that when (4.50) holds with equality, it defines by the implicit function theorem a function $\tilde{\chi}_H(\chi_F)$ with $\tilde{\chi}_H'(\chi_F)<0$. The proof requires to show that the derivatives of Γ with respect to χ_H and χ_F are positive. This is clear for χ_F, but requires a more detailed analysis for χ_H. First note that Γ can be rewritten as:

$$\Gamma=\Delta\left[\left(E\left[(\eta_H\beta_{Ht}\chi_H+\eta_F\beta_{Ft}\tau^{1-\epsilon}\chi_F)^{1-C}\right]\right)^{\frac{1}{1-C}}-\chi_H\eta_H\left(E\left(\beta_{Ht}^{1-C}\right)\right)^{\frac{1}{1-C}}\right]-f_x \tag{4.51}$$

so that:

$$\begin{aligned}\frac{1}{\Delta}\frac{\partial\Gamma}{\partial\chi_H}= & \left(\left(E\left[\left(\eta_H\beta_{Ht}\chi_H+\eta_F\beta_{Ft}\tau^{1-\epsilon}\chi_F\right)^{1-C}\right]\right)^{\frac{C}{1-C}}\right.\\ \times & \left. E\left[\eta_H\beta_{Ht}\left(\eta_H\beta_{Ht}\chi_H+\eta_F\beta_{Ft}\tau^{1-\epsilon}\chi_F\right)^{-C}\right]\right)\\ - & \eta_H\left(E\left(\beta_{Ht}^{1-C}\right)\right)^{\frac{1}{1-C}}\end{aligned} \tag{4.52}$$

By the reverse Holder's inequality (since $1-C<1$):

$$\frac{1}{\Delta}\frac{\partial\Gamma}{\partial\chi_H}>0 \tag{4.53}$$

This completes the proof that $\tilde{\chi}_H'(\chi_F)<0$.

There exists a $\check{\check{\chi}}_F$ such that $\tilde{\chi}_H(\check{\check{\chi}}_F)=0$, which is given by:

$$\check{\check{\chi}}_F=\frac{f_x\tau^{\epsilon-1}}{\Delta\eta_F}\left(E\left(\beta_{Ft}^{1-C}\right)\right)^{\frac{1}{C-1}} \tag{4.54}$$

It can be immediately noted that $\check{\check{\chi}}_F<\check{\chi}_F$. This states that if a firm has a taste parameter on its domestic market of $\chi_H=0$, the taste parameter on the export market which is necessary to make strategy G profitable in comparison to no profits is larger than if compared to the H strategy. This is due to the fact that the additional costs compared to the H strategy are only the fixed costs of exporting f_x while the additional costs compared to the N strategy are both fixed costs: $f+f_x$. The necessary taste parameter to cover the sum of these costs is therefore larger than that needed to cover the sole f_x.

I now turn to the determination of the intersection between the curves $\check{\chi}_H(\chi_F)$ and $\check{\check{\chi}}_H(\chi_F)$. These will cross at (χ_H',χ_F') for which (4.48) and (4.51) both hold with equality. It is straightforward to show that this happens for $\chi_H'=\chi_H^*$. Since $\check{\chi}_H(\chi_F)$ and $\check{\check{\chi}}_H(\chi_F)$ are strictly decreasing functions of χ_F, there is only one intersection between the two curves.

I further define: $\tilde{\tilde{\chi}}_F\equiv\lim_{\chi_H\to\infty}\tilde{\chi}_H^{-1}(\chi_H)$. This limit can be zero if the difference between the two strategies as χ_H increases goes to ∞, or can be bounded away from zero. Figure 4.1 summarises the argument.

