Risk Aversion in the Bond Market

The Case of Redemption Lottery Bonds

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Chapter 1

Introduction

Market participants' risk preferences and probability beliefs are the two subjective components in neoclassical asset pricing. Evidence on the empirical magnitude of risk aversion is essential in understanding the behavior of asset prices. Mehra and Prescott (1985) presented a momentous estimate of relative risk aversion (RRA) in the stock market. The estimate's magnitude was hard to explain and generated an extensive literature addressing the equity premium puzzle.[1] A related subject in the credit risk literature is the credit spread puzzle. This puzzle relates to the relatively large part of the bond yield spread generally interpreted as risk premium, which cannot be explained by expected default loss.[2] Both the equity premium and the credit spread puzzle approach the problem of a reasonable size of investors' risk aversion.

Even assuming that asset prices are determined by the investment and consumption decisions of a representative agent, the estimation of risk aversion is impeded by the fact that observed prices depend on probability beliefs. Hence, most RRA estimates are conditional on distributional assumptions about the state variables. The German bond market had a segment of bond issues that

[1] For comprehensive surveys, see Kocherlakota (1996) and Mehra and Prescott (2003).

[2] See Amato and Remolona (2003) for a literature overview pertaining to the credit spread puzzle. Elton et al. (2001), Collin-Dufresne et al. (2001), and Driessen (2005) employ statistical approaches to decompose credit spreads, while Huang and Huang (2003) provide an overview of structural approaches to disentangling credit spreads.

were redeemed by a sequence of lotteries (*Tilgungsanleihen*), providing us with an exceptional environment to study investors' risk preferences independent of subjective probability beliefs. These bonds, subsequently referred to as lottery bonds, were typically issued by the Federal Republic of Germany, German states, and government-owned enterprises and can be considered free of default risk. The issuer was obligated to redeem a certain fraction of the outstanding debt at predefined dates and redemption values. According to the redemption amounts, the total issue was split into series equal in size. At each redemption date, one series was randomly drawn by a lottery with the probability of one over the number of outstanding series.

It is important to note that there is only one price for the lottery bond and no individual price for each series, as the maturity of the outstanding series has the same uniform probability distribution. Therefore, prices of the outstanding series usually jump at the drawing date due to the following reasoning. The holder of a bond series has a lottery in his portfolio. If his bond belongs to the series that is drawn, he receives the face value and the coupon at the next redemption date. In addition, the series no longer takes part in future lotteries. The traded bond after a drawing date consists of the non-drawn series only. If, for example, the lottery bond trades below the discounted redemption value immediately before the drawing date, its price will drop, as the bond no longer has the chance of being repaid at the next redemption date. This chance is delayed until the next drawing date, typically by one year. Another important feature is that the probability of a price change caused by the redemption lottery is objectively known but cannot be arbitraged, even though it is public information. Hence, the segment of German default-free lottery bonds presents an exceptionally clean market setting to implicitly estimate risk preferences from observed transaction prices. Redemption risk is the only difference between a lottery bond and a simultaneously traded straight default-free German government, state, or government-owned enterprise bond.

Our contribution to the literature is twofold. On the theoretical side, we develop a fully specified dynamic equilibrium model to price lottery bonds. In this neoclassical model framework, a representative investor maximizes his utility of terminal wealth under the well established power utility function. The equilibrium price and the implied RRA coefficient are determined recursively by standard dynamic programming techniques. Comparative static results for the RRA coefficient are provided which are in line with standard asset pricing theory. On the empirical side, we employ the valuation model to estimate implied RRA coefficients from transaction prices of German lottery bonds. The estimates are based on a unique dataset containing transaction prices of 83 lottery bonds and 483 lotteries traded between 1974 to 1987. Our most important empirical findings address the magnitude and evolution of risk aversion in the bond market.

To the best of our knowledge, this study is the first to estimate implied RRA coefficients from lottery bond prices in a dynamic equilibrium setting. One of the first papers dealing explicitly with lottery bonds is Schilbred (1973). He studies annuities issued by an Italian government enterprise that were redeemed by lottery. The study focuses on estimating the market price of risk in a mean-variance equilibrium model. Green and Rydqvist (1997) evaluate Swedish lottery bonds for which the coupons and not the maturity are determined by lotteries. By construction, the lottery risk is idiosyncratic and should not result in a risk premium. Nevertheless, the authors find empirical evidence that transaction prices include a premium for lottery risk. In two follow-up studies, Green and Rydqvist (1999) and Florentsen and Rydqvist (2002) analyze abnormal ex-day returns of Swedish and Danish coupon lottery bonds and find tax clientele effects. Our study is closest to the paper by Ukhov (2005). Ukhov uses price data of two coupon and redemption lottery bonds issued by the Russian government between 1864 and 1866 to estimate the Arrow-Pratt measure of absolute risk aversion with respect to the coupon lottery risk. In contrast to our study, Ukhov does not employ a dynamic asset pricing framework and estimates risk aversion under the assumption that bond prices after the lottery are perfectly known immediately before the lottery.

The importance of RRA estimates in financial economics can be judged by the numerous publications over the last four decades attempting to determine the magnitude of risk aversion. Table 1.1 gives a fragmentary overview of the range of RRA coefficients reported in previous studies. A large body of empirical literature attempts to extract risk preferences through direct assessments in hypothetical environments or from cross-sectional household survey data. Most of these studies

Table 1.1: **Range of RRA Coefficients**

This table shows a selection of RRA coefficients reported in literature. For further details, see (1) Arrow (1970), p. 98, (2) Friend and Blume (1975), pp. 920, (3) Wolf and Pohlman (1983), p. 848, Table II, (4) Barsky et al. (1997), p. 563, Table XI (harmonic mean (4.2) and arithmetic mean (12.1)), (5) Holt and Laury (2002), p. 1649, Table 3, (6) Grossman and Shiller (1981), p. 226, (7) Hansen and Singleton (1983), p. 258, Table I, (8) Ferson (1983), p. 492, Table 5, (9) Brown and Gibbons (1985), p. 374, Table III, (10) Mehra and Prescott (1985), p. 155, Footnote 5, (11) Grossman et al. (1987), p. 324, Table 5 (datasets 1 to 4), (12) Weil (1989), p. 413, Table 1 and 2, (13) Constantinides (1990), p. 532, Table 1, (14) Epstein and Zin (1991), pp. 277, Tables 2 to 5, (15) Ferson and Constantinides (1991), pp. 216, Table 4, (16) Kandel and Stambaugh (1991), pp. 50, (17) Mankiw and Zeldes (1991), p. 109, Table 6, (18) Cochrane and Hansen (1992), p. 124, Figure 1, (19) Jorion and Giovannini (1993), pp. 1092, Table 2 and 3, (20) Cecchetti et al. (1994), p. 135, Table II and p. 149, (21) Campbell and Cochrane (1999), p. 244, (22) Guo and Whitelaw (2006), pp. 1447, Tables II, IV, V, and VI, (23) Bartunek and Chowdhury (1997), p. 121, Table I, (24) Aït-Sahalia and Lo (2000), p. 35, (25) Bliss and Panigirtzoglou (2004), p. 431, Table VI (all observations and power utility).

	Study	RRA Coefficient
Direct Assessments and Survey Data		
1	Arrow (1970)	1
2	Friend and Blume (1975)	~ 2
3	Wolf and Pohlman (1983)	2 to 4.5
4	Barsky et al. (1997)	4.2 or 12.1
5	Holt and Laury (2002)	0.3 to 0.5
Consumption-based Asset Pricing		
6	Grossman and Shiller (1981)	~ 4
7	Hansen and Singleton (1982, 1983)	0 to 2
8	Ferson (1983)	-1.4 to 5.4
9	Brown and Gibbons (1985)	0 to 7
10	Mehra and Prescott (1985)	55
11	Grossman et al. (1987)	> 20
12	Weil (1989)	45
13	Constantinides (1990)	2.8
14	Epstein and Zin (1991)	0.4 to 1.4
15	Ferson and Constantinides (1991)	0 to 12
16	Kandel and Stambaugh (1991)	29
17	Mankiw and Zeldes (1991)	35
18	Cochrane and Hansen (1992)	40 to 50
19	Jorion and Giovannini (1993)	5.4 to 11.9
20	Cecchetti et al. (1994)	≥ 6
21	Campbell and Cochrane (1999)	≥ 60
22	Guo and Whitelaw (2006)	1.6 to 7.8
Option Data		
23	Bartunek and Chowdhury (1997)	0 to 1
24	Aït-Sahalia and Lo (2000)	1 to 60
25	Bliss and Panigirtzoglou (2004)	2 to 9.5

suggest low levels of risk aversion between zero to about four. A second body of literature employs the framework of consumption-based asset pricing models introduced by Lucas (1978) and Breeden (1979) and their extensions. These studies rely on time series data on asset returns and aggregate consumption reporting RRA estimates ranging from zero to above 60. Furthermore, e. g. Campbell and Cochrane (1999) and Kandel and Stambaugh (1990, 1991) argue that high risk aversion is not as implausible as commonly believed. The heterogeneity is caused by the varying characteristics of the underlying valuation model in particular with respect to the preference specification, alternative econometric methods, and differing properties of the underlying dataset. A third body of literature employs option price data to obtain market risk preferences. For example, Bliss and Panigirtzoglou (2004) assume a parametric stationary form for the utility function, which they use to adjust the risk-neutral probability distribution function to deduce implied risk preferences.

Our empirical findings contribute to the dispute in literature on the reasonable level of risk aversion. We obtain a robust, pooled, implied RRA estimate of 1.78, indicating a moderate level of risk aversion. The magnitude is in line with RRA estimates reported by studies based on direct assessments, cross-sectional survey data, and option price data. However, our results contrast sharply with those of many studies relying on consumption-based asset pricing models to provide RRA estimates ranging above 20. Based on our representative agent expected utility model and data from an exceptionally clean bond market environment, we provide arguments for a moderate level of the pooled, overall RRA coefficient below two and find no evidence of "puzzling" risk aversion. We also obtain results concerning the dynamics of implied risk aversion and the relation between risk aversion and macroeconomic factors. Implied risk aversion is time dependent and attains maxima in 1980 and 1981, reflecting the challenging economic situation after the second oil crisis in 1979. Our results provide further evidence that severe economic crises coincide with periods of high risk aversion and suggest a structural break in the relation between changes in risk aversion and macroeconomic factors in 1977.

The outline of this thesis is as follows. Chapter 2 describes the payment structure as well as institutional facts for German redemption lottery bonds and gives a conclusive market overview. Chapter 3 introduces a dynamic equilibrium valuation model for redemption lottery bonds and provides comparative static results. Chapter 4 presents the dataset and briefly discusses the estimation procedure for the risk-free term structures of interest rates. In Chapter 5, we analyze the ex-day price behavior of lottery bonds and determine risk premia within a preliminary setting. We estimate equilibrium RRA coefficients in Chapter 6. Estimates are reported for several segmentations of the data panel, tested for robustness, and analyzed from a time series perspective. Chapter 7 contains concluding remarks.

Chapter 2

German Redemption Lottery Bonds

"[... I]n der Regel ist es nie der Klugheit angemessen, die Rückzahlung eines bedeutenden Anlehens auf einen bestimmten Termin festzusetzen [...]. Man setzt sich der Gefahr aus, daß unvorhergesehene Zufälle, die mit dem Rückzahlungstermin zusammen treffen, die Erfüllung der eingegangenen Verbindlichkeit erschweren, und bereitet sich oft für die Zukunft Verlegenheiten, die man ohne besondere Opfer hätte vermeiden können. Diese Gefahr wird in dem nämlichen Grade vermindert, als die Termine vervielfältigt werden, und nicht nur die Zinsen, sondern auch das Bedürfniß zur successiven Rückzahlung in bestimmten Einkünften ihre Deckung erhalten. [...] Da ein großes Anlehen unter viele Gläubiger vertheilt wird; so läßt sich aber eine successive Zurückzahlung im Einzelnen, ohne große Schwierigkeiten und Weitläufigkeiten, nicht vollziehen. Daher die Einrichtung, daß Partial-Obligationen von einem durchaus gleichen, oder verschiedene Klassen solcher Obligationen von gleichem Betrage ausgefertigt, und diejenigen Nummern, zu deren Tilgung die festgesetzte Rückzahlungssumme in jedem Verfalltermine verwendet werden soll, durch das Loos bestimmt werden."

Nebenius (1829), pp. 334.

2.1 Historical Aspects and Classification

Due to political and economical instability during the 19th and early 20th centuries, bond issuers feared that unforeseen circumstances could coincide with a single predefined redemption date. Redemption lottery bonds, repaid in constant fractions, were employed to reduce the refinancing risk associated with repayment at a single date.[1] Lottery bonds soon became an important long term government financing instrument in the German Reich and Prussia, surviving the Weimar Republic as well as the Second World War. During the early years of the Federal Republic of Germany, lottery bonds were the major instrument of federal government and state bond issues.[2]

Building on the historical motives for redemption lottery bonds, we provide arguments for their eventual disappearance.[3] We determine bond investors' risk preferences and find significant evidence of risk aversion. Investors demanded compensation for the redemption risk, which was by institutional facts systematic. Lottery bonds were more expensive than straight bonds, and similar payment structures could be achieved without introducing extraneous redemption risk, e. g. by issuing several individual bonds with different maturities. Aversion to redemption risk appears to be the fundamental motive for replacing lottery bonds with straight bonds in the 1970s. However, from a theoretical perspective, lottery bonds provide an exceptional environment for studying investors' risk preferences because they disbursed uncertain payoffs by an objective probability distribution.

As redemption lottery bonds are the primary focus of this thesis, we start with a brief classification of bonds with regard to the redemption feature.[4] We distinguish perpetuity or consol bonds with an unlimited lifetime from redemption bonds repaid according to a preset redemption schedule. The redemption schedule

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[1] See e. g. Nebenius (1829), pp. 333.

[2] See Müncks (1972), pp. 34.

[3] Smith and Villamil (1998) motivate the existence of lottery bonds within a theoretical framework and provide explanations for the utilization of apparently expensive government bonds with extraneously randomized returns.

[4] See Freund (1907), pp. 177, Dieben and Ebert (1958), pp. 50, Dreißig (1981), pp. 107, and Rothacker (1986), pp. 6 for detailed classifications of bonds with regard to redemption features.

obliges the issuer to retire a certain fraction of outstanding debt at predefined dates and redemption values. Redemption bonds are either repaid once at maturity or in several fractions prior to maturity. We differentiate between three types of bonds redeemed in fractions: (i) serial bonds, (ii) redemption lottery bonds, and (iii) sinking fund bonds.

Serial bonds are composed of individual series, and each series matures at a definite date.[5] Generally, individual series of a serial bond have distinct issue prices and listings in the secondary market. A variety of the serial bond is a debt contract composed of series, from which repayments are deducted proportionally.[6] This variety is also free of redemption risk for the creditor. In contrast to serial bonds and their varieties, the creditors of redemption lottery or sinking fund bonds are exposed to redemption risk.

Redemption lottery bonds are split into series approximately equal in size. In accordance with the redemption schedule, series are drawn by lottery and repaid. Hence, the bearer of a single series does not know when his bond will be repaid. Lottery bonds are classified according to the form of the redemption schedule. The redemption schedule either sets constant repayment rates or repayments that increase over time such that the amortization and interest burden is constant for the issuer. Lottery bonds characterized by a constant amortization and interest burden are denoted annuity bonds.[7]

Sinking fund bonds are also split into series and oblige the issuer to amortize

[5] The bond issues by the Free State of Bavaria of 1925 (6.5%), the Free State of Oldenburg of 1925 (7%), and the Free State of Anhalt of 1926 (7%) are examples for German serial bonds. German serial bonds were also issued in the course of the Gesetz zur endgültigen Regelung der Liquidations- und Gewaltschäden as of 1928 and the Polenschädenverordnung as of 1930 in conjunction with the Gesetz zur Durchführung der Entschädigung auf Grund des deutsch-polnischen Liquidationsabkommens as of 1930. See Dieben and Ebert (1958), pp. 52.

[6] The bond issues by the Free State of Prussia of 1931 (7%, second issue), the German Reich of 1944 (3.5%), and the state of Hesse of 1953 (5%, first and second issue) are examples for such varieties of serial bonds. See Dieben and Ebert (1958), pp. 52.

[7] The bond issues by the German Reich of 1923 (Schatzanweisungen K), the Free Hanseatic City of Bremen of 1925 (7%), and the Free State of Prussia of 1926 (6.5%) are examples for German annuity bonds. See Dieben and Ebert (1958), pp. 53.

a predefined portion of the debt prior to maturity.[8] Generally, the issuer has the option to either purchase the required quantity of bonds in the open market to offset redemption drawings or to call series at the predefined redemption value. When the issuer chooses to call bonds for redemption, series are drawn by lottery and repaid. The issuer will purchase the required quantity in the open market if bond prices are below the respective redemption value and will call bonds if prices are above the value. Most US-American corporate indentures are sinking fund bonds.[9]

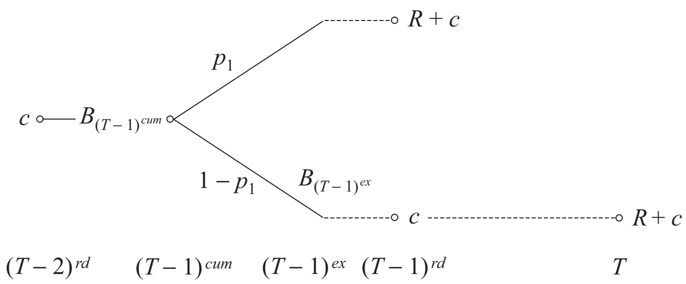
In addition to the redemption schedule, debt contracts contain various other redemption options. We differentiate between the following three early or increased redemption options: (i) issuer call options, (ii) increased redemption provisions, and (iii) open market repurchases. **Issuer call options** allow the borrower to call the entirety of outstanding series prior to maturity. If the embedded option is exercised, all outstanding series are repaid at the predefined redemption value, and the issue is delisted from the secondary market. Issuer call options were a common feature of public German debt contracts, whereas creditor put options were rarely granted. **Increased redemption provisions** enable the issuer to repay more than the scheduled series at predefined redemption values and dates. Series repaid in excess of the redemption schedule are deductable from future repayments. If the bond indenture allows for **open market repurchases**, the issuer has the privilege to repurchase bonds in the market to fulfill its redemption requirements. Repurchased bonds are again deductable from future redemption obligations. Several indentures contain more than one redemption option. However, most of the redemption options specify a time interval within which the options can be exercised and limit the number of series that can be additionally redeemed.

[8] We refer to sinking fund bonds repaid according to a redemption schedule.

[9] See e. g. Jen and Wert (1966), p. 697 and Ho and Singer (1984), p. 315.

Figure 2.1: **Structure of Last Redemption Lottery**

This figure depicts the time and informational structure of the last redemption lottery. We consider a lottery bond with annual coupon payments and redemption lotteries. The time structure is as follows: (i) at $(T-2)^{rd}$, the last coupon before the lottery is paid, (ii) $(T-1)^{cum}$ is the last date to trade the lottery bond before the lottery, (iii) at $(T-1)^{l}$, between $(T-1)^{cum}$ and $(T-1)^{ex}$, the lottery takes place, (iv) $(T-1)^{ex}$ is the first trading date after the lottery of those series not drawn, (v) at $(T-1)^{rd}$, the lottery bond series drawn in the lottery is redeemed, and the first coupon after the lottery is paid, (vi) at T, the last outstanding series is redeemed, and the last coupon is paid. The lottery bond price before the lottery is denoted by $B_{(T-1)^{cum}}$ and immediately after the lottery by $B_{(T-1)^{ex}}$. The redemption probability is $p_1 = 1/2$, the annual coupon of the lottery bond is c, and the redemption value is R.



2.2 Payment Structure and Institutional Facts

German redemption lottery bonds are fixed coupon bonds redeemed by lotteries. Before issuance, a bond is split into series of equal size identified by a series number. About three months before a redemption payment, the series to be redeemed is determined by a random drawing of the series numbers. The series that are not drawn for redemption participate in the following lottery. A sequence of lotteries is conducted until all but one series are drawn. A trustee monitors the indenture and calls the drawn bonds at the prearranged call price, which is usually the face value. Figure 2.1 characterizes the time and informational structure of the last redemption lottery.

For the issuer of a lottery bond, the redemption payments are deterministic. The bearer of a single bond series, however, does not know when his bond will be redeemed. Therefore, the maturities of outstanding bond series are uncertain until the last redemption date. However, the bearer knows the objective redemption probabilities of outstanding bond series at all future redemption dates.[10] These probabilities can easily be determined by calculating the ratio of the actual number of series to be redeemed to the total number of outstanding series. Hence, lottery bonds disburse uncertain payoffs by an observable, and thus objective, probability distribution. Lottery bonds provide an exceptional environment to study investors' tastes as summarized by their preference relation, since investors' beliefs about redemption risk are objectified. Furthermore, redemption risk is by construction independent of other random variables in the economy.

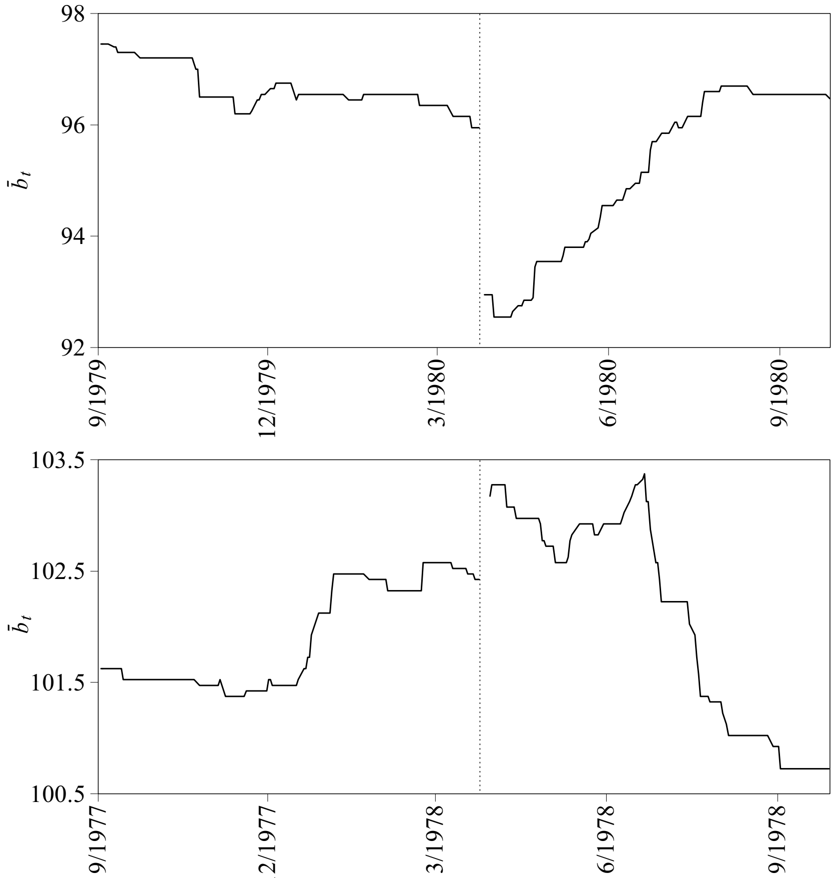
It is important to note that individual series were not traded on organized exchanges. As a consequence, there is only one price for a lottery bond, rather than a price for each outstanding series. The bearer of a lottery bond faces the risk or chance that his series will be drawn before the final maturity of the bond. If his bond belongs to the series that is drawn, he will receive the face value of the bond plus the coupon at the next coupon date. In addition, his bond will no longer take part in future lotteries. After a drawing date, only non-drawn series of the bond remain traded. If a lottery bond is trading below the discounted redemption value immediately before the drawing date, the price of the undrawn series will jump downwards, since the chance of being repaid at the subsequent redemption date is zero. Instead, the chance is postponed until the next drawing date, typically by one year. The upper graph in Figure 2.2 shows a typical downward price jump of a lottery bond issued by the Federal Republic of Germany around the last redemption lottery. Similarly, the price of the undrawn series of a lottery bond trading above the discounted redemption value immediately before the drawing date will jump upwards, since the risk of being repaid at the subsequent redemption date no longer exists. The lower graph in Figure 2.2 shows a typical upward price jump of the lottery bond around the next to last redemption lottery.

A basic problem for our study refers to the question whether lottery risk can be diversified. The following rules applied during the period in which lottery bonds were traded in Germany. In the primary market, the issuer assigned perfectly

[10] The redemption probabilities are conditional probabilities in the sense that they only apply to undrawn series.

Figure 2.2: **Price Jumps**

This figure shows the time series of the clean price in German Mark for a lottery bond issued by the Federal Republic of Germany (WKN 110022) in 1963. The upper graph shows a time interval including the last redemption lottery on March 21, 1980 (upper dotted line). The lower graph shows a time interval including the next to last redemption lottery on March 23, 1978 (lower dotted line). The lottery bond paid an annual coupon rate of 6% and was redeemed at par. Redemption lotteries were conducted biennially.



diversified portfolios of lottery bond series to retailers, i. e. portfolios with identical numbers of bonds from each series. Focusing on the secondary market, German government bonds were mainly traded in the over-the-counter market, and only marginal volumes were settled on organized exchanges such as the Frankfurt Stock Exchange.[11] Neither in the over-the-counter market nor on organized exchanges, lottery bonds were traded as diversified portfolios, and series-specific orders were not processed either.[12] This observation is of great importance for our study, as the available prices result from transactions in the secondary market. One explanation for this lack of diversification possibilities is the behavior of traders in banks who channelled orders to the market and who wanted to avoid research costs for individual series. As a consequence, an investor ordering a lottery bond was uncertain about the specific series he would receive until after the trade. The absence of trading possibilities for individual series impeded diversification efforts. Hence, we assume that lottery risk was systematic and had to be priced in equilibrium.[13]

The taxation of interest income in Germany was enforced by a withholding tax. Coupons of straight bonds and lottery bonds were subject to such a tax. Taxes were retained directly by the bank disbursing the coupon payments and were regarded as an advance income tax payment. Capital gains from government bonds, however, were tax exempt.[14] Therefore, a gain from the redemption lottery, which was considered a capital gain, was tax-free. Furthermore, a capital loss could not be used to reduce the tax base. In the following, we abstract from tax effects.

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[11] See Deutsche Bundesbank (2000), p. 17.

[12] Lottery bond traders of a large German bank affirmed that during the period of our analysis, transactions for individual series were not processed for reasonable order sizes.

[13] Bühler and Rothacker (1986) and Rothacker (1986) abstract from these frictions and assume German redemption lottery risk to be idiosyncratic. They develop a continuous-time no-arbitrage valuation model for redemption lottery bonds and analyze the performance of the model based on German bond market data.

[14] See paragraph 20(1.4) and paragraph 23(2.2) of the Einkommensteuergesetz (EStG) as of 1975. Paragraph 23(2.2) EStG was in place for the entire period of our study.

2.3 Market Size and Issuers

We consider the relevance of German redemption lottery bonds for public sector financing between 1971 and 1987 by focusing on the outstanding nominal volume.[15] Public sector bonds include securities issued by the Federal Republic of Germany, German states, German cities, the Deutsche Bundesbahn, and the Deutsche Bundespost. In 1971, the outstanding nominal volume of public sector lottery bonds (not due en bloc) was DEM 13.36 billion and DEM 18.30 billion for straight bonds (due en bloc). The volume of public sector lottery bonds reached its maximum at DEM 14.43 billion in 1973 and decreased to DEM 0.03 billion in 1987. In the same time interval, the volume of straight bonds increased twentyfold and reached DEM 383.33 billion in 1987. Figure 2.3 depicts the time series of the outstanding nominal volume of public sector bonds which are partially repaid or repaid once at maturity. The time series results illustrate the declining relevance of lottery bonds. In 1971, about 42% of the outstanding nominal volume of public bonds were partially repaid, whereas the remaining bonds were repaid once at maturity. However, in 1973, one of the last lottery bonds was issued by the public sector, and by 1987 lottery bonds had disappeared from the market.

Lottery bonds were issued by the public sector, financial agencies, and supranational institutions. We focus on lottery bond issuers and consider the number of lottery bond issues and the aggregate nominal issue volume. Table 2.1 compiles the aggregate number of issues and volume data segmented by issuer groups from 1955 onwards. Altogether, 238 lottery bonds were issued. The Federal Republic of Germany issued a total of eleven lottery bonds. German states issued 71 and German municipalities 16 lottery bonds. Additionally, 28 lottery bonds were issued by government-owned enterprises such as the Deutsche Bundesbahn and the Deutsche Bundespost, and 33 bonds were issued by financial agencies such as the IKB Industriekreditbank AG and Kreditanstalt für Wiederaufbau (KfW). Supranational institutions such as the Council of Europe Resettlement Fund, European Investment Bank, or World Bank issued another

[15] The data on the outstanding nominal volume were obtained from Deutsche Bundesbank (1971-1987), Statistical Supplements to the Monthly Report. Before 1971, the outstanding nominal volume of lottery bonds is not available.

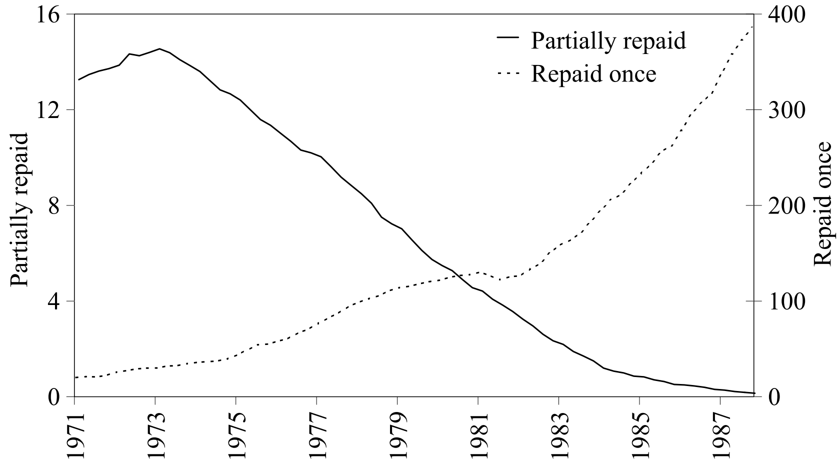
Table 2.1: **Lottery Bond Issuers and Issue Volume**

This table shows the number of lottery bond issues and the aggregate nominal issue volume segmented by issuer groups from 1955 onwards. The aggregate nominal issue volume is reported in billion German Mark. The data were obtained from Deutsche Finanzdatenbank (DFDB).

Issuers	Issues	Issue Volume
Federal Republic of Germany	11	3.80
German States	71	7.79
Baden-Württemberg	4	0.63
Bavaria	7	1.41
Berlin	6	0.60
Bremen	5	0.35
Hamburg	5	0.39
Hesse	5	0.70
Lower Saxony	9	1.23
North Rhine-Westphalia	1	0.20
Rhineland-Palatinate	10	0.88
Saarland	8	0.53
Schleswig-Holstein	11	0.87
German Municipalities	16	1.18
Cologne	2	0.20
Düsseldorf	1	0.05
Essen	1	0.06
Munich	7	0.55
Stuttgart	4	0.31
Wiesbaden	1	0.01
Government Enterprises	28	6.64
Deutsche Bundesbahn	13	3.48
Deutsche Bundespost	15	3.16
Financial Agencies	33	3.43
IKB Industriekreditbank	21	1.36
Kreditanstalt für Wiederaufbau (KfW)	12	2.07
Supranational Institutions	79	10.44
Council of Europe Resettlement Fund	23	2.54
Eurofima	10	0.68
European Coal and Steel Community	20	2.64
European Economic Community	5	0.93
European Investment Bank	11	1.39
Inter-American Development Bank	2	0.16
World Bank	8	2.10
Total	238	33.28

Figure 2.3: **Time Series of Outstanding Nominal Volume**

This figure shows the time series of the outstanding nominal volume of public sector bonds that were partially repaid (solid line) and that were repaid once at maturity (dotted line). The outstanding nominal volume is reported in billion German Mark. The data were obtained from the Deutsche Bundesbank (1971-1987), Statistical Supplements to the Monthly Report.



79 lottery bonds. The aggregate nominal issue volume for these issuer groups amounts to DEM 33.28 billion. The mean nominal volume per issue is largest for the Federal Republic of Germany with DEM 345 million followed by government enterprises with DEM 237 million. The mean nominal volume per issue for the remaining issuer groups is between DEM 73 million and DEM 132 million.

Figure 2.4 depicts the absolute frequency of bond issues per year segmented for the six issuer groups. From 1959 until 1965, the Federal Republic of Germany issued between one to four lottery bonds per year. German states, municipalities, government enterprises, and financial agencies issued their last lottery bonds either in 1972 or 1973. Most lottery bonds were issued by supranational institutions. However, about 54 issues were placed after 1973, when supranational institutions were the only remaining issuers of lottery bonds.

In addition to the scheduled redemption by lottery, debt contracts contained embedded redemption options such as issuer call features, increased redemption provisions, and open market repurchases. Table 2.2 compiles the number of lottery bonds that contained these redemption options segmented by issuer

Figure 2.4: **Issue Years**

This figure shows the absolute frequency of bond issues per year segmented for issuer groups. The data were obtained from Deutsche Finanzdatenbank (DFDB).

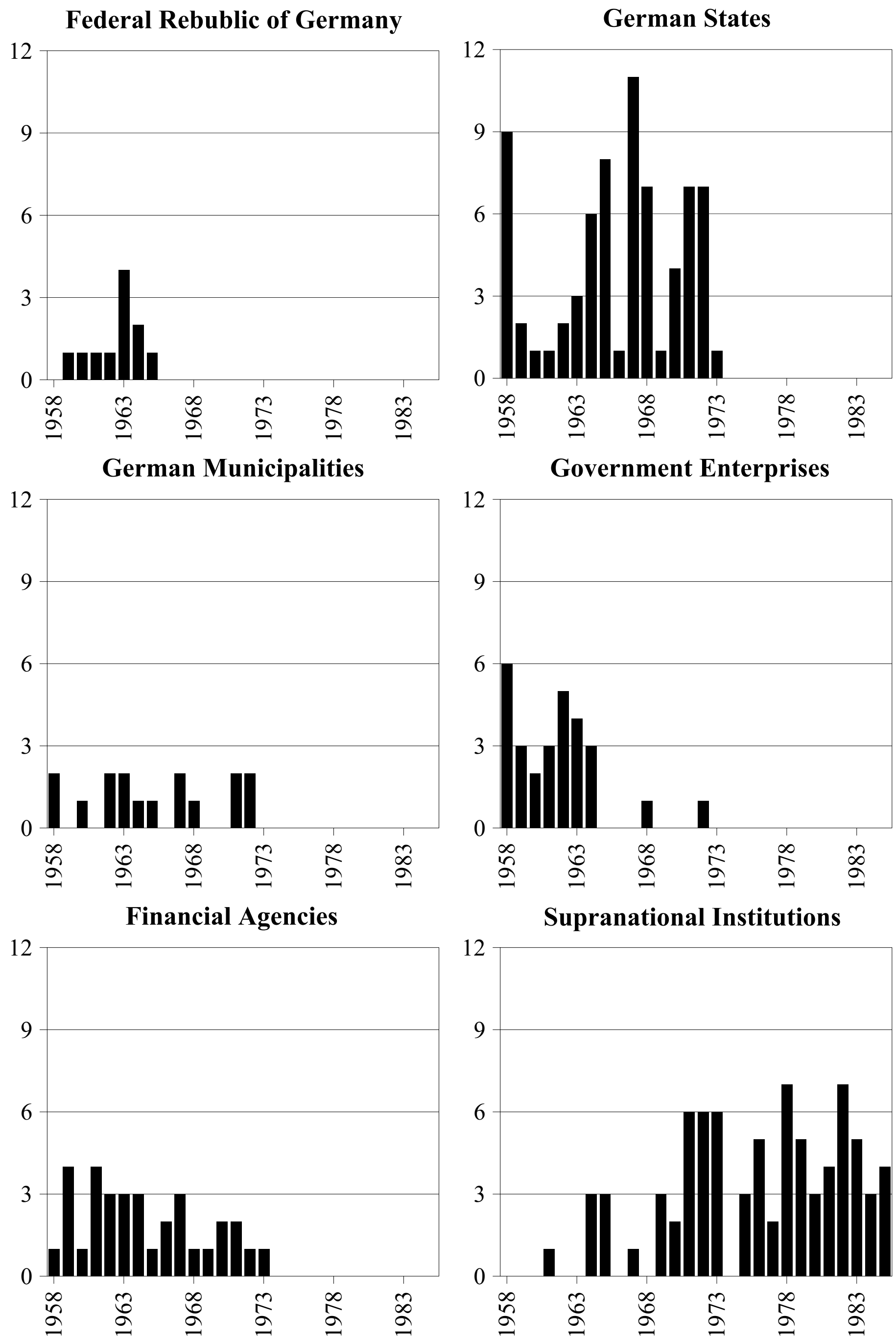


Table 2.2: **Redemption Options**

This table shows the number of lottery bond issues that contained early or increased redemption options. The options are segmented by the following issuer groups: Federal Republic of Germany (FRG), German states (GS), German municipalities (GM), government enterprises (GE), financial agencies (FA), and supranational institutions (SI). The data were obtained from Deutsche Finanzdatenbank (DFDB).

Redemption Options	FRG	GS	GM	GE	FA	SI
Issuer Call Feature						
Not Callable	2	25	3	6	0	12
Callable, but Not Exercised	9	44	13	22	33	44
Callable and Exercised	0	2	0	(2)[a]	0	23
Increased Redemption Provision						
Not Possible	2	26	5	6	1	71
Possible, but Not Exercised	9	38	11	22	32	8
Possible and Exercised	0	7	0	(2)[a]	0	0
Open Market Repurchases						
No Offsetting	11	53	15	28	0	51
Offsetting, but Not Exercised	0	18	1	0	32	26
Offsetting and Exercised	(1)[b]	0	0	0	1	2

[a] For two bond issues by the Deutsche Bundespost (WKN 116001, 106002), either the embedded issuer call feature or the increased redemption provision was exercised in October 1963 and in May 1964, respectively.

[b] One bond indenture issued in 1990 by the FRG (WKN 117018, DEM-Fundierungsschuld) is excluded because the issuer used the privilege to repurchase bonds in the open market to fulfill their redemption requirements. However, the indenture is classified as a sinking fund rather than a redemption lottery bond, since the issuer used the option to either call series by lottery or purchase the required quantity of redeemable bonds in the open market.

groups. Of the 238 German lottery bond issues, 190 indentures contained an embedded issuer call feature, 127 contained an increased redemption provision, and 80 allowed for open market repurchases. The large majority of lottery bond indentures (about 80%) were equipped with an embedded issuer call feature. After an initial call-free period of several years, the call feature enabled the issuer to redeem the lottery bond before the last scheduled redemption payment. Callable bonds permitted early redemption either at a coupon or at a scheduled redemption date.[16] The announcement period for the exercise of the option

[16] Only two lottery bond issues by the state of Schleswig-Holstein (WKN 179009, 179010) were callable either at the end or the beginning of the month.

covered at least three months.[17] The call price was usually equal to the face value of the lottery bond plus a premium that decreased to zero as the maturity of the lottery bond approached. About 25 lottery bonds were called before the last redemption date. Of these, two bonds were issued by German states, and the remaining 23 bonds were issued by supranational institutions. The restrictive call policies of all issuer groups besides supranational institutions are consistent with the findings of Bühler and Schulze (1993, 1999). They show that, between 1960 and 1988, only two straight coupon bonds by the Federal Republic of Germany and no bonds by the Deutsche Bundesbahn or the Deutsche Bundespost were called early. Both bonds were called on March 8, 1978 when interest rates in the German bond market dropped to their lowest level within the period we analyze.

More than half of the lottery bond indentures were equipped with an increased redemption provision. For all issuer groups other than supranational institutions, increased redemption provisions were a common feature. Most of the provisions allowed for the unrestricted deduction of additionally redeemed series from future scheduled repayments. Six provisions restricted the offsetting to the last scheduled repayments, and one issue by the state of Bavaria (WKN 105025) contained the option to redeem at most one additional series per lottery date. Seven lottery bond issues by German states actually exercised the increased redemption provision.

About one-third of the lottery bond indentures was equipped with the option to purchase bonds in the open market to offset redemption drawings. Bond issues by the Federal Republic of Germany and government enterprises did not allow for open market repurchases, but all issues by financial agencies were equipped with such an option. Most of the open market repurchases allowed for offsetting only when the respective series was drawn for redemption. However, six issues allowed for unrestricted offsetting, and the issuer had the option to either call series by lottery or to purchase the required quantity of redeemable bonds in the open market. Hence, these bonds are rather classified as sinking fund than redemption lottery bonds.[18] Open market repurchases have been recorded for only three

[17] Only one lottery bond issue by the municipality of Düsseldorf (WKN 118003) had an announcement period of two months.

[18] See Section 2.1 for the classification of redemption lottery and sinking fund bonds.

lottery bond issues by financial agencies and supranational institutions implying that repurchases were a rather unimportant redemption feature.[19]

2.4 Other Selected Lottery Bond Indentures

So far, we have focused on German redemption lottery bond indentures. However, lottery bonds have also been issued by other European governments and government institutions. We distinguish redemption lottery bonds from coupon lottery bonds and give a brief overview of selected European government lottery bond markets.

First, we consider redemption lottery bond markets. The **Belgian Government** issued redemption lottery bonds until 1975. Of 32 Belgian government bonds (*Emprunts Belge*) issued between 1961 and 1975, 26 indentures contained a redemption lottery with similar characteristics to the German redemption feature.[20] Furthermore, several issues by state enterprises were also repayable by lottery. On behalf of the Belgian Government, the Féderation des Coopératives pour Dommages de Guerre issued seven peculiar redemption lottery bonds (*Emprunts à Lots*) between 1921 and 1941 with maximum maturities ranging from 60 to 90 years.[21] The first two lottery bonds matured in 2001 and 2008, and the last three bonds will mature in 2011, 2012, and 2013. The issues are annuity bonds paying an annual coupon of 4%. Before issuance, the bonds were divided into series that are redeemed by a two-tier redemption lottery defined in the redemption schedule.[22] The redemption schedule fixes the lottery dates, the number of series to be redeemed, and the redemption payments. The number of series to be redeemed increases over time such that the amortization and

[19] Reiter (1967), p. 275, also characterizes open market repurchases as a redemption feature that is only rarely used.

[20] See Banque Nationale de Belgique (2008).

[21] The setup of *Emprunts à Lots* is exemplarily discussed for the issue of 1922 (ISIN Code: BE000402140).

[22] Between 1956 and 1959, the original lottery bond indentures were converted, and nominal values, coupons, and redemption schedules were adjusted.

interest burden remains constant. Unlike the German redemption lottery bonds, both the redemption date of an individual series and its repayment value are stochastic. However, the distribution of the repayments at all future lottery dates can be determined from the redemption schedule. In the first monthly redemption lottery, about 25 series are repaid at up to 800 times their face value. In addition, a second annual redemption lottery determines series that are repaid at a premium of about 20% above face value. Overall, less than 0.14% of the series are redeemed in the first prize drawing, whereas the large majority of series is drawn in the second redemption lottery. Moreover, the indentures contain an embedded issuer call option, which has historically never been exercised. The bonds are traded on Euronext Brussels, and in August 2008 a nominal volume of EUR 12.3 million remained outstanding.