Appendix B: First order Taylor approximation of the covariance between g_{GFt} and g_{GHt}

Noting that $\frac{\partial g_{Git}(\mu_H,\mu_f)}{\partial \beta_{Ht}} = -\frac{\partial g_{Git}(\mu_H,\mu_f)}{\partial \beta_{Ht-1}} \equiv g_i^H$ and $\frac{\partial g_{Git}(\mu_H,\mu_f)}{\partial \beta_{Ft}} = -\frac{\partial g_{Git}(\mu_H,\mu_f)}{\partial \beta_{Ft-1}} \equiv g_i^F$, the first order Taylor expansion of g_{Git} is given by:

$$g_{Git} \approx g_i(\mu_H, \mu_F) + g_i^H(\beta_{Ht} - \beta_{Ht-1}) + g_i^F(\beta_{Ft} - \beta_{Ft-1}) \quad (4.55)$$

The covariance between the growth of domestic sales g_{GH} and the growth of exports of an exporter g_{GF} can therefore be approximated as follows:

$$\begin{aligned} Cov(g_{GH}, g_{GF}) &= E[(g_{GH} - E(g_{GH}))(g_{GF} - E(g_{GF}))] \\ &\approx E[(\beta_{Ht} - \beta_{Ht-1})^2 g_H^H g_F^H + (\beta_{Ft} - \beta_{Ft-1})^2 g_H^F g_F^F] \\ &+ E\left[(\beta_{Ht} - \beta_{Ht-1})(\beta_{Ft} - \beta_{Ft-1})(g_H^H g_F^F + g_F^H g_H^F)\right] \end{aligned}$$

Since β_H and β_F are i.i.d., the above expression reduces to:

$$Cov(g_{GH}, g_{GF}) \approx 2g_H^H g_F^H \sigma_H^2 + 2g_H^F g_F^F \sigma_F^2 < 0 \quad (4.56)$$

where the inequality comes from (4.32) and (4.33), and their counterparts for g_{GF}. This proves that the approximations of the growth rates of domestic sales and of exports are negatively correlated.

Appendix C: Existence of price indices

The price indices in H and F must solve a system of the form:

$$P_H = \Phi_H(P_H, P_F) \quad (4.57)$$
$$P_F = \Phi_F(P_H, P_F) \quad (4.58)$$

I define $P \in \mathbb{R}^{+2}$ as the vector (P_H, P_F) and the system of equations above as the function $\Phi : \mathbb{R}^{+2} \to \mathbb{R}^{+2}$. In order to apply Brouwer's fixed point theorem to the above system I need to restrict attention to a non-empty, convex, compact set, i.e. to show that there exists a set $\Theta = [\underline{P}_H, \overline{P}_H] \times [\underline{P}_F, \overline{P}_F]$ such that if $P \in \Theta$, then $\Phi(P) \in \Theta$.

To do so, the proof will be structured as follows. In a first step, I show that there exist uniform strictly positive lower bounds $\underline{P}_H$ and $\underline{P}_F$ such that: $\Phi_H(P) \geq \underline{P}_H$ and $\Phi_F(P) \geq \underline{P}_F$ for all P.

In a second step, I show that there exist uniform finite upper bounds $\overline{P}_H$ and $\overline{P}_F$ such that $\Phi_H(P) \leq \overline{P}_H$ and $\Phi_F(P) \leq \overline{P}_F$ for all $P \in [\underline{P}_H, \infty) \times [\underline{P}_F, \infty)$.

This guarantees that for any $P \in \Theta$, $\Phi(P)$ is also an element of Θ. In a third step, I apply Brouwer's fixed point theorem on the compact set Θ to show that P has at least one fixed point.

In order to obtain more insight on the form of the functions Φ_H and Φ_F, remember that the price index is defined from (4.4) as the integral over a function of the prices of different varieties, where the integral is taken to the power $\frac{1}{1-\epsilon}$. I define for expositional clarity the functions ϕ_H and ϕ_F such that: $(\phi_H(P))^{1-\epsilon} = \Phi_H(P)$ and $(\phi_F(P))^{1-\epsilon} = \Phi_F(P)$.

Define:

$$\tilde{p}_H(\chi_H, \chi_F) = \int_{\underline{\beta}_H}^{\overline{\beta}_H} (\chi_H \beta_{Ht})^{\epsilon} (p_H(\varphi_t))^{1-\epsilon} dG_H(\beta_{Ht}) \quad (4.59)$$

Using (4.22), this can be rewritten as:

$$\tilde{p}_H(\chi_H, \chi_F) = \chi_H^{\epsilon} \frac{\int_{\underline{\beta}_H}^{\overline{\beta}_H} \beta_{Ht}^{\epsilon - C} dG_H(\beta_{Ht})}{\left[\int_{\underline{\beta}_H}^{\overline{\beta}_H} \beta_{Ht}^{1-C} dG_H(\beta_{Ht})\right]^{\frac{\alpha-1}{\alpha}}} \equiv \chi_H^{\epsilon}\kappa_H \tag{4.60}$$

where κ_H is a strictly positive and bounded constant, independent of P.