On behalf of the **Italian Government**, the Instituto Mobiliare Italiano issued 14 redemption lottery bonds between 1945 and 1963.[23] The indentures were annuity bonds paying a semi-annual coupon between 5% to 6%. Redemption lotteries were conducted either once or twice a year, and the maximum maturity of the issues ranged between 14 to 20 years. Lottery bonds were traded on the Italian stock exchange. Until 1971 bonds were exempt from taxes on interest payments and capital gains.

Coupon lottery bonds are debt contracts paying coupons determined by lotteries. Generally, the total amount of interest paid on any given coupon date is fixed in the coupon schedule. However, the allocation of coupon payments across bonds within the issue is determined by lottery. For the issuer of a coupon lottery bond, interest payments are deterministic, whereas the size of coupon payments is stochastic for the bearer of a single bond. Notwithstanding, the bearer knows the objective probability distribution of the coupon prizes at all future coupon dates. Coupon schedules can be designed such that a proportion of the coupon payment is guaranteed for the bondholders.

Coupon lottery bonds appear in various forms and structures and have historically

[23] See Schilbred (1973, 1974).

been issued by governments in most of the principal European countries.[24] We give a brief overview of selected coupon lottery bond markets. The **Imperial Russian Government** issued lottery bonds in 1864 and 1866.[25] Russian lottery bonds contained a combined redemption and coupon lottery. Bond series were redeemed by lottery according to a redemption schedule with a maximum maturity of 60 years. The allocation of coupon payments across bonds was also determined by lottery. A semi-annual coupon of 2.5% was guaranteed, and additional cash prizes preset in the coupon schedule were randomly drawn. Lottery bonds were exempt from taxes on interest payments, capital gains, and lottery proceeds. In 1870, the nominal outstanding volume of Russian government debt was RUB 300 million, of which RUB 200 million was attributed to lottery bonds. The two Russian lottery bond issues eventually defaulted.

The **Swedish Government** has issued coupon lottery bonds since 1918.[26] Coupon lotteries have a two-tier structure and are organized two or three times a year. Large prizes are randomly drawn from all bonds in the issue, and smaller prizes are drawn from a sequence of bonds. Lottery bonds are traded in mixed and sequenced form on the Stockholm Stock Exchange. The sequenced form guarantees a proportion of the coupon payment to bond holders. Lottery proceeds are subject to a lottery tax at a flat rate of 20% and are exempt from income tax. Between 1977 and 1997, the Swedish Treasury issued one to three lottery bonds per year with five to ten years to maturity. About 46 issues were outstanding over the period 1986 to 1997. In 2007, the nominal issue volume of Swedish lottery bonds was SEK 9.1 billion, and the total outstanding nominal volume amounted to SEK 38.2 billion.[27]

The **Danish Government** has issued coupon lottery bonds since 1948.[28] The construction of the coupon lotteries is similar in Denmark and Sweden. Coupon lotteries are organized once or twice a year. Most bonds receive no prizes, but a

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[24] See Lévy-Ullmann (1896) for details on the historical dispersion of coupon lottery bonds in Europe.

[25] See Ukhov (2005).

[26] See Green and Rydqvist (1997, 1999).

[27] See Swedish National Debt Office (2007), pp. 18.

[28] See Florentsen and Rydqvist (2002).

few winning bonds receive a coupon payment up to 10,000 times the face value, ranging from DKK 50 to 200. Lottery proceeds are subject to a lottery tax at a flat rate of 15% for payments above DKK 200 and are exempt from income tax. Between 1948 and 1980, the Danish Treasury floated seven issues with ten to 25 years initial maturity. Until 1998, maturing issues were prolonged such that the bonds were effectively perpetuities. Bonds that have been prolonged contain creditor put options, and bonds issued after 1968 contain issuer call options. Lottery bonds are traded on the Copenhagen Stock Exchange. The aggregate nominal issue volume of Danish lottery bonds is DKK 1.2 billion.

On behalf of the **British Government**, National Savings and Investments has issued premium bonds since 1956.[29] Similarly to coupon lottery bonds, British premium bonds pay coupons determined by lottery. The interest rate used to calculate the prize fund is set by National Savings and Investments in advance. For each British Pound invested, bondholders are assigned a chance to win. Prize drawings are organized at the beginning of each month. Prizes range from GBP 50 to 1,000,000 and are exempt from income and capital gains tax. The major differences from coupon lottery bonds are that British premium bonds are not transferable and that their face values are guaranteed. A premium bond can be exchanged against its face value in cash at any time. Hence, premium bonds are not traded in a financial market and are similar to lottery-linked deposit accounts studied by e. g. Guillén and Tschoegl (2002). The minimum amount that can be invested in premium bonds is GBP 100, and investments are limited to GBP 30,000 per investor. In 2007, total funds invested in premium bonds amounted to GBP 35.3 billion.[30]

The **German Government** issued a premium bond, the so called baby-bond, in 1951.[31] The construction of the German premium bond was similar to the British one. The interest amount was channelled into a fund from which coupon payments were determined by lottery. Coupon lotteries were organized four times a year. At each lottery date, DEM 625,000 were allocated to 270 randomly drawn

[29] See Lobe and Hölzl (2007).

[30] See National Savings and Investments (2007), p. 22.

[31] See Achterberg and Muthesius (1951a, 1951b), Dieben and Ebert (1958), pp. 54, Reiter (1967), pp. 161.

bonds. Prizes were spread from DEM 50 to 50,000 and exempt from income and capital gains tax. The face value of the bonds was DEM 10. Premium bonds matured in 1956 and were repaid at face value. The nominal issue volume of the German premium bond was DEM 50 million. Premium bonds were not accepted by investors, and only a nominal volume of about DEM 38 million was distributed until 1954. Hence, the German Treasury withdrew a volume of DEM 12 million from the market. Ever since, there have been no further attempts to issue premium bonds in Germany.

Chapter 3

Valuation of Lottery Bonds and Implied Risk Aversion

3.1 One-period Model Framework

3.1.1 Model Setup and Assumptions

In this chapter, we address the problem of optimal portfolio selection and equilibrium asset pricing within a dynamic discrete-time model framework.[1] We consider a pure exchange economy without market frictions like transaction costs, short-selling constraints, or taxes. The market consists of one issue of a typical lottery bond with annual redemption lotteries, annual coupon payments c, longest maturity T, and redemption value R, as well as risk-free zero-bonds with multiple maturities. Interest rates are assumed to be risk-free.[2] Hence, the only risk in our economy is redemption risk. For ease of notation, we assume a flat risk-free

[1] Samuelson (1969) was one of the first to develop a dynamic model in discrete-time examining lifetime planning of consumption and investment decisions. Deviating from Samuelson, our modeling approach does not allow for interim consumption. As central results remain unchanged and notation is simplified, we focus on terminal consumption.

[2] Note that the integration of interest rate risk would introduce a subjective probability component and equilibrium prices as well as implied risk preferences would depend on probability beliefs.

term structure of interest rates at the interest level r throughout Chapter 3.[3]

Contrary to standard asset pricing models, we assume that there are as many homogeneous investor groups with identical risk aversion as there are series of the lottery bond. Each series is traded by one investor group in a separate market segment, and individual investor groups have access only to their group specific segment. This assumption ensures that lottery risk is of systematic nature and reflects the institutional setting that an individual investor can generally not trade multiple series in order to diversify lottery risk. Note that the assumption does not imply that an investor in one specific group knows when the series he trades will be drawn. Except for the differing series numbers, lottery bonds traded in all market segments are identical and will have the same equilibrium price. Hence, it is sufficient to focus on one segment. Whenever a specific series is drawn, the corresponding segment consists of zero-bonds only. Investors in such a segment are forced to invest their entire wealth in zero-bonds.

We assume that there exists a representative investor with a power utility function maximizing his terminal wealth and focus on the last redemption lottery first. At $(T-1)^{cum}$, shortly before the drawing, the investor optimally distributes his wealth among the lottery bond series and the risk-free instrument. If the agent invests in the lottery bond, he has to consider two disjoint states of the world at $(T-1)^{ex}$, shortly after the drawing. In state d, the investor holds the lottery bond series which has been drawn and in state n, he holds the series which has not been drawn.

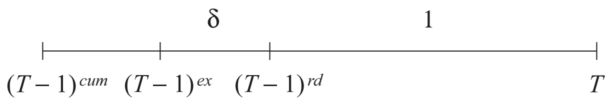
Event dates and the time structure of the one-period model are reported in Figure 3.1. The length of the time interval between $(T-1)^{cum}$ and $(T-1)^{ex}$ is denoted by ε, and the length of the interval between $(T-1)^{ex}$ and the redemption date $(T-1)^{rd}$ is equal to δ. The difference between $(T-1)^{rd}$ and final maturity T is one year.

We denote v_1^d the present value at $(T-1)^{ex}$ of future cash flows from the lottery bond given that the series is drawn at the lottery date $(T-1)^l$. Analogously, v_1^n

[3] In the empirical Chapters 5 and 6, we employ the risk-free term structure of interest rates estimated from daily market prices of straight coupon bonds and money market rates. See Section 4.2.2 for details on the term structure estimation.

Figure 3.1: **Time Structure**

This figure depicts the time structure of the one-period model. We consider a lottery bond with annual coupon payments and redemption lotteries. The time structure is as follows: (i) $(T-1)^{cum}$ is the last trading date of the lottery bond before the lottery, (ii) $(T-1)^{ex}$ is the first trading date after the lottery of those series not drawn, (iii) at $(T-1)^{rd}$, the series drawn in the lottery is redeemed, and the first coupon after the lottery is paid, (iv) at T, the last outstanding series is redeemed, and the last coupon is paid. We denote the length of the time interval between $(T-1)^{cum}$ and $(T-1)^{ex}$ by ε and the interval between $(T-1)^{ex}$ and $(T-1)^{rd}$ by δ. The difference between $(T-1)^{rd}$ and final maturity T is one year.



denotes the present value at $(T-1)^{ex}$ of future cash flows given that the series is not drawn at all. Considering the typical lottery bond with annual coupon and redemption frequency, the two expressions specify:[4]

$$v_1^d = \frac{R+c}{(1+r)^\delta}, \tag{3.1}$$

$$v_1^n = \frac{c}{(1+r)^\delta} + \frac{R+c}{(1+r)^{1+\delta}}$$

Note that, for $c/R = r$, i. e. $v_1^d = v_1^n$, the economy is free of redemption risk and the representative agent is indifferent between investing in the lottery bond or risk-free instrument. For ease of notation, we assume $c/R \neq r$ throughout this chapter.

[4] The model framework can easily be adapted for lottery bonds with biennial redemption frequency or semi-annual coupon payments.

3.1.2 No-arbitrage Bounds

Before proceeding with the analytical deduction of the optimal portfolio choice and the equilibrium price, we develop upper and lower bounds for the dirty price B_{cum} (price including accrued interest) of the lottery bond immediately before the drawing.[5] Traded lottery bonds consist of non-drawn series only. Hence, after the last drawing, the lottery bond corresponds to a straight coupon bond and its dirty price at $(T-1)^{ex}$ is equal to v_1^n.

At $(T-1)^{cum}$, the bearer of a lottery bond knows that, at $(T-1)^{ex}$, his series is either worth v_1^d in state d or traded at v_1^n in state n. Under the assumption of non-satiation, the following bounds hold:

$$\min\left[\frac{v_1^d}{(1+r)^{\varepsilon}}, \frac{v_1^n}{(1+r)^{\varepsilon}}\right] < B_{cum} < \max\left[\frac{v_1^d}{(1+r)^{\varepsilon}}, \frac{v_1^n}{(1+r)^{\varepsilon}}\right], \tag{3.2}$$

where the value terms are discounted at the risk-free rate to date $(T-1)^{cum}$. The bounds are independent of risk preferences and correspond to the minimum and maximum discounted value of the lottery bond. The no-arbitrage bounds can be transformed to bounds for the clean lottery bond price b_{cum} by adjusting v_1^d and v_1^n for accrued interest.

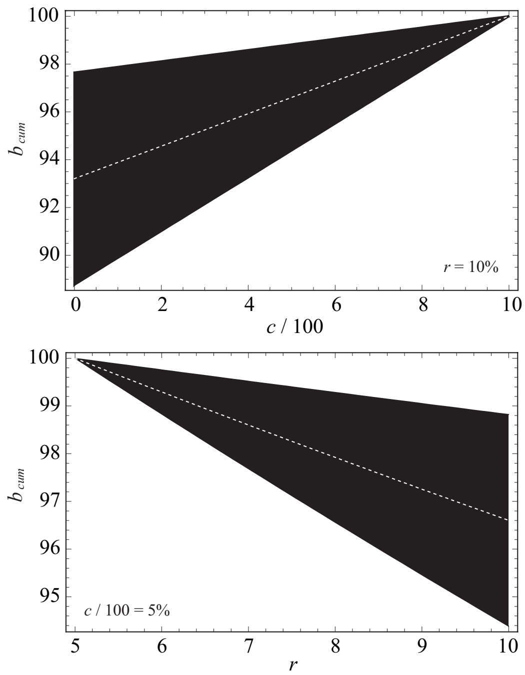
Figure 3.2 depicts the no-arbitrage bounds for below par clean lottery bond prices b_{cum} as a function of the coupon c and the risk-free rate r. Lottery bond prices that can be explained by no-arbitrage are located within the bounds characterized by the black surface. The larger the difference between c and r, the larger is the no-arbitrage price interval. Results are analogous for above par prices.

If we further assume that the investor is risk-averse, we can deduce a sharper upper bound on B_{cum}. Under risk neutrality, the lottery bond price is equal to

[5] For ease of notation, we omit the time index and use the superscripts (cum, ex, etc.) only, e. g. B_{cum} instead of $B_{(T-1)^{cum}}$. Unless stated otherwise, the superscript refers to $(T-1)$.

Figure 3.2: **No-arbitrage Bounds for Clean Lottery Bond Prices**

This figure plots the no-arbitrage bounds for below par clean lottery bond prices b_{cum} against the annual coupon c and the risk-free rate r. A lottery bond issue with two outstanding series and one outstanding lottery is considered. The redemption lottery is conducted annually, and drawn series are redeemed at face value. The time structure is as follows: ε is zero, and δ is equal to 90 days. In the upper graph, the risk-free rate is $r = 10\%$. In the lower graph, the coupon rate of the lottery bond is $c/100 = 5\%$. The black surface contains the clean lottery bond prices b_{cum} which can be explained by no-arbitrage. The dotted line characterizes the clean risk-neutral lottery bond price.



the expected value

$$B_{cum}^{e} = p_1 \cdot \frac{v_1^d}{(1+r)^{\varepsilon}} + (1-p_1) \cdot \frac{v_1^n}{(1+r)^{\varepsilon}}, \tag{3.3}$$

where $p_1 = 1/2$ is the objective probability of early redemption for the last redemption lottery. A risk-neutral investor is willing to pay at most B_{cum}^{e} for the lottery bond at $(T-1)^{cum}$. Hence, for a risk-averse investor, we obtain the following bounds:

$$\min\left[\frac{v_1^d}{(1+r)^{\varepsilon}}, \frac{v_1^n}{(1+r)^{\varepsilon}}\right] < B_{cum} < B_{cum}^{e}$$

The black surface below the dotted lines in Figure 3.2 characterizes clean lottery bond prices consistent with no-arbitrage and risk-averse preferences.

3.1.3 Optimal Portfolio Choice

We continue with the problem of optimal portfolio selection immediately before the last redemption lottery. Throughout this section, lottery bond prices are assumed to be exogenous and restricted to values within the no-arbitrage bounds defined by Inequalities (3.2). The representative investor has a state-independent power utility function defined for terminal wealth at T. The power utility function is given by

$$u\left(w_T^s\right) = \frac{\left(w_T^s\right)^{1-\gamma}}{1-\gamma} \quad \text{for } \gamma \neq 1,\ s \in \{d, n\}, \tag{3.4}$$

where γ is the RRA coefficient and w_T^s is investor's wealth at T in either state d or n.[6] The power utility function is characterized by linear risk tolerance decreasing absolute and constant relative risk aversion.[7] RRA coefficients indicate risk aversion if $\gamma > 0$, risk-seeking preferences if $\gamma < 0$, and risk neutrality if $\gamma = 0$.

The investor's utility maximization problem at $(T-1)^{cum}$ is characterized by the following expression:

$$\max_{x_{cum}} \mathbb{E}_{cum}\left[u\left(w_T^s\right)\right] = \tag{3.5}$$
$$\max_{x_{cum}} \left\{p_1 \cdot u\left(w_{ex}^d \cdot (1+r)^{1+\delta}\right) + (1-p_1) \cdot u\left(w_{ex}^n \cdot (1+r)^{1+\delta}\right)\right\},$$

where $\mathbb{E}_{cum}$ is the expectation operator conditional on the information given at $(T-1)^{cum}$ and x_{cum} is the proportion of wealth at $(T-1)^{cum}$ invested in the lottery bond. Between $(T-1)^{ex}$ and T the lottery bond is risk-free, and the wealth terms w_{ex}^s are transformed to wealth at T by risk-free compounding.

Expression (3.5) is maximized subject to the investor's budget constraint resulting in the following characterization of wealth at $(T-1)^{ex}$

$$w_{ex}^s = w_{cum} \cdot \left(x_{cum} \cdot \frac{v_1^s}{B_{cum}} + (1-x_{cum}) \cdot (1+r)^{\varepsilon}\right), \quad s \in \{d, n\}, \tag{3.6}$$

where w_{cum} is investor's wealth immediately before the lottery. Risk-free holdings are compounded at the risk-free rate from $(T-1)^{cum}$ to $(T-1)^{ex}$.

The first order condition of the general utility maximization problem (3.5) to

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[6] In the case of $\gamma = 1$, $u(w_T^s) = \log[w_T^s]$. Unless stated otherwise, $\gamma \neq 1$ throughout this chapter.

[7] See e.g. Pratt (1964), Yaari (1969), and Arrow (1970).

(3.6) is

$$\frac{\partial \mathbb{E}_{cum}\left[u\left(w_T^s\right)\right]}{\partial x_{cum}} \stackrel{!}{=} 0 \Leftrightarrow \frac{\left(\frac{v_1^d}{B_{cum}\cdot(1+r)^{\varepsilon}}-1\right)}{\left(x_{cum}\cdot\left(\frac{v_1^d}{B_{cum}\cdot(1+r)^{\varepsilon}}-1\right)+1\right)^{\gamma}} + \frac{\left(\frac{v_1^n}{B_{cum}\cdot(1+r)^{\varepsilon}}-1\right)}{\left(x_{cum}\cdot\left(\frac{v_1^n}{B_{cum}\cdot(1+r)^{\varepsilon}}-1\right)+1\right)^{\gamma}} = 0 \quad \text{for } \gamma \neq 0. \tag{3.7}$$

Solving Equation (3.7) for x_{cum}, we obtain the optimal portfolio composition

$$x_{cum}^* = \frac{B_{cum}\cdot(1+r)^{\varepsilon}}{B_{cum}\cdot(1+r)^{\varepsilon} - v_1^d + \frac{v_1^n - v_1^d}{\left(\frac{v_1^n - B_{cum}\cdot(1+r)^{\varepsilon}}{B_{cum}\cdot(1+r)^{\varepsilon} - v_1^d}\right)^{\frac{1}{\gamma}} - 1}} \quad \text{for } \gamma \neq 0. \tag{3.8}$$

The optimal portfolio composition x_{cum}^* is a function of: (i) the RRA coefficient γ, (ii) the exogenous lottery bond price B_{cum}, (iii) the coupon c, (iv) the redemption value R, and (v) the risk-free rate r.

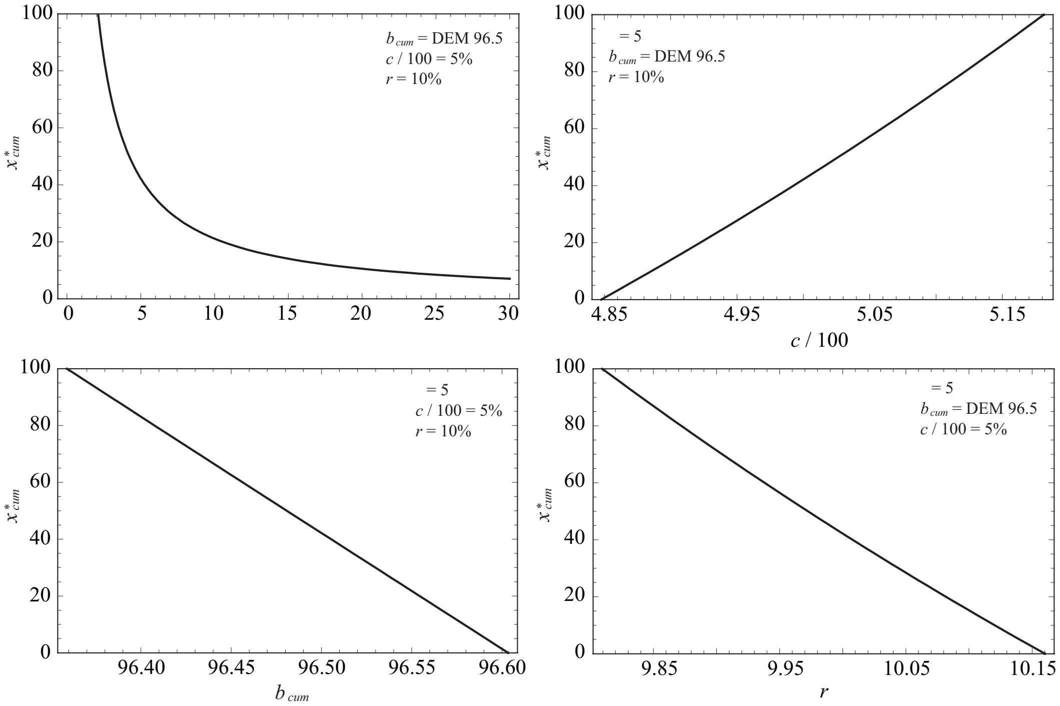
Comparative static results for x_{cum}^* are depicted in Figure 3.3.[8] For positive RRA coefficients, x_{cum}^* is decreasing in γ. The more risk-averse the investor is, the smaller the proportion of wealth invested in the risky lottery bond. For γ approaching infinity, x_{cum}^* converges to zero and the investor holds only risk-free zero-bonds. However, if γ converges to zero, x_{cum}^* approaches infinity and the investor short sells the zero bond and leverages his position in the lottery bond. For negative RRA coefficients, x_{cum}^* is increasing in γ.

The optimal portfolio composition x_{cum}^* is decreasing in the lottery bond price b_{cum}, increasing in the coupon c, and decreasing in the risk-free rate r. As expected, the demand for the lottery bond is negatively related to its price and opportunity cost and positively related to its payout. The results for the

[8] The sign of $\partial x_{cum}^*/\partial\gamma$ is derived in Appendix A.3.1.

Figure 3.3: **Comparative Statics for Portfolio Composition**

This figure plots the optimal fraction of wealth invested in the lottery bond x^*_{cum} against the RRA coefficient γ, the exogenous clean lottery bond price b_{cum}, the annual coupon c, and the risk-free rate r. We consider a lottery bond with two outstanding series equal in size. The redemption lottery is conducted annually, and drawn series are redeemed at face value. The time structure is as follows: ε is zero, and δ is equal to 90 days. In the upper and lower right graphs, b_{cum} is constant at DEM 96.5. In the upper left and lower graphs, $c/100 = 5\%$. In the upper and lower left graphs, $r = 10\%$. In the upper right and lower graphs, $\gamma = 5$. The optimal portfolio composition x^*_{cum} is reported in percentage points. All considered parameter combinations yield arbitrage-free lottery bond prices.



redemption value R are analogous to those of the coupon c. Exemplarily, we consider the relation between x^*_{cum} and the clean arbitrage-free lottery bond price in further detail. For b_{cum} equal to the clean risk-neutral lottery bond price, the fraction of wealth invested in the lottery bond is zero. If, however, b_{cum} converges to the lower or upper no-arbitrage bound defined in Inequality (3.2), x^*_{cum} approaches plus and minus infinity, respectively. These results persist for positive RRA coefficients and multiple combinations of the remaining parameters within a reasonable range. For negative RRA coefficients, the relation between x^*_{cum} and b_{cum}, c, and r is no longer monotonic. Relative to the results for positive RRA coefficients, the relations are inverted if γ is sufficiently negative.

3.1.4 Equilibrium Prices and RRA Coefficients

Based on the problem of optimal portfolio selection, we derive the equilibrium lottery bond price. We assume that the lottery bond is in unit-net supply. Therefore, in equilibrium, the condition

$$x_{cum} \equiv 1$$

has to hold. Solving the first order condition (3.7) with market clearing at B_{cum}, we obtain the equilibrium price

$$B^*_{cum} = \frac{\left(v_1^d\right)^{1-\gamma} + \left(v_1^n\right)^{1-\gamma}}{\left(v_1^d\right)^{-\gamma} + \left(v_1^n\right)^{-\gamma}} \cdot \frac{1}{(1+r)^{\varepsilon}} \quad \text{for } \gamma \neq 0. \tag{3.9}$$

For $\gamma = 0$, Equation (3.9) is not defined but can be complemented by the risk-neutral price B^e_{cum}, defined in Equation (3.3), such that B^*_{cum} is continuous and differentiable in γ. The equilibrium price B^*_{cum} is a function of: (i) the RRA coefficient γ, (ii) the coupon c, (iii) the redemption value R, and (iv) the risk-free rate r.

In Section 3.1.2, we derived no-arbitrage bounds for the lottery bond price. It is shown in Appendix A.3.2.1 that for γ approaching infinity, the equilibrium price is equal to the left-hand side of the no-arbitrage bound in Inequality (3.2) and, for γ approaching minus infinity, it is equal to its right-hand side.

By adjusting B^*_{cum} for accrued interest, we obtain the clean equilibrium price b^*_{cum}. Comparative static results for b^*_{cum} are depicted in Figure 3.4. The clean equilibrium price b^*_{cum} is decreasing in the RRA coefficient γ.[9] As expected, the higher the level of risk aversion, the lower is the equilibrium price of the risky lottery bond. Furthermore, b^*_{cum} is increasing in the coupon c and decreasing in the risk-free rate r. The relation for the redemption value R is analogous to that of the coupon c. The results persist for multiple parameter combinations within a reasonable range.

It is important to note that, for a fixed c, r, and R, the relation between the equilibrium price B^*_{cum} and γ is strictly monotonic decreasing. Hence, Equation (3.9) is a one-to-one mapping of the equilibrium price to the RRA coefficient. Solving the equilibrium pricing Equation (3.9) at a fixed arbitrage-free price B_{cum} for γ, we obtain the implied RRA coefficient

$$\gamma = -\frac{\log\left[\frac{B_{cum}-\frac{v_1^n}{(1+r)^{\varepsilon}}}{\frac{v_1^d}{(1+r)^{\varepsilon}}-B_{cum}}\right]}{\log\left[\frac{v_1^d}{v_1^n}\right]}. \tag{3.10}$$

The sensitivity of the implied RRA coefficient γ is of major importance throughout the empirical Chapter 6. The objective of the subsequent comparative static analysis is to provide theoretical evidence on the robustness of implied RRA coefficients helping us to ensure the quality and precision of our estimations. Therefore, we examine the implied RRA coefficient, the partial derivative of Equation (3.10), and the elasticity of the RRA coefficient with respect to the clean lottery bond price b_{cum} in further detail.

Figure 3.5 depicts the comparative static results for γ with respect to the coupon c and risk-free rate r. For an exogenous and fixed clean lottery bond price b_{cum},

[9] The sign of $\partial B^*_{cum}/\partial\gamma$ is derived in Appendix A.3.2.2.

Figure 3.4: **Comparative Statics for Equilibrium Price**

This figure plots the clean equilibrium lottery bond price b^*_{cum} against the RRA coefficients γ, the annual coupon c, and the risk-free rate r. We consider a lottery bond with two outstanding series equal in size. The redemption lottery is conducted annually, and drawn series are redeemed at face value. The time structure is as follows: ε is zero, and δ is equal to 90 days. In the upper graph, the coupon rate is $c/100 = 5\%$, and the risk-free rate is $r = 10\%$. Therefore, $v_1^d =$ DEM 102.53, $v_1^n =$ DEM 98.09, and accrued interest is DEM 3.71. The lower dashed line depicts the lower no-arbitrage bound for b^*_{cum}, and the upper dashed line depicts the maximum price a risk-neutral investor is willing to pay for the lottery bond. In the middle and lower graph, the RRA coefficient is $\gamma = 5$. In the middle graph, the risk-free rate is $r = 10\%$ and in the lower graph, the coupon rate is $c/100 = 5\%$.

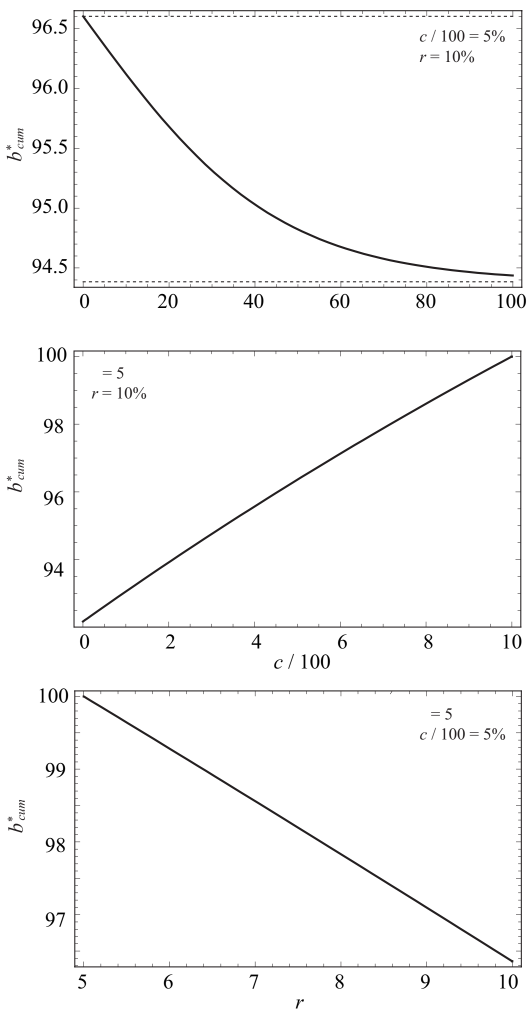
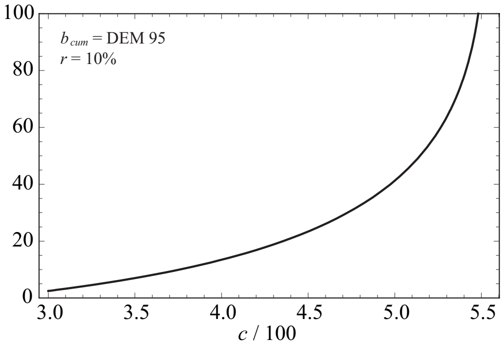
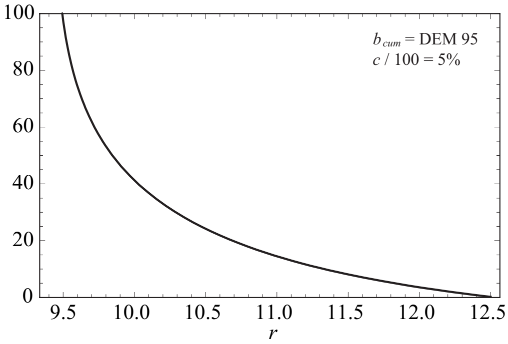


Figure 3.5: **Comparative Statics for RRA Coefficient**

This figure plots the RRA coefficient γ against the annual coupon c and the risk-free rate r. We consider a lottery bond with two outstanding series equal in size. The redemption lottery is conducted annually, and drawn series are redeemed at face value. The time structure is as follows: ε is zero, and δ is equal to 90 days. In both graphs, the clean lottery bond price b_{cum} is exogenous and constant at DEM 95. In the upper graph, the risk-free rate is $r = 10\%$ and in the lower graph, the coupon rate is $c/100 = 5\%$.





the implied RRA coefficient is increasing in the coupon c and decreasing in the risk-free rate r. The relation for the redemption value R is analogous to that of the coupon c, and results persist for multiple parameter combinations within a reasonable range.

The partial derivative $\partial\gamma/\partial b_{cum}$ is denoted $\gamma_{b_{cum}}$, and the elasticity measuring the responsiveness of γ to a change in b_{cum} in relative terms is defined

$$\eta_{\gamma,b_{cum}} = \gamma_{b_{cum}} \cdot \frac{b_{cum}}{\gamma}. \tag{3.11}$$

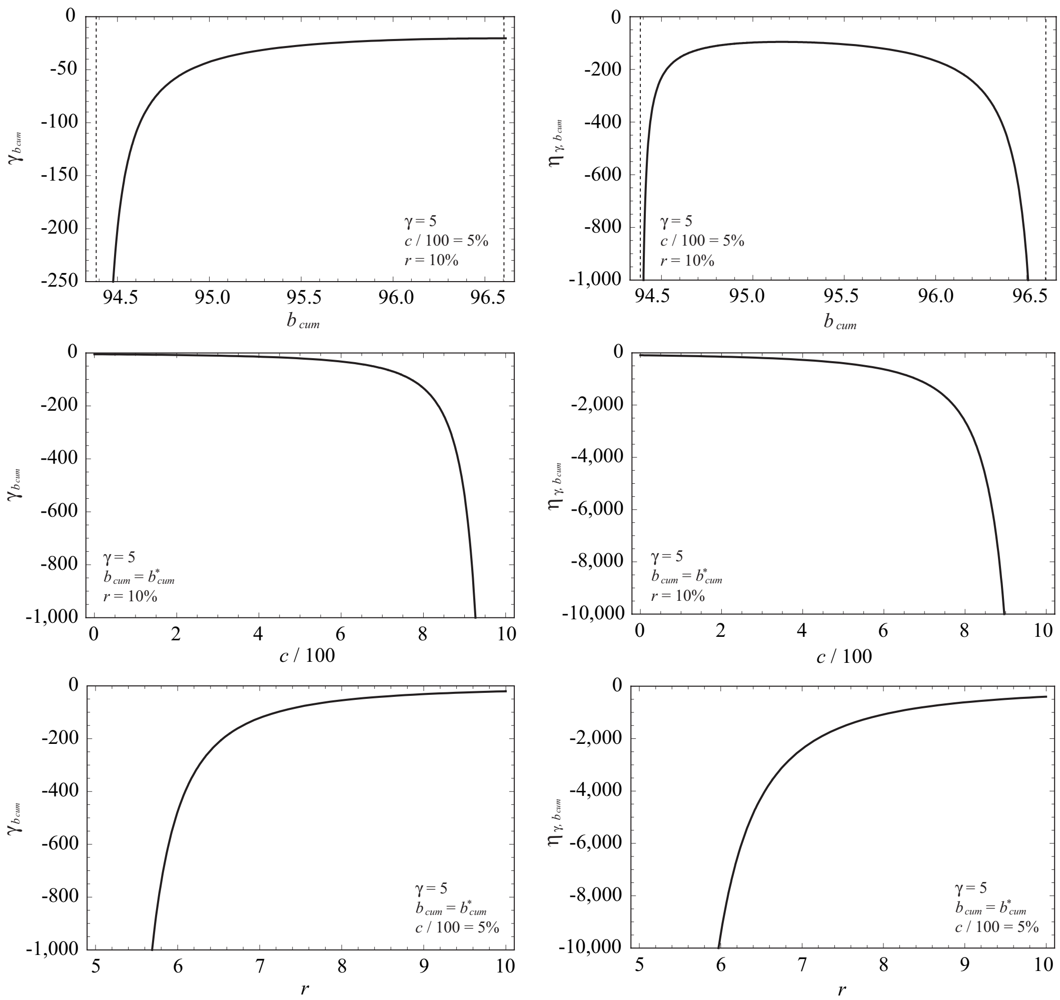
Figure 3.6 depicts the comparative static results for the partial derivative $\gamma_{b_{cum}}$ as well as elasticity $\eta_{\gamma,b_{cum}}$. First, we regard the comparative static results for $\gamma_{b_{cum}}$ and $\eta_{\gamma,b_{cum}}$ with respect to the exogenous and arbitrage-free clean lottery bond price b_{cum} in the upper graphs. For b_{cum} approaching the lower no-arbitrage bound defined by Inequalities (3.2) and characterized by the left dashed line, $\gamma_{b_{cum}}$ converges to minus infinity. The RRA coefficient is most sensitive to price changes close to the no-arbitrage bounds and least sensitive to price changes close to the risk-neutral price defined by Equation (3.3) and characterized by the right dashed line.[10] Note that the no-arbitrage bounds and risk-neutral lottery bond price are independent of γ. Accordingly, for b_{cum} approaching the no-arbitrage bounds or risk-neutral lottery bond price, $\eta_{\gamma,b_{cum}}$ converges to minus infinity implying a perfectly elastic RRA coefficient in b_{cum}.

Second, we consider the comparative static results for $\gamma_{b_{cum}}$ and $\eta_{\gamma,b_{cum}}$ with respect to the coupon c as well as to the risk-free rate r in the middle and lower graphs. The larger the spread between c and r the less sensitive is the RRA coefficient to a price change. For r approaching c/R, $\gamma_{b_{cum}}$ and $\eta_{\gamma,b_{cum}}$ converge to minus infinity.

[10] A small price change in close distance to the no-arbitrage bounds results in a distinct response of the RRA coefficient such that we control for the distance of price observations to the no-arbitrage bounds in the empirical Chapter 6.

Figure 3.6: **Derivative and Elasticity of RRA Coefficient**

This figure plots the derivative $\gamma_{b_{cum}}$ and elasticity $\eta_{\gamma,b_{cum}}$ against the exogenous and arbitrage-free clean lottery bond price b_{cum}, the annual coupon c and the risk-free rate r. We consider a lottery bond with two outstanding series equal in size. The redemption lottery is conducted annually, and drawn series are redeemed at face value. The time structure is as follows: ε is zero, and δ is equal to 90 days. The left dashed lines in the upper graphs depict the lower no-arbitrage bound for the clean lottery bond price, and the right dashed lines depict the maximum price a risk-neutral investor is willing to pay for the lottery bond. In all six graphs, the RRA coefficient is $\gamma = 5$. In the upper and middle graphs, the risk-free rate is $r = 10\%$, in the upper and lower graphs, the coupon rate is $c/100 = 5\%$, and in the middle and lower graphs, $b_{cum} = b^*_{cum}$.



3.2 Dynamic Model Framework

3.2.1 Model Setup and Assumptions

In this section, the one-period model is extended to a multi-period setting. Trading possibilities exist at every point in time from issuance until maturity of the lottery bond at T. First, we deduce equilibrium lottery bond prices at cum-lottery dates. Second, we show that ex-lottery prices are determined by simply adjusting the subsequent cum-price for coupon payments and risk-free discounting. Since the general structure of equilibrium prices between the ex- and the subsequent cum-lottery date is equal to that of the ex-lottery price, it is sufficient to characterize equilibrium prices at cum- and ex-lottery dates.

We denote $v^d_{i,j}$ the present value at $(T-i)^{ex}$ of future cash flows from the lottery bond given that the series is drawn at one of the future drawing dates $(T-j)^l$, $\forall i,j \in \mathbb{N}$, $1 \leq i < T$, and $i \geq j$. Analogously, $v^n_{i,1}$ denotes the present value at $(T-i)^{ex}$ of future cash flows given that the series is never drawn. The terms are applied to define the dynamic no-arbitrage bounds and the risk-neutral lottery bond price. Considering the typical lottery bond with annual coupon and redemption frequency, the expressions specify[11]

$$v^d_{i,j} = \sum_{k=0}^{i-j-1} \frac{c}{(1+r)^{k+\delta}} + \frac{R+c}{(1+r)^{i-j+\delta}}, \tag{3.12}$$

$$v^n_{i,1} = \sum_{k=0}^{i-1} \frac{c}{(1+r)^{k+\delta}} + \frac{R+c}{(1+r)^{i+\delta}}. \tag{3.13}$$

For ease of notation, we denote $v^d_{i,i}$, accumulating cash flows for one period, by v^d_i. The term is equal to v^d_1 defined in Equation (3.1). Analogously, we denote $v^n_{1,1}$ by v^n_1.

We define C_i as the present value at $(T-i)^{cum}$ of all cash flows the lottery bond

[11] The degenerated sum expression is defined $\sum\limits_{k=m}^{z} \equiv 0$ for $z < m$.

pays between $(T-i-1)^{ex}$ and $(T-i)^{cum}$. Considering a typical lottery bond with annual coupon and redemption frequency, the expression specifies

$$C_i = c \cdot (1+r)^{1-\varepsilon-\delta}\,, \tag{3.14}$$

where the difference between the last coupon and the redemption date $(T-i-1)^{rd}$ and $(T-i)^{cum}$ is equal to $(1-\varepsilon-\delta)$.

Note that, for $c/R = r$, the term $v_{i,j}^d$ simplifies

$$\begin{aligned} v_{i,j}^d &= R \cdot \left(\sum_{k=0}^{i-j-1} \frac{r}{(1+r)^{k+\delta}} + \frac{1}{(1+r)^{i-j-1+\delta}} \right) \\ &= R \cdot (1+r)^{1-\delta}\,. \end{aligned}$$

Analogously, the term $v_{i,1}^n$ simplifies to $R \cdot (1+r)^{1-\delta}$. Since $v_{i,j}^d = v_{i,1}^n$, the economy is free of redemption risk. Hence, for ease of notation we continue to assume $c/R \neq r$.

Figure 3.7 characterizes the time structure in the dynamic model framework. In the i-period model, we distinguish $i+1$ states of the world. The lottery bond series is either drawn with probability $p_i = 1/(i+1)$ in the first redemption lottery after the valuation date, or drawn in one of the $i-1$ subsequent lotteries, or not drawn at all.

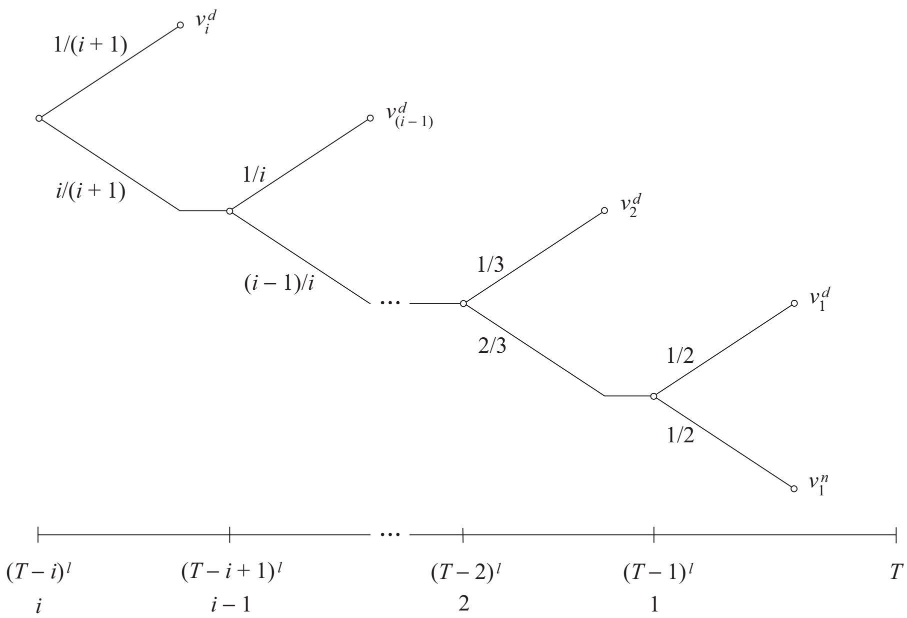
3.2.2 No-arbitrage Bounds

Before deducing analytical lottery bond prices in the dynamic model framework, we develop upper and lower bounds for the dirty prices $B_{(T-i)^{cum}}$.