I also define:

$$\tilde{p}_{GH}(\chi_H, \chi_F) = \int_{\underline{\beta}_H}^{\overline{\beta}_H} \int_{\underline{\beta}_F}^{\overline{\beta}_F} (\chi_H \beta_{Ht})^{\epsilon} (p_{GH}(\varphi_t))^{1-\epsilon} dG_F(\beta_{Ft}) dG_H(\beta_{Ht}) \tag{4.61}$$

which, using (4.23), can be rewritten as:

$$\tilde{p}_{GH}(\chi_H, \chi_F) = \Lambda^{1-\epsilon} \frac{\int_{\underline{\beta}_H}^{\overline{\beta}_H} \int_{\underline{\beta}_F}^{\overline{\beta}_F} (\chi_H \beta_{Ht})^{\epsilon} (\eta_H \beta_{Ht} \chi_H + \tau^{1-\epsilon} \eta_F \beta_{Ft} \chi_F)^{-C} dG_F(\beta_{Ft}) dG_H(\beta_{Ht})}{\left[\int_{\underline{\beta}_H}^{\overline{\beta}_H} \int_{\underline{\beta}_F}^{\overline{\beta}_F} (\eta_H \beta_{Ht} \chi_H + \tau^{1-\epsilon} \eta_F \beta_{Ft} \chi_F)^{1-C} dG_F(\beta_{Ft}) dG_H(\beta_{Ht})\right]^{\frac{-C}{1-C}}} \tag{4.62}$$

The function ϕ_H integrates piecewise $\tilde{p}_H(\chi_H, \chi_F)$, $\tilde{p}_{GH}(\chi_H, \chi_F)$ and its counterpart for prices of imported varieties, which I define as $\tilde{p}_{GFH}(\chi_H, \chi_F)$, over χ_F and χ_H. The domain of (χ_H, χ_F) over which each $\tilde{p}$ applies is defined by the different areas in figure 4.1 and its counterpart for the foreign country. I now proceed along the steps outlined above.

- First step: There exist uniform strictly positive lower bounds $\underline{P}_H$ and $\underline{P}_F$ such that: $\Phi_H(P) \geq \underline{P}_H$ and $\Phi_F(P) \geq \underline{P}_F$ for all P

Define:

$$X \equiv \frac{\eta_H \chi_H}{\eta_F \chi_F} \tag{4.63}$$

An upper bound of $\tilde{p}_{GH}(\chi_H, \chi_F)$ is given by taking the limits of the domain of integration of β_H and β_F which maximise the right hand side of (4.62). Therefore:

$$\tilde{p}_{GH}(\chi_H, \chi_F) < \chi_H^{\epsilon} \Lambda^{1-\epsilon} \overline{\beta}_H^{\epsilon} \left(\frac{\overline{\beta}_H X + \tau^{1-\epsilon} \overline{\beta}_F}{\underline{\beta}_H X + \tau^{1-\epsilon} \underline{\beta}_F} \right)^C \tag{4.64}$$

It can easily be shown that the term in bracket on the right hand side has a finite maximum over X if $\underline{\beta}_H$, $\underline{\beta}_F$, $\overline{\beta}_H$ and $\overline{\beta}_F$ are bounded away from zero and ∞. There exists therefore a finite constant $\overline{\kappa}_{GH}$, such that $\tilde{p}_{GH}(\chi_H, \chi_F) < \chi_H^{\epsilon} \overline{\kappa}_{GH}$ for all P. A similar reasoning can be applied to the counterpart of $\tilde{p}_{GH}(\chi_H, \chi_F)$ for imported varieties, $\tilde{p}_{GFH}(\chi_H, \chi_F)$. There exists therefore a $\overline{\kappa}$ positive and bounded such that $\chi_H^{\epsilon} \overline{\kappa}$ is a finite upper bound of $\tilde{p}_{GH}(\chi_H, \chi_F)$, $\tilde{p}_H(\chi_H, \chi_F)$ and $\tilde{p}_{GFH}(\chi_H, \chi_F)$ for all P.