In the i-period model, no-arbitrage bounds are determined from $i+1$ present

Figure 3.7: **Dynamic Time Structure**

This figure depicts the time structure in the dynamic model framework. We consider a lottery bond with annual redemption lotteries. Redemption probabilities p_i and present value terms are reported in the binomial lattice. The first row under the time line shows the lottery dates and the terminal date T. The second row shows the number of remaining lotteries at the corresponding cum-dates.



value expressions. For $i > 1$, the following bounds for $B_{(T-i)^{cum}}$ hold:[12]

$$B_{(T-i)^{cum}} > \min\left[v_i^d, v_{i,(i-1)}^d, ..., v_{i,1}^d, v_{i,1}^n\right] \cdot \frac{1}{(1+r)^{\varepsilon}}, \qquad (3.15)$$

$$B_{(T-i)^{cum}} < \max\left[v_i^d, v_{i,(i-1)}^d, ..., v_{i,1}^d, v_{i,1}^n\right] \cdot \frac{1}{(1+r)^{\varepsilon}}$$

If we further assume that the investor is risk-averse, we can deduce a sharper upper bound on $B_{(T-i)^{cum}}$. Under risk neutrality, the lottery bond price is equal to the expected value

$$B^e_{(T-i)^{cum}} = \frac{1}{i+1} \cdot \left(v_i^d + v_{i,(i-1)}^d + ... + v_{i,1}^d + v_{i,1}^n\right) \cdot \frac{1}{(1+r)^{\varepsilon}}. \qquad (3.16)$$

A risk-neutral investor is willing to pay at most $B^e_{(T-i)^{cum}}$ for the lottery bond. Hence, for a risk-averse investor, we obtain the following bounds:

$$\min\left[v_i^d, v_{i,(i-1)}^d, ..., v_{i,1}^d, v_{i,1}^n\right] \cdot \frac{1}{(1+r)^{\varepsilon}} < B_{(T-i)^{cum}} < B^e_{(T-i)^{cum}}$$

Analogously, the no-arbitrage bounds for the ex-lottery price $B_{(T-i)^{ex}}$ are determined by i present value expressions. For $i > 1$, the following bounds for $B_{(T-i)^{ex}}$ hold:

$$B_{(T-i)^{ex}} > \min\left[v_{i,(i-1)}^d, ..., v_{i,1}^d, v_{i,1}^n\right], \qquad (3.17)$$

$$B_{(T-i)^{ex}} < \max\left[v_{i,(i-1)}^d, ..., v_{i,1}^d, v_{i,1}^n\right]$$

If we again assume that the investor is risk-averse, we can deduce a sharper upper bound on $B_{(T-i)^{ex}}$. Under risk neutrality, the lottery bond price is equal to the

[12] No-arbitrage bounds for $B_{(T-1)^{cum}}$ are given by Inequalities (3.2).

expected value

$$B^e_{(T-i)^{ex}} = \frac{1}{i} \cdot (v^d_{i,(i-1)} + ... + v^d_{i,1} + v^n_{i,1}). \tag{3.18}$$

A risk-neutral investor is willing to pay at most $B^e_{(T-i)^{ex}}$ for the lottery bond. Hence, for a risk-averse investor, we obtain the following bounds:

$$\min\left[v^d_{i,(i-1)}, ..., v^d_{i,1}, v^n_{i,1}\right] < B_{(T-i)^{ex}} < B^e_{(T-i)^{ex}}$$

For $i = 1$, after the last drawing, the ex-lottery price $B_{(T-1)^{ex}}$ is deterministic and equals v^n_1.

3.2.3 Equilibrium Prices at Cum-dates

In the multi-period setting, equilibrium lottery bond prices are determined by backward induction using standard dynamic programming techniques.[13] As before, the representative investor has a state-independent power utility function as specified in Equation (3.4), which is defined for terminal wealth at T. First, we formulate investor's optimization problem and budget constraint. We assume a functional form of the indirect utility function and deduce the first order condition. Employing market clearing, we assess the equilibrium lottery bond price.

The investor's utility maximization problem at a point in time $(T-i)^{cum}$, $\forall i \in$

[13] See e. g. Ingersoll (1987), pp. 235 or Huang and Litzenberger (1988), pp. 179 for a comprehensive treatment of discrete-time inter-temporal portfolio selection and equilibrium valuation problems.

$\mathbb{N},\ 1 \leq i < T$, is characterized by the following expression:

$$\max_{x_{(T-i)^{cum}}} \mathbb{E}_{(T-i)^{cum}} \left[u \left(w_T^s \right) \right] = \tag{3.19}$$

$$\max_{x_{(T-i)^{cum}}} \left\{ p_i \cdot u \left(w^d_{(T-i)^{ex}} \cdot (1+r)^{i+\delta} \right) + (1-p_i) \cdot J_{(T-i)^{ex}} \left(w^n_{(T-i)^{ex}} \right) \right\},$$

where $p_i = 1/(1+i)$ is the objective probability of being drawn at $(T-i)^l$, and $x_{(T-i)^{cum}}$ is the proportion of wealth at $(T-i)^{cum}$ invested in the lottery bond. The indirect utility function $J_{(T-i)^{ex}}(\cdot)$ is the optimal value of investor's utility maximization problem at $(T-i)^{ex}$.

Expression (3.19) is maximized subject to the investor's budget constraint resulting in the following characterizations of wealth at $(T-i)^{ex}$ if the lottery bond series is drawn in the redemption lottery at $(T-i)^l$ and not drawn, respectively:

$$w^d_{(T-i)^{ex}} = w^n_{(T-i)^{cum}} \cdot \left(x_{(T-i)^{cum}} \cdot \frac{v_i^d}{B_{(T-i)^{cum}}} + \left(1 - x_{(T-i)^{cum}} \right) \cdot (1+r)^{\varepsilon} \right), \tag{3.20}$$

$$w^n_{(T-i)^{ex}} = w^n_{(T-i)^{cum}} \cdot \left(x_{(T-i)^{cum}} \cdot \frac{B^*_{(T-i)^{ex}}}{B_{(T-i)^{cum}}} + \left(1 - x_{(T-i)^{cum}} \right) \cdot (1+r)^{\varepsilon} \right),$$

where $w^n_{(T-i)^{cum}}$ is investor's wealth immediately before the lottery at $(T-i)^l$ given that his series was not drawn in any of the previous lotteries.

The functional form of the indirect utility function $J_{(T-i)^{ex}}(\cdot)$ is given by[14]

$$J_{(T-i)^{ex}}\left(w^n_{(T-i)^{ex}}\right) = \frac{1}{i} \cdot \left(w^n_{(T-i)^{ex}}\right)^{1-\gamma} \cdot \frac{a_{(T-i)^{ex}}}{\left(B^*_{(T-i)^{ex}}\right)^{1-\gamma}}, \tag{3.21}$$

where $a_{(T-i)^{ex}}$ is defined[15]

$$a_{(T-i)^{ex}} \equiv \sum_{j=1}^{i-1} u\left(v^d_j \cdot (1+r)^{j+\delta} \cdot \prod_{k=j}^{i-1} y_{(T-k)^{cum}}\right) + \tag{3.22}$$

$$u\left(v^n_1 \cdot (1+r)^{1+\delta} \cdot \prod_{k=1}^{i-1} y_{(T-k)^{cum}}\right),$$

$y_{(T-k)^{cum}}$ is defined

$$y_{(T-k)^{cum}} \equiv 1 + \frac{C_k}{B^*_{(T-k)^{cum}}}, \tag{3.23}$$

and C_k is defined in Equation (3.14).

For $i > 1$, the indirect utility function $J_{(T-i)^{ex}}(\cdot)$ depends on wealth $w^n_{(T-i)^{ex}}$ and future equilibrium prices $B^*_{(T-k)^{cum}}$, $\forall k \in \mathbb{N}$, and $1 \leq k < i$. In equilibrium, the

[14] It is shown in Appendix A.3.3.1 that:

$$J_{(T-i)^{ex}}\left(w^n_{(T-i)^{ex}}\right) = \max_{x_{(T-i)^{ex}}} \left\{J_{(T-i+1)^{cum}}\left(w^n_{(T-i+1)^{cum}}\right)\right\},$$

where the indirect utility function $J_{(T-i+1)^{cum}}(\cdot)$ is the optimal value of investor's utility maximization problem at $(T-i+1)^{cum}$. See e. g. Ingersoll (1987), pp. 235 for a thorough derivation of the functional form of the indirect utility function in the classical portfolio selection framework.

[15] Throughout this section the degenerated sum and product expressions are defined as

$$\sum_{j=m}^{z} v^d_j \equiv 0 \text{ and } \prod_{k=m}^{z} y_{(T-k)^{cum}} \equiv 1 \text{ for } z < m.$$

lottery bond is in unit-net supply, and the term $y_{(T-k)^{cum}}$ characterizes the gross growth in wealth between $(T-k-1)^{ex}$ and $(T-k)^{cum}$.

The first order condition of the general utility maximization problem (3.19) to (3.20) with respect to the portfolio composition $x_{(T-i)^{cum}}$ is

$$\begin{aligned}
&\frac{\partial\left\{\mathbb{E}_{(T-i)^{cum}}\left[u\left(w_T^s\right)\right]\right\}}{\partial x_{(T-i)^{cum}}} \stackrel{!}{=} 0 \Leftrightarrow \\
&p_i \cdot u'\left(w^d_{(T-i)^{ex}}\right) \cdot \left((1+r)^{i+\delta}\right)^{1-\gamma} \cdot \left(\frac{v_i^d}{B_{(T-i)^{cum}}} - (1+r)^{\varepsilon}\right) + \\
&(1-p_i) \cdot J'_{(T-i)^{ex}}\left(w^n_{(T-i)^{ex}}\right) \cdot \left(\frac{B^*_{(T-i)^{ex}}}{B_{(T-i)^{cum}}} - (1+r)^{\varepsilon}\right) = 0 \qquad \text{for } \gamma \neq 0.
\end{aligned}$$

We assume that the lottery bond is in unit-net supply and the following condition

$$x_{(T-i)^{cum}} \equiv 1$$

holds. Solving the first order condition with market clearing at $B^*_{(T-i)^{cum}}$, we obtain the equilibrium price

$$\begin{aligned}
B^*_{(T-i)^{cum}} = &\frac{1}{(1+r)^{i+\varepsilon+\delta}} \cdot \\
&\frac{\sum_{j=1}^{i}\left(v_j^d \cdot (1+r)^{j+\delta} \cdot \prod_{k=j}^{i-1} y_{(T-k)^{cum}}\right)^{1-\gamma} + \left(v_1^n \cdot (1+r)^{1+\delta} \cdot \prod_{k=1}^{i-1} y_{(T-k)^{cum}}\right)^{1-\gamma}}{\sum_{j=1}^{i}\left(v_j^d \cdot (1+r)^{j+\delta} \cdot \prod_{k=j}^{i-1} y_{(T-k)^{cum}}\right)^{-\gamma} + \left(v_1^n \cdot (1+r)^{1+\delta} \cdot \prod_{k=1}^{i-1} y_{(T-k)^{cum}}\right)^{-\gamma}},
\end{aligned} \tag{3.24}$$

for $\gamma \neq 0$. The equilibrium price $B^*_{(T-i)^{cum}}$ is a function of the discounted present value expressions v_j^d and v_1^n, which depend on the coupon of the lottery bond c, the redemption value R, and the risk free rate r, as well as the future equilibrium prices $B^*_{(T-k)^{cum}}$, $\forall k \in \mathbb{N}$ and $1 \leq k < i$ contained in the $y_{(T-k)^{cum}}$ terms.

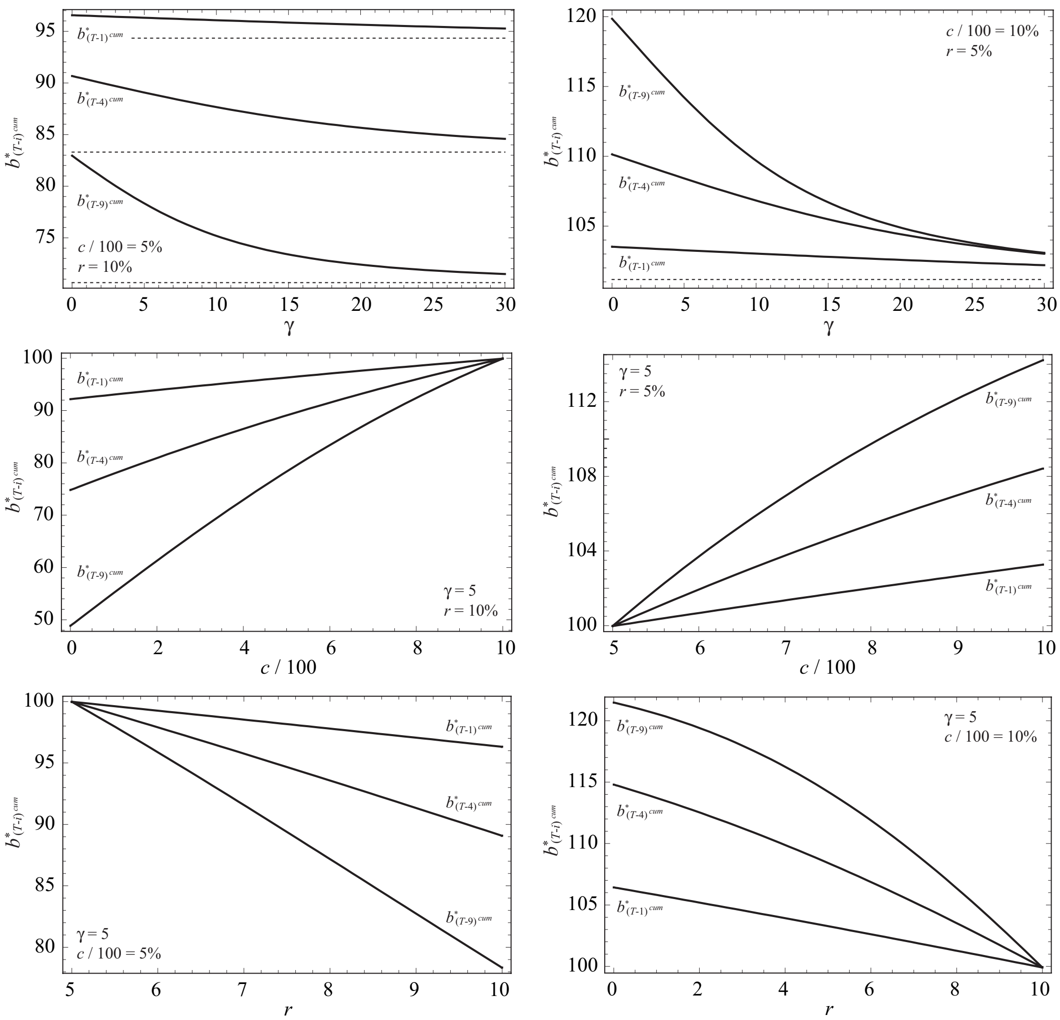
For $\gamma = 0$, Equation (3.24) is not defined but can be complemented by the risk-neutral price $B^e_{(T-i)^{cum}}$, defined in Equation (3.16), such that $B^*_{(T-i)^{cum}}$ is continuous and differentiable in γ. Furthermore, Equation (3.24) simplifies to the equilibrium pricing Equation (3.9) deduced in the one-period framework for i equal to one.

Comparative static results for the clean equilibrium prices $b^*_{(T-1)^{cum}}$, $b^*_{(T-4)^{cum}}$, and $b^*_{(T-9)^{cum}}$ with respect to the RRA coefficient γ, the coupon c, and the risk-free rate r are depicted in Figure 3.8. Corresponding to the results obtained in the one-period context, dynamic equilibrium prices are decreasing in γ, increasing in c, and decreasing in r. The relation for the redemption value R is analogous to that of the coupon. The results persist for multiple parameter combinations within a reasonable range.

We consider the comparative static results for clean cum-day equilibrium prices with respect to the RRA coefficient in further detail. First, we regard the results for below par prices in the upper graph on the left-hand side. Below par equilibrium prices decrease in the time-to-maturity, as future coupon and redemption payments are discounted over a longer period at the risk-free rate r which is larger than the coupon rate $c/100$. The general no-arbitrage bounds for dirty lottery bond prices given by Inequality (3.15) can be transformed to bounds for clean prices by adjusting the terms for accrued interest. The adjusted lower bounds are characterized by the dashed lines in the graph. Since it is favorable to be drawn in the redemption lotteries, equilibrium prices converge to the respective clean price of a straight coupon bond as γ approaches infinity. Note that the price of the straight below par coupon bond, equal to the lower no-arbitrage bound, is strictly decreasing in the time-to-maturity. Furthermore, we can numerically show that, for γ approaching minus infinity, equilibrium prices converge to the upper no-arbitrage bound equal to the redemption payment $v^d_i/(1+r)^{\varepsilon}$ adjusted for accrued interest. The adjusted redemption payment at $(T-i)^{cum}$ is equal $\forall i \in \mathbb{N}$, $1 \leq i < T$. Therefore, the difference between the constant upper and the lower no-arbitrage bound rises, the more redemption lotteries are outstanding, and the range of no-arbitrage equilibrium prices increases. In addition, the graph shows that the higher the RRA coefficient, the larger is the spread between

Figure 3.8: **Comparative Statics for Equilibrium Prices at Cum-dates**

This figure plots the comparative static results for the clean equilibrium lottery bond prices $b^*_{(T-1)^{cum}}$, $b^*_{(T-4)^{cum}}$, and $b^*_{(T-9)^{cum}}$ against the RRA coefficient γ, the annual coupon c, and the risk-free rate r. We consider a lottery bond with two, five, and ten outstanding series equal in size. The redemption lottery is conducted annually, and drawn series are redeemed at face value. The time structure is as follows: ε is zero, and δ is equal to 90 days. The graphs on the left-hand side depict the comparative static results for below par prices ($c/100 < r$), and the graphs on the right-hand side depict the results for above par prices ($c/100 > r$). The dashed lines in the upper graphs depict the lower no-arbitrage bound for the respective lottery bond price. If applicable, the values of γ, $c/100$, and r are reported in the graphs.



equilibrium prices for different maturities.

Second, we regard the results for above par prices in the upper graph on the right-hand side. Above par equilibrium prices increase in the time-to-maturity as future coupon and redemption payments are discounted over a longer period at the risk-free rate r, which is smaller than the coupon rate $c/100$. Again, the adjusted lower no-arbitrage bound is characterized by the dashed line in the graph. Since it is unfavorable to be drawn in the redemption lotteries, equilibrium prices converge to the adjusted redemption payment identical for all maturities as γ approaches infinity. Furthermore, we can numerically show that equilibrium prices converge to the respective clean price of a straight coupon bond as γ approaches minus infinity. Note that the price of the straight above par coupon bond, equal to the upper no-arbitrage bound, is strictly increasing in time-to-maturity. Therefore, the difference between the constant lower and the upper no-arbitrage bound rises, the more redemption lotteries are outstanding and the range of no-arbitrage equilibrium prices increases. In addition, the graph shows that the lower the RRA coefficient, the larger is the spread between equilibrium prices for different maturities.

Next, we consider the comparative static results for the clean cum-day equilibrium prices with respect to the coupon. First, we regard the results for below par prices in the middle graph on the left-hand side of Figure 3.8. If $c/100$ approaches r, the investor becomes indifferent with respect to the outcome of the redemption lotteries, and equilibrium lottery bond prices for all maturities converge to the clean price of a straight coupon bond. For $c/100$ smaller than r, equilibrium prices decrease in the time-to-maturity. The lower c, the higher is the spread between equilibrium prices for different maturities. Second, we regard the results for above par prices in the middle graph on the right-hand side. Again, if $c/100$ approaches r, equilibrium lottery bond prices for all maturities converge to the clean price of a straight coupon bond. For $c/100$ larger than r, equilibrium prices increase in time-to-maturity. The larger c, the higher is the spread between above par equilibrium prices for different maturities.

Lastly, we consider the comparative static results for the clean cum-day equilibrium prices with respect to the risk-free rate. We regard the results for below par prices in the lower graph on the left-hand side of Figure 3.8 first. If r

approaches $c/100$, equilibrium lottery bond prices for all maturities converge to the clean price of a straight coupon bond. For r larger than $c/100$, equilibrium prices decrease in the time-to-maturity. The larger r, the higher is the spread between below par equilibrium prices for different maturities. Second, we regard the results for above par prices in the lower graph on the right-hand side. Again, if r approaches $c/100$, equilibrium lottery bond prices for all maturities converge to the clean price of a straight coupon bond. For r smaller than $c/100$, equilibrium prices decrease in the time-to-maturity. The lower r, the higher is the spread between above par equilibrium prices for different maturities.

Analogous to the equilibrium pricing equation in the one-period framework, Equation (3.24) is a one-to-one mapping of the equilibrium price to the RRA coefficient.[16] Hence, each price corresponds to a unique RRA coefficient. If we numerically solve the equilibrium pricing Equation (3.24) at a fixed arbitrage-free price $B_{(T-i)^{cum}}$ for γ, we obtain the implied RRA coefficient.

3.2.4 Equilibrium Prices at Ex-dates

Having deduced and analyzed equilibrium prices at cum-lottery dates, we proceed by showing that equilibrium prices at ex-lottery dates are determined by simply adjusting the subsequent cum-lottery price for coupon payments and risk-free discounting.

We focus on the investor's utility maximization problem at $(T-i)^{ex}$, $\forall i \in \mathbb{N},\ 1 < i \leq (T-1)$, characterized by the following expression:

$$\max_{x_{(T-i)^{ex}}} \mathbb{E}_{(T-i)^{ex}}\left[u\left(w_T^s\right)\right] = \max_{x_{(T-i)^{ex}}} \left\{ J_{(T-i+1)^{cum}}\left(w^n_{(T-i+1)^{cum}}\right)\right\}, \tag{3.25}$$

where the indirect utility function $J_{(T-i+1)^{cum}}(\cdot)$ is the optimal value of investor's utility maximization problem at $(T-i+1)^{cum}$.

Expression (3.25) is maximized subject to the investor's budget constraint

[16] It can be numerically shown that $B^*_{(T-i)^{cum}}$ is strictly monotonic decreasing in γ.

resulting in the following characterizations of wealth at $(T-i+1)^{cum}$ if the lottery bond series has not been drawn in the redemption lottery at $(T-i)^{l}$

$$w^n_{(T-i+1)^{cum}} = w^n_{(T-i)^{ex}} \cdot \left(x_{(T-i)^{ex}} \cdot \frac{B^*_{(T-i+1)^{cum}} + C_{i-1}}{B_{(T-i)^{ex}}} + \left(1 - x_{(T-i)^{ex}}\right) \cdot (1+r)^{1-\varepsilon} \right). \tag{3.26}$$

The functional form of the indirect utility function $J_{(T-i+1)^{cum}}(\cdot)$ is given by[17]

$$J_{(T-i+1)^{cum}}\left(w^n_{(T-i+1)^{cum}}\right) = \frac{1}{i} \cdot \left(w^n_{(T-i+1)^{cum}}\right)^{1-\gamma} \cdot \frac{a_{(T-i+1)^{cum}}}{\left(B^*_{(T-i+1)^{cum}}\right)^{1-\gamma}}, \tag{3.27}$$

where $a_{(T-i+1)^{cum}}$ is defined

$$a_{(T-i+1)^{cum}} \equiv \sum_{j=1}^{i-1} u\left(v^d_j \cdot (1+r)^{j+\delta} \cdot \prod_{k=j}^{i-2} y_{(T-k)^{cum}}\right) + u\left(v^n_1 \cdot (1+r)^{1+\delta} \cdot \prod_{k=1}^{i-2} y_{(T-k)^{cum}}\right). \tag{3.28}$$

We determine the first order condition of the general utility maximization problem (3.25) to (3.26) for the portfolio composition $x_{(T-i)^{ex}}$. Solving the first order condition with $x_{(T-i)^{ex}} \equiv 1$ for $B_{(T-i)^{ex}}$, the equilibrium price at $(T-i)^{ex}$ is given

[17] It is shown in Appendix A.3.3.2 that $J_{(T-i+1)^{cum}}(\cdot)$ is the optimal value of the adjusted general utility maximization problem (3.19) to (3.20).

by

$$
\begin{aligned}
B^*_{(T-i)^{ex}} &= \frac{B^*_{(T-i+1)^{cum}} + C_{i-1}}{(1+r)^{1-\varepsilon}} \\
&= \frac{B^*_{(T-i+1)^{cum}}}{(1+r)^{1-\varepsilon}} + \frac{c}{(1+r)^{\delta}}.
\end{aligned}
\tag{3.29}
$$

Equilibrium prices at $(T-i)^{ex}$ are the sum of the discounted equilibrium price $B^*_{(T-i+1)^{cum}}$ and the present value of interim coupon payments.

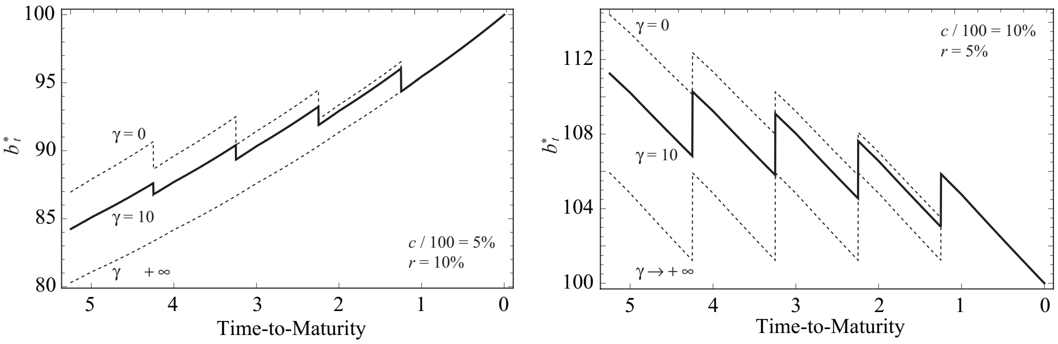
The general structure of equilibrium prices in the interval $[(T-i)^{ex}, (T-i+1)^{cum})$ is equal to that of $B^*_{(T-i)^{ex}}$ defined in Equation (3.29). After the redemption lottery at $(T-i)^l$ and before the subsequent lottery, the economy is risk-free. Hence, equilibrium prices are determined by simply accumulating the cum-lottery price $B^*_{(T-i+1)^{cum}}$ and interim coupon payments and discounting the cash flows at the risk-free rate to the valuation date.

Employing the equilibrium pricing Equations (3.24) and (3.29), we conduct a comparative static analysis of the clean equilibrium price b^*_t with respect to the time-to-maturity of the bond. Figure 3.9 shows the price path of a lottery bond with initial time-to-maturity of 5.25 years. We consider the path of below par prices in the graph on the left-hand side first. After each of the four redemption lotteries, the below par price drops because of the foregone chance to receive the face value at the next redemption date. Between lottery dates, the equilibrium price evolves like the price of a comparable straight coupon bond. The more risk-averse the representative investor, the smaller are the price drops caused by the lottery. As γ approaches infinity, the equilibrium price converges to the clean price of a straight coupon bond which is independent of redemption risk.

Next, we consider the path of above par prices in the graph on the right-hand side of Figure 3.9. After each redemption lottery, the above par price rises because of the foregone risk to receive the face value at the redemption dates. The more risk-averse the representative investor, the larger are the positive price jumps caused by the lottery. As γ approaches infinity, the equilibrium price converges to the

Figure 3.9: **Dynamics of Equilibrium Prices**

This figure plots the clean equilibrium lottery bond price b_t^* against time-to-maturity for different RRA coefficients γ. Coupons are paid annually, and the redemption lottery is conducted on an annual basis. Drawn series are redeemed at face value. The time structure is as follows: ε is zero, and δ is equal to 90 days. The graph on the left-hand side depicts the comparative static results for clean below par prices ($c/100 < r$), and the graph on the right-hand side depicts the results for clean above par prices ($c/100 > r$). In the graph on the left, the coupon rate is $c/100 = 5\%$, and the risk-free rate is $r = 10\%$. In the graph on the right, the coupon rate is $c/100 = 10\%$, and the risk-free rate is $r = 5\%$. The upper and lower dashed lines depict the lottery bond price for $\gamma = 0$ and $\gamma \to +\infty$, respectively. The middle line shows the lottery bond price for $\gamma = 10$.



present value of the series which will be drawn in the subsequent redemption lottery. Hence, jumps are at their maximum and equal for all redemption probabilities. In the subsequent section, we consider the ex-day behavior of equilibrium prices in further detail.

3.3 Equilibrium Ex-day Price Behavior

3.3.1 Definition and Bounds

We have deduced equilibrium prices for points in time $(T-i)^{cum}$, Equation (3.24), and for $(T-i)^{ex}$, Equation (3.29). Based on these pricing equations, we focus on the equilibrium price reactions around lottery dates.

We analyze the ex-day behavior of equilibrium prices caused by redemption lotteries and focus on the difference between the discounted ex-lottery price $B^*_{(T-i)^{ex}}$ and the cum-lottery price $B^*_{(T-i)^{cum}}$ given by

$$\begin{aligned} \Delta B^*_i &= \frac{B^*_{(T-i)^{ex}}}{(1+r)^{\varepsilon}} - B^*_{(T-i)^{cum}} \qquad (3.30) \\ &= \frac{B^*_{(T-i+1)^{cum}}}{1+r} + \frac{c}{(1+r)^{\varepsilon+\delta}} - B^*_{(T-i)^{cum}}. \end{aligned}$$

Substituting the ex-lottery price by pricing Equation (3.29), we obtain the second equality. In the case of ε equal to zero, ΔB^*_i remains unchanged independent of whether clean or dirty prices are used, as accrued interest cancels out.

The price difference ΔB^*_i characterizes the equilibrium price behavior attributable to the realization of redemption risk. The larger $|\Delta B^*_i|$, the more pronounced is the reaction of equilibrium prices with respect to the redemption drawings.

Before proceeding with the comparative static analysis for the ex-day behavior of equilibrium prices, we consider the upper and lower bounds for ΔB_i^*.[18] First, we focus on the bounds for ΔB_i^* if $c/R < r$ that specify:

$$\frac{v_{i,(i-1)}^d - v_i^d}{(1+r)^\varepsilon} < \Delta B_i^* < 0, \tag{3.31}$$

where $v_{i,(i-1)}^d$, defined by Equation (3.12), denotes the present value at $(T-i)^{ex}$ of future cash flows from the lottery bond given that the series is drawn at $(T-i+1)^l$ and v_i^d denotes the present value given that the series is drawn at $(T-i)^l$.

Appendix A.3.4 shows that it is sufficient to consider the difference between the equilibrium prices $B^*_{(T-i)^{ex}}$ and $B^*_{(T-i)^{cum}}$ at their upper bounds and their lower bounds, respectively, when determining the bounds for ΔB_i^*.[19] The supremum of the equilibrium price $B^*_{(T-i)^{cum}}$, equal to the respective dirty price of a straight coupon bond, is *ceteris paribus* equal to the discounted supremum of $B^*_{(T-i)^{ex}}$, and ΔB_i^* approaches zero. Furthermore, the infimum of $B^*_{(T-i)^{cum}}$, equal to the redemption payment $v_i^d/(1+r)^\varepsilon$, is *ceteris paribus* larger than the discounted infimum of $B^*_{(T-i)^{ex}}$, equal to the redemption payment $v_{i,(i-1)}^d/(1+r)^\varepsilon$, such that ΔB_i^* attains its infimum. Because of the foregone chance of an early redemption at face value, ex-prices drop, and the ex-day behavior is negative.

Second, we focus on the bounds for ΔB_i^* if $c/R > r$ that specify:

$$0 < \Delta B_i^* < \frac{v_{i,(i-1)}^d - v_i^d}{(1+r)^\varepsilon} \tag{3.32}$$

The infimum of the equilibrium price $B^*_{(T-i)^{cum}}$, equal to the respective dirty price of a straight coupon bond, is *ceteris paribus* equal to the discounted infimum of $B^*_{(T-i)^{ex}}$, and ΔB_i^* approaches zero. The supremum of $B^*_{(T-i)^{cum}}$, equal to the redemption payment $v_i^d/(1+r)^\varepsilon$, is *ceteris paribus* lower than the discounted

[18] The bounds for ΔB_i^* are derived in Appendix A.3.4.

[19] Note that, due to the *ceteris paribus* assumptions, it is not reasonable to intersect upper and lower bounds.

supremum of $B^*_{(T-i)^{ex}}$, equal to the redemption payment $v^d_{i,(i-1)}/(1+r)^\varepsilon$, such that ΔB^*_i attains its supremum. Because of the foregone risk of early redemption at face value, ex-prices rise, and the ex-day behavior is positive.

As already indicated in Figure 3.9, the ex-day behavior of equilibrium prices is reversed for clean below par and above par prices. In the case of c/R approaching r, the ex-day behavior becomes zero, as the lottery bond is equal to a straight coupon bond, and the economy is free of redemption risk.

3.3.2 Comparative Static Analysis

We derive further properties of the equilibrium ex-day price behavior by conducting a comparative static analysis of ΔB^*_i. Figure 3.10 depicts the comparative static results for ΔB^*_1, ΔB^*_4, and ΔB^*_9 with respect to the RRA coefficient γ, the coupon c, and the risk-free rate r. The results in Figure 3.10 persist for multiple parameter combinations within a reasonable range.

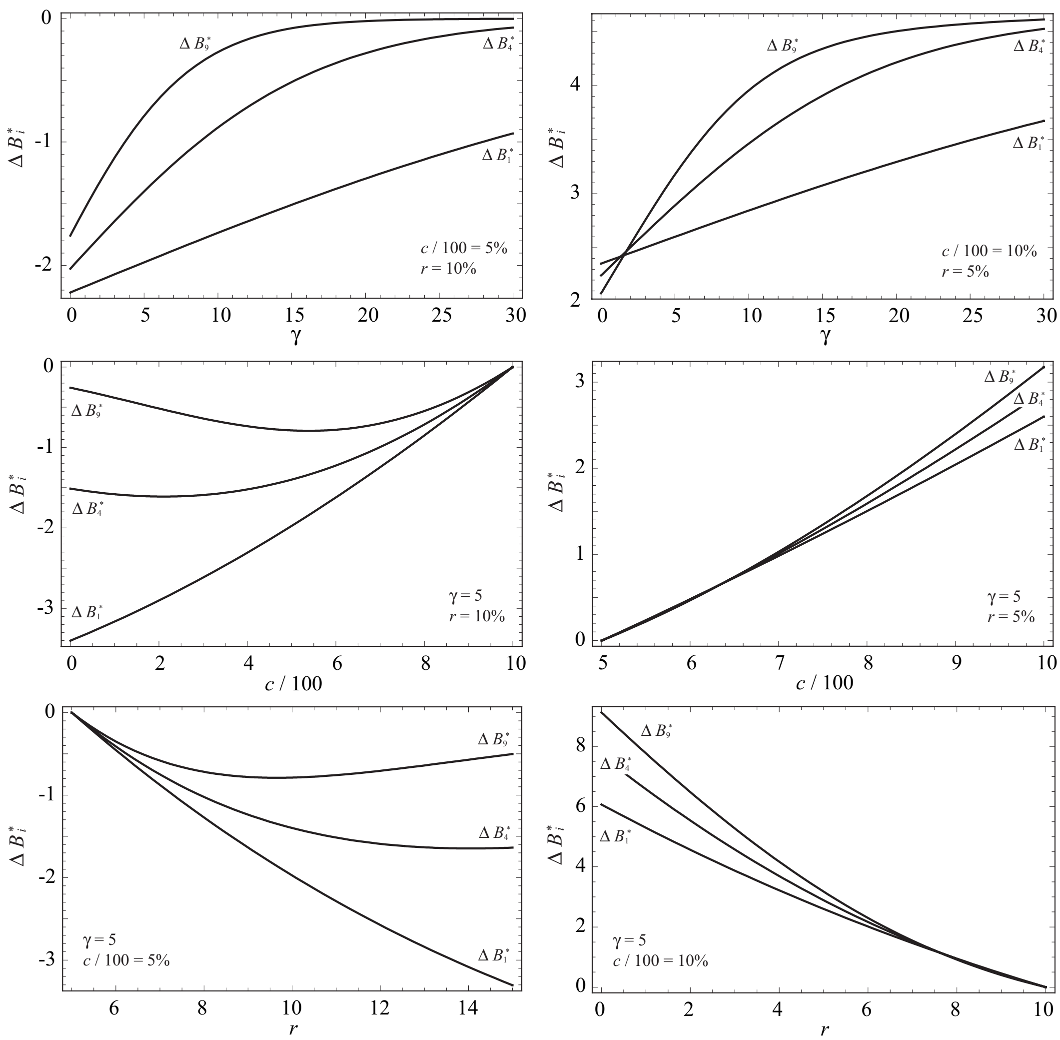
We focus on the comparative static results for the ex-day price behavior with respect to the RRA coefficient. First, we regard the price drops for below par prices ($c/100 < r$) in the upper graph on the left-hand side. Price drops are decreasing in γ. The comparative static results for equilibrium prices at cum-dates in Figure 3.8 indicate that the more redemption lotteries are outstanding, the larger is the sensitivity of the equilibrium price with respect to γ.[20] Hence, if γ increases, the decline of the cum-lottery price is larger than the decline of the ex-lottery price and the ex-day price drop decreases. For γ approaching infinity, ΔB^*_i is zero as equilibrium prices converge to the price of a straight coupon bond. We can numerically show that, for γ approaching minus infinity, price drops are equal $\forall i \in \mathbb{N},\ 1 \leq i < T$ and at their supremum. Dirty equilibrium prices converge to $v^d_i/(1+r)^\varepsilon$ and $v^d_{i,(i-1)}$, respectively, while ΔB^*_i converges to the lower no-arbitrage bound defined in Inequality (3.31). The supremum price drop for the above parameter specifications is DEM 4.44.

[20] It can be numerically shown that the following holds:

$$\frac{\partial B^*_{(T-i)^{cum}}}{\partial \gamma} < \frac{\partial B^*_{(T-i)^{ex}}/(1+r)^\varepsilon}{\partial \gamma} < 0$$

Figure 3.10: **Comparative Statics for Ex-day Price Behavior I**

This figure plots the comparative static results for the ex-day price behavior ΔB_1^*, ΔB_4^*, and ΔB_9^* against the RRA coefficient γ, the annual coupon c, and the risk-free rate r. The redemption lottery is conducted annually, and drawn series are redeemed at face value. The time structure is as follows: ε is zero, and δ is equal to 90 days. The graphs on the left-hand side depict the comparative static results for below par cum-prices ($c/100 < r$), and the graphs on the right-hand side depict the results for above par cum-prices ($c/100 > r$). If applicable, the values of γ, $c/100$, and r are reported in the graphs.



Second, we regard the positive price jumps for above par prices ($c/100 > r$) in the upper graph on the right-hand side. Positive price jumps are increasing in the RRA coefficient. If γ increases, the decrease of the cum-lottery price is larger than the decrease of the ex-lottery price and also the ex-day price jump increases. For γ approaching infinity, ΔB_i^* is constant and at its supremum. Dirty equilibrium prices converge to $v_i^d/(1+r)^\varepsilon$ and $v_{i,(i-1)}^d$, respectively, while ΔB_i^* converges to the upper no-arbitrage bound defined in Inequality (3.32). The supremum price rise for the above parameter specifications is DEM 4.70. For γ approaching minus infinity, ΔB_i^* becomes zero, as equilibrium prices converge to the price of a straight coupon bond.

Next, we consider the comparative static results for the ex-day price behavior with respect to the coupon. First, we regard the price reactions for below par prices ($c/100 < r$) in the middle graph on the left-hand side of Figure 3.10. Price drops are not necessarily monotonic in c. A variation of c has two opposing effects on ΔB_i^*. It influences ΔB_i^* directly via the coupon term $C_{i-1} = c/(1+r)^{\varepsilon+\delta}$ contained in the ex-day price $B^*_{(T-i)^{ex}}$. If c increases, the coupon term rises and *ceteris paribus* the price drop falls. Furthermore, the variation influences ΔB_i^* indirectly via the equilibrium prices. The comparative static results for below par equilibrium prices at cum-dates in Figure 3.8 indicate that the lower c, the larger is the spread between $B^*_{(T-i)^{cum}}$ and $B^*_{(T-i+1)^{cum}}$. Since $B^*_{(T-i)^{cum}} \leq B^*_{(T-i+1)^{cum}}$, an increase in c results in a decrease of $B^*_{(T-i+1)^{cum}}/(1+r) - B^*_{(T-i)^{cum}}$ and *ceteris paribus* the price drop rises. The fewer redemption lotteries are outstanding and the larger the coupon, the more the direct effect prevails. For $c/100$ approaching r, the price drop becomes zero, as the equilibrium price converges to the price of a straight coupon bond.

Second, we regard the ex-day price behavior for above par prices ($c/100 > r$) in the middle graph on the right-hand side. Positive price jumps are increasing in the coupon. The larger the spread between $c/100$ and r, the higher is ΔB_i^*. The comparative static results for above par equilibrium prices at cum-dates indicate that the higher c, the larger is the spread between $B^*_{(T-i)^{cum}}$ and $B^*_{(T-i+1)^{cum}}$. Hence, the direct and indirect effect on ΔB_i^* have the same direction. Again, for $c/100$ approaching r, the price jump becomes zero.

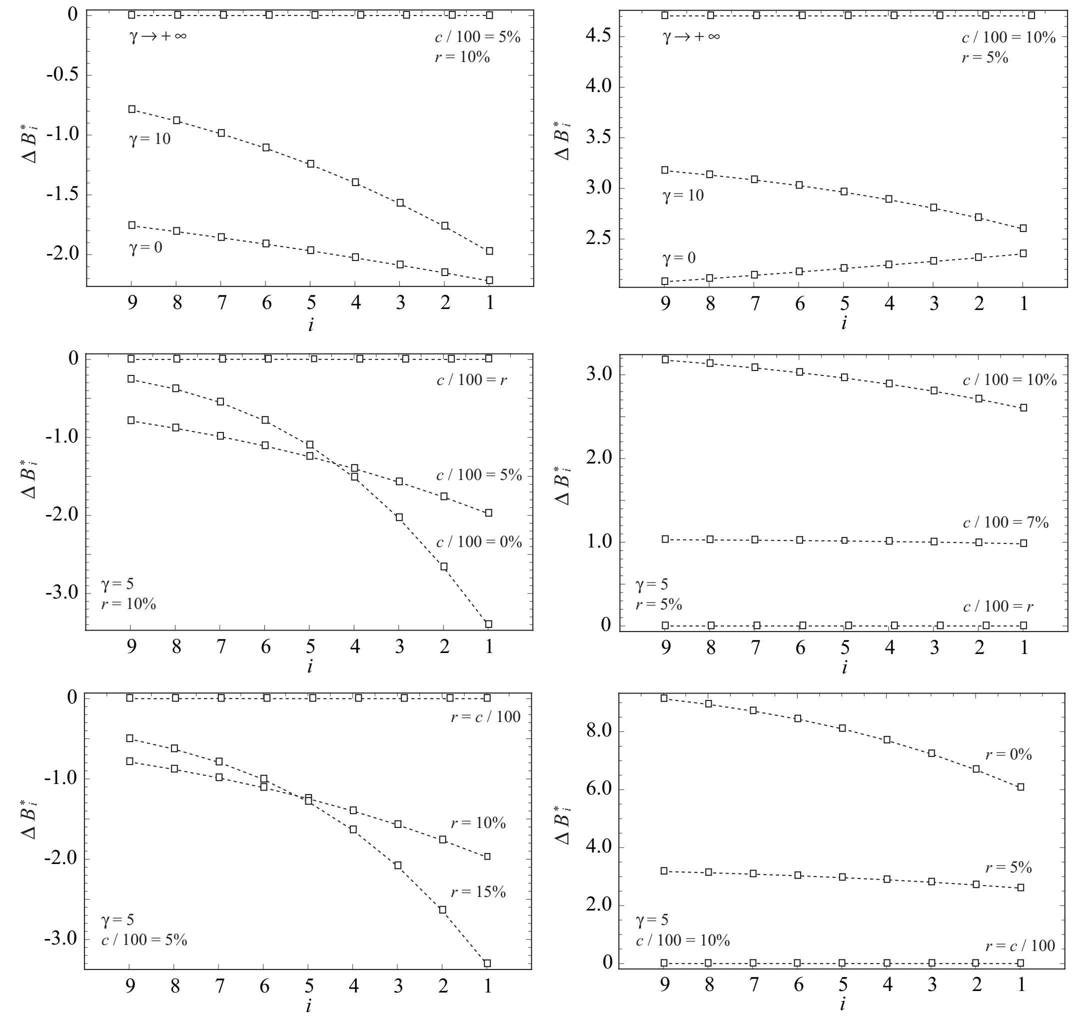
Lastly, we consider the comparative static results for the ex-day price behavior with respect to the risk-free rate. We first regard the price reactions for below par prices ($c/100 < r$) in the lower graph on the left-hand side of Figure 3.10. Price drops are not necessarily monotonic in r. A variation of r has two opposing effects on ΔB_i^*. The variation influences ΔB_i^* directly over the coupon term $C_{i-1} = c/(1+r)^{\varepsilon+\delta}$ contained in the ex-day price $B^*_{(T-i)^{ex}}$ and the discount factor $1/(1+r)$ of $B^*_{(T-i+1)^{cum}}$. If r increases, the coupon term and discount factor fall and *ceteris paribus* the price drop rises. Furthermore, the variation influences ΔB_i^* indirectly over the equilibrium prices. The comparative static results for below par equilibrium prices at cum-dates in Figure 3.8 indicate that the higher r, the larger is the spread between $B^*_{(T-i)^{cum}}$ and $B^*_{(T-i+1)^{cum}}$. Since $B^*_{(T-i)^{cum}} \leq B^*_{(T-i+1)^{cum}}$, an increase in r results in an increase of $B^*_{(T-i+1)^{cum}} - B^*_{(T-i)^{cum}}$ and *ceteris paribus* the price drop falls. The fewer redemption lotteries are outstanding and the smaller the risk-free rate, the more the direct effect prevails. For r approaching $c/100$, the price drop becomes zero, as the equilibrium price converges to the price of a straight coupon bond.

Second, we regard the ex-day price behavior for above par prices ($c/100 > r$) in the lower graph on the right-hand side. Positive price jumps are decreasing in the risk-free rate. The larger the spread between r and $c/100$, the higher is ΔB_i^*. The comparative static results for above par equilibrium prices at cum-dates indicate that the lower r, the larger is the spread between $B^*_{(T-i)^{cum}}$ and $B^*_{(T-i+1)^{cum}}$. Hence, the direct and indirect effect on ΔB_i^* have the same direction. Again, for r approaching $c/100$, the price jump becomes zero.

In a next step, we focus on the ex-price reaction segmented by the i lottery dates. Figure 3.11 depicts ΔB_i^* at the i lottery dates for selected RRA coefficients γ, coupons c, and risk-free rates r. We consider the relation between ΔB_i^* and the RRA coefficient and regard the price drops for below par equilibrium prices ($c/100 < r$) in the upper graph on the left-hand side first. Price drops are decreasing in the RRA coefficient and zero for γ approaching infinity. We analyze the relation between ΔB_i^* and the number of outstanding redemption lotteries i. It can easily be shown that, under risk neutrality, price drops fall in the number of outstanding redemption lotteries. For positive RRA coefficients, this relation remains negative. However, we can show that for sufficiently negative RRA coefficients the price reaction reverses and price drops rise in the number

Figure 3.11: **Comparative Statics for Ex-day Price Behavior II**

This figure plots the comparative static results for the ex-day price behavior ΔB_i^* at the i lottery dates for selected RRA coefficients γ, annual coupons c, and risk-free rates r. The redemption lottery is conducted annually, and drawn series are redeemed at face value. The time structure is as follows: ε is zero, and δ is equal to 90 days. The graphs on the left-hand side depict the comparative static results for below par prices ($c/100 < r$), and the graphs on the right-hand side depict the results for above par prices ($c/100 > r$). If applicable, the values of γ, $c/100$, and r are reported in the graphs.



of outstanding redemption lotteries such that they are not monotonic in i.

Second, we regard the positive price jumps for above par equilibrium prices ($c/100 > r$) in the upper graph on the right-hand side. Price jumps are increasing in the RRA coefficient and zero for γ approaching minus infinity. Again, we analyze the relation between ΔB_i^* and i. It can easily be shown that, under risk neutrality, positive price jumps fall in the number of outstanding redemption lotteries. For sufficiently high RRA coefficients, this relation reverses and positive price jumps rise in i. However, for negative RRA coefficients, the relation remains negative. Reverting to the comparative static results for ΔB_i^* with respect to γ in Figure 3.10, the intersection of ΔB_1^*, ΔB_4^*, and ΔB_9^* already indicated that the relation between ΔB_i^* and i is not monotonic.

Lastly, we consider the relation between ΔB_i^* and the coupon as well as the risk-free rate. First, we regard the price drops for below par equilibrium prices ($c/100 < r$) in the middle and lower graph on the left-hand side of Figure 3.11. Price drops are non-monotonic in the coupon as well as the risk-free rate and become zero for $c/100$ approaching r. For positive RRA coefficients, the relation between ΔB_i^* and i is positive. Second, we regard the price jumps for above par equilibrium prices ($c/100 > r$) in the middle and lower graph on the right-hand side. Price jumps are increasing in the coupon, decreasing in the risk-free rate, and become zero for $c/100$ approaching r. For a sufficiently high RRA coefficient and coupon, respectively low risk-free rate, the relation between ΔB_i^* and i is positive.

3.4 Valuation under Perfect Foresight

3.4.1 Model Setup and Assumptions

As a benchmark to the fully specified dynamic equilibrium model derived in Section 3.2, we determine lottery bond prices and implied RRA coefficients in a simple static one-decision setting under the assumption of perfect foresight.

Contrary to the dynamic framework, we assume that lottery bond prices after the redemption drawing are exogenous and perfectly known to agents immediately before the drawing (perfect foresight). We consider an one-decision problem at a point in time $(T-i)^{cum}$, $\forall i \in \mathbb{N},\ 1 \leq i < T$, for a representative investor with a state-independent power utility function and investment horizon ϵ maximizing his terminal wealth at $(T-i)^{ex}$, shortly after the drawing. The agent optimally distributes his wealth at $(T-i)^{cum}$ among the lottery bond series and the risk-free instrument.

The investor has to consider two disjoint states of the world: d and n. In state d, realized with the objective probability p_i, the agent receives the redemption payment v_i^d defined in Equation (3.12). In state n, realized with the objective probability $(1-p_i)$, the agent holds a series which has not been drawn and is still traded. The series is directly sold, and the cash flow, given that the series is not drawn, specifies

$$v_i^n = B_{(T-i)^{ex}}. \tag{3.33}$$

Throughout this section, the lottery bond price $B_{(T-i)^{ex}}$ is assumed to be exogenous and restricted to values within the no-arbitrage bounds defined by Inequalities (3.17). Perfect foresight implies that $B_{(T-i)^{ex}}$ is already known before the redemption lottery at $(T-i)^{cum}$. However, note that before the lottery, the investor does not know whether his series is drawn or not.

3.4.2 Perfect Foresight Prices and RRA Coefficients

We focus on the investor's utility maximization problem under perfect foresight at $(T-i)^{cum}$, $\forall i \in \mathbb{N},\ 1 \leq i < T$, characterized by the following expression:

$$\max_{x_{(T-i)^{cum}}} \mathbb{E}_{(T-i)^{cum}} \left[u \left(w^s_{(T-i)^{ex}} \right) \right] = \tag{3.34}$$

$$\max_{x_{(T-i)^{cum}}} \left\{ p_i \cdot u \left(w^d_{(T-i)^{ex}} \right) + (1-p_i) \cdot u \left(w^n_{(T-i)^{ex}} \right) \right\},$$

where $w^s_{(T-i)^{ex}}$ is the investor's terminal wealth at $(T-i)^{ex}$ in either state d or n and $x_{(T-i)^{cum}}$ is the proportion of wealth at $(T-i)^{cum}$ invested in the lottery bond.

Expression (3.34) is maximized subject to the investor's budget constraint resulting in the following characterization of wealth at $(T-i)^{ex}$

$$w^s_{(T-i)^{ex}} = w_{(T-i)^{cum}} \cdot \left(x_{(T-i)^{cum}} \cdot \frac{v_i^s}{B_{(T-i)^{cum}}} + \left(1 - x_{(T-i)^{cum}}\right) \cdot (1+r)^{\varepsilon} \right). \quad (3.35)$$

The first order condition of the general utility maximization problem (3.34) to (3.35) with respect to the portfolio composition $x_{(T-i)^{cum}}$ is

$$\begin{aligned}
&\frac{\partial \mathbb{E}_{(T-i)^{cum}} \left[u \left(w^s_{(T-i)^{ex}} \right) \right]}{\partial x_{(T-i)^{cum}}} \stackrel{!}{=} 0 \Leftrightarrow \\
&p_i \cdot u' \left(w^d_{(T-i)^{ex}} \right) \cdot \left(\frac{v_i^d}{B_{(T-i)^{cum}}} - (1+r)^{\varepsilon} \right) + \\
&(1-p_i) \cdot u' \left(w^n_{(T-i)^{ex}} \right) \cdot \left(\frac{B_{(T-i)^{ex}}}{B_{(T-i)^{cum}}} - (1+r)^{\varepsilon} \right) = 0 \quad \text{for } \gamma \neq 0.
\end{aligned}$$

We assume that the lottery bond is in unit-net supply. Solving the first order condition with $x_{(T-i)^{cum}} \equiv 1$ for $B_{(T-i)^{cum}}$, we obtain the perfect foresight price

$$B^f_{(T-i)^{cum}} = \frac{p_i \cdot \left(v_i^d\right)^{1-\gamma} + (1-p_i) \cdot \left(B_{(T-i)^{ex}}\right)^{1-\gamma}}{p_i \cdot \left(v_i^d\right)^{-\gamma} + (1-p_i) \cdot \left(B_{(T-i)^{ex}}\right)^{-\gamma}} \cdot \frac{1}{(1+r)^{\varepsilon}} \quad \text{for } \gamma \neq 0. \quad (3.36)$$

The perfect foresight price $B^f_{(T-i)^{cum}}$ is a function of (i) the RRA coefficient γ, (ii) the coupon c, (iii) the redemption value R, (iv) the risk-free rate r, (v) the redemption probability p_i, and (vi) the exogenous ex-day price $B_{(T-i)^{ex}}$.

For $\gamma = 0$, Equation (3.36) is not defined but can be complemented by the risk-neutral price under perfect foresight,

$$B^{e,f}_{(T-i)^{cum}} = p_i \cdot v_i^d + (1 - p_i) \cdot v_i^n,$$

such that $B^{f}_{(T-i)^{cum}}$ is continuous and differentiable in γ. Furthermore, Equation (3.36) simplifies to the equilibrium pricing Equation (3.9) for i equal to one and $B_{(T-1)^{ex}}$ equal to v_1^n defined in Equation (3.1).

For a reasonable choice of $B_{(T-i)^{ex}}$, the relation between the perfect foresight price $B^{f}_{(T-i)^{cum}}$ and γ is strictly monotonic decreasing, and Equation (3.36) is a one-to-one mapping of the cum-price to the RRA coefficient. Solving Equation (3.36) at a fixed arbitrage-free price $B_{(T-i)^{cum}}$ for γ, we obtain the implied RRA coefficient

$$\gamma = -\frac{\log\left[\frac{\left(\frac{1}{p_i}-1\right)\cdot\left(B_{(T-i)^{cum}}-\frac{B_{(T-i)^{ex}}}{(1+r)^{\varepsilon}}\right)}{\frac{v_i^d}{(1+r)^{\varepsilon}}-B_{(T-i)^{cum}}}\right]}{\log\left[\frac{v_i^d}{B_{(T-i)^{ex}}}\right]}. \tag{3.37}$$

Appendix to Chapter 3

A.3.1 Comparative Statics for Optimal Portfolio Composition with respect to RRA Coefficient

We analytically derive the relation between the optimal portfolio composition x^*_{cum} and the RRA coefficient γ by considering the partial derivative of Equation (3.8) with respect to γ:

$$\frac{\partial x^*_{cum}}{\partial \gamma} = B_{cum} \cdot \frac{\left(\frac{\frac{v_1^n}{(1+r)^\varepsilon} - B_{cum}}{B_{cum} - \frac{v_1^d}{(1+r)^\varepsilon}}\right)^{\frac{1}{\gamma}} \cdot \left(\frac{v_1^d}{(1+r)^\varepsilon} - \frac{v_1^n}{(1+r)^\varepsilon}\right) \cdot \log\left[\frac{\frac{v_1^n}{(1+r)^\varepsilon} - B_{cum}}{B_{cum} - \frac{v_1^d}{(1+r)^\varepsilon}}\right]}{\left(\left(\frac{v_1^n}{(1+r)^\varepsilon} - B_{cum}\right) \cdot \left(1 + \left(\frac{\frac{v_1^n}{(1+r)^\varepsilon} - B_{cum}}{B_{cum} - \frac{v_1^d}{(1+r)^\varepsilon}}\right)^{\frac{1}{\gamma} - 1}\right)\right)^2 \cdot \gamma^2} \tag{3.38}$$

for $\gamma \neq 0$. The denominator of the second product term of Equation (3.38) is positive for B_{cum} located inside the no-arbitrage bounds defined by Inequalities (3.2). Since we are interested in the sign of the partial derivative, we neglect the first product term and the denominator and focus on the numerator of the second product term, which is equal to

$$\left(\frac{\frac{v_1^n}{(1+r)^\varepsilon} - B_{cum}}{B_{cum} - \frac{v_1^d}{(1+r)^\varepsilon}}\right)^{\frac{1}{\gamma}} \cdot \left(\frac{v_1^d}{(1+r)^\varepsilon} - \frac{v_1^n}{(1+r)^\varepsilon}\right) \cdot \log\left[\frac{\frac{v_1^n}{(1+r)^\varepsilon} - B_{cum}}{B_{cum} - \frac{v_1^d}{(1+r)^\varepsilon}}\right]. \tag{3.39}$$

Applying the no-arbitrage bounds characterized by Inequalities (3.2), we obtain the following relation between the lottery bond price B_{cum} and the present value terms v_1^s, $s \in \{d, n\}$. For $v_1^d > v_1^n$, the relations specify

$$B_{cum} < \max\left[\frac{v_1^d}{(1+r)^{\varepsilon}}, \frac{v_1^n}{(1+r)^{\varepsilon}}\right] = \frac{v_1^d}{(1+r)^{\varepsilon}} \tag{3.40}$$

$$B_{cum} > \min\left[\frac{v_1^d}{(1+r)^{\varepsilon}}, \frac{v_1^n}{(1+r)^{\varepsilon}}\right] = \frac{v_1^n}{(1+r)^{\varepsilon}},$$

and, for $v_1^d < v_1^n$, the relations specify

$$B_{cum} < \max\left[\frac{v_1^d}{(1+r)^{\varepsilon}}, \frac{v_1^n}{(1+r)^{\varepsilon}}\right] = \frac{v_1^n}{(1+r)^{\varepsilon}} \tag{3.41}$$

$$B_{cum} > \min\left[\frac{v_1^d}{(1+r)^{\varepsilon}}, \frac{v_1^n}{(1+r)^{\varepsilon}}\right] = \frac{v_1^d}{(1+r)^{\varepsilon}}.$$

The first product term of Expression (3.39) is positive, as $\frac{v_1^n}{(1+r)^{\varepsilon}} - B_{cum}$ and $B_{cum} - \frac{v_1^d}{(1+r)^{\varepsilon}}$ have identical signs. Therefore, the second and third product term characterize the sign of Equation (3.39).

We distinguish two cases and analyze positive and negative RRA coefficients separately.

Case 1 We consider a risk-averse investor with $\gamma > 0$.

A risk-averse investor is willing to pay at most $B^e_{cum} = \frac{1}{2} \cdot (\frac{v_1^d}{(1+r)^{\varepsilon}} + \frac{v_1^n}{(1+r)^{\varepsilon}})$ for the lottery bond. For $\gamma > 0$, the following limit on B_{cum} holds

$$B_{cum} < \frac{1}{2} \cdot \left(\frac{v_1^d}{(1+r)^{\varepsilon}} + \frac{v_1^n}{(1+r)^{\varepsilon}}\right). \tag{3.42}$$

Rearranging Inequality (3.42) and employing Inequalities (3.40) and (3.41), we obtain

$$\frac{\frac{v_1^n}{(1+r)^\varepsilon}-B_{cum}}{B_{cum}-\frac{v_1^d}{(1+r)^\varepsilon}} < 1 \qquad \text{if} \;\; v_1^d > v_1^n$$

$$\frac{\frac{v_1^n}{(1+r)^\varepsilon}-B_{cum}}{B_{cum}-\frac{v_1^d}{(1+r)^\varepsilon}} > 1 \qquad \text{if} \;\; v_1^d < v_1^n.$$

Hence, if the second product term within Expression (3.39) is positive, the log-term is negative, as its argument is smaller than one. However, if the second product term is negative, the log-term is positive, as its argument is larger than one.

For $\gamma > 0$, the first product term of Expression (3.39) is positive and the second and third product terms have opposite signs. Hence, the sign of Equation (3.38) is negative, and we obtain $\partial x^*_{cum}/\partial\gamma < 0$, $\forall\gamma > 0$.

Case 2 We consider a risk-seeking investor with $\gamma < 0$.

A risk-seeking investor is willing to pay at least B^e_{cum} for the lottery bond. For $\gamma < 0$, the following limit on B_{cum} holds

$$B_{cum} > \frac{1}{2}\cdot\left(\frac{v_1^d}{(1+r)^\varepsilon}+\frac{v_1^n}{(1+r)^\varepsilon}\right). \tag{3.43}$$

Rearranging Inequality (3.43), we obtain

$$\frac{\frac{v_1^n}{(1+r)^\varepsilon}-B_{cum}}{B_{cum}-\frac{v_1^d}{(1+r)^\varepsilon}} > 1 \qquad \text{if} \;\; v_1^d > v_1^n$$

$$\frac{\frac{v_1^n}{(1+r)^\varepsilon}-B_{cum}}{B_{cum}-\frac{v_1^d}{(1+r)^\varepsilon}} < 1 \qquad \text{if} \;\; v_1^d < v_1^n.$$

Hence, if the second product term within Expression (3.39) is positive, the log-term is positive, as its argument is larger than one. However, if the second product term is negative, also the log-term is negative, as its argument is less than one.

For $\gamma < 0$, the first product term of Expression (3.39) is positive and the second and third product terms have identical signs. Hence, the sign of the entire fraction is positive, and we obtain $\partial x^*_{cum}/\partial\gamma > 0$, $\forall\gamma < 0$. ■

A.3.2 Comparative Statics for Equilibrium Price in One-period Model Framework

A.3.2.1 Bounds for Equilibrium Price

We prove that the equilibrium lottery bond price defined in Equation (3.9) is located within the no-arbitrage bounds characterized by Inequalities (3.2). Two cases are distinguished, and v_1^d smaller and larger than v_1^n are analyzed separately.

Case 1 We consider $v_1^d > v_1^n$.

The equilibrium price B_{cum}^* in Equation (3.9) is defined:

$$B_{cum}^* = \frac{\left(v_1^d\right)^{1-\gamma} + \left(v_1^n\right)^{1-\gamma}}{\left(v_1^d\right)^{-\gamma} + \left(v_1^n\right)^{-\gamma}} \cdot \frac{1}{(1+r)^{\varepsilon}}$$

First, we analyze the limit of B_{cum}^* for $\gamma \rightarrow +\infty$. The equilibrium price is rearranged, and the value of the limit is determined. Since $0 < \frac{v_1^n}{v_1^d} < 1$ and $\lim\limits_{\gamma \rightarrow +\infty} (\frac{v_1^n}{v_1^d})^{\gamma} = 0$, the following holds:

$$\lim_{\gamma \rightarrow +\infty} \frac{v_1^d \cdot \left(\frac{v_1^n}{v_1^d}\right)^{\gamma} + v_1^n}{\left(\frac{v_1^n}{v_1^d}\right)^{\gamma} + 1} \cdot \frac{1}{(1+r)^{\varepsilon}} = \frac{v_1^n}{(1+r)^{\varepsilon}} \tag{3.44}$$

Second, we take the limit of B_{cum}^* for $\gamma \rightarrow -\infty$. The equilibrium price is rearranged, and the value of the limit is determined. Since $\frac{v_1^d}{v_1^n} > 1$ and

$\lim_{\gamma \to -\infty} (\frac{v_1^d}{v_1^n})^\gamma = 0$, the following holds:

$$\lim_{\gamma \to -\infty} \frac{v_1^d + v_1^n \cdot \left(\frac{v_1^d}{v_1^n}\right)^\gamma}{1 + \left(\frac{v_1^d}{v_1^n}\right)^\gamma} \cdot \frac{1}{(1+r)^\varepsilon} = \frac{v_1^d}{(1+r)^\varepsilon} \tag{3.45}$$

Case 2 We consider $v_1^d < v_1^n$.

First, we take the limit of B^*_{cum} for $\gamma \to +\infty$. The equilibrium price is rearranged, and the value of the limit is determined. Since $0 < \frac{v_1^d}{v_1^n} < 1$ and $\lim_{\gamma \to +\infty} (\frac{v_1^d}{v_1^n})^\gamma = 0$, the following holds:

$$\lim_{\gamma \to +\infty} \frac{v_1^d + v_1^n \cdot \left(\frac{v_1^d}{v_1^n}\right)^\gamma}{1 + \left(\frac{v_1^d}{v_1^n}\right)^\gamma} \cdot \frac{1}{(1+r)^\varepsilon} = \frac{v_1^d}{(1+r)^\varepsilon} \tag{3.46}$$

Second, we take the limit of B^*_{cum} for $\gamma \to -\infty$. The equilibrium price is rearranged, and the value of the limit is determined. Since $\frac{v_1^n}{v_1^d} > 1$ and $\lim_{\gamma \to -\infty} (\frac{v_1^n}{v_1^d})^\gamma = 0$, the following holds:

$$\lim_{\gamma \to -\infty} \frac{v_1^d \cdot \left(\frac{v_1^n}{v_1^d}\right)^\gamma + v_1^n}{\left(\frac{v_1^n}{v_1^d}\right)^\gamma + 1} \cdot \frac{1}{(1+r)^\varepsilon} = \frac{v_1^n}{(1+r)^\varepsilon} \tag{3.47}$$

As we show in Appendix A.3.2.2, the equilibrium price B^*_{cum} is strictly monotonic decreasing in γ. Hence, the limit of B^*_{cum} for $\gamma \to -\infty$ is an upper bound, and the limit for $\gamma \to +\infty$ is a lower bound on the equilibrium price. Considering

Expressions (3.44) to (3.47), the bounds for B^*_{cum} specify:

$$\min\left[\frac{v_1^d}{(1+r)^\varepsilon}, \frac{v_1^n}{(1+r)^\varepsilon}\right] < B^*_{cum} < \max\left[\frac{v_1^d}{(1+r)^\varepsilon}, \frac{v_1^n}{(1+r)^\varepsilon}\right]$$

■

A.3.2.2 Comparative Statics with respect to RRA Coefficient

We analytically derive the relation between the equilibrium lottery bond price B^*_{cum} and the RRA coefficient γ by considering the partial derivative of Equation (3.9) with respect to γ:

$$\frac{\partial B^*_{cum}}{\partial \gamma} = -\frac{\left(v_1^d - v_1^n\right) \cdot \left(\log\left[v_1^d\right] - \log\left[v_1^n\right]\right)}{\left(\left(v_1^d\right)^\gamma + \left(v_1^n\right)^\gamma\right)^2} \cdot \frac{\left(v_1^d\right)^\gamma \cdot \left(v_1^n\right)^\gamma}{(1+r)^\varepsilon} \quad \text{for } \gamma \neq 0 \qquad (3.48)$$

We assume that the risk-free rate r and the present value terms v_1^s, $s \in \{d, n\}$, are positive. Because we are only interested in the sign of the partial derivative, we neglect the positive denominator of the first product term as well as the entire second product term and focus on the numerator of the first fraction which is equal to

$$-\left(v_1^d - v_1^n\right) \cdot \left(\log\left[v_1^d\right] - \log\left[v_1^n\right]\right). \qquad (3.49)$$

If $(v_1^d - v_1^n)$ is positive, the second product term within Expression (3.49) is positive. If $(v_1^d - v_1^n)$ is negative, the second product term is negative. Hence, for $v_1^d \neq v_1^n$, Expression (3.49) is negative resulting in a negative sign of Equation (3.48). We obtain $\frac{\partial B^*_{cum}}{\partial \gamma} < 0$, $\forall \gamma \neq 0$. ■

A.3.3 Derivation of Indirect Utility Functions

A.3.3.1 Indirect Utility Function at $(T-i)^{ex}$

We subsequently show that equality

$$J_{(T-i)^{ex}}\left(w^n_{(T-i)^{ex}}\right) = \max_{x_{(T-i)^{ex}}} \left\{ J_{(T-i+1)^{cum}}\left(w^n_{(T-i+1)^{cum}}\right)\right\} \tag{3.50}$$

holds, where $J_{(T-i)^{ex}}(\cdot)$ and $J_{(T-i+1)^{cum}}(\cdot)$ are given by Equations (3.21) and (3.27) respectively.

We evaluate the right-hand side of Equation (3.50). The expression is maximized subject to the investor's budget constraint resulting in the characterization of wealth $w^n_{(T-i+1)^{cum}}$ defined in Equation (3.26).

Substituting the wealth expression $w^n_{(T-i+1)^{cum}}$ and the market clearing condition $x_{(T-i)^{ex}} \equiv 1$ into the right-hand side of Equation (3.50), we obtain the solution of the maximization problem given by

$$\frac{1}{i} \cdot \left(w^n_{(T-i)^{ex}} \cdot \left(\frac{B^*_{(T-i+1)^{cum}} + C_{i-1}}{B^*_{(T-i)^{ex}}} \right) \right)^{1-\gamma} \cdot \frac{a_{(T-i+1)^{cum}}}{\left(B^*_{(T-i+1)^{cum}}\right)^{1-\gamma}}. \tag{3.51}$$

Rearranging Expression (3.51), we obtain

$$\frac{1}{i} \cdot \left(w^n_{(T-i)^{ex}} \right)^{1-\gamma} \cdot \left(1 + \frac{C_{i-1}}{B^*_{(T-i+1)^{cum}}} \right)^{1-\gamma} \cdot \frac{a_{(T-i+1)^{cum}}}{\left(B^*_{(T-i)^{ex}}\right)^{1-\gamma}}. \tag{3.52}$$

Reverting to the expressions $a_{(T-i)^{ex}}$ and $a_{(T-i+1)^{cum}}$ defined in Equations (3.22) and (3.28) and employing the term $y_{(T-i+1)^{cum}}$ defined in Equation (3.23), the following is a straightforward result:

$$a_{(T-i)^{ex}} = \left(y_{(T-i+1)^{cum}}\right)^{1-\gamma} \cdot a_{(T-i+1)^{cum}} \tag{3.53}$$

Hence, substituting $(y_{(T-i+1)^{cum}})^{1-\gamma} \cdot a_{(T-i+1)^{cum}}$ with $a_{(T-i)^{ex}}$, Expression (3.52) simplifies to

$$\frac{1}{i} \cdot \left(w^n_{(T-i)^{ex}}\right)^{1-\gamma} \cdot \frac{a_{(T-i)^{ex}}}{\left(B^*_{(T-i)^{ex}}\right)^{1-\gamma}}, \tag{3.54}$$

which is equal to the indirect utility function $J_{(T-i)^{ex}}(w^n_{(T-i)^{ex}})$ defined in Equation (3.21). ■

A.3.3.2 Indirect Utility Function at $(T-i+1)^{cum}$

We subsequently show that equality

$$\begin{aligned} &J_{(T-i+1)^{cum}}\left(w^n_{(T-i+1)^{cum}}\right) = \\ &\max_{x_{(T-i+1)^{cum}}} \left\{p_{(i-1)} \cdot u\left(w^d_{(T-i+1)^{ex}} \cdot (1+r)^{i-1+\delta}\right) + \right. \\ &\qquad \left. \left(1-p_{(i-1)}\right) \cdot J_{(T-i+1)^{ex}}\left(w^n_{(T-i+1)^{ex}}\right)\right\} \end{aligned} \tag{3.55}$$

holds, where $J_{(T-i+1)^{cum}}(\cdot)$ is given by Equation (3.27).

We evaluate the right-hand side of Equation (3.55). The expression is maximized subject to the investor's budget constraint resulting in the characterization of wealth $w^s_{(T-i+1)^{ex}}$, $s \in \{d, n\}$ which is defined[21]

$$w^d_{(T-i+1)^{ex}} = w^n_{(T-i+1)^{cum}} \cdot \left(x_{(T-i+1)^{cum}} \cdot \frac{v^d_{(i-1)}}{B_{(T-i+1)^{cum}}} + \left(1 - x_{(T-i+1)^{cum}}\right) \cdot (1+r)^{\varepsilon} \right),$$

$$w^n_{(T-i+1)^{ex}} = w^n_{(T-i+1)^{cum}} \cdot \left(x_{(T-i+1)^{cum}} \cdot \frac{B^*_{(T-i+1)^{ex}}}{B_{(T-i+1)^{cum}}} + \left(1 - x_{(T-i+1)^{cum}}\right) \cdot (1+r)^{\varepsilon} \right).$$

The functional form of the indirect utility function $J_{(T-i+1)^{ex}}(\cdot)$ and the term $a_{(T-i+1)^{ex}}$ are given by[22]

$$J_{(T-i+1)^{ex}}\left(w^n_{(T-i+1)^{ex}}\right) = \frac{1}{(i-1)} \cdot \left(w^n_{(T-i+1)^{ex}}\right)^{1-\gamma} \cdot \frac{a_{(T-i+1)^{ex}}}{\left(B^*_{(T-i+1)^{ex}}\right)^{1-\gamma}},$$

$$a_{(T-i+1)^{ex}} \equiv \sum_{j=1}^{i-2} u\left(v^d_j \cdot (1+r)^{j+\delta} \cdot \prod_{k=j}^{i-2} y_{(T-k)^{cum}} \right) + u\left(v^n_1 \cdot (1+r)^{1+\delta} \cdot \prod_{k=1}^{i-2} y_{(T-k)^{cum}} \right).$$

Substituting the wealth expressions $w^s_{(T-i+1)^{ex}}$ and the market clearing condition $x_{(T-i+1)^{cum}} \equiv 1$ into the right-hand side of Equation (3.55), we obtain the solution

[21] The wealth terms $w^s_{(T-i+1)^{ex}}$ correspond to the time adjusted terms defined in Equation (3.20).

[22] The terms $J_{(T-i+1)^{ex}}(\cdot)$ and $a_{(T-i+1)^{ex}}$ correspond to the time adjusted terms defined in Equations (3.21) and (3.22).

of the maximization problem given by

$$p_{(i-1)} \cdot \frac{\left(w^n_{(T-i+1)^{cum}} \cdot \frac{v^d_{(i-1)} \cdot (1+r)^{i-1+\delta}}{B^*_{(T-i+1)^{cum}}}\right)^{1-\gamma}}{1-\gamma} + \tag{3.56}$$
$$\left(1 - p_{(i-1)}\right) \cdot \frac{1}{i-1} \cdot \left(w^n_{(T-i+1)^{cum}} \cdot \frac{B^*_{(T-i+1)^{ex}}}{B^*_{(T-i+1)^{cum}}}\right)^{1-\gamma} \cdot \frac{a_{(T-i+1)^{ex}}}{\left(B^*_{(T-i+1)^{ex}}\right)^{1-\gamma}}.$$

Substituting $p_{(i-1)} = \frac{1}{i}$ and rearranging Expression (3.56), we obtain

$$\frac{1}{i} \cdot \left(w^n_{(T-i+1)^{cum}}\right)^{1-\gamma} \cdot \frac{\frac{\left(v^d_{(i-1)} \cdot (1+r)^{i-1+\delta}\right)^{1-\gamma}}{1-\gamma} + a_{(T-i+1)^{ex}}}{\left(B^*_{(T-i+1)^{cum}}\right)^{1-\gamma}}. \tag{3.57}$$

Reverting to $a_{(T-i+1)^{cum}}$ defined in Equation (3.28) and the term $a_{(T-i+1)^{ex}}$, the following is a straightforward result:

$$a_{(T-i+1)^{cum}} = \frac{\left(v^d_{(i-1)} \cdot (1+r)^{i-1+\delta}\right)^{1-\gamma}}{1-\gamma} + a_{(T-i+1)^{ex}} \tag{3.58}$$

Hence, substituting the right-hand side of Equation (3.58) with $a_{(T-i+1)^{cum}}$, Expression (3.57) simplifies to

$$\frac{1}{i} \cdot \left(w^n_{(T-i+1)^{cum}}\right)^{1-\gamma} \cdot \frac{a_{(T-i+1)^{cum}}}{\left(B^*_{(T-i+1)^{cum}}\right)^{1-\gamma}}, \tag{3.59}$$

which is equal to the indirect utility function $J_{(T-i+1)^{cum}}(w^n_{(T-i+1)^{cum}})$ defined in

Equation (3.27). ■

A.3.4 Derivation of Bounds for Equilibrium Ex-day Price Behavior

We derive bounds for the equilibrium ex-day price behavior ΔB_i^* defined in Equation (3.30). Two cases are distinguished, and $c/R < r$ and $c/R > r$ are analyzed separately.

Case 1 We consider $c/R < r$.

First, we derive bounds for $B^*_{(T-i)^{cum}}$. The general bounds are given by Inequalities (3.15). It is unfavorable not to be drawn in the redemption lotteries, and the supremum equilibrium price is equal to $v^n_{i,1}/(1+r)^\varepsilon$ corresponding to the respective price of a straight coupon bond. However, it is favorable to be drawn in the earliest redemption lottery, and the infimum equilibrium price is equal to $v^d_i/(1+r)^\varepsilon$.

Hence, if $c/R < r$, the bounds for $B^*_{(T-i)^{cum}}$ simplify to

$$\frac{v^n_{i,1}}{(1+r)^\varepsilon} < B^*_{(T-i)^{cum}} < \frac{v^d_i}{(1+r)^\varepsilon}. \tag{3.60}$$

The present value terms v^d_i and $v^n_{i,1}$, defined in Equations (3.12) and (3.13), specify

$$v^d_i = \frac{R+c}{(1+r)^\delta}, \tag{3.61}$$

$$v^n_{i,1} = \sum_{k=0}^{i-1} \frac{c}{(1+r)^{k+\delta}} + \frac{R+c}{(1+r)^{i+\delta}}.$$

Second, we derive bounds for $B^*_{(T-i)^{ex}}$. The general bounds are given by Inequalities (3.17). Analogously, it is unfavorable not to be drawn in the redemption lotteries, and the infimum equilibrium price is equal to $v^n_{i,1}$, which corresponds to the respective price of a straight coupon bond. However, it is favorable to be drawn in the earliest redemption lottery, and the supremum equilibrium price is equal to $v^d_{i,(i-1)}$.

Hence, if $c/R < r$ and $i > 1$, the bounds for $B^*_{(T-i)^{ex}}$ simplify to

$$v^n_{i,1} < B^*_{(T-i)^{ex}} < v^d_{i,(i-1)}. \tag{3.62}$$

For $i = 1$, the equilibrium price $B^*_{(T-1)^{ex}}$ is deterministic and equal to v^n_1.

The present value term $v^d_{i,(i-1)}$ specifies

$$v^d_{i,(i-1)} = \frac{c}{(1+r)^{\delta}} + \frac{R+c}{(1+r)^{1+\delta}}. \tag{3.63}$$

We focus on the bounds for the equilibrium ex-day price behavior ΔB^*_i. The comparative static results for equilibrium lottery bond prices at cum-dates in Figure 3.8 show that prices are strictly monotonic decreasing in γ. In addition, it can be numerically shown that, for $c/R \neq r$, the following holds:

$$\frac{\partial B^*_{(T-i)^{cum}}}{\partial \gamma} < \frac{\partial \frac{B^*_{(T-i)^{ex}}}{(1+r)^{\varepsilon}}}{\partial \gamma} < 0$$

Hence, it is sufficient to evaluate Equation (3.30) firstly at the infimum equilibrium prices and secondly at the supremum equilibrium prices defined by Inequalities (3.60) and (3.62). The smaller result is equal to the lower bound on ΔB^*_i, while the larger result is equal to the upper bound.

In the following, we regard ΔB^*_i at infimum equilibrium prices:

$$\frac{\inf\left[B^*_{(T-i)^{ex}}\right]}{(1+r)^{\varepsilon}} - \inf\left[B^*_{(T-i)^{cum}}\right]$$

Evaluating the infimum expressions, we obtain

$$\frac{v_{i,1}^n}{(1+r)^{\varepsilon}} - \frac{v_{i,1}^n}{(1+r)^{\varepsilon}} = 0. \tag{3.64}$$

Next, we regard ΔB_i^* at supremum equilibrium prices:

$$\frac{\sup\left[B^*_{(T-i)^{ex}}\right]}{(1+r)^{\varepsilon}} - \sup\left[B^*_{(T-i)^{cum}}\right]$$

Evaluating the supremum expressions, we obtain

$$\frac{v_{i,(i-1)}^d - v_i^d}{(1+r)^{\varepsilon}} \tag{3.65}$$

simplifying to

$$\frac{\frac{R+c}{1+r} - R}{(1+r)^{\varepsilon+\delta}}. \tag{3.66}$$

For $c/R < r$, Equation (3.66) is negative such that the no-arbitrage bounds for ΔB_i^* specify

$$\frac{v_{i,(i-1)}^d - v_i^d}{(1+r)^{\varepsilon}} < \Delta B_i^* < 0. \tag{3.67}$$

Case 2 We consider $c/R > r$.

First, we derive bounds for $B^*_{(T-i)^{cum}}$. It is unfavorable to be drawn in the earliest redemption lottery, and the infimum equilibrium price is equal to $v_i^d/(1+r)^\varepsilon$. However, it is favorable not to be drawn in the redemption lotteries, and the supremum equilibrium price is equal to $v_{i,1}^n/(1+r)^\varepsilon$.

Hence, if $c/R > r$, the bounds for $B^*_{(T-i)^{cum}}$ simplify to

$$\frac{v_i^d}{(1+r)^\varepsilon} < B^*_{(T-i)^{cum}} < \frac{v_{i,1}^n}{(1+r)^\varepsilon}. \tag{3.68}$$

Second, we derive bounds for $B^*_{(T-i)^{ex}}$. Analogously, it is unfavorable to be drawn in the earliest redemption lottery, and the infimum equilibrium price is equal to $v_{i,(i-1)}^d$. However, it is favorable not to be drawn in the redemption lotteries, and the supremum equilibrium price is equal to $v_{i,1}^n$.

Hence, if $c/R > r$ and $i > 1$, the bounds for $B^*_{(T-i)^{cum}}$ simplify to

$$v_{i,(i-1)}^d < B^*_{(T-i)^{cum}} < v_{i,1}^n. \tag{3.69}$$

We focus on the bounds for ΔB_i^*. Again, it is sufficient to evaluate Equation (3.30) firstly at the infimum equilibrium prices and secondly at the supremum equilibrium prices defined by Inequalities (3.68) and (3.69).

We evaluate ΔB_i^* at infimum equilibrium prices and obtain

$$\frac{v_{i,(i-1)}^d - v_i^d}{(1+r)^\varepsilon}. \tag{3.70}$$

Next, we evaluate ΔB_i^* at supremum equilibrium prices and obtain

$$\frac{v_{i,1}^n}{(1+r)^\varepsilon} - \frac{v_{i,1}^n}{(1+r)^\varepsilon} = 0. \tag{3.71}$$

For $c/R > r$, Equation (3.70) is positive such that the no-arbitrage bounds for ΔB_i^* specify

$$0 < \Delta B_i^* < \frac{v_{i,(i-1)}^d - v_i^d}{(1+r)^\varepsilon}. \tag{3.72}$$

Note that, for c/R approaching r, the present value terms $v_{i,j}^d$ and $v_{i,1}^n$ converge, and the economy is free of redemption risk. Therefore, the equilibrium lottery bond prices approach the prices of straight coupon bonds, and the equilibrium ex-day price behavior B_i^* becomes zero. ■

Chapter 4

Descriptive Data Analysis

4.1 German Redemption Lottery Bonds

4.1.1 Basic Characteristics

The empirical study covers the period from January 1974 until December 1987. The analysis starts in 1974, when daily bond price data become available. It ends in 1987, since the last lottery of the considered issuer groups was conducted in that year.

We focus on lottery bonds issued by the Federal Republic of Germany (FRG), German states (GS), and government-owned enterprises (GE). The issuer structure within these groups is assumed to be sufficiently homogeneous with respect to credit risk, taxation, and liquidity. Bond issues by the FRG and GE are presumably free of credit risk. The bonds of the Deutsche Bundesbahn and the Deutsche Bundespost have been state-guaranteed. The guarantees are specified in paragraph 2(2) of the Poststrukturgesetz as of June 8, 1989[1] and paragraph 3(2) of the Bundesbahngesetz as of December 13, 1951[2] and were in place for the entire period of our analysis. Bond issues by GS are assumed to

[1] See e. g. Fangmann et al. (1990), pp. 39.

[2] See e. g. Finger (1982), pp. 75.

contain only limited credit risk. However, term structures of interest rates are determined separately for the FRG, GS, and GE issuer segments. We disregard bond issues by German municipalities, financial agencies, and supranational institutions. Financial agencies as well as German municipalities may contain non-negligible credit risk, and the segment of supranational institutions is rather illiquid.

To ensure the homogeneity of our dataset, we consider only lottery bonds denoted in German Mark paying fixed, non-zero, and regularly taxed coupons. Table 2.1 in Section 2.3 showed that a total of 110 lottery bonds were issued by the FRG, GS, and GE. Altogether, 15 bonds are excluded because they matured before 1974. Ten lottery bonds from GS are excluded because of non-standard redemption lotteries due to either the exercise of early or increased redemption options or by irregularities in the redemption plan.[3] Lastly, two lottery bonds by the FRG (WKN 110002, 110003) are excluded because price data could not be obtained. Hence, our dataset contains 83 lottery bonds for which data are available. For these issues, 483 redemption lotteries were played. Table 4.1 compiles the sample of lottery bonds segmented by issuer groups.

For our empirical study, we collected detailed data on German lottery bond characteristics. The data were obtained from Deutsche Finanzdatenbank (DFDB)[4] and Hoppenstedt Rentenführer[5]. Table 4.2 gives an overview of the basic lottery bond characteristics segmented by issuer groups. The importance of the lottery bond indenture as an instrument of the FRG, GS, and GE bond issues decreased over time. Figure 2.4 indicated that the FRG issued its last lottery bond in 1965, and GS and GE issued their last lottery bonds in 1973 and 1972, respectively. Daily bond price data do not become available until 1974 such that we disregard lotteries drawn beforehand. The mean nominal volume per issue is DEM 392.86 million for FRG lottery bonds, DEM 114.45 million for GS bonds, and DEM 241.90 million for GE bonds included in our sample. The range is largest for GS, where the state of Saarland has the lowest mean volume per

[3] See Table 4.4, Note c, and Table 4.5, Notes a and b, for further details on the excluded bonds.