The function ϕ_H integrates piecewise these three elements over χ_H and χ_F, and the domain on which each of the $\tilde{p}$ applies depends on P_H and P_F[32]. Under the assumption that $\int_c^{\infty} \chi_H^{\epsilon} dG_{\chi}(\chi_H)$ is uniformly bounded for any $c > 0$[33], the integral of $\chi_H^{\epsilon} \overline{\kappa}$ over χ_H on any subset of $(0,\infty)$ has a uniform finite upper

[32] see figure 4.1 and the proof of Proposition 4.1.

[33] This is similar to the standard assumption made in models with heterogeneously productive firms to ensure that the price index is defined.

bound. Since the function $\phi_H(P)$ consits of such an integral, it therefore has a uniform finite upper bound. There exists therefore a $\underline{P}_H$ such that $\Phi_H(P) \geq \underline{P}_H$ for all P. A symmetric reasoning applies for the foreign country.

- Second step: There exist uniform finite upper bounds $\overline{P}_H$ and $\overline{P}_F$ such that $\Phi_H(P) \leq \overline{P}_H$ and $\Phi_F(P) \leq \overline{P}_F$ for all $P \in [\underline{P}_H, \infty) \times [\underline{P}_F, \infty)$

Note that by the reverse Hölder's inequality, for two random variables Y and Z:

$$E\left[Y^{\epsilon} Z^{-\frac{C}{1-C}}\right] \geq \left(E\left[Y^{\epsilon(1-C)}\right]\right)^{\frac{1}{1-C}} (E[Z])^{-\frac{C}{1-C}} \tag{4.65}$$

Applying this result to (4.62) shows that:

$$\tilde{p}_{GH}(\chi_H, \chi_F) \geq \Lambda^{1-\epsilon} \chi_H^{\epsilon} \left(\int_{\underline{\beta}_H}^{\overline{\beta}_H} \beta_{Ht}^{\epsilon(1-C)} dG_H(\beta_{Ht}) \right)^{\frac{1}{1-C}} \equiv \chi_H^{\epsilon} \underline{\kappa}_{GH} \tag{4.66}$$

A similar reasoning applies to $\tilde{p}_{GFH}(\chi_H, \chi_F)$, so that there exists a positive $\underline{\kappa}$, bounded away from zero such that $\chi_H^{\epsilon}\underline{\kappa}$ is a uniform lower bound of $\tilde{p}_{GH}(\chi_H, \chi_F)$, $\tilde{p}_H(\chi_H, \chi_F)$ and $\tilde{p}_{GFH}(\chi_H, \chi_F)$ for all P.

The next step consists in proving that the mass of firms supplying the Home market is strictly positive and bounded away from zero for all $P \in [\underline{P}_H, \infty) \times [\underline{P}_F, \infty)$. If P_H and P_F would approach zero, the conditions on both market would be so difficult for any firm that none would find profitable to pay the fixed costs of production and of exports, implying a zero mass of firm. However, if $P_H \geq \underline{P}_H$ and $P_F \geq \underline{P}_F$ there exists a strictly positive uniform lower bound to the mass of firms supplying the Home market. There exists therefore a strictly positive uniform lower bound to $\phi_H(P)$ for all $P \in$ and therefore a finite uniform upper bound $\overline{P}_H$ such that $\Phi \leq \overline{P}_H$ for all P. A similar reasoning holds for the foreign country.

- Third step: Brouwer's fixed point

For any $P \in \Theta$, $\Phi_(P) \in \Theta$. Θ is a non-empty, compact, convex set. By the continuity of Φ, Brouwer's fixed point theorem establishes that P has at least one equilibrium.

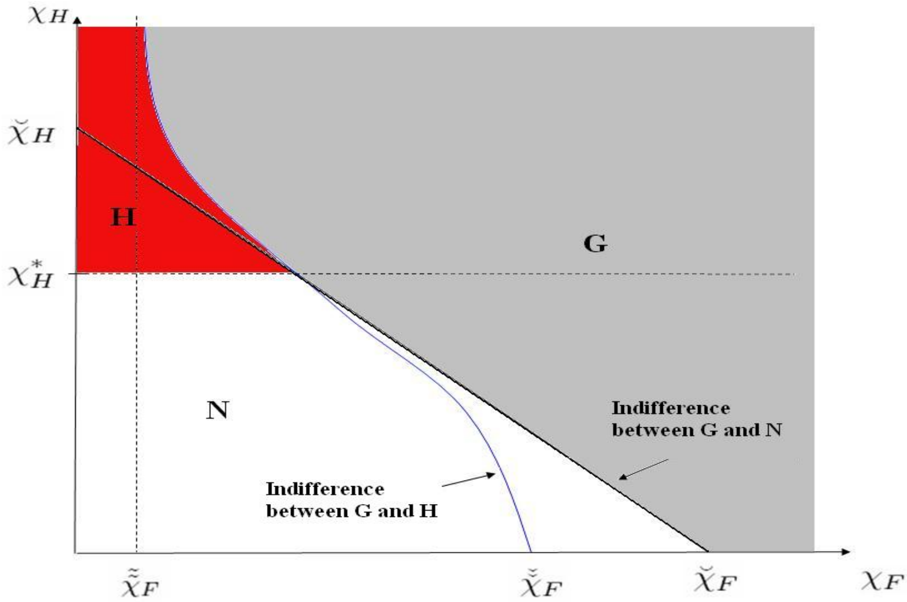

Figure 4.1: Optimal strategies as a function of the χ_H and χ_F parameters. The grey area represents firms choosing strategy G (exporting firms), the red area are H firms (domestic firms). Other firms do not produce (N).

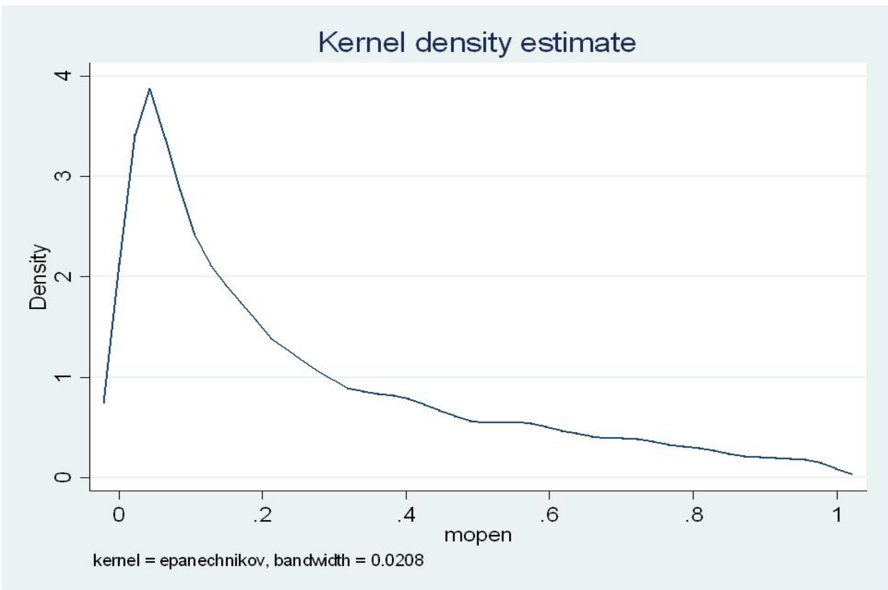

Figure 4.2: Density function of the mean openness of continuous exporters.

Table 4.3: Test of Prediction 1: Pooled OLS. Dependent variable: growth rate of exports.

	POLS1	POLS2	POLS3
growth rate of dom. sales	**-.0373124***	**-.0507679****	**-.0507741****
	(.0191496)	(.0229481)	(.0229485)
growth rate of ta. fixed assets	.0781183***	.078922***	.0788295***
	(.0085353)	(.0105278)	(.010528)
log(mean sales)	-.0313396***	-.0265245***	-.0264709***
	(.0053823)	(.0065261)	(.0065257)
log(mean ta. fixed assets)	.0056683	.0040802	.0040088
	(.0045221)	(.0055304)	(.0055299)
mean openness	-.430428***	-.4189875***	-.418301***
	(.0154411)	(.0183369)	(.018314)
growth rate of employment		.0065993	.0066124
		(.0071594)	(.0071584)
mean growth of domestic sales			.0000785*
			(.0000473)
mean growth of exports			.0008781**
			(.0003805)
constant	.4883268***	.4281124***	.4276375***
	(.0542609)	(.0569455)	(.0569374)
year dummies	Yes	Yes	Yes
sector dummies	Yes	Yes	Yes
N	91324	60902	60902
R^2	.0194588	.0199032	.0199721