[4] For further information on the DFDB, see Bühler et al. (1993).

[5] See Hoppenstedt (1974–1987).

Table 4.1: **Lottery Bond Issuers**

This table reports the number of lottery bonds issued by the Federal Republic of Germany, German states, and government enterprises as well as the number of lotteries related to these bonds.

Issuers	Bonds	Lotteries
Federal Republic of Germany	7	27
German States	55	361
Baden-Württemberg	3	18
Bavaria	4	35
Berlin	5	35
Bremen	3	26
Hamburg	4	28
Hesse	3	24
Lower Saxony	9	57
North Rhine-Westphalia	1	3
Rhineland-Palatinate	9	58
Saarland	8	51
Schleswig-Holstein	6	26
Government Enterprises	21	95
Deutsche Bundesbahn	10	46
Deutsche Bundespost	11	49
Total	83	483

issue of DEM 65.63 million and the state of Bavaria has the highest mean volume per issue of DEM 240.00 million. During the same period (until 1973), straight bonds have slightly higher mean nominal volumes per issue of DEM 437.83 million for FRG bonds, DEM 167.67 million for GS bonds, and DEM 293.75 million for GE bonds.

Lottery bonds were long term debt contracts with maturities between ten to 25 years and an average maturity of 16.37 years. About one third of the lottery bonds paid annual coupons, and the remainder paid coupons semi-annually. Coupon rates ranged from 5% to 9% and averaged 6.55%.

4.1.2 Redemption Features

Each lottery bond contained a redemption schedule specifying the lottery dates, repayment dates, the number of series to be redeemed, and the redemption values. Tables 4.3 and 4.4 detail the composition of lottery bonds regarding these

Table 4.2: **Basic Lottery Bond Characteristics**

This table shows descriptive statistics for the basic lottery bond characteristics of issues by the Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE). The first column reports the characteristics. The second to fourth columns report the absolute frequencies for bonds. The fifth to seventh columns report the absolute frequencies for lotteries.

Characteristics	Bonds			Lotteries		
	FRG	GS	GE	FRG	GS	GE
Issue Years						
1958 to 1960	0	6	6	0	28	36
1961 to 1965	7	15	13	27	65	47
1966 to 1970	0	19	1	0	151	9
1971 to 1973	0	15	1	0	117	3
Issue Volume (mDEM)						
< 50	0	4	0	0	19	0
50 to < 100	0	14	0	0	76	0
100 to < 200	0	28	3	0	195	15
200 to < 300	0	8	14	0	62	60
300 to < 400	2	1	1	5	9	3
400 to 500	5	0	3	22	0	17
Maximum Maturity (years)						
10 to < 15	3	32	9	7	209	19
15 to < 20	4	20	10	20	132	58
20 to 25	0	3	2	0	20	18
Coupon Frequency						
Annual	5	18	6	17	133	23
Semi-annual	2	37	15	10	228	72
Coupon Rate						
5 to < 6	0	4	7	0	21	36
6 to < 7	6	25	12	24	146	52
7 to < 8	1	15	1	3	106	4
8 to 9	0	11	1	0	88	3

Table 4.3: **Redemption Features I**

This table shows descriptive statistics for the redemption features of lottery bonds issued by the Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE). The first column shows the features. The second to fourth columns report the absolute frequencies for bonds. The fifth to seventh columns report the absolute frequencies for lotteries.

Redemption Features	Bonds			Lotteries		
	FRG	GS	GE	FRG	GS	GE
Redemption Value						
Par	–	–	–	27	336	95
Above Par	–	–	–	0	25	0
Redemption Frequency						
Annual	2	50	15	10	345	80
Biennial	5	5	6	17	16	15
Lag between Issuance and First Lottery (years)						
3 to < 5	0	7	1	0	53	3
5 to < 6	2	29	10	10	190	46
6 to < 10	2	16	4	5	101	8
10 to 12	3	3	6	12	17	38
Lag between Lottery and Redemption (days)						
< 100	–	–	–	2	39	10
100 to < 110	–	–	–	19	101	76
110 to < 130	–	–	–	6	153	9
130 to < 150	–	–	–	0	47	0
150 to 200	–	–	–	0	21	0

redemption features. All redemption lotteries by the FRG and GE and about 90% of the lotteries by GS were redeemed at par. The remaining GS lotteries were redeemed at either 101, 102, or 103. Redemption lotteries were conducted on an annual or a biennial basis. More than 70% of the lottery bonds by the FRG had biennial drawings, whereas more than 90% of the lotteries by GS and more than 70% of the lotteries by GE were conducted annually. After issuance, three to twelve years passed until the first redemption lottery was played. The average lag between issuance and the first drawing was 6.61 years. The lag between the lottery and the redemption payments ranged from 68 to 186 calendar days and averaged 115 days.

The majority of the lottery bonds was split into five to 15 series, and the average bond was composed of twelve series. At each redemption date, one series was

Table 4.4: **Redemption Features II**

This table shows descriptive statistics for further redemption features of lottery bonds issued by the Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE). The first column reports the feature. The second to fourth columns report the absolute frequencies for bonds. The fifth to seventh columns report the absolute frequencies for lotteries.

Redemption Features	Bonds			Lotteries		
	FRG	GS	GE	FRG	GS	GE
Number of Series[a]						
3 to < 5	0	2	1	0	5	3
5 to 9	5	8	6	17	31	15
10	1	35	5	2	268	18
11 to 15	1	6	8	8	31	55
16 to 20	0	2	1	0	15	4
100	0	2	0	0	11	0
Redemption Probabilities						
1/2	–	–	–	7	55	21
1/3	–	–	–	7	52	19
1/4	–	–	–	5	51	15
1/5	–	–	–	4	45	12
1/6	–	–	–	1	39	8
1/7	–	–	–	1	37	6
1/8	–	–	–	1	33	6
1/9	–	–	–	1	29	5
1/10	–	–	–	0	20	3
< 1/10	–	–	–	0	(6)[b]	0
Irregular	–	–	–	0	(8)[c]	0

[a] For three lottery bonds, more than one series is regularly redeemed. The issues by the state of Hamburg (WKN 136510, 136511) consisted of 100 series that were redeemed in 12 and 15 stages, respectively. The issue by the Deutsche Bundesbahn (WKN 115003) consisted of 20 series that were redeemed in 14 stages.

[b] Six lottery observations of two bond issues by the state of Bavaria (WKN 105024, 105025) with redemption probabilities below 1/10 are excluded to simplify our analysis.

[c] One lottery bond (eight lotteries) by the state of Schleswig-Holstein (WKN 179005) is excluded because of non-standard redemption probabilities caused by irregularities in the redemption plan.

drawn by lottery and repaid. Only two lottery bonds by GS were amortized in more than 15 stages.[6]

Since all series of one bond issue were approximately of the same size, the redemption probabilities can easily be determined by calculating the ratio of the actual number of redeemed series to the total number of outstanding series. Within the empirical analysis, we consider the first nine redemption probabilities (1/2 to 1/10). Lotteries with higher probabilities appear more frequently, since lottery bonds were issued between 1958 and 1973, and we disregard lotteries before 1974. To clarify, we consider a typical lottery bond issued in 1965. The bond was divided into ten series of equal size. After five redemption-free years, the lottery bond was redeemed by ten annual lotteries. Hence, only the redemption lotteries with probabilities 1/2 to 1/6 are included. Lotteries with lower probabilities are disregarded as they were conducted before 1974 and daily lottery bond prices are not available. Next, we focus on the distribution of redemption lotteries over time. Figure 4.1 depicts the absolute frequency of lotteries per drawing year segmented by the nine redemption probabilities. The histograms illustrate the varying distribution of lottery dates. The higher the redemption probability, the more recent the mean lottery date and the larger the range of lottery dates. For probability 1/2, lottery dates range from 1974 to 1987 with a standard deviation of 1,140 days, whereas, for probability 1/10, lottery dates range from 1974 to 1979 with a standard deviation of only 601 days.

Apart from the scheduled redemption by lottery, most of the indentures contained early or increased redemption options. Table 4.5 compiles these embedded options. About 75% of the lottery bond indentures in our sample are equipped with an embedded issuer call option. However, only two call features out of 64 were exercised by GS within the period of our empirical analysis.[7] As already mentioned, the restrictive call policy of the FRG, GS, and GE is consistent with the findings of Bühler and Schulze (1993, 1999), who analyze issuer call features for straight FRG and GE bonds. More than 70% of the lottery bond indentures are equipped with increased redemption provisions. Out of the 66 lottery bond

[6] One issue by the state of Bavaria (WKN 105024) was redeemed in 16 stages, and one issue by the state of Lower Saxony (WKN 159011) was redeemed in 20 stages.

[7] For details on issues that exercised the call feature, see Table 4.5, Note a.

Figure 4.1: **Lottery Years**

This figure shows the absolute frequency of lotteries per drawing year segmented by redemption probabilities.

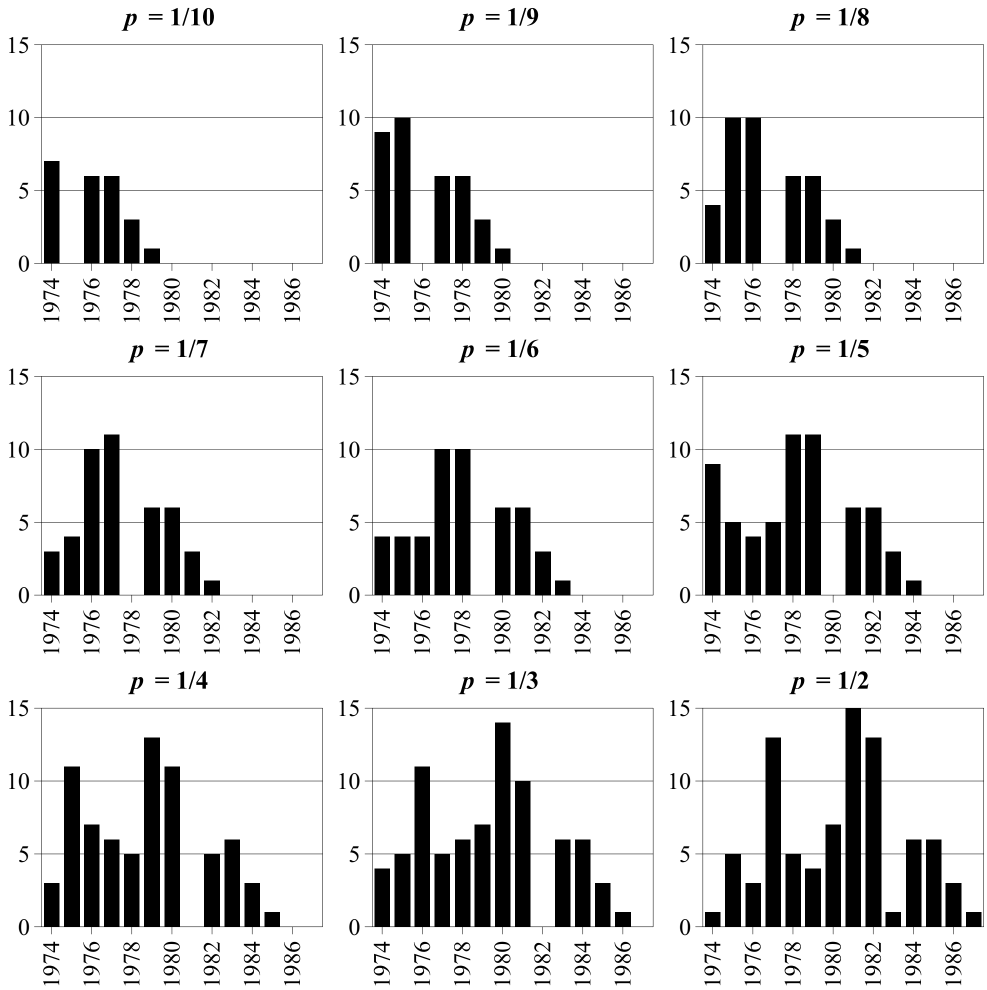


Table 4.5: **Redemption Options**

This table shows the descriptive statistics for early or increased redemption options of lottery bonds issued by the Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE). The first column reports the redemption option. The second to fourth columns show the absolute frequencies for bonds. The fifth to seventh columns show the absolute frequencies for lotteries.

Redemption Options	Bonds			Lotteries		
	FRG	GS	GE	FRG	GS	GE
Issuer Call Feature						
Not Callable	0	20	1	0	142	3
Callable, but Not Exercised	7	35	20	27	219	92
Callable and Exercised	0	(2)[a]	0	0	(3)[a]	0
Increased Redemption Provision						
Not Possible	0	23	1	0	172	3
Possible, but Not Exercised	7	32	20	27	189	92
Possible and Exercised	0	(7)[b]	0	0	(32)[b]	0
Open Market Repurchases						
No Offsetting	7	42	21	27	289	95
Offsetting, but Not Exercised	0	13	0	0	72	0
Offsetting and Exercised	(1)[c]	0	0	0	0	0

[a] Two lottery bond issues (three lotteries) are excluded from the dataset because the embedded call feature was exercised. A first issue by the state of Bremen (WKN 108010) was called on April 1, 1977, and a second issue also by the state of Bremen (WKN 108012) was called before its first lottery on July 1, 1978.

[b] Seven lottery bond issues (32 lotteries) are excluded from the dataset because increased redemption provisions were exercised: (i) within the bond indenture by the state of Baden-Württemberg (WKN 104011), three additional series were redeemed on November 2, 1978, (ii) within the bond indenture by the state of Bavaria (WKN 105023), one additional series was redeemed on April 1, 1978, (iii) within the bond indenture by the state of Hamburg (WKN 136512), one additional series was redeemed on November 1, 1978, (iv) within the bond indenture by the state of Hesse (WKN 138003), one additional series was redeemed on July 1, 1978, (v-vii) the state of Schleswig-Holstein exercised increased redemption provisions for three issues. Within a first bond indenture (WKN 179010), three additional series were redeemed on August 1, 1978, within a second bond indenture (WKN 179011), two additional series were redeemed on April 1, 1978, and within a third bond indenture (WKN 179012), one additional series was redeemed on April 1, 1978.

[c] One bond indenture issued in 1990 by the FRG (WKN 117018, DEM-Fundierungsschuld) is excluded because the issuer used the privilege to repurchase bonds in the open market to fulfill its redemption requirements. However, the indenture is classified as a sinking fund rather than a redemption lottery bond, since the issuer used the option to either call series by lottery or purchase the required quantity of redeemable bonds in the open market.

issues that allowed for increased redemption, seven provisions were exercised by GS.[8] All of the early and increased redemption options were employed between 1977 and 1978, when interest rates were at a historical low. Lastly, about 13 lottery bond indentures by GS are equipped with the option to purchase bonds in the open market to offset redemption drawings. All but one of the options allowed for offsetting only when the respective series was drawn for redemption. One issue by North Rhine-Westphalia (WKN 159501) allowed for offsetting without restrictions. However, open market repurchases were not recorded for lottery bonds in our sample, and repurchases are classified as a rather unimportant redemption feature.[9]

During the entire period of our analysis the FRG and GE did not exercise any of the early or increased redemption options embedded in lottery bond indentures. Since the issuer groups refrained from exercising redemption options for several decades, this policy evolved into a market convention. Hence, investors did not expect the FRG or GE, which feared a loss of reputation, to deviate from their historical redemption policies.[10] However, the FRG called two straight coupon bonds in 1978, and the exercise of early or increased redemption options was recorded for eight lottery bonds by GS between 1977 and 1978. Within the subsequent empirical analysis, we account for the early and increased redemption options by focusing on clean bond prices quoting below the discounted redemption value. We assume that these prices are not influenced by the out-of-the-money redemption options. A total of 18 lottery bonds (130 lotteries) by GS and one issue (three lotteries) by GE do not contain early or increased redemption options. These lottery bonds will be used as a control group to test for possible biases caused by redemption options.

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[8] For details on issues that exercised increased redemption provisions, see Table 4.5, Note b.

[9] One bond issue by the FRG repurchased bonds in the open market to fulfill redemption requirements. However, the bond is of the sinking fund type and was issued 17 years after the last lottery bond. See Table 4.5, Note c.

[10] See Bühler and Schulze (1999), pp. 248.

4.1.3 Official Market Prices

German lottery bonds and straight coupon bonds were traded on organized exchanges and over-the-counter. Our empirical analysis is based on bond market prices (*amtlicher Kurs*) fixed once each trading day by official exchange brokers at organized exchanges such as the Frankfurt Stock Exchange.[11] Based on the order book, the brokers determined the official market price in line with the principle of maximum execution. The principle ensured the maximum daily turnover of the bond at the exchange. All orders received during the trading day had to be included when fixing the price.

Brokers marked prices with an addendum that characterized to which extent orders were executed.[12] The addenda specified whether: (i) all orders could be executed and markets cleared, (ii) buy or sell orders could not be entirely executed such that there was additional demand or supply, or (iii) no trades were recorded and only buy or sell offers existed. A price addendum also specified whether prices were suspended due to redemption drawings. Prior to the drawings lottery bonds were not quoted on the stock exchange and trade was suspended for on average two trading days. According to German bond market conventions, lottery bond contracts were settled on the second trading day after the execution such that all trades closed before the lottery were already settled by the drawing date.[13] On each trading day the official prices and price addenda were published by the organized exchange in the official price list (*amtliches Kursblatt*). Daily clean market prices and price addenda for most of the outstanding lottery bonds are available as of January 2, 1974. The prices are restricted to changes in discrete ticks of DEM 0.05.

Table 4.6 shows descriptive statistics for the trading activity of lottery bonds segmented by issuer groups. The table reports the aggregate number of trading days, the number of active trading days with positive trading volumes, and the

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[11] See Börsenordnung für die Frankfurter Wertpapierbörse (1987), Section V, Paragraphs 24 to 32, and Deutsche Bundesbank (2000), pp. 49, for details on the fixing of the official bond market price.

[12] See Table 4.6, Note a, for the definition of the official addenda used at price fixings.

[13] See Oppermann and Degner (1983), p. 35.

number of trading days with zero trading volume. Our dataset contains 12,953 price observations of lottery bond issues by the FRG, 122,787 prices of issues by GS, and 35,071 prices of issues by GE. On average, each lottery bond accounts for about 2,058 price observations.

We distinguish active trading days with positive trading volumes from days with zero trading volume. First, we focus on days with positive trading volumes in which the official market prices are transaction prices. The dataset contains 9,605 transaction prices from the FRG, 54,477 transaction prices from GS, and 19,169 transaction prices from GE. About 74.2% of the aggregate price observations from the FRG, 44.4% of the prices from GS, and 54.7% of the prices from GE are transaction prices. On more than 90% of the active trading days, all orders were executed and markets cleared. On the remaining days, orders could only be partially executed.

Second, we focus on days with zero trading volume. We distinguish days with sell or buy orders only in which the official market prices are quotes from days with cancelled or suspended quotes. The dataset contains 3,276 quotes for FRG lottery bonds, 67,019 quotes for GS lottery bonds, and 15,554 quotes for GE lottery bonds. On more than 97% of the days without trading activity, only buy orders existed. It is important to note that, by order and for the account of the bond issuer, the Deutsche Bundesbank engaged in price management operations at organized exchanges.[14] The major objective of these operations was to ensure the liquidity of the secondary bond market. The Deutsche Bundesbank held funds and a substantial volume of bonds for price management purposes. The predominance of buy offers on days without trading activity documents that the Deutsche Bundesbank was frequently engaged as a liquidity provider from the demand side. The bank placed buy orders and guaranteed that lottery bonds remained tradable. A further objective of the price management operations was to establish market-related prices at the official price fixings. The Deutsche Bundesbank frequently claimed that on trading days with zero trading volume,

[14] See Barocka (1959), Bösch (1959), and Reiter (1967), pp. 397, for details on the price management activities of the Deutsche Bundesbank in the bond market. Price management operations for bonds issued by the Deutsche Bundesbahn were conducted by the Deutsche Verkehrs-Kreditbank AG.

Table 4.6: **Trading Activity**

This table shows descriptive statistics for the trading activity of lottery bonds by the Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE). The second row reports the aggregate number of trading days. The third to fifth rows report the aggregate number of active trading days. We distinguish days on which all orders were executed from days on which orders were only partially executed. The sixth to tenth rows report the aggregate number of trading days on which no trade was recorded. We distinguish days on which only buy or sell offers were reported from days on which prices were cancelled or suspended due to a redemption drawing.

Trading Activity[a]	FRG	GS	GE
Trading Days	12,953	122,787	35,071
Active Trading Days[b]	9,605	54,477	19,169
All Orders Executed	8,788	51,747	18,055
Orders Partially Executed	817	2,730	1,114
Days w/ Zero Trading Volume[c]	3,348	68,310	15,902
Buy Offers Only	3,276	67,018	15,553
Sell Offers Only	0	1	1
Cancelled	18	464	148
Drawing	54	827	200

[a] Official exchange brokers at the Frankfurt Stock Exchange used the following addenda at price fixings to specify to what extent orders were executed: (**b**) paid: all orders were executed and markets cleared, (**G**) bid: no trades were recorded and only buy orders existed, (**B**) ask: no trades were recorded and only sell orders existed, (**bG**) paid, bid: buy orders were not necessarily executed in full such that there was additional demand, (**bB**) paid, ask: sell orders were not necessarily executed in full such that there was additional supply, (**ebG**) partially paid, bid: only a small portion of the buy orders could be executed, (**ebB**) partially paid, ask: only a small portion of the sell orders could be executed, (**ratG**) scaling down, bid: buy orders were only partially executed, (**ratB**) scaling down, ask: sell orders were only partially executed, (**-**) quotation cancelled: no price was determined, (**-T**) price estimate was cancelled, (**-Z**) quotation cancelled, drawing: quotation was suspended due to a redemption drawing, (*****) small amounts were not traded.

[b] The segment *All Orders Executed* comprises prices marked with the addendum (b). The segment *Orders Partially Executed* comprises prices marked with the addenda (bG), (bB), (ebG), (ebB), (ratG), (ratB), and (*).

[c] The segment *Buy Offers Only* comprises prices marked with the addenda (G) and (G*). The segment *Sell Offers Only* comprises prices marked with the addenda (B) and (B*). The segment *Cancelled* comprises prices marked with the addenda (-) and (-T). The segment *Drawing* comprises prices marked with the addendum (-Z).

their buy orders were actually adjusted reflecting current market conditions. Nevertheless, we consider throughout our empirical analysis only transaction prices from active trading days and disregard price quotes from days with zero trading volume.

We examine the trading volume and number of transactions per lottery bond in further detail. Trading volume is measured in face value per traded bond. The data include trades on the Frankfurt Stock Exchange only. In January 1987, the official definition of trading volume changed such that we restrict our analysis to volumes and transactions in the interval January 1974 to December 1986. Figure 4.2 shows the time series of the monthly trading volume and the number of transactions per lottery bond segmented by issuer groups. For issues by the FRG, monthly trading volumes and transactions per lottery bond attain their maxima at DEM 10.21 million in November 1976 and at 83 in October 1975, respectively. The maxima for GS issues are reached at DEM 1.02 million in April 1975 and at 43 in July 1975, respectively. Finally, the maxima for GE issues are attained at DEM 6.93 million in July 1975 and at 52 in October 1975, respectively. The time series demonstrate that trading volume and number of transactions per lottery bond decreased over time.

Table 4.7 reports the means and standard deviations of the monthly trading volume per lottery bond, the monthly number of transactions per lottery bond, and the volume per transaction and lottery bond segmented by issuer groups. The mean monthly trading volume per lottery bond is DEM 1.61 million for issues by the FRG, DEM 0.22 million for issues by GS, and DEM 1.18 million for issues by GE. The mean number of monthly transactions per lottery bond is 42 for issues by the FRG, 18 for issues by GS, and 22 for issues by GE. Hence, the measures are largest for issues by the FRG followed by issues by GE and distinctly lower for GS lottery bonds. Table 4.6 already indicated more trading activity in the FRG and GE market segments relative to the GS segment. The magnitudes of the monthly trading volume and the number of transactions per lottery bond are not very pronounced. One reason is the historical definition of trading volume differing from the current convention and counting the face value per traded bond only once. In addition, transactions between banks and transactions between exchange brokers were historically not counted. The low

Figure 4.2: **Monthly Trading Volume and Transactions**

This figure shows the time series of the mean monthly trading volume and the number of transactions per lottery bond segmented by issuer groups. Trading volume is measured in face value per traded bond and reported in million German Mark. Between January 3, 1983 and April 5, 1983 as well as between October 3, 1983 and November 1, 1983, trading volume and transaction data are not available.

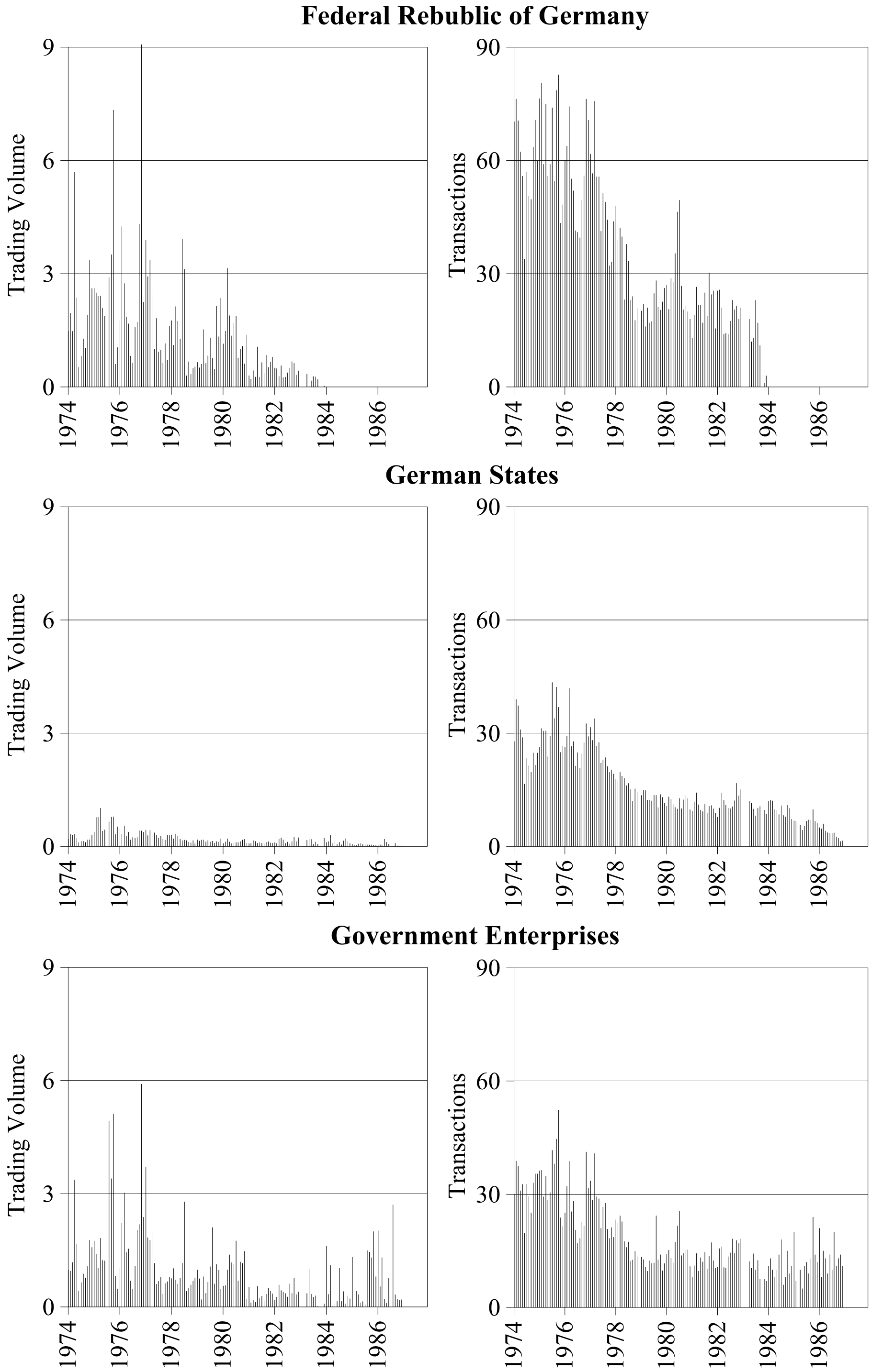


Table 4.7: **Monthly Trading Volume and Transactions**

This table shows the means and standard deviations of the monthly trading volume per lottery bond, the monthly number of transactions per lottery bond, and the volume per transaction and lottery bond. The results are reported for the Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE) separately. Trading volume and volume per transaction are measured in face value per traded bond and are given in thousand German Mark. Between January 3, 1983 and April 5, 1983 and between October 3, 1983 and November 1, 1983, trading volume and transaction data are unavailable.

Volume and Transactions	FRG	GS	GE
Trading Volume (tDEM)			
Mean	1,613	221	1,175
Std. Dev.	728	197	733
Transactions			
Mean	42	18	22
Std. Dev.	11	15	11
Volume per Transaction (tDEM)			
Mean	35	13	46
Std. Dev.	9	7	18

magnitude also reflects that the majority of trading in German debt securities took place in over-the-counter markets.

Even though the trading activity in the secondary bond market is rather low, the quality of the official market price data is notable. First of all, the prices were fixed by official exchange brokers who included all orders received during the trading day when fixing the price. Second, the Deutsche Bundesbank ensured that the official price fixings were market-related. Therefore, we claim that transaction prices from active trading days are suitable for our further empirical analysis.

4.2 Straight German Bonds and Money Market Rates

4.2.1 Basic Characteristics

The certainty alternative to lottery bonds are straight coupon bonds with a fixed maturity date issued by the FRG, GS, and GE. We estimate term structures of interest rates from straight coupon bonds separately for each of the three issuer groups. To ensure a sufficient number of observations in the short-term maturity segment, we replenish our dataset with money market rates for banks' cash deposits. Before estimating the term structures, we focus on the characteristics of straight coupon bonds and money market rates.

Daily clean market prices for straight bonds are available as of January 2, 1974 such that we include straight bond issues outstanding between January 1974 and December 1987. We disregard issues with embedded call or put options and without regularly taxed coupon payments to ensure the homogeneity of the dataset. Four straight bonds by the FRG (WKN 102512, 102516, 102520, and 102524, all Ausgleichsfonds) and one bond by the state of Bremen (WKN 108011) are excluded because their nominal issue volumes are distinctly below DEM 100 million. We also disregard price observations of issues with time-to-maturities above ten years or below six months and observations from the first six months after issuance to guarantee the quality of data and a sufficient liquidity of prices.

Our dataset contains a total of 277 straight coupon bonds for which data are available. Table 4.8 compiles the straight bond issues and nominal volume data segmented by issuer groups. A total of 124 bonds are from the FRG, 61 bonds are from GS, and 92 bonds are from GE. The aggregate nominal issue volume of straight bonds is DEM 229.90 billion.

Table 4.9 gives an overview of the basic bond characteristics. Unlike that of lottery bond indentures, the importance of straight coupon bonds as an instrument of bond issues by the FRG, GS, and GE increased over time. No more than 30 straight bonds were issued between 1963 and 1969, and about 81 such bonds were issued between 1980 and 1984. Nominal volumes per issue range

Table 4.8: **Straight Coupon Bond Issuers**

This table reports the number and nominal issue volume of straight bonds issued by the Federal Republic of Germany, German states, and government enterprises. The aggregate nominal issue volume is reported in billion German Mark.

Issuers	Bonds	Issue Volume
Federal Republic of Germany	124	148.96
German States	61	24.88
Baden-Württemberg	5	1.95
Bavaria	9	4.60
Berlin	7	1.25
Bremen	3	0.85
Hamburg	7	2.30
Hesse	4	1.28
Lower Saxony	9	2.50
North Rhine-Westphalia	10	8.00
Rhineland-Palatinate	1	0.60
Saarland	1	0.15
Schleswig-Holstein	5	1.40
Government Enterprises	92	56.06
Deutsche Bundesbahn	53	29.98
Deutsche Bundespost	39	26.08
Total	277	229.90

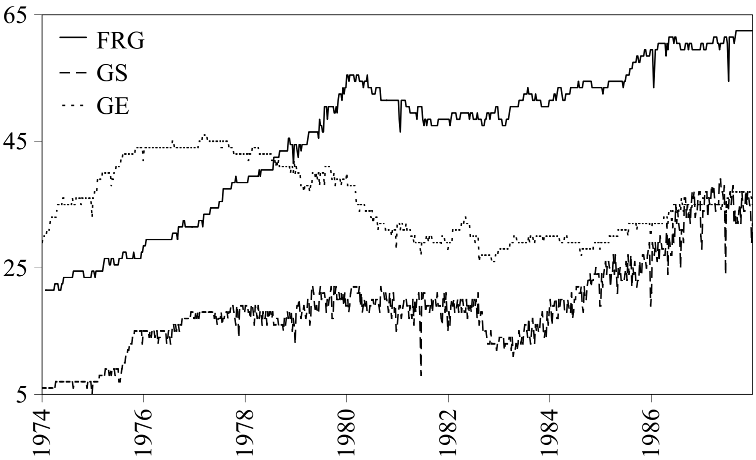
Table 4.9: **Straight Coupon Bond Characteristics**

This table shows the basic characteristics of straight coupon bonds issued by the Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE). The first column reports the characteristic. The second to fourth columns report the absolute frequencies.

Characteristics	FRG	GS	GE
Issue Years			
1963 to 1969	13	2	15
1970 to 1974	17	7	30
1975 to 1979	43	17	14
1980 to 1984	35	21	25
1985 to 1987	16	14	8
Issue Volume (bnDEM)			
0.10 to < 0.50	24	42	41
0.50 to < 0.75	30	8	14
0.75 to < 1.00	13	11	25
1.00 to < 2.00	33	0	11
2.00 to < 3.00	16	0	1
3.00 to 4.00	8	0	0
Initial Maturity (years)			
5 to < 10	38	21	23
10	76	39	54
> 10 to 15	10	1	15
Coupon Frequency			
Annual	116	58	82
Semi-annual	8	3	10
Coupon Rate			
5 to < 6	9	2	1
6 to < 7	28	15	23
7 to < 8	32	18	17
8 to < 9	35	20	27
9 to < 10	10	3	8
10 to < 11	10	3	16

Figure 4.3: **Number of Outstanding Straight Coupon Bonds**

This panel shows the number of outstanding straight coupon bonds for which transaction price data are available. The time series is based on weekly observations reported for the Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE).



from DEM 100 million to DEM 4,000 million. The mean nominal issue volume is DEM 1,201 million for issues by the FRG, DEM 408 million for issues by GS, and DEM 609 million for issues by GE. Maturities range from five to 15 years and average 9.37 years. Only 21 of the straight bonds paid semi-annual coupons, whereas the remainder paid coupons annually. Coupon rates range from 5% to 10.75% and average 7.74%.

The total number of straight coupon bond price observations available for the term structure estimation varies over time and across issuer groups. Only transaction prices from active trading days are considered.[15] Figure 4.3 shows the number of outstanding straight coupon bonds over time for which transaction price data are available. Between 1974 and 1987, the number of bond price observations for the FRG ranges from 21 to 62 and averages 45.17 per observation date. The number of bond price observations for GS ranges from five to 39 and averages 19.10 per observation date, whereas the number of observations for GE ranges from 26 to 46 and averages 35.47. The mean number of outstanding

[15] According to Section 4.1.3, transaction prices are fixed by official exchange brokers on active trading days. See Table 4.6, Footnote b, for the definition of an active trading day.

bonds is lowest for GS, but at the end of 1975 a total of 15 issues were already outstanding. The jumps in the time series result from the exclusion of prices from days with zero trading volume.

Money market rates are considered to ensure a sufficient number of observations in the short-term maturity segment. Daily money market rates for 1-month and 3-month cash deposits are obtainable as of January 2, 1974 from the Deutsche Bundesbank. As of January 2, 1975 daily money market rates for 1-month, 3-month, 6-month, and 1-year are obtainable from Datastream. Mid rates are calculated from banks' bid and ask quotes for cash deposits in Deutsche Mark in the German money market. We disregard the overnight and 1-week money market rates, as they are highly volatile and lead to additional and unexplainable noise in the estimation.

4.2.2 Estimation of Term Structure of Interest Rates

There is a large body of literature dealing with purely descriptive approaches to fitting the term structures of interest rates to a set of cross-sectional bond price observations.[16] First, papers on term structures relate the internal rates of return of coupon bonds to their time-to-maturity. For example, Cohen et al. (1966) and Fisher (1966) approximate yield-to-maturity curves using ordinary least squares regression analysis. However, the yield-to-maturity of a coupon bond, which effectively contains information regarding various interest rates of the term structure, is only a rough approximation of the interest rate at the given time-to-maturity. One of the first approaches for fitting the discount function to coupon bond prices goes back to McCulloch (1971). The basic idea was to approximate the relation between discount factors and bond maturity using a specified functional form fitted to price data. McCulloch uses a piecewise polynomial spline

[16] See Anderson et al. (1996) for a comprehensive overview of purely descriptive curve fitting approaches. We distinguish these approaches from spot-rate or forward-rate factor models based on either equilibrium or no-arbitrage considerations. See e. g. Vasicek (1977), Brennan and Schwartz (1982), Cox et al. (1985), Ho and Lee (1986), or Heath et al. (1992) for classical spot-rate or forward-rate factor models.

technique, whereas subsequent studies employ exponential or cubic splines.[17] Spline estimates are highly flexible and allow for the approximation of complex shapes of the term structure, even though the flexibility has the disadvantage of imparting shapes that are economically unreasonable.[18]

Nelson and Siegel (1987) pursue an alternative approach for fitting the term structure based on a parsimonious parametrization of the discount function. The discount function is of exponential form with only four unknown parameters. Svensson (1995) increases the flexibility of the original Nelson and Siegel formulation by adding two additional parameters allowing for a second hump in the spot-rate curve. According to the Deutsche Bundesbank, both models are sufficient to reflect the typical data constellations observed in the German bond market and are a reasonable compromise between flexibility and smoothness.[19] Term structure estimates are robust and relatively independent of outliers also under critical data constellations, as is the case for the GS issuer segment between 1974 and 1976.[20] An additional appealing property of the parsimonious models is that the parameters have an economic interpretation. Since our primary purpose for estimating term structures is to extract investors' risk preferences, we are interested in sufficiently robust and smooth estimates. Hence, we refrain from spline-based approximations and employ the Svensson approach to estimate term structures of interest rates.

The Nelson and Siegel or Svensson term structure models are widely used by central banks and practitioners.[21] As of October 1997, the Deutsche Bundesbank officially estimates term structures using the Svensson approach. The estimations are based on clean market prices of straight coupon bonds and five-year special bonds by the FRG as well as Treasury notes with a time-to-maturity of at least

[17] See e. g. McCulloch (1975), Vasicek and Fong (1982), Shea (1984, 1985), Steeley (1991), and Fisher et al. (1994).

[18] See e. g. Shea (1984) who addresses difficulties encountered by spline-based term structure approximation techniques.

[19] See Deutsche Bundesbank (1997), pp. 64.

[20] See Schich (1997), p. 23.

[21] See e. g. Bank for International Settlements (2005).

three months.[22] Term structures for the GS and GE issuer segments are not explicitly determined. Since August 1997, term structure parameters of the Svensson function are provided on a daily basis. In addition, the Deutsche Bundesbank supplies backward looking parameter estimates on a monthly basis since October 1972. However, as the short-term maturity segment was historically thinly represented, these parameters lead to implausibly volatile and in parts negative interest rates for short maturities.[23] Within the subsequent empirical sections, term structures are used to discount over short time intervals such that we are interested in sufficiently robust short-term interest rates. To ensure a sufficient number of observations in the short term segment, we replenish the straight coupon bond data with 1-month, 3-month, 6-month, and 1-year money market rates and estimate straight term structures for the FRG, GS, and GE separately.

The subsequent paragraphs briefly describe the term structure model by Svensson. We assume the following functional form for the relation between spot-rates at date t and the time-to-maturity m:[24]

$$\begin{aligned} r\left(\mathbf{b}_t, m\right) =& \beta_{0,t} + \beta_{1,t} \cdot \left(\frac{1 - \exp\left[-\frac{m}{\tau_{1,t}}\right]}{\frac{m}{\tau_{1,t}}} \right) + \\ & \beta_{2,t} \cdot \left(\frac{1 - \exp\left[-\frac{m}{\tau_{1,t}}\right]}{\frac{m}{\tau_{1,t}}} - \exp\left[-\frac{m}{\tau_{1,t}}\right] \right) + \\ & \beta_{3,t} \cdot \left(\frac{1 - \exp\left[-\frac{m}{\tau_{2,t}}\right]}{\frac{m}{\tau_{2,t}}} - \exp\left[-\frac{m}{\tau_{2,t}}\right] \right), \end{aligned} \tag{4.1}$$

where $\mathbf{b}_t$ denotes the Svensson parameter vector consisting of $\beta_{0,t}$, $\beta_{1,t}$, $\beta_{2,t}$, $\beta_{3,t}$,

[22] See Deutsche Bundesbank (1997) and Schich (1997) for a detailed description of the estimating procedure of the Deutsche Bundesbank.

[23] See e. g. Schich (1997), p. 23, Footnote 12.

[24] See Svensson (1995), p. 18, Equation (11).

$\tau_{1,t}$, and $\tau_{2,t}$ to be estimated. We consider the influence of time-to-maturity m on the spot-rate curve $r(\mathbf{b}_t, m)$ for a given $\mathbf{b}_t$. If the time-to-maturity m approaches infinity, $r(\mathbf{b}_t, m)$ converges asymptotically towards $\beta_{0,t}$, interpreted as the long-term interest rate. If, however, m approaches zero, $r(\mathbf{b}_t, m)$ converges towards the parameter combination $\beta_{0,t} + \beta_{1,t}$, interpreted as the short-term interest rate. The remaining parameters identify the shape of the spot-rate curve.[25]

The term structure of spot-rates $r(\mathbf{b}_t, m)$ implies the structure of forward-rates $f(\mathbf{b}_t, m, \lambda)$ at date t for the time interval $[m, (m + \lambda)]$, defined by

$$f\left(\mathbf{b}_t, m, \lambda\right) = \frac{\left(1 + r\left(\mathbf{b}_t, m + \lambda\right)\right)^{1+\frac{m}{\lambda}}}{\left(1 + r\left(\mathbf{b}_t, m\right)\right)^{\frac{m}{\lambda}}} - 1, \tag{4.2}$$

where $m, \lambda \geq 0$.

The objective of the term structure estimation is to determine the parameter vector $\mathbf{b}_t$ that minimizes the sum of the squared errors of the implicit and the observed yield-to-maturity. At each observation date, $\mathbf{b}_t$ is determined from the cross-section of straight coupon bonds and money market instruments. Yield errors rather than price errors are minimized because we want to attribute higher weights to the short end of the term structure.[26]

The starting point of the estimation is the definition of the implicit price of a straight coupon bond compatible with the functional form of the term structure of spot-rates defined in Equation (4.1). We assume a bond with an annual coupon frequency such that the implicit dirty price at date t is defined as

$$B_{n,t}\left(\mathbf{b}_t\right) = \sum_{i=0}^{T_n - \lceil t \rceil} \frac{c_n}{\left(1 + r\left(\mathbf{b}_t, i + \lceil t \rceil - t\right)\right)^{i+\lceil t \rceil - t}} + \frac{R_n}{\left(1 + r\left(\mathbf{b}_t, T_n - t\right)\right)^{T_n - t}}, \tag{4.3}$$

[25] See e. g. Schich (1997), pp. 15, and Bolder and Stréliski (1999), pp. 6, for a detailed decomposition of the spot-rate curve.