Robust s.e. in brackets. * indicates significance at the 10%, ** at the 5% and *** at the 1% levels

Table 4.4: Test of Prediction 1: Fixed effects

	FE1	FE2	FE3
growth rate of dom. sales	**-.1371386*****	**-.1473734*****	**-.1392106*****
	(.0215998)	(.0215279)	(.0260214)
growth rate of ta. fixed assets		.0606609***	.0612941***
		(.0088906)	(.0111031)
growth rate of employment			.0045252
			(.0055351)
constant	.224847***	.2227856***	.2143745***
	(.0173259)	(.0173333)	(.0197142)
year dummies	Yes	Yes	Yes
N	91324	91324	60902
R^2	.0052366	.0059883	.0050393

Robust s.e. in brackets. * indicates significance at the 10%, ** at the 5% and *** at the 1% levels

Table 4.5: Test of Prediction 2(i). Dependent variable: Log(VARD)

	OLS1	OLS2	OLS3
log(mean sales)	-.1225686***	-.0889617***	-.0767453***
	(.0184016)	(.0190918)	(.0212343)
log(mean tangible fixed assets)	-.0830879***	-.1148908***	-.1078658***
	(.0147298)	(.0154159)	(.016422)
mean openness	**2.35824*** **	**2.259767*** **	**2.126179*** **
	(.1921928)	(.1962087)	(.2058212)
mean openness squared	**.6440096*** **	**.7463503*** **	**.8144485*** **
	(.2485645)	(.2550576)	(.2694457)
log(mean leverage)		.0824691***	.0872893***
		(.0094883)	(.0101451)
mean growth of ta. fixed assets		.4221969***	.4391171***
		(.0451156)	(.0490924)
log(mean intangible fixed assets)			-.0088976
			(.0074837)
constant	-2.866947***	-3.011657***	-3.132578***
	(.1962882)	(.1979403)	(.209084)
sector dummies	Yes	Yes	Yes
N	12096	11288	10374
R^2	.276553	.2950427	.2863211

Robust s.e. in brackets. * indicates significance at the 10%, ** at the 5% and *** at the 1% levels

Table 4.6: Test of Prediction 2(ii). Dependent variable: Log(VARX)

	OLS1	OLS2	OLS3
log(mean sales)	-.2541498***	-.2240573***	-.2122094***
	(.0254741)	(.0263901)	(.029517)
log(mean tangible fixed assets)	-.0406837**	-.0707281***	-.0587949**
	(.0205939)	(.0214126)	(.0232941)
mean openness	**-7.504125*** **	**-7.727016*** **	**-7.935433*** **
	(.2627027)	(.2722108)	(.2855309)
mean openness squared	**5.052641*** **	**5.298732*** **	**5.574069*** **
	(.3061496)	(.3182103)	(.3371414)
log(mean leverage)		.090397***	.096044***
		(.0139363)	(.0148034)
mean growth of ta. fixed assets		.3639069***	.394522***
		(.0610891)	(.0674345)
log(mean intangible fixed assets)			-.0301429***
			(.0108011)
constant	2.178777***	2.079732***	1.910093***
	(.2265579)	(.2343461)	(.2424976)
sector dummies	Yes	Yes	Yes
N	12096	11288	10374
R^2	.261425	.2722451	.2765859

Robust s.e. in brackets. * indicates significance at the 10%, ** at the 5% and *** at the 1% levels

Table 4.7: Test of Prediction 2 - Robustness

Dependent variable (log)	VARD	VARX
log(mean sales)	-.0195427	-.2102026***
	(.0238831)	(.0328531)
log(mean tangible fixed assets)	-.1030818***	-.058627**
	(.0163996)	(.0233539)
log(mean intangible fixed assets)	-.007704	-.030101***
	(.0074619)	(.010806)
mean openness	**3.793896*** **	**-7.876926*** **
	(.3619207)	(.4643293)
mean openness squared	**1.087704*** **	**5.583655*** **
	(.2773498)	(.3475967)
mean openness*log(mean sales)	**-.2178906*** **	**-.0076441**
	(.0410114)	(.0507589)
log(mean leverage)	.0873825***	.0960473***
	(.0101245)	(.0148046)
mean growth of ta. fixed assets	.436154***	.3944181***
	(.0490281)	(.0674297)
constant	-3.691835***	1.890473***
	(.2359735)	(.2824627)
sector dummies	Yes	Yes
N	10374	10374
R^2	.288599	.2765872

Robust s.e. in brackets. * indicates significance at the 10%, ** at the 5% and *** at the 1% levels

Table 4.8: Test of Prediction 3. Dependent variable: Log(VAR)

	OLS1	OLS2	OLS3
log(mean sales)	-.1144376***	-.0529987***	-.0287562*
	(.0146851)	(.0150672)	(.0164468)
log(mean tangible fixed assets)	-.0719439***	-.1169046***	-.1142548***
	(.0115292)	(.01184)	(.0118468)
log(mean intangible fixed assets)	.0039578	-.0037559	-.0028927
	(.0051219)	(.005175)	(.0051819)
continuous exporter dummy	**-.4794881*** **	**-.4309709*** **	**-.4392635*** **
	(.0218848)	(.0220607)	(.0221941)
mean openness	**2.202789*** **	**2.054253*** **	**2.984926*** **
	(.1529615)	(.1549856)	(.2814168)
mean openness squared	**-1.23946*** **	**-1.077056*** **	**-.8622708*** **
	(.2034569)	(.2073217)	(.2164599)
log(mean leverage)		.1048659***	.105376***
		(.0071001)	(.0070944)
mean growth of ta. fixed assets		.5234478***	.5215606***
		(.0328849)	(.032854)
mean openness*log(mean sales)			-.1269159***
			(.0327126)
constant	-2.607563***	-2.96776***	-3.191064***
	(.1448286)	(.15295)	(.1644912)
sector dummies	Yes	Yes	Yes
N	21919	20374	20374
R^2	.1520038	.1783488	.1790683

Robust s.e. in brackets. * indicates significance at the 10%, ** at the 5% and *** at the 1% levels

Table 4.9: Robustness: Volatility of value added and employment

Dependent variable (volatility of:)	Sales	Value added	Employment
log(mean sales)	-.0529987***	.0518578***	-.2253362***
	(.0150672)	(.0187402)	(.0204413)
log(mean tangible fixed assets)	-.1169046***	-.0771628***	-.1215709***
	(.01184)	(.0147386)	(.0161611)
log(mean intangible fixed assets)	-.0037559	.022241***	.0345253***
	(.005175)	(.0058555)	(.007026)
continuous exporter dummy	**-.430971*** **	**-.279983*** **	**-.265733*** **
	(.0220607)	(.0259414	(.030859)
mean openness	**2.054253*** **	**1.080673*** **	**.7009602*** **
	(.1549856)	(.1846767)	(.2032166)
mean openness squared	**-1.077056*** **	**-.114389**	**-.4686044* **
	(.2073217)	(.2567542)	(.2604102)
log(mean leverage)	.1048659***	.1452433***	.132627***
	(.0071001)	(.0094657)	(.0097551)
mean growth of ta. fixed assets	.5234478***	.5011943***	.6588011***
	(.0328849)	(.038767)	(.0481745)
constant	-2.96776***	-3.502734***	-1.560567***
	(.15295)	(.1666263)	(.2052668)
sector dummies	Yes	Yes	Yes
N	20374	18648	15585
R^2	.1783488	.1184317	.1524033

Robust s.e. in brackets. * indicates significance at the 10%, ** at the 5% and *** at the 1% levels. The volatilty of a variable is defined as the log variance of its growth rate.

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Eidesstattliche Erklärung

Hiermit erkläre ich, dass ich die Dissertation selbstandig angefertigt und mich anderer als der in ihr angegebenen Hilfsmittel nicht bedient habe, insbesondere, dass aus anderen Schriften Entlehnungen, soweit sie in der Dissertation nicht ausdrücklich als solche gekennzeichnet und mit Quellenangaben versehen sind, nicht stattgefunden haben.

Mannheim, 20.04.2009

Gonzague Vannoorenberghe