[26] Alternatively, price errors could be adjusted by employing a weighting term based on the inverse of the duration of the individual bond. See e. g. Bolder and Stréliski (1999), pp. 11.

where c_n is the coupon, R_n is the redemption value, and T_n is the time-to-maturity in years of security n. The ceiling function $\lceil t \rceil$ returns the next higher integer for t.[27] The implicit price is equal to the present value of future coupon and redemption payments. Cash flows are discounted with the term structure of spot-rates $r(\mathbf{b}_t, m)$. For bonds with semi-annual coupon payments, the relation can easily be adjusted. We apply German bond market conventions by discounting annually and using the 30/360 day count rule.

Implicit and observed yield-to-maturities are determined by numerically solving the following equations for the implicit yield $y_{n,t}$ and the observed yield $\bar{y}_{n,t}$, respectively:

$$\sum_{i=0}^{T_n - \lceil t \rceil} \frac{c_n}{(1 + y_{n,t})^{i + \lceil t \rceil - t}} + \frac{R_n}{(1 + y_{n,t})^{T_n - t}} = B_{n,t}\left(\mathbf{b}_t\right)$$

$$\sum_{i=0}^{T_n - \lceil t \rceil} \frac{c_n}{(1 + \bar{y}_{n,t})^{i + \lceil t \rceil - t}} + \frac{R_n}{(1 + \bar{y}_{n,t})^{T_n - t}} = \bar{b}_{n,t} + c_n \cdot (1 - \lceil t \rceil + t)\,,$$

where $B_{n,t}(\mathbf{b}_t)$ is the implicit dirty price defined in Equation (4.3) and $\bar{b}_{n,t}$ is the observed clean market price of security n at date t. To obtain a dirty bond price, $\bar{b}_{n,t}$ is adjusted for accrued interest. Accrued interest is defined as $c_n \cdot (1 - \lceil t \rceil + t)$, where $(1 - \lceil t \rceil + t)$ is the fraction of the year between the last coupon date and the estimation date t.

Money market instruments are interpreted as zero-bonds, and their implicit price is defined as $B_{n,t}(\mathbf{b}_t) = 100/(1 + r(\mathbf{b}_t, m))^m$. Hence, the implicit yield-to-maturity is equal to $y_{n,t}(\mathbf{b}_t) = r(\mathbf{b}_t, m)$. Observed yield-to-maturities are determined from money market rates in the following steps: (i) we observe the rate and calculate the price by interpreting the security as a zero-bond and employing German money market conventions, (ii) we determine the observed yield-to-maturity by applying German bond market conventions.[28] We do so, as the estimated term

[27] The ceiling function of t is defined by $\lceil t \rceil \equiv \min\{n \in \mathbb{N} : n \geq t\}$.

[28] The German money market conventions required linear discounting and the act/360 day count rule.

structures are used to price securities in the bond market.

Constraints ensure that the long-term interest rate $\beta_{0,t}$ and the short-term interest rate $\beta_{0,t}+\beta_{1,t}$ are no more than three percentage points above or below the average of the observed yield-to-maturities of the three bonds with the longest time-to-maturity and shortest time-to-maturity, respectively. Additionally, $\beta_{0,t}$, the parameter combination $\beta_{0,t} + \beta_{1,t}$, $\tau_{1,t}$, and $\tau_{2,t}$ are restricted to positive values. For the remaining parameters, the lower and upper limits are set to -30 and 30, respectively.[29] The starting value of $\beta_{0,t}$ is set equal to the average of the observed yield-to-maturities of the three bonds with the longest time-to-maturity. Accordingly, the starting value for $\beta_{1,t}$ is set equal to the difference between the averages of the observed yield-to-maturities of the three bonds with the shortest and longest time-to-maturity. The starting values of $\beta_{2,t}$ and $\beta_{3,t}$ are set to minus one, while $\tau_{1,t}$ and $\tau_{2,t}$ are set to one.[30]

The underlying estimation routine is non-linear least squares identifying $\hat{\mathbf{b}}_t$ as

$$\hat{\mathbf{b}}_t = \arg\min_{\mathbf{b}_t} \left\{ \sum_n \left(y_{n,t}\left(\mathbf{b}_t\right) - \bar{y}_{n,t} \right)^2 \right\}. \tag{4.4}$$

The sum of squared errors of the implicit and observed yield-to-maturity is minimized in the cross-section, and we obtain the estimate of the Svensson parameter vector at each observation date t. By inserting $\hat{\mathbf{b}}_t$ into Equation (4.1), we obtain the estimated term structure of spot rates $r(\hat{\mathbf{b}}_t, m)$ at observation date t.

We consider the quality of 731 term structure estimations of the FRG, GS, and GE issuer segment between January 1974 and December 1987 employing bond market data at a weekly frequency using Wednesday observations. The estimation

[29] All restrictions on the Svensson parameter vector $\mathbf{b}_t$, except the constraints on the parameter combination $\beta_{0,t} + \beta_{1,t}$, are similar to that employed by the Deutsche Bundesbank. See Schich (1997), pp. 19, Footnote 9 and 12.

[30] Control estimations showed that the error terms and estimation results were robust with respect to changes in starting values.

error is measured by the mean absolute error (MAE) defined as

$$\mathrm{MAE}_t = \frac{1}{N_t} \cdot \sum_n \left| y_{n,t}\left(\mathbf{b}_t\right) - \bar{y}_{n,t} \right| , \tag{4.5}$$

where N_t is the number of included price observations at the estimation date t.[31]

Figure 4.4 depicts the time series of the MAE and the absolute frequency of the 731 weekly estimations per MAE segmented by the three issuer groups. Including all term structure estimations, the MAE ranges from 2 to 41 basis points. Less than 5% of the estimations have a MAE above 20 basis points. The overall MAE is 11 basis points for the FRG, 13 basis points for the GS, and 10 basis points for the GE issuer segment. The overall standard deviation of the MAE is 4 basis points for the FRG and GE and 6 basis points for the GS issuer segment. The statistics of the error term are similar to those found in studies using a comparable dataset of straight German government bonds to estimate the term structure of interest rates.[32] The second oil crisis in 1979 resulted in a severe recession in Germany between 1981 and 1982. Interest rates were at a historical high and became increasingly volatile. Hence, in 1981, error terms are at their maximum. In the second half of the 1980s, the MAE becomes less volatile and decreases below 10 to 15 basis points.

Table 4.10 reports descriptive statistics for selected spot-rates segmented by issuer groups. Mean spot-rates between January 1974 and December 1987 for all three issuer groups are strictly increasing in time-to-maturity. Standard deviations are strictly decreasing in maturity, and spot-rates for short maturities exhibit lower minima and higher maxima. For the GS issuer group, standard deviations marginally increase from 1.39% to 1.41% between eight to ten years time-to-maturity. However, the long-term maturity segment for straight GS bonds is only weakly populated.

[31] Alternatively, the estimation error was measured by the root mean squared error (RMSE). The magnitude of the RMSE and the conclusions are similar to that of the MAE.

[32] See e. g. Schulze (1996), pp. 96, Schich (1997), and Uhrig and Walter (1997).

Figure 4.4: **Term Structure Estimation Errors**

This figure shows descriptive statistics for the mean absolute error (MAE), reported in basis points, of the term structure estimations segmented by issuer groups. The graphs on the left-hand side report the time series of the MAE. The graphs on the right-hand side report the absolute frequencies of the 731 weekly estimations per MAE.

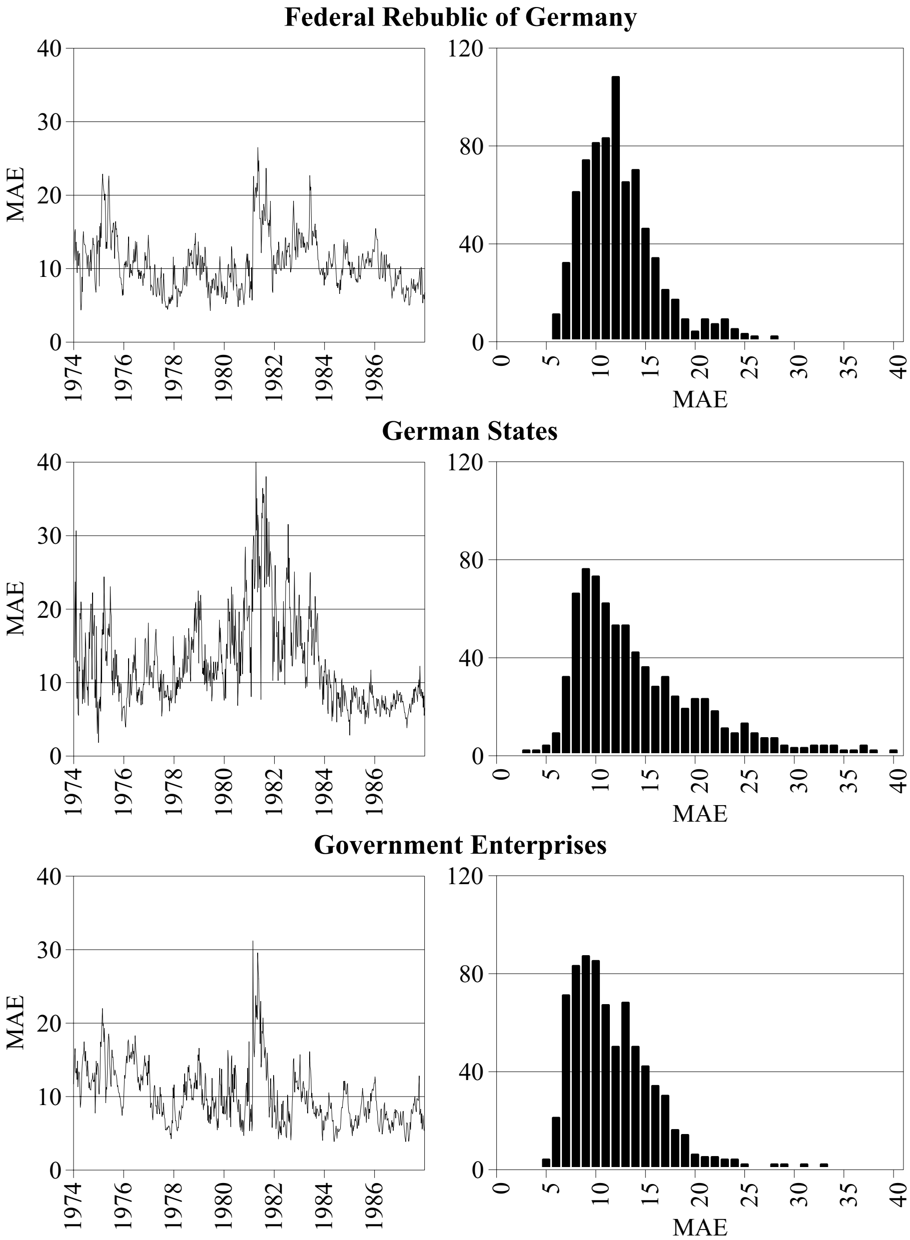


Table 4.10: **Descriptive Statistics for Term Structure**

This table shows descriptive statistics for selected spot-rates segmented by issuer groups. The statistics include the mean, standard deviation, minimum, and maximum over all 731 weekly term structure estimations. All statistics are reported in percentage points. Time-to-maturities are given in years, except in the column headed o/n, reporting statistics for spot-rates with a maturity of one day.

Maturity in Years	o/n	1	2	3	4	5	6	7	8	9	10
Federal Republic of Germany											
Mean	6.20	6.59	6.93	7.22	7.44	7.62	7.76	7.87	7.96	8.04	8.10
Std. Dev.	2.73	2.28	1.98	1.78	1.65	1.56	1.49	1.44	1.41	1.38	1.35
min	1.52	3.60	4.04	4.44	4.80	5.08	5.31	5.50	5.64	5.75	5.80
max	15.25	12.83	12.31	12.00	11.73	11.47	11.22	10.98	11.11	11.29	11.45
German States											
Mean	6.17	6.77	7.13	7.40	7.60	7.76	7.89	7.99	8.08	8.15	8.21
Std. Dev.	2.76	2.37	1.98	1.72	1.56	1.47	1.42	1.40	1.39	1.40	1.41
min	1.74	3.66	4.21	4.62	5.00	5.28	5.45	5.57	5.65	5.54	5.42
max	15.47	13.54	13.10	12.64	12.06	11.52	11.08	11.23	11.45	11.65	11.93
Government Enterprises											
Mean	6.19	6.59	6.96	7.25	7.48	7.65	7.79	7.90	7.99	8.06	8.13
Std. Dev.	2.75	2.25	1.96	1.77	1.64	1.54	1.47	1.41	1.36	1.32	1.28
min	1.55	3.60	4.05	4.47	4.86	5.11	5.30	5.46	5.59	5.70	5.80
max	15.15	13.03	12.49	11.97	11.63	11.37	11.15	10.96	10.95	11.04	11.12

Figure 4.5 depicts the average term structure of spot-rates and one year forward-rates for the FRG issuer group (upper panel) as well as the spread between the average term structures of the spot-rates of the GS and GE, respectively, and the FRG (lower panel). First, we consider the upper panel. The average term structure of spot-rates ranges from 6.2% to 8.1% and is strictly increasing in the time-to-maturity. The term structure of forward-rates is located above the spot-rate curve and ranges from 6.6% to 8.7%. Spot- and one year forward-rates at one year time-to-maturity are by definition equal.[33]

Next, we consider the lower panel in Figure 4.5, depicting the spread between the average term structures of the spot-rates of the GS and GE, respectively, and the FRG. The average spread is positive, except for short maturities. Since the estimation samples for all three issuer groups are replenished by money market rates, the spread is lowest in the short-term maturity segment. For the GS issuer group, the spread rises up to 20 basis points at 1.5 years time-to-maturity and levels at about 11 basis points for the long-term maturity segment. For the GE issuer group, the spread rises up to 4 basis points at 3.5 years time-to-maturity and levels at about 3 basis points for the long-term maturity segment. We obtain a mean spread of 14 basis points for the GS and 2 basis points for the GE over all maturities. The positive spread could be attributed to differences in liquidity and credit risk. It is minor for bond issues by GE that were state-guaranteed as well as free of default risk and more pronounced for the GS issuer segment.

Figure 4.6 depicts the term structure of spot-rates for the FRG issuer group between January 1974 and December 1987 for maturities between one day and ten years. The figure shows that, between January 1974 and December 1987, most term structures are increasing in maturity. Flat and inverted term structures are rarely observed and appear over rather short time periods. In 1974 and between 1979 and 1982, we observe inverted term structures caused by the first oil crisis in 1973 and by the second oil crisis in 1979, which resulted in a severe recession in Germany between 1981 and 1982.

Unless stated otherwise, we use the respective risk-free term structure determined

[33] From the definition of the term structure of forward rates in Equation (4.2), we obtain that $f(\mathbf{b}_t, m, 0)$ is equal to $r(\mathbf{b}_t, m)$.

Figure 4.5: **Average Term Structure and Issuer Spreads**

The upper panel shows the average term structure of spot-rates (solid line) and one year forward-rates (dotted line) in percentage points for the FRG issuer group. The lower panel shows the average spreads in basis points between the term structures of the spot-rates of the GS (solid line) and the GE (dotted line), respectively, and the FRG issuer group. Averages are calculated from the 731 weekly observations between January 1974 and December 1987. Time-to-maturity m is given in years.

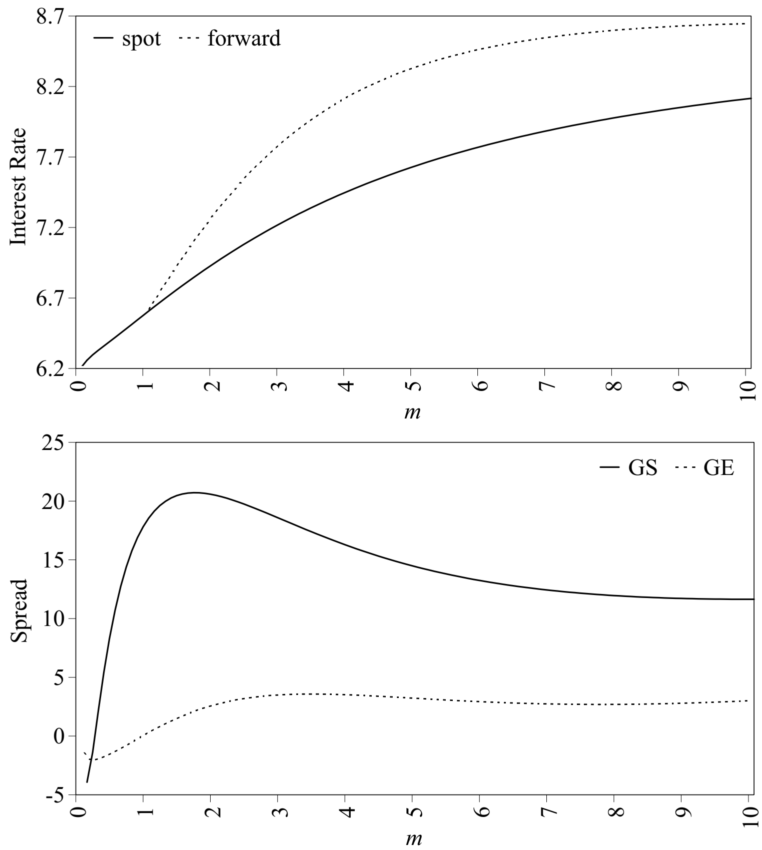
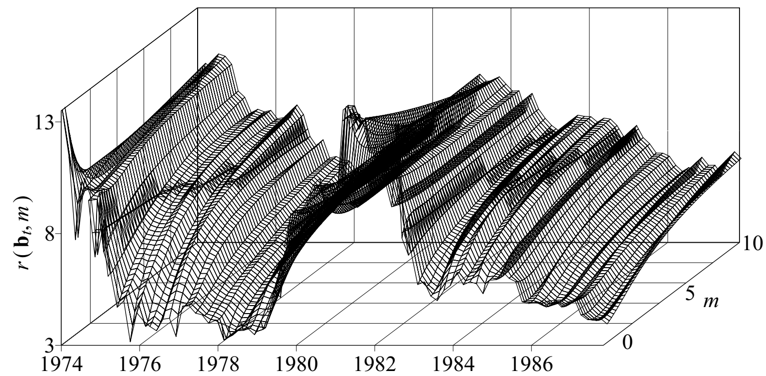


Figure 4.6: **Term Structure for FRG**

This figure shows the term structure of spot-rates $r(\mathbf{b}_t, m)$ for the FRG issuer group between January 1974 and December 1987 for maturities m between one day and ten years. Interest rates are given in percentage points.



from straight coupon bonds for discounting purposes in the subsequent chapters. Hence, we assume deterministic interest rates in the sense that the implied forward rates are equal to the spot rates at the forward dates. We abstract from interest rate risk as well as forward premia and focus on the analysis of the risk caused by redemption lotteries.

Chapter 5

Empirical Analysis of Redemption Risk in Prices

5.1 Framework and Hypotheses

Within a standard event study framework, we empirically test hypotheses derived from the analysis of the equilibrium ex-day price behavior in Section 3.3. Based on transaction prices of German redemption lottery bonds around drawing dates, we determine the ex-day price reaction and bond market participants' risk preferences. We assume that price formation across lotteries is mutually independent and consider each lottery individually. Focusing on the influence of redemption risk, we disregard additional sources of risk such as interest rate risk. Furthermore, we abstract from market frictions such as transaction costs, short-selling constraints, or taxes.

Our primary focus is the price behavior between the event dates $(T-i)^{cum}$ and $(T-i)^{ex}$, $\forall i \in \mathbb{N},\ 1 \leq i < T$. The dirty lottery bond price shortly before the redemption drawing is denoted $B_{(T-i)^{cum}}$, and the price shortly after the drawing is denoted $B_{(T-i)^{ex}}$. Reflecting the institutional setting, we assume that redemption risk is of systematic nature. Hence, investors are restricted to trade only one type of lottery bond series. We think of $B_{(T-i)^{cum}}$ as a participation fee for the subsequent Bernoulli redemption lottery and distinguish two disjoint states of the world: d and n. In state d, realized with the objective probability p_i, the investor receives the redemption payment, usually the face value plus coupon. We denote v_i^d the present value at $(T-i)^{ex}$ of future cash flows from the lottery bond given that the series is drawn. Considering a typical lottery bond with annual coupon and redemption frequency, the expression specifies:[1]

$$v_i^d = \frac{R+c}{(1+r)^{\delta}}, \tag{5.1}$$

where c is the coupon, R is the redemption value, r is the risk-free rate, and δ is the time span between $(T-i)^{ex}$ and the early redemption date $(T-i)^{rd}$.

In state n, realized with the objective probability $(1-p_i)$, the investor holds a series which has not been drawn and is still traded. If the series is directly sold, the cash flow, given that the series is not drawn, specifies:

$$v_i^n = B_{(T-i)^{ex}} \tag{5.2}$$

The gain from the redemption lottery is state dependent and given by

$$g_i^s = \frac{v_i^s}{(1+r)^{\varepsilon}} - B_{(T-i)^{cum}}, \quad s \in \{d, n\}, \tag{5.3}$$

where ε is the time span between $(T-i)^{cum}$ and $(T-i)^{ex}$ and the cash flow terms are discounted to $(T-i)^{cum}$.

It turns out to be practical to classify lotteries according to the gain in state d and to distinguish drawings resulting in a redemption gain from those resulting in a redemption loss. The investor realizes a redemption gain if $B_{(T-i)^{cum}}$ is smaller than the risk-free discounted cash flow term v_i^d. Referring to the dynamic equilibrium valuation model derived in Chapter 3, a lottery resulting in a redemption gain is corresponding to $c/R < r$. Analogously, the investor

[1] The cash flow term is equal to v_i^d defined in Equation (3.12).

realizes a redemption loss if $B_{(T-i)^{cum}}$ is larger than the discounted cash flow term v_i^d. A lottery resulting in a redemption loss is corresponding to $c/R > r$ in Chapter 3.

Subsequent to a redemption lottery, only the series not drawn remain traded. For these series, the risk or chance of being repaid at face value is postponed until the next lottery such that we expect an ex-day price reaction. We consider the influence of redemption drawings on lottery bond prices and derive hypotheses with respect to the ex-day price behavior. Focusing on the actual ex-day price reaction, we assume for the calculation of g_i^n that the time span ε is zero and refrain, at this point, from discounting $B_{(T-i)^{ex}}$ to $(T-i)^{cum}$. In the case of ε equal to zero, the difference between ex-lottery and cum-lottery prices is independent of whether clean or dirty prices are used, as accrued interest cancels out. Hence, g_i^n reduces to the difference between the clean ex-price $b_{(T-i)^{ex}}$ and the clean cum-price $b_{(T-i)^{cum}}$.

First, we consider lotteries resulting in a gain if drawn. Due to the binomial structure of the lottery and no-arbitrage considerations, a redemption gain implies a capital loss if not drawn and thus $b_{(T-i)^{ex}} < b_{(T-i)^{cum}}$. The result is summarized in our first hypothesis:

Hypothesis 1
Redemption lotteries resulting in a capital gain if drawn ($g_i^d > 0$) imply a drop of the clean ex-day price ($b_{(T-i)^{ex}} < b_{(T-i)^{cum}}$).

Hypothesis 1 corresponds to the right-hand side of the no-arbitrage condition for the equilibrium ex-day price behavior if $c/R < r$, as defined in Inequality (3.31).

Second, we consider the ex-day behavior for lotteries resulting in a loss if drawn. A redemption loss implies a capital gain if not drawn and thus $b_{(T-i)^{ex}} > b_{(T-i)^{cum}}$. The result is summarized in our second hypothesis:

Hypothesis 2
Redemption lotteries resulting in a capital loss if drawn ($g_i^d < 0$) imply a positive jump of the clean ex-day price ($b_{(T-i)^{ex}} > b_{(T-i)^{cum}}$).

Hypothesis 2 corresponds to the left-hand side of the no-arbitrage condition for the equilibrium ex-day price behavior if $c/R > r$, as defined in Inequality (3.32).

Accordingly, lotteries resulting in a zero gain if drawn imply a zero capital gain if not drawn, and the ex-day price reaction is zero. The result is consistent with the equilibrium ex-day price behavior for c/R approaching r in Section 3.3.

Third, we consider the relation between the ex-day price behavior and redemption probabilities for lotteries resulting in a gain if drawn. The comparative static analysis for the equilibrium ex-day price behavior in Section 3.3.2 implies that, for $c/R < r$ and a fixed positive RRA coefficient γ, price drops are increasing in the redemption probability p_i.[2] The result is summarized in our third hypothesis:

Hypothesis 3
For investors with constant risk-aversion, redemption lotteries resulting in a capital gain if drawn ($g_i^d > 0$) imply an increasing ex-day price drop ($|b_{(T-i)^{ex}} - b_{(T-i)^{cum}}|$) in the redemption probability p_i.

Note that, for lotteries resulting in a gain if drawn in combination with risk-seeking preferences and for lotteries resulting in a loss if drawn in combination with risk-averse preferences, the relation between the ex-day price behavior and redemption probabilities is ambiguous.

Next, we consider whether observed lottery bond prices are consistent with risk-neutral preferences or include risk premia compensating for redemption risk. A risk-neutral investor values a lottery bond series by simply weighting the cash flow terms v_i^s in states d and n defined by Equations (5.1) and (5.2) with the respective objective state probabilities. Risk-averse investors are willing to trade the bond below the risk-neutral price. We analyze the difference between the risk-neutral and the observed lottery bond price and assess whether the valuation is consistent with the expected wealth setup. Changing our analysis' focus to determining risk preferences, we henceforth refrain from assuming that ε is zero when calculating the gain if not drawn g_i^n.

[2] See Figure 3.11 for the comparative static analysis of the equilibrium ex-day price behavior with respect to redemption probabilities.

Analogous to Schilbred (1973) we choose an intuitive definition of the risk premium π_i on price level, and define:

$$\begin{aligned}\pi_i &= \frac{p_i \cdot v_i^d + (1-p_i) \cdot v_i^n}{(1+r)^{\varepsilon}} - B_{(T-i)^{cum}} \\ &= p_i \cdot g_i^d + (1-p_i) \cdot g_i^n,\end{aligned} \tag{5.4}$$

where π_i is equal to the difference between the discounted expected lottery bond value and $B_{(T-i)^{cum}}$. The difference is equivalent to the sum of the probability weighted redemption gains in state d and n. A positive risk premium corresponds to risk-aversion, whereas a negative or zero premium indicates risk-seeking or risk-neutral preferences, respectively.[3]

The only difference between a lottery bond and a simultaneously traded risk-free straight bond is the redemption lottery. We expect bond market participants to exhibit risk-averse preferences. Hence, investors should charge a premium for taking redemption risk implying that lottery bonds trade below their expected and risk-free discounted value. This prospect is summarized in our fourth hypothesis:

Hypothesis 4
Bond market participants exhibit risk-averse preferences implying a positive risk premium ($\pi_i > 0$).

The relation between the risk premium and the underlying redemption risk σ_i, defined as

$$\sigma_i = \left(p_i \cdot \left(g_i^d\right)^2 + (1-p_i) \cdot (g_i^n)^2\right)^{\frac{1}{2}}, \tag{5.5}$$

[3] Note that the classification of risk preferences corresponds to the results for the valuation model under perfect foresight in Section 3.4. For $\gamma > 0$, the perfect foresight price $B^f_{(T-i)^{cum}}$ defined in Equation (3.36) is smaller than the risk neutral price $B^{e,f}_{(T-i)^{cum}}$, while for $\gamma < 0$, $B^f_{(T-i)^{cum}}$ is larger than $B^{e,f}_{(T-i)^{cum}}$.

is characterized by π_i/σ_i and denoted price of risk.

5.2 Ex-day Price Behavior

Based on the 483 German redemption lotteries by the FRG, GS, and GE between January 1974 and December 1987, we empirically test whether observed prices react according to Hypotheses 1 to 3. Due to the structure of the hypotheses, we segment our observations and examine lotteries resulting in a redemption gain separately from those resulting in a redemption loss.

Throughout the empirical study, we consider transaction prices.[4] Applying German bond market conventions in place during the research period, we employ the 30/360 day count rule. Risk-free rates are determined from the issuer specific term structures of spot rates defined according to the Svensson method described in Section 4.2.2.

We start by analyzing the time series of daily returns of observed clean lottery bond prices in a ten trading day interval around the drawing dates. The return at date t is defined

$$\frac{\bar{b}_t - \bar{b}_{t-1}}{\bar{b}_{t-1}}.$$

Negative returns indicate negative price jumps, while positive returns indicate positive price jumps. Table 5.1 depicts the mean returns, standard deviations, minima, and maxima in an interval spanning five trading days before the lottery-related price suspension and five trading days after the lottery drawing.[5] On average, trading was suspended for two days prior to the redemption drawing. Hence, the return at trade resumption is strictly speaking a three-day return. Furthermore, we employ previous transaction prices at non-active trading days

[4] According to Section 4.1.3, transactions prices are fixed by official exchange brokers on active trading days. See Table 4.6, Footnote b, for the definition of an active trading day.

[5] Transaction prices are not available for 106 redemption lotteries in the considered interval around the drawing date. About 90% of these observations are by GS.

to prevent distinct variation in the number of observations.[6]

Daily mean returns shortly after the drawing are significantly negative for lotteries resulting in a capital gain if drawn. The mean return at the drawing date is -26 basis points. Prices continue to fall significantly over the three subsequent days with means of -10, -8, and -4 basis points. We detect significant and reverse price changes prior to the lotteries. However, the ex ante changes are small, between 2 and 3 basis points. According to German bond market conventions, lottery bond contracts were settled the second trading day after the order was executed. Two days prior to the redemption lottery trade of the respective bond was suspended such that transactions closed before the lottery were settled before the drawing.[7] Hence, lottery-related price reactions realized after the series number of the drawn series have been published. These results provide a first evidence in support of Hypothesis 1 indicating that price drops caused by the drawing occur with a lag of up to three trading days.

Daily mean returns shortly after the drawing are significantly positive for lotteries resulting in a capital loss if drawn. Note that the mean return at the drawing date is 1 basis point and insignificant. However, prices rise significantly over the two subsequent days with means of 7 and 6 basis points. Overall, price reactions are less significant and pronounced relative to the redemption gain segment. Nonetheless, we find a first evidence in support of Hypothesis 2.

Having identified significant ex-day price reactions up to three trading days after the redemption drawing, we focus on the cross sectional price reactions in further detail. We define $\bar{b}_{(T-i)^{cum}}$ as the last transaction price available before the lottery-related price suspension and $\bar{b}_{(T-i)^{ex}}$ as the first transaction price available two

[6] Note, however, that central results remain unchanged if observations, except those at trade resumption, for which the previous or current transaction price is unavailable, are excluded.

[7] Gørtz and Balling (1977) and Ankerstjerne and Møller (1982) study the price effect of Danish lottery bond redemptions and find evidence for lottery-related price reactions prior to the drawing. However, Danish lottery bonds are not excluded from trade before the redemption lotteries.

Table 5.1: **Time Series of Daily Returns**

This table shows daily returns of observed, clean lottery bond prices around lottery dates. Results are segmented according to redemption gains and losses. Mean returns, standard deviations, minima, and maxima are reported in basis points for the interval spanning five trading days before the lottery-related price suspension and five trading days after the lottery drawing. The first row depicts the lag in trading days, where lag -5 equals five days before the price suspension and lag 1 the date of trade resumption. The results are based on 252 to 266 observations per lag in the redemption gain and on 106 to 111 observations per lag in the redemption loss segment. The t-values are given in parentheses. We report the level of significance based on the two-sided t-test for time lags -5 to -1 and based on the one-sided t-test for time lags 0 to 5. *** denotes significance at the 1% level, ** denotes significance at the 5% level, and * denotes significance at the 10% level.

Time Lag	−5	−4	−3	−2	−1	1	2	3	4	5
Gain if Drawn										
Mean Return	3**	2*	1	2*	1	−26***	−10***	−8***	−4**	−2
t-value	(2.59)	(1.78)	(0.98)	(1.71)	(0.75)	(−8.08)	(−3.80)	(−3.22)	(−2.46)	(−1.06)
Std. Dev.	17	19	16	18	16	53	44	38	29	26
min	−52	−152	−78	−110	−110	−323	−312	−244	−220	−178
max	137	113	105	111	77	184	153	197	102	108
Loss if Drawn										
Mean Return	2**	−1	−1	0**	−2**	1	7**	6*	−1	1
t-value	(1.98)	(−0.96)	(−0.59)	(−0.05)	(−2.24)	(0.33)	(2.46)	(1.95)	(−0.99)	(0.60)
Std. Dev.	13	16	10	15	11	22	31	31	16	16
min	−24	−74	−50	−74	−49	−70	−70	−24	−117	−53
max	69	57	63	71	25	99	118	235	35	89

trading days after the lottery, at $(T-i)^l$.[8] This ensures that prices reflect the accrued effects of the redemption lotteries.

Table 5.2 reports the number of lottery observations resulting in a redemption gain and those resulting in a redemption loss. Observations are reported for the overall dataset and for segments corresponding to issuer groups and redemption probabilities. We exclude 88 lottery observations resulting in a redemption gain and 35 lottery observations resulting in a redemption loss for which the time lag between $\bar{b}_{(T-i)^{cum}}$ and $\bar{b}_{(T-i)^{ex}}$ is larger than ten trading days.[9] More than 90% of the excluded lotteries are by GS, while the remaining observations are by GE. The filtered sample contains 256 lottery observations resulting in a redemption gain, whereof 21 observations are on the FRG issuer group level, 168 observations are on the GS level, 67 observations are on the GE level, and between 11 and 39 observations are on the redemption probability level. In addition, the sample contains 104 lottery observations resulting in a redemption loss, whereof 6 observations are on the FRG issuer group level, 82 observations are on the GS level, 16 observations are on the GE level, and between 7 and 16 observations are on the redemption probability level. After the adjustment the overall mean cum-ex lag is 6.5 trading days. The filtered sample is employed for the cross sectional analysis of the ex-day price behavior in this section.

We consider the relation between the ex-day price reaction (measured by g_i^n) and the difference between the discounted redemption payment and the dirty cum-price (measured by g_i^d). Figure 5.1 shows the distribution of the ex-day price reaction segmented by issuer groups. Hypotheses 1 and 2 refer to lotteries resulting in a redemption gain or loss, respectively, and state that observations in quadrant four and two of the panels are arbitrage-free, while observations in quadrants one and three violate no-arbitrage conditions under the assumption of perfect foresight.

We first examine observations resulting in a capital gain if drawn. For the FRG

[8] We define $\bar{b}_{(T-i)^{ex}}$ as the first available price two instead of three trading days after the lottery because price reactions with lag three are marginal at -4 and -1 basis points, respectively, and only partly significant.

[9] Our empirical results are robust with respect to varying choices of the cut-off point.

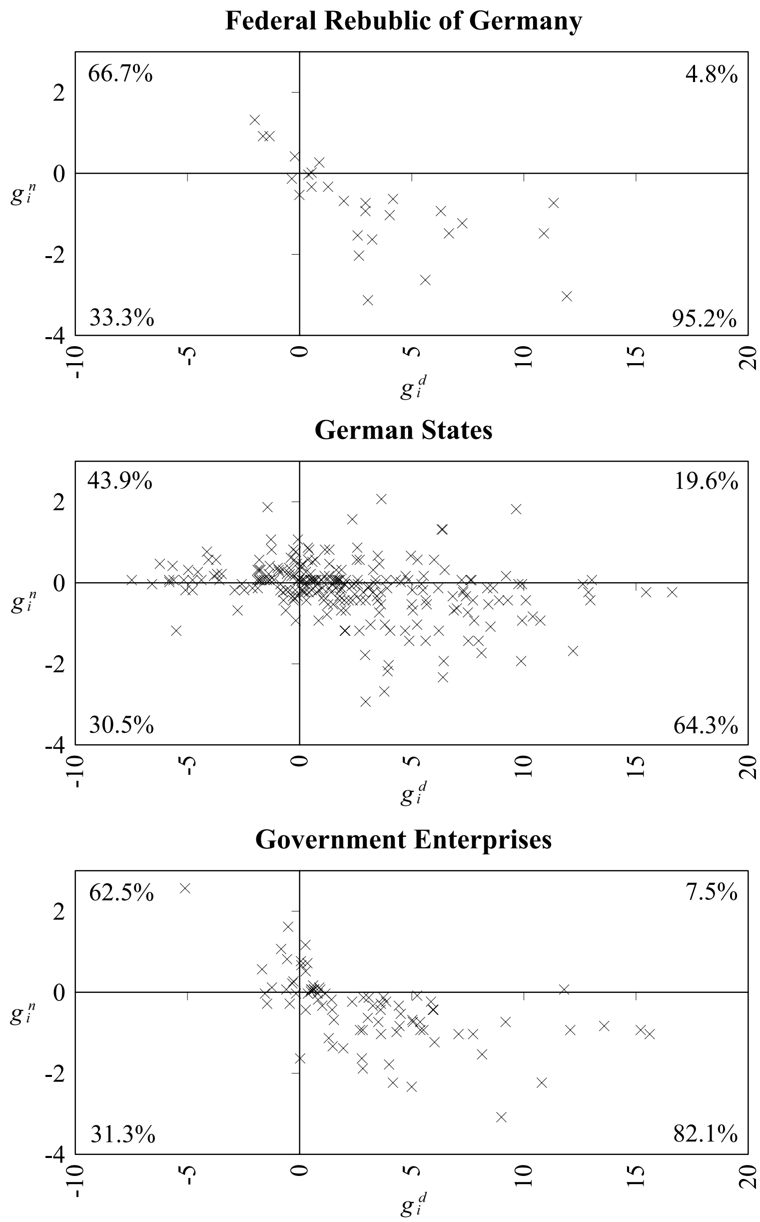
Table 5.2: **Number of Lottery Observations I**

This table shows the number of lottery observations resulting in a redemption gain ($g_i^d > 0$) and those resulting in a redemption loss ($g_i^d < 0$). Observations are given for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). The second, third, and sixth column report the total number of observations and the number of observations resulting in a redemption gain, respectively redemption loss. The fourth and seventh column reports the number of observations in the redemption gain segment, respectively redemption loss segment, excluded because the time lag between the cum-price and ex-price is larger than ten trading days. The fifth and eighth column reports the number of observations used in the estimations.

		$g_i^d > 0$			$g_i^d < 0$		
	Total	Red. Gain	Cum-ex Lag	Clean	Red. Loss	Cum-ex Lag	Clean
Overall	483	344	(88)	256	139	(35)	104
Issuer Group							
FRG	27	21	(0)	21	6	(0)	6
GS	361	247	(79)	168	114	(32)	82
GE	95	76	(9)	67	19	(3)	16
Redemption Probability							
1/2	83	60	(21)	39	23	(11)	12
1/3	78	55	(19)	36	23	(7)	16
1/4	71	53	(20)	33	18	(4)	14
1/5	61	43	(10)	33	18	(5)	13
1/6	48	33	(6)	27	15	(5)	10
1/7	44	34	(4)	30	10	(1)	9
1/8	40	32	(3)	29	8	(1)	7
1/9	35	23	(5)	18	12	(1)	11
1/10	23	11	(0)	11	12	(0)	12

Figure 5.1: **Distribution of Ex-day Price Reaction**

This figure shows the distribution of the reaction of clean lottery bond prices at lottery dates segmented by issuer groups. The ordinate depicts the gain if not drawn, g_i^n, measuring the ex-day reaction of observed clean prices. The abscissa depicts the lottery gain if drawn, g_i^d, measuring the difference between the discounted redemption payment and the observed dirty cum-price. Both terms g_i^d and g_i^n are expressed in German Mark. In the corner of each quadrant, we report the fraction of lottery observations resulting in a redemption gain and loss that coincide with a negative or positive gain if not drawn, respectively.



and GE issuer groups, 95.2% and 82.1% of the observations, respectively, are located in quadrant four supporting Hypotheses 1. A positive ex-day price jump coincides with 4.8% and 7.5% of the observations, respectively, and GE prices remain constant for 10.4% of the observations. The results are less distinct for GS: 64.3% of the observations are located in quadrant four, while 19.6% coincide with a positive ex-day price jump and 16.1% with a zero ex-day price reaction. Considering the distribution for the entire sample, 71.5% of the observations are located in quadrant four, while 15.2% coincide with a positive ex-day price jump and 13.3% with a zero ex-day price reaction. Applying a standard binomial sign test, the results are highly significant for the entire sample and the three issuer groups. Hence, the distribution of price reactions in the redemption gain segment strongly supports Hypothesis 1.

Next, we examine observations resulting in a capital loss if drawn. For the FRG and GE issuer groups, 66.7% and 62.5% of the observations, respectively, are located in quadrant two. An ex-day price drop coincides with 33.3% and 31.3% of the observations, respectively, and GE prices remain constant for 6.2% of the observations. For GS, 43.9% of the observations are located in quadrant two, while 30.5% coincide with an ex-day price drop and 25.6% with a zero ex-day price reaction. If we again consider the distribution for a combination of all three issuer groups, 48.1% of the observations are located in quadrant two, while 30.8% coincide with an ex-day price drop and 21.2% with a zero ex-day price reaction. Applying a standard binomial sign test, the results are neither significant for the entire sample nor for the issuer groups. Hence, the distribution of price reactions in the redemption loss segment does not support Hypothesis 2.

Furthermore, g_i^d and g_i^n are negatively correlated: -0.67 for the FRG, -0.30 for GS, and -0.56 for GE. The larger the redemption gain, the more pronounced is the ex-day price drop.

Table 5.3 considers the robustness of the distribution of price reactions for the entire dataset. We report the number of lottery observations opposing Hypotheses 1 and 2 for varying tolerance thresholds. For $g_i^d > 0$, out of 256 observations 29% are in conflict with Hypothesis 1. About half of the misclassifications coincide with a zero ex-day price reaction, while 39 observations coincide with a positive ex-day price jump. However, only 20 observations exhibit an opposing price

Table 5.3: **Robustness of Ex-day Price Reaction**

This table shows the number of lottery observations opposing Hypotheses 1 and 2. We report the number of misclassifications for varying tolerance thresholds. The thresholds for $|g_i^n|$ require a minimum price reaction of DEM 0.05, 0.25, and 0.50. For lotteries resulting in a redemption gain or loss, the thresholds for g_i^d require a minimum redemption gain or loss of DEM 0.25, 0.50, and 1.00, respectively. Observations outside the thresholds are not considered. In addition and as a benchmark, we report the number of unrestricted misclassifications.

Misclassifications	Unrestr.	$\lvert g_i^n \rvert >$ 0.05	$\lvert g_i^n \rvert >$ 0.25	$\lvert g_i^n \rvert >$ 0.50
Gain if Drawn				
Unrestr.	73	39	29	20
$g_i^d > 0.25$	66	33	24	17
$g_i^d > 0.50$	56	28	21	15
$g_i^d > 1.00$	43	24	20	14
Loss if Drawn				
Unrestr.	54	31	23	8
$g_i^d < -0.25$	45	27	19	6
$g_i^d < -0.50$	38	20	14	4
$g_i^d < -1.00$	29	12	7	2

reaction larger than DEM 0.50. Only 14 misclassifications are attributed to lotteries with a redemption gain larger than DEM 1.00 coinciding with an ex-day price jump larger than DEM 0.50. For $g_i^d < 0$, out of 104 observations 52% are in conflict with Hypothesis 2. About 40% of the misclassifications coincide with a zero ex-day price reaction, while 31 observations coincide with an ex-day price drop. However, only eight observations exhibit an opposing price reaction smaller than DEM -0.50. Only 3.7% of the misclassifications are attributed to lotteries with a redemption loss larger than DEM 1.00 coinciding with an ex-day price reaction smaller than DEM -0.50. Hence, Table 5.3 qualifies violating observations which cluster in a narrow interval around the zero ex-day price reaction.

We continue our analysis of Hypotheses 1 to 3 by considering the magnitude of the ex-day reaction of clean lottery bond prices at lottery dates. Tables 5.4 and 5.5 report the mean price reaction segmented by lotteries resulting in a redemption gain or loss as well as issuer groups and redemption probabilities, respectively. For $g_i^d > 0$, the mean price reaction is negative and highly significant for the overall sample, issuer group and redemption probability segments. The average price drop is DEM 0.49 for the overall sample, DEM 1.22 for the FRG, DEM 0.69 for GE, and DEM 0.32 for GS. Considering the redemption probability segments, average price drops reach from DEM 0.19 for 1/6 to DEM 0.65 for 1/10. The results confirm Hypothesis 1 at the 1% level of significance, except for redemption probability segment 1/6, where the hypothesis is confirmed at the 10% level. According to Hypothesis 3, the ex-day price reaction is increasing in the redemption probability if investors have constant and risk-averse preferences. However, Table 5.5 reports a hump shaped relation between the ex-day price reaction and redemption probabilities, and we find no evidence supporting Hypothesis 3. Referring to the comparative static analysis of the equilibrium ex-day price behavior in Chapter 3.3, the non-monotonic relation can be rationalized by time varying risk preferences.

For $g_i^d < 0$, the mean price reaction is positive. The average price jump is DEM 0.13 for the overall sample, DEM 0.42 for the FRG, DEM 0.34 for GE, and DEM 0.07 for GS. Considering the redemption probability segments, average price reactions reach from a drop of DEM 0.15 for 1/10 to a rise of DEM

Table 5.4: **Magnitude of Ex-day Price Reaction I**

This table shows the ex-day reaction of clean lottery bond prices at lottery dates segmented by lottery observations resulting in a redemption gain and loss. The price reaction is defined as the difference between the clean ex-price $\bar{b}_{(T-i)^{ex}}$ and the clean cum-price $\bar{b}_{(T-i)^{cum}}$. Results are given for the overall dataset, the Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE) separately. The table reports the mean price reaction, standard deviation, minimum, and maximum in German Mark. The t-values are given in parentheses. We report the level of significance based on the one-sided t-test, where *** denotes significance at the 1% level, ** denotes significance at the 5% level, and * denotes significance at the 10% level.

Price Reaction	Overall	FRG	GS	GE
Gain if Drawn				
Mean Reaction	-0.49^{***}	-1.22^{***}	-0.32^{***}	-0.69^{***}
t-value	(-9.72)	(-5.92)	(-5.57)	(-7.53)
Std. Dev.	0.81	0.95	0.75	0.75
min	-3.20	-3.20	-3.00	-3.15
max	2.00	0.20	2.00	1.10
Loss if Drawn				
Mean Reaction	0.13^{***}	0.42	0.07^{*}	0.34^{*}
t-value	(2.38)	(1.45)	(1.42)	(1.47)
Std. Dev.	0.56	0.71	0.43	0.92
min	-1.70	-0.60	-1.25	-1.70
max	2.50	1.25	1.80	2.50

Table 5.5: **Magnitude of Ex-day Price Reaction II**

This table shows the ex-day reaction of clean lottery bond prices at lottery dates segmented by lottery observations resulting in a redemption gain and loss. The price reaction is defined as the difference between the clean ex-price $\bar{b}_{(T-i)^{ex}}$ and the clean cum-price $\bar{b}_{(T-i)^{cum}}$. Results are given for redemption probabilities. The table reports the mean price reaction, standard deviation, minimum, and maximum in German Mark. The t-values are given in parentheses. We report the level of significance based on the one-sided t-test, where *** denotes significance at the 1% level, ** denotes significance at the 5% level, and * denotes significance at the 10% level.

Price Reaction	1/10	1/9	1/8	1/7	1/6	1/5	1/4	1/3	1/2
Gain if Drawn									
Mean Reaction	−0.65***	−0.60***	−0.47***	−0.40***	−0.19*	−0.58***	−0.38***	−0.60***	−0.60***
t-value	(−3.59)	(−4.44)	(−3.35)	(−2.73)	(−1.62)	(−3.34)	(−3.29)	(−5.63)	(−3.36)
Std. Dev.	0.60	0.58	0.76	0.81	0.61	1.00	0.67	0.64	1.12
min	−1.80	−1.75	−3.15	−2.75	−1.60	−3.10	−2.25	−2.40	−3.20
max	0.00	0.50	0.75	1.75	1.25	1.50	2.00	0.40	1.10
Loss if Drawn									
Mean Reaction	−0.15*	0.09	−0.11	−0.04	0.14	−0.03	0.29	0.27**	0.52***
t-value	(−1.63)	(0.97)	(−0.50)	(−0.34)	(1.14)	(−0.30)	(1.17)	(1.93)	(3.23)
Std. Dev.	0.32	0.29	0.56	0.39	0.40	0.42	0.91	0.56	0.55
min	−1.00	−0.25	−1.25	−0.75	−0.35	−0.75	−1.70	−0.50	−0.40
max	0.15	0.70	0.40	0.50	1.00	0.85	2.50	1.80	1.55

0.52 for 1/2. The mean price reaction is positive and highly significant on the aggregate level and for redemption probabilities 1/2 and 1/3. It is, however, lowly significant or insignificant for the issuer group and remaining redemption probability segments.[10] Overall, we interpret these results as a weak affirmation of Hypothesis 2.

The quality of our results depends strongly on whether the lotteries considered result in a redemption gain or in a loss. Statistical evidence for lotteries resulting in a redemption loss is less distinct and of lower significance relative to lotteries resulting in a redemption gain. One might expect that embedded redemption options provide an explanation for this discrepancy. According to Table 4.5, all of the FRG, 95% of GE, and about 65% of GS lottery bond issues are equipped with issuer call options and increased redemption provisions. Lotteries resulting in a redemption loss from investor's perspective imply a redemption gain for the issuer, who is able to refinance his debt under more favorable conditions. Hence, early or increased redemption options of lotteries resulting in a redemption loss are in-the-money. Over the entire period of our analysis, the FRG and GE did not exercise any of the embedded options, while GS exercised nine options. We are left with 33 lottery observations resulting in a redemption loss if we limit our analysis to bonds without redemption options. The fraction of observations coinciding with a positive gain if not drawn rises only marginally, from 48.1% to 48.5%, and the restriction on observations without redemption options does not imply a considerable improvement in the classification results. This suggests the existence of an unknown factor not contained in our simple framework. The inclusion of lotteries resulting in a redemption loss would introduce unexplainable noise into our estimations. Throughout the subsequent empirical sections, we therefore disregard lotteries resulting in a redemption loss and focus on the redemption gain segment.

Lottery observations resulting in a redemption gain provide strong evidence for Hypothesis 1. The results are most convincing for the FRG and GE issuer groups. The fraction of observations coinciding with a positive ex-day price reaction violating the price hypothesis is limited to 4.8% for the FRG and 7.5% for GE.

[10] The t-statistics for the FRG and GE issuer segment and the redemption probability segments should be interpreted with care, as they are lowly populated.

Overall, results for the highly populated GS segment are more volatile. However, the mean price reaction is significantly negative at the 1% level for all three issuer groups.

5.3 Risk Premia

Within an intuitive setting, we provide first evidence of bond market participants' risk preferences.[11] Based on German redemption lotteries resulting in a drawing gain, we determine risk premia and test Hypothesis 4.

Table 5.6 reports the number of lottery observations resulting in a redemption gain for the overall dataset and for segments corresponding to issuer groups and redemption probabilities. Compared to the filtration in Table 5.2, we in addition exclude 69 lottery observations resulting in a redemption gain which violate the adjusted price Hypothesis 1 and thus the no-arbitrage condition under perfect foresight.[12] According to the definition of the risk premium in Equation (5.4), including observations violating the price hypothesis would result in larger risk premia. Our filtered sample contains 187 lottery observations on the aggregate level, 20 observations on the FRG issuer group level, 112 observations on the GS level, 55 observations on the GE level, and between 10 and 30 observations on the redemption probability level.

In Tables 5.7 and 5.8 we report the mean lottery gain in states d and n, the mean risk premium, and the mean price of risk for lottery observations resulting in a redemption gain. Results are given for the overall dataset as well as for issuer group and redemption probability segments. We first consider the results on the aggregate and issuer group level. The mean redemption gain is DEM 4.69, and the mean loss in state n is DEM 0.80. Due to the filtration of the sample, the sign of the gain in states d and n is by definition positive and negative, respectively.

[11] For a detailed analysis of risk preferences, we refer to Chapter 6, where we estimate RRA coefficients within the dynamic equilibrium valuation model derived in Chapter 3.

[12] Note that in contrast to Section 5.2, we refrain from assuming that ε is zero when calculating g_i^n. Hence, the number of observations violating against the adjusted price Hypothesis 1 differs compared to Table 5.3.

Table 5.6: **Number of Lottery Observations II**

This table shows the number of lottery observations resulting in a redemption gain ($g_i^d > 0$) for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). The second and third column report the total number of observations and the number of observations resulting in a redemption gain. The fourth column reports the number of observations in the redemption gain segment excluded because the time lag between the cum-price and ex-price is larger than ten trading days. The fifth column reports the number of observations excluded because of a violation against the adjusted price Hypothesis 1 in Section 5.1. The sixth column reports the number of filtered observations used in the estimations.

		$g_i^d > 0$			
	Total	Red. Gain	Cum-ex Lag	Price React.	Clean
Overall	483	344	(88)	(69)	187
Issuer Group					
FRG	27	21	(0)	(1)	20
GS	361	247	(79)	(56)	112
GE	95	76	(9)	(12)	55
Redemption Probability					
1/2	83	60	(21)	(16)	23
1/3	78	55	(19)	(6)	30
1/4	71	53	(20)	(8)	25
1/5	61	43	(10)	(6)	27
1/6	48	33	(6)	(11)	16
1/7	44	34	(4)	(11)	19
1/8	40	32	(3)	(8)	21
1/9	35	23	(5)	(2)	16
1/10	23	11	(0)	(1)	10

Table 5.7: **Risk Premia and Prices of Risk I**

This table shows mean gains in states d and n, mean risk premia, and mean prices of risk for lottery observations resulting in a redemption gain. Results are given for the overall dataset, the Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE). The gains in states d and n as well as the risk premia are reported in German Mark. We report t-values in parentheses for risk premia and prices of risk as well as standard deviations, minima, and maxima. The level of significance is based on the one-sided t-test, where *** denotes significance at the 1% level, ** denotes significance at the 5% level, and * denotes significance at the 10% level.

Risk Measures	Overall	FRG	GS	GE
Gain in State d	4.69	4.39	4.71	4.74
Gain in State n	−0.80	−1.30	−0.66	−0.90
Risk Premium	0.28***	0.24***	0.28***	0.27***
t-value	(7.02)	(2.91)	(5.03)	(4.34)
Std. Dev.	0.54	0.38	0.60	0.47
min	−1.87	−0.15	−1.87	−1.68
max	2.30	1.35	2.30	1.29
Price of Risk	0.11***	0.13***	0.09***	0.13***
t-value	(4.98)	(2.77)	(2.91)	(4.15)
Std. Dev.	0.30	0.21	0.33	0.24
min	−0.88	−0.22	−0.88	−0.67
max	0.69	0.69	0.66	0.59

The mean risk premium is DEM 0.28, and the mean price of risk is 0.11.[13] Both measures are positive and highly significant. Mean risk premia range from DEM 0.24 for the FRG to DEM 0.28 for GS, and the mean risk premium is DEM 0.04 higher for GS relative to FRG issues. The mean prices of risk range from 0.09 to 0.13 and are of moderate magnitude compared to e. g. Schilbred (1973) who estimates a market price of risk within a mean-variance equilibrium model in a bond market setting on the order of ITL 0.50 per unit of variance and Hansen and Jagannathan (1991) or Cogley and Sargent (2008) who infer a Sharpe ratio from security market data, which is on the order of 0.23. The results provide strong evidence for Hypothesis 4 and suggest that investors are averse to redemption risk.

Next, we consider the results for the redemption probability segments. The mean redemption gain ranges from DEM 9.48 for $p_i = 1/10$ to DEM 2.08 for $p_i = 1/2$ and documents the pull-to-par effect as bond maturity approaches. The mean loss in state n ranges from DEM 0.58 to 1.27 and is largest for $p_i = 1/2$. Mean risk premia and prices of risk are significantly positive for redemption probabilities above 1/7, and mean prices of risk tend to rise in the redemption probability. Mean risk premia are consistently positive, while we observe one negative mean price of risk for $p_i = 1/10$.

The magnitude and significance of risk premia in Tables 5.7 and 5.8 confirm Hypothesis 4. Bond market participants are risk-averse and demand compensation for redemption risk. Hence, lottery bond prices cannot be determined by simply weighting the future cash flows in states d and n by the respective objective state probabilities. Rather, lottery bond prices should be determined within an expected utility framework for risk averse investors.

The results have direct policy implications and provide a fundamental motive for substituting lottery bonds with straight bonds in the 1970s. From the issuers' perspective, lottery bonds were more expensive than straight coupon bonds, and similar payment structures could be achieved without introducing extraneous redemption risk, e. g. by issuing several individual bonds with different maturities.

[13] To facilitate the interpretation of risk premia, we report the mean observed dirty lottery bond price $\bar{B}_{(T-i)^{cum}}$. It is equal to DEM 97.55 for the 187 observations on the aggregate level.

Table 5.8: **Risk Premia and Prices of Risk II**

This table shows mean gains in states d and n, mean risk premia, and mean prices of risk for lottery observations resulting in a redemption gain. Results are segmented by redemption probabilities. The gains in states d and n as well as the risk premia are reported in German Mark. We report t-values in parentheses for risk premia and prices of risk as well as standard deviations, minima, and maxima. The level of significance is based on the one-sided t-test, where *** denotes significance at the 1% level, ** denotes significance at the 5% level, and * denotes significance at the 10% level.

Risk Measures	1/10	1/9	1/8	1/7	1/6	1/5	1/4	1/3	1/2
Gain in State d	9.48	8.73	6.20	5.12	6.13	4.69	2.85	2.37	2.08
Gain in State n	−0.72	−0.73	−0.71	−0.78	−0.58	−0.90	−0.58	−0.77	−1.27
Risk Premium	0.30	0.32**	0.15	0.06	0.54***	0.22***	0.28***	0.28***	0.41***
t-value	(1.34)	(2.14)	(1.06)	(0.40)	(3.16)	(2.70)	(3.05)	(4.20)	(4.63)
Std. Dev.	0.71	0.60	0.66	0.68	0.68	0.42	0.45	0.36	0.42
min	−0.76	−0.52	−1.68	−1.87	−0.42	−0.73	−0.78	−0.53	−0.16
max	1.32	1.29	1.13	1.00	2.30	1.06	1.22	0.94	1.35
Price of Risk	−0.02	0.00	0.05	0.00	0.12**	0.08*	0.10*	0.18***	0.31***
t-value	(−0.18)	(0.00)	(0.98)	(0.02)	(1.88)	(1.51)	(1.39)	(3.61)	(5.52)
Std. Dev.	0.27	0.31	0.25	0.28	0.26	0.28	0.36	0.28	0.27
min	−0.46	−0.88	−0.65	−0.64	−0.62	−0.84	−0.78	−0.67	−0.18
max	0.31	0.33	0.33	0.37	0.39	0.36	0.50	0.53	0.69

Our findings suggest that public sector lottery bond indentures were dominated by straight coupon bonds.

We proceed with a more detailed analysis of RRA coefficients in the subsequent chapter, where we estimate bond market participants' risk preferences within the dynamic equilibrium valuation model derived in Chapter 3.

Chapter 6

Estimation of Implied RRA Coefficients

"It purports that bond evaluation should be a rewarding field for empirical studies of investors' treatment of risk and uncertainty."

Schilbred (1968), p. 43.

6.1 Estimation Procedure

Standard neoclassical asset pricing models yield equilibrium prices depending on market participants' risk preferences and probability beliefs. Preferences are generally viewed as fixed, primitive characteristics of economic agents, whereas probability beliefs are subjective and unstable.[1] In most real-world settings, it is impossible to disentangle preferences and beliefs. German redemption lottery bonds disburse uncertain payoffs according to an observable probability distribution and provide an exceptional environment for studying investors' risk preferences independent of subjective probability beliefs.

We use the dynamic equilibrium valuation model for redemption lottery bonds derived in Chapter 3 to extract implied risk preferences from bond market data.

[1] See Allen (1980), p. 344.

The comparative static analysis demonstrated that equilibrium prices are strictly monotonic in the RRA coefficient. Each equilibrium price is a one-to-one mapping to γ and corresponds to a unique risk preference.

The pooled, implied RRA estimate is obtained by minimizing the sum of squared deviations between the clean equilibrium prices b_t^* and the clean observed market prices $\bar{b}_t$.[2] The underlying estimation routine is non-linear least squares identifying $\hat{\gamma}$ as

$$\hat{\gamma} = \arg\min_{\gamma} \left\{ \sum_{n,t} \left(b_{n,t}^* - \bar{b}_{n,t} \right)^2 \right\}, \tag{6.1}$$

where n specifies the lottery bond issue and t the observation date. Conditional on the observation date, we revert to pricing Equation (3.24) or (3.29) to assess $b_{n,t}^*$.

In addition, we estimate pooled, implied RRA estimates under the assumption of perfect foresight. We employ the clean perfect foresight prices $b_{(T-i)^{cum}}^f$ obtained by adjusting $B_{(T-i)^{cum}}^f$, defined in Equation (3.36), for accrued interest. Note that RRA estimates under perfect foresight are restricted to cum-days.

After each estimation, we apply an outlier detection rule to avoid model misspecifications resulting in distorted RRA estimates and invalid inferences. Observations are classified as outliers if

$$e_n - \mathrm{ME} > 3 \cdot \mathrm{SE},$$

where e_n is the price residual for observation n, ME is the mean price residual, and SE is the standard deviation of the price residuals. Outliers are excluded

[2] Transaction prices are by convention clean prices. We adjust theoretical prices for accrued interest and base our estimations on clean prices. Alternatively, we could have adjusted transaction prices for accrued interest and based our estimations on dirty prices.

from the sample, and the estimation routine is run again.[3]

We estimate implied RRA coefficients for various segmentations of the lottery bond data panel. First, we determine pooled, implied RRA estimates based on cum-day observations and compare dynamic equilibrium estimates to those under perfect foresight. The restriction to cum-days, however, limits the number of observations and leads to unstable estimation results. Since lottery bond prices contain information on risk preferences not only at cum-days but also at trading days before the lotteries, we then estimate pooled, implied RRA coefficients based on the entire sample of observed market prices. We analyze the robustness of our estimations by examining subsamples covering distinct intervals before the redemption lotteries and by controlling for the proximity of price observations to the no-arbitrage bounds. Lastly, we focus on the time series properties of implied RRA estimates and consider the relation between changes in risk aversion and macroeconomic factors.

6.2 Pooled, Implied RRA Coefficients based on Cum-days

6.2.1 Equilibrium RRA Estimates

We restrict our attention to the sample of German lottery bond observations resulting in a drawing gain and determine pooled, implied RRA estimates based on equilibrium pricing Equation (3.24). We focus on cum-lottery dates $(T-i)^{cum}$, $\forall i \in \mathbb{N}$, $1 \leq i < T$, and extract risk preferences from the last clean transaction prices $\bar{b}_{(T-i)^{cum}}$ available before the lottery-related price suspensions.

Table 6.1 reports the number of cum-day observations for the overall dataset and for segments corresponding to issuer groups and redemption probabilities. The estimation of RRA coefficients within the dynamic equilibrium valuation model

[3] We restrict the number of re-estimation loops and apply the outlier detection rule at most ten times. However, only for the 1/6 redemption probability segment based on the entire sample of bond price observations this limit was attained.

Table 6.1: **Number of Cum-day Observations**

This table shows the number of cum-day lottery observations for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). The second and third column report the total number of observations and the number of observations resulting in a redemption gain. The fourth column reports the number of observations in the redemption gain segment excluded because the cum-lottery transaction price is located outside the no-arbitrage bounds defined by Inequalities (3.15). The fifth column reports the number of observations classified as outliers. The last column reports the number of filtered observations used in the estimations.

	Total[a]	Red. Gain	No-arb.	Outl.	Clean
Overall	465	306	(19)	(5)	282
Issuer Group					
FRG	27	21	(0)	(0)	21
GS	343	212	(16)	(9)	187
GE	95	73	(3)	(2)	68
Redemption Probability					
1/2	83	49	(8)	(4)	37
1/3	78	51	(5)	(0)	46
1/4	69	47	(3)	(0)	44
1/5	58	39	(2)	(0)	37
1/6	46	29	(0)	(0)	29
1/7	41	32	(0)	(0)	32
1/8	37	28	(1)	(0)	27
1/9	32	21	(0)	(0)	21
1/10	21	10	(0)	(0)	10

[a] Due to an incomplete redemption schedule, we exclude 18 observations by GS from the original sample.

requires the complete schedule of future redemption dates. Due to an incomplete redemption schedule and missing data about future lotteries, we exclude 18 price observations by GS. A lottery observation is classified as a drawing gain if the observed clean price $\bar{b}_{(T-i)^{cum}}$ is smaller than the present value term $v_i^d/(1+r)^{\varepsilon}$ adjusted for accrued interest.[4] Due to the dynamic structure of the equilibrium model, an observation at $(T-i)^{cum}$ is classified as a drawing gain if, in addition, $v_j^n < v_j^d$, $\forall j \in \mathbb{N}$, $j < i$. Since RRA coefficients can only be extracted from pricing Equation (3.24) for transaction prices located inside the no-arbitrage bounds, we exclude 19 price observations, 16 of which are by GS and three by GE, located outside the no-arbitrage bounds defined by Inequalities (3.15). Furthermore, we disregard between zero and nine observations classified as outliers. Our filtered and pooled sample contains 282 lottery observations on the aggregate level, 21 observations on the FRG issuer group level, 187 observations on the GS level, 68 observations on the GE level, and between 10 and 46 observations on the redemption probability level.

Figure 6.1 depicts the absolute frequency of cum-day price observations per month. Focusing on lotteries resulting in a redemption gain, we limit observations in times of relatively low interest rates, e. g. between 1977 and 1978, when risk-free spot rates reached a historical low. Table 6.2 shows the varying distribution of price observations per year across issuer groups and redemption probabilities. For the overall dataset the mean observation date is April 1978. At the issuer group level, the mean observation date is February 1977 for the FRG, September 1978 for GS, and August 1977 for GE. At the redemption probability level, mean observation dates tend to rise with the redemption probability. For probability 1/10, the mean observation date is June 1975, whereas, for probability 1/2, the mean observation date is August 1980.

We proceed to analyze the estimation results compiled in Table 6.3. The overall least squares RRA estimate for the pooled estimation is 1.15 with a mean absolute error between theoretical and observed prices of DEM 0.57.[5] On the issuer group level, the RRA estimates are 0.11 for the FRG, 2.99 for GS, and 0.17 for GE.

[4] See Equation (3.12) for the definition of the present value term.

[5] To facilitate the interpretation of the error terms, we report the mean lottery bond price (clean) based on the 282 cum-day observations, which is DEM 95.60.

Table 6.2: **Observation Dates for Cum-day Observations**

This table shows the relative frequency of filtered cum-day price observations per observation year. Results are given for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). Relative frequencies are reported in percentage points.

Observation Year	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985
Overall	14.2	16.0	15.2	6.4	0.7	10.3	12.4	14.2	7.4	1.8	1.1	0.4
Issuer Group												
FRG	14.3	23.8	19.0	14.3	–	4.8	9.5	14.3	–	–	–	–
GS	11.8	13.4	14.4	4.8	1.1	11.8	13.9	15.5	9.6	2.1	1.1	0.5
GE	19.1	19.1	16.2	8.8	–	8.8	10.3	10.3	4.4	1.5	1.5	–
Redemption Probability												
1/2	2.7	5.4	5.4	2.7	–	8.1	13.5	35.1	24.3	–	2.7	–
1/3	6.5	13.0	8.7	8.7	–	13.0	26.1	21.7	–	–	–	2.2
1/4	6.8	18.2	20.5	4.5	–	18.2	18.2	–	4.5	6.8	2.3	–
1/5	21.6	16.2	10.8	2.7	–	16.2	–	13.5	13.5	2.7	2.7	–
1/6	13.8	13.8	13.8	13.8	6.9	–	3.4	20.7	10.3	3.4	–	–
1/7	9.4	12.5	31.3	15.6	–	3.1	15.6	9.4	3.1	–	–	–
1/8	14.8	37.0	29.6	–	–	7.4	7.4	3.7	–	–	–	–
1/9	42.9	38.1	–	4.8	–	9.5	4.8	–	–	–	–	–
1/10	60.0	–	30.0	–	–	10.0	–	–	–	–	–	–

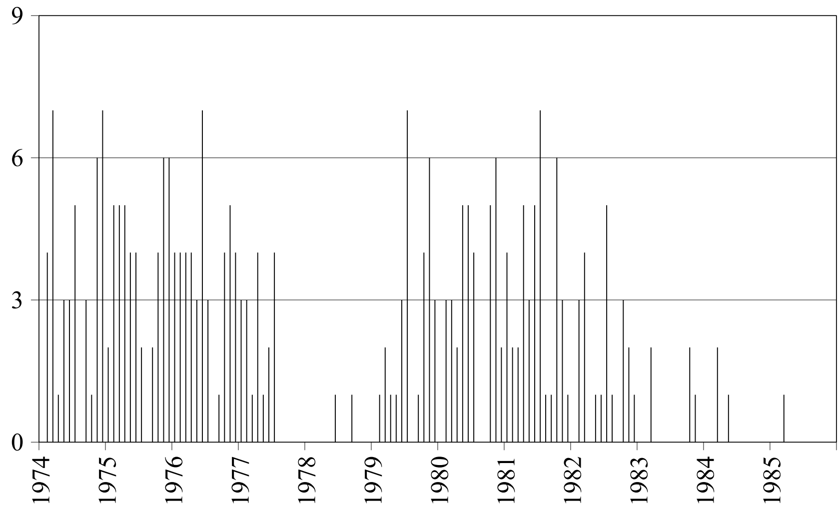
Table 6.3: **Implied Cum-day RRA Estimates**

This table shows the pooled, implied RRA estimates based on cum-day observations. The statistics are given for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). The second column reports the least squares RRA estimate $\hat{\gamma}$ of the respective segment. Columns three to seven report the mean price residuals ME, the standard deviation of the price residuals SE, the minimum and maximum price residual min and max, and the mean absolute error MAE. All error terms are reported in German Mark.

	$\hat{\gamma}$	ME	SE	min	max	MAE
Overall	1.15	0.26	0.67	−1.76	2.13	0.57
Issuer Group						
FRG	0.11	0.07	0.49	−1.21	1.05	0.37
GS	2.99	0.30	0.60	−1.29	1.88	0.53
GE	0.17	0.02	0.60	−1.44	1.27	0.47
Redemption Probability						
1/2	6.99	0.07	0.22	−0.47	0.57	0.17
1/3	2.78	0.19	0.47	−0.88	1.23	0.40
1/4	3.06	0.19	0.60	−1.46	1.06	0.46
1/5	−0.01	0.16	0.61	−1.04	1.62	0.51
1/6	−0.01	0.49	0.67	−0.97	1.85	0.66
1/7	5.26	0.12	0.77	−1.45	1.59	0.66
1/8	1.33	0.10	1.20	−3.22	1.83	0.95
1/9	0.90	−0.09	1.13	−2.84	1.74	0.81
1/10	0.43	0.14	0.78	−1.06	1.46	0.60

Figure 6.1: **Observation Dates for Cum-day Observations**

This figure shows the absolute frequency of the 282 filtered cum-day price observations per observation month.



The mean absolute errors on bond price level are DEM 0.37 for the FRG, DEM 0.53 for GS, and DEM 0.47 for GE. Our results show that agents investing in the state-guaranteed FRG and GE lottery bond segments are risk-averse at a level below one. Agents investing in the GS issuer segment exhibit a higher level of risk aversion. Estimates for the redemption probability segments are rather volatile and range between -0.01 and 6.99.[6]

We examine the robustness of the cum-day estimations by considering the bootstrapped standard errors of the pooled RRA estimates. Table 6.4 reports the results. The standard deviation of the bootstrap estimation on the aggregate level is 0.23. From the quantiles we conclude that the pooled RRA estimate is significantly positive for the overall sample. However, results on the issuer group level indicate that RRA estimates are rather unstable. The standard deviations range from 0.27 for GE to 0.35 for GS. The 1% quantile for the FRG and GE is negative such that we cannot rule out risk-seeking preferences. For the redemption probability segments standard deviations are even higher ranging between 0.36 and 1.69. The volatility of the estimates might be caused by the

[6] Causes for the heterogeneity across segments are considered in further detail in Sections 6.3.1 and 6.3.2.

Table 6.4: **Bootstrap Statistics for Cum-days**

This table shows the bootstrapped standard errors of pooled, implied RRA estimates based on cum-day observations. Bootstrap statistics are reported for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). Between 500 and 1,000 bootstrap samples are randomly drawn with replacement from the original dataset, and implied RRA coefficients are estimated. The size of the bootstrap samples corresponds to the respective number of filtered observations in the original sample specified in Table 6.1. Columns three to eight report the mean, standard deviation, minimum, maximum, 1% quantile, and 99% quantile of the estimated RRA coefficients. We exclude observations classified as outliers when estimating the bootstrap statistics.

	Bootstr. Sample	Mean	Std. Dev.	min	max	1% Quant.	99% Quant.
Overall	1,000	1.18	0.23	0.47	2.00	0.71	1.74
Issuer Group							
FRG	500	0.12	0.33	−2.10	2.11	−0.68	0.79
GS	1,000	3.04	0.35	2.25	4.71	2.38	4.00
GE	500	0.16	0.27	−1.13	1.25	−0.42	0.85
Redemption Probability							
1/2	500	7.05	0.85	4.71	10.13	5.28	9.46
1/3	500	2.80	1.14	−0.75	6.91	0.66	5.64
1/4	500	3.28	1.69	−0.99	7.65	−0.63	6.91
1/5	500	−0.07	0.43	−1.68	1.79	−1.27	0.93
1/6	500	0.25	0.85	−0.37	8.16	−0.30	3.93
1/7	500	5.38	1.04	2.49	11.12	3.08	8.14
1/8	500	1.36	0.89	−2.11	4.74	−1.17	3.52
1/9	500	0.84	0.44	−0.97	2.02	−0.32	1.82
1/10	500	0.50	0.36	−0.20	2.82	−0.03	1.89

limited number of observations. In Section 6.3, we therefore use the entire lottery bond data panel to estimate pooled, implied RRA estimates.

6.2.2 RRA Estimates under Perfect Foresight

We continue with the analysis of cum-day price observations and determine the pooled, implied RRA estimates under the assumption of perfect foresight employing pricing Equation (3.36). Estimations are based on the last clean transaction prices $\bar{b}_{(T-i)^{cum}}$ available before the lottery-related price suspensions.

Table 6.5 reports the number of cum-day observations under perfect foresight for the overall dataset and for segments corresponding to issuer groups and redemption probabilities.[7] We exclude 88 price observations for which the time difference between $\bar{b}_{(T-i)^{cum}}$ and $\bar{b}_{(T-i)^{ex}}$ is larger than ten trading days as well as 69 observations violating the adjusted price Hypothesis 1 derived in Section 5.1 and thus the no-arbitrage condition under perfect foresight. Since RRA coefficients can only be extracted from pricing Equation (3.36) for cum-day transaction prices located inside the no-arbitrage bounds, we disregard ten price observations by GS located outside the no-arbitrage bounds defined by Inequalities (3.15).[8] Furthermore, we exclude between zero and three observations classified as outliers. Our filtered and pooled sample contains 174 lottery observations on the aggregate level, 20 observations on the FRG issuer group level, 101 observations on the GS level, 54 observations on the GE level, and between 10 and 29 observations on redemption probability level.

Table 6.6 reports the estimation results for the overall dataset and for segments corresponding to issuer groups and redemption probabilities. The overall least squares RRA estimate of the pooled estimation is 5.15 with a mean absolute error

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[7] The numbers complement Tables 5.2 and 5.6 in Section 5.3.

[8] Within a control estimation, we also consider the no-arbitrage bounds for ex-day prices given by Inequalities (3.17), which results in a further exclusion of 35 observations, 20 of which are from the $p_i = 1/2$ redemption probability segment. The results of the control estimation are comparable to those of the unrestricted estimation.

Table 6.5: **Number of Perfect Foresight Observations**

This table shows the number of cum-day price observations under perfect foresight for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). The second and third column report the total number of observations and the number of observations resulting in a redemption gain. The fourth column reports the number of observations in the redemption gain segment excluded because the time lag between the cum-price and ex-price is larger than ten trading days. The fifth column reports the number of observations excluded because of a violation against the adjusted price Hypothesis 1 in Section 5.1. The sixth column reports the number of observations excluded because the cum-lottery transaction price is located outside the no-arbitrage bounds defined by Inequalities (3.15). The fifth column reports the number of observations classified as outliers. The last column reports the number of filtered observations used in the estimations.

	Total	Red. Gain	Cum-ex Lag	Price React.	No-arb.	Outl.	Clean
Overall	483	344	(88)	(69)	(10)	(3)	174
Issuer Group							
FRG	27	21	(0)	(1)	(0)	(0)	20
GS	361	247	(79)	(56)	(10)	(1)	101
GE	95	76	(9)	(12)	(0)	(1)	54
Redemption Probability							
1/2	83	60	(21)	(16)	(3)	(0)	20
1/3	78	55	(19)	(6)	(1)	(0)	29
1/4	71	53	(20)	(8)	(4)	(0)	21
1/5	61	43	(10)	(6)	(1)	(0)	26
1/6	48	33	(6)	(11)	(0)	(0)	16
1/7	44	34	(4)	(11)	(0)	(0)	19
1/8	40	32	(3)	(8)	(1)	(0)	20
1/9	35	23	(5)	(2)	(0)	(0)	16
1/10	23	11	(0)	(1)	(0)	(0)	10

Table 6.6: **RRA Estimates under Perfect Foresight**

This table shows the pooled, implied RRA estimates under perfect foresight. The statistics are given for the overall dataset and for segments corresponding to the issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). The second column reports the least squares RRA estimate $\hat{\gamma}$ of the respective segment. Columns three to seven report the mean price residuals ME, the standard deviation of the price residuals SE, the minimum and maximum price residual min and max, and the mean absolute error MAE. All error terms are reported in German Mark.

	$\hat{\gamma}$	ME	SE	min	max	MAE
Overall	5.15	0.04	0.45	−1.31	0.98	0.35
Issuer Group						
FRG	2.34	0.06	0.36	−0.92	0.95	0.26
GS	5.93	0.02	0.51	−1.37	0.92	0.40
GE	4.12	0.09	0.38	−0.71	1.02	0.32
Redemption Probability						
1/2	6.30	0.08	0.37	−0.74	0.74	0.28
1/3	6.09	0.11	0.38	−0.79	0.78	0.33
1/4	8.02	0.10	0.46	−1.27	0.86	0.34
1/5	1.51	0.10	0.43	−0.92	0.95	0.35
1/6	9.32	−0.02	0.43	−0.99	0.60	0.33
1/7	1.93	−0.03	0.67	−1.96	0.82	0.48
1/8	1.74	0.05	0.68	−1.94	0.95	0.49
1/9	4.67	−0.08	0.47	−0.97	0.76	0.36
1/10	3.86	−0.10	0.56	−1.04	0.76	0.45

between theoretical and observed prices of DEM 0.35.[9] On the issuer group level, the RRA estimates are 2.34 for the FRG, 5.93 for GS, and 4.12 for GE. The mean absolute errors on bond price level are DEM 0.26 for the FRG, DEM 0.40 for GS, and DEM 0.32 for GE. Our results indicate that agents investing in the state-guaranteed FRG and GE lottery bond segments are risk-averse at a level between two and four, while agents investing in the GS issuer segment are risk-averse at a level about six. Estimates for the redemption probability segments are rather volatile, ranging between 1.51 and 9.32.

We examine the robustness of the estimations under perfect foresight by considering the bootstrapped standard errors of the pooled RRA estimates for the overall sample and issuer group segments. Table 6.7 reports the results. The standard deviation of the bootstrap estimation on the aggregate level is 0.75 and standard deviations on issuer group level range from 1.20 for GE to 1.74 for the FRG segment. From the quantiles, we conclude that the pooled RRA estimates are significantly positive on the aggregate level, as well as on the issuer group level, ruling out risk-seeking preferences. However, the 1% and 99% quantiles span from 3.40 to 6.87 on the aggregate level, and from 0.13 to 6.74 for the FRG, from 3.09 to 8.88 for GS, and from 2.34 to 7.87 for the GE segment. For the redemption probability segments, standard deviations are even higher ranging between 1.31 and 5.24. The volatility of the estimates might again be caused by the limited number of observations.

Next, we compare the RRA estimates implied by the dynamic equilibrium valuation model reported in Table 6.3 with the results under perfect foresight.[10] The overall RRA estimate under perfect foresight is 4.5 times larger than the RRA estimate implied by the dynamic model. On the issuer group level, the RRA estimates under perfect foresight are 21.3 times larger for the FRG, 2.0 times

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[9] To facilitate the interpretation of the error terms, we report the mean lottery bond price (clean) based on the 174 cum-day observations, which is DEM 94.82.

[10] Note that the filtration of the sample in Table 6.5 is based on the assumption of perfect foresight. Hence, the samples characterized by Tables 6.1 and 6.5 have a different scope. However, control estimations employing the dynamic equilibrium valuation model, which are based on the sample characterized by Table 6.5, show that results are robust with respect to the filtration of the sample.

Table 6.7: **Bootstrap Statistics under Perfect Foresight**

This table shows the bootstrapped standard errors of pooled, implied RRA estimates under perfect foresight. Bootstrap statistics are reported for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). Between 500 and 1,000 bootstrap samples are randomly drawn with replacement from the original dataset, and implied RRA coefficients are estimated. The size of the bootstrap samples corresponds to the respective number of filtered observations in the original sample specified in Table 6.5. Columns three to eight report the mean, standard deviation, minimum, maximum, 1% quantile, and 99% quantile of the estimated RRA coefficients. We exclude observations classified as outliers when estimating the bootstrap statistics.

	Bootstr. Sample	Mean	Std. Dev.	min	max	1% Quant.	99% Quant.
Overall	1,000	5.11	0.75	2.88	7.68	3.40	6.87
Issuer Group							
FRG	500	2.95	1.74	−0.18	7.29	0.13	6.74
GS	1,000	5.97	1.22	1.33	10.18	3.09	8.88
GE	500	4.37	1.20	2.12	10.74	2.34	7.87
Redemption Probability							
1/2	500	6.20	1.63	0.89	10.27	1.82	9.28
1/3	500	6.42	3.01	−0.45	27.57	1.43	14.57
1/4	500	9.08	5.24	−2.35	40.15	0.86	25.20
1/5	500	1.85	1.31	−0.93	10.52	−0.23	5.56
1/6	500	9.55	2.79	2.10	20.69	2.62	16.70
1/7	500	2.30	3.51	−7.55	16.23	−4.74	12.79
1/8	500	2.31	3.28	−6.12	14.31	−4.85	9.93
1/9	500	4.90	1.71	0.43	12.17	1.57	9.42
1/10	500	3.99	2.14	−8.62	14.87	−0.27	9.25

larger for GS, and 24.2 times larger for GE. On the redemption probability level, the difference is most pronounced for probability 1/6, where the RRA estimate under perfect foresight is 9.32 relative to -0.01 implied by the dynamic model.

In order to explain the difference between RRA estimates implied by the dynamic equilibrium model and those under perfect foresight, we reconsider the theoretical framework deduced in Section 3.3. The comparative static results for the equilibrium ex-day price behavior showed that the larger the ex-day price drop, the lower is the implied RRA coefficient.[11] The same result holds for the ex-day behavior under perfect foresight. From this property follows that the theoretical ex-day price drop entering our dynamic equilibrium valuation model is larger than the observed drop considered under perfect foresight. The dynamic equilibrium model represents a more realistic valuation setup than the perfect foresight model and leads to distinctly lower implied RRA estimates. Hence, our results imply that estimations relying on the perfect foresight assumption overestimate RRA coefficients.

6.3 Pooled, Implied RRA Coefficients based on Entire Sample

6.3.1 Equilibrium RRA Estimates

Lottery bond prices contain information on risk preferences at cum-days as well as at trading days prior to the redemption lotteries. Before $(T-1)^{ex}$, market prices reflect the value of future cash flows from the bond indenture which depends on the outcome of the lotteries. Based on lottery bond observations within the interval $[(T-10)^{ex}, (T-1)^{cum}]$, we estimate pooled, implied RRA coefficients.

Henceforth, we employ bond market data at a weekly frequency using clean Wednesday transaction prices.[12] Moving to daily price data does not improve

[11] See Figure 3.10 for the comparative static results of the equilibrium ex-day price behavior.

[12] If a Wednesday transaction price is missing, it is replenished by the next available transaction price within the calendar week or marked unavailable.

our estimation results, but increases the computational effort and introduces unexplainable noise. Table 6.8 reports the number of weekly observations for the overall dataset and for segments corresponding to issuer groups and redemption probabilities. For the final estimation, we only include lottery bond prices resulting in a redemption gain and located inside the no-arbitrage bounds defined by Inequalities (3.15) and (3.17).[13] We exclude 971 observations due to no-arbitrage violations. On the aggregate level, 417 observations are classified as outliers and, on the issuer group level, we disregard 19 observations by the FRG, 319 observations by GS, and 122 observations by GE. Our filtered and pooled sample contains 12,019 observations on the aggregate level, 1,539 observations from the FRG, 7,133 observations from GS, 3,304 observations from GE, and between 364 and 2,114 observations on the redemption probability level.

Figure 6.2 depicts the absolute frequency of price observations per month and illustrates that restricting our analysis to observations resulting in a redemption gain limits data in times of relatively low interest rates, e. g. between 1977 and 1978. Table 6.9 shows the varying distribution of price observations per year across issuer groups and redemption probabilities. For the overall dataset the mean observation date is September 1977. At the issuer group level, the mean observation date is December 1976 for the FRG, February 1978 for GS, and February 1977 for GE. At the redemption probability level, mean observation dates tend to rise with the redemption probability. For probability 1/10, the mean observation date is September 1975, whereas, for probability 1/2, the mean observation date is March 1979.

We analyze the data panel by estimating pooled, implied RRA coefficients for various segments of the dataset. Table 6.10 summarizes the estimation results. The overall least squares RRA estimate is 1.78 with a mean absolute error between theoretical and observed prices of DEM 0.55.[14] At the issuer group level, the RRA estimate is 1.17 with a mean absolute error of DEM 0.37 for the FRG, 2.93 with a mean absolute error of DEM 0.60 for GS, and 0.98 with a mean absolute error of DEM 0.44 for GE. RRA estimates for the FRG and GE are of similar magnitude,

[13] See Section 6.2.1 for a compilation of the filtration motives.

[14] To facilitate the interpretation of the error terms, we report the mean lottery bond price (clean) based on the entire sample of 12,019 observations, which is DEM 95.14.

Table 6.8: **Number of Observations for Entire Sample**

This table shows the number of price observations for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). The second and third column report the total number of observations and the number of observations resulting in a redemption gain. The fourth column reports the number of observations in the redemption gain segment excluded because the transaction price is located outside the no-arbitrage bounds defined by Inequalities (3.15) or (3.17). The fifth column reports the number of observations classified as outliers. The last column reports the number of filtered observations used in the estimations.

	Total	Red. Gain	No-arb.	Outl.	Clean
Overall	18,612	13,407	(971)	(417)	12,019
Issuer Group					
FRG	1,832	1,610	(52)	(19)	1,539
GS	12,462	8,164	(712)	(319)	7,133
GE	4,318	3,633	(207)	(122)	3,304
Redemption Probability					
1/2	3,368	2,464	(375)	(34)	2,055
1/3	3,231	2,367	(225)	(28)	2,114
1/4	2,872	2,267	(202)	(34)	2,031
1/5	2,238	1,530	(55)	(14)	1,461
1/6	1,743	1,286	(30)	(94)	1,162
1/7	1,629	1,315	(25)	(59)	1,231
1/8	1,504	1,099	(20)	(41)	1,038
1/9	1,222	701	(28)	(14)	659
1/10	805	378	(11)	(3)	364

Table 6.9: **Observation Dates for Entire Sample**

This table shows the relative frequency of filtered price observations per observation year. Results are given for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). Relative frequencies are reported in percentage points.

Observation Year	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985
Overall	16.0	18.3	18.0	8.4	2.3	9.2	10.3	9.1	5.6	1.5	1.2	0.2
Issuer Group												
FRG	17.9	22.0	20.7	9.6	4.4	11.6	10.2	3.5	–	–	–	–
GS	12.9	16.6	17.0	7.8	1.9	9.1	10.9	12.2	8.2	1.9	1.4	0.1
GE	21.2	20.2	18.8	9.2	2.2	8.4	9.2	5.5	2.7	1.1	1.2	0.3
Redemption Probability												
1/2	4.2	11.5	13.8	4.1	1.5	9.8	23.0	20.3	9.7	1.1	0.6	0.3
1/3	14.4	14.7	16.5	9.0	3.3	16.3	13.2	5.8	1.9	1.2	3.3	0.5
1/4	13.7	24.9	18.4	7.2	3.7	11.8	5.8	3.6	4.2	4.0	2.4	0.2
1/5	24.7	14.9	9.9	8.8	3.8	7.7	1.7	9.0	16.0	2.7	0.8	–
1/6	7.1	7.1	27.8	20.0	2.6	1.1	4.7	20.2	8.4	0.9	–	–
1/7	5.7	24.9	31.9	10.7	0.7	2.4	15.7	7.1	0.9	–	–	–
1/8	28.6	35.2	14.8	1.8	0.6	10.3	7.7	1.0	–	–	–	–
1/9	56.3	22.6	9.3	3.8	–	6.2	1.8	–	–	–	–	–
1/10	31.0	25.5	25.8	14.3	–	3.3	–	–	–	–	–	–

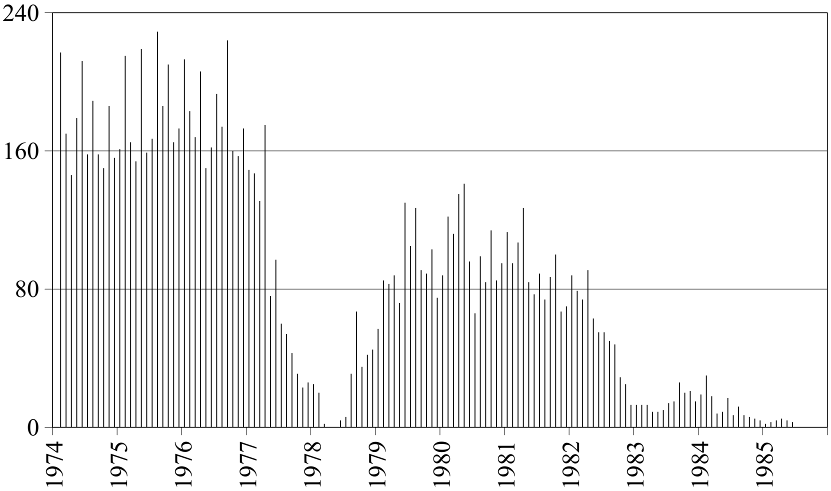
Table 6.10: **Implied RRA Estimates for Entire Sample**

This table shows the pooled, implied RRA estimates based on observations from the entire sample. The statistics are given for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). The second column reports the least squares RRA estimate $\hat{\gamma}$ of the respective segment. Columns three to seven report the mean price residuals ME, the standard deviation of the price residuals SE, the minimum and maximum price residual min and max, and the mean absolute error MAE. All error terms are reported in German Mark.

	$\hat{\gamma}$	ME	SE	min	max	MAE
Overall	1.78	0.23	0.66	−1.76	2.22	0.55
Issuer Group						
FRG	1.17	0.14	0.45	−1.19	1.43	0.37
GS	2.93	0.28	0.70	−1.82	2.37	0.60
GE	0.98	0.11	0.54	−1.51	1.58	0.44
Redemption Probability						
1/2	6.17	0.09	0.36	−0.98	1.15	0.29
1/3	2.68	0.15	0.47	−1.27	1.55	0.40
1/4	1.75	0.16	0.60	−1.59	1.83	0.49
1/5	0.87	0.21	0.73	−1.96	2.22	0.59
1/6	6.37	0.17	0.72	−2.03	2.35	0.59
1/7	5.08	0.12	0.80	−2.27	2.47	0.66
1/8	2.04	0.18	0.94	−2.65	2.42	0.77
1/9	0.96	0.01	1.10	−3.28	2.64	0.85
1/10	0.61	0.33	0.90	−1.98	2.18	0.80

Figure 6.2: **Observation Dates for Entire Sample**

This figure shows the absolute frequency of the 12,019 filtered price observations per observation month.



while agents investing in the GS issuer segment exhibit a higher risk aversion. Error terms are smaller for the state-guaranteed FRG and GE lottery bonds relative to the GS bonds. Estimates for the redemption probability segments are dispersed and range between 0.61 and 6.37. The smaller the probability, the wider is in general the span of arbitrage-free equilibrium prices and the larger are the mean absolute price error, the maximum price residual, and the absolute value of the minimum price residual.

We address potential causes of the heterogeneity of implied RRA estimates across issuer groups and redemption probabilities. First, we examine implied RRA estimates for issuer group and redemption probability subsegments. Table 6.11 reports the estimation results. The results show that the overall and issuer group RRA estimates follow a similar pattern across redemption probabilities. For all subsegments, RRA estimates for GS are higher than FRG and GE estimates. We find neither evidence that the different level of issuer group estimates is caused by redemption probabilities nor that the dispersion of redemption probability estimates is driven by issuer groups.

Second, we consider the varying distribution of observation dates. Table 6.8 documented that the distribution is similar across issuer groups, but differs

Table 6.11: **Implied RRA Estimates for Probability Subsegments**

This table shows the pooled, implied RRA estimates for issuer group and redemption probability subsegments. RRA estimates are reported for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)). Columns two to twelve report the least squares RRA estimate $\hat{\gamma}$ based on the entire sample and redemption probability subsegments (1/2 to 1/10). The numbers of filtered observations are given in parentheses.

	Entire	1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9	1/10
Overall	1.78	6.17	2.68	1.75	0.87	6.37	5.08	2.04	0.96	0.61
	(12,019)	(2,055)	(2,114)	(2,031)	(1,461)	(1,162)	(1,231)	(1,038)	(659)	(364)
Issuer Group										
FRG	1.17	4.29	1.96	1.58	0.81	2.29	3.97	1.03	0.82	–
	(1,539)	(465)	(433)	(330)	(161)	(48)	(50)	(49)	(9)	
GS	2.93	9.48	6.63	2.15	2.32	8.11	6.81	3.44	1.34	1.02
	(7,133)	(825)	(994)	(1,077)	(925)	(869)	(938)	(748)	(498)	(301)
GE	0.98	5.42	2.22	1.80	0.70	1.47	1.16	0.26	−0.56	0.21
	(3,304)	(765)	(694)	(621)	(346)	(246)	(268)	(247)	(166)	(62)

distinctly for redemption probabilities. In Section 6.4, we show that implied RRA estimates are time dependent and argue that the underlying dynamic equilibrium model predicts a higher dispersion of implied RRA estimates in times of relatively low interest rates. Note that 50.4% and 43.3% of the price observations for the probability segments 1/6 and 1/7, respectively, are located in the time interval 1976 to 1978, characterized by historically low interest rates. RRA estimates for these probability segments are rather volatile. Besides, 43.3%, 24.9%, and 22.8% of the price observations for the probability segments 1/2, 1/6, and 1/7, respectively, are located in the time interval 1980 to 1981, characterized by a severe recession in Germany after the second oil crisis in 1979. RRA estimates for these probability segments tend to be relatively high. Hence, a major part of the heterogeneity across probability segments is driven by the varying distribution of price observations over time.

Within additional estimations, we control for varying bond characteristics, e. g. the redemption value, or embedded early or increased redemption options and find no systematic effects explaining the heterogeneity of implied RRA estimates across issuer groups. One could ascribe the heterogeneity to differences in liquidity and credit risk.[15] However, we control for the difference in trading activity by focusing on price observations from active trading days and by disregarding price quotes from days with zero trading volume. Furthermore, we control for differences in credit risk by applying issuer specific risk-free term structures of interest rates. Therefore, our analyses suggest that the heterogeneity of implied RRA estimates across issuer groups is systematic implying a higher risk aversion for GS compared to FRG and GE investors.

6.3.2 Robustness Analysis

As a first robustness check, we consider the bootstrapped standard errors of the pooled RRA estimates based on the entire sample. Table 6.12 reports

[15] Tables 4.6 and 4.7 in Section 4.1.3 showed a distinct difference in the trading activity and mean trading volumes between GS and the FRG as well as GE, respectively. In Section 4.2.2, we obtained a mean credit spread for straight coupon bonds of 14 and 12 basis points between the term structures of spot rates of GS and the FRG as well as GE, respectively.

the bootstrap statistics and reinforces the robustness of our estimations. The standard deviations of the bootstrap estimation are marginal, ranging between 0.03 and 0.05 for the overall sample and issuer group segments. For the redemption probability segments standard deviations are higher and more heterogenous ranging between 0.05 and 0.27. From the quantiles, we conclude that: (i) the pooled RRA estimates on the aggregate and issuer group level are significantly positive, (ii) RRA estimates for the FRG and GE segments are of similar magnitude at a level below two, (iii) relative to the FRG and GE results, RRA estimates for the GS segment are significantly higher.

By employing the entire sample of filtered market prices, we include observations for which the next lottery is due between three trading days up to two years. We consider whether the distance of price observations to the subsequent redemption lotteries has a systematic effect on pooled, implied RRA estimates and examine subsamples covering distinct intervals before the redemption lotteries. Table 6.13 reports the results for control estimations based on price observations 180, 90, and 30 calendar days prior to the redemption lotteries.

First, we compare the results based on the entire sample with the cum-day estimates. The overall cum-day RRA estimate is 1.6 times smaller than the RRA estimate based on the entire sample. On the issuer group level, the cum-day RRA estimates are 10.6 times smaller for the FRG, of similar magnitude for GS, and 5.8 times smaller for GE. On the redemption probability level, the difference is most pronounced for probability 1/6, where the cum-day RRA estimate is -0.01 relative to 6.37 for the entire sample. We observe neither abnormal returns nor trading volumes prior to lottery dates and see no evidence of a diverse investor or market structure at cum-days relative to the entire sample, which could explain the difference. However, the bootstrap statistics for cum-day RRA estimates in Table 6.4 suggest that the difference is partly caused by the limited number of cum-day observations. The standard deviations of the bootstrap estimation are distinctly lower for the entire sample compared to cum-day observations.

Second, we focus on the difference between RRA estimates based on the entire sample and the 180-day, 90-day, and 30-day intervals. Overall RRA estimates are relatively robust with respect to variation in the length of the estimation intervals and range from 1.18 for the 30-day interval to 2.02 for the 180-day interval. At

Table 6.12: **Bootstrap Statistics for Entire Sample**

This table shows the bootstrapped standard errors of pooled, implied RRA estimates based on observations from the entire sample. Bootstrap statistics are reported for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). Between 500 and 1,000 bootstrap samples are randomly drawn with replacement from the original dataset, and implied RRA coefficients are estimated. The size of the bootstrap samples corresponds to the respective number of filtered observations in the original sample specified in Table 6.8. Columns three to eight report the mean, standard deviation, minimum, maximum, 1% quantile, and 99% quantile of the estimated RRA coefficients. We exclude observations classified as outliers when estimating the bootstrap statistics.

	Bootstr. Sample	Mean	Std. Dev.	min	max	1% Quant.	99% Quant.
Overall	1,000	1.78	0.03	1.66	1.89	1.71	1.86
Issuer Group							
FRG	500	1.18	0.04	1.07	1.30	1.08	1.27
GS	1,000	2.93	0.05	2.72	3.09	2.80	3.05
GE	500	0.98	0.04	0.86	1.15	0.89	1.08
Redemption Probability							
1/2	500	6.17	0.23	5.43	6.83	5.67	6.75
1/3	500	2.69	0.12	2.32	3.21	2.41	2.99
1/4	500	1.75	0.07	1.56	1.97	1.59	1.92
1/5	500	0.88	0.05	0.73	1.04	0.77	1.00
1/6	500	6.38	0.27	5.61	7.53	5.78	6.95
1/7	500	5.09	0.17	4.56	5.61	4.71	5.51
1/8	500	2.05	0.09	1.83	2.29	1.85	2.26
1/9	500	0.97	0.08	0.65	1.22	0.77	1.13
1/10	500	0.60	0.10	0.31	0.93	0.40	0.83

Table 6.13: **Robustness with respect to Estimation Intervals**

This table shows the pooled, implied RRA coefficients for various estimation intervals. RRA estimates are reported for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). Columns two to six report the least squares RRA estimate $\hat{\gamma}$ based on the entire sample, intervals containing observations 180, 90, and 30 calendar days prior to the redemption lotteries, and cum-day observations only. The numbers of filtered observations are given in parentheses.

	Entire	180 days	90 days	30 days	cum-days
Overall	1.78	2.02	1.89	1.18	1.15
	(12,019)	(5,665)	(2,807)	(961)	(282)
Issuer Group					
FRG	1.17	1.20	1.23	1.01	0.11
	(1,539)	(498)	(256)	(100)	(21)
GS	2.93	3.13	3.22	3.21	2.99
	(7,133)	(3,616)	(1,785)	(554)	(187)
GE	0.98	0.84	0.40	0.21	0.17
	(3,304)	(1,536)	(761)	(285)	(68)
Redemption Probability					
1/2	6.17	5.41	4.87	4.46	6.99
	(2,055)	(727)	(354)	(109)	(37)
1/3	2.68	3.16	3.07	2.78	2.78
	(2,114)	(908)	(434)	(151)	(46)
1/4	1.75	4.17	4.87	4.27	3.06
	(2,031)	(867)	(432)	(136)	(44)
1/5	0.87	1.06	0.63	0.43	−0.01
	(1,461)	(696)	(371)	(134)	(37)
1/6	6.37	2.10	2.90	0.13	−0.01
	(1,162)	(633)	(310)	(112)	(29)
1/7	5.08	5.66	5.89	5.17	5.26
	(1,231)	(719)	(361)	(120)	(32)
1/8	2.04	2.48	2.43	1.94	1.33
	(1,038)	(603)	(293)	(99)	(27)
1/9	0.96	0.90	0.75	1.12	0.90
	(659)	(397)	(208)	(74)	(21)
1/10	0.61	0.32	0.17	0.25	0.43
	(364)	(198)	(90)	(34)	(10)

the issuer group level, estimates span from 1.01 to 1.23 for the FRG, from 2.93 to 3.22 for GS, and from 0.21 to 0.98 for GE. At the redemption probability level, results are relatively robust, except for the 1/4 and 1/6 probability segments. Hence, in case the underlying sample is sufficiently large, variation in the length of the estimation interval does not systematically alter our inferences on implied risk preferences.

According to the dynamic equilibrium valuation model derived in Chapter 3, implied RRA coefficients are most sensitive to price changes close to the no-arbitrage bounds.[16] Hence, a small price change in close distance to the no-arbitrage bounds defined by Inequalities (3.15) and (3.17) causes a distinct response of the RRA coefficient. Note, in addition, that price observations adjoining no-arbitrage bounds imply extreme RRA coefficients. We control for these model related characteristics by requiring a minimum distance between the observed market price and both, the upper and lower no-arbitrage bound.

Table 6.14 reports the results for a minimum distance to the no-arbitrage bounds of DEM 0.25, 0.50, 1.50, and 2.50. Overall RRA estimates are relatively robust with respect to variation in the no-arbitrage span and range from 1.44 for a minimum distance of DEM 2.50 to 1.78 for the unrestricted sample. At the issuer group level, estimates range from 1.13 to 1.17 for the FRG, from 1.97 to 2.93 for GS, and from 0.86 to 0.98 for GE. The stronger the no-arbitrage restriction, the smaller become the RRA coefficients. Results are most remarkable at the redemption probability level, where estimates become less dispersed, the larger the minimum no-arbitrage span. Only the implied estimates for redemption probability 1/7 are unaffected by a variation in the span and remain in the range of five. Excluding price observations adjoining no-arbitrage bounds causes RRA estimates for the GS issuer group and the probability segments to approach the overall estimate.

Our pooled results provide evidence of a moderate risk aversion in the bond market. We find no evidence of the “puzzling” RRA coefficients found by e. g. Mehra and Prescott (1985) based on equity, bond, and consumption indices. On

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[16] See Figure 3.6 for the comparative static results of the partial derivative and elasticity of the RRA coefficient with respect to the lottery bond price.

Table 6.14: **Robustness with respect to No-arbitrage Bounds**

This table shows the pooled, implied RRA estimates for various minimum spans between observed market prices and both, upper and lower no-arbitrage bounds. RRA estimates are given for the overall dataset and for segments corresponding to issuer groups (Federal Republic of Germany (FRG), German states (GS), and government enterprises (GE)) and redemption probabilities (1/2 to 1/10). Columns two to six report the least squares RRA estimate $\hat{\gamma}$ based on the unrestricted sample and on samples containing observations with a minimum distance to the no-arbitrage bounds of DEM 0.25, 0.50, 1.50, and 2.50. The numbers of filtered observations are given in parentheses.

	Unrestr.	DEM 0.25	DEM 0.50	DEM 1.50	DEM 2.50
Overall	1.78	1.74	1.69	1.57	1.44
	(12,019)	(11,350)	(10,599)	(7,637)	(5,461)
Issuer Group					
FRG	1.17	1.16	1.16	1.15	1.13
	(1,539)	(1,473)	(1,401)	(998)	(770)
GS	2.93	2.89	2.81	2.65	1.97
	(7,133)	(6,634)	(6,096)	(4,269)	(3,080)
GE	0.98	0.97	0.97	0.93	0.86
	(3,304)	(3,182)	(3,033)	(2,289)	(1,603)
Redemption Probability					
1/2	6.17	5.90	5.70	4.41	2.68
	(2,055)	(1,787)	(1,514)	(561)	(97)
1/3	2.68	2.69	2.67	2.43	1.98
	(2,114)	(1,928)	(1,728)	(972)	(512)
1/4	1.75	1.72	1.70	1.67	1.62
	(2,031)	(1,915)	(1,768)	(1,338)	(1,024)
1/5	0.87	0.87	0.87	0.85	0.81
	(1,461)	(1,420)	(1,362)	(1,018)	(710)
1/6	6.37	6.17	5.91	1.57	1.24
	(1,162)	(1,135)	(1,111)	(947)	(606)
1/7	5.08	5.06	5.03	5.02	4.87
	(1,231)	(1,204)	(1,179)	(1,018)	(874)
1/8	2.04	2.03	1.96	1.88	1.88
	(1,038)	(1,022)	(1,012)	(953)	(891)
1/9	0.96	0.96	0.96	1.00	0.96
	(659)	(654)	(653)	(634)	(592)
1/10	0.61	0.61	0.60	0.56	0.55
	(364)	(350)	(325)	(269)	(210)

the contrary, our estimations suggest a level of risk aversion of about one for the FRG and GE and of about three for GS. Compared to Mehra and Prescott, our RRA estimates are independent of probability beliefs and determined from individual bond price data. The magnitude is in accordance with the results reported by several studies based on direct assessments, cross-sectional survey data, and option price data as compiled in Table 1.1. RRA estimates are robust across the length of estimation intervals and for restrictions on the proximity of price observations to the no-arbitrage bounds.

6.4 Time Series Properties of Implied RRA Coefficients

6.4.1 Annual Equilibrium RRA Estimates

Having examined pooled, implied RRA estimates for segments corresponding to issuer groups and redemption probabilities, we focus on the time series properties of risk aversion. We analyze implied RRA estimates for disjoint and overlapping time intervals and consider the relation between risk aversion and macroeconomic factors. The estimations that follow are based on the entire filtered sample characterized in Table 6.8.

We estimate implied RRA coefficients for disjoint annual time intervals. Intervals span January 1 through December 31 of the respective year. By restricting our analysis to price observations resulting in a redemption gain, we limit data in times of relatively low risk-free interest rates. Note that the restriction biases estimation results and that RRA estimates for periods with low interest rates have to be interpreted with caution. For price observations resulting in a redemption gain, risk-free rates are in general larger than the coupon rate c/R. In Chapter 3, we have shown that the span of arbitrage-free lottery bond prices defined by Inequalities (3.15) and (3.17) falls, as r approaches c/R. Furthermore, we have shown that small price changes in close distance to the no-arbitrage bounds cause a distinct response of the RRA coefficient. Hence, the equilibrium model predicts a higher dispersion of implied RRA estimates in times of relatively low interest rates. We disregard estimates for the years 1978 and 1983 to 1987, as between 1983 to 1987 at most 186 observations per year and in 1978 only 308 observations resulting in a redemption gain are available and estimation results are unstable.

Table 6.15 reports annual implied RRA estimates ranging from −3.20 in 1977 to 8.65 and 5.81 in 1980 and 1981, respectively. Mean absolute errors between theoretical and observed prices are smallest in 1977 and 1979 when interest rates are relatively low and the span of arbitrage-free lottery bond prices is narrow. We consider the robustness of annual RRA estimates by controlling for the distance between the observed market price and both, the upper and lower no-arbitrage bound. Table 6.16 reports the results for a minimum distance to the no-arbitrage bounds of DEM 0.25, 0.50, 1.50, and 2.50. Relatively unaffected by the no-arbitrage adjustment, annual RRA estimates remain at a level of about one for 1974 and 1979, at a level between two and three for 1975 and 1976, at a level of minus three for 1977, and at a level above four for 1980 to 1982. In 1977, we obtain a robust negative RRA estimate implying risk-seeking behavior. This could partly be attributed to the recovery of the German economy from the first oil crisis coinciding with historically low spot interest rates. In 1980 and 1981, implied RRA estimates reflect the challenging economic situation after the second oil crisis in 1979, which resulted in a severe recession in Germany and historically high interest rates.[17] The relation is consistent with the results by Kumar and Persaud (2002) and Coudert and Gex (2008) implying that risk aversion is a leading indicator of financial crises that coincide with periods of high risk aversion.[18]

6.4.2 Time Series Analysis

The variability of implied RRA estimates with respect to the estimation year indicates time dependence of risk aversion. We do not explicitly capture time-varying risk aversion in our dynamic equilibrium valuation model. However, in our empirical study, we consider the time behavior of RRA coefficients by

[17] See Figure 6.4 for the time series of the adjusted CRB Spot Price Index Fats & Oils.

[18] See e. g. Gai and Vause (2004), Deutsche Bundesbank (2005), or Illing and Meyer (2005) for a comprehensive overview of risk aversion indicators.

Table 6.15: **Annual Implied RRA Estimates**

This table shows implied RRA estimates for disjoint annual time intervals from January 1 until December 31. We only list time intervals with more than 500 observations. The second column reports the least squares RRA estimate $\hat{\gamma}_t$ for the respective time interval. Columns three to eight report the number of observations, the mean price residuals ME, the standard deviation of the price residuals SE, the minimum and maximum price residual min and max, and the mean absolute error MAE. All error terms are reported in German Mark.

Time Interval	$\hat{\gamma}_t$	Obs.	ME	SE	min	max	MAE
1974	1.20	2,154	0.18	0.73	−1.98	2.14	0.60
1975	2.99	2,268	0.25	0.83	−2.21	2.47	0.70
1976	2.18	2,122	0.25	0.77	−2.05	2.21	0.65
1977	−3.20	882	0.12	0.36	−0.95	1.15	0.30
1979	1.22	1,117	0.06	0.37	−1.05	1.09	0.30
1980	8.65	1,230	0.12	0.41	−1.09	1.34	0.34
1981	5.81	1,083	0.18	0.75	−2.07	2.42	0.59
1982	4.27	590	0.19	0.45	−1.14	1.53	0.39

Table 6.16: **Robustness of Annual Implied RRA Estimates**

This table shows implied RRA estimates for disjoint annual time intervals from January 1 until December 31 for various minimum spans between observed market prices and both, upper and lower no-arbitrage bounds. We only list RRA estimates based on more than 500 price observations. Columns two to six report the least squares RRA estimate $\hat{\gamma}$ based on the unrestricted sample and on samples containing observations with a minimum distance to the no-arbitrage bounds of DEM 0.25, 0.50, 1.50, and 2.50. The numbers of filtered observations are given in parentheses.

Time Interval	Unrestr.	DEM 0.25	DEM 0.50	DEM 1.50	DEM 2.50
1974	1.20	1.20	1.20	1.19	1.18
	(2,154)	(2,154)	(2,153)	(2,118)	(1,986)
1975	2.99	2.89	2.88	2.64	1.92
	(2,268)	(2,196)	(2,135)	(1,896)	(1,608)
1976	2.18	2.11	2.08	1.93	1.66
	(2,122)	(2,030)	(1,964)	(1,533)	(1,120)
1977	−3.20	−3.30	−3.46	–	–
	(882)	(790)	(691)		
1979	1.22	1.33	1.26	–	–
	(1,117)	(1,037)	(905)		
1980	8.65	8.52	7.66	–	–
	(1,230)	(1,119)	(979)		
1981	5.81	5.88	5.60	4.69	–
	(1,083)	(1,034)	(984)	(647)	
1982	4.27	4.13	–	–	–
	(590)	(538)			

estimating $\hat{\gamma}_t$ from disjoint monthly time intervals and annual forward looking rolling windows at a monthly frequency.[19] We consistently employ data at a weekly frequency using Wednesday transaction prices and rates.

Figure 6.3 depicts the time series of implied RRA estimates $\hat{\gamma}_t$ and the three-month money market rate r_t as a benchmark.[20] RRA estimates for the disjoint monthly time intervals between January 1974 and December 1987 range from -20.99 in November 1978 to 25.02 in June 1980. The mean RRA estimate is 1.95 and significantly positive at the 1% level with a standard deviation of 7.72. RRA estimates for the annual forward looking rolling windows at a monthly frequency have a smaller range of -15.26 in April 1978 to 9.56 in December 1979. The mean RRA estimate is 2.82 and significantly positive at the 1% level with a standard deviation of 4.44. Visual inspection of the panels indicates that implied RRA estimates and the risk-free rate are negatively related between 1974 until 1976. Between 1977 and 1978, risk-free spot rates reached a historical low, and due to narrow no-arbitrage bounds and the limited number of observations, RRA estimates are strongly volatile over this period. From 1979 to 1980, RRA estimates and interest rates generally rise together. Relative to the implied RRA estimates, which reach a maximum in June 1980, spot interest rates attain a maximum in March 1981. For the entire interval, RRA estimates and the risk-free rate are positively correlated at 49.03% in the upper panel and at 63.58% in the lower panel.[21]

The dynamic equilibrium valuation model derived in Chapter 3 uses the risk-free term structure of interest rates as an exogenous variable. In our comparative static analysis, we assumed a flat term structure and showed that the equilibrium RRA coefficient is strictly decreasing in the risk-free rate. Nonetheless, we employ the entire German term structure for the estimation of RRA coefficients, and the

[19] Compared to the implied disjoint monthly interval estimate at e. g. December 1974 based on all price observations from December 1974, the respective implied annual rolling window estimate is based on all price observations from December 1974 until October 1975.

[20] We report the three-month money market rate instead of the current yield of public sector debt securities, as the short end of the term-structure of interest rates is generally more reactive to changes in the overall economic environment.

[21] For the calculation of the correlation coefficients, we omit missing values.

Figure 6.3: **Time Series of Implied RRA Estimates**

The panels show the time series of implied RRA estimates (solid lines) and the three-month money market rate (dotted lines). In the upper panel, we report RRA coefficients for disjoint monthly time intervals and end of the month money market rates. We only include RRA coefficients which are estimated using more than 50 observations. In the lower panel, RRA coefficients and money market rates are determined using annual forward looking rolling windows at a monthly frequency. We only include RRA coefficients which are estimated using more than 500 observations.

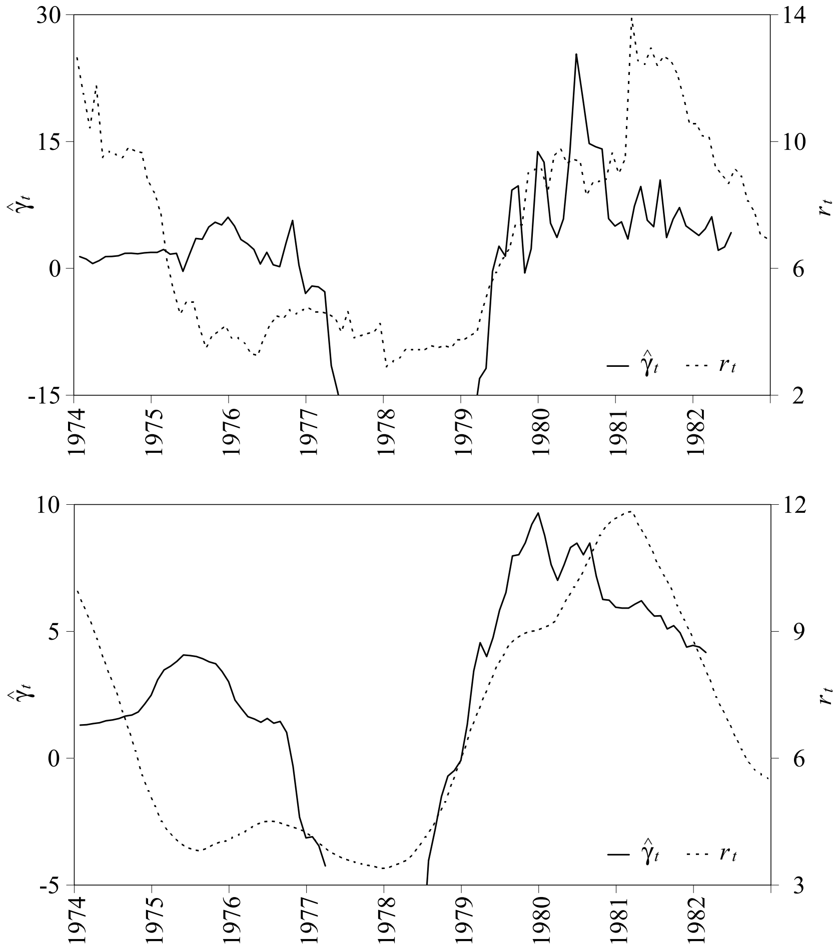
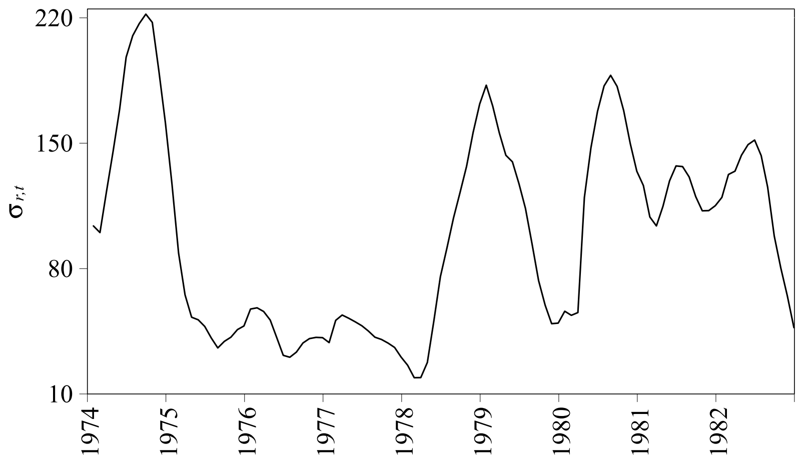
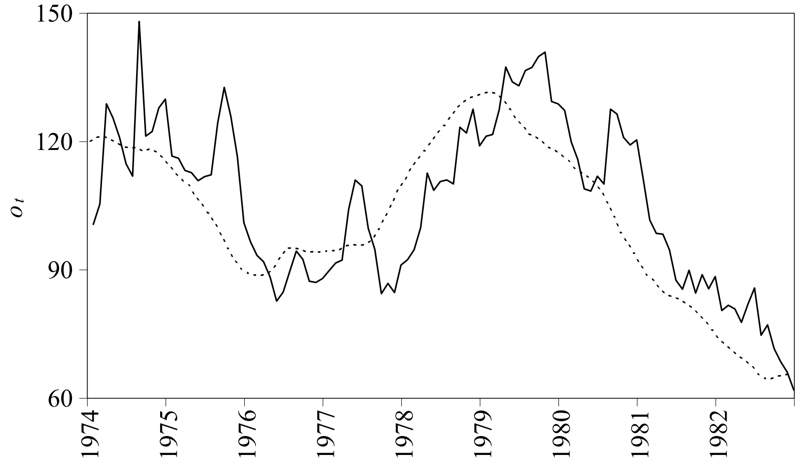


Figure 6.4: **Time Series of Macroeconomic Factors**

The upper panel shows the time series of the 52-week (forward looking) volatility of the three-month money market rate in basis points. The lower panel shows the time series of the CRB Spot Price Index Fats & Oils in German Mark normalized to January 2, 1974. The solid line refers to end of the month values, and the dotted line refers to values which are determined using annual forward looking rolling windows at a monthly frequency.





relation between risk aversion and risk-free rates does not remain unambiguous for non-parallel shifts of the term-structure.

Figure 6.4 focuses on further macroeconomic factors and depicts the time series of the volatility of the three-month money market rate $\sigma_{r,t}$ as well as the CRB Spot Price Index Fats & Oils in German Mark o_t between January 1974 and December 1982. The volatility of the money market rate is determined from 52-week forward looking intervals and attains maxima in September 1974 (bp 221), January 1979 (bp 181), and August 1980 (bp 186). The oil price index is normalized to January 2, 1974 and attains maxima in August 1974 (147%) and October 1979 (140%) reflecting the first and second oil crisis. The second oil crisis resulted in a severe recession lasting until 1981. Implied RRA estimates attain a global maximum about eight months and the money market rate about 17 months after the peak of the oil price index in 1979. For the entire interval, the oil price index and RRA estimates, respectively risk-free rate, are correlated at -7.05% or -5.48% for end of the month values and at -14.29% or -6.07% for annual forward looking rolling windows. The volatility of the risk-free rate and RRA estimates, respectively oil price index, are correlated at 7.28% or 26.44% for end of the month values and at 26.67% or 30.38% for annual forward looking rolling windows.

Next, we analyze in further detail how changes in macroeconomic factors are related to a change in risk aversion. We consider the relation using ordinary least squares regressions. As the dependent variable, we employ the first difference of RRA estimates $\Delta\hat{\gamma}_t$. Independent variables are the first difference of the three-month money market rate Δr_t, the volatility of the money market rate $\sigma_{r,t}$, and the first difference of the adjusted CRB Spot Price Index Fats & Oils Δo_t. We work with first differences in order to control for the non-stationarity of the input variables.[22] All variables are determined from annual forward looking rolling

[22] Employing the Augmented Dickey-Fuller (ADF) test, we reject the hypothesis of a unit root for $\Delta\hat{\gamma}_t$, Δr_t, Δo_t, and the level variable $\sigma_{r,t}$.

windows. The three regression equations are given by

$$\Delta\hat{\gamma}_t = \alpha + \beta \cdot \Delta r_t + e_t, \tag{6.2}$$

$$\Delta\hat{\gamma}_t = \alpha + \beta \cdot \sigma_{r,t} + e_t, \tag{6.3}$$

$$\Delta\hat{\gamma}_t = \alpha + \beta \cdot \Delta o_t + e_t, \tag{6.4}$$

where α is a constant and e_t is the error term.

Table 6.17 reports the regression results. The regressions are either based on Equation (6.2), (6.3), or (6.4). Regressions 1 to 3 are performed for 84 observations in the interval [1974, 1982]. Considering Regressions 1 and 3, the coefficients of Δr_t and Δo_t are positive at the 1% significance level, respectively 5% level, implying a positive relation between changes in risk aversion and the money market rate as well as the oil price index. The explanatory power of Δr_t is 7% compared to 14% for Δo_t. Regression 2 shows that $\sigma_{r,t}$ has an insignificant coefficient and almost no explanatory power.

We use 1977 as a natural structural break and perform six additional regressions for the subintervals [1974, 1977] and [1978, 1982]. First, we focus on Regressions 4 to 6 performed for 38 observations in the interval [1974, 1977]. Considering Regressions 4 and 5, the coefficient of Δr_t is negative and the coefficient of $\sigma_{r,t}$ is positive, both at the 1% significance level. This implies a negative relation between $\Delta\hat{\gamma}_t$ and the money market rate and a positive relation between $\Delta\hat{\gamma}_t$ and the volatility of the money market rate between 1974 and 1977. The explanatory power of Δr_t is 20% compared to 16% for $\sigma_{r,t}$. The negative coefficient of Δr_t agrees with the economic intuition that higher risk aversion is associated with a rising demand for the risk-free asset, and, hence, with a lower risk-free rate. Regression 6 shows that Δo_t has an insignificant coefficient and almost no explanatory power in the first subinterval. Last, we focus on Regressions 7 to 9 performed for 46 observations in the interval [1978, 1982]. Analogous to the results for the entire interval, Regression 7 and 9 detect a positive relation between the dependent variable and Δr_t as well as Δo_t at the 1% significance level. However, the explanatory power of Δo_t is 38% compared to 9% for Δr_t. Since the second subinterval was strongly affected by the second oil crisis, the oil price index is a driving factor of changes in risk aversion. The crisis resulted in

Table 6.17: **Implied RRA Estimates and Macroeconomic Factors**

This table presents the results of nine regressions of the first difference of implied RRA estimates $\Delta\hat{\gamma}_t$ against the first difference of the three-month money market rate Δr_t, the volatility of the money market rate $\sigma_{r,t}$, and the first difference of the adjusted CRB Spot Price Index Fats & Oils Δo_t. RRA coefficients are estimated from annual rolling windows at a monthly frequency and are included if more than 500 observations per estimation are available. Independent variables are also determined from annual forward looking rolling windows. Regressions 1 to 3 are performed for 84 observations in the interval $[1974, 1982]$, Regressions 4 to 6 for 38 observations in the subinterval $[1974, 1977]$, and Regressions 7 to 9 for 46 observations in the subinterval $[1978, 1982]$. Regressions are calculated using ordinary least squares and include a constant. Problems arising from heteroscedasticity are accounted for by applying the White correction. The body of the table lists the coefficients and t-values. The row titled F-statistic reports the value of the F-test statistic for the null hypothesis that the coefficients of the explanatory variables are all zero. The last line reports the R^2. *** denotes significance at the 1% level, ** denotes significance at the 5% level, and * denotes significance at the 10% level.

Dependent Variable	$\Delta\hat{\gamma}_t$								
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Δr_t	0.98^{***}			-1.04^{***}			1.44^{***}		
t-value	(2.64)			(-4.62)			(3.59)		
$\sigma_{r,t}$		0.002			0.003^{***}			-0.004	
t-value		(1.47)			(3.57)			(-0.73)	
Δo_t			0.28^{**}			-0.05			0.54^{***}
t-value			(2.12)			(-1.26)			(2.93)
Constant	0.18	-0.08	0.40^{*}	-0.31^{***}	-0.40^{***}	-0.18^{*}	0.28^{*}	0.92	0.96^{***}
t-value	(1.55)	(-0.32)	(1.98)	(-3.17)	(-3.02)	(-1.98)	(1.77)	(1.13)	(3.10)
F-statistic	5.68^{**}	1.18	13.51^{***}	8.99^{***}	6.80^{**}	0.77	4.26^{**}	0.72	26.70^{***}
R^2	0.07	0.01	0.14	0.20	0.16	0.02	0.09	0.02	0.38

historically high interest rates, and the relation between $\Delta\hat{\gamma}_t$ and Δr_t becomes positive. Regression 8 shows that $\sigma_{r,t}$ has an insignificant coefficient and almost no explanatory power in the second subinterval.

The regression analysis suggests a structural break in the relation between changes in risk aversion and macroeconomic factors in 1977. We observe a negative relation between the RRA coefficient and money market rate between 1974 and 1977 contrasting with a positive relation between 1978 and 1982. However, the second subinterval was strongly affected by the second oil crisis, and we identify the oil price index as a driving factor of changes in risk aversion. Overall, the time series results suggest that severe economic crises coincide with periods of high risk aversion.

Chapter 7

Concluding Remarks

Both the equity premium puzzle and the credit spread puzzle address the problem of a reasonable size of investors' risk aversion. The estimation of risk aversion parameters is impeded by the fact that observed prices depend on risk preferences and probability beliefs. The market for default-free German redemption lottery bonds constitutes an exceptional environment to estimate risk aversion coefficients from transaction prices, as the probabilities of price changes caused by redemption lotteries are objectively known.

We develop a dynamic expected utility model using the well established power utility function over terminal wealth to extract the representative agent's RRA coefficient. The equilibrium price and implied RRA coefficient are determined recursively by standard dynamic programming techniques. Implied RRA estimates are obtained by minimizing the sum of squared deviations between theoretical lottery bond prices, calculated within the equilibrium framework, and transaction prices.

Using a unique dataset, containing transaction prices of 83 redemption lottery bonds traded between 1974 and 1987, we analyze risk preferences in the German bond market. Our empirical results describe the magnitude and evolution of risk aversion in the bond market. The pooled, implied RRA estimates are consistent with the moderate level of risk aversion found in most of the recent studies.[1] We

[1] See Table 1.1 for an overview of RRA coefficients reported in the literature.

obtain a robust pooled, implied RRA estimate of 1.78 and find no evidence of the extreme level of risk aversion suggested by Campbell and Cochrane (1999) and Kandel and Stambaugh (1990, 1991). Rather, the estimations indicate that the pooled, overall RRA coefficient is below two and robust across the length of estimation intervals as well as for restrictions on the proximity of price observations to the no-arbitrage bounds. We also obtain results on the dynamics of implied risk aversion and the relation between risk aversion and macroeconomic factors. Implied risk aversion is time-dependent and attains its maximum in 1980 and 1981, reflecting the challenging economic situation after the second oil crisis in 1979. Our time series results suggest a structural break in the relation between changes in risk aversion and macroeconomic factors in 1977 and provide further evidence that severe economic crises coincide with periods of high risk aversion.

